SHIELDED SURFACE WAVEGUIDES

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C E R T I F I C A T E

CERTIFIED that the dissertation entitled " SHIELDED SURFACE WAVE GUIDES" which is being submitted by Shri S. Vijaya Raghavan in partial fulfilment for the award of the DEGREE OF MASTER OF ENGINEERING in ADVANCED ELECTRONICS, Department of Electronics and Communication Engineering of University of Roorkee, Roorkee is a record of the Student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other Degree or Diploma.

This is further to certify that he has worked for about 8 months from January to August, 1968 in preparing this thesis for the Master of Engineering at this University.

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A C K N O W L E D G E M E N T

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SYNOPSIS

Recently a large number of papers have been published on the topic of Shielded Surface Waveguides. H.M. Barlow has been one of the earliest workers in this field. In his first paper on the topic (3), he poin-ted out the possibility of a screened surface waveguide and followed it up with another paper (4), where he presented an analysis of both the parallel plate and coaxial waveguide structures with dielectric coated guiding surfaces. Especially, he talks about the hybrid - TEM dual surface wave, which he considers to be the natural mode in two conductor transmission lines, the usual TEM mode being present only when the guiding surfaces are perfect conductors. Barlow maintains that this hybrid mode, under suitable circumstances, suffers an attenuation which is roughly one half of that exper-ienced by the usual TEM mode. To support this mode, one is to enhance the surface reactance by coating the surface with a practically loss-free dielectric. Through his experiments, whose results appeared in the literature (6,7,8) Barlow has been able to substantiate his observations. He has found that to reduce the attenuation one has to have an asymmetry in the field distribution, that is, the surfaces should be of un-equal reactances and further, only one of the surfaces should be coated with dielectric, the thickness of coating being optimum.

Barlow's work thus revealed the possibility of propagating, in the usual transmission line structures, a surface wave mode, which is for less attenuated compared to the usual TEM mode. This naturally aroused the curiosity of other workers and Wait came up with his theory of Shielded Surface Wave guides[5]. His conclusions are rather interesting and they stimulated the present work.

It is in order to mention that this work is an extension of Wait's work. We have ventured to make a thorough investigation of the various modes which can exist in a parallel plate guide, with reactive guiding surfaces. Wait has merely indicated the approach and what we have done is, to pursue this course to its logical end. We have analysed the symmetric case (i.e., where the two surfaces are of equal reactances) and the unsymmetric case (i.e. surface reactances unequal). Also we have considered the case where one surface is inductive and the other is capacitive.

In the course of these investigations we have come up with some new and interesting results. Mostly, the results have supported Wait's conclusions. But on some counts we differ with him. A significant point of departure is in the fact, emerging out of this work, that the modes are not orthogonal in the sense of Wait, for the general unsymmetric case. Further, Wait has not mentioned TE type modes existing in a guide with inductive guiding surfaces. It is shown that we would have TE type waveguide modes in such a structure though no TE type surface wave can exist. Another significant result is that there are two surface waves in general, of which, one vanishes at a pacticular value of surface reactance called the threshold value and turns into a zero order waveguide mode; Wait had said that this zero order mode would split into two surface waves at a particular value of reactance. Finally it is shown that one of the surface waves which stays on till zero frequency coincides with the type Barlow has been considering. Thus this work links the works of Barlow and Wait. Also a few new problems are brought to light as a result of this work. They are mentioned at the end.

While writing this dissertation, we have divided it into four Chapters. The first Chapter gives a brief but comprehensive introduction to surface waves This Chapter draws its material chiefly from references (1) and (2). The second is devoted to a review of the past work in the field of Shielded Surface Waveguides. The third constitutes the author's contribution. Here all the results of calculations performed on the IBM 1620 computer, along with fairly detailed anslysis and discussions are presented. The last Chapter, after presenting in outline, the new results which have come to light after the present work, goes on to unfold areas for further work.

Before we close, it is to be pointed out that much work is in progress on this topic. Recently papers have appeared which are critical of Barlow's appraach and his conclusions. Especially , the plausibility of reduced attenuation is being debated. Millington, Brown and Cullen have conducted theoretical investigations wherein they have examined the possibility of reduced attenuation. Their results seem to diverge from Barlow's conclusions. As yet the last word has not been written on this topic. Already much absorbing work is going on and scientific world is awaiting the results.

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CHAPTER I

AN INTRODUCTION TO SURFACE WAVES

1.1.

A surface wave may be called as "an em wave that propagates without radiation along an interface between two different media".[1,2] When the media have finite losses, the main stream of energy directed along the interface will be required to supply these losses. When we say that the surface wave does not radiate, we mean to say that energy is not absorbed from the wave independently of the media supporting the wave. Thus, when we say that the main stream has to supply energy to meet the losses in the supporting media, we are not violating the definition of a surface wave. We are interested in surface wave propagation because of its non-radiating characteristic and thus a surface wave can be employed for on efficient transmission of h.f. energy from one point to another, except in so far as demands are made on that energy to compensate for the losses occurring in the media. Usually one of the two media is a loss free dielectric, say air. We may note that the interface must be straight in the direction of propagation, though it may take a variety of forms in the transverse direction. The boundary of the medium surrounded by the loss-free dielectric is usually called the supporting surface. One can have flat surfaces or transversely cylindrical surfaces to support a surface wave. One would have encountered the Brewster angle in optics., i.e., the angle of incidence for which no reflection takes place. One may put it in another way, that there is no radiation away from the surface under the circumstances. Hence one may expect to derive a surface wave by allowing a wave of the required field configuration to be incident at the Brewster angle on the surface. This turns out to be the case and this can be established for any flat surface. Since the power flow is normal to the wavefront, defined as an equiphased surface, it follows that the field distribution is to be evanescent over that surface, suffering a decay with increase of distance away from the surface. This follows from the requirement that there shall be no radiation away from the surface. It is to be noted that both E-modes and H- modes can propagate as surface waves. But the conditions for the support of E-modes are more readily met than for the H-modes. Hence usually one is interested in the E- modes. Another feature of surface waves existing on the interface between two homogeneous media, is that they do not experience any cut-off. This is because there is only one finite boundary condition to be satisfied.

There are three distinctive forms of surface waves namely,

1. The zenneck or in homogeneous plane wave supported by a flat surface.

2. The radial cylindrical wave also supported by a flat surface.

3. The Sommerfeld - Goubau wave supported by a transversely cylindrical surface. This is also called the axial cylindrical surface wave.

Now we have to distinguish another 'surface wave' encountered in the theory of ground wave propagation over a flat earth due to Sommerfeld. This is not exactly the same as the surface wave under consideration. The confusion is unfortunate and is partly because Sommerfeld was responsible for some of the early work on the true surface wave. In his discussion of the problem of radiation from a vertical dipole over a flat earth, Sommerfeld divided the ground wave into two parts:
1. The space wave and 2. The surface wave.

The surface wave part is respresented by one of the terms in the analysis of the total field and its particular feature is that it tends to predominate near the earth's surface. Both parts are required simultaneously to satisfy Maxwell's equations. Sommerfeld was able to identify the surface wave part of the solution with the true surface wave, because the expressions describing the two were similar in form. As yet there is a controversy over the interpretation of the various terms appearing in the analysis of the total field for the above radiation problem. Hence we rest by saying that, one has to distinguish between the true surface wave under consideration and the wave appearing as part of the solution of the radiation problem mentioned above. We give the field patterns for the three kinds of surface wave in Fig. 1.

Now a brief outline of the features of the three types of surface wave follows:

(a) THE ZENNECK WAVE

Refer to fig. 1a for the field distribution of this wave. This wave is a particular solution of Maxwell's equations and it can be described as "a wave that travels without change of pattern over a flat surface bounding two homogeneous media of different permittivity and conductivity"(2). This is an inhomogeneous plane wave, for the field decays (exponentially in this case) over the wave front with increase of distance from the surface.

In Fig. 1, a and 1 and 2 refer to the two media below and above the surface respectively.

The surface lies in the X-Z plane at $y = 0$, the media on both sides being homogeneous. For a wave propagating along the X- axis the three components satisfying the 2- dimensional wave equation are

H_z , Ex and Ey

Where H and E stand for the magnetic and electric fields respectively.

The general form of the field components is as follows:

$$\left.\begin{aligned} H_{z1} &= A\, e^{jwt}\, e^{u_1 y}\, e^{-\gamma x} \\ E_{x1} &= A \frac{u_1}{(\sigma_1 + j w \epsilon_1)}\, e^{jwt}\, e^{u_1 y}\, e^{-\gamma x} \\ E_{y1} &= A \frac{\gamma}{(\sigma_1 + jw \epsilon_1)}\, e^{jwt}\, e^{u_1 y}\, e^{-\gamma x} \end{aligned}\right] \quad Y \leq 0$$

These exist in medium 1.

σ_1 , ϵ_1 are parameters of medium 1. $\gamma = \alpha + j\beta$ is the longitudinal propagation constant.

$u_1 = a_1 + j b_1$ is the transverse propagation constant, where a_1 represents attenuation and b_1 stands for phase change for the wave travelling inward from the surface.

α = attenuation constant β = phase constant as usual.

In Medium 2

That is, above the surface, we have

$$\left.\begin{aligned} H_{z2} &= A\, e^{jwt}\; e^{-u_2 y}\; e^{-\gamma x} \\ E_{x2} &= A \frac{u_2}{(jwE_o)}\; e^{jwt}\; e^{u2y}\; e^{-\gamma x} \\ E_{y2} &= A \frac{\gamma}{(jw(E_o))}\; e^{jwt}\; e^{-u2y}\; e^{-\gamma x} \end{aligned}\right] \quad y \geq 0$$

Here $u_2 = a_2 - j b_2$

For the wave not only suffers an 'attenuation at the rate of a_2 with increasing distance from the surface but also a progressive phase change b_2 as it travels towards the surface. These characteristics are in accordance with the specification of a surface wave for which the power flow has two components one representing the main stream along the interface and subject to the usual attenuation α and phase change β while the other, usually a minor one, is directed into the surface to supply the losses. No radiation therefore occurs.

(b) THE RADIAL CYLINDRICAL WAVE

Unlike the plane wave the wave front is limited in the horizontal direction. Refer Fig. 1b , for field distribution. The components in this case are H_θ , E_y and E_x

Inside the Surface

$$\left.\begin{aligned}
H_{\theta_1} &= A\, e^{jwt}\, e^{u_1 y} H_1^{(2)}(-j\Upsilon r) \\
E_{r1} &= -A \frac{u_1}{(\sigma_1 + j w E_1)} e^{jwt} e^{u_1 y} H_1^{(2)}(-j\Upsilon r) \\
E_{y1} &= A \frac{j\Upsilon}{(\sigma_1 + j w E_1)} e^{jwt} e^{u_1 y} H_o^{(2)}(-j\Upsilon r)
\end{aligned}\right]\quad y \leq 0$$

Outside the Surface.

$$\left.\begin{aligned}
H_{\theta 2} &= A\, e^{jwt}\, e^{-u_2 y} H_1^{(2)}(-j\Upsilon r) \\
E_{r2} &= A \frac{u_2}{jw E_o} e^{jwt} e^{-u_2 y} H_1^{(2)}(-j\Upsilon r) \\
E_{y2} &= A \frac{\Upsilon}{E_o} e^{jwt} e^{-u_2 y} H_o^{(2)}(-j\Upsilon r)
\end{aligned}\right]\quad y \geq 0$$

One sees that the transverse variation is the same as for the plane wave case. But along the radial direction it decays according to a Henkel function which becomes at large distances as $\dfrac{e^{-\Upsilon r}}{\sqrt{r}}$. One would later see that this wave and the plane zenneck wave have much in common.

(c) AXIAL CYLINDRICAL WAVE

Sommer feld was the first to point out that a transversely cylindrical surface could support a surface wave. Goubau developed the idea in its application to a wave guide consisting of a metal wire having a dielectric - coated or corrugated surface. When the radius of the cylindrical surface becomes infinite, the Goubau wave becomes identical with the Zenneck wave. See Fig. 1.c for field distribution.

For the Components

(a) Inside the Surface.

$$H_{Q1} = A \frac{(\sigma_1 + j w E_1)}{j u_1} e^{jwt} e^{-\gamma x} J_1 (j u_1 r)$$

$$E_{x_1} = A e^{jwt} e^{-\gamma x} J_o (j u_1 r)$$

$$E_{r_1} = A \left(\frac{\gamma}{j u_1} \right) e^{-\gamma x} e^{jwt} J_1 (j u_1 r)$$

$r \lesssim S$

(b) Outside the Surface

$$\left.\begin{aligned} H_{Q2} &= A \frac{w E_o}{u_2} e^{jwt} e^{-\gamma x} H_1^{(1)} (ju_2 r) \\ E_{x2} &= A e^{jwt} e^{-\gamma x} H_o^{(1)} (ju_2 r) \\ E_{r2} &= A \left(\frac{\gamma}{ju_2}\right) e^{jwt} e^{-\gamma x} H_1^{(1)} (ju_2 r) \end{aligned}\right] r \geq S$$

1.2. SURFACE IMPEDANCE CONCEPT AND ITS SIGNIFICANCE

The surface impedance Z_s is defined as the ratio of the tangential components of the electric and magnetic field at the surface. In general Z_s is complex.

i.e. $Z_s = R_s + j X_s$ where R is the surface resistance and X_s is the surface reactance. For any medium constituting a surface of finite conductivity and a thickness exceeding the Skin depth, R_s and X_s cannot be separated physically. The existance of R_s implies the existance of X_s due to the penetration of the field. But in loss-free media it is possible to have X_s without R_s and this is approximated by a polythene coated smooth copper surface. If the radius of the surface is large compared to the skin depth one can add the reactance due to the dielectric to the reactance due to finite conductivity. As R_s increases the inclination of the wavefront toward the surface as measured from the normal, increases and this causes an increased phase velocity along the interface. In general an inductive surface slows down the wave whereas a capacitive surface speeds it up.

(a) SURFACE IMPEDANCE FOR ZENNECK AND RADIAL CYLINDRICAL WAVES.

$$Z_s = \frac{E_{x2}}{H_{z2}} \quad \text{at } y = 0 \text{ for Zenneck wave.}$$

$$Z_s = \frac{-E_{r2}}{H_{\theta 2}} \quad \text{at } y = 0 \text{ for the radial wave.}$$

On substituting the values of the field components for the two cases, we get the result that Z_s is identical for both the waves and this is given by

$$Z_s = \frac{1}{w E_o}(b_2 + j a_2) \text{ for both the waves.}$$

$$\therefore X_s = \frac{a_2}{w E_o} \qquad \text{and } R_s = \frac{b_2}{w E_o}$$

Thus a_2, representing the rate of decay with distance from the surface, is directly proportional to the reactance of the surface and b_2 the phase factor depends only on the resistance.

Now we may consider some methods of increasing the surface reactance.

(1) DIELECTRIC COATING OF THE METAL SURFACE

See Fig. 2. We perform the analysis assuming that a Zenneck wave is supported by the flat surface considered. Then we find that a standing wave exists in the dielectric.

On solving for the resistance and reactance at the interface between dielectric and air, we get

$$R_s = \frac{w \mu_o \Delta}{2}$$

$$X_s = w \mu_o \left[\frac{\Delta}{2} + \frac{E_2 - E_o}{E_2} \cdot 1 \right]$$

Where Δ = skin depth.

Assuming that a loss free dielectric has been coated, we know that the reactance contributed by it (owing to [redacted]. the storage of energy in the standing wave that exists in it) can be added to the reactance of the metal surface. Thus the additional component in the total reactance is

$$w \mu_o 1 \left[\frac{E_2 - E_o}{E_2} \right]$$

. If we choose

$E_2 > E_o$, one can increase the reactance above the original reactance of the metal surface. Also one can increase the reactance by increasing the thickness of the coating. But this usually raises the surface resistance, because of the added losses in the dielectric.

For a surface wave , We require that the decay factor in air surrounding the supporting surface be positive and finite . This can be satisfied for the Zenneck wave if we coat a metal surface with a dielectric.

Higher the surface reactance and higher the frequency, greater is the decay factor and the wave clings to the surface more and more closely.

(2) We can also corrugate the interface to increase the reactance. The pitch of the corrugations must be smaller than the wave length along the interface, to provide a uniformly distributed supplementary reactance.

(B) <u>SURFACE IMPEDANCE OF THE AXIAL CYLINDRICAL WAVE</u>

$$Z_s = \left. \frac{E_{x2}}{H_{\theta 2}} \right|_{r=s} = \frac{u_2}{w\, E_o} \left[\frac{H_o^{(1)}\ (j\ u_2\ S)}{H_1^{(1)}\ (j u_2\ s)} \right]$$

As $s \longrightarrow \infty$ one finds that $Z_s \longrightarrow \frac{j\ u_2}{w\ E_o}$

as for the Zenneck wave. Usually $|j\ u_2\ s| < 0.05$.
Hence we can apply the small argument approximation to the Hankel functions in Z_s to get

$$Z_s = R_s + j\ X_s = \frac{-j\ u_2^2 s}{w\ E_o} \log_e (0.89\ u_2\ s)$$

with $u_2 = a_2 - j\ b_2$ as before

On an examination of the behavior of R_s and X_s with varying values of the decay factor a_2 with the phase factor b_2 fixed, one arrives at the following conclusions.

For values of a_2 larger than that of the Smooth metal surface, we must have an enhanced inductive reactance at the surface and for a_2 smaller, we require a capacitive reactance at the surface. Also the net surface reactance can be positive, Zero or negative while still providing for finite positive values for both a_2 and b_2 necessary to support the wave This is in contrast with the requirement for the Zenneck wave where X_s must be always positive to keep a_2 positive.

At a large radial distance from the x- axis one can show by applying the large argument approximation to the Hankel functions that the wave impedance looking toward the wire for a given radial propagation constant u_2 is $\frac{j u_2}{w\ E_o}$, whatever is the radius of the wire. Hence for wires of large diameter the impedance at the surface is the same as that for a plane surface supporting a Zenneck wave with the same value of u_2.

For wires of small diameter the curvature of the equiphase surfaces near the wire has an important effect on the wave impedance. This curvature retards the phase of E_x while advancing that of H_θ so that the wave impedance may change from being inductive at a large distance from the wire to capacitive near the wire. Thus in general for a cylindrical surface wave, the wave impedance is inductive at large distances from the surface.

1.3. ATTENUATION AND PHASE VELOCITY ALONG THE SURFACE IN THE DIRECTION OF PROPAGATION.

The attenuation constant α i.e., the real part of the longitudinal propagation constant γ can be written as

$$\alpha \simeq \frac{V}{w} (a_2 b_2)$$

Where V = velocity of light in free space and a_2, b_2 are respectively the real and imaginary parts of u_2 the transverse propagation constant.

Now to reduce attenuation one must keep as much of the field as possible outside the guide so as to reduce the losses in the guide and accommodate it in the surrounding air. But for practical purposes we have to confine the field close to the surface. This means a_2 will be large and so this implies a larger attenuation α.

PHASE VELOCITY OF ZENNECK AND RADIAL SURFACE WAVES

These waves behave identically as far as phase velocity and attenuation are concerned.

$$V_p = \text{phase velocity} = \frac{w}{\beta}$$

This can be shown to be

$$V_p \simeq \frac{V}{\left[1 - \frac{7}{16}\frac{V^2}{w^2}\left(b_2^{\,2} - a_2^{\,2}\right)\right]}$$

This is for Smooth Metal surface for which $b_2^2 > a_2^2$. Hence $V_p > V$. But by a dielectric coating one can enhance the reactance thereby increasing a_2. With added dielectric a_2 increases faster than b_2 (Because the dielectric has only small losses). Hence we can make $a_2^2 > b_2^{\,2}$ after coating. Thus one can slow down the wave i.e. $V_p < V$. This is true for inductively loaded surfaces. Thus for such surface one can have V_p greater than, equal to or .....

less than V depending on the loading applied.

GOUBAU WAVE

For this wave supported by a smooth metal surface $\frac{a_2}{b_2} \approx 2.25$. Hence it is seen that $V_p < V$. Inductive loading further enhances the ratio a_2 / b_2 beyond 2.25. Hence the wave is slowed down further. With capacitive loading one can increase V_p to V or even beyond V.

If one measures V_p one gets an opportunity for measuring a_2 for a highly reactive surface for which $a_2 \gg b_2$.

1.4. THE COMPLEX BREWSTER ANGLE AND ITS RELATION TO THE ZENNECK SURFACE WAVE

Let us consider a homogeneous plane wave travelling in the direction of PQ and incident at an angle ψ on the flat surface of a lossless medium See Fig.3. The Magnetic field component of such a wave above the surface is given by

$$H_{z2} = Ae^{jwt} e^{-j\beta_o (x \sin \psi - y \cos \psi)}$$

$$\beta_o = w\sqrt{\mu_o \varepsilon_o} \qquad ..(1)$$

We know that there exists a certain critical angle of of incidence. For which there is no reflected wave, this angle of incidence being the Brewster angle. If ψ coincides with the Brewster angle then

$$\tan \psi = \sqrt{E_r} \qquad \ldots (2)$$

$$E_r = \frac{E_1}{E_o}$$ (Where E_1 is the permittivity of the lossless medium 1.)

Now ψ is real.

If the medium 1 has losses then E_r becomes complex. i.e., we have to replace E_r by $\underline{E_r - j E'}$ where $E' = \frac{\sigma_1}{w E_o}$

Since the analysis will be true for complex as well as real values of impedances of the two media, relation (2) giving the Brewster angle is still valid. The only modification is that the Brewster angle becomes complex. That is instead of ψ, we have $(\psi - j\chi)$.

Hence (2) becomes,

$$\tan (\psi - j\chi) \frac{Z_o}{Z_1} = \sqrt{E_r - j E'} \qquad ..(3)$$

If you replace ψ by $(\psi - j\chi)$ in (1) we get the physical meaning of the complex Brewster angle.

Since there is no reflected wave, we see that the field expression (1) in which ψ has been replaced by $(\psi - j\chi)$ must represent the total field above the surface.

This is

$$H_{z2} = A\, e^{jwt} \cdot e^{-\beta_o y} \left[\sin\psi \;\sinh \chi - j \cos\psi \cosh \chi \right]$$

$$\cdot\ \bar{e}^{\beta_o \chi} \left[\cos\psi \sinh \chi + j \sin\psi \cosh \chi \right] \qquad \ldots (4)$$

The field is seen to decay exponentially in amplitude and suffers a progressive advance in phase with increasing distance above the surface. Moreover the wave is attenuated along χ direction and there is a progressive lag in phase along the interface. These are the characteristics of a Zenneck wave supported by a flat surface.

Thus it is clear that one can establish the Zenneck wave by having a inhomogeneous plane wave incident at the Brewster angle.

1.5. EQUIPHASE AND EQUIAMPLITUDE SURFACES

For a homogeneous plane wave these two surfaces coincide. But there are waves for which these do not coincide. These are inhomogeneous waves. The Zenneck and the Sommerfeld waves belong to this category.

We now give the form of these surfaces for the Zenneck wave.

Referring to (4) above, one sees that the equiamplitude surfaces are planes given by :

$$\chi \cos\psi + Y \sin\psi = \text{constant} \qquad \ldots (5)$$

The equiphase surfaces are also planes given by

$$X \quad \sin\psi - Y \cos\psi = \text{Constant} \quad \ldots (6)$$

We note that they are orthogonal to each other, if you consider the medium above the surface to be loss-free . See Fig. 4.

The direction of propagation is normal to the wavefront and hence is inclined at an angle ψ with respect to the normal, where ψ is the real part of the complex Brewster angle.

The wave can be said to travel with out attenuation in a direction normal to the wavefront. The decrease in amplitude with increase in X can be interpretted as arising from the exponential variation of amplitude across a wavefront as the wave sinks through the plane $y = 0$. See Fig. 5.

Since there is no reflected wave above the surface one can regard the medium below the surface to provide a nortched termination to the incident inhomogeneous wave.

1.6. THE EVANESCENT STRUCTURE OF THE FIELD OVER THE WAVEFRONT OF SURFACE WAVES

Since surface waves propagate without radiation it follows that the field structure across the wavefront must be evanescent. We wish to illustrate this further, by proceeding in another way. Refer to Fig. 6.

Here the usual X-Y coordinate axes have been rotated by an angle ($\frac{\pi}{2} - \psi$) in the clockwise direction. The direction of power flow is inclined at an angle ψ to the normal .

In Fig. 7 we have shown an evanescent E_{o1} mode between two parallel perfectly conducting plates separated by d and of infinite extent, represented by the planes $X' = \pm \frac{d}{2}$.

In Fig. 6 , the field expression for the wave travelling in the direction normal to the wavefront i or equivalently parallel to the X' - axis is given by

$$H_{z2} = A\, e \quad e^{-\beta_o Y' \operatorname{Sin} hX} \quad e^{-j\beta_o X' \operatorname{Cos} h X} \qquad \ldots (7)$$

We see that the wave travels without attenuation in the X' direction, which is normal to the wave front.

For the evanescent E_{o1} mode in Fig. 7., the field is given by,

$$H_z = 2A\, e^{-\alpha_c Y'} e^{-\alpha_c Y'} \operatorname{Cos} \frac{\pi X'}{d} \longrightarrow \qquad \ldots (8)$$

$$= A e^{jwt} \left[e^{-\alpha_c Y'} e^{-j \frac{\pi X'}{d}} + e^{-\alpha_c Y'} e^{+j \frac{\pi X'}{d}} \right] \qquad \ldots (9)$$

with $\alpha_c^2 = -\beta_o^2 + \left(\frac{\pi}{d}\right)^2$

Thus the evanescent mode consists of a standing wave pattern formed by a pair of inhomogeneous plane waves travelling back and forth along the X' - axis, between the plates without having any net forward progress.

If you compare (7) with (9) we see that (7) is identical to the first term in (9) if

$$\frac{\pi}{d} = \beta_o \operatorname{Cosh} X$$

and $$\alpha_c = \beta_o \operatorname{Sinh} X$$

Thus the Zenneck wave is identical with the correspondingly directed component wave of the evanescent E_o, mode between two parallel perfectly conducting planes.

Since $d < \frac{\lambda_o}{2}$ necessarily, we see that the separation of the equiphase planes in a Zenneck wave must be less than $\frac{\lambda_o}{2}$. Any plane wave complying with these conditions must have an evanescent structure in the direction parallel to its equiphase planes.

From figure 6 it is also plain that the Zenneck wave progressively sinks into the plane Y = 0, as it advances along X.

1.7. TRANSMISSION LINE MODES AND THEIR RELATION TO SURFACE WAVES

In a parallel plate transmission line, we usually assume that the plates are perfect conductors and the separation is small compared to a wavelength. Then the anly propagating mode is the simple TEM wave. Once there are losses we must have a longitudinal componen t of the electric field, to supply the losses. Hence the mode is not strictly TEM.

Fig. 8 a shows the field lines for the lossy plates with small separation.

A simple analysis shows that as the distance of separation is progressively increased the TEM wave gradually approaches the Zenneck wave.

As we increase the separation, γ , the propagation constant of the wave approaches that of the Zenneck wave. The field components are stronger near the plates and they resemble the Zenneck waves closely. See Fig. 8 b . In the central overlap region there is some interaction between, the individual Zenneck wave type fields associated with the two plates and it is the magnitude and phase of the interaction that determines how far the γ of the system departs from that for a time Zenneck wave.

For a clear understanding of the above, consider how the TEM type of wave between plates of finite conductivity varies as the distance of separation is varied. There are three types of propagation which can be distinguished, as 2d varies in relation to the length $\frac{\Delta^3}{\lambda_o^2}$ and $\frac{\lambda_o^2}{\Delta}$ (Δ = skin depth).

(a) <u>Small separation:</u>

i.e. $2d \ll \frac{\Delta^3}{\lambda_o}$. Then the wave is essentially of the TEM pattern within the metal. Practically no energy travels in the narrow air gap between the plates.

(b) $\frac{\Delta^3}{\lambda_o} \ll 2d \lesssim \frac{\lambda_o^2}{\Delta}$ .. The usual TEM wave assumed in Transmission line analysis. Attenuation is generally small, phase velocity is slightly greater than that of Light, most of the energy travels in the region between the plates.

(c) $2d \gg \frac{\lambda_o^2}{\Delta}$

The wave tends to separate into two isolated Zenneck waves one associated with each plate. Most of the energy travels between the plates and close to the plates.

Actually these three regions are not sharply defined. One passes continuously from one type of propagation to another as the distance of separation is increased.

Similarly for parallel wire transmission lines, the TEM wave reduces to two axial cylindrical surface waves each of which is supported by one of the conductors, as the distance is increased. Thus the surface waves are the limiting cases to TEM waves on 2-conductor systems, as the distance of separation is increased indefinitely. This is another link between surface waves and other known types of propagating e.m. waves.

1.8. LAUNCHING OF SURFACE WAVES

The aim in launching is to convert a high percentage of energy in the source, which illuminates the launcher, into the surface wave. Usually in a design of the launcher, one arranges the geometry so that the higher order spurious modes get attenuated rapidly within a small distance from the launcher so as to have a reasonably pure surface wave at large distances from the launcher. The range of the surface wave depends on the launching arrangement. Workers have advanced several arrangements claiming large launching efficiences. (i.e., the ratio of surface wave to total power used). One of the earliest launching schemes employed the principle of the complex Brewster angle to launch the Zenneck wave.

Here an inhomogeneous plane wave is incident at an angle ψ, the real part of the complex Brewster angle, on a flat surface. This produces the Zenneck wave.

The idea of using a horn to launch the surface wave has been tried.

Since these early attempts many schemes for launching have been elaborated in the literature. Further, it is problem altogether different from what we propose to consider. Hence we do not consider the launching problem in any detail. Those who are interested in this aspect of the problem are referred to the ample literature existing on the subject.

CHAPTER II

REVIEW OF PAST WORK ON SHIELDED SURFACE WAVEGUIDES

As we know, surface waves are e.m. waves which travel along an interface between two different media without radiation. And the structures usually used to guide such waves are open or unbounded. Because of this feature there is only one finite boundary condition to be met and as a consequence these waves do not have any cut off.

What is a shielded surface waveguide ? . The term can be taken to mean that a confining screen has been placed above the guiding structure. As we know , the surface wave field extends theoretically to infinity in the transverse direction. Hence if we are interested in limiting the extent of the field we contemplate the utilization of a screen. Then it follows that we have to match the screen to the field so as not to cause unwanted reflection.

The theory of such a shielded guide was first advanced by H.M. Barlow (3) . He considers both the planar and cylindrical guides. The above mentioned matching problem is tackled by him in two ways.

The first approach is to examine the wave impedance, at the location of the screen, in the transverse direction. This turns out to be having a capacitive reactance and a small negative resistance. So the surface needed cannot be passive but has to be active, which means the introduction of a generator of power at the operating frequency. If the losses are negligible, a tolerable match can be obtained by having a dielectric coated metal surface. But with significant dissipation in the media, we require an auxiliary source of power along with the passive reactive surface.

The second method advanced by Barlow is interesting. It is by setting up what Barlow calls as the dual surface wave, that we seek to match the boundaries. This involves the setting up of a second surface wave on the inside of the screen, in addition to the main surface wave guided by the guiding structure. Under suitable circumstances, this wave would serve to limit the spread of the primary wave. Further we have simultaneously satisfied the boundary conditions at the screen, in the process of setting up of the secondary surface wave on its inside surface. Now, the question may arise that the presence of the two surface waves might lead to a standing wave pattern across the guide. But we know that there can be no reflection of a surface wave at its guiding surface.

This is true for both the waves at their respective guiding surfaces. Hence there can be no standing wave across the guide as would be the case for a normal waveguide mode. This fact enforces the conclusion that the said sombination of the two surface waves experiences no cut off. It is in order, to observe that the primary wave carries the major portion of the power, the secondary merely serving to limit the primary wave field.

A closer examination of the problem brings out the similarities between the dual surface wave and the TEm wave usually assumed to be present in such geometries. (i.e., planar and coaxial). Both have no cut off . Further the field patterns stemming from the dual wave solution exhibit features of the TEm wave field patterns. In fact this wave combines the features of the TEM wave and the surface wave. So Barlow terms it as the hybrid TEm-dual surface wave mode. Barlow establishes that this is the natural mode that exists in parallel wire or coaxial transmission lines. The usual TEM wave solution is obtaining in the ideal situation.

To arrive at the field components Barlov proceeds as follows. He assumes that the solution of the 2- dimensional wave equation is representable as a linear combination of two surface waves. (One guided by each of the two surfaces.) For the planar geometry, the components are E_x , E_y . and H_z considering the E mode solution. It is to be noted that Barlow has tacitly assumed inductively loaded surfaces. For only such surfaces can support an E-type surface wave mode. The transverse variation for the planar case is exponential where-as for the cylindrical case it is governed by Hankel functions.

Barlow's solution can be presented as follows: Namely,

$$H_z = \left[A_1 e^{-uy} + A_2 e^{uy} \right] e^{-\gamma X} e^{jwt}$$

Where γ = longitudinal propagation constant

$= \alpha + j \beta$

u = Transverse propagation constant

$= a - jb$

Thus the field is seen to be a superposition of two Z-enneck type surface waves each of which is guided by one of the surfaces.

Once H_z is thus assumed, we may get E_x and E_y readily from it.

Now Barlow makes a further stipulation that $E_x = 0$ at $y = m$ i.e. at some level in the transverse direction the longitudinal component of the electric field vanishes. This leads us to infer that the two surfaces waves are in anti-phase, resulting in the cancellation of the longitudinal component. The level at which such a cancellation occurs depends on the relative amplitudes of the two surface waves. If they are equal in amplitude, then this level is at $y = d/2$ i.e. the central plane of symmetry. For the case of unequal amplitudes this occurs at a level nearer to one of the surfaces. It is to be noted that for the case of equal amplitudes, the two waves also travel with the same phase velocity and they are anti-phase. Hence we get a complete cancellation of the component E_x at the central plane. For the case of unequal amplitudes, one of the waves travels faster than the other. Hence we cannot say that $E_x = 0$ at $y = m$. We can only say that E_x is minimum at this plane. This is because, we cannot get complete cancellation owing to the differing phase velocities of the two waves.

We can say the same thing in terms of the impedances of the surfaces. If the two plates have identical reactances we get the two amplitudes equal. Hence $E_x = 0$ at the central plane. This we can call the symmetric case. If the two reactances are different, the wave amplitudes are different and $E_x = 0$ at a level off the middle. This is the asymmetric case.

Barlow has taken $E_x = 0$ at $y = m$ for the asymmetric case. This is not strictly correct as we have seen above. However, one can take it that the minimum is close to zero, if the phase velocities do not differ appreciably. Thus the field patterns which we derive on the assumption that $E_x = 0$, at $y = m$, are approximate. However they throw considerable light on the actual situation obtaining in the geometry.

Barlow has obtained the equation of the electric field lines in the x- y plane. We show his results in a qualitative manner in Fig. 9. Referring to this figure, we see that the surface wave feature is present alongside the TEM wave features. One may note that as the surface reactance is reduced, we diminish the surface wave aspect and in the limit the field reduces to the TEM type, i.e. at zero surface reactance. The field pattern before us in Fig. 9, is a distorted version of the TEM wave. Thus we can get the hybrid wave by a progressive increase of the surface reactance which accentrates the surface wave feature in the total field . And to enhance the surface reactance, we can coat the metal surface with a film of dielectric or introduce transverse corrugations in the metal surface.

Barlow has extended this method of analysis to the coaxial case, by introducing suitable modifications warranted by a change of geometry from planar to cylindrical. His solution for this case is given by

$$E_x = e^{-\gamma x}\left[C_1 H_o^{(1)}(hr) + C_2 H_o^{(2)}(hr)\right]$$

and the components E_r and H_θ are easily got from Maxwell's equations. Once again we see the superposition of two surfaces waves in the solutiob. Barlow lets $E_x = 0$ at $r = r_m$ in the interspace between the two conductors.

Thus we see that the calculations for the coaxial case wa-re simple extensions of these in the planar case. The only difficulty is in the manipulation of the Hankel functions while calculating the various quantities of interst like power density, for example, Barlow has obtained expressions for the impedances at the different interfaces.

In an example which he considers in the same paper he has compared the two matching methods, and arrives at the conclusion that the setting up of the dual surface wave is the better alternative. In his second paper on the same topic(4) Barlow establishes that the hybrid mode is the natural mode whereas the TE^M mode is a close approximation to the truth. For, at high frequencies we have resistance as well as reactance exhibited by the supporting surfaces and so it is the hybrid mode that will be supported by the transmission line. By making the surfaces deliberately reactive, we can alter the distribution of power flow across the cross section and th-is, in suitable circumstances will

will reduce the attenuation significantly as compared to the TEM mode.

That the TEM mode can exist only in ideal conditions is clear because, we need a component of the field in the direction of propagation to supply for the losses in the supporting surfaces. Thus, the hybrid mode with a longitudinal electric field component is seen to fulfil the requirement better than the TEM wave. The two parts of this wave supported by the inner and outer conductor provide for a component of power toward the associated surface, thus over coming the surface losses. The degree of symmetry of this wave depends on the relative values of the two impedances. By deliberately enhancing the surface reactance, we can accentuate the surface wave feature and thus make for reduced attenuation.

Barlow analyses the stripline and coaxial transmission line where the surfaces have been coated with thin films of dielectric. He assumes the hybrid mode as the only mode in the geometry, the waveguide modes being cut off. Further he assumes that the longitudinal component E_x goes to zero at a certain level in the transverse plane.

With the solution as assumed above, the field components are readily evaluated and one can obtain the eigen value equation for the problem by matching the tangential field components at the various interfaces.

But the resulting equation is found to be complex. Hence Barlow assumes the transverse propagation constant. Then for a given frequency the given parameters of the different media and the given separation between the plates, Barlow obtains the necessary thicknesses of coating t_o and t_d, for a range of values of m (t_o corresponds to the thickness of coating on the lower plate and t_d to that on the upper plate) from the eigen value equation with the transverse attenuation as parameter. One can then obtain the longitudinal attenuation, for each transverse attenuation, as a function of m.

We show the results in Fig. 10. We see from this figure that for the symmetric case the attenuation is maximum. Thus to reduce attenuation one must interoduce asymmetry in the wave. Actually a minimum attenuation about half the maximum can be achieved when the zero level of the longitudinal component falls near one of the surfaces. Also there is a particular value for the thickness of coating corresponding to the minimum of attenuation.'

Thus it is established that an accentuation of the surface wave feature and an introduction of asymmetry in the wave make for reduced losses or equivalently reduced attenuation which by a proper choice of the thickness of coating can be made to be half that for the TEM wave.

These results are once again confirmed for the coaxial case as well.

Now these conclusions are to be valid at all frequencies. Hence, in particular at 50 c/s, we should be able to reap the advantage of reduced attenuation. But the usual dielectric coatings cannot result in large reactances at 50 c/s. Barlow has employed Ferrite coated conductors. Actually Ferrite rings are incorporated in one of the conductors. This is to circumvent the difficulty presented by the brittle nature of Ferrites. It has been experimentally verified by Barlow and Sen[7,8] that it results in minimum attenuation when only one of the conductors is coated with the optimum thickness of dielectric and not while both are coated. Thus one can use these ferrite loaded conductors for a transmission line and benefit from the low attenuation of the hybrid mode. This has been already experimentally demonstrated by Barlow and Sen. These results apply in the case of coaxial lines as well. Millington in his paper[9] has expressed some doubts about the method of solution adopted by Barlow. The calculations of Barlow show that for a given $t_o + t_d$ the attenuation along the guide for the symmetrical case $t_o = t_d$

is equal to or greater than that for the quasi TEM mode $t_o = t_d = 0$. But there is a marked decrease in attenuation as $t_o + t_d$ is distributed more and more unevenly among the two surfaces. But as $t_o + t_d$ is increased the attenuation rapidly increases. Millington carrying out his own calculations finds that the attenuation is independent of the ratio t_o/t_d at least for the case considered by Barlow. Further there is not much variation in attenuation as $t_o + t_d$ is varied. Millington also wonders how such large values of transverse attenuation can result from such thin films of dielectric ehich figure in Barlow's calculations. Further the conclusion obtained by Barlow that the attenuation is critical with the thickness of coating, is, to Millington, rather surprising. He believes that these anomalies arise due to the manner of solution adopted.

The direct method of attack would be for the assumed frequency, the constants of the media, the separation and the thicknesses of coating to and t_d one has to solve the eigen value equation so as to obtain the transverse attenuation. This is dasily done by numerical techniques. Then from the known transverse attenuation one is to obtain the longitudinal attenuation and from here the nature of the field is to be found out.

But Barlow's method seems to be the reverse of the above. Barlow assumes $E_x = 0$ at a given level $y = m$ and the transverse attenuation as well. Then he solves the equations for the thicknesses. Thus the calculations become critical with the conditions assumed. Even though the assumption to start with is not far from the truth, Millington believes that the computation however accurate would diverge from the true solution.

Thus Barlow's procedure a side from being laborious is basically unsound, according to Millington.

In a reply to Millington's observations, Barlow demonstrates the possibility of reducing the attenuation by enhancing the surface reactance, but he does not fully answer Millington's objections.

But theoretical procedures apart, the experimental work done by Barlow and Sen[6,7,8] seem to uphold Barlow's predictions. They have measured the Q of a coaxial cavity resonator as a function of the thickness of coating on either conductor. This demonstrated the existance of a particular value of coating thickness for which the loaded Q is a maximum. The attenuation due to the coaxial part of the resonator registers a minimum for the same value of thickness of coating. A measurement to determine the field distribution in the coaxial line supporting the hybrid mode clearly illustrates the vanishing of the longitudinal component E_x at a certain radius which depends on the loading of the two surfaces. Further in another experiment they have found that we can get the minimum of attenuation provided we load one or the other surface and not both simultaneously. The loading of course has to be optimum.

In a recent paper[10] Barlow has attempted to explain the behavior of the hybrid mode in physical terms. Calculations show that the hybrid mode contains 20% more power than the quasi TEM mode for the same current in the inner conductor. Also the hybrid wave produces much less loss in the outer conductor, than the TEM wave.

The calculations also show that the hybrid mode suffers less attenuation. This is brought about primarily by the redistribution of power across the cross section following the coating of the inner conductor.

Because of the highly reactive inner conductor the field at the outer conductor is weak. So the losses there are reduced. If the thickness is optimum, the added losses due to the dielectric are more than compensated and there is a reduction in attenuation.

Thus the experimental evidence is seen to support all that has been derived by Barlow. Especially the existence of a minimum of attenuation much lower than that for the quasi TEM mode, and a corresponding thickness of coating of dielectric, the vanishing of the longitudinal component E_x at a certain level in the transverse plane have been confirmed. This leaves us in no doubt as to the existence of the hybrid mode. The reduced attenuation is a physical reality and the method to achieve this, namely by accentuating the surface wave feature which is in turn achieved by enhancing the surface reactances, has been vindicated.

Another person who has contributed to the theory of Shielded surface waveguides is J. R. Wait (5). It is in order to observe that the present work is an extension of Wait's work.

In a later Chapter we will have occasion to trace the steps formulated by Wait in his paper, wherein he has attempted to obtain the various modes that would exist in a parallel plate waveguide with reactive guiding surfaces. Hence we rest by enumerating the conclusions obtained by Wait on his analysis of the problem.

Wait assumes a TM type of solution comprising of the field components E_x, E_y and H_z. Since we have a bounded structure, Wait proposes a series solution for the wave equation. This series is made up of an orthogonal set of modes. (The orthogonality property is assumed by Wait). That is he takes

$$H_z = \sum_n a_n \; f_n \; (y) \; e^{-i \lambda_n x}$$

Where a_n = amplitude of nth mode

$f_n(y)$ = Transverse distribution of the nth mode.

λ_n = longitudinal wave number.

$f_n(y)$ is of the form

$$f_n(y) = C_1 \; e^{-u_n y} \; + C_2 \; e^{u_n y}$$

where u_n is the transverse wave number of the nth mode. This form is obtained on substituting the series of solution in the wave equation and solving the resulting differential equation in $f_n(y)$.

Wait then gets the modal equation on imposing the boundary conditions at the two supporting surfaces on the field components. The modal equation is in variable u_n. Thus only those u_n which satisfy the modal equation can characterize the allowed modes in the structure. The real roots of the equation correspond to surface wave modes and the imaginary roots denote waveguide type modes. Complex roots are not considered if we restrict our attention to lossless guides.

Wait's conclusions are :

1. There exists two surface waves in both the symmetric and asymmetric geometries.

2. In the symmetric case, the field distribution are to be of even and odd types.

3. There are an infinite number of waveguide modes.

4. The zero order waveguide mode is expected to degenerate into the two surface waves.

In his consideration of the real and imaginary roots Wait has restricted himself to very special cases, perhaps for simplicity. Hence his results are not of much significance.

On the assumption of the orthogonality of the different modes, Wait has solved the excitation problem. For the arrangement where a voltage has been set up across a slot at $y = y_s$ in the transverse plane, he has derived the expression for the amplitude of the nth mode and from there the power carried by the nth mode. The assumption of orthogonality really simplifies these conclusions. He obtains the ratio of surface power to the power in the nth wave guide mode as

$$\frac{P}{P_n} = e^{kpd} \quad \text{where}$$

P = Surface wave power.

P_n = Power in nth waveguide mode.

p   parameter relating free space impedance to surface impedance.

k   free space wave number

d   separation

Thus if kd is large one can have more power in the surface wave.

Then he proceeds to consider the lossy case. For this he merely extends the analysis for the loss-free case. He replaces parameters p and q (q is the parameter similar to p which is defined above) by $p(1-i\delta_o)$ and $q(1 - i \delta d)$ respectively (to take into account the surface losses) in all his previous results.

Hence Wait concludes that there are not only surface wave modes present but also the enclosed waveguide modes, once we resort to shielding. So we have to take extra care to prevent contamination of the surface wave mode by the other waveguide modes.

In the present work, to be presented in the next Chapter, the author has tried to verify Wait's conclusions and especially his speculations. Accordingly a detailed analysis of the model equation with the help of the computer was made and this has confirmed some of Wait's results and has contradicted him on some points. A case in point is that the zero order mode does not degenerate into two surface waves but into only one surface wave. There is a surface wave which exists down to zero frequency and the other one turns into the waveguide mode of order zero as the frequency is varied. Further evidence on hand negates the type of orthogonality mentioned by Wait. This renders the excitation problem much more difficult. Lastely it has been possible to link Barlow's results to Wait's results. The wave which exists down to zero frequency is of the Barlow type and thus in Wait's language Barlow's wave is got when there is a large interaction between the two component surface waves. For no interaction we have two surface waves disappeared and becomes a waveguide mode. By a suitable choice of frequency and parameters of the system, we can make this waveguide mode to be cut off. So we are left with only one surface wave which propagates and this is exactly the hybrid mode discussed by Barlow. It has also been shown that proceeding similar to Barlow's approach we can arrive at the existance of a second surface wave. This is shown to be the type for which $\frac{d\,E_x}{dy} = 0$ at $y = m$, Thus this work has bridged the two approaches.

CHAPTER III

INVESTIGATION OF THE MODES IN A PARALLEL PLATE GUIDE WITH REACTIVE GUIDING SURFACES

3.1. The problem to be solved is as described below.

Given a system of cartesian coordinates, x, y, z, we consider two parallel plates aligned as shown in Fig. 11. The lower of the two plates is defined by the plane $y = -d/2$ and the upper one is defined by the plane $y = +d/2$. Further the lower plate has a surface impedance Z_o while the upper one has a surface impedance Z_d. The distance of separation measured along the Y - axis is d. The plates extend to infinity in both x and z directions.

We are interested in finding the $\bar{E}$ and $\bar{H}$ configurations that can be supported by the two plates.

In the solution we are trying to seek TM and TE type of field configurations which would meet the boundary conditions.

The TM type of field solution is made up of the three components E_x E_y and H_z while the TE type solution has E_z, H_x and H_y for its components.

Even though the surface impedances Z_o, z_d may in general , be complex we restrict our attention to the situation where they are both purely reactive.

The dielectric which separates the two plates is assumed to be air and hence can be taken to be practically loss-free. Thus the whole structure is loss free.

We express the surface reactances in terms of the free space wave impedance η (i.e. 120π Ω). We mean,

$$Z_o = j\,\eta\,p \quad ; \quad Z_d = j\,\eta\,q$$

$$= j\left(\frac{k}{WE}\right)p \qquad = j\left(\frac{k}{W\,E}\right)q$$

Here k = free space wave number = $\frac{2\pi}{\lambda_o}$

λ_o = free space wave length .

$W = 2\pi f$

f = frequency.

E = free space permittivity.

p and q are positive real parameters which signify the comparative magnitude of the impedances Z_o , Z_d with reference to the free space impedance. Further we are not concerned how the reactances are produced. We assume them to be already there.

Now we outline the procedure to arrive at the TM and TE type of solutions. Here we follow Wait's approach as found in his paper[5].

<u>The Solution</u>:

We begin with the two Maxwell's equation of interest.

$$\left.\begin{aligned} \nabla \times \bar{H} &= jW\varepsilon\bar{E} \\ \nabla \times \bar{E} &= -jW\mu\bar{H} \end{aligned}\right] \qquad \ldots(1)$$

$\bar{E}$, $\bar{H}$ refer to the electric and magnetic field intensities. E, μ are the parameters of free space, with their usual significance.

j = imaginary Number.

For the TM type solution one starts with H_z, the z- component of the magentic field, which is uniform in the z- direction.

For the TE type solution, we start with E_z the z- component of the electric field, once again uniform in the z- direction.

The uniformity of E_z and H_z in the z- direction is a natural consequence of the supposed infinite extent of the plates in the z- direction. Even though this is not practicable one may choose the z- dimension to be several times the y- dimension.

We treat the two solutions parallelly, for this would bring out the duality between the two solutions.

Wait utilizes a universal principle valid for all bounded structures in the course of his analysis. That is the field solution pertaining to such a structure can be expressed as a discrete spectrum of modes that are mutually orthogonal. Or in other words one is obtaining a series solution for Maxwell's equations. The orthogonality property makes for the independent carriage of power by the different modes and this in turn helps one to expressthe total power carried as the sum of the powers in the individual modes.

The form of the series solution proposed by Wait is,

$$H_z = \sum a_n f_n (y) e^{-j \lambda_n x} \qquad \ldots (2.)$$

Here a_n = amplitude coefficient of the nth mode.

$f_n(y)$ = Transverse variation of the nth mode.

λ_n = longitudinal wave number of the nth mode.

The separability of the wave equation in a cartesian coordinate system has been taken advantage of while writing H_z as a product of two functions one of which depends on y only and the other depends on X only. There is no z- variation because H_z

is uniform in that direction. This is because of the essentially 2- dimensional nature of the problem.

From H_z, we easily get E_x and E_y as

$$E_x = \frac{1}{jw\,E} \frac{\partial H_z}{\partial y}$$

$$E_y = - \frac{1}{jw\,E} \frac{\partial H_z}{\partial x} \qquad \ldots (3)$$

So to completely determine the field all one has to do is to find the following, i.e.,

a_n, f_n (y) and λ_n

As one would see later, $f_n(y)$ is to be determined by inserting the form of H_z into the 2-dimensional wave equation written for the region

$$-\frac{d}{2} \leq y \leq \frac{d}{2}$$

When we impose the boundary conditions on the fields, we get the modal equation i.e., the equation to be satisfied by any field configuration that is to exist in the space between the two plates. Since the boundaries are defined by the planes $y = \pm d/2$, one naturally expects these boundary conditions to determine the transverse variation of the fi-elds. So these constraints would determine the transverse wave number U_n. From a knowledge of u_n , λ_n can be easily derived.

To determine a_n one has to know the nature of the excitation. Given this it is easy to calculate the individual amplitudes if the modes are orthogonal. Wait assumes, that they would be orthogonal. But during the present investigation, evidence has been obtained which points to the contrary i.e., the modes are not orthogonal in the sense of Wait. Still the power carried by the individual modes might be independent. For there are different kinds of orthogonality relations. It still remains to be established that precisely is the kind of orthogonality that prevails in the system considered. We discuss this more fully in a latter section.

Out object is to determine the form of $f_n(y)$

Now H_z has to satisfy the wave equation in the region $(-d/2 \, < y < d/2)$

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2 \right] H_z = 0 \qquad \ldots (4)$$

Inserting the form of H_z as given in (2) we have the following differential equation for $f_n(y)$ i.e.,

$$f''_x (y) + (k - \lambda_n^2) f_n(y) = 0 \qquad \ldots (5)$$

Write $u_n = (\lambda_n^2 - k^2)^{1/2}$

Then $f''_n (y) - u_n^2 \; f_n(y) = 0 \qquad \ldots \qquad \ldots . (6)$

is the differential equation.

From this $f_n(y)$ can be easily deduced to be

$$f_n(y) = C_1 e^{u_n y} + C_2 e^{-u_n y} \quad \ldots(7)$$

Derivation of the Model Equation.

For this we impose the boundary conditions,

$$\left.\frac{E_x}{H_z}\right|_{y=-\frac{d}{2}} = Z_o \qquad \left.\frac{E_x}{H_z}\right|_{y=+\frac{d}{2}} = -Z_d \quad \ldots(8)$$

i.e. the wave impedance looking in the direction of the plate in question must equal to the surface impedance of that plate, at that plate. This is what we term as matching the field to the plate.

Now,

$$\frac{E_x}{H_z} = \frac{1}{jw\,E}\;\frac{f'_n(y)}{f_n(y)} \quad \ldots(9)$$

Where $f'_n(y) = \dfrac{d\,(f_n(y))}{dy}$

Now $f_n(y) = C_1 e^{u_n y} + C_2 e^{-u_n y}$

and $f'_n(y) = u_n \left[C_1 e^{u_n y} - C_2 e^{-u_n y} \right]$

The first boundary condition in (8) can be shown to result in the equation,

$$C_1 \left[u_n - j w E Z_o \right] - C_2 e^{u_n d} \left[u_n + j wE Z_o \right] = 0 \qquad \ldots (10)$$

The second condition in (8) similarly yields the equation

$$C_1 \left[u_n + j w E Z \right] - C_2 e^{-u_n d} \left[u_n - j w E Z_d \right] = 0 \qquad \ldots (11)$$

For non trival values of C_1 and C_2 the determinant of the system of homogeneous equations in C_1 and C_2 (10,11) must vanish.

This yields,

$$(u_n - j w E Z_o) (u_n - j w E Z_d) e^{-u_n d} = (u_n + j w EZ_o) (u_n + j w E Z_d) e^{u_n d}$$

which in turn reduces to the identity,

$$\frac{u_n - j w E Z_o}{u_n + j w E Z_o} \quad \frac{u_n - j w E_d Z}{u_n + j w E Z_d} e^{-2 u_n d} = 1 \qquad \ldots . (12)$$

(12) is called the modal equation for the system. Each value of u_n that would be a solution of (12) characterizes an allowed mode.

We will consider real and imaginary roots of (12). Complex roots are not of interest because the structure we take up is to be loss-free.

As mentioned earlier the real roots signify surface wave modes and the imaginary roots give the waveguide type of modes. As the root changes from real to imaginary we go over from a surface wave mode into a waveguide mode.

The total field is thus composed of a combination of surface wave and waveguide modes. By waveguide modes we mean those modes which are common to parallel plate waveguides with perfectly conducting guide surfaces. We note that for Zenneck type of surface wave modes to be supported the surface has to be inductively reactive (we are having TM type of modes in our minds). Thus when the walls are perfectly conducting we cannot have Zenneck type surface waves. Then, only the waveguide modes exist. As we enhance the reactance of the surface we start having the surface wave modes as well .

In (12) let us put

$$Z_o = j\,\eta\,p = j\,(k/WE)\,p$$

$$Z_d = j\,\eta\,q = j\,(k/WE)\,q$$

where the symbols their pre-assigned meanings.

(12) becomes on this substitution,

$$\left[\frac{u - kp}{u + kp}\right]\left[\frac{u - kq}{u + kq}\right] = e^{-2ud} \qquad \ldots.(13)$$

(13) is the characteristic equation for the loss-less case we are investigating.

We now show that for the TE type solution, the model equation is identical in form with that corresponding to the TM case.

<u>TE Type Solution:</u>

We start with E_z here and proceed exactly as we have done for the TM case.

So we let

$$E_z = \sum a_n f_n(y) e^{-i\lambda_n x} \qquad \ldots(14)$$

with symbols having their usual meanings.

$$H_x = \frac{j}{w\mu} \frac{\partial E_z}{\partial x} \qquad \ldots(15)$$

The corresponding boundary conditions are

$$\left.\begin{aligned} \frac{E_z}{H_x} &= -Z_o \quad \text{at} \quad y = -d/2 \\ \frac{E_z}{H_x} &= Z_d \quad \text{at} \quad y = +d/e \end{aligned}\right] \qquad \ldots(16)$$

An examination shows that $f_n(y)$ is still the same as in the TM case.

An application of the conditions (16) on the field components is found to yield the model equation for this case as

$$\left[\frac{u_n - \dfrac{j\,w\,\mu}{Z_o}}{u_n + \dfrac{j\,w\,\mu}{Z_o}}\right]\left[\frac{u_n \;\; \dfrac{j\,w\,\mu}{Z_d}}{u_n + \dfrac{j\,w\,\mu}{Z_d}}\right] e^{-2\,u_n d} = 1 \qquad \ldots(17)$$

(17) is the dual of (12) got for the TM case.

However one expects TE modes to be supported by capacitive surfaces. So defining

$$Z_o = -\,j\,\eta\,p = -\,j\,(w\,\mu/k)\,p$$

$$Z_d = -\,j\,\eta\,q = -\,j\,(w\,\mu/k)\,q$$

and inserting these in (17), we obtain

$$\frac{\left[u_n - \dfrac{k}{p}\right]}{\left[u_n + \dfrac{k}{p}\right]} \qquad \frac{\left[u_n - \dfrac{k}{q}\right]}{\left[u_n + \dfrac{k}{q}\right]} = e^{-2\,u_n d} \qquad \ldots(18)$$

Letting $\frac{1}{p} = p'$ and $\frac{1}{q} = q'$, we write (18) as,

$$\frac{u_n - k\,p'}{u_n + kp'} \qquad \frac{u_n - k\,q'}{u_n - kq'} = e^{-2\,u_n\,d} \qquad \ldots(19)$$

which is identical to (13) except for a change of p to p' and q to q'. But the behaviour of the roots would be the same.

Hence the roots, real and imaginary, of (13) for the TM case with inductive boundaries, is identical with the roots real and imaginary of (19) which relates to the TE case with capacitive boundaries.

This is as expected. For the TM case is the dual of the TE case and the inductive boundary is the dual of the capacitive boundary. So from a consideration of duality TE type solutions are also possible.

Now it is interesting to consider whether it is possible for TM type surface wave and wave guide type modes to be supported by capacitive surfaces.

For this, replace Z_o and Z_d by $-j\eta p$ and $-j\eta q$ respectively in (13) we get

$$\left[\frac{u_n - kp}{u_n + kp}\right]\left[\frac{u_n - kq}{u_n - kq}\right] = e^{2u_n d} \quad ..(20)$$

which shows that we do not have any real roots. But we do have imaginary roots.

So one concludes that TM type surface wave modes cannot be supported by capacitive surfaces. But we can have TM type waveguide modes supported by the same surfaces.

Hence we can infer that TE type waveguide modes would be supported by inductive surfaces though not the TE surface waves.

So one might sum up the types of modes one expects in the geometry considered.

We consider the case of inductive boundaries. For the case of capacitive boundaries the same conclusions hold except that we have to read TE for TM and vice versa.

Hence for an inductive boundary, we have,

i. TM type surface waves (real roots of equation (13))

ii. TM type waveguide modes (imaginary roots of equation (13)).

iii. TE type waveguide modes (imaginary roots of equation (20)).

3.2. BEHAVIOR OF THE REAL ROOTS OF THE MODEL EQUATION (FOR THE TM CASE)

We recall that the model equation of interest is given by (13).

$$\frac{u_n - kp}{u_n + kp} \; \frac{u_n - kq}{u_n + kq} = e^{-u_n d}$$

We normalize the equation and write it as

$$\left[\frac{X - P}{X + P}\right] \left[\frac{X - Q}{X + Q}\right] = e^{-2X} \quad \ldots (2*)$$

Where $X = u_n d$
$P = kpd$
$Q = kqd$

We plot equation (21) graphically in Fig. 12. The real roots are got by the intersection of the exponential e^{-2x} with the curve $y = \frac{X - P}{X+P} \quad \frac{X - Q}{X + Q}$

As we vary P and Q we can have different shapes for this curve and the intersections can be modified.

We restrict our attention to positive values of X only, for reasons which are apparent. In general we have two intersections, that stand for two surface wave modes.

The roots can be expressed generally as

$$X_1 = P - \delta_1 \quad , X_2 = Q + \delta_2$$

where δ's are variable quantities depending on the values of P and Q.

For large values of P and Q $\delta_1 \approx \delta_2 \approx 0$ i.e., the roots become nearly

$X_1 \approx P$; $X \approx Q$. This is the case when P and Q both exceed 5. We always take $Q > P$.

For values of P, $2 < P < 5$, one sees that the δ's are small quantities. This is there is only a small departure from the previous case.

For P still smaller, the δ's become fairly large say of the order of 0.25 to 0.5, and the departure is still more marked. From the figure it is clear that the root X_1 moves toward zero more rapidly than

X_2 does as the values of P and Q are decreased. In fact at a certain value of P , for a given Q, X_1 becomes zero, while X_2 has still some positive value. It turns out that X_2 is different from zero as long as Q is non zero and becomes zero when Q equals zero.

This is what can be called as the threshold phenomenon. That is, there is a critical value of P , for a given Q , only above which there are two positive real roots. Below this value , there is only one positive root (that is X_2). In physical terms, above this critical value of P (for a fixed Q) we have two surface waves propagating and below this one of them disappears.

Wait has called the case, wherein P and Q are quite large (say, in particular > 5) as the case of 'no interaction'. Here the two surface waves do not interact with each other, or in other words they cling to their respective surfaces, and propagate with different phase velocities, one of them travelling faster than the other.

For P and Q , moderately large, we have the case of 'small interaction', and the third case, with P sufficiently small, becomes the case of ' Large interaction'. And it is only in this case of large interaction that we witness the threshold phenomenon i.e. wherein one of the surface wave disappears.

Whenever there are two surface waves propagating in the geometry, the one attached to the lower plate travels faster than the other wave. Hence this wave attains the velocity of light first. At this point it disappears, while the the other wave still travels slower than light.

We show in Fig. (13) the occurrence of the threshold graphically. For a particular choice of P and Q the curve $Y = \frac{X - P}{X + P} \quad \frac{X - Q}{X + Q}$ takes the form (1), in the figure and now it has two intersections with the exponential. For a second choice of P and Q the curve becomes as in (2) in the same figure and now there is only one intersection with the exponential. The number of intersections depend on the critical slope of the curve at $(X = 0 , y = 1)$.

Thus there should be a gradual transition from the one extreme to the other as this slope at (0,1) is varied. From the figure it is clear that if the slope of the curve at the said point is smaller than that of the exponential at the same point we have two intersections and if that slope is greater we have only one. So the case when the two slopes are equal, should define the transition point between the two extremes. At this point one of the roots just goes to zero. This is readily seen to be the threshold phenomenon.

The slope of the curve is determined by P and Q . Hence the threshold condition is to be in terms of P and Q . Thus one may get the threshold condition by equating the slopes of the curve and the exponential at (0,1) . i.e.,

$$\frac{d}{dx}\left[\frac{X-P}{X+P}\;\frac{X-Q}{X+Q}\right]_{X=0} = \left[\frac{d}{dx}\,e^{-2x}\right]_{X\neq 0}$$

which on evaluation reduces to

$$\frac{1}{P} + \frac{1}{Q} = 1 \qquad \ldots.(22)$$

(22) is the threshold equation.

If $Q = P$,

We find that the threshold value becomes,

$\underline{P_{th} = 2}$ (where P_{th} = threshold value of P)

For $Q \gg P$

$$P_{Th} \longrightarrow 1$$

Thus the threshold value of P range from 1 to 2 as Q takes different values with $Q > P$ always .

Hence we see that under all circumstances we have one root, i.e., $X_2 = Q + \Delta(Q)$, which vanishes only when $Q = 0$. $\Delta(Q)$ is the increment depending on the value of Q. $\Delta(Q)$ varies inversely with Q. Hence we have one surface wave which propagates down to zero frequency and its phase velocity is always smaller than that of light. And above the threshold value of P, which depends on the value of Q, there are two surface waves.

3.3. BEHAVIOUR OF REAL ROOTS FOR THE SYMMETRIC CASE i.e. WHEN P = Q

The characteris-tic equation becomes

$$\frac{X - P}{X + P} = \pm e^{-X} \qquad \ldots (23)$$

In Fig. 14, we show the intersection of the curves $Y = \dfrac{X - P}{X + P}$ with $Y = \pm e^{-X}$. In general, two intersections are possible with the exponentials $y = e^{-X}$ and $y = -e^{-X}$ of the two $y = +e^{-X}$ always has an intersection with the curve, whereas the other curve $(-e^{-X})$ may or may not produce an intersection. Thus, at one point, depending on the value of P, there will be no intersection with the $-e^{-X}$ curve. So once again we have the threshold in evidence.

That the curve will have an intersection with the negative exponential depends on the slope of the curve at (0,-1) in relation to that of the $-e^{-X}$ curve at the same point. If it is smaller compared to that of the exponential then we have an intersection and if it is larger there can be no intersection. Thus when the two slopes are equal at (0,-1), one of the roots just goes to zero, as in the previous case.

As the slope is determined by P, the condition of equal slopes determines the threshold value of P

i.e.

$$\frac{d}{dx}\left(-e^{-X}\right)\Bigg|_{X=0} = \frac{d}{dx}\,\frac{X-P}{X+P}\Bigg|_{X=0}$$

This yields

$P_{Th} = 2$ and this confirms our previous calculations.

In Fig. (15) we demonstrate the threshold condition graphically.

We see when P = 2, the curve is tangential to $-e^{-X}$ at X = 0 and this defines the transition point when one of the roots just vanishes.

a. <u>Behaviour of Real Roots as P is Changed</u>

For large P, say in excess of 10, the roots are identical i.e. we have the repeated roots

$$X = P, \quad P.$$

This means that there are two non-interacting surface waves, each travelling with the same phase velocity. They cling to their respective surfaces.

(b) For moderately large, P, well below 10 but above 2, there are two distinct roots repeesented by

$$X_1 = P - \Delta(P)$$

$$X_2 = P + \Delta(P)$$

where $\Delta(P)$ is a quantity which varies inversely with P. $\Delta(P)$ signifies the amount of interaction. For large P it is nearly zero. Hence it describes the case of no interaction. For moderately large P it is significant, though small. This is the case of small interaction.

(c) $P < 2$

For this case only one surface wave exists and it is characterized by the root

$X_2 = P + \Delta(P)$. This is the case of large interaction.

Thus all the phenomena observed in the unsymmetric case are seen to occur for the symmetric case as well. It should be so, for it is only a special case of the more general unsymmetric case.

Using the computer, the roots of the characteristic equations were calculated for both the symmetric and unsymmetric cases. For the symmetric case the variation of the roots with P was determined. For the unsymmetric case the variation of the roots with P, was determined while Q/P was kept fixed. Further, the calculations were repeated for different values of Q/P.

We display the results of the calculations for the symmetric case in Fig. 16. This figure confirms all our predictions.

We see three distinct regions named, A, B, C in the figure. A is the region of large interaction B is that of small interaction while C is that of no interaction. Wait has confined his attention to regions B and C while we have extended the calculations to the region of large interaction. Here in region A we see the threshold phenomenon occurring, or in other words one of the roots vanishes. It is worthwhile to point out that Barlow has been considering region A when one surface wave has already disappeared.

The threshold value of P i.e. $P_{Th} = 2$ has been confirmed by calculation. This is in accordance with the threshold condition got for the symmetric case earlier. Thus our ideas about threshold have been verified.

Next we show in Figs. 17 and 18 the results of calculations for the unsymmetric case.

Here the variation of the two distinct roots with P (while Q/P is fixed) is shown. All our conclusions from the threshold equation (22) are confirmed by the calculations. This is evident from the figures. The threshold values as got from (22) for different Q/P values agree with the calculated values. Thus the threshold equation is established. The existence of three distinct regions of interaction is once again confirmed.

3.4. INVESTIGATION OF THE IMAGINARY ROOTS OF THE CHARACTERISTIC EQUATION FOR THE SYMMETRIC CASE

For this one makes the substitution

$u = j\,k\,C'$ (where C is a positive real parameter)

in the characteristic equation.

For the symmetric case, on making this

substitution for u, one gets

$$\left[\frac{C - j p}{C + j p}\right]^2 = e^{2 j k c d} \qquad ..(24)$$

Wait replaces 1 on the right hand side by $e^{-2\pi im}$ with m taking the values 0,1,2,3...... It is found that this assumption is by no means necessary. In fact it does not yield anything new. So we keep 1 as it is

$$\therefore \frac{C - j p}{C + jp} = \pm \; e^{j k cd} \qquad ..(25)$$

This is actually two equations, one resulting when we consider the positive sign and the other arising when we take the negative sign. We consider in particular

$$\frac{C - jp}{C + jp} = e^{jkcd}$$

On some manipulation this can be written as

$$\frac{jp}{C} = \frac{1 - e^{jkcd}}{1 + e^{jkcd}}$$

$$= - j \tan \frac{kcd}{2}$$

$$\text{or } p/C = - \tan \frac{kcd}{2} \qquad ....(26)$$

Similarly the second equation can be reduced to

$$p/C = \text{Cot} \frac{kcd}{2} \qquad .....(27)$$

(26) and (27) yield the imaginary roots of the modal equation for the symmetric case and these roots in turn represent TM type waveguide modes that exist in the geometry.

We can have TE type waveguide modes as well. Proceeding on similarlines we obtain the equations which define the imaginary roots of the equation(20) These represent the TE type waveguide modes that can exist in the geometry.

These are

$$\frac{p'}{C} = \tan\left(\frac{kcd}{2}\right) \qquad \ldots(28)$$

$$\frac{p'}{C} = \cot\left(\frac{kcd}{2}\right) \qquad \ldots(29)$$

where $p' = 1/p$.

Now we turn our attention to the TM case. (26) and (27) yield each one of them an infinite number of waveguide modes. We represent the equation (26) graphically in Fig. 19.

We normalize the equations (26) and (27) before representing them on the graph.

That is we have

$$-K_1 / X = \tan X \quad \text{and} \quad K_1/X = \cot X$$

for the two equations.

Here $K_1 = \frac{kpd}{2} = \frac{P}{2}$

and $X = \frac{kcd}{2}$

We confine our attention to the intersections on the right of the origin. This is because of symmetry considerations. These intersections on the left, do not give anything new.

In Fig. 19, the positive roots lie in the interval

$$\left[\frac{(2n-1)\pi}{2} , \quad n\pi \right] \qquad n= 1,2....etc.$$

For large P , the roots are dose to $\frac{(2n - 1)\pi}{2}$ and for small P they are close to $n\pi$.

$n = 1$ gives the first order mode and $n = 2$ and so on give the higher order modes.

Now in Fig. (20) , we show equation (27).

Now also there is an infinite number of waveguide modes. But these are distinct from the former set in that they have different cut off frequencies and they transverse variation is different.

We notice from the above figure that we may have an intersection in the interval $(0,\pi/2)$ or we we may not have. This depends solely on the parameter K_1 . In particular one notices that if K_1 is > 1 the curve K_1/X lies above the cotangent curve so there is no intersection possible . For $K_1 < 1$ the curve K_1/X lies below the cotangent curve and yields an inersection in the range $(0, \pi/2)$.

If one remembers the inequality $\tan\theta \geq \theta$ (the equality obtaining for small θ) one would be able to comprehend the above argument. From the above inequality it follows that $\cot\theta \lesssim 1/\theta$ (equality holding for small θ) . Thus K_1/θ with $K_1 > 1$ is definitely greater than $\cot\theta$ for all θ in the range $(0, \pi/2)$. Hence the corresponding K_1/θ curve lies entirely above the cotangent curve yielding no intersection. Then K_1/θ with $K_1 \leqslant 1$ would be below $\cot\theta$ for a certain range of θ $(0, \theta_1)$, where $\theta_1 > 0$ is the point of intersection between the cotangent curve and the hyperbola K_1/θ . θ_1 being $< \pi/2$ at the same time. Thus with $K_1 > 1$ we do not have an intersection and with $K_1 < 1$ we have an intersection in the interval $(0, \pi/2)$. So $K_1 = 1$ should be the transition point where the intersection just goes to zero.

As K_1 is gradually increased from 0 to 1 the intersection point gradually moves from $\pi/2$ to zero and beyond $K_1 = 1$ there is no intersection. Thus we have a threshold phenomenon.

We call that mode represented by the intersection in the range $(0, \pi/2)$ as the zero order mode. This zero order mode exists if $K_1 < 1$ and it does not exist for $K_1 > 1$.

$K_1 = 1$ means $P = 2$. Thus the threshold condition for the zero order waveguide mode is also given by $P = 2$. We recall that $P \doteq 2$ was the point at which one of the surface wave disappeared.

Now if we say that the zero order mode becomes the surface wave at $P = 2$ we would be able to explain the results. As the reactance of the surface is gradually increased from zero, the zero order mode which is the dominant mode gradually tends toward the surface wave. The cut off frequency of this waveguide mode gradually approaches zero as P tends to 2. Then the mode can not propagate as a waveguide mode; so it turns into a surface wave mode. With the computer calculations were made for the imaginary root in the interval $(0, \pi/2)$ and the variation of this root with P was observed. The plot appears in Fig. 21. This figure confirms the above observations.

We see from this curve that $X \to 0$ as the reactance is increased. $X \to 0$ means $k\,C \to 0$

Now $C = 1$ specifies the cut off condition.

So $k_c \longrightarrow 0$ as $X \to 0$

or $2\pi / \lambda_c \longrightarrow 0$ as $X \to 0$ or $\lambda_c \longrightarrow \infty$

This shows that as the reactance is increased, λ_c the cut - off wavelength of the zero order mode tends to infinity. But a TM type waveguide mode existing in a bounded structure cannot have this property. So it becomes a surface wave at the threshold.

It has also been verified that there is only one intersection in the interval

$$0 < |X| < \pi/2$$

For TE type modes which may also exist we have to consider the equations

$$\frac{K_1}{X} = \text{Tan } X \qquad \ldots.(30)$$

$$-\frac{K_1}{X} = \text{Cot } X \qquad \ldots.(31)$$

as derived earlier.

Equation (30) is depicted in Fig. 22 once again we consider only positive roots. These are seen to be infinite in number. They lie in the interval

($n\pi$, $(2n+1)\pi/2$) $n = 0,1,2....$etc. Thus we have a zero order mode here. This is found to exist for all values of P including zero. So there is no threshold phenomenon. This is as it should be. For we have no real roots for the corresponding characteristic equation, implying the absence of TE surface waves. Hence the waveguide mode cannot become a surface wave or there is no threshold.

Equation (31) is shown in Fig. (23) . Here the positive roots lie in the interval ($(2n-1)\pi/2$, $n\pi$) . They are once again infinite in number. But the modes represented by these, are different from the modes got from equation (30) in that their cut off, frequencies differ and their transverse variation is different.

Thus one finds that one has

1. Two TM type surface waves.
2. Two types of TM waveguide modes.
3. Two types of TE waveguide modes.

The waveguide modes being infinitely many.

3.5. THE CHARACTERISTICS OF THE MODES EXISTING IN THE SYMMETRIC ARRANGEMENT (i.e. P = Q)

We now consider in more detail the transverse variation $f_n(y)$ for the symmetric case.

We know

$$f_n(y) = C_1 e^{u_n y} + C_2 e^{-u_n y}$$

we recall equation (10) i.e.

$$C_1(u_n - jw E Z_o) - C_2 e^{u_n d} (u_n + jw E Z_o) = 0$$

This is got on the insertion of the Boundary conditions as specified in (8)

$$\text{Now } C_2 = \left[\frac{u_n - j w E Z_o}{u_n + j w E Z_o} \right] e^{-u_n d} C_1$$

$$= R_o e^{-u_n d} C_1$$

$$\text{Where } R_o = \frac{u_n - j w E Z_o}{u_n + j w E Z_o}$$

For the reactive surfaces we are considering

$$R_o = \frac{u_n + kp}{u_n - k p} \qquad \ldots (32)$$

Now $f_n(y)$ can be written as,

$$f_n(y) = C_1 \left[e^{u_n y} + (R_o e^{-u_n d}) e^{-u_n y} \right]$$

represents the odd type of surface wave which sin h (uy) for its transverse variation. We remember that this real root vanished for a particular value of P.

The other real root corresponding to the equation

$$\frac{u_n - kp}{u_n + kp} = e^{-u_n d}$$

represents the even type of surface wave having Cosh (uy) for its transverse variation. This wave travels slower than the odd wave and does not vanish until P = 0 .

Considering imaginary roots we put $u_n = jk\,C$ in (33).

This becomes

$$R_o \; e^{-jkcd} = \pm 1$$

or

$$\frac{C - jp}{C + jp} = \pm e^{j\,k\,cd}$$

which coincides with the equations previously obtained while we were dealing with the imaginary roots.

Thus each imaginary root of the characteristic equation represents either an even or an odd mode, the even mode having a cosine distribution while the odd one has a sine distribution. Further it is easily seen that the odd waveguide mode of zero order, Where C_2 has been replaced by $R_o e^{-u_n d} C_1$

Say that we wish to have an odd or even distribution in the transverse plane. That means $f_n(y)$ should be either an odd or even function of y

Clearly this is possible whenever

$$R_o e^{-u_n d} = \pm 1 \qquad \text{....(33)}$$

From (32) we can write (33) as

$$\frac{u_n + kp}{u_n + kp} \cdot e^{-u_n d} = \pm 1$$

or

$$\frac{u_n - kp}{u_n + kp} = \pm e^{-u_n d}$$

This is nothing but the characteristic equation for the symmetric case. That is every u_n which satisfies the characteristic equation would represent a mode of either even or odd symmetry.

Thus for real roots we have even and odd types of surface waves and the imaginary roots give even and odd type of waveguide modes.

In particular the real root which satisfies the equation.

$$\frac{u_n - kp}{u_n + kp} = - e^{-u_n d}$$

represents the odd type of surface wave which sin h (uy) for its transverse variation. We remember that this real root vanished for a particular value of P.

The other real root corresponding to the equation

$$\frac{u_n - kp}{u_n + kp} = e^{-u_n d}$$

represents the even type of surface wave having Cosh (uy) for its transverse variation. This wave travels slower than the odd wave and does not vanish until P = 0 .

Considering imaginary roots we put $u_n = jk\,C$ in (33).

This becomes

$$R_o \; e^{-jkcd} = \pm 1$$

or

$$\frac{C - jp}{C + jp} = \pm e^{j\,k\,cd}$$

which coincides with the equations previously obtained while we were dealing with the imaginary roots.

Thus each imaginary root of the characteristic equation represents either an even or an odd mode, the even mode having a cosine distribution while the odd one has a sine distribution. Further it is easily seen that the odd waveguide mode of zero order, obtained while considering the equation

$$\frac{C - jp}{C + jp} = - e^{j\,kcd}$$

experiences the threshold phenomenon, and goes over into the odd type of surface wave at the threshold value of P (i.e. $P = 2$). It is thus in keeping with our expectation that the odd type of waveguide mode should become the odd type of surface wave and vice versa.

For the TE case similarly we have odd and even types of waveguide modes, exactly akin to the types we encounter for the TM case.

A physical reasoning would demand that for the symmetric case such even and odd types of modes exist. In fact Wait has predicted such a thing in his paper. It is satisfying to note that the actual conclusions support the physical reasoning.

3.6. IMAGINARY ROOTS FOR THE UNSYMMETRIC CASE

We start with the relations,

$$\frac{C - jp}{C + jp} \cdot \frac{C - jq}{C + jq} = e^{2\,jkcd} \qquad \ldots (33)$$

This was the result of putting $u = jkc$ in the appropriate characteristic equation (13).

Taking square roots on both sides of (33)

we have,

$$\sqrt{\frac{(C-jp)(C-jq)}{(C+jp)(C+jq)}} = \pm e^{jkcd} \qquad \ldots(34)$$

Taking first the positive sign before the exponential, we have, after effecting some algebraic manipulations:

$$\frac{\sqrt{(C^2 - pq) + j\,(p+q)\,C} - \sqrt{(C^2 - pq) - j\,(p+q)C}}{\sqrt{(C^2-pq) + j\,(p+q)C} + \sqrt{(C^2-pq) - j\,(p+q)C}} = -j \tan\frac{kcd}{2} \qquad \ldots\ldots(35)$$

writing $a = C^2 - pq$

$b = C(p+q)$

We write (35) as

$$\frac{\sqrt{a + jb} - \sqrt{a - jb}}{\sqrt{a + jb} + \sqrt{a - jb}} = -j \tan\frac{kcd}{2} \qquad \ldots(36)$$

Simplifying LHS,

we have $$\frac{a - \sqrt{a^2 + b^2}}{b} = \tan\frac{kcd}{2} \qquad \ldots(37)$$

Repeating the above for the negative sign before the exponential, we have for the second equation,

$$\frac{a - \sqrt{a^2 + b^2}}{b} = - \text{Cot} \quad \frac{kcd}{2} \qquad \ldots (38)$$

Thus the imaginary roots are given by the following set of equations,

$$\frac{(C^2 - pq) - \sqrt{(C^2 + p^2)(C^2 + q^2)}}{C(p + q)} \left.\begin{array}{ll} & = \tan \dfrac{kcd}{2} \\ \text{or} & = - \text{Cot } kcd/2 \end{array}\right] \qquad \ldots (39)$$

To verify the accuracy of the above calculation we put $p = q$ in (39) . Then we get,

$$\left.\begin{array}{l} \dfrac{p}{c} = - \tan \dfrac{kcd}{2} \\ \quad \text{or } \cot \dfrac{kcd}{2} \end{array}\right]$$

This is the set of equations we had got for the symmetric case earlier when we were finding the imaginary roots for that case . Thus we have reason to be satisfied that, our present approach to the unsymmetric case is valid.

We wish to normalize the set (39) . Thus inserting in (39) $X = kcd/2$, and simplifying, We have for the L.H.S. of (39)

$$\text{L.H.S.} = \frac{(4X^2 - PQ) - \sqrt{(4X^2+P^2)}\sqrt{(4X^2+Q^2)}}{2X\ (P+Q)}$$

Where $P = kpd$

$Q = k\,qd$

Finally inserting $Q = \beta P$ and $\alpha = P/2$,

we have the final form of (39) as,

$$\left.\begin{aligned} &\frac{(X^2 - \beta\alpha^2) - \sqrt{(X^2+\alpha^2)(X^2+\beta^2\alpha^2)}}{\alpha(\beta+1)\,X} \\ &\qquad = \tan kcd/2 \\ \text{or} &\qquad = -\operatorname{Cot}\frac{kcd}{2} \end{aligned}\right] \quad \ldots(40)$$

Now we wish to investigate the LHS of (40).

For small values of X, we can easily show that the LHS reduces to a form given below :

$$\text{L.H.S.} = \frac{-2\beta\alpha}{(\beta+1)X} \quad \ldots(41)$$

And for large values of X, one can show that,

$$\text{L.H.S.} = \frac{-\alpha(\beta+1)}{2X} \quad \ldots(42)$$

Thus in both cases it behaves like a rectangular hyperbola.

In (41) if we put $\beta = 1$, we get

$$\text{L.H.S.} = -\alpha / X$$

Therefore the equation (40), reduces to

$$+\frac{\dot{\phi}}{X} = -\tan\left(\frac{kcd}{2}\right)$$

$$\text{or} \quad = \text{Cot}\left(\frac{kcd}{2}\right)$$

for small values of X. This is seen to coincide with the equations already got for the symmetric case. Thus, this checks our calculations.

We proceed to establish that this curve does not cross the X- axis for any real X.

We are examining the function

$$\frac{(X^2 - \beta\,\alpha^2) - \sqrt{(X^2 + \alpha^2)\,(X^2 + \beta^2\alpha^2)}}{\alpha(\beta + 1)\,X}$$

This is seen to be an odd function of X and it is symmetric with respect to the origin.

We are looking for a zero on the X- axis. Hence equate numerator of the above expression to zero.

i.e. $X^2 - \beta\,\alpha^2 = \sqrt{(X^2 + \alpha^2)\,(X^2 + \beta^2\alpha^2)}$

or $(X^2 - \beta\,\alpha^2)^2 = (X^2 + \alpha^2)\,(X^2 + \beta^2\alpha^2)$

which finally reduces to the condition,

$(\beta + 1)^2 = 0$

If we are to have a zero on the X-axis, then the above condition is to be true.

But $\beta \geq 1$ always . Hence $(\beta +1)^2$ is always positive. Hence we can never satisfy the above condition, which means that the curve defined by the said function can never cross the X- axis.

Now we can trace the LHS of (40) as a curve. This is shown in Fig. (24).

In figs, 25 a and 25 b we represent the set of equations (40).

From Fig. 25 a, it is seen that we have an infinite number of intersections, corresponding to an infinite number of wave guide modes. We restrict ourselves to the positive roots only. It is also seen that, the curve may or may not intersect with the cotangent curve in the interval $(0, \pi/2)$. Thus a zero order mode may or may not exist.

In Fig. 25 b , we see that once again there are an infinite number of roots giving an infinite number of waveguide modes. There is no intersection possible in $(0, \pi/2)$, for this case. So we have only the first and higher order modes and there is no mode of zero order. The modes in this case have cut off frequencies differing from those of the modes considered in Fig. 25 a. Also they have a different transverse variation.

From Fig. (25 a) it is clear that the threshold phenomenon is occurring. To find the condition for threshold, we use the small argument approximation for the L.H.S. of (40).

The small argument approximation desired is given by (41)

i.e. $\text{L.H.S.} = \dfrac{-2\beta\alpha}{X(\beta + 1)}$ for small X

where $\alpha = P/2$ and $\beta = Q/P$

Now this is a rectangular hyperbola of the form $-K/X$ and we have to examine its intersection with the $-\text{Cot}\,X$ curve. From our previous experience in this matter we are in a position to say that we have an intersection when the coefficient of $(1/X)$, is smaller than unity and there is no intersection when it is larger, than unity. When the coefficient of $1/X$, equals unity the root will just vanish. Thus the threshold condition can be got by equating the coefficient of $1/X$ to unity. We have discussed this matter in an elaborate way in the section devoted to the behavior of the imaginary roots in the symmetric case. Refer to Section IV.

So for the threshold condition, we must have,

$$\frac{2\beta\alpha}{(\beta+1)} = 1 \quad \text{or} \quad \frac{\beta P}{(\beta+1)} = 1$$

This reduces to $1/P + 1/Q = 1$.

This agrees with the threshold condition we got for the real roots. That is when the real root just goes to zero, we have the emergence of the imaginary root. i.e., the surface wave disappearing to yield its place for the zero order waveguide mode.

These observations agree with what we encountered in the symmetric case. With the computer actual calculations were performed to check the threshold behavior. The calculations were done with Q/P as parameter. In each case the results confirmed the predicted behaviour of the roots and further they established the threshold condition to be valid.

We show the results in Fig. (26). We have plotted the variation of the root, in the interval $(0,\pi/2)$ with P when Q/P is kept as a fixed parameter. This root corresponds to the first intersection of the curves given in the equation below :

$$\frac{(X^2 - \beta\alpha^2) - \sqrt{(X^2 + \alpha^2)(X^2 + \beta^2\alpha^2)}}{\alpha(\beta \mp 1)X} = -C\alpha X.$$

or in other words, it describes the zero order waveguide mode. We know that there is a particular P, called the threshold P, (for each value of Q/P) only below which we have the zero order mode. The curves in Fig. (26) show this threshold behavior. The threshold values obtained coincide with what we had already predicted.

Thus it is established that the surface wave which disappears at the threshold becomes the zero order wave guide mode.

We can do the same calculations for the TE case as well. The procedure is identical and we will get the equations as ,

$$\left.\begin{aligned} \frac{(X^2 - \beta\alpha^2) - \sqrt{(X^2 + \alpha^2)(X^2 + \beta^2\alpha^2)}}{\alpha(\beta + 1)X} &= -\tan X \\ \text{or} \qquad &= \operatorname{Cot} X \end{aligned}\right] \quad \ldots (43)$$

Thus there will result two types of TE waveguide modes, disti nguished by their transverse variation and different cut off frequencies. As we know already, there will not be any TE surface curve.

3.7. THE DISTINCTION BETWEEN THE TWO SURFACE WAVES

We know that there are two surface waves which may exist in the geometry. What is the distinction between them? This question we proceed to answer.

For the symmetric case, we got the transverse variations of the two waves as, $f_1(y) = \text{Cosh}(u y)$ and $f_2(y) = \text{Sinh}(uy)$. For the first wave, E_{x_I} has a y - distribtion given Sinh (uy) . So one easily sees that

$$E_{X_I} = 0 \quad \text{at} \quad y = 0 .$$

For the second wave $E_{X_{II}}$ has a y - distribution given by cosh (uy) or

$$\frac{d\, E_{X_{II}}}{dy} = 0 \qquad \text{at } y = 0 .$$

Because of symmetry, we expect the electric field to be zero or maximum at the central plane. Thus there are the two posibilities. The two surface waves which we have got, represent the two cases.

This leads us to expect the following field patterns for the two surface waves. See Fig. (27).

In one case the field lines oppose each other at the control plane, whereas for the other case, they aid each other at the control plane.

Now Barlow has discussed the TE^M dual surface wave (Ref. 3) with $(A_1 e^{-uy} + A_2 e^{uy})$ for its transverse distribution. He imposes the condition that $E_x = 0$ at $y = m$ for this wave.

(He is discussing the unsymmetric case).

From our experience with the symmetric case, we expect that there might be a second surface wave for the unsymmetric case which is characterized by

$$\frac{d\,E_x}{dy} = 0 \quad \text{at} \quad y = m$$

The only departure from the symmetric case, is that the plane of symmetry of the fields is shifted to a plane different from the central plane. It is to be noted that the field lines are still symmetrical about a certain plane $y = m$, though $m \neq 0$.

Trying this idea we expect two waves for the unsymmetric case also.

One of them characterized by

$$E_x = 0 \quad \text{at } y = m,$$

while the other is given by

$$\frac{d\,E_x}{dy} = 0 \quad \text{at } y = m$$

Let us see what these conditions mean.

We know $f(y) = (A_1 e^{-uy} + A_2 e^{uy})$ in Barlow's notation.

Now $E_x \propto (A_1 e^{-uy} - A_2 e^{uy})$

$\therefore$ $E_x = 0$ at $y = m$, gives

$$A_1 / A_2 = e^{2um} \qquad \ldots I)$$

$$\frac{d E_x}{dy} \propto \left(A_1 e^{-uy} + A_2 e^{uy} \right)$$

$$\therefore \frac{d E_x}{dy} = 0 \quad \text{at } y = m, \quad \text{gives,}$$

$$\frac{A_1}{A_2} = - e^{2\,um} \qquad \ldots\ldots II$$

Hence the corresponding H_z components for the two waves would be ,

$$\left.\begin{aligned} H_{z1} &= A_2 \left[e^{uy} + e^{-u(y-2m)} \right] e^{-\gamma x} \\ H_{z2} &= A_2 \left[e^{uy} - e^{-u(y-2m)} \right] e^{-\gamma x} \end{aligned}\right] \qquad \ldots.(44)$$

Putting in (44) m = 0 , we get the symmetric case as it should be . Now we know that Wait has assumed for f(y),

$$f(y) = \left[e^{uy} + R_o e^{-uy} \right]$$

In Barlow's notation it would be

$$f(y) \propto \left[e^{uy} + \frac{A_1}{A_2} e^{-uy} \right]$$

or we see R_o corresponds to A_1/A_2 .

$$R_o = \frac{u + kp}{u - kp}$$ for reactive surfaces. We find that

R_o is positive real for one surface wave and it is negative real for the other surface wave. This is because one root is smaller than kp while the other is larger than kp. Thus R_o is in complete correspondence with A_1 / A_2 which is positive for one wave and negative for the other.

Thus Barlow has been considering one of the two waves discussed by Wait.

We recall that one of the two surface waves experiences the threshold phenomenon and vanishes, at the threshold value of P_o . The other stays in the geometry down to zero frequency or in other words till P is reduced to zero. For this wave R_o is positive.

Now for the wave considered by Barlow A_1 / A_2 is positive. Since A_1 / A_2 is nothing but Ro , the wave is of the same character as we have encountered before. Thus it is seen that Barlow has been dealing with the case where one of the two surface waves has disappeared and only one remains. In Wait's language, he has been dealing with the region of large interaction.

So we have established a link between the investigations of Wait and Barlow.

Now we have an idea of the field patterns for the unsymmetric case mainly drawing from our previous association with the symmetric case. We display them in Fig. (28). We can get the equation to the field lines in the x-y plane. For this we have to solve the differential equation.

$$\frac{R_e\ (E_x)}{R_e\ (E_y)} = \frac{dx}{dy} \qquad \ldots (45)$$

We know the H_z components for the two waves. (See (44)). From these we can easily get the electric field components. Then we substitute them in 45 and integrate the resulting simple differential equation. In our evaluation we assure $\underline{\gamma = j\beta}$. The results of this integration are,

$$\left[e^{uy} + e^{-u(y-2m)} \right] \sin \beta x = \text{const} \quad \ldots (46)$$

$$\left[e^{uy} - e^{-u(y-2m)} \right] \sin \beta x = \text{const.} \quad \ldots (47)$$

(46) gives the pattern for the first wave and (47) describes the pattern for the second wave (The first wave has $E_x = 0$ at $y = m$ and the second has $\frac{d\,Ex}{dy} = 0$ at $y = m$). (46) and (47) can be rewritten as,

$$\text{Cosh} \left[u\,(y-m) \right] \sin \beta x = \text{Const.} \quad \ldots (48)$$

$$\text{Sinh} \left[u\,(y-m) \right] \sin \beta x = \text{Const.} \quad \ldots\ldots (49)$$

which clearly show symmetry about the plane $y = m$. Setting $m = 0$ in these equation, we get the field lines for the symmetric case, as

$$\text{Cosh}\,(uy) \sin \beta x = \text{Const.} \quad \ldots. (50)$$

$$\text{Sinh}\,(uy) \sin \beta x = \text{Const.} \quad \ldots. (51)$$

These four equations were plotted for different values of the constant appearing on the RHS of the equations and the graphs confirm our expectations. See Fig. 29, 30, 31, 32.

Barlow calls his wave as the hybrid TEM dual surface wave. Now we are in a position to justify this name. We show previously that he was discussing the surface wave which remain down to zero frequency. See Fig. 29. Where the field pattern is given for this wave.

One sees that it is the distorted version of the TEM wave. In the first chapter we had pointed out how the TEM wave gets distorted as we increase the separation between the two parallel plates. As the distance goes on increasing the field pattern incorporates more and more of the surface wave features. We can produce a similar effect by enhancing the surface reactance. By doing this, the wave attains a state which can be described as the superposition of two surface waves each associated with one of the surfaces. As Barlow terms it, we introduce more and more of the surface wave as we increase the reactance. Since it combines the features of the TEM and the surface waves it is called the hybrid TEM dual surface wave mode. The term 'dual surface wave' describes the presence of two surface waves each linked to one of the surfaces. Also it covers the fact that the field solution is described by the superposition of two surface waves (according to our original assumption).

When we see Fig. 30, we see the field pattern resembling a waveguide mode. That is, no lines cross the plane of symmetry $y = m$. Thus we can appreciate the fact that it is this surface wave which becomes a waveguide mode of order zero.

In Fig. 29, the plane $y = m$ is located near the lower surface. Now at $y = m$, $Ex = 0$. This yields $Z_s = 0$ at $y = m$. And as we move from the plane $y = m$, Z_s gradually increases. We have assumed the lower surface to be of a smaller reactance than the upper surface. Since Z_s increases with distance aways from the plane $y = m$, it is logical that this plane is located nearer to the lower surface than to the upper one.

Similarly in Fig. 30, we see $y = m$ is located near the upper surface. Now E_x is maximum at $y = m$ for this case. or Z_s is maximum at $y = m$, and it falls off as we proceed away from the plane. Since once again the upper surface has a larger reactance than the lower one, it is to be expected that the plane $y = m$ lies nearer to the upper surface than to the lower one.

Fig. 31 and 32 display the field patterns for the symmetric case. There is no difference exhibited except that the plane of symmetry is transferred from $y = m$ to $y = 0$.

Thus we note with satisfaction that all our expectations have been substantiated by the calculations for both the symmetric and unsymmetric cases.

3.8. It was decided to explore another case wherein one of the Surfaces has a capacitive reactance while the other has an inductive reactance.

3.8. AN INVESTIGATION OF THE MODES WHEN ONE OF THE SURFACES IS CAPACITIVELY REACTIVE AND THE OTHER IS INDUCTIVELY REACTIVE.

So we take for this case

$$Z_d = -j \left(\frac{k}{w\epsilon} \right) q$$

$$Z_o = +j \left(\frac{k}{w\epsilon} \right) p$$

We consider the TM type modes first. The characteristic equation reduces to,

$$\frac{X - P}{X + P} \quad \frac{X + Q}{X - Q} = e^{-2X} \qquad \ldots (52)$$

Where $X = ud$

$P = kpd$

$Q = kqd$

We show the plot of this equation in Fig. 33.

We see from the Figure, there is only one real intersection and this is only for positive X. For large values of P $X \simeq P$ is the only root. This is so for values of $P \geqslant 5$.

For moderately large P, we have the root given by,

$$X = P - \Delta(P)$$

Where $\Delta(P)$ is a quantity which inversely varies with P.

So we have one TM type surface wave.

A look at the figure tells us that there is a threshold value for P for a given Q , only above which we can have the real root. Analogous to the previous arguments we can get the threshold condition, by equating the slopes of the curve and the exponential at the point (0,1).

Doing this we get the condition as

$$\frac{1}{P} - \frac{1}{Q} = 1 \qquad \ldots(53)$$

If $Q \longrightarrow P$, then $P_{Th} \longrightarrow 0$ m which means that if $Q \gtrsim P$, then the wave does not experience the threshold phenomenon, as long as $P > 0$. For very large Q, $P_{Th} \longrightarrow 1$.
So the threshold values vary from 0 to 1

If we set $Q = P$ in the characteristic equation we have

$$1 = e^{-2X} \qquad \ldots.(54)$$

The only real solution of which is X = 0 . But this does not give any meaningful result. So if the surfaces are of equal and opposite reactances, no surface wave exists.

Imaginary Roots:

For this, we have to take the equation,

$$\sqrt{\frac{C - jp}{C + jp} \cdot \frac{C + jq}{C - jq}} = \pm e^{jkcd} \quad \ldots (55)$$

Proceeding exactly as we did in Section VI, we have the final normalized set of equations as

$$\left.\begin{aligned} \frac{(X^2 + \beta\alpha^2) - \sqrt{(X^2 + \alpha^2)(X^2 + \beta^2\alpha^2)}}{\alpha X(1 - \beta)} &= \tan X \\ \text{or} \quad &= -\operatorname{Cot} X \end{aligned}\right] \quad \ldots (56)$$

Where

$\beta = Q / P$

$\alpha = P/2$

We see that by putting $\beta = -\beta$ in equation (40) we get equation (56) . The same applies for the threshold condition. We replace Q by - Q (The same thing as β being replaced by $-\beta$) , in equation (22) to get the threshold equation for the present case as given in (53).

Now we examine the LHS of (56) for large and small X . The results are

For large X

$$\text{L.H.S.} = \frac{\alpha(\beta - 1)}{2X} \qquad \ldots (57)$$

For small X

$$\text{L.H.S.} = \frac{X(\beta - 1)}{2\alpha\beta} \qquad \ldots (58)$$

Thus it behaves like a straight line $y = mx$, for small X and behaves like a rectangular hyperbola for large X. The function as given in L H. S. of (56) is odd and thus has symmetry about the origin. Further it is easily shown that it does not cross the X- axis except at the origin.

If we let $\beta = 1$ in L.H.S. of (56) we get it as the ratio $0/0$. Evaluating this indeterminate quantity by L' Hospital's rule, we see that this tends to zero as β tends to 1. Hence the curve reduces to the X- axis for $\beta = 1$.

Now the curve approximates a straight line near the origin and rectangular hyperbola near infinity, we expect it to attain a maximum in between. This maximum is more and more pronounced as you increase β beyond 1. We show the curve in Fig. (34).

If Figs. 35(a), 35(b), we show the plot of equation (56).

Seeing Fig. 35 b, we conclude that there is no possibility of threshold, whereas from Fig. 35a, we can see that the zero order mode may or may not exist. This can be readily seen if we recall the inequality, tan X > X except for small X. We know that for small X, the curve behaves like

$$\frac{X \quad (\beta - 1)}{\beta P} \quad \text{i.e. a straight line.}$$

Now if the slope $\frac{\beta - 1}{\beta P}$ of the line is greater than or equal to 1 then there is a possibility of intersection. If, on the other hand, it is less than 1, the tangent curve would lie always above the curve and no intersection would result. Thus the slope equated to 1 would give the threshold condition.

$$\therefore \quad \frac{\beta - 1}{\beta P} = 1 \quad \text{is the condition.}$$

If $P > P_{Th}$, $\frac{\beta - 1}{\beta P}$ is less than 1.

So no intersection is possible. or the zero order mode cease to exist if $P > P_{Th}$ and we have the surface wave. On the other hand if $P < P_{Th}$,

$$\frac{\beta - 1}{\beta P} > 1 \quad \text{and we have an intersection}$$

or in other words, we have the zero order mode existing and there is no surface wave. This is as it should be.

At $P = P_{Th}$ the surface wave mode becomes the Zero order wave guide mode.

The existance of TE modes .

Considering the characteristic equation for the TE case, (17) we put

$$Z_d = -j \left(\frac{\mu w}{k}\right) q$$

$$Z_o = j \left(\frac{w \mu}{k}\right) p$$

This results in the equation

$$\frac{X - Q'}{X + Q'} \qquad \frac{X + P'}{X - P'} = e^{-2X} \qquad \ldots (59)$$

Where $Q' = k'q'd$ $\quad P' = k\,p'd$ and

or $q' = 1/q$ and $p' = 1/p$

Now $Q' < P'$ (for $q > p$) . So (59) is identical with equation (52) which was obtained for the TM case. So we have the same conclusions as in the TM case.

Thus for each TM wave we have a corresponding TE wave.

Hence in all there exist,

1. A TM surface wave resembling a waveguide mode and which goes into the TM zero order waveguide mode at the threshold.

2. A TE surface wave resembling a waveguide mode and going into the TE zero order waveguide mode at the threshold.

3. Two sets of TM waveguide modes.

4. Two sets of TE waveguide modes.

The Case when $P = Q$

Here no surface wave exists but the waveguide modes are given by the roots of the equation

$$1 = \pm e^{ikcd}$$

which reduces to

$$\left.\begin{aligned} 0 &= \tan\frac{kcd}{2} \\ 0 &= \cot\frac{kcd}{2} \end{aligned}\right] \quad \cdot\cdot (60)$$

Thus the roots are nothing but the zeroes of tangent and cotangent functions. So we have two types of TM waveguide modes and similarly two types of TE waveguide modes.

It can be seen that the cut off wavelengths are given by

$$\lambda_c = \frac{d}{2n} \quad \text{for the 1st set} \quad n=0,1,2,\ldots\ldots$$

$$\lambda_c = \frac{2d}{2n+1} \quad \text{for the IInd set} \quad n = 0,1,2.\ldots.$$

For the TM Type Modes

A little reflection shows that the TE modes have identical cut off wavelengths. That is, there is degeneracy; we have for each TM mode a TE mode having the same cut-off frequency.

For the case $P \neq Q$ calculations were made, using the computer, of the real and imaginary roots. The behavior of these roots as P was varied, for a fixed Q, was exactly as predicted. The threshold condition was established to be valid. Though the calculations were made for the TM case they hold for the TE case as well.

We show the results in Figs. 36 a and 36 b.

3.9. ORTHOGONALITY

In his paper[5] Wait has talked about the orthogonality of the modes comprising the series solutions. In particular he has said that the integral

$$\frac{1}{d} \int_0^d f_n(y)\, f_{n'}(y) = 0 \quad \text{for } n \neq n' \quad \ldots (61)$$

Where $f_n(y)$ is the transverse variation of the nth mode and is given by ,

$$f_n(y) = e^{u_n y} + R_o \; e^{-u_n y}$$ Where

$$R_o = \frac{u_n + kp}{u_n - kp}$$

Now we take up two modes characterized by the transverse wave numbers u_1 and u_2 respectively Then using the usual form of $f_n(y)$ we compute the integral in (61). This is got as

$$I = \frac{1}{d} \left[\int_0^d \frac{1}{[u_1 + u_2]} \left[e^{(u_1 + u_2)y} - R_{o1} R_{o2} e^{-(u_1 + u_2)y} \right] + \frac{1}{(u_2 - u_1)} \left[e^{(u_2 - u_1)y} - R_{o2} e^{-(u_2 - u_1)y} \right] \right]$$

Where R_{o1} and R_{o2} are the different R_o' s for the two modes.

To show $I = 0$, we have to utilize the information provided by the characteristic equation which determines the u's.

It has been found that even after utilizing the fact that the u's are solutions of the modal equation we are not in a position to show that $I = 0$. The expressions which arise in the calculation are practically incapable of reduction. On the other hand if really the orthogonality principle in our above sense did hold, the expressions would have reduced and we would have easily shown I = 0 .

It was decided, therefore, to verify, for a few cases where the u's had been derived from the modal equation by an actual computation, whether the integral vanishes. Since it was suspected that the said kind of orthogonality may not hold, the argument was as follows. If even for one case the integral does not vanish, then Wait's assumption of orthogonality is incorrect.

Accordingly the intergral was calculated for three specific cases. These were three sets of real roots of the equation

$$\frac{X - P}{X + P} \quad \frac{X - Q}{X + Q} = e^{-2X}$$

i.e., the characteristic equation for the unsymmetric case. The roots were

	X_1	X_2	P	Q
a	1.8648	10.0000	2	10.0
b	1.6910	4.0080	2	4.0
c.	2.9200	4.5055	3	4.5

The integral values were, to slide rule accuracy, as follows: a . I = 20.52

I = -1650

c. I = -13890

So the integral clearly does not vanish.

However for the symmetric case we see that the orthogonality of the above kind is found to hold, as the integral

$$\frac{1}{d}\int_{-d/2}^{d/2} \text{Cosh }(u_1 y)\ \text{Sinh }(u_2 y) = 0$$

Where the fact that the modes are of even and odd symmetry has been used.

Thus for the general unsymmetric case, which is of practical significance, this orthogonality principle does not hold. This means the excitation problem becomes rather tough. In fact we have come face to face with another problem, i.e. the problem of excitation. For we cannot enjoy the advantages of the situation wherein the orthogonality principle is valid.

But still we may have independent carriage of power by the different modes. This is possible because, we have many kinds of orthogonality relations and our problem (which is once again a crucial one) would be to ascertain pre-cisely the kind of relations which would be valid for the present case.

Perhaps Wait has surmised that in keeping with the usual cases of bounded structures, the modes should be orthogonal. It is thought that he has not actually verified the vanishing of the concerned integral.

But when one applies the Sturm-Liouville theory to the differential equation determining $f_n(y)$, one is surprised to find that, at least apparently there is nothing to prevent the orthoganality of the modes. But why is it that we do not get the modes to be orthogonal on an actual evaluation ? Obviously there is something wrong somewhere in the application of the Sturm. Liouville Theory to this problem. Perhaps there is one condition which we overlook while applying the theory to this problem. The author is of the opinion, that the fact that the guiding surfaces are reactive may be responsible for this anomaly. At least, to the author's knowledge, it is true that the Sturm - Liouville theory has not been developed for such boundary conditions as we encounter in this problem. Hence we are not at liberty to apply the usual Sturm - Liouville theory to our problem and utilize its conclusions.

In any case we come up with a new problem. That is to determine the general orthogonality relations valid for impedance boundaries. The problem is complex and little work has been done in this direction. However it is beyond the scope of this work to discuss any more of orthogonality and we rest by pointing out the problem that has emerged before us.

3.10. CONCLUSIONS

We have investigated the types of field configurations that may exist in a parallel plate waveguide which is having purely reactive guiding surfaces. We considered the following cases.

I. i. Both the surfaces inductively reactive and of equal reactance.

ii. Both the surfaces capacitively reactive and of equal reactance.

iii. One of the surfaces inductive, the other surface capacitive but the reactances being of equal magnitude.

II. i. Surfaces with unequal inductive reactances.

ii. Surfaces with unequal capacitive reactances.

iii. One surface with inductive reactance, the other with capacitive reactance but the reactances being of unequal values.

One may wonder why the case of equally reactive surfaces has been separately considered when the more general case of unequal reactances has been studied. It is also natural to expect that the results which hold for the general unsymmetric case will naturally follow for the particular case of equal reactances. But it is to be pointed out that the characteristic equations which one obtains for the symmetric case are markedly different from those obtained for the unsymmetric case. Though ultimately, we record the same behavior of modes here as in the unsymmetric case, that such a thing is true is not readily obvious from a cursory glance at the modal equations for the symmetric acase. We just expect, from physical reasoning, that what holds for the unsymmetric case must be true for the special case of equal reactances. But that is just an intuitive guess. Also since the defining equations for the symmetric case are decidedly different from the corresponding ones for the other case, one has to make sure by an actual calculation whether ultimately we get the same results (qualitatively) for the two cases. This is our justification for having devoted our time for the symmetric case as well.

Now we summarise the results.

I. i. For this case we find that in general there are two surface waves of the even and odd type. One of them has $E_x = 0$ at the central plane of symmetry and for the other E_x is maximum at the same plane.

The former is of the even type and has a field pattern which mixes the features of the TEM and surface waves. Hence it is called the hybrid TEM dual surface wave. The second wave has a field pattern resembling a waveguide mode. We also find that there is a critical value of the surface reactance, for which , this second wave disappears and turns into the zero order waveguide mode which is of the odd type. This is what we have called the threshold phenomenon. For values of reactance above the threshold value, the surface wave of the odd type propagates along with the surface wave of the even type while for reactances less than the threshold reactance the zero order mode of the odd type exists. Also it is found that the cut-off frequency of the zero-order waveguide mode which is of the odd type becomes zero at the threshold. Hence this mode can no longer propagate as a TM type waveguide mode and consequently it becomes a surface wave mode of the odd type. The even type of surface wave is found to propagate till the reactance is reduced to zero.

We also find that there are an infinite number of waveguide modes and they possess either even or odd symmetry. Of these, in the odd type of modes, the zero order mode may or may not exist depending on the value of reactance. The critical value of reactance for which this zero order mode just disappears coincides with the critical value of reactance at which the surface wave of the odd type just appears.

So far we have been talking about TM type waveguide modes. It is found that TE type waveguide modes can exist in the geometry, though surface waves of the TE type cannot exist. Once again we have even and odd TE waveguide modes. The zero order mode of the TE type is even while the corresponding TM mode is odd. The cut off frequencies of the TE modes do not coincide with those of the TM modes. Since there are no TE surface waves, we find that there is no threshold phenomenon observed.

I ii. This is the dual of the above case. If we substitute TE for TM and vice versa we can have the conclusions for this case. We summarise them as:

1. Two TE surface waves. One is even, while the other is odd. The odd type becomes the zero order TE waveguide mode of the odd type.

2. TE type waveguide modes of even and odd types.

3. TM type waveguide modes of even and odd types.

It is to be noted that in the field patterns the roles of electric field in the TM case are played the magnetic field in the TE case. We replace H_z , E_x , E_y of the TM case by E_z , H_x, H_y in the TE case. Thus the electric field pattern in TM case is identical with the magnetic field pattern for the TE case. Thus the TE case is a perfect dual of the TM case. But we are not very much interested in this case. For, practically speaking, it is easier to provide surfaces of inductive reactances than capacitively reactive surfaces. So the TM solution is of more practical utility.

I iii. Here there are no surface waves propagating. But there are two sets of TM and two other sets of TE waveguide modes. The essential feature is that for each TE mode there is a corresponding TM mode of the same cut-off frequency. Thus there is degeneracy. The cut off frequencies are given by the zeroes of tangent and cotangent functions for both the cases.

Even this case is not of practical interest.

II. i. All the results which were obtained for the symmetric case I(i) are found to be true for this case also., except that the modes are no longer even or odd. But they are symmetric about a plane $y = m$ instead of $y = 0$ for the symmetric case.

Here one of the surface waves is characterized by $E_x = 0$ at $y = m$ and the other is described by $\frac{d E_x}{dy} = 0$ at $y = m$. The latter mode resembles a waveguide mode and becomes the zero order waveguide mode at the threshold value of reactance. The former mode propagate till the reactances are reduced to zero. Once again this mode is the hybrid TEM dual surface wave mode.

There are two types of TM and two types of TE waveguide modes. These are non-degenerate. The threshold condition is found to be

$\frac{1}{P} + \frac{1}{Q} = 1$. Thus it depends on the relative magnitudes of the two reactances.

II ii. We do not discuss this case in any detail, for it is simply the dual of the above case. Further it is not of much practical interest. In any case the results of I (ii) follow with the modification that the modes are asymmetric.

II. iii. Here we have one TM surface wave and one TE surface wave. Both possess field patterns resembling a waveguide mode pattern. Both experience the threshold phenomenon and turn into corresponding zero order waveguide modes. The threshold condition is given by

$$\frac{1}{P} - \frac{1}{Q} = 1$$

Also Two sets of TM and two sets of TE waveguide modes exist. These are non degenerate.

CHAPTER IV

CONCLUDING REMARKS AND A FEW SUGGESTION FOR FUTURE WORK

Thus by this work we have been able to substantiate a majority of the conclusions arrived at by Wait in his paper[5]. At the same time we have found occasions to differ with Wait. The first occasion arises in connection with the threshold phenomenon. Wait does indicate the possibility of a waveguide mode turning into a surface wave. But he speculates that the zero order waveguide mode would degenerate into the two surface waves at a suitable value of reactance. But this has not been found to be the case. On the contrary, this zero order waveguide mode which is also the dominant mode turns into only one surface wave and does not split into two surface waves. This is a significant point of departure from Wait's con-clusions.

Further Wait has not worked out the precise relation for the threbhold conditiob. He, merely, using his intuition, guesses the possibility of a threshold. And we have been able to formulate the threshold equaations for all the cases we have considered and these have been amply supported by our conclusions performed with the computer.

The second occasion arises when we consider the defining equations to be used for the evaluation of the imaginary roots. Wait has considered only the unsymmetric case. And there too the approach he adopts, in the author's opinion, is not veey s-ound. For he makes a few assumptions which one is not compelled to make. It has been demonstrated in this work, that these assumptions are by no means necessary . The procedure adopted here has beenvindicated by the results obtained . Wait does not talk of two types of waveguide modes, while infact they seem to exist in reality. He restricts himself to a single infinite set of modes. Further the results he has obtained for cut off frequency etc., are not of much significance because they have been derived for a very special case. In the analysis given in Chapter III, we have not restricted ourselves to any particular case. Thus the form of the defining equations for the waveguide modes is the most general, and the solution of these equations would give us exact results. Once again our calculations have demonstrated the validity of the adopted procedure, and we were able to go to the symmetric case from the unsymmetric case by putting P = Q in the equations for the unsymmetric case.

Another occasion arises when we consider the possibility of the TE type modes co-existing with the

TM type modes. Though TE surface waves are ruled out, we can have TE waveguide modes, once again two sets of them, coexisting with two sets of TM waveguide modes. Wait does not mention the possibility of TE modes at all. Once again this is considered to be an important omission by Wait, for while we are interested in isolating the surface wave from the contaminating waveguide modes, it is of paramount concern to know precisely all types of waveguide modes that may exist. Only on such knowledge can we design a suitable excitation arrangement, which would help eliminate the undesired modes.

While talking about the degeneration of waveguide mode of zero order into a surface wave, we also must emphasize that the other surface wave exists for all values of reactance greater than zero. Below the threshold, this surface wave will coexist with the dominant zero order waveguide mode. And by a suitable choice of dimensions of the guide, and the operating frequency, we can make the zero order mode to be cut-off and thus having only one surface wave in the geometry. This possibility has not been mentioned by Wait, perhaps because he has not carried out a detailed analysis of the problem.

Lastly we have yet another occasion to differ with Wait when he assumes the orthogonality of the modes. Our calculations point to the contrary.

Hence it is suspected that, the principle may not hold for the ansymmetric case though it is valid for the symmetric case . Perhaps Wait has assumed orthogonality on the basis of Sturm-Liouville theory and on the strength of experience with bounded straatures in general. It is thought that the Sturm -Liouville theory may not be applicable for a system with reactive boundaries.

Now we come to the practical aspect of the problem . This means we are to consider the unsymmetric case where the surfaces are inductively reactive and that brings us in contact with the TM type of modes.

Barlow in his experiements has found it expedient to have one of the surfaces highly reactive while the other is only slightly so. Now the former surface becomes the main guiding surface while the latter merely serves as a shield which primarily limits the spread of the main wave field. Since he has succeeded in supporting the hybrid mode under these circumstances and has obtained some useful results, it is worthwhile to consider this case more closely.

Now coming to the problem, we find that there are two surface waves of the TM type, two sets of TM waveguide modes and two sets of TE waveguide modes.

First of all we eliminate the TE type modes by choosing a proper type of source. Then we are left with two TM surface waves and two sets of TM waveguide modes. By a proper choice of frequency and a suitable choice of dimensions, one can do away with all the higher order waveguide modes.

Then we have two surface waves. But it is desirable to have only one of them. So we choose the reactance below the threshold value. This makes the surface wave with E_x maximum at the plane y = m go into the zero order wave guide mode of the TM type. By choosing a small enough reactance for the lower surface, well below the threshold value, one can raise the cut off frequency of the zero order mode, which is also the dominant mode. This makes it simpler to eliminate the zero order mode also. For one can adjust the dimensions and the operating frequency so that this zero order mode is cut off.

Thus we are left with only one surface wave which is the hybrid- TEM dual surface wave of Barlow. That what we have said so far is a physical reality has been amply proved by Barlow's success in having only the hybrid mode in the guide system. That stable operation has been possible with this mode is also quite satisfying. Further Barlow has experimentally determined the field distribution of this mode and this coincides with our description of the same mode. Also Barlow has proved that this mode, when supported by suitably chosen reactances, has an attenuation which is almost half of what a normal quasi - TEM wave would experience.

It is to be noted that we should have only small reactances for the supporting surfaces if we can have a pure surface wave uncontaminated by others. It is found that Barlow has used only small reactances for the support of the hybrid mode. This supports our previous arguments.

We propose to illustrate the above by a numerical example.

We choose the case where

$P = 0.25$ $\quad f_o = 1$ Gc/s

$Q = 0.50$ $\quad d = 10$ cm $\quad \lambda_o = 30$ cms.

$P = kpd$ $\quad Q = k\,qd$

For this case $k = 2\pi/\lambda_o = 0.209\ cm^{-1}$ $\quad d = 10$ cms

$\therefore\ p = 0.119.$

Now $|Z_o| = \eta\, p = 120\,\pi \times 0.119 = \underline{45}\ \Omega$

$|Z_d| = 2\ Z_o = \underline{90}\ \Omega$

Z_o and Z_d are surface impedances. We find the cut off wavelength of the zero order waveguide mode. This is given by the root of equation (40) in the interval $(0, \pi/2)$. For the assumed values of P and Q, this first root is

$$X = \frac{k\,c\,d}{2} = 1.442$$

$$k\,c\,d = 2.884$$

Now C = 1 gives the cut off condition.

$$\therefore \quad \frac{2\pi d}{\lambda_c} = 2.884$$

$$\therefore \quad \lambda_c = \underline{2.18\ d} = \underline{21.8}\ \text{cms.}$$

So 21.8 cms is the cut-off wavelength of the zero order mode. Since the free space wavelength is 30 cms, this mode is cut off.

So we have only one surface wave mode that is propagating and it is the hybrid mode.

It is of interest to calculate the phase velocity of the wave.

$$V_p = \frac{w}{\sqrt{u^2 + k^2}}$$

The transverse wave number u, can be got from the real root of the characteristic equation (13).

This is got as

$$ud = 0.905$$

or $u = 0.0905\ \text{cm}^{-1}$ for d = 10 cm.

$$V_p = \frac{2\pi \times 10^9}{(0.0082 + 0.044)^{1/2}} = 2.75 \times 10^{10}\ \text{cm/sec.}$$

This is 91.7 % of the velocity of light. Reducing d = to 5 cms, we find that the zero order mode is still cut off , but the phase velocity reduces to 76% of the velocity of light. Finally, if d = 1 cm. with all other things the same, we get a phase velocity which is only 22.5% of the velocity of light.

So by choosing a low enough frequency of operation and a very small distance of separation one can reduce the phase velocity to any desired fraction of the velocity of light. However in practice difficulties in realizing purely reactive surfaces will come in the way of this reduction in phase velocity. So there will be a limit beyond which we cannot reduce the phase velocity. Since our boundary conditions are hypothetical we got the result that there is no apparent limit to the extent of reduction of the phase velocity.

Even though the possibility of having slow waves in the structure would point towards an application in the design of slow - wave structures used in travelling wave tubes, one has to be cautious in his pronouncements. For we need really slow waves in the case of travelling wave tubes, (i.e waves which travel at a frew percent of the velocity of light) and it is doubtful if we can realize this in practical guiding structures. Further there may be other considerations in the travelling wave interaction which would render the structure considered, unsuitable. But at least this can be used as a phase shifter. When used as a delay line, we need only a smaller length as compared to the usual line lengths for we have slow wave propagating in the structure.

It might be anticipated that many of the results deduced for the parallel plate case may also hold for the coaxial case, which is more suited for practical applications. Already Barlow has proved that the hybrid mode can be propagated in the coaxial system. Also this mode is such that $E_x = 0$ at $r = r_m$, a certain radius. This is experimentally established. Hence we might expect that there might be two surface waves one of the hybrid TEM type, the other of the coaxial waveguide mode type. There might be a threshold value of reactance below which the surface wave of the waveguide mode type becomes the lowest order coaxial mode. Once again by a proper choice of frequency, the surface reactances and of the geometrical proportions, it is possible to propagate only the hybrid mode. That this is possible has been proved by Barlow in his experiments. Thus the entire analysis of the parallel plate system can be extended to the coaxial system with suitable modifications on account of the changed geometry. Since the coaxial system is most used in practice, we are satisfied that the experience gained in respect of the parallel plate system, will stand us in good stead.

Thus one area of extension of the present work will be to determine the various quantities for the coaxial case. Further one can determine the various wave impedances, power densities etc., for the parallel plate case itself. Also the cut off frequencies of the different wave guide modes can be found out from the imaginary roots of the characteristic equation.

One can consider the effect of surface losses on the results obtained. Also the attenuation of the various modes can be computed, while travelling through a lossy dielectric that separates the two plates.

T-hen there is the excitation problem i.e., the proper choice of source and its location so that we might get maximum power in the surface wave. In this direction already some work has been done by several persons. But still there is further scope, because we have to design the launcher so that TE modes, which are a definite possibility, do not get excited.

Closely linked with the aboves problem, is the determination of the orthogonality relations, if any that are valid in the present case. Also we would like to have the set of orthogonality relations that hold for the general case of impedance boundaries. This problem has

has not been solved satisfactorily as yet.

Thus we came to a close after pointing out a few areas for further work.

-.-.-.-

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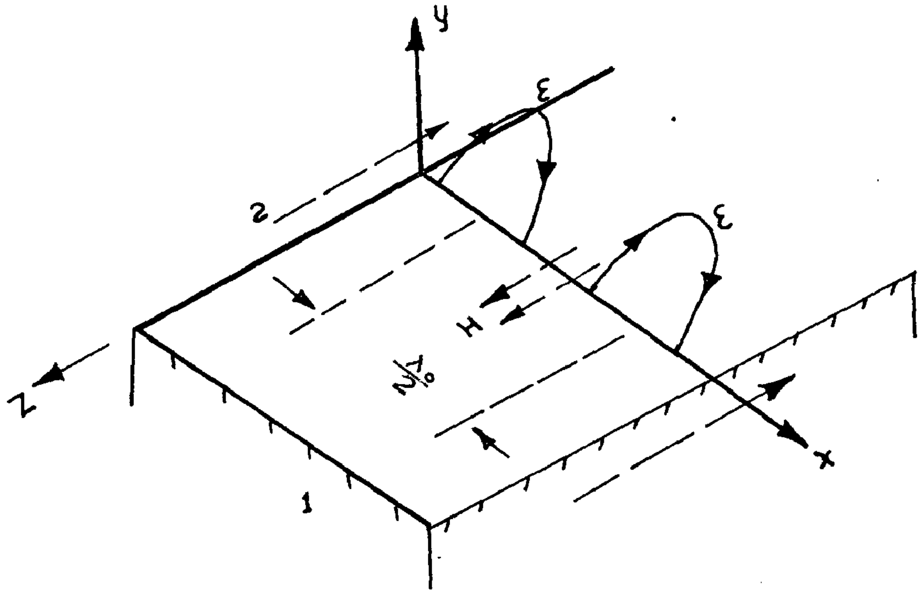


FIG. 1 a - ZENNECK WAVE

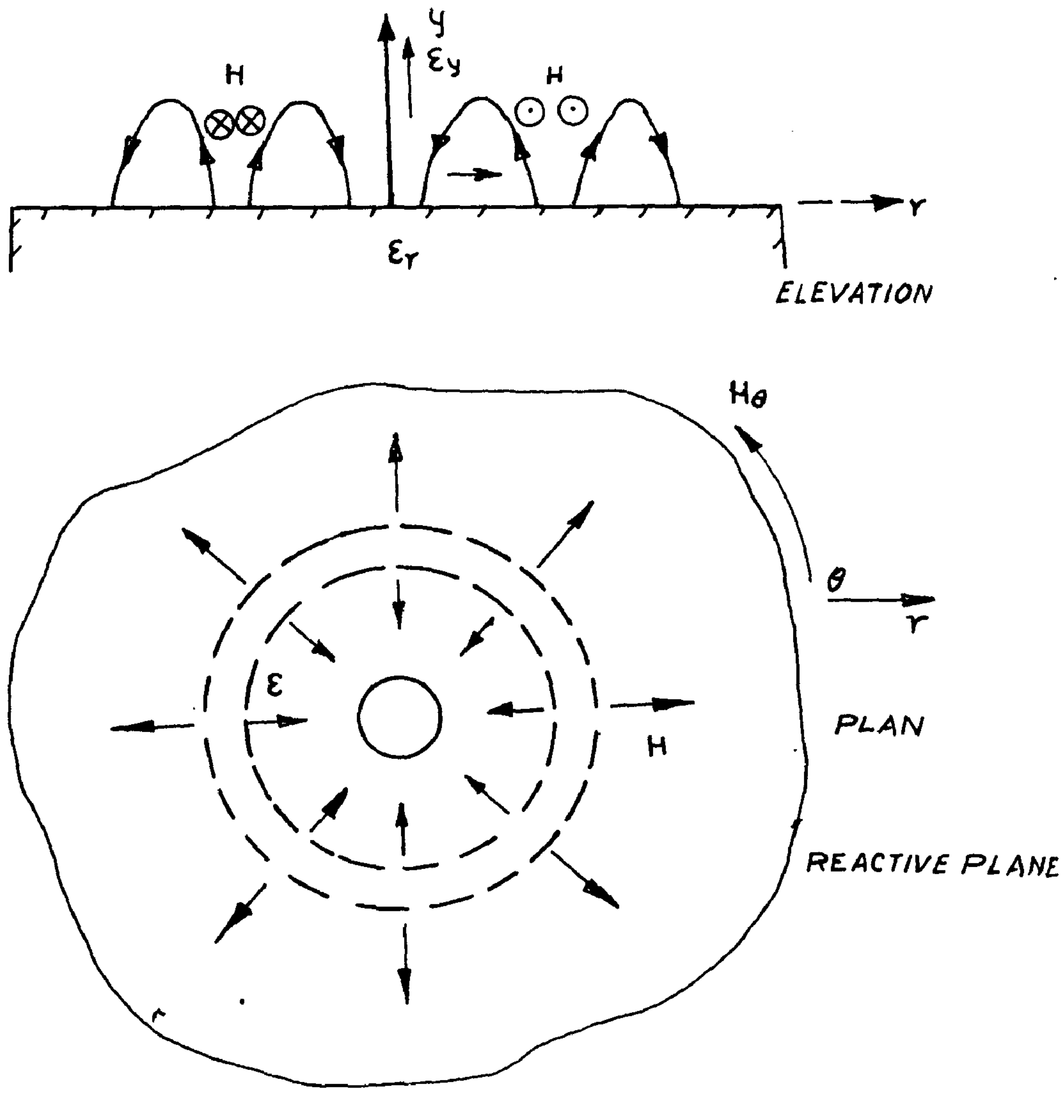
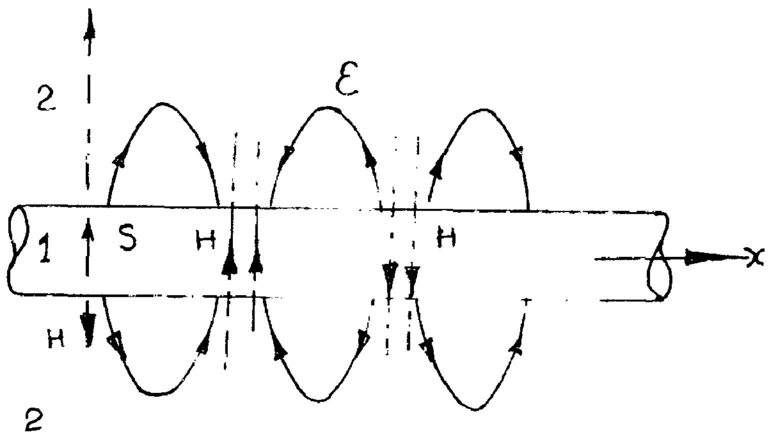
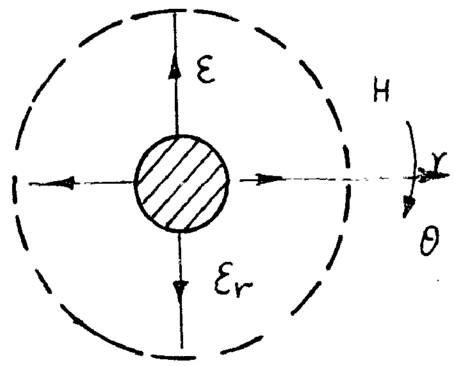


FIG. 1 b - RADIAL CYLINDRICAL WAVE



SIDE ELEVATION



END ELEVATION

FIG.1 C – AXIAL CYLINDRICAL WAVE

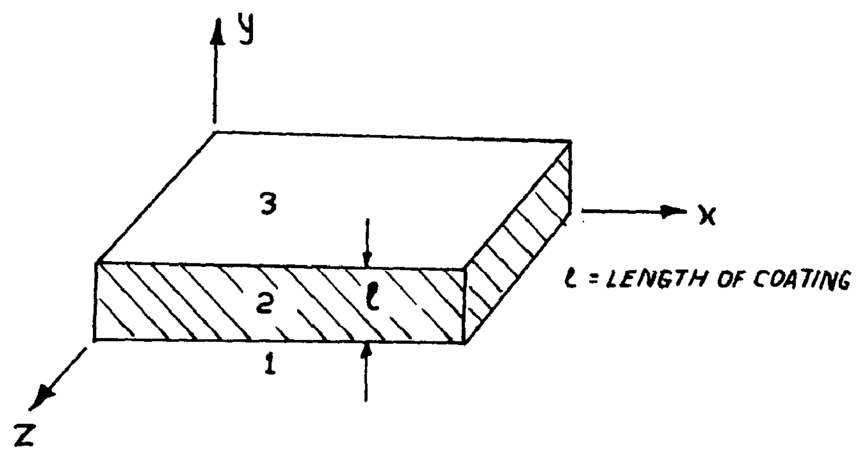

y
3
2
1
ℓ
x
Z
ℓ = LENGTH OF COATING

FIG. 2

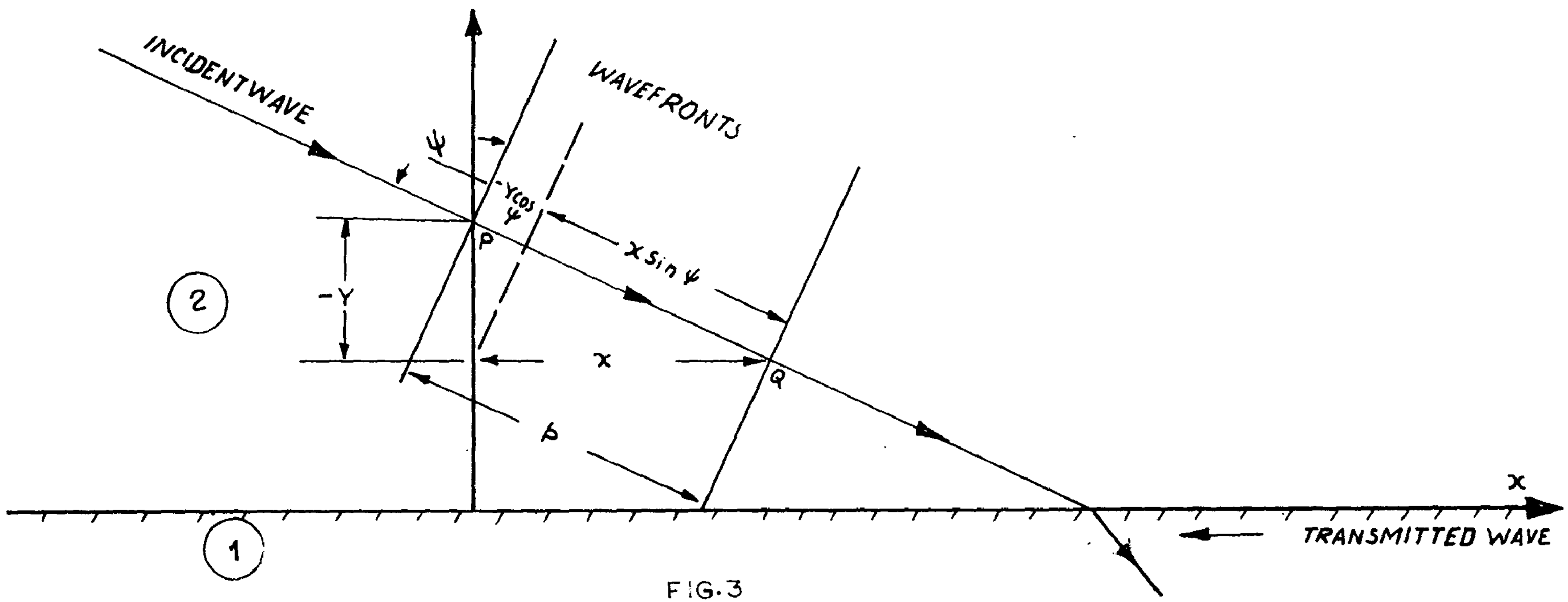

INCIDENTWAVE
WAVEFRONTS
ψ
-Y cos ψ
P
x sin ψ
Q
-Y
x
p
x
2
1
TRANSMITTED WAVE

FIG.3

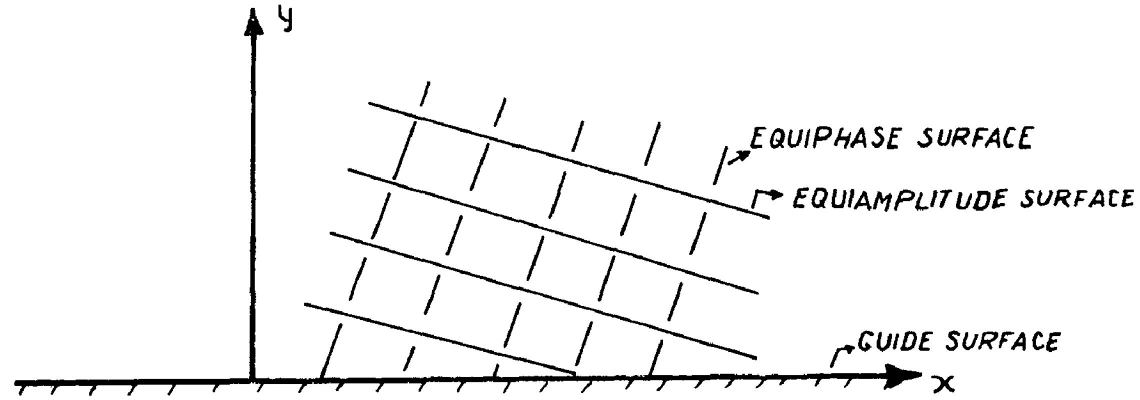


FIG. 4

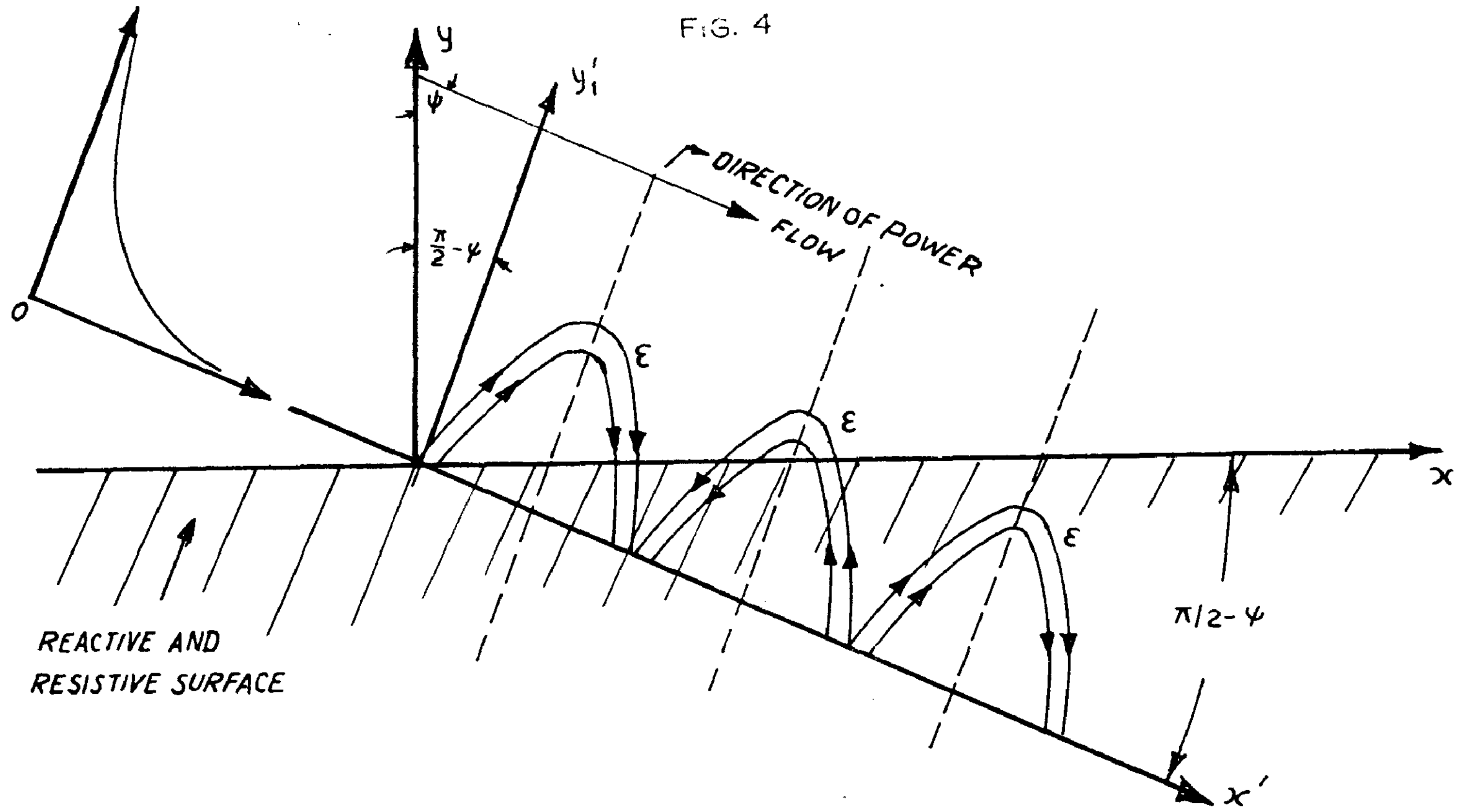


FIG. 6 - THE ZENNECK WAVE SINKING INTO A LOSSY SURFACE

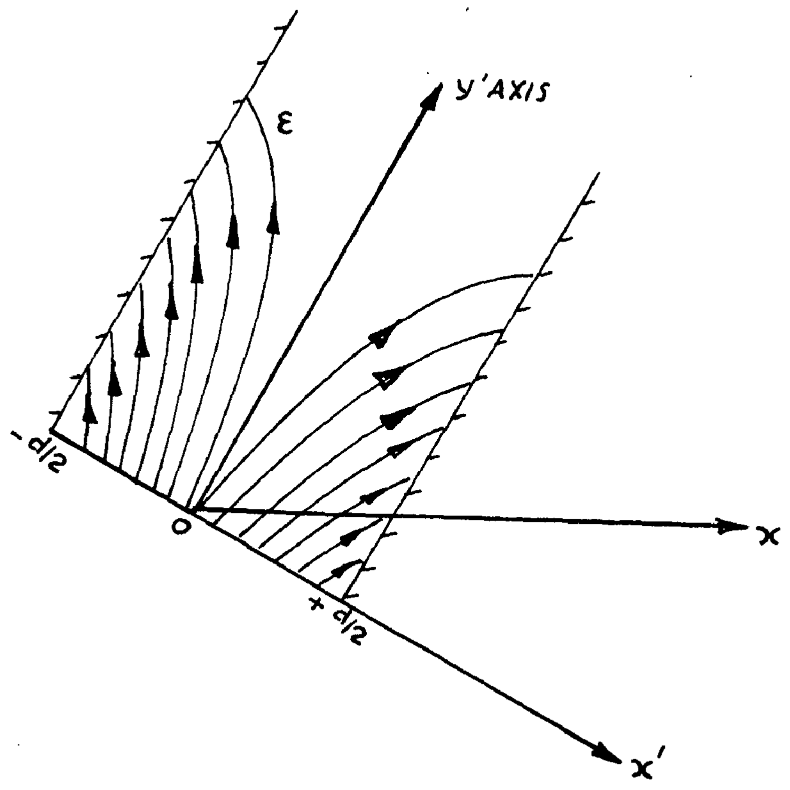


FIG. 7- EVANESCENT E_{01} MODE

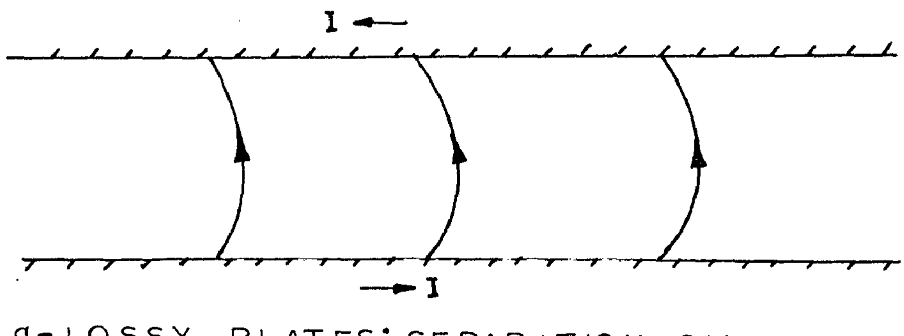


FIG. 8 a- LOSSY PLATES: SEPARATION SMALL

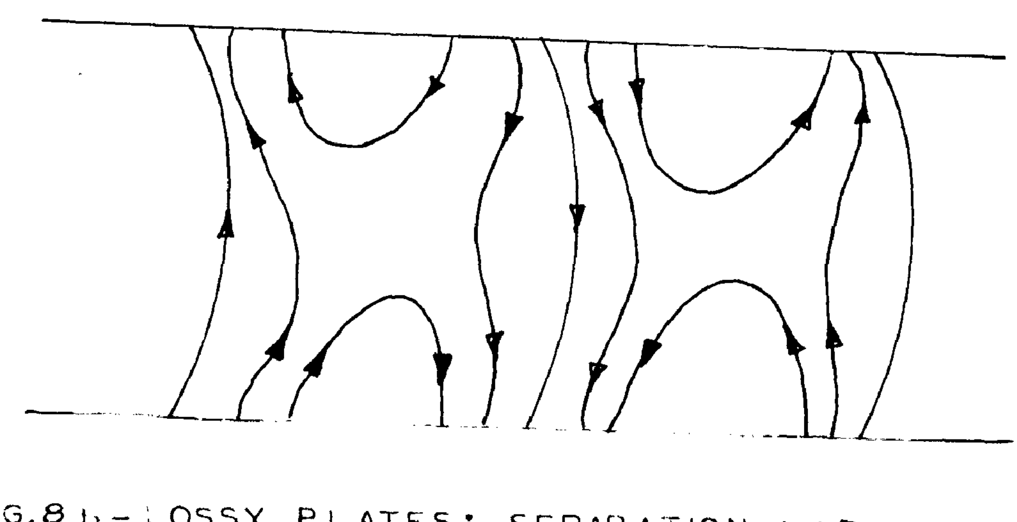


FIG. 8 b- LOSSY PLATES: SEPARATION LARGE

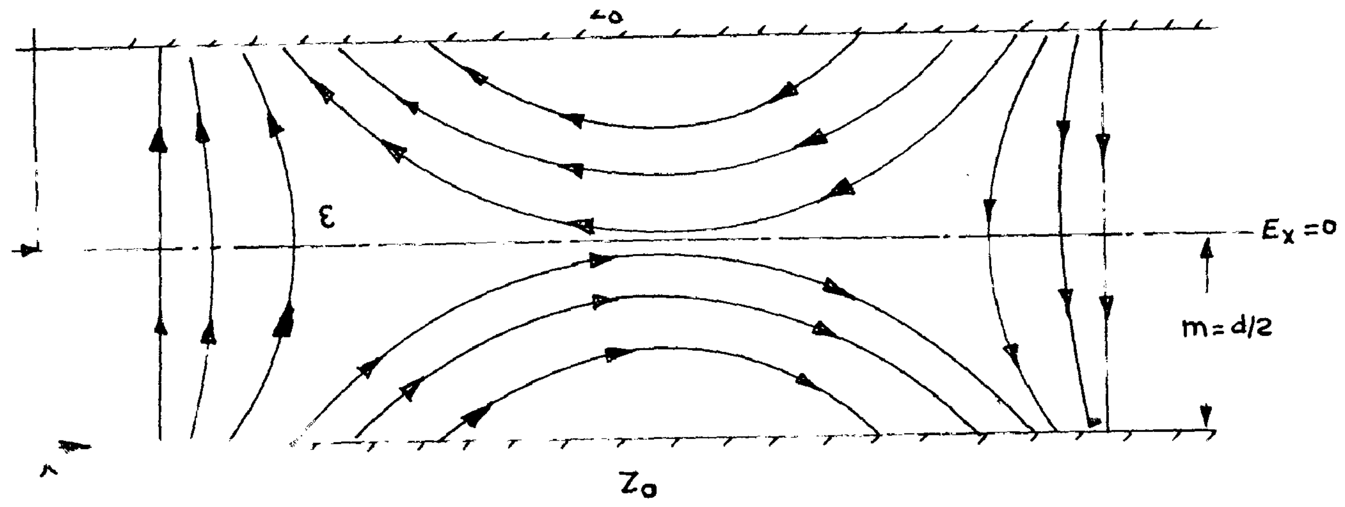


FIG. 9a- SYMMETRIC CASE:- ELECTRIC FIELD LINES IN THE X-Y PLANE

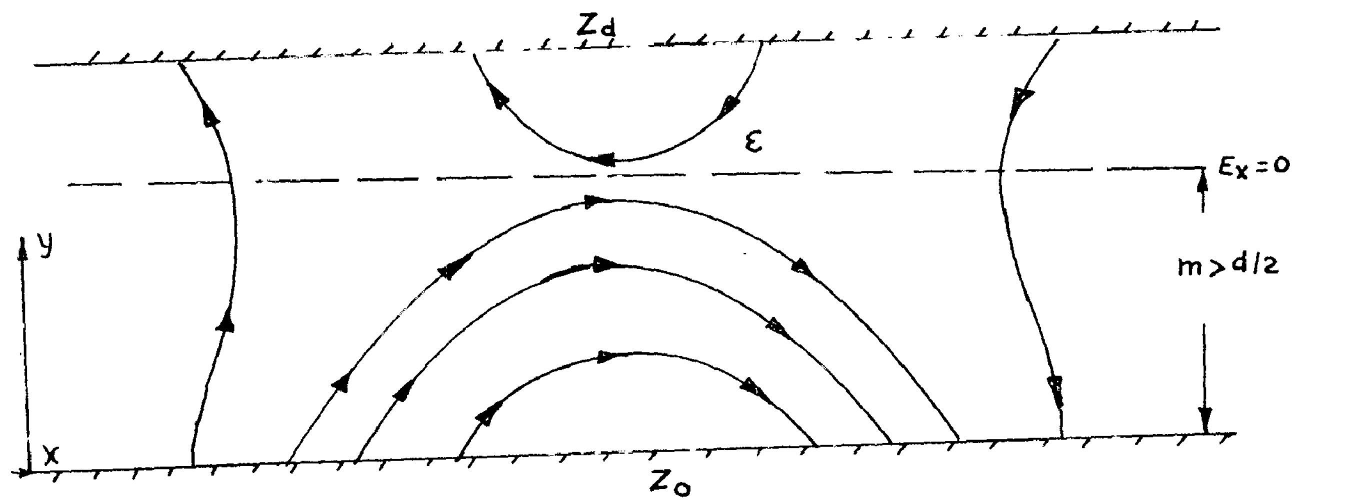


FIG. 9b- UNSYMMETRIC CASE:- ELECTRIC FIELD LINES IN THE X-Y PLANE

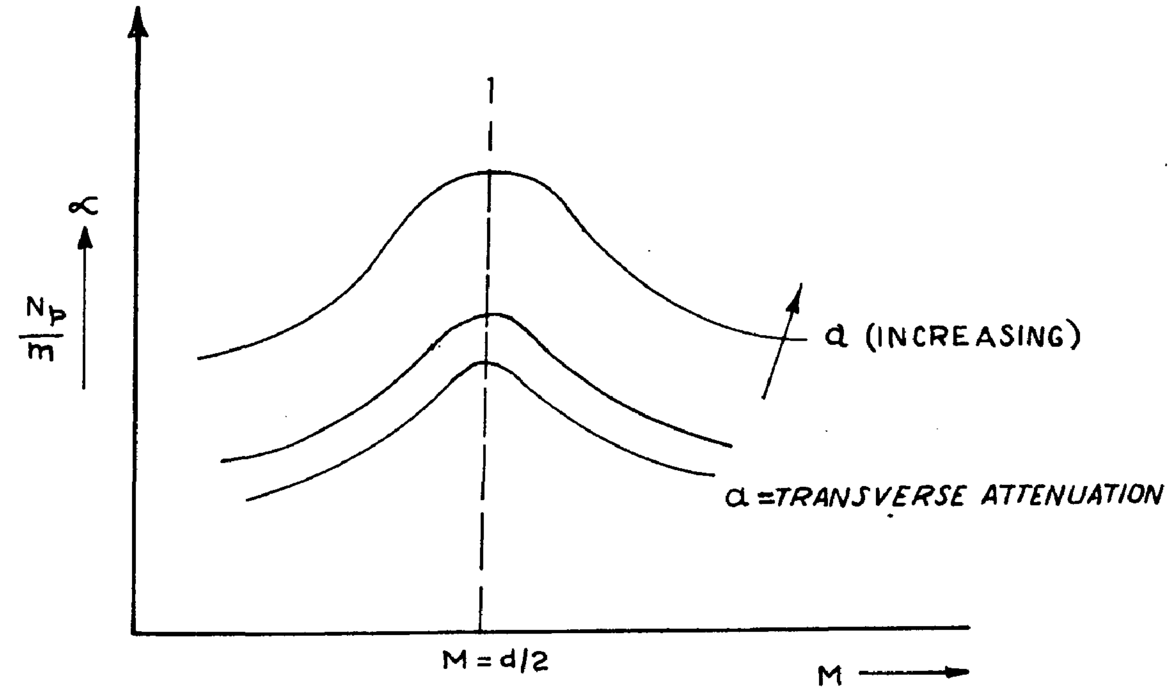


FIG.10 – VARIATION OF α WITH M

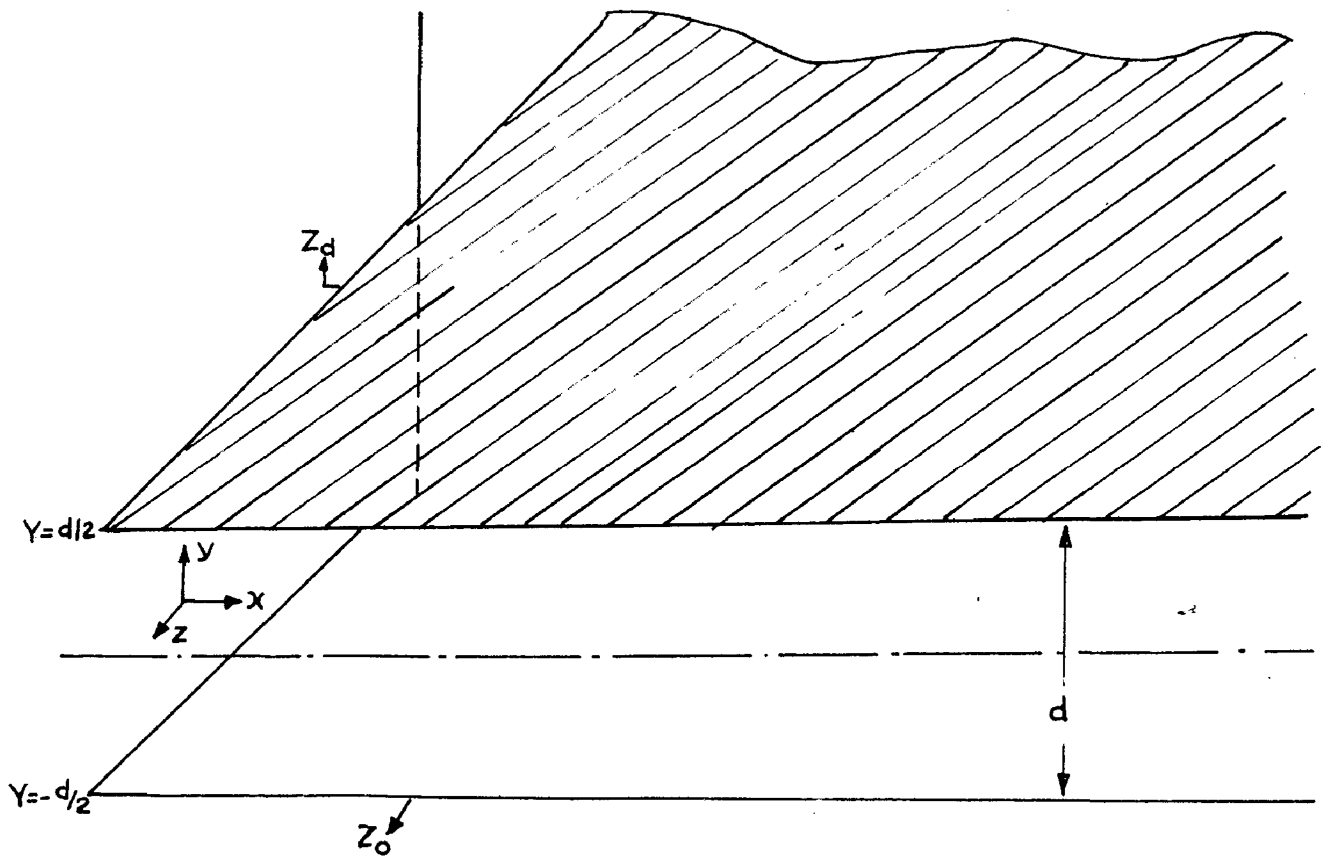
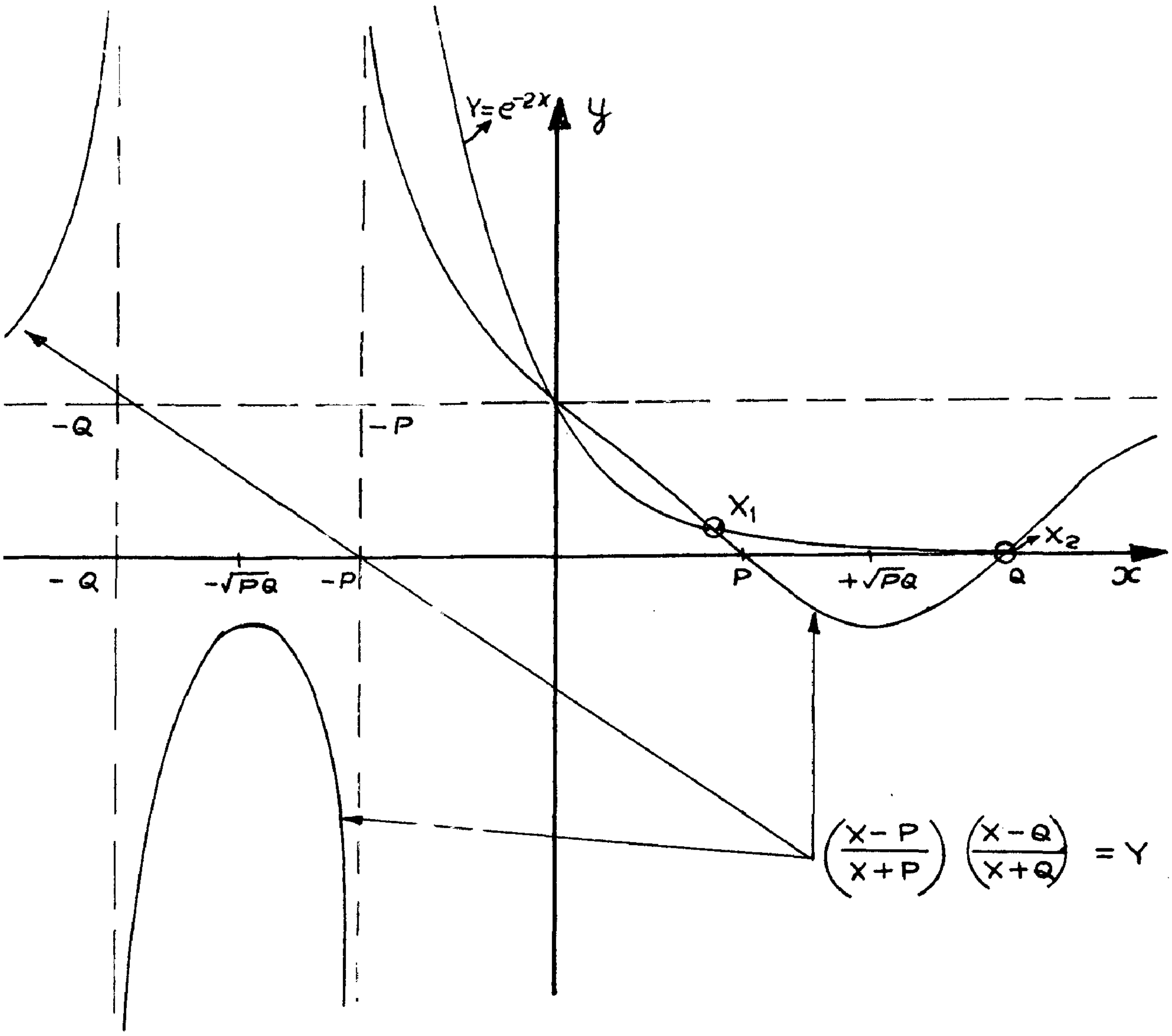
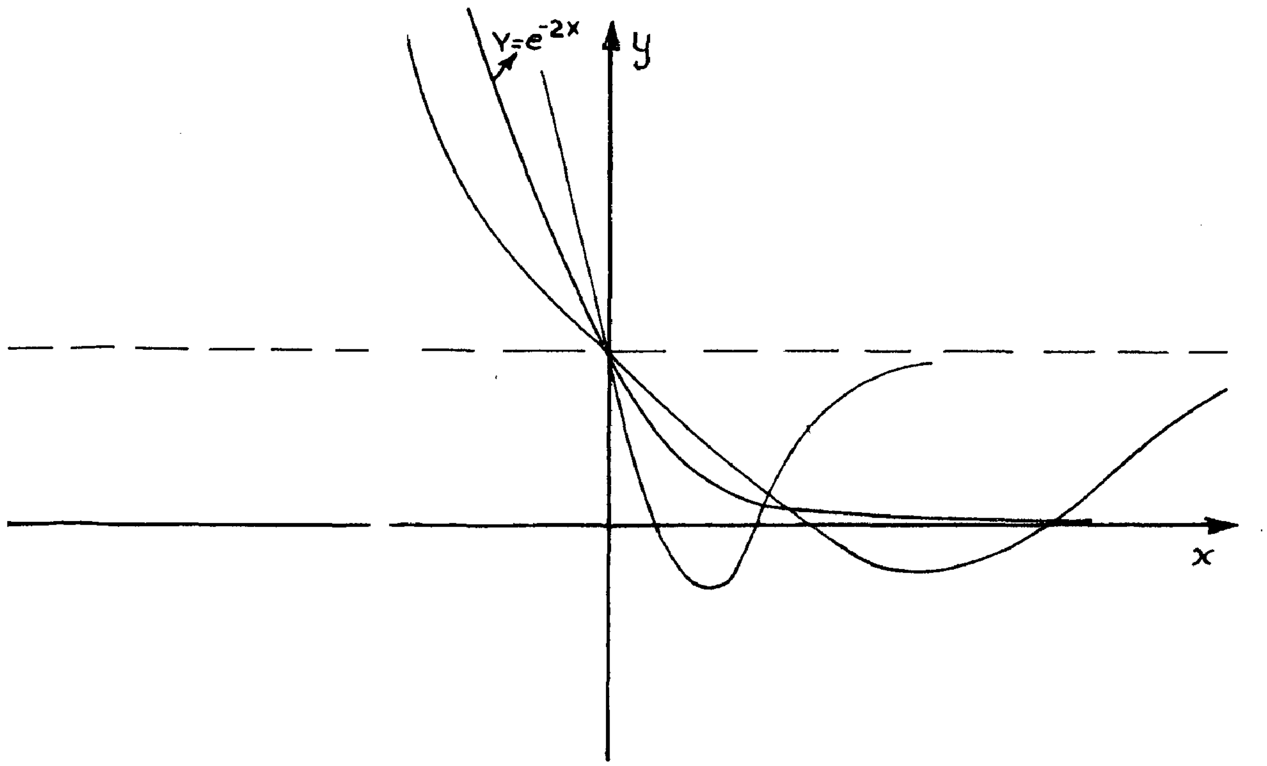


FIG. 11 – PARALLEL PLATE GUIDE WITH REACTIVE GUIDING SURFACES



$Y=e^{-2x}$
y
$-Q$
$-P$
X_1
X_2
$-Q$
$-\sqrt{PQ}$
$-P$
P
$+\sqrt{PQ}$
Q
x
$\left(\frac{X-P}{X+P}\right)\left(\frac{X-Q}{X+Q}\right)=Y$

FIG. 12



$Y=e^{-2x}$
y
x

FIG. 13

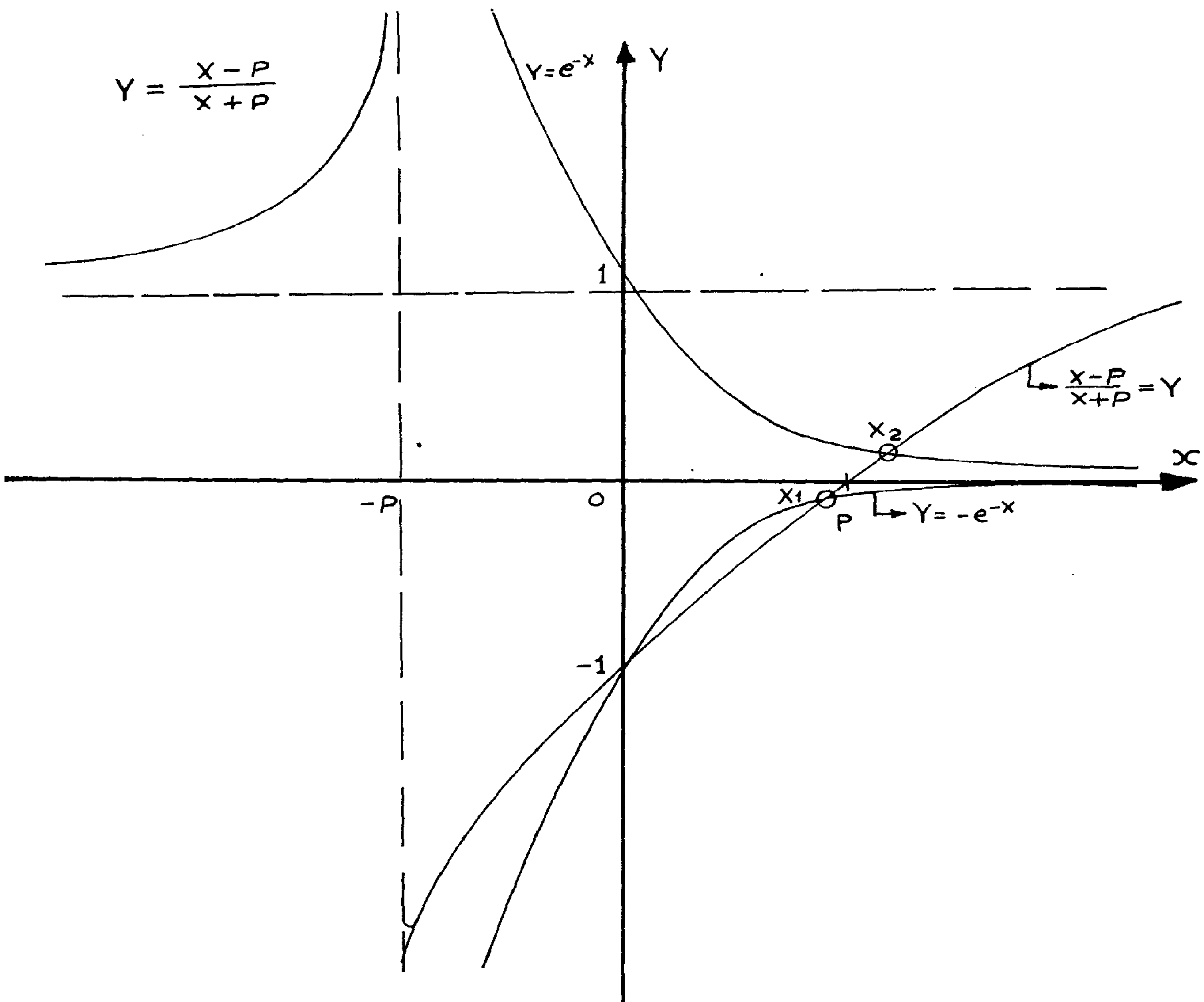


FIG.14

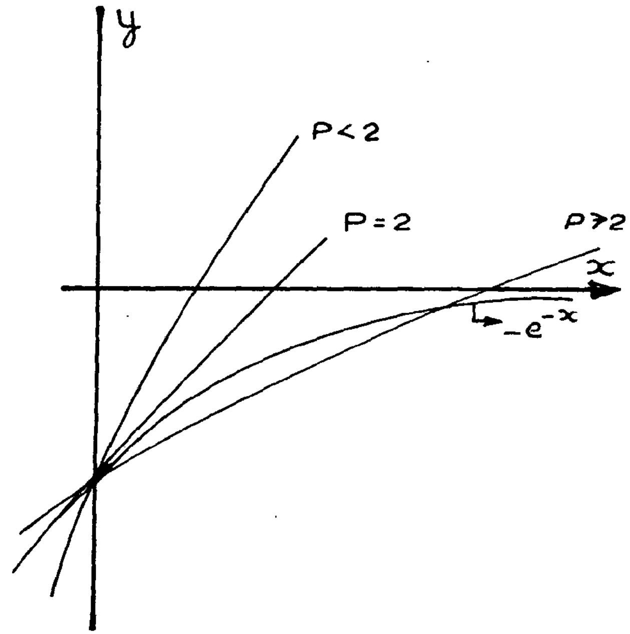


FIG.15

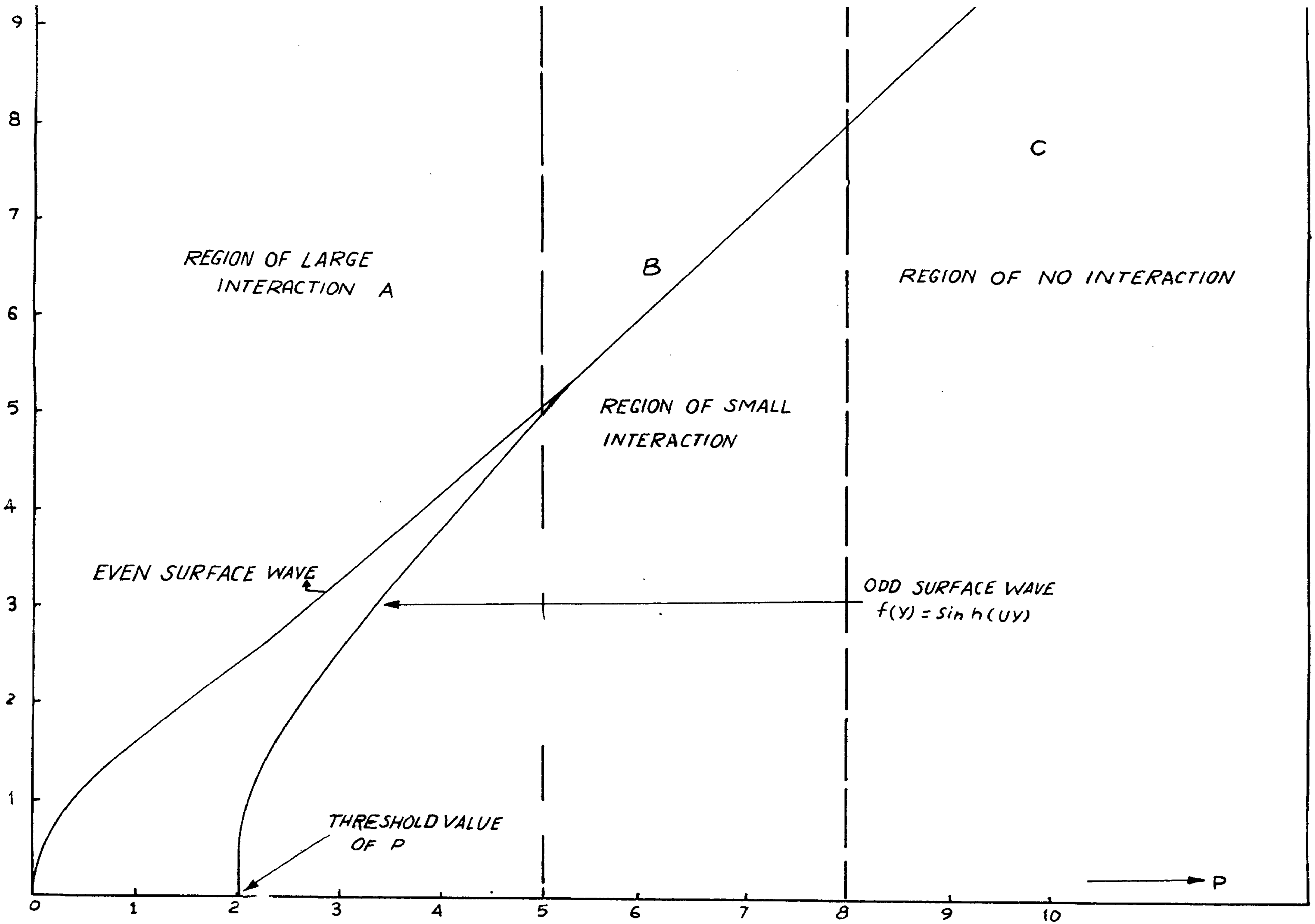
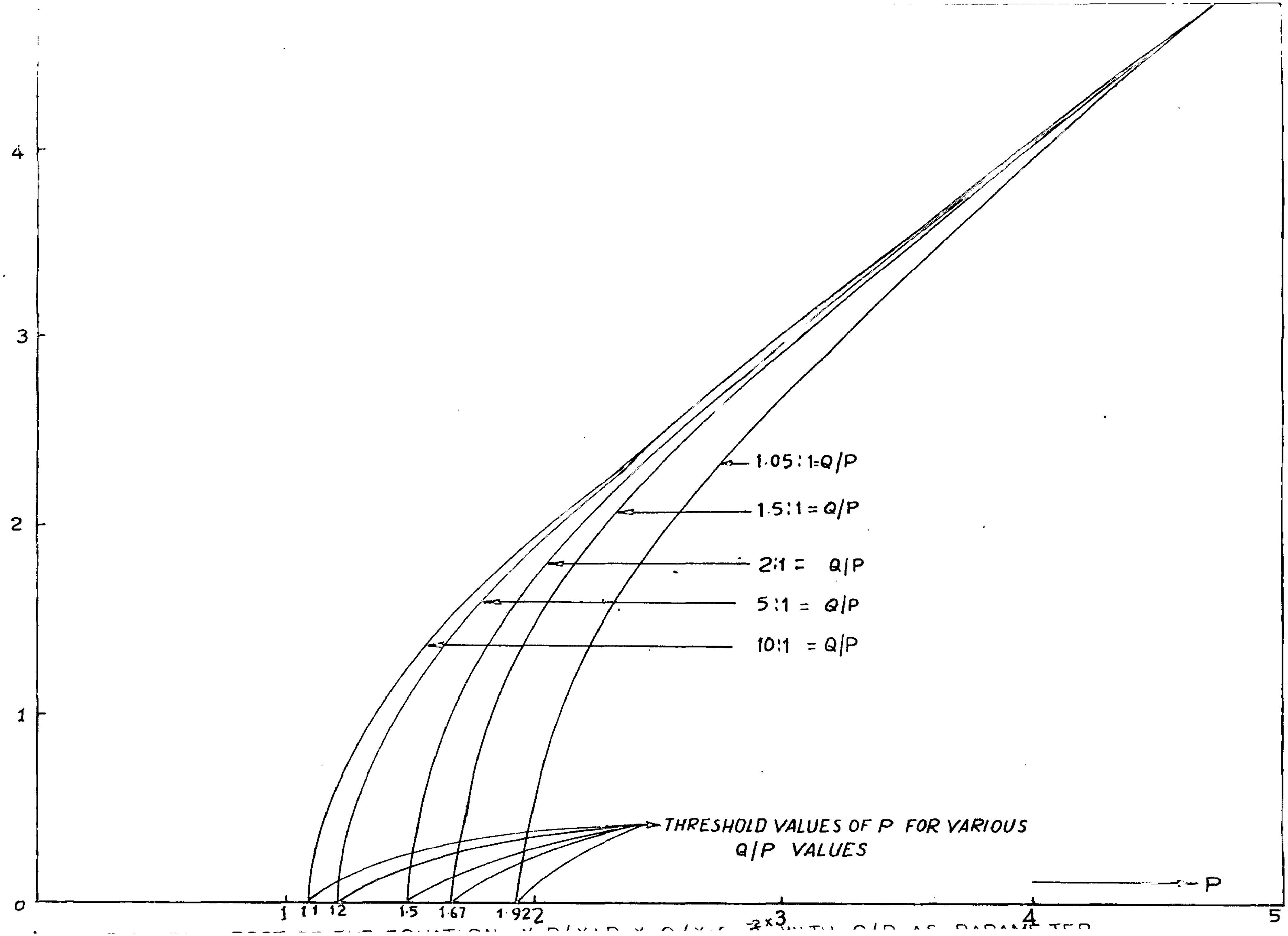


FIG.16 - PLOT OF REAL ROOTS OF THE EQUATION $X-P/X+P=\mp e^{-X}$


1·05:1=Q/P
1·5:1 = Q/P
2:1 = Q/P
5:1 = Q/P
10:1 = Q/P
THRESHOLD VALUES OF P FOR VARIOUS Q/P VALUES
P
0
1
2
3
4
5
1·1
1·2
1·5
1·67
1·92

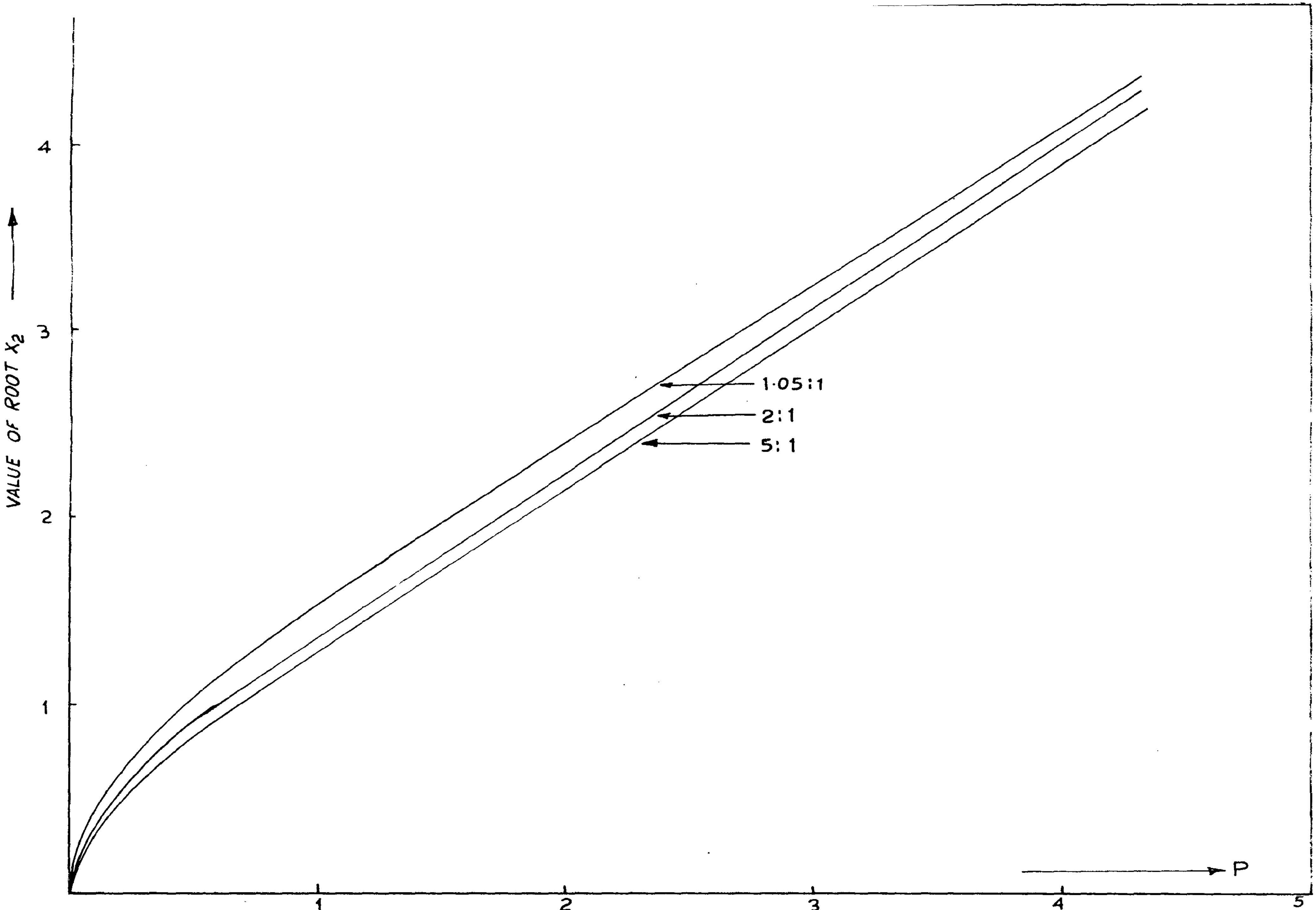


FIG.18-PLOT OF REAL ROOT OF EQUATION

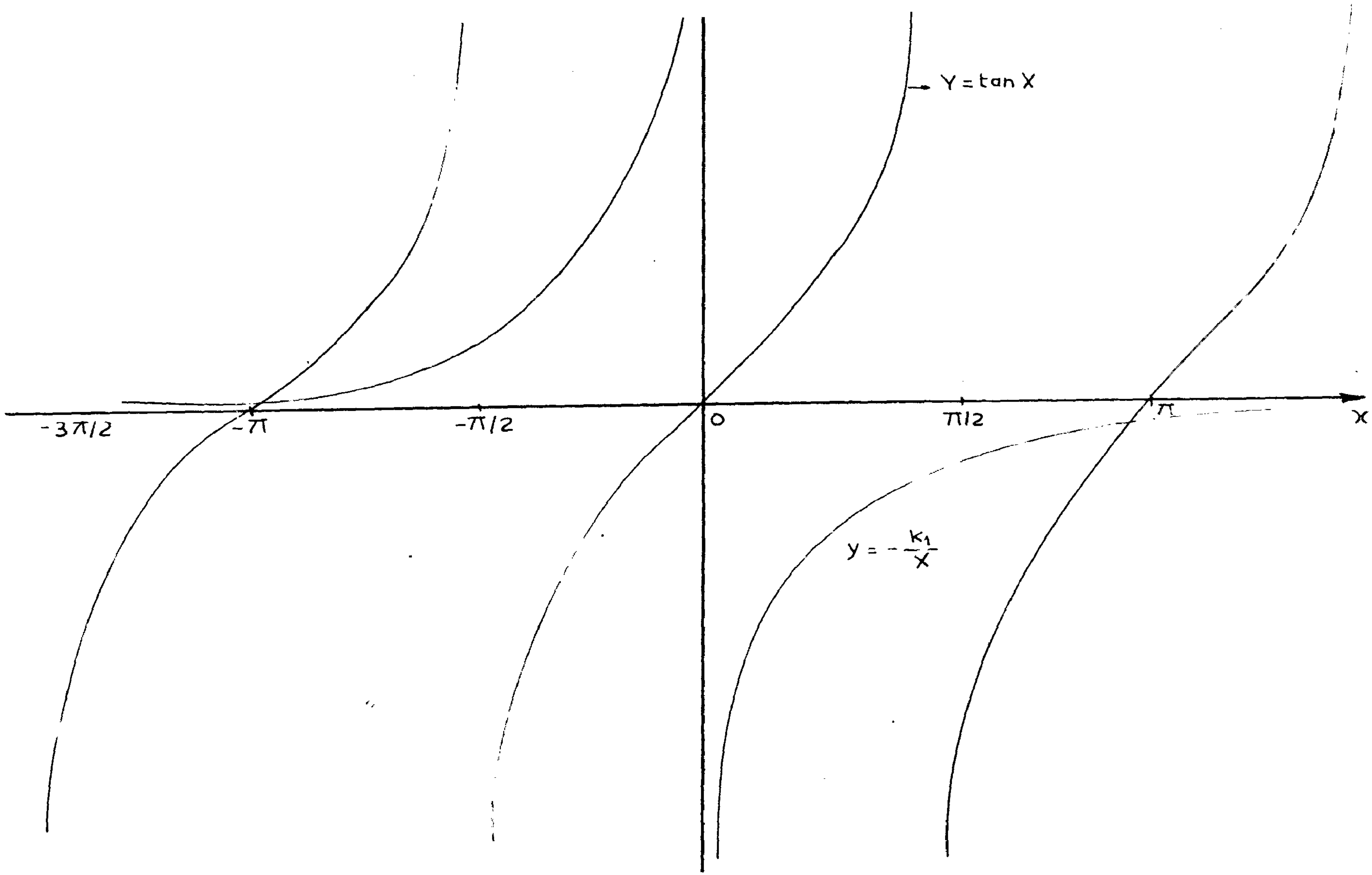


FIG. 19

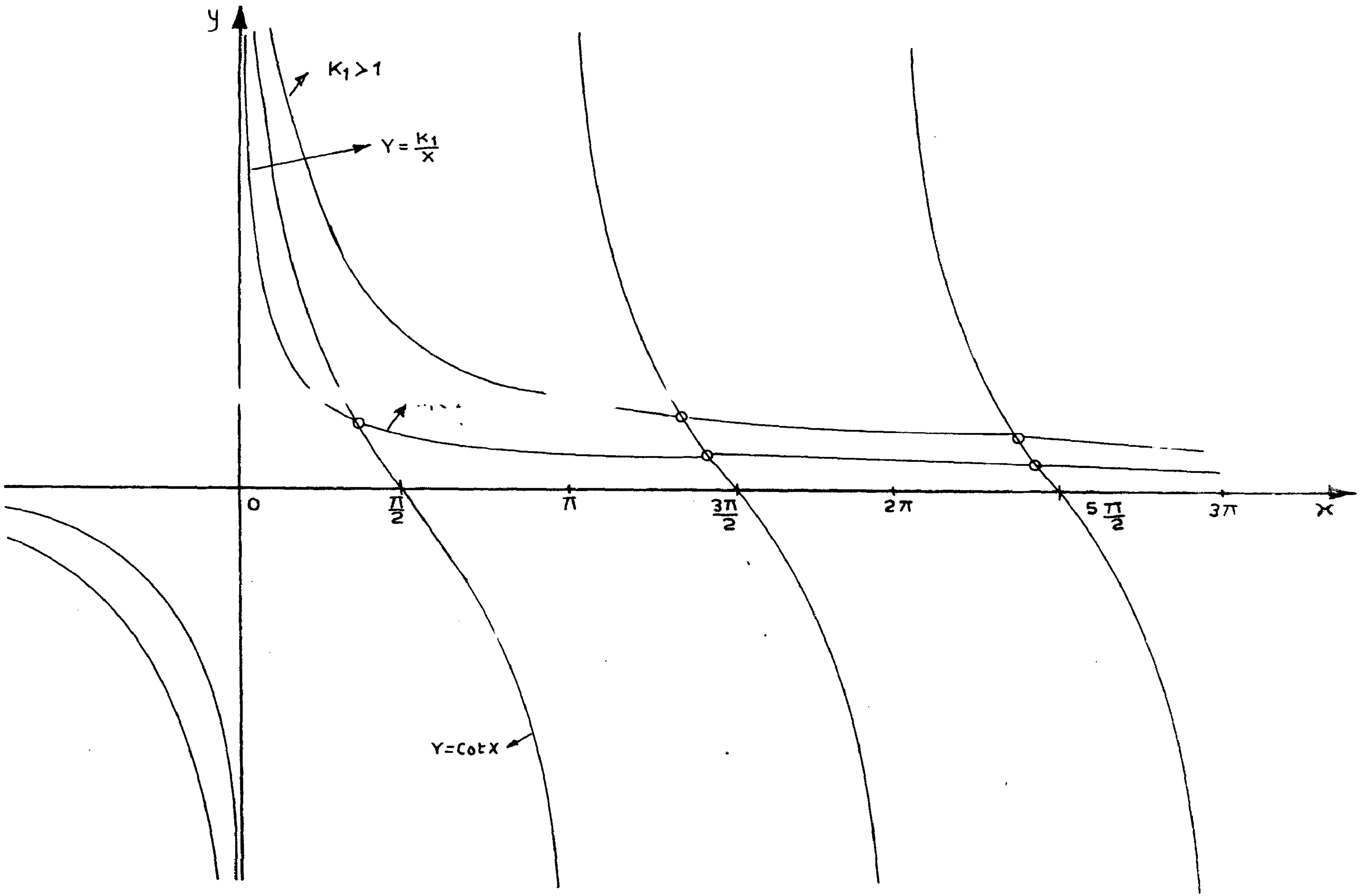

y
$K_1 > 1$
$Y = \frac{K_1}{X}$
0
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
$5\frac{\pi}{2}$
3π
X
Y=Cot X

FIG. 20

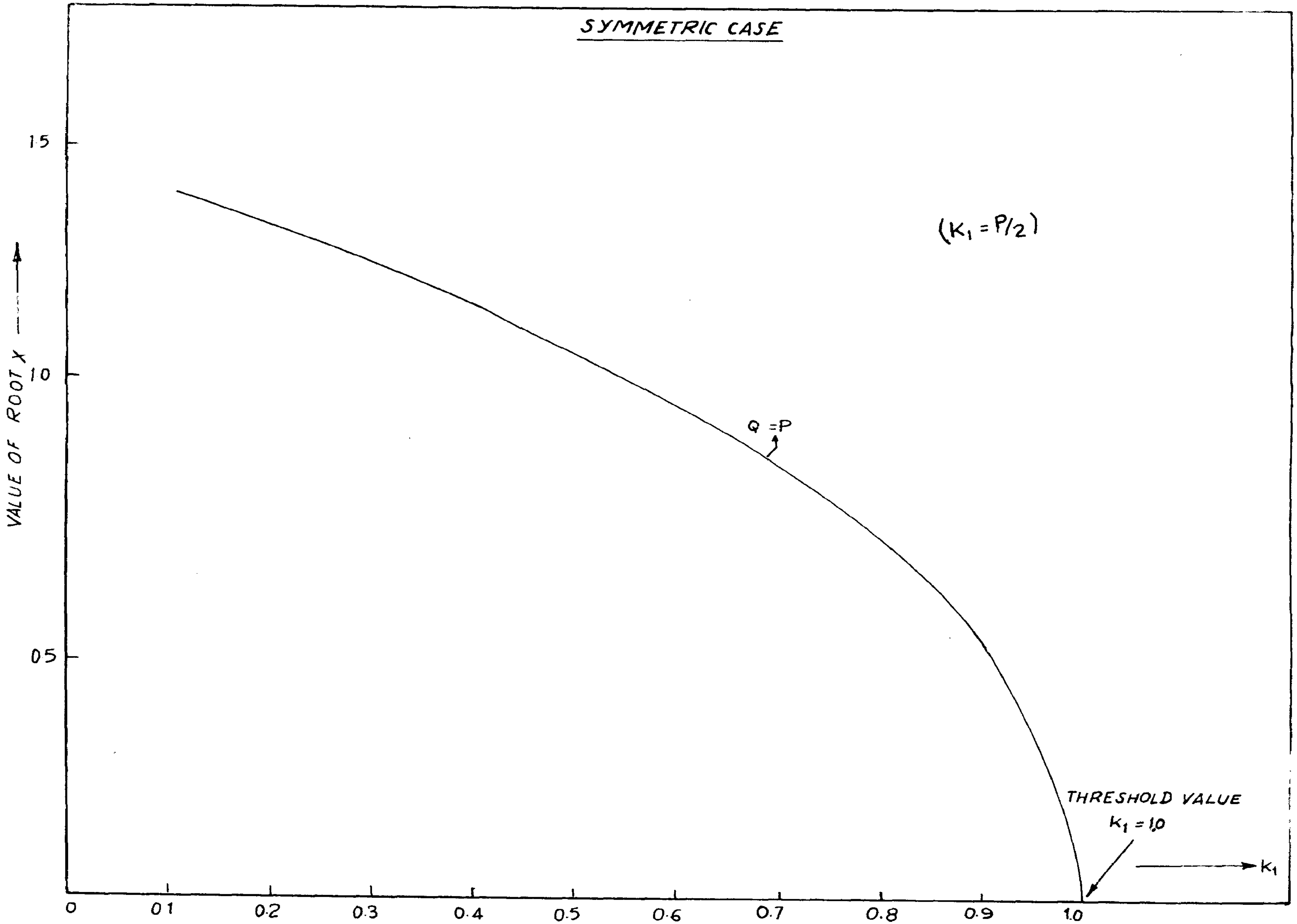


FIG.21- PLOT OF THE ROOT OF EQUATION

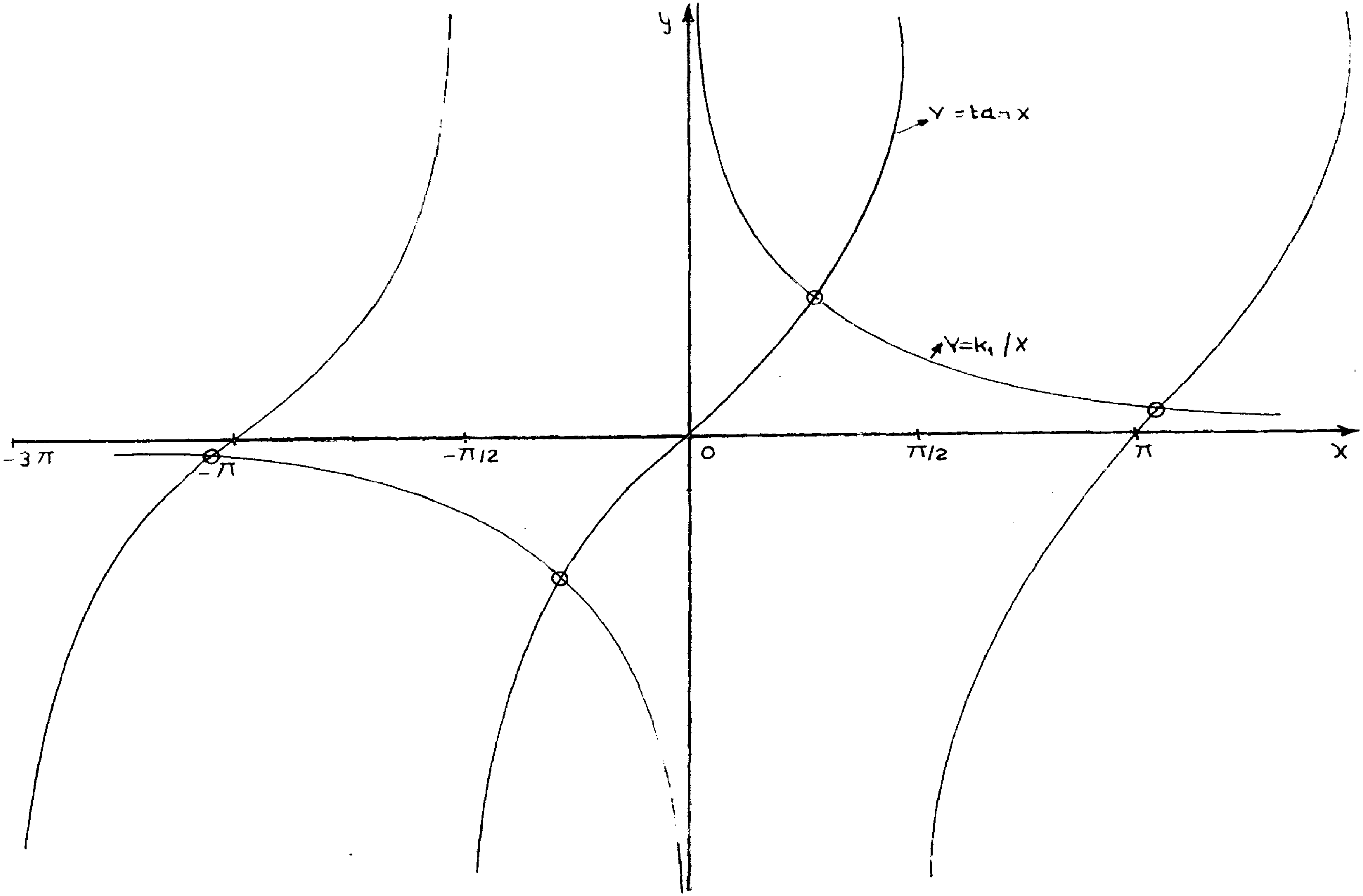


FIG. 22

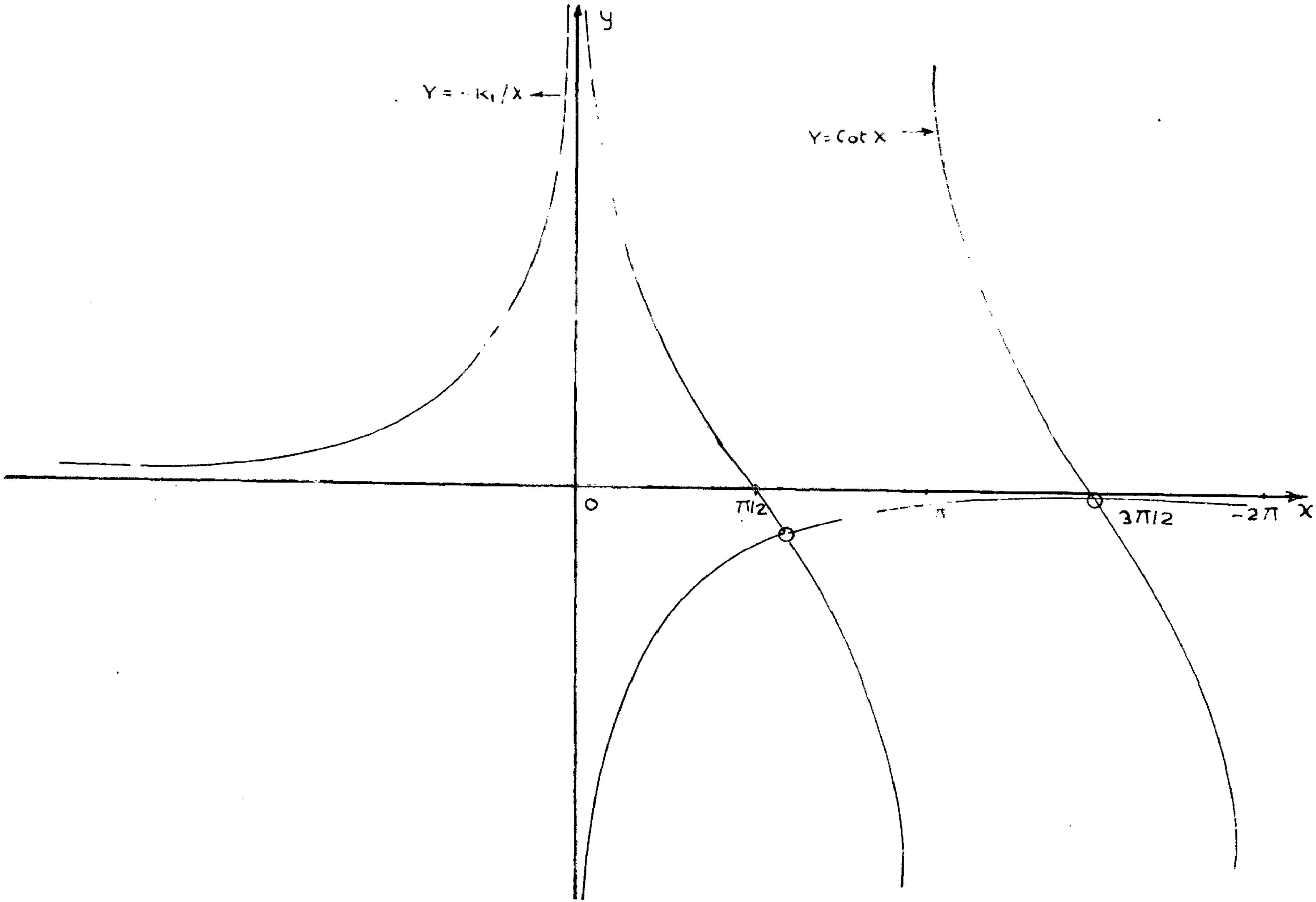


FIG. 23

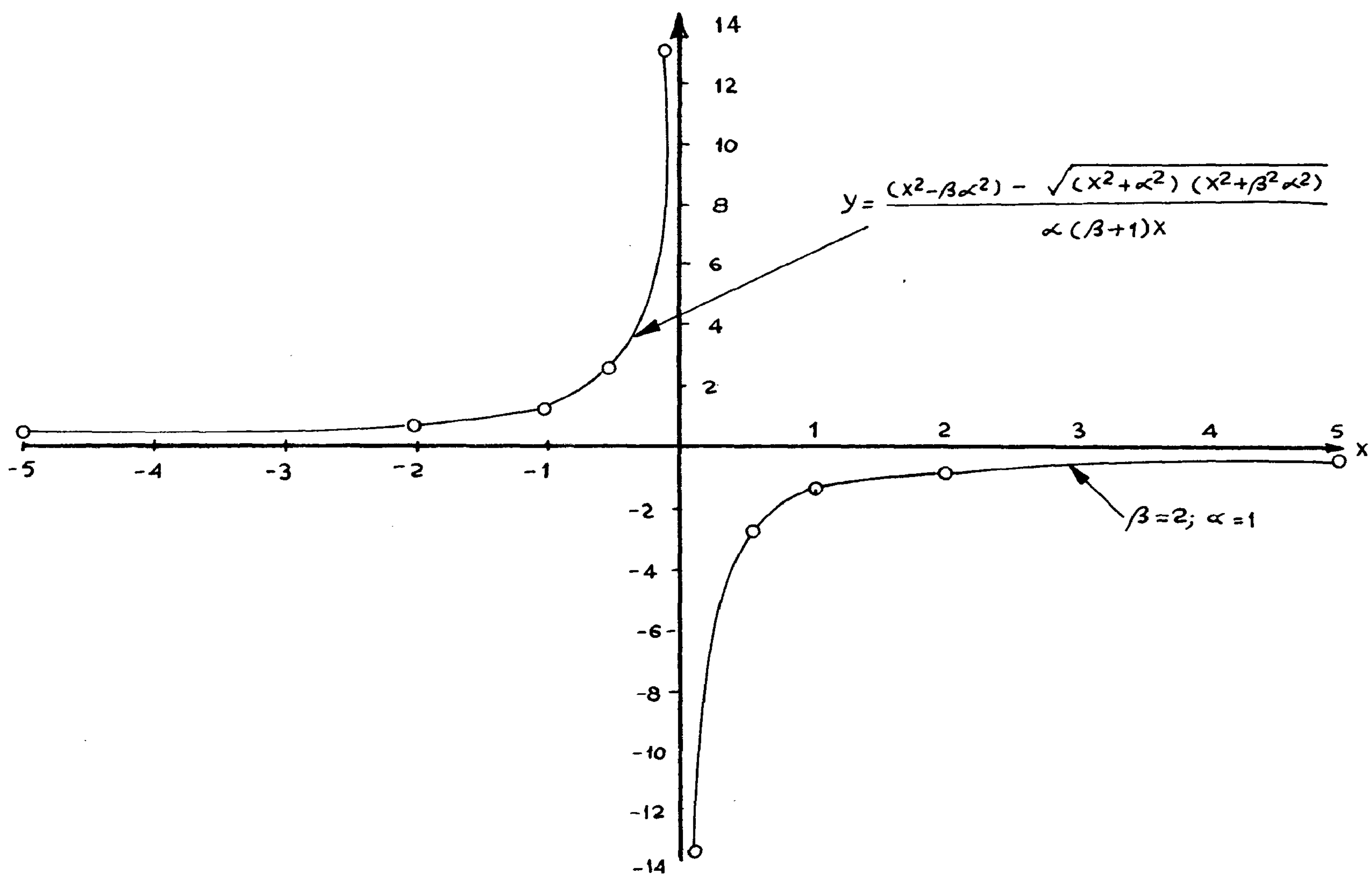

y = (x²-βα²) - √((x²+α²)(x²+β²α²)) / α(β+1)x
β=2; α=1
x
14
12
10
8
6
4
2
-2
-4
-6
-8
-10
-12
-14
-5
-4
-3
-2
-1
1
2
3
4
5

FIG. 24

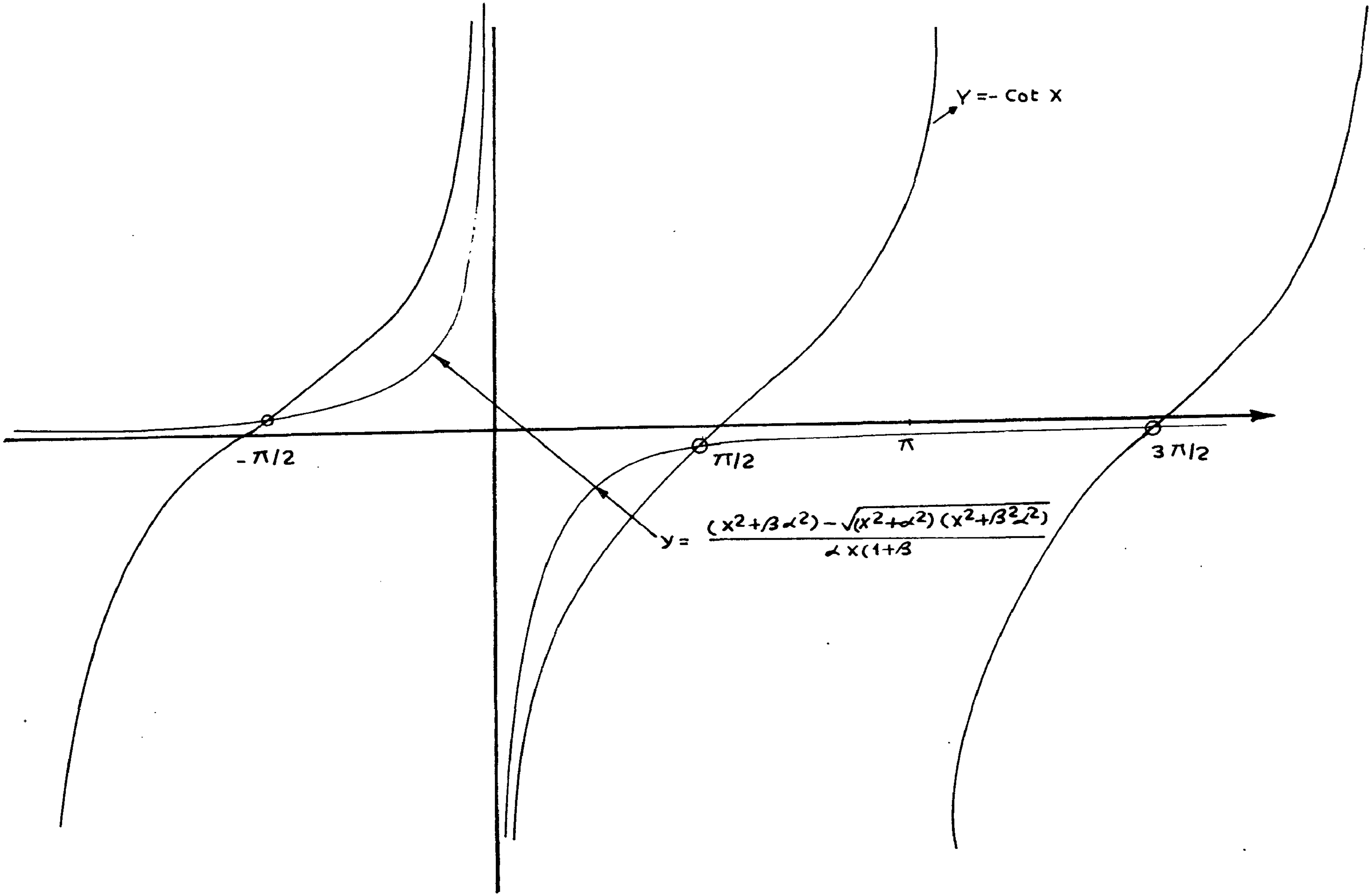

$Y = -\cot X$
$-\pi/2$
$\pi/2$
π
$3\pi/2$
$y = \frac{(x^2+\beta\alpha^2) - \sqrt{(x^2+\alpha^2)(x^2+\beta^2\alpha^2)}}{\alpha x(1+\beta}$

FIG. 25a

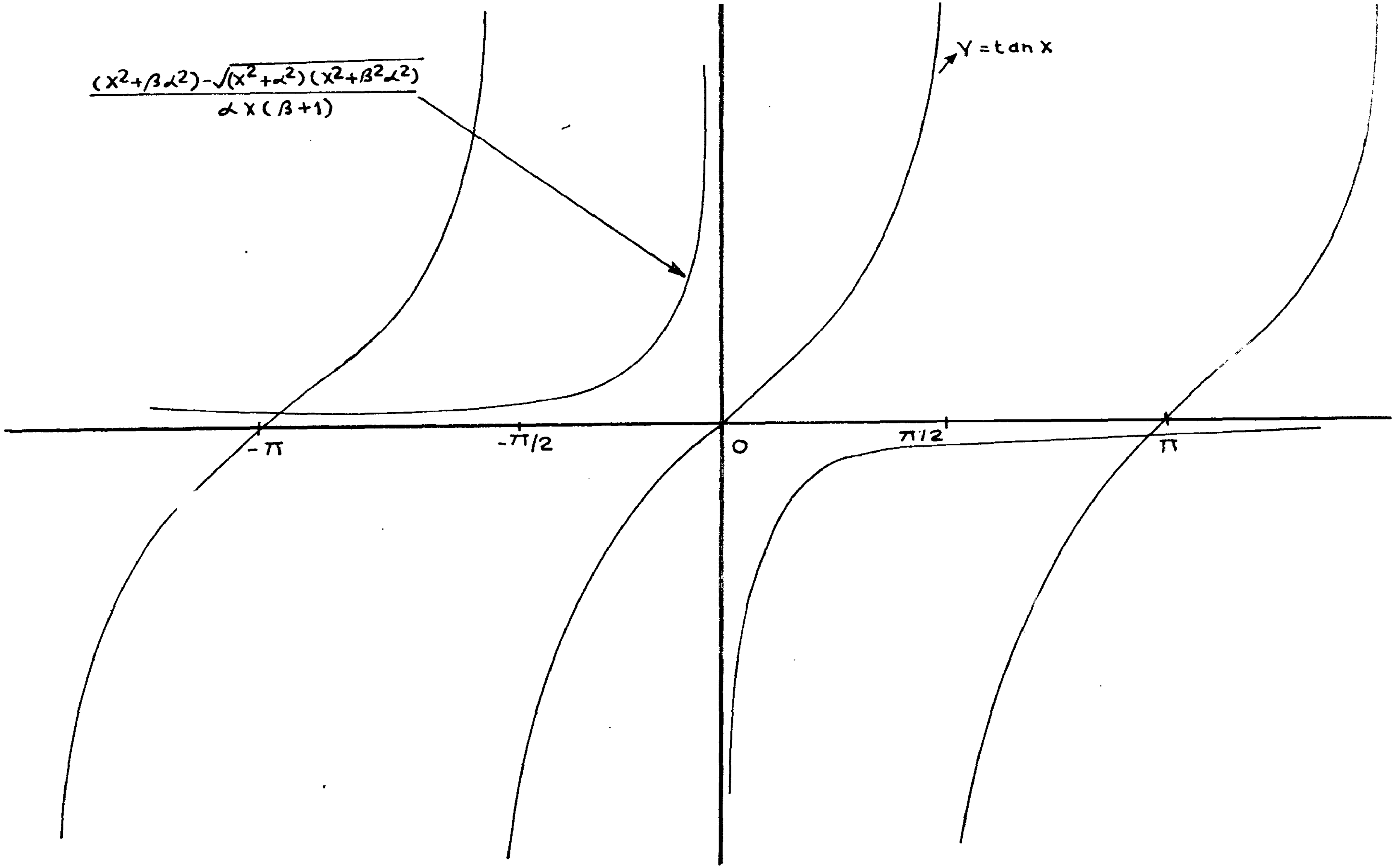
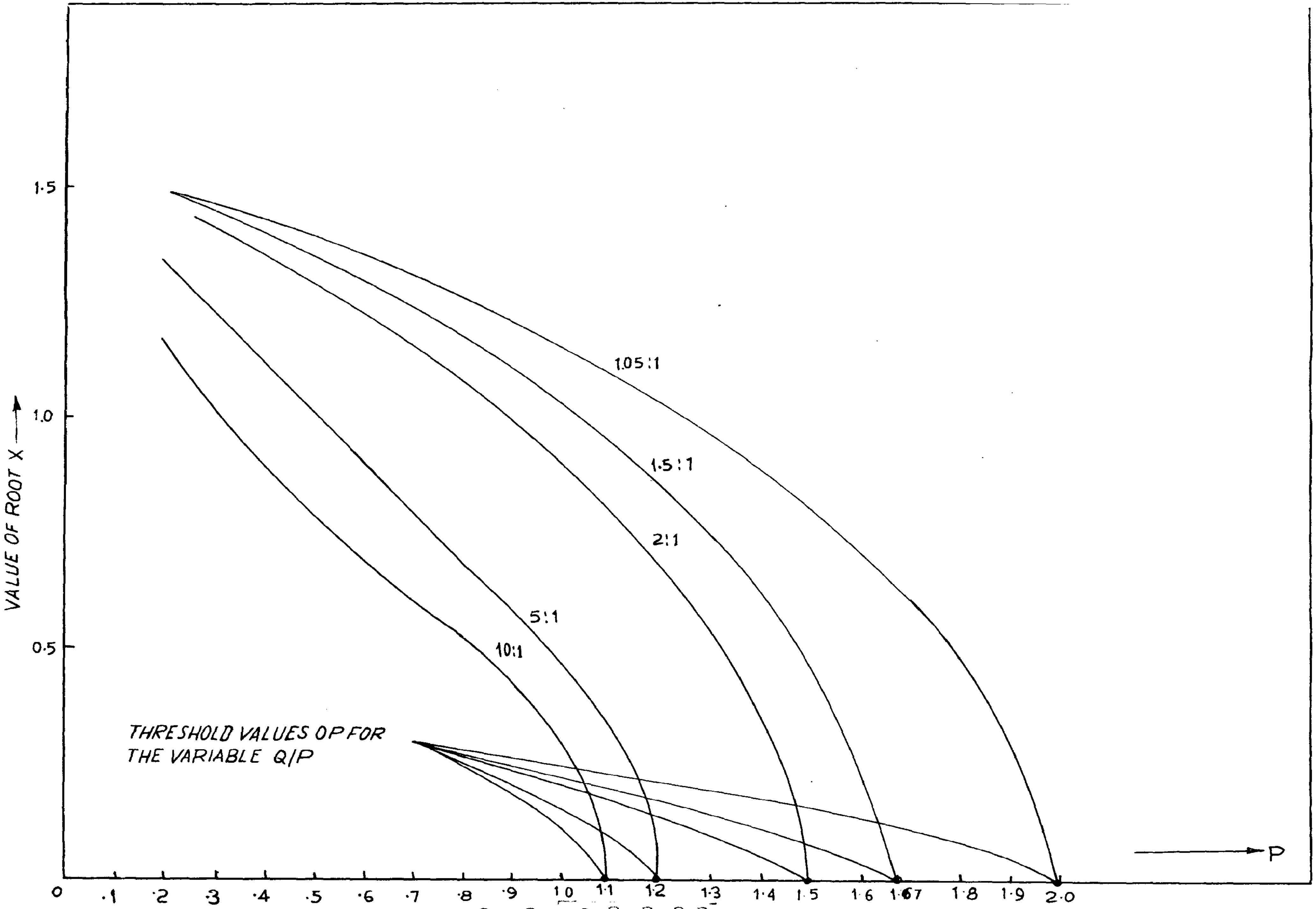

$\frac{(X^2+\beta\alpha^2)-\sqrt{(X^2+\alpha^2)(X^2+\beta^2\alpha^2)}}{\alpha X(\beta+1)}$
Y = tan X
-π
-π/2
0
π/2
π

FIG. 25 b



1.05:1
1.5:1
2:1
5:1
10:1
THRESHOLD VALUES OP FOR
THE VARIABLE Q/P
VALUE OF ROOT X
P
1·5
1.0
0·5
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1·0
1·1
1·2
1·3
1·4
1·5
1·6
1·67
1·8
1·9
2·0

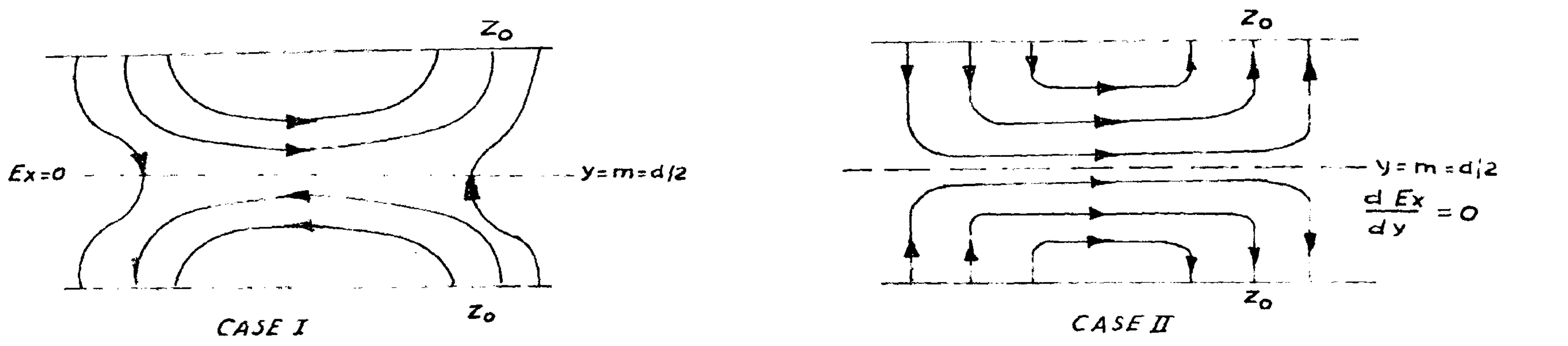

Zo
Ex=0
y=m=d/2
Zo
CASE I
Zo
y=m=d/2
d Ex / dy = 0
Zo
CASE II

FIG. 27

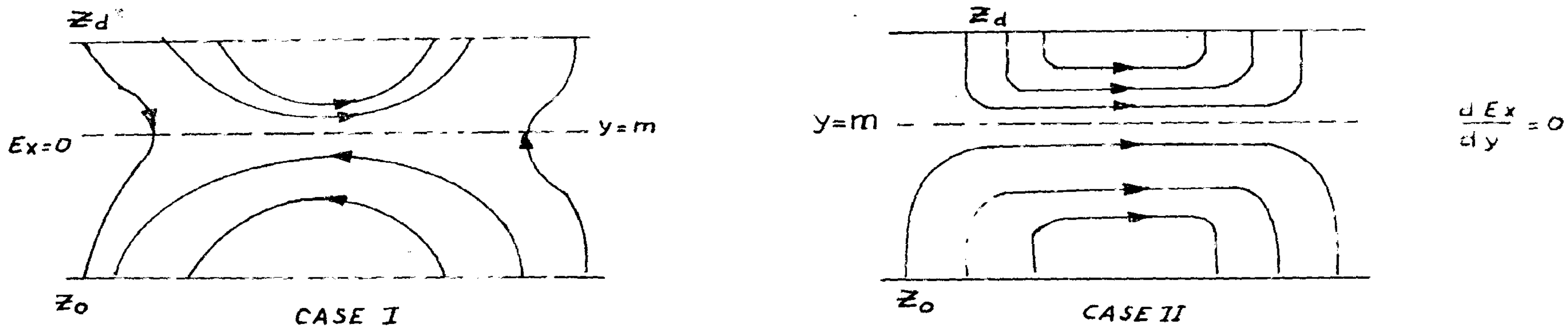
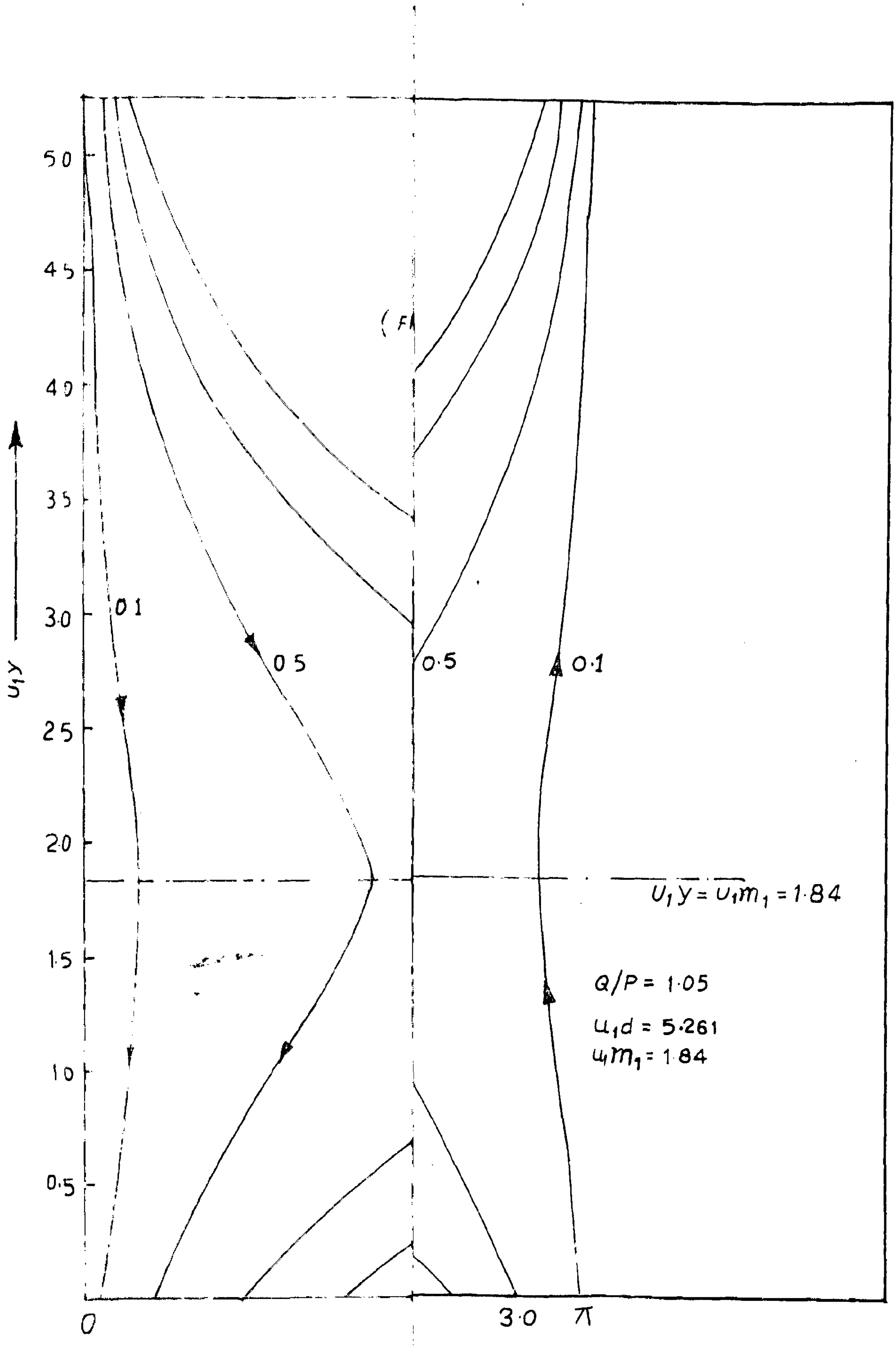

Zd
Ex=0
y=m
Zo
CASE I
Zd
y=m
d Ex / dy = 0
Zo
CASE II

FIG. 28



5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
u₁y
0.1
0.5
0.5
0.1
(F
U₁y = u₁m₁ = 1.84
Q/P = 1.05
u₁d = 5.261
u₁m₁ = 1.84
0
3.0
π

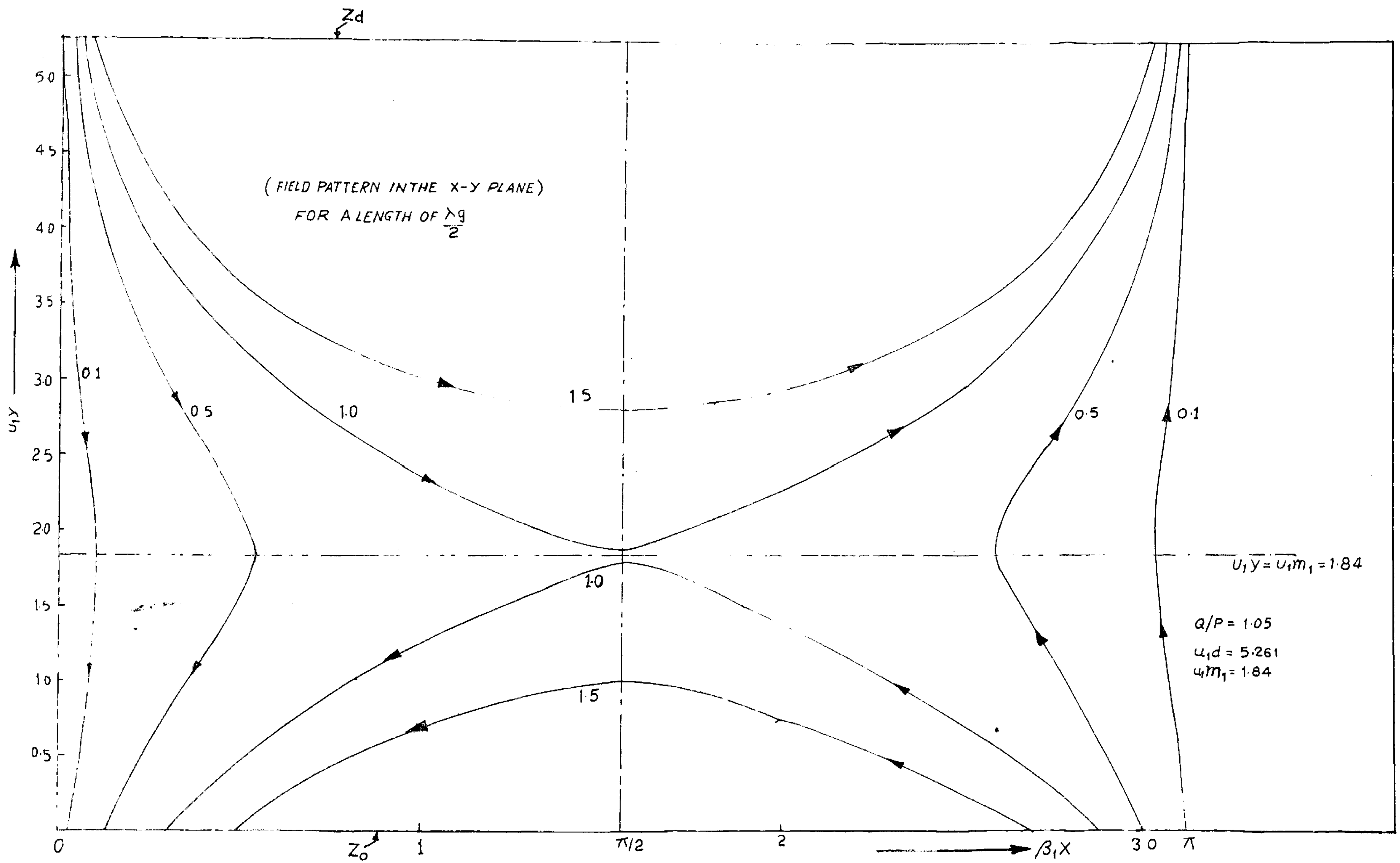


FIG. 2 9- PLOT OF COSH $u_1(y-m_1) = \frac{K}{\sin \beta_1 X}$

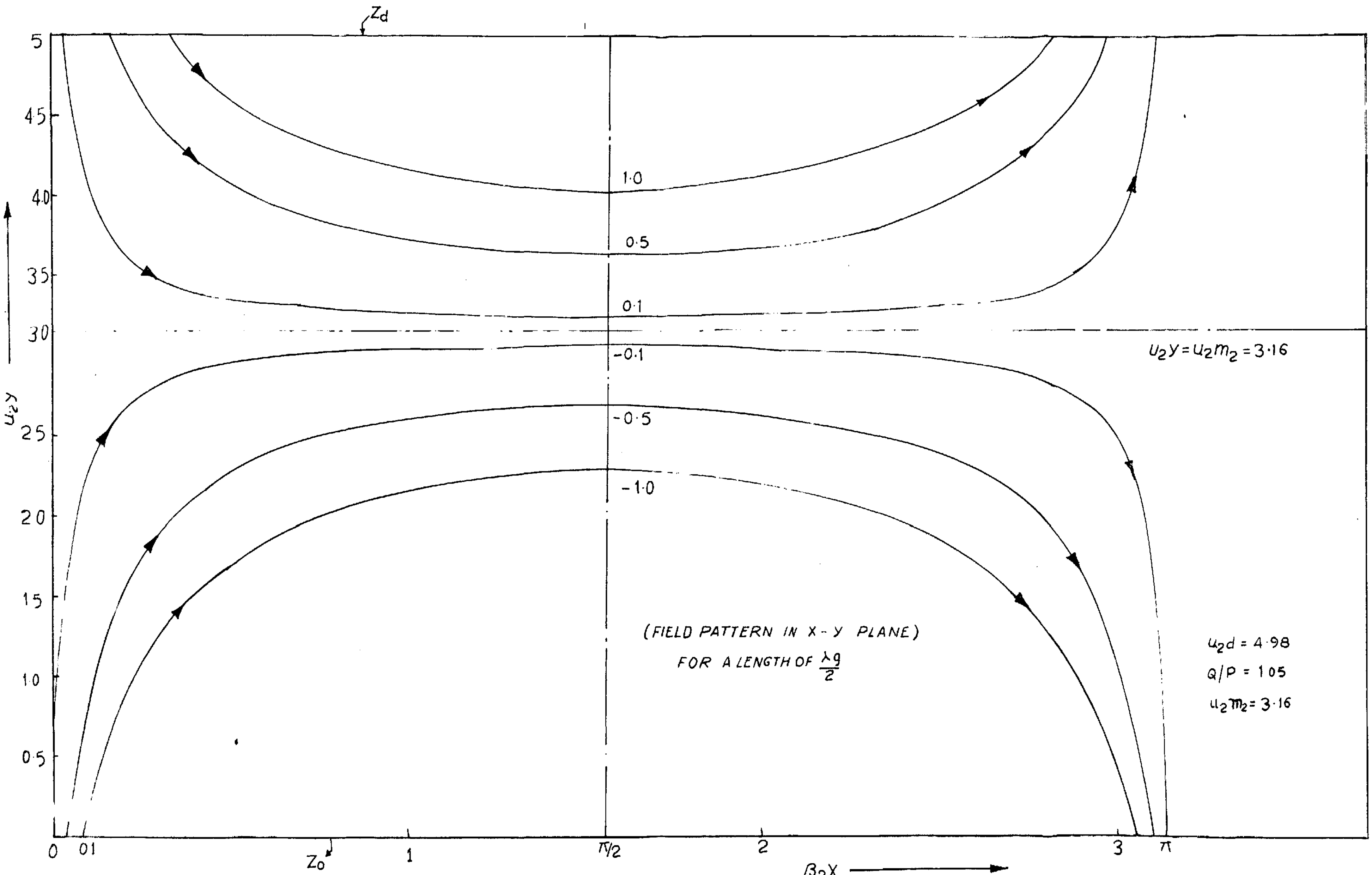


FIG. 30- PLOT OF $\sinh[u_2(y-m_2)] = \frac{K}{\sin \beta_2 X}$

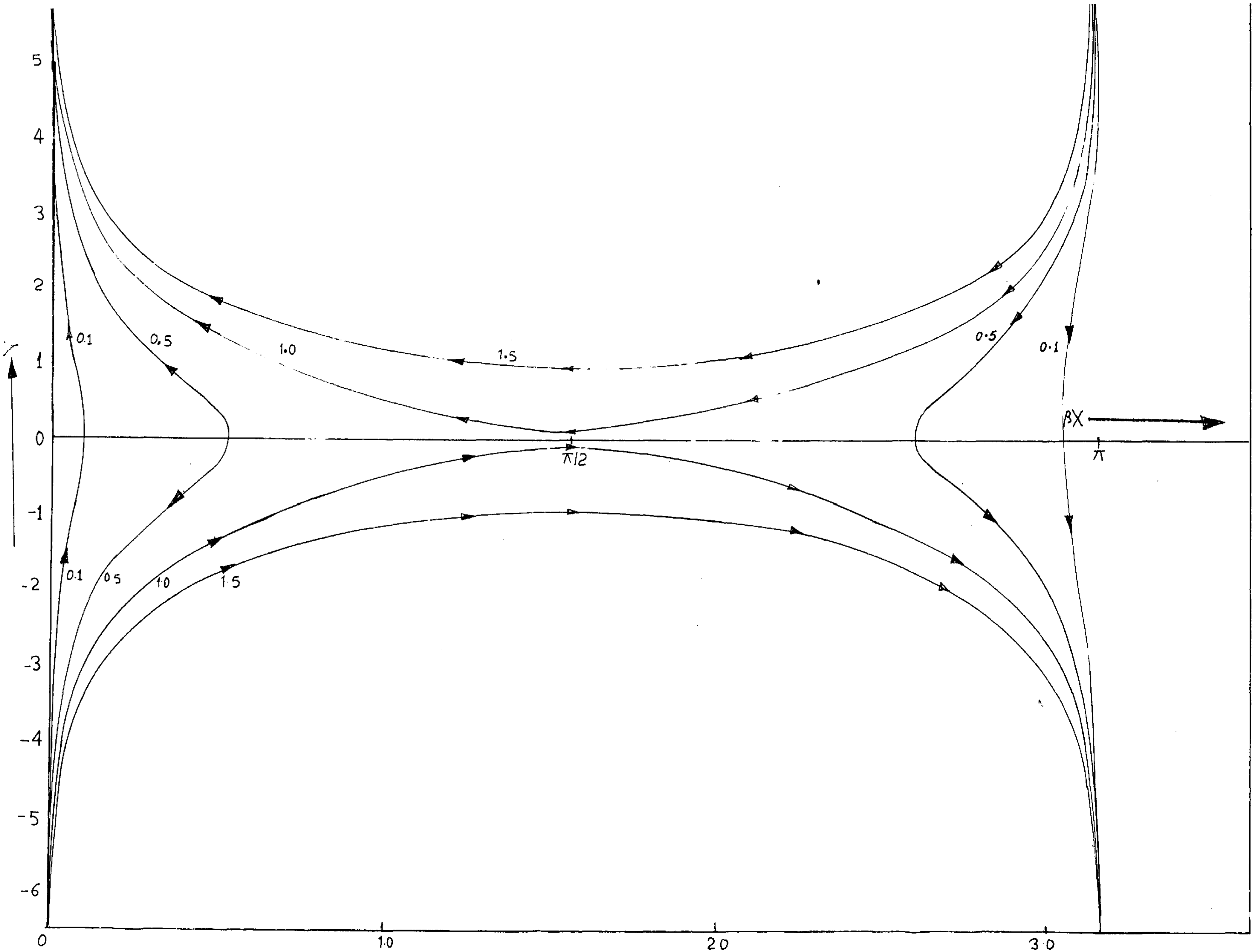


FIG. 31- FIELD PATTERN IN THE X-Y PLANE FOR THE EVEN TYPE OF SURFACE WAVE $\left(\cosh uy = \frac{K}{\sin \beta x}\right)$ OVER A LENGTH OF $\frac{\lambda g}{2}$

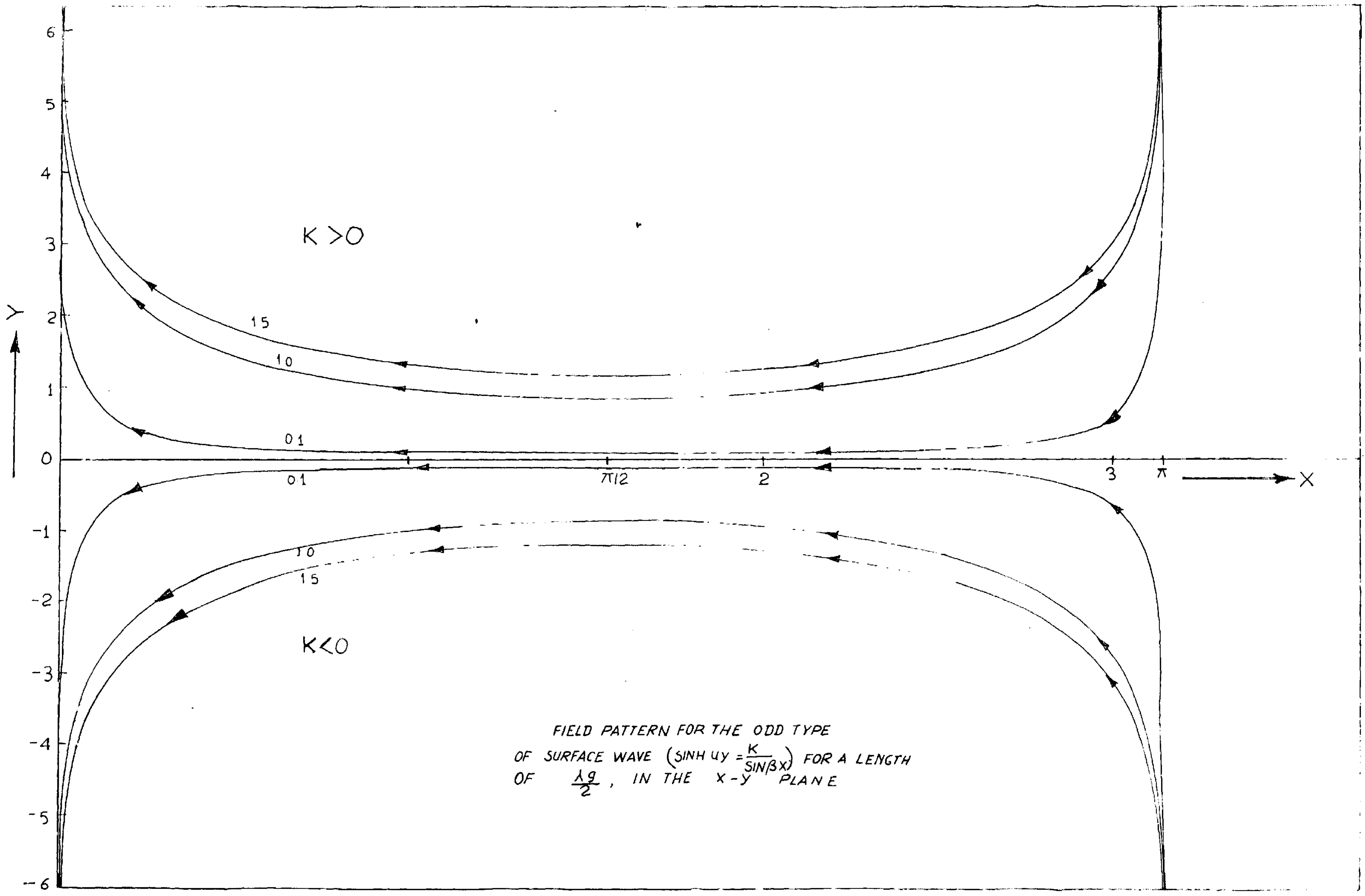

Y
X
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
π/2
2
3
π
K >0
K<0
15
10
0.1
0.1
10
15
FIELD PATTERN FOR THE ODD TYPE
OF SURFACE WAVE $\left(\sinh uy = \frac{K}{\sin \beta x}\right)$ FOR A LENGTH
OF $\frac{\lambda g}{2}$, IN THE X-Y PLANE

FIG. 32.

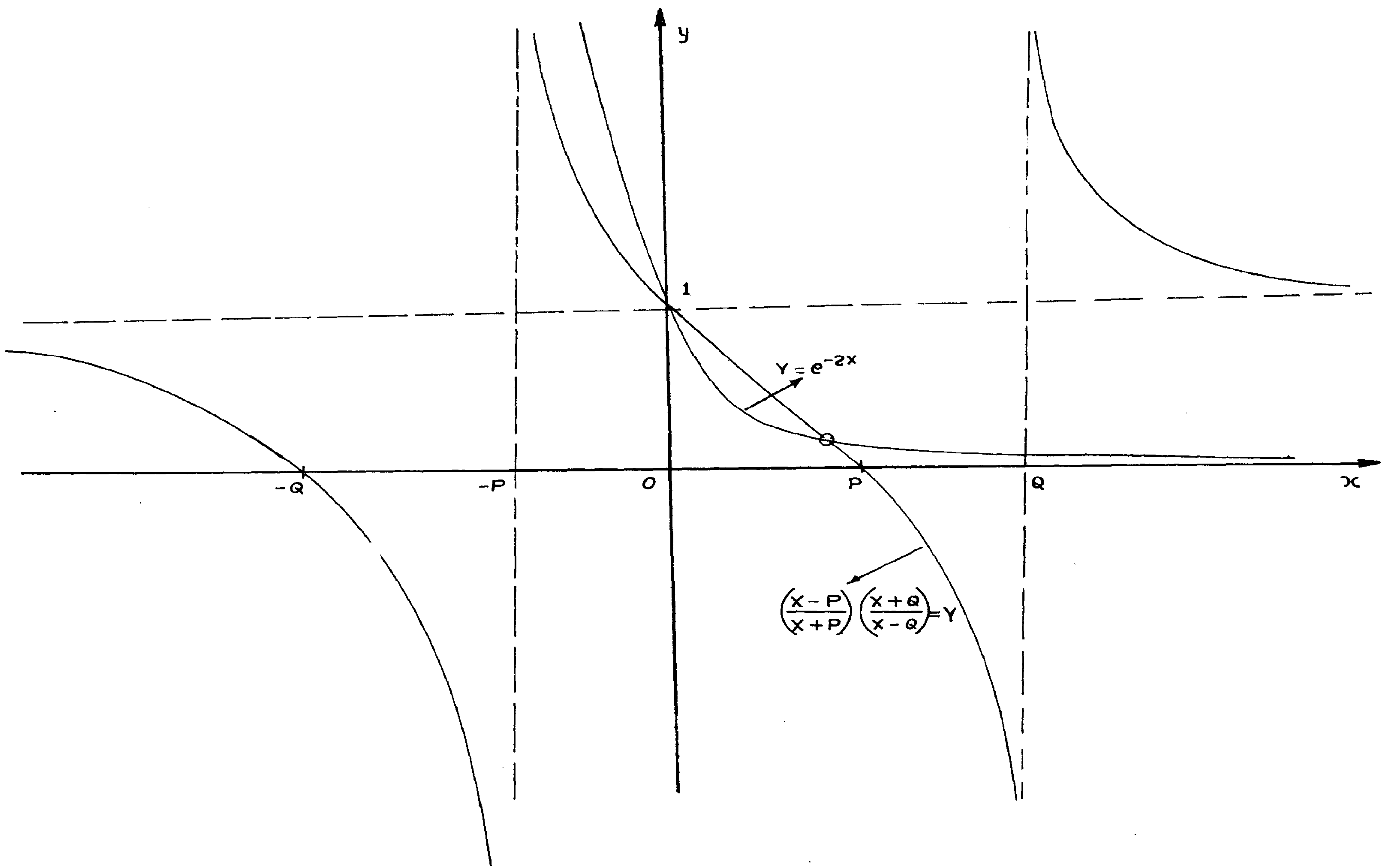
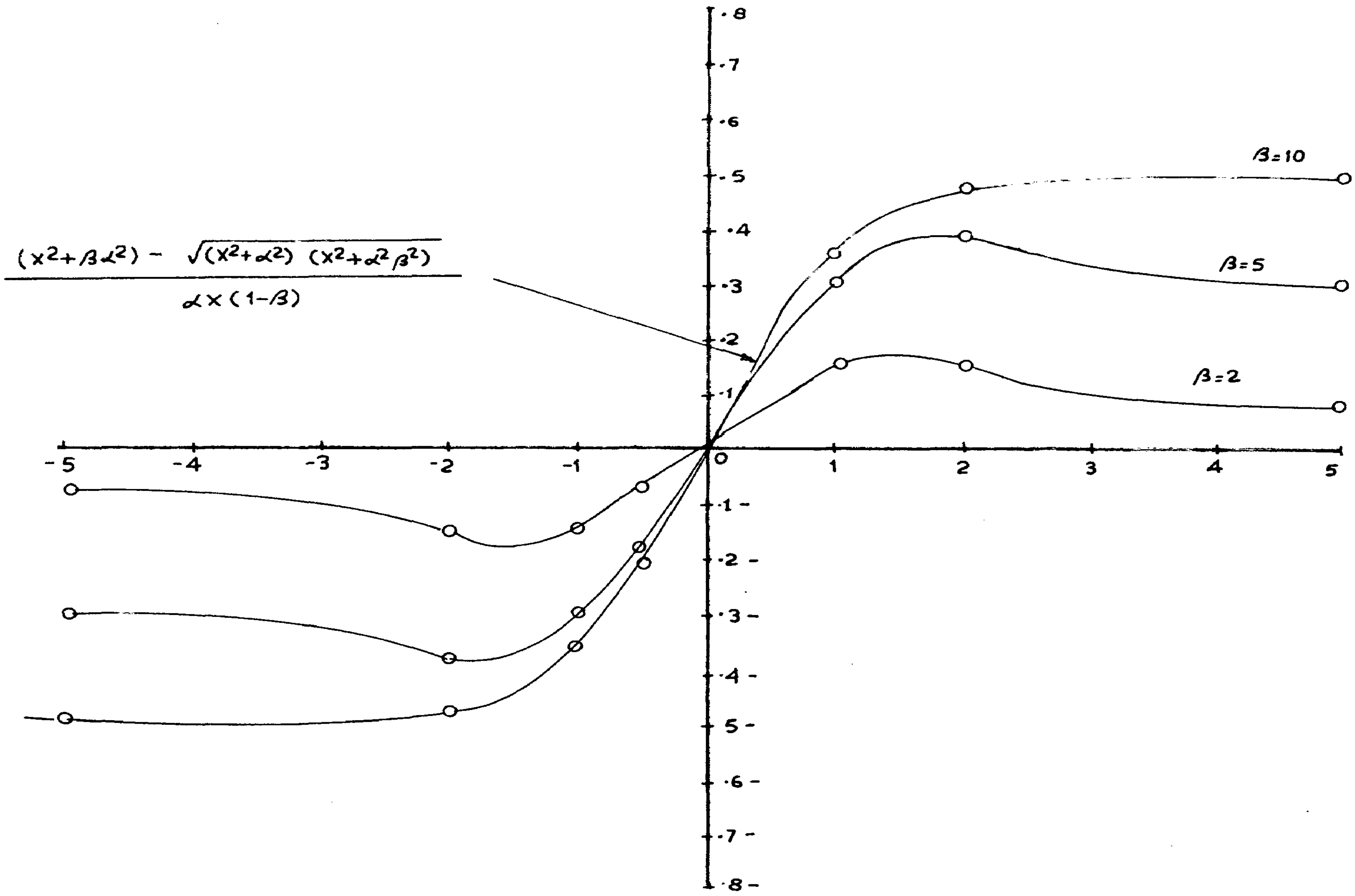


FIG. 33



$\frac{(x^2+\beta\alpha^2) - \sqrt{(x^2+\alpha^2)\,(x^2+\alpha^2\beta^2)}}{\alpha x(1-\beta)}$
β=10
β=5
β=2
·8
·7
·6
·5
·4
·3
·2
·1
0
-5
-4
-3
-2
-1
1
2
3
4
5

FIG. 34

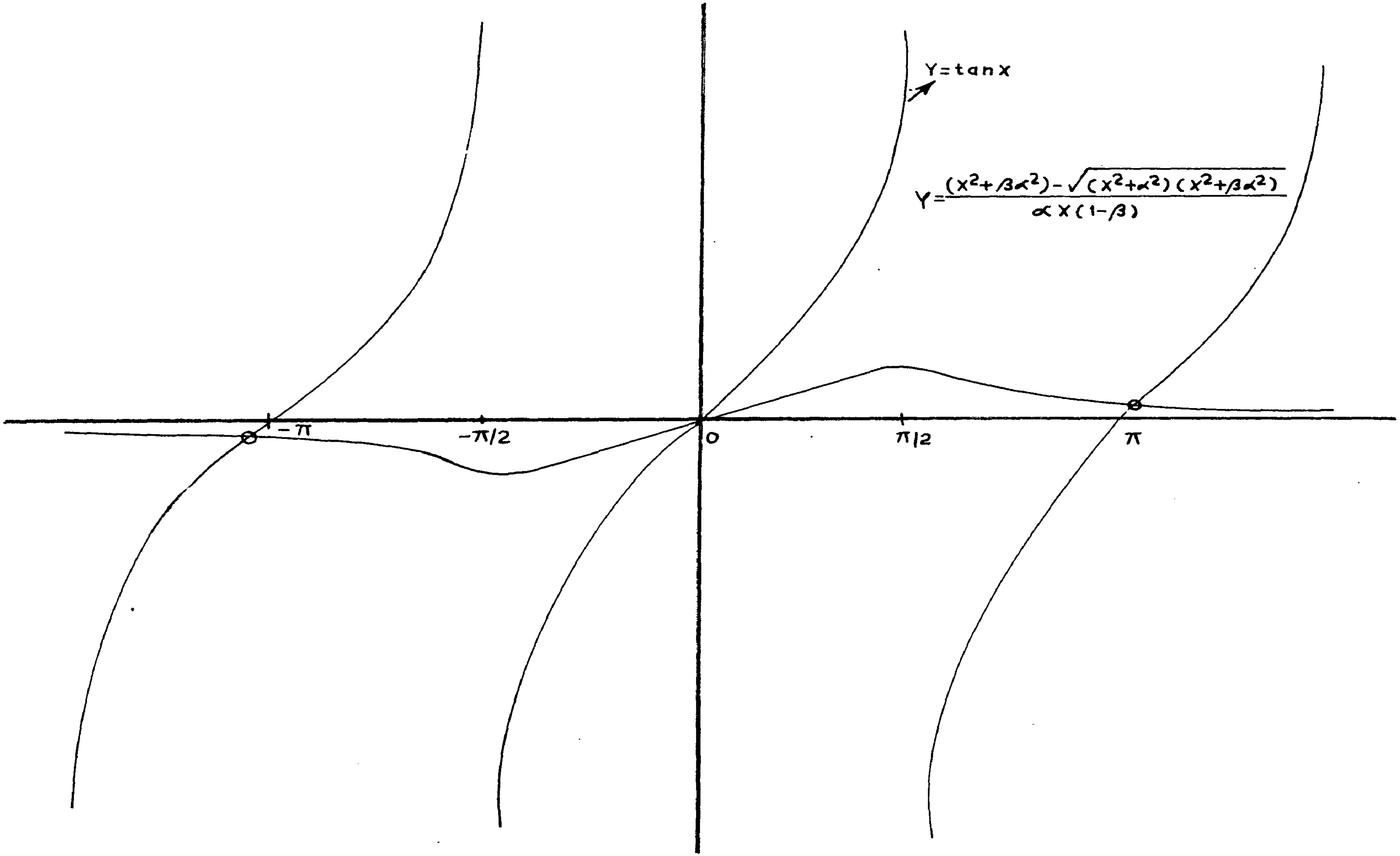

Y=tanX
$Y=\frac{(X^2+\beta\alpha^2)-\sqrt{(X^2+\alpha^2)(X^2+\beta\alpha^2)}}{\alpha X(1-\beta)}$
-π
-π/2
0
π/2
π

FIG. 35 a

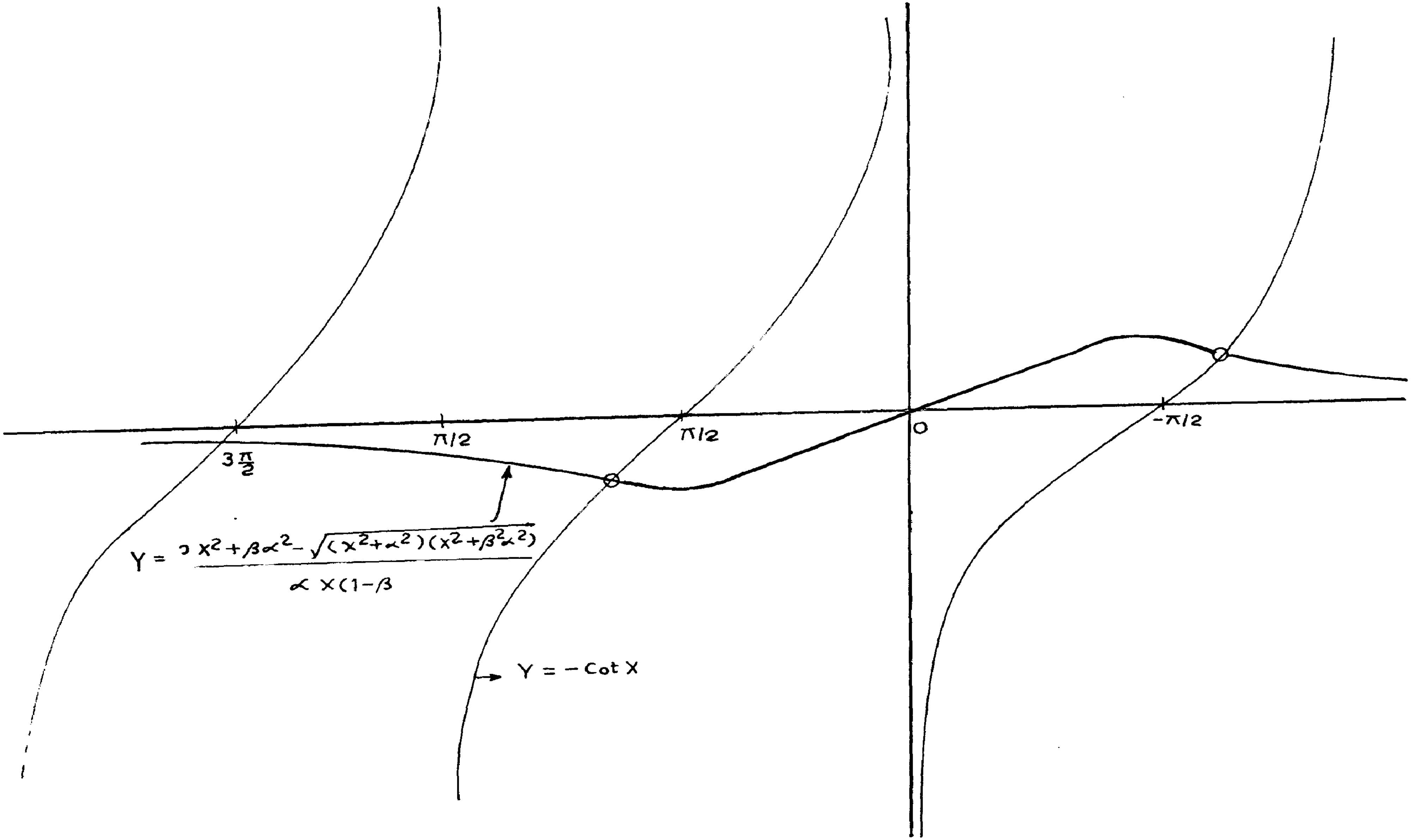


FIG. 35.b

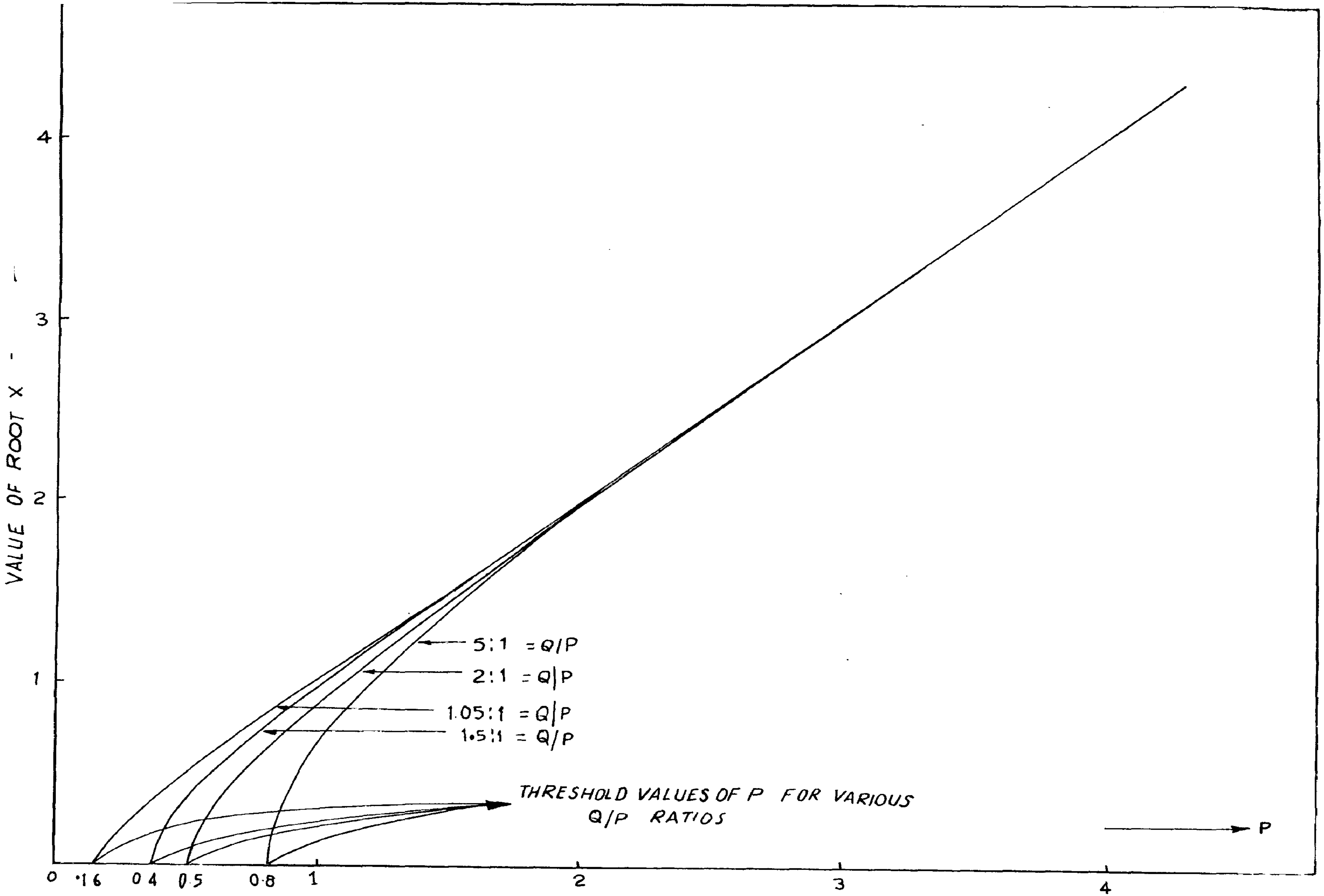


FIG 36 a PLOT OF REAL ROOT OF THE EQUATION $X-P/X+P \cdot X+Q/X-Q = e^{-2X}$ WITH Q/P AS PARAMETER

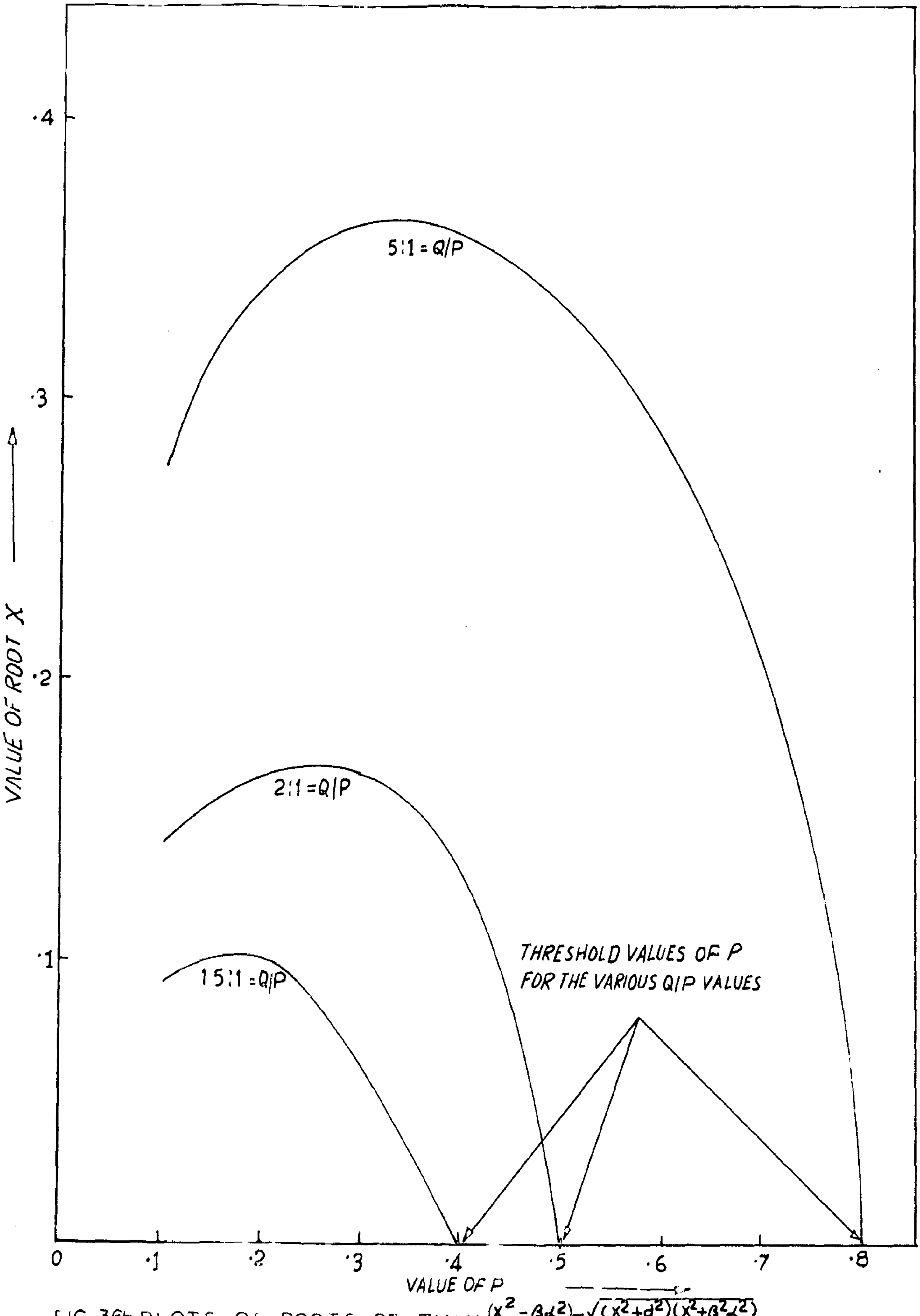


FIG 36b PLOTS OF ROOTS OF $\tan X \frac{(X^2-\beta\alpha^2)-\sqrt{(X^2+\alpha^2)(X^2+\beta^2\alpha^2)}}{\alpha(1-\beta)X}$ IN THE INTERVAL WITH $0 < X < \pi/2$ WITH Q/P AS PARAMETER ($X = Kcd/2$; $\beta = Q/P$; $\alpha = P/2$)