ACQUISITION PROPERTIES OF COMBINATION SEQUENCES

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(COMMUNICATION SYSTEMS)

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CERTIFICATE

Certified that the thesis entitled 'ACQUISITION PROPERTIES OF COMBINATION SEQUENCES', which is being submitted by Sri R.VENKATESWARA RAO in partial fulfilment of the requirements for the award of the DEGREE OF MASTER OF ENGINEERING in COMMUNICATION SYSTEMS, DEPARTMENT OF ELECTRONICS AND COMMUNICATIONS ENGINEERING at the University of Roorkee, is a record of the candidate's own work carried out by him under the supervision and guidance of the undersigned. The matter embodied in this thesis has not been submitted for the award of any other degree or diploma.

This is further certified that he has worked for a period of 6 months from 1st of January to 10th of July'79, for preparing this thesis at this University.

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ABSTRACT

The various acquisition schemes have been studied. The combination of sequences is one of the methods for faster acquisition. The correlation properties of various combination sequences have been computed for use in ranging and multiple-access communication applications. The logical functions that have been used in forming these combination sequences are 'Majority-Logic', 'AND' and 'Mod-2' for 2-component sequences and 'Majority-logic' , 'OR' and 'A1+A2.A3' for 3-component sequences.

Further, the acquisition time performance in a multiple-access communication environment has been discussed. It has been shown that as the number of users increases, the probability of acquisition error increases correspondingly for a fixed integration time. A few examples are done to substantiate this fact.

CONTENTS

Chapter-I

INTRODUCTION

One of the basic problems associated with any communication system [2] is 'How to locate and connect to the target'. The problem of locating and connecting up with the proper communication target is called the 'Acquisition Problem'.

In this chapter, we discuss the acquisition problem that comes into picture in ranging and multiple-access communication systems.

In section 1.1, we discuss the acquisition problem in ranging applications and in section 1.2 the same acquisition problem in a multiple-access communication system.

1.1 The Ranging Problem [1],[2]

The problem of measuring the range of a distant cooperative vehicle arises in a variety of problems, e.g., tracking a deep space vehicle, missile tracking, trajectory determination of artificial satellites, navigation, surveillance systems etc. For these applications the problem of determining range is equivalent to that of measuring the distance between two points; however , it should be noted that the way in which range data is obtained differs from application to application.

The classical radar technique for ranging is to transmit a pulse and measure the time until the return of the reflected pulse. The elapsed time multiplied by the propagation velocity in the medium is twice the range. As one attempts to measure larger and larger ranges, as in deep space applications, it becomes increasingly difficult to detect the reflected pulse, even if a transponder is used to enhance the echo. One may resort to transmitting many pulses or even sending a square wave or sine wave and apply correlation methods to detect the returned signal. However, this leads to an ambiguity in the range measurement if the repetition period of the pulses or period of the square wave or sine wave is less than the time required for the signal to travel to the target and back. In situation such as these, one often turns to the transmission of a binary sequence of digits (called ranging code) as a signal whose period for certain classes of codes can be designed to be arbitrarily long; hence, range ambiguity presents no problem. On the otherhand, the use of a digital ranging signal brings with it the problem of range resolution, in that the range measurement on the returned signal using correlation techni-

-ques can be resolved only to within one code-digit time interval. Clearly a trade off exists here, since the finer the range resolution (that is, the shorter the duration of a code digit) the longer the code sequence must be to avoid range ambiguity. If the range measurement is performed by sliding (in one code digit increments) a replica of the transmitted code sequence past the received signal and searching for the maximum correlation, then, long code sequences necessarily imply large acquisition times. In fact, since one usually has no 'a priori' information about the range to be determined, cross-correlating the received signal with the transmitted signal replica shifted one digit at a time would, in cases of distant range, lead to unacceptable acquisition time performance. From this discussion, one can summarize that a ranging code should have the following four characteristics: [1]

1) To avoid ambiguity in range measurement, the length of one complete code cycle (code digits per period times the duration of one code symbol) should be greater than the maximum anticipated round-trip transit time.

2) The code symbol repetition rate must be sufficiently high to meet the desired resolution or accuracy of the range measurement.

3) The auto-correlation function of the code should be

of the two level type: the sequence should have a maximum correlation when compared and correlated with itself and uniform degree of mismatch when compared with its k-digit shifts (k, any number).

4. To improve efficiency in transmission, it is desirable to provide a balanced use of power in the carrier sidebands by requiring the ranging code to have nearly the same number of ones and zeros within one complete period of the sequence.

Ranging codes possessing the above desirable properties are commonly formed from a class of binary sequences known as pseudo-noise (PN) sequences. The word 'pseudo' is used to denote the resemblance between the auto-correlation function of these sequences and that of band limited white random noise with uniform spectral density over a wide range of frequencies.

1.2 Multiple-Access Communication [3],[4],[5],[6],[11]

The use of radio communications has grown enormously in the past few decades to provide a broad range of services required by modern civilization. One consequence of this growth is that portions of the radio spectrum are very crowded with users, and yet there is a requirement to meet legitimate needs of still more users. This presents more problems in developing procedures for sharing, allocating and assigning the spectrum so as to satisfy the need to accommodate even more use of spectrum.

One possibility of alleviating some of the demands for spectrum is, of course, to move to ever higher frequencies where there is less, or no, current usage. However, the use of those higher frequencies may not permit the kind of system performance required, or equipment and techniques for operating at higher frequencies may not exist or are too costly for the service required. Thus, other solutions must be found.

The principal one employed today is that of reducing the bandwidth used to provide a service, either as a result of improved techniques for reducing necessary bandwidth or a forced reduction of assigned bandwidth by administrative decision. But reducing the assigned bandwidth also has obvious limitations, for the bandwidth assigned cannot continually be reduced without degrading system performance quality or requiring more costly equipment. So in recent years, there has been increased interest in a class of multiple-access techniques known as Code-Division multiple-Access (CDMA). The CDMA techniques are those multiple-access methods in which the multiple-access capability is due primarily to coding and in which - unlike traditional time problems in developing procedures for sharing, allocating and assigning the spectrum so as to satisfy the need to accommodate even more use of spectrum.

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The most common form of CDMA is Spread Spectrum Multiple Access (SSMA) where a certain allocated frequency band is shared by a number of users, each of whom occupies the whole band (as opposed to frequency division multiple access, FDMA, where each user is restricted to operate within some fraction of the band) with a duty cycle of 100 per cent (as opposed to time division multiple-access, TDMA, where each user has exclusive use of the medium for a certain fraction of time).

In SSMA, many systems will be operating in the same neighbourhood and each user will be assigned a distinct maximal encoding sequence usually a pseudonoise (PN) code. Each active user then modulates his message onto his pseudonoise (PN) carrier and transmits it through the satellite repeater to the receiving terminals. Each receiving station employs a phase coherent correlator capable of locking onto any one of the transmitted signals while rejecting the others. Once the receiver is locked onto one of the PN carriers, the message can be recovered by a correlation detector.

In general, the cross-correlation function between maximal sequences may be relatively large. Thus different systems operating in the same neighbourhood can interfere with the successful attainment and maintenance of proper synchronization by having the receiver of one communication link lock onto the cross correlation peaks obtained by correlating with the encoding sequence of a different communication link. Thus the successful use of spread spectrum communication systems in multiplexing applications depends upon the design of the codes which have good auto-correlation properties for each individual user, while at the same time the cross correlation between the codes of any two different users must be uniformly low.

From the above discussion we can conclude that the acquisition problem in spread spectrum systems involves the design of a large number of codes which have good auto-correlation properties when correlated with itself, while at the same time the cross correlation between the codes of any two different users must be uniformly low. Gold[11] has described a method to generate a large family of encoding sequences, which have low cross-correlation values. Instead of having each communication link employ a different maximal sequence, we assign to each link a member of the encoding family.

1.3 Organization of the thesis

The organization of the thesis is as follows:

In Chapter-II, we have studied the various approaches to the acquisition problem. Also, the use of combination sequences and their applications in acquisition problems have been discussed.

In Chapter-III, the properties of the combination sequences have been discussed at length. And, computer results for partial and full correlations of 2 and 3 component combination sequences are tabulated and are drawn in figs. Also, the partial and full cross-correlation of combination sequences which are useful in studying the acquisition time performance, have been tabulated. In the same chapter, the acquisition time performance in a multiple access communication environment has been analysed. A few examples are done in this regard.

In Chapter-IV the summary and conclusions are drawn up.

In Appendix, the programmes used for obtaining the above said computer results are given.

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Chapter-II

ACQUISITION SCHEMES

2.1 Introduction [8],[9]

A recurring communication systems problem involves the determination, with relatively high precision, of the phase of a periodic, noise-corrupted signal. Perhaps the most commonly proposed signals for such purposes are pseudo-random sequences. Indeed, such signals known to be optimum, at least when the noise is white and Gaussian, in the sense of minimizing the time needed to find the correct phase with a specified reliability.

In particular, suppose the received signal is known to be of the form $y(t) = x(t-\delta \Delta T) + n(t)$, with $x(t)$ a signal, periodic with period T, and $n(t)$ additive white Gaussian noise. The problem is to determine as reliably and rapidly as possible the phase δ of the received signal.

Many present day communication systems utilize pseudo-noise (PN) spread-spectrum modulation for various reasons, such as ranging, interference protection, power flux density reduction, low detectability, or multiple-access capability . All these systems must be able to synchronize the received code with the receiver code by searching over the code phase, and in addition, sometimes over a frequency uncertainty region. Then once having 'acquired' the code, the system must track the code with small error.

In the next section, we discuss the various approaches to the acquisition problem. These are :

(i) Stepping Correlation method.

(ii) Holmes and Chen method.

(iii) RASE system .

(iv) RARASE system.

2.2 Different Approaches to Acquisition

(i) Stepping correlation method [8],[14]

A common requirement in the receivers of spread spectrum systems is to set the initial state and maintain the phase of a receiver - generated replica of the transmitted PN signal in synchronism with the received PN signal. This acquisition process is most simply done by stepping the phase of the replica while cross-correlating the replica against the received PN signal until the cross-correlation output rises, i.e., if the transmitted sequence is $x=\left[\varsigma_1,\varsigma_2,\ldots,\varsigma_N\right]$ where N is the sequence length , the desired information is obtained by ascertaining which of the cyclic permutations x^i of x is being received, where $x^i = \left[\varsigma_{i+1}, \varsigma_{i+2}, \ldots,\varsigma_{i+N}\right]$, the subscripts interpreted modulo-N. In the absence of any equipment constraints , the optimum procedure for determining i is well known. If the additive noise is white and Gaussian, the optimum receiver would simultaneously form for each value of j (j=0,1,...N-1) the correlation $I_j = \sum_i c_{i+j} y_i$, where

$$y_i = \int_{iT/N}^{(i+1)T/N} y(t)\, dt,$$

with y(t) denoting the received signal and T the sequence period. As soon as a suitable number of terms y_i are available, the j_o is accepted as indicating the phase of the received sequence, where $\max_j \left[I_j\right] = I_{jo}$. Typically, however, N is large and equipment limitations preclude the simultaneous determination of all N of the correlations $\left[I_j\right]$. Indeed, often only one such correlation can be determined at a time. This method is slow because it takes no account of what the phase of the incoming PN signal appears to be.

The integration time over which the correlation is to be performed to find out the phase of the sequence, is dependent upon the probability of detection error. If we want to keep the probability of error low, the integration time would be more.

In this method, the acquisition time of a code is relatively large because of two reasons :

a) the integration time of the correlation should be large in order to keep the probability of error low and

b) the number of correlations that has to be performed is large because the phase of the incoming PN signal is unknown.

In the next method discussed, the integration time is done twice in order to estimate the phase of the incoming sequence, the result is a reduction in acquisition time.

(ii) Holmes and Chen method [9]

The next approach we discuss, was given by Holmes et al. The acquisition system which they studied is called a double dwell time system since, two integration times (or dwell time τ_D) is utilised in the code search process. The first integration is done to search the code phases quickly (τ_D is small), and the second to provide a 'better' estimate of whether the 'in-synch' code phase has been found. The basic idea is to apportion some of the false alarm protection in the first integration and place the remaining (usually greater) protection in the second integration.

Acquisition model [9] :- Consider the model shown in Fig.1. This model (assuming no Doppler) is as follows: Assume that there are q-cells to be searched (q may be equal to the length of the PN code to be searched or some multiple of it).

For example, if the update size is one-half chip, q will be twice the code length to be searched. Typically, the search starts by advancing the reference code phase to the extremity of the code search range ambiguity region and then a trial integration of τ_{D_1} seconds is made on the received signal plus noise. If the threshold is not exceeded, then the reference (system) code is delayed, for example, by one-half chip and again the dwell integration time is τ_{D_1} seconds. The process continues in this manner until a hit (threshold is exceeded) occurs. Then without changing the code phase the integration time is increased to τ_{D_2} seconds. This dwell provides both a higher probability of detection and a lower probability of false alarm. If the second threshold is exceeded, then typically the code loop is activated and a third integration of the input signal plus noise is performed (in practice it could be the same duration as τ_{D_2}). If the threshold were not exceeded at this point the search would continue. This final integration period forms the basis of a lock detector. Briefly, the idea is to detect say, three consecutive times in which the output does not exceed the threshold. When this event occurs, the code loop is declared to be out of lock and the search is started over.

(iii) RASE System [13]

The next acquisition method, Rapid Acquisition by Sequential Estimation (RASE), was given by Ward [13]. It is useful for moderate input signal-to-noise ratio (SNR), a common condition of many high-quality tracking systems. For SNR's down to -10 to -15 dB, the RASE system can give significantly shorter acquisition times, than swept systems or cross correlation stepping acquisition systems.

In this system, the first n received bits are to be loaded into the receiver sequence generator and let the generator start from that initial condition. It will then continue to produce a sequence which is approximately in-phase with the incoming sequence. A tracking loop can maintain the phase from that time on, following any variations of Doppler due to target dynamics or other causes.

The RASE system makes its best estimate of the first n received bits, loads the receiver sequence generator with that estimate, and starts operation of the sequence generator and the tracking circuits. If the correct estimate is loaded, tracking will occur. At the same time, a cross-correlation is performed between the input signal and the signal from the local sequence generator. If the cross-correlation indicates the receiver to contain the correct sequence and tracking to be occurring, no further action is taken. If the cross-correlation indicates that an incorrect estimate was made, a new estimate of the input is made, loaded and tracked. This process is continued until the correct estimate is finally obtained.

Next, an improved method of this system, given by [14] has been discussed . It is named 'Rapid Acquisition by Recursion-Aided Sequential Estimation '(RARASE).

(iv) RARASE System [14]

An improved method of acquisition of pseudonoise signals is discribed by Ward and Yiu[14]. This method , named Rapid Acquisition by Recursion-Aided Sequential Estimation (RARASE) is an outgrowth of the RASE method.

This method uses the recursion relation of the PN signal for improving the initial phase estimate. In this method the incoming PN signal is observed for a time much longer than the minimum needed to determine the phase in the absence of noise. The additional bits estimated are used to determine whether the estimate appears to be error-free and therefore worth attempting to track. A high proportion of incorrect initial state estimates can be discarded with relatively simple logic. The estimates for which tracking is attempted have a high probability of being correct. It provides a better performance than RASE, but does so with a moderate increase in logic hardware.

Description [14] :- The recursion aided system is based on the use in the receiver PN generator of identical feedback logic to that used in the transmitter (the recursion logic) and the use of a device which we call a sync worthiness indicator (SWI) to find regions of the input data stream which are probably error free.

The simplest RARASE implementation is shown in Fig. 2. There, a five-stage m-sequence generator identical to that used in the transmitter feeds back the outputs of flip-flops 3 and 5 through a mod-2 adder to the input to produce the sequence when the load/track logic has the switch in the upper position. The same flip-flop outputs and the present received bit are compared in the SWI, in this case a single 3-input mod-2 adder, in order to determine when the load /track switch should be raised.

The sequence of operations controlled by the load/ track logic is as follows: Five incoming estimated bits are allowed to fill the shift register. Then as more bits flow through the register the output of the SWI is permitted to raise the switch as soon as mod-2 agreement among the three SWI inputs is observed. A trial track and in-lock correlation is then performed while the switch is maintained in the upper position and while the shift register continues to produce a replica of the incoming sequence with initial phase corresponding to the contents of the shift register when the switch was raised. If a not-in-lock decision is made at the end of the trial period, the switch is lowered and the process is repeated. This continues until the contents of the shift register when the switch is raised are error-free. Then the phase of the signal produced by the shift register matches that being received and the in-lock detector prevents the load/track logic from loading another estimate.

2.3 Use of Combination Sequences [2],[7],[10],[16].

As has already been discussed to achieve fine range resolution, one desired a high pulse repetition rate. For long ranges this implies a long sequence. Long ranges also imply a low returned signal level, and therefore a long integration time to detect the signal. One can use correlation to determine the phase of a PN sequence, by successive trial and error; that is, one chooses a phase and tries a correlation . If the phase choosen is not correct, one has no better choice than to try the next one . If the sequence were a million digits long, one might have to try all of the million possible phases to find the right one. However, from an information theory standpoint one should have to ask for only $\log_2 10^6$ yes - or - no questions instead of 10^6 questions. Sequences have been found which have the property that the phase can be determined by correlation using far fewer than the full number of trials. These sequences, called combination sequences or composite sequences are formed by combining several short PN sequences digit by digit[8]. If the periods of the several sequences have no common divisors, the period of combination sequence is the product of the periods of several sequences. It is possible to determine the phase of the combination sequence by determining separately the phases of the component sequences. This requires at most

$$p_1 + p_2 + \dots + p_n$$

trial correlations to determine the phase of a sequence whose length is $p_1 \, p_2 \dots p_n$ when the p_i are the lengths of the component sequences.

Easterling [16] has described one interesting means for combining an odd number of binary sequences; the sequences are simply combined in a majority logic. For example, the composite code c(i) can be the majority vote combination of a number of shorter sequences $a_k(i)$ of period p_k:

$$c(i) = \text{sgn} \left[\sum_{k=1}^{N} a_k(i) \right]$$

for N odd, where $a_k(i) = \pm 1$.

The model used by Milstein [7] has been taken from Titsworth. Given m sequences of lengths n_i, $i = 1, 2, \dots, m$, where the n_i are pairwise relatively prime, a composite sequence is formed of length $N = \prod_{i=1}^{m} n_i$. Each chip of the composite sequence is a logical combination of one chip from each of the component sequences. For example, suppose the component sequences were given by 110 and 00101 and that the logical function was 'union'. The composite sequence would then be as follows:

A = First component	1 1 0 1 1 0 1 1 0 1 1 0 1 1 0
B = Second component	0 0 1 0 1 0 0 1 0 1 0 0 1 0 1
C = A∪B = Composite	1 1 1 1 1 0 1 1 0 1 1 0 1 1 1

That means, by starting with sequences of lengths 3 and 5, a sequence of length 3 x 5 = 15 was generated.

To acquire such a sequence, it is necessary to cross correlate the composite sequence with each of its components to determine at what relative shift, the correlation 'peaks up'. In particular, the i^{th} component requires up to n_i such correlations and therefore atmost $\sum_{i=1}^{m} n_i$ correlations are required to acquire a sequence of length $\prod_{i=1}^{m} n_i$. This is in contrast to a PN sequence where for a sequence of length n, one requires up to n correlations to acquire the code.

The choice of what Boolean function to use can be made by considering the auto-correlation properties. Tittsworth proved that to maximize the difference between the in-phase correlation value of a composite sequence with one of its component sequences and any out- of -phase correlation value, the Boolean function to use is that of majority logic.

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It should be noted that not all Boolean functions lead to codes whose components can be separated. For example, mod-2 addition of components will not lead to a code which is acquirable on a component - by - component basis.

Because of the long time required to calculate the full correlation between two long binary sequences, the partial correlation is sometimes used as an approximation to the full correlation. In fact, the whole objective of combination sequences is to permit more rapid search and acquisition. That is, for the sequences $\{U_i\}$ and $\{V_i\}$, rather than calculate

$$R = \frac{1}{N} \sum_{i=1}^{N} U_i V_i ,$$

one settles for

$$R_{M_o} = \frac{1}{M_o} \sum_{i=1}^{M_o} U_i V_i$$

for some M_o much smaller than N.

The full and partial correlation properties of combination sequences that are useful in ranging and multiple-access communication systems, are discussed in detail, in the next chapter.

Chapter-III

CORRELATION PROPERTIES AND ACQUISITION TIME PERFORMANCE OF COMBINATION SEQUENCES

In this chapter, the partial and full correlation properties of combination sequences that are useful in ranging and multiple-access communication applications have been discussed. Computer results are obtained for the full and partial correlations of 2 and 3 component combination sequences which are formed by using various logical functions and these correlations are compared. Finally , the acquisition time performance in multiple-access communication systems has been discussed. A few examples are given to calculate the acquisition time and probability of acquisition error in a multiple-access environment.

3.1 Properties of Combination sequences [6],[10]

As we have already discussed in section 2.3, combination sequences come into picture when there is a need for longer sequences. In this section we will discuss the correlation properties of the combination sequences.

For a combination sequence to be useful for rapid acquisition, it must correlate well with each of the subsequences. That is, the cross-correlation

$$R_{ca_k}(j) \triangleq \frac{1}{N}\sum_{i=1}^{N} c(i)\ \ a_k(i+j) \qquad ..(1)$$

should be maximised at $j = 0$ for all subsequences a_k. Tausworthe and Braverman have shown, as given by Spilker[10], that the majority logic composite code maximizes this cross-correlation and is the same for all of the component sequences a_k, i.e.,

$$\rho(M) \triangleq R_{ca_1}(0) = R_{ca_2}(0) = \ldots. = R_{ca_M}(0)$$

Some combinations of sequences have correlation functions which turn out to be of special interest. There are two points that should be noted about the autocorrelation function of the combination sequences. First, there are peaks for all values of τ (phase shifts) which are divisible by the period of either component. Second , the correlation, when both components are out of phase, is not $-1/n_1 n_2$ where n_1, n_2 are the lengths of the component sequences . This is a consequence of the fact that the combination sequence is very unbalanced.

Not all combination sequences have auto-correlation functions with peaks, at the multiples of the periods of component sequences.

The value of $\rho(M)$ has been computed from eqn(1) as

$$1 > \rho(M) = \left[\begin{matrix} M-1 \\ \frac{M-1}{2} \end{matrix} \right] / 2^{M-1} \simeq \frac{(2/\pi)^{1/2}}{\sqrt{M-1}} ; M >> 1$$

Values of $P(M)$ for a majority vote of M code subsequence components are given in [10].

As has already been discussed in Chapter-I, in a CDMA system, we have a number of users, each using an independent combination code sequence. In this type of situations the cross correlations of one combination sequence with the components of other combination sequences will be received as noise by that particular combination sequence's user. Milstein[4] developed simple bounds for the cross correlation functions.

Consider any two composite sequences X and Y. Assume that each one was generated by combining two shorter sequences. That is, letting Z_i denote the i^{th} chip of Z,

$$X_i = \text{sgn}\,(A_{1_i} + A_{2_i})$$

and

$$Y_i = \text{sgn}\,(B_{1_i} + B_{2_i})$$

where $\left[A_{j_i}\right]$ and $\left[B_{j_i}\right]$ are independent random binary sequences of length n_{j_i}, $j = 1,2$ and $i = 1,2,\ldots N$.

Assume it is desired to acquire X by first correlating with A_1. Since X and Y will be received simultaneously, the (unwanted) contribution from Y will be given by

$$R = \frac{1}{N}\sum_{i=1}^{N} A_{1_i}\,\text{sgn}\,(B_{1_i} + B_{2_i})$$

Implicit in the above equation are the assumption that all the signals have been converted to the ± 1, that all signals are arriving with equal strengths and that they are all arriving synchronously. Milstein has derived that the variance of the out-of-phase cross correlation is bounded by the equation

$$\sigma^2 = \frac{1}{N} + \frac{n_2 - 1}{2N} \simeq \frac{1}{2n_1}$$

for $n_1 \gg 1$ and $n_2 \gg 1$.

If three component sequences are used in forming the combination sequences, it can be shown that

$$\sigma^2 = \mathrm{var}(R) = \frac{1}{4n_1}\left[1 + \frac{1}{n_2} + \frac{1}{n_3} + \frac{1}{n_2 n_3}\right] \simeq \frac{1}{4n_1}$$

for $n_1 \gg 1$, $n_2 \gg 1$ and $n_3 \gg 1$.

Also, if M component sequences are used, where M is an odd number, then, when $n_i \gg 1$ for all $i = 1, 2, \ldots M$, the variance of the cross correlation between the composite sequence and the first component sequence of a different composite sequence is approximated by

$$\sigma^2 \simeq 1/2^{(2M-2)} \left[\frac{(M-1)!}{\left[\left(\frac{M-1}{2}\right)!\right]^2}\right]^2 \frac{1}{n_1} \simeq \frac{2}{\pi(M-1)\,n_1}$$

for $M \gg 1$.

Upto this point, only the unwanted correlation has been discussed.

Consider now the correlation between a combination sequence and one of its own components. The most important property of such a correlation is the expected value when the two are in phase. Under such conditions the following results can be shown to hold.

(a) If M = 2,

$$E\left[\frac{1}{N}\sum_{i=1}^{N} A_{1_i} \operatorname{sgn}(A_{1_i} + A_{2_i})\right] = \frac{1}{2}$$

(b) If M = 2k+1 , k = 1,2,...

$$E\left[\frac{1}{N}\sum_{i=1}^{N} A_{1_i} \operatorname{sgn}(A_{1_i} + A_{2_i} + \cdots + A_{2k+1})\right] = 2^{1/2k}\binom{2k}{k}$$

The reason for the emphasis on the M=2 case (besides the simplicity), is that it leads to the largest component length n_1 for a given composite length N (assuming that with r components, each component has length approximately $(N)^{1/r}$). But one disadvantage with M = 2 is that it leads to unbalanced combination sequences; i.e. the number of ones is not equal to the number of minus ones. Because of this fact, these composite sequences are more vulnerable to certain types of jamming then those in which the number of ones equals the number of minus ones. With such jamming, it makes sense to use an odd number of components, hence an odd number of M.

The whole object of using combination sequences is to permit more rapid search and acquisition. And, because of the long time required to calculate the full correlation between two long binary sequences, the partial correlation is used as an approximation to the full correlation. So the partial-correlation properties of the combination code are more important than the full-period correlation , since the partial correlations result in rapid search and acquisition.

For two sequences $[U_i]$ and $[V_i]$, rather than calculate the full period correlation

$$R = \frac{1}{N} \sum_{i=1}^{N} U_i V_i \ ,$$

one satisfies with the partial correlation

$$R_{N_0} = \frac{1}{N_0} \sum_{i=1}^{N_0} U_i V_i$$

for some N_0 much smaller than $N = n_1 n_2$.

If the U_i in the above equations are the chips of one component of a combination sequence and the V_i are the chips of a complete combination sequence, then the following results, derived by Milstein [7], hold for the two component combination sequences :

(a) If the U sequence is one of the components (A_1) of V (i.e., V_i = sgn ($A_{1_i} + A_{2_i}$) where $U_i = A_{1_i}$), then the in-phase value of R_k has a variance bounded by

$$\sigma^2_{R_{N_0}} \leq \frac{1}{4N_0} \left(2[\frac{N_0}{n_1}] - \langle [\frac{N_0}{n_1}] - 1 \rangle + \right.$$

$$\left. + 2 [\frac{N_0}{n_2}] - \langle [\frac{N_0}{n_2}] - 1 \rangle + 3 \right)$$

where [X] denotes the greatest integer less then X (i.e. [2.3] = 2, [3] = 2), and where

$$\langle X \rangle = \begin{cases} X & X > 0 \\ 0 & X \leq 0 \end{cases}$$

(b) If the U_i are one of the components of V_i, but the partial correlation is performed over an out-of-phase relative shift or if the U is a component of a combination sequence other than V, then the variance of R_{N_0} is bounded by

$$\sigma^2_{R_{N_0}} \leq \frac{1}{2N_0} (2 + [\frac{N_0}{n_1}])$$

where n_1 is the fundamental period of U.

(c) The expected value of R_k for any of the above conditions equals the expected value of R.

Milstein [7] has tabulated as shown below, the various bounds on the variance of combination sequences.

Table-1. Bounds on the variance of combination sequences:

Type of correlation	Total correlation	Partial correlation
Correlation of one composite with component of different composite	$\frac{n_2 + 1}{2N}$	$\leq \frac{1}{2N_o}\left[2 + \left[\frac{N_o}{n_1}\right]\right]$
Correlation of one composite with in-phase value of one of its own components	$\frac{n_1 + n_2 + 3}{4N}$	$\leq \frac{1}{4N_o}\left[2\left[\frac{N_o}{n_1}\right] - \left\langle\left[\frac{N_o}{n_1}\right]-1\right\rangle + 2\left[\frac{N_o}{n_2}\right] - \left\langle\left[\frac{N_o}{n_2}\right]-1\right\rangle + 3\right]$
Correlation of one composite with out-of-phase value of one of its own components.	$\frac{n_2 + 1}{2N}$	$\leq \frac{1}{2N_o}\left[2 + \left[\frac{N_o}{n_1}\right]\right]$

The properties of combination sequences, which we have discussed up to this point, have been verified with the practical results obtained by using computer. This is done in the next section.

3.2 Computer Results

Combination sequences for various lengths of 2 components and 3 components have been found by using different logical operations. Auto-correlation values of these combination sequences for all possible phases of each component have been calculated. Also, the partial correlations over different partial lengths of the sequences have been computed. And also, the cross-correlations, and partial correlations of combination sequences with components of other combination sequences are calculated.

(a) Full Correlations :

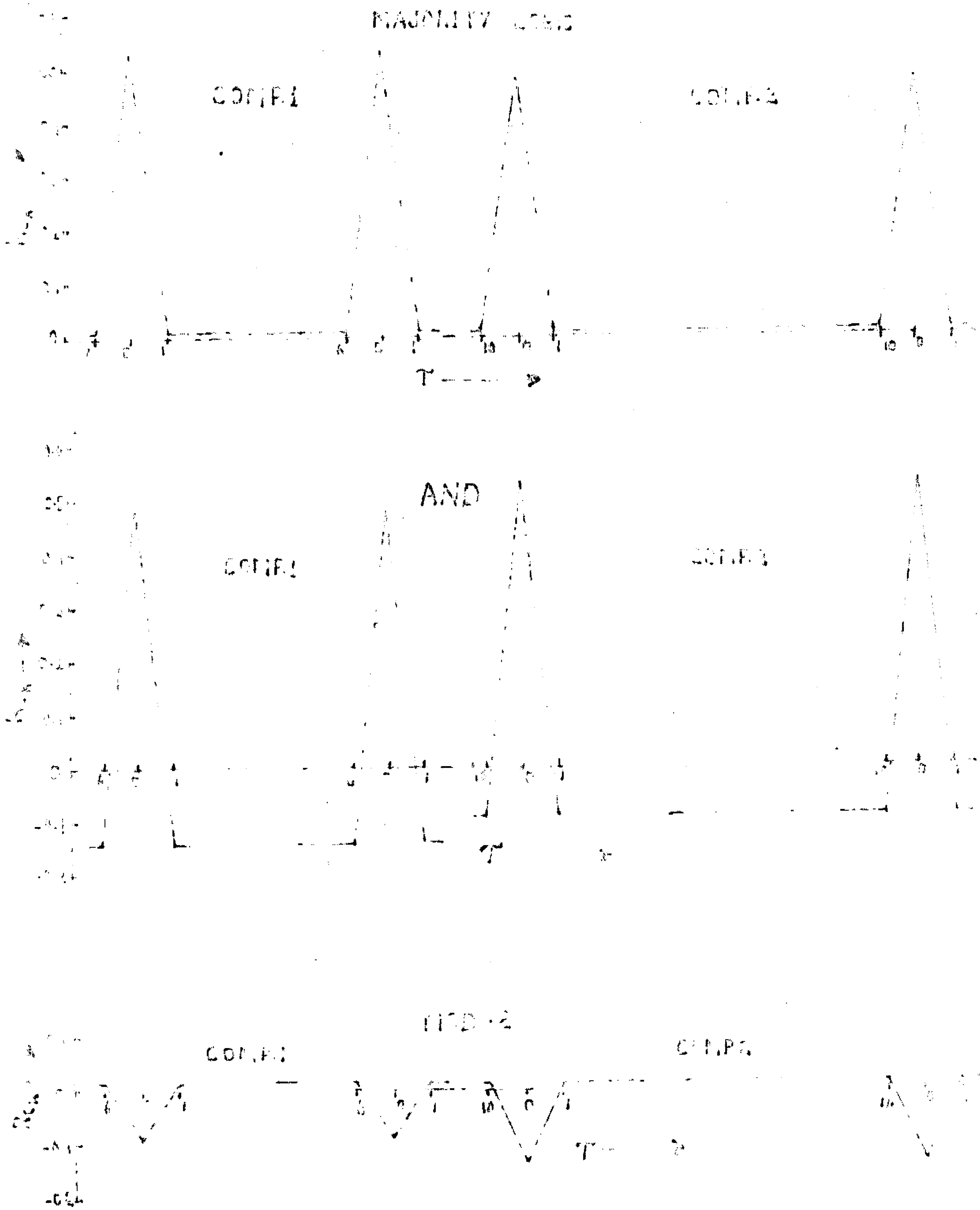
2 Component combination sequences :- For these combination sequences the various logical operations that have been carried upon are 'Majority Logic'(which is the same as the OR operation for 2 components), 'AND' and 'Mod-2'. The results are arranged in the tabular form and are drawn in Figs. 3 and 4. The programme for this is given in Appendix - Program A-1.

(i) Auto correlations of 2 components combination sequences using components of lengths 3 and 7.

First component (3°) = 1 -1 1

Second component (7) = 1 -1 -1 1 -1 1 1

[o The figures in the brackets indicate the lengths of the sequences].



[illegible] ACF OF COMB. SIGNALS USING COMPS. OF LENGTH 7,11

Phase	Auto-correlation coefficients		
	Majority logic	AND	Mod-2
First component			
Phase τ = 1	4.761904E-2	-3.333333E-1	4.761904E-2
2	4.761904E-2	-3.333333E-1	4.761904E-2
0	6.190476E-1	4.285714E-1	-1.428571E-1
Second component			
Phase τ = 1	4.761904E-2	-1.428571E-1	4.761904E-2
2	4.761904E-2	-1.428571E-1	4.761904E-2
6	4.761904E-2	-1.428571E-1	4.761904E-2
0	4.285714E-1	6.190476E-1	-3.333333E-1

(ii) Using components of lengths 7 and 11.

First component (7) = 1 -1 -1 1 -1 1 1

Second component(11)= 1 1 -1 1 1 1 -1 -1 -1 1 -1

Phase	Auto-correlation coefficients		
	Majority Logic	AND	Mod-2
First component			
Phase τ = 1	1.298701E-2	-1.428571E-1	1.298701E-2
6	1.298701E-2	-1.428571E-1	1.298701E-2
0	5.324675E-1	4.805196E-1	-9.090908E-2
Second component			
Phase τ = 1	1.298701E-2	-9.090908E-2	1.298701E-2
10	1.298701E-2	-9.090908E-2	1.298701E-2
0	4.805196E-1	5.324675E-1	-1.428571E-1

From these tables and the graphs it can be concluded that Majority - logic combination of 2 component sequences gives the maximum difference between the in-phase and out-of-phase correlations. So in ranging applications, where this difference should be as large as possible, the Majority-logic combination of 2 component sequences finds good application. The next best logical function is the AND operation. The Mod-2 logical operation, which gives the minimum difference between the in-phase and out-of-phase correlations of 2 component combination sequences , is not useful for ranging purposes.

<u>3 Component combination sequences</u> :- For these combination sequences the various logical functions that have been used a are Majority - Logic , OR , and A1 + A2.A3 (A1, A2, and A3 are the three components). The results are arranged in the tabular form as shown below and are drawn in Figs.5 and 6. The programme for this is given in Appendix-Programme A-3.

(i) Auto-correlations of 3 component combination sequences using components of length 3,7 and 7.

First component (3) = 1 -1 1

Second component(7) = 1 -1 -1 1 -1 1 1

Third component (7) = 1 -1 -1 1 1 1 -1

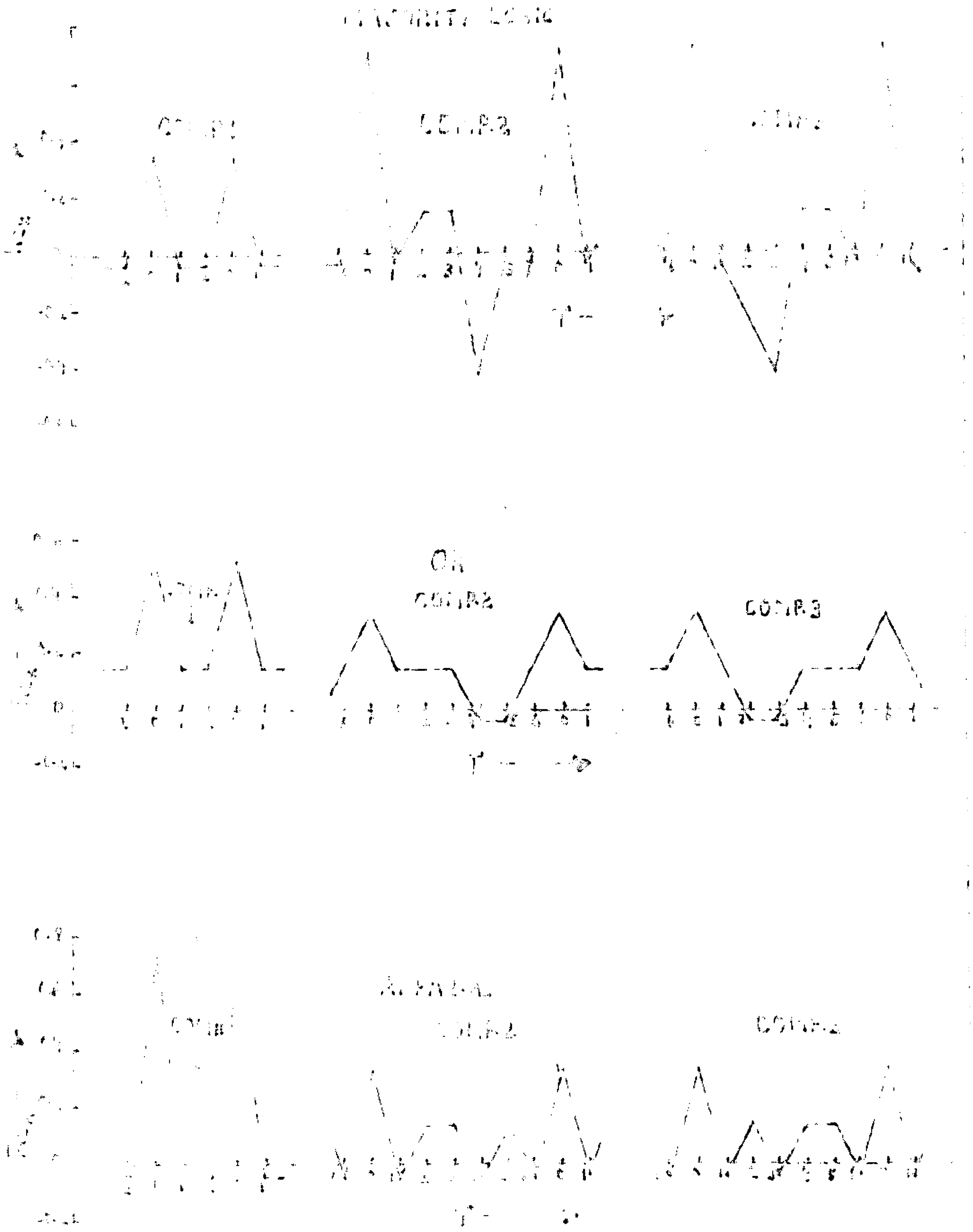


FIG. [illegible] USING [illegible] OF [illegible]

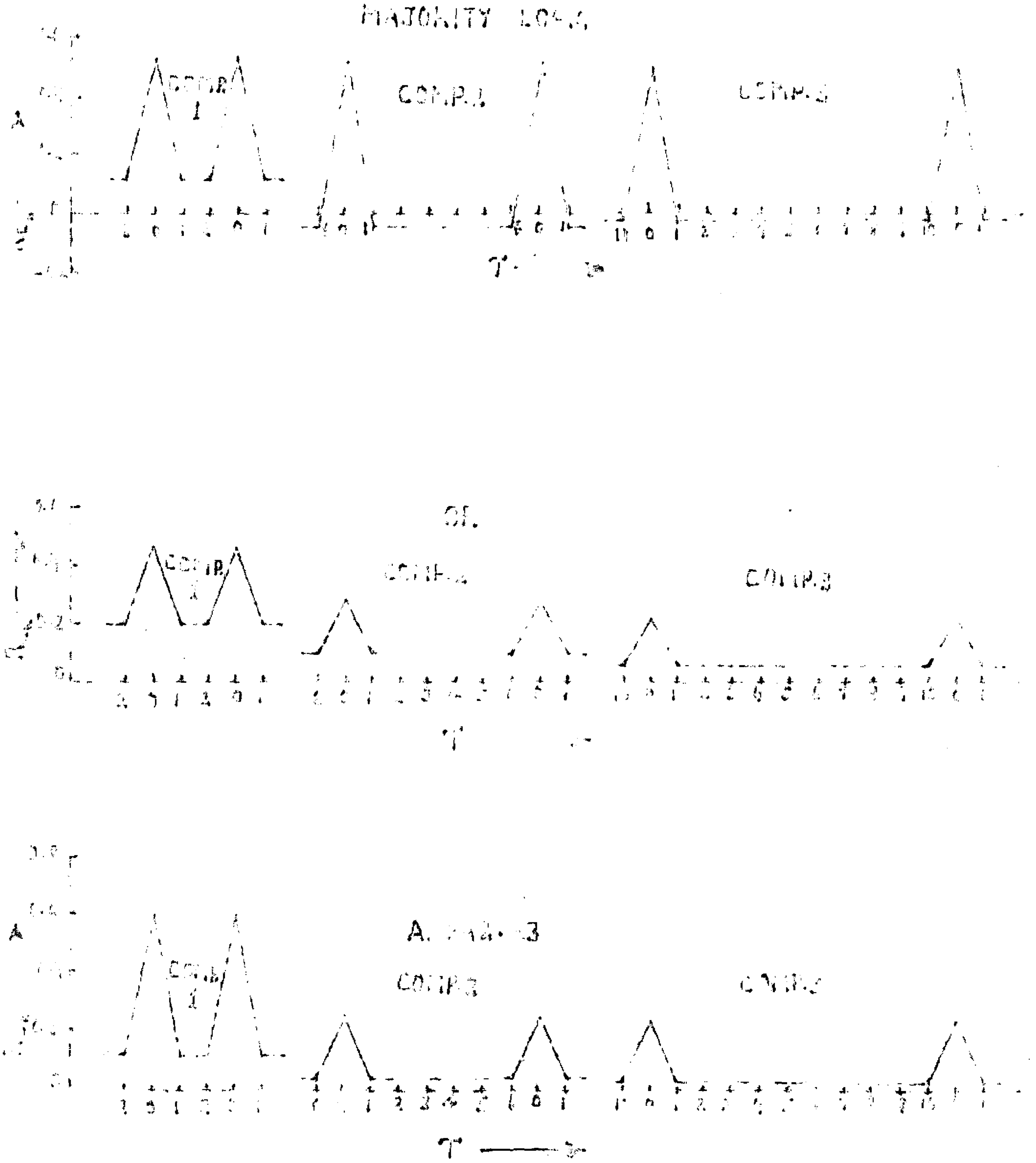


FIG-4. ACF OF COMB. SEQUENCE USING COMPS. OF LENGTHS 3,7,11

Phase	Auto-correlation coefficients		
	Majority-Logic	OR	A1+A2.A3
First component			
Phase τ = 1	-4.761905D-2	1.428571D-1	-4.761905D-2
2	-4.761905D-2	1.428571D-1	-4.761905D-2
0	3.333333D-1	5.238095D-1	7.142856D-1
Second component			
Phase τ = 1	-4.761905D-2	1.428571D-1	-4.761905D-2
2	1.428571D-1	1.428571D-1	1.428571D-1
3	1.428571D-1	1.428571D-1	1.428571D-1
4	-4.285714D-1	-4.761905D-2	-4.761905D-2
5	-2.380952D-2	-4.761905D-2	1.428571D-1
6	-4.761905D-2	1.428571D-1	-4.761905D-2
0	7.142856D-1	3.333333D-1	3.333333D-1
Third component			
Phase τ = 1	-4.761905D-2	1.428571D-1	-4.761905D-2
2	-2.380952D-1	-4.761905D-2	1.428571D-1
3	-4.285714D-1	-4.761905D-2	-4.761905D-2
4	1.428571D-1	1.428571D-1	1.428571D-1
5	1.428571D-1	1.428571D-1	1.428571D-1
6	-4.761905D-2	1.428571D-1	-4.761905D-2
0	7.142856D-1	3.333333D-1	3.333333D-1

(ii) Using components of lengths 3, 7 and 11.

First component (3) = 1 -1 1

Second component (7) = 1 -1 -1 1 -1 1 1

Third component (11) = 1 1 -1 1 1 1 -1 -1 -1 1 -1

Phase	Auto-correlation coefficients		
	Majority - Logic	OR	A1+A2.A3
First component			
Phase τ = 1	-1.255411D-1	2.034632D-1	-1.255411D-1
2	-1.255411D-1	2.034632D-1	-1.255411D-1
0	5. 324675D-1	4.632034D-1	7.922077D-1
Second component			
Phase τ = 1	-3.896104D-2	9.956708D-2	4.761904D-2
6	-3.896104D-2	9.956708D-2	4.761904D-2
0	5.151515D-1	2.727273D-1	2.554112D-1
Third component			
Phase τ = 1	-2.164502D-2	6.493506D-2	3.030303D-2
10	-2.164502D-2	6.493506D-2	3.030303D-2
0	4.978355D-1	2.207792D-1	2.380952D-1

From these tables and graphs we can conclude that with longer component sequences, the Majority logic combination of 3 component sequences gives a better performance than the ~~performance~~ remaining two logical functions used , even though it is somewhat inferior with shorter component sequences. The A1+A2.A3 logical operation gives the next best performance. It can also be observed from these graphs that the Auto-correlation function, of 3-component combination sequences with shorter components, is not a two-level function.

(b) <u>Partial Correlations</u>

As has already been discussed in the previous chapters, partial correlations of combination sequences find extensive use in ranging applications. Partial correlations over different fractions of lengths of various combination sequences for all possible phases of each component have been performed and these results are tabulated.

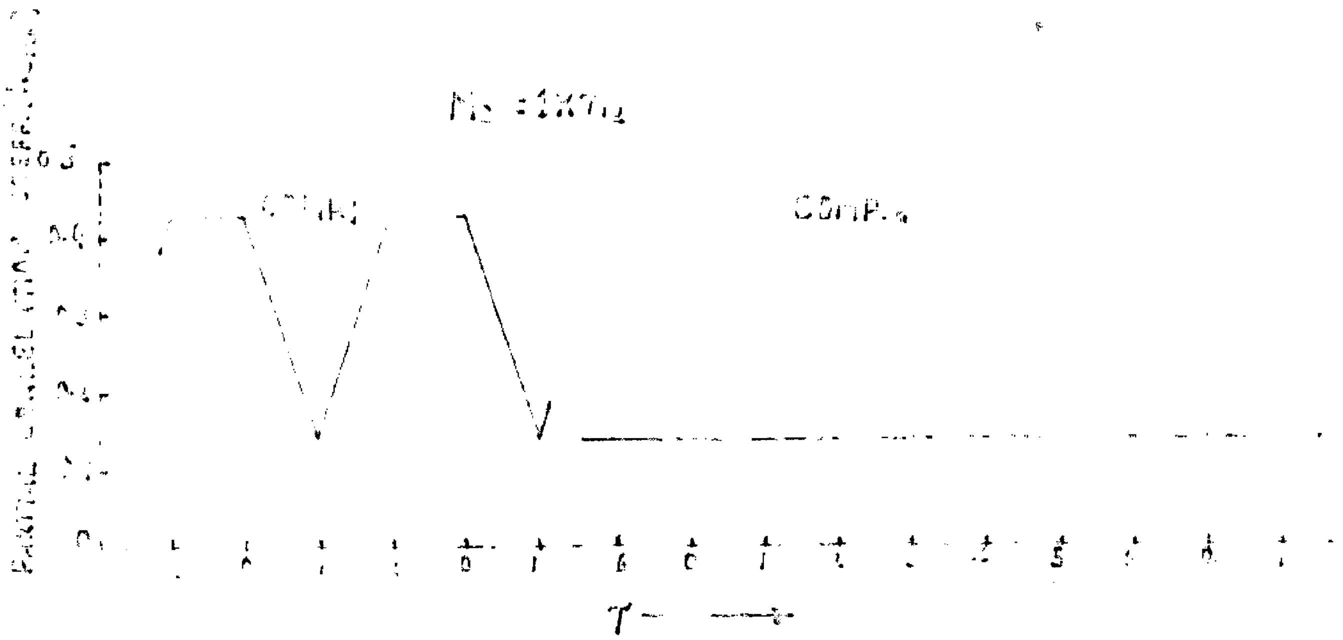
<u>2-Component combination sequences</u> :- The logical function that has been used in forming the combination sequences is the Majority logic since we found from the previous results that Majority logic operation is the best suitable for ranging purposes. The programme for this is given in

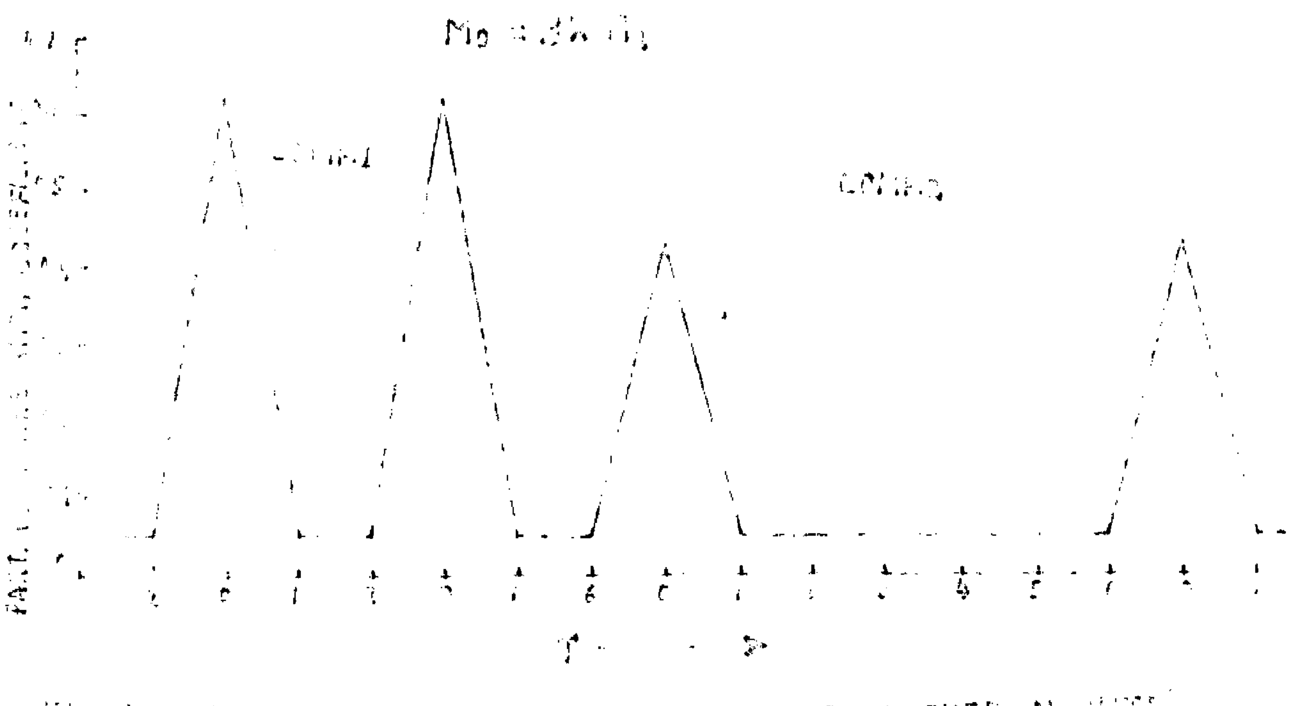
Appendix-Programme A-2. The results are tabulated as shown below and are drawn in Figs.7 and 8.

(i) Partial correlations of 2 component combination sequence using components of lengths 3 and 7.

The data is the same as that used for full correlations.

Phase	Partial correlations over N_0 Bits	
	$N_0 = 1 x n_2$	$N_0 = 3 x n_2 = n_1 n_2$
First component		
Phase τ = 1	1.428571E-1	4.761904E-2
2	4.285714E-1	4.761904E-2
0	4.285714E-1	6.190476E-1
Second component		
Phase τ = 1	1.428571E-1	4.761904E-2
6	1.428571E-1	4.761904E-2
0	1.428571E-1	4.285714E-1





[illegible] SEQUENCE ([illegible] 3,7) OVER [illegible] BITS

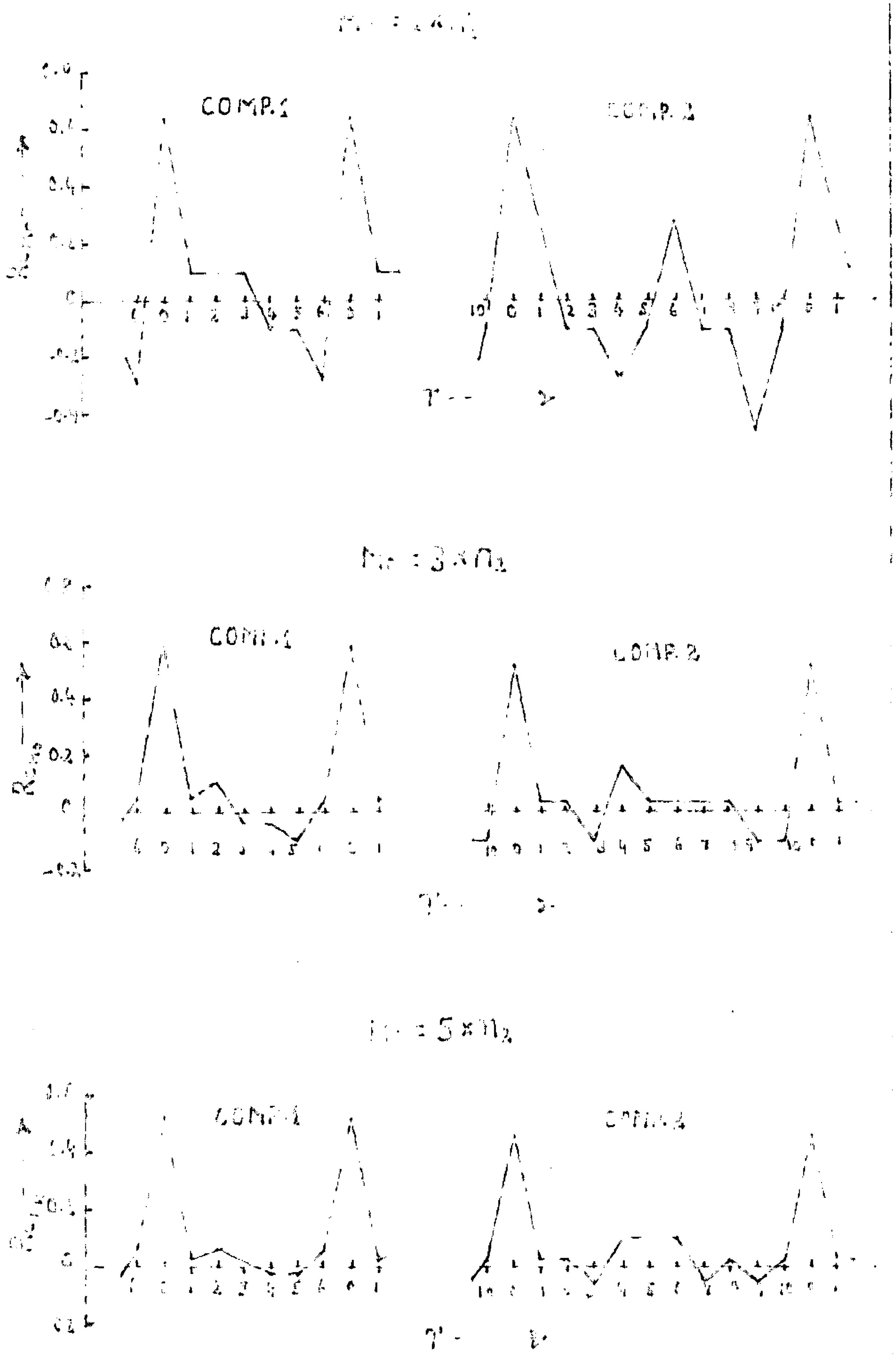


FIG. 8. PG. OF CORR. FG. (COMPS. I,II) OVER [illegible]

(ii) Using components of lengths 7 and 11.

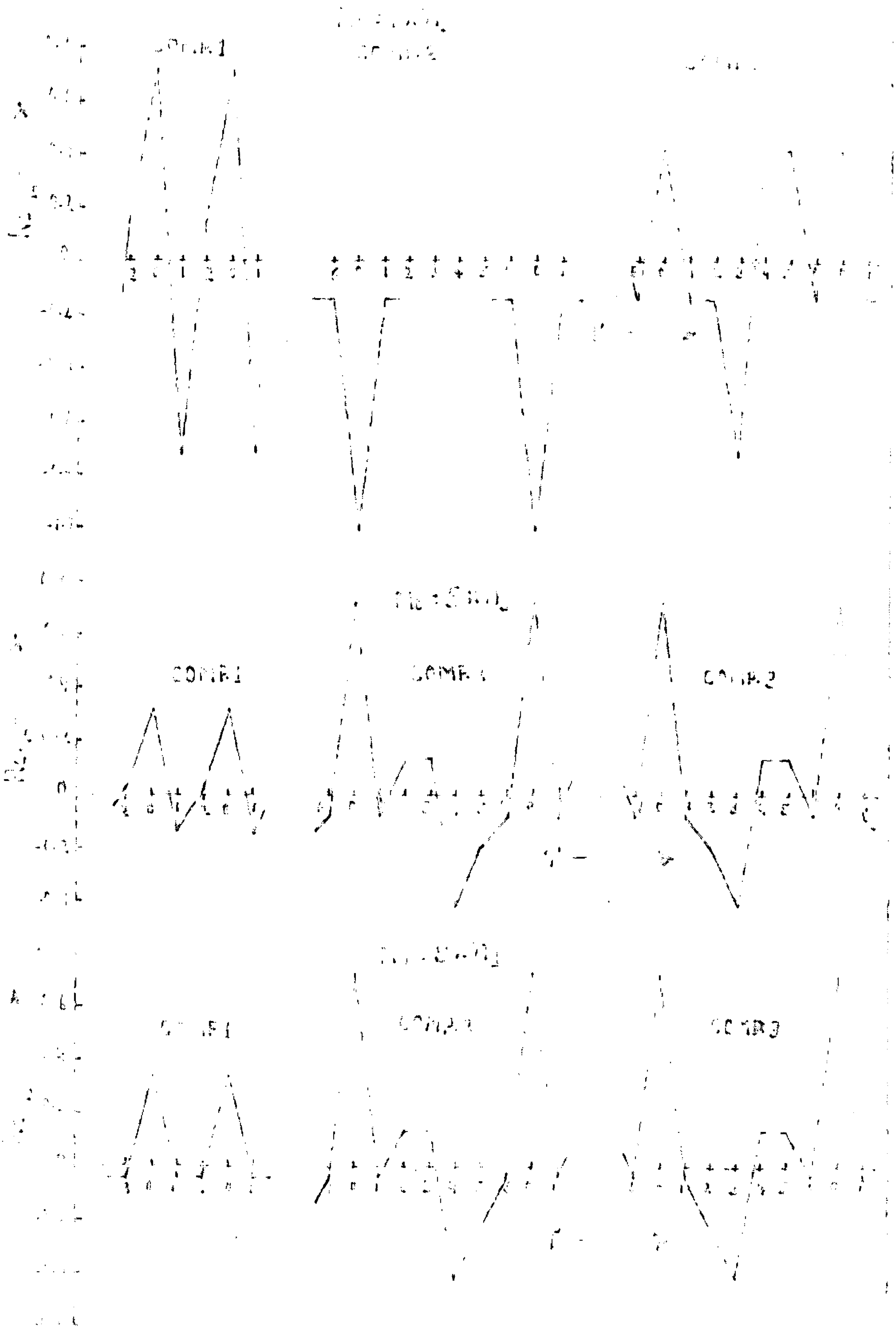
Phase		Partial correlations over M_0 Bits		
		$M_0=1m_2$	$M_0=3m_2$	$M_0=5m_2$
First component				
Phase τ =	1	9.090908E-2	3.030303E-2	1.818181E-2
	2	9.090908E-2	9.090908E-2	5.454545E-2
	3	9.090908E-2	-3.030303E-2	1.818181E-2
	4	-9.090908E-2	-3.030303E-2	-1.818181E-2
	5	-9.090908E-2	-9.090908E-2	-1.818181E-2
	6	-2.727273E-1	3.030303E-2	5.454545E-2
	0	6.363636E-1	5.757575E-1	5.272727E-1
Second component				
Phase τ =	1	2.727273E-1	3.030303E-2	1.818181E-2
	2	-9.090908E-2	3.030303E-2	1.818181E-2
	3	-9.090908E-2	-9.090908E-2	-5.454545E-2
	4	-2.727273E-1	1.515151E-1	9.090908E-2
	5	-9.090908E-2	3.030303E-2	9.090908E-2
	6	2.727273E-1	3.030303E-2	9.090908E-2
	7	-9.090908E-2	3.030303E-2	-5.454545E-2
	8	-9.090908E-2	3.030303E-2	1.818181E-2
	9	-4.545454E-1	-9.090908E-2	-5.454545E-2
	10	-9.090908E-2	-9.090908E-2	1.818181E-2
	0.	6.363636E-1	5.151515E-1	4.545454E-1

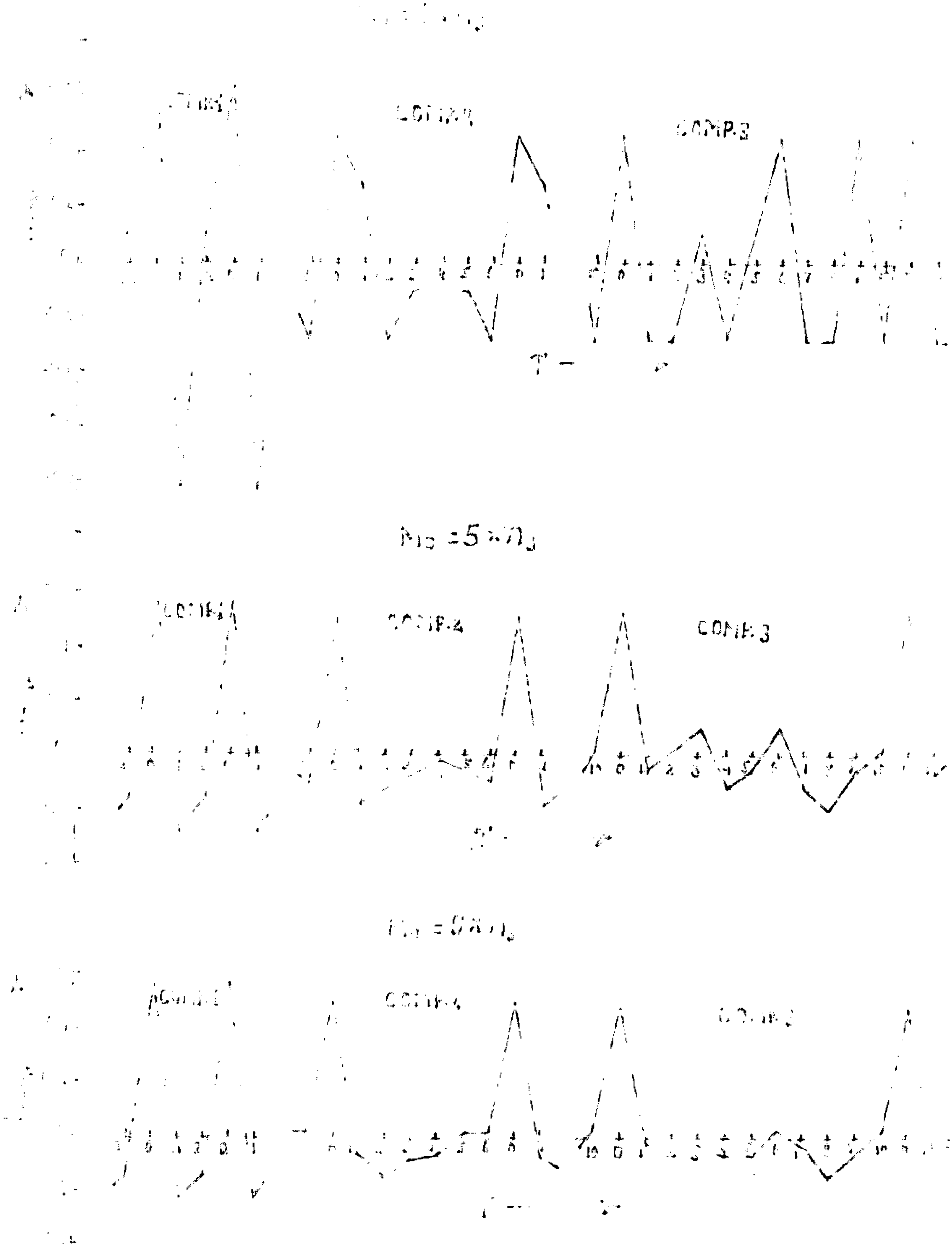
3-Component combination sequences:- Partial correlations of 3-component combination sequences have been calculated over various lenghts of sequences. Majority logic operation is performed in forming the combination sequences. The data that are used here is the same as that used for full correlations.

The results are tabulated as shown below and are drawn in Figs.9 and 10. The programme for these results is given in Appendix-ProgrammeA-4.

(i) Using components of lengths 3,7 and 7.

Phase	Partial correlations over N_0 Bits		
	$N_0 = n_3$	$N_0 = 5 \times n_3$	$N_0 = 9 \times n_3$
First component			
Phase τ = 1	-7.142855D-1	-1.428571D-1	-4.761904D-2
2	1.428571D-1	2.857143D-2	-4.761904D-2
0	7.142855D-1	3.142857D-1	3.333333D-1
Second component			
Phase τ = 1	-1.428571D-1	-8.571430D-2	-4.761904D-2
2	-1.428571D-1	1.428571D-1	1.428571D-1
3	-1.428571D-1	1.428571D-1	1.428571D-1
4	-1.428571D-1	-4.285714D-1	-4.285714D-1
5	-1.428571D-1	-2.000000D-1	-2.380952D-1
6	-1.428571D-1	-8.571430D-2	-4.761904D-2
0	-1.000000D+0	7.142856D-1	7.142855D-1
Third component			
Phase τ = 1	-1.428571D-1	-8.571430D-2	-4.761904D-2
2	-1.428571D-1	-2.000000D-1	-2.380952D-1
3	-7.142855D-1	-4.285714D-1	-4.285714D-1
4	4.285714D-1	1.428571D-1	1.428571D-1
5	4.285714D-1	1.428571D-1	1.428571D-1
6	-1.428571D-1	-8.571430D-2	-4.761904D-2
0	4.285714D-1	7.142856D-1	7.142855D-1


COMB1
COMB2



[illegible] (COMPS. 3,7 [illegible]

(ii) Using components of lengths 3, 7 and 11.

Phase	Partial correlations over N_0 Bits		
	$N_0=n_3$	$N_0=5n_3$	$N_0=9n_3$
First component			
Phase τ= 1	-8.181818D-1	-2.727273D-1	-2.121212D-1
2	9.090909D-2	-1.636363D-1	-1.313131D-1
0	6.363636D-1	5.636363D-1	5.555555D-1
Second component			
Phase τ= 1	2.727273D-1	-1.818181D-2	-5.050504D-2
2	-2.727273D-1	-1.272727D-1	-1.313131D-1
3	-9.090903D-2	-5.454545D-2	-7.070703D-2
4	-9.090908D-2	-1.818181D-2	-5.050504D-2
5	-9.090908D-2	-5.454545D-2	1.010101D-2
6	-2.727273D-1	-9.090903D-2	1.010101D-2
0	4.545454D-1	4.909090D-1	4.949494D-1
Third component			
Phase τ= 1	-2.727273D-1	-5.454545D-2	-1.010100D-2
2	-2.727273D-1	1.818181D-2	-1.010100D-2
3	9.090903D-2	9.090903D-2	-1.010100D-2
4	-2.727273D-1	-1.272727D-1	-5.050504D-2
5	9.090903D-2	-5.454545D-2	-5.050504D-2
6	4.545454D-1	9.090903D-2	3.030303D-2
7	-2.727273D-1	-1.272727D-1	-1.010100D-2
8	-2.727273D-1	-2.000000D-1	-1.313131D-1
9	4.545454D-1	-5.454545D-2	-5.050504D-2
10	-2.727273D-1	1.818181D-2	3.030303D-2
0	4.545454D-1	5.272727D-1	4.747474D-1

From these tables for partial correlations, we can observe that the partial correlations over less number of bits do not give much difference between in-phase and out-of-phase correlations. Indeed, sometimes the out-of-phase correlation is as large as the in-phase correlation. As the number of bits , over which the partial correlations are performed increases the performance is seen to be improving.

In this part of the computer results, we have calculated the correlations of a combination sequence with its own components and correlations of the same combination sequence with the components of other combination sequences. These correlations are useful in analysing the acquisition time performance in a multiple-access environment.

Results are obtained for two examples. In one example, we have taken 15 and 31 as the lengths of the component sequences and in the second example, 31 and 63 are taken as the component lengths. Since we have only two different sequences [12] of length 15 and six of length 31 and 63, we can form only two different combination sequences of length 15x31 = 465 in the first example and six different sequences of length 31x63 = 1953 in the second example. The logical function that has been used in forming the combination sequences is the Majority logic. These results are tabulated as shown below. The programme for this is given in Appendix - Programme A-5.

(i) The data that have been used in this first example are :

15 : 1 -1 -1 -1 1 1 1 1 -1 1 -1 1 1 -1 -1

31 : 1 -1 -1 -1 -1 1 -1 1 -1 1 1 1 -1 1 1 -1 -1 -1 1 1 11 1 -1 -1 1 1 -1 1 -1 -1

15 : 1 -1 -1 -1 1 -1 -1 1 1 -1 1 -1 1 1 1

31 : 1 -1 -1 -1 -1 1 -1 -1 1 -1 1 1 -1 -1 1 1 1 1 1 -1 -1 -1 1 1 -1 1 1 1 -1 1 -1

Table: Partial and full correlations of a combination sequence with its own components.

Correlation over N_o bits	first component	second component
$2n_2$	5.806451E-1	6.129031D-1
$4n_2$	5.161290E-1	5.322580E-1
$6n_2$	4.838709D-1	5.161291D-1
$8n_2$	4.677419D-1	5.080645D-1
$10n_2$	4.774193E-1	5.161291E-1
n_1n_2	4.838709D-1	5.182795D-1

Table: Partial and full correlations of a combination sequence with the components of other combinations.

Correlation over N_o bits	first component	second component
$2n_2$	-6.451612D-2	-3.225805D-2
$4n_2$	-4.838709D-2	-3.225806D-2
$6n_2$	-5.376345D-2	-3.225806D-2
$8n_2$	-5.645162D-2	-3.225806D-2
$10n_2$	-5.806452D-2	-3.225806D-2
n_1n_2	-6.666668E-2	-3.225806D-2

(ii) In the second example, we have used five different sequences of lengths 31 and 63 . It has been observed from the computer results that the cross correlations of one composite sequence with the components of any other composite sequence are coming same. So here we have given the data of only two different sequences of lengths 31 and 63 each.

31 : 1 -1 -1 -1 -1 1 -1 1 -1 1 1 1 -1 1 1 -1 -1 -1 1 1 1 1 1
-1 -1 1 1 -1 1 -1 -1

63 : 1 -1 -1 -1 -1 -1 1 1 1 1 1 1 -1 1 -1 1 -1 11 -1 -1 1 1
-1 1 1 1 -1 1 1 1 -1 1 -1 -1 1 -1 -1 1 1 1 -1 -1 -1 1 -1
1 1 1 1 -1 -1 1 -1 1 -1 -1 -1 1 1 -1 -1 -1 -1

31 : 1 -1 -1 -1 -1 1 -1 -1 1 -1 1 1 -1 -1 1 1 1 1 1 -1 -1 -1
1 1 1 -1 1 1 1 -1 1 -1

63 : 1 -1-1 -1 -1 -1 1 1 1 -1 -1 -1 -1 1 -1 -1 1 -1 -1 -1 1 1
-1 1 1 -1 -1 1 -1 1 1 -1 1 -1 1 1 1 -1 1 1 1 1 -1
-1 1 1 -1 -1 -1 1 -1 1 -1 1 -1 -1 1 1 1 1 1 1 -1

Table : Partial and full correlations of a combination sequence with its own components.

Correlation over N_o bits	first component	second component
$2n_2$	4.761904D-1	5.555554D-1
$4n_2$	5.238094D-1	5.317460D-1
$6n_2$	5.343915D-1	5.449735D-1
$8n_2$	5.198411D-1	5.317460D-1
$10n_2$	4.952381D-1	5.079365D-1
n_1n_2	5.084486D-1	4.920635D-1

Table : Partial and full correlations of a combination sequence with the components of other combinations.

Correlation over N_o bits	first components	second component
$2n_2$	-3.174603D-2	-1.587301D-2
$4n_2$	-2.380952D-2	-1.587301D-2
$6n_2$	-2.645502D-2	-1.587301D-2
$8n_2$	-2.380952D-2	-1.587301D-2
$10n_2$	-2.539682D-2	-1.587301D-2
n_1n_2	-3.225806D-2	-1.587301D-2

From these results we can observe that the correlation coefficients of one combination with the first component of other combinations ared different for different lengths of correlations even though these are same in the case of second component. Further, it has been observed that the correlation coefficients of a combination sequence with its own components are different for different lengths of correlations.

3.3 Acquisition time performance [15],[8]

As has already been discussed in the first chapter, spread-spectrum methods are used to obtain multiple-access capability. In a multiple-access communication system many users are operating in the same environment . So a receiver receives its own signal alongwith those of other users, which are added as noise. As the number of users increases, this noise , called interference power, increases and as a result, the acquisition time of the signals increases.

Here, a procedure has been given for calculating the acquisition time in a multiple-access environment.

The transmitted signal is formed by modulating source data by a user code (usually PN code). Source data rate and user code rate will be denoted by $1/T$ and $1/T_c$ binary digits / second , respectively. It is convenient to assume that source data and user code are synchronized so that only whole chip of the user code fall into a source signal interval, T. The signal interval, T, therefore contains

$$N = T/T_c \qquad .. (2)$$

chips , so that, in effect, source data is being coded into an $(N,1)$ code with a corresponding bandwidth increase over the base-band bandwidth by a factor of N.

Now the interference power received by a receiver is:

$$N_o' = PT(L-1)\,(R'_{k\ell})^2 \qquad .. (3)$$

where P = Signal power received,

T = Period of a data bit (also called Integration time),

L = Number of users,

R'_{k_i} = Cross correlation coefficient of one combination sequence with i^{th} component of other combination sequences.

[Note:- Here we have assumed the maximum value of R'_{k_i} as the cross correlation of one combination sequence with i^{th} component of all other combination sequences which is a worst case.]

This interference power will be added to the additive white Gaussian noise (N_o) already present there. Now the total noise power will become

$$N' = N_o + N_o' \qquad .. (4)$$

As a result of this increased noise power, the signal-to-noise ratio, (SNR), available at the receiver will be reduced to

$$SNR = \frac{PTR_{k_i}^2}{N'} \qquad .. (5)$$

where R_{k_i} = Auto-correlation of a combination sequence with its own i^{th} component.

And, for every SNR available we can get the corresponding probability of acquisition error (P_e) from the tables given by Lindsey and Simon [1].

The acquisition time of i^{th} component now be calculated as

$$T_{acq} = n_i \times T \qquad .. (6)$$

where n_i = length of the i^{th} component sequence and

T = integration time corresponding to that probability of error .

Now we shall do two examples to study the acquisition time performance.

(a) Using components of lengths 15 and 31

Assumptions made are :

Chip period (T_c) $= 1 \times 10^{-6}$ sec.

Length of the combination sequence (N) $= n_1 n_2 = 15 \times 31$
$= 465.$

Maximum SNR available at the receiver (PT/N_o)
$= 15$ dB $= 31.62$ (ratio)

Spectral density of white Gaussian noise

$$(N_o) = k\,t$$

where k = Boltzmann's constant $= 1.37 \times 10^{-23}$ J/^oK

t = Absolute temperature = 300 ^oK

So $N_o = 1.37 \times 10^{-23} \times 300 = 4.11 \times 10^{-21}$

Now

Data bit period (integration time T) $= N \times T_c = 0.465 \times 10^{-3}$ sec.

$$\text{Signal power } (P) = \frac{4.11 \times 10^{-21} \times 31.62}{0.465 \times 10^{-3}} = 2.795 \times 10^{-16} \text{ W.}$$

The probability of error (P_e) has been calculated for different integration times and for different number of users, using the formulae given in the beginning of this section.

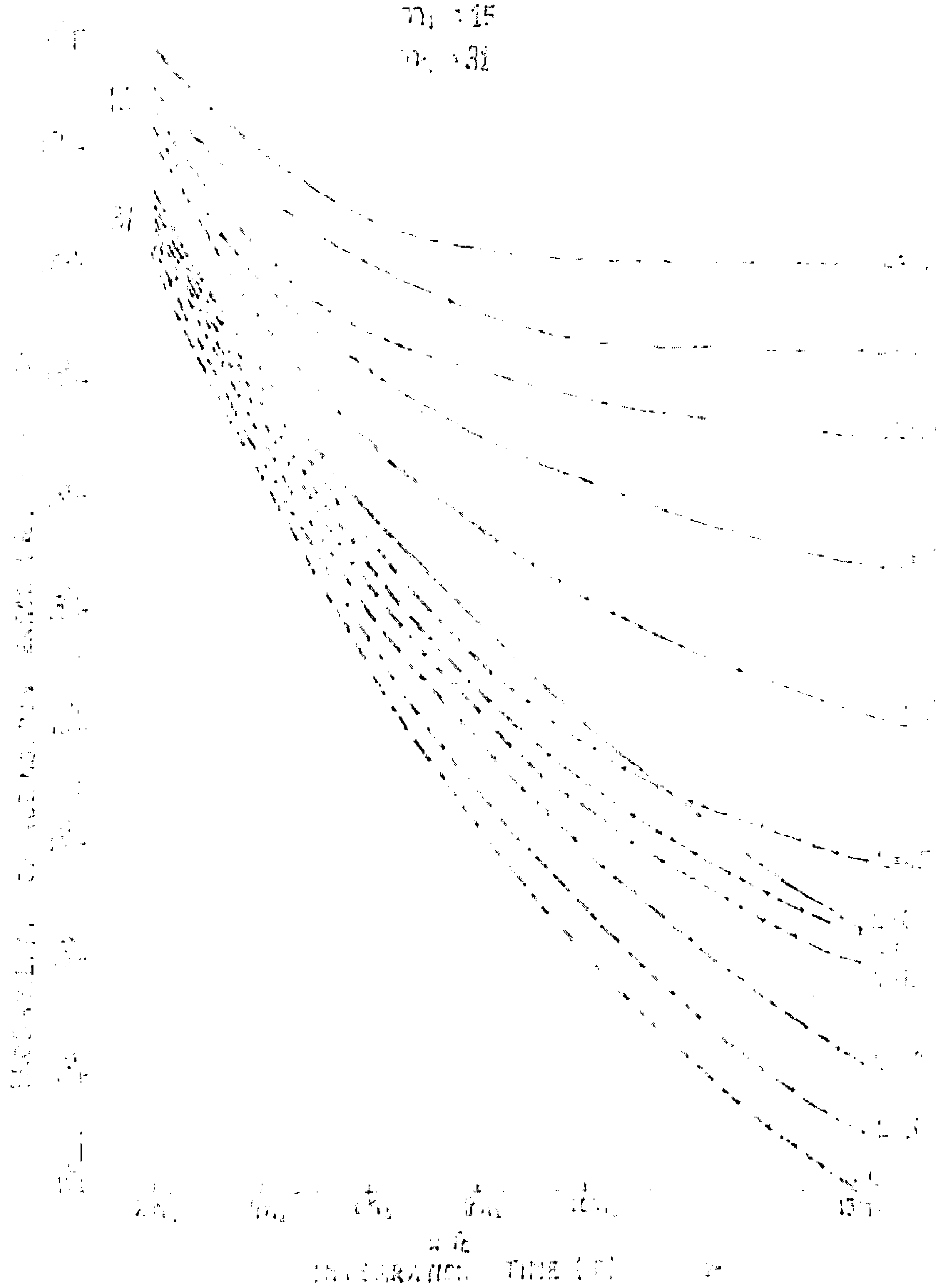
The results are tabulated as shown below and are drawn in Fig.11.

For finding the acquisition time of a component with some given probability of acquisition error, we obtain the corresponding integration time of that component from Fig.11 and using Eq.(6) we can calculate the acquisition time of that component.

[Note:- The values of R_{k_1} and R'_{k_1} are taken from the computer results which are tabulated in the previous section.]

(i) When the number of users (L) = 1

Component length n_i	Integration time (T) (sec.)	SNR available (dB)	Probability of acquisition error (P_e)
15	0.062×10^{-3}	1.523	6.4×10^{-2}
31		1.997	1.7×10^{-2}
15	0.124×10^{-3}	3.514	1.2×10^{-3}
31		3.783	2×10^{-4}
15	0.186×10^{-3}	4.713	9×10^{-5}
31		5.276	4.5×10^{-6}
15	0.248×10^{-3}	5.669	1.5×10^{-5}
31		6.389	2.6×10^{-7}
15	0.31×10^{-3}	6.817	1.4×10^{-6}
31		7.494	1.4×10^{-8}
15	0.465×10^{-3}	8.695	3×10^{-8}
31		9.251	1.5×10^{-10}



(ii) When the number of users° (L) = 5.

Component Length n_i	Integration time (T) (sec)	SNR available (dB)	Probability of acquisition error (P_e)
15	0.062×10^{-3}	1.228	1.1×10^{-1}
31		1.911	2.1×10^{-2}
15	0.124×10^{-3}	3.191	2.2×10^{-3}
31		3.635	2.7×10^{-4}
15	0.186×10^{-3}	4.119	3.5×10^{-4}
31		5.053	5.0×10^{-6}
15	0.248×10^{-3}	4.823	8.0×10^{-5}
31		6.089	5.0×10^{-7}
15	0.310×10^{-3}	5.724	1.2×10^{-5}
31		7.131	3.5×10^{-8}
15	0.465×10^{-3}	6.753	1.5×10^{-6}
31		8.751	6.0×10^{-10}

[° Actually there are only two different sequences of length 15 and six of length 31. So we can generate only two different combination sequences of length 15 x 31 = 465. Here, we have assumed, for the sake of convenience that there are a large number (up to 25) of different combinations generated. Indeed, we can generate 25 different comb. sequences by using components of length 255 and above.]

(iii) When L = 10

n_1	T (sec)	SNR (dB)	P_0
15	0.062×10^{-3}	0.897	2.0×10^{-1}
31		1.811	2.6×10^{-2}
15	0.124×10^{-3}	2.821	4.8×10^{-3}
31		3.455	4.5×10^{-4}
15	0.186×10^{-3}	3.471	1.3×10^{-3}
31		4.788	1.5×10^{-5}
15	0.248×10^{-3}	3.96	5×10^{-4}
31		5.741	1.3×10^{-6}
15	0.310×10^{-3}	4.647	1.2×10^{-4}
31		6.716	1.0×10^{-7}
15	0.465×10^{-3}	5.139	4.2×10^{-5}
31		8.16	2.8×10^{-9}

(iv) When L = 15

n_1	T (sec)	SNR (dB)	P_0
15	0.062×10^{-3}	0.584	3.5×10^{-1}
31		1.723	3.2×10^{-2}
15	0.124×10^{-3}	2.483	1.0×10^{-2}
31		3.29	6.8×10^{-4}
15	0.186×10^{-3}	2.903	4.0×10^{-3}
31		4.550	2.8×10^{-5}
15	0.248×10^{-3}	3.237	2.1×10^{-3}
31		5.423	2.8×10^{-6}
15	0.465×10^{-3}	3.965	5.0×10^{-4}
31		7.64	1.0×10^{-8}

(v) When L = 20

n_i	T (sec)	SNR (dB)	P_e
15	0.062×10^{-3}	0.289	5.7×10^{-1}
31		1.64	3.8×10^{-2}
15	0.124×10^{-3}	2.159	1.8×10^{-2}
31		3.123	1.0×10^{-3}
15	0.186×10^{-3}	2.412	1.0×10^{-2}
31		4.303	5.4×10^{-5}
15	0.248×10^{-3}	2.617	7.2×10^{-3}
31		5.125	6.3×10^{-6}
15	0.310×10^{-3}	3.073	3.0×10^{-3}
31		5.982	7.8×10^{-7}
15	0.465×10^{-3}	3.042	3.2×10^{-3}
31		7.175	3.0×10^{-8}

(vi) When L = 25

n_i	T (sec)	SNR (dB)	P_e
15	0.062×10^{-3}	0.013	9.0×10^{-1}
31		1.554	4.8×10^{-2}
15	0.124×10^{-3}	1.91	3.0×10^{-2}
31		2.963	1.6×10^{-3}
15	0.186×10^{-3}	1.967	2.6×10^{-2}
31		4.079	9.5×10^{-5}
15	0.248×10^{-3}	2.075	2.2×10^{-2}
31		5.658	1.3×10^{-6}
15	0.310×10^{-3}	2.458	1.0×10^{-2}
31		5.658	1.6×10^{-6}
15	0.465×10^{-3}	2.282	1.4×10^{-2}
31		6.755	8.5×10^{-8}

(b) Using components of lengths 31 and 63

Here also, we can have only six different combination sequences because there are only six different sequences of lengths 31 and 63.

The assumptions made are :

Chip period (T_c) = 1×10^{-6} sec.

Length of combination sequence (N) = $n_1 \times n_2$ = 31 x 63 = 1953.

Max. SNR ~~transmitted~~ (PT/N_o) = 20 dB = 100 (ratio).

Spectral density of Gaussian noise (N_o) = kt = 4.11×10^{-21}

Now

Data bit period (T) = $N \times T_c$ = 1.953×10^{-3} sec.

$$\text{Signal power (P)} = \frac{4.11 \times 10^{-21} \times 100}{1.953 \times 10^{-3}} = 2.104 \times 10^{-16} W$$

The probability of acquisition error (P_c) has been calculated for different integration times and for different number of users. The results are tabulated as shown below and are drawn in Fig.12.

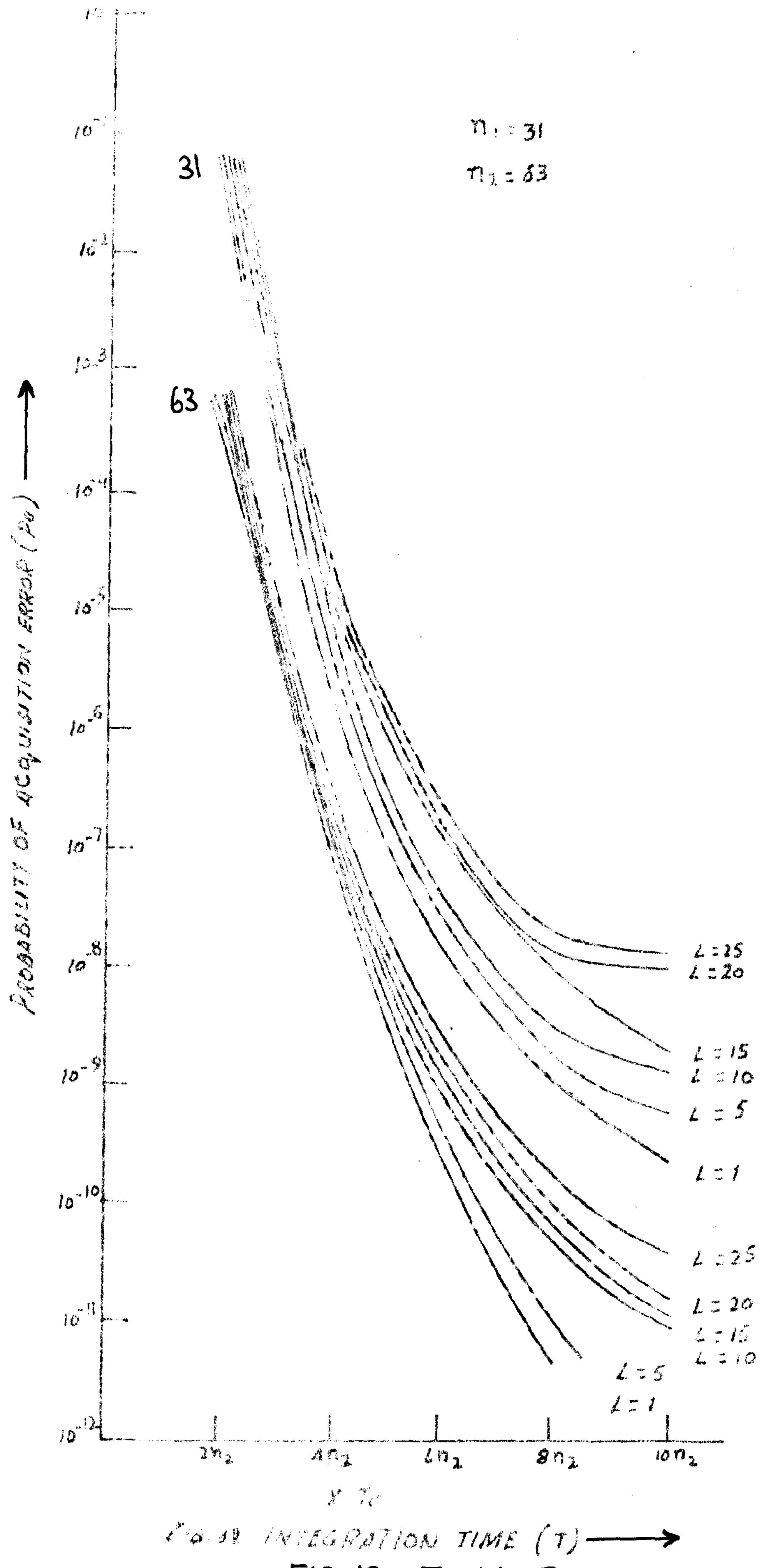

PROBABILITY OF ACQUISITION ERROR (Pe)
INTEGRATION TIME (T)
31
63
n1 = 31
n2 = 63
L = 25
L = 20
L = 15
L = 10
L = 5
L = 1
L = 25
L = 20
L = 15
L = 10
L = 5
L = 1
2n2
4n2
6n2
8n2
10n2

FIG-12. T Vs. Pe

(i) When $L = 1$

n_1	T (sec)	SNR (dB)	P_o
31	0.126×10^{3}	1.648	3.8×10^{-2}
63		2.997	6.1×10^{-4}
31	0.252×10^{-3}	5.49	2.5×10^{-6}
63		5.62	2.2×10^{-7}
31	0.378×10^{-3}	7.41	1.9×10^{-8}
63		7.60	4.4×10^{-10}
31	0.504×10^{-3}	8.44	1.3×10^{-9}
63		8.64	7.5×10^{-12}
31	0.63×10^{-3}	8.98	3.5×10^{-10}
63		9.2	—*

(ii) When $L = 5$

n_1	T (sec)	SNR (dB)	P_e
31	0.126×10^{-3}	1.52	6.0×10^{-2}
63		2.97	7×10^{-4}
31	0.252×10^{-3}	5.37	4.0×10^{-6}
63		5.56	2.6×10^{-7}
31	0.378×10^{-3}	7.19	2.9×10^{-8}
63		7.51	6.0×10^{-10}
31	0.504×10^{-3}	8.19	2.5×10^{-9}
63		8.52	2.0×10^{-11}
31	0.630×10^{-3}	8.63	8.0×10^{-10}
63		9.06	—

[* The values of P_o are not given for high SNR's in Lindsey and Simon's tables.]

(iii) When $L = 10$

n_i	T (sec.)	SNR (dB)	P_e
31	0.126×10^{-3}	1.40	7.0×10^{-2}
63		2.94	6.0×10^{-4}
31	0.252×10^{-3}	5.21	5.0×10^{-6}
63		5.49	3.0×10^{-7}
31	0.378×10^{-3}	6.91	6.0×10^{-8}
63		7.41	8.0×10^{-10}
31	0.504×10^{-3}	7.90	5.0×10^{-9}
63		8.30	7.0×10^{-11}
31	0.630×10^{-3}	8.24	2.5×10^{-9}
63		8.90	1.0×10^{-11}

(iv) When $L = 15$

n_i	T (sec.)	SNR (dB)	P_e
31	0.126×10^{-3}	1.27	9.0×10^{-2}
63		2.90	7.0×10^{-4}
31	0.252×10^{-3}	5.07	8.0×10^{-6}
63		5.43	4.0×10^{-7}
31	0.378×10^{-3}	6.66	1.3×10^{-7}
63		7.31	1.2×10^{-9}
31	0.504×10^{-3}	7.63	1.0×10^{-8}
63		8.16	1.0×10^{-10}
31	0.630×10^{-3}	7.87	3.0×10^{-9}
63		8.67	2.0×10^{-11}

(v) When L = 20

n_1	T (sec.)	SNR (dB)	P_o
31	0.126×10^{-3}	1.14	1.1×10^{-1}
63		2.86	9.0×10^{-4}
31	0.252×10^{-3}	4.93	1.2×10^{-5}
63		5.36	5.0×10^{-7}
31	0.378×10^{-3}	6.42	2.3×10^{-7}
63		7.21	1.5×10^{-9}
31	0.504×10^{-3}	7.37	2.2×10^{-8}
63		8.04	1.3×10^{-10}
31	0.630×10^{-3}	7.53	1.2×10^{-8}
63		8.50	3.0×10^{-11}

(vi) When L = 25

n_1	T (sec.)	SNR (dB)	P_o
31	0.126×10^{-3}	1.02	1.5×10^{-1}
63		2.825	1.1×10^{-3}
31	0.252×10^{-3}	4.79	1.5×10^{-5}
63		5.10	1.0×10^{-6}
31	0.378×10^{-3}	6.19	4.0×10^{-7}
63		7.12	2.2×10^{-9}
31	0.504×10^{-3}	7.13	3.6×10^{-8}
63		7.91	2.2×10^{-10}
31	0.630×10^{-3}	7.22	2.9×10^{-8}
63		8.33	6.5×10^{-10}

From Figs. 11 and 12 it can be observed that the probability of acquisition error decreases as the integration time increases which automatically means an increase in acquisition time. So it is a point of compromisation between probability of acquisition error and the acquisition time - i.e. the smaller the integration time the higher the probability of acquisition error.

It can also be observed that as the length of the component sequences increases, the probability of acquisition error goes down. Further, we can observe that the probability of acquisition error keeps going up for larger number of users.

It can be concluded from the above that, in a multiple-access communication system, where a number of users are operating simultaneously, it is advisable to use longer component ~~f~~sequence~~z~~s to give a better performance (i.e. low P_c) for a fixed integration time.

-00-

Chapter-IV

SUMMARY AND CONCLUSIONS

For any digital sequence to be useful in ranging and spread spectrum applications, it should be quickly acquirable .

To achieve quick acquisition, the first thing considered is the formation of combination sequences by using a number of shorter sequences. The next problem is how to form the combination sequences. Combination sequences have been formed by using various logical functions such as Majority logic, AND and Mod-2 for two component sequences and Majority logic, OR and A1+A2.A3 (where A1, A2, A3 are the components) for three component sequences. And it has been observed that the majority logic combination of sequences gives the best results.

The second thing that has been considered to achieve quick acquisition is the partial correlations which greatly reduces the acquisition time. As the number of bits over which the correlations are taken decreases, the properties of the sequences become worse, and the probability of error larger. So it is a compromise as to over how many bits the partial correlations are to be taken. The next thing that affects the acquisition time is the method of acquisition. Various acquisition methods have been discussed in this regard.

Finally, the acquisition of PN signals in a multiple access communication environment has been considered. It has been shown that as the number of users increases, the interference noise increases proportionately, thereby reducing the SNR available at the receiver. As a result, the probability of acquisition error increases. On the other hand, if we want to acquire the signals with the same probability of error, the integration time of the signals increases correspondingly.

In order to allot an independent sequence to each user, a number of different sequences of same length must be required. Not so many different sequences of shorter lengths can be generated now a days. A means must be provided to generate a large number of different sequences of shorter lengths.

So far, the pseudo noise sequences that have been considered have all been binary or two-level codes, with binary levels +1 and -1 . Since the most applicable of these is the binary maximal length sequence, it is interesting to note that the binary maximal length sequence is but one class of the more general q-level [17] maximum length sequences(where q is a prime number) of period $N=q^n-1$. One may therefore, by using a value of q other than 2 (e.g. q=3 or 5) can generate a PN signal suitable for ranging and spread-spectrum applications.

APPENDIX

COMPUTER PROGRAMMES IN FORTRAN

Programme A-1:

```
C:AUTO CORRELATIONS OF 2 COMPONENT COMB . SEQUENCES RVRAO
DIMENSION A1(40),A2(40),SUM1(40),SUM2(40),RK1(30),RK2(30)
NK=0
READ K1,K2
DO 10 I=1,K1
READ A1(I)
10: CONTINUE
DO 20 J=1,K2
READ A2(J)
20:CONTINUE
K=K1*K2
22:DO 25 J1=1,K1
SUM1(J1)=0
25: CONTINUE
DO 26 J2=1,K2
SUM2(J2)=0
26: CONTINUE
DO 180 I=1,K
N1=I/K1
I1=I-N1*K1
IF (I1) 30,30,40
30:I1=I1+K1
GO TO 50
40:I1=I1
50: N2=I/K2
I2=I-N2*K2
IF (I2) 60,60,70
60: I2=I2+K2
GO TO 80
70:I2=I2
80:P=A1(I1)+A2(I2)
IF (NK-1) 81,82,83
81:IF (P) 90,100,100
82:IF (P) 90,90,100
83: IF (P) 90,100,90
90: C=-1
GO TO 110
100: C=1
110:DO 140 J1=1,K1
I11=I1+J1
IF (I11-K1) 120,120,130
120:I11=I11
GO TO 135
130:I11=I11-K1
135:SUM1(J1)=SUM1(J1)+A1(I11)* C
140: CONTINUE
DO 180 J2=1,K2
I22=I2+J2
IF (I22-K2) 150,150,160
150: I22=I22
GO TO 165
160: I22=I22-K2
165: SUM2(J2)=SUM2(J2)+ A2(I22)* C
180: CONTINUE
AK=K
```

```
DO 200 J1=1,K1
RK1=SUM1(J1)/AK
WRITE 190 ,RK1
190:FORMAT (/,"     ",E)
200: CONTINUE
DO 220 J2=1,K2
RK2=SUM2(J2)/AK
WRITE 210,RK2
210:FORMAT (/,"     ",E)
220: CONTINUE
NK= NK+1
GO TO (22,22,230),NK
230: STOP
END
```

Programme A-2:

```
C:PARTIAL CORRELATIONS OF 2 COMPONENT COMB . SEQUENCES RVRAO
DIMENSION A1(40),A2(40),SUM1(40),SUM2(40),RK1(30),RK2(30)
DO 230 LL=1,4
READ K1,K2
DO 10 I=1,K1
READ A1(I)
10: CONTINUE
DO 20 J=1,K2
READ A2(J)
20: CONTINUE
K11=(K1+3)/2
DO 220 NK=1,K11,2
K=NK*K2
DO 25 J1=1,K1
SUM1(J1)=0
25: CONTINUE
DO 26 J2=1,K2
SUM2(J2)=0
26: CONTINUE
DO 180 I=1,K
N1=I/K1
I1=I-N1*K1
IF (I1) 30,30,40
30:I1=I1+K1
GO TO 50
40:I1=I1
50:N2=I/K2
I2=I-N2*K2
IF (I2) 60,60,70
60:I2=I2+K2
GO TO 80
70:I2=I2
```

```
80:P=A1(I1)+A2(I2)
IF (P) 90,100,100
90:C=-1
GO TO 110
100: C=1
110:DO 140 J1=1,K1
I11=I1+J1
IF (I11-K1) 120,120,130
120:I11=I11
GO TO 135
130:I11=I11-K1
135:SUM1(J1)=SUM1(J1)+A1(I11)*C
140:CONTINUE
DO 180 J2=1,K2
I22=I2+J2
IF (I22-K2) 150,150,160
150:I22=I22
GO TO 165
160:I22=I22-K2
165:SUM2(J2)=SUM2(J2)+A2(I22)* C
180:CONTINUE
AK=K
DO 200 J1=1,K1
RK1=SUM1(J1)/AK
WRITE 190 ,RK1
190:FORMAT (/,"    ",E)
200: CONTINUE
DO 220 J2=1,K2
RK2=SUM2(J2)/AK
WRITE 210,RK2
210:FORMAT (/,"    ",E)
220: CONTINUE
230:CONTINUE
STOP
END
```

Programme A-3:

```
C: AUTO CORRELATIONS OF 3 COMPONENT COMB . SEQUENCES RVRAO
DIMENSION A1(40),A2(40),A 3(40),SUM1(30),SUM2(30),SUM3(30)
DIMENSION RK1(30),RK2(30),RK3(30)
DO 250 LL=1,4
NK=0
READ K1,K2,K3
DO 10 I=1,K1
READ A1(I)
10: CONTINUE
DO 20 J=1,K2
READ A2(J)
20: CONTINUE
DO 21 L=1,K3
READ A3(L)
21: CONTINUE
K=K1*K2*K3
22:DO 25 J1=1,K1
SUM1(J1)=0
25: CONTINUE
DO 26 J2=1,K2
SUM2(J2)=0
26: CONTINUE
DO 27 J3=1,K3
SUM3(J3)=0
27: CONTINUE
DO 185 I=1,K
N1=I/K1
I1=I-N1*K1
IF (I1) 30,30,40
30:I1=I1+K1
40:N2=I/K2
I2=I-N2*K2
IF (I2) 60,60,70
60:I2=I2+K2
70:N3=I/K3
I3=I-N3*K3
IF (I3) 81,81,82
81:I3=I3+K3
82:IF (NK-1) 83,88,89
83:P=A2(I2)+A3(I3)
IF (P) 84,84,85
84:B=-1
GO TO 86
85: B=1
86: Q=A1(I1)+ B
IF (Q) 90,100,100
88:P=A1(I1)+A2(I2)+A3(I3)
PQ=P+2.
IF (PQ) 90,100,100
89:P=A1(I1)+A2(I2)+A3(I3)
```

```
IF (P) 90,90,100
90:C =-1
GO TO 110
100:C=1
110:DO 140 J1=1,K1
I11=I1+J1
IF (I11-K1) 120,120,130
120:I11=I11
GO TO 135
130:I11=I11-K1
135:SUM1(J1)=SUM1(J1)+A1(I11)* C
140:CONTINUE
DO 180 J2=1,K2
I22=I2+J2
IF (I22-K2) 150,150,160
150:I22=I22
GO TO 165
160:I22=I22-K2
165:SUM2(J2)=SUM2(J2)+A2(I22)*C
180: CONTINUE
DO 185 J3=1,K3
I33=I3+J3
IF (I33-K3) 181,181,182
181:I33=I33
GO TO 183
182:I33=I33-K3
183:SUM3(J3)=SUM3(J3)+A3(I33)* C
185:CONTINUE
AK=K
DO 200 J1=1,K1
RK1=SUM1(J1)/AK
WRITE 190,RK1
190:FORMAT (/,"    ",E)
200:CONTINUE
DO 220 J2=1,K2
RK2=SUM2(J2)/AK
WRITE 210,RK2
210:FORMAT (/,"    ",E)
220:CONTINUE
DO 240 J3=1,K3
RK3=SUM3(J3)/AK
WRITE 230,RK3
230:FORMAT (/,"    ",E)
240:CONTINUE
NK= NK+1
GO TO (22,22,250),NK
250:CONTINUE
STOP
END
```

```
I22=I22
GO TO 165
160:I22=I22-K2
165: SUM2(J2)=SUM2(J2)+A2(I22)* C
180: CONTINUE
DO 185 J3=1,K3
I33=I3+J3
IF (I33-K3) 181,181,182
181:I33=I33
GO TO 183
182:I33=I33-K3
183:SUM3(J3)=SUM3(J3)+ A3(I33)* C
185:CONTINUE
AK=K
DO 200 J1=1,K1
RK1=SUM1(J1)/AK
WRITE 190,RK1
190:FORMAT (/,"    ",E)
200:CONTINUE
DO 220 J2=1,K2
RK2=SUM2(J2)/AK
WRITE 210,RK2
210:FORMAT (/,"    ",E)
220:CONTINUE
DO 240 J3=1,K3
RK3=SUM3(J3)/AK
WRITE 230,RK3
230:FORMAT (/,"    ",E)
240:CONTINUE
250:CONTINUE
STOP
END
```

```
IF (P) 90,90,100
90:C =-1
GO TO 110
100:C=1
110:DO 140 J1=1,K1
I11=I1+J1
IF (I11-K1) 120,120,130
120:I11=I11
GO TO 135
130:I11=I11-K1
135:SUM1(J1)=SUM1(J1)+A1(I11)* C
140:CONTINUE
DO 180 J2=1,K2
I22=I2+J2
IF (I22-K2) 150,150,160
150:I22=I22
GO TO 165
160:I22=I22-K2
165:SUM2(J2)=SUM2(J2)+A2(I22)*C
180: CONTINUE
DO 185 J3=1,K3
I33=I3+J3
IF (I33-K3) 181,181,182
181:I33=I33
GO TO 183
182:I33=I33-K3
183:SUM3(J3)=SUM3(J3)+A3(I33)* C
185:CONTINUE
AK=K
DO 200 J1=1,K1
RK1=SUM1(J1)/AK
WRITE 190,RK1
190:FORMAT (/,"    ",E)
200:CONTINUE
DO 220 J2=1,K2
RK2=SUM2(J2)/AK
WRITE 210,RK2
210:FORMAT (/,"    ",E)
220:CONTINUE
DO 240 J3=1,K3
RK3=SUM3(J3)/AK
WRITE 230,RK3
230:FORMAT (/,"    ",E)
240:CONTINUE
NK=NK+1
GO TO (22,22,250),NK
250:CONTINUE
STOP
END
```

Programme A-4:

```
C:PARTIAL CORRELATIONS OF 3 COMPONENT COMB . SEQUENCES RVRAO
DIMENSION A1(40),A2(40),A3(40),SUM1(30),SUM2(30),SUM3(30)
DIMENSION RK1(30),RK2(30),RK3(30)
DO 250 LL=1,4
READ K1,K2,K3
DO 10 I=1,K1
READ A1(I)
10: CONTINUE
DO 20 J=1,K2
READ A2(J)
20:CONTINUE
DO 21 L=1,K3
READ A3(L)
21:CONTINUE
DO 250 NK=1,11,2
K=NK*K3
DO 25 J1=1,K1
SUM1(J1)=0
25:CONTINUE
DO 26 J2=1,K2
SUM2(J2)=0
26:CONTINUE
DO 27 J3=1,K3
SUM3(J3)=0
27: CONTINUE
DO 185 I=1,K
N1=I/K1
I1=I-N1*K1
IF (I1) 30,30,40
30:I1=I1+K1
40: N2=I/K2
I2=I-N2*K2
IF (I2) 60,60,70
60:I2=I2+K2
70: N3=I/K3
I3=I-N3*K3
IF (I3) 81,81,82
81:I3=I3+K3
82:P=A1(I1)+A2(I2)+A3(I3)
IF (P) 90,90,100
90: C=-1
GO TO 110
100: C=1
```

```
110:DO 140 J1=1,K1
I11=I1+J1
IF (I11-K1) 120,120,130
120:I11=I11
GO TO 135
130:I11=I11-K1
135:SUM1(J1)=SUM1(J1)+ A(I11)* C
140: CONTINUE
DO 180 J2=1,K2
I22=I2+J2
IF (I22-K2) 150,150,160
150:I22=I22
GO TO 165
160:I22=I22-K2
165: SUM2(J2)=SUM2(J2)+A2(I22)* C
180: CONTINUE
DO 185 J3=1,K3
I33=I3+J3
IF (I33-K3) 181,181,182
181:I33=I33
GO TO 183
182:I33=I33-K3
183:SUM3(J3)=SUM3(J3)+ A3(I33)* C
185:CONTINUE
AK=K
DO 200 J1=1,K1
RK1=SUM1(J1)/AK
WRITE 190,RK1
190:FORMAT (/,"    ",E)
200:CONTINUE
DO 220 J2=1,K2
RK2=SUM2(J2)/AK
WRITE 210,RK2
210:FORMAT (/,"    ",E)
220:CONTINUE
DO 240 J3=1,K3
RK3=SUM3(J3)/AK
WRITE 230,RK3
230:FORMAT (/,"    ",E)
240:CONTINUE
250:CONTINUE
STOP
END
```

Programme A-5:

```
C:CROSS CORRELATIONS OF COMB. SEQUENCES WITH COMPS. OF
OTHER COMB. SEQUENCES  RVRAO
DIMENSION A(160),B(320),P(6),C(6)
READ N,K1,K2
DO130 M=1,N
DO10 I=1,K1
J1=(M-1)*K1+I
READ A(J1)
10: CONTINUE
DO 20 I=1,K2
J2=(M-1)*K2+I
READ B(J2)
20: CONTINUE
K=K1*K2
SUM1=0
SUM2=0
DO 100 I=1,K
N1=I/K1
I1=I-N1*K1
IF (I1) 30,30,40
30:I1=I1+K1
40: N2=I/K2
I2=I-N2*K2
IF (I2) 50,50,60
50:I2=I2+K2
60:J3=(M-1)*K1+I1
J4=(M-1)*K2+I2
P(M)=A(J3)+B(J4)
IF (P(M)) 70,80,80
70: C(M)=-1
GO TO 90
80:C(M)=1
90:SUM1=SUM1+A(J3)*C(1)
SUM2=SUM2+B(J4)*C(1)
100:CONTINUE
AK=K
RK1=SUM1/AK
RK2=SUM2/AK
WRITE 110,RK1,RK2
110:FORMAT (/,"    ",E,"    ",E)
130:CONTINUE
STOP
END
```

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