MATHEMATICAL MODELLING OF SPRINGFLOW

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By

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Gratis

CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled **"Mathematical Modelling of Springflow"** in fulfilment of the requirement for the award of the degree of Doctor of Philosophy and submitted in the Department of Earth Sciences of the University of Roorkee is an authentic record of my own work carried out during the period March, 1989 to Jan., 1996 under the supervision of Dr. G.C. Mishra, Dr. B.B.S. Singhal and Dr. P.K. Gupta.

The matter presented in this thesis has not been submitted by me for the award of any other Degree of this or any other University.

A. K. Bhar

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This is to the certify that the above statement made by the candidate is correct to the best of our knowledge.

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SPRINGFLOW COULD BE THE LIFELINE

ABSTRACT

A spring is a natural outlet for concentrated discharge of groundwater either on land surface or into a body of surface water. Springs have been used as dependable and ready source of water in many parts of the world. Springs could be of various sizes from small trickles to large streams under both water table and artesian conditions. An active spring can be treated as a flowing well with constant head. This feature could be used conveniently in the mathematical modelling of springflow. In the analysis of regional groundwater flow, a spring can serve as a boundary condition of Dirichlet type.

The physical process of release of spring water from groundwater storage can be compared with the lower portion of the recession part of a flood hydrograph in a river and can be simulated by a linear reservoir. A linear reservoir is a conceptual reservoir in which outflow is linearly proportional to the storage. Combination of this postulation with continuity equation gives the equation for base flow,

$$Q_t = Q_0 k_r^t$$

where k_r is the recession constant or depletion factor and is equal to $\exp(-1/\tau_0)$. τ_0 is a parameter of the spring and is designated as the depletion time.

Bear (1979) suggested a simple mathematical model to simulate the unsteady flow of a spring over the recession period for a lumped recharge. Bear model assumes a linear relationship between springflow and storage. A springflow model has been developed using Bear's model and the convolution technique for simulating springflow for a known time variant recharge and aquifer parameters. However, the time variant recharge is not known. The Newton-Raphson iterative method for solving non-linear equation has been used to compute the time variant recharge, and the model parameter, i.e., depletion time (τ_0) from the springflow. The model has been tested on three springs. The springs are: (i) Sulkovy Pramney springs, Czechoslovakia emerging from sandstone strata (a third magnitude spring) (ii) Kirkgoz spring, Turkey emerging from Karstic aquifer (a first magnitude spring) and (iii) White Rock spring, Nevada from perched waters in volcanics tuffs (a eighth magnitude spring). For the Kirkgoz spring, Turkey, the added up monthly recharge for each year matched with the annual recharge for 6 years computed by an earlier investigator using Bear's model.

In Bear's model, the logarithm plot of springflow with time during the period of recession follows a straight line. It is found that during the process of recession, the variation of logarithm of flow with time follows a straight line, provided the springflow domain is a closed one. A closed flow domain implies that all the recharge will appear as springflow.

The Bear model assumes that an unsteady state is the succession of the steady state conditions and there is no time lag between onset of recharge and emergence of springflow at the spring's threshold. But, in case, the transmission zone of the spring in the flow domain is long and the hydraulic diffusivity is low, there would be a time lag between the onset of recharge and its appearance as springflow at the spring's threshold due to the storage and translation effect in the transmission zone. In order to simulate springflow for such a geohydrological system, a mathematical model has been developed considering an unsteady state for simulating springflow for a known time variant aquifer recharge. Starting from the basic solution given by Carslaw and Jaegar for flow in an aquifer of finite length and using convolution technique, the unit pulse response function coefficients for outflow due to unit recharge in the recharge zone has been obtained. Using the unit pulse response function coefficient and convolution technique, springflow has been computed for the time varying recharge. The storativity of the transmission zone reduces the magnitude of peak springflow and it causes delay in the appearance of peak springflow. When storage in the transmission zone is small, the springflows simulated by the two models compare well.

With an initial guess of the range of values of the model parameters i.e.,ϕ_1 (specific yield), ϕ (storage coefficient), T (Transmissivity), LW_R/W_S (a linear dimension representing recharge area and spring width), l (length of transmission zone), the time variant recharge and model parameters are computed by random search technique. The recharge computed by the random jump technique compares well with those obtained by the Newton-Raphson technique.

The Bear's model and the model with long transmission zone deal with one dimensional flow. However, the flow processes associated with springflow will be two dimensional. Therefore, using basic solution given by Hantush for the evolution of piezometric surface due to recharge from a rectangular basin, a two dimensional springflow model has been developed. The response of the spring for unit pulse recharge through the rectangular recharge zone of the spring has been obtained. Using these unit response function coefficients, springflow for any time variant recharge can be computed.

The variation of logarithm of springflow with time during recession, does not follow a straight line. Only towards the latter part of recession, the variation is approximately linear.

Using the random jump technique and the springflow model for an open flow domain, recharge area, spring opening, distance of the spring from the recharge area, transmissivity and storativity of the transmission zone and the recharge have been estimated from observed springflow data. Since the domain is an open one, the recharge computed by the model which is based on Hantush's solution, is found higher than those computed using the model for a closed system.

NOTATIONS

The following notations have been used in this thesis. In chapter 2, which deals with review of literature, attempts have been made to retain the original notations and are explained therein. The notations used in the other chapters are described herein.

Notations	Description	Dimension
A	Area of the recharge zone	L^2
Δh	Fall in groundwater in the recharge zone	L
Δt	Time interval, sampling period	T
$\delta(\Delta t,n)$	Discrete kernel coefficient for springflow	T^{-1}
$\delta_0(\Delta t,N)$	Discrete kernel coefficient for inflow to the transmission zone per unit width of the transmission zone	L
$\delta_v(\Delta t,N)$	Discrete kernel coefficient for volume of inflow into the transmission zone per unit width of the transmission zone	LT
δ_D	Dirac delta function	--
$\delta_1(\Delta t,N)$	Discrete kernel coefficient for springflow per unit width of spring	L
$\delta(\Delta t,N)$	Discrete kernel coefficient for springflow	T^{-1}
$\delta(2L,W_R;X_1,X_2;x_1,x_2;m)$	Discrete kernel coefficient for rise in piezometric surface	--
ϕ_1	Specific yield	--

ϕ	Storage coefficient of the transmission zone	--
$h(t)$	Water table height in the recharge zone	L
h_2	Level of spring's threshold, initial level of groundwater table in the recharge zone (in Chapter-3); initial level of the piezometric surface (in Chapter-4)	L
$h_3(0)$	Water table height in the recharge zone at t=0 due to unit impulse recharge per unit area	L
h_4	Constant boundary head in the recharge zone	L
$k(t)$	Flow of a spring due to unit impulse recharge through entire recharge area (in Chapter-3);	T^{-1}
$k(N\Delta t)$	Unit impulse response coefficient for springflow (in Chapter-4)	T^{-1}
$K(t)$	Unit step response coefficient for flow (in Chapter-3)	--
$K(t)$	Unit step response coefficient for spring outflow (in Chapter-4)	--
l	Length of the transmission zone	L
N	Impulse recharge per unit area	L
$Q(t)$	Discharge of the spring at time t	L^3/T
$Q_B(t)$	Component of springflow due to perturbation prior to time origin	L^3/T
$Q(0)$	Spring discharge at time origin	L^3/T
$Q_1(t)$	Simulated springflow	L^3/T
$Q_0(t)$	Flow per unit width	L^2/T

$q(n\Delta t)$	Springflow rate at time $n\Delta t$	L^3/T
$R_u(\gamma)$	Pulse recharge per unit area	L
$r(t)$	Time varying recharge rate through entire recharge area	L^3/T
R	Total impulse recharge	L^3
$R(\gamma)$	Pulse recharge during γth time step	L^3
s	Rise in piezometric surface	L
T	Transmissivity	L^2/T
τ_0	Depletion time	T
τ	Dummy variable	T
$V_V(N)$	Cumulative volume of inflow to the transmission zone upto Nth time step per unit length of transmission zone	L^2
$V_C(t)$	Volume of water that enters to the transmission zone per unit width upto time t for constant head in the recharge zone	L^2
$V_V(t)$	Volume of water that enters to the transmission zone per unit width for variable head in the recharge zone	L^2
W_R	Width of the recharge zone	L
W_S	Width of the spring	L
W_1, W_2, W_3	Weightage factors for numerical integration	--
z_1	Height of the spring's threshold above datum	L

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A.K. Bhar

CONTENTS

CHAPTER-1

INTRODUCTION

A spring is a natural outlet for concentrated discharge of groundwater either on land surface or into a body of surface water. Springs have been used as dependable and ready source of water and had supported many ancient civilisations. In the Roman empire, water was supplied from springs through elaborate aqueducts and even presently the major source of water supply of Rome is from two karst springs viz., Peschiera and Capore having a total discharge of about 15 m^3s^{-1} (Ozis,1987). In the eastern Mediterranean area, spring water was often diverted to run small water turbines before its use for irrigation and domestic purposes. In 800 B.C., the water from a powerful spring (about 2-3 m^3s^{-1} capacity) was conveyed by a 56 km. long canal and was supplied to a town of eastern Anatolia in Turkey (Garbrech,1987). Many towns and villages in Saudi Arabia depended on spring water for major portion of their drinking and agricultural needs till sometime back (Bazuhair and Sen,1994). Great rivers like Cauvery in the southern part of India and Jhelum in the northern part of India originate from springs and are shrouded with many religious and mythological folklores which are popular even today. Springs could be of various sizes from small trickles to large streams and emerge under both water table and artesian conditions. Springs in carbonate rocks have high discharge. The largest karst spring, viz., Krasnyi Klyuch in the western Ural area of Russia can meet the total water requirement of Moscow city in springtime when the snow melts. The springtime discharge of this spring varies between 30 to 52 $m^3 s^{-1}$ (Klimentov, 1983).

In India, springs are used as source of water supply in various regions, such as in the Himalayas, in the Western Ghats, in the Khasi-Jaintia hills in the north east, in the Vindhyan formation in central India and in many other places where it is usually logistically difficult to create storage for water. Exploitation of forested area for food, fibre, minerals and urbanisation lead to deforestation and changes in the watershed characteristics. This human interference in turn reduces infiltration and increases runoff. As a consequence the springflow diminishes which may lead to drying of the springs (Valdiya 1987; Bahuguna, 1990; and Dewan 1990). Acharya (1986) reported the rejuvenation of the hilly springs in Palampur, Himachal Pradesh due to the restoration of the vegetative cover on the soil. Study of springflow in a hilly forested area could serve as a diagnostic tool for watershed management. As such the study of springflow has relevance to rural water supply especially in the hilly region.

The fact that springs flow freely with no obvious source had made them to appear mysterious. Prior to the later part of the seventeenth century, it was generally believed that the spring water could not be derived from the rain. Greek mathematician Thales (7th century B.C.) stated that springs draw water from ocean. He believed that sea water is driven into the rocks by winds and is elevated to the mountains by the pressure of the rocks. Plato (427-347 B.C.) also stated in his discourse "Phaedon" that water that forms the springs comes from vast caverns known as Tartarus and all these waters return through various routes to Tartarus. Aristotle (384-322 B.C.) developed a theory of subterranean condensation drawing the parallel from the condensation of atmospheric water vapour (Kulandaiswamy,1994). The Roman architect Marcus Vitrivius (15 B.C. to 58 A.D.) realised that the mountains receive large amounts of water from melting snow that seeps through the rock strata and emerges as springs at lower elevations(Walton,1970). Varahamihira, the famous Indian mathematician (6th century A.D.), in his celebrated Sanskrit work "Brhatsmhita" suggested a scientifically based system of exploring underground springs on the basis of naturally occurring specific signatures in the flora, fauna, rocks, soil and minerals of the terrain (Rao,1971). It is of interest to note that verse 21 of the thirty ninth chapter of the Quran (recorded between A.D. 611 and 622) is very explicit in stating that rainwater infiltrates into the ground and appears as springs (Kashef, 1955 vide Kashef, 1986).

The first hydrologic study of spring was conducted by French scientist Pierre Perrault during the middle of seventeenth century. After studying part of the Seine river basin during 1668 to 1670, he invalidated an age-old assumption that precipitation does not support springflow. Because of his study and those conducted by others afterwards, it was established that the springs are natural outlets of concentrated groundwater flow on the surface and the ultimate source of springwater is precipitation (Tarbuck and Lutgens, 1990).

An active spring can be treated as a flowing well with constant head. This feature could be used in the mathematical modelling of springflow. In the analysis of regional groundwater flow,the spring can serve as a boundary condition of Dirichlet type.

Study of springs help in the evaluation of groundwater potential of an area. The location and magnitude of springs give a good indication about the presence and possible potential of the aquifer in a region. Emergence of large springs confined to valley bottoms indicates the existence of aquifers of high permeability and greater depth to watertable whereas abundant small springs on valley sides and slopes of a hill are indicative of a shallow watertable with a shallow circulation of subsurface water in aquifers of poor permeability (Davis and Dewiest, 1966). A few large springs may indicate thick, transmissive aquifers, whereas frequent small springs indicate thin aquifer of low transmissivity (Bouwer,1978). Large flow from springs is usually associated with aquifers with high permeability like cavernous limestone, porous basalt and well sorted gravel. Variability of springflow is usually low in volcanic and sandstone formations (Balek,1989). He also related the occurrence of stable and perennial springs with extensive aquifers. Presence of several horizons of springs along the valley slope is due to stratification controlled by lithology or structure (Karanth,1989). Conditions necessary to produce springs are many and are related to different conditions of geologic, hydrologic, hydraulic, pedologic, climatic and even biological controls.

The process of release of spring water from groundwater storage can be compared with the lower portion of the recession part of a flood hydrograph in a river and can be simulated by a linear reservoir. A linear reservoir is a conceptual reservoir in which outflow is linearly proportional to the storage. Combination of this postulation with continuity equation gives the equation for base flow

$$Q_t = Q_0 k_r^t$$

where k_r is the recession constant or depletion factor (Chow,1964) and is equal to $\exp(-1/\tau_0)$. τ_0 is a parameter of the spring and is designated as the depletion time. Singh (1989) reported wide use of this equation for analysing base flow recession. Suggested value of k_r is more than 0.9 but is less than unity.

Based on the above mentioned theoretical background, a few conceptual linear one dimensional mathematical models have been developed by various investigators for the modelling of springflow and evaluation of the dynamic storage inside the spring flow domain (Mero,1963; Bear,1979; Mandel and Shiftan,1981). These models assume that the springflow is linearly proportional to the dynamic storage in the springflow domain and an exponential form of springflow which is same as that used for baseflow has been derived as

$$Q(t+\Delta t)=Q(t)\exp(-\Delta t/\tau_0)$$

where, $Q(t)$=springflow at time t during recession, and Δt is the time increment.

Bear's model (1979) is one such conceptual model to analyse unsteady flow of a spring during recession period. The spring flow domain has been hydrologically decomposed in two parts;-i)a recharge zone, and ii) a transmission zone. The model is based on the assumption that the springflow is linearly proportional to the dynamic storage in the spring flow domain. The flow pattern is two dimensional in the vertical plane. Dupuit- Forchheimer assumptions are assumed to be valid in the spring's transmission zone and the flow in transmission zone, as such, is in horizontal direction. The model simulates the unsteady state flow by assuming that an unsteady state is succession of steady state conditions. He introduced a coefficient in the solution which takes care of the aquifer characteristics and physical dimensions of flow domain and the spring. Bear's model and the other existing springflow models do not account for the time variant recharge and assume one time recharge in the beginning. Therefore, Bear's model should be suitably adapted to simulate springflow for a time variant recharge.

Flow from a spring and the various boundary conditions that may exist in the springflow domain have not been related. For a spring having a long transmission zone with low hydraulic diffusivity, there will be a time lag between the onset of recharge and its appearance as springflow at the spring's outlet due to storage and translation effect in the transmission zone. Springs with such configuration and geohydrological conditions require a mathematical model which considers an unsteady state flow in the flow domain of the spring to simulate springflow for a time variant recharge.

Further, the groundwater flow processes associated with springflow are two dimensional. As such, a two dimensional springflow model is required to be developed. Such model will be of use to simulate springflow for a spring or a group of springs emerging from hill slopes and valley bottom.

In the existing springflow model, the dynamic storage of groundwater at any time during recession is assumed to be equal to the product of depletion time and discharge of the spring at that time. It is yet to be verified that spring discharge from an aquifer conforming to a linear system would follow strictly an exponential decay curve. Thus, there is a need to establish the true relationship between springflow and dynamic storage which has been assumed to be linear in all the existing models.

The raison d'etre of this study is to address the gaps in springflow modelling to the possible extent within the scope of groundwater modelling. The method of presentation in the thesis is as follows:

Chapter 2 gives the literature survey pertaining to three important aspects of springflow studies, viz.,occurrence of the springs in various geologic formations and their classification; mathematical modelling of springflow, and statistical analyses of springflow data.

Chapter 3 describes a springflow model which has been developed using Bear's model and the Duhamel's approach. The model provides an expression for springflow for variable recharge. A procedure has also been presented to compute the time variant recharge and depletion time from the given springflow as the inverse problem with the help of Newton-Raphson method for solving the nonlinear equations. The procedure has been applied to three springs. The springs are: (i) Sulkovy Pramney spring in Czechoslovakia which emanates from sandstone aquifer (a third magnitude spring), (ii) Kirkgoz spring, Turkey which emerges from karstic rocks (a first magnitude spring), and (iii) White Rock spring, southern Nevada - a relatively much smaller spring emerging from volcanic tuffs.

Chapter 4 describes a model which is based on solution of Boussinesq's equation given by Carslaw and Jaegar for flow in an aquifer of finite length and addresses to the requirement of an unsteady state flow in a springflow domain conforming to a geohydrological system wherein the transmission zone of the spring is long and is of low hydraulic diffusivity. With an initial guess of the range of values of the model parameters, the time variant recharge and model parameters are computed by random search technique.

Chapter 5 deals with a springflow model that has been developed by considering groundwater flow processes pertaining to a spring as two dimensional. Hantush's basic solution for the evolution of piezometric surface due to recharge from a rectangular basin and convolution technique have been used in the model. The expression for the springflow has been given in terms of unit pulse response function coefficients. Using these coefficients, the springflow for any time variant recharge can be computed. It has been observed that during the recession period, the springflow is not linearly proportional to the dynamic storage in the spring. The inverse problem has been solved using random search technique, and recharge and model parameters have been determined.

The general conclusions based on the study are presented in Chapter 6.

CHAPTER-2

REVIEW OF LITERATURE

2.0 GENERAL

The literature reviewed has been arranged under three main aspects of springflow studies; - (i) hydrogeology of springs depicting the variations in their occurrence and their classifications, (ii) hydrological modelling of springflow and movement of water in the springflow domain, and (iii) statistical analysis of springflow data.

2.1 HYDROGEOLOGY OF SPRINGS

Occurrence of springs is mostly controlled by the local geology, geomorphology and the drainage characteristics. Large springs are usually associated with highly permeable aquifers like cavernous limestone, porous basalt, and well sorted gravel, whereas small springs may occur in all types of rock. Bouwer (1978) provided the order of magnitudes of spring discharge from various geological formations as given in Table 2.1.

Further, depending upon the various hydrogeological controls and geomorphological conditions, the springs could occur in different configurations. They could occur either as a single spring or as a group of springs. A group of springs could be

Table 2.1 Expected springflow from various geological formations (Bouwer,1978)

Sl.No.	Geological formation	Expected springflow (m^3 day^{-1})
1.	Sorted or coarse sand gravels, porous basalts	1000-20,000
2.	Cavernous limestone	500-5000
3.	Sand and gravel mixture, sandstones	100-2000
4.	Fractured and weathered rocks	10-500

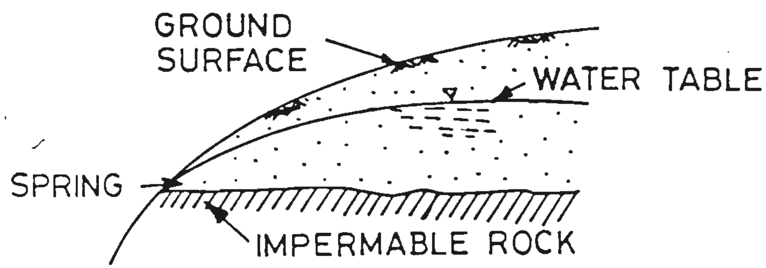
either along a line at the same elevation or at various elevations. There are a variety of geological controls which give rise to different types of springs with respect to their occurrences, locations, magnitudes, and configurations. Lithology, structure, and topography are the main and most important geological controls that influence the occurrence of springs as discussed below.

2.1.1 Lithological Control

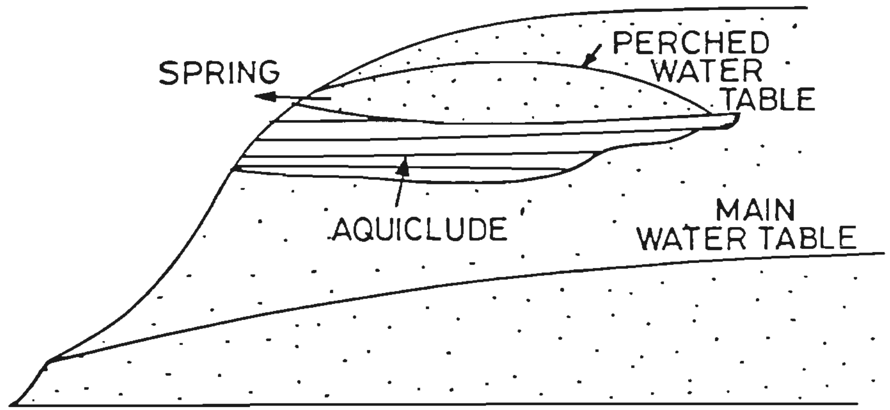
Springs occur where downgradient parts of aquifer or other water bearing materials with their lower boundary are exposed to the surface like at hill sides, canyons or dissection due to erosional channel (Fig.2.1a). It is common to find line of springs where permeable sandstone or limestones form high ground and rest on less permeable rocks such as shales or clay (Price, 1985). Davis and Dewiest (1966) stated that such vertical variations of permeability associated with layered sedimentary rocks are the cause of larger and permanent springs.

Blocking of downward movement of groundwater by an aquiclude forces water to move laterally and a line of springs results at the outcrop of the permeable layer in a valley (Fig.2.1b). Outcrop of bedrock or nearness of bedrock to surface is the controlling factor. If this aquiclude is situated above the main water table, a portion of the percolated water is intercepted by it and thereby creates a localised zone of saturation - a perched water table. This could give rise to springs if the aquiclude outcrops in the valley (Fig.2.1c).

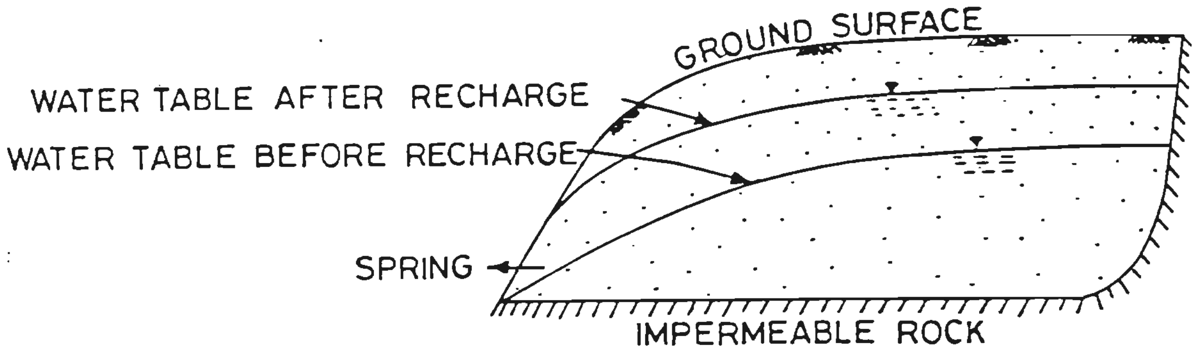
Springs are also developed as a result of lateral variation in the geological material and ground slope as manifested in the upper parts of the alluvial fans. The upper portion of the alluvial fan consisting of coarsest grades of deposits has steep slope, high permeability, porosity, and high water table gradient. At the downslope, the aquifer is fine grained and laterally merges into clay and silt formations, the latter forming confining beds. Therefore, a part of the groundwater which is recharged in the upper part of the fan is discharged on the ground surface where the water table intercepts the land surface. The upper limit of



(a) GRAVITY SPRING (UNESCO, 1972)



(b) PERCHED SPRING (TARBUCK & LUTGENS,1990)



(c) OVER FLOWING SPRING (PINNEKER ,1980)

FIG.2·1–TYPES OF SPRING FROM VARIOUS LITHOLOGICAL CONFIGURATIONS.

this zone of rejected recharge of groundwater is marked by a series of springs along the topographic contour as spring line (Karanth, 1989). Intersection of the water table with the land surface is the controlling feature in the location of those springs (Fig.2.2). As the alluvial material is very thick, the deep underlying bed rock does not have any control over the flow of these springs. Such situation occurs at the foot of the Siwalik hills in Nainital district, India at the boundary between Bhabar and Tarai. The northern boundary of Bhabar belt is in contact with the Siwalik hill range and southern limit is the spring line which defines the northern limit of the Tarai sediments (Pandey, Rao and Raju, 1968). Discharge of such springs is usually small. Similarly, springs occur when there is a compositional change in the alluvial deposits. If an alluvial sand grades into a sandy clayey sequence, the groundwater flow retards. As a result groundwater level rises and springs occur at topographical low (Klimentov, 1983).

Large sub-marine springs often discharge groundwater through opening to the sea in coastal areas that contain limestone and volcanic rock aquifer. Such sub-marine springs are found along the borders of the Mediterranean and Adriatic seas, in Hawaii, in New Zealand and elsewhere (Todd, 1980).

2.1.1.1 Springs from Carbonate Rocks

Carbonate rocks springs represent constricted discharge at widely separated outlets. In the beginning, a number of solution activity in these rocks of weak discharging springs may

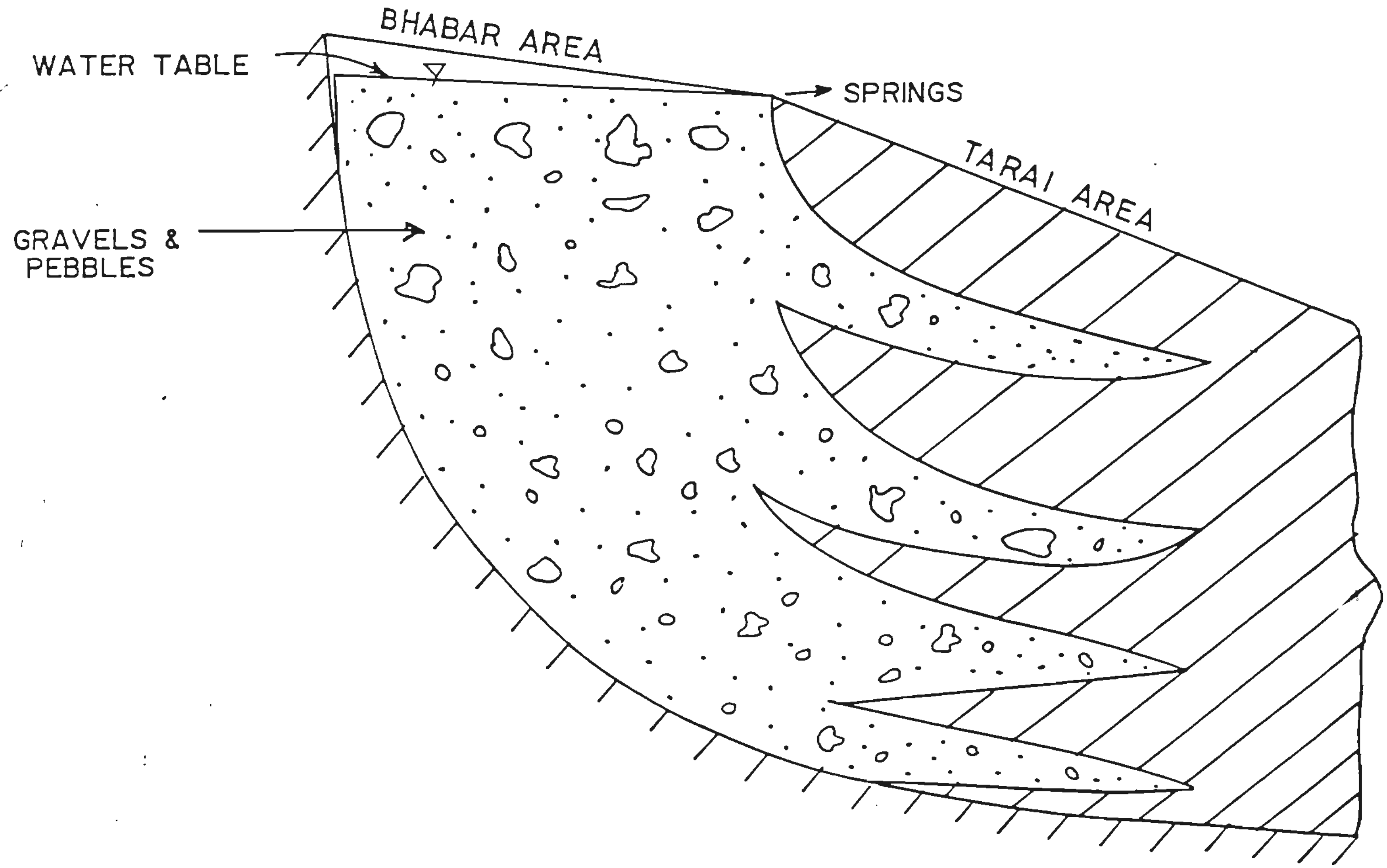


FIG.2·2-SPRINGS EMERGING FROM ALLUVIAL FAN DEPOSIT
(AFTER PANDEY ET. AL., 1968).

develop. The largest spring progressively captures the water feeding its neighbours and the smaller springs become dry. This is specially so in a strongly developed karstic rock. A karst indicates the development of chemically enlarged openings in soluble carbonate rocks and in other non carbonate rocks like gypsum (Fig.2.3). The karst area is characterised by the scarcity of surface stream. It is well established that groundwater flow in karst aquifers is of two principal types, viz., diffuse and conduit. The separation of spring discharge into baseflow (diffuse) and storm flow (conduit) component can be made by means of hydrograph analysis (Atkinson, 1977 vide Crowther, 1983). In a typical karst region, drilling of wells is avoided due to existence of large underground cavities at different levels and the karst water as such is not filtered. Erosion of a karstic valley may reach so deep that it reaches the groundwater level. Lines of groundwater flow converge on the valley and dissolve bedrock at a faster rate than for adjacent uplands. Less permeable residual soils and floodplain sediments promote the formation of line of springs at the contact of bedrock and valley fill sediments and is illustrated in Fig.2.4 (Parizek, 1975).

The majority of important springs in karsts are located along the perimeter of the erosion base. A common characteristic of these springs, whether permanent or temporary, is the direct dependence between precipitation and their outflows. It is possible to have two closely spaced springs with greatly different discharges. The two springs of the Pliva river near Jajce (in

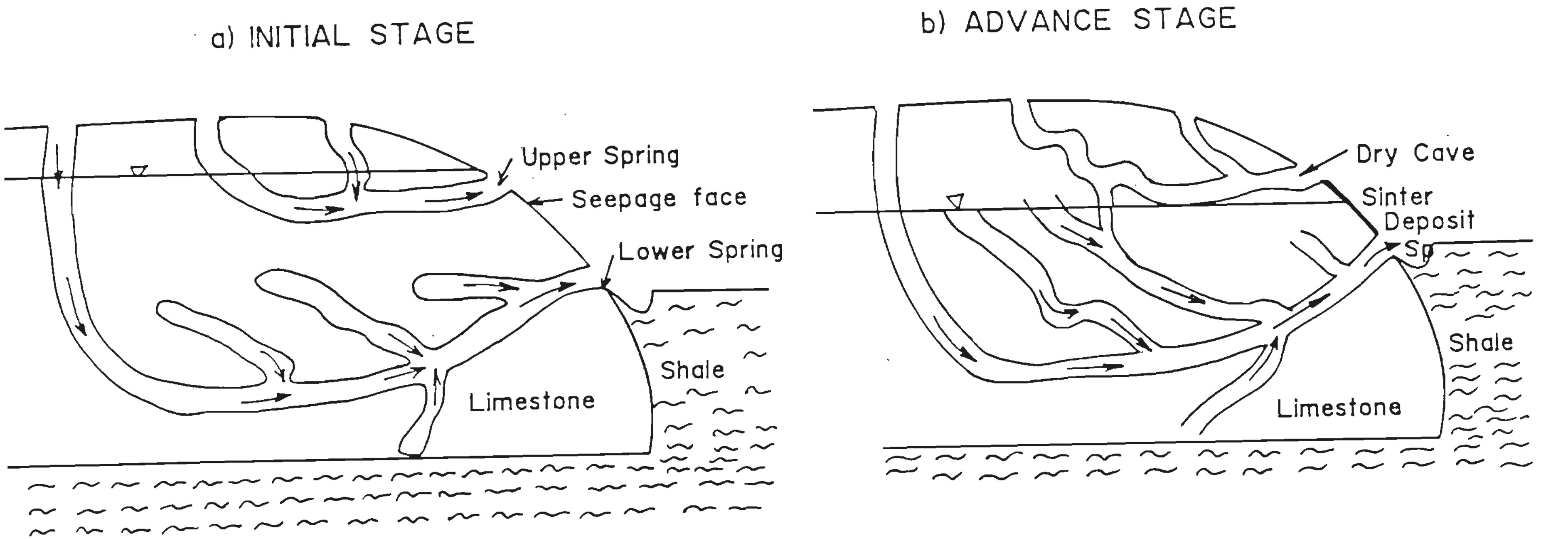


FIG 2·3-DEVELOPMENT OF SOLUTION CHANNEL (AFTER MANDEL & SHIFTAN, 1981).

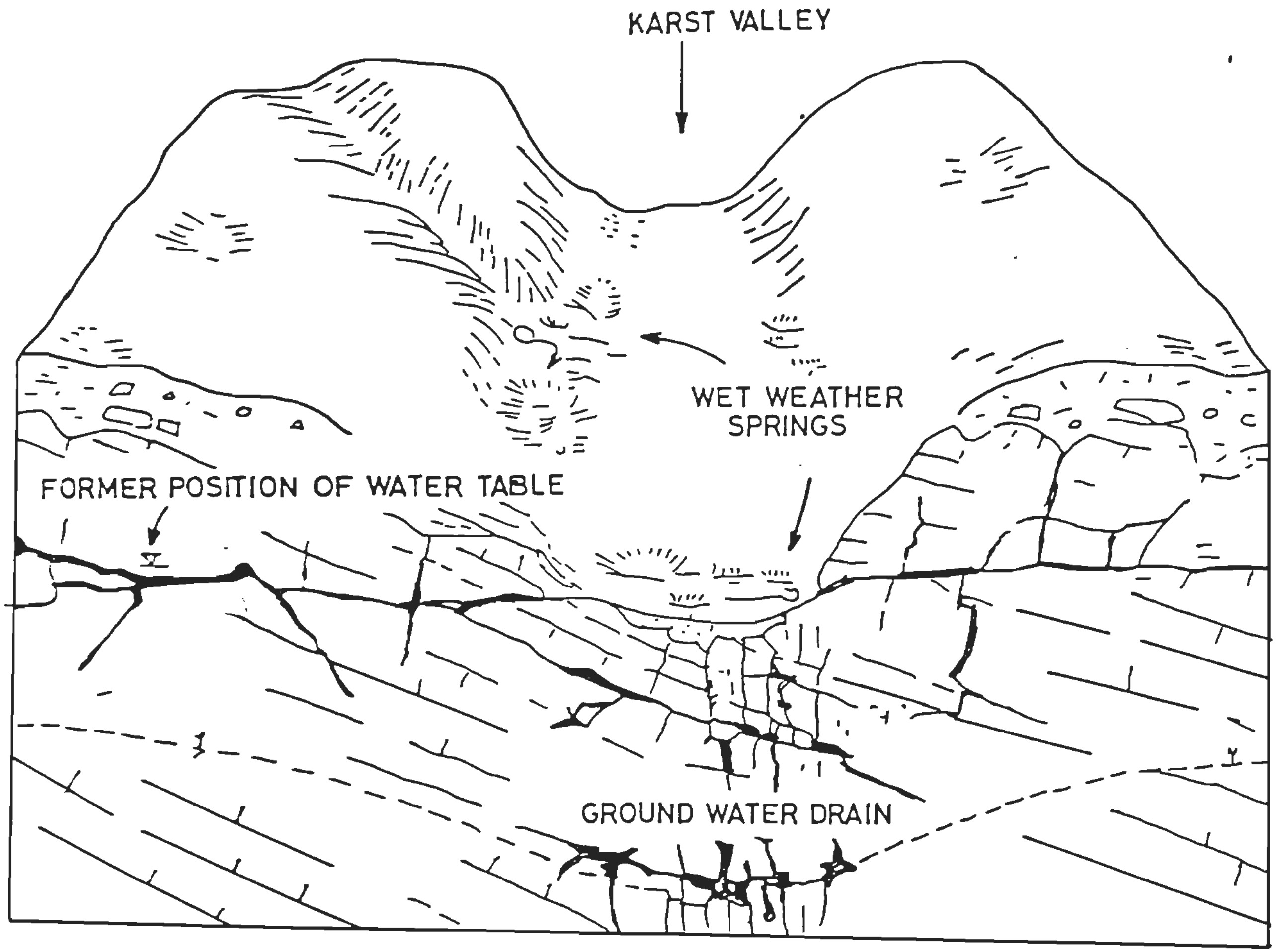


FIG. 2·4–ERODED KARST VALLEY UPTO WATERTABLE (AFTER PARIZEK, 1975).

former Yugoslavia) are good example of such occurrence of springs (Milanovic, 1981).

Submarine springs and springs at the sea surface were formed during continental phase when the base of erosion was lower than at present. These springs are active only during the wet season. At this time they discharge substantial quantities of fresh water into the sea changing the salinity and temperature of the sea water in the coastal belt. Their main characteristic is their considerable variation in flow. More than 50 localities with submarine springs have been discovered along the Adriatic coast in Crotia. The submarine shelf, between Florida and the Bahama Islands, is composed of karstified limestone and is covered with thousands of sinkholes. Submarine springs are active in many of them (Milanovic, 1981). There are quite a few submarine springs off the coast of Nelson, New Zealand (Williams, 1977). Using thermal infrared scanning, Gandinc and Tonelli (1983) identified more than 700 fresh water springs spread over 1500 km of Italian shorelines with a total yield of about 150 m^3 s^{-1}.

Probably ,the largest known spring in the world is Ras -el-Ain, Syria which is on a tributary of the river Euphrates. It emerges from a massif of karstified limestone with an average flow rate of approximately 40 $m^3 s^{-1}$. The spring is situated in a region where the annual average rainfall is only 280 mm (Davis and Dewiest, 1966; Price, 1985). Karanjac and Gunay (1980) declared Dumanli spring in Turkey as the largest spring of the world issuing from one single orifice which has dry period flow rate of about 36 m^3s^{-1}. The spring is in the Manavgat river canyon of Turkey. It emerges from upper Cretaceous limestone on the thrusted flank of an anticline. The spring discharge varies from 25 to more than 100 m^3s^{-1} with a mean discharge of about 50 m^3s^{-1}. The spring had been known from the Roman time but now is under the submergence of high Oymapinar Dam, Turkey. The only outlets of the large limestone aquifer in the coastal area of Israel are two springs;- the Yarkon spring and the Taninim spring (Bear, 1979). Mandel and Shiftan (1981) reported the drying up of the first spring and reduction of discharge to less than half for the second spring due to large scale withdrawl of groundwater through boreholes.

In the study of Gaula catchment in Nainital district, U.P. (Kumaun Himalayas), the investigators of Kumaun University (1988) observed that few springs emerge from carbonate rocks characterised by cavities and solution channels. The yield of the springs of the karst belt is very high being of the order of 3760 m^3d^{-1}. The dolomitic limestone has a good network of joints and there is a good deal of infiltration of water. Selective solution along these fractures and joints has created a network of subsurface water courses leading to almost complete lack of perennial surface streams. The springs in the Nainital township mainly the Parda spring and Sipahi spring emerge from the carbonate rocks of Nainital area. The maximum and minimum discharges as reported by Sharma (1990) are 3125 and 760 m^3d^{-1} for Parda spring, and 40190 and 30745 m^3d^{-1} for Sipahi spring respectively. Coward et al., (1972) studied five major karstic springs within a distance of 25 km in the Anantnag area in the vale of Kashmir. All the five springs occur at the limestone/alluvium contacts at an altitude of 1620 to 1940 m. The geology, chemistry and hydrology of each spring was investigated. At each spring site the rocks dip (30^0-65^0) was towards the hillside and water moves predominantly along bedding planes. The springs are at the lowest hydrologic points of the limestone blocks which suggests that the limestone groundwater is well integrated. The five springs are Kukarnag(1.8), Verinag(2.8), Achhhabal(2.5), Bawan(1.4), and Anantnag(0.4); the figures in the bracket are their respective mean annual discharges in m^3s^{-1}. Verinag spring is the source of river Jhelum and is of vauclusian resurgence type. The spring pool is about 60 m in diameter and is 18 m deep. The spring discharges clear water and has its peak flow in May and June. Karst springs also exist in the Late Proterozoic Upper Vindhyan formations in Central India.

The Vaucluse spring in France is well known worldwide. It has a recharge area of over 1600 km^2 and emerges from strongly jointed and karstified Neocomian limestone. This spring is a periodically flowing one which works as a siphon and is termed as siphon spring (Fig.2.5). The spring emerges from the huge grotto found in the deep canyon. The annual average yield is 17 m^3s^{-1} whereas the maximum recorded yield during springtime is 152 m^3s^{-1}. The area receives an average annual precipitation of 550 mm (Klimentov, 1983). Big permanent Vauclusian springs are usually connected to the lowest erosion base or sea level. Outflow from some of the major vauclusian springs are given in Table 2.2 (Bogali, 1980):

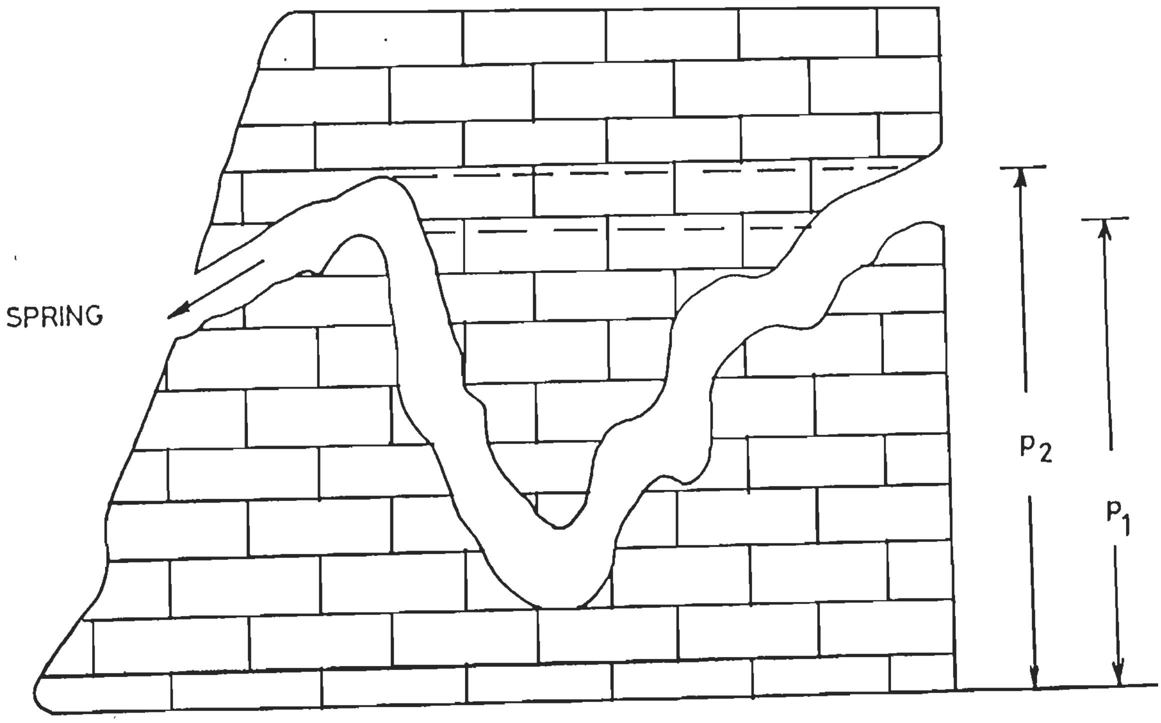


FIG. 2·5-SIPHON SPRING OF VAUCLUSE TYPE (AFTER KLIMENTOV, 1983).

Table.2.2 Discharge from some of the major Vauclusian springs (after Bogali, 1980)

Sl.No.	Name of the spring	Extreme values (m^3s^{-1})	Average (m^3s^{-1})
1.	Aachquelle Spring (Germany)	1.3-24.1	8.8
2.	Areusequelle (Switzerland)	0.18-100	4.0
3.	Fontaine De Vaucluse (France)	4.5-200	29.0
4.	Silver Spring (Florida, USA)	Practically no variation	23.0
5.	Peschiera(Italy)	-	18.0
6.	Big Spring (Ozarks,USA)	-	12.0

2.1.1.2 Springs from Basalt

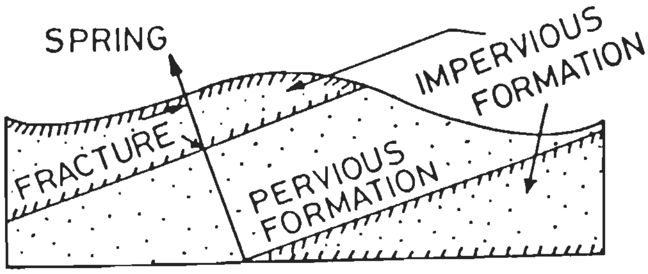
Out of all hard rock formations, basalts are amongst the most productive because of numerous openings. Basalts are spread all over the world : 500,000 km^2 in India, the trap rocks of Deccan; 650,000 km^2 in North America; 900,000 sq,km in Brazil; 100,000 km^2 in Ireland and U.K. and in other places. They represent extensive sources of groundwater (UNESCO, 1972). Springs emerging from basalts or from associated gravel are among the largest kncwn and compare in importance with those issuing from carbonate rocks. The flow from the basalt springs may be relatively constant where it is sustained by the large waterbody and where the fissures are narrow and ash is present. On the other hand, the flow may be variable if wide fissures are well developed and abundant. Many springs issue from the porous zones between successive lava beds, forming well defined spring horizons along dip scarps formed by the lava. Some of the spectacular basalt springs are in the Hawaiian islands, Oregon, Washington, Nevada and Idaho in USA. Average discharge of most of these springs is over 14 m^3s^{-1} (Karanth, 1989). One of the largest series of basalt springs are in Idaho,U.S.A. These springs emerge from pillow structure formed due to submarine volcanic activity. The springs above the canyon walls between Milner and King Davis have a discharge of about 110 m^3s^{-1}. The famous thousand springs along the Snake river in Southern Idaho which rise from the jointed and vesicular basaltic lava flows yield 15 to 20 m^3s^{-1} (Meinzer, 1927; Tarbuck & Lutgens, 1990).

One group of basalt springs in California and Oregon, USA has a combined discharge of 40 m^3s^{-1} of which Datta spring alone supplies 1.4 to 3 m^3s^{-1}. The discharge of springs at Oahu and Kaluaoopu islands in Pearl Harbour, Hawaii varies between 0.4 to 0.7 m^3s^{-1} (UNESCO, 1972). Ground water confined in between dikes in basaltic terrain in oceanic island at times gives rise to emergence of springs. Fragmentary material known as pyroclastic material such as pumice, tuff etc. are associated with basalt. Such pyroclastic material has high permeability provided the material is not folded. In some areas like in Java, Indonesia and in Japan, rivers are fed substantially by springs emerging from pyroclastic material.

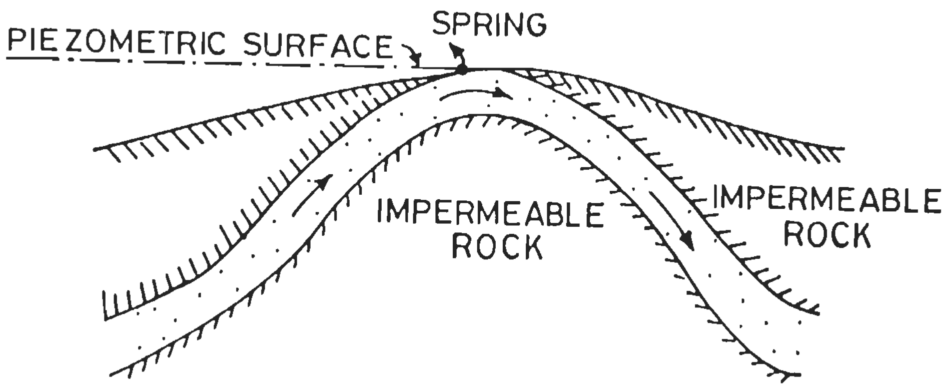
As stated in the report published by Kumaun University (1988), the Bhimtal volcanics in Nainital district of Uttar Pradesh, India, are vesicular and associated tuffite have in addition vertical and horizontal joints and shear planes. The depth of weathering varies from 60 cm to 2.5 m. About 9% of the springs in the Gaula river catchment are located in Bhimtal volcanics. An intrusive body within the Bhimtal volcanics act as a barrier to the groundwater movement causing emergence of springs. Like in Bhimtal volcanics and elsewhere , dikes, sills, layers of tuff, and buried sills commonly control the location of springs in volcanic rocks (Davis and Dewiest, 1966).

2.1.2 Structural Control

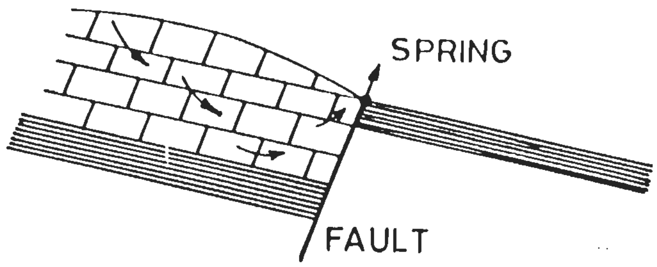
Springs are formed where discontinuties like faults, tectonic fractures, dikes etc. create hydraulic barriers and force groundwater to flow upward along the weak zones across confining layers. Dikes, sills, layers of tuff and buried soils commonly control the location of springs in volcanic rocks. Groundwater confined in between dikes in basaltic terrain in oceanic island at times give rise to the occurrence of spring. Earth movements cause tilting and folding which bring permeable or impermeable beds near the surface and springs emanate from a permeable bed. Joints and joint systems commonly are responsible for small springs. This is particularly true for exfoliation joints in massive granitic rocks. Structural variability in rocks produces changes in permeability and localises the occurrence of the springs. Fig.2.6 shows some of structural controls on the formation of springs.



(a). SPRING EMERGING FROM A FRACTURE (TODD, 1980)



(b) ANTICLINAL SPRING (PINNEKER, 1980)



(c) ARTESIAN SPRING IN CARBONATE ROCK (UNESCO, 1972)

FIG.2·6-SPRINGS SHOWING VARIOUS STRUCTURAL CONTROLS.

Investigators from Kumaun University (1988) observed emergence of springs on the north and north east sloping flanks of hills in different sub-catchments of Gaula river in Nainital district of Uttar Pradesh, India. This essentially indicates structural (dip) and topographical controls on the movement of groundwater and the formation of springs. Highly fractured crystalline rocks located on the lineaments and occupied by first or second order streams in the Gaula river catchment give higher discharge than the springs emerging from crystalline rocks with less or negligible fracturing. In Bhimtal area in the Kumaun Himalayas, springs emerge from the contact of the country rock and the dolerite dike. Sharma (1990) reported the emergence of many springs from the synclinally folded Krol belt which are mostly active in the Sher ka danda slope towards the north eastern side of Nainital lake. These springs generally follow faults and major fractures and discharge very small amount of water (Sharma, 1990). The springs in and around Shimla town in Himachal Pradesh, India are well distributed and are controlled by the lithological contacts, faults, shears,and prominent regional joints (Singh, 1990).

In Jeloya- Moss area of southern Norway, there are springs which emerge from the fissured rock aquifers where the groundwater surface is below the overlying Quaternary deposits. The springs have very well defined water outlets because of the continuation of the drainage fissures upto the spring. Englund and Meyer (1980) studied five such springs. The average discharge of the five springs studied is 60 m^3d^{-1}.

2.1.3 Topographical Control

When an aquifer having groundwater under hydrostatic pressure is exposed on a sloping ground surface, the groundwater emerges as a concentrated discharge, and the spring so formed is controlled by topography. The role of topographic control on the emergence of springs is evident especially at the hill slopes. Most of springs described in Figs.2.1, 2.2, and 2.6 have topographical control associated with them. However, all the three main control could play complimentary role in the emergence of a spring or a group of springs. For example, in a basaltic terrain if the topography is steep and if the water table cuts across the topography, springs emerge. So, lithology and topography both could influence the emergence of a spring. Likewise emergence of springs between the dikes in a basaltic island is the manifestation of all the three controls. All the springs of the Kumaun region discussed in the preceding paragraphs have some kind of topographical control.

2.1.4 Classification of Springs

One of the earliest classification of springs was given by Keilhack (1935) based on ascending (artesian) and descending (water table) water flow in the spring. Descending type of springs are usually periodically flowing. Discharging points of ascending type of springs are more varied. They form erosion barrier and structural springs (Ovchinnikov, 1968). However, Keilhack's classification is based on the aquifer geometry only. Springs can be classified according to the important controlling factors related to geology, hydrology, and geomorphology. Some of these are: (i) character of the water bearing formations, (ii) quantity of water discharged, (iii)uniformity and periodicity of the rate of discharge, (iv) temperature of water, and (v) chemical quality of water. Chow (1964) compiled the various important classifications of the springs made by different investigators in a comprehensive way. These are stated herein in Table 2.3.

Davis and Dewiest (1966) argued that there could be at most a few hundred first magnitude springs in the world as combination of large rainwater infiltration, large drainage area and favourable geologic structure at a place that are needed to produce a first magnitude spring, will be rare. Almost all first magnitude springs issue from lava, limestone, boulder or gravel aquifers. But small springs of magnitude 7 or 8 can be found in all types of rocks e.g., loess, dolomite, graywacke, gypsum, serpentine etc. Most springs of magnitude 8 are springs which flow only for a short time following a period of precipitation.

Todd (1980) reported the relation of catchment area and annual recharge to average spring discharge. This is depicted in Fig.2.7. A spring with a few hectares of recharge area can supply the needs of a single family whereas large recharge area with high rainfall is necessary to produce a first magnitude spring.

Table 2.3 Classification of springs (after Chow, 1964)

A. Classification based on the type of water-bearing formation or type of opening (after Tolman, 1937)

I. Spring issuing from permeable veener formations overlying relatively impermeable bedrock with irregular surface; bedrock outcrop or near approach to surface is controlling feature; includes contacts and so-called gravity springs, perched, talus, pocket and barrier springs. Usually are small, but some are large, and discharge may vary periodically.

II. Springs issuing from thick permeable formations; water movement unaffected by deeply lying impermeable bedrock. Controlling feature is intersection of water table with the land surface. Includes all water-table springs, among special types of which are channel, valley, cliff, valley, dimple, and boundary, or alluvial-slope springs. Discharge is usually small.

III. Springs issuing from interstratified permeable and impermeable formations; aquifers are stratiform and may be structurally deformed. Control of springs is outcrop of aquifer. Springs may draw on confined water (artesian springs) or water may be unconfined. Include contact, monoclinal, synclinal, anticlinal and unconformity springs. Discharge of springs usually small, but some may be large.

continued...Table 2.3

IV. Springs issuing from solution openings formed primarily along fractures and bedding planes in carbonate rocks. Discharge of these springs is often large and fluctuates considerably.

V. Springs issuing from fractures and tubes in lava and from thin interbedded porous strata. Discharge is often large and usually steady.

VI. Springs issuing from fractures intersecting permeable materials and impermeable materials and fractures supplied in part by waters of deep-seated unknown origin. For the most part, discharge of such springs is small, but some may be large.

B. Classification of springs according to magnitude of discharge (after, Meinzer, 1923)

Magnitude	Discharge
First	100 cfs or more (0.245×10^6 m^3d^{-1} or more)
Second	10 to 100 cfs (0.245×10^5 to 0.245×10^6 m^3d^{-1})
Third	1 to 10 cfs (2450 t0 24500 m^3d^{-1})

continued Table 2.3

Magnitude	Discharge
Fourth	100 gpm to 1 cfs(448.8gpm) (545 to $2450\ m^3d^{-1}$)
Fifth	10 to 100 gpm (54 to $545\ m^3d^{-1}$)
Sixth	1 to 10 gpm (5.4 to $54\ m^3d^{-1}$)
Seventh	1 pt.* to 1 gpm (1.3 to $5.4\ m^3d^{-1}$)
Eighth	less than 1 pt./min (less than $1.3\ m^3d^{-1}$)

* 1 gallon = 8 pts.

C. Classification according to variability and permanence of discharge (after, Meinzer, 1923)

I. Perennial springs (springs that discharge continuously)

a) Constant - springs with a variability of not more than 25 per cent

b) Subvariable - springs with a variability of more than 25 per cent but not more than 100 per cent

c) Variable - springs with a variability of more than 100 per cent

continued...Table 2.3

N.B.:variability of a spring is defined by Meinzer as the ratio of the discharge fluctuation to its average within a given period of record. Thus, V = 100 (a-b)/c ,where V is the variability in per cent, a the maximum, b the minimum, and c the average discharge.

II. Intermittent springs (all are variable since they discharge only during certain periods and are dry at other times.)

D.Classification according to temperature (after, Meinzer, 1923)

I. Nonthermal springs

a.Ordinary springs whose waters have temperatures closely approximating the local mean annual temperature of the atmosphere

b.Cold springs whose waters have temperatures appreciably below the local mean annual temperature

II. Thermal springs

a.Hot springs whose waters have temperatures higher than 98^0F (36.6^0C)

b.Warm springs whose waters have temperatures higher than the local mean annual temperature but lower than 98^0F (36.6^0C)

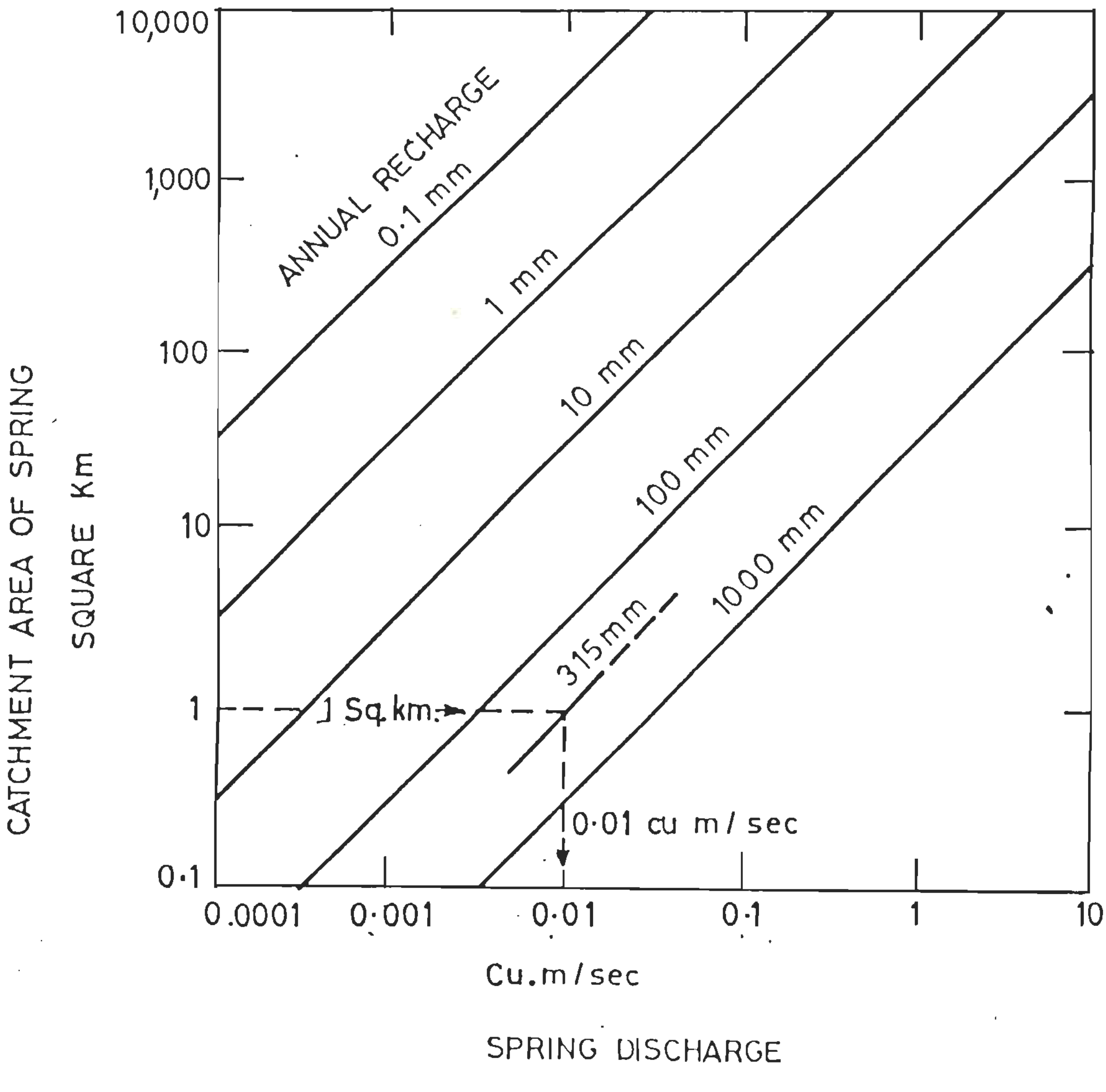


Fig. 2·7- RELATION OF CATCHMENT AREA AND ANNUAL RECHARGE TO AVERAGE SPRING DISCHARGE (AFTER TODD, 1980).

Low variability of discharge is normally found in volcanic rocks and sandstone formations. High altitude springs may exhibit fluctuation of flow which are associated with rainfall regime like in Sulkovy Pramney spring in the hilly region of central Czechoslovakia (Balek, 1989). He suggested a classification for these springs emanating from sandstone aquifer on the basis of $Q_{10\%}/Q_{90\%}$ ratio of the springflow. $Q_{10\%}$ and $Q_{90\%}$ are the spring discharges at 10% and 90% of the relative cumulative frequencies respectively.

Kumaun University conducted geohydrological investigations of the Gaula catchment (1988). The investigators studied the formation of the springs in the catchment. Based on the genesis, nature of water bearing formations and conditions governing the formation of the springs, the springs of the Gaula river catchment are classified into 7 categories: i) Fracture-joint related springs, ii) Lineament-fault related springs, iii) Colluvial springs, iv) Springs related to fluvial deposits, v) Bedding plane related spring, vi) Dike related springs, and vii) Karst springs.

2.2 Hydrological Modelling of Springflow

An active spring can be considered as a flowing well with constant head and this feature could be used conveniently in the hydrological modelling of springflow. In the analysis of regional groundwater flow, a flowing spring can serve as a boundary condition of Dirichlet type. The elevation of the spring threshold or outlet is considered as of fixed boundary head. But, when the water table in the vicinity of the spring drops below the spring threshold at some point of time (which is a priori unknown), the spring ceases to exist and the boundary condition at the spring does not prevail.

The discharge of a spring depends on (i) the difference between the elevation of the water table (or piezometric head) in the aquifer in the vicinity of the spring, and the elevation of spring threshold, (ii) the geometry of the aquifer, and (iii) aquifer parameters. During a dry season, the spring discharge is derived from water stored in the aquifer. Consequently, the water table in the aquifer gradually falls and spring discharge declines. During the precipitation period, the aquifer gets recharged and water table rises and spring discharge increases. The fall and rise of springflow go on in a cyclic pattern. The relationship between the discharge of dry and wet seasons and time depends on dynamic storage characteristics of the aquifer and transmissivity of the aquifer. Fig.2.8 shows a typical portion of a spring hydrograph. On a semilog paper (with time on the linear scale), the recession curve follows approximately a straight line.

2.2.1 Springflow Recession Curve

The lowest portion of the recession curve in a flood hydrograph depicts the depletion of ground water storage and can be simulated by a conceptual linear reservoir, in which outflow is linearly proportional to the storage. Integration of the storage-outflow relation with continuity equation and solution of the first order differential equation gives the relation (Chow, 1964)

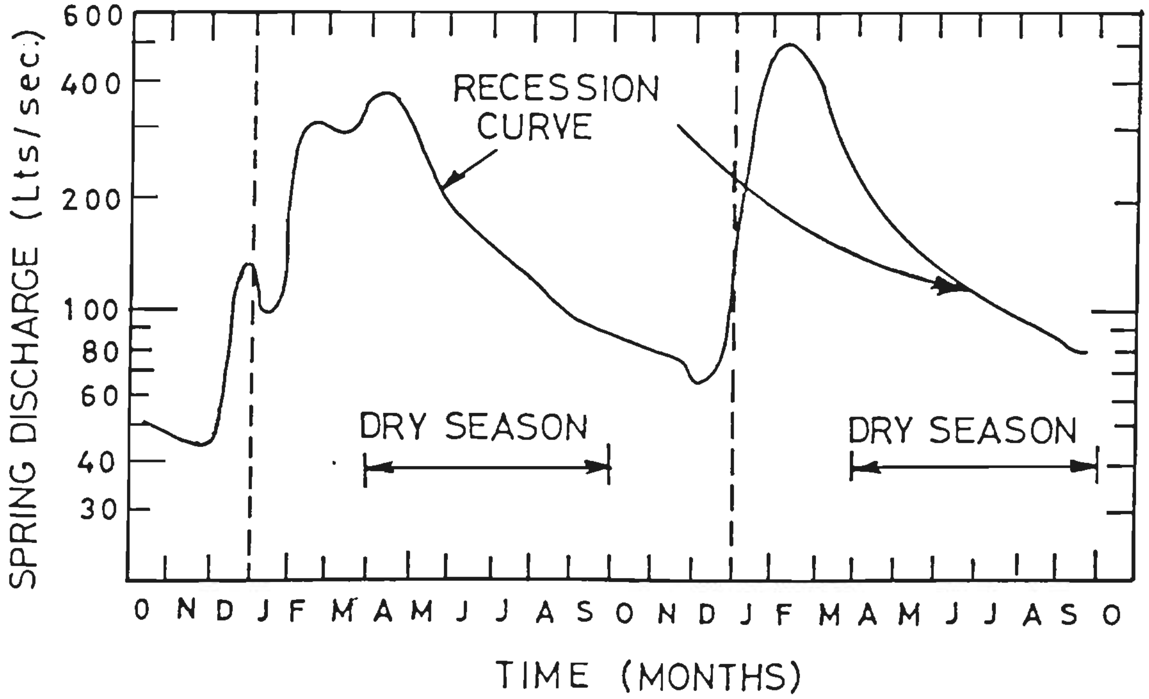


FIG. 2·8 - A TYPICAL SPRING HYDROGRAPH WITH SEASONAL FLUCTUATION (AFTER BEAR, 1979).

$$Q_t = Q_0 k_r^t \qquad \ldots(2.1)$$

where Q_0 is springflow at any arbitrary time origin and Q_t is the springflow at any time t. k_r is the recession constant or depletion factor which is normally greater than 0.9 but always less than 1. The recession constant k_r has also been expressed as $\exp(-1/\tau_0)$ where τ_0 is the depletion time. Anderson and Burt (1980) quoted Barnes (1939) stating that Eq.(2.1) is capable of describing the recession of individual components of runoff, i.e., overland flow, throughflow and groundwater flow. Nutbrown and Downing (1976) studied recession flow in rivers in U.K. and opined the same. They suggested different values of depletion factor for modelling the different components of the recession portion of a flood hydrograph following a storm. They suggested the values of 1 and 0.99 as the depletion factor for the direct run-off and the baseflow recession, respectively.

The analysis of recession curve defines the regime of flow of a spring. The recession curve characterises the storage depletion or base flow from an aquifer during the period of zero or negligible precipitation (Karanjac and Altug, 1980). Therefore, recession curve of a springflow is used to estimate the dynamic storage inside a spring flow domain which is subsequently discharge as springflow. The physical process of release of water from groundwater storage is a phenomenon which can be described by an exponential law which is same as that used for baseflow (Chow, 1964; Singh, 1989). One of the convenient ways to express the exponential law is

$$Q(t+\Delta t)=Q(t)\exp(-\Delta t/\tau_0) \quad ...(2.2)$$

where Q(t)=springflow at time t during recession, Δt is the time increment, and τ_0= a parameter of the spring and is designated as depletion time and has dimension of time.

Eq.(2.2) is the most widely used equation derived by Boussinesq as early as 1877 (vide Singh, 1989). Maillet(1905, vide Ford and Williams, 1989) suggested Eq.(2.2) for the analysis of spring's recession hydrograph. Karanjac and Altug (1980) stated that recession curve analysis of a springflow hydrograph using the exponential law is quite appropriate especially for relatively large systems with no secondary groundwater accretion and use of more sophisticated formula is not essential.

Either of the Eqs.(2.1) and (2.2) has been widely used for the analysis of the recession curve of a springflow hydrograph all over the world particularly in Europe and in the Mediterranean area for karstic as well as for other type of springs. (Yevjevich, 1959 vide Knisel, 1972; Faulkner, 1975; Torbarov, 1975; Milanovic, 1975; Magnin, 1975 vide Ford and Williams, 1989; Atkinson, 1977; Karanjac and Altug, 1980; Karanjac and Gunay, 1980; Korkmaz, 1990; Bonacci, 1995).

2.2.2 Depletion Time (τ_0)

According to Eq.(2.2) the variation of logarithm of springflow with time is linear. The reciprocal of the product of negative of the slope of the straight line ($\log_{10}Q$ versus t) and

2.3, is designated as the depletion time in time unit. Depletion time is a characteristic parameter for a groundwater flow domain and can be treated as model parameter for mathematical models for springflow. Any change in the slope of the line is indicative of interference in the groundwater system. A progressive flattening of the slope indicates the replenishment of the aquifer in the dry season (probably due to return flow of irrigation/urban effluent or seepage from reservoirs) and steepening of the slope indicates groundwater abstraction from the aquifer and reduction in natural recharge. Occurrence of earthquake can have effect on spring discharge and on the slope of the time discharge line considerably.

In a recent study of six major karstic springs in Greece by Soulis (1991), it was observed that each of the aquifers, from which a spring emerges, was unique in respect of boundary conditions and their internal organisations and consequently different recession curves were obtained. The investigator recommended the use of the characteristics of these different recession curves for the classification of the aquifer system from which the springs emanate. The depletion time representing recession characteristic depends on the geology and geomorphology of the basin. For a spring whose recharge area is small or the aquifer from which it emerges has high permeability and low porosity, the τ_0 will be small. If the volume of water that is stored in the flow domain of the spring is large, or its drainage is slower, the τ_0 will be large. Karanjac and Altug (1980) studied a number of springs in Turkey and suggested some values of depletion time according to type of porosity (Table 2.4). They showed that the dynamic storage inside the flow domain of a spring during recession is $Q_0 . \tau_0$ where Q_0 is the springflow at the onset of recession.

Table 2.4 Range of values of depletion time (τ_0) (after Karanjac and Altug, 1980)

Sl.No.	Type of porosity	Order of magnitude of Depletion Time (months)
1.	Flow is primarily through interbedding joints and fissures (3rd Group of openings)	33
2.	Flow is primarily in larger fractures, faults etc. (2nd Group of openings)	3.3
3.	Massive Karstified limestone terrains with primary drainage through large flow channels, interconnected solutional features and other privileged ways (1st Group of openings)	0.33

Englund and Meyer (1980) in their study of four springs which have supplied drinking water for several decades in Norway observed that springs emanating from strongly fractured rocks, e.g., limestone, have depletion time in the range of 0.7-13 months while sandstones with minor fracturing give values around 13-33 months. The orders of magnitude of these values of τ_0 are comparable with the values given in sl.no.3 and 1 respectively of the Table 2.4. They further observed that a spring with longer depletion time will be more stable and will emanate water with higher concentration of dissolved solids during the period of no recharge.

Martin (1973, vide Ford and Williams, 1989) suggests that a concept equivalent to the half life in nuclear physics would be appropriate and defined a half-flow period ($\tau_{0.5}$) as the time required for the baseflow of the spring to halve. Hence, by definition

$$2Q_{\tau_{0.5}} = Q_0 \quad \ldots(2.3)$$

Using Eq.(2.1),

$$Q_{\tau_{0.5}} = Q_0 \exp(-\tau_{0.5} / \tau_0)$$

or $$Q_0 = Q_{\tau_{0.5}} \exp(\tau_{0.5} / \tau_0)$$

or $$2Q_{\tau_{0.5}} = Q_{\tau_{0.5}} \exp(\tau_{0.5}/\tau_0)$$

or $$2 = \exp(\tau_{0.5}/\tau_0)$$

or $$\tau_{0.5} = \tau_0 \log_e 2$$

or $$\tau_{0.5} = -\log_e 2 / \log_e k_r \quad (2.4)$$

The parameter $\tau_{0.5}$ has the following properties:

(a) It is independent of Q_0 and Q_t and of the time elapsed between them.

(b) It can take values in the range of zero to infinity.

(c) It can be easily evaluated from equation $\tau_{0.5}$=Const./$\log_e k_r$.

(d) It is a direct measure of rate of recession and therefore can be used as a means of characterising exponential baseflow recessions.

When k_r is large, i.e.,when $\tau_{0.5}$ is small, the recession is steep, indicating rapid drainage of conduits and little underground storage. But if k_r is small and $\tau_{0.5}$ is large (meaning conduit function is less), then very slow drainage of aquifer is indicated from an extensive fissure or porous network and the spring seems to have a large storage capacity and high resistance to recharge throughout.

2.2.3 Non Linearity of Storage-Outflow Relationship

Mandel and Shiftan, (1981) stated the possibility of depletion lines being composed of several linear segments with different slopes. This could happen due to following reasons:

1. There is diversion of water upstream from the spring discharge measuring point;

2. There is vertical variability of the product of plan

area of the aquifer and aquifer storativity which the model assumes to be constant;

3. Delayed runoff (interflow) enters the spring flow domain through soil mantle;

4. More than one underground reservoir (common in fissured and karstic rocks) contribute to the same spring.

During recession, the baseflow component, Q(t), of a river partially penetrating an aquifer is equal to the total release from groundwater storage and can be estimated from the solution of the two-dimensional equation of ground water flow. Further, with Dupuit's assumption of negligible vertical flow, Nutbrown (1975 vide Nutbrown and Downing, 1976) has shown that

$$Q(t) = \sum_{i=1}^{n} A_i \, K_i^t \qquad \ldots(2.5)$$

where, $K_1, K_2, \ldots K_n$ are depletion factors of groundwater and $A_1, A_2 \ldots A_n$ are constant coefficients. Although theoretically the summation in Eq.(2.5) includes infinite number of terms, in practice, only a few of them will dominate at any particular time. With this assumption, log Q(t) in t plane is fitted with several straight lines of decreasing slope. The implication of Eq.(2.5) is that the exhibition of these successive straight line segments does not necessarily be the characteristic of a complex aquifer structure. An aquifer may be perfectly uniform, with no particularly unusual features, and still may exhibit the behaviour implied by Eq.(2.5) in its baseflow contribution to the flow.

It has been noted in the analysis of baseflow recession curves for many partially penetrating streams in U.K. that the semi-log plot of baseflow against time is not a straight line but rather a curve, even for uniform values of aquifer parameters. The composite nature of discharge-recession curve could be attributed to the release and subsequent discharge of water from different parts of the aquifer under constantly varying heads. The complete baseflow recession curve expressed as $\log_{10} Q$ vs t has an initial steep slope followed by a flatter portion and there is a further steep portion as the stream dries up. Nutbrown et al. (1976), inferred that deviation of the plot from a single straight line is due to the dynamics of ground water flow and normally not due to the complex hydrological structure. It is quite normal that the average catchment value of storage coefficient could fluctuate markedly with baseflow decline even for a simple aquifer. Singh and Stall (1971 vide Nutbrown et al., 1976), studied recession characteristic of baseflow in rivers which partially penetrates an aquifer and defined a dimensionless parameter τ as aquifer response time and $\tau = Tt/SL^2$ where T, S are transmissivity and storage coefficient of the aquifer, respectively, t is the time elapsed from the start of the recession and L is the distance between the outflow point and the watershed divide. Under a wide range of conditions, they concluded that for $\tau > 0.2$ plot of log Q vs t will be a straight line and for $\tau < 0.2$ the plot curves away from the straight line. Knisel (1972) while studying Goodenough springs near Comstock, Texas, found that there were changes of slope several times in the log q vs t plot. He attributed these changes of slope to variable porosity throughout the thickness of saturated mantle. A steep portion of the recession indicates a low porosity layers, whereby the hydraulic gradient would decrease rapidly and causes a large change in discharge. On the other hand, high porosity layers would have little change in hydraulic gradient for a given discharge and the recession slope would be flatter relative to low porosity layers. However, each layer in the mantle is affected by the underlying layers and the spring discharge represents an integration of flow through the system. As the observed recession may be due to obscured stratification, Knisel termed his arguments for the change of slope as conjectural rather than factual.

If the resulting recession curve is found to be nonlinear, a double exponential form of equation could be used to represent the curve better. A double exponential equation which was first suggested by Horton (1933, 1935) and had been used by Yevjevich (1963), Toebes and Strang (1964), and Toebes et al. (1969) (vide Singh, 1989) is

$$Q_t = Q_0 \exp(-t^n/\tau_0) \qquad \ldots(2.6)$$

Eq.(2.6) in logarithm form is

$$\mathrm{Log}_{10}\{\log_{10}(Q_0/Q_t)\} = n \log_{10} t + \log_{10}(1/\tau_0) - 0.369222$$

and represents a straight line in log $\{\log(Q_0/Q_t)\}$ vs log t plot.

Alternatively, the equation developed by Boussinesq (1904), for a case where a stream is located on an horizontal impermeable lower boundary draining an aquifer with an initial curvilinear water table and zero water table elevation in the stream, could be used for springflow. The equation is given as (vide Singh, 1989, and vide Ford and Williams, 1989)

$$Q_t = Q_0 / (1+Ct)^n \qquad \ldots(2.7)$$

where C is a constant and is equal to $(1/t)\ \{(Q_0\ /\ Q_t)^{1/n} -1\}$ and the exponent n usually lies in the range of 0.5 and 2.

This equation has been used in Europe for estimation of spring discharge by Maillet (1905), Hall (1968) and Toebes et al. (1969) (vide Singh, 1989). One difficulty with many recession curves is that although they are non-linear they do not fit Eq.(2.7). Maillet (1905) and Boussinesq (1904) (vide Hall, 1968) solved this problem by assuming two components or sources of base flow, one constant and one declining either as

$$Q = (Q_0 - B)/(1+Ct)^2 + B \qquad \ldots(2.8)$$

or

$$Q = (Q_0 - B)\ \exp\ (-\ \alpha\ t) + B \qquad \ldots(2.9)$$

Boussinesq (1904 vide Hall, 1968) showed that a recession fitted by Eq.(2.8) could also be expressed by:

$$Q = Q_1\ \exp(-\alpha_1 t) + Q_2 \exp(-\alpha_2 t) \qquad \ldots(2.10)$$

Eqs.(2.9) and (2.10) are examples of principle of superposition valid for linear system and are relatively easy to be handled. Dooge (1960) and Kraijenhoff van de Leur (1958) (vide Hall, 1968) have shown the advantages of using linear solutions to approximate non linear systems. Dooge (1960) (vide Knisel, 1972) proposed two equations to describe the rising and falling limbs of a hydrograph from a linear groundwater system. The suggested discharge equations for the recharge and recession periods respectively are

$$q = r\{1 - \exp(-t/K)\}, \qquad 0 < t < T \qquad ...(2.11)$$

$$q = r\{\exp(T/K) - 1\}\exp(-t/K), \qquad T < t < \infty \qquad ...(2.12)$$

where r is the rate of recharge, T is the duration of uniform recharge, t is the time, and K is the storage delay time of an element.

2.2.4 Existing Springflow Models

Though the first reported work on springflow is almost a century old, the efforts on mathematical modelling of springflow is somewhat limited. Some of them are discussed earlier. Some conceptual linear mathematical models have been developed during last one decade or so to estimate springflow and to assess the dynamic storage inside a spring flow domain (Bear, 1979; Mandel and Shiftan, 1981; Kovacs, 1981). These models assume that during the recession period, the springflow is linearly proportional to the dynamic storage inside the spring flow domain. These models are similar to each other and can accept only a lumped recharge in the beginning. The models simulate an unsteady state flow as a succession of steady state flow. In the first model, i.e., in Bear's model, the recession portion of a spring hydrograph could be simulated for an one time recharge in the beginning. He interpreted the recession constant in terms of hydraulic diffusivity and the geometry of the aquifer. In the second model developed by Mandel and Shiftan, the flow domain is conceptualised as a tank having an outlet at the lower portion and the recharge takes place at the open surface of the tank. The model, as such, is known as Unicell model. These two models are essentially for geological formation which has primary porosity. A third model by Kovacs demonstrates the influence of fracture (secondary porosity) on springflow. These three models are described herein.

Bear's Model

Bear (1979) suggested a simple tank model to analyse unsteady flow of a spring (Fig.2.9). Assuming that at anytime during recession period, the discharge $Q = \alpha_1 h$, where α_1 is a constant, and h is the potential difference causing flow, the decline in dynamic storage from the spring flow domain during the recession period is

$$Q(t)\ dt = \alpha_1 h\ dt = -\phi\ A\ dh \qquad \text{...(2.13)}$$

where ϕ is storage coefficient, and A is the plan area of the flow domain. Rearranging Eq.(2.13) as

$$\frac{\alpha_1 dt}{-A\phi} = \frac{dh}{h} \qquad \text{...(2.14)}$$

Integrating, and applying the initial conditions, i.e., at $t = t_0$, $h=h_0$, and $Q = Q_0 = \alpha_1 h_0$, the solution of Eq.(2.14) is

$$(t-t_0)=(\phi A/\alpha_1) \ln(h_0/h) = (\phi A/\alpha_1) \ln(Q_0/Q)$$

or $$Q(t) = Q_0 \exp\{ - \frac{\alpha_1}{\phi A} (t - t_0)\} \qquad \ldots(2.15)$$

The variation of Q(t) with t will plot as a straight line on a semilogarithm paper (Q on logarithmic scale).

Bear suggested another simple model of a spring draining an unconfined aquifer (Fig.2.10) with a view to giving an interpretation of α_1.

The unconfined flow in Zone I has been approximated to follow Dupuit's conditions and flow rate at any time has been expressed as

$$Q=WK(h_L^2 - h_0^2)/2 = WK(h_L + h_0)/2 * (h_L - h_0)/L = WT(h_L - h_0)/L \quad \ldots(2.16)$$

where Q = rate of flow from spring, K = permeability of the aquifer, T = average transmissivity of the aquifer, $(h_L - h_0)$ = difference of head, L = length of transition zone and W = width of the spring's threshold.

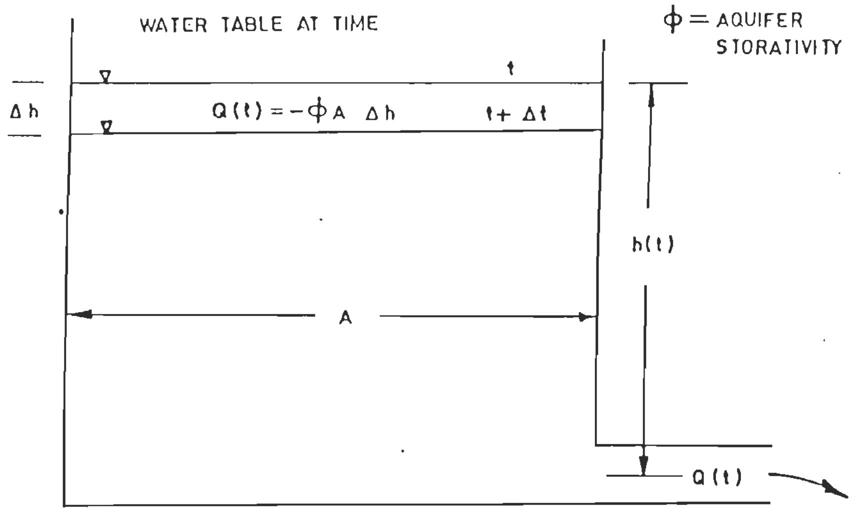


FIG.2·9-A SIMPLE MODEL OF SPRING FLOW (BEAR,1979).

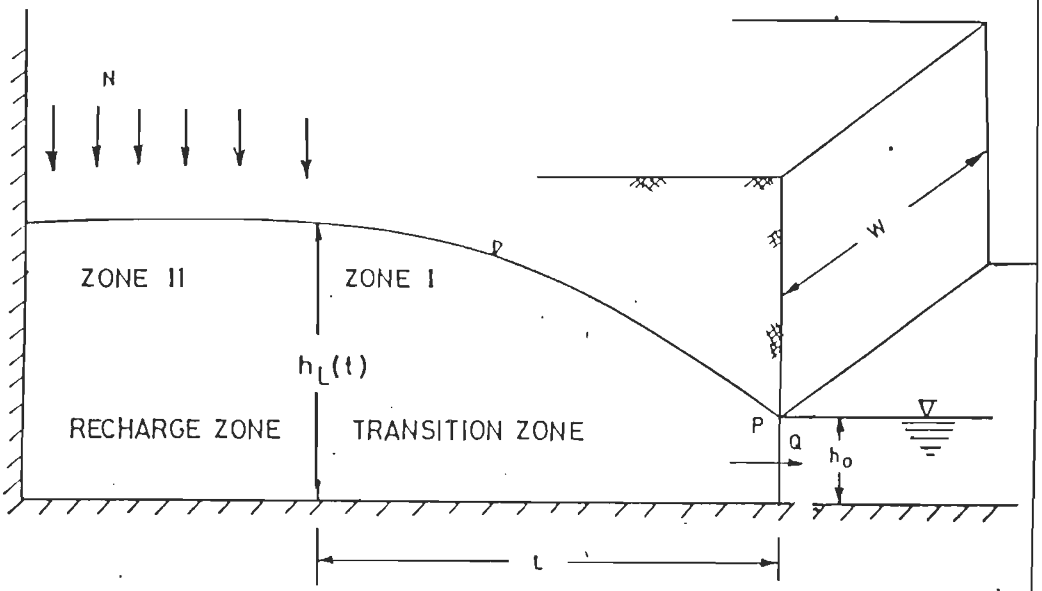


FIG. 2·10-ANOTHER SIMPLE MODEL OF SPRING FLOW (BEAR, 1979).

As the spring discharge is linearly proportional to the head available, therefore, α_1 = (WT/L) and we have

$$Q(t) = Q_0 \exp [-\{WT/(AL\phi)\} (t-t_0)] \quad ...(2.17)$$

Thus, the depletion time τ_0 is equal to $AL\phi/(WT)$.

If the aquifer contributing to the springflow is made up of several separate subregions, then the each subregion will have its own characteristic depletion time, τ_0.

The coefficient, τ_0, or any other coefficient in one form or other appearing in the expression like Eq.(2.17) describing a spring recession curve, is related to the aquifer's geometry, transmissivity, and storativity. Therefore, as an inverse problem, it is possible to investigate about these aquifer properties by the analysis of the hydrograph of a spring discharge. It is assumed in the above mentioned analysis that pumpage or recharge does not take place during the recession period of the springflow.

Unicell Model

The Unicell model (Mandel and Shiftan, 1981) interprets the time series of spring discharge data and predicts spring discharge. It is assumed that the spring is perennial and has a well defined outlet, and the flow of the spring is fed from a thick aquifer. The flow domain of the spring is conceptualised as a tank, with vertical walls filled with porous material. The tank has a spout at the bottom (Fig.2.11).

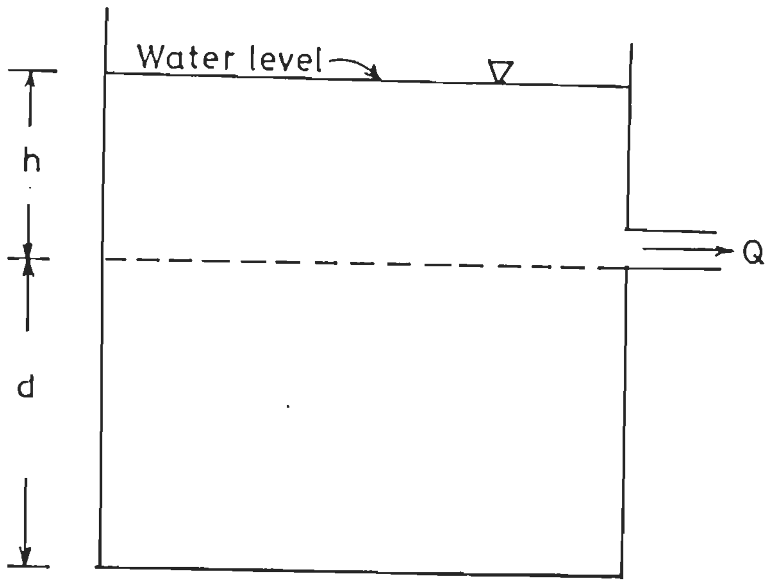


FIG. 2·11 - UNICELL MODEL FOR SPRINGFLOW (MANDEL & SHIFTAN, 1981).

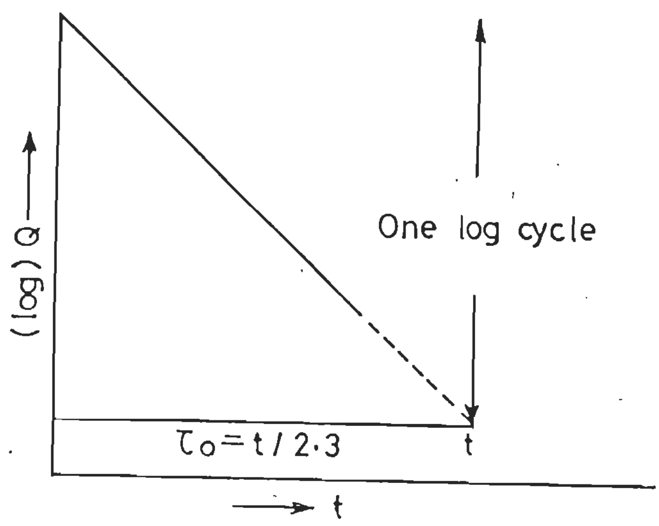


FIG. 2·12 - DETERMINATION OF DEPLETION TIME τ_o.

The flow of the spring during the dry season, is computed from the model. A description of model is given below.

For $d >> h$, $h+d \simeq b =$ Constant

$$V(t) = A\phi h(t) \quad \text{...(2.18)}$$

$$Q(t) = KbCh(t) \quad \text{...(2.19)}$$

$$Q(t) = -\frac{dV}{dt} \quad \text{...(2.20)}$$

where $h(t)$ is the elevation of the water level above the outlet, V is the volume of water stored above the outlet, d the aquifer thickness below the outlet, A is the base area of the tank, ϕ, and K are the storativity and permeability of the aquifer, respectively, and C is a dimensionless parameter representing the flow pattern.

The elimination h (t) from Eqs.(2.18) and (2.19) gives

$$V(t) = A\phi/(KbC) * Q(t) = Q(t) * \tau_0 \quad \text{...(2.21)}$$

and substitution of the derivative $\frac{dV}{dt}$ in Eq.(2.20) yields

$$Q(t) + \tau_0 \frac{dQ}{dt} = 0 \quad \text{...(2.22)}$$

Solving Eq. (2.22) and applying the initial condition that at t=0, $Q=Q_0$, we have

$$Q(t) = Q_0 \exp(-t/\tau_0) \quad \ldots(2.23)$$

or

$$\log Q(t) = \log Q_0 - (1/2.3)* (t/\tau_0) \quad \ldots(2.24)$$

where τ_0 is depletion time.

The variation of $\log_{10} Q(t)$ vs t plots a straight line on a semi logarithmic scale (Fig.2.12).

Aquifer replenishment between the end of one dry season and the beginning of the next dry season can be estimated by Eq. (2.21) with the aid of principle of continuity. Mass balance for the period, t_1 to t_2, during which the replenishment to the spring flow domain occurs, is

$$Q(t_1)\ \tau_0 + \int_{t_1}^{t_2} A\ R(t)\ dt = \int_{t_1}^{t_2} Q\ (t)\ dt + Q(t_2).\tau_0 \quad \ldots(2.25)$$

where R is the replenishment (LT^{-1}), A is the replenishment area of the spring (L^2).

For a thin aquifer, (permeable veener formation overlying an impermeable bedrock) the spout is assumed at the extreme bottom and therefore, the aquifer thickness is a function of time.

So

$$d = 0, \text{ and } b=h(t) \quad \ldots(2.26)$$

$$Q(t) = K\,C\,h^2(t) \qquad \ldots(2.27)$$

Combining Eq.(2.27) with Eqs. (2.18) and (2.19), yields

$$A\,\phi\,\frac{dh}{dt} = -\,K\,C\,h^2(t)$$

and solution of which results in

$$h(t+\Delta t) = h(t)/\ [1+\ KC\ \Delta t\ h(t)/(A\phi)]$$

Squaring both sides of the above expression and then multiplying both sides by KC, the solution becomes

$$KCh^2(t+\Delta t) = KC\ h^2(t)/\ [1+\ \{KC\ \Delta t\ h^2(t)/(A\phi\ h(t))\}]^2$$

or

$$Q(t+\Delta t) = \frac{Q(t)}{(1+\ \Delta t/\tau_0)^2} \qquad \ldots(2.28)$$

where $\tau_0 = V(t)/Q(t)$.

The depletion time τ_0 is determined using Eq.(2.28) by computing the ratio $Q_{(t+\Delta t)}/\ Q_t$ for successive time intervals and averaging the resulting values. The Eq. (2.28) is essentially applicable for intermittent springs.

Model Depicting Influence of Fractures

Kovacs (1981) showed the influence of different sized fractures on springflow. The model reported is very simple. It is conceptualised as the depletion of a cylindrical reservoir. The water leaves through a pipe, which is filled with porous material. The length and area of the cross section of the pipe are l and f respectively and the hydraulic conductivity of material filling the pipe is K. (Fig.2.13a).

At a point of time t, the amount of the outflowing water is equal to the change of storage.

Hence $$-F.dh = Q.dt = fK\ (h/l).dt \qquad \ldots(2.29)$$

$$t = -Fl/(fK)\ln h + C \qquad \ldots(2.30)$$

$$h = h_0 \exp(-a(t-t_0) \qquad \ldots(2.31)$$

$$Q = Q_0 \exp(-a(t-t_0) \qquad \ldots(2.32)$$

where initial flow rate at a point of time t_0 (i.e., $Q_0=h_0Kf/l$) depends on the height of the water level (h_0) at the same point of time. The constant $a = fK/(lF)$ is inversely proportional to the storage capacity of the system.

Now, if two reservoirs are drained through the same outlet (Fig.2.13b), their flow rates have to be added to get the instantaneous discharge at a point of time t.

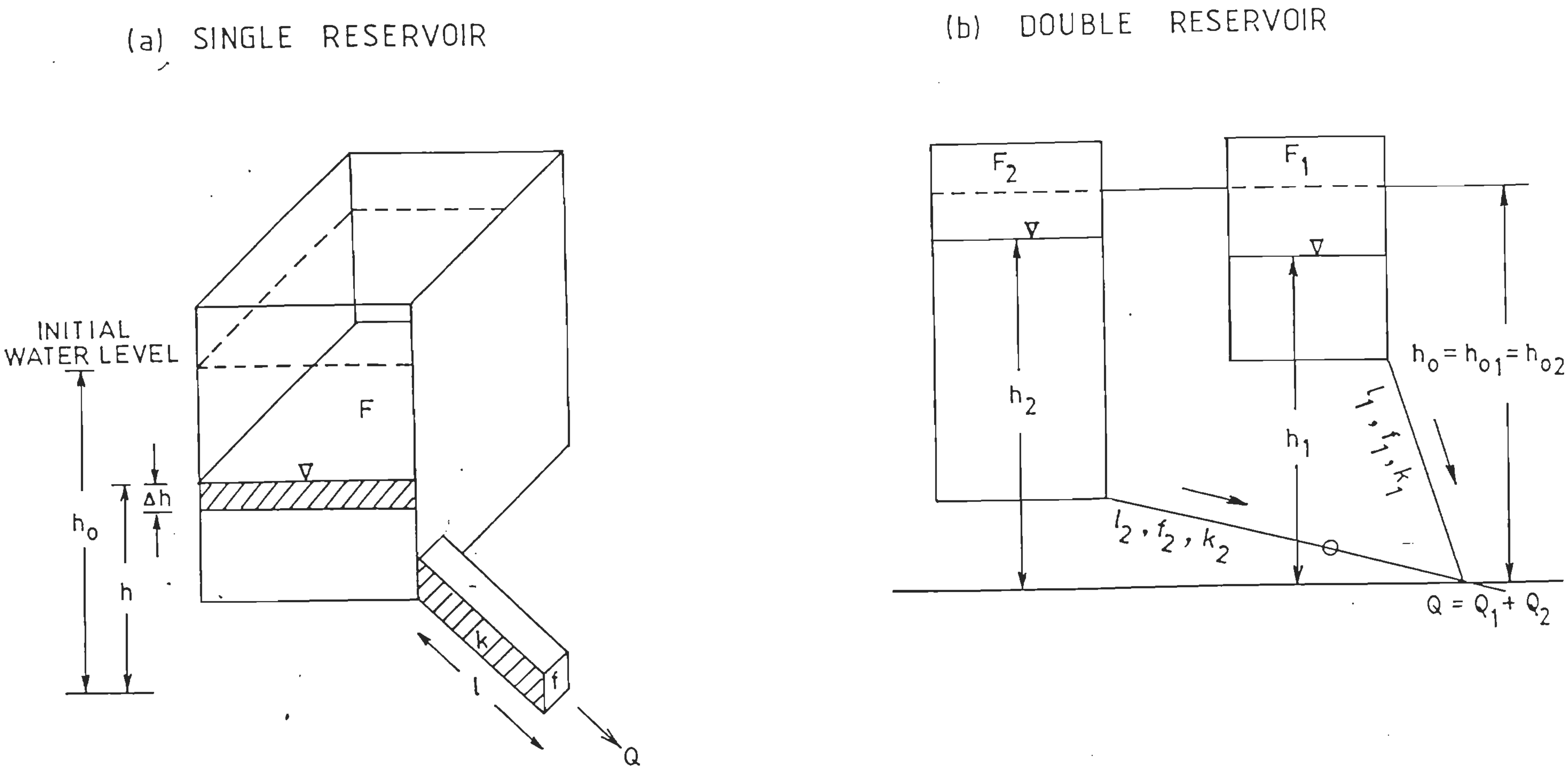


FIG. 2.13 MODEL DEPICTING INFLUENCE OF FRACTURES (AFTER KOVACS, 1981).

Hence $$Q = Q_{01}\exp[-a_1(t-t_0)] + Q_{02}\exp[-a_2(t-t_0)] \qquad \ldots(2.33)$$

The comparison of the springflow estimated by the mathematical model with observed yields from karstic springs proves the accuracy of such approximation of the recession curve and the constants can be calculated from the observed springflow data.

2.3 STATISTICAL ANALYSES OF SPRINGFLOW

2.3.1 Karstic Springs

Almost one third of Turkey is underlain by carbonate rock formations which are mostly karstified. Many a karst spring has emerged from these carbonate rock formations. Ozis and Keloglu (1975) studied a group of karstic springs named Sarikiz springs in Western Turkey for autocorrelation, spectral analysis, lag cross correlation. A comparison of peaks of the springflow with that of precipitation revealed a time lag of approximately 2.5 months for the first and 7.5 months for the second peak. On the basis of topographical and geological conditions, it was assumed that the first lag was indicative of the water travel through karst formations while the second lag represented the flow through alluvium from the flooded surface areas. These large time lags were indicative of extremely large underground water retention. The random element of the annual data is fitted satisfactorily as a normal probability distribution.

Knisel (1972) made time series analyses of 3 Karstic springs in USA. The three springs are from the limestone area of Texas and Missouri, namely, i) the Goodenough spring near Comstock, Texas ii) San Folipe spring at Deh Rio, Texas, and iii) the Big spring near Van Buren, Missouri. Because of rapid response of limestone aquifers to recharge, he inferred that the 1 day time series would be inadequate and suggested the analyses of 12-hour and 6-hr time series.

2.3.2 Springs from Sandstone

The Sulkovy Prameny spring together with certain other springs, form the main drainage of the Lower Turonian sandstone strata in Czechoslovakia. These sandstone strata have large supplies of groundwater through fissures. Monthly discharge data of Sulkovy Prameny spring are available from 1901-70 (Kriz, 1973). The water year for the area is from November to October. The maximum observed discharge of the Sulkovy Prameny springs was 0.2422 m^3s^{-1} (on March 12, 1941) while the minimum observed discharge was 0.0357 m^3s^{-1} (on June 16, 1954) within this 70-year duration. In these 70 years Over one half of the annual maxima, occur in the spring months (March to May) while the remaining are evenly distributed over all the remaining months. The minimum annual discharge of the springs was registered mainly from October to January.

In the statistical treatment of the observed data for 70 years for Sulkovy Pramney springs (Kriz, 1973), the 3658 weekly discharge values of the springs were divided into 22 classes by arranging the weekly discharge values in descending order between 240.1 and 40 litres per sec with a class interval of 10 litres per second. Cumulative frequency for each class of discharge was determined. The cumulative frequency curve of weekly discharge of the springs for the period of 1901-70 was obtained by plotting the mid values of each of the 22 intervals against the relative cumulative frequency expressed in percentage.

Further, Kriz (1973) classified these springs on the basis of ratio of values of discharge read from the cumulative frequency curve at 10% and 90% of the relative cumulative frequency ($Q_{10\%}$ /$Q_{90\%}$) according to a 5-unit scale, namely, (i) extraordinary balanced (1 to 2.5), (ii) well balanced (2.6 to 5), (iii) balanced (5.1 to 7.5), (iv) unbalanced (7.6 to 10.0), and (v) extraordinary unbalanced (more than 10). The range of values of the ratio is given in the brackets for each unit of the scale. The values of discharge for the Sulkovy Pramney springs at 10% and 90% relative cumulative frequency are 148 and 68 litres per second respectively. The Sulkovy Prameny spring is, as such, ranked as "extraordinary balanced" with a ratio value of 2.2. Kriz also classified the water years 1901 to 1970 on the basis of probability of exceedence of a discharge during a year over the annual average discharge for the same year into five classes. The classes are: (i) extraordinary yielding, for a probability lower than 11%, (ii) yielding, for the probability range of 11 to 40%, (iii) average yielding, for the probability range of 41 to 60%, (iv) dry years, for the probability 61 to 90%, and (v) extraordinary dry years if the probability exceeds 90%. On the basis of the classification, out of 70 years, 11 years were "extraordinarily high", 20 years were "high", 11 years were "average", 20 years were "dry", and remaining 8 years could be termed as "extraordinarily dry". He concluded that the variation in the annual average discharge of the springs repeated in a 3 to 6-year cycle. It was found that a 3-year running average of discharge data smoothed out the periodicity of fluctuations in them.

2.4 CONCLUSIONS

From the review of literature following conclusions are made:

(i) Depending upon geology and geomorphology, springs could have various types of flow domain and geohydrological boundary conditions. Only a few of them have so far been investigated and mathematically modelled.

(ii) An active spring has a constant head at the outlet and could serve as a boundary condition of Dirichlet type in mathematical modelling of groundwater.

(iii) The existing models for the simulation of springflow assume one time recharge in the beginning. So, the models should be suitably adapted such that the springflow could be simulated for time variant recharge.

(iv) Assumption that an unsteady state is the succession of steady state conditions in the flow domain of the spring in the existing models, inhibit the simulation of springflow from a spring which have a long transmissive zone with low diffusivity. Hence, there is a need to develop a mathematical model for unsteady flow of a spring.

(v) The flow processes associated with springflow are two-dimensional. The presently available models deal with one-dimensional groundwater flow. As such, in order to simulate springflow from a spring or from a group of springs emerging from the slopes and foothills, a two-dimensional mathematical model is required to simulate springflow.

(vi) Most of the existing models assume that the springflow is linearly proportional to the dynamic storage inside the spring flow domain. This assumption needs verification.

CHAPTER-3

ANALYSIS OF SPRINGFLOW BY BEAR'S MODEL

3.0 INTRODUCTION

Bear's springflow model (1979) simulates the recession portion of a spring hydrograph. The model is based on the assumption that the springflow is linearly proportional to the dynamic storage in the springflow domain. Bear also has interpreted the recession constant to be a product of hydraulic diffusivity of the aquifer and a coefficient representing the geometry of the aquifer. The Bear's conceptual model which simulates springflow during recession and the model which interpretes recession constant have been shown in Fig.2.9 and 2.10 respectively in the earlier chapter. In this chapter integrating the two models of Bear an unsteady flow from a spring is analysed for variable recharge.

3.1 STATEMENT OF THE PROBLEM

Let h_2 be the initial level of groundwater table which is equal to the level of the spring's threshold. Let the spring be inactive before the onset of recharge. The spring flow domain has been conceptualised to be consisting of two parts:- i) a recharge zone, and ii) a transmission zone. The length of the transmission zone is l. A time varying recharge, r(t), through the entire recharge area commences at time t=0 due to which the groundwater table in the recharge zone rises to h(t). Consequently, immediately after the onset of recharge, the springflow emerges from the spout of the spring of width W_s (Fig.3.1). It is required to find the variation of the springflow with time.

Assumptions:

(a) As implied in the Bear's model, the flow in the recharge zone is in the vertical direction, and the flow in the transmission zone is in the horizontal direction. The Dupuit Forchheimer's assumptions are valid in the transmission zone.

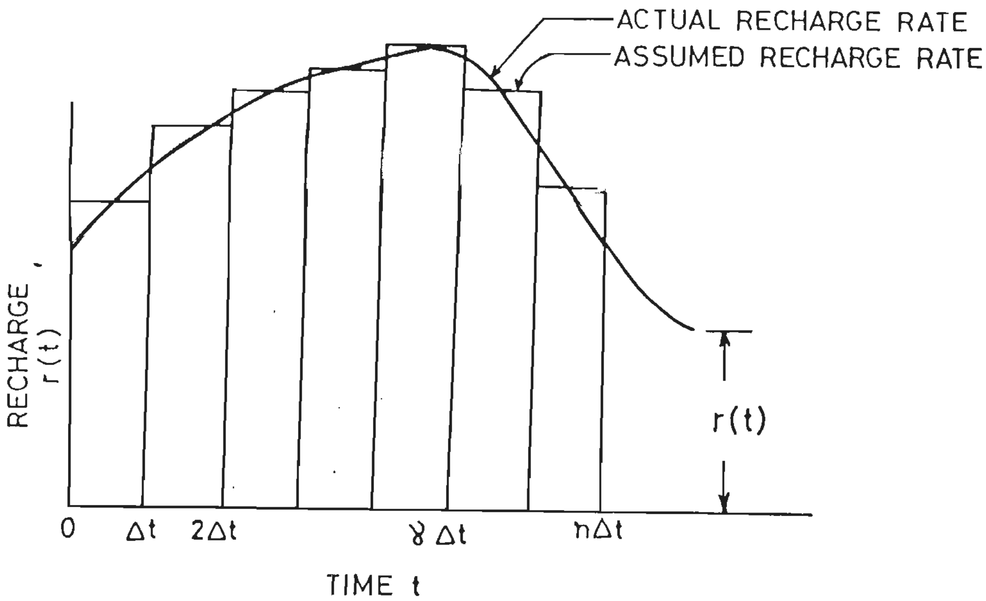
(b) Springflow occurs immediately after a recharge.

(c) In the transmission zone, the unsteady state flow is equivalent to succession of steady state flows.

The assumption mentioned at (b) implies that there is no storage effect in the transmission zone and the flow in the transmission zone is pressure flow.

3.2 ANALYSIS

First an expression for springflow has been derived for a unit impulse recharge and using the response of the spring aquifer system to the unit impulse recharge, the response to a unit pulse perturbation has been derived. The springflow for a time varying input has been obtained using convolution technique and the unit pulse response coefficients.



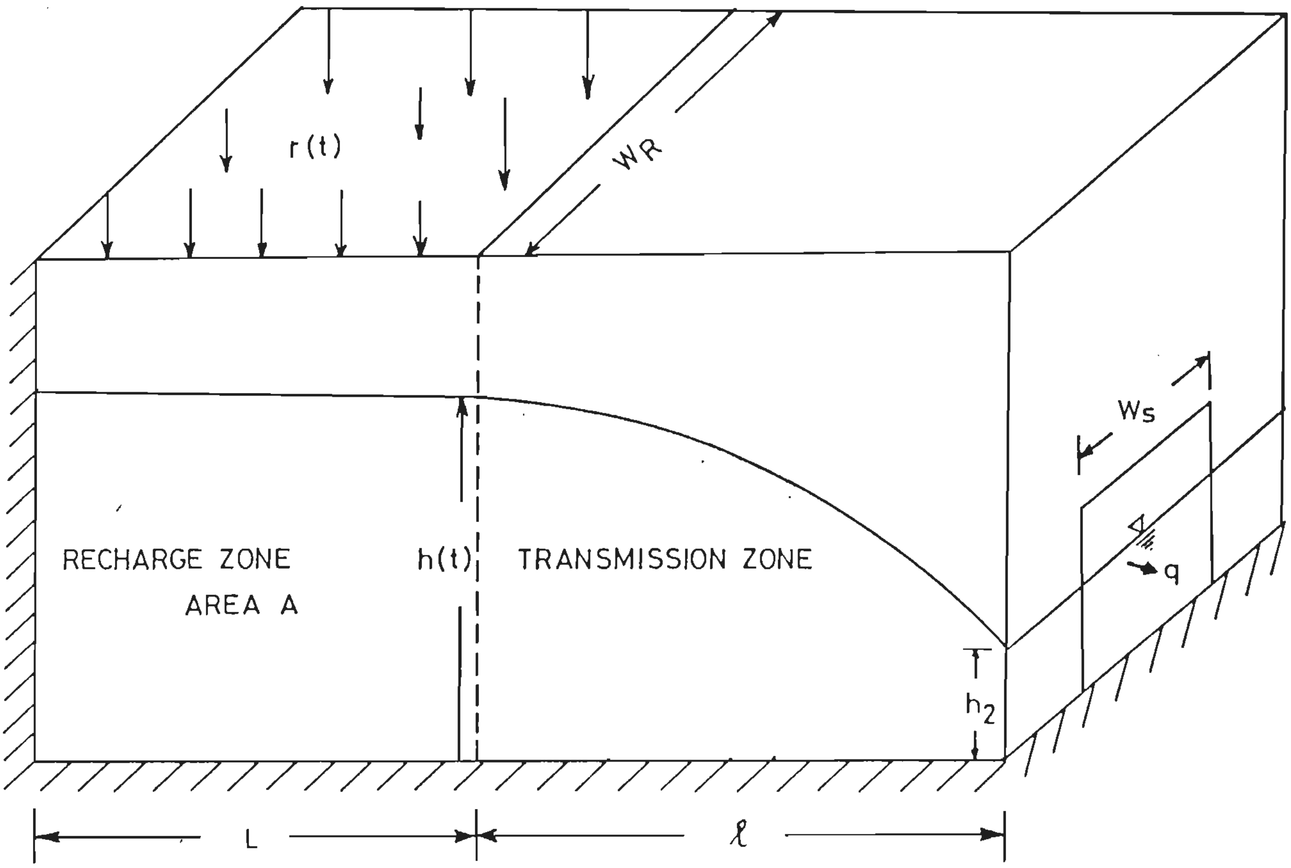


FIG. 3·1 - PROPOSED MODEL.

time t=0. Let the spring be inactive during the time $t < 0$. Since, it has been assumed that there is no storage effect in the transmission zone, and the unsteady flow in the transmission zone is succession of steady state flows, the flow at any section of the transmission zone is equal to the discharge, Q(t), of the spring. The depletion rate from the recharge zone, in turn, is equal to the springflow. A water balance equation for the recharge zone over the time period t to $t+\Delta t$ is

$$\int_{t}^{t+\Delta t} Q(\tau)d\tau \simeq 0.5[Q(t)+Q(t+\Delta t)]\Delta t$$

$$= -\Delta h\ \phi_1 A \qquad \ldots(3.1)$$

where Δh is fall in groundwater table in the recharge zone during the time interval Δt, A is area of the recharge zone and $A=LW_R$, W_R=width of the recharge zone and ϕ_1 is the specific yield of the aquifer in the recharge zone. When $\Delta t \rightarrow 0$, Eq.(3.1) reduces to

$$Q(t) = -\phi_1 A \frac{dh}{dt} \qquad \ldots(3.2)$$

According to the Darcy's law, the flow in the transmission zone is given by

$$Q(t) = W_S T\{h(t)-h_2\}/l \qquad \ldots(3.3)$$

where T = average transmissivity of the aquifer. Combining Eq. (3.2) and Eq. (3.3)

$$\frac{dh}{dt} = - \frac{W_S T}{Al\phi_1} \{h(t) - h_2\} \qquad \ldots(3.4)$$

The watertable height in the recharge zone at time $t \longrightarrow 0$, for the impulse recharge of N per unit area at t=0, is given by

$$h(0) = h_2 + N/\phi_1 \qquad \ldots(3.5)$$

Solution of the linear differential Eq.(3.4) for the initial condition given by Eq.(3.5) is

$$h(t) = h_2 + (N/\phi_1) e^{-\frac{W_S Tt}{Al\phi_1}} \qquad \ldots(3.6)$$

Substituting the expression of h(t) in Eq.(3.3) and defining the term $Al\phi_1/(W_S T) = \tau_0$,

$$\begin{aligned} Q(t) &= \{W_S TN/(l\phi_1)\}\ e^{-t/\tau_0} \\ &= \{W_S TAN/(Al\ \phi_1)\}\ e^{-t/\tau_0} \\ &= R/\tau_0\ e^{-t/\tau_0} \end{aligned} \qquad \ldots(3.7)$$

where R = AN and R is the total recharge in m^3 occuring instantaneously at t=0. The springflow at time t=0 is

$$Q(0) = R/\tau_0 \qquad ...(3.8)$$

Thus, flow Q(t) can be written as

$$Q(t) = Q(0) \quad e^{-t/\tau_0} \qquad ...(3.9)$$

Eq (3.9) is the Bear's equation for springflow. The parameter, τ_0, is the depletion time and is equal to $Al\phi_1/(W_S T)$ as derived by Bear. Eq.(3.4) being linear, the Duhamel's principle can be applied to derive the expression for discharge of a spring due to variable recharge.

Let k(t) be defined as the flow of a spring due to a unit impulse recharge through the entire recharge area, A, taken place at t=0. Therefore, k(t)= Q(t) when R=1. Hence, putting R=1 in Eq.(3.7)

$$k(t) = (1/\tau_0) \; e^{-t/\tau_0} \qquad ...(3.10)$$

For a varying recharge rate r(t), the corresponding springflow rate, q(t), can be expressed as

$$q(t) = \int_0^t k(t-\tau) \; r(\tau) \; d\tau \qquad ...(3.11)$$

Let the time span be discretised by uniform time steps of size Δt. Thus, $t=n\,\Delta t$ where n is a positive integer. The springflow rate at the end of n th time step is

$$q(n\Delta t) = \int_0^{\Delta t} k(n\Delta t-\tau)\ r(\tau)\ d\tau + \int_{\Delta t}^{2\Delta t} k(n\Delta t-\tau)\ r(\tau)\ d\tau + \ldots\ldots +$$

$$\int_{(\gamma-1)\Delta t}^{\gamma\Delta t} k(n\Delta t-\tau)\ r(\tau)\ d\tau + \ldots + \int_{(n-1)\Delta t}^{n\Delta t} k(n\Delta t-\tau)\ r(\tau)\ d\tau \qquad \ldots(3.12)$$

Let the recharge be represented by a train of pulses. Let $R(\gamma)$ be the total recharge during γ th time step. Hence, the recharge rate, $r(\tau)$, during γ th time step $=R(\gamma)/\Delta t$. Assuming that the recharge rate is constant during a time step, but it varies from time step to time step, Eq. (3.12) simplifies to

$$q(n\Delta t) = \sum_{\gamma=1}^{n} R(\gamma)/\Delta t \int_{(\gamma-1)\Delta t}^{\gamma\Delta t} k(n\Delta t-\tau)\ d\tau$$

$$= \sum_{\gamma=1}^{n} R(\gamma)/\Delta t \int_{(\gamma-1)\Delta t}^{\gamma\Delta t} (1/\tau_0) e^{-(n\Delta t-\tau)/\tau_0}\ d\tau$$

$$= \sum_{\gamma=1}^{n} R(\gamma)/\Delta t \int_0^1 (\Delta t/\tau_0)\ e^{-\Delta t\{(n-\gamma+1)-v\}/\tau_0}\ dv$$

$$= \sum_{\gamma=1}^{n} R(\gamma)\{-e^{-(n-\gamma+1)\ \Delta t/\tau_0} + e^{-(n-\gamma)\Delta t/\tau_0}\}/\Delta t \qquad \ldots(3.13)$$

Equation (3.13) could be further simplified in terms of discrete kernel coefficients in the following manner.

Let a discrete kernel coefficient $\delta(\Delta t,n)$ be defined as

$$\delta(\Delta t,n) = \frac{1}{\Delta t}\int_0^{\Delta t} k(\,n\Delta t-\tau\,)\, d\tau \qquad \ldots(3.14)$$

$\delta(\Delta t,n)$ is the response of the spring aquifer system due to a unit pulse perturbation imparted during the first time step. Incorporating the expression of k(t) in Eq.(3.14)

$$\delta(\Delta t,n) = \frac{1}{\Delta t}\int_0^{\Delta t} (1/\tau_0)\, e^{-(n\Delta t-\tau)/\tau_0}\, d\tau \qquad \ldots(3.15)$$

Let $\tau = \nu\,\Delta t$ and $d\tau = \Delta t\; d\nu$. Making these substitutions in Eq.(3.15) and integrating

$$\delta(\Delta t,n) = \frac{1}{\Delta t}\int_0^{1} (\Delta t/\tau_0)\; e^{-\Delta t(n-\nu)/\tau_0}\, d\nu$$

$$= \frac{1}{\Delta t}\,(\Delta t/\tau_0)\; e^{-n\Delta t/\tau_0}\int_0^{1} e^{\nu\Delta t/\tau_0}\, d\nu$$

$$= \{-e^{-n\Delta t/\tau_0} + e^{-(n-1)\Delta t/\tau_0}\}/\Delta t \qquad \ldots(3.16)$$

In terms of discrete kernel coefficients, Eq.(3.13) simplifies to

$$q(n\Delta t) = \sum_{\gamma=1}^{n} R(\gamma)\ \delta(\Delta t, n-\gamma+1) \qquad \ldots(3.17)$$

If the depletion time, τ_0, is in month unit, and Δt is in month unit, the springflow rate, $q(n\Delta t)$ is in the unit of m^3 per month.

An observed springflow at any time n consists of two parts; one part is the response to the recharge that has occurred since the time origin and the other part is the response to the perturbation prior to the time origin. The component $q(n\Delta t)$ corresponds to the recharge taken place since the time origin. Thus, using the convolution technique and the Bear's model, the springflow for a time variant recharge can be simulated.

3.3 THE INVERSE PROBLEM

In the inverse problem, the unknowns are the time varying recharge to the spring flow domain and the depletion time, τ_0. Using the measured discharge of the spring during recharge period and atleast one value of the springflow during the non recharge period, the unknowns can be computed.

Let K(t) be the unit step response function of a spring. The unit step response function K(t), can be evaluated by integrating k(t), the response function for a unit impulse recharge. k(t) has been defined in Eq.(3.10).

Hence

$$K(t) = \int_0^t k(\tau)\,d\tau = \int_0^t (1/\tau_0)\exp(-\tau/\tau_0)\,d\tau$$

$$= 1 - e^{-t/\tau_0}$$

or $K(n\Delta t) = 1 - \{\exp(-n\Delta t/\tau_0)\}$

$$= 1 - y^n \qquad \text{...(3.18)}$$

where $y = \exp(-\Delta t/\tau_0)$ and Δt = unit time step.

Further, the discrete kernel coefficient, $\delta(\Delta t, n)$, can be expressed in terms of a unit step response function, $K(n\Delta t)$, as

$$\delta(\Delta t, n) = \{K(n\Delta t) - K(n\Delta t - \Delta t)\}/\Delta t$$

Hence

$$\delta(\Delta t, 1) = K(\Delta t)/\Delta t = (1-y)/\Delta t$$

$$\delta(\Delta t, 2) = \{K(2\Delta t) - K(\Delta t)\}/\Delta t = (y - y^2)/\Delta t$$

... ...

$$\delta(\Delta t, n) = \{K(n\Delta t) - K(n\Delta t - \Delta t)\}/\Delta t = (y^{n-1} - y^n)/\Delta t$$

...(3.19)

The component of the springflow, $Q_B(n\Delta t)$, which is the response to the perturbation prior to the time origin is given by

$$Q_B(n\Delta t) = Q(0)\ \exp(-n\Delta t/\tau_0) = Q(0)\ y^n \quad \ldots(3.20)$$

where Q(0) is the springflow at time t=0.

Hence

$$\begin{aligned} Q_B(\Delta t) &= Q(0)\ y \\ Q_B(2\Delta t) &= Q(0)\ y^2 \\ \ldots &\quad \ldots \\ Q_B(\gamma\Delta t) &= Q(0)\ y^\gamma \\ \ldots &\quad \ldots \\ Q_B(n\Delta t) &= Q(0)\ y^n \end{aligned} \quad \ldots(3.21)$$

where $Q_B(\gamma\ \Delta t)$ is the component of springflow rate at $t=\gamma\Delta t$ to perturbation before time t=0.

Let $Q(\Delta t)$, $Q(2\Delta t)$, $Q(3\Delta t)\ldots,Q\{(n-1)\Delta t\}$, $Q(n\Delta t)$ are the springflow rate and R(1), R(2), R(3),....R(n-1) are the values of corresponding recharge which are unknowns. The recharge R(n) is equal to zero.

The springflow rate at the end of n th time step, $Q(n\Delta t)$, is given by

$$\sum_{\gamma=1}^{n} R(\gamma)\ \delta(\Delta t, n-\gamma+1) + Q(0)e^{-n\Delta t/\tau_0} = Q(n\Delta t)$$

or
$$\sum_{\gamma=1}^{n} R(\gamma)\ \delta(\Delta t, n-\gamma+1) + Q(0)e^{-n\Delta t/\tau_0} - Q(n\Delta t) = 0$$

or
$$\sum_{\gamma=1}^{n} R(\gamma)\ (y^{n-\gamma} - y^{n-\gamma+1})/\Delta t + Q(0)y^{n} - Q(n\Delta t) = 0$$
...(3.22)

Let $r(\gamma)=R(\gamma)/\Delta t$. The equations for each of the n observations are

$r(1)(1-y) + Q(0)y - Q(\Delta t) = 0$

$= f_1[r(1), r(2), r(3), \ldots\ldots\ldots\ldots r(n-1), y]$

$r(1)(y-y^2) + r(2)(1-y) + Q(0)y^2 - Q(2\Delta t) = 0$

$= f_2[r(1), r(2), r(3), \ldots\ldots\ldots\ldots r(n-1), y]$

$r(1)(y^2-y^3)+r(2)(y-y^2)+r(3)(1-y)+Q(0)y^3-Q(3\Delta t) = 0$

$= f_3[r(1), r(2), r(3), \ldots\ldots\ldots\ldots r(n-1), y]$

... ...

$$r(1)(y^{n-1} - y^{n}) + r(2)(y^{n-2} - y^{n-1}) + r(\gamma)(y^{n-\gamma} - y^{n-\gamma+1}) + \ldots +$$

$$r(n-1)\{y^{n-(n-1)} - y^{n-(n-2)}\} + Q(0)y^{n} - Q(n\Delta t) = 0$$

$$= f_n[r(1),\ r(2),\ r(3),\ \ldots\ldots\ldots\ldots r(n-1),\ y] \qquad \ldots(3.23)$$

In Eq.(3.23), r(1), r(2), r(3),..,r(n-1) and y are the n unknowns which are to be solved from the n equations. The procedure for solving the set of nonlinear equations by Newton Raphson iterative technique is well documented (Carnahan, 1969) and is adopted here for finding the unknowns.

Let, $\mathbf{r} = \bar{\mathbf{r}}$ be a solution of the nonlinear system of Eqs.(3.23) in which the vectors, $\mathbf{r}$ and $\bar{\mathbf{r}}$ are

$$\mathbf{r} = [r(1),\ r(2),\ \ldots\ldots\ldots\ldots,r(n-1),\ y]^T$$

$$\bar{\mathbf{r}} = [\bar{r}(1),\ \bar{r}(2),\ \ldots\ldots\ldots\ldots\bar{r}(n-1),\ y\ ]^T$$

This means $f_1(\bar{\mathbf{r}}) = f_2(\bar{\mathbf{r}}) = \ldots\ldots\ldots = f_n(\bar{\mathbf{r}}) = 0$

If $\mathbf{r}$ approximates $\bar{\mathbf{r}}$, the increment from $\mathbf{r}$ to $\bar{\mathbf{r}}$ is

$$\Delta\mathbf{r} = \bar{\mathbf{r}} - \mathbf{r} = \begin{bmatrix} \bar{r}(1) - r(1) \\ \bar{r}(2) - r(2) \\ \ldots\ \ldots \\ \bar{r}(n-1) - r(n-1) \\ \bar{y} - y \end{bmatrix} = \begin{bmatrix} \Delta r(1) \\ \Delta r(2) \\ \ldots \\ \Delta r(n-1) \\ \Delta y \end{bmatrix}$$

The problem is to find the vector $\Delta \mathbf{r}$, i.e., to find the direction and distance to move from $\mathbf{r}$ to $\bar{\mathbf{r}}$. Rather than seeking the exact increment $\Delta \mathbf{r}$ that satisfies $f_i(\mathbf{r}+\Delta \mathbf{r})=0$ for all i, the Newton-Raphson technique suggests to find an approximate increment, **dr**, that satisfies more easily the linear approximation of

$$f_i(\mathbf{r}+\mathbf{dr})=0 \qquad \text{for} \quad i= 1, 2, \ldots n$$

in which $\mathbf{dr} = [dr(1), dr(2), \ldots . dr(n-1), dy]^T$

This means that

$$f_1(r) + \frac{\partial f_1}{\partial r(1)} dr(1) + \frac{\partial f_1}{\partial r(2)} dr(2) + \ldots + \frac{\partial f_1}{\partial r(n-1)} dr(n-1) + \frac{\partial f_1}{\partial y} dy = 0$$

$$f_2(r) + \frac{\partial f_2}{\partial r(1)} dr(1) + \frac{\partial f_2}{\partial r(2)} dr(2) + \ldots + \frac{\partial f_2}{\partial r(n-1)} dr(n-1) + \frac{\partial f_2}{\partial y} dy = 0$$

$$\ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots$$

$$f_{n-1}(r) + \frac{\partial f_{n-1}}{\partial r(1)} dr(1) + \frac{\partial f_{n-1}}{\partial r(2)} dr(2) + \ldots + \frac{\partial f_{n-1}}{\partial r(n-1)} dr(n-1) + \frac{\partial f_{n-1}}{\partial y} dy = 0$$

$$f_n(r) + \frac{\partial f_n}{\partial r(1)} dr(1) + \frac{\partial f_n}{\partial r(2)} dr(2) + \ldots + \frac{\partial f_n}{\partial r(n-1)} dr(n-1) + \frac{\partial f_n}{\partial y} dy = 0$$

...(3.24)

In matrix notation the Eqs.(3.24) can be written as

$$\begin{bmatrix} \frac{\partial f_1}{\partial r(1)} & \frac{\partial f_1}{\partial r(2)} & \cdots & \frac{\partial f_1}{\partial r(n-1)} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial r(1)} & \frac{\partial f_2}{\partial r(2)} & \cdots & \frac{\partial f_2}{\partial r(n-1)} & \frac{\partial f_2}{\partial y} \\ \cdots & & \cdots & & \cdots \\ \frac{\partial f_{n-1}}{\partial r(1)} & \frac{\partial f_{n-1}}{\partial r(2)} & \cdots & \frac{\partial f_{n-1}}{\partial r(n-1)} & \frac{\partial f_{n-1}}{\partial y} \\ \frac{\partial f_n}{\partial r(1)} & \frac{\partial f_n}{\partial r(2)} & \cdots & \frac{\partial f_n}{\partial r(n-1)} & \frac{\partial f_n}{\partial y} \end{bmatrix} \begin{bmatrix} dr(1) \\ dr(2) \\ \cdots \\ dr(n-1) \\ dy \end{bmatrix} = - \begin{bmatrix} f_1 \\ f_2 \\ \cdots \\ f_{n-1} \\ f_n \end{bmatrix}$$

or $$[a][b] = [c] \qquad \ldots(3.25)$$

where [a] is the left hand side square matrix and is known as Jacobian matrix, [b] is the left hand side column matrix, and [c] is the right hand side column matrix.

In Eqs.(3.25), the partial derivatives, $\frac{\partial f_i}{\partial r(j)}$, and f_i are evaluated at currently known values of $\mathbf{r}=\mathbf{r}^*$.

The elements, a(i,j), of the Jacobian matrix for the present problem are

$a(1,1) = 1-y^*$; $a(1,j)=0$ for $j = 2, \ldots, (n-1)$; and

$a(1,n) = -r^*(1)+Q(0)$

$a(2,1) = y^* - y^{*2}$; $a(2,2)=1-y^*$; $a(2,j)=0$ for $j = 3, \ldots; (n-1)$, and

$a(2,n) = -r^*(2)+r^*(1)(1-2y^*) + 2Q(0)y^*$

$a(n,1) = y^{*(n-1)} - y^{*n}$; $a(n,2) = y^{*(n-2)} - y^{*(n-1)}$;

$a[n,(n-1)] = y^* - y^{*2}$; and

$$a(n,n) = r^*(1)[(n-1)y^{*(n-2)} - ny^{*(n-1)}]$$
$$+r^*(2)[(n-2)y^{*(n-3)} - (n-1)y^{*(n-2)}]$$
$$+r^*(3)\ [(n-3)y^{*(n-4)} - (n-2)y^{*(n-3)}]+\ldots$$
$$+r^*(n-1)(1-2y^*)+nQ(0)y^{*(n-1)} \qquad \ldots(3.26)$$

Following matrix inversion

$$[b]=[a]^{-1}[c] \qquad \ldots(3.27)$$

To start with, an initial guess of $r^*(1)$, $r^*(2)$, ..., y^* is made and **dr** is solved using Eq.(3.27). Improved $\mathbf{r}^*$ values are then obtained by adding the increment **dr** to the earlier $\mathbf{r}^*$. The iteration is continued till the modulus of the difference between two successive iterated values is less than a small prescribed value.

3.4 RESULTS AND DISCUSSIONS

Bear's model, which can simulate springflow for a time variant recharge, is based on solution of a linear differential equation. This implies that the spring aquifer system in Bear's model is a linear system. Therefore, it will be appropriate to recheck that the springflow computed using Bear's model follows the principles of proportionality and superposition.

For verification of the principle of proportionality, the simulated springflow corresponding to a pulse recharge of 50 million cubic meter in the first month was compared with the simulated springflow for a pulse recharge of 100 million cubic meter in the first month. The springflow for the latter case is found to be twice that of the former case. The respective springflow values simulated for the two cases are presented in the Table 3.1. In order to verify the principle of superposition, springflow for three different recharge scenarios, viz., (i) 100 million cubic meter of recharge in the first month and no recharge thereafter, (ii) 50 million cubic meter of recharge in the second month and no recharge prior and afterwards, and (iii) 100 and 50 million cubic meter of recharge in the first and the second month, respectively and no recharge afterwards, is compared. The comparison of the simulated springflow for these three cases is presented in Table 3.2. The summation of monthly springflow simulated for the case (i) and case (ii) is equal to the monthly springflow simulated in case (iii). A depletion time of 8 month has been used to compute the springflow. From the results shown in Tables (3.1) and (3.2), it is verified that the Bear's springflow model is a linear flow model as expected.

Table 3.1 Springflow values for two different pulse recharge computed using a depletion time, τ_0 = 8 month, Δt=1 month

Time (month)	Springflow in million cubic meter per month for a pulse recharge of 50 million cu m during the first time step	Springflow in million cubic meter per month for a pulse recharge of 100 million cu m during the first time step
1	5.875	11.750
2	5.185	10.370
3	4.576	9.152
4	4.038	8.076
5	3.563	7.126
6	3.145	6.290
7	2.775	5.550
8	2.449	4.898
9	2.161	4.322
10	1.907	3.814
11	1.683	3.366
12	1.485	2.970

Table.3.2 Simulated springflow for different pulse recharges during different time steps, τ_0 = 8 month, Δt=1 month

Time (month)	Springflow in million cubic meter per month for		
	Recharge(1) =100 million cubic meter	Recharge(1)=0 Recharge(2) =50 million cubic meter	Recharge(1) =100 M cu m Recharge(2) =50 million cubic meter
1	11.750	0.000	11.750
2	10.370	5.875	16.245
3	9.151	5.185	14.336
4	8.076	4.576	12.652
5	7.127	4.038	11.165
6	6.289	3.563	9.852
7	5.550	3.145	8.695
8	4.898	2.775	7.673
9	4.323	2.449	6.772
10	3.815	2.161	5.976
11	3.366	1.907	5.273
12	2.971	1.683	4.654

The unit pulse response coefficients, $\delta(\Delta t,n)$, are the springflow due to a unit pulse recharge taken place during the first time step and is defined in Eq.(3.14). Two sets of coefficients are generated using Eq.(3.16) for τ_0= 8 and 16 month and the variations of $\delta(\Delta t,n)$ with time are presented in Fig.3.2. $\delta(\Delta t,n)$ being the springflow due to a unit pulse recharge imparted during the first time step through the entire recharge zone, area under each of the graphs is one. The plot of logarithm of $\delta(\Delta t,n)$ values with n, presented in Fig.3.3, follows a straight line beyond n=1 and the depletion time estimated from the slope of the straight line is equal to the depletion time used to generate $\delta(\Delta t,n)$. The linear variation of log $\delta(\Delta t,n)$ with n during recession verifies the linear relationship of springflow with dynamic storage during recession period.

Assuming τ_0= 8 month, springflow corresponding to a set of time varying recharge is computed and presented in Fig.3.4. The time varying recharge assumed is indicated in the figure. It is noticed from the graph that if during two or more consecutive time steps, slope of the graph does not change from negative to positive, and magnitude of the slope remains the same, then during these periods there is neither recharge to the springflow domain nor abstraction from it. Further, it is observed that the plot of logarithm of simulated springflow versus time, for a time varying recharge and abstraction, follows a straight line only during the period of no recharge and no abstraction. The depletion time computed from the slope of the straight line during period of no recharge and no abstraction is same as the depletion time

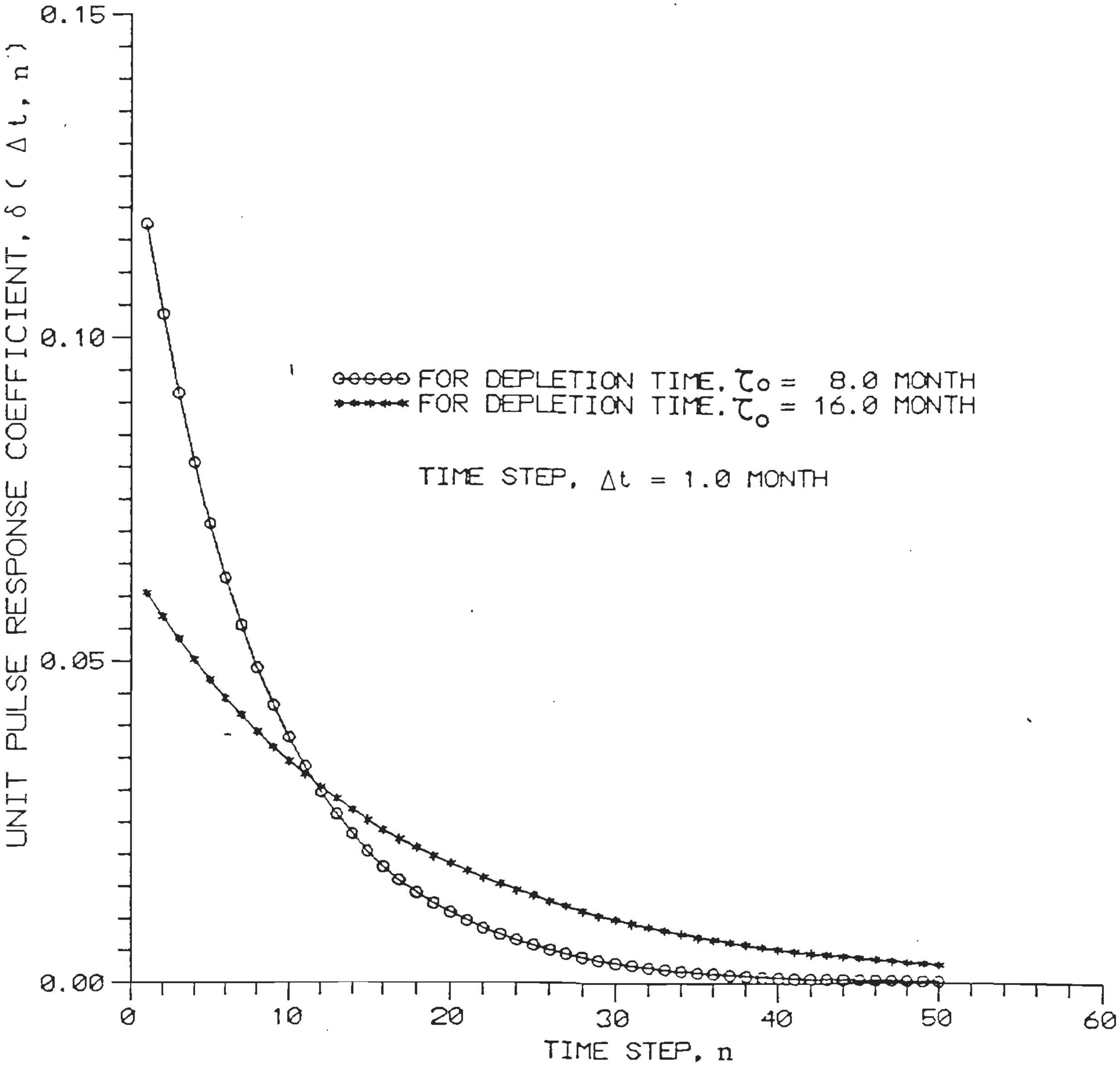


FIG. 3.2 - VARIATION OF UNIT PULSE RESPONSE COEFFICIENTS WITH TIME STEP.

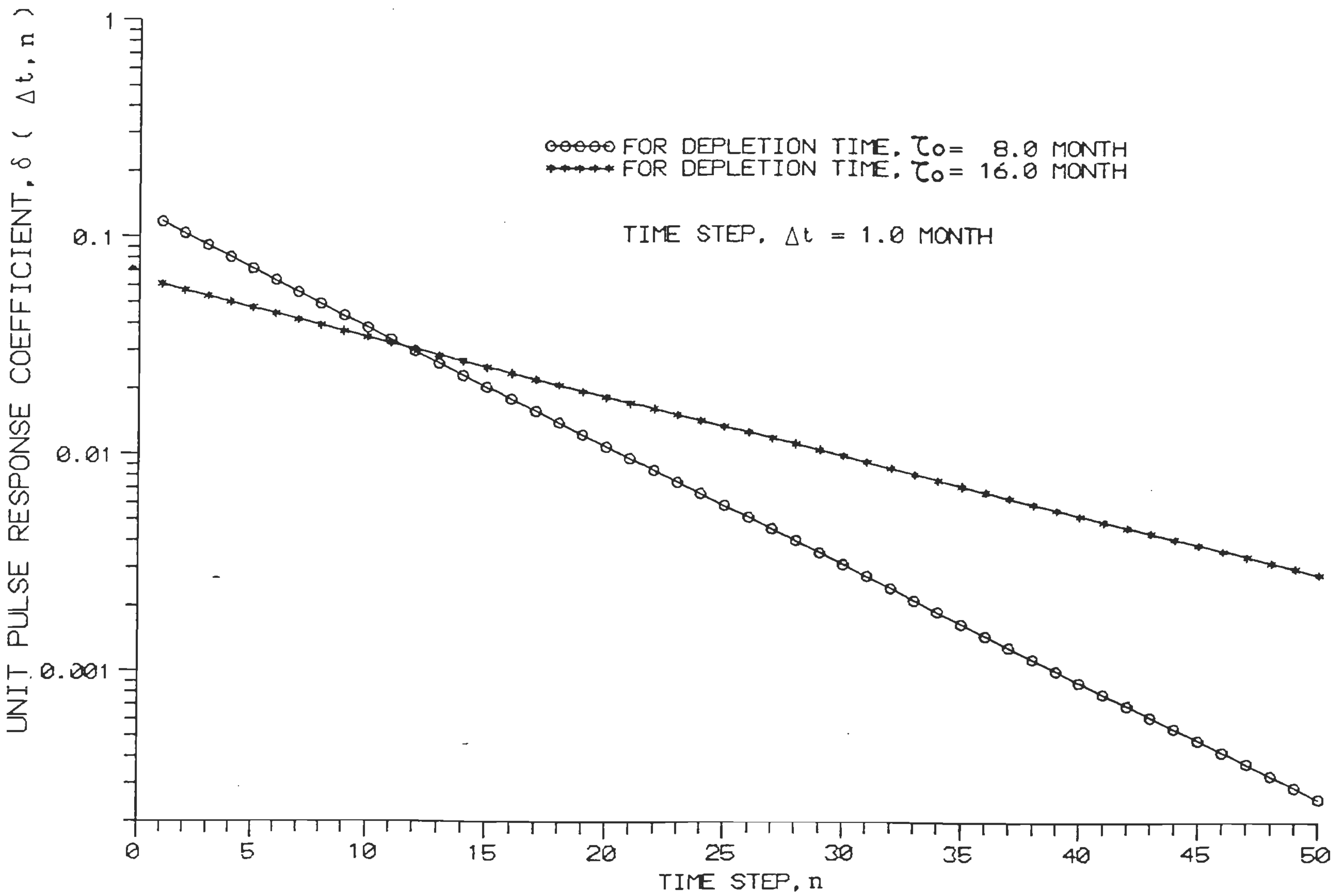


FIG. 3.3 - VARIATION OF LOGARITHM OF UNIT PULSE RESPONSE COEFFICIENTS WITH TIME STEP.

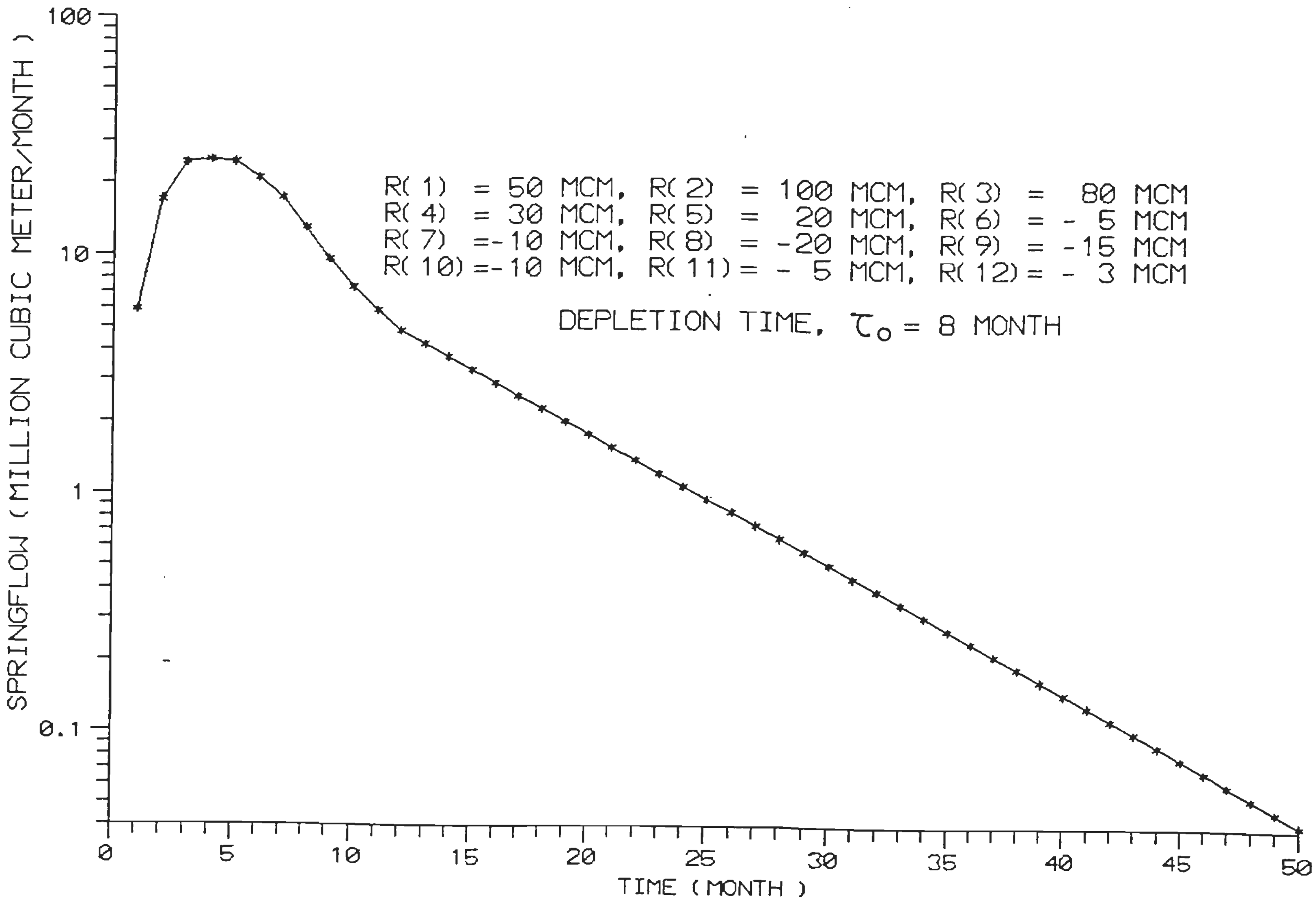


FIG. 3.4 - PLOT OF LOGATHRIM OF SPRINGFLOW WITH TIME.

parameter used to simulate the springflow by the model. As such the periods of no recharge and no abstraction, could be ascertained from the slope of the graph of logarithm of the observed springflow versus time. The slope, α_2, of the graph $\log_{10}Q(t)$ versus time t can be expressed as

$$\alpha_2=(\log_{10}Q(t+\Delta t) - \log_{10}Q(t))/\Delta t \qquad ...(3.28)$$

For recession period, the slope is negative and the negative sign can be used to segregate recession period from the recharge period.

Estimation of Recharge and Depletion Time by the Newton-Raphson Method

The Bear's springflow model has been applied to three springs for which reasonably long springflow records are available. The three springs are: (i) Sulkovy Pramney spring, Czechoslovakia, emerging from sandstone strata (a third magnitude spring) (Kriz, 1973), (ii) Kirkgoz spring, Turkey, emerging from karstic aquifer (a first magnitude spring) (Korkmaz, 1990), and (iii) White Rock spring, Nevada from perched waters in volcanics tuffs (a eighth magnitude spring). From the available springflow data, the respective periods of springflow which have been used in the analyses are: (i) Sulkovy Pramney spring- March, 1961 to July,1969 (101 months), (ii) Kirkgoz spring- October, 1973 to May,1981 (92 months), and (iii) White Rock spring- October, 1982 to July, 1986 (46 months).

At the outset,it has been assumed that all the springflow data are free from any error. Since monthly springflow data are reported, the time step size has been taken as one month. From the springfflow data for each spring, the slope of the graph of logarithm of springflow versus time for each time step is calulated using Eq.(3.28). The consecutive time steps in which the slopes do not change their sign and do not differ much, say about 20% or so, are the time steps of no recharge or no abstraction from the spring flow domain.

After ascertaining the period of no recharge and no abstraction from the the spring flow domain, the Newton-Raphson method for solving the set of non-linear equations has been used to compute the time varying recharge and to estimate the depletion time parameter, τ_0, for each of the three springs. The springflow values for each spring have been so arranged to form a number of sets such that the last springflow value in each set corresponds to a period of no recharge and no abstraction. The selection of the last value of the springflow in a set is a critical task and recharge or abstraction must be zero at that time step. However, there may be other time steps in the set where the recharge or abstraction is zero. The following guidelines are followed to select the last value of a set.

1. There should be two or more consecutive negative values of the slopes.
2. The magnitude of these negative slopes should not differ much.

3. If the Jacobian does not exist for a set, springflow values adjacent to the last time step should be tried.
4. The last value of the set could be the lowest springflow value in a set.
5. Abstraction rate and depletion time obtained for a selected set should not be abnormal.

Sets of springflow values for each of the three springs are formed with these guidelines. The first value of springflow in a set of data has been used to estimate the component of springflow due to perturbations prior to the time origin. The last value of a set is taken as the first value of springflow for the succeeding set. Initial guess of depletion time parameter, τ_0, and recharge are made. It has been found that the Newton-Raphson method performs well with any initial guess of the model parameter, τ_0, and the recharge values. The iteration continued till the difference between two successive iterated values of $\mathbf{R}^*$ became less than 0.001.

The depletion times computed by all sets for a spring are averaged and the recharge is computed for the average depletion time. The computed recharge and depletion time parameters by the Newton-Raphson method and the recharge computed with the average depletion time by the Bear's model for each of the springs are presented in Tables 3.3, 3.4, and 3.5. The selected sets of springflow for the Newton-Raphson method are indicated in the Tables.

For the Kirkgoz spring, Turkey, the summations of monthly recharge for water years 1974 to 1980 (October to September), are compared with the annual recharge for these water years computed by Korkmaz (1990). It is found that the recharge values computed by the model are of comparable magnitudes with the annual recharge values estimated by Korkmaz, except for the water year 1978. The comparison is presented in Table 3.6.

Table 3.3(a) Observed springflow, recharge and depletion time computed by Newton Raphson method, for Sulkovy Pramney spring, Czechoslovakia

Time (month)	Observed Springflow (10^6 cu m per month)	Slope (α_2) (month^{-1})	Pulse recharge computed by: Newton-Raphson method (10^6 cu m)	Pulse recharge computed by: Bear's Model for average τ_0=25.8 month (10^6 cu m)
0(3/61)	.16615			
1	.17680	.1412	.49	.45
2	.24470	.0628	2.25	1.96
3	.28280	-.0183	1.41	1.25
4	.27110	-.0042	-.08	-.02
5	.26850	.0083	.19	.20
6	.27370	-.0176	.42	.41
7	.26280	.0331	-.06	-.01
8	.28360	-.0318	.89	.81
9	.26360	-.0081	-.32	-.24
10	.25870	.1318	.11	.13
11	.35040	.0083	3.06	2.67
12	.35720	-.0685	.55	.53
13	.30510	.0268	-1.23	-1.01
14	.32450	.0028	.89	.82
15	.32660	-.0501	.38	.38
16	.29100	-.0157	-.76	-.61
17	.28070	-.0134	-.02	.02
18	.27220	-.0144	.02	.06
19	.26330	.0184	.00	.04
			τ_0=30.06 month	

Table 3.3(b) Observed springflow, recharge and depletion time computed by Newton Raphson method, for Sulkovy Pramney spring, Czechoslovakia

Time	Observed Springflow	Slope (α_2)	Pulse recharge computed by	
			Newton-Raphson method	Bear's Model for average τ_0=25.8 month
(month)	(10^6 cu m per month)	($month^{-1}$)	(10^6 cu m)	(10^6 cu m)
19	.263300			
20	.27470	-.0070	.46	.56
21	.27030	-.0144	.19	.16
22	.26150	-.0038	.11	.04
23	.25920	.0057	.22	.20
24	.26260	.0138	.32	.35
25	.27110	.0243	.41	.49
26	.28670	-.0112	.55	.68
27	.27940	-.1048	.15	.09
28	.21950	-.0586	-.79	-1.30
29	.19180	.0063	-.27	-.51
30	.19460	-.0436	.24	.27
31	.17600	-.0307	-.13	-.29
32	.16400	-.0209	-.04	-.14
33	.15630	-.0296	.03	-.03
34	.14600	.0856	-.03	-.11
35	.17780	.0347	.71	.98
36	.19260	.1628	.44	.57

continued...Table 3.3(b)

Time	Observed Springflow	Slope (α_2)	Pulse recharge computed by	
			Newton-Raphson method	Bear's Model for average τ_0=25.8 month
(month)	(10^6 cu m per month)	($month^{-1}$)	(10^6 cu m)	(10^6 cu m)
37	.28020	-.1819	1.75	2.50
38	.18430	-.0364	-1.43	-2.24
39	.16950	-.0778	-.08	-.20
40	.14170	.0218	-.32	-.56
41	.14900	-.0513	.27	.33
42	.13240	.0460	-.15	-.29
43	.14720	.0053	.39	.52
44	.14900	.0194	.17	.19
45	.15580	-.0232	.27	.33
46	.14770	-.0791	.02	-.06
47	.12310	.2215	-.29	-.50
48	.20500	.1324	1.59	2.28
49	.27810	-.0622	1.51	2.13
50	.24100	.2108	-.38	-.70
51	.39160	-.0656	2.93	4.20
52	.33670	-.0962	-.59	-1.05
53	.26980	-.0248	-.86	-1.42
54	.25480	-.0036	.00	-.12
55	.25270	-.0504	.21	.20
56	.22500	-.0232	-.24	-.47

continued...Table 3.3(b)

Time (month)	Observed Springflow (10^6 cu m per month)	Slope (α_2) ($month^{-1}$)	Pulse recharge computed by: Newton-Raphson method (10^6 cu m)	Pulse recharge computed by: Bear's Model for average τ_0=25.8 month (10^6 cu m)
57	.21330	-.0037	.02	-.08
58	.21150	.1297	.18	.16
59	.28510	.0343	1.52	2.14
60	.30850	-.0191	.70	.90
61	.29520	-.0264	.07	-.04
62	.27780	-.0415	-.02	-.16
63	.25250	.0488	-.17	-.38
64	.28253	.1691	.78	1.04
65	.41705	-.0778	2.68	3.82
66	.34862	-.0532	-.80	-1.38
67	.30845	-.0397	-.37	-.71
68	.28149	.0188	-.17	-.40
69	.29393	.0027	.50	.61
70	.29575	.1342	.32	.34
71	.40280	.0116	2.21	3.11
72	.41368	-.0036	.59	.69
73	.41031	-.0460	.35	.33
74	.36910	-.0203	-.32	-.67
75	.35225	-.0250	.07	-.07
76	.33255	-.0430	.00	-.17
			τ_0=17.38 month	

Table 3.3(c) Observed springflow, recharge and depletion time computed by Newton Raphson method, for Sulkovy Pramney spring, Czechoslovakia

Time	Observed Springflow	Slope (α_2)	Pulse recharge computed by	
			Newton-Raphson method	Bear's Model for average τ_0=25.8 month
(month)	(10^6 cu m per month)	(month^{-1})	(10^6 cu m)	(10^6 cu m)
76	.33255			
77	.30119	-.0094	-.62	-.49
78	.29471	-.0497	.10	.13
79	.26283	-.0100	-.67	-.54
80	.25687	-.0486	.08	.10
81	.22965	.0646	-.57	-.46
82	.26646	.0665	1.35	1.20
83	.31052	.0516	1.60	1.43
84	.34966	.0058	1.50	1.34
85	.35433	-.0573	.48	.47
86	.31052	-.0966	-.97	-.80
87	.24857	-.0749	-1.57	-1.32
88	.20917	.0531	.95	-.79
89	.23639	-.0160	1.03	.93
90	.22784	-.0100	-.02	.01
91	.22265	-.0046	.07	.09
92	.22032	-.0193	.15	.16
93	.21073	-.0065	-.07	-.03
94	.20762	.0389	.11	.13
95	.22706	.0657	.80	.72
96	.26412	.0593	1.35	1.20
97	.30275	-.0421	1.44	1.28
98	.27475	-.0116	-.54	-.43
99	.26749	-.0145	.05	.08
100	.25868	.0048	.00	.03
			τ_0=29.89	

Table 3.4(a) Observed springflow, recharge and depletion time computed by Newton Raphson method, for Kirkgoz spring, Turkey

Time (month)	Observed Springflow (10^6 cu m per month)	Slope (α_2) ($month^{-1}$)	Pulse recharge computed by: Newton-Raphson method (10^6 cu m)	Pulse recharge computed by: Bear's Model for average τ_0=6.1 month (10^6 cu m)
0(10/73)	28.12	-.0017		
1	28.01	-.0075	26.86	27.44
2	27.53	.0584	22.52	24.76
3	31.49	.1933	72.78	53.76
4	49.14	-.0060	233.18	148.23
5	48.47	-.0089	41.48	44.69
6	47.49	-.0088	37.27	41.96
7	46.53	-.0365	36.51	41.19
8	42.79	-.0632	3.79	21.75
9	36.99	-.0851	-23.48	04.47
10	30.41	-.0431	-38.20	-06.53
11	27.54	-.0398	-02.38	11.42
12	25.13	.0350	00.00	11.58
			τ_0=10.92 month	

Table 3.4(b) Observed springflow, recharge and depletion time computed by Newton Raphson method, for Kirkgoz spring, Turkey

Time (month)	Observed Springflow (10^6 cu m per month)	Slope (α_2) ($month^{-1}$)	Pulse recharge computed by Newton-Raphson method (10^6 cu m)	Pulse recharge computed by Bear's Model for average τ_0=6.1 month (10^6 cu m)
12	25.13			
13	27.24	-.0024	39.13	39.08
14	27.09	.0490	26.24	26.26
15	30.33	.0458	48.58	48.50
16	33.70	.0383	52.69	52.61
17	36.81	.0380	54.33	54.27
18	40.18	.0343	59.17	59.09
19	43.48	-.0192	62.07	62.03
20	41.60	-.1146	31.00	31.05
21	31.95	-.0240	-22.43	-22.22
22	30.23	-.0742	20.53	20.57
23	25.48	-.0710	-01.28	-1.16
24	21.64	.0762	00.00	00.08
			τ_0 = 6.12 month	

Table 3.4(c) Observed springflow, recharge and depletion time computed by Newton Raphson method, for Kirkgoz spring, Turkey

Time (month)	Observed Springflow (10^6 cu m per month)	Slope (α_2) (month^{-1})	Pulse recharge computed by: Newton-Raphson method (10^6 cu m)	Pulse recharge computed by: Bear's Model for average τ_0=6.1 month (10^6 cu m)
24	21.64			
25	25.79	.0849	45.83	49.09
26	31.36	.1321	58.26	62.63
27	42.51	.1115	96.37	105.08
28	54.95	-.0352	115.04	124.79
29	50.67	-.0720	29.99	26.67
30	42.93	.0348	5.53	-00.51
31	46.52	-.0353	63.86	66.62
32	42.89	-.1097	25.35	22.52
33	33.32	-.0700	-12.91	-20.40
34	28.36	-.0396	4.39	00.51
35	25.89	-.0816	13.95	12.02
36	21.45	.0086	00.00	-03.44
			τ_0 = 5.31 month	

Table 3.4(d) Observed springflow, recharge and depletion time computed by Newton Raphson method, for Kirkgoz spring, Turkey

Time (month)	Observed Springflow (10^6 cu m per month)	Slope (α_2) ($month^{-1}$)	Pulse recharge computed by: Newton-Raphson method (10^6 cu m)	Pulse recharge computed by: Bear's Model for average τ_0=6.1 month (10^6 cu m)
36	21.45			
37	21.88	.0846	23.82	24.28
38	26.58	.0966	47.86	52.99
39	33.20	.0392	63.17	70.37
40	36.34	.0167	50.55	53.95
41	37.77	-.0509	44.24	45.77
42	33.59	.0300	14.66	10.17
43	36.00	-.0257	46.91	49.48
44	33.92	-.1525	24.50	22.30
45	23.88	-.0867	-21.57	-32.50
46	19.56	.0590	00.00	-04.71
			τ_0 = 5.01 month	

Table 3.4(e) Observed springflow, recharge and depletion time computed by Newton Raphson method, for Kirkgoz spring, Turkey

Time (month)	Observed Springflow (10^6 cu m per month)	Slope (α_2) (month^{-1})	Pulse recharge computed by: Newton-Raphson method (10^6 cu m)	Pulse recharge computed by: Bear's Model for average τ_0=6.1 month (10^6 cu m)
46	19.56			
47	22.40	-.1280	31.93	38.38
48	16.68	.0383	-02.53	-15.41
49	18.22	.0407	23.39	26.85
50	20.01	.2779	26.02	30.07
51	37.95	.2257	98.20	138.64
52	63.81	-.0089	150.67	208.98
53	62.52	-.0166	58.18	55.27
54	60.17	-.0175	52.27	46.97
55	57.79	-.0880	49.79	44.41
56	47.18	-.0683	11.54	-12.34
57	40.32	-.1131	17.27	01.77
58	31.07	-.0470	00.00	-20.83
			τ_0=3.84 month	

Table 3.4(f) Observed springflow, recharge and depletion time computed by Newton Raphson method, for Kirkgoz spring, Turkey

Time (month)	Observed Springflow (10^6 cu m per month)	Slope (α_2) (month^{-1})	Pulse recharge computed by: Newton-Raphson method (10^6 cu m)	Pulse recharge computed by: Bear's Model for average τ_0=6.1 month (10^6 cu m)
58	31.07			
59	27.88	-.0246	12.75	10.00
60	26.35	.0028	19.09	17.72
61	26.52	.0414	27.32	27.46
62	29.17	.1723	41.73	44.04
63	43.36	.2599	110.64	123.07
64	78.88	-.0561	247.29	278.29
65	69.33	-.1313	24.04	15.68
66	51.24	-.0391	-34.53	-50.28
67	46.83	.0579	25.92	22.07
68	53.51	-.0831	85.18	90.99
69	44.19	-.0485	00.00	-08.10
			τ_0 = 5.22 month	

Table 3.4(g) Observed springflow, recharge and depletion time computed by Newton Raphson method, for Kirkgoz spring, Turkey

Time (month)	Observed Springflow (10^6 cu m per month)	Slope (α_2) ($month^{-1}$)	Pulse recharge computed by: Newton-Raphson method (10^6 cu m)	Pulse recharge computed by: Bear's Model for average τ_0=6.1 month (10^6 cu m)
69	44.19			
70	39.53	-.0414	12.84	13.32
71	35.93	-.1406	15.31	15.73
72	25.99	.1694	-30.93	-29.80
73	38.39	.0391	109.40	107.98
74	42.01	.0936	62.74	62.34
75	52.12	-.0287	110.01	108.86
76	48.78	-.0074	29.65	30.05
77	47.95	-.0059	43.19	43.30
78	47.30	-.0106	43.57	43.67
79	46.17	-.0203	39.69	39.80
80	44.06	-.0699	31.97	32.25
81	37.51	-.0428	00.00	00.74
			τ_0 = 6.21 month	

Table 3.4(h) Observed springflow, recharge and depletion time computed by Newton Raphson method, for Kirkgoz spring, Turkey

Time (month)	Observed Springflow (10^6 cu m per month)	Slope (α_2) ($month^{-1}$)	Pulse recharge computed by: Newton-Raphson method (10^6 cu m)	Pulse recharge computed by: Bear's Model for average τ_0=6.1 month (10^6 cu m)
81	37.51			
82	33.99	-.0370	14.38	14.22
83	31.22	-.1160	15.79	15.65
84	23.90	.0348	-16.86	-17.19
85	25.89	.0787	36.97	37.10
86	31.04	.2122	59.72	59.90
87	50.60	.0460	159.53	160.39
88	56.25	.0724	87.71	87.97
89	66.45	-.0664	123.26	123.70
90	57.02	-.0717	4.50	04.13
91	48.34	-.0035	00.00	-00.40
			τ_0=6.05 month	

Table 3.5(a) Observed springflow, recharge and depletion time computed by Newton Raphson method, for White Rock spring, Nevada

Time (month)	Observed Springflow (cu m per month)	Slope (α_2) (month^{-1})	Pulse recharge computed by: Newton-Raphson method (cu m)	Pulse recharge computed by: Bear's Model for average τ_0=6.43 month (cu m)
0	3.90(10/82)			
1	3.90	.0110	3.90	3.90
2	4.00	.1222	4.80	4.59
3	5.30	.1270	15.70	13.03
4	7.10	.1078	21.50	17.80
5	9.10	.0744	25.10	20.99
6	10.80	-.0422	24.40	20.90
7	9.80	-.1162	01.80	3.86
8	7.50	.0058	-10.90	-06.17
9	7.60	.0057	08.40	8.19
10	7.70	.0273	08.50	8.29
11	8.20	.0500	12.20	11.17
12	9.20	-.0946	17.20	15.14
13	7.40	-.0304	-07.00	-03.30
14	6.90	-.0395	02.90	3.93
15	6.30	-.0512	01.50	2.73
16	5.60	-.0406	00.00	1.44
			τ_0 = 8.49 month	

Table 3.5(b) Observed springflow, recharge and depletion time computed by Newton Raphson method, for White Rock spring, Nevada

Time (month)	Observed Springflow (cu m per month)	Slope (α_2) ($month^{-1}$)	Pulse recharge computed by: Newton-Raphson method (cu m)	Pulse recharge computed by: Bear's Model for average τ_0=6.43 month (cu m)
16	5.60			
17	5.10	-.0263	03.27	2.13
18	4.80	.0000	03.70	3.02
19	4.80	-.0280	04.80	4.80
20	4.50	.1663	03.40	2.72
21	6.60	.1805	14.25	19.08
22	10.00	-.0655	22.38	30.21
23	8.60	-.1216	03.50	0.28
24	6.50	-.1053	-01.15	-05.98
25	5.10	.1185	00.00	-03.22
			τ_0 = 4.14 month	

Table 3.5(c) Observed springflow, recharge and depletion time computed by Newton Raphson method, for White Rock spring, Nevada

Time (month)	Observed Springflow (cu m per month)	Slope (α_2) (month^{-1})	Pulse recharge computed by: Newton-Raphson method (cu m)	Pulse recharge computed by: Bear's Model for average τ_0=6.43 month (cu m)
25	5.10			
26	6.70	.0982	17.21	16.21
27	8.40	-.0212	19.57	18.50
28	8.00	-.1178	05.37	5.62
29	6.10	-.0611	-06.38	-05.19
30	5.30	-.0615	-00.04	.55
31	4.60	-.0293	00.00	.44
			τ_0 = 7.04 month	

Table 3.5(d) Observed springflow, recharge and depletion time computed by Newton Raphson method, for White Rock spring, Nevada

Time (month)	Observed Springflow (cu m per month)	Slope (α_2) (month^{-1})	Pulse recharge computed by: Newton-Raphson method (cu m)	Pulse recharge computed by: Bear's Model for average τ_0=6.43 month (cu m)
31	4.60			
32	4.30	-.0314	02.50	2.52
33	4.00	-.0580	02.20	2.22
34	3.50	-.0669	00.50	.53
35	3.00	-.0147	00.00	.03
			τ_0 = 6.48 month	

Table 3.5(e) Observed springflow, recharge and depletion time computed by Newton Raphson method, for White Rock spring, Nevada

Time (month)	Observed Springflow (cu m per month)	Slope (α_2) (month^{-1})	Pulse recharge computed by: Newton-Raphson method (cu m)	Pulse recharge computed by: Bear's Model for average τ_0=6.43 month (cu m)
35	3.00			
36	2.90	.0561	02.35	2.31
37	3.30	.0256	05.50	5.68
38	3.50	-.1127	04.60	4.69
39	2.70	.0310	-01.70	-2.05
40	2.90	.0817	04.00	4.09
41	3.50	.0580	06.80	7.07
42	4.00	-.1249	06.75	6.97
43	3.00	-.0621	-02.50	-2.94
44	2.60	-.0726	00.40	0.22
45	2.20	-.0414	00.00	-.18
			τ_0 = 5.98 month	

Table 3.6 Annual recharge to the Kirkgoz spring, Turkey

Water year	Recharge estimated by Korkmaz on annual basis (10^6 cu m)	Recharge estimated by the model on monthly basis (10^6 cu m)
1974	426.8	474.4
1975	349.3	393.8
1976	439.5	458.6
1977	317.7	347.7
1978	604.6	500.1
1979	564.0	609.2
1980	486.8	500.4

3.5 CONCLUSIONS

Based on the study presented in this chapter, the following conclusions have been made:

(i) The flow domain of a spring in Bear's model performs as a linear system.

(ii) The linearity assumption between springflow and the dynamic storage in the spring flow domain is valid for the Bear's model in which the flow domain is a closed system. A closed flow domain implies that all the recharge will appear as springflow.

(iii) The logarithm plot of observed springflow with time follows a straight line only during the period of no recharge and no abstraction. The slope of the straight line during the periods of no perturbation equals the negative of the reciprocal of 2.3 times the depletion time.

(iv) The inverse problem can be solved using the Bear's model. The time variant recharge and the depletion time parameter are the unknowns. The unknowns could be computed by the Newton-Raphson iterative method by solving a set of nonlinear equations with any initial guess of model parameters to begin with.

CHAPTER-4

A ONE DIMENSIONAL GROUNDWATER FLOW MODEL FOR SPRING

4.0 INTRODUCTION

The Bear's springflow model, that has been used in chapter 3 to simulate springflow for variable recharge, assumes that an unsteady state flow is succession of steady state flows. The implication of this assumption is that there is no time lag between onset of recharge and change in springflow. In reality, there will be a time lag between onset of recharge and consequent change in springflow because of the storativity of the transmission zone. If the transmission zone of the spring flow domain is long and the hydraulic diffusivity is low, the time lag between the perturbation to the spring aquifer system and response of the system will be more pronounced. In order to simulate springflow in such a geohydrological system, an unsteady state flow condition needs to be considered. A mathematical model for a spring which has a long transmission zone with low diffusivity is described herein to predict the springflow for variable recharge. The configuration of the model is given in Fig.4.1.

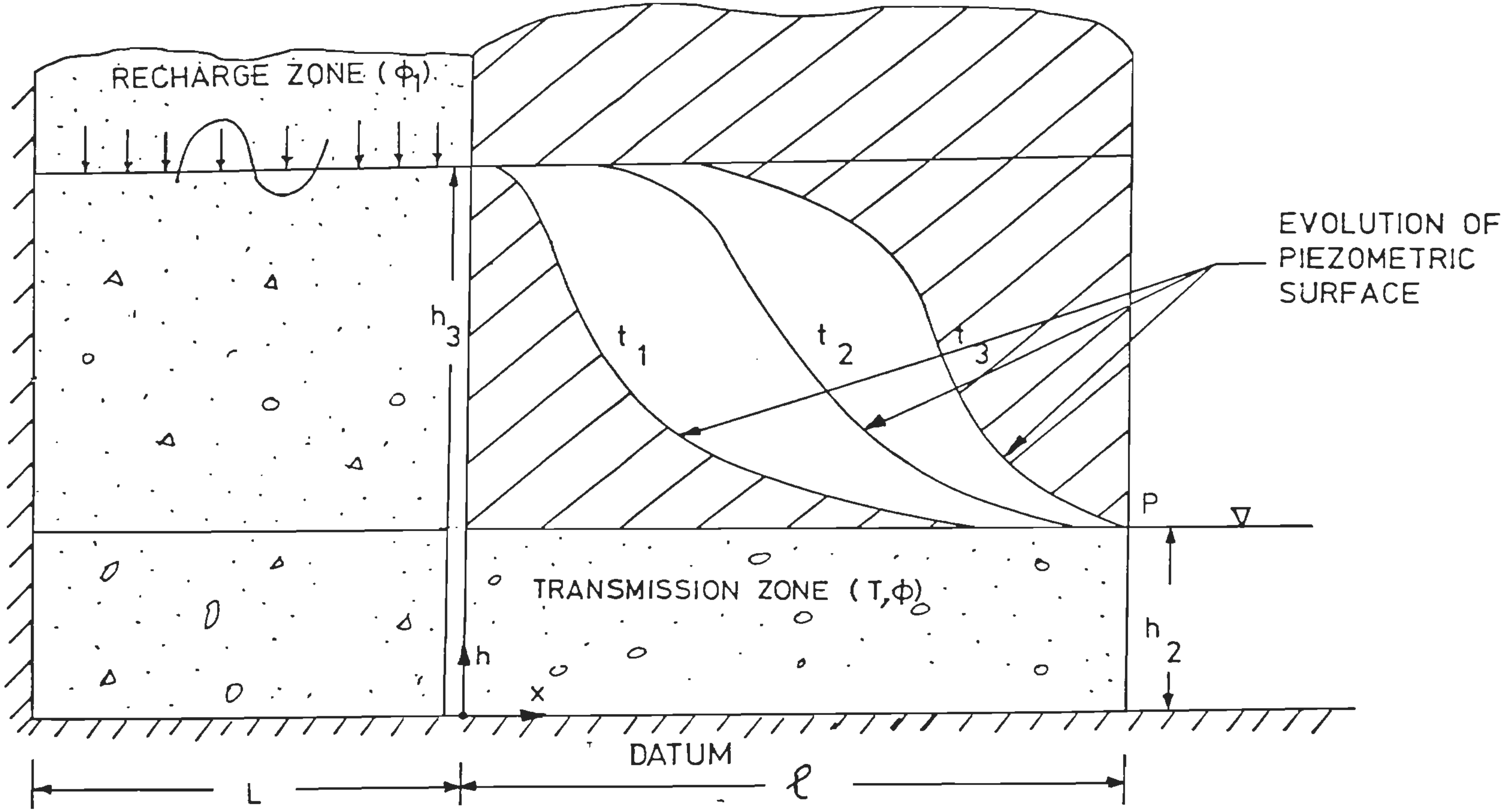


FIG. 4·1 - CONCEPTUALISED SPRINGFLOW DOMAIN FOR THE PROPOSED MODEL.

4.1 STATEMENT OF THE PROBLEM

The flow domain of the spring has been hydrologically decomposed into i) a recharge zone and ii) a transmission zone. Let L and l be the length of recharge zone and transmission zone of the spring, respectively. In the recharge zone, the flow is in the vertical direction and in the trasmission zone, the flow is in the horizontal direction.

Let the base of the aquifer be chosen as the datum. Let h_2 be the height of the outlet point (P) of the spring from the datum. Let the spring be initially at rest condition. Hence, before the onset of recharge, the level of water in the recharge zone is equal to h_2. Let T and ϕ be the transmissivity and storage coefficient of the homogeneous, isotropic confined aquifer which is acting as transmission zone for the spring. Let the specific yield of the recharge zone be designated as ϕ_1. Let the water table in the recharge zone rise to a height $h_3(0)$ instantaneously at t= 0 due to a unit impulse recharge per unit area of the recharge zone. $h_3(0)$ is given by the relation

$$\{h_3(0)-h_2\}\phi_1=1$$

or $$h_3(0)=h_2+1/\phi_1 \qquad \ldots(4.1)$$

It is aimed to find i) the flow from the spring in response to the unit impulse recharge, and ii) the unit pulse response function coefficients and the discharge from the spring due to time variant recharge.

4.2 ANALYSIS

The differential equation which governs the unsteady flow in the transmission zone is the one-dimensional Boussinesq's equation and is given by

$$\frac{\partial^2 h}{\partial x^2} = \frac{\phi}{T} \frac{\partial h}{\partial t} \qquad \ldots(4.2)$$

where h = height of the piezometric surface above the datum, ϕ = storage coefficient, and T = transmissivity.

The pertinent boundary conditions to be satisfied are

$$h(0,t) = h_3(t)$$

$$h(1,t) = h_2 \qquad \ldots(4.3)$$

The initial condition to be satisfied is

$$h(x,0) = h_2 \qquad \ldots(4.4)$$

The variation of $h_3(t)$ with time is not known, only $h_3(0)$ is known from Eq.(4.1). Determination of $h_3(t)$ is part of the solution being sought.

The solution for a varying head boundary condition is obtained from the solution for a constant head boundary condition. Let h(0,t) be equal to h_4. The solution of the Boussinesq's equation for the initial condition stated in Eq.(4.4) and constant head boundary conditions, i.e., $h(0,t)=h_4$; and $h(l,t)=h_2$; is given by (Carslaw and Jaeger, 1959)

$$h = h_2\left[\frac{x}{l} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(n\pi x/l)\exp\{-n^2\pi^2 Tt/(\phi l^2)\}\right]$$

$$+h_4\left[1 - \frac{x}{l} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(n\pi x/l)\exp\{-n^2\pi^2 Tt/(\phi l^2)\}\right]$$

...(4.5)

The rate of flow from the recharge zone into unit width of the transmission zone of the spring is given by

$$Q_o(t) = -T\frac{\partial h}{\partial x}\bigg|_{x=0} \qquad ...(4.6)$$

Differentiating Eq. (4.5) with respect to x and evaluating the differential at x= 0, and substituing it in Eq.(4.6), the rate of flow into unit width of the transmission zone for constant head boundary condition is derived as

$$Q_o(t) = -h_2\left[\frac{T}{l} + \frac{2T}{l}\sum_{n=1}^{\infty}\exp\{-n^2\pi^2 Tt/(\phi l^2)\}\right]$$

$$+ h_4\left[\frac{T}{l} + \frac{2T}{l}\sum_{n=1}^{\infty}\exp\{-n^2\pi^2 Tt/(\phi l^2)\}\right] \qquad ...(4.7)$$

According to Duhamel's principle, the rate of flow into unit width of the transmission zone for variable head boundary condition is given by

$$Q_o(t) = -h_2 \left[\frac{T}{l} + \frac{2T}{l} \sum_{n=1}^{\infty} \exp\{-n^2\pi^2 Tt/(\phi l^2)\}\right]$$

$$+ h_3(0) \left[\frac{T}{l} + \frac{2T}{l} \sum_{n=1}^{\infty} \exp\{-n^2\pi^2 Tt/(\phi l^2)\}\right]$$

$$+ \int_0^t \frac{\partial h_3(\tau)}{\partial \tau} \left[\frac{T}{l} + \frac{2T}{l} \sum_{n=1}^{\infty} \exp\{-n^2\pi^2 T(t-\tau)/(\phi l^2)\}\right] d\tau \qquad \ldots(4.8)$$

Let, the time parameter be discretised by uniform time steps of size Δt and let t be equal to $N\Delta t$, where N is an integer. Let the slope of the temporal variation of the water table height in the recharge zone, $\frac{\partial h_3(\tau)}{\partial \tau}$, be assumed to be constant within a time step Δt but be varying from time step to time step as shown in Fig.4.2.

Let a discrete kernel coefficient $\delta_0(\Delta t, N)$ be defined as

$$\delta_0(\Delta t, N) = \int_0^{\Delta t} \left[\frac{T}{l} + \frac{2T}{l} \sum_{n=1}^{\infty} \exp\{-n^2\pi^2 T(N\Delta t-\tau)/(\phi l^2)\}\right] d\tau \qquad \ldots(4.9)$$

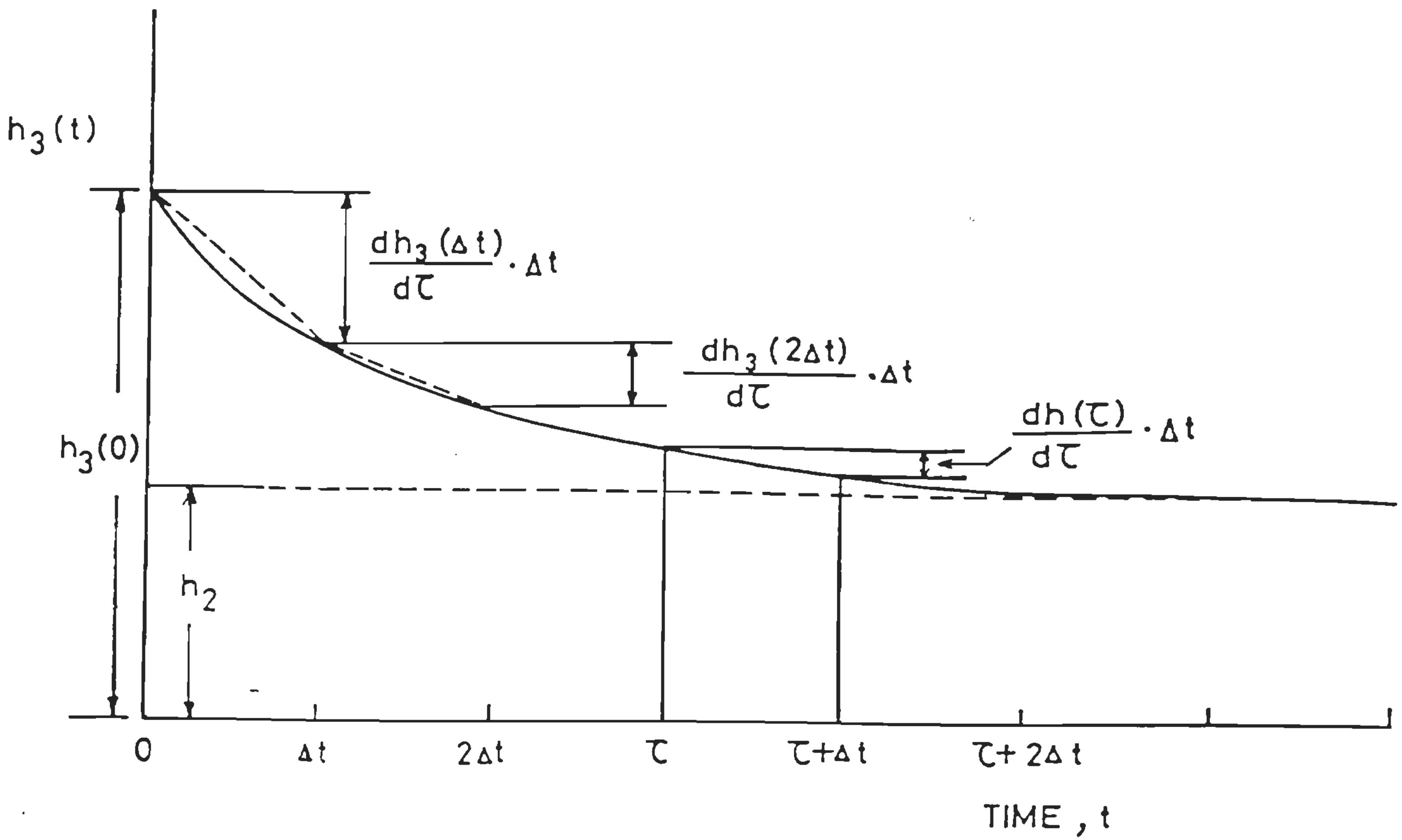


FIG. 4.2 - VARIABLE HEAD IN THE RECHARGE ZONE.

Integrating

$$\delta_0(\Delta t, N) = \frac{T\Delta t}{l} + \frac{2\phi l}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} [\exp\{-n^2\pi^2 T\Delta t(N-1)/(\phi l^2)\}$$

$$- \exp\{-n^2\pi^2 TN\Delta t/(\phi l^2)\}] \qquad \ldots(4.10)$$

The rate of flow from the recharge zone into unit width of the transmission zone for varying boundary head can be expressed as

$$Q_0(N\Delta t) = -h_2 [\frac{T}{l} + \frac{2T}{l} \sum_{n=1}^{\infty} \exp\{-n^2\pi^2 TN\Delta t/(\phi l^2)\}]$$

$$+h_3(o) [\frac{T}{l} + \frac{2T}{l} \sum_{n=1}^{\infty} \exp\{-n^2\pi^2 TN\Delta t/(\phi l^2)\}]$$

$$+\sum_{\gamma=1}^{N} \delta_0(\Delta t, N-\gamma+1)\{h_3(\gamma) - h_3(\gamma-1)\}/\Delta t \qquad \ldots(4.11)$$

In particular, at the end of first time step, the rate of flow is

$$Q_0(\Delta t) = -h_2 [\frac{T}{l} + \frac{2T}{l} \sum_{n=1}^{\infty} \exp\{-n^2\pi^2 T\Delta t/(\phi l^2)\}]$$

$$+h_3(o) [\frac{T}{l} + \frac{2T}{l} \sum_{n=1}^{\infty} \exp\{-n^2\pi^2 T\Delta t/(\phi l^2)\}]$$

$$+\delta_0(\Delta t, 1)\{h_3(1) - h_3(0)\}/\Delta t \qquad \ldots(4.12)$$

Starting from Eq.(4.7), the volume of water, $V_c(t)$, that enters into unit width of the transmission zone from the recharge zone upto time t for fixed head boundary condition, i.e., $h(0,t)=h_4$ and $h(L,t)=h_2$, is obtained as

$$V_c(t) = \int_0^t Q_0(\tau)\, d\tau$$

$$= -h_2 \int_0^t \Big[-\frac{T}{l} + \frac{2T}{l} \sum_{n=1}^{\infty} \exp\{-n^2\pi^2 T\tau/(\phi l^2)\}\Big] d\tau$$

$$+ h_4 \int_0^t \Big[\frac{T}{l} + \frac{2T}{l} \sum_{n=1}^{\infty} \exp\{-n^2\pi^2 T\tau/(\phi l^2)\}\Big] d\tau$$

$$= -h_2 \Big[\frac{Tt}{l} + \frac{2\phi l}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \{1-\exp(-n^2\pi^2 Tt/(\phi l^2))\}\Big]$$

$$+ h_4 \Big[\frac{Tt}{l} + \frac{2\phi l}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \{1-\exp(-n^2\pi^2 Tt/(\phi l^2))\}\Big] \quad \ldots(4.13)$$

The volume of water, $V_v(t)$, that will enter into unit width of the transmission zone from the recharge zone for varying boundary head, $h_3(t)$, is given by

$$V_v(t)=-h_2\left[\frac{Tt}{l}+\frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\{1-\exp(-n^2\pi^2Tt/(\phi l^2))\}\right]$$

$$+h_3(0)\left[\frac{Tt}{l}+\frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\{1-\exp(-n^2\pi^2Tt/(\phi l^2))\}\right]$$

$$+\int_0^t\frac{\partial h_3(\tau)}{\partial\tau}\left[\frac{T(t-\tau)}{l}+\frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\{1-\exp(-n^2\pi^2T(t-\tau)/(\phi l^2))\}\right]d\tau$$

...(4.14)

Let a discrete kernel coefficient, $\delta_v(\Delta t,N)$, be defined as

$$\delta_v(\Delta t,N)=\int_0^{\Delta t}\left[\frac{T(N\Delta t-\tau)}{l}+\frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\{1-\exp(-n^2\pi^2T(N\Delta t-\tau)/(\phi l^2))\}\right]d\tau$$

Integrating

$$\delta_v(\Delta t,N)=\left[\frac{T\Delta t^2(2N-1)}{2l}+\frac{2\phi l\Delta t}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\{1+\frac{\phi\ l^2}{n^2\pi^2T\Delta t}(\exp(-n^2\pi^2TN\Delta t/(\phi l^2))-\exp(-n^2\pi^2T\Delta t(N-1)/(\phi l^2)))\}\right]$$

...(4.15)

Applying Duhamel's principle, the cumulative volume of water that leaves the recharge zone and enters into unit width of the transmission zone upto N th time step is expressed in terms of discrete kernel coefficients as

$$V_v(N) = -h_2\left[\frac{TN\Delta t}{l} + \frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\{1-\exp(-n^2\pi^2 TN\Delta t/(\phi l^2))\}\right]$$

$$+h_3(0)\left[\frac{TN\Delta t}{l} + \frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\{1-\exp(-n^2\pi^2 TN\Delta t/(\phi l^2))\}\right]$$

$$+\sum_{\gamma=1}^{(N-1)}\delta_v(\Delta t, N-\gamma+1)\{h_3(\gamma)-h_3(\gamma-1)\}/\Delta t$$

$$+\delta_v(\Delta t,1)\{h_3(N)-h_3(N-1)\}/\Delta t \qquad \ldots(4.16)$$

The cumulative volume of water that enters upto (N-1) th time step into unit width of the transmission zone is

$$V_v(N-1) = -h_2\left[\frac{T(N-1)\Delta t}{l} + \frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\{1-\exp(-n^2\pi^2 T(N-1)\Delta t/(\phi l^2))\}\right]$$

$$+h_3(0)\left[\frac{T(N-1)\Delta t}{l} + \frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\{1-\exp(-n^2\pi^2 T(N-1)\Delta t/(\phi l^2))\}\right]$$

$$+\sum_{\gamma=1}^{(N-1)}\delta_v(\Delta t, N-\gamma)\{h_3(\gamma)-h_3(\gamma-1)\}/\Delta t \qquad \ldots(4.17)$$

The volume of water that leaves the recharge zone through unit width of the transmission zone during N th time step is given by

$$V_v(N)-V_v(N-1)=-h_2\left[\frac{TN\Delta t}{l}+\frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\{1-\exp(-n^2\pi^2TN\Delta t/(\phi l^2))\}\right]$$

$$+h_3(0)\left[\frac{TN\Delta t}{l}+\frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\{1-\exp(-n^2\pi^2TN\Delta t/(\phi l^2))\}\right]$$

$$+\sum_{\gamma=1}^{(N-1)}\delta_v(\Delta t,N-\gamma+1)\{h_3(\gamma)-h_3(\gamma-1)\}/\Delta t$$

$$+\delta_v(\Delta t,1)\{h_3(N)-h_3(N-1)\}/\Delta t$$

$$+h_2\left[\frac{TN\Delta t}{l}-\frac{T\Delta t}{l}+\frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\{1-\exp(-n^2\pi^2T(N-1)\Delta t/(\phi l^2))\}\right]$$

$$-h_3(0)\left[\frac{TN\Delta t}{l}-\frac{T\Delta t}{l}+\frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\{1-\exp(-n^2\pi^2T(N-1)\Delta t/(\phi l^2))\}\right]$$

$$-\sum_{\gamma=1}^{(N-1)}\delta_v(\Delta t,N-\gamma)\{h_3(\gamma)-h_3(\gamma-1)\}/\Delta t$$

...(4.18)

Equating depletion rate in the recharge zone with inflow rate to the transmission zone, the following expression is obtained.

$$-\{h_3(N)-h_3(N-1)\}L\,W_R\phi_1 = \{V_v(N)-V_v(N-1)\}W_S$$

$$\text{or } -\phi_1\{h_3(N)-h_3(N-1)\}LW_R/W_S = -h_2\left[\frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\{1-\exp(-n^2\pi^2 TN\Delta t/(\phi l^2))\}\right]$$

$$+h_2\left[-\frac{T\Delta t}{l}+\frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\{1-\exp(-n^2\pi^2 T(N-1)\Delta t/(\phi l^2))\}\right]$$

$$+h_3(0)\left[\frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\{1-\exp(-n^2\pi^2 TN\Delta t/(\phi\, l^2))\}\right]$$

$$-h_3(0)\left[-\frac{T\Delta t}{l}+\frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\{1-\exp(-n^2\pi^2 T(N-1)\Delta t/(\phi l^2))\}\right]$$

$$+\sum_{\gamma=1}^{(N-1)}\delta_v(\Delta t, N-\gamma+1)\{h_3(\gamma)-h_3(\gamma-1)\}/\Delta t$$

$$-\sum_{\gamma=1}^{(N-1)}\delta_v(\Delta t, N-\gamma)\{h_3(\gamma)-h_3(\gamma-1)\}/\Delta t$$

$$+\delta_v(\Delta t,1)\{h_3(N)-h_3(N-1)\}/\Delta t \qquad \ldots(4.19)$$

where W_R and W_S are the width of the recharge zone and width of the spring's outlet, respectively.

Rewriting Eq.(4.19)

$$h_3(N) = h_3(N-1)$$

$$- \frac{1}{\{L\phi_1 W_R/W_S + \delta_v(\Delta t,1)/\Delta t\}} [-h_2\{\frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}(1-\exp(-n^2\pi^2 TN\Delta t/(\phi l^2)))\}$$

$$+h_2\{-\frac{T\Delta t}{l} + \frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}(1-\exp(-n^2\pi^2 T(N-1)\Delta t/(\phi l^2)))\}$$

$$+h_3(0)\{\frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}(1-\exp(-n^2\pi^2 TN\Delta t/(\phi l^2)))\}$$

$$-h_3(0)\{-\frac{T\Delta t}{l} + \frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}(1-\exp(-n^2\pi^2 T(N-1)\Delta t/(\phi l^2)))\}$$

$$+ \sum_{\gamma=1}^{(N-1)}\delta_v(\Delta t, N-\gamma+1)\{h_3(\gamma)-h_3(\gamma-1)\}/\Delta t$$

$$- \sum_{\gamma=1}^{(N-1)}\delta_v(\Delta t, N-\gamma)\{h_3(\gamma)-h_3(\gamma-1)\}/\Delta t]$$

...(4.20)

In particular for N = 1, Eq.(4.20) becomes

$$h_3(1)=h_3(o)-\{h_3(o)-h_2\}[\{\frac{2\phi l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}(1-\exp(-n^2\pi^2T\Delta t/(\phi l^2)))\}+T\Delta t/l]/$$

$$\{L\phi_1 W_R/W_S+\delta_v(\Delta t,1)/\Delta t\}$$

...(4.21)

The flow from the transmission zone at the exit is given by

$$Q_1(t) = -W_S T\left.\frac{\partial h}{\partial x}\right|_{x=l} \qquad ...(4.22)$$

where $Q_1(t)$ is the springflow from the entire width of the spring.

Differentiating Eq.(4.5) with respect to x and evaluating the differential at x=l and substituting it in the Eq.(4.22), the springflow, $Q_1(t)$, for constant head boundary is obtained as

$$Q_1(t)=W_S Th_2[-\frac{1}{l}-\frac{2}{l}\sum_{n=1}^{\infty}(-1)^n\exp\{-n^2\pi^2Tt/(\phi l^2)\}]$$

$$+W_S Th_4[\frac{1}{l}+\frac{2}{l}\sum_{n=1}^{\infty}(-1)^n\exp\{-n^2\pi^2Tt/(\phi l^2)\}]$$

...(4.23)

Applying Duhamel's principle for the variable boundary condition, the springflow is

$$Q_1(t) = -W_s h_2 \left[\frac{T}{l} + \frac{2T}{l} \sum_{n=1}^{\infty} \{(-1)^n \exp(-n^2\pi^2 Tt/(\phi l^2))\}\right]$$

$$+W_s h_3(0) \left[\frac{T}{l} + \frac{2T}{l} \sum_{n=1}^{\infty} \{(-1)^n \exp(-n^2\pi^2 Tt/(\phi l^2))\}\right]$$

$$+W_s \int_0^t \frac{\partial h_3(\tau)}{\partial \tau} \left[\frac{T}{l} + \frac{2T}{l} \sum_{n=1}^{\infty} \{(-1)^n \exp(-n^2\pi^2 T(t-\tau)/(\phi l^2))\}\right] d\tau$$

...(4.24)

Let a discrete kernel $\delta_1(\Delta t, N)$ be defined as

$$\delta_1(\Delta t, N) = \int_0^{\Delta t} \left[\frac{T}{l} + \frac{2T}{l} \sum_{n=1}^{\infty} (-1)^n \exp\{-n^2\pi^2 T(N\Delta t-\tau)/(\phi l^2)\}\right] d\tau$$

Integrating

$$\delta_1(\Delta t, N) = \frac{T\Delta t}{l} + \frac{2\phi l}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \{\exp(-n^2\pi^2 T(N-1)\Delta t/(\phi l^2))$$

$$-\exp(-n^2\pi^2 TN\Delta t/(\phi l^2))\}$$

...(4.25)

So, for variable head boundary condition, the springflow is

$$Q_1(N\Delta t) = -W_S\ h_2\left[\frac{T}{l} + \frac{2T}{l}\sum_{n=1}^{\infty}(-1)^n \exp\{-n^2\pi^2 TN\Delta t/(\phi l^2)\}\right]$$

$$+W_S\ h_3(0)\left[\frac{T}{l} + \frac{2T}{l}\sum_{n=1}^{\infty}\{(-1)^n \exp(-n^2\pi^2 TN/(\phi l^2))\}\right]$$

$$+W_S \sum_{\gamma=1}^{N}\delta_1(\Delta t, N-\gamma-1)\{h_3(\gamma)-h_3(\gamma-1)\}/\Delta t \qquad \ldots(4.26)$$

Let $k(N\Delta t)$ be defined as the flow from the spring due to a unit impulse recharge taken place at t=0 through the recharge area of the spring. Hence, from Eq.(4.26) the expression for $k(N\Delta t)$ is

$$k(N\Delta t) = [-h_2\{\frac{T}{l} + \frac{2T}{l}\sum_{n=1}^{\infty}((-1)^n \exp(-n^2\pi^2 TN\Delta t/(\phi l^2)))\}$$

$$+h_3(0)\{\frac{T}{l} + \frac{2T}{l}\sum_{n=1}^{\infty}((-1)^n \exp(-n^2\pi^2 TN\Delta t/(\phi l^2)))\}$$

$$+\sum_{\gamma=1}^{N}\delta_1(\Delta t, N-\gamma+1)\{h_3(\gamma)-h_3(\gamma-1)\}/\Delta t]\,W_S/(LW_R)$$

$$\ldots(4.27)$$

Let K(t) be the unit step response function of a spring. The unit step response function, K(t), can be evaluted by integrating the unit impulse response function, k(t), which has been defined in Eq.(4.27). The unit pulse response function coefficient, $\delta(\Delta t, N)$ is given by

$$\delta(\Delta t,N)=\frac{1}{\Delta t}\ [K(N\Delta t)-K\{(N-1)\Delta t\}]$$

$$=\frac{1}{\Delta t}\ [\int_0^{N\Delta t} k(t)dt - \int_0^{(N-1)\Delta t} k(t)dt]$$

$$=\frac{1}{\Delta t}\int_{(N-1)\Delta t}^{N\Delta t} k(t)dt \qquad \ldots(4.28)$$

For the present case, the integration $\int_{(N-1)\Delta t}^{N\Delta t} k(t)$ is to be numerically integrated since a functional relation of $h_3(t)$ with time is not defined. It was assumed that within two time intervals, i.e., $(N-2)\Delta t \le t \le N\Delta t$, $k(N\Delta t)$ approximates to a second order polynomial. Accordingly (Bhargava,1992), the integration within the lower limit $(N-1)\Delta t$ and the upper limit $N\Delta t$ is given by

$$\int_{(N-1)\Delta t}^{N\Delta t} k(t)dt=[W_1 k\{(N-2)\Delta t\}+W_2 k\{(N-1)\Delta t\}+W_3 k(N\Delta t)]\Delta t \qquad \ldots(4.29)$$

in which W_1=-1/12, W_2=2/3, and W_3= 5/12

Hence, $\delta(\Delta t,N)= -1/12\ k\{(N-2)\Delta t\}+2/3\ k\{(N-1)\Delta t\}+ 5/12\ k\ (N\Delta t)$ ...(4.30)

The unit pulse response coefficient $\delta(\Delta t,1)$ is derived as follows. Applying Simpson's rule

$$\int_0^{2\Delta t} k(t)dt= \frac{\Delta t}{3}\ \{k(0)+4k(\Delta t)+k(2\Delta t)\} \qquad \ldots(4.31)$$

From Eq.(4.29)

$$\int_{\Delta t}^{2\Delta t} k(t)dt = \{W_1 k(0) + W_2 k(\Delta t) + W_3 k(2\Delta t)\}\Delta t$$

Therefore, $\delta(\Delta t,1) = \frac{1}{\Delta t} \int_0^{\Delta t} k(t)dt$

$$= \frac{1}{\Delta t} \{\int_0^{2\Delta t} k(t)dt - \int_{\Delta t}^{2\Delta t} k(t)dt\}$$

$$= \frac{1}{\Delta t} [\Delta t\{\frac{1}{3} k(0) + \frac{4}{3} k(\Delta t) + \frac{1}{3} k(2\Delta t)\}$$

$$- \Delta t\{-\frac{1}{12} k(0) + \frac{2}{3} k(\Delta t) + \frac{5}{12} k(2\Delta t)\}]$$

$$= \frac{2}{3} k(\Delta t) - \frac{1}{12} k(2\Delta t) \qquad ...(4.32)$$

Assuming the recharge to be a train of pulse recharge, the springflow, $q(N\Delta t)$, at time $N\Delta t$ could be computed from the known time variant pulse recharge, $R(\gamma)$, $\gamma = 1,2,...,N$, using the convolution

$$q(N\Delta t) = \sum_{\gamma=1}^{N} R(\gamma)\,\delta(\Delta t, N-\gamma+1) \qquad ...(4.33)$$

4.3 RESULTS AND DISCUSSIONS

The developed model has five parameters. These are: (i) specific yield of the recharge zone (ϕ_1), (ii) storage coefficient of the transmission zone (ϕ), (iii) transmissivity of the transmission zone (T), (iv) a linear dimension of the recharge zone (LW_R/W_S), and (v) length of the transmission zone (l).

If the purpose is to find the spring aquifer system parameters, and the recharge to the springflow domain from the observed springflow data, discrete kernel coefficients $\delta(\Delta t,N)$ are required to be generated using a time step size Δt, which is equal to the uniform sampling period at which observations are available. The computation of $\delta(\Delta t,N)$ is simplified by using a value of transmissivity per unit time step size and $\Delta t=1$. If transmissivity of an aquifer is 10000 sq meter per month and discrete kernel coefficients are required to be generated for $\Delta t=1/10$ th of a month, the same could be achieved using T as 1000 sq meter per one tenth of a month, and $\Delta t=1$.

Discrete kernel coefficients for a coarser time step size can be computed making use of discrete kernel coefficient computed with a finer step using a convolution technique. For example, discrete kernel coefficients for $\Delta t=1$ month can be computed from discrete kernel coefficients generated with time step size 1/10 th of a month using the relation

$$\delta(1\text{ month}, N) = \sum_{\gamma=1}^{10N} R(\gamma)\,\delta(1/10\text{ month}, 10N-\gamma+1) \qquad \ldots(4.34)$$

where $R(\gamma)=1/10$, for $\gamma=1,2,\ldots\ldots,10$ and $R(\gamma)=0$, for $\gamma>10$.

In Table 4.1, a comparison has been made of $\delta(\Delta t,N)$ computed with different time step size. It could be seen that beyond second time step, the coefficients are almost equal. Thus, the first two coefficients, $\delta(\Delta t,1)$ and $\delta(\Delta t,2)$, should be compuetd using a finer time step size, $\Delta t/10$.

A sample computation of $h_3(t)$, the head in the recharge zone, due to a unit impulse recharge per unit area at t=0, is presented in Fig.4.3 for an assumed set of aquifer parameters and geometry. The head decreases rapidly in the beginning starting from $h_3(0)$=15m and decreases monotonically in the latter time merging finally with h_2. The inflow to the aquifer corresponding to the unit impulse recharge at t=0, is presented in Fig.4.4.

The discrete kernel coefficients, $\delta(\Delta t,N)$, which are the springflow due to a unit pulse recharge in the entire recharge zone during the first time step, are presented in Fig.4.5. The area under the graph should be 1. The area under the graph computed numerically upto the time step 50, is found to be 0.96. Unlike the variation of unit response function coefficients with time depicted in the Bear's model, the response function coefficient starts from zero, reaches a maximum and then decreases with time due to the storage effect of the transmission zone. For an assumed set of recharge and abstraction, and spring aquifer parameters, the springflow has been computed and is presented in Fig.4.6 in a semilog plot for different values of storage coefficient. It is seen that during the periods of no recharge and no abstraction, the graphs are straight lines. Higher the values

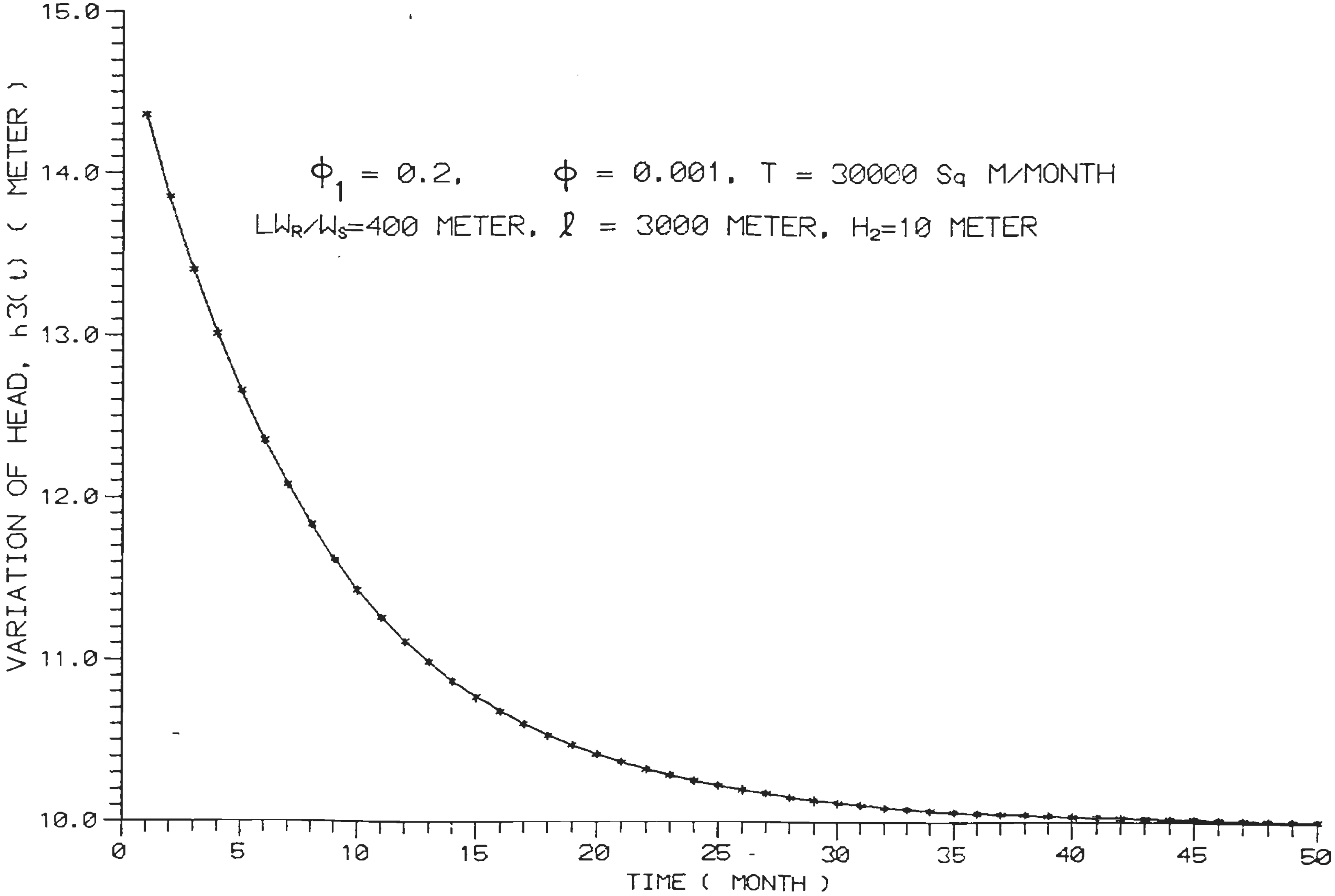


FIG. 4.3 - VARIATION OF HEAD IN THE RECHARGE ZONE DUE TO A UNIT IMPULSE RECHARGE PER UNIT AREA.

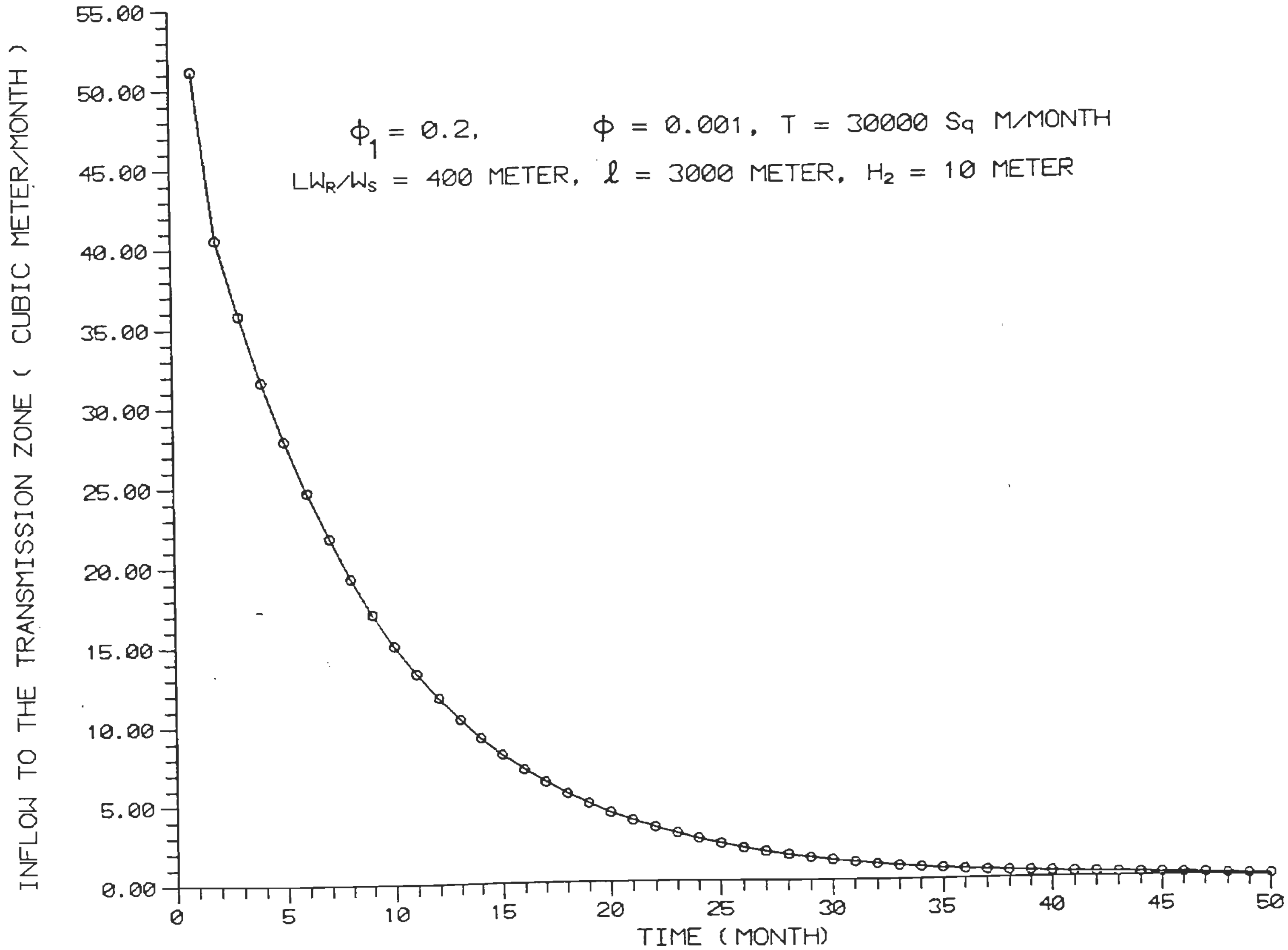


FIG. 4.4 - INFLOW PER UNIT WIDTH OF THE TRANSMISSION ZONE DUE TO UNIT IMPULSE RECHARGE PER UNIT AREA OF THE RECHARGE ZONE.

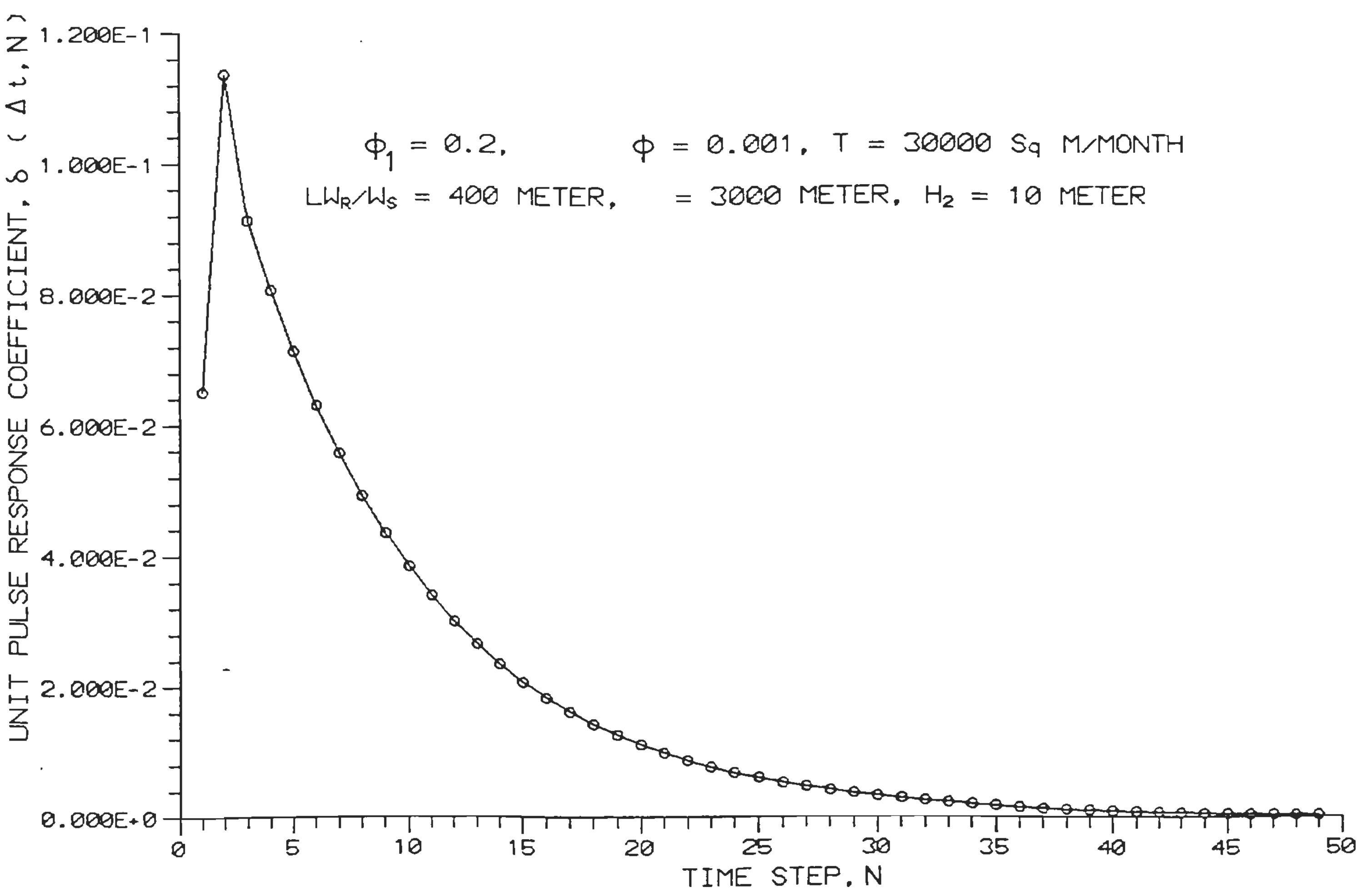


FIG. 4.5 - PLOT OF UNIT PULSE RESPONSE COEFFICIENTS.

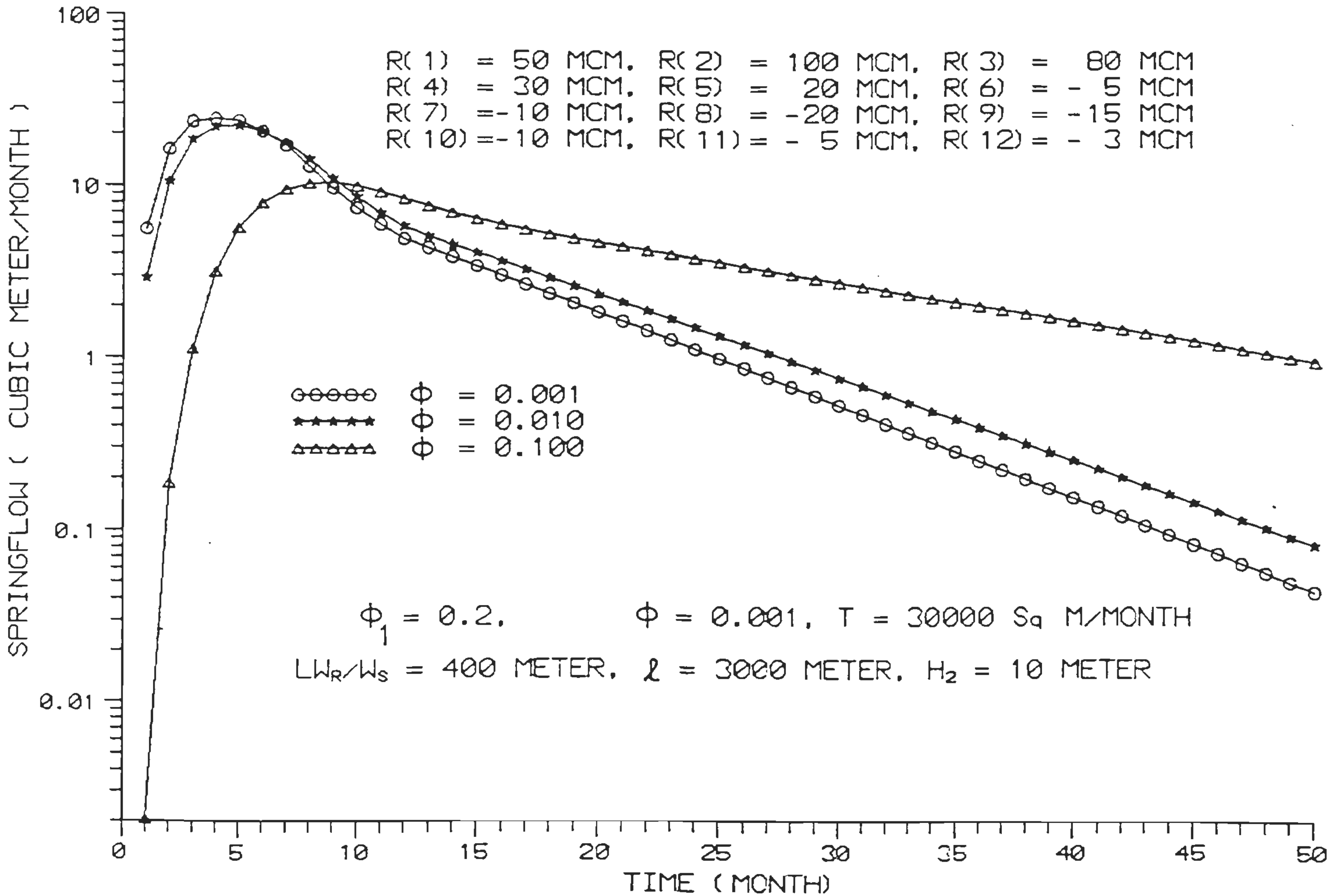


FIG. 4.6 - PLOT OF LOGARITHM OF SPRINGFLOW WITH TIME.

Table 4.1 $\delta(\Delta t,N)$ computed using a time step size Δt=1 month and $\delta(\Delta t,N)$ computed from $\delta(\Delta t/10,N)$ and $\delta(\Delta t/20,N)$; ϕ_1=0.2, ϕ=0.001, T=30000 sq meter per month, LW_R/W_S=400 meter, l=3000 meter, H_2=10 meter

Time step	$\delta(\Delta t,N)$ Δt=1 month	$\delta(\Delta t,N)$ computed from $\delta(\Delta t/10,N)$	$\delta(\Delta t,N)$ computed from $\delta(\Delta t/20,N)$
1	0.065125	0.111610	0.111340
2	0.113610	0.103280	0.103280
3	0.091199	0.091284	0.091286
4	0.080601	0.080682	0.080684
5	0.071229	0.071312	0.071314
6	0.062947	0.063030	0.063032
7	0.055628	0.055710	0.055711
8	0.049160	0.049240	0.049241
9	0.043444	0.043521	0.043522
10	0.038392	0.038467	0.038468
11	0.033928	0.033999	0.034000
12	0.029983	0.030051	0.030052

of ϕ, flatter the slope during recession. As ϕ increases, the appearance of peak springflow is delayed. Also, higher the ϕ, lower the peak springflow. The lean flow of spring is higher for transmission zone with higher storativity. As the storativity of the transmission zone decreases the model tends to the Bear's model and slope of the graph during recession in semilog plot tends to $- W_S T/(2.3\phi_1 lLW_R)$.

THE INVERSE PROBLEM

APPLICATION TO THE KIRKGOZ SPRING, TURKEY

In the inverse problem, the five model parameters and the recharge are unknowns which are to be found from the observed springflow data. The component of springflow due to the perturbation prior to the time origin is $Q_0 \exp(-\alpha N\Delta t)$, where Q_0 is the springflow at time origin, α is a decay constant, and N is a positive integer, Δt is sampling period which is uniform. This component of flow is subtracted from the observed springflow and the remaining component of observed springflow has been used to compute the unknowns. The deacy constant α, as such, is also considered as one of the unknowns.

The Newton Raphson iterative method is tried upon for the parameter estimation of the model and to compute the recharge for the Kirkgoz spring, Turkey. However, the method has been found to be unsuitable for this model probably because of the large variation in the quantitative values amongst the parameters. A random search method has been used for the estimation of the parameters. There are some well known random search methods and the method based on random jumping technique (Rao,1984) is used.

In the method, the problem is to find the minimum of $f(\mathbf{v})$ in the n-dimensional hypercube defined by

$$v_{li} \le v_i \le v_{ui} \qquad \ldots(4.35)$$

where v_{li} and v_{ui} are the lower and the upper bounds on the variable v_i. Sets of n random numbers $(r_1, r_2, \ldots, r_n)$, that are uniformly distributed between 0 and 1, are generated. Each set of these numbers, is used to find a point, $\mathbf{v}$, inside the hypercube defined by Eq.(4.35) as

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} v_{l1}+r_1(v_{u1}-v_{l1}) \\ v_{l2}+r_2(v_{u2}-v_{l2}) \\ \vdots \\ v_{ln}+r_n(v_{un}-v_{ln}) \end{bmatrix} \qquad \ldots(4.36)$$

and the value of the function is evaluated at this point $\mathbf{v}$. The objective function is evaluated at each of these large numbers of generated points. The least value of $f(\mathbf{v})$ is accepted as the desired minimum objective function.

The summation of square of the difference between the computed and observed springflow for all the time steps during which there is no recharge and no abstraction, is the objective function, $f(\mathbf{v})$, which is to be minimised by the random jumping technique. Model parameters and recharge estimated for the minimum objective function are the desired values of the model parameters and recharge for the spring. The following steps are followed to find the model parameters and recharge.

1. Specific yield of the recharge zone (ϕ_1), storage coefficient (ϕ) and transmissivity of the transmission zone (T), linear dimension of the recharge zone (LW_R/W_S), length of the transmission zone (l), and decay constant α are the variables which constitute the vector **v**. An initial random vector has been chosen by

$$\mathbf{v}^{(0)} = \mathbf{v}_l^{(0)} + [\mathbf{v}_u^{(0)} - \mathbf{v}_l^{(0)}]\mathbf{R}_n \qquad \ldots(4.37)$$

where $\mathbf{v}_l$ and $\mathbf{v}_u$ are the lower and the upper bounds of **v**, respectively, and $\mathbf{R}_n$ is a uniformly distributed random number lying between 0 and 1; the subscript $_0$ denotes the initial value.

2. With these initial values of $\mathbf{v}^{(0)}$, the unit pulse response coefficients, $\delta(\Delta t,N)$ are generated. The first six coefficients have been generated with a finer time step of 1/20th of a month. The remaining coefficients have been generated with $\Delta t=1$ month. Recharge is estimated in succession starting with first time step using the relation

$$R(N) = [\{Q_S(N\Delta t) - Q_0 \exp(-\alpha N\Delta t)\} - \sum_{\gamma=1} R(\gamma)\delta(\Delta t,n-\gamma+1)]/\delta(\Delta t,1)$$

where $Q_S(N\Delta t)$ is the observed springflow at time $N\Delta t$. The time steps wherein there is no recharge and no abstraction are known a priori from the straight line portion of the graph of logarithm of springflow with time. Recharge is set to zero for these time steps and also for those timesteps wherein recharge or abstraction has been computed as negligible with the help of Newton-Raphson method by Bear's model.

3. Springflow is computed for those time steps wherein the recharge or abstraction has been set to zero. The summation of square of the difference between the computed and observed springflow for all these time steps, wherein there are no recharge and no abstraction, is calculated as the objective function f(**v**), which is to be minimised. At all other time steps where recharge has not been assumed to be zero, but computed, the observed springflow and the computed springflow would tally.

4. The process is repeated for another feasible **v**. If the f(**v**) is less than the previously obtained value of f(**v**), the present vector is retained by naming it as $\mathbf{v}_s$. The process is repeated for a number of times to obtain **v** for the minimum f(**v**). The search is refined by reducing the range of **v** by

$$\mathbf{v}_l^{(r+1)} = \mathbf{v}_S^{(r)} - 0.45[\mathbf{v}_u^{(r)} - \mathbf{v}_l^{(r)}] \qquad ...(4.38)$$

$$\mathbf{v}_u^{(r+1)} = \mathbf{v}_S^{(r)} + 0.45[\mathbf{v}_u^{(r)} - \mathbf{v}_l^{(r)}] \qquad ...(4.39)$$

where r= the number of cycles. The process is repeated for several cycles till the f(**v**) between two successive cycle is small. It has been observed that the procedure converges within a few cycles.

The observed springflow values for the Kirkgoz spring, Turkey for the period from October, 1973 to May, 1981 (92 months) have been used for the estimation of model parameters and the monthly recharge. The October, 1973 springflow value has been considered as Q_0. The following initial guess of the upper and the lower bounds of model parameters has been made : ϕ_{1u}=0.2, ϕ_{1l}=0.03; ϕ_u =0.001, ϕ_l =0.0001; T_u =40000 Sq m/month, T_l =10000 Sq m/month; l_u =4000m, l_l =1000m; $(LW_R/W_S)_u$=2000m, $(LW_R/W_S)_l$=1000m; α_u=0.250 month^{-1} , α_l= 0.167 month^{-1}.

With this initial guess, fifteen random search is made after which the upper bound and the lower bound are improved. This cycle is repeated ten times. The estimated model parameters for the Kirkgoz spring which minimised the objective function are:

Specific yield of the recharge zone (ϕ_1) = 0.06445
Storage coefficient of the transmission zone (ϕ) = 0.0009548
Transmissivity of the transmission zone (T) = 37960 Sq m/month
linear dimension of the recharge area (LW_R/W_S) = 2369 meter
Length of the transmission zone (l) = 1496 meter
Decay constant (α) = 0.185 month^{-1}

The successive rapid decrease of f(**v**) after end of each cycle is presented in Table 4.2. From the Table, it could be seen that the objective function attains the minimum at the end of third cycle. The number of successful search are more in the first cycle and after the third cycle, there is no further reduction in the value of objective function. The recharge computed using the parameters obtained through optimisation by random jump technique, is presented in Table 4.3. For small values of ϕ , the parameter τ_0 in the Bear's model, is equal to $lLW_R\phi_1/(W_ST)$. Substituting the values of l, LW_R/W_S, T and ϕ_1, which have been obtained through optimisation , τ_0 is found to be 6.01. Using the Bear's model, the recharge is computed for $\tau_0 = 6.01$ and is also presented in Table 4.3. It is found that the sets of recharge computed by the two models, compare well since the value of ϕ is very small.

Table 4.2 Successive reduction of the objective function in different cycle of the random jump technique

Cycle	Number of times the objective function is reduced	Objective function after each successful search (cubic meter per month)^2
0	-	$10.0x10^{33}$
1	5	$1.77x10^{14}$
		$1.44x10^{14}$
		$4.65x10^{13}$
		$1.38x10^{13}$
		$8.86x10^{12}$
2	2	$8.32x10^{12}$
		$7.13x10^{12}$
3	2	$3.21x10^{12}$
		$2.81x10^{12}$
4	0	$2.81x10^{12}$
10	0	$2.81x10^{12}$

TABLE 4.3 Computed monthly recharge for Kirkgoz spring, Turkey.

Month	Recharge computed using the parameters found by Random jump technique (x 10^6 cu m)	Recharge computed by the Bear's model, $\tau_0 = lLW_R \phi_1 / (W_S T)$ =6.01 month (x 10^6 cu m)
1	30.86	27.45
2	27.30	24.80
3	56.01	53.44
4	150.40	146.82
5	44.59	44.74
6	43.25	42.03
7	42.24	41.27
8	22.60	22.05
9	05.28	04.93
10	-05.55	-06.01
11	12.44	11.65
12	12.23	11.77
13	39.82	38.91
14	26.40	26.27
15	48.97	48.24
16	52.73	52.34
17	54.35	54.02
18	59.19	58.82
19	62.08	61.77
20	30.86	31.20
21	-22.30	-21.45
22	21.52	20.71
23	00.00	-00.78

continued...Table 4.3

Month	Recharge computed using the parameters found by Random jump technique ($\times 10^6$ cu m)	Recharge computed by the Bear's model, $\tau_0 = 1LW_R\phi_1/(W_S T)$ =6.01 month ($\times 10^6$ cu m)
24	00.00	00.39
25	49.38	48.76
26	62.65	62.19
27	105.20	104.19
28	124.50	123.80
29	25.47	27.01
30	00.00	00.10
31	67.31	66.34
32	22.24	22.81
33	-20.38	-19.64
34	00.00	00.90
35	13.60	12.22
36	00.00	-03.09
37	21.89	24.25
38	53.27	52.62
39	70.30	69.84
40	53.60	53.70
41	45.68	45.65
42	09.88	10.50
43	50.09	49.29
44	22.05	22.46
45	-32.62	-31.70
46	-00.00	-04.37

continued...Table 4.3

Month	Recharge computed using the parameters found by Random jump technique (x 10^6 cu m)	Recharge computed by the Bear's model, $\tau_0 = lLW_R \phi_1 / (W_S T)$ =6.01 month (x 10^6 cu m)
47	35.58	38.15
48	-15.83	-14.96
49	27.54	26.73
50	30.07	29.92
51	139.30	137.21
52	208.70	206.92
53	53.09	55.38
54	47.10	47.16
55	44.64	44.60
56	-12.55	-11.50
57	02.64	02.32
58	-20.40	-20.09
59	10.77	10.26
60	18.10	17.84
61	27.74	27.45
62	44.22	43.82
63	123.50	121.94
64	278.70	275.45
65	12.30	16.44
66	-50.13	-48.83
67	23.56	22.42
68	91.81	90.46

continued...Table 4.3

Month	Recharge computed using the parameters found by Random jump technique (x 10^6 cu m)	Recharge computed by the Bear's model, $\tau_0 = 1LW_R\phi_1/(W_S T)$ =6.01 month (x 10^6 cu m)
69	00.00	-07.36
70	06.65	13.69
71	16.20	16.02
72	-29.85	-29.00
73	109.60	106.99
74	61.51	62.05
75	109.10	108.06
76	29.10	30.32
77	43.67	43.36
78	43.74	43.72
79	40.02	39.89
80	32.37	32.42
81	00.00	01.27
82	15.47	14.50
83	15.96	15.87
84	-17.17	-16.61
85	37.90	36.94
86	60.10	59.49
87	160.90	158.83
88	86.62	87.52
89	123.80	122.89
90	00.00	04.88
91	00.00	00.29

The observed and the simulated springflow through optimisation for the periods of no recharge and no abstraction, is presented in Table 4.4 which shows that the simulated values tally very well with the observed springflow. The simulated springflow during the recharge period will automatically tally with the observed springflow.

Table 4.4 Observed and simulated springflow for the periods of no recharge and no abstraction

Month	Observed springflow (10^6 cu m)	Simulated springflow (10^6 cu m)
23	25.11	25.36
24	21.33	21.55
30	42.83	43.04
34	28.31	28.21
36	21.42	22.00
46	19.56	20.21
69	44.20	45.69
81	37.51	37.52
90	57,02	56.79
91	48.34	48.28

4.4 CONCLUSIONS

From the study presented in this chapter, the following conclusions have been drawn:

(i) The logarithm plot of springflow with time during recession follows a straight line.

(ii) If the storativity of the transmission zone decreases, the model tends to the Bear's model and slope of the graph during recession in semilog plot tends to - $W_S T/(2.3\phi_1 lLW_R)$.

(iii) The storativity the transmission zone controls the time of occurence of peak flow as well as the magnitude of the peak flow of the spring. It also controls springflow during recession.

(iv) The random search technique can predict the aquifer parameters and geometry of the spring flow domain as well as the time distribution of recharge to the spring flow domain.

CHAPTER-5

A TWO DIMENSIONAL GROUNDWATER FLOW MODEL FOR SPRING

5.0 INTRODUCTION

The Bear's springflow model and the model with long transmission zone, assume the flow to be one-dimensional. However, in reality, the flow pertaining to a spring is three-dimensional. Using the Dupuit-Forchheimer assumptions, some three-dimensional flow process could be dealt as two-dimensional. A springflow domain can be visualised to have a recharge area which may not be well defined and a discharge area which acts as the spring. Hantush (1967) has given solution for the rise of piezometric surface due to uniform recharge at a constant rate from a rectangular basin. The shape of the recharge area for a spring can be considered as rectangular. Similarly, a rectangular shape can be assumed for the spring's opening. Using the Hantush's basic solution for the rise of piezometric surface due to recharge from a rectangular area, a two-dimensional springflow model has been developed in this chapter.

5.1 STATEMENT OF THE PROBLEM

A schematic configuration of a spring flow domain is shown in Fig.5.1(a). The corresponding idealised flow domain adopted for the development of the model, is shown in Fig.5.1(b). The recharge area of the spring is assumed to be a rectangle of size LxW_R and the spring opening conforms to a rectangle of size axb. The aquifer which transmits water to the spring is homogeneous, isotropic, and

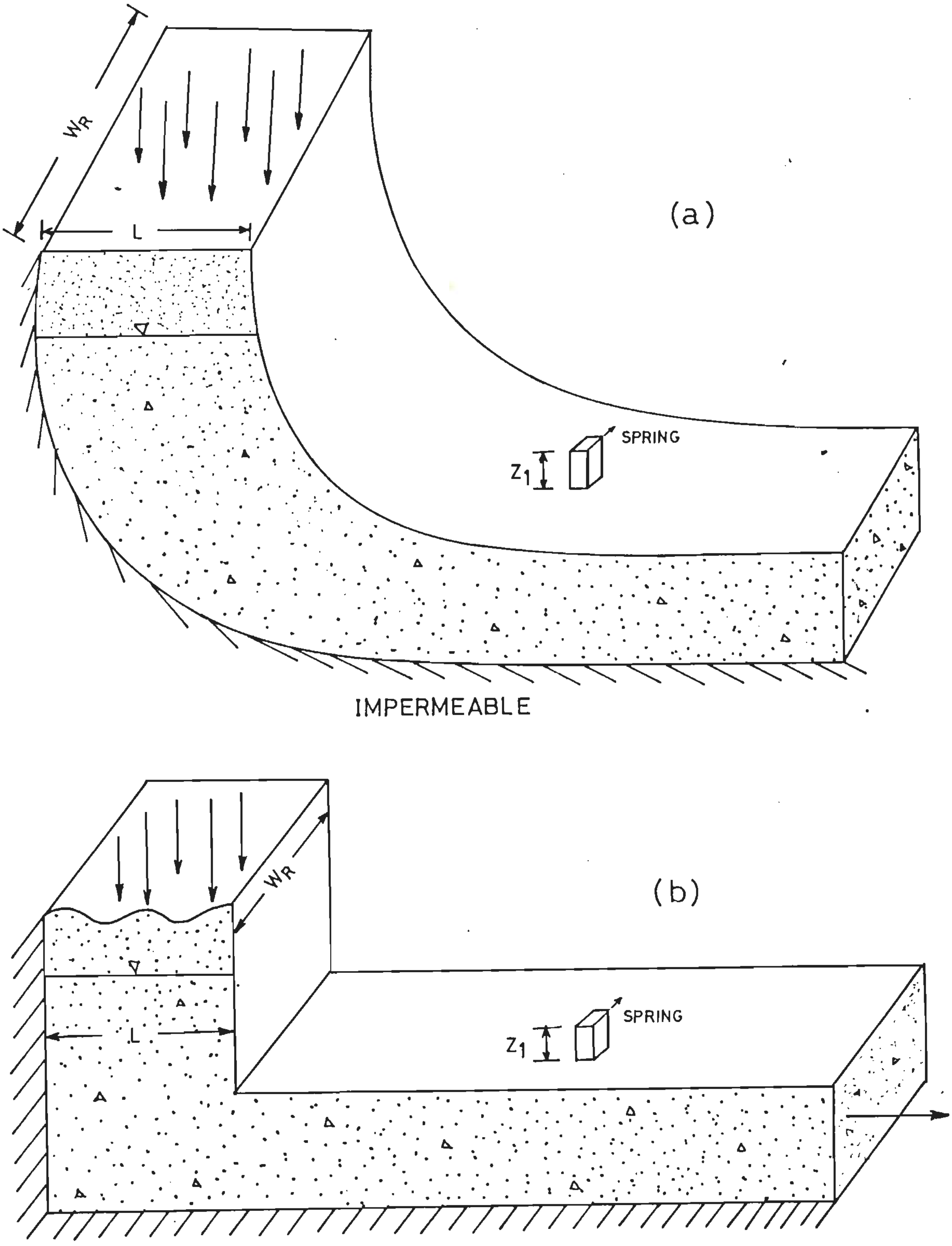


FIG. 5·1(a) & (b)-SCHEMATIC AND IDEALISED FLOW DOMAIN OF A SPRING.

has semi-infinite areal extent. It is aimed to find the temporal variation of the springflow due to time variant recharge through the entire recharge zone.

5.2 DEVELOPMENT OF THE MODEL

The basic saturated flow equation describing the flow in the spring aquifer system is the Boussinesq's equation:

$$\phi \frac{\partial s}{\partial t} - T \frac{\partial^2 s}{\partial x^2} - T \frac{\partial^2 s}{\partial y^2} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} r(\xi, \vartheta, t)\ \delta_D(\xi - x, \vartheta - y)\, d\xi\ d\vartheta \qquad \text{...(5.1)}$$

where s is the rise in piezometric surface , t is time, x and y are the horizontal cartesian coordinates, ϕ is the storage coefficient,T is the transmissivity, $r(\xi,\vartheta,t)$ is the recharge or discharge rate per unit area (positive for recharge and negative for discharge) and $\delta_D(\xi-x,\vartheta-y)$ is a Dirac delta function singular at the point of coordinates x, and y. The level of the initially rest piezometric surface coinciding with top of the aquifer is taken as the datum.

The required solution to the differential Eq.(5.1) for the spring needs to satisfy an initial condition $s(x,y,0)=0$. The boundary conditions to be satisfied are:

$$\frac{\partial s}{\partial x}\bigg|_{x=0} = 0 \qquad \ldots(5.2)$$

$$s(x, \pm\infty, t) = 0 \qquad \ldots(5.3)$$

$$s(\infty, y, t) = 0 \qquad \ldots(5.4)$$

A spring gets activated when the piezometric surface tends to rise above its threshold. Once a spring gets activated, the rise in piezometric surface at the location of the spring remains invariant till the springflow becomes zero. Therefore, the other boundary condition to be satisfied is:

$$s(x_1, y_1, t) = z_1, \quad t > t_1 \qquad \ldots(5.5)$$

where x_1, y_1 are the coordinate of the spring, t_1 is time of activation of the spring, z_1 is height of the threshold of the spring above the initially rest piezometric surface.

The method of image is applied to convert the finite flow domain into an infinite one. The boundary condition stated in Eq.(5.2) is thereby satisfied. The system of image and real springs is shown in Fig.5.2. Hantush's basic solution being used in the present analysis, the boundary conditions stated in Eq.(5.3) and Eq.(5.4) and the initial condition are automatically satisfied.

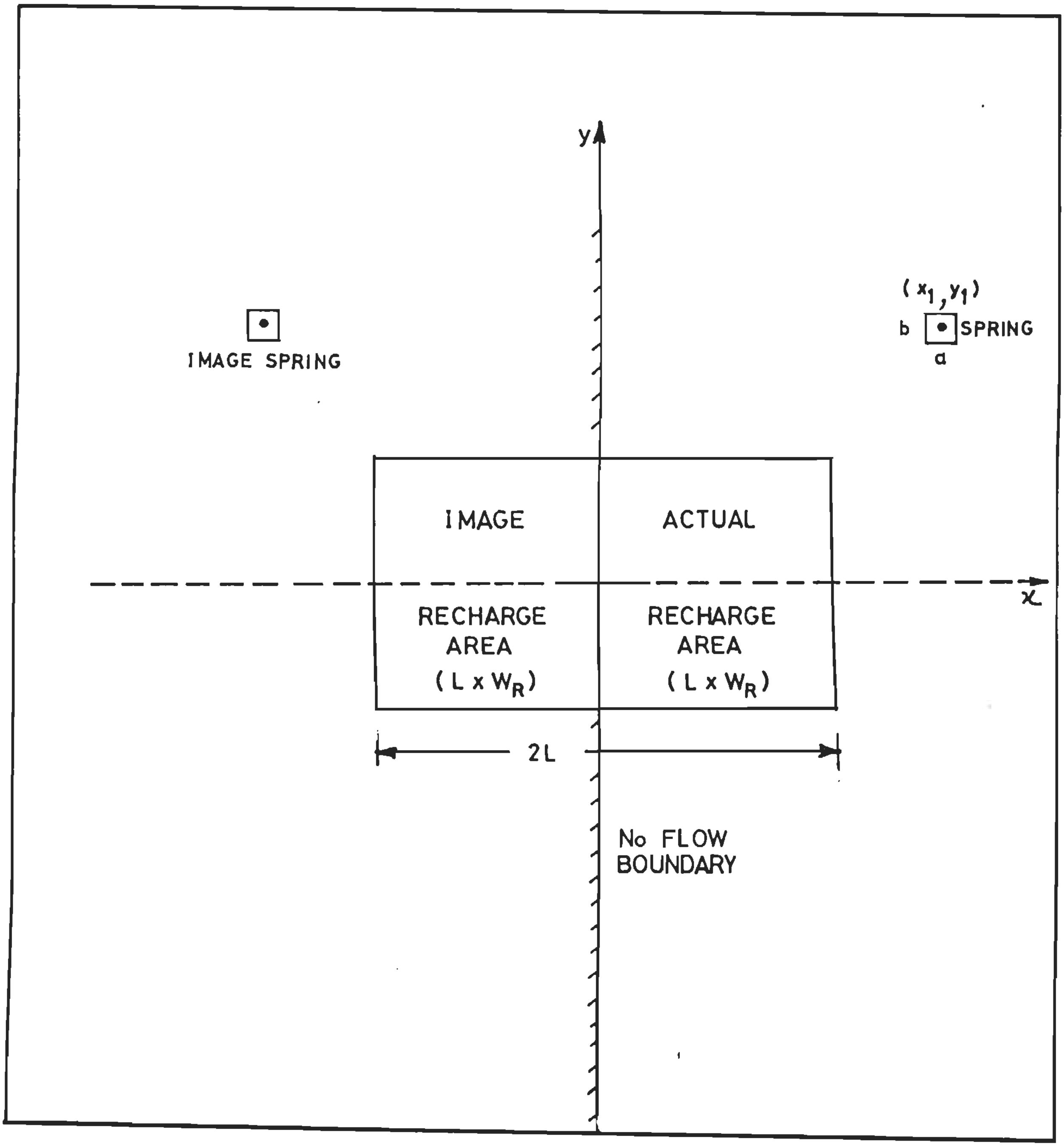


FIG. 5·2 - FLOW DOMAIN OF THE PROPOSED MODEL BASED ON THEORY OF IMAGE.

Let the time span be discretised into uniform time steps of size Δt. Let during a time step, γ, the pulse recharge per unit area be $R_u(\gamma)$, and the pulse spring discharge per unit area of spring opening be $q(\gamma)$. However, $q(\gamma)$ and $R_u(\gamma)$ may vary from one time step to next.

The rise in piezometric surface, $s(x_1, y_1, n\Delta t)$, at the spring at time $t=n\Delta t$, due to the time variant pulse recharge, $R_u(\gamma)$, $\gamma = 1,2,\ldots,n$, through the recharge zone in the equivalent flow domain until the spring gets activated is given by

$$s(x_1, y_1, n\Delta t) = \sum_{\gamma=1}^{n} R_u(\gamma)\,\delta(2L, W_R; x_1, y_1; \Delta t; n-\gamma+1) \qquad \ldots(5.6)$$

in which $\delta(A,B;X,Y;\Delta t;m)$ is a discrete kernel coefficient for rise in piezometric surface; A and B are length and width of the excitation zone; X, Y are the coordinate of the point of observation, the coordinate being measured from a local origin chosen at the centre of the zone of perturbation ; Δt is the time step size; 2L and W_R are length and width of the recharge area in the equivalent flow domain. The discrete kernel coefficients for rise in piezometric surface are the rise in piezometric surface at an observation point due to a unit pulse perturbation per unit area given to the system during the first unit time period. In the present problem, the zone of perturbation is either the area through which recharge takes place or the spring's opening.

Let the spring get activated during Nth time step and the rising piezometric surface touches the spring's threshold at

$t=(N-1)\Delta t$. Hence

$$\sum_{\gamma=1}^{N-1} R_u(\gamma)\ \delta(2L, W_R; x_1, y_1; \Delta t;\ N-\gamma\ ) = z_1 \qquad \ldots(5.7)$$

The time of activation of the spring can be predicted from Eq.(5.7) using an iteration procedure. As the spring gets activated at n=N, therefore, $q(\gamma)=0$ for $\gamma = 1,2,\ldots,N-1$.

The expression for rise in piezometric surface at $t=n\Delta t$ at the spring after its activation is given by

$$\begin{aligned} s(x_1, y_1, n\Delta t) &= \sum_{\gamma=1}^{n} [\ R_u(\gamma)\ \delta(2L, W_R; x_1, y_1; \Delta t; n-\gamma+1)\ ] \\ &- \sum_{\gamma=1}^{n} [q(\gamma)\{\delta(a,b;0,0;\Delta t;n-\gamma+1)+\delta(a,b;2x_1,0;\Delta t;n-\gamma+1)\}] \end{aligned} \qquad \ldots(5.8)$$

The dimension a and b of the spring are in x and y direction, respectively.

Since after activation of the spring, $s(x_1, y_1, n\Delta t) = z_1$, hence

$$\sum_{\gamma=1}^{n} [R_u(\gamma)\ \delta(2L, W_R; x_1, y_1; \Delta t; n-\gamma+1)\ ]$$

$$-\sum_{\gamma=1}^{n} [q\ (\gamma)\{\delta(a,b;0,0;\Delta t;n-\gamma+1)+\delta(a,b;2x_1,0;\Delta t;n-\gamma+1)\}]=z_1 \qquad \ldots(5.9)$$

Splitting the second temporal summation into two parts, one part containing the summation up to (n-1)th terms, and the other part the n th term, Eq.(5.9) is simplified to

$$\sum_{\gamma=1}^{n} [R_u(\gamma)\ \delta(2L,W_R;x_1,y_1;\Delta t;n-\gamma+1)]$$

$$-\sum_{\gamma=1}^{n-1} [q(\gamma)\{\delta(a,b;0,0;\Delta t;n-\gamma+1)+\delta(a,b\ ;2x_1,0;\Delta t;n-\gamma+1)\}]$$

$$-\ q(n)\{\delta(a,b;0,0;\Delta t;1)+\ \delta(a,b\ ;2x_1,0;\Delta t;1)\}=\ z_1 \qquad \ldots(5.10)$$

Solving for q(n)

$$q(n) = [\sum_{\gamma=1}^{n}\{R_u(\gamma)\delta(2L,W_R;x_1,y_1;\Delta t;n-\gamma+1)\}$$

$$-\sum_{\gamma=1}^{n-1}\{q(\gamma)(\delta(a,b;0,0;\Delta t;n-\gamma+1)$$

$$+\delta(a,b;2x_1,0;\Delta t;n-\gamma+1))\}-z_1]/$$

$$[\delta(a,b;0,0;\Delta t;1)+\delta(a,b;2x_1,0;\Delta t;1)] \qquad \ldots(5.11)$$

Since the spring gets activated during N th time step, $q(\gamma)=0$ for $\gamma=1,2,..,N-1$. q (n), $n \geq N$, can be solved in succession starting from time step N. For time step N Eq.(5.11) reduces to

$$q(N)=[\sum_{\gamma=1}^{N}\{R_u(\gamma)\delta(2L,W_R;x_1,y_1;\Delta t;N-\gamma+1)\}-z_1]/$$

$$[\delta(a,b;0,0;\Delta t;1)+\delta(a,b;2x_1,0;\Delta t;1)] \qquad ...(5.12)$$

The discrete kernel coefficients for rise of piezometric surface can be obtained from Hantush's solution for the rise of piezometric surface due to uniform recharge at a constant rate from a rectangular basin (Fig.5.3). Making use of Hantush's solution, $\delta(A,B;X,Y;\Delta t;m)$ is found to be

$$\delta(A,B;X,Y;\Delta t;m)=\frac{m}{4\phi}\,[F\{(A/2+X)\eta_1,(B/2+Y)\eta_1\}+F\{(A/2+X)\eta_1,(B/2-Y)\eta_1\}$$

$$+F\{(A/2-X)\eta_1,(B/2+Y)\eta_1\}+F\{(A/2-X)\eta_1,(B/2-Y)\eta_1\}]$$

$$-\frac{(m-1)}{4\phi}\,[F\{(A/2+X)\eta_2,(B/2+Y)\eta_2\}+F\{(A/2+X)\eta_2,(B/2-Y)\eta_2\}$$

$$+F\{(A/2-X)\eta_2,(B/2+Y)\eta_2\}+F\{(A/2-X)\eta_2,(B/2-Y)\eta_2\}]$$

...(5.13)

where

m = time step ,

$\eta_1 = (4Tm\Delta t/\phi)^{-0.5}$

$\eta_2 = \{4T(m-1)\Delta t/\phi\}^{-0.5}$

$$F(\Phi,\Psi) = \int_0^1 \mathrm{erf}(\Phi\,\tau^{-0.5}).\mathrm{erf}(\Psi\,\tau^{-0.5})\,d\tau$$

where $\Phi=(A/2\pm X)\eta_i$, and $\Psi=(B/2\pm Y)\eta_i$, $i=1,2$

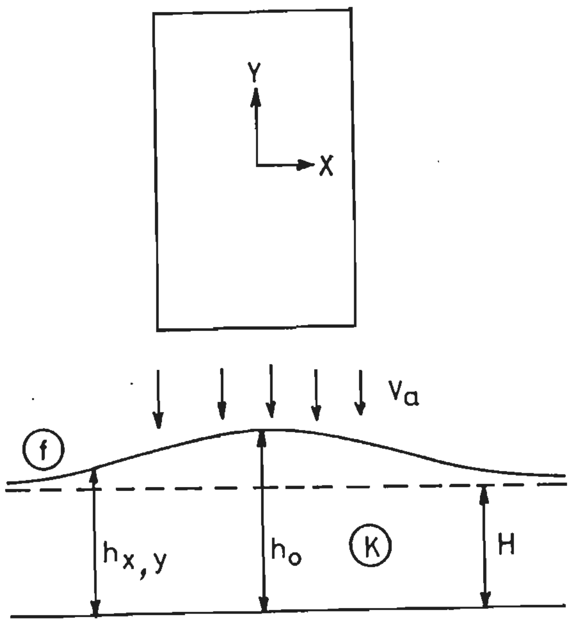


FIG. 5·3-RECHARGE AREA AND GROUNDWATER MOUND (AFTER HANTUSH VIDE BOUWER, 1978).

5.3 RESULTS AND DISCUSSIONS

Assuming aquifer parameters and its geometry, springflow has been generated for a set of time variant recharge. The elevation z_1 has been assumed to be zero. The variation of $\log_{10}Q(t)$ versus time, is presented in Fig.5.4. As seen from the Figure, the graph during recession does not follow a straight line; the slope of the graph changes with time. For the assumed set of recharge, the recession starts from seventh month. The slope at time step 7 is -0.0408. The slope decreases with time and reaches a minimum at time step 8 and then increases. The variation of slope with time is shown in Table 5.1. The slope changes because the spring flow domain is not a closed system. In the example presented, a total of one meter recharge per unit area takes place in a span of six month. The actual recharge area is 1 km x 1 km, which means that 10^6 cubic meter of water has been recharged. It is found that at the end of 120th time step, only 6.36% of recharge appears as springflow. The remaining recharge has flown out as regional groundwater flow.

Using the random jump technique and springflow of the Kirkgoz spring, Turkey, aquifer parameters for the spring are estimated and are given below. The following initial guess of the upper and the lower bounds of the model parameters has been made: $2L_u$=3000meter, $2L_l$= 1000 meter; W_{Ru} = 3000 meter, W_{Rl} = 1000 meter; a_u=15 meter, a_l= 2 meter; b_u=15 meter, b_l= 2 meter; x_{lu} = 5000 meter, x_{ll}= 4000 meter; T_u= 40000 sq m/month, T_l= 20000 sq m/month; and, ϕ_u= 0.001, ϕ_l= 0.0001. The decay constant has been assumed to be equal to 1/6 $month^{-1}$ and y_1=0. The springflow for the month of December, 1973, i.e., 27.53 cu meter per month has

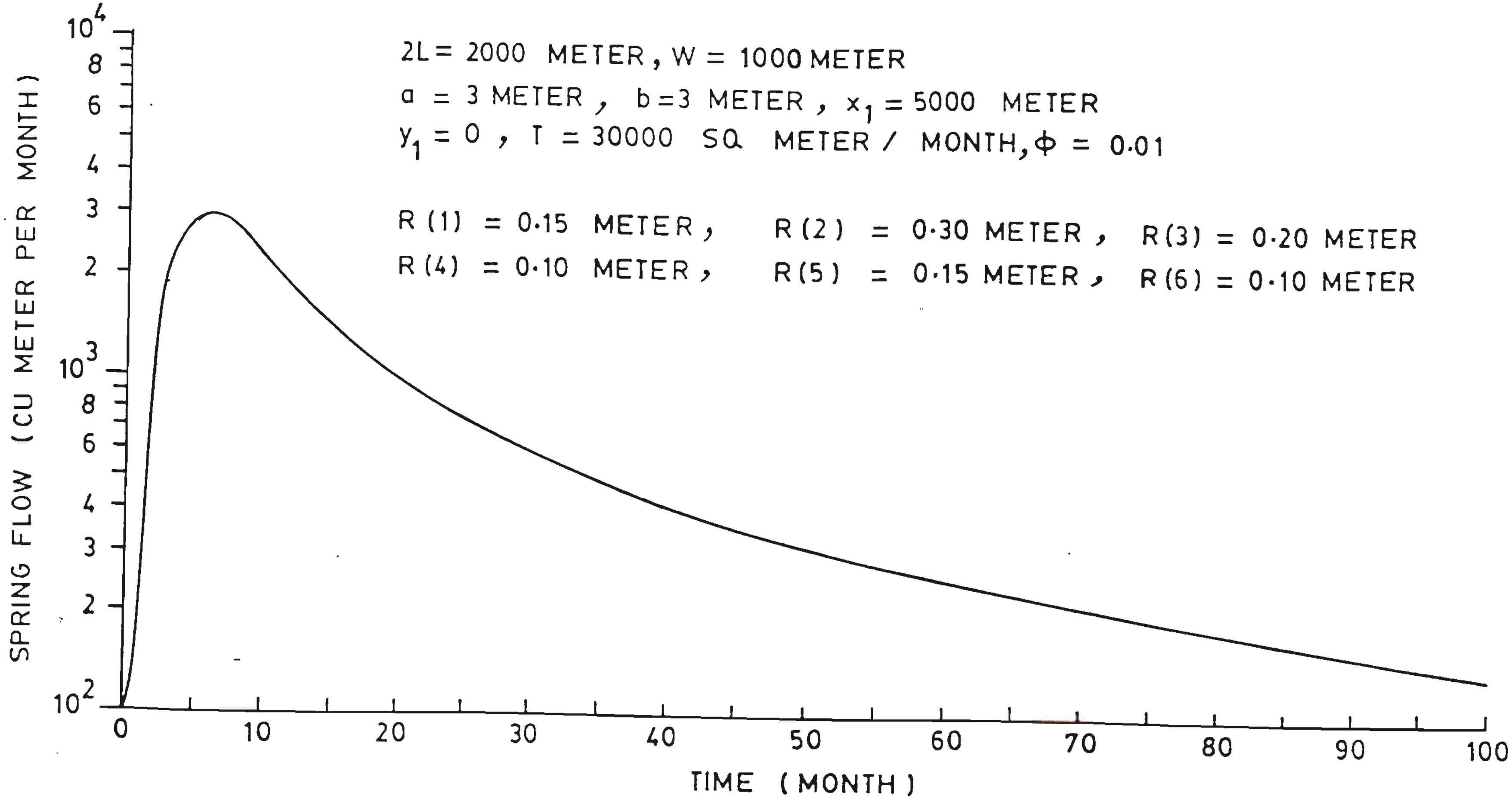


FIG. 5.4 – VARIATION OF SPRING FLOW WITH TIME FOR A SET OF ASSUMED MODEL PARAMETERS AND AN ASSUMED RECHARGE PER UNIT AREA.

Table 5.1 Variation of the slope of the graph of logarithm of computed springflow with time, 2L=2000 meter, W=1000 meter, a=3 meter, b= 3 meter, x_1 = 5000 meter, T= 30000 sq meter per month, ϕ=0.01

Time step	Recharge (cubic meter per sq meter)	Computed springflow (cubic meter per month)	Slope (α_2) ($month^{-1}$)
1	0.15	168.25	0.7157
2	0.30	874.24	0.3285
3	0.20	1862.44	0.1213
4	0.10	2462.66	0.0468
5	0.15	2742.55	0.0387
6	0.10	2998.26	0.0013
7	zero	3007.25	-0.0408
8	onwrds	2737.49	-0.0489
9		2446.06	-0.0480
10		2190.02	-0.0454
11		1972.68	-0.0425
12		1788.60	-0.0398
13		1631.79	-0.0374
14		1497.15	-0.0352
15		1380.62	-0.0332
16		1279.99	-0.0314
17		1189.73	-0.0298
18		1110.82	-0.0283
19		1040.66	-0.0270
20		977.92	-0.0258
21		921.55	-0.0247
22		870.68	-0.0236
23		824.56	-0.0227
24		782.60	-0.0218
25		744.28	-0.0210
26		709.18	-0.0202
27		676.91	-0.0195
28		647.17	-0.0188
29		619.68	-0.0182

continued ...Table 5.1

Time step	Recharge (meter per sq meter)	Computed springflow (cubic meter per month)	Slope (α_2) ($month^{-1}$)
30		594.21	-0.0176
31		570.56	-0.0171
32		548.54	-0.0166
33		527.99	-0.0161
34		508.79	-0.0156
35		490.81	-0.0152
36		473.94	-0.0148
37		458.09	-0.0144
38		443.16	-0.0140
39		429.10	-0.0137
40		415.81	-0.0133
41		403.26	-0.0130
42		391.37	-0.0127
43		380.10	-0.0124
44		369.41	-0.0121
45		359.25	-0.0118
46		349.58	-0.0116
47		340.36	-0.0113
48		331.61	-0.0111

been used to compute the springflow due to perturbation prior to the time origin. The estimated model parameters for the spring for which the objective function is the minimum are :

Length of the recharge zone (L) = 1155.50 meter
Width of the recharge zone (W_R) = 4046.00 meter
Length of the spring's opening (a) = 30.68 meter
Width of the spring's opening (b) = 30.68 meter
Distance of the spring from the no-flow boundary (x_1) = 4186 meter
Transmissivity of the aquifer in the flow domain (T) = 20590 m^2/month
Storage coefficient of the aquifer in the flow domain (ϕ) = 0.0013

The successive rapid decrease of the objective function in the search technique after end of each cycle are presented in Table 5.2.

Using these parameters, the recharge has been computed by the model and is presented in Table 5.3. As expected, the recharge which is estimated in this model, would be more than the recharge estimated by the model presented in Chapter-4. This is due to the reason that the previous model has a closed flow domain and in the present model, the flow domain is open and all recharge does not appear as springflow; part of the recharge flows out as regional groundwater flow.

The observed and the simulated springflow for the periods of no recharge and no abstraction, is similar in magnitude and is presented in Table 5.4.

Table 5.2 Successive reduction of the objective function in different cycle

Cycle	Number of times the objective function is reduced	Objective function (cubic meter per month)^2
0	-	10.0×10^{33}
1	8	4.79048×10^{10}
		2.00133×10^{10}
		1.10646×10^{10}
		5.95567×10^{9}
		3.21393×10^{9}
		1.39925×10^{9}
		8.44934×10^{8}
		7.38658×10^{8}
2	9	7.11159×10^{8}
		6.44212×10^{8}
		4.10417×10^{8}
		3.08260×10^{8}
		3.06065×10^{8}
		3.02576×10^{8}
		2.75378×10^{8}
		2.17338×10^{8}
		2.10443×10^{8}
3	9	1.92741×10^{8}
		1.87884×10^{8}
		1.86065×10^{8}
		1.71854×10^{8}
		1.68859×10^{8}
		1.65229×10^{8}
		1.59233×10^{8}
		1.55749×10^{8}
		1.49561×10^{8}

continued ...Table 5.2

4	9	1.49294×10^8
		1.48742×10^8
		1.46143×10^8
		1.45085×10^8
		1.37593×10^8
		1.34069×10^8
		1.29092×10^8
		1.24265×10^8
		1.24261×10^8
5	10	1.20508×10^8
		1.17643×10^8
		1.17435×10^8
		1.16395×10^8
		1.16392×10^8
		1.14708×10^8
		1.14480×10^8
		1.12569×10^8
		1.10442×10^8
		1.09304×10^8

Table 5.3. Computed monthly recharge for Kirkgoz spring, Turkey

Month	Recharge computed using parameters found by random jump technique (cubic meter)	Month	Recharge computed using parameters found by random jump technique (cubic meter)
1	.4655E+08	25	.1144E+09
2	.1403E+09	26	.1479E+09
3	.8339E+08	27	.8083E+08
4	.8155E+08	28	.4729E+08
5	.7815E+08	29	.8944E+08
6	.5986E+08	30	.5584E+08
7	.3915E+08	31	.1296E+08
8	.2104E+08	32	.1542E+08
9	.2726E+08	33	.1823E+08
10	.0000E+00	34	.0000E+00
11	.6158E+08	35	.2358E+08
12	.4084E+08	36	.4887E+08
13	.6138E+08	37	.7283E+08
14	.6989E+08	38	.6956E+08
15	.7641E+08	39	.6674E+08
16	.8455E+08	40	.3684E+08
17	.9100E+08	41	.6294E+08
18	.6794E+08	42	.4242E+08
19	.1746E+08	43	-.7968E+07
20	.3779E+08	44	.2372E+05
21	.1622E+08	45	.3109E+08
22	.0000E+00	46	.0000E+00
23	.5508E+08	47	.1514E+08
24	.6800E+08	48	.2640E+08

continued...Table 5.3

Month	Recharge computed using parameters found by random jump technique (cubic metre)	Month	Recharge computed using parameters found by random jump technique (cubic metre)
49	.1238E+09	70	.0000E+00
50	.2127E+09	71	.7851E+08
51	.1193E+09	72	.7014E+08
52	.1052E+09	73	.1173E+09
53	.9297E+08	74	.6439E+08
54	.3690E+08	75	.6923E+08
55	.3059E+08	76	.6621E+08
56	-.1609E+06	77	.6126E+08
57	.1330E+08	78	.5254E+08
58	.0000E+00	79	.2201E+08
59	.3607E+08	80	.2360E+08
60	.4250E+08	81	.2014E+08
61	.1163E+09	82	.0000E+00
62	.2722E+09	83	.1954E+08
63	.1003E+09	84	.5056E+08
64	.2377E+08	85	.1468E+09
65	.5079E+08	86	.1156E+09
66	.1016E+09	87	.1536E+09
67	.2796E+08	88	.6253E+08
68	.3234E+08	89	.0000E+00
69	.2691E+08		

Table 5.4. Observed and simulated springflow for the periods of no recharge and no abstraction

Month	Observed springflow after deducting effect of prior perturbation (cu meter/month)	Simulated springflow (cu meter/month)
10	.1992E+08	.1559E+08
22	.2093E+08	.1900E+08
34	.2134E+08	.2085E+08
46	.1666E+08	.1820E+08
58	.2634E+08	.2334E+08
70	.2599E+08	.2886E+08
82	.2389E+08	.2579E+08
89	.4833E+08	.4119E+08

5.4. CONCLUSIONS

(i) The graph $\log_{10} Q(t)$ versus t during recession does not follow a straight line.

(ii) Parameters of the model can be estimated by the random jump technique.

(iii) The model based on Hantush's basic solution assumes the flow domain to be infinite. The Bear's model, with or without storage effect of the transmission zone, assumes that the flow domain of the spring is a closed one. Because of this difference in the characteristics of the flow domain, the recharge computed by the model which is based on Hantush's solution is more than those which is computed by either of the Bear's model. In the Bear's model, all recharge appears as springflow, whereas in the model based on Hantush's solution, only part of the recharge appears at the spring.

GENERAL CONCLUSIONS

Two springflow models have been developed. The first model assumes the flow domain of the spring aquifer system to be a closed one. This implies that all the recharge to the spring flow domain would appear as springflow. Starting from the basic solution given by Carslaw and Jaeger (1959) to Boussinesq's equation for one-dimensional unsteady flow in a finite aquifer bound by two streams, and applying Duhamel's principle, the springflow for time variant recharge has been obtained. It is found that the variation of logarithm of springflow versus time strictly follows a straight line during the period of recession. This model in which the storativity of the transmission zone has been considered, behaves like the Bear's springflow model for very low storativity of the transmission zone. The storativity of the transmission zone reduces the magnitude of peak springflow and it causes delay in the appearance of peak springflow.

The second model is based on the Hantush's solution for two-dimensional groundwater flow due to recharge from a rectangular basin. In this model, the spring aquifer system is an open system. Therefore, all the recharge does not appear as springflow. The variation of logarithm of springflow with time during recession, does not follow a straight line. Only towards the latter part of recession, the variation is approximately linear.

In this thesis, both the direct and the inverse problems have been solved.

Bear's springflow model is a one parameter model, the parameter being the depeletion time. The recharge and the depletion time have been found for three springs by Newton-Raphson technique. It is found that for any initial guess of depletion time and recharge, unique solution is obtained after finite iterations.

Using the random jump technique, and the springflow model for a closed flow domain, the transmissivity, storativity, specific yield of the recharge zone, length of transmission zone, a linear dimension representing recharge area and spring width, and the recharge have been successfully computed from observed springflow data. The recharge computed by the random jump technique compare well with those obtained by Newton-Raphson technique.

Also, using the random jump technique and the springflow model for an open flow domain, recharge area, spring opening, distance of the spring from the recharge area, transmissivity and storativity of the transmission zone and the recharge have been estimated from observed springflow data.

Since the domain is an open one, the recharge computed by the model which is based on Hantush's solution, is found higher than those computed using the model for a closed system.

The Bear's one parameter springflow model which contains all the parameters except the storativity of transmission zone can still be regarded as an appropriate model to deal direct and inverse problems relating to springflow.

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