# BANG BANG CONTR OR AN AUTOMATIC PHASE LOCK SYSTEM

*By*
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Last but not the least, thanks are to all those in the Department who helped in various ways in this dissertation.

A K Mehrotra

( A K MEHROTRA )

# CERTIFICATE

Certified that the dissertation entitled "BANG BANG CONTROL FOR AN AUTOMATIC PHASE LOCK SYSTEM" which is being submitted by Sri A K Mehrotra in partial fulfilment for the award of Master of Engineering in "Applied Electronics and Servomechanism" of University of Roorkee is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other Degree or Diploma.

This is to further certify that he has worked for a period of seven and half months from 1st January 1965 to 15th August 1965 for Master of Engineering Degree at the University.

( P V INDIRESAN)

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# SUMMARY

The desirable characteristics of the APC system are the high value of lock and capture range togather with low value of noise bandwidth. These three are not compatable with each other i.e. the increase of the lock and capture range of the system is accompanied with an increase of noise bandwidth of the system. However, after the system is locked the noise bandwidth can be reduced without affecting the other desirable characteristics. A two position servomechanism has been developed which incorporates this idea. Initially, the system starts with a broad band loop and after it has synchronised it automatically switches over to a narrow band condition thereby, reducing the noise bandwidth. It has been shown theoritically and experimentally that this switching action does not introduce any instability.

## LIST OF SYMBOLS

$A_1$ = Maximum amplitude of the reference signal

$A_2$ = Maximum amplitude of the controlled output

$K_1$ = Sensitivity of the Phase detector in mv/radian

$K_2$ = Sensitivity of the reactance tube oscillator combination in radians/sec x volt.

k = $K_1K_2$, overall gain constant of the system

$\omega_1$ = Frequency of the incoming signal.

$\omega_0$ = Free running frequency of the socillator

$\Phi_1$ = Phase of the referance signal in radians

$\Phi_2$ = Phase of the controlled output in radians

$\Phi$ = Steady state phase error = $\Phi_1 - \Phi_2$ in radians

$\Omega$ = The difference between the frequency $\omega_1$ of the reference source and the free running frequency $\omega_0$ of the oscillator.

F(p) = Transfer function of the interconnecting network.

$\xi$ = Damping factor of the system

$N_1$ = Noise at the main input of the system

$N_2$ = Noise at the reactance tube oscillator combination.

$H(j\omega)$ = Closed loop transfer function of the system

$\tau_1, \tau_2$ = Time constants of the interconnecting network

$\tau$ = Time constant of the Low pass filter.

$\omega_c$ = Cut-off frequcncy of the system

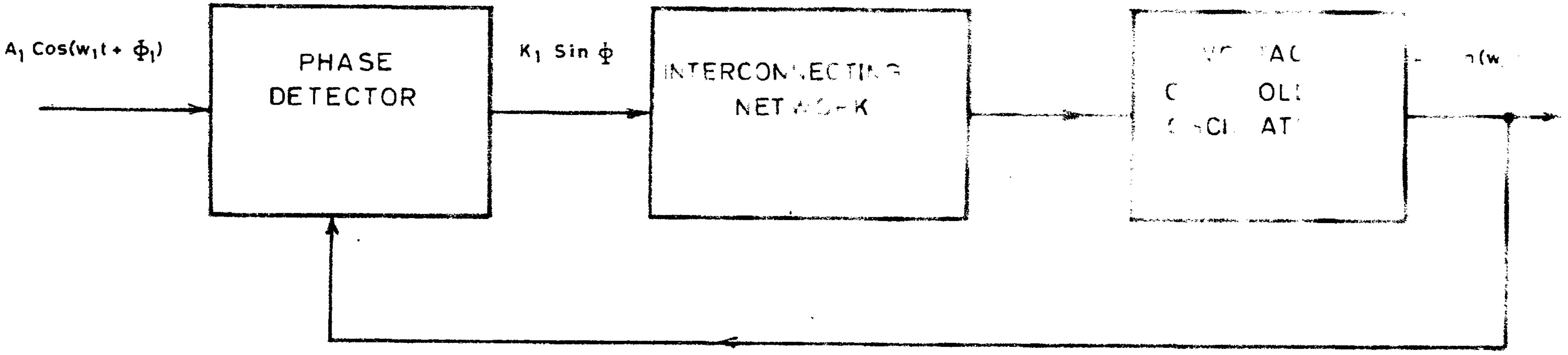


FIG 1- BLOCK DIAGRAM OF THE APC SYSTEM

using an automatic two position switching system. Initially, the system has a wide bandwidth and after locking the system is switched over to a narrow bandwidth position Thus a large pull-in range is obtained initially and after the circuit has synchronised the noise bandwidth is reduced by the switching action. Basically, such a system is a relay or Bang Bang servomechanism and hence it has been called Bang Bang control of an APC loop.

## 2.0 Characteristics of the APC system:

The desirable characteristics of the APC system are (a) a small steady state error. (b) an adequate pull-in and the lock range (c) small pull-out range (d) a small noise bandwidth.

## 2.1 Steady state error:

The steady state solution of the differential equation governing the APC system as derived in appendix 1 is $KF(p)\mathrm{Sin}\phi = \Omega / KF(p)$

where $\Omega$ = difference between the frequency of the source and the free running frequency of the oscillator

$K$ = Overall gain constant of the system

$F(p)$ = Transfer function of the interconnecting network.

from the above it is seen that for a small steady state error $\Omega \ll KF(p)$ i.e. the amount of initial mistuning should be as small as possible or in other words for a given mistuning the value of gain constant K should be large.

## 2.2 Lock Range

Lock range (1) is the total change in the unlocked oscillator frequency which can be obtained by the system. In other words the lock range is limited to the total variation of frequency which is obtainable from the V.C.O. circuit. The lock range for a linear reactance tube oscillator combination and a linear phase detector is $\pi K_1 K_2$ radians/sec. where:

$K_1$ = The sensitivity constant of the phase detector in volts/radian.

$K_2$ = Sensitivity of the reactance tube oscillator combination in radians/sec.volt

The phase detector output does not vary linearly with the phase difference $\Phi$ but the Sine of $\Phi$. It can be shown (1) that this non-linearity reduces the lock range to $2K_1K_2$ radians/sec.volt. The lock range can also be ascertained alternatively from the steady state solution of the fundamental differential equation of the APC system which is of the form:

$$\dot{\Phi} + KF(p)\sin\Phi = \Omega$$

for $F(p) = 1$, the steady state solution of the above differential equation is:

$$\Phi = \arcsin \Omega/K$$

As the maximum value of $\sin\Phi$ is limited to ± 1 the maximum difference frequency between the incoming signal and the local oscillator when the system can lock is ± K radians/sec. Therefore, the range of operation is limited to 2K radians/sec. For $K > \Omega$ a hysteresis or a pulling effect exists and the system locks into synchronisation with the input signal. For $K < \Omega$ the system is at asynchronous (2) state and no locking is possible.

## .3 Capture Range or the Pull-in range of the APC system:

Let, originally, the system be unlocked and $\Omega_o$ be the frequency difference between the free running oscillator and the input signal. As $\Omega_o$ is reduced slowly by varying the oscillator tuning there will be a difference frequency $\Omega_1$ radians/sec. at which the system just locks. This $\Omega_1$ the largest value of $\Omega$ at which locking can take place is known as capture range. (1,2) In other words the pull in range of synchronization or the capture range is the maximum value of for which irrespective of the initial condition of the system the phase difference $\Phi$ between the reference and the controlled output reaches the steady state value. (4,5) The variation of $\Phi$ in the pull-in range is in many radians and as such the capture or the pull-in range of synchronization can only be found out by drawing the phase potrait and finding out the singularities in it. It has been shown by Gruen that the pull in range of synchronization is approximately equal to $\sqrt{2\zeta\omega_n K}$ when $\omega_T/K \rightarrow 0$.

In the above expression $\zeta$ = damping constant of the system.

K = Open loop gain of the system

$\omega_n$ = Resonance frequency of the system in absence of damping $= \sqrt{K/\tau_2} = \omega_c$ for $\zeta = 0.5$

McAleer also shows that the lock range is equal to capture range when there is no interconnecting network between the phase detector and the V.C.O. circuit.

2.4 <u>Pull-out Range of Synchronization:</u>

Suppose the system is initially locked at an angle $\Phi = \arcsin \Omega/K$ (where $\Phi$ is the phase difference between the reference signal and the free running frequency of the oscillator K the overall gain of the APC loop) and an additional perturbation of say $\Omega_1$ comes up in the system then the pull-out range(2) of the system is defined as the maximum value of $\Omega_1$ which will not throw the system out of synchronization. When the phase potrait is drawn from the general nonlinear equation of the APC loop and the two equlibria points $\Phi_\infty$ and $\pi - \Phi_\infty$ are ascertained, where $\Phi_\infty$ is the stable equilibrium point while $\pi - \Phi_\infty$ is an unstable equlibrium point as shown in appendix no. 2. Then if the perturbation(2) is such that $\pi - \Phi_\infty$ is not traversed there can be no pull-out. and the system remains in synchronism.

The pull-out range is a complex function of the frequency of the disturbing signal its amplitude, the amount by which the desired signal is mistuned and to the system parameters. The phase potrait indicates critically the nature of the system performance to undesired interferance signal. Ingeneral, its magnitude decreases as the system bandwidth decreases.

## 2.5 Noise Bandwidth of the APC system:

The objective of the present thesis is to minimise the noise bandwidth of the system. The unwanted noise can enter the system either at the main input or at any other grid point in the whole loop. The noise configuration of the APC system is shown in fig. 2.

The APC system acts as a low pass filter (1) to noise $N_1$ at the system input terminals and therfore, the low frequency components of the input noise will be attenuated by the circuit. But the low pass nature of the APC loop makes the high frequency components of $N_2$ to appear amplified at the output while the low frequency components will get cancilled at the V.C.O. input due to negative feedback. Therefore, if the high frequency components of $N_2$ ( i.e. due to oscillator instability ) are objectionable in the circuit the gain $K_2$ of the reactance tube oscillator combination should be made as low as practicable. The cut off frequency of the interconnecting network is also ascertained by knowing the exact purpose of the system i.e. whether oscillator instability has to be reduced or the input

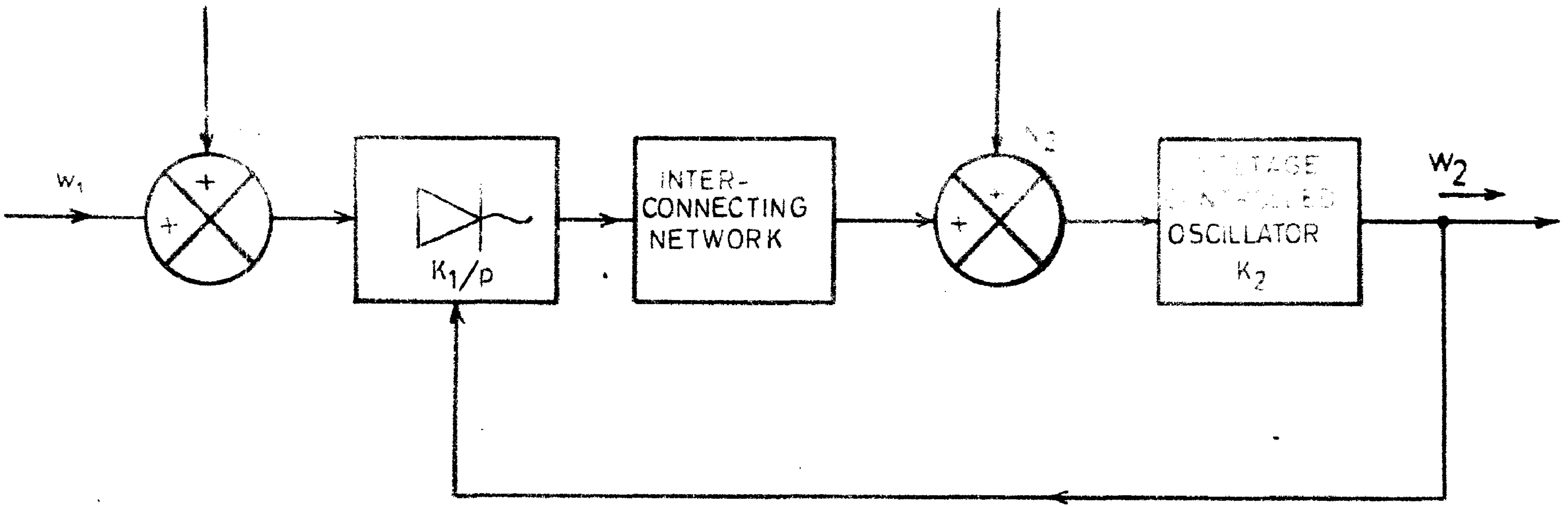


FIG 2_NOISE CONFIGURATION OF THE APC SYSTEM

noise has to be suppressed.

The effect of noise can be measured by knowing the integrated noise bandwidth (1,4,6) of the system. This noise bandwidth is defined mathematically as:

$$B = 1/2\pi \; j\int_{-\infty}^{+\infty} |H(j\omega)|^2 \, d\omega$$

where $H(J\omega)$ is the closed loop transfer function of the system. (2) The limits in the above expression is from $-\infty$ to $+\infty$ as most of the noise at the output is attributed due to $N_1$. The noise bandwidth has been found out to be $\pi K$ (1,2,4) when the interconnecting network is not employed. The noise bandwidth remains unaltered even when the simple low pass filter is used as an interconnecting network. Therefore, in both of these cases the noise bandwidth rises linearly with the overall gain constant of the system. In case a double time constant interconnecting network is used and the gain of the system is so adjusted such that $\omega_n/K \rightarrow 0$ the integrated noise bandwidth has been found (1,4) to be independent of the gain constant of the system and that it has a value $2\pi\omega_n$. Thus in this case by properly adjusting the time constants of the interconnecting network the noise bandwidth can be suitably adjusted. (4) On the other hand if $\omega_n/K \rightarrow 1$ the noise bandwidth remains as $\pi K$ radians/sec. The nonlinear analysis of the APC system (2) indicates that the input noise $N_1$ does not effect the phase potrait much but when the initial frequency error is large and if the signal is contaminated with noise the system requires a different number of cycles for locking.

## 2.6 Comments on the Characteristics of the APC system:

From the above table it is seen that with a simple low pass filter in the circuit the lock range, capture range noise bandwidth all depend upon the gain constant of the system. Thus if the gain of the loop is made high for making the lock and capture range high noise bandwidth also rises proportionately. Thus in practice a compromise has to be made between the lock and capture or the pull-in range on one hand and the noise bandwidth on the other.

Table I also shows that the noise bandwidth remains same when there is no interconnecting network as well as when the simple low pass filter is employed. The noise bandwidth of the system is defined (1,4,6) as:

$$B = 1/2\pi j \int_{-\infty}^{+\infty} |H(j\omega)|^2 d\omega$$

From the above expression the noise bandwidth is the area under the square of the frequency response curve of the APC system. With the use of the low pass filter as the system bandwidth is reduced the damping factor $\xi$ decreases (1) so that the overshoot in the transient response of the system increases. Correspondingly the height of the frequency response curve increases such that the noise bandwidth remains unaltered.

However, the noise bandwidth can be reduced by the use of double time constant network with a transfer

## TABLE 1.

Characteristics of the APC system:

| Item F(p) | $1$ | $1/1 + p\tau$ | $1 + p\tau_1/1 + p\tau_2$ | Remarks. |
|---|---|---|---|---|
| Lock Range | $\pi K$ | $\pi K$ | $\pi K$ | When the range of operation of the phase detector is $\pi$ |
|  | $2K$ | $2K$ | $2K$ | Phase detector is nonlinear. |
| Capture range |  | $\sqrt{K/\tau}$ | $\sqrt{2\xi\omega_n K}$ |  |
| Noise Bandwidth | $\pi K$ | $\pi K$ |  |  |
| " " |  |  | $2\pi\omega_n$ | when $\omega_n/K \to 0$ |
| " " |  |  | $\pi K$ | when $\omega_n/K \to 1$ |

function of the form $1 + p\tau_1/1 + p\tau_2$ at the same time the capture range also is appreciably reduced. For instance, for $K = 100{,}000$ radians/sec. and $\omega_n = 1{,}000$ radians/sec. the noise bandwidth of the system is equal to 1,000 cycles/sec. while the capture range is 2,000 cycles/sec.

## 3.0 Transient and the Frequency Response of the APC systems:

The transient response of the system could not be experimentally varified due to the shortage of the instruments. However, the theoritical analysis for transient and the frequency response of the system have been given in the appendix no. 3.

## 4.0 Modified APC system:

As discussed earlier the desirable characteristics of the APC system are (a) higher lock and capture range (b) lower pull-out range and (c) lower noise bandwidth. It was seen that the noise bandwidth (1,2,4) of the system can be made independent of the gain constant of the system if a double time constant network is employed and if $\omega_n/K \to 0$. The reduction in $\omega_n$ with the double time constant network reduces the (1) capture range also by about the same order of magnitude. The capture range however, should be made such that the system locks for the range of frequencies of interest. Thus the noise bandwidth cannot be made small independently of the value of the capture range. Fortunately, a high value of capture range is needed only initially until the system is locked. and once the locking has been achieved the bandwidth can be

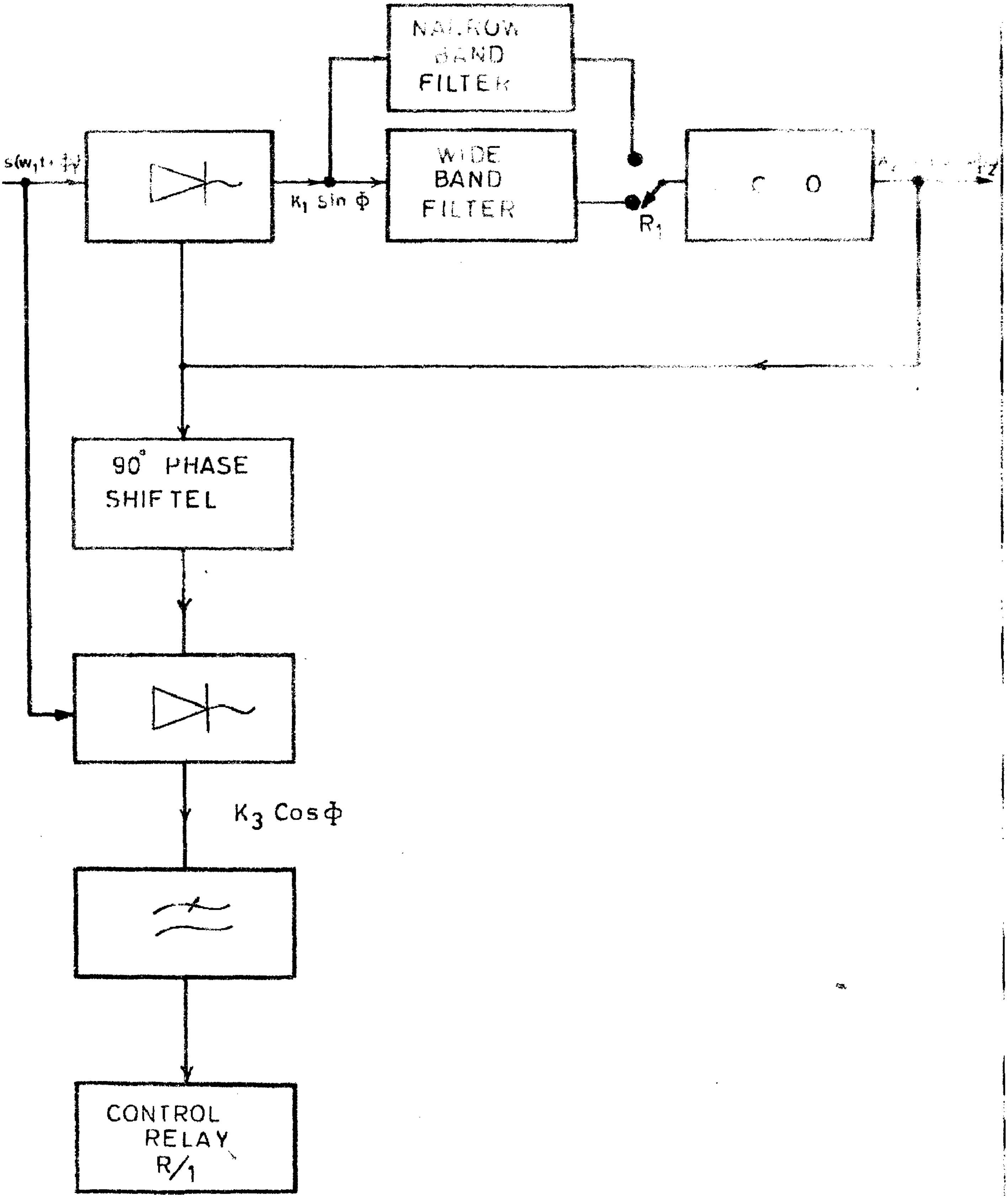


FIG. 3 - MODIFIED APC SYSTEM

made very small just large enough to follow any circuit drift. Thus the bandwidth can be lowered once the system is locked. This has been achieved by a two positon servomechanism shown in fig. no. 3. Initially, the switch is kept to the wide band position inorder to obtain a high capture range and after the system is locked the control circuit automatically switches the loop to the narrow band position. The relay operates from the controlling signal obtained as shown in the fig. no.3 by a quadrature phase detector which develops a voltage proportional to Cos$\Phi$. The relay operates only after the loop has locked in. The indication for this is the development of a d.c. error voltage at the output of the phase detector (when the circuit is not synchronised the error signal will be an a.c.)Unfortunately, the main phase detector output which is proportional to Sin$\Phi$ is zero or near to zero at synchronism (fig.4,5) and hence will not be adequate or suitable for the operation of the relay. Hence an auxiliary quadrature phase loop is employed. Its output is proportional to Cos$\Phi$ and as seen from fig. nos. 4 and 5 this has a large value near the range of interest. and can be used to operate the relay. When the circuit is not locked there is no d.c. input to the relay, the switch is in the wideband position and the loop has a desirable large capture range. Once, the circuit is locked the resultant d.c. output of the quadrature phase detector operates the relay; the switch transfers to narrow band position to yield a correspondingly low noise bandwidth. This does not affect the lock range at all and hence the circuit will remain in synchronism so long as the oscillator

does not drift beyond this value.

## 5.0 Bang Bang affect on the AFC system:

As stated earlier the switch S is closed when once the system is locked. The network comprising $R_3$ and $C_3$ lowers the cut-off frequency of the overall loop. The theoritical analysis is as shown in appendix no. 4.

The transformed output equation is:

$$V_2(s) \quad A/s \; \frac{(R_1 + R_2 + 1/C_2 s)(R_3 + 1/C_3 s)}{(R_3 + 1/C_3 s)(R_1 + R_2 + 1/C_2 s) + R_1(R_2 + 1/C_2 s)}$$

The solution of the above equation in the time domain is:

$$V_2(t) = A\left[1 + 0.044e^{-.85x10^3 t} - 0.042e^{-7.85x10^3 t}\right]$$

for $R_1 = 15x10^3$ $R_2 = 47x10^3$ $R_3 = 15x10^3$

$C_3 = 0.01x10^{-6}$ Fd. $C_2 = 0.02x10^{-6}$ Fd.

The table no. 2 shows the voltage variation with time. As the variation is not much during switching period the circuit will not be unlocked by the Bang Bang operation.

## 6.0 Response of the AFC system with various types of input signal:

### 6.1 Response due to AM input:

The output of the phase detector depends upon the product of the magnitude of the input and the output of the phase locked system. Thus if an amplitude modulated signal is applied at the input of the AFC system the output of the phase detector will vary from instant to instant according to the

# TABLE NO. 2

## Bang Bang affect on the APC system:

The output voltage equation is:

$$V_2(t) = A\left[1 + 0.044e^{-3.85x10^3t} - 0.042e^{-7.85x10^3t}\right]$$

The equation can be put more conveniently in the form:

$$V_2(t) = A\left[1 + B + C\right]$$

$$\text{Where } B = 0.044e^{-3.85x10^3 t}$$

$$C = 0.042e^{-7.85x10^3 t}.$$

The value of $V_2(t)$ at different times will be as follows:

| t in $\mu$ sec | B | C | $V_2(t)$ |
|---|---|---|---|
| 0 | 0.044 | 0.042 | 1.002 |
| 1 | 0.0438 | 0.0417 | 1.0021 |
| 2 | 0.0433 | 0.0413 | 1.0020 |
| 10 | 0.0410 | 0.0388 | 1.0022 |
| 100 | 0.0221 | 0.0191 | 1.0030 |
| 1000 | 0.0004[illegible]4 | 0.000162 | 1.000302 |
|  | 0 | 0 | 1.000000 |

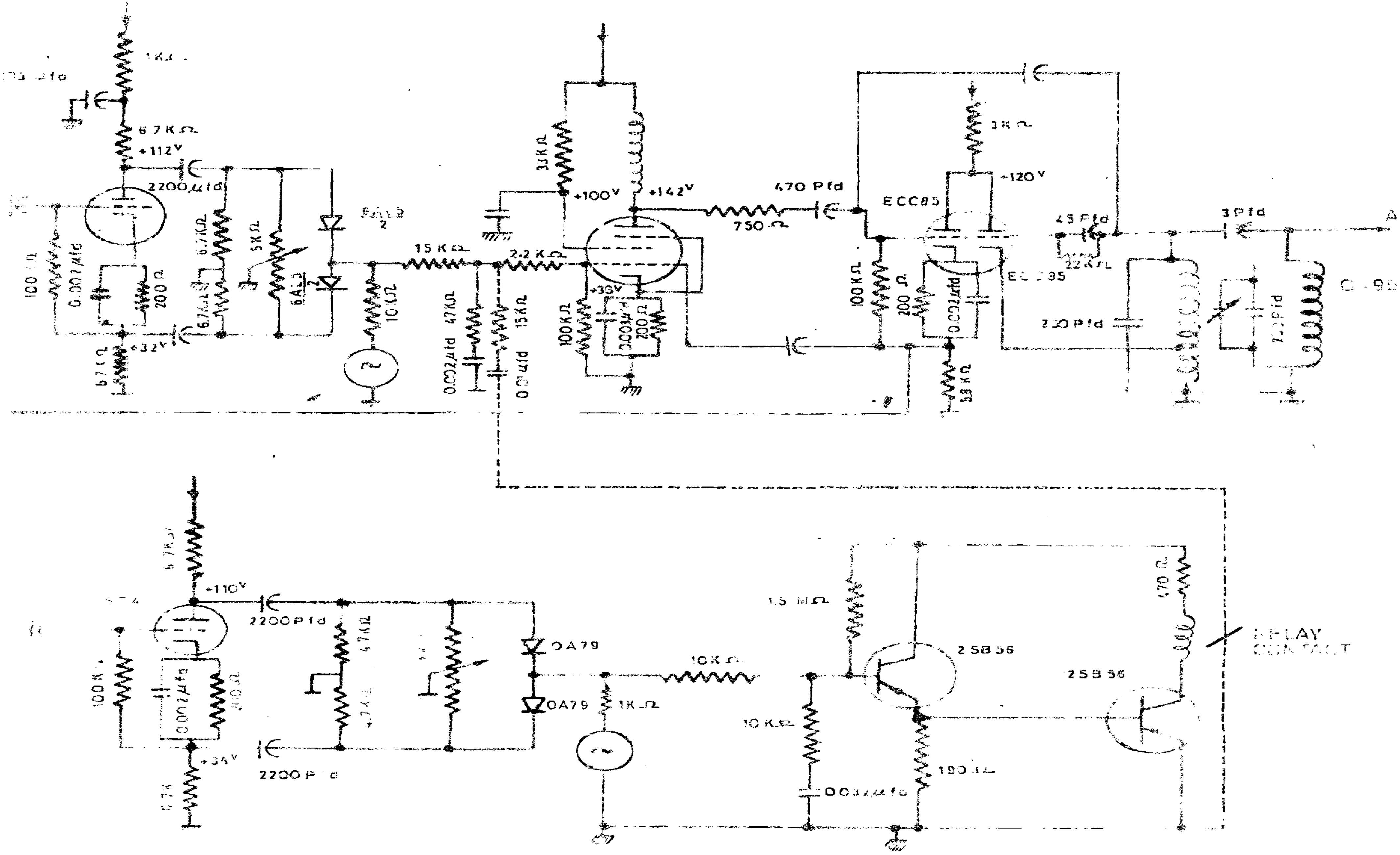


FIG. 4_ CIRCUIT DIAGRAM OF AN APC SYSTEM

amplitude of the modulated signal. This varying voltage when applied to the reactance tube oscillator combination will frequency modulate the output signal.

6.2 Response due to FM input:

In response to a step frequency input Rt the steady state error is equal to $e_{ss}$ defined as:

$$e_{ss} = \frac{R}{\mathrm{Lit.}_{p\to 0}\, p\, G(p)}$$

$$= \frac{R}{\mathrm{Lit.}_{p\to 0}\, \frac{p}{p} K_1 K_2 (1+\tau_1 p)(1+\tau_2 p)}$$

$$= \frac{R}{K_1 K_2}$$

Thus the steady state value of the error will be the magnitude of the ramp function divided by the gain constant of the system. This error in frequency can be minimised by increasing the gain constant of the loop. Thus the output FM signal will follow the input with some steady state error. Moreover, the error in phase will be a function of time.

## 7.0 Experimental Results:

### 7.1 Electronic Circuit Description:

The electronic circuit for the APC system is shown schematically in fig. no. 4. The phase detector is of the shunt type employing 6C4 as phase splitter and 6AL6 as the diodes for the bridge circuit. The phase splitter as well as the diodes can effectively be used at high frequencies. Moreover, as the amplitude of the applied voltage in the quadrature phase splitter was not high OA79 was employed. in the bridge circuit of the quadrature phase detector. The amplitude of the switching voltage should be at least 3-4 times the signal amplitude of the reference source so as to minimize the leakage of the reference frequency at the output.

The two time constants of the integral plus proportional type of interconnectin network are $94\times10^{-6}$ and $124\times10^{-6}$. This makes the cut-off frequency $\omega_n$ of the loop equal to 20 Kc/s. The hgih $g_m$ tube 6AC7 was employed as the modulator. This high value of $g_m$ makes the control more affective for a given variation of the grid voltage. The oscillator portion of the V.C.O circuit employs ECC85 tube. To avoid loading of the output circuit due to feed back one section of ECC85 is used as a buffer cathode follower stage. The capactance affect due to measuring instruments are avoided due to this isolation of the output circuit.

The controlling signal is obtained by a switching transistor. Its base voltage is so adjusted that ordinarly the transistor does not conduct. The base voltage required to drive it to conduction is approximately -150mv. The bandwidth alteration was done mechanically by a Siemens relay connected to the transistor load circuit. As the operating coil resistance of this relay is only 70 ohms it does not appreciably affect the operating condition of the switching transistor

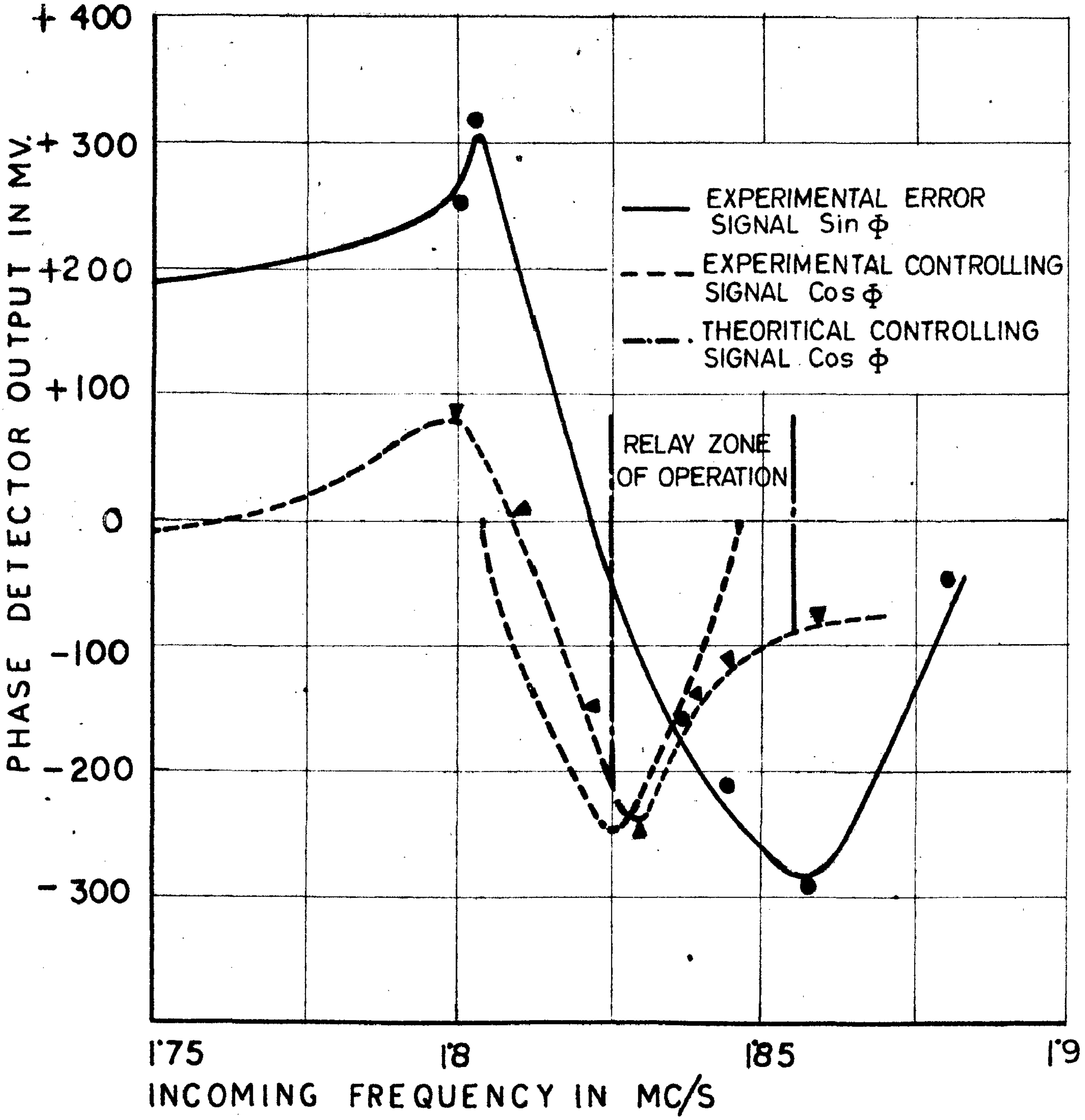


FIG. 5_ CONTROLLING SIGNAL VOLTAGE AS A FUNCTION OF FREQUENCY; INPUT VOLTAGE=5·2 VOLTS. (FREQUENCY INCREASING)

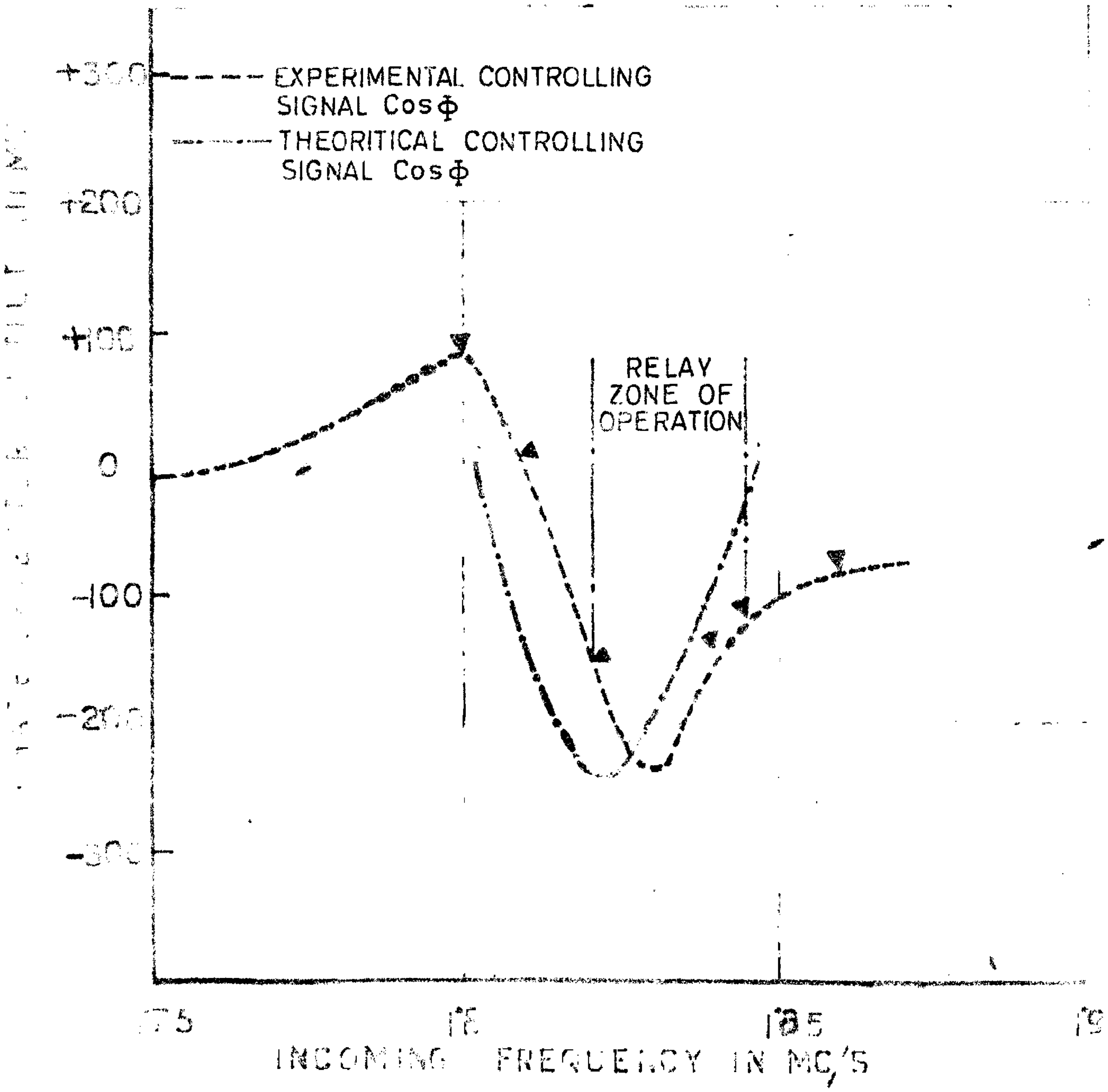


FIG. [illegible] CONTROLLING SIGNAL VOLTAGE AS A
FUNCTION OF FREQUENCY, INPUT [illegible] E 5.2 VOLT

The switching circuit would have caused some small but finite change in the d.c. conditions of the circuit. This would upset the phse lock mechanism and make the system unstable. The mechanical switching time is appreciably short -2 msec. and the stability of the circuit is excellient.

### 7.2 Alignment of the Circuit:

The bridge circuit f the phase.detector was adjusted so as to give as small a d.c. output as possible when the circuit was unlocked. The output could not be made exactly zero due to difference in the characteristics of the two diodes and due to bridge circuit unbalance. It was tried to make the oscillator lock range symmetrical ro nd its free running frequency by adjusting the bias of the reatance tube but the circuit could not be made symmetrical probably due to the lack of the proper adjustment of the plate circuit of the reactance tube. The phase shifter was adjusted to produce a phase shift as near to 90°as possible. But the output of the phase detector drastically reduced as the phase approached 90°. Thus a compromise was made for the output voltage on one hand and the phase shift on the other.

### 7.3 Various Measurements:

Three important characteristics of the APC system namely the lock range, capture range' or the pull-in range, noise bandwidth along with the output of the phase detectors were measured. The experimental curve for the phase detectors output are as shown in fig. nos. 5 & 6. These were found out by knowing the d.c. output at various frequencies.

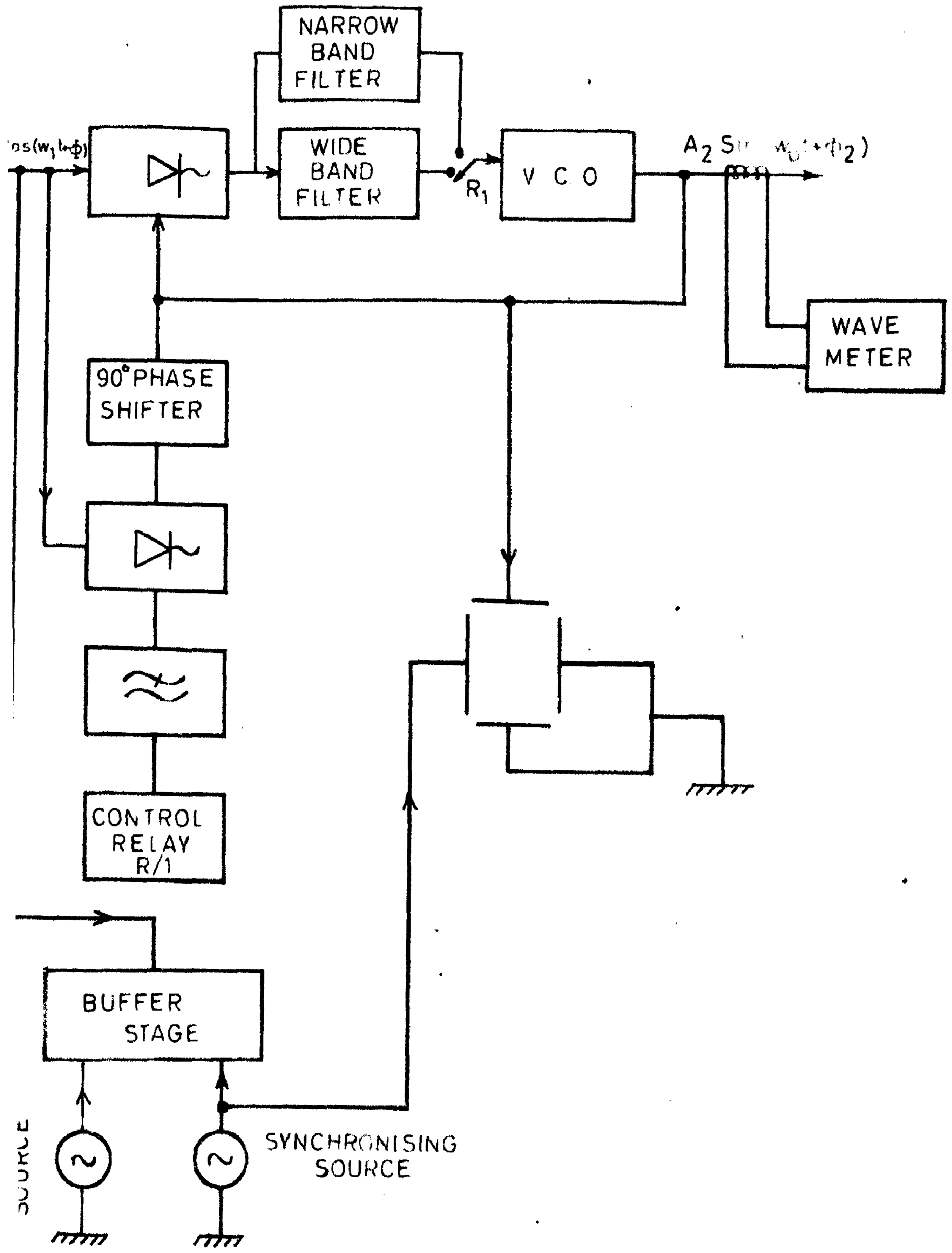


FIG. 7. SETUP FOR MEASURING THE NOISE BANDWIDTH OF THE APC SYSTEM

The lock and the capture range were found out by knowing the end frequencies between which the output is synchronised when $F(p) = 1$ and when $F(p) = 1 + \tau_1 p / 1 + \tau_2 p$. The lock and the capture range remained same when the frequencies was raised from a lower initial value as well as when the frequency was lowered from a higher value.

The noise bandwidth was measured as shown schematically in fig. 7. Due to nonavability of noise generator a sinusoidal oscillator was employed for noise bandwidth measurements. Firstly the system was locked to the input signal frequency until theoutpt of the interferance oscillator was kept at zero. Wavemeter at this instance measured the locked frequency of the system. Then the interferance voltage was increased and the frequency of the interferance oscillator was slowly brought closer to the locked frequency until the wavemeter shows changes in its value. The variation in the reading of the wavemeter increases as the frequency of the oscillator is brought closer to the locked frequency until at some other frequency where the needle of the wavemeter comes to the original position. The difference between this reading and the original where the interferance was noted firstly is the measure of the pull out range of the system and it is close measure of the noise bandwidth.

## 8.0 Comments on the Experimental Results:

### 8.1 Controlling signal Cos$\Phi$:

For lowering the noise bandwidth of the AFC system a controlling signal proportional to Cos$\Phi$ is produced.

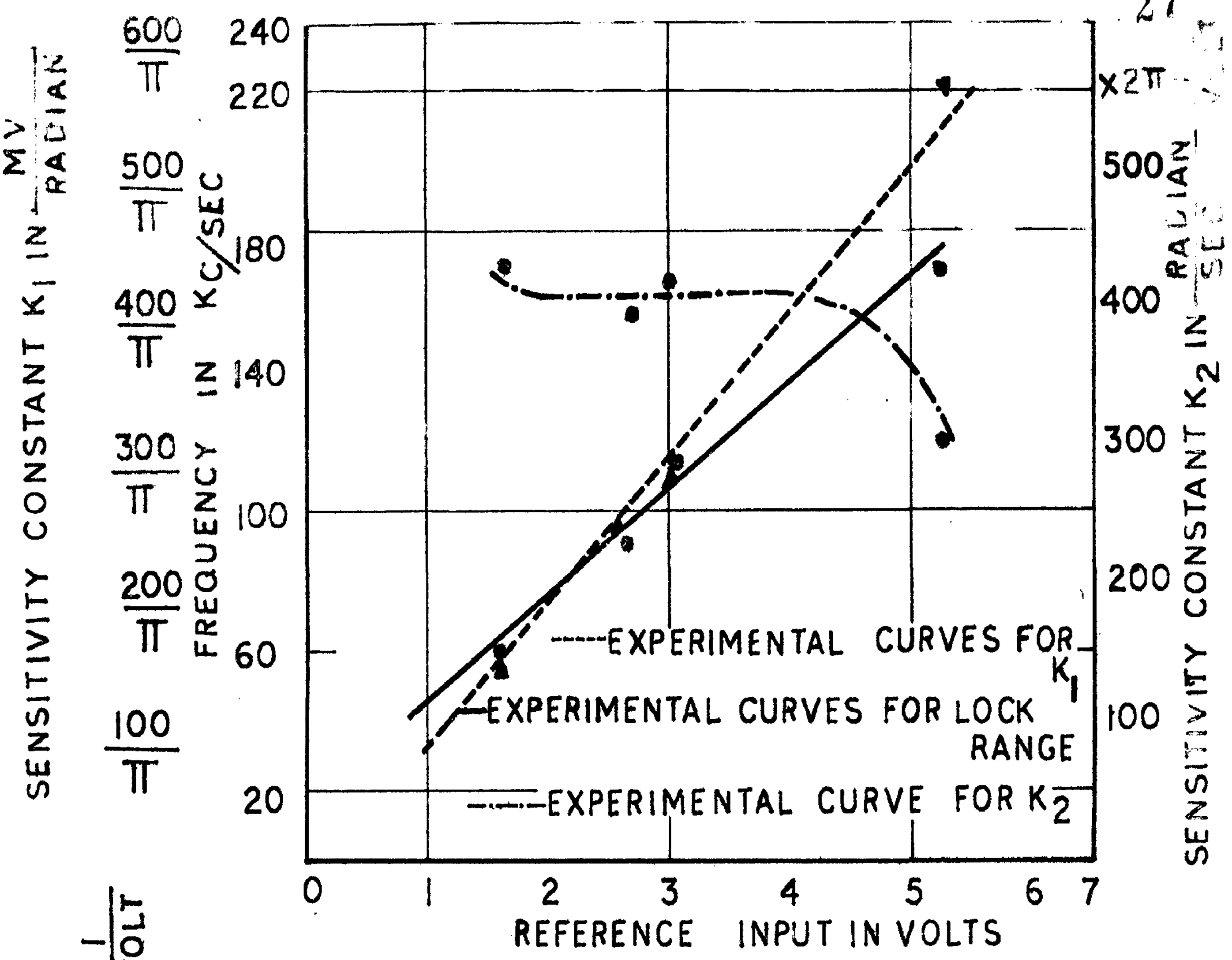


FIG.8 LOCK RANGE MEASUREMENT

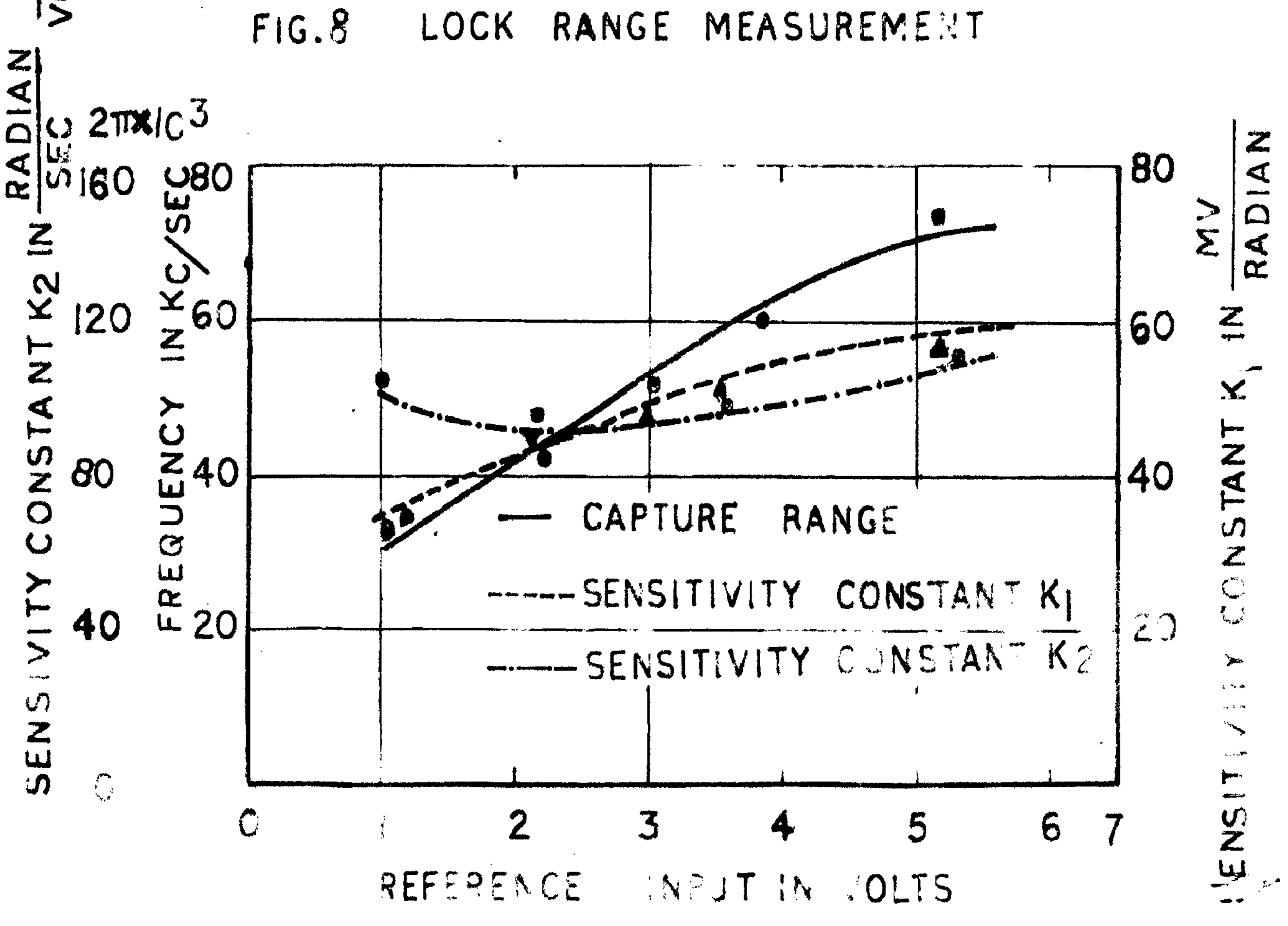


FIG. [illegible] CAPTURE RANGE MEASUREMENT

This $\cos\Phi$ curve is not exactly out of phase with the $\sin\Phi$ curve. This is mainly because as the $\cos\Phi$ circuit was brought in quadrature the output decreased so much that the controlling action was not possible. Thus a compromise was made between the phase shift on one hand and the maximum range of relay operation on the other. This makes the circuit assymetrical so that the relay range of operation is different as shown in fig. nos.5 & 6. This is 33 Kc/s. when the frequency is increased from a low initial value and 22 Kc/s. when the frequency is decreased from a high initial value. This unequality is also partially due to unequal pull in and pull out torque of the relay.

## 8.2 Lock Range:

From the graph nos. 8 it is seen that the lock range ( $\pi K_1 K_2$) increases nearly linearly with the reference input voltage. This is due to the increase in $K_1$ the phase detector sensitivity with increase in input voltage. $K_2$ the reactance tube sensitivity is uneffected by the magnitude of the input voltage.

## 8.3 Capture Range:

Capture range as seen before is $\sqrt{2\xi\omega_n K}$, where $\omega_n^2 = K/\tau_2$

and $\xi$ = damping ratio = $1/2\tau_2\omega_n + K\,\tau_1/\tau_2 2\omega_n$

Thus the capture range is = $AK^{3/4} = (AK_1K_2)^{3/4}$

The measured value of the capture range as seen from graph no. 9 increases as the reference input according to this 3/4 power law.

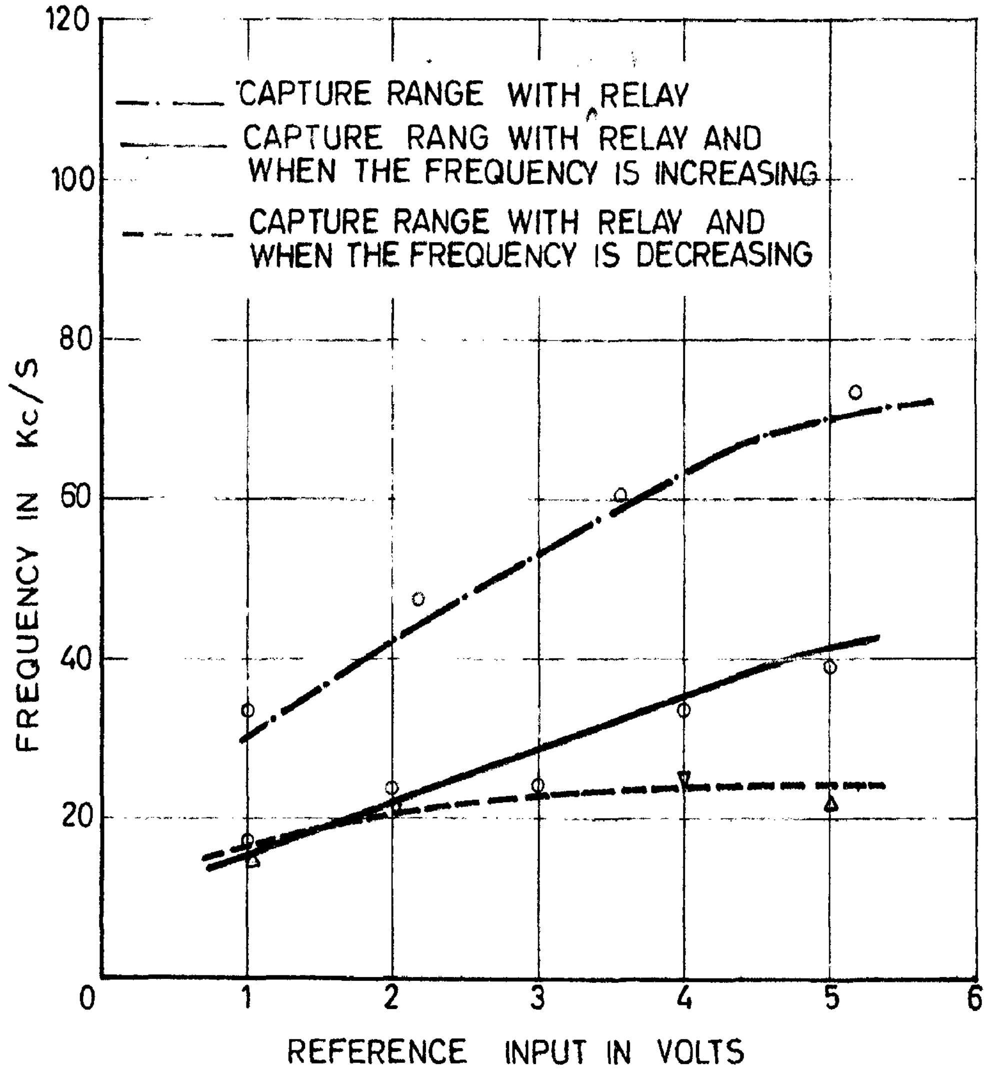


FIG.10-CAPTURE RANGE WITH AND WITHOUT RELAY

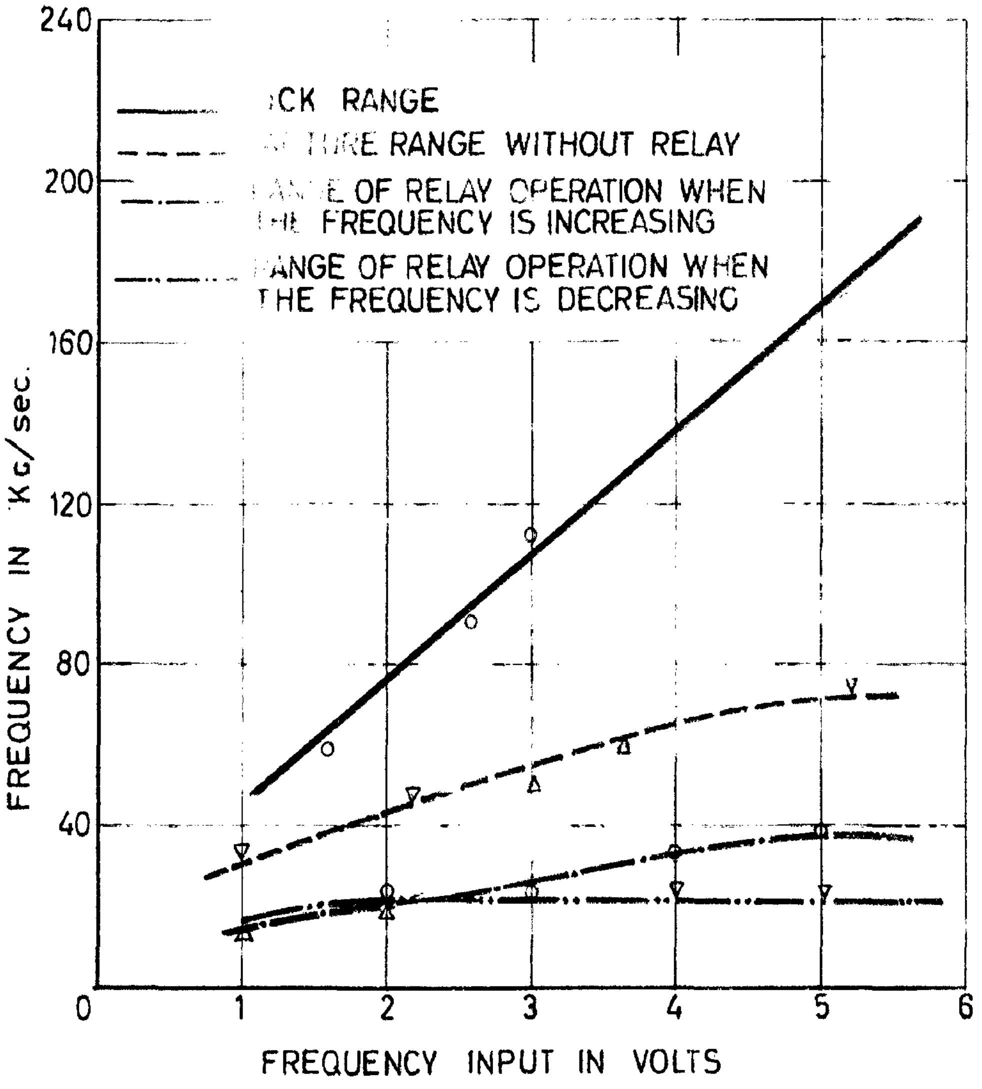


FIG.11. LOCK RANGE, CAPTURE RANGE, AND RANGE OF RELAY OPERATION

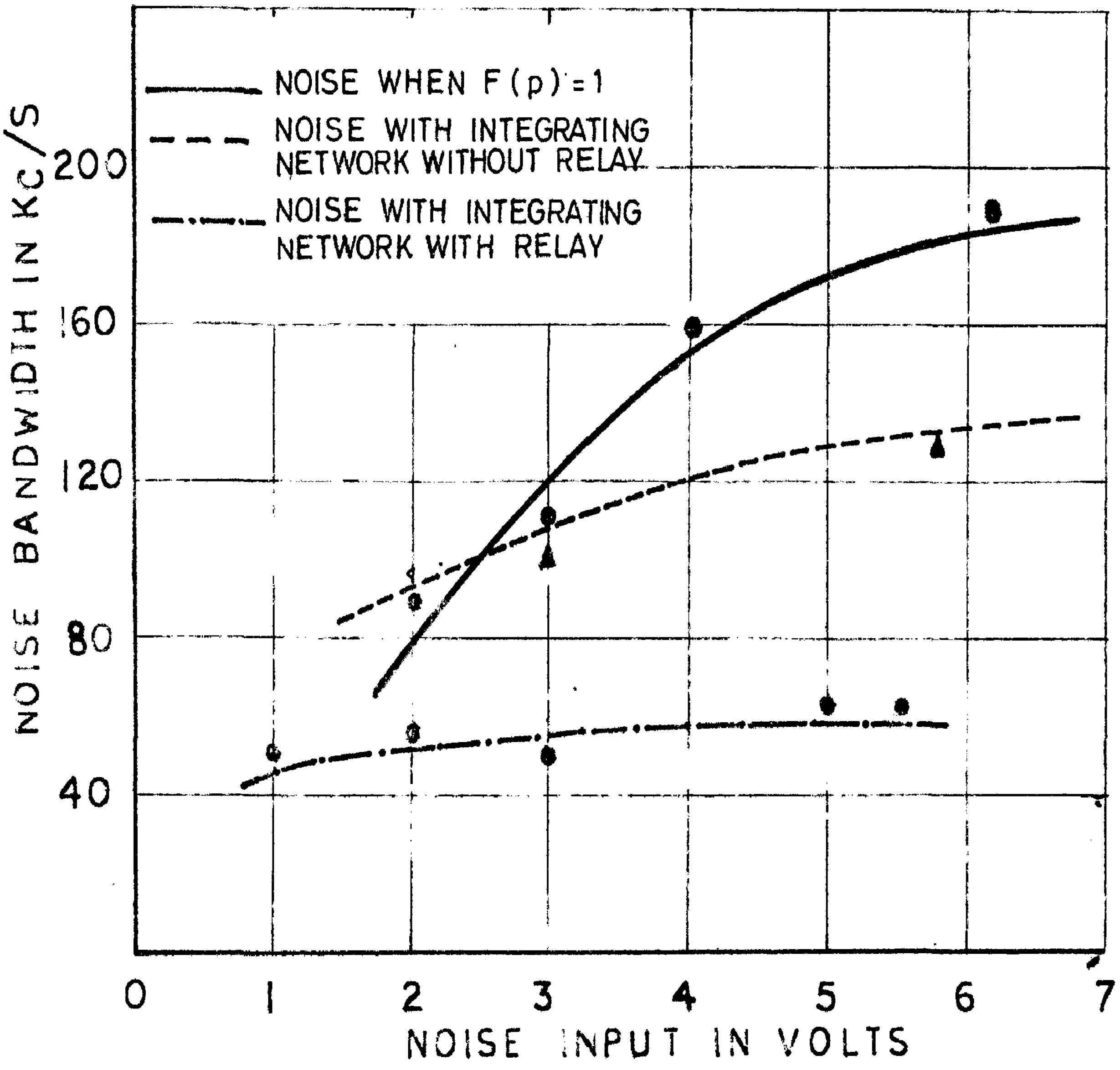


FIG.12. MEASUREMENT OF NOISE BANDWIDTH AT 4V REFERENCE INPUT.

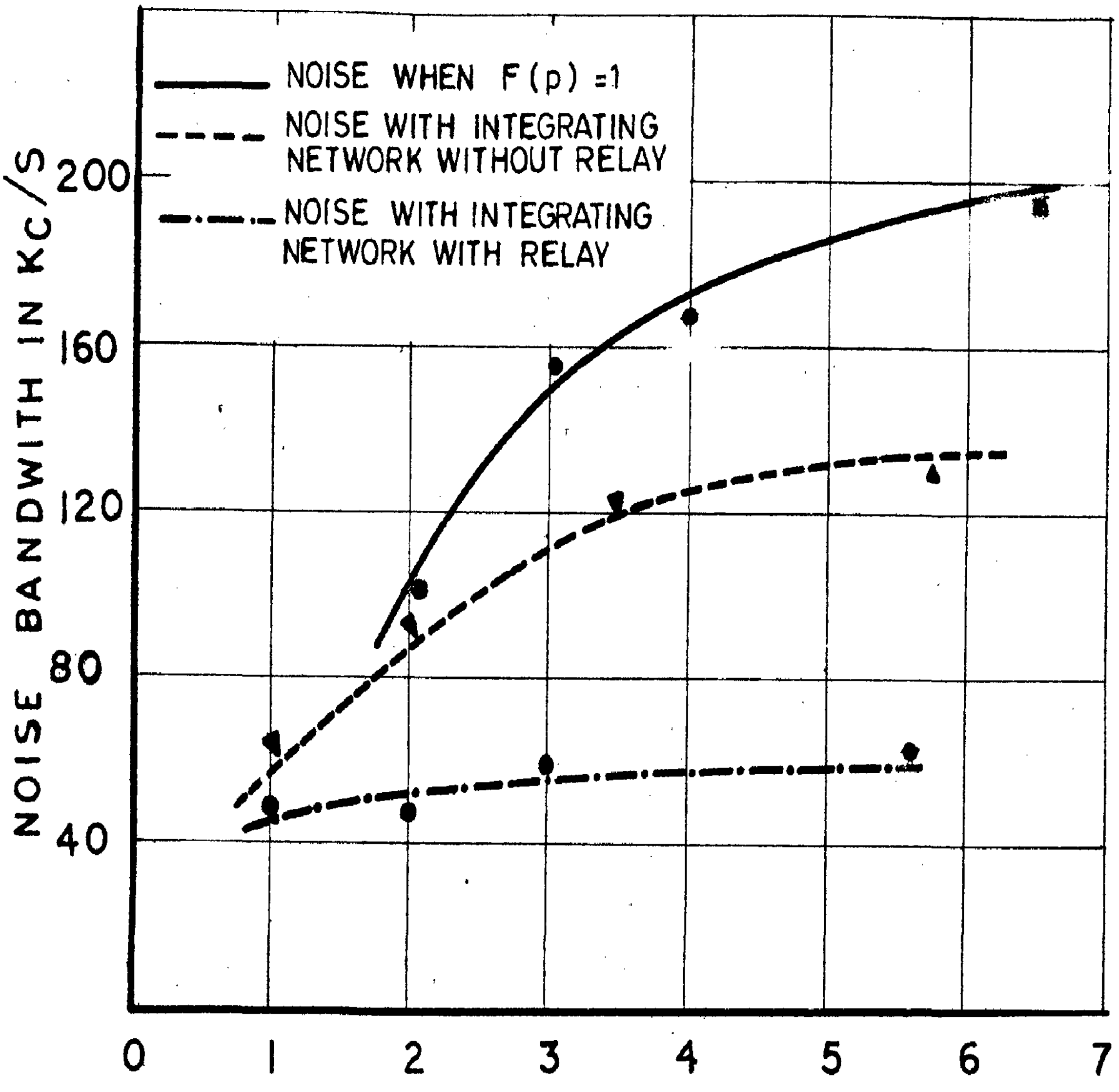


FIG.13. MEASUREMENT OF NOISE BANDWITH AT 3·0V REFERENCE INPUT.

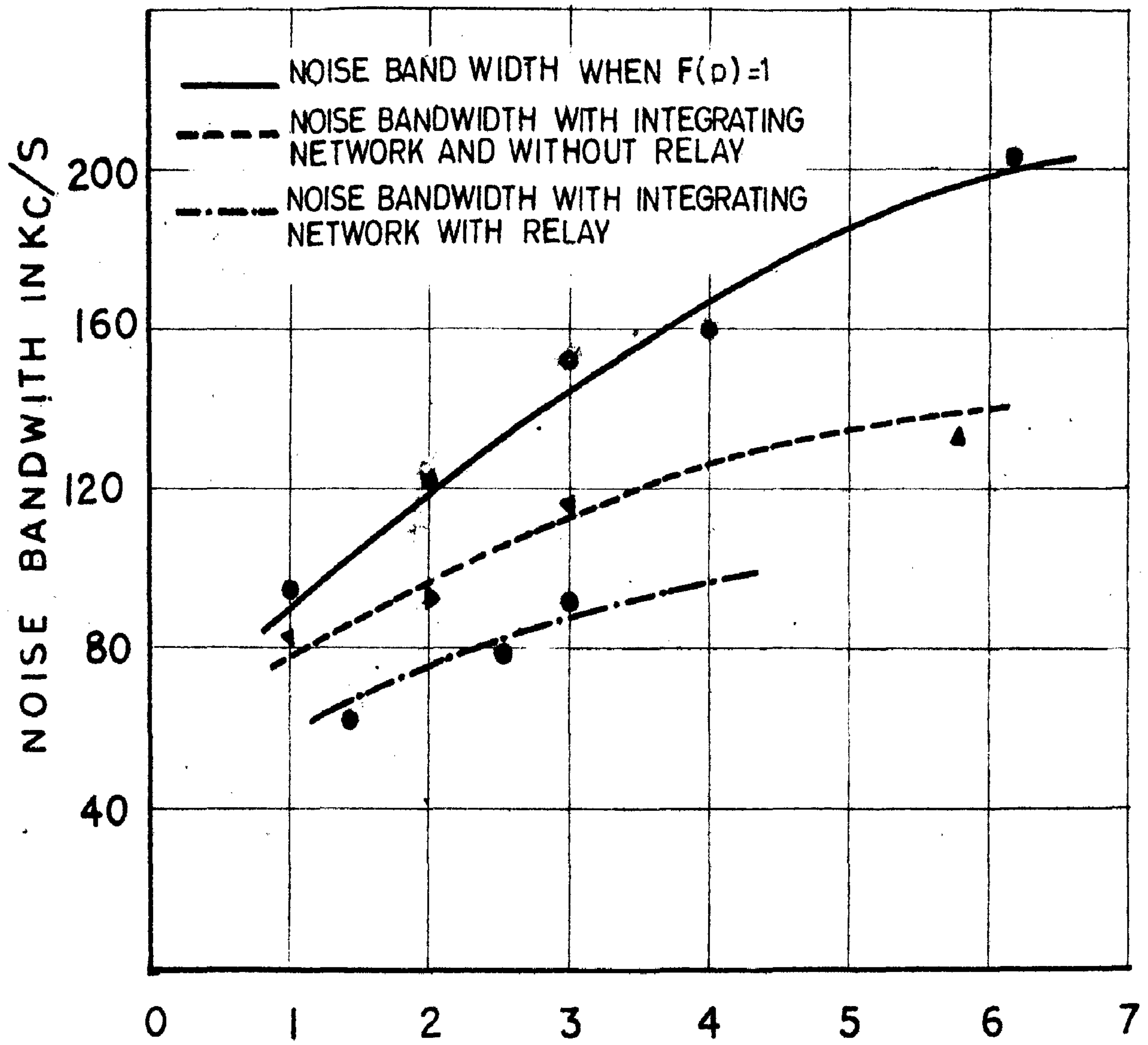
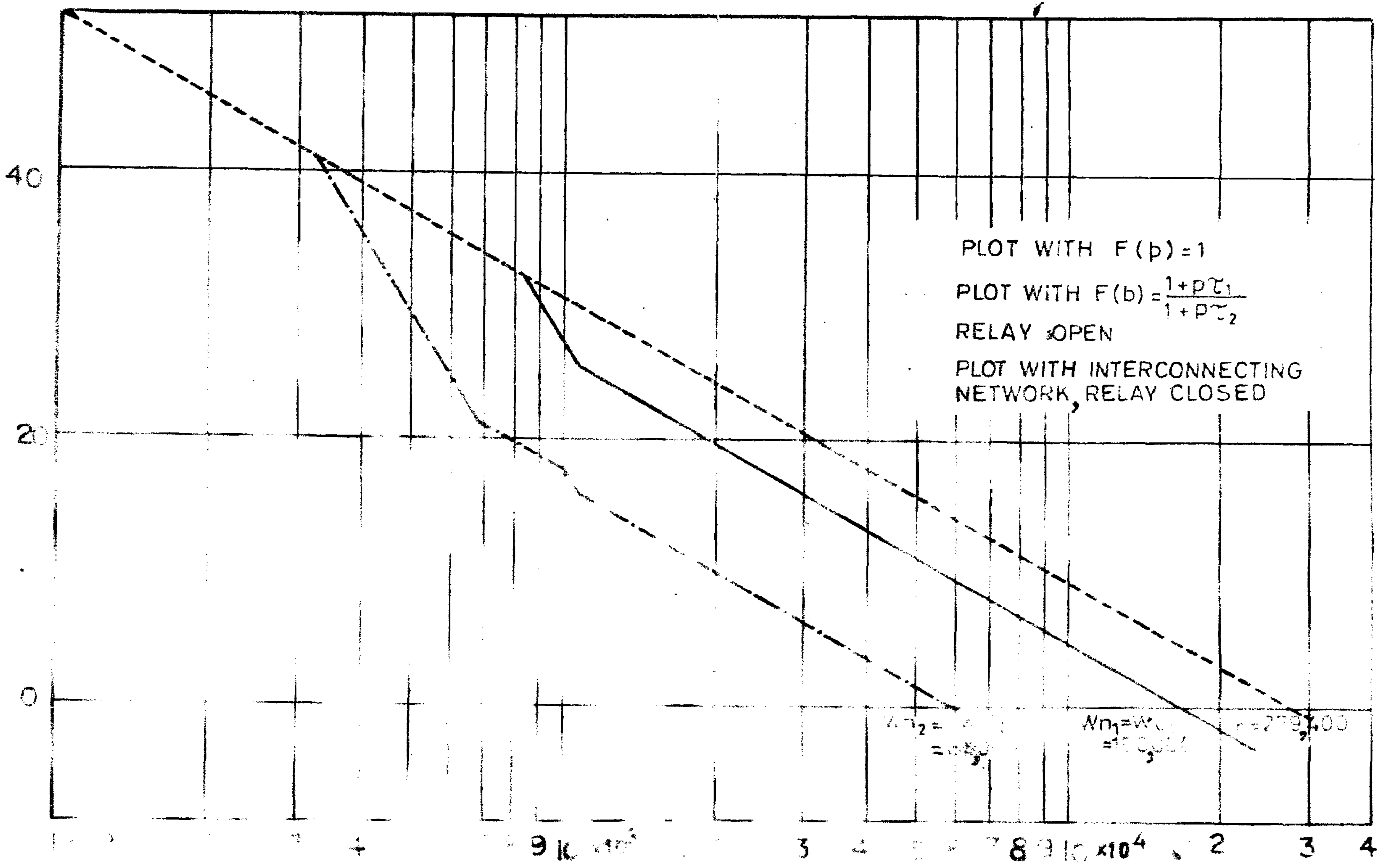


FIG.14. NOISE BANDWITH MEASUREMENT AT $2^V$ REFERENCE INPUT.

40
20
0
PLOT WITH F(p)=1
PLOT WITH F(b)=$\frac{1+p\tau_1}{1+p\tau_2}$
RELAY OPEN
PLOT WITH INTERCONNECTING NETWORK, RELAY CLOSED
$Wn_2$=
$Wn_1$=
4
9 10 ×10³
3
4
8 9 10 ×10⁴
2
3
4

F [illegible] LC [illegible] OR [illegible] SY [illegible]

## 8.4 Capture Range when the Relay is closed:

The capture range is uneffected by the closing of the relay contact. However, the relay contact is only closed on some point of the capture range. The zone of action of the relay is different when the frequency is raised from a lower initial value than when the frequency is decreased from a high initial value. This is shown in fig. no.10. The unequality in the relay zone of action is due to the unequal pull-in and the pull-out torque.

## 8.5 Noise Bandwidth:

Noise bandwidth as from graph nos. 12,13,14 rises as the amplitude of the interferance signal is increased. In case when F(p) = 1 the increase of noise bandwidth with the higher magnitude of interferance signal is due to its dependence on $K_1$ which as shown in appendix 1 increases as the magnitude of the noise input signal is increased. In case when a double time constant network is employed the affect is indirect. From the Bode plot no. 15 it is seen as K increased the 1/S line shifts its position. As $\tau_1$ and $\tau_2$ are constant for the network the value of the frequency where the plot intersects the 0 db. axis varies with $K_1K_2$. The resultant affect is to increase noise bandwidth when the input noise voltage is increased. The measured value of noise bandwidth agrees closely with these obtained from the Bode plot for all the three cases (a) when F(p) =1, (b) $F(p) = 1 + p\tau_1/1 + \tau_2 p$ without relay (c) $F(p) = 1 + \tau_1 p/1 + \tau_2 p$ with relay. (For details of calculation see appendix no. 5)

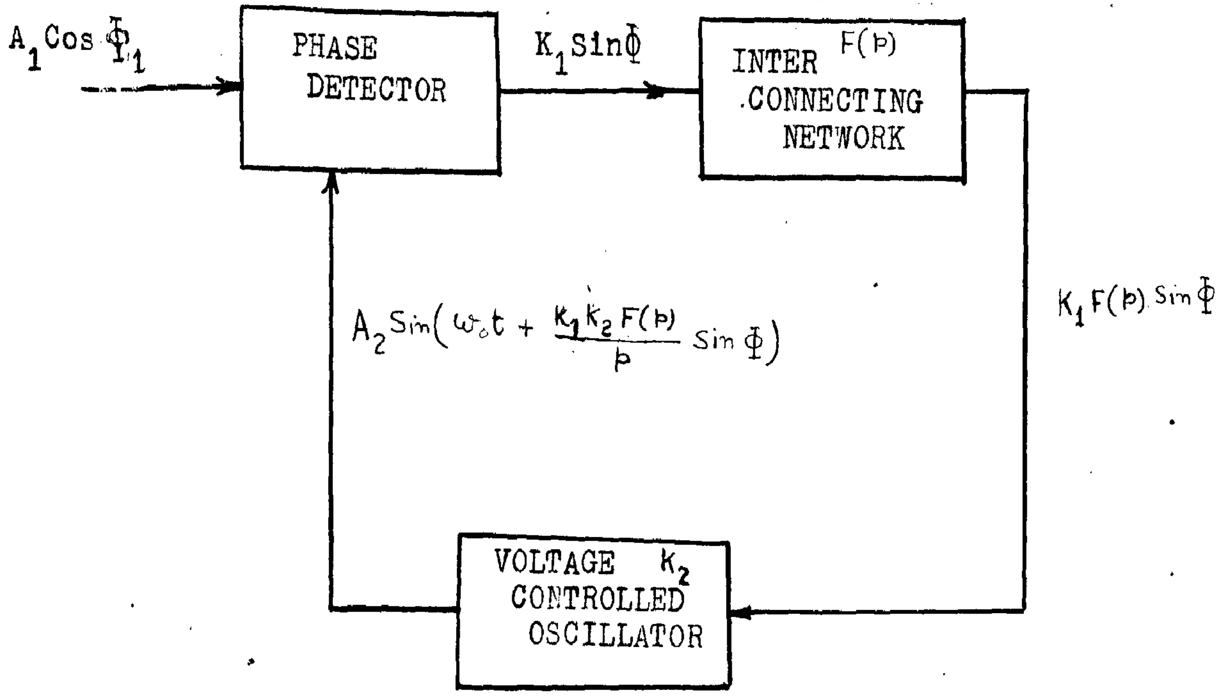


FIG. - 16 BLOCK DIAGRAM OF THE APC SYSTEM

# APPENDIX 1.

## Differential equation of the APC system:

Fig. 16 gives the configuration of the APC system In the loop the output of the phase detector $K_1 \mathrm{Sin}\Phi$ after passing through the interconnecting network is applied to the reactance tube oscillator combination which changes the free running frequency of the oscillator by the ammount (7) $K_1K_2F(p)\mathrm{Sin}\Phi$. The free running output of the V.C.O in this case is assumed to be in quadrature with the reference signal. Equating the output of the phase detector with the sine of the difference phase we get:

$$K_1 \mathrm{Sin}\Phi = \tfrac{1}{2}A_1A_2 \mathrm{Sin}\left[\Phi_1 - \omega_o t - K_1K_2F(p)/p\ \mathrm{Sin}\Phi\right]$$

$$K_1 = A_1A_2/2$$

$$\text{and } \Phi = \Phi_1 - \omega_o t - K_1K_2F(p)/p\ \mathrm{Sin}\Phi$$

$$\text{or } p\Phi = p\Phi_1 - p\omega_o t - K_1K_2F(p)\ \mathrm{Sin}\Phi$$

$$\text{or } \dot{\Phi} = \dot{\Phi}_1 - \omega_o - K_1K_2F(p)\ \mathrm{Sin}\Phi$$

$$\text{or } \dot{\Phi} + KF(p)\mathrm{Sin}\Phi = \Phi_1 - \omega_o = \Omega$$

at steady state $\dot{\Phi} = 0$. Therefore, the solution of the above equation under steady state conditions is:

$$KF(p)\mathrm{Sin}\Phi = \Omega$$

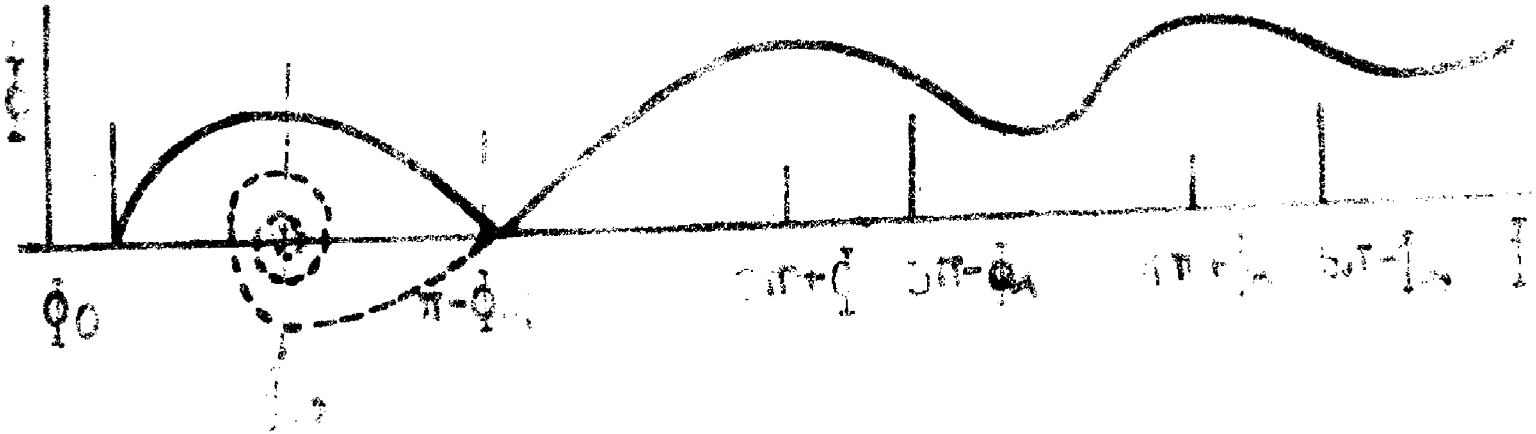


FIG. [illegible] - PHASE PORTRAIT OF THE [illegible] SYSTEM

[illegible]

## APPENDIX 2.

Pull-out range of the APC system:

The phase potrait when drawn from the differential equation of the APC system will appear as shwon in fig. no. 16. In the phase trajectory $\Phi_\infty$ is the stable point while $\pi - \Phi_\infty$ (2) is the unstable equilibrium point. Now if the perturbation be such that $\pi - \Phi_\infty$ point is reached the value of $\dot{\Phi}$ will increase and the system will unlock.

## APPENDIX 3.

Transient and the frequency response ofthe APC system can be ascertained from the fundamental differential equation of the loop which is:

$$\dot{\Phi} + KF(p)\text{Sin}\Phi = \Omega = p\Phi_1 - \omega_0 \tag{4}$$

The above equation can further be simplified if the origin of the phase angle measurements is shifted to $\omega_0$ the free running oscillator frequency. Then the above equation reduces to:

$$p\Phi + KF(p)\text{Sin}\Phi = p\Phi_1$$

expressing the above equation in terms of $\Phi_1$ and $\Phi_2$ and linearizing ( i.e. $\text{Sin}\Phi = \Phi$ ) we get:

$$p\Phi_2 + KF(p)\Phi_2 = KF(p)\Phi_1$$

Case I

When no interconnecting network is used inbetween the phase discriminator and the V.C.O circuit i.e. when F(p) = 1 we have:

$$p\Phi_2 + K\Phi_2 = K\Phi_1$$

$$\text{or} \quad \Phi_2/\Phi_1(j\omega) = 1/1 + j\frac{\omega}{K}$$

Therefore, the APC loop once synchronised behaves like a low pass filter filter as shown in fig. no. 17.

The transient response for a step input i.e. $O_1(t) = U(t)$ is

$$\Phi_2(p) = K/p(p + +K)$$

$$\text{or} \quad \Phi_2(t) = U(t) - e^{-Kt} \qquad (4)$$

when the initial detuning $\Phi_2(t)$ is zero.

Therefore, if the cut off frequency of the system is reduced the phase shift at the frequency at which the ooop gain is unity gets closer to 180° and the system transient response to a step input develops overshoot.

Case II

When the interconnecting network is proportional plus integral type:

$$F(p) = 1 + p\tau_1/1 + \tau_2 p$$

As the fundamental differential equation of the APC sytem is :

$$\dot{\Phi} + KF(p)\mathrm{Sin}\Phi = \Omega$$

substituting the value of F(p) in the above equation we get:

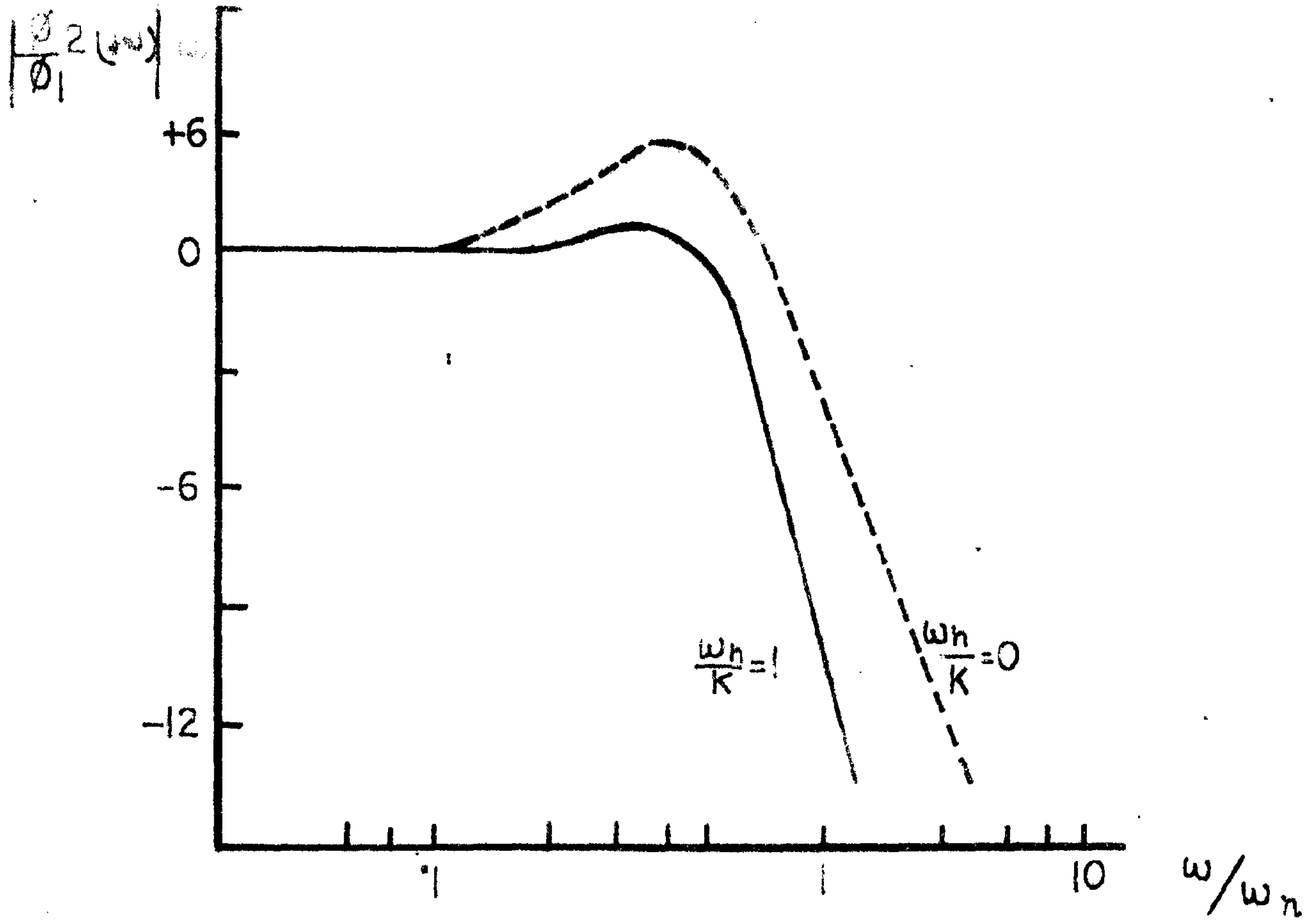


FIG. 1[illegible] FREQUENCY RESPONSE FOR $\xi = 0{\cdot}5$

(SKETCH)

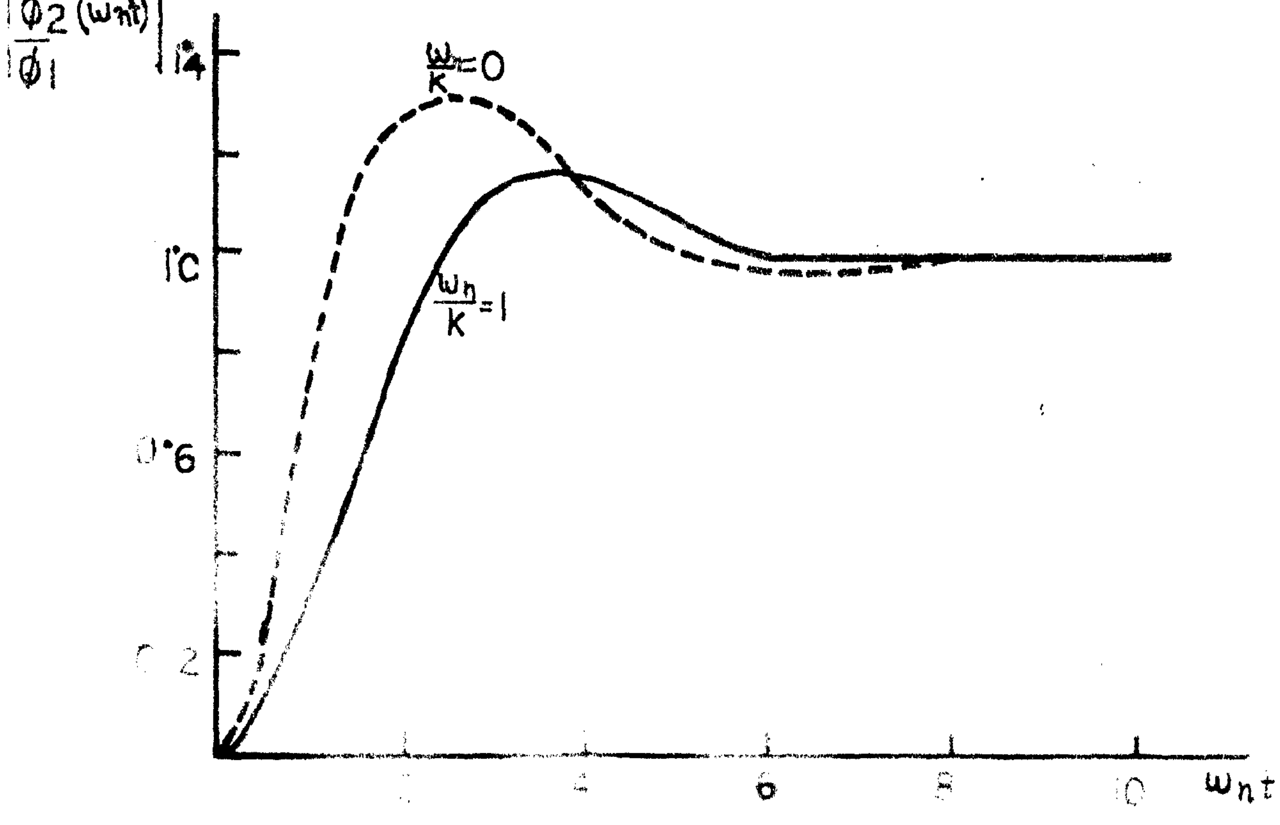


FIG. 9 TRANSIENT RESPONSE FOR $\xi = 0{\cdot}5$

(SKETCH)

$$p\Phi_2 + K\frac{1+\tau_1 p}{1+\tau_2 p}\Phi_2 = K\frac{1+\tau_1 p}{1+\tau_2 p}\Phi_1$$

or $$p^2\Phi_2 + \Phi_2 p(1/\tau_2 + K\frac{\tau_1}{\tau_2}) + K/\tau_2\Phi_2 = K/\tau_2\Phi_1 + K\frac{\tau_1}{\tau_2}p\Phi_1$$

to convert the above equation in the conventional 2nd order form let us assume:

$$2\xi\omega_n = 1/\tau_2 + K\tau_1/\tau_2$$

and $\omega_n^2 = K/\tau_2$, the natural frequency of the system when damping is equal to zero.

The above equation reduces to

$$p^2\Phi_2 + 2\xi\omega_n p\Phi_2 + \omega_n^2\Phi_2 = \Phi_1\omega_n^2 + (2\xi\omega_n - \omega_n^2/K)p\Phi_1$$

Therefore, the frequency response becomes equal to

$$\frac{\Phi_2}{\Phi_1}(j\omega) = \frac{1 + 2j\xi\frac{\omega}{\omega_n}\left(1 - \frac{\omega_n}{2\xi K}\right)}{1 + 2j\xi\frac{\omega}{\omega_n} - \left(\frac{\omega}{\omega_n}\right)^2}$$

As the time constant of the network can not be negative or at best it can only be equal to zero in which case the proportional plus integral control network is reduced to simple low pass filter. Thus when $\tau_1 = 0$

$$2\xi\omega_n - \omega_n^2/K = \tau_1 = 0 \qquad \text{or} \qquad \omega_n/K = 2\xi$$

This is also the maximum value of $\omega_n/K$ as for all positive values of $\tau_1$, $\omega_n/K$ will be less than $2\xi$. The minimum value of $\omega_n/K = 0$. This happens when for fixed value of $\omega_n$, K is made to approach infinity. Thus for these limiting value of $\omega_n/K$ and $\xi = 0.5$ the frequency response curve for the interconnecting network is as shown in fig. no. 18.

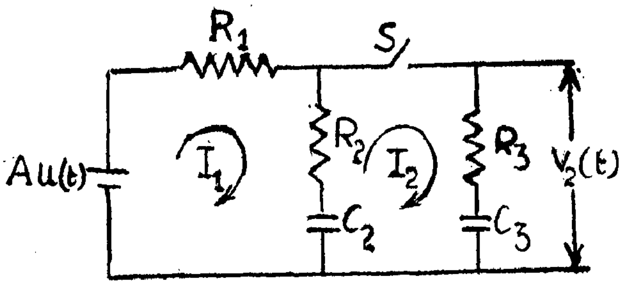
The transient response for a step input will be:

$$\Phi_2/\Phi_1(t) = 1 - e^{-\xi\omega_n t}\left[\cos\sqrt{1-\xi^2}\,\omega_n t - \frac{\xi - \frac{\omega_n}{K}}{\sqrt{1-\xi^2}}\sin\sqrt{1-\xi^2}\,\omega_n t\right]$$

Transient response for $\xi = 0.5$ and for two limiting values of $\omega_n/K$ is as shown in fig. no. 19.

### A P P E N D I X 4.

<u>Bang Bang Effect on the APC system:</u>



The equation of the above network in the matrix form is:

$$\begin{bmatrix} R_1 + R_2 + 1/C_2 s & -R_2 - 1/C_2 s \\ -R_2 - 1/C_2 s & R_2 + R_3 + 1/C_2 s + 1/C_3 s \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} \begin{bmatrix} 0 \\ U(t)/s \end{bmatrix}$$

solving the above we have:

$$V_2(s) = U(t)A/s\,(R_3 + 1/C_3 s)\,\frac{\Delta_{22}}{\Delta}$$

Where $\Delta = (R_3 + 1/C_3 s)(R_1+R_2+1/C_2 s) + (R_2 + 1/C_2 s)R_1$

$$\Delta_{22} = R_1 + R_2 + 1/C_2 s$$

$$V_2(s) = A/s\,\frac{(R_1 + R_2 + 1/C_2 s)(R_3 + 1/C_3 s)}{(R_3 + 1/C_2 s)(R_1+R_2+1/C_2 s) + R_1(R_2+1/C_2 s}$$

# APPENDIX 5.

Calculations for nois bandwidth:

The measured value of lock range at 5.2 volts = 171.5 Kc/s.

The measured value of lock range at 3.0 volts = 113.2 Kc/s.

Thus the value of lock range at 4.0 volts = 139.7 Kc/s.

$$\text{Therefore, } K_1K_2 = 2\pi \times 139.7 \times 10^{3} \text{ radians/sec.}$$

$$K_1K_2 = 279,4000 \text{ radians/sec.}$$

The two time constants of the network are:

$$47\times10^{3}\times0.002\times10^{-6} = 94\times10^{-6}$$

$$62\times10^{3}\times0.002\times10^{-} = 124\times10^{-6}$$

Thus the lower and the higher cut-off frequencies are:

10,620 radians/sec. and 8060 radians/sec.

The cut-off frequency obtained from the graph is 155,000 radians/sec.

Therefore, the theoritical value of the noise bandwidth is

$$\frac{150,000\times2\pi}{2\pi}$$

$$= 150 \text{ Kc/s.}$$

The measured value of noise bandwidth when the reference signal is 4.0 volts is 129,00 Kc/s.

Calculations when the extra network has been incorporated in the circuit:

The transfer function of the network is:

$$F(p) \quad \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{s^2\left[RC_1C_2(R_1 + R_2) + \tau_1\tau_2\right] + s\left[R(C_1 + C_2) + \tau_1 + \tau_2\right] + 1}$$

Therefore, the corner frequencies are:

10,600 6,667 3,1000 9,900

The cut-off frequency from the Boda plot is 60,000 radians/sec.

Therefore, the noise bandwidth is equal to 60.00 Kc/s.

The measured value of noise bandwidth is 59.6 Kc/s.

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