SURFACE-WAVEGUIDES COMPRISING UNIDIRECTIONALLY CONDUCTING SCREENS

A DISSERTATION
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C E R T I F I C A T E

CERTIFIED that the dissertation entitled "Surface Waveguides Comprising Unidirectionally Conducting Screens" which is being submitted by Smt. Manju Lata Guha in partial fulfilment for the award of the degree of Master of Engineering in Microwaves, Department of Electronics and Communication Engineering of the University of Roorkee, Roorkee is a record of the student's own work carried out by her under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is to further certify that she has worked for a period of ten months from January, 1970 to October, 1970 for preparing this dissertation at this University.

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A C K N O W L E D G E M E N T S

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SYNOPSIS

A unidirectionally conducting screen is seen to be capable of supporting surface waves; however, it has the drawback that energy travels along the direction of conduction while phase propagates obliquely. A pair of parallel U.C. screens has been found to be advantageous in that the energy travels parallel to the sides of the structure reducing the possibility of end-effects caused by limiting the width of the structure for a physically realisable system. This system in free-space had been studied by C.K. Arora and has also been briefly reviewed in this thesis.

However, to construct U.C. screens in free-space has been found to be a rather difficult problem. Dielectric sheets to support the U.C. screens have been suggested and an analytical as well as a practical study of this system has been attempted.

As an extension of Arora's work, a determinantal equation of this new system and the field expressions have been derived. The results inferred herefrom have been compared with those obtained in case of U.C. screens in free-space.

A waveguide comprising a pair of parallel U.C. screens with bakelite sheets to support them, has also been constructed and an attempt has been made to launch surface waves onto it in the laboratory. Discrepancies, however, are encountered due to crude measuring techniques and the choice of improper dielectric material which have been discussed in the text.

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CHAPTER -I

SURFACE WAVES-A BRIEF SURVEY

1.1 INTRODUCTION

Electromagnetic energy can be supported, and by careful launching can be made to predominate, over a class of open boundary structures, such as in the case of closed cylindrical conducting tube and the conventional TEM wave-transmission line. The wave, thus launched, propagates without radiation along an interface between two media with different physical properties. The e.m. field, having the usual propagation function $e^{-j\beta z}$ along the axis of the structure, extends to infinity in the transverse direction but the energy density decreases with distance so that, in practice, most of the energy of the wave is constrained to flow in the immediate neighbourhood of the structure.

To meet these requirements of a surface wave, the interface between two different media must be straight in the direction of propagation of the wave. If the two media concerned have finite losses, the main stream of energy must supply these losses as well as provide for any power transmitted. The radiation, thus, is construed to mean energy absorbed from the wave independently of the media supporting it. To comply with the essential condition that the only flow of energy away from the interface is that required to supply the losses in the media concerned, either the electric-or magnetic-field component, tangential to the supporting surface must have an evanescent distribution over the corresponding equiphase surface.

Consequently, it is necessary that tangential electric- or magnetic-field should have an evanescent distribution over the supporting surface. This is a vital characteristic of a surface wave.

Although an equiphase surface can and does have, in case of a surface wave, some form of evanescent structure, the converse of it is not true. That is, it is not necessary that, all fields having evanescent distribution over a particular equi-phase surface constitute surface waves. waves. [1,2,3]

1.11 Brewster-Angle and Surface Waves

Most interesting feature of surface waves is their non-radiating property which they exhibit when allowed to progress along a straight interface in the direction of propagation. Brewster angle, in electromagnetism, is the angle of incidence with the normal to the surface for which no reflection takes place. Alternatively, one can construe it as that in such circumstances there is no outward radiation from the surface. It might therefore be expected that a surface wave is simply a wave of the required field configuration incident on the surface at the Brewster angle. On a flat surface this condition can be established analytically. The energy flow is normal to the wave-front, an equiphase surface, and suffers a decay in amplitude with distance from the surface. [6]

1.1.2 Surface-wave Mode Types

General waves of interest are E-modes. However, in special cases mixed-modes do occur. For homogeneous media only single interface is concerned, and hence there is only one finite boundary condition to satisfy so that the corresponding surface waves do not exhibit any cut-off phenomenon.

There exist three distinct forms of surface waves, viz.,

(a) The Zenneck or inhomogeneous plane-wave supported by a flat surface,

(b) The radial cylindrical wave also supported by a flat surface.

(c) The Sommerfeld-Goubau or axial cylindrical wave, associated with a transversely cylindrical surface.

The field-distributions of the three forms of surface-waves are shown in Figs. 1.1, 1.2 and 1.3 respectively [1,2,6].

1.1.3 Outstanding Differences of Surface-waves from Conventional Wave-guide Modes.

Although surface waves have several features similar to those of conventional wave-guide modes, they differ from them in following aspects:

(i) The possibility of a surface-wave mode of propagation with no low-frequency cut-off.

(ii) The non-existence of an infinite number of discrete modes of propagation at a given frequency.

(iii) The existence of a finite number of discrete modes, together with an eigen-function solution with a continuous eigen value spectrum.

(iv) The possibility of mode solutions with a phase-velocity less than that of light.

1.1.4 Launching and Support

The surface-waves described so far are known to satisfy Maxwell's equations. This is a necessary condition for a wave to occur in practice, but is not a sufficient condition. For instance, a homogeneous plane wave is not physically realizable, because it extends to infinite distance, entailing a radiating aperture of infinite area and infinite amount of power. At a sufficient distance from any finite aperture the field produced in free space must take the form of an outward-travelling spherical wave, known as the 'radiation' field. This field represents a leakage of energy which we wish to guide by the surface wave. The problem of excitation is to avoid radiation as far as possible. Many papers and reports have been published on this subject. The efficiency of an antenna in exciting a surface-wave mode is defined as the ratio of the power radiated as a surface wave to the total power radiated. Several investigators have shown that launching efficiencies of 80 percent and more can be obtained.

A typical launching device is a flared horn, the aperture field of which can be chosen much like the transverse field of the surface wave. Slots in conducting planes, dipoles, and line-sources have also been studied and provide fairly good efficiencies when properly oriented with respect to the surface-wave guide. [3,8,11,12]

For a pure surface wave, the supporting surface must be straight in the direction of propagation of the wave. If there exists a solution to Maxwell's equations representing a field distribution of the surface-wave form, the corresponding surface wave can be supported by the surface in question.

When the supporting surface is curved or tapered in the direction of propagation, another important factor is introduced because in those circumstances the smooth progress of the wave is disturbed, tending to set up radiation and causing a departure from the pure surface wave field. If a sudden discontinuity is introduced along the length of the guide, the same effect is produced. This effect can be minimized by using a guide of high surface reactance . This, however, ensures that a large proportion of the energy of the wave is stored within the surface, some unavoidable radiation is inherent.

1.1.5 Definition of Surface Wave

Since surface wave entails a straight interface, between two homogeneous media, in the direction of propagation for its most characteristic feature, viz., its non-radiating property, we define a surface-wave in the following way.

" A surface wave is one that propagates along an interface between two different media without radiation; such radiation being construde to mean energy converted from the surface-wave field to some other form." [6]

1.2 FIELD COMPONENTS [1,6]

1.2.1 The Zenneck Wave

Zenneck, in 1907, described the behaviour of a wave that travelled without the change of pattern over a flat surface between two homogeneous media having different conductivity and permittivity. This wave was also a solution of Maxwell's equations. The field distribution of this surface wave is shown in Fig.1.1. It is an inhomogeneous plane wave because it decays in amplitude over the wave front with increasing distance from the surface.

Let the surface lies in the x-z plane at y = 0 and the media on each side of the interface are homogeneous. For a wave travelling along the interface in the positive x-direction with a propagation coefficient

$$\gamma = \alpha + j\beta \qquad \ldots (1.1)$$

the three components of the field required to satisfy the two dimensional wave equation are

(i) <u>Below the Surface</u> - Medium 1 ($\mu_1 = \mu_0, \sigma_1, \epsilon_1$) i.e., for $y \leq 0$.

$$\left.\begin{aligned} H_{z1} &= A\, e^{j\omega t}\, e^{u_1 y}\, e^{-\gamma x} \\ E_{x1} &= A\left(\frac{u_1}{\sigma_1 + j\omega\epsilon_1}\right) e^{j\omega t}\, e^{u_1 y}\, e^{-\gamma x} \\ E_{y1} &= A\left(\frac{\gamma}{\sigma_1 + j\omega\epsilon_1}\right) e^{j\omega t}\, e^{u_1 y}\, e^{-\gamma x} \end{aligned}\right\} \qquad \ldots (1.2)$$

where A is a constant and the propagation coefficient along the y-axis is

$$u_1 = a_1 + jb_1 \qquad \ldots (1.3)$$

representing an attenuation a_1 and phase change b_1 for a wave travelling inwards from the surface.

(ii) Above the Surface- Medium 2 assuming air ($\mu_1 = \mu_0, \epsilon_1 = \epsilon_0, \sigma_2 = 0$), i.e., for $y \geq 0$.

$$\left.\begin{aligned} H_{z2} &= A\, e^{j\omega t}\, e^{-u_2 y}\, e^{-\gamma x} \\ E_{x2} &= -A\left(\frac{u_2}{j\omega\epsilon_0}\right) e^{j\omega t}\, e^{-u_2 y}\, e^{-\gamma x} \\ E_{y2} &= A\left(\frac{\gamma}{j\omega\epsilon_0}\right) e^{j\omega t}\, e^{-u_2 y}\, e^{-\gamma x} \end{aligned}\right\} \qquad \ldots (1.4)$$

$\ldots (1\text{-}5)$

where, $u_2 = a_2 + jb_2$

Here Eqn.(1.5) shows that the field decays at the rate a_2 with increasing distance and also suffers a progressive phase change b_2 for a wave travelling towards the surface. This is in accordance with the usual definition of surface wave for which the power flow has two components, one representing the main stream along the interface and subject to the attenuation α and phase change β, while the other , the minor one, is directed into the surface to supply the losses. This is same as to say that no radiation takes place.

On the two sides of the interface we have,

$$\gamma^2 + u^2 = K^2 = j\omega\mu_o(\sigma + j\omega\epsilon) \qquad .. (1-6)$$

Within the surface

$$\gamma^2 + u_1^2 = K_1^2 = j\omega\mu_1(\sigma_1 + j\omega\epsilon_1) \qquad .. (1-7)$$

In the air outside the surface

$$\gamma^2 + u_2^2 = K_2^2 = -\omega^2\mu_o\epsilon_o \qquad .. (1-8)$$

1.2.2 The Radial Cylindrical Surface Wave

This wave supported by a flat surface differs from the plane wave in that the wave-front here is finite in extent in the horizontal direction. The field distribution is shown in Fig.1.2, and defining this in cylindrical coordinates for a medium with constants μ_o, ϵ_1, σ_1 within the surface, assumed to be surrounded by air, we get the following field components:

(i) <u>Inside the Surface,</u> i.e. for $y \leqslant 0$,

$$\left.\begin{aligned} H_{\phi 1} &= A\, e^{j\omega t}\, e^{u_1 y}\, H_1^{(2)}(-j\gamma r) \\ E_{r1} &= -A\left(\frac{u_1}{\sigma_1 + j\omega\epsilon_1}\right) e^{j\omega t}\, e^{u_1 y}\, H_1^{(2)}(-j\gamma r) \\ E_{y1} &= A\left(\frac{j\gamma}{\sigma_1 + j\omega\epsilon_1}\right) e^{j\omega t}\, e^{u_1 y}\, H_o^{(2)}(-j\gamma r) \end{aligned}\right\} \quad .. \ (1-9)$$

with equations (1-3) and (1-7) as before.

(ii) <u>Outside the Surface,</u> i.e., for $y \geqslant 0$

$$\left.\begin{aligned} H_{\phi 2} &= A\, e^{j\omega t}\, e^{-u_2 y}\, H_1^{(2)}(-j\gamma r) \\ E_{r2} &= A\left(\frac{u_2}{j\omega\epsilon_o}\right) e^{j\omega t}\, e^{-u_2 y}\, H_1^{(2)}(-j\gamma r) \\ E_{y2} &= A\left(-\frac{\gamma}{\epsilon_o}\right) e^{j\omega t}\, e^{-u_2 y}\, H_o^{(2)}(-j\gamma r) \end{aligned}\right\} \quad .. \ (1-10)$$

with (1-5) and (1-8) as before.

Comparing Eqns.(1-2) and (1-9) or (1-4) and (1-10), radial form of surface wave has the same field distribution in y-direction as the corresponding Zenneck wave. In the radial direction field decays according to a Hankel-function.

1.2.3 The Sommerfeld-Goubau or Axial Cylindrical Surface Wave.

Sommerfeld, at the beginning, pointed out that a transversely cylindrical surface can support a surface wave. Goubau extended its application to a wave-guide consisting of a metal wire with dielectric coating or corrugated surface. The field distribution of such a wave is shown in Fig. 1.3. From the field distribution it will be seen that, when the radius of the cylindrical surface is increased to infinity, the Sommerfeld-Goubau wave becomes identical in form with the Zenneck wave. If a surface, having constants μ_o, ε_1, σ_1 is surrounded by air, the field components can be represented as follows

(i) Inside the Surface when $r \leqslant s$

$$\left.\begin{aligned} H_{\theta 1} &= A\left(\frac{\sigma_1 + j\omega\varepsilon_1}{j\,u_1}\right) e^{j\omega t}\, e^{-\gamma x}\, J_1(ju_1 r) \\ E_{x1} &= A\, e^{j\omega t}\, e^{-\gamma x}\, J_o(ju_1 r) \\ E_{r1} &= A\left(\frac{\gamma}{ju_1}\right) e^{j\omega t}\, e^{-\gamma x}\, J_1(ju_1 r) \end{aligned}\right\} \quad .. \text{(1-11)}$$

with Eqns.(1-3) and (1-7) as for the Zenneck wave.

(ii) Outside the Surface when $r \geqslant s$

$$\left.\begin{aligned} H_{\theta 2} &= A\left(\frac{\omega\varepsilon_o}{u_2}\right) e^{j\omega t}\, e^{-\gamma x}\, H_1^{(1)}(ju_2 r) \\ E_{x2} &= A\, e^{j\omega t}\, e^{-\gamma x}\, H_o^{(1)}(ju_2 r) \end{aligned}\right\} \quad .. \text{(1-12)}$$

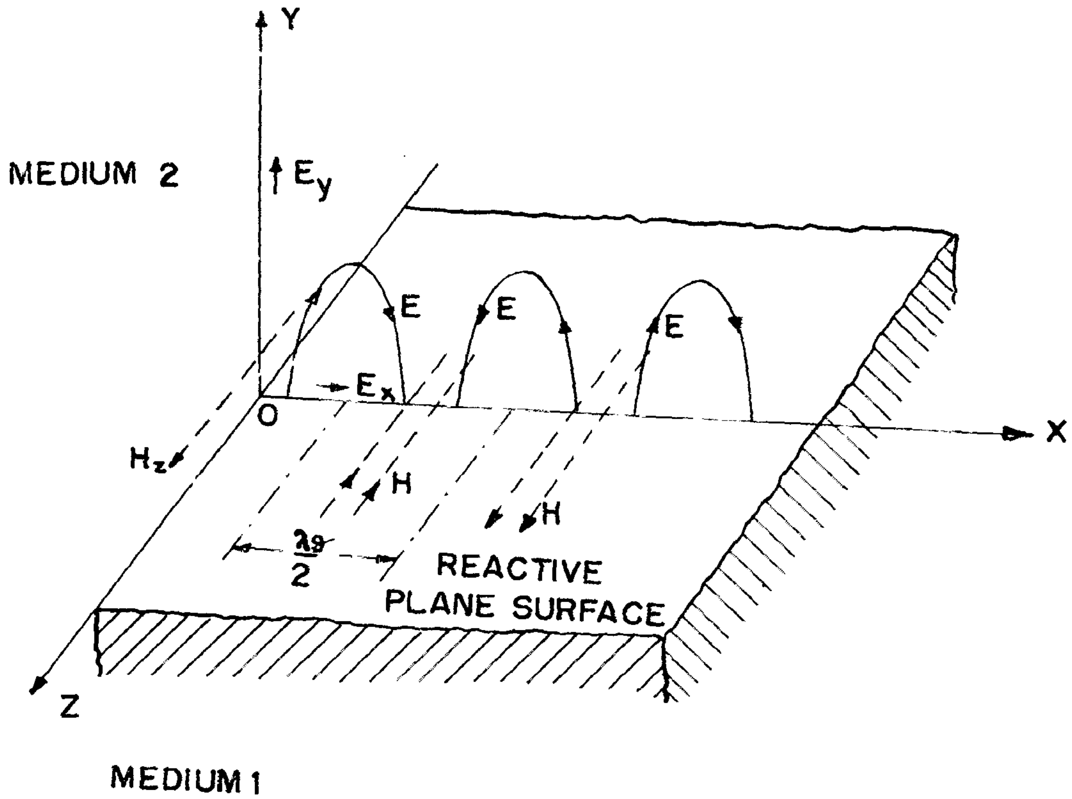

FIG. 1·1 ZENNECK WAVE, MEDIUM 1 — $\mu_1 = \mu_0, \epsilon_1, \sigma_1$
MEDIUM 2 — $\mu_2 = \mu_0, \epsilon_2, \sigma_2 = 0$

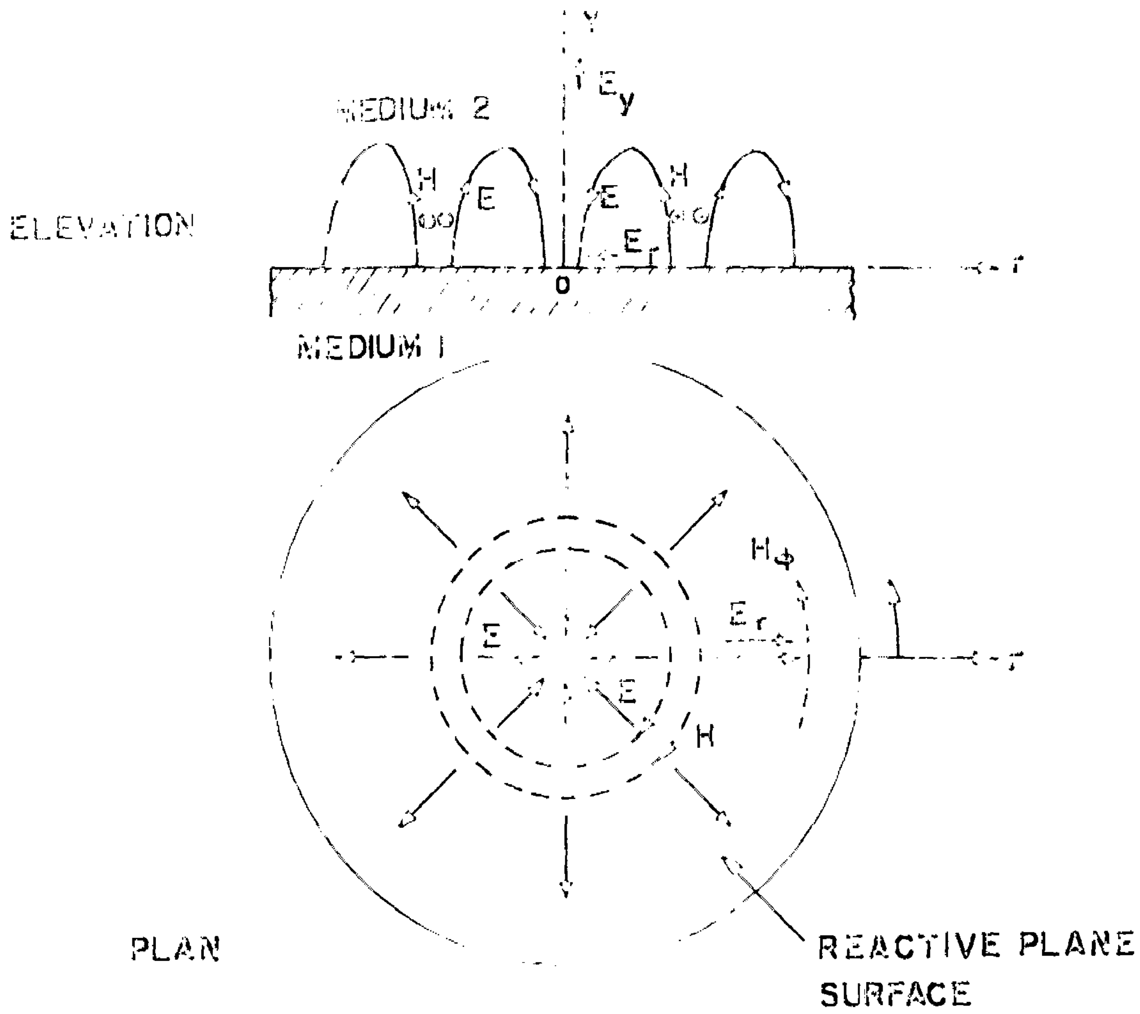

FIG. 1-2 RADIAL CYLINDRICAL SURFACE WAVE

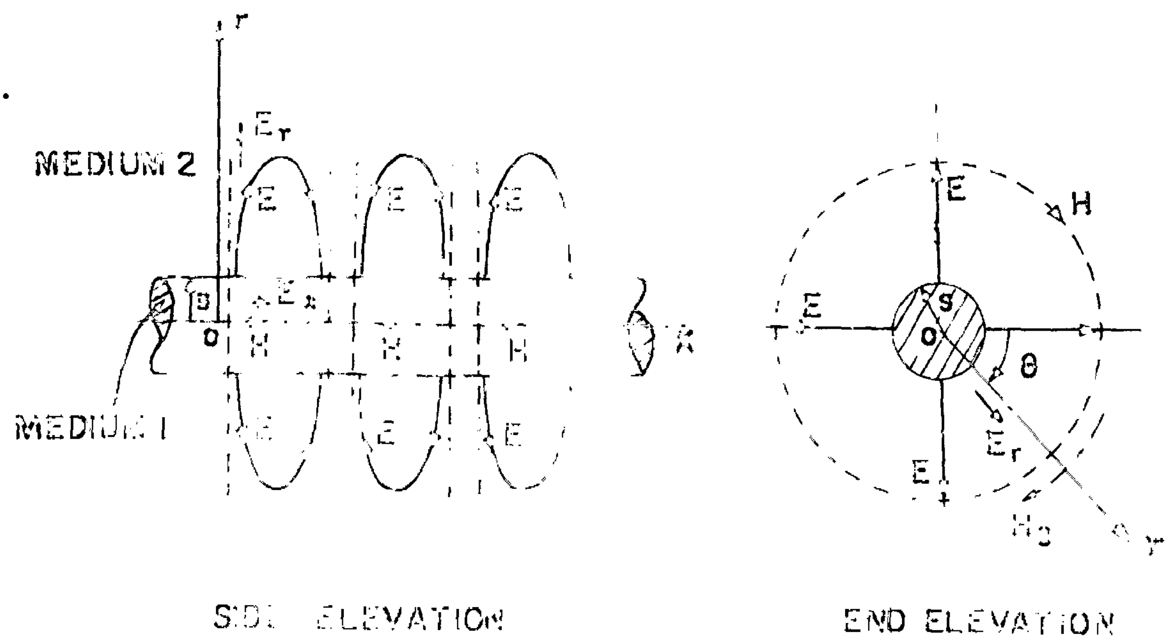

FIG. 1-3 SOMMERFELD-GOUBAU WAVE

$$E_{r2} = A\left(\frac{\gamma}{ju_2}\right) e^{j\omega t} e^{-\gamma x} H_1^{(1)}(ju_2 r)$$

with Eqns.(1-5) and (1-8) as for the Zenneck wave.

1.3 SURFACE IMPEDANCE AND ITS EFFECTS ON FIELD DISTRIBUTION [1,6]

1.3.1 Surface Impedance

Some times it is very convenient to specify the properties of guiding structure in terms of surface impedances because it may describe the behaviour of surface wave outside the surface without knowing the actual constitution of the supporting surface. The surface impedance Z_s is defined as the ratio of the tangential components of electric and magnetic field at the surface. In general Z_s is complex and can be represented as

$$Z_s = R_s + jX_s \qquad \text{... (1-13)}$$

Obviously, for a given structure there are two possible values for this impedance, one depending on the transverse electric field and the longitudinal magnetic field and the other on the transverse magnetic field and longitudinal electric field.

Since surface impedance is defined in terms of field components its value would depend both on physical properties of guiding structure and the nature of the field being investigated. This way, some structures can support several surface waves of different types, and the value of surface impedances may be different for each surface wave.

1.3.2 Effect of Surface Impedance on the Field Distribution outside the Surface

The idea of a surface impedance is widely used in calculating the attenuation in wave-guides, the effect of the losses in the conducting walls being included by imposing an impedance boundary condition at the surface of each wall.

For any medium, having finite conductivity and thickness greater than the skin depth, R_s and X_s can not be separated physically, since presence of R_s entails a corresponding X_s quantity arising from the penetration of the field. In any good conductor R_s is slightly larger than X_s, although these may be assumed to be equal for most practical cases. In a loss free media with polythene coated smooth copper surface X_s can be obtained without having R_s. Thus the reactance arising from the finite conductivity of the metal can be increased by coating it with a thin layer of dielectric or by making the radius of the cylindrical surface large compared with the skin depth. Any increase in R_s increases the inclination of the wave front from the normal, and this, in turn, increases the phase velocity along the interface. Also by analogy with electric circuits it can be anticipated that the corresponding phase velocity would be reduced by an inductive surface and increased by a capacitive one. We shall show these effects of surface-impedance by considering the cases of Zenneck and Radial Cylindrical Surface Waves as follows.

Zenneck and Radial Cylindrical Waves

Looking into the surface supporting a Zenneck wave we have

$$Z_s = \frac{E_{x2}}{H_{z2}} \qquad \ldots (1\text{-}14)$$

and for radial cylindrical surface wave

$$Z_s = -\frac{E_{r2}}{H_{\phi_2}} \qquad \ldots (1\text{-}15)$$

The values of Z_s, here, are defined for y = 0 and are independent of the distance from the surface.

Thus

$$Z_s = -\left(\frac{u_2}{j\omega\epsilon_o}\right)$$

or $$Z_s = \frac{1}{\omega\epsilon_o}(b_2+ja_2) \qquad \ldots (1\text{-}16)$$

Therefore,

$$B_s = \frac{b_2}{\omega\epsilon_o} \qquad \ldots (1\text{-}17)$$

and $$X_s = \frac{a_2}{\omega\epsilon_o} \qquad \ldots (1\text{-}18)$$

This shows that for both of these wave forms the quantity a_2, representing the rate of decay of the field with distance from the surface, is directly proportional to the surface reactance, while, b_2, representing the phase factor depends only on the surface resistance and corresponds to the attenuation of surface wave. Our interest, therefore lies in surfaces having low surface resistance.

CHAPTER -2

STUDY OF SURFACE WAVES ON UNIDIRECTIONALLY CONDUCTING SCREEN

2.1 INTRODUCTION

As long back as 1956 Toraldo-di-Francia defined a unidirectionally conducting screen. R.A. Hurd (1960) studied the problem of diffraction by a unidirectionally conducting half-plane and observed that a surface wave could exist on this wire structure. In 1957 Karp also solved the same problem but the transform technique used by Hurd was independent and much simpler than Karp's method. In 1961 Rumsey found a new way of solving Maxwell's equations which was most suitable to study the behaviour of surface waves on anisotropic sheets like a U.C. screen. In 1965 Arora studied the modes of propagation on a U.C. screen shielded on either side by parallel metal planes and solved the problem of bifurcation of a parallel plate waveguide by such a screen [9].

Seshadri (1962), Karal and Karp (1963-64), Felsen and Hessel (1966) and several other author's have studied the problem of excitation of surface waves on U.C. screen. Since we are much interested in studying the behaviour of surface waves we will confine ourselves to this problem.

2.1.1 Definition of U.C. Screen

A unidirectionally conducting screen is one on which the induced electric currents are constrained to flow along specified directions. Such an idealized structure may be physically approximated by a grid of tightly packed, insulated, thin wires whose orientation define the direction of conduction.

If the wave-length of the electromagnetic field is large compared to the element spacing, the boundary conditions on such a surface require the vanishing of the electric field component parallel to the wires whereas the perpendicular components of electric field and parallel components of magnetic field are continuous through the surface.

The interesting feature of a unidirectionally conducting screen is its ability to support surface waves which carry energy along the surface and have an evanescent field in the perpendicular direction, thereby making this structure a useful prototype for certain surface wave applications. Several methods have been employed by different authors to calculate the fields, and in particular the surface waves, excited by elementary current distributions. [7,11]

2.12 Description of the Problem

We assume a unidirectionally conducting screen, infinitely wide in y-direction and occupying a region z = 0 in (x,y,z) space. A second rectangular co-ordinate system (x',y',z') may be set up, such that

$$x = x'\cos\alpha - y'\sin\alpha$$
$$y = x'\sin\alpha + y'\cos\alpha$$
$$z = z$$

where α is the angle between the positive x and positive x' directions, $-\pi/2 \leq \alpha \leq \pi/2$. In x'-direction conductivity is supposed to be infinite and in y' direction it is taken to be zero.

The components of an incident plane electromagnetic wave, then, can be given by

$$\bar{E} = \bar{A}\, e^{-j(\bar{K}.\bar{r} - \omega t)}$$

where,

$$\bar{K} = \left[k_1, k_2, k_3\right]$$

$$\bar{r} = \left[x, y, z\right]$$

The problem is to find $\bar{E}$, $\bar{H}$, which satisfy the following boundary conditions

$E'_x = 0$ on the screen,

E'_y = continuous across the screen, and

H'_x = continuous across the screen.

[4,7]

2.2 PROPERTIES OF SURFACE WAVES ON UNIDIRECTIONALLY CONDUCTING SCREEN

Rumsey(1961) found a general solution of Maxwell's equations at a single frequency and expressed it as the combination of two types of solutions. Each of the solutionsis characterized by an electric vector which is equal to the magnetic vector times the intrinsic impedance of free space but a quarter cycle out of phase. He expressed these two solutions by means of a single scalar function by using Hertz potential technique to the corresponding fields [5] .

The particular method can roughly be described as a decomposition of the field into right-handed and left-handed circularly polarized parts. This approach is equivalent, in generality in a rough way, to the usual microwave technique of solving Maxwell's equations, namely, the decomposition of the field into transverse electric (TE) and transverse magnetic (TM) parts. This type of analysis has proved to be advantageous in solving problems involving propagation over anisotropic sheets such as a U.C. screen shown in Fig. 2.1.

In the following two sections, the behaviour of surface waves on U.C. screen and a graphical picture of the same given by Rumsey using his Hertz potential technique, will be described briefly.

2.2.1 Solution of Maxwell's Equations by Bunsoy's Method

To start with we write Maxwell's equations, for a loss-free region, in the form

$$\left.\begin{aligned} \nabla \times H &= j\omega\epsilon E \\ \text{and} \quad \nabla \times E &= -j\omega\mu H \end{aligned}\right\} \quad \text{.. (2-1)}$$

with $e^{j\omega t}$ to be the assumed time factor.

Solutions of (2-1) have been considered for which

$$E = CH \quad \text{.. (2-2)}$$

Substitution of (2-2) in (2-1) , gives

$$C = \pm j\eta \quad \text{.. (2-3)}$$

where η is the intrinsic impedance given by the relation

$$\eta = \sqrt{\mu/\epsilon} \quad \text{.. (2-4)}$$

Thus two types of solutions, denoted by E_1 and E_2, are

$$\left.\begin{aligned} E_1 &= j\eta H_1 \\ E_2 &= -j\eta H_2 \end{aligned}\right\} \quad \text{.. (2-5)}$$

Instead of working with the three scalar functions which constitute three components of E_1, the whole field is expressed by means of only one scalar function. This has been done by applying Hertz potential technique to E_1. The results are found to be as

$$E_1 = \nabla \times \nabla \times \hat{z}U_1 - \beta\nabla \times \hat{z}\, U_1 \quad \text{.. (2-6)}$$

or $$E_1 = \nabla\left(\frac{\partial U_1}{\partial z}\right) - \beta\nabla \times \hat{z}\, U_1 + \beta^2 \hat{z}\, U_1 \qquad \ldots (2\text{-}7)$$

where $\hat{z}$ is a unit vector, U_1 and U_2 are Hertz potential functions and $\beta^2 = \omega^2\mu\epsilon$. The formulae for E_2 are obtained by reversing the sign of β. The function U_1 from which E_1 is derived is any solution of the scalar wave equation

$$\nabla^2 f + \beta^2 f = 0 \qquad \ldots (2\text{-}8)$$

The total field E, then, can be found by putting

$$E = E_1 + E_2 \qquad \ldots (2\text{-}9)$$

so that on substitution from (2-6) in (2-9) gives

$$E = \nabla \times \nabla \times \hat{z}\,(U_1 + U_2) + \beta\nabla \times \hat{z}\,(U_2 - U_1) \qquad \ldots (2\text{-}10)$$

Here, (U_1+U_2) and (U_2-U_1) are the TM and TE Hertz potential functions for E and the representation (2-9) is complete. It can readily be shown that E_1 and E_2 are orthogonal to each other [5].

2.2.2 Application to W.G. Screen

The solution of the type $E = jY_i H$ complies with the boundary conditions of a W.G screen where E parallel to the wires must be zero and H parallel to the wires must be at least continuous, since the discontinuity in tangential H is perpendicular to the current. This requires that H parallel to the wires should also be zero, so that boundary conditions on E and H are same.

This has been expressed more explicitly by letting a field of type E_1 for $z > 0$ and of type E_2 for $z < 0$, where E_1 and E_2 are expressed as in (2-7). Let

$$\left. \begin{aligned} U_1 &= -U_2 \\ \text{and} \quad \frac{\partial U_1}{\partial z} &= \frac{\partial U_2}{\partial z}, \quad \text{at } z = 0 \end{aligned} \right| \qquad \text{.. (2-11)}$$

From (2-7) it is found that this makes tangential H discontinuous by the amount

$$J = 2\hat{z} \times H_1 = \frac{2\hat{z} \times E_1}{j\gamma_1} \qquad \text{.. (2-12)}$$

If

$$E.\hat{K} = 0 \text{ on } z = 0 \qquad \text{.. (2-13)}$$

J should be parallel to $\hat{K}$, where $\hat{K}$ is some vector in x-y plane. This can be interpreted, physically, as that there exists an electric current sheet J on z = 0 flowing in perfectly conducting direction, $\hat{K}$, in which the filaments of the screen point.

Now let us take

$$U_1 = e^{\gamma_x x + \gamma_y y + \gamma_z z} \quad (z > 0) \qquad \text{.. (2-14)}$$

$$U_2 = -e^{\gamma_x x + \gamma_y y - \gamma_z z} \quad (z < 0) \qquad \text{.. (2-15)}$$

where

$$\gamma_x^2 + \gamma_y^2 + \gamma_z^2 = -\beta^2 \qquad \text{.. (2-16)}$$

From Eqn. (2-7), we get

$$j\gamma_1 H_1 = E_1 = (\bar{\gamma}\gamma_z + \hat{z} \times \bar{\gamma}\omega + \hat{z}\,\beta^2)U_1 \qquad \text{.. (2-17)}$$

where,

$$\bar{Y} = \hat{x}\, Y_x + \hat{y}\, Y_y + \hat{z}\, Y_z \qquad .. \ (2\text{-}18)$$

On solving, (2-17) gives

$$E.E = 0 \qquad .. \ (2\text{-}19)$$

This means E and consequently H, are circularly polarized. The component of electric field in the direction of the wires would be zero, if

$$E_x = Y_x Y_z - Y_y \beta = 0 \qquad .. \ (2\text{-}20)$$

Substituting (2-20) in (2-16) and (2-17) following results are obtained

$$\left.\begin{aligned} Y_x &= \pm j\beta \\ Y_z &= \pm j Y_y \end{aligned}\right\} \qquad .. \ (2\text{-}21)$$

$$\text{and } \left.\begin{aligned} E_z &= (\beta^2 - Y_y^2) U_1 \\ E_y &= \pm j E_z \end{aligned}\right\} \qquad .. \ (2\text{-}22)$$

Hence, Y_y fixes the solution and may be considered to represent the mode of excitation.

With

$$Y_y = j\beta_y \qquad .. \ (2\text{-}23)$$

from (2-16) and (2-22), it can be shown that the instantaneous electric field, which is the real and imaginary parts of E at t = 0 and $\omega t = -\pi/2$, is tangential to the curves

$$\beta_y z \pm \log \sin (y\, \beta_y \pm \beta\, x) = \text{constant} \qquad .. \ (2\text{-}24)$$

These curves have been plotted and are shown in Figs. 2.2 and 2.3. A three dimensional picture of the same is shown in Fig. 2.4. It may be noted that the phase vector lies in the x-y plane and has components ($\pm \beta$, β_y), while electric vector lies in the y-z plane. Consequently, the waveform moves obliquely over the U.C. screen with a phase velocity less than the velocity of light. Also the Poynting vector, found to be

$$\bar{P} = \pm \hat{x}(\beta^2+\beta_y^2)^2 \, e^{\pm 2\beta_y z} \qquad .. \; (2\text{-}25)$$

is along the x-axis which is the direction of conduction. If the sign of β_y has been changed and signs of $\bar{P}$ in (2-25) have been chosen such that field decreases exponentially with distance, the sense of polarisation remains unchanged. [5]

Thus an outlined behaviour of surface waves on U.C. screen is that

(i) They are slow waves having phase velocity less than the velocity of light.

(ii) They are circularly polarized in the plane normal to the wires.

(iii) They travel along the wires with the velocity of light.

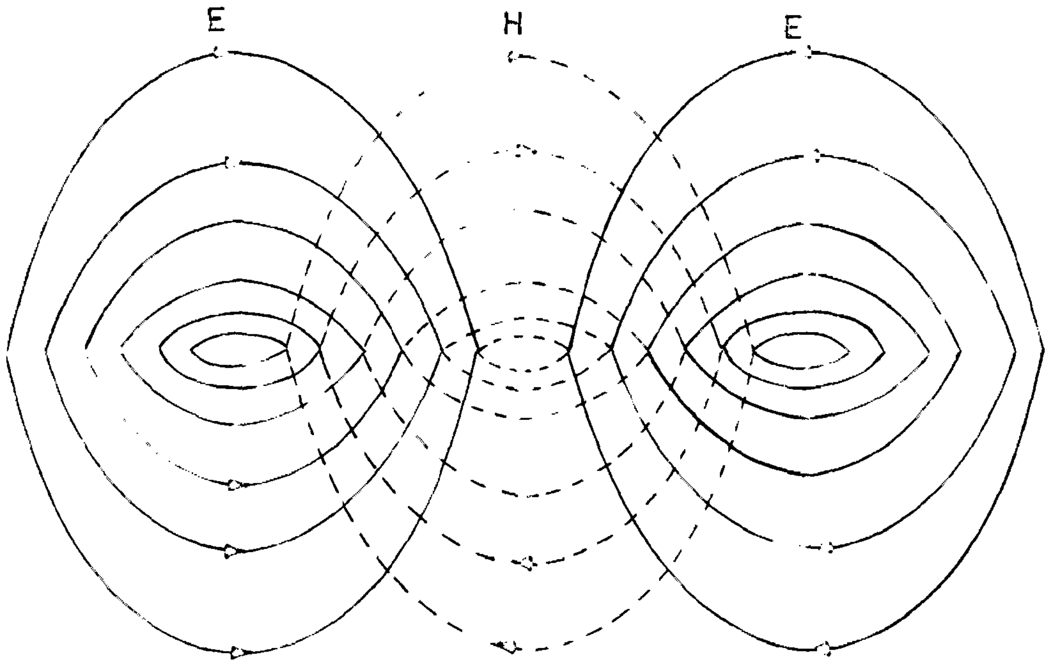

FIG. 2·2 THE INSTANTANEOUS FIELD OF A CIRCULARLY POLARIZED SURFACE WAVE TRAVELING OVER A u.c. SCREEN.

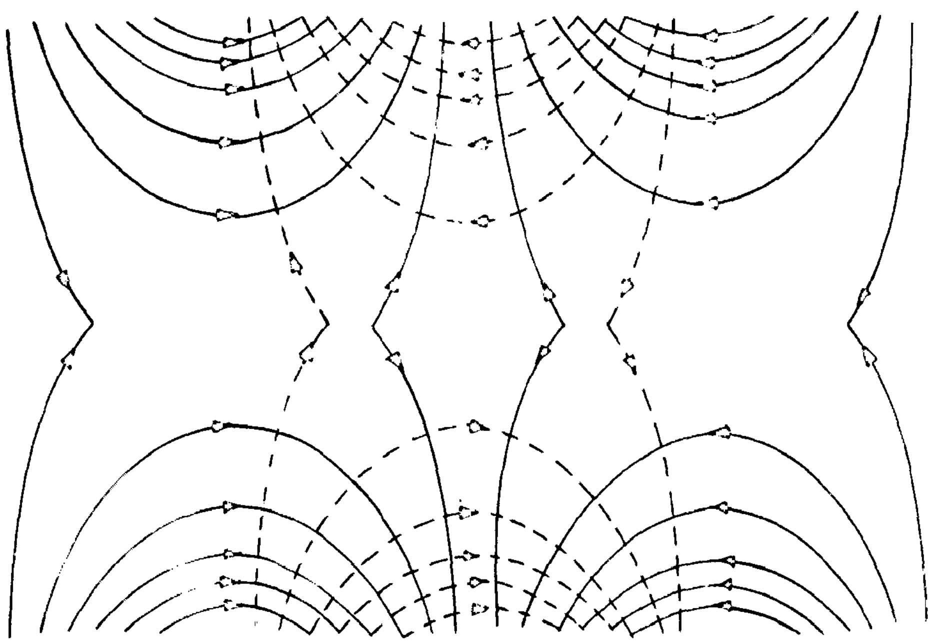

FIG. 2·3 THE FIELD OF A CIRCULARLY POLARIZED WAVE, INCREASING EXPONENTIALLY WITH DISTANCE FROM THE SCREEN.

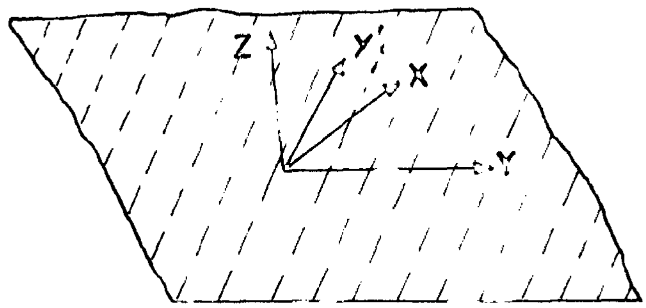

FIG. 2·1 ENERGY TRAVELS ALONG Y'-DIRN. WHICH IS THE DIRECTION OF CONDUCTION WHILE PHASE PROPAGATES OBLIQUELY IN Y-DIRN. OVER A u.c. SCREEN.

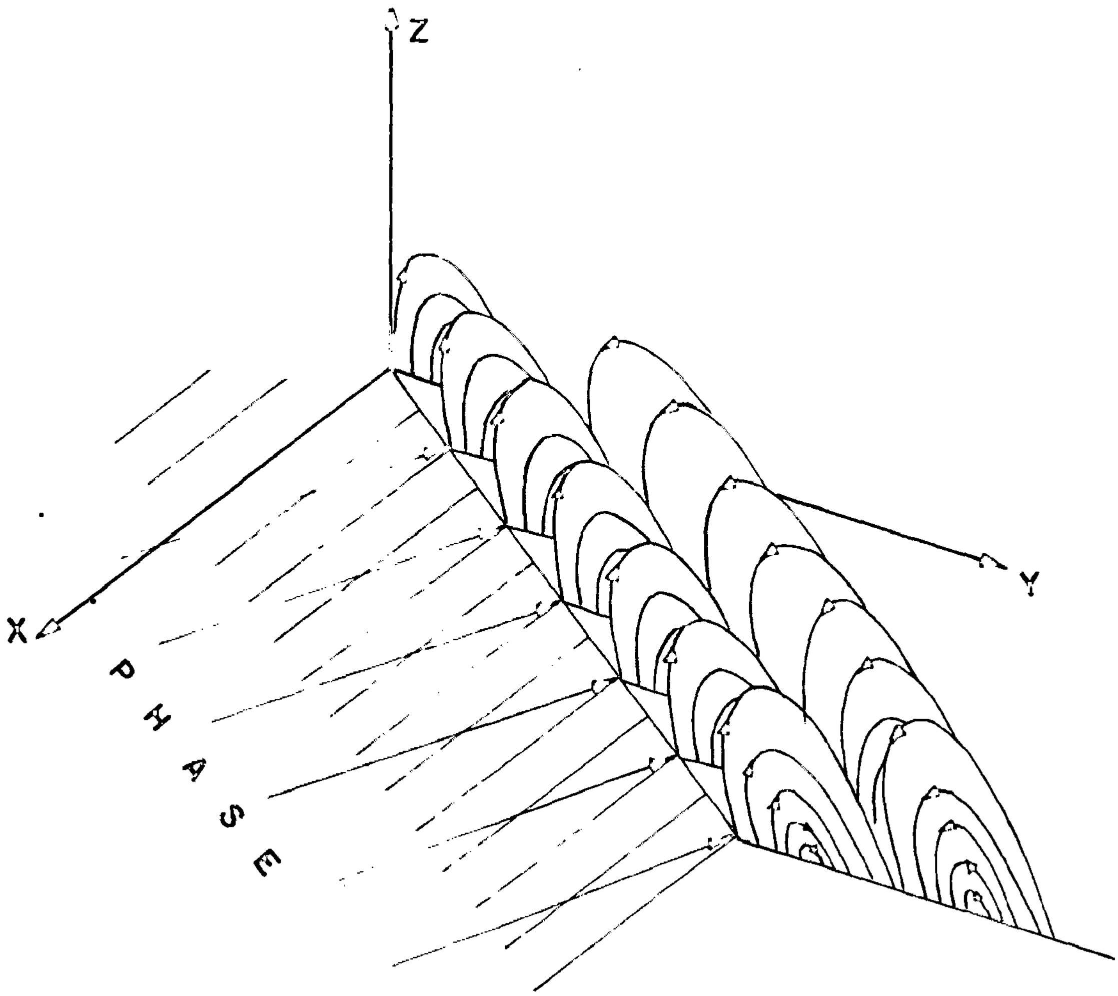

FIG. 2·4 THE WAVEFORM MOVES OBLIQUELY OVER THE u.c. SCREEN IN THE DIRECTION OF THE PHASE-VECTOR.

CHAPTER -3

PAIR OF UNIDIRECTIONALLY CONDUCTING SCREENS IN FREE SPACE

3.1 NEED OF A PAIR OF U.C. SCREENS FOR THE PROPAGATION OF SURFACE WAVES

It has been seen in the previous chapter that a single unidirectionally conducting screen is capable of supporting surface waves. However, Arora in 1966 pointed out the limitation of the single screen as a practical surface-wave guide. As has been described by Rumsey, the surface wave energy, on a U.C. screen, always travels in the direction of conduction while the wave-front advances obliquely. In a practical system where dimensions are always finite in transverse direction, energy transport of a surface wave has components perpendicular to the sides. Thus limiting the width of the practical surface-wave guide causes much more serious side effects than when the direction of energy transport is parallel to the sides.

Arora in 1966 suggested a surface-wave guide, consisting of a pair of parallel unidirectionally conducting screens, which was free from this drawback. He analysed the problem and found the propagation properties and the power transmitted along such a guide. In a second communication, also in 1966, he investigated the problem of surface-wave excitation by means of a line source oriented parallel to the y-axis and determined the power in the surface wave and the radiation pattern and also calculated the surface-wave launching efficiency. [12]

3.2 DESCRIPTION OF PAIR OF U.C. SCREENS

The orientation of a pair of parallel unidirectionally conducting screens is shown in Fig. 3.1. Here x is the direction of propagation and z is the transverse direction. The screens are supposed to be infinite in extent in y-direction. x' and x'' are the directions of perfect conduction of top and bottom screens, respectively, where x' makes an angle α with the x-axis and x'' makes an angle $-\alpha$ with the same. The screens are perfectly insulating in y' and y'' directions which are perpendicular to x' and x'', respectively. The distance between two screens is $2a$ in free-space.

Two distinct surface-wave modes were found to exist, one of which was termed transverse-symmetric and other longitudinal-symmetric according to the symmetry of the transverse and longitudinal components, respectively, about the $z = 0$ plane.

From the symmetry of the structure it can readily be anticipated that the direction of phase propagation and that of the energy transport are same. Thus, limiting this configuration in y-direction, will not much affect the surface wave propagating away from the ends. [10,12]

The behaviour of surface waves on a pair of U.C. screens will be studied and a few results obtained in Arora's work will be discussed in the following sections.

3.3 FIELD COMPONENTS AND DETERMINANTAL EQUATION [10]

The solution of wave equation for a structure comprising a pair of parallel unidirectionally conducting screens and assumed to be infinitely wide in y-direction, when is subjected to proper boundary conditions, gives the y-components of electric and magnetic fields as well as the determinantal equation of the system, which is

$$e^{2ua} = \pm \frac{u^2+k^2\tan^2\alpha}{u^2-k^2\tan^2\alpha} \qquad .. \ (3\text{-}1)$$

where,

$$u^2 = \beta^2 - k^2 \ , \qquad .. \ (3\text{-}2)$$

$$k^2 = \omega^2\mu_o\epsilon_o \ , \qquad .. \ (3\text{-}3)$$

the decay coefficient , u, being real and positive. Corresponding to + and - signs there are two positive-real values of u, and therefore eqn. (3-1) leads to the conclusion that there are two types of surface waves that can be supported by this particular structure.

Using subscript 1 for + sign in (3-1), for mode 1, determinantal equation reduces to

$$\tan h\, u_1 a = \frac{1}{u_1^2} k^2 \tan^2 \alpha \qquad .. \text{ (3-4)}$$

It is apparent from this equation that $u_1 a$ increases both with k and α. When k approaches infinity or α approaches 90°, decay coefficient u_1 approaches infinity which means that the wave will degenerate into two different surface waves, one bound to the top screen and other to the bottom one. $u_1 a$ against ka/π has been plotted by Arora and is shown in Fig. 3.2. The phase change coefficient, β, can be computed with the help of eqn.(3-2). Variation of $\beta_1 a$ against ka/π is shown in Fig. 3.3.

Choosing −sign in eqn.(3-1) and using subscript 2 for this mode, the determinantal equation reduces to

$$\cot h\, u_2 a = \frac{1}{u_2^2} k^2 \tan^2 \alpha \qquad .. \text{ (3-5)}$$

Variation of $u_2 a$ and that of $\beta_2 a$ against ka/π are shown by dotted curves in Figs.(3.2) and (3.3), respectively. In this case also, the wave will degenerate into two surface waves when α approaches 90° or k approaches infinity. Variations of k tan α and k sec α are also indicated in Figs. 3.2 and 3.3 so as to compare the two surface wave modes. It can be seen that $u_1 a$ and $\beta_1 a$ are the straight line curves while $u_2 a$ and $\beta_2 a$ are below them. This is expected from the inequality $\tan h\, u_1 a < 1$, and in consequence from eqn.(3-4) $\beta_1 a > ka \sec \alpha$.

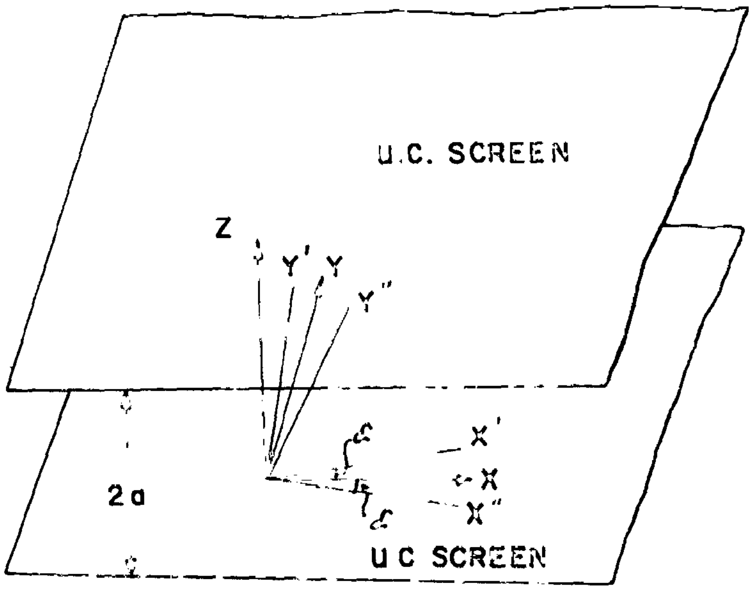

FIG. 3·1 A PAIR OF PARALLEL u.c. SCREENS, X IS THE DIRECTION OF PROPAGATION; X' IS THE DIRECTION OF CONDUCTION OF TOP SCREEN AND X'' IS THE DIRECTION OF CONDUCTION OF BOTTOM SCREEN. SCREENS ARE PERFECTLY INSULATING IN Y' & Y'' DIRECTIONS.

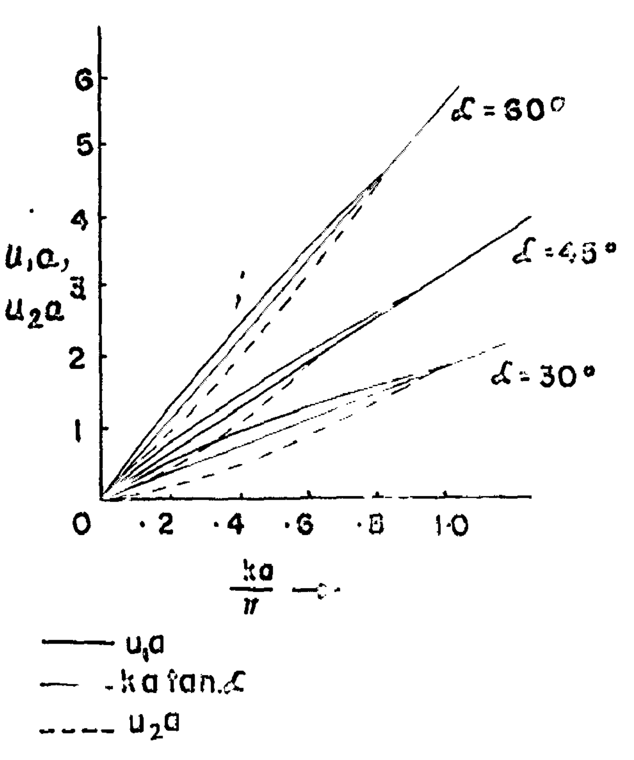

FIG. 3·2 u_1a AND u_2a AS FUNCTIONS OF FREQUENCY

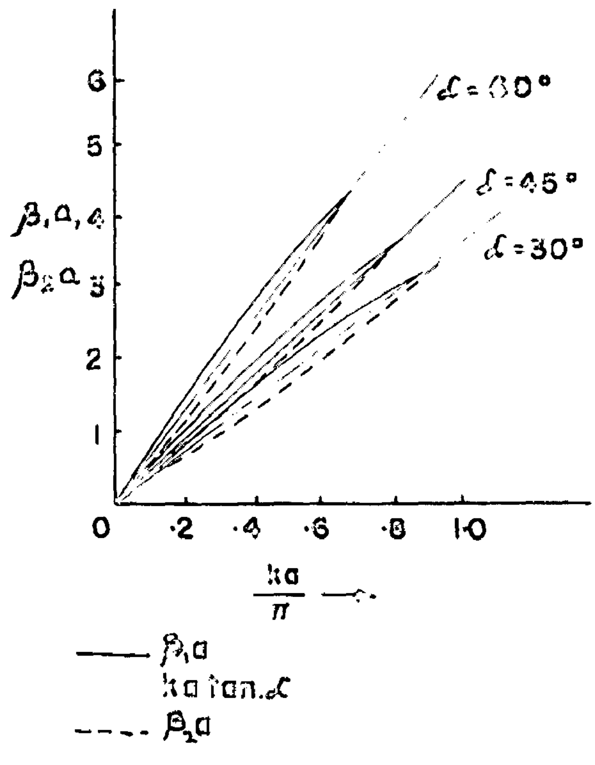

FIG. 3·3 β_1a AND β_2a AS FUNCTIONS OF FREQUENCY

The following expressions for the y-components of electric and magnetic fields for the two types of surface waves have been obtained.

Mode 1:-

$$E_y = \begin{cases} A\, e^{-u_1(z-a)}\, e^{-j\beta_1 x} & z \geq a \\ A\, \dfrac{\cosh u_1 z}{\cosh u_1 a}\, e^{-j\beta_1 x} & -a \leq z \leq a \\ A\, e^{u_1(z+a)}\, e^{-\beta_1 x} & z \leq -a \end{cases} \quad \text{.. (3-6)}$$

$$H_y = \begin{cases} -\dfrac{j\omega\varepsilon_o}{u_1} \tan\alpha A\, e^{-u_1(z-a)}\, e^{-j\beta_1 x} & z \geq a \\ \dfrac{j\omega\varepsilon_o}{u_1} \tan\alpha A\, \dfrac{\cosh u_1 z}{\sinh u_1 a}\, e^{-j\beta_1 x} & -a \leq z \leq a \\ -\dfrac{j\omega\varepsilon_o}{u_1} \tan\alpha A\, e^{u_1(z+a)}\, e^{-j\beta_1 x} & z \leq -a \end{cases} \quad \text{.. (3-7)}$$

Mode 2:-

$$E_y = \begin{cases} A\, e^{-u_2(z-a)}\, e^{-j\beta_2 x} & z \geq a \\ A\, \dfrac{\sinh u_2 z}{\sinh u_2 a}\, e^{-j\beta_2 x} & -a \leq z \leq a \\ -A\, e^{u_2(z+a)}\, e^{-j\beta_2 x} & z \leq -a \end{cases} \quad \text{.. (3-8)}$$

$$H_y = \begin{cases} -\dfrac{j\omega\epsilon_a}{u_2} \tan\alpha\, A\, e^{-u_2(z-a)}\, e^{-j\beta_2 x} & z \geq a \\ \dfrac{j\omega\epsilon_o}{u_2} \tan\alpha\, A\, \dfrac{\sinh u_2 z}{\sinh u_2 a}\, e^{-j\beta_2 x} & -a \leq z \leq a \\ \dfrac{j\omega\epsilon_a}{u_2} \tan\alpha\, A\, e^{u_2(z+a)}\, e^{-j\beta_2 x} & z \leq -a \end{cases} \quad ..(3\text{-}9)$$

Using Maxwell's equations and the fact that the structure is infinite in y-direction , i.e.,

$$\frac{\partial}{\partial y} = 0 \qquad ... (3\text{-}10)$$

all the other field components can be obtained using following expressions

$$E_x = \frac{j}{\omega\epsilon_o} \frac{\partial H_y}{\partial z} \qquad .. (3\text{-}11)$$

$$E_z = -\frac{j}{\omega\epsilon_o} \frac{\partial H_y}{\partial x} \qquad .. (3\text{-}12)$$

$$H_x = -\frac{j}{\omega\mu_o} \frac{\partial E_y}{\partial z} \qquad .. (3\text{-}13)$$

$$H_z = \frac{j}{\omega\mu_o} \frac{\partial E_y}{\partial x} \qquad .. (3\text{-}14)$$

From the above expressions for the field components for the two surface waves it,can be seen that for surface-wave mode 1, transverse components are symmetrical about z = 0 plane while longitudinal components are antisymmetrical about the same, and for surface-wave mode 2. Longitudinal components are symmetrical while transverse components are antisymmetrical about $z = 0$ plane . Hence, these modes of propagation of surface wave over U.C. screens have been named 'transverse symmetric' and 'longitudinal symmetric' modes, respectively.

For mode 1, the angle which the zero-electric field component makes with the x-axis varies from $+\alpha$ in the region $z \geqslant a$ to $-\alpha$ in the region $z \leqslant a$ and passes through $\alpha = 0$ at $z = 0$. For mode 2, this angle varies from $+\alpha$ in the region $z \geqslant a$ to $\pi - \alpha$ in the region $z \leqslant a$ and passes through $\alpha = 0$ at $z = 0$.

Both of these surface-wave types are elliptically polarized [10].

CHAPTER -4

SURFACE WAVES ON A PAIR OF U.C. SCREENS SUPPORTED BY DIELECTRIC PLATES

4.1 INTRODUCTION

In the previous chapter the behaviour of surface waves on a pair of parallel unidirectionally conducting screens in free-space, studied thoroughly by Arora, has been reviewed. Although the analysis, and the results obtained theoretically are perfect in their form, it was interesting to observe and prove them practically. Since to construct a unidirectionally conducting screen in free-space was rather difficult, a thin dielectric sheet having dimensions of the screen was suggested to mount equally spaced parallel conducting wires. Another possibility was to have a printed wire mesh on a thin dielectric sheet. In any case, a dielectric support was necessary to have a practical surface-wave guide comprising U.C. screens. Consequently, it became a topic of further interest to analyse this new problem mathematically and then to observe the practical results. In the following sections an exact treatment of the problem following Arora's approach will be given. A comparative study of the pair of U.C. screens in free-space and that with dielectric supports will also be done.

4.2 DESCRIPTION OF THE STRUCTURE

A diagrammatical view of the pair of parallel unidirectionally conducting screens supported by dielectric sheets with respect to coordinate axes is shown in Fig. 4.1. The distance between two dielectric sheets is 2a. The thickness of one dielectric sheet is then

$$t = a - b \qquad .. (4-1)$$

The top and bottom screens are conducting in x' and x'' directions, respectively, while in perpendicular directions, y' and y'', they are perfectly insulating. The angle between x' and x'' directions is 2α. The x-axis, taken to be the direction of phase propagation of the surface wave, bisects this angle such that direction of conduction on top screen makes an angle $+\alpha$ and that on bottom screen makes an angle $-\alpha$ with x-axis. The dielectric is assumed to be loss-less. By the symmetry of the structure (about the $z = 0$ plane) the direction of phase propagation and that of the Poynting vector averaged over the cross-section will coincide as in case of parallel U.C. screens in free-space.

4.3 GENERAL SOLUTION OF WAVE EQUATION AND APPLICATION TO BOUNDARY CONDITIONS

The screens may be assumed to be infinite in y-direction, so that

$$\frac{\partial}{\partial y} = 0 \qquad .. (4-2)$$

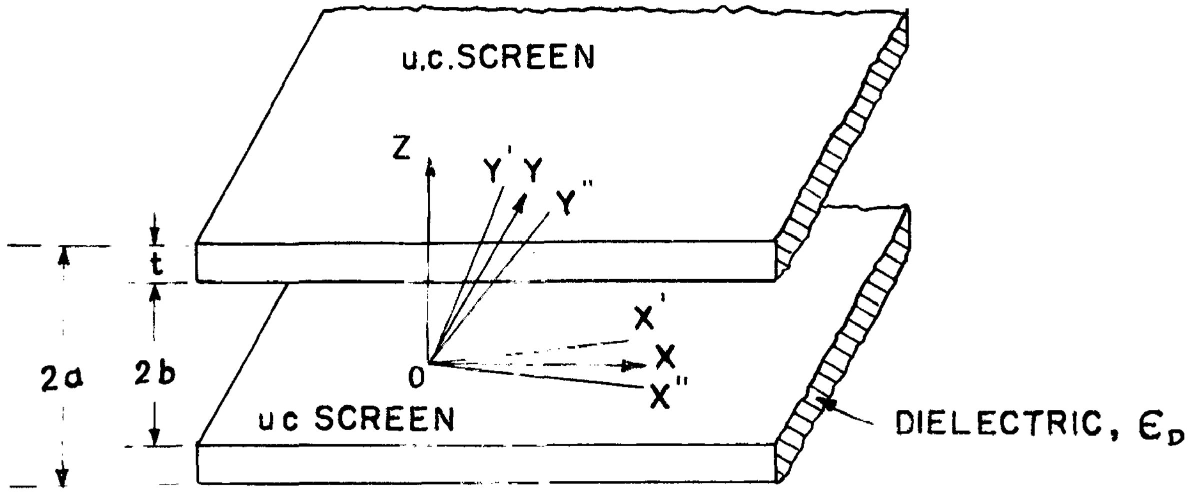

FIG. 4·1 A PAIR OF PARALLEL u.c. SCREENS WITH DIELECTRIC SUPPORTS. DIRECTIONS OF PERFECT CONDUCTION AT TOP AND BOTTOM SCREENS ARE X' AND X'' RESPECTIVELY. X IS THE DIRECTION OF PROPAGATION. SCREENS ARE PERFECTLY INSULATING IN Y' & Y'' DIRECTIONS. (DIELECTRIC IS LOSS-LESS) and three dimensional wave equation reduced to a two dimensional one. Assuming $e^{j\omega t}$ time dependence, the general expressions for the y-components of electric and magnetic fields for a surface wave travelling in +ve x-direction may be written as

$$E_y = \begin{cases} A\, e^{-u_1(z-a)}\, e^{-j\beta x} & z \geqslant a \\ \left[B\, e^{u_2 z} + C\, e^{-u_2 z}\right] e^{-j\beta x} & b \leqslant z \leqslant a \\ \left[D\, e^{u_1 z} + E\, e^{-u_1 z}\right] e^{-j\beta x} & -b \leqslant z \leqslant b \\ \left[F\, e^{u_2 z} + G\, e^{-u_2 z}\right] e^{-j\beta x} & -b \geqslant z \geqslant -a \\ H\, e^{u_1(z+a)}\, e^{-j\beta x} & z \leqslant -a \end{cases} \qquad \text{.. (4-3)}$$

and

$$H_y = \begin{cases} I\, e^{-u_1(z-a)}\, e^{-j\beta x} & z \geqslant a \\ \left[K\, e^{u_2 z} + L\, e^{-u_2 z}\right] e^{-j\beta x} & b \leqslant z \leqslant a \\ \left[M\, e^{u_1 z} + N\, e^{-u_1 z}\right] e^{-j\beta x} & -b \leqslant z \leqslant b \\ \left[O\, e^{u_2 z} + P\, e^{-u_2 z}\right] e^{-j\beta x} & -b \geqslant z \geqslant -a \\ Q\, e^{u_1(z+a)}\, e^{-j\beta x} & z \leqslant -a \end{cases} \qquad \text{.. (4-4)}$$

where u_1 and u_2 are the transverse decay coefficients for surface wave in free-space region and in dielectric region, respectively, and are given as,

$$u_1^2 = \beta^2 - k_1^2 , \qquad k_1^2 = \omega^2 \mu_0 \epsilon_0 \qquad \ldots (4\text{-}5)$$

$$u_2^2 = \beta^2 - k_2^2 , \qquad k_2^2 = \omega^2 \mu_0 \epsilon_D \qquad \ldots (4\text{-}6)$$

Here, both u_1 and u_2 are real and positive. By Maxwell's equations and using eqn.(4-2) following relations are obtained which will give all other field components,

$$E_x = \frac{j}{\omega \epsilon_0} \frac{\partial H_y}{\partial z} \qquad \ldots (4\text{-}7)$$

$$E_z = - \frac{j}{\omega \epsilon_0} \frac{\partial H_y}{\partial x} \qquad \ldots (4\text{-}8)$$

$$H_x = - \frac{j}{\omega \mu_0} \frac{\partial E_y}{\partial z} \qquad \ldots (4\text{-}9)$$

$$H_z = \frac{j}{\omega \mu_0} \frac{\partial E_y}{\partial x} \qquad \ldots (4\text{-}10)$$

From the continuity of tangential components of electric and magnetic fields at the boundary, following 16 conditions at the 4 boundaries should be applied to the field equations

(i) $E'_x = E_x \cos \alpha + E_y \sin \alpha = 0$ ... (4-11)

on $z = +a$

(ii) $E''_x = E_x \cos \alpha - E_y \sin \alpha = 0$ ... (4-12)

on $z = -a$

(iii) E_x should be continuout across $z = +a$ ... (4-13)

(iv) E_x should be continuous across $z = -a$ ... (4-14)

(v) E_y should be continuous across $z = +a$ ... (4-15)

(vi) E_y should be continuous across $z = -a$ .. (4-16)

(vii) E_x should be continuous across $z = +b$ .. (4-17)

(viii) E_x should be continuous across $z = -b$ .. (4-18)

(ix) E_y should be continuous across $z = +b$ .. (4-19)

(x) E_y should be continuous across $z = -b$ .. (4-20)

(xi) H_x should be continuous across $z = +b$ .. (4-21)

(xii) H_x should be continuous across $z = -b$ .. (4-22)

(xiii) H_y should be continuous across $z = +b$ .. (4-23)

(xiv) H_y should be continuous across $z = -b$ .. (4-24)

(xv) $H'_x = H_x \cos\alpha + H_y \sin\alpha$ continuous across $z = +a$ .. (4-25)

(xvi) $H''_x = H_x \cos\alpha - H_y \sin\alpha$ continuous across $z = -a$ .. (4-26)

Applying these conditions to the field expressions following sixteen equations are obtained

$$I = \frac{\varepsilon_0}{j u_1} A \tan\alpha \qquad .. (4\text{-}27)$$

$$Q = \frac{\varepsilon_0}{j u_1} H \tan\alpha \qquad .. (4\text{-}28)$$

$$-\frac{j u_1}{\varepsilon_0} I = \frac{j u_2}{\varepsilon_D}\left[K e^{u_2 a} - L e^{-u_2 a}\right] \qquad .. (4\text{-}29)$$

$$\frac{j u_1}{\omega \varepsilon_0} Q = \frac{j u_2}{\omega \varepsilon_D}\left[O e^{-u_2 a} - P e^{u_2 a}\right] \qquad .. (4\text{-}30)$$

$$A = B e^{u_2 a} + C e^{-u_2 a} \qquad .. (4\text{-}31)$$

$$H = F e^{-u_2 a} + G e^{u_2 a} \qquad .. (4\text{-}32)$$

$$\frac{ju_2}{\varepsilon_D}\left[K\,e^{u_2 b} - L\,e^{-u_2 b}\right] = \frac{ju_1}{\varepsilon_o}\left[M\,e^{u_1 b} - N\,e^{-u_1 b}\right] \quad .. \; (4\text{-}33)$$

$$\frac{ju_2}{\varepsilon_D}\left[O\,e^{-u_2 b} - P\,e^{u_2 b}\right] = \frac{ju_1}{\varepsilon_o}\left[M\,e^{-u_1 b} - N\,e^{u_1 b}\right] \quad .. \; (4\text{-}34)$$

$$B\,e^{u_2 b} + C\,e^{-u_2 b} = D\,e^{u_1 b} + E\,e^{-u_1 b} \quad .. \; (4\text{-}35)$$

$$F\,e^{-u_2 b} + G\,e^{u_2 b} = D\,e^{-u_1 b} + E\,e^{u_1 b} \quad .. \; (4\text{-}36)$$

$$-\frac{ju_2}{\mu_o}\left[B\,e^{u_2 b} - C\,e^{-u_2 b}\right] = -\frac{ju_1}{\mu_o}\left[D\,e^{u_1 b} - E\,e^{-u_1 b}\right] \quad .. \; (4\text{-}37)$$

$$-\frac{ju_2}{\mu_o}\left[F\,e^{-u_2 b} - G\,e^{u_2 b}\right] = -\frac{ju_1}{\mu_o}\left[D\,e^{-u_1 b} - E\,e^{u_1 b}\right] \quad .. \; (4\text{-}38)$$

$$K\,e^{u_2 b} + L\,e^{-u_2 b} = M\,e^{u_1 b} + N\,e^{-u_1 b} \quad .. \; (4\text{-}39)$$

$$O\,e^{-u_2 b} + P\,e^{u_2 b} = M\,e^{-u_1 b} + N\,e^{u_1 b} \quad .. \; (4\text{-}40)$$

$$-\frac{ju_1}{\omega\mu_o} A\cos\alpha + I\sin\alpha = -\frac{ju_2}{\mu_o}\left[B\,e^{u_2 a} - C\,e^{-u_2 a}\right]\cos\alpha + \left[K\,e^{u_2 a} + L\,e^{-u_2 a}\right]\sin\alpha \quad .. \; (4\text{-}41)$$

$$-\frac{ju_1}{\omega\mu_o} H\cos\alpha - Q\sin\alpha = -\frac{ju_2}{\mu_o}\left[F\,e^{-u_2 a} - G\,e^{u_2 a}\right]\cos\alpha - \left[O\,e^{-u_2 a} + P\,e^{u_2 a}\right]\sin\alpha \quad .. \; (4\text{-}42)$$

4.4 DETERMINANTAL EQUATION AND FIELD-COMPONENTS

4.4.1 Determinantal Equation

Since, to solve equations(4-27) to(4-42) is tedious, by the symmetry of the structure following assumptions have been made

(i) For symmetry

$$D = L, \quad M = N \qquad .. \ (4\text{-}43)$$

(ii) For antisymmetry

$$D = -L, \; M = -N \qquad .. \ (4\text{-}44)$$

Solving eqns.(27-44), relative values of 16 amplitude coefficients have been obtained and determinantal equations for the two cases have been determined, which are

(i) For symmetry

$$u_2 \frac{1+\frac{u_1}{u_2}\tanh u_1 b - \left(1-\frac{u_1}{u_2}\tanh u_1 b\right) e^{-2u_2 t}}{1+\frac{u_1}{u_2}\tanh u_1 b + \left(1-\frac{u_1}{u_2}\tanh u_1 b\right) e^{-2u_2 t}}$$

$$-\frac{k_2^2}{u_2}\left[\frac{1+\frac{u_1}{u_2}\epsilon_r \tanh u_1 b + \left(1-\frac{u_1}{u_2}\epsilon_r \tanh u_1 b\right) e^{-2u_2 t}}{1+\frac{u_1}{u_2}\epsilon_r \tanh u_1 b - \left(1-\frac{u_1}{u_2}\epsilon_r \tanh u_1 b\right) e^{-2u_2 t}}\right]\tan^2\alpha$$

$$= -u_1 + \frac{k_1^2}{u_1}\tan^2\alpha \qquad .. \ (4\text{-}45)$$

(ii) for antisymmetry

$$u_2' \frac{1+\frac{u_1'}{u_2'}\coth u_1'b-(1-\frac{u_1'}{u_2'}\coth u_1'b)e^{-2u_2't}}{1+\frac{u_1'}{u_2'}\coth u_1'b+(1-\frac{u_1'}{u_2'}\coth u_1'b)\,e^{-2u_2't}}$$

$$-\frac{k_2^2}{u_2'}\left[\frac{1+\frac{u_1'}{u_2'}\epsilon_r\coth u_1'b+(1-\frac{u_1'}{u_2'}\epsilon_r\coth u_1'b)\,e^{-2u_2't}}{1+\frac{u_1'}{u_2'}\epsilon_r\coth u_1'b-(1-\frac{u_1'}{u_2'}\epsilon_r\coth u_1'b)\,e^{-2u_2't}}\right]\tan^2\alpha$$

$$= -u_1' + \frac{k_1^2}{u_1'}\tan^2\alpha \qquad .. \; (4\text{-}46)$$

where primed decay coefficients correspond to the second surface wave, and $\epsilon_r = \epsilon_D/\epsilon_o$ (ϵ_D, being the permittivity of the dielectric material). Eqns.(4-45) and (4-46), as a particular case in which

$$\left.\begin{array}{l} t = 0 \\ \epsilon_r = 1 \\ u_1 = u_2,\; u_1' = u_2' \end{array}\right\} \qquad .. \; (4\text{-}47)$$

reduce to the following forms, respectively,

$$\tanh u_1 a = k^2\tan^2\alpha/u_1^2 \qquad .. \; (4\text{-}48)$$

and $$\coth u_1' a = k^2\tan^2\alpha/u_1'^2 \qquad .. \; (4\text{-}49)$$

These are the same equations which were obtained for surface waves on a pair of parallel U.C. screens in free-space for transverse-symmetric and longitudinal-symmetric cases, respectively, in Chapter-3. Hence, the assumptions (4-43) and (4-44) are correct.

The values of β, satisfying eqn.(4-48) and (4-46), have been calculated for $\alpha = 20^{\circ}$ and b/λ varying in the range $0 \leq b/\lambda \leq 1$. The results have been plotted and are shown in Fig.4.2. From the graph and the determinantal equations it can be seen that β increases with increasing values of α and b/λ. In eqns. (4-48) and (4-46) if α approaches 90° or b/λ approaches infinity, the transverse decay coefficients u_1 and u_2 approaches infinity and therefore, the wave will degenerate into two different surface waves, one bound to the top screen and other to the bottom one.

Calculating β for a particular value of α from eqn.(4-48) and that from eqn.(4-48) it can be shown that value of β increases with increasing values of ϵ_r. It can be concluded, therefore, that the phase velocity of the transverse-symmetric surface-wave will be decreased for a dielectric supported surface-wave guide comprising a pair of parallel U.C. screens. Calculating β from (4-49) and that from (4-46), it can again be shown that for a longitudinal-symmetric surface wave also phase velocity decreases with dielectric supports.

A graph between b/λ and β has been plotted for both symmetric and antisymmetric cases and is shown in Fig.4.2. The particular values of the parameters chosen are

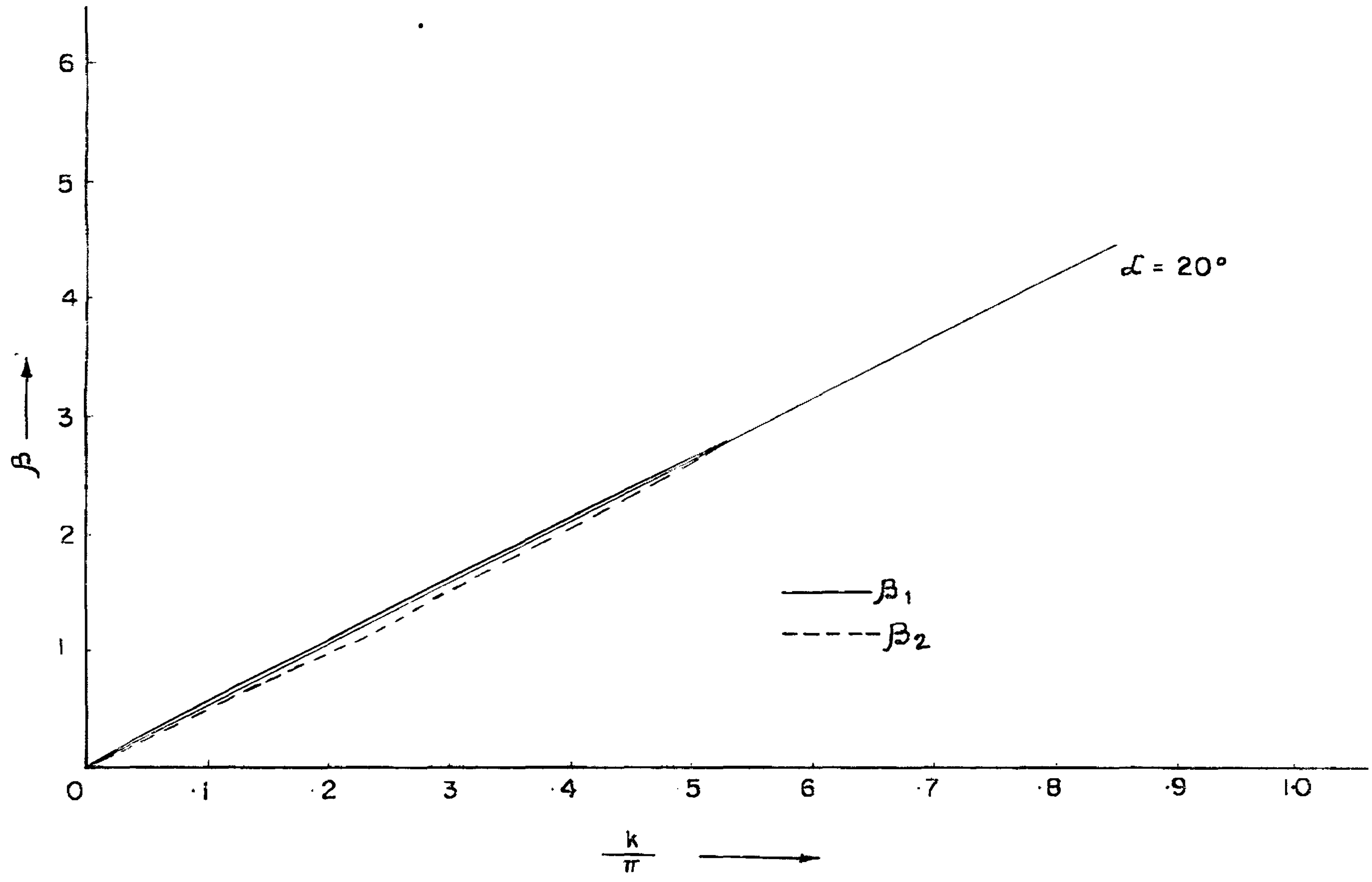

FIG. 4·2 VARIATIONS OF β_1 AND β_2 WITH FREQUENCY.

$t = 0.3$ cm,

$b = 1$ cm.

$\alpha = 20^{\circ}$

$\epsilon_r = 2.55$

$f = 10$ GHz.

4.4.2 Field Components

(i) Symmetrical Case

The y-components of electric-and magnetic-fields of this surface-wave mode are given below.

$$E_y = A\, e^{-u_1(z-a)}\, e^{-j\beta_1 x} \qquad z \geq a$$

$$= \frac{A\left[\left(1+\frac{u_1}{u_2}\tanh u_1 b\right) e^{u_2(z-a)} + \left(1-\frac{u_1}{u_2}\tanh u_1 b\right) e^{-u_2(z+a-2b)}\right] e^{-j\beta_1 x}}{1+\frac{u_1}{u_2}\tanh u_1 b + \left(1-\frac{u_1}{u_2}\tanh u_1 b\right) e^{-2u_2 t}} \qquad b \leq z \leq a$$

$$= \frac{2A\, e^{-u_2 t} \cosh u_1 z\, e^{-j\beta_1 x}}{\cosh u_1 b\left[1+\frac{u_1}{u_2}\tanh u_1 b + \left(1-\frac{u_1}{u_2}\tanh u_1 b\right) e^{-2u_2 t}\right]} \qquad -b \leq z \leq b$$

$$= \frac{A\left[\left(1-\frac{u_1}{u_2}\tanh u_1 b\right) e^{u_2(z-a+2b)} + \left(1+\frac{u_1}{u_2}\tanh u_1 b\right) e^{-u_2(z+a)}\right] e^{-j\beta_1 x}}{1+\frac{u_1}{u_2}\tanh u_1 b + \left(1-\frac{u_1}{u_2}\tanh u_1 b\right) e^{-2u_2 t}}, \qquad -b \geq z \geq -a$$

$$= A\, e^{u_1(z+a)}\, e^{-j\beta_1 x} \qquad z \leq -a$$

... (4-50)

and

$$H_y = \frac{\omega\epsilon_o}{ju_1} A \tan\alpha\, e^{-u_1(z-a)} e^{-j\beta_1 x} \qquad z \geq a$$

$$= \frac{j\omega\epsilon_o}{u_2} \frac{A\tan\alpha\left[(1+\frac{u_1}{u_2}\epsilon_r \tanh u_1 b)e^{u_2(z-a)} + (1-\frac{u_1}{u_2}\epsilon_r\tanh u_1 b)e^{-u_2(z+a-2b)}\right]e^{-j\beta_1 x}}{1+\frac{u_1}{u_2}\epsilon_r \tanh u_1 b-(1-\frac{u_1}{u_2}\epsilon_r\tanh u_1 b)e^{-2u_2 t}} \qquad b \leq z \leq a$$

$$= \frac{j\omega\epsilon_o}{u_2} \frac{2A\tan\alpha\, e^{-u_2 t}\cosh u_1 z\, e^{-j\beta_1 x}}{\cosh u_1 b\left[1+\frac{u_1}{u_2}\epsilon_r\tanh u_1 b-(1-\frac{u_1}{u_2}\epsilon_r\tanh u_1 b)e^{-2u_2 t}\right]} \qquad -b \leq z \leq b$$

$$= \frac{j\omega\epsilon_o}{u_2} \frac{A\tan\alpha\left[(1-\frac{u_1}{u_2}\epsilon_r\tanh u_1 b)e^{u_2(z-a+2b)} + (1+\frac{u_1}{u_2}\epsilon_r\tanh u_1 b)e^{-u_2(z+a)}\right]e^{-j\beta_1 x}}{1+\frac{u_1}{u_2}\epsilon_r\tanh u_1 b - (1-\frac{u_1}{u_2}\epsilon_r\tanh u_1 b)e^{-2u_2 t}} \qquad -b \geq z \geq -a$$

$$= \frac{\omega\epsilon_o}{ju_1} A\tan\alpha\, e^{u_1(z+a)} e^{-j\beta_1 x} \qquad z \leq -a$$

.. (4-51)

It can again be seen in this case that transverse components of electric and magnetic fields are symmetric about $z = 0$ plane while the longitudinal components are antisymmetric about it. The angle which the direction of zero-electric field-components makes with the x-axis varies from + α in the region z > a to -α in the region z < a and passes through α = 0 at z = 0. This wave is also elliptically polarized similar to the case of U.C. screens in free-space.

(ii) Antisymmetric Case

$$E_y = A\, e^{-u_1'(z-a)} e^{-j\beta_2 x}$$

$$= \frac{A\left[\left(1+\frac{u_1'}{u_2'}\coth u_1'b\right) e^{u_2'(z-a)} + \left(1-\frac{u_1'}{u_2'}\coth u_1'b\right) e^{-u_2'(z+a-2b)}\right] e^{-j\beta_2 x}}{1+\frac{u_1'}{u_2'}\coth u_1'b + \left(1-\frac{u_1'}{u_2'}\coth u_1 b\right) e^{-2u_2' t}} \qquad b \leq z \leq a$$

$$= \frac{2A\, e^{-u_2' t} \sinh u_1' z\, e^{-j\beta_2 x}}{\sinh u_1 b\left[1+\frac{u_1'}{u_2'}\coth u_1'b + \left(1-\frac{u_1'}{u_2'}\coth u_1'b\right) e^{-2u_2' t}\right]} \qquad -b \leq z \leq b$$

$$= -\frac{A\left[\left(1-\frac{u_1'}{u_2'}\coth u_1'b\right) e^{u_2'(z-a+2b)} + \left(1+\frac{u_1'}{u_2'}\coth u_1'b\right) e^{-u_2(z+a)}\right] e^{-j\beta_2 x}}{1+\frac{u_1'}{u_2'}\coth u_1'b + \left(1-\frac{u_1'}{u_2'}\coth u_1'b\right) e^{-2u_2' t}} \qquad -b \geq z \geq -a$$

$$= -A\, e^{u_1'(z+a)} e^{-j\beta_2 x} \qquad z \leq -a$$

$$\ldots (4\text{-}52)$$

is the expression for y-component of electric-field, and the y-component of magnetic-field is

$$H_y = \frac{\epsilon_0}{ju_1'} A \tan\alpha\, e^{-u_1'(z-a)} e^{-j\beta_2 x} \qquad z \geq a$$

$$= \frac{j\omega\epsilon_0}{u_2'} \frac{A\tan\alpha\left[(1+\frac{u_1'}{u_2'}\epsilon_r \coth u_1'b)e^{u_2'(z-a)} + (1-\frac{u_1'}{u_2'}\epsilon_r \coth u_1'b)e^{-u_2'(z+a-2b)}\right]e^{-j\beta_2 x}}{1+\frac{u_1'}{u_2'}\epsilon_r \coth u_1'b - (1-\frac{u_1'}{u_2'}\epsilon_r \coth u_1'b)e^{-2u_2't}} \qquad b \leq z \leq a$$

$$= \frac{j\omega\epsilon_0}{u_2'} \frac{A\tan\alpha\, e^{-u_2't}\sinh u_1'z \quad e^{-j\beta_2 x}}{\sinh u_1'b\left[1+\frac{u_1'}{u_2'}\epsilon_r \coth u_1'b - (1-\frac{u_1'}{u_2'}\epsilon_r \coth u_1'b)e^{-2u_2't}\right]} \qquad -b \leq z \leq b$$

$$= -\frac{j\omega\epsilon_0}{u_2'} \frac{A\tan\alpha\left[(1-\frac{u_1'}{u_2'}\epsilon_r \coth u_1'b)e^{u_2'(z-a+2b)} + (1+\frac{u_1'}{u_2'}\epsilon_r \coth u_1'b)\, e^{-u_2'(z+a)}\right]e^{-j\beta_2 x}}{1+\frac{u_1'}{u_2'}\epsilon_r \coth u_1'b - (1-\frac{u_1'}{u_2'}\epsilon_r \coth u_1'b)\, e^{-2u_2't}} \qquad -b \geq z \geq -a$$

$$= -\frac{\omega\epsilon_0}{ju_1'} A \tan\alpha\, e^{u_1'(z+a)} e^{-j\beta_2 x} \qquad z \leq -a$$

.. (4-53)

For its symmetry properties this mode may be called longitudinal-symmetric mode. Polarization in this case is same as in transverse-symmetric case. The angle which the direction of zero-electric field makes with x-axis varies from $+\alpha$ in region $z > a$ to $\pi - \alpha$ in the region $z < a$ passing through y-axis at $z = 0$ plane.

4.5 PRACTICAL SURFACE WAVE-GUIDE COMPRISING U.C. SCREENS

An attempt has been made to fabricate in the laboratory a surface waveguide comprising a pair of parallel U.C. screens with bakelite sheets to support them. Surface waves were excited and received with the help of wave-guide and horn at a free-space frequency of 10 GHz. A few measurements were taken and results obtained are given in the following sections.

4.5.1 Design Considerations

Since at the time of fabrication of the screens the problem of U.C. screens with dielectric supports had not been analysed and the characteristic equation for the same was not known, the following approximate design assuming free space conditions has been done.

For a pair of parallel U.C. screens in free space, using the relations

$$\beta = k \sec \alpha \quad .. (4\text{-}54)$$

and $$u_1 = k \tan \alpha \quad .. (4\text{-}55)$$

surface-wave-lengths and the distances, at which field would decay to 1/e of its original value, at different angles have been calculated and are tabulated on the next page.

Table -1

α (degrees)	$\lambda_{surface}$ (cm.)	Distance at which field will decay to 1/e of its original value
10°	2.955	2.7
15°	2.9	1.78
20°	2.82	1.31
25°	2.715	1.02
30°	2.6	0.83

From the practical point of view field should not decay too rapidly in the transverse direction and at the same time surface-wave-length should be distinguishable from the free space-wave-length. Having these considerations in mind

(i) $\alpha = 20^\circ$

has been selected as the angle which the direction of conduction makes with the direction of propagation.

Since for a U.C. screen element spacing should be much less than the wave-length

(ii) 24-SWG wire, and

(iii) element spacing = 0.2 cm.

have been chosen.

The spacing between the screens should be of the order of wave-length, hence,

(iv) 2a = 1-4 cms., has been kept variable.

(v) Since the field decays to 1/e of its original value at a distance 1.31 cm. from the screen a distance 4 times larger has been kept between lower screen and the ground.

(vi) To avoid end effects, screen dimensions should be larger than the wave length, these are taken as 38x30 cms.

4.5.2 Construction

Tooth and slots were cut in a rectangular angle-aluminium frame to stretch wire with proper tension and element spacing was maintained to the designed value. Araldite had been used as an adhesive to fix up the wire onto a 38x30 cm. bakelite sheet. After giving proper time for adhesion the screen was separated out of the aluminium frame. Another screen was fabricated in the same way and the two were kept under pressure for several days. A wooden base with prespect supports was fabricated to put the screens at proper spacings so as to form a desired surface-waveguide.

4.5.3 Performance and Results

A block diagram of the practical set up is shown in Fig. 4.3. The wave was launched by a wave-guide section excited by a Klytron oscillator and received by a crystal detector and meter. Approximate measurements of wave-lengths at $\alpha = 20^{o}$ and also by turning down the set-up to 90^{o}, i.e. making $\alpha = 70^{o}$, where being taken. Free-space wave-length was also being measured in the same way.

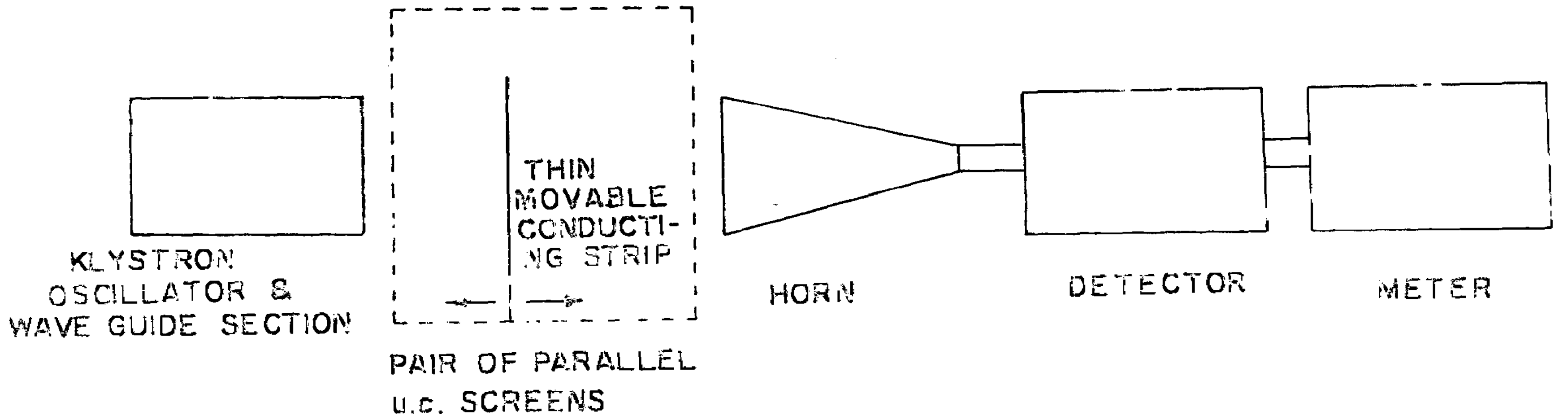

FIG. 4.3 BLOCK SCHEMATIC TO MEASURE WAVELENGTH OF A SURFACE-WAVE OVER A PAIR OF PARALLEL u.c. SCREENS.

wave-lengths, at the two angles were also being calculated from the theoretical results and a comparison is shown in the following table.

Table -2

	Wave-length at $\alpha = 20^o$ (cm)	Wave-length at $\alpha = 70^o$ (cm)	Free-space wave length (cm)
Theoretical	2.8	1.01	3.08
Practical	3.1	2.94	

An exact calculation of surface wave-length on a pair of parallel W.G. screens with dielectric support having a relative permittivity, $\epsilon_r = 2.55$, was being done using eqn.(4-45) and Fig. 4.2 which gave a theoretical wave-length, at $\alpha = 20^o$, 1.8 cms.

4.5.4 Discussion

A comparison, observed from Table 2 and the theoretical wave-length calculated from eqn.(4-45), shows that there was some discrepancy either in the practical set-up, measurement technique or in the design of the wave-guide itself. It can also be seen from the table 2 that measured wave-lengths for $\alpha = 20^o$ as well as for $\alpha = 70^o$ are almost same and nearly equal to the free-space wave-length rather than the theoretical surface-wave-length. It has already been observed that the characteristic equations for the surface-waveguide comprising U.C. screens in free-space and that having dielectric supports are different and latter is highly affected by the dielectric-constant of the material used. The theoretical results obtained in two cases at a frequency of 10 GHz and for $\alpha = 20^{0}$ are being tabulated below for a comparison.

Table -3

	β_1	Transverse decay coefficient u_1 (cm^{-1})	Distance at which field decays to 1/e of its original strength (cm.)
U.C. screens in free-space	2.24	0.763	1.31
U.C. screens with dielectric sheets(ϵ_r=2.05)	3.5	2.8	0.358

It can be seen that the value of β, and hence the transverse decay coefficient u is larger in case of dielectric supported waveguide. The wave therefore is much confined to the surface rather than predominating in the space nearby. It might, therefore, be possible that while making measurements the free-space-wave predominated over the surface-wave since the latter was fast decaying over the screen in transverse direction.

A bend in the screens in the direction of wave propogation also occurred when they were kept freely in space for long and this might have caused unnecessary reflections to disturb the wave.

Although bakelite has been used to construct the U.C. screens, it is rather a poor microwave material with exact properties not known. Prespect might have been used instead, but it was found to be too soft to take the tension of tightly packed copper wires. Losses in bakelite might account for the fact that experimental results obtained did not comply with the theoretical ones. A good microwave material, like teflon, probably, could give better results.

Another drawback was with the measurement technique itself, which was being limited by the available facilities. A precise measurement technique would definitely improve the results.

CHAPTER - 5

BRIEF SUMMARY AND CONCLUDING REMARKS

5.1 SUMMARY

Surface waves in general and in particular those propagating on a pair of parallel unidirectionally conducting screens have been studied and their behaviour has been discussed during this course of work. These have been defined as the waves that propagate along an interface between two different media without radiation, and the radiation if there is any being construde to mean energy converted from surface wave to some other form.

It has been observed by various authors that surface waves can be supported by a U.C. screen, however, the surface-wave energy on such a screen travels along the wires while the wave-front advances obliquely to this direction. To avoid the end effects caused by limiting the width of this structure, Arora suggested a symmetrical structure comprising a pair of parallel unidirectionally conducting screens in which energy travelled along the sides of the screens. This situation has been thoroughly studied by him and has been briefly discussed in this thesis as well.

During the course of study a practical surface-waveguide comprising of a pair of parallel u.c.screens has been proposed to watch the behaviour of these surface-waves in reality. However, a U.C. screen in free-space has been found to be difficult to construct in the laboratory. It has, therefore, been suggested to construct U.C. screens using thin dielectric sheets to support them. This led to an interesting problem of analysing this system.

The determinantal equation for the pair of parallel U.C. screens with thin dielectric sheets to support them has been derived following Dr. Arora's approach. The phase-change coefficient, β, as a function of frequency has been computed using the determinantal equation and a graph has been plotted in order to show this variation. A comparative study of surface-wave behaviour on this structure, therefore, has been done in this presentation.

It has been observed that the phase-velocity of surface-waves on this structure is much reduced and the wave is more confined to the surface as compared to the surface wave on U.C. screens in free space. These effects increase with an increased value of permittivity of the dielectric material used.

A waveguide comprising a pair of parallel U.C. screens with bakelite sheets to support them has been fabricated in the departmental workshop and an attempt has been made to excite surface waves along this guide in the laboratory. A few measurements as to see the surface-wave behaviour on U.C. screens have also been taken, however, due to imperfections in design as well as in measurement techniques, the results obtained are not compatible with theory.

6.2 CONCLUSIONS AND SOME SUGGESTIONS FOR FURTHER WORK

The pair of parallel U.C. screens with thin dielectric supports have been seen capable of supporting two surface-wave modes, namely, transverse symmetric and longitudinal symmetric-waves. Both of these waves are of 'slow-wave' type. The results of the analysis have shown that thin dielectric sheets lead to a large increase in the concentration of field near the surface.

It may be interesting to evaluate the attenuation constant of such a surface wave by finding the power dissipated in the dielectric and the unidirectionally conducting screens. This constant might considerably be decreased by properly choosing the parameters of the dielectric sheets.

To investigate the practical behaviour of surface waves propagating onto it, a suitable design of surface-waveguide composed of U.C. screens would be interesting to do. This can be done by using the determinantal equation derived in this thesis.

A suitable measurement technique should also be contrived in order to investigate the behaviour of surface waves in the close vicinity of the screens.

Further, excitation problem of this structure may yield some interesting results. To investigate the radiation properties and launching efficiencies, therefore, may be a topic of further research.

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