DESIGN OF NON-UNIFORMLY SPACED ANTENNA ARRAYS

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by
N.C.V. KRISHNAMACHARYULU

DEPTT. OF ELECTRONICS & COMMUNICATION ENGINEERING
UNIVERSITY OF ROORKEE
ROORKEE U.P.
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CERTIFICATE

CERTIFIED that the dissertation entitled "DESIGN OF NON-UNIFORMLY SPACED ANTENNA ARRAYS" which is being submitted by Shri N. C. V. KRISHNAMACHARYULU in partial fulfilment of the award of the Degree of MASTER OF ENGINEERING in ADVANCED ELECTRONICS, Department of Electronics and Communication Engineering of the University of Roorkee, Roorkee, is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other Degree or Diploma.

This is further to certify that he has worked for a period of Eight months from January to August, 1967 in preparing this thesis for Master of Engineering Degree at the University.

R K Arora

(R. K. ARORA)
Reader in
Electronics and Communication Engineering Deptt;
University of Roorkee
Roorkee U. P.

Roorkee
Dated September 11, 1967.

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PREFACE

In recent years non-uniformly spaced antenna arrays have received great attention as witnessed by a large amount of publication in this area. The main interest seems to be in seeking a way to reduce the number of elements, or , equivalently, seeking a way to broaden the bandwidth and the scanning range of the array The work in this field has been started since 1960, but no rigorous theory has been developed yet. The work so far done in the field is reviewed in Chapter II.

Dynamic programming is studied in Chapter III and IV as an optimizing technique in synthesizing unequally spaced ,symmetrical linear arrays. The criterion of optimization is to find an element combination which has the highest sidelobe level over a specified angular interval less than the highest sidelobe of any other combination.

A 25 element array is synthesized with aperture length 50 λ , and spacing quantization λ /2 . The synthesis technique is given in Chapter V . The results obtained are quite encouraging.

The calculations were performed on an IBM 7044 Computer.

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CHAPTER I

INTRODUCTION

Linear arrays with variable interelement spacings have received increasing attention in recent years. The reason for this interest is primarily that a considerable saving in the number of array elements is possible in large directional antenna arrays where high resolution is important, as in the fields of Radar, Satellite communication, Radio Astronomy. Further, by changing the interelement phase the main beam of the radiation pattern can often be steered through a wider angle and over a much larger frequency bandwidth than in possible with equispaced arrays. Both equally and unequally spaced linear arrays are very simple to analyse. Well developed methods are also available for designing linear antenna arrays with equispaced elements that will produce a desired radiation pattern with reasonable accuracy. Most of these methods, theoretical as well as experimental, make use of the fact that the radiation pattern of an array of equispaced elements can be expressed as polynomial, the coefficients of which are used to determine the array excitation coefficients. Dolph designed a theoretical optimum broadside array with equispaced elements making use of the properties of the Tchebyscheff polynomials.

DuHamel extended Dolph's method to the case of an end-fire array with equispaced elememts. For an arbitrary pattern, Woodward and Lawson have given a method of designing a linear array of equispaced elements which will produce a radiation pattern that exactly equals a desired radiation pattern in a number of directions in space which are chosen equidistant in Sin Θ , Θ being the angle between the normal to the array axis and the direction of observation.

The radiation pattern of an array of equispaced elements is a periodic function. It is thus generally necessary to chose the interelement spacing not larger than one half wavelength in order to avoid more than one period of the radiation pattern appearing in visible space In special cases where more than one period of radiation pattern can be allowed in visible space, a larger spacing can be chosen close to one wave length for a broadside array, before several pencil beams will appear.

Spacings of less than one half wavelength are not very practical, with such small spacings, the coupling between the elements of the array will be strong and the precribed excitation coefficients of the array may be hard to realize . Therefore, a linear array of equispaced elements can seldon be used to cover a large bandwidth.

For arrays with variable interelement spacings, a prescribed radiation pattern can be approximated more closely than with constant spacings. The reason for this is that by chosing the spacings as independent variables, an additional degree of freedom is gained which can be used to control the radiation pattern. This has been discussed by Unz (1960). Very often, however, a good approximation to a radiation pattern that is given in advance can still be obtained only when the average interelement spacing is not larger than one half wavelength. As an example, an array where the elements are spaced according to the zeros of Legendre polynomial of the same order as the number of elements can be made to approximate a given radiation pattern very closely. However, an average interelement spacing must be less then one half wavelength.

When the array to be used as a directional antenna with high resolution, the unequally spaced array will often be much superior to the equispaced arrays. The equally spaced array requires fewer elements to produce a certain resolution, and the main beam can be steered over a larger frequency bandwidth, than is possible with an equispaced array. However, there is a lower limit to the sidelobe level attainable.

Although in recent years unequally spaced arrays have shown to be useful, a precise mathematical theory has not been fully developed. These arrays have been considered by methods involving a larger number of simultaneous equations, by perturbation methods, by computations for trial sets of element spacings and iterative procedures, and by approximating continuous aperture illuminations. These methods are by and large empirical and generally make use of the modern high-speed digital computers. The antenna arrays have been analysed by two sets of parameters, namely 1) Variable spacings in uniformly illuminated linear arrays and 2) Variable spacings and amplitudes in linear and planner arrays. In the past considerable attention has been paid to the former without considering the later , and the following conclusions have been derived.

1. The sidelobe level is closely related to the number of elements and to a much lesser degree to the aperture dimension. Extremely high reduction can be achieved with very few elements . On the other hand for a given number of elements higher and higher resolution can be obtained by spreading the elements at random over a large aperture.
2. The 3 db beam-width of the main lobe depends primarily on the length of the array.

3. The product of bandwidth and steerability can be made much larger than for conventional equispaced arrays.

The following is the outline of the discussions in the coming Chapters. A brief but thorough account of uniformly spaced array with Dolph-Tchebyscheff optimisation to show its inferiority in a large antenna design is discussed. The work so far done in the field of non-uniformly spaced antenna arrays and their synthesis techniques is reviewed. A good discussion about the non-uniformly spaced antenna array with special reference to side-lobe level, length and gain considerations is given, in which it has been shown that there is a saving in the number of elements.

Dynamic programming is studied as an optimising technique in the synthesis of unequally spaced symmetrical linear antenna arrays. The criterion of optimization is to find an element combination which has the highest side-lobe level over a specified angular interval, less then the highest sidelobe level of any other combination. A 25 element array is synthesized with aperture length 50 λ and spacing quantization $\lambda/2$ using an IBM 7044 computer. The results obtained have established conclusively that the dynamic programming method, if properly used, can yield excellent results, as is amply brought out by the fact that the results obtained are considerably superior to those reported by other investigators using different techniques.

CHAPTER II

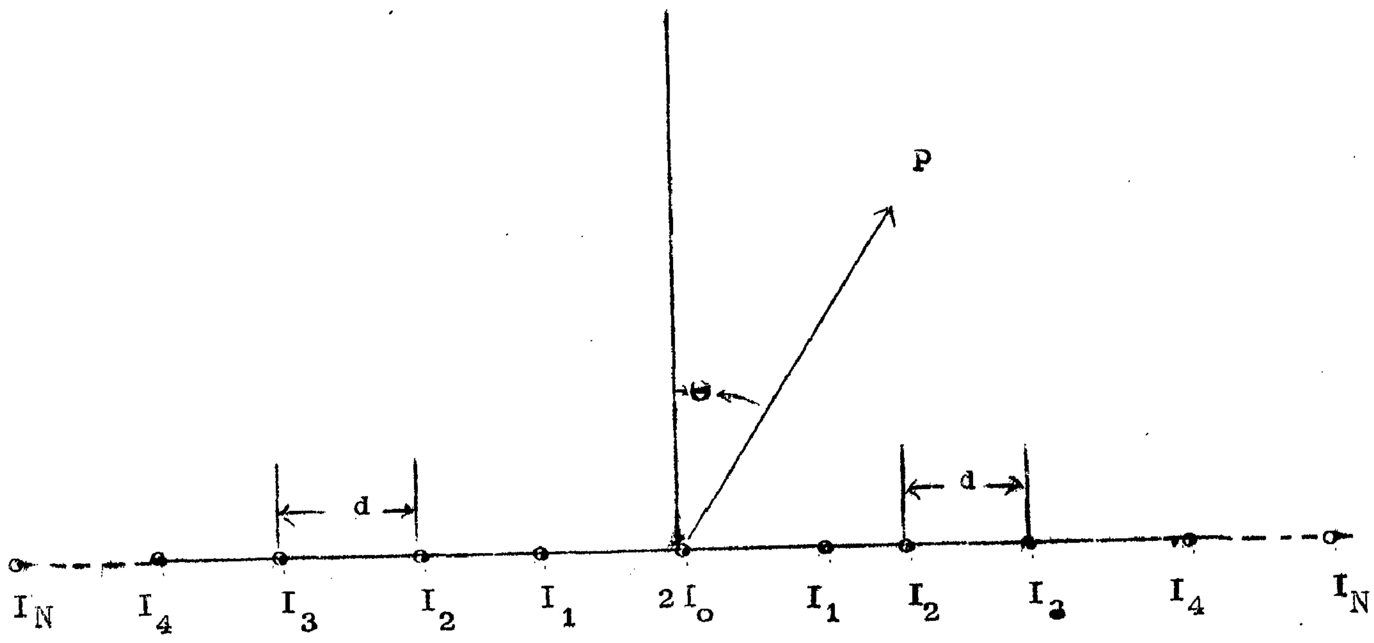
REVIEW OF THE CONTRIBUTIONS TO THE NON UNIFORMLY SPACED ANTENNA ARRAYS AND THEIR SYNTHESIS TECHNIQUES

2.1. EQUALLY SPACED ARRAYS

Before giving a detailed review of the contributions to the non-uniformly spaced antenna arrays, a brief review of Dolph's derivation for the equispaced broadside arrays will be given. It is shown that the Tchebyscheff current distribution" may be calculated after either the sidelobe level or the position of the first null is specified. The "Tchebyscheff pattern" resulting from this current distribution is optimum in the sense that a) if the sidelobe level is specified, the beam width of the resultant pattern is minimum, or b) if the beam width is specified, the sidelobe level will be a minimum.

The radiation pattern of a linear equispaced broadside symmetric array of point sources as shown in the Figure No. 1 is proportional to

$$\left| E_{2N-1} (\Theta) \right| = \left| \sum_{k=1}^{N} I_k \, \text{Cos} \left[\frac{2k-1}{2} \left(\frac{2\pi d}{\lambda} \right) \text{Sin} \, \Theta \right] \right| \quad \ldots.(1)$$



REFERENCE SYSTEM FOR AN ARRAY OF (2N + 1) POINT SOURCES

FIG. No. 1

$$\left| E_{2N}(\theta) \right| = \left| \sum_{k=0}^{N} I_k \cos \left[k \left(\frac{2\pi d}{\lambda} \right) \sin \theta \right] \right| \qquad \ldots (1')$$

Where (1) and (1') apply to an even number (2N) and an odd number (2N+1) of elements respectively. I_k represents the current in the k th element from the centre of the array. The above equations are valid only if all the currents are in phase along the array. Only the even case will be discussed here.

Introducing the new variable

$$U = \frac{\pi d \sin \theta}{\lambda}$$ simplifies (1) to,

$$F_{2N-1}(U) = \sum_{k=1}^{N} I_k \cos (2k-1) U \qquad \ldots (2)$$

where, henceforth, only the absolute values of all pattern expressions will be considered so that the absolute value signs may be omitted.

A term of the form cos (nU) may be expanded into a polynomial in powers of Cos U wherever n is an integer :

$$\cos (2k-1) U = \sum_{q=1}^{k} A_{2q-1}^{2k-1} \; x^{2q-1} \qquad \ldots (3)$$

Where

$$A_{2q-1}^{2k-1} = (-1)^{k-q} \sum_{p=k-q}^{k} \binom{p}{p-k+q} \binom{2k-1}{2p}$$

$x = \cos U$ and $\binom{n}{m} = \frac{n!}{m!\,(n-m)!}$

When (3) is substituted into (2) and the summation signs arranged, the pattern equation, $F_{2N-1}(U)$, takes the polynomial form,

$$G_{2N-1}(x) = \sum_{q=1}^{N} \sum_{k=q}^{N} I_k \, A^{2k-1}_{2q-1} \, x^{2q-1} \qquad \ldots.(4)$$

Where 'x' is restricted to $|x| = |\cos U| \leq 1$

It will now be shown that with suitable values of the currents (I_k) the antenna pattern described by the polynomial (4) may be made to coincide with the pattern of an appropriate Tchebyscheff polynomial, which in turn possesses all the previous mentioned optimum properties. The normalized Tchebyscheff polynomials are defined by

$$T_n(z) = \cos(n \text{ arc } \cos z) \;; \quad |z| \leq 1 \qquad \ldots.(5)$$

Where 'n' is an integer. Clearly the maxima and the nulls of (5) are given by

$$|T_n(z)| = 1 \quad \text{for } z = \cos \frac{k\pi}{n} \;; \quad k = 0,1,2,\ldots.n \qquad \ldots(6)$$

$$|T_n(z)| = 0 \text{ for } z = \cos (2k-1)\frac{\pi}{2n}, \quad k = 1,2,\ldots n$$

$T_n(z)$ is also of the form $\cos n\phi$, where $\phi = \text{arc}\cos z$ and n is an integer. Therefore, it may be converted into a polynomial in powers of $\cos\phi = \cos(\text{arc} \cos z) = z$

Expansion of $T_{2N-1}(z)$ using (3) yields

$$T_{2N-1}(z) = \cos\left[(2N-1)\,\text{arc} \cos z\right]$$

$$= \sum_{q=1}^{N} A^{2N-1}_{2q-1} \, z^{2q-1} \qquad \ldots.(7)$$

Forms (5) and (7) of Tchebyscheff polynomial are equivalent. The two polynomials may be made to correspond exactly by restricting the variable in (7) to $z \leq z_o$, where z_o is an arbitrary parameter, and setting $x = \cos U = z / z_o$

Equation (7) may als now be written as

$$T_{2N-1}(z_o x) = \sum_{q=1}^{N} A^{2N-1}_{2q-1} \, z_o^{2q-1} \, x_o^{2q-1} \qquad ..(8)$$

Where $|x| \leq 1$. Equations (8) representing Tchebyscheff polynomial limited to the region within $\pm z_o$ and (4), representing the antenna pattern, are now in the same form. Corresponding coefficients may now be equated and solved for the currents. Thus

$$\sum_{k=q}^{N} I_k \, A^{2k-1}_{2q-1} = A^{2N-1}_{2q-1} \, z_o^{2q-1}, \quad q=1,2,3\ldots N$$

$$\ldots..(9)$$

Whence

$$I_q = \frac{1}{A^{2q-1}_{2q-1}} \left[A^{2N-1}_{2q-1} \; z_o^{2q-1} - \sum_{k=q+1}^{N} I_k \; A^{2k-1}_{2q-1} \right] \qquad \ldots(10)$$

If I's are computed from (10), the resultant field pattern given by (4) will agree with Tchebyscheff pattern shown in (8). The sidelobes and the nulls of the antenna pattern will coincide with the maxima and minima of the Tchebyscheff pattern given by (6) and will occur in the region $|z_o x| \leq 1$. In the region $1 \leq |z_o x| \leq z_o$, the Tchebyscheff Polynomial rises very steeply. This portion will represent the main lobes whose shape may be deduced from the polynomial form of $T_N(z_o x)$. It was shown by Dolph that the Tchebyschaff pattern yields a minimum beam width when the sidelobe levels are known and a minimum sidelobe level when the beam width is specified. The adjustable parameter z_o may be calculated when either the side lobe level or the beam width (position of first nulls) is given. In the first case z_o must satisfy the equation $T_{2N-1}(z) = \Upsilon$, where $\Upsilon/1$ is the specified main beam to side lobe ratio. Since $\Upsilon > 1$, z_o must be evaluated from the polynomial form of $T_{2N-1}(z_o)$.

$$z_o = \frac{1}{2} \left[(\Upsilon + \sqrt{\Upsilon^2 - 1})^{1/2N-1} + (\Upsilon - \sqrt{\Upsilon^2 - 1})^{1/2N-1} \right] \qquad \ldots(11)$$

or more easily from

$$z_o = \text{Cosh}\left(\frac{\text{arc Cosh } \Upsilon}{2N-1}\right)$$

From (6) the nulls of $T_{2N-1}(z_o x)$ are at

$$z_o x = \text{Cos}\left(\frac{2k-1}{2}\,\frac{\pi}{2N-1}\right)$$

Where

$$z_o x_1^0 = \text{Cos}\,\frac{\pi}{2(2N-1)}$$

defines the position of the first null when Θ_o is specified as the angular position of the first null, z_o may be deduced from the relations

$$z_o = \frac{1}{x_1^0}\ \text{Cos}\ \frac{\pi}{2(2N-1)}$$

$$x_1^0 = \text{Cos } U_1^0 = \text{Cos }\left(\left(\frac{\pi d}{\lambda}\right)\text{Sin }\Theta_o\right)$$

It is evident that the numerical work involved in calculating the current distribution from (10) and z_o from (11) can become extremely tedious as the number of elements increases. A simple method of calculating I's is given by Barbiere (1952).

2.2. UNEQUALLY- SPACED ARRAYS

Considerable work has been done in recent years to develop synthesis techniques for the design of linear non-uniformly spaced arrays. The work so far done in this field has been reviewed in this Chapter.

H. Unz (1960) discussed a linear array with general arbitrary distributed elements. He deduced a matrix relationship between the elements of an array and its far zone pattern. As pointed out by various authors, it is difficult to make use of this matrix relationship to yield useful numerical results.

D. D. King et. al, (1960) gave the requirements for a broad-band, steerable linear array, and discussed the limitations due to grating lobes of an equally spaced array. After studying several different unequally spaced arrays they showed that such arrays have two advantages over the equally spaced arrays (1) They require fewer elements for comparable beam width, (2) Grating lobes and minor lobes are replaced by sidelobes of unequal amplitude which are less then the main beam. Further, they developed a scheme for controlling the cosine arguments in the radiation pattern formula which resulted in one of the best patterns of this study of unequally spaced arrays. The array synthesized by this scheme is capable of steering a beam $\pm 90^{\circ}$ over a 2 to 1 frequency band with nosidelobes above -5 db. It uses 21 elements, compared to 78 for an equally spaced array of similar beam width. The results obtained indicated that further study of the cosine method and unequal spacing in general should result in better pattern characteristics. They computed the data by means of the formula.

$$E = 20 \log_{10} \left[\frac{C + 2 \sum_{k=1}^{n} \operatorname{Cos} \left(2\pi U \frac{x_k}{\lambda} \right)}{2n + C} \right]$$

Where

E = The magnitude of the pattern factor in db.

$2n+C$ = Number of elements in the array.

$C = 1$, for an odd number of elements.

$C = 0$, for an even number of elements.

$\frac{x_k}{\lambda}$ = distance in wave length from the centre of the array.

$U = d_{min} (\operatorname{Sin} \theta - \operatorname{Sin} \theta_o)$

d_{min} = The smallest of the set of unequal spacings in wavelengths.

θ_o = The angle to which the beam is steered.

θ = The azimuth angle measured from the broadside direction.

$2U\left(\frac{x_k}{\lambda}\right)$ = Arguments for the cosine term for k = 1,2,3..n

Directive gain was computed from relative power pattern data by numerical integration with the interval chosen to be less then half of the half-power beam width.

Comparasion of array factors of two arrays of equal aperture is given in Table No. 1.

S. S. Sandler (1960) formulated a general analytical expression for unequally spaced arrays. These relations allowed for the analysis of the non-uniformly spaced arrays in terms of its equdvalent uniformly spaced array. He discussed the inherent broad band qualities of the nonuniformly spaced arrays. Some equivalence was observed between the amplitude and the spatial variation with uniformly and nonuniformly spaced array. He discussed the general synthesis problem and also considered an array with monotonically increasing interelement spacing.

R. F. Harrington (1961) presented a perturbational procedure for reducing the sidelobe level of discrete linear arrays with uniform amplitude excitation by using non-uniform element spacing. The calculation of the required element spacings is quite simple. The method can reduce the sidelobe level to about 2/N times the field intensity of the main lobe, where N is the total number of elements,

	Array with spacings chosen by controlled cosines method	Array with equal spacing.
Spacing	Smallest spacing equals $\lambda/2$ at the lowest frequency	All spacings equal $\lambda/2$ at the lowest frequency
Aperture length	$19.3\,\lambda$	$19.5\,\lambda$
Total number of elements in the aperture	21	40
	Bandwidth sacrified for low sidelobes	
Bandwidth	1.09 to 1.0	1.3 to 1.0
Beam steering range	$\pm 30^{\circ}$	$\pm 30^{\circ}$
Maximum sidelobe level	-9 db	- 13 db.
	Band sidelobe level sacrificed for Bandwidth.	
Bandwidth	2 to 1	2 to 1
Beam steering range	$\pm 90^{\circ}$	Two zero db grating lobes when steered at $\pm 90^{\circ}$
Maximum sidelobe level	-5 db	One zero db grating lobe with beam broadside.

Table No.1

without increasing the beam width of the main lobe.

J. D. Bruce and H. Unz (1962) predicted that non-uniformly spaced antenna arrays are less sensitive to changes in frequency. They determined mathematically the condition for minimum sensitivity. Also they deduced an alternative method for obtaining the maximum broadband performance.

A. L. Maffett (1962) formulate an algorithm to describe the construction of arrays whose individual antenna are to be distributed nonuniformly over an aperture. From the distribution to which the algorithm is equivalent, a distribution of array factor values is inferred. He pointed out that an array of antenna elements can be non-uniformly distributed so as to produce an array factor with a single major lobe from one fourth of the elements required by a uniform distribution at a sacrifice of 5 db in sidelobe and no sacrifice in main beam-width.

M. G. Anderson (1962) designed a variety of arrays with widely and variably spaced elements using both analog and digital computer techniques. All those arrays have many fewer elements than Dolph - Tchebyscheff arrays with the same beam width and sidelobe level. One of the arrays he designed has 21 elements and is 76 wavelengths long when used as a broadside array. The 3 db beamwidth is 0.74 degrees, the sidelobe level is -7.4 db. The array has a perfect steerability in a 1.8 : 1 bandwidth with no interelement spacing smaller then one-half wavelength in this band. The data of initial arrays and of the arrays synthesized from the initial array's are presented in the following table No. 2. The initial arrays determined by the method of controlled cosines.

Robert E. Willey (1962) presented a simplified theory of space tapered arrays along with methods of designing arrays for a given gain, beamwidth, and side lobe level using graphical techniques and simple mathematics. He indicated that the reduction in the number of elements of from 50 to 90 percent for moderate and large size planar arrays is possible while retaining good pattern characteristics. He further mentioned that space tapering allows seperate transmitting and receiving elements to be placed in a single aperture.

F. W. Brown (1962) considered a 4- element symmetric array compared a Tchebyscheff 5 element, $\lambda/2$ spaced, 20 db sidelobe array with a similar array non-uniformly spaced, and noted that slightly asymmetrical arrays offer little prospect of producing desirable patterns.

Array	No. of Elements	Average spacing in wave-length	Bandwidth x steerability $B(1+\sin\theta_o)$			beamwidth of main lobe.	in db.	with same pattern data	
					db			No. of Elements	Gain db
Initial array.	11	3.0	-	0.723	-2.8	-	-	-	-
Synthesized array.	11	3.75	4.0	0.558	-5.1	1.4^{o}	10.4	26	7.7
Initial array	21	3.0	-	-0.547	-5.2	-	-	-	-
Synthesized array	21	3.78	3.6	0.425	-7.4	0.74^{o}	13.2	53	10.0
Initial array	51	3.0	-	0.356	-9.0	-	-	-	-
Synthesized Array	51	3.19	3.0	0.300	-10.5	0.37^{o}	17.1	125	-

TABLE No. 2.

H. Unz (1962) considered nonuniformly spaced arrays with spacings larger than one wavelength and deduced a formula using the asymptotic series expansion of Bessel functions. The theory is used to find the maximum average spacing ($\simeq 4\lambda$) acceptable for arbitrary pattern synthesis.

J. D. Bruce and H. Unz (1962) synthesized nonuniform arrays having prescribed field patterns with different beamwidths and sidelobe levels using mechanical quadratures.

A. Ishimaru (1962) presented a new approach to the unequally spaced array problem, based on the use of Poisson's sum formula and introducing a new function, " the source position function" . By appropriate transformation, the original radiation pattern is converted into a series of integrals, each of which is equivalent to the radiation from a continuous source distribution whose amplitude and phase distribution clearly exhibit the effects of the unequal spacings. They showed that an unequally spaced array of uniform amplitude with any desired sidelobe level may be designed by this method. Three examples are shown to illustrate the effectiveness of this method.

M. T. Ma (1963) presented a contribution to the perturbation method of pattern calculation of linear arrays consisting of non-uniformly spaced, equiamplitude inphse elements. He showed that the analysis holds good when the total number of elements is either even or odd . He calculated field patterns for 7 and 8 element arrays.

Y. T. Lo (1963) compared the sidelobe level of various nonuniformly spaced antenna arrays using two methods; (i) Systematic design with patterns being computed in each case, (ii) probabilistic estimates. He concluded that since the agreement between the results obtained by the two methods is close, there is no essential difference between the various non-uniform spacings, unless they are specifically chosen for a low sidelobe level. He illustrated this by considering the Benelux-Mills -Crosstelescope, each arm of which has a dimension $10^2 \lambda \times 10^4 \lambda$ and one minute of arc beamwidth.

Maher and Cheng (1963) studded the problem of random removal of elements in a uniformly spaced array Their assumption that the removal any element is statistically independent of the removal of others seems to preclude the validity of their analysis for a more interesting case when a large number of elements are to be removed.

Snover and Ferroro (1964) discussed the preliminary results obtained by synthesizing closely spaced multi-element arrays by systematic variation of spacing in a computer programme in which the reduction of sidelobe level is the point of interest. They employed both current tapering (nonuniform current distribution) and non-tapering and presented the numerical results in two tables. They compared the results with tapered Tchebyscheff arrays of equivalent sidelobes level.

Skolnik et al. (1964) described the application of the optimization technique known as dynamic programming to the design of 'thinned arrays with unequally spaced elements. A tinned array is one in which the number of elements is significantly less than the number of elements in a 'filled' array with elements spaced every half-wavelength. Dynamic progtamming is a systamatic procedure for efficiently utilizing the capacbilities of modern high-speed digital computers to find solutions to problems not computationally feasible by conventional means. They applied it to the design of linear arrays of 25 elements spaced within a 50 wavelength aperture.

Y.T. Lo (1964) studied various probabilities properties of a large antenna array with randomly spaced elements. He found that for almost all cases of interest the required number of elements is closely related to the desired sidelobe level and is almost independent of the aperture dimension, the resolution dependsmainly on the aperture dimension and the directive gain in proportional to the number of elements used if the average spacing is large. He stated that starting with a given number of elements and a given aperture size, it is possible to improve the resolution by a factor of ten, a hundred or more by spreading these elements over a large aperture with little risk in obtaindng a much higher sidelobe level and a lower directive gain. He further stated that in addition, this analysis also gives a simple estimate of the sidelobe level of most non-uniformly spaced antenna arrays.

Sherman and Skolnik (1964) obtained an upper bound for the sidelobes of an unequally spaced array by applying a result from number theory known as Vander Corputs method. When the number of elements is large the sidelobe level is proportional to $N^{1/2}$ where $2N+1$ is the total number of elements in the array.

Skolnik et al. (1964) considered the design of 'thinned' planar antenna arrays in which the density of the elements located within the aperture is made proportional to the amplitude of the aperture illumination of a conventional "filled array". They indicated that density tapering permits good sidelobe performance from arrays of equal radiating elements.

Janis and Galejs (1964) developed a method of minimizing the sidelobes of uniformly excited space tapered linear arrays. He indicated that it is possible to design space tapered linear arrays by representing the element positions by a polynomial and by formally minimizing the sidelobe energy averaged over a finite frequency band.

V. Galindo (1964) introduced the idea of non-linear arrays; that is the elements of a uniformly spaced array are displaced perpendicularly from their usual positions along a straight line. He discussed that such an array possessed some of the non-resonent properties of a linear non uniformly spaced array and hence has wide band frequency or scanning properties. Further this array has the unique advantage of having equal lateral spacing. Hence the problem of packaging phase shifters and other auxiliary networks between elements is greatly simplified.

Ishimaru and Chen (1965) presented a theory for designing a thinned or broad band antenna array by means of unequal spacings. They expressed the patterns in a series of Anger functions and its sidelobe level is shown to decrease approximately as $N^{-0.5}$ or $N^{-0.4}$ where N is the total number of elements, and the gain is approximately equal to N. They verified that the sidelobe level can be improved by varying the amplitude distribution.

C. H. Tang (1965) gave a design procedure for non uniformly spaced linear arrays for which the pattern approximates to that of an equivalent uniformly spaced arrays or continuous source with piece-wise uniform excitation. He indicated that the approximation is best in the main lobe region and discussed the effect on the sidelobe levels for several typical arrays with reductions of upto half the number of elements used in the uniformly spaced arrays.

Bulter and Unz (1965) introduced Fourier transform method for obtaindng the radiation pattern of a non uniformly spaced array and also to synthesize approximately any arbitrary pattern.

Y.L. Chow (1965) showed that the exponential spacing function is optimum in the sense that the plateau becomes flat. The pattern of a non-uniformly spaced array is in general an almost periodic function and as a result the grating beams are spread out into plateaux. For uniformly illuminated elements the envelop of the plateau is flat if the element spacing increasing exponentially. Because of the characteristic flatness and low intensities of these plateaux, they optimized the array factor with respect to its grating platooux. He also developed the theory of space factor gain of nonuniformly spaced arrays through the use of Parseval's theorem.

Lo and Lee (1965) used an array of N isotropic elements placed at a prescribed positions in space as a model for the derivation of optimum S/N ratio for a non uniformly distributed noise. They also proposed two new methods to determine the sidelobe levels of nonuniformly spaced antenna arrays (1) Estimation of sidelobe level by solving. Diophantine equation (2) Estimation of sidelobe level by Triangular Function approximation.

C. H. Tang (1965) described a method of synthesizing the patterns of a non-uniformly spaced array based on considering the pattern of the array as an approximation of that due to a continuous source. He achieved in designing specific sidelobe levels. He further indicated that in order to get sidelobe levels better then those of a uniformly spaced array the non-uniformly spaced array must be thinned at the end.

M. T. Ma(1965) proposed another method for synthesizing the non-uniformly spaced arrays. He applied Haar's theorem, by varying both the amplitude excitations and the element spacings, but a rigorous theory has not been developed.

Larson et al. (1965) discussed the minimization of the grating lobes produced when the array elements are many wavelengths long, by a special type of (linearly) nonuniform array with equal power division between the elements. He summarised the results for a number of cased in the form of graphs and figures.

H. Unz (1966) described a method of designing a non uniformly spaced array using Schmidt orthogonalisation procedure. He indicated that this method can be performed by a digital computer, and avoids the inversion of large matrices and is applicable to asymmetric as well as to symmetric non-uniform arrays.

Lo and Lee (1966) made an exhaustive study on a few small arrays and came to a conclusion that among a large number of possible arrangements, only very few yield resonably low sidelobe level. They made some statistical studies in order to relate the sidelobe level to the element arrangement. Further they made a comparative study on some designs which are proposed by a few authors and concluded that non of them are turly optimum. Also they discussed the optimization of directivity and S/N ratio of an arbitrary antenna array.

C. H. Tang (1966) presented numerical results on the beamwidth and the operating region of patterns for the arrays for which a numerical approximation method of symthesis has previously been given (1965). He also presented on the results obtained on the gain characteristies of non-uniformly spaced arrays and the excitation coefficients for the optimum gain for the arrays.

A. Meyer (1966) discussed the use of convolution theorem and the generalised sampling theorem in evaluating arbitrary arrays.

Chow and Yen (1966) studied a class of non-uniformly spaced planar arrays in which the elements are located on a lattice derivable from a conformal mapping of a uniform lattice. They formulated the array space factor in a two dimensional Poission's sum, and determined the grating plateaux from a stationary phase integration. They applied an optimization process to make the grating plateaux flat. They concluded that the array derived is the conformal exponentially spaced array having characteristics very similar to those of the linear exponentially spaced array A numerical example is included to justify the various approximations they used in the analysis.

Lo and Rimcoe (1967) conducted an experimental investigation on the planar array with randomly spaced elements using diffection techniques. They tested two sample arrays, each consisting of 210 elements over a circular aperture of about 56 wavelengths in diameter at 71.25 GHz. They verified that the measured sidelobes -12.8 db and -13 db were in excellent agreement with the theory which predicted below -12.8 db with 90 percent probability and -13.3 db with 50 percent probability Further they indicated that one may consider the pattern in each plane cutting through the antenna as that of a linear random array, and thus one may study the sample distribution of the sidelobe levels of as many linear random arrays as cuts. They obtained results which are in nerely perfect agreement with the theory, despite the fact that in the theory the mutual coupling effect was neglected altogether.

Thus different techniques of optimization have been attempted by many authors and non of them formulates a rigorous mathematical theory. As that there is no perfect theory available to date, the optimization problem becomes a real challenge. It can also be said that there is little possibility in obtaining an array with the lowest sidelobe level, unless a true optimization procedure is found . Thus, one should not be surprised to find that there may be little difference between many pseudo-optimum and trial- and - miss methods.

CHAPTER III

DYNAMIC PROGRAMMING

3.1. INTRODUCTION

Dynamic programming theory by Richard Bellman (1957) is one of the various branches of modern mathematics. It is a simple but powerful concept for the treatment of many novel and interesting problems both in this new discipline and in various parts of classical analysis. One of its various applications is in solving multistage decision problems.

The adjective, " dynamic" indicates that time is a significant variable and the order of operations may be crucial. However, many static processes can also be reinterpreted as dynamic processes in which time can be artificially introduced.

The mathematical advantages of dynamic programming are :

1. It reduces the dimensionality of the process to a convenient level, thus making the problem computationally simpler.
2. The reduced form obtained by techniques has a property like, " monotonicity of convergence", and therefore is well studied to applications.

In dynamic programming a very difficult or unsolvable problem is transformed into a class of simpler solvable problems which are easy to handle.

3.2. PRINCIPLE OF OPTIMALITY

An optimum system design problem is visualised as a multistage decision problem; these multistage decision problems are best solved by means of "functional equation approach". In each process the functional equation governing the process is obtained by an application of the following intuitive :

"An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision".

By repeated application of the functional equation, the optimum decisions for a multistage process can be obtained. For illustration let us consider that a state of physical system is transformed from x' to x^2 by the transformation;

$$x^2 = y\,(x', m_1) \qquad \ldots (1)$$

This operation will yield an output or return

$$R_1 = r\,(x', m_1) \qquad \ldots (2)$$

Where m_1 = decision number to be taken.

The decision which yields the maximum value of the return or criterion function, is referred to as the optimum decision or optimal control strategy.

The maximum return for this one stage decision process is given by

$$f_1(x') = \underset{m_1}{\text{Maximum}} \left[r\,(x', m_1) \right] \quad \ldots(3)$$

Consider now the case of a two stage decision process. From first transformation we have,

$$x^2 = y\,(x', m_1)$$

This is further transformed into x^3

$$x^3 = y\,(x^2, m_1) \quad \ldots(4)$$

The sequence of operation results in a total return

$$R_2 = r\,(x', m_1) + r\,(x^2, m_2) \quad \ldots(5)$$

And the maximum return is given by

$$f_2(x') = \underset{m_1, m_2}{\text{Maximum}} \quad r\,(x', m_1) + r\,(x^2, m_2) \quad \ldots(6)$$

The total result is maximized over the policy (m_1, m_2) and the policy which maximized R_2 is called optimal policy.

In general for an N-stage decision process, the problem is to choose a N-stage policy, $(m_1, m_2 \ldots m_N)$

So as to maximize the total return,

$$f_N(x') = \underset{[m_j]}{\text{maximum}} \left[\sum_{j=1}^{N} r\,(x^j, m_j) \right]$$

Where $[m_j]$ forms an N-stage control policy. This functional equation so obtained can be solved by conventional techniques available.

CHAPTER IV

APPLICATION OF DYNAMIC PROGRAMMING TO ANTENNA ARRAY SYNTHESIS

4.1. DESCRIPTION

A brief qualitative description of dynamic programming and its application to unequally spaced array antennas is given. Dynamic programming is a step by step method by which a multistage decision process is reduced to a sequence of single stage decision processes. The possibility of application of this method to the design of antenna arrays with unequally spaced elements was originally proposed by Skolnik et al (1964). Dynamic programming is a systematic procedure for efficiently utilizing the capabilities of modern high speed digital computers to find optimum solutions to certain problems not solvable by conventional means. It is used here to determine solutions which approximate the optimum configuration of element spacings for achieving a desired radiation pattern.

One possible method of designing an array with unequal spacings is that of total enumeration. In this approach all possible combinations of spacings are examined, the radiation pattern is computed for each combination, and the one which yields the best pattern is selected. Although it is possible in principle to carry out such a brute-force procedure, it is generally not practical to do so except in the simplest of cases. If each of the N elements of an array can occupy any one of m possible positions within the aperture, there are a total of m^N combinations that must be examined. Ten elements, each capable of occupying ten different possible positions, result in a total of 10^{10} combinations. Even with modern high speed computers, the brute-force approach generally is not practical.

The advantage of dynamic programming is that it drastically reduces the number of combinations that must be examined but nevertheless finds a set of spacings with a satisfactory radiation pattern nearly optimal. This is accomplished by converting a single N-dimensional optimization problem into a sequence of N one dimensional optimization problems. In stead of the m^N cases required for the brute-force approach, approximately $(N-1)m^2$ cases need be examined with dynamic programming.

4.2. OPTIMIZATION CRITERION

In order to get desirable radiation pattern, some criterion for optimization is to be established. There is little value in utilizing the main beam parameters as a design criterion since the shape of the main beam and the maximum intensity are relatively unaffected by the precise arrangement of a given number of elements within a given aperture. The sidelobes however, are significantly dependent on the arrangement of elements. Thus it seems reasonable to establish the criterion on the basis of sidelobes properties. The criterion best suited for our problem is that the optimum radiation pattern is one whose highest sidelobe peak over a specified angular interval is less than that of any other pattern. This is a special case of general criterion of minimizing the maximum deviations.

4.3. DERIVATION OF RADIATION PATTERN

The radiation pattern of a linear array containing an odd number (2N+1) of isotropic elements symmetrically arranged about the centre as shown in Fig. No.2. can be derived as follows:

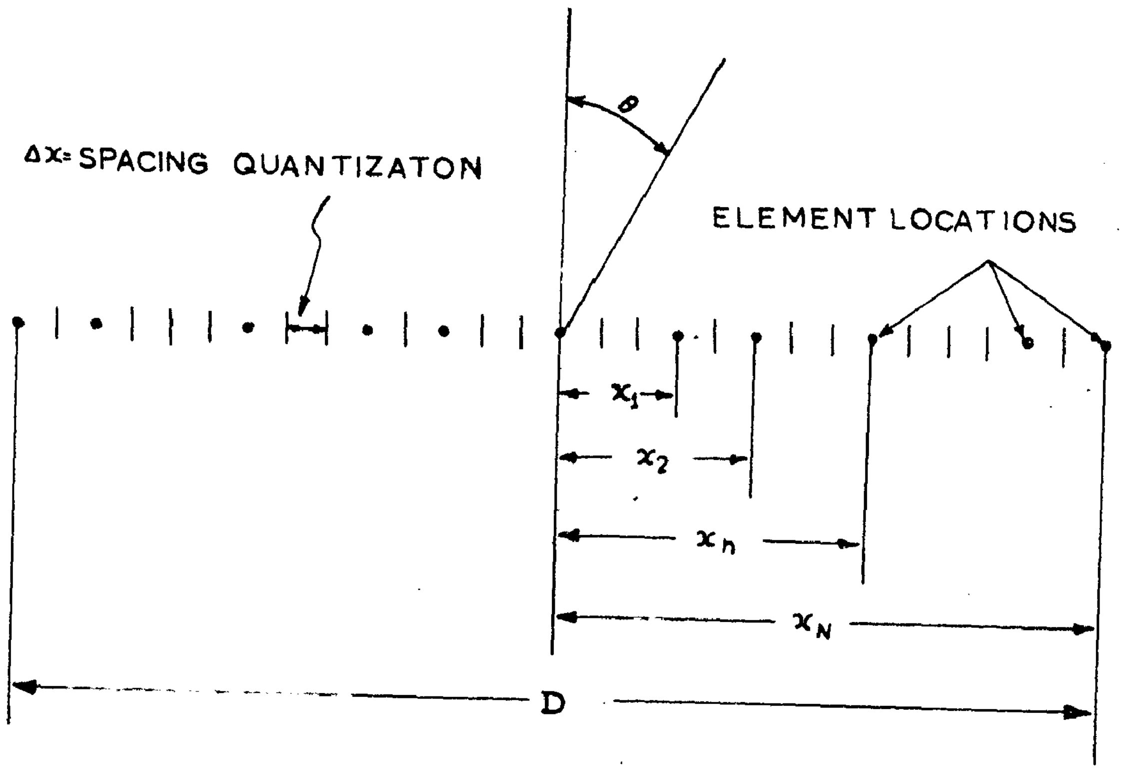
Let x_n = the distance of the nth pair of elements measured in wavelengths from the centre of the array.

D = array length

Δx = spacing quantization

x_N = spacing of Nth pair of elements ($D=2x_N$)

θ = Angle with respect to array normal.



GEOMENTARY OF THE UNEQUALLY SPACED ARRAY SYMETRICALLY ARRANGED IN PAIRS ABOUT THE CENTRE

FIG. NO. 2

The elements are energized so as to get main lobe in any specified direction θ_o. Also the elements are allowed to occupy positions whose location from the array centre is an integral number of some specified value Δx.

Considering only the centre element and two other elements symmetrically placed about the centre, the radiation pattern is given by

$$E(x_1, \theta) = A_o \left[1 + e^{(\alpha + 2\pi x_1 \sin\theta)} + e^{-(\alpha + 2\pi x_1 \sin\theta)} \right] \quad ...(1)$$

where

x_1 = distance of the first element pair from the centre element in wavelengths.

A_o = Current excitation

λ = wavelength

α = Progressive phase shift; leading from left to right.

Equation (1) can be written as

$$E(x_1, \theta) = A_o \left[1 + 2 \cos(\alpha + 2\pi x_1 \sin\theta) \right] \quad ...(2)$$

at beam maximum, $E(x_1, \theta)$ is maximum and $\theta = \theta_o$

Therefore $\cos(\alpha + 2\pi x_1 \sin\theta) = 1$

$$\text{or} \quad \alpha + 2\pi x_1 \sin\theta_o = 0$$

$$\text{or} \quad \alpha = -2\pi x_1 \sin\theta_o \qquad \ldots(3)$$

Substituting equation (3) in (2) we get,

$$E(x_1, \theta) = A_o \left[1 + 2 \cos 2\pi x_1(\sin\theta - \sin\theta_o) \right] \qquad \ldots(4)$$

Defining $u = \sin\theta - \sin\theta_o$ (the angular coordinate) and taking A_o as the unit current excitation, the equation (4) becomes

$$E(x_1, u) = 1 + 2 \cos 2\pi x_1 u$$

For $(2N+1)$ number of elements, the radiation pattern is

$$E(x_1, x_2, x_3, \ldots x_N, u) = 1 + 2 \sum_{n=1}^{N} \cos 2\pi x_n u \qquad \ldots(5)$$

If the distance of the nth pair of elements is measured in half wavelengths instead of wavelengths the expression (5) becomes,

$$E(x_1, x_2, x_3, \ldots\ldots x_N, u) = 1 + 2 \sum_{n=1}^{N} \cos \pi x_n u \qquad \ldots(6)$$

4.4. SYMMETRY IN THE RADIATION PATTERN

If the radiation pattern given by equation (5) is to be symmetrical about some value of $u = u_o$, then $E(u_o + \Delta u) = E(u_o - \Delta u)$; thus we must have

$$\cos 2\pi x_n (u_o + \Delta u) = \cos 2\pi x_n (u_o - \Delta u)$$

Expanding the cosine terms we get

$$\cos 2\pi\, x_n u_o \cos 2\pi x_n \Delta u - \sin 2\pi x_n u_o \sin 2\pi x_n \Delta u$$

$$= \cos 2\pi\, x_n u_o \cos 2\pi\, x_n \Delta u + \sin 2\pi x_n u_o \sin 2\pi x_n \Delta u$$

The equation holds good if $\sin 2\pi\, x_n u_o = 0$, or if

$$2\pi\, x_n u_o = 0,\ \pm\pi,\ \pm 2\pi \ \ldots\ldots \pm k\pi \ldots$$

or $$x_n u_o = 0,\ \pm\frac{1}{2},\ \pm 1,\ \pm\frac{3}{2},\ \ldots\ldots \pm\frac{k}{2} \ldots$$

When the distance of the element pair from the centre element is a multiple of $\lambda/2$ that is whenever x_n is a multiple of 1/2, then $u_o = 1$ and the pattern is ssymmetrical about $u_o = 1$.

When the distance of the element pair from the centre element is a multiple of λ that is whenever x_n is an integer, then $u_o = 1/2$, and the pattern is symmetrical about $u_o = 1/2$.

Similarly whenever x_n is a multiple of 1/4 then $u_o = 2$ and the pattern is symmetrical about $u_o = 2$, and so on.

4.5. METHOD OF OPTIMIZATION

Applying the dynamic programming technique we have to to locate a vector $x = (x_1, x_2, \ldots\ldots x_N)$ such that over some region of u, the maximum value of the summation

$$E(x_1, x_2, x_3, \ldots x_N, u) = 1 + 2\sum_{n=1}^{N} \text{Cos}\ (2\pi\ x\ u)$$ is a

minimum;

The given input conditions are :

1. Each x_n has an upper and lower bound

   $a \leq x_n \leq b$ which is variable with n

2) The region of u over which the expression

   is to be evaluated is $u_{min} \leq u \leq u_{max}$.

With these values the optimization is carried out, so that the value of x_n's are determined, which minimizes the maximum value of this summation over the required value of u.

CHAPTER V

DESIGN OF LINEAR, SYMMETRIC, NON-UNIFORMLY SPACED ANTENNA ARRAYS OF 25 ELEMENTS. (SPACED WITHIN A 50 WAVELENGTH APERTURE)

5.1. REQUIREMENTS

The main object in the design of non uniformly spaced antenna arrays is to control the radiation so as not to produce objectionably high sidelobes. The following characteristics were assumed while synthesizing the array.

1) The array has a single narrow main beam steerable to θ_o^o from the end fire.

2) The array has all the sidelobes below the main beam level.

3) All the elements to be isotropic radiators.

4) Each element has unit amplitude illumination.

5) The current fed to the nth pair of elements has a phase angle $\pm\ 2\pi x_n \sin\theta_o$ with respect to the centre element.

6) The array is symmetrical about the centre.

7) The coupling effect between the elements is neglected.

The array geometry is shown in Fig. (2). The elements are allowed to occupy positions whose locations from the array centre are integral numbers of some prespecified value of Δx. That is, the element locations are quantized. This not only makes the computations easier but is consistent with the practical array design. The spacing of Nth pair of elements is fixed by the aperture dimension so that $2x_N = D$, where D is the aperture length in wavelengths. Thus it remains to find the N-1 value of x_n. For example, in the 9 element array there are 3 spacings that must be determined while in 25 element array there are 11 spacings to be determined.

Further the aperture length is quantized into 2m + 1 locations by dividing it into 2 m equal parts. No consecutive elements may have a spacing between them which is less then the length of one interval. This simplifies the optimization process and results in pattern symmetry about some value of u as stated in (4.4). The problem is to determine a set of element locations for which the peak sidelobe level is smaller than that for any other element combination for a given odd number of elements, over a preassigned angular interval, $u_{min} \leq u \leq u_{max}$

5.2. SYNTHESIS TECHNIQUE

In dynamic programming the optimization process is carried out in the stages M.Nath (1966). This reduces considerably the computational work that would be involved if the optimization were to be achieved by trying all possible combinations of elements. The expression for radiation pattern as already derived is

$$E\,(x_1, x_2, x_3, \ldots \ldots x_N , u) = 1 + 2 \sum_{n=1}^{N} \cos 2\pi x_n u \qquad \ldots.(1)$$

<u>Stage 1.</u>

The first and second pair of elements are considered while remaining elements are supposed to be absent, except for the central (zeroth) element and the outer most pair of elements. For any particular value of x_2 say x_2' there is a corresponding value of x_1, say x_1', for which the peak sidelobe level is smaller then that for any other value of x_1 in the range $x_{1(min)} \leq x_1 \leq x_{1(max)}$

We express this by means of the functional equation

$$E(x_2', x_1', \bar{u}) = \min_{\substack{x_2(min) \leq x_2 \leq x_2(max) \\ x_1(min) \leq x_1 \leq x_1(max)}} \left[\max_{u_{min} \leq u \leq u_{max}} E\,(x_N, x_2, x_1, u) \right] \qquad \ldots(2)$$

This equation provides

$x_{2(max)} - x_{2(min)} + 1$ "Optimal" pairs of values of x_2 and x_1 expressed concisely as $x_1(x)$. These are computed and stored in the computer memory.

Stage 2: The next step is to determine the best combination of x_1 and x_2 for any chosen value of x_3. For every pair of chosen values of x_2 and x_3 we assume that the optimal value of x_1 is the same as was obtained in the previous stage for the same value of x_2. In other words we assume that the principle of optimality is valid for our problem. There may seem to be no valid justification for this assumption, despite that the technique was applied and it was found that the results obtained compare favourably with those obtained by other design procedures. The results of this stage may be expressed by means of functional equation:

$$E(x_2', x_3', \bar{u}) = \underset{\substack{x_{3(min)} \leq x_3 \leq x_{3(max)} \\ x_{2(min)} \leq x_2 \leq x_{2(max)} \\ x_1(x_2)}}{\text{Min}} \left[\underset{u_{min} \leq u \leq u_{max}}{\text{Max}} E(x_N, x_3, x_2, x_1, u) \right] \qquad \ldots (3)$$

This will yield $x_{3(max)} - x_{3(min)} + 1$ values of $x_2(x_3)$ which are again stored in the computer memory.

Stage 3 : Continuing the same process, the results of this stage may be expressed by means of the equation :

$$E(x_4', x_3', \bar{u}) = \underset{\substack{x_{4(min)} \leq x_4 \leq x_{4(max)} \\ x_{3(min)} \leq x_3 \leq x_{3(max)} \\ x_2(x_3) \\ x_1(x_2(x_3))}}{\text{Min}} \left[\underset{u_{min} \leq u \leq u_{max}}{\text{Max}} E(x_N, x_4, x_3, x_2, x_1, u) \right] \quad \ldots (4)$$

This will yield $x_{4(max)} - x_{4(min)} + 1$ value of x_3 (x_4) which are stored in the computer memory.

Stage i (General Stage): In the same manner we obtain for the general ith) stage)

$$E(x'_{i+1}, x'_i, \bar{u}) = \underset{\substack{x_{i+1(min)} \leq x_{i+1} \leq x_{i+1(max)} \\ x_{i(min)} \leq x_i \leq x_{i(max)} \\ x_{i-1}(x_i) \\ x_{i-2}(x_{i-1}(x_i)) \\ \vdots \quad \vdots \quad \vdots \\ x_1(x_2(x_3(\ldots x_i)\ldots))}}{\text{Min}} \left[\underset{u_{min} \leq u \leq u_{max}}{\text{Max}} E(x_N, x_{i+1}, x_i \ldots x_1, u) \right] \quad \ldots\ldots (5)$$

which will yield $x_{i+1(max)} - x_{i+1(min)} + 1$ value of $x_i(x_{i+1})$.

Final Stage : This corresponds to the optimum location of (N-2) the element for every (N-1) th element. The functional equation is

$$E(x_N', x_{N-1}', \bar{u}) = \underset{\substack{x_{N-1(min)} \leq x_{N-1} \leq x_{N-1(max)} \\ x_{N-2(min)} \leq x_{N-2} \leq x_{N-2(max)} \\ x_{N-3}(x_{N-2}) \\ x_{N-4}(x_{N-3}(x_{N-2})) \\ \vdots \\ x_1(x_2(x_3(\ldots x_i \ldots (x_{N-2}) \ldots)))}}{\text{Min}} \left[\underset{u_{min} \leq u \leq u_{max}}{\text{Max}} E(x_N, x_{N-1}, \ldots x_i \ldots x_2, x_1, u) \right]$$

and this equation yields the optimum combination. The corresponding radiation pattern is obtained by using this optimum combination in equation (1).

The number of stages depends upon the number of elements and is equal to N-2 . For example in the 9 element array the number of locations to be determined is 3 and therefore the number of stages is 2 . In the 25 element array the number of locations is 11 and the number of stages is 10.

5.3. FIXATION OF DESIGN PARAMETERS

The input parameters play an important role in the optimization role. They are discussed in detail in the following article:

i. Spacing Quantization Δx

A convienent choice of Δx is $\lambda/2$, where λ is the free-space wavelength, that is the element locations are quantized into $\lambda/2$ intervals. This choice of Δx results in pattern symmetry about $u = 1.0$

ii. Possible Element Locations m

Each pair of elements can be located any where within the aperture subject to the following two constraints : a) no two adjacent elements may be closer than a predetermined spacing, in this case half-wavelength, and b) the number of possible positions an element can occupy is limited by quantizing the aperture into discrete increments in this case half-wavelength intervals. Both of these constraints are consistent with practical array design.

The array length considered is 50λ and the number of elements is 25. Since the centre element and the end pair of elements is fixed, eleven optimum element locations have to be computed to fix the locations of the remaining eleven element pairs. Since the array length on either side of the centre element is 25 λ, quantized into $\lambda/2$ intervals and since there can be no more than one element at a particular location, the first element can occupy any positions from 1 to 39, the second element from 2 to 40, the third element 3 to 41 and so on and finally the last (eleventh) element from 11 to 49. Therefore m = 39.

iii. Angular Coordinate u

The flexibility of dynamic programming can be employed to determine how the radiation pattern is affected by varying the input conditions. The angular region or the u- region, is of practical importance because in many applications increased sidelobes may be permitted over some angular sector if reduced sidelobes can be achieved within some specified sector.

a. u_{min}

Generally the angular region over which the sidelobes are to be optimized should not include the main beam. If u_{min} is too small it might include a portion of the main beam and not give the optimum design. A u_{min} that is too large might cause the sidelobe region in the vicinity of the main beam to be higher than desired, that is it should not be so large as to miss any of the sidelobes which occur near the main lobe. An approximate value of u_{min} can be estimated as follows :-

The expression for the radiation pattern as derived previously is

$$E(x,u) = 1 + 2 \sum_{n=1}^{N} \text{Cos } 2\pi \; x_n \; u.$$

For 25 element case $N = 12$ with $\Delta x = \lambda/2$

The last term of the summation is $2 \text{ Cos } 2\pi \, x_{12} \, u.$

$$\text{Where} \quad x_{12} = \frac{D}{2\lambda} = \frac{50\lambda}{2\lambda} = 25$$

Therefore $2 \text{ Cos } 2\pi \, x_{12} \, u = 2 \cos 50\pi \; u$. This is a periodic term having the highest frequency of all the terms in the summation. u_{min} is taken at that point where the first minimum occurs in the term $\text{Cos } 50\pi \; u.$

$$\text{i.e.} \quad \text{Cos } 50\pi \; u_{min} = -1$$

$$\text{or} \quad 50\pi \; u_{min} = \pi$$

$$\text{or} \quad u_{min} = 1/50 = 0.02$$

It is not possible to predict the precise location of u_{min} in an unequally spaced array. Infact it is sometimes necessary to vary u_{min} to determine that value which just excludes the main beam. In any case it should not be less than 0.02 in our problem, but it can be greater. In fact the array was synthesized first by taking $u_{min} = 0.02$ and then $u_{min} = 0.04$ and it was found that $u_{min} = 0.04$ is the most suitable value, since it very nearly fulfils the above requirements. u_{min} is largely governed by the number of elements in the array, since as the number of elements increases the main lobe becomes narrower and u_{min} is accordingly reduced.

b. u_{max}

The value of u_{max} is determined by the range over which optimization is desired, or if optimization is desired over the entire space, is set equal to the value about which the pattern is symmetrical, which is governed by the spacing quantization.

Since the spacing quantization is fixed at $\lambda/2$ the pattern is symmetrical about $u = 1$. With $u_{min} = 0.04$, and $u_{max} = 1.0$, the pattern is optimized over the region $0.04 \leq u \leq 1.96$. This covers the entire space outside the main beam. $u_{max} = 0.5$ corresponds to the angular region 30° to either side of the main beam when it points in the broadfire direction. If the spacing quantization is reduced from $\lambda/2$ to $\lambda/4$,

u_{max} increases from 1 to 2, since the pattern is symmetrical now about $u = 2$.

c. Increment in u, Δu:

If fact this does not come under the input parameters but it is worthwhile to discuss it here itself. The value of Δu is so chosen that while calculating the radiation pattern over the sidelobe region of interest it should not miss any of the peaks of the sidelobes. Δu also on the total aperture dimension and the total region of u over which the pattern is to be optimized. With the limitations in the storage capacity of the computer Δu was taken to be 0.0025. With this the programme has to compute the pattern for about 400 discrete values of u in order to determine a particular configuration of elements. The perdentage error incurred while chosing Δu as 0.0025 can approximately be calculated as follows:

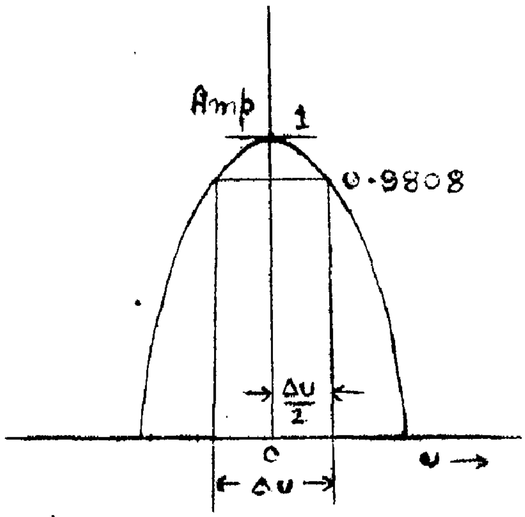
Once again the highest frequency term in the radiation pattern is $\cos 50\pi u$. When $u = 0$

maximum amplitude = 1

When $u = \dfrac{\Delta u}{2} = 0.00125$

maximum amplitude $= \cos 50\pi \times 0.00125$

$= 0.9808.$



Therefore the fall in peak

amplitude $= \dfrac{1-0.9808}{1} \times 100$

$= 1.92\ \%$

With $\Delta u = 0.0025$ error $= 1.92$ % and discrete u calculations $\simeq$ 400 .

For obtaining an error of 1 % the increment u should be

$$\Delta u = \frac{2 \times \cos^{-1} 0.99}{50 \times 180} = 0.0018$$, and the

discrete u calculation $\simeq$ 560 . It is verified that $\Delta u = 0.0025$ gives sufficiently accurate results. The case for $\Delta u = 0.002$ was also considered and further discussed in a later chapter.

5.4. COMPUTATIONS AND RESULTS

The application of dynamic programming to antenna array synthesis necessitates the use of a modern high-speed digital computer. Before proceeding to the synthesis of the 25 element array, a 9 element array which was already synthesized by M. Nath (1966) was attempted again to make sure that the procedure followed was correct. A programme was written for the IBM 1620 digital Computer for the 9 element array case. The various input parameters taken are $u_{min} = 0.1$, $u_{max} = 1.0$, $\Delta x = \lambda/2$, $m = 16$, $\Delta u = 0.01$ and $D = 19\lambda$. The programme differ somewhat from that of Nath, but the final results obtained were exactly the same. The optimum element locations in half wave lengths obtained ar

$x_0 = 0$, $x_1 = 1$, $x_2 = 5$, $x_3 = 8$ and $x_4 = 10$. The peak sidelobe level was - 5.64 db below the main beam.

The problem of 25 element array case is just an extension of the 9 element array case and a programme is written for the IBM 7044 digital computer with the capacity of determining the optimum spacings. of upto 11 pairs of elements (25 elements total). The IBM 7044 computer has a storage capacity of nearly 33,000 words which is roughly 8 times higher than that of IBM 1620 and is also 100 times faster then IBM 1620.

Qualitatively the dynamic programming procedure can be discribed as follows. The first element(or element pair of a symmetrical array) can be placed in any one of m possible locations likewise the second element can be placed in any one of m possible locations. In our problem the first element can occupy locations 1,2,3,......39 while the second element can occupy locations 2,3,4,... ,40 . These possible locations are clearly overlapping. The only restriction is that adjacent elements may not be placed closer than a predetermined spacing, in this case $\lambda/2$. For each location of the second element say ath all possible locations $1,2,......a-1$, of the first element are examined and the contribution of each to the radiation pattern is computed. The central element (zeroth) and the last element are always taken into consideration while computing the radiation pattern. For each location of the second element there will be a particular location of the first element which produces the best results (meeting the criterion already defined). The best location of the first element for each particular location of the second element is noted and is stored in the computer memory. All other combinations are discarded. Thus we have assumed that the optimal position of the first element depends only upon the position of the second element. This assumption is not readily justifeable and only approximates the actual optimum. The above procedure can further be illustrated as follows :-

The radiation pattern over $u_{min} \leq u \leq u_{max}$ is

$$|E(u)| = \left|1 + 2(\cos \pi x_1 u + \cos \pi x_2 u + \cos 50 \pi u)\right| \ldots\ldots$$

where x_1 and x_2 are numbers giving the distance of the 1st and 2nd element pairs in half-wavelengths. The absolute value of peak sidelobe level for each x_2 and all possible x_1's is calculated from the above expression and stored in the computer memory. Then the best x_1 associated with each x_2 is selected by comparing the peak sidelobe levels of each combination and taking that combination having the minimum of the maximum side-lobe level. The best combinations are stored in the computer memory and all other combinations are discarded.

The total number of cases considered here

$$= 39 \times \frac{40}{2} = 780$$

The next step is to consider the 3rd element which can be placed in any one of the locations 3, 4, 5, ... 41. For each location of the 3rd element it is necessary to determine the best location of the second element and of the first element. However part of this problem has been solved since the optimum location of first element for every location of second element was determined in the previous stage. This is the saving afforded by attacking a multistage problem by dynamic programming. Each location of 3rd element will result in an optimum location of 2nd element and hence an optimum location of 1st element. The best location of 2nd element for each particular location of the 3rd element is noted and is stored in the computer memory. All other combinations are discorded. Again the second step is further illustrated as follows :

The radiation pattern in this step is

$$\left| E(u) \right| = \left| 1 + 2(\cos \pi x_1 u + \cos \pi x_2 u + \cos \pi x_3 u + \cos 50 \pi u). \right|$$

Where x_1, x_2 and x_3 are members giving the distance of the 1st, 2nd and 3rd element pairs from the centre element in half-wavelengths. The absolute value of peak sidelobe level for each x_3 and all possible x_2's and best x_1's is calculated from the above expression and is stored in the computer memory. The best x_2 associated with each x_3 is selected by comparing the peak sidelobe level of each of the combinations and taking that combination having the minimum of the maximum sidelobe level. The best combinations are strored in the computer memory and all other are discarded.

Once again the total number of cases considered here $= \dfrac{39 \times 40}{2} = 780.$

The procedure is repeated in turn for each of the remaining elements. The calculation is made for various locations of the (n-1) st element with each possible location of (n-2) nd element. No further calculations with (n-3)rd , (n-4)th, etc. elements are necessary since their positions as a function of position of the (n-2)nd element only were determined in the previous stages. It should be noted that each stage of the process does not determine the location of a particular element. It only specifies that if a certain location is chosen for the (n-1)st element the location of (n-2)nd element is determined, which then determines that of (n-3)rd element, and so on. The precise configuration is not given until the last but one element is examined and its optimum location is found. Thus the design of complete array is built up from successive designs of partial arrays.

Among the 25 elements the two end elements and the central element are kept fixed so the remaining 22 elements are to be located along the aperture length 50λ. Since the array is symmetrical about the central element computations are done to find the optimum locations of 11 elements, and the remaining 11 elements can be placed symmetrically about the central elements. The precise configuration will be obtained only after considering the 11th element and the total number of cases considered until now will be

$$= 780 \times 10 = 7800.$$

That is total number of cases examined with dynamic programming $= \dfrac{m\,(m+1) \times (N-2)}{2}$

Total number of cases to be examined with brute-force approach $= m^{N-1} = 39^{11}$

This shows clearly the advantage of dynamic programming technique.

A programme is written for IBM 7044 is given in Appendix 'A'. The computer will select those element locations which give the best pattern meeting the criterio specified already. The programme for calculating the radiation pattern is given in Appendix 'B'. Three different cases have been considered.

First Case: The various input parameters taken are: $D = 50\lambda$, $\Delta x = 0.5\lambda$, $m = 39$, $u_{min} = 0.02$, $u_{max} = 1.0$ and $\Delta u = 0.0025$. The programme has to compute 393 discrete values of u in determining a particular configuration of elements over the region of interest $u_{min} \leqslant u \leqslant u_{max}$. Because of the symmetry the pattern is optimized over the region $0.02 \leqslant u \leqslant 1.98$. The optimum spacings measured in half wavelengths from the centre of the array, of each pair of elements as found by the computer are

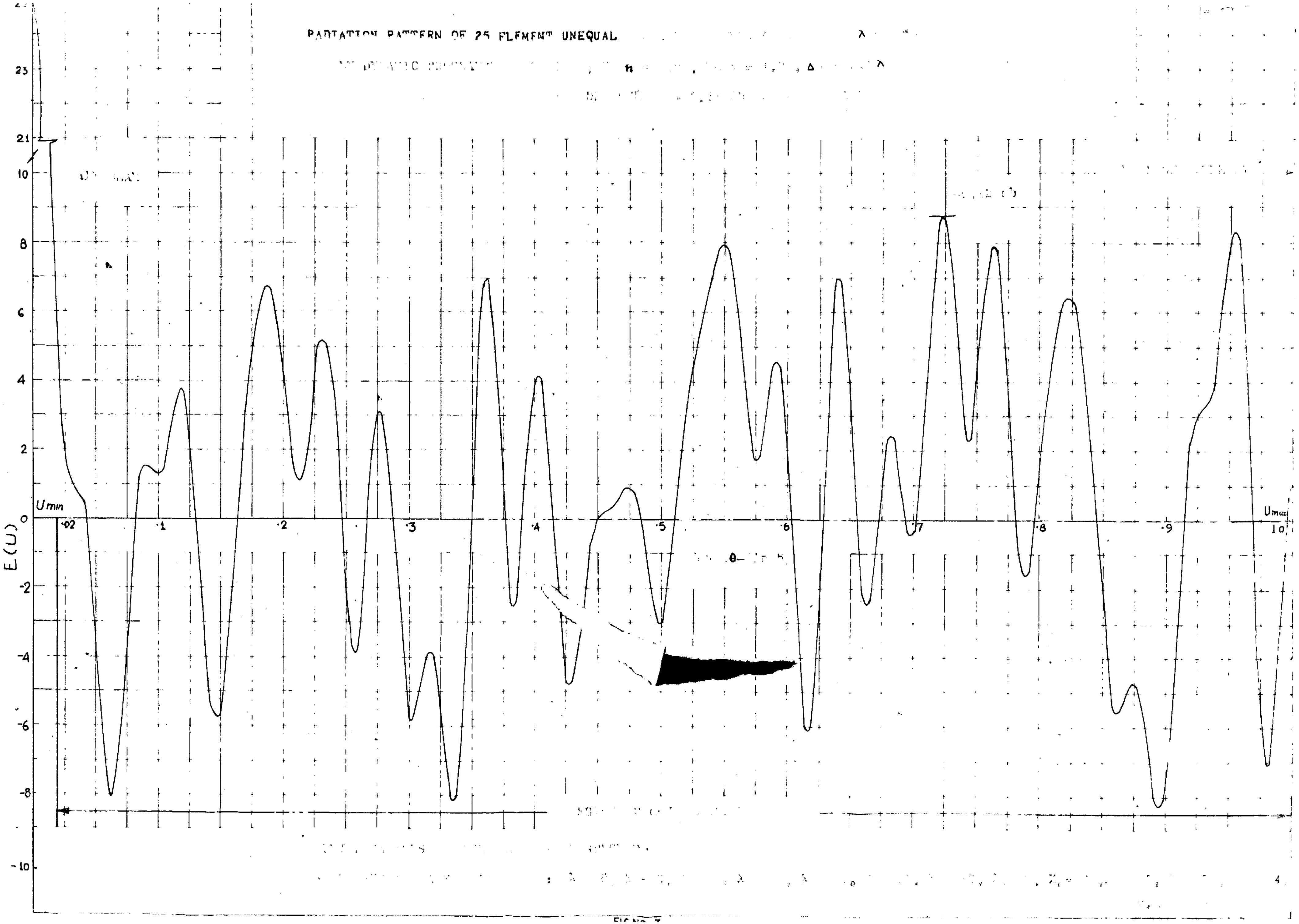
$x_1 = 3, x_2 = 8$, $x_3 = 10$, $x_4 = 11$, $x_5 = 15$ $x_6 = 17$ $x_7 = 19$, $x_8 = 21$, $x_9 = 22$, $x_{10} = 34$ $x_{11} = 44$, along with $x_0 = 0$ and $x_{12} = 50$.

The radiation pattern for the above spacings is calculated and plotted in figure No. 3 from points spaced of increments of $\Delta u = 0.0025$. The maximum sidelobe level is 9.1 db below the main beam. Since the pattern is symmetrical about $u = 1$, the region from $u = 1$, to $u = 2$ is not plotted. The various element combinations obtained by the computer along with the absolute peak side lobe level are shown in Table No. 3. The run time for this programme is 15 minutes. Intermediate prints have been introduced in the programme at the end of th each

NO	ELEMENT LOCATIONS IN HALF WAVELRNGTHS SYMMETRICAL WITH RESPECT TO X = 0 UMIN = 0.04, UMAX = 1.0 DELTA X = 0.5 LAMBDA, D = 50 LAMBDA													MAXIMUM SIDELOBE AMPLITUDE	SIDELOBE LEVEL WHEN MAIN BEAM AMPLITUDE IS UNITY
	0	5	6	7	8	9	10	13	15	17	21	26	50	7.780362	0.31121418
	0	5	6	7	8	9	10	13	15	17	21	41	50	8.188244	0.32752976
	0	5	6	7	8	9	10	13	15	17	21	32	50	8.271950	0.33087800
	0	5	6	7	8	9	10	13	15	17	21	24	50	8.339328	0.33357321
	0	5	6	7	8	9	10	13	15	17	19	22	50	8.369837	0.33479348
	0	5	6	7	8	9	10	13	15	17	19	37	50	8.378109	0.33512436
	0	5	6	7	8	9	10	13	15	17	19	36	50	8.391637	0.33566548
	0	5	6	7	8	9	10	13	15	17	21	31	50	8.407403	0.33629612
	0	5	6	7	8	9	10	13	15	17	21	39	50	8.423352	0.33693408
	0	5	6	7	8	9	10	13	15	16	17	21	50	8.433839	0.33735356
	0	5	6	7	8	9	10	13	15	17	21	27	50	8.455521	0.33822084
	0	5	6	7	8	9	10	13	15	17	19	34	50	8.496565	0.33986260
	0	5	6	7	8	9	10	13	15	17	21	33	50	8.521352	0.34085408
	0	5	6	7	8	9	10	13	15	17	19	23	50	8.530307	0.34121228
	0	5	6	7	8	9	10	13	15	17	21	47	50	8.617916	0.34471664
	0	5	6	7	8	9	10	13	15	17	21	25	50	8.618034	0.34472136
	0	5	6	7	8	9	10	13	15	17	21	40	50	8.618034	0.34472136
	0	5	6	7	8	9	10	13	15	17	21	46	50	8.617916	0.34472136
	0	5	6	7	8	9	10	13	15	17	19	35	50	8.628932	0.34515728
	0	5	6	7	8	9	10	13	15	17	19	42	50	8.644721	0.34578884
	0	5	6	7	8	9	10	13	15	17	19	30	50	8.648761	0.34595044
	0	5	6	7	8	9	10	13	15	17	19	38	50	8.655483	0.34611932
	0	5	6	7	8	9	10	13	15	17	21	46	50	8.668321	0.34673528
	0	5	6	7	8	9	10	13	15	17	19	20	50	8.679398	0.34717552
	0	5	6	7	8	9	10	13	15	18	24	43	50	8.746396	0.34985584
	0	5	6	7	8	9	10	13	15	18	24	44	50	8.763917	0.35055668
	0	3	8	10	11	12	13	17	18	23	27	29	50	8.796426	0.35185704
	0	5	6	7	8	9	10	13	15	16	18	19	50	8.917588	0.35670352
	0	5	6	7	8	9	10	13	15	17	19	49	50	8.947208	0.35788832
	0	5	6	7	8	9	10	13	15	17	19	48	50	8.985638	0.35942552
	0	5	6	7	8	9	10	13	15	18	24	28	50	9.106205	0.36424820
	0	5	6	7	8	9	10	13	14	15	16	18	50	9.358913	0.37435652
	0	5	6	7	8	9	10	13	14	15	16	17	50	9.534455	0.38137820
	0	4	5	6	7	8	9	10	12	14	15	16	50	9.925904	0.39703616
	0	4	5	6	7	8	9	10	12	13	14	15	50	11.206920	0.44827680
	0	4	5	6	7	8	9	10	11	12	13	14	50	13.009469	0.52037876
	0	3	4	5	6	7	8	9	10	11	12	13	50	13.878994	0.55515776
	0	2	3	4	5	6	7	8	9	10	11	12	50	15.941714	0.63766964
	0	1	2	3	4	5	6	7	8	9	10	11	50	17.800390	0.71201560

TABLE NO.4

RADIATION PATTERN OF 25 ELEMENT UNEQUAL



stage to verify the execution of the computer. Had the intermediate prints not been there, the run time would have been about 12 minutes.

From the radiation pattern Fig. No. 3, it is found that $u_{min} = 0.02$ does include a portion of the main beam. For this value of u_{min} the amplitude is 6.7 which does not fulfil the requirements laid down in art 5.3(iii). Hence we may suspect that the results obtained are truly optimum, but still somewhat better than what M. L. Skolnik et al. (1964) have obtained.

Second Case: u_{min} is changed from 0.02 to 0.04 the other input parameters remaining the same. The programme now computed 385 discrete values of u in determining a particular configuration of elements over the region of interest $u_{min} \leq u \leq u_{max}$. The pattern is optimized over the region $0.04 \leq |u| \leq 1.96$. The optimum spacings measured in half wavelengths from the array centre of each pair of elements as found from the computer are :

$x_1 = 5$, $x_2 = 6$, $x_3 = 7$, $x_4 = 8$, $x_5 = 9$, $x_6 = 10$

$x_7 = 13$, $x_8 = 15$, $x_9 = 17$, $x_{10} = 21$, $x_{11} = 26$

along with $x_0 = 0$ and $x_{12} = 50$.

The radiation pattern for the above spacings is plotted in Fig. No. 5 of at increments of $\Delta u = 0.0025$.

Because of the pattern symmetry about $u = 1$ the region from $u = 1$ to $u = 2$, is not plotted. The maximum sidelobe level is 10.14 db below the main beam, a remarkable improvement over that obtained by the previous investigators. The various element combinations obtained by the computer along with the absolute peak side lobe level are shown in Table No. 4. It is interesting to see that three different element combinations (S. No. 16, 17, 18) have the same peak side-lobe level. The run time for this programme is about 15 minutes.

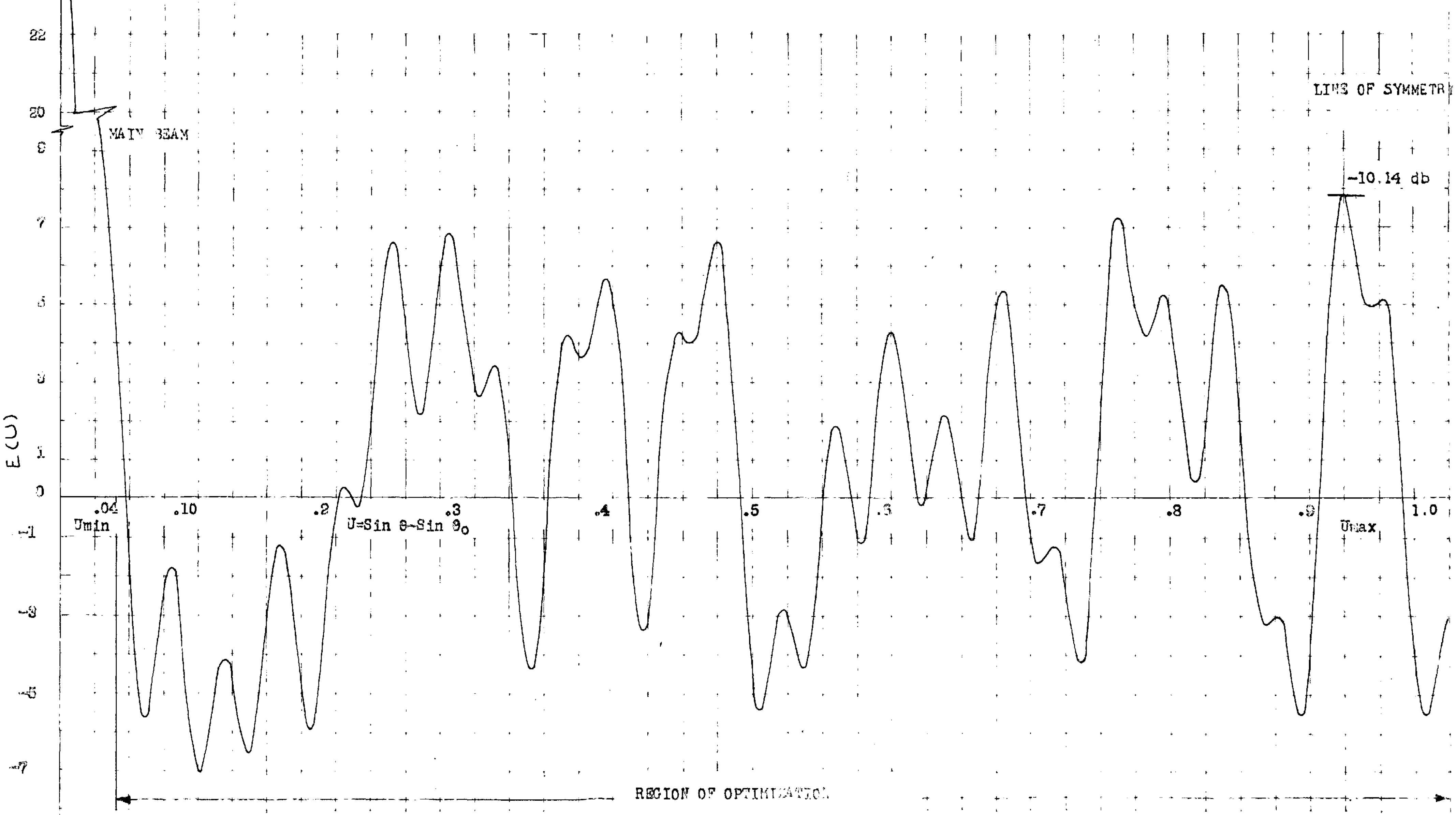
From the radiation pattern it is seen that $u_{min} = 0.04$ still includes only a very little portion of the main beam but is closest to the first null and hence fulfils the requirements laid down in art 5.3(iii). The results of the second case are further discussed in the last Chapter.

Third Case: The effect of scanning of the radiation pattern over a smaller angle is verified here. Other input parameter remaining the same u_{max} is taken as 0.5 keeping $u_{min} = 0.02$. This corresponds to the angular region 30^o to either side of the main beam of an unscanned array.

N	ELEMENT LOCATIONS IN HALF WAVELENGTHS SYMMETRICAL WITH RESPECT TO X = 0 UMIN = 0.02, UMAX = 1.0 DELTA X = 0.5 LAMBDA, D = 50 LAMBDA													MAXIMUM SIDELOBE AMPLITUDE	SIDELOBE LEVEL WHEN MAIN BEAM AMPLITUDE IS UNITY
	0	3	8	10	11	15	17	19	21	22	34	44	50	8.752245	0.35008980
	0	3	8	10	11	15	17	19	21	22	34	35	50	8.754927	0.35019708
	0	3	8	9	13	20	22	23	26	29	31	33	50	8.866946	0.35467784
	0	3	8	10	11	15	17	19	21	22	34	49	50	8.896218	0.35584872
	0	3	8	10	11	15	17	19	21	24	38	41	50	8.919292	0.35677168
	0	3	8	10	11	15	17	19	22	23	32	34	50	8.924178	0.35696712
	0	3	8	10	11	15	17	19	20	21	35	38	50	8.947821	0.35791284
	0	3	8	10	12	13	19	20	24	25	26	28	50	9.000000	0.36000000
	0	3	8	10	11	15	17	19	20	21	36	37	50	9.007296	0.36029184
	0	3	8	10	12	13	19	20	24	25	27	39	50	9.023040	0.36092160
	0	3	8	10	11	15	17	19	20	21	35	42	50	9.038295	0.36153180
	0	3	8	9	13	20	22	23	26	34	41	45	50	9.049490	0.36197960
	0	3	8	10	11	15	17	19	20	21	35	43	50	9.079233	0.3631693
	0	3	8	10	12	13	19	20	24	25	27	46	50	9.084659	0.3633863
	0	3	8	9	13	20	22	23	26	28	30	32	50	9.100102	0.3640040
	0	3	8	10	11	15	17	19	20	21	35	36	50	9.101286	0.3640514
	0	3	8	10	11	15	17	19	21	22	34	40	50	9.115643	0.3646257
	0	3	8	9	13	20	22	23	26	35	42	47	50	9.116827	0.3646730
	0	3	8	10	12	13	19	20	24	25	27	31	50	9.312800	0.3725120
	0	3	8	10	11	15	17	19	20	21	33	48	50	9.423503	0.3769401
	0	3	8	10	11	15	17	19	21	22	24	27	50	9.498632	0.3799452
	0	3	8	10	12	13	19	20	24	25	27	29	50	9.603656	0.3841462
	0	3	8	9	13	20	22	23	26	28	29	30	50	9.693265	0.3877306
	0	3	8	10	11	15	17	19	21	24	25	26	50	9.782430	0.3912972
	0	3	8	10	11	15	17	19	21	22	23	25	50	10.365898	0.4146359
	0	3	8	10	11	15	17	19	21	22	23	24	50	10.672184	0.4268873
	0	3	8	10	11	15	17	19	20	21	22	23	50	11.662957	0.4665182
	0	3	8	10	11	12	16	17	18	19	20	22	50	12.458943	0.4983577
	0	3	8	10	11	12	16	17	18	19	20	21	50	12.756384	0.5102553
	0	3	8	10	12	13	14	16	17	18	19	20	50	13.761580	0.5504732
	0	3	8	10	11	12	13	14	15	16	17	19	50	13.925108	0.5570043
	0	3	8	10	11	12	13	14	15	16	17	18	50	14.040417	0.5616166
	0	3	8	9	10	11	12	13	14	15	16	17	50	14.877515	0.5871006
	0	5	6	8	9	10	11	12	13	14	15	16	50	15.711099	0.6284439
	0	3	6	7	8	9	10	11	12	13	14	15	50	16.449099	0.6579639
	0	4	5	6	7	8	9	10	11	12	13	14	50	17.210695	0.6884278
	0	3	4	5	6	7	8	9	10	11	12	13	50	17.900421	0.7160168
	0	2	3	4	5	6	7	8	9	10	11	12	50	18.515556	0.7406223
	0	1	2	3	4	5	6	7	8	9	10	11	50	19.053673	0.7421469

TABLE NO. [illegible]

RADIATION PATTERN OF 25 ELEMENT UNEQUALLY SPACED ANTENNA ARRAY IN 50 LAMBDA APERTURE DESIGNED BY DYNAMIC PROGRAMMING TECHNIQUE $U_{min} = 0.04$, $U_{max} = 1.0$, $\Delta x = 0.5\ \lambda$
HIGHEST SIDE LOBE IS – 10.14 db BELOW THE MAIN BEAM



ELEMENT LOCATIONS $X_0 = 0$, $X_1 = 5$, $X_2 = 6$, $X_3 = 7$, $X_4 = 8$, $X_5 = 9$, [illegible] 10, [illegible] 17, $X_{10} = 21$, $X_{11} = 23$, $X_{12} = 50$

[illegible] TO ARRAY [illegible]

The programme now computes 193 discrete values of u in determining the particular configuration of elements over the region of interest $0.02 \leq u \leq 0.50$. The location of the element pair for the optimum case measured in half-wave lengths from the centre of the array as obtained by the computer are :

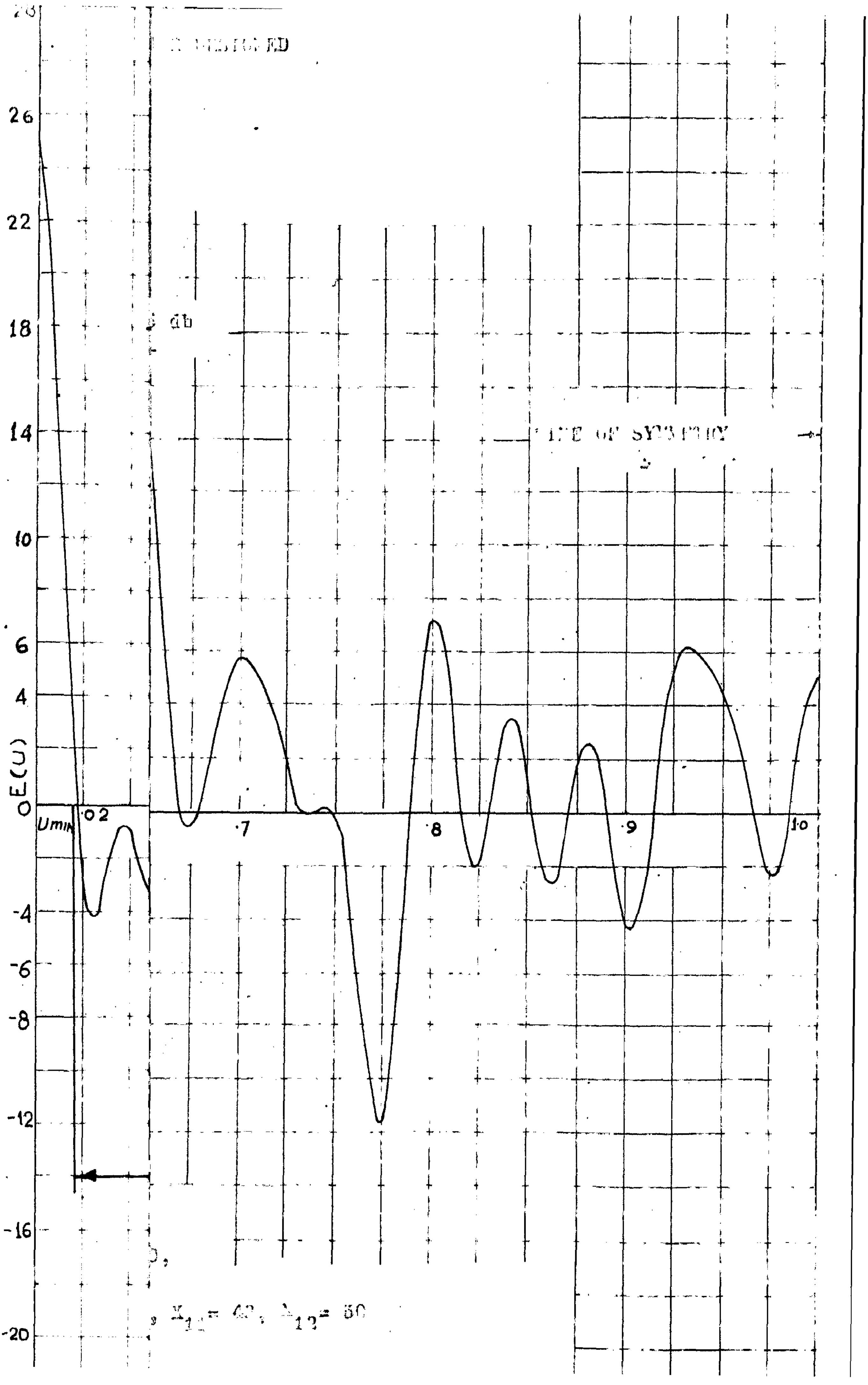
$x_1 = 6$, $x_2 = 9$ = , $x_3 = 12$, $x_4 = 15$, $x_5 = 19$
$x_6 = 22$, $x_7 = 25$, $x_8 = 28$, $x_9 = 32$, $x_{10} = 40$
$x_{11} = 43$ along with $x_o = 0$ and $x_{12} = 50$.

The radiation pattern for the above spacings is plotted in Fig. No. 5 at increments of $\Delta u = 0.0025$. Because of symmetry about u = 1 , the region from u =1, to u = 2 is not plotted. The maximum sidelobe level in the region of optimization is 13.54 db below the main lobe, a 3.4 db improvement over that obtained for the scanned array of Fig. No. 4 . In the remaining visible portion of the u region, however, the side lobe increases to a value of -3.15 db. The various element combinations obtained by the computer along with the absolute peak sidelobe level are shown in Table No.5. The run time for this programme is 10 minutes.

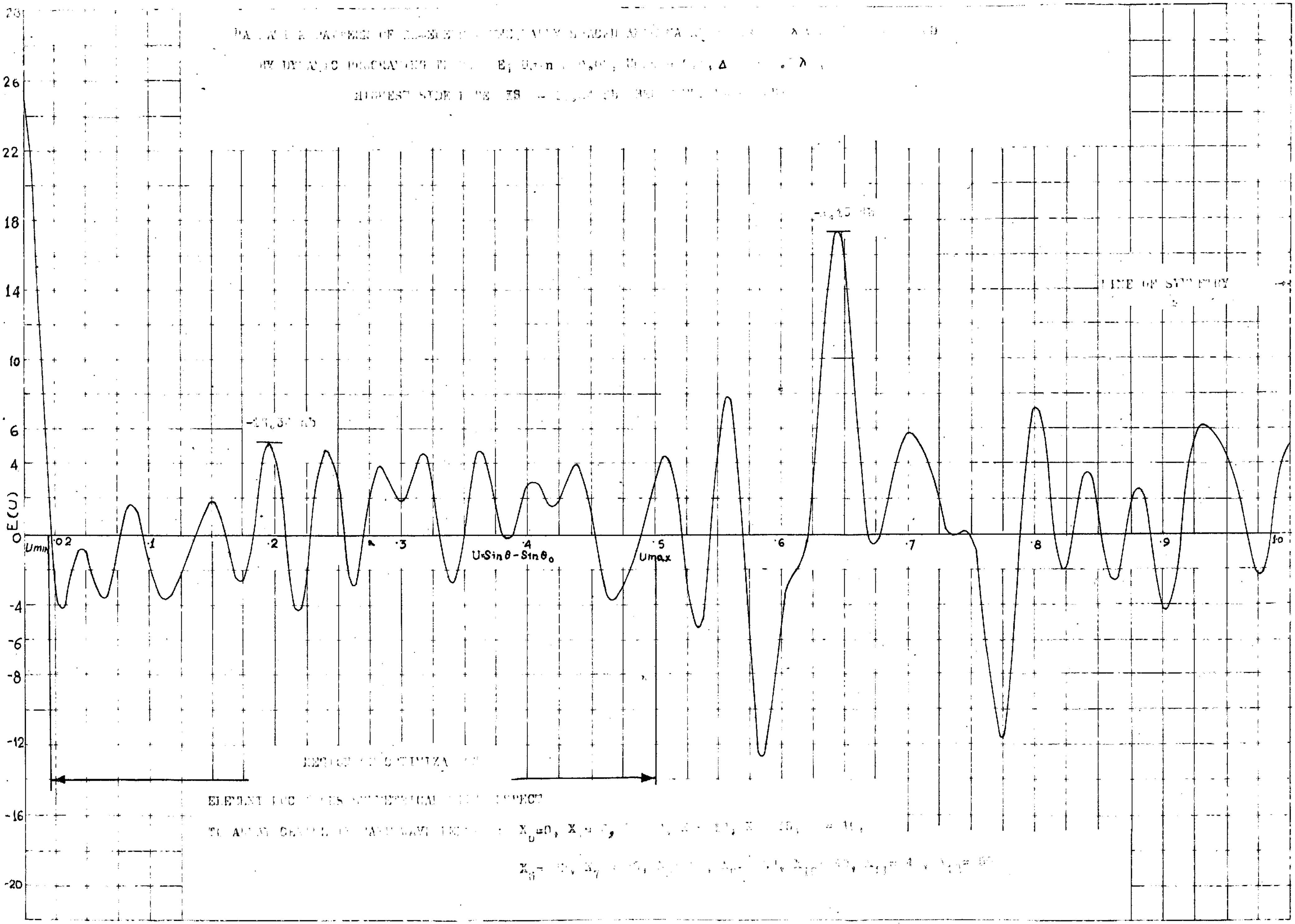
From the radiation pattern Fig. No. 5 it is seen that $u_{min} = 0.02$ does not include the main lobe

S.No	ELEMENT LOCATIONS IN HALF WAVELENGTHS SYMMETRICAL WITH RESPECT TO X = 0 UMIN = 0.02, UMAX = 0.5 DELTA X = 0.5 LAMBDA, D = 50 LAMBDA													MAXIMUM SIDELOBE AMPLITUDE	SIDELOBE LEVEL WHEN MAIN BEAM AMPLITUDE IS UNITY
1	0	6	9	12	15	19	22	25	28	32	40	43	50	5.258899	0.21035596
2	0	6	9	12	15	19	22	25	28	32	35	39	50	5.361791	0.21447164
3	0	6	9	12	15	19	22	25	28	32	35	38	50	5.464385	0.21857540
4	0	6	9	12	15	19	22	25	28	34	37	40	50	5.695960	0.22783840
5	0	6	9	12	15	19	22	25	28	32	40	47	50	5.756215	0.23024860
6	0	6	9	12	15	19	22	25	28	32	34	36	50	5.784729	0.23138916
7	0	6	9	12	15	19	22	25	28	32	35	44	50	5.851580	0.23406320
8	0	6	9	12	15	19	22	25	28	34	37	45	50	6.113975	0.24455900
9	0	6	9	12	15	19	22	25	28	32	35	37	50	6.120516	0.24482064
10	0	6	9	12	15	19	22	25	28	31	39	46	50	6.155243	0.24620972
11	0	6	9	12	15	19	22	25	26	28	31	42	50	6.171448	0.24685792
12	0	6	9	12	15	19	22	25	28	38	41	48	50	6.259138	0.25036552
13	0	6	9	12	15	19	22	25	28	32	34	35	50	6.306362	0.25225448
14	0	6	9	12	15	19	22	25	26	28	32	34	50	6.527634	0.26110536
15	0	6	9	12	15	19	22	25	28	32	35	41	50	6.538016	0.26152064
16	0	6	9	12	15	19	22	25	28	38	41	49	50	6.911409	0.27645636
17	0	6	9	12	15	19	22	25	26	28	31	33	50	7.031434	0.28125736
18	0	6	9	12	15	19	22	25	26	27	28	32	50	7.691211	0.30764844
19	0	6	9	12	15	19	22	25	26	27	28	31	50	8.138665	0.32554660
20	0	6	9	12	15	19	22	25	26	27	28	30	50	8.528853	0.34115412
21	0	6	9	12	15	19	22	25	26	27	28	29	50	8.844890	0.35379560
22	0	6	9	12	15	19	22	23	25	26	27	28	50	9.433110	0.37732440
23	0	6	8	9	12	13	14	17	21	23	25	27	50	9.863588	0.39454352
24	0	6	8	9	12	13	14	17	21	23	24	26	50	10.114255	0.40457020
25	0	6	8	9	12	13	14	17	21	23	24	25	50	10.239836	0.40959344
26	0	6	8	9	12	13	14	17	21	22	23	24	50	10.660188	0.42640752
27	0	6	8	9	12	13	14	17	20	21	22	23	50	11.705852	0.46833440
28	0	6	8	9	12	13	14	17	19	20	21	22	50	12.706504	0.50826016
29	0	6	8	9	12	13	14	17	18	19	20	21	50	13.572016	0.54288064
30	0	5	7	8	9	12	13	14	15	16	17	20	50	14.083681	0.56334724
31	0	5	7	8	9	12	13	14	15	16	17	19	50	14.201896	0.56807584
32	0	5	7	8	9	12	13	14	15	16	17	18	50	14.317205	0.57254820
33	0	5	7	8	9	11	12	13	14	15	16	17	50	15.006673	0.60026692
34	0	4	6	7	8	9	10	12	13	14	15	16	50	16.014779	0.64059116
35	0	4	6	7	8	9	10	11	12	13	14	15	50	16.484152	0.67936608
36	0	4	5	6	7	8	9	10	11	12	13	14	50	17.210695	0.68842780
37	0	3	4	5	6	7	8	9	10	11	12	13	50	17.900421	0.71601684
38	0	2	3	4	5	6	7	8	9	10	11	12	50	18.515556	0.74062224
39	0	1	2	3	4	5	6	7	8	9	10	11	50	19.053673	0.76214692

TABLE NO.5



26
22
18
14
10
6
4
OE(U)
0
-4
-6
-8
-12
-16
-20
UMIN
·02
·7
·8
·9
1·0
db


E(U)
26
22
18
14
10
6
4
0
-4
-6
-8
-12
-16
-20
Umin
·02
·1
·2
·3
·4
·5
·6
·7
·8
·9
1·0
U=Sin θ - Sin θ₀
Umax

and is almost at the first null and hence it exactly fulfils the requirements laid down in art. 5.3(iii).

Another case was tried in which Δu is taken as 0.002 to have more percentage accuracy, sacrificing at the same time the element locations from 39 to 29. The modified programme was written for IBM 7044, and a slightly different of element locations was obtained. However, the peak sidelobes in the two cases were within a tenth of a db, hence the results have not been given here.

The peak sidelobe level obtained by various design techniques available so far for the 25 element array is given in the Table No. 6.

Synthesis Techniques	Peak sidelobe level when the main beam amplitude is unity.	
By Ishimaru and Chen's formula	$(2N+1)^{-0.4}$	0.480 (−6.19 db)
	$(2N+1)^{-0.5}$	0.348 (−9.17 db)
Dynamic programming technique as applied by Skolnik et al	$U_{max} = 1$	0.363 (−8.8 db)
	$U_{max} = 0.5$	0.240 (−12.6 db)
Statistical method of Lo and Lee	$U_{max} = 1$	0.358 (−8.9 db)
	$U_{min} = 0.5$	-
Dynamic programming as applied here.	$U_{max} = 1$	0.311 (−10.14 db)
	$U_{min} = 0.5$	0.210 (−13.4 db)

Table No. 6.

CHAPTER VI

CONCLUSIONS AND DISCUSSIONS

The optimization problem considered here is very complex and is very difficult to find a general analytical expression which can predict the sidelobe level without much computations. The method of dynamic programming to the synthesis of a 25 element array located within a 50 wave-length aperture was first applied by Skolnik et. al.(1964). However, the results obtained were not truly optimum as slightly better results were reported by Lo and Lee(1966) using the method of space topering and total enumaration. Their statistical study indicated that the dynamic programming technique was not very efficient in searching for an element arrangement producing low sidelobes. The use of dynamic programming is reinvestigated here considering the same example as that of Skolnik. The results obtained are very much superior to those obtained by the previous investigators (See Table No. 6 , Chapter V) For the 9 element case the results are infact truly optimum as they coincide with those obtained by total enumeration . It seems that the calculations of the original authors namely Skolnik et, al. are in error . Once again from Table 4 Chapter 5 it is seen that out of 39 sets of element combinations obtained by the computers as many as 30 sets give peak side-lobe levels which are lower then those obtained by previous investigators. The best element combination of Skolnik et al. for a similar array has a peak sidelobe level 8.8. db below the main beam; that of Lo and Lee has a peak sidelobe level 8.9 db below the main beam.

Dynamic programming may be used to explore the properties of array antennas by varying the input parameters, examining the result, and making proper deductions as to the array behaviour. It does not yield closed form answers. But it has the important advantage that it can supply useful answers where other more elegant techniques fail to provide practical solutions.

Computational difficulties might be encountered using dynamic programming if the number of elements become too large. However other techniques suffer from the same limitations. The computer programme that generated the results reported here can be extended to enlarge the scope of the investigation. This programme was performed using only the rapid access storage of the computer and involved no additional storage. The upper limit of array complexity that dynamic programming can economically handle is a subject of future exploration, but it can be said that it is practical to design considerably large arrays than described here.

Dynamic programming has proved to be a useful tool for the design of one class of antenna arrays and promises to be of value for other antenna array problems. In conclusion it can be said that with confidence that out of various synthesis methods available so far, dynamic programming technique is the best and the results obtained by this method will be very close to optimum if not truly optimum.

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APPENDIX-A

PROGRAM FOR THE DESIGN OF NON-UNIFORMLY SPACED ANTENNA ARRAYS OF 25 ELEMENTS SPACED WITHIN A 50 WAVELENGTH APERTURE

```
JOB             OCG026
IBJOB
IBFTC MAIN      NODECK
      NA CORRESPONDS TO POSITIONS ALLOTED TO 1ST ELEMENT (1 TO 39)
      NB CORRESPONDS TO POSITIONS ALLOTED TO 2ND ELEMENT (2 TO 40)
      NC CORRESPONDS TO POSITIONS ALLOTED TO 3RD ELEMENT (3 TO 41)
      ND CORRESPONDS TO POSITIONS ALLOTED TO 4TH ELEMENT (4 TO 42)
      NE CORRESPONDS TO POSITIONS ALLOTED TO 5TH ELEMENT (5 TO 43)
      NF CORRESPONDS TO POSITIONS ALLOTED TO 6TH ELEMENT (6 TO 44)
      NG CORRESPONDS TO POSITIONS ALLOTED TO 7TH ELEMENT (7 TO 45)
      NH CORRESPONDS TO POSITIONS ALLOTED TO 8TH ELEMENT (8 TO 46)
      NO CORRESPONDS TO POSITIONS ALLOTED TO 9TH ELEMENT (9 TO 47)
      NP CORRESPONDS TO POSITIONS ALLOTED TO 10TH ELEMENT (10 TO 48)
      NQ CORRESPONDS TO POSITIONS ALLOTED TO 11TH ELEMENT (11 TO 49)
      JLAST REFERS TO POSITION OF FIXED ANTENNA (50)
      KLAST IS SUCH THAT FOR SOME VALUE OF K GIVES U MAX
      INVE REFERS TO INITIAL POSITION FOR THE 2ND ELEMENT (2)
      HI IS SUCH THAT K = 1 GIVES U MIN
      H REFERS TO INCREMENT IN U (0.0025)
      ABE(J) CORRESPONDS TO ABSOLUTE SUM
      DIMENSION Z(50,393)
      DIMENSION X(50),Y(393),ABE(393),A(39),M(40),AD(40),B(40),KA(41)
      DIMENSION AF(41),C(41),KB(42),BF(42),VA(43),KC(43),FA(43),VB(43)
      DIMENSION FB(44),JD(44),VC(44),JE(45),FC(45),VD(45),JF(46),FD(46)
      DIMENSION VE(46),FE(47),JH(47),VF(47),JO(48),FF(48),JP(49),FG(49)
      DIMENSION VG(48),MPG(48)
  700 READ 100,HI,H,JLAST,KLAST,INVE
  100 FORMAT (2F10.6,I2,I3,I2)
      PY = 3.1415927
      DO 60 I = 1,JLAST
      XX=I
   60 X(I) = PY*XX
      DO 61 K = 1,KLAST
      YY = K
   61 Y(K) = HI+H*YY
      DO 900 I = 1,JLAST
      DO 900 K = 1,KLAST
      S = COS(X(I)*Y(K))
  900 Z(I,K) = S+S
```

```
      STAGE 1
      DO 1 NB = INVE,40
      N1 = NB-1
      DO 2 NA = 1,N1
      DO 3 J = 1,KLAST
    3 ABE(J) = ABS(1.+Z(NA,J)+Z(NB,J)+Z(JLAST,J))
      AB = ABE(1)
        A(NA)=AB
      DO 4 K = 2,KLAST
      IF(AB-ABE(K))5,4,4
    5 AB = ABE(K)
      A(NA) = AB
    4 CONTINUE
    2 CONTINUE
      AC = A(1)
      AD(NB) = AC
      M(NB) = 1
      IF(N1-1)11,11,42
   11 GO TO 1
   42 DO 9 NA = 2,N1
      IF(AC-A(NA))9,10,10
   10 AC = A(NA)
      AD(NB)= AC
      M(NB) = NA
    9 CONTINUE
    1 PRINT 30,NB,M(NB),AD(NB)
   30 FORMAT (1XI2,I4,F10.6)

      STAGE 2
      DO 12 NC = 3,41
      I1 = NC-1
      DO 13 NB = 2,I1
      MN = M(NB)
      DO 14 J = 1,KLAST
   14 ABE(J) = ABS(1.+Z(NC,J)+Z(NB,J)+Z(MN,J)+Z(JLAST,J))
      AE = ABE(1)
      B(NB) = AE
      DO 16 K = 2,KLAST
      IF(AE-ABE(K))15,16,16
   15 AE = ABE(K)
      B(NB) = AE
   16 CONTINUE
   13 CONTINUE
      AF1 = B(2)
      AF(NC) = AF1
      KA(NC) = 2
      KANC = 2
      IF(I1-2) 51,51,52
   51 GO TO 12
   52 DO 18 NB = 3,I1
      IF(AF1-B(NB)) 18,19,19
   19 AF1 = B(NB)
      AF(NC) = AF1
      KA(NC) = NB
      KANC = KA(NC)
   18 CONTINUE
   12 PRINT 300,NC,KA(NC),M(KANC),AF(NC)
  300 FORMAT (1XI2,2I4,F10.6)
```

```
      STAGE 3
      DO 6 ND = 4,42
      J1 = ND-1
      DO 7 NC = 3,J1
      MA = KA(NC)
      MB = M(MA)
      DO 8 J = 1,KLAST
    8 ABE(J) = ABS(1.+Z(ND,J)+Z(NC,J)+Z(MA,J)+Z(MB,J)+Z(JLAST,J))
      AC2 = ABE(1)
      C(NC) = AC2
      DO 23 K = 2,KLAST
      IF(AC2-ABE(K)) 22,23,23
   22 AC2 = ABE(K)
      C(NC) = AC2
   23 CONTINUE
    7 CONTINUE
      AF2 = C(3)
      BF(ND) = AF2
      KB(ND) =3
      KB1 = 3
      KD = KA(KB1)
      IF(J1-3) 24,24,25
   24 GO TO 6
   25 DO 27 NC = 4,J1
      IF(AF2-C(NC)) 27,28,28
   28 AF2 = C(NC)
      BF(ND) = AF2
      KB(ND) = NC
      KB1 = KB(ND)
      KD =KA(KB1)
   27 CONTINUE
    6 PRINT 29,ND,KB(ND),KA(KB1),M(KD),BF(ND)
   29 FORMAT (1XI2,3I4,F10.6)

      STAGE 4
      DO 31 NE = 5,43
      I2 = NE-1
      DO 32 ND = 4,I2
      MC = KB(ND)
      MD = KA(MC)
      ME = M(MD)
      DO 33 J = 1,KLAST
   33 ABE(J)=ABS(1.+Z(NE,J)+Z(ND,J)+Z(MC,J)+Z(MD,J)+Z(ME,J)+Z(JLAST,J)
      AC3 = ABE(1)
      VA(ND) = AC3
      DO 34 K= 2,KLAST
      IF(AC3-ABE(K)) 35,34,34
   35 AC3 = ABE(K)
      VA(ND) = AC3
   34 CONTINUE
   32 CONTINUE
      AF3 = VA(4)
      FA(NE) = AF3
      KC(NE) = 4
      KB2 = 4
      KP = KB(KB2)
      KQ = KA(KP)
      IF(I2-4) 37,37,38
   37 GO TO 31
   38 DO 39 ND = 5,I2
```

```
      IF(AF3-VA(ND)) 39,40,40
   40 AF3 = VA(ND)
      FA(NE) = AF3
      KC(NE) = ND
      KB2 = KC(NE)
      KP = KB(KB2)
      KQ = KA(KP)
   39 CONTINUE
   31 PRINT 41,NE,KC(NE),KB(KB2),KA(KP),M(KQ),FA(NE)
   41 FORMAT (1XI2,4I4,F10.6)

      STAGE 5
      DO 43 NF = 6,44
      I3 = NF-1
      DO 44 NE = 5,I3
      MF = KC(NE)
      MG = KB(MF)
      Mh = KA(MG)
      MO = M(MH)
      DO 45 J = 1,KLAST
   45 ABE(J)=ABS(1.+Z(NF,J)+Z(NE,J)+Z(MF,J)+Z(MG,J)+Z(MH,J)+Z(MO,J)+
     1Z(JLAST,J))
      AC4 = ABE(1)
      VB(NE) = AC4
      DO 46 K = 2,KLAST
      IF(AC4-ABE(K)) 47,46,46
   47 AC4 = ABE(K)
      VB(NE)=AC4
   46 CONTINUE
   44 CONTINUE
      AF4 = VB(5)
      FB(NF) = AF4
      JD(NF) = 5
      KB3 = 5
      KR = KC(KB3)
      KS = KB(KR)
      KT = KA(KS)
      IF(I3-5) 50,50,49
   50 GO TO 43
   49 DO 53 NE = 6,I3
      IF(AF4-VB(NE)) 53,54,54
   54 AF4 = VB(NE)
      FB(NF) = AF4
      JD(NF) = NE
      KB3 = JD(NF)
      KR = KC(KB3)
      KS = KB(KR)
      KT = KA(KS)
   53 CONTINUE
   43 PRINT 55,NF,JD(NF),KC(KB3),KB(KR),KA(KS),M(KT),FB(NF)
   55 FORMAT (1XI2,5I4,F10.6)
```

```
C      STAGE 6
       DO 56 NG = 7,45
       I4 = NG-1
       DO 57 NF = 6,I4
       MP = JD(NF)
       MQ = KC(MP)
       MR = KB(MQ)
       MS = KA(MR)
       MT = M(MS)
       DO 58 J = 1,KLAST
    58 ABE(J)=ABS(1.+Z(NG,J)+Z(NF,J)+Z(MP,J)+Z(MQ,J)+Z(MR,J)+Z(MS,J)+
      2Z(MT,J)+Z(JLAST,J))
       AC5 = ABE(1)
       VC(NF) = AC5
       DO 62 K = 2,KLAST
       IF(AC5-ABE(K)) 59,62,62
    59 AC5=ABE(K)
       VC(NF) = AC5
    62 CONTINUE
    57 CONTINUE
       AF5 = VC(6)
       FC(NG) = AF5
       JE(NG) = 6
       KB4 = 6
       KU = JD(KB4)
       KV = KC(KU)
       KW = KB(KV)
       KX = KA(KW)
       IF(I4-6) 64,64,65
    64 GO TO 56
    65 DO 67 NF = 7,I4
       IF(AF5-VC(NF)) 67,68,68
    68 AF5=VC(NF)
       FC(NG) = AF5
       JE(NG) = NF
       KB4 = JE(NG)
       KU = JD(KB4)
       KV = KC(KU)
       KW = KB(KV)
       KX = KA(KW)
    67 CONTINUE
    56 PRINT 69,NG,JE(NG),JD(KB4),KC(KU),KB(KV),KA(KW),M(KX),FC(NG)
    69 FORMAT (1XI2,6I4,F10.6)

C      STAGE 7
       DO 70 NH = 8,46
       I5 = NH-1
       DO 71 NG = 7,I5
       MU = JE(NG)
       MV = JD(MU)
       MW = KC(MV)
       MX = KB(MW)
       MY = KA(MX)
```

```
      MZ = M(MY)
      DO 72 J = 1,KLAST
   72 ABE(J) =ABS(1.+Z(NH,J)+Z(NG,J)+Z(MU,J)+Z(MV J)+Z(MW,J)+Z(MX,J)+
     3Z(MY,J)+Z(MZ,J)+Z(JLAST,J))
      AC6 = ABE(1)
      VD(NG) = AC6
      DO 73 K = 2,KLAST
      IF(AC6-ABE(K)) 74,73,73
   74 AC6 = ABE(K)
      VD(NG) = AC6
   73 CONT+NUE
   71 CONTINUE
      AF6 = VD(7)
      FD(NH) = AF6
      JF(NH) = 7
      KB5 = 7
      IA = JE(KB5)
      IB = JD(IA)
      IC = KC(IB)
      ID = KB(IC)
      IE = KA(ID)
      IF(I5-7) 76,76,77
   76 GO TO 70
   77 DO 78 NG = 8,I5
      IF(AF6-VD(NG)) 78,79,79
   79 AF6 = VD(NG)
      FD(NH) = AF6
      JF(NH) = NG
      KB5 = JF(NH)
      IA = JE(KB5)
      IB = JD(IA)
      IC = KC(IB)
      ID = KB(IC)
      IE = KA(ID)
   78 CONTINUE
   70 PRINT 80,NH,JF(NH),JE(KB5),JD(IA),KC(IB),KB(+C),KA(ID),M(IE),
     4FD(NH)
   80 FORMAT (1XI2,7I4,F10.6)

C     STAGE 8
      DO 81 NO = 9,47
      I6 = NO-1
      DO 82 NH = 8,I6
      LD = JF(NH)
      LE = JE(LD)
      LF = JD(LE)
      LG = KC(LF)
      LH = KB(LG)
```

```
      LO = KA(LH)
      LP = M(LO)
      DO 83 J = 1,KLAST
   83 ABE(J) =ABS(1.+ Z(NO,J)+Z(NH,J)+Z(LD,J)+Z(LE,J)+Z(LF,J)+Z(LG,J)+
     8Z(LH,J)+Z(LO,J)+Z(LP,J)+Z(JLAST,J))
      AC7 = ABE(1)
      VE(NH) = AC7
      DO 84 K = 2,KLAST
      IF(AC7-ABE(K)) 85,84,84
   85 AC7 = ABE(K)
      VE(NH) = AC7
   84 CONTINUE
   82 CONTINUE
      AF7 =VE(8)
      FE(NO) = AF7
      JH(NO) = 8
      KB6 = 8
      IO = JF(KB6)
      IP = JE(IO)
      IQ = JD(IP)
      IR = KC(IQ)
      IS = KB(IR)
      IT = KA(IS)
      IF(I6-8) 87,87,88
   87 GO TO 81
   88 DO 89 NH = 9,I6
      IF(AF7-VE(NH)) 89,90,90
   90 AF7 = VE(NH)
      FE(NO) = AF7
      JH(NO) = NH
      KB6 = JH(NO)
      IO = JF(KB6)
      IP = JE(IO)
      IQ = JD(IP)
      IR = KC(IQ)
      IS = KB(IR)
      IT = KA(IS)
   89 CONTINUE
   81 PRINT 91,NO,JH(NO),JF(KB6),JE(IO),JD(IP),KC(+Q),KB(IR),KA(IS),
     5M(IT),FE(NO)
   91 FORMAT (1XI2,8I4,F10.6)
C     STAGE 9
      DO 92 NP = 10,48
      I7 = NP-1
      DO 93 NO = 9,I7
      LQ = JH(NO)
      LR = JF(LQ)
      LS = JE(LR)
```

```
      LT = JD(LS)
      LU = KC(LT)
      LV = KB(LU)
      LW = KA(LV)
      LX = M(LW)
      DO 94 J = 1,KLAST
   94 ABE(J) =ABS(1.+Z(NP,J)+Z(NO,J)+Z(LQ,J)+Z(LR,J)+Z(LS,J)+Z(LT,J)+
     6Z(LU,J)+Z(LV,J)+Z(LW,J)+Z(LX,J)+Z(JLAST,J))
      AC8 = ABE(1)
      VF(NO) = AC8
      DO 95 K = 2,KLAST
      IF(AC8-ABE(K)) 96,95,95
   96 AC8=ABE(K)
      VF(NO)=AC8
   95 CONTINUE
   93 CONTINUE
      AF8 = VF(9)
      FF(NP) = AF8
      JO(NP) = 9
      KB7 = 9
      IA1 = JH(KB7)
      IB1 = JF(IA1)
      IC1 = JE(IB1)
      ID1 = JD(IC1)
      IE1 = KC(ID1)
      IF1 = KB(IE1)
      IG1 = KA(IF1)
      IF(I7-9) 98,98,99
   98 GO TO 92
   99 DO 101 NO = 10,I7
      IF(AF8-VF(NO)) 101,102,102
  102 AF8=VF(NO)
      FF(NP) = AF8
      JO(NP) = NO
      KB7 = JO(NP)
      IA1 = JH(KB7)
      IB1 = JF(IA1)
      IC1 = JE(IB1)
      ID1 = JD(IC1)
      IE1 = KC(ID1)
      IF1 = KB(IE1)
      IG1 = KA(IF1)
  101 CONTINUE
   92 PRINT 103,NP,JO(NP),JH(KB7),JF(IA1),JE(IB1),JD(IC1),KC(ID1),
     9KB(IE1),KA(IF1),M(IG1),FF(NP)
  103 FORMAT (1XI2,9I4,F10.6)
```

```
      STAGE 10
      DO 104 NQ = 11,49
      I8 = NQ-1
      DO 105 NP = 10,I8
      MAL = JO(NP)
      MBM = JH(MAL)
      MCN = JF(MBM)
      MDO = JE(MCN)
      MEP = JD(MDO)
      MFQ = KC(MEP)
      MGR = KB(MFQ)
      MHS = KA(MGR)
      MNT = M(MHS)
      DO 106 J = 1,KLAST
  106 ABE(J) =ABS(1.+Z(NQ,J)+Z(NP,J)+Z(MAL,J)+Z(MBM,J)+Z(MCN,J)+
     1Z(MDO,J)+Z(MEP,J)+Z(MFQ,J)+Z(MGR,J)+Z(MHS,J)+Z(MNT,J)+Z(JLAST,J))
      AC9 = ABE(1)
      VG(NP) = AC9
      MPG(NP) = 1
      DO 107 K = 2,KLAST
      IF(AC9-ABE(K)) 108,107,107
  108 AC9=ABE(K)
      VG(NP)=AC9
      MPG(NP) = K
  107 CONTINUE
  105 PRINT 109,NQ,NP,JO(NP),MPG(NP),VG(NP)
  109 FORMAT (1XI2,2I4,I6,F10.6)
      AF9 = VG(10)
      FG(NQ) = AF9
      JP(NQ) = 10
      KB8 = 10
      MA1 = JO(KB8)
      MB1 = JH(MA1)
      MB2 = JF(MB1)
      MB3 = JE(MB2)
      MB4 = JD(MB3)
      MB5 = KC(MB4)
      MB6 = KB(MB5)
      MB7 = KA(MB6)
      IF(I8-10) 110,110,111
  110 GO TO 104
  111 DO 112 NP = 11,I8
      IF(AF9-VG(NP)) 112,113,113
  113 AF9=VG(NP)
      FG(NQ) = AF9
      JP(NQ) = NP
      KB8 = JP(NQ)
```

```
      MA1  =  JO(KB8)
      MB1  =  JH(MA1)
      MB2  =  JF(MB1)
      MB3  =  JE(MB2)
      MB4  =  JD(MB3)
      MB5  =  KC(MB4)
      MB6  =  KB(MB5)
      MB7  =  KA(MB6)
  112 CONTINUE
  104 PRINT 114,NQ,JP(NQ),JO(KB8),JH(MA1),JF(MB1),JE(MB2),JD(MB3),
     2KC(MB4),KB(MB5),KA(MB6),M(MB7),FG(NQ)
  114 FORMAT (1XI2,10I4,F10.6)
      GO TO 700
  800 STOP
      END
$ENTRY
.0175      .0025      50393 2
.0375      .0025      50385 2
.0175      .0025      50193 2
```

APPENDIX-B

PROGRAM FOR RADIATION PATTERN

```
$JOB            OCG027
$IBJOB
$IBFTC MAIN     NODECK
      DIMENSION X(50),Y(401),Z(50,401),FOXU(401)
  900 READ 20,NA,NB,NC,ND,NE,NF,NG,NH,NO,NP,NQ
   20 FORMAT (11I2)
      PY = 3.1415927
      DO 60 I = 1,50
      XX = I
   60 X(I) = PY*XX
      DO 61 K = 1,401
      YY = K-1
   61 Y(K) = 0.0025*YY
      DO 62 I = 1,50
      DO 62 K = 1,401
      S = COS(X(I)*Y(K))
   62 Z(I,K) = S+S
      DO 6 K = 1,401
      FXA = 1.+Z(NA,K)+Z(NB,K)+Z(NC,K)+Z(ND,K)+Z(NE,K)+Z(NF,K)+Z(NG,K)
      FXB = Z(NH,K)+Z(NO,K)+Z(NP,K)+Z(NQ,K)+Z(50,K)
    6 FOXU(K) = FXA+FXB
      PRINT 30,(Y(K),FOXU(K),K=1,401)
   30 FORMAT (3XF8.5,F10.5,F8.5,F10.5,F8.5,F10.5,F8.5,F10.5,F8.5,F10.5)
      GO TO 900
   50 STOP
      END
$ENTRY
0308101115171921223444
0506070809101315172126
0609121519222528324043
```