NONLINEAR BEHAVIOUR OF BRICK MASONRY SUBJECTED TO IN-PLANE LOADING

A THESIS

submitted in fulfilment of the requirements for the award of the degree of

DOCTOR OF PHILOSOPHY

in

EARTHQUAKE ENGINEERING

by

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OCTOBER, 1995

Gratis

CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled **"Nonlinear Behaviour of Brick Masonry Subjected to In-plane loading"** in fulfilment of the requirement for the award of the Degree of Doctor of Philosophy and submitted in the department of Earthquake Engineering of the University of Roorkee is an authentic record of my own work carried out during a period from October, 1989 to October, 1995 under the supervision of Dr. A.R. Chandrasekaran and Dr. D.K. Paul.

The work presented in this thesis has not been submitted by me for the award of any other degree of this or any other University.

Ajit Lal Guha

This is to certify that the above statement made by the candidate is correct to the best of our Knowledge.

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8 Summary and Conclusions

ABSTRACT

This thesis presents a comprehensive material model which is capable of simulating the in-plane behaviour of masonry from elastic range to final failure for any angle of bed joint orientation. The final element model and computer code are developed for the analysis of masonry infilled frame and shear walls to demonstrate the suitability and accuracy of the material model. The proposed model accounts for most of the observed sources of material non-linearity in biaxial stress state.

The proposed elasto-plastic model includes the generalised anisotropic quadratic failure criterion in three dimensional stress space to take into account the directional strength properties of brick masonry at failure. The magnitude of interaction between stresses like other failure theories is not constant. It is restrained in such a way that the shape of the failure surface is ellipsoidal.

The accuracy of the failure criterion depends upon the interaction strength parameter. To determine this parameter stability condition must be checked. Sensitivity analysis is carried out to select the type of test result to be used for the determination of the interaction strength parameter, because a small inaccuracy in test results may change the value to a large extent such that the failure surface may become hyperboloid instead of ellipsoid. Associated flow rule is used for modelling the elasto-plastic behaviour of brick masonry.

Smeared crack approach is employed to model the tensile behaviour of masonry. Maximum stress criterion is used for initiation and propagation of cracks. Tensile strain softening rule is employed to model the post cracking behaviour of masonry. Closing and reopening of cracks are allowed following the secant path.

The material nonlinearity due to slip and/or cracking in the mortar joint at the interface between the frame and the infill and at the

interface between masonry wall is incorporated in the finite element model.

The results of the finite element analysis of masonry frame and shear walls are compared with the experimental results to validate the accuracy and versatility of the proposed material model. A good agreement between the analytical and experimental results is observed.

The model has also been used to analyse shear walls to study the influence of horizontal to vertical load ratio, effect of vertical stress level and the influence of strength properties of brick masonry on the behaviour of shear walls.

It is revealed from studies that the computational model developed and implemented in the computer code is able to reproduce accurately the non-linear behaviour of masonry structures subjected to in-plane loads.

A micro mechanical brick masonry model is proposed to determine strength and elastic properties of brick masonry in terms of those of its constituents. The formulae have been proposed for computing the stress distribution in the bricks and mortar joints for known stresses in masonry. So the finite element analysis at micro-level is not necessary to find out the stresses in the bricks and mortar joints in the brick masonry.

Micromechanics investigations are carried out to achieve desired strength and stiffness of brick masonry in terms of strength and elastic properties of brick and mortar so that brick masonry can be analysed based on macro level approach. Thus economy in cost and time can be achieved for the analysis and design of masonry structures.

LIST OF FIGURES

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LIST OF NOTATIONS

Main notations used in this thesis are listed below. Sometimes a notation may have alternative meaning. In such case, the context is sufficient to aviod confusion.

a_n	Strength parameter related to stress normal to bed joint
a_p	Strength parameter related to stress parallel to bed joint
a_1, a_2, b_1 and b_2	Strength parameter used in failure criterion of mortar joint
b	Width of brick
$\underline{C}^{me}$	Elastic compliance matrix of mortar
$\underline{C}^{be}$	Elastic compliance matrix of brick
$\underline{C}^{e}$	Elastic compliance matrix for brick masonry
$\underline{D}_{ep}$	Elasto-plastic material property matrix
$\underline{D}$	Elasticity matrix
E^b	Elastic modulus of brick
E^m	Elastic modulus of mortar
E_u^*	Upper bound of elastic modulus
E_L^*	Lower bound of elastic modulus
E_u	Modified upper bound
E_L	Modified lower bound
E	Effective modulus
E_f	Elastic modulus of the frame material
E'	Reduced elastic modulus
E_j	Elastic modulus of mortar at interface between masonry infill and the frame or interface between masonry wall and the supporting base
f_{pc}	Compressive of masonry parallel to bed joint
f_{nc}	Compressive strength normal to bed joint
f_{bc}	Biaxial compressive strength
f^s	Uniaxial compressive strength of steel
G_j	Shear modulus of mortar
H	Hardenig parameter
L_t	Critical length of brick
L	Length of brick

$\underline{T}$ Tranformation Matrix

t^b Thickness of brick

t^m Thickness of mortar joint

V^b Volume fraction of brick

V^m Volume fraction of mortar

V Volume of brick masonry

W Strain energy function

α_l Contact length along beam

α_h Contact length along column

α_m Tension stiffening parameter

$d\lambda$ Fluidity parameter

ε^b_{tm} Maximum tensile strain in brick

ϵ^p Plastic strain in brick masonry

ϵ^{mp} Plastic strain in mortar

J'_2 Second deviatoric strain invariant

ε_u Uniaxial ultimate strain

$\epsilon^{\wedge m}$ Modified elastic strain in mortar

μ Co-efficient of friction

υ^m Poisson's ratio of mortar

υ^b Poisson's ratio of brick

σ^b_t Tensile stress in the brick

τ_o Shear stress without precompression

τ Shear stress

σ^b_{ta} Average tensile stress in brick

σ^b_{tm} Maximum tensile stress in brick

σ^m Maxmum compressive stress in mortar

σ^m_{ca} Average compressive stess in mortar.

σ_y Normal stress in brick masonry in direction

$\psi(V^m)$ Proportionality factor

σ^m_{ph} Normal stress in mortar parallel bed joint

σ^m_{nn} Normal stress in mortar bed joint normal to bed joint

τ^m_{nph} Shear stress in bed joint

σ_p Normal stress in brick masonry parallel to bed joint

σ_n Normal stress in brick masonry normal to bed joint

σ^b_p Normal stress in brick normal to bed joint

τ^b_{np} Shear stress in brick

σ^m_{pv} Normal stress in vertical joint parallel to bed joint

σ^m_{nv} Normal stress in vertical joint normal to bed joint

τ^m_{npv} Shear stress in vertical joint

CHAPTER 1

INTRODUCTION

1.1 Prelude

Brick masonry is the first man made composite material. It has been widely used throughout the world as a traditional building material. Design and analysis of masonry structures have not kept pace with the development of material technology. Until nineteenth century, the "rule of thumb" methods were used for the design of brick masonry. These rules were based on stability criterion and load bearing capacity of the masonry walls. As a result, walls were excessively thick which were causing a wasteage of space and materials. One of the most important examples of this method is the much quoted Monadnock building in Chicago (1891). Its 6-ft(1829mm) thick masonry walls at the base of the building provided the required stability against wind loads, but these walls occupied one fifth of the total area of the building. Its very massiveness prohibited further use of masonry walls in high rise buildings. Around this period, steel or concrete framed multi-storey buildings became the most common and economical form of construction. For the next fifty years masonry was mainly used as an infill material for multi-storey framed structures.The situation has changed in a number of countries after the second world war with the introduction of the structural brick masonry concept, wherein an important consideration in the analysis and design of masonry structures is its ability to with stand in-plane loads i.e., lateral loads induced by earthquake or wind in addition to vertical loads. This requires the consideration of brick masonry as a structural material.

To use brick masonry as a structural material a thorough knowledge of its strength and deformation characteristics are essential. Unfortunately, research into the basic properties of brick masonry has not kept pace with these developments. Most of the research has been directed towards development of the design procedure and construction standards. These have involved the study of ultimate strengths of walls, piers and small specimens in order to correlate strength properties with that of constituent materials. Tests on full size masonry structures were also carried out to develop the design and construction Codes, even though these tests are expensive and difficult to perform. In many countries, present day design codes continue, in many respects, to be based on this empirical procedure.

This empirical procedure provides the basis of design of masonry structures, but it is not altogether satisfactory as it contains inherent inconsistencies such as unrealistically low values of allowable stresses and arbitrarily chosen limits of certain design parameters, resulting poor material utilization. This procedure is also inadequate to determine the ultimate strength. To achieve economy in the brick masonry construction, without compromising on structural capability, the principles of structural engineering are required for the development of rational design procedures. This has prompted the resurgence in brick masonry research.

From the early sixties, comprehensive research work was undertaken in a number of countries. The purpose of some of the investigations was

- o to predict the strength and deformation characteristics of brick masonry in terms of those of its constituent materials and to collate them with experimental results;
- o to determine the effects of creep, shrinkage and workmanship on the strength of masonry and to select the values of allowable stresses;
- o to develop a material model which can trace the deformation characteristics from elastic range to failure load;
- o to develop computational procedures for the analyses of in-plane behaviour of masonry structures.

1.2 Properties of the Brick Masonry and its Constituents

It is observed that compressive strength of the bricks and mortar determined in accordance with the methods recommended by codes of different countries are higher due to the restraining effects of platens of the testing machine. Real strengths were determined either by using a interface friction reducing system [Thomas and O'Leary(1970), Page(1978), McNary and Abrams(1985) among others] or by increasing the aspect ratio of the specimens [Page(1978), Khoo and Hendry(1973), Baba *et al.*(1985) among others]. Elastic properties were also determined from the stress strain relation obtained from compression tests.

Attempts have been made to predict the strength of masonry in terms of that of its constituents by Francis *et al.*(1970), Khoo and Hendry(1973), Atkinson and Noland(1983) with partial success. Sahlin(1971) first attempted to predict the strength and elastic properties of brick masonry prism based on the composite material concept without complete success. Andam and Pande(1986) also attempted to predict both the uniaxial and biaxial strength of reinforced and unreinforced brick masonry walls based on an "equivalent material" approach. The analytical data were not compared with the experimental results. Liang *et al.*(1990) developed an analytical method to predict the elastic properties of brick masonry walls using an'equivalent material' approach. The analytical predictions of strength and elastic properties are not in good agreement with those of the experimental results.

In the present investigation, an attempt has been made to develop an analytical method based on micromechanics for an accurate prediction of strength and elastic properties of brick masonry, in terms of strength and elastic properties of its constituents.

1.3 Computational Model

Masonry structures subjected to multi-axial state of stresses exhibit anisotropic strength and deformation characteristics due to the influence of mortar joints. Thus, a realistic material model is necessary to define the total response of the structural system from isotropic linear elastic to anisotropic nonlinear bahavior. Until the seventies linear elastic analyses, based on a macro level approach, had been carried out to analyze the in-plane behaviour of masonry structures like composite wall beam due to a lack of detailed information on fundamental properties of masonry and a sound computational technique. These include finite difference method [Wood(1952), Rosenhaupt and Sokal(1965)], variational method [Coull(1966)], shear lag method [Yettram and Hirst(1971)]. The limitations of computational techniques have been eliminated after the introduction of finite element method(FEM), though elastic analyses were carried out using the FEM due to the lack of a suitable material model. In the last two decades, a number of material models suitable for FEM analyses based on both macro and micro level approaches have been proposed to simulate the in-plane behaviour of masonry.

In micro level approach, these include the most simplified model, the nonlinear deformation characteristics of joints with limited capacity in shear and tension [Page(1978) among others] and more refined model, allowing for the nonlinear deformations and the failure of the bricks and the mortar joints [Ali and Page(1988)]. As a large number of elements are needed even for a small problem, this approach is not applicable for field problems.

In macro level approach, linear elastic fracture models have been used by many investigators. Ignoring the influence of normal stress parallel to the bed joints, Mohr-Coulomb type failure criterion with material nonlinearity due to cracking has been used for the analysis of shear walls and infill frames. This model has been further, improved by using a biaxial failure criterion, derived from tests on masonry panels [Samarasinghe *et al.*(1982)]. Nonlinear deformation and failure due to biaxial compression state of stresses and failure of mortar joints were not considered in this model. A material model incorporating the nonlinear deformation in compression region and tensile cracking has been proposed by Dhanasekar(1985) and was developed from biaxial tests on masonry panels. In this model, yield stresses in both the direction, normal and parallel to the bed joint were chosen in lieu of employing the yield criterion and nonlinear deformation was assumed in a limited biaxial compression region. Plastic strain components parallel and normal to the bed joint are assumed to be independent and a function of the corresponding stress in that direction. The normality condition has not been checked as the yield surface has not been considered. The failure surface consisting of three truncated elliptical cones derived from the biaxial tests is not comprehensive.

The present investigation is an attempt to remove these deficiencies by developing a realistic computational model using a comprehensive yield(or failure) surface suitable for finite element analysis of in-plane behaviour of brick masonry for the complete loading range.

1.4 Objective and the Scope of the Study

The objectives of the investigation are:

- o to critically review the literature on the in-plane macro and micro behaviour of the brick masonry, its constituent materials, experimental studies, and the analytical models used for it.

- to determine experimentally the strength and elastic properties of commonly used clay bricks, mortar, brick masonry prisms and walls.
- to predict analytically the strength and elastic properties of brick masonry in terms of those of its constituent materials and to collate them with the experimental results.
- to propose a failure surface which can be employed to determine ultimate strength of brick masonry for multi-axial state of stresses and different bed joint angles.
- to develop material model using the proposed failure surface simulating nonlinear phenomena like material nonlinearity due to deformation and cracking, tension stiffening and the possibility of crack opening and closing, suitable for finite element analysis of in-plane behaviour of brick masonry.
- to develop a computer program which embodies the above formulations and is capable of tracing the complete structural behaviour from elastic regime to failure load.
- to study the in-plane behaviour of masonry shear walls and infill frames and to check the accuracy of the material model by comparing the analytical data with the experimental results.

1.5 Layout of the Thesis

The thesis is divided into 8 chapters. Chapter 2 presents the review of literature pertaining to strength and deformation characteristics of brick masonry and its constituents. Further, Codal methods have also been critically reviewed.

Chapter 3 contains detailed information of laboratory investigations for the determination of strength and elastic properties of brick masonry and its constituent materials based on the methods employed to obtain the actual values and the Codal methods.

Chapter 4 presents the prediction of the strength and the elastic properties of the brick masonry based on a micromechanics approach. Thus, the most economical form of brick masonry for a particular application can be designed with the required elastic properties. It also highlights the importance of an accurate evaluation of the strength parameters and the modulus of elasticity of the bricks and the mortar.

In Chapter 5, first the state of the art of the material models is critically reviewed, then, the material model developed for the modelling of nonlinear behaviour of masonry is presented. The material model accounts for most of the observed sources of material nonlinearity under compressive, tensile or a combined state of compressive and tensile stresses.

In Chapter 6, two dimensional finite element model is presented for the analysis of masonry structures subjected to in-plane loading. Isoparametric finite elements suitable for the analysis of masonry structures are described. The nonlinear solution technique employed in the study is presented.

In Chapter 7, the analysis of masonry infilled frame and shear walls are carried out. The analytical data are compared with the experimental results reported in the literature to check the validity and accuracy of the material model presented in Chapter 5. The influence of varying vertical to horizontal load ratio and masonry properties on the behaviour of masonry shear wall are investigated. The state of the art of masonry infilled frame and shear walls is reviewed.

Chapter 8 epitomizes the results of the study. Some important conclusions drawn from the studies carried out together with suggestions for further research in this field are given.

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CHAPTER 2

LITERATURE REVIEW ON THE PROPERTIES OF BRICK MASONRY AND ITS CONSTITUENTS

2.1 Introduction

Brick masonry is a widely used building material possessing excellent properties in terms of its aesthetic appeal, durability and cost in comparison with alternatives. To use brick masonry as a structural material a through knowledge of the mechanical properties of the brick masonry and its constituent materials are essential. The strength and stiffness properties of the brick masonry primarily depend on the strength and deformation characteristics of the brick and the mortar. Other factors such as geometry of the brick, thickness and orientation of the joint, brickwork bonding, workmanship, curing, environment and age also affect the properties of brick masonry. All factors are not of equal significance.

The available literature covering mechanical properties of the brick masonry, its constituent materials and other factors directly affecting the strength and deformation characteristics are critically reviewed so that the problem later investigated can be placed in proper perspective. The accuracy of the numerical modelling for nonlinear analysis of masonry mainly depends on the consideration of failure theory. So the emphasis is placed on the literature pertaining to the behaviour of the brick masonry under biaxial stresses.

This investigation also involves the study of the behaviour of the masonry shear walls and the infilled frames. The literature pertaining to these topic are reviewed in Chapters 6. An extensive review of brick masonry structure subjected to in-plane load has been carried out by Guha et al.(1990).

2.2 Material Behaviour

The behaviour of the brick masonry depends upon the properties of the constituent materials, i.e., the bricks, mortar and the interaction between them. The properties of the brick, mortar and their composite action are reviewed in the following sections.

2.2.1 Brick

Fired clay bricks must be free from deep and extensive cracks, damage to edges, corners, and also from expansive particle of lime. The bricks are normally classified based on the compressive strength laid down in the relevant national standards, for example in India, IS:1077. Physical properties such as effloresence, frost resistance, thermal movement, moisture movement, sulphate attack, fire resistance, permeability, dimensional variations, etc., have a significant influence on the durability and satisfactory performance of brick masonry structures. Initial rate of absorption(I.R.A.) or brick suction significantly affects the bond strength of the brick masonry. So it should also conform to the specification of physical properties as per relevant national Codes. In many countries the bricks are classified according to physical properties. In United Kingdom the bricks are also classified according to their resistance to frost and the maximum soluble salt content. The clay brick typically exhibit elastic brittle behaviour. The bricks are not necessarily isotropic material, hand molded bricks are isotropic while extruded bricks are orthotropic material. Bricks of various strengths are available to suit a wide range of architectural and engineering requirements. For low-rise masonry buildings bricks of 5.2 N/mm^2 is sufficient. For reinforced and pre-stressed brick masonry, it is highly unlikely that the brick strength lower than 20 N/mm^2 will be used.

(a) Compressive Strength: The bricks are generally classified on the basis of minimum average compressive strength evaluated by uniaxial tests on a random sample in their normal orientation in the wall. The strength of the clay bricks depends on the percentage of clay, method of forming and the firing history of the bricks. The clay or concrete bricks are brittle material. Brittle material fails in compression by separation of cross-sections perpendicular to the axis of the specimen. But this type of failure does not occur in compression tests due to complex state of stress which is far from the assumed uniform state of stress. The actual stress distribution is much more complicated, even if the surfaces are in perfect contact and the load is uniformly applied.

Owing to the friction on the surfaces of contact between the specimen and the platens, the lateral expansion is prevented at these surfaces. As a result high stress concentration develops near the edges of the surface. The magnitude of the stress concentration depends on the shape of the edges. The material at the edges of the surfaces under compression is crushed while inner material of the surface remains unaffected. So the compressive strength determined is higher than the real value. The specimen fails by subdividing into plates parallel to one of the lateral sides provided the platen resistance is relieved either by using proper capping material or increasing aspect ratio. The full implications of these with reference to method of testing and specification laid down in various Codes and recommendations of many researchers are discussed in detail in the following sections.

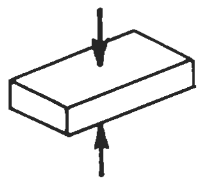
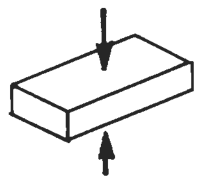
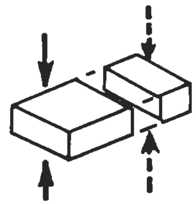
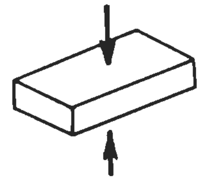
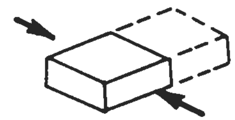
(i) Test Methods and Specifications in Accordance with Codes

Test methods for determining compressive strength in various Codes and specifications are listed in Table 2.1. In the case of clay units, the whole shape is recommended by Indian, British and Australian standards brick Codes. In this test, the bricks are compressed in their normal orientation in the walls between the platens. Plywood capping is used on the top and bottom faces between the platen and the bricks. The brick exhibits shear failure due to lateral strain and the strength estimated is larger than the real value under actual conditions. Despite these deficiencies of this standard test, the nominal compressive strength provides a good form of quality control. On the other hand, the true strength of the bricks is essential for structural design of masonry structures.

(ii) Test Methods and Specifications Suggested by Investigators

The effects of the testing machine platens can be overcome either by the use of larger height to width ratio or by using capping material on the specimen. The stiffness of capping material markedly influence the strength of the specimen. The apparent strength may increase or decrease depending on the relative stiffness of the capping material and the specimen. For example, a highly flexible capping material will tend to expand laterally to a greater extent than the specimen and induce lateral tensile stress may result in premature vertical splitting while a stiff capping material induces lateral compressive stresses which delay the failure. The stiffness of the capping material depends upon its thickness and the deformation properties under normal and shear stresses.

Table 2·1–Test methods for determining the Compressive Strength of Masonry Units

Code	IS:3495 (1976) (Part I) Standard Clay Brick	BS 3921(1974) Clay Bricks and Blocks	ASIM C 67 (1981) Sampling and Testing Clay Masonary Units	IOS/TC/179/SC3/N19A	JIS R 1250 (1981) Standard Clay Bricks
Shape	 Full Size	 Full Size	 Half Size (quarter)	 Full Size	 Half Size
Condition of Specimen	o Wet (24 hrs) o Filling Frog and • All Voids in Bed face stored under Dampjute for 24 hrs & then for 3 days	o Wet (more than 24 hrs) o Saturated under Vacuum o Saturated by Boiling	Absolute Dry (105 C – 115 C and Cool 24+8C) (30 – 70 % RH)	Grounding of loading Surface if Capping not used	—
Capping	o Plywood (3 mm) • Gypsum (3 mm)	o Mortar o Plywood Sheet	• Gypsum o Sulphur o Mortar	• Gypsum o No Capping	o No Capping o Paper o Rubber

To minimize the effect of platen restraint, the unevenness of the loaded surfaces and to maintain the uniform stress distribution, flexible steel brush platen was used by Page(1978). The compressive strength with brush platen was found to be almost half of that with solid platens.

For compression tests of bricks, Bhandari(1982) suggested small size specimens by using brick core of 44.5 mm diameter and height as can be extracted from a full brick thickness. Tests were performed after grinding the loaded surfaces. The test results were corrected for the desired height to diameter ratio effect, adopting the recommendation for correction for height to diameter ratio of concrete cylinders in accordance with I.S:516(1959). The observed compressive strength was almost half of that of the whole shape tests as per I.S:3495(partI) although there is large variation in test results.

Baba *et al.*(1985) critically examined the test methods for determination of compressive strength for various types of concrete bricks according to Codes of different countries. They recommended small specimens cut from concrete masonry units such as coupon shape(3x3x6 cm) with gypsum capping at the two faces under compression.

To use the small capacity compressive testing machine, many researchers prefer small size specimens. For high strength bricks, such as concrete and extruded clay bricks, small size specimens are suitable. But high capacity compressive testing machine is not required for hand moulded clay bricks because the compressive strength is very low as compared to that of concrete bricks. Hand molded bricks are also not uniformly pressed and flaws of varying sizes and their random distributions are observed due to the manufacturing process and can not be avoided. As a result large variations in strength is observed if small specimen size is used. The results of compressive tests of bricks carried out by Page (1978) also support the above observation. So standard size clay bricks should be used as test specimens with proper capping materials. The mechanical properties of four capping materials used in compression testing: ply-wood, hard board(masonite), fiber board(caneite), and particle board were tested by Kleeman and Page (1990). They recommended hard board as a capping material for compression testing.

The brick compressive strength has been correlated with brick masonry strength for different types of mortar in various codes for designing the masonry structures. Although this correlationship is a very useful guide for designers, its limitations should be realized. It makes no allowance for variations in thickness of joints, workmanship and does not really represent the behaviour of brick itself in the wall. In the standard compression test, the brick exhibits a shear failure due to restraining effects of the platens. In real situation in a wall, the brick usually fails in a vertical tension split.

(b) Tensile Strength: The tensile strength of masonry unit is also an important characteristics. The masonry wall under compression load fails due to the formation and propagation of vertical cracks by tensile splitting of bricks. In shear or diagonal tensile mode of failure of masonry shear wall, tensile splitting of bricks may also occur. If the bond strength of the mortar is sufficiently high, diagonal cracks indiscriminately cross bricks and mortar joints. So it is necessary to predict the tensile strength with reasonable accuracy.

There are experimental difficulties in evaluating the actual tensile strength of bricks. Direct tension test was carried out by Thomas and O'Leary(1970). The results were extremely variable and no definite trends were noted. Because of the difficulties of holding the specimens and stress concentration induced by the gripping devices, direct tensile test method is rarely used in determining the tensile strength of bricks.

The tensile strength may be measured by transverse strength tests. The strength obtained by this method is more than the actual strength because the bending tensile stress is not uniform through out the section.

Indirect tensile splitting tests have been widely used to determine the tensile strength of brittle material, concrete in particular. In this test, cylindrical specimens are placed horizontally with packing strip on top and bottom through which compression is applied uniformly along two opposite generators. The tensile strength, f_t^b is given by

$$f_t^b = \frac{2P}{\pi DL} \qquad (2.1)$$

where P is the applied compressive load at failure, D and L are the diameter and length of the cylinder respectively.

Many investigators have suggested the use of cubes and similar prisms as a more practical alternative to cylinders for measuring indirect tensile strength. Francis *et al.*(1970) carried out the indirect tensile test using full size bricks instead of cylinders made from brick and evaluated the tensile strength by aforementioned expression while Thomas and O'Leary(1970) estimated the tensile strength from the relationship derived by Rosenhaupt *et al*(1957) using photo elastic method given by

$$f_t^b = \frac{0.648P}{DL} \tag{2.2}$$

where P is the applied compressive load at failure, D and L are the equivalent diameter of the cross section and length of bricks respectively. Split tests on brick cores, 1.7"(43.18 mm) dia., cut from the bricks were also carried out to compare the tensile strength with that obtained on whole bricks.

Attempts have been made to derive a suitable relationship between the brick tensile strength and masonry wall strength. But a reliable relation could not be developed due to the wide scatter in results of clay bricks.

(c) Strength under Combined Stresses: Bricks in masonry assemblages under in plane loads are normally subjected to triaxial stresses, vertical compression and biaxial tension. Most of the experimental programmes were limited to biaxial compression-tension tests. The strength of brick determined by tests for biaxial stress state [Khoo and Hendry(1973), McNary and Abrams(1985)] was given by

$$\left[\frac{\sigma_c^b}{f_c^b}\right]^n + \left[\frac{\sigma_t^b}{f_t^b}\right] = 1 \tag{2.3}$$

where f_c^b and f_t^b are the uniaxial compressive and tensile strength. σ_c^b and σ_t^b are the compressive and tensile stresses in the brick and n is a constant. The value of n determined by Khoo and Hendry(1973) was 0.5456, while that determined by McNary and Abrams(1985) is 0.58.

(d) Elastic Properties: Early research was devoted to estimate average properties of brick masonry based on experimental results of complete structural elements to develop the design principles. Elastic properties have rarely been reported. Hilsdorf(1965) observed linear stress-strain relationship up to the failure of the bricks. Poisson's ratio was found to increase from 0.2 to 0.35 with the increase of load. Sahlin(1971) suggested a relationship between compressive strength and modulus of elasticity as shown below:

$$E^b = 300 f_c^b \tag{2.4}$$

where E^b and f_c^b are the elastic modulus and the crushing strength of bricks in kg/cm^2.

Page(1978) also observed linear stress-strain relation for clay bricks. Bricks were marginally stiffer in normal direction to the bed joint which is the direction of pressing during manufacture. A constant value of Poisson's ratio, 0.20 was obtained for brick. Large variation of elastic properties were noticed even when taken from the same batch. (Coefficient of variation more than 20%).

Baba *et al.*(1985) evaluated elastic properties of different types of bricks as a part of material research programme, US-Japan earthquake research on masonry structures. Young's modulus of the one third secant was evaluated as a function of ultimate compressive strength, f_c^b as follows:

For concrete units,

$$E^b = 13100\sqrt{f_c^b} \tag{2.5}$$

For clay units,

$$E^b = 340\, f_c^b \tag{2.6}$$

where E^b and f_c^b are the Young's modulus and the crushing strength of bricks in kg/cm^2. Young's modulus and Poisson's ratio were also expressed in terms of uniaxial strain as follows:

$$E^b = E_o^b\,(1 - a\,\varepsilon_1) \tag{2.7}$$

$$\mu^b = \mu_o^b \frac{b}{\varepsilon_1 - \varepsilon_\alpha} \tag{2.8}$$

where

E^b = Young's modulus

E_o^b = Young's modulus, at initial stress

ε_1 = Uniaxial strain

μ^b = Poisson's ratio

a, b, E_o^b, μ_o^b, ε_α - constants determined experimentally

Brick properties varies from region to region depending largely on the quality of local clay. Sufficient data in India are not available for elasticity and Poissons' ratio. Ghosh *et al.*(1969) reported the modulus of elasticity ranging from 400 to 900 MPa for clay bricks available in different parts of India. No attempt has been made by them to determine the Poisson's ratio.

(e) Initial Rate of Absorption (IRA) or Water Absorption

Initial rate of absorption of water of the bricks significantly influences the brick masonry strength. It plays an important role in the achievement of bond between the brick and the mortar. Both very dry and fully saturated bricks lead to low bond strength (Fig. 2.1). Hence bricks should have an optimum value of IRA of about three quarter of full saturation to maximize the bond strength of masonry.

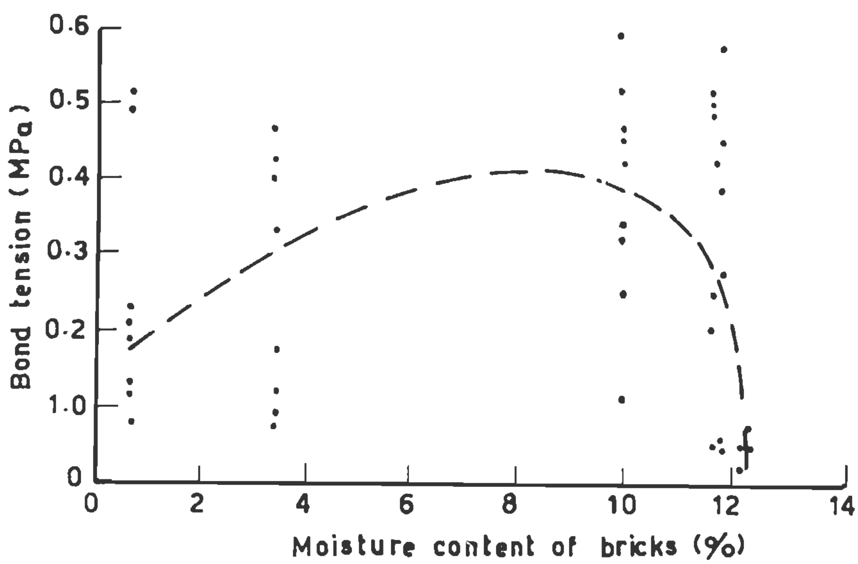


Fig.2.1 Relationship Between Moisture Content of Bricks and Tensile Bond Strength of Brick Masonry Couplets [Sinha(1967)]

Measurement of IRA or Water Absorption: Methods of determination of IRA has been specified by relevant Code of the country. In India, water absorption may be determined either by immersing a sample for 24 hours and determining the amount of water absorbed or by immersing a sample for 5 hours in boiling water and determining the amount of water (IS:3495). In case of ASTM(1967, part 12), the IRA is measured as the amount of water absorbed by immersing a dry unit for one minute to a depth of 3mm (1/8 inch).

Water absorption of the bricks in different region in India was investigated by Ghosh *et al.*(1969). They observed the absorption, for 24 hours immersion in water, varies from 5.5 to 22.3 percent of dry weight. It is also observed that water absorption after one hour immersion is very close to that after 24 hours immersion due to the high porosity of bricks.

(f) Other Properties: There are other properties such as frost resistance, thermal movement, moisture movement, sulphate attack etc. which have a significant influence on the durability and satisfactory performance of brick masonry structures. Depending on the use, bricks are classified in United Kingdom(BS:3921). Hendry *et al.*(1981) summarised all these properties.

2.2.2. Mortar: The effects of different joint materials on the strength of the brick masonry have been studied by many investigators [Monk(1967), Morsy(1968), Astbury and West(1969), Francis *et al.*(1970)]. These have been reviewed in detail by Hendry(1981). It is observed that there is an eight fold increase in the prism compressive strength with the substitution of steel for rubber in the bed joints. In the case of rubber, the brick fails in tension as a result of tensile stress induced by the deformation of the rubber. On the other hand, steel in the bed joints induce biaxial compressive stress in bricks as a result bricks fail by crushing. Mortar has properties intermediate between these extremes, causing splitting rather than crushing failure of bricks. Low strength mortar is normally used in brick masonry construction so that in failure of masonry subjected to lateral load, diagonal cracks pass through only mortar joints. Thus the complete collapse of masonry assemblage can be avoided at the same time it is comparatively easy to repair. In fact, the use of high strength mortars with low strength bricks results in negligible increase in strength of masonry. The choice of different types of mortar depends on the strength required for different types of masonry structures. When the strength of masonry is more or less immaterial, a weak mortar like lime-sand mortar may be chosen. In load bearing walls, where the direct compressive stresses are usually high, cement sand mortar may be chosen. In shear walls, where shear stresses are high, cement lime sand mortar with bricks of low suction may be useful. The choice of mortar also depends upon various situation such as climatic exposure, frost and chemical attack, e.g., soluble sulphate, movement due to settlement, temperature and moisture changes, moisture content of the soil for foundation work, etc., from the point of durability. Hendry *et al.*(1981) recommended the use of different types of mortar for various situation.

(a) Physical Properties of Mortar: Mortar also plays an important role to ensure good quality of masonry for satisfactory structural performance. In order to ensure good quality of brick masonry, the properties of mortar to be considered are as follows:

- o the ability to spread easily for laying of bricks and sealing the joints,
- o development of reasonable strength in order to prevent excessive racking movements of newly laid bricks,
- o proper development of bond with the bricks,
- o resistance to cracking and rain penetration.

To perform these functions, mortar should possess various properties in green and hardened state. In the green state, the properties required are good workability, water retentivity, consistency, and early stiffening.

The workability of the freshly mixed mortar depends on the mortar and brick properties, in particular the water retentivity of mortar and the initial rate of water absorption of bricks. It is measured by flow test. A flow of 110 to 120 percent is recommended by IS:2250(1981) for good workability.

The good water retaintivity is needed to resist the brick suction, and to prevent bleeding of water from mortar, so that sufficient water is retained in the mortar joint for hydration of cement, resulting in proper development of bond between brick and mortar. Water retentivity is indirectly a measure of the workability of mortars. It is measured by the flow of mortar when tested on a standard flow table before and after application of partial vacuum equivalent to 50mm of mercury to the mortar sample. The detail procedure of measurement of water retentivity is given in the code IS:2250(1981). The minimum value allowed by the IS:2250(1981) for water retention is 70%.

Mortar should be well consistent to prevent the segregation of aggregates from the cementitious material. The consistency practically judged by the masons during application. The consistent mortar should have good workability and water retentivity. The quantity of water to maintain required consistency also depends upon the joint thickness, for example thinner joints will require greater fluidity. The consistency is measured by penetration test, given in IS:2250(1981). The depth of penetration for laying walls with solid bricks is 90 to 130 mm as specified in IS:2250(1981).

The rate of hardening of mortar should be such that it causes no delay in the progress of the work maintaining plastic properties for sufficient interval of time till the initial shrinkage of the mortar is complete. This will minimize the possibility of formation of cracks. Extra time shall be allowed when the atmospheric temperature during construction is very low. The rate of hardening of lime mortars is slower than that of cement mortars but is satisfactory for most of the building works. The rate of hardening of lime mortar can be increased by using cement or pozzolana without appreciably reducing workability.

(b) Mortar Constituents: Mortar contains some or all of the constituents discussed below.

Aggregate: The aggregates used for the mortar are sand, broken bricks(surki), cinder, etc. Aggregates must conform to specification laid down in the relevant national codes, for example in India, IS:2116(1980) for sand, IS:3182(1975) for surki, and IS:2686(1977) for cinder. The size and shape of the particles in the aggregates have a direct influence on the workability, shrinkage and ultimate strength. Workability and strength will increase with larger particles. As a general rule the maximum particle size should not be more than $\frac{1}{3}$ or $\frac{1}{2}$ of thickness of the joint. Most of the national codes have a fairly broad specification for aggregate grading with the provision of minimum particle size. The limits of the grading should be chosen to produce a dense mass of the aggregate, requiring a minimum amount of cementitious material for a given strength. At the same time, creep and shrinkage are minimised, and the mortar is economical in use. Natural sand with small adjustment are normally used in mortar. All the national codes have specified the limits of the finer particles. The finer particles in natural sand will usually be clay, which may increase the workability to some extent. On the other hand it will reduce the strength and increase shrinkage and setting times.

Cementitious Material: There are many cementitious materials used for making mortars. The selection of the cementitious material depends upon the structural performance and durability of masonry. Various types of Portland cement are used in masonry mortar. Sulphate-resistant cement should be used in situations where brickwork is expected to remain wet for prolonged periods or where it is susceptible to sulphate attack.

Hydraulic lime is also used as a cementitious material. Lime surki, lime sand and lime cinder mortar are useful for non-engineered building, compound wall, etc. Lime mortar and weak cement mortar have the ability to accommodate movement due to settlement, temperature and moisture changes. Non-hydraulic or semi- hydraulic lime is added to cement mortar to improve the workability, water retention. The water retentivity property of lime is particularly important in situation where high water absorption bricks or dry brick are used. Addition of lime increases bond strength but reduces compressive strength.

Masonry cement which consists of approximately 75% ordinary Portland cement, an inert mineral filler, and an air-entraining agent, is also recommended for brickwork where the strength of the brickwork is not so important, like non-engineered building partition wall, filler wall in R.C. framed structures, because mortar made from masonry cement will have lower strength compared to a Portland cement mortar of similar mix. It is made to give a mortar that combines the desirable properties of lime mortar and cement mortar.

Water: Potable water is generally used for making masonry mortar. The limits of quantities of deleterious materials is also specified by IS:456(1978). The quantity of water to be added to the mortar shall be such that the required consistency is obtained.

Plasticiser and additives: To reduce the cement content and to improve the workability, plasticiser, which entrains air, may be used. At the same time plasticised mortar should not be used with highly absorptive bricks as it reduces the bond strength due to poor water retention properties. Excessive use of plasticiser will have a detrimental effect on strength, so manufacturer's instructions and Codes should be strictly followed.

Accelerating additives such as calcium chloride have sometimes been used in mortar. It is normally not allowed for masonry, since it attracts water and cause a dampness in a wall. There is a risk of steel corrosion and staining in reinforced masonry. Retarding agents are sometimes used to delay the setting time of the mortar in situations where the temperature is very high.

Pulverized brick bats known as pozzolana are abundant and cheap. Hamid, et al.(1986) observed that the addition of pozzolana conforming ASTM:C593(1969) to lime sand mortar would give good workability, water retentivity, and reasonable strength.

(c) Mechanical Properties Of Mortar: When the mortar is in the hardened state, the effects of mortar properties are of paramount importance. The properties of mortar depend on various factors, mainly proportion of ingredients, grading of fine aggregates, water cement ratio, lime to cement ratio, additives, etc. As already discussed, the accurate determination of strength and deformation properties are necessary for the analysis of the brick masonry and for optimum use of material capabilities to meet a particular structural requirement. The available literature covering the strength and other properties are discussed below.

Uni-axial Compressive Strength: The compressive strength of mortar cubes is used as a means of classifying mortars. The size of the mortar cube is specified by the relevant national code, for example in India, 5cm mortar cube is specified by IS:2250(1981). It should be noted that compression tests on mortar cubes overestimate the true compressive strength due to platen restraint. It therefore, serves a useful purpose in quality control. Water/cement ratio also affects the compressive strength of the mortar(Fig.2.2). In practice, the water/cement ratio for a given mix will be determined by workability. Hendry, et al.(1981) suggested that the structural engineer should specify the water/cement ratio to achieve the optimum workability for mortar to be used for structural brickwork. The practice of adding water to partly set mortar to restore workability should be prevented.

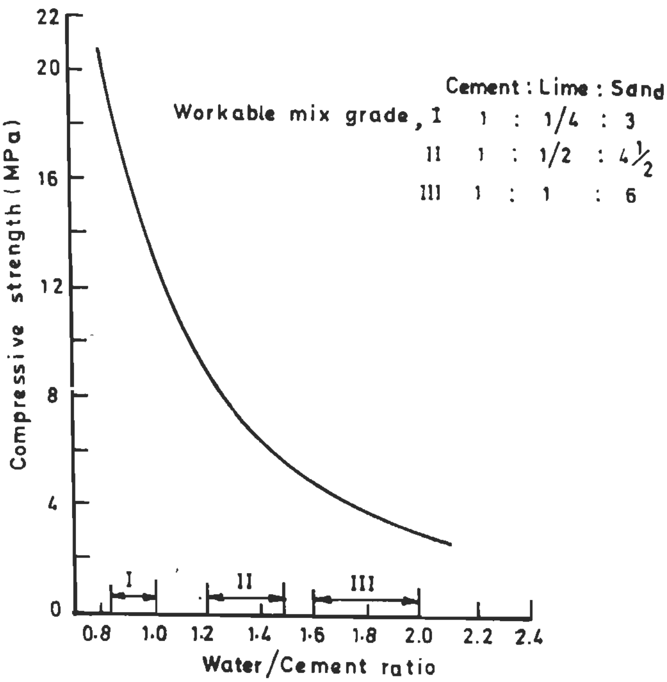


Fig.2.2 Effect of Water/Cement Ratio on Compressive Strength of Mortar [Sinha, (1967)]

Hilsdorf (1965) performed compressive tests on cubes of mortar of different kinds. Hoath *et al.*(1970) used mortar cube strength to compare the effects of mortar strength on brick masonry strength. The highest strength reported is about 40 times the lowest strength. Grimm (1975) expressed the compressive strength of mortar as a function of shape, curing, age, air content and initial flow rate of mortar. Page (1978) also used 7cm mortar cube strength, in accordance with Australian brickwork code, AS:1640(1974), for quality control. To minimize the effects of the platen restraint, Page(1978) carried out compression tests on mortar cylinders of 10cm diameter and 20cm long.

In early research, the attempt has been made to correlate the strength of brick and mortar with that of brickwork without considering the contribution of other factors, in particular adhesion and the effect of air entraining agentt. But the experimental results of Hoath *et al.*(1970) shows that although lime cement mortar may not have greatest strength, but the ratio of brickwork strength to mortar strength is always higher for cement lime mortar than for pure cement mortar. This is due to the difference in bond strength between lime and non-lime mortars. It therefore, shows that the compressive strength of mortar is not a reliable means for predicting the strength of brickwork. Sahlin(1971) also suggested that the quality of mortar should not be judged only by its compressive strength due to the variation of other properties with the ratio of lime to cement. It may also be mentioned that the strength of masonry assemblages in shear and flexutre depends on the bond strength of mortar.

Triaxial Compression Strength: Mortar in brick masonry assemblages, is able to withstand higher compressive stress due to biaxial or triaxial compressive stress induced in them. To reproduce the deformation characteristics of mortar in brick masonry triaxial compression tests should be carried out.

Khoo and Hendry (1973) carried out triaxial compression tests for type M(1:1/4:3) and type N(1:1:6) mortars on cylinders of $1\frac{1}{2}''$ (38.1 mm) diameter and 4"(101.6 mm) long. In triaxial compression tests, higher ultimate strength and increased ultimate strain were observed due to confining pressure on mortar specimen. The principal stress relationship may be defined by the expression:

$$\frac{\sigma_1^m}{f_c^m} = 1 + 2.911 \left(\frac{\sigma_3^m}{f_c^m} \right)^{0.805} \qquad (2.9)$$

where f_c^m is the uniaxial compressive strength and σ_1^m and σ_3^m are the major and minor principal stresses.

McNary and Abrams(1985) carried out triaxial compressive tests using four types of ASTM designated mortar subjected to different lateral confining stresses. The result of the triaxial tests shows that the behaviour of mortar is dependent on the confining pressure and type of mortar. The axial stress and axial strain at failure increased with the increase of confining pressure. The ultimate lateral strain normally decreased with the increased confining pressure. Both the weak and strong mortars exhibited a brittle type behaviour under low confining pressures. At high confining pressures, type O(1:2:9) mortar exhibited substantial ductility and failed at nearly three times the ultimate strain of type M(1:1/4:3) mortar. The results of tests of M and O types are shown in Fig.2.3.

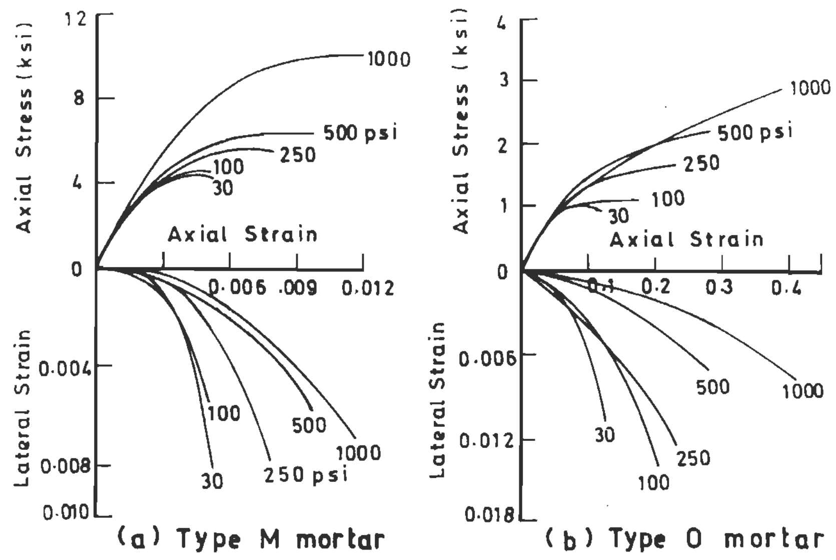


Fig.2.3 Stress-Strain Relationships of Mortar for Different Confining Stresses [McNary and Abrams (1985)]

Elastic Properties: Elastic properties of mortar have rarely been reported as few attempts have been made to correlate properties of mortars with that of brick masonry. Hilsdorf(1965) investigated the stress-strain relation for different types of mortars as shown in Fig.2.4. The wide variation in deformation under loading is striking. The highest modulus of elasticity, tangent modulus at initial stress, reported is about 200 times the lowest.

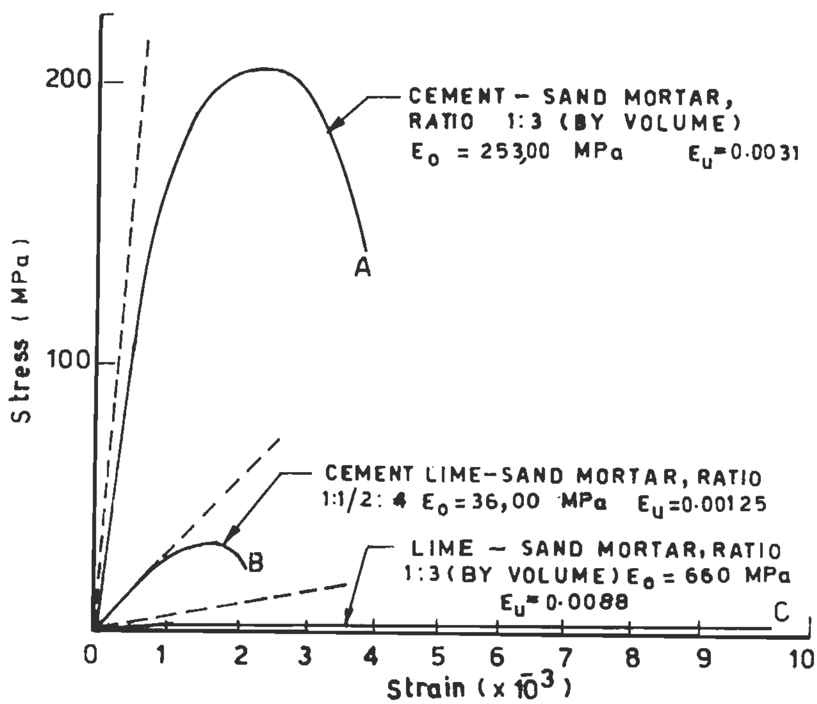


Fig.2.4 Stress-Strain Diagrams of Mortars of Different Kinds [Hilsdorf (1965)].

Page(1978) determined stress-strain relation of mortar loaded in uniaxial compression. Tests were carried out on mortar cylinders of 100mm diameter and 200mm length. Nonlinear stress-strain relation was observed. But Page(1978) modelled brick masonry using stress-strain relation of mortar obtained after deducting stress of brick masonry from that of bricks.

From the triaxial tests results [Khoo and Hendry (1973), McNary and Abrams(1985)], it was observed that Young's modulus and Poisson's ratio at low axial stress level are not significantly influenced by the confining stresses but at higher axial stress level they are markedly influenced by lateral confining stresses.

2.2.3 Brick Masonry

To determine properties of brick masonry as a composite material and to correlate its strength properties with that of its constituents, early investigations have been carried out on walls, piers and small specimens. Thus the analysis and design of brick masonry has been based on empirical rules. Present day design Codes continue, in many respects, to be based on empirical approaches. They frequently contain inherent inconsistency and often do not reflect the clear understanding of the composite behaviour of brick masonry. In recent years attempts have been made to develop failure theories for brick masonry in direct compression and under biaxial stress states. In this section basic strength properties of brick masonry subjected to uniaxial and biaxial state of stresses are reviewed.

(a) Compressive Strength: Until nineteenth century, "rule of thumb" methods were used for the design and construction of brick masonry structures. These rules were based on stability requirement and load bearing capacity of the structures. From late nineteenth century, investigations were carried out to study the ultimate strengths of walls and piers, to develop suitable procedure for the design of masonry structures based on strengths determined from simple uniaxial tests. These formulae are mainly in terms of the primary variables, brick and mortar strength. The well known formula derived by Thomas(1953) is shown in Fig.2.5. A few investigators also included height and density of bricks. Literature pertaining to these studies were critically reviewed by many investigators [Monk(1967), Sahlin(1971), Hendry(1981)].

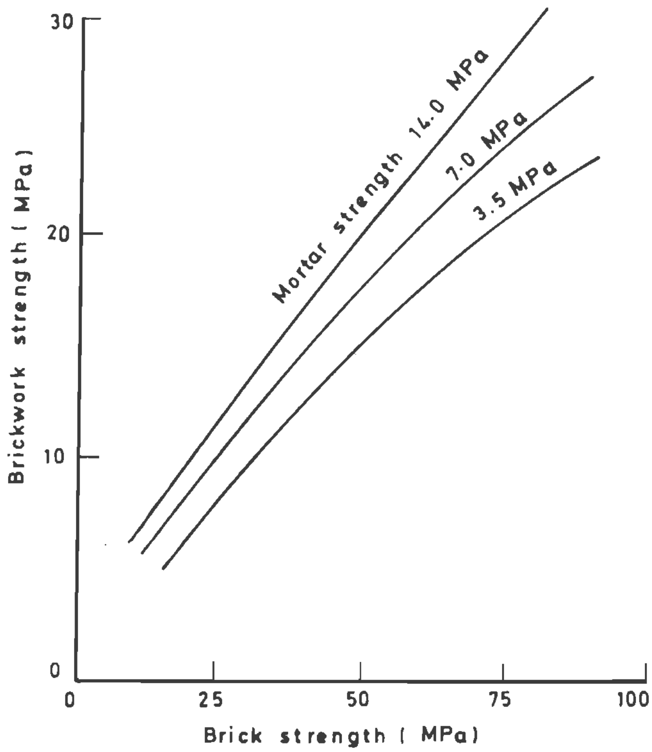


Fig.2.5 Crushing Strength of Brick Masonry Walls and Piers [Thomas (1953)]

From early fifties fresh research was undertaken to collate the analytical data with the experimental results. Investigations have also been performed to examine the variables affecting brick masonry strength. The implications of this research with reference to the methods of determination of brick masonry strength laid down in various Codes are discussed in detail.

(i) Methods of Determination of Compressive Strength in Accordance with Code: The compressive strength of brick masonry can be determined either from the empirical relationship among brick strength, mortar type and brick masonry strength or from tests on brick masonry specimens. These methods are discussed in detail.

Empirical Relationship among Brick Strength, Mortar Type and Masonry Strength: Most of the Codes including Indian Code have suggested a relationship between the brick strength and brick masonry strength for different types of mortar. These relationships have been derived from extensive laboratory tests. For comparison, variation of masonry strength against brick strength for 1:1/4:3 (cement:lime:sand) mortar has been graphically shown in Fig.2.6. These graphs are similar to those proposed by Thomas (1953). These empirical relations do not reflect the effects of variations in aspect ratio (height:length) and thickness of mortar joints. These relationships should also be interpreted carefully because no allowance has been made for variations in workmanship.

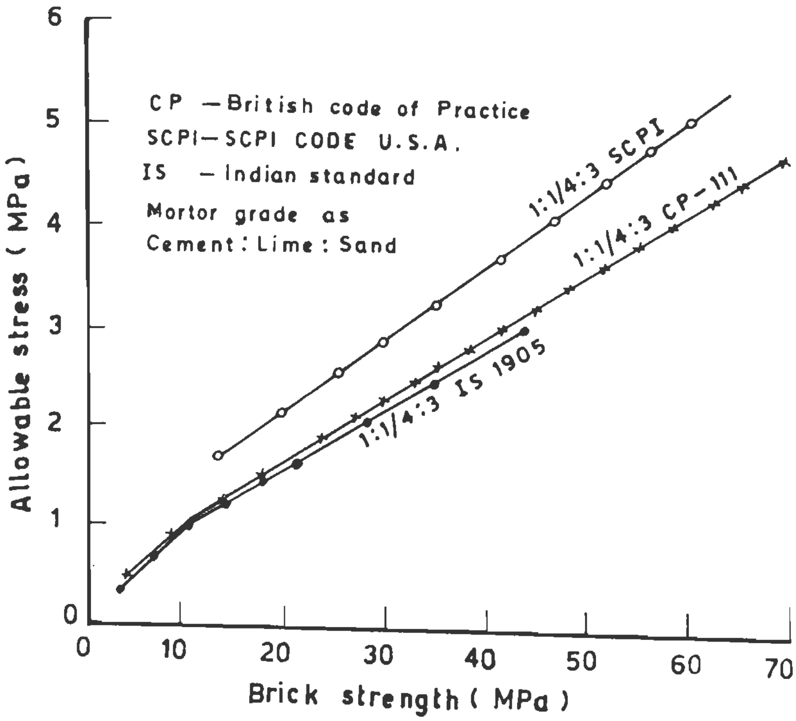


Fig.2.6 Allowable Stresses in Brick Masonry for Different Types of Brick Strengths According to the Code of Practice of Different Countries.

Experimental Method: The compressive strength may be determined experimentally from tests on brick masonry specimens. Many countries including Australia recommend masonry prism tests. Finally wall strength is predicted once allowance is made due to the difference between prism strength and wall strength. As for example, in the Australian brickwork Code, AS:1640-1974 the ultimate compressive strength of the wall, f_c is estimated from masonry prism tests using the following relation:

$$f_c = 0.75(f_c^p - 0.36R) \qquad (2.10)$$

where f_c^p is the average prism compressive strength in MPa and R is the range of prism compressive strength in MPa.

Indian Code recommends that the test specimen should be of the same type as the wall actually used in the structure. But prism test is more reliable, cost effective and easier to perform than the test of an actual wall specimen and should be preferred.

(ii) Factors Affecting the Compressive Strength: The effect of parameters such as strength of the bricks and the mortars, geometry of the bricks, deformation characteristics of the brick and the mortar, joint thickness, rate of absorption of the bricks, water retentivity of the mortar, brickwork bonding, wall thickness, slenderness ratio, workmanship, etc., have been investigated by many researchers and are reviewed by many investigators [Sahlin(1971), Page(1978), Hendry(1981)]. Some of these parameters such as strength and geometry of bricks, are determined by the manufacturing process, while others such as mortar properties, joint thickness, etc., can be controlled during construction of the brick masonry. Not all parameters are of equal significance. Only some of the more relevant aspects of the research will be cited here.

Strength of the Bricks and the Mortar: The primary variables affecting the brick masonry strength are the strength of the brick and the mortar. The weaker mortar is normally used in the brick masonry. Thus, the mortar is restrained from expanding due to the bond and the friction at the mortar-brick interface, and as a result, a triaxial compressive stress develops in the mortar, while a biaxial lateral tensile stress and vertical compressive stress in the brick. The lateral tension produced will eventually cause failure in the bricks (Fig.2.7). Most of the uniaxial strength theories developed till date is based on weaker mortar. The strength theories of the brick masonry in terms of strength of the bricks and the mortar will be reviewed in the next section.

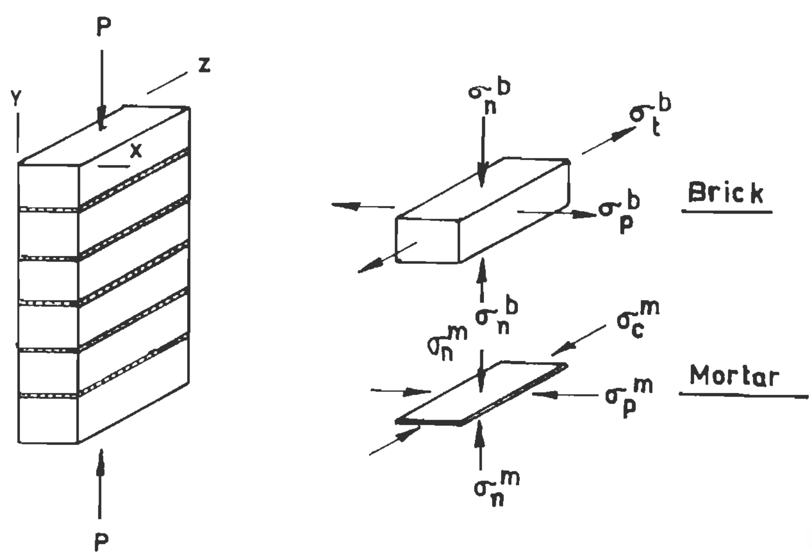


Fig.2.7 Stresses in the Brick and the Mortar due to Compressive Loading

Joint Thickness: In order to study the effect of joint thickness on masonry strength, most of the investigators [Monk(1967), Francis *et al.*(1970), Khoo and Hendry(1973)] determined strength of masonry prism with various combination of brick and mortar strengths for different thicknesses (Fig.2.8). Houston and Grimm(1972) carried out tests on the same material but with different brick height to joint thickness ratios.

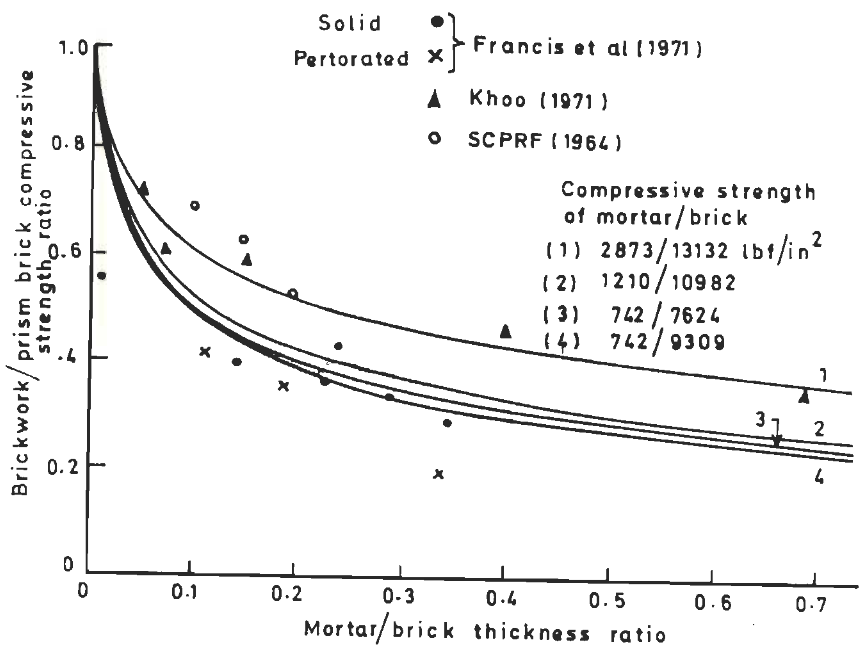


Fig.2.8 Effect of Mortar Joint/Brick Thickness Ratio on the Brick Masonry Strength.

Monk(1967) suggested a linear reduction in compressive strength with an increase in joint thickness. Sahlin (1971) suggested that the brick masonry strength decreases by about 15% for every $\frac{1}{8}''$ (3.175 mm) increase in bed joint thickness and vice versa, the normal value being at a normal joint thickness of $\frac{3}{8}''$ (9.53mm). Hendry (1981) has observed from the results of different investigators that the excessively thick bed joints, say 16-19 mm, may be expected to reduce the strength of the brick masonry by about 30%, as compared to normal 10mm thick joint.

Workmanship: In common with most engineering materials, the strength of the brickwork is affected by site workmanship. Brick masonry being labour intensive, is very sensitive to workmanship factors. Unfortunately, brickwork is commonly done under very little engineering inspection. Consequently, it is frequently regarded with some suspicion as a structural material and carries very large safety factors in design Codes without taking into account the effects of different factors involved during masonry construction. It is therefore, needed to assess the effects of workmanship factors. The most important defects in workmanship are as follows:

- Variation in proportioning and water cement ratio,
- Incorrect adjustment of absorption rate of bricks,
- Variation in joint thickness,
- Imperfect laying and filling of mortar joints,
- Deviation from verticality and alignment,
- Improper curing of brick masonry and protection of fresh masonry from weathering effects.

All these factors are reviewed in detail by Hendry(1981). The effect of all the factors are investigated separately and in combination in many countries [McDowell *et al.*(1966), James(1973) in Australia, Gross et al., (1969) in USA]. The combined effect of outside curing, deep bed furrowing, unfilled perpends, 16mm bed joints and 12mm bow was to reduce the wall strength by 61%. US National Bureau of Standards also found reductions of the same order of magnitude. If the same level of supervision is applied to brickwork as required for concrete, brickwork will be quite as reliable as concrete.

(b) Failure Criterion of Brick Masonry Subjected to Uniaxial Load

From early fifties principles of structural engineering have been adopted to design masonry structures. Investigators therefore, showed their interest to study the behaviour of brick masonry. Failure theories based on elastic analysis were proposed by a number of investigators. The well known failure theory proposed by Francis *et al.*(1970) is derived by considering the brick masonry prism subjected to an axial compressive stress, as shown in Fig.2.9a. Assuming a linear relationship between the ultimate longitudinal compressive stress and lateral tensile stress the failure criterion is derived by considering the force equilibrium, i.e, the total lateral tensile force in the brick is equal to the total compressive force in the mortar.

Shrive and Jessop(1980) also proposed failure theory based on elastic analysis. The lateral tensile stress is obtained by considering the strain compatibility and the force equilibrium in lateral direction. Utilising a linear failure envelope for the brick in the compression-tension quadrant and the predicted brick tensile stress at the failure stress, the compressive strength is derived. The predicted strength was found to be much less than that of the experimental result. They recognised the conditions of lateral compatibility and uniformity of lateral stresses can not hold true at the mortar brick boundary but they considered that the zone is limited in extent as observed in the finite element study by Khoo and Hendry (1973).

An alternative approach, failure criterion based on the strength of brick and mortar under multi-axial stress, was suggested by Hilsdorf(1969) based on the assumed linear relationship between lateral biaxial tensile stress and compressive stress for the brick as well as mortar. The average compressive strength of brick masonry is determined by dividing the compressive strength by a coefficient of nonuniformity, U_u which Hilsdorf established experimentally for various brick-mortar combinations. Referring to Fig.2.9b, the line A is a biaxial tension compression stress failure envelope of the brick. When the compression load is applied to the brick masonry, the lateral tensile stresses developed in the bricks follow some lines such as B_1. The local crack is developed in the brick masonry when this line intersects the failure envelope. Further local cracks will appear on subsequent increase of compressive load. But the failure of masonry will take place when the line defining the triaxial strength of the mortar, line C, intersects the failure line for the brick. Hilsdorf assumed the triaxial strength of the mortar which was originally obtained for concrete. In this criterion the deformation characteristics are not considered. For the failure of brick masonry the prior assumption of mortar failure imposes a condition which is not proven to exist.

Hilsdorf's theory was modified by Khoo and Hendry(1973) who derived a biaxial tension compression stress failure criterion experimentally for brick and a triaxial compression stress failure criterion for mortar. The failure strength of the brick masonry for a given brick and mortar can be obtained by superimposing the failure envelope of the mortar derived from triaxial compressive stress tests on the biaxial failure envelope of the bricks at a lateral tensile strain of 225 micron (Fig. 2.9c).

Atkinson and Noland(1983) proposed a nonlinear failure theory. This theory is derived by considering the strain compatibility and equilibrium of lateral forces in brick and mortar like that of Francis *et al.*(1970). In this theory the nonlinear characteristics of mortar is considered by expressing the Young's modulus and the Poisson's ratio as functions of the major principal stress, σ_1 and the confining stress,

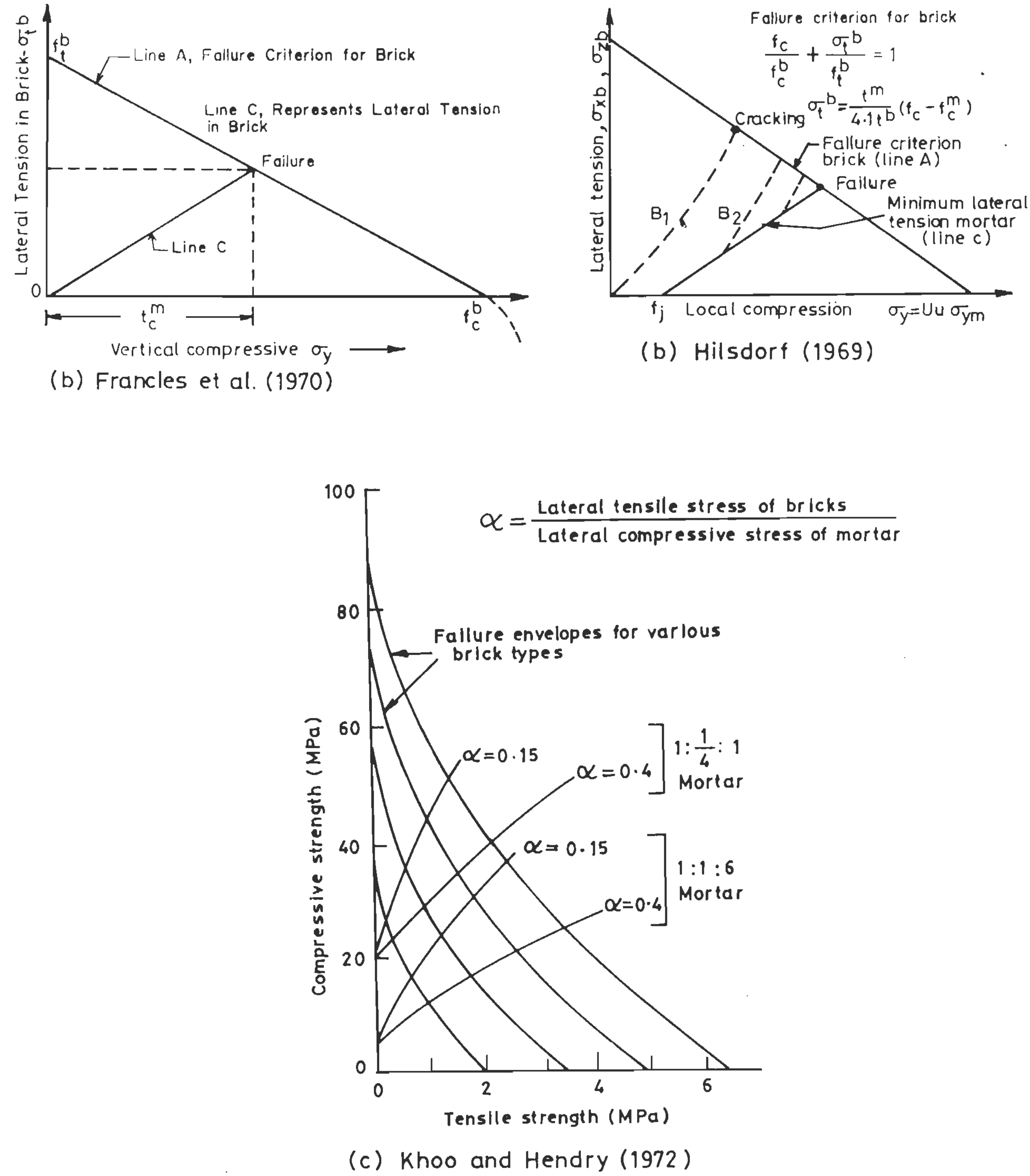


Fig.2.9 Failure Theories of Brick Masonry

σ_3. In order to validate the analytical data with the experimental results, McNary and Abrams(1985) performed the brick masonry prism tests. It was observed that the calculated strength was in the range of 60 to 70 percent of the experimentally observed strength.

All the strength criteria attempted only to predict the prism strength. The predicted values are found to be much less than the experimental results. Micro-mechanics approach has been employed to predict the strengths of prisms. This is discussed in Chapter 4.

(c) Tensile Strength: The tensile strength of the brick masonry is also important from the design point of view of masonry structures. Its value is very low and hence becomes the controlling parameter when tensile stresses are developed. Tensile stresses may occur due to either in-plane or transverse loads. In-plane loads produce tensile stresses such as uplift due to wind loads whereas transverse loads normal to the surface of the masonry panel produce tensile stresses through the thickness of wall. Regardless of loading direction, the tensile strength is dependent on the brick-mortar characteristics, particularly bond strength between them. Grandet(1973) observed that the mechanical bond is developed between the brick and the mortar due to the formation of a micro-layer of ettringite($3CaSO_4$, Al_2O_3, $3CaO$, $31H_2O$). The mean diameter of the pore size of the brick should be more than .05mm for the formation of micro-crystal of ettringite. The brick should have sufficient moisture content for the proper hydration of cement behind the ettringite layer. The results of the experiment of Sinha(1967) also confirmed that the moisture content of the bricks at the time of laying has maedrk influence on the tensile bond strength(Fig.2.1). Although it is difficult to define the relationship between moisture content and tensile bond strength, it will be noted that tensile bond strength reduces as the brick approaches its saturation moisture content.

Tensile Strength under In-plane Loading: Tensile stresses may occur either normal or parallel to the bed joint depending on the direction of the in-plane loads. Investigators employed different types of tests (Fig.2.10) to determine the tensile strength of masonry. A simple brick couplet test was used by Sinha(1967). Modulus of rupture tests on the brick masonry prism was used by Grenley *et al.*(1969). Diametrical splitting tests developed by Johnson and Thompson(1969) has been used by many investigators to determine the direct tensile strength of masonry. By rotating the orientation of the bed joint to the splitting force, various tensile stress states can be induced in the masonry. The masonry wall splitting tests were used by Ali and Page(1987).

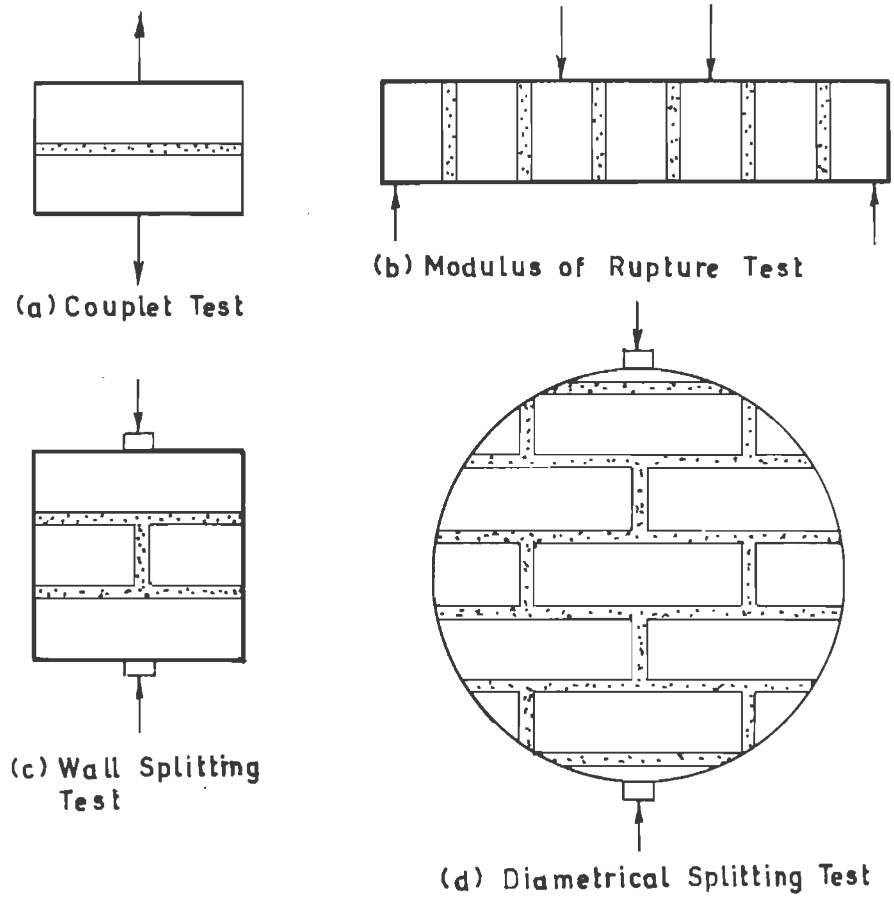


Fig.2.10 Tensile Test Arrangements

Tensile Strength under Transverse loading: Transverse loading produces flexural tensile stress through the thickness of the wall. If the precompression is large, the tensile stresses will be offset by the compressive stresses and the strength of the wall will be guided by compressive stress. When the precompression is small, the strength of the panel will be restricted by flextural tensile strength of masonry. The flextural tensile strength was determined by many investigators[Satti and Hendry(1973), Sinha and Hendry(1975)]. The flextural tensile strength parallel to the bed joint is greater than normal to the bed joint. Their ratio is in the range of 2 to 7. It is revealed that this ratio decreases markedly with increase in the flexural tensile stress normal to the bed joint. Studies by Baker(1980) and Lawrence(1975) showed that the strength ratio is a function of the tensile bond strength of the joints and the modulus of rupture of the brick unit.

(d) Elastic Properties

A knowledge of elastic properties are of fundamental importance as the masonry, now a days, is used as a structural material. These properties of the brick masonry are mainly affected by the elastic properties of both the masonry constituents: the brick and the mortar. Other factors such as workmanship, aspect ratio of the bricks, thickness of the mortar joint, stress level, etc., also influence these properties. Sahlin(1971) critically reviewed the literature pertaining to elastic properties of brick masonry and factors influencing them. He suggested a relationship between compressive strength and modulus of elasticity as given below:

for clay brick masonry,

$$E = 700\ f_c \tag{2.11}$$

and for concrete brick masonry,

$$E = 1000\ f_c \tag{2.12}$$

where E and f_c are Young's modulus and crushing strength of masonry.

These purely empirical relations are suitable for medium strength masonry. At low stress level, these empirical relations can be used for the determination of initial tangent modulus of the masonry. With the increase in compressive stress the modulus normally decreases.

Turnsek and Cacovic(1970) derived experimentally stress-strain relationship for the brick masonry. The compressive deformations were measured up to 80% of the ultimate load. The complete stress-strain relations including unloading portion have been determined by Powell and Hodkinson(1976), using solid and perforated bricks. The nondimensional stress-strain diagram(Fig.2.11) derived by Turnsek and Cacovic(1970) for the brick masonry in compression is in good agreement with that obtained by Powell and Hodkinson (1976).

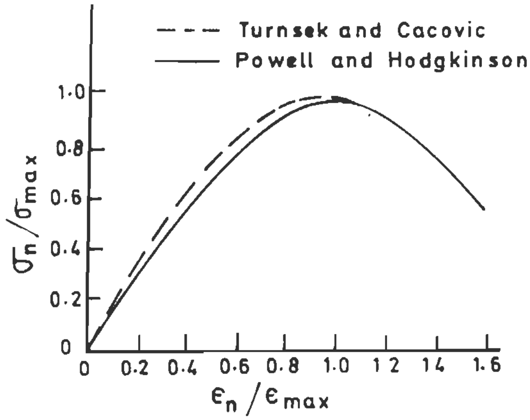


Fig.2.11 Stress-Strain Curve of Brick Masonry

Nonlinear stress strain relationships have been suggested by some researchers[Warren and Lenczner(1981), Sinha and Pedreschi(1983)]. Pedreschi and Sinha(1982) carried out investigation on two types of prisms built from four types of bricks. They suggested nondimensional polynomial stress-strain relationship for brick masonry loaded parallel to the bed joints. Hodkinson and Davis(1982) studied the stress-strain relationship for stresses normal and parallel to the bed joint. The values of the modulus of elasticity are typically lower in prisms loaded parallel to the bed joint than that in prisms loaded normal to the bed joint. The reduction was approximately 20% for prisms with froged bricks and 60% for prisms with perforated bricks.

Attempts have been made by investigators to determine analytically elastic properties of the masonry in terms of elastic properties of brick and mortar. An "equivalent material approach" has been used for computing the elastic properties of masonry by Liang *et al.*(1990). The brick masonry with two sets of bed and head joints is represented by an equivalent homogeneous orthorhombic elastic material. Elastic properties of this equivalent material is derived in terms of the constituent materials and their relative thicknesses. Elastic properties derived by this approach have not been validated with experimental results.

It is experimentally established that the brick masonry behaves as an isotropic material up to the development of the first crack provided that the bricks used are isotropic.

(e) Shear Strength

An important consideration in the analysis and design of masonry structures is its ability to withstand lateral loads induced by earthquake or wind in addition to gravity loads. The resistance to such loads is predominantly by in-plane shear resistance of masonry structures. The shear strength of brick masonry is a function of the bond strength of the masonry which in turn depends upon the degree of saturation of the brick at the time of laying and water retentivity of the mortar. Workmanship also affect the shear strength significantly.

In most cases, the in-plane failure of unreinforced masonry walls is dominated by shear or diagonal tension. It is characterized by diagonal cracks. At low compression, failure tends to develop step wise along the brick mortar joint on an approximately 45° angle(Fig.2.12). As the compression increases the diagonal crack indiscriminately crosses masonry units and mortar joints.

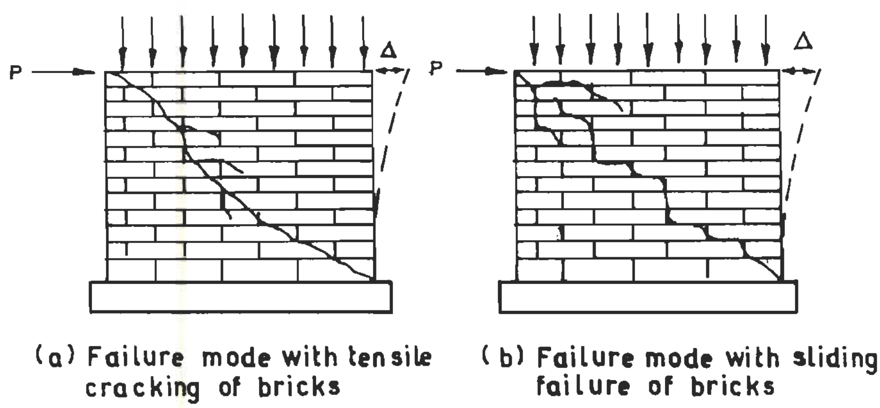


Fig.2.12 Failure Behaviour of Shear Walls

Both the experimental and theoretical evaluation of in-plane shear resistance has been the object of study for many years. The resulting criterion for shear failure is a function of average bond strength and the frictional resistance of brick mortar interface. The criterion can be formulated as

$$\tau_f = \tau_o + \mu \, \sigma_n \qquad (2.13)$$

where τ_f = ultimate shear stress of the masonry evaluated by dividing the horizontal loads by the area of the bed joint.

τ_o = initial shear strength of the masonry

σ_n = compression stress normal to bed joints evaluated by dividing the vertical load by the area of the bed joint.

μ = coefficient of friction

The above relationship is very simple and applicable upto $\sigma_n \leq 2N/mm^2$. Indian Code of practice has also given similar type of a relation to calculate the in-plane shear strength of masonry. The shear strength of brick masonry may be varied from $0.1N/mm^2$ in the absence of precompression to a maximum of $0.5N/mm^2$.

A study of test results reported by many investigators [Hendry and Sinha(1971), Chinwah(1972), Pieper and Trautsch (1970), Yorulmaz and Atan(1977)], reveals that the value of τ_o varied from 0.14 MPa to 0.70 MPa and μ varied from 0.30 to 1.04 . The wide variation of test results is influenced not only by the variation of properties of materials, but also varying test conditions which will produce different stress distribution within the test specimen. Different types of tests used to determine shear strength are shown in Fig.2.13. Pieper and Trautsch(1970) observed that the shear strength decreases with increase in the aspect ratio whereas it increases with increase in thickness. It also depends on the suction rate of the bricks at the time of laying.

The use of Coulomb type failure criterion has been questioned by many researchers on the basis that diagonal tension failure occurs generally due to the attainment of critical value of principal tensile stress. Borchelt(1970), Turnsek and Cacovic(1970) suggested that the mean shear stress value causing the first crack may be calculated by the formula:

$$\frac{\tau_f}{\sigma_t} = \sigma_t \sqrt{1+ \frac{\sigma_n}{\sigma_t}} \qquad (2.14)$$

where τ_f = ultimate shear strength of masonry

σ_n = precompression normal to bed joint

σ_t = principal tensile stress at failure

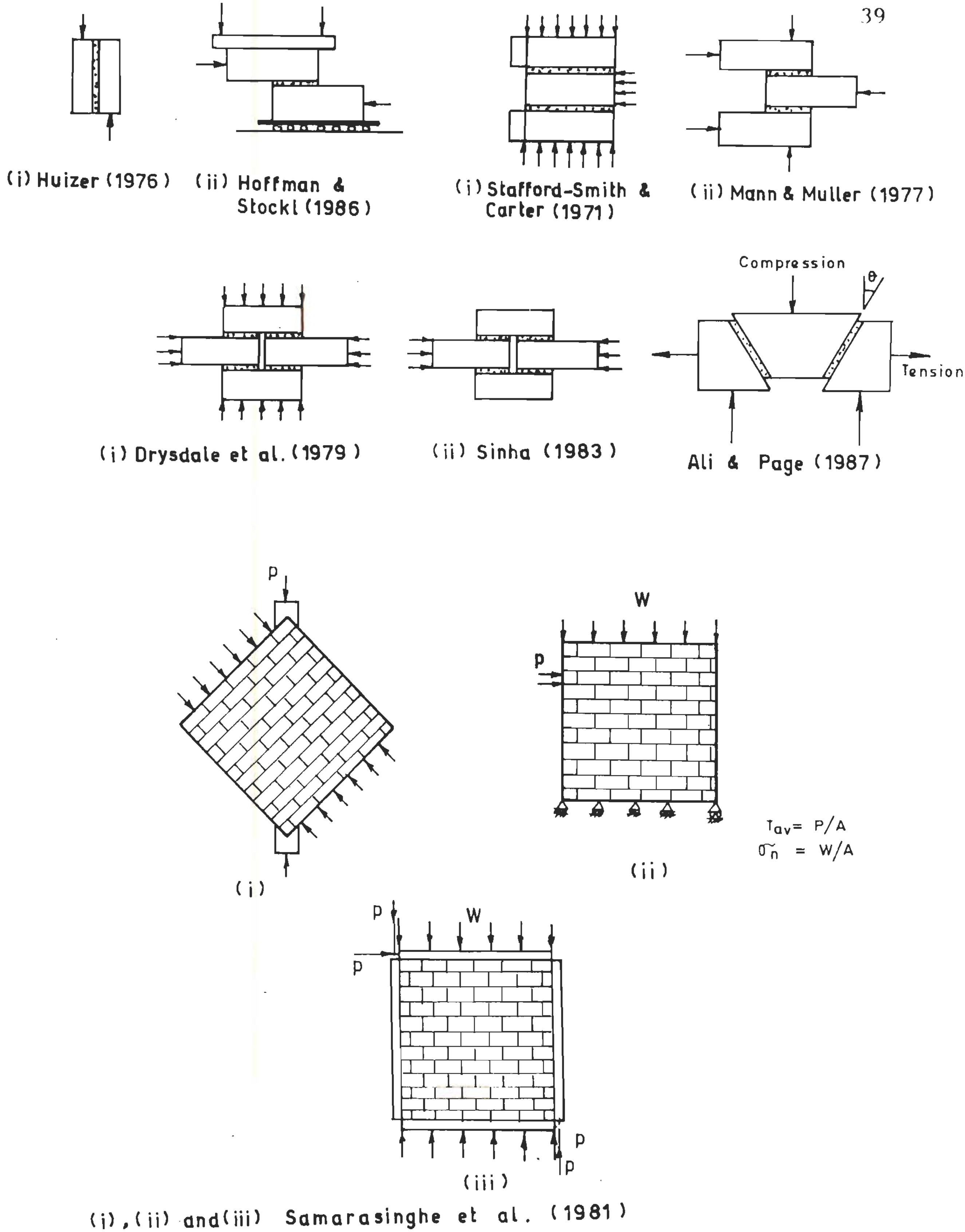


Fig.2.13 Shear Test Arrangements

The value of the principal tensile stress at failure increases with the increase in σ_n and Chinwah(1972) and Schineider(1976) have derived a relationship of the type:

$$\sigma_t = \sigma_{to} + 0.05\ \sigma_n \tag{2.15}$$

where σ_{to} is the value of the principal tensile stress at failure when the precompression, σ_n is zero.

As the principal tensile stress at failure, σ_{to}, is equal to the ultimate shear stress, τ_o, in pure shear condition, using Eqn.(2.14) and (2.15), Hendry(1978) derived the following expression to calculate the shear strength.

$$\tau^2 = (\ \tau_o^2 + 1.1\ \tau_o\ \sigma_n + 0.053\ \sigma_n^2\ ) \tag{2.16}$$

Shear strength criteria discussed above are of semi-empirical nature. The effect of normal stress, σ_p, parallel to the bed joints is completely ignored in this Coulomb type failure criterion. The effect of σ_p on the failure criterion is significant specially under a tension-compression stress state. To completely define masonry failure, a biaxial failure criterion is required.

(f) Behaviour of Brick Masonry Subjected to Biaxial Stresses

Biaxial state of stress exists in masonry structures subjected to in-plane loading, such as shear walls infilled frames. The behaviour of brick masonry subjected to biaxial state of stress has been studied for the last two decades. The failure modes under different biaxial state of stresses are shown in Fig.2.14.

Samarasinghe et al.(1982) carried out biaxial tension-compression tests on masonry panels by using one-sixth scale bricks(compressive strength =30.5 MPa) with 1:0.25:3 (cement:lime:sand) mortar (25mm cube strength = 9.6 MPA) with varying bed joint angle relative to the applied load for different ratios of principal stresses. The effect of the orientation of bed joint to the applied load is quite marked. The failure surface derived is shown in Fig.2.15a.

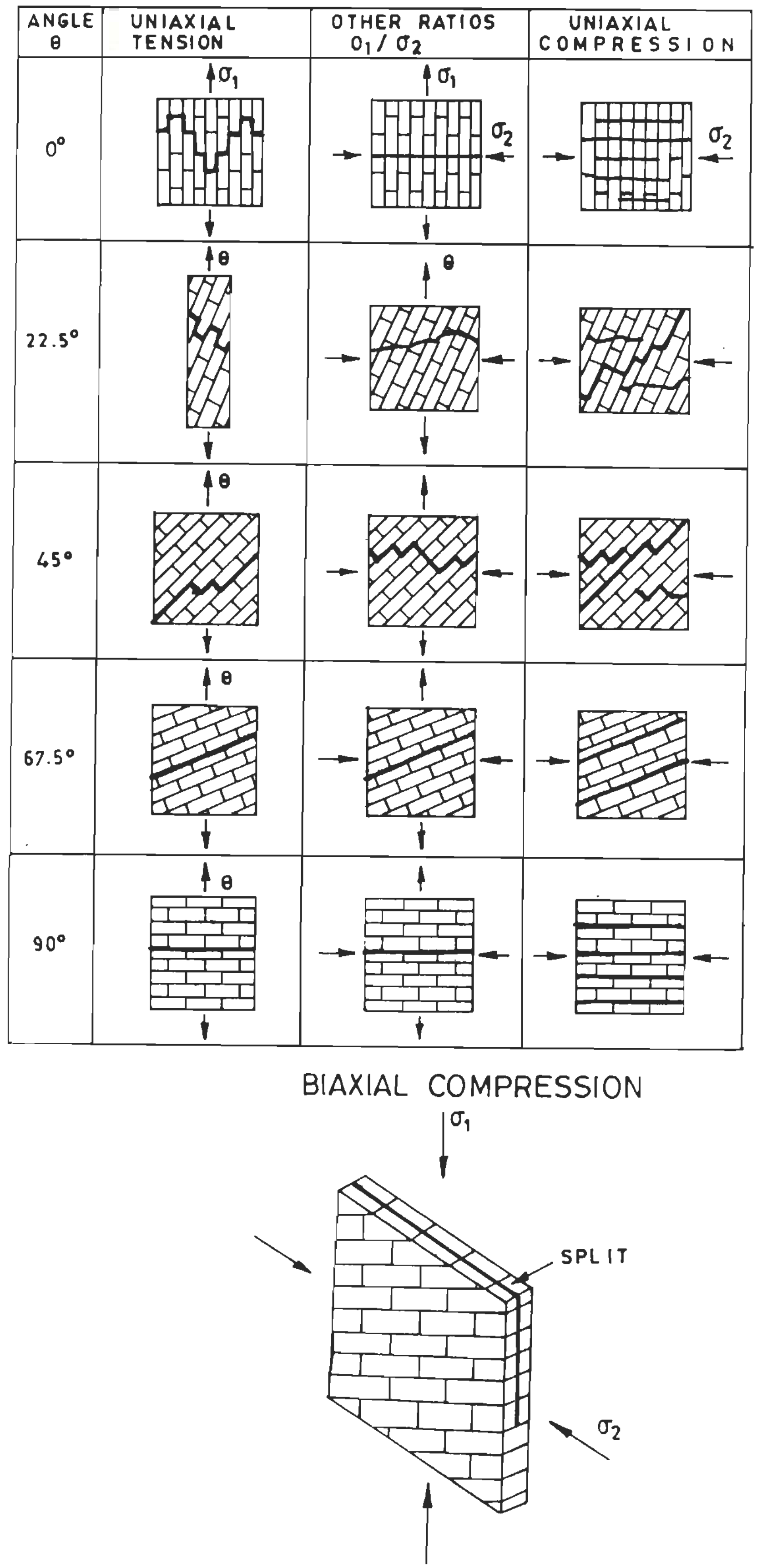


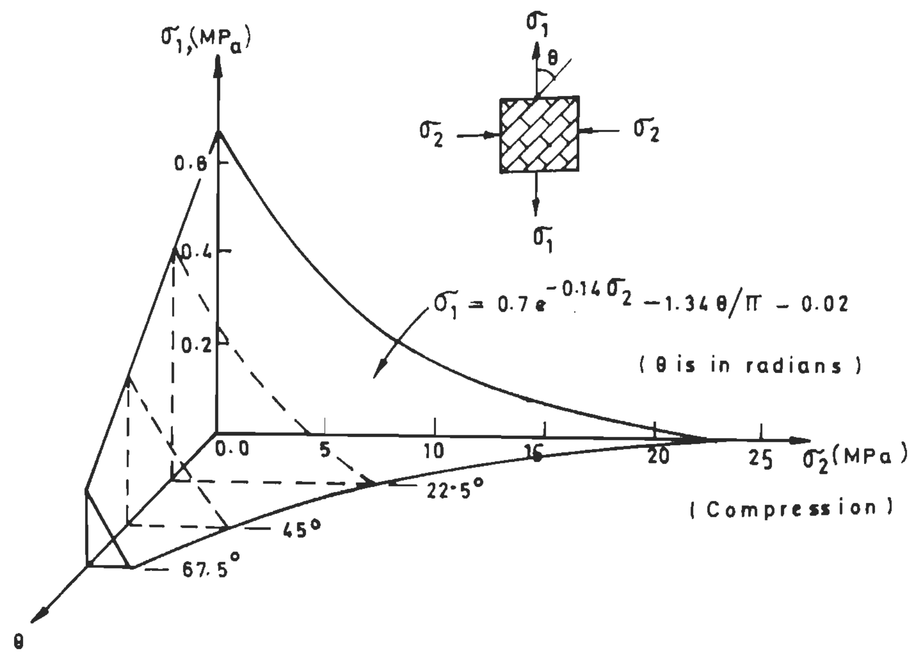
Fig.2.14 Modes of Failure of Brick Masonry Subjected to various State of Stress

Page(1983) derived experimentally failure surfaces for the brick masonry subjected to biaxial tension-compression and compression-compression state of stress with different bed joint angle relative to the applied loading. For biaxial tension-compression state of stress, it is evident that the strength of the brick masonry is significantly influenced by the orientation of the bed joint angle(Fig.2.15b), while the bed joint angle has little effect on the strength of the masonry under biaxial compression-compression state of stress(Fig.2.16). This is consistent with the observed failure modes. Failures occurred by splitting in a plane parallel to the free surface of the specimen at mid thickness, regardless of the bed joint angle. Biaxial tension-tension tests have not been reported till date. Page(1980), however, proposed a failure surface for brick masonry subjected to biaxial tension-tension. The failure surface (Fig.2.17) is derived by using a finite element model considering joint failure only. A shear bond to tensile bond ratio of 0.83 was used.

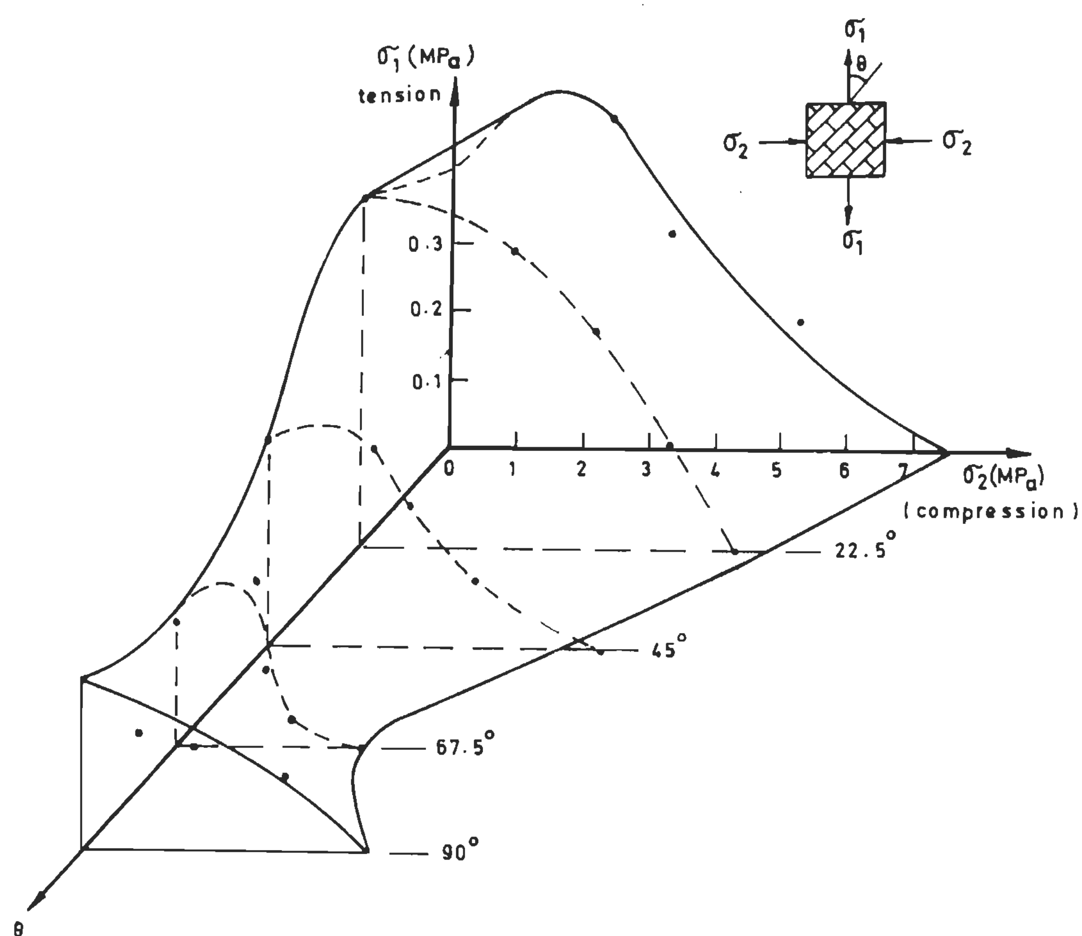
Dhanasekar(1985) proposed a failure surface using the biaxial test results of Page(1981,1983). The failure surface is defined in σ_n, σ_p ,τ stress space where σ_n ,σ_p are the stresses in the direction of normal and parallel to the bed joint and τ is the shear stress. Three truncated elliptic cone are used to define the failure surface(Fig.2.18a).

Ganz and Thurliman(1983) proposed a failure surface for hollow brick masonry subjected to biaxial stresses. The failure surface in σ_n, σ_p, τ space consists of four failure criteria. Two of the criteria, namely a limit on the maximum principal compressive stress and a no-tension limit, provide conical segments. The third criterion, a Mohr-Coulomb sliding limit provides a plane segment parallel to the σ_p axis, whereas the fourth criterion associated with splitting of the webs provides a cylindrical segment with generators parallel to the σ_n axis. The failure surface is shown in Fig.2.18b.

Hamid and Drysdale(1982) proposed a failure criterion taking into account the anisotropic nature of brick masonry subjected to biaxial compressive loads. The maximum stress theory, modified to account for the strength variation normal and parallel to the bed joints, was used to express failure due to splitting. An experimental investigation on the behaviour of concrete masonry subjected to biaxial stresses was carried by Hegemier *et al.*(1978).



(a) Samarasinghe *et al.* (1982)



(b) Page (1983)

Fig.2.15 Failure Surface under Biaxial Tension-Compression Stresses

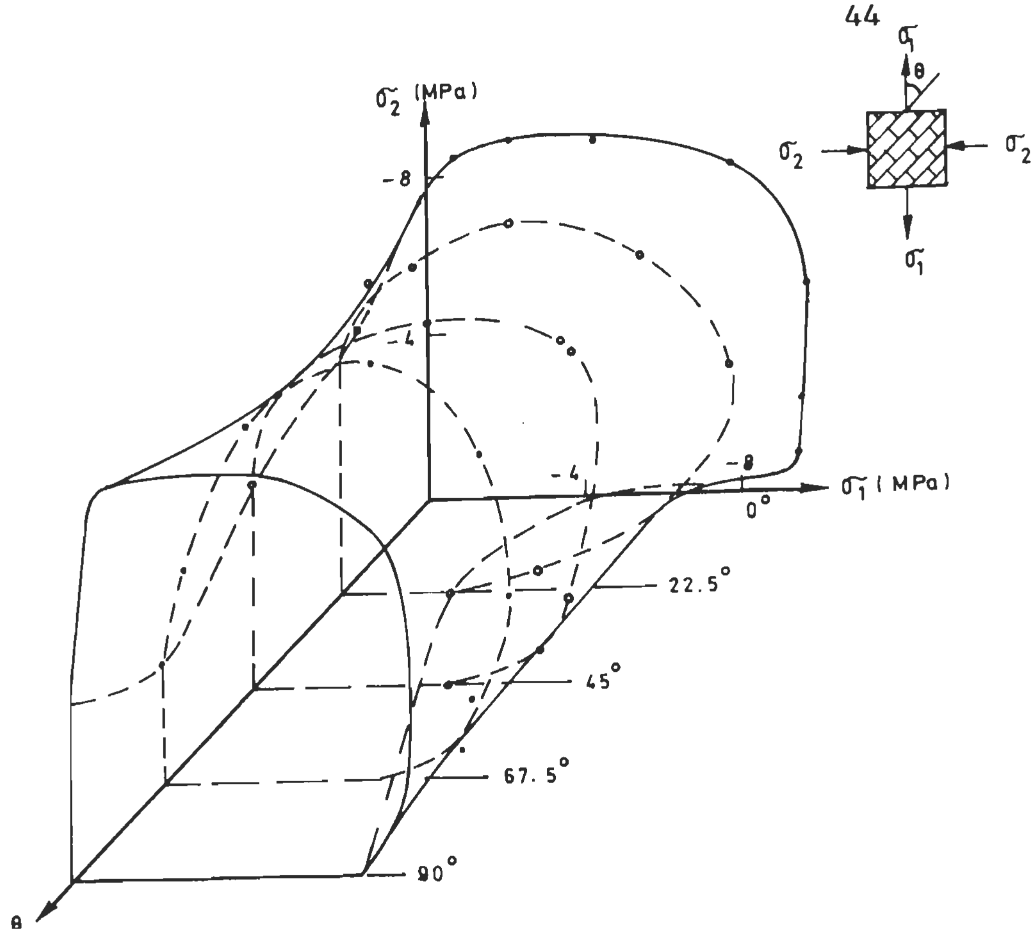


Fig.2.16 Failure Surface under Biaxial Compression-Compression Stresses [Page (1981)]

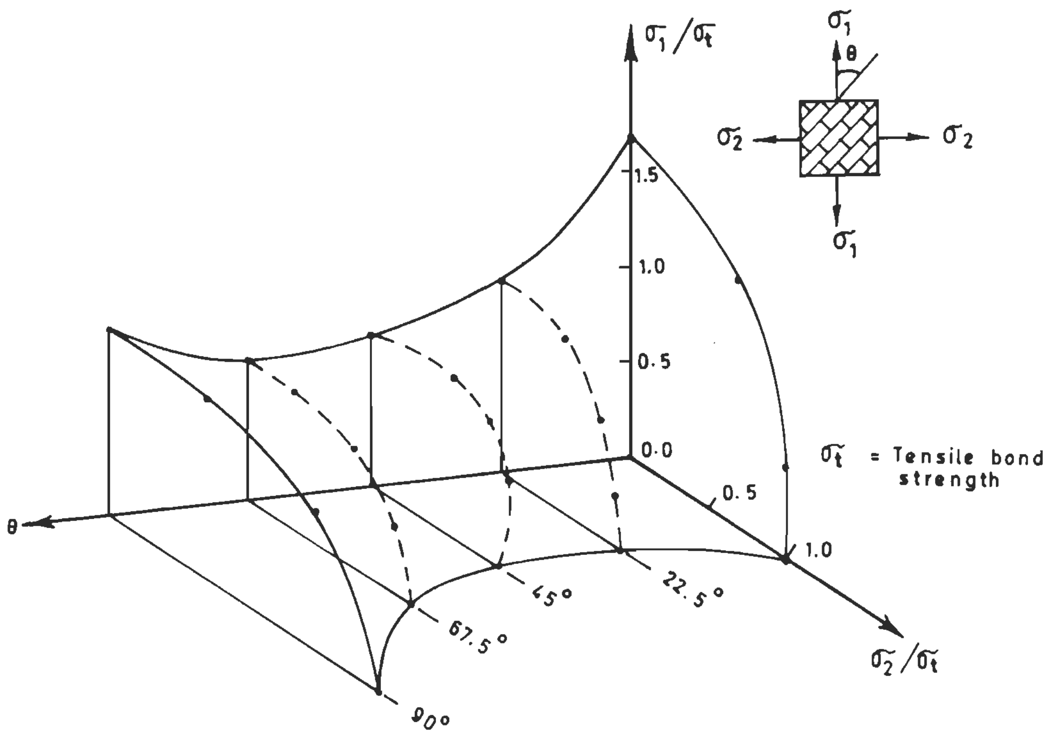
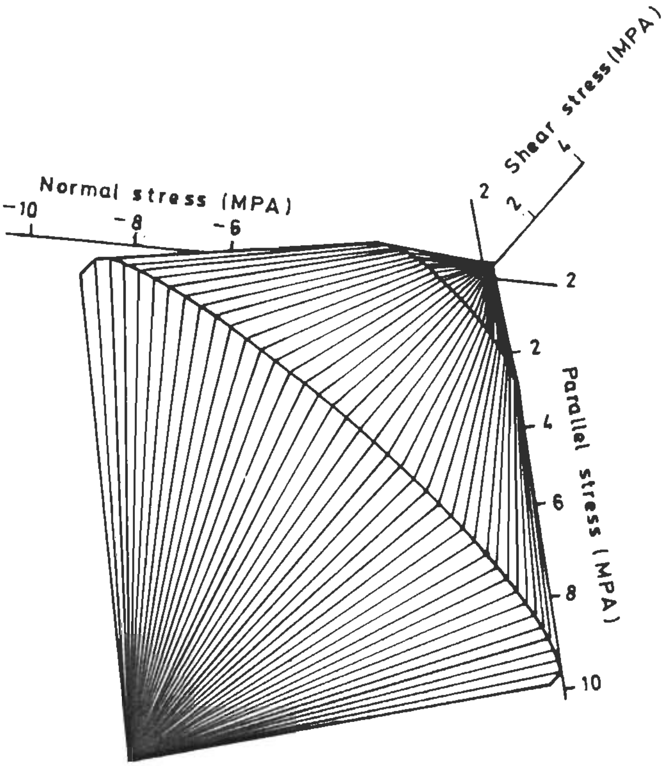
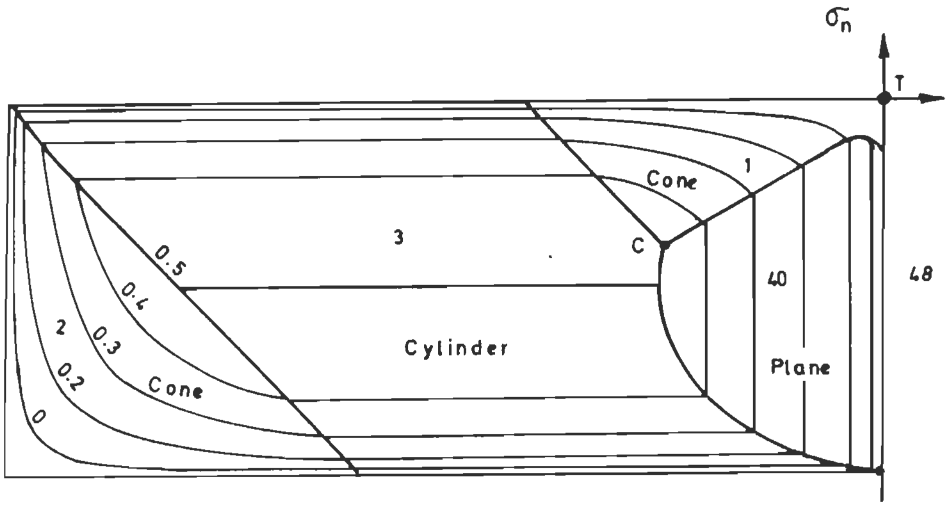


Fig.2.17 Failure Surface under Biaxial Tension-Tension Stresses



(a) Dhanasekar(1985)



(b) Ganz and Thurlimann(1983)

Fig.2.18 Failure Surface in Stresses (σ_n, σ_p, τ) Space

2.3 Conclusions

The state of the art of the properties of the brick masonry and its constituent materials relevant to the present investigation has been critically reviewed in this chapter.

- The Indian Standard Code testing procedures overestimate the compressive strengths of bricks and mortar. The apparent increase in the compressive strength is due to the restraining effect of the platens of the testing machine. To overcome this problem the methods suggested by many investigators have been discussed.

- The biaxial tension-compression strength of bricks markedly affects the behaviour of brick masonry subjected to in-plane loading. If the tensile strength of the brick is less than the bond strength of the brick masonry, the failure of brick masonry will occur by a failure of bricks by tensile splitting. This is not desirable as it may lead to the collapse of the masonry structure. Besides this, it can not be easily repairable.

- The stress-strain relation of the brick upto failure is linear, while that of the mortar is nonlinear. The degree of nonlinearity depends upon the proportion of its constituents. The highest value of initial tangent modulus reported in the literature is about 200 times the lowest.

- The properties of brick masonry and the factors influencing it have been studied in depth. The Indian Code like all Codes of many other countries has recommended an empirical relation in terms of compressive strength of bricks for different grades of mortar to compute the compressive strength for the design of masonry structures. The effects of workmanship, variation in joint thickness, etc., have not been taken into account, resulting in wastage of material as large safety factors are prescribed.

- To obtain an accurate value of compressive strength, Indian Code has recommended tests on the same type of wall that will be used in the masonry structures. But Codes of many countries have recommended masonry prism test as the basic test for the determination of the compressive strength of brick masonry. Tests on masonry prisms and walls are performed in order to define the relation between the two. The details of these tests are described in Chapter 3

- Attempts have been made by many investigators to derive the uniaxial compressive strength of brick masonry in terms of strength and elastic properties of brick and mortar with partial success. The strength and elastic properties of brick masonry derived in terms of those of brick and mortar based on micromechanics approach has been described in Chapter 4.

- The failure theory for clay brick masonry developed from biaxial tests is not comprehensive. An anisotropic failure theory is employed to define the failure behaviour of brick masonry. The strength prameters are defined using biaxial test results reported in the literature. The failure theory and the material model are described in Chapter 5.

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CHAPTER - 3

EXPERIMENTAL INVESTIGATIONS OF PROPERTIES OF BRICK, MORTAR AND BRICK MASONRY

3.1 Introduction

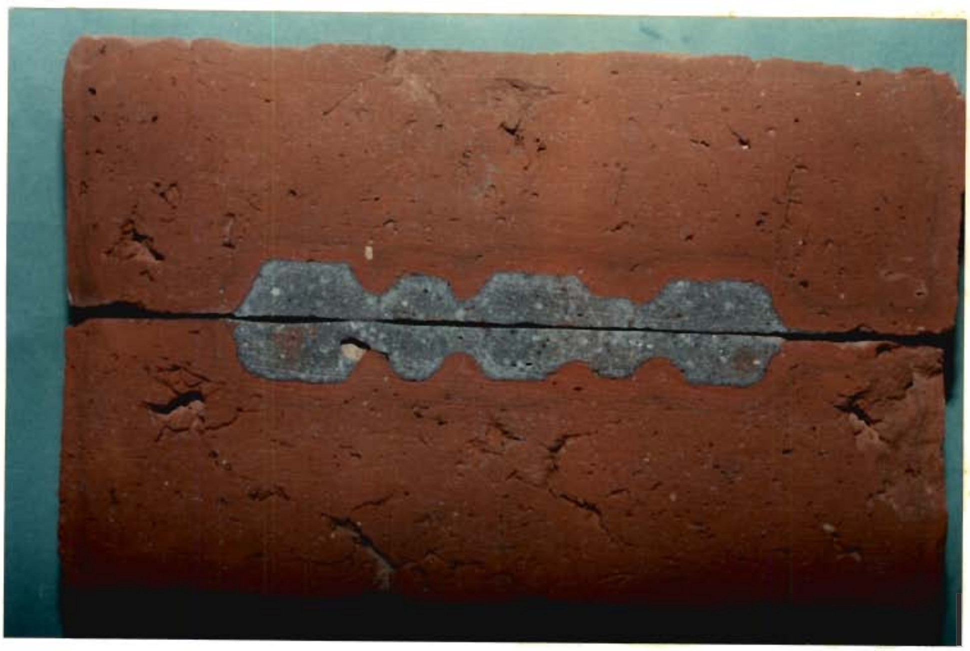
Strength and deformation properties of brick masonry and its constituents are necessary to define an analytical model which considers brick masonry as a composite material. The available data related to all these properties for Indian conditions are insufficient and not well documented. So experimental investigations are carried out to determine strength and deformation properties of brick masonry and its constituents.

3.2 Brick

Conventional clay bricks are widely used in India. These bricks are manufactured by a hand moulding process and fired in rudimentary kilns. Bricks, thus obtained have distortions in shape and size, surface cracks and flaws of different sizes and these are randomly distributed as shown in Fig.3.1.

Large samples of bricks from the same batch were procured at the beginning of this investigation. The bricks were grounded using carborandum powder of grades 300-800 to eliminate the distorted shapes and to obtain smooth faces. Dimensions were measured using slide calipers. The variation of dimensions are shown in Table 3.1.

(a) Brick Specimen No. 1



(b) Brick Specimen No. 2

Fig.3.1 Randomly Distributed Flaws of Different Shapes and Sizes in Brick Sections

Table 3.1 Dimension of Bricks

Brick Type	No.of samples	Quantity	Observations				
			Maximum in mm	Minimum in mm	Average in mm	Standard Deviation	Coeff. of Var.
Solid clay bricks	20	Length	235.5	222.50	230.700	3.460	1.5
		Width	114.0	106.25	111.275	2.346	2.1
		Thickness	74.0	65.00	68.210	1.816	2.7

3.2.1 Properties of Brick

In the present investigation strength and deformation characteristics such as compressive and tensile strength, elastic modulus and Poisson's ratio are determined. Biaxial test data are needed to model the behaviour of bricks in shear walls, and infill frames for a micro level analysis to study the effects of brick on the behaviour of masonry. Due to a lack of facilities these tests could not be performed.

Most of the physical properties do not significantly affect the mechanical properties of the brick masonry. Initial rate of absorption (I.R.A.) or water absorption plays an important role in the laying of bricks as well as in the development of bond strength and as such affects the strength properties of the brick masonry. This property was determined in accordance with the procedure laid down in Indian Code of practice, IS:3495(Part I)-1976. The average **water absorption** was found to be **13.39 percent**.

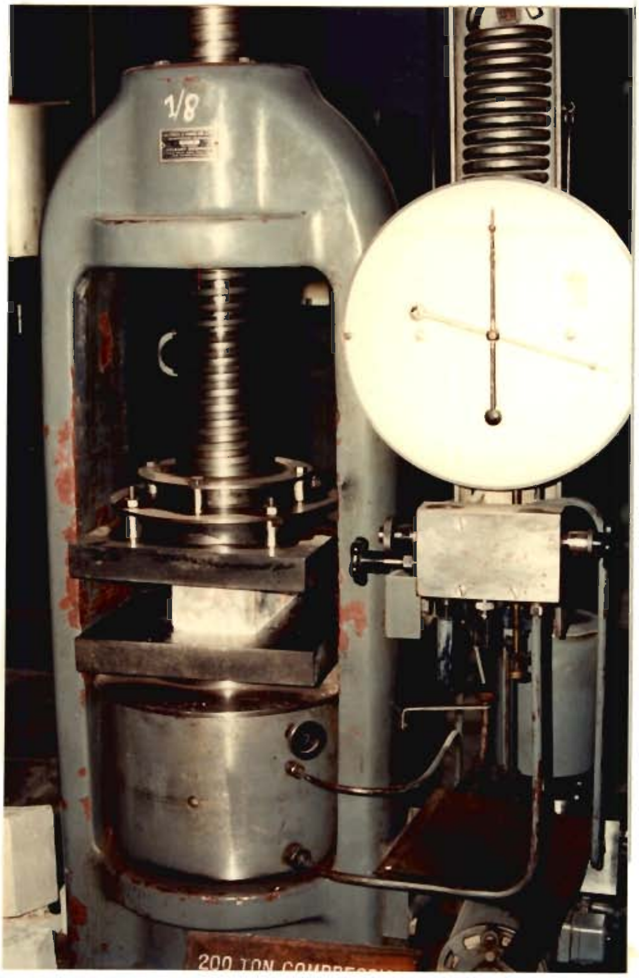
(a) Compressive Strength: All faces of the bricks were grounded, after filling the frog, surface cracks and holes, if any, with 1:1 cement sand mortar, to remove unevenness and distortion of shapes to minimise uneven distribution of the applied load. Two types of compressive tests were carried out: (i) IS.code method and (ii) a method employed to obtain actual compressive strength.

(i) Indian Standard Code Procedure: In the standard compression test bricks were loaded in their normal orientation as in the walls. The strength was evaluated using the standard Code procedure except that the bricks were tested in the dry condition. Both plywood and gypsum(3mm thick) are used as capping materials, as laid down in the Code, IS:3495(Part I) -1976 to find out the effect of these two different capping materials on the strength of the bricks. The variation in the test results were insignificant. The aspect ratio (height/width) of the bricks was of the order of 0.7, as a result of apparent strengthening due to the restraining effect of the platens. The brittle mode failure of bricks was prevented by a complex state of stress arising due to the restraining effect of the platens(Fig.3.2). Thus this method can be used for quality control. The tests results are summarized in the Table 3.2.

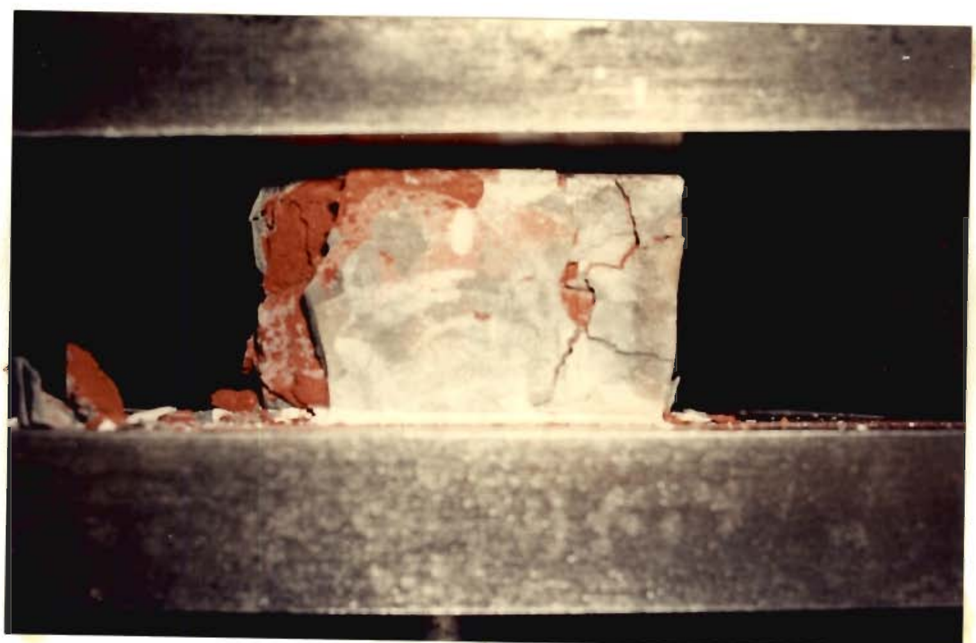
To evaluate the actual compressive strength of the brick, the restraining effect can be minimized by employing interface friction reduction(IFR) method. In this method, a capping material which reduces the interface friction is used. McNary and Abrams(1985) placed polished steel plates between the specimen and the platens. A greased teflon sheet was also placed between the steel plate and the gypsum cap on the surface of the specimen. The induced interface friction will also be minimum if the stiffness of the capping material is approximately equal to the stiffness of the specimen. For this purpose steel brushes were used in compressive tests of concrete by Kupfer *et al.*(1969) and in compressive tests of bricks by Thomas and O'Leary(1970), Page(1978) among others. This technique involves the use of steel brushes placed between the machine platen and the specimen.

The platens restraining effect can also be minimized by increasing the aspect ratio of the test specimen. This method is cost effective and simple to perform and was employed to determine the compressive strength of the bricks.

(ii) Proposed Method Based on Increase in Aspect Ratio: The brick specimens were placed vertically(bed faces parallel to the direction of loading) between loading platens, resulting in an increase of aspect ratio (Length : width) of the order of 2.1(Fig.3.3). A thin layer of gypsum(3mm thick) was applied on the loading surfaces an hour before testing. The tests results are summarized in the Table 3.2. The strength obtained in this method is about 60% of the strength obtained by the procedure suggested in the IS code. The details of the tests results are given in Appendix A.



(a) Setup for Compression Tests for Bricks



(b) Shear failure of bricks

Fig.3.2 Compressive Tests for Bricks
[IS:3495(Part I)-1976]

Table 3.2 Compressive Strength of Solid Clay Bricks using Codal Method and Proposed Method

No. of specimen	IS:3495(part-I)-1976			Proposed method			
	Mean (MPa)	Stand. Dev.	Coeff. of Var.	Mean (MPa)	Stand. Dev.	Coeff. of Var.	$\frac{\text{Mean, Codal Method}}{\text{Mean, Prop. Meth.}}$
20	25.26	1.547	6.17	15.45	1.682	10.97	1.6488

(b) Tensile Strength: Tensile strength is an important property of the brick. Bricks in a wall under normal load fail from tensile splitting due to lateral biaxial tension. In shear or diagonal mode of failure, splitting of bricks may occur if the bond strength is high.

(i) Determination of Tensile Strength: The direct tensile test method has rarely been used, due to the difficulties of holding the specimen and the stress concentration induced by the gripping devices. The results are extremely variable and no definite trends are noted. The strength obtained from the modulus of rupture test is more than the actual strength.

Indirect tensile splitting test was employed to evaluate the tensile strength. This method has been widely used by many invertigapesto evaluate tensile strength of brittle materials. The tensile strength of brick was determined by using the equation suggested by Rosenhaupt *et al.*(1957).

$$f_t^b = \frac{0.648P}{DL} \tag{3.1}$$

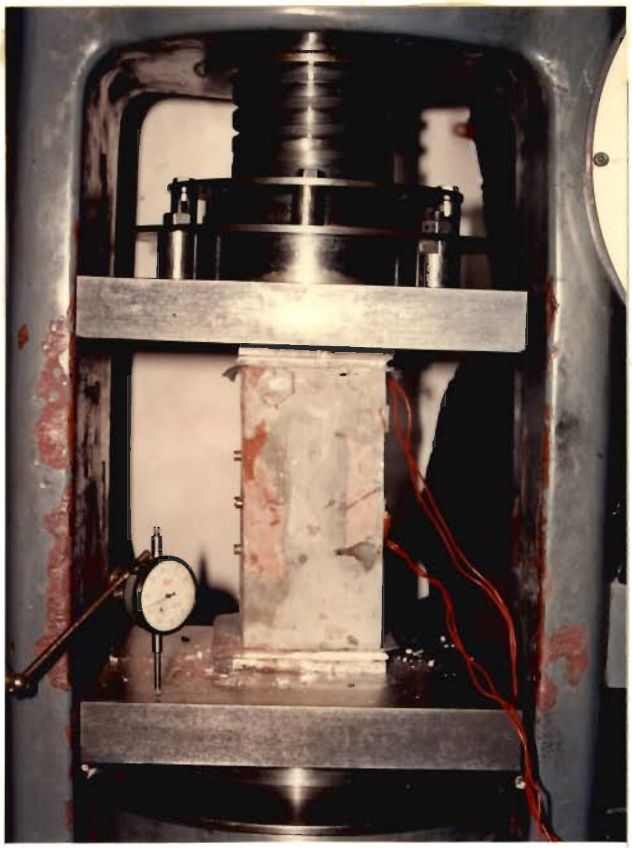
where P is the applied compressive load at failure, D and L are the equivalent diameters of the cross-section and the length of the brick respectively.

The frog of the specimen brick was filled with 1:1(cement:sand) mortar and cured for 28 days before testing. The average **tensile strength** is found to be **0.512 MPa** with a coefficient of variation of 15.52 percent.

(c) Elastic Properties: Uniaxial compression tests were used to investigate elastic properties of the bricks. Two types of tests were performed, one with load applied normal to the bed faces and the other with load applied parallel to the bed faces of the bricks. The tests are briefly described in this section.

(i) Load Parallel to Bed Joint: After filling the frog with cement mortar, bricks were cured for 28 days. Both the loading faces were grounded and a uniform thin layer(3mm) of gypsum was applied one hours before testing. Bricks were then placed vertically between the platens of an universal testing machine(Fig.3.3). Two 55mm long bakelite base strain gauges were fixed on a vertical face. One was fixed along the loading direction and the other was fixed in the transverse direction. A dial gauge was fixed on the platen to measure the total deformation of the specimen. Load was applied at a rate of 500N/min. The results are summarized in Table 3.3.

(ii) Load Normal to Bed Joint: Uniaxial compression tests were performed with load applied normal to bed face. Both the faces were grounded after curing of the filled up frogs with 1:1 cement sand mortar for 28 days. To eliminate the restraining effects of the platens and to prevent adhesion of the joint material to the bed faces of the brick, the specimen brick was wrapped in a plastic sheet and placed of the centre of a three brick high stack bonded brick prism(Fig.3.4). The prisms were constructed using gypsum as a joint material. The end faces of the prisms were also capped with gypsum for uniform distribution of the applied load. Two 55mm long bakelite base strain gauges were fixed at the central portion to measure the longitudinal and lateral strains in the brick. Stress-strain curves are shown in Fig 3.5. The elastic properties are summarized in Table 3.3.

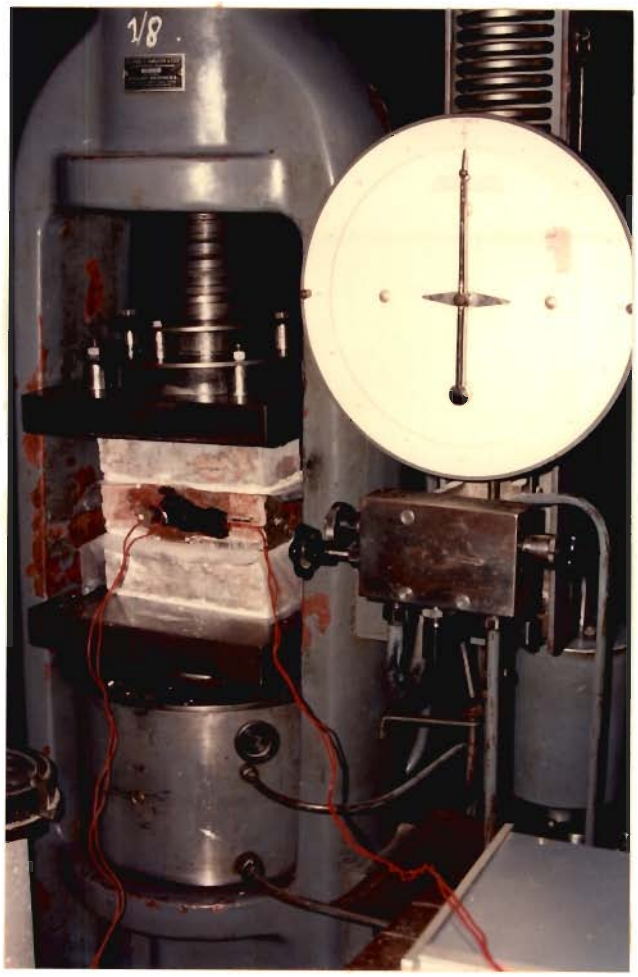


(a) Brick in Vertical Position
Load Parallel To Bed Joint

(b) Typical Splitting Failure of Brick

Fig.3.3 Proposed Method for Compression Test for Bricks

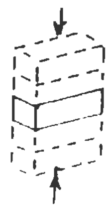
(a) Brick Specimen Placed in a Three Brick High Brick Prism



(b) Load Normal to the Bed Face

Fig.3.4 Prism Test for Bricks

Table 3.3 Modulus of Elasticity and Poisson's Ratio of Clay Bricks

No of specimen	Loading direction	Modulus of elasticity(N/mm^2)			Poisson's ratio
		Mean	Stand. Dev.	Coeff. of Var.	Mean
5	Load parallel to bed joint 	5088	499.7	9.80	0.14
5	Load normal to bed joint 	5180	582	11.23	0.15

Significant variations in the brick properties were reported by Page(1978) and Bhandari(1982). In the present investigation, these variations were reduced considerably by removing distortions in shape, unevenness of the loading faces and filling the surface cracks(Coefficient of variation was found to be of the order of 10 percent). The variation in the properties can be further reduced by improving the manufacturing process. The bricks are observed to be marginally stiffer in the direction normal to bed joint, as it is the direction of pressing during its manufacture. Thus the conventional clay bricks can be considered as isotropic material.

3.3 Mortar

Mortar plays an important role for satisfactory performance of brick masonry as a structural material. The choice of different types of mortar is governed by strength requirements, type of masonry structure, situation of use, strength of bricks, workability, water retintivity, exposure to weather, resistance to frost, chemical attack and fire etc. Depending on the various functions and strength requirement of brick masonry, Sahlin(1971), Hendry *et al.*(1981) among others recommended different types mortar grades. In masonry structures such as shear wall, infill frames, cement-lime-sand mortar should be used as the required shear and tensile bond stresses are quite high. Low strength mortar is normally used in brick masonry so that failure of masonry subjected to in-plane loading may occur only the in mortar joints. Thus, the complete collapse of masonry structures can be avoided and at the same time it can be easily repaired. To minimize creep and shrinkage and to obtain the maximum strength of the brick masonry for a particular type of mortar, a dense mix of the ingredients is necessary. It has been observed that sand contains approximately $\frac{1}{3}$ voids. So typical mortar mix specifications of $1:\frac{1}{2}:4\frac{1}{2}$, 1:1:6 or 1:2:9 etc. based on the loose volume have been evolved.

Keeping in view these factors, the mortar used throughout the investigation was 1:1:6(cement: lime:sand) by volume. Ordinary Portland cement and hydrated lime were used. The sieve analysis of the local sand carried out in accordance with IS: 2116-1980 is given in the Appendix B. To eliminate the variations in material bulk density, the weights of the measured volumes were taken and there after batching by weight was made. The ingredients for each batch were mixed dry until a uniform colour of the mix was obtained. A measured quantity of water was added to produce a flow of 115% as per Code for good workability and water retaintivity.

3.3.1 Properties of Mortar

The uniaxial compressive strength, elastic modulus, and Poisson's ratio are determined. Due to a lack of facilities, triaxial compressive strength could not be determined.

(a) Compressive Strength: Compressive strength of mortar was determined by two procedures:(i) Codal procedure and (ii) increase in aspect ratio.

Indian Standard Code Procedure: The compressive strength of mortar was determined as per the procedure laid down in IS:2250-1981. The tests were carried out on 5cm cube specimens. Nine specimens were made as per the procedure laid down in the Code and cured for 28 days before testing. The average **compressive strength** was found to be **5.70MPa** with a **coefficient of variation of 6.5 percent**. The brittle failure of mortar, like that in brick, was prevented and an increase in compressive strength was observed due to the restraining effect of the platens of the testing machine as the aspect ratio of the specimens was 1. The restraining effect was minimized by using the specimens of increased

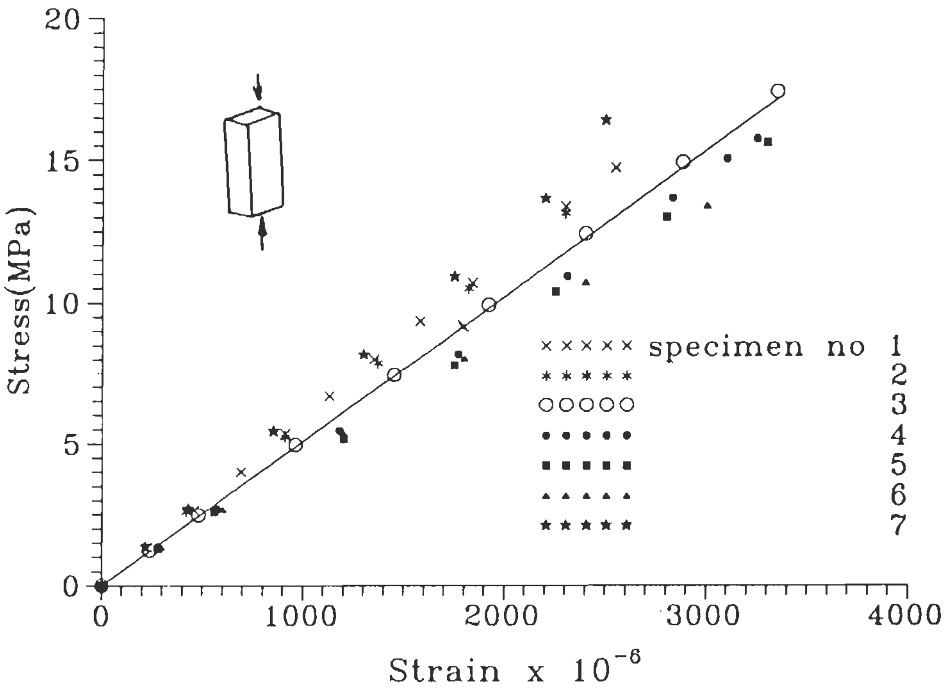


Fig.3.5 Stress Strain Curve for Bricks Subjected to Uniaxial Compression

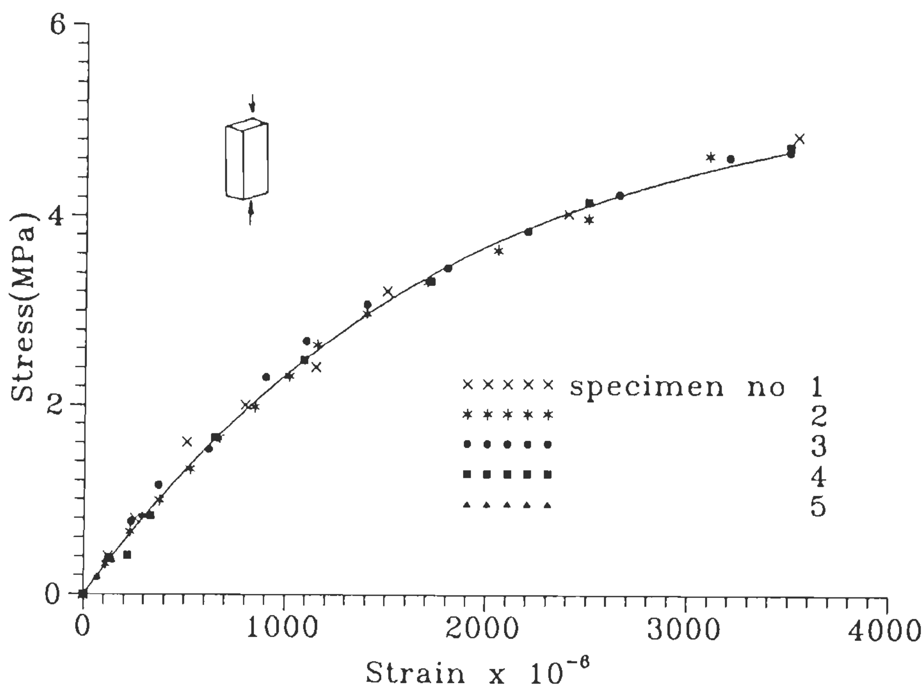


Fig.3.6 Stress-Strain Curve for Mortar subjected to uniaxial compression

aspect ratio of the order of 2. However, the Codal procedure was used for quality control throughout the test programme.

Method Based on Increase in Aspect Ratio: To determine more accurately, the compressive strength, tests were performed on mortar prisms, 5cm x 5cm x 10cm long. Nine specimens were made using 1:1:6 mortar and tests were performed after curing for 28 days. The average compressive strength of the mortar prism was 4.84MPa with a coefficient of variation 6.3 percent.

(b) Elastic Properties: The same mortar prisms were used to determine the elastic properties of mortar. Two bakelite based strain gauges(10mm), parallel and normal to the applied compressive load, were fixed on one side of the prism. A dial gauge was mounted to measure the deformation of the specimen. This was used to check the strain measurements. Strains were measured during the compressive test. Strains after the ultimate stress could not be measured. The stress-strain curve was drawn using average strains at successive stress levels(Fig.3.6).

3.4 Brick Masonry: Strength and elastic properties of brick masonry were also determined. In deriving these properties, brick masonry was assumed to be continuum possessing average properties. This allows the material properties to be used in the analysis at macro level. For the nonlinear analysis of masonry structures such as shear walls, infill frames etc. which are in a biaxial state of stress, yield and failure criterion in terms of biaxial stresses must be determined. In view of the difficulty in carrying out biaxial tests, the biaxial test data reported by Page(1981, 1983) was used to evaluate parameters of the yield and failure criterion.

In this section, the details of investigations for strength and elastic properties, determined from tests on brick masonry pier, are discussed. In addition to these tests, the uniaxial compression tests on 5 bricks high masonry prisms were also carried out. The Stress-strain relationship was also determined and compared with that derived from tests on masonry pier. The data obtained from micro mechanics formulation are used to collate with the experimental results.

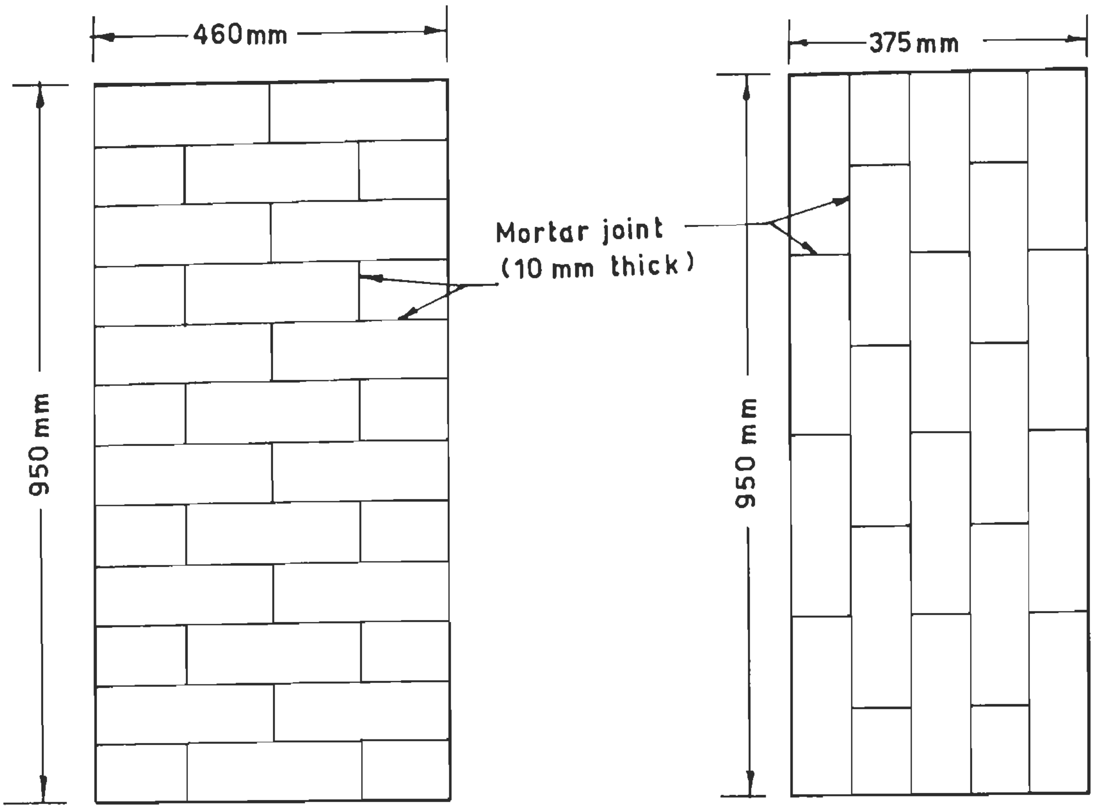
3.4.1 Compressive Strength and Elastic Properties

Brick Masonry Piers: The elastic properties and compressive strength of masonry piers were determined for the two distinct directions of loading, normal and parallel to the bed joint. The dimensions of piers so tested are shown in Fig.3.7. A set of 3 piers were tested for each loading case. The piers were constructed in stretcher bond with 10mm thick 1:1:6 mortar joints. The bricks were soaked in water up to 75% of w+ater absorption as recommended by Sinha(1967), to obtain maximum bond strength. All the piers were cured for 21 days by covering with wet gunny bags and then left uncovered for drying in the natural conditions. Tests were carried out after 28 days. To ensure a uniform contact between the bottom surface and the loading girder, a thin layer of sand was laid on the girder before placing the pier in position. A thin layer (3mm) of gypsum was also applied on the top surface of the pier before placing the joist in position. The Gypsum layer is allowed to set for two hours before loading the piers.

The strains were measured in the directions normal and parallel to the bed joints using a Huggenberger extensometer over a gauge length of 200mm(8"). To accomplish this, eight brass studs were fixed in the central portion of the front face of the piers. Dial gauges were also used to measure the overall deformation both in vertical and lateral directions. The dial gauges which measure vertical deformation help to check uneven distribution of loading. The loading arrangement and the instrumentation details are shown in Fig.3.8.

Before taking the initial readings of the extensometer and dial gauges, a 2 tonnes load (4 kg/cm^2) was applied on the piers and then released. The load was increased by 2 tonnes in each step till failure of the pier. A typical failure of piers is shown in Fig.3.9. It was observed that for compression load normal to bed joints, the failure occured by vertical tensile splitting of the bricks. For uniaxial compression parallel to the bed joint, the failure occured initially by splitting of the vertical bed joints. On further loading, the pier was split into columns of brick masonry, resulting in a failure of the pier due to the collapse of brick masonry columns. Although the resulting columns failed at higher load, the pier is regarded as having failed at the initial splitting of the bed joints.

The average stress-strain curve obtained for loading normal to the bed joints is shown in Fig.3.10. The stress-strain curve is nonlinear in nature. Due to the method of strain measurement and the loading device, strains after maximum loading could not be recorded. However, it is expected that it would be similar to that obtained by Powell and Hodkinson (1976), who obtained complete stress-strain curves for brick masonry piers(Fig.2.11). The strength and elastic properties are summarized in Table 3.4.



(a) Load Normal to Bed Joint (12 Courses High)

(b) Load Paralell to Bed Joint (4 Courses High)

Note :

- Brick Dimension - 230 x 110 x 70 mm
- Panel Thickness - 110 mm

Fig.3.7 Brick Masonry Specimens for Uniaxial Compression Test

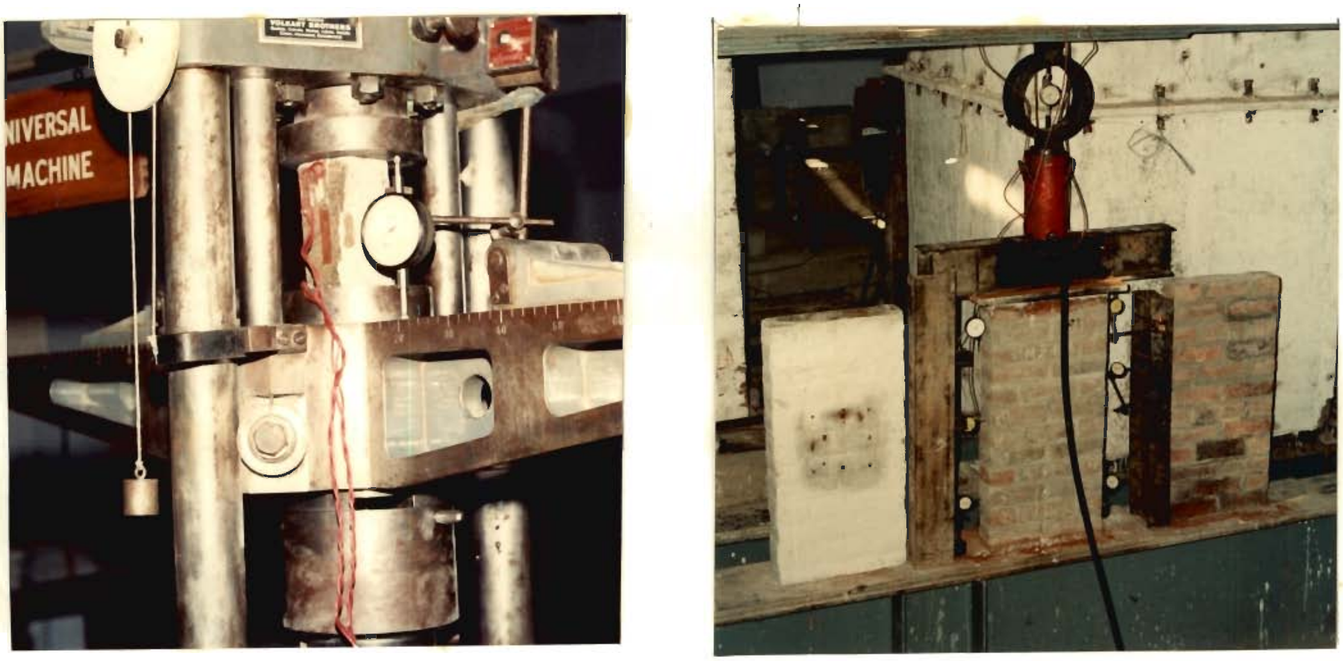


Fig.3.8 Setup for Compressive Test for Mortar and Brick Masonry

Fig.3.9 Failure of Brick Masomry Piers subjected uniaxial Loading

Brick Masonry Prism: Many countries adopted compressive strength of brick masonry prism as basic strength to determine the compressive strength of masonry, because of the difficulty in performing compression tests on masonry piers. Prism strength can also be used for quality control and to compare the strength of different types of brick masonry.

In the present investigation, the compressive strength tests were performed on 5 bricks high stack-bonded prisms, made of the same type of bricks and 10mm thick 1:1:6(cement:lime:sand) mortar. The stress-strain relation (Fig.3.11) is derived to compare it with that obtained from masonry pier tests. The variation in elastic modulus is not significant. This confirms the observation of Sahlin(1971).

The first crack occured in the central region of the prism due to by tensile splitting of brick. On further loading, cracks propagated vertically towards the edge (Fig.3.12). The strength and elastic properties of masonry prism is summaraized in Table 3.4. The details of the test results are given in Appendix-C. The masonry wall strength in terms of prism strength is given by the expression

$$f_c = 0.75\, f_c^p \tag{3.1}$$

where f_c and f_c^p are the compressive strengths of masonry wall and prism respectively.

Table: 3.4 Strength and Elastic Properties of Brick Masonry

Description of Item	Compressive Strength (MPa)			Elastic Modulus (MPa)			Poiss. Ratio
	Mean	Std. Dev.	Coeff. of Var.	Mean	Std. Dev.	Coeff. of Var.	
Brick Masonry							
a. Prism	7.01	0.92	13%	4541.65	195.60	4.30%	0.16
b. Wall							
Load Normal to Bed Joint	5.31	0.30	5.72%	4465.25	272.50	6.10%	0.17
Load Parallel to Bed Joint	3.07	0.32	10.48%	4371.72	385.82	8.82%	0.19

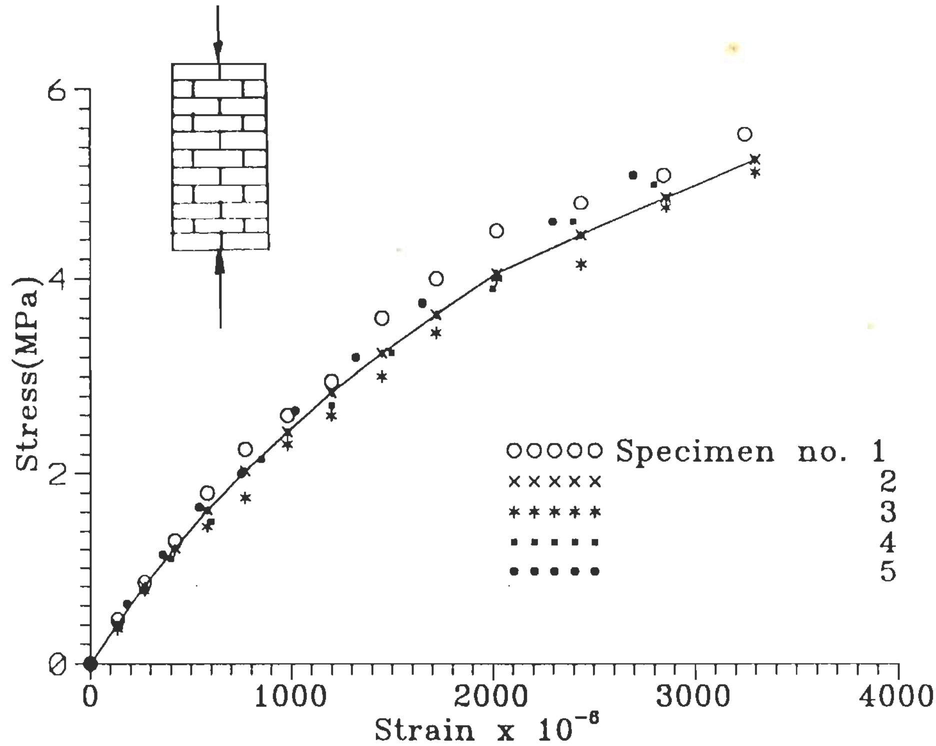


Fig.3.10 Stress-Strain Curve for Brick Masonry Piers under Uniaxial Compressive Loads

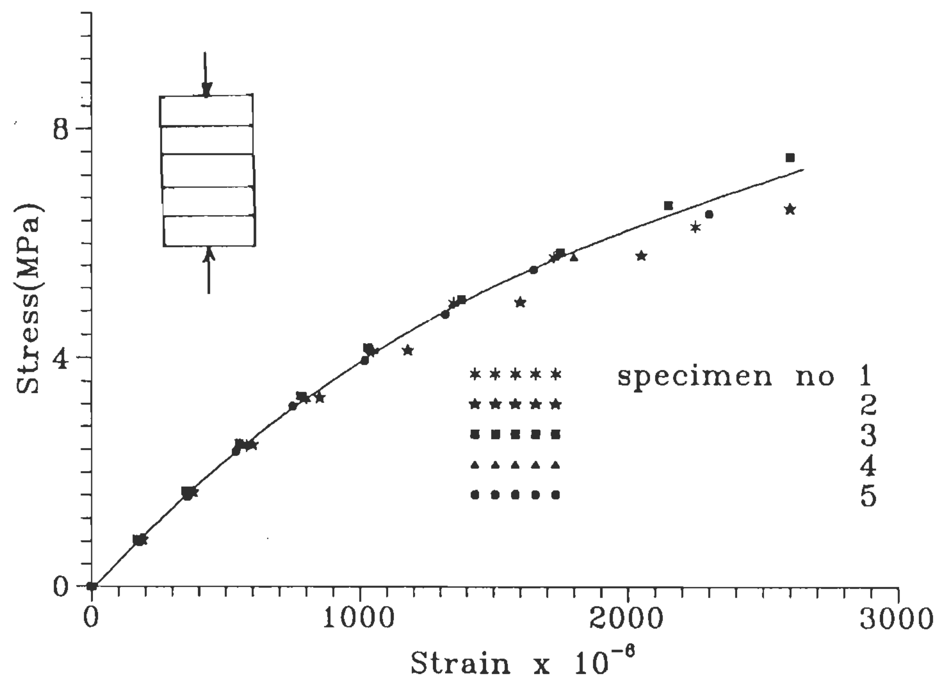


Fig.3.11 Stress-Strain Curve for Brick Masonry Prism under Uniaxial Compressive Loading

(a) Setup for Compression Test for Masonry Prism

(b) Typical Failure of Masonry Prism under Uniaxial Load

Fig. 3.12 Stress-Strain Curve for Brick Masonry Prism under Uniaxial Compressive Loading

Appendix A

Compressive Strength of Solid Clay Bricks

Specimen No.	Method of test	Compressive Strebgth(MPa)	Remarks
S-1	IS:3495(part I)	22.015	Mean strength = 25.26 MPa Std. dev. = 1.547 Coeff.of var. = 6.17%
S-2		26.950	
S-3		24.028	
S-4		26.148	
S-5		27.220	
S-6		24.330	
S-7		25.040	
S-8		24.320	
S-9		25.748	
S-10		26.798	
P-1	Proposed method	16.76	Mean strength = 15.45 MPa Std..dev. = 1.68 Coeff.of var. = 10.97%
P-2		17.03	
P-3		14.58	
P-4		12.66	
P-5		17.72	
P-6		15.33	
P-7		15.98	
P-8		13.00	
P-9		15..64	
P-10		15.84	

Grading of Sand

Seivesize (μm)	Percentage Passing *			Fineness
	IS: 2116(1980)		Local Sand	
	Upper	Lower		
4750	100	100	100.0	1.97
2360	100	90	100.0	
1180	100	70	91.3	
600	100	40	68.5	
300	70	5	33.0	
150	15	0	10.5	

* Allowable limit specified by IS: 2116-1980, mortar for unreinforced brick masonry.

Notes:

- Mortar Mix - 1:1:6 (Cement:Lime:Sand)
- Flow before suction 115%
- Typical mix properties are

  Cement - 240 gms

  Lime - 100 gms

  Sand - 2340 gms

  Water - 510 cc

  (weights of volume batched ingradients)

Appendix C

Compressive Strength of Brick Masonry Prism

Specimen No.	Dimension (mm)	Compressive Strebgth(MPa)	Remarks
MP-1	225x108x372.5	6.296	
MP-2	224x108x376	6.05	
MP-3	222x108x376	8.341	
MP-4	228x108x375	4.87*	Mean strength
MP-5	225x110x379	6.569	= 7.07MPa
MP-6	222x105x376	8.58	Std. dev. = 0.92
MP-7	225x108x385	6.584	
MP-8	230x110x380	6.64	Coeff.of var. = 13%
MP-9	225x113x385	6.804	
MP-10	220x110x385	7.79	

* not considered

Compressive Strength of Brick Masonry Wall

Specimen No.	Loading Direction	Dimension (mm)	Compressive Strength(MPa)	Remark
WN-1		475x110x945	5.72	
WN-2		460x107x930	5.26	Mean = 5.27 MPa
WN-3	Load Normal	463x104x936	5.13	Std. Dev. =0.28
WN-4	to Bed Joint	467x108x940	5.38	Coeff. of Var.
WN-5		454x108x865	4.88	= 5.27%
WP-1		376x110x960	2.90	
WP-2		370x108x950	3.45	Mean = 3.07 MPa
WP-3	Load Paralle	350x104x915	3.02	Std. Dev. =0.32
WP-4	to Bed Joint	375x110x930	3.39	Coeff. of Var.
WP-5		365x106x940	2.58	= 10.48%

CHAPTER-4

PROPERTIES OF BRICK MASONRY: A MICROMECHANICS APPROACH

4.1 Introduction

Brick masonry is a two phase composite material. Like all other composite materials it can also be studied from two points of view: **micromechanics** and **macromechanics**. Micromechanics is the study of composite behaviour wherein the interaction of the constituent materials is examined in a microscopic scale. While macromechanics is the study of composite behaviour wherein the material behaviour is presumed homogeneous and the effects of the constituents are detected only as **averaged or equivalent** properties of the composite.

In a macromechanics study the equivalent properties are used for the design and analysis and can be determined experimentally but these would be applicable only for the material so tested. Micromechanics investigations, on the other hand, are carried out in order to gain an appreciation of the properties of the constituents and their proportion to achieve required strength and stiffness. Use of both the concepts of macromechanics and micromechanics allows the tailoring of masonry to meet the particular requirement with little waste of material capacity.

In this chapter, the derivation of uniaxial strength criterion and elastic properties of brick masonry based on the micromechanics approach has been presented. Firstly the analytical methods developed till date for the determination of strength and elastic properties are reviewed.

4.2 Review of Brick Masonry Properties: Analytical Methods

In early sixties, the effects of various bedding materials on brick masonry strength have been investigated. These have encouraged investigators to study the behaviour of brick masonry for the estimation of strength and elastic properties. The brick masonry prism strength is considered as the basic strength for the determination of strength of the brick masonry walls under compression. Thus most of the early works were devoted to develop the strength theories of brick masonry based on the prism strength. These have included, (a) strength criteria based on elastic analysis consisting of the compatibility of the lateral strains and the equilibrium of the lateral forces in the brick and the mortar joint with the biaxial tension-compression strength criterion of the brick [Francis *et al.*(1970), Shrive and Jessop(1980)] and (b) the strength approach consisting of the equilibrium of the lateral forces in the brick and the mortar joint with biaxial tension-compression strength criterion of the brick and the triaxial compression strength criterion of the mortar [Hilsdorf(1969), Khoo and Hendry(1972)].

Sahlin(1971) considered brick masonry as a composite material and employed the "rule of mixture" to predict the elastic properties and compressive strength of the brick masonry prism without complete success. Andam and Pande(1986) determined uniaxial and biaxial strength of reinforced brick masonry based on multi-laminate rheological analogy with the elasto-viscoplastic material model. The analytical data were not compared with the experimental results. Based on the micromechanics approach, Liang *et al.*(1990) derived elastic properties of the brick masonry in terms of the elastic properties of the bricks and the mortar. Having obtained the elastic properties and the stress distribution in the brick masonry, the stresses in the bricks and mortar joints are also calculated. The analytical data do not agree well with the experimental results.

4.3 Proposed Micromechanical Brick Masonry Model

The stiffness and strength characteristics are necessary for the design and analysis of brick masonry structures. In order to determine these properties it is required that the brick masonry should be as uniform as possible so as to decrease the number of weaker areas.

The uniformity in material properties may be obtained if both the mortar and the bricks have uniform stiffness and strength characteristics. Therefore, the mortar should be uniformly mixed and completely cured, and the bricks should be, as far as possible, free from flaws. Secondly a high degree of uniformity can be achieved by careful construction of brick masonry. The defects in workmanship should be minimised as far as possible so that a high degree of bonding is produced between the mortar and the bricks.

These ideals lead to the following assumptions required in developing micromechanical model of brick masonry.

- Both bricks and mortar are homogeneous and isotropic materials.
- Bricks are uniform, regularly spaced and perfectly aligned.
- The bricks are perfectly elastic before cracking.
- The mortar is elasto-perfectly plastic material.
- The brick masonry is macroscopically homogeneous and perfect bonding exists between the bricks and the mortar joints.

4.3.1 Compressive Strength of Brick Masonry Prism

When the brick masonry prism is subjected to the compressive loads, the lateral stresses are induced by shear mechanism between the bricks and the mortar joints. The difference in strains due to the difference in elastic moduli and Poisson's ratios of the bricks and the mortar induces a shear stress along the brick-mortar interface (Fig. 4.1). The variation of shear stress along the interface is small. So it is assumed to be constant.

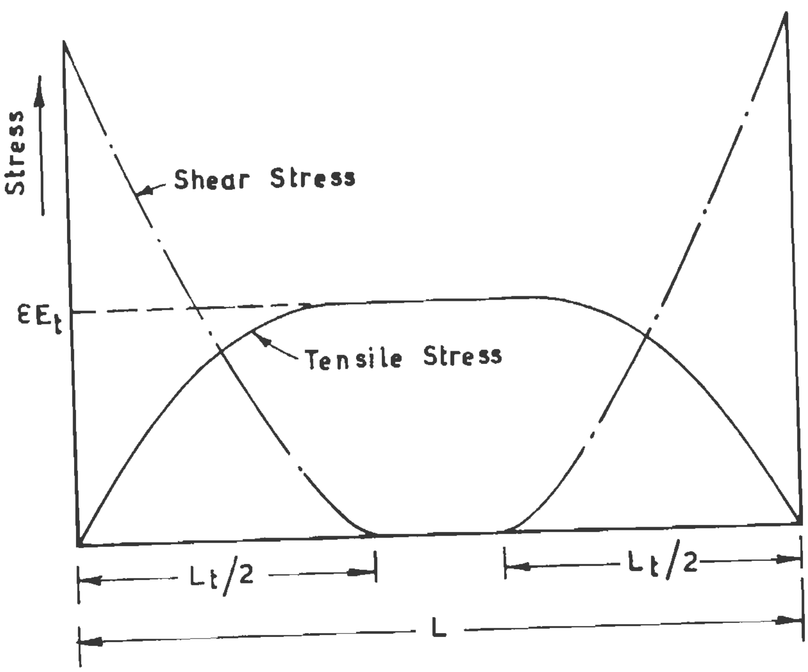


Fig. 4.1 Theoretical variations of tensile and shear stresses along the length of brick, the length being greater than critical length

The mortar used in brick masonry is normally weaker than the bricks. As a result the lateral tensile stresses are developed in the brick while lateral compression stresses are induced in the mortar. Thus the brick is in axial compression-lateral tension and the mortar is in a triaxial compression state of stress. Considering the equilibrium of lateral forces acting on an element of a brick(Fig.4.2)

$$b\,t^b\left(\sigma_t^b + \frac{d\sigma_t^b}{dx}\,dx\right) - b\,t^b\,\sigma_t^b - 2\,b\,\tau\,dx = 0 \tag{4.1}$$

which on simplification gives

$$\frac{d\sigma_t^b}{dx} = \frac{2\tau}{t^b} \tag{4.2}$$

where σ_t^b = tensile stress in the brick

τ = shear stress at the brick-mortar interface assumed to be constant and can be written as $\tau = \tau_o + \mu\,\sigma_y$

τ_o = shear stress without precompression

t^b = thickness of the brick

b = width of the brick

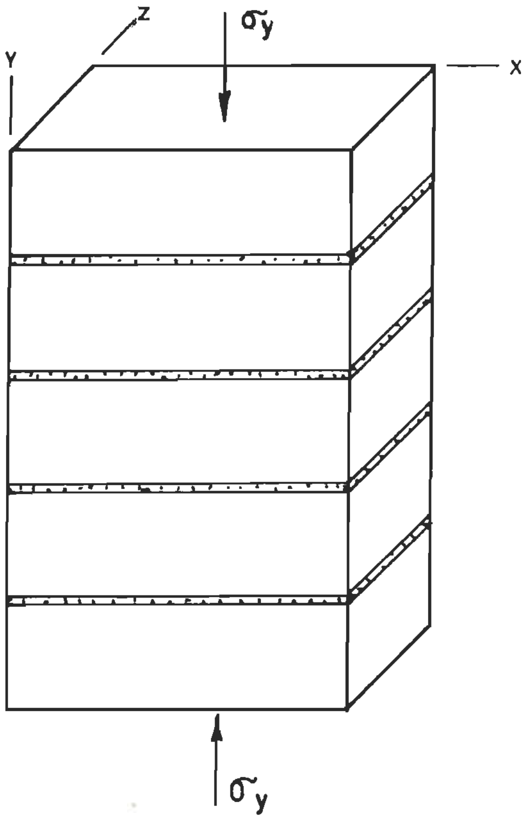
μ = coefficient of friction

At $x = 0$, $\sigma_t^b = 0$ and the integration of the Eqn.(4.2) with respect to x gives

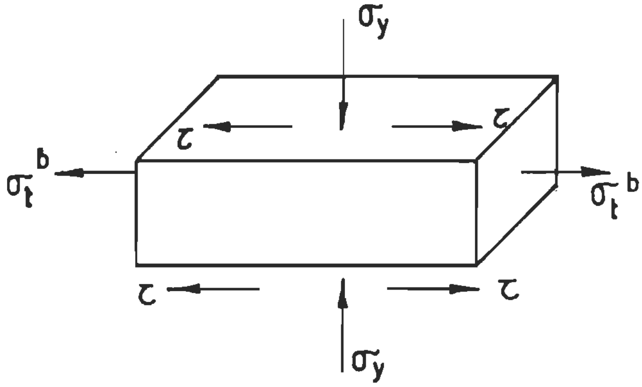
$$\sigma_t^b = \frac{2\tau}{t^b}\,x \tag{4.3}$$

The tensile stress in the brick is not uniform. As x increases, it builds up linearly from zero to the maximum value at $x = L_t/2$. The maximum stress that can be achieved at a given load is

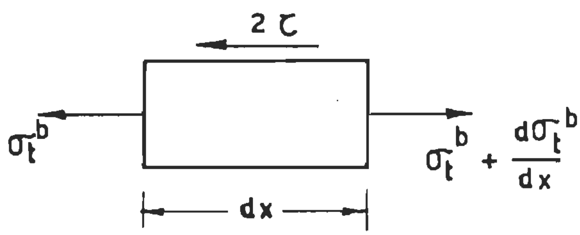
$$\sigma_{tm}^b = \frac{\tau\,L_t}{t^b} \tag{4.4}$$



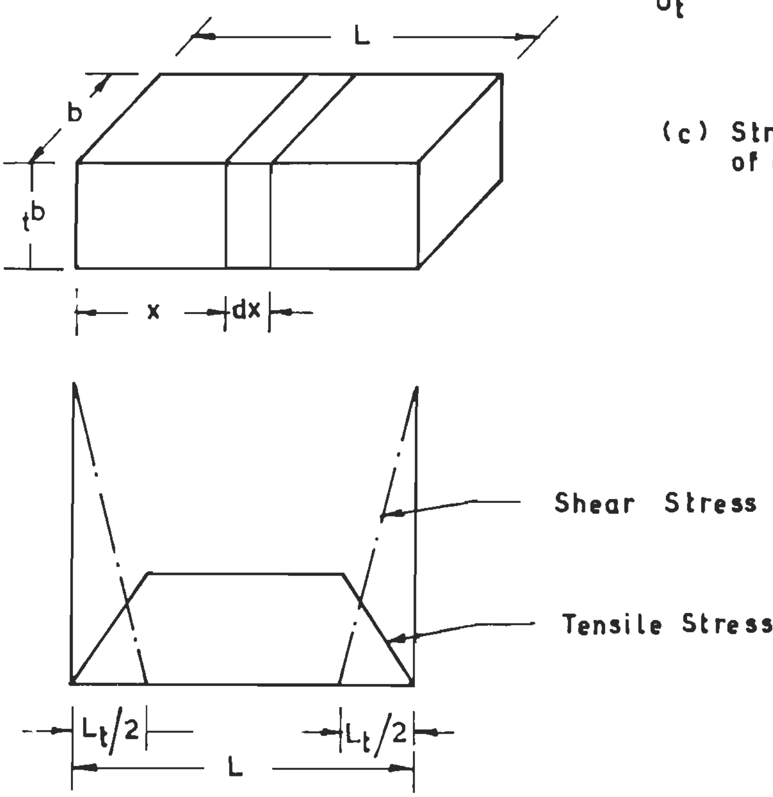
(a) Brick Masonry Prism



(b) Stresses in the Brick due to Compressive Loading



(c) Stresses acting on an Element of a Brick in the x direction



(d) Variation of Tensile and Shear Stresses along the Length of the Brick

Fig.4.2 Brick Masonry Prism subjected to Uniaxial Load where L_t = critical length of the brick to attain maximum tensile stress

The average stress is given by

$$\sigma_{ta} = \sigma^{b}_{tm} \left[1 - \frac{\sigma^{b}_{tm} t^{b}}{2 \tau L} \right] \tag{4.5}$$

The maximum lateral strain in the brick is given by

$$\varepsilon^{b}_{tm} = \frac{1}{E^{b}} \left[\sigma^{b}_{tm} + \nu^{b} \sigma_{y} \right] \tag{4.6}$$

Similarly in the mortar joints

$$\varepsilon^{m}_{cm} = \frac{1}{E^{m}} \left[- \sigma^{m}_{cm} + \nu^{m} \sigma_{y} \right] \tag{4.7}$$

where E^{b} and E^{m} are the elastic moduli of the brick and mortar respectively and ν^{b} and ν^{m} the corresponding Poisson's ratios. σ^{m}_{cm} is the maximum lateral stress in the mortar.

Considering the equilibrium of the lateral forces in the brick and the mortar joint

$$\sigma^{m}_{ca} = \alpha \, \sigma^{b}_{ta} \tag{4.8}$$

where $\alpha = t^{b}/t^{m}$, and, σ^{b}_{ta} and σ^{m}_{ca} are the average lateral stresses in the brick and the mortar respectively. As the lateral strains in the brick and the mortar are same, Eqns.(4.6) to (4.8) give

$$\sigma_{ta}^{b} = \sigma_y \, P\left[\, 1 - \sigma_y \, p \, Q \right] \tag{4.9}$$

where
$$P = \frac{\left[\beta \, \nu^{m} - \nu^{b}\right]}{1 + \alpha \, \beta}$$

$$Q = \frac{t^{b}}{2 \, \tau \, L}$$

and
$$\beta = \frac{E^{b}}{E^{m}}$$

Khoo and Hendry(1973) derived experimentally the biaxial tensile-compressive strength criterion given by

$$\left[\frac{\sigma_t}{f_t^{b}}\right] = 0.9968 - 2.0264\left[\frac{\sigma_c}{f_c^{b}}\right] + 1.2781\left[\frac{\sigma_c}{f_c^{b}}\right]^2 - 0.2487\left[\frac{\sigma_c}{f_c^{b}}\right]^3 \tag{4.10}$$

where f_t^b and f_c^b are the uniaxial tensile and compressive strength of the brick respectively. Putting the value of $\sigma_t = \sigma_{ta}^b$ and $\sigma_c = \sigma_y$ in the Eqn.(4.10), the value of σ_y is determined.

Similarly lateral stress in the mortar is determined from Eqn.(4.7) and (4.8). Using Von Mises strength criterion the value of σ_y is determined. The lesser of the two values is the strength of the brick masonry prism.

4.3.2 Elastic Properties of Brick Masonry

To develop a mathematical model for the accurate determination of elastic properties of brick masonry, the detailed features of the microstructure as well as the nature of stress-strain field need to be explicitly specified. This problem can be avoided by directing attention to upper and lower bounds of the elastic properties of the composite material. The results from the bounding approach can serve as guides to material behaviour provided the upper and lower bounds are sufficiently close together so as to bracket the true behaviour within the experimental error.

Explicit relationship for the bound can be obtained through the application of Saint-Venant's principle by proposing statically equivalent trial strain and stress fields for the use in the strain and complementary energy computations. The selection of the appropriate trial stress and strain fields is the central issue of the bounding approach. The earliest upper bound of elastic properties of composite material is derived by using constant strain field in the representative volume element(RVE). This bound is identical to the classical Voigt(1910) estimate given by

$$E_u^* = V^b E^b + V^m E^m \tag{4.11}$$

Similarly, the lower bound is obtained by employing constant stress field in the RVE. This bound is identical to the classical Reuss(1929) estimate as follows

$$E_l^* = \frac{E^m E^b}{V^b E^m + V^m E^b} \tag{4.12}$$

where V is the volume fraction and the superscripts, m and b refer to mortar and brick respectively and the subscripts u, and l indicate the upper and lower bounds of the brick masonry, respectively. Whitney and McCllough (1981) summarised the derivation.

Closer bounds have been proposed by many investigators [Hashin and Shtrikman(1963), Hill(1963), Walpole(1966)]. Further improved bounds have also been proposed assuming the phase geometry of packing [Hashin(1962), Hashin and Rosen(1964)]. These improved bounds are suitable for the composite materials whose elastic properties can be determined precisely. Besides this the draw back of the improved bounds is the need to specify the phase geometry packing. For some composite materials the phase geometry of packing is difficult to describe.

For the masonry composite, the closer bounds can not enclose scatter results. Indeed the best bounds are the early ones presented by Voigt(1910) and Reuss(1929), although for large values of the ratio of elastic modulus of brick and mortar, these bounds are not close enough to give any useful estimate of the effective elastic properties within the margin of experimental error. The effective elastic properties of the masonry can be computed by using an approach based on the series expansion in terms of the difference between the upper and lower bounds of elastic properties.

From the Eqns.(4.11) and (4.12) it is observed that if the volume of mortar is zero the elastic properties will be same as those of the bricks. But it is observed that the elastic properties of the prism without mortar joint is less than those of bricks. To take care of the above observation, Eqns (4.11) and (4.12) are modified as follows.

$$E_u = E^m V^m + k E^b V^b \tag{4.13}$$

and

$$E_l = \frac{k E^m E^b}{E^m + V^m (k E^b - E^m)} \tag{4.14}$$

The masonry behaviour tends towards the upper bound, E_u, as the volume fraction of mortar, V^m, approaches zero; on the other hand, as V^m approaches unity, it will be closer to the lower bound, E_l. Thus the difference between the upper and lower bounds serves as a measure of influence of the phase geometry and the properties of the components. So it is reasonable to assume that the effective elastic properties may be modelled as a series expansion in terms of the difference between the upper and lower bounds [Whitney and McCllough(1981), Phani(1993)]. Retaining only the linear terms effective elastic modulus, E, is given by

$$E = E_u - \psi (V^m) (E_u - E_l) + \ldots \tag{4.15}$$

where $\psi(V^m)$, is a proportionality factor. Since the effective elastic properties must lie between the extremes of the upper and lower bounds, the value of $\psi(V^m)$ must satisfy the condition $0 < \psi(V^m) < 1$. This condition is assured by requiring that $\psi(0) = 0$ and $\psi(1) = 1$. Assuming that $\psi(V^m)$ can be adequately approximated by a truncated series expansion in V^m given by

$$\varphi(V^m) = \xi + \eta\, V^m + \gamma\, V_m^2 \tag{4.16}$$

The above two conditions give

$$\psi(o) = \xi = 0 \tag{4.17}$$

$$\psi(1) = \xi + \eta + \gamma = 1 \tag{4.18}$$

Thus

$$\psi\,(V^m) = (\,1-\gamma)\,(V^m) + \gamma\,(V^m)^2 \tag{4.19}$$

Substituting Eqn (4.19) in Eqn. (4.15)

$$E = (1 - V^m)\,(1 + \gamma\, V^m)\,E_u + V^m\,(1 - \gamma + \gamma\, V^m)\,E_l \tag{4.20}$$

The parameter γ is evaluated by using the phase geometry of the constituent materials of brick masonry.

The thickness of the mortar joint is normally 10mm. Let the volume fraction of mortar be V_c^m for 10mm thickness of mortar joint. In the neighborhood of $V^m = V_c^m$, the behaviour of masonry tends toward the limits of the upper bound, while in the vicinity of $V^m = 1- V_c^m$, the behaviour will tend toward the limits of lower bound. Thus

$$\Big[\,E\,\Big]_{V^m = V_c^m} = \Big[\,E_l\,\Big]_{V^m = V_c^m} + \epsilon \tag{4.21}$$

$$\Big[\,E\,\Big]_{V^m=1-V_c^m} = \Big[\,E_l\,\Big]_{V^m=1-V_c^m} - \epsilon \tag{4.22}$$

Using Eqns. (4.13), (4.14) and (4.20), these two equations can be solved to obtain the value of γ. The value of γ is given by

$$\gamma = \frac{\left(1 - 2V_c^m\right)\left(E^b - E^m\right)}{\left(1 - V_c^m\right)\left(E^b + E^m\right)} \tag{4.23}$$

Once γ is known the value of elastic properties of brick masonry can be determined from Eqn. (4.20).

The volume fraction for brick and mortar can be calculated by the following expression.

In case of masonry prism,

the volume fraction of brick, $V^b = \dfrac{t^b}{t^m + t^b}$ (4.24)

and

volume fraction of mortar, $V^m = \dfrac{t^m}{t^m + t^b}$ (4.25)

In case of masonry wall,

the volume fraction of brick, $V^b = \dfrac{L\, t^b}{(L + t^m)\,(t^m + t^b)}$ (4.26)

and

volume fraction of mortar $V^m = 1 - V^b$ (4.27)

Poisson's ratio of brick masonry can be calculated by Eqn. (4.20) replacing elastic modulus by Poisson's ratio. Eqn. (4.11) can be used as the difference of Poisson's ratio of the two material is small. Thus Poisson's ratio of masonry can be calculated as follows

$$\nu = V^m \nu^m + V^b \nu^b \tag{4.28}$$

4.4 Distribution of Stresses in the Masonry Constituents

The masonry structures such as shear walls, infilled frames etc., are analysed based on macro-level approach. Once the stresses at a point in the masonry are known, the stresses in the masonry unit, head and bed mortar joint can be obtained. The stresses in the masonry units, and mortar joint are given below.

Brick Masonry Prism: Stresses in the brick and the mortar joint of the brick masonry prism in the matrix form are given as follows.

$$\underline{\sigma}^{i} = \underline{\alpha}^{i}\ \underline{\sigma} \qquad (4.29)$$

or

$$\begin{Bmatrix} \sigma_{p}^{i} \\ \sigma_{n}^{i} \\ \tau_{np}^{i} \end{Bmatrix} = \begin{bmatrix} \frac{E^{i}}{E} & \frac{\nu^{i}E - \nu E^{i}}{E} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \sigma_{p} \\ \sigma_{n} \\ \tau_{np} \end{Bmatrix} \qquad (4.30)$$

where $\underline{\sigma}^{i}$ is the stress vector of the constituents, brick and mortar joint. $\underline{\sigma}$ is the stress vector for the brick masonry prism.

Brick Masonry Wall: Stresses in the brick and the mortar joint of the brick masonry wall in the matrix form are given as follows.

Stresses in the Bed Joint

$$\begin{Bmatrix} \sigma_{ph}^{m} \\ \sigma_{nh}^{m} \\ \tau_{nph}^{m} \end{Bmatrix} = \begin{bmatrix} \frac{E^{m}}{E} & \frac{\nu^{m}E - \nu E^{m}}{E} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \sigma_{p} \\ \sigma_{n} \\ \tau_{np} \end{Bmatrix} \qquad (4.31)$$

Stresses in the Brick

$$\begin{bmatrix} \sigma_p^b \\ \sigma_n^b \\ \tau_{np}^b \end{bmatrix} = \begin{bmatrix} \frac{E^b}{E} & \frac{\nu^b E - \nu E^b}{E} & 0 \\ \frac{E^b}{E} & \frac{\nu^b E - \nu E^b}{E} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sigma_p \\ \sigma_n \\ \tau_{np} \end{bmatrix} \qquad (4.32)$$

Stresses in the Vertical Joint

$$\begin{bmatrix} \sigma_{pv}^m \\ \sigma_{nv}^m \\ \tau_{npv}^m \end{bmatrix} = \begin{bmatrix} \frac{E^b}{E} & \frac{\nu^b E - \nu E^b}{E} & 0 \\ \frac{E^m}{E} & \frac{\nu^m E - \nu E^m}{E} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sigma_p \\ \sigma_n \\ \tau_{np} \end{bmatrix} \qquad (4.33)$$

4.5 Constitutive Equations for Brick Masonry

The constitutive equations of brick masonry are derived from brick and mortar properties by applying micromechanics theory. The bricks are assumed to be linear elastic material. The mortar can behave as a nonlinear elastic material.

The stress field in a representative volume element(RVE), V, of brick masonry is denoted as $\underline{\sigma}_v$. The volume average stresses $\underline{\sigma}$ of the brick masonry are defined by,

$$\underline{\sigma} = \frac{1}{V} \int_V \underline{\sigma}_v \, dv = \frac{1}{V} \left[\int_{V^m} \underline{\sigma}_v \, dv + \int_{V^b} \underline{\sigma}_v \, dv \right] \qquad (4.34)$$

where V^m and V^b are the mortar and brick volumes, respectively. The volume average stresses $\underline{\sigma}^m$ and $\underline{\sigma}^b$ of the mortar and bricks, respectively are defined by,

$$\underline{\sigma}^{m} = \frac{1}{V^{m}} \int_{V^{m}} \underline{\sigma}_{v} \, dv \tag{4.35}$$

$$\underline{\sigma}^{b} = \frac{1}{V^{b}} \int_{V^{b}} \underline{\sigma}_{v} \, dv \tag{4.36}$$

Similar to the stresses, by introducing ϵ^{m} and ϵ^{b} as the volume average strains of the mortar and bricks, respectively, the average stresses $\underline{\sigma}$ and the strain $\underline{\epsilon}$ of brick masonry can be expressed as

$$\underline{\sigma} = V^{m} \underline{\sigma}^{m} + V^{b} \underline{\sigma}^{b} \tag{4.37}$$

$$\underline{\epsilon} = V^{m} \underline{\epsilon}^{m} + V^{b} \underline{\epsilon}^{b} \tag{4.38}$$

The superscripts m and b indicate the mortar and the bricks. V^{m} and V^{b} are the volume fractions of the mortar and the bricks respectively.

For the constitutive equations of the constituents, the incremental stress-strain relationship for the mortar is

$$d\underline{\epsilon}^{m} = d\underline{\epsilon}^{me} + d\underline{\epsilon}^{mp} \tag{4.39}$$

$$= \underline{C}^{me} \, d\underline{\sigma}^{m} + d\underline{\epsilon}^{mp}$$

and the linear elastic stress-strain relationship for the bricks

$$d\underline{\epsilon}^{b} = \underline{C}^{be} \, d\underline{\sigma}^{b} \tag{4.40}$$

The superscripts e and p denote the elastic and plastic components. $\underline{C}^{me}$ and $\underline{C}^{be}$ are the elastic compliance matrices of the mortar and the bricks.

For the plane stress case,

$$\underline{C}^{me} = \begin{bmatrix} \dfrac{1}{E^m} & -\dfrac{\nu^m}{E^m} & 0 \\ -\dfrac{\nu^m}{E^m} & \dfrac{1}{E^m} & 0 \\ 0 & 0 & \dfrac{2(1+\nu^m)}{E} \end{bmatrix} \tag{4.41}$$

$$\underline{C}^{be} = \begin{bmatrix} \dfrac{1}{E^b} & -\dfrac{\nu^b}{E^b} & 0 \\ -\dfrac{\nu^b}{E^b} & \dfrac{1}{E^b} & 0 \\ 0 & 0 & \dfrac{2(1+\nu^b)}{E^b} \end{bmatrix} \tag{4.42}$$

In addition to the above constitutive equations, micromechanics theory is employed to relate the bricks and material stresses and strains. Based on this theory, the strains of the bricks and the mortar along the direction parallel to bed joint are given by

$$\epsilon_p^m = \epsilon_p^b \tag{4.43}$$

The mortar stress σ_n^m in the direction normal to bed joint, n, can be related to brick stress σ_n^b as follows.

$$\sigma_n^m = \eta_n \, \sigma_n^b + \eta_{np} \, \sigma_p^b \tag{4.44}$$

Where η_n and η_{np} are stress partitioning factors measuring the fraction of the brick stresses, σ_n^b and σ_p^b, respectively. Further, the shear stress of mortar, σ_s^m, is related to the shear stress, σ_s^b, and can be expressed as

$$\sigma_s^m = \eta_s \, \sigma_s^b \tag{4.45}$$

where the subscript, s, denotes the shear. η_s is the stress partitioning factor measuring the fraction of the brick shear stress, σ_s^b, η_n and η_s can be determined from the experimental data. However, η_{np} can be determined from the symmetry condition of the composite elastic compliance, $\underline{C}^e$, which will be defined later. Eqns.(4.43 - 4.45) can be written in the matrix form

$$\underline{R}^m \, d\underline{\sigma}^m = \underline{R}^b \, d\underline{\sigma}^b - \underline{S} \, d\underline{\epsilon}^{mp} \tag{4.46}$$

where, $\underline{R}^m$, $\underline{R}^b$ and $\underline{S}$ are defined as,

$$\underline{R}^m = \begin{bmatrix} 1 & -\nu^m & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4.47}$$

$$\underline{R}^b = \begin{bmatrix} \dfrac{E^m}{E^b} & -\dfrac{E^m \nu^b}{E^b} & 0 \\ \eta_{np} & \eta_n & 0 \\ 0 & 0 & \eta_s \end{bmatrix} \tag{4.48}$$

$$\underline{S} = \begin{bmatrix} E^m & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4.49}$$

The stress-strain relationship of brick masonry can be expressed in an incremental form as

$$d\underline{\epsilon} = \underline{C}^{e} d\underline{\sigma} + d\underline{\epsilon}^{p} \qquad (4.50)$$

where $\underline{C}^{e}$ and $d\underline{\epsilon}^{p}$ are the elastic compliance and the incremental plastic strains of brick masonry. By combining Eqns. (4.37 - 4.40) and (4.46), the expressions for $\underline{C}^{e}$ and $d\underline{\epsilon}^{p}$ in terms of the mortar and brick properties are as follows.

$$\underline{C}^{e} = \frac{\nu^{b}\, \underline{C}^{be}\, (\underline{R}^{b})^{-1} + \nu^{m}\, \underline{C}^{me}\, (\underline{R}^{m})^{-1}}{\nu^{b}\, (\underline{R}^{b})^{-1} + \nu^{m}\, (\underline{R}^{m})^{-1}} \qquad (4.51)$$

$$d\underline{\epsilon}^{p} = \frac{\nu^{m}\, \underline{I} + \nu^{m}\, (\underline{C}^{e} - \underline{C}^{me})}{\underline{R}^{m}}\, d\epsilon^{mp} \qquad (4.52)$$

Where $\underline{I}$ is the identity matrix. From the symmetry condition of $\underline{C}^{e}$ in Eqn. (4.43), namely $\underline{C}^{e}_{np} = \underline{C}^{e}_{pn}$

$$\eta_{np} = \frac{\nu^{m} - \nu^{b}}{\frac{E^{b}}{E^{m}}\left(1 - \frac{E^{m}}{E^{b}}\right) + \nu^{m}\left(\nu^{b} - \nu^{m}\frac{E^{b}}{E^{m}}\right)} \left(1 - \eta_{n}\right) \qquad (4.53)$$

Using the constitutive Eqn. (4.50) the stress $\underline{\sigma}$ of brick masonry can be calculated. $\underline{\epsilon}^{p}$ can be expressed in terms of the mortar plastic strains, $\underline{\epsilon}^{mp}$, which is function of the unknown mortar stresses, $\underline{\sigma}^{m}$. Combining Eqns. (4.37 - 4.40) and (4.46)

$$d\hat{\underline{\epsilon}}^{m} = \hat{\underline{C}}^{me} d\underline{\sigma}^{m} + d\underline{\epsilon}^{mp} \qquad (4.54)$$

Compared with Eqn. (4.39), $d\hat{\underline{\epsilon}}^{m}$ and $\hat{\underline{C}}^{me}$ can be called the "modified" strains and elastic compliance of mortar and can be expressed as

$$d\hat{\underline{\epsilon}}^{m} = \frac{d\underline{\epsilon}}{\nu^{m}\,\underline{I} + \nu^{b}\,\underline{C}^{be}(\underline{R}^{b})^{-1}\,\underline{I}} \tag{4.55}$$

$$\hat{\underline{C}}^{me} = \frac{\nu^{m}\,\underline{C}^{me} + \nu^{b}\underline{C}^{be}(\underline{R}^{b})^{-1}\,\underline{R}^{m}}{\nu^{m}\,\underline{I} + \nu^{b}\,\underline{C}^{be}\,(\underline{R}^{b})^{-1}\,\underline{I}} \tag{4.56}$$

It is obseved in Eqns(4.54- 4.56), if the masonry strains $\underline{\epsilon}$ are known, the mortar stresses $\underline{\sigma}^{m}$ and plastic strains, $\underline{\epsilon}^{mp}$ can be calculated. Then the masonry plastic strains $\underline{\epsilon}^{p}$ can be calculated by using Eqn.(4.52) and finally the masonry stresses $\underline{\sigma}$ in Eqn.(4.50).

4.6 Validation of the Proposed Formulae

Prism Strength: To demonstrate the validation of the strength theory, the analytical data are compared with the experimental results of Khoo and Hendry(1972) and the present experimental investigation described in Chapter 3. The material properties used in the study are presented in Table 4.1.

The strength of brick masonry prism determined experimentally by Khoo and Hendry(1972), and the experimental and analytical results of the present investigation are as summarised in the Table 4.2. A good agreement is observed between the analytical and experimental results.

Elastic Properties

Elastic properties of masonry prism and wall are computed using the proposed formulae. The upper and lower bounds and effective elastic modulus are plotted in Fig.4.3. Experimental and analytical results of elastic moduli and Poisson's ratios are shown in Table 4.2. A close agreement is observed between the two results.

Table 4.1 Material Properties for Brick and Mortar Joint

Serial No.	Property		Khoo and Hendry	Present Exp.
Brick	Young's modulus(MPa)	E^b	5700.00	5130.00
	Poisson's ratio	ν^b	0.16	0.145
	Uniaxial compressive strength(MPa)	f_c^b	39.30	15.45
	Uniaxial tensile strength(MPa)	f_t^b	1.88	0.51
	Length(mm)	l	73.00*	230.00
	Width(mm)	b	35.00*	110.00
	Thickness(mm)	t^b	22.30*	67.00
Mortar	Young's modulus(MPa)	E^m	2500.00	2250.00
	Poisson's ratio	ν^m	0.24	0.25
	Uniaxial compressive strength(MPa)	f_c^m	4.78	4.84
	Thickness(mm)	t^m	3.35**	10.00
	Bond strength	τ_o	0.25	0.25

* Brick - one third scale, ** $t^m/t^b = 0.15$

Table 4.2 Comparison of Experimental Results with the Analytical Data

Serial No.	Item	Experimental Results: Khoo and Hendry (i)	Experimental Results: Present Exp. Study (ii)	Analyses based on Proposed Theory (i)	Analyses based on Proposed Theory (ii)
1	Prism strength (MPa)	22.42	7.07	20.93	6.79
2	Young's modulus(MPa)	-	4541	-	4756
	a. Masonry prism	-	4541	-	4548
	b. Masonry wall (average)	-	4430	-	4422
3	Poisson's ratio				
	a. Masonry prism	-	0.16	-	0.158
	b. Masonry wall (average)	-	0.18	-	0.162

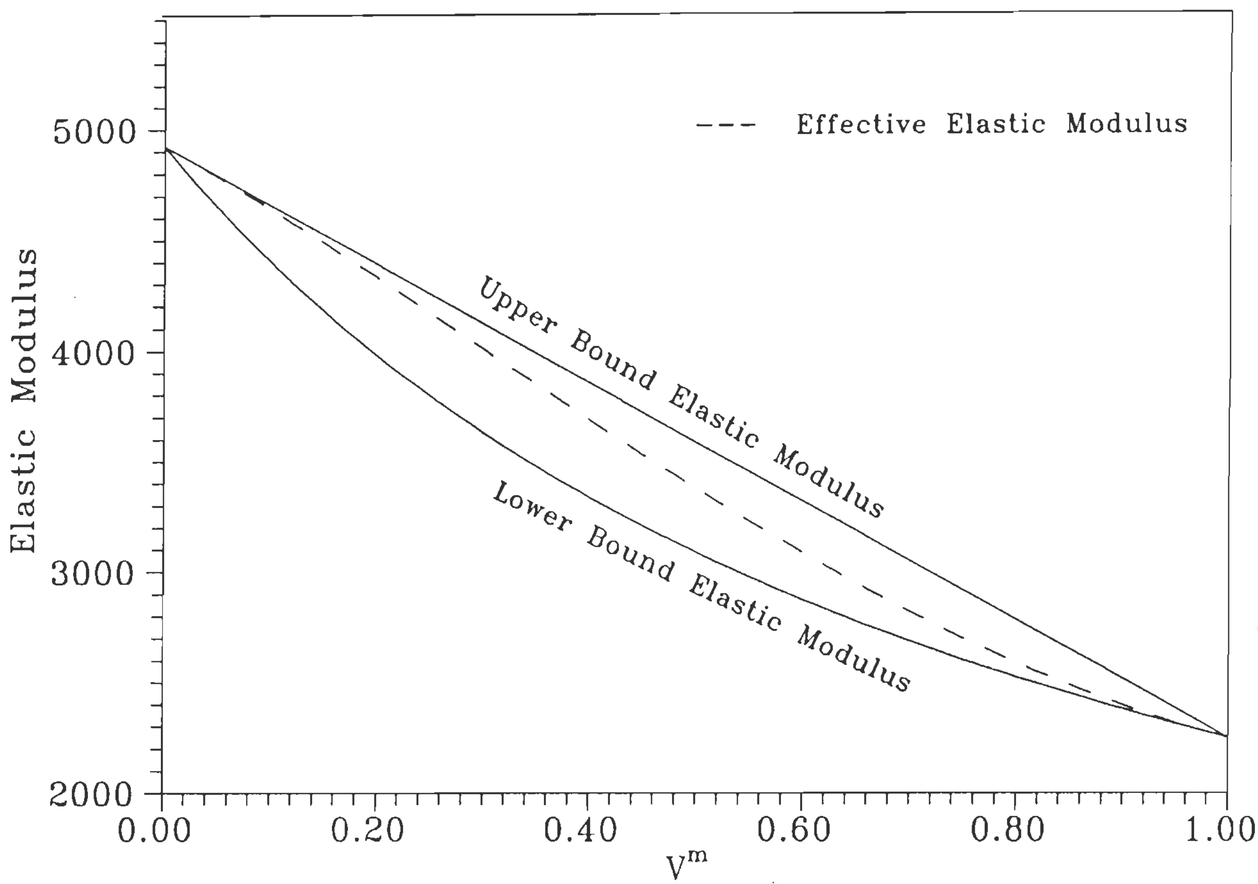


Fig.4.3 Upper and Lower Bounds and Effective Elastic Modulus of Brick Masonry

Distribution of stresses in the Brick and Mortar Joints

To determine stresses in bricks and mortar a 4" thick masonry wall subjected to uniformly distributed load 800 lbs, was analysed using finite element method by Stafford-Smith and Caster (1970). The wall was discritised by four noded elements such that one element encompasses only one material. The element subdivisions and properties of brick and mortar are shown in Fig. 4.4.

From the result of finite element analysis it is observed that the stresses in brick and vertical joint is 8.7344 psi and 2.933 psi respectively and 8.33 psi in bed joint. On the other hand, according to proposed stress distribution formulae, the stresses in the brick and vertical mortar joint are 8.594 psi and 2.873 psi respectively. Stress in the bed joint is 8.33 psi. Thus a good agreement between the results of finite element analysis and proposed formulae is observed.

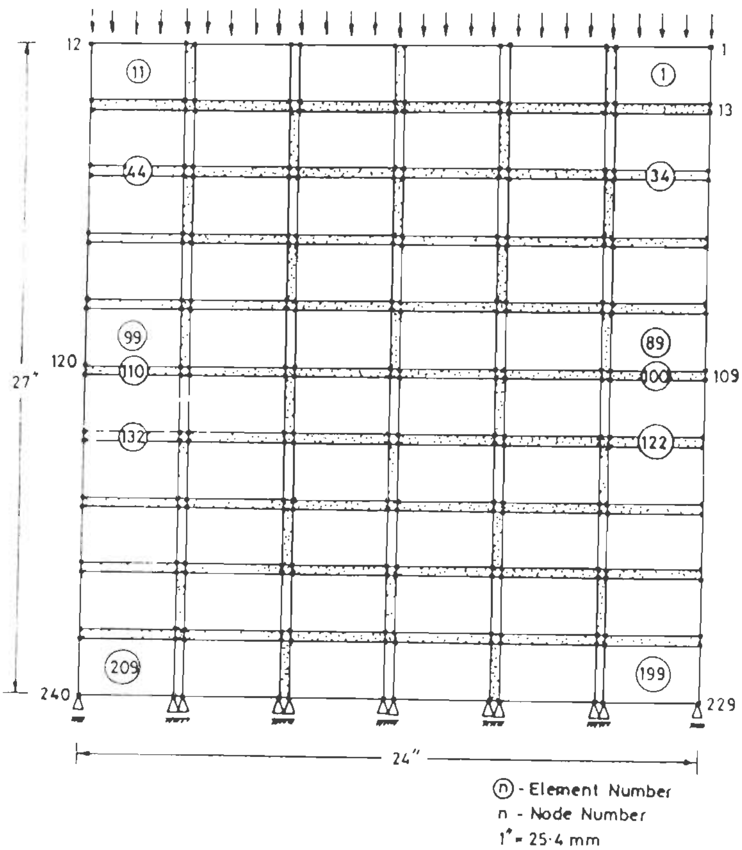


Fig. 4.4 Finite Element Subdivisions for Masonry Wall

4.7 Conclusions

The conclusions drawn from the study presented in this Chapter are:

- Literature pertaining to mathematical modelling for determination of strength and elastic properties is critically reviewed.
- A Mathematical model for the determination of strength of the brick masonry prism, based on micromechanics, has been developed. The strength of masonry prism computed from the existing strength theories does not agree well with the experimental results. In most cases, the computed strength is of the order of 60 to 75 percent of the experimental strength. The proposed model can determine the strength of the masonry prism to within 95 percent of the experimental strength.
- Formulae have also been developed to determine the equivalent elastic properties of the brick masonry in terms of those of brick and mortar. The analytical data agrees well with the experimental results as shown in the Table 4.2. These elastic properties can be used to develop the constitutive equations for use in finite element analysis. Thus the desired properties of masonry can be designed and produced with sufficient accuracy for the design and analysis of brick masonry structures.
- After obtaining the stresses in the brick masonry by finite element analysis or by any other method, the stresses in the brick, bed joint, and head joint can be calculated from the proposed formulae given in section 4.4. The results based on the proposed formulae have been compared with that of the finite element analysis at micro-level, where each element encompasses only one material. The results obtained using these formulae show very good agreement with those of the finite element analysis.
- Constitutive equations for brick masonry have also been developed in terms of the elastic properties of the brick and mortar, for use in finite element analysis. In this approach it is not necessary to obtain the equivalent elastic properties of the brick masonry.
- Finite element analysis at micro-level is also not necessary to calculate the actual stresses in the brick and the mortar. Thus both cost and computing time can be saved without a loss in accuracy of results. However, this approach could not be used in the present investigation because of nonavailability of experimental results.

4.7 References

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CHAPTER 5

MATERIAL MODEL FOR BRICK MASONRY

5.1 Introduction

In the recent years the increase in structural brick masonry research is quite marked. A large number of research efforts have been devoted to study the in-plane behaviour of masonry walls. Mathematical simulation of its behaviour is rather difficult as its accurate and rational analysis requires satisfactory material model for brick masonry. Despite the sophistication of the structural idealisation and computational techniques, material modelling has been the biggest hurdles for structural analysts for the validation of nonlinear analysis.

Brick masonry is a composite material consisting of elastic bricks set in inelastic mortar matrix. At failure load masonry exhibits direction-dependent deformation and strength characteristics. Its behaviour becomes exceedingly complicated under multi-axial state of stresses. The additional problem of scarce or incomplete experimental data, specially for multi-axial state of stresses is a major source of difficulty in the development of material model. Also, considerable variation is observed in experimental results.

In this chapter, first, a brief review of the available material models is presented. Then, the proposed material model for the analysis is described. The model takes into account direction-dependent strength characteristics and material nonlinearity and local failure.

5.2 Review of Material Models

A large number of material models have been proposed in the last two decades for the analysis of masonry structures. A summary of models and its application can be found in a number of references[Page(1978), Dhanasekar(1985)]. All these models have certain inherent advantages and disadvantages which depend to a large extent on their particular application. The existing models have been critically evaluated within the context of their use in the numerical analysis of masonry structures. These models are based on two approaches:

- micro-level modelling, considering the distinct properties of the components(units, grout and joint mortars, reinforcements), well fitted for local effect analysis, and

- macro-level modelling, considering an equivalent homogeneous material, suitable for overall structural analysis.

For both approaches a number of theories for modelling masonry behaviour under various state of stresses reported in the literature are as follows:

- Linear and nonlinear elasticity theories
- Plasticity and viscoplasticity theories

5.2.1 Linear and Nonlinear Elasticity Theories

Early analyses were based on macro-level models. Assuming masonry as a linear elastic continuum, the in-plane behaviour of composite wall beam has been studied employing finite difference method [Wood(1952), Rosenhaupt and Sokal(1965)]. In order to allow possible variations in the horizontal and vertical directions, orthotropic linear elastic model has been incorporated in variational approach for the analysis of composite wall beam problem [Coull(1966)]. A linear elastic finite element analysis of composite wall beam problem was also carried out by many investigators[Male and Arbon(1969) and Saw(1974)among others].

An improvement in the analysis was obtained by using micro-level approach. The influence of mortar joint acting as a plane of weakness, ignored in the macro-level of analysis, has been included in this analysis. The linear elastic analyses with bricks and joints being modelled separately were carried out by many investigators [Stafford-Smith and Carter(1970), Stafford-Smith and Rahman(1972), Ali and Page(1985)]. However, for monotonic loading, the linear elastic model can be significantly improved by employing nonlinear elastic formulations: Hyperelastic and Hypoelastic formulations.

In hyperelastic (Green) material, elastic response function is restricted by the existence of an elastic strain energy function, W which is a function of strain components, ε_{ij} such that

$$\sigma_{ij} = \frac{\partial W}{\partial \varepsilon_{ij}} \tag{5.1}$$

This ensures that no energy can be generated through load cycles and thermodynamic laws are satisfied. This model is based on finite(or total) material characterisation in the form of secant formulation. Hyperelastic formulation approximate a path independent reversible process with no memory.

In hypoelastic(incremental) models, the state of stress is a function of current state of strain as well as of stress. The hypoelastic formulation approximates a path dependent irreversible process with limited memory. Thus for a hypoelastic material the constitutive equation is expressed as

$$\sigma_{ij} = F_{ij}(\varepsilon_{kl}, \sigma_{mn}) \tag{5.2}$$

where σ_{ij} = stress (or increment) tensor,

ε_{ij} = strain(or increment) tensor,

F_{ij} = elastic response function.

Although the use of both hyperelastic and hypoelastic formulations in brick masonry has not been reported till to-day, yet both the formulations have been used for modelling the behaviour of concrete by many investigators [Kupfer and Gerstle(1973), Kotsovos and Newman(1978), Darwin and Pecknold(1977), Elwi and Murray(1979) and, Gerstle(1981)]. These formulations are not suitable for unloading behaviour of concrete as well as in brick masonry.

Linear elastic theory coupled with failure criterion has been used for material modelling of brick masonry. This type of model is generally used to model the behaviour of the material subjected to tensile-compression state of stress. Linear elastic fracture model based on macro-level has been developed and used by many investigators [Samarasinghe *et al.*(1982) and Dhanasekar(1985)] to study the nonlinear response of shear walls and infill frames. This type of model has also been used in micro-level of analysis by many investigators [Page(1978), Hegemier *et al.*(1978), Ali and Page(1987)].

5.2.2 Plasticity and Viscoplasticity Theories

Brittle materials like concrete, brick masonry subjected to compression state of stresses exhibit nonlinear characteristics. Perfect plasticity models are often used to account for the plastic flow before crushing. The description of such a model requires (a) the definition of constitutive relation prior to yielding (b) a yield surface and (c) a potential surface. Normality principle determines the direction of the plastic strain increment vector. Most of the models use linear elastic constitutive laws before yielding and an associative flow rule i.e. plastic strain increment vector is assumed to be normal to the yield surface.

An improvement over the perfect plasticity can be obtained by incorporating work or strain hardening. In the strain hardening plasticity model, the yield and failure surfaces are two different surfaces. When the material is stressed beyond the yield surface, this surface expands creating a family of 'loading surfaces' and associative or nonassociative flow rules may be used to obtain the plastic strain. Expansion of the loading surfaces continues until the failure surface is reached. Strain softening effects can not be incorporated in such models as the laws of thermodynamics rule out such behaviour for continuum material. However, fracture theory can take into account the decrease in stress at increasing strain. Both the theories can be combined to obtain a constitutive relation that is incrementally linear. The model requires a fracture surface defined in stress space in terms of the strain invariants and a hardening parameter. Such models are able to take into account degradation of elastic moduli, inelastic dilatency due to micro cracking.

Based on macro-level approach a failure theory has been developed by Dhanasekar *et al.*(1985) from biaxial tests data of clay brick masonry. Plastic strain components are assumed to be a function of the corresponding stress in that direction. The hardening parameter was obtained from stress-plastic strain curves and was employed in the incremental constitutive laws. However, this formulation is not applicable to whole biaxial compression-compression region. Saw(1975) used modified Von Mises criterion to model the nonlinear behaviour of masonry.

In micro-level approach, the nonlinear behaviour of the masonry units and the mortar has been modelled by many investigators. The nonlinear deformation of mortar with no provision of failure of bricks has been considered by and Page(1978). A bilinear failure criterion has been used for the mortar joints. Nonlinear behaviour of the concrete bricks and mortar have been taken into account by Ali and Page(1987) for the concrete brick masonry. Incremental stress-strain relation has been used for concrete bricks while nonlinear stress-strain formulation, similar to that proposed by Dhanasekar *et al.*(1985) has been used to model the behaviour of mortar. Von Mises failure criterion with tension cut off has been used for the brick and the mortar.

5.3 Proposed Material Model

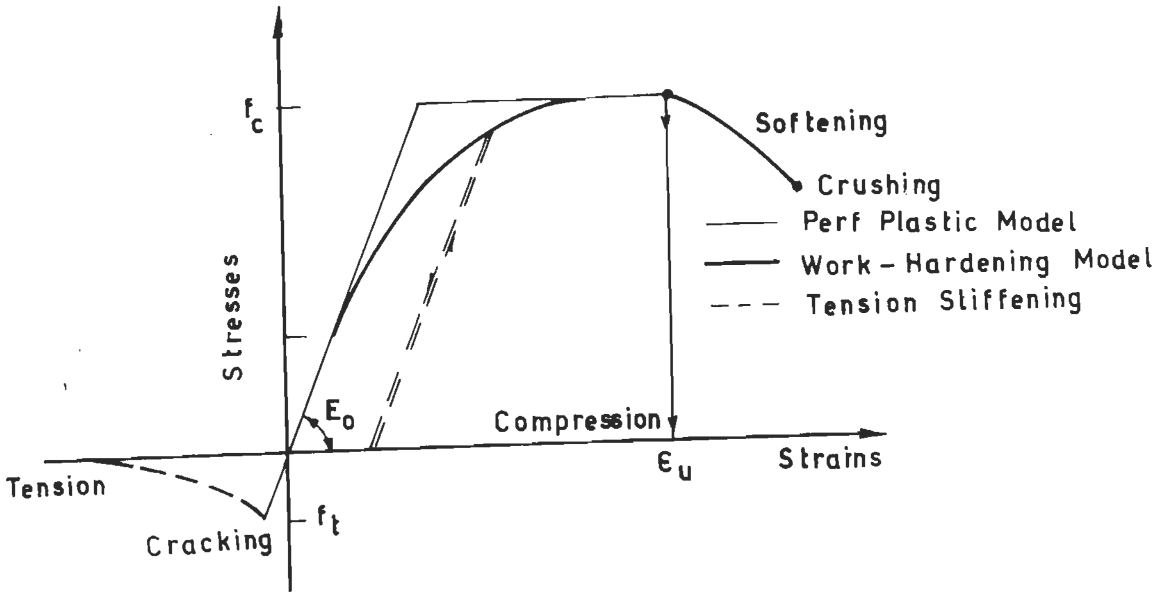
For the development of material model due consideration is given to (a)accurate and rational simulation of material response, (b) simplicity and economy, and (c) minimum number of parameters which can be evaluated with desired degree of accuracy and reliability. The behaviour of brick masonry in different stress regimes is modelled in a form suitable for numerical computation. Various stress regimes are summarised in Fig.5.1 In the present analysis elasto-perfectly plastic approach has been employed to model the compression behaviour. Linear elasticity is used for the recoverable part of strain and a plasticity approach is employed for irrecoverable part of the deformation. The description of the model is consisting of the following items:

- o Yield and Failure criteria,
- o Flow rule,
- o Crushing condition and
- o Cracking of masonry.

5.3.1 Failure Criterion

Brick masonry exhibits anisotropic strength properties due to the presence of mortar joints acting as a plane of weakness and to model the failure behaviour, an anisotropic failure criterion is necessary. Thus, a general theory of strength for anisotropic material proposed by Tsai and Wu(1971) is employed to model the nonlinear behaviour of masonry. Strength criteria proposed by Hill(1950), Hoffman(1967) for anisotropic material and Von Mises(1928) for isotropic materials can also be derived form the proposed model. The strength criterion, defining the initiation of material degradation in the stress space is given by:

$$f(\underline{\sigma}) = a_i \sigma_i + a_{ij} \sigma_i \sigma_j - 1 = 0 \tag{5.3}$$



One Dimensional Representation of the Constitutive Model

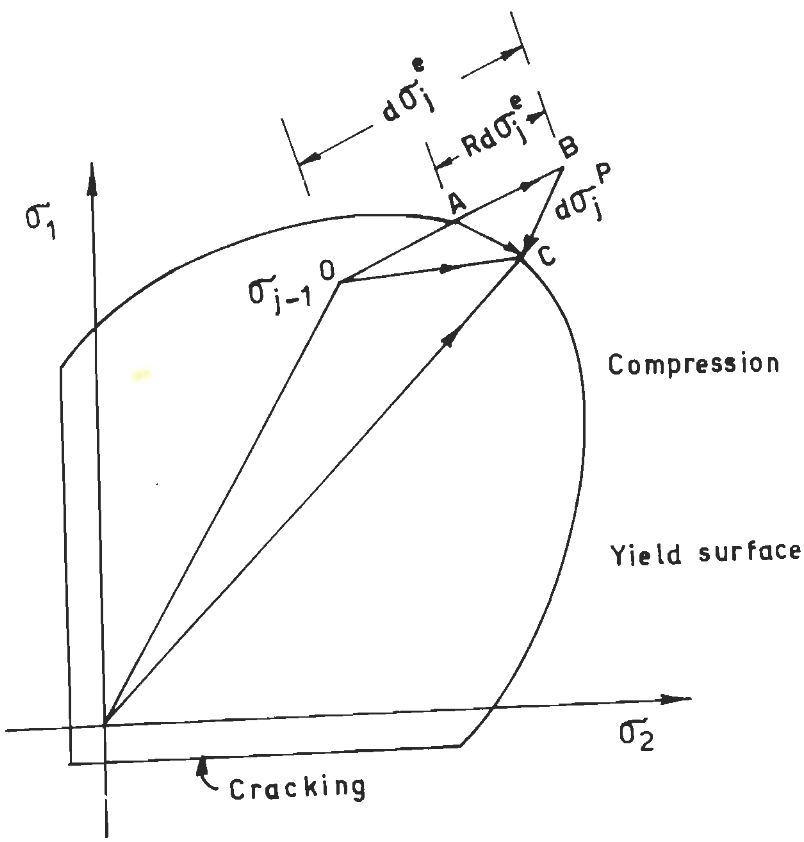


Fig. 5.1 Modelling of Various Stress Regimes for Brick Masonry

where the contracted notation is used and i, j= 1,2,...6 and parameters of anisotropy, a_i and a_{ij} are second and fourth order tensors. The linear terms accounts for the differences in tensile and compressive strengths and the quadratic terms define an ellipsoid in the stress space. Certain stability condition are incorporated in the strength parameters to ensure that the shape of the surface will be ellipsoidal [Tsai and Wu(1971)]. The magnitude of the interaction terms are constrained by the following condition:

$$a_{ii}\, a_{jj} - a_{ij}^2 \geq 0 \qquad (5.4)$$

where repeated indices are not summations for this equation. The specific advantages of this theory include:

- o invariant under rotation of co-ordinates,
- o transformation relations according to tensorial laws,
- o interactions among stress components are independent of material properties, and
- o symmetry properties of strength tensors similar to those of well established properties of anisotropic materials, such as the elastic compliance matrix.

Equation (5.3) in long-hand form is:

$$\begin{aligned} f(\underline{\sigma}) = {} & a_1\sigma_1 + a_2\sigma_2 + a_3\sigma_3 + a_4\sigma_4 + a_5\sigma_5 + a_6\sigma_6 \\ & + a_{11}\sigma_1^2 + 2a_{12}\sigma_1\sigma_2 + 2a_{13}\sigma_1\sigma_3 + 2a_{14}\sigma_1\sigma_4 + 2a_{15}\sigma_1\sigma_5 + 2a_{16}\sigma_1\sigma_6 \\ & + a_{22}\sigma_2^2 + 2a_{23}\sigma_2\sigma_3 + 2a_{24}\sigma_2\sigma_4 + 2a_{25}\sigma_2\sigma_5 + 2a_{26}\sigma_2\sigma_6 \\ & + a_{33}\sigma_3^2 + 2a_{34}\sigma_3\sigma_4 + 2a_{35}\sigma_3\sigma_5 + 2a_{36}\sigma_3\sigma_6 \\ & + a_{44}\,\sigma_4^2 + 2a_{45}\sigma_4\sigma_5 + 2a_{46}\sigma_4\sigma_6 \\ & + 2a_{55}\sigma_5^2 + 2a_{56}\sigma_5\sigma_6 \\ & + a_{66}\sigma_6^2 - 1 = 0 \end{aligned} \qquad (5.5)$$

The subscript 1, 2, 3 refer to the directions of three principal axes of anisotropy. As the principal axes are the axes of symmetry, anisotropic parameters a_4, a_5, a_6, a_{14}, a_{15}, a_{16}, a_{24}, a_{25}, a_{26}, a_{34}, a_{35}, and a_{36} will be zero. Since the failure surface is employed only in compression zone, the linear terms are omitted. For the plane stress problem, the Eqn. (5.5) reduces to

$$f(\underline{\sigma}) = a_{11}\sigma_1^2 + 2a_{12}\sigma_1\sigma_2 + a_{22}\sigma_2^2 + a_{66}\sigma_6^2 - 1 = 0 \tag{5.6}$$

In the matrix form Eqn. (5.6) can be written as

4

$$f(\underline{\sigma}) = \underline{\sigma}_{1,2}^T \, \underline{A} \, \underline{\sigma}_{1,2} \tag{5.7}$$

where $\underline{\sigma}_{1,2}^T = \left[\sigma_1 \ \sigma_2 \ \sigma_6 \right]$ and $\underline{A}$ is the matrix of anisotropic parameters,

$$\underline{A} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{12} & a_{22} & 0 \\ 0 & 0 & a_{66} \end{bmatrix} \tag{5.8}$$

The reference axes are normal to the bed joint, n and parallel to the bed joint, p. The principal stresses are transformed to stresses with respect to reference axes n and p using the following relationship

$$\underline{\sigma}_{1,2} = \underline{T} \, \underline{\sigma}_{n,p} \tag{5.9}$$

where $\underline{T}$ is transformation matrix defined by

$$\underline{T} = \begin{bmatrix} \mathrm{Cos}^2\theta & \mathrm{Sin}^2\theta & \mathrm{Sin}\theta \ \mathrm{Cos}\theta \\ \mathrm{Cos}^2\theta & \mathrm{Sin}^2\theta & \mathrm{Sin}\theta \ \mathrm{Cos}\theta \\ 2\mathrm{Sin}\theta \ \mathrm{Cos}\theta & -2\mathrm{Cos}\theta \ \mathrm{Sin}\theta & (\mathrm{Cos}^2\theta - \mathrm{Sin}^2\theta) \end{bmatrix}$$

and $$\underline{\sigma}_{n,p}^T = \left[\sigma_n \ \sigma_p \ \tau_n \right]^T \tag{5.10}$$

The yield or failure surface, expressed with respect to reference axes n, p is then

$$f(\underline{\sigma}) = a_n \sigma_n^2 + a_p \sigma_p^2 + 2a_{np} \sigma_n \sigma_p + s_{np} \tau_{np}^2 = 1$$

$$= \underline{\sigma}_{n,p}^T \underline{A}' \underline{\sigma}_{n,p} \tag{5.11}$$

where $\underline{A}'$ is the matrix of the anisotropic parameter with respect to reference axes written as

$$\underline{A}' = \underline{T}^T \underline{A} \underline{T} = \begin{bmatrix} a_n & a_{np} & 0 \\ a_{np} & a_p & 0 \\ 0 & 0 & s_{np} \end{bmatrix}$$

(a) Determination of Strength Parameters: The strength parameters of the failure theory are determined from the experimental data obtained by Page(1981,83). Strength parameters such as a_n , a_p are to be determined from the tests results of uniaxial compression in the direction normal and parallel to the bed joint. The strength parameter, s_{np} is to be determined from shear strength. This test has not been carried out by Page(1983). Biaxial compressive strength with (σ_{pc}/σ_{nc} = 4 and θ =45) is used to evaluate parameter, s_{np}. The determination of strength parameter, a_{np}, can be achieved through infinite number of combined-stress states. The type of test is to be selected in such a manner that the magnitude of a_{np} does not exceed the limit as indicated in Eqn.(5.4). If the value of a_{np} exceeds the limit as indicated by the Eqn.(5.4), the failure surface will become hyperboloid [Tsai and Wu(1971)]. The value of a_{np} has been derived from off-axis and other biaxial stress test and the off-axis and biaxial strengths are also computed for each value of a_{np}. It is observed that the strengths have been blown up when the unrestricted values of a_{np} are substituted in the criterion. So the stability requirement must be checked for the determination of the value of a_{np}.

The type of test is to be chosen such that the inaccuracy of the test results does not induce appreciable change in the value of a_{np}. The study on the sensitivity of a_{np} on the off-axis or other combined stress tests are carried out. The effect of a_{np} on some off-axis and biaxial tests are shown in Fig.5.2. The curve such as U_{45} (45 degree off-axis uniaxial strength) is nearly horizontal which means that this test should not be used for the determination of a_{np}. A small inaccuracy in the value of 45 degree off-axis uniaxial strength can induce a large change in a_{np}. For negative value of a_{np} biaxial compression-compression test results should be used and for positive value of a_{np} 45 degree off-axis shear strength should be used.

The following four types of experimental results are used to determine strength parameters:

- Unixaial compression strength parallel to bed joint, f_{pc}
- Uniaxial compression strength normal to bed joint, f_{nc}
- Biaxial compression of strength with $\sigma_{pc}/\sigma_{nc} = 4$ at $\theta = 45$ degree (the loading direction makes an angle, $\theta = 45^o$ with the material axis normal to the bed joint).
- Equal biaxial compression strength, f_{bc}

The experimental strength values used to determine the parameters of the model are:

$f_{pc} = 4.3275$ MPa, $\qquad f_{nc} = 7.564$ MPa,

at $\theta = 45$ degree $\qquad f_{bc} = 8.3500$ MPa

$\sigma_{nc} = 2.04$ MPa

$\sigma_{pc} = 8.31$ MPa

The values of parameters are

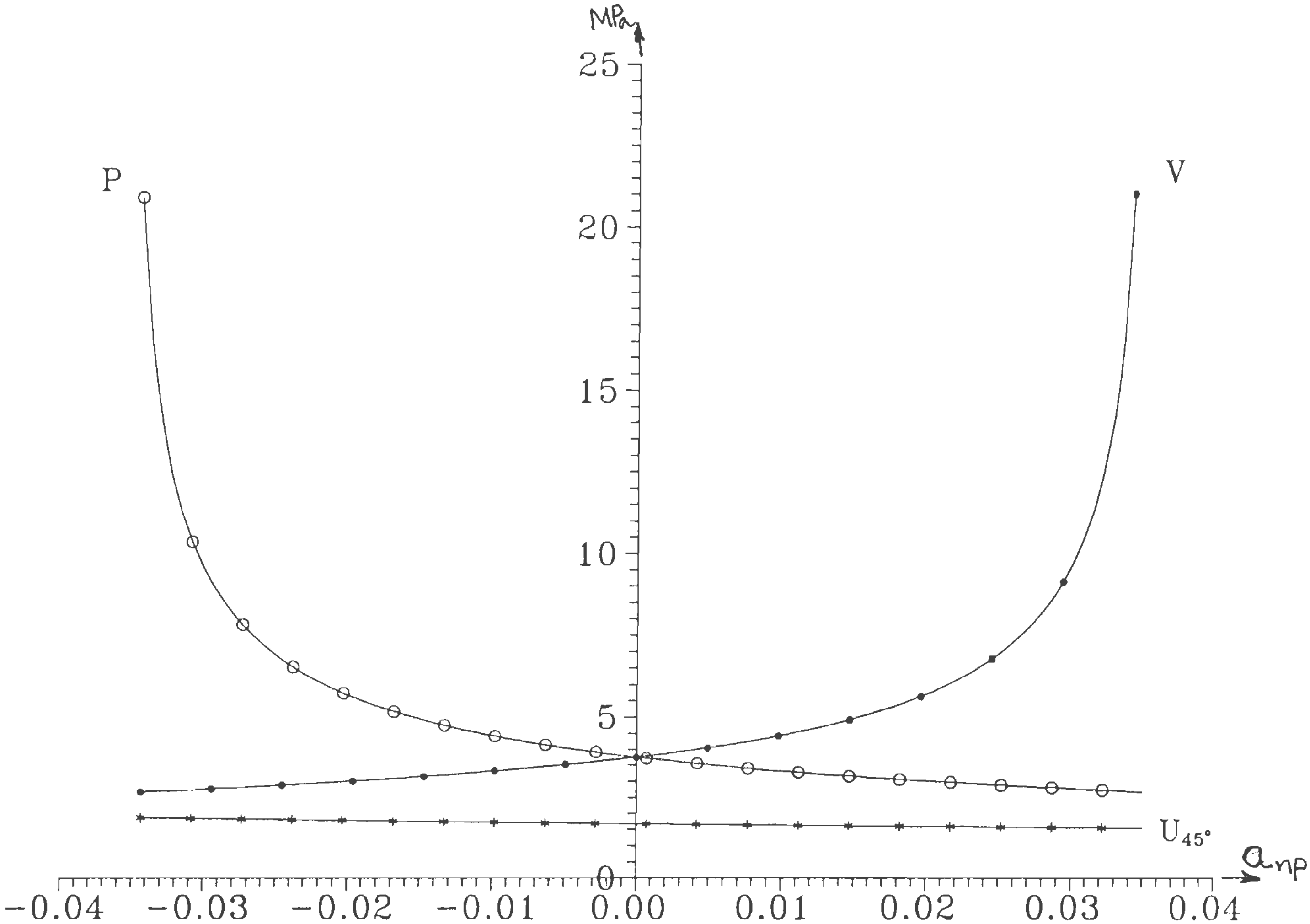
$a_p = 0.0533981, \qquad a_n = 0.0174782,$

$a_{np} = -0.0282668, \qquad s_{np} = 0.0626904$

The failure criterion is compared in Fig.5.3 with the experimental failure envelope [Page(1981)].

5.3.2 Yield Surface and Elasto-plastic Considerations

The yield surface defines the onset of plastic behaviour. If the

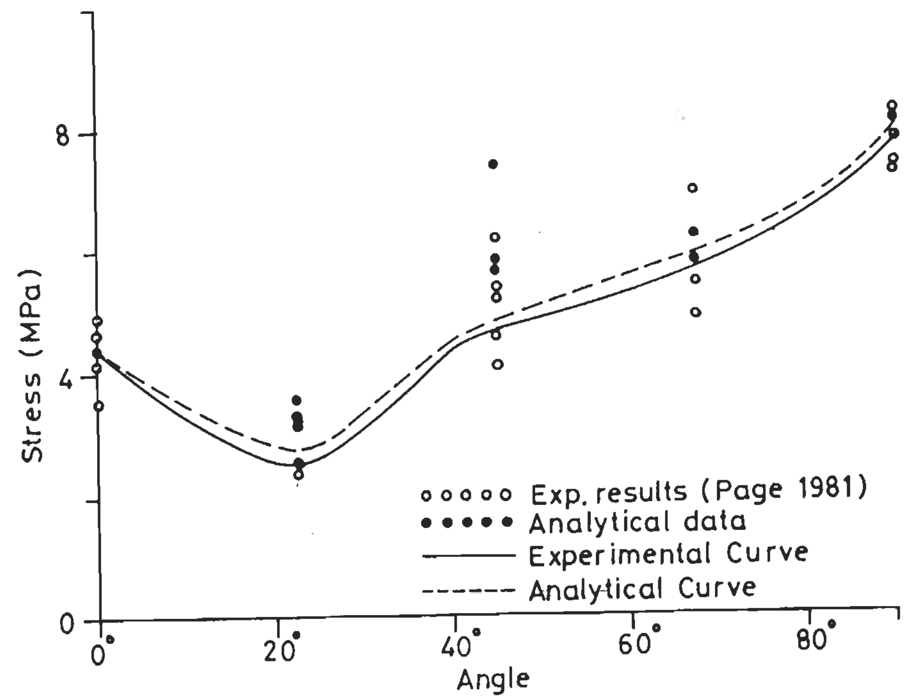


P is the Equal Biaxial Compressive Stress

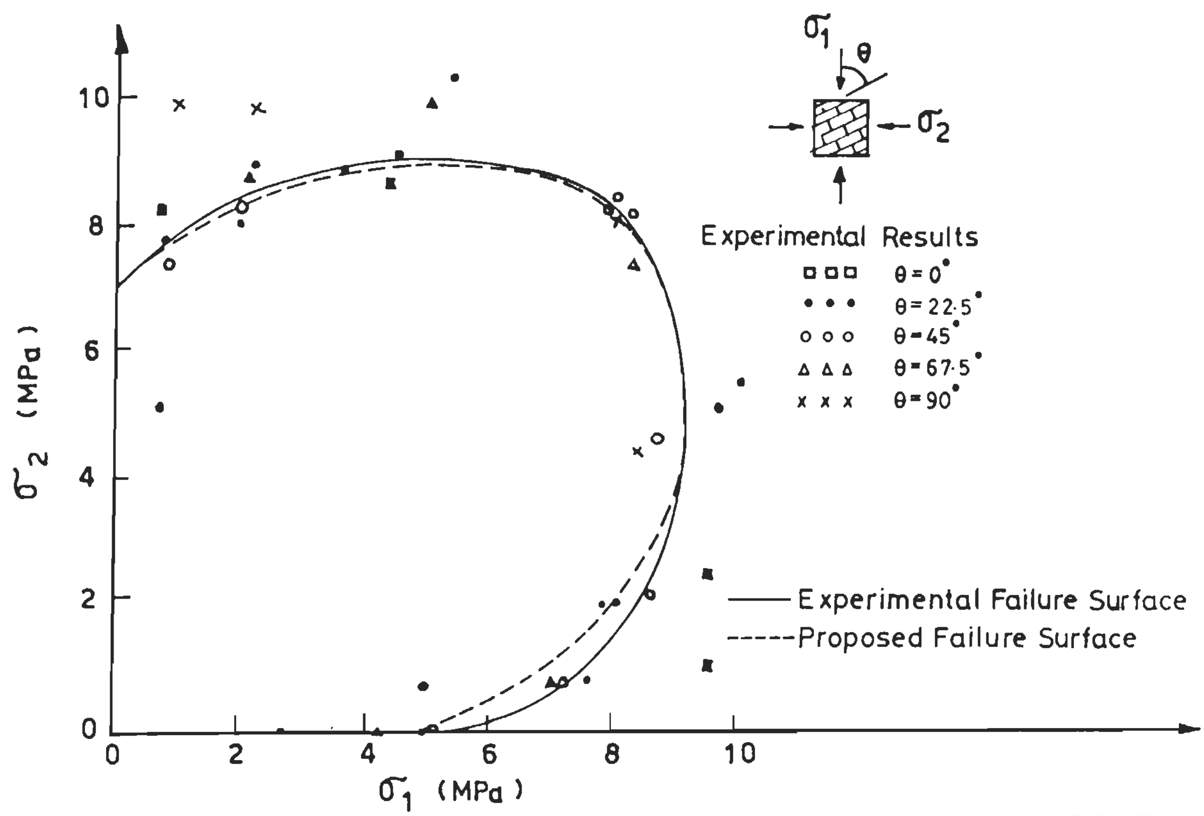
V is the Shear Strength at 45^0 off -axis.

U_{45} is the 45^o off-axis uniaxial Compressive stress

Fig. 5.2 Effect of Interaction Strength Parameter, a_{np}, on Biaxial State of Stress



(a) Failure Curve for Brick Masonry under uniaxial compressive load



(b) Failure Surface for Brick Masonry Subjected to Biaxial Compressive Load

Fig. 5.3 Failure Surface for Brick Masonry subjected to Compressive Loading stress path remains inside the yield surface, the stress-strain relation is linearly elastic. When the yield surface is reached, the inelastic deformation begins. Further increase of loading causes the expansion of yield surface which is monitored by using a hardening rule. The expanded yield surface is defined as 'loading surface'. In this way a family of loading surfaces is formed. Unloading inside the current loading surface occurs elastically. The loading surface continues to expand until the failure surface is reached. The failure surface is only a monitoring device to define the initiation of failure and the loading surface begins to shrink according to the post failure dissipated energy.

In this work no hardening is considered. In such a model the response is elastic until the effective stress reaches the failure strength. After that plastic flow continues until the crushing surface is reached in the strain space. Axis parallel to the bed joint is considered as reference axis. Thus the effective stress at yield (or failure) will be the yield(or failure) stress in the direction parallel to bed joint of brick masonry. Accordingly, the strength computed parameters (Eqn.5.12) are modified.

5.3.3 Flow Rule

By using the associated flow rule, the yield function is taken as the plastic potential function from which the incremental plastic strain can be defined as

$$d\underline{\varepsilon}^{p} = d\lambda \frac{\partial f(\underline{\sigma})}{\partial \underline{\sigma}} \qquad (5.12)$$

where $d\lambda$ is a proportionality constant which determines the magnitude of the plastic strain increment. The current stress function $f(\underline{\sigma})$ is the yield function or subsequent loading functions in the strain hardening model.

The yield function derivatives which define the flow vector $\underline{a}$ computed for the present yield surface are expressed as

$$\underline{a}^{T} = \left[\frac{\partial f}{\partial \sigma_{n}}, \frac{\partial f}{\partial \sigma_{p}}, \frac{\partial f}{\partial \sigma_{np}} \right] = \left[\frac{\partial f}{\partial \underline{\sigma}} \right] \qquad (5.13)$$

$$a_1 = \frac{\partial f}{\partial \sigma_n} = (a_n \sigma_n + a_{np} \sigma_p)/f$$

$$a_2 = \frac{\partial f}{\partial \sigma_p} = (a_n \sigma_p + a_{np} \sigma_n)/f$$

$$a_3 = \frac{\partial f}{\partial \tau_{np}} = s_{np} \tau_{np} /f$$

where $\quad f = (a_n \sigma_n^2 + a_p \sigma_p^2 + 2 a_{np} \sigma_n \sigma_p + s_{np} \tau_{np}^2)^{1/2}$

The elasto-plastic incremental stress-strain relation is given by

$$d\underline{\sigma} = \underline{D}_{ep} \, d\underline{\varepsilon} \tag{5.14}$$

with the elasto-plastic material matrix defined by

$$\underline{D}_{ep} = \underline{D} - \frac{\underline{D}\,\underline{a}^T \underline{a}\,\underline{D}}{H + \underline{a}^T \underline{D}\,\underline{a}} \tag{5.15}$$

where $\quad$ H is the hardening parameter,

$\underline{D}$ is the elasticity matrix, and

$d\underline{\varepsilon}$ is the total strain incremental vector.

5.3.4 Crushing Condition

Inelastic deformation continues until crushing occurs. The crushing type of failure is strain controlled phenomenon. Due to the lack of available experimental data for ultimate deformation capacity under multiaxial state of stress, a crushing surface is assumed in the strain space which is related to ultimate strain of uniaxial test.

$$3J_2' = \varepsilon_u^2 \tag{5.16}$$

where J_2' is the second deviatoric strain invariant and ε_u is an ultimate strain obtained from uniaxial test result. When ε_u reaches the value specified ultimate strain, the material is assumed to loose strength and rigidity.

5.3.5 Tensile Behaviour of Brick Masonry

The tensile weakness of masonry and its consequent cracking is the dominant cause of nonlinear behaviour in masonry structures. The accurate representation of this cracking is essential if the realistic behaviour of the masonry is to be simulated. Brick masonry when subjected to tensile stress state, behaves as a linear elastic material until the fracture surface is reached and the strength in the direction of tension reduces to zero due to cracking. Three different approaches are now available for modelling of cracks. These are (a) Discrete cracking model, (b) Smeared cracking model with fixed or rotating cracks, and (c) Fracture mechanics model. The particular model to be selected depends on the purpose of analysis. If overall load deflection behaviour is of interest, discrete cracking model is useful. For the special class of problem in which fracture is the appropriate tool, fracture mechanics model is employed.

(a) Discrete Cracking Model: In this model, the cracking response is mesh dependent as the cracks are assumed to form at the element boundaries (Fig.5.4). The topology of mesh changes and the updating procedures are complex and time consuming. Adaptive model used to refine the model is also very expensive because of remeshing and the introduction of new elements. These difficulties have resulted in a very limited acceptance of this model in general structural application. However, the method seems appropriate for those problems involving only a few dominant cracks.

(b) Smeared Cracking Model: In this model, brick masonry is considered to remain continuum ignoring the real discontinuities in the mesh(Fig.5.4). The cracked masonry is assumed to become orthotropic or transversely isotropic with one of the material principal axes oriented along the direction of cracking. The Young's modulus is reduced to zero in the direction normal to the crack plane and Poisson's effect is usually neglected. The shear modulus in the cracked plane is also reduced.

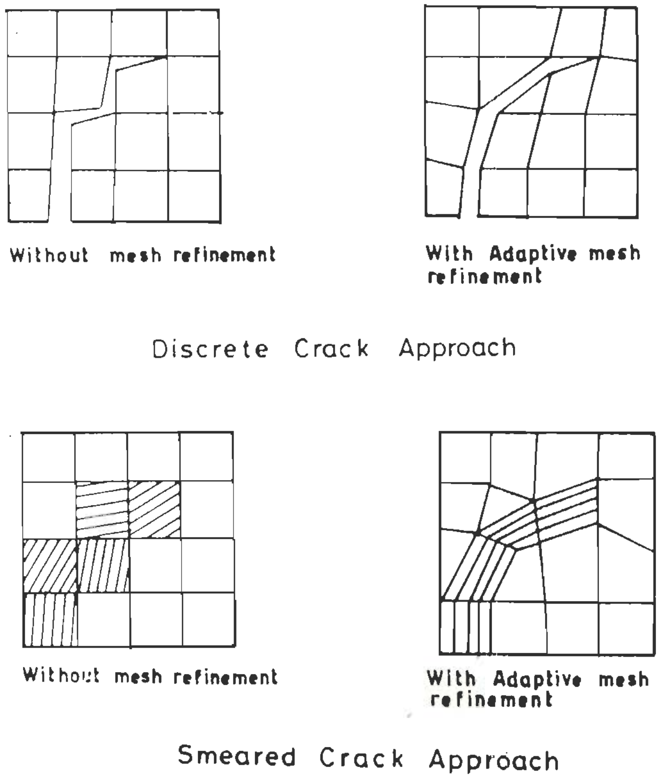


Fig. 5.4 Two Alternative Approach for Crack Modelling

This model is computationally simple and more attractive as the updating of node number or remeshing is not involved. Only stress-strain relation need to be modified when cracking occurs. This model has been used by many researchers to model the behaviour of concrete [(Valliappan and Doolan(1972), Schnobrich(1977), Zienkiewicz *et al.*(1983), Owen *et al.*(1983), Owen and Figueiras(1984), Damjanic(1985)] and brick masonry [Dhanasekar(1985) , Ali and Page(1987)]. This model can be refined with mesh updating procedure as suggested by Gustafsson(1985).

(c) Fracture Mechanics Approach: Stress concentrations at crack tips, crack width, bond, etc can be modelled more accurately by fracture mechanics method. This method has been employed in concrete by several researchers [(Hillemier and Hilsdorf(1977), Bazant and Cedolin(1979)]. Use of this approach in brick masonry is not reported in the available literature.

Smeared crack model with fixed crack direction has been employed for the modellinig of cracked behaviour of brick masonry. The complete description of the model requires the following:

- o cracking criterion, and
- o strain softening rule.

(d) Cracking Criterion: Brick masonry in tension is modelled as linear elastic-strain softening material. The maximum stress criterion extended to predict the strength of anisotropic materials[Jenkins(1920) is employed to predict the cracking behaviour of brick masonry under in-plane static loading. The stresses acting in masonry are resolved into direction of material symmetry(σ_n, σ_p, τ_{np}). It is then postulated that failure will occur when one (or all) of these stresses attains a respective ultimate strength f_{nt}, f_{pt}, s_{np}. These strengths are obtained experimentally under uniaxial loading. This criterion can be stated that failure will not occur as long as the following prevail.

$$f_{nt} > \sigma_n , \qquad f_{pt} > \sigma_p , \qquad s_{np} > \tau_{np} \tag{5.17}$$

where f_{nt}, f_{pt}, s_{np} are normal tensile and shear strengths of brick masonry.

If the brick masonry is subjected off-axis tensile stresses, the failure will not occur as long as the following prevail.

$$\frac{f_{nt}}{\cos\theta} > \sigma_n , \qquad \frac{f_{pt}}{\sin\theta} > \sigma_p , \qquad \frac{s_{np}}{\cos\theta\sin\theta} > \tau_{np} \tag{5.18}$$

The limiting value required to define the onset of cracking is established as follows:

(i) In the biaxial tension stress state,

$$\begin{aligned} \sigma_{no} &= f_t \\ \sigma_{po} &= f_t \end{aligned} \tag{5.19}$$

(ii) In tension compression stress state, linearly decreasing tensile strength expressions are used as given below:

$$\sigma_{po} = f_t\left(1 + \frac{\sigma_n}{f_{nc}}\right) \qquad \sigma_n \le 0 \tag{5.20}$$

or $\sigma_{no} = f_t\left(1 + \frac{\sigma_p}{f_{pc}}\right) \qquad \sigma_p \le 0$ (5.21)

where f_{nc} and f_{pc} are the uniaxial compressive stresses normal and parallel to the bed joint.

These expressions incorporate the fact that compression in one direction favours micro cracking in the orthogonal directions, thus reducing tensile capacity.

The elastic stress-strain matrix($\underset{\sim}{D}$) is modified according to mode of failure as defined in Table 5.1. In cracking criterion tensile cracking normal and parallel to bed joints and shear failure along the bed joint are considered.

Tensile Failure: If tensile failure occurs, the stresses normal to the crack and the shear stress are reduced but the stress parallel to the crack are not altered. Poission's ratio effect is neglected for cracked brick masonry.

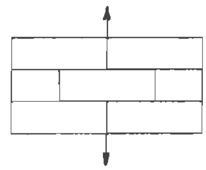
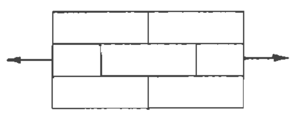
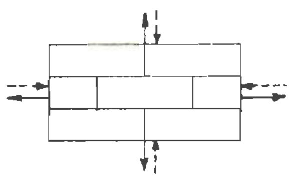
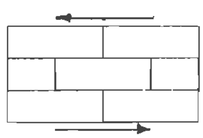
Shear Failure: If the shear failure occurs, stresses normal and parallel to bed joint are not altered. But the shear stress is reduced to a constantnt value so that ultimate stress is not allowed to increase for subsequent loading. While calculating the element stiffness matrices, the $\underset{\sim}{D}$ matrix is modified by reducing the shear modulus to one tenth of its original value.

(e) Strain Softening Rule: The use of brittle collapse model with the onset of cracking results in immediate complete redistribution of the tensile stresses in the cracked region. This sudden redistribution of stresses results in premature failure of masonry panel. So the computed strength is always less than the experimental strength.

Amongst several approaches a gradual release of the stress component normal to the crack plane is adopted in this investigation(Fig. 5.5c). The normal stress σ_n (and or σ_p)is given by

$$\sigma_n = \alpha_m\, f_t\left(1 - \frac{\varepsilon_n}{\varepsilon_m}\right) \qquad \text{for } \varepsilon_t \le \varepsilon_i \le \varepsilon_m \qquad (5.22)$$

Table 5.1 Stress-strain Matrices for Different Modes of Failure of Brick Masonry

No.	Mode of Failure	$\underline{D}$
1	Tension Failure Normal to Bed Joint 	$\begin{matrix} E & 0 & 0 \\ 0 & E' & 0 \\ 0 & 0 & \phi G \end{matrix}$
2	Tension Failure Parallel to Bed joint 	$\begin{matrix} E' & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & \phi G \end{matrix}$
3	Biaxial Tension or Biaxial Compression failure 	$\begin{matrix} E' & 0 & 0 \\ 0 & E' & 0 \\ 0 & 0 & \phi G \end{matrix}$
4	Shear Failure. 	$\begin{matrix} \frac{E}{1-\nu^2} & \frac{E\nu}{1-\nu^2} & 0 \\ \frac{E}{1-\nu^2} & \frac{E\nu}{1-\nu^2} & 0 \\ 0 & 0 & \psi G \end{matrix}$

E' = reduced value of modulus of elasticity

ϕ =0.005 and ψ = 0.01

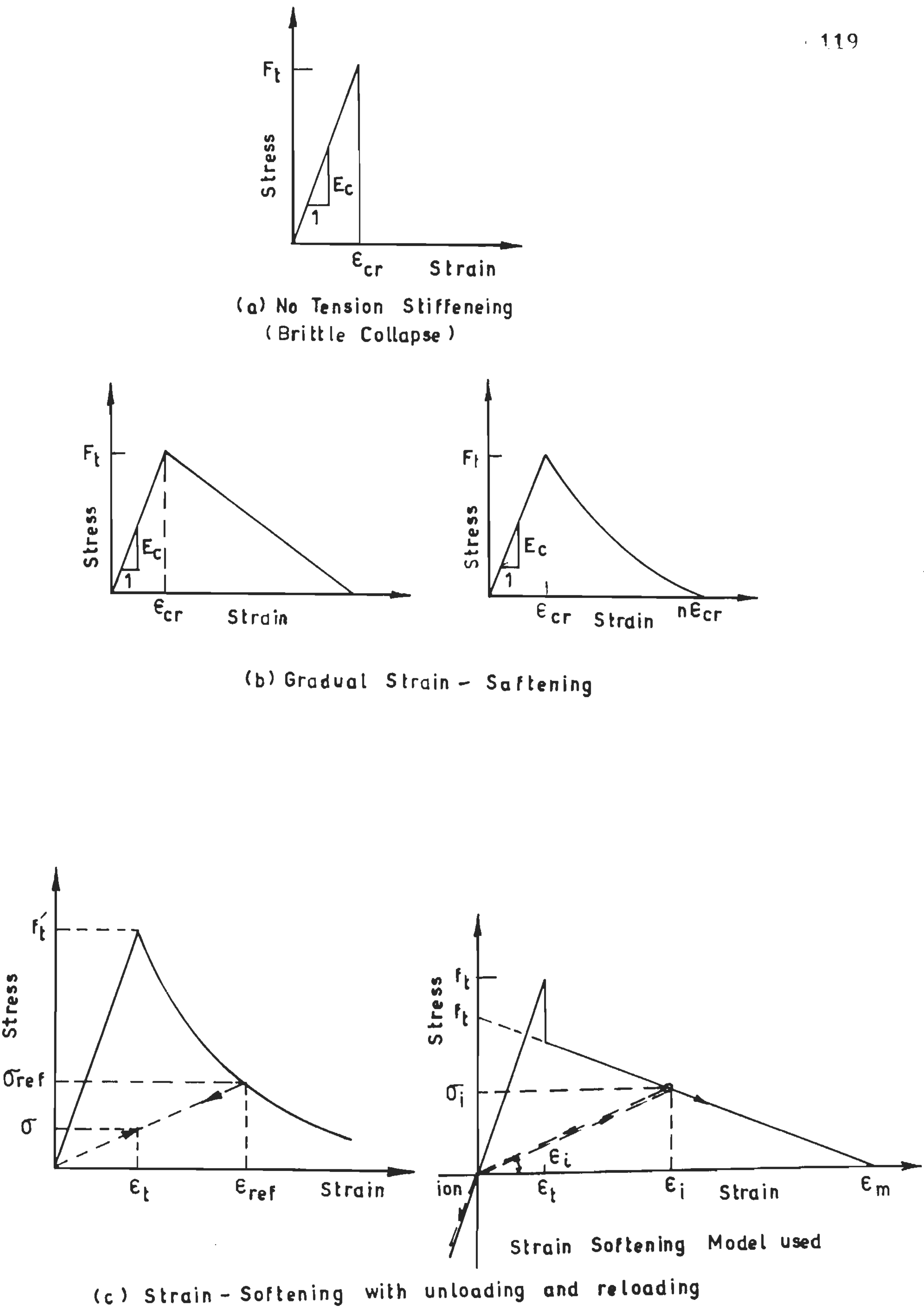


Fig. 5.5 Typical Strain-Softening Models

or by $\sigma_n = \sigma_i \dfrac{\varepsilon_n}{\varepsilon_i}$ for $\varepsilon_n < \varepsilon_i$ (5.23)

where f_t is the ultimate tensile strength of brick masonry. ε_i is the current tensile strain in the material direction n. α_m and ε_m are the tension stiffening parameters. The reduced elastic modulus, E', is given by

$$E' = \alpha f_t(1- \varepsilon_i/ \varepsilon_m)/\varepsilon_i \qquad \varepsilon_t \le \varepsilon_i \le \varepsilon_m$$

Unloading and reloading of cracked masonry is assumed to follow the linear behaviour shown in Fig.5.5c

The redistribution of stresses due to cracking in other sampling points or further loading may close previously open cracks. If the crack closes i.e. if the strain component normal to the crack plane becomes negative, the masonry behaves as uncracked in the corresponding direction, but the crack direction and the maximum tensile strain continue to be stored. The value ε_i is modified to simulate bond deterioration during reloading.

5.4 Conclusions

The proposed two-dimensional material model is suitable for finite element analysis of brick masonry subjected to in-plane loading The state of the art of the material model is also critically reviewed. The salient features of the proposed model are given below.

- The failure behaviour of brick masonry in various stress regimes is modelled using an anisotropic failure criterion. The generalised three-dimensional failure criterion [Tsai and Wu(1971)] is modified for a two-dimensional situation. The linear term is removed as the failure criterion is used in a compressive stress regime. The failure criterion is transformed to the material axes, normal and parallel to the bed joint, although this criterion can be used with reference to the other axes also. The strength parameters have been obtained from the biaxial experimental results of Page(1981). The sensitivity analysis is carried out to select the particular type of test to be used to determine the value of the strength parameter related to the interaction between the stresses, normal and parallel to the bed joint.

- Strain hardening in compression state of stress of brick masonry is discussed, although it is considered as elasto-perfectly plastic material.

- The smeared cracking model has been used to model the cracking behaviour of brick masonry. The maximum tensile stress criterion extended to anisotropic material has been used to predict the tensile strength and shear strength of masonry. In biaxial tension-compression state of stress, the effect of compressive stress on the reduction of tensile strength is considered, although the effect of tensile stress on the reduction of compressive strength is neglected as the effect is very small.

- The brick masonry in tension is considered as a linear elastic material. Tension softening model is used to allow the gradual release of stresses in the fracturaed region. Closing and reopening for cracks is allowed in the model following the secant path.

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FINITE ELEMENT MODEL FOR THE IN-PLANE BEHAVIOUR OF BRICK MASONRY

6.1 Introduction

Since its inception in the late fifties, tremendous advances have been made both on the mathematical foundations and generalisation of the method to solve problems in various areas of engineering analysis. The fast development in computer hardware has extended the applications of this method to problems involving geometrical and material nonlinearities, nonconservative loading etc. A detail description of this method is hardly necessary in view of large number of text books on the topic are now available. However, for completeness some points relevant to the present study are discussed in brief.

In this Chapter an incremental, iterative finite element model for the analysis of brick masonry structures subjected to in-plane loading is developed incorporating the material model for brick masonry described in Chapter 5. Lastly details of nonlinear program is presented.

6.2 Nonlinear Finite Element Model

The modelling is carried out at "macro" level with each element encompassing several bricks and joints, so that complete structural units can be analysed with greater computational efficiency and minimum cost. The material characteristics developed in Chapter 5 are incorporated into the finite element model to reproduce nonlinear behaviour of masonry. The material characteristics of the mortar are adopted to model the nonlinear behaviour due to slip and tensile cracking. For the present analysis the elastic behaviour of frame is considered since the aim of the analysis is to demonstrate the adequacy of the material model for the masonry. However the provision is made for adopting the material nonlinearity of the frame. For the analysis of infilled frames and shear walls elements capable of simulating the behaviour of masonry, frame and the joints between them are required. The characteristics of each of these elements are shown in Fig.6.1 and described below.

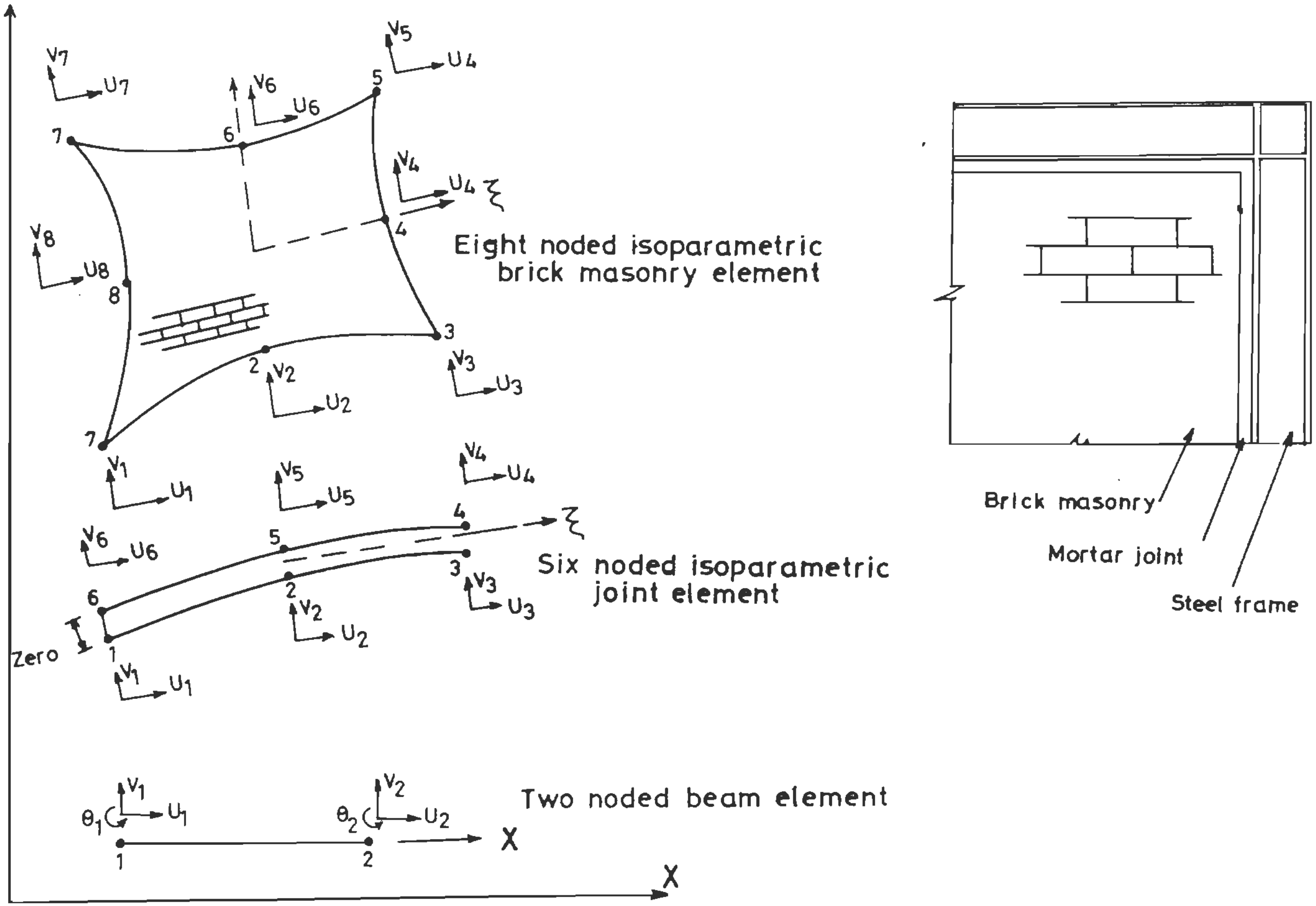

Fig. 6.1 Various Elements Used for Modelling Infilled Frame

6.2.1 Types of Element Used

In the present study elements used are as follows:

- o Eight noded isoparametric elements for modelling of masonry
- o six noded isoparametric interface elements for modelling the mortar joints between the frame and masonry infill and the mortar joint between the masonry wall and supporting base.
- o Two noded beam elements for modelling the frame.

Masonry Elements: Most brick masonry structures are rectangular in configuration and in a biaxial state of stress under in-plane loading. To model such structures eight noded isoparametric plane stress elements(Fig.6.2)with 2x2 Gaussian integration have been used. The shape functions in the natural co-ordinate system for the element are written as follows.

(a) For corner nodes

$$N_i^e = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i)(\xi\xi_i + \eta\eta_i - 1)$$

$$i = 1,\ 3,\ 5,\ 7 \qquad (6.1)$$

(b) For mid-side nodes

$$N_i^e = \frac{\xi_i^2}{2}(1+\xi\xi_i)(1-\eta^2) + \frac{\eta_i^2}{2}(1+\eta\eta_i)(1-\xi^2)$$

$$i = 2,\ 4,\ 6,\ 8 \qquad (6.2)$$

Using the shape functions the strain matrix, $\underline{B}$, can be evaluated in ξ-η co-ordinate system and then transformed to x-y co-ordinate system using the Jacobian matrix. The details of the mathematical procedure of the evaluation of element stiffness are given in Appendix I.

Two Noded Beam Element: Two noded beam elements with three degrees of freedom(Fig.6.3) at each node have been used for the modelling of frame. The stiffness matrix has been modified to take into account the eccentricity between the centre line of the frame and the outer edge of

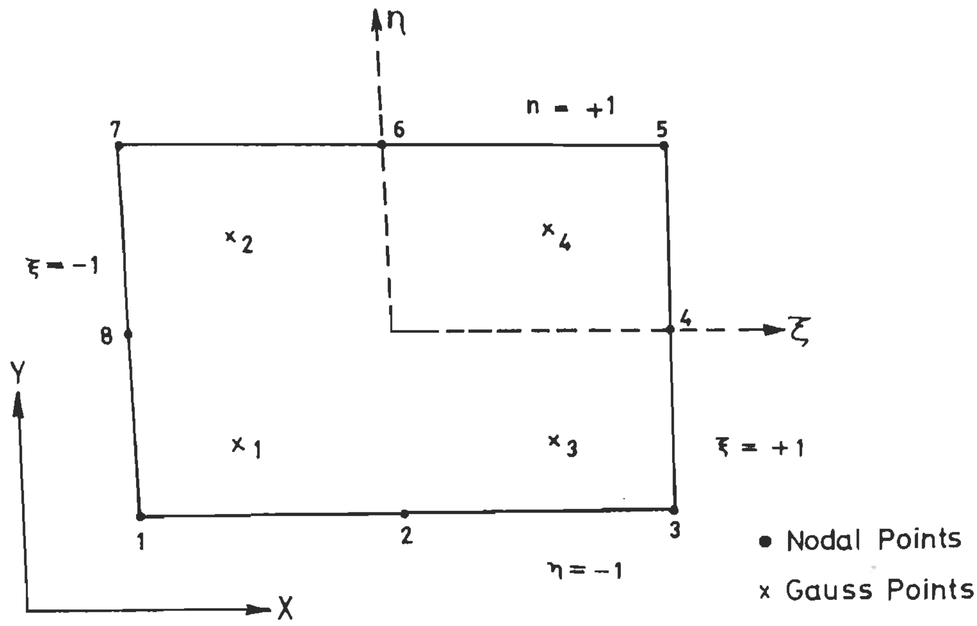

Fig. 6.2 Eight Noded Plane Stress Isoparametric Element

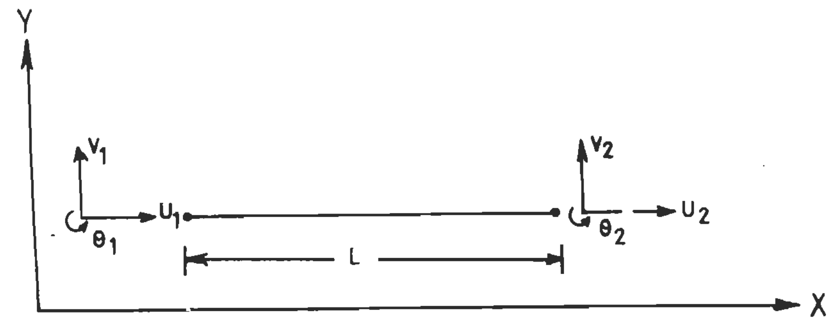

Fig. 6.3 Two Noded Beam Element

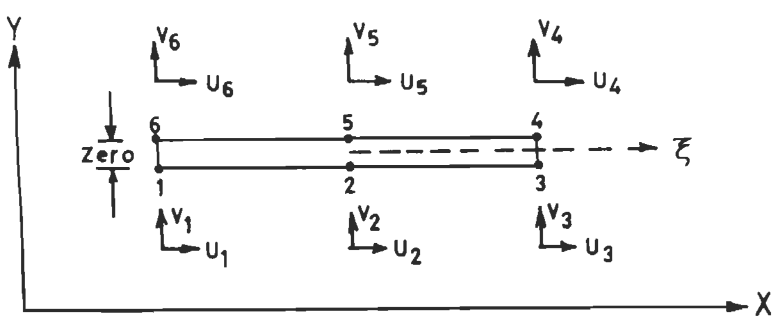

Fig. 6.4 Six Noded Isoparametric Interface Element

the mortar joints. The evaluation of the modified stiffness matrix is given in Appendix II.

Material Model: The linear elastic behaviour of the frame is considered as the main objective is to demonstrate the accuracy and effectiveness of material model for masonry. The beam elements have therefore, been assumed to remain elastic throughout the loading history. In this investigation a steel frame with masonry infill is analysed considering the elastic properties specified by the manufacturer.

Interface Elements: Conventional finite element method is inadequate to model the behaviour of the mortar joint between infill and the frame. Special elements known as 'interface' elements are employed in the analysis. This concept was originally proposed by Goodman *et al.*(!968) and later used by many investigators. A number of improved model is now reported in the literature. Comprehensive reviews and applications of these models are summarised by Desai(1981), Wolf(1985) and Sharma and Desai(1992) among others.

In the present study six noded interface elements (Fig.6.4)with 2x1 Gaussian integration are used to model the mortar joints. The element is assumed to posses only normal and shear stiffness. Thickness of the element is assumed to be negligible. The element Stiffness formulation is given in Appendix III. The stress strain relation is given below.

$$\begin{Bmatrix} \tau \\ \sigma \end{Bmatrix} = \begin{bmatrix} G_j & 0 \\ 0 & E_j \end{bmatrix} \begin{Bmatrix} \gamma \\ \epsilon \end{Bmatrix} \qquad (6.3)$$

where G_j and E_j are shear modulus and Young,s modulus of the interface elements.

Material Model: The failure of the mortar joint at the interface occurs due to separation of masonry from the frame and the slip between the two. This behaviour has been modelled by assuming mortar to be linear elastic material till failure. A Coulomb type trilinear failure criterion with tension cut off has been used for initiation of separation or slip between the frame and the infill as the tensile bond strength between the steel frame and the masonry is negligible. The stress strain relation and failure criterion are shown in Fig.6.5. For masonry shear wall a bilinear Coulomb type failure criterion is used as shown in Fig.6.5. The strength parameters are derived experimentally. If failure initiates or tension failure followed by shear failure

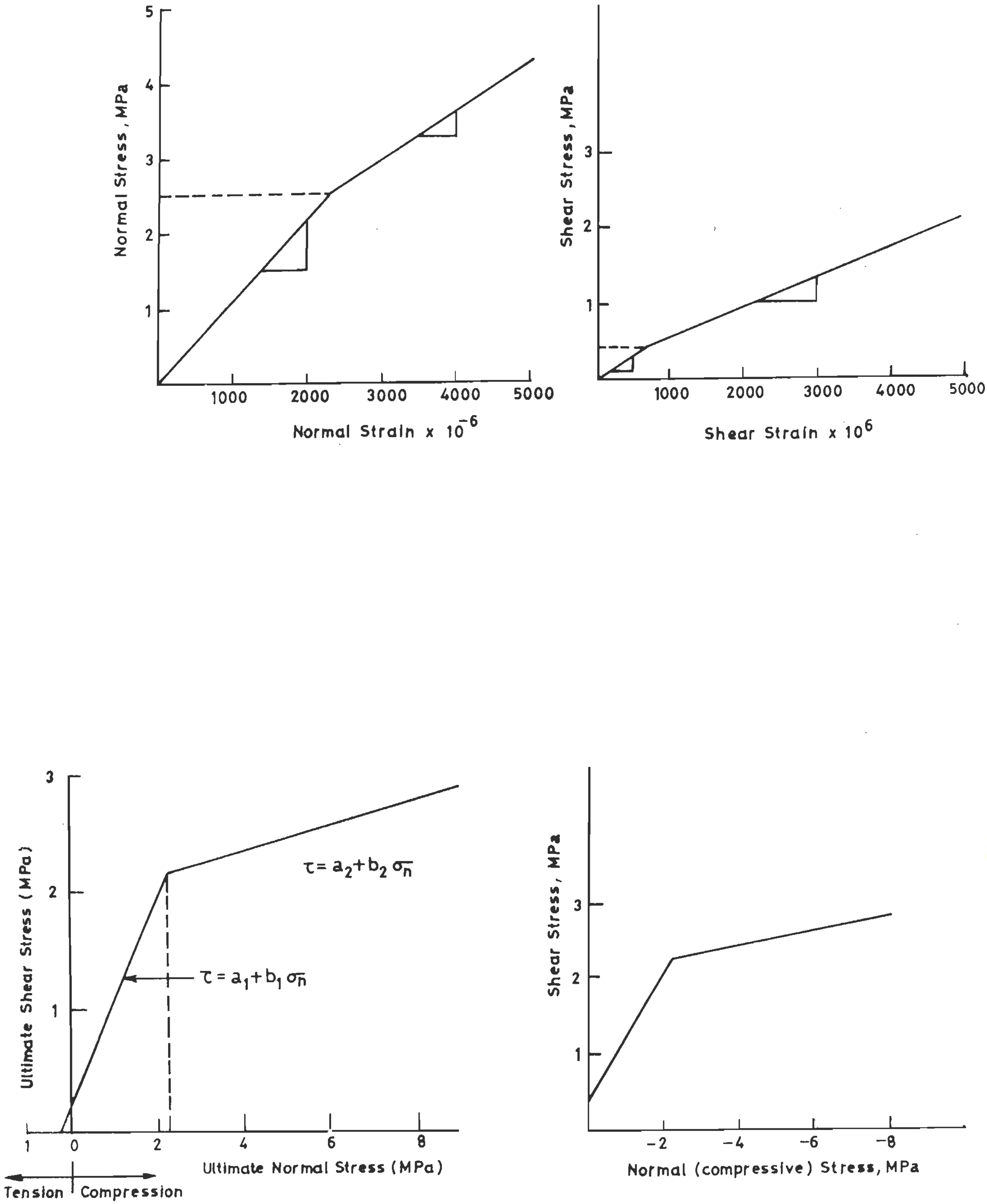

Fig. 6.5 Material Model for Interface Element

occurs at a Gauss point, the stress strain matrix is modified as follows:

$$\begin{bmatrix} D_j \end{bmatrix} = \begin{bmatrix} \phi G_J & 0 \\ 0 & \phi E_j \end{bmatrix} \tag{6.4}$$

where ϕ = 0.005

The shear failure occurs when the joint is subjected to compression (i.e. a sliding failure), the stress strain matrix ,$\underline{D}_j$, is modified by reducing the shear modulus as given by

$$\begin{bmatrix} D_j \end{bmatrix} = \begin{bmatrix} \psi G_J & 0 \\ 0 & E_j \end{bmatrix} \tag{6.5}$$

where ψ = 0.01

Apart from modifying the stiffness properties of the element, the stresses in the Gauss points are also adjusted according to the mode of failure. When a tension failure occurs, both the normal and shear stresses are reduced to a negligibly small value (o.05 MPa). In the case of a shear failure, the stress is reduced to a level consistent with failure but the shear stress is constrained to that value for all subsequent loading.

6.2.2 Compatibility of Displacements

Three types of elements are used in the finite element analysis of the masonry infilled frame. Beam elements have three degrees of freedom as against the two degrees of freedom of the interface and and masonry elements. To achieve compatibility between the elements at their common nodal points, dummy rotational degrees of freedom have been introduced at the nodes of the interface and masonry elements. This is achieved by introducing zero coefficients corresponding to the dummy degrees of freedom into the stiffness matrices of the joint and brick masonry elements.

In the case of interface element, the strain-displacement relation with a dummy degree of freedom is written as,

$$\begin{Bmatrix} \gamma \\ \\ \epsilon \end{Bmatrix} = \begin{bmatrix} \partial/\partial x & 0 & 0 \\ & & \\ 0 & \partial/\partial y & 0 \end{bmatrix} \begin{Bmatrix} u \\ v \\ \theta \end{Bmatrix} \qquad (6.6)$$

where θ is the rotational dummy degree of freedom.

Similarly, for the masonry element, the strain-displacement relation is given as

$$\begin{Bmatrix} \epsilon_p \\ \epsilon_n \\ \gamma \end{Bmatrix} = \begin{bmatrix} \partial/\partial p & 0 & 0 \\ 0 & \partial/\partial n & 0 \\ \partial/\partial n & \partial/\partial p & 0 \end{bmatrix} \begin{Bmatrix} u \\ v \\ \theta \end{Bmatrix} \qquad (6.7)$$

Obviously the rotations corresponding to the dummy degrees of freedom are zero and hence the equations corresponding to those degrees of freedom are not considered during the solution process.

6.2.3 Solution of Nonlinear Equilibrium Equations

Using finite element procedure , the discrete equations governing the nonlinear behaviour of a structure can be derived from the principle of virtual work. The resulting nonlinear equations of equilibrium can be written as

$$\psi(\underline{d}) = \underline{f} - \underline{p}(\underline{d}) = 0 \qquad (6.8)$$

where, $\psi(\underline{d})$ is the residual force vector; $\underline{f}$ is the external force vector; $\underline{p}(\underline{d})$ is the internal force vector; $\underline{d}$ is the vector of nodal displacements.

Due to the nonlinear nature of Eqn.(6.8), an incremental solution procedure is usually adopted in order to trace the response of the structure. But pure incremental method is very much unreliable because no check is performed on the global equilibrium, and the method can lead to a progressive drift off from the true equilibrium path. Its performance gets improved when iteration is performed at the end of increment for balancing the residual forces. During the typical load increment, the linearised equations to be solved for each iteration (say i), have the form

$$\underline{K}_i \ \delta\underline{d}_i = \underline{\psi}_i \tag{6.9}$$

where, $\delta\underline{d}_i$ is the incremental nodal displacement during the ith iteration. In the finite element model, the profile solver is used for the solution of the Eqn.(6.9).

6.2.4 Modified Newton Raphson Method

To overcome the difficulty of having to solve a completely new system of equations at each iteration stage as in Newton Raphson method, stiffness matrix is occasionally updated. If the stiffness matrix is never updated after the initial stiffness is evaluated, the method is called the initial stress method. This is clearly more economical at each step but the large number of iterations are necessary for convergence if no accelerators are used. In many cases, however, this process has an overall economy. Better convergence properties can be achieved if the stiffness is updated once per increment, or more efficiently after some load increments and at a particular iteration, which has been adopted in the present work.

6.2.5 Convergence Criterion

A convergence criterion is required for determining when the current solution is close enough to the true or equilibrating solution to terminate the iteration. The convergence criterion and tolerances must be carefully chosen, so as to provide accurate and economic solutions. A loose convergence criterion leads to inaccurate results. Whereas, a tight criterion requires too much effort in obtaining unnecessary accuracy.

The convergence criteria usually employed in the nonlinear analysis of structure are based on displacements, residual forces or energy. The displacement based criterion is not advisable as it can be misleadingly satisfied by a slow convergence rate. Far more reliable are the residual forces based criteria, as they check that the equilibrium of forces has been achieved within a specified tolerance in the current increment. An overall check for all degrees of freedom can be expressed as

$$\frac{\| \psi \|}{\| f \|} \le \varepsilon_t \tag{6.10}$$

where, $\| \psi \| = \left[\sum_{j=1}^{N_d} \psi^2 \right]^{1/2}$

$$\| f \| = \left[\sum_{j=1}^{N_d} \underline{f}^2 \right]^{1/2}$$

where, N_d is the total number of degrees of freedom; and ε_t is the specified tolerance; vector f contains the total external loads i.e., the external applied loads plus the computed reactions. The addition of the contribution of the reactions to the total external loads is essential to monitor convergence when there are zero applied loads, for instance in problems involving only prescribed displacements or thermal loading.

6.3 THE FINITE ELEMENT PROGRAM (PROGRAM "BMAL-2D")

The computer program "BMAL-2D" has been developed by modifying the nonlinear plane stress finite element program presented by Owen and Hinton(1980) incorporating the deformation characteristics and failure criterion described in Chapter 5. The general structure of the program is illustrated in the flow chart given in Fig.6.6. Variable names used in the program are similar to those used by Owen and Hinton(1980). For consistency new variable names have also been made five characters in length.

The program is capable of analysing brick masonry structures such as masonry shear walls and infilled frames. Elastic analysis, if desired, can also be carried out by omitting the material nonlinearity and local failure.

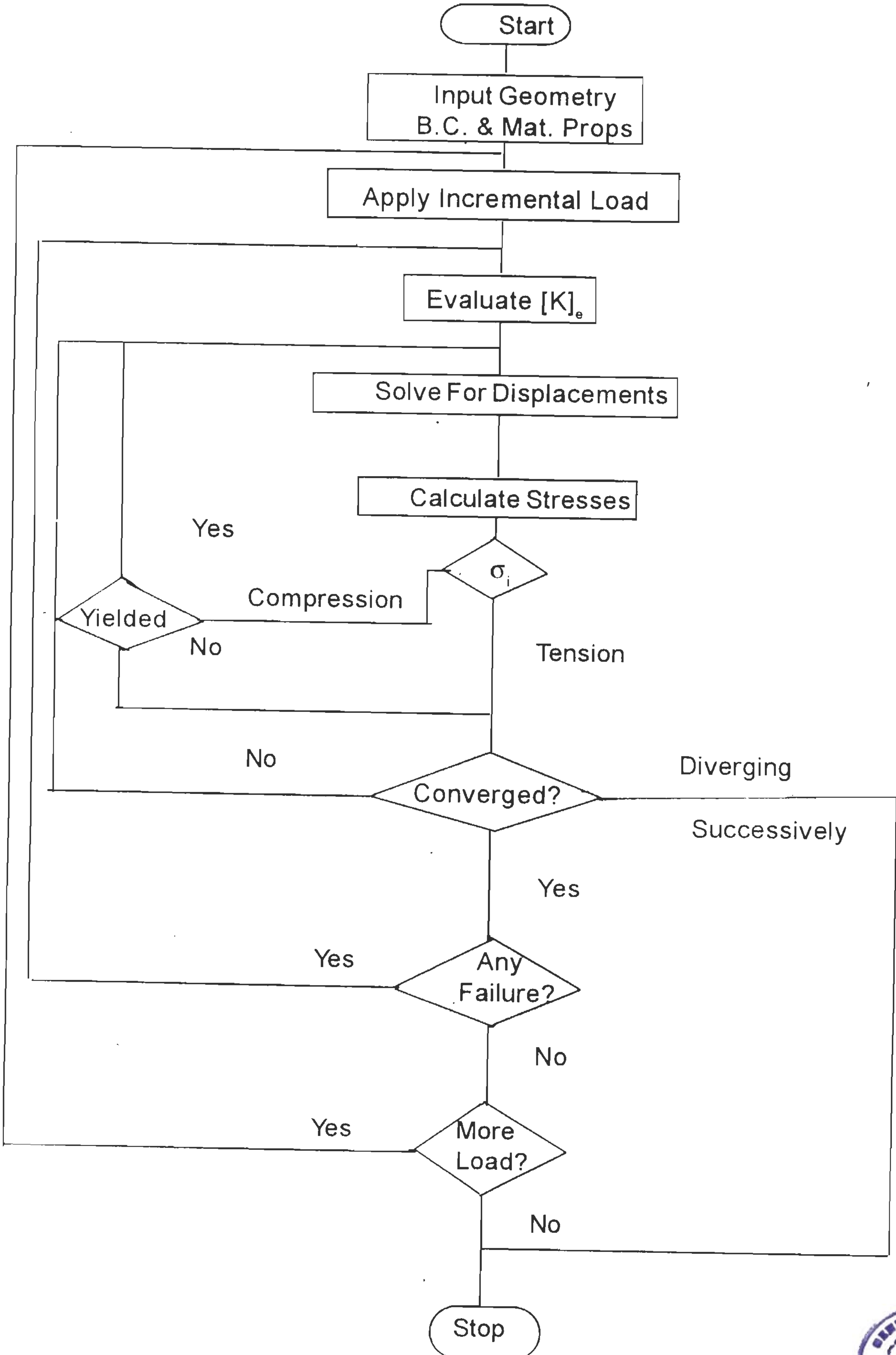

Fig. 6.6 A Flow Chart for Finite Element Model

Elastic behaviour is assumed for the first (small) load increment. Stresses are calculated at each Gauss point, and if any are tensile, the local behaviour is assumed to be elastic brittle; for other cases non-linear material behaviour is assigned.

At a given load level, iteration continues until the unbalanced nodal forces associated with material non-linearities are less than a chosen tolerance. The stresses are then checked for violation of the failure surface. If failure is indicated at a Gauss point, then its elasticity matrix, $\underline{D}$, is modified in accordance with its mode of failure. With tension or shear-tension cracks, Young's modulus normal to the crack is gradually reduced as per tension stiffening rule employed in the present investigation. Stresses are also gradually released using the same tensile stiffening rule. Shear modulus and shear stress are also reduced to a small value. For shear failure in joints in which the direct stress is compressive, shear stresses maintained constant to simulate residual friction in the joint.

The procedure is repeated until convergence occurs at the load level under consideration. The applied load is then increased to the next increment and the process repeated. Final collapse is assumed when the solution fails to converge.

6.4 Conclusions

- In this Chapter a finite element model for the analysis of brick masonry structures subjected to in-plane loads has been described. The model incorporates the material model for brick masonry described in Chapter 5. Since the influence of the mortar joints on the properties of brick masonry has been considered in the material model relatively large continuum elements encompassing several bricks and joints have been used. The use of a coarser mesh of this type leads to considerable savings in computer time. The finite element program is incremental and iterative, with iterations being performed for material non-linearity and progressive cracking.
- Beam and interface elements are also incorporated in the finite element program for the analysis of masonry infilled frame and shear walls. The slip and tensile cracking at the interface is modelled using interface elements. The nonlinear behaviour of the mortar joint due to slip and cracking is taken into account by employing Coulomb type failure criterion.

- o In Chapter 7, one infilled frame and several masonry shear walls are analysed using the finite element program and the results compared with to experimental results.

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Eight Noded Isopramétric P lane Stress Element

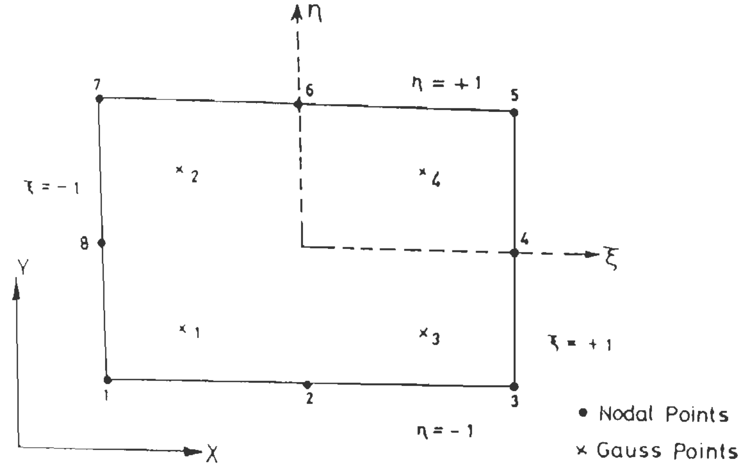

Fig. (I.1) Eight Noded Plane Stress Element

The shape functions of the nodes are given by:

$$N_1 = \frac{1}{4} (1 - \xi) (1 - \eta) (-1 - \xi - \eta) \quad (I.1)$$

$$N_2 = \frac{1}{2} (1 - \xi^2) (1 - \eta) \quad (I.2)$$

$$N_3 = \frac{1}{4} (1 + \xi) (1 - \eta) (-1 + \xi - \eta) \quad (I.3)$$

$$N_4 = \frac{1}{2} (1 + \xi) (1 - \eta^2) \quad (I.4)$$

$$N_5 = \frac{1}{4} (1 + \xi) (1 - \eta) (-1 + \xi + \eta) \quad (I.5)$$

$$N_6 = \frac{1}{2} (1 - \xi^2) (1 + \eta) \quad (I.6)$$

$$N_7 = \frac{1}{4} (1 - \xi) (1 + \eta) (-1 - \xi + \eta) \quad (I.7)$$

$$N_8 = \frac{1}{2} (1 - \xi) (1 - \eta^2) \quad (I.8)$$

Displacements $u(\xi, \eta)$ and $v(\xi, \eta)$ at any point within the element may be calculated using the shape functions

$$\begin{Bmatrix} u(\xi, \eta) \\ v(\xi, \eta) \end{Bmatrix} = \sum_{i=1}^{8} \begin{bmatrix} N_i(\xi, \eta) & 0 \\ 0 & N_i(\xi, \eta) \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \end{Bmatrix} \quad (I.9)$$

Derivations of the displacement function with respect to the natural co-ordinates may be determined is follows:

$$\begin{Bmatrix} \epsilon_\xi \\ \epsilon_\eta \\ \gamma_{\xi\eta} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial\xi} & 0 \\ 0 & \frac{\partial}{\partial\eta} \\ \frac{\partial}{\partial\eta} & \frac{\partial}{\partial\xi} \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \end{Bmatrix}$$

$$= \sum_{i=1}^{8} \begin{bmatrix} \frac{\partial N_i}{\partial\xi} & 0 \\ 0 & \frac{\partial}{d\eta} \\ \frac{\partial N_i}{\partial n} & \frac{\partial N_i}{\partial\xi} \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \end{Bmatrix} \qquad (I.10)$$

To convert (ϵ_ξ, ϵ_η, $\gamma_{\xi\eta}$) to (ϵ_x, ϵ_y, γ_{xy}), the terms $\frac{\partial N_i}{\partial\xi}$, $\frac{\partial N_i}{\partial\eta}$ should be transforoed to $\frac{\partial N_i}{\partial x}$, $\frac{\partial N_i}{\partial y}$. This is achieved as

$$\begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{Bmatrix} = \underline{J}^{-1} \begin{Bmatrix} \frac{\partial N_i}{\partial\xi} \\ \frac{\partial N_i}{\partial\eta} \end{Bmatrix} \qquad (I.11)$$

$$\text{where} \quad \underline{J} = \sum_{i=1}^{8} \begin{bmatrix} \frac{\partial N_i}{\partial\xi} & \frac{\partial N_i}{\partial\xi} \\ \frac{\partial N_i}{\partial\eta} & \frac{\partial N_i}{\partial\eta} \end{bmatrix} \begin{Bmatrix} x_i \\ y_i \end{Bmatrix} \qquad (I.12)$$

where x_i, y_i are co-ordinates of the nodes of the element with respect to the x,y co-ordinate system. Knowing [J], $[J]^{-1}$ may easily be worked out and hence $\partial N_i/\partial x$ and $\partial N_i/\partial y$ may be determined. Then the strains (ϵ_x, ϵ_y, γ_{xy}) may be expressed as

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \end{Bmatrix} \tag{I.13}$$

In shorthand form

$$\underline{\epsilon} = \underline{B}\ \underline{d} \tag{I.14}$$

knowing $\underline{B}$ and the material constitutive matrix $\underline{D}$ expressed for plane stress conditices as

$$\underline{D} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{I.15}$$

So $\underline{k}^e = \int_V \underline{B}^T \underline{D}\ \underline{B}\ dv$

$\int_V B^T D\ B\ dv$ may be evaluated using numerical integration, Gaussian quadrature of $(B^T DB)$ is evaluated at the four Gauss points shown in Fig. I.1 and added together to form the stiffness matrix of the element. For a 2 x 2 Gaussian integration,

$$\underline{K}^e = t \sum_{p=1}^{2} \sum_{q=1}^{2} \sum_{i=1}^{8} \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & \frac{\partial N_i}{\partial y} \\ 0 & \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \end{bmatrix} \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \sum_{j=1}^{8} \begin{bmatrix} \frac{\partial N_j}{\partial x} & 0 \\ 0 & \frac{\partial N_j}{\partial y} \\ \frac{\partial N_j}{\partial y} & \frac{\partial N_j}{\partial x} \end{bmatrix} |J| W_p\ W_q \tag{I.16}$$

APPENDIX II

Two Noded Modified Beam Element

The procedure of transforming the stiffness matrix of a typical beam element shown at the bottom beam of the infilled frame in Fig. II.1 is explained in the Appendix. The element is redrawn for clarity in Fig. II.2.

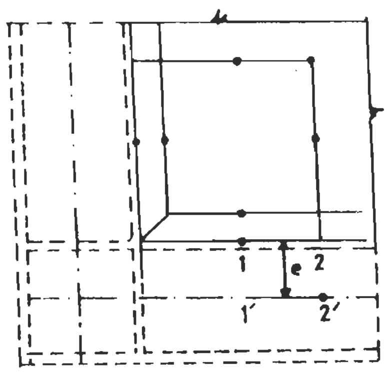

Fig. (II.1) Modelling of Frame

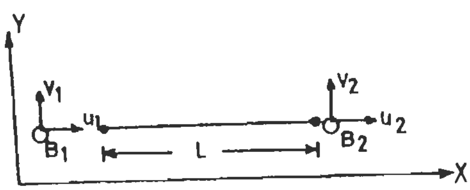

Fig. (II.2) Two Noded Beam Element

The stiffness matrix of the beam element at the 1'-2' location is written as

$$\underline{K}_{1',-2'} = \frac{E}{L^3}\left[\begin{array}{ccc:ccc} AL^2 & 0 & 0 & -AL^2 & 0 & 0 \\ 0 & 12I & 6IL & 0 & -12I & 6IL \\ 0 & 6IL & 4IL^2 & 0 & -6IL & 2IL^2 \\ \hdashline -AL^2 & 0 & 0 & AL^2 & 0 & 0 \\ 0 & -12I & -6IL & 0 & 12I & -6IL \\ 0 & 6IL & 2IL^2 & 0 & -6IL & 4IL^2 \end{array}\right] \qquad (II.1)$$

The displacements (u_1', v_1', θ_1') and (u_2', v_2', θ_2') are related to (u_1, v_1, θ_1) and (u_2, v_2, θ_2) as follows

$$\begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ \hdashline u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} = \left[\begin{array}{ccc:ccc} 1 & 0 & -e & & & \\ 0 & 1 & 0 & & [0] & \\ 0 & 0 & 1 & & & \\ \hdashline & & & 1 & 0 & -e \\ & [0] & & 0 & 1 & 0 \\ & & & 0 & 0 & 1 \end{array}\right] \begin{Bmatrix} u_1' \\ v_1' \\ \theta_1' \\ \hdashline u_2' \\ v_2' \\ \theta_2' \end{Bmatrix} \qquad \text{(II.2)}$$

In shorthand form

$$\underset{\sim}{u} = \underset{\sim}{T}\ \underset{\sim}{u}' \qquad \text{(II.3)}$$

using the transformation matrix $\underset{\sim}{T}$, $\underset{\sim}{K}_{1',-2'}$ may be transformed to $\underset{\sim}{K}_{1-2}$ as follows

$$\underset{\sim}{K}_{1-2} = \underset{\sim}{T}^T\ \underset{\sim}{K}_{1',-2'}\ \underset{\sim}{T} \qquad \text{(II.4)}$$

After the transformation, $\underset{\sim}{K}_{1-2}$ may be written as

$$\underset{\sim}{K}_{1-2} = \left[\begin{array}{ccc:ccc} \frac{EA}{L} & 0 & \frac{-EAe}{L} & \frac{-EA}{L} & 0 & \frac{EAe}{L} \\ & \frac{12EI}{L^2} & \frac{6EI}{L^2} & 0 & \frac{12EI}{L^2} & \frac{6EI}{L^2} \\ & & \left(\frac{EAe^2}{L}+\frac{4EI}{L}\right) & \frac{EAe}{L} & \frac{6EI}{L^2} & \left(\frac{EAe^2}{L}+\frac{2EI}{L}\right) \\ \hdashline & & & \frac{EA}{L} & 0 & \frac{-EAe}{L} \\ & [symm] & & & \frac{12EI}{L^2} & \frac{-6EI}{L^2} \\ & & & & & \left(\frac{EAe}{L}+\frac{4EI}{L}\right) \end{array}\right]$$

(II.5)

The $\underset{\sim}{K}_{1-2}$ matrix given in expression (II.5) defines the completely transformed matrix for the element in the bottom beam. However for the two vertical columns and for the top beam additional rotational transformation is necessary. Such a transformation is shown schemeically is Figure II.3.

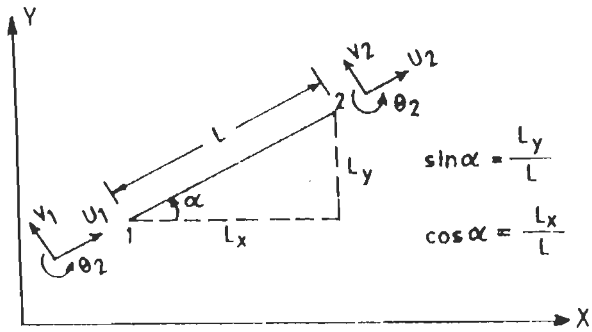

Fig. (II.3) Procedure of Transformation

The transformation matrix $\underline{T}_r$, in this case, is

$$\underline{T}_r = \left[\begin{array}{ccc:ccc} \cos\alpha & -\sin\alpha & 0 & & & \\ \sin\alpha & \cos\alpha & 0 & & [0] & \\ 0 & 0 & 1 & & & \\ \hdashline & & & \cos\alpha & -\sin\alpha & 0 \\ & [0] & & \sin\alpha & \cos\alpha & 0 \\ & & & 0 & 0 & 1 \end{array}\right] \qquad \text{(II.6)}$$

To account for various orientations, pre and post multiplications of the stiffness matrix is carried out as $\underline{T}_r^T\ \underline{K}_{1-2}\ \underline{T}_r$.

Appendix-III

Six Noded Isoparametric One Dimensional Interface Element

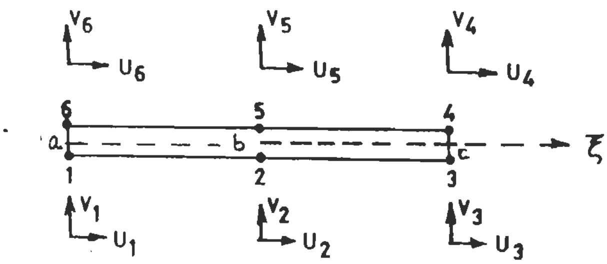

Fig. III.1 Six Noded Interface Element

The interface element nodes, (1, 6), (2, 5), (3, 4), and the centre line nodes are defined by the same coordinates (Fig. III.1). The global relative displacements, Δu_a and Δv_a are defined as

$$\Delta u_a = u_6 - u_1 \tag{III.1}$$

$$\Delta v_a = v_6 - v_1 \tag{III.2}$$

where u_1, u_6, v_1, v_6 are nodal displacements in x and y directions.

$$\Delta a = \begin{Bmatrix} \Delta u_a \\ \Delta v_a \end{Bmatrix} \tag{III.3}$$

$$\Delta\, u_a = u_6 - u_1 \tag{III.4}$$

and
$$\Delta\, v_a = v_6 - v_1 \tag{III.5}$$

where u_1, u_6, v_1 and v_6 are nodal displacements in x and y directions

$$\begin{Bmatrix} \Delta u_a \\ \Delta v_a \end{Bmatrix} = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_6 \\ v_6 \end{Bmatrix} \tag{III.6}$$

$$= \begin{bmatrix} -I_2 & I_2 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_6 \end{Bmatrix} \tag{III.7}$$

$$= \underset{\sim}{T}_a\, \delta_a$$

where, $\underline{T}_a$ transformation matrix

In general the differential displacement can be expressed function of nodal displacement δ^e,

$$\left\{\Delta_a\right\}^e = \begin{bmatrix} \Delta u_a \\ \Delta v_a \\ \Delta u_b \\ \Delta v_b \\ \Delta u_c \\ \Delta v_c \end{bmatrix} = \underline{T}\ \underline{\delta}^e \tag{III.8}$$

(i) Relative displacement at any point

As in the case of isoparametric elements, the relative dispalcements at any point can be expressed in terms of nodal dispalcements.

$$\Delta u \text{ at any point} = \sum N_i(\xi)\ \Delta\ u_i \tag{III.9}$$

$$= N_a \Delta u_a + N_b \Delta u_b + N_c \Delta u_c \tag{III.10}$$

and $\Delta v = N_a \Delta v_a + N_b \Delta v_b + N_c \Delta v_c$

For six noded element

$$N_a = \frac{1}{2}\ \xi\ (\xi - 1) \tag{III.11}$$

$$N_b = (1 - \xi^2) \tag{III.12}$$

$$N_c = \frac{1}{2}\ \xi\ (\xi + 1) \tag{III.13}$$

$$\partial N_a/\partial \xi = \xi - 0.5, \quad \partial N_b/\partial \xi = -2\xi \quad \partial N_c/\partial \xi = \xi + 0.5 \tag{III.14}$$

For four noded element

$$N_a = \frac{1}{2}(1 - \xi) \tag{III.15}$$

$$N_b = \frac{1}{2}(1 + \xi) \tag{III.16}$$

$$\partial N_a/\partial \xi = -\ 0.5, \quad \partial N_b/\partial \xi = 0.5 \tag{III.17}$$

$$\begin{Bmatrix} \Delta u \\ \Delta v \end{Bmatrix} = \left[- I_2N_a,\ I_2N_a,\ -I_2N_b,\ I_2N_b,\ -I_2N_c,\ I_2N_c \right] \tag{III.18}$$

$$= \underline{N}\ \underline{T}\ \underline{\delta}^e$$

$$= \underline{N}_j\ \underline{\delta}^e$$

where $\underline{N}_j = \left[- I_2N_a,\ I_2N_a,\ -I_2N_b,\ I_2N_b,\ -I_2N_c,\ I_2N_c \right]$ (III.19)

and $\underline{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(ii) Strain at any point

The strains at any point are defined by the local relative displacement, $\Delta u'$ (the sliding part) and the local normal relative displacement $\Delta v'$ as

$$\begin{Bmatrix} \varepsilon_s \\ \varepsilon_n \end{Bmatrix} = \frac{1}{t} \begin{Bmatrix} \Delta u' \\ \Delta v' \end{Bmatrix} \tag{III.20}$$

The relations between the global relative dispalcements at any point and the corresponding local relative displacements 4u',4v' are obtained as under

$$\begin{Bmatrix} \Delta u' \\ \Delta v' \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} \Delta u \\ \Delta v \end{Bmatrix} \tag{III.21}$$

ds can be expressed as,

$$ds = \left[\left(\frac{dx}{d\xi}\right)^2 + \left(\frac{dv}{d\xi}\right)^2 \right]^{1/2} d\xi = M\ d\xi \tag{III.22}$$

where $M = \dfrac{ds}{d\xi}$

for isoparametric element, co-ordinates of any point is given by

$$x = \sum_{i=1}^{3} N_i\ x_i \tag{III.23}$$

$$y = \sum_{i=1}^{3} N_i \; y_i \tag{III.24}$$

Now,
$$\cos\theta = \frac{dx}{d\xi} \Big/ \frac{ds}{d\xi} = \frac{1}{M} \; \frac{dx}{d\xi} \tag{III.25}$$

$$\sin\theta = \frac{dy}{d\xi} \Big/ \frac{ds}{d\xi} = \frac{1}{M} \; \frac{dx}{d\xi} \tag{III.26}$$

$$\begin{Bmatrix} \Delta u' \\ \Delta v' \end{Bmatrix} = \frac{1}{M} \begin{bmatrix} dx/d\xi & dy/d\xi \\ -dy/d\xi & dx/d\xi \end{bmatrix} \begin{Bmatrix} \Delta u \\ \Delta v \end{Bmatrix} \tag{III.27}$$

$$= \underline{R} \begin{Bmatrix} \Delta u \\ \Delta v \end{Bmatrix} \tag{III.28}$$

where,

$$R = \frac{1}{M} \begin{bmatrix} dx/d\xi & dy/d\xi \\ -dy/d\xi & dx/d\xi \end{bmatrix} \tag{III.29}$$

is a rotation matrix that transfers global to local strain and is given by the slope of the curve of the centre line of the interface element.

then,

$$\begin{Bmatrix} \varepsilon_s \\ \varepsilon_n \end{Bmatrix} = \frac{1}{t} \begin{Bmatrix} \Delta u' \\ \Delta v' \end{Bmatrix} \tag{III.30}$$

$$= \frac{1}{t} \underline{R} \begin{Bmatrix} \Delta u \\ \Delta v \end{Bmatrix} = \frac{1}{t} \underline{R} \; \underline{N} \; \underline{T} \; \underline{\delta}^e \tag{III.31}$$

For interface elements having negligible thickness, t is taken as unity.

$$\begin{Bmatrix} \varepsilon_s \\ \varepsilon_n \end{Bmatrix} = \frac{1}{t} \; \underline{R} \; \underline{N}_j \; \underline{\delta}^e \tag{III.32}$$

$$= \underline{B}_j \; \underline{\delta}^e$$

Here $\underline{B}_j$ is the strain displacement matrix of the joint

(iii) Element stiffness matrix

The element stiffness matrix can be written as

$$\underline{K}_{12x12} = \int \underline{B}_j^T \, \underline{D}_j \, \underline{B}_j \, ds \tag{III.33}$$

$\underline{D}_j$ is the elasticity matrix of the joint

$$ds = M \, d\xi$$

$$\underline{K}^e_{12x12} = \int \underline{N}_j^T \, \underline{R}^T \, \underline{D}_j \, \underline{R} \, \underline{N}_j \, \underline{M} \, d\,\xi \tag{III.34}$$

This equation when written in numerically integrable form becomes,

$$\underline{K}^e_{12x12} = \sum_{i=1} (\underline{N}_j^T \, \underline{R}^T \, \underline{D}_j \, \underline{R} \, \underline{N}_j \, \underline{M})_i w_i \tag{III.35}$$

where w_i is a weighting function

(iv) Stress

normal stress, $\sigma_n = K_{nt} \, \Delta v'$ (III.36)

shear stress, $\tau = K_{st} \, \Delta u'$ (III.37)

where

$\Delta v'$ = average relative normal displacement across the element.

$\Delta u'$ = average relative shear displacement along the interface of element.

CHAPTER - 7

NONLINEAR ANALYSIS OF BRICK MASONRY SHEAR WALLS AND INFILLED FRAMES

7.1 Introduction

An important consideration in the analysis and design of masonry structures is its ability to withstand lateral loads induced due to earthquakes or arising from wind loading. Multistorey masonry buildings consist of different types of shear walls (Fig. 7.1). Resistance of a structure as a whole depends predominantly on the in-plane shear resistance of such assemblages. Consequently the strength of masonry shear walls have become the subject of investigation.

Brick masonry infilled frames are widely used all over the world even in regions of high seismicity for economy in construction and to meet the architectural and functional requirements. The behaviour of the infilled frames is highly indeterminate in nature. The strength and deformation characteristics depend on the interaction of the frame and the infill. The degree of interaction controls the stress distribution in the infill and therefore affects the strength and modes of failure of the infilled frame.

Generally the designers tend to ignore the contribution of masonry on strength and stiffness of the infilled frame due to complexities in the analysis, presuming such an omission to be safe and conservative. Such an approach is also likely to lead to less safer designs due to improper distribution of lateral load among the frames, induction of unintended shear and axial forces in frame members. The main reason for disregarding the contribution of the masonry infill lies in the complexity associated with its analysis, since it is in a state of biaxial stress.

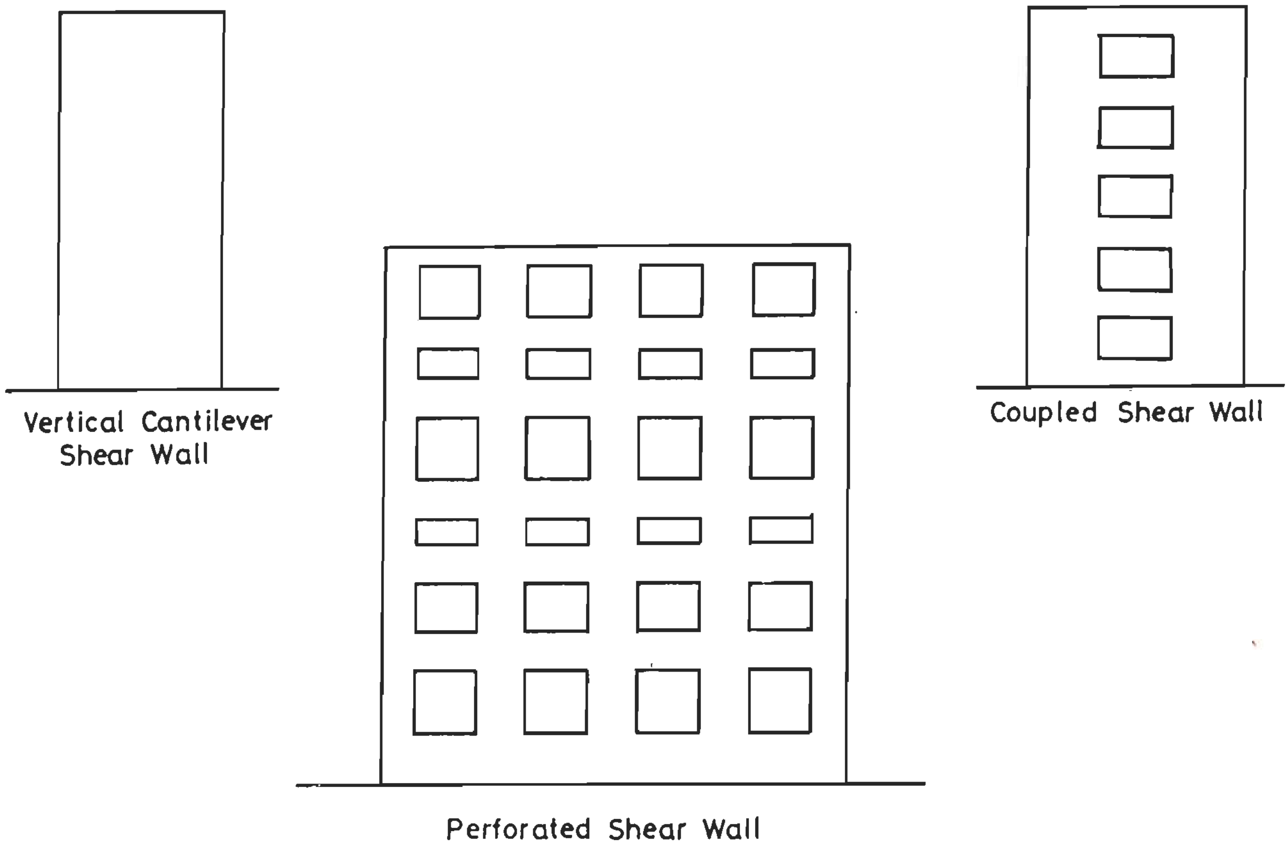

Fig. 7.1 Typical Shear Walls

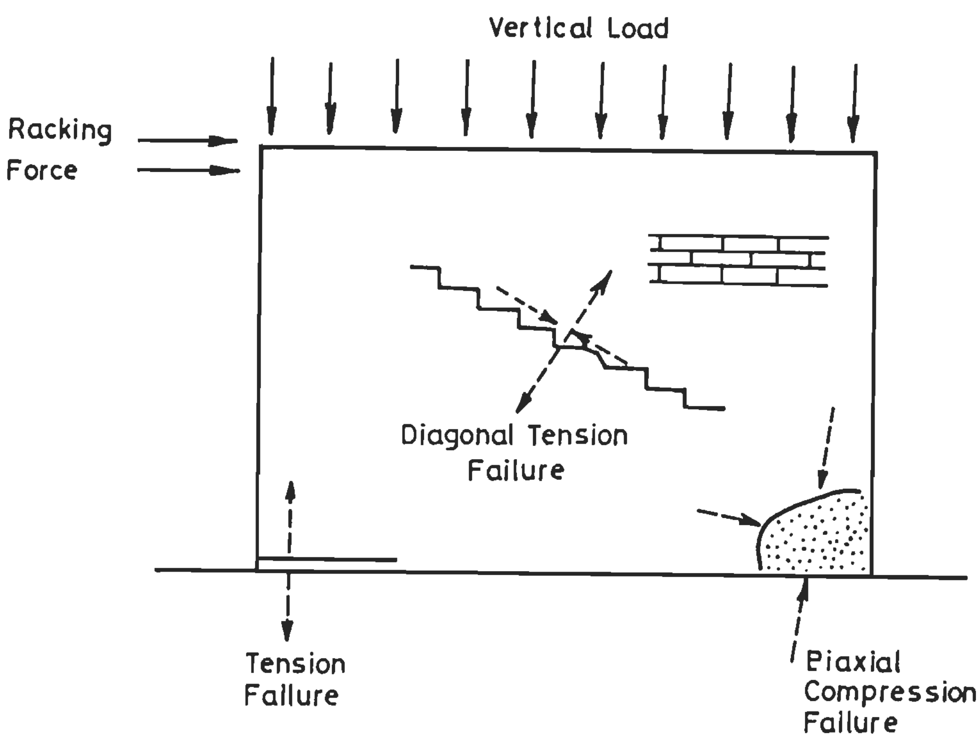

Fig. 7.2 Behaviour of Masonry Shear Walls

The behaviour of infilled frames and masonry shear walls subjected to racking load has been studied by many investigators. Experimental methods have been extensively used to estimate strength and stiffness of the infill frames and shear walls. Recently finite element method has also been used for the realistic analysis of such masonry structures. Unfortunately the analysis has been hampered due to the lack of suitable material model. At present there is no comprehensive material model which can realistically reproduce the behaviour of brick masonry.

The present material model is able to eliminate this deficiency. The finite element analysis incorporating the proposed material model has been employed to study the in-plane behaviour of the masonry shear walls and in-filled frames to demonstrate the suitability of the material model, structural idealisation and numerical techniques by comparing the analytical data with the experimental results.

To place this investigation in context, the state of art on the in-plane behaviour of masonry walls and infilled frames is presented.

7.2 Behaviour of Masonry Shear Walls and Infill Frames

7.2.1 Behaviour of Masonry Shear Walls

Masonry walls are generally designed to resist racking load in addition to vertical load. A typical shear wall subjected to in-plane load is shown in Fig.7.2. The behaviour of masonry walls depends upon the ratio of racking to compressive loads. Besides this, it will be influenced by aspect ratio, support conditions and properties of the masonry. At a very small racking load, tensile cracks will develop at the interface between the base and bottom of the masonry wall at the heel in the biaxial tensile stress zone. Diagonal cracking will develop at higher loading in the central region due to biaxial tension-compression state of stress. Several such cracks may develop with further increase in load. Failure may finally result from the gradual degradation of stiffness of masonry as a result of these cracks or due to crushing in the region at the toe, provided the vertical load is very high in comparison to the racking load.

For modelling the behaviour of shear walls, different failure modes, stiffness degradation, closing and opening of cracks, tensile stiffening and crushing condition should be taken. The proposed material model is capable of reproducing more accurately all these characteristics.

7.2.2 Behaviour of Infilled Frames

Brick masonry is commonly used as an infill material in many steel and reinforced concrete frames. The behaviour of the infilled frames depend upon the composite action of the frame and the infill. The general behaviour of the structural interaction between the frame and infill is shown in Fig. 7.3.

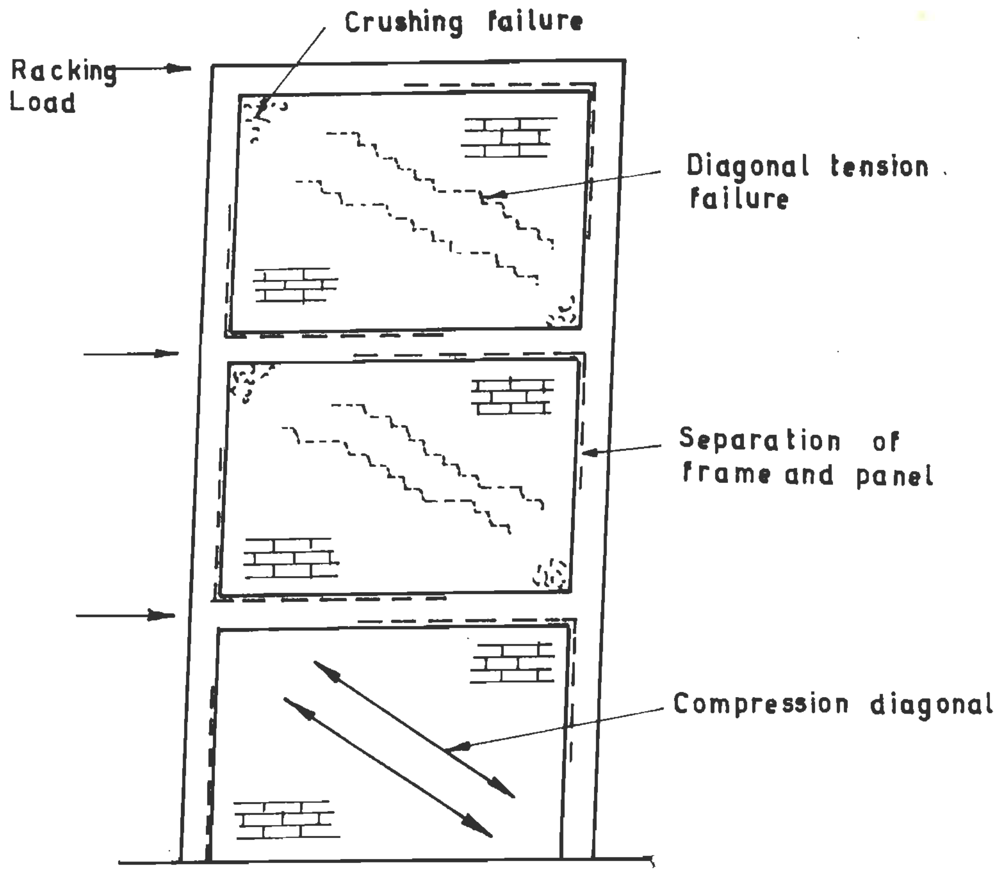

Fig. 7.3 Behaviour of Masonry Infilled Frames Subjected to Racking Loads

On first application of racking load there may be full composite action between the frame and infill panel provided these are bonded together. At a comparatively early stage, cracks will develop at the interface of the frame and infill panel except in the vicinity of the two corners where the infill panel will lock into the frame and there will be transmission of compressive forces into the masonry panel. At this stage, the panel acts as a diagonal strut within the frame, the effective width of which depends on the relative stiffness of the two components and the aspect ratio of the panel. This action continues until a shear failure starts near the centre of the panel. Several parallel cracks of this type may develop with further increase in load and failure may finally occur due to the loss of rigidity of the infill, as a result of these cracks or due to local crushing of masonry in the compression region. Thus for the modelling of the behaviour of masonry infilled frame the following behavioural stages should be incorporated.

- Slipping and cracking at the interface of the frame and masonry panel.
- Closing and opening of cracks at the interface of the frame and panel.
- Failure of masonry panel due to cracking and crushing.
- Closing and opening of cracks in the masonry panel.
- Failure of the frame.

7.3 Review of Literature on the Behaviour of Masonry Shear Walls and Infilled Frames

7.3.1 Masonry Shear Walls

To develop a suitable method for the prediction of strength of the wall, investigations of in-plane behaviour of brick masonry have been carried out in many countries. The literature pertaining to these studies have been summarised by Hendry(1978). In the last three decades, attempts have been made by many investigators to model the nonlinear behaviour of masonry.

Semi-Empirical Method of Analysis: From the results of the experiments Coulomb type failure criterion has been suggested by many investigators to estimate the shear strength of masonry walls. The shear strength of masonry wall has been described in detail in Chapter 2. However some of the important points are presented here.

A curve defining the failure of masonry shear wall, for full range of normal stress, has been derived experimentally by many investigators [Yorulmaz and Atan(1977), Hamid and Drysdale(1980), Dhanasekar(1985)]. It is observed that Coulomb type failure criterion is no longer valid for normal stress greater than 2 N/mm^2 (Fig.7.4).

The use of such an empirical relation has been questioned by many researchers [Stafford-Smith and Carter(1970), Trunsek and Cacovic(1971), Borchelt(1970)]. It is pointed out that failure of the shear wall occurs due to principal tensile stress. The value of principal stress is also not constant. It increases with increase of normal compressive stress

[Chinwah(1972), Schneider(1976)]. A modified Coulomb type failure criterion has been proposed Hendry(1978) as an extension of the work of Trunsek and Cacovic(1970), Chinwah(1972) and Schneider(1976)].

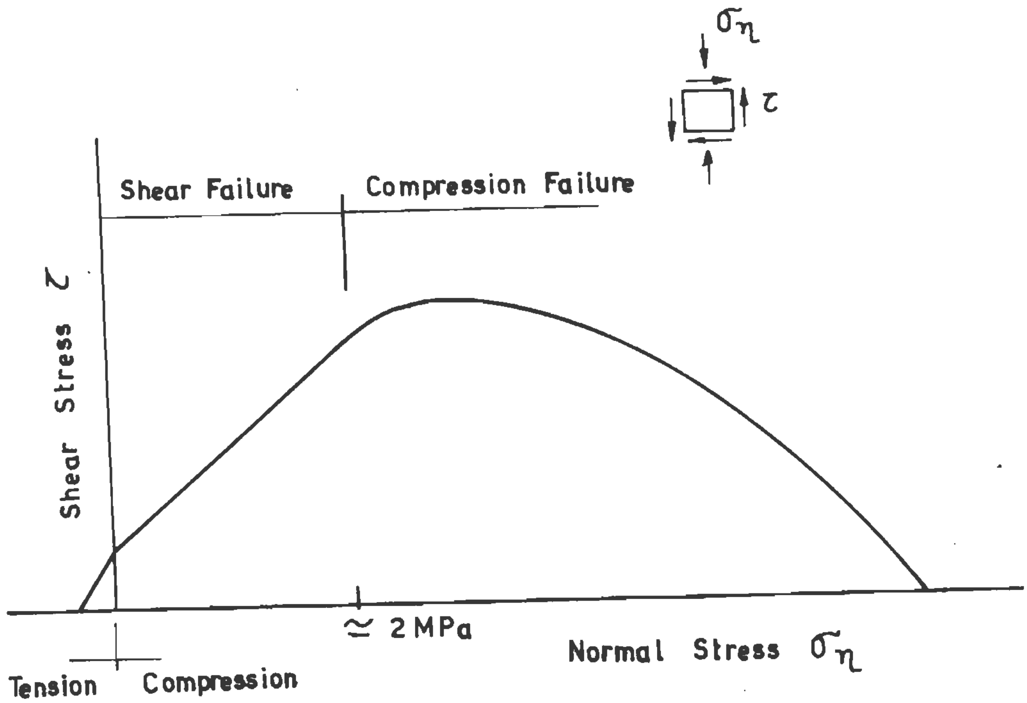

Fig. 7.4 Typical Failure Surface for Masonry Shear Wall in (σ_n, τ) Stress Space

Finite Element Analysis: Strength criteria as discussed above are semi-empirical in nature. In view of the limitations of these criteria, many investigators have attempted to develop a biaxial, stress failure criterion. Samarasinghe and Hendry(1980) proposed biaxial tension-compression stress failure criterion for the prediction of strength of masonry walls. This criterion has been incorporated into the finite element model in which non-linear effects produced by progressive cracking of zones subjected to biaxial tension-compression were considered. Dhanasekar(1985) proposed a biaxial failure criterion for solid masonry. This failure criterion is not comprehensive. The failure surface defined in stress space (σ_n, σ_p and τ) consists of three elliptical truncated cones (Fig.2.18 a). This failure criterion has been incorporated in the finite element model.

7.3.2 Infilled Frames: Begining in 1949 experimental investigations on the behaviour of infilled frames have been carried out in many countries. Different test techniques used for the study of infilled frames are shown in Fig.7.5. The literature pertaining to these studies during the fifties were critically reviewed by Benjamin and

Williams(1957,1958). Tests on infilled frames consisting of steel and reinforced concrete using both concrete and brick masonry as infill materials have also been carried out. They proposed formulae to estimate strength and stiffness of masonry infilled frames assuming full contact between the frame and the infill.

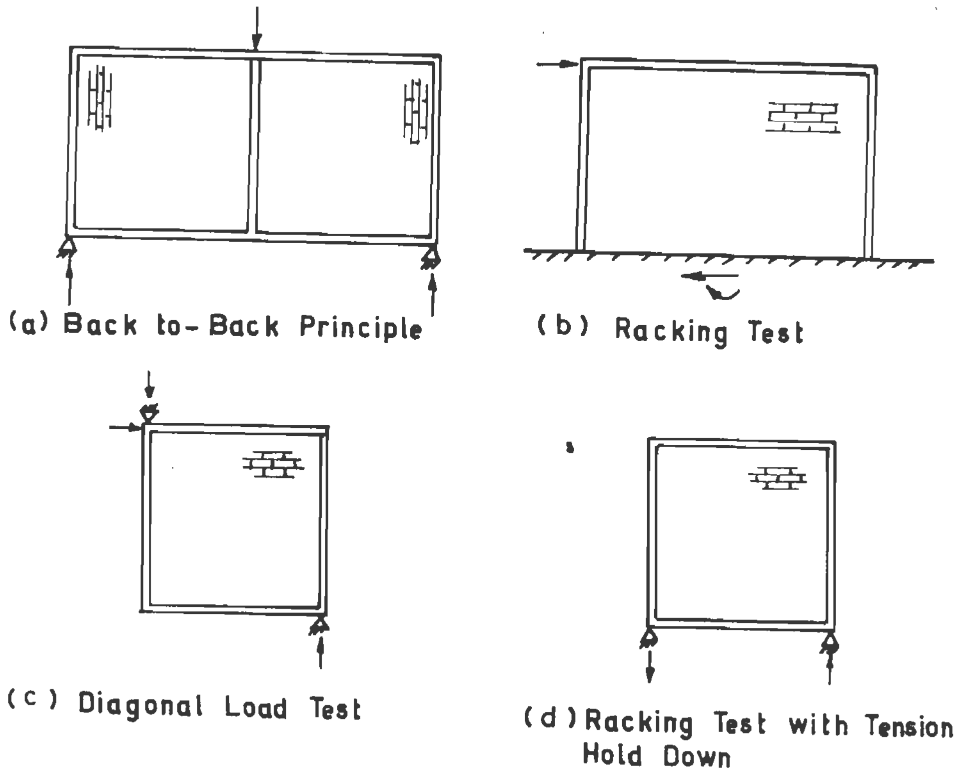

Fig. 7.5 Test Techniques for Infilled Frames

Polyakov(1956) first proposed the "equivalent strut approach" for the design of infilled frames. Subsequently Stafford-Smith (1962,1966,1967) and Stafford-Smith and Carter(1969) developed this approach. It is shown that the width of the equivalent strut depends on the relative stiffness of the frame and panel and aspect ratio of the panel. The effective width of the strut may be taken as

$$W = \frac{1}{2}\sqrt{\alpha_l^2 + \alpha_h^2} \tag{7.1}$$

where α_l and α_h are contact lengths along the beam and column respectively. The contact length in the case of the column of the frame is given by

$$\alpha_h = \frac{\pi}{2\lambda_h} \tag{7.2}$$

where

$$\lambda_h = \sqrt[4]{\frac{E\ t\ \sin 2\theta}{4E_f I_h\ h}} \tag{7.3}$$

and the contact length in the case of beam is given by

$$\alpha_l = \frac{\pi}{\lambda_l} \tag{7.4}$$

where

$$\lambda_l = \sqrt[4]{\frac{E\ t\ \sin 2\theta}{4E_f I_l\ l}} \tag{7.5}$$

in which

E = elastic modulus of the panel material
E_f = elastic modulus of the frame material
I_h = second moment of inertia of the column
I_l = second moment of inertia of the beam
h = height of the infill panel
l = length of the beam
t = thickness of the infill panel

Kadir and Hendry(1977) carried out tests on 43 one-third scale infilled frames consisting of steel frames with brick masonry infill. The strength of the infilled frames was in good agreement with that estimated from the formula of Stafford-Smith and Carter(1969).

Liauw(1970, 1972) and Liauw and Lee(1977) computed the stress distribution in infilled frames using Airy's stress function expressed in the form of Fourier Series and compared the results with experimental ones. Liauw also presented an approximate method of analysis of infilled frames.

Mainstone (1971) conducted tests on large scale models of infilled frames using concrete and masonry infills and predicted the equivalent width of the strut as a function of column-infill, relative stiffness and geometry of panel.

Smolira (1973) presented an approximate analytical formulation of infilled shear walls based on force displacement method in which the statically indeterminate variables were taken as forces (or moments) and displacements. The analysis included the effect of tightly fitted infill, interface, spaces and stress concentrations at contact lengths on the over all response of a shear wall.

Riddington (1984) conducted tests on six full scale models of steel frame infilled with block masonry and observed that even relatively small initial gaps adversely affect the structural behaviour. Riddington recommended that initial gaps should be avoided wherever possible.

Infilled Frames with Opening: The behaviour of infilled frame with openings such as windows or doors in the infill has been studied by many researchers [Benjamin and Williams (1958), Thomas (1953), Wood (1958), Liauw and Lee (1977)]. It was concluded that the effect of such openings was to reduce the strength of infill. An extensive series of tests on infilled frames with openings were carried out by Mallick and Garg (1971). It was concluded that the performance of the infilled frames would improve if the openings were moved away from the compression diagonal as far as practicable. Liauw (1972) proposed equivalent frame concept in which the shape of the cross sections of the members of the equivalent frame were modified to account for the infill and the frame. This method also overestemate the stiffness specially when the area of the opening was less than 50% of the infill area.

Finite Element Analysis: Karamanski (1967) for the first time applied the concept of finite element to the analysis of infilled frames and worked out the stress distribution in the infill.

Mallick and Severn(1967) described a method based on the concept of finite element which can find out the points of separation between frame and infill and the stress distribution at the contact surfaces. Slip between frame and infill was also taken into account.

Franklin(1970) studied the behaviour of infilled frames, taking into account the material nonlinearity, cracking of plain concrete infill, separation and slip between the frame and the infill

Riddington and Smith(1977) presented a series of elastic finite element stress analysis results of infilled frames. They considered the separation between frame and the infill and loss of friction along the

Barua and Mallick(1977) conducted finite element analysis of infilled frames taking into account the axial deformation and slip at the interface. They computed the length of contact at various stages of lateral load and concluded that the ignorance of axial deformation resulted in a marginally conservative value of lateral stiffness.

King and Pandey(1978) developed a procedure for nonlinear analysis of infilled frames and demonstrated that the interface between the infill and the frame can be modelled with accuracy using friction elements.

Liauw and Kwan(1984) examined the nonlinear behaviour of nonintegral infilled frames using the finite element method. The nonlinearities of material, structural interface, effects of initial lack of fit and friction at the interface were taken into account. It was shown that the stress redistributions towards collapse were significant and the strength of nonintegral infilled frame was very much dependent on the flexural strength of the frame.

May and Ma(1984) used a nonlinear finite element model for the analysis of infilled frames. An eight noded isoparametric membrane element with two degrees of freedom at each node was used to model the infill. Two noded bar element with two degrees of freedom per node were used to model the behaviour at the infill frame interface. Theoretical results were in good agreement with those obtained experimentally.

Dhanasekar(1985) presented finite element analysis of infilled frames. Brick masonry was modelled using eight noded isoparametric element. The material nonlinearity and cracking of masonry has been incorporated in the finite element model. Plastic strain components are assumed to be function of the stress in that direction. The failure surface is defined by using three truncated cones. The material model can not predict accurately the behavior of brick masonry.

Papia(1988) used boundary element method to model the behaviour of the infill frame. A comparison of the results with those of the equivalent strut model was made.

May and Naji(1991) carried out nonlinear analysis of the infilled frames under monotonic and cyclic loading using finite element method.

The frame was modelled with three noded beam element and the panel with eight noded isoparametric element. A six noded interface element was used to model the interface between the frame and the infill. The predicted strength is less than the experimental strength.

Haddad(1991) carried out nonlinear analysis of masonry infilled frames using the finite element method and fracture mechanics. The model takes into account the effect of crack length and its location, relative stiffness of infilled frame, aspect ratio and contact length.

From the literature survey it is revealed that the analysis of infilled frames and shear walls has been hampered due to a lack of suitable material model which can realistically model the behaviour of masonry.

7.4 Validation Test

The validity of the proposed formulation and the computer program are demonstrated in this section. Nonlinear analysis of four test problems has been carried out to compare the analytical data with the experimantal results. The test problems consist of one masonry infilled frame and three shear walls.

7.4.1 Masonry Infilled Frame

Dhanasekar(1985) performed tests on four infilled frames consisting of steel frames with brick masonry infill. Out of these four frames one infilled frame has been analysed. The test arrangement is shown in Fig.7.6. Half scale size bricks of dimension 110 x 50 x 35 mm and 1:1:6 (cement:lime:sand)mortar were used in the masonry panel. The thickness of the mortar joint was 5mm. The panel consists of twenty five courses of bricks with nine bricks in each course. The detail dimensions (centre to centre) of the infill frame is shown in Fig. 7.7.

Material Properties and Loading: The material properties used in the study is presented in Table 7.1. The load was applied horizontally near the top of one side of the frame and reacted at a horizontal support near the bottom of the other side. The in-plane rotation of the frame was prevented by roller supports at the top of the frame near the loaded corner and at the bottom of the frame at the opposite corner. The load was applied by means of a hydraulic jack. The increment of load was 5 kN

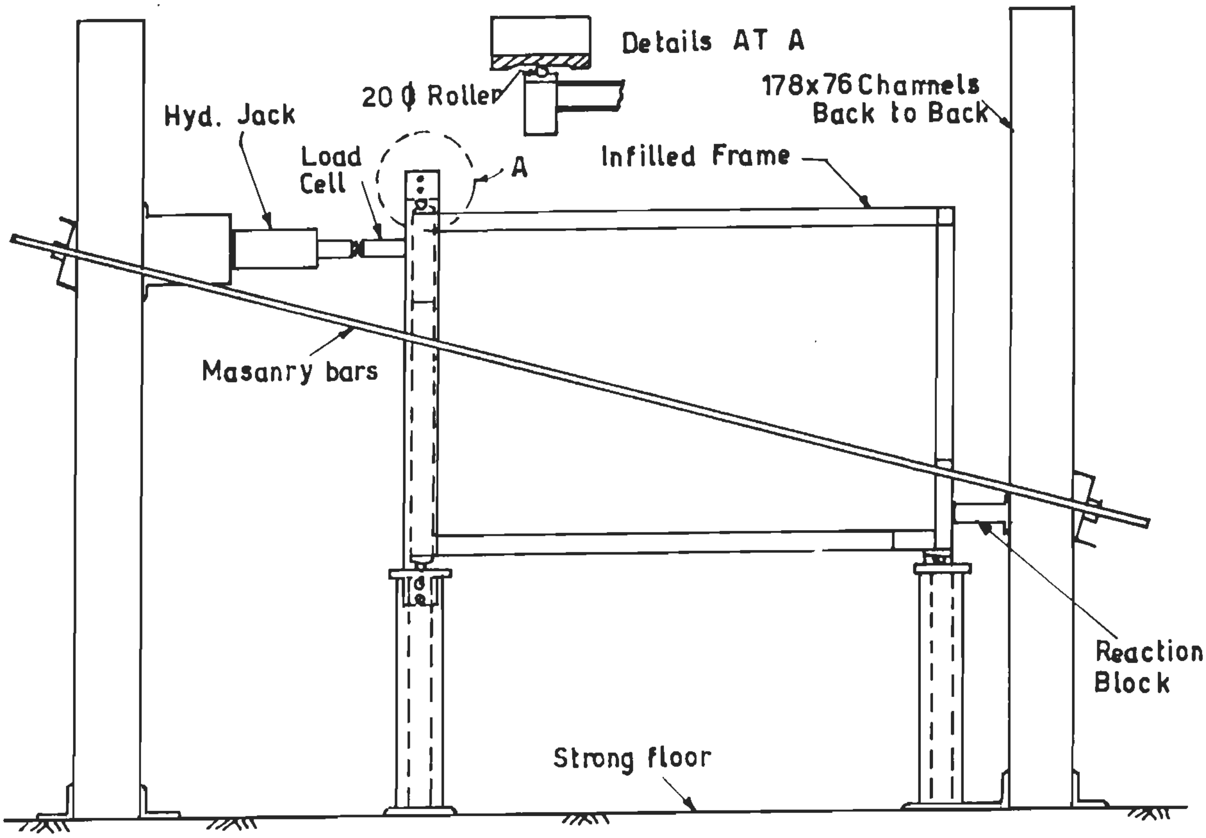

Fig. 7.6 Test Arrangement of Masonry Infilled Frame

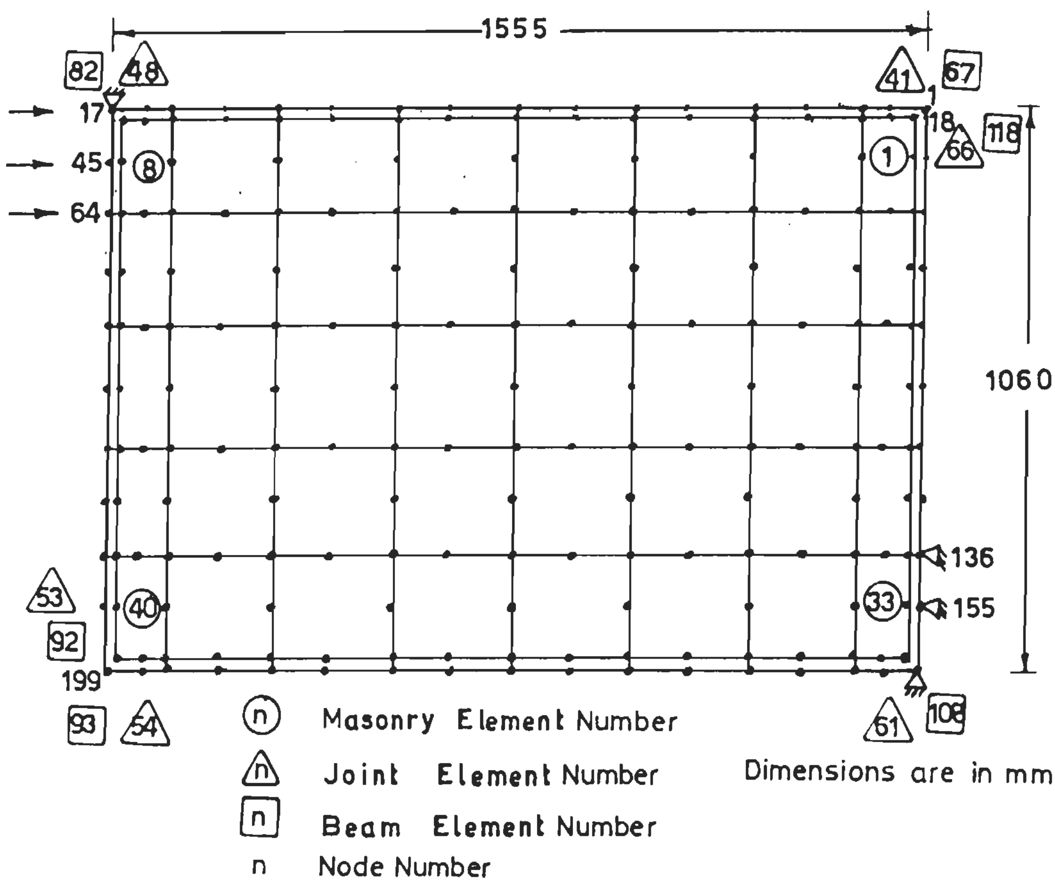

Fig. 7.7 Finite Element Subdivisions of the Infilled Frame

Table 7.1: Material Properties for Brick Masonry Infilled Frame [Dhanasekar(1985)]

Material	Property	Symb.	Value
Brick masonry	Young's Modulus (MPa)	E	5700.00
	Poisson's ratio	ν	0.19
	compressive strength parallel to bed joint (MPa)	f_{pc}	4.33
	Compressive strength normal to bed joint (MPa)	f_{nc}	7.56
	Biaxial compressive strength (MPa) with $\theta = 0^{o}$(MPa)	f_{bc}	8.15
	Biaxial compressive strength (MPa) with $\theta = 45^{o}$, and $\sigma_2/\sigma_1 = 4$	σ_1	2.33
		σ_2	9.40
	Tensile strength	f_t	0.40
	Maximum tensile strain	ϵ	0.0025
Mortar	Young's modulus (MPa)	E^m	1150.00
	Shear modulus (MPa)	G^m	660.00
	Strength parameter of failure criterion for mortar joint (strength parameters are valid for vertical compressive stress only)	a_1	0.3
		b_1	0.85
		a_2	1.92
		b_2	0.11

Procedure of Analysis: The applied load has been simulated by loads applied at nodes 17, 45 and 64. These loads are multiplied by a load factor and applied incrementally. Load factor is not uniform. The load factor has been reduced from 0.1 to .05 when the diagonal crack developed.

The masonry panel is discretised by eight noded isoparametric masonry elements and the frame is discrectised by two noded beam elements. Six noded interface elements are used at the interface of the frame and the masonry panel. The element subdivisions are shown in Fig. 7.7.

At node 183 both horizontal and vertical displacements are constrained; node 155 is constrained against horizontal displacement and node no. 17 is constrained against vertical displacement(Fig.7.7).

The same trilinear failure criterion used by Dhanasekar(1985) has been adopted to model the separation of the masonry panel from the frame by considering the failure due to slipping or cracking of the interface element. The frame is assumed to be linearly elastic.

Observations: The observations of nonlinear analysis of the masonry infilled frame are presented below.

o The separation of the masonry panel in the region of the unloaded corners started at 5kN racking load and a stable "contact length" has been obtained at 12.0 kN racking load (Fig.7.8).

o At 45.0 kN racking load, failure has occurred at the central region of the panel and then propagated in a diagonal direction towards the loaded corner. The failure of masonry infill is shown in Fig.7.8.

o The load-deflection curve is in good agreement with that obtained from experimental results as shown in Fig. 7.9.

o The failure load obtained from the stress-strain curve is 53 kN and experimental failure load is 55.4 kN. Thus a very good agreement between the analytical and experimental strength is observed.

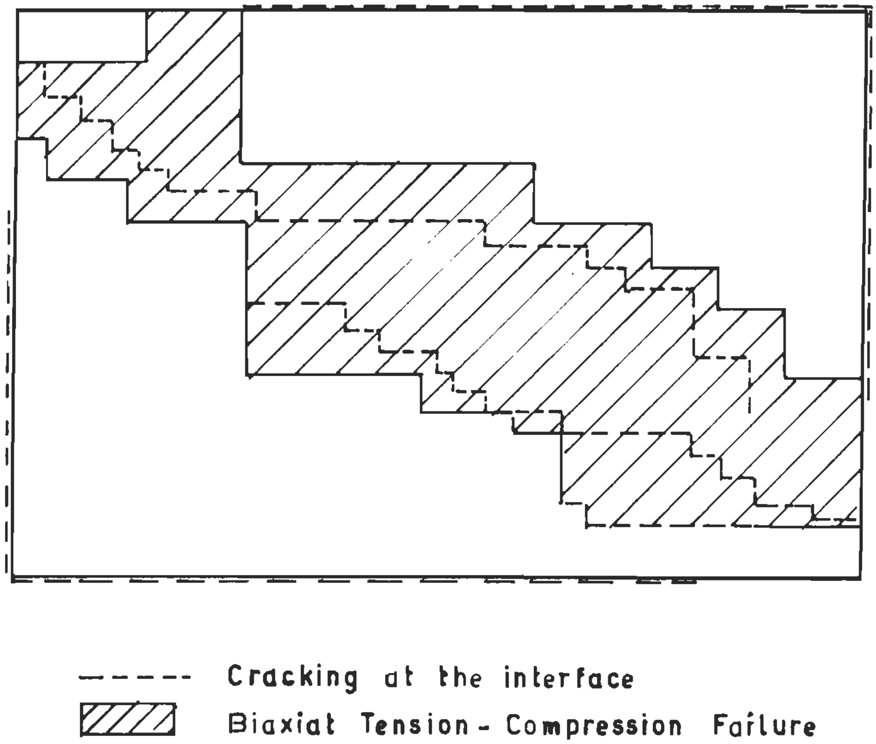

Fig. 7.8 Mode of Failure of Masonry Infilled Frame

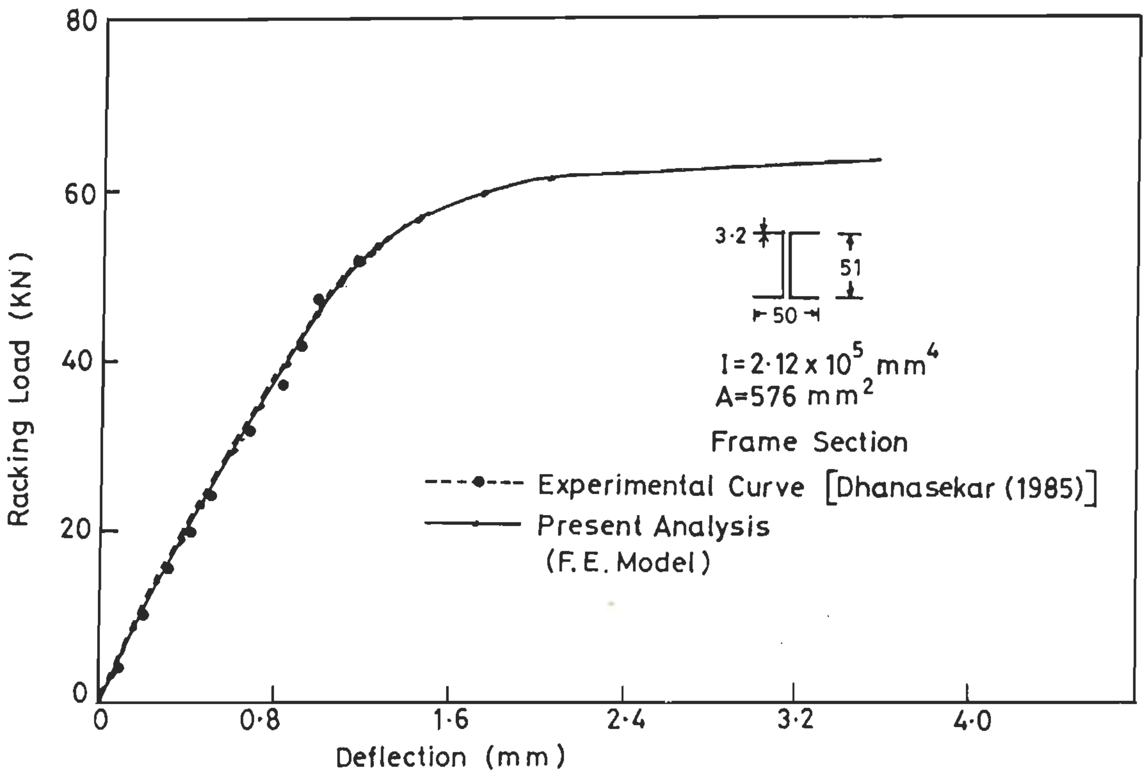

Fig. 7.9 Load-deflection Curve for Infilled Frame

7.4.2 Masonry Shear Wall

Nonlinear analysis has been carried out on three types of masonry shear walls: (a) solid walls (b) wall with a door opening and (c) wall with a door and window openings.

(a) Solid Masonry Shear Wall: The behaviour of the shear wall has been experimentally studied by Bhargava(1978). The wall was constructed using 230 x 110 x 67mm size clay bricks and 1:6 cement mortar. The dimensions of the shear wall are shown in Fig.7.10.

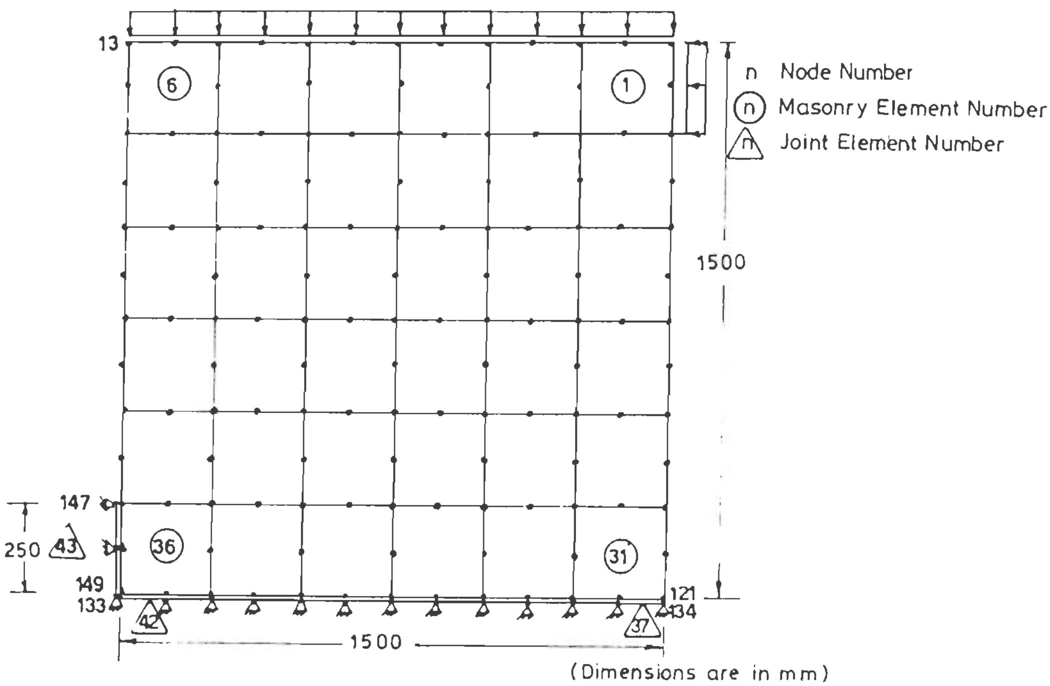

Fig. 7.10 Finite Element Subdivisions of a Solid Masonry Wall

Material Properties and Loading: The material properties used in the study are presented in Table 7.2. The wall was restrained at one corner and the racking load was applied over a length of 250 mm near the top of the wall. Uniformly distributed load was applied on the top of the wall. The load was increased maintaining a constant load ratio, 2:1 (vertical to horizontal).

Procedure of Analysis: Brick masonry has been modelled using eight noded isoparametric element. Six noded interface element has been used between the support and the wall in order to allow slip or tensile failure at the base. The finite element subdivisions and boundary conditions are shown in Fig.7.10.

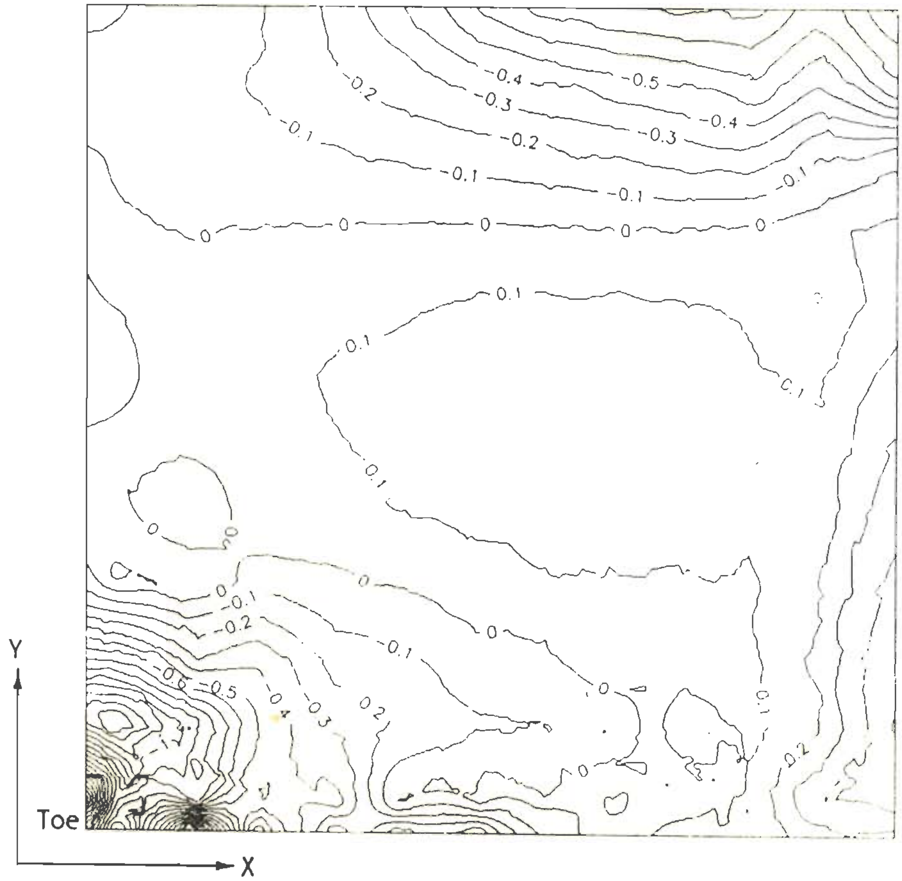

Fig. 7.11 Principal Stress Contours at Failure Load for Solid Masonry Shear Wall

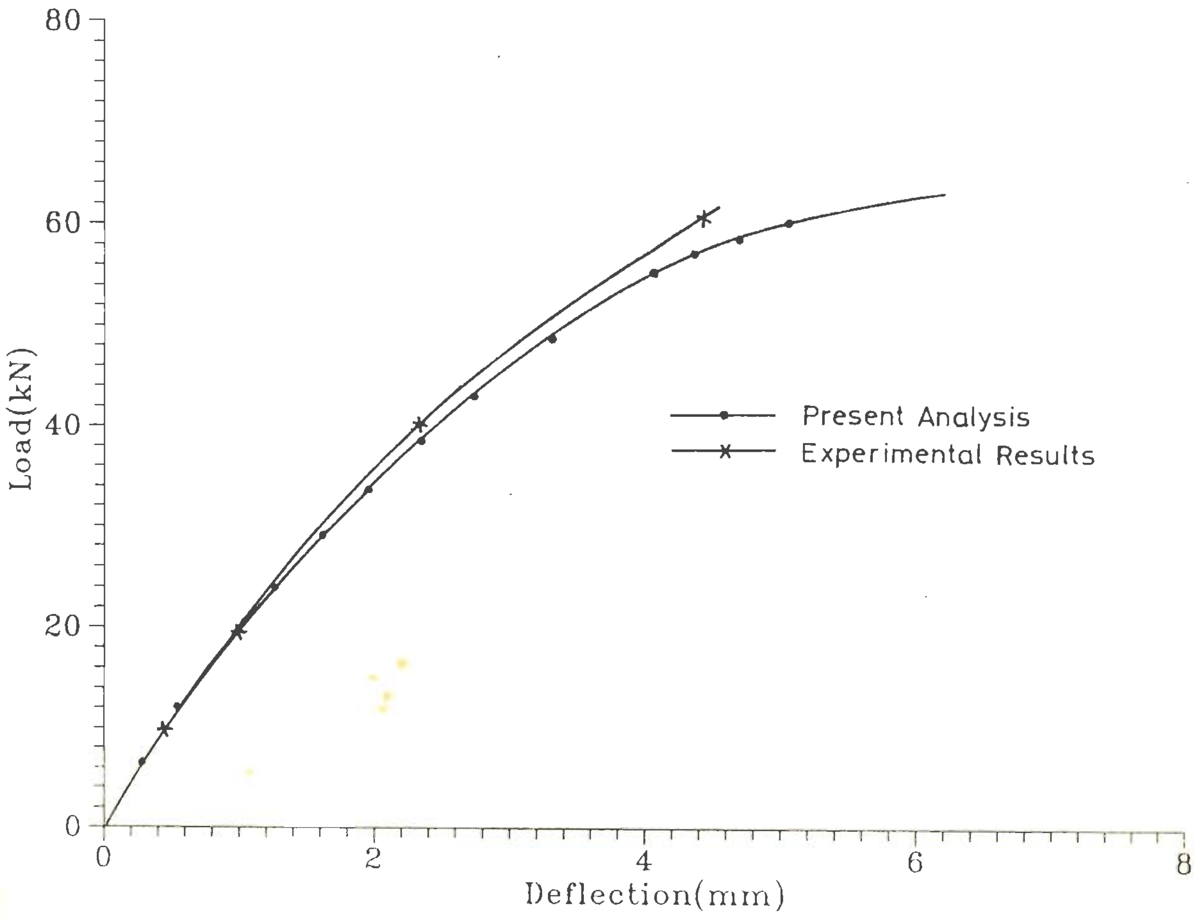

Fig. 7.12 Load-deflection Curve for Solid Masonry Shear Wall

The applied loads are simulated by considering the forces acting on the nodes as shown in Fig.7.10. The loads have been multiplied by a load factor and applied incrementally. Load factor is not uniform throughout the analysis. The load factor has been reduced from 0.2 to 0.1 when a diagonal tensile crack has developed in the brick masonry.

Observations: The main features of the nonlinear analysis of the masonry shear wall are presented below.

- o At an early stage of loading (20 percent of ultimate strength) tensile crack has occurred in the mortar joint at the heel and with subsequent increase in load this crack has propagated towards the toe till a tensile crack occurs in brick masonry. Closing of the crack is observed when cracks develop near the heel in the tension compression region.

- o Tensile crack in masonry first occurs at 75 percent of failure load. Several such cracks occur with subsequent increase in load. Finally failure of masonry occurs due to degradation of stiffness as a result of these cracks.

- o Major principal stress contours, at failure load are plotted in Fig.7.11. These contours show the failure pattern in the masonry wall.

- o Analytical and experimental load-deflection curves are drawn in Fig.7.12. The analytical curve bears a close resemblance to the experimental one. The nonlinearity observed in the load deflection curve is mainly due to tensile cracking.

(b) Masonry Shear Wall with Door Opening: Bhargava(1978) carried out test on a masonry wall with door opening. The wall was constructed using the same material as in the case of the test specimen described above. The dimensions are shown in Fig. 7.13.

Material Properties and Loading: The material properties are the same as in the previous case and are given in Table 7.2. Load was applied following the same procedure as in the above analysis.

Procedure of Analysis: Eight node isoparametric elements and six noded interface elements, like in the previous analysis have been used.

The element subdivisions and boundary conditions are shown in Fig.7.13. The applied loads are simulated considering the same procedure as discussed in the previous analysis.

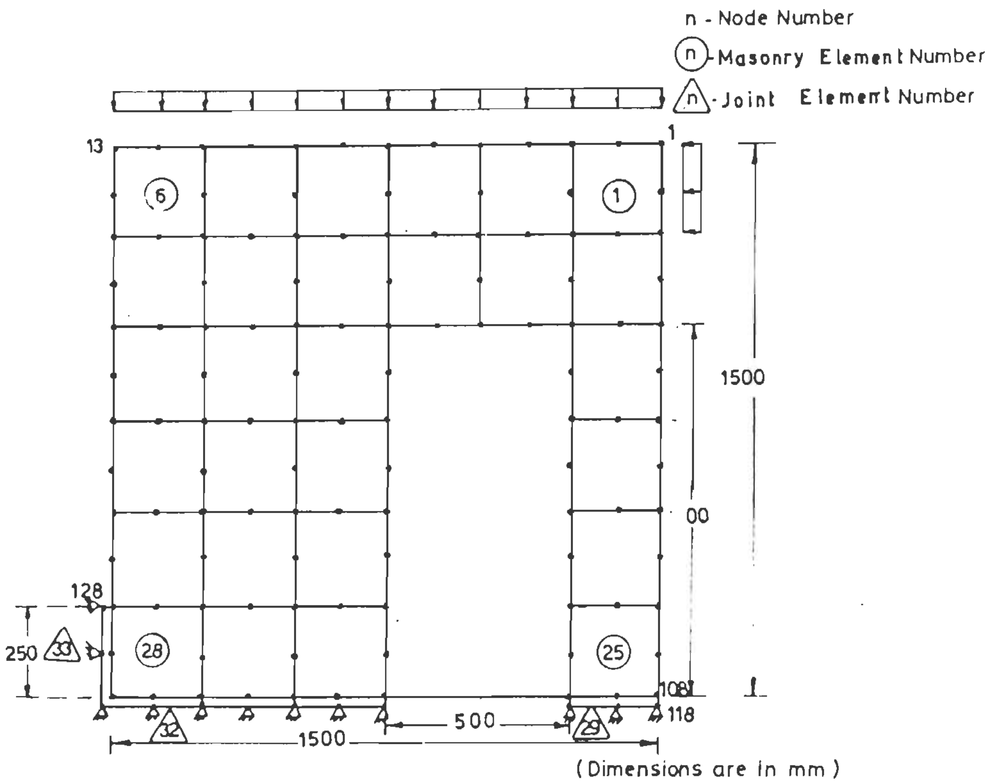

Fig. 7.13 Finite Element Subdivisions of Masonry Shear Wall with Door openig

Observations: The observations from the results of the nonlinear analysis are summarised below.

- At an early stage of loading tensile crack, like in the previous analysis, occurs in the mortar joint at the heel at a racking load of 0.71 ton.

- The tensile crack first occurs in the loaded side of pier at a racking load of 2.81 tons and several such cracks have developed with increase of loads. Failure of wall finally occurs mainly due to the failure of masonry pier on the loaded side.

- The maximum principal stress contours, at failure load, are shown in Fig.7.14. It shows that the failure occurred mainly due to the failure of pier.

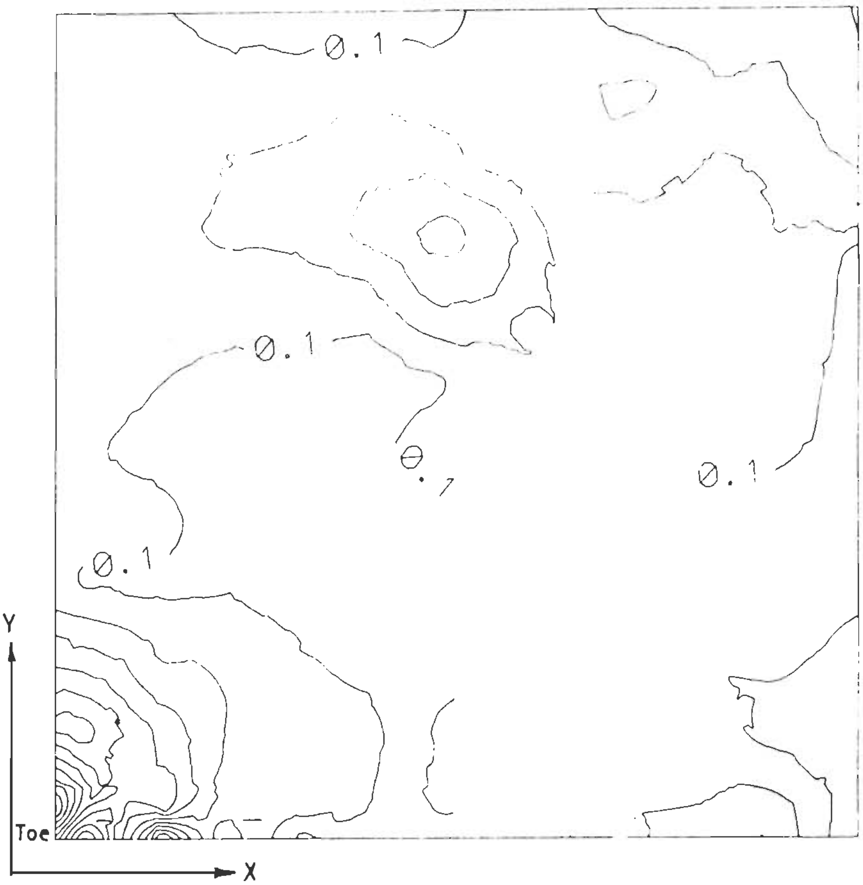

Fig. 7.14 Maximum Principal Stress Contours at Failure Load for Masonry wall with Door openig

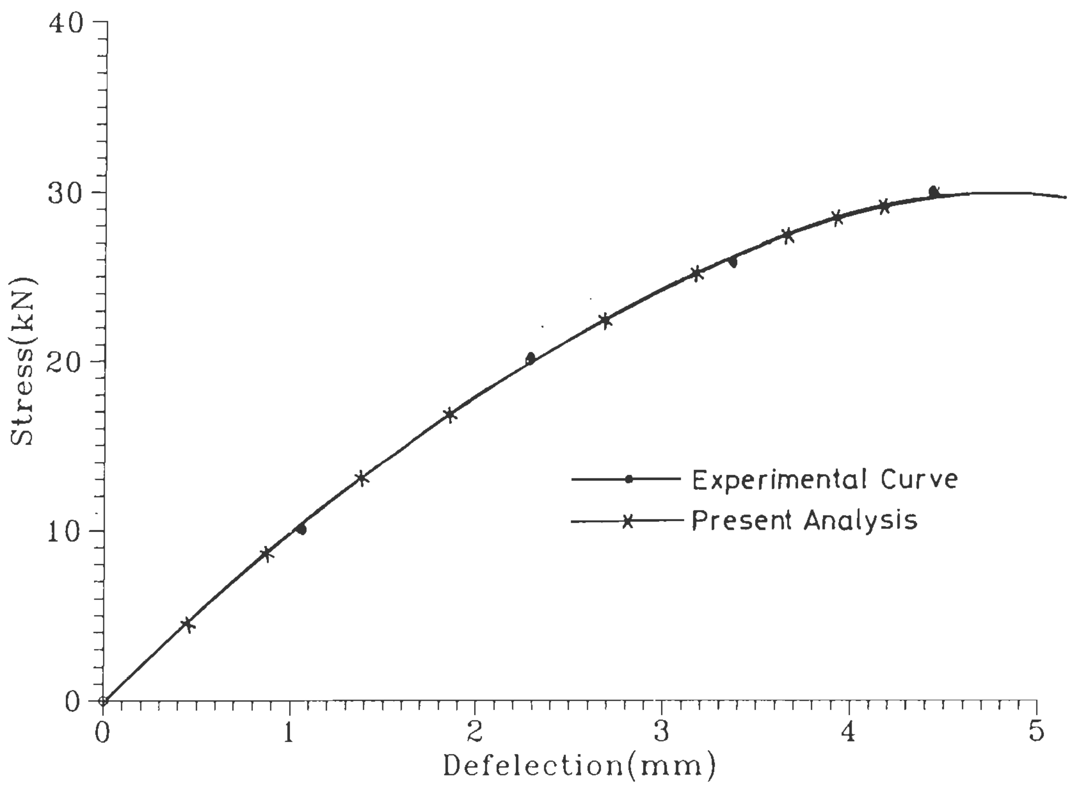

Fig. 7.15 Load-deflection Curve for Masonry Wall with a Door Opening

- The net area of the wall is reduced to 67 percent of the gross area due to the door opening, but the load carrying capacity is reduced to 50 percent of that of the solid wall. The analytical and experimental failure loads are shown in Table 7.3. The ultimate strength is 3.1 tons which is 95 percent of the experimental strength.

- The load -deflection curve has been plotted in Fig.7.15. The predicted deflection is very close to the experimental value.

(c) Masonry Wall with Door and Window Openings: Kanungo(1966) performed tests on unreinforced masonry wall with door and window openings. The model was made of 3 x 1.5 x 1 inch size bricks with 1:6 (by weight) cement sand mortar (Fig. 7.16). The model was scaled down to 1/4th size of the prototype. To satisfy the conditions of similitude, a load of 496 lbs was distributed on the top of the wall. A shear Jig was used to apply the racking load. The load was applied 2.5 inch below the top of the wall through a proving ring placed between the screw jack and the wall. The load was increased till the ultimate load was reached.The material properties are shown in Table 7.2.

Procedure of Analysis: Brick masonry is modelled using eight noded isoparametric element and six node interface element is used to model the mortar joint between the base and the wall. The element subdivisions and boundary conditions are shown in Fig.7.17. The applied load has been simulated by forces at nodes shown in Fig.7.17. The racking load is multiplied by a load factor. The load is increased till the failure is reached. The load factor is not constant throughout the analysis. Rate of increment is reduced after the 50 percent of failure load is reached. Coulomb type failure criterion derived experimentally by Kanungo(1966) has been used to model the slip and cracking behavior of the interface element.

Observations: The salient points of the results of the nonlinear analysis are described below.

- At a load of 2.56 kN(565 lbs) a tensile crack suddenly developed in the tension compression zone and during iteration several such cracks occur and finally failure has occurred as a result of stiffness degradation due to these cracks. Cracks at the base occur suddenly at this load. The ultimate loads obtained analytically and

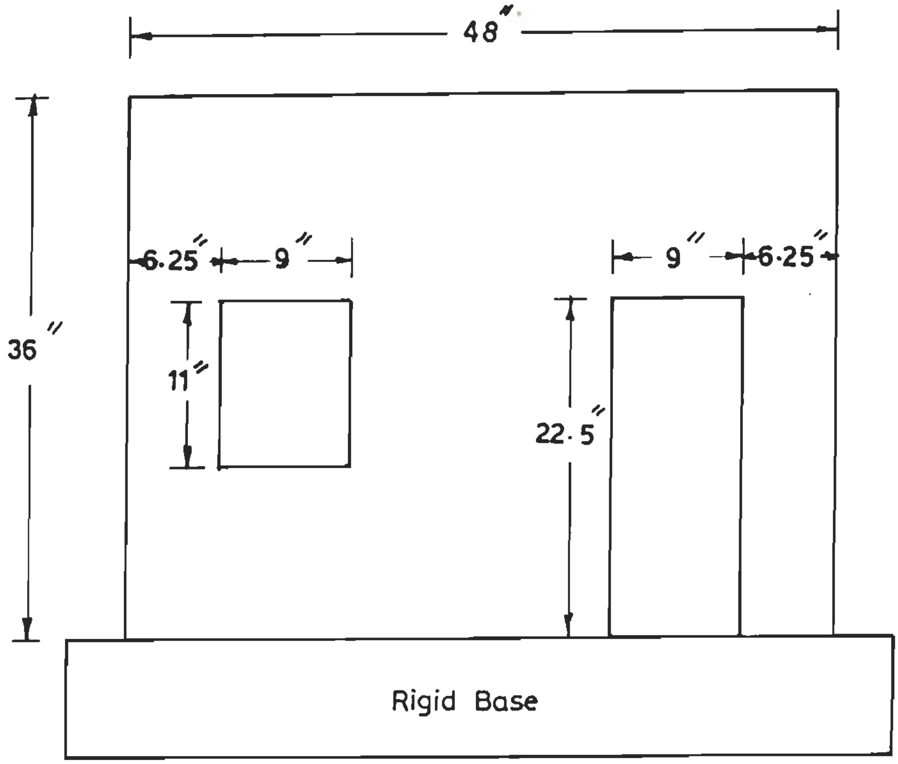

Fig. 7.16 Brick Masonry Wall with Door and Window Openings

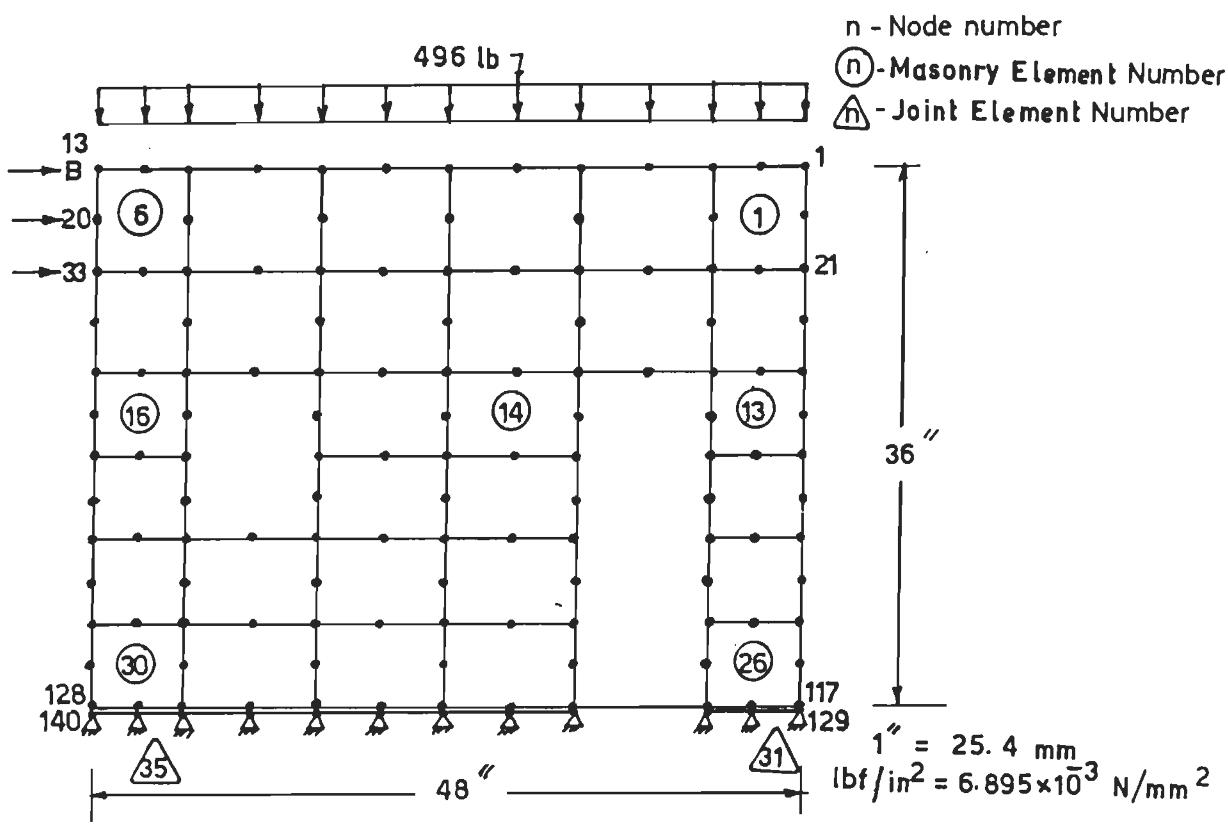

Fig. 7.17 Finite Element Subdivisions for Brick Masonry Wall with Door and Window Openings

Table 7.2: Material Properties of Masonry Shear Walls

Material	Property	Symb.	Bhargava (1978) (MPa)	Kanungo (1966) (MPa)
Brick masonry	Young's Modulus (MPa)	E	560.0	1172.0 (170000)
	Poisson's ratio	ν	0.20	0.20
	compressive strength parallel to bed joint (MPa)	f_{pc}	2.63	2.63 (382.42)
	Compressive strength normal to bed joint (MPa)	f_{nc}	3.15	453.15 (456)
	Biaxial compressive strength (MPa) with $\theta = 0^{o}$ (MPa)	f_{bc}	3.50	500.00 (3.50)
	Shear strength	τ_{o}	0.40	200.0 (1.4)
	Tensile strength	f_{t}	0.20	35.5 (0.245)
	Maximum tensile strain	ϵ_{m}	.0025	.0025
Mortar	Young's modulus (MPa)	E^{m}	600.0	900.0 (130527)
	Shear modulus (MPa)	G^{m}	250.0	400.0 (58012)
	Strength parameter of failure criterion for mortar joint	a_{1}	0.2	1.38 (200)
		b_{1}	0.5	.008 (1.1)
		a_{2}	0.0	0.0
		b_{2}	0.0	0.0

• values in the bracket are in psi.

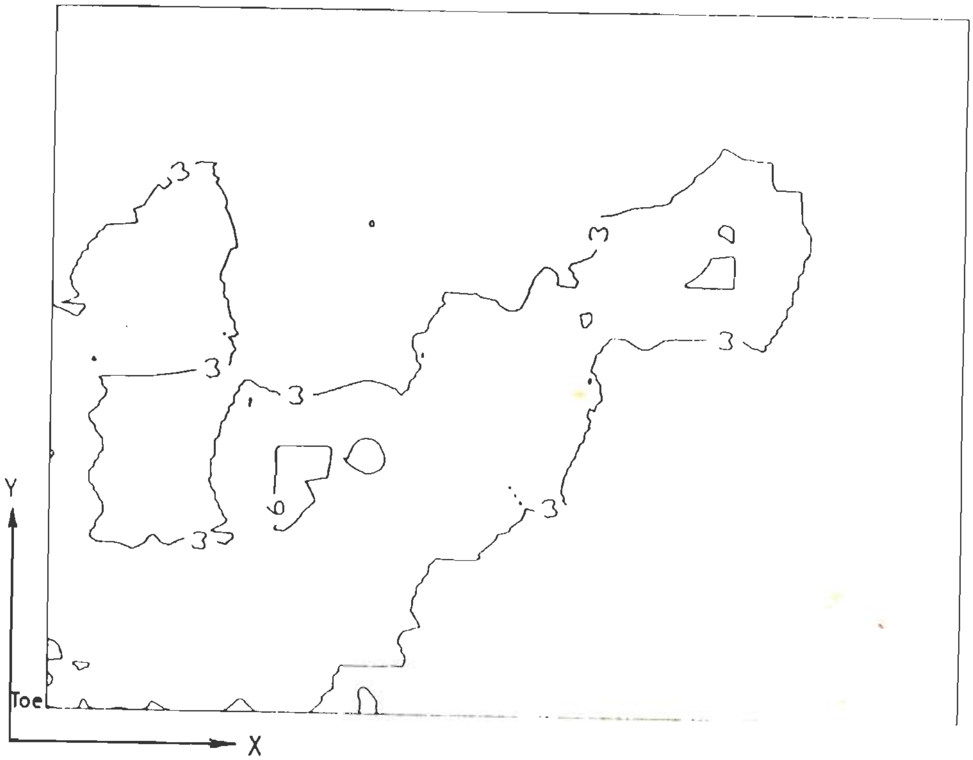

Fig. 7.18 Maximum Principal Stress Contours at Failure
Load for Masonry Wall with Door and Window Openings

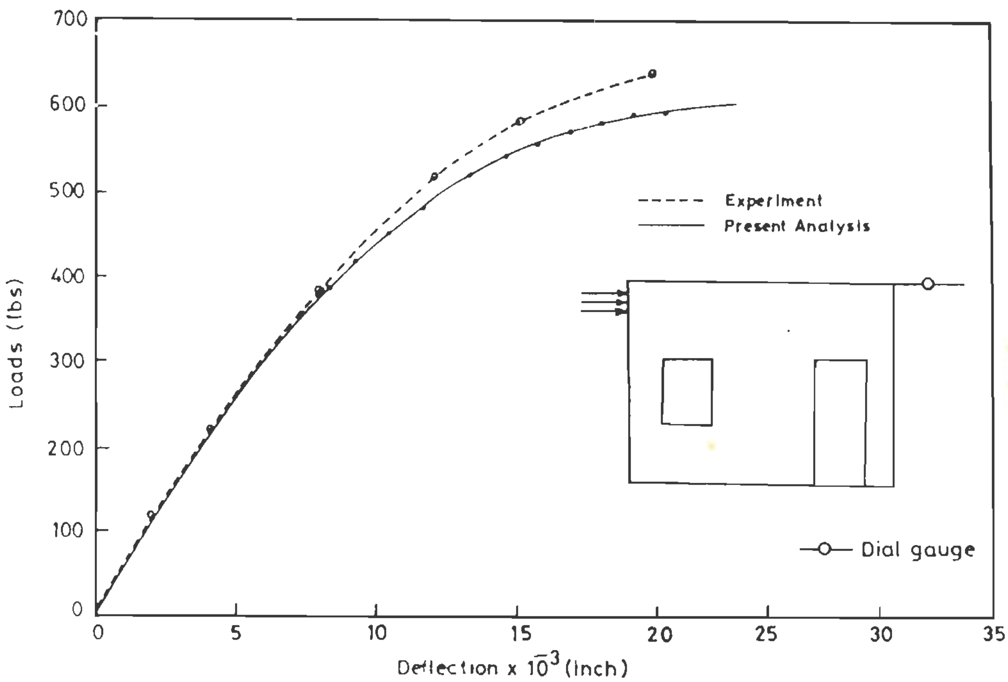

Fig. 7.19 Load - Deflection Curve for Brick Masonry
Wall With Door and Window Openings

- o Major principal stress contours at failure load are plotted in Fig. 7.18. This shows stresses in different zone after failure. Analysing the magnitude of stress contours it is observed that the failure has occurred due to loss of stiffness.

- o The load-deflection curve has been compared with that obtained experimentally as shown in Fig.7.19. A good agreement is observed between the two curves.

Table 7.3: Comparison of Strength and Deflection for Masonry Walls

Type of Wall	Lateral Loads		Vertical Loads		Deflection	
	Experiment. (kN)	Predict. (kN)	Experiment. (kN)	Predicted (kN)	Exp. (mm)	Pred. (mm)
Solid wall	62.0	60.0	140.0	140.0	*	4.2
Wall with a door op.	32.5	31.0	60.00	57.0	*	5.5
Wall with a door and window op.	2.85 (628)	2.56 (565)	2.25 (496)	2.25 (496)	0.4	0.45

* Deflection could not be measured at failure load
** Figures within brackets are in lbs.

7.5 Analysis of Masonry Shear Walls

Dhanasekar(1985) analysed ten shear walls with varying lateral to vertical load ratios. To prevent overturning failure about the toe, the position of the horizontal load, H, was varied along the height of the wall depending upon the intensity of the vertical load. A shear wall of dimension 1500 x 1000 x 50mm made of half scale size bricks was analysed. The dimension of the shear wall is shown in Fig.7.20. The wall was restrained at one corner and free to slide along its base.

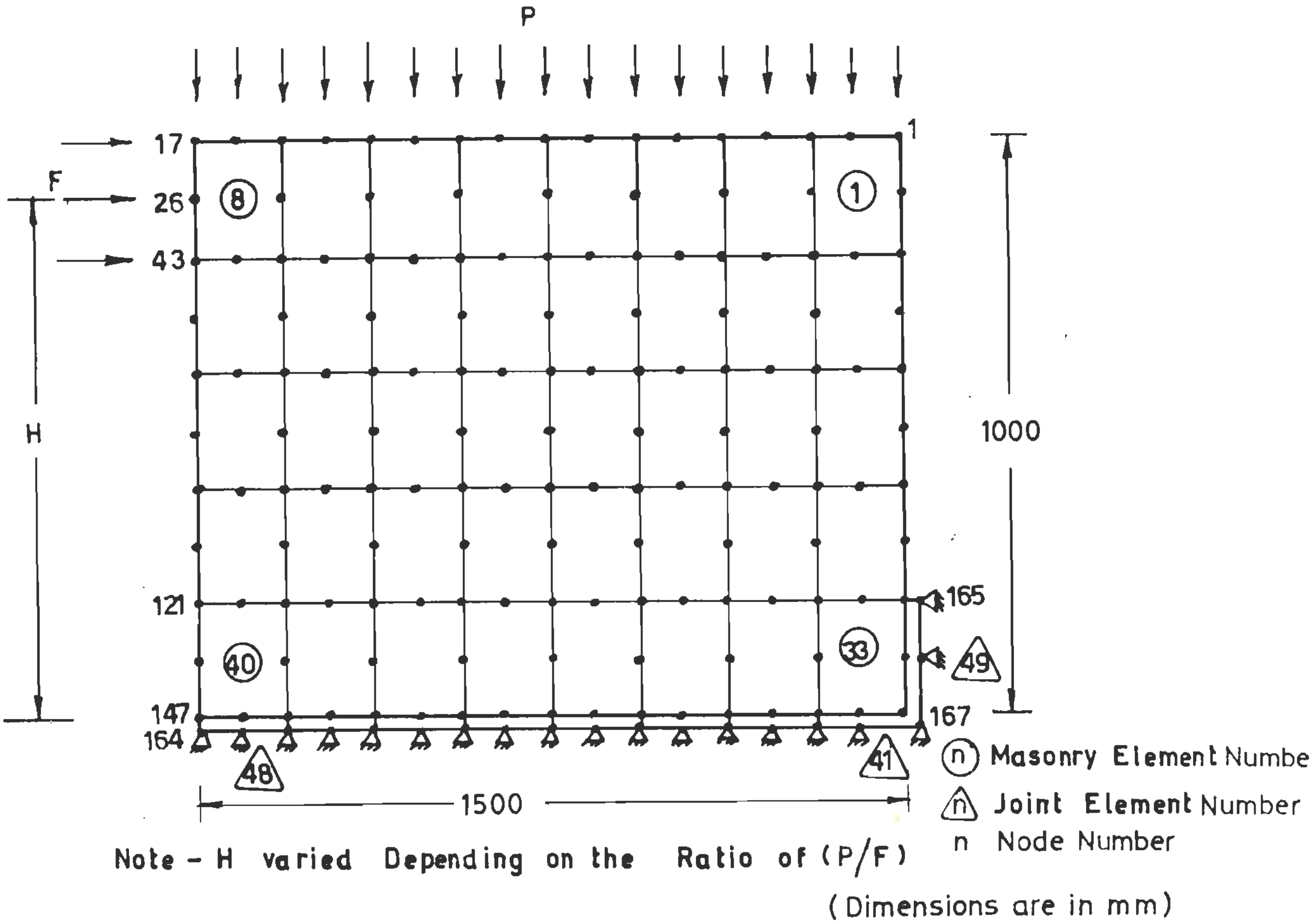

Fig. 7.20 Finite Element Subdivisions of Masonry Wall

Material Properties and Loading: The vertical compressive load has been distributed uniformly on the top of the wall. The racking load has been distributed over a small portion of the vertical face of the wall. The load ratio is kept constant during the analysis. The material properties of the masonry wall are shown in Table 7.4.

Procedure of Analysis: Brick masonry has been modelled using eight noded isoparametric element. Six noded interface element has been used between the support and the wall. The element subdivisions and boundary conditions are shown in Fig.7.20. A bilinear Coulomb type failure criterion, described in the previous Chapter, has been used to model the mortar joint at the interface between the wall and the base.

Table 7.4: Material Properties for Masonry Shear Wall.
[Dhanasekar (1985)]

Material	Property	Symb.	Value
Brick masonry	Young's Modulus (MPa)	E	5700.00
	Poisson's ratio	ν	0.19
	compressive strength parallel to bed joint (MPa)	f_{pc}	4.33
	Compressive strength normal to bed joint (MPa)	f_{nc}	7.56
	Biaxial compressive strength (MPa) with $\theta = 0^{o}$(MPa)	f_{bc}	8.15
	Biaxial compressive strength (MPa) with $\theta = 45^{o}$, and $\sigma_2/\sigma_1 = 4$	σ_1	2.33
		σ_2	9.4
	Tensile strength	f_t	0.79
	Maximum tensile strain	ϵ	.0025
Mortar	Young's modulus (MPa)	E^m	1150
	Shear modulus (MPa)	G^m	660
	Strength parameter for failure criterion of mortar joint	a_1	.85
		b_1	.24
		a_2	.11
		b_2	1.92

Observations: From the results of this analysis it is observed that

- o In all cases, at an early stage of loading tensile crack first has occurred in the mortar joint at the heel between the wall and the base and with an increase of load this has propagated towards the toe. The extent of cracking of mortar joint is shown in Fig.7.21. The length of this crack has varied depending upon the position and intensity of the racking load.

- o In most of the cases tensile crack has developed in brick masonry at a load of 60 percent of the failure load. Several such cracks have occurred with increase of load. The failure of shear walls have occurred due to the stiffness degradation as a result of these cracks(Fig.7.21).

- o From the Fig.7.21 it is revealed that at low compressive stresslevel (less than 1 MPa) the shear strength of masonry wall increases with the increase in compressive stress.

- o The failure mode of brick masonry is significantly influenced by the vertical to horizontal load ratio, α. For low value of α the failure zone generally extended across the wall from the point of racking loading to the toe. Closing of cracks also observed in analysis no.3, 5, 6 and 8.

- o The ultimate shear stress is plotted against ultimate normal compressive stress in Fig.7.22. It is observed that for values of normal stress greater than 1 MPa, the shear strength decreases. It compares favourably with a similar curve reported by Dhanasekar(1985) as shown in Fig.7.22.

- o The relationship for allowable shear stress as suggested by Indian Code is also plotted in Fig.7.22. For a factor of safety 3 the Code values seems to be reasonably representative of this investigation.

7.6 Influence of Masonry Properties on the Behaviour of Shear Walls

The influence of properties of masonry is studied by analysing the masonry wall of the previous section. The wall is analysed for different values of tensile and shear strength, keeping compressive strength constatnt. Procedure of increment of load is similar to the previous analysis.

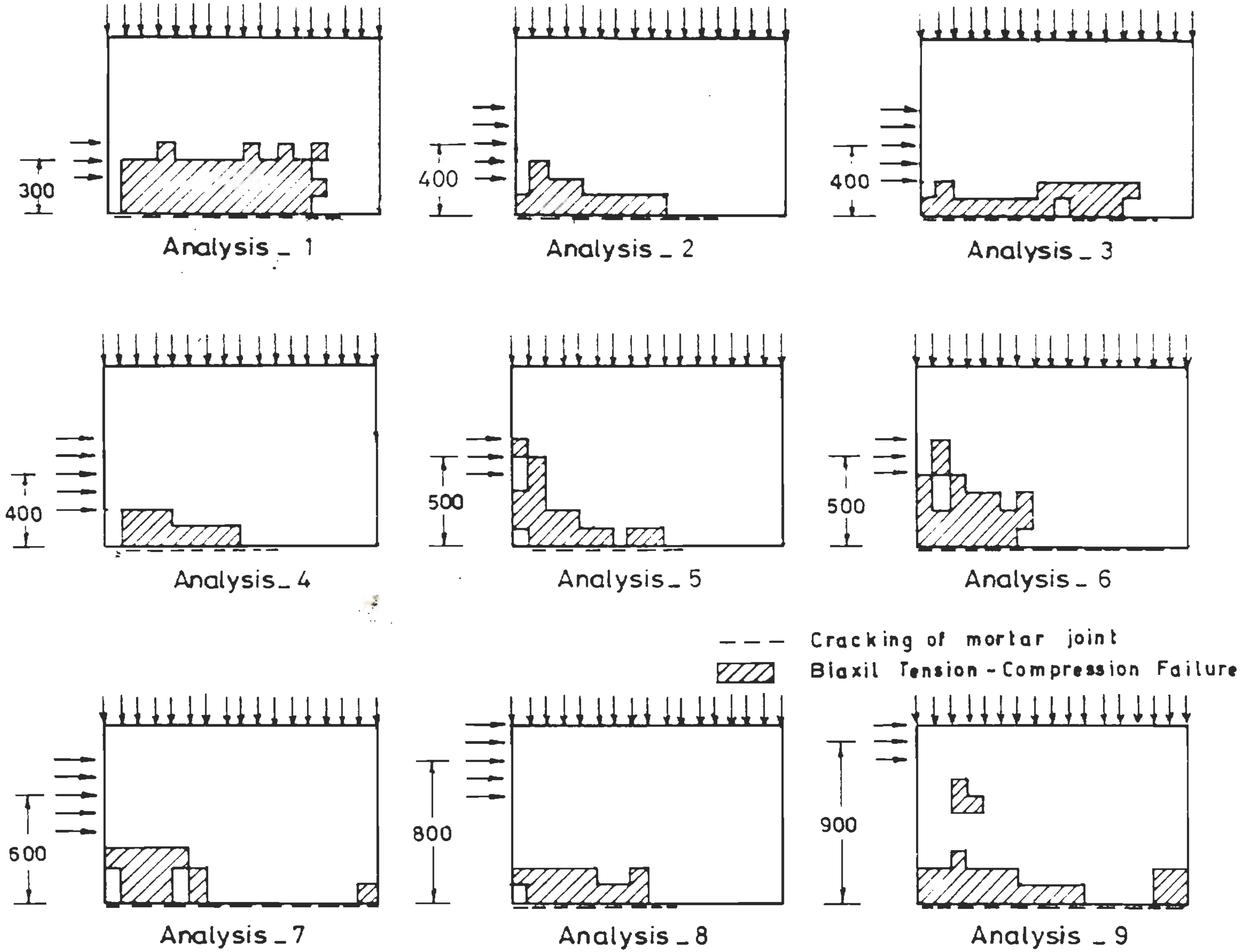

Fig. 7.21 Mode of Failure of Masonry Shear Walls

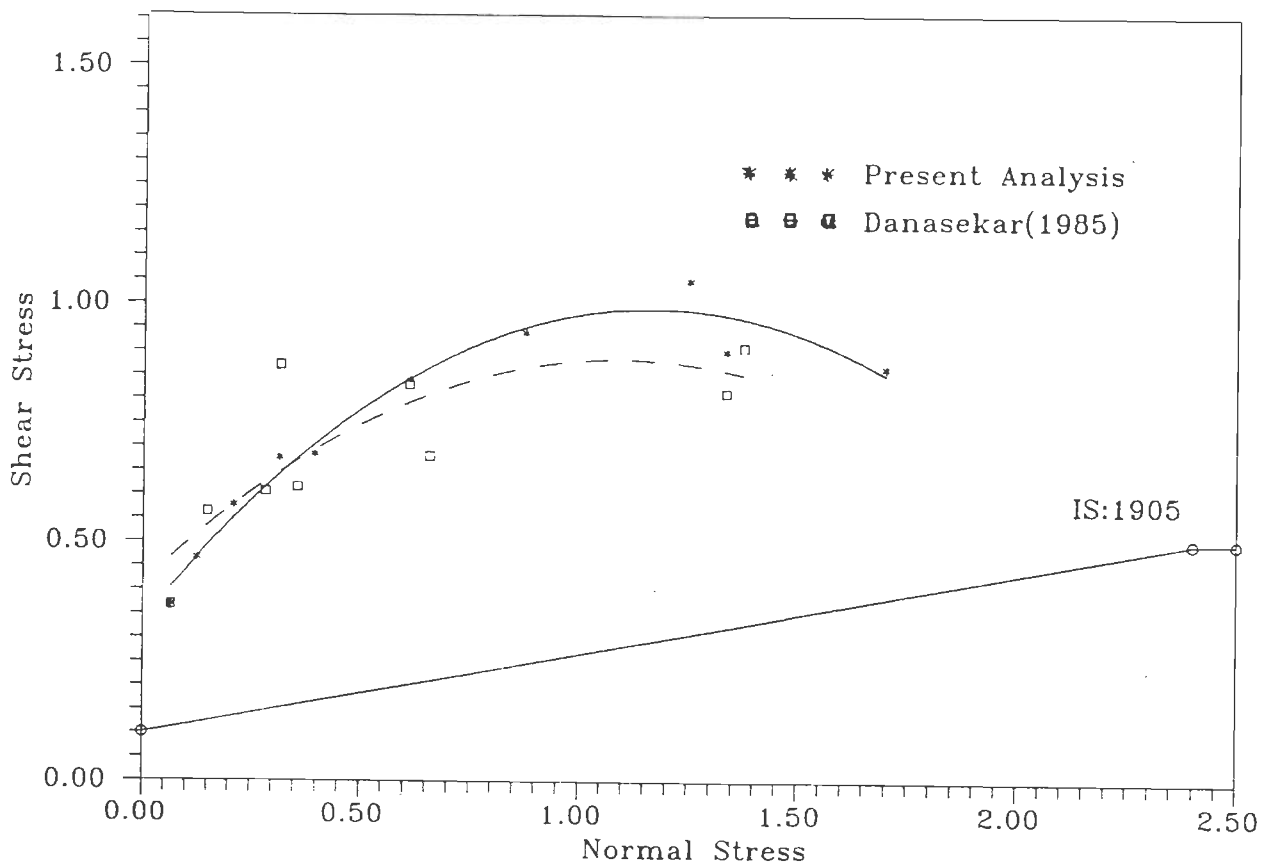

Fig. 7.22 Failure of Masonry Shear Walls Expressed in Terms of Average Normal and Shear Stresses

Observations From the studies of the effects of strength parameters of masonry on the behaviour of shear wall, it is revealed that

- The ultimate strength is not changed significantly when the compressive strength is increased twice of it original strength. The failure occurs due to tensile cracking.

- The ultimate strength is also not changed when compressive strength is reduced to half of the original strength. The failure mode is the combination of tensile cracking and crushing.

- The ultimate strength is increased significantly when the tensile bond strength is increased to twice of its original value. The failure has occurred due to tensile cracking.

7.7 Conclusions

The conclusions drawn from the study presented in this Chapter are as follows:

- A review of the previous work on the behaviour of masonry shear walls and infilled frames reveals that the analysis of masonry structures has been hampered due to the lack of realistic material model.

- To demonstrate the accuracy and versatility of the material model, structural idealisations nonlinear analysis of a infilled frame and masonry shear walls have been carried out. A good agreement between the analytical and experimental results is observed. Thus the material model can be used to analyse masonry infilled frames and shear walls from elastic range to failure load.

- The ultimate load carrying capacity is significantly affected by the ratio of vertical to horizontal load. For low value of this ratio lateral load carrying capacity increases with increase in vertical load.

- The ultimate strength load is significantly influenced by the strength properties of brick masonry. The ultimate strength will increase with the increase of tensile and bond strength. When the compressive strength is increased without changing in tensile strength, the ultimate strength will not change significantly but crushing failure is prevented. Similarly, if the compressive strength is reduced, failure may occur due to crushing of masonry. Thus tensile and bond strength significantly influence the ultimate strength while compressive strength affect the mode of failure of brick masonry walls.

- Brittle failure of masonry shear walls occurs at higher load if the strength of mortar used at the base and masonry wall is high. Diagonal tensile crack may initiate in the brick masonry before the tensile crack may occur at the base. Finally failure may occur due to tensile cracking and crushing of brick masonry.

- A failure envelope based on the ultimate shear stress and the corresponding vertical stress has been plotted. For low level of compressive stress, the ultimate shear stress increases with an increase in compressive stress.

- The main aim of the investigation reported in this Chapter is to demonstrate the accuracy of the finite element analysis using the proposed material model. However, further analysis and experimental studies are necessary to recommend the design and construction procedure of brick masonry structures.

7.8 References

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CHAPTER 8

SUMMARY AND CONCLUSIONS

8.1 Introduction

The structural properties of brick masonry were not investigated in detail as it was used only for the nonengineered constructions. For the last thirty years attempts have been made without complete success (a) to determine strength and elastic properties of brick masonry in terms of those of its constituents, (b) to develop a material model to predict the deformation characteristics due to material nonlinearity and cracking and (c) to develop computational procedures for the analysis of brick masonry structures subjected to in-plane loads.

This thesis presents (a) a comprehensive material model which can reproduce the deformation characteristics from elastic range to final failure for any state of stress and any angle of bed joint orientation, (b) finite element model and computer code for the analysis of masonry infilled frame and shear walls to demonstrate the accuracy and versatility of the material model, (c) a micromechanical brick masonry model to analyse and design (i) the strength and deformation characteristics of brick masonry in terms of those of its constituents and (ii) to determine the stress distribution in the brick and the mortar joints induced due to the stresses in brick masonry, (c) experimental investigations of the strength and elastic properties of brick masonry and its constituents so that analytical results based on micro-mechanics approach can be collated with the experimental results. The significant conclusions on the basis of the above studies are summarised in the subsequent Sections.

8.2 Properties of Brick Masonry and its Constituents

Experimental Method: The strength and elastic properties of the brick and mortar are experimentally determined using Codal procedure. The results are far from the actual values. Investigators have suggested various methods to determine more accurately the strength and elastic properties of the material. Out of these, a method based on increased aspect ratio has been selected for the determination of actual strength and elastic properties of bricks and mortar. The method is cost effective and simple to perform.

To determine the strength and elastic properties of masonry wall, Indian code has suggested testing of specimens (at least 400 mm high) of the same type as the structure to be designed. In many countries, the strength and elastic properties of masonry prism are used to determine these properties for brick masonry. So the strength and elastic properties are determined for both the masonry prism and the wall. A definite relation is observed between the strengths of the masonry prism and the wall. The difference in elastic properties, elastic modulus in particular, is insignificant. Thus the strength and elastic properties of masonry prism can be utilised for the determination of those for brick masonry wall.

Analytical Method: To determine compressive strength of masonry prism a strength theory has been developed. The proposed theory can determine the strength of the masonry prism within 95 percent of the experimental strength.

Formulae have been derived to determine the elastic properties of brick masonry in terms of those of the bricks and the mortar. The predicted elastic properties are very close to the experimental results.

Formulae have been derived to compute the stress distribution induced in the bricks and mortar for known masonry stresses. Stresses in the bricks and mortar computed from the proposed formulae agree well with those obtained from a micro-level finite element analysis in which the bricks and mortar joints are discretised in such a way that each element can encompass only one material.

The constitutive equations for brick masonry have been developed in terms of the properties of the brick and mortar assuming a linear elastic behaviour of the brick and material nonlinearity for mortar.

Micromechanics investigations are carried out in order to achieve required strength and stiffness of brick masonry in terms of strength and elastic properties of brick and mortar so that brick masonry can be analysed based on macro level approach. Thus economy in cost and time can be achieved for the analysis and design of masonry structures.

8.3 Material Model

The brick masonry is assumed to be elasto-perfectly plastic material under biaxial compressive stresses. To account for the directional strength properties at failure a generalised anisotropic quadratic failure criterion in three dimensional stress space has been used to model the nonlinear behaviour of the masonry. The specific advantage of this theory includes (1) invariant under rotation of coordinates, (2) transformation relations according to tensorial law, (3) the magnitude of the interaction term like that of the other failure theories is not constant. It is restrained in such a way that the shape of the failure surface is ellipsoidal.

The strength parameters are evaluated using the uniaxial and biaxial test results. The accuracy of the failure criterion depends upon the interaction strength parameter. To determine this parameter stability criterion must be checked. The influence of the test result on the variation of the strength parameter is also to be determined. As small inaccuracy in test results may change the value to a large extent. Sensitivity analysis has therefore, been carried out to select the type of test results to be used for the determination of the interaction strength parameter.

Smeared crack approach with fixed crack angle is adopted for modelling of the cracking behaviour of brick masonry. Maximum stress criterion for anisotropic material has been used for initiation and propagation of cracks. Tensile strain softening is employed for the gradual release of tensile stress after cracking. Closing and reopening of cracks are allowed in this model following the secant path.

8.4 In-Plane Behaviour of Masonry Infilled Frame and Shear Wall

Nonlinear analysis of masonry infilled frame and shear walls has been carried out to check the accuracy and versatility of the proposed finite element model and the computer code.

Eight noded isoparametric elements and two noded beam elements are used to model the behaviour of masonry and the frame. Six noded interface elements are used between the frame and the infill to model the mortar joint between them. For shear walls interface elements are used to model the mortar joint. For modelling the interface Coulomb type failure criterion is used for the separation and slip between the frame and the infill, and for shear walls tensile cracking and slip at the base.

The predicted ultimate strength of masonry infilled frame are in very good agreement with the experimental results. The predicted load-deflection curve closely resembles the experimentally derived load-deflection curve.

Masonry shear walls with and without door and window openings are also analysed. The predited ultimate strengths are in very good agreement with the experimental results. The predicted load-deflection curves closely resemble the experimentally derived load-deflection curves. The maximum principal stress contours at failure load are plotted to show that the failure occurred due to stiffness degradation of brick masonry.

Several shear walls with varying ratios of compressive load to racking load have been analysed and their failure behaviour have been studied. The study indicates that if the compressive strength of masonry is very high, the failure of shear wall will occur due to tensile cracking. On the other hand failure of shear wall will occur due to the crushing or combination of both tensile cracking and crushing. The failure load of the shear walls is significantly influenced by the shear and tensile bond strengths. At low compressive stress level the load carrying capacity is increased with increase in compressive load.

8.5 Scope of Future Investigations

The validity and versatility of the material model presented in this study has been amply demonstrated. The material model can be employed to study the behaviour of reinforced brick masonry and other types of building materials such as hollow brick and block masonry.

Further investigations are necessary to use micromechanical approach to achieve most economical form for the desired strength and stiffness of reinforced brick masonry. Use of both the concepts of micromechanics and macromechanics will help to achieve economy in time and cost for the analysis and design of reinforced masonry structures.