APPLICABILITY OF LINEAR SYSTEM THEORY FOR SIMULATING FLOW TO TILE DRAIN

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree
of
MASTER OF TECHNOLOGY
in
HYDROLOGY

By

UPENDER.L

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in this Dissertation entitled, "**APPLICABILITY OF LINEAR SYSTEM THEORY FOR SIMULATING FLOW TO TILE DRAIN**", in the partial fulfillment of requirement for the award of the degree of **Master of technology** in **Hydrology**, submitted in the **Department of Hydrology, Indian Institute of Technology, Roorkee**, is an authentic record of my own work carried out during the period from July 2010 to June 2011 under the supervision of **Dr. M. Perumal**, Professor, Department of Hydrology, Indian Institute of Technology, Roorkee and **Dr. N.C. Ghosh**, Scientist 'F', National Institute of Hydrology, Roorkee.

I have not submitted the matter embodied in this dissertation for award of any degree of this or any other institutes.

(Upender. L)

Date: 30/06/2011

Place: Roorkee.

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UPENDER.L

ABSTRACT

This thesis is documented to test the one parameter linear system theory model to predict the subsurface tile-drain flow for a given hyetograph and soil characteristics. A parameter K (storage coefficient) has been estimated by trial and error for which the estimated tile drain hydrograph best matches with observed hydrograph. The model comparisons with the WQFS observed hydrographs were promising, the field station is located at the Agronomy Centre for Research and Education in West Lafayette, Indiana (USA). It was noticed that the selection of events favored those situations where the model was likely to be successful. In other words, by satisfying the mass balance criterion, there was some reassurance that the total volume of rainfall had infiltrated the soil and runoff was not generated. Further linear system theory model is justified by analyzing the flux movement by using the Richard's equation. The relation between total moisture content and flux at the lower boundary is linear for the initial condition: saturated moisture content, and upper boundary condition soil moisture content at all the times. Finally, it concluded that the simple linear system theory can predict the tile drain hydrographs for a given recharge, irrespective of subsurface zones.

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LIST OF SYMBOLS AND ABBREVIATIONS

I	Inflow rate
Q	Outflow rate
K	Storage coefficient
S	Storage
T	Time duration
H	Total hydraulic head
H	Soil water pressure head
q	Flux
z	Vertical distance from the soil surface
θ	Moisture content
θ_s	Saturated moisture content
θ_R	Residual moisture content
C	Specific water capacity
D	Soil water diffusivity
Δz	Thickness of layer
Δt	Time step
g	Acceleration due to gravity
f	Relative humidity
M	Molecular weight
k, K_s	Hydraulic conductivity at saturation

S_e	Normalized soil water content
H_0	Initial height of ponded water in the infiltrometer
$\Delta\theta$	Difference between saturated and initial water content
R	Universal gas constant (8.314 x 10 erg/mole/K)
WQFS	Water Quality Field Station

CONTENTS

CHAPTER -1

INTRODUCTION

1.0 General

Sub-surface tile drainage is a common agricultural water management practice in areas with shallow groundwater or seasonally perched water tables. The purpose of agricultural drainage is to remove excess water from the soil in order to enhance crop production. In some soils, the natural drainage processes are sufficient for growth and production of agricultural crops, but in many other soils, artificial drainage is needed for efficient agricultural production.

Tile drainage is an artificial drainage system that is practiced in many countries for removing excess water above from field capacity of soil subsurface caused due to over irrigation or prevent the rising water table to reach root zone. Irrigation is normally practiced to add additional water when soil is naturally dry; sub-surface drainage brings soil moisture levels down for optimal crop growth. While surface water can be drained via pumping and/or open ditches, tile drainage is often the best recourse for subsurface water. Too much subsurface water can be counterproductive to agriculture by preventing root development, and inhibiting the growth of crops.

The presence of tiles drain systems can affect the hydrology of a watershed significantly depending on its soil type, storm characteristics, and topography. Tiles drains can increase or decrease the peak flow at the outlet of a watershed that depends on its grid layout and size. In general, tile drains facilitate infiltration from rainfall, which, in turn, reduces the surface runoff. In soils with large cracks or macro pores, tile drains may significantly contribute to stream flow (Nicholson, 1953; Trafford and Rycroft, 1973). Since tile-drain systems can have a significant effect on watershed hydrology and groundwater quality, it is essentially important to study the hydrological and hydro-geological prospects of tile drain and derive simple method to compute the flow generated from a tile drain for diverse hydrological perturbations.

For designing of a tile drain network to removing excess water from soil sub-surface of an agricultural field, one has to thoroughly analyze the hydrological components which are associated with the soil-water movement in the sub-surface. The quantity of water that is to be drained from the sub-surface, and the rates at which soil will transmit water from its storage to the drain, are essentially to be known *a priori* to control and manage the soil-moisture condition favorable for the health of plants growth. The rate at which soil will transmit water from its storage will depend on soil properties, soil textures and porosity, suction head, field capacity and wilting point of the soil, and rate of infiltration of water from rainfall or irrigation application. These characteristics indicate that the sub-surface drainage outflows are time-variant component linked to several hydro-geological factors and variables. An accurate estimation of the drainage outflows is thus a complicated task involving requirement of number of parameters. Having known the measured quantities of inflows to the sub-surface and outflows at the tile drains outlets; and employing the lumped linear reservoir theory, the responses of the system can be simulated treating it as an inverse problem. The present dissertation is thus aimed at to conceptualize the sub-surface drainage processes through tile drain by applying the routing equation based on linear reservoir model with the capability to simulate tile drain hydrographs as responses of varied stress conditions that develop on the soil sub-surface.

1.1 Scope of the study

Tile drainage system studied earlier by Stillman et al. (2006) involving a mathematically involved procedure describe a semi-analytical method to simulate tile drain hydrograph due to different storms which produced negligible surface runoff. The modeling approach advocated by Stillman et al. (2006) even though is physically based, it still requires some initial estimates of parameters on event to event basis.

Simulation of tile drain hydrographs on event to event basis and with same level of accuracy as that of the semi-analytical approach may also be obtained using linear system theory concept suitable for application to groundwater modeling studies based on the following consideration: Groundwater and subsurface movement are characterized by very small flow velocity unlike that of the surface water. Therefore, considering the outflow from a groundwater system to be equivalent to outflow from a linear reservoir system may not be grossly in error with the reality. Considering this view point, it is proposed to apply linear reservoir system theory widely applied in surface hydrological modeling studies, for tile hydrograph modeling and evaluate the adequacy of this approach using past observed hydro graphs. The methodology of applying linear reservoir system modeling for groundwater recharge and subsequent outflow had been presented by Dooge (1960).

1.2 Objectives of the study

The present investigations are aimed at to achieve the following objectives:

(i) To apply the linear reservoir system theory based routing equation developed by Dooge (1960) to simulate the time-varying responses of the sub-surface drainage system using tile drain in an agricultural field.

(ii) To verify whether linear system theory can substantiate the responses of the subsurface drainage system represented by tile drains by studying the soil moisture movement process governed by the Richard's equation using the linear system theory.

1.3 Organization of the Thesis

The thesis has been organized as follows:

Chapter-1: Introduction

This chapter explains the relevance of the study, and scope of the study followed by the objectives of the present steady.

Chapter-2: Literature review

This chapter covers an overview of the linear system theory, the various models developed on linear system theory, study of tile-drain flow and the subsurface flow movement by the governing Richard's equation and its solution.

Chapter-3: Application of linear system theory to tile drain

The methodology to solve the problem using Dooge (1960) and the procedure for verifying the applicability of linear system theory to the subsurface flow model has been discussed in this chapter.

Chapter-IV: Results and Discussions

This chapter analyzes and compares the result of computed tile-drain flow using linear system theory with the observed flow for selected events. And also analyzes the result of moisture movement through the sub-surface by using the Richard's equation.

Chapter-V: Conclusions

This chapter summarizes the salient findings of the investigation, and also brings out the conclusions. It also suggests the future work to be carried out regarding to the problem.

CHAPTER-2

LITERATURE REVIEW

2.0 General

Subsurface drainage is the practice of placing perforated pipe at a specified slope at some depth below the soil surface. Excess water from the crop root zone can enter the pipe through the perforations and flow away from the field to a ditch or other outlet. Figure 2.1 shows the physical model of the sub subsurface drainage system. Subsurface improves the productivity of poorly drained soils by lowering the water table, providing greater soil aeration, and enabling faster soil drying and warming in the spring season. It also provides a better environment for crop emergence and early growth, and can reduce soil compaction. Subsurface drainage provides conducive environment for the growth of a crop by greatly reducing the risk of crop water stress from excessive rainfall. Figure-2.2 shows the subsurface drainage system.

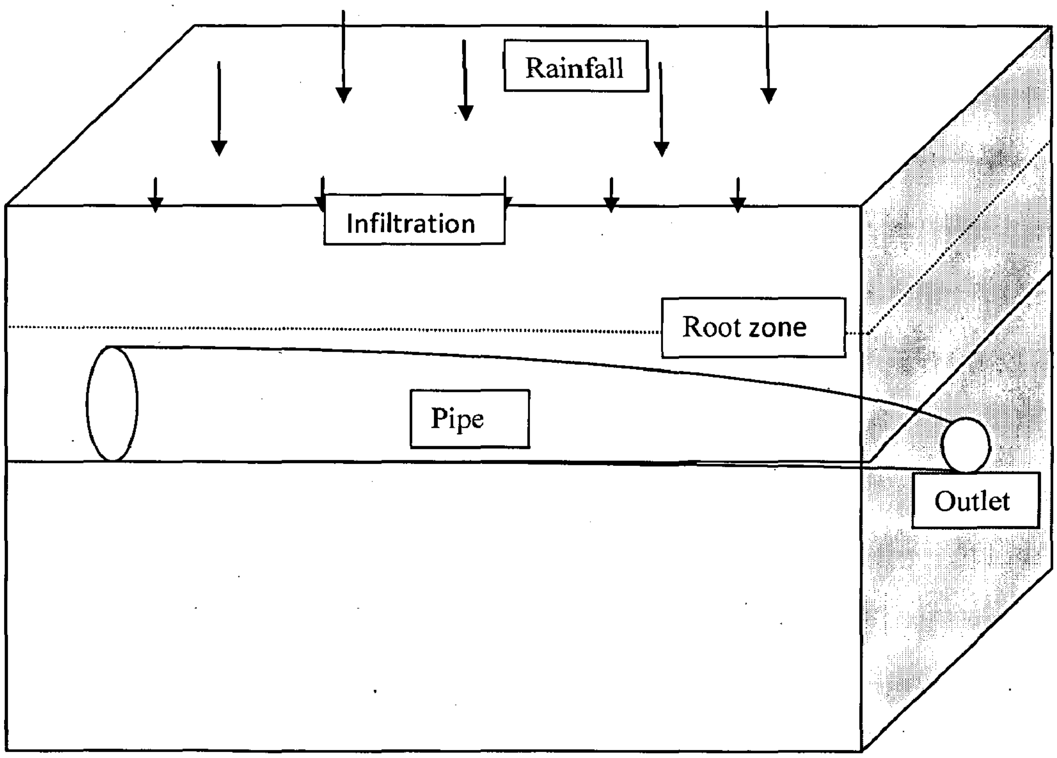

Figure 2.1: Physical model of Tile-Drain flow.

Figure 2.2: Outlet of the Tile-Drain system.

The reviews of related literature are presented as follows:

1. Introduction to linear system theory
2. Linear reservoir model
3. Soil water flow
4. Semi-analytical technique to simulate the tile drains flow.

2.1 Introduction to linear system theory

A system, as defined by Dooge (1973), is any structure, device, scheme, or procedure, real or abstract, that interrelates in a given time reference, an input, cause or stimulus, of matter, energy or information, and an output, effect, or response, of information, of energy or matter. When the components are isolated from the 'real' system and provide the state variables, the result is an 'idealized' system since it excludes some of the parameters or characteristics found in the environment. If this was not done, the task would either be impossible or so complex that it would be economically infeasible. Most of the hydrologic models which are widely used in practice are based on system theory. The Instantaneous Unit Hydrograph (IUH) is the primary elaboration of rainfall-runoff modeling has also been derived based on linear system theory. The linear system theory has successfully been used in the past, and continued its applicability in the present to simulate hydrologic responses. A schematic of a hydrologic system is shown in Figure 2.3.

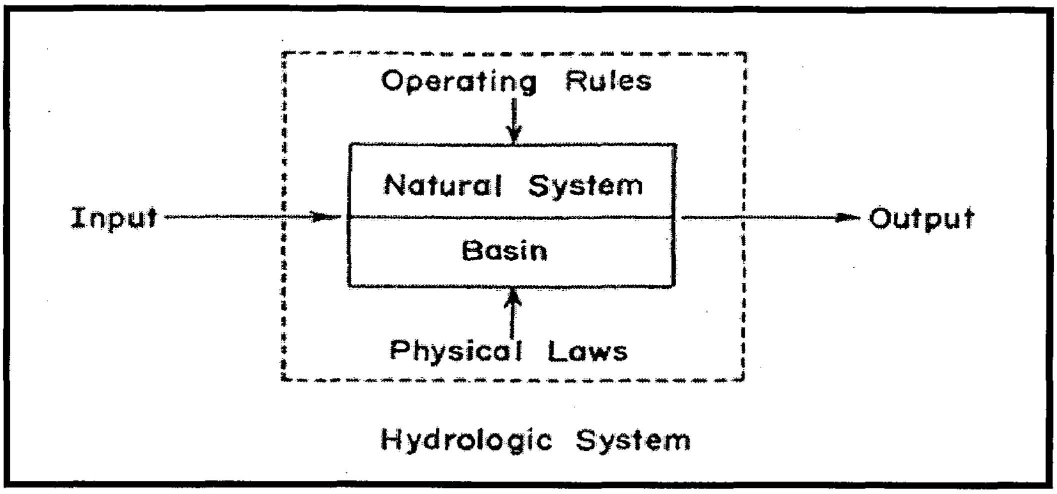

Figure 2.3: A schematic of a hydrologic system.

The characteristic function of a linear system, e.g., IUH can be of two types, one being time invariant and the other being time variant. If the system is time invariant, i.e., the parameters of the system are independent of time, then the system can be represented by a differential equation of the following type (Evan, 1972) :

$$I(t) = A_n \frac{d^n Q(t)}{dt^n} + A_{n-1} \frac{d^{n-1} Q(t)}{dt^{n-1}} + \ldots\ldots\ldots + A_0\, Q(t) \quad \ldots\ldots\ldots\ldots 2.1$$

where, I (t) is time varying input; Q (t) is time varying output; $A_n, A_{n-1}, \ldots\ldots A_0$ are the parameters of the system; superscript, n is the order of the differential equation, for n = 1, it is first order differential equation; and subscript, n is an integer number.

The difference between a time invariant system and a time variant system is that, the coefficients of a time variant system are time dependent. The behavior of a typical linear time invariant system is shown in Figure 2.4. This figure also represents the use of the convolution integral (or Duhamel's Integral) for a causal system.

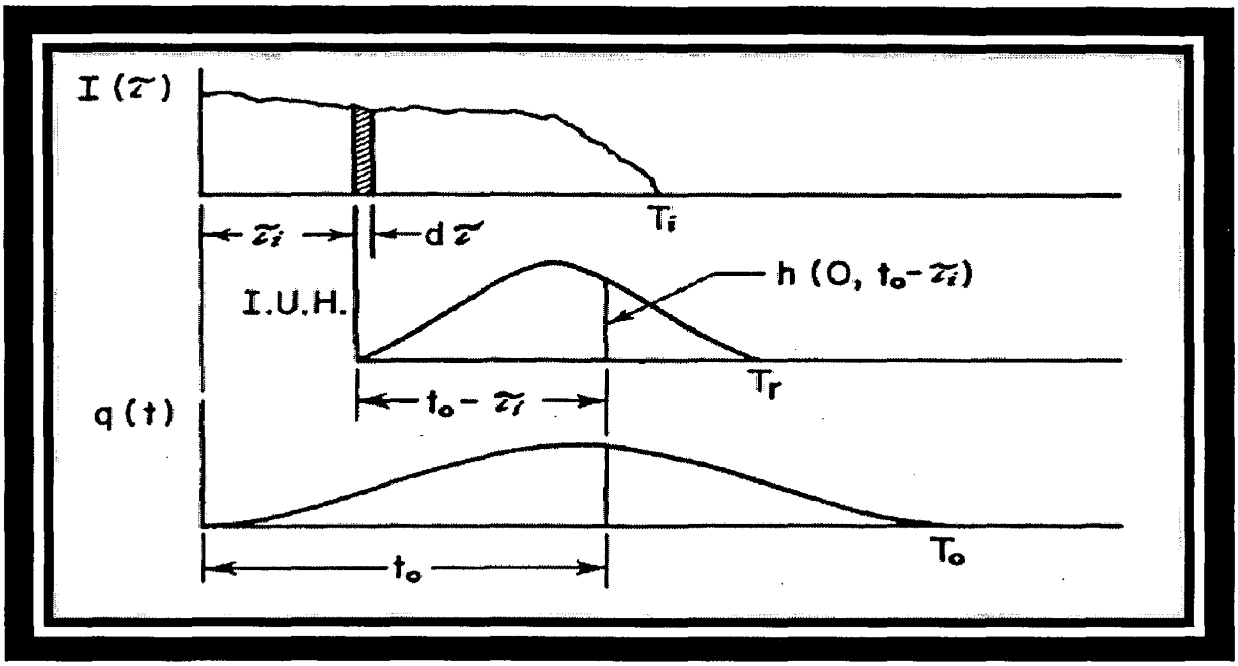

Figure 2.4: A behavior of a typical linear time invariant system.

A linear system approach in deriving an outflow hydrograph can be represented by the following convolution equation:

$$Q\ (t) = \int_0^t I\ (\tau)\ h^1(t-\tau)\ d\tau \qquad \text{.............................2.2}$$

where, I (τ) is the time varying inflow rate; Q (t) is the time varying system's outflow rate; h^1 (t-τ) is the impulse response function of the system; τ is a dummy time variable; and t is the time.

The characteristic function (Equation 2.1) may consist of one, two or three parameters which may be required to represent the system responses to an input function. The subsequent section describes one of the basic hydrologic models, the linear reservoir model, widely used for modeling of a linear hydrologic system. Further discussion on the linear system theory is restricted to the matter related to the study.

2.2 Linear Reservoir Model

The linear reservoir model is a single parameter model. the parameter K characterizes the storage-outflow relationship expressed as

S (t) = K Q (t) 2.3

where, K is the storage coefficient; S (t) is the storage and Q (t) is the outflow of linear reservoir. A schematic representation of a conceptual storage reservoir is shown in Figure 2.5.

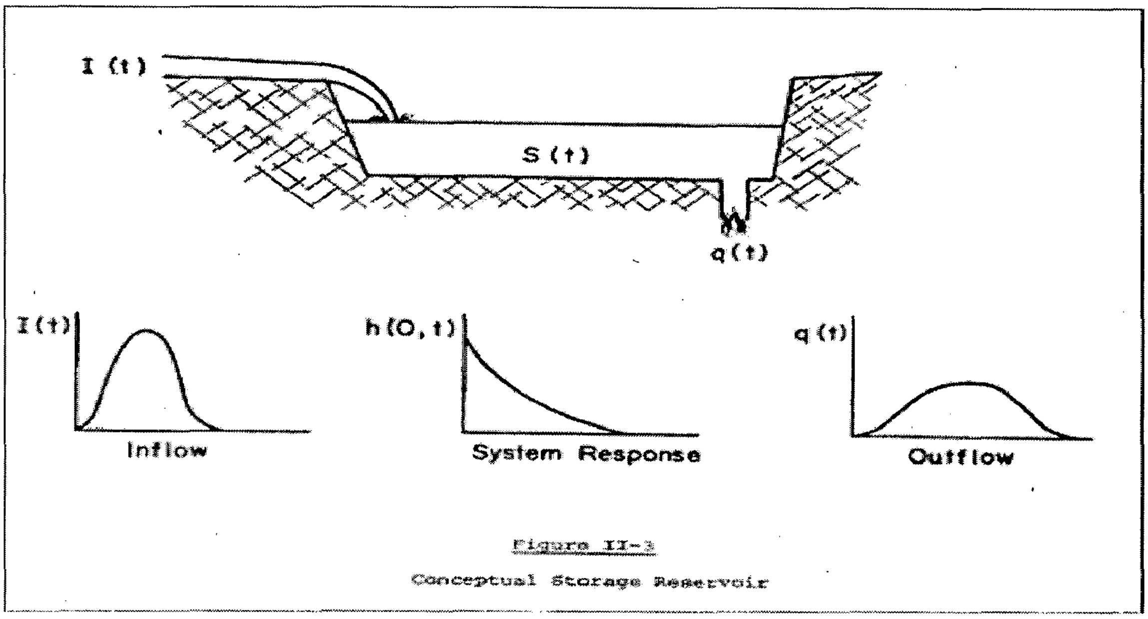

Figure 2.5: Schematic of a conceptual linear reservoir and its responses.

Figure 2.6 depicts the response of a linear reservoir to a pulse input of duration T.

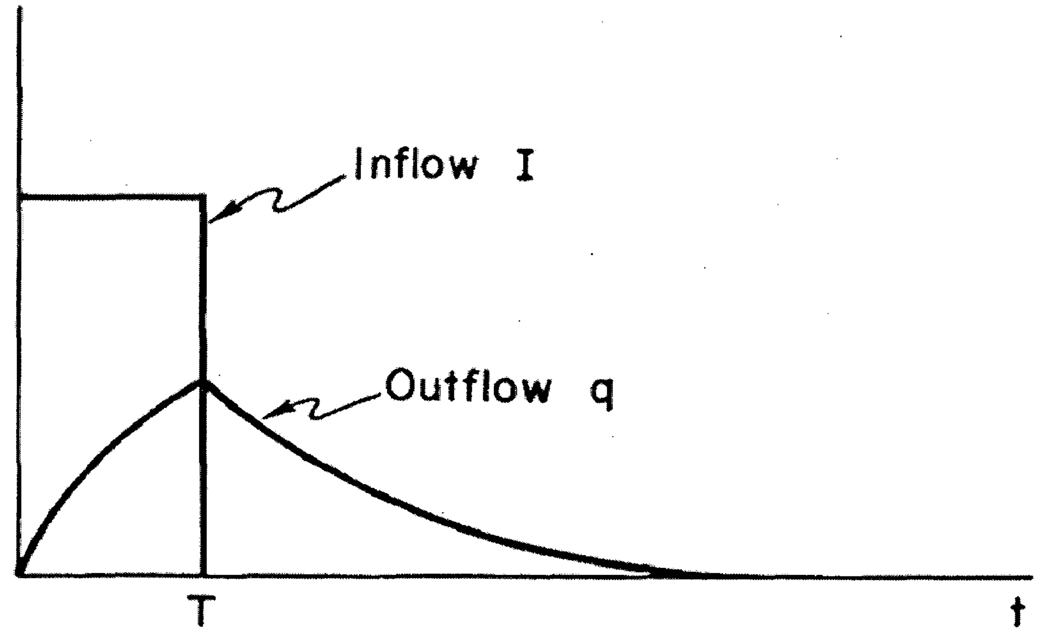

Figure 2.6: Outflow Hydrograph Resulting from a constant input to Linear Reservoir

Dooge (1960) successfully applied the linear reservoir concept for the estimation of groundwater outflow Q (t) when the groundwater recharge R (t) is completely known. For the sake of brevity, the time operator t is dropped. Dooge (1960) recognized that the groundwater flow is very small and, therefore, the transmission effect can be neglected and the behavior of the groundwater reservoir is essentially one that of a linear reservoir. Dooge (1960) developed the storage routing equation of a groundwater reservoir on the following lines:

The continuity equation for the storage reservoir is given by

$$I - Q = \frac{dS}{dt} \qquad \text{......2.4}$$

Using Equation (2.3) in Equation (2.4), we get the governing equation describing flow motion in groundwater reservoir as

$$I - Q = K\frac{dQ}{dt} \qquad \text{......2.5}$$

Using two consecutive pulse recharge rates: I_n and I_{n-1} applied to groundwater system, the response Q_n using the solution of Equation (2.5) at nT, where T is the routing time interval, can be expressed in recursive form as

$$\mathbf{Q_n = C_0 . I_n + C_1 . Q_{n-1}} \qquad \text{......2.6}$$

Where, $C_0 = 1 - e^{\frac{-T}{K}}$

$C_1 = e^{\frac{-T}{K}}$ or $C_0 = 1 - C_1$

2.3 Soil Water Flow

Subsurface formations containing water may be divided vertically into several horizontal zones according to how large a portion of the pore space is occupied by water. Essentially, we have a zone of saturation in which all the pores are completely filled with water, and an overlaying zone of aeration in which the pores contain both gases (mainly air and water vapor) and water. The latter zone is called the unsaturated zone. Sometimes the term soil water is used for the water in this zone.

For analytical studies on soil moisture regime, critical review and accurate assessment of the different controlling factors is necessary. The controlling factors of soil moisture may be classified under two main groups' viz. climatic factors and soil factors. Climatic factors include rainfall characterized by different values, rainfall intensity, storm duration, inter storm period, temperature of soil surface, relative humidity, radiation, evaporation, and evapotranspiration. The soil factors include soil matric potential and water content relationship, hydraulic conductivity and water content relationship of the soil, saturated hydraulic conductivity, and effective medium porosity. Besides these factors, the information about depth to water table is also required.

Most of the processes involving soil-water interactions in the field, and particularly the flow of water in the root zone of most crop plants, occur while the soil is in an unsaturated condition. Unsaturated flow processes are in general complicated and difficult to describe quantitatively, since they often entail changes in the state and content of soil water during flow. Such changes involve complex relations among the variable soil wetness, suction, and conductivity, whose inter-relations may be further complicated by hysteresis. The formulation and solution of unsaturated flow problems very often require the use of indirect methods of analysis, based on approximations or numerical techniques. For this reason, the development of rigorous theoretical and experimental methods for treating these problems was rather late in coming. In recent decades, however, unsaturated flow has become one of the most important and active topics of research and the findings of this research have resulted in significant theoretical and practical advances.

The one-dimensional partial differential equation, which describes the movement of moisture through unsaturated porous media, subject to appropriate boundary and initial conditions has many field applications in the water environment. In hydrology, it describes the infiltration process that links the surface and sub-surface waters on land. In soil physics, it describes the capillary rise as well as drainage and evaporation of moisture in soils. In environmental pollution, it describes the longitudinal dispersion of pollutants in water courses. Therefore, the problem of seeking solutions to this equation has become a subject of concern for investigators from many different disciplines.

Downward infiltration into an initially unsaturated soil generally occurs under the combined influence of suction and gravity gradients. As the water penetrates deeper and the wetted part of the profile lengthens, the average suction gradient decreases, since the overall difference in pressure head (between the saturated soil surface and the unwetted soil inside the profile) divides itself along an ever-increasing distance. This trend continues until eventually the suction gradient in the upper part of the profile becomes negligible, leaving the constant gravitational gradient in effect as the only remaining force moving water downward. Since the gravitational head gradient has the value of unity (the gravitational head decreasing at the rate of 1 cm with each centimeter of vertical depth below the surface), it follows that the flux tends to approach the hydraulic conductivity as a limiting value. In a uniform soil (without crust) under prolonged ponding, the water content of the wetted zone approaches saturation. However, in practice, because of air entrapment, the soil-water content may not attain total saturation but some maximal value lower than saturation which has been called 'satiation'. Total saturation is assured only when a soil sample is wetted under vacuum.

The dynamics of soil water is cast in the form of mathematical expressions that describe the hydrological relations within the system. The governing equations define a mathematical model. The entire model has usually the form of a set of partial differential equations, together with auxiliary conditions. The auxiliary conditions must describe the system's geometry, the system parameters, the boundary conditions and, in case of transient flow, also the initial conditions. Operations with such a mathematical model are called simulation.

If the governing equations and auxiliary conditions are simple, an exact analytical solution may be found. Otherwise, a numerical approximation is applicable. The numerical simulation models are by far the most applied ones.

2.3.1 Constitutive Equations

The relationships that govern the flow of water in unsaturated soil are quasi-linear equations of the parabolic type. Since the coefficients in these equations are functions of the dependent variables, exact analytical solutions for specific boundary conditions are extremely difficult to obtain.

Darcy's equation for vertical flow is

$$q = -k\frac{\partial H}{\partial z} = -k\frac{\partial(h-z)}{\partial z} \qquad \text{2.7}$$

where, 'q' is the flux, 'H' the total hydraulic head, 'h' the soil water pressure head, 'z' the vertical distance from the soil surface downward (i.e., the depth), and 'k' the hydraulic conductivity. At the soil surface, q = i, the infiltration rate. In an unsaturated soil, 'h' is negative. Combining this formulation of Darcy's equation (II.17) with the continuity equation $\partial\theta/\partial t = -\partial q/\partial z$ gives the general flow equation

$$\frac{\partial\theta}{\partial t} = \frac{\partial(k\frac{\partial H}{\partial z})}{\partial z} = \frac{\partial(k\frac{\partial h}{\partial z})}{\partial z} - \frac{\partial k}{\partial z} \qquad \text{2.8}$$

If soil moisture content θ and pressure head h are uniquely related, then the left-hand side of Equation (2.8) can be written as:

$$\frac{\partial\theta}{\partial t} = \frac{d\theta}{dh}.\frac{\partial h}{\partial t} \qquad \text{2.9}$$

Which transforms Equation (2.8) into

$$C\frac{\partial h}{\partial t} = \frac{\partial(k\frac{\partial h}{\partial z})}{\partial z} - \frac{\partial k}{\partial z} \qquad \text{2.10}$$

Where, C (= dθ/dh) is defined as the specific (or differential) water capacity (i.e., the change in water content in a unit volume of soil per unit change in matric potential).

Alternatively, we can transform the right-hand side of equation (2.8) once again using the chain rule to render

$$\frac{\partial h}{\partial z} = \frac{dh}{d\theta}.\frac{\partial\theta}{\partial z} = \frac{1}{C}.\frac{\partial\theta}{\partial z}$$

Thus we obtain,

$$\frac{\partial\theta}{\partial t} = \frac{\partial(D.\frac{\partial\theta}{\partial z})}{\partial z} - \frac{\partial k}{\partial z} \quad \text{...2.11}$$

Where, 'D' is the soil water diffusivity. Equations (2.8), (2.10) and (2.11) can all be considered as forms of the Richards equation.

Note that the above three equations contain two terms on their right-hand sides, the first term expressing the contribution of the suction (or wetness) gradient and the second term expressing the contribution of gravity. Whether the one or the other term predominates depends on the initial and boundary conditions and on the stage of the process considered. For instance, when infiltration takes place into an initially dry soil, the suction gradients at first can be much greater than the gravitational gradient and the initial infiltration rate into a horizontal column tends to approximate the infiltration rate into a vertical. On the other hand, when infiltration takes place into an initially wet soil, the suction gradients are small from the start and become negligible much sooner. The effects of ponding depth and initial wetness can be significant during early stages of infiltration, but decrease in time and eventually tend to vanish in a very deeply wetted profile.

The following simplifications can be introduced to find analytical solutions: k is an analytical function of θ or h; hysteresis is neglected; the medium is homogeneous and isotropic; the flow is considered to be stationary or a succession of steady-state situations (quasi-stationary approach); the gravity force is neglected. The first two assumptions linked with the third one have resulted in a great number of analytical solutions. The gravity force is often neglected in describing the infiltration process in originally dry soil, resulting in analytical solutions as derived by e.g. Philip (1957, 1958).

The classical Richards-flow theory (Richards, 1931), upon which most simulation models are based, holds for stable flow conditions only. Yet instability of flow has been observed under a wide variety of circumstances such as abrupt and gradual increases of hydraulic conductivity with depth, compression of air ahead of the wetting front and water repellency of the solid phase. Another example of non-Richards type of flow is the preferential flow through non-capillary macro-pores. With classical flow theories one may then underestimate the velocity and depth of water/solute transport.

2.3.2 Numerical Approach

With the advance of digital computers, emphasis has shifted drastically from the classical approach of analytical solutions to the rapidly developing field of numerical analysis. At present, numerical approximations are possible for complex, compressible, nonhomogeneous and anisotropic flow regions having various boundary configurations.

Numerical methods are based on subdividing the flow region into finite segments bounded and represented by a series of nodal points at which a solution is obtained. This solution depends on the solutions of the surrounding segments and on an appropriate set of auxiliary conditions. In recent years, a number of numerical methods have been introduced. The methods most appropriate to the problem of soil water dynamics are finite difference method, finite element method and boundary element method. The finite difference method has been discussed below.

2.3.2.1 Finite Difference Methods

Finite difference methods (Remson et al., 1971), either explicit or implicit, belong to the most frequently used techniques in modeling unsaturated flow conditions. The simplest type of finite differencing, the explicit one, orders the differencing operators in such a manner that the resulting finite difference equation contains only one unknown, and consequently, may be solved simply and directly. The explicit method is computationally simple but it has one serious drawback. In order to attain reasonable accuracy, the length of the interval in space must be kept small. To get a stable solution, the time step has to be small compared with the space interval. Thus, it is necessary to have a large number of time steps when using the simple explicit method.

Implicit solution methods generally use much larger time steps than explicit ones, but their stability depends upon the degree of nonlinearity of the differential equation. There are a great number of methods to solve an implicit set of algebraic equations, such as linearization, predictor-corrector or iteration methods.

The advantage of the finite difference method is its simplicity and efficiency in treating the time derivatives. On the other hand, the method is rather incapable to deal with complex geometries of flow regions. A slow convergence, a restriction to bilinear grids and difficulties in treating moving boundary conditions are other serious drawbacks of the method.

2.3.2.2 Discretization Schemes

Different discretization schemes can be used using explicit or implicit methods. In the explicit method, a series of linearized independent equations are solved directly, while in the implicit method, a system of simultaneous linear algebraic equations (involving tri-diagonal coefficient matrix with zero elements outside the diagonals) has to be solved. For a given grid point at a given time, the values of the coefficients k(h) or k(θ) and C(h) or D(θ) can be expressed either from their values at the preceding time step (explicit linearization) or from a prediction at time (t+1/2Δt) using a method described by Douglas and Jones, 1963 (implicit linearization).

The following discretization schemes (Equation 2.11) can be used for the various models.

Model 1: Explicit Scheme Solved Directly

$$h_i^{j+1} = h_i^j + \frac{\Delta t}{C_i^j \Delta z}\left[k_{\frac{i+1}{2}}^j\left(\frac{h_{i+1}^j - h_i^j}{\Delta z} - 1\right) - k_{\frac{i-1}{2}}^j\left(\frac{h_i^j - h_{i-1}^j}{\Delta z} - 1\right)\right] \ldots\ldots\ldots 2.12$$

Where j refers to time, and i refers to depth and

$$k_{\frac{i+1}{2}}^j = \frac{k_{i+1}^j + K_i^j}{2};\ k_{\frac{i-1}{2}}^j = \frac{k_i^j + k_{i-1}^j}{2}$$

Defining D_{max} as the maximum value of the soil water diffusivity in the soil profile at time t, the scheme is stable when (Haverkamp et al., 1977)

$$\Delta t < \frac{r(\Delta z)^2}{D_{max}} \qquad \text{2.13}$$

Where, 'ΔZ' is the layer thickness and 'r' an arbitrary chosen coefficient.

The method is limited by extensive use of computer time when the water content approaches saturation and 'Δt' becomes very small (D_{max} becomes large).

2.3.2.3 Initial and Boundary Conditions

Initial conditions must be defined when transient soil water flow is modeled. Usually values of matric head or soil water content at each nodal point within the soil profile are required. However, when these data are not available, water contents at field capacity or those in equilibrium with the ground water table might be considered as the initial ones.

2.3.2.3.1 Upper boundary conditions

While the potential evapotranspiration rate from a soil depends only on crop and atmospheric conditions, the actual flux through the soil surface and the plants is limited by the ability of the soil matrix to transport water. Similarly, if the potential rate of infiltration exceeds the infiltration capacity of the soil, part of the water runs off, since the actual flux through the top layer is limited by moisture conditions in the soil. Consequently, the exact boundary conditions at the soil surface cannot be estimated a priori and solutions must be found by maximizing the absolute flux. The minimum allowed pressure head at the soil surface, h^{lim} (time dependent) can be determined from equilibrium conditions between soil water and atmospheric vapor.

The possible effect of ponding has been neglected so far. In case of ponding, usually the height of the ponded water as a function of time is given. However, when the soil surface is at saturation then the problem is to define the depth in the soil profile where the transition from saturation to partial saturation occurs. In most of the dynamic transient models, the surface nodal point is treated during the first iteration as a prescribed flux boundary and matric head h is computed.

If $h^{lim} \leq h \leq 0$, the upper boundary condition remains a flux boundary during the whole iteration. If not, the surface nodal point is treated as a prescribed pressure head in the following iteration.

Then in case of infiltration, h = 0 and in case of evaporation $h = h^{lim}$. The actual flux is then calculated explicitly and is subject to the condition that actual upward flux through the soil-air interface is less than or equal to potential evapotranspiration (time dependent). If the relative humidity (f) and the temperature of the air (T^1) as a function of time are known, and if it may be assumed that the pressure head at the soil surface is at equilibrium with the atmosphere, then h(0,t) can be derived from the thermodynamic relation (Edlefson and Anderson, 1943):

$$h(0,t) = \frac{RT^1(t)}{Mg} \ln [f(t)] \qquad 2.14$$

Where, 'R' is the universal gas constant (8.314 x 10 erg/mole/K), 'T^1' is the absolute temperature (K), 'g' is acceleration due to gravity (980.665 cm/s), 'M' is the molecular weight of water (18 gm./mole), 'f' is the relative humidity of the air (fraction) and h is in bars. Knowing h (0,t), θ (0,t) can be derived from the soil water retention curve.

2.3.2.3.2 Lower boundary conditions

The phreatic surface (place, where matric head is atmospheric) is usually taken as lower boundary of the unsaturated zone in the case where recorded water table fluctuations are known a priori. Then the flux through the bottom of the system can be calculated. In regions with a very deep ground water table, a Neumann type of boundary condition is used.

2.3.2.3.3 Required Input Data

Simulation of water dynamics in the unsaturated zones requires input data concerning the model parameters, the geometry of the system, the boundary conditions and, when simulating transient flow, initial conditions. With geometry parameters, the dimensions of the problem domain are defined. With the physical parameters, the physical properties of the system under consideration are described. With respect to the unsaturated zone, it concerns the soil water characteristic, $h(\theta)$ and the hydraulic conductivity, $k(\theta)$.

For a proper description of the unsaturated flow, a correct description of the two hydraulic functions, $k(\theta)$ and $h(\theta)$, is important. The hydraulic conductivity, $k(\theta)$, decreases strongly as the moisture content θ, decreases from saturation. The experimental procedure for measuring $k(\theta)$ at different moisture contents is rather difficult and not very reliable. Alternative procedures have been suggested to derive the $k(\theta)$ function from more easily measurable characterizing properties of the soil. In many studies, the hydraulic conductivity of the unsaturated soil is defined as product of a non-linear function of the effective saturation, and hydraulic conductivity at saturation. The relation is given by

$$k(\theta) = k_s(\frac{\theta-\theta_r}{\theta_s-\theta_r})^n \qquad \text{2.15}$$

Where,

k_s = hydraulic conductivity at saturation;

θ_s = saturated water content; and

θ_r = residual water content.

The value of n is found to be 3.5 for coarse textured soils. n will vary with soil type. In literature, established empirical correlation between n and soil characteristic is available.

The relationship between the soil water pressure head $h(\theta)$ and moisture content θ, usually termed as the water retention curve or the soil moisture characteristic, is basically determined by the textural and the structural composition of the soil. Also the organic matter content may have an influence on the relationship. A characteristic feature of the water retention curve is that suction head (-h) decreases fairly rapidly with increasing moisture content. Hysteresis effects may appear, and, instead of being a single valued relationship, the h-θ relation consists of a family of curves. The actual curve will have to be determined from the history of wetting and drying.

2.4 Semi-analytical model for transient flow to a subsurface tile drain

Stillman et al. (2005), developed a semi-analytical model to predict the tile-drain flow in the form of hydrograph given only the rainfall hyetograph and soil hydrologic properties. A sharp-front redistribution theory has been used to describe water movement in the vadose zone, which was then combined with a semi-analytical solution of the Boussinesq equation to arrive at tile drain flow. A Fourier cosine series is used to represent the dimensionless of water table (Govindaraju, 2003). Figure 2.7 represents the physical model of the problem.

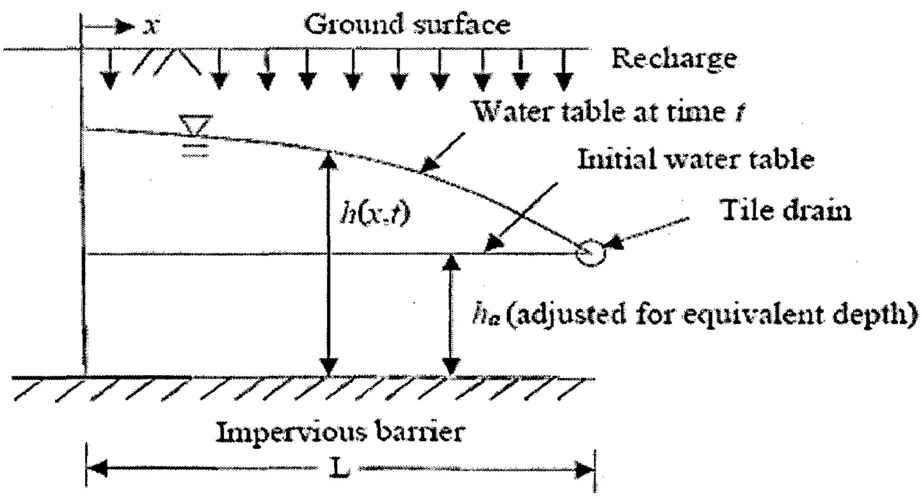

Figure 2.7: Schematic representation of the tile drain problem.

The Boussinesq equation is used to estimate the subsurface saturated flow. The equation can be written as (Brutsaert, 1994)

$$S\frac{\partial z}{\partial t} = \frac{\partial}{\partial x}\left(k.z\frac{\partial z}{\partial x}\right) + i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.15$$

Where, S is storativity; k is saturated hydraulic conductivity; and i is recharge rate. A sharp-front theory was used to determine the depth to the wetting front during redistribution and to estimate the water flux to the water table. The unsaturated soil hydraulic conductivity was described by a power-law relationship (Charbeneau, 2000)

$$k\,(S_e) = k.S_e^{\varepsilon} \quad \text{.......................................2.16}$$

Where, k is the saturated hydraulic conductivity including effects of macro-pores, and the exponent ε is a model parameter and S_e is a normalized soil water content defined by the ratio

$$S_e = \frac{\theta-\theta_{initial}}{\theta_s-\theta_{initial}} \quad \text{..2.17}$$

In the above expression, θ is the volumetric soil water content, $\theta_{initial}$ is the soil water content at which drainage by macropore flow will cease, θ_s is the saturated water content, just before water redistribution starts.

The Green-Ampt equation is used to estimate the saturated hydraulic conductivity k. The various parameter values of Green-Ampt infiltration equation are given by Rawls et al. (1983), the equation is

$$t = \frac{1}{k}\{\frac{H_0-z}{1-\Delta\theta} - \frac{(H_0-z)\Delta\theta}{(1-\Delta\theta)^2}\ln\left[1+\frac{(1-\Delta\theta)(H_0-z)}{(H_0+h)\Delta\theta}\right] \quad \text{..........................2.18}$$

Where, k is saturated hydraulic conductivity; z is depth of ponded water in the infiltrometer; h is net positive soil suction head; H_0 is initial height of ponded water in the infiltrometer; and $\Delta\theta$ is the difference between saturated and initial water content. Equation 2. is the time for a volume of water F = (H_0 -z) to infiltrate into the soil using Green–Ampt theory. A slightly different form of equation 2.18 is presented in Selker et al. (1999) and was originally developed by Polubarinova-Kochina (1962).

The Purdue Water Quality Field Station (WQFS) data has been used to simulate the tile drain flow which is located at the Agronomy Center for Research and

Education in West Lafayette, Indiana (USA). The same data has been used in this present model, by digitalizing the published paper.

From equation 2.16 and 2.18 the parameters soil water condition $\Delta\theta$ and ε are unknowns for the model, which have been estimated by trial and error method. Once calibrated, the same value of ε was used for the remaining validation events. However, the value of $\Delta\theta$ is somewhat dependent on antecedent soil water conditions and had adjusted on an event basis.

The results of this study suggests that a semi-analytical model reasonably sufficient to predict the tile drain flow, also from the study, the model predicts the tile drain flow hydrograph for those events which have the single burst of hyetograph. And in those cases in which the multiple bursts occur the model fails to generate the subsequent peaks.

CHAPTER-3

APPLICATION OF LINEAR SYSTEM THEORY TO TILE DRAIN

3.0 General

Linear system theory inter-relates input and output. While output in the present study is tile drain flow which is easily measurable, the input in the form of recharge is not directly measured, but indirectly estimated using the information on infiltration, initial soil moisture condition and soil matrix characteristics. No attempt has been made in this study to arrive at the recharge estimates of the events using unsaturated flow modeling.

The present application of linear system theory on observed rainfall and tile hydrograph data from the Purdue Water Quality Field Station (WQFS), located at the Agronomy Center for Research and Education in West Lafayette, Indiana (USA). The soil at the WQFS is a silty clay loam with glacial till at approximately 2m below the surface (Stillman et al., 2006). The station contains 48 plots; the area of each plot is $240m^2$. The subsurface flow in each plot is directed to a single tile drain, about 1m below the soil surface. Flow from each tile drain is routed to a collection station equipped with tipping bucket flow meters and data loggers. Flow volumes are measured in hourly basis. Rainfall hyetograph data are also collected using a rainfall gauge at the site. Five events (hyetograph and recorded tile-drain flow) were selected from the available WQFS data which satisfies the water balance criterion of the transformation process. Since the separation of rainfall into infiltration and runoff would introduce extra uncertainty into the model, the analysis was limited to those events in which the total amount of rainfall infiltrated into the soil, which was subsequently collected as tile-drain flow. The criterion for this condition was that the overall water balance must be satisfied between the total amount of rainfall that fell on the plot and the volume under the observed tile-hydrograph within an acceptable error of less than 20% by volume (Stillman et al., 2006) Here, in some cases some amount of initial hour's rainfall has been deducted, which was estimated by trial and error, to satisfy the initial moisture deficit.

3.1 Methodology

In order to achieve the major objective, of computation of tile drain flow and to verify the applicability of linear system theory to tile drain flow simulation the following approach was followed:

1. Simulation of tile drain flow based on linear system theory applied on Purdue Water Quality Field Station (WQFS) data.
2. Verifying the appropriateness of applying linear system theory to subsurface flow modeling.

3.1.1 Simulation of tile drain flow based on linear system theory applied on WQFS data

To compute the tile drain flow from the plots of Purdue Water Quality Field Station, the routing equation of single linear reservoir developed by Dooge (1960) based on linear system theory has been used. The routing equation is:

$$Q_n = C_0^1 . R_n + C_1^1 . R_{n-1} + C_2^1 . Q_{n-1} \qquad \text{3.1}$$

Where, $C_0^1 = 1 - (1 - e^{-\frac{T}{K}})\frac{K}{T}$;

$C_1^1 = (1 - e^{-\frac{T}{K}})\frac{K}{T} - e^{-\frac{T}{K}}$;

$C_2^1 = e^{-\frac{T}{K}}$.

Here, R_n and R_{n-1} are the varying rate of recharges at times nT and (n-1)T respectively. Q_n and Q_{n-1} are the corresponding outflow at the outlet. T is the computational routing interval. Considered as T = 1hour throughout this study which is also the duration of the rainfall pulse input. K is storage coefficient which is unknown for each plot of the region. The parameter K has been calibrated by trial and error to estimate the best tile drain flow hydrograph for each plot.

The product of rainfall and plot area of $240m^2$ gives the recharge rate per hour. Considering initially, the system is at rest, i.e., $Q_0 = 0$, and considering the recharge rate to be constant over the duration of routing interval, one can express Equation (3.1) as

$$Q_n = C_0 . R_n + C_1 . Q_{n-1} \quad \text{.......................................} 3.2$$

Where, $C_0 = 1 - e^{\frac{-T}{K}}$

$C_1 = e^{\frac{-T}{K}}$ or $C_0 = 1 - C_1$

Using equation (3.2) recursively the tile drain hydrograph can be estimated for the considered K value. The best K value is the one which closely estimates the observed tile drain hydrograph. By observing the simulated and observed hydrographs of each plot it has been noticed that there is a delay time of 1 or 2hours to respond the hydrograph-plot in the form of hydrograph for a given input, therefore considered a parameter 'dt' in the Dooge (2006) model which represents the delay time. The delay time is the time at which the tile drains plot responses after the recharge starts.

Appendix I presents the C++ program used to compute the flow for a given recharge, time duration, delay time and storage coefficient K.

3.1.2 Verifying the appropriateness of applying linear system theory to subsurface flow modeling.

The applicability of linear system theory to subsurface flow model was verified by using the Richard's equation. The verification has done by using the following methodology:

Analyze the subsurface characteristics such as residual soil moisture content (θ_R), saturated soil moisture content (θ_s), soil moisture content at field capacity (θ_{fc}) and saturated hydraulic conductivity (k) for the site; the constant coefficients used in Equation 2. 15 are fixed for the site. In this present study these values are

$\Theta_s = 0.396$; $\theta_{fc} = 0.35$; $\theta_R = 0.131$; $k=0.048$ cm/Δt; Δt = 1/50 part of 1hour; Δz = 20 cm thickness; $n = 2.06$; $m = 1-(1/n)$; $\alpha = 0.0042$; $\gamma = 0.5$. α, γ are Van Genuchten parameters.

The total depth 10 m. from subsurface to phreatic region was taken for analyze the movement of flux. In order to solve the Richard's equation Explicitly, the total depth divided into 50 nodes

Here, Richard's equation is used to analyze the flow pattern in subsurface. The

$$q = -K\frac{\partial H}{\partial z} = -K\frac{\partial (h-z)}{\partial z}$$

Where, 'q' is the flux, 'H' the total hydraulic head, 'h' the soil water pressure head, 'z' the vertical distance from the soil surface downward (i.e., the depth), and 'K' the hydraulic conductivity. At the soil surface, q = i, the infiltration rate. In an unsaturated soil, 'h' is negative.

The solution of the Richard's equation is given in chapter-2. To analyze the subsurface flow pattern, total depth (10m) of the subsurface is divided into number (50) of nodes; each node has an equal thickness (20cm), the soil characteristics of a particular region has been taken input to find the solution of Richard's equation. The relation between storage (i.e. moisture content) and output at the last node, moisture movement has been analyzed. The variation of moisture content with respect to depth at each node has been analyzed for3000hours with the initial condition θ_s (i.e. moisture content at saturated condition) of the value 3.96, and boundary conditions are θ_s at lower boundary and θ_f (theta at field capacity) of the value 3.5 at upper boundary. The analysis has been done by keeping 100% relative humidity all the times. By using the Equation 2.12 we can estimate the soil water pressure head in terms of depth, then by applying the Darcy's law we can estimate the flux at the lower boundary. Then by plotting graph between flux at the lower boundary and total moisture content, we can analyze the behavior of subsurface model in actual process.

A FORTRAN program has been given in ANNEXURE-III for solving the Richard's equation by using Explicit Scheme.

CHAPTER-4

RESULTS AND DISCUSSIONS

4.1 Simulation of Tile drain flow

A set of observed events (hyetographs and the corresponding tile-drain flow hydrographs) were selected from the available data sets collected at the Purdue Water Quality Field Station (WQFS) for the study using the proposed methodology. Since the separation of rainfall into infiltration and runoff would introduce extra uncertainty into the model, the analysis was limited to those events in which the hyetograph almost infiltrated into the soil, and was subsequently collected as tile-drain flow, and negligible surface runoff was generated. Selection of such events assured that the overall water balance must be satisfied between the total amount of rainfall that fell on the plot and the volume under the observed tile-hydrograph within an acceptable error (less than 20% by volume). Five events were selected from those data sets collected during the years 1997 and 2002. These events comprised a mixture of single-burst and multiple burst rainfall hyetographs. For this analysis, a burst was defined as a series of consecutive hourly rainfall pulses. It was observed that the tile drains had no significant base flow, and generated flow only in response to rainfall events, thus justifying an event based analysis. An event hyetograph was further subdivided into bursts to account for small periods of rainfall hiatus. Since the tile plots satisfy the water mass balance criterion, the total volume of rainfall for each burst was used for the infiltration volume. A burst of rainfall may take place over several hours; the simple model treats the volume of the burst as a single impulse applied instantaneously to the soil surface. Once completed, the hydrographs from individual bursts in an event were superimposed to create a single tile-drain hydrograph for the event.

The soil moisture content in the subsurface has been assumed almost saturated condition or the value is somewhat dependent on antecedent soil water conditions and had to be adjusted on an event basis before recharge starts. Parameter K has been tuned in this case due to in sufficiency of data, and was observed that there was a delay time of 1 or 2 hours to system response. When tuning the parameter it was observed that, if K value increases then the peak is reduced and the hydrograph is elongated, and if the value of K is reduced to small hours, then the peak is increased and hydrograph base is reduced in all cases, which is basically the characteristic of a linear reservoir model. For the given parameters K and system response delay time, the trend and the peak of all the computed hydrographs were matched with the observed hydrographs. The results of all events are discussed below with the tables and figures. For each of the events simulated, hydrograph the values of storage coefficient K, and the delay time are mentioned. A computational routing interval of T = 1hour was used in all the simulations studied.

To simulate the P14 hydrograph, the recharge rate was obtained by multiplying the hydrograph plot area (i.e. 240 m^2) and the rainfall intensity, and the parameter K was tuned as 5.2hours. It was observed that the system response delay time (i.e. 'dt' in C++ program parameter) was 1hour. It can be observed from Table 4.1 (Appendix-II) that the mass balance was satisfied. The computed hydrograph coincided well with the observed hydrograph with efficiency above 94%, as shown in Figure 4.1.

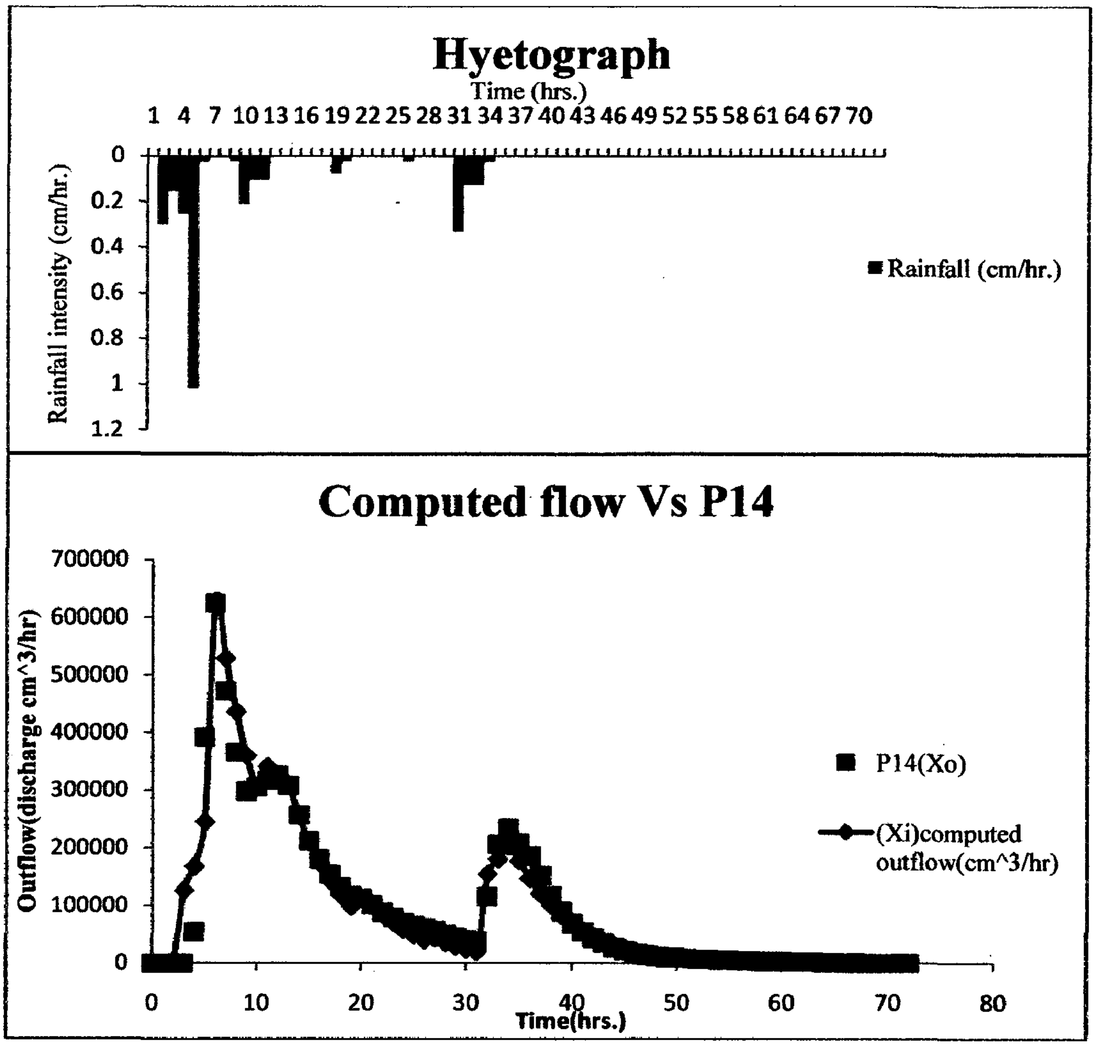

Figure 4.1: Comparison of observed and predicted hydrographs after calibration for event-1

It is seen from Figure 4.1 that the computed flow almost overlaps with the observed flow, except the initial hours. This deviation has been observed in every tile drain hydrograph.

The storage coefficient K was estimated by trial and error as 7.7hours and the delay time is 1hr. The estimated tile drain hydrograph matches well with the observed flow for the event-1 of hydrograph plot P33. Figure 4.2 shows the comparison of observed and predicted hydrographs. The efficiency of the computed flow is above 96%.

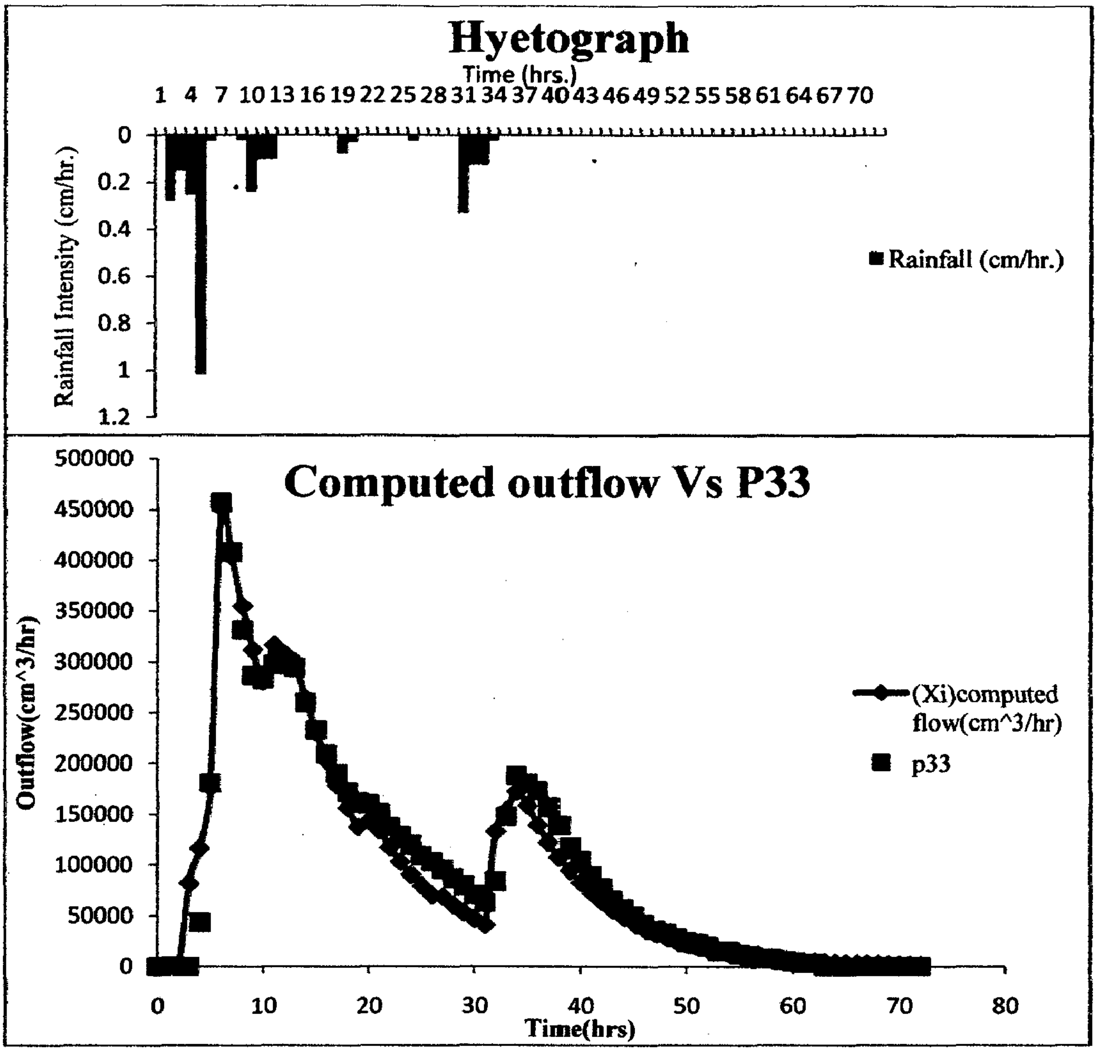

Figure 4.2: Comparison of observed and predicted hydrographs after calibration for event-1

Table 4.3 (see in Appendix-II) shows the computational details of the tile drain flow for single burst event-2, of hydrograph plot P18. Figure 4.3 shows that the estimated drain flow hydrograph well matches with the observed hydrograph P18 of event-2, with the efficiency of above 96%. The parameter K is 4.5hours and the delay being 1hr.

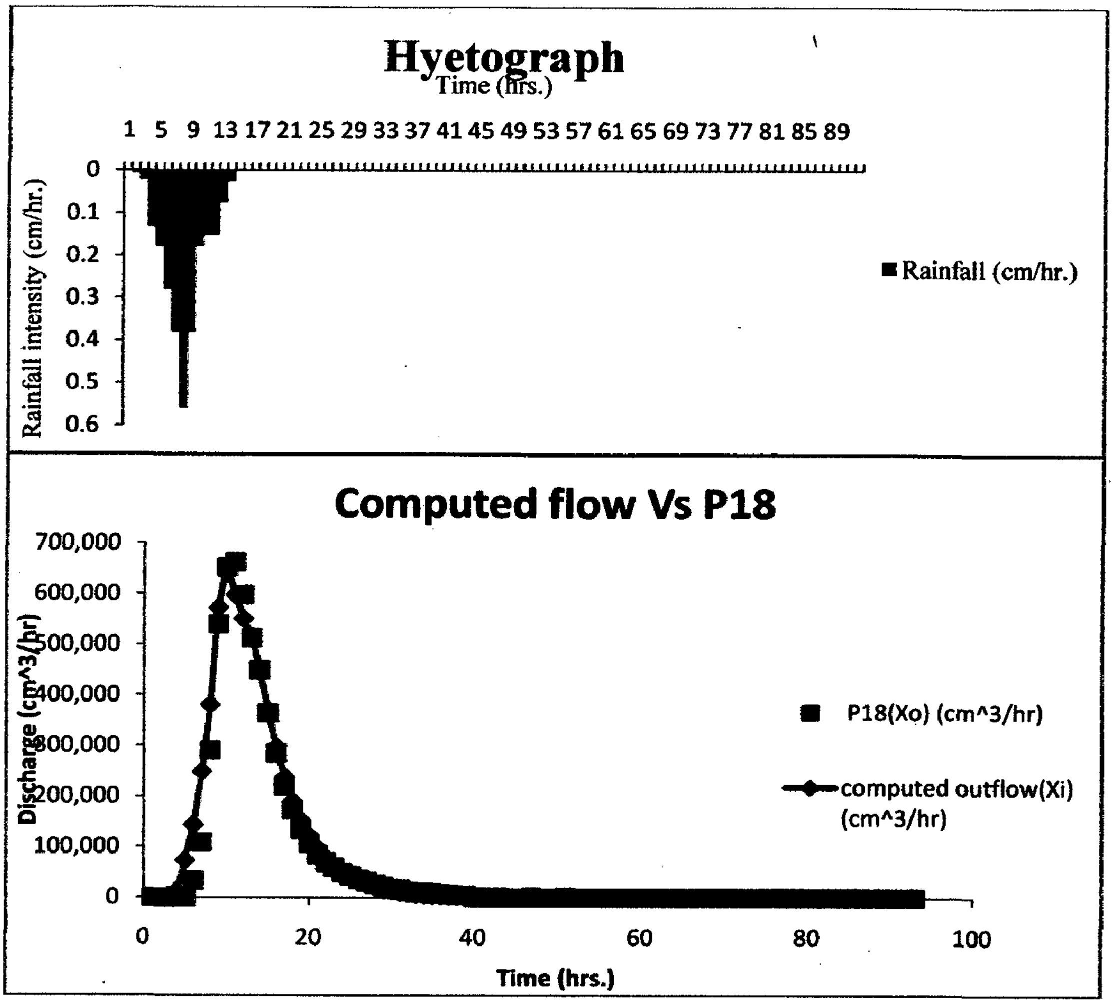

Figure 4.3: Comparison of observed and predicted hydrographs after calibration for event-2

The computational flow has little bit more variation in terms of efficiency with above 88% as shown in Table 4.4 (Appendix-II), in compared to previous hydrograph plots. But as shown in Figure 4.4 the computational flow follows the trend of observed flow. In this hydrograph plot, the parameter K and delay time are 4.4hours and 1hr. respectively. From the figure we can observe that the observed hydrograph P13 has no outflow up to 8hours from the recharge takes place for the event-2, thereafter at 9thhr it has been observed the flow is about 550,000 cm^3, whereas the peak flow is about 700,000 cm^3. Therefore, here we can say observed error has been occurred. Due to these deviations between observed and computed hydrographs at the initial hours the efficiency is diminished, whereas for the remaining hours the computed hydrograph matches with the observed hydrograph.

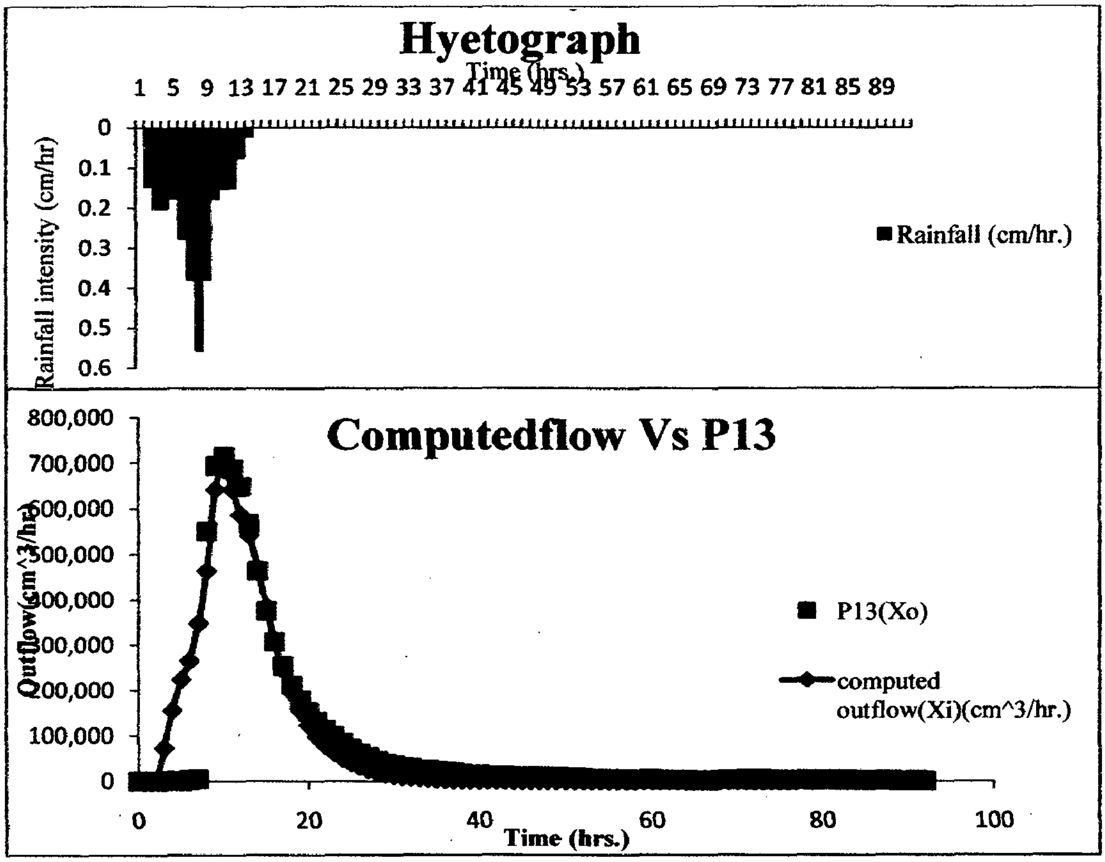

Figure 4.4: Comparison of observed and predicted hydrographs after calibration for event-2

While simulating hydrograph plot P12 it was noticed that some deviation occurred with the observed flow for event-3 and event-4, both are having multiple bursts. The efficiency of the tile drain hydrograph of estimated was 83.6% for event-3. The tuned parameter K (storage coefficient) and delay time are 5.5hours and 2hous respectively. It can be observed from Figure 5.5 that the peak of the computed flow is lesser than the observed flow.

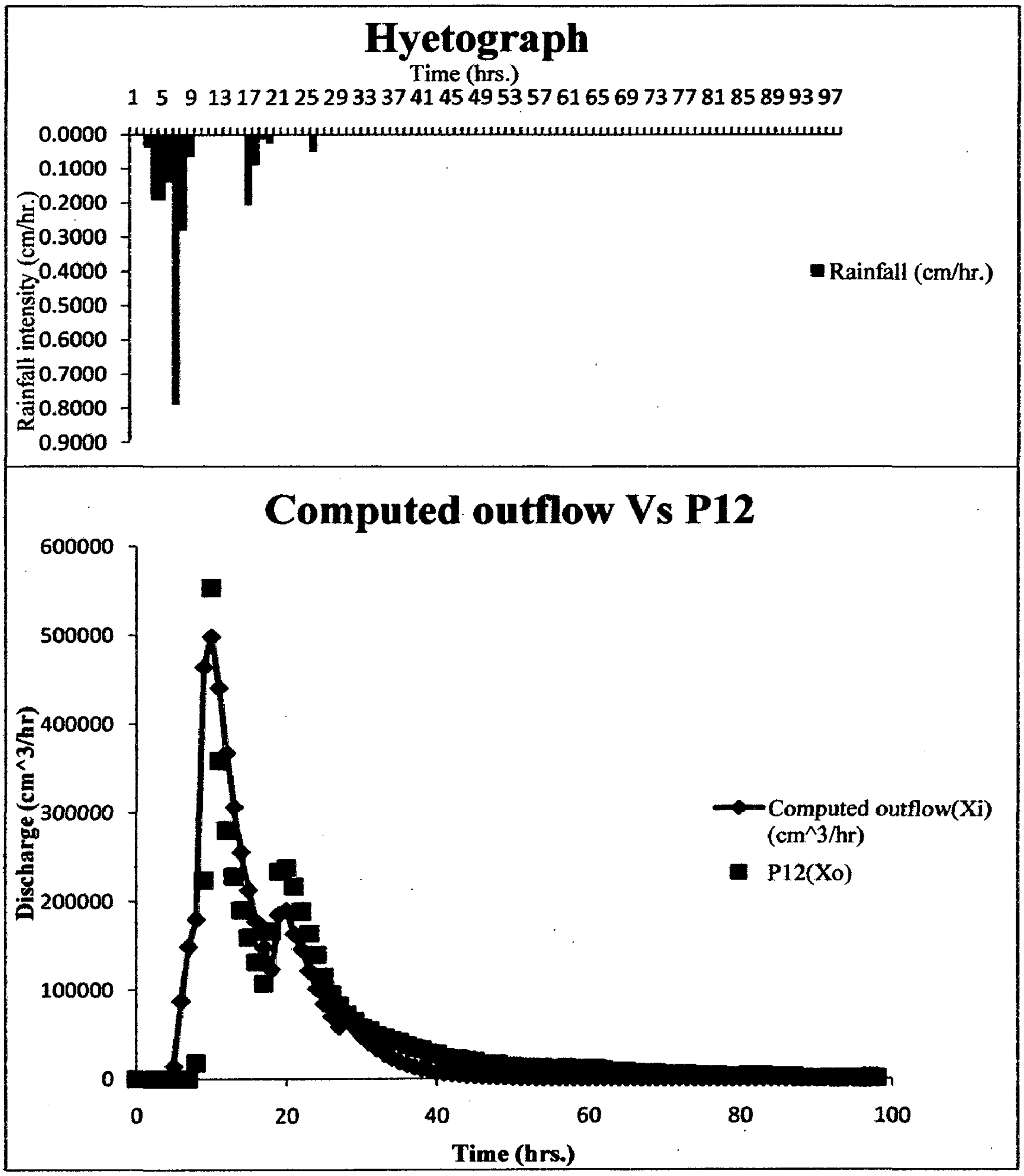

Figure 4.5: Comparison of observed and predicted hydrographs after calibration for event-3

Actually, event-4 has multiple bursts. When computing the tile drain flow for the hydrograph plot P12 it was observed that for best trial value of K and delay time, it was unable to produce the hydrograph close to the observed hydrograph. It was also observed that the peak of the estimated hydrograph was high due to the first burst, and the peak due to the second burst was lower than that of the corresponding peak of the observed hydrograph. Therefore, the first peak of the P12 hydrograph produced due to the first burst was removed and the hydrograph produced by the second burst only was considered for analysis.

It can be observed from Table 4.6 presenting the estimated hydrograph due to the analysis of second burst of storm event-4 of P12 hydrograph plot that the volume of this estimated hydrograph was found to be lower than that of the observed hydrograph, though the observed hydrograph has been produced with an efficiency of about 90%. The plausible reason for this lower volume of the reproduced hydrograph may be attributed to the release of accumulated water in the unsaturated zone of the soil due to the first burst of storm event-4 over and above the field capacity and the same got released with the observed hydrograph of second burst of storm event-4. The best trial values of K and delay time which resulted in the reproduction efficiency of about 90% were 2.3 and 1hr. respectively.

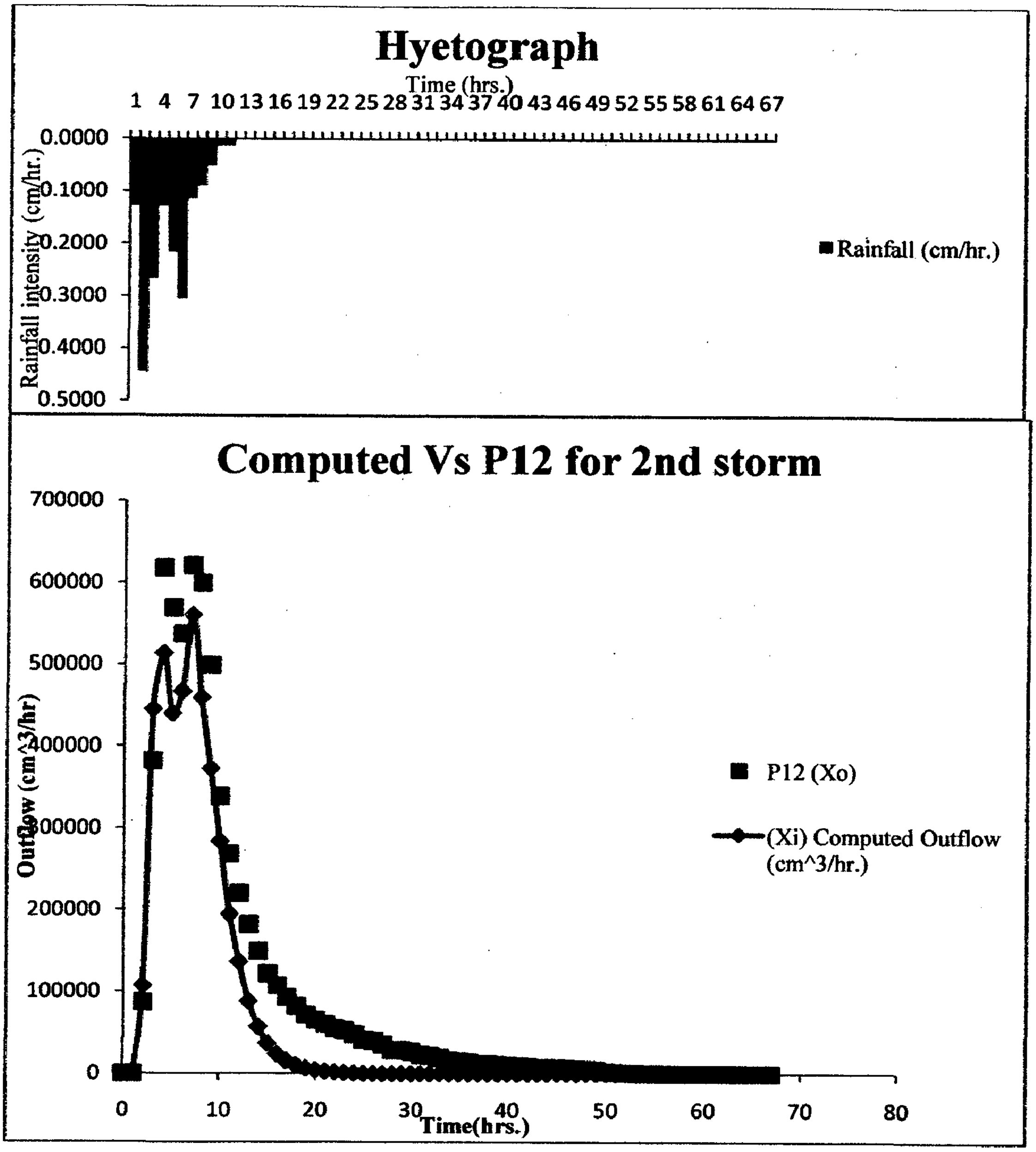

Figure 4.6: Comparison of observed and predicted hydrographs after calibration for event-4

Table 4.7 presents the reproduction results of plot P11 due to storm event-5. It can be observed from the estimated hydrograph that there an immediate response of second peak, though small in magnitude, due to a small second burst. This proves that the linear system considered for the analysis of recharge and the corresponding tile drain hydrograph from the considered agriculture plot is sensitive to subsequent storm bursts, however, small it may be.

Here, the best trial values of K and delay time were estimated to be 2.75 hours and 1hour and it can be observed that the estimated flow for hydrograph P11 matches well with the observed tile drain hydrograph in terms of peak and base time. The efficiency of computed flow was about 91%.

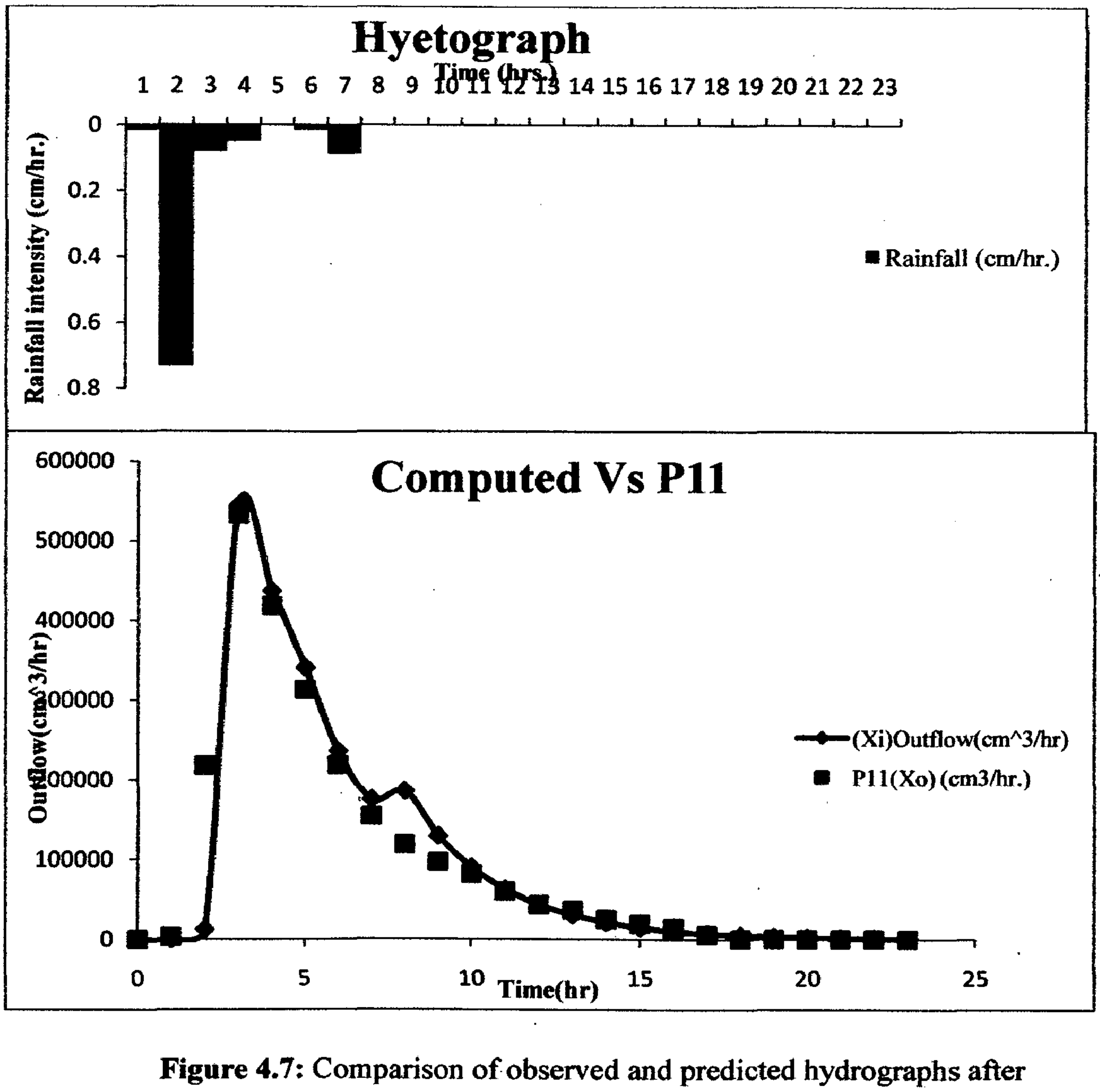

Figure 4.7: Comparison of observed and predicted hydrographs after calibration for event-5

Based on the analysis of all the results considered in this study that those events with the volume of the tile drain hydrograph less than the volume of the causative storm bursts could produce the observed tile drain hydrograph closely by the single linear reservoir model. Therefore, it can be seated that single linear reservoirs may be considered as a suitable model for the tile drain hydrographs from agricultural plots of the size studied herein.

Table 4.0: Computation of Tile drain flow Results.

Event	1		2		3	4	5
Date	2/20/1997		3/13/1997		4/8/2002	5/11/2002	5/9/2002
Total precipitation (cm)	3.05		2.97		2.39	3.16 (4.1) 1.4 (4.2) 1.76	1.14
No. of bursts	3		1		2	2nd burst	1
Observed hydrograph	P14	P33	P13	P18	P12	P12	P11
Considered ppt For recharge (cm)	2.923	2.9348	2.9186	2.5235	2.1	1.76	0.983
Recharge Volume (cm^3)	7015680	7043520	7004626.7	6056318.3	4971360	4236720	2360160
Computed Volume (cm^3)	7015000.23.	7034146.7	7004626.42	6055817.7	4971358.3	4236720.06	2358346.1
K/T	5.2	7.7	4.4	4.5	5.5	2.3	2.75
Observed Volume (cm^3)	7019892.93	7336999	6986161	5658272	4951204	6268348	2360456
Efficiency (%)	94.24	96.38	88.14	96.4	83.55	89.75	91.15
Delay Time (hrs.)	1	1	1	1	2	1	1

4.2 Verification of applicability of linear system theory subsurface flow modeling

While the simulation of tile drain hydrograph study described in section 5.2 demonstrates the applicability of linear system theory for modeling tile drain hydrograph using linear reservoir model, an alternative way of strengthening the verification of the suitability of applying linear system theory for soil water flux movement can be carried out by studying it using Richard's equation which governs the flow movement in unsaturated soil domain.

To demonstrate this approach, the soil water flux a hypothetical 10m depth of uniform characteristics soil was studied. The entire 10m thickness of soil mass is divided into 50 equal thickness layers. The behavior of the soil water flux at the outlet of the bottom most soil layer is studied by relating the flux with the soil water storage of the last layer.

The following conditions are assumed to prevail in the considered 10m soil layer immediately prior to the study of flux movement:

i. Initially the entire soil layer is under saturated condition.
ii. There is no ponding of water on the subsurface of soil due to rain or artificial irrigation.
iii. The moisture condition of the top of the upper soil layer (i.e. the ground surface always under field capacity condition) Relative humidity of 100% prevails immediately above the top of soil layer.
iv. Water table forms the boundary condition at the bottom of the 10m thick soil layer and , therefore, saturated moisture condition prevails always at the bottom layer.

With the above initial and boundary conditions, the Richard's equation was applied to this 10m soil layer to study the soil water flux throughout the soil which has been divided into 50 equal thickness soil layers. The water movement was studied for 3000hours. The soil water flux nearer to the upper soil layer is taking place in the upper direction while at the bottom of the soil layer is in the downward direction.

The soil moisture condition simulated up to a time of 3000hours of study is shown in Figure 4.9. it can be seen from this figure that the soil moisture in the entire soil column is approaching towards the condition of field capacity when the top of the upper soil layer is maintained at the field capacity condition throughout the study.

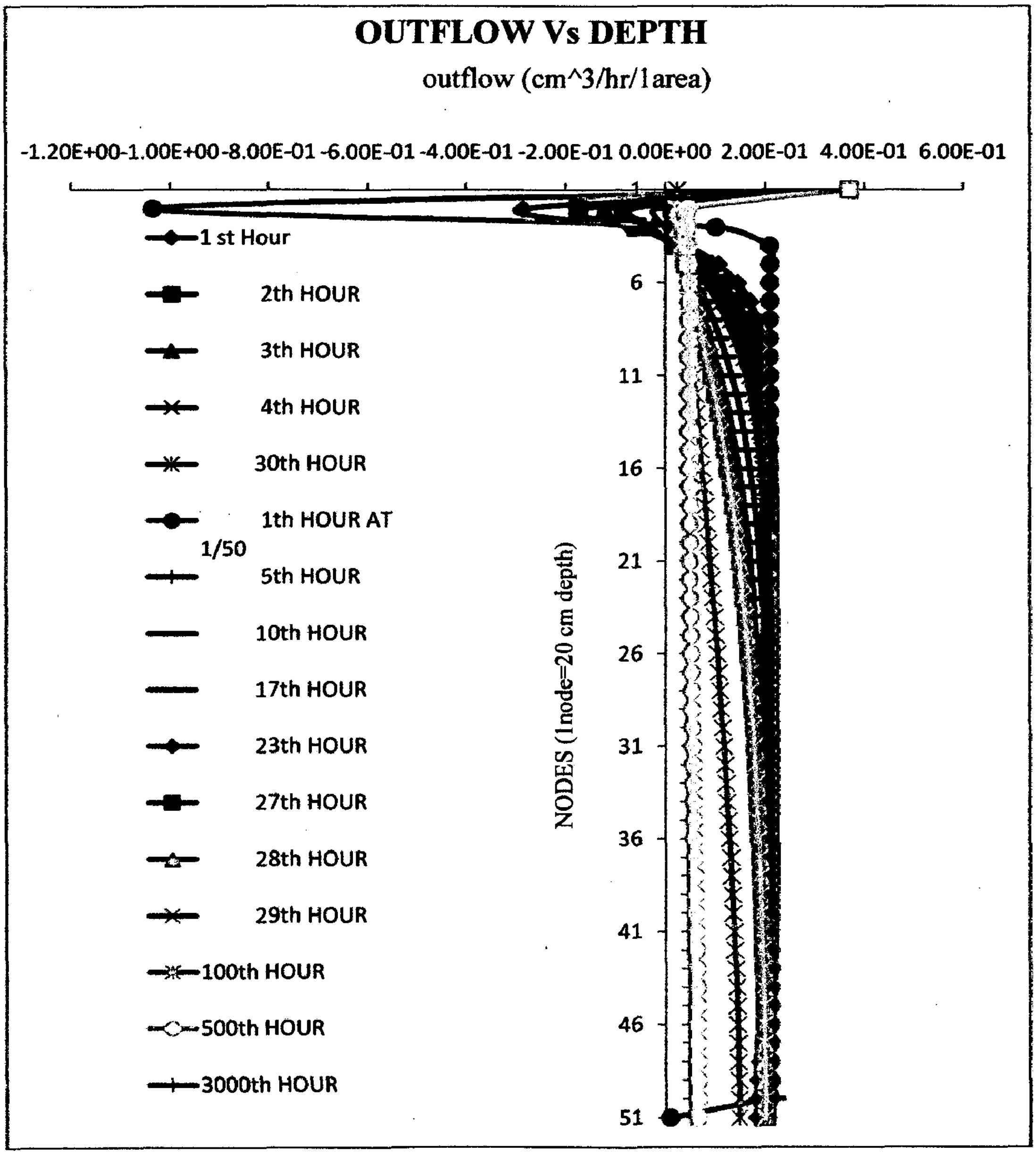

Figure 4.8: Movement of flux

Figure 4.9 shows the variation of soil water flux in the entire 10m soil thickness studied using 50 equal thickness soil layers at different simulation times.

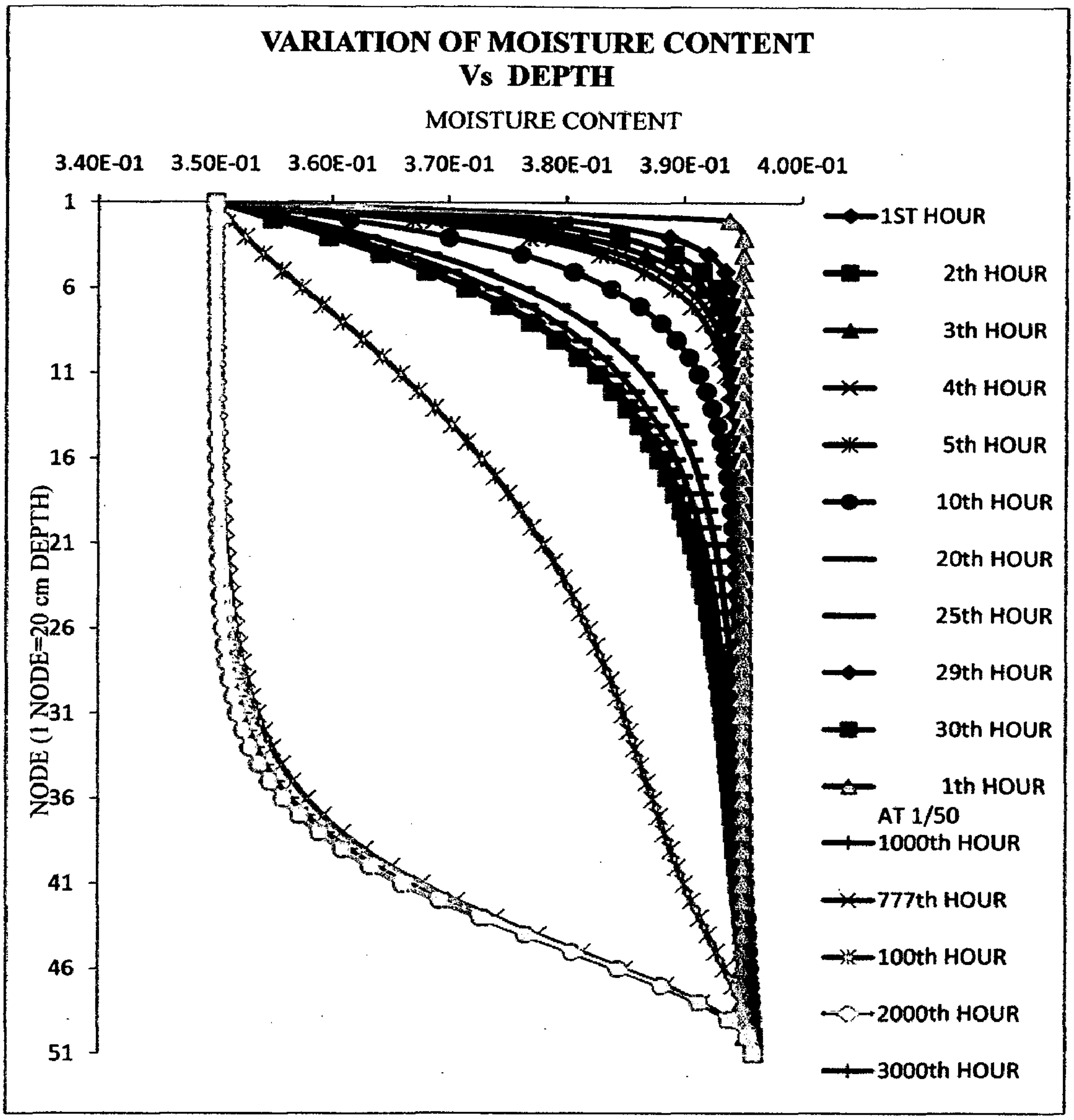

Figure 4.9: Variation of moisture content with respect to depth

In order to verify the applicability of linear system theory to soil water flux, the flow movement through the bottom most layer of 10m thickness of soil is studied. While the soil water flux of the bottom most soil layer is in the downward direction through the lower surface of this soil layer. Figure 4.10 shows the relationship between the soil water flux and the corresponding total soil moisture storage that prevails in the 10m soil thickness. This relationship shown in this figure was simulated for a period of 500hours. It can be seen that except for few hours at the beginning of simulation period, the relation between soil water flux and the soil moisture storage within the considered 10m thickness of soil layer is perfectly linear with the upper boundary condition: moisture content at field capacity. It can be inferred from this study, as demonstrated by figure 4.10 that the linear system theory application for flow movement study in the subsequent modeling is appropriate. This appropriates was confirmed from the tile hydrograph simulation studies illustrated in section 4.1.

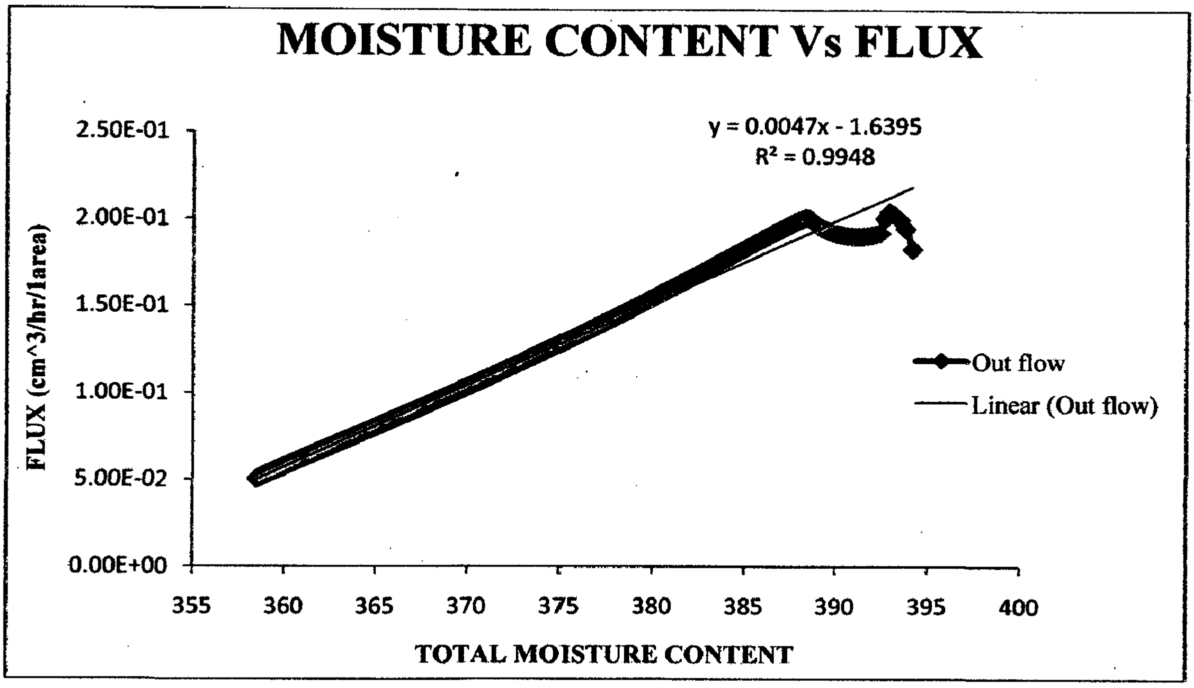

Figure 4.10: Relation between Flux and Total moisture content

CHAPTER-5

CONCLUSION

This study was carried out with the objective of evaluating the appropriateness of modeling the groundwater flow system using linear system theory. To achieve this objective, two types of analysis were taken up:

1) By reproducing the tile drain hydrograph for given rainfall bursts and analyzing them by applying a linear reservoir model, and
2) Substantiating the use of linear reservoir model for tile drain hydrograph simulation by establishing the linear storage-discharge relationship for hypothetical scenarios of generating flow-storage relationships of a soil media using the Richard's equation, which governs the flow in unsaturated soil strata under specified initial and boundary conditions.

Both these analysis confirm the appropriateness of using the linear reservoir model for groundwater flow modeling. However, an extensive analysis under varied initial and boundary conditions need to be carried out for the wide application of linear system theory for groundwater modeling.

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ANNEXURE-I

/* A PROGRAM FOR COMPUTING SPRING/TILE DRAIN FLOW USING LINEAR SYSTEM THEORY FOR A GIVEN RECHARGE */

```
#include<iostream>

#include<math.h>

#include<fstream>

using namespace std;

int main()

{

   double R[150],Q[150],C1,C2,KT;

   int i,nhours,dt;

   ifstream infile("recharge.txt");

   ofstream outfile("outflow.txt");

   cout<<"enter K/T, nhours & delay time<10hrs. respectively\n";

   cin>>KT>>nhours>>dt;

   C1=exp(-1/KT);

   C2=1-C1;

   for(i=1;i<=nhours;i++)

   {

      infile>>R[i];

   }

   outfile<<"for given  K/T="<<KT<<", delay time="<<dt<<", C1="<<C1<<",
C2="<<C2<<" outflow is in cm^3/hr\n\n";
```

```
for(i=0;i<=dt;i++)
{
 Q[i]=0;
 outfile<<Q[i]<<endl;
}
for(i=1;;i++)
{
   if(i>=nhours)R[i]=0;
   Q[i+dt]=C2*R[i]+C1*Q[i+dt-1];


   outfile<<Q[i+dt]<<endl;
   if(i>=nhours-dt)
   {
    if(Q[i+dt]<=1000||i==140)break;
   }
 }
 system("pause");
 return 1;
}
```

ANNEXURE-II

COMPUTATION OF TILE DRAIN FLOW BY USING LINEAR SYSTEM THEORY FOR A GIVEN HYETOGRAPHS

Table 4.1: Simulated tile drain flow for the event-1 of hydrograph plot P14.

Time (hr.)	Rainfall (cm/hr.)	Recharge rate (cm^3/hr.)	(Xi) computed outflow (cm^3/hr.)	observed flow P14 (Xo)	(Xi-Xo)^2	(Xi-X_{mean})^2
0	0	0	0	0	0	9505960016
1	0.002	4800	0	0	0	9505960016
2	0.3002	720480	839.746	0	705173.3445	9342917217
3	0.1524	365760	126739	0	16062774121	855006086.4
4	0.254	609600	168555	54372	13037766461	5049024360
5	1.016	2438400	245714	392424	21523740898	21967830619
6	0.0254	60960	629318	625278	16325250.87	2.82832E+11
7	0	0	529886	472800	3258850402	1.86959E+11
8	0	0	437184	366420	5007581168	1.15386E+11
9	0.024	57600	360700	299046	3801242361	69275022817
10	0.2094	502560	307673	306138	2356904.164	44173315032
11	0.1016	243840	341768	317958	566927040.8	59667582333
12	0.1016	243840	324636	325050	171201.5781	51591438051
13	0	0	310501	308502	3996892.277	45370059515
14	0	0	256180	257676	2237458.947	25179814351
15	0	0	211362	212760	1953974.176	12964893697
16	0	0	174385	180846	41742832.42	5911531900
17	0	0	143877	154842	120228771.3	2150964067
18	0	0	118706	132384	187085067.2	449757509.5
19	0.0762	182880	97938.6	117018	364020277.8	193676.6644
20	0.0254	60960	112799	112290	259163.606	234104905.8
21	0	0	103730	101652	4318389.272	38831431.6
22	0	0	85582.6	88650	9408549.787	141988980
23	0	0	70610.2	79194	73680640.04	722981370.1
24	0	0	58257.1	69738	131809907.7	1539888486
25	0	0	48065.2	65010	287124655.1	2443652423
26	0.0254	60960	39656.3	61464	475573842.2	3345721592
27	0	0	43383.4	56736	178290831.9	2928445443
28	0	0	35793.6	49644	191832586.5	3807496275

29	0	0	29531.6	44916	236678764.7	4619501248
30	0	0	24365.1	40188	250363245.4	5348496081
31	0.3302	792480	20102.5	37824	314050593.6	5990142811
32	0.127	304800	155228	115836	1551736258	3332693682
33	0.127	304800	181395	206850	647949415.7	7038620549
34	0.0254	60960	202984	234036	964216201.6	11127187991
35	0	0	178138	209214	965708380.2	6502726881
36	0	0	146973	186756	1582676352	2447724875
37	0	0	121260	152478	974556644.9	564608269.7
38	0	0	100046	117018	288045913.9	6489690.576
39	0	0	82543.5	89832	53121286.05	223652410.5
40	0	0	68102.7	68556	205435.9818	864113815.5
41	0	0	56188.4	54372	3299451.688	1706525427
42	0	0	46358.4	43734	6887641.231	2615311146
43	0	0	38248.1	35460	7773644.489	3510611428
44	0	0	31556.7	27186	19103190.21	4348322687
45	0	0	26036	21276	22657746.36	5106890749
46	0	0	21481.1	16548	24335593.58	5778647062
47	0	0	17723	14184	12524593.54	6364132457
48	0	0	14622.4	11820	7853493.63	6868450088
49	0	0	12064.3	9456	6803264.534	7299004732
50	0	0	9953.66	9456	247672.2764	7664101268
51	0	0	8212.3	8274	3806.15227	7972027812
52	0	0	6775.58	7092	100118.3734	8230650552
53	0	0	5590.21	7092	2255357.812	8447136140
54	0	0	4612.22	5910	1684221.844	8627863407
55	0	0	3805.33	5910	4429617.833	8778412520
56	0	0	3139.6	4728	2523003.707	8903604442
57	0	0	2590.33	4728	4569618.423	9007563179
58	0	0	2137.16	3546	1984822.926	9093787625
59	0	0	1763.27	3546	3178117.117	9165236731
60	0	0	1454.79	3546	4373148.548	9224396707
61	0	0	1200.28	2364	1354240.263	9273349658
62	0	0	990.296	2364	1887057.987	9313835927
63	0	0	817.046	2364	2393061.393	9347306040
64	0	0	674.107	1182	257954.4319	9374965576
65	0	0	556.174	1182	391657.1132	9397817069
66	0	0	458.873	0	210564.4301	9416691710
67	0	0	378.594	1182	645461.2008	9432278645

68	0	0	312.36	0	97568.7696	9445148314
69	0	0	257.714	0	66416.5058	9455772969
70	0	0	212.628	0	45210.66638	9464543399
71	0	0	175.429	0	30775.33404	9471782658
72	0	0	144.738	0	20949.08864	9477757485
Total=	2.9232	7015680	7015000.235	7019892.928	73325302356	1.27299E+12
			X mean =	97498.51289		
			efficiency =	1-res var/total variance	0.942399216	94.23992163

Table 4.2: Simulated tile drain flow for the event-1 of hydrograph plot P33.

Time (hr.)	Rainfall (Cm/hr.)	Recharge rate (cm^3/hr.)	(Xi) Computed flow (cm^3/hr.)	(Xo) Observed flow (cm^3/hr.) P33	$(Xi-Xo)^2$	$(Xi-X_{mean})^2$
0	0	0	0	0	0	8204779136
1	0	0	0	0	0	8204779136
2	0.2802	672480	0	0	0.000196953	8204779136
3	0.1524	365760	81901.7	0	6707888463	75316985.37
4	0.254	609600	116473	43448	5332658981	670435232.7
5	1.016	2438400	176531	180896	19051858.49	7387533849
6	0.0254	60960	452005	457286	27885198.44	1.30628E+11
7	0	0	404380	407984	12986505.05	98470291949
8	0	0	355130	331790	544767996.7	69986577685
9	0.023	55200	311879	286970	620469897.6	48973142992
10	0.24	576000	280618	282488	3496040.07	36114351784
11	0.1016	243840	316592	297428	367268123	51081317512
12	0.1016	243840	307732	300416	53527410.12	47154888652
13	0	0	299950	294440	30362729.19	43835698119
14	0	0	263419	260078	11163709.32	29873238376
15	0	0	231337	233186	3418082.453	19812466640
16	0	0	203163	209282	37439994.36	12674878772
17	0	0	178419	189860	130892751	7715648479
18	0	0	156690	171932	232313989.7	4370500909
19	0.0762	182880	137606	161474	569674621.7	2211422489
20	0.0274	65760	143120	159980	284254831.3	2760426812
21	0	0	133698	151016	299908450.2	1859141581
22	0	0	117415	137570	406218977.1	720104564.3
23	0	0	103115	128606	649785026.9	157120311
24	0	0	90556.7	119642	845948145.2	553.9385888
25	0	0	79527.8	109184	879483987.5	122156339.3
26	0.0254	60960	69842	103208	1113283256	430074428.2
27	0	0	68760.3	95738	727791171.6	476109602.7
28	0	0	60385.9	86774	696327149.8	911697920.4
29	0	0	53031.5	79304	690239888.5	1409907568

30	0	0	46572.8	70340	564876152.5	1936654414
31	0.3322	797280	40900.6	62870	482651405.7	2468066223
32	0.127	304800	133020	83786	2423995260	1801133577
33	0.127	304800	153942	148028	34976966.58	4014713150
34	0.0254	60960	172315	188366	257629402.7	6680571662
35	0	0	158753	180896	490305516.8	4647525765
36	0	0	139418	173426	1156533784	2385127202
37	0	0	122438	156992	1193969292	1014917133
38	0	0	107527	139064	994574402.4	287192813.5
39	0	0	94430.8	118148	562500301.5	14826843.89
40	0	0	82930	104702	474015565.3	58526109.33
41	0	0	72830	89762	286689553.2	315070874.5
42	0	0	63960	77810	191820227.4	708636959.4
43	0	0	56170.2	65858	93852046.51	1184050571
44	0	0	49329.2	56894	57225186.4	1701647963
45	0	0	43321.4	49424	37240975.73	2233397571
46	0	0	38045.3	41954	15277496.86	2759919490
47	0	0	33411.7	35978	6585629.732	3268241497
48	0	0	29342.5	32990	13303893.99	3750060298
49	0	0	25768.8	28508	7502962.334	4200522223
50	0	0	22630.4	24026	1947578.833	4617180199
51	0	0	19874.3	22532	7063145.503	4999329371
52	0	0	17453.8	19544	4368769.064	5347475627
53	0	0	15328.1	15062	70828.74539	5662883958
54	0	0	13461.3	15062	2562122.986	5947330274
55	0	0	11821.8	12074	63587.41657	6202891225
56	0	0	10382	10580	39190.7487	6431757041
57	0	0	9117.59	7592	2327520.371	6636162677
58	0	0	8007.15	7592	172375.5174	6818314515
59	0	0	7031.96	6098	872337.7443	6980314406
60	0	0	6175.53	4604	2469798.154	7124154378
61	0	0	5423.41	3110	5351995.695	7251684997
62	0	0	4762.89	3110	2732138.14	7364616857
63	0	0	4182.82	122	16490469.5	7464513474
64	0	0	3673.39	122	12612554.96	7552799864
65	0	0	3226	0	10407076	7630762530
66	0	0	2833.11	0	8026512.272	7699558104
67	0	0	2488.06	0	6190442.564	7760231455
68	0	0	2185.04	0	4774399.802	7813710658
69	0	0	1918.92	0	3682253.966	7860828937
70	0	0	1685.22	0	2839966.448	7902323852
71	0	0	1479.97	0	2190311.201	7938857383
72	0	0	1299.73	0	1689298.073	7971008734
Total=	2.9348	7043520	7034146.69	7336999.109	30742977932	8.48906E+11
			X_{mean} =	90580.23592		
			efficiency =	1-residual var/total variance	0.963785186	96.37851861

Table 4.3: Simulated tile drain flow for the event-2 of hydrograph plot P18.

Time (hr.)	Rainfall (cm/hr.)	Recharge rate (cm^3/hr.)	Outflow (Xi) (cm^3/hr.)	Observed flow P18 (Xo) (cm^3/hr.)	(Xi-Xo)^2	(Xi-X_{mean})^2
0	0	0	0	0	0	3782614089
1	0	0	0	0	0	3782614089
2	0.007	16800	0	0	0	3782614089
3	0.0211	50640	3347.61	0	11206492.71	3382044734
4	0.13319	319656	12771.2	1,196	133985275	2374784433
5	0.17779	426696	73921.9	33,488	1634902226	154230070.7
6	0.279399	670557.6	144217	108,836	1251820726	6841612413
7	0.38099	914376	249097	290,628	1724806518	35191523844
8	0.5587989	1341117.36	381662	540,592	25258620737	1.02502E+11
9	0.380999238	914398.1712	572846	653,016	6427153242	2.61472E+11
10	0.177799644	426719.1466	640904	662,584	470001640.7	3.35706E+11
11	0.152399695	365759.2685	598225	598,000	50819.63385	2.88071E+11
12	0.152399695	365759.2685	551903	513,084	1506943545	2.40492E+11
13	0.076199848	182879.6342	514812	449,696	4240135774	2.05489E+11
14	0.025399949	60959.87808	448670	364,780	7037576324	1.49898E+11
15	0	0	371414	285,844	7322260248	96044852714
16	0	0	297405	220,064	5981654878	55649772476
17	0	0	238143	173,420	4189082950	31201703731
18	0	0	190690	133,952	3219211628	16689291304
19	0	0	152693	106,444	2138977115	8315623395
20	0	0	122267	84,916	1395101785	3692268557
21	0	0	97903.7	69,368	814289035.1	1325013873
22	0	0	78395.1	59,800	345779351	285344393.8
23	0	0	62773.9	49,036	188730870	1615288.484
24	0	0	50265.4	43,056	51975896.95	126282754.8
25	0	0	40249.4	35,880	19091882.93	451713812.7
26	0	0	32229.2	31,096	1284193.165	856953024.5
27	0	0	25807.1	26,312	254904.8114	1274194421
28	0	0	20664.7	21,528	745260.0316	1667763480
29	0	0	16547	16,744	38804.23317	2021038340
30	0	0	13249.8	15,548	5281671.601	2328367450
31	0	0	10609.6	11,960	1823556.82	2590134092
32	0	0	8495.52	10,764	5145966.223	2809788695
33	0	0	6802.68	9,568	7646956.465	2992120632
34	0	0	5447.16	8,372	8554653.638	3142252714
35	0	0	4361.75	7,176	7919973.877	3265117880
36	0	0	3492.61	5,980	6187087.516	3365200707
37	0	0	2796.67	4,784	3949466.789	3446428486
38	0	0	2239.4	4,784	6474971.567	3512169544
39	0	0	1793.17	1,196	356613.0411	3565259022
40	0	0	1435.86	1,196	57533.23418	3608056502
41	0	0	1149.74	0	1321902.068	3642511164
42	0	0	920.643	1,196	75821.00152	3670217133
43	0	0	737.193	0	543453.5192	3692478439
44	0	0	590.298	0	348451.7288	3710352392
45	0	0	472.674	0	223420.7103	3724695809
46	0	0	378.488	1,196	668324.4571	3736201077

47	0	0	303.069	0	91850.81876	3745426658
48	0	0	242.679	0	58893.09704	3752822028
49	0	0	194.322	0	37761.03968	3758749093
50	0	0	155.601	1,196	1082428.281	3763498456
51	0	0	124.596	0	15524.16322	3767303567
52	0	0	0	0	0	3782614089
53	0	0	0	0	0	3782614089
54	0	0	0	0	0	3782614089
55	0	0	0	0	0	3782614089
56	0	0	0	0	0	3782614089
57	0	0	0	0	0	3782614089
58	0	0	0	0	0	3782614089
59	0	0	0	0	0	3782614089
60	0	0	0	0	0	3782614089
61	0	0	0	0	0	3782614089
62	0	0	0	0	0	3782614089
63	0	0	0	0	0	3782614089
64	0	0	0	0	0	3782614089
65	0	0	0	0	0	3782614089
66	0	0	0	0	0	3782614089
67	0	0	0	0	0	3782614089
68	0	0	0	0	0	3782614089
69	0	0	0	0	0	3782614089
70	0	0	0	0	0	3782614089
71	0	0	0	0	0	3782614089
72	0	0	0	0	0	3782614089
73	0	0	0	0	0	3782614089
74	0	0	0	0	0	3782614089
75	0	0	0	0	0	3782614089
76	0	0	0	0	0	3782614089
77	0	0	0	0	0	3782614089
78	0	0	0	0	0	3782614089
79	0	0	0	0	0	3782614089
80	0	0	0	0	0	3782614089
81	0	0	0	0	0	3782614089
82	0	0	0	0	0	3782614089
83	0	0	0	0	0	3782614089
84	0	0	0	0	0	3782614089
85	0	0	0	0	0	3782614089
86	0	0	0	0	0	3782614089
87	0	0	0	0	0	3782614089
88	0	0	0	0	0	3782614089
89	0	0	0	0	0	3782614089
90	0	0	0	0	0	3782614089
91	0	0	0	0	0	3782614089
92	0	0	0	0	0	3782614089
Total=	2.52346597	6056318.327	6055817.733	5,658,272	75423548404	2.0932E+12
			Xmean=	61502.95556		
			Efficiency=	1-RV/TV	0.963967365	96.39673651

Table 4.4: Simulated tile drain flow for the event-2 of hydrograph plot P13.

Time (hr.)	Rainfall (cm/hr.)	Recharge rate (cm^3/hr.)	Computed outflow (Xi) (cm^3/hr.)	Observed flow P13(Xo)	(Xi-Xo)^2	$(Xi-X_{mean})$^2
0	0	0	0	0	0	5766356588
1	0	0	0	0	0	5766356588
2	0.15	360000	0	0	0	5766356588
3	0.203199594	487679.0246	73186.8	1,402	5.153E+09	7561015.073
4	0.203199594	487679.0246	157452	0	2.479E+10	6644771849
5	0.177799644	426719.1466	224586	0	5.044E+10	22096664931
6	0.279399441	670558.6589	265679	2,804	6.91E+10	36002204922
7	0.380999238	914398.1712	347989	4,206	1.182E+11	74012546433
8	0.558798882	1341117.318	463138	549,584	7.473E+09	1.49925E+11
9	0.380999238	914398.1712	641628	693,989	2.742E+09	3.20007E+11
10	0.177799644	426719.1466	697082	715,019	321753309	3.85822E+11
11	0.152399695	365759.2685	642118	686,980	2.013E+09	3.20561E+11
12	0.152399695	365759.2685	585935	647,724	3.818E+09	2.60098E+11
13	0.076199848	182879.6342	541174	566,408	636734101	2.16446E+11
14	0.025399949	60959.87808	468334	462,660	32198070	1.53976E+11
15	0	0	385516	374,334	125043173	95839448245
16	0	0	307142	307,038	10862.196	53455969358
17	0	0	244701	253,762	82098398	28481446334
18	0	0	194954	211,702	280490380	14165158165
19	0	0	155321	178,054	516783439	6301894077
20	0	0	123745	152,818	845232908	2285649804
21	0	0	98587.8	131,788	1.102E+09	513080032.6
22	0	0	78545.2	113,562	1.226E+09	6805159.169
23	0	0	62577.2	99,542	1.366E+09	178471698
24	0	0	49855.5	84,120	1.174E+09	680220125.9
25	0	0	39720.1	72,904	1.101E+09	1311629802
26	0	0	31645.1	61,688	902573162	1961730771
27	0	0	25211.8	53,276	787597161	2572998234
28	0	0	20086.3	44,864	613932811	3119248191
29	0	0	16002.8	39,256	540709991	3592051992
30	0	0	12749.5	35,050	497311171	3992600760
31	0	0	10157.6	30,844	427926223	4326867632
32	0	0	8092.58	29,442	455796826	4602801552
33	0	0	6447.38	25,236	353011556	4828741968
34	0	0	5136.65	25,236	403983137	5012623008

35	0	0	4092.39	22,432	336340700	5161580452
36	0	0	3260.42	21,030	315757433	5281816965
37	0	0	2597.59	19,628	290034382	5378600120
38	0	0	2069.51	16,824	217694616	5456336644
39	0	0	1648.78	15,422	189701282	5518669800
40	0	0	1313.59	14,020	161452598	5568583174
41	0	0	1046.54	11,216	103417752	5608510602
42	0	0	833.785	11,216	107790220	5640422307
43	0	0	664.279	11,216	111338645	5665911771
44	0	0	529.233	8,412	62137920	5686260441
45	0	0	421.642	9,814	88216256	5702498310
46	0	0	335.924	9,814	89833790	5715451628
47	0	0	267.632	9,814	91133007	5725782125
48	0	0	213.223	8,412	67219845	5734019223
49	0	0	169.876	9,814	93008991	5740585858
50	0	0	135.34	7,010	47260880	5745820405
51	0	0	107.826	8,412	68959205	5749992350
52	0	0	85.9055	7,010	47943014	5753317237
53	0	0	68.4412	5,608	30686667	5755966898
54	0	0	54.5274	5,608	30841013	5758078319
55	0	0	43.4421	5,608	30964260	5759760791
56	0	0	34.6105	5,608	31062625	5761101384
57	0	0	27.5743	5,608	31141106	5762169555
58	0	0	21.9685	4,206	17506094	5763020648
59	0	0	17.5024	4,206	17543487	5763698752
60	0	0	13.9442	4,206	17573306	5764239035
61	0	0	11.1094	2,804	7800226.6	5764669493
62	0	0	8.85091	2,804	7812847.1	5765012452
63	0	0	7.05155	2,804	7822909.3	5765285698
64	0	0	5.618	2,804	7830930.5	5765503397
65	0	0	4.47588	1,402	1953070.8	5765676843
66	0	0	3.56595	1,402	1955615	5765815029
67	0	0	2.841	1,402	1957643.1	5765925125
68	0	0	2.26344	2,804	7849716.4	5766012838
69	0	0	1.80329	2,804	7852295	5766082721
70	0	0	1.43669	2,804	7854349.7	5766138396
71	0	0	1.14461	4,206	17680783	5766182754
72	0	0	0.911917	2,804	7857291.4	5766218094
73	0	0	0.726527	4,206	17684299	5766246249

74	0	0	0.578827	2,804	7859158.9	5766268681
75	0	0	0.461153	2,804	7859818.7	5766286552
76	0	0	0.367402	2,804	7860344.4	5766300790
77	0	0	0.292711	2,804	7860763.2	5766312134
78	0	0	0.233204	2,804	7861096.9	5766321171
79	0	0	0.185794	1,402	1965080.2	5766328371
80	0	0	0.148023	2,804	7861574.5	5766334108
81	0	0	0.11793	2,804	7861743.3	5766338678
82	0	0	0.0939555	1,402	1965337.7	5766342319
83	0	0	0.0748547	2,804	7861984.9	5766345220
84	0	0	0.059637	1,402	1965433.9	5766347531
85	0	0	0.047513	1,402	1965467.9	5766349372
86	0	0	0.0378538	1,402	1965495	5766350839
87	0	0	0.0301582	1,402	1965516.6	5766352008
88	0	0	0.0240272	1,402	1965533.8	5766352939
89	0	0	0.0191425	0	0.0003664	5766353681
90	0	0	0.0152509	0	0.0002326	5766354272
91	0	0	0.0121505	1,402	1965569.9	5766354743
92	0	0	0.00968031	0	9.371E-05	5766355118
Total=	2.918594463	7004626.711	7004626.415	6,986,161	3.004E+11	2.53156E+12
			X_{mean}=	75936.5321		
			efficiency=	1-Rv/TV	0.881342	88.134203

Table 4.5: Simulated tile drain flow for the event-3 of hydrograph plot P12.

Time (hr.)	Rainfall (cm/hr.)	Recharge Rate (cm^3/hr)	Outflow(Xi) (cm^3/hr.)	Observed flow P12 (cm^3/hr.)	(Xi-Xo)^2	(Xi-Xavg)^2
0	0.0	0	0	0	0	2552521996
1	0.0000	0	0	0	0	2552521996
2	0.0000	0	0	0	0	2552521996
3	0.0362	86880	0	0	0	2552521996
4	0.1905	457200	0	0	0	2552521996
5	0.1905	457200	14443.5	0	208614692.3	1301693519
6	0.1397	335280	88050.5	0	7752890550	1408351535
7	0.7874	1889760	149421	0	22326635241	9780915280
8	0.2794	670560	180319	18226	26274140649	16847134008

9	0.0635	152400	464509	224320	57690755721	1.71385E+11
10	0.0	0	498764	553790	3027860676	2.0092E+11
11	0.0	0	441182	358912	6768352900	1.52615E+11
12	0.0	0	367837	280400	7645228969	1.00688E+11
13	0.0	0	306685	228526	6108829281	65619231530
14	0.0	0	255700	190672	4228640784	42097810610
15	0.0	0	213190	159828	2847503044	26460718810
16	0.0	0	177748	131788	2112321600	16186330395
17	0.2064	495360	148198	107954	1619579536	9540505254
18	0.0889	213360	123560	166838	1872985284	5334477867
19	0.0127	30480	185371	234134	2377830169	18184120649
20	0.0254	60960	190024	238340	2334435856	19460671292
21	0.0	0	163500	217310	2895516100	12763917766
22	0.0	0	146453	189270	1833295489	9202662749
23	0.0	0	122106	164034	1757957184	5124198904
24	0.0	0	101806	140200	1474099236	2629998398
25	0.0	0	84881.2	114964	904974855.8	1180520953
26	0.0508	121920	70769.9	95336	603493269.2	409957611.7
27	0.0	0	59004.6	82718	562325339.6	71946190.05
28	0.0	0	69464.1	72904	11832912.01	358784589.4
29	0.0	0	57915.9	65894	63650079.61	54662511.43
30	0.0	0	48287.6	57482	84536991.36	4994733.312
31	0.0	0	40259.9	54678	207881607.6	105320753.5
32	0.0	0	33566.8	49070	240349210.2	287495423.4
33	0.0	0	27986.4	46266	334143776.2	507875352.5
34	0.0	0	23333.8	43462	405144435.2	739224863.9
35	0.0	0	19454.6	42060	511004109.2	965213789.1
36	0.0	0	16220.3	37854	468016975.7	1176640239
37	0.0	0	13523.7	35050	463381591.7	1368910461
38	0.0	0	11275.5	33648	500528756.3	1540326224
39	0.0	0	9400.95	29442	401643685.1	1690981052
40	0.0	0	7838.07	28040	408117975.7	1821959711
41	0.0	0	6535.01	23834	299255055	1934898397
42	0.0	0	5448.58	22432	288436554.9	2031657363
43	0.0	0	4542.77	22432	320024550	2114134651
44	0.0	0	3787.55	21030	297302082	2184154617
45	0.0	0	3157.88	19628	271264852.8	2243406280
46	0.0	0	2632.89	16824	201387603	2293413788
47	0.0	0	2195.18	16824	214002374.6	2335528892

48	0.0	0	1830.24	16824	224812838.9	2370935210
49	0.0	0	1525.97	14020	156100785.6	2400658972
50	0.0	0	1272.28	14020	162504365.2	2425583185
51	0.0	0	1060.77	11216	103128696.4	2446461745
52	0.0	0	884.418	12618	137676946.6	2463938192
53	0.0	0	737.386	11216	109801351.4	2478556580
54	0.0	0	614.798	12618	144076858.3	2490777721
55	0.0	0	512.589	11216	114563007	2500990198
56	0.0	0	427.373	11216	116394472.5	2509520747
57	0.0	0	356.323	12618	150348722.9	2516644311
58	0.0	0	297.086	11216	119222682.9	2522591207
59	0.0	0	247.696	11216	120303692.6	2527554912
60	0.0	0	206.517	11216	121208715.9	2531697139
61	0.0	0	172.184	11216	121965871.8	2535153314
62	0.0	0	143.559	9814	93517429.13	2538036689
63	0.0	0	119.693	8412	68762355.38	2540441945
64	0.0	0	99.7943	7010	47750942.82	2542448242
65	0.0	0	83.2038	8412	69368846.14	2544121592
66	0.0	0	69.3714	7010	48172325.36	2545517176
67	0.0	0	57.8386	7010	48332548.13	2546681041
68	0.0	0	48.2231	5608	30911119.18	2547651619
69	0.0	0	40.2062	7010	48578025.61	2548460978
70	0.0	0	33.522	5608	31074804.97	2549135890
71	0.0	0	27.9491	5608	31136968.05	2549698661
72	0.0	0	23.3026	5608	31188845.05	2550167928
73	0.0	0	19.4286	5608	31232130.29	2550559211
74	0.0	0	16.1987	4206	17554434.93	2550885461
75	0.0	0	13.5057	2804	7786858.438	2551157495
76	0.0	0	11.2604	4206	17595840.31	2551384316
77	0.0	0	9.3884	4206	17611548.92	2551573433
78	0.0	0	7.82761	2804	7818580.035	2551731116
79	0.0	0	6.52629	2804	7825859.158	2551862590
80	0.0	0	5.44131	2804	7831930.741	2551972208
81	0.0	0	4.53671	4206	17652293.78	2552063605
82	0.0	0	3.7825	2804	7841218.047	2552139807
83	0.0	0	3.15367	4206	17663917.27	2552203343
84	0.0	0	2.62938	2804	7847677.351	2552256317
85	0.0	0	2.19225	2804	7850126.668	2552300485
86	0.0	0	1.8278	2804	7852169.038	2552337309

87	0.0	0	1.52393	2804	7853872.123	2552368013
88	0.0	0	1.27058	1402	1962042.908	2552393612
89	0.0	0	1.05935	1402	1962634.705	2552414955
90	0.0	0	0.883237	1402	1963128.184	2552432750
91	0.0	0	0.736402	1402	1963539.671	2552447587
92	0.0	0	0.613977	1402	1963882.785	2552459957
93	0.0	0	0.511905	0	0.262046729	2552470271
94	0.0	0	0.426802	1402	1964407.429	2552478870
95	0.0	0	0.355848	1402	1964606.329	2552486039
96	0.0	0	0.296689	0	0.088024363	2552492017
97	0.0	0	0.247365	2364	5587326.519	2552497001
98	0.0	0	0.206242	1182	1396636.486	2552501156
Total=	2.1	4971360	4971358.305	4951204	1.73882E+11	1.05681E+12
			Xavg=	50522.49		
			efficient=	1-Rv/Tv	0.83546609	83.54660902

Table 4.6: Simulated tile drain flow for the event-4 of hydrograph plot P12.

time (hr.)	Rainfall (cm/hr.)	Recharge (cm^3/hr.)	(Xi) Outflow (cm^3/hr.)	observed flow (cm^3/hr) P12 (Xo)	(Xi-Xo)^2	(Xi-X_{mean})^2
0	0.0000	0	0	0	0	1.2118E+10
1	0.1270	304800	0	0	0	1.2118E+10
2	0.4445	1066800	107471	87326	405821025	6825626.51
3	0.2667	640080	445725	382148	4042034929	1.1266E+11
4	0.1270	304800	514254	618086	1.0781E+10	1.6335E+11
5	0.2159	518160	440401	569222	1.6595E+10	1.0911E+11
6	0.3048	731520	467819	537574	4865760025	1.2797E+11
7	0.1143	274320	560798	621292	3659524036	2.0314E+11
8	0.0889	213360	459788	599664	1.9565E+10	1.2229E+11
9	0.0508	121920	372899	499318	1.5982E+10	6.9072E+10
10	0.0127	30480	284405	339088	2990230489	3.0388E+10
11	0.0127	30480	194872	268988	5493181456	7189074470
12	0.0000	0	136908	220516	6990297664	719548972

13	0.0000	0	88635.3	182260	8765584450	460029144
14	0.0000	0	57383	149612	8506188441	2777352186
15	0.0000	0	37150	122170	7228400400	5319308550
16	0.0000	0	24051.1	107748	7005171070	7401589336
17	0.0000	0	15570.8	93526	6077013207	8932667474
18	0.0000	0	10080.6	82310	5217086224	1.0001E+10
19	0.0000	0	6526.26	71994	4286024981	1.0724E+10
20	0.0000	0	4225.14	65584	3764909700	1.1206E+10
21	0.0000	0	2735.38	60678	3357347212	1.1524E+10
22	0.0000	0	1770.9	55668	2904897388	1.1732E+10
23	0.0000	0	1146.49	53268	2716651805	1.1867E+10
24	0.0000	0	742.243	48150	2247495424	1.1956E+10
25	0.0000	0	480.532	41250	1662149521	1.2013E+10
26	0.0000	0	311.099	39950	1571242472	1.205E+10
27	0.0000	0	201.407	35746	1263418092	1.2074E+10
28	0.0000	0	130.392	28806	822290494	1.209E+10
29	0.0000	0	84.4166	28938	832529275	1.21E+10
30	0.0000	0	54.6517	27538	755334434	1.2106E+10
31	0.0000	0	35.3818	23532	552091067	1.2111E+10
32	0.0000	0	22.9064	22228	493066182	1.2113E+10
33	0.0000	0	14.8297	19628	384676449	1.2115E+10
34	0.0000	0	9.60084	16642	276636702	1.2116E+10
35	0.0000	0	6.21564	15280	233288489	1.2117E+10
36	0.0000	0	4.02404	14020	196447582	1.2118E+10
37	0.0000	0	2.60518	12618	159148186	1.2118E+10
38	0.0000	0	1.68661	12618	159171364	1.2118E+10
39	0.0000	0	1.09192	11216	125774163	1.2118E+10
40	0.0000	0	0.706915	10216	104352213	1.2118E+10
41	0.0000	0	0.45766	9814	96305613.3	1.2118E+10
42	0.0000	0	0.296292	8814	77681373.1	1.2118E+10
43	0.0000	0	0.191821	7814	61055598.3	1.2118E+10
44	0.0000	0	0.124186	7412	54935903.1	1.2118E+10
45	0.0000	0	0.0803986	7014	49195068.2	1.2118E+10
46	0.0000	0	0.0520505	6710	45023401.5	1.2118E+10
47	0.0000	0	0.0336978	6010	36119695	1.2118E+10
48	0.0000	0	0.0218161	5710	32603850.9	1.2118E+10
49	0.0000	0	0.0141239	4608	21233533.8	1.2118E+10
50	0.0000	0	9.14E-03	2706	7322386.51	1.2118E+10
51	0.0000	0	5.92E-03	1806	3261614.62	1.2118E+10

52	0.0000	0	3.83E-03	1304	1700406	1.2118E+10
53	0.0000	0	2.48E-03	902	813599.524	1.2118E+10
54	0.0000	0	1.61E-03	502	252002.387	1.2118E+10
55	0.0000	0	1.04E-03	504	254014.952	1.2118E+10
56	0.0000	0	6.73E-04	0	4.5329E-07	1.2118E+10
57	0.0000	0	4.36E-04	302	91203.7367	1.2118E+10
58	0.0000	0	2.82E-04	0	7.9631E-08	1.2118E+10
59	0.0000	0	1.83E-04	0	3.3376E-08	1.2118E+10
60	0.0000	0	1.18E-04	0	1.3989E-08	1.2118E+10
61	0.0000	0	7.66E-05	0	5.8633E-09	1.2118E+10
62	0.0000	0	4.96E-05	0	2.4575E-09	1.2118E+10
63	0.0000	0	3.21E-05	0	1.03E-09	1.2118E+10
64	0.0000	0	2.08E-05	0	4.3172E-10	1.2118E+10
65	0.0000	0	1.35E-05	0	1.8095E-10	1.2118E+10
66	0.0000	0	8.71E-06	0	7.5841E-11	1.2118E+10
67	0.0000	0	5.64E-06	0	3.1787E-11	1.2118E+10
Total=	1.7653	4236720	4236720.058	6268348	1.6353E+11	1.5948E+12
			X_{mean} =	108074.9655		
			efficiency=	1-Rv/Tv	0.89746508	89.75%

Table 4.7: Simulated tile drain flow for the event-5 of hydrograph plot P11.

Time (hr)	Rainfall (cm)	Recharge Rate (cm^3/hr)	(Xi) Outflow (cm^3/hr)	Observed flow P11(Xo) (cm^3/hr.)	(Xi-Xo)^2	($Xi-X_{mean}$)^2
0	0	0	0	0	0	1.9279E+10
1	0.0171	41040	0	2,764	7639696	1.9279E+10
2	0.733	1759200	12511.3	218,356	4.2372E+10	1.5962E+10
3	0.0802	192480	5.45E+05	533,452	133356304	1.6496E+11
4	0.0508	121920	437532	418,746	352913796	8.9211E+10

5	0	0	341316	313,714	761870404	4.0992E+10
6	0.0154	36960	237264	219,738	307160676	9685244538
7	0.0869	208560	176200	154,784	458645056	1394995608
8	0	0	186065	118,852	4517587369	2229222230
9	0	0	129342	96,740	1062890404	90408909.9
10	0	0	89911.3	81,538	70112152.9	2395031594
11	0	0	62501.3	59,426	9457470.09	5829178963
12	0	0	43447.4	42,842	366509.16	9101724777
13	0	0	30202.2	35,932	32830608	1.1804E+10
14	0	0	20994.9	24,876	15062937.2	1.389E+10
15	0	0	14594.5	19,348	22595762.3	1.544E+10
16	0	0	10145.2	13,820	13504155	1.6565E+10
17	0	0	7052.41	5,528	2323825.85	1.7371E+10
18	0	0	4902.44	0	24033918	1.7942E+10
19	0	0	3407.9	0	11613782.4	1.8345E+10
20	0	0	2368.98	0	5612066.24	1.8627E+10
21	0	0	1646.78	0	2711884.37	1.8825E+10
22	0	0	1144.75	0	1310452.56	1.8963E+10
23	0	0	795.767	0	633245.118	1.9059E+10
Total=	0.9834	2360160	2358346.127	2,360,456	5.0186E+10	5.6724E+11
			average flow=	138850.3529		
			efficiency=	1-Rv/Tv	0.91152503	

ANNEXURE-III

A PROGRAM FOR SOLUTION OF RICHARD'S EQUATION BY USING EXPLICIT SCHEME

```
C        SOLUTION OF RICHARD'S EQUATION, IN TERMS OF HC
C    EXPLICIT SCHEME SOLVED DIRECTLY (HAVERKAMP ET AL) MODEL 2
         DIMENSION RAIN(3000),RH(3000)
         DIMENSION THETA(65,3000,0:100),HC(65,3000,0:100),
   1AK(65,3000,0:100),CC(65,3000,0:100),SE(65,3000,0:100),
   2 DSMP(3000),Q(65,0:3000,0:100),THEAD(65,3000,0:100),
   3 TTHETA(65,3000,100),SMP(0:3000)
C        NO.OF NODES=10,EACH NODE AT 20 CM APART
         OPEN(UNIT=1,FILE='INPUT.DAT',STATUS='OLD')
         OPEN(UNIT=2,FILE='UPR.OUT',STATUS='UNKNOWN')
C ************ INPUT DATA ***************************
         PI=3.14159
         KTIME=50
C        A HOUR IS DISCRETIZED TO 50 KTIME STEP; HYDRAULIC CONDUCTIVITY IS EXPRESSED
C    IN UNIT OF CM PER 1/50 HOUR, ACCORDINGLY DELT=1
     WRITE(*,*)'ENTER NO. OF HOURS RAINFALL DATA'
         READ(*,*)NHOURS
         DO J=1,NHOURS
         READ(1,*)RAIN(J),RH(J)
         END DO

C        XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
C        VAN GENUCHTEN MODEL PARAMETERS
C
         AN=2.06
         AM=1.-1./AN
         ALPHA1=0.00423
         GAMMA=0.5
         AKKTIME=KTIME
         DELZ=20.
         DELT=1.
C
C        SOIL MOISTURE PARAMETERS
C
         THETAS=0.396
         THETAR=0.131
         THETAFC=0.35
```

```
      AKS=0.248/AKKTIME
      WRITE(2,*)'THETAR=',THETAR,'THETAS=',THETAS
      WRITE(2,*)'AKS (CM/UNIT KTIME) =',AKS
C   AKS IS IN CM/UNIT KTIME PERIOD
C       NUMBER OF NODES=10
   WRITE(2,4)
      IMAX=51
C*********************************************************************
C       GOTO 333
C****** CHECKING VAN GUNCHTEN EQUATION FOR VARIOUS THETA **************
C****** DETERMINATION OF SPACE GRID SIZE DELZ *************************

   DTHETA=(THETAS - THETAR)/100
      THETAA=THETAR
      DO I=1,99
      THETAA=THETAA+DTHETA
      SEE=(THETAA-THETAR)/(THETAS-THETAR)
      HCTHETA=(SEE**(-1./AM)-1.)**(1./AN)/ALPHA1
      TERM1=(1.-SEE**(1./AM))**AM
      AKK1=AKS*SEE**GAMMA*(1.-TERM1)**2
C       AKK1=FUNCTION OF MOISTURE CONTENT K(THETA)
      TERM1=(1.+(ALPHA1*HCTHETA)**AN )**(AM/2.)
      TERM2=(1.+(ALPHA1*HCTHETA)**AN )**(-AM)
      TERM3=(ALPHA1*HCTHETA)**(AN-1)
      TERM4=(1.-TERM3*TERM2)**2
      AKK2=AKS*TERM4/TERM1
C       AKK2=FUNCTION OF HC K(h):UNSATURATED HYDRAULIC COND.
      CCC=-ALPHA1*AM*(THETAS-THETAR)*SEE**(1./AM)*
  1(1.-SEE**(1./AM))**AM/(1.-AM)
C       CCC=DTHETABYDHC =SPECIFIC WATER CAPACITY
      THETAA1=THETAR+(THETAS-THETAR)/(1.+(ALPHA1*HCTHETA)**AN )**AM
      DIFF=-AKK2/CCC
      DELZM=(2*DIFF)**0.5
      DtMAX=(.5*DELZM**2)/(AKK1/CCC)
      END DO
      WRITE(2,*)'AFTER CHECKING VAN GUNCHTEN Eqn HCTHETA=',HCTHETA,
  1'KTHETA=',AKK1,' Kh=',AKK2,' Ch=',CCC,' dZ=',DELZM,' dtMAX=',DtMAX
C 22    FORMAT(4E10.5,/)
C       WRITE(2,*)THETAS,THETAA,AKS,AKK
C*********************************************************************
C       WRITE(2,*)'INITIAL CONDITION, J=1, K=0 '
C
      DO I=1,IMAX
      J=1
      K=0
```

```
      THETA(I,J,K)=.395
      SE(I,J,K)=(THETA(I,J,K)-THETAR)/(THETAS-THETAR)
      HC(I,J,K)= ((SE(I,J,K)**(-1./AM)-1.)**(1./AN))/ALPHA1
      TERM1=(1.-SE(I,J,K)**(1./AM))**AM
      AK(I,J,K)=AKS*(SE(I,J,K)**GAMMA)*(1.-TERM1)**2
C     CC(I,J,K)=DTHETABYDHC, AK=K(THETA), HC=H(THETA)
      CC(I,J,K)=-ALPHA1*AM*(THETAS-THETAR)*SE(I,J,K)**(1./AM)*
     1(1.-SE(I,J,K)**(1./AM))**AM/(1.-AM)
      END DO
C*********************************************************************
C     WRITE(2,*)' UPPER BOUNDARY CONDITION'
C
      I=1
      DO J=1,NHOURS
      DO K=0,KTIME
      IF(RAIN(J).GT.2.5)THETA(I,J,K)=0.9999999*THETAS
     IF(RAIN(J).LE.2.5)THETA(I,J,K)=ACOS(1.-2.*RH(J)/100.)*(THETAFC/PI)
     HC(I,J,K)=((((THETA(I,J,K)-THETAR)/(THETAS-THETAR))**(-1./AM)-1.)
     1**(1./AN))/ALPHA1
C      HC(I,J,K)=83140000.*294.15*LOG(.99)*1019.8/(18.*980.665)
      SE(I,J,K)=(THETA(I,J,K)-THETAR)/(THETAS-THETAR)
      TERM1=(1.-SE(I,J,K)**(1./AM))**AM
      AK(I,J,K)=AKS*(SE(I,J,K)**GAMMA)*(1.-TERM1)**2.
      CC(I,J,K)=-ALPHA1*AM*(THETAS-THETAR)*SE(I,J,K)**(1./AM)*
     1(1.-SE(I,J,K)**(1./AM))**AM/(1.-AM)

      END DO
      END DO

C*********************************************************************
C     WRITE(2,*)' LOWER BOUNDARY CONDITION APPLIED'

      I=IMAX
      DO J=1,NHOURS
    DO K=0,KTIME
    THETA(I,J,K)=THETAS
      SE(I,J,K)=(THETA(I,J,K)-THETAR)/(THETAS-THETAR)
      HC(I,J,K)=((SE(I,J,K)**(-1./AM)-1.)**
     1(1./AN))/ALPHA1
      TERM1=(1.-SE(I,J,K)**(1./AM))**AM
      AK(I,J,K)=AKS*(SE(I,J,K)**GAMMA)*(1.-TERM1)**2.

      CC(I,J,K)=-ALPHA1*AM*(THETAS-THETAR)*SE(I,J,K)**(1./AM)*
     1(1.-SE(I,J,K)**(1./AM))**AM/(1.-AM)
```

```
C        DELTAMAX(I,J,K)=0.5*DELZ**2./TERMD

         END DO
         END DO
C        GOTO 999
C*********************************************************************
C        SOLUTION OF RICHARD'S EQUATION CONSIDERING SINK TERM

         DO J=1,NHOURS
         DO K=1,KTIME
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
         DO I=2,IMAX-1
         TERM2=0.5*(AK(I+1,J,K-1)+AK(I,J,K-1))
         TERM3=0.5*(AK(I,J,K-1)+AK(I-1,J,K-1))
         TERM4=(HC(I+1,J,K-1)-HC(I,J,K-1))/DELZ+1.
         TERM5=(HC(I,J,K-1)-HC(I-1,J,K-1))/DELZ+1.
         TERM6=(TERM2*TERM4-TERM3*TERM5)/(DELZ*CC(I,J,K-1))

         HC(I,J,K)=-TERM6*DELT+HC(I,J,K-1)
         THETA(I,J,K)=THETAR+(THETAS-THETAR)/(1+(ALPHA1*HC(I,J,K))**AN)**
   1 (1-1./AN)


         SE(I,J,K)=(THETA(I,J,K)-THETAR)/(THETAS-THETAR)
         TERM1=(1.-SE(I,J,K)**(1./AM))**AM
         AK(I,J,K)=AKS*((SE(I,J,K)**GAMMA))*(1.-TERM1)**2.

         CC(I,J,K)=-ALPHA1*AM*(THETAS-THETAR)*SE(I,J,K)**(1./AM)*
   1(1.-SE(I,J,K)**(1./AM))**AM/(1.-AM)

     END DO
C30   CONTINUE
CXXXXXXXXXXXX SOLUTION NOT COMPLETED XXXXX CONTINUED XXXXXX
C        WRITE(2,*)
C 31   CONTINUE
      END DO
      WRITE(2,4)
C     WRITE(2,*)'NODE   HOUR  HC(END)   DHC(END)'
C      WRITE(2,4)
C     DO K=0,KTIME
C        DO I=1,IMAX
C        IF(I.EQ.1)DHC(I,J,K)=HC(I,J,K)
C        IF(I.GT.1)DHC(I,J,K)=HC(I,J,K)-HC(I-1,J,K)
C        IF(DHC(I,J,KTIME).LT.0)DHC(I,J,KTIME)=0-1.*DHC(I,J,KTIME)
```

```
C        WRITE(2,111)I,J,HC(I,J,KTIME),DHC(I,J,KTIME)
C 111  FORMAT(2X,2I4,2X,3E10.3,/)
C        END DO
C        END DO
         WRITE(2,3)
3        FORMAT(1X,/)
C        WRITE(2,*)

         DO I=2,IMAX-1
         HC(I,J+1,0)=HC(I,J,KTIME)
         AK(I,J+1,0)=AK(I,J,KTIME)
         CC(I,J+1,0)=CC(I,J,KTIME)
         END DO
   END DO
C*****************************************************************
C32   CONTINUE
C   CHANGE IN SOIL MOISTURE
C
C
C        WRITE(2,*) 'THETA(I,1,0)=',THETA(I,1,0)
C        WRITE(2,*) 'THETA(IMAX,1,0)=',THETA(IMAX,1,0)
C
    DO IIMAX=1,IMAX
         SUM1=0.
         DO I=1,IIMAX,2
         SUM1=SUM1+2.*THETA(I,1,0)
         END DO
         SUM1=SUM1- THETA(1,1,0)-THETA(IIMAX,1,0)
C
C
         SUM2=0.
         DO I=2,IIMAX-1,2
         SUM2=SUM2+4.*THETA(I,1,0)
         END DO
         SMP(0)=(SUM1+SUM2)*DELZ/3.
         WRITE(2,*)'SMP(0)=',SMP(0)

    DO J=1,NHOURS
         SUM1=0.
         DO I=1,IIMAX,2
         SUM1=SUM1+2.*THETA(I,J,KTIME)
         END DO
         SUM1=SUM1- THETA(1,J,KTIME)-THETA(IIMAX,J,KTIME)
C
         SUM2=0.
```

```
      DO I=2,IIMAX-1,2
      SUM2=SUM2+4.*THETA(I,J,KTIME)
      END DO
      SMP(J)=(SUM1+SUM2)*DELZ/3.
      TTHETA(IIMAX,J,KTIME)=SMP(J)
   DSMP(J)=SMP(J)-SMP(J-1)
      END DO
      END DO
C***************OUTPUT OF THETA w.r.t DEPTH***************************
      WRITE(2,4)
      WRITE(2,*)'MOISTURE CONTENT HOURLY VARIATION WITH DEPTH'
      DO J=1,NHOURS
      WRITE(2,*)J,'th HOUR'
      DO I=1,IMAX
      WRITE(2,33)THETA(I,J,KTIME)
      END DO
      END DO
33    FORMAT(4E20.15)
C**********OUTPUT OF THETA OF 1st Hr. w.r.t DEPTH**********************
      J=1
      WRITE(2,*)J,'TH HR MOISTURE CONTENT VARIATION'
      WRITE(2,*)J,'th HOUR AT 1/50'
      DO I=1,IMAX
      WRITE(2,*)THETA(I,J,1)
      END DO
      WRITE(2,*)J,'th HOUR AT 10/50'
      DO I=1,IMAX
      WRITE(2,*)THETA(I,J,10)
      END DO
      WRITE(2,*)J,'th HOUR AT 20/50'
      DO I=1,IMAX
      WRITE(2,*)THETA(I,J,20)
      END DO
      WRITE(2,*)J,'th HOUR AT 30/50'
      DO I=1,IMAX
      WRITE(2,*)THETA(I,J,30)
      END DO
   WRITE(2,*)J,'th HOUR AT 40'
      DO I=1,IMAX
      WRITE(2,*)THETA(I,J,40)
      END DO
   WRITE(2,*)J,'th HOUR AT 49'
      DO I=1,IMAX
      WRITE(2,*)THETA(I,J,49)
      END DO
```

```
      WRITE(2,*)J,'th HOUR AT 50/50'
      DO I=1,IMAX
      WRITE(2,*)THETA(I,J,50)
      END DO
C*********OUTPUT OF THETA w.r.t TIME***********************************
      WRITE(2,*)'MOISTURE CONTENT HOURLY VARIATION WITH TIME'
      DO I=1,IMAX
      WRITE(2,*)I,'th NODE'
      DO J=1,NHOURS
      IF(J.EQ.1)WRITE(2,*)THETA(I,J,1)
      WRITE(2,*)THETA(I,J,KTIME)
      END DO
      END DO
C***************** VARIATION OF THETA w.r.t TIME OF 1st HOUR***********
C
C     J=1
C     WRITE(2,*)J,'TH HR MOISTURE CONTENT VARIATION'
C     WRITE(2,*)J,'th HOUR AT 1/50'
C     DO I=1,IMAX
C     WRITE(2,*)THETA(I,J,1)
C     END DO
C     WRITE(2,*)J,'th HOUR AT 10/50'
C     DO I=1,IMAX
C     WRITE(2,*)THETA(I,J,10)
C     END DO
C     WRITE(2,*)J,'th HOUR AT 20/50'
C     DO I=1,IMAX
C     WRITE(2,*)THETA(I,J,20)
C     END DO
C     WRITE(2,*)J,'th HOUR AT 30/50'
C     DO I=1,IMAX
C     WRITE(2,*)THETA(I,J,30)
C     END DO
C   WRITE(2,*)J,'th HOUR AT 40'
C     DO I=1,IMAX
C     WRITE(2,*)THETA(I,J,40)
C     END DO
C  WRITE(2,*)J,'th HOUR AT 49'
C     DO I=1,IMAX
C     WRITE(2,*)THETA(I,J,49)
C     END DO
C     WRITE(2,*)J,'th HOUR AT 50/50'
C     DO I=1,IMAX
C     WRITE(2,*)THETA(I,J,50)
C     END DO
```

```
C******************************************************
C   WRITE(2,*)'FLOW AT 1/50 TIME USING DHC'
C        DO I=1,IMAX
C   Q(I,1,1)=AK(I,1,1)*AKKTIME*(DHC(I,1,1)/DELZ+1.)
C        WRITE(2,*)Q(I,1,1)
C        END DO
C        WRITE(2,*)'HEAD DHC'
C        DO I=1,IMAX
C        WRITE(2,*)DHC(I,1,1)
C        END DO
C********* FINDING TOTAL HEAD **************************************
         DO I=1,IMAX
         DO J=1,NHOURS
         DO K=0,KTIME
         THEAD(I,J,K)=-HC(I,J,K)-(I-1.)*20.0
         END DO
         END DO
         END DO
C**************VARIATION OF HYDRAULIC HEAD w.r.t DEPTH***************
         WRITE(2,*)'VARIATION OF HYDRAULIC HEAD W.R.T DEPTH'
         DO J=1,NHOURS
         WRITE(2,*)J,'th HOUR'
         K=KTIME
         DO I=1,IMAX
         THEAD(I,J,K)=-HC(I,J,K)-(I-1.)*20.
         WRITE(2,*)THEAD(I,J,K)
         END DO
         END DO
         J=1
         WRITE(2,*)J,'th HOUR AT 1/50 TIME'
         DO I=1,IMAX
         THEAD(I,J,1)=-HC(I,J,1)-(I-1.)*20.0
         WRITE(2,*)THEAD(I,J,0)
         END DO
C******************************************************************

CXXXXXXXXXXX              CALICULATION              OF              FLOW
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
         WRITE(2,*)'OUT FLOW'
C   WRITE(2,*)' J SMP(J) DSMP(J)  RAIN(J)  RH(J)   K(h)   Q(J)'
         WRITE(2,4)
 4       FORMAT(/)
C
         DO J=1,NHOURS
         WRITE(2,*)J,'th HOUR'
```

```
      DO I=1,IMAX
      K=KTIME
C     Q(I,0,KTIME)=0
  IF(I.EQ.1)Q(I,J,K)=-AK(I,J,K)*AKKTIME*(THEAD(I,J,K)/DELZ)
  Q(I,J,K)=-AK(I,J,K)*AKKTIME*((THEAD(I,J,K)-THEAD(I-1,J,K))/DELZ)
C     Q(J)=Q(J-1)+Q(J)
      WRITE(2,18)Q(I,J,K)
C     WRITE(2,*)J,' ',Q(J),' ',SMP(J)
C     WRITE(2,18)J,SMP(J),DSMP(J),RAIN(J),RH(J),AKK2,Q(J)
 18   FORMAT(6E10.4)
      END DO
      END DO
C*****************************************************************
      J=1
      WRITE(2,*)J,'th HOUR AT 1/50 '
      DO I=1,IMAX
C     Q(I,0,KTIME)=0
      K=1
  IF(I.EQ.1)Q(I,J,K)=-AK(I,J,K)*AKKTIME*(THEAD(I,J,K)/DELZ)
  Q(I,J,K)=-AK(I,J,K)*AKKTIME*((THEAD(I,J,K)-THEAD(I-1,J,K))/DELZ)
C     Q(J)=Q(J-1)+Q(J)
      WRITE(2,*)Q(I,J,K)
C     WRITE(2,*)J,' ',Q(J),' ',SMP(J)
C     WRITE(2,18)J,SMP(J),DSMP(J),RAIN(J),RH(J),AKK2,Q(J)
C18   FORMAT(6E10.4)
      END DO

      J=1
      WRITE(2,*)J,'th HOUR AT 10/50 '
      DO I=1,IMAX
C     Q(I,0,KTIME)=0
      K=10
  IF(I.EQ.1)Q(I,J,K)=-AK(I,J,K)*AKKTIME*(THEAD(I,J,K)/DELZ)
  Q(I,J,K)=-AK(I,J,K)*AKKTIME*((THEAD(I,J,K)-THEAD(I-1,J,K))/DELZ)
C     Q(J)=Q(J-1)+Q(J)
      WRITE(2,*)Q(I,J,K)
C     WRITE(2,*)J,' ',Q(J),' ',SMP(J)
C     WRITE(2,18)J,SMP(J),DSMP(J),RAIN(J),RH(J),AKK2,Q(J)
C18   FORMAT(6E10.4)
      END DO

      J=1
      WRITE(2,*)J,'th HOUR AT 20/50 '
      DO I=1,IMAX
C     Q(I,0,KTIME)=0
```

```
      K=20
   IF(I.EQ.1)Q(I,J,K)=-AK(I,J,K)*AKKTIME*(THEAD(I,J,K)/DELZ)
   Q(I,J,K)=-AK(I,J,K)*AKKTIME*((THEAD(I,J,K)-THEAD(I-1,J,K))/DELZ)
C     Q(J)=Q(J-1)+Q(J)
      WRITE(2,*)Q(I,J,K)
C     WRITE(2,*)J,' ',Q(J),' ',SMP(J)
C     WRITE(2,18)J,SMP(J),DSMP(J),RAIN(J),RH(J),AKK2,Q(J)
C18   FORMAT(6E10.4)
      END DO

      J=1
      WRITE(2,*)J,'th HOUR AT 30/50 '
      DO I=1,IMAX
C     Q(I,0,KTIME)=0
      K=30
   IF(I.EQ.1)Q(I,J,K)=-AK(I,J,K)*AKKTIME*(THEAD(I,J,K)/DELZ)
   Q(I,J,K)=-AK(I,J,K)*AKKTIME*((THEAD(I,J,K)-THEAD(I-1,J,K))/DELZ)
C     Q(J)=Q(J-1)+Q(J)
      WRITE(2,*)Q(I,J,K)
C     WRITE(2,*)J,' ',Q(J),' ',SMP(J)
C     WRITE(2,18)J,SMP(J),DSMP(J),RAIN(J),RH(J),AKK2,Q(J)
C18   FORMAT(6E10.4)
      END DO

      J=1
      WRITE(2,*)J,'th HOUR AT 40/50 '
      DO I=1,IMAX
C     Q(I,0,KTIME)=0
      K=40
   IF(I.EQ.1)Q(I,J,K)=-AK(I,J,K)*AKKTIME*(THEAD(I,J,K)/DELZ)
   Q(I,J,K)=-AK(I,J,K)*AKKTIME*((THEAD(I,J,K)-THEAD(I-1,J,K))/DELZ)
C     Q(J)=Q(J-1)+Q(J)
      WRITE(2,*)Q(I,J,K)
C     WRITE(2,*)J,' ',Q(J),' ',SMP(J)
C     WRITE(2,18)J,SMP(J),DSMP(J),RAIN(J),RH(J),AKK2,Q(J)
C18   FORMAT(6E10.4)
      END DO

      J=1
   WRITE(2,*)J,'th HOUR AT 49/50 '
      DO I=1,IMAX
C     Q(I,0,KTIME)=0
      K=49
   IF(I.EQ.1)Q(I,J,K)=-AK(I,J,K)*AKKTIME*(THEAD(I,J,K)/DELZ)
   Q(I,J,K)=-AK(I,J,K)*AKKTIME*((THEAD(I,J,K)-THEAD(I-1,J,K))/DELZ)
```

```
C          Q(J)=Q(J-1)+Q(J)
           WRITE(2,*)Q(I,J,K)
C          WRITE(2,*)J,' ',Q(J),' ',SMP(J)
C          WRITE(2,18)J,SMP(J),DSMP(J),RAIN(J),RH(J),AKK2,Q(J)
C18        FORMAT(6E10.4)
           END DO

           J=1
           WRITE(2,*)J,'th HOUR AT 50/50 '
           DO I=1,IMAX
C          Q(I,0,KTIME)=0
           K=50
      IF(I.EQ.1)Q(I,J,K)=-AK(I,J,K)*AKKTIME*(THEAD(I,J,K)/DELZ)
      Q(I,J,K)=-AK(I,J,K)*AKKTIME*((THEAD(I,J,K)-THEAD(I-1,J,K))/DELZ)
C          Q(J)=Q(J-1)+Q(J)
           WRITE(2,*)Q(I,J,K)
C          WRITE(2,*)J,' ',Q(J),' ',SMP(J)
C          WRITE(2,18)J,SMP(J),DSMP(J),RAIN(J),RH(J),AKK2,Q(J)
C18        FORMAT(6E10.4)
           END DO
C************************************************************************

           WRITE(2,*)'FLOW AT UPPER BOUNDARY w.r.t TIME'
           I=1
           DO J=1,NHOURS
C          IF(J.EQ.1)WRITE(2,*)Q(I,J,1)
C          WRITE(2,*)Q(I,J,KTIME)
           END DO
C !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
           WRITE(2,*)'FLOW at LOWER BOUNDARY w.r.t TIME'
           I=11
           DO J=1,NHOURS
           IF(J.EQ.1)WRITE(2,*)Q(I,J,1)
           WRITE(2,*)Q(I,J,KTIME)
           END DO
C          XXXXXXXXXXXXXXXX     OUTPUT     FOR     Q     vs     TTHETA
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      WRITE(2,*)' OUTFLOW vs TTHETA CALICULTION'
      DO I=1,IMAX
      WRITE(2,*)I,'th NODE'
      DO J=1,NHOURS
     WRITE(2,*)TTHETA(I,J,KTIME)
C           WRITE(2,*)Q(I,J,KTIME)
      END DO
      END DO
```

```
STOP
END
```