ANALYSIS OF UNSTEADY FLOW TO A LARGE-DIAMETER WELL USING DISCRETE KERNEL APPROACH

A THESIS

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By

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.... to dear MANISHA for sparing her Papa for completing this task.

CANDIDATE'S DECLARATION

I hereby certify that the work, which is being presented in the thesis entitled **ANALYSIS OF UNSTEADY FLOW TO A LARGE-DIAMETER WELL USING DISCRETE KERNEL APPROACH** in fulfilment of the requirement for the award of **Degree of Doctor of Philosophy** submitted in the Department of Earth Sciences of the University is an authentic record of my own work carried out during a period from Jul. 1984 **to May 1989**, under the supervision of Dr. G.C. MISHRA and Dr.(Prof.) B.B.S. SINGHAL.

The matter embodied in this thesis has not been submitted by me for the award of any other degree.

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ABSTRACT

Large-diameter wells are extensively used in many parts of the world. The low cost and simplicity of their construction and operation are the main reasons for their extensive use. Another important advantage of these wells is that thay are suitable for shallow aquifers with low transmissivity. In India and in other South Asian countries, people have been using large-diameter wells tapping mostly the phreatic and in some areas, the shallow semi-confined aquifers near to the surface since ancient times. Dug wells continue to be the primary source of groundwater in rural India. As reported by Ghosh (1987), of the total 14.84 million approximate number of wells in India about 67 percent are dug wells with large-diameter.

Accounting for well storage, Papadopulos and Cooper (1967) have analysed unsteady flow to a large-diameter well, which taps an aquifer of infinite areal extent. The solution has been obtained by integral transform technique. Results for drawdown in the piezometric surface due to continuous pumping at constant rate have been presented by them. Since then many investigators have contributed to this field. For aquifer with low transmissivity, it may so happen that more water may flow from the aquifer to the well during recovery phase than during pumping. In such hydrogeological condition the analysis of flow to a large-diameter well during recovery is quite important. Most of the analyses of flow to a large-diameter well made so far, are based on the assumption that the pumping rate is independent of drawdown at the well. However, if a centrifugal pump is used for abstraction of water from the well, it is not possible to pump at a constant rate independent of the drawdown at the well. Another assumption, that the aquifer is of infinite areal extent, may not be valid for hard rock areas. Considering these facts and limitations analysis of unsteady flow to a large-diameter well has been

carried out in the present thesis by discrete kernel approach. The discrete kernel coefficients are the response of a linear system to a unit pulse excitation. In the discrete kernel approach, the time parameter is discretised by uniform time-steps; the excitation and the response are assumed to be piece wise constants within each time-step; the response of the linear system to a time-dependent excitation is predicted making use of the discrete kernel coefficients. Desired accuracy in the results can be achieved with selection of appropriate time-step size. The methodology provides tractable solution.

In order to have a better understanding of the flow mechanism associated with the large-diameter wells in different hydrogeological and physical conditions, the following analyses have been carried out in the present thesis :

(i) Analysis of flow to a large-diameter well during the recovery period.

(ii) Analysis of unsteady flow to a large-diameter well due to abstraction that varies linearly with drawdown at the well.

(iii) Analysis of flow to a large-diameter observation well due to pumping of a large-diameter production well.

(iv) Analysis of unsteady flow to a large-diameter well experiencing well loss.

(v) Analysis of flow to a large-diameter well in a finite aquifer.

Analysis of Flow to a Large-Diameter Well During the Recovery Period

Analysis of flow to a large-diameter well during pumping has been carried out by several researchers. Foremost among the solutions is that of Papadopulos and Cooper (1967), who have presented the type curves for estimating aquifer parameters. The evaluation of aquifer response by Papadopulos and

Cooper's method requires numerical integration of an improper integral involving Bessel's function. The numerical integration therefore involves large computations. Although a unique value of transmissivity can be obtained with the type curves given by Papadopulos and Cooper, the evaluation of storage coefficient from a short duration pump test data is questionable. According to Papadopulos and Cooper, for accurate determination of storage coefficient, the well should be pumped beyond the time $t = 25\ r_c^2/T$, where r_c and T are the radius of the well casing and aquifer transmissivity respectively. In case of aquifer with low transmissivity, it may not be possible to pump upto the required time as the well may go dry due to abstraction from well storage during pumping. Under such circumstances, evaluation of aquifer parameters with the help of recovery data is appropriate. In the present thesis analysis of unsteady flow to a large-diameter well both during pumping and recovery periods has been done using discrete kernel approach. A family of type curves has been presented for different durations of pumping. These type curves provide a fairly accurate means of determining aquifer parameters from data of pump tests conducted in large-diameter wells. The replenishment of well storage at various times after the cessation of pumping has been estimated. The sensitivity of the solution to the time time-step size has been studied.

Analysis of Unsteady Flow to a Large-Diameter Well due to Abstraction that Varies Linearly with Drawdown at the Well

It has been found that if a centrifugal pump is used for abstraction from a dug well, there is a gradual decline in discharge because the height of water stored above the footvalve of the pump declines with pumping. The variation in discharge rate with time in several dug wells in basaltic terrains have been investigated by Athavale et al. (1983). It has been reported by them that the discharge rate may be either a linear or a nonlinear function of the drawdown. In the present study unsteady flow to a large-diameter well induced by a drawdown-dependent time-variant pumping has been analysed using discrete kernel approach. A linear relationship between pumping rate and drawdown at the well has been assumed to hold good. Tractable analytical expressions have been derived for determining the aquifer contribution, well storage contribution and drawdown at any point in the aquifer. It is shown that with an average pumping rate, it will not be possible to simulate the drawdown and aquifer response that would evolve due to drawdown-dependent time-dependent pumping of a large-diameter well.

Analysis of Flow to a Large-Diameter Observation Well due to Pumping of a Large-Diameter Production Well

A large-diameter well can also serve as an observation well if a pumping test is conducted in a production well of negligible diameter. Storage associated with large-diameter production or observation well modifies and causes delay in the aquifer response. Barker (1984), has identified that, if both the production well and the observation well have storages, a tractable solution for the drawdown at any point in the aquifer is yet to be known. In the present study a generalised discrete kernel approach has been described to analyse the combined effect of the production and the observation well storages on drawdown at any point in the aquifer during pumping and recovery phases of a pumping test . The nondimensional time-drawdown graphs have been presented for four different combinations of production and observation wells located at a distance, r_1, apart which may or may not have storage. The contribution of observation well storage to the aquifer during pumping and the replenishment of observation well storage during recovery have been presented both for different distances between the production and observation wells and for different radii of well casings. It has been verified that the drawdown in an observation well with negligible storage due to pumping in a large-diameter well is same if the roles of the wells are reversed. It is seen that the influence of the observation well storage on drawdown at the production well during recovery is more pronounced than during abstraction phase. The production well storage controls the drawdown at the production well during pumping irrespective of the observation well storage.

Analysis of Unsteady Flow to a Large-Diameter Well Experiencing Well Loss

The concept of step-drawdown test in a water well was first presented by Jacob (1947) as a means to separate the components of drawdown pertaining to laminar and turbulent flow regimes. Jacob assumed that the laminar component is directly proportional to the discharge rate and that the turbulent component is a second-order function of well discharge. This assumption is widely used in practice. Since then significant contributions were made by several investigators towards the development of the techniques for collection and analysis of the step drawdown test data to find the flow components and aquifer parameters. Although many researchers have dealt with step drawdown test and estimation of well losses, no attempt was made to take into account of the well storage. In the present study unsteady flow to a large-diameter well in a confined aquifer has been analysed taking into account the well losses. The effect of well storage on well loss component and on the specific drawdown has been investigated. It is found that, if well storage effect is accounted for, the variation of specific drawdown with pumping rate is nonlinear. However, for small and large pumping rates, the variation tends to be linear. The well loss component can be greately reduced by providing well storage.

Analysis of Flow to a Large-Diameter Well in a Finite Aquifer

In hard rock areas, the weathered and the fractured zones form an aquifer. Therefore, the aquifers in a hard rock area are likely to be of finite areal extent and the hydrologic boundary is likely to be a no-flow boundary. In the present thesis, using discrete kernel approach, unsteady flow to a large-diameter well located at the centre of a finite aquifer of circular shape has been analysed during pumping and recovery phases. The nondimensional time-drawdown graphs at specific locations in the aquifer have been presented. The recovery characteristics of well storage has also been analysed. It is found that well storage contribution is little affected by the presence of the barrier boundary where as the drawdown characteristics during pumping as well as during recovery are influenced significantly by the barrier boundary.

It is shown that various problems of unsteady flow to a large-diameter well in a homogeneous isotropic and confined aquifer during pumping as well as during recovery, can be solved with ease by discrete kernel approach. The solutions obtained by discrete kernel approach are tractable for numerical computations.

LIST OF FIGURES

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LIST OF NOTATIONS

The following notations have been used in the thesis. This also includes notations of original papers reviewed in Chapter 2.

Notation	Description of Notation	Dimension
A^*	cross sectional area of the dug well	L^2
a^*	length of reach	L
a	constant	-
a_1	radial distance between the centre of the weil and the impervioуis boundary	L
B	aquifer loss factor	TL^{-2}
B^*	constant	-
b^*	breadth of reach	L
b	constant	-
C	well loss factor	T^2L^{-5}
c	constant	-
C_n	$n \pi \rho_w/2$	-
D	depth of water column in the well prior to pumping	L
$E_1(.)$	an exponential integral $= \int_x^\infty \frac{e^{-y}}{y} dy$	-
G	depth to water level	L
H	initial saturated thickness of the aquifer	L
H_a	average thickness of the aquifer	L
h	thickness of the aquifer below water table	L
I	an integer/index	T
$J_o(\ )$	Bessel function of first kind of zero order	-
$J_1(\ )$	Bessel function of first kind of first order	-
$K_0(\ )$	modified Bessel function of second kind of zero order	-

$K_1(\ )$	modified Bessel function of second kind of first order	-
K_v	permeability of aquifer in vertical section	LT^{-1}
K_h	permeability of aquifer in horizontal direction	LT^{-1}
K	hydraulic conductivity	LT^{-1}
l	distance from water table to bottom of unlined part of abstraction well	L
l'	l/h	-
l_1	distance from water table to top of unlined part of abstraction well	L
l_1'	l_1/h	-
M	total number of wells	-
m	time step/integer	T
n	time step/integer	T
P_l	lateral permeability of the aquifer	LT^{-1}
p	parameter of transformation	-
Q	constant rate of pumping	L^3T^{-1}
Q_I	initial maximum rate of pumping	L^3T^{-1}
Q'_W	average discharge from well storage	L^3T^{-1}
Q_W	instantaneous discharge from well storage	L^3T^{-1}
Q_i	inflow discharge from the aquifer	L^3T^{-1}
Q_a	instantaneous discharge from the aquifer	L^3T^{-1}
$Q_p(n)$	quantity of water pumped from the well at the end of nth time step	L^3T^{-1}
$Q_A(n)$	quantity of water with drawn from aquifer storage at the end of nth time step	$L^3\ T^{-1}$
$Q_W(n)$	quantity of water withdrawn from well storage at the end of nth time step	$L^3\ T^{-1}$
$Q_o(n)$	quantity of recharge taking place from the observation well storage at the end of nth time step	$L^3\ T^{-1}$

q	is equal to $(p\phi/T)^{\frac{1}{2}}$	TL^{-2}
R	conditional radius of influence of the well	L
r_w	radius of the well screen	L
r_c	radius of the well casing	L
r_1	distance between production well and observation well	L
r_2	distance between the point under consideration and the centre of the observation well	L
r_{wp}	radius of the production well screen	L
r_{cp}	radius of the production well casing	L
r_{wo}	radius of the observation well screen	L
r_{co}	radius of the observation well casing	L
r	distance measured from the centre of the production well to any specific point in the aquifer	L
r_{cm}	casing radius of observation well	L
r_{cw}	casing radius of discharging well	L
r_m	distance between discharging and observation well	L
ΔS	drawdown/recovery in an observation well at distance r from the abstraction well	L
$S_W(t)$	drawdown in the well at time t	L
$S_W(n)$	drawdown in the abstraction well at the end of nth time step due to withdrawal from well storage	L
$S_A(n)$	drawdown in the aquifer at the abstraction well face at the end of nth time step due to withdrawal from aquifer storage	L
$S_r(n)$	drawdown in the aquifer at any distance r from the centre of the abstraction well at the end of nth time step	L
S_F	maximum drawdown at which the pumping rate would diminish to zero	L
$S_{Wp}(n)$	drawdown in the water surface at production well due to abstraction from production well storage at the end of nth time step	L

$S_{Wo}(n)$	drawdown in the water surface in the observation well due to recharge taking place from observation well storage to the aquifer at the end of nth time step	L
$S_{Ap}(n)$	drawdown in the piezometric surface in the aquifer at the production well face at the end of nth time step due to withdrawal from aquifer storage through the production well and recharge from the observation well storage to aquifer	L
$S_{Ao}(n)$	drawdown in the piezometric surface in the aquifer at the observation well face at the end of nth time step due to contribution of aquifer storage to pumping	L
S'	drawdown in the aquifer adjacent to the well face	L
ΔS	incremental drawdown in the well	L
S_m	drawdown at the observation well	L
s_o	maximum drawdown attained in the well when pumping is stopped	L
s	residual drawdown	L
T	transmissivity of the aquifer	L^2/unit time
t	time	T
t_p	total duration of pumping	T
t_i	time correction	T
Δt	incremental time	T
t'	time after the stoppage of pumping	T
U	drawdown function	L^2
V	volume of cone of depression plus storage volumes of discharging and observation well	L^3
x_n	argument of exponential integral	-
x_w	argument of exponential integral	-
$Y_o(\ )$	Bessel function of the second kind of zero order	-
$Y_1(\ )$	Bessel function of the second kind of first order	-

$\delta_{r_2}(I)$ discrete kernel coefficient and is equal to

$$-\frac{1}{4\pi T}\left[E_1\left(\frac{\phi r_2^2}{4TI}\right) - E_1\left\{\frac{\phi\, r_2^2}{4T(I-1)}\right\}\right] \qquad L/L^3/T$$

$\delta_{r_{wp}}(I)$ discrete kernel coefficient and is equal to

$$-\frac{1}{4\pi T}\left[E_1\left(\frac{\phi r_{wp}^2}{4TI}\right) - E_1\left\{\frac{\phi\, r_{wp}^2}{4T(I-1)}\right\}\right] \qquad L/L^3/T$$

$\delta_{r_{wo}}(I)$ discrete kernel coefficient and is equal to

$$\frac{1}{4\pi T}\left[E_1\left(\frac{\phi r_{wo}^2}{4TI}\right) - E_1\left\{\frac{\phi\, r_{wo}^2}{4T(I-1)}\right\}\right] \qquad L/L^3/T$$

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(A.G CHACHADI)

CONTENTS

CHAPTER 1

INTRODUCTION

The rapid expansion of population, industry and agriculture in recent years, throughout much of the world, has brought about a steep increase in water demand. The current per capita world demand for water has been estimated (Frits Van der Leeden, 1975) at about 1.59 m^3 per day made up of a domestic supply component of 0.15 m^3, an industrial requirement of 0.12 m^3 and an agricultural use of 1.32 m^3. The requirements have to be met mostly from available surface water and groundwater resources. However as the number of suitable surface storage sites have already been made use of, greater stress on use of groundwater appears inevitable to meet the ever increasing water demand. In most countries of the temperate region, groundwater represents a significant and in places predominant proportion of the available water resources. The specific nature of groundwater as a water resource is initially the result of the physical conditions under which it occurs, of its distribution, and of its regime within the natural environment. Groundwater basins are typically endowed with a stock of water that has built up over a time from a relatively small flow or recharge to the subsurface reservoir. This water is generally in the process of movement through the permeable aquifer materials from a place of recharge to a place of discharge. On an average, the rate of discharge from the aquifer, over long periods of time, is equal to the rate of input, so that, under natural conditions, prior to human interference in the form of continued developmental activity, aquifers are in a state of average dynamic equilibrium. Wise exploitation of groundwater contributes substantially in satisfying water requirements, particularly in relation to potable water for human consumption. Commonly easy and economical to exploit, as well as much sought after because of its advantages over surface water groundwater nonetheless represents a resource sensitive to the risks of excessive exploitation and to qualitative degradation as demographic and economic growth advances in many of the developing countries.

The two types of regional aquifers which are of particular importance, are those in sedimentary basins and those associated with basement shield rocks. Aquifers in sedimentary basins in arid regions contain mainly pluvial water and current recharge is always a small proportion of the total volume in storage. Aquifers in sedimentary basins in humid areas receive significant recharge annually. Basement shield aquifers are regionally extensive but of low permeability and storage, they occur within the weathered overburden, and are more discontinuous. Their current development on a large scale is mainly for rural water supply. New techniques of abstraction and improved methods for locating high yielding areas require that a better understanding be acquired of the aquifer behaviour.

Vast areas of Arfica, South America and Asia are floored by crystalline basement rocks and although the associated aquifers are not highly productive they are of considerable importance, particularly for rural water supply. Aquifers generally occur in the weathered overburden or in the fractured bed rock and they are now being developed extensively by boreholes and dug wells mainly fitted with centrifugal pumps and other low cost withdrawal devices. The wide use of large-diameter dug wells for groundwater abstraction especially in hard rock areas calls for a thorough understanding of the flow dynamics in these wells for better management and development of groundwater resources.

Although the origin of groundwater had been understood centuries ago, the understanding of the behaviour of water bearing formations (aquifers) when pumped is relatively of recent times. Dupuit (1863) is the first scientist to analyse steady state flow of groundwater to a well. Flow towards wells and galleries was analysed by A. Thiem (1870). G. Thiem (1906) developed a field method for determining permeability of aquifers using a pumping well and the resultant drawdowns in observation wells. De Glee (1930) studied the steady-state flow towards a well in a leaky-confined aquifer replenished by an overlying formation. A bench mark study was conducted by Theis (1935) who gave the solution for unsteady flow to a well in confined aquifer. Hantush and Jacob (1955) incorporating De Glee's concept of recharge to the pumped aquifer from another aquifer through intervening semipermeable layer (aquitard), analysed the unsteady flow to a well in a leaky-confined aquifer. The other important study in the field of well hydraulics is that of Boulton (1963) who gave a mathematical solution for evaluation of drawdown due to pumping of an unconfined aquifer having delayed yield characteristics.

Analytical solutions of unsteady flow to a well considering well storage have been developed by several research workers (Papadopulos and Cooper, 1967; Lai et al., 1973; Lai and Wusu, 1974; Boulton and Streltsova, 1976; Fenske, 1977; Rushton and Holt, 1981; Herbert and Kitching, 1981; Basak, 1982; Patel and Mishra, 1983; Rushton and Singh, 1983; 1987 etc.).

If transmissivity and storage coefficient of an aquifer are small, neither drawdown nor recovery in large-diameter well conform to the Theis model. No account of manipulation of the Theis equation will produce valid results unless the storage in the well can be accounted for.

Many a time it may not be practicable to use the domestic wells for carrying out pumping tests. In the absence of other test wells, measurements of the well reponses to pumping for normal well usage may have tobe made.

When a well of negligible diameter is pumped for a very small duration with a small discharge rate, much of the pumped water is taken from well-bore storage and hence the aquifer response is quite local. If a small volume of water is pumped over a small duration, the response of the well is more likely to resemble that of a finite-diameter well than the infinitesimal-well assumed in the Theis method (Booth, 1988). The response of an aquifer during recovery phase is quite significant for large-diameter well. Therefore, aquifer properties play a significant role during the recovery phase than during the pumping phase. Hence, recovery data of pumping test in a large-diameter well are more useful than the data of the pumping phase.

Groundwater hydrology is a quantitative science and mathematics is its important dialect. Mathematical tools have enabled analysis of many complex groundwater flow problems. Discrete Kernel approach is comparatively new within its ambit. The discrete kernel coefficients are response of a linear system to a unit pulse excitation given to the system during the first unit time period. In an unsteady flow problem the time parameter can be conveniently discretised and within each time step the input to the system can be assumed to be constant but it can vary from time step to time step. Knowing the response of the system for a unit pulse excitation, solution to the initial value problem can be conveniently obtained. The problem becomes simple when the flow domain is homogeneous. This method is recognised as discrete kernel method and tractable solutions for many complex initial value problems have been obtained by this approach. In the last two decades, many complex groundwater flow problems have been analysed by the discrete kernel approach (Maddock, 1972; Morel-Seytoux, 1975; Morel-Seytoux and Daly, 1975; Patel and Mishra, 1983; Mishra, et al., 1985). The advantages in solving groundwater flow problems by discrete kernel approach has been highlighted by Morel-Seytoux, (1975).

The discrete kernel approach is not limited to the rare situations when the pumping kernel function is known analytically. For heterogeneous aquifers, of finite size and intersected by a stream, the methodology has already been developed and implemented on the computer (Morel-Seytoux and Daly, 1975).

The advantage of the methodology results from the following facts :

(a) A finite difference model is used only to generate basic response functions to specialized excitations in an aquifer. Once these basic response functions have been calculated for a particular aquifer and saved, simulation of the aquifer behaviour to any pumping pattern is obtained without even making use any longer of the numerical model.

(b) Because the finite difference model is used only to generate the response functions smaller grid sizes and time increments can be used to calculate accurately the influence coefficients than is usually feasible when performing a large number of simulation runs under many varied pumping patterns. Also with this procedure the accuracy of the calculations for an actual simulation remains that with which the influence coefficients were obtained. On the other hand in typical simulation approaches the accuracy of the finite-difference model is usually tested with an analytical solution using small time and space increments. When the simulator is used on an actual aquifer, vastly different time and space increments are used and the accuracy of the results is to a large degree unknown.

(c) Because the response functions are known explicitly in terms of the controllable (decision) variables many management problems can be solved through the efficient algorithm associated with a well structured Mathematical Programming formulation.

Using discrete kernel approach unsteady flow to a large-diameter well has been studied under different hydrogeological situations and the results are presented in the thesis. The scheme of presentation in the thesis is as follows :

Chapter 2 deals with the review of literature pertaining to flow to a large-diameter well in different hydrogeological conditions and application of discrete kernel approach to groundwater flow problems. In Chapter 3 an efficient method has been described to generate discrete kernels for drawdown and recovery phases in a large-diameter well in a confined aquifer. Type curves have been presented for determining aquifer parameters using pumping test data from large-diameter wells.

In Chapter 4 analysis of flow to a large-diameter well is presented for a case in which the abstraction rate from the well is linearly dependent on drawdown at the well. Analysis of flow to a large-diameter observation well has been described in Chapter 5. Chapter 6 deals with the problem of unsteady flow to a large-diameter well considering well loss component. Analysis of flow to a large-diameter well located at the centre of a finite aquifer of circular shape has been presented in Chapter 7.

Besides these, the general conclusions of the study are brought out in Chapter 8 .

CHAPTER 2

REVIEW OF LITERATURE

2.0 INTRODUCTION

Provision of adequate water supplies to meet established needs is a problem of major concern to communities located in semi-arid and arid-regions. The combined effects of steady population growth, competing demands of agricultural and industrial users, and the scarcity of available water resources have often resulted in imbalance between sustainable water supply and demand. The future possibility of meeting increased water requirements depends upon the technical and economic feasibility of developing potential supplies.

In India over 70 percent of population lives in villages whose main occupation is agriculture. Over 90 percent of the utilisable water resources are consumed by irrigation of which nearly 40 percent of the groundwater is extracted through dug wells of large-diameter. At present there are over 9 million dug wells in the country and 4 million shallow tube wells besides more than one million deep tube wells (Ghosh , 1987). From the above statistics it is obvious that the dug wells are the most common groundwater extraction structures in India. Dug wells of large-diameter are the primary source of groundwater extraction not only in India but also in other central and south-east Asian countries where the crystalline rocks predominate the aquifer system.

Hard rocks (crystalline rocks) such as granites, gneisses, schists, basalts, and indurated pre-cambrian sediments cover approximately 65 percent of the total area of the Indian continent.

The wide use of large-diameter wells is mainly due to the low cost of their construction, and simplicity of maintenance and operation. Besides, these types of wells are quite suitable for shallow aquifers with low transmissivity. The volume of water which gets stored within the well acts as a reservoir from which a large proportion of pump discharge is withdrawn. During recovery phase the well storage gets replenished slowly . This is how it becomes possible to exploit low transmissivity aquifers. For assessment of groundwater resources and estimation of yield of a well, it is necessary to have an accurate knowledge of the aquifer parameters such as transmissivity (T) and storage coefficient (ϕ). Among the various methods available for the determination of the aquifer parameters, pumping tests are the most suitable as the insitu aquifer parameters can be determined by analysing the pumping test data. Over more than three decades, considerable work has been carried out on methodology relating to analysis of pumping test data from large-diameter wells.

The analysis of the test data from large-diameter wells pose special problems. These problems arise due to low groundwater inflow into the well during the abstraction phase relative to the abstraction from well storage, and significant discharge from the aquifer to the well during the recovery phase. The storage capacity of the well retards restoration of the piezometric level in the aquifer. Regime of groundwater flow into a large-diameter well differs considerably from that of a bore well of negligible diameter. The aquifer contribution to pumping is time dependent; it increases as pumping continues, attains a maximum value equal to the pumping rate and when pumping is discontinued the aquifer contribution to well storage continues at a decreasing rate. Besides, the time dependent abstraction from aquifer storage, there are other problems such as seepage face in large-diameter wells in unconfined aquifer, partial penetration, anisotropic nature of the aquifer and observation well bore storage.

Due to the very significant effect of the well storage on drawdown, the conventional methods based on Theis (1935) equation are not suitable for analysing flow to a large-diameter wells. Analytical and numerical solutions of steady and unsteady flow to a well considering well storage have been developed by several researchers. In this chapter review of literature has been carried out pertaining to groundwater flow to large-diameter wells. The various techniques of analysis of test pumping data from large-diameter wells have also been reviewed.

2.1 LARGE-DIAMETER WELL IN CONFINED AQUIFER

In the following paragraphs analysis of unsteady flow to a large-diameter well in a confined aquifer has been reviewed :

Papadopulos and Cooper (1967) have presented a method which predicts the drawdown in a confined aquifer due to pumping from a large-diameter well. The analytical solution takes into account the well storage and determines the drawdowns which occur both in the well and in the aquifer while the well is pumped at constant rate. Assuming that well losses are negligible, expression for the drawdown distribution in and around the well has been found solving the following differential equations by Laplace transform technique :

$$\frac{\partial^2 S}{\partial r^2} + \left(\frac{1}{r}\right)\frac{\partial S}{\partial r} = \frac{\phi}{T}\frac{\partial S}{\partial t} \qquad r \geq r_w \qquad \ldots(2.1)$$

Satisfying the conditions :

$$S(r_w, t) = S_w(t) \qquad \ldots(2.2)$$

$$S(\infty, t) = 0 \qquad \ldots(2.3)$$

$$S(r, 0) = 0 \qquad r \geq r_w \qquad \ldots(2.4)$$

$$S_w(0) = 0 \qquad \ldots(2.5)$$

$$2\pi r_w T \frac{\partial S(r_w,t)}{\partial r} - \pi r_c^2 \frac{\partial S_w(t)}{\partial t} = -Q \qquad t > 0 \qquad \text{...(2.6)}$$

where,

S = drawdown in the aquifer at distance r at time t,

S_w = drawdown in the well at time t,

r = distance from the centre of well,

r_w = effective radius of well screen or open hole,

r_c = radius of well casing in the interval over which the water level declines,

t = time since well begins to discharge,

ϕ = coefficient of storage of aquifer,

T = transmissivity of aquifer, and

Q = constant discharge of well.

The solution, which has been derived by Papadopulos and Cooper, is

$$S = \frac{2Q\alpha}{\pi^2 T} \int_0^\infty (1-e^{-\beta^2/4u_w}) \{ J_o(\frac{\beta r}{r_w}) [\beta Y_o(\beta)-2\alpha Y_1(\beta)]-Y_o(\frac{\beta r}{r_w})$$

$$[\beta J_o(\beta) - 2\alpha J_1(\beta)] \} [\frac{1}{\beta^2 \Delta\beta}] d\beta \qquad \text{...(2.7)}$$

where,

$$\alpha = \frac{r_w^2 \phi}{r_c^2}, \quad u_w = \frac{r_w^2 \phi}{4Tt}, \quad \text{and}$$

$$\Delta(\beta) = [\beta J_o(\beta) - 2\alpha J_1(\beta)]^2 + [\beta Y_o(\beta) - 2\alpha Y_1(\beta)]^2$$

The drawdown S_w at the well face, which has been obtained by substituting $r = r_w$ in equation (2.7), has been expressed as

$$S_w = \frac{Q}{4\pi T} F(u_w, \alpha) \qquad \text{...(2.8)}$$

where,

$$F(u_w, \alpha) = \frac{32\alpha^2}{\pi^2} \int_0^\infty \frac{(1-e^{-\beta^2/4u_w})}{\beta^3 \Delta(\beta)} d\beta \qquad \text{...(2.9)}$$

The value of the function $F(u_w, \alpha)$ are computed by numerical integration. Plots of the well function $S_w/[Q/(4\pi T)]$ versus u_w on log-log paper for different values of α form the family of type curves that have been provided by Papadopulos and Cooper which could be used for determination of aquifer parameters. The method requires that the time-drawdown data be plotted on log-log scale. This plot is then compared with a family of type curves drawn on the same scale as that of the time-drawdown graph. The family of type curves given by Papadopulos and Cooper contains straight line portions which are parallel. These straight line portions of the type curves correspond to the period when most of the water is pumped from the well storage. If a short duration pumping test is conducted in a large-diameter well, the time-drawdown curve matches with any of the straight line portions of the type curves. Although a unique value of transmissivity can be obtained, the evaluation of the storage coefficient using such short duration pump test data is questionable as the storage coefficient would change by an order of magnitude when the data plot is moved from one type curve to another. According to Papadopulos and Cooper, the well storage dominates the time-drawdown curve upto a time t given by

$$t = 25 r_c^2 / T$$

For accurate determination of storage coefficient the well is required to be pumped beyond this time which is quite long for aquifer with low transmissivity.

Lai and Su (1974) using Laplace transform technique have obtained a theoretical solution for non-steady flow induced by an arbitrary time dependent pumping rate in a large-diameter well that penetrates a leaky artesian aquifer. The effect of well storage on the drawdown is found to be significant when the time of pumping is not large or if aquifer diffusivity $(\frac{T}{\Phi})$ is small. Though the analysis of Lai and Su takes care of the effect of linear abstraction rate, it is often not possible to represent satisfactorily the variation of abstraction rate that actually occurs in practice. Evaluation of drawdown in their method requires numerical integration of improper integral involving Bessel's functions. The numerical integration therefore involves large computations. Boulton and Streltsova (1976) have criticised the solution of Lai and Su on the basis that an error exists in the solution given by Lai and Su as the singularity has been neglected.

Fenske(1977) has extended Theis equation to remove the requirement that the discharging well and the observation well have infinite-simal diameter and there by has considered the effects of the production and the observation well storage. Fenske's analysis is based upon the simple relationship that the volume of the region of the aquifer and all of the wells or other storages at any instant in time that are emptied by the discharging well divided by the average discharge, is equal to the time required to develop that volume and its associated drawdown. The assumption that has been made by Fenske is that the water stored in the observation well recharges the aquifer instantenously with the drop in piezometric head in the adjacent aquifer. Although any number and type of storages in the radial well field of the discharging well can be considered in the mathematical procedure given by Fenske, the mathematical derivations have been made considering one observation well storage besides the discharging well storage.

The total volume of the cone of depression, discharging well, and observation well at any instant in time has been expressed as

$$V = 2\phi\pi \int_{r_w}^{\infty} Sr\,dr + \pi r_{cw}^2 S_w + (1-\phi)\pi r_{cm}^2 S_m \qquad ...(2.10)$$

in which, r = radius to any location on cone of depression, r_{cm} = radius of the casing of observation well, r_{cw} = radius of casing of discharging well, S = drawdown at any location in cone of depression, S_m = drawdown at the observation well, S_w = drawdown at the discharging well and V = volume of cone of depression plus storage volumes of discharging and observation wells.

Assuming that,

$$S = \frac{Q_a}{4\pi T}\left\{E_1\left(\frac{\phi r^2}{4Tt}\right)\right\},$$

Fenske has integrated equation (2.10) and has obtained the following expression for V :

$$V = \frac{Q_a r_w^2 \phi}{4T}\left[(r_w e^{x_w})^{-1} - \left(1-\frac{1}{\alpha}\right) E_1(x_w) + \frac{1}{\beta_m} E_1(x_m)\right] \qquad ...(2.11)$$

in which,

x_m = (ar_m^2) = $(r_m/r_w)^2 x_w$, argument of exponential integral referred to the observation well,

r_m = distance between discharging well and observation well,

x_w = (ar_w^2), argument of exponential integral at discharging well,

$a = \phi/(4Tt)$

$\alpha = (\frac{r_w}{r_{cw}})^2 \phi$,

$\beta_m = (\frac{r_w}{r_{cm}}) \phi/(1 - \phi)$.

Q_a = instantaneous discharge from the aquifer,

r = radial distance,

T = transmissivity,

ϕ = storage coefficient of the aquifer, and

t = time

Assuming that $t = V/Q$ where Q is the constant pumping rate, and t is the time required to develop the instantaneous volume V, the following equation has been obtained from equation (2.11).

$$\frac{4Tt}{r_w^2 \phi} = \frac{Q_a}{Q} \left[(x_w e^{x_w})^{-1} - (1 - \frac{1}{\alpha}) E_1(x_w) + \frac{1}{\beta_m} E_1 (x_m) \right] \quad ...(2.12)$$

Also,

$$\frac{4 \pi T\phi}{Q} = \frac{Q_a}{Q} E_1(x) \quad ...(2.13)$$

According to Fenske equations (2.12) and (2.13) linked together by the argument of the exponential integral describes the dimensionless drawdown versus dimensionless time at any location in the radial well field. Fenske has stated that Q_a/Q is time dependent. In that case while deriving the total volume of the cone of depression the use of $S = [Q_a/(4 \pi T)] E_1 \{(r^2 \phi/(4Tt)\}$ is questionable for a varying Q_a.

Rushton and Holt (1981) have presented an elegant digital simulation approach for analysing abstraction and recovery phase data of pumping test in a large-diameter well which tap either a confined aquifer or an unconfined aquifer. The existence of the seepage face in the abstraction well, variable abstraction rate and well losses have been included in the digital model. A very high transmissivity value and a storage coefficient value equal to one are assigned in the free water region inside the large-diameter well to simulate the well storage. A region of low permeability is assigned for the aquifer just adjacent to the discharging well to simulate the effect of seepage face. It has been found by Rushton and Holt that for different combinations of aquifer permeability and the extent to which the permeability can artificially be reduced in the region close to the well, one may possibly obtain near identical drawdown in the well. Therefore, to get a unique values of aquifer parameters, the field and computed results of drawdown at additional observation wells in the aquifer would also need to be matched.

Patel and Mishra (1983) have analysed unsteady flow to a large-diameter well by discrete kernel approach considering well storage. The variation of drawdown with time has been obtained at the well face and at a point in the aquifer. The validity of the method has been verified by comparing the drawdown at the well that has been computed by discrete kernel approach with the drawdown given by Papadopulos and Cooper (1967). The method proposed by Patel and Mishra is simple and involves inversion of only a 2 x 2 matrix. On the other hand the evaluation of the aquifer response by Papadopulos and Cooper's method requires numerical integration of an improper integral involving Bessel's function, which involves large computations.

Rushton and Singh (1983) have developed type curves using numerical approach, for both constant and variable abstraction rates from a large-diameter well. However, it has been stated that the estimation of storage coefficient by the numerical approach is questionable. The assumed linear variation of well discharge with drawdown may introduce error in the analysis because in field, discharge variation are not strictly linear.

Barker (1984) has derived an expression for a drawdown in a large-diameter observation well near a pumping well of negligible diameter. The analysis provides an estimate of the delay in response of an observation well with finite storage capacity. The solution is derived using the Laplace transform technique. It is shown in the analysis that the drawdown in a large-diameter observation well in response to pumping of a production well of negligible diameter is identical to the drawdown that would be observed if the roles of the wells were reversed. The solution does not provide an expression for the drawdown in the aquifer other than at the single observation well. Therefore, it is not possible to use the solution to determine the extent of the effect of the observation well storage on the response of the aquifer.

Mucha and Paulikova (1986) have studied the effect of storage of large-diameter observation well as well as production well on a piezometric head at any point in the aquifer. The approximate expression for calculating drawdown at any point in the aquifer caused by pumping in a large-diameter well has been expressed as

$$S = \sum_{i=1}^{n} \frac{Q_i - Q_{i-1}}{4 \pi T} \quad W \left[\frac{\phi r^2}{4T(t_i - t_{i-1})} \right]$$

The storage in the pumping well is included in the analysis by considering the appropriate aquifer discharges Q_1 in t_1, Q_2 in t_2, ... ,and Q_n in t_n, where n is the number of discrete time steps and Q_i is the quantity of water withdrawn from aquifer storage at time t_i.

Chachadi and Mishra(1986) have derived expressions for the drawdown in a large-diameter well for variable abstraction rates. A quadratic relationship between pumping rate and drawdown has been assumed in the derivation of the expressions for drawdown in a large-diameter well in a confined aquifer of infinite areal extent. A comparison of the drawdowns computed for average constant pumping rate with those computed for variable pumping rate showed considerable difference and hence it was suggested that an average pumping rate cannot substitute the drawdown dependent variable abstraction rate.

2.2 LARGE-DIAMETER WELL IN UNCONFINED AQUIFER

Literature review pertaining to flow to large-diameter well in unconfined aquifer has been reviewed in the following paragraphs :

Zdankus (1974) has reported a method of pump test data analysis applicable for dug wells in hard rock areas in which the hydraulic conductivity decreases linearly with depth. The hydraulic conductivity has been assumed to be maximum at the static water level and zero at the bottom of the aquifer. A drawdown function U has been worked out for the estimation of average hydraulic conductivity (K) and a conditional radius of influence (R). The approximate equations that have been developed by Zdankus to determine the radius of influence and the average hydraulic conductivity are :

$$R = 1.5 \sqrt{\{\beta (t + t_i)\}} \qquad ...(2.14)$$

in which R is the conditional radius of influence of the well at instant 't' that is reckoned since the start of pumping, β is the ratio of the transmissivity, T, to the specific yield, ϕ_y, of the aquifer, the transmissivity is the product of the hydraulic conductivity and an average thickness of the aquifer equal to H-S'/2, and t_i is a time correction introduced because of the finite radius of the well, r_w, and is given by $t_i = (r_w^2/2.25\beta)$; and

$$\frac{K}{\ln (R/r_w)} = \frac{Q_i}{2\pi U} \qquad ...(2.15)$$

in which,

Q_i = the discharge from the aquifer, and

U = the drawdown function given by

$U = S' [H - (S'/2)]$, where S' is the drawdown in the aquifer adjacent to the well face and H is the initial saturated thickness of the aquifer. During the abstraction phase Q_i is computed as $Q_i = Q - [\pi r_w^2 (\Delta S/\Delta t)]$, and during the recovery phase Q_i is computed as $Q_i = \pi r_w^2 (\Delta S/\Delta t)$, in which, Q is the pumping rate.

In the above two expressions, ΔS is the change in water level in the well during an interval, Δt, between two time instants. Q_i is the discharge from the aquifer at a time which is at the middle of the two time instants. The values of the hydraulic conductivity of the aquifer K and the conditional radius of influence R for each discrete time interval have been obtained using equations (2.14) and (2.15) by a trial and error method. While using equation (2.14) an assumption has to be made on the specific yield value of the aquifer depending on the rock type at well site. As concluded by Zdankus, this method of analysis is based on approximate equations and the accuracy of the estimated aquifer parameters may not be high. The drawback is that the drawdown adjacent to well face is difficult to measure. However, the equations are useful to analyse flow during recovery phase because during recovery the drawdown in the well is approximately equal to the drawdown in the aquifer at the well face especially towards the later part of the recovery phase.

Boulton and Streltsova (1976) have presented an analytical solution for flow to a partially penetrating large-diameter well in an unconfined aquifer. The anisotropy of the aquifer in respect of hydraulic conductivity has been taken into account in the solution. The method relies on curve matching of early time-drawdown data. Since this method takes into account the compressibility and anisotropy of the aquifer, and partial penetration of the well, it offers a more realistic model for analysing unsteady flow in a well in a hard rock area. The drawdown equation that has been derived by Boulton and Streltsova is

$$S = \frac{Q}{4\pi T} \sum_{n=1,3,5,\ldots}^{\infty} G_n \sin \frac{n\pi y'}{2} \left[\frac{\pi K_o(\xi_n r/r_w)\{1 - e^{-\alpha_n \theta_w/4}\}}{K_1(\xi n)[4\phi(1'-1'_1)\xi n\{1-\phi(1'-1'_1)\} + \alpha_n^2/\xi n} \right.$$

$$\left. + \int_0^{\infty} \frac{P_2 J_o(\beta_1 r/r_w) - P_1 Y_o(\beta_1 r/r_w)][1-e^{-\frac{\theta_w}{4}(\beta_1^2 + c_n^2)}]\beta_1 d\beta_1}{(P_1^2 + P_2^2)(\beta_1^2 + c_n^2)} \right]$$

in which

$$G_n = \frac{32\phi}{\pi^2}\left[\frac{1}{n}\left(\cos\frac{n\pi 1'_1}{2} - \cos\frac{n\pi 1'}{2}\right)\right]$$

$$P_1 = (\beta_1^2 + c_n^2)\, J_o(\beta_1) - 2\,(1'-1'_1)\,\phi\,\beta_1 J_1(\beta_1)$$

$$P_2 = (\beta_1^2 + c_n^2)\, Y_o(\beta_1) - 2(1'-1'_1)\,\phi\,\beta_1 Y_1(\beta)$$

$$c_n = n\,\pi\,\rho_w/2$$

$$\theta_w = 4Tt/(r_w^2 \phi)$$

ξ_n is the positive root of : $c_n^2 - \xi_n^2 = 2\phi(1'-1'_1)\dfrac{\xi_n K_1(\xi_n)}{K_o(\xi_n)} = 0$

α_n = $c_n^2 - \xi_n^2$

K_o = modified Bessel function of the second kind and of the zero order

K_1 = modified Bessel function of the second kind of the first order

J_o = Bessel function of the first kind of the zero order

J_1 = Bessel function of the first kind of the first order

l = distance from water table to bottom of unlined part of abstraction well

l' = l/h, and h is the depth of aquifer below the water table

l_1 = distance from water table to top of unlined part of abstraction well

Q = constant volume of water per unit time discharged from abstraction well

r = horizontal distance from abstraction well axis to any point

r_w = radius of abstraction well

S = drawdown of hydraulic head at any point in the aquifer

ϕ = coefficient of storage for compressible aquifer

t = time reckoned from start of pumping

T = transmissivity of aquifer

y = depth of any point below water table

y' = y/h

Y_o = Bessel function of the second kind of zero order

Y_1 = Bessel function of the second kind of first order

ρ_w = $\mu r_w/h$

μ = $(K_v/K_h)^{\frac{1}{2}}$

K_v = permeability of aquifer in vertical direction

K_h = permeability of aquifer in horizontal direction

β_1 = a dummy variable for integration

l_1' = l_1/h

Owing to the large number of parameters involved in the solution, it is generally not possible to construct the whole set of type curves and as such there is no complete set of type curves available for use. The very complexity of the solution allows too many options to be selected for the curve matching process. Therefore, it is also clear that the well function involves too many parameters and becomes unwieldy for field use. The solution fails to provide an unique value of storage coefficient since the well function is non-linear in ϕ. Assumptions have to be made for those parameters which are not available, which in turn lead to erroneous parameter estimation.

Herbert and Kitching (1981) have proposed approximate expressions for finding the transmissivity of an unconfined aquifer. Two expressions have been derived : one using 50 percent recovery and other using 90 percent recovery of a large-diameter partially penetrating well. Singh (1982) while using the expressions derived by Herbert and Kitching for estimation of aquifer parameters from pump test data of large-diameter well have found that the expressions do not provide reasonable estimates of the aquifer parameters. The transmissivity estimated may be in error by a factor of 2 which may be either multiplying or a dividing one.

Narahari (1983) has critically analysed the well function proposed by Boulton and Streltsova (1976) and presented a modified model incorporating relevant field conditions. The modified model allows a faster computation of the well function for specified values of parameters.

Rajagopalan (1983) has presented a mathematical model for analyzing the recovery in a large-diameter well. Approximate equations to determine drawdown during the recovery phase have been derived on the assumption that the partial derivative of hydraulic head with respect to radius along the well face is linearly related to the drawdown in the large-diameter well. Using the equations, a parameter of the form $P_r B^*$, in which P_r is the lateral permeability of the aquifer and B^* is a constant, can be determined and the time required for the large-diameter well to recoup fully can be predicted.

The expression for the parameter $P_r B^*$ is written as

$$P_r B^* = 2.303\ A^*/(2\pi r_w D \Delta t')$$

where

A^* is the cross-sectional area of the dug-well

r_w is the radius of the dug-well.

D is the depth of water column in the well prior to pumping, and

$\Delta t'$ is the time difference for one log cycle of the residual drawdown in a semilog plot of S versus t', where S is the residual drawdown at time t' after the stoppage of pumping

The time taken for complete recuperation of the large-diameter well has been expressed as

$$t'_{rec} = \frac{2.303 A^*}{2\pi r_w D P_r B^*} \log \left(\frac{100\ S_o}{D}\right)$$

where S_o is the maximum drawdown attained when pumping is stopped

The maximum drawdown S_o in a well can be obtained by variety of ways which would give rise to different rates of recovery for the same maximum drawdown in a well and consequently there would be different $P_r B^*$ values for the same well. Therefore the contribution of aquifer to flow is a function of the discharge from the well and this in turn reflects in the different recovery rates. Rajagopalan has suggested that an experiment in the dug well can be designed to obtain $P_r B^*$ values for different discharges from the well that cause same maximum drawdown at the end of pumping. An empirical relationship between $P_r B^*$ and the discharge rate Q can be derived from the analysis of such experimental data.

Unlike Slitcher's (1906) formula the expressions derived by Rajagopalan take care of the effect of variable discharges on the rate of recovery and hence should provide useful means of parameter estimation from large-diameter wells.

Rushton and Singh (1987) have developed a method of analysing the pumping and recovery phases of large-diameter wells based on a kernel function approach. A consideration is given to include the effect of the seepage face which occurs when large-diameter wells in unconfined aquifers are pumped. It has been found by the authors that ignoring the seepage face generally leads to an underestimation of the transmissivity and storage coefficient of the aquifer.

2.3 ANALYSIS OF FLOW TO WELL IN FINITE AQUIFER

Generally the solutions presented for analysing unsteady flow to a well are based on the assumption that the aquifer is of infinite areal extent. Although such aquifers do not exist, many aquifers are of such wide extent that for all practical purposes they can be considered infinite. Others however are of limited extent because of the presence of an impervious barrier or a recharge boundary. If an aquifer is being pumped near a recharge or an impervious boundary, the effect of the hydrologic boundary must be considered in the analysis.

Analysis of unsteady flow to a well in an aquifer of finite areal extent has been done by Muskat (1937) and Kuiper (1972). The solution to the problem has been obtained by Laplace transform technique. However, the effect of well storage has not been considered by them.

Zekai Sen (1981) using the concept of depression cone volume and image well theory, derived type curves for large-diameter well in aquifer of finite areal extent limited by an impervious straight barrier boundary. The solution is based on the joint use of the groundwater movement equation (Darcy's law) and the continuity equation for large-diameter wells.

Basak (1982) has reported an approximate analytical solution for unsteady flow to a large-diameter well during recovery phase in a finite aquifer. A very elegant method of solving a particular class of partial differential equations describing transient groundwater flow has been used to arrive at the approximate solutions. However, the method developed by Basak has the following limitations :

The assumption of restricting the aquifer to a finite extent in the radial directions has been probably made under a notion that the cone of depression stops expanding as soon as pumping is discontinued. This is true only when the discharge into the well from the aquifer storage during recovery is negligible. However, in case of large-diameter wells the discharge from the aquifer into the well is significant during the recovery phase (Zdankus, 1974).

Chachadi and Mishra (1985) have analysed unsteady flow to a large-diameter well located near a river and a no-flow boundary using discrete kernel approach. Expressions for drawdown at any point in the aquifer have been derived using image well theory and method of superposition.

2.4 CONCLUSIONS

Based on the review of literature on flow to large-diameter wells the following conclusions have been made :

(i) The Laplace transform technique for obtaining solution to unsteady flow to a large-diameter well is rigorous but it presents solutions which are intractable for numerical computations.

(ii) There is a scope for analysing unsteady flow to a large-diameter well by discrete kernel approach.

(iii) There is a need for analysing unsteady flow to a large-diameter well considering well loss, finite aerial extent of the aquifer and variable pumping rate.

(iv) The response of an aquifer with low transmissivity is more significant during the recovery phase than the response during the pumping phase in case of large-diameter well. Therefore, solution techniques should be developed giving more weightage to the recovery phase.

(v) If the production well and the observation well possess storages a tractable solution for analysing unsteady flow needs to be developed.

CHAPTER 3

ANALYSIS OF FLOW TO A LARGE-DIAMETER WELL DURING THE RECOVERY PERIOD

3.0 INTRODUCTION

Analysis of flow to a large-diameter well during pumping has been carried out by several investigators. Foremost among the solutions is that of Papadopulos and Cooper (1967). According to Papadopulos and Cooper, the well storage dominates the time-drawdown curve up to a time 't' given by $t = (25\ r_c^2)/T$, where r_c is radius of the well casing, and T is the transmissivity of the aquifer. For accurate determination of the storage coefficient, the well should be pumped beyond this time which is quite long for an aquifer with low transmissivity. Large-diameter wells are generally constructed in shallow aquifers with low transmissivity and long duration pumping tests in such wells are therefore not practicable (Herbert and Kitching, 1981). Under these circumstances, evaluation of aquifer parameters with the help of recovery data needs due consideration. Rushton and Holt (1981) and Herbert and Kitching (1981) used numerical methods to analyse flow to a large-diameter well during the abstraction phase and the recovery phase. Patel and Mishra (1983) have analyzed flow to a large-diameter well during pumping using a discrete kernel approach. In the present chapter, the application of discrete kernel theory has been extended for analysing unsteady flow to a large-diameter well during recovery phase.

3.1 STATEMENT OF THE PROBLEM

A schematic cross section of a large-diameter well in a homogeneous, isotropic, confined aquifer of infinite areal extent is shown in Fig. (3.1). It is assumed that the aquifer prior to pumping was at rest condition. The

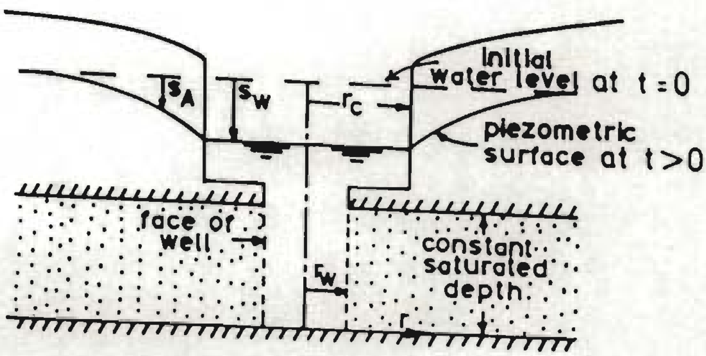


FIG. 3.1 - Schematic cross section of a large-diameter well

radius of the well screen is r_w, and that of the well casing r_c. Pumping is carried out at a uniform rate up to time t_p. It is necessary to determine the drawdown in piezometric surface at the well face and at any distance, r, from the center of the well during the recovery period.

3.2 ANALYSIS

The following assumptions have been made in the analysis :

(i) The time parameter is discrete.

(ii) Within each time step, the aquifer response and well storage response are separate constants, but they vary from step to step.

The Boussinesq's partial differential equation, which describes the evolution of piezometric surface in a homogeneous isotropic confined aquifer, for an axially-symmetric radial flow, onset by pumping of a well, is given by

$$\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r} = \frac{\phi}{T}\frac{\partial S}{\partial t}, \quad r > r_w \qquad \text{...(3.1)}$$

in which, r_w = radius of the well screen, S = drawdown in piezometric surface at distance 'r' from the well at time 't', T = transmissivity and ϕ = storage coefficient of the aquifer. To account for the well storage effect, a solution to the above equation has to satisfy the following boundary condition.

$$2\pi r_w T \left.\frac{\partial S}{\partial r}\right|_{r = r_w} - \pi r_w^2 \frac{\partial S_W}{\partial t} = -Q_p(t) \qquad \text{...(3.2)}$$

in which,

S_W = drawdown in the well,

$Q_p(t)$ = pumping rate at time, t, and

$Q_p(t)$ is equal to zero during recovery. The other boundary conditions to be satisfied are : $S(\infty, t) = 0$ for an aquifer of infinite areal extent and $S(r_w, t) = S_W(t)$, in which $S(r_w, t)$ = drawdown in the aquifer at the well face at time t. The initial condition to be satisfied is $S(r, 0) = 0$, $r > r_w$.

An exact solution to equation (3.1) has been given by Papadopulos and Cooper (1967). An alternate solution using discrete kernel approach has been given here for the recovery phase. In a discrete kernel approach the time parameter is discretised and during each discrete time interval the excitation and response are treated piecewise constants (Morel Seytoux, 1975). An accurate generation of such approximation is only possible through proper selection of time discretisation. Let the large-diameter well be pumped at a constant rate Q. In response to this pumping let $Q_A(\gamma)$, $\gamma = 1,2,\ldots,n$, be the discharges of the aquifer which are assumed to be piecewise constants.

If a well with negligible storage is pumped at rates which are constant within each period, Morel Seytoux (1975), starting from the solution given by Carslaw and Jaeger (1959) has derived the following solution to equation (3.1) :

$$S(r,n) = \sum_{\gamma=1}^{n} Q_A(\gamma)\ \delta_r(n-\gamma+1) \qquad \ldots(3.3)$$

In the above equation S(r,n) is the drawdown in the piezometric surface at the end of n^{th} unit time step at radial distance r from the well. $\delta_r(I)$ is the discrete kernel coefficient defined as :

$$\delta_r(I) = \frac{1}{4\pi T}\left[E_1\left(\frac{r^2}{4\beta I}\right) - E_1\left(\frac{r^2}{4\beta(I-1)}\right)\right] \qquad \ldots(3.4)$$

in which, the exponential integral

$E_1(X) = \int_x^{\infty} \frac{e^{-Y}}{Y} dY$, I = an integer, and β, the hydraulic diffusivity $= T/\phi$. The discrete kernel coefficient, $\delta_r(I)$, is the response of a linear system at the end of I^{th} unit time step consequent to a unit pulse excitation given to the system during the first unit time step. In the coefficient $\delta_r(I)$, 'I' is an index and it has no dimension. But the term 'I', that appears in the exponential integral $E_1 \{r^2/(4 \beta I)\}$, is an integer having the dimension of time. For computing the dimensionless term $[r^2/(4\beta I)]$, a transmissivity value, T, per unit time step size is to be used. In both the terms, $\delta_r(I)$ and $[r^2/(4 \beta I)]$, values of I are numerically equal. Identical methods for evaluating the response of a aquifer to variable pumping rates have also been described by Stallman (1962), Moench (1971), and Maddock (1972). $Q_A(\gamma)$ is known a priori in case of a well with negligible storage. In case of a large-diameter well the discharges of the aquifer are unknown. A methodology for determination of $Q_A(\gamma)$, $\gamma = 1,2,....,n$, during recovery is described here.

Let the total time of pumping be discretised to m units of equal time steps. The quantity of water pumped during any time step 'n' can be written as :

$$Q_A(n) + Q_W(n) = Q_p(n) \qquad ...(3.5)$$

in which, $Q_A(n)$ = water withdrawn from aquifer storage, and $Q_W(n)$ = water withdrawn from well storage. For $n > m$, $Q_p(n) = 0$. Otherwise $Q_p(n) = Q$, where Q is the pumping rate for unit time period. The boundary condition stated at equation (3.2) is satisfied through equation (3.5).

Drawdown, $S_W(n)$, in the well at the end of time step 'n' is given by

$$S_W(n) = \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n} Q_W(\gamma) \qquad \ldots(3.6)$$

where $Q_W(\gamma)$ represents rate of withdrawal from well storage or replenishment at time step γ. $Q_W(\gamma)$ values are unknown a priori. A negative value of $Q_W(\gamma)$ means there is replenishment of well storage that occurs during the recovery period.

Drawdown in the aquifer at the well face at the end of time step 'n' due to abstraction from aquifer storage is given by (Morel Seytoux, 1975)

$$S_A(n) = \sum_{\gamma=1}^{n} Q_A(\gamma)\, \delta_{rw}(n-\gamma+1) \qquad \ldots(3.7)$$

where,

$$\delta_{rw}(I) = \frac{1}{4\pi T} \left[E_1\left(\frac{\phi r_w^2}{4TI}\right) - E_1\left\{\frac{\phi r_w^2}{4T(I-1)}\right\}\right], \qquad \ldots(3.8)$$

Because $S_W(n) = S_A(n)$,

$$\sum_{\gamma=1}^{n} Q_A(\gamma)\, \delta_{rw}(n-\gamma+1) = \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n} Q_W(\gamma) \qquad \ldots(3.9)$$

$Q_A(n)$ and $Q_W(n)$ can be solved in succession starting from time step one using the two linear algebraic equations (3.5) and (3.9) for known values of T, ϕ, r_w, r_c, and $Q_p(n)$. Once $Q_A(n)$ values are known, the drawdown, $S_r(n)$, in the aquifer at any distance 'r' from the center of the well can be found using equations (3.3) and (3.4).

In the preceding analysis the direct problem of calculating the drawdown in and around a large-diameter well has been considered when the aquifer parameters T and ϕ are known. The inverse problem of calculating T and

ϕ from field measurement of drawdown is also equally important. The aquifer parameters can be evaluated making use of the drawdown $S_W(n)$, observed in the large-diameter well in response to a known pumping rate $Q_p(n)$ in the following manner : For known $S_W(n)$, $Q_W(n)$ can be found in succession starting from time step 1 with the help of equation (3.6). Knowing $Q_W(n)$, $Q_A(n)$ can be solved from equation (3.5). Recalling that $S_A(n) = S_W(n)$ and since $Q_A(n)$ values have been evaluated, $\delta_{rw}(n)$ can be found in succession starting from the step 1 from equation (3.9). With any two values of $\delta_{rw}(n)$, T and ϕ can be known by an iteration procedure making use of equation (3.8).

3.3 RESULTS AND DISCUSSION

The discrete kernel coefficients, $\delta_{rw}(n)$, have been generated using equation (3.8) for known values of transmissivity, storage coefficient, and radius of the well screen. The exponential integral, $E_1(X)$, which appears in equation (3.8) has been evaluated making use of the polynomial and rational approximations given by Gautschi and Cahill (1964). The computational efficiency of these approximations has been brought out by Huntoon (1980). After generating the discrete kernel coefficients, $Q_A(n)$ and $Q_W(n)$ are solved using equations (3.5) and (3.9) for known values of r_c and m. The drawdown at the well face is then obtained with the help of equation (3.6).

In order to analyse the sensitiveness of drawdown to time step size and hence to the number of time steps, the accuracies in drawdown at a particular time, calculated with different sizes of uniform time steps, have been compared. The drawdowns at the end of the first and the second day during pumping at the well face are presented in Table (3.1). These drawdowns have been calculated with time step size varying from 1/288th of a day to one day. The corresponding exact drawdowns have been determined using

TABLE 3.1 **Drawdown at Well Face During Pumping Computed with Different Sizes of Time Step and Percentage Errors in Drawdown**
[$Q = 100\ m^3/day$, $T = 50\ m^2/day$, $\phi = 0.004$, $r_w = 0.1m$ and $r_c = 2m$]

Time step size in days	Drawdown at the end of 1st day (m)	% Error	Drawdown at the end of 2nd day (m)	% Error
1	1.8220	16.50	2.3048	4.30
1/2	2.0033	8.20	2.3716	1.50
1/4	2.0996	3.80	2.3940	0.58
1/8	2.1445	1.70	2.4023	0.23
1/24	2.1707	0.50	2.4061	0.07
1/48	2.1764	0.26	2.4069	0.04
1/144	2.1798	0.10	2.4073	0.03
1/288	2.1805	0.07	2.4074	0.02

the values of well function given by Papadopulos and Cooper (1967), and the percentage errors in drawdown have been ascertained. As seen from Table (3.1) for any assumed size of time step, the percentage error diminishes with time. For example, with a time step size of $(1/8)^{th}$ of a day, the percentage error in computation of drawdown at the end of the first day is 1.70. With the same time step size, the percentage error in drawdown at the end of the second day is 0.23. The percentage error also decreases as the number of time steps used to calculate the drawdown increases. If the number of time step is increased from 8 to 288, the error in drawdown computation for the first day decreases from 1.70 percent to 0.07 percent.

The perc-entage errors in drawdown at the end of the first unit time step are given in Table (3.2) for various time step sizes. It can be seen from this table that as the time step size increases from 1/200th to 1/2.5th of a day, the percentage error increases from 0.86 to 18.94. A further in - crease in time step size results in reduction of error. If the transmissivity and radius of the well casing have values equal to 50 m^2/day and 2m. respectively the well storage will predominate the drawdown for two days since pumping starts. The percentage error in drawdown at the end of the first time step will decrease with the increase in time step size provided the time step size is more than the period during which well storage contribution is significant.

The percentage errors in drawdown at the end of the second day during the recovery period is presented in Table (3.3) for a case in which the well has been pumped for the first day. It could be seen from the table that the computation of drawdown for the recovery period is vulnerable to time step size. The time step size thus greatly influences the accuracy of drawdown computation in the beginning of pumping and recovery period during

TABLE 3.2 **Percentage Errors in Drawdown at Well Face at the End of First Unit Time Step [Q = 100 m^3/day, T = 50 m^2/day, ϕ = 0.004, r_w = 0.1m and r_c = 2m]**

Time step size in day	% Error in drawdown at the end of Ist unit time step
1/200	0.86
1/100	1.93
1/50	3.48
1/25	6.08
1/10	11.35
1/5	15.96
1/2.5	18.94
1	16.50
2	11.10
4	2.55

TABLE 3.3 **Drawdown at Well Face During Recovery Computed with Different Sizes of Time Step and Percentage Errors in Drawdown [Q = 100 m^3/day, Duration of pumping = 1 day, T = 50 m^2/day, ϕ = 0.004, r_w = 0.1m and r_c = 2m]**

Time step size in day	Drawdown at the end of 2nd day (m)	% Error
1	0.48276	113.60
1/2	0.36830	62.96
1/4	0.29492	30.49
1/24	0.23538	4.15
1/48	0.23048	1.98
1/144	0.22755	0.68
1/288	0.22689	0.39

which the well storage predominates. The appropriate size of the time step cannot be estimated mathematically. The following procedure may therefore be adopted to find the suitable size of the time step :

1. Assume an initial time step size (say 10 minutes) and obtain the results at the end of various time steps.

2. Reduce the time step size and obtain the results for the first few time steps during pumping as well as during recovery period.

3. Compare the results of these two time step sizes.

4. If the discrepancy in the results is significant, the first time step size should be discarded.

5. Repeat the procedure to obtain the appropriate time step size.

The variation of $S_W(t)/[Q/(4\pi T)]$ with $4Tt/(\phi r_w^2)$ for 'm' equal to 2,5,10, 25,50,100,250, and 500 are shown in Figures [3.2(a)] through [3.2(f)] for different values of α, where $\alpha = \phi(r_w/r_c)^2$. $S_W(t)$ is the drawdown at the well face at time t and $S_W(t)/[Q/(4\pi T)]$ can be regarded as the well function for a large-diameter well. The type curves in Figures [3.2(a)] through [3.2(f)] contain the response of an aquifer during the abstraction as well as recovery phase. Each of the recovery curves is characterized by a nondimensional time factor $4Tt_p/(\phi r_w^2)$, at which it deflects from the time-drawdown curve of the abstraction phase. The nondimensional time parameter $4Tt_p/(\phi r_w^2)$ corresponds to the duration of pumping. The nondimensional time factor, $4Tt_p/(\phi r_w^2)$, can be used to check the accuracy of the aquifer parameters determined by curve matching.

The variations of $S_r(t)/[Q/(4\pi T)]$ with $4Tt/(\phi r^2)$ are shown in Figures [3.3(a)] through [3.3(f)] for an observation point located at a distance of $10r_w$

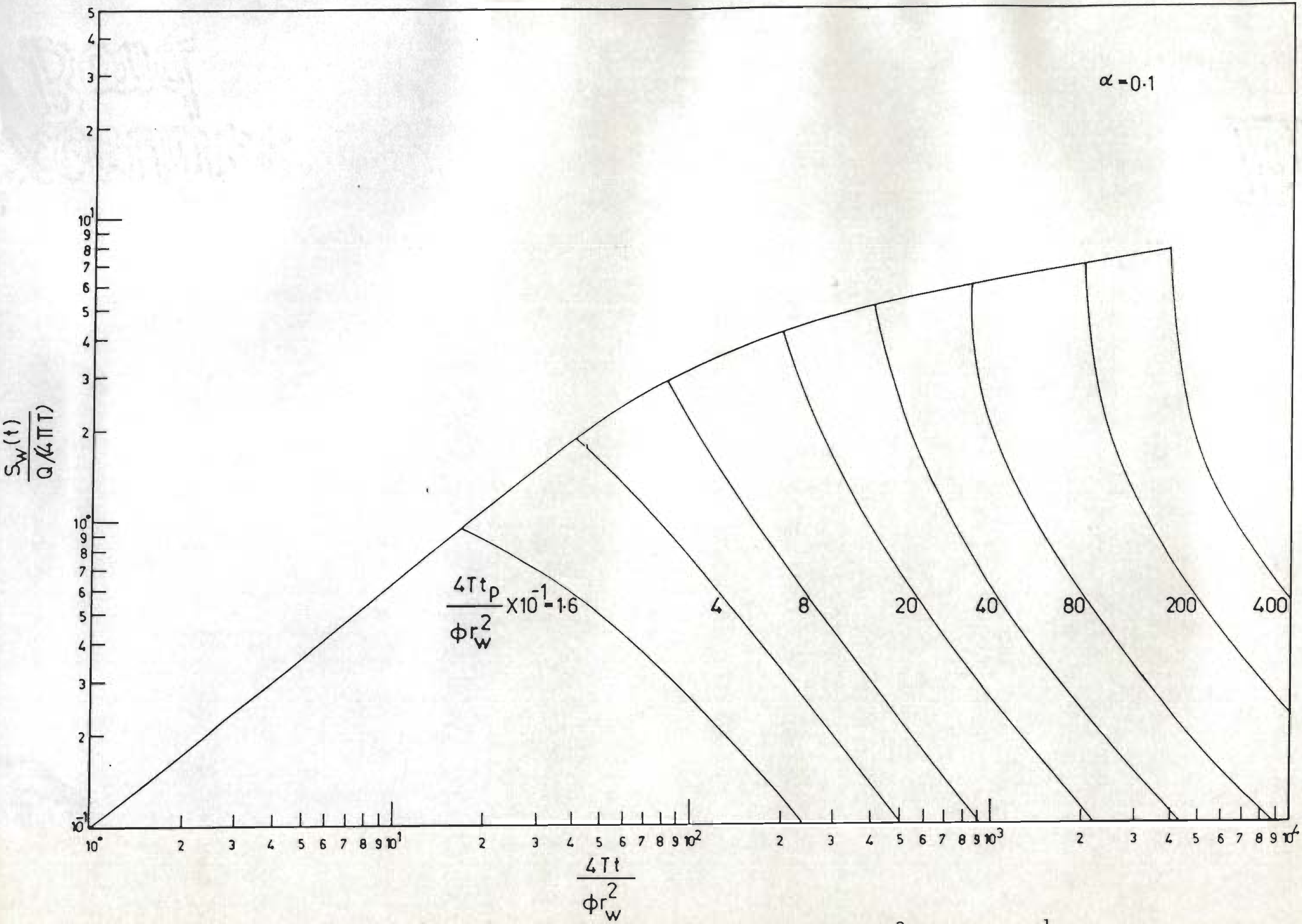


FIG. 3.2(a) - Family of type curves : $S_W(t)/(Q/4\pi T)$ versus $4Tt/(\phi\ r_w^2)$ for $\alpha = 10^{-1}$

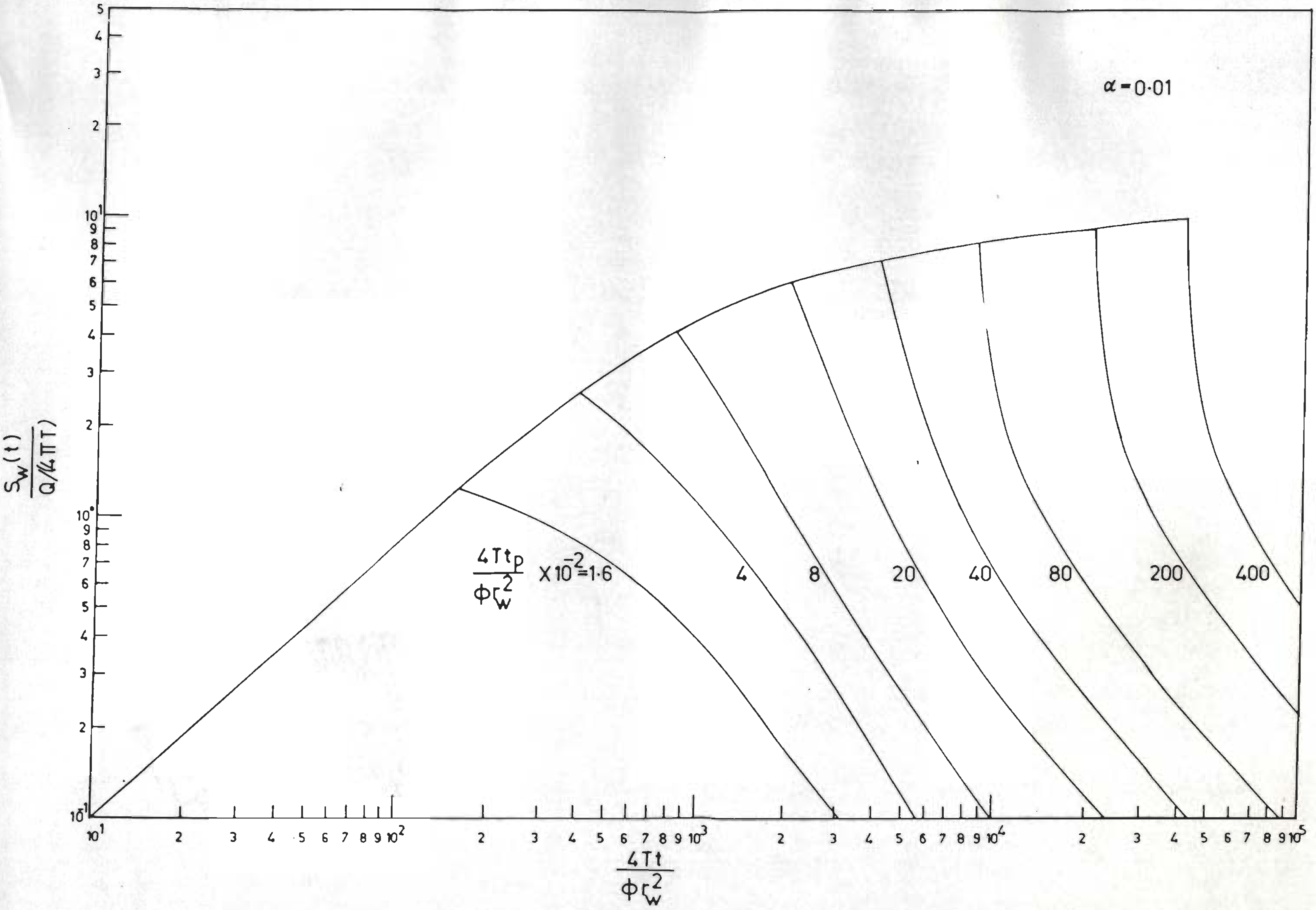


FIG. 3.2(b) - Family of type curves : $S_W(t)/(Q/4\pi T)$ versus $4Tt/(\phi\ r_w^2)$ for $\alpha = 10^{-2}$.

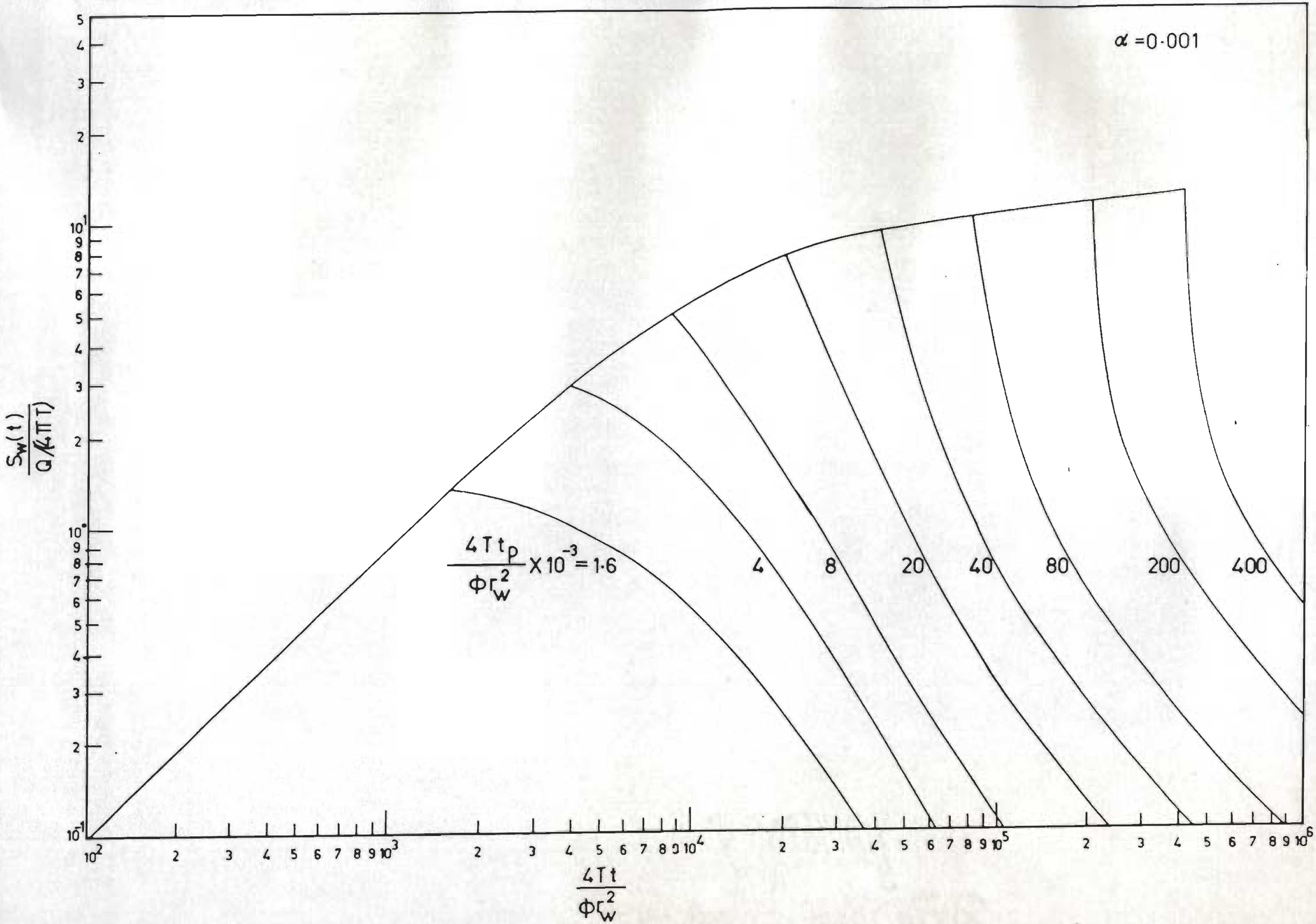


FIG. 3.2(c) - Family of type curves : $S_W(t)/(Q/4\pi T)$ versus $4Tt/(\phi r_w^2)$ for $\alpha = 10^{-3}$.

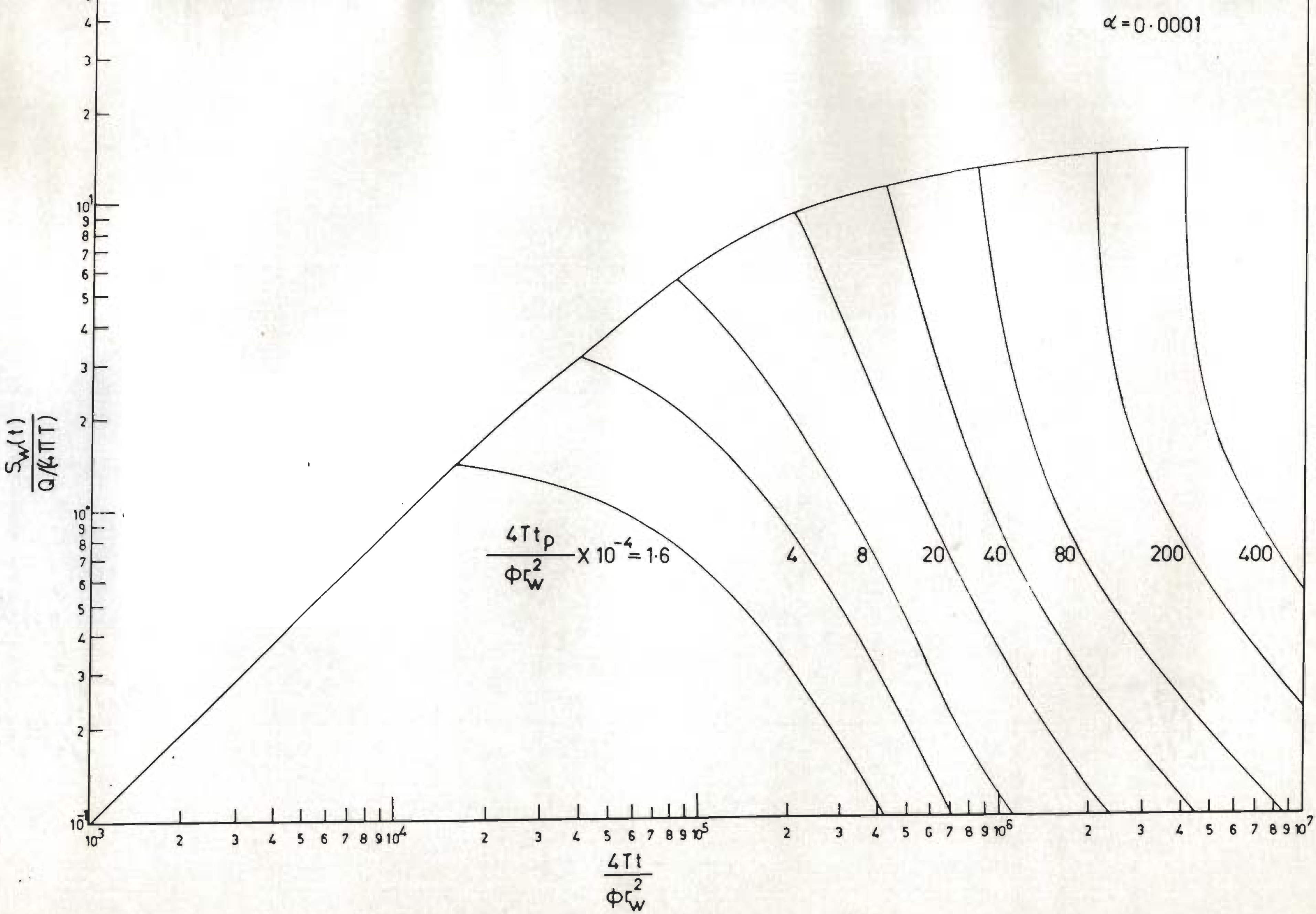


FIG. 3.2(d) - Family of type curves : $S_W(t)/(Q/4\pi T)$ versus $4Tt/(\phi r_w^2)$ for $\alpha = 10^{-4}$.

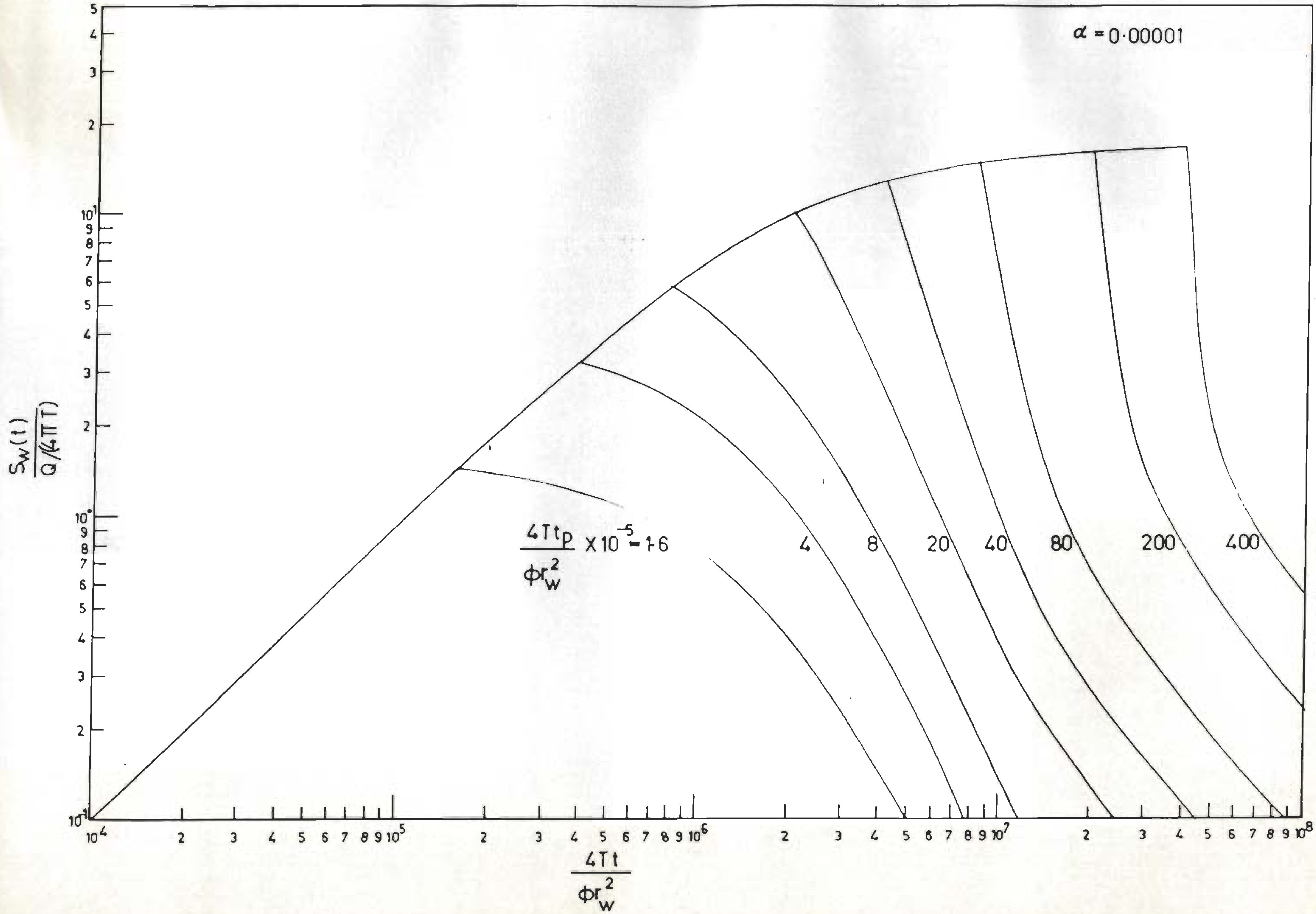


FIG. 3.2(e) - Family of type curves : $S_W(t)/(Q/4\pi T)$ versus $4Tt/(\phi r_w^2)$ for $\alpha = 10^{-5}$.

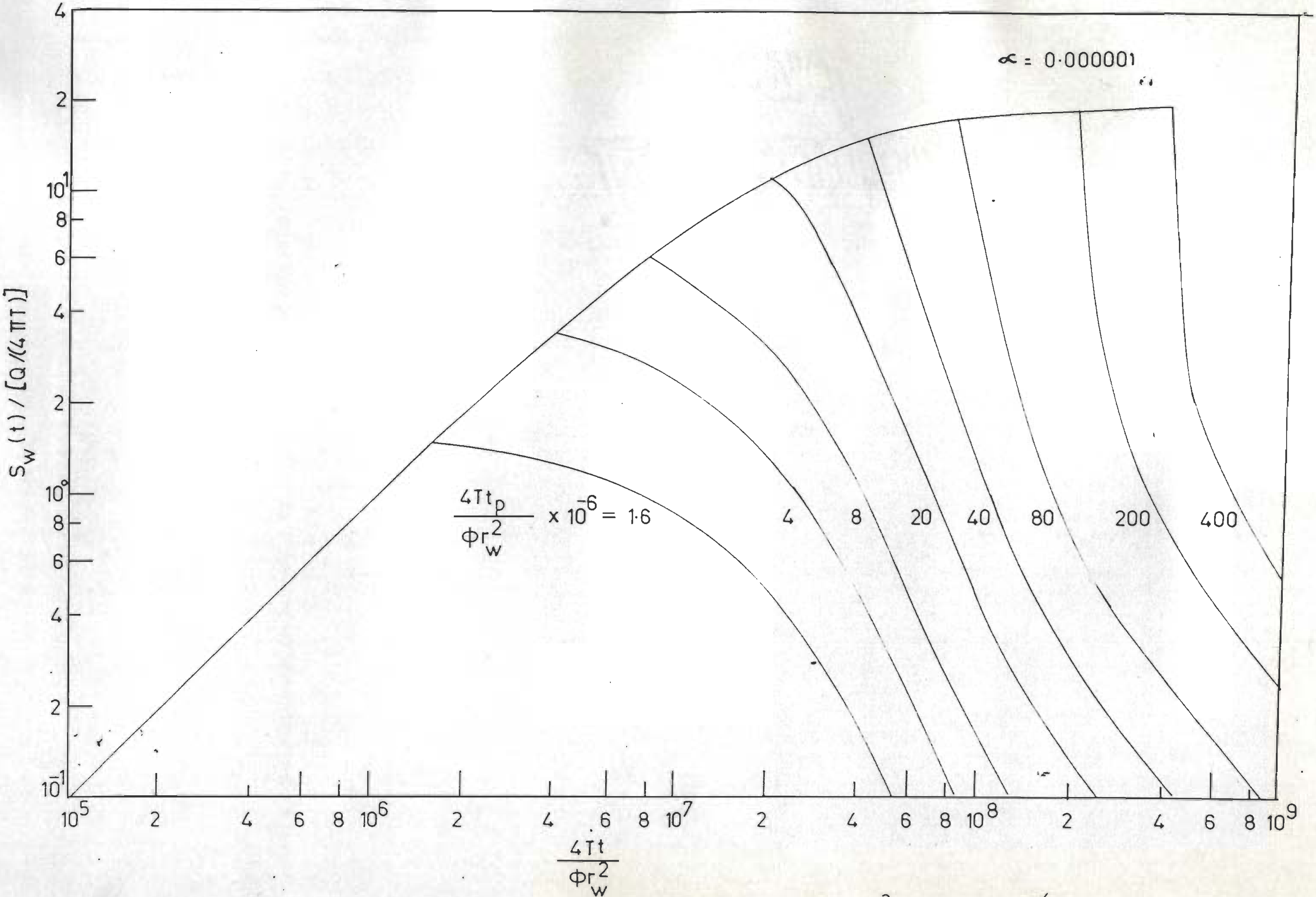


FIG. 3.2(f) - Family of type curves : $S_w(t)/(Q/4\pi T)$ versus $4Tt/(\phi r_w^2)$ for $\alpha = 10^{-6}$.

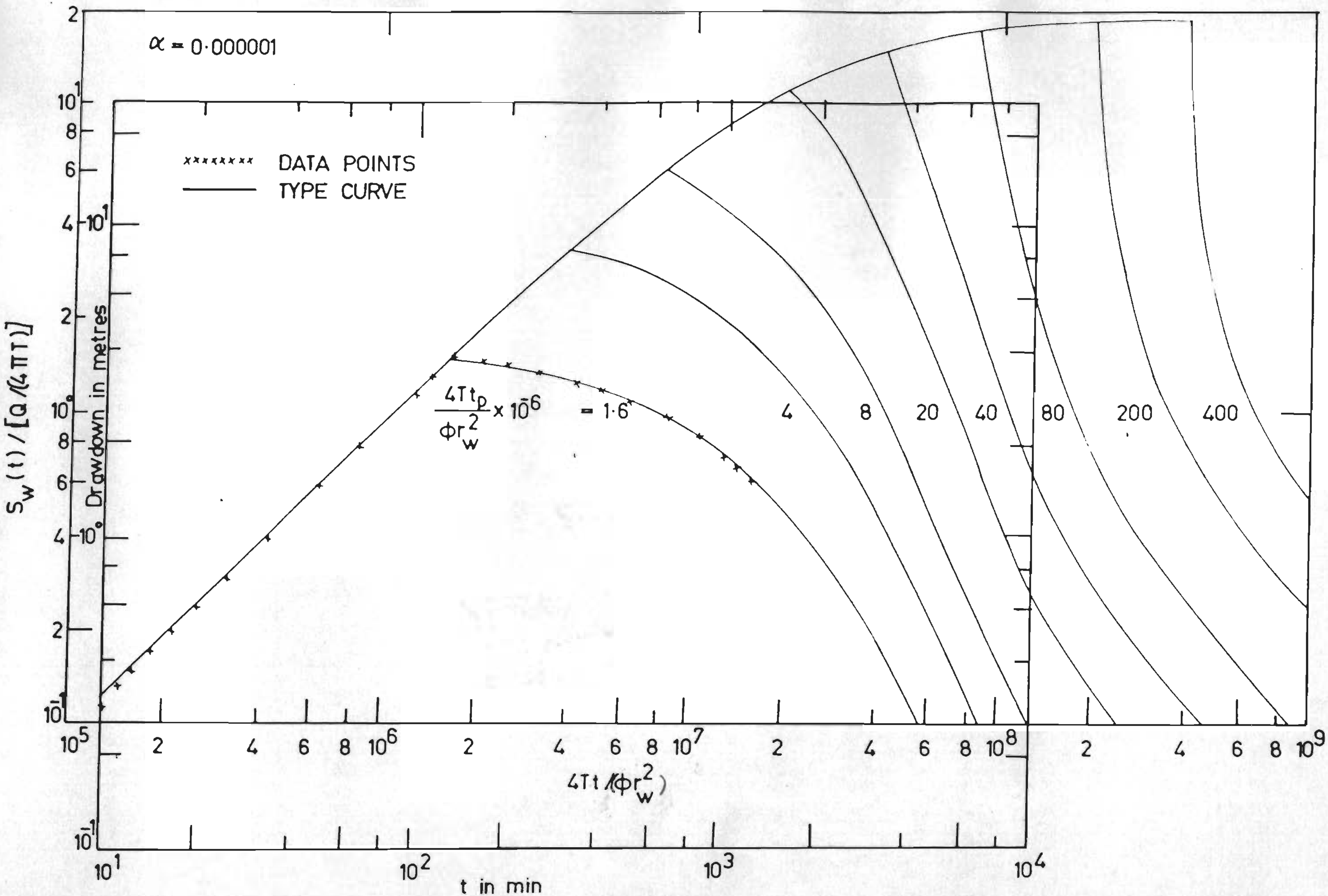


FIG. 3.2(g) - Matching of pumping test data from large-diameter abstraction well with the type curve

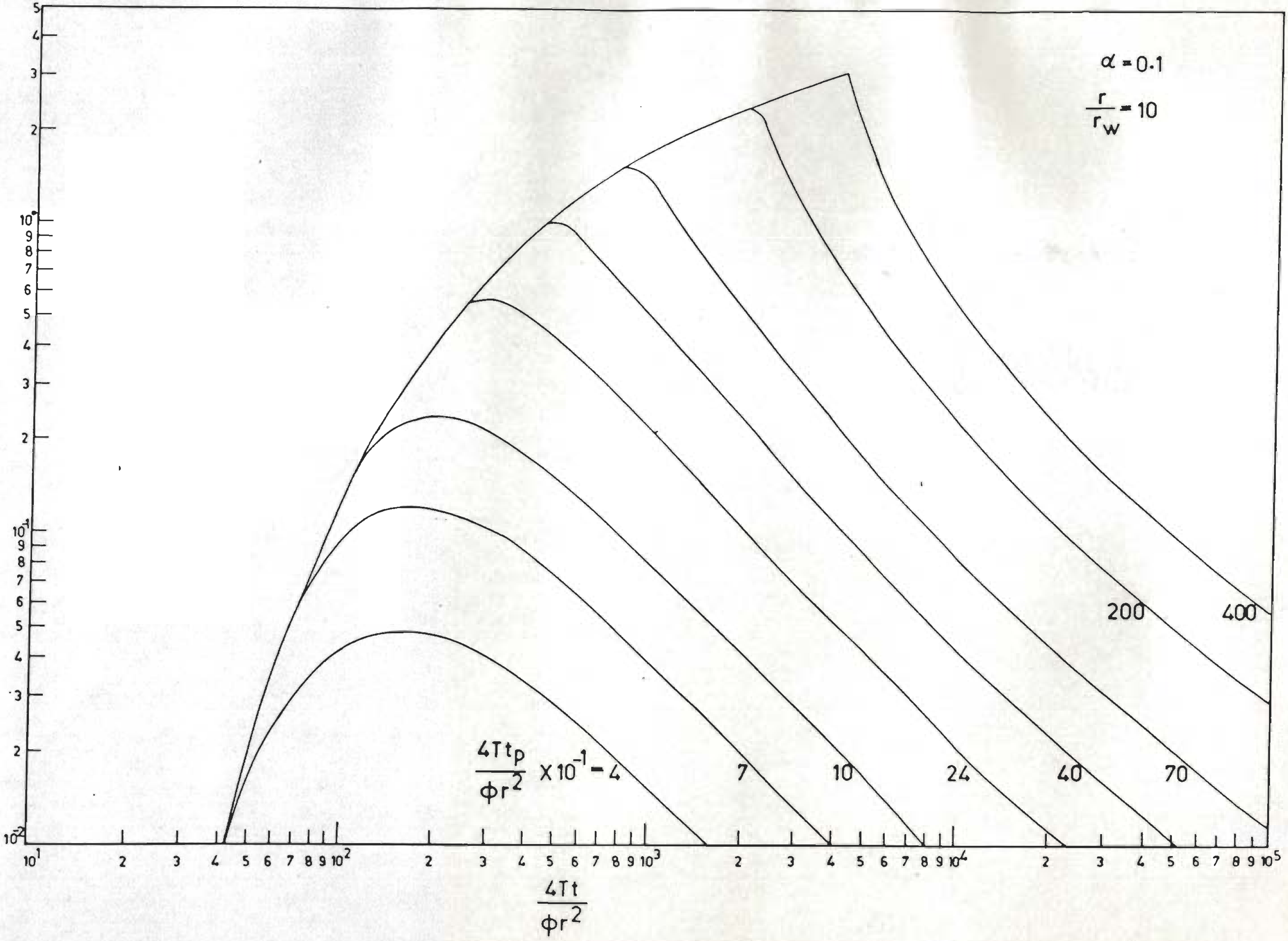


FIG. 3.3(a) - Variation of $S_r(t)/(Q/4\pi T)$ with $4Tt/(\phi r^2)$ for r/r_w = 10 and $\alpha = 10^{-1}$.

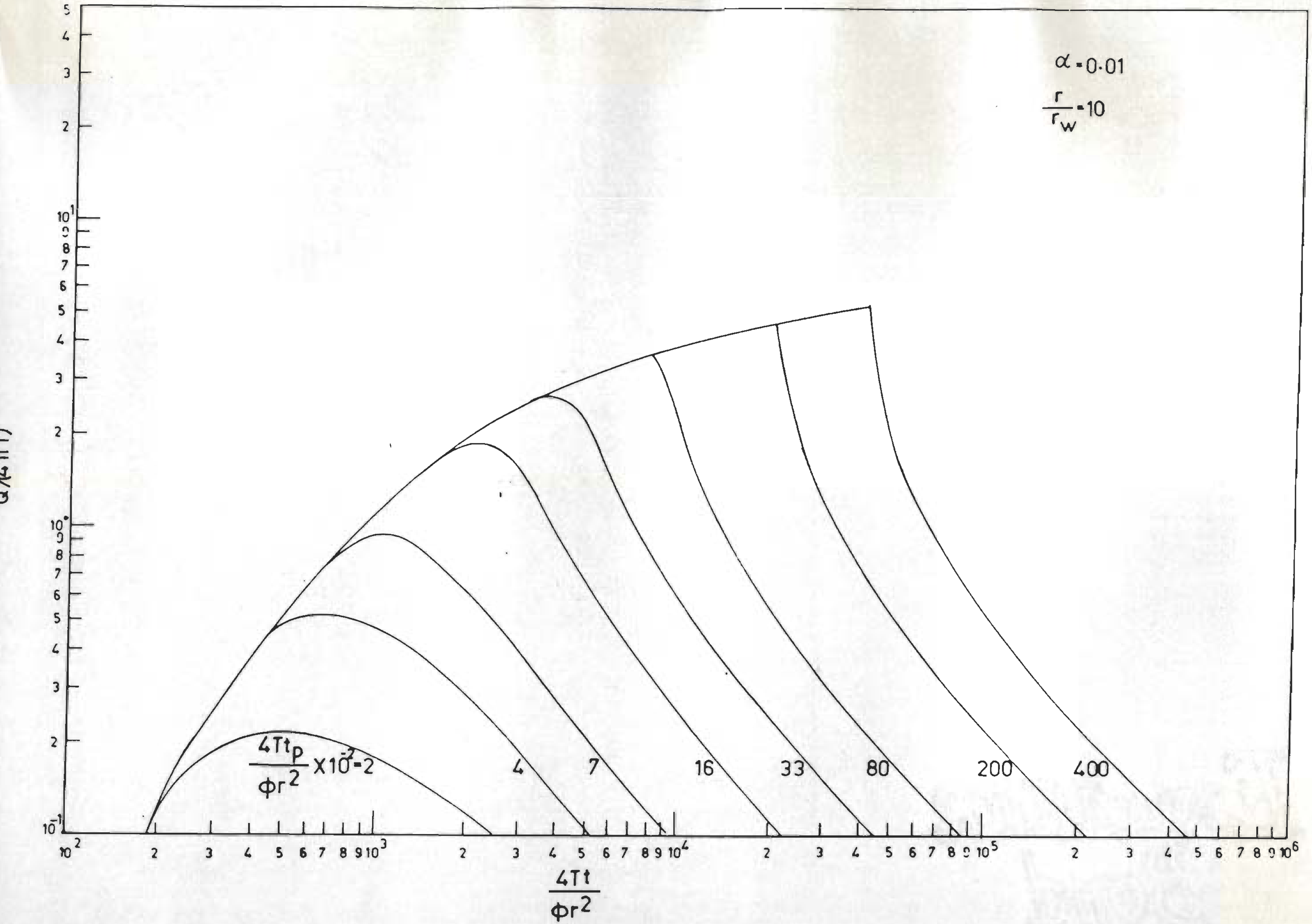


FIG. 3.3(b) - Variation of $S_r(t)/(Q/4\pi T)$ with $4Tt/(\phi r^2)$ for $r/r_w = 10$, and $\alpha = 10^{-2}$.

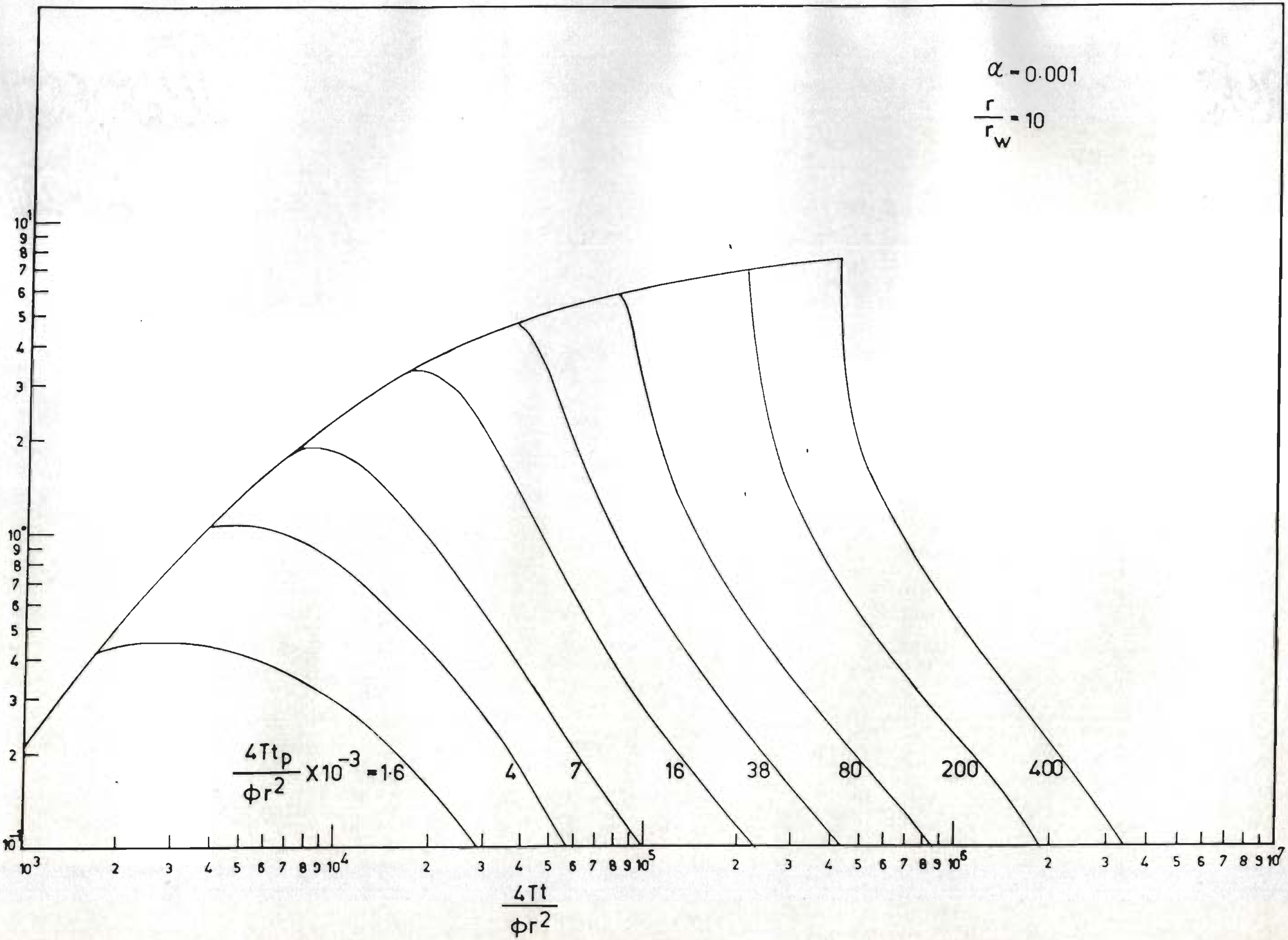


FIG. 3.3(c) - Variation of $S_r(t)/(Q/4\pi T)$ with $4Tt/(\phi r^2)$ for r/r_w = 10, and $\alpha = 10^{-3}$.

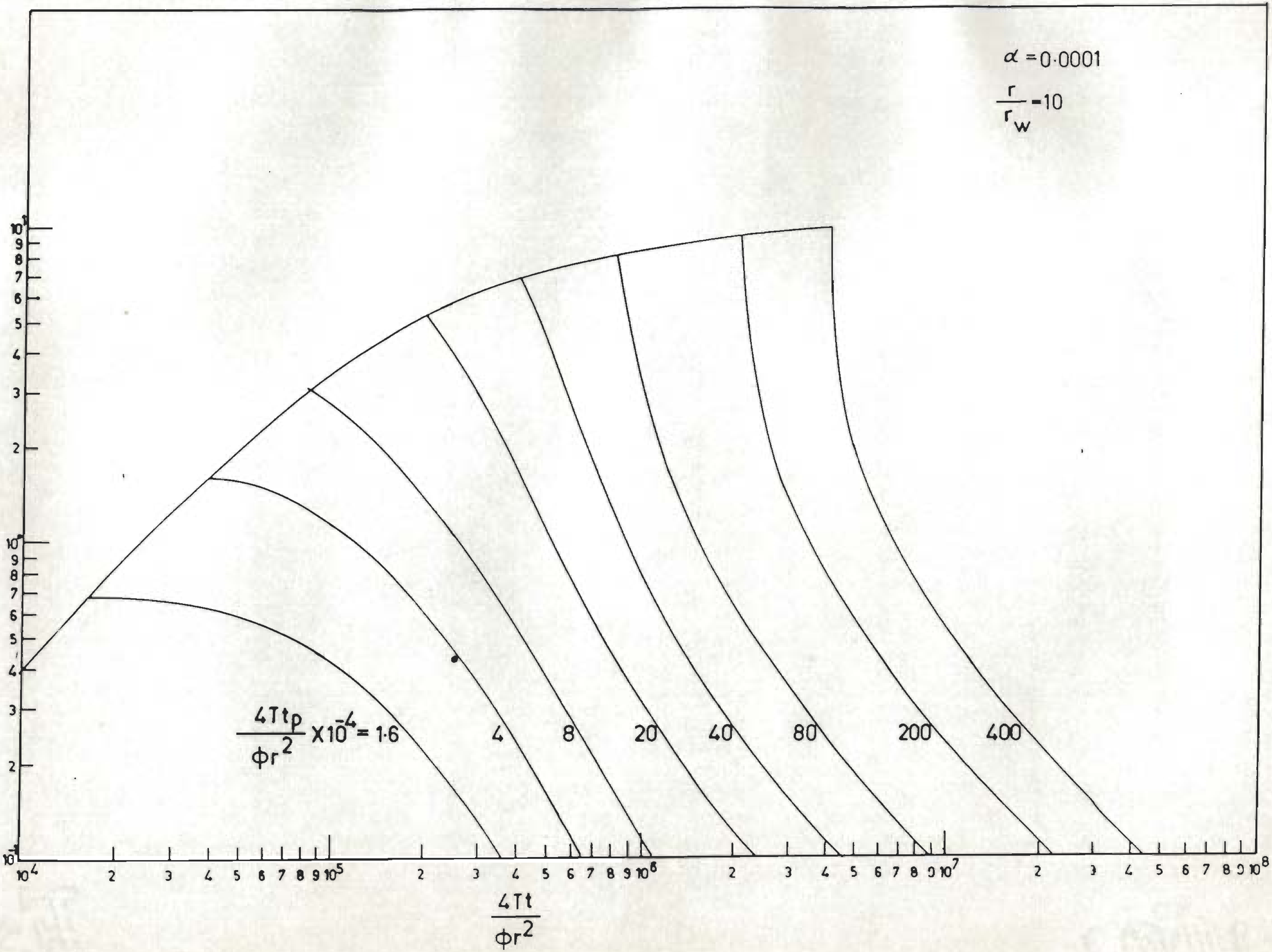


FIG. 3.3(d) - Variation of $S_r(t)/(Q/4\pi T)$ with $4Tt/(\phi r^2)$ for r/r_w = 10, and $\alpha = 10^{-4}$.

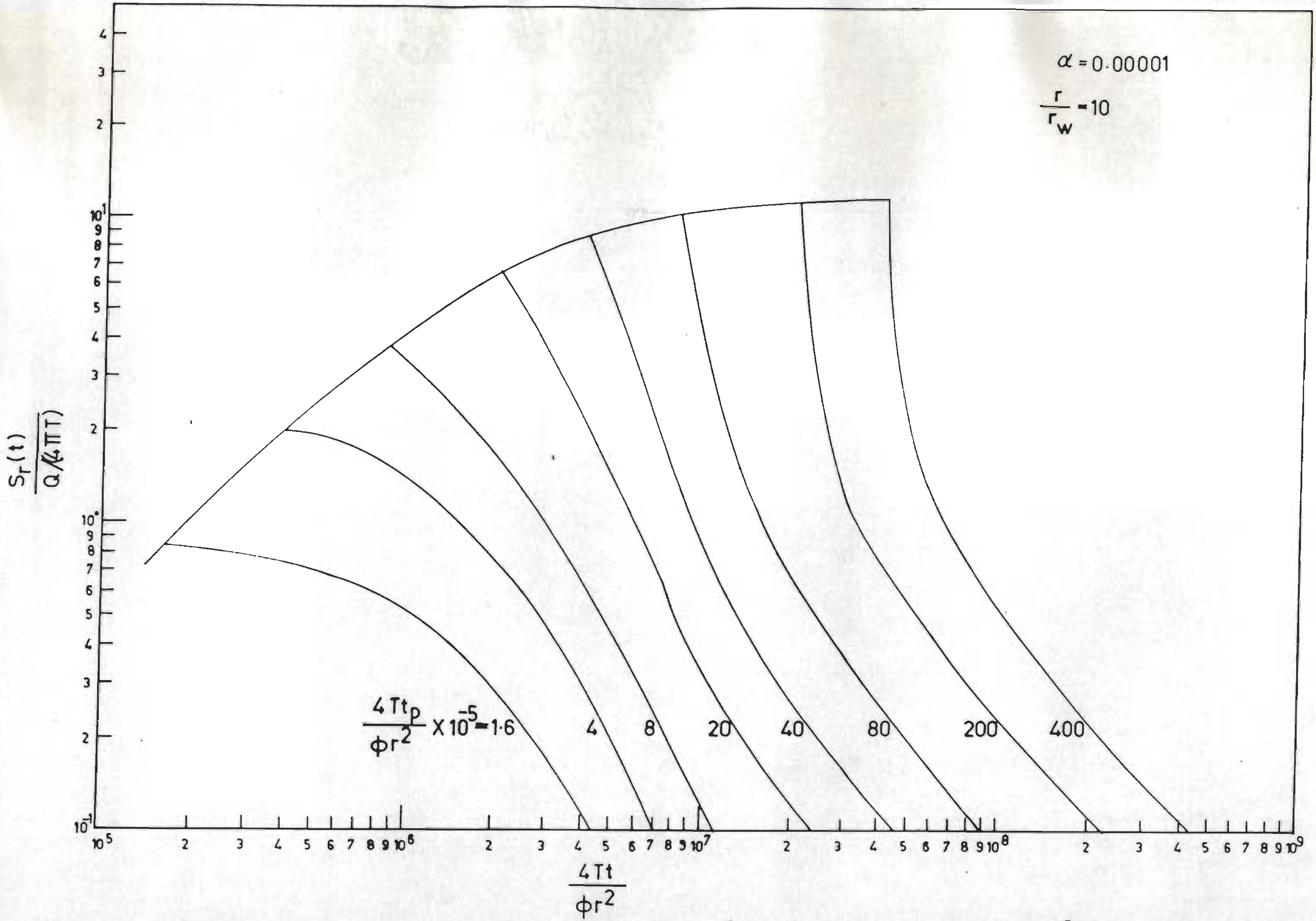


FIG. 3.3(e) - Variation of $S_r(t)/(Q/4\pi T)$ with $4Tt/(\phi r^2)$ for $r/r_w = 10$, and $\alpha = 10^{-5}$.

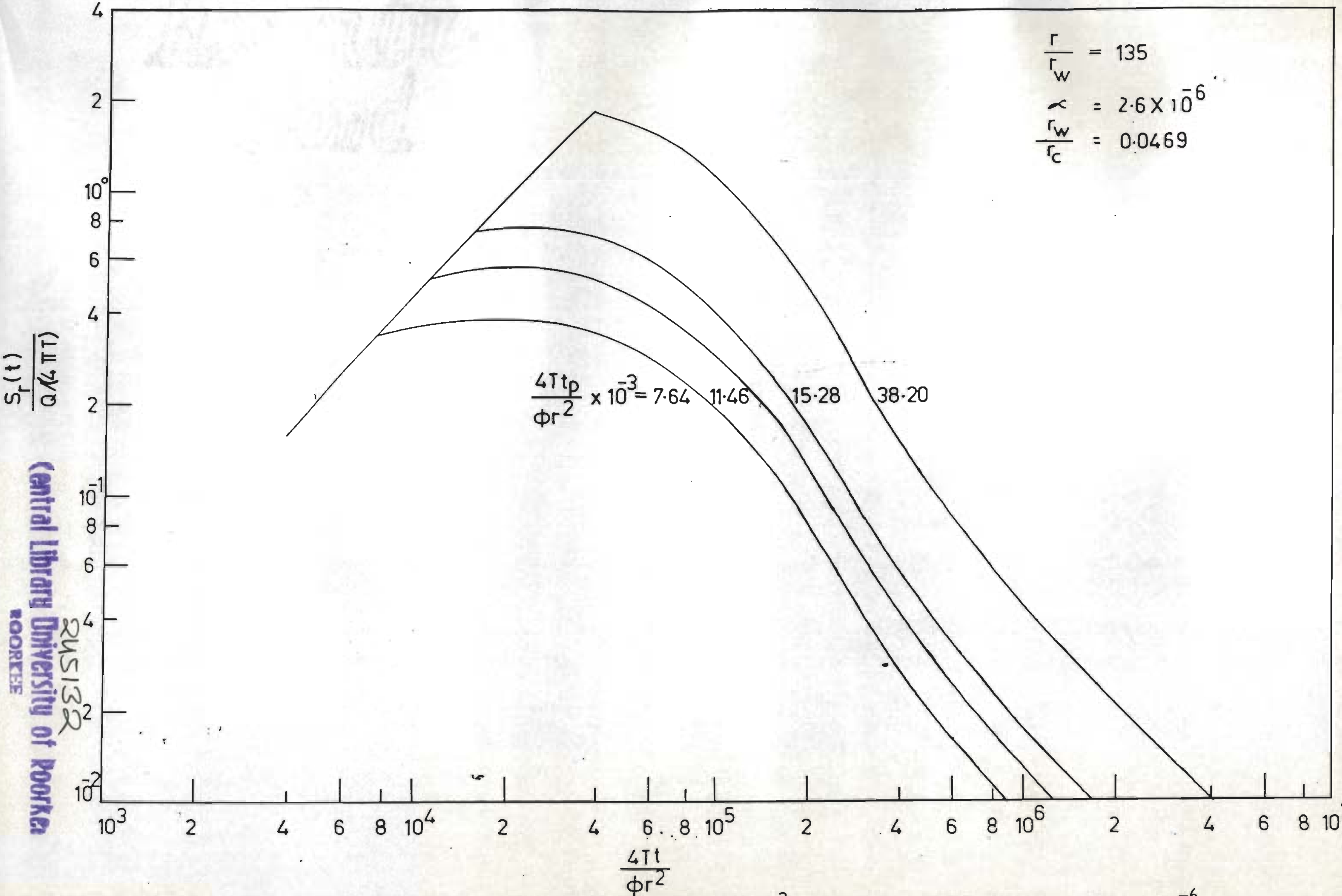


FIG. 3.3(f) - Variation of $S_r(t)/(Q/4\pi T)$ with $4Tt/(\phi r^2)$ for r/r_w = 135 and $\alpha = 2.6 \times 10^{-6}$.

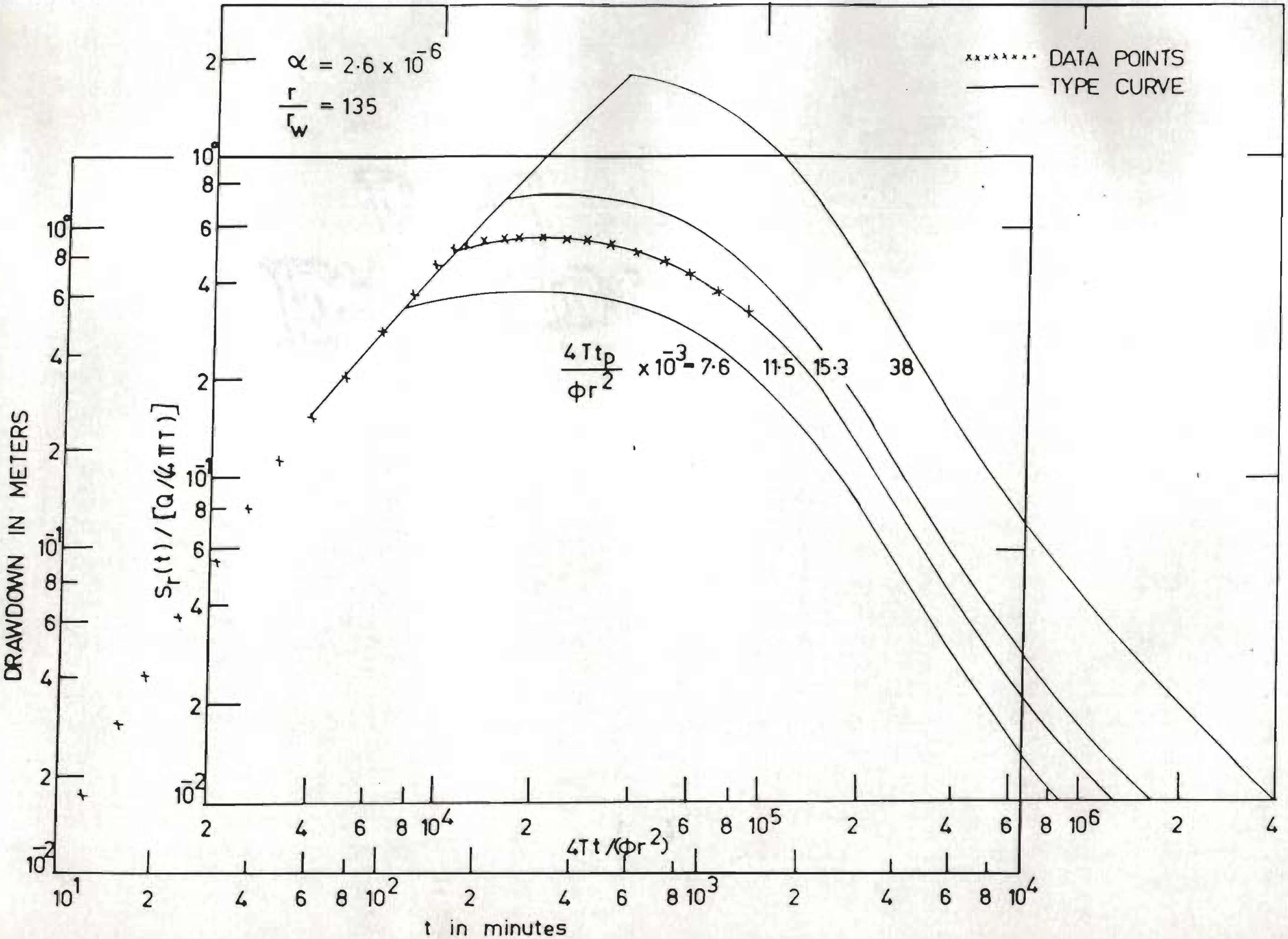


FIG. 3.3(g) - Matching of pumping test data from an observation well located near a large-diameter abstraction well with the type curve

from the center of the well for r_w/r_c = 10 and for different values of α . The results have been given for various durations of pumping. The nondimensional time factor $4Tt_p/(\phi r^2)$ at which the pumping discontinued, have been indicated in the figures. It can be seen from the figures that water level continues to fall at the observation point after the abstraction ceased. Such phenomenon occurs due to the fact that the aquifer continues to supply water to refill the well even after pumping is discontinued.

The value of well function, $S_W(t)/[Q/(4\pi T)]$, will tend to zero when recovery becomes almost complete. As the recovery continues, the type curves will show a reducing slope which tends to zero. This fact can be verified by plotting $S_W(t)/[Q/(4\pi T)]$ versus $4Tt/(\phi r_w^2)$ in either natural scale or in a semilog scale.

The family of type curves presented in Figures [3.2(a)] through [3.2(f)] and [3.3(a)] through [3.3(f)] provide an accurate means of determining parameters of a confined aquifer. Rushton and Holt (1981) have estimated aquifer parameters for a large-diameter well using numerical technique. They have used the drawdown data of abstraction phase, and the recovery phase at the well point and at an observation point in the vicinity of the well. These data have been used for estimating aquifer parameters by curve matching techniques with the help of the type curves presented herein. The time-drawdown curve at the well face matches with the type curve corresponding to α = .000001 and $4Tt_p/(\phi r_w^2) = 1.6 \times 10^6$ which has been presented in Figure [3.2(g)]. The duration of pumping, t_p, obtained through matching is 136.9 minutes. The true pumping period reported by Rushton and Holt is 135 minutes. A proper matching ensures an agreement between true duration of pumping and the duration estimated through curve matching. Sufficient recovery data are necessary to have a unique match. The time-drawdown curve at the observation point matches closely with the type curve corresponding to $\alpha = 2.6 \times 10^{-6}$ and $4Tt_p / (\phi r^2) = 11.5 \times 10^3$. The matching has been shown in Figure [3.3(g)]. Table (3.4) shows the values estimated by Rushton and Holt and the values evaluated with the help of the type curves. The drawdowns at the well face and at the observation point calculated by discrete kernel approach using the estimated aquifer parameters are shown in Figures (3.4) and (3.5) respectively. The observed drawdowns also have been plotted in these figures. But for the last part of the time-drawdown curve during recovery, the observed and calculated drawdown fairly match.

TABLE 3.4 Comparison of Aquifer Parameters Obtained by Numerical Method and Discrete Kernel Approach

Method	Data from	T(m^2/day)	ϕ
Numerical method (Rushton and Holt)	-	24 to 29	0.0006 to 0.001
Discrete kernel approach	Discharging well	22	0.00045
	Piezometer	30	0.0012

The drawdown in a large-diameter well during recovery and the corresponding Theis recovery values are presented in Table (3.5) for the purpose of comparison. The Theis recovery for the assumed values of aquifer parameters and well geometry differs from the recovery of the large-diameter well by 72.25% at the end of 120 minutes since pumping stopped. Thus, there is considerable difference in the recovery values of a large-diameter well and Theis recovery. Therefore, calculation of drawdown during recovery by Theis recovery formula is not valid for a large-diameter well.

The quantity of water withdrawn from well storage during pumping and the replenishment that occurs during recovery are presented in Figures [3.6(a)]

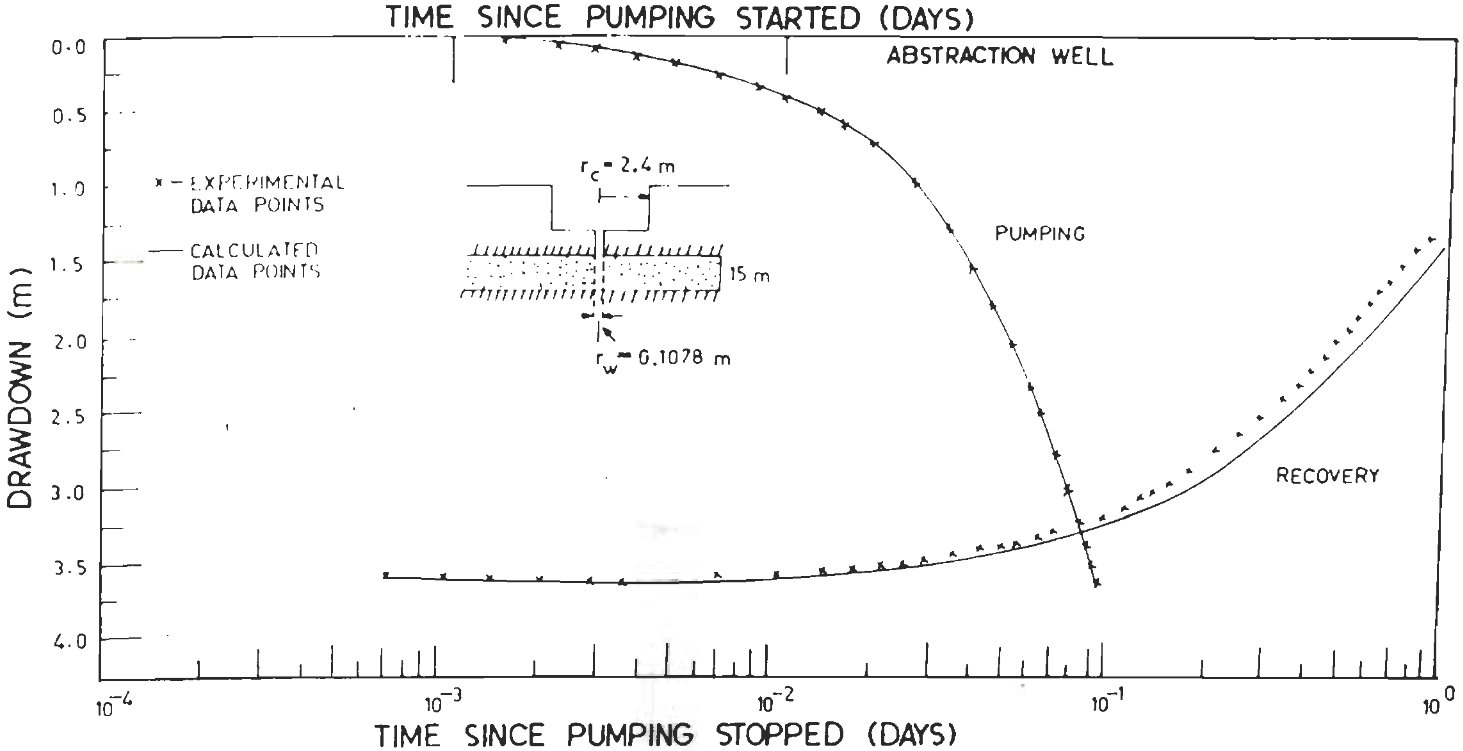


FIG. 3.4 - Comparison of observed and computed drawdowns at the abstraction well face

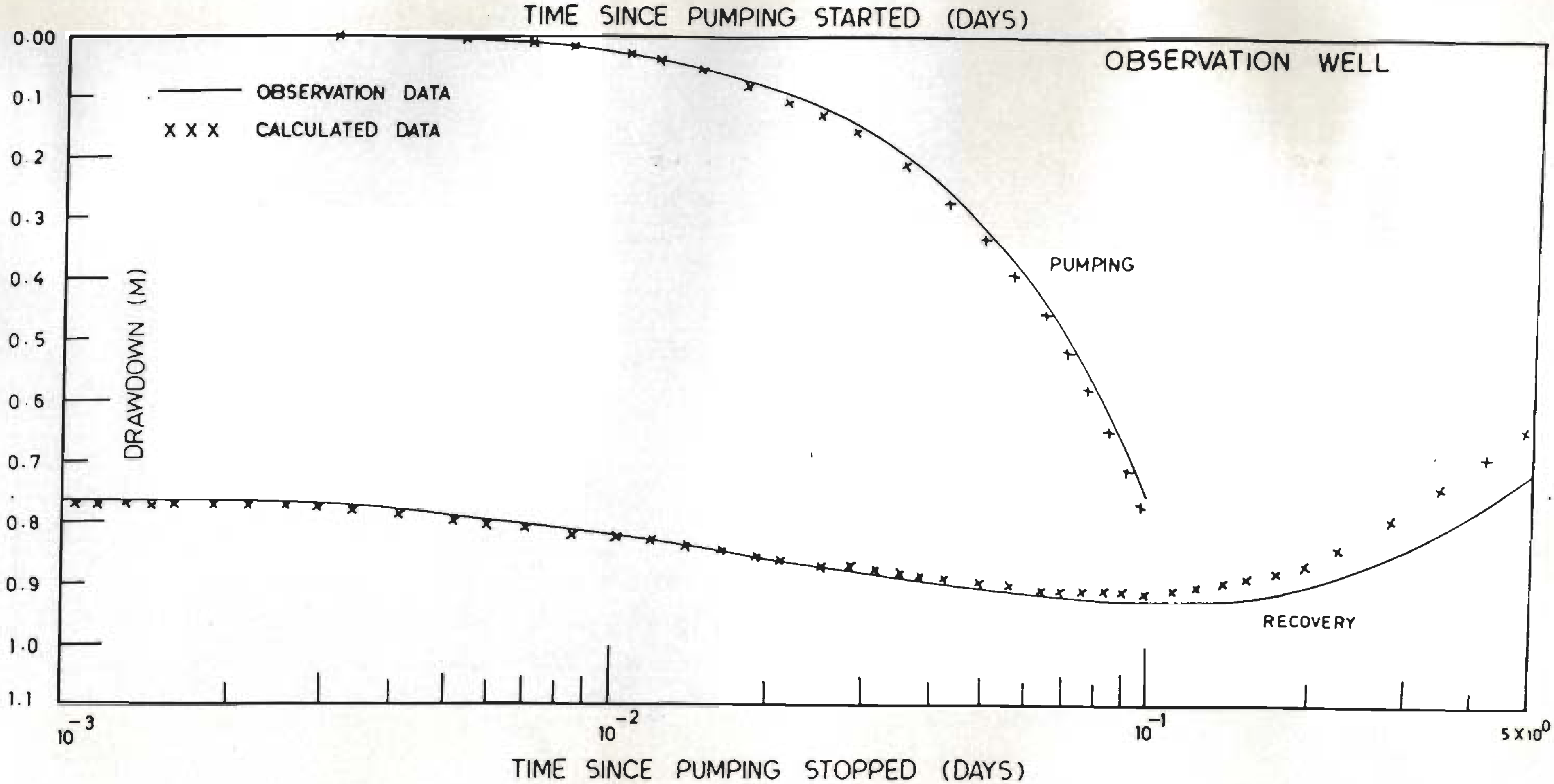


FIG. 3.5 - Comparison of observed and computed drawdowns at the observation well

TABLE 3.5 Comparison of Large-Diameter Well Recovery with Theis Recovery [$Q = 100\ m^3/day$, $T = 50\ m^2/day$, $\phi = 0.004$, $r_w = 0.1m$ and $r_c = 2m$]

Time since pumping stopped (min.)	Drawdown in large-diameter well during recovery (m)	Theis recovery (m)
5	.54183	.51229
10	.53407	.40822
15	.52650	.34970
20	.51910	.30970
25	.51186	.27977
30	.50478	.25615
40	.49104	.22063
50	.47784	.19477
60	.46512	.17485
80	.44105	.14583
100	.41860	.12549
120	.39761	.11032

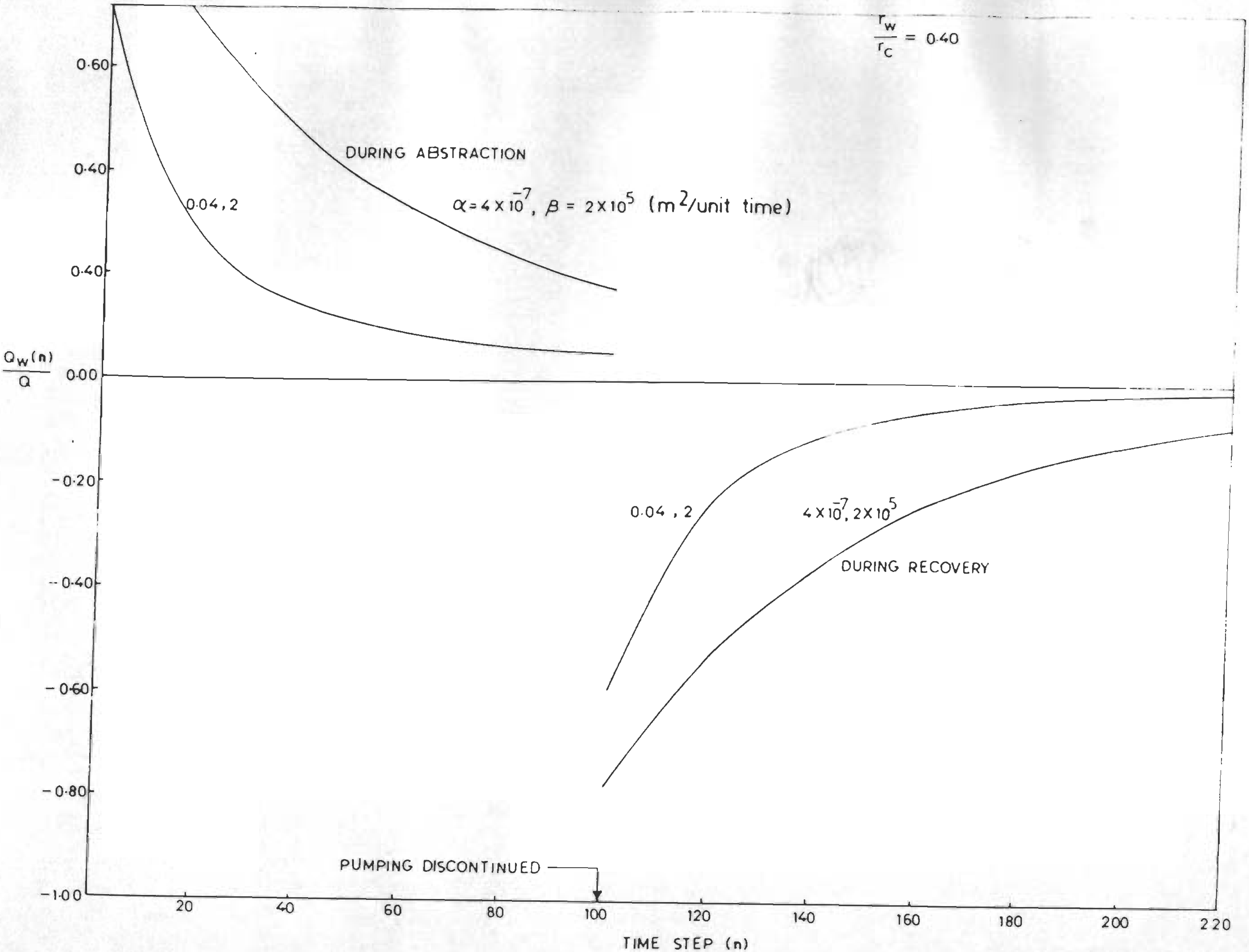


FIG. 3.6(a) - Variation of contribution from well storage to pumping during abstraction and replenishment during recovery for $r_w/r_c = 0.4$.

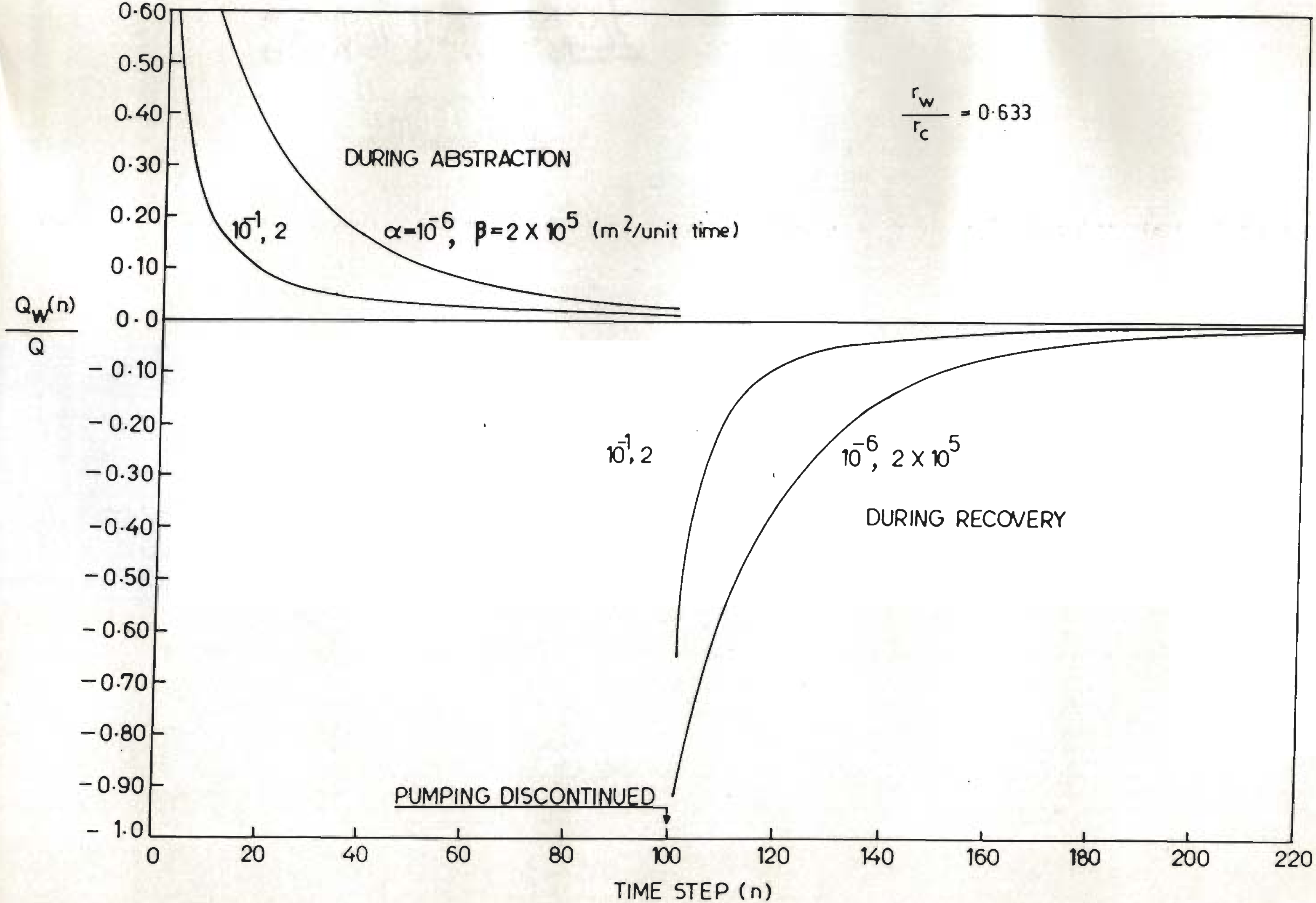


FIG. 3.6(b) - Variation of contributions from well storage to pumping during abstraction and replenishment during recovery for r_w/r_c = 0.633.

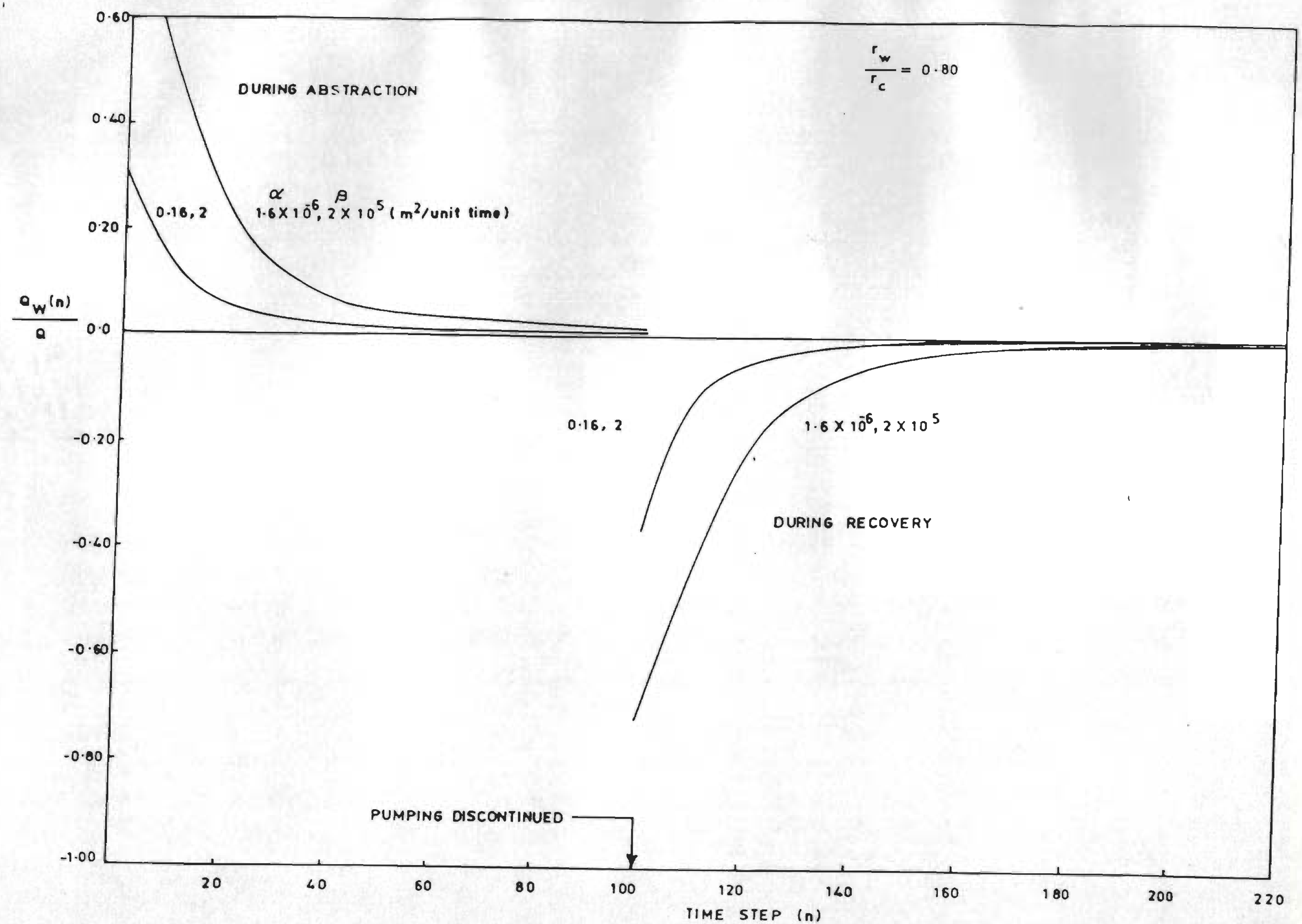


FIG. 3.6(c) - Variation of contribution from well storage to pumping during abstraction and replenishment during recovery for r_w/r_c = 0.8.

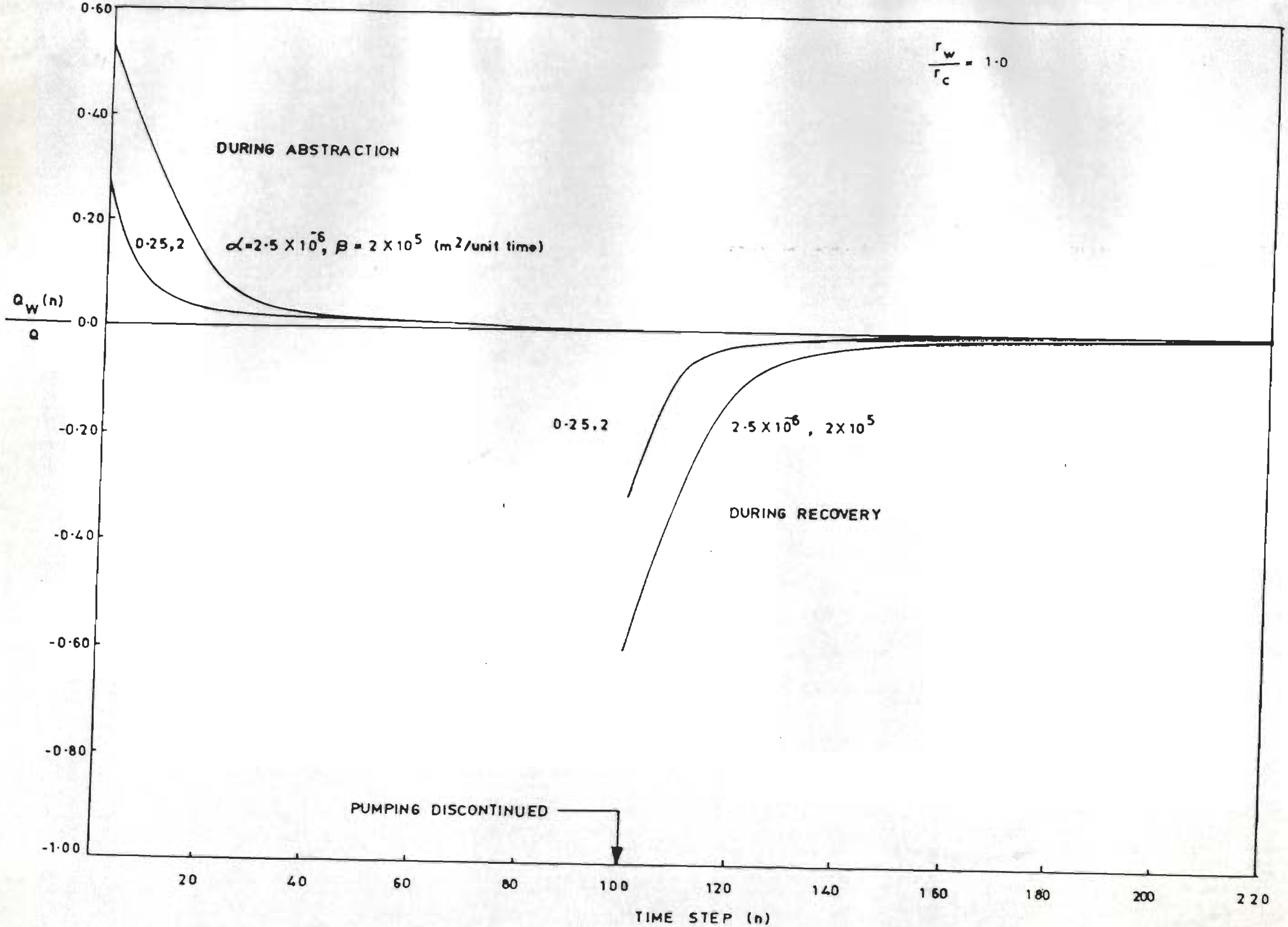


FIG. 3.6(d) - Variation of contributions from well storage to pumping during abstraction and replenishment during recovery for $r_w/r_c = 1.0$

through [3.6(d)] for a large-diameter well with r_w/r_c ratio equal to 0.4, 0.633, 0.8 and 1. The pumping has been discontinued at the end of the 100th time step. The results have been given for two sets of aquifer parameters in which only the value of storage coefficient differ. For example $\alpha = 0.1$ refers to $\phi = .25$, and $\alpha = .000001$ corresponds to $\phi = .0000025$. The value of transmissivity has been assumed to be 0.5 m^2 per unit time period. In the figures, β refers to hydraulic diffusivity which is equal to T/ϕ . It can be seen from the figures that more water is withdrawn from the storage of that well which has been constructed in the aquifer having a lower storage coefficient. The rate of replenishment of well storage is found to be more in the aquifer with a lower storage coefficient as more water is to be replenished.

The variation of $[-\sum_{\gamma=m+1}^{n} Q_W(\gamma) / \sum_{\gamma=1}^{m} Q_W(\gamma)]$ with $4Tt'/(\phi r_w^2)$ is shown in Fig. (3.7) for different values of ϕ and m. The time t' is measured since stoppage of pumping. $\sum_{\gamma=1}^{m} Q_W(\gamma)$ represents the total quantity of water with - drawn from well storage during pumping. $-\sum_{\gamma=m+1}^{n} Q_W(\gamma)$ represent the total quantity of water recouped upto time step n. It can be seen from the figure (3.7) that the time of 90 percent recovery of a well storage is nearly same for different durations of pumping. Smaller the value of storage coefficient longer will be the duration for 90 percent recovery. For example from figure (3.7), for t_p = 6 hours, T = 150 m^2/day, $\phi = 0.01$ the value of t' for 90 percent recovery of well storage is 8.8×10^{-3} hours. For $\phi = 0.001$, for corres- ponding value is 11.21×10^{-3} hours.

3.4 CONCLUSIONS

Based on the study the following conclusions are drawn :

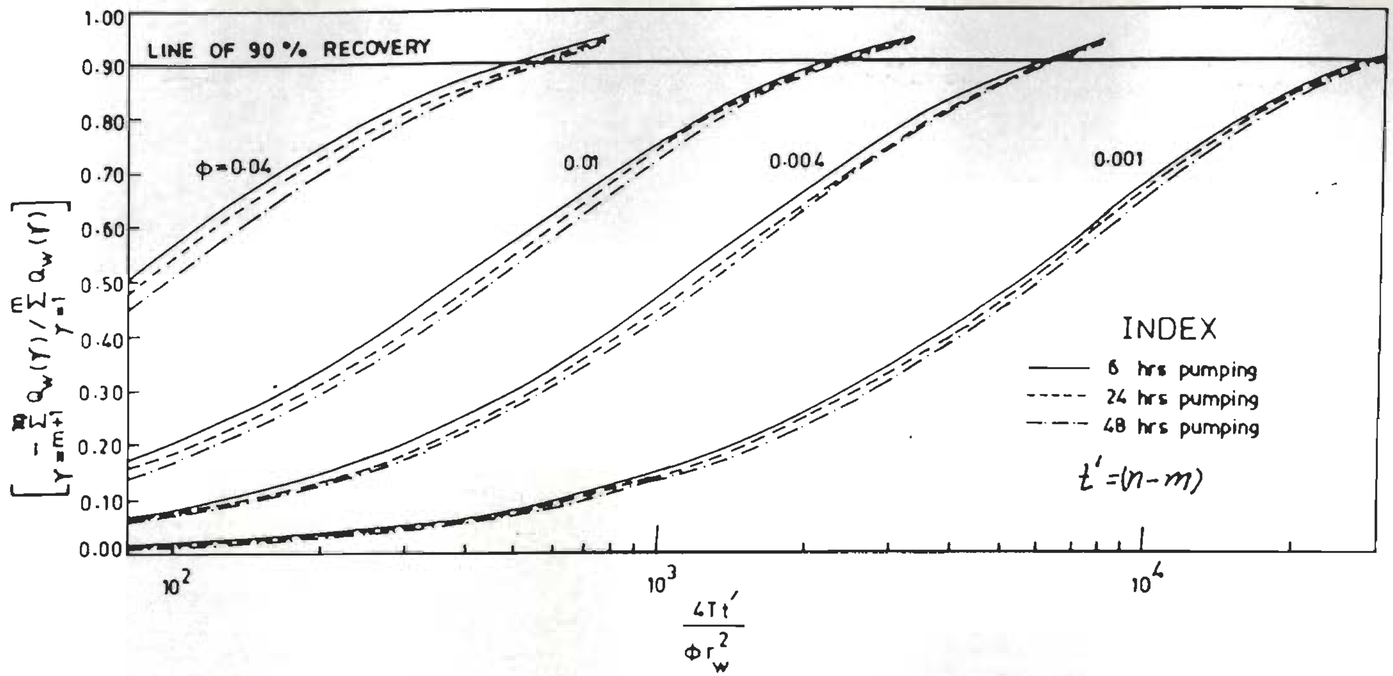


FIG. 3.7 - Rate of replenishment of well storage with time for ϕ = 0.04. 0.01, 0.004, 0.001 and t_p = 6, 24 and 48 hours.

(i) Computation of drawdown during the early stages of pumping and during recovery is sensitive to the time step size.

(ii) Accuracy in the computation of drawdown for any time step size improves with increase in the number of time steps used for computation.

(iii) Rate of contribution of well storage to pumping and rate of replenishment during recovery are higher for aquifers with lower storage coefficient.

(iv) Calculation of drawdown during recovery using Theis recovery formula is not valid for a large-diameter well.

(v) The type curves which incorporates the response of the aquifer during recovery can provide an accurate means of determining aquifer parameters.

(vi) The duration of pumping, t_p, computed from the non-dimensional time factor $4Tt_p/(\phi r_w^2)$ through type curve matching and its comparison with actual duration of pumping recorded in the aquifer test helps in selecting appropriate type curve for matching.

Based on the work reported in this chapter, the following paper has been published :

Mishra, G.C. and A.G. Chachadi. (1985), Analysis of flow to a large-diameter well during the recovery period. Ground Water, V. 23, No. 5, pp. 646-651.

CHAPTER 4

ANALYSIS OF UNSTEADY FLOW TO A LARGE-DIAMETER WELL DUE TO ABSTRACTION THAT VARIES LINEARLY WITH DRAWDOWN AT THE WELL

4.0 INTRODUCTION

4.0.1 Analysis for Variable Pumping Rate

The solution given by Papadopulos and Cooper (1967) for analysing pumping test data from a large-diameter well is for a constant abstraction rate. Therefore, when a constant abstraction rate cannot be maintained, which is often the case, if centrifugal pumps are used, the type curves of Papadopulos and Cooper are not applicable. To overcome the problem of variable abstraction rates, Lai and Su (1974) have given an equation for the drawdown in and around a well of large-diameter in a leaky artesian aquifer induced by an arbitrary time-dependent pumping rate using Laplace transform techniques. The effect of the storage capacity of the well on the drawdown is found to be significant when the time of pumping is not large or the ratio of the transmissivity of the aquifer to its storage coefficient is small. Though the analysis of Lai and Su takes care of the effect of linearly and exponentially variable abstraction rates, it is often not possible to represent satisfactorily the variation of abstraction rate that actually occurs in practice. Evaluation of drawdown in their method requires numerical integration of an improper integral involving Bessel's functions. The numerical integration therefore, involves large computations.

Rushton and Holt (1981) have presented an elegant numerical solution for analysis of pumping test data from large-diameter well both during abstraction as well as during recovery phases. The existence of the seepage face in the abstraction well, variable abstraction rate and well losses can also be included in the numerical model. The model simulates the water levels in a confined aquifer quite accurately, however, the results for unconfined aquifer are not quite satisfactory.

Rushton and Singh (1983) have developed type curves using numerical approach. These type curves are given both for constant as well as variable abstraction rates. With these type curves it would be possible to obtain reasonable estimate of the transmissivity value. The storage coefficient values computed by this method may not be reliable.

4.0.2 Specific Capacity for Wells

In many cases, especially during reconnaissance type of groundwater investigations and for water balance studies it may not be economical to construct test wells and conduct the time consuming aquifer tests for estimation of hydrogeological parameters. Also, some of the modern quantitative techniques such as those for which electric analog models or mathematical models are contemplated, a sufficiently large number of T and ϕ values are required. In all such cases, quick and approximate methods may have to be resorted to, for the determination of hydrogeological parameters. These properties can be estimated with reasonable accuracy by some of the indirect methods based on analysis of water level fluctuations, specific capacity data of wells, and well logs etc.

The productivity of a well is often expressed in terms of the specific capacity, which is defined as $Q_p(t)/S_W(t)$, where $Q_p(t)$ is the pumping rate and $S_W(t)$ is the drawdown at time t. In other words specific capacity is the discharge per unit drawdown and it is time variant. The theoretical specific capacity of a well discharging at a constant rate in a homogeneous, isotropic, nonleaky artesian aquifer of infinite areal extent is given by the following expression (Walton, 1970) :

$$\frac{Q_p}{S_W} = \frac{4 \pi T}{2.30 \log_{10} \{ 2.25 \ Tt/(r_w^2 \phi) \}}$$

in which,

S_W = drawdown in a 100 percent efficient pumped well in metres,

r_w = radius of the pumped well in metres,

Q_p/S_W = specific capacity in m^3/day/metre of drawdown,

Q_p = rate of discharge in m^3/day,

T = transmissivity in m^2/day,

ϕ = dimensionless storage coefficient, and

t = time after pumping started in days.

The above equation assumes that : (1) the production well has full penetration and the well is uncased in the entire depth of aquifer, (2) the well loss is negligible, and (3) the effective radius of the production well has not been affected during drilling and development of the production well and is equal to the nominal radius of the production well. The storage coefficient value can be estimated either from well log data or from study of water level data. As the specific capacity varies with the logarithm of $1/\phi$, large error in assumed storativity value results in comparatively small error in transmissivity estimated using the above relation. Specific capacity decreases with the period of pumping because the drawdown continuously increases with time as the cone of influence of the well expands till the steady state conditions are arrived at. For this reason it is important to state the duration of the pumping period at which a specific capacity is computed.

The relationship between the specific capacity and transmissivity for artesian and water table conditions has been given for different durations of pumping (Walton, 1970). These graphs can be used to obtain rough estimates of the transmissivity from specific capacity data provided approximate value of storage coefficient is known. The transmissivity -specific capacity relationship given by Walton is for a constant pumping rate and negligible well storage.

In the present chapter using discrete kernel approach the unsteady flow to a large-diameter well induced by time dependent pumping has been analysed. Transmissivity Vs specific capacitiy relationship for known values of storativity has been developed similar to that of Walton (1970) taking well storage into consideration for linear variation of discharge with drawdown.

4.1 STATEMENT OF THE PROBLEM

Figure (4.1) shows a schematic cross section of a large-diameter well in a homogeneous isotropic confined aquifer of infinite areal extent which was initially at rest condition. The radius of the well screen is r_w and that of the unscreened part is r_c. Pumping is carried out upto time t_p and the rate of pumping depends on the drawdown. It is necessary to determine the drawdown in piezometric surface at the well face and at any distance 'r' from the center of the well at time 't' after the onset of pumping.

4.2 ANALYSIS

The following assumptions have been made in the analysis :

(i) At any time the drawdown in the aquifer at the well face is equal to that in the well.

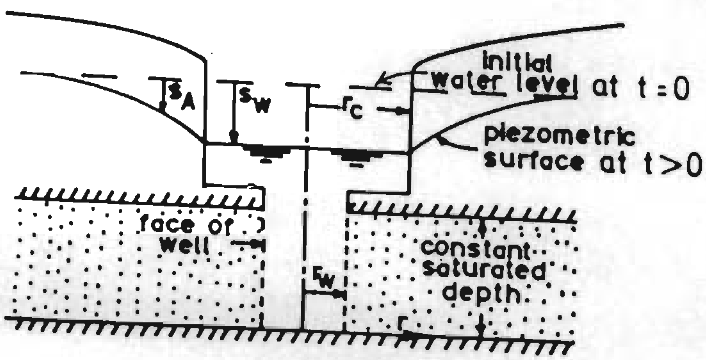


FIG. 4.1 - Schematic cross section of a large-diameter well

(ii) The time parameter is discrete. Within each time step, the abstraction rate of water derived from well storage and that from aquifer storage are separate constants.

Let the total time of pumping, t_p, be discretised to m units of equal time steps. The quantity of water pumped during any time step 'n' can be written as :

$$Q_A(n) + Q_W(n) = Q_p(n) \quad ...(4.1)$$

in which,

$Q_A(n)$ = water withdrawn from aquifer storage, and

$Q_W(n)$ = water withdrawn from well storage.

For $n > m$, $Q_p(n) = 0$. Otherwise $Q_p(n)$ is equal to rate of pumping per unit time period. When centrifugal pump is used for abstraction the pumping rate decreases with the increase in drawdown. A typical variation of discharge with drawdown at the well face is shown in Figure (4.2). In the present analysis a linear relationship between pumping rate and drawdown has been assumed to be valid. The pumping rate is expressed by :

$$Q_p(n) = [1-S_W(n)/S_F]Q_I \quad ...(4.2)$$

in which,

$S_W(n)$ is the drawdown at the well face at the end of time step n ,

S_F and Q_I have been explained in the figure. S_F is the maximum drawdown at which pumping rate would diminish to zero and Q_I is the initial maximum withdrawal rate.

Drawdown, $S_W(n)$, at the well face at the end of time step 'n' is given by

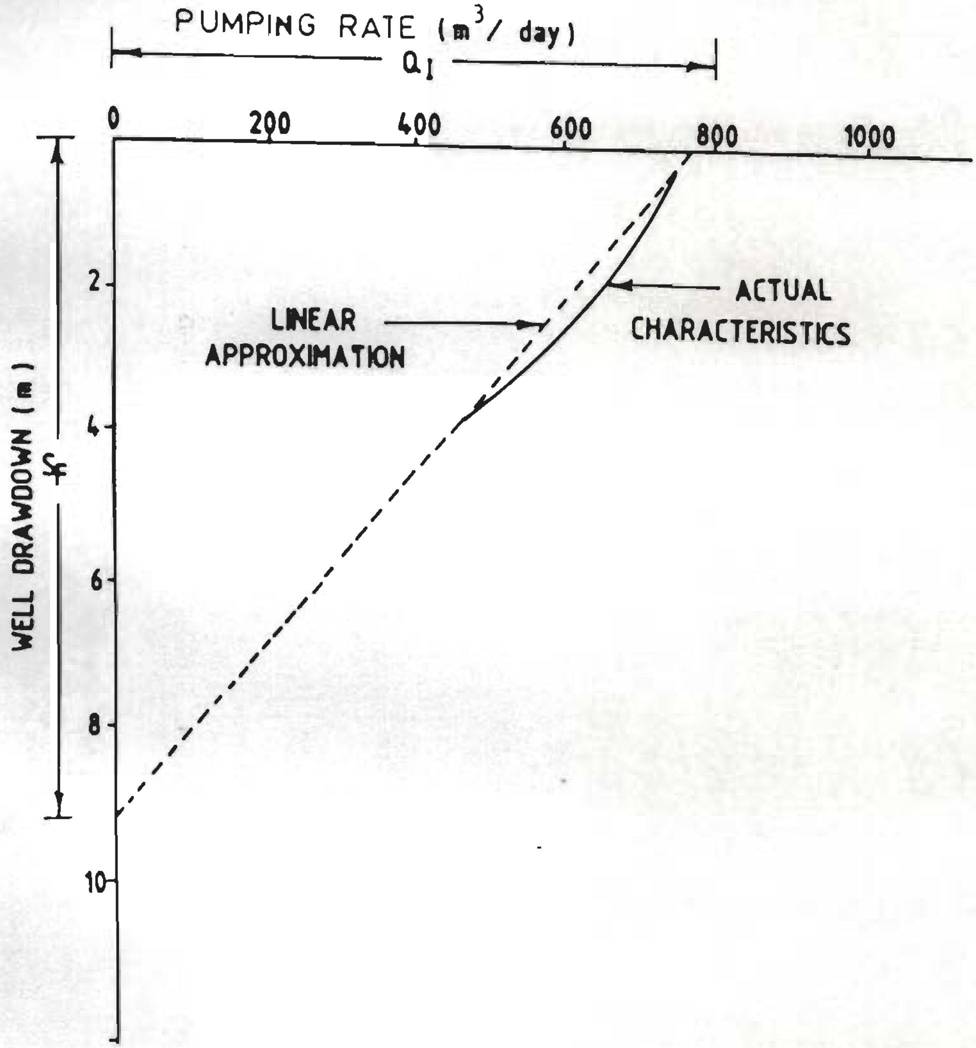


FIG. 4.2 - A typical variation of abstraction rate with draw-down in a large-diameter well fitted with centrifugal pump

$$S_W(n) = \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n} Q_W(\gamma) \qquad \ldots(4.3)$$

where $Q_W(\gamma)$ represents rate of withdrawal from well storage or replenishment at time step γ. $Q_W(\gamma)$ values are unknown a priori. A negative value of $Q_W(\gamma)$ means there is replenishment of well storage which occurs during recovery period. Making use of equations (4.1), (4.2) and (4.3) the following expression is obtained:

$$Q_A(n)+Q_W(n) = [1 - \frac{1}{S_F \pi r_c^2} \sum_{\gamma=1}^{n} Q_W(\gamma)]Q_I \qquad \ldots(4.4)$$

or

$$Q_A(n)+Q_W(n)(1 + \frac{Q_I}{S_F \pi r_c^2}) = [1 - \frac{1}{S_F \pi r_c^2} \sum_{\gamma=1}^{n-1} Q_W(\gamma)]Q_I \qquad \ldots(4.5)$$

Drawdown at the well face at the end of time step 'n' due to abstraction from aquifer storage is given by (Morel-Seytoux, 1975),

$$S_A(n) = \sum_{\gamma=1}^{n} Q_A(\gamma)\ \delta_{rw}(n-\gamma+1) \qquad \ldots(4.6)$$

Where,

$$\delta_{rw}(I) = \frac{1}{4\pi T} [E_1 (\frac{\phi r_w^2}{4TI}) - E_1 \left\{\frac{\phi r_w^2}{4T(I-1)}\right\}] \qquad \ldots(4.7)$$

Because $S_W(n) = S_A(n)$,

$$\sum_{\gamma=1}^{n} Q_A(\gamma)\ \delta_{rw}(n-\gamma+1) = \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n} Q_W(\gamma) \qquad \ldots(4.8)$$

Rearranging, the following relation is obtained,

$$\delta_{rw}(1)Q_A(n) - Q_W(n)\frac{1}{\pi r_c^2} = \frac{1}{\pi r_c^2}\sum_{\gamma=1}^{n-1} Q_W(\gamma)$$

$$- \sum_{\gamma=1}^{n-1} Q_A(\gamma)\ \delta_{rw}(n-\gamma+1) \qquad ...(4.9)$$

Equations (4.5) and (4.9) can be written in the following matrix form :

$$[A].\ [B] = [C] \qquad ...(4.10)$$

where,

$$[A] = \begin{bmatrix} 1, & 1 + \dfrac{Q_I}{S_F \pi r_c^2} \\ \delta_{rw}(1), & -\dfrac{1}{\pi r_c^2} \end{bmatrix}$$

$$[B] = \begin{bmatrix} Q_A(n) \\ Q_W(n) \end{bmatrix}$$

and

$$[C] = \begin{bmatrix} Q_I - \dfrac{Q_I}{S_F \pi r_c^2}\displaystyle\sum_{\gamma=1}^{n-1} Q_W(\gamma) \\ \dfrac{1}{\pi r_c^2}\displaystyle\sum_{\gamma=1}^{n-1} Q_W(\gamma) - \sum_{\gamma=1}^{n-1} Q_A(\gamma)\ \delta_{rw}(n-\gamma+1) \end{bmatrix}$$

Hence,

$$[B] = [A]^{-1}.\ [C] \qquad ...(4.11)$$

In particular for the first time step

$$[C] = \begin{bmatrix} Q_I \\ o \end{bmatrix}$$

$Q_A(n)$ and $Q_W(n)$ can be solved in succession starting from time step one using equation (4.11). Once $Q_A(n)$ values are known, the drawdown $S_r(n)$, in the aquifer at any distance 'r' from the centre of the abstraction well at the end of n^{th} time step can be found using the relation

$$S_r(n) = \sum_{\gamma=1}^{n} Q_A(\gamma) \quad \delta_r(n-\gamma+1) \qquad \ldots(4.12)$$

where,

$$\delta_r(I) = \frac{1}{4\pi T} [E_1 (\frac{\phi r^2}{4TI}) - E_1 \left\{ \frac{\phi r^2}{4T(I-1)} \right\}]$$

4.3 RESULTS AND DISCUSSION

The discrete kernel coefficients, $\delta_{rw}(n)$, have been generated using equation (4.7) for known values of transmissivity, storage coefficient and radius of the well screen. After generating the discrete kernels, $Q_A(n)$ and $Q_W(n)$ are computed in succession starting from the first time step using equation (4.11) for known values of m, Q_I and S_F. The drawdown at the well face is then obtained using equation (4.3). The values of $Q_A(n)$, $Q_W(n)$, $Q_p(n)$ and $S_W(n)$ at different time steps are presented in Table [4.1(a)] for variable abstraction rate. The values of S_F and Q_I adopted correspond to an actual pumping test. For the corresponding average constant abstraction rate, the values of $Q_A(n)$, $Q_W(n)$, and $S_W(n)$ at different time steps are given in Table [4.1(b)]. It could be seen from Table [4.1(a)] that during 18th time step after which the pumping is discontinued the aquifer contribution is 3.0840

TABLE 4.1(a) **Aquifer and Well Storage Contributions and Drawdown for the Pumping Rate that is Linearly Dependent on Drawdown**

(T = 2.1875 m^2/10 min., ϕ = 0.001, $r_w = r_c$ = 5.4 m,

Q_I = 9.44 m^3/10 min., S_F = 1.8 m, m = 18)

Time Step (n)	$Q_A(n)$	$Q_W(n)$	$Q_p(n)$	$S_W(n)$
1	.4951	8.4645	8.9596	.0923
2	.8756	7.6461	8.5217	.1758
4	1.4593	6.3042	7.7635	.3203
6	1.8936	5.2408	7.1344	.4402
8	2.2273	4.3818	6.6091	.5403
10	2.4875	3.6809	6.1684	.6243
12	2.6923	3.1048	5.7970	.6951
14	2.8541	2.6288	5.4829	.7550
16	2.9823	2.2339	5.2162	.8058
18	3.0840	1.9050	4.9890	.8491
19	2.8566	-2.8566	0.0	.8179
20	2.6789	-2.6789	0.0	.7887
22	2.3922	-2.3922	0.0	.7350
24	2.1596	-2.1596	0.0	.6866
26	1.9627	-1.9627	0.0	.6427
28	1.7923	-1.7923	0.0	.6027
30	1.6426	-1.6426	0.0	.5661
32	1.5100	-1.5100	0.0	.5324
34	1.3916	-1.3916	0.0	.5014
36	1.2852	-1.2852	0.0	.4728

TABLE 4.1(b) **Aquifer and Well Storage Contributions and Drawdown for Average Constant Rate of Pumping**

(T = 2.1875 m^2/10 min., ϕ = 0.001, $r_w = r_c$ = 5.4 m, Q = 7.2812 m^3/10 min., m = 18)

Time Step (n)	$Q_A(n)$	$Q_W(n)$	$S_W(n)$
1	.4023	6.8789	.0750
2	.7312	6.5500	.1465
4	1.2845	5.9968	.2803
6	1.7519	5.5293	.4035
8	2.1602	5.1211	.5175
10	2.5227	4.7585	.6233
12	2.8480	4.4332	.7218
14	3.1420	4.1393	.8137
16	3.4091	3.8722	.8997
18	3.6528	3.6284	.9802
19	3.3645	-3.3645	.9433
20	3.1447	-3.1447	.9092
24	2.5176	-2.5176	.7899
28	2.0817	-2.0817	.6923
32	1.7495	-1.7495	.6108
36	1.4863	-1.4863	.5418

$m^3/10$ minutes and the drawdown at the well face is 0.8491 m. For the corresponding average constant pumping rate the aquifer contribution and drawdown at the well face as could be seen from Table [4.1(b)] are 3.6528 $m^3/10$ minutes and 0.9802 m respectively. There is significant difference in the drawdown values as well as in the values of aquifer contribution and therefore an average pumping rate can not substitute for the variable abstraction rate.

The specific capacity values at the end of pumping have been determined for known values of transmissivity, storage coefficient, initial pumping rate, well dimensions, and duration of pumping for the case in which the pumping rate is a linear function of drawdown at the well. Graphs between transmissivity and specific capacity at the end of pumping have been plotted in Figs. [4.3(a)] through [4.3(g)] for different values of ϕ. These graphs can be used to find the approximate value of transmissivity if the storage coefficient is known. Storage coefficient can be determined either by water level fluctuation method or using well log data.

Variation of specific capacity with time for a set of T, ϕ, Q_I, S_F and r_w values are presented for different values of r_c in Fig. (4.4) to depict the effect of well storage on specific capacity. It is seen from the figure that the specific capacity decreases with increase in the time of pumping. For the purpose of comparison, the specific capacity values for the well having negligible storage and for the well having considerable storage, for constant and for drawdown dependent abstractions, are presented in Table (4.2).

4.4 CONCLUSIONS

Based on the study the following conclusions are drawn :

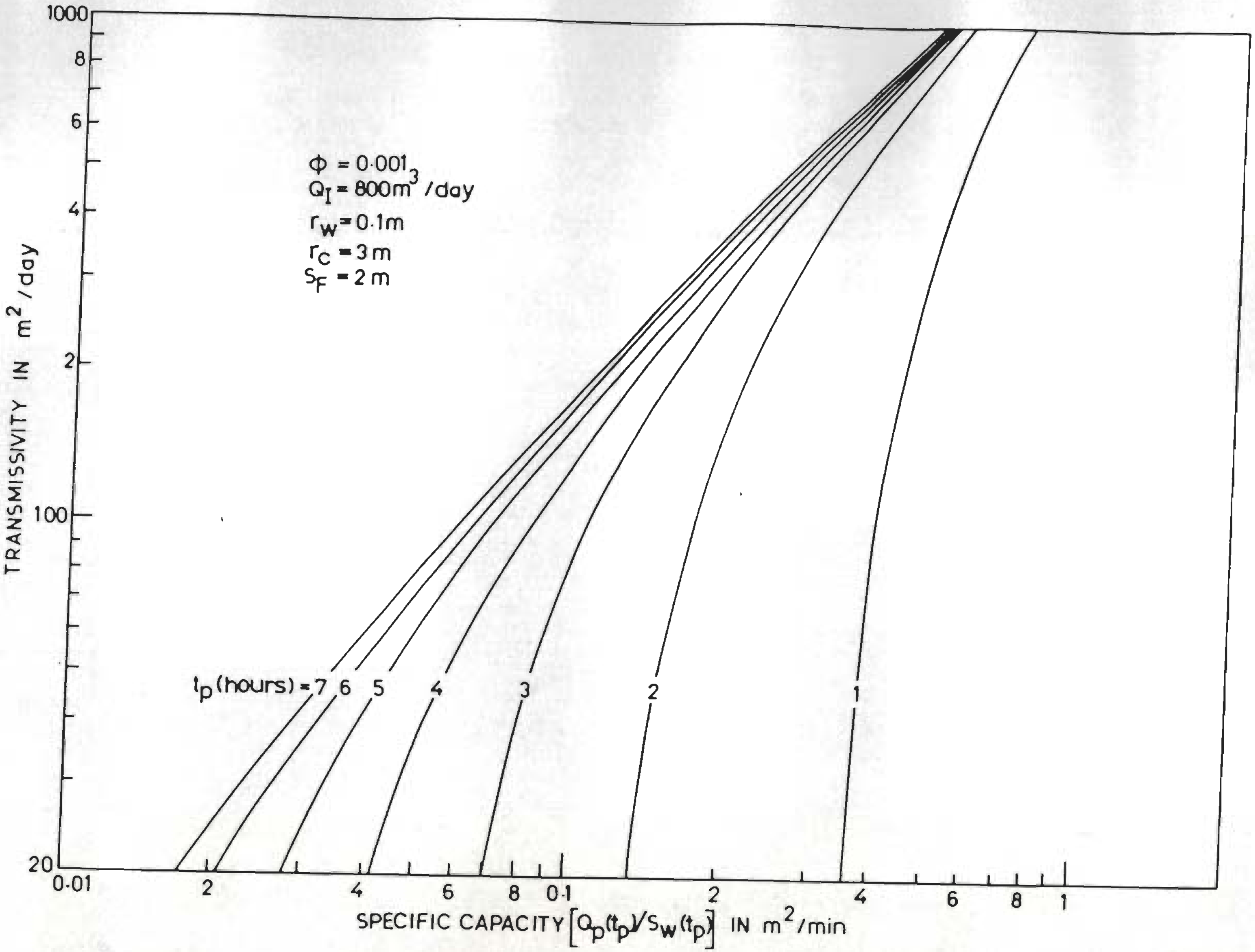


FIG. 4.3(a) - Relation between transmissivity and specific capacity of a large-diameter well for different durations of pumping and for $\phi = 0.001$.

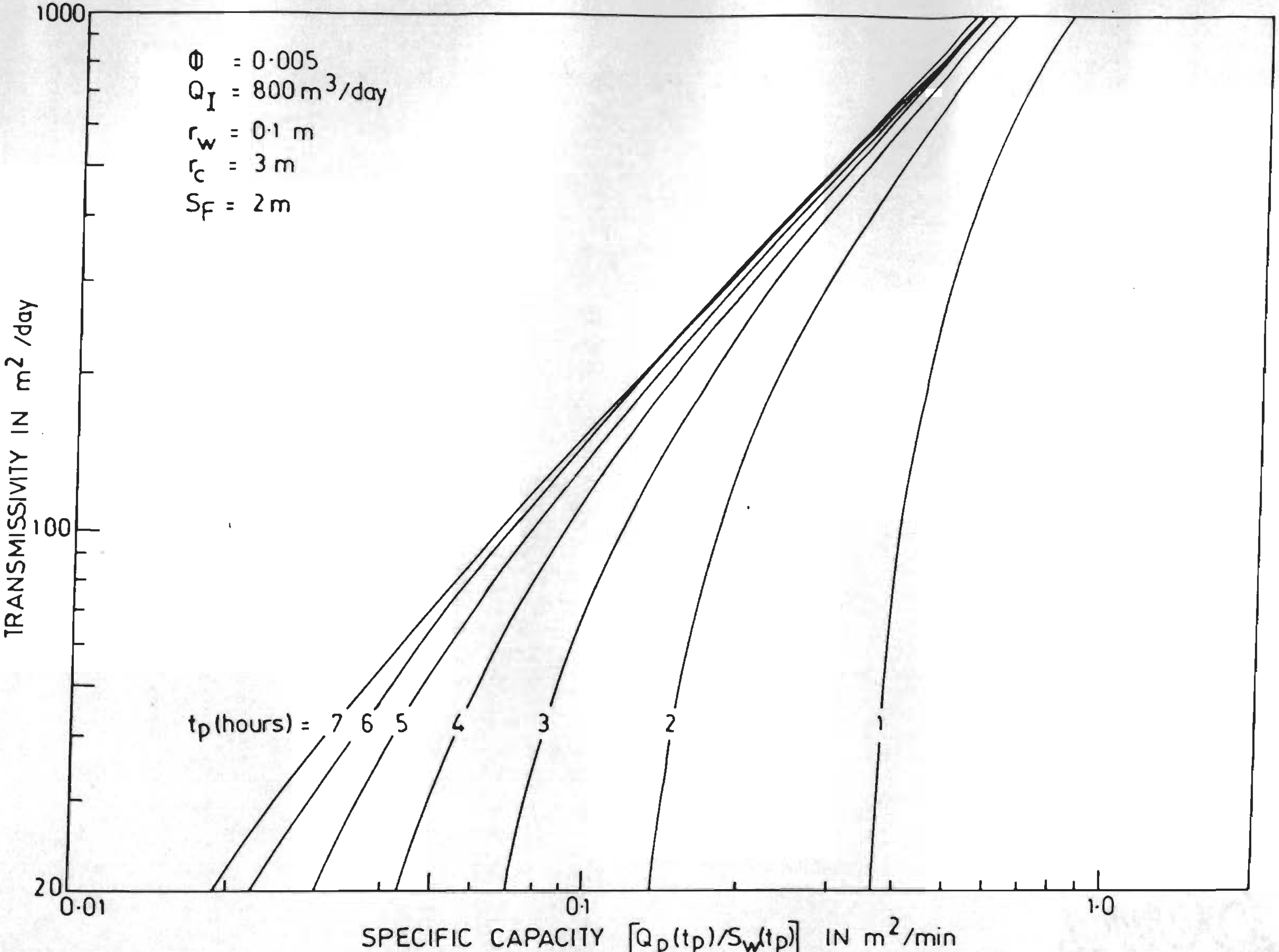


FIG. 4.3(b) - Relation between transmissivity and specific capacity of a large-diameter well for different durations of pumping and for $\phi = 0.005$.

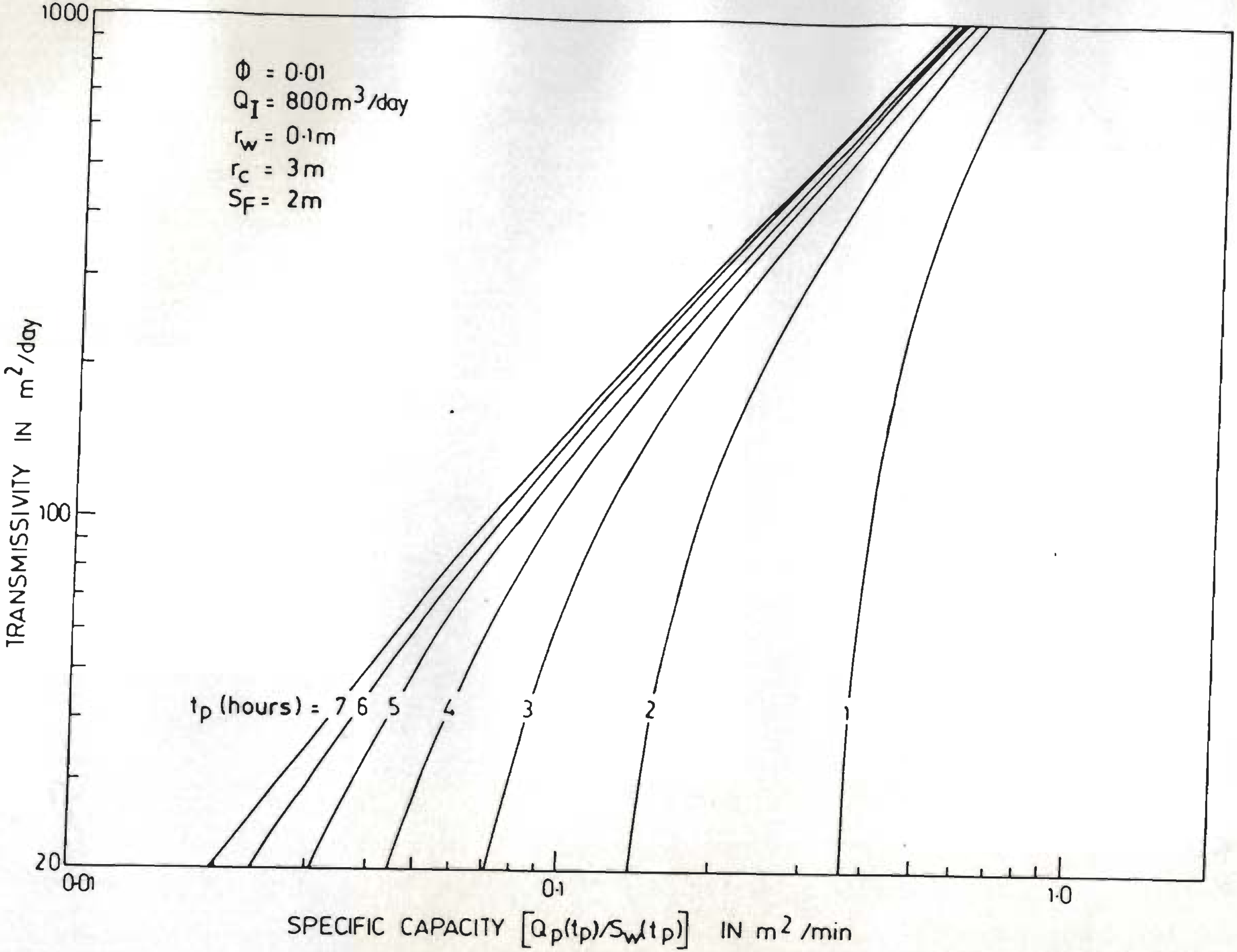


FIG. 4.3(c) - Relation between transmissivity and specific capacity of a large-diameter well for different durations of pumping and for ϕ = 0.01.

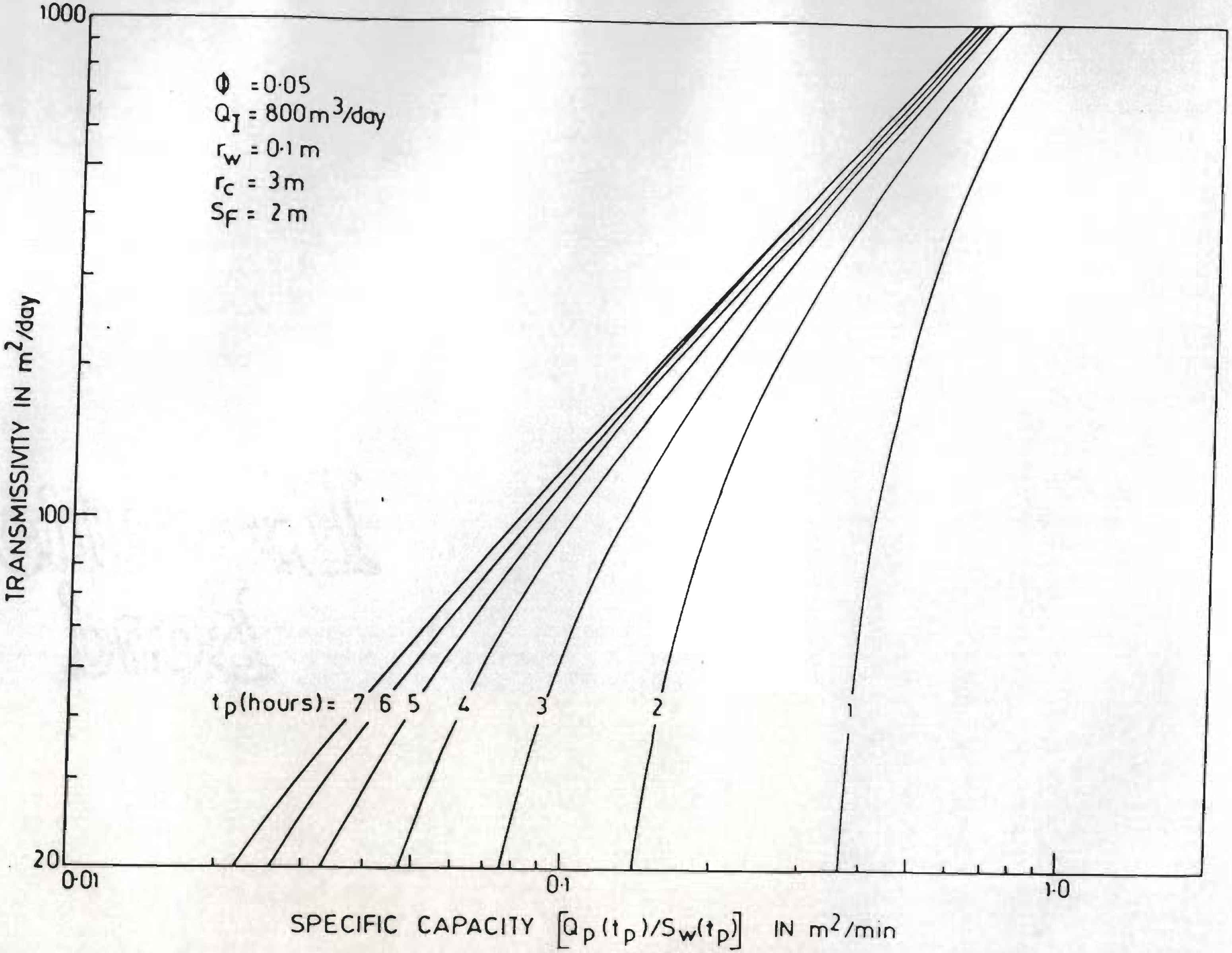


FIG. 4.3(d) - Relation between transmissivity and specific capacity of large-diameter well for different durations of pumping and for $\phi = 0.05$.

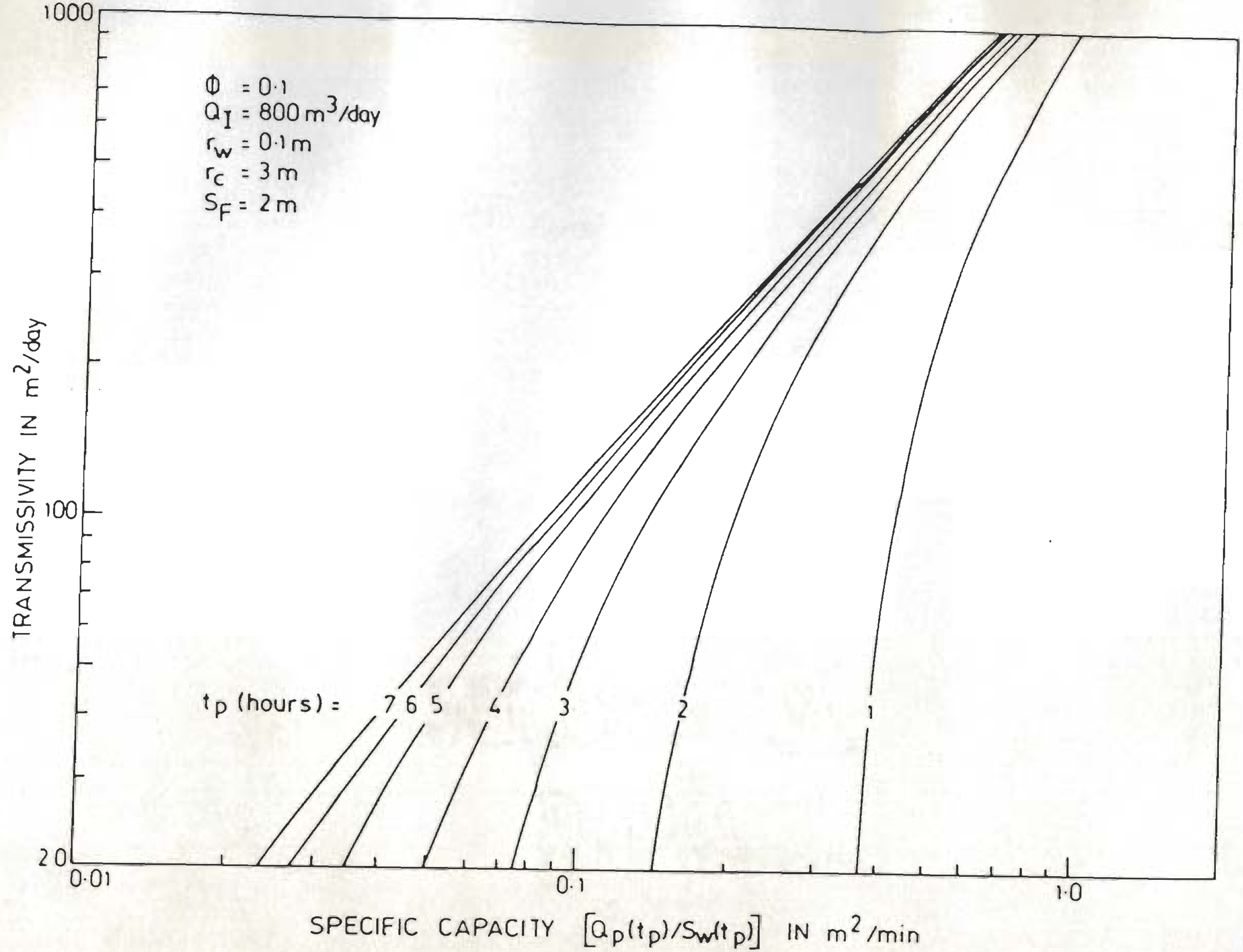


FIG. 4.3(e) - Relation between transmissivity and specific capacity of a large-diameter well for different duratrions of pumping and for ϕ = 0.1.

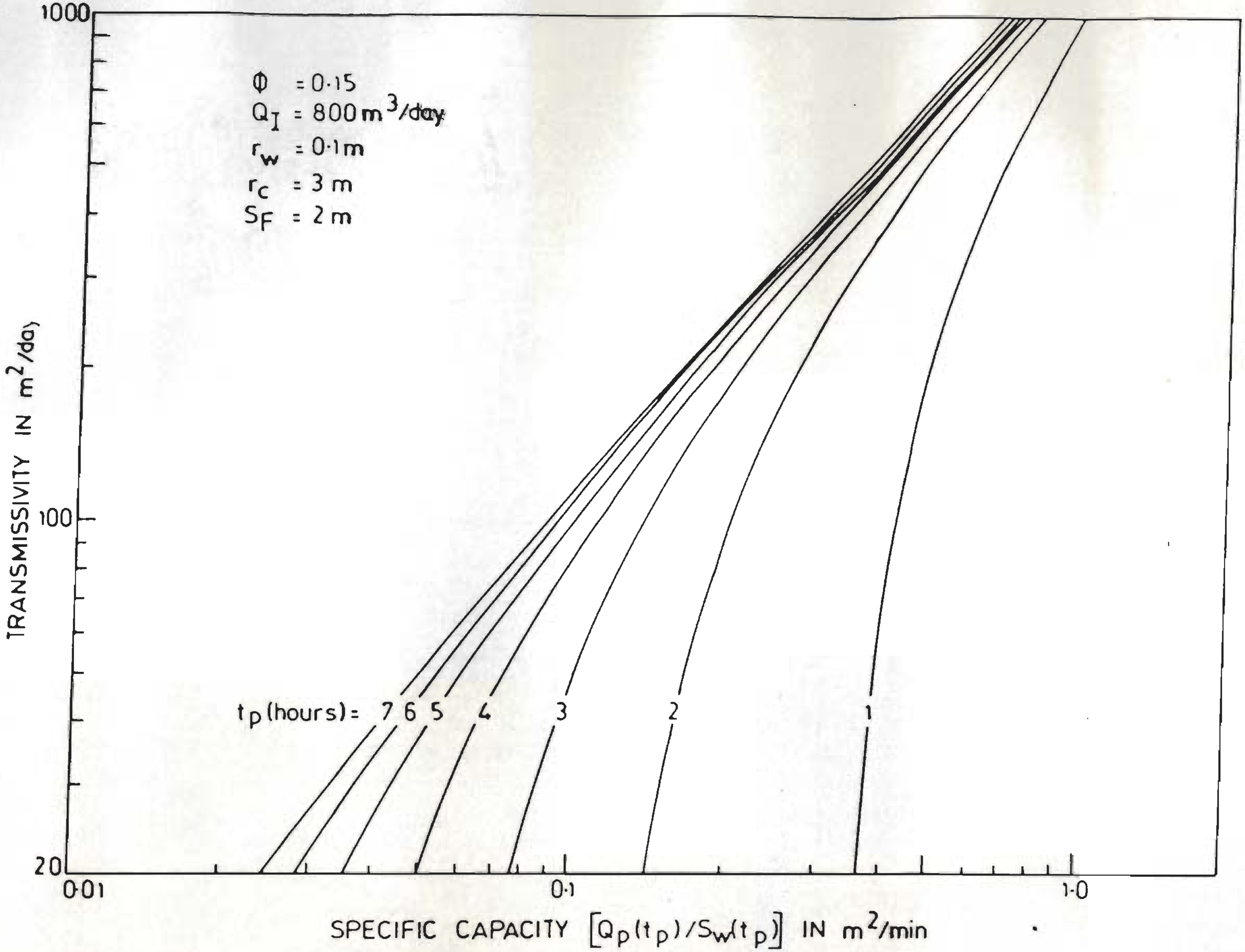


FIG. 4.3(f) - Relation between transmissivity and specific capacity of a large-diameter well for different durations of pumping and for Φ = 0.15.

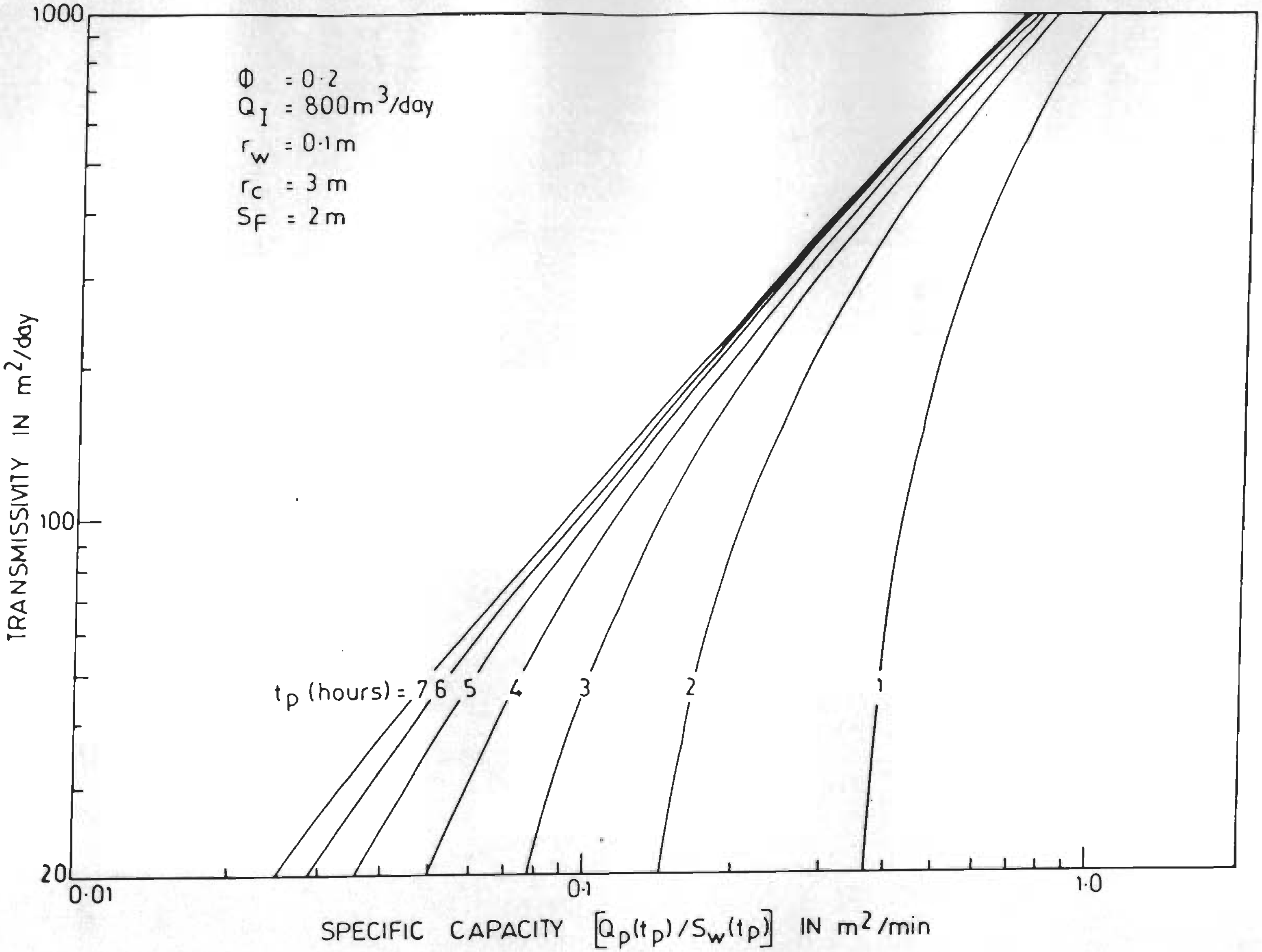


FIG. 4.3(g) - Relation between transmissivity and specific capacity of a large-diameter well for different durations of pumping and for ϕ = 0.2.

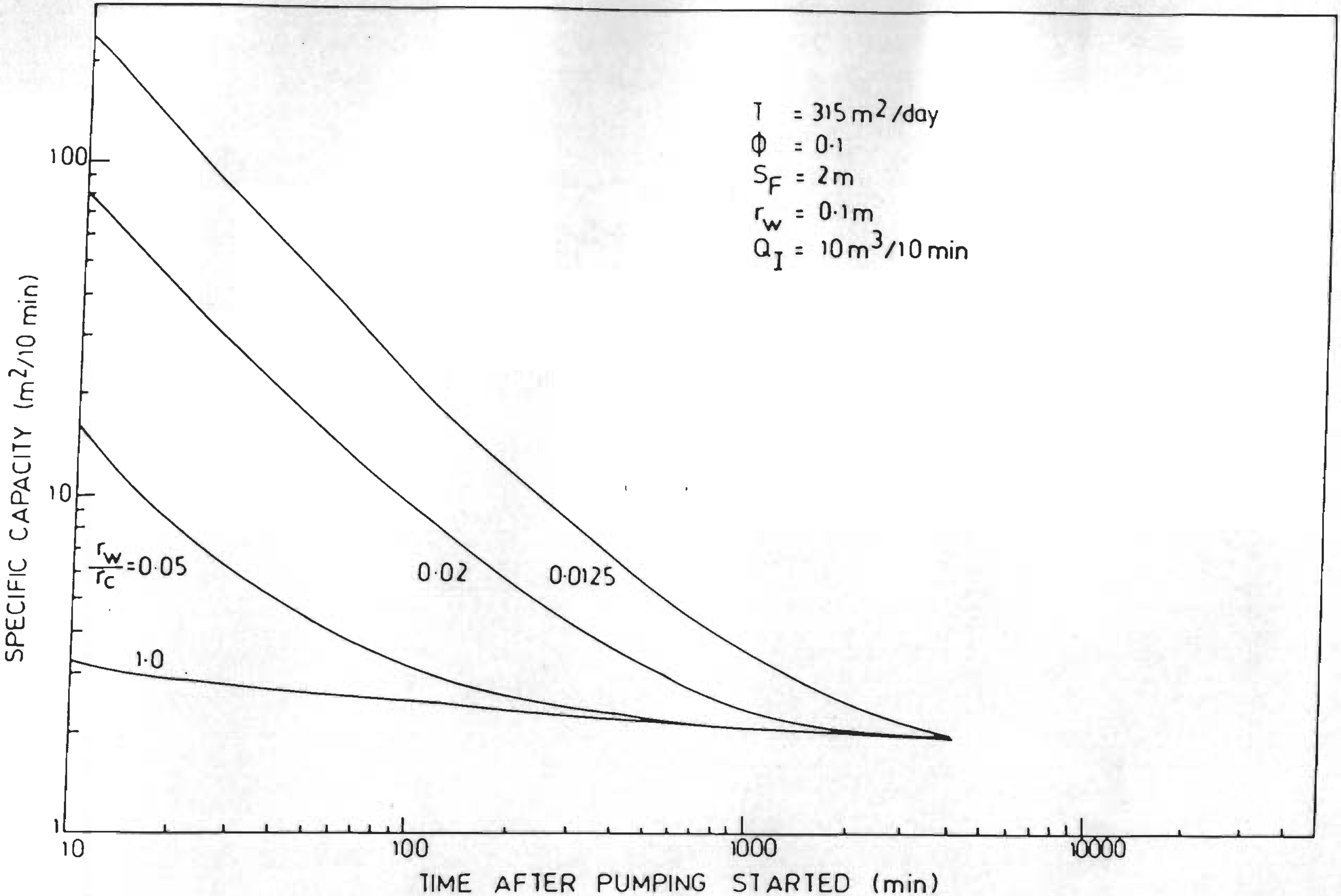


FIG. 4.4 - Variation of specific capacity of a large-diameter well with time of pumping during which the pumping rate varies linearly with drawdown in the well.

TABLE 4.2 **Comparison of Specific Capacity Values at Different Durations of Pumping [T = 100 m^3/day, ϕ = 0.001, Q = 800 m^3/day, Q_I = 800 m^3/day, r_w = 0.1 m, r_c = 3 m and S_F = 2 m].**

Time Since Pumping (hours)	SPECIFIC CAPACITY VALUES (m^2/day)		
	For a well of negligible storage (Constant discharge)	For a well of considerable storage (Constant discharge)	For a well of considerable storage (Discharge varies with drawdown)
1	91.40	763.20	576.00
2	87.01	406.08	249.12
3	84.64	288.00	154.08
4	83.03	227.52	120.96
5	81.93	192.96	100.80
6	80.86	16.56	93.60
7	80.02	15.12	86.40

(i) The graphs showing variations of transmissivity with specific capacity have been provided for the case in which the pumping rate is linearly dependent on drawdown in the well. These specific capacity graphs can be used to find the transmissivity of the aquifer provided the storage coefficient is known a priori.

(ii) An average constant pumping rate cannot substitute for the drawdown dependent abstraction rate for finding the well storage contribution, aquifer contribution, and drawdown in the aquifer.

Based on the work reported in this chapter, the following paper has been published :

Mishra, G.C. and A.G. Chachadi. (1985). Analysis of unsteady flow to a large-diameter well. Proc. International Workshop on Rural Hydrogeology & Hydraulics in Fissured Basement Zones, University of Roorkee, Roorkee, India, pp. 139-145.

CHAPTER 5

ANALYSIS OF FLOW TO A LARGE-DIAMETER OBSERVATION WELL DUE TO PUMPING OF A LARGE-DIAMETER PRODUCTION WELL

5.0 INTRODUCTION

The aquifer response during pumping test in a large-diameter well may be recorded either in the large-diameter well itself or at a nearby observation well of small-diameter. Papadopulos and Cooper (1967) have analysed unsteady flow to a large-diameter production well in a confined nonleaky aquifer. Using the solution of Papadopulos and Cooper the aquifer response can be estimated at the production well and at other observation wells which have negligible storage. Fenske (1977) has derived a set of equations based on Theis solution for finding the aquifer response when both the observation well and the production well possess storage. To account for the effect of storage in observation well, Fenske assumed that the water stored in the observation well recharges the aquifer instantaneously with drop in piezometric head in the adjacent aquifer. A large-diameter well can also serve as an observation well if an aquifer test is conducted in a production well of negligible diameter. Barker (1984) has shown that if a pumping test is conducted in a production well of negligible diameter, the drawdown in large-diameter observation well is identical to the drawdown in an observation well if roles of the wells are reversed. Barker has identified that if both the production well and the observation well have storages the solution for the drawdown is yet to be known. Mucha and Paulikova (1986) have presented an approximate equation for finding the response of the observation well possessing storage during pumping of a large-diameter production well. Storage associated with large-diameter production or observation well modifies and causes delay in the aquifer response. Therefore, storage effect should be duly considered while solving a direct or an inverse problem. In the present study a generalised discrete kernel approach has been described to analyse effect of both production well and observation well storages on drawdown at any point in the aquifer.

5.1 STATEMENT OF THE PROBLEM

The four combinations at a production well and a single observation well located at distance 'r_1' apart which may or may not have storage are shown in Fig. (5.1). The radii of the screened and unscreened parts of the production well are r_{wp} and r_{cp} and that of the observation well are r_{wo} and r_{co} respectively. The confined aquifer is homogeneous, isotropic, infinite in areal extent and is initially at rest condition. Pumping is continued up to time t_p. The rate of pumping is constant or it may vary with time. It is required to determine the drawdown in the piezometric surface at the large-diameter observation well, at the production well and at any distance 'r' from the centre of the production well during pumping and recovery periods.

5.2 ANALYSIS

The following assumptions have been made in the analysis :

a) The time parameter is discrete. Within each time step the abstraction rates from well storages and that from aquifer storage are separate constants.

b) At any time, drawdown in the piezometric surface in the aquifer at the well face is equal to drawdown in the water surface in the well. This assumption is true both for the production well as well as for the observation well.

The basic differential equation for an axially-symmetric, radial unsteady

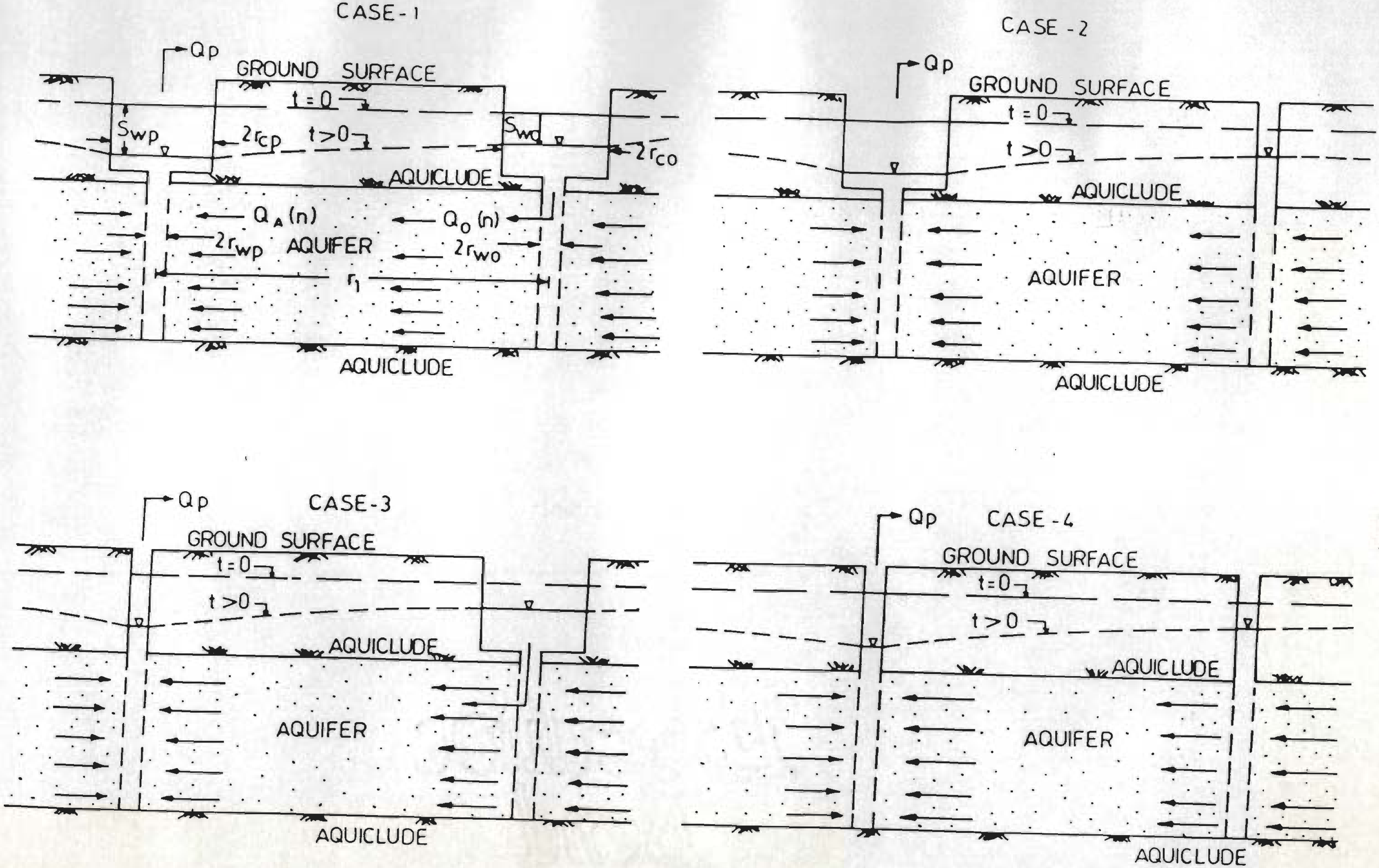


FIG. 5.1 - Schematic diagrams of production and observation wells with or without storage.

groundwater flow in a homogeneous, isotropic, confined aquifer of uniform thickness is given by

$$\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r} = \frac{\phi}{T}\frac{\partial S}{\partial t} \qquad \ldots(5.1)$$

Where, S = drawdown, r = distance measured from the centre of the well, t = time, ϕ = storage coefficient, and T = transmissivity of the aquifer.

For the initial condition $S(r,0) = 0$ and boundary condition $S(\infty, t) = 0$, solution to the above differential equation, when a unit impulse quantity of water is withdrawn from the aquifer storage through a well with negligible radius, is given by (Carslaw and Jaeger, 1959).

$$S(r,t) = \frac{e^{-\frac{r^2}{4\beta t}}}{4\pi Tt}, \quad \beta = \frac{T}{\phi} \qquad \ldots(5.2)$$

Defining a unit impulse kernel

$$k(t) = \frac{e^{-\frac{r^2}{4\beta t}}}{4\pi Tt} \qquad \ldots(5.3)$$

drawdown caused by a variable abstraction rate can be found using the expression (Morel-Seytoux, 1975).

$$S(r,t) = \int_0^t Q_A(\tau)\, k(t-\tau)\, d\tau \qquad \ldots(5.4)$$

Where $Q_A(\tau)$ is the variable abstraction rate from the aquifer storage at time τ.

Dividing the time span into discrete time steps and assuming that the aquifer discharge is constant within each time step, but varies from step to step, drawdown at the end of time step n can be written as (Morel-Seytoux, 1975)

$$S(r,n) = \sum_{\gamma=1}^{n} \delta_r(n-\gamma+1)\, Q_A(\gamma) \qquad \text{...(5.5)}$$

in which the discrete kernel coefficient $\delta_r(I)$ is given by

$$\delta_r(I) = \int_0^1 k(I-\tau)\,d\tau = \frac{1}{4\pi T}\left[E_1\left(\frac{r^2}{4\beta I}\right) - E_1\left\{\frac{r^2}{4\beta(I-1)}\right\}\right] \qquad \text{...(5.6)}$$

The large-diameter observation well acts as a recharge well in response to pumping in the production well. When several wells operate simultaneously the resulting drawdown can be found by summing up the drawdown caused by pumping of individual well since equation (5.1) is linear and method of superposition is valid for a linear system.

Let the total time of pumping, t_p, be discretised to m units of equal time steps. The quantity of water, $Q_p(n)$, pumped during any time step, n, can be written as :

$$Q_A(n) + Q_W(n) = Q_p(n) \qquad \text{...(5.7)}$$

in which,

$Q_A(n)$ = water withdrawn from aquifer storage at the production well during time step n, and

$Q_W(n)$ = water withdrawn from production well storage during time step n.

For $n > m$, $Q_p(n) = 0$, otherwise $Q_p(n)$ is equal to rate of pumping during time step n.

Drawdown, $S_{Wp}(n)$, in the water surface at the production well due to abstraction from the production well storage at the end of time step 'n' is given by

$$S_{Wp}(n) = \frac{1}{\pi r_{cp}^2} \sum_{\gamma=1}^{n} Q_W(\gamma) \qquad \ldots(5.8)$$

where $Q_W(\gamma)$ represents rate of withdrawal from production well storage or the replenishment during time step γ. $Q_W(\gamma)$ values are unknown a priori. A negative value of $Q_W(\gamma)$ means that there is replenishment of well storage which occurs during recovery period.

Similarly drawdown in the water surface at the observation well at the end of time step 'n' due to recharge taking place from the observation well storage to the aquifer is given by

$$S_{Wo}(n) = \frac{1}{\pi r_{co}^2} \sum_{\gamma=1}^{n} Q_0(\gamma) \qquad \ldots(5.9)$$

in which, $Q_0(\gamma)$ is the recharge rate from the observation well storage during time step γ.

Drawdown in the piezometric surface in the aquifer at the production well face at the end of time step 'n' due to abstraction from aquifer through the production well and recharge from the observation well storage is given by

$$S_{Ap}(n) = \sum_{\gamma=1}^{n} Q_A(\gamma)\ \delta_{r_{wp}}(n-\gamma+1) - \sum_{\gamma=1}^{n} Q_0(\gamma)\ \delta_{r_1}(n-\gamma+1) \qquad \ldots(5.10)$$

where the discrete kernel coefficients $\delta_{r_{wp}}(I)$ and $\delta_{r_1}(I)$ are given by

$$\delta_{r_{wp}}(I) = \frac{1}{4\pi T}\left[E_1\left(\frac{\phi r_{wp}^2}{4TI}\right) - E_1\left\{\frac{\phi r_{wp}^2}{4T(I-1)}\right\}\right] \qquad \ldots(5.11)$$

and

$$\delta_{r_1}(I) = \frac{1}{4\pi T}\left[E_1\left(\frac{\phi r_1^2}{4TI}\right) - E_1\left\{\frac{\phi r_1^2}{4T(I-1)}\right\}\right], \text{ respectively} \qquad \ldots(5.12)$$

Drawdown in the piezometric surface in the aquifer at the observation well face is given by

$$S_{Ao}(n) = \sum_{\gamma=1}^{n} Q_A(\gamma)\,\delta_{r_1}(n-\gamma+1) - \sum_{\gamma=1}^{n} Q_0(\gamma)\,\delta_{r_{wo}}(n-\gamma+1) \ldots(5.13)$$

where the discrete kernel coefficient $\delta_{r_{wo}}(I)$ is given by

$$\delta_{r_{wo}}(I) = \frac{1}{4\pi T}\left[E_1\left(\frac{\phi r_{wo}^2}{4TI}\right) - E_1\left\{\frac{\phi r_{wo}^2}{4T(I-1)}\right\}\right] \qquad \ldots(5.14)$$

Because $S_{Ap}(n) = S_{Wp}(n)$,

$$\sum_{\gamma=1}^{n} Q_A(\gamma)\,\delta_{r_{wp}}(n-\gamma+1) - \sum_{\gamma=1}^{n} Q_0(\gamma)\,\delta_{r_1}(n-\gamma+1) = \frac{1}{\pi r_{cp}^2}\sum_{\gamma=1}^{n} Q_W(\gamma)$$

$$\ldots(5.15)$$

Rearranging,

$$Q_A(n)\,\delta_{r_{wp}}(1) - \frac{1}{\pi r_{cp}^2} Q_W(n) - Q_0(n)\,\delta_{r_1}(1) = \frac{1}{\pi r_{cp}^2}\sum_{\gamma=1}^{n-1} Q_W(\gamma)$$

$$+ \sum_{\gamma=1}^{n-1} Q_0(\gamma)\ \delta_{r_1}(n-\gamma+1) - \sum_{\gamma=1}^{n-1} Q_A(\gamma)\ \delta_{r_{wp}}(n-\gamma+1) \qquad ...(5.16)$$

Also,

$S_{Ao}(n) = S_{Wo}(n)$, therefore,

$$\sum_{\gamma=1}^{n} Q_A(\gamma)\ \delta_{r_1}(n-\gamma+1) - \sum_{\gamma=1}^{n} Q_0(\gamma)\ \delta_{r_{wo}}(n-\gamma+1) = \frac{1}{\pi r_{co}^2} \sum_{\gamma=1}^{n} Q_0(\gamma) \qquad ...(5.17)$$

Rearranging,

$$Q_A(n)\ \delta_{r_1}(1) - Q_0(n)\left[\delta_{r_{wo}}(1) + \frac{1}{\pi r_{co}^2}\right] = \frac{1}{\pi r_{co}^2}\sum_{\gamma=1}^{n-1} Q_0(\gamma)$$

$$+ \sum_{\gamma=1}^{n-1} Q_0(\gamma)\ \delta_{r_{wo}}(n-\gamma+1) - \sum_{\gamma=1}^{n-1} Q_A(\gamma)\ \delta_{r_1}(n-\gamma+1) \qquad ...(5.18)$$

Equations (5.7), (5.16) and (5.18) can be expressed in the following matrix form :

$$\begin{bmatrix} 1 & , & 1 & , & 0 \\ \delta_{r_{wp}}(1) & , & -\frac{1}{\pi r_{cp}^2} & , & -\delta_{r_1}(1) \\ \delta_{r_1}(1) & , & 0 & , & -\left[\delta_{r_{wo}}(1) + \frac{1}{\pi r_{co}^2}\right] \end{bmatrix} \begin{bmatrix} Q_A(n) \\ Q_W(n) \\ Q_0(n) \end{bmatrix} =$$

$$\begin{bmatrix} Q_p(n) \\ \frac{1}{\pi r_{cp}^2} \sum_{\gamma=1}^{n-1} Q_W(\gamma) + \sum_{\gamma=1}^{n-1} Q_0(\gamma)\,\delta_{r_1}(n-\gamma+1) - \sum_{\gamma=1}^{n-1} Q_A(\gamma)\,\delta_{r_{wp}}(n-\gamma+1) \\ \frac{1}{\pi r_{co}^2} \sum_{\gamma=1}^{n-1} Q_0(\gamma) + \sum_{\gamma=1}^{n-1} Q_0(\gamma)\,\delta_{r_{wo}}(n-\gamma+1) - \sum_{\gamma=1}^{n-1} Q_A(\gamma)\,\delta_{r_1}(n-\gamma+1) \end{bmatrix} \quad \ldots(5.19)$$

or $[A].[B] = [C]$

in which [A] is the left hand side square matrix, [B] is the column matrix with unknown elements and, [C] is the right hand side column matrix whose elements are known at any time step.

In particular for time step 1, the right-hand side column matrix

$$[C] = [Q_p(1),\ 0,\ 0]',$$

$Q_A(n)$, $Q_W(n)$, and $Q_0(n)$ can be solved in succession starting from time step 1 using the relation $[B] = [A]^{-1}.[C]$. Once $Q_A(n)$, $Q_W(n)$ and $Q_0(n)$ are found the drawdown at any point in the aquifer which is at a distance 'r' from the production well can be known using the relation

$$S(r,n) = \sum_{\gamma=1}^{n} \delta_r(n-\gamma+1)\, Q_A(\gamma) - \sum_{\gamma=1}^{n} \delta_{r_2}(n-\gamma+1)\, Q_0(\gamma) \quad \ldots(5.20)$$

in which, r_2, is the distance between the point under consideration and the large-diameter observation well, and the discrete kernel coefficient is given by

$$\delta_{r_2}(I) = \frac{1}{4\pi T}\left[E_1\left(\frac{\phi r_2^2}{4\,TI}\right) - E_1\left\{\frac{\phi\, r_2^2}{4T(I-1)}\right\}\right].$$

5.3 RESULTS AND DISCUSSION

The discrete kernel coefficients $\delta_{r_{wp}}(I)$, $\delta_{r_1}(I)$ and $\delta_{r_{wo}}(I)$ have been generated using equations (5.11), (5.12) and (5.14) respectively for known values of transmissivity, storage coefficient, radii of the production and observation well screens and the distance between the production and observation wells. After generating the discrete kernel coefficients, $Q_A(n)$, $Q_W(n)$ and $Q_0(n)$ are solved in succession starting from the first time step from the matrix equation (5.19) for known values of r_{cp}, r_{co}, r_1 and $Q_p(n)$. It is only required to inverse the matrix [A] once. The column vector [C] is required to be evaluated each time for finding the unknowns, $Q_A(n)$, $Q_W(n)$ and $Q_0(n)$. The drawdowns at the production well and at the observation well are then obtained with the help of equations (5.8) and (5.9). An appropriate time step size has been used for the numerical computation.

The variation of $S_{Wp}(t)/[Q_p/(4\pi T)]$ with $4Tt/(\phi\, r_{wp}^2)$ for the production well and $S_{Wo}(t)/[Q_p/(4\pi T)]$ with $4Tt/(\phi\, r_1^2)$ for the observation well are shown in Figs. (5.2) through (5.5) for different values of α_p and α_o where $\alpha_o = \phi(r_{wo}/r_{co})^2$ and $\alpha_p = \phi\,(r_{wp}/r_{cp})^2$. The parameters α_o and α_p quantify the observation and production well storages respectively. $S_{Wp}(t)$ and $S_{Wo}(t)$ are the drawdowns at the production well and at the observation well respectively at time t and $S_{Wp}(t)/[Q_p/(4\pi T)]$ and $S_{Wo}(t)/[Q_p/(4\pi T)]$ can be regarded as the well functions for the large-diameter production well and observation well respectively. The curves in Figures (5.2) through (5.5) contain the response of the aquifer during the abstraction phase as well as during

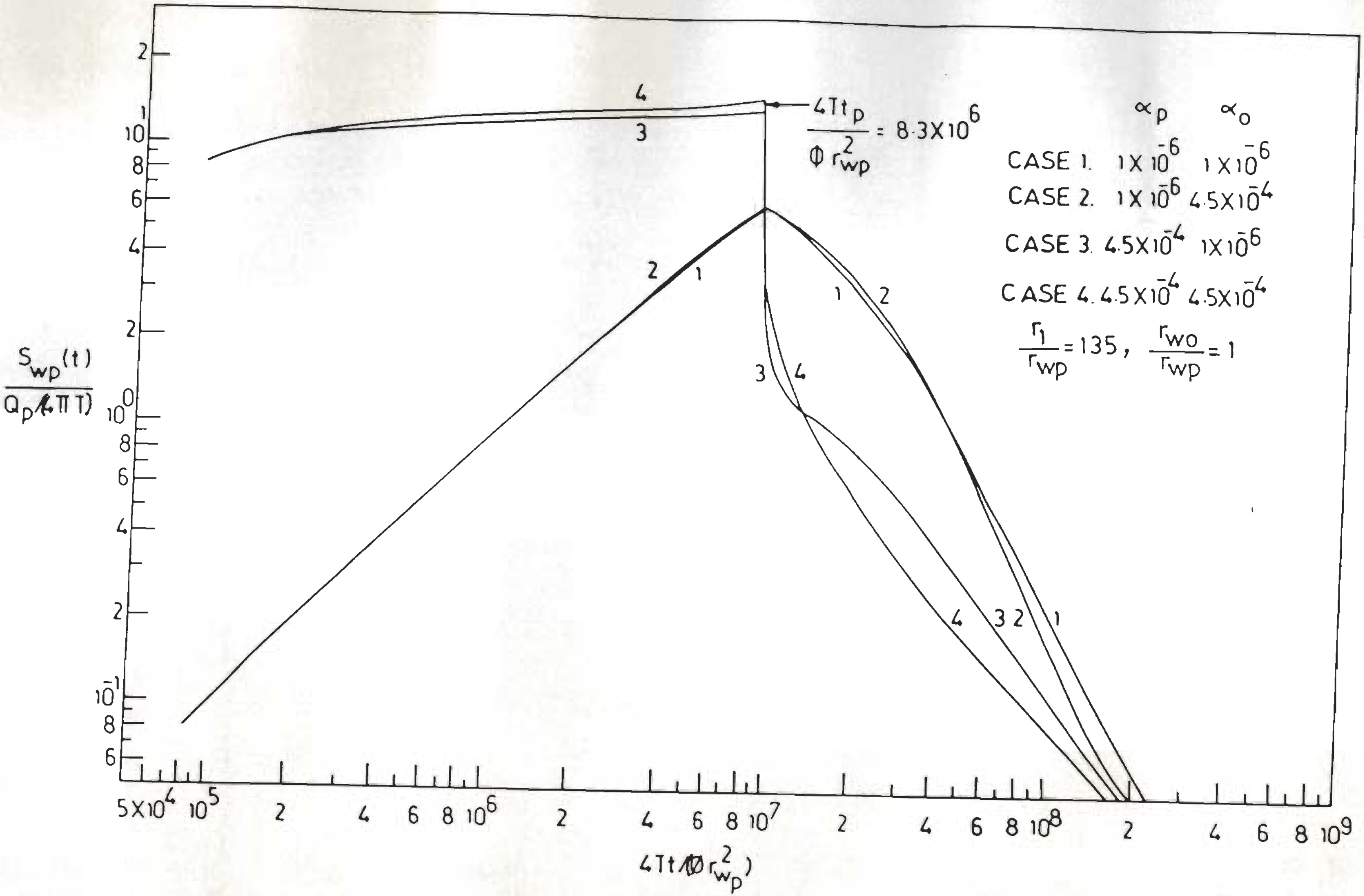


FIG. 5.2 - Variation of $S_{Wp}(t)/(Q_p/(4\pi T)$ with $4Tt/(\phi r_{wp}^2)$ at production well.

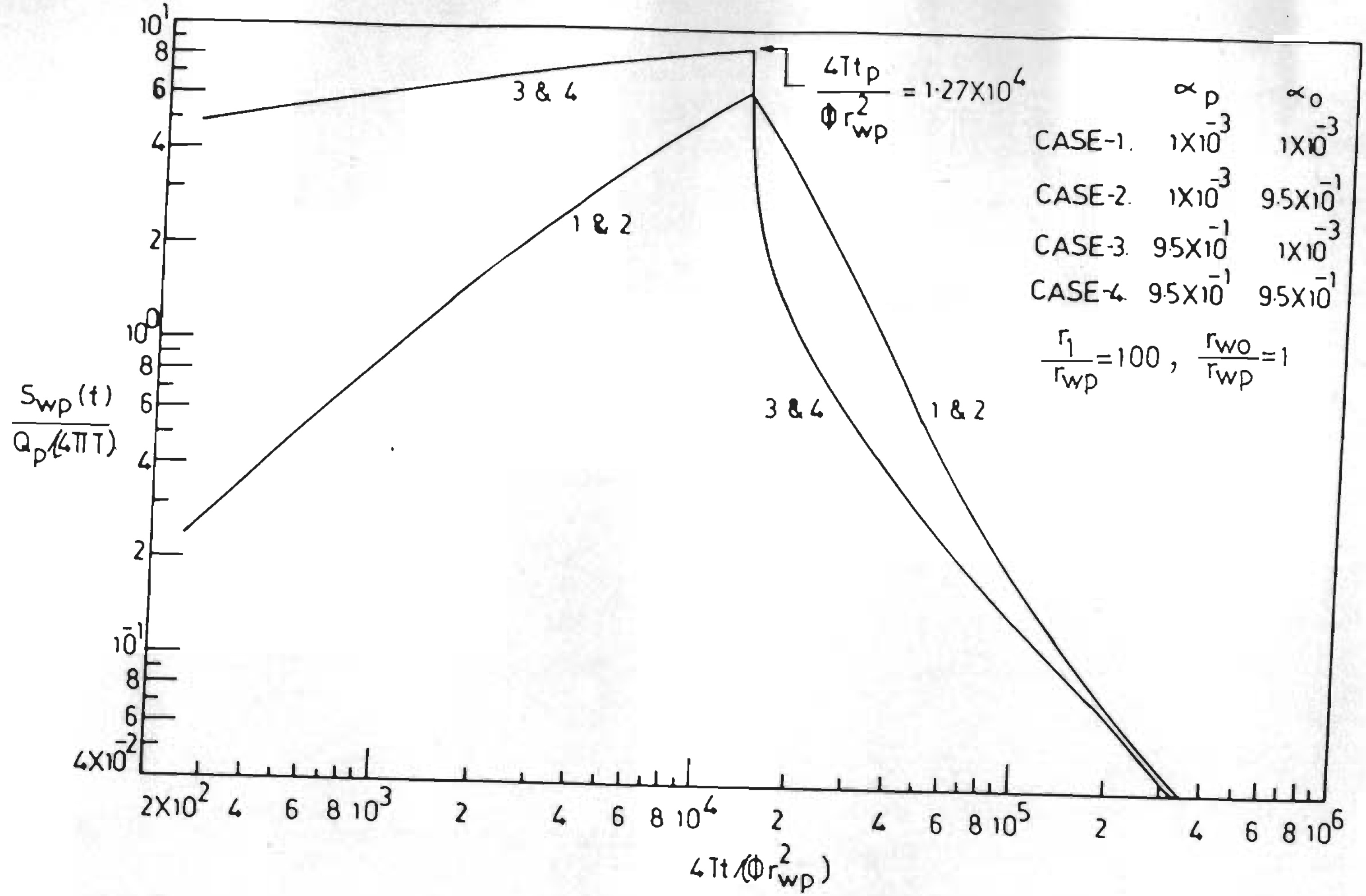


FIG. 5.3 - Variation of $S_{Wp}(t)/(Q_p/4\pi T)$ with $4Tt(\phi\ r_{wp}^2)$ at production well.

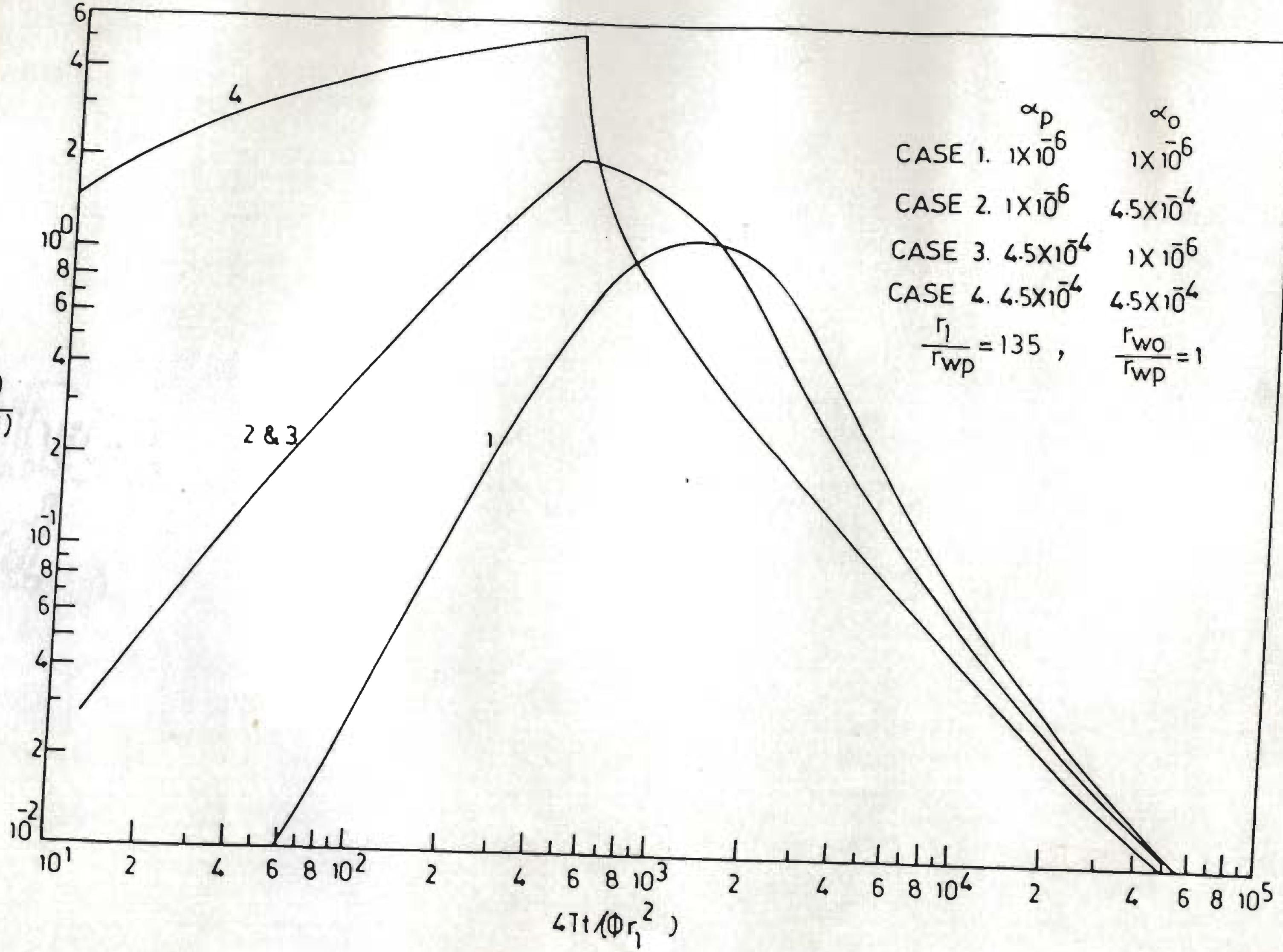


FIG. 5.4 - Variation of $S_{wo}(t)/(Q_p/4\pi T)$ with $4Tt(\phi\ r_1^2$) at observation well.

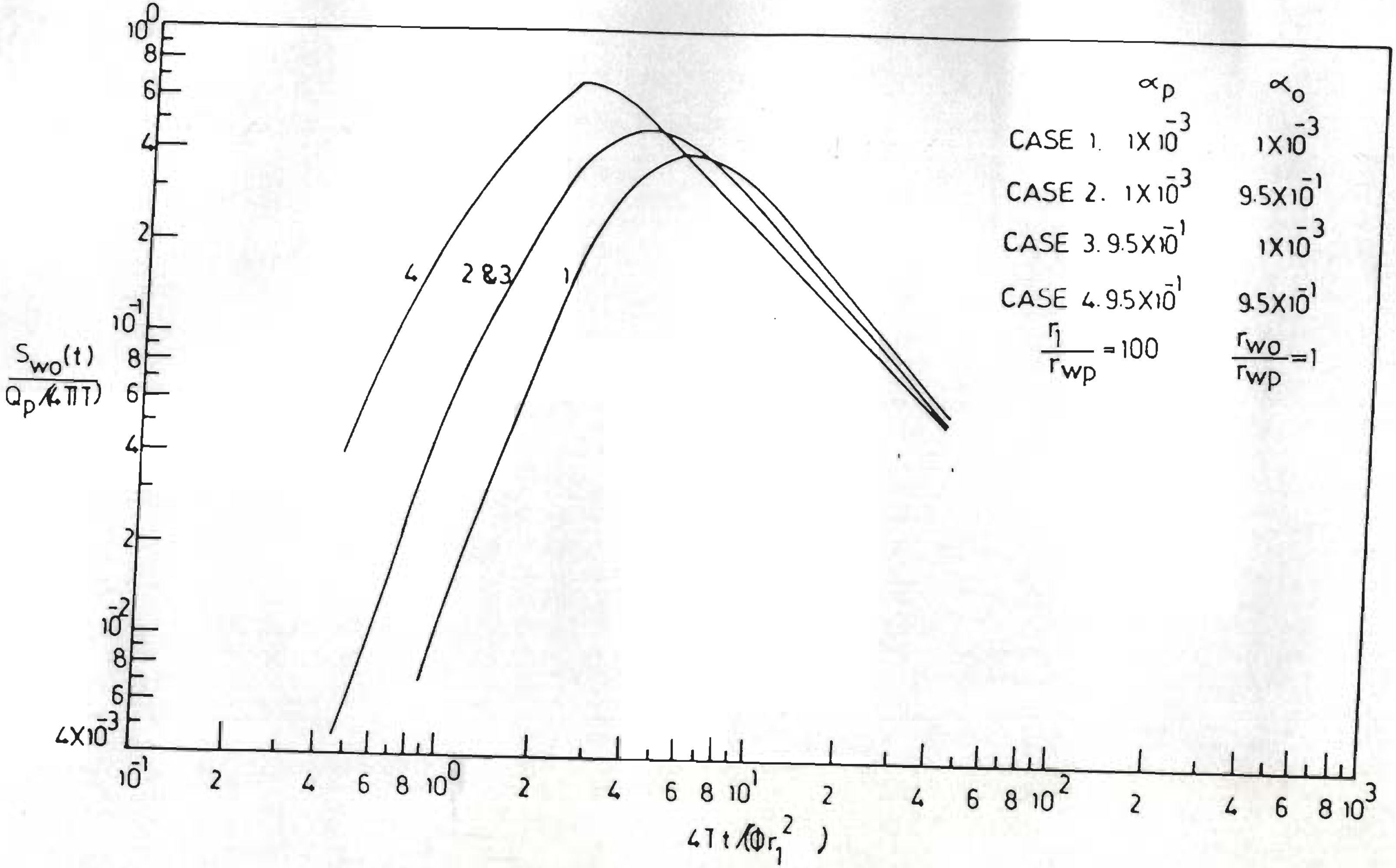


FIG. 5.5 - Variation of $S_{Wo}(t)/(Q_p/4\pi T)$ with $4Tt(\phi\ r_1^2)$ at observation well

the recovery phase. Each of the recovery curves is characterised by a non-dimensional time factor, $4Tt_p/(\phi r_{wp}^2)$ at which it deviates from the time-drawdown curve of the abstraction phase. In an inverse problem, the non-dimensional time factor $4Tt_p/(\phi r_{wp}^2)$ can be used to check the accuracy of the aquifer parameters while they are determined by curve matching.

The behaviours of the nondimensional time-drawdown plots for different cases have been discussed in the following paragraphs.

CASE 1 : In this case both the production well and the observation well are of large-diameter and have considerable storages in them. The production well has been pumped with a constant rate Q_p and the changes in water levels both during pumping and recovery periods at the production well and the observation well have been observed.

The nondimensional time-drawdown plot at the production well and at the observation well for Case 1 are shown in Figs. (5.2) and (5.4) respectively for values of α_p and $\alpha_o = 1 \times 10^{-6}$, and $[4Tt_p/(\phi \; r_{wp}^2)] = 8.3 \times 10^6$. The manifestation of the near straight-line portions of the time-drawdown graphs both at the production well and at the observation well during pumping in Figs (5.2) and (5.4) is due to the effect of individual well storage. During the initial stages of pumping most part of the pumped water comes from the production well storage, as contribution of production well storage decreases, the contributions of aquifer and observation well storage increase starting from zero. The rate of contribution of observation well storage would reach a maximum after which it would decrease till the time the storage in the observation well in finished. During the later part of the pumping phase, Q_p, is mainly derived from the aquifer storage.

After the cessation of pumping the production well soon starts recovering. On the other hand the water level in the observation well continues to fall for sometime after cessation of pumping but at a decreasing rate [Fig. (5.4)]. This is due to the fact that when pumping is stopped there is a difference between the water levels at the production and the observation wells. The water level in the observation well being at higher level, the flow of water towards the pumping well continues till the piezometric heads are at same level in both the wells. At this juncture the observation well also starts recovering its storage. During the initial period of recovery of the production well, water is derived both from the observation well storage as well as from the aquifer storage. Therefore, the recovery rate of production well during the period immediately after the cessation of pumping is higher when compared to the recovery rates during later period when both the wells start recovering. This is because the water derived from the aquifer is distributed to refill both the wells during later part of the recovery period.

CASE 2 : In this case only the production well is of large-diameter having storage and the observation well is of small-diameter with negligible storage. The large-diameter well has been pumped at a constant rate Q_p and the change in water level both during pumping and recovery periods have been observed at the production as well as at the observation well.

The plot of non-dimensional time-drawdown curve at the production and the observation wells are shown in Figs. (5.2) and (5.4) (Case 2) for values of α_p and α_o equal to 1×10^{-6} and 4.5×10^{-4} respectively, and $[4Tt/(\phi\, r_{wp}^2)] = 8.3 \times 10^6$. From Figs. (5.2) and (5.4), it is seen that the near straight line portions of the plot during pumping phase is due to the effect of well storage in the production well. During the early part of pumping phase most of the water is derived from the production well storage. When pumping continues for a longer period the well storage gets depleted and aquifer contribution becomes dominant.

After the cessation of pumping the production well soon starts recovering. On the other hand the recovery in the observation well is delayed because of water level gradients towards the production well [Fig. (5.4)]. When the water levels are equalised in both the wells the recovery starts faster in the observation well as compared to production well, as the observation well has a little storage capacity.

CASE 3 : In this case the production well is of negligible diameter and the observation well is of large-diameter having storage. The production well has been pumped at a constant rate Q_p and the changes in water level both during pumping and recovery phases have been observed at the production well as well as at the observation well.

The plots of nondimensional time-drawdown curve at the production and at the observation well are shown in Figs. (5.2) and (5.4) (Case 3) for values of α_p and α_o equal to 4.5×10^{-4} and 1×10^{-6} respectively, and $4Tt_p/(\phi\, r_{wp}^2)$ $= 8.3 \times 10^6$. From Fig. (5.2) it is seen that the effect of the observation well storage on drawdown in the production well is to reduce the drawdown. On the other hand, the plot of nondimensional time-drawdown curve at the observation well (Fig. 5.4) during pumping is a straight line indicating the effect of the observation well storage.

After the cessation of pumping, the production well soon starts recovering. The recovery rate is very fast during the early period due to contribution from the observation well storage to the aquifer. The moment the observation well storage ceases to contribute to aquifer the recovery rate in the production well becomes slow (Fig. 5.2). During the later period of recovery phase the rate of recovery in the production well again becomes faster. On the other hand, the early period recovery in the observation well is rather slow because of the storage effect in the well.

An important observation that is made from Fig. (5.4) is that the time-drawdown responses for cases 2 and 3, both during pumping and recovery, are identical. It indicates that when a production well of large-diameter is pumped, the drawdown response in a well of negligible diameter is same as that when the roles of the wells are reversed. This fact was also brought out by Barker (1984).

CASE 4 : In this case both the production and the observation wells are of small diameter. The production well is pumped at a constant rate Q_p and the changes in water level have been measured both during pumping and recovery phases at both the wells.

The plots of nondimensional time-drawdown curves have been presented in Figs. (5.2) and (5.4)(Case 4) both at the production as well as at the observation wells. The values of α_p and α_o are equal to 4.5×10^{-4} and $4Tt_p/(\phi r_{wp}^2) = 8.3 \times 10^6$. The effect of the well storages on both drawdown and recovery is negligible at both the wells.

The nondimensional time-drawdown plots for different a set of α_p, α_o, and $4Tt_p/(\phi r_{wp}^2)$ have been presented in Figs. (5.3) and (5.5) for all the four cases. From these plots it is seen that the effect of the well storage on drawdown increases as the size of the well increases.

The quantity of water that is going into the aquifer from large-diameter observation well storage during pumping and the replenishment that occurs during recovery are presented in Figs. [5.6(a)] through [5.6(c)] for all the four cases for different values of α_p and α_o. The pumping has been discontinued at the end of the 500th time step. It can be seen from the figures that during early period of pumping larger quantity of observation well storage goes into the aquifer in Case 3 as compared to Case 1. The replenishment of observation well storage starts early and is faster in Case 3 than in Case 1.

5.4 CONCLUSIONS

Based on the study presented in this chapter the following conclusions are made :

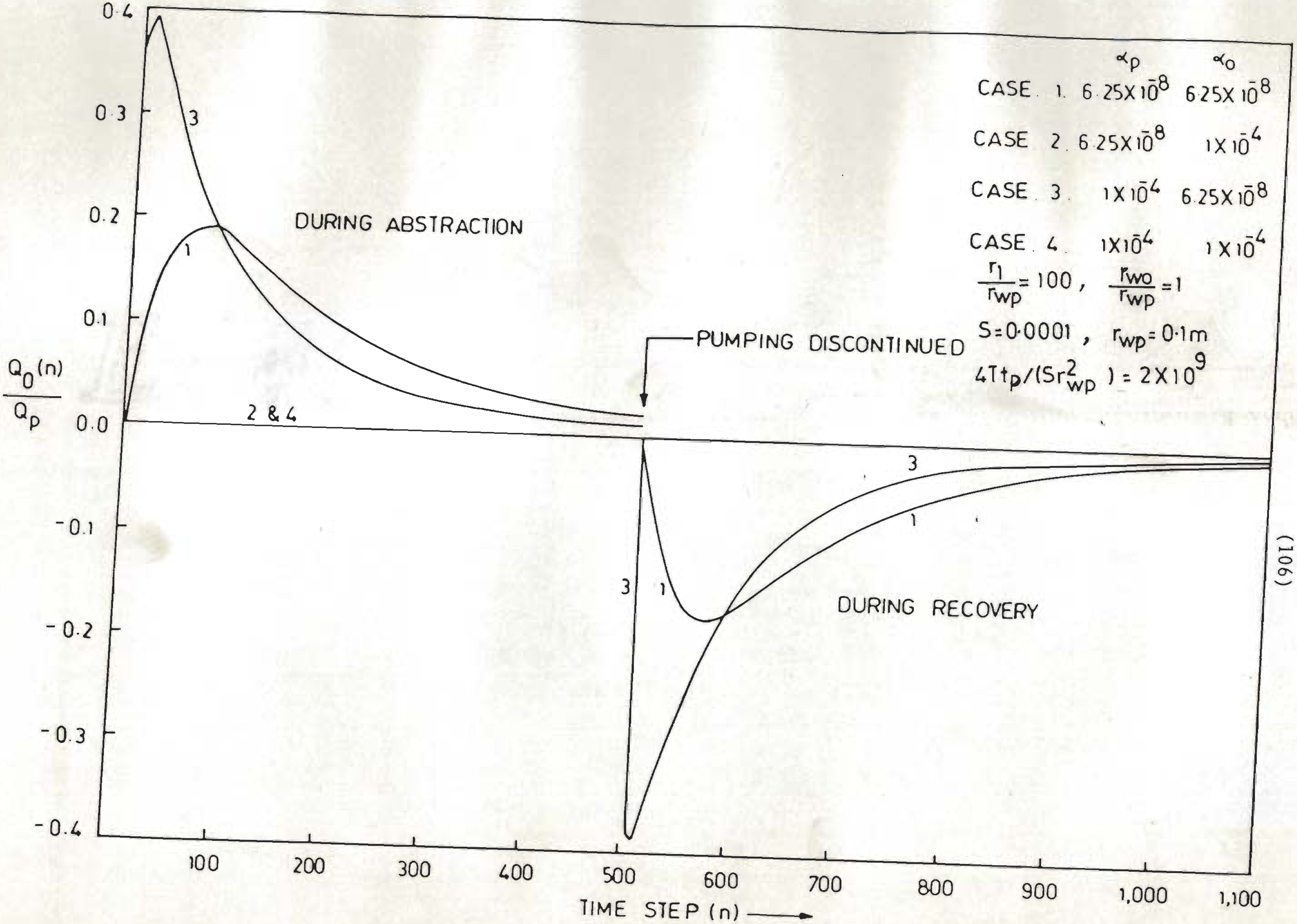
(i) Tractable analytical expressions have been derived for analysing the effect of production and observation well storages on drawdown at any point in the aquifer.

(ii) It has been found that the influence of the observation well storage on drawdown is more pronounced during recovery than during abstraction phase.

(iii) The effect of observation well storage on drawdown in the aquifer increases with increase in the diameter of observation well.

(iv) It has also been confirmed that the drawdown in an observation well of negligible diameter due to pumping in a large-diameter well is same if the roles of the wells are reversed.

(v) The contribution from the observation well storage to the aquifer during abstraction is a function of dimensions of the production and the observation wells and the time since pumping. The contribution of observation well storage increases initially from zero to a maximum value during pumping and then decreases as pumping is continued. Similar trends have been observed during the recovery phase in respect of the replenishment of observation well storage.



(106)

FIG. 5.6(a) - Contributions of observation well storage to aquifer during pumping and replenishment during recovery

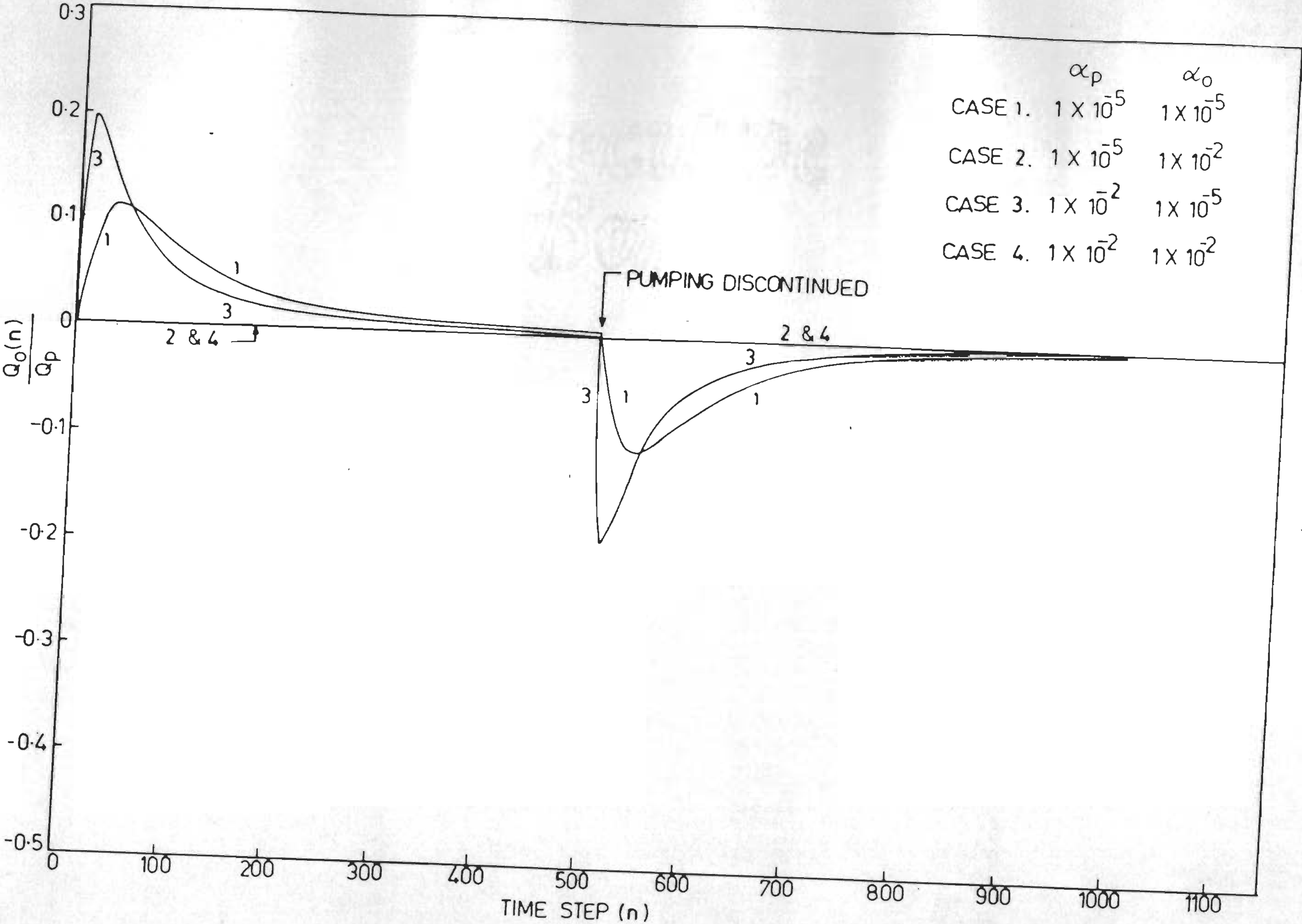


FIG. 5.6(b) - Contribution of observation well storage to aquifer during pumping and replenishment during recovery.

(107)

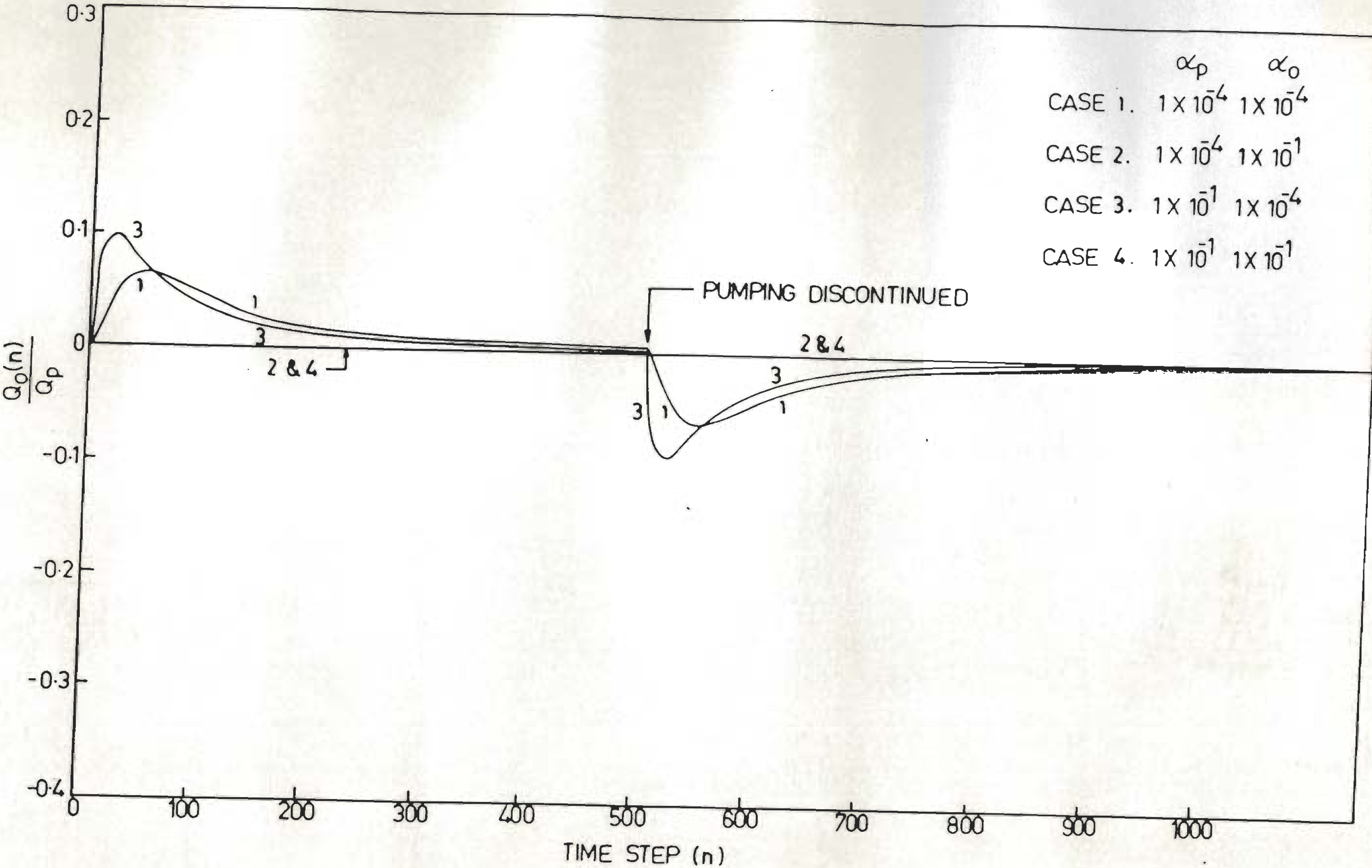


FIG. 5.6(c) - Contributions of observation well storage to aquifer during pumping and replenishment during recovery.

CHAPTER 6

ANALYSIS OF UNSTEADY FLOW TO A LARGE-DIAMETER WELL EXPERIENCING WELL LOSS

6.0 INTRODUCTION

The drawdown at a pumped well comprises the formation loss, which is attributed to the aquifer, and, the well loss encountered at the well screen and in the well bore by the flowing water. The formation loss is a function of the time of pumping, and it can be expressed as a product of the pumping rate, and a formation loss factor independent of the pumping rate. The well loss, which is caused due to resistance to flow of water into and inside the well, may result from laminar or turbulent flow conditions. Components of laminar flow well loss may be the result of screen blockage, partial penetration and screen location in the aquifer, all of which vary with the first power of the pumping rate (Sheahan, 1971). Well loss due to turbulent flow conditions at the well screen and inside the well bore can be resonably assumed as a product of a turbulent flow well loss factor and square of the pumping rate (Jacob, 1947). Rorabaugh (1953) has pointed out that the exponent of the pumping rate can deviate significantly from two. An exact value for the exponent cannot be stated due to differing well characteristics. However, the exponent has been assumed to be two for the most cases.

The concept of a step-drawdown test in a water well was first presented by Jacob (1947) as a means to separate the laminar and turbulent components of drawdown. Jacob assumed that the laminar component is directly proportional to the discharge rate and the turbulent component is a second-order function of well discharge. Rorabaugh (1953) noted that treatment of discharge as second order variable in the turbulent component term of the Jacob equation was over restrictive, and suggested a more general form in which turbulent loss is assigned an n^{th} order dependence on discharge. A trial-and-error method of solution for the values of C, the turbulent flow well loss factor, and 'n' has been proposed by Rorabaugh. The analysis of step-drawdown test data has been fully described by Lennox (1966). A significant contribution was made by Sheahan (1971) with the introduction of a type-curve solution technique for step-drawdown test data analysis. Analysis of step-drawdown test data has been further made by several investigators (Eden and Hazel, 1973, Labadie and Helweg, 1975, Clark, 1977, Miller et al. 1983).

Although many authors have dealt with step-drawdown test and estimation of well losses, no attempt has been made to take into account the well storage effect. In the present study unsteady flow to a large-diameter well in a confined aquifer has been analysed taking into account the well losses.

6.1 STATEMENT OF THE PROBLEM

Figure (6.1) shows a schematic cross section of a large-diameter well in a homogeneous, isotropic and confined aquifer of infinite areal extent. It is assumed that the aquifer prior to pumping was at rest condition. The radius of the well screen is r_w, and that of well casing is r_c. Pumping is carried out at a uniform rate upto time t_p. It is required to determine the components of drawdown at the well face owing to well loss and aquifer loss. It is also required to find the drawdown at a distance 'r' from the well during the pumping as well as during the recovery periods.

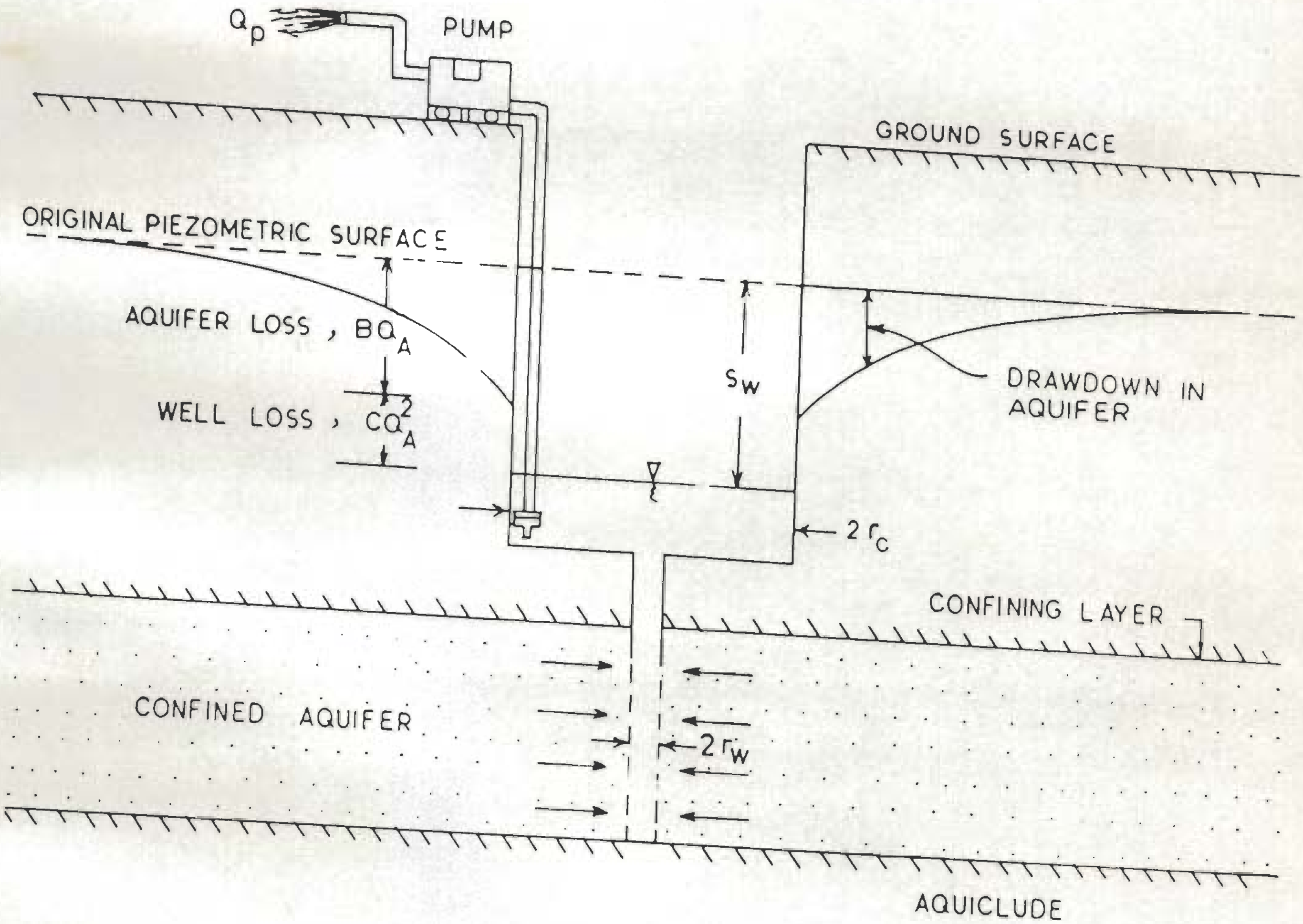


FIG. 6.1 - Schematic cross section of a large-diameter well showing well loss and aquifer loss components.

6.2 ANALYSIS

The following assumptions have been made in the analysis : The time parameter is discrete. Within each time step the abstraction rate of water derived from well storage and that from aquifer storage are separate constants.

Let the total time of pumping, t_p, be discretised to 'm' units of equal time steps. The quantity of water pumped during any time step 'n' can be written as

$$Q_A(n) + Q_W(n) = Q_p(n) \qquad \ldots(6.1)$$

in which,

$Q_A(n)$ = water withdrawn from aquifer storage, and

$Q_W(n)$ = water withdrawn from well storage.

For $n > m$, $Q_p(n) = 0$. Otherwise $Q_p(n)$ is equal to the rate of pumping.

The drawdown, $S_W(n)$, in the well at the end of time step 'n' is given by

$$S_W(n) = \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n} Q_W(\gamma) \qquad \ldots(6.2)$$

where $Q_W(\gamma)$ represents rate of withdrawal from the well storage or its replenishment at time step γ. $Q_W(\gamma)$ values are unknown a' priori. A negative value of $Q_W(\gamma)$ means replenishment of well storage that occurs during time step γ in recovery phase.

According to Jacob's finding, the drawdown in a pumped well can be expressed as

$$S_W(n) = BQ + CQ^2 \qquad \ldots(6.3)$$

in which B is the laminar aquifer loss factor, C is the well loss factor, Q is the rate of flow, BQ is the aquifer loss, and CQ^2 is the well loss component.

Substituting the value of $S_W(n)$ from equation (6.2) in equation (6.3) and replacing Q by $Q_A(n)$, the total drawdown in the well at n^{th} time is written as

$$\frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n} Q_W(\gamma) = S_A(n) + CQ_A^2(n) \qquad ...(6.4)$$

The component of drawdown at the well face at the end of n^{th} unit time step due to aquifer loss, $S_A(n)$, is given by

$$S_A(n) = \sum_{\gamma=1}^{n} Q_A(\gamma)\ \delta_{rw}(n-\gamma+1) \qquad ...(6.5)$$

where,

$$\delta_{rw}(I) = \frac{1}{4\pi T} \left[E_1 \left(\frac{\phi r_w^2}{4TI}\right) - E_1 \left(\frac{\phi r_w^2}{4T(I-1)}\right)\right] \qquad ...(6.6)$$

$$E_1(X) = \int_X^{\infty} \frac{e^{-y}}{y}\, dy,$$

T = transmissivity of the aquifer,

ϕ = storage coefficient, and

I = an integer.

$\delta_{rw}(I)$ is known as discrete kernel coefficient.

Replacing $S_A(n)$ in equation (6.4) by equation (6.5) and rearranging

$$\sum_{\gamma=1}^{n} Q_A(\gamma)\ \delta_{rw}(n-\gamma+1) + CQ_A^2(n) = \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n} Q_W(\gamma) \qquad ...(6.7)$$

Using the relation $Q_W(n) = Q_p(n) - Q_A(n)$ in equation (6.7), splitting the temporal summation into two parts and rearranging

$$\delta_{rw}(1)\ Q_A(n) + CQ_A^2(n) - \frac{1}{\pi r_c^2}[Q_p(n) - Q_A(n)]$$

$$= \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n-1} Q_W(\gamma) - \sum_{\gamma=1}^{n-1} Q_A(\gamma)\ \delta_{rw}(n-\gamma+1)$$

or

$$[C]Q_A^2(n)+[\delta_{rw}(1) + \frac{1}{\pi r_c^2}]\ Q_A(n) + [\sum_{\gamma=1}^{n-1} Q_A(\gamma)\delta_{rw}(n-\gamma+1)$$

$$- \frac{1}{\pi r_c^2} Q_p(n) - \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n-1} Q_W(\gamma)] = 0 \qquad \ldots(6.8)$$

Equation (6.8) is a quadratic equation in $Q_A(n)$. The solution for $Q_A(n)$ is given by

$$Q_A(n) = -[\delta_{rw}(1) + \frac{1}{\pi r_c^2}]/(2C)+ \surd\{[\delta_{rw}(1) + \frac{1}{\pi r_c^2}]^2$$

$$-4C[\sum_{\gamma=1}^{n-1} Q_A(\gamma)\ \delta_{rw}(n-\gamma+1) - \frac{1}{\pi r_c^2} Q_p(n) - \frac{1}{\gamma r_c^2} \sum_{\gamma=1}^{n-1} Q_W(\gamma)]\}\ /(2C)$$

$$\ldots(6.9)$$

At any time step, n, all terms but $Q_A(n)$ are known in equation (6.9), and $Q_A(n)$ can be solved in succession starting from time step 1. In particular for time step 1, $Q_A(1)$ is given by

$$Q_A(1) = -[\delta_{rw}(1) + \frac{1}{\pi r_c^2}]/(2C)+\surd\{[\delta_{rw}(1) + \frac{1}{\pi r_c^2}]^2$$

$$-4C[-\frac{1}{\pi r_c^2} Q_p(n)]\}\ /(2C) \qquad \ldots(6.10)$$

Once $Q_A(n)$ is solved $Q_W(n)$ can be known from equation (6.1). The drawdown, $S_r(n)$, in the aquifer at any distance, r, from the centre of the well can be found, using the relation

$$S_r(n) = \sum_{\gamma=1}^{n} Q_A(\gamma)\ \delta_r(n-\gamma+1) \qquad \ldots(6.11)$$

where

$$\delta_r(I) = \frac{1}{4\pi T}\left[E_1\left(\frac{\phi r^2}{4TI}\right) - E_1\left(\frac{\phi r^2}{4T(I-1)}\right)\right].$$

6.3 RESULTS AND DISCUSSION

The discrete kernel coefficients, $\delta_{rw}(I)$, are generated using equation (6.6) for known values of T, ϕ, and r_w. After generating the discrete kernel coefficients, $Q_A(n)$ values are computed using equation (6.9) for known values of Q_p, C, and r_c. The values of $Q_W(n)$ are then computed using equation (6.1). $Q_A(n)$ and $Q_W(n)$ are solved in succession starting from the first time step to m^{th} time step. In the recovery phase i.e., for $n > m$, $Q_p(n) = 0$. The values of $Q_A(n)$ and $Q_W(n)$ during recovery period are found using equations (6.9) and (6.1) with $Q_p(n) = 0$.

The drawdown at the end of 10^{th} day of continuous pumping have been evaluated for T = 200 m^2/day, ϕ = 0.1, and C = 0.00001 $(day)^2/m^5$ for different pumping rates and the variation of drawdown with Q_p is shown in Fig.(6.2). From the figure it is seen that the well loss component can be substantial fraction of total drawdown when pumping rates are large. For example, for Q_p = 400 m^3/day, the well loss component at 1/10th of a day is 47.3 % of the total drawdown. Without well storage, the corresponding well loss component is 48.5 % of the total drawdown at the well.

The variation of specific drawdown at the end of 6th hour of continuous pumping with Q_p has been presented for C = 0.001 and 0.0005 $(hr)^2/m^5$,

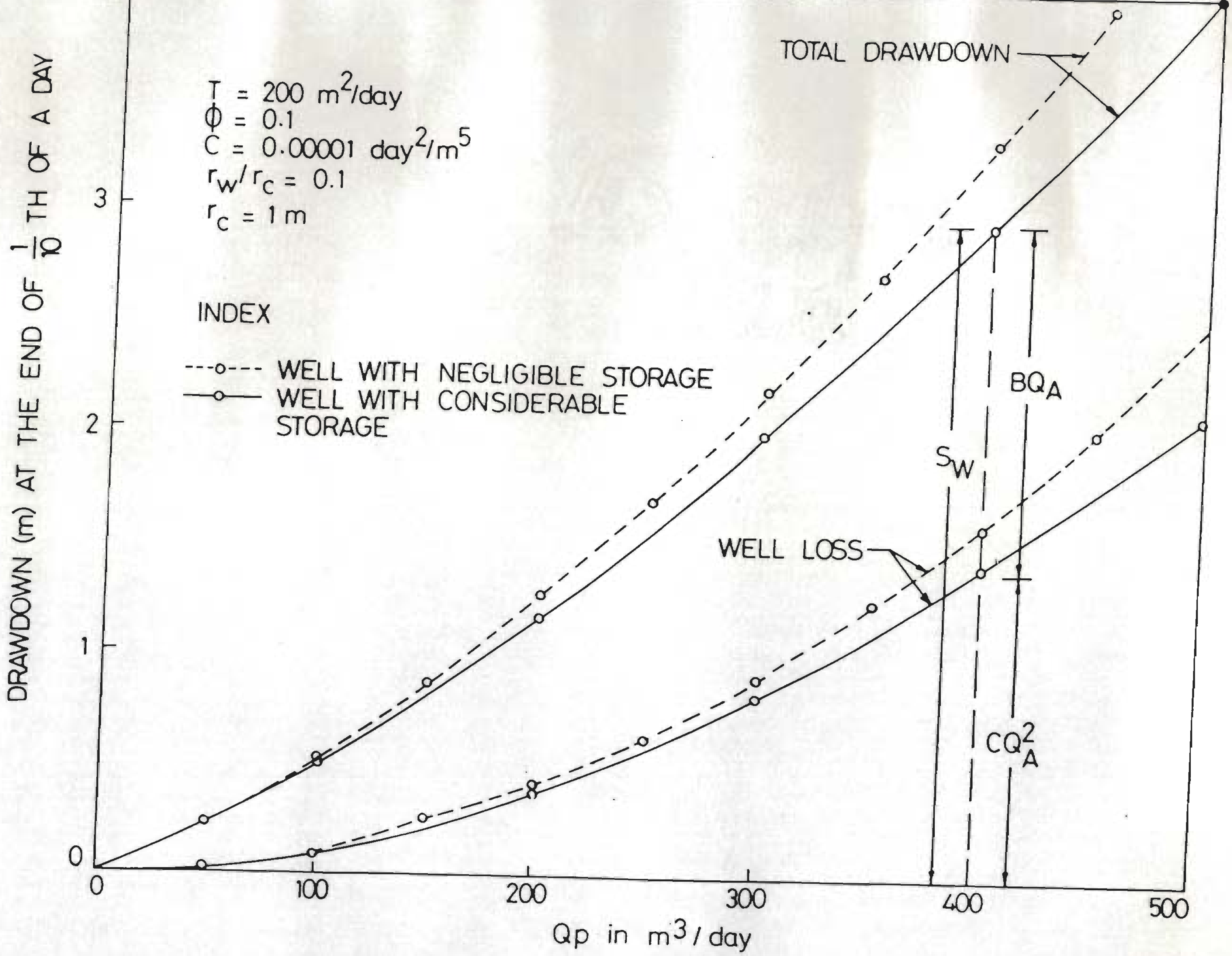


FIG. 6.2 - Variation of total-drawdown, S_W, aquifer loss, BQ_A, and well loss, CQ_A^2, with well discharge, Q_p, for a large-diameter well at the end of 1/10th day of pumping.

in Fig. (6.3). If well loss component is negligible, the relation between specific drawdown and Q_p is linear for large-diameter well. Also, if well loss is prominent, but well storage is negligible, the relation between specific drawdown and Q_p is linear (Todd, 1980). The plots shown in Fig. (6.3) depicts that the relation between specific drawdown and Q_p is linear in the beginning for small values of Q_p, and also linear for large values of Q_p. At small values of Q_p, the well losscomponent being small, the relation between specific drawdown and Q_p is linear. At large values of Q_p, the well storage contribution to pumping rate being small in comparison to well discharge Q_p, the relationship is linear. The variations of specific drawdown with Q_p corresponding to different time during pumping are shown in Fig. (6.4), for $T = 5\ m^2/hr.$, $\phi = 0.1$, $C = 0.001\ (hr)^2/m^5$, and $r_w = 0.1m$. It could be seen from the figure that as time of pumping increases the relation between specific drawdown and Q_p becomes linear. At large time the relationship is linear because of negligible contribution of well storage towards well discharge Q_p. Thus after well storage effect becomes negligible, the relationship between specific drawdown and Q_p is linear. Since in the beginning of pumping it is the well storage which contributes to pumping, the specific drawdown at $t \to 0$ is given by $[Q_p/(\pi r_c^2)]/Q_p = 1/(\pi r_c^2)$. Also for small values of Q_p as water will be taken from well storage the specific drawdown will be equal to $1/(\pi r_c^2)$. Therefore as seen from Figs.(6.3) and (6.4) the specific drawdown graphs does not pass through origin.

Variations of the nondimensional well loss component $CQ_A^2(t)/[Q_p/(4\pi T)]$ and total drawdown $S_W(t)/[Q_p/(4\pi T)]$ with nondimensional time $4Tt/(\phi r_w^2)$ for different pumping rates, Q_p, are shown in Figs. [6.5(a)] through [6.5(c)] for different values of α, where $\alpha = (r_w/r_c)^2 \phi$. These results have been evaluated for $C = 0.001(hr)^2/m^5$. It is seen from the figures that the relationship between $S_W(t)/[Q_p/(4\pi T)]$ and $4Tt/(\phi r_w^2)$ and between $CQ_A^2(t)/[Q_p/(4\pi T)]$ and $4Tt/(\phi r_w^2)$ is not unique and depends upon the pumping rate as well as on α. The well loss component and the total drawdown

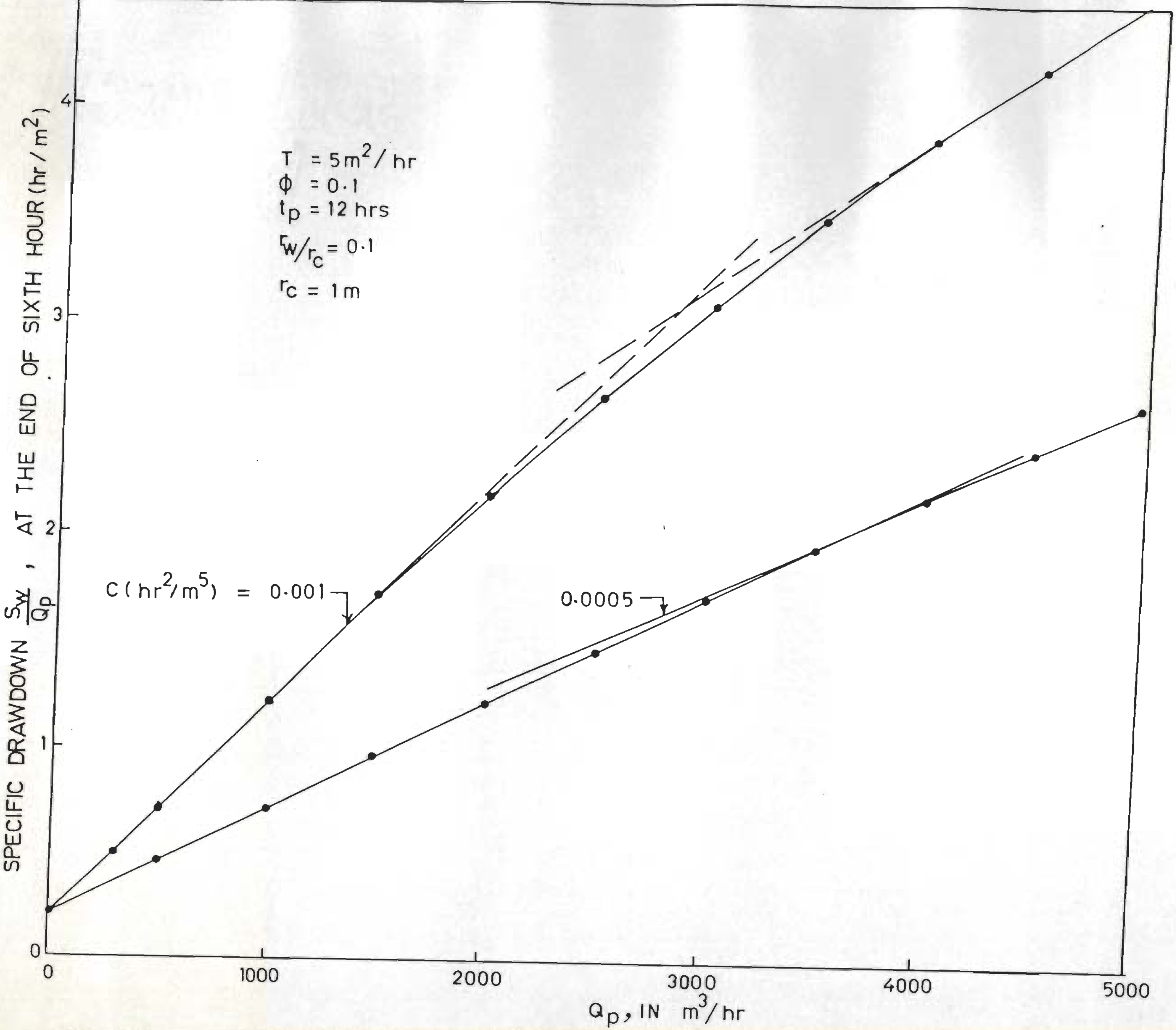


FIG. 6.3 - Variation of specific drawdown, $S_W(t)/Q_p$, with well discharge, Q_p, at the end of 6th hour of pumping

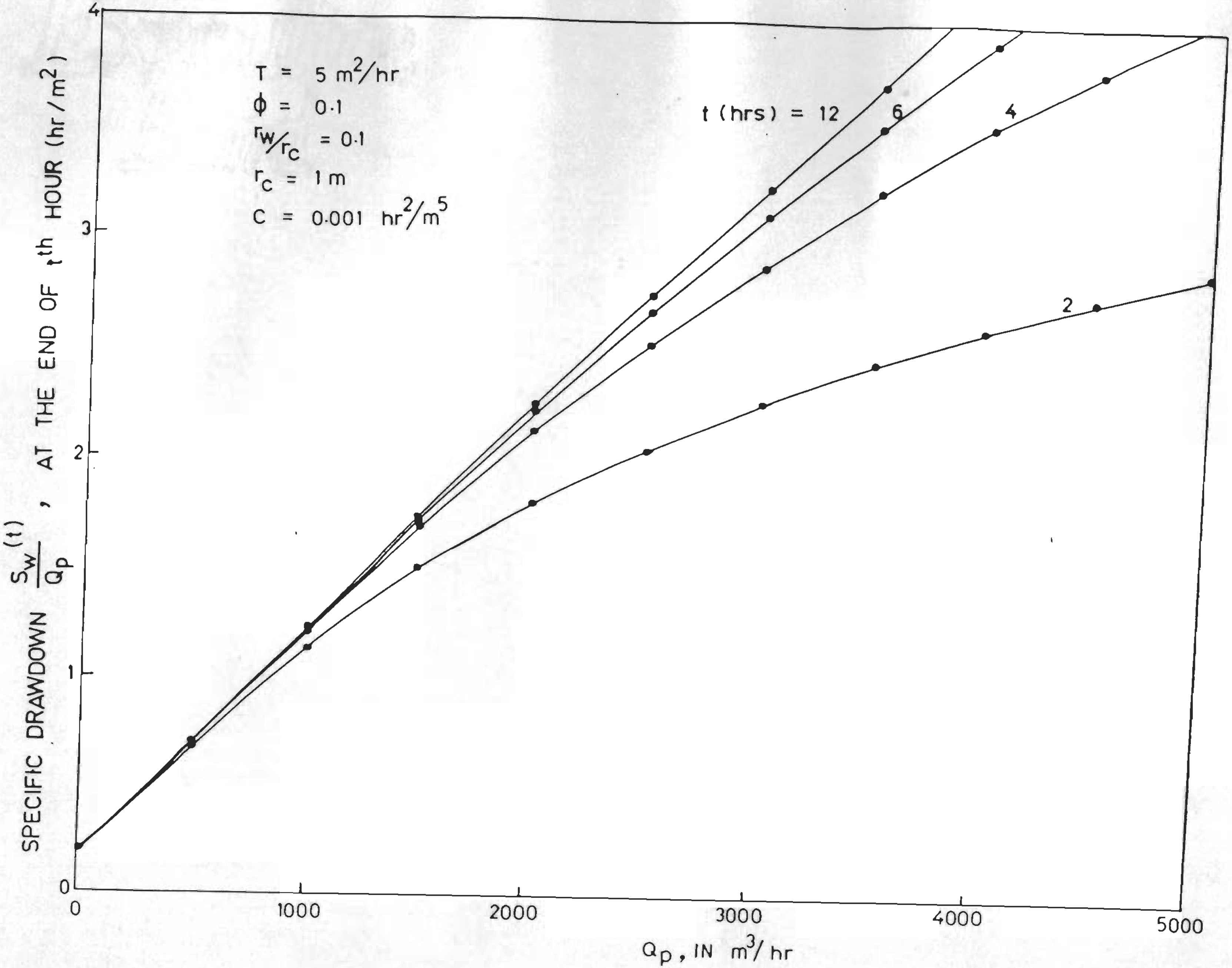


FIG. 6.4 -Variation of specific drawdown, $S_W(t)/Q_p$, with well discharge, Q_p, at the end of 2nd, 4th, 6th and 12th hour of pumping.

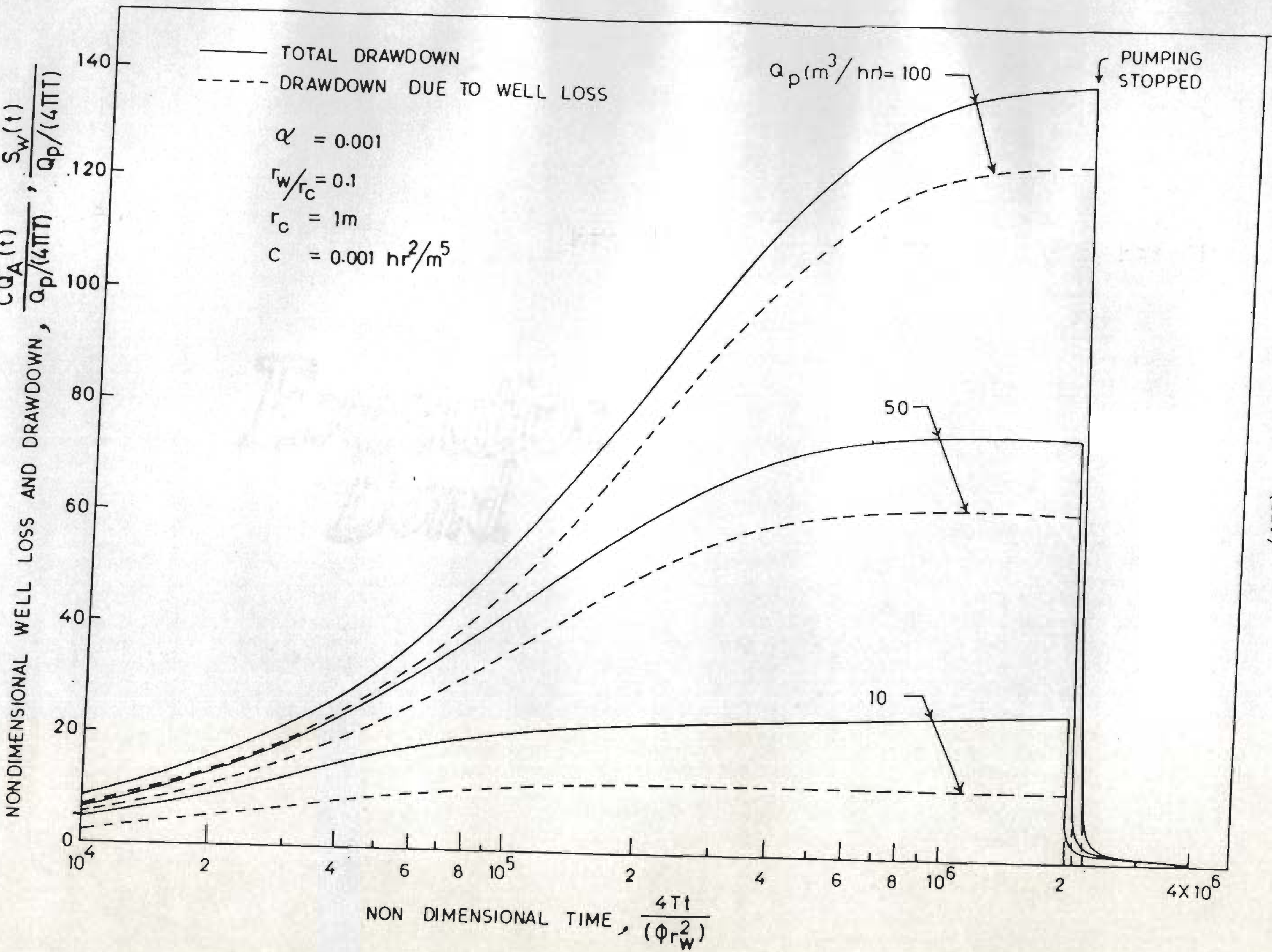


FIG. 6.5(a) - Variation of nondimensional well loss and drawdown with nondimensional time for α = 0.001

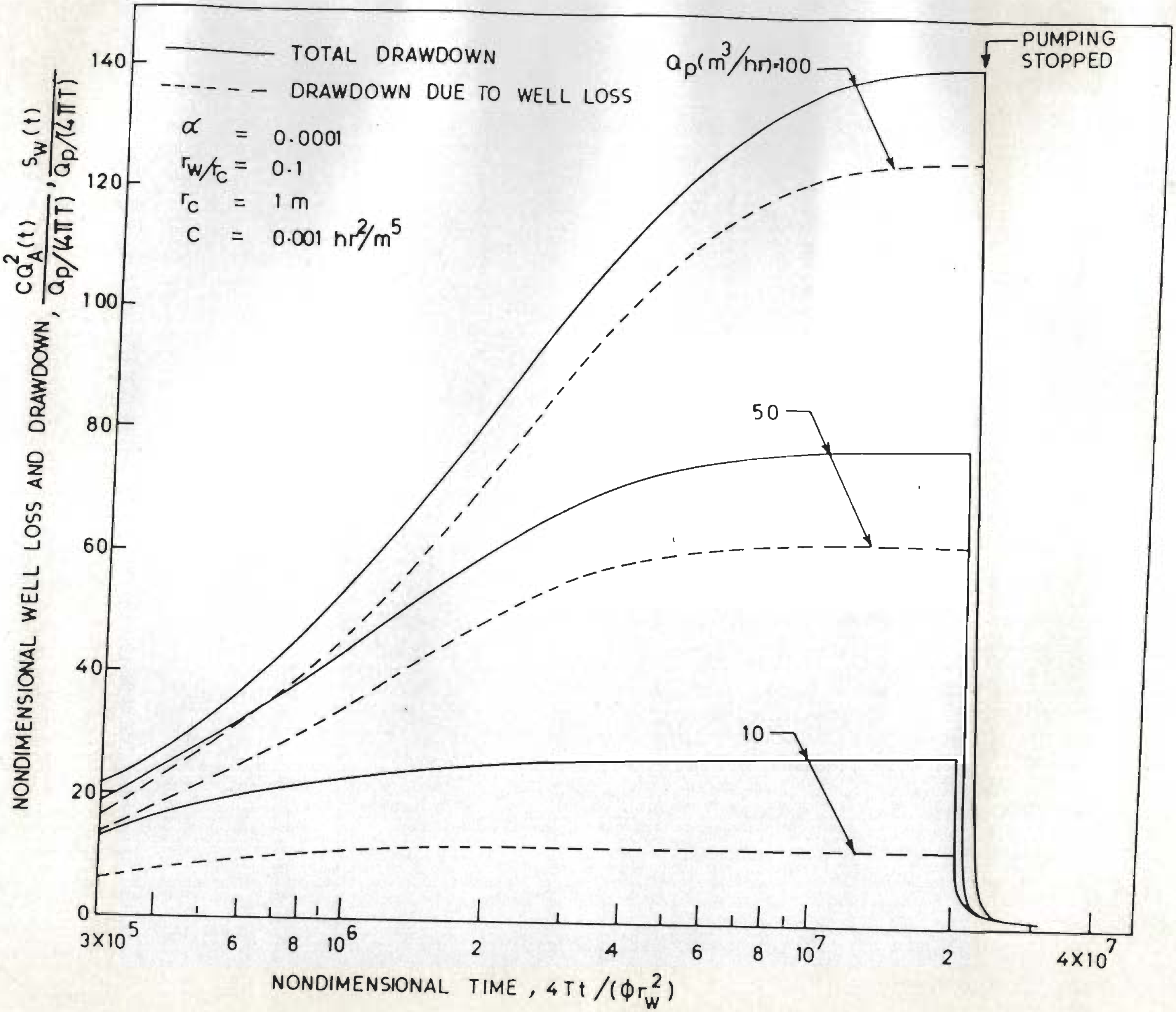


FIG. 6.5(b) - Variation of nondimensional well loss and drawdown with nondimensional time for $\alpha = 0.0001$.

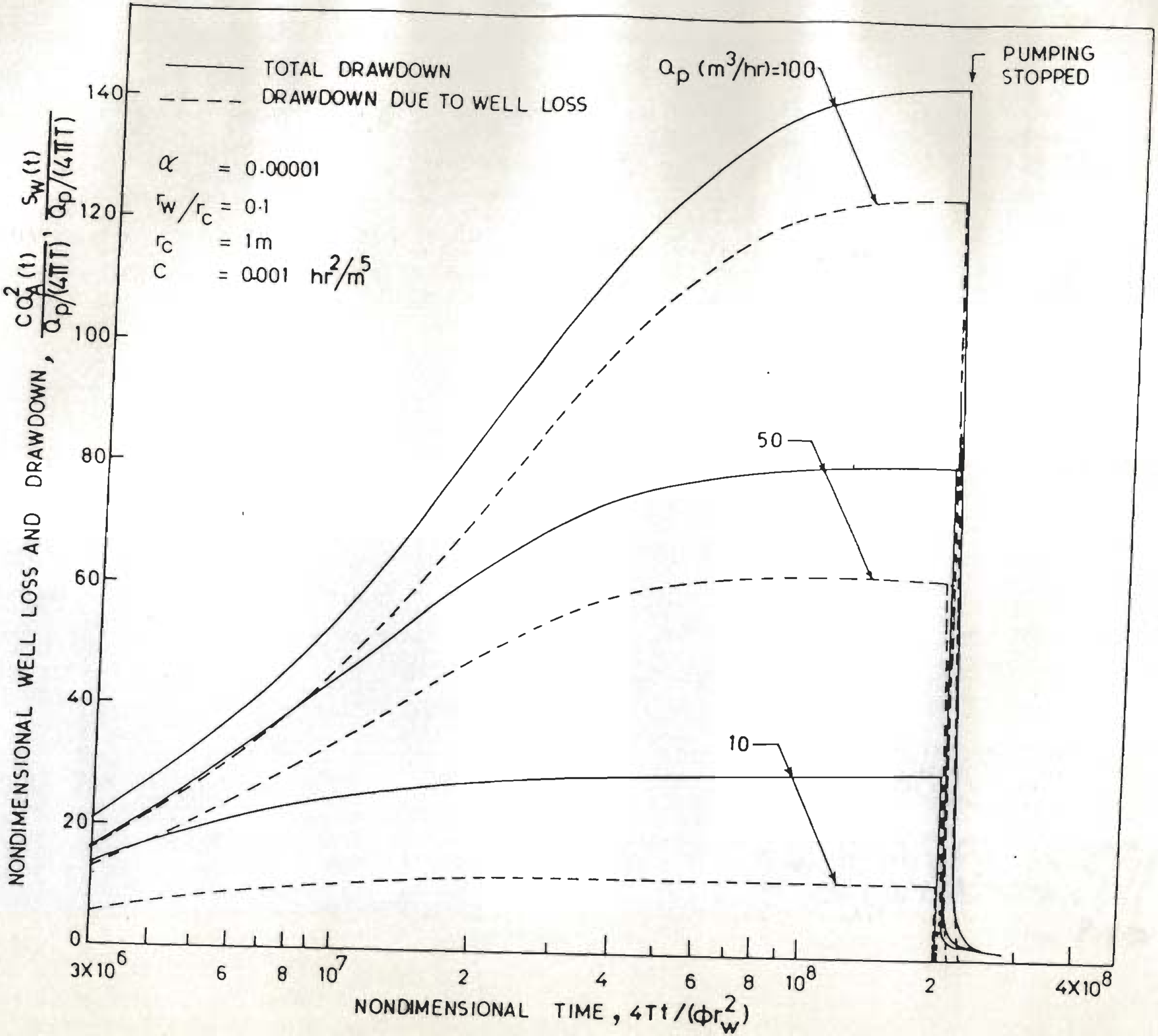


FIG. 6.5(c) - Variation of nondimensional well loss and drawdown with nondimensional time for α = 0.00001.

attain near steady state conditions under continuous pumping. Lesser the pumping, sooner the well loss component and total drawdown at the well attain the near steady state condition. It is also seen from the figures that there is no proportionate increase in the drawdown and well loss component with increase in Q_p. For example for Q_p = 10 m^3/hour, α = 0.001, at nondimensional time $4Tt/(\phi r_w^2) = 10^6$, the nondimensional well loss component and drawdown are 13 and 25.5, respectively. For Q_p = 100 m^3/hour, the corresponding characteristics are 123 and 136 respectively.

The effect of storage coefficient on well loss component has been presented in Table (6.1). It could be seen from the table that for different values of storage coefficient the difference in the values of well loss component becomes small as pumping continues. For example at time step 1, for ϕ = 0.1 and 0.00001, the well loss component are 4.66 m and 3.59 m respectively. At the end of 10th time step, the well loss components are 9.93 m and 9.91 m respectively. In the beginning of pumping, well in an aquifer with smaller storage coefficient exhibits lower value of well loss component. The effect of storage coefficient on total drawdown at well point is given in in Table (6.2). It is seen from the table that the total drawdown at the well at a particular time is higher for lower storage coefficient. For example for ϕ = 0.1 the drawdown at the end of the 10th time step is 19.643 m., the corresponding drawdown for ϕ = 0.00001 is 26.885 m.

The variations of nondimensional aquifer loss, $BQ_A(t)/(Q/4\pi T)$, with nondimensional time $4Tt/(\phi r_w^2)$ are presented in Figs. [6.6(a)] through [6.6(c)] for different values of Q_p and α. It is seen from the figures that for a given value of α, the relationship between the dimensionless aquifer loss and dimensionless time is not unique. At any particular time, higher the

TABLE 6.1 - WELL LOSS COMPONENT $CQ_A^2(t)$ FOR DIFFERENT VALUES OF STORAGE COEFFICIENT

[$T = 10\ m^2/hr$, $C = 0.001\ hr^2/m^5$, $Q_p = 100\ m^3/hr$, and $r_w/r_c = 0.1$]

S.No.	Time since pumping (hrs.)	STORAGE COEFFICIENT VALUES (ϕ)				
		0.1	0.01	0.001	0.0001	0.00001
1	1	4.6597000	4.3548000	4.0767000	3.8226000	3.5900000
2	2	7.3747000	7.1314000	6.8940000	6.6629000	6.4387000
3	3	8.6708000	8.5127000	8.3515000	8.1882000	8.0236000
4	4	9.2959000	9.2002000	9.0991000	8.9933000	8.8834000
5	5	9.6037000	9.5477000	9.4866000	9.4209000	9.3507000
6	6	9.7595000	9.7272000	9.6910000	9.6510000	9.6073000
7	7	9.8412000	9.8226000	9.8013000	9.7773000	9.7504000
8	8	9.8861000	9.8753000	9.8628000	9.8483000	9.8318000
9	9	9.9122000	9.9058000	9.8984000	9.8896000	9.8795000
10	10	9.9283000	9.9245000	9.9200000	9.9146000	9.9084000
	Pumping stopped					
11	11	1.6453000	1.8247000	1.9979000	2.1649000	2.3256000
12	12	0.1562200	0.2105800	0.2710500	0.3367200	0.4067100
13	13	0.0126910	0.0204800	0.0310090	0.0444960	0.0610550
14	14	0.0016290	0.0026300	0.0041820	0.0064470	0.0095930
15	15	0.0004140	0.0005800	0.0008470	0.0012575	0.0018680
16	20	0.0000250	0.0000270	0.0000284	0.0000304	0.0000328
17	25	0.0000062	0.0000064	0.0000066	0.0000068	0.0000070

TABLE 6.2 - TOTAL DRAWDOWN AT WELL POINT FOR DIFFERENT VALUES OF STORAGE COEFFICIENT

[$T = 10\ m^2/hr$, $C = 0.001\ hr^2/m^5$, $Q_p = 100\ m^3/hr$, and $r_w/r_c = 0.1$]

S.No.	Time since pumping (hrs.)	STORAGE COEFFICIENT VALUES (ϕ)				
		0.1	0.01	0.001	0.0001	0.00001
1	1	10.1020	10.8260	11.5070	12.1510	12.7590
2	2	14.5980	15.7760	16.9090	17.9990	19.0480
3	3	16.7890	18.2380	19.6570	21.0270	22.3670
4	4	17.9300	19.5350	21.1190	22.6720	24.1960
5	5	18.5670	20.2660	21.9460	23.6070	25.2470
6	6	18.9530	20.7030	22.4420	24.1670	25.8780
7	7	19.2060	20.9870	22.7600	24.5240	26.2780
8	8	19.3880	21.1860	22.9790	24.7660	26.5470
9	9	19.5280	21.3360	23.1450	24.9420	26.7390
10	10	19.6430	21.4570	23.2690	25.0780	26.8850
	Pumping stopped					
11	11	6.7311	7.8596	9.0408	10.2680	11.5350
12	12	2.7526	3.2405	3.8002	4.4269	5.1156
13	13	1.6186	1.7999	2.0277	2.3036	2.6284
14	14	1.2123	1.2835	1.3767	1.4954	1.6426
15	15	1.0076	1.0410	1.0838	1.1384	1.2075
16	20	0.5926	0.5989	0.6055	0.6125	0.6200
17	25	0.4276	0.4306	0.4337	0.4368	0.4401

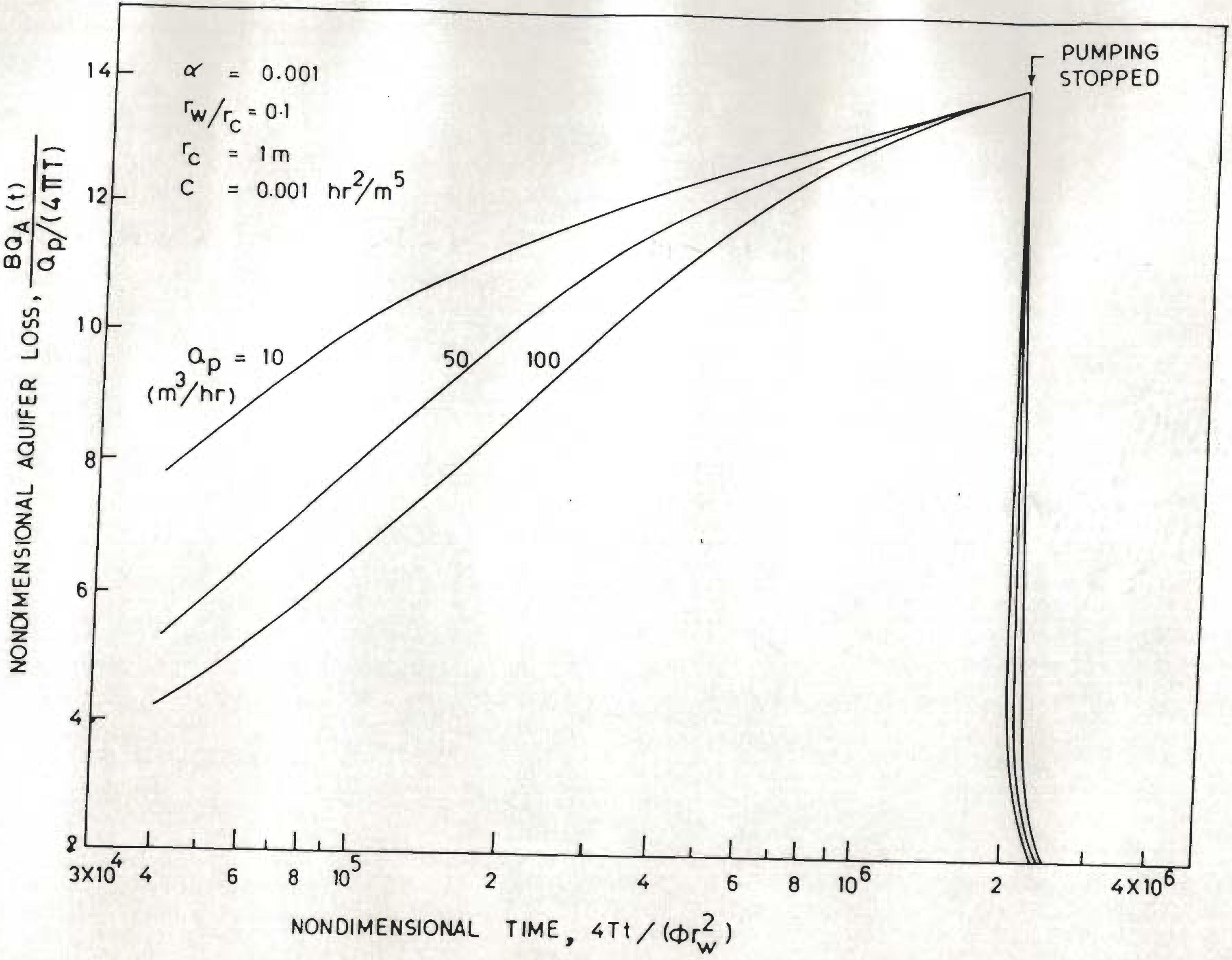


FIG. 6.6(a) - Variation of nondimensional aquifer loss with non-dimensional time for $\alpha = 0.001$.

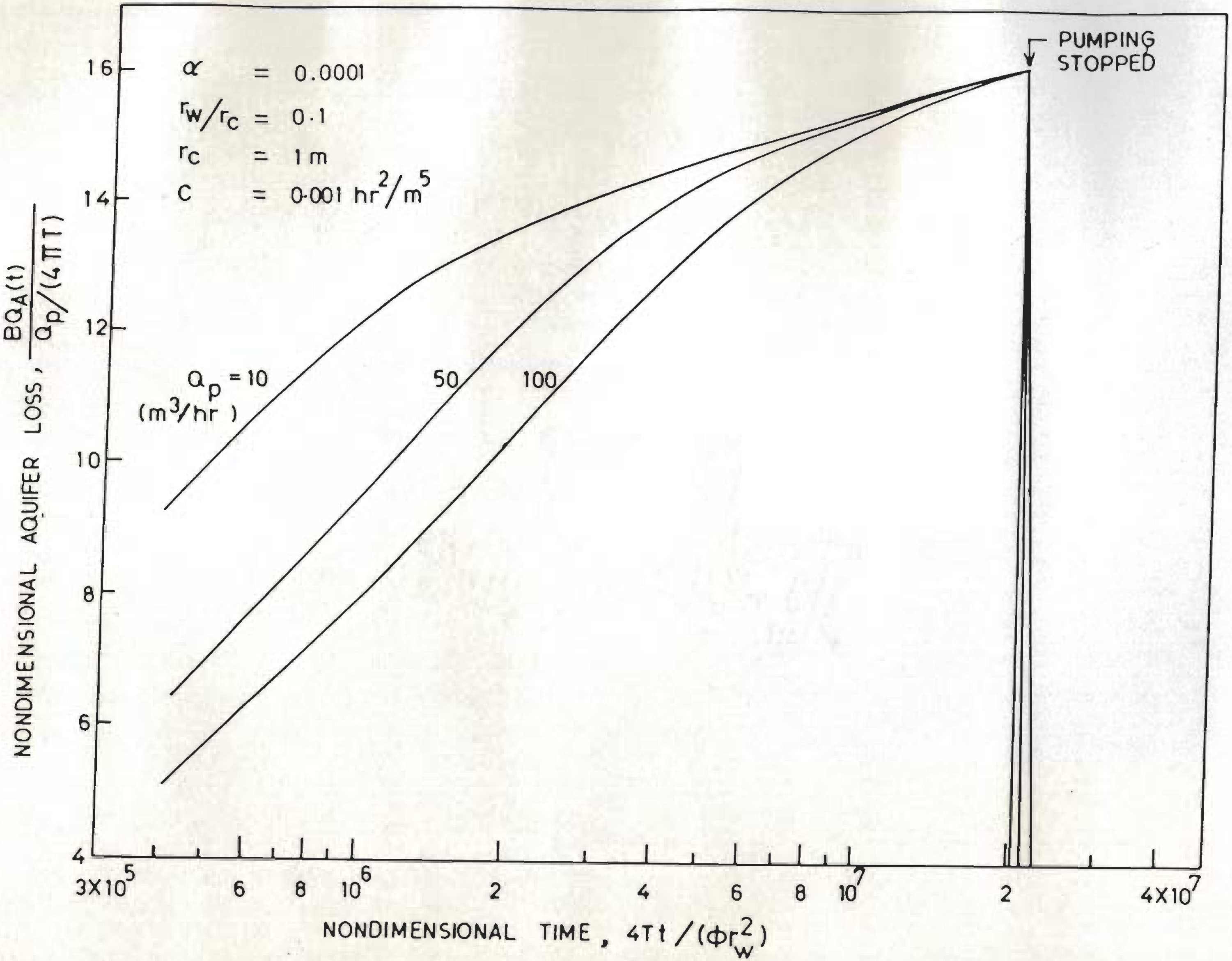


FIG. 6.6(b) - Variation of nondimensional aquifer loss with nondimensional time for $\alpha = 0.0001$.

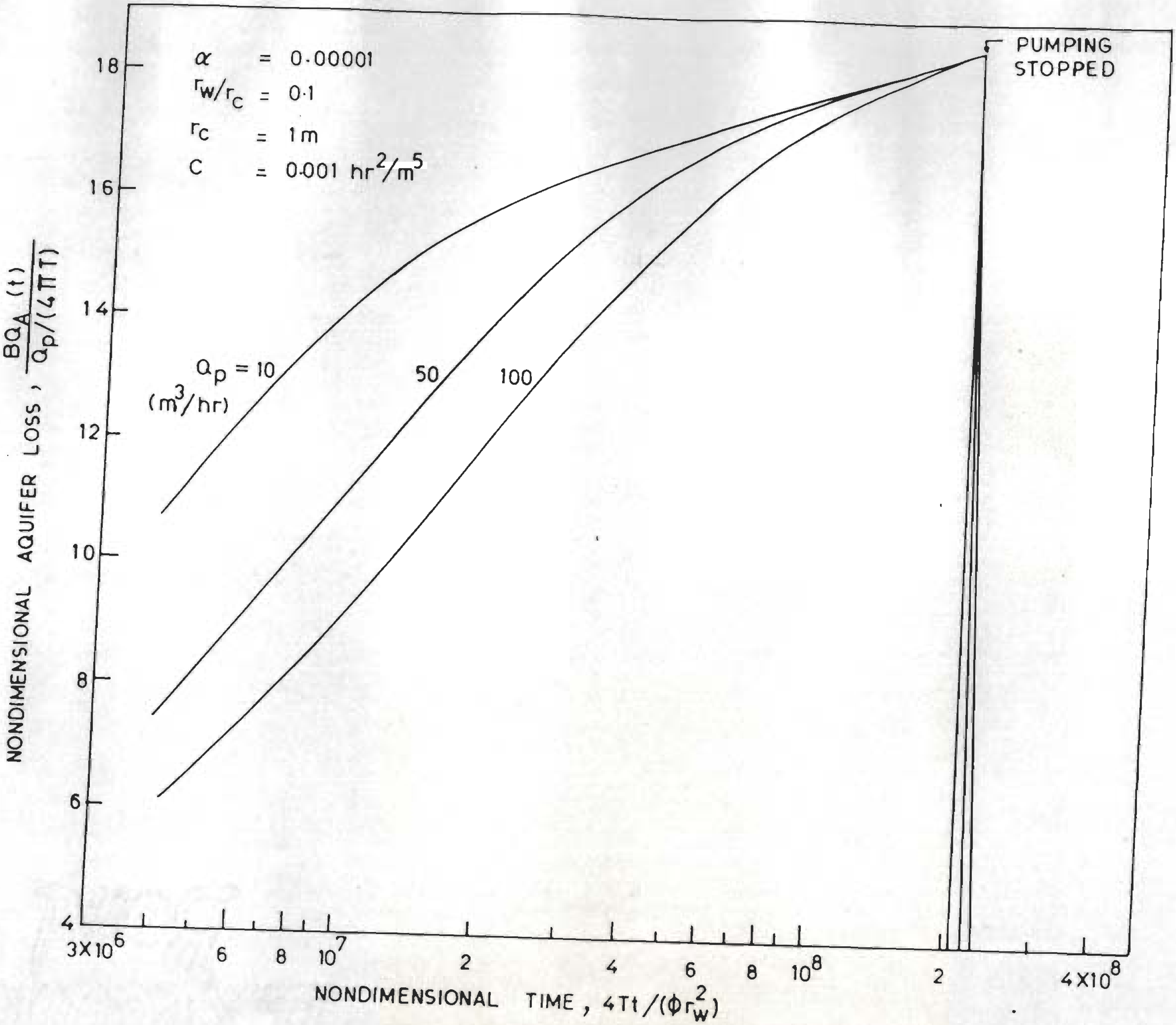


FIG. 6.6(c) - Variation of nondimensional aquifer loss with nondimensional time for $\alpha = 0.00001$.

pumping rate is, lower will be aquifer loss. For example at $4Tt/(\phi r_w^2) = 10^5$, and $\alpha = 0.001$, for $Q_p = 10$ m^3/hour, the dimensionless aquifer loss = 10.2. For $Q_p = 50$ and 100 m^3/hour, the corresponding aquifer losses are 7.9 and 6.5 respectively. As pumping continues, the difference in aquifer loss due to difference in Q_p decreases. At dimensionless time $4Tt/(\phi r_w^2) = 2 \times 10^6$, the difference in aquifer loss components pertaining to the three pumping rates are zero [Fig. 6.6(a)].

The well loss component is a indicator of well sickness. Higher the well loss component, more will be the energy loss during pumping for a fixed withdrawal. The energy consumed upto any time t, during pumping will be proportional to $\int_0^t Q_p(\tau)[S_W(\tau) + G]d\tau$ where, G is the depth to water level before the onset of pumping. The component $\int_0^t Q_p(\tau)S_W(\tau)d\tau$ is a variable component which will change depending upon field conditions. The term, $\int_0^t Q_p(\tau)S_W(\tau)d\tau$ evaluated with well loss component and without well loss component would indicate the extra energy consumed due to well sickness. In figure [7(a)] the variation of the dimensionless term $\sum_{i=1}^{n} S_W(i)/[Q_p/(4\pi T)]$ with r_c/r_w is presented for duration of pumping equal to 6 hours, and $r_w = 0.1$ m. The term, $\sum_{i=1}^{n} S_W(i)/[Q_p/(4\pi T)]$, is a measure of variable energy loss under constant pumping. It could be seen from figure [6.7(a)] that with increase in the radius of the casing, r_c, the energy loss due to well sickness is reduced. For example for $C = 10^{-2}(hr)^2/m^5$, $r_c/r_w = 10$, and $t_p = 6$ hours, the extra energy loss index, $\sum_{i=1}^{n} S_W(i)/[Q_p/(4\pi T)]$, due to well loss component is 58, where as for $r_c/r_w = 50$, the corresponding quantity is zero. When the duration of pumping is increased, the extra energy loss due to well sickness will be increased. For example in Fig. [6.7(b)] for $t_p = 12$ hours, $C = 10^{-2}(hr)^2/m^5$, $r_c/r_w = 10$, the extra energy

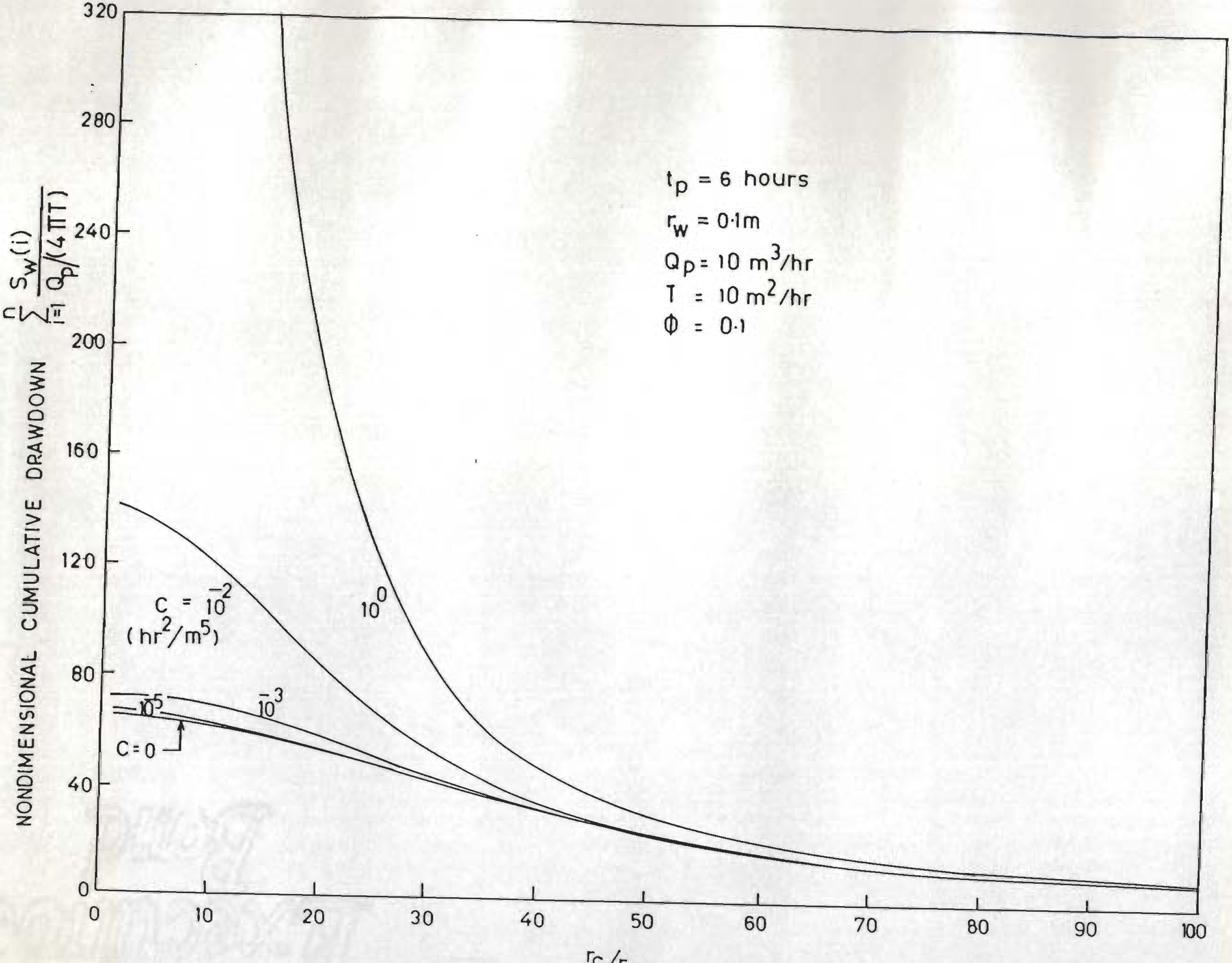


FIG. 6.7(a) - Variation of nondimensional cumulative drawdown with r_c/r_w for t_p = 12 hours.

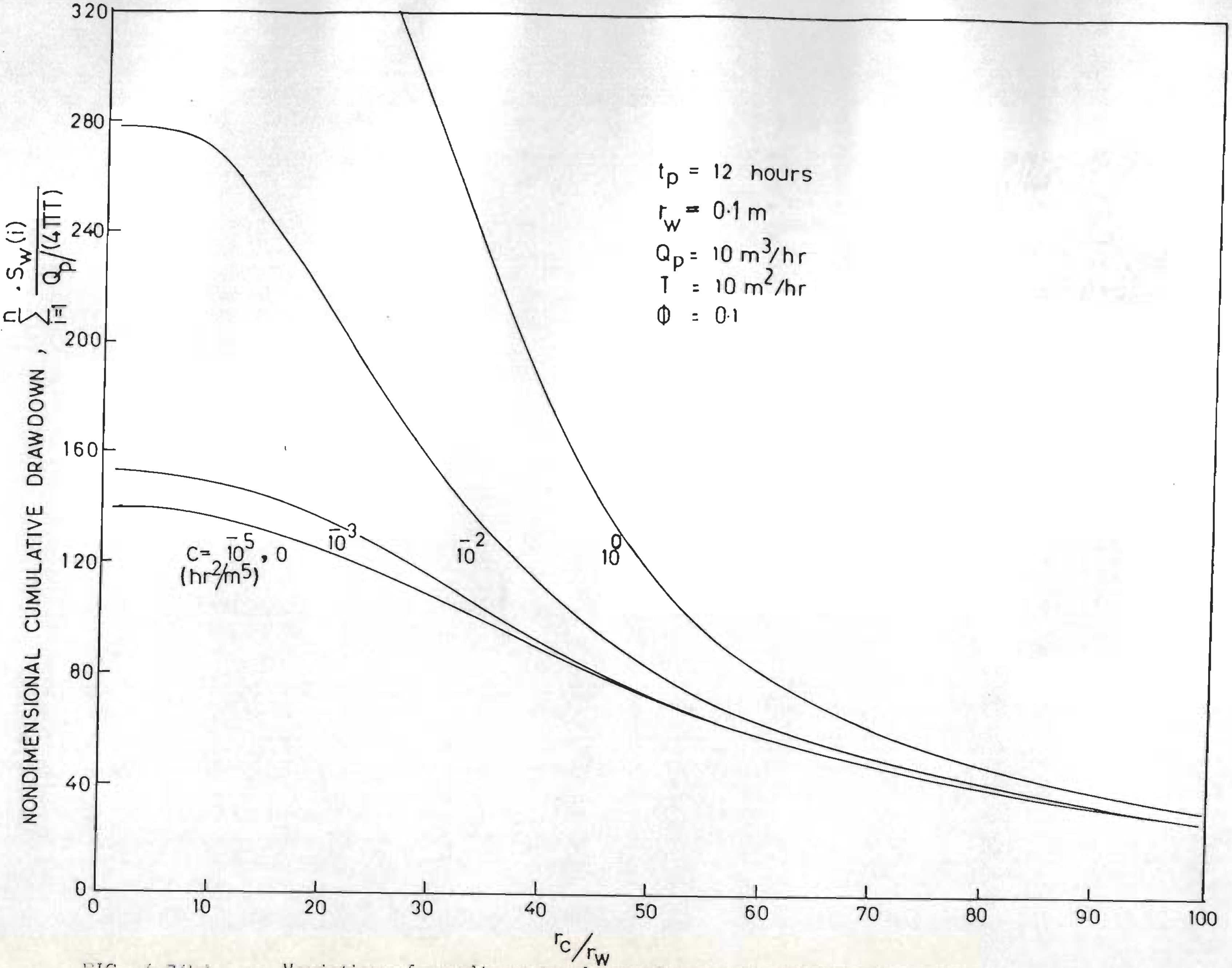


FIG. 6.7(b) - Variation of nondimensional cumulative drawdown with r_c/r_w for t_p = 6 hours.

loss index $\sum_{i=1}^{n} S_W(i)/[Q_p/(4\pi T)] = 132$, and for $r_c/r_w = 50$, the corresponding quantity is 10.

6.4 CONCLUSIONS

Based on the study presented, the following conclusions are made:

(i) Tractable analytical expressions have been derived for determination of aquifer contribution, well storage contribution and drawdown at the well face and at any point in the aquifer considering well loss effect in a large-diameter dug-cum-bore well.

(ii) The study of influence of well loss component on energy loss during pumping shows that with increase in the radius of the well casing r_c, the energy loss due to well sickness is reduced.

(iii) The well loss component is less sensitive to changes in aquifer storativity for higher duration of pumping.

(iv) The relation between specific drawdown and pumping rate is nonlinear for large-diameter well experiencing well loss.

CHAPTER 7

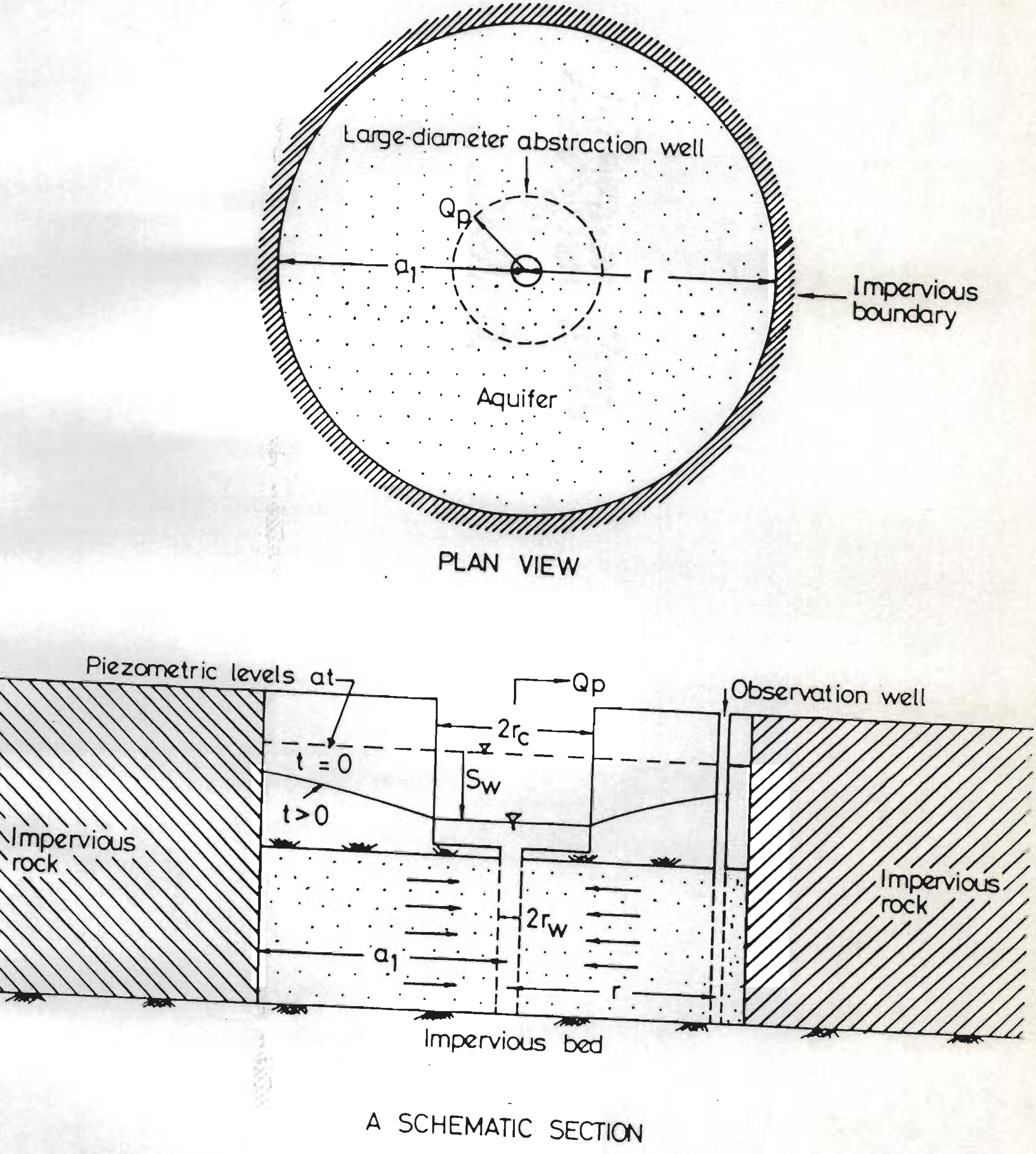
ANALYSIS OF UNSTEADY FLOW TO A LARGE-DIAMETER WELL IN A FINITE AQUIFER

7.0 INTRODUCTION

Most of the solutions presented for analysing flow to a well are based on the assumption that the aquifers are of infinite areal extent. Although aquifers of infinite areal extent do not exist, many aquifers are of such wide extent that for all practical purposes they can be considered to be infinite. Some aquifers however are of limited areal extent because of the presence of an impervious barrier or a recharge boundary. Analysis of unsteady flow to a well having negligible storage in an aquifer of finite areal extent has been given by Muskat (1937), and Kuiper (1972). Zekai Sen (1981), Basak (1982), Mishra and Chachadi (1984) and Chachadi and Mishra (1985) have presented analyses of unsteady flow to a large-diameter well with storage in an aquifer of finite areal extent. The hydrological boundaries in these analyses have been assumed to be straight and fully penetrating the aquifer. In the present study a solution for analysing unsteady flow to a large-diameter well located at the centre of an aquifer, which is limited by a circular barrier boundary, has been presented.

7.1 STATEMENT OF THE PROBLEM

A schematic plan view and a section of a large-diameter well in a homogeneous, isotropic and confined aquifer of finite areal extent is shown in Fig. (7.1). It is assumed that the aquifer prior to pumping was at rest condition. The well is located at the centre of the aquifer limited by a circular barrier boundary at a distance 'a_1' from the centre of the well. The radius of the well screen is r_w, and that of well casing is r_c. Pumping is carried



G. 7.1 - Plan view and schematic section of a large-diameter well in a finite aquifer.

out at a uniform rate upto time t_p. It is required to determine the drawdown at the well face, at the barrier boundary and at any point in the aquifer during pumping and recovery periods. It is also required to find the aquifer contribution and well storage contribution in response to a constant pumping rate.

7.2 ANALYSIS

The following assumptions have been made in the analysis :

(i) At any time the drawdown in the aquifer at the well face is equal to that in the well.

(ii) The time parameter is discrete.

(iii) Within each time step, the aquifer contribution and well storage contribution are separate constants, but they vary from step to step.

The Boussinesq's partial differential equation, which decribes the evolution of piezometric surface in a homogeneous isotropic confined aquifer, for an axially-symmetric radial flow onset by pumping of a well is given by

$$\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r} = \frac{\phi}{T}\frac{\partial S}{\partial t}, \quad r > r_w \qquad \ldots(7.1)$$

in which, r_w = radius of the well screen, S = drawdown in piezometric surface at distance ,r, from the well at time t, T = transmissivity and ϕ = storage coefficient of the aquifer. To account for the well storage effect, a solution to the above differential equation has to satisfy the following boundary conditions :

$$2\pi r_w T \left.\frac{\partial S}{\partial r}\right|_{r=r_w} - \pi r_w^2 \frac{\partial S_W}{\partial t} = -Q_p(t) \qquad \ldots(7.2)$$

$$S_W(r_w,t) = S_W(t) \qquad \ldots(7.3)$$

in which,

$S_W(t)$ = drawdown in the well, and

$Q_p(t)$ = pumping rate at time 't', $Q_p(t)$ is equal to zero during recovery.

To account for the existence of the noflow boundary at the radial distance 'a_1', the other boundary condition to be satisfied is

$$\left.\frac{\partial S}{\partial r}\right|_{r = a_1} = 0 \qquad \ldots(7.4)$$

The initial condition required to be satisfied is

$$S(r,0) = 0, \qquad r > r_w \qquad \ldots(7.5)$$

Discretising the time parameter by uniform time steps and assuming that the excitation and response of the system are piecewise constants in each time step, the alternate form of the boundary conditions stated in equation (7.2) is

$$Q_A(n) + Q_W(n) = Q_p(n) \qquad \ldots(7.6)$$

in which, $Q_A(n)$ = aquifer contribution to pumping during nth time step,

$Q_W(n)$ = well storage contribution to pumping during nth time step,

and $Q_p(n)$ = pumping rate during nth time step.

$Q_A(\gamma)$, and $Q_W(\gamma)$, for $\gamma = 1,2 \ldots\ldots\ldots,n$ are unknown a priori.

Solution to differential equation (7.1) for negligible well storage i.e. for $r_c = r_w$ and for small value of r_w has been given by Muskat (1937) and Kuiper (1972) for a constant continuous pumping of a well in the finite aquifer.

The solution is given by

$$S = \frac{Q}{2\pi T}\left[\frac{3}{4} + \log_e\left(\frac{r}{a_1}\right) - \frac{1}{2}\left(\frac{r}{a_1}\right)^2 + \frac{4Tt}{\phi a_1^2}\right.$$

$$+ 2\sum_{m=1}^{\infty}\{\alpha_m J_o(\alpha_m a_1)^{-2}\}\; J_o(\alpha_m r)$$

$$\left. .\exp\{-(\alpha_m)^2\, Tt/(\phi a_1^2)\}\right] \quad \ldots(7.7)$$

This solution satisfies the boundary condition stated in equation (7.4), and the initial condition stated in equation (7.5). It also satisfies the boundary condition stated in equation (7.2) for $r_c = r_w$ and $r_w \longrightarrow 0$. $(\alpha_m a_1)$ values for m=1,2,3 ... are zeros of J_1, the Bessel's function of the first kind and of the first order. $(\alpha_m a_1)$ values have been tabulated for values of m upto 20 (Abramowitz and Stegun, 1970). $\alpha_m a_1$ values for higher values of m can be evaluated using the following formula of McMahon's expansions for large zeros :

$$(\alpha_m a_1) \simeq \eta - \frac{\mu-1}{\eta} - \frac{4(\mu-1)(7\mu-31)}{3(8\eta)^3} - \frac{32(\mu-1)(83\mu^2-982\mu+3779)}{15(8\eta)^5}$$

$$- \frac{64(\mu-1)(6949\mu^3 - 153855\mu^2 + 1585743\mu - 6277237)}{105(8\eta)^7} \quad \ldots(7.8)$$

in which,

$$\eta = (m + \frac{1}{4}) \pi , \quad \text{and}$$

$$\mu = 4.$$

Let K(t) be the drawdown in piezometric surface of the confined aquifer of finite areal extent at a radial distance 'r' from the well due to a unit step excitation. Substituting Q by 1 expression for K(t) can be obtained from equation (7.7). Let $\delta_r(I)$ be the aquifer response at the end of time step 'I' due to a unit pulse excitation given during the first time step. $\delta_r(I)$, the discrete kernel coefficient for drawdown in a finite aquifer of circular shape, the pumping well being at the centre of the aquifer, is related to the unit step response function and is given by

$$\delta_r(I) = K(I) - K(I-1) \qquad \ldots(7.9)$$

Substituting K(I) and K(I-1) by their respective expressions in equation (7.9) and simplifying, the following expression for discrete kernel coefficient for drawdown for a confined aquifer of finite areal extent is obtained :

$$\delta_r(I) = \frac{I}{\pi \phi a_1^2} - \frac{1}{\pi T} \sum_{m=1}^{\infty} \{ (\alpha_m a_1) J_o(\alpha_m a_1) \}^{-2} J_o(\alpha_m r).$$

$$\exp \{\frac{-(\alpha_m)^2 TI}{\phi}\} - \frac{(I-1)}{\pi \phi a_1^2} + \frac{1}{\pi T} \{ \sum_{m=1}^{\infty} \{(\alpha_m a_1) J_o(\alpha_m a_1)\}^{-2}$$

$$J_o(\alpha_m r). \exp \{\frac{-(\alpha_m)^2 T(I-1)}{\phi}\} , \quad I > 1 \qquad \ldots(7.10)$$

For I = 1, $\delta_r(1)$ is given by

$$\delta_r(1) = - \frac{1}{2\pi T} \left[\frac{3}{4} + \ln\left(\frac{r}{a_1}\right) - \frac{1}{2}\left\{\left(\frac{r}{a_1}\right)^2 + \frac{4T}{\pi a_1^2}\right\} + 2 \sum_{m=1}^{\infty} \left\{(\alpha_m a_1) J_o(\ _m a_1)\right\}^{-2} J_o(\alpha_m r).\exp\left\{-(\alpha_m)^2 \frac{T}{\phi}\right\}\right] \quad ...(7.11)$$

Having obtained the discrete kernel coefficients for a finite aquifer, the solution for the large-diameter well problem can be obtained as follows :

The drawdown in the piezometric surface at the well face in the confined aquifer at the end of nth unit time step is given by

$$S_A(r_w,n) = \sum_{\gamma=1}^{n} Q_A(\gamma)\ \delta_{rw}(n-\gamma+1) \quad ...(7.12)$$

The drawdown in the water level in the well at the end of nth unit time step due to withdrawal from well storage upto nth time step is given by

$$S_W(n) = \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n} Q_W(\gamma) \quad ...(7.13)$$

Since $S_A(r_w,n) = S_W(n)$, equations (7.12) and (7.13) shall be equal. Hence,

$$\frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n} Q_W(\gamma) = \sum_{\gamma=1}^{n} Q_A(\gamma)\ \delta_{rw}(n-\gamma+1)$$

Splitting the summations into parts and collecting the unknowns

$$Q_A(n)\ \delta_{rw}(1) - \frac{1}{\pi r_c^2} Q_W(n) = \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n-1} Q_W(\gamma) - \sum_{\gamma=1}^{n-1} Q_A(\gamma)\ \delta_{rw}(n-\gamma+1) \quad ...(7.14)$$

$Q_W(n)$ and $Q_A(n)$ are solved from equation (7.6) and (7.14) and they are given by the following expressions :

$$Q_A(n) = \frac{Q_p(n) + \sum_{\gamma=1}^{n-1} Q_W(\gamma) - \pi r_c^2 \sum_{\gamma=1}^{n-1} Q_A(\gamma)\, \delta_{rw}(n-\gamma+1)}{1 + \pi r_c^2\, \delta_{rw}(1)} \quad \ldots(7.15)$$

$$Q_W(n) = \frac{[\pi r_c^2\, \delta_{rw}(1) Q_p(n) + \pi r_c^2 \sum_{\gamma=1}^{n-1} Q_A(\gamma)\, \delta_{rw}(n-\gamma+1) - \sum_{\gamma=1}^{n-1} Q_W(\gamma)]}{1 + \pi r_c^2\, \delta_{rw}(1)}$$

...(7.16)

For time step 1

$$Q_A(1) = Q_p(1)/[1 + \pi r_c^2\, \delta_{rw}(1)]$$

$$Q_W(1) = \pi r_c^2\, \delta_{rw}(1)\, Q_p(1)/[1 + \pi r_c^2\, \delta_{rw}(1)]$$

Using equations (7.15) and (7.16) $Q_A(n)$ and $Q_W(n)$ can be found in succession starting from time step 1. If the pumping period is discretised to m' units of equal time steps, $Q_p(n) = Q_p$ for $n \leq m'$ and $Q_p(n) = 0$ for $n > m'$. Thus from equations (7.15) and (7.16) the response of the aquifer and well storage replenishment can be known for the pumping as well as for the recovery periods.

7.3 RESULTS AND DISCUSSION

The discrete kernel coefficients $\delta_r(I)$, are generated making use of equations (7.10) and (7.11) for assumed values of aquifer parameters T and ϕ and radii r_w and a_1. The converging series that appears in equation (7.10) has been truncated after the 100^{th} term. The large zeros, after the 10^{th} one, of the Bessel's function have been evaluated using McMahon's formula given at equation (7.8). Values of small zeros have been taken from the tabulated values (Abramowitz & Stegun, 1970). For given radius of well casing, r_c, pumping rate Q_p, and duration of pumping, t_p, $Q_A(n)$ and $Q_W(n)$ are found in succession starting from the first time step.

The contributions of aquifer and well storage towards a continuous constant well discharge are shown in Tables (7.1) to (7.3) for values of a_1 ranging from 100 m to 1000 m. It could be seen from the tables that at nondimensional time $4Tt/(\phi r_w^2) = 80 \times 10^4$, for a_1 equal to 100 m, 500 m and 1000 m, values of Q_W/Q_p are 0.22108, 0.22089 and 0.22087 respectively. There is practically no difference amongst Q_W/Q_p values. Thus there is negligible influence of the finite barrier boundary on the contribution of well storage towards pumping, though contribution of a large-diameter well in an aquifer of less areal extent is higher than that of a well in an aquifer of large areal extent.

Variation of $Q_W(n)/Q_p$ and $Q_A(n)/Q_p$ with nondimensional time factor $4Tt/(\phi r_w^2)$ are shown in Figures 7.2(a) and 7.2(b) for a_1/r_w = 5000 and 10,000 respectively. $Q_W(n)/Q_p$ being equal to $1-Q_A(n)/Q_p$ during pumping, the variation of $Q_A(n)/Q_p$ with nondimensional time is the image of the variation of $Q_W(n)/Q_p$ with the nondimensional time. $Q_W(n)/Q_p$ being equal to $-Q_A(n)/Q_p$, during recovery phase there is symmetry about the time axis during recovery.

From the figures it is seen that both for a_1/r_w = 5000, and 10,000 at about $4Tt/(\phi r_w^2) = 9.5 \times 10^3$ the well storage contribution and aquifer contribution are equal.

Table 7.1 Nondimensional Withdrawals from Aquifer and Well Storages and Drawdowns at Different Points for r_c = 2m, r_w = 0.1m, a_1 = 100m, T = 100 m^3/day, ϕ = 0.01, and t_p = 2 days.

Nondimensional time $\frac{4Tt}{(\phi r_w^2)} \times 10^4$	$\frac{Q_A}{Q_p}$	$\frac{Q_W}{Q_p}$	Nondimensional Drawdown at r/r_w =		
			1	500	1000
2	0.05535	0.94465	0.4827	0.01705	0.00000
4	0.10123	0.89877	0.9411	0.06231	0.00000
6	0.14262	0.85738	1.3779	0.12194	0.00000
8	0.18073	0.81927	1.7950	0.18979	0.00000
10	0.21620	0.78380	2.1939	0.26268	0.00000
20	0.36421	0.63579	3.9521	0.65476	0.00017
30	0.47736	0.52264	5.3889	1.04340	0.00224
40	0.56626	0.43374	6.5753	1.40540	0.00944
50	0.63721	0.36279	7.5642	1.73550	0.02437
60	0.69442	0.30558	8.3943	2.03390	0.04827
70	0.74091	0.25909	9.0958	2.30320	0.08133
80	0.77892	0.22108	9.6925	2.54640	0.12308
Pumping Stopped					
82	0.73030	-0.73030	9.3183	2.57510	0.13241
84	0.69091	-0.69091	8.9652	2.57480	0.14205
86	0.65576	-0.65576	8.6305	2.55920	0.15200
88	0.62363	-0.62363	8.3123	2.53460	0.16224
90	0.59393	-0.59393	8.0095	2.50410	0.17277
100	0.47164	-0.47164	6.6915	2.31280	0.22932
108	0.39625	-0.39625	5.8301	2.14860	0.27781
120	0.30844	-0.30844	4.7839	1.91600	0.35103

Table 7.2 **Nondimensional Withdrawals from Aquifer and Well Storages and Drawdowns at Different Points for r_c = 2m, r_w = 0.1m, a_1 = 500m, T = 100 m^2/day, ϕ = 0.01, and t_p = 2 days.**

Nondimensional time $\frac{4Tt}{(\phi r_w^2)} \times 10^4$	$\frac{Q_A}{Q_p}$	$\frac{Q_W}{Q_p}$	Nondimensional Drawdown at r/r_w =		
			1	2500	5000
2	0.05535	0.94465	0.4827	0.000002	0.0000004
4	0.10123	0.89877	0.9411	0.000007	0.0000008
6	0.14261	0.85739	1.3779	0.000100	0.0000011
8	0.18073	0.81927	1.1795	0.000506	0.0000014
10	0.21620	0.78380	2.1939	0.001489	0.0000017
20	0.36421	0.63579	3.9521	0.020168	0.0000029
30	0.47735	0.52265	5.3886	0.062984	0.0000039
40	0.56627	0.43373	6.5753	0.124230	0.0000046
50	0.63722	0.36278	7.5642	0.197510	0.0000052
60	0.69444	0.30556	8.3942	0.277940	0.0000057
70	0.74099	0.25901	9.0957	0.362030	0.0000060
80	0.77911	0.22089	9.6921	0.447370	0.0000063
Pumping Stopped					
82	0.73053	-0.73053	9.3178	0.464430	0.0000060
84	0.69116	-0.69116	8.9645	0.481470	0.0000057
86	0.65605	-0.65605	8.6296	0.498370	0.0000054
88	0.62397	-0.62397	8.3113	0.514920	0.0000052
90	0.59431	-0.59431	8.0083	0.530840	0.0000049
100	0.47232	-0.47232	6.6891	0.595660	0.0000039
108	0.39724	-0.39724	5.8260	0.628360	0.0000033
120	0.31003	-0.31003	4.7759	0.651660	0.0000025

Table 7.3 **Nondimensional Withdrawals from Aquifer and Well Storages and Drawdowns at Different Points for r_c = 2m, r_w = 0.1m, a_1 = 1000m, T = 100 m^2/day, ϕ = 0.01 and t_p = 2 days**

Nondimensional time $\frac{4Tt}{(\phi r_w^2)} \times 10^4$	$\frac{Q_A}{Q_p}$	$\frac{Q_W}{Q_p}$	Nondimensional Drawdown at r/r_w = 1	5000	10,000
2	0.05507	0.94493	0.4829	0.0000339	0.0000064
4	0.10102	0.89898	0.9413	0.0000299	0.0000142
6	0.14244	0.85756	1.3782	0.0000275	0.0000212
8	0.18059	0.81941	1.7955	0.0000256	0.0000276
10	0.21608	0.78392	2.1944	0.0000242	0.0000335
20	0.36416	0.63584	3.9528	0.0001017	0.0000582
30	0.47734	0.52266	5.3894	0.0010733	0.0000770
40	0.56627	0.43373	6.5761	0.0044152	0.0000917
50	0.63723	0.36277	7.5650	0.0112280	0.0001035
60	0.69446	0.30554	8.3950	0.0219490	0.0001130
70	0.74101	0.25899	9.0963	0.0365210	0.0001207
80	0.77913	0.22087	9.6927	0.0546100	0.0001270
Pumping Stopped					
82	0.73083	-0.73083	9.3182	0.0585770	0.0001217
84	0.69140	-0.69140	8.9649	0.0627000	0.0001150
86	0.65624	-0.65624	8.6298	0.0669360	0.0001090
88	0.62413	-0.62413	8.3115	0.0712820	0.0001036
90	0.59445	-0.59445	8.0084	0.0757330	0.0000987
100	0.47239	-0.47239	6.6889	0.0993660	0.0000783
108	0.39728	-0.39728	5.8256	0.1192200	0.0000658
120	0.31005	-0.31005	4.7755	0.1452900	0.0000513

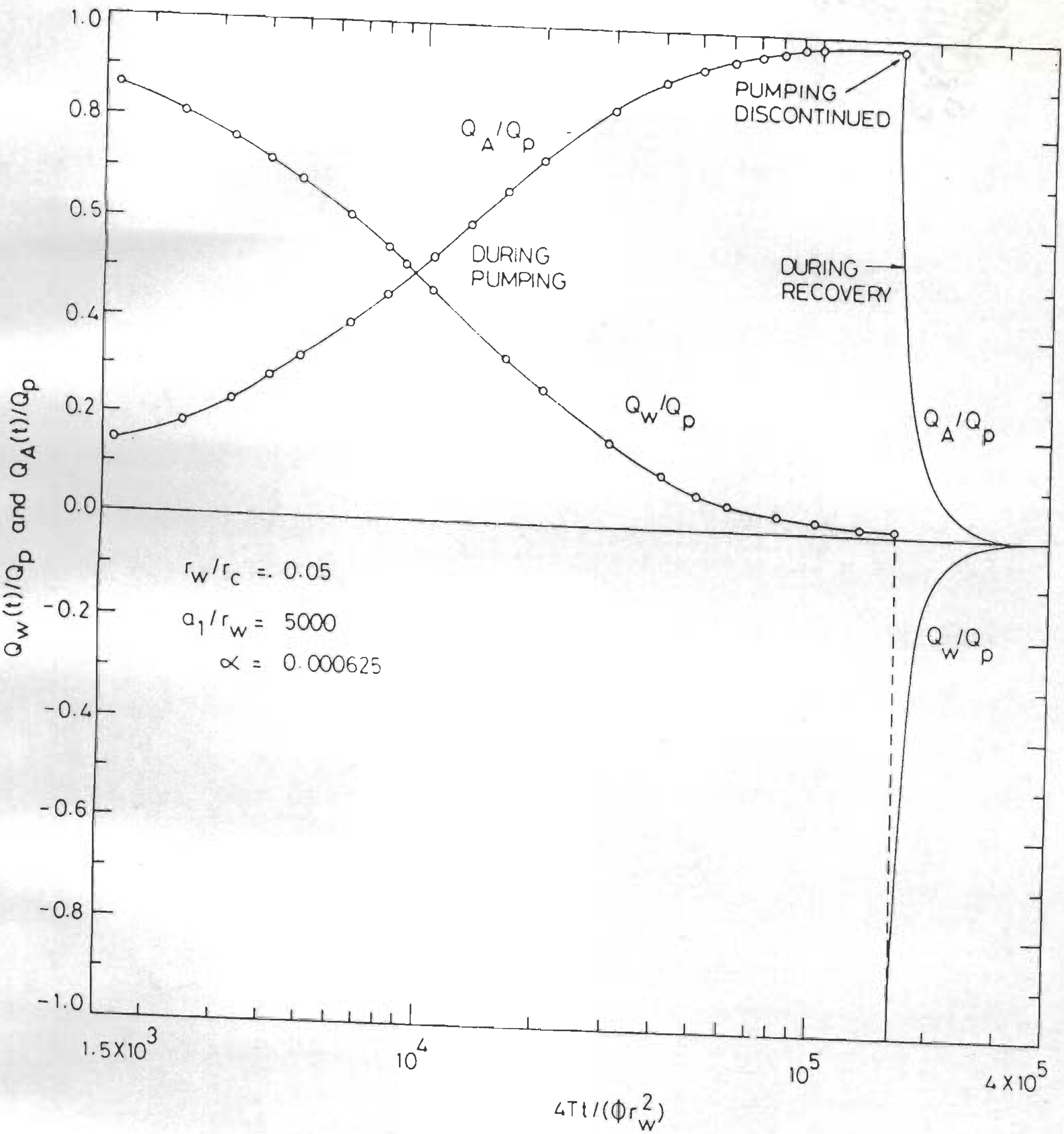


FIG. 7.2(a) - Variation of $Q_W(t)/Q_p$ and $Q_A(t)/Q_p$ with $4Tt/(\phi\ r_w^2)$ for a_1/r_w = 5000.

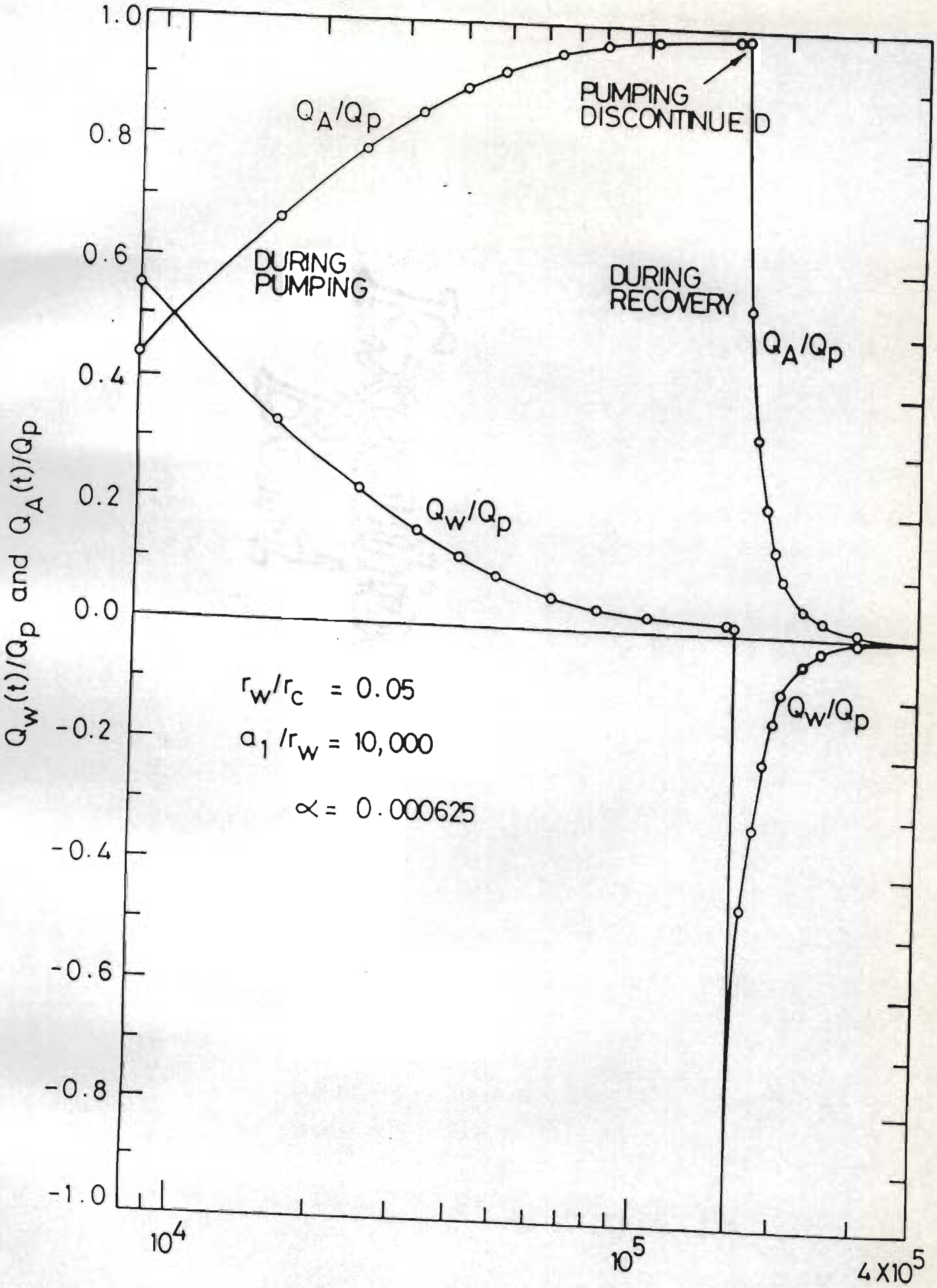


FIG. 7.2(b) - Variations of $Q_W(t)/Q_p$ and $Q_A(t)/Q_p$ with $4Tt/(\phi\ r_w^2)$ for $a_1/r_w = 10,000$.

This indicates that the barrier boundary has no influence on the performance of well storage.

Variations of nondimensional drawdown $S_r(t)/[Q_p/(4\pi T)]$ with dimensionless time $4Tt/(\phi r_w^2)$ at the pumping well and at the barrier boundary are shown in Fig. (7.3) both for pumping and for recovery phases for different durations of pumping. It could be observed that immediately after cessation of pumping the drawdown at the well decreases with time but the drawdown continues to increase at the barrier boundary, even after the pumping is discontinued, because of the aquifers contribution to well storage. There is a permanent drawdown for each pumping operation because of the finite extent of the aquifer. Some time after the cessation of pumping the drawdowns at the well and at the barrier boundary become equal indicating that the aquifer has come to a rest condition after the stoppage of pumping.

The variations of dimensionless drawdown, $S_r(t)/[Q_p/(4\pi T)]$, at an observation well, which is located at a distance of $r/r_w = 100$ from the pumping well, with nondimensional time parameter, $4Tt/(\phi r_w^2)$, have been presented in Figs. [7.4(a)] to [7.4(c)] for values of α ranging from 0.01 to 0.0001 for different values of a_1/r_w . These results have been obtained for low values of a_1/r_w in order to know the response of the bounded aquifer system to pumping of small duration. It could be seen from the figures that the dimensionless drawdown is influenced significantly by the location of the barrier boundary. For example in Fig. [7.4(a)], for $a_1/r_w = 500$, at $4Tt/(\phi r_w^2) = 10^5$ the dimensionless drawdown is 3.15, whereas for $a_1/r_w = 250$, the corresponding value is 4.75. Thus the drawdown is increased by about 51 percent if the value of a_1/r_w is changed from 500 to 250. With cessation of pumping it could be

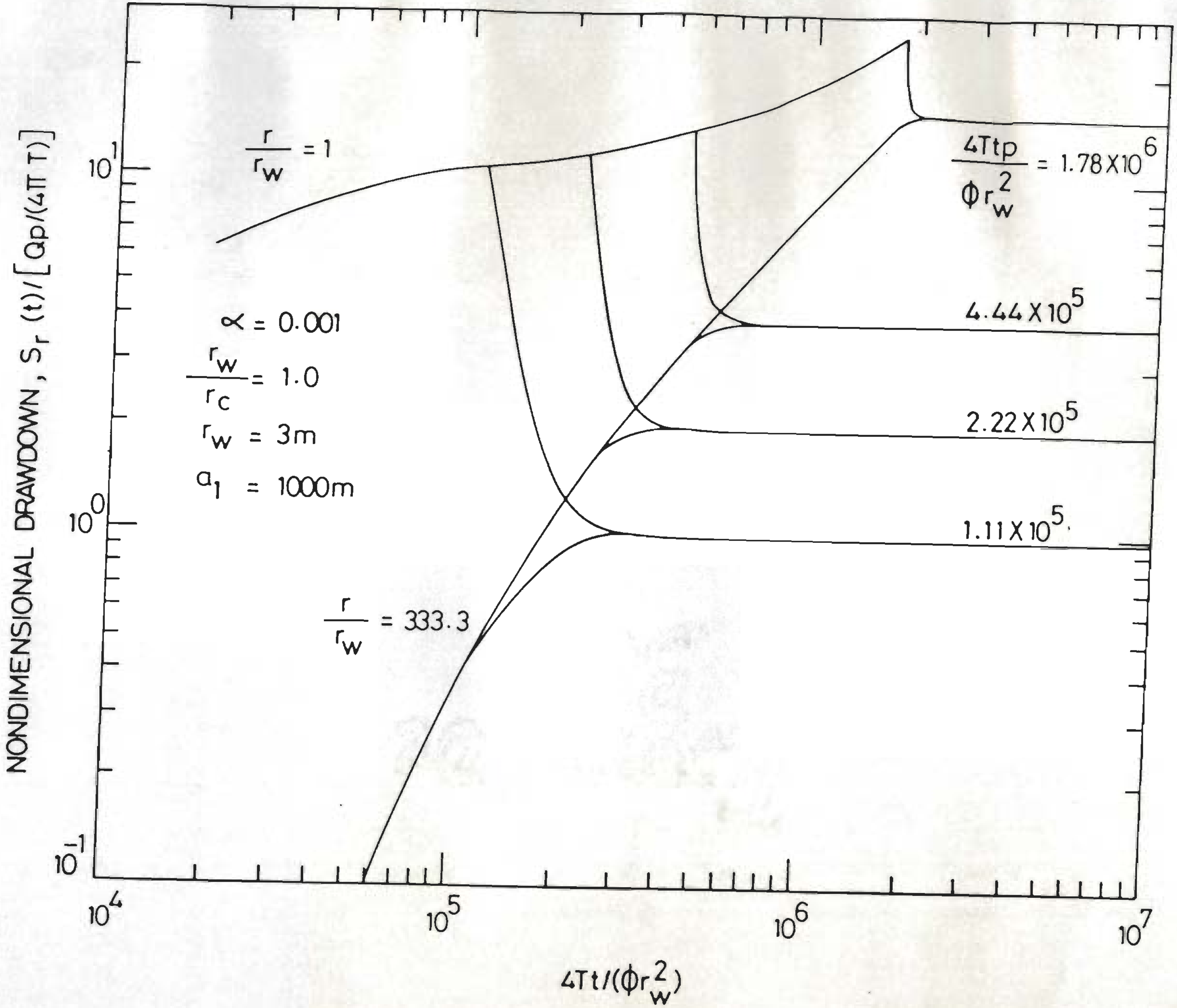


FIG. 7.3 - Variation of nondimensional drawdown $S_r(t)/[Q_p/(4\pi T)]$ with $4Tt/(\phi r_w^2)$ for $r/r_w = 1$, 333.33 and $\alpha = 0.001$.

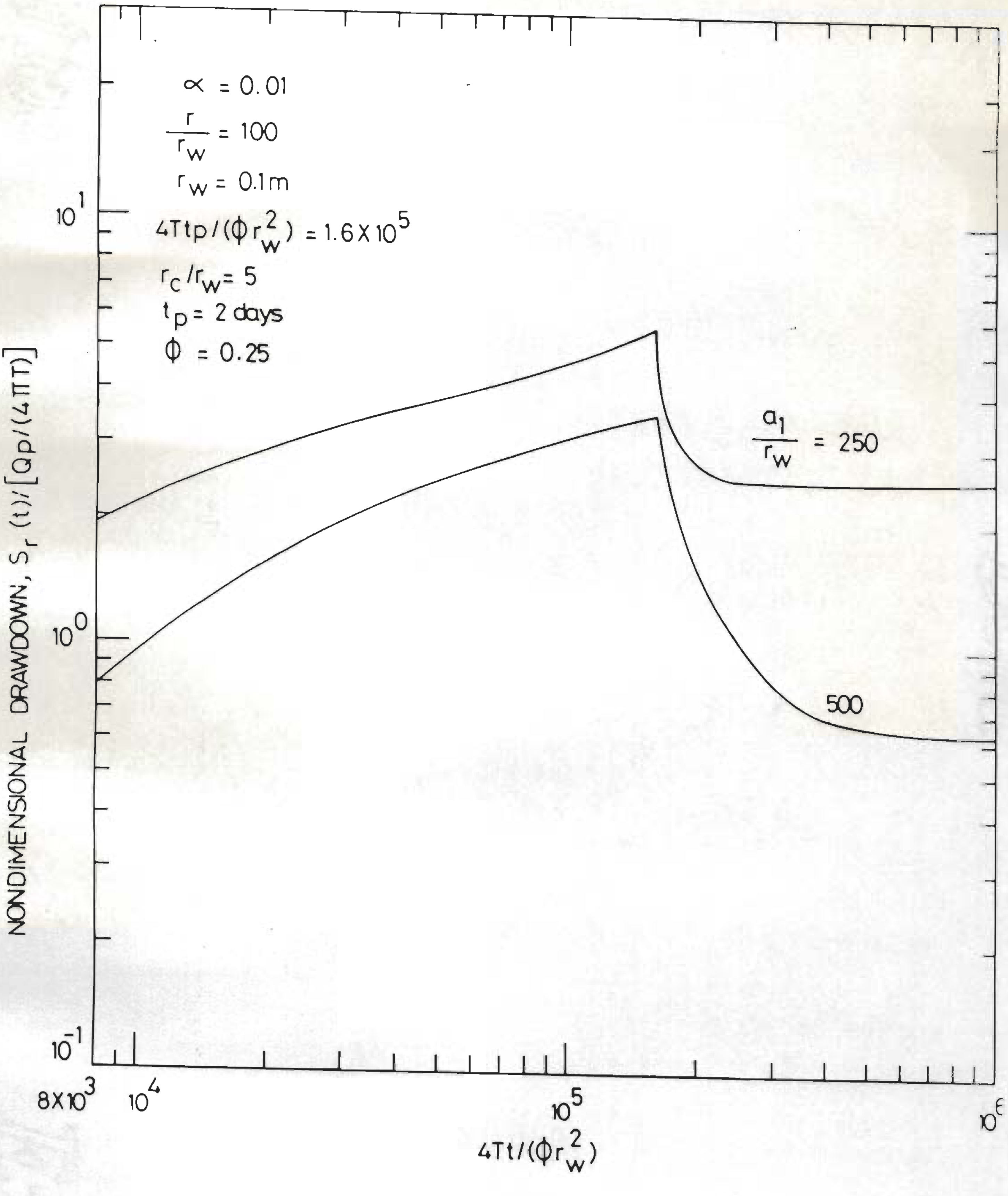


FIG. 7.4(a) - Variation of nondimensional drawdown $S_r(t)/[Q_p/(4\pi T)]$ with $4Tt/(\phi r^2)$ for $r/r_w = 100$ and $\alpha = 0.01$.

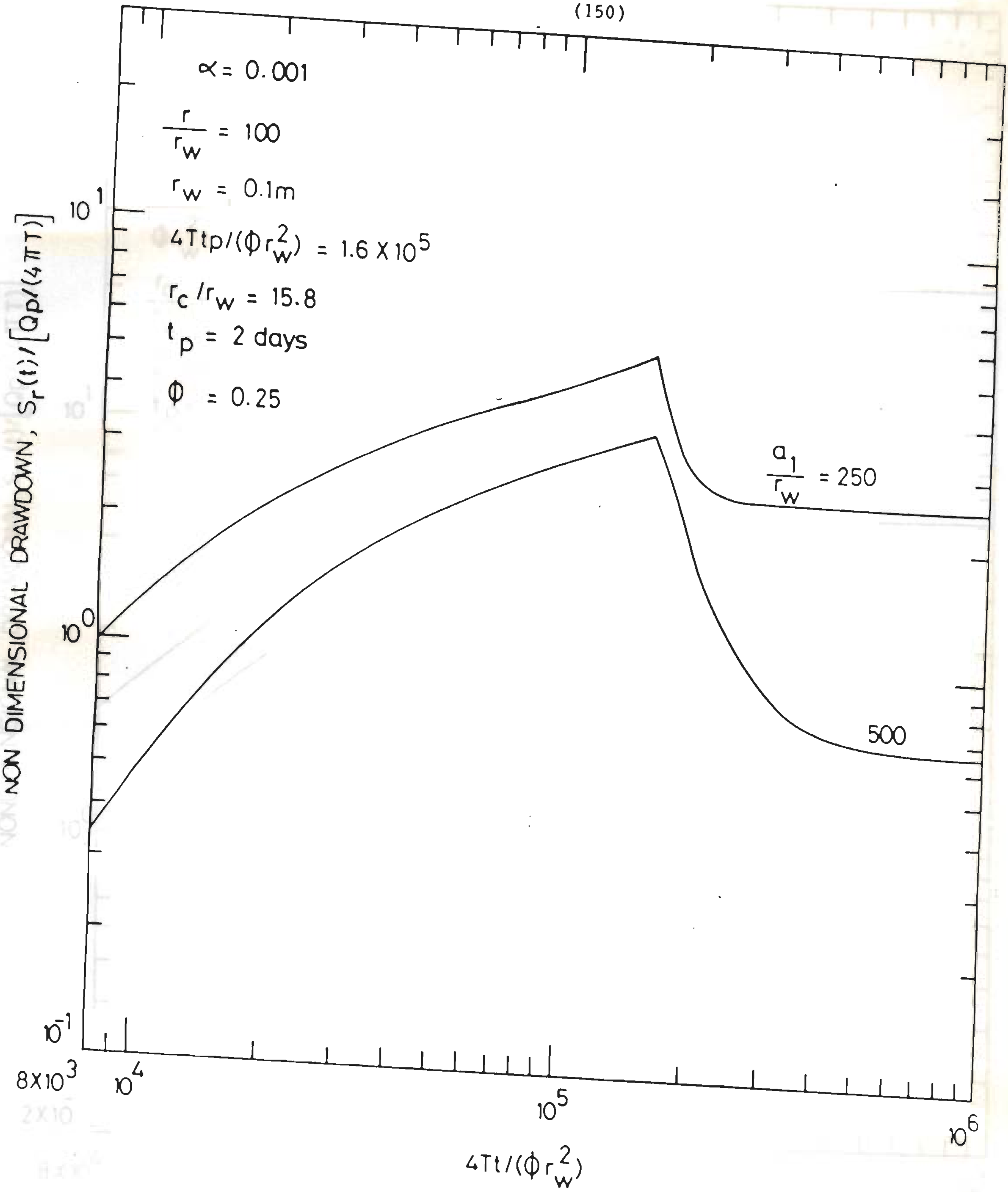


FIG. 7.4(b) - Variation of nondimensional drawdown $S_r(t)/[Q_p/(4\pi T)]$ with $4Tt/(\phi\ r_w^2)$ for r/r_w = 100 and α = 0.001.

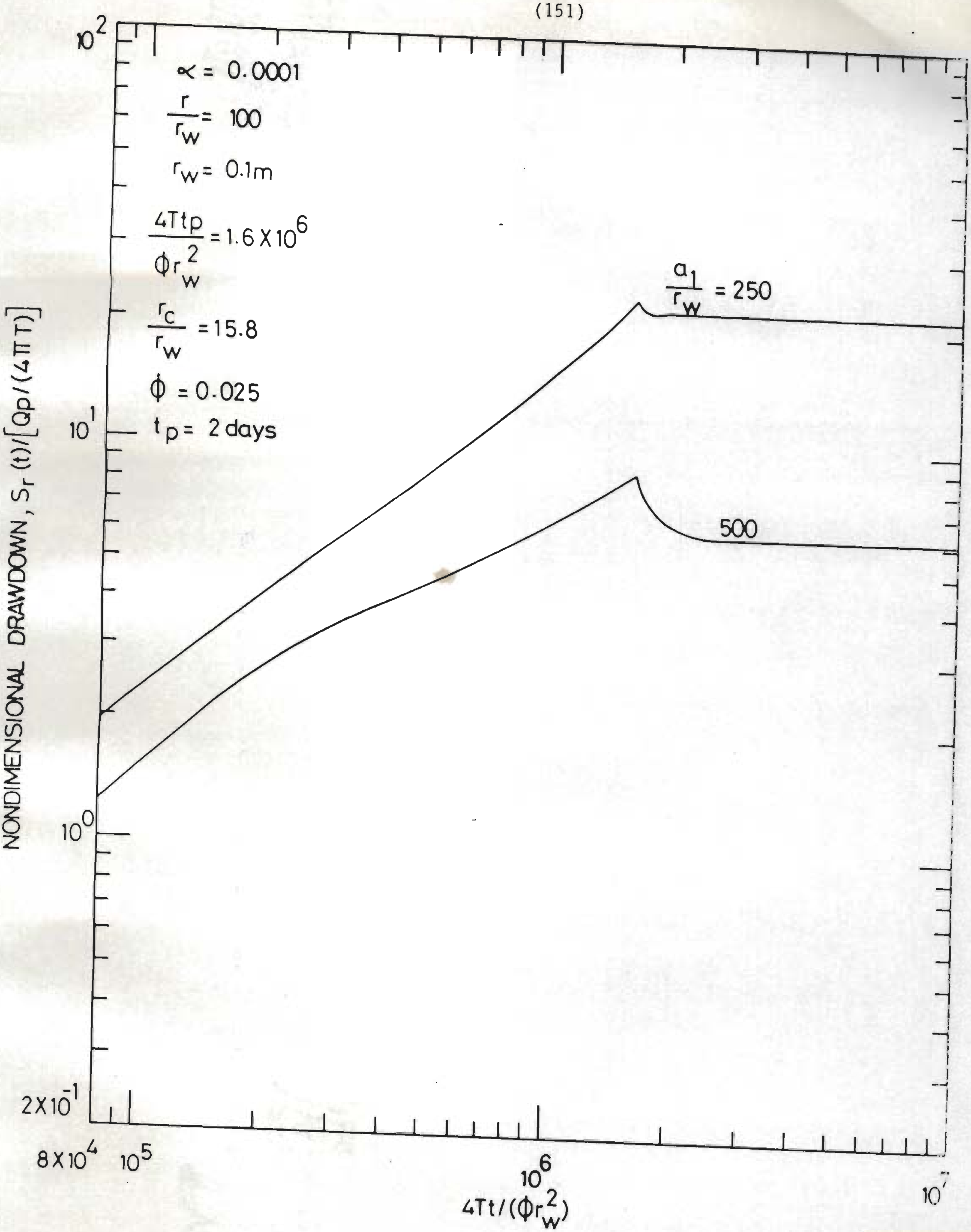


FIG. 7.4(c) - Variation of nondimensional drawdown $S_r(t)/[Q_p/(4\pi T)]$ with $4Tt/(\phi\ r_w^2)$ for $r/r_w = 100$ and $\alpha = 0.0001$.

seen from the figure that water level rises more quickly at the observation well in an aquifer having less areal extent. However, for a finite aquifer, consequent to any pumping operation, there would be a permanent drawdown in the piezometric surface every where in the aquifer. For smaller value of a_1/r_w, the aquifer returns back to rest condition more quickly after the cessation of pumping. For example in Fig. [7.4(a)] for $a_1/r_w = 250$, and $4Tt_p/(\phi r_w^2) = 1.6x10^5$, when pumping is discontinued, the nondimensional drawdown at the observation well has decreased from 5.7 at dimensionless time $4Tt/(\phi r_w^2) = 1.6x10^5$ to 2.55 at nondimensional time $4Tt/(\phi r_w^2) = 3x10^5$. For $a_1/r_w = 250$, and $4Tt_p/(\phi r_w^2) = 1.6x10^5$ the permanent drawdown is also 2.55 and thus the aquifer has attained rest condition at nondimensional time $3x10^5$. On the other hand for $a_1/r_w = 500$, the dimensionless drawdown decreases from 3.55 at nondimensional time $1.6x10^5$ to 0.84 at nondimensional time $3x10^5$. For $a_1/r_w = 500$, and $4Tt_p/(\phi r_w^2) = 1.6x10^5$, the corresponding permanent dimensionless drawdown is 0.64, which will be attained at a larger nondimensional time of 10^6.

From Fig. 7.4(c) it is seen that for $a_1/r_w = 250$, the drawdown variation with time is having a linear trend. This is because for small values of storage coefficient, and for small value of a_1/r_w, the finite aquifer would behave like a tank for which the rate of increase in the dimensionless drawdown with dimensionless time during pumping will be given by

$$\frac{\Delta S'}{\Delta t'} = \frac{1}{(\frac{a_1}{r_w})^2 + [\frac{1}{\phi} - 1] \frac{r_c^2}{r_w^2}} .$$

in which S' is the dimensionless drawdown and t' is the dimensionless time.

For $a_1/r_w = 250$, $r_c/r_w = 15.8$, $\phi = .025$, $\frac{\Delta S'}{\Delta t'}$ is found to be 0.000014. Value of $\Delta S'/\Delta t'$ from the slope of the straight line in Fig. 7.4(c) is also found to be same. The variations of dimensionless drawdown at the pumping well, at the barrier boundary and at an intermediate observation well with nondimensional time parameter have been presented in Fig. [7.5(a)] to [7.5(c)] for different values of well storage parameter, α , and for two pumping durations, for a specific case in which $r_w = r_c$. The graph for $r/r_w = 1$, corresponds to the pumping well and the graph for $r/r_w = 500$ corresponds to an observation point located at the barrier boundary. It could be seen from the figure that during recovery the time-drawdown graphs at all the three observation points converge to a value equal to the permanent drawdown that occurs because of the finite areal extent of the aquifer. The permanent lowering of the piezometric surface in the finite aquifer depends on the duration of pumping. The graphs presented in Figs. [7.5(a)] to [7.5(c)] can be regarded as well function for a large-diameter production well and for an observation well without storage. The flat slope of the time-drawdown graph during recovery would indicate presence of the barrier boundary. The recovery characteristics are predominantly influenced by the storage coefficient of the aquifer. It could be seen that the permanent drawdown has been attained within one log cycle of time after stoppage of pumping for $\alpha = 0.001$ and 0.0001, where as for $\alpha = 0.01$ which corresponds to a higher value of storage coefficient the permanent drawdown has not been attained in one log cycle of time after the discontinuation of pumping.

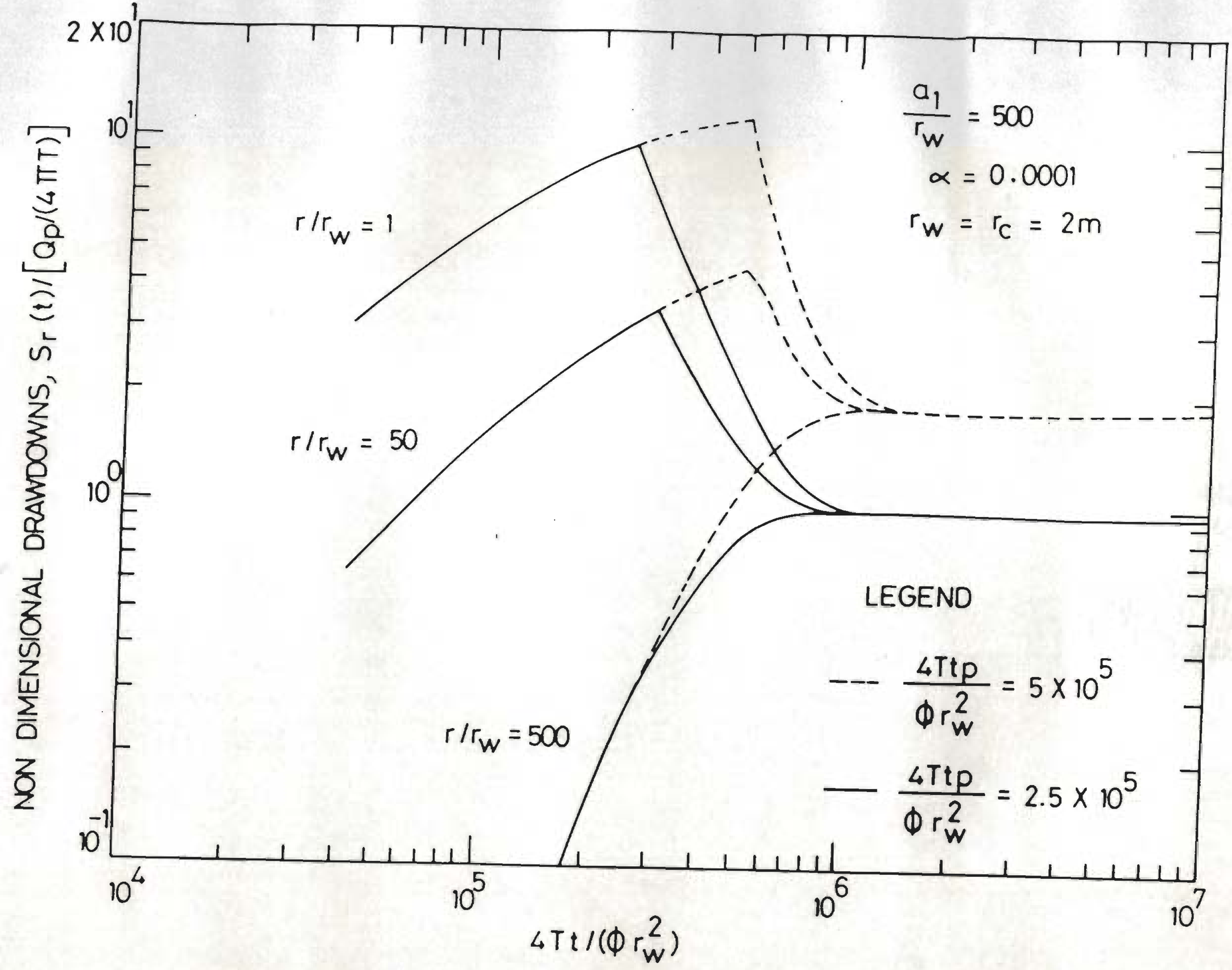


FIG. 7.5(a) - Variation of nondimensional drawdown $S_r(t)/[Q_p/(4\pi T)]$ with $4Tt/(\phi r_w^2)$ for $a_1/r_w = 500$, $r_w = r_c$, and $\alpha = 0.0001$.

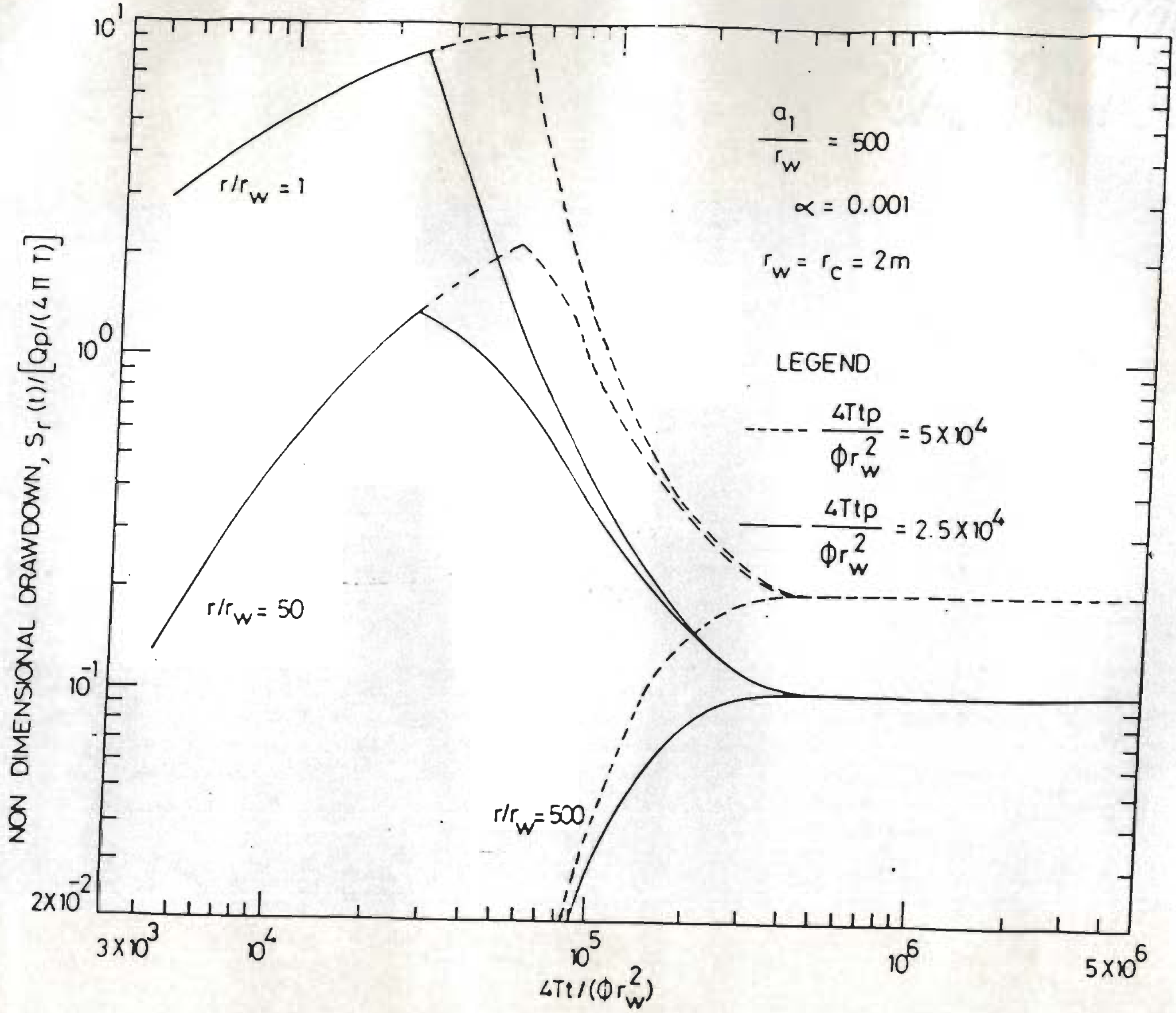


FIG. 7.5(b) - Variation of nondimensional drawdown $S_r(t)/[Q_p/(4\pi T)]$ with $4Tt/(\phi\ r_w^2)$ for $a_1/r_w = 500$, $r_w = r_c$ and $\alpha = 0.001$.

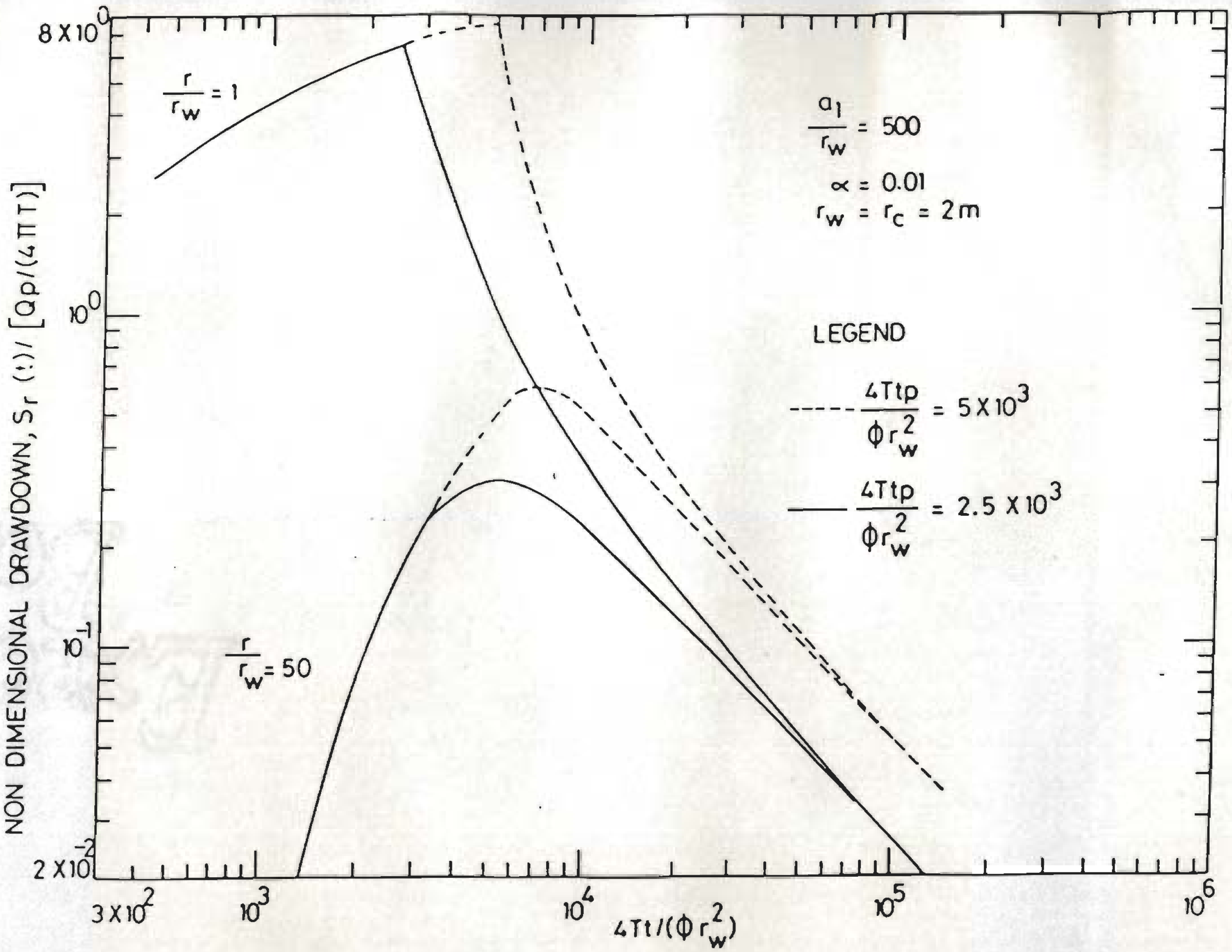


FIG. 7.5(c) - Variation of nondimensional drawdown $S_r(t)/[Q_p/(4\pi T)]$ with $4Tt/(\phi\ r_w^2)$ for a_1/r_w = 500, r_w = r_c and α = 0.01.

7.4 CONCLUSIONS

Based on the study presented in this chapter the following conclusions are made :

(i) Tractable analytical expressions have been derived for determination of aquifer contribution, well storage contribution and drawdown at the well face and at any point in the aquifer for a large-diameter well located in a finite aquifer bounded by circular barrier boundary.

(ii) The influence of finite barrier boundary on the contribution of well storage towards pumping is negligible during the initial time.

(iii) The dimensionless drawdown is influenced significantly by the location of the barrier boundary.

(iv) For a nearer location of the barrier boundary from the pumping well the aquifer attains rest condition more quickly after the cessation of pumping.

(v) The time-drawdown graphs during recovery at observation wells in an aquifer of low storativity are distinctly characterised by a permanent draw-down due to the finite areal extent of the aquifer.

CHAPTER 8

GENERAL CONCLUSIONS

A study on transient flow to large-diameter well is relevant to groundwater abstraction from aquifer of low transmissivity. In the present study analysis of unsteady flow to a large-diameter well in a homogeneous isotropic and confined aquifer has been carried out using discrete kernel approach. The discrete kernel coefficients are the response, of a linear system to a unit pulse excitation. In the discrete kernel approach, the time parameter is discretised by uniform time steps; the excitation and the response are assumed to be piece-wise constants within each time step; the response of the linear system to a time-variant excitation is predicted making use of the discrete kernel coefficients. Desired accuracy in the results can be achieved with selection of appropriate time step size. The methodology provides tractable solution. It has been shown that solution for unsteady flow to a large-diameter well in a homogeneous isotropic aquifer can be obtained with ease by discrete kernel approach. Solutions to the following problems have been obtained in the present study :

(i) Analysis of flow to a large-diameter well during the recovery period,

(ii) Analysis of unsteady flow to a large-diameter well due to abstraction that varies linearly with drawdown at the well,

(iii) Analysis of flow to a large-diameter observation well due to pumping of a large-diameter production well,

(iv) Analysis of unsteady flow to a large-diameter well experiencing well loss, and

(v) Analysis of flow to a large-diameter well in a finite aquifer.

Based on the study the following conclusions are made :

(1) In a discrete kernel approach accuracy in the computation of drawdown at any time t, improves with the increase in the number of time steps used for computations. It is found that the computation of drawdown during early stages of pumping and recovery is sensitive to the time step size. A maximum time step size of t/10 could be used to obtain results with reasonable accuracy at any time t.

(2) Rate of contribution of well storage to pumping and rate of its replenishment during recovery are higher for aquifer with lower storage coefficient.

(3) Comparison of drawdowns at a large-diameter production well during recovery with those of a production well of negligible diameter has shown that calculation of drawdown during recovery using Theis recovery formula is not appropriate for a large-diameter well.

(4) The type curves which incorporate the response of an aquifer during recovery can provide an accurate means of determining aquifer parameters from a short duration pump test data.

(5) A comparison of the duration of pumping computed independently from type curve matching with the actual duration of pumping recorded in an aquifer test helps in perfect matching of time-drawdown graph with the appropriate type curve.

(6) A set of graphs depicting variation of specific capacity with transmissivity for given values of storage coefficient has been developed for

different well storages pertaining to abstraction rate that is linearly dependent on drawdown at the well. These specific capacity graphs can be used to find the transmissivity of an aquifer in case pumping is carried out by a centrifugal pump and the storage coefficient is known a priori.

(7) An average constant pumping rate can not simulate the evolution of piezometric surface pertaining to drawdown dependent abstraction rate.

(8) A tractable analytical expressions have been derived for analysing the effect of production and observation well storage on drawdown at any point in the aquifer. It is found that the influence of the observation well storage on drawdown at the production well is more pronounced during recovery period than during abstraction phase.

(9) The effect of observation well storage on drawdown in the aquifer increases with increase in observation well diameter.

(10) The drawdown in an observation well of negligible diameter due to pumping in a large-diameter well is same if the roles of the wells are reversed.

(11) The contribution from the observation well storage to the aquifer during abstraction is a function of the production and the observation well storages and the time since pumping. The contribution of observation well storage to aquifer increases initially from zero to a maximum value during pumping and then decreases as pumping continues.

(12) Tractable analytical expressions have been derived for determination of aquifer contribution, well storage contribution and drawdown at the well face and at any point in the aquifer considering well loss effect

in a large-diameter dug-cum-bore well. It is found that the relation between specific drawdown and pumping rate is nonlinear for a large-diameter well experiencing well loss.

(13) With increase in the well casing radius, r_c, the expense of extra energy during pumping due to well loss is reduced.

(14) Tractable analytical expressions have been derived for determination of aquifer contribution, well storage contribution and drawdown at the well face and at any point in the aquifer for a large-diameter well located in a finite aquifer bounded by circular barrier boundary.

(15) The influence of the barrier boundary on the contribution of well storage towards pumping is negligible. However, the dimensionless drawdown is influenced significantly by the location of the barrier boundary.

(16) An aquifer returns back to rest condition more quickly after the cessation of pumping for smaller distance of the barrier boundary from the pumping well, and for lower storage coefficient. The time-drawdown graphs at a large-diameter well and at other observation wells during recovery are distinctly characterised by a permanent drawdown because of the finite areal extent of the aquifer. The permanent drawdown that would occur consequent to any pumping operation is attained at all observation points more quickly for finite aquifer with lower storage coefficient.

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