ON THE NATURE OF EQUIVALENCE CLASSES & CIRCUITS FOR THREE VARIABLE BOOLEAN FUNCTIONS

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CERTIFICATE

Certified that the dissertation entitled "ON THE NATURE OF EQUIVALENCE CLASSES AND CIRCUITS FOR 3- VARIABLE BOOLEAN FUNCTION" which is being submitted by Sri G. Ramachandra Naidu in partial fulfilment for the award of the Degree of Master of Engineering in Applied Electronics and Servomechanism of the University of Roorkee is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other Degree or Diploma.

This is to further certify that he has worked for a period of 7 months from December, 1966 to June, 1967 for preparing this thesis for Master of Engineering at the University.

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S Y N O P S I S

Lee Hillerman[3] has given 78 equivalent networks to realize 256 Boolean functions in three variable case. In this paper an attempt is made to reduce these equivalent networks still further.

78 Networks are reduced to 32 possible minimal equivalent networks either by complimenting the output of a given network or grouping the functions which are having equal α-numbers weather they are totally symmetric or partially symmetric and making certain changes at the input terminals. Also it is reduced to 19 networks if we are not aimed at the minimal networks.

C O N T E N T S

CHAPTER I

INTRODUCTION

INTRODUCTION

The logic designer aims at minimum number of logical elements required to perform a given logical function. This is due to economy and reliability of the circuit. Many authors tried to get the minimal implementation.

Quite some work has been done to work out the implementation by a regular process, rather than an hit and trial method. Hillerman[3] has given the circuits, 78 in number, which are minimal in nature for the 256 functions that occure in the three variables. Hillerman[3] got these minimal circuits by feeding the data to a computer.

Hillerman[3] suggested an equivalence table in the same paper. By changing the input literals the 256 functions in three variables are brought down to 78 functions. The table which gives the 78 functions and the permutations to be used to get all the 256 functions is called Hillerman's[3] equivalence table and is given in the Table 3.1 (Appendix ii).

Figures written in parenthesis denotes the serial number of Bibliography given at the end.

78 functions which represents the remaining all the functions are called leaders. The properties of symmetric functions explained in Chapter II are used to bring down the number of leaders still further. E. J. McCluskey[4] Jr. has suggested the decomposition principle. This is explained in the Chapter III. By the help of this principle different functions are classified into symmetric and partially symmetric functions. These functions are expressed in terms of its α - numbers. The functions which are having same α - numbers and variables of symmetry are classified into one group. To make the variables of symmetry to be same, for the functions whose α- numbers are same, some changes in the variables are suggested. By this method the 78 leaders are reduced to 38 leaders. These groups and leaders are given in the Appendix II Table 3.2.

For each leader the complementary function is found out. If you complement one leader and make some changes to the input variables then we will get another leader. Thus the 78 leaders are reduced into 30 leaders. The relationship between the leaders and changes to be made are given in the Appendix II Table 3.3.

By the help of tables 3.2 and 3.3 (Appendix II) the leaders are still reduced. From the Hillerman's[3] catalogue each network is taken and the possibility of getting a minimal network for another function is studied. Changes are made according to the table 3.2 and 3.3. (Appendix II). Thus the 78 circuits are reduced to 32 circuits. These 32 circuits are main minimal circuits. By making the changes according to the (Appendix II)table 4 we will get all the 78 minimal circuits. The main 32 networks are given in the Appendix II. Table 4.

If we are not aiming at the minimal networks, we can reduce the number of equivalence networks still further. As explained in Chapter V we reduce the 78 minimal equivalence networks to 18 networks as given in Appendix II Table 5.

(4)

CHAPTER II

DEFINITION AND PROPERTIES OF TOTALLY SYMMETRIC AND PARTIALLY SYMMETRIC FUNCTIONS

DEFINITION AND PROPERTIES OF TOTALLY SYMMETRIC AND PARTIALLY SYMMETRIC FUNCTIONS

2.1. DEFINITION

C.E. Shannon(6) first introduced the word 'Symmetric function' in his paper on relays and switching circuits in 1938. He defined the symmetric function in general. In his words, " A function of the n variables $x_1, x_2, \ldots, x_n$ is said to be symmetric in these variables if any interchange of the variables leaves the function identically the same ". But most of the generality of concept has been lost in subsequent treatment of the subject. For example S.H. Washburn(7) defines the symmetric relay contact net work as "One in which the conditions for closing a particular input to output path are given .... in terms of the number of relays operated and unoperated.". This particular type of net works are derived from a restricted class of symmetric functions.

There are different types of symmetric functions, depending upon the nature of symmetry or nature of variables . Let us define the different types of symmetric function systematically as follows:

2.1.1. A function of $(x_1, x_2 \ldots\ldots x_i \ldots\ldots x_j \ldots x_n)$ is called symmetric in the pair of variables x_i, x_j if the function remains invariant under an interchange of the variables, x_i, x_j.

That is if

$$f(x_1, x_2, \ldots\ldots x_i, \ldots\ldots x_j \ldots\ldots x_n)$$

$$= f(x_1, x_2 \ldots x_j \ldots x_i \ldots\ldots x_n)$$

For example the function

$$f(x_1, x_2, x_3) = x_1 x_2^2 x_3 + x_1^2 x_2 x_3$$

is symmetric in x_1 and x_2

2.1.2 A function of $f(x_1, x_2, \ldots\ldots x_n)$ is called a totally symmetric function if it is symmetric in all pairs of variables

x_i, x_j, $1 \leq i, j \leq n$

Since any permutation of variables can be obtained by successive interchange of pairs of variables, it immediately follows that a totally symmetric function is invariant under any permutation of variables.

We shall often simply refer to the totally symmetric function as 'symmetric function'. For example the function

$$f(x_1, x_2, x_3) = x_1x_2 + x_1x_3 + x_2x_3 \qquad \ldots (2.1)$$

is totally symmetric in x_1, x_2 and x_3

If the function is not symmetric in all the pairs, but symmetric in some pairs, the function is called partially symmetric. In usual practice we find many of the partially symmetric functions. For example the function

$$f(x_1, x_2, x_3) = \bar{x}_1\bar{x}_2 + x_1x_2x_3 \qquad \ldots (2.2)$$

is symmetric only in one pair x_1x_2 only but not in other pairs. Hence it is called partially symmetric.

The variables for which a function is totally symmetric are called variables of symmetry. In expression (2.1) the variables x_1, x_2 and x_3 are called the variables of symmetry.

Depending upon the variables of symmetry there are two classes of symmetric functions. The first class has the variables of symmetry which are either all unprimed or all primed. Expression (2.1) is an example. In the second class some of the variables of symmetry are primed, but not all. Hence it is designated as symmetric function with mixed variables. For example the function

$$f(x_1x_2x_3) = \bar{x}_1\bar{x}_2x_3 + x_1x_2x_3 + x_1\bar{x}_2x_3 \quad \ldots(2.3)$$

is a totally symmetric function. It is not symmetric in x_1, x_2 and x_3, but it is symmetric in $\bar{x}_1$, x_2, $\bar{x}_3$

$$\therefore f(x_1, x_2x_3) = \bar{x}_1\bar{x}_2x_3 + x_1x_2x_3 + x_1\bar{x}_2\bar{x}_3$$

2.2. ALGEBRAIC PROPERTIES OF SYMMETRIC FUNCTIONS

For the analysis and realisation of symmetric functions we must know some of the important properties of these functions. First Shannon[6] in 1938 explained some of the properties in terms of hindrance functions without algebraic proof. In 1963 R.F. Arnold and M.A. Harrison[1] proved some of the properties mathematically by the help of set theory and group theory. To prove these properties they introduced new functions namely α - functions and - symmetric functions. To avoid higher mathematics, some of the properties are prooved in the following lines analytically.

Theorem 2.1. (Shannon's[6] Theorem or α- numbers theorem).

A function $f(x_1, x_2, \ldots x_n)$ is totally symmetric in the variables $(x_1, x_2, \ldots x_n)$ if and only if it may be specified by a set of numbers $(\alpha_1, \alpha_2, \alpha_3 \ldots \alpha_n)$ such that $f(x_1, x_2, \ldots x_n) = 1$

if and only if exactly α_j ($j = 1, 2, \ldots k$) of the variables are equal to 1 and others remaining variables are equal to 0 and not otherwise.

<u>Proof:</u>

Assume the function $f(x_1, x_2 \ldots x_n)$ is totally symmetric and $f(x_1, x_2 \ldots x_n) = 1$ when the first α_j variables are equal to 1 and other variables are equal to 0. Then $f = 1$ when any set of exactly α_j variables are equal to 1, since f is invariant under any permutation of variables.
For example

$$f(x_1, x_2, x_3) = x_1x_2 + x_2x_3 + x_3x_1$$

Now $f = 1$ if and only if two or three variables equal to 1. For this function the set of numbers ($\alpha_1, \alpha_2, \ldots \alpha_k$) is the set (2, 3). This function can be represented by $S_{2,3}(x_1, x_2, x_3)$ where 'S' stands for a totally symmetric function, 2, 3 stands for the set of numbers, ($\alpha_1, \alpha_2, \ldots \alpha_k$). Usually these set of numbers are called α numbers of the symmetric function, and ($x_1, x_2, \ldots x_n$) are called the variables of symmetry. The notation

$$S_{\alpha_1, \alpha_2 \ldots \alpha_k}(x_1, x_2, \ldots x_n)$$

is commonly used to represent a totally symmetric function.

The α - numbers for n- variable function can be selected from the numbers 0 to n. From this it follows that there are exactly 2^{n+1} symmetric functions of n variables, when these variables are taken as variables of symmetry. If the set of α- numbers includes all the integers 0 to n, this Boolean function is the trivial Boolean function which is always equal to 1. Similarly, if the set of α- numbers is empty the function is again trivial, that is always equal to zero. Thus there are $2^{n+1} - 2$ non trivial symmetric Boolean functions of n - variables. For $n = 2$, there are $2^{2^2} = 16$ functions of two variables. But there are $2^{2+1} - 2 = 6$ non trivial symmetric functions. They are

$S_0 = \bar{x}_1\bar{x}_2$ $\qquad$ $S_1 = x_1\bar{x}_2 + \bar{x}_1 x_2 = x_1 \odot x_2$

$S_{0,1} = \bar{x}_1 + \bar{x}_2$ $\qquad$ $S_{1,2} = x_1 + x_2$

$S_{0,2} = x_1 x_2 + \bar{x}_1\bar{x}_2$ $\qquad$ $S_2 = x_1 x_2$

Theorem 2.2. The union of two symmetric functions g and h having the same variables of symmetry, is a totally symmetric function of the same variables of symmetry whose set of α- numbers is the union of the sets of α- numbers for g and h.

Proof: $f = g + h$

$g = S_{\alpha_1, \alpha_2, \ldots \alpha_k}(x_1, x_2, \ldots x_n)$ ..(2.4)

$h = S_{\alpha'_1, \alpha'_2, \ldots \alpha'_k}(x_1, x_2, \ldots x_n)$ ...(2.5)

$g = 1$, when a set of $[\alpha_j]$ variables are equal to 1 ...(2.6)

$= 0$ when a set of $[\alpha_j]$ variables are not equal to 1 ...(2.7)

$h = 1$ when a set of $[\alpha'_j]$ variables are equal to 1 ...(2.8)

$h = 0$ when a set of $[\alpha'_j]$ variables are not equal to 1 ...(2.9)

$f = g + h$

As we know from Boolean algebra (Appendix I)

$1 + 0 = 1$, or $0 + 1 = 1$

$f = 1$, when either a set of $[\alpha_j]$ variable are equal to one or a set of $[\alpha'_j]$ variables are equal to one. Hence f is a symmetric function of the same variables having a set of α- numbers equal to $[\alpha_j]$ and $[\alpha'_j]$

$$S_{\alpha_1, \alpha_2 \ldots \alpha_k}(x_1, x_2 \ldots x_n) + S_{\alpha'_1, \alpha'_2 \ldots \alpha'_k}(x_1, x_2 \ldots x_n)$$

$$= S_{\alpha''_1, \alpha''_2 \ldots \alpha''_p}(x_1, x_2, \ldots x_n)$$

where $\alpha''_1, \alpha''_2 \ldots \alpha''_p = (\alpha_1, \alpha_2 \ldots \alpha_k) + (\alpha'_1, \alpha'_2 \ldots \alpha'_k)$

For example

$$S_{1,2,3,}(x_1, x_2 \ldots x_6) + S_{2,3,5}(x_1, x_2 \ldots x_6)$$

$$= S_{1,2,3,5}(x_1, x_2, \ldots x_6)$$

<u>Theorem 2.1.</u>

The product of two totally symmetric functions g and h having the same variables of symmetry, is a totally symmetric function of the same variables whose set of α- numbers is the set of numbers appearing in both sets of α- numbers for g and h.

<u>Proof.</u>

As we know

$$\left.\begin{array}{ll} 0.1 & = 0 \\ 1.0 & = 0 \\ 1.1 & = 1 \end{array}\right] \text{ From Boolean algebra (Appendix I)}$$

f = g. h.

If f is symmetric f must be equal to 1. To get f = 1, from the expressions 2.6 to 2.9 we require the set of $[\alpha_j]$ and $[\alpha'_j]$ numbers must be same,

$$S\,\alpha_1, \alpha_2 \ldots\alpha_k\,(x_1, x_2, \ldots x_n)\,.\,S\,\alpha'_1, \alpha'_2 \ldots \alpha'_k$$

$$(x_1, x_2, \ldots x_n)$$

$$= S\,\alpha''_1, \alpha''_2 \ldots \alpha''_p\,(x_1, x_2, \ldots x_n)$$

Where $\alpha''_1, \alpha''_2, \ldots \alpha''_p$ are the common α - numbers in $\alpha'_1, \alpha_2 \ldots \alpha_n$ and $\alpha'_1, \alpha'_2, \ldots \alpha'_k$.

For example

$$S_{1,2,3}(x_1, x_2, \ldots x_6) \cdot S_{2,3,5}(x_1, x_2, \ldots x_6)$$

$$= S_{2,3}(x_1, x_2, \ldots x_6)$$

From the above theorem it can be noted that the product of two symmetric functions, which are not having even single common α - number is not a symmetric function.

Theorem 1.4.

The complement of a symmetric function of n variables is a symmetric function of these variables having α - numbers from 0 to n which are not the α numbers of the given function.

Proof:

This theorem can be proved by using α - number theorem. The α - numbers of the given symmetric function for the numbers of the variables which make the given function equal to 1 when these variables are 1. Therefore, the α - numbers from 0 to n which are not the α - numbers of the given function will make the given function 0 . Thus , a symmetric function with these α - numbers and with the same variables of symmetry is the complement of the given function.

$$\bar{S}_{\alpha_1, \alpha_2, \ldots, \alpha_k}(x_1, x_2, x_3, \ldots, x_n)$$

$$= S_{\alpha'_1, \alpha'_2, \ldots, \alpha'_k}(x_1, x_2, \ldots, x_n)$$

Where $\alpha'_1, \alpha'_2, \ldots, \alpha'_k = (0, 1, 2, \ldots, n) - (\alpha_1, \alpha_2, \ldots \alpha_k)$

For example the complement of the function f is shown below:

$$\bar{f} = S_{2,3}(x_1, x_2, x_3)$$

$$= S_{0,1}(x_1, x_2, x_3)$$

Theorem 1.8

A symmetric function of n variables is equal to the symmetric function in which each of the original variables is complemented and each α- number α_i of the original function is replaced by the α- numbers $n - \alpha_i$.

This theorem can be deduced from α- numbers theorem. A symmetric function is equal to 1 when α_i variables of symmetry are equal to 1 or conversely when $n - \alpha_i$ variables of symmetry are equal to 0 (i.e., when $n - \alpha_i$ complemented variables of symmetry are equal to 1). This proves the above theorem.

$S_{\alpha_1, \alpha_2 \ldots \alpha_k}(x_1, x_2, \ldots, x_n)$

$= S_{n-\alpha_1, n-\alpha_2, \ldots n-\alpha_k}(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n)$

For example

$$S_{2,3}(x_1, x_2, x_3) = S_{0,1}(\bar{x}_1, \bar{x}_2, \bar{x}_3)$$

<u>Theorem 2.6</u>

<u>Shannon's Expansion Theorem</u>

A totally symmetric function

$S_{\alpha_1, \alpha_2 \ldots \alpha_k}(x_1, x_2, \ldots, x_n)$

can be expressed in expanded forms as

$S_{\alpha_1, \alpha_2 \ldots \alpha_k}(x_1, x_2, \ldots, x_n)$

$= \bar{x}_1 S_{\alpha_1, \alpha_2 \ldots \alpha_k}(x_2, x_3, \ldots, x_n)$

$+ x_1 S_{\alpha_1-1, \alpha_2-1, \ldots \alpha_k-1}(x_2, x_3 \ldots x_n)$

Where α_{j-1} and α_j are eliminated in the expression if $\alpha_j = 0$ and $\alpha_j = n$ respectively.

<u>Proof:</u> This theorem can be proved by the expansion theorem given in Appendix I and also theorem 2-2

<u>Theorem 2.7</u>

A function $f(x_1, x_2 \ldots x_n)$ is totally symmetric in the variables $x_1, x_2, \ldots x_n$ if and only if

$$f(x_1, x_2, \ldots, x_n) = f(x_2, x_1, x_3, \ldots, x_n)$$

and

$$f(x_1, x_2, \ldots, x_n) = f(x_2, x_3, \ldots, x_n, x_1)$$

Proof:

A constructive verification of this theorem is given below instead of a precise proof.

$$f(x_1, x_2, \ldots, x_n) = f(x_2, x_1, x_3, \ldots, x_n) \quad \ldots.(2.10)$$

is obtained directly from the definition 2-1.1

If it is totally symmetric f will be invariant under all permutations of variables . Hence

$$f(x_1, x_2, \ldots, x_n) = f(x_2, x_3, \ldots, x_n, x_1) \quad \ldots.(2.11)$$

To prove the converse we must show that all interchanges of pairs of variables may be obtained using the two permutations (2.10) and (2.11). Suppose we wish to interchange x_i and x_j , $i < j$. Then repeat (2.11) until we obtain the function of the form

$$f(x_i, x_{i+1}, \ldots, x_j, x_{j+1}, \ldots, x_n, x_1, x_2, \ldots, x_{i-1})$$

Applying (2-10) we get

$$f(x_{i+1}, x_i, x_{i+2}, \ldots, x_j, x_{j+1}, \ldots, x_n, x_1, x_2, \ldots, x_{i-1})$$

By the application of (2.11) followed by the application of (2.10) we get

$$f(x_{i+2}, x_i, x_{i+3}, \ldots x_j, x_{j+1}, \ldots x_n, x_1, x_2 \ldots x_{i-1}, x_{i+1})$$

This can be repeated until x_i is moved to a position just preceding x_{j+1}. This gives

$$f(x_j, x_i, x_{j+1}, \ldots x_n, x_1, x_2 \ldots x_{i-1}, x_{i+1} \ldots x_{j-1})$$

Now by repeating (2.11) we obtain the form

$$f(x_{j-1}, x_j, x_i, x_{j+1}, \ldots x_n, x_1, x_2, \ldots x_{i-1}, x_{i+1}, \ldots x_{j-2})$$

Then (2.10) moves x_j left one place. By repeated alteration of (2.11) and (.210) and finally using (2.11) we will have the desiredform.

$$f(x_1, x_2, \ldots x_{i-1}, x_j, x_{i+1}, x_{j-1}, x_i, x_{j+1}, \ldots x_n)$$

and the theorem is verified.

By applying the theorem let us verify an example :

$$f(x_1, x_2, x_3) = \bar{x}_1 \bar{x}_2 x_3 + x_1 x_2 x_3 + x_1 \bar{x}_2 \bar{x}_3$$

It is a symmetric function in the variables $\bar{x}_1, x_2, \bar{x}_3$

If we replace $\bar{x}_1$, by x_2 and x_2 by $\bar{x}_1$, we get

$$f(x_2, \bar{x}_1, \bar{x}_3) = x_2 x_1 x_3 + \bar{x}_2 \bar{x}_1 x_3 + \bar{x}_2 x_1 \bar{x}_3$$

$$f(\bar{x}_1, x_2, \bar{x}_3) = f(x_2, \bar{x}_1, \bar{x}_3)$$

By replacing $\bar{x}_1$ by x_2, x_2 by $\bar{x}_3$ and $\bar{x}_3$ by $\bar{x}_1$ we get

$$f(x_2, \bar{x}_3, \bar{x}_1) = x_2 x_3 x_1 + \bar{x}_2 \bar{x}_3 x_1 + \bar{x}_2 x_3 \bar{x}_1$$

$$\therefore \quad f(\bar{x}_1, x_2, \bar{x}_3) = f(x_2, \bar{x}_3, \bar{x}_1)$$

Hence the function is totally symmetric in the variables $\bar{x}_1$, x_2, $\bar{x}_3$.

CHAPTER III

EQUIVALENCE TABLES

EQUIVALENCE TABLES

3.1. DEFINITION OF EQUIVALENCE FUNCTIONS

Two functions are said to be equivalent if and only if one function can be obtained from other by permutation of inputs.

Let us take an example and examine the equivalence. If we have minterm $\sum 0, 1, 4, 6$ what will be the equivalent if we change the input B A C. The example below is shown for that purpose

$$f_{abc} = \sum 0, 1, 4, 6$$

$$= \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + a\bar{b}\bar{c} + ab\bar{c}$$

If we now change the inputs to BAC i.e. we interchange B and A we get a function

$$f' = \bar{b}\bar{a}\bar{c} + \bar{b}\bar{a}c + b\bar{a}\bar{c} + ba\bar{c}$$

$$= \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + \bar{a}b\bar{c} + ab\bar{c}$$

$$f' = \sum_{abc} 0, 1, 2, 6$$

$\therefore$ It shows that there is no need to have two circuits to obtain the functions f and f'. The circuit which gives f, by inputs ABC will give f' if we change the inputs to BAC.

The relation between minterms when input literals are changed are given in the table given below :

S.No.	abc	acb	bac	bca	cab	cba
1	0	0	0	0	0	0
2	1	2	1	4	2	4
3	2	1	4	1	4	2
4	3	3	5	5	6	6
5	4	4	2	2	1	1
6	5	6	3	6	3	5
7	6	5	6	3	5	3
8	7	7	7	7	7	7

By making use of the above table we can obtainé the equivalence function.

3.2. HILLERMAN'S EQUIVALENCE TABLE

Hillerman (3) partitioned all the 256 logical functions of three variables in to 80 equivalence classes. All the logical functions are expressed in terms of octal numbers. This is done by converting each minterms function to octal number, as explained below:

Each minterms present is written down as 'one' and the ones not present as 'zero'. Hence m_i can take any value 1 or 0 depending upon weather the minterm is present or not. Now we bunch these ones and 0's in three's (starting from right to left) and each bunch gives us a number. Hence we get the number for the whole function as three digital number and this called the octal number for the functional expression. For example

$$f = \sum 0, 1, 2, 3, 6, 7$$

m_7	m_6	m_5	m_4	m_3	m_2	m_1	m_0
1	1	0	0	1	1	1	1
3			1			7	

Hence octal number for $f = \sum 0, 1, 2, 3, 6, 7$ is 317.

This type of designation has one to one correspondence i.e., for any function there is only one octal number.

Hillerman (2) selected 80 leaders and he expressed all the function in terms of these 80 leaders. He explained the procedure how to read the table. For example the function to be designed has the number 321. The entry for this number in the equivalence class table is - $000213. This tells us our original function is equivalent to function 213. We note that the higher order digit of the entry, 5, tells us which permutation to apply to 213 in order to obtain original function 321.

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The permutation numbers and their meanings are as follows :

Number	Permutation	Changes in circuit figures
1	1	None
2	(abc)	atob , b to c, c to a
3	(acb)	atoc, c to b , b to a
4	(bc)	btoc , c to b
5	(ac)	atoc , c to a
6	(ab)	ato b, b to a

Hillerman's[3] equivalence classes of functions of 3 variable is given in the Appendix II (Table 3.1)

3.3. REDUCTION OF EQUIVALENCE CLASSES BY DECOMPOSITION PRINCIPLE

The number of leaders given in the Hillerman's[3] table are still reduced, by making use of the properties of symmetric and partially symmetric functions. The principle of decomposition, suggested by McCluskey[4] is used. Each Hillerman's[3] leader is tested and determined wether the function is totally symmetric or partially symmetric or assymmetric. For this purpose McCluskey method is used.

3.3.a. E. J. McCluskey Jr. Method for Testing the Group Invariance of Function

For a given function write down the transmission matrix. The function is symmetric if and only if , it satisfy the following conditions.

1. a. The ratio of the number of 1's to the number of 0's must be the same for each column if the variables of symmetry are not mixed.

   b. If the variables of symmetry are mixed, the reciprocal ratio will be found under those columns which represents the complimentary variables.

2. The weight of each row must be the same for all rows (terms) representing a given α - number.

3. The number of rows for the same α - number must be that given by the formula n_{C_k} . Where n is the number of columns of the standard matrix (number of variables) and k is the weight of the rows (α - number of the terms).

$$n_{C_k} = \frac{n!}{k!\ (n-k)!}$$

Example:

f = 177 (octal number)

$$= \frac{1}{01}\quad \frac{7}{111}\quad \frac{7}{111}$$

$$f = \sum m_0 + m_1 + m_2 + m_3 + m_4 + m_5 + m_6$$

Write down the transmission matrix for the given function. Find out the ratio of 1's and 0's for each column and also the weight of each row. Partion the matrix such that the columns much and rows of equal weight are grouped together. Count the number of rows of equal weight and check up weather these rows will satisfy the condition (3) or not.)

Standard matrix of the given function, is given below :

a	b	c		
0	0	0	=	0
0	0	1	=	1
0	1	0	=	1
0	1	1	=	2
1	0	0	=	1
1	0	1	=	2
1	1	0	=	2
3/4	3/4	3/4		

Partition the above matrix such that the rows of equal weight are grouped togehter.

a	b	c		
0	0	0	=	0
0	0	1	=	1
0	1	0	=	1
1	0	0	=	1
0	1	1	=	2
1	0	1	=	2
1	1	0	=	2
3/4	3/4	3/4		

The given function satisfy all the above necessary and sufficient conditions. Hence it is totally symmetric. The function is expressed as

$$f = S_{0,1,2}(abc)$$

The meaning of α - numbers, and its relationship and some of the necessary properties of symmetric functions are explained in the Chapter II.

The relationship between α - numbers and number of terms to be present in a symmetric function, inthe case of two variable and three variable is given in the following table.

2 - Variable

α - numbers k	no. of terms n_{α_k}
0	1
1	2
2	1

3 Variable

α number k	no. of terms n_{α_k}
0	1
1	3
2	3
3	1

3.3.b. Partially Symmetric Function

If the function is not totally symmetric it does not satisfy the above conditions. Then partion the matrix, and group the columns of either the equal ratio or equal to the reciprocal of the ratio.

Expand the function, about the variable which is not having the equal ratio. The expansion of the given function, about a pabticular variable is done by the help of expansion theorem given in the appendix I. Each residue is tested separately for symmetry. If all the residues are symmetric, then the function is partially symmetric about those variables.

Example:

$$f = 3 \quad 5 \quad 2$$

$$= 11 \; , \; 101 \; , \; 010$$

$$= \sum m_1 \; , m_3 , \; m_5 \; , m_6 \; , m_7$$

Write down the transmission matrix

a	b	c
0	0	1
0	1	1
1	0	1
1	1	0
1	1	1
3/2	3/2	4/1

Ratio of first two columns is same. Hence the function is expanded about the variable c and again the transmission matrix is written

c	a	b		
1	0	0	=	0
1	0	1	=	1
1	1	0	=	1
1	1	1	=	2
	2/2	2/2		
0	1	1	=	2
	1/0	1/0		

Each residue is tested for symmetry The ratio of the columns is same and also they are satisfying the α- number condition. Both residues are symmetric. Hence the given function is partially symmetric, in the variables ab. The function is written as below :

$$f = c\, S_{0,1,2}(ab) + \bar{c}\, S_2(ab)$$

All the leaders are tested individually for symmetry and partially symmetry. Also all leaders are expressed in symmetric and partially symmetric form and in terms of its corresponding α- numbers.

First, the leaders of equal number of terms are grouped together. Again the leaders, weather they are totally symmetric or partially symmetric are grouped together, irrespective of its variables. Certain permutations are made in the variables, and the variables of symmetry and brought down to be equal. Now, as the variables of symmetry are same and also the α - numbers are same , the functions in each group are equivalent.

<u>Example</u>: N = no. of terms = 5

The functions 75, 233 and 78 are tested as explained earlier and found that they are partially symmetric. They are expressed as below:

Function	Expression
75	$\bar{c}\, S_{0,1}(ab) + c\, S_2(ab)$
78	$c\, S_{0,1}(ab) + \bar{c}\, S_1(ab)$
233	$\bar{a}\, S_{0,1}(b\bar{c}) + a\, S_1(b\bar{c})$

For our convenience the variables 'bc' are taken as variables of symmetry. The leader, the original expression, the changes to be made and the final expressions are given below:

Function	Original Expression	Changes to be made	Final expression
75	$\bar{c}\, S_{0,1}(ab) + c\, S_1(ab)$	ac	$\bar{a}\, S_{0,1}(bc) + a\, S_1(bc)$
76	$\bar{c}\, S_{0,1}(ab) + \bar{c}\, S_1(ab)$	$c\bar{c}$-ac	$\bar{a}\, S_{0,1}(bc) + a\, S_1(bc)$
233	$\bar{a}\, S_{0,1}(b\bar{c}) + a\, S_1(b\bar{c})$	$c\bar{c}$	$\bar{a}\, S_{01}(bc) + a\, S_1(bc)$

Changes ac means a is changed to c and c is changed to a

$c\bar{c}$ means c is changed to $\bar{c}$ and $\bar{c}$ to c. The changes are made inserial order as given in the table 3.2

All the leaders are tested as explained earlier and tabular form is made. It is shown in the Appendix II table 3.2. By the help of this table the 78 equivalence functions are reduced to 38 equivalence functions.

3.4. FURTHER REDUCTION BY THE HELP OF COMPLIMENTARY FUNCTIONS

Every Boolean function is a complementary to another function. Hence if we know our leader, we can get another leader by complementing it. For example the function 350 is the complementary function for the function 27. Hence if we know any one function we can get the/another function by complementing the output.

The complementary function is arrived by changing intersection (.) in to union (+) and union (+) into intersection (.), and also the barred variable into unbarred variables and unbarred variables into barred variables. For example

$$f = \bar{a}b + a\bar{b}$$

is the given function. Its complementary function is $\bar{f}$

$$\bar{f} = (a + \bar{b})(\bar{a} + b)$$

$$= \bar{a}\bar{b} + ab$$

If a function is expressed either in octal number or in m-interms we can found out the complementary function very easily. Suppose the function is expressed in minterms. If we take the minterms, which are not given for the given function, it will give the complementary function for the given function.

Example:

$$f = \Sigma m_0, m_1, m_2, m_4$$

$$\bar{f} = \Sigma m_3, m_5, m_6, m_7$$

If the function is given in the octal number, we can get the octal number of the complimentary function by substracting the given o-ctal number from 377. For example the above given function 'f' is 27. Itsis complementary function '$\bar{f}$' is 350. It is obtained by substracting 27 from 377.

For some of the leaders, the complementary functions are not the leaders directly. For such functions the leaders are found out by the help of the Hillerman's[2] Table.

In the above method the 78 leaders are reduced to 30 leaders. The leaders and its complementary leaders are given in Appendix II table 3.3.

CHAPTER IV

NAND LOGIC MINIMAL CIRCUITS FOR THE EQUIVALENCE FUNCTION

NAND LOGIC MINIMAL CIRCUITS FOR THE EQUIVALENCE FUNCTIONS

4.1. HILLERMAN'S EQUIVALENT NET WORKS CATALOGUE

Lee Hillerman(3) found out the minimal NOR and NAND circuits for all the 78 leaders he has given in his equivalense table. These minimal NOR and NAND circuits are obtained by feeding data to IBM 7090 digital computer.

He has given a catalogue for all the leaders and minimal circuits obtained. Each function is expressed as a sum of elementary products. A four digit decimal number gives certain information about each circuit in the catalogue. The number is denoted by TCL, where T and L are single digits and c is two digits. The information given is

T ---- number of transistors (Blocks)

C ---- number of connections

L ---- number of levels.

In this Chapter an attempt is made to reduce the minimal equivalense circuits less than the number 78. Before proceeding further let us define the minimal circuit and equivalense circuit. A minimal circuit is one which satisfy the following conditions:

1. The number of logic blocks of the circuit is least possible for performing the function.
2. The levels at which the function is performed must be the minimum possible.
3. The number of connections in the circuit (total number of inputs) is least possible, subject to the condition that the circuit satisfies the first two conditions.

In addition the circuits satisfy certain reasonable restrictions on fan in and fan out. In three variable case the fan in and fan out is restricted to three.

Two networks belong to the same class (Equivalence) if and only if the connection matrix of one can be obtained from the connection matrix of the other, by permutation of the input variables. The equivalence networks perform only equivalent functions. Since implementation of any member of a function class serves (after suitable variable permutations) to implement all members of the class, it is sufficient to evaluate only one network from each equivalence class of networks.

4.2. REDUCTION IN NUMBER OF EQUIVALENCE NETWORKS BY COMPLEMENTING THE NETWORK OUTPUTS

The 78 equivalence groups, suggested by Hillerman (3) are reduced still further. For this purpose the tables 3.2

and Table 3.3 are used. By complementing the output of one network we will get another function. But the complemented network must be the minimal network.

The minimal network for each leader, given by Hillerman,[3] is taken and the output is complemented The minimal possible networks are taken. If the complemented network is not minimal the original network is taken. For example, the function 350 has got a minimal network. By complementing the output we get the function 27. This is a minimal network. Here if we know any one network, we can get another network.

Another network is given for the function 150. By complementing the output of this network we will get the function 237. But the complemented network is not minimal. Hence the original network is retained.

A network is tested weather it is minimal or not by comparing TCL of the network to the TCL given in Hillerman's[3] catalogue. The outp ut of a network is complemented by putting another NAND block at the output. If a input variable is to be complemented a NAND block is placed at the input terminal. For some leader we cannot get the leader directly by complementing the output of the network. In that case we will get the equivalence function. Then the leader can be obtained by the help of Hillerman's[3] table.

4.3. FURTHER REDUCTION IN THE NUMBER OF EQUIVALENCE NETWORKS

The number of equivalence networks are further reduced by the help of α - numbers of totally symmetric and partially symmetric functions.

The leaders, weather totally symmetric or partially symmetric, are grouped togehter and consider as one equivalence class if they are having equal α - numbers. All the functions which are having same α - numbers can be represented by a single network if the variables of symmetry are same. To make the variables of symmetry to be same in each group, we make some permutations in the input variables. We have to complement some of the input variables also. The input variable can be complemented by placing a NAND block at the input terminal.

By complementing some of the input variables and making some permutations at the input variables, we can reduce to 78 equivalence classes into 34 equivalence classes. If we have a minimal network for one functions, all the functions in that class can be realised, by bringing some changes at input terminals. While reducing the Hillerman's[3] equivalence classes we have to see weather the network which we will get from the equivalent network for each function is minimal or not.

For example from Table (3.2) we can see that 78, 76 and 233 comes under one group. But we can not deduce a minimal network for one function from the minimal network of the other function. Hence they are realised independently.

Either complementing the output or by making certain changes at the input terminals for the functions which are having equal α- numbers we can reduce the minimal networks to a smaller number. By each method it is checked that the network obtained is a minimal one or not. If the minimal network is not obtained then that function is realised independently.

By the above method the Hillerman's[3] 78 networks are reduced to 33. The leaders of each equivalence group and the changes to be made to get other Hillerman's leaders are given in the Appendix II Table 4.7. The TCL of the network we obtain and TCL of Hillerman aregiven for comparison. The equivalent networks are also given in Appendix III.

CHAPTER V

MINIMUM POSSIBLE EQUIVALENCE NAND LOGIC CIRCUITS FOR THE EQUIVALENCE FUNCTIONS

MINIMUM POSSIBLE EQUIVALENCE NAND LOGIC CIRCUITS FOR THE EQUIVALENCE FUNCTIONS

In the previous Chapter - IV we were aiming to find out the least possible minimal NAND logic circuits. If we are not aiming at the minimal networks we can still reduce the number of equivalence logic circuits.

Each leader will have a complementary leader or equivalent complementry function. By complementing each network we will arrive at another network. Hence we can get 78 networks from 39 networks itself.

As explained in Chapter III and shown in Table 3.2, some of the equivalence functions can be grouped together. All the functions of one group can be obtained from a single network.

Hence from a single network we can realise all the functions in one group shown in the Table 3.2. Also the complementary functions of this group also can be realised.

To reduce the number of networks we have followed the method, explained in the articles 4.2 and 4.3. Bu this method we reduced the 78 equivalence minimal networks to 19 equivalence networks. The networks obtained from these equivalence network may mot be minimal.

The catalogue of these networks and the changes to be made to obtain the equivalence function are given in the Appendix II (Table 8). Equivalence networks are given in Appendix III.

CONCLUSION

While reducing number of Hilleman's[3] equivalent networks only the NAND method is used. Also it is assumed that the complementary variables are not avilable. If the complemented variables are available we can reduce the number of equivalent functions still further. Only the complementary of a function is taken, but not the dual of the function. If we take to dual of the function also we can still reduce the number of equivalent networks. For this purpose we have to find out a single logic by which we can found out either the complementary of a function or to dual of a function.

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2.

APPENDICS

APPENDIX I

BASIC THEOREMS IN BOOLEAN ALGEBRA

1\. a. $A + \bar{A} = 1$

b $A \cdot \bar{A} = 0$

2 a. $A + 1 = 1$

b $A \cdot 1 = A$

3 a. $A + 0 = A$

b $A \cdot 0 = 0$

4 a $1 + 1 = 1$

b $1 \cdot 1 = 1$

5 a $1 + 0 = 1$

b $1 \cdot 0 = 0$

6 a $\overline{A + B} = \bar{A} \cdot \bar{B}$

b $\overline{A \cdot B} = \bar{A} + \bar{B}$

7\. $f(x_1, x_2 \ldots x_n) = f(1, 1, 1 \ldots 1)(x_1, x_2, x_3 \ldots x_n)$

$+ f(0, 1, 1, 1 \ldots 1)(\bar{x}_1, x_2, x_3, \ldots x_n)$

$+ \ldots\ldots\ldots\ldots\ldots\ldots$

$+ f(0, 0, 0 \ldots 0)(\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots \bar{x}_n)$

(Expansion Theorem)

8\. $AB = (A \mid B) \mid (A \mid B)$

$A+B = (A \mid A) \mid (B \mid B)$

$\bar{A} = (A \mid A)$

'|' indicates NAND Logic

APPENDIX II

Table 3.1

HILLERMAN'S EQUIVALENCE CLASSES OF FUNCTIONS OF 3 - VARIABLES

320	-3000212	-3000213	-3000232	-3000233	-3000216	-3000217	-3000236	-3000237
330	-3000254	-3000255	-3000274	-3000275	-3000256	-3000257	-3000276	-3000277
340	-2000250	-2000251	-2000254	-2000255	-3000254	-3000255	-2000274	-2000275
350	1000350	1000351	1000352	1000353	-3000352	-3000353	-1000356	1000357
360	-2000252	-2000253	-2000256	-2000257	-3000256	-3000257	-2000276	-2000277
370	-2000352	-2000353	-2000356	-2000357	-3000356	-3000357	1000376	-1000377

APPENDIX II

Table 3.1

HILLEMAN'S EQUIVALENCE CLASSES OF CUNCTIONS OF 3 - VARIABLES

	0	1	2	3	4	5	6	7
0	-1000000	1000001	1000002	-1000003	-3000002	-3000005	1000006	1000007
10	1000010	1000011	-1000012	1000013	-4000012	-4000013	1000016	-1000017
20	-2000006	-2000005	-2000006	-2000007	-3000006	-3000007	1000026	1000027
30	1000030	1000031	1000032	1000033	-4000032	-4000033	1000036	1000037
40	-2000010	-2000011	-6000012	-6000013	-2000030	-2000031	-6000032	-6000033
50	1000050	1000051	1000052	1000053	1000054	1000055	1000056	1000057
60	-2000012	-2000013	-2000016	-2000017	-2000032	-2000033	-2000036	-2000037
70	-6000054	-6000055	-6000056	-6000057	-1000074	1000075	1000076	-1000077
100	-3000010	-3000011	-3000030	-3000031	-3000012	-3000013	-3000032	-3000033
110	-3000050	-3000051	-4000054	-4000055	-3000052	-3000053	-4000056	-4000057
120	-5000012	-5000013	-5000032	-5000033	-3000016	-3000017	-3000036	-3000037
130	-3000054	-3000055	-3000074	-3000075	-3000056	-3000057	-3000076	-2000077
140	-2000050	-2000051	-2000054	-2000055	-5000054	-5000055	-2000074	-2000075
150	1000150	1000151	1000152	1000153	-3000152	-3000153	1000156	1000157
160	-2000052	-2000053	-2000056	-2000057	-5000056	-5000057	-2000076	-2000077
170	-2000152	-20000153	-2000156	-2000157	-3000156	-3000157	1000176	1000177
200	1000200	1000201	1000202	1000203	-3000202	-3000203	1000206	1000207
210	-1000210	1000211	1000212	1000213	-4000212	-4000213	1000216	1000217
220	-2000202	-2000203	-2000206	-2000207	-3000206	-3000207	1000226	1000227
230	1000230	-1000231	1000232	1000233	-4000232	-4000233	1000236	1000237
240	-2000210	-2000211	-6000212	-6000213	-2000230	-2000231	-6000232	-6000233
250	1000250	1000251	-1000252	1000253	1000254	1000255	1000256	-1000257
260	-2000212	-2000213	-2000216	-2000217	-2000232	-2000233	-2000236	-2000237
270	-6000254	-6000255	-6000256	-6000257	1000274	1000275	1000276	1000277
300	-3000210	-3000211	-3000230	-3000231	-3000212	-3000213	-3000232	-3000233
310	-3000250	-3000251	-4000254	-4000255	-3000252	-3000253	-4000256	-4000257
320	-5000212	-5000213	-5000232	-5000233	-3000216	-3000217	-3000236	-3000237
330	-3000254	-3000255	-3000274	-3000275	-3000256	-3000257	-3000276	-3000277
340	-2000250	-2000251	-2000254	-2000255	-5000254	-5000255	-2000274	-2000275
350	1000350	1000351	1000352	1000353	-3000352	-3000353	-1000356	1000357
360	-2000252	-2000253	-2000256	-2000257	-5000256	-5000257	-2000276	-2000277
370	-2000352	-2000353	-2000356	-2000357	-3000356	-3000357	1000376	-1000377

EXPLANATORY EXAMPLE

Class of 321 is given by word at intersection of row 320 and Column 1, = 500213

This says 321 is in class of 213 by permutation 5.

Negative permutation 1 means the function is degenerative.

Permutation 1 is the identity

Permutation 2 is (ABC)

Permutation 3 is (ACB)

Permutation 4 is (BC)

Permutation 5 is (AC)

Permutation 6 is (AB)

GROUPING OF EQUIVALENCE FUNCTIONS BY DECOMPOSITION PRINCIPLE

Function	Functions expressed in α - numbers	Changes to be made	Functions expressed in α - numbers and with same variables of symmetry.
	Degenerative Function		
	N = 2		
3	$a' S_{e}(ab) + aS_{e}(ab)$	ac	$a'S_{e}(bc) + aS_{e}(bc)$
15	$b'S_{2}(a'c) + bS_{2}(a'c)$	a'b	$a'S_{2}(bc) + aS_{2}(bc)$]
210	$a'S_{2}(bc) + aS_{2}(bc)$	-	$a'S_{2}(bc) + aS_{2}(bc)$]
	N = 4		
17	$a'S_{e12}(bc)$	aa'	$aS_{e12}(bc)$]
252	$aS_{012}(ab)$	ac	$aS_{012}(bc)$]
74	$a'S_{1}(ab) + aS_{1}(ab)$	ac	$a'S_{1}(bc) + aS_{1}(bc)$
231	$a'S_{02}(bc) + aS_{02}(bc)$	-	$a'S_{e2}(bc) + aS_{e,2}(bc)$
	N=6		
77	$a'S_{01}(ab) + a\, S_{01}(ab)$	ac	$a'S_{01}(bc) + aS_{01}(bc)$]
237	$b'S_{01}(ac') + bS_{01}(ac')$	ab - ac'	$a'S_{01}(bc) + a\, S_{01}(bc)$]
336	$a'S_{12}(bc) + a\, S_{12}(bc)$	-	$a'\, S_{12}(bc) + a\, S_{12}(bc)$

Contd...../..

Table 3.2 Contd..

Function	Functions expressed in α - numbers	Changes to be made	Functions expressed in α - numbers and with same variables of symmetry.
		Non-degenerative Functions	
		M = 2	
30	$S_{03}(a'bc)$	aa'	$S_{03}(abc)$]
201	$S_{03}(abc)$	-	$S_{03}(abc)$]
6	$a'S_1(bc)$	aa'	$a\,S_1(bc)$]
50	$eS_1(ab)$	ae	$aS_1(bc)$]
11	$a'\,S_{02}(ba)$	aa'	$aS_{02}(bc)$]
202	$e\,S_{02}(ba)$	ae	$a\,S_{02}(bc)$]
		M = 3	
26	$S_1(abc)$	-	$S_1(abc)$]
31	$S_1(abc')$	cc'	$S_1(abc)$]
150	$S_2(abc)$	-	$S_2(abc)$]
200	$S_2(a'bc)$	aa'	$S_2(abc)$]
7	$a'\,S_{01}(bc)$	-	$a'\,S_{01}(bc)$]
13	$a'\,S_{01}(bc')$	cc'	$a'\,S_{01}(bc)$]
52	$eS_{01}(ab)$	ae - aa'	$a'S_{01}(bc)$]

Contd....

Table 3.2. Contd..

Function	Functions expressed in α – numbers	Changes to be made.	Functions expressed in α – numbers and with same variables of symmetry
16	$a' S_{12}(bc)$	–	$a' S_{12}(bc)$
213	$c S_{12}(a'b)$	$a'c$	$a' S_{12}(bc)$
250	$c S_{12}(ab)$	$ac = aa'$	$a' S_{12}(bc)$
31	$a' S_{02}(bc) + a S_0(bc)$	–	$a' S_{02}(bc) + a S_0(bc)$
32	$b' S_{02}(ac') + b S_0(ac')$	$ab = cc'$	$a' S_{02}(bc) + a S_0(bc)$
203	$c' S_{02}(ab + c S_{02}(ab)$	$cc' = ac$	$a' S_{02}(bc) + a S_0(bc)$
84	$c' S_2(a'b) + c S_{02}(a'b)$	$a'c$	$a' S_{02}(bc) + a S_2(bc)$
211	$a' S_{02}(bc) + a S_2(bc)$	–	$a' S_{02}(bc) + a S_2(bc)$
230	$a' S_2(bc) + a S_{02}(bc)$	aa'	$a' S_{02}(bc) + a S_2(bc)$
		M = 4	
27	$S_{01}(abc)$	–	$S_{01}(abc)$
53	$S_{01}(abc')$	cc'	$S_{01}(abc)$
151	$S_{02}(abc)$	–	$S_{02}(abc)$
216	$S_{23}(a'bc)$	aa'	$S_{23}(abc)$
350	$S_{23}(abc)$	–	$S_{23}(abc)$

Contd...

Table 2.2 Contd..

Function	Functions expressed in α - numbers	Changes to be made	Functions expressed in α - numbers and with same variables of symmetry
226	$S_{13}(abc)$	-	$S_{13}(abc)$
30	$a'S_{12}(bc) + aS_0(bc)$	-	$a'S_{12}(bc) + aS_0(bc)$
55	$a'S_{12}(bc') + aS_0(bc')$	cc'	$a'S_{12}(bc) + aS_0(bc)$
232	$cS_{12}(a'b) + c'S_0(a'b)$	aa'	$a'S_{12}(bc) + aS_0(bc)$
251	$c'S_0(ab) + cS_{12}(ab)$	cc ↔ ac	$a'S_{12}(bc) + aS_0(bc)$
152	$cS_{01}(ab) + c'S_2(ab)$	cc' ↔ ac	$a'S_{01}(bc) + aS_2(bc)$
207	$a'S_{01}(bc) + aS_2(bc)$	-	$a'S_{01}(bc) + aS_2(bc)$
33	-	-	-
96	-	-	-
213	-	-	-
254	-	-	-
		N = 3	
153	$S_{013}(abc')$	cc'	$S_{013}(abc)$
227	$S_{013}(abc)$	-	$S_{013}(abc)$
236	$S_{023}(a'bc)$	aa'	$S_{023}(abc)$
351	$S_{023}(abc)$	-	$S_{023}(abc)$

Contd....

Table 3.2 Contd..

Function	Functions expressed in α - numbers	Changes to be made	Functions expressed in α - numbers and with same variables of symmetry
75	$c' S_{01}(ab) + c S_1 (ab)$	ac	$a' S_{01}(bc) + aS_1 (bc)$
76	$c S_{01}(ab) + c' S_1 (ab)$	ac↔ac	$a' S_{01}(bc) + aS_1 (bc)$
233	$a' S_{01} (bc') + aS_1(bc')$	cc'	$a' S_{01} (bc) + a S_1(bc)$
150	$a' S_{12}(bc) + a S_1 (bc)$	-	$a' S_{12}(bc) + a S_1(bc)$
255	$b' S_1(a'c) + b S_{12}(a'c)$	a'b	$a' S_{12}(bc) + a S_1(bc)$
274	$c' S_1 (ab) + c S_{12}(ab)$	ac↔ac	$a'S_{12}(bc) + a S_1(bc)$
37	$a'S_{012}(bc) + a S_0(bc)$	aa'	$a' S_0 (bc) + a S_{012}(bc)$
253	$c' S_0 (ab) + c S_{012} (ab)$	ac	$a' S_0 (bc) + a S_{012}(bc)$
238	$c' S_0 (ab') + c S_{012} (ab')$	ac→bb'	$a' S_0 (bc) + a S_{012} (bc)$
217	$a' S_{012}(bc) + a S_2(bc)$	aa'	$a' S_2(bc) + a S_{012} (bc)$
353	$c' S_2 (ab) + c S_{012} (ab)$	ac	$a' S_2(bc) + a S_{012}(bc)$
57	-	-	-

N = 6

Function	Functions expressed in α - numbers	Changes to be made	Functions expressed in α - numbers and with same variables of symmetry
176	$S_{12}(abc)$	-	$S_{12}(abc)$
275	222 $S_{12}(abc')$	cc'	$S_{12}(abc)$

Contd.../..

Table 3.2 Contd..

Function	Functions expressed in α - numbers	Changes to be made.	Functions expressed in α - numbers and with same variables of symmetry
157	$a' S_{012}(bc) + a S_1(bc)$	aa'	$a' S_1(bc) + a S_{012}(bc)$
376	$e' S_1(ab) + e S_{012}(ab)$	ac	$a' S_1(bc) + a S_{012}(bc)$
337	$a' S_{012}(bc) + a S_{02}(bc)$	aa'	$a' S_{02}(bc) + a S_{012}(bc)$
383	$e' S_{02}(ab) + e S_{012}(ab)$	ac	$a' S_{02}(bc) + a S_{012}(bc)$
	M = 7		
177	$S_{012}(abc)$	-	$S_{012}(abc)$
277	$S_{012}(abc')$	cc'	$S_{012}(abc)$
357	$S_{123}(a'bc)$	aa'	$S_{123}(abc)$
376	$S_{123}(abc)$	-	$S_{123}(abc)$

<u>Table 3.3</u>

RELATIONSHIP BETWEEN EQUIVALENCE FUNCTIONS AND ITS COMPLEMENTARY FUNCTIONS

Function	Complementary function	Function	Complementary function
	Degenerative Functions		
3	374 - 356 (-3)	36	341 - 261 (-2)
12	365 - 257 (-5)	37	340 - 250 (-2)
17	360 - 352 (-2)	50	327 - 237 (-3)
74	303 - 231 (-3)	51	326 - 236 (-3)
77	300 - 210 (-3)	52	325 - 217 (-3)
	Non- Degenerative Functions	53	324 - 216 (-3)
1	376	54	323 - 233 (-5)
2	375 - 357 (-3)	55	322 - 232 (-5)
6	371 - 353 (-2)	56	321 - 213 (-5)
7	370 - 352 (-2)	57	320 - 212 (-5)
10	367 - 277 (-2)	75	302 - 230 (-3)
11	366 - 276 (-2)	76	301 - 211 (-3)
13	364 - 256 (-5)	150	227
16	361 - 253 (-2)	151	226
26	351	152	225-207 (-3)
27	350	153	224- 206 (-3)
30	347 - 275 (-2)	156	221- 203 (-2)
31	346 - 274 (-2)	157	220-202 (-2)
32	345 - 255 (-5)	176	201
33	344 -254 (-5)	177	200

<u>Table No. 4</u>

THE CATALOG OF MINIMAL EQUIVALENCE NAND LOGIC CIRCUITS OF THREE VARIABLE FUNCTIONS

Function	Circuit No.	Changes to be made	New function arrived	T	C	L	Hillerman's T	Hillerman's C	Hillerman's L
		Degenerative Functions							
74	4D	-	-		-		4	05	3
231	5D	-	-		-		5	06	2
252	1D	-	-		-		0	01	0
		ca'	17	1	01	1	1	01	1
257	2D	-	-		-		2	03	2
		ac-ce	13	3	04	3	3	04	3
		c'b	77	1	02	1	1	02	1
		ac'-ab-ce	210	2	03	2	2	03	2
356	3D	~~[illegible]~~	-		-		3	04	2
		ac-ce	3	4	05	3	4	05	3
		Non degenerative Functions							
26	26	-	-		-		7	15	3
		cc'	51	7	15	3	7	15	4or5
30	16	-	-		-		5	11	3
		aa'	301	6	11	3	6	11	3
		ac	275	6	12	4	6	11	4
36	23	-	-		-		6	14	3

Contd..

Table 4 Contd...

Function	Circuit no.	Changes to be made	New function arrived	T	C	L	Hilleman's T	C	L
50	9	-	-	-			4	10	3
		ca-aa'	8	5	11	3	5	11	3
53	20	-	-	-			6	10	3
55	22	-	-	-			6	11	4
		ac'	251	6	12	4	6	12	4
		aa'-ac	232	5	11	4	5	11	4
56	7	-	-	-			4	08	3
57	1	-	-	-			3	05	3
		ac-cc	212	4	06	4	4	06	4
76	17	-	-	-			5	11	2
150	19	-	-	-			5	15	3
		aa'	208	6	13	3	6	15	3ord
151	25	-	-	-			7	16	5
152	16	-	-	-			4	10	3
		ac'-ac	207	5	10	3	5	10	3ord
156	8	-	-	-			4	08	3
		aa'-ac	274	5	10	4	5	10	4
		a'b	253	6	11	4	6	10	4
170	18	-	-	-			5	12	3

Contd..../..

Table 4 Contd....

Function	Circuit No.	Changes to be made	New function arrived.	T	C	L	Hillerman's T	C	L
203	13	-	-	-			5	09	3
		ac'-ac	31	6	10	3	6	10	3
		bc'	32	5	10	3	5	10	3
		bb'	54	4	09	3	4	09	3
		aa'-bb'-cc	75	6	10	4	6	10	4
		ac-aa'-bb'-cc'	211	6	10	3	6	20	3
		ac-cc'-bb'	230	5	09	3	5	09	3
213	11	-	-	-			5	08	3
226	27	-	-	-			7	20	4
227	21	-	-	-			6	14	3
		cc'	153	7	15	4	7	14	4
233	14	-	-	-			5	09	3
236	24	-	-	-			6	15	4
		aa'	351	7	15	4	7	15	4
237	12	-	-	-			5	09	3
		aa'-ac	353	6	10	3	6	10	3
250	3	-	-	-			3	06	2
		ac-aa'	16	4	07	3	4	07	3
		aa'	212	4	07	3	4	06	4
254	4	-	-	-			4	07	3
		ac-cc	33	5	08	4	5	08	4

Contd....

Table 4 Contd.

Function	Circuit no.	Changes to be made	New function arrived	T	C	L	Hellerman's T	C	L
270	15	-	-		-		5	10	3
		ac↔cc	11	0	11	4	0	11	4
		cc'↔cc	159	4	00	3	4	00	3
		cc'↔cc↔cc	203	5	10	4	5	10	4
309	0	-	-		-		4	00	2
		cc	37	5	10	3	5	10	3
		ca'	310	5	10	2	5	10	2 or 4
352	3	-	-		-		3	03	3
		ac↔cc	7	4	00	3	4	06	3
		ac↔cc'↔cc	13	5	07	4	5	07	4
		ac↔ac'↔bb'↔cc'	37	4	03	3	4	00	3
		cc'↔cc	03	3	03	3	3	03	3
		cc'↔ac	217	3	04	3	3	04	3
		aa'↔bb'	263	5	07	3	5	07	3
		cc'	350	4	03	3	4	00	3
376	5	-	-		-		4	00	2
		cc	1	5	07	3	5	07	3
		cc'↔cc	3	4	00	3	4	00	3
		bb'↔cc'↔cc	10	3	03	3	3	03	3
		aa'↔bb'↔cc'	177	1	03	1	1	03	1
		aa'↔bb'↔cc'↔cc	300	3	04	3	3	04	3
		aa'↔bb'	377	3	04	3	3	04	3
		aa'	367	3	03	2	3	03	3

Note: cg = complement the output

The changes are to be made in serial order given in the table.

Table 8

THE CATALOG OF EQUIVALENCE NAND LOGIC CIRCUITS OF THREE VARIABLE FUNCTIONS

Function	Circuit no.	Changes to be made	New Function arrived	T C L	Hillerman's T C L
Degenerative Function					
74	4D	-	-	-	4 08 3
		ac - c0	231	5 09 4	5 08 3
252	1D	-	-	-	0 01 0
		ca'	17	1 01 1	1 01 1
257	2D	-	-	-	2 03 2
		ac - cc	12	3 04 3	3 04 3
		bc'	77	1 02 1	1 02 1
		ac'-ab - cc	310	2 03 2	2 03 2
256	3D	-	-	-	3 04 3
		ac - cc	3	4 05 3	4 05 3
Non - degenerative functions					
26	26	-	-	-	7 15 3
		cc' - cc	51	7 15 3	7 15 4or5
		cc	351	8 16 4	7 15 4
		aa' - cc	236	8 16 4	6 14 3

Contd.../

Table 5 Contd..

Function	Circuit no.	Changes to be made	New function arrived	T	C	L	Hillerman's T	C	E
30	16	-	-		-		5	11	3
		aa'-cc	176	7	12	4	5	12	3
		aa'	201	6	11	3	6	11	3
		ac -cc	275	6	12	4	6	11	4
50	9	-	-		-		4	10	3
		ac -aa'	6	5	11	3	5	11	3
		ac-cc	237	5	11	4	5	09	3
		ac'-cc	353	6	12	4	6	10	3
85	22	-	-		-		6	11	4
		ac'	36	7	12	4	6	14	3
		aa' -ac	232	5	11	4	5	11	4
		ac'	251	6	12	4	6	12	4
86	7	-	-		-		4	08	3
		ac-cc	213	5	09	4	5	08	3
150	19	-	-		-		5	13	3
		aa' -ac	153	7	16	4	7	14	4
		aa'	206	6	15	3	6	15	3or4
		cc	227	6	14	4	6	16	3
152	10	-	-		-		4	10	3
		cc'-ac	207	5	10	3	5	10	4
203	13	-	-		-		6	10	4
		cc'-ac	31	6	10	3	6	10	3
		bc'	32	5	10	3	5	10	3
		bb'	54	4	09	3	4	09	3

Table 5 Contd..

Function	Circuit no.	Changes to be made	New function arrived	T	C	L	Hillerman's T	C	L
		aa'-bb'-cc	75	6	10	4	6	10	4
		ac-cc	150	7	11	5	4	08	3
		ac-aa'-cc'-ccl	211	6	10	3	5	10	3
		ac-cc'-bb'	230	5	09	3	5	09	3
		bb'-ac-cc	233	5	10	4	5	08	3
		bc'-ac-cc	255	6	11	4	6	10	4
		cc'-cc	274	7	11	4	5	10	4
280	27	-	-	-			7	10	4
		cc	151	6	11	5	7	10	5
250	3	-	-	-			3	06	2
		ac-aa'	16	4	07	3	4	07	3
		aa'-ac-cc	37	5	08	4	3	05	3
		aa'	212	4	07	3	4	06	4
254	4	-	-	-			4	07	3
		ac-cc	33	5	08	4	5	08	4
276	18	-	-	-			5	10	3
		ac-cc	11	6	11	4	6	11	4
		ac'-ac	157	4	09	3	4	09	3
		cd-ac-cc	202	5	10	4	6	10	4
350	6	-	-	-			4	09	2
		cc	27	5	10	3	5	10	3
		aa'-ac-cc	63	5	11	4	6	10	3
		aa'	215	5	10	3	5	10	4

Contd...

Table 8 Contd..

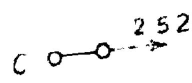
Function	Circuit No.	Changes to be made	New Function arrived	T	C	L	Hillerman's T	C	L
352	3	-	-		-		3	05	2
		ac-cc	7	4	06	3	4	06	3
		ac-ac↔cc	13	5	07	4	5	07	4
		ac-aa↔bb↔cc'	37	4	06	3	4	06	3
		cc↔cc	52	3	05	3	3	05	3
		cc↔ac	217	2	04	2	2	04	2
		aa↔bb'	253	5	07	3	5	07	3
		ac'	250	4	06	3	4	06	3
376	5	-	-		-		4	06	2
		cc	1	5	07	3	5	07	3
		cc↔cc	2	4	06	3	4	06	3
		bb↔cc↔cc	10	3	05	3	3	05	3
		aa↔bb↔cc'	177	1	03	1	1	03	1
		aa↔bb↔cc↔cc	200	2	04	2	2	04	2
		aa↔bb'	377	2	04	2	2	04	2
		aa'	357	3	05	2	3	05	2

Note : cc = compliment the output

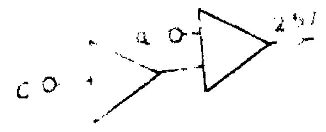
The changes are to be made in serial order given in the table.

APPENDIX III

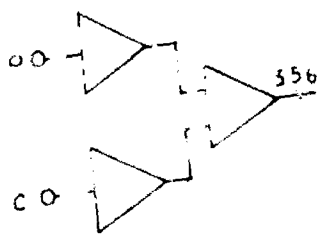
EQUIVALENCE NAND LOGIC CIRCUITS



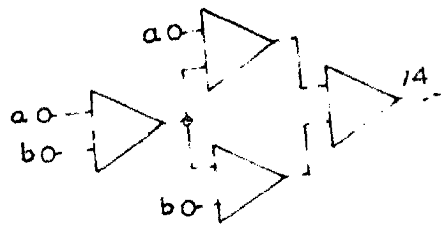
CIRCUIT-1D



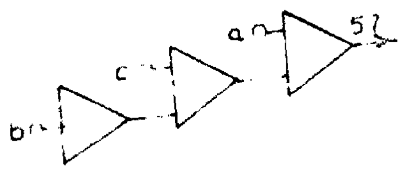
CIRCUIT-2D



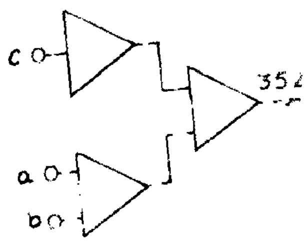
CIRCUIT-3D



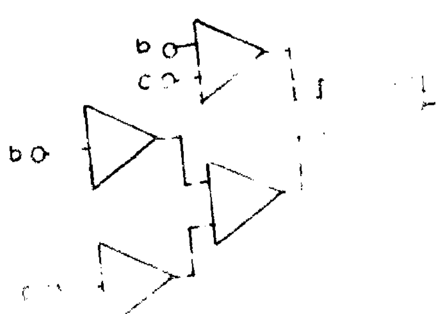
CIRCUIT-4D



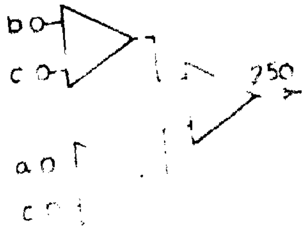
CIRCUIT-1

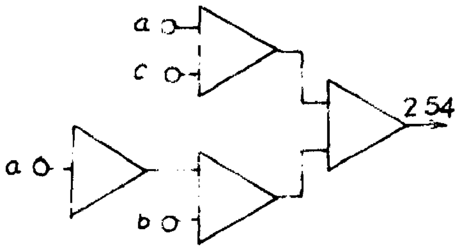


CIRCUIT - 2

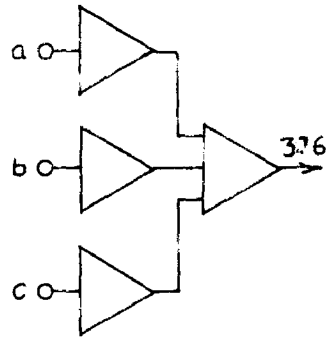


CIRCUIT-5D

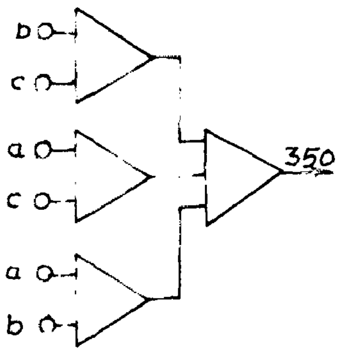




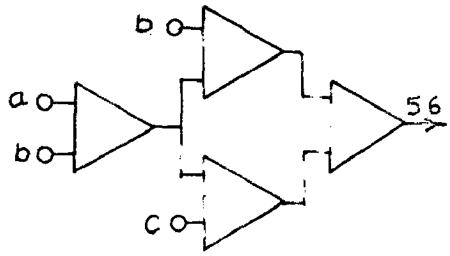
CIRCUIT - 4



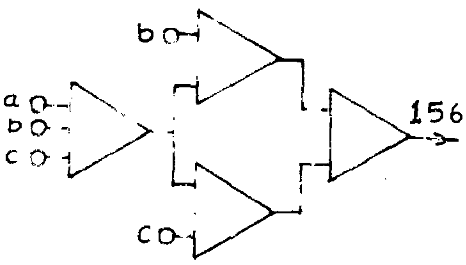
CIRCUIT - 5



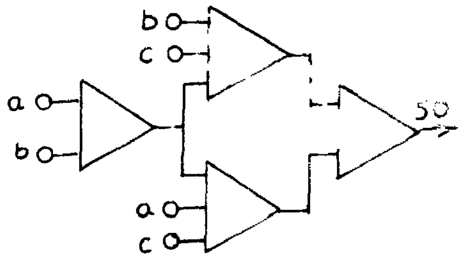
CIRCUIT - 6



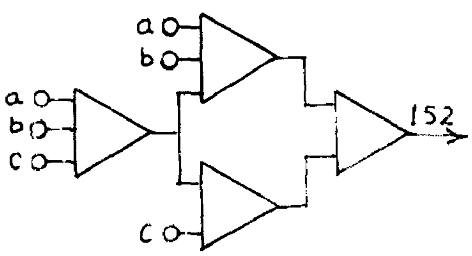
CIRCUIT - 7



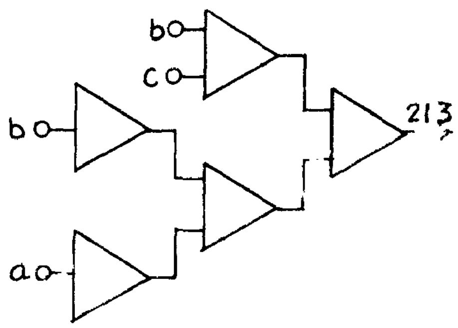
CIRCUIT - 8



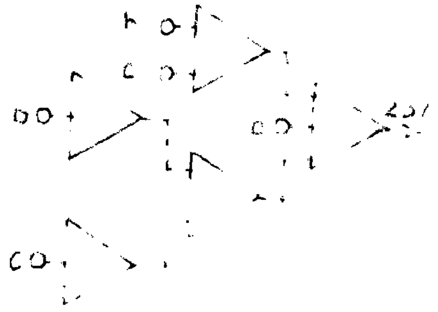
CIRCUIT - 9



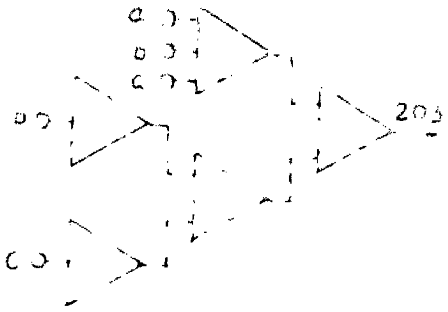
CIRCUIT - 10



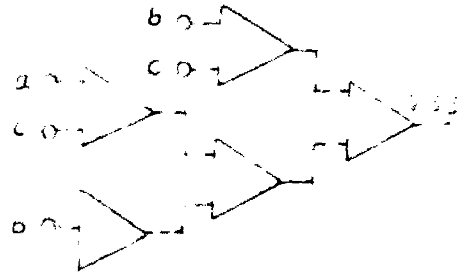
CIRCUIT - 11



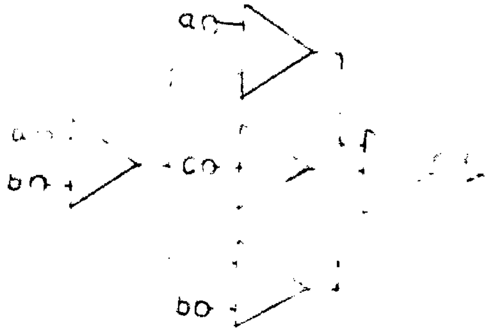
CIRCUIT 12



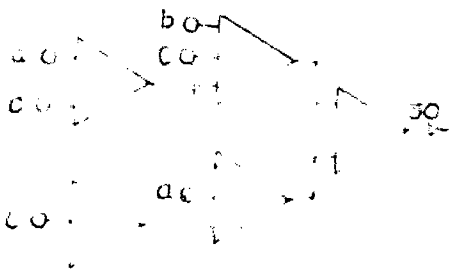
CIRCUIT - 13



CIRCUIT - 14

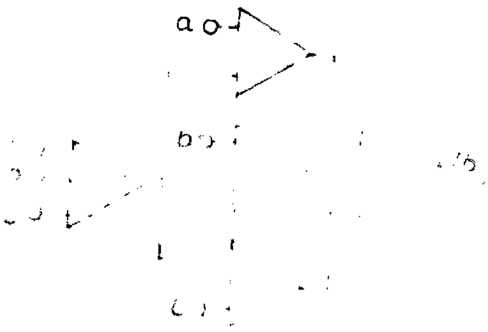


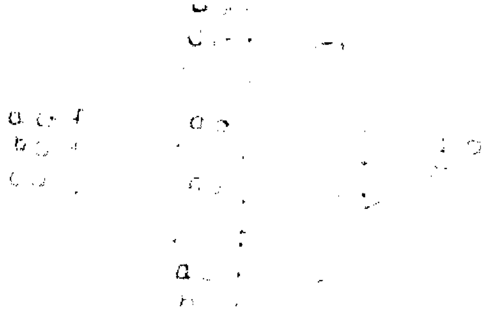
CIRCU[illegible] 15

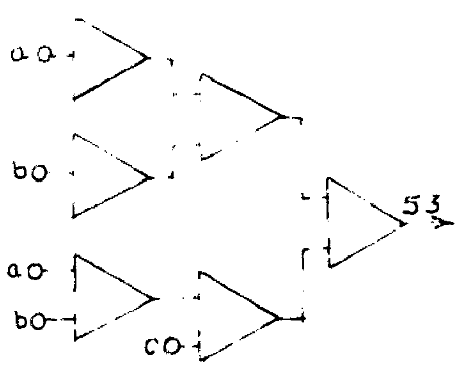


CIRCUIT - 16

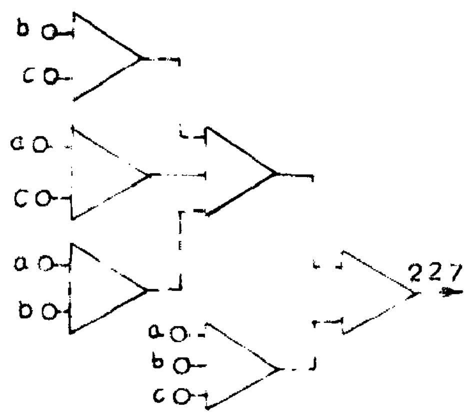
CIRCUIT 17



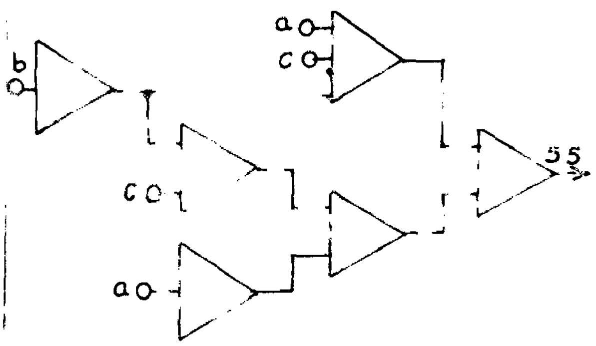




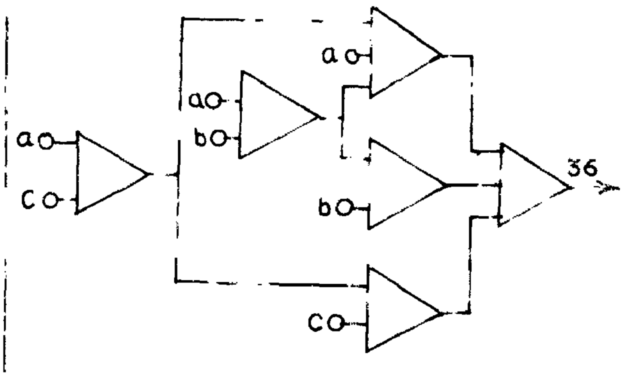
CIRCUIT - 20



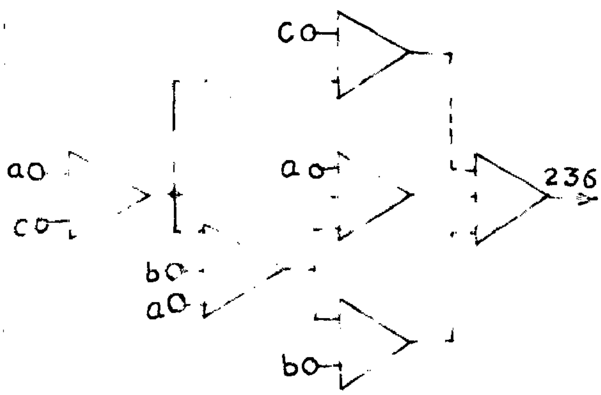
CIRCUIT-21



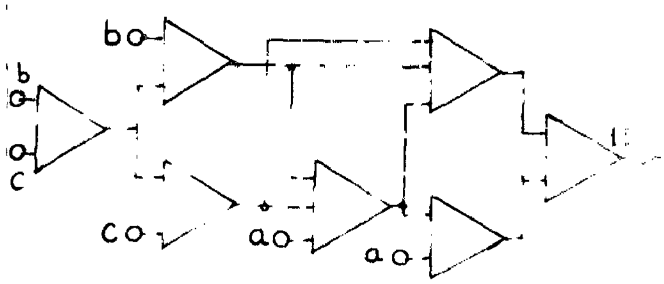
CIRCUIT - 22



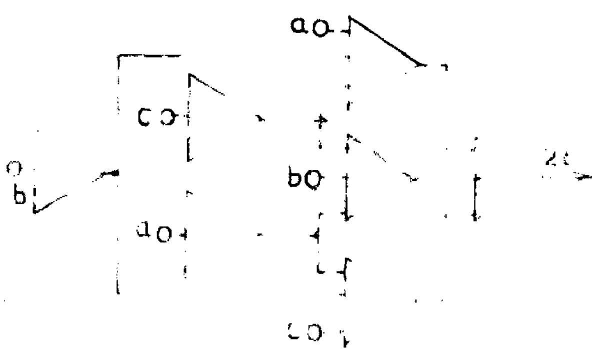
CIRCUIT-23



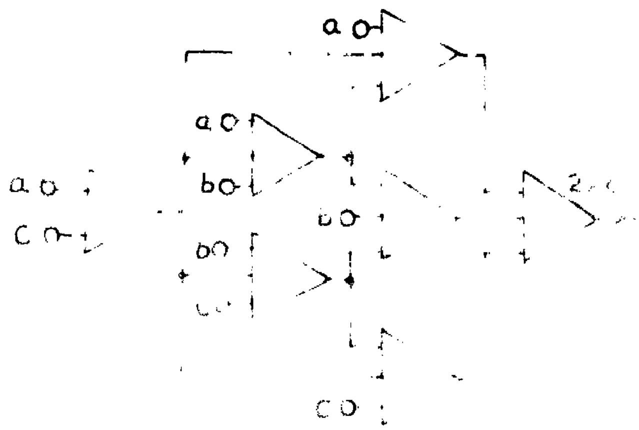
CIRCUIT - 24



CIRCUIT-25



CIRCUIT - 26



CIRCUIT-27