BEARING CAPACITY AND THE DESIGN OF FOOTINGS

by

D. BABU RAO, B. E.

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BEARING CAPACITY
AND
THE DESIGN OF FOOTINGS

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IN

SOIL MECHANICS AND FOUNDATION ENGINEERING

by

D. BABU RAO, B. E.

DEPARTMENT OF CIVIL ENGINEERING

University Of Roorkee

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CERTIFICATE

This is to certify that the Dissertation entitled "Bearing Capacity and the Design of Footings", which is being submitted by Sri D. Babu Rao, as a partial fulfilment for the degree of Master of Engineering (Soil Mechanics and Foundation Engineering) of the University of Roorkee, is a record of bonafide work carried out by him under my supervision and guidance.

He has worked for a period of four and half months at this University for the Dissertation. This Dissertation has not been submitted previously for the award of any degree or diploma or for publication in a journal.

Roorkee.

(A.N. Harkauli)
Reader in Civil Engineering
University of Roorkee .
ROORKEE.

A C K N O W L E D G E M E N T S

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CONTENTS

N O T A T I O N

The symbols used in this work conform generally to those suggested in 1941 by the American Society of Civil Engineers (Soil Mechanics Nomenclature, Manual of Civil Engineering Practice No. 22), although exceptions have been made wherever necessary to avoid confusion.

A	area of the base of the footing
B	breadth of footing
C	constant
C_c	compression index
C_s	coefficient of settlement
c	cohesion
D	depth of footing
E	modulus of elasticity
e_o	void ratio in loosest state
G_s	factor of safety
H	thickness of stratum
K_p	coefficient of passive earth pressure
L	length of footing
M_z	modulus
N	dimensionless factor (N_c, N_r, and N_q bearing capacity factors); number of blows on sampling spoon during standard penetration test

N'	number of blows on sampling spoon for very fine sands below water table
P_p	passive earth pressure. May be subdivided into P_p' which depends on unit weight of the soil, and P_p'' which depends on cohesion and surcharge
p	pressure or normal stress
p_i	intrinsic pressure
p_o	initial pressure
Q	concentrated load
Q_d	critical load on footing
Q_{dr}	bearing capacity of circular footing
Q_{ds}	bearing capacity of square footing
q	surcharge per unit of area
q_a	allowable soil pressure
q_d	ultimate bearing capacity
q_u	unconfined compressive strength
r	radius
S	settlement
s	shearing resistance
x, y, z	cartesian coordinates
α	spread angle, inclination of load
β	slope angle

SYNOPSIS

Footings undoubtedly represent the oldest form of foundation. Bearing capacity and the settlement are the two important factors in the study of footings. In the present work, various theories of bearing capacity are presented in detail. The concepts have been analyzed by theoretical considerations of idealized, isolated footings. The settlement aspect of the problem has been approached similarly. The allowable soil pressure and its determination have been theoretically investigated.

The above concepts have been developed to explain their use in the actual design of footings. A critical review of the present knowledge available on the subject has revealed that both the fundamental research and the procedure for adopting the theoretical concepts to the practical requirements need further work. Accumulation of well-documented field records is absolutely necessary for development of the subject.

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γ	unit weight of soil
Δ	increment
θ	angle
σ	direct stress
τ	shearing stress

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I. I N T R O D U C T I O N

1.1. HISTORICAL REVIEW :-

During the Middle Ages foundation construction used to employ mats of stone blocks laid in over-lapping courses on a levelled ground surface. When greater use of widely spaced walls and individual columns developed, the mats were separated to form spread footings. No particular rules for design were followed. When the underlying soil was hard, the footing was often no longer than the member it supported; in such a case it served merely as a leveling course of masonry on the soil or rock. When the soil was soft, the footing was enlarged by corbelling it outward from the lines of column or wall. The size of footings was seldom related to the column load; instead it was dictated by the space available or the shape of the column or wall it supported. When failures occured, the offending member was enlarged until it carried the load adequately. Often the footings were built without mortar. When very soft ground was encountered, mats of brush several feet thick were spread on the surface to provide a support for the masonry footings. Needless to say, the settlements of such footings were often excessive.

The Industrial Revolution brought about a number of changes in Civil Engineering, both in theory and technique, but these did not extend to foundation design. In general, foundations were designed and constructed during the Eighteenth and Nineteenth cen centuries in as much the same way as they had been during the Middle Ages. The slow mechanization of construction brought about some improvements in the technique for building foundations but only a few real changes in the design. Builders occasionally pre-loaded a building site so that part of settlement would take place before the structure was built. This was not widely practised and the basic principles behind it were not understood, however.

The construction of higher and heavier buildings during the later part of the last century resulted in numerous cases of foundation difficulties and an awakening interest in designs. For example, in constructing a corbelled masonry foundation, each foot of width beyond the limits of the column or wall required an additional foot of footing depth. Therefore, when the footings became wider (as was required to support the heavier loads) they also became deeper and heavier until the foundation weight alone became a major part of the structural load.

A significant advance in the understanding of foundation behaviour was the concept that the areas of foundation should be proportional to the loads and that the centre of the load should be x aligned over the centre of the foundation. These ideas, first put in writing in the United States by F. Baumann of Chicago in 1873, were to guide foundation engineers for over half a century.

The development of highly competitive industry led to a demand for large but inexpensive buildings. The types that developed were more sensitive to differential settlement than their predecessors. Furthermore, many of the most desirable sites for industrial buildings were located in regions that had previously been avoided because of notoriously bad conditions. Hence, designers found themselves in need of a reliable procedure, applicable under soil conditions, for proportioning the footings of a given building in such a manner that they would all experience the same settlement. One result was the formulation of rules for maximum foundation pressure on various types of soils. A second result was the development of field loading tests to evaluate the bearing capacity of soil in place. Although, both of these innovations are considered to be inadequate today, they were significant forward steps in the scientific approach to the foundation design.

1.2. PRESENT STATUS :

Following World War I, foundation engineering progressed more rapidly. The greatest step was the development of science of Soil Mechanics. Much of the credit for this advance belongs to Karl Terzaghi, who in 1925 published his treatise "Edbaumeckanik" This furnished the first integrated analysis of the mechanical behaviour of soils and particularly of settlement under the load and opened the way for a rational approach to the solution of foundation problems. Studies of bearing capacity and of stresses beneath foundations were made by Kogler and Scheidg in Germany, Fellenius in Sweden, Housel in U.S., Skempton, Meyerhof and Wilson in United Kingdom. Since 1930, progress in developing rational methods of foundation analysis and design has been remarkable. During the same period, there were significant advances in the art of foundation construction. The increased use of reinforced concrete made it possible to built thin footings. Construction in areas of weak soil and high water is made possible by improved techniques for unwatering, soil stabilization and excavation.

Since World War II, there has been a growing realization among foundation engineers that a scientific analysis of foundation performance is unsufficient because of the extreme complexity of actual Soil conditions compared with the assumptions made in the solution. The result has been the accumulating of emperical knowledge, guided by theory, to temper the analytical methods. This process is continuing and forms the basis for the modern methods of design.

2. GENERAL REQUIREMENTS OF FOOTINGS

Like any other part of a structure, the footing foundation must meet certain requirements. These requirements are based on the needs of the structure supported, because in the over all picture, the footing (including the soil beneath it) and the superstructure form an integrated unit and act together under the influence of loads applied to it.

The three basic requirements are :

1. The footing must be properly located with respect to any future influence which could adversely affect its performance.
2. The footing must be stable or safe from failure.
3. The footing must not settle or deflect sufficiently to damage the structure or impair its usefulness.

These requirements should ordinarily be considered in the order named. The first is rather nebulous. It involves many different factors, some of which cannot be evaluated analytically, but which must be determined by engineering judgement. The second is specific; it is analogous to the requirement that a beam in the superstructure must be safe against breaking under its working load. The third requirement is both specific and vague. It is analogous to the requirement that a beam in the superstructure should not deflect enough to be objectionable - the amount of deflection is specific, but how much is objectionable cannot always be defined accurately. These three requirements are independent of one another and each must be satisfied, the fact that only two out of the three have been met still means that the foundation is inadequate.

2.1. FOOTING LOCATION AND DEPTH :

The location and depth of a footing are governed by a number of different and unrelated factors. These include the following :-

1. Depth of frost action (or of thawing in perma-frost regions)
2. Depth of seasonal volume changes
3. Adjacent structures, property lines, excavations and future construction operations
4. Ground water level
5. Underground defects such as faults, caves and mines.

2.1.1. FROST ACTION :

In any region where the air temperature falls below 32^{0} deg. F. for more than a few days, the ground freezes and the heave of the soil may occur. Footings placed above this zone of heave may be slowly lifted during the cold weather and then suddenly dropped when the frozen mass thaws. In the U.S. the zone of ice-layers during heavy frost extends as deep as 8 feet beneath the ground surface. To be free of frost heave, footings should be placed at a depth equal to 3/4 of the maximum penetration. In particularly susceptible soils such as saturated silty sands and silts, the full depth is recommended, while in gravels and dry sands even less than 3/4 the maximum depth may be adequate. Local experience is the best guide to check frost penetration

Figure 1 shows the location of the footing of the outside column below the level to which frost may cause perceptible heave.

2.1.2. SEASONAL VOLUME CHANGES :

Clays, particularly those with high plasticity, shrink greatly upon drying and swell upon the addition of moisture. In geographic regions which have well defined reasons of high and low moisture, such clays swell and shrink in regular cycles, often causing severe damage to structures which they support. Black cotton soil of India is a typical example. The outside walls move up and down with the swelling and shrinking. However, the central parts of huge structures shelter the soil from both sum and rain and minimize the volume changes. The interior parts of the structure, therefore, suffer less than the outside walls and this causes severe damage due to differential settlement.

In arid regions where the soil is normally dry, the problem is somewhat different. Added moisture from leakage of pipes, watering lawns, or the reduction of evaporation caused by the presence of a building or a pavement can bring about swelling and heave of several inches.

In humid regions the Soils are ordinarily moist. Severe desiccation may cause high volume changes. Unusual prolonged periods of drought have brought about settlement of structures which have stood for years without any sign of distress.

Accelerated dedccation accompanied by rapid and irregular settlement may be caused by many local conditions such as heat from boilers, ovens and furnaces, that are inadequately insulated from the ground. Footings should be placed as far possible from all sources of heat and below the depth of desiccation, particularly if the soil has a potentially high volume change.

In numerous instances moisture used by vegetation has resulted in accelerated soil desiccation and settlement. Large trees and even some shrubs and field crops are capable of removing large amounts of moisture and causing settlement of footings placed above or adjacent to their major root systems. In such cases foundations should be placed well below large roots and as far from them as possible.

2.1.3. ADJACENT STRUCTURES, PROPERTY LINES AND FUTURE CONSTRUCTION OPERATIONS :

The location of adjacent structures, property limes, and the possibility of future construction are important factors in the location and depth of footings. Expensive law suits have arisen when footings extended into adjoining property even though the building wall was well inside the line.

The construction of new foundations can cause damage to existing ones due to vibration, shock of blasting, undermining by excavation, or the lowering of ground water table. The deeper the new foundation and the nearer to the old it is located, the greater the damage is likely to be. A rule of thumb is that the minimum horizontal spacing between old and new footings should be equal to the width of the larger one. Further, a line drawn outward at a 45 deg. angle from the edge of the base of the higher one should not intersect the base of the lower one, as shown in Fig. 2.

Foundation depth must be selected with future nearby excavation in mind. This is particularly true close to the property lines where only limited legal control may be possible over the construction operations on the adjoining site. For example, the American Standard Code for Excavation requires that a person making an excavation adjacent to a property line provide support for the adjoining property only when the excavation is deeper than 10 feet. Under such conditions, a minimum footing depth of 10 ft. would be prudent. When future deep excavations are planned, such as for an addition having a basement, the foundations for the initial part of the structure should be placed deep enough that they will be unaffected by the addition. If this is not done, under-pinning may be required in future.

Basement floors are commonly located well below the minimum depth required for footings of buildings without basements. Hence, under normal conditions the minimum depth of foundations located within the boundaries of a basement, C and d in Figure 1, is governed solely by structural requirements.

2.1.4. GROUND WATER LEVEL :

The level of ground water table is a factor in foundation depth in three ways. First, construction below the water level often presents difficulties. In cohesionless sands and silts, for example, upward flow of water into a footing excavation can create a quicksand condition and make construction impossible without pre-drainage. Second, the presence of the water-table establishes the bearing capacity of the footing. The submerged weight is about onehalf of the saturated weight. Hence, it may be concluded that a rise of the water table from a depth greater than about B below the base of the footing upto the top of the surcharge would have the effect of reducing the bearing capacity to about one half of its value for saturated sand. Third, when the water table is above the lowest floor, waterproofing and resistance against hydrostatic uflift become serious considerations. Ordinarily spread footings are placed above the highest ground water level unless the additional expnse of greater depth is well justified.

2.1.5. UNDERGROUND DEFECTS :

The presence of underground defects such as faults, caves, mines and man-made discontinuities such as sewers, underground cables, and utilities influences both footing location and depth. Minor faults occur frequently in bedrock and when they are active, the entire sturucture should be placed on one side or the other of the fault line.

Man-made discontinuities such as old wells, sewers, cables and utilities are frequently encountered in cities and established industrial sites. If these are in use, they should be relocated or the structure moved away from them because maintenance and repair will be complicated if they are buried beneath a structure. Further more, they present a hazard to foundations because the back-fill over them is usually rather loose and because they often are structurally inadequate. No footing should be located over such a discontinuity or on its backfill unless both are known to be capable of carrying the load. The same rule for footing depth as is used for adjacent foundations and excavations is often applied to underground utility lines to minimize damage from their presence. Unfortunately, in many areas the location of underground utilities is not accurately known and this leads to costly changes during construction. For example, (Ref. 40), the position of a 14 ft. sewer was determined as accurately as possible from old plans and from an internal inspection by means of man holes. However, during construction it was found that the sewer was laid on the sweeping curve, which brought it directly beneath several of the footings. The columns had to be relocated and the footings lowered to the level of the bottom of the sewer. A very careful survey of the local conditions can do much to minimize this hazard.

2.2. BEARING CAPACITY AND SETTLEMENT :

On the assumption that it is practicable to construct a given type of footing in the light of the above considerations, the probable performance of the footing must be judged with respect to two types of unsatisfactory behaviour. On the one hand, the entire footing or any of the elements of which it is composed may break into the ground because the soil or rock is incapable of supporting the load without failure. On the other hand, the supporting soil or rock may not fail, but the settlement of the structure may be so great or so uneven that the superstructure may become cracked or damaged. These two types of unsatisfactory behaviour have almost independent causes and can usually be investigated separately. The first is a function of the strength of the supporting soil or rock, and is known as a bearing capacity failure. The second depends upon the stress-deformation characteristics of the soil or rock, and is known as 'detrimental settlement'. In each of the following chapters, they will be treated at length with the aim of developing the concepts in the design of footings.

3. BEARING CAPACITY

3.1. INTRODUCTION :

The subject of bearing capacity is perhaps one of the important subjects in Soil Engineering. It is generally believed that bearing capacity is an absolute inherent property of the soil, just as cohesion and internal friction and this has consequently led to wrong interpretation, sometimes leading to a false sense of security. A true understanding of the factors upon which it depends and an explanation of the use of the bearing capacity concepts in the design of footings needs the presentation of certain fundamental ideas. Numerous factors enter into the complex problem of determining bearing capacities for buildings which rest on many spread footings. However, some of the most important of these may be explained without difficulty by theoretical considerations of idealized, simple cases of isolated footings. The bearing capacities of all types of soils, ranging from cohesionless to highly cohesive, will be discussed. The most important variables on which the bearing capacity is dependent in any given soil are the dimensions of the footings, and the investigations into the relationships between the bearing capacity and the breadth and shape of the footing below ground surface will be studied in detail. All soils considered in these studies are assumed to be homogeneous unless otherwise stated.

Relationships will be obtained which are good for soils in general. The formulas for such cases contain two soil characteristics, the friction angle and the unit cohesion; and when these two soil characteristics appear it may be concluded that the expression applies to soils in general. In other studies the considerations are limited to the extreme or limited cases of cohesionless and very highly cohesive soils. Cohesionless soils may be defined as those in which the shearing strength depends entirely on intergranular pressures which are caused by the footing load and the overburden Highly cohesive soils are those in which the strength is primarily caused by intrinsic pressure and in which the strength, therefore, does not vary with the depth below ground surface.

The dimension used to express the size of the footing is the breadth. This dimension is equal to the diameter of a round footing, and the smaller side of a rectangular footing. Cases covered herein will in general be limited to long footings, square footings and round footings. The designation long footing applies to such cases as wall footings wherein the length is very large in comparison to the breadth.

3.2. FUNDAMENTAL CONCEPTS :

If a load is applied on a limited area on or below the surface of the soil, the loaded area settles. If the settlements due to a steady increase of the load are plotted as ordinates against the load per unit of area we obtain a settlement diagram. The settlement curve may have any shape intermediate between those represented by the curves C_1 and C_2 in Figure 3a. It is seen that at the lower values of loading intensity, where the diagram is an approximation of a straight line, the settlement for a given area is roughly proportional to the loading. If it is commonly assumed that when this is the case, settlement is due primarily to compression rather than lateral displacement of the soil beneath the footing. As settlement per increment of loading increases and curvature of the diagram becomes pronounced, it is considered that soil rupture is taking place and that the footing is sinking into the ground as a result of lateral displacement of supporting soil.

There is seldom a clear demarcation of the two sections of the diagram. In a all probability, soil rupture is a progressive rather than an abrupt development and it may be initiated well before the break in curvature is reached. It is convenient, however, for purposes of discussion to assume that ranges of loading can be identified in which settlement can be attributed either to Soil compression or to soil displacement. Thus in Figure 3b,

A is the settlement due chiefly to Soil Compression,

B is the settlement due to combination of soil compression and lateral displacement &

C is the settlement due to lateral displacement.

If the settlement curve passes fairly abrupty into a vertical tangent (curve C_1 in figure 3a), we identify the failure of the earth support with the transition of the curve into the vertical tangent. On the other hand, Terzaghi (43) states that if the settlement curve continues to descend on a slope, as shown by the curve C_2 we specify arbitrarily, but in accordance with current conceptions, that the earth support has failed as soon as the curve passes into a steep and fairly straight tangent.

The area covered by the load is called the 'bearing area'. The load required to produce the failure of the soil support is called the 'critical load' or the 'total bearing capacity'. The average critical load per unit of area, q_d or q'_d (fig. 3a) is called the 'bearing capacity of the soil'. It may be defined as the largest intensity of pressure which may be applied by a structure or a structural member to the soil which supports it without causing excessive settlement or danger of failure of the soil in shear.

3.2.1. <u>THE PRESSURE BULB</u> :

The pressure bulb is a common term used to represent the zone below a footing within which appreciable stresses are caused by the footing load. The concept of a pressure bulb is a valuable one, and the bulb should be pictured simply as a stressed zone within a homogeneous mass.

Soil characteristics and pressure below footings are not well enough known to allow an accurate plotting of contours of stress in the presence bulb. However, stresses below a cricular loaded area on the surface of an elastic mass of infinite extent may be determined from the theory of elasticity, and from plots of stress contours for the elastic case a general picture is given which may be accepted as valid, in a roughly qualitative sense for footings on soil. The compressive stresses on the horizontal plane are shown in Fig. (4a) by stress contours for all points below a round uniformly loaded area of the surface of an elastic mass. The maximum shearing stresses at all points is similarly represnted in (b). It may be seen that all concepts of the size of the pressure bulb depend on an arbitrary choice of the magnitude of stress at which values are considered to pass from appreciable to inappreciable. If direct stresses are considered to be of inappreciable magnitude when they are smaller than 10 per cent of the intensity of the applied stress at the surface, in general, the pressure bulb will have a depth a depth of roughly 1.5 times the breadth of the loaded area.

THE ACTION WITHIN THE PRESSURE BULB :

The outward picture of the action of a footing is limited to the concepts that a footing is loaded and therefore settles. However, the action within the pressure bulb is more complicated. The settlement of the footing is due to the vertical strains which occur within the height of the pressure bulb. These vertical strains are due in part to shearing strains or change of shape and in part to volumetric strains or decreases in void ration. In Fig. 5

the full lines show a square footing before loading and the zone in which its idealized pressure bulb will form when load is applied. The original position of a small element of soil at the centre of the bulb is also indicated. The displaced positions of these lines after the load is applied are shown by dashed lines. With the magnitude of changes considerably exaggerated. If the settlement is due mainly to the squeezing out of soil from under the footing, as in a relatively dense sand which is loaded nearly to failure, the bulb and the element are distorted with little change of volume, as the figure shows. If the settlement is due mainly to compression of the soil, as in a very compressible soil subjected to a load that is small compared to the load causing failure the changes in positions of horizontal lines would be about as shown by the figure but the changes in positions of vertical lines would be only a small fraction of those shown.

In Figure 5 the deashed lines representing the width of the bulb after loading are not shown near ground surface because strains may be large in this zone. A rigid surface footing on sand, when carrying even a very small load, will develop a plastic zone within the surrounding soil. Plastic zones of this type are shown in Fig. 6 for a long wall footing on the surface and beneath the surface level of a cohesionless soil; the zones shown in this figure are according to concepts developed by O.K. Frohlich Zones I are plastic under a small loading and enlarge to Zones II under greater loading. Qualitatively similar shapes of plastic zones exist below the edges of square and round footings.

3.3. THEORIES OF BEARING CAPACITY :

No exact mathematical approach has been devised for the analysis of bearing capacity failure. Many methods have been formulated but all involve some simplifying approximations regarding the Soil properties and the movements which take place that are incompatible with the observed facts. Inspite of these short comings, comparisons between the ultimate bearing capacity computed by the best of these methods and the observed ultimate bearing capacity of both model and full sized footings show that the range of error is little greater than for problems of structural stability in other materials.

3.3.1 PRANDTL'S ANALYSIS :

Many modern analyses of the problem of bearing capacity are based on a solution by Prandtl (Ref.35). The Prandtl plastic equilibrium theory presents an expression for the ultimate bearing capacity of long loaded areas of breadth B on ground surface. Prandtl investigated the plastic failures of metals. A special case of his general solution is applicable to foundations. Since Prandtl was mainly concerned with the penetration of punches into metals, where movement of these punches was guided, a basic assumption of his solution is that a loaded footing of width B and very great length L will sink vertically downward into the underlying material, thereby producing shear failures on both sides of the footing. Figure 7 shows the three zones which, according to Prandlt, exist after the failure is reached. The following assumptions are made :-

1. The soil is homogeneous, isotropic and weightless.
2. The wedge - shaped soil zone ABC immediately beneath the footing moves downward without any deformation together with the footing. Soil zones AFC and BCD are assumed to be in a plastic state and to push soil zones AFG and BED upward as units. Theremainder of the loaded medium is essentially unaffected by the load.

   NOTE: The actual angles and lengths in the diagram are not assumed but are derived in the course of the analysis.
3. The line of rupture (envelope to the Mohr circles) for the soil is a straight line.
4. In the plastic sectors AFC and BCD the stresses along any radius vector such as BX are constant but they vary from radius vector to radius vector.

This Zone I is similar to the unsheared conical zone at the top of a cylindrical compression test specimen., Zone II plastic, in this zone all radial planes through points A and B are failure planes, and the curved boundary is a logarithmic spiral. Zone III is forced by passive pressure upward and outward as a unit. It may be noted that all failure planes are at $(45^\circ \pm \phi/2)$ to principal planes. The section is symmetrical upto the point of failure, with an equal chance of occuring as shown, or along the similar failure surface shown at the left by dashed times.

Prandtl considers the equilibrium of the plastic zones. The boundary conditions are :

that the major principal stress on the boundaries AC and BC is q_d, while the minor principal stress on the boundaries AF and BD is zero. On the basis of the assumption that the shearing strength of any soil may be expressed by

$$S = c + \sigma \tan \phi$$

and that c is a constant, Prandtl shows that the ultimate bearing capacity of any soil is

$$q_d = \frac{c}{\tan\phi}\left[\frac{1+\sin\phi}{1-\sin\phi}\cdot e^{\pi\tan\phi} - 1\right] \qquad (3.1)$$

The analysis covered varying values of the angle of internal friction ϕ. For $\phi = 0$, the solution needs Calculus, since the substitution of $\phi = 0$ gives the product of infinity and zero. The final solution is

$$q_d = 5.142C = 2.571\, q_u \qquad (3.2)$$

For ϕ greater than zero q_d increases rapidly with the value of ϕ, as shown by the following table.

Table 3.1 Values of q_d according to Prandtl.

ϕ	B'/B	q_d/q_u
0	1.000	2.571
10	1.572	3.499
20	2.530	5.194
30	4.290	8.701
40	8.462	17.560

It will be seen that, if C = 0, Equation (3.1) reduces to zero. This would mean that a cohesionless soil such as dry sand has no bearing capacity. Actually this is not so, and the assumption chiefly responsible for this discrepency is that the soil is weightless. But the consideration of the material complicates the situation very considerably. At given values of C and ϕ it increases the critical load and it changes the shape of the surfaces of sliding within both the zones II and III. Thus for instance in the zone of radial shear, (Zone II) the radial lines of shear are not straight as shown in figure 7 , but curved. There are two alternative corrections to Prandtl's formula due to Terzaghi and Taylor respectively. Of these the first is preferred for accuracy but the second is much more easy to calculate.

Terzaghi's Correction for the Weight of the Material :

To C in the original formula (3.1) add C' where :-

$$c' = h\gamma \tan\phi$$

$$h = \frac{\text{Area of wedges and sectors in Fig. 7.}}{\text{Length GE}}$$

The formula then becomes

$$q_d = \frac{c + c'}{\tan\phi} \left[\frac{1 + \sin\phi}{1 - \sin\phi} \cdot e^{\pi \tan\phi} - 1 \right] \qquad (3.3)$$

Taylor's Correction for the Weight of the Material :-

To $\dfrac{c}{\tan\phi}$ in the original formula add

$$B\gamma \cot\left(\frac{\pi}{4} - \frac{\phi}{2}\right).$$

The formula then becomes

$$q_d = \left[c \cdot \cot\phi + B\gamma \cot\left(\frac{\pi}{4} - \frac{\phi}{2}\right) \right] \cdot \left[\frac{1 + \sin\phi}{1 - \sin\phi} \cdot e^{\pi \tan\phi} - 1 \right] \qquad (3.4)$$

Effect of Surcharge on Prandtl's Formula :-

The derivation of the formula (3.1) was made for loading at the surface;it is not applicable when the footing applied its load at a depth below the surface level. Allowance can, however, be made for this by increasing the bearing pressure for surface loading by the over burden pressure. If a vertical pressure, p , is applied to the outer rigid wedges AFG and BDE, then the bearing capacity is increased from that given by the preceding Prandtl formula by an additional

$$p\left[\frac{1+\sin\phi}{1-\sin\phi}\cdot e^{\pi\tan\phi}-1\right]$$

This surcharge effect can only apply for values of p considerably less than q_d for otherwise there results merely an increase in the loaded width. In practice p will usually be small compared with q_d but nevertheless on granular soils, where ϕ may be between 35 and 40 degrees, marked increase in bearing capacity is produced. On cohesive soils, where ϕ is of the order of 0 to 10 degrees, surcharge produces little effect.

DISCUSSION.

The accuracy and value of any of the bearing capacity theory depend on the extent to which the assumed shape of the surfaces of failure approach reality. It is in this respect that Prandtl's analysis is considered to be the most reliable, since his assumed mode of failure agrees quite well with observations made on both granular and cohesive soils. Terzaghi has restricted the validity of the above equations to foundations with a perfectly smooth base.in contact with the soil. Shearing stresses along a rough base are believed to exert a restraining effect on the soil and thus coefficient in equation (3.2) is increased to 5.7 from Prandtl's value of 5.14.

The application to all foundation designs of the above suggestions to increase to this extent the original values obtained by Prandtl, according to Tschebotarioff, appears questionable in the light of the following considerations. In a great many cases the nature of the possible downward movement of a foundation is not restrained in any manner, so that the foundation is free to rotate about any one of its edges. Thus the basic assumption of the Prandtl solution, illustrated by fig.7 does not necessarily hold in all cases. Actual records of shear failure of the clay underlying large foundations usually indicate rotational displacements of the soil. Further the clay deposit cannot always be absolutely homogeneous to such an extent that a shear failure would develop in it simultaneously on both sides of the foundation. It is likely to be somewhat weaker on one side than on the other, so that a rotational failure, as investigated by Fellenius, would result.

Inspite of the above shortcomings, it remains that the present concepts of the nature of soil failure under footings stem mainly from the analysis of Prandtl.

3.3.2. BELL - TERZAGHI ANALYSIS :

For a soil possessing cohesion as well as friction on analytical solution for the relationship between bearing capacity and depth was first derived by A.L. Bell (1915). This was later extended by Terzaghi. The analysis approximates the curved surface of failure with a pair of planes as shown in fig.8. The average stresses on the planes at failure are computed from the Rankine Theory, and from these the bearing capacity is obtained. The derivation assumes an infinitely long foundation of width B placed at a depth of D below the ground surface of a homogeneous soil.

The average pressure exerted by the foundation at the instant of failure produces shear in a prism of Soil, I, immediately beneath th footing whose width is B and whose depth is B tan (45 + ϕ /2). The lateral bulging of this prism produces shear in a pair of similar prisms beside the first and an upward bulging of the ground above them. The second prism, II, sustains a vertical stress or minor principal stress of q' due to the wt. of soil above the footing level plus an average stress due to the weight of the prism of $\frac{1}{2}\gamma B \tan(45^{\circ}+\frac{\phi}{2})$. Therefore,

$$q' = \gamma D$$

$$\sigma_{3-II} = q' + \frac{1}{2}\gamma B \tan(45+\phi/2)$$

The horizontal stress (major principal stress) required to produce failure on prism II can be computed graphically by Mohr's circle as shown in fig.9. The major principal stress on prism II is equal to the minor principal stress on prism I. Similarly the major principal stress (vertical stress) on prism I can be found by Mohr's circle as in Fig.9. The ultimate bearing capacity is, therefore, equals to this stress minus the average stress caused by the weight of the soil in the prism.,

$$\frac{1}{2}\gamma B \tan(45+\phi/2) \quad \text{as in Fig. 9.}$$

If the Mohr envelope for the soil is a straight line then the bearing capacity may be computed analytically. The result is as follows :

$$q_d = \frac{\gamma B}{2}\left[\tan^5(45+\frac{\phi}{2}) - \tan(45+\frac{\phi}{2})\right] + c\left[2\tan(45+\frac{\phi}{2}) + 2\tan^3(45+\frac{\phi}{2})\right] + q'\tan^4(45+\frac{\phi}{2}).$$

This expression may be rewritten in a simple form as a bearing capacity equation :-

$$q_d = \frac{1}{2}\gamma B N_\gamma + c N_c + \gamma D N_q \qquad (3.5)$$

where N_γ, N_c and N_q are bearing capacity factors which are functions of the angle of internal friction of the soil.

For strip loading on purely cohesive soil, the ultimate bearing capacity according to Bell's theory of conjugate stresses,

$$q_d = 4c.$$

DISCUSSION:

The conjugate stress method of Bell does not agree with the results of the experiments. It does not make allowance for shear strength of clay above foundation level, and this fact makes the method unduly conservative. Thus in the case of footing failure at Kippen, Scottland(Ref.37) Bell's approach gives a value of 1,4000 psf where as the actual ultimate bearing capacity at the time of failure, as investigated by Skempton (1941) was about 2,500 psf.

Elastic failure of the loaded soil, as assumed by Terzaghi in his extension of Bell's method is possible only in clay soils with moisture close to plastic limit (Baidya, 1961). In fact the stress strain curves as obtained during laboratory compression test satisfy elastic failure only approximately.

3.3.3. TERZAGHI'S ANALYSIS :

Terzaghi has presented a solution for the ultimate bearing capacity of long footings which is of more general nature than any other theores. This method contains various assumptions which cannot be presented without going into great detail. Although this approach is not the most rigorous possible, all assumptions that are used are quite reasonable, and the results should be sufficiently accurate for most uses.

FAILURE BY LOCAL AND BY GENERAL SHEAR :

All soils are covered in Terzaghi's approach by two cases which are designated as general and local shear. Before the load on a footing is applied, the soil located beneath the level of the base of the footing is in a state of elastic equilibrium. When the load on the footing is increased beyond a certain critical value, the soil gradually passes into a State of plastic equilibrium. During this process of transition both the distribution of the soil reactions over the base of the footing and the orientation of the principal stresses in the soil beneath the footings change. The transition starts at the outer edges of the base and spreads as indicated in Figure 6 a for a continuous footing which rests on the horizontal surface of a homogeneous mass of sand and in Fig. 6b for a footing whose base is located at some depth beneath the surface. If the mechanical properties of the soil are such that the strain which procedes the failure of the soil by plastic flow is very small the footing does not sink into the ground until a slate of plastic equilibrium similar to that assumed by Prandtl and illustrated by Fig.7 has been reached. The corresponding relation between load and settlement is shown by the solid curve C_1 in Fig. 7. The failure occurs by sliding in the two outward directions. In Fig. 10C the time def represents one of these surfaces. This type of failure will be called a 'general shear failure'.

On the other hand, if the mechanical properties are such that the plastic flow is preceded by a very important strain, the approach to the general shear failure is associated with a rapidly increasing settlement and the relation between load and settlement is approximately as indicated in Fig. 11a by the dashed curved C_2. The criterion for the failure of the soil support, represented by a conspicuous increase of the slope of the settlement curve, is satisfied before the failure spreads to the surface. Hence, this type of failure will be called 'Local shear failure'.

CONDITIONS FOR GENERAL SHEAR FAILURE OF SOIL SUPPORT OF SHALLOW, CONTINUOUS FOOTINGS :

The term 'shallow footing' is applied to footings whose width B is equal to or greater than the vertical distance D between the surface of the ground and the base of the footing. If this condition is satisfied the shearing resistance of the soil located above the base of the footing may be neglected. In other words, we can replace the soil with a unit weight γ, located above this level, by a surcharge $q = \gamma D$ per unit of area. This substitution simplifies the computations very considerably On the other hand, if the depth D is considerably greater than the width B (deep footings), it is necessary to take the shearing strength of the soil located above the level of the base into consideration.

If the soil has thus been replaced by a surcharge, per unit area, the base of the footing represents a loaded strip with a uniform width B located on the horizontal surface of a semi-infinite mass. The state of plastic equilibrium produced by such a load is illustrated by Fig. 7. In order to produce such a state of stress at the base of a continuous footing it would be necessary to eliminate completely the friction and the adhesion between the base and the soil. Fig. 10a has been plotted on the basis of the same assumption. The soil located within the central zone I spreads laterally and the section through this zone undergoes the distortion indicated in the figure. If the load is transmitted on to the ground by means of a continuous footing with a rough base as shown in Fig. 9b, the tendency of the soil located within the zone I to spread is counteracted by the friction and adhesion.

COMPUTATION OF BEARING CAPACITY :

Fig. 10c is a section through a shallow continuous footing whose base is located at a depth D. At the instant of failure, the pressure on each of the surfaces ad and bd is equal to the resultant of the passive earth pressure P_p and the cohesion force C_a. Since slip occurs along these faces, the resultant earth pressure acts at an angle ϕ to the normal on each face and, as a consequence in a vertical direction. If the weight of the soil within adb is disregarded, the equilibrium of the footing requires that

$$Q_d = 2P_p + 2C_a \sin\phi = 2P_p + Bc\tan\phi \qquad (3.6)$$

The problem, therefore, is reduced to determining the passive earth pressure P_p. The passive earth pressure required to produce a slip on def can be divided into two parts, P_p' and P_p''. The force P_p' represents the resistance due to the weight of the mass adef. The point of application of P_p' is located at the lower third-point of ad. The second part P_p'' of the passive earth pressure can itself be resolved into two parts. One part P_c, is due to the surcharge $q = \gamma D$. Since both pressures P_c and P_q are uniformly distributed, their point of application is located at the mid-point of the contact face ad in Fig. 10c.

Hence, the value of the bearing capacity may be calculated by replacing P_p in equation 3.6 by $P_p' + P_c + P_q$, Thus,

$$Q_d = 2\left(P_p' + P_c + P_q + \frac{1}{2} Bc \tan\phi\right)$$

By introducing into this equation the symbols,

$$N_c = \frac{2P_c}{Dc} + \tan\phi$$

$$N_q = \frac{2P_q}{\gamma DB},$$

$$N_\gamma = \frac{4P_p'}{\gamma B^2}$$

We obtain

$$Q_d = B\left(cN_c + \gamma D N_q + \frac{1}{2}\gamma B N_\gamma\right) \qquad (3.7)$$

The quantities N_c , N_q , and N_γ are called the 'bearing capacity factors'. They are dimensionless quantities that depend on the value of ϕ . Therefore, they can be computed once for all and plotted in a chart. The solid curves in Fig. 11b represent the relation between the bearing acapacity factors and the values of ϕ .

CONDITIONS FOR LOCAL SHEAR FAILURE :

The stress conditions for the failure of a cohesive soil are approximately determined by the equation

$$\sigma_I = 2c \tan(45 + \frac{\phi}{2}) + \sigma_{III} \tan^2(45 + \frac{\phi}{2})$$

Wherein σ_I is the major principal stress and σ_{III} is the minor principal stress. Fig. 11a shows the relation between the stress difference $\sigma_I - \sigma_{III}$ and the corresponding linear strain in the direction of the major principal stress σ_1 for two different soils. If the stress strain relations are such as indicated by the dashed curve C_2, the lateral compression required to spread the state of plastic equilibrium as far as the outer edge f on the wedge aef (Fig. 10.c) is greater than the lateral compression produced by the sinking of the footing. Hence, in this case the soil support fails by local shear. In order to obtain information on the lower limit for the corresponding critical load Q_d, curve C_2 is replaced by a broken line Ocd. It represents the stress strain relation for an ideal plastic material whose shear values c' and ϕ' are smaller than the shear values c and ϕ for the material represented by the curve C_2. The available data on stress strain relations suggest that it is justified in assigning to c' and ϕ' the lower Limiting values

$$c' = \frac{2}{3} c$$

and

$$\tan \phi' = \frac{2}{3} \tan \phi$$

If the angle of shearing resistance is ϕ' instead of

ϕ, the bearing capacity factors assume values N_c', N_q' and N_r'. These values are given by the dash curves in Fig.10.b. The bearing capacity is then obtained from the equation

$$Q_d' = B\left(\frac{2}{3} c N_c' + \gamma D N_q' + \frac{1}{2}\gamma B N_\gamma'\right) \qquad (3.8)$$

Table 3.2, given below gives the values of various bearing capacity factors for varying values of ϕ.

Table 3.2 - Relation between ϕ and bearing capacity factors

ϕ	N_c	N_q	N_r	N_c'	N_q'	N_r'
0	5.7	1	0	5.7	1	0
10	9	3	1	5	2	0
20	17	7	5	12	4	2
25	25	13	10	15	5	3
30	35	22	20	18	8	6
35	53	43	40	24	14	10

If the stress strain relations for a soil are intermediate between the two extremes represented by the curves C_1 and C_2 in Fig. 11a, the critical load is intermediate between Q_d and Q'_d.

DISCUSSION.

Terzaghi's subdivision of the problem into two types of shear is an arbitrary one, since two cases cannot cover the wide range of conditions which necessiate the recognition of two expressions as different as equations 3.7 and 3.8. In practice the conditions for the general shear failure illustrated by Fig. 10c are never completely satisfied, because the horizontal compression of the soil located immediately below the level of the base of the footing on both sides of the base, is not great enough to produce the state of plastic equilibrium within the entire upper part of the zone aef. Therefore one has to expect a failure similar to that illustrated by Fig. 10.d. On account of inadequate lateral compression the shear failure occurs while the upper most part of the zones of potential plastic equilibrium is still in a state of elastic equilibrium. In cohesive soils the surface of sliding terminates at the boundary of the zone of elastic equilibrium. In the proximity of free surface of such soils one may find instead of a zone of shear a set of discontinuous tension cracks. In the theory of general shear failure these discrepencies between theory and reality are disregarded.

In connection with the method of determining the pressure P_p it should be ~~determined~~ remembered that the surface of sliding represents only an approximation to the real surface of sliding because the method is not rigorous. Therefore, the surface of sliding obtained by means of the spiral or the friction circle method does not necessarily start at point d in Fig. 10c with a vertical tangent. However, the error due to this discrepency between the real and the approximate surface of sliding is unimportant.

Experience has shown that even uniformly loaded foundations always fail by tilting. This fact, however, does not invalidate the reasoning of Terzaghi's analysis. It merely demonstrates that there are no perfectly uniform subgraded. With increasing load the settlement above the weakest part of the subgrade increases more rapidly than that above the rest. Because of the tilt, the center of gravity of the structure shifts towards the weak part and increases the pressure on that part, whereas the pressure on the stronger parts decreases. These factors almost exclude the possibility of a failure without tilting.

The two equations 3.7 and 3.8 are intended only as expressions which are approximate and conservative; they give estimates which are of much practical value but which must, in their application be tempered with considerable judgement.

3.3.4 MEYERHOF'S ANALYSIS :

A theory of bearing capacity has been developed by G.G. Meyerhof (Ref. 26, 1951) on the basis of plastic theory, by extending the previous analysis for surface footings to shallow and deep foundations in a uniform cohesive material with internal friction. As in Terzaghi's method, the theoretical results are represented by bearing capacity factors in terms of mechanical properties of the soil and the physical characteristics of the foundation.

For a deep footing, the Terzaghi's method suffers from the difficulty that when the failure surface no longer reaches the ground level, the height over which the shearing strength of the soil is mobilized becomes very uncertain and must be assumed. According to Meyerhof's theory, which has been extended to overcome such limitations, the zones of plastic equilibrium incres with foundation depth (Fig.12). For a given depth the size of these zones varies with the roughness of foundation and for a perfectly smooth footing, two symmetrical plane shear zones are formed below the base. The extent of these zones is largely governed by the shape of the foundation, and is a minimum for a circular footing.

At the ultimate bearing capacity the region above the composite failure surface is, in general, assumed to be divided into two main zones on each side of the central zone ABC, namely a radial shear zone BCD and a mixed shear zone BDEF in which the shear varies between the limits of radial and plane shear. To simplify the analysis, the resultant of the forces on the foundation shaft BF and the weight of the adjacent soil wedge BEF are replaced by the equivalent stresses p_o and S_o, normal and tangential respectively to the plane BE. This plane may be considered as an 'equivalent free surface', subjected to 'equivalent free stresses' p_o and S_o. The inclination of the surface increases with the foundation depth and together with the stresses, forms a parameter of that depth.

On this basis the bearing capacity can approximately be represented by

$$q_d = c\,N_c + p_o\,N_q + \frac{1}{2}\gamma B N_\gamma \tag{3.9}$$

This expression is of the same form as that given by Terzaghi, but N_c N_q and N_γ are now the general bearing capacity factors which depend on the depth and shape of the foundation as well as ϕ and roughness of the base. It will be convenient to express the resultant bearing capacity by the relation

$$q_d = c\,N_{cq} + \frac{1}{2}\gamma B N_{\gamma q} \tag{3.10}$$

where one term represents the influence of the cohesion and the other represents the influence of the weight of the material. The above expression gives only the base resistance of the foundation; to this must be added any skin friction along the shaft to obtain the total bearing capacity.

Meyerhof has obtained the factors on the basis of analytical and semi-graphical treatment, and to avoid determining them in every case, they have been calculated for the lower limit of zero shearing stress on the equivalent free surface (m = 0) and for the upper limit of full mobilization of the shearing strength (m = 1) within practical limits of β and ϕ. They are presented in the form of charts.

DISCUSSION :

The above theory is based on a number of simplifying assumptions, relating mainly to the deformation characteristics of the material and the method of installing the footing, the effect of which on bearing capacity can at present only be taken into account on the basis of empirical evidence. The need for checking against experimental data is particularly important for materials with internal friction, owing to the major influence of earth pressure coefficient on the shaft. Analysis of the main results of the laboratory and field loading tests on buried and driven foundations in clay has shown reasonable agreement with the theory in the case of shallow footings. For deep footings, the actual base resistance is less than estimated, on account of local shear failure and empirical compressibility factor is introduced in the theory by which the shearing strength is reduced.

3.3.5. SLIP SURFACE METHODS :

The slip surface methods are semi-graphical methods wherein a probable shape for the slip surface is assumed. The slip surface is assumed in different methods to be a circular arc (Fellenius), a circular arc with tangent (Krey) or a lograthmic spiral.

FELLENIUS' METHOD:

Also known as the circular arc method, it is the best known method, originally proposed by Fellenius (1929) for a strip load applied at the ground surface. While the method can be adapted to frictional soil, it is most suitable for cohesive soil ($\phi = 0$).

A cylindrical slip surface is chosen with centre at O (Fig.13 a) and the total cohesion C along the surface is calculated. By equating the moment of the applied load Q_d about O to the moments of W and C the value of Q_d is determined. The process is repeated for several other trial surfaces and the ultimate load is taken as the minimum value of Q_d.

This method has been extended to footings founded below ground surface. G. Wilson (1941) found that the net value of Q_d by this method has an almost exactly linear variation with the depth breadth ratio upto depths of 1.5 times the breadth. The expression furnished by Wilson's results, for long footings below the surface of highly cohesive soils is

$$q_d' = 5.5\,c\left(1 + 0.38\,\frac{D}{B}\right) \qquad (3.11)$$

where q'_d denotes the ultimate net bearing capacity at depth D. The process of trial surfaces has been shortened by calculatir from the geometry of the problem the coordinates of the centre of critical slip surface. Thus in Fig. 13a,

$$B q_d \left(x - \frac{B}{2}\right) = c R (\theta + \alpha) R$$
$$= c R^2 \left(\tan^{-1}\frac{x}{y} + \cos^{-1}\frac{y - D}{\sqrt{(x^2 + y^2)}}\right).$$

From this equation the centre of circle for various ratios of D/B can be found. The results are plotted in Fig.13b.

DISCUSSION :

The criteria by which any method for the determination of bearing capacity of a purely cohesive material should be judged are two : that reasonable agreement with Prandtl's solution should exist for the case of surface loading and that the bearing capacity should increase with the depth of the footing. For surface loading the circular arc method gives an ultimate bearing capacity of 5.5.c which differs from that obtained by Prandtl by only 7.4 per cent and the critical circle agrees closely with the Prandtl failure surface for this case. The rate of increase in bearing capacity with depth is in good agreement with that obtained by Terzaghi.

In addition to its simplicity, the method is particularly useful when the properties of the soil vary within the zone of general shear failure, in which case Wilson's coordinates should be used for the first trial centre, and several other trial circles drawn with centres near the first.

KREY'S METHOD :

The most widely known graphical method is that due to Krey which assumes that the surface of failure consists of a cylindrical surface passing through one edge of bearing strata and a tangent plane making an angle of $(45 - \frac{\phi}{2})$ with the horizontal, as illustrated in Fig. 14. The Krey method may be applied to either of two different cases. In the first, a foundation carrying a specified load is analysed to determine the factor of safety which applies. In the second case, a foundation is analyzed to determine the load which it can safely support with a specified factor of safety.

In the first case, a trial surface of failure is established by arbitrarily choosing a centre A of the circular arc BC, as shown in Fig. 14. The surface of failure is completed by drawing the line CD that is tangent to the circle and makes an angle $(45^\circ - \frac{\phi}{2})$ with the ground surface. It is considered that the foundation load plus the weight of the soil prism BCEF tends to cause rotation along the arc BC. This tendency produces an active horizontal thrust at the plane CE. This thrust is resisted by the passive resistance pressure of the triangle CDE; and the ratio of the ultimate passive resistance at the limit of equilibrium to the active thrust is the factor of safety against failure of the foundation soil.

In the second case utilizing the Krey method, a trial centre point A is chosen, the failure surface is drawn, and the value of the resultant passive resistance pressure is determined. The quotient obtained by dividing this by the factor of safety represents the value of the active horizontal thrust, which must not be exceeded. Thus the problem is simplified to find the load on the foundation which will produce this value of active thrust. As in the case of retaining wall pressures, it is rather common practice to neglect the cohesion of the soil when making computations for safe bearing capacity. However, the Krey method can be extended to include cohesion, if desired.

DISCUSSION:

Wilson has shown for footings at depths greater than half the width, bearing capacity according to Krey's method is less than at the surface. At the surface $q_d = 6.047c$, at $D = 0.5B$, $q_d = 6.20\ c$ and thereafter it decreases. Such a result is contrary to reason. This is probably due to Krey's assumption that the axes of the cylinder of failure lies in the plane of base. This is avoided in modified Krey's method, often adopted in U.S. This modification gives $q_d = 5.41\ c$ at the surface, a figure closer to Prandtl's value.

OTHER METHODS :

If it assumed that the rotational axis of the cylindrical failure surface coincides with the edge 0 (Fig. 15.1), limit equilibrium will require the following approximate relationship of moments in respect to 0 :

$$q_d \frac{B^2}{2} = c\left(\pi B^2 + DB\right) + \gamma D \frac{B^2}{2}$$

$$q_d = c\left(2\pi + \frac{2D}{B}\right) + \gamma D$$

$$= 6.28\, c\left(1 + 0.32\frac{D}{B} + 0.16\frac{\gamma}{c} D\right)$$

When the foundation rests on the ground surface (D = 0),

$$q_d = 6.28\ C$$

This value is somewhat too high, as compared to the value of Fellenius.

Another method consists in assuming the centre of rotation O at the edge of the footing, as shown in figure 15.2. The rotational equilibrium of the sector OAB is assumed to be provided in part by the passive lateral resistance of the adjoining day. For $\phi = 0$ the passive resistance will equal $2C + \gamma D$. Equilibrium will then require

$$q_d \frac{B^2}{2} = C \frac{\pi B}{2} B + 2C \frac{B^2}{2} + \gamma D \frac{B^2}{2}$$

$$q_d - \gamma D = C(\pi + 2) = 5.14\ C$$

for surface loading (D = 0) this result is identical with that of the original Prandtl solution.

Figure 15.3 illustrates the assumption of plane surfaces of failure, on which, some early analyses by Terzaghi were based. Equilibrium along the plane OB will require

$$q_d - q_u = q_u + \gamma D$$

or

$$q_d - \gamma D = 2 q_u = 4.0\ C$$

It will be noted that in the preceding analyses of rotation stability the weight of the rotating cylindrical soil sectors could be neglected, since it is in approximate equilibrium in respect to the centre of rotation. Shearing stresses along a rough base AO, Fig.

Figure 15, cannot affect the stability in respect to O.

All the preceding equations are valid for very long footings The resistance to rotation of somewhat shorter fundations will be increased by the shearing strength of the soil on vertical planes beneath the two ends of the footing strip. The increased bearing capacity can then be roughly estimated, as illustrated by Fig. 16.

BEARING CAPACITY OF SHORT FOOTINGS :

When the shearing resistance on the cylindrical surface ADBGEF (Fig. 16) of radius B reaches its maximum value of C, the conservative assumption can be made that at some smaller distance ρ from the axis of rotation OO the unit shearing resistance C on a vertical plane through the short ends AB or EF on the rectangular footing will be reduced in direct proportion to the distance from O, so that

$$c_\rho = c\,\frac{\rho}{B}$$

The rotational sresistance dR around O of a ring $d\rho$ thick, located at a distance ρ from O, will then be

$$dR = c\,\frac{\rho}{B}\,\pi\rho^2\,d\rho$$

and of the entire sector ADF

$$R = \frac{\pi c}{B}\int_{\rho=0}^{\rho=B}\rho^3\,d\rho = \frac{\pi c}{B}\left[\frac{\rho^4}{4}\right]_{\rho=0}^{\rho=B} = 0.25\,\pi c B^3 \qquad (3.12)$$

By dividing Eq. 3.12 by the resistance $\pi cb^2 L$ offered by the cylindrical surface ADBGEF to rotation around the axis OO_1 we obtain 0.25B/L as an expression for the increment of rotational resistance offered by one end sector ADB, if the resistance of the cylindrical surface is taken to be equal unity for both end surfaces this value becomes 0.50 B/L. The three dimensional failure surface may be somewhat smaller than the one assumed on the basis of the preceding simplified two dimensional analysis. Hence the value 0.50 B/L may be reduced at least to read 0.44 B/L By adding this value to Eq. 3.11 we obtain the following general expression for the approximate value of bearing capacity on clays :

$$q_d = 5.52\, c \left(1 + 0.38 \frac{D}{B} + 0.44 \frac{B}{L}\right) \quad (3.13)$$

For a square footing (B = L) on the surface of the ground,

$$qd = 7.95\ C$$

Tezaghi and Peak (Ref. A-6) suggest for a square footing $qd = 7.4C$.

3.3.6. HOUSEL'S THEORY :

W.S. Housel has suggested a practical method of determining bearing capacity by means of load bearing tests. This is particularly applicable in cases where the soil is reasonably homogeneous in depth. In this method, the footing load is assumed to be transmitted to the soil as the sum of two components. One is that which is carried by the soil column directly beneath the footing; and the other is that which is carried by the soil around the perimeter of the foundation. The first of these components is a function of the area, and the second is a function of the perimeter of the foundation. This Perimeter Shear concept is expressed by the formula

$$Q = nA + mP \qquad (3.14)$$

wherein A is the area and P is the perimeter of the footing; n designates the unit compressive strength of the pressure bulb, and m the unit perimeter shear. The unit perimeter shear may be defined as the load carrying ability per foot of perimiter, furnished on the vertical cylindrical surface which passes through the perimeter of the footing by the shearing resistance developed when the footing and the soil below it settle relative to the soil outside.

METHOD :

The unit values m and n may be determined by loading two or more test plates or footings which have different areas and different perimeter lengths. These test plates should be placed on the soil at the same elevation as that of the proposed foundation, and should be loaded until the maximum allowable settlement is developed. The load on each test plate which is required to produce this settlement is recorded. Then appropriate values of Q, A, and P are substituted in eq. 3.14 for each test plate. This gives two or more simultaneous equations from which m and n may be determined. With these values, the allowable load on the actual foundation may be computed.

DISCUSSION :

The unit perimeter shear is of small magnitude in sands. In highly cohesive soils the second term of Eq. 3.14 predominates, indicating that most of the load is in this case carried by perimeter shear. It may be claimed that concepts relative to the pressure bulb and concepts concerning the bearing capacity criteria, with the detailed considerations of stress and strain, offer a more complete understanding of the problem than can be obtained from the perimeter shear concept. However, the two approaches are similar in principle, their differences being mainly in terminology.

3.3.7. OTHER THEORIES :

There are various other theories and methods to estimate the bearing capacity. Some of them are extensions of the theories already discussed and some are of classical interest, open to a number of serious objections. They will be presented here in brief.

RANKINE'S METHOD :

In Rankine's well-known theory for the minimum depth of foundations in cohesionless soil the vertical downward pressure of the footing is considered as a maximum principal stress, and the lateral or minimum principal stress is the corresponding active earth pressure. This lateral stress is, for particles just beyond the edge of the footing, considered as a maximum principal stress, which in turn brings into play a vertical minimum principal stress. Rankine's evaluation of the principal stress causing shear failure in a cohesionless soil is illustrated in Fig. 17. The bearing pressure q_d produces a lateral pressure p at the base of the footing, and, according to Rankine's relationship between conjugate stresses,

$$p = q_d \, \frac{1 - \sin\phi}{1 + \sin\phi}$$

At a point clear of the footing the lateral pressure p produces a conjugate (vertical) stress which, for equilibrium, cannot exceed the weight of the superincumbent soil. If, as is usually assumed in this solution, the point concerned is on the same level as the base of the footing,

$$\gamma D = p \, \frac{1 - \sin\phi}{1 + \sin\phi}$$

Thus

$$q_d = \gamma D \left[\frac{1 + \sin\phi}{1 - \sin\phi}\right]^2 = \gamma D \tan^4 (45^\circ + \phi/2)$$

or, using the coefficient of passive pressure K_p,

$$q_d = \gamma D \, K_p^2$$

The same result may be obtained by drawing Mohr's circles for the two elements shown in Fig. 17.

DISCUSSION :

This method is of classical interest and it always gives results lower than those found from tests. An abrupt change in stress conditions is implied below the edge of the footing and this is contrary to the facts. The bearing pressures thus calculated are independent of the size and shape of the footing, a result which again, for frictional soils, conflicts with actual conditions.

HENCKY'S METHOD :

H. Hencky working on the lines of Prandtl, has solved the problem for a rigid circular footing and in this case, he finds that the failure occurs when

$$q_d = 5.64\,S$$

Unfortunately, the Hencky's method has not been extended to footings below the surface, and approximations must therefore be made. As a rough estimate, Skempton (Ref. 37) suggests that bearing capacity is increased by full fractional resistance which can be developed along the sides of the footing. This is not an upper limit but it is probably the maximum increase which would be allowed in design. Hencky's equation is hence modified to the form :

$$q_d = 5.64\,S + \frac{F}{A}\,S'$$

where F is the area of the side of footing in contact with clay of skin friction $S' = 0.75\,S$ and A is the area of base of footing.

Though original Hencky formula is conservative, when modified to allow for friction on the sides of footing, it is of adequate practical reliability. Thus in the case of Kippen footing failure (Ref. 37), Hencky's method gives a value of 2,000 psf, the modified value is 2,600, the actual value of ultimate bearing capacity being 2,500 psf.

RITTER'S METHOD :

Ritter assumes the soil to have no cohesion and the formula he presented is

$$q_d = \left[\gamma D + \gamma \frac{B}{4} \tan\left(45^\circ + \phi/2\right)\right] \cdot \left[\tan^4\left(45^\circ + \phi/2\right) - 1 + \gamma D\right]$$

It is obvious that the above formula cannot be used for cohesive soils. However, for cohesive soils it was later amended to include the term

$$\frac{2c}{\tan\left(45^\circ - \phi/2\right) \sin^2\left(45^\circ - \phi/2\right)}$$

3.4. EFFECTS OF SOIL PROPERTIES AND FOOTING DIMENSIONS ON BEARING CAPACITY :

8 As can be seen by an examination of the general bearing capacity equation (3.7), the bearing capacity depends on the properties of the soil and the dimensions of the footing :

1. The angle of internal friction

2. The unit weight of soil

3. The cohesion

4. The footing width

5. The roughness of base of the footing

6. The surcharge.

The angle of internal friction has by far the greatest influence on all the three bearing capacity factors. All increase at a rapidly increasing rate when ϕ becomes larger. However, if ϕ becomes very small, as in the case of saturated clays, the last term approaches zero, values of N_γ for friction angles of 32^o and 40^o are nearly 8 and 100. Thus the ultimate bearing capacity of dense cohesionless soil is represented as being roughly ten times that of loose cohesionless soil.

Both the second and the last term of Eq. 3.7. vary in direct proportion to the unit weight of the soil. When the footing is above water table, a distance of 1.5B, the full unit weight is used in the computation. When the water table is at the level of the base of the footing, the submerged unit weight is used in the last term. The effect is to reduce that term to about half its previous value.

If the water table is above the base of the footing, the surcharge weight is similarly affected. In a cohesionless soil where the cohesion term is zero, a water table rising to the ground surface can, therefore, have the disastrous effect of cutting the soil bearing capacity to approximately half.

Eastwood (9) concludes from his experiments with narrow footings on sand that the ultimate bearing capacity of dry sand is reduced by less than 20 percent if the sand is submerged. This discrepency from the usually assumed value of 50 percent is, according to him, because of the wrongly assumed mechanics of failure in the work of Prandtl, Terzaghi and Krey.

The cohesion influences only the first term. While the angle of internal friction is zero, as in the case of saturated clays, the cohesion term becomes the major part of the bearing capacity. If a soil has both c and ϕ, the bearing capacity is likely to be very high because N_c increases rapidly with ϕ.

To give some idea of the magnitude of the changes in bearing capacity which may be brought about by changes in the shearing strength of a soil with variations in the values of internal friction and cohesion, the following figures of Table 3.3 have been calculated based on equation 3.7.

TABLE 3.3 EFFECT OF VARIATION OF C AND ϕ ON THE BEARING CAPACITY

Ultimate bearing capacity in T/Sft.

ϕ (Degrees)	2' Square footing Cohesion			2' Wide Strip footing Cohesion		
	200 psf	500 psf	1000 psf	200 psf	500 psf	1000 psf
0	0.8	1.75	3.4	0.6	1.4	2.65
5	0.95	2.20	4.25	0.8	1.75	3.5
10	1.45	3.1	5.85	1.2	2.45	4.6

Note : The base of the footing is presumed 2' below ground level and unit weight of soil is taken as 125 lbs/cft.

The last term of the bearing capacity equation varies in direct proportion to the footing width. Therefore a wide footing on a soil with a high angle of internal friction, such as gravel or sand, will have a very high bearing capacity, while a narrow footing on the same soil will have a much lower value. This Variation of Bearing capacity with footing width for sands and gravel gives rise to many apparent. incongruities. For example, the narrow foundations of a small light structure on sand may fail while those of anadjacent heavy structure are safe, although both are designed for the same bearing pressure. Footing width has no influence on the bearing capacity of soils with no internal friction. Therefore, footings of different sizes will be equally safe at the same bearing pressure.

Whereas the procedure for simple soil and foundation conditions is fairly well established and sufficiently reliable estimates can be made in many cases in practice, the methods of analysis for special conditions are still controversial. Thus according to Terzaghi (Ref. 43, 1943) the bearing capacity of a rough based strip footing on clay is somewhat greater than that of a smooth base. According to Meyerhof's analysis (Ref. 25, 1951) , however, the bearing capacity of a perfectly smooth footing on c-less material is one-half that for a perfectly rough base, and the ultimate load of a strip footing on purely cohesive soil is not affected by roughness of base. The bearing capacity of a weightless material is independent of base friction and that of a material with weight increases with the roughness of the base.

The surcharge influences only the second term. Its contribution to bearing capacity may be negligible for soils with a small ϕ. For soils with high friction angles, a small amount of surcharge produces a large increase in bearing capacity. For example, increasing the depth of a footing by one foot in a sand weighing 110 pcf and having ϕ = 35 degrees, will increase the ultimate bearing capacity by ($N_q \times r \times 1$) = (42 x 110 x 1) = 4600 psf. Table 3.4 given below indicates the effect of increasing the depth of the footing on bearing capacity. These figures presume that, owing to cracking near the surface of the soil, cohesion is ineffective for a depth of 2 feet.

Table 3.4. EFFECT OF INCREASE IN DEPTH ON BEARING CAPACITY

ϕ (Degrees)	Ultimate bearing capacity in T/sf with C = 100 psf.					
	2' square footing			2' wide strip footing		
	Depth			Depth		
	0'	2'	4'	0'	2'	4'
0	3.3	3.4	3.8	2.55	2.65	3.3
5	4.15	4.25	4.75	3.2	3.3	4.1
10	5.55	5.85	7.00	4.25	4.6	6.2
15	7.4	7.95	9.75	5.7	6.25	8.75
20	10.2	11.00	13.6	7.9	8.75	12.35

3.5 RECTANGULAR, SQUARE AND CIRCULAR FOOTINGS.

The various methods of bearing capacity analysis are based on the assumption of an infinitely long footings of width B, which simplifies the actual problem to two dimensions. When the length of the footing is of the same order of magnitude as the width, the failure involves three dimensional shear. No general method of analysis for rectangular, square and circular footings has been developed which fully considers this shear condition.

On the basis of experiments the following semiempirical equation has been derived by Terzaghi (Ref. 43) for the bearing capacity Q_{dr} of a circular footing with a radius r resting on a faily dense or stiff soil.

$$q_{dr} = \pi r^2 \; (1.3 \; c \; N_c + \gamma D_f \; N_q + 0.6 \; \gamma \; r \; N_\gamma) \quad (3.15)$$

or $$q_{dr} = 1.3 \; c \; N_c + \gamma \; D_f \; N_q + 0.6 \; \gamma \; r \; N_\gamma$$

The corresponding value for square footings, B x B, on dense or stiff soil is

$$q_{ds} = 1.3 \; c \; N_c + \gamma \; D_f \; N_q + 0.4 \; \gamma \; B \; N_\gamma \quad (3.16)$$

The values of N are given by the ordinates of the solid curves in Fig. 11 b. If c is greater than zero, $\phi = 0$, and D = 0 we obtain for the bearing capacity the value,

$$q_{dr} = q_{ds} = 7.4 \; c = 3.7 \; q_u$$

which is considerably greater than the value $q_d = 5.70$ c, for a continuous footing. On the other hand, if c = 0 and D = 0, the bearing capacity q_{dr} per unit of area is considerably smaller than q_d for a continuous footing with a width equal to the diameter of the circular footing.

If the supporting soil is fairly loose or soft, the values of N must be replaced by the values N', determined from the dashed curves in Fig. 11 b, and the value of c must be replaced by c'.

The ultimate bearing capacity of a rectangular or oblong footing with width B and length L, is according to Terzaghi and Peck, (Ref. op cite) roughly equal to

$$q_{do} = 2.85 \; q_u \; (1 + 0.3 \frac{B}{L})) \quad (3.17)$$

For a square footing, L = B and for a strip footing $L \rightarrow \infty$; hence the bearing capacity of a square footing is 30 pc greater than that of a strip footing.

If It is assumed that ϕ is zero, then N_q is unity, N_r is zero and N_c is 5.7. Under this condition, the ultimate bearing capacity of a footing on clay reduces to

$$q_d = c N_c + \gamma d \qquad (3.18)$$

Skempton (Ref. 38), after study of experimental data both from laboratory tests and from full-scale observation, forms the general conclusion that for cohesive soil the coefficient N_c increases with depth upto maximum of about 7.5 for depths exceeding $2\frac{1}{2}$ times the width of the footing, and his suggested values are plotted in Fig.18. It will be noted that the curve for strip footings starts at Prandtl's value of 5.14 c for surface loading. He suggests that the rules given in Table 3.5 can be easily remembered and employed in the absence of graphical data, by substituting for N_c in eq. 3.18.

Table 3.5 Valuesof N_c for various footing depths

Depth D	N_c
D = 0	N_{co} = 5 for a continuous footing
	= 6 for a square or cirular footing.
D/B $<$ 2.5	$(1 + 0.2\ D/B)\ N_{co}$
D/B $>$ 2.5	$1.5\ N_{co}$
Any value of D	$(1 + 0.2\ B/L)\ N_c$ (strip)

If the soil support of a continuous footing yields, all the soil particles move parallel to a plane which is perpendicular to the center line of the footing. Therefore, the problem of computing the bearing capacity of such footings is a problem of plane deformation. On the hand if the soil support of a square or circular footing yields, the soil particles move in radial and not in parallel planes. Hence mathematical difficulties involved allow no rigorous solution and until the results of successful or of adequate experimental investigations are available, we are obliged to estimate the bearing capacity from the above mentioned formulae, based on limited experience. From the somewhat conflicting results of these limited data, it is possible to determine empirical corrections for the factors N_r and N_c in the general bearing capacity equation 3.7. These are given in the following table (Ref.40), and are to be multiplied by the Bearing Capacity Factors of Terzaghi.

Table 3.6 <u>CORRECTIONS FOR BEARING CAPACITY FACTORS, RECTANGULAR AND CIRCULAR FOOTINGS AT SHALLOW DEPTHS</u>

Shape of Footing	Correction for N_c	Correction for N_r		
		$\phi = 45^\circ$	$\phi = 40^\circ$	$\phi = 30^\circ$ or less
Square, L/B = 1	1.25	0.80	0.85	0.90
Rectangular L/B = 1.5	1.17			
L/B = 2	1.12	0.85	0.90	0.95
L/B = 3	1.08			
L/B = 5	1.05	0.90	0.95	0.98
L/B = 10	1.02	1.00	1.00	1.00
Circular	1.20	0.70	0.80	0.90.

3.6 ECCENTRIC AND INCLINED LOADS :

Footings are frequently subjected to eccentric and inclined loads due to bending moments and horizontal thrusts acting in conjunction with the vertical loading. These conditions have been analyzed by Meyerhof (Ref. 27, 1953), as an extension to his bearing capacity theory under central vertical load.

Bearing Capacity of a Footing with Eccentric Load :

When a footing carries an eccentric load, it tilts towards the side of eccentricity, and the contact pressure below the base is generally taken to decrease linearly towards the heel from a maximum at the toe. At the ultimate bearing capacity, the distribution of contact pressure is not even approximately linear and a very simple solution of the problem is obtained by assuming that the contact pressure distribution is identical to that of a centrally loaded footing, but of reduced width. Thus the edge of the footing farthest from the point of load application no longer contributes to the bearing capacity. In other words, the real width of the footing B is reduced to an equivalent width B', the amount of reduction is equal to 2 e and

$$B' = B - 2e$$

e being the eccentricity of the load. This reduced width must be used in eqn. 3.10.

Bearing Capacity of Footing with Inclined Load :

Under the central load inclined at an angle α to the vertical, the central shear zone shown in Fig. 12 is tilted and the adjacent zones are modified accordingly. Two main cases may be considered, namely, footings with a horizontal base and footings with base normal to the load. The bearing capacity factors depend on ϕ , D/B and α . Reduction factors have been derived by Meyerhhf for various inclinations. The vertical component of bearing capacity can be found by multiplying the appropriate factor by the reduction factor. The horizontal component may be found by multiplying the vertical lines the tangent of the angle of inclination. It is of interest to note that for a given angle, an inclined footing has a greater bearing capacity than a horizontal base, which supports the practice of designing shallow foundations with a base normal to resultant load, if possible.

When a footing carries an eccentric, inclined load, the bearing capacity can be estimated by combining the above methods of analysis. Results of laboratory tests conducted by Meyerohf (Op.cit) indicate that on clay, the average bearing capacity decreases linearly, with increase in eccentricity, whereas on sand, bearing capacity of the footings decreases approximately parabolically, with increase in eccentricity. G.S. Dhillon (7.1) concludes that the decrease in the bearing capacity of an eccentrically loaded footing is small with the eccentricity in the longitudinal axis, compared to an equivalent eccentricity in the shorter axis. He has found the theory of Meyerhof to err on the unsafe side.

3.7 FOOTINGS ON SLOPES :

Meyerohf's theory of bearing capacity has been extended and combined with the theory of stability of slopes to cover the stability of footings on slopes. The footing may be located either on the face or on top of a slope. In the former case, when the footing is loaded to failure, the zones of plastic flow in the soil on the side of the slope are smaller than those of similar footing on level ground and ultimate bearing capacity is correspondingly reduced.

The resultant bearing factors N_{cq} and N_{rq} have been correlated with the inclination of the slope β (Ref. 29). The factors decrease with greater inclination of the slope to a minimum for $\beta = 90^o$ on purely cohesive material and $\beta = \phi$ on c -less soil, when the slope becomes unstable. For inclinations of slopes used in practice ($\beta < 90^o$) the decrease in bearing capacity is small in the case of clays but can be considerable for sands and gravels because the bearing capacity of cohesionless soils is found to decrease approximately parabolically with the increase in slope angle.

In cohesive material with a small or no angle of shearing resistance the bearing capacity may be limited by the stability of the whole slope with a slip surface intersecting the base or toe of the slope. For slopes in practice in purely cohesive soil of great depth, base failure of an unloaded slope occurs along a critical mid-point circle so that footings below the mid-point section increase the overall stability of the slope and vice versa.

For footings located on the top of the slope, beyond a distance of about 2 to 6 times the footing width, bearing capacity is independent of inclination of slope and is same as that of a footing on an extensive horizontal ground surface. For a given height and inclination, bearing capacity factor N_{rq} increases with greater footing distance from the edge of slope and beyond a distance of 2 to 4 times the height, bearing capacity is independent of slope angle.

Except for the observations of Peyniricioglu (Ref.33), no published information in practice is available for checking the above said conclusions. The theoretical mechanism of failure, assumed by Meyerohf, is supported by these observations of soil movements below model footings.

3.8 STRATIFIED SOILS :

All of the theoretical analysis are based on the assumption that the soil is homogeneous throughout the zone of soil shear. When the soil is non-homogeneous, these methods are not strictly applicable. The effect of a non-homogeneous soil is to distort the shear pattern. The area of that portion of the rupture surface in the weaker material will tend to increase while that in the stronger material will decrease.

In frictionless soils, the method of Fellenius (Art. 3.3.5) may be employed. Solutions for the case of a footing on the surface of a two layered saturated clay have been developed by J.S. Button (Ref. 3, 1953) from the Fellenius method and the results expressed graphically, Fig. 19. The analysis shows that bearing capacity factor is changed depending on the ratio of the strength of the lower to the upper layer, c_2/c_1, and the ratio of the layer thickness d to the footing width. When the upper layer is harder than the lower, the bearing capacity increases with the thickness of the upper layer; when the upper layer is softer, the bearing capacity decreases as its thickness increases. When the upper layer is much softer than the lower and is thick, the shear surface becomes tangent to the hard layer. The strength of the hard layer does not influence the bearing capacity other than to fix the shear surface. This can be seen by the horizontal lines of unchanged N_c on the right side of the figure.

For soils having internal friction, and for more complex conditions of non-homogeneity, similar solutions are calculated. As an approximation where the soil strengths do not vary more than 50 percent throughout a depth below the footing equal to 1.5B, a weighted average of the soil properties is computed. This may be used in the bearing capacity analysis based on homogeneous soils without serious error.

3.9 BURIED STRATA :

A subsoil condition which required careful consideration is that of a buried stratum with a bearing capacity which is much less than that of the deposit above it. When footings are founded at or near the surface of a good stratum which overlies a poor stratum, the pressures applied at the footing level spread out with increasing depth, and thus the induced pressures reaching the poorer stratum are of considerably small magnitude . The most unfavourable stresses in the buried stratum are at its surface, and whether the danger is from excessive compression or possibly from lateral flow of the clay, the problem is conservatively handled if the stress at the surface of the buried stratum is limited to the bearing capacity which would be reasonable on this soil on the ground surface.

A number of approximate methods are available for obtaining stresses at the surfaces of buried deposits. Formulae from the theory of elasticity might be used, although question regarding their validity im soils makes them no more dependable than simpler approaches. The distribution curves obtained from the Westergaard and Boussinesq elastic solutions, are illustrated in Fig.20a.

A simpler method is to assume that the stress spreads with depth to a larger area, defined by lines through the edges of the surface area at angle α to the vertical and that on this larger area the stress is uniformly distributed as shown in Fig. 19b. The uniform stress is, of course, not the true picture but as a measure of the degree to which the surface of the buried stratum is stressed, this simple approach is may be satisfactory. The expression for the case shown in (b) for square or round footings, is

$$\frac{q}{q_0} = \left[\frac{\frac{B}{D}}{\frac{B}{D} + 2\tan\alpha}\right]^2$$

and for a long footing the relationship is

$$q/q_o = \frac{B}{D} \Big/ \left(\frac{B}{D} + 2 \tan \alpha \right)$$

where q and q_o are respectively, the stresses at the surfaces of the buried stratum and α is the spread angle. The spread angle is commonly assumed to be equal to 30 degrees or more. In the Boston Code, it is taken as 30 degrees.

Another simple approach, advanced by Kogler (Ref. 24, 1929), is shown in Fig. 19c. The stress on the surface of the buried stratum is assumed to be uniform below the loaded surface area, and outside it is assumed to vary linearly to zero at a distance defined by spread angle β . The equation for Kogler's method for square or round footings is

$$\frac{q_k}{q_o} = \frac{\left(\frac{B}{D}\right)^2}{\left(\frac{B}{D}\right)^2 + 2\left(\frac{B}{D}\right)\tan\beta + \frac{4}{3}\tan^2\beta}$$

and for a long footing the expression is

$$\frac{q_k}{q_o} = \frac{B/D}{\frac{B}{D} + \tan\beta}$$

wherein q_k is the stress on the central portion of the buried stratum and the spread angle β recommended by Kogler is 55 degrees.

For square and round footings, the stress on the buried stratum is about one fifth that at the surface when the breadth-depth ratio is 1. Boussinesq formula gives relatively large values of q/q_o but the other three approaches are in reasonable agreement with each other. Use of anyone of these three methods is probably conservative and sufficiently accurate for the rough indications usually desired from such a method.

3.10 PARTIAL BEARING CAPACITY FAILURE.

When the actual footing pressure is very close to the ultimate bearing capacity, a partial bearing capacity failure may result depending on the soil properties and the footing structure, two different modes of failure can occur :

1. Initial rapid movement which eventually stops.
2. Slow, continued movement; constant, slowly increasing, or slowly decreasing in rate.

The first takes place when a footing moves downward sufficiently during failure so that it finds increased bearing capacity at its new level. When a soil has high angle of internal friction, greater depth means increased bearing capacity. This mode of failure is likely to occur with very shallow, narrow footings on cohesionless sand. The low initial bearing capacity, caused by both lack of surcharge and small width, is increased materially by as little as to inches of movement of the footing into the ground. The same type of partial failure may occur when the footing rests on a thin layer of very weak soil which in turn rests on much stronger soil. Failure of the weak soil allows the foundation to move downward and come to rest on the stronger soil which is capable of supporting the load safely. This condition often occurs when rainfall or ground water is allowed to soften the soil in the bottom of the footing excavation or when loose soil is not removed from the excavation before the footing concrete is poured.

The second mode of partial failure is a progressive shearing. It may occur in sensitive soils where partial failure produces more failu and an increasing rate of movement. It may occur in clays that tend to creep or distort plastically at a low constant rate. It also occurs in very loose soils which shear slowly and in doing so increase in density and become stronger. In this case the rate of movement becomes less as it goes on. This last condition may develop in poorly compacted soils.

Movements due to partial bearing capacity failure are sometimes confused with settlement. However, if an adequate factor of safety is employed in design, partial failure will not occur.

3.11 RESEARCH ON BEARING CAPACITY.

Research on bearing capacity of footings has been reported in recent years by Golder (Ref.11), Peynircioglu (Ref. 33), Meyerhof, (Ref. 26 to 30), Skempton (Ref. 37, 38) and Eastwood (Ref.9). Study of this work reveals that, given ideal loading conditions, a symmetrical indentation (Prandtl, Terzaghi) is obtainable in the laboratory, although asymmetrical failure is more common. Nevertheless, none of the experimenters interpreted their results as slip surface failures. Both theory and experiment show that shearing failure in the soil can be expected to develop first in zones near the edges of a foundation and this state of affairs does not constitute a stress condition in the slip surface methods. The kinematics of the problem as one of rotation require a perfectly cylindrical rupture surface and this rarely, if ever appears to be attained. It seems that the actual mode of failure is satisfied better by the assumption of rupture zones than by the assumption of a rupture surface although the true solution is likely to be composite, with rupture zones adjacent to the footing and a rupture surface breaking ground surface. Apart from these considerations, a great advantage of the surface methods is that they are well adapted to graphical solutions where the shearing strength of the soil varies. On the other hand, the indentation analyses are more universal and more readily applied. In practice asymmetrical failure can be anticipated because of intentional and unintentional eccentricity of loading, and non-uniformity of soil and constructional materials, although beams and other structural units commonly afford sufficient restraint to prevent appreciable lateral drift of a footing. The application of the indentation analysis to practical problems is not invalidated by slight lateral drift although it must obviously affect bearing capacity to some extent.

3.12. SUMMARY AND CONCLUSIONS :

In Table 3.7 are reported the results of study of bearing capacity of soils at Rudrapur, Uttar Pradesh (Ref. 6). The bearing capacity at different sites has been calculated using various formulae, discussed in the above articles. From the Table it is observed that the values of bearing capacity as calculated by Terzaghi, Bell-Terzaghi and amended Ritter's formulae are quite comparable with each other. The values obtained by Prandtl's formula are generally on the higher

S.No.	Type o
1.	Silty Clay
2.	Clay
3.	Sandy Loam
4.	Loam
5.	Clay
6.	Sand
7.	Loam
8.	Sandy Loam

REFERENCES :

1. Prandtl : t

2. Prandtl-Taylor

3. Bell-Terzaghi :

side. Further, the values of Tschebotarioff and Hencky are on the lower side. Thus it may be inferred that a very judicious selection of the formula is necessary for calculation as the results differ greatly in some formulae.

Reviewing the various theories, approaches and methods of estimating the ultimate bearing capacity of footings, the following conclusions emerge :

METHODS :

(i) The conjugate stress methods of Rankine and Bell do not agree with the results of experiments.

(ii) The plastic equilibrium theory as modified by Terzaghi and Skempton appears to give ultimate bearing pressures which agree very closely with practical observations. Model tests, however, indicate that the form of the outer part of the surface of slip is usually a flat curve rather than a straight line.

For frictional and C – ∅ soils reliable values of the ultimate bearing capacity can be found from Terzaghi's formula:

$$q_d = CN_C + \gamma DN_q + \frac{1}{2}\gamma BN_\gamma$$

using the values of coefficients from Fig.

For cohesive soil (∅ = 0)

$$q_d = C N_C + D$$

and the value of N_C may be taken from Figure. 11.

(iii) The circular arc graphical method is useful for footings where the strength of the soil varies with depth.

TABLE 3.7: Table Showing Bearing Capacity of Eight Soils at Rudrapur by different Formulas (Ref. 6)

S.No.	Type of Soil	F.S. used	See Ref. 1 to 7 below						
			1	2	3	4	5	6	7
1.	Silty Clay Loam	3	0.988	1.353	0.7913	0.674	0.306	0.289	0.7932
2.	Clay	3.5	0.7763	0.9045	0.564	0.4878	0.328	0.31	0.5636
3.	Sandy Loam	3	1.4	1.971	0.958	0.8455	0.306	0.2897	0.982
4.	Loam	3	1.484	1.855	1.034	0.8921	0.459	0.4345	1.051
5.	Clay	3.5	0.9951	1.191	0.73	0.6338	0.393	0.3723	0.731
6.	Sand	2.5	0.84	1.626	0.876	0.7725	0.1873	0.1738	0.88
7.	Loam	3	0.495	0.854	0.537	0.4561	0.1561	0.1448	0.5382
8.	Sandy Loam	3	0.7	1.325	0.719	0.611	0.1561	0.1448	0.715

REFERENCES :

1. Prandtl : $\dfrac{c}{\tan\phi}\left[\dfrac{1+\sin\phi}{1-\sin\phi}\cdot e^{\pi\tan\phi}-1\right]$

2. Prandtl-Taylor $\left[c\cot\phi + B\gamma\cot\left(\dfrac{\pi}{4}-\dfrac{\phi}{2}\right)\right]\cdot\left[\dfrac{1+\sin\phi}{1-\sin\phi}\cdot e^{\pi\tan\phi}-1\right]$.

3. Bell-Terzaghi: $\dfrac{\gamma B}{2}\left[\tan^5\left(45+\dfrac{\phi}{2}\right)-\tan\left(45+\dfrac{\phi}{2}\right)\right]+c\left[2\tan\left(45+\dfrac{\phi}{2}\right)+2\tan^3\left(45+\dfrac{\phi}{2}\right)\right]+q'\tan^4\left(45+\dfrac{\phi}{2}\right)$.

4. Terzaghi : $\dfrac{2}{3}cN_c' + \gamma DN_q' + \dfrac{1}{2}\gamma BN_\gamma'$

5. Tschebotarioff : $5.52c\left(1+0.38\dfrac{D}{B}+0.44\dfrac{B}{L}\right)$

6. Hencky : $5.64S + \dfrac{F}{A}S'$

7. Ritter : $\left[\gamma D + \dfrac{\gamma B}{4}\tan^4\left(45+\dfrac{\phi}{2}\right)\right]\cdot\left[\tan^4\left(45+\dfrac{\phi}{2}\right)-1+\gamma D\right]+2c/\tan\left(45-\dfrac{\phi}{2}\right)\sin^2\left(45-\dfrac{\phi}{2}\right)$.

side. Further, the values of Tschebotarioff and Hencky are on the lower side. Thus it may be inferred that a very judicious selection of the formula is necessary for calculation as the results differ greatly in some formulae.

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(i) The conjugate stress methods of Rankine and Bell do not agree with the results of experiments.

(ii) The plastic equilibrium theory as modified by Terzaghi and Skempton appears to give ultimate bearing pressures which agree very closely with practical observations. Model tests, however, indicate that the form of the outer part of the surface of slip is usually a flat curve rather than a straight line.

For frictional and $C - \phi$ soils reliable values of the ultimate bearing capacity can be found from Terzaghi's formula:

$$q_d = CN_c + \gamma DN_q + \frac{1}{2}\gamma BN_\gamma$$

using the values of coefficients from Fig.

For cohesive soil ($\phi = 0$)

$$q_d = C N_c + D$$

and the value of N_c may be taken from Figure. 11.

(iii) The circular arc graphical method is useful for footings where the strength of the soil varies with depth.

COHESIONLESS SOILS :

(iv) In cohesionless soils the bearing capacity is proportional to the breadtb of the footing, provided the material is in a relatively loose state. If the sand is compacted the rate of increase of bearing capacity decreases with increase of width. The bearing capacity of square or round footings appears to be about the same as for a strip footing of the same width.

(v) For bearing capacity below the surface of cohesionless soils, Terzaghi's approach furnishes the following formula :

$$q_{dD} = q_{do}\left(1 + C\,\frac{D}{B}\right)$$

in which q_{dD} and q_{do} are the ultimate bearing capacities at depth D and zero, respectively, the coefficient C being equal to N_q / N_γ in general shear and $2\,N_q' / N'_\gamma$ in local shear. A conservative expression for the depth factor is $(1 + 2\,D/B)$.

COHESIVE SOILS :

(vi) For strip loading the ultimate bearing capacity is independent of the width. The theoretical values for footings at the surface are :

Bell	4 C
Prandtl	5.14 C
Terzaghi	
General Shear	5.7 C
Local Shear	3.8 C
Fellenius	5.5 C

For footings below the surface the coefficient of C increases with the depth to a maximum of 1.5 times the surface value at a depth of 2.5 B.

(vii) For circular footings on cohesive soil Hencky gives 5.64 C as the ultimate bearing capacity. Experiments on square and circular footings have shown ultimate values twenty to twenty five perient greater than for strip loading.

(vii) Skempton suggests that for a rectangular loaded area of width B and length L the coefficient N_c is found by multiplying the appropriate value of N_c for a square footing by $(0.84 + 0.16 \ B/L)$.

4. SETTLEMENT ANALYSIS AND CONTACT PRESSURE

4.1. INTRODUCTION :

It is generally agreed that the objective of determining the soil bearing capacity is elimination of the possibility of rupture, reduction of gross settlement to a tolerable value and elimination of differential settlement. Current analytical methods for establishing the bearing value place major emphasis on the first criterion, namely elimination of the possibility of rupture. Development of convenient, practical procedures for estimating settlement has to some extent been neglected. There has been a tacit assumption that with a satisfactory factor of safety against rupture or shear failure, settlement in many cases does not require analysis.

While analysis to insure against soil rupture may well appear to be over-riding consideration and hence to deserve the attention which it has received, under practical conditions, there is actually much less chance of soil rupture due to structural loading than has been imagined. In particular, if a reasonably careful site investigation is made and if code regulations are complied with, as is mandatory in many cases, it is believed (19) that the chances of soil rupture are nil whereas chances of differential settlement due to soil compression may well remain. Thus the need for the development and application of rupture theory diminishes in importance, whereas settlement analysis especially for differing conditions of footing size, depth and surcharge becomes correspondingly greater Rupture

Rupture theory has no value whatever in settlement analysis.

The above considerations lead to the conclusion that in the design of footings, equal if not more attention should be given to settlement analysis as to bearing capacity theories. The point remains that even if the bearing capacity is not exceeded, the amount of differential settlement is liable to change the entire design. The purpose of a settlement forecast is to obtain a reliable conception of the differential settlement in order to determine whether or not the foundation layout under consideration is satisfactory.

4.2. PLATE BEARING TEST :

Plate bearing rest or loading test is often employed to obtain information on the bearing capacity and the settlement characteristics of the soil at a given site. It is a good medium to undetstand certain concepts regarding the settlement analysis and as such it will first be presented here.

The test is made by increasing the load on a bearing plate by small increments and measuring the corresponding settlements. The bearing plate rests on the bottom of a pit, at the level of the base of the footing. Depending on the preference of the engineer who makes the test, the plate may be surrounded by a box and the pit backfilled to final grade or a hole may be made in the pit. The test results are represented by load settlement curves similar to the those shown in Figure 3. Two of the most common methods for performing the tests and interpreting the results will be described here.

The first method consists of loading a square or circular bearing block of any dimensions chosen by the investigator, as big plate as possible being preferred. The allowable load q_a per unit of area is taken as some fraction, such as one half, of the average pressure on the block at the time of failure. This procedure is objectionable for several reasons. In the first place, if the load settlement curve resembles C_2 in Fig. 3a, there is no definite failure load. Second, the size of the loaded area, which is optional, may have large influence on the ultimate bearing capacity per unit of area. Hence, by using this first procedure two different investigators can obtain very different values of q_a for the same soil.

The second method consists of loading a bearing block covering an area of one foot square. The allowable load q_a is arbitrarily defined as one half the load at which the settlement of the bearing block is 0.5 in. (In countries using the metric system the area of contact is customarily taken as 0.1 sq m, and the settlement as 1 cm. This procedure, though arbitrary, is preferable because two different investigators will at least obtain the same value of q_a for the same soil.

DISCUSSION :

Whatever, the method of testing may be, the test results reflect the character only of the soil located within a depth of less than twice the width of the bearing plate, whereas the settlement of the footings depends on the properties of a much thicker soil stratum

As a consequence, if the characer of the soil changes below a depth of about twice the width of the bearing plate, as it commonly does, the test results are certain to be misleading. Since it is also almost universal practice to select the allowable soil pressure without regard for the size of the footings, the type of superstructure and other vital characteristics of the proposed foundation, it is not surprising that increasing recourse to load tests has not significantly reduced the frequency of faulty footing design. In fact, several complete footing failures have occured in spite of the conscientious performance of load tests. To reduce the risk of faulty design, the allowable soil pressure must be chosen in accordance not only with the results of load tests or their equivalent, but also with the character of the soil profile and of the foundation itself.

The full-sized footing will settle much more than would be anticipated on the basis of the load test. The reason is illustrated by Figure 20. This figure represents a vertical section through a stratified subgrade. A is a bearing block covering an area of 1 ft. square, and B is a full sized footing. The load on both A and B has the same intensity q. Beneath A and B are shown curves of equal vertical pressure in the subsoil. The load on A increases the average vertical pressure in stratum C beneath the loaded area by about 0.02 q, whereas the footing B increases it by 0.50 q . If stratum C is very compressible, the settlement of B may be very large. If C is hard, the settlement of B may be very small. Yet, the result of the load test is practically independent of the compressibility of C, because the increase of the pressure in stratum C due to the load on the bearing plate is negligible.

Thus the loading tests which are correctly interpreted offer a truly scientific attack to the problem but unless the scientific aspect extends to the interpretation, the use of the test may be more harmful than helpful.

4.3 COEFFICIENT OF SETTLEMENT :

A definite characteristics of many loading test plots is the early straight line portion extending to intensities of roughly one third or one half of the ultimate intensity. This straight line occurs in a surprisingly large percentage of loading tests. If the early portion of the curve is a straight line, the ratio between the stress and the settlement at points on the line has a definite constant value. This ratio is called the 'Coefficient of Settlement' although in tests on highway and airport subgrades it is usually called the 'Coefficient of Subgrade Reaction'. When there is deviation from the straight line, no standardized definition has been chosen for this coefficient, and reciprocates of slopes at arbitrarily chosen points or reciprocals of slopes of chords are generally used. The slope reciprocal at the point where the plot has the least curvature is perhaps the most logical choice. Herein the stress-settlement ratio is called the coefficient of settlement and is designated by C_s. In the metric system the units are usually kilograms per cubic centimeter. In the English system its most convenient units are tons per square foot per inch of settlement or pounds per cubic inch.

Approximate General Expression for the Coefficient of Settlement for any Soil :

This analysis covers the effects of both size and depth of the footing in any homogeneous soil and furnishes a general expression for the coefficient of settlement in terms of two soil properties. Its scope is limited to the straight line portion of the loading test curve, however, and it does not include ultimate bearing capacity considerations.

The ratio between the direct stress σ_Z on the horizontal plane, and the vertical compressive strain e_Z, at a point at any depth below the surface of a homogeneous soil deposit, is a stress strain modulus, M_Z. In highly cohesive material, in which there is a constant interinsic pressure p_i, the modulus is constant. In cohesionless soils the pressure depends on the weight of the overlying soil and to a smaller degree on pressures caused by the footing load; therefore the modulus in such a soil is proportional to the unit weight γ, and at any given depth it is approximately proportional to the depth. In a soil which falls between the classifications of cohesionless and highly cohesive, the modulus may be expressed by

$$M_Z = C_a \gamma Z + C_b p_i$$

where C_a and C_b are constants for the given soil.

An approximate relationship between loading intensity, settlement, and depth and breadth of footing may be obtained by using average values for stress, strain, and modulus within the pressure bulb. On this basis,

$$\frac{(\sigma_z)_{av.}}{(e_z)_{av.}} = (M_z)_{av.}$$

Figure 21 represents the general case under consideration. The average stress is designated by $C_c q$ and the average vertical strain may be expressed as the settlement S divided by the bulb depth. If the shape of the bulb is assumed to be cube, the average strain is S/B. The average modulus is the value holding at the mid point of the bulb, where Z equals D + (B/2). Inserting these average values in the above equation gives

$$\frac{C_c q}{S/B} = C_a \gamma \left(D + \frac{B}{2}\right) + C_b p_i$$

whence

$$\frac{q}{S} = \left(\frac{C_a \gamma}{2 C_c}\right)\left(1 + \frac{2D}{B}\right) + \left(\frac{C_b p_i}{C_c}\right)\frac{1}{B}$$

If γ, pi, Ca, Cb and C_c are assumed to be constant, the relationship which is valid for any soil, may be written

$$\frac{q}{S} = C_1\left(1 + \frac{2D}{B}\right) + \frac{C_2}{B} \qquad (4.1)$$

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where q/S is the coefficient of settlement and C_1 and C_2 are soil constants.

In highly cohesive soils, C_1 is inappreciably small and the coefficient is inversely proportional to the breadth; for cohesionless soil, C_2 equals zero and for a surface footing ($D = 0$) the coefficient has the same value for all breadths.

For a material which conforms approximately to one of the limiting cases of cohesionless or highly cohesive soil, the coefficient of settlement for any size of footing may be estimated from the results of a single loading test. + For soils, in general, however, loading tests on at least two breadths of footings must be available. With these data constants C_1 and C_2 in Eq. 4.1 can be evaluated and coefficient of settlement for any footing at any depth is obtained.

DISCUSSION :

The many bold assumptions and extreme degree of extrapolation, adopted in the above analysis greatly affect the accuracy of the results. After such factors as the possibility of disturbance to the soil during excavation and the effect of loading and unloading caused by the lowering and raising of the water table during the construction have been recognized, it is obvious that there is much question regarding the amount of dependability that can be attached to the numerical value. This indicates the desirability of using such analyses with descretion and, if possible, using the average data of two or more tests for each breadth tested in an investigation of this type.

4.4. FOOTINGS ON SAND :

The settlement of footings on sand is governed by the stress-strain characteristics of the material. The rigidity of the sand increases markedly with increase in relative density and is approximately proportional to the confining pressure.

The confining pressure in a mass of sand is at least roughly proportional to the vertical pressure and is, therefore, also roughly proportional to the unit weight of the sand immediately beneath and beside the footing. The most important factor that has an influence upon the unit weight of sand is the position of the water table. If the water table is near the ground surface, the effective vertical pressure in the sand is due only to its submerged weight. Hence, if the water table is raised to ground surface from below the pressure bulb, the settlement of a footing is likely to be approximately doubled. This leads to the conclusion that, for a given soil pressure, the settlement of a footing on sand depends upon the relative density and on the position of the water table. Various theoretical investigations, show that the settlement for a given soil pressure also increases with increasing width of footing. This is shown by the plain curve in fig. 22a. In accordance with this theoretical conclusion, the results of experiments and observations indicate that the settlement increases with the width B of the footing approximately as shown in fig. 22b. The empirical data were derived (46) from small scale load tests on artificially compacted sands, from load tests on relatively homogeneous sand strata, and from settlement observations on buildings. In this figure, S_1 is the settlement of a loaded area 1 ft. square under a given load q per unit of area, and S is the settlement at the same load per unit of area of a footing with a width B. The relation between S, S_1 and B is given approximately by the equation,

$$S = S_1 \left(\frac{2B}{B+1} \right)^2 \qquad (4.2)$$

in which S and S_1 are expressed in inches and B in feet. This relationship, when generalized takes the form,

$$S_B = S_T \cdot \left[\frac{B\ (T+1)}{T\ (B+1)} \right]^2$$

where S T and SB are the settlements of loaded areas of widths T and B respectively.

There is no significant difference between the settlements of square and continuous footings having the same width B, because the effect of stressing the sand to a greater depth below a continuous footing is compensated by the restraint that keeps the sand from being displaced in directions parallel to the footing. According to figure 23b, the settlement of a large footing, greater than about 20 ft square, exceeds that of a small footing 4 or 5 ft. square by roughly 30 percent, provided the soil pressures are equal. At a given width B of the footing, the settlement decreases to some extent with increasing values of the depth ratio, D/B. Yet, even under extreme conditions involving a foundation on footings with very different sizes and depth ratios, Fig. 1, the differential settlement is unlikely to exceed 75 percent of the maximum settlement. Normally it is very much smaller.

Most ordinary structures, such as office buildings, apartment houses, or factories, can withstand a differential settlement between adjacent coolumns of three quarters of an inch. As indicated above, this settlement will not be exceeded if the soil pressure is selected such that the largest footing would settle 1 inch even if it rested on the most compressible part of the sand deposit. Therefore, the allowable soil pressure for the design of footings of such structures can be assumed equal to the pressure that will cause the largest footing to settle 1 in. An approximate method is described below for selecting the allowable soil pressure on sand in accordance with this assumption. If a differential settlement of d S of more than 3/4 in. can be tolerated, the allowable soil pressure can be multiplied by 4 dS/3.

ALLOWABLE PRESSURE ON DRY AND ON MOIST SAND :

The settlement of a footing on dry or moist sand depends primarily on the relative density of the sand and the width of the footing. The relative density can be judged adequately on the basis of th results of any of the sounding methods, provided the relation between relative density and penetration resistance has been determined previously by means of suitable calibration tests. When test boring data include the standard penetration test, the penetration resistance may be used to extrapolate the allowable pressure.

In order to select allowable soil pressure on the above basis, it is necessary to estimate very roughly the width B of the largest footings. Between the level of the base of the footings and a depth B below this level one standard penetration test should be performed for every 2½ ft. of depth. The average value of number of blows, N, for this depth indicates the relative density of the sand within the seat of settlement of the footing. The value of the allowable soil pressure is then obtained by means of the chart, Fig. 24, in which the curves represent the relation between B and the soil pressure required to produce a settlement of the footing of 1 in., provided the footing rests on a sand for which the number of blows N has the value inscribed on the curve. If the pressure corresponding to some other amount of settlement is desired, it may be computed by linear interpolation between the curves.

If the water level is above the base of the footing, the pressure corresponding to a 1-in. settlement should be taken as half the value given by the chart. For intermediate positions, proper values may be obtained by interpolation . If the subsoil consists of very fine sand below the water table, the values of N, referred to as N' may be too great. In such a case, the equivalent value of N may be obtained from the expression.

$$N = 15 + \frac{1}{2} (N' - 15)$$

The chart, Fig. 24 was prepared on the basis of knowledge concerning the relation between N, the results of surface loading tests, and equation 4.2. If B is the width of the largest footing supporting a structure, and if all the footings are proportioned in accordance with the allowable soil pressure corresponding to B, the maximum settlement of the footing should not exceed 1 in., and the differential settlement $\frac{3}{4}$ in for important concrete buildings and $\frac{1}{2}$ in for ordinary buildings.

Even if very low soil pressures are used in design, footings on the sand are likely to settle excessively if the sand is subject to high frequency vibrations. The statement applies to saturated as well as to moist or dry sands.

4.5 FOOTINGS ON CLAY :

If the footings rest on normally loaded clay, the magnitude of both the total and differential settlement can be very large. This can be demonstrated by computing the ultimate settlement of continuous footings of different widths resting on soft normally loaded clay. In this context, it is essential to distinguish between the consolidation settlement and the immediate settlement. The results of consolidation computation are shown in Fig.24. The soil pressure on the base of the footings was taken as 1000 psf. In addition, it was assumed that the depth of foundation was 5ft., that within this depth the effective unit weight of the soil 100 lbs per cuft, that the liquid limit of the clay was 40 percent, and that the settlement of the footings was caused solely by consolidation. The compression index C_c is estimated by means of laboratory tests or by using the equation

$$C_c = 0.009 \; (L_w - 10\%)$$

The settlement is computed by

$$S = H \frac{C_c}{1+e_0} \log_{10} \frac{p_0 + \Delta p}{p_0}$$

Wherein H is the thickness of clay layer and p_0 and Δp are the original intergranular pressure and increase in pressure due to footing load respectively.

The curve that represents the relation between the immediate settlement and the width of the footing resembles the dash-dotted line in Fig.23. The trend of the curve indicates that the settlement of footings on clay, in contrast to that of footings on sand, increases in almost direct proportion to the width of the footings. Fig. 25 shows that the settlement of continuous uniformly loaded footings of constant width on a uniform deposit of normally loaded clay can be very large and that the settlement of footings with different widths can be very different. Furthermore, the settlement of footings with the same width can also be very non-uniform, because the compressibility of natural clay strata may vary considerably in horizontal directions. Fortunately, footing foundations on normally loaded clays are rare exceptions. In most localities even soft clays are precompressed, either by desiccation or temporary lowering of the water table. Medium and stiff clays beneath a shallow overburden are always precompressed. ~~Cince the allowable soil pressures rarely exceed the precompression~~

Since the allowable soil pressures rarely exceed the precompression pressure, the differential settlement on footing foundations on such clays rarely exceeds that of adequately designed footings on sand. The maximum setlement, however, is likely to be greater.

In the few regions where structures must be built above normally or almost normally loaded clays differential settlements of several inches or even a half foot are commonly considered unavoidable. Attempts to reduce the settlement by reducing the allowable soil pressures are ineffective and wasteful. Hence the designer must choose between two alternatives. Either he designs the footings at the risk of large unequal settlements, or else he provides the structure with another type of foundation.

If it is doubtful whether or not the settlement of the proposed footings with width B will be excessive, load tests should be made at the level of the base of the footings, on bearing plates 2 ft. square at the bottom of test pits 6 ft. square. If the consistency of the clay varies considerably between this level and a depth B, load tests must be made at two or three different levels within this depth. The number of load tests or sets of tests that are required depends primarily on the degree of homogeneity of the clay stratum and the number of footings. After the application of each load increment, the load should be kept constant until further settlement becomes imperceptible.

In accordance with the relation represented by the dash-dotted line in Fig. 23a, it can be assumed that the immediate settlement S of a footing with width B will very roughly be equal to the value,

$$S = S_o \frac{B}{B_o}$$

where S_o is the settlement of the bearing plate under the design load per unit of area, and B_o is the width of the bearing plate.

4.6. EFFECTS OF SETTLEMENT :

The settlement of a homogeneous, compressible soil deposit acted upon by a uniform flexible loading forms a saucer shaped depression which extends beyond the limits of the loaded area. The central part of the saucer is concave upward and the edges tilt toward the centre of the loading. The effect that settlement has on a structure depends on where the structure is located in the depression and on how the movements there influence the performance of the structure. Three aspects of settlement must be considered; the maximum amount of settlement, the differential settlement between adjacent parts which result in tilting, and the differential settlement which results in curvature or distortion. Depending on the structure itself, any one of these may have a serious consequences.

The amount of settlement which a structure can undergo is large provided it is relatively uniform. The National Palace of Fine Arts in Mexico City, for example, has settled over 12 ft. Since its completion in 1909. It is still in operation, and the building itself shows little effects from this great movement. Even, uniform settlement can result in trouble, however. First a building sitting in a depression has a poor appearance. The access might be .

impaired; utility connections might be damaged; drainage often proves a serious problem.

Unequal or differential settlement has far more serious consequences. Tilting occurs in the parts of the structure that are outside the centre of depression, that are unequally loaded or underlain by nonuniform soils. Instances occur of the tilting of a tall building when one side settles more than the other. The centre of gravity is shifted, the load on the base becomes ecentric, and the bearing pressure at one edge is increased. The pressure may eventually increase sufficiently to cause shear failure in the soil.

Thus it is seen that the distribution of the settlement is far more important than the maximum value. In general, however, the differential settlements are largest when the average settlements are largest, and, on the assumption that the magnitude of the settlemen may be accepted as a measure of the amount of probable differential settlement, the settlement requirement is frequently expressed in the form of a maximum allowable settlement.

4.7. LIMITING SETTLEMENTS :

The amount of settlement a structure can tolerate has been subject to much argument, particularly by architects and structural engineers. Ordinarily, the settlements are computed only for representative parts of the structure: the centre, the edge and the corners of a uniformly loaded structure; the largest, smallest and typical columns of irregularly loaded structures. Studies have been made of the cracking of existing structures in many locations. The limitations given in Table 4.1, (40) are based on the structural considerations and on the effect of settlement on the building contents.

TABLE 4.1 LIMITING SETTLEMENTS

Type of Settlement	Limiting Factor	Maximum Settlement in inches.
Total Settlement	Drainage	6 - 12
	Access	12 - 14
	Probability of non-uniform settlement :	
	Masonry walled structure	3 - 4
	Framed Structure	4
	Connection to smokestacks, soils, rigid structures	12
Differential Settlement	Brick wall cracking :	
	L / H = 3 or less	0.0004L
	L / H = 5 or more	0.0007L
	One storey masonry mill building wall cracking	0.0001L - 0.002L
	Plaster cracking	0.001 L
	Reinforced Concrete-	0.0025 L -

building frame	0.004L
R.C. building curtain walls	0.004L

Note : L is the distance between adjacent columns that settle different amounts, or between any two points that settle differently, and H is the wall height.

4.8 CONTACT PRESSURE :

The term contact pressure indicates the normal stress at the surface of contact between a footing and the supporting earth. It is important in the design of footings as it determines the distribution of moment and shear within it. The distribution of pressure is very different below footings on cohesionless soil from that below footings on cohesive soil. The distribution also depends greatly on the rigidity of the footing. The concepts arrived at in the following paragraphs are valid for square, round, or long footings.

A flexible footing on the surface of a cohesionless soil, carrying a uniformly distributed load is considered first. Since the footing is completely flexible the uniform distribution of pressure also acts on the surface of the soil. The soil just outside the edge of the footing is not under pressure and has no strength. Therefore, the outer edge of the footing undergoes a relatively large settlement. Below the centre of the footing the soil develops strength and rigidity as fast as it is loaded from above and from surrounding points, and because of this the settlement is relatively small. Fig. 26a, shows the uniform loading diagram for this case, with the curve of settlement shown by heavy dashed lines.

For a rigid footing resting on the surface of a cohesionless soil the settlement must be uniform. Under uniform settlement the high resistance to compression in the soil below the centre of the footing, as compared to the lack of resistance below the edge, must result in a relatively large pressure under the centre and no pressure at the edge. This case with constant settlement and an approximately parabolic pressure distribution is shown in Fig. 26b. If the average pressure is relatively small, or if the width of the footing is large, this pressure distribution is somewhat flatter over the central portion of the footing as shown in Fig. C, being nearer ellipsoidal than parabolic in shape but still having zero pressure at the edges.

For rigid footings founded below the surface of a cohesionless deposit there is some strength below the edge of the footing and, therefore, the pressure is not zero at the edge but is more like that shown in the distribution curve in Fig. 26d.

A uniformly loaded flexible footing on highly cohesive soil gives conditions that can best be visualized by considering the stresses and strains caused in a typical thin horizontal layer of soil within the height of the pressure bulb. The uniform surface distribution transmits a bell shaped distribution of pressure as shown in Fig. 27a. All horizontal layers below ground surface similarly show maximum compression below the centre of the footing, and thus the surface settlement must have the dished pattern shown, with a much greater settlement under the centre than under the edge of the footing.

A rigid footing on highly cohesive soil must undergo uniform settlement. The layer shown in (b) is at a depth of slightly less than B/2 and may be accepted as representative of the average of all such layers. If the compression of this layer is nearly as large at point B as at A, the pressure at this level must be nearly as large at B as at A, and the pressure distribution curve at this level must be about as shown. For an elastic material of infinite strength, the distribution shown by the theory of elasticity is indicated in Fig. c by a light dashed curve, this curve shows an infinite stress at the edge of the footing. Actually an infinite stress cannot occur, but the stress at the edges may be much larger than that at the centre.

Numerical values of pressures for the variable distributions in Figs. 26 and 27 cannot be given because the actual magnitudes depend on numerous factors. The usual assumption made in the design of a footing is that the contact pressure is uniform. For footings on sand this is conservative, but for soils such as clays, with contact pressure highest at the outside edges this may be unsafe. Ordinarily the factor of safety is adequate to take care of the condition.

4.9 SUMMARY

(i) Except for narrow footings on loose saturated sand, the allowable bearing values for sand are governed only by settlement considerations, because it can be taken for granted that the factor of safety with respect to a base failure is adequate. The rules suggested for choosing these values satisfy the condition that the maximum settlement is unlikely to exceed 1 in and the differential settlement $\frac{3}{4}$ in.

(ii) On routine jobs the allowable soil pressure on dry and moist sand can be determined by means of the chart, Fig. 24, on the basis of the results of standard penetration tests.

(iii) If the water table is located close to or above the base of the footings, the depth ratio D/B must be considered. If the ratio is very small, the values obtained from the chart must be reduced by 50 per cent; if it is close to unity, the values need be reduced only by one third.

(iv) On large jobs the plate bearing test may be employed. However, it is expensive and cumbersome, and, if it is not expertly planned and executed, the results may be very misleading.

(v) If the clay is normally loaded, the settlement is likely to be excessive, and a type of foundation other than a footing foundation may be indicated. On the other hand, if the clay is precompressed, the differential settlement is likely to be tolerable. In doubtful cases the loading test may be used.

(vi) The pattern of distribution of contact pressure is studied. An assumption that may be used in the design of rigid footings is that the pressure is uniform, and no definite recommendation for a better procedure can be given.

5. DESIGN OF FOOTINGS

5.1 OBJECTIVES AND GENERAL APPROACH :

The design of a footing foundation consists of determining the elevation, size, shape and structural design of the cheapest foundation which will meet the three basic requirements: sufficient depth, safety against failure, and freedom from objectionable settlement, outlined in the earlier chapters. Like any other problem of design, this is an art. It makes use of scientific analysis of bearing capacity, settlement, contact pressure and structural stresses. The final choice, however, is governed by the considerations such as the time required, the space and materials available, the skill of the builder, and above all cost. The aspect of structural design is beyond the scope of the present work, hence it will not be dealt with.

The elevation of the footing structure depends on a number of considerations. First, there is a minimum depth requirement which was discussed in Chapter 2. Second, additional depth may be necessary depending on the bearing capacity and the settlement of the various soil strata below the minimum depth. Third, it may be desirable to limit the depth because of such conditions as a high ground water level, the presence of rock, and the presence of adjacent structures which might be endangered by deep excavations.

The size and shape of the footing depend on the magnitude and configuration of loads imposed on it by the superstructure the bearing capacity, settlement and contact pressure resulting from these loads and on the space available for the foundation itself. The greatest average foundation pressure that may be employed without exceeding the safe bearing capacity and without producing excessive settlement is 'Allowable foundation or bearing pressure'. It can be equal to the safe bearing capacity when the soils are incompressible, but it is often considerably lower because of the limitations imposed by settlement. The determination of allowable soil pressure is the most critical step in the design process.

There are a number of different approaches to footing design. The first is the time-honoured procedure setforth in most building codes and in many hand books where the allowable pressure is estimated on the basis of soil description. The second approach is based on a plate load test, described under Article 4.2. The third is a rational approach, based on the determination of soil bearing capacity, settlement, contact pressure, and the actual needs and limitations of the superstructure.

5.2. STEPS IN DESIGN.

The first step in designing a footing is to compute the total effective load that will be transferred to the subsoil at the base of the footing. The second step is to determine the allowable bearing value of the soil. The area of the footing is then obtained by dividing the total effective load by the allowable bearing value while the actual length and width are selected to fit the limitations of space and layout. Finally, the bending moments and shears in the footing are computed, and the structural design of the footing is carried out.

5.3 DESIGN LOADS (46)

The total effective or excess load Q_t transferred to the subgrade may be expressed by the equation,

$$Q_t = (Q - W_s) + Q_l = Q_{dn} + Q_l$$

in which

Q = permanent or dead load on the base of the footing, including the weight of the footing and the soil located above the footing. If the water table is higher than the base of the footing, the hydrostatic uplift on the submerged part of the body of soil and concrete should be deducted.

W_s = effective weight of the soil (total weight of soil reduced by hydro-static uplift) that was located above the base of the footing prior to excavation. However, in connection with basement footings such as c and d in Fig.1, the weight of the soil previously located above the basement floor should not be deducted, because the soil was removed not only above the base but also above the area adjoining at least one side of the base.

$Q_{dn} = Q - W_s$ = net dead load

Q_l = live load on footing, including that due to wind and snow.

In any discussion of live load, a distinction must be made between the normal live load and the maximum live load. The normal live load Q_{ln} is that part of the live load which acts on the foundation at least as often as once a year; the maximum live load Q_{lmax} acts only during the simultaneous occurence of several exceptional events. For instance, the normal live load in a tall office building includes only the weight of the equipment and the furniture of the persons who normally occupy the building on week days, and of a normal snow load. The maximum live load is the sum of the weights of furniture and equipment, of the maximum number of persons who may crowd into the building on exceptional occasions, combined with the maximum snow and the wind load. The total excess load on a footing at normal live load will be indicated by the symbol,

$$Q_{tn} = Q_{dn} + Q_{ln}$$

and at maximum live load by

$$Q_{tmax} = Q_{dn} + Q_{lmax}$$

Because of the exceptional character of the maximum live load and the low probability that the foundation will ever be called on to sustain it, it is customary to design footings in such a manner that the soil pressure produced by the normal total load Q_{tn} is the same for all the footings. However, sound engineering also requires that even the maximum load Q_{tmax} should not cause irreparabledamage to the structure. The procedure for complying with this requirement without excessive expenditure depends on the type of subsoil.

5.4 ALLOWABLE SOIL PRESSURE:

Beginning in the late Nineteenth Century engineers in large cities, particularly Chicago, began assembling records of foundation success and failure correlating them with the character of the soil on which the foundation rested and the pressure exerted by the foundation on the soil. Since then similar empirical correlations have been derived by a number of municipalities, state agencies and by many engineering organizations. All have a similar form: a soil description and a corresponding allowable foundation pressure. The allowable bearing pressure is also termed as 'presumptive bearing pressure', because it is presumed that the soil can support that load with safety and without undue settlement on the basis of its past performance. Typical allowable pressures are given in Table 5.1., American Civil Engineers Handbook values and American Standards Association values are presented.

The presumptive bearing pressures are best applied to the design of small structures such as dwellings and very light industrial buildings with simple soil conditions where the cost of evaluation of soil bearing capacity exceeds the cost of over designing the foundations. It can be seen from the examination of Table that many significant factors in the design of foundations are omitted. First the compressibility of the soil is ignored, which may not be serious with the lightly loaded structures but which can be disastrous with heavy ones. Second, the character of the structure itself, including its loads and its ability to withstand settlement is not mentioned. Third, the determination of which value to use is largely visual and consequently is a crude estimate rather than a sound basis for good design. Furthermore the allowable soil pressure is believed to be that pressure under which the settlements of various footings would not exceed reasonable values. However, it is known that different footings beneath the same structure are not likely to settlement the same amount even under the same soil pressure.

Table 5.1. PRESUMPTIVE BEARING PRESSURE OF SOIL IN psf

Soil Description	American Civil Engineering Hand Book.	American Standards Association.
Fill or silt	-	0
Hard clay	4000 - 10,000	-
Loose sand gravel, loose coarse sand, compact fine sand.	4000 - 6000	6000
Loose gravel, Compact coarse sand	6000 - 12000	8000
Compact sand, gravel.	–	12000
Hardpan, cemented gravel sand.	12000 - 20000	20000
Massive bed rock (Granite, Diorite, Trap).	60000	200000

Most building codes contain tables of allowable soil pressure. These can be a helpful guide to local practice but often they lead to trouble. The tendency of the designers to create false confidence in a poor design is serious: many designers are satisfied if they design in accordance with the code values regardless of the peculiarities of soil or the requirements of structure. According to G.F. Sowers (40) most of the footing failures, he has investigated were designed strictly in accordance with the applicable code. The use of code values does not relieve the engineer of proper design, and when the soil conditions are bad or the structure critical, lower pressures must be used.

5.5 CONVENTIONAL PROCEDURE OF PROPORTIONING FOOTINGS :

In its simplest form, the method of proportioning footings according to an allowable soil pressure is as follows : The load acting at the base of each column is determined. The weight of the footing is then estimated and added to the column load. The total load is divided by the allowable soil pressure to determine the area required for the footing. After the size of the footing has been determined, its weight is calculated and the value assumed in the computation is revised if necessary. After the dimensions of the footing are established, the footing is designed.

Except for the choice of allowable soil pressure, the most difficult step in the procedure is the determination of load for which the footing should be proportioned. It has been generally believed that a settlement of a footing is caused primarily by the dead load plus only the amount of live load that acts on the footing for an extensive period of time. Under these circumstances, equal settlement would be achieved by choosing the areas of the footings in proportion to the dead load plus a fraction of the design live liad. However, it is also generally believed that the allowable soil pressure should not be exceeded beneath any footing, even if the maximum probable wind and live loads should act upon the footing for a short time. These two requirements lead to the following procedure which represents conventional practice at the present time :

1. Determine the dead load for each column, including the estimated weight of the footing.

2. Determine the maximum live load, including wind load, that may act on the footing. This value is usually established by the building code.

3. Determine the ratio of maximum live load to dead load for each footing.

4. Select the footing for which this ratio is the largest, and determine the area of this footing by dividing the sum of the dead load and maximum live load by the allowable soil pressure.

5. To the dead load of this same footing add the live load that will actually be present to govern the settlement. This live load is termed the 'reduced live load'.

6. Divide the sum of the dead load and the reduced live load on this footing by the area of the footing to obtain the reduced allowable soil pressure.

7. Use the reduced allowable soil pressure for determining the area of all other footings, considering the dead load and reduced live load for these footings.

If the prodedure is used, the soil pressure will be the same beneath all footings for dead load plus the reduced live load. According to the concepts on which the procedure is based, this should lead to equal settlement of the footings. Furthermore, the allowable soil pressure will not be exceeded, even if the wind load and the maximum live load specified by the building code should act on any footing, because the reduced allowable soil pressure is chosen for the footing having the maximum ratio of wind and live load to dead load.

Certain modifications of the conventional procedure are also in common use. For example, it is sometimes specified that the allowable soil pressure may be increased by an arbitrary percentage when the dead load, wind load, and maximum live load are assumed to act simultaneously. Yet the essence of the procedure will not be altered.

5.6. RATIONAL DESIGN :

The rational approach to footing design is essentially the same as for the design of the other parts of the structure: trial, followed by evaluation and revision. A trial design is assumed on the basis of experience. It is analyzed to determine how it meets the requirements of depth, safety and deflection, and an estimate is made of its cost. The method corresponds to limit design in the structure since it is based on the limits of bearing capacity and settlement.

The use of the method requires extensive, accurate information regarding to the structures to be joined by the foundation viz. the soil and the superstructure. Without this information the design becomes little more than a guess. With it the risks to the structure can be minimized and the cost reduced. Since the cost of foundation is often one-tenth of the total for the structure, the extra time and expense required to obtain the information and the trouble involved in utilizing it in design can yield substantial savings.

If the footings rest on sand, an increase of load produces an almost simultaneous increase of settlement, but it can be assumed that the factor of safety with respect to a foundation failure remains adequate. In order to eliminate the possibility of serious damage due to the maximum live load, the designer should estimate, the greatest differential settlement dS in excess of the normal value of 3/4 in. that, in his judgement, the structure can stand without serious injury. An additional differential settlement of dS would correspond to a maximum settlement of 1.33 dS plus the normal maximum value of 1 in.

If all the footings were designed on the basis of a maximum settlement of 1 in. at normal live load, the maximum live load would increase the maximum settlement to

$$S_{max} = 1" \times \frac{Q\,t_{max}}{Q\,tn}$$

If S_{max} is smaller than the tolerable maximum of $(1.33\ dS + 1")$ the maximum live load can be disregarded. On the other hand, if S_{max} is larger than $(1.33\ d\,S + 1")$, the footings should be designed so that the soil pressure at normal live load is

$$q_a' = q_a \frac{1.33\ dS + 1"}{S_{max}}$$

The value of q_a' is commonly different for different footings. The smallest value should be used for proportioning all the footings; it corresponds to the footing for which the ratio Q_{tmax} / Q_{tn} is greatest.

If the footings rest on clay, the allowable soil pressure is determined by the conditions that under the normal total load the factor of safety against failure should be equal to 3, but under no circumstances should it be less than 2. If the factor of safety G_s at normal total load is equal to 3, the factor of safety G_s' at maximum total load is

$$G_s' = 3 \frac{Q_{tn}}{Q_{t\,max}}$$

If G_s' is equal to 2 or more, the maximum live load can be disregarded and all the footings can be proportioned for normal line load on the basis of $G_s = 3$. On the other hand, if Gs' is less than 2, the allowable soil pressure must be so chosen that the factor of safety at normal live load is equal to $6/G_s'$.

5.7 REDESIGN ALTERNATIVES :

When the trial design is found to be either unsafe or to result in excessive settlement, it is necessary to redesign the footing to meet the required limits. This can often be accomplished by a change in footing size. However, changing the size is not always the economical solution and in some cases it is impossible. Numerous alternatives can be employed, and while all are not applicable to each particular situation, they point out what can be done to obtain a satisfactory, economical foundation under adverse conditions.

Three alternatives are possible :

1. Change the foundation
2. Change the superstructure
3. Change the soil

Changes in the superstructure are not always possible because of the limitations imposed by its function, changes in the soil may not be possible because of limited techniques available. Changing the foundation itself is usually the simplest expedient for both the soil engineer and the designer of the superstructure. The first change ordinarily considered is to reduce the bearing pressure by increasing the footing size. This is very effective when the factor of safety against bearing capacity failure is inadequate. In homogeneous saturated clays the factor of safety increases in direct proportion to the footing area and in cohesionless soils it increases with the product of the area and the footing width. For example, with a constant column load and a square footing on cohesionless soil the factor of safety increases as the cube of width. Increasing the footing size is not always effective in reducing the settlement however. And there is a limit to how big the footings can be made. The limit for interior footings will be reached when they meet. For exterior footings the limit will be imposed by property lines, adjacent structures, and utilities. In all cases cost imposes a final limit.

A second possibility is to join the foundations to make a continuous foundation structure, such as a combined footing. This enables the foundation to bridge over small erratic soft areas which would reduce the safety of individual footings; and the increased total width and area provides a higher factor of safety than individual footings of smaller size.

The settlement of loaded areas of similar shape but different size increases at a given intensity of load with increasing width of the area. If the footings of a structure differ greatly in size, the differential settlement due to this cause can be important.

In such instance it may be justifiable to adapt the pressure on the base of the footings to some extent to the size of the footings. If the subsoil consists of sand, the differential settlement can be reduced by decreasing the size of the smallest footings, because even after the reduction the factor of safety G_s of these footings with respect to breaking into the ground is likely to be adequate. The application of this procedure to footing foundations on clay would reduce the value of Gs for the smallest footings to less than 3, which is not admissible. Hence, the differential settlement of footing foundations on clay can be reduced only by increasing the size of the largest footings beyond that required by allowable soil pressure. However, sound judgement is required to make such adjustments with prospects for success, because periodic and exceptional changes in the loading conditions must be considered.

5.8 LAYOUT OF FOOTINGS AND COMPUTATION OF MOMENTS :

It is customary to lay out each footing so that the resultant load Q_{tn}, passes through the centroid of the area covered by the footing. The bending moments are then computed on the assumption that the soil pressure is distributed uniformly over the base. In reality, the contact pressure against footings on sand decreases from the center toward the rim, and the real bending moments are usually less than the computed ones. On the other hand, if the footings are very rigid, and they rest on soft or medium day, the contact pressure may increase toward the rim, and the real moments may exceed the computed ones. However, the difference is amply covered by the margin of safety customarily provided in structural design.

The columns that support crane runway in industrial buildings are subject to largeeecentric loads whenever the crane operates near by, but during the rest of the time they carry ordinary dead and live loads. It is customary to design the connections between the columns and the footings for the ecentric loads. As a consequence, the moments are transmitted to the base of the footings. If the footings rest on clay, the allowable soil pressure q_a should not be exceeded under the toe of any footing when all the loads including that due to the crane, are acting. The centroid of the base of every footing should be made to coincide with the resultant of the net dead load, the normal live load, and a small fraction, such as 25 per cent, of the crane load; and all the footings should be proportioned for the same soil pressure under this resultant load. On the other hand, if the footings rest on sand, they should be laid out so that the soil pressure is uniform and equal to q_a under the net dead load, the normal live load, and the maximum crane load that can be expected under ordinary operating conditions. Under no conceivable combination of loads should the pressure 1.5 q_a be exceeded.

5.9 PRECUATIONS DURING CONSTRUCTION :

All footing foundations are inevitably designed on the assumption that the soil beneath the footings is in approximately the same state as that disclosed by whatever borings or load tests were made. If the soil contains soft packets not encountered by the borings, or if the soil structure is disturbed during excavation, the settlement will be larger and more unequal than the designer anticipated. To avoid such a risk a simple penetration test should be made at the site of each footing afte r the excavation is completed. One of the several practicable methods is merely to count the number of blows per foot required for driving a sounding rod into the ground by means of a drop weight. If exceptionally soft pockets are encountered within the seat of settlement of any one footing, this footing should be redesigned. Such a procedure is more economical than subsequent repair.

Disturbance of the structure of the subsoil during construction is especially likely to occur under two conditions commonly encountered in the field. If the subsoil contains chiefly of silt or fine sand, it can be radically disturbed by pumping from open sumps. The disturbance is likely to be associated with serious damage to adjoining property due to loss of ground. Hence, if footings on such soils require excavation below the water table, the site should be drained by

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pumping from well points and not from open sumps.

If the subsoil consists of clay, the top layer of the exposed clay is likely to become soft because of the absorption of moisture from puddles and the kneading effect of walking on it. Therefore, footings on clay should be concreted and back-filled immediately after the excavation is completed. If this cannot be done, the last 4 to 6 in. of clay should not be removed until preparations for placing the concrete are complete.

5.10. SUMMARY AND CONCLUSIONS :

On account of the complexity of relations involved, scientific research in the realm of footings did not yield any results of immediate practical usefulness. However, it cleared the field of deep-rooted superstitions, and disclosed the type and relative importance of the factors which determine the failure and settlement of footings. Expedient and yet adequate procedures for footing design were subsequently developed by radical simplification of the real relationships.

Nevertheless, the present review work points out certain conspicuous differences between various theories and analyses of bearing capacity. The shape of the failure surface has been a much controversial issue, as discussed in Chapter 3. Evaluation of settlement for both cohesive and cohesionless soils, on a rigorous basis is needed. Current analytical methods for establishing bearing value place major emphasis on the criterion of elimination of rupture. Development of convenient, practical procedures for estimating settlement especially in constratified soils has to some extent been neglected.

The necessity for future fundamental research on bearing capacity and settlement of footings being as stated above, the procedure for adapting the theoretical knowledge to the practical requirements needs more light to be thrown on it. The development work can be carried in the field in connection with foundation jobs, and the relative value of results obtained can be judged only on the basis of well documented case records.

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