BIAXIAL BENDING OF RESTRAINED I—COLUMNS

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CERTIFICATE

CERTIFIED that the dissertation entitled " BIAXIAL [illegible] OF [illegible] I-COLUMNS" which is being submitted by [illegible] KUMAR in partial fulfilment for the award of the Degree of Master of Engineering in STRUCTURAL ENGINEERING of University of Roorkee is a record of student's own work, carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other Degree or Diploma.

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S Y N O P S I S

The elastic behaviour of columns with lateral supports provided in one flange of the column has been investigated. The lateral supports have been assumed to provide rigid lateral restraint at the same eccentricity but no torsional restraint.

An elastic design method for columns restrained laterally in one flange and subjected to axial and end moments about the major and minor axes has been evolved. The present practice of designing such columns neglects the effect of the lateral supports and is therefore more conservative.

Tests have been conducted and the test results have been compared with theoretical work. The variation of stresses due to minor axis bending along the length of the column has also been studied in the tests. After comparison of test results with theoretical work, conclusions have been made pertaining to the relative merits of the design method evolved with the existing design methods.

N_x	Magnification factor for major axis moment.
N_y	Magnification factor for minor axis moment.
p	Axial stress in column
P	Axial load in column
P_E	Euler buckling load
P_T	Torsional buckling load
r_o	Polar radius of gyration about restrained longitudinal axis
r_p	Polar radius of gyration about longitudinal axis through centroid
r_x	Radius of gyration about major axis
r_y	Radius of gyration about minor axis
t_f	Flange thickness
t_w	Web thickness
T	$\dfrac{AGKa_x^2}{I_x^2}$
u	Lateral displacement of centroid in x-direction.
v	Displacement of centroid in y-direction
z	Distance of a section from one end of the column
Z_x	Elastic section modulus about the major axis
Z_y	Elastic section modulus about the minor axis
α	$\dfrac{(GK - P r_o^2)\, l^2}{E\, I_y\, (a^2 + d^2/4)}$
α_o	Maximum initial twist
β	Ratio of smaller terminal moment to larger terminal moment.
β_1	M_1/M (See Fig. 2.3)

γ $\frac{k\, l^4}{E\, I_y (a^2 + d^2/4)}$

μ Factor for equivalent uniform moment of restrained column

ϕ Twist of column at any section

σ Longitudinal stress at any element of the cross section.

NOTATIONS

The following notations have been used throughout the dissertation. Some other notations have been used locally and defined wherever they appear first.

a	Distance of restrained longitudinal axis from the centroid of the section.
a_x	Distance of extreme fibre of a section from major axis.
a_y	Distance of extreme fibre of a section from minor axis.
A	Area of cross-section
b	Width of the flange
c_1	Spacing of lateral supports
C_w	Warping constant
C_{wo}	Warping constant about the restrained axis of twist.
d	Distance between the centres of flanges.
E	Young's Modulus of Elasticity.
f	Total stress due to axial load, major and minor axis bending and initial imperfections in the column.
f_L	Yield point stress of steel
f_o	Longitudinal stress due to minor axis bending caused by initial imperfections in the column on application of axial load and major axis moment.
f_x	Bending stress due to major axis moment
f_{xm}	Magnified f_x due to axial load.

f_y	Bending stress due to minor axis moment
f_{yy}	Magnified f_y due to axial load and major axis moment
G	Modulus of rigidity
I_o	Polar moment of inertia about the restrained longitudinal axis.
I_p	Polar moment of inertia about longitudinal axis through centroid
I_x	Moment of inertia of the section about major axis
I_y	Moment of inertia of the section about minor axis
k	Equivalent uniform torsional restraint (K_o/c_1)
K	St. Venant's torsional constant
K_o	Torsional restraint provided by one lateral support
l	Length of column
m	Number of segments of Column (l/c_1)
m_1	$\dfrac{Pr_o^2 + 2M_x a - GK}{EI_y(a^2 + d^2/4)}$
M	Larger end moment about major axis.
M_1	Moment in the column at the lateral support nearest to the end with the larger end moment
M_E	Critical moment for torsional buckling about the restrained longitudinal axis.
M_x	Moment about major axis
M_y	Moment about minor axis
n	Number of half waves in torsional mode of buckling

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CHAPTER - 1

INTRODUCTION AND REVIEW OF RESEARCH

Industrial buildings such as warehouses; aircraft hangers, large scale assembly plants and large garages are often fabricated as single-storey steel shed type structures with a travelling overhead crane spanning their full width. Rows of steel columns uncased by concrete or masonry are interconnected by angle sections running horizontally, attached to the outer flanges of the columns. The cladding is supported by these angles or sheeting rails. For the sake of rigidity, crossed counterbracing is done between the end pairs of columns or in intermediate bays so that flexure of the columns in a plane parallel with the wall is under considerable restraint. The shear resistance of the cladding assisted by crossed bracing is sufficient to provide complete lateral support at the point of attachment to the column but the column does not have complete restraint against twisting.

The mode of failure of such laterally restrained columns is likely to be a combined flexural-torsional mode with the axis of twist in the plane of the sheeting rails. The lateral torsional buckling assumes importance in the columns under axial load and end moments about both the axes, if the section is an open thin walled section as used in aricraft design or the column is so supported laterally that pure lateral buckling is prevented.

Wagner[3] was the first to investigate torsional buckling of thin walled sections. Goodier[4] put forward analysis of the unsymmetrical open sections under axial compression, moments about major and minor axes and torsional loads. Trahair[5] and Massey[6] found the effect of lateral restraints at ends and intermediate points. Lay and Galambos[7] investigated the rotation of closely braced steel columns under uniform moment and found that the intermediate restraints at compression flange were fully effective in resisting lateral buckling.

Dooley[8] showed that an axially loaded column attached to sheeting rails which prevent displacement of the attached flange at these points will adopt an instability trend towards torsional failure about the attached flange and this may be analysed by representing the restraint as continuous. The analysis of the more general problem with eccentric thrust offset from both principal axes and different end conditions is thus facilitated.

Dooley[9] further extended the analysis to full range of loading and eccentricity about both axes of the section, using this simplified idealisation. He concluded that the current practice of designing eccentrically loaded columns (BSS 449 - 1959)[17] is overconservative for columns restrained against flexure of one flange.

Ajmani[10] investigated the elastic, inelastic and post-buckling behaviour of columns with lateral supports connected to one flange of the column and providing full lateral restraint and elastic torsional restraint to the column. He found a general solution to the problem in the elastic range. Finite difference method was found to be more suitable for the case when side rails provide elastic lateral and torsional restraint at different eccentricities. Energy method could be used with advantage for solution of the problem when the side-rails were provided at the same eccentricity. He found a general numerical solution for the column subjected to axial load and unequal terminal moments and constrained to rotate about longitudinal axis.

Ajmani[10] also developed the concept of equivalent uniform moment. He found that the unequal terminal moments could be replaced, by an equivalent uniform moment, thereby simplifying the more general solution. This could be done as the critical moment is the same for uniform and non-uniform moment. He also developed the criteria for the completeness of lateral supports. Criteria was developed to determine for any loading condition and given spacing of lateral supports, the minimum torsional restraint which causes buckling to occur between supports rather than in an overall mode.

In the present investigation, the elastic behaviour of restrained column with lateral supports at the same eccentricity has been studied. The present practice of designing such column ignores the effect of lateral supports in one flange and is therefore more conservative. Attempt has been made to evolve an elastic design method for a restrained column subjected to axial load and end moments about major and minor axes by considering the effect of lateral supports. The magnification of stresses due to minor axis bending on application of axial load and the major axis moment has been considered. The column has certain initial imperfections and these are magnified by the application of the axial loads and bending moments. The stresses resulting from the initial imperfections in the column have also to be taken into account while evolving a design criterion.

Tests were conducted to study the stresses occurring at mid span of the column on application of load and the results were compared with the theoretical work. The variation of stresses due to minor axis bending along the length of the column was also studied. Chapter 4 describes the testing apparatus. Test results and their comparison with theory have been given in Chapter 5.

CHAPTER - 3

LATERAL TORSIONAL BUCKLING

3.1. UNRESTRAINED I- COLUMN

3.1.1. Unrestrained I-Columns Subjected to Uniform Moments About Major and Minor Axes and Axial Load.

When an unrestrained column is subjected to major and minor axis terminal moments, the maximum stress always occurs at the mid height of the column. Consider a column, shown in Fig. 3.1, subjected to an axial load P and equivalent uniform moments M_x and M_y about the major axis and minor axis respectively. Representing the applied equivalent uniform moment M_y by the equivalent Fourier half range series :

$$\frac{4}{\pi} M_y \sum_{n=0}^{n=\infty} \frac{1}{(2n+1)} \sin (2n+1) \frac{\pi z}{l} \qquad \ldots(3.1)$$

The deflections produced by the minor-axis bending moment in the X'O' Z plane, in the absence of the axial load P and the major axis moment M_x are obtained by integrating twice as,

$$\frac{4}{\pi^3} \frac{M_y l^2}{EI_y} \sum_{n=0}^{n=\infty} \frac{1}{(2n+1)^3} \sin (2n+1)\frac{\pi z}{l}$$

The value of deflection at mid height ($z = l/2$) considering the first term only is given by,

$$\frac{4}{\pi^3} \frac{M_y l^2}{EI_y} \quad \text{Sin} \quad \frac{\pi z}{l}$$

or $0.129 \quad \frac{M_y l^2}{EI_y}$

The accurate value of the central deflection is $0.125 \frac{M_y l^2}{EI_y}$, so that by taking the first term only a reasonably safe result is obtained. The initial imperfections in the column may be allowed for by assuming the longitudinal axis to have an initial curvature in a plane perpendicular to the web (plane X'O'Z in Fig. 2.1). Since it is not possible to have an exact knowledge of the imperfections in the column, it can be assumed that the initial curvature which represents them is so arranged that the most rigorous conditions are produced. When the loads are such that the maximum stress will occur at or near the column length, the bending moment there, due to initial curvature, will have its greatest value, when the curvature is sinusoidal.

Tests conducted by Horne[12] showed that ϵ, the versed sine of the initial curvature could be taken as $\epsilon = 0.0015 \frac{l \; r_y}{a_y}$. The displacements in the direction X'O' can therefore be represented by $\epsilon \text{ Sin } \frac{\pi z}{l}$

The total deflections in the X'O'Z plane in the absence of the axial load P and the major axis bending moment M_x are given by

or

$$u = \left[e + \frac{4}{\pi^3} \frac{M_y l^2}{EI_y} \right] \sin \frac{\pi z}{l} \qquad \ldots(2.2)$$

The governing differential equation for the case of the column subjected to an axial load P and terminal moment M_x can be obtained as,

$$\left.\begin{aligned} EI_y \frac{d^2(u_1-u_0)}{dz^2} &= -Pu_1 - M_x \phi_1 \\ (GK - Pr_p^2)\frac{d(\phi_1-\phi_0)}{dz} - EI_y \frac{d^2}{4}\frac{d^3(\phi_1-\phi_0)}{dz^3} &= M_x \frac{du_1}{dz} \end{aligned}\right| \qquad \ldots(2.3)$$

Where,

u_0 = Initial displacement of shear centre at any section in the column in the X direction before P and M_x are applied.

u_1 = Displacement of shear centre at any section in the column in the X direction after P and M_x are applied.

ϕ_0 = Initial twist of the Column at any section before P and M_x are applied.

ϕ_1 = Twist of the column at any section after P and M_x are applied.

Let,

$$u_o = U \sin \frac{\pi z}{l}$$

$$\phi_o = 0$$

$$u_1 = U_1 \sin \frac{\pi z}{l}$$

$$\phi_1 = \Phi \sin \frac{\pi z}{l}$$

Where,

U = Displacement of shear centre at mid span of the column before P and M_x are applied.

U_1 = Displacement of the shear centre at mid span of the column after P and M_x are applied

Φ = Twist of the column at mid span after P and M_x are applied.

Substituting these in the equations (2.3) we get,

$$- EI_y \frac{\pi^2}{l^2} (U_1 - U) \sin \frac{\pi z}{l} = -PU_1 \sin \frac{\pi z}{l} - M_x \Phi \sin \frac{\pi z}{l}$$

$$(GK - Pr_p^2) \frac{\pi}{l} \Phi \cos \frac{\pi z}{l} + EI_y \frac{d^2}{4} \frac{\pi^3}{l^3} \Phi \cos \frac{\pi z}{l} = M_x U_1 \frac{\pi}{l} \cos \frac{\pi z}{l}$$

or

$$U_1 (P_E - P) - M_x \Phi = U P_E$$

$$U_1 M_x - \Phi \frac{M_E^2}{P_E} = 0$$

as $P_E = \dfrac{\pi^2 E I_y}{l^2}$

and $$M_E^2 = \frac{\pi^2 EI_x}{l^2}\left[(GK - Pr_p^2) + \frac{\pi^2 EI_y}{l^2}\;\frac{d^2}{4}\right]$$

We now obtain,

$$U_1(P - P)\frac{M_E^2}{P_E} - U_1 M_x^2 = UM_E^2$$

or $$U_1 = \frac{U}{1 - \left[\frac{P}{P_E} + \frac{M_x^2}{M_E^2}\right]}$$

$$U_1 = \frac{U}{1 - \gamma_y} \qquad \ldots(2.4)$$

Where $$\gamma_y = \frac{P}{P_E} + \left(\frac{M_x}{M_E}\right)^2 \qquad \ldots(2.5)$$

It is now possible to calculate the minor axis bending moment at mid height. The value of this bending moment is M_y before the application of P and M_x. When the loads P and M_x are applied, there is an increase in the value of M_y equal to

$$\left[- EI_y \frac{d^2 (U_1 - U)}{dz^2}\right]$$

The new central bending moment is,

$$M_y = - EI_y \frac{d^2}{dz^2}\left[\frac{U}{1 - \gamma_y} - U\right] + M_y$$

$$= \frac{\pi^2 EI_y}{l^2}\left(\frac{\gamma_y}{1 - \gamma_y}\right)\zeta\,.\,\mathrm{Sin}\,\frac{\pi z}{l} + \left[1 + \frac{4}{\pi}\left(\frac{\gamma_y}{1 - \gamma_y}\right)\right] M_y \mathrm{Sin}\frac{\pi z}{l}$$

$$(M_y)_{1/2} = \frac{\pi^2 EI_y}{l^2}(\gamma_y / 1 - \gamma_y) \leftarrow + \left[1 + \frac{4}{\pi}\frac{\gamma_y}{1-\gamma_y}\right] M_y \qquad ...(2.6)$$

The central bending moment about the major axis, $(M_x)_{1/2}$ can similarly be obtained.

The deflections v in the plane ZO'Y' in the absence of the axial load P are given approximately by

$$v = \frac{4}{\pi^3}\frac{M_x l^2}{EI_x} \sin \frac{\pi z}{l} \qquad ...(2.7)$$

On the application of the axial load P, this deflection is magnified to v_1 such that

$$\frac{v_1}{v} = \frac{1}{1 - (P/P_E)}$$

Where P_E = Euler's critical load for the column treated as a pin ended strut buckling about the major axis.

$$= \frac{\pi^2 EI_x}{l^2}$$

$$v_1 = \frac{v}{1 - \gamma_x} \qquad ... \qquad ...(2.8)$$

$$\text{Where } \gamma_x = P/P_E = \frac{Pl^2}{\pi^2 EI_x} \qquad ...(2.9)$$

The value of the central bending moment about the major axis in the presence of the axial load is,

$$\left[M_x - EI_x \frac{d^2(v_1 - v)}{dz^2}\right] \quad \text{When } z = l/2$$

Hence, $$\left[M_x\right]_{l/2} = \left[M_x + EI_x \cdot \frac{4\, l^2\, M_x}{\pi^3\, EI_x} \;\; \text{Sin}\; \frac{\pi z}{l}\right]_{z=l/2}$$

or

$$\left[M_x\right]_{l/2} = \left[1 + \frac{4}{\pi}(\gamma_x / 1 - \gamma_x)\right] M_x \qquad \ldots(2.10)$$

The value of the maximum stress at the mid-height of the column can now be calculated. Equating this stress to the lower yield stress f_L, the failure criterion is obtained as follows :

$$\frac{P}{A} + \left[M_x\right]_{l/2} \cdot \frac{a_x}{I_x} + \left[M_y\right]_{l/2} \frac{a_y}{I_y} = f_L \quad \ldots(2.11)$$

Let the mean stress $P / A = p$

$$f_x = M_x \frac{a_x}{I_x}$$

$$f_y = M_y \frac{a_y}{I_y}$$

So that f_x, f_y are externally applied bending stresses about the major and minor axis respectively. It follows from equations (2.6), (2.10) and (2.11) that

$$p + N_x f_x + N_y f_y = f_L - f_o = f \qquad \ldots(2.12)$$

Where $$N_x = 1 + (4/\pi)(\gamma_x / 1 - \gamma_x) \qquad \ldots(2.13)$$

$$N_y = 1 + (4/\pi)(\gamma_y / 1 - \varphi_y) \qquad \ldots(2.14)$$

and $$f = f_L - \frac{a_y}{I_y}\frac{\pi^2 EI}{l^2}(\gamma_y / 1 - \gamma_y)\ (-$$

or

$$f = f_L - 0.0015\,\pi^2 E \left[\frac{r_y}{l}\right]\left[\frac{\gamma_y}{1 - \gamma_y}\right] \quad \ldots(2.15)$$

Equation (2.8) can be expressed in the form,

$$Y_x = \frac{1}{\pi^2 E} (1 / r_x)^2 \; p$$

Neglecting the resistance of the member to warping,

$$M_E^2 = \frac{\pi^2}{l^2} EI_y GK$$

Equation (2. 5) can be written as,

$$Y_y = \frac{pl^2}{\pi^2 EI_y} + \left[\frac{M_x^2 \, l^2}{\pi^2 \; EI_y \; GK}\right]$$

$$= \left[\frac{l}{r_y}\right]^2 \frac{1}{\pi^2 E} \left[p + f_x^2 \frac{I_x^2}{A \; GK \; a_x^2} \right]$$

$$\text{or } \; Y_y = \frac{1}{\pi^2 E} \left[\frac{l}{r_y}\right]^2 \left[p + \frac{f_x^2}{T}\right]$$

Where $T = \frac{AGK \; a_x^2}{I_x^2}$ ...(2.16)

There $Y_y = \frac{1}{\pi^2 E} \left[\frac{l}{Y_y}\right]^2 p'$ ...(2.17)

Where $p' = p + \frac{f_x^2}{T}$ ...(2.18)

It is seen that N_y is the same function of l/r_y and p' as is N_x of l/r_x and p. Also the value of f given by equation (2.15) is a function of l/r_y and p'. The values of N_x, N_y and that of f can be obtained from Charts 34 ,35 and 36 of BCSA Publication[13] No.23 of 1964. The values of f_L and E have been taken as 15.25 tons/in^2 and 13,000 tons/in^2 respectively. It may be noted that the figures in parentheses give an alternat pair of scale for (p,p') and (l/r_x , l/r_y) .

The value of K is obtained by

$$K = \sum \frac{1}{3} B t^3 \qquad \ldots(2.19)$$

Where B is the larger dimension. To check the ability of the column to carry a given axial load and uniform applied moments about the two axes, stresses, p, f_x and f_y are first calculated. The values of N_x and N_y are obtained from the chart using $p' = p + \frac{f_x^2}{T}$

The column will be safe provided,

$$p + N_x f_x + N_y f_y \leq f \qquad \ldots(2.20)$$

2.1.2. Equivalent Uniform Moment

When the applied bending moments vary along the length of the column, it becomes difficult to use a general design method. For simple columns with unequal terminal bending moments but no lateral support, Salvadori(14) and Horne(15) solved the problem by replacing the unequal terminal moments by equivalent uniform moment.

Horne used the energy procedure to calculate the flexural-torsional buckling load. He used the process of successive approximations to obtain a close approximation of the deflected form. He further assumed lateral deflection to obtain approximate twist, which again he used to get a better approximation of lateral deflection and so on. Finally he gave the equivalent uniform bending moment for the following two cases :-

a. torsional rigidity neglected

b. warping rigidity neglected

If M_x' and M_x'' are the unequal terminal moments acting on a column, they are equivalent to equal terminal moment $M_x = \mu M_x'$, bending the column in single curvature. The value of μ depends on the ratio of the unequal terminal moments $\beta = \frac{M_x'}{M_x''}$, $M_x' < M_x''$.

The value of μ can be found knowing the value of p from Chart 33 of BCSA Publication[13] No.23 of 1964.

Tee equivalent uniform moment M_y can similarly be found for unequal terminal moments M_y' and M_y'' acting about the minor axis.

The concept of equivalent uniform moment thus facilitates the use of simple design method as shown in Art 2.1.3. and enables us to avoid complicated procedure to solve the problem.

2.1.3. Method of Design for Unequal Terminal Moments about Major and Minor Axes

Equations (2.13) to (2.20) may be used to ensure that the maximum stress does not exceed the yield value provided that at both the ends of the column the stress is less then the yield stress. If the yield stress is reached at either end before it occurs elsewhere the instability equations are not to be used and the condition may be checked by elementary methods.

The following conditions must therefore be satisfied to ensure that the yield stress is not exceeded anywhere in the section.

$$\left.\begin{array}{lll} p + f'_x + f'_y & = & f_L \\ p + f''_x + f''_y & = & f_L \\ p + N_x f_x + N_y f_y & \leq & f \end{array}\right] \qquad \ldots (2.21)$$

Where

f'_x, f''_x and f_x = Extreme fibre stresses corresponding to major axis bending moments M'_x, M''_x and M_x respectively.

f'_y, f''_y, and f_y = Extreme fibre stresses corresponding to minor axis bending moments M'_y, M''_y and M_y respectively

2.2. RESTRAINED COLUMNS

A general solution for the stability of a column subjected to axial load and unequal terminal moments about major and minor axes and supported laterally by side rails was found by Ajmani[10]. The lateral supports were assumed to be connected to the column at different eccentricities and provided different torsional and lateral restraints. He obtained the differential equations of equilibrium for the general case. The energy procedure could not be used as it is difficult to assume any suitable deflected shape if the lateral supports are provided at random. The exact solution of the differential equations incorporating the boundary conditions and continuity conditions at supports being difficult, the finite difference method was used to advantage in solving the differential equations. The critical bending moment for a given axial load could be obtained by solving the finite difference equations.

In the case of restrained column with lateral supports at the same eccentricity, the energy method was used. Dooley[8] solved the problem of an unsymmetrical I-section subjected to an axial load only. Ajmani[10] found the critical buckling loads using the energy method for (a) axial load only (b) end moment only.

2.2.1. Completeness of Lateral Supports

A column subjected to axial load and/or uniform moment will buckle either by twisting about the restrained longitudinal axis or by flexural-torsional buckling between the lateral wupportw. Ajmani[11] has given the criteria for the completeness of lateral supports so that a column subjected to axial load, uniform moment or non-uniform moment buckles between the two consecutive supports.

The critical load (axial load and/or moment) for a laterally supported column of given length and section depends on :-

(i) Spacing of lateral supports, c_1

(ii) Eccentricity of lateral supports, a

(iii) Torsional restraint of lateral supports, k

Depending on these three factors, the column will buckle in one of the following two modes:-

(a) Torsion about the laterally supported longitudinal axis, the critical load being given by P_T of M_T

(b) Flexural or lateral torsional buckling between the supports, with no lateral or torsional displacement at the supported section, the critical load being given by P_E or M_E .

P_T and M_T depend on the nature of lateral supports as defined by a and k but are indpendent of the spacing c_1 provided the torsional restraint of each lateral support is adjusted to keep k constant. P_E and M_E depend on c_1 and are independent of a and k .

The 'complete supports' will be defined as the supports which force the column to buckle by flexural or lateral torsional buckling between the supports, with no lateral or torsional displacement at the supported sectio . The critical load will then be P_E or M_E with effective length equal to c_1 .

2.2.1.1. Critérion of Complete Support for the case of Axial Load and Uniform Moment

Ajmani[11] has investigated the completeness of supports in the case of a restrained column subjected to axial load and uniform moment. If the column with lateral supports is subjected to axial load and uniform moment and it buckles by twisting about the restrained longitudinal axis, the critical moment is given by the equation,

$$P + \frac{2\,M\,a}{r_o^2} = P_T \qquad ..(2.22)$$

When there is flexural torsional buckling between the lateral supports the critical moment is given by the equation

$$(P_E - P)\,(r_p^2 P_E P - M_E^2) + P_E M^2 = 0 \qquad ..(2.23)$$

For torsional restraint equal to zero, the condition for supports to be just complete was obtained by Ajmani[11] by equating equations (2.22) and (2.23)

$$\left(1 - \frac{P}{P_E}\right)\left[\frac{4\,GKl^2}{\pi^2 EI_y\,d^2}\,\frac{1}{m^2} + 1 - \frac{P}{P_E}\right]$$

$$= \left[\frac{1}{m^2}\left\{\frac{4\,GKl^2}{\pi^2 EI_y d^2}\,\frac{d}{4a} + \frac{d}{4a} + \frac{a}{d}\right\} - \frac{P}{P_E}\left\{\frac{a}{d} + \frac{d}{4a}\right\}\right]^2 \qquad ..(2.24)$$

Equation (2.24) is used to plot the relation between m and $\frac{4\,GKl^2}{\pi^2 EI_y d^2}$ for given values of a/d and P/P_E . Fig. 2.4 shows this relation for the case when a/d = 0.75 and γ = 0 For a given value of m and P/P_E the support is complete if the torsional rigidity is greater than given by Fig.2.4 and the column fails by lateral-torsional buckling between the supports. If however the torsional rigidity is less, the support is incomplete.

2.2.2. Equivalent Uniform Moment

The equivalent uniform moment concept developed by Horne(15) in the case of unrestrained columns was extended to the case of restrained column by Ajmani(10) While investigating the stability of restrained column under axial load and non-uniform moment, it was found convenient to replace the non-uniform moment by equivalent uniform moment. This also facilitated the study of the criterion of the completeness of support in the case of non-uniform moment. Also it is found convenient when establishing design criteria(16) to replace unequal terminal moment by equivalent uniform moment.

In the case of a restrained column subjected to axial load and non-uniform moment, energy method can be used with advantage to compute numerically the buckling loads as an explicit analytical expression for critical non-uniform moment cannot be found. It is assumed that the column will undergo torsional buckling about the restrained axis of twist.

Consider a column of length l as shown in Fig. 2.3, restrained laterally by supports at spacing c_1 and at an eccentricity of a from the centre line of the column. The column is subjected to an axial load P and unequal terminal moments M and β M, so that $-1 \leq \beta \leq 1$. The larger terminal moment M produces compression in the unsupported flange.

Discrete torsional restraint is replaced in the analysis by the uniformly distributed torsional restraint $k = K_s / c_1$.

The buckled shape is expressed as the infinite series

$$\phi = \sum \phi_n \text{Sin} \frac{n\pi z}{l} \qquad ..(2.25)$$

where n = number of half wavew in torsional mode of Buckling.

In this series each term satisfies the boundary condition for the column. The arbitrary constants ϕ_1, ϕ_2 ...... are so chosen that the total potential energy of the system is stationary for variations in all values of ϕ_n

i.e. $$\frac{\partial (U + V)}{\partial \phi_n} = 0 \qquad ..(2.26)$$

Let , U = Strain energy of the system

V = Change in the potential energy of the loads.

$$U = \frac{1}{2} GK \int (\phi')^2 \, dz + \frac{1}{2} E C_{wo} \int (\phi'')^2 \, dz + \frac{1}{2} k \int (\phi)^2 dz$$

$$= \frac{\pi^2 GK}{4l} \sum n^2 \phi_n^2 + \frac{\pi^4 EI_y}{4 l^3} (a^2 + d^2/4) \sum n^4 \phi_n^2 + \frac{kl}{4} \sum \phi_n^2 \qquad \ldots (2.27)$$

$$V = - \int_0^l \int_A \frac{1}{2} \sigma (dw/dz)^2 \, dA \, dz$$

Where, w = Total displacement of an elemental area dA of the cross section at initial coordinates, x, y

σ = Longitudinal stress on area dA

$$\text{As} \quad w = \phi \sqrt{\left[x^2 + (a + y)^2\right]}$$

$$\sigma = \frac{P}{A} + \frac{My}{I_x} \left[1 - (1 - \beta) z/l\right]$$

We have,

$$V = - \int_0^l \int_A \frac{1}{2} \left[\frac{P}{A} + \frac{My}{I_x} \left\{1 - (1 - \beta) z/l\right\}\right] \left[(a+y)^2 + x^2\right] (\phi')^2 d$$

$$= - \frac{1}{2} (Pr_o^2 + 2 Ma) \int_0^l (\phi')^2 \, dz + \frac{Ma(1-\beta)}{l} \int_0^l z \, (\phi')^2 dz$$

Substituting the value of ϕ we get,

$$V = -\frac{1}{2}\,(Pr_o^2 + 2\,Ma\,)\;\frac{\pi^2}{2l}\;n^2\,\phi_n^2$$
$$+\frac{1-\beta}{l}\;M_a\left[\frac{\pi^4}{4}\;\sum\;n^2\,\phi_n^2 - \sum\sum mn\,\phi_m\,\phi_n\left\{\frac{1}{(m+n)^2}+\frac{1}{(m-n)^2}\right\}\right]$$

...(2.28)

Where the summation extends over all values of n and the double summation also extending over all values of m when m + n is odd.

Using the non-dimensional quantities

$$\alpha = \frac{(GK - Pr_o^2)\;l^2}{EI_y(a^2 + d^2/4)}$$
$$\lambda = \frac{Ma\;l^2}{EI_y(\,a^2 + d^2/4)} \qquad \text{...(2.29)}$$
$$\gamma = \frac{kl^4}{\pi^4\;EI_y(a^2 + d^2/4)}$$

and substituting the value of $\{U + V\}$ in equation (2.26) for stationary potential energy it can be shown[11] that for each n

$$\left[\frac{1+\beta}{\alpha + \pi^2 n^2 + \pi^2\gamma/n^2} - \frac{1}{\lambda}\right]\phi_n + \frac{8}{\pi^2}\;\frac{1-\beta}{\alpha + \pi^2 n^2 + \pi^2\gamma/n^2}$$
$$\sum\;\frac{m}{n}\;\frac{m^2 + n^2}{(m^2 - n^2)^2}\;\phi_m$$
$$= 0$$

...(2.30)

Equation (2.30) represents a system consisting of an infinite number of homogeneous linear equations in ϕ_1, ϕ_2 .... The set of equations is obtained by putting n = 1, and m = 2,4,6,....; n = 2 and m = 1,3,5.. and so on.

The obvious and trivial solution is obtained by putting $\phi_1 = \phi_2 = \ldots = 0$. We can obtain by equating the determinants of the coefficients of ϕ_1, ϕ_2 .... to zero an expression of the form,

$$f(\alpha, \beta, \gamma, \lambda) = 0 \qquad \ldots(2.31)$$

A numerical solution can be obtained for any finite number of equations. For given values of α, β and γ equation (2.31) leads to a standard eigenvalue problem

$$\left| \bar{A} - \frac{1}{\lambda} \bar{B} \right| = 0 \qquad \ldots(2.32)$$

The lowest value of λ gives the least value of the larger end-moment at which buckling will occur. Ajmani[11] has solved the eigenvalue problem of equation (2.32) by programming on the computer. The relations between α and λ for various values of β were obtained by him. Although the relation between α and λ can be graphically represented it is desirable, when establishing design criteria[16] to replace the unequal terminal moment by equivalent uniform moment.

Let the critical value of λ for $\beta = 1$ be λ_1 and for any other value of β be λ. If M_{cr} is the larger critical terminal moment for any value of β and μM_{cr} is the equivalent uniform moment to cause buckling of the column :

$$\mu = \frac{\mu M_{cr}}{M_{cr}} = \frac{\lambda_1}{\lambda}$$

The values of μ can be calculated for various values of β and α. Ajmani(10) has plotted the relation between μ and β for various values of α as shown in Fig. 2.5. This represents the case where $\gamma = 0$ i.e. there is full lateral restraint but no torsional restraint.

Thus more charts can be drawn for various other values of α with different values of γ and the value of μ can be found for given case thereby enabling the determination of equivalent uniform moment for that case.

CHAPTER - 3

ELASTIC DESIGN OF RESTRAINED COLUMN

3.1. DESIGN CRITERION

In the case of a column provided with lateral support at the same eccentricity and subjected to axial load and uniform terminal moments about the major and minor axes, the elastic design criterion will be the development of first yield at the mid- height of the column as the plastic hinge is not formed at the end of the column. Let M_x and M_y be the equivalent uniform moments about the major and minor axis respectively and P the axial load to which the column is subjected.

The failure criterion is

$$p + f_{xx} + f_{yy} + f_o = f_L$$

or,

$$p + N_x f_x + N_y f_y + f_o = f_L \qquad \ldots(3.1)$$

or,

$$p + N_x f_x + N_y f_y = f_L - f_o = f$$

Where p = Axial stress in column

f_x, f_y = Extreme fibre stresses corresponding to bending moments M_x, M_y respectively.

f_{xx} = Magnified f_x due to the magnification caused by axial load P

f_{yy} Magnified f_y due to the magnification caused by axial load P and major axis moment M_x .

f_L Yield point stress of steel

f_o Longitudinal stress due to minor axis bending caused by initial imperfections in the column on application of P and M_x .

N_x Magnification factor of major axis moment on application of the axial load P

N_y Magnification factor of minor axis moment on application of axial load P and major axis moment M_x .

3.2. MAGNIFICATION OF MAJOR AXIS MOMENT

Consider the case of a restrained column subjected to axial load P and major axis uniform moment M_x The governing flexural equation of equilibrium about the major axis $x - x$ is given by,

$$EI_x \frac{d^2v}{dz^2} = -Pv - M_x \qquad \ldots(3.2)$$

$$\text{or} \quad v'' + k_1^2 v = -\frac{M_x}{E I_x}$$

$$\text{Where} \quad k_1^2 = \frac{P}{E I_x}$$

The solution is given by $v = A_1 \cos k_1 z + A_2 \sin k_1 z - \dfrac{M_x}{EI_x k_1^2}$

Using the boundary condition $v = 0$, at $z = 0$ and $z = l$

$$A_1 = \frac{M_x}{EI_x\, k_1^2}$$

$$A_2 = \frac{M_x}{E\, I_x\, k_1^2} \left[\frac{1 - \cos k_1 l}{\sin k_1 l} \right]$$

At mid-height of the column, the magnified moment is given by $- EI_x\,(v'')$. Substituting the value of the constants A_1 and A_2 in v'', the magnification factor for moments at mid-height can be obtained as,

$$N_x = \cos \frac{k_1 l}{2} + \frac{1 - \cos k_1 l}{\sin k_1 l} \sin \frac{k_1 l}{2}$$

$$= \cos \frac{k_1 l}{2} + \tan \frac{k_1 l}{2} \sin \frac{k_1 l}{2}$$

$$\text{or } N_x = \sec \frac{k_1 l}{2}$$

$$\text{Where } k_1^2 = \frac{p}{E\, I_x}$$

or

$$N_x = \sec \sqrt{\frac{p}{4E} \left[\frac{1}{r_x} \right]^2} \qquad \ldots(3.3)$$

Equation (3.3) has been used to plot a relation between p and N_x [13] for given value of $1 / r_x$. Fig. (3.1) gives the value of N_x for given values of p and $1/r_x$.

3.3. MAGNIFICATION OF MINOR AXIS MOMENTS

The minor axis moment M_y is magnified on application of the axial load and the major axis moments. The lateral displacement or twist is due to the moment M_y. The effect of P and M_x is to increase or reduce this displacement. The twist ϕ can be found by framing the differential equation of equilibrium.

Consider a fibre of area dA in the top flange at a distance of x and y from minor and major axis respectively. Let its distance from the point of lateral support be r. On application of the axial load P and uniform end moments M_x and M_y the section will twist by an angle ϕ as shown in Fig. 3.2

The moment M at any point in the buckled fibre is given by

$$M = (\sigma \, dA)\,(r\phi)$$

Where σ = longitudinal stress on the fibre.

$$\text{Shear force} = (\sigma \, dA)\,\frac{d\,(r\phi)}{dz}$$

$$\text{Twisting moment} = (\sigma \, dA)\,\frac{d\,(r\phi)}{dz}\,.\,r$$

$$\text{Total twisting Moment } M_t = \int_A \sigma r^2 \,\frac{d\phi}{dz}\, dA$$

$$= \int_A \left[\frac{P}{A} + \frac{M_x}{I_x}\,y + \frac{M_y}{I_y}\,x\right](x^2 + y^2 + 2ay + a^2)\,\frac{d\phi}{dz}\,dA$$

as $r^2 = x^2 + y^2 + 2\,ay + a^2$

and $\sigma = \frac{P}{A} + \frac{M_x}{I_x} y + \frac{M_y}{I_y} x$

$$M_t = \left[\frac{d\phi}{dz}\right]\left[\frac{P}{A} I_y + \frac{P}{A} I_x + Pa^2 + 2\,M_x \cdot a\right]$$

$$= \left[\frac{d\phi}{dz}\right]\left[P r_y^2 + P r_x^2 + P a^2 + 2\,M_x\,a\right]$$

$$= \left[\frac{d\phi}{dz}\right]\left[P r_o^2 + 2\,M_x\,a\right] \qquad \ldots (3.4)$$

as

$$r_o^2 = r_x^2 + r_y^2 + a^2$$

The total twisting moment can now be equated to the resisting moment due to torsional and warping rigidity

$$M_t = GK \frac{d(\phi - \phi_o)}{dz} - E\,C_{wo} \frac{d^3 (\phi - \phi_o)}{dz^3} \qquad \ldots (3.5)$$

Where C_{wo} = Warping constant for restrained column

$= I_y \left(a^2 + \frac{d^2}{4}\right)$ for doubly symmetric I-sectio

ϕ_o = Initial twist

ϕ = Final twist.

Hence we obtain the differential equation by equating (3.4) and (3.5) .

$$GK \frac{d(\phi - \phi_o)}{dz} - EI_y (a^2 + d^2/4) \frac{d^3(\phi - \phi_o)}{dz^3}$$

$$= \frac{d\phi}{dz} (Pr_o^2 + 2 M_x a)$$

Differentiating with respect to z we get.

$$GK \frac{d^2(\phi - \phi_o)}{dz^2} - EI_y (a^2 + d^2/4) \frac{d^4(\phi - \phi_o)}{dz^4}$$

$$= \frac{d^2 \phi}{dz^2} (Pr_o^2 + 2 M_x a) \quad \ldots(3.6)$$

$$\text{or} \quad EI_y(a^2 + d^2/4) \frac{d^4\phi}{dz^4} - (GK - Pr_o^2 - 2 M_x a) \frac{d^2\phi}{dz^2}$$

$$= EI_y(a^2 + d^2/4) \frac{d^4\phi_o}{dz^4} - GK \frac{d^2\phi_o}{dz^2}$$

Let. $\phi_o = \alpha_o \operatorname{Sin} \pi z / l \quad \ldots(3.7)$

Where α_o = Maximum initial twist.

Substituting the value of ϕ_o we get.

$$\frac{d^4\phi}{dz^4} + \frac{Pr_o^2 + 2 M_x a - GK}{EI_y (a^2 + d^2/4)} \frac{d^2\phi}{dz^2}$$

$$= \frac{\pi^4}{l^4} \alpha_o \operatorname{Sin} \frac{\pi z}{l} + \frac{\pi^2}{l^2} \frac{GK}{EI_y(a^2 + d^2/4)} \operatorname{Sin} \pi z / l$$

$$\text{or} \quad \frac{d^4\phi}{dz^4} + m_1^2 \frac{d^2\phi}{dz^2} = \frac{\pi^4}{l^4} \alpha_o \left[1 + \frac{GK\, l^2}{\pi^2 EI_y (a^2 + d^2/4)} \right] \operatorname{Sin} \frac{\pi}{l}$$

$$\ldots(3.8)$$

$$\text{where} \quad m_1^2 = \frac{Pr_o^2 + 2 M_x a - GK}{EI_y (a^2 + d^2/4)} \quad \ldots(3.9)$$

The solution for ϕ is given by.

or $\left[\phi - \phi_o\right]''_{z=0} = -\dfrac{M_y}{(a + d/2)\, EI_y}$

$$= - A_1 m_1^2$$

or $A_1 = \dfrac{M_y}{(a + d/2)\, EI_y\, m_1^2}$

and $A_4 = -\dfrac{M_y}{(a + d/2)\, EI_y\, m_1^2}$

(iii) $\frac{1}{2} EI_y \,(a + d/2)\, (\phi - \phi_o)''_{z=l} = -\frac{1}{2} M_y$

$$(\phi - \phi_o)''_{z=l} = -\frac{M_y}{(a + d/2)\, EI_y}$$

Substituting the value of A_1 we get.

$$A_2 = \frac{M_y}{(a + d/2)\, EI_y\, m_1^2} \tan \frac{m_1 l}{2}$$

(iv) $\left[\phi\right]_{z=l} = 0 = A_1 \operatorname{Cos} m_1 l + A_2 \operatorname{Sin} m_1 l + A_3 l + A_4$

Substituting the values of A_1, A_2 and A_4 we get

$$A_3 = 0.$$

Substituting the value of the constants in the value of ϕ we get.

$$\phi = \frac{M_y}{EI_y (a+d/2) m_1^2} \operatorname{Cos} m_1 z + \frac{M_y}{(a+d/2) m_1^2\, EI_y} \tan \frac{m_1 l}{2} \operatorname{Sin} m_1 z$$

$$- \frac{M_y}{(a+d/2) EI_y m_1^2} + A_5 \operatorname{Sin} \frac{\pi z}{l} \qquad ..(3.11)$$

$$- \left| E a_y \, (a + d/2) \, (\phi'' - \phi_o'') \right|_{z = 1/2}$$

$$= - E a_y (a + d/2) \left[\frac{M_y}{(a+d/2) E I_y} \left(- \operatorname{Cos} \frac{m_1 \, l}{2} - \tan \frac{m_1 l}{2} \cdot \operatorname{Sin} \frac{m_1 l}{2} \right) - \frac{\pi^2}{l^2} A_5 + \frac{\pi^2}{l^2} \alpha_o \right]$$

$$= E a_y \, (a+d/2) \left[\frac{M_y}{(a+d/2) \, E I_y} \operatorname{Sec} \frac{m_1 \, l}{2} + \frac{\pi^2}{l^2} \, (A_5 - \alpha_o) \right]$$

$$= \frac{M_y}{I_y} a_y \operatorname{Sec} \frac{m_1 \, l}{2} + \frac{\pi^2}{l^2} E a_y (a+d/2) \, (A_5 - \alpha_o)$$

$$= f_y \operatorname{Sec} \frac{m_1 \, l}{2} + f_o$$

$$\text{or } f_{yy} = f_y N_y + f_o \qquad \ldots (3.13)$$

Where, N_y is the magnification factor of minor axis moment.

$$N_y = \operatorname{Sec} \frac{m_1 \, l}{2} \qquad \ldots (3.14)$$

$$f_o = \frac{\pi^2}{l^2} E a_y (a + d/2) \, (A_5 - \alpha_o) \qquad \ldots (3.15)$$

Thus N_y can be found from the Fig. 3.1 provided $1/r_x$ is replaced by $1/r_y$ and p is replaced by an equivalent p' in the chart.

$$\text{Since } p = \frac{E I_x \, k_1^2}{A} \, , \; p' = \frac{E I_y \, m_1^2}{A}$$

$$\text{or } p' = \frac{E I_y}{A} \; \frac{P r_o^2 + 2 M_x a - GK}{E I_y (a^2 + d^2/4)} \qquad \ldots (3.16)$$

as $r^2 = x^2 + y^2 + 2\,ay + a^2$

and $\sigma = \frac{P}{A} + \frac{M_x}{I_x}\,y + \frac{M_y}{I_y}\,x$

$$M_t = \left[\frac{d\phi}{dz}\right]\left[\frac{P}{A}\,I_y + \frac{P}{A}\,I_x + Pa^2 + 2\,M_x\cdot a\right]$$

$$= \left[\frac{d\phi}{dz}\right]\left[Pr_y^2 + Pr_x^2 + Pa^2 + 2\,M_x\,a\right]$$

$$= \left[\frac{d\,\phi}{dz}\right]\left[Pr_o^2 + 2\,M_x\,a\right] \qquad \ldots(3.4)$$

as

$$r_o^2 = r_x^2 + r_y^2 + a^2$$

The total twisting moment can now be equated to the resisting moment due to torsional and warping rigidity

$$M_t = GK\,\frac{d(\phi - \phi_o)}{dz} - E\,C_{wo}\,\frac{d^3\,(\phi - \phi_o)}{dz^3} \qquad \ldots(3.5)$$

Where C_{wo} = Warping constant for restrained column

$= I_y(\,a^2 + \frac{d^2}{4})$ for doubly symmetric I-section

ϕ_o = Initial twist

ϕ = Final twist.

Hence we obtain the differential equation by equating (3.4) and (3.5)

$$GK \frac{d(\phi - \phi_o)}{dz} - EI_y (a^2 + d^2/4) \frac{d^3(\phi - \phi_o)}{dz^3}$$

$$= \frac{d\phi}{dz} (Pr_o^2 + 2 M_x a)$$

Differentiating with respect to z we get,

$$GK \frac{d^2(\phi - \phi_o)}{dz^2} - EI_y (a^2 + d^2/4) \frac{d^4(\phi - \phi_o)}{dz^4}$$

$$= \frac{d^2 \phi}{dz^2} (Pr_o^2 + 2 M_x a) \qquad \ldots(3.6)$$

$$\text{or} \quad EI_y(a^2 + d^2/4) \frac{d^4\phi}{dz^4} - (GK - Pr_o^2 - 2 M_x a) \frac{d^2\phi}{dz^2}$$

$$= EI_y(a^2 + d^2/4) \frac{d^4\phi_o}{dz^4} - GK \frac{d^2\phi_o}{dz^2}$$

Let, $\phi_o = \alpha_o \operatorname{Sin} \pi z / l$ ...(3.7)

Where α_o = Maximum initial twist.

Substituting the value of ϕ_o we get,

$$\frac{d^4\phi}{dz^4} + \frac{Pr_o^2 + 2 M_x a - GK}{EI_y (a^2 + d^2/4)} \frac{d^2\phi}{dz^2}$$

$$= \frac{\pi^4}{l^4} \alpha_o \operatorname{Sin} \frac{\pi z}{l} + \frac{\pi^2}{l^2} \frac{GK}{EI_y(a^2 + d^2/4)} \operatorname{Sin} \pi z / l$$

$$\text{or} \quad \frac{d^4\phi}{dz^4} + m_1^2 \frac{d^2\phi}{dz^2} = \frac{\pi^4}{l^4} \alpha_o \left[1 + \frac{GK\, l^2}{\pi^2 EI_y (a^2 + d^2/4)} \right] \operatorname{Sin} \frac{\pi z}{l}$$

...(3.8)

$$\text{where} \quad m_1^2 = \frac{Pr_o^2 + 2 M_x a - GK}{EI_y (a^2 + d^2/4)} \qquad \ldots(3.9)$$

The solution for ϕ is given by,

$$\phi = A_1 \mathrm{Cos}\, m_1 z + A_2 \mathrm{Sin}\, m_1 z + A_3 z + A_4 + A_5 \mathrm{Sin} \frac{\pi z}{l}$$

Substituting the value of ϕ in the equation (3.8),

$$\frac{\pi^4}{l^4} A_5 - m_1^2 \frac{\pi^2}{l^2} A_5 = \frac{\pi^4}{l^4} \alpha_o \left[1 + \frac{GK\, l^2}{\pi^2 EI_y (a^2 + d^2/4)} \right]$$

$$A_5 = \frac{\alpha_o \frac{\pi^4}{l^4} \left[1 + \frac{GK\, l^2}{\pi^2 EI_y (a^2 + d^2/4)} \right]}{\frac{\pi^4}{l^4} - \frac{\pi^2}{l^2} \left[\frac{Pr_o^2 + 2 M_x a - GK}{EI_y (a^2 + d^2/4)} \right]}$$

or

$$A_5 = \alpha_o \left[\frac{\frac{\pi^2}{l^2} EI_y (a^2 + d^2/4) + GK}{\frac{\pi^2}{l^2} EI_y (a^2 + d^2/4) + GK - (Pr_o^2 + 2 M_x a)} \right] \qquad \ldots (3.10)$$

Now,

$$\phi'' = - A_1 m_1^2 \mathrm{Cos}\, m_1 z - A_2 m_1^2 \mathrm{Sin}\, m_1 z - A_5 \frac{\pi^2}{l^2} \mathrm{Sin} \frac{\pi z}{l}$$

Using the boundary conditions,

(i) $\left[\phi \right]_{z=0} = 0 = A_1 + A_4$

$$A_1 = - A_4$$

(ii) $\frac{1}{2} EI_y (a + d/2) (\phi - \phi_o)''_{z=0} = - \frac{1}{2} M_y$

or $\left[\phi - \phi_o\right]''_{z=0} = - \dfrac{M_y}{(a + d/2)\, EI_y}$

$= - A_1 m_1^2$

or $A_1 = \dfrac{M_y}{(a + d/2)\, EI_y m_1^2}$

and $A_4 = - \dfrac{M_y}{(a + d/2)\; EI_y m_1^2}$

(iii) $\frac{1}{2} EI_y (a + d/2) (\phi - \phi_o)''_{z=l} = - \frac{1}{2} M_y$

$$(\phi - \phi_o)''_{z=l} = - \frac{M_y}{(a + d/2)\, EI_y}$$

Substituting the value of A_1 we get,

$$A_2 = \frac{M_y}{(a + d/2)\, EI_y m_1^2} \tan \frac{m_1 l}{2}$$

(iv) $\left[\phi\right]_{z=l} = 0 = A_1 \cos m_1 l + A_2 \sin m_1 l + A_3 l + A_4$

Substituting the values of A_1, A_2 and A_4 we get

$$A_3 = 0.$$

Substituting the value of the constants in the value of ϕ we get,

$$\phi = \frac{M_y}{EI_y(a+d/2)m_1^2} \cos m_1 z + \frac{M_y}{(a+d/2)m_1^2\; EI_y} \tan \frac{m_1 l}{2} \sin m_1 z$$

$$- \frac{M_y}{(a+d/2)EI_y m_1^2} + A_5 \sin \frac{\pi z}{l} \qquad ..(3.11)$$

The initial imperfection causes bending moment about the minor axis. If the initial twist ϕ_o due to imperfection is magnified to ϕ by the application of the axial load and the major axis bending moment, the minor axis bending moment M_η is given by

$$\left| (M_\eta) \right| = \left| EI_y \, a \left(\frac{d^2\phi}{dz^2} - \frac{d^2\phi_o}{dz^2} \right) \right|$$

The bending stress f_{y1} caused by M_η is given by

$$f_{y1} = \left| \frac{M_\eta}{I_y} \right| a_y$$

$$= \left| E \, a_y \, a \, (\phi'' - \phi_o'') \right|$$

In addition to f_{y1}, longitudinal stress will also be caused by differential flange bending. This additional longitudinal stress f_{y2} is given by.

$$f_{y2} = \left| \frac{1}{2} EI_y \cdot \frac{d}{2} (\phi'' - \phi_o'') \right| \frac{2 a_y}{I_y}$$

$$= \left| Ea_y \cdot \frac{d}{2} (\phi'' - \phi_o'') \right|$$

The total longitudinal stress f_{yy} due to minor axis bending is given by,

$$f_{yy} = f_{y1} + f_{y2} = \left| E \, a_y \cdot (a + d/2) (\phi'' - \phi_o) \right| \qquad \ldots (3.12)$$

At mid-height, value of f_{yy} is given by.

It is seen from equation (3.14) that if $Pr_o^2 + 2 M_x a > GK$ ie the load is greater than pure torsional load, $N_y > 1$ i.e. f_{yy} at mid span is greater than at ends and the twist in the column due to M_y is magnified by P and M_x

If $Pr_o^2 + 2 M_x a < GK$, $N_y < 1$,

f_{yy} at mid-span is less than at the ends and twist in the column is smaller than what is due to M_y alone. However for the purpose of design, in such case N_y is taken as unity as this is conservative and on the safer side. At the transition load(9), $Pr_o^2 + 2 M_x a = GK$ $N_y = 1$, f_{yy} at mid-span is the same as at ends and twist in the column is same as given by M_y alone.

3.3.1. Stress Due to Initial Imperfections (f_o)

The column has some initial imperfections which get magnified on the application of loads. The initial lateral displacement in the column is increased on the application of P and M_x. Thus there exists a stress f_o due to minor axis bending on application of loads and this is due to the initial imperfections in the column.

From equation (3.18) we get,

$$f_o = \frac{\pi^2}{l^2} Ea_y (a + d/2) (A_3 - \alpha_o)$$

Substituting the value of A_3 from equation (3.10), we get,

$$f_o = \frac{\pi^2}{l^2} Ea_y(a+d/2) \left[\frac{\alpha_o \frac{\pi^2}{l^2} EI_y (a^2+d^2/4) + GK}{\frac{\pi^2}{l^2} EI_y(a^2+d^2/4) + GK - (Pr_o^2+2M_x a)} - \alpha_o \right]$$

$$= \frac{\pi^2}{l^2} Ea_y(a+d/2)\alpha_o \left[\frac{Pr_o^2 + 2 M_x a}{\frac{\pi^2}{l^2} EI_y(a^2+d^2/4)+GK - (Pr_o^2+2M_x a)} \right]$$

...(3.17)

Horne[12] has assumed the value of initial lateral displacement due to initial imperfections for a column with uniform moment as,

$$u_o = 0.0015 \frac{lr_y}{a_y} \text{Sin} \frac{\pi z}{l}$$

Taking the same initial lateral displacement at the centre of the unsupported flange of the restrained column,

$$\phi_o = \frac{u_o}{a + d/2} = \frac{0.0015 \; lr_y}{a_y(a+d/2)} \text{Sin} \frac{\pi z}{l}$$

$$= \alpha_o \text{ Sin } \pi z/l$$

$$\alpha_o = \frac{.0015 \; lr_y}{a_y(a + d/2)}$$

...(3.18)

Substituting the value of α_o in equation (3.17)

$$f_o = \frac{0.0015\,\pi^2\,E\,r_y}{l}\left[\frac{P r_o^2 + 2\,M_x\,a}{\frac{\pi^2}{l^2}EI_y(a^2 + d^2/4) + GK - (P r_o^2 + 2M_x\,a)}\right]$$

...(3.19)

Thus the value of f_o can be found for given P and M_x .

The parameters fixed were as follows :-

(a) Section : $1\frac{3}{4}$" x 4" RSJ 0 lbs.

(b) End conditions: both ends were free to rotate about the major and minor axes but not free to rotate about the longitudinal axis or displace laterally.

(c) Spacing of supports = 2' 3" i.e. short enough so that buckling occurs by column twisting about restrained axis passing through the points of support.

(d) Eccentricity of lateral supports = 3.5 "

(e) Torsional support provided by lateral supports -Nil.

4.2. REQUIREMENTS OF TEST RIG

A testing rig had to be designed to meet the following requirements for testing restrained I-column :-

(a) Length of the column : 0'

(b) Axial load upto 3 tons.

(c) End moments about the axes upto 10 in tons.

(d) Axial load and the major axis bending moment could be kept constant and the minor axis bending moment could be varied independently.

(e) The moments about both the axes to be uniform along the length of the column.

(f) End Conditions

(i) Column is free to rotate about both the major and minor axes.

(ii) No lateral displacement of the column is possible.

(iii) No rotation about the longitudinal axis of the column is possible.

4.3. TEST RIG

4.3.1. Loading Arrangement

The column was to be subjected to constant axial load and major axis moment with varying minor axis moment. This was achieved by fixing levers to the ends of the column. and applying loads by turnbuckle to the levers by tensioning rods passing through the levers .

For this purpose , a 4" x 4" box section was obtained by welding two 2" x 2" channels. Two t-ees (5'x2'4") were made out of the box sections by welding as shown in Fig. 4.1 and photograph 1 . The tees had 7/8" dia holes at 1" c/c , the first hole being 5" from the centre of the tee. The two tees were welded at the two ends of the 8' long I-column so as to provide levers. The tensioning rods passing through the three levers were 7/8" dia. These rods were secured at one end of the column by a 3" long hexagonal ended nut of 1.5" dia. and at the other end, the rod could be tightened by turning another similar nut , which acted as a turn-buckle. The rods were secured to the tees at both ends through a special nut of 1.5" external dia and 1" in length resting on a groove in a 4" x 4" plate separating the turnbuckle nut from the tee. The details of loading arrangement are shown in Fig. 4.1 and Photograph 1.

For measuring the load in the tensioning rod, a special load measuring device was connected to the rod in the centre as shown in Photograph 2. The device consisted of a 16" long , 7/8" dia mild steel rod tapered to 3/4" dia in the central 10" of its length. The ends of the rod were threaded so that it could be connected to the main tensioning rod.

Two strain gauges were pasted diametrically opposite on the central portion of the rod and a calibration chart was prepared by taking strain gauge readings when the rod was tested in a universal testing machine. Fig. 4.2 gives the calibration chart for the load measuring device. Thus the load in the tensioning rod could be measured by directly reading off from the calibration chart.

4.3.2. Lateral Supports

The lateral supports were provided at the ends of the column and at two other equally spaced points in the column. The lateral supports were provided in both flanges of the column at the ends and in only one flange at other points . Two restraining rods, 3/8" diameter were connected to a 2" x 2" plate welded to the column at the required point of lateral support at an eccentricity of 3.5" as shown in Fig. 4.1 and photograph 3. At the other end, the restraining rods were allowed to slide in a 3/4" groove on testing frame made by welding two channels 1" x 1" at 3/4" spacing. The special nut securing the restraining rods to the channels had a recess cut in it so that the nut could slide vertically up and down.

Thus the mechanism of lateral support provided the following features:-

(a) No horizontal movement of the column at the point of support

(b) Vertical movement of the restraining rods was permissible at both ends.

(c) At the point of restraint the column could rotate about 15° without interference from the restraining rod.

(d) No torsional restraint was provided by the lateral support.

4.4. TESTING PROCEDURE

The aim of the testing was to measure the stress at various sections of the free flange when the column was subjected to axial load and terminal moments about major and minor axes. As the stresses were to remain in elastic region only, the yield stress of the column material was determined by taking out tension specimens from the web of the column and testing them on the tension testing machine. The axial load and terminal moments were so chosen that the total stress did not exceed 80% of the yield stress of column.

The restraining rods were tightened and made horizontal. Loads were applied to the tensioning rods by tightening the turnbuckle. The minor axis moment could be varied by varying the load on the two tensioning rods passing through the horizontal levers of the tee.

Strains were measured from strain gauges at various sections for the selected loads on the column and the total stress at the centre and other sections was found. The brittle lacquer coat applied in the free flange of the column was constantly observed to ensure there were no cracks thereby keeping the stresses well within elastic limit.

CHAPTER - 5

TEST RESULTS AND COMPARISON

Tests were conducted to investigate the behaviour of the restrained I - column in the elastic range only. The column was subjected to the following loads and tests:

(a) Loading case 'A' , P = 4 Tons.

(b) Loading case 'B' , P = 4.5 Tons

(c) Loading case 'C' , P = 5.0 tons.

In each loading case, the minor axis moments were varied for the same value of major axis moments in each test.

The yield stress of column from the tension test was found to be 21.8 tons/in^2 . The values of the total longitudinal stress for various sets of loads in each loading case have been tabulated in Tables 5.1, 5.2 and 5.3. The values of the theoretical stress have also been listed in these tables. Charts showing the variation of total stress depending on the variation in minor axis moment are drawn in Fig. 5.1, 5.2 and 5.3.

TABLE 5.1

LOADING CASE 'A' P = 4 TONS

Test No	M_x (in tons)	M_y (in tons)	Actual 'f' (tons/in^2)	Theoretical 'f' (tons/in^2)
T-1	5.0	0.0	5.8	6.3
		1.0	7.5	8.2
		2.0	9.4	10.1
		3.0	10.9	12.0
T-2	7.0	0.0	7.0	7.8
		1.0	9.4	10.5
		2.0	12.0	13.2
		3.0	14.4	15.9
T-3	8.0	0.0	8.1	8.6
		1.0	10.8	11.9
		2.0	13.9	15.2
		3.0	16.4	18.5
T-4	9.0	0.0	8.9	9.4
		1.0	12.5	13.4
		1.5	14.0	15.4
		2.0	16.2	17.4

TABLE - 5.2

LOADING CASE 'B' P = 4.5 TONS

Test No.	M_x (in tons)	M_y (in tons)	Actual 'f' (tons/in^2)	Theoretical 'f' (tons/in^2)
T-5	5.0	0.0	6.1	6.8
		1.0	8.1	9.1
		2.0	10.0	11.4
		3.0	12.0	13.7
T-6	7.0	0.0	8.0	8.4
		1.0	10-9	11.7
		2.0	13.5	15.0
		3.0	16.6	18.3
T-7	8.0	0.0	8.8	9.3
		1.0	12.2	13.4
		1.5	14.1	15.5
		2.0	16.0	17.5
T-8	9.0	0.0	9.9	10.3
		0.5	11.8	12.9
		1.0	14.0	15.5
		1.5	16.0	18.1

TABLE - 5.3

LOADING CASE 'C' P = 5 TONS

Test No.	M_x (in tons)	M_y (in tons)	Actual 'f' (tons/in^2)	Theoretical 'f' (tons/in^2)
T-9	5.0	0.0	6.9	7.4
		1.0	9.1	10.2
		2.0	11.3	13.0
		3.0	14.3	15.8
T-10	6.0	0.0	7.6	8.2
		1.0	10.5	11.5
		2.0	13.3	14.8
		3.0	16.7	18.1
T-11	7.0	0.0	8.5	9.1
		1.0	12.0	13.3
		1.5	13.7	15.4
		2.0	15.5	17.5

In all the tests, the actual values of stress are much less than the theoretical values. This is due to the attual end conditions in the experiment being different than theoretical end conditions. The lovers are welded to the ends whereas the column is taken as simply supported. This tend to reduce the stresses. The strain gauges also do not give accurate measurement of strains throughout the experiment.

We cannot take into consideration the exact effect of the initial crookedness of the column on the stresses when the column is loaded. Horne(12) has assumed the initial imperfection so that initial lateral displacement

$$u_o = 0.0015 \frac{l r_y}{a_y} \sin \frac{\pi z}{l}$$

and this has been used to find the stress due to initial imperfections. The actual effect of the initial imperfections in the column under test however cannot be exactly considered and this may cause variation in the theoretical value of the stress.

The variation of the total longitudinal stress along the length of the column was found for the same set of loads. For sake of comparison the values of the axial load and minor axis bending moment were kept constant and the value of major axis bending moment was varied for each loading case so as to give different magnifications of the stress at the mid-span. The results are summarised in Table 5.4.

Charts showing the variation of the longitudinal stress due to minor axis bending along the length for each loading case are drawn in Figs. 5.4, 5.5 and 5.6 It is clear that the change in the stress level between the centre of the column and the end of the column is gradual and follows a pattern. Also the stress level at the centre is dependent on the magnification produced by the axial load and major axis bending moment For smaller values of axial load and major axis bending moment ($Pr_o^2 + 2 M_x a < CK$), the stress level at centre is less than at the ends of the column.

As the axial load and the major axis moments increase, the magnification factor also increases till the stress level at the centre is higher than at the ends ($Pr_o^2 + 2 M_x a > CK$). Thus the axial load and the major axis moments are the two parameters which affect the magnification of stress level at the centre.

--

Test No	P (Tons)	M_x (in. tons)	Theoretical M_y	M_y (in tons)	Actual f_{yy} at distances from one end (tons/in^2)				Actual f_{yy}/f_y at distances from one end.		
					0 stress = f_{yy}	1/6	1/3	1/2	1/6	1/3	1/2
T-1	4.0	5.0	0.59	2.0	5.8	4.5	3.9	3.6	0.78	0.68	0.62
T-2	4.0	7.0	0.82	2.0	5.9	5.3	4.8	5.0	0.90	0.82	0.85
T-3	4.0	8.0	1.0	2.0	6.1	5.7	5.7	5.8	0.94	0.94	0.95
T-4	4.0	9.0	1.23	2.0	6.1	6.6	6.9	7.3	1.08	1.13	1.20
T-5	4.5	5.0	0.70	1.0	3.1	2.4	2.2	2.0	0.78	0.70	0.65
T-6	4.5	7.0	1.0	1.0	3.1	3.0	2.9	2.9	0.97	0.94	0.94
T-7	4.5	8.0	1.24	1.0	2.9	3.1	3.2	3.4	1.08	1.10	1.17
T-8	4.5	9.0	1.41	1.0	3.0	4.1	4.4	5.1	1.36	1.48	1.70
T-9	5.0	5.0	0.84	2.0	5.9	5.1	4.8	4.5	0.86	0.82	0.76
T-10	5.0	6.0	1.0	2.0	6.0	5.6	5.7	5.7	0.93	0.95	0.95
T-11	5.0	7.0	1.26	2.0	5.9	6.4	6.8	7.0	1.08	1.15	1.19

CHAPTER - 6

C O N C L U S I O N S

The elastic behaviour of a restrained I-column with lateral supports at some eccentricity in one flange and subjected to axial load and terminal moments about major and minor axes was studied by conducting tests and compared with the theoretical results. An elastic method of design of restrained column which considers the effect of lateral supports on the stresses has been described in Chapter 3. This is a rational method of elastic design of restrained column. At present there is no suitable method existing for designing restrained column with lateral supports in one flange. The present practice of using the interaction formula (IS Code 800-1962)(10) ignores the effect of lateral supports and is therefore more conservative.

The two methods have been compared in Appendix 'A' and the following conclusions can be drawn :-

(a) The effective length of the restrained column for finding out the slenderness ratio is much greater than the actual effective length. This results in reducing the permissible average compressive stress F_a and the design becomes more conservative.

(b) There is a magnification of stresses due to minor axis bending when the axial load and major axis moments are applied. This magnification of moments is considered in the method evolved whereas in the interaction formula it is neglected.

(c) The initial imperfections in the column cause stresses when the load is applied. These stresses are neglected in the interaction formula whereas they are taken into consideration in this method.

(d) The magnification of minor axis bending stress is dependent on the magnitude of the axial load and the major axis moment. The magnification is greater than, equal to or less than unity depending on as $Pr_o^2 + 2 M_x a$ is greater than equal to or less than the torsional rigidity GK of the column.

However the magnification is taken as unity for design purpose in case it is less than unity. This is more conservative and on the safer side. Also the value of stresses at the end of the column is automatically checked when N_y is taken as unity for values less than unity.

(e) The interaction formula approach for design of restrained columns neglects the effect of the lateral supports and is more conservative. The design method evolved considers the beneficial effect of the lateral supports.

The test results conducted to verify the new design approach are satisfactory as the actual stresses occurring at mid-span were less than theoretical stress which is on the safer side. Figures 5.1, 5.2 and 5.3 show the variation of total longitudinal stress with the minor axis bending moment. The design method evolved is therefore gives us a workable approach to the design of restrained columns.

Tests were conducted to determine the variation of longitudinal stresses due to minor axis bending along the length of the column with the variation of minor axis terminal moments. The variation of these stresses along the length was found to be gradual and smooth. The magnification of stress level at the centre was found to depend on the axial load and the major axis moments and was greater than equal to or less than unity depending on as $Pr_o^2 + 2 M_x a$ was greater than equal to or less than the torsional rigidity GK. Fig. 5.4 and 5.5 shows the variation of the stress with the minor axis bending moment.

--

APPENDIX 'A'

COMPARISON OF METHODS FOR ELASTIC DESIGN OF RESTRAINED I - COLUMNS

PROBLEM

A 1.75 " x 4" RSJ of length 8' carries an axial load of 0.5 tons and end moments of 2 in tons about major axis and 1 in tons about minor axis. The column is restrained by lateral supports in one flange only at the ends and at two other equally spaced intermediate points. The lateral supports are provided at an eccentricity of 3.5 " and give no torsional restraint . The end conditions are such that both ends are free to rotate about the major and minor axes but not free to rotate about the longitudinal axes or displace laterall-y. Check the safety of the column by using interaction formula (IS Code 800-1962) and by the elastic design method considering the effect of lateral restraints.

Method of Design Based on Interaction Formula (IS Code 800-62)

The interaction formula for a column subjected to bending about both axes is given by

$$\frac{f_a}{F_a} + \frac{f_{b1}}{F_{b1}} + \frac{f_{b2}}{F_{b2}} \leq 1$$

Where,

f_a = Average compressive stress

F_a = Allowable average compressive stress for concentrically loaded member, as determined by maximum slenderness ratio.

f_{b1} = Actual bending stress due to moment about major axis.

F_{b1} = Permissible compressive stress for bending about major axis taking account of lateral stability.

f_{b2} = Actual bending stress due to moment about minor axis.

F_{b2} = Permissible compressive stress for bending about minor axis

$$= \frac{\text{Yield point stress}}{\text{Factor of safety}}$$

From the steel tables, the sectional properties of 1.75" x 4" RSJ are:-

$A = 1.47\ in^2$ $\qquad r_y = 0.36\ in$

$d = 3.76\ in$ $\qquad r_x = 1.58\ in$

$t_f = 0.24\ in$ $\qquad b = 1.75\ in$

$Z_x = 1.83\ in^2$ $\qquad t_w = 0.17\ in$

$I_x = 3.66\ in^4$ $\qquad Z_y = 0.21\ in^2$

$$f_a = \frac{0.5}{1.47} = 0.34\ tons/in^2$$

$$f_{b1} = 2/1.83 = 1.20\ tons/in^2$$

$$f_{b2} = 1/0.21 = 4.8\ tons/in^2$$

Slenderness ratio $= 96/0.36 = 266$

From Table II of IS Code 800-1962

$F_a = 127$ Kg/cm^2 $= 0.81$ tons/in^2

Torsional constant $K = \frac{1}{3} \sum bt^3$

$= 0.0218$

$$C_s = 1.2 \times \frac{\pi^2 E}{2} \frac{I_y d}{Z_x l^2} \sqrt{1 + \frac{.162 K l^2}{I_y d^2}}$$

$= 11.8$ tons per sq. in.

Where,

C_s = Critical bending stress at which lateral buckling occurs.

From Table IV of IS Code 800-1962 for the above value of C_s, $F_{b1} = 734$ Kg/cm^2 $= 4.7$ tsi

$$F_{b2} = \frac{\text{Yield Point stress}}{\text{Factor of safety}} = 10 \text{ tsi}$$

Now,

$$\frac{f_a}{F_a} + \frac{f_{b1}}{F_{b1}} + \frac{f_{b2}}{F_{b2}} = \frac{0.34}{0.81} + \frac{1.1}{4.7} + \frac{4.8}{10}$$

$$= 0.42 + 0.23 + 0.48$$

$$= 1.13 > 1$$

Thus the section is unsafe for the given loading using the IS Code approach of interaction formula.

Method of Elastic Design of Restrained Column

As shown in Chapter 3, the design criterion for the restrained column is given by :-

$$p + N_x f_x + N_y f_y + f_o < f_L$$

$$p = \frac{0.5}{1.47} = 0.34 \text{ tsi}$$

$$1/r_x = 96 / 1.58 = 61.$$

From the chart given in Fig. 3.1,

$$N_x = 1.01 \quad (\text{say } 1.0)$$

$$m_1^2 = \frac{Pr_o^2 + 2 M_x a - GK}{EI_y (a^2 + d^2/4)}$$

$$Pr_o^2 + 2 M_x a = 21.5 \text{ ton in}^2$$

$$GK = 116.0 \text{ ton in}^2$$

As $Pr_o^2 + 2 M_x a < GK$, m_1^2 will be negative and

$$N_y = \text{Sec} \frac{m_1 l}{2} < 1$$

We will take $N_y = 1$ for the purpose of design as it is more conservative.

$$f_o = 0.0015 \frac{\pi^2}{l} \times Er_y \left[\frac{Pr_o^2 + 2 M_x a}{\frac{\pi^2}{l^2} EI_y (a^2 + d^2/4) + GK - (Pr_o^2 + 2M_x a)} \right]$$

$$= 0.11 \text{ tsi}$$

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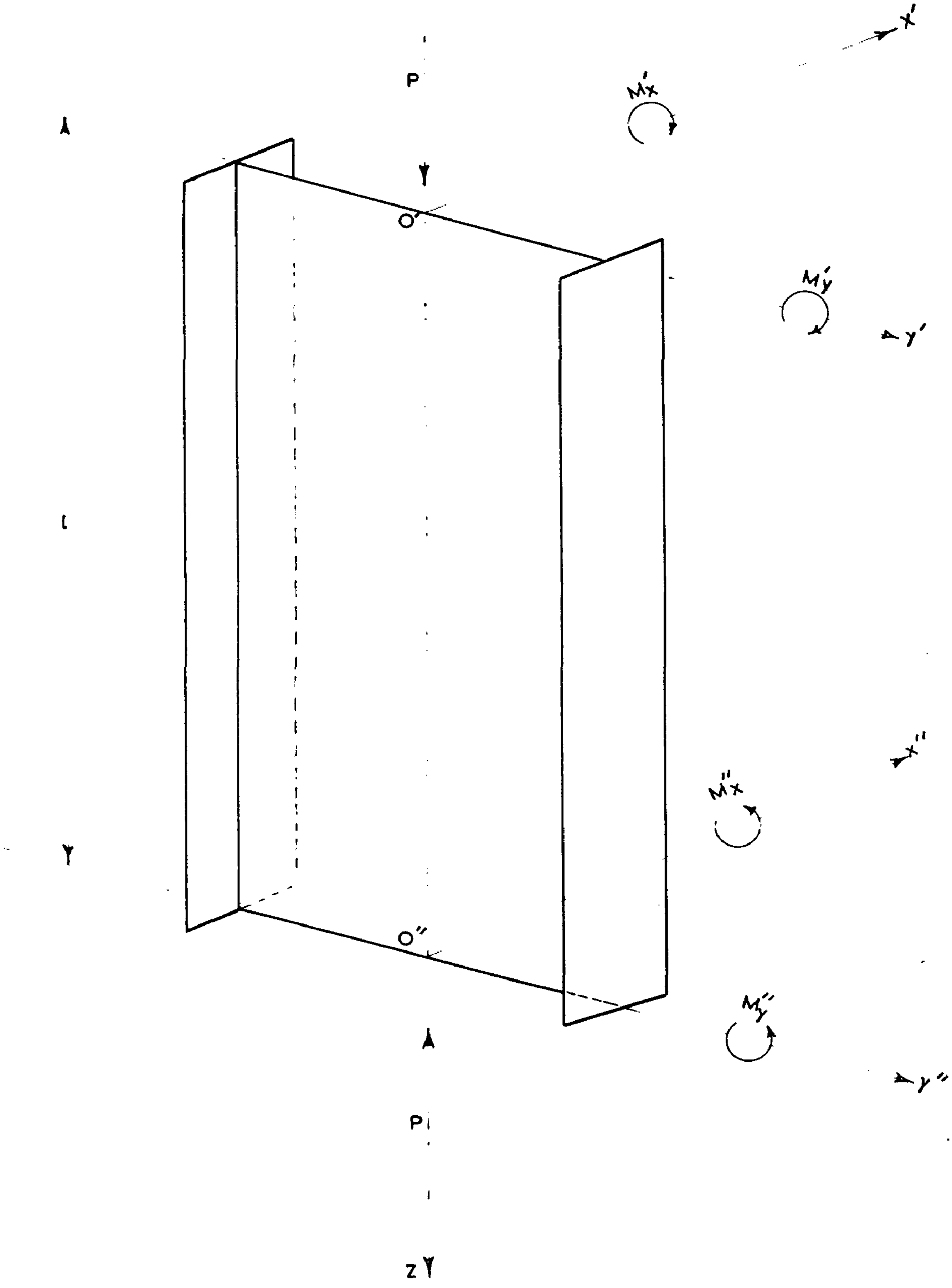


FIG. 2·1

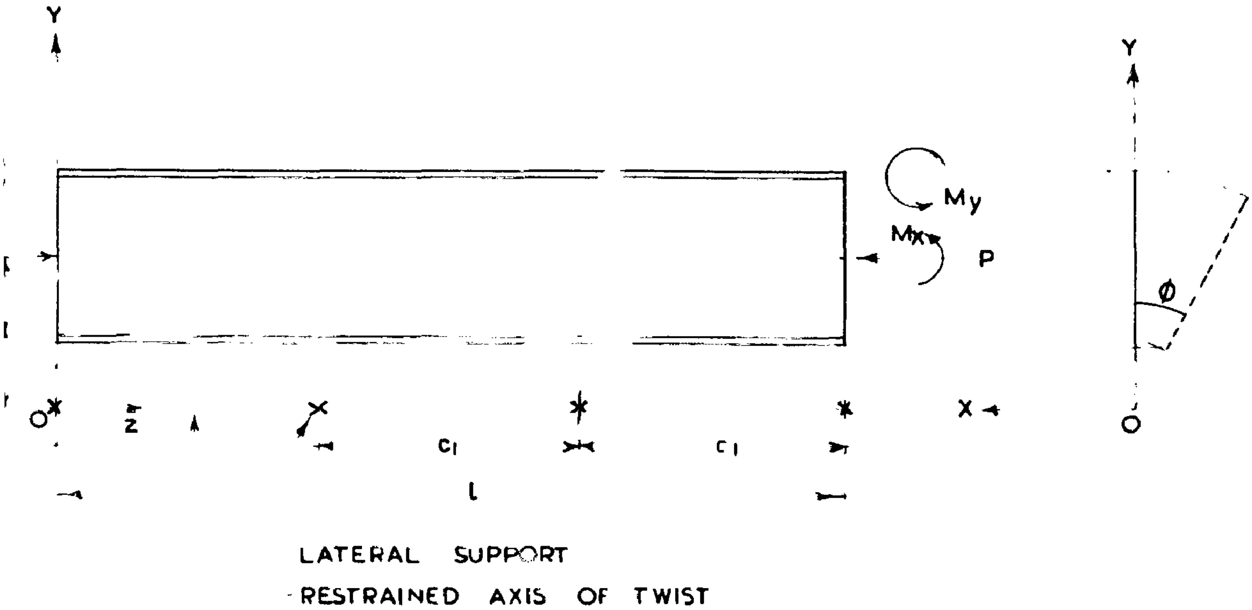

Y
My
Mx
P
O
Z
C1
C1
X
L
Y
Ø
O
LATERAL SUPPORT
RESTRAINED AXIS OF TWIST

FIG. 2·2

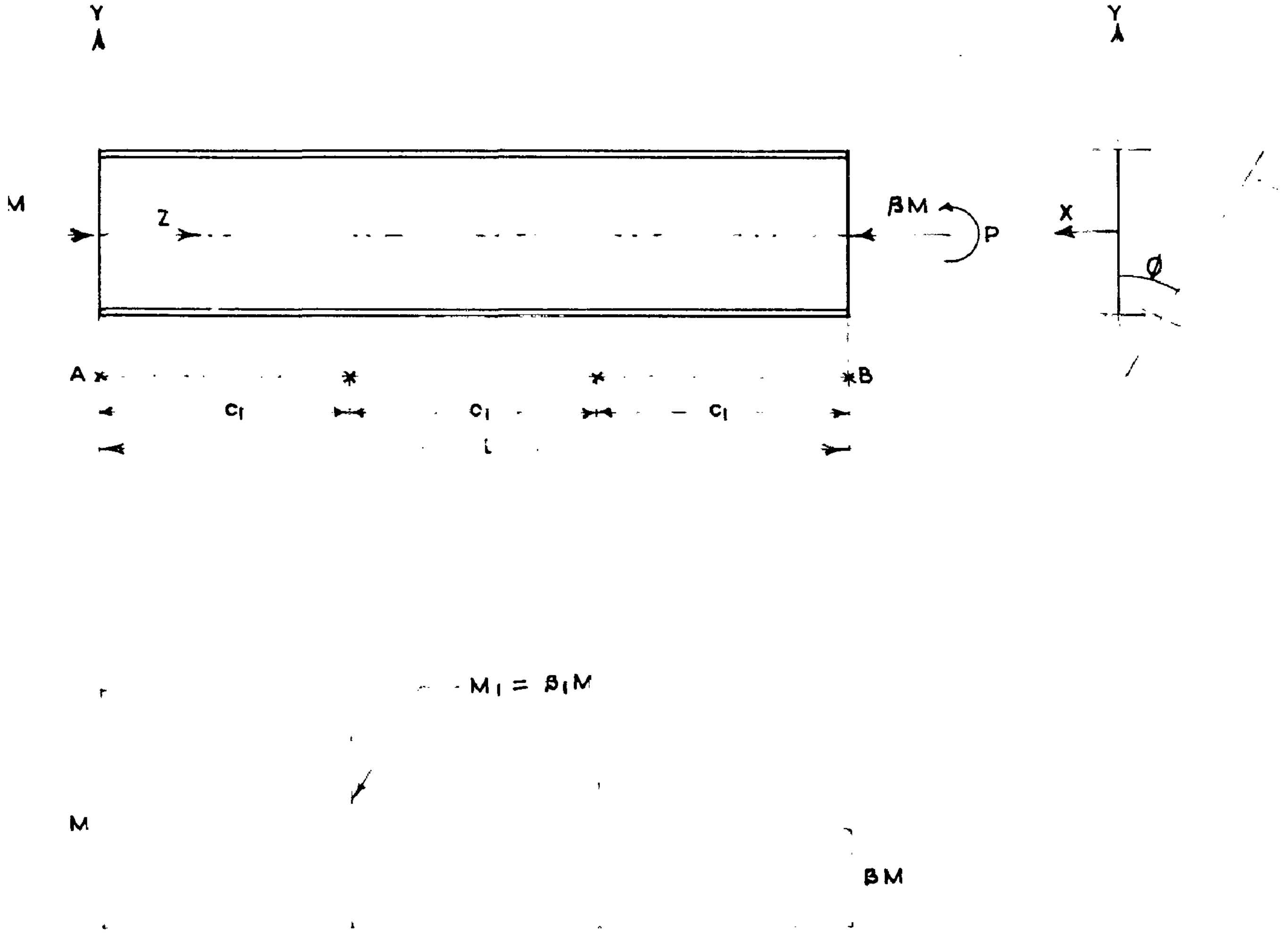
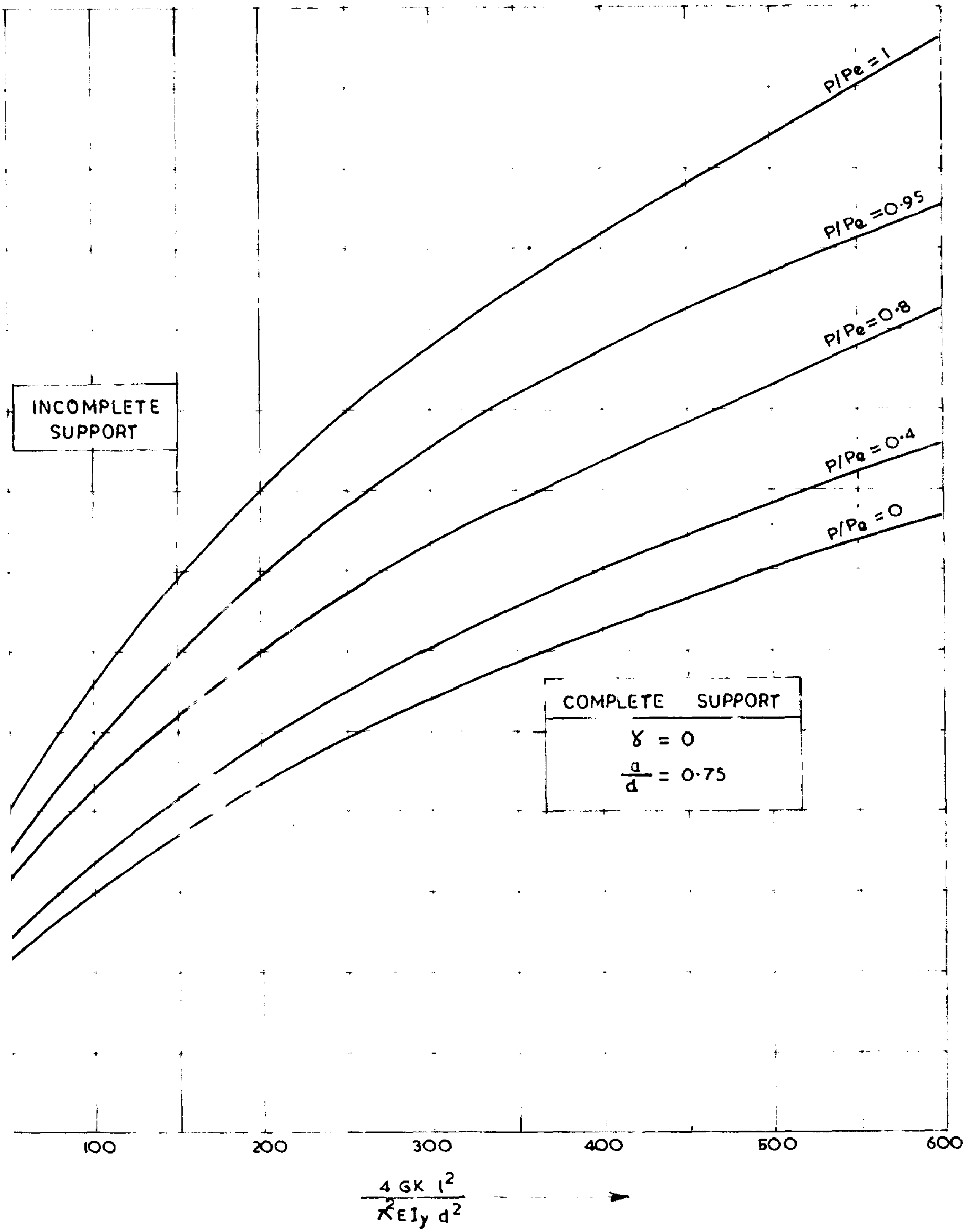


FIG. 2·3



ART GIVING CRITERION OF COMPLETE SUPPORT FOR A COLUMN
UBJECTED TO AXIAL LOAD AND UNIFORM MOMENT

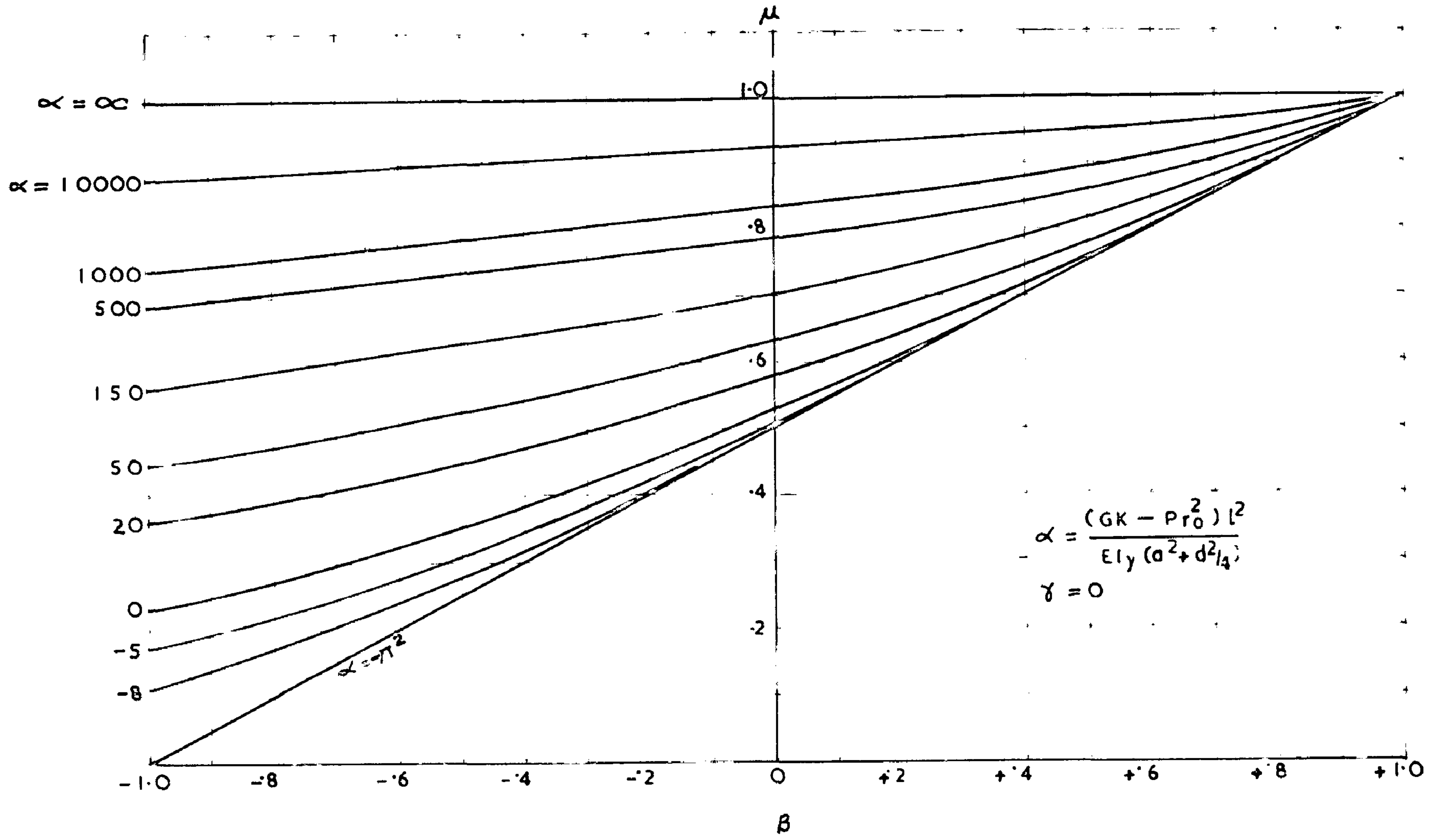

μ
1·0
·8
·6
·4
·2
α = ∞
α = 10000
1000
500
150
50
20
0
-5
-8
α = -π²
α = (GK − Pr₀²) l² / EIy (a² + d²/4)
γ = 0
-1·0
-·8
-·6
-·4
-·2
0
+·2
+·4
+·6
+·8
+1·0
β

FIG. 2·5

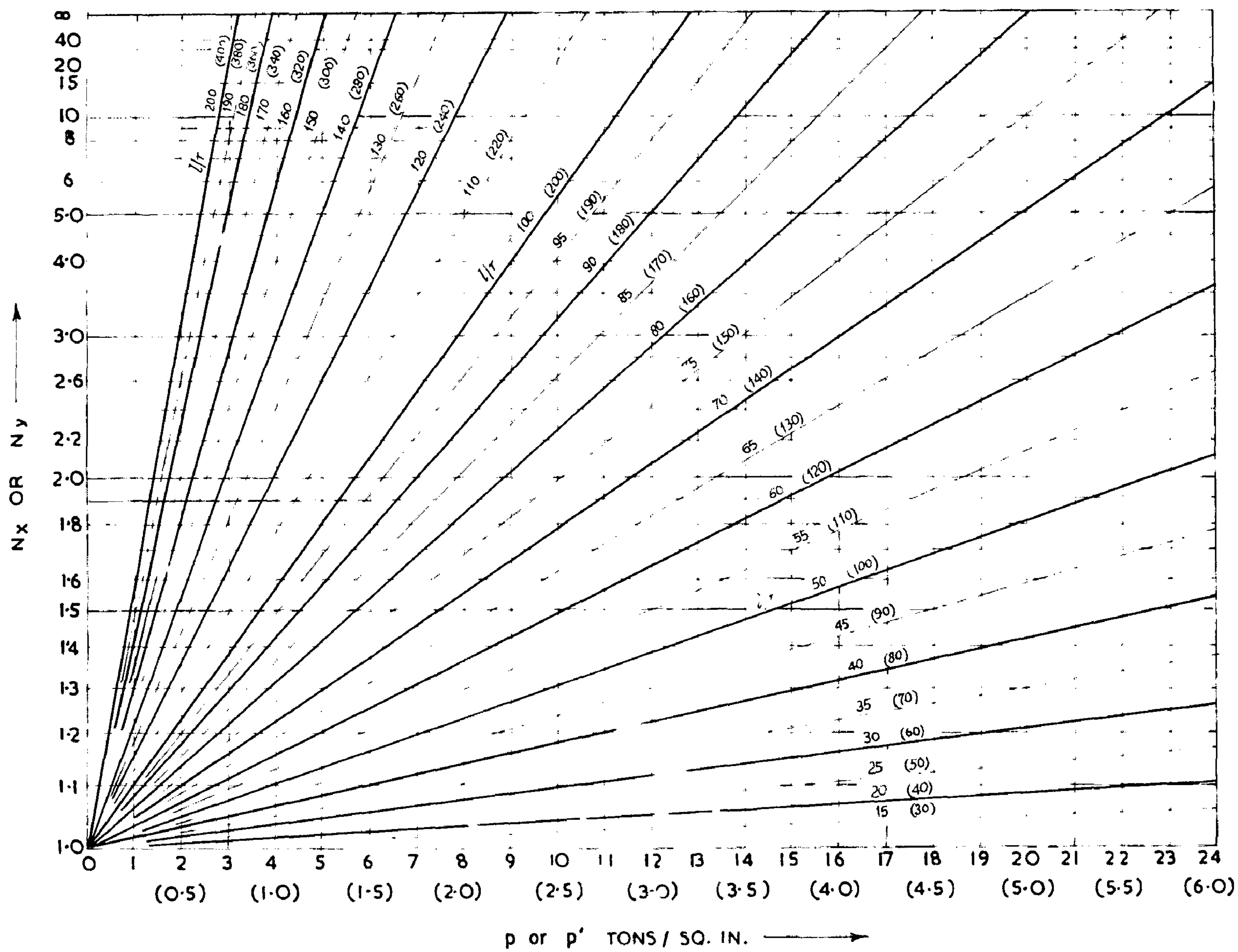
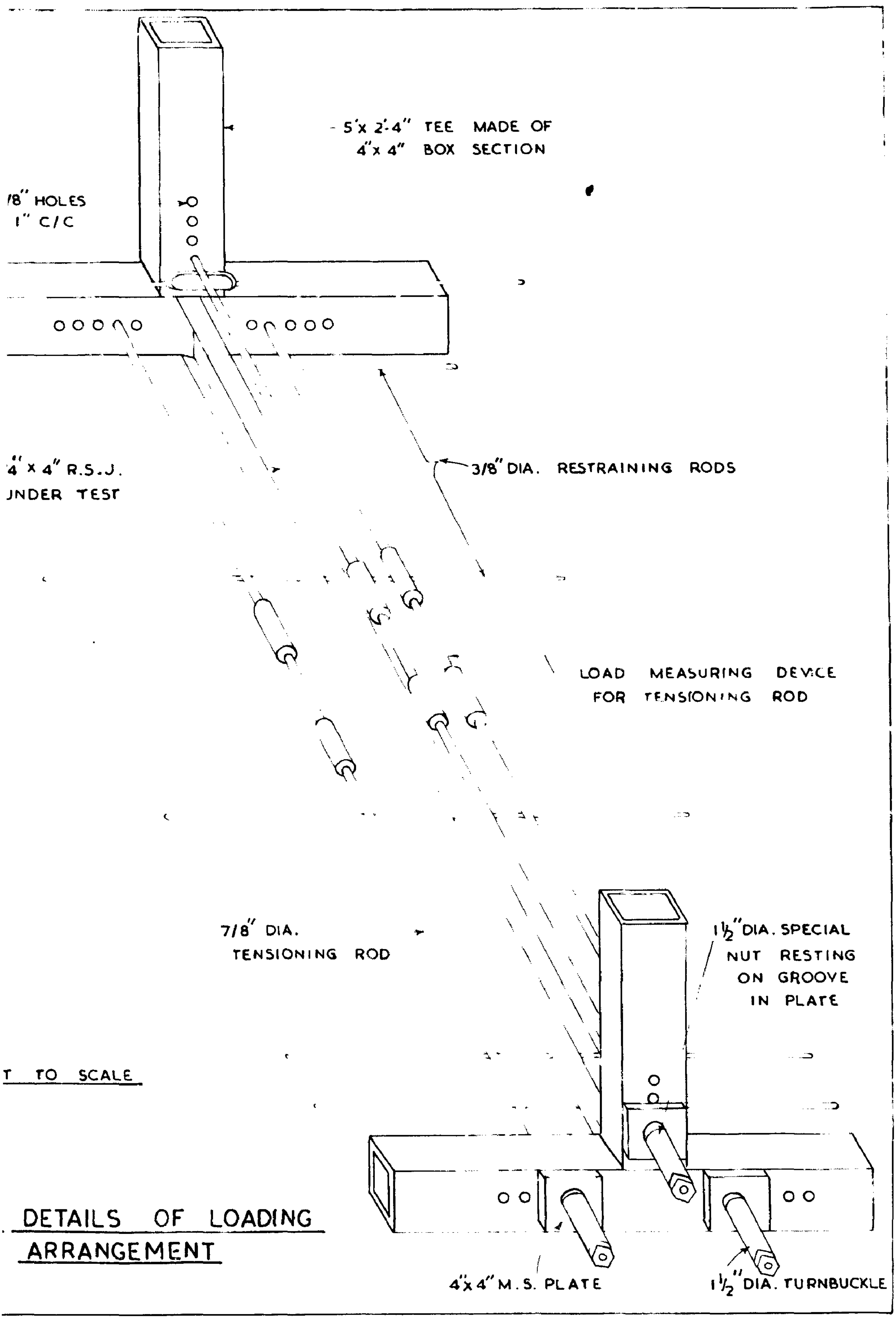


FIG. 3·1



DETAILS OF LOADING ARRANGEMENT

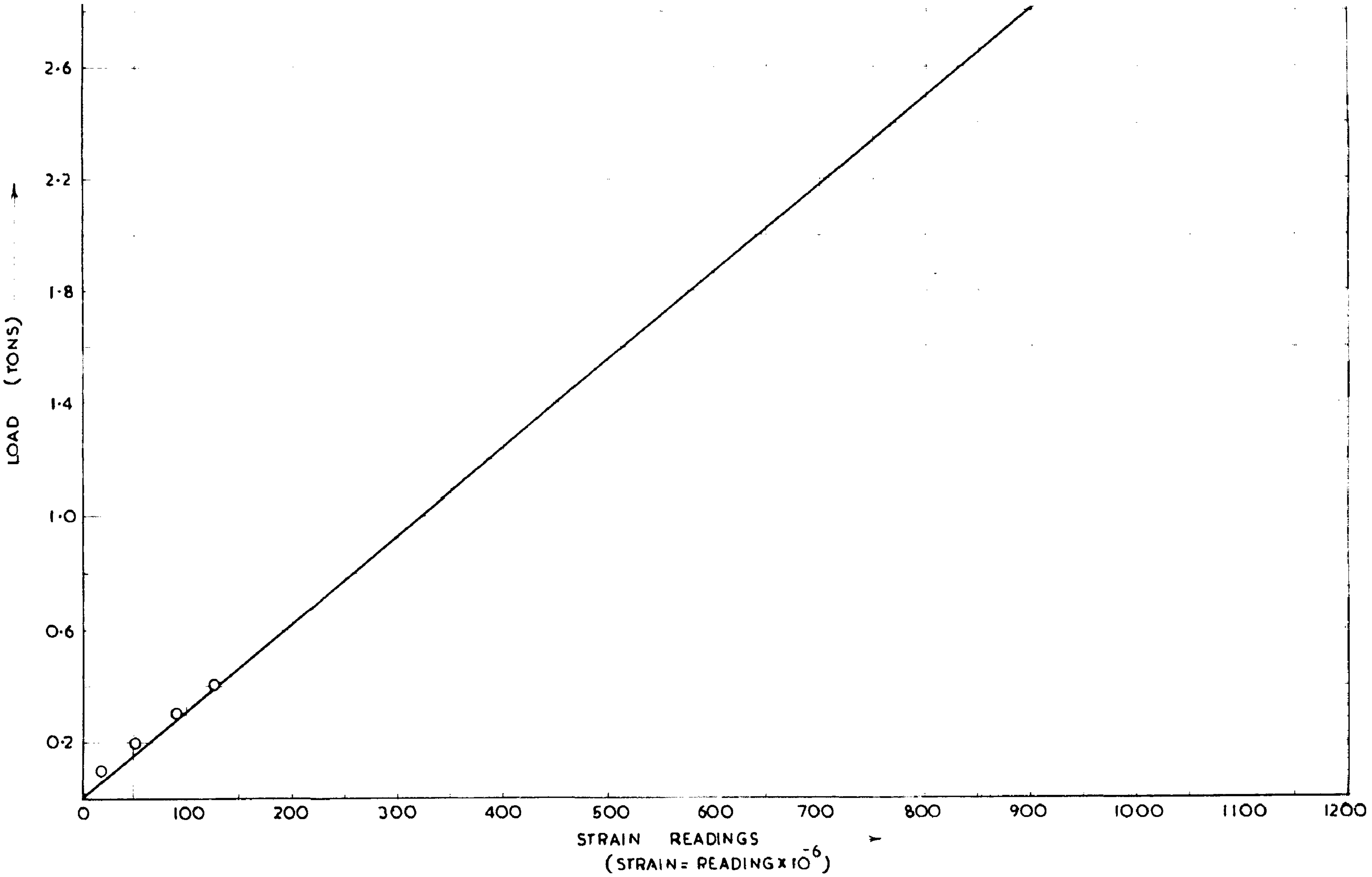


FIG. 4·2 CALIBRATION CHART FOR LOAD MEASURING DEVICE

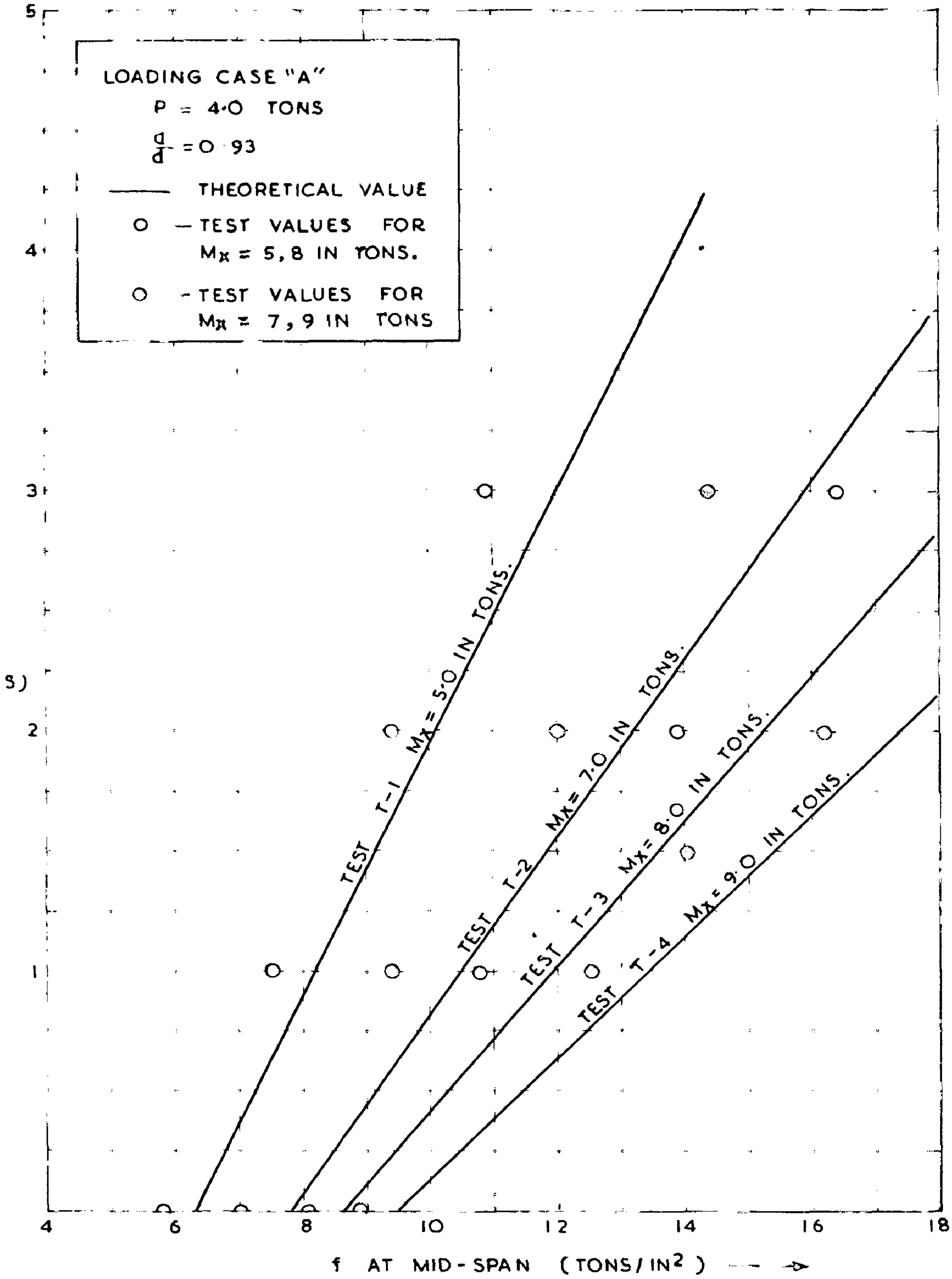

LOADING CASE "A"
P = 4·0 TONS
$\frac{a}{d}$ = 0·93
THEORETICAL VALUE
O — TEST VALUES FOR M_x = 5, 8 IN TONS.
O — TEST VALUES FOR M_x = 7, 9 IN TONS
TEST T-1 M_x = 5·0 IN TONS.
TEST T-2 M_x = 7·0 IN TONS.
TEST T-3 M_x = 8·0 IN TONS.
TEST T-4 M_x = 9·0 IN TONS.
5
4
3
2
1
4
6
8
10
12
14
16
18
f AT MID-SPAN (TONS/IN²) →

FIG. 5·1

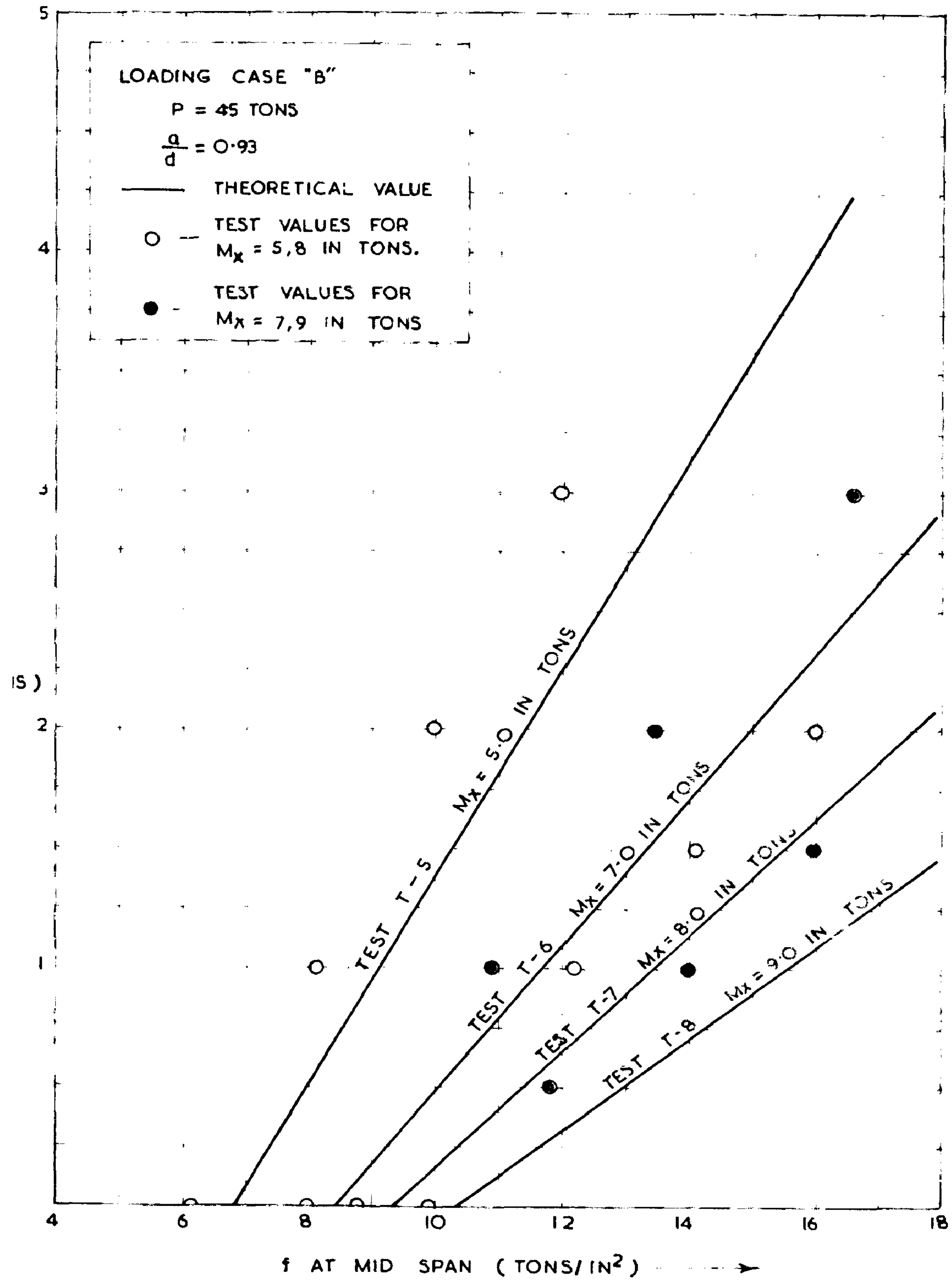
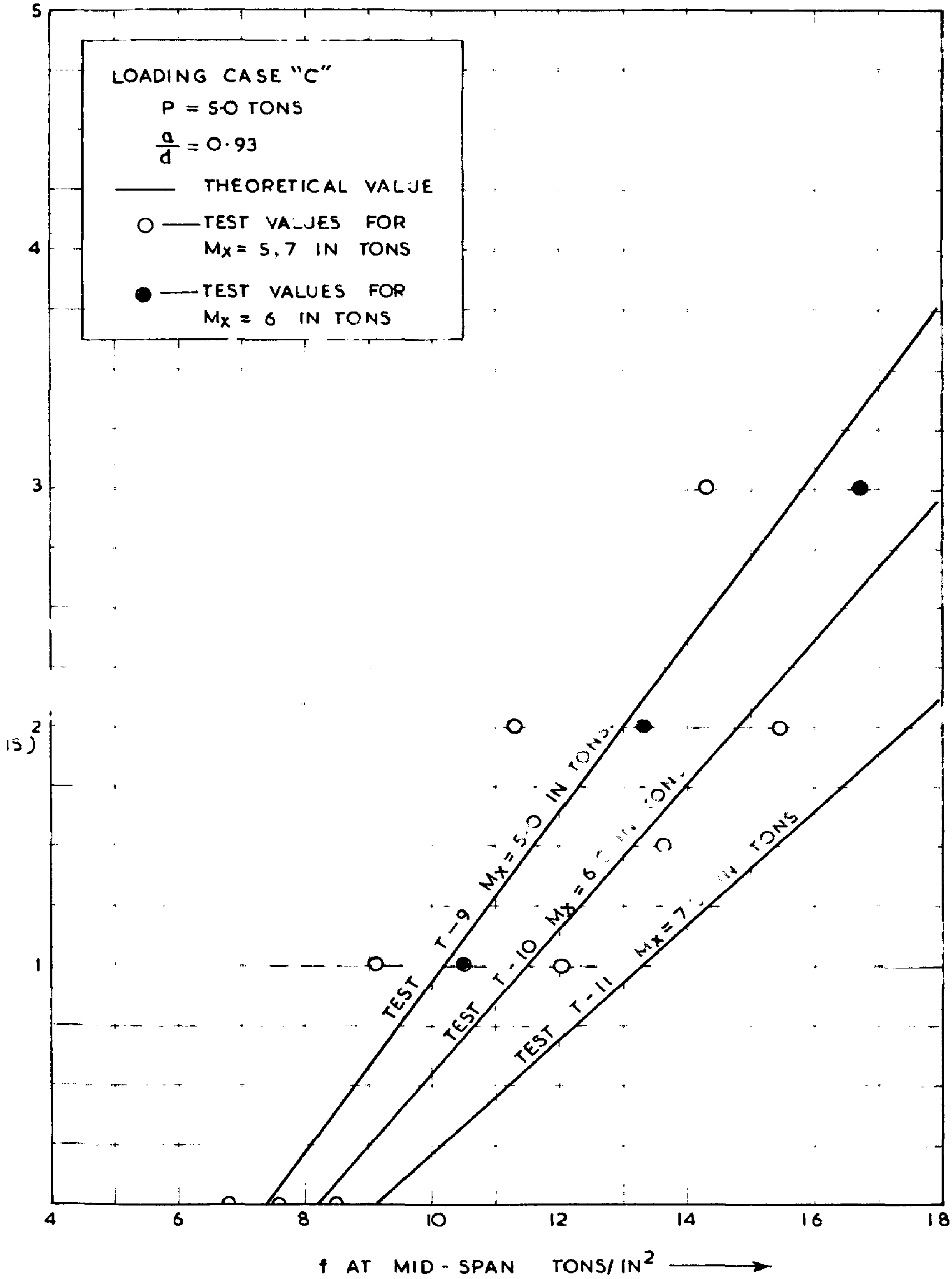
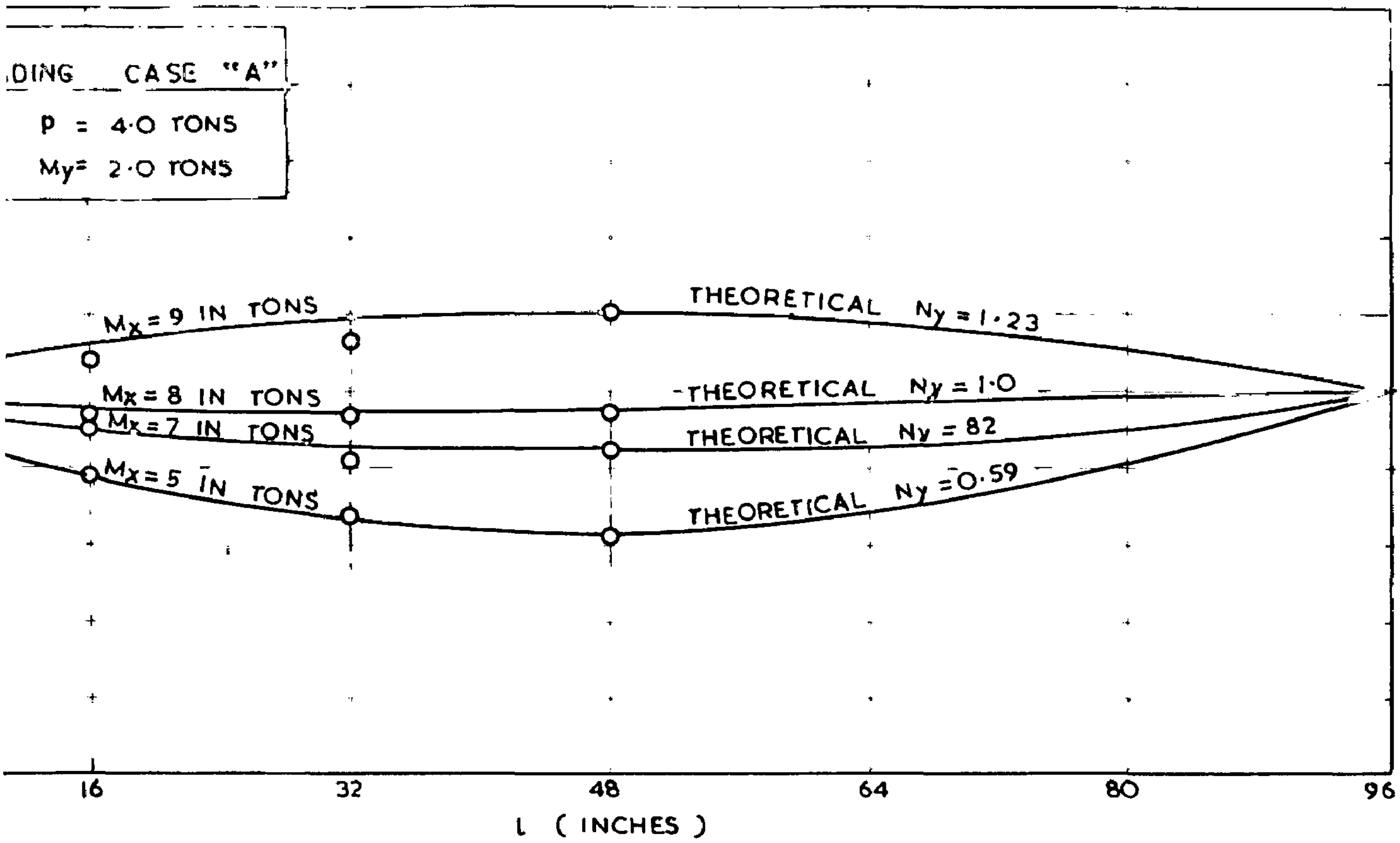

LOADING CASE "B"
P = 45 TONS
a/d = 0·93
THEORETICAL VALUE
TEST VALUES FOR Mx = 5,8 IN TONS.
TEST VALUES FOR Mx = 7,9 IN TONS
TEST T-5 Mx = 5·0 IN TONS
TEST T-6 Mx = 7·0 IN TONS
TEST T-7 Mx = 8·0 IN TONS
TEST T-8 Mx = 9·0 IN TONS
f AT MID SPAN (TONS/IN²)
4
6
8
10
12
14
16
18
1
2
3
4
5

FIG. 5·2



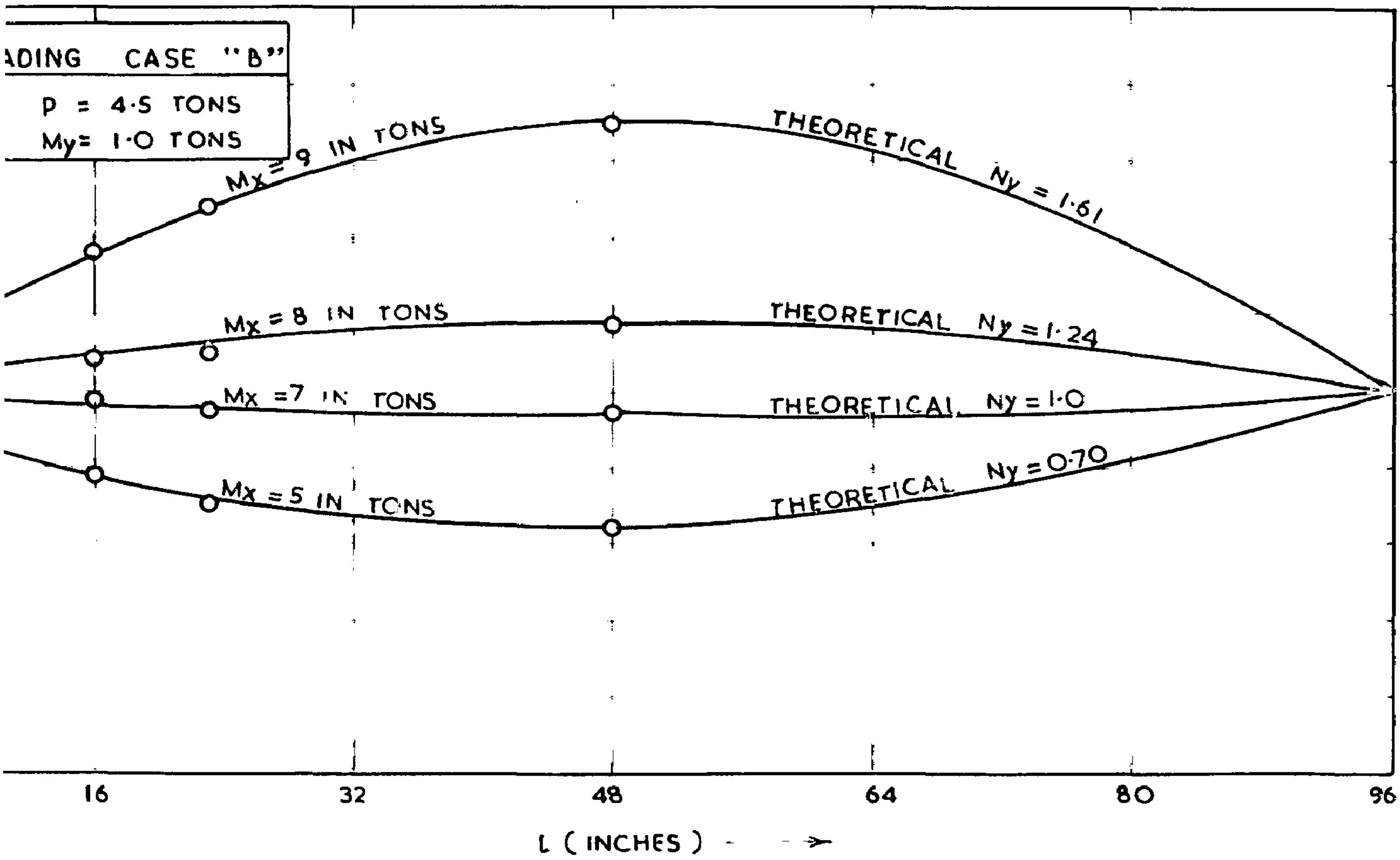
LOADING CASE "C"
P = 5·0 TONS
a/d = 0·93
THEORETICAL VALUE
TEST VALUES FOR Mx = 5, 7 IN TONS
TEST VALUES FOR Mx = 6 IN TONS
TEST T-9
TEST T-10
TEST T-11
5
4
3
2
1
4
6
8
10
12
14
16
18
f AT MID-SPAN TONS/IN²

FIG. 5·3



DING CASE "A"
P = 4·0 TONS
My= 2·0 TONS
Mx = 9 IN TONS
THEORETICAL Ny = 1·23
Mx = 8 IN TONS
THEORETICAL Ny = 1·0
Mx = 7 IN TONS
THEORETICAL Ny = 82
Mx = 5 IN TONS
THEORETICAL Ny = 0·59
16
32
48
64
80
96
L (INCHES)

FIG. 5·4



DING CASE "B"
P = 4·5 TONS
My= 1·0 TONS
Mx = 9 IN TONS
THEORETICAL Ny = 1·61
Mx = 8 IN TONS
THEORETICAL Ny = 1·24
Mx = 7 IN TONS
THEORETICAL Ny = 1·0
Mx = 5 IN TONS
THEORETICAL Ny = 0·70
16
32
48
64
80
96
L (INCHES)

FIG. 5·5

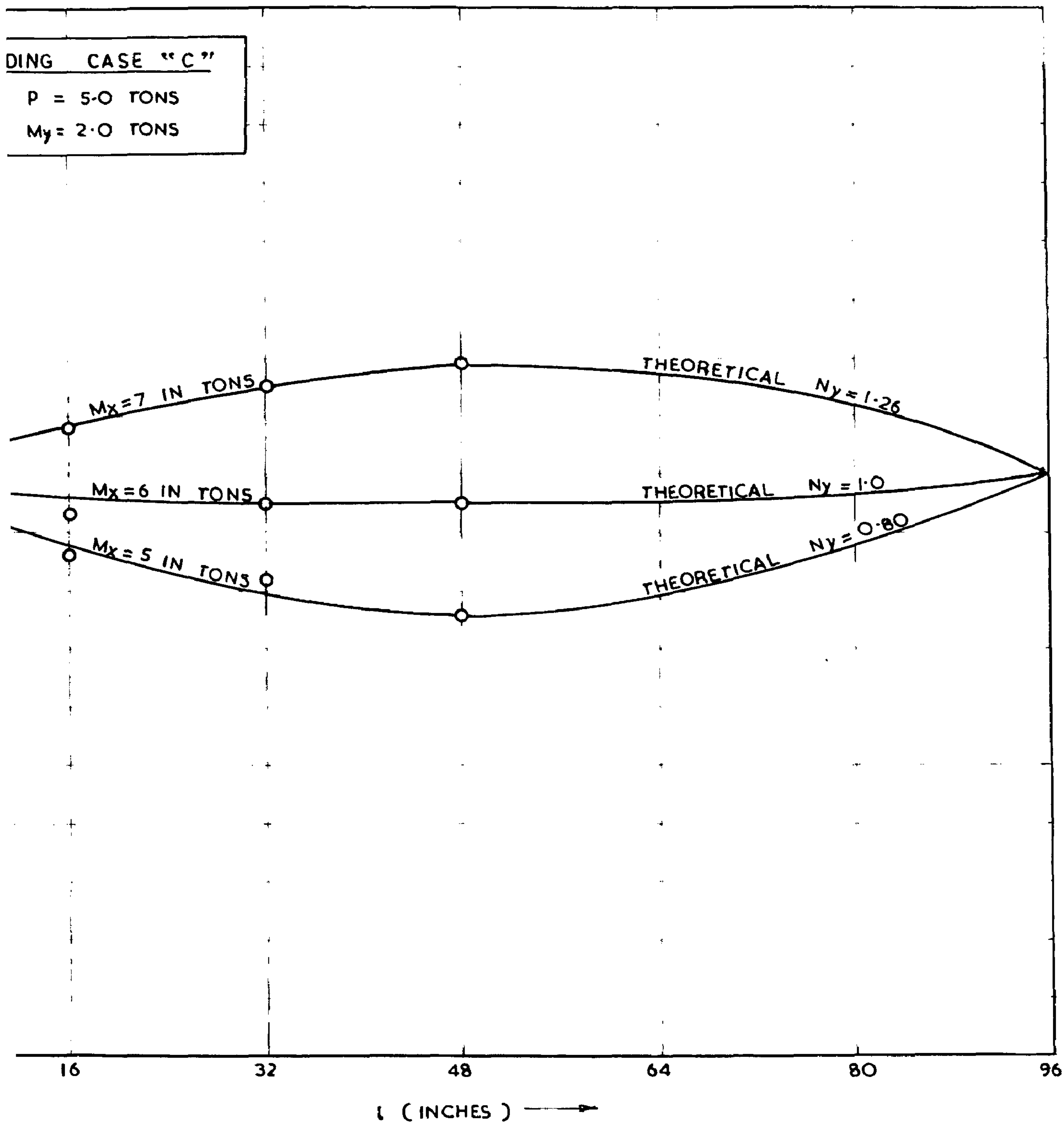
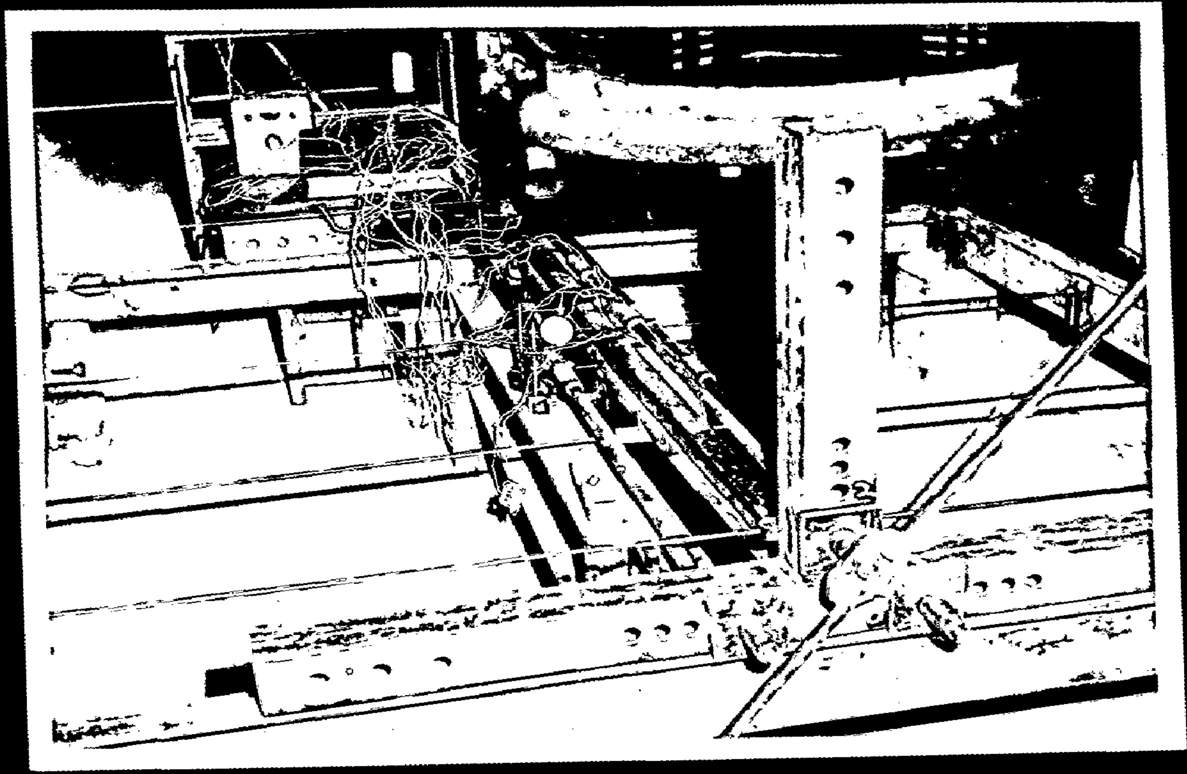
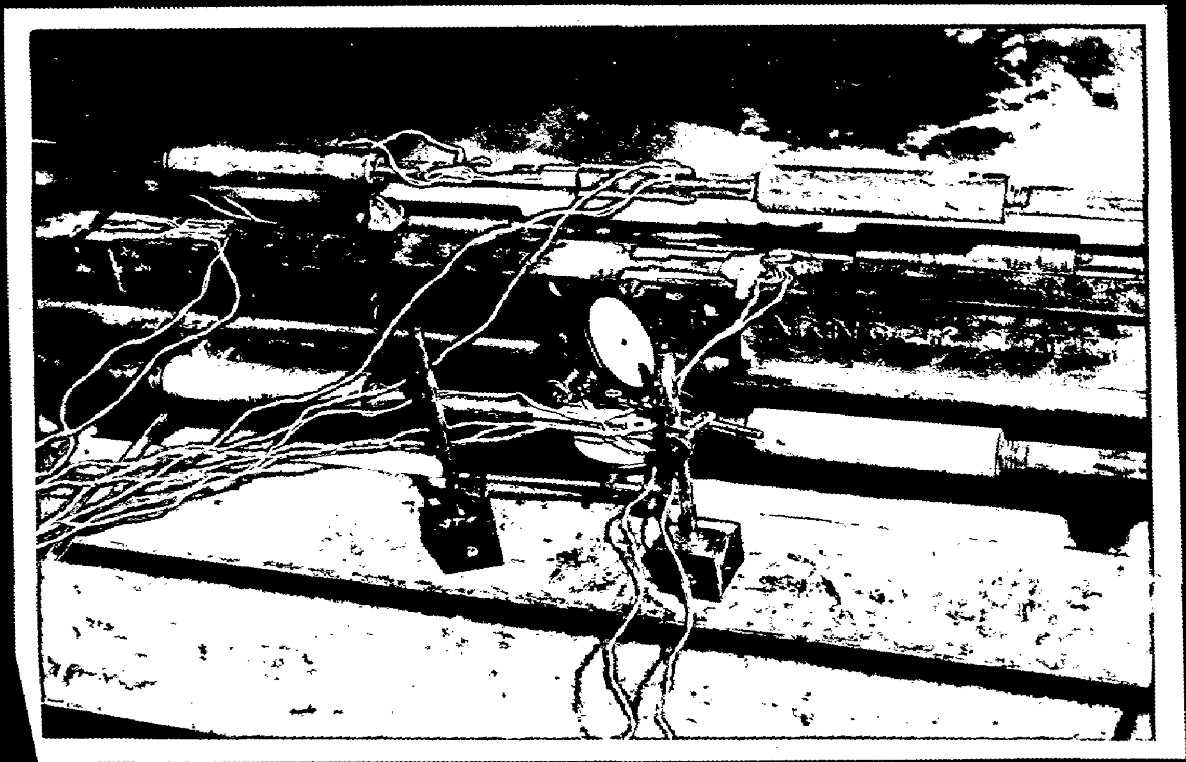

DING CASE "C"
P = 5·0 TONS
My = 2·0 TONS
Mx = 7 IN TONS
THEORETICAL Ny = 1·26
Mx = 6 IN TONS
THEORETICAL Ny = 1·0
Mx = 5 IN TONS
THEORETICAL Ny = 0·80
16
32
48
64
80
96
l (INCHES)

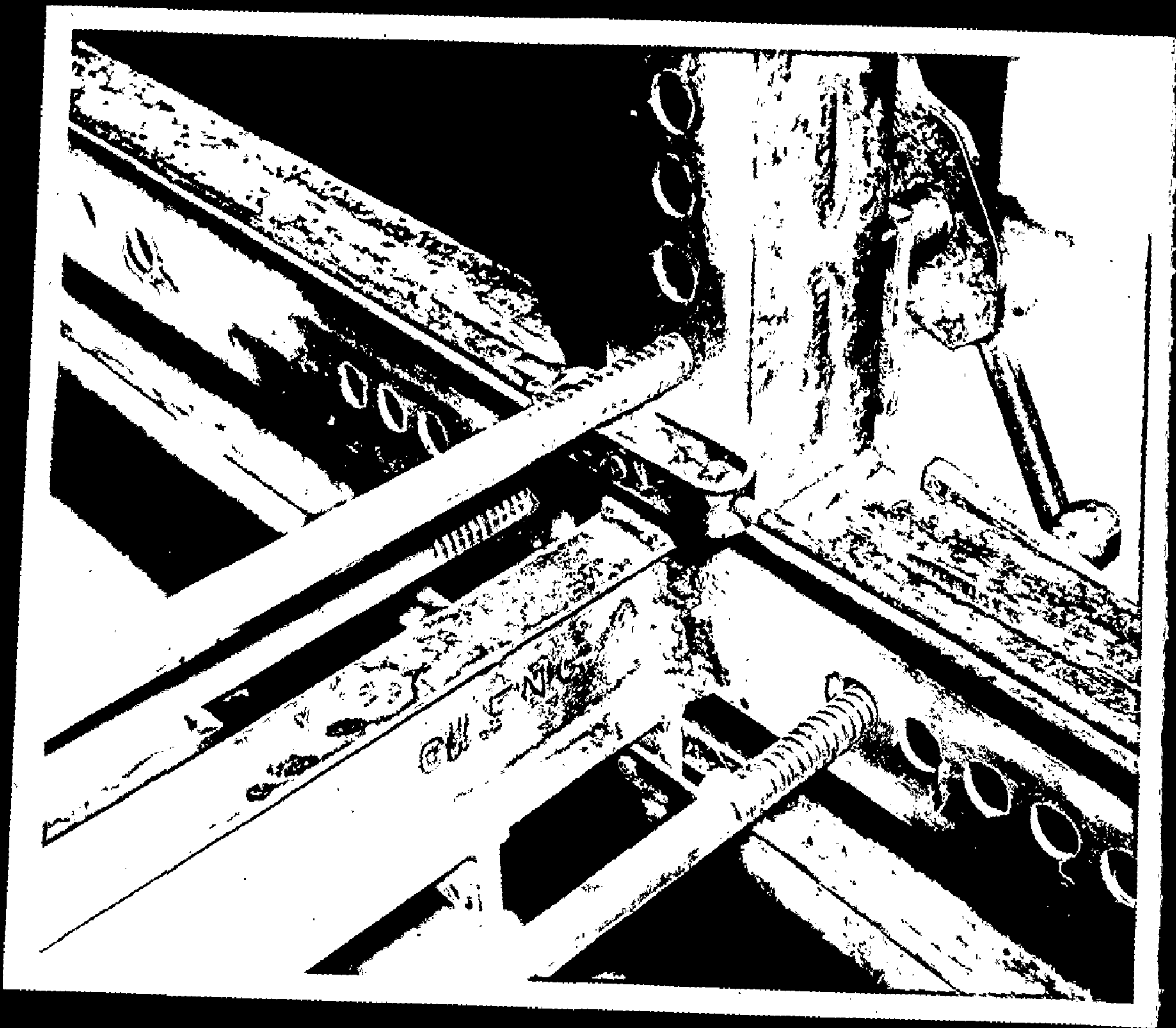
FIG. 5·6



Photograph - 1
General View of the Testing Apparatus



Photograph - 2
The Load Measuring Device Connected to the Tensioning Rods



Photograph - 3
View of the Lateral Support and its Connection to the Flange of the Column