ENERGY CONSERVATION STUDIES FOR THE MULTIPLE EFFECT EVAPORATOR HOUSE OF PULP AND PAPER MILLS

A THESIS

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled *"ENERGY CONSERVATION STUDIES FOR THE MULTIPLE EFFECT EVAPORATOR HOUSE OF PULP AND PAPER MILLS "* in fulfilment of the requirements for the award of the Degree of **DOCTOR OF PHILOSOPHY** and submitted in the *Department of Chemical Engineering* of the University is an authentic record of my own work carried out during a period from August, 1982 to October 1992 under the supervision of Professor B.S. Varshney.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other university.

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This is to certify that the above statement made by candidate is correct to the best of my knowledge.

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ABSTRACT

The present investigation deals with the theoretical studies pertaining to concentration of weak kraft black liquor in multiple effect evaporators in the pulp and paper mills for recovery of chemicals with the distinct aim to make the evaporation system achieve high gains in energy efficiency. Thus, in essence, it is an attempt to provide short term measures to conserve steam energy in the evaporator house of the mills without incorporating any hardware which necessitates capital input burden on the part of management. In other words, this investigation considers the variations in input parameters, viz., the liquor feed temperature, liquor feed concentration, steam temperature, temperature in the last effect, and, liquor feed rate, to evaluate their influence on steam consumption, steam economy along with heating surface area requirements. The effect of choice of the liquor flow sequence has also been investigated on steam energy requirements. Hence, it represents, to some extent, a short term measure to solve the complicated issue of energy requirements in black liquor evaporators.

Equations based on material- and energy- balances and heat transfer rate have been formulated for this purpose. For calculation of overall heat transfer coefficient, the Gudmundson's model is employed. It is important to note that these equations are of general nature and not necessarily applicable to black liquor of any set of physico-thermal properties.

The above equations are capable of computing quantities like; boiling point rise, temperature and concentration of liquor in each effect, steam consumption, steam economy, heating surface area requirements, etc. of the evaporators having any number of effects.

For the solution of these equations, Newton-Raphson method has been employed. A set of computer programme in FORTRAN 77 has been developed and executed using an IBM PC/AT 486 machine.

General applicability of the equations to solve for various quantities has been supported by an excellent agreement between the predicted quantities and the reported values of McDonald in the available literature.

Sensitivity analysis has been performed to screen out qualitatively the input parameters having appreciable effect on the output parameters, viz., saturation temperature and concentration of each effect, steam consumption, steam economy and heating surface area of evaporators for improved design and control strategies of multiple effect evaporators.

The investigation has categorized the liquor flow sequences into two main groups, namely; backward-and mixed-feeds. As regards the backward feed, it has been further classified into sequences with and without split of liquor feed and steam. This facilitates a systematic study of the effect of liquor flow sequence on the steam consumption, steam economy and heating surface area of evaporators.

Finally, it may be mentioned that this investigation suggests the use of sextuple effect evaporator with split of liquor feed evenly among 5th and 6th effects and steam in the 1st
5
effect, leading to a sequence >4- 3- 2- 1B- 1A. This
6
provides the best steam economy, along with the smallest heating surface area requirements for concentrating the weak kraft black liquor from its concentration lying between 0.12 and 0.20 to a final liquor product concentration of 0.45. This observation holds good practically for the entire range of each operating parameter used in the mills, as given in Table 3.3

A comparison between the above sequence and the Scandinavian sequence also suggests that the latter is inferior to the former as regards the steam consumption, steam economy and requirement of heating surface area.

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CONTENTS

Page

LIST OF FIGURES

LIST OF TABLES

NOMENCLATURE

A	Heat transfer area of effect, m^2
a	Fractional heating area of the effect defined by $A_i = a_i(F/50)$, sm^2/kg
BPR	Boiling point rise, ^oC
b	Exponent in Eq.(4.2) (= 0.98)
C	Annual cost; T, total; f, fixed cost; s, steam; r-m, repair and maintenance; c, cleaning; c-w, condensing and cooling; l, labour and supervision, Rs/year
C_p	Specific heat of liquor; v, vapour; w, water, kJ/kg K
C_{me}	Capital cost of multiple effect evaporator, Rs
c	cost; w, replacement water; pi, water injection spray; pa, air handling; pc spray cooling of water, Rs/kg of water
c_c	Cost of cleaning per unit area, Rs/m^2
c_e	Cost of evaporator including its installation and interest on capital cost, Rs/m^2
c_s	Cost of steam per unit weight, Rs/kg
D	Number of working days per year
E_T	Total amount of water evaporated per day in the evaporator, kg/day
e	Evaporation coefficient, defined as evaporation in an effect per hour per unit area, kg/hr m^2
F	Liquor feed rate, kg/s ; fixed charges as fraction of the capital cost of evaporator
FD	Mass flowrate per unit cross-sectional area of the tube;kg/sm^2
f	Fraction of capital cost of evaporator for repair and maintenance cost; functional notation in Eqs. (5.7) & (5.8)
g_i	A function defined as $f_i/F\lambda_o$

H	Specific enthalpy of the vapour, kJ/kg
h	Specific enthalpy of liquor kJ/kg; heat transfer coefficient, W/m^2 K
K	relative cost factor for cleaning; i, any body
K_1	Constant in Eq. (4.6)
k	Thermal conductivity, W/m K
L	Liquor flow rate from the effect, kg/s
l	Fractional liquor flow rate defined by $L_i = l_iF$
m	Constant in Eq. (4.6)
n	Number of effects in an evaporator; exponent in Eq.(4.7)
P	Pressure, N/m^2
p1-p31	Constants used in Eq. (2.18) to (2.34)
Q	Rate of heat transfer across the tube from the steam/water vapour to the liquor, W
q	Heat flux, W/m^2
R	Time between successive cleanings (for first two bodies), days
r	Replacement cum make-up water factor
S	Steam economy
SC	Steam consumption, kg/s
T	Temperature,^{o}C; saturation temperature of water at pressure p
t_F	temperature, ^{o}F
T_{boil}	Boiling temperature of liquor, ^{o}C
TS	Total solids content of liquor, %
U	Overall heat transfer coefficient, W/m^2 K
u	Fractional temperature defined by $T_i = u_iT_o$
V	Vapour flow rate from the effect, kg/s
v	Fractional vapour flow rate from the effect defined by $V_i = v_iF$

W	Amount of water required per kg of vapour condensed
x	Mass fraction of solute in the liquor

GREEK LETTERS

ρ	Density of liquor, kg/m^3
μ	Viscosity of liquor, cP
ν	Kinematic viscosity
λ	Latent heat of vaporization of water at its saturation temperature T_i and pressure p_i, kJ/kg

SUBSCRIPTS (EXCEPT AS ABOVE)

c-c	Condensing and cooling
f	Feed
i	Input; effect number (i = 1,2,3, ,n)
l	Last effect; labour & supervision
m	Average
o	Live steam; output
o1	Live steam entering body No. 1A
o2	Live steam entering body No. 1B
p	Final product
s	Steam; solids; saturation
T	Total
v	Vapour
w	Water

DIMENSIONLESS NUMBERS

Re	Reynolds number ($Dv\rho/\mu$)
Pr	Prandtl number ($C_p\mu/k$)

CHAPTER - 1

INTRODUCTION

Multiple effect evaporators are widely used in pulp and paper, sugar, food processing, chemical and many other allied process industries with a distinct aim to conserve energy.

Currently, in the pulp and paper industry, the 'kraft' process dominates among the chemical pulping processes. The chemical recovery from black liquor is an integral part of the kraft process as the cost of the chemicals is too high for a once - through operation. Importance of a suitably designed recovery system has been stressed by Gundersby [G.9], since about 35 per cent of the capital cost of a modern pulp and paper mill is atributable to it. The weak kraft black liquor contains dissolved solids ranging from 12 to 20 per cent. This liquor is corrosive, viscous, foaming, scaling and strongly ionic, and also exhibits some boiling point rise. Because of its fuel-value, it is always employed as an effective energy resource. Grant [G.6] has pointed out that spent liquor contributed 39.2 per cent of the total energy consumed by pulp and paper industry in U.S.A. in the year 1986. In Indian industry, it is generally concentrated to a solids content of about 45 to 50 per cent in multiple effect evaporators.

Due to the rapid increase in its viscosity with concentration in the vicinity of 45 per cent, the use of forward feed evaporators poses serious operational problems. Though, the backward feed evaporators have been quite popular and eliminate the above problem, they consume electrical energy for pumping the liquor from effect to effect. To reduce it, the mixed feed evaporation system appears to be a feasible solution. Even in this system, it is important to know a priori, as to which effect of the evaporator the liquor feed should be introduced and also to decide upon the liquor flow sequence to be adopted for minimizing the steam consumption, thereby maximizing in the steam economy or the energy efficiency gain. In practice, however, it entirely depends on the suppliers of the evaporator system, amounting to a tailor-fit liquor flow sequence. The variations in physico-thermal properties of black liquor with changes in its temperature and composition could also be accounted and the existing design practice improved to make it more reliable by incorporating such variations and through proper analysis of the alternative liquor flow sequences.

Rao and Kumar [R.2] have pointed out that the black liquor multiple effect evaporator alone consumes around 24 to 30 per cent of the total steam energy required in a large paper mill, and also that the total steam energy consumption for various operations in Indian mills is quite large and varies from 10.5 to 17.4 tonne per tonne of paper. Therefore, it calls for a thorough probe into various possibilities of reducing the steam energy consumption in such highly energy intensive units and achieving the highest possible evaporation capacity per unit area of heating surface. Obviously, this can be attained by using optimum number of effects in an evaporator, use of proper flow sequences of the liquor and steam, conceptual changes in system design and extensive scanning of alternative designs by using sophisticated computer softwares.

It is also important to point out here that there exists a difference in the number of effects used in North American and Scandinavian paper mills. The mills in North America employ sextuple effect evaporators, whereas in Scandinavia quintuple effect evaporators are used. As regards the Indian mills, they make use of quadruple as well as quintuple effect evaporators. However, recent trend in India is for adoption of sextuple effect evaporators. This transition, calls for an extensive scientific techno-economic evaluation to check the validity of this trend.

Foaming in the black liquor evaporators is controlled by splitting the feed evenly and then introducing it in the last two effects. Thus, it shall be desirable to carry out material-and energy-balances for determination of the effect of splitting the liquor feed on the steam energy consumption, steam economy and heating area of the evaporators.

With regard to liquor flow sequences in evaporators, there is no hard and fast rule which dictates the sequence. In this context, it is interesting to note that Hough [H.11] has pointed out that with the same terminal conditions of liquor, steam, and pressure in the last effect, the North American evaporator plants employing sextuple effect evaporators with liquor flow sequence liquor flow sequence 5- 6- 4- 3- 2- 1 claim to have achieved slightly higher evaporation capacity, but lesser steam economy when compared to the standard sequence 3- 4- 5- 1A- 1B- 2, as practiced in Scandinavia. Based on industrial data on Indian integrated paper mills, it has been reported by Rao and Kumar [R.2], that by shifting the liquor feed entry from third to fourth effect in quintuple effect evaporators steam economy can be increased by about 12.7%. This result is contrary to earlier findings and mill practices. Thus, it necessitates a fresh examination of this prime issue of energy conservation which, in turn, would need more sophisticated computer software for evaporator calculations. In fact, the liquor flow sequence in the evaporators should be such that it leads to the highest possible evaporation capacity per unit area of heating surface, alongwith a high steam economy/energy efficiency gain.

Another consideration which merits attention is the type of evaporators used in pulp and paper mills in India. Some of the mills use short tube vertical (STV) evaporators, whereas others employ long tube vertical (LTV) evaporators. To mention a few Indian mills under the former category are Ballarpur Industries Sri Gopal Unit, Yamuna Nagar; Andhra Pradesh Paper Mills, Rajamundhary; Titagarh Paper Mills, Titagarh; and Bengal Paper Mills, Raniganj. The latter category includes Star Paper Mills, Saharanpur; Seshayee Paper Mills Ltd., Erode; Century Paper Mills, Nainital; Orient Paper Mills, Brajrajnagar; etc. However, the LTV evaporators may be either climbing film-or falling film-type. Most of the units use climbing film LTV evaporators. It is important to note that the LTV evaporators provide higher capacity per unit area compared to STV evaporators and can be changed to forced circulation easily. They are the most versatile and well-established units and therefore used worldwide. Hence, it shall be of immediate industrial importance if the present investigation attempts an analysis relevant to climbing film LTV evaporators.

Parameters like; liquor feed temperature, liquor feed concentration, steam pressure, pressure in the last body and flow rate of liquor feed influence the steam consumption, steam economy and heating surface area of the evaporators. Although some information is available on the effects of these parameters, yet it cannot be considered sufficiently exhaustive and reliable for its use by designers. Therefore, there is a need to study these in detail for specific flow sequences in multiple effect evaporators.

For optimum design involving adequate control, it is necessary to know the input parameters having appreciable effects on the output parameters. Sensitivity analysis is used to yield qualitative information about them.

There are many variations noticed with regard to the estimation of physico-thermal properties of black liquor. It is true that some of the parameters like viscosity, specific heat and boiling point rise are very sensitive as far as heat balance and computation of heat transfer coefficients are concerned.

These properties must, therefore, be properly checked before applying them to design equations.

WORK PLAN

Keeping the above considerations in view, the present investigation envisages the following major objectives to provide short term measures for economising the steam energy requirements in evaporator house :

1. To estimate the optimum number of effects in an evaporator required for concentration of weak kraft black liquor for a meaningful analysis, especially in view of the prevalent variations in the present industrial practice.
2. To develop mathematical models for different liquor flow sequences in optimum multiple effect evaporators based on mass-and energy-balances and heat transfer rate equations, taking into account the due variations in physico-thermal properties and overall heat transfer coefficients with temperature and concentration of the kraft black liquor.
3. To develop appropriate computer programmes based on FORTRAN 77 for solution of the above models.
4. To perform sensitivity analysis for identification of the most significant input parameter, in relation to

its effect on the output parameters used in this study, and finally,

5. To estimate the steam energy consumption, steam economy and heating surface area of the optimum multiple effect evaporator as a function of input parameters to evaporators, namely; liquor feed temperature, liquor feed concentration, steam temperature, temperature (pressure) in the last effect, and liquor feed rate for various potential liquor flow sequences in order to finally establish the sequence corresponding to the maximum energy efficiency gain.

CHAPTER - 2

PHYSICO-THERMAL PROPERTIES OF BLACK LIQUOR AND HEAT TRANSFER COEFFICIENT

Physical properties of black liquors are required for striking material-and energy-balances and for predicting the heat transfer coefficient from available empirical correlations. Use of reliable and accurate values of the physical properties of black liquors would ensure an effective and economic design of process equipment involved, including multiple effect evaporator. Regestad [R.7] has pointed out that several of the procedures available for estimating properties are valid for pure compounds and simple solutions. However, such simple techniques cannot be easily applied to the complex black liquor from pulp and paper mills. The important properties depend upon the nature and concentration of both the organic and inorganic constituents of black liquors and are functions of temperature as well. Commercial black liquors exhibit a wide range of properties due to the diversity of cellulosic raw materials and pulping conditions employed. Grace and Malcolm [G.5] pointed out that the raw materials include softwoods, hardwoods, and non-wood fibres (such as straw, bagasse, bamboo esparto) containing 27-33 per cent, 16-24 per cent and 11-22 per cent lignin, respectively. The pulping chemicals cause degradation and dissolution of the non-cellulosic portions. As regards the organic compounds in black liquor, they include low molecular weight alkali lignin and thiolignin, sugar residues as isosaccharinic acids, low molecular weight polysaccharides and extraneous compounds such as resin and fatty acids. The inorganic chemicals in black liquor include sodium hydroxide, sodium sulphide, sodium carbonate, and small amounts of sodium sulphate, sodium thiosulphate and sodium chloride depending upon the make-up chemicals used and the recycle practices. Silica from bamboo, bagasse and grasses enters the black liquor as sodium silicate. The weak black liquor from the washers normally has a dissolved solids content varying from 12 to 20 per cent, the value actually depending upon the alkali charge, liquor to wood ratio, pulp yield, chip moisture content and water added during pulp washing. The weak black liquor may also contain some fibre fines and other colloidal particles which may be removed partially by suitable screening and filtration equipment.

The important physico-thermal properties include density, viscosity, specific heat, thermal conductivity, boiling point rise and surface tension.

2.1 PHYSICO-THERMAL PROPERTIES

Available literature on some of the important physico-thermal properties is summarised below :

Hough [H.11], Hultin [H.12], Passinen [P.2], Koorse et al. [K.6, K.7] have studied physical and engineering properties and/or composition and discussed their relevance in design, development and opeation of kraft chemical recovery units.

2.1.1 Density

Han [H.1] has reported that the density characteristics of different black liquors are very similar. The specific gravity vs concentration isotherms are more or less linear. The higher the inorganic content, the steeper is the initial slope of these isotherms. Temperature effects are minor compared to the effect of changes in solids content. Regestad [R.7] has presented the following useful equation relating the density of black liquor to solids content :

$$\rho = 1007 + 6.0(TS) - 0.495\,T \qquad \ldots.\ (2.1)$$

Koorse et al. [K.6] have determined experimentally the specific gravity of black liquors from pulping indigenous fibrous raw materials such as bamboo, bagasse, bamboo + 10% salai, eucalyptus and mixed hardwoods, using commercial samples and the results have been presented in the form of graphs with concentration as the correlating variable at 70°C. Black liquors from eucalyptus, bamboo, bamboo + 10% salai, and bagasse exhibit increasing specific gravity in that order.

Hultin [H.12], based on 77 determinations, has plotted density vs dry solids content of black liquors at a temperature of 90°C and the linear relationships developed are as follows :

For dry solids content between 10 and 25% :

$$TS = 177\,(\rho_{90^\circ C} - 963) \qquad \ldots.(2.2)$$

and for solids content between 50 and 65% :

$$TS = 146\,(\rho_{90^{\circ}C} - 920) \qquad \ldots.(2.3)$$

2.1.2 Viscosity

Black liquor is reasonably Newtonian at solids levels below 50 per cent, but becomes appreciably non-Newtonian at higher solids content. The high viscosity of strong black liquor can adversely affect the heat transfer rates and limit the number of units in a multiple effect evaporator. Viscosity data on black liquors as a function of solids content (upto about 50-55%) and temperature (upto 100°C) have been reported by Passinen [P.2] and Venkatesh and Nguyen [V.4]. Representative data have been plotted by Grace and Malcolm [G.5] for a softwood kraft black liquor.

Kinematic viscosities for some Finnish black liquors based on about 100 determinations have been plotted as function of temperature and dissolved solids content by Hultin [H.12]. Deviations from the linear plots are normally less than 5% for solids content upto 30%, and about 25% at solids content of 60%. Most of the data were determined at 50 and 90°C. No measurements were made at temperature higher than 100°C. The plots were extrapolated upto 150°C using the following equation, after Cornelissen and Waterman [C.10]

$$\ln \nu = \frac{A}{T^3} + B \qquad \ldots.(2.4)$$

where, T is expressed in (K) and A and B are constants for a given solids content. However, this expression showed quite crude estimation.

Kobe and McCormack [K.3] have reported the viscosity data for various black liquors.

Koorse et al. [K.6, K.7] have determined experimentally the influence of pulping species on viscosity of black liquor. They have presented plots of viscosity of black liquor at 70°C vs solids content for various raw materials such as bamboo, bamboo + 10% salai, eucalyptus, bagasse and pine and have shown that bagasse black liquor has the highest viscosity while the pine black liquors exhibit the lowest values among the systems considered by them. At 45% concentration, viscosity of bagasse black liquor is 10 times the value for bamboo, and 100 times the value for pine black liquors at 70°C. At lower concentrations, these ratios are somewhat smaller. Eucalyptus black liquor can be expected to have nearly same values of viscosity as bamboo black liquor. Addition of 10% salai (hardwood) to bamboo would tend to decrease the viscosity. The higher viscosity of bagasse black liquor may be attributed to the higher pentosan content compared to the other species.

Several workers, viz., Co et al. [C.6], Leonard [L.5], Sandquist [S.1] and Wennberg [W.3] have published data on the rheological behaviour of black liquor at high solids contents. Both the apparent viscosity and the degree of shear thinning decrease with increasing temperature. In the temperature range of 110-120°C, black liquors of upto 70-75% solids show Newtonian behaviour. Wennberg [W.3] also found little shear dependence at higher temperatures.

Kulkarni et al. [K.8] observed that the apparent viscosity of black liquor can be strongly affected by the residual alkali content. Low residual alkali causes high viscosity and non-Newtonian behaviour to set in at lower solids contents or to persist at higher temperatures. Black liquor oxidation has been known to result in greatly increased viscosity, and this has been shown by Hermans and Grace [H.8] to be due to the reduction in alkalinity associated with oxidation. Lower viscosity can be achieved with higher level of residual active alkali by charging more chemicals during cooking or addition of sodium hydroxide externally above 35 per cent concentration.

Viscosity can be reduced significantly by removal of polymeric and gummy materials from the black liquor through molecular separation or gel filtration before evaporation.

Addition of selective chemicals like Busperse-47 and Chemosperse-47 in a dosage of about 25 ppm to the black liquor is helpful in bringing down the viscosity. Data for hardwood- as well as softwood-liquors show that respective reduction by about 17 per cent and 33 per cent, in the viscosity of these liquors can be achieved. However, detailed experimental evaluation is essential before any final conclusion can be drawn.

In general, soda liquors exhibit higher viscosity than kraft liquors. The following equations have been recommended in the

Tappi Monograph, cited by Ray et al. [R.6] for the various solids contents shown against each:

For (TS) > 40%,

$$\mu = 0.06198889 \exp[-0.0032563(t_F - 460) + 0.10178\,(TS) + 37.287 - 6(t_F - 460)^2 + 0.0018003(TS)^2 + 0.00049515(t_F - 460)(TS)] \quad ....(2.5)$$

For (TS) < 40%,

$$\mu = \exp[-8.3\text{x}10^{-3} - 6.55\text{x}10^{-3}(TS/100)^2 + 5.62\text{x}10^{-2}(TS/100)^2(t_F - 660) + 5.7(TS/100) - 1.307] \quad ....(2.6)$$

and, for the concentration range, from 0.10 to 0.50, and temperature range from 20 to 40°C,

$$\log\mu = 0.015 + 0.079\,(TS) - 0.0003125\,T\,(TS) - 5.75 \text{ x } 10^{-3}\,T \quad ....(2.7)$$

Gudmundson [G.7, G.8] arrived at the following correlations making use of experimental data of Davis [D.1] and Kobe and McCormack [K.3] :

$$\mu = \exp\,[a + b.\,(TS) + c.\,(TS)^2 + d.\,(TS)^3] \quad ....(2.8)$$

where,

$$a = 0.4717 - 0.02472\,T + 0.7059 \text{ x } 10^{-5}\,T^2 \quad ....(2.8a)$$

$$b = 0.06973 - 0.5452\text{x}10^{-3}\,T + 0.1656\text{x}10^{-5}\,T^2 \quad ....(2.8b)$$

$$c = 0.002046 + 0.3183\text{x}10^{-4}\,T + 0.9761\text{x}10^{-7}\,T^2 \quad ....(2.8c)$$

and,

$$d = 0.5793\times10^{-4} - 0.6129\times10^{-6}\,T + 0.1837\times10^{-8}\,T^{2} \quad \ldots.(2.8d)$$

Value of viscosity in Eq. (2.8) is expressed in centipoise.

2.1.3 Specific Heat

Hultin [H.12] has proposed the following equation for specific heat, to fit the experimental data of Kobe, Sorensen and Basberg,

$$C_p = 0.96 - 0.45\times10^{-2}\,(TS) \quad \ldots.(2.9)$$

where, C_p is expressed in kcal/kg °C,

The product $C_p\rho$, the specific heat per unit volume, is nearly constant at 0.94 $\mathrm{Mcal/m^3\,^oC}$ for black liquor at concentrations between 10 and 60 per cent dissolved solids and at temperatures between 50 and 120°C.

In the absence of actual data, the following approximation has been recommended in Tappi Monogragh (Cited by Grace and Malcolm [G.5]) :

$$C_p = 1.0 - (1 - C_{p,s})\times10^{-2}\,(TS) \quad \ldots.(2.10)$$

where, $C_{p,s}$ is the specific heat of the black liquor solids and its value is assumed to be in the range of 0.3 - 0.5 kcal/kg °C. For the value of $C_{p,s} = 0.5$, the estimated black liquor values correspond closely with those of Harvin and Brown [H.5] at 90°C.

Koorse et al. [K.6] have plotted specific heat as a function of per cent solids at a temperature of 80°C for various black liquors.

Veeramani [V.2, V3] has proposed the following equations for the specific heat of non-wood black liquors:

For bamboo and pine black liquors, with 15 - 50% dissolved solids content,

$$C_p = (1.8 \times 10^{-3}T - 0.54) \times 10^{-2}\,(TS) + 1.0 \qquad ....(2.11)$$

and, for bagasse and straw black liquors,

$$C_p = (1.04 \times 10^{-3}T - 1.26) \times 10^{-2}\,(TS) + 1.0 \qquad ....(2.12)$$

The dependence of specific heat of kraft black liquor on total solids content of the liquor as suggested by Regestad [R.7] is :

$$C_p = [1.0 - 0.0054\,(TS)] \times 4187 \text{ W s/kg K} \qquad ....(2.13)$$

Regestad considers the dependence of specific heat on the temperature to be negligible.

2.1.4 Thermal Conductivity

Hunter et al. [H.13] have experimentally determined thermal conductivity of black liquor. However, the method employed has not been considered very accurate to yield reliable data.

The only complete set of data for thermal conductivity of kraft black liquor has been reported by Harvin and Brown [H.5] and their results have been plotted. The isotherms are linearly decreasing function of the solids content and terminate at the thermal conductivity of water. The slopes vary linearly with temperature according to the following expression,

$$\text{Slope} = 0.21 - 3.38 \times 10^{-4}\ T \qquad \text{....(2.14)}$$

where, T is in ^{o}F.

Hultin [H.12] has reported that for practical calculations, one can use the same values for kraft black liquors as for sulphite liquors.

Koorse et al [K.6] have plotted thermal conductivity of black liquors versus per cent solids at 70^{o}C and have concluded that it has a very small dependence on pulping species.

The thermal conductivity of black liquor increases with an increase in temperature. The combined effect of the solids content in the black liquor and the temperature on thermal conductivity is given by the following equation:

$$k = [0.504 - 0.282\text{x}10^{-2}\ (TS) + 1.35\text{x}10^{-3} T]\text{x}1.163\ \text{W/mK} \qquad \text{....(2.15)}$$

This is valid over the range of solids content between 15-55 per cent and temperature range of 45 - 90^{o}C. The deviation in values is + 2% for wood and non-wood fibres.

2.1.5 Boiling Point Rise

The boiling point rise (BPR) is the difference between the boiling temperature of black liquor and that of pure water at the same pressure. It is a strong function of solids content of the liquor and only a very weak function of pressure and hence temperature. The BPR increases more rapidly at higher solids contents. Hultin [H.12] has shown that the BPR can be expressed as follows :

$$\mathrm{BPR} = K \frac{(\mathrm{TS})}{[110 - (\mathrm{TS})]} \qquad \ldots.(2.16)$$

The constant K is just equivalent to the BPR at 50% solids content. Typically K varies between 5 and 8 °C. Increased inorganic content in the black liquor increases BPR.

The fraction (TS)/[100-(TS)] is the solids to water ratio in the black liquor. Above about 50% solids, sodium carbonate and sodium sulphate begin to precipitate out of black liquor and the BPR begins to rise less steeply. This has been substantiated by Frederick et al. [F.2].

Kumana [K.9] has defined the terms Apparent BPR, True BPR, Effective BPR, and Excess BPR. The amount of liquid superheat represents "excess" boiling point rise, over and above that caused by the presence of dissolved solids. Forced circulation evaporators are routinely designed with the liquor being introduced under the liquid surface in the vapour body. This can result in liquid superheat. Excess BPR due to liquid superheat is more widely prevalent than generally recognized.

The Tappi correlation for BPR is:

$$\mathrm{BPR} = 41.4\,[(\mathrm{TS}/100) + 0.1]^2 \qquad \ldots.(2.17)$$

2.1.6 Surface Tension

Very little published information is available on the surface tension of kraft black liquor. Data for bamboo and bagasse black liquors upto concentrations of 30% solids and temperature range of 30 to 95 °C, show that the effect of temperature on surface tension of liquor is similar to that on the surface tension of water. The organic constituents of black liquor are all surface active and contribute to the observed decrease in surface tension. However, the effect of inorganics is comparatively small. Grace and Malcolm [G.5] have pointed out that data on surface tension should be used with caution because time dependent effects may be important. These are believed to be due to the time required for surfactants in the black liquor to migrate to the surface.

2.1.7 Relevant Properties of Water

For striking enthalpy balance around different effects of the evaporators, the temperature dependence of latent heat of vapourisation and saturation pressure of water are required. Using Data tabulated in Steam Tables [S.7], and, least square method, the following polynomial expressions are arrived at for these,

For latent heat of vapourisation of water,

$$\lambda = 2519.5 - 2.653\ T \quad \ldots.(2.18)$$

and for saturation pressure of water,

$$P = (2.1278 - 0.064\ T + 0.0005296\ T^2) \times 10^5 \quad \ldots.(2.19)$$

2.2 OVERALL HEAT TRANSFER COEFFICIENT

One of the most important parameters in the design of evaporators is the overall heat transfer coefficient and hence has been studied extensively.

Several workers, viz.,Bergles et al. [B.2], Butterworth et al. [B.9], Harvey and Foust [H.4], Kern [K.1], Lavin and Young [L.3], McDonald [M.3], McQuillan and Whally [M.4], Mishima and Ishil [M.6], Wallis [W.2] and Zinemanas et al. [Z.1] have studied and analysed the problem of heat transfer in two phase flow systems; Bolmstedt et al. [B.4] has made simulation studies and Collier [C.9] studied convective boiling and condensation. Lavery et al. [L.2] studied slug and churn flow heat transfer, Olauson [O.2, O.3] studied and simulated heat transfer in climbing film evaporators, Ray [R.6] studied short tube multiple effect evaporators, Sarma et al. [S.2] the vertical thermosiphon reboilers, whereas Spoelstra [S.6] compared the single pass vs recirculation in evaporators.

Reported data after Kern [K.1], McDonald [M.3], Whitney [W.5], Bolmstedt et al. [B.4, B.5], Harrison [H.3], Clay [C.5], Ulrich [U.1], Rosenbled [R.10], Koorse et al. [K.6, K.7] and Arhippainen et al. [A.1], on this parameter are based on various plant parameters and do not lead to any final conclusion about their reliability.

Based on these studies, several workers, also including Liem [L.6] and Sandquist [S.1], have proposed models for theoretical prediction of overall heat transfer coefficients. However, the model developed by Gudmundson [G.7, G.8] is quite comprehensive and empirical in nature. Therefore for the present investigation the model of Gudmundson has been selected for the prediction of heat transfer coefficient of LTV evaporator.

2.2.1 Gudmundson's Model

In long tube vertical evaporators, various regions related to non-boiling, nucleate boiling and annular film boiling exist along the tube length. As regards the boiling section, it consists of bubble flow, slug flow and annular flow.

Based on the extensive experimental data obtained on full scale as well as pilot plant scale climbing film evaporator with 8.5m long stainless steel tubes having OD/ID as 51/48mm and using black liquor as feed, Gudmundson [G.8] has proposed an empirical model to determine the overall heat transfer coefficient in a LTV evaporator. The model makes use of parameters like; heat flux, liquor feed rate, subcooling or superheating of the liquor feed, boiling point, viscosity of the liquor and total solids content of the liquor feed. Predictions from the empirical equations of this model are within ± 15 per cent of the measured data for 95 per cent of the cases investigated. Details about the model are available in [G.8]. For the sake of presentation, various equations needed in the model for computation of overall heat transfer coefficient are reproduced in the following sub-section.

2.2.1.1 Functional Relationships Between Overall Heat Transfer Coefficient and Independent Parameters

In fact, U depends upon heat flux, q. The following polynomial equation is suggested when T_{boil} is 100°C and liquor feed enters at the same temperature, i.e. 100°C,

$$U = p1.\ C^{p2} \times [p3 \times q + p4 \times q^2 + p5 \times q^3] \quad ....(2.20)$$

where, C is a correction factor and is calculated as follows :

$$C = C_{\mu} \times C_T \times C_F \quad ....(2.21)$$

$$C_{\mu} = p6 + p7 \times \mu_m + p8 \times \mu_m{}^2 + p9 \times \mu_m{}^3 \quad ....(2.22)$$

$$C_T = p10 + p11 \times T_{boil} + p12 \times (T_{boil})^2 + p13 \times (T_{boil})^3 \quad ....(2.23)$$

$$C_F = Z + [p14 + p15 \times Z] \times (FD) \quad ....(2.24)$$

where, C_{μ}, C_T and C_F are the correction factors for viscosity, departure of boiling point from 100°C and liquid feed rate in combination with liquid feed temperature and heat flux respectively. The value of Z is obtained from the following equation:

$$Z = 1.0 + p16 \times \text{Tanh}\ [(q - \text{Bound 1})/p17] \times (T_{sub})^{p18} \quad ....(2.25)$$

Bound 1 is a limiting value of q and is found to be dependent on the viscosity and boiling point of the liquor. It is calculated as follows :

$$\text{Bound 1} = p19 + p20 \times \log \mu_m + p21 \times T_{boil} \quad ....(2.26)$$

For the liquor feed temperature higher than the boiling point, the liquor will flash at the inlet to the tubes of the evaporator. This is equivalent to an increase in the value of heat flux, q, which is calculated by the following equation :

$$\Delta q = T_{sup} \times C_p \times FD \times \frac{\pi D^2}{4} \times Ceff/A \qquad ....(2.27)$$

where, T_{sup} is calculated from,

$$T_{sup} = T_{in} - T_{max} \qquad ....(2.28)$$

T_{max} is the maximum temperature of the bulk liquor temperature profile along the tube and can be calculated from,

$$T_{max} = p22 + [p23 + p24 \times T_{boil}] \times (FD) + [p25+p26 \times T_{boil}] \times \log \mu_m + p27 \times T_{boil} \qquad ....(2.29)$$

C_{eff} is an efficiency factor which shows the efficiency of the flashed vapour in comparison with the corresponding increase of the heat flux. The value of this coefficient lies between 0 and 1. In the present investigation it has been taken as 0.61[G.8]

At extremely low heat flux where q < Bound 2, the value of U is computed as follows :

$$\text{Bound } 2 = p28 + p29 \times \log \mu_m + p30 \times T_{boil} \qquad ....(2.30)$$

When the heat flux is zero (q = 0), the heat transfer coefficient of the liquid side, h, can be computed from the Dittus Boelter correlation :

$$h = 0.023 \,.\, Re^{0.8}\, Pr^{0.4} \,.\, k/D \qquad ....(2.31)$$

For the condition q = 0, the resistances due to the wall and the condensate film can be neglected.

Hence $U = h$ ....(2.32)

Now, when q is greater than zero and less than Bound 2, U is supposed to be directly proportional to the heat flux. Thus,

$$U = [h + (U_{bound} - h) \times q/\text{Bound } 2] \times C_{sub} \quad ....(2.33)$$

U_{bound} is the value of U with existing parameter values but with q assumed to be equal to Bound 2.

C_{sub} is the correction factor due to the subcooling of the liquid feed as given below :

$$C_{sub} = 1.0 + p31 \times T_{sub} \cdot [(\text{Bound } 2 - q) / \text{Bound } 2]^2 \quad ....(2.34)$$

Values of p1 to p31 as used in Gudmundson's model are computed from experimental data using least square technique. Values of these constants are given in Table 2.1.

TABLE 2.1

VALUES OF THE CONSTANTS p1 TO p31

1. p1 = 0.981	12. p12 = -0.3640E-03	22. p22 = 15.0
2. p2 = 1.000	13. p13 = 0.946E-06	23. p23 = 0.12
3. p3 = 0.2334	14. p14 = -0.0515	24. p24 = -0.001
4. p4 =- 0.1006E-04	15. p15 = 0.0505	25 p25 = 3.0
5. p5 = 0.1362E-09	16. p16 = 0.01	26. p26 = -0.025
6. p6 = 1.000	17. p17 = 21800	27. p27 = -0.12
7. p7 = -0.07858	18. p18 = 0.4	28. p28 = 4300
8. p8 = 0.002735	19. p19 = 19500	29. p29 = 100
9. p9 =-0.1092E-04	20. p20 = 2000	30. p30 = -14.0
10.p10 = -1.020	21. p21 = -95	31. p31 = 0.04
11.p11 = 0.0476		

To compute overall heat transfer coefficient, U, from Gudmundson's model [G.8], viscosity, specific heat, density, and thermal conductivity of black liquor are determined using Eq.(2.8), Eq. (2.13), Eq.(2.1) and Eq.(2.15), respectively.

2.2.1.2 Algorithm

The empirical correlations for determination of U for non-foaming flow by using values of p1 to p31, presented in Table 2.1, and based on Gudmundson's model [G.8] are described in an algorithm form below :

Step - 1 : Determine physical properties μ_m, C_p, ρ , k based on liquor concentration and temperature

Step - 2 : Determine liquor feed rate per unit cross sectional area, FD, kg/sm^2

Step - 3 : Determine the value of q, W/m^2 based on evaporation rate from a body

Step - 4 : Determine the values of the correction factors C_μ and C_T

For $0 < \mu_m < 15$

$$C_\mu = 1.0 - 0.07858 \times \mu_m + 0.002735 \times (\mu_m)^2 - 0.1092 \times 10^{-4} \times (\mu_m)^3 \quad \text{....(2.35)}$$

and for $\mu_m > 15$

$$C_\mu = 1.576 - 0.095 \times \log \mu_m \quad \text{....(2.36)}$$

$$C_T = -1.02 + 0.0476 \times T_{boil} - 0.364 \times 10^{-3} \times (T_{boil})^2 + 0.946 \times 10^{-6} \times (T_{boil})^3 \qquad \ldots.(2.37)$$

Step - 5 : Determine the value of T_{max}

$$T_{max} = 15.0 + [0.12 - 0.001 \times T_{boil}] \times FD + [3.0 - 0.025 \times T_{boil}] \times \log \mu_m - 0.12 T_{boil} \qquad \ldots.(2.38)$$

Step - 6 : Compute Bound 1 and Bound 2

$$\text{Bound } 1 = 19500 + 2000 \log \mu_m - 95 T_{boil} \qquad \ldots.(2.39)$$

if $\mu_m < 1.0$, put $\mu_m = 1.0$ in the equation of Bound 1.

$$\text{Bound } 2 = 4300 + 100 \log \mu_m - 14.0 T_{boil} \qquad \ldots.(2.40)$$

if $\mu_m < 1$ put $\mu_m = 1$ in the equation of Bound 2

Step - 7 : Determine the value of T_{sub}

$$T_{sub} = T_{max} - T_{in} \qquad \ldots.(2.41)$$

Step - 8 : Determine the value of C_F and C

$$Z = 1.0 + 0.01 \times \operatorname{Tanh}[(q - \text{Bound } 1)/21800] \times (T_{sub})^{0.4} \qquad \ldots.(2.42)$$

if $T_{sub} < 0$, put $T_{sub} = 0$ in the calculation of Z

$$C_F = Z + [-0.0515 + 0.0505 \times Z] \times FD \qquad \ldots.(2.43)$$

$$C = C_\mu \times C_T \times C_F$$

Step - 9 : If T_{sub} is negative, calculate T_{sup} and the subsequent increase in q, Δq and correct the q calculated in Step-3 by making $q = q + \Delta q$

Superheated liquid feed contributes to a virtual increase in heat flux :

$$T_{sup} = T_{in} - T_{max} \quad \text{....(2.44)}$$

$$\Delta q = T_{sup} \times C_p \times FD \times 0.00181 \times 0.61/1.321 \quad \text{....(2.45)}$$

Step - 10 : Check whether q lies in the range Bound 2 $\le q \le$ 20,000. If q is greater than 20,000 take q = 20,000, calculate the overall heat transfer coefficient

For Bound2 $\le q \le 20000$ W/m^2

$$U = 0.981 x C^{1.0} [0.2334 x q - 0.1006 x 10^{-4} x q^2 + 0.1362 x 10^{-9} x q^3] \quad \text{....(2.46)}$$

and for q > 20000, U =U(at q=20000) ....(2.47)

Step - 11 : If q is less than Bound 2, calculate the overall heat transfer coefficient as given below :

$$C_{sub} = 1.0 + 0.04 \times T_{sub} \times [\text{Bound } 2 - q)/\text{Bound } 2]^2 \quad \text{....(2.48)}$$

if $T_{sub} < 0$, put $T_{sub} = 0$ in this equation.

$$U = [h + (U_{bound} - h) \times q/ \text{Bound } 2] \times C_{sub} \quad \text{....(2.49)}$$

Here h and U_{bound} are calculated by Eqs.(2.31), and (2.33), respectively.

CHAPTER - 3

EVAPORATION OF WEAK KRAFT BLACK LIQUOR

Before arriving at a methodology for the reduction in steam energy consumption to concentrate kraft black liquor in multiple effect evaporators, it is essential to describe some related salient aspects of the kraft process, kraft black liquor and its constituents, multiple effect evaporator, liquor flow sequences in evaporators, and, ranges of operating parameters for evaporators.

3.1 DESCRIPTION OF THE KRAFT PROCESS

The kraft process used for pulping the wood species, according to Arhippainen et al. [A.1], Casey [C.3], Grace, and Malcolm [G.5], is currently the dominant chemical pulping process. In this process, a mixture of sodium sulphide and sodium hydroxide is used to make `white liquor'. Sodium sulphide in the cooking liquor accelerates delignification. In fact, the kraft process can be advantageously employed for pulping of all types of wood species, especially for which conventional sulphite process does not suit. Further, the chips are exposed to the hot alkali for a shorter period of time than in the soda process and consequently this produces a pulp of considerably superior strength.

Figure 3.1 provides a brief but overall view of the various equipment of the pulping and recovery sections in a kraft mill.

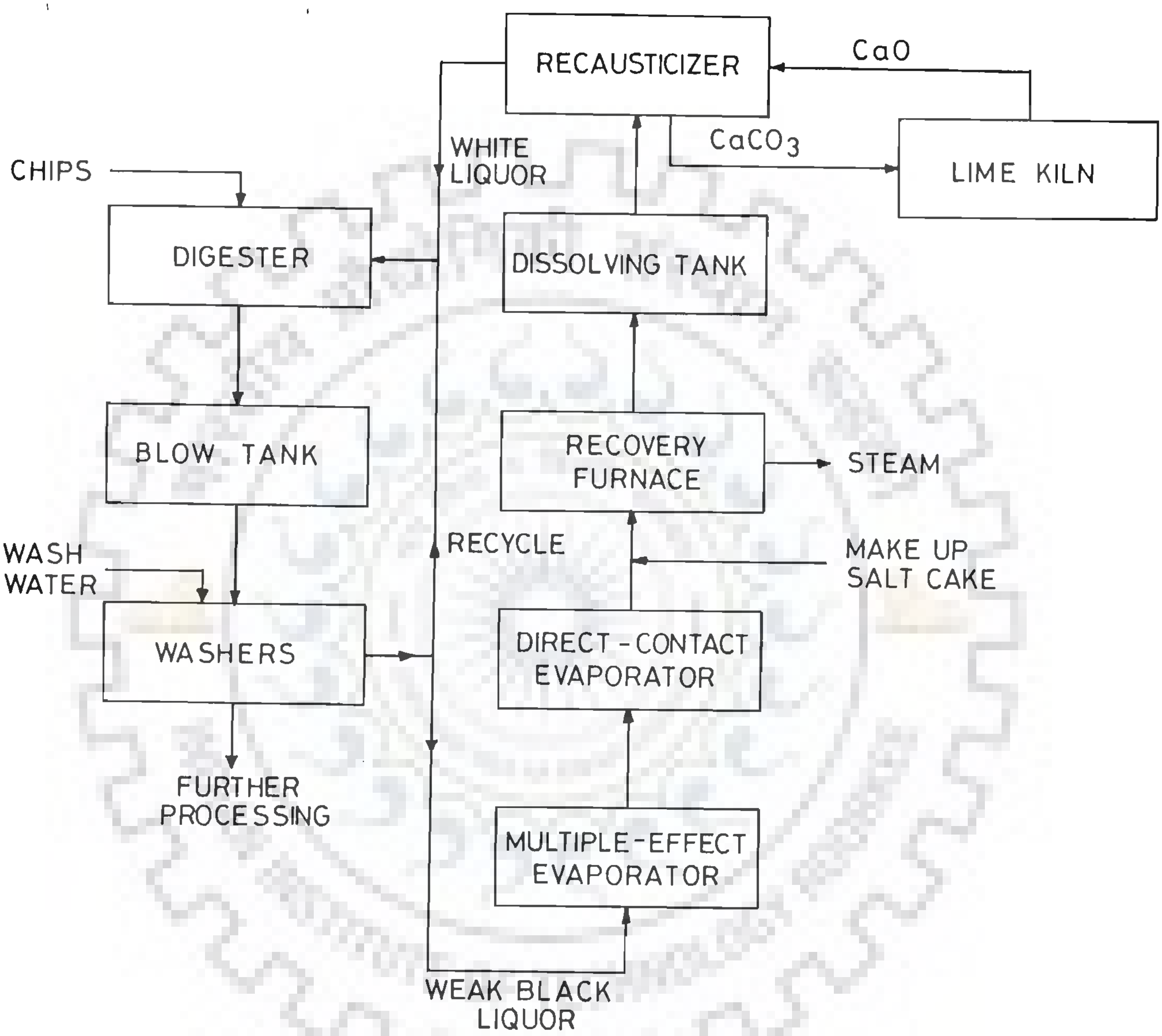

FIG. 3.1 SCHEMATIC FLOW DIAGRAM OF THE KRAFT PROCESS

It essentially consists of a digester, to which wood chips and cooking liquor are charged in such proportions that the chemical to wood ratio is maintained at the required level. The digester is maintained in the temperature range of 160 to 180°C by means of either direct or indirect steam-heating. Cooked chips are discharged to a blow -tank alongwith the spent liquor, also known as black liquor, and then to washers, where they are separated. The wood pulp is sent for its further processing to obtain bleached/unbleached paper. Now a part of the black liquor is recycled to the digester and the remainder is passed on to the chemical recovery section consisting of multiple effect evaporator, direct contact evaporator, recovery furnace, dissolving tank and recausticizer (for converting sodium carbonate into sodium hydroxide), coupled with lime kiln for the regeneration of calcium oxide. The first step in the recovery section is concentration of the weak kraft black liquor to roughly 60 to 64 per cent solids in evaporator house. To achieve this goal, multiple effect evaporator using steam, followed by a direct contact evaporator, employing the furnace flue gases are used. This concentrated black liquor is termed as `strong black liquor'. This liquor is introduced to recovery furnace, where the organic matter present in the black liquor undergoes combustion and thereby raises steam to be employed in the mill itself for various uses including that for evaporation. As regards the inorganic matter in the black liquor, it is discharged in a molten state called `smelt'. This, in turn, is dissolved in water to produce a solution called `green liquor'.

The green liquor is treated with lime to obtain white liquor which is used in digester. The calcium carbonate formed there is calcined to get lime for causticization of green liquor.

Before describing the multiple effect evaporator of the evaporator house, it is worthwhile to discuss some related aspects of kraft black liquor.

3.2 WEAK KRAFT BLACK LIQUOR AND ITS CONSTITUENTS

Koorse et al. [K.6, K.7] have presented details of the composition of black liquor from the kraft process which, it is evident , is exceedingly complex. It is a mixture of organic and inorganic chemicals. The proportion of organic compounds [K.6] in the liquor ranges from 50 to 70 per cent. It depends upon the chemical composition of the species used for pulping, in addition to digester operating conditions . The alkali charge, liquor to wood ratio, pulp yield, chip moisture content and water added during pulp washing control the concentration of dissolved solids in black liquor. The usual concentration of dissolved solids in weak liquor lies between 12-20 per cent, whereas its pH value generally ranges between 10.5 and 13.5 [K.7]. Table 3.1, reproduced from Koorse paper [K.6], shows the chemicals in a typical Indian black liquor :

TABLE 3.1

WEAK KRAFT BLACK LIQUOR CONSTITUENTS

	Organic Compounds	
1.	Alkali lignin and thiolignin	
2.	Isosaccharinic acid	
3.	Low molecular weight polysaccharides	
4.	Resin and fatty acid soaps	
5.	Sugars	
	Inorganic Compounds,	**gpl**
1.	Sodium hydroxide	4-8
2.	Sodium sulphide	6-12
3.	Sodium carbonate	6-15
4.	Sodium thiosulphate	1-2
5.	Sodium polysulphides	small
6.	Sodium sulphate	0.5-1
7.	Elemental sulphur	small
8.	Sodium sulphite	small

The available data indicate that the multiple effect evaporators consume a sizeable portion of the total steam requirements of the mill. This, in fact, ranges from 24 to 30 per cent thereby indicating that the evaporation process in the mill is highly energy-intensive. Before a methodology for the reduction in steam energy consumption in the evaporator is attempted, a review of the multiple effect evaporators is quite in order.

The application of multiple effect evaporators for kraft black liquor in pulp and paper industry has been extensively studied for its development and proper engineering process design for economic operation and reviewed by several workers,viz., Britt [B.7]. Casey [C.3], Coulson et al. [C.12], Davis [D.1], Grace et al. [G.5], Harrison [H.3], Logsdon [L.7], McCabe et al. [M.2], Perry et al. [P.3], Reid and Sherwood [R.8], Veeramani [V.2, V.3], Venkatesh and Nguyen [V.4], Walker et al. [W.1], Wennberg [W.3] and Whitney [W.5]. Typical studies include the one , from the viewpoint of developing new operational strategies for controlling the scaling problem,by Harrison [H.3]; study of the problems during evaporation and burning of the black liquors by Kulkarni et al. [K.8] and therefore planning new strategies; analysis of the improvements and recent trends based on energy conservation and pollution control viewpoints , by Rao and Ram [R.2] and Sittig [S.4], and trouble-shooting by Ulrich [U.1].

3.3 MULTIPLE EFFECT EVAPORATORS

A review of the evaporation plants by Arhippainen [A.1], Britt [B.7], Gundersby [G.9], Hough [H.11], Rao et al.[R.2], Rastogi et al. [R.3], for concentrating the weak kraft black liquor in India, U.S.A. and Scandinavia shows that there exists a variation in number of effects and the liquor flow sequences.

In U.S.A. the mills employ sextuple effect evaporators, whereas use of quintuple effect evaporators is the practice in

Scandinavia.Contrary to the above, Indian mills employ quadruple/quintuple effect evaporators. However, the recent trend in India is to adopt the sextuple effect evaporators.

As regards the sequences, they can be either forward (co-current) or backward (counter-current).

It will not be out of place to emphasize that due to typical physico-thermal properties and the behaviour of organic and inorganic constituents of the black liquor as a consequence of the changes in temperature and solids concentration of the liquor during evaporation, the possibility of forward (co-current) feed sequence is ruled out. Hence, the backward (counter-current) feed sequence is being employed in the mills world over. It is equally important to point out that splitting of the feed amongst the last two effects is helpful in reducing the foaming tendency of the black liquor in the evaporators. Yet another feed sequence exists where the feed is introduced to some intermediate effect and then liquor flow is co-current, followed by the counter-current one for the remaining effects. Splitting of steam leads to higher steam economy of the multiple effect evaporators. Therefore, it is also practised in some of the mills.

Keeping the above facts in view alongwith the mill practices in North America and Scandinavia, it is desirable to consider various potential liquor flow sequences in evaporators in order to establish the one which yields the highest possible evaporation capacity per unit area of heating surface,together with an enhanced steam energy efficiency. In fact, the number of effects and their heating surface area, and steam/liquor flow sequences are important in determining energy usage and process economics of a particular multiple effect evaporator configuration. Further, it is important to point out that in USA, the evaporator houses generally employ a 5- 6- 4- 3- 2- 1 liquor flow sequence in sextuple effect evaporators, whereas in Scandinavia, most plants operate with `intermediate' feeding sequence, with weak black liquor fed to the third effect and steam split in the first effect. This sequence can be denoted as 3- 4- 5- 1A- 1B- 2. As regards the Indian evaporator houses, Rao et al. [R.2] have pointed out that these generally employ different number of effects and sequences. However, the recent design practice shows a shift towards the one used in North America.

3.4 LIQUOR FLOW SEQUENCES IN EVAPORATORS

As stated above, there can be several ways in which liquor flows from effect to effect in evaporators. As regards the effects, they are numbered according to the flow of vapour/steam through the set-up. The first effect is the one which receives live steam or the one operating at the highest pressure and in the subsequent effects the pressure decreases progressively. Thus the last effect is at the lowest pressure/vacuum. For improving energy efficiency, vapours emerging out of the first effect are used as heating medium in the second effect and so on. In fact, this is the only possible way to employ vapours from different effects for the most effective heating in multiple effect evaporators.

However, the liquor flow sequence can be anything from forward (co-current) to backward (counter-current). It has been recommended that the liquor flow sequence may be represented by indicating the bodies in the direction of increasing solids content.

To understand the above without any ambiguity, some of the liquor flow sequences for the sextuple effect evaporators are shown in Figures 3.2 through 3.5 and are also presented in Table 3.2. In the sequence **a** (Figure 3.2 (a)), the liquor feed enters the 6th effect and from there, the liquor is sent to the 5th effect and so on. As regards sequence **b** (Figure 3.2(b)), it actually represents an equal split of the feed before it enters the 5th and the 6th effects. Similarly for sequence **d** (Figure 3.3(a)), the feed is introduced into the 5th effect, and consequently passes through the 6th effect, 4th effect and so on. It would be important to mention that for sequence **c**(Figure 3.5 (a)), bodies 1A and 1B indicate that the first effect has been split into two bodies and the live steam enters into them separately in different quantities depending on their requirements.

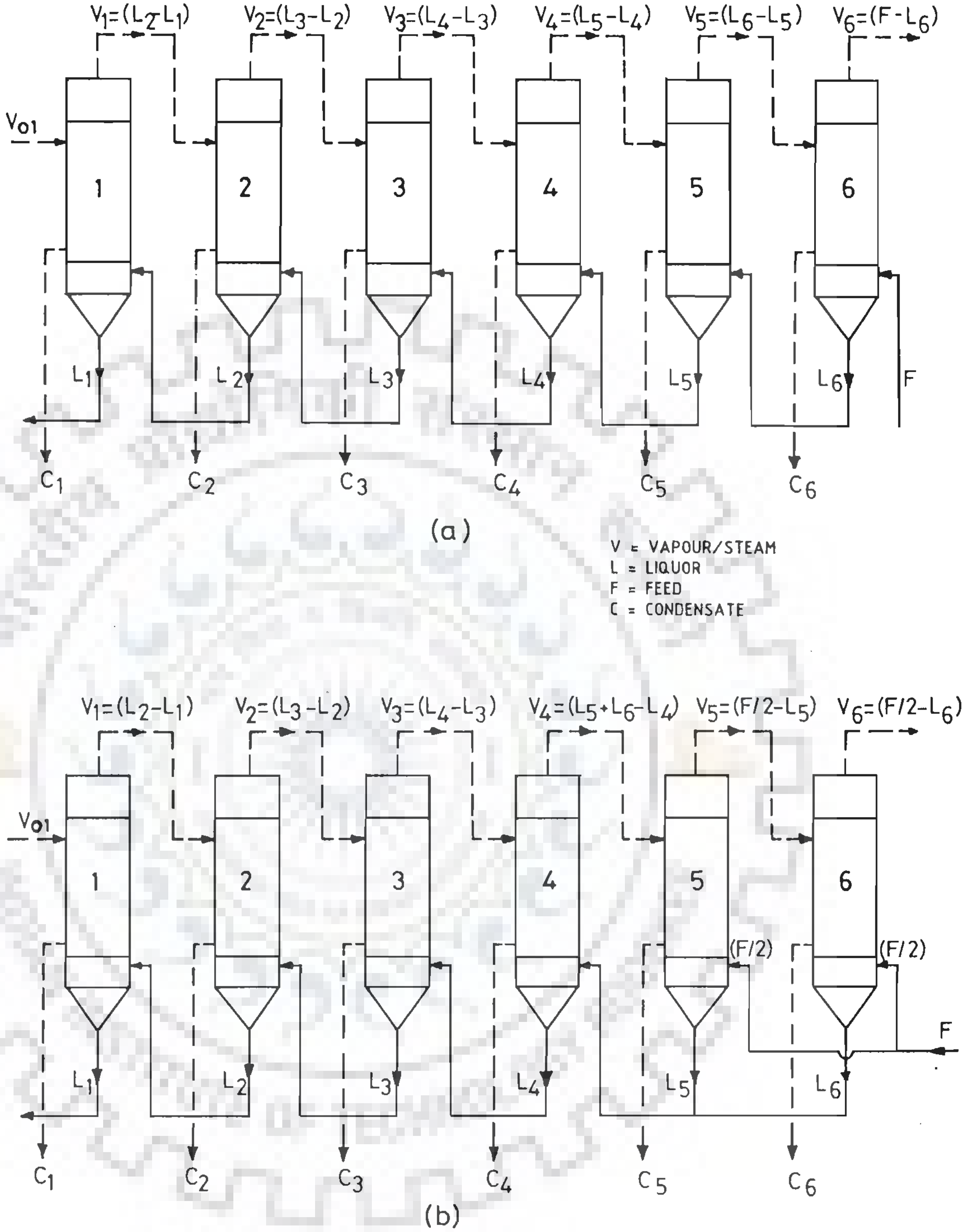

FIG.3.2(a)SEXTUPLE EFFECT EVAPORATOR WITH BACKWARD FEED
(b)SEXTUPLE EFFECT EVAPORATOR WITH BACKWARD FEED
(WITH FEED SPLIT AMONG 5TH & 6TH EFFECTS)

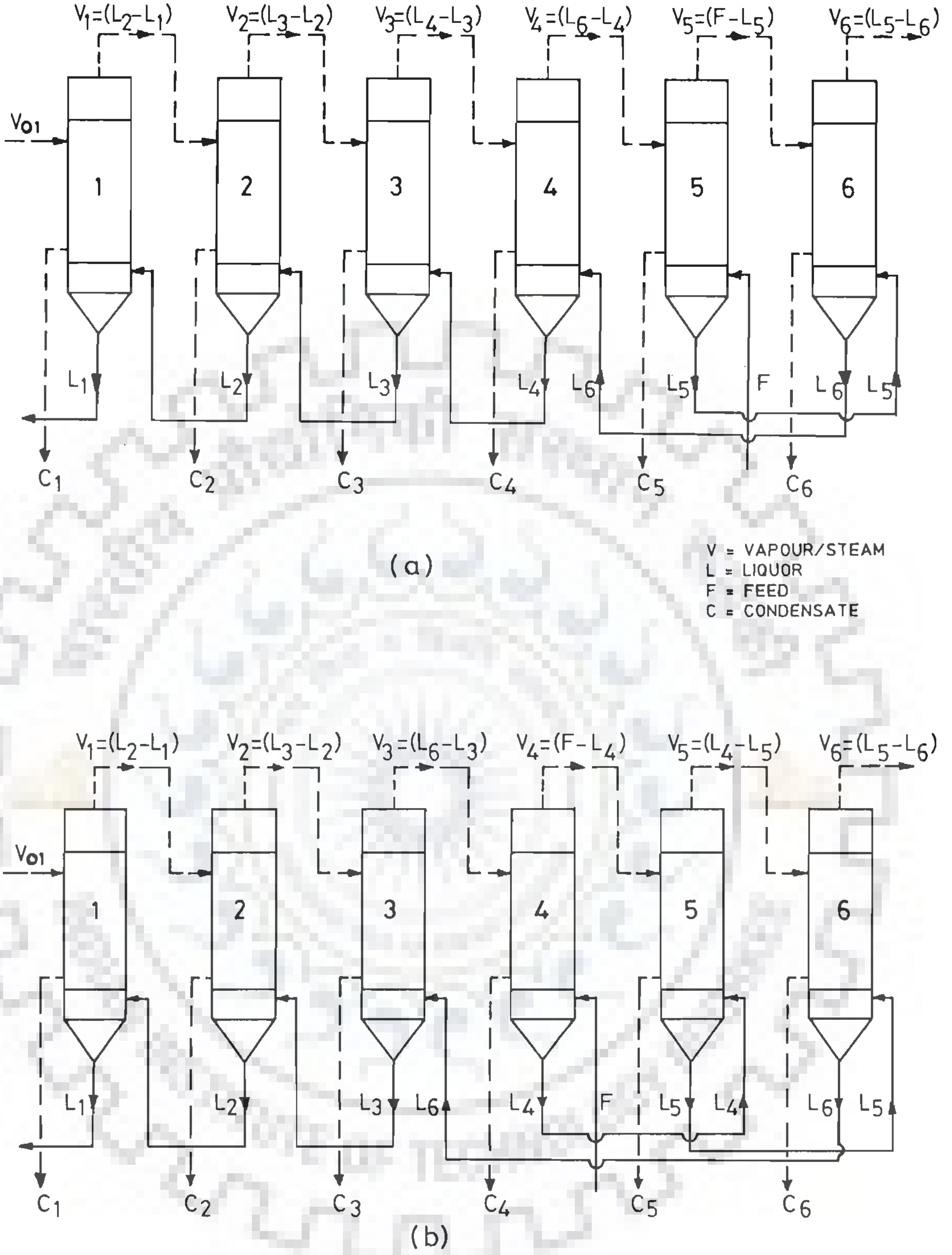

FIG. 3.3 (a) SEXTUPLE EFFECT EVAPORATOR WITH SEQUENCE 5→6→4→3→2→1

(b) SEXTUPLE EFFECT EVAPORATOR WITH SEQUENCE 4→5→6→3→2→1

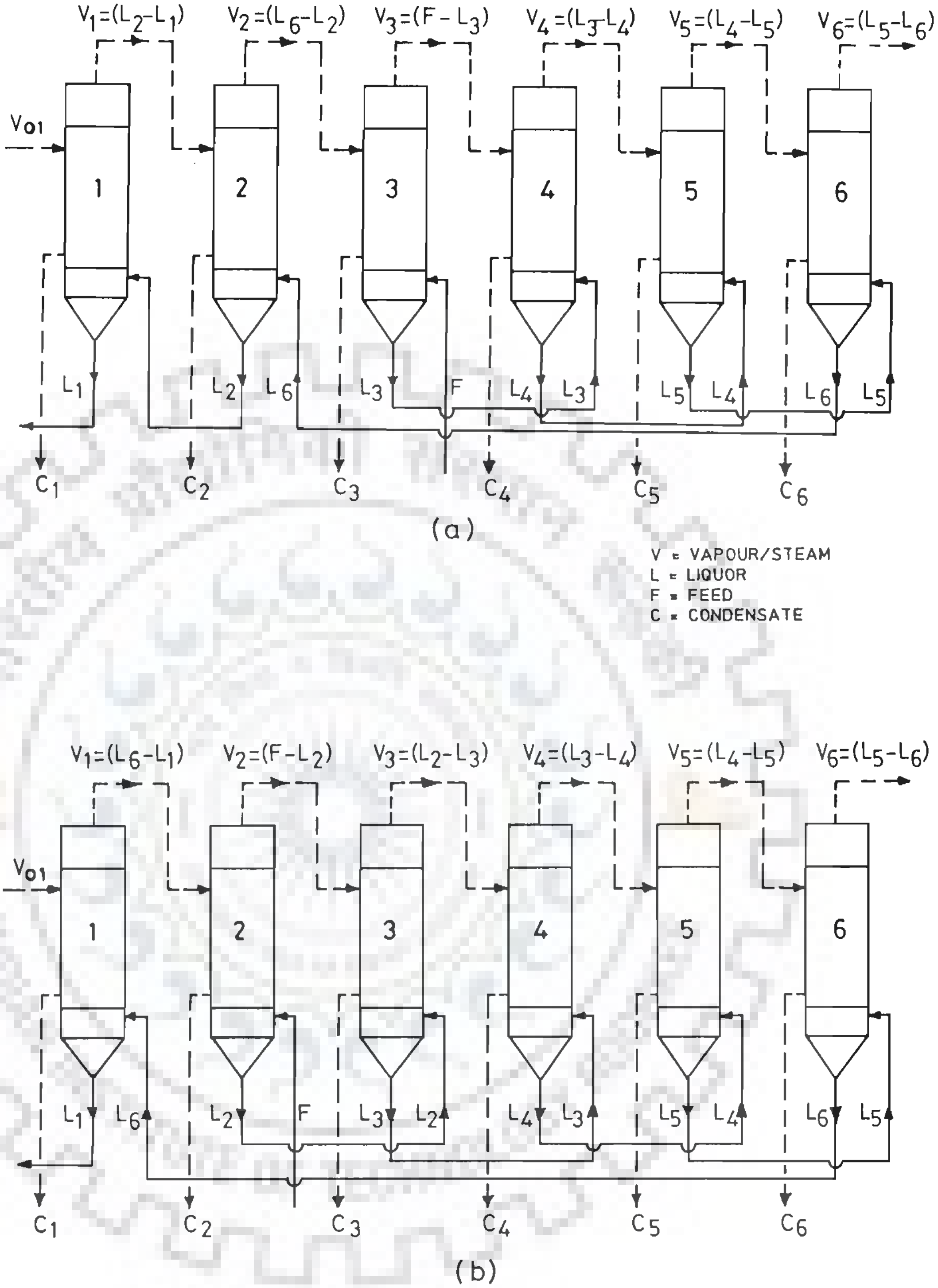

FIG. 3·4 (a) SEXTUPLE EFFECT EVAPORATOR WITH SEQUENCE 3→4→5→6→2→1
(b) SEXTUPLE EFFECT EVAPORATOR WITH SEQUENCE 2→3→4→5→6→1

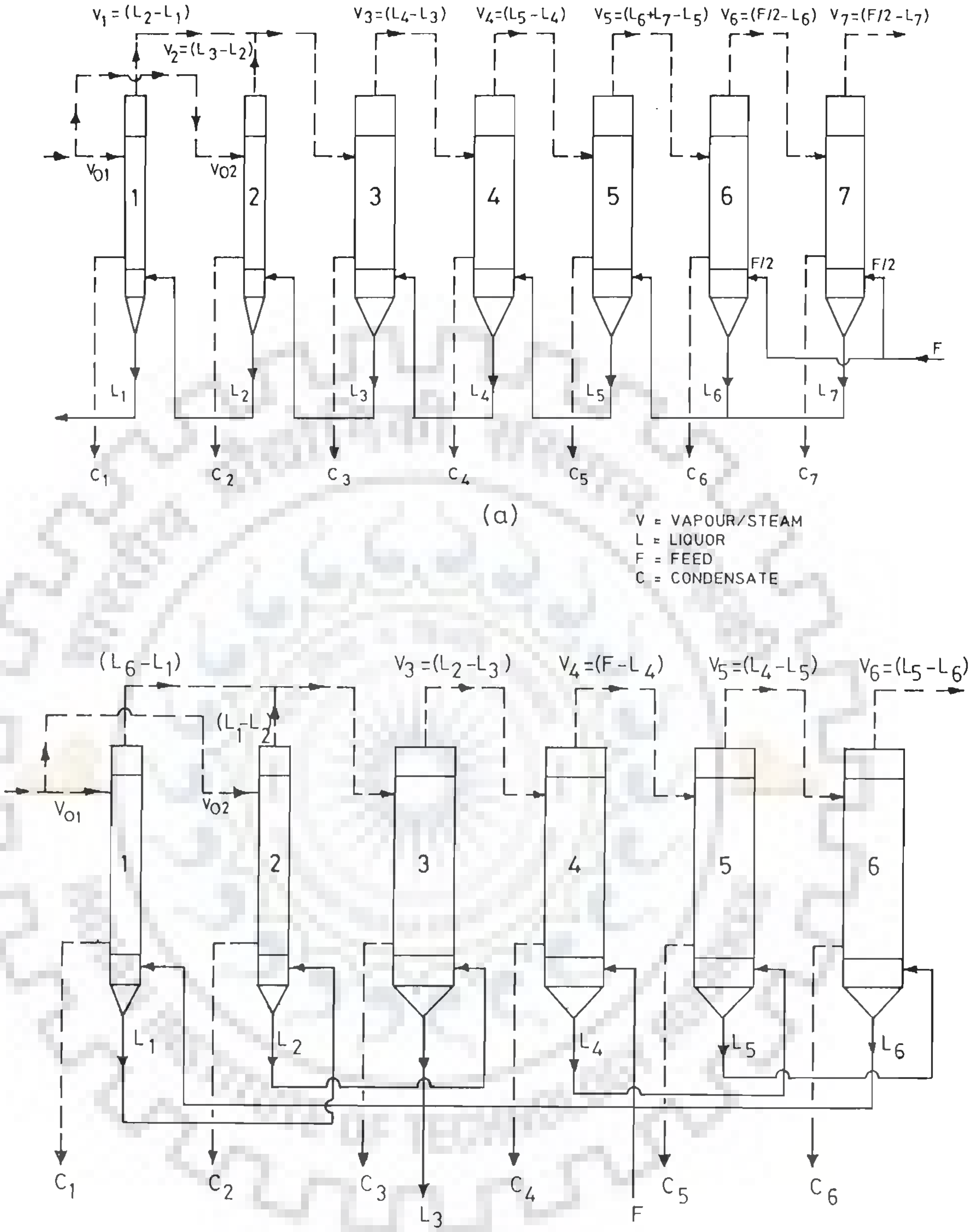

FIG.3·5 (a) SEXTUPLE EFFECT EVAPORATOR WITH SPLIT OF FEED AND STEAM FOR THE SEQUENCE $\genfrac{}{}{0pt}{}{5}{6}$>4-3-2-IB-IA
FOR CONVENIENCE IT HAS BEEN SHOWN AS $\genfrac{}{}{0pt}{}{6}{7}$>5-4-3-2-1

(b) SCANDINAVIAN SEQUENCE 3-4-5-IA-IB-2 FOR CONVENIENCE IT HAS BEEN SHOWN AS 4-5-6-1-2-3

TABLE 3.2

LIQUOR FLOW SEQUENCES IN SEXTUPLE EFFECT EVAPORATOR

Sequence		Figure	Remarks
a	6- 5- 4- 3- 2- 1	3.2 (a)	Backward, feed to 6th effect
b	5 >4- 3- 2- 1 6	3.2 (b)	Backward,feed split among 5th and 6th effects
c	5 >4- 3- 2- 1B- 1A 6	3.5 (a)	Backward,feed split among 5th and 6th effects and steam split in 1st effect
d	5- 6- 4- 3- 2- 1	3.3 (a)	Mixed, feed to 5th effect
e	4- 5- 6- 3- 2- 1	3.3 (b)	Mixed, feed to 4th effect
f	3- 4 -5- 6- 2- 1	3.4 (a)	Mixed, feed to 3rd effect
g	2- 3- 4- 5- 6- 1	3.4 (b)	Mixed, feed to 2nd effect
Scan	Scandinavian Sequence 3- 4- 5- 1A-1B- 2	3.5 (b)	Mixed feed, Quintuple effect, Steam split in first effect

3.5 RANGES OF OPERATING PARAMETERS IN EVAPORATORS

Over the period of years of operation of evaporator plants, it has been possible to know the ranges of operating parameters in Indian mills. They are presented in Table.3.3.

TABLE 3.3

OPERATING PARAMETERS IN EVAPORATORS

Sl. No.	Parameter	Range	Interval
1.	Liquor feed temperature, °C	50-90	10
2.	Liquor feed concentration, mass fraction solids	0.12-0.20	0.02
3.	Steam Temperature, °C	110-140	10
4.	Vacuum in the last effect, mm Hg (Saturation temperature, °C)	600-650 (50-60)	25 5
5.	Liquor feed rate, kg/s	18-42	8

The above values shall be used for the computation of steam consumption, steam economy and heating area of evaporators in the present work.

CHAPTER - 4

A MODEL FOR OPTIMUM NUMBER OF EFFECTS OF A BACKWARD FEED EVAPORATOR

For concentrating weak kraft black liquor, the multiple effect evaporators are universally used. With regard to liquor flow sequence, backward feed is desirable due to the typical problems associated with the viscosity of concentrated liquor. However, for better steam energy efficiency, the liquor feed is split evenly and fed to the last two effects of the evaporator. For the same reason the evaporators with split steam have been in operation.

It is equally important to emphasize that there exist different practices as to the number of effects in a multiple effect evaporator employed for concentrating kraft black liquor. For example, the American and Scandinavian mills use sextuple and quintuple effect evaporators, respectively. Hence, it is important to determine the optimum number of effects for a given set of operating conditions for a cost conscious design.

As already indicated in Chapter 3, multiple effect evaporators are employed to achieve repeated use of the energy present as latent heat in the vapour. For this, the vapours coming from one effect serve as heating medium to the next and so on, thereby the steam economy increases with the number of effects. This has prompted use of more and more number of effects in a given evaporator set-up. But there is a limit to the increase in the number of effects. This is governed by the fact that there is a point of diminishing return at higher number of effects. Though the steam economy improves with the number of effects, the other associated cost elements like capital cost of the evaporator, cost of repair and maintenance, and cost of cleaning etc. increase with increase in number of effects. Thus, for any cost optimization of evaporators, it is necessary to express all the cost elements in terms of number of effects. The cost equation thus developed is solved graphically for optimum number of effects.

Several techno-economic studies including analysis of cost structure, improvement of design and reduction of heating surface area requirements of evaporator houses of pulp and paper mills have been reported in literature. Thus, Britt [B.7], Casey [C.3], Chilton [C.4], Coulson et al. [C.12], Schweyer [S.3], Davis [D.1], Peters and Timmerhaus [P.4], Ulrich [U.1] have reviewed cost-analysis. Bonilla [B.6] has studied design of multiple effect evaporator for minimum area costs, Caldwell et al. [C.1] have analysed cost factor in evaporator design; Coston et al. [C.11] have reviewed solution and application of evaporator equipment; Ghosh et al. [G.1, G.2] have studied decision on optimum number of effects; Leonard [L.5] has studied maintenance costs; Newton and Aries [N.6] have presented preliminary estimation of operating costs; Farin [F.1] has studied low cost evaporation methods saving energy by reusing heat energy; Rosenblad [R.10] has described three evaporator flow diagrams that adopt to energy cost and emission requirements. Reinhold and Connelly [R.9] have discussed how to get the optimum number of effects in multiple effect evaporators. Kohlins et al. [K.4] analysed cost factor in evaporator design, and Kerr [K.2] the capacity and economy of multiple effect evaporators.

From experience, it is established that the contributing cost elements are capital cost of evaporator (C_f), repair and maintenance cost (C_{r-m}), cleaning cost (C_c), steam cost (C_s), operating cost due to condensing water, air handling and spray cooling of water (C_{c-w}), and labour and supervision cost (C_l). It has been found that the fixed cost of condensers and air handling pumps are negligible in comparison with other cost elements and hence are neglected in this cost analysis. Another factor that leads to loss in evaporation capacity or loss of heating area is boiling point rise, BPR. The influence of BPR has been considered during development of model. The value of quantity `e', which appears in various cost equations, is determined from the model and hence accounts for the effect of BPR. As such, effect of BPR is not included separately in the set of cost equations.

In view of the above facts, the annual total cost of evaporation of black liquor in a multiple effect evaporator can be written as follows :

$$C_T = C_f + C_{r-m} + C_c + C_s + C_{c-w} + C_l \qquad ....(4.1)$$

In the following sections, the relationships for the various cost elements in terms of number of effects of evaporators have been developed.

4.1 ANNUAL FIXED COST OF EVAPORATOR

Increasing the number of effects results in an increase in the capital cost of the evaporator. To obtain an expression for the capital cost of a multiple effect evaporator, let E_T represent total amount of water evaporated per day in kg and 'e' the amount of vapour generated in single effect in kg per hour per unit surface area. If equal surface areas for all effects are considered, then the surface area for each effect is given by $(E_T/24e)$. If the cost of evaporator including its installation and interest on capital cost is c_e (Rs/m^2), then the cost of each effect shall be $c_e\ (E_T/24e)$. In the present study, stainless steel has been chosen as the material of construction of the evaporator.

Since there are 'n' effects in the evaporator and they are of the same shape and size, the capital cost of multiple effect evaporator, C_{me}, will be somewhat lesser than 'n' times that of the single effect. It is logical to consider it as $(n)^b$ times the cost of the single effect, which therefore, can be represented by the following relationship :

$$C_{me} = c_e\ (E_T/24e)\ n^b \qquad \ldots (4.2)$$

where, the exponent 'b' is recommended to be 0.98, according to Ray et al. [R.4]. Let the annual depreciation on the fixed cost of evaporator be a fraction of its capital cost represented by 'F'. Hence the annual fixed cost of the evaporator C_f is as follows :

$$C_f = c_e\ (E_T/24e)\ n^b\ F \qquad \ldots\ (4.3)$$

4.2 ANNUAL REPAIR AND MAINTENANCE COST

In the absence of any procedure to calculate this cost element, it is expressed as some fraction of capital cost of evaporator. This, in fact, has been suggested by Tyler [T.1], Aries and Newton [A.2], and Leonard [L.5]. Hence, it can be calculated by the following relationship :

$$C_{r-m} = c_e\ (E_T/24e)\ n^b\ f \qquad \ldots\ (4.4)$$

4.3 ANNUAL CLEANING COST

It depends upon the scaling tendency of the liquor in question. The frequency of cleaning, the cleaning time and boiling time are also dependent on the nature of the liquor. Scale formation is not uniform in all the bodies of the evaporator. It changes with the concentration of liquor. With the backward flow sequence, the scale formation is much more in the first two bodies and comparatively much less in the subsequent bodies in progressively decreasing order. Hence the costs of cleaning of first two bodies are to be separated from that of the remaining bodies. As regards these costs, they can be found out through experience and judgement. A fair and generally adopted relationship between C_c and 'n' [R.4] is as follows :

$$C_c = [(n-2) + 2/K]\ (D/R)\ (E_T K\ c_c/24e) \qquad \ldots\ (4.5)$$

4.4 ANNUAL STEAM COST

The major component of the operating charges is due to steam cost. It depends upon feed temperature, its concentration, the liquor flow sequence, the pressure of steam, and the degree of flashing of the condensate. However, the major factor which determines the steam cost is the number of effects in the evaporator. In fact, the number of effects increases the steam economy.

The annual steam cost can then be written as follows :

$$C_s = [E_T/(K_1\ (n)^m)]\ D\ .\ c_s \qquad \ldots .(4.6)$$

where, K_1 and `m' are constants whose values have been determined as 1.062 and 0.813, respectively in the present investigation.

4.5 ANNUAL OPERATING COST DUE TO CONDENSING WATER, AIR HANDLING AND SPRAY COOLING WATER

To calculate this cost, one should consider costs due to water for condensing, spray cooling and power requirement for air and water handling and the replacement cum make-up water.

As regards the cooling water, it is also proportional to the amount of steam used for evaporation. The quantity of water is also influenced by the vacuum in the last body of the evaporator and the type of condenser. In the industry, where spray ponds are used, make-up and blow-down demand of water may be computed as a fixed fraction of the pond holding capacity.

Based on the above facts, Ray et al. [R.4] have developed a relationship for the calculation of the annual operating cost due to condensing water, air handling and spray cooling water, as given below :

$$C_{c-w} = \frac{E_T \; D \; W}{(n)^m} \left(c_{pi} + c_{pa} + c_{pc} + r \; c_w\right) \cdot (Y)^n \qquad \ldots.(4.7)$$

where, Y is the fraction vapour produced in an effect/vapour fed to the same effect [U.1].

4.6 ANNUAL LABOUR AND SUPERVISION COST

For this, Ulrich [U.1] has recommended that 0.3 operator per evaporator body per shift is needed. One spare operator is provided over the three shifts for every three operators. Thus the total number of operators is n . (0.3 x 4). The annual labour cost comes out n . (0.3 x 4) . c_l, where c_l represents labour cost in terms of Rs/operator. He has also suggested the cost of supervision as 15% of labour cost. Thus the total annual cost of labour and supervision is as follows :

$$C_l = (1 + 0.15) \cdot n \cdot (0.3 \times 4) \; c_l \qquad \ldots (4.8)$$

Values of various cost terms and constants used in Eqs. (4.2 through 4.8) are presented in Table 4.1.

TABLE 4.1

VALUES OF COST TERMS AND CONSTANTS USED IN EQS. (4.2 THROUGH 4.8)

c_e	: Rs. 15000/- per sq.m	c_c	: Rs. 9/sq.m
b	: 0.98	r	: 0.1
F	: 0.45	f	: 0.05
D	: 330 days/yr.	K_1	: 1.062
m	: 0.813	W	: 50 kg/kg
K	: 0.15	R	: 15
Y	: 0.85	c_{pi}	: Rs. 3.55E-05/kg of water
c_{pa}	: Rs. 1.63 E-05/kg of water	c_s	: Rs. 0.25/kg
c_{pc}	: Rs. 3.42 E-05/kg of water	c_w	: Rs. 4.0 E-04/kg
c_l	: Rs.12000/year	e	: 115 kg/m^2hr

4.7 A REMARK ON THE PRESENT TOTAL COST MODEL

Eqs. (4.2 through 4.8) developed in the present work can be used to calculate the respective cost elements in terms of number of effects. It can be convincingly stated that the absolute values of cost elements stated above will depend upon the liquor flow rate, composition of liquor from individual effects, steam temperature, pressure in the last effect, and water evaporated from individual effects. Therefore, it is important to specify the values of these parameters before one uses the total cost model, i.e., Eq. (4.1) to determine optimum number of effects. The normally adopted values of operating parameters including the above quantities in Indian mills are given in Table 4.2.

TABLE 4.2

OPERATING PARAMETERS

Sl. No.	Parameter	Value
1.	Liquor feed, kg/s	18.144
2.	Liquor feed composition	0.15
3.	Liquor feed temperature, °C	70.00
4.	Final liquor composition	0.45
5.	Steam temperature, °C	138.00
6.	Last effect temperature, °C	51.00
7.	Water evaporated from each effect, kg/hr m²	115.00
8	Heat transfer area, m²	379.00

From the data presented in Table 4.2, one can calculate E_T. This comes out to be equal to 1,045,095 kg/day. Now using the values of various quantities from Table 4.1 in Eqs. (4.2 through 4.8) the annual cost of individual elements are determined and the sum of these as a function of the number of effects, n, is given in Table 4.3.

Based on the data of Table 4.3, a plot has been drawn between annual total cost, C_T and number of effects, n, as shown in Figure 4.1. From the shape of the curve, it is clearly noted that the curve is steeper on the left hand side than on the right hand side and presents an unimodal nature. Hence, the graphical solution to determine optimum number of effects is an appropriate choice.

TABLE 4.3

ANNUAL TOTAL COST, C_T OF A BACKWARD FEED EVAPORATOR

Sl. No.	n	C_T (Rs./Year)
1.	3	4.24×10^7
2.	4	3.80×10^7
3.	5	3.63×10^7
4.	6	3.58×10^7
5.	7	3.63×10^7
6.	8	3.72×10^7
7.	9	3.86×10^7

The plot reveals that the optimum number of effects are a little less than six. Hence, for the concentration of the weak kraft black liquor, the use of sextuple effect evaporator as used in American mills seems to be justified.

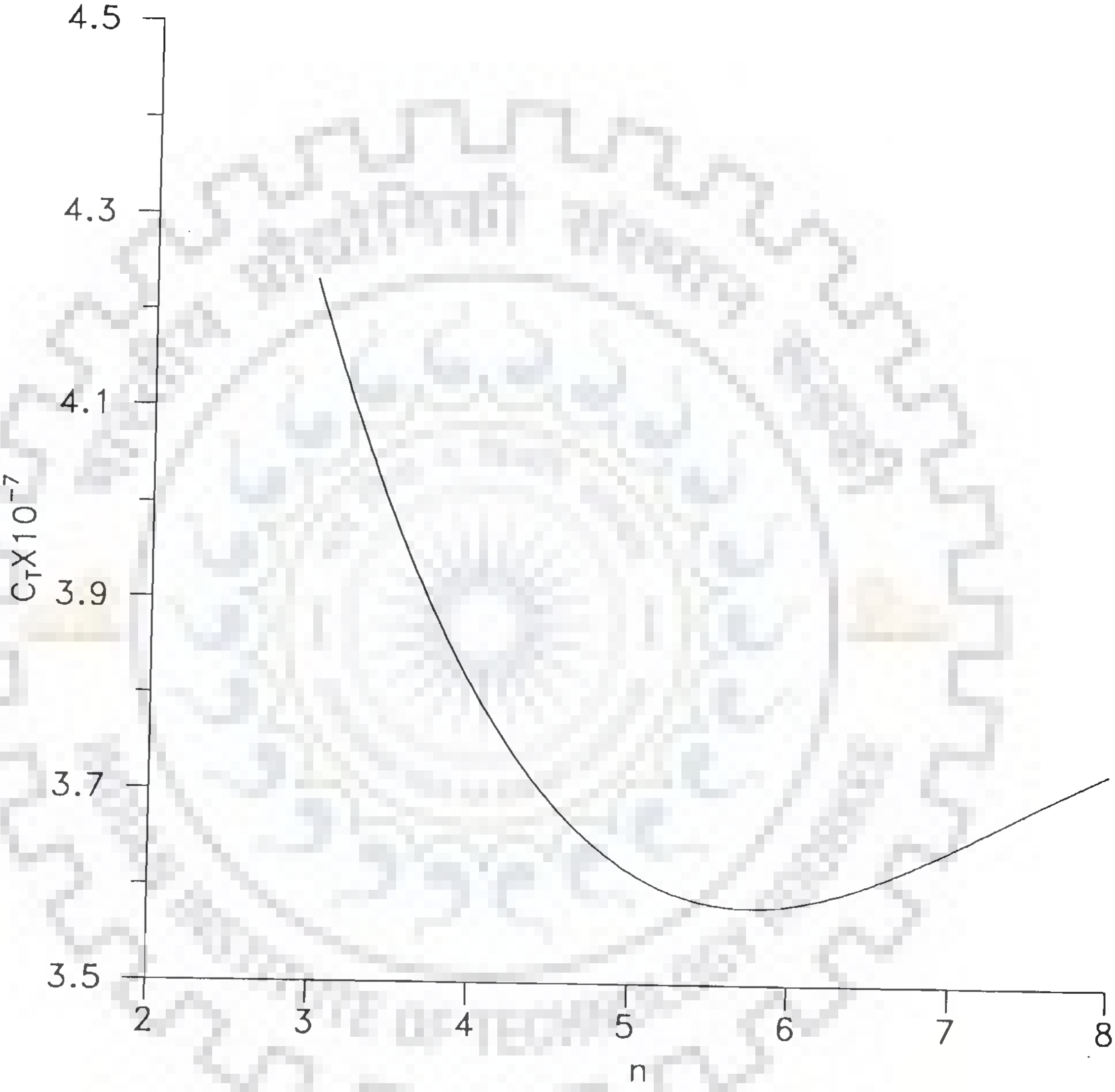

Fig. 4.1 Total annual cost vs number of effects

CHAPTER - 5

PROBLEM FORMULATION AND MATHEMATICAL MODELLING

In Chapter 4, it has been concluded that for concentrating weak kraft black liquor the sextuple effect evaporators are the most economical. Hence, the mathematical models related to various liquor flow sequences have been developed for sextuple effect evaporators in the following sections.

Several researchers ,viz., Asantila et al. [A.3], Ayangbile et al. [A.4], Beveridge et al. [B.3], Carnahan and Ray [C.2], Holland [H.10], Koko and Joye [K.5], Lambert et al. [L.1], Lavery et al. [L.2], Logsdon [L.7], Merek [M.1], Militzer [M.5], Neewell and Fisher [N.5], Olauson [O.3] have worked on development of material-and energy-balance - models for multiple effect evaporators. Some others , viz. , Bolmstedt and Jernqvist [B.4, B.5], Burdett and Holland [B.8], Narshimhan [N.1], Wiklund [W.6] have worked on dynamic aspect of modelling of multiple effect evaporators. Others , viz., Clay [C.5], Coates [C.8], Oden [O.1] and Wise [W.7] have developed simplified calculation procedures for multiple effect evaporators using analytical and graphical approaches. Yet another group of workers,viz.,Farin [F.1], Freund [F.3], Friedmann [F.4], Gonzalez Maiz [G.3], Ray [R.4], Rosenblad [R.10] and Slota [S.5] have suggested methods including splitting and bleeding for improvement of energy efficiency of evaporators, while a few have made use of `thermodynamic approaches' and `pinch method' for better energy integration of evaporators.

5.1 Model for a Sextuple Effect Evaporator with Backward Feed (6- 5- 4- 3- 2- 1)

The schematic flow diagram for a sextuple effect evaporator with backward feed is shown in Figure 3.2(a). For the purpose of illustration, the equations around the first effect of the evaporator are derived hereunder.

5.1.1 Material-and Energy-Balance Equations Around First Effect

a. Total material balance:

$$L_2 = V_1 + L_1 \qquad \ldots (5.1)$$

b. Solute material balance:

$$L_2 x_2 = L_1 x_1 \qquad \ldots (5.2)$$

c. Enthalpy balance :

$$L_2 h_2 + Q_1 - V_1 H_1 - L_1 h_1 = 0 \qquad \ldots (5.3)$$

d. Enthalpy balance on the steam:

$$Q_1 = V_{01} \lambda_0 \qquad \ldots (5.4)$$

e. Heat transfer rate:

$$Q_1 = U_1 A_1 (T_0 - T_1) \qquad \ldots (5.5)$$

Substitution of values of V_1 and Q_1 from Eqs. (5.1) and (5.4) in equation (5.3) gives

$$L_2 h_2 + V_{01} \lambda_0 - (L_2 - L_1) H_1 - L_1 h_1 = 0$$

This can be represented by following functional notation:

$$\begin{aligned} f_1 &= L_2 h_2 + V_{01} \lambda_0 - (L_2 - L_1) H_1 - L_1 h_1 \\ &= L_2 h_2 - L_2 h_1 + V_{01} \lambda_0 + L_2 h_1 - (L_2 - L_1) H_1 - L_1 h_1 \\ &= L_2 (h_2 - h_1) + V_{01} \lambda_0 - (L_2 - L_1)(H_1 - h_1) \qquad \ldots .(5.6) \end{aligned}$$

Now letting $h = CpT_{boil}$, and putting $T_{boil} = (T+BPR)$ in Eq.(5.6);

$$f_1 = L_2[Cp_2(T_2+BPR_2)-Cp_1(T_1+BPR_1)] + V_{01}\lambda_0$$
$$- (L_2-L_1)(\lambda_1+Cp_vBPR_1) \qquad \text{....(5.7)}$$

Similarly, writing Eq.(5.5) in functional notation:

$$f_2 = U_1A_1(T_0 - T_1) - V_{01}\lambda_0 \qquad \text{... (5.8)}$$

A scaling procedure is used to reduce the magnitude of the terms appearing in the functional equations and matrices. For computational purposes, it is desirable to have terms with magnitudes near unity. The following scaling procedure has been used :

1. Each functional equation is divided by the product $F\,\lambda_0$ and the new functional expression so obtained is denoted by g_i $(1 \leq i \leq 6)$; where

$$g_i = \frac{f_i}{F\,\lambda_0}$$

2. All the flow rates are expressed as a fraction of the feed rate F, i.e., $L_i = l_iF$ and $V_i = v_iF$.

3. All the temperatures are expressed as a fraction of the steam temperature as follows: $T_i = u_iT_0$, which defines the fractional temperature u_i.

4. The area of each effect is expressed as a fraction of a term proportional to the feed rate in the following manner: $A_i=a_i(F/50)$, which defines the fractional area a_i.

With this scaling procedure, Eqs.(5.7) and (5.8) assume the following respective forms:

$$g_1 = \frac{l_2 T_O}{\lambda_o}\{Cp_2(u_2 + \frac{BPR_2}{T_O}) - Cp_1(u_1 + \frac{BPR_1}{T_o})\} + v_{O1} - \frac{(l_2-l_1)}{\lambda_o}(\lambda_1 + Cp_v BPR_1) \quad ....(5.9)$$

$$g_2 = \frac{U_1 T_O a_1}{50\ \lambda_O}\{1.0 - (u_1 + \frac{BPR_1}{T_O})\} - v_{O1} \quad ....(5.10)$$

Similarly equations for the second effect and onwards have been derived and are given below:

Second Effect

$$g_3 = \frac{l_3 T_O}{\lambda_o}\{Cp_3(u_3 + \frac{BPR_3}{T_O}) - Cp_2(u_2 + \frac{BPR_2}{T_o})\} + \frac{(l_2-l_1)}{\lambda_O}(\lambda_1+Cp_v BPR_1) - \frac{(l_3-l_2)}{\lambda_o}(\lambda_2+Cp_v BPR_2) \quad ....(5.11)$$

$$g_4 = \frac{U_2 T_O a_2}{50\ \lambda_O}\{u_1 - (u_2+\frac{BPR_2}{T_O})\} - \frac{(l_2-l_1)}{\lambda_O}(\lambda_1+Cp_v BPR_1) \quad ....(5.12)$$

Third effect

$$g_5 = \frac{l_4 T_O}{\lambda_o}\{Cp_4(u_4 + \frac{BPR_4}{T_O}) - Cp_3(u_3 + \frac{BPR_3}{T_o})\} + \frac{(l_3-l_2)}{\lambda_O}(\lambda_2+Cp_v BPR_2) - \frac{(l_4-l_3)}{\lambda_o}(\lambda_3+Cp_v BPR_3) \quad ....(5.13)$$

$$g_6 = \frac{U_3 T_o a_3}{50\,\lambda_o}\{u_2 - (u_3 + \frac{BPR_3}{T_o})\} - \frac{(l_3 - l_2)}{\lambda_o}(\lambda_2 + Cp_v BPR_2) \quad ....(5.14)$$

Fourth effect

$$g_7 = \frac{l_5 T_o}{\lambda_o}\{Cp_5(u_5 + \frac{BPR_5}{T_o}) - Cp_4(u_4 + \frac{BPR_4}{T_o})\}$$

$$+ \frac{(l_4 - l_3)}{\lambda_o}(\lambda_3 + Cp_v BPR_3) - \frac{(l_5 - l_4)}{\lambda_o}(\lambda_4 + Cp_v BPR_4) \quad ....(5.15)$$

$$g_8 = \frac{U_4 T_o a_4}{50\,\lambda_o}\{u_3 - (u_4 + \frac{BPR_4}{T_o})\} - \frac{(l_4 - l_3)}{\lambda_o}(\lambda_3 + Cp_v BPR_3) \quad ....(5.16)$$

Fifth effect

$$g_9 = \frac{l_6 T_o}{\lambda_o}\{Cp_6(u_6 + \frac{BPR_6}{T_o}) - Cp_5(u_5 + \frac{BPR_5}{T_o})\}$$

$$+ \frac{(l_5 - l_4)}{\lambda_o}(\lambda_4 + Cp_v BPR_4) - \frac{(l_6 - l_5)}{\lambda_o}(\lambda_5 + Cp_v BPR_5) \quad .....(5.17)$$

$$g_{10} = \frac{U_5 T_o a_5}{50\,\lambda_o}\{u_4 - (u_5 + \frac{BPR_5}{T_o})\} - \frac{(l_5 - l_4)}{\lambda_o}(\lambda_4 + Cp_v BPR_4) \quad .....(5.18)$$

Sixth effect

$$g_{11} = \frac{T_o}{\lambda_o}\{Cp_F(u_F) - Cp_6(u_6 + \frac{BPR_6}{T_o})\} + \frac{(l_6 - l_5)}{\lambda_o}(\lambda_5 + Cp_v BPR_5)$$

$$- \frac{(1.0 - l_6)}{\lambda_o}(\lambda_6 + Cp_v BPR_6) \quad .....(5.19)$$

$$g_{12} = \frac{U_6 T_o a_6}{50\,\lambda_o}\{u_5 - (u_6 + \frac{BPR_6}{T_o})\} - \frac{(l_6 - l_5)}{\lambda_o}(\lambda_5 + Cp_v BPR_5) \quad ....(5.20)$$

5.2 Models for Other Liquor Sequences of Sextuple Effect Evaporator and Scandinavian Evaporator

The other liquor sequences, which have been considered for the present investigation are given in Table 3.2. Equations for all these sequences of sextuple effect evaporator have been derived in a manner explained above. In addition, the sequence (3-4--5-1A-1B-2), as practiced in Scandinavia for quintuple effect evaporators, has also been included for the sake of comparison of results, and the equations have been arrived at in an analogous manner. All the equations are listed as under in the following sub-Sections:

5.2.1 Model for a Sextuple Effect Evaporator with Backward Feed, the Feed Split among the 5th and 6th Effects (Sequence $\left.\begin{matrix}5\\6\end{matrix}\right\rangle 4-3-2-1$)

$$g_1 = \frac{l_2 T_o}{\lambda_o}\{Cp_2(u_2 + \frac{BPR_2}{T_o}) - Cp_1(u_1 + \frac{BPR_1}{T_o})\} + v_{o1} - \frac{(l_2-l_1)}{\lambda_o}(\lambda_1 + Cp_v BPR_1) \quad ....(5.21)$$

$$g_2 = \frac{U_1 T_o a_1}{50\,\lambda_o}\{1.0 - (u_1 + \frac{BPR_1}{T_o})\} - v_{o1} \quad ....(5.22)$$

$$g_3 = \frac{l_3 T_o}{\lambda_o}\{Cp_3(u_3 + \frac{BPR_3}{T_o}) - Cp_2(u_2 + \frac{BPR_2}{T_o})\} + \frac{(l_2-l_1)}{\lambda_o}(\lambda_1+Cp_v BPR_1) - \frac{(l_3-l_2)}{\lambda_o}(\lambda_2+Cp_v BPR_2) \quad ....(5.23)$$

$$g_4 = \frac{U_2 T_o a_2}{50\,\lambda_o}\{u_1-(u_2+\frac{BPR_2}{T_o})\} - \frac{(l_2-l_1)}{\lambda_o}(\lambda_1+Cp_v BPR_1) \quad ....(5.24)$$

$$\begin{aligned} g_5 &= \frac{l_4 T_O}{\lambda_o}\left\{Cp_4\left(u_4 + \frac{BPR_4}{T_O}\right) - Cp_3\left(u_3 + \frac{BPR_3}{T_o}\right)\right\} \\ &\quad + \frac{(l_3-l_2)}{\lambda_o}(\lambda_2 + Cp_v BPR_2) - \frac{(l_4-l_3)}{\lambda_o}(\lambda_3 + Cp_v BPR_3) \qquad \ldots.(5.25) \end{aligned}$$

$$g_6 = \frac{U_3 T_O a_3}{50\,\lambda_O}\left\{u_2 - \left(u_3 + \frac{BPR_3}{T_O}\right)\right\} - \frac{(l_3-l_2)}{\lambda_O}(\lambda_2 + Cp_v BPR_2) \qquad \ldots.(5.26)$$

$$\begin{aligned} g_7 &= \frac{l_5 T_O}{\lambda_o}\left\{Cp_5\left(u_5 + \frac{BPR_5}{T_O}\right) - Cp_4\left(u_4 + \frac{BPR_4}{T_o}\right)\right\} \\ &\quad + \frac{l_6 T_O}{\lambda_o}\left\{Cp_6\left(u_6 + \frac{BPR_6}{T_O}\right) - Cp_4\left(u_4 + \frac{BPR_4}{T_O}\right)\right\} \\ &\quad + \frac{(l_4-l_3)}{\lambda_O}(\lambda_3 + Cp_v BPR_3) - \frac{(l_5+l_6-l_4)}{\lambda_O}(\lambda_4 + Cp_v BPR_4) \qquad \ldots.(5.27) \end{aligned}$$

$$g_8 = \frac{U_4\ T_O\ a_4}{50\ \lambda_O}\left\{u_3 - \left(u_4 + \frac{BPR_4}{T_O}\right)\right\} - \frac{(l_4-l_3)}{\lambda_O}(\lambda_3 + Cp_v\ BPR_3) \qquad \ldots.\ (5.28)$$

$$\begin{aligned} g_9 &= \frac{T_O}{2\ \lambda_O}\left\{Cp_F(u_F) - Cp_5\left(u_5 + \frac{BPR_5}{T_O}\right)\right\} + \frac{(l_5+l_6-l_4)}{\lambda_O}(\lambda_4 + Cp_v BPR_4) \\ &\quad - \frac{(0.5-l_5)}{\lambda_O}(\lambda_5 + Cp_v BPR_5) \qquad \ldots.(5.29) \end{aligned}$$

$$g_{10} = \frac{U_5 T_O a_5}{50\ \lambda_O}\left\{u_4 - \left(u_5 + \frac{BPR_5}{T_O}\right)\right\} - \frac{(l_5+l_6-l_4)}{\lambda_O}(\lambda_4 + Cp_v BPR_4) \qquad \ldots(5.30)$$

$$\begin{aligned} g_{11} &= \frac{T_O}{2\ \lambda_O}\left\{Cp_F(u_F) - Cp_6\left(u_6 + \frac{BPR_6}{T_O}\right)\right\} + \frac{(0.5-l_5)}{\lambda_O}(\lambda_5 + Cp_v BPR_5) \\ &\quad - \frac{(0.5-l_6)}{\lambda_O}(\lambda_6 + Cp_v BPR_6) \qquad \ldots.(5.31) \end{aligned}$$

$$g_{12} = \frac{U_6 T_o a_6}{50\ \lambda_o}\{u_5 - (u_6 + \frac{BPR_6}{T_o})\} - \frac{(0.5 - l_5)}{\lambda_o}(\lambda_5 + Cp_v BPR_5) \qquad \ldots . (5.32)$$

5.2.2 Model for a Sextuple Effect Evaporator with Mixed Feed Sequence 5-6-4-3-2-1

$$g_1 = \frac{l_2 T_o}{\lambda_o}\{Cp_2(u_2 + \frac{BPR_2}{T_o}) - Cp_1(u_1 + \frac{BPR_1}{T_o})\} + v_{o1} - \frac{(l_2 - l_1)}{\lambda_o}(\lambda_1 + Cp_v BPR_1) \qquad \ldots . (5.33)$$

$$g_2 = \frac{U_1 T_o a_1}{50\ \lambda_o}\{1.0 - (u_1 + \frac{BPR_1}{T_o})\} - v_{o1} \qquad \ldots . (5.34)$$

$$g_3 = \frac{l_3\ T_o}{\lambda_o}\{Cp_3(u_3 + \frac{BPR_3}{T_o}) - Cp_2(u_2 + \frac{BPR_2}{T_o})\} + \frac{(l_2 - l_1)}{\lambda_o}(\lambda_1 + Cp_v BPR_1) - \frac{(l_3 - l_2)}{\lambda_o}(\lambda_2 + Cp_v\ BPR_2) \qquad \ldots . (5.35)$$

$$g_4 = \frac{U_2\ T_o\ a_2}{50\ \lambda_o}\{u_1 - (u_2 + \frac{BPR_2}{T_o})\} - \frac{(l_2 - l_1)}{\lambda_o}(\lambda_1 + Cp_v\ BPR_1) \qquad \ldots . (5.36)$$

$$g_5 = \frac{l_4\ T_o}{\lambda_o}\{Cp_4\ (u_4 + \frac{BPR_4}{T_o}) - Cp_3\ (u_3 + \frac{BPR_3}{T_o})\} + \frac{(l_3 - l_2)}{\lambda_o}(\lambda_2 + Cp_v\ BPR_2) - \frac{(l_4 - l_3)}{\lambda_o}(\lambda_3 + Cp_v\ BPR_3) \qquad \ldots . (5.37)$$

$$g_6 = \frac{U_3\ T_o\ a_3}{50\ \lambda_o}\{u_2 - (u_3 + \frac{BPR_3}{T_o})\} - \frac{(l_3 - l_2)}{\lambda_o}(\lambda_2 + Cp_v\ BPR_2) \qquad \ldots . (5.38)$$

$$g_7 = \frac{l_6 \; T_o}{\lambda_o} \left\{ Cp_6 \left(u_6 + \frac{BPR_6}{T_o} \right) - Cp_4 \left(u_4 + \frac{BPR_4}{T_o} \right) \right\} + \frac{(l_4 - l_3)}{\lambda_o} (\lambda_3 + Cp_v \; BPR_3) - \frac{(l_6 - l_4)}{\lambda_o} (\lambda_4 + Cp_v \; BPR_4) \qquad \ldots (5.39)$$

$$g_8 = \frac{U_4 \; T_o \; a_4}{50 \; \lambda_o} \left\{ u_3 - \left(u_4 + \frac{BPR_4}{T_o} \right) \right\} - \frac{(l_4 - l_3)}{\lambda_o} (\lambda_3 + Cp_v \; BPR_3) \qquad \ldots . (5.40)$$

$$g_9 = \frac{T_o}{\lambda_o} \left\{ Cp_F \; (u_F) - Cp_5 \left(u_5 + \frac{BPR_5}{T_o} \right) \right\} + \frac{(l_6 - l_4)}{\lambda_o} (\lambda_4 + Cp_v \; BPR_4) - \frac{(1.0 - l_5)}{\lambda_o} (\lambda_5 + Cp_v \; BPR_5) \qquad \ldots . (5.41)$$

$$g_{10} = \frac{U_5 \; T_o \; a_5}{50 \; \lambda_o} \left\{ u_4 - \left(u_5 + \frac{BPR_5}{T_o} \right) \right\} - \frac{(l_6 - l_4)}{\lambda_o} (\lambda_4 + Cp_v \; BPR_4) \qquad \ldots .. (5.42)$$

$$g_{11} = \frac{l_5 \; T_o}{\lambda_o} \left\{ Cp_5 \left(u_5 + \frac{BPR_5}{T_o} \right) - Cp_6 \left(u_6 + \frac{BPR_6}{T_o} \right) \right\} - \frac{(1.0 - l_5)}{\lambda_o} (\lambda_5 + Cp_v \; BPR_5) - \frac{(l_5 - l_6)}{\lambda_o} (\lambda_6 + Cp_v \; BPR_6) \qquad \ldots . (5.43)$$

$$g_{12} = \frac{U_6 \; T_o \; a_6}{50 \; \lambda_o} \left\{ u_5 - \left(u_6 + \frac{BPR_6}{T_o} \right) \right\} - \frac{(1.0 - l_5)}{\lambda_o} (\lambda_5 + Cp_v \; BPR_5) \qquad \ldots .. (5.44)$$

5.2.3 Model for a Sextuple Effect Evaporator with Mixed Feed Sequence 4-5-6-3-2-1

$$g_1 = \frac{l_2 \; T_o}{\lambda_o} \left\{ Cp_2 \left(u_2 + \frac{BPR_2}{T_o} \right) - Cp_1 \left(u_1 + \frac{BPR_1}{T_o} \right) \right\} + v_{O1} - \frac{(l_2 - l_1)}{\lambda_o} (\lambda_1 + Cp_v \; BPR_1) \qquad \ldots . (5.45)$$

$$g_2 = \frac{U_1 \; T_O \; a_1}{50 \;\; \lambda_O} \{1.0 - (u_1 + \frac{BPR_1}{T_O})\} - v_{O1} \qquad \ldots.(5.46)$$

$$g_3 = \frac{l_3 \;\; T_O}{\lambda_o} \{Cp_3 \; (u_3 + \frac{BPR_3}{T_O}) - Cp_2 \; (u_2 + \frac{BPR_2}{T_o})\}$$

$$+ \frac{(l_2 - l_1)}{\lambda_O} (\lambda_1 + Cp_v \; BPR_1) - \frac{(l_3 - l_2)}{\lambda_o} (\lambda_2 + Cp_v \; BPR_2) \qquad \ldots.(5.47)$$

$$g_4 = \frac{U_2 \; T_O \; a_2}{50 \;\; \lambda_O} \{u_1 - (u_2 + \frac{BPR_2}{T_O})\} - \frac{(l_2 - l_1)}{\lambda_O} (\lambda_1 + Cp_v \; BPR_1) \qquad \ldots. (5.48)$$

$$g_5 = \frac{l_6 \;\; T_O}{\lambda_o} \{ Cp_6 \; (u_6 + \frac{BPR_6}{T_O}) - Cp_3 \; (u_3 + \frac{BPR_3}{T_o})\}$$

$$+ \frac{(l_3 - l_2)}{\lambda_O} (\lambda_2 + Cp_v \; BPR_2) - \frac{(l_6 - l_3)}{\lambda_O} (\lambda_3 + Cp_v \; BPR_3) \qquad \ldots(5.49)$$

$$g_6 = \frac{U_3 \; T_O \; a_3}{50 \;\; \lambda_O} \{u_2 - (u_3 + \frac{BPR_3}{T_O}) \} - \frac{(l_3 - l_2)}{\lambda_O} (\lambda_2 + Cp_v \; BPR_2) \qquad \ldots.(5.50)$$

$$g_7 = \frac{T_O}{\lambda_o} \{Cp_F \; (u_F) - Cp_4 \; (u_4 + \frac{BPR_4}{T_O})\} + \frac{(l_6 - l_3)}{\lambda_O} (\lambda_3 + Cp_v \; BPR_3)$$

$$- \frac{(1.0 - l_4)}{\lambda_O} (\lambda_4 + Cp_v \; BPR_4) \qquad \ldots.(5.51)$$

$$g_8 = \frac{U_4 \; T_O \; a_4}{50 \;\; \lambda_O} \{u_3 - (u_4 + \frac{BPR_4}{T_O})\} - \frac{(l_6 - l_3)}{\lambda_O} (\lambda_3 + Cp_v \; BPR_3) \qquad \ldots.. \; (5.52)$$

$$g_9 = \frac{l_4 \;\; T_O}{\lambda_o} \{Cp_4 \; (u_4 + \frac{BPR_4}{T_O}) - Cp_5 \; (u_5 + \frac{BPR_5}{T_o})\}$$

$$+ \frac{(1.0 - l_4)}{\lambda_o} (\lambda_4 + Cp_v \; BPR_4) - \frac{(l_4 - l_5)}{\lambda_O} (\lambda_5 + Cp_v \; BPR_5) \qquad \ldots.(5.53)$$

$$g_{10} = \frac{U_5 \; T_o \; a_5}{50 \;\; \lambda_o} \{u_4 - (u_5 + \frac{BPR_5}{T_o})\} - \frac{(1.0 - l_4)}{\lambda_o} (\lambda_4 + Cp_v \; BPR_4) \qquad .... \; (5.54)$$

$$g_{11} = \frac{l_5 \; T_o}{\lambda_o} \{Cp_5 \; (u_5 + \frac{BPR_5}{T_o}) - Cp_6 \; (u_6 + \frac{BPR_6}{T_o})\} + \frac{(l_4 - l_5)}{\lambda_o} (\lambda_5 + Cp_v \; BPR_5) - \frac{(l_5 - l_6)}{\lambda_o} (\lambda_6 + Cp_v \; BPR_6) \qquad .....(5.55)$$

$$g_{12} = \frac{U_6 \; T_o \; a_6}{50 \;\; \lambda_o} \{u_5 - (u_6 + \frac{BPR_6}{T_o})\} - \frac{(l_4 - l_5)}{\lambda_o} (\lambda_5 + Cp_v \; BPR_5) \qquad ..... \; (5.56)$$

5.2.4 Model for a Sextuple Effect Evaporator with Mixed Feed Sequence 3-4-5-6-2-1

$$g_1 = \frac{l_2 \; T_o}{\lambda_o} \{Cp_2 \; (u_2 + \frac{BPR_2}{T_o}) - Cp_1 \; (u_1 + \frac{BPR_1}{T_o})\} + v_{o1} - \frac{(l_2 - l_1)}{\lambda_o} (\lambda_1 + Cp_v \; BPR_1) \qquad .....(5.57)$$

$$g_2 = \frac{U_1 \; T_o \; a_1}{50 \;\; \lambda_o} \{1.0 - (u_1 + \frac{BPR_1}{T_o})\} - v_{o1} \qquad .....(5.58)$$

$$g_3 = \frac{l_6 \; T_o}{\lambda_o} \{Cp_6 \; (u_6 + \frac{BPR_6}{T_o}) - Cp_2 \; (u_2 + \frac{BPR_2}{T_o})\} + \frac{(l_2 - l_1)}{\lambda_o} (\lambda_1 + Cp_v \; BPR_1) - \frac{(l_6 - l_2)}{\lambda_o} (\lambda_2 + Cp_v \; BPR_2) \qquad .....(5.59)$$

$$g_4 = \frac{U_2 \; T_o \; a_2}{50 \;\; \lambda_o} \{u_1 - (u_2 + \frac{BPR_2}{T_o})\} - \frac{(l_2 - l_1)}{\lambda_o} (\lambda_1 + Cp_v \; BPR_1) \qquad .... \; (5.60)$$

$$g_5 = \frac{T_o}{\lambda_o} \{Cp_F\ (u_F) - Cp_3\ (u_3 + \frac{BPR_3}{T_o})\} + \frac{(l_6 - l_2)}{\lambda_o} (\lambda_2 + Cp_v\ BPR_2) - \frac{(1.0 - l_3)}{\lambda_o} (\lambda_3 + Cp_v\ BPR_3) \qquad .....(5.61)$$

$$g_6 = \frac{U_3\ T_o\ a_3}{50\ \lambda_o} \{u_2 - (u_3 + \frac{BPR_3}{T_o})\} - \frac{(l_6 - l_2)}{\lambda_o} (\lambda_2 + Cp_v\ BPR_2) \qquad .....\ (5.62)$$

$$g_7 = \frac{l_3\ T_o}{\lambda_o} \{Cp_3\ (u_3 + \frac{BPR_3}{T_o}) - Cp_4\ (u_4 + \frac{BPR_4}{T_o})\} + \frac{(1.0 - l_3)}{\lambda_o} (\lambda_3 + Cp_v\ BPR_3) - \frac{(l_3 - l_4)}{\lambda_o} (\lambda_4 + Cp_v\ BPR_4) \qquad .....(5.63)$$

$$g_8 = \frac{U_4\ T_o\ a_4}{50\ \lambda_o} \{u_3 - (u_4 + \frac{BPR_4}{T_o})\} - \frac{(1.0 - l_3)}{\lambda_o} (\lambda_3 + Cp_v\ BPR_3) \qquad ....\ (5.64)$$

$$g_9 = \frac{l_4\ T_o}{\lambda_o} \{Cp_4\ (u_4 + \frac{BPR_4}{T_o}) - Cp_5\ (u_5 + \frac{BPR_5}{T_o})\} + \frac{(l_3 - l_4)}{\lambda_o} (\lambda_4 + Cp_v\ BPR_4) - \frac{(l_4 - l_5)}{\lambda_o} (\lambda_5 + Cp_v\ BPR_5) \qquad ....(5.65)$$

$$g_{10} = \frac{U_5\ T_o\ a_5}{50\ \lambda_o} \{u_4 - (u_5 + \frac{BPR_5}{T_o})\} - \frac{(l_3 - l_4)}{\lambda_o} (\lambda_4 + Cp_v\ BPR_4) \qquad ....\ (5.66)$$

$$g_{11} = \frac{l_5\ T_o}{\lambda_o} \{Cp_5\ (u_5 + \frac{BPR_5}{T_o}) - Cp_6\ (u_6 + \frac{BPR_6}{T_o})\} + \frac{(l_4 - l_5)}{\lambda_o} (\lambda_5 + Cp_v\ BPR_5) - \frac{(l_5 - l_6)}{\lambda_o} (\lambda_6 + Cp_v\ BPR_6) \qquad ....(5.67)$$

$$g_{12} = \frac{U_6 \; T_O \; a_6}{50 \;\; \lambda_O} \{u_5 - (u_6 + \frac{BPR_6}{T_O})\} - \frac{(l_4 - l_5)}{\lambda_O} (\lambda_5 + Cp_v \; BPR_5) \qquad \ldots . \; (5.68)$$

5.2.5 Model for a Sextuple Effect Evaporator with Mixed Feed Sequence 2-3-4-5-6-1

$$g_1 = \frac{l_6 \; T_O}{\lambda_o} \{Cp_6 \; (u_6 + \frac{BPR_6}{T_O}) - Cp_1 \; (u_1 + \frac{BPR_1}{T_o})\} + v_{O1} - \frac{(l_6 - l_1)}{\lambda_o} (\lambda_1 + Cp_v \; BPR_1) \qquad \ldots . \; (5.69)$$

$$g_2 = \frac{U_1 \; T_O \; a_1}{50 \;\; \lambda_O} \{1.0 - (u_1 + \frac{BPR_1}{T_O})\} - v_{O1} \qquad \ldots . \; (5.70)$$

$$g_3 = \frac{T_O}{\lambda_o} \{Cp_F \; (u_F) - Cp_2 \; (u_2 + \frac{BPR_2}{T_O})\} + \frac{(l_6 - l_1)}{\lambda_O} (\lambda_1 + Cp_v \; BPR_1) - \frac{(1.0 - l_2)}{\lambda_o} (\lambda_2 + Cp_v \; BPR_2) \qquad \ldots . \; (5.71)$$

$$g_4 = \frac{U_2 \; T_O \; a_2}{50 \;\; \lambda_O} \{u_1 - (u_2 + \frac{BPR_2}{T_O})\} - \frac{(l_6 - l_1)}{\lambda_O} (\lambda_1 + Cp_v \; BPR_1) \qquad \ldots . \; (5.72)$$

$$g_5 = \frac{l_2 \; T_O}{\lambda_o} \{Cp_2 \; (u_2 + \frac{BPR_2}{T_O}) - Cp_3 \; (u_3 + \frac{BPR_3}{T_o})\} - \frac{(1.0 - l_2)}{\lambda_o} (\lambda_2 + Cp_v \; BPR_2) - \frac{(l_2 - l_3)}{\lambda_O} (\lambda_3 + Cp_v \; BRP_3) \qquad \ldots .. \; (5.73)$$

$$g_6 = \frac{U_3 \; T_O \; a_3}{50 \;\; \lambda_O} \{u_2 - (u_3 + \frac{BPR_3}{T_O})\} - \frac{(1.0 - l_2)}{\lambda_O} (\lambda_2 + Cp_v \; BPR_2) \qquad \ldots .. \; (5.74)$$

$$g_7 = \frac{l_3 \; T_o}{\lambda_o} \{Cp_3 \; (u_3 + \frac{BPR_3}{T_o}) - Cp_4 \; (u_4 + \frac{BPR_4}{T_o})\}$$

$$+ \frac{(l_2 - l_3)}{\lambda_o} (\lambda_3 + Cp_v \; BPR_3) - \frac{(l_3 - l_4)}{\lambda_o} (\lambda_4 + Cp_v \; BPR_4) \qquad \ldots\ldots \; (5.75)$$

$$g_8 = \frac{U_4 \; T_o \; a_4}{50 \;\; \lambda_o} \{u_3 - (u_4 + \frac{BPR_4}{T_o})\} - \frac{(l_2 - l_3)}{\lambda_o} (\lambda_3 + Cp_v \; BPR_3) \qquad \ldots\ldots \; (5.76)$$

$$g_9 = \frac{l_4 \; T_o}{\lambda_o} \{Cp_4 \; (u_4 + \frac{BPR_4}{T_o}) - Cp_5 \; (u_5 + \frac{BPR_5}{T_o})\}$$

$$+ \frac{(l_3 - l_4)}{\lambda_o} (\lambda_4 + Cp_v \; BPR_4) - \frac{(l_4 - l_5)}{\lambda_o} (\lambda_5 + Cp_v \; BPR_5) \qquad \ldots\ldots \; (5.77)$$

$$g_{10} = \frac{U_5 \; T_o \; a_5}{50 \;\; \lambda_o} \{u_4 - (u_5 + \frac{BPR_5}{T_o})\} - \frac{(l_3 - l_4)}{\lambda_o} (\lambda_4 + Cp_v \; BPR_4) \qquad \ldots\ldots \; (5.78)$$

$$g_{11} = \frac{l_5 \; T_o}{\lambda_o} \{Cp_5 \; (u_5 + \frac{BPR_5}{T_o}) - Cp_6 \; (u_6 + \frac{BPR_6}{T_o})\}$$

$$+ \frac{(l_4 - l_5)}{\lambda_o} (\lambda_5 + Cp_v \; BPR_5) - \frac{(l_5 - l_6)}{\lambda_o} (\lambda_6 + Cp_v \; BPR_6) \qquad \ldots\ldots \; (5.79)$$

$$g_{12} = \frac{U_6 \; T_o \; a_6}{50 \;\; \lambda_o} \{u_5 - (u_6 + \frac{BPR_6}{T_o})\} - \frac{(l_4 - l_5)}{\lambda_o} (\lambda_5 + Cp_v \; BPR_5) \qquad \ldots\ldots \; (5.80)$$

5.2.6 Model for a Sextuple Effect Evaporator with Backward Feed, the Feed Split among the 5th and 6th Effects and the Steam Split in the First Effect (Sequence $\begin{matrix}5\\ \\6\end{matrix}\!>4-3-2-1B-1A$)

It may be noted that the splitted portions of the first effect are designated as 1A and 1B, respectively. For the sake of convenience the sequence has been renumbered as $\begin{matrix}6\\ \\7\end{matrix}\!>5-4-3-2-1$, as shown in Fig. 3.5(a).

$$g_1 = \frac{l_2\ T_O}{\lambda_o}\ \{Cp_2\ (u_2 + \frac{BPR_2}{T_O}) - Cp_1\ (u_1 + \frac{BPR_1}{T_O})\} + v_{O1}$$

$$- \frac{(l_2 - l_1)}{\lambda_o}\ (\ \lambda_1 + Cp_v\ BPR_1) \qquad \ldots.\ (5.81)$$

$$g_2 = \frac{U_1\ T_O\ a_1}{50\ \lambda_O}\ \{1.0 - (u_1 + \frac{BPR_1}{T_O})\} - v_{O1} \qquad \ldots..\ (5.82)$$

$$g_3 = \frac{l_3 T_O}{\lambda_o}\ \{Cp_3\ (u_3 + \frac{BPR_3}{T_O}) - Cp_2\ (u_2 + \frac{BPR_2}{T_O})\} + v_{O2}$$

$$- \frac{(l_3 - l_2)}{\lambda_o}\ (\ \lambda_2 + Cp_v\ BPR_2) \qquad \ldots.\ (5.83)$$

$$g_4 = \frac{U_2\ T_O\ a_2}{50\ \lambda_O}\ \{1.0 - (u_2 + \frac{BPR_2}{T_O})\} - v_{O2} \qquad \ldots.\ (5.84)$$

$$g_5 = \frac{l_4\ T_O}{\lambda_o}\ \{Cp_4\ (u_4 + \frac{BPR_4}{T_O}) - Cp_3\ (u_3 + \frac{BPR_3}{T_O})\}$$

$$+ \frac{(l_2 - l_1)}{\lambda_o}\ (\ \lambda_1 + Cp_v\ BPR_1) + \frac{(l_3 - l_2)}{\lambda_o}\ (\ \lambda_2 + Cp_v\ BPR_2)$$

$$- \frac{(l_4 - l_3)}{\lambda_o}\ (\ \lambda_3 + Cp_v\ BPR_3) \qquad \ldots..\ (5.85)$$

$$g_6 = \frac{U_3\ T_o\ a_3}{50\ \lambda_o} \{u_2 - (u_3 + \frac{BPR_3}{T_o})\} - \frac{(l_2 - l_1)}{\lambda_o} (\lambda_1 + Cp_v\ BPR_1) - \frac{(l_3 - l_2)}{\lambda_o} (\lambda_2 + Cp_v\ BPR_2) \quad \ldots\ldots (5.86)$$

$$g_7 = \frac{l_5\ T_o}{\lambda_o} \{Cp_5\ (u_5 + \frac{BPR_5}{T_o}) - Cp_4\ (u_4 + \frac{BPR_4}{T_o})\} + \frac{(l_4 - l_3)}{\lambda_o} (\lambda_3 + Cp_v\ BPR_3) - \frac{(l_5 - l_4)}{\lambda_o} (\lambda_4 + Cp_v\ BPR_4) \quad \ldots\ldots (5.87)$$

$$g_8 = \frac{U_4\ T_o\ a_4}{50\ \lambda_o} \{u_3 - (u_4 + \frac{BPR_4}{T_o})\} - \frac{(l_4 - l_3)}{\lambda_o} (\lambda_3 + Cp_v\ BPR_3) \quad \ldots\ldots (5.88)$$

$$g_9 = \frac{l_6\ T_o}{\lambda_o} \{Cp_6\ (u_6 + \frac{BPR_6}{T_o}) - Cp_5\ (u_5 + \frac{BPR_5}{T_o})\} + \frac{l_7\ T_o}{\lambda_o} [Cp_7\ (u_7 + \frac{BPR_7}{T_o}) - Cp_5\ (u_5 + \frac{BPR_5}{T_o})] + \frac{(l_5 - l_4)}{\lambda_o} (\lambda_4 + Cp_v\ BPR_4) - \frac{(l_6 + l_7 - l_5)}{\lambda_o} (\lambda_5 + Cp_v\ BPR_5) \quad \ldots\ldots (5.89)$$

$$g_{10} = \frac{U_5\ T_o\ a_5}{50\ \lambda_o} \{u_4 - (u_5 + \frac{BPR_5}{T_o})\} - \frac{(l_5 - l_4)}{\lambda_o} (\lambda_4 + Cp_v\ BPR_4) \quad \ldots\ldots (5.90)$$

$$g_{11} = \frac{T_o}{2\ \lambda_o} [Cp_F\ (u_F) - Cp_6\ (u_6 + \frac{BPR_6}{T_o})] + \frac{(l_6 + l_7 - l_5)}{\lambda_o} (\lambda_5 + Cp_v\ BPR_5) - \frac{(0.5 - l_6)}{\lambda_o} (\lambda_6 + Cp_v\ BPR_6) \quad \ldots\ldots (5.91)$$

$$g_{12} = \frac{U_6\ T_o\ a_6}{50\ \lambda_o} \{u_5 - (u_6 + \frac{BPR_6}{T_o})\} - \frac{(l_6 + l_7 - l_5)}{\lambda_o} (\lambda_5 + Cp_v\ BPR_5) \quad \ldots\ldots (5.92)$$

$$g_{13} = \frac{T_o}{2\,\lambda_o}\left[Cp_F\,(u_F) - Cp_7\left(u_7 + \frac{BPR_7}{T_o}\right)\right] + \frac{(0.5-l_6)}{\lambda_o}\,(\lambda_6 + Cp_v\,BPR_6) - \frac{(0.5-l_7)}{\lambda_o}\,(\lambda_7 + Cp_v\,BPR_7) \qquad \ldots\ldots (5.93)$$

$$g_{14} = \frac{U_7\,T_o\,a_7}{50\,\lambda_o}\left\{u_6 - \left(u_7 + \frac{BPR_7}{T_o}\right)\right\} - \frac{(0.5-l_6)}{\lambda_o}\,(\lambda_6 + Cp_v\,BPR_6) \qquad \ldots\ldots (5.94)$$

5.2.7 Model for the Scandinavian Quintuple Effect Evaporator (with Mixed Feed and Steam Split in the First Effect , Sequence 3-4-5-1A-1B-2)

For convenience in computations the sequence has been renumbered as 4-5-6-1-2-3 as shown in Fig. 3.5 (b).

$$g_1 = \frac{l_6\,T_o}{\lambda_o}\left\{Cp_6\left(u_6 + \frac{BPR_6}{T_o}\right) - Cp_1\left(u_1 + \frac{BPR_1}{T_o}\right)\right\} + v_{o1} - \frac{(l_6-l_1)}{\lambda_o}\,(\lambda_1 + Cp_v\,BPR_1) \qquad \ldots\ldots (5.95)$$

$$g_2 = \frac{U_1\,T_o\,a_1}{50\,\lambda_o}\left\{1.0 - \left(u_1 + \frac{BPR_1}{T_o}\right)\right\} - v_{o1} \qquad \ldots\ldots (5.96)$$

$$g_3 = \frac{l_1\,T_o}{\lambda_o}\left\{Cp_1\left(u_1 + \frac{BPR_1}{T_o}\right) - Cp_2\left(u_2 + \frac{BPR_2}{T_o}\right)\right\} + v_{o2} - \frac{(l_1-l_2)}{\lambda_o}\,(\lambda_2 + Cp_v\,BPR_2) \qquad \ldots\ldots (5.97)$$

$$g_4 = \frac{U_2\,T_o\,a_2}{50\,\lambda_o}\left\{1.0 - \left(u_2 + \frac{BPR_2}{T_o}\right)\right\} - v_{o2} \qquad \ldots\ldots (5.98)$$

$$g_5 = \frac{l_2\ T_o}{\lambda_o} \{Cp_2 \ (u_2 + \frac{BPR_2}{T_o}) - Cp_3 \ (u_3 + \frac{BPR_3}{T_o})\} + \frac{(l_6-l_1)}{\lambda_o} (\lambda_1 + Cp_v \ BPR_1) + \frac{(l_1-l_2)}{\lambda_o} (\lambda_2 + Cp_v \ BRP_2) - \frac{(l_2-l_3)}{\lambda_o} (\lambda_3 + Cp_v \ BPR_3) \qquad \ldots\ldots \ (5.99)$$

$$g_6 = \frac{U_3\ T_o\ a_3}{50\ \ \lambda_o} \{u_2 -(u_3 + \frac{BPR_3}{T_o})\} - \frac{(l_6-l_1)}{\lambda_o} (\lambda_1 + Cp_v \ BPR_1) - \frac{(l_1-l_2)}{\lambda_o} (\lambda_2 + Cp_v \ BPR_2) \qquad \ldots\ldots \ (5.100)$$

$$g_7 = \frac{T_o}{\lambda_o} \{Cp_F \ (u_F) - Cp_4 \ (u_4 + \frac{BPR_4}{T_o})\} + \frac{(l_2-l_3)}{\lambda_o} (\lambda_3 + Cp_v \ BPR_3) - \frac{(1.0-l_4)}{\lambda_o} (\lambda_4 + Cp_v \ BPR_4) \qquad \ldots\ldots \ (5.101)$$

$$g_8 = \frac{U_4\ T_o\ a_4}{50\ \ \lambda_o} \{u_3 - (u_4 + \frac{BPR_4}{T_o})\} - \frac{(l_2-l_3)}{\lambda_o} (\lambda_3 + Cp_v \ BPR_3) \qquad \ldots\ldots \ (5.102)$$

$$g_9 = \frac{l_4\ T_o}{\lambda_o} \{Cp_4 \ (u_4 + \frac{BPR_4}{T_o}) - Cp_5 \ (u_5 + \frac{BPR_5}{T_o})\} + \frac{(1.0-l_4)}{\lambda_o} (\lambda_4 + Cp_v \ BPR_4) - \frac{(l_4-l_5)}{\lambda_o} (\lambda_5 + Cp_v \ BPR_5) \qquad \ldots\ldots \ (5.103)$$

$$g_{10} = \frac{U_5\ T_o\ a_5}{50\ \ \lambda_o} \{u_4 - (u_5 + \frac{BPR_5}{T_o})\} - \frac{(1.0-l_4)}{\lambda_o} (\lambda_4 + Cp_v \ BPR_4) \qquad \ldots\ldots \ (5.104)$$

$$g_{11} = \frac{l_5\ T_o}{\lambda_o}\ \{Cp_5\ (u_5 + \frac{BPR_5}{T_o}) - Cp_6\ (u_6 + \frac{BPR_6}{T_o}\ )\}$$

$$+ \frac{(l_4 - l_5)}{\lambda_o}\ (\ \lambda_5 + Cp_v\ BPR_5) - \frac{(l_5 - l_6)}{\lambda_o}\ (\ \lambda_6 + Cp_v\ BPR_6) \quad \ldots\ldots (5.105)$$

$$g_{12} = \frac{U_6\ T_o\ a_6}{50\ \ \lambda_o}\ \{u_5 - (u_6 + \frac{BPR_6}{T_o})\} - \frac{(l_4 - l_5)}{\lambda_o}\ (\ \lambda_5 + Cp_v\ BPR_5) \quad \ldots\ldots (5.106)$$

5.3 REMARKS ON PRESENT MODEL

The equations developed for the various liquor flow sequences are based on the following assumptions :

(i) Heating surfaces in all the effects of the evaporator are taken equal

(ii) Heat loss from the evaporator to the surroundings is neglected

(iii) Fouling is not appreciable so as to influence the overall heat transfer coefficient.

(iv) It is presumed that identical increases in steam economy are expected for all flow sequences under investigation through the inclusion of condensate and product flash in the model equations. As such, non-inclusion of the flash in the development of the model equations is not likely to affect appreciably the comparative study made on different liquor flow sequences.

5.4 SOLUTION PROCEDURE

To demonstrate the solution of the set of non-linear equations developed for the design of sextuple effect evaporator let us consider the case of backward feed (sequence **a** of Table 3.2) with equal area of all the effects ($a_i = a$, $1 \le i \le 6$) . The various variables involved are categorized as specified and unknown variables. These are given hereunder :

Specified variables : F, x_f, $x_p (=x_1)$, T_f, T_0, P_o, P_1 (or T_6), U_1, U_2, U_3, U_4, U_5, U_6, BPR1, BPR2, BPR3, BPR4, BPR5, and BPR6

Unknown variables : v_{o1}, u_1, u_2, u_3, u_4, u_5, l_2, l_3, l_4, l_5, l_6, a

The twelve independent non-linear equations (Eq. (5.9) to Eq. (5.20)) can be solved simultaneously for the twelve unknowns v_{o1}, u_1, u_2, u_3, u_4, u_5, l_2, l_3, l_4, l_5, l_6 and a. In addition to the above unknowns, the concentration terms x_2, x_3, x_4, x_5 and x_6 can be calculated from the component material balance for solute:

$$x_f - l_i x_i = 0 \quad (i = 2,3,4,5,6\ ) \qquad \ldots\ldots \quad (5.107)$$

The specified variables are supplied to the Eq. (5.9 through 5.20) from different correlations, operational considerations and models as detailed below :

The values of boiling point rise in various effects are obtained from Eq. (2.17), specific heat from Eq. (2.13); and the latent heat of vapourization, from Eq. (2.18). As regards the pressure in the last effect (P_1), it is fixed by the operational considerations.

It is essential to note that the overall heat transfer coefficient in the individual effects, U_i (i = 1,2,...6), of the multiple effect evaporator does not remain constant over the range of variations of different parameters under study. In fact, it strongly depends upon heat-flux, temperature, concentration, flow rate and various physico-thermal properties of the liquor entering the evaporator tubes. Therefore, any estimation of the parametric effects of various variables on the steam consumption, steam economy and heating surface area can never be based on a wrong premise of constancy of U_i. Hence an empirical model developed by Gudmundson and discussed in Sec. 2.2.1, has been used.

Eqs. (5.9 through 5.20) form a set of twelve non-linear algebraic equations. It is required to solve these equations simultaneously.

For solution of simultaneous linear and non-linear equations developed for analysis of multiple effect evaporators, several workers,viz.,Hassett [H.7], Freund [F.3], Militzer [M.5], Sun and Stadtherr [S.9], Wiklund [W.6] have developed analytical techniques, Paloschi [P.1] used a quasi-Newton method for non-linear equations, and Vazquez-Esparragoza [V.1], employed a combination of Newton-Raphson and functional transformation method. Oden [O.1] and Wise [W.7] proposed graphical methods. Bolmstedt et al. [B.4], however, pointed out that the methods in many cases are short-cut and therefore not suitable for accurate calculations.

In the present investigation, Newton-Raphson method has been selected as it is considered quite comprehensive. It may be mentioned that this method has not posed any convergence problem in this investigation as the initial values can be guessed accurately because of the known boundary of input parameters.

5.4.1 Newton-Raphson Method

The functions g_1, g_2 ..... g_{12} described in Eqs. (5.9 through 5.20) are linearized using Taylor series as given below :

$$
\begin{aligned}
0 = g_j &+ \frac{\partial g_j}{\partial v_{o1}} \Delta v_{o1} + \frac{\partial g_j}{\partial u_1} \Delta u_1 + \frac{\partial g_j}{\partial u_2} \Delta u_2 + \frac{\partial g_j}{\partial u_3} \Delta u_3 \\
&+ \frac{\partial g_j}{\partial u_4} \Delta u_4 + \frac{\partial g_j}{\partial u_5} \Delta u_5 + \frac{\partial g_j}{\partial l_2} \Delta l_2 + \frac{\partial g_j}{\partial l_3} \Delta l_3 + \frac{\partial g_j}{\partial l_4} \Delta l_4 \\
&+ \frac{\partial g_j}{\partial l_5} \Delta l_5 + \frac{\partial g_j}{\partial l_6} \Delta l_6 + \frac{\partial g_j}{\partial a} \Delta a + \ldots, (j=1,2,3 \ldots 12)
\end{aligned}
\qquad \ldots.(5.108)
$$

where,

$\Delta v_{o1} = v_{o1,(k+1)} - v_{o1,k}$

$\Delta u_1 = u_{1,(k+1)} - u_{1,k}$

$\Delta u_2 = u_{2,(k+1)} - u_{2,k}$

$\Delta u_3 = u_{3,(k+1)} - u_{3,k}$

$\Delta u_4 = u_{4,(k+1)} - u_{4,k}$

$\Delta u_5 = u_{5,(k+1)} - u_{5,k}$

$\Delta l_2 = l_{2,(k+1)} - l_{2,k}$

$\Delta l_3 = l_{3,(k+1)} - l_{3,k}$

$\Delta l_4 = l_{4,(k+1)} - l_{4,k}$

$$\Delta l_5 = l_{5,(k+1)} - l_{5,k}$$

$$\Delta l_6 = l_{6,(k+1)} - l_{6,k}$$

$$\Delta a = a_{,(k+1)} - a_{,k}$$

and the subscripts k and (k+1) denote k th and (k+1) th trials. The twelve equations may be stated in compact form by means of the matrix equation given below :

$$J_k \Delta X_k = - g_k \qquad \text{....(5.109)}$$

where J_k is the square jacobian matrix of order 12 and X_k and g_k are conformable column vectors. For simplicity the subscript k is omitted from the elements of J_k, X_k and g_k

$$J_k = \begin{bmatrix} \frac{\partial g_1}{\partial v_{o1}} & \frac{\partial g_1}{\partial u_1} & \frac{\partial g_1}{\partial u_2} & \cdots\cdots & \frac{\partial g_1}{\partial a} \\ \vdots & \vdots & \vdots & \cdots\cdots & \vdots \\ \vdots & \vdots & \vdots & \cdots\cdots & \vdots \\ \vdots & \vdots & \vdots & \cdots\cdots & \vdots \\ \frac{\partial g_{12}}{\partial v_{o1}} & \frac{\partial g_{12}}{\partial u_1} & \frac{\partial g_{12}}{\partial u_2} & \cdots\cdots & \frac{\partial g_{12}}{\partial a} \end{bmatrix} \qquad \text{....(5.110)}$$

$$\Delta X_k = X_{(k+1)} - X_k \qquad \text{....(5.111)}$$

$$X_k = [v_{o1}, u_1, u_2, u_3 \, \ldots\ldots\, u_5, l_2, l_3 \, \ldots\ldots\, l_6, a]^T \qquad \text{....(5.112)}$$

$$\Delta X_k = [\Delta v_{o1}, \Delta u_1, \Delta u_2, \, \ldots\ldots\, \Delta u_5, \Delta l_2, \Delta l_3 \, \ldots\ldots\, \Delta l_6, \Delta a]^T \qquad \text{....(5.113)}$$

$$g_k = [g_1, g_2, g_3 \, \ldots\ldots\, g_{12}]^T \qquad \text{....(5.114)}$$

On the basis of an assumed set of values of the column vector $X_k (=X_o)$, the corresponding values of the elements of J_k and g_k are computed. The computed values of J_k and g_k thus obtained are used to calculate the values of X_k, by solving Eq.(5.109) using Gauss elimination method with partial pivoting supplemented with LU decomposition. Subsequently, X_{k+1} is obtained from Eq.(5.111) for its use in the next trial. This procedure is repeated until the desired accuracy in the values of unknown variables is obtained.

5.4.2 Sufficient Conditions for the Convergence of Newton-Raphson Method

Let the solution set of the independent variables be represented by the column vector β

$$\beta = [\beta_1, \beta_2, \beta_3 \ldots\ldots \beta_{12}]^T \qquad \ldots.(5.115)$$

It implies that

$$g_i (\beta) = 0 \quad (i=1,2,3\ldots,12) \qquad \ldots.(5.116)$$

or

$$g (\beta) = 0$$

Let the elements of g_k and J_k be continuous over all starting vectors X_o and all vectors X_k generated thereafter by use of the Newton-Raphson method. Further, if the determinate of J_k is non zero ($|J_k| \neq 0$) for all values of X_k, including X_o, it can be demonstrated that when the starting vector X_o is "close" to the solution vector β, the Newton-Raphson method converges to the solution vector β. "Close" means that the product of the

Hilbert norms of the matrices required to relate the erro vectors $\mathbf{E_k}$ and $\mathbf{E_o}$ is less than unity, where the error vectors are given as; $\mathbf{E_k}=\mathbf{X_k}-\beta$ and $\mathbf{E_o}= \mathbf{X_o} -\beta$.

For the present work, the Jacobian matrix is generate numerically. For this a perturbation of the magnitude 0.01 i column vector $\mathbf{X_k}$ is applied and subsequent values of $\mathbf{g_k}$ compute to form $\mathbf{J_k}$. The tolerance selected for variables v_{o1}, u_1,u_2,u_3, u_4, u_5, l_2, l_3, l_4, l_5, l_6, a, and the functions g_1 to g_{12} is taken to be 0.000001 for an acceptable solution.

CHAPTER - 6

RESULTS AND DISCUSSION

In this Chapter, the various results obtained in the present investigation are discussed :

6.1 SCOPE OF THE PRESENT INVESTIGATION

Based on the cost optimization, it has been established in Chapter 4 that the sextuple effect evaporators are the most economical for the concentration of weak black liquor. In an attempt to further improve the economy of a sextuple effect evaporator, investigations are carried out for different liquor flow sequences, including feed and steam splitting. For this purpose, the five input parameters presented in Table 3.3, with their indicated operating ranges, have been chosen to analyse their effects on steam consumption, steam economy and requirement of heating surface area to arrive at the best liquor flow sequence of Table 3.2.

In view of the fact that many Indian mills employ quintuple effect evaporators, and that the Scandinavian sequence is also well-established, it is thought worthwhile to evaluate the status of Scandinavian sequence vis-a-vis the best liquor flow sequence for a sextuple effect evaporator evaluated as above.

The sensitivity analysis is used to evaluate the sensitivity of different input parameters, i.e., the extent of variation of output parameters resulting from a predetermined perturbation in the value of input parameter. This information can be utilized in meeting specified performance criteria in design of a multiple effect evaporator. This can also be used to arrive at the optimum operating conditions by adjustment of the input parameters.

In this Chapter the results have been presented and discussed in the following steps:

(i) validation of model

(ii) sensitivity analysis of sextuple effect evaporator employing backward feed

(iii) detailed analysis of the effect of operating parameters on SC, S and A for a sextuple effect evaporator employing backward feed

(iv) comparison of the three sequences **a**, **b** and **c** of backward feed of Table 3.2 to arrive at the best of them

(v) comparison among the various mixed feed sequences **d**, **e**, **f** and **g** shown in Figs. 3.3(a), 3.3 (b), 3.4(a) and 3.4(b), respectively to choose the best out of them

(vi) comparison of the best sequences obtained in steps (iv) and (v) and finally,

(vii) comparison of the better of the two from Step (vi) with the Scandinavian sequence to estimate the losses, if any, due to adoption of the latter in industrial practice.

6.2 MODELS

The mathematical equations, detailed in Sections 5.1 and 5.2 have been solved for the ranges of the input parameters of Table 3.3 by using the solution technique, discussed in Section 5.4. For this, a series of computer programmes in FORTRAN 77 have been developed and run on an IBM PC/AT-486, using a MS FORTRAN 5.0 compiler. The approximate computation time required is 122 seconds for one set of input data. Details of the computer programme for backward feed are presented in Appendix A, while the sample data and results appear in Appendix B.

6.3 VALIDATION OF THE MODEL

Before obtaining results from the models developed in Chapter 5, it has been considered necessary to test their validity with the available data in the literature. McDonald [M.3] has published data related to boiling point rise (BPR), total solids concentration (TS), liquor temperature , liquor flow rate from different effects in a sextuple effect LTV evaporator. His analysis is for the liquor flow sequence, represented by sequence **c** of Table 3.2.

For this flow sequence, Eqs. (5.81 through 5.94) have been derived in the present investigation. Using these equations, computations have been carried out corresponding to the values of input parameters specified in McDonald's study [M.3]. The calculated values are presented along with those reported by McDonald in Table 6.1 for a comparison .

TABLE 6.1

DATA PREDICTED IN THE PRESENT INVESTIGATION VS THOSE OF McDONALD

Parameter		Evaporator body						
		1A	1B	2	3	4	5	6
BPR °F	McDonald	13	11	7	5	4	3	3
	Present	13.28	10.23	7.54	5.07	3.74	2.85	3.18
Conc.	McDonald	0.511	0.423	0.342	0.269	0.211	0.178	0.189
	Present	0.511	0.423	0.340	0.258	0.211	0.178	0.190
Liq. Temp. °F	McDonald	252	250	225	203	181	158	128
	Present	250	246	221	200	178	157	128
Liq. Flow Rate lb/hr	McDonald	90605	109775	135825	178875	219585	130475	122520
	Present	90415	109296	136037	179442	219392	129999	121624

Since the values reported by McDonald are in FPS system of units, the calculated values in the present investigation have also been expressed in the same system for the sake of immediate comparison and appreciation.

From the above Table, it is noted that there is an excellent agreement between the values of McDonald and those predicted, making use of the equations derived in the present investigation. This clearly shows that the set of Eqs.(5.81 through 5.94), the solution technique employed, and Gudmundson's Model [G.7 and G.8] for calculation of overall heat transfer coefficient, are capable of predicting the various parameters related to evaporators. Based on this, it may be concluded that the similarly developed sets of equations for other liquor flow sequences in the present investigation are also equally reliable and accurate for the design calculations of these evaporators.

It is important to point out that for the calculation of density, viscosity, specific heat and thermal conductivity equations Eqs. (2.1), (2.8), (2.13), and (2.15), respectively have been used.

6.4 DESIGN CALCULATIONS OF EVAPORATORS

The mathematical equations developed in the present study facilitate design calculations of multiple effect evaporators either as a design problem in which the final liquor concentration, x_p is specified and area of the evaporator along with other unknown parameters are to be determined or as an evaluation problem for existing evaporator, in which the heat transfer area of each effect, A is specified and the product concentration, x_p is determined along with other unknown parameteres.

In the present investigation, x_p has been kept constant at 0.45 (as per the prevailing practice in Indian mills) and input parameters are changed. A change in a given input parameter of an evaporator is likely to affect its steam consumption, SC; steam economy (energy efficiency gain), S; and heating surface area, A. These parameters have been termed as output parameters in subsequent sections. To study the effect of variations of input parameters on the value of aforesaid output parameters of sextuple effect evaporators, a given input parameter is changed while others are held constant, as per standard set of parameters given in Table 6.2. These values are based on operating data of Indian paper mills.

TABLE 6.2

STANDARD SET OF VALUES OF INPUT PARAMETERS

Sl. No.	Parameter	Value
1.	Liquor Feed temperature, T_f	70°C
2.	Liquor Feed concentration, x_f	0.15
3.	Steam temperature, T_s	138°C
4.	Temperature in last effect, T_l	51°C
5.	Liquor Feed rate ,F	18.144 kg/s

It is worth-mentioning that the mathematical models developed in the present investigation are versatile and can be used for the design calculations of the multiple effect evaporators for different black liquors obtained from varying raw materials and pulping processes. For this, one has to replace the existing equations for physico-thermal properties (μ, C_p, ρ, k, and BPR) by those for the black liquor under study.

6.5 SENSITIVITY ANALYSIS

Analysis of the multiple effect evaporators is quite complex because of a large number of input and output parameters involved. In order to assist design engineers in meeting design objectives, performance criteria and also to evolve a plan for control strategy, it is important that the sensitivity analysis should be carried out which can give the order of magnitude of variations in the output parameters due to perturbation in a given input parameter.

Sensitivity Coefficient

The sensitivity coefficient, S_{x-p} is defined, mathematically, as follows :

$$S_{x-p} = (\Delta P/P)/(\Delta x/x) \qquad \text{....(6.1)}$$

where, S_{x-p} is the sensitivity coefficient of a given output parameter P due to perturbation in an input parameter x.

To calculate the above, a typical study has been made as follows : a +5 per cent perturbation is introduced in an input parameter and the variation in output parameter is noted. The sensitivity coefficient is then computed from Eq. 6.1. The input and output parameters for the sensitivity analysis are given in Table 6.3.

Sensitivity coefficients for the five input parameters, T_s, T_f, T_1, x_f and F, and fifteen output parameters, listed in Table 6.3 for a sextuple effect evaporator employing backward feed sequence are computed at the standard set of values of input parameters shown in Table 6.2.

TABLE 6.3

INPUT AND OUTPUT PARAMETERS FOR SENSITIVITY ANALYSIS

Sl. No.	Input parameters	Output parameters
1.	Liquor feed temperature, T_f	Liquor concentration in each effect (x_i, i = 1, ...6)
2.	Liquor feed concentration, x_f	Heating surface area of each effect, A, or the product concentration, x_p
3.	Steam temperature, T_s	Steam consumption, SC
4.	Temperature in the last effect, T_l	Steam economy, S
5.	Liquor feed rate, F	Saturation temperatures in each effect (T_i, i=1,....6)

These computations are done under two cases :

Case-I : where the product concentration, x_p, is fixed at 0.45 (The results are tabulated in Table 6.4).

Case-II : where heating surface area of each effect is kept constant at 392.6 m^2. (The results are tabulated in Table 6.5).

TABLE 6.4
SENSITIVITY COEFFICIENTS FOR INPUT OUTPUT PARAMETERS
(x_p = 0.45)

	PARAMETERS														
Input	Output														
	Saturation temperature of effects						Liquor concentration								
	1	2	3	4	5	6	1	2	3	4	5	6	A	SC	S
T_s	.85520	.74856	.61406	.44776	.22646	.00000	.00000	-.13880	-.17525	-.16148	-.11527	-.05377	-1.83402	.60188	-.58211
T_f	.00000	-.00310	-.00530	-.00410	-.00249	.00000	.00000	.04758	.07381	.09096	.10248	.11061	-.15291	-.15377	.15528
T_l	.12740	.22457	.34384	.49967	.72203	1.00046	.00000	.01948	.01451	-.00360	-.02947	-.05840	.84751	-.13060	.13173
x_f	.00000	-.01773	-.03141	-.03909	-.02841	.00000	.00000	.39271	.61521	.75888	.85999	.93317	-.30023	-.30420	-.19883
F	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	1.00000	.99999	.00000

TABLE 6.5

SENSITIVITY COEFFICIENTS FOR INPUT OUTPUT PARAMETERS

($A = 392.6\ m^2$)

Input	PARAMETERS														
	Output														
	Saturation temperature of effects						Liquor concentration								
	1	2	3	4	5	6	1	2	3	4	5	6	A	SC	S
T_s	.73308	.62385	.53511	.42699	.24883	.00000	3.21223	1.70231	.97827	.56298	.30545	.13561	......	.93035	.33130
T_f	-.00686	-.00932	-.00884	-.00410	-.00249	.00000	.21026	.17875	.15944	.14578	.13494	.12524	......	-.11891	.21814
T_l	.15917	.25916	.36284	.49967	.70774	1.00046	-1.09573	-.67475	-.44248	-.29732	-.20177	-.13709	......	-.32995	-.23073
x_f	-.00697	-.02565	-.03594	-.03648	-.02525	.00000	.35870	.63064	.77520	.86419	.92251	.96137	......	-.23096	-.06610
F	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	......	.99999	.00000

It is to be noted that a negative sign associated with a sensitivity coefficient indicates a decrease in output parameter with an increase in input parameter. An examination of Table 6.4 reveals that out of the input parameters investigated, the perturbation in steam temperature T_s affects the output parameter significantly. The sensitivity coefficient for the heating surface area, S_{Ts-A}, has the maximum value among the values for all the other output parameters listed in first row of Table 6.4. In other words, it has the maximum sensitivity to the perturbation in T_s. The next sensitive parameters in decreasing order are saturation temperatures in the first, second and third effects, steam consumption, steam economy followed by saturation temperatures in fourth and fifth effects. The least affected parameters are liquor concentrations in various effects.

It is interesting to note that the sensitivity coefficients, S_{Ts-xi}, are negative thereby indicating that concentrations in second to sixth effects decrease with increase in steam temperature T_s. This is due to the fact that with increase in steam temperature, area of each effect decreases (S_{Ts-A} =-1.83402) and hence the outlet concentration in each effect decreases, except in the first effect (the concentration of black liquor in first effect is x_p and is constant at a value equal to 0.45). As a result, the concentration of liquor entering the first effect goes down significantly and thus the evaporation load on the first effect increases.

A perusal of Table 6.4 reveals that the sensitivity coefficient of any input parameter with the product concentration ($x_p = x_1$) is zero. This is so because x_p is held constant and perturbation in the input parameters are unable to cause any variations in it. Further, the sensitivity coefficients of all the input parameters except T_1 on the temperature of the last effect T_6 are zero, as the vacuum in the last effect is also kept fixed.

However, S_{T1-T6} assumes a non-zero value, because this causes the temperature of the last effect to vary , instead of being maintained constant.

An observation of Table 6.4 shows that T_s affects almost all the output parameters except x_1 and T_6 (which are kept constant from operational view- points). It can also be seen that T_s

affects T_1 to the maximum extent and this influence diminishes as one moves forward from effect to effect, coming to a value zero in the sixth effect. It can also be seen that T_1 affects the value T_6 the largest and the influence diminishes as one moves from effect to effect towards the first.

It can be concluded that the sensitivity coefficient due to any input parameter is more pronounced at the point of entry of that parameter to the multiple effect evaporator and decreases progressively as one moves away from the point of entry.It can be seen that the feed concentration x_f affects the concentrations of all the effects to a large extent. However, this influence diminishes as one moves from the sixth effect (which is also the entry point of feed) towards the first effect.

A further observation is that the feed flow rate, F does not influence any of the output parameters except SC and A. The values of S_{F-A} and S_{F-SC} are close to unity , indicating that a certain percentage increase in F increases SC and A by the same percentage.

Table 6.5 shows that the sensitivity coefficient for all the input parameters in relation to x_1 ($x_p = x_1$) hold different non-zero values. This is quite expected as in this case the area A is maintained fixed and x_p is allowed to vary. Further, the sensitivity coefficients for all the input parameters except T_1, with respect to the output parameter T_6 are zero. This is so because T_6 ($T_1 = T_6$) is maintained at a fixed value due to the fact that a constant vacuum in the last effect is always maintained in a mill. However, once again S_{T1-T6} has a non-zero value due to the reasons discussed earlier for case-I.

Values of the sensitivity coefficient due to T_1 on the saturation temperatures of the effects 6,5 and 4 are identical for the case-I and case-II, as can be seen from Tables 6.4 and 6.5.

Finally, it can be concluded that the perturbation in steam temperature, T_s produces considerable changes in almost all the output parameters and thus can be effectively used as a tool to meet design objectives and performance criteria.

6.6 EFFECT OF INPUT PARAMETERS ON STEAM CONSUMPTION, STEAM ECONOMY AND HEATING SURFACE AREA OF THE EVAPORATOR EMPLOYING BACKWARD FEED

As has been discussed earlier under Section 6.5, the output parameters SC, S, and A are quite sensitive to variations in the temperature of steam, T_s. Therefore, yet another look can be taken to understand the dependence of SC, S and A on T_s.

Figures 6.1, 6.2 and 6.3 have been prepared to show the effects of T_s on SC, S and A, respectively with T_1 as parameter. These plots are for some specified values of x_f, x_p, F and T_f. The following observations can be made from Figures 6.1 to 6.3 :

(a) With an increase in T_s, there is increase in the value of SC for all the values of T_1. This behaviour is attributed to the reduction in latent heat of condensation of steam

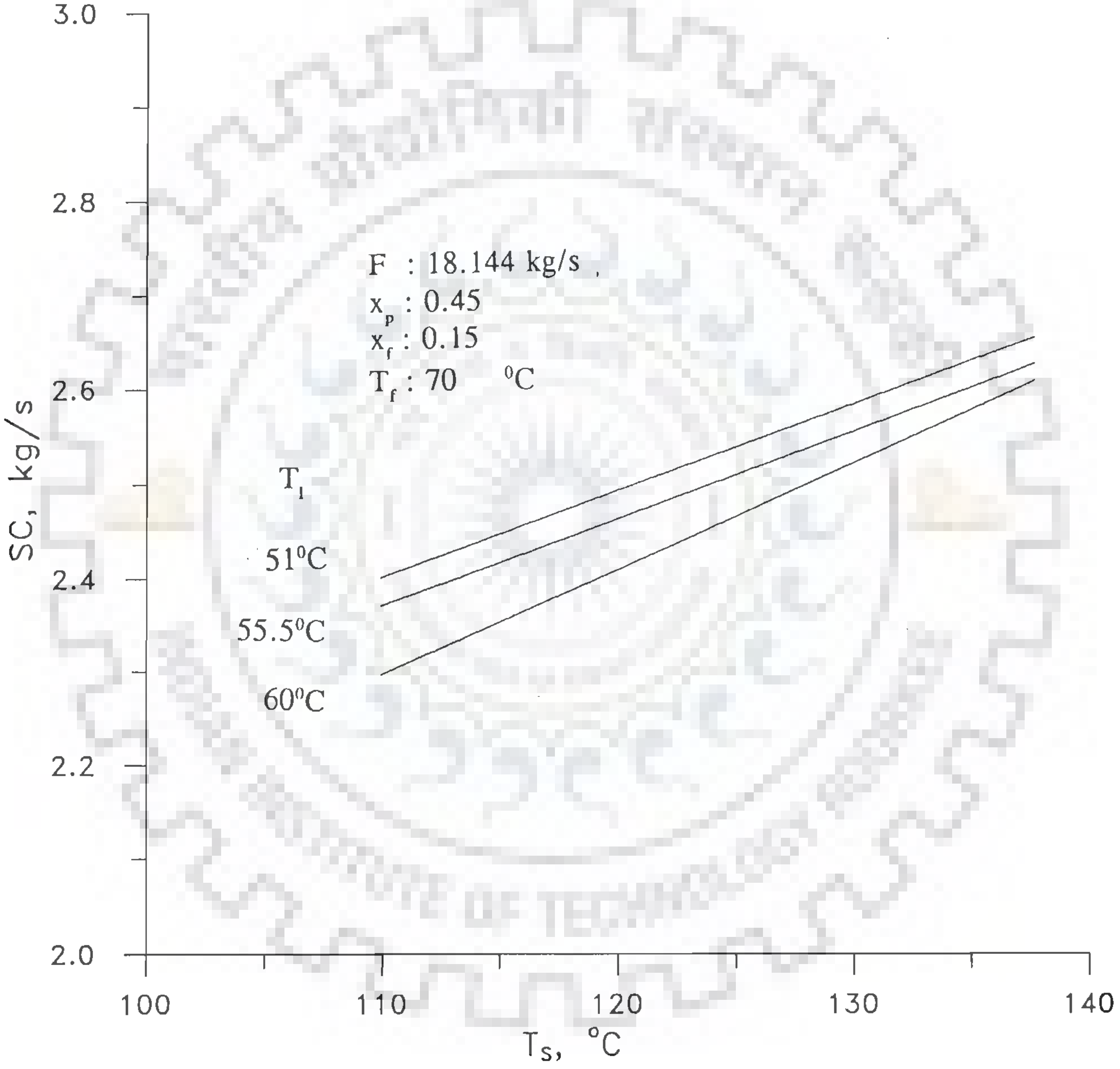

Fig. 6.1 Effect of T_S on SC with T_l as parameter for backward feed

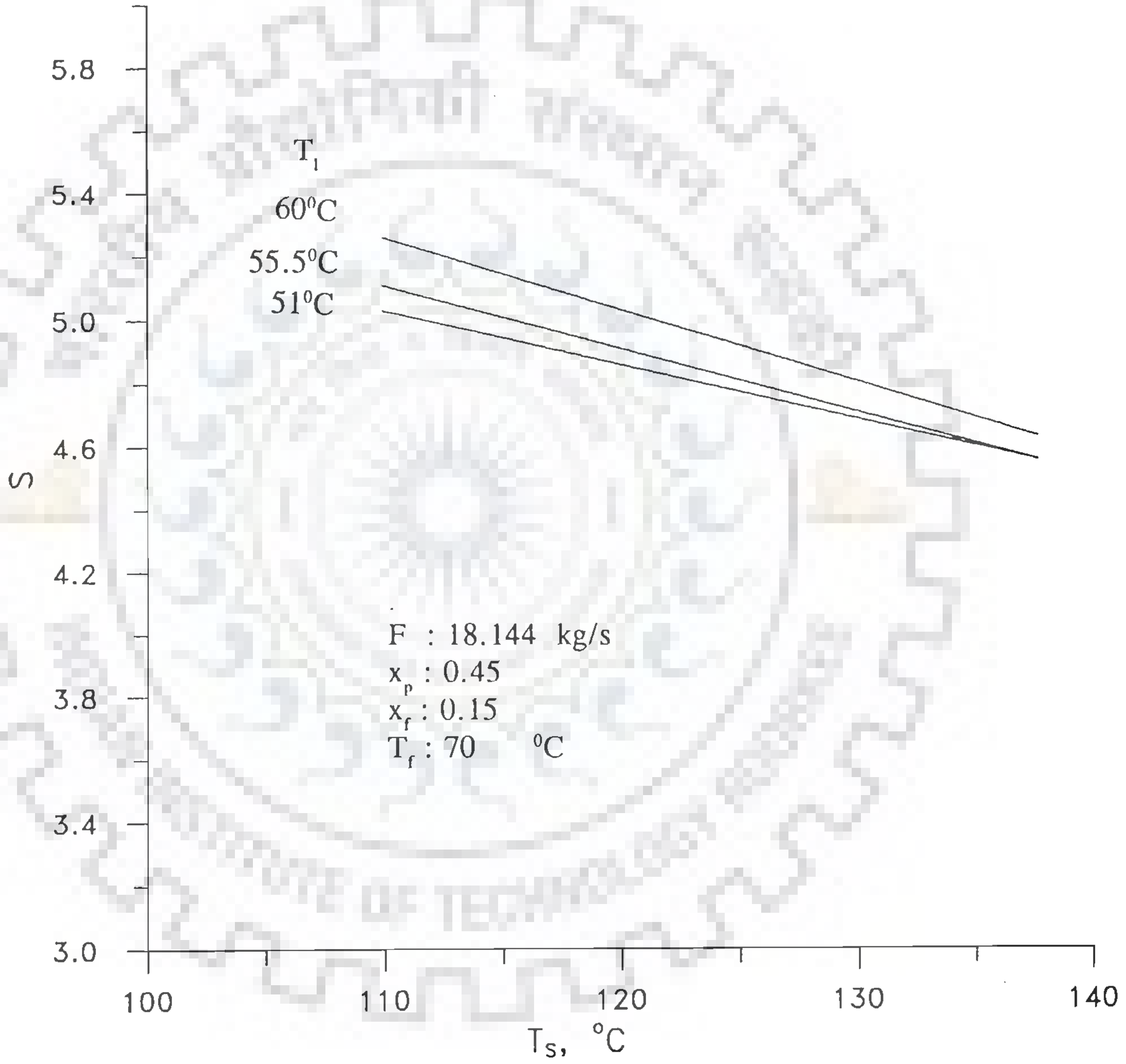

Fig. 6.2 Effect of T_S on S with T_l as parameter for backward feed

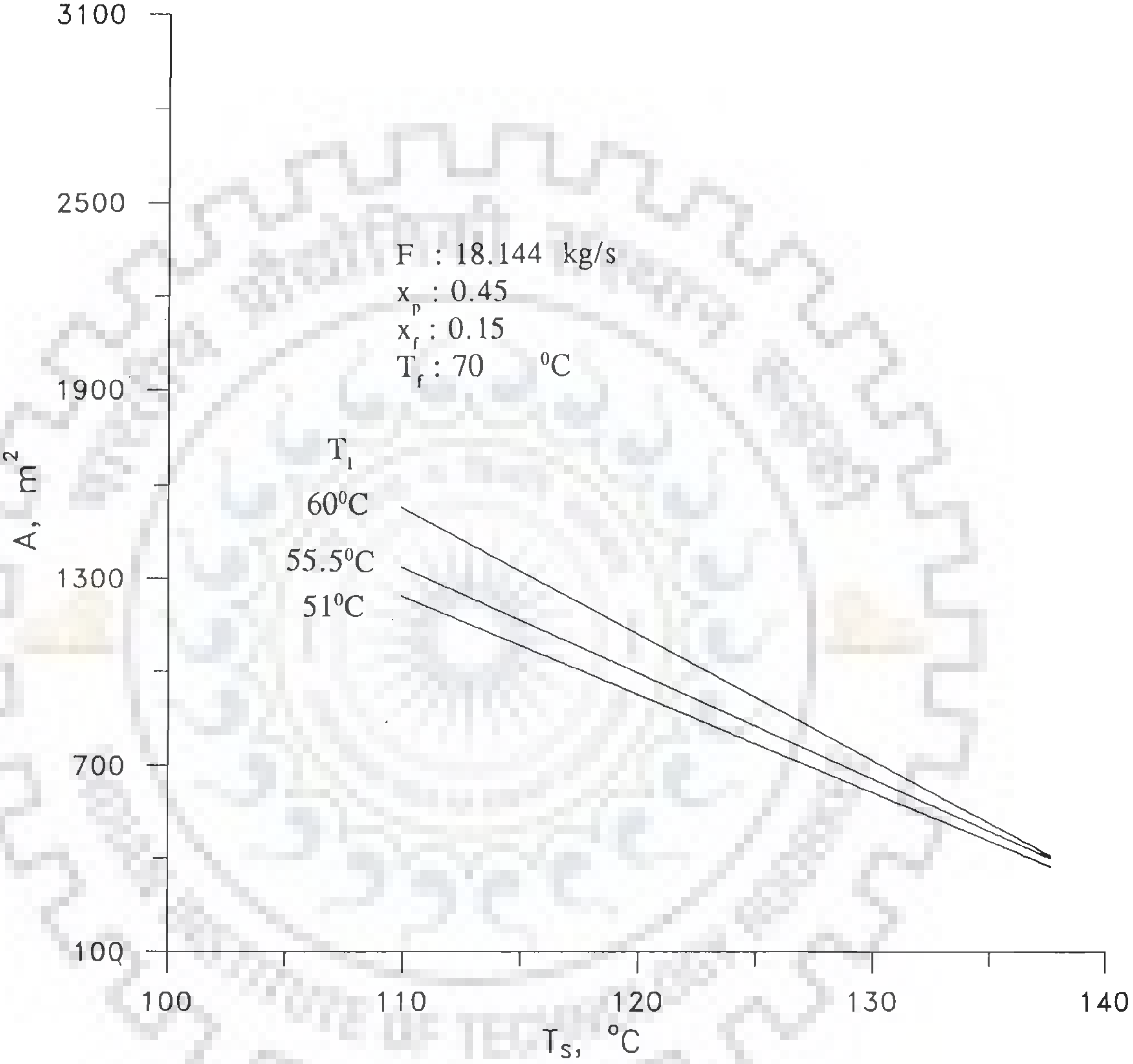

Fig. 6.3 Effect of T_S on A with T_l as parameter for backward feed

with the increase in its saturation temperature, increase in temperature difference between steam and liquor and decrease in heating surface area which raises the evaporation load on first effect, as discussed in Section 6.5. Hence, for the evaporation of a given quantity of water from the black liquor, the steam consumption should normally increase.

(b) When T_l is raised, the value of SC decreases (but A increases) for the entire range of T_s, which means that higher the temperature in the last effect, the smaller is the steam consumption.

Values of the sensitivity coefficients S_{Ts-SC} and S_{Tl-SC} are 0.60188 and - 0.13060, respectively which indicate that SC increases steeply with an increase in T_s, when compared to its decrease with increase in T_l. This supports the nature of plots in Figure 6.1.

(c) For a given quantity of evaporation of water, as would be expected, there is reduction in S with rise in T_s for all values of T_l. The increase in S with increase in the value of T_l for a given T_s is also an expected behaviour in view of the nature of plots in Figure 6.1.

(d) The heating area increases with decrease in T_s and increase in T_l values. Thus, any increase in requirements of heating surface by employing live steam at slightly lower saturation temperature can be countered by decrease in the value of T_l. This needs maintenance of increased vacuum in the last effect. However, as can be seen from Table 6.4, the sensitivity coefficients S_{Ts-A} and S_{Tl-A}, have the values -1.83402 and +0.84751, respectively. The smaller absolute value of S_{Tl-A} explains the comparative nearness between the different curves.

Figures 6.4, 6.5 and 6.6 have been prepared to show quantitative effect of T_s on SC, S and A, respectively vis-a-vis that of T_f.

There is a drop in the value of SC with decrease in T_s and increase in T_f. This is also on the expected lines, as increase in T_f raises the enthalpy of the feed entering the evaporator, thereby reducing its total heat requirements and hence the resultant SC. The values of S_{Ts-S} and S_{Tf-S} are -0.58211 and +0.15528, respectively in Table 6.4. Thus, in Figure 6.5, the more steep decrease in steam economy with increase in T_s than its trend of increase with increasing T_f values is expected one. An increase in T_f value by 10°C, in the upper range of 80-90°C, boosts the steam economy much more appreciably than what is achieved for a similar increment of T_f in the lower ranges.

Figure 6.6 brings out very vividly the role of a low value of S_{Tf-A} when compared to value of S_{Ts-A}. This is seen both from the proximity among the different curves and their comparatively higher slopes.

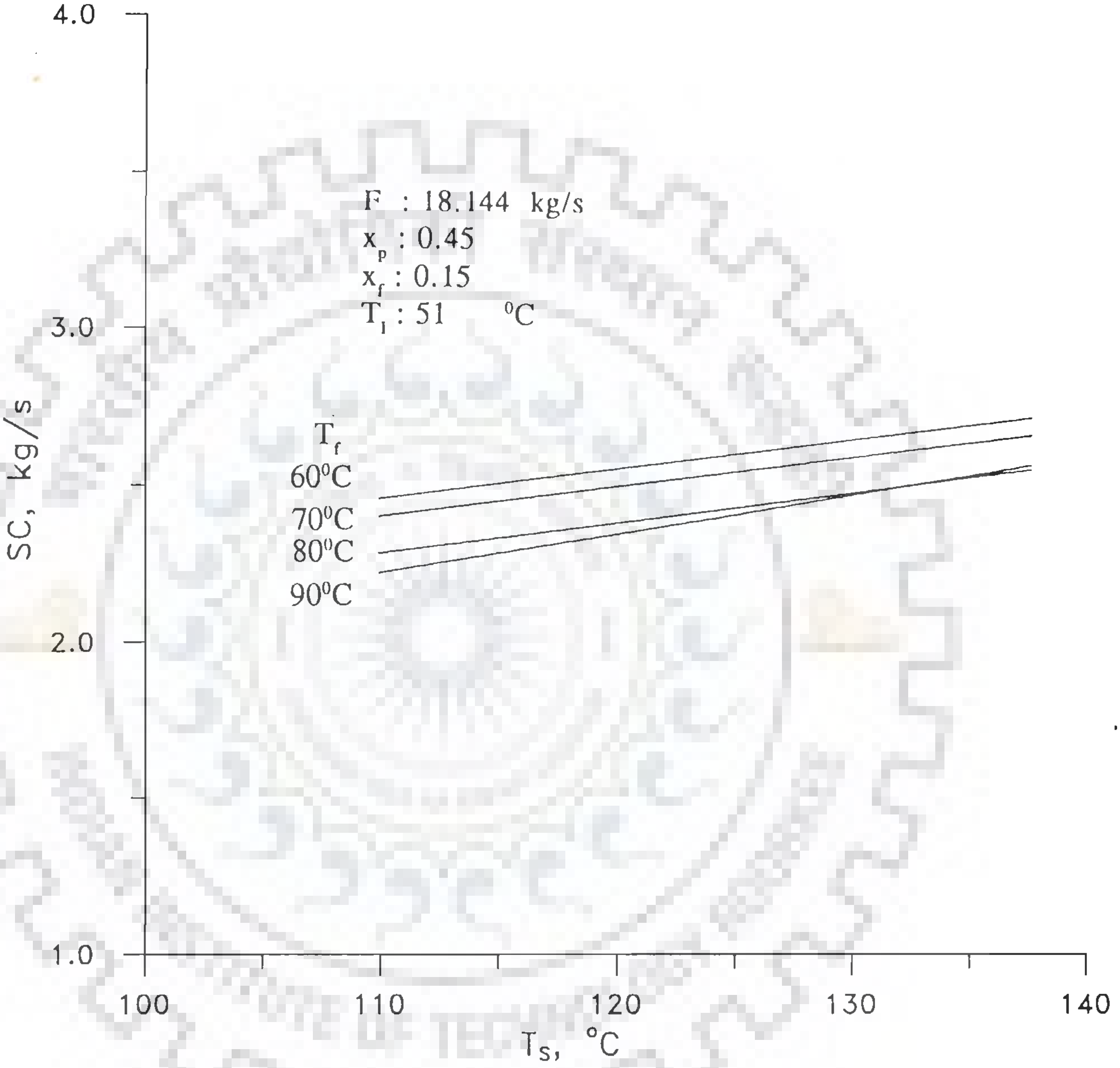

Fig. 6.4 Effect of T_S on SC with T_f as parameter for backward feed

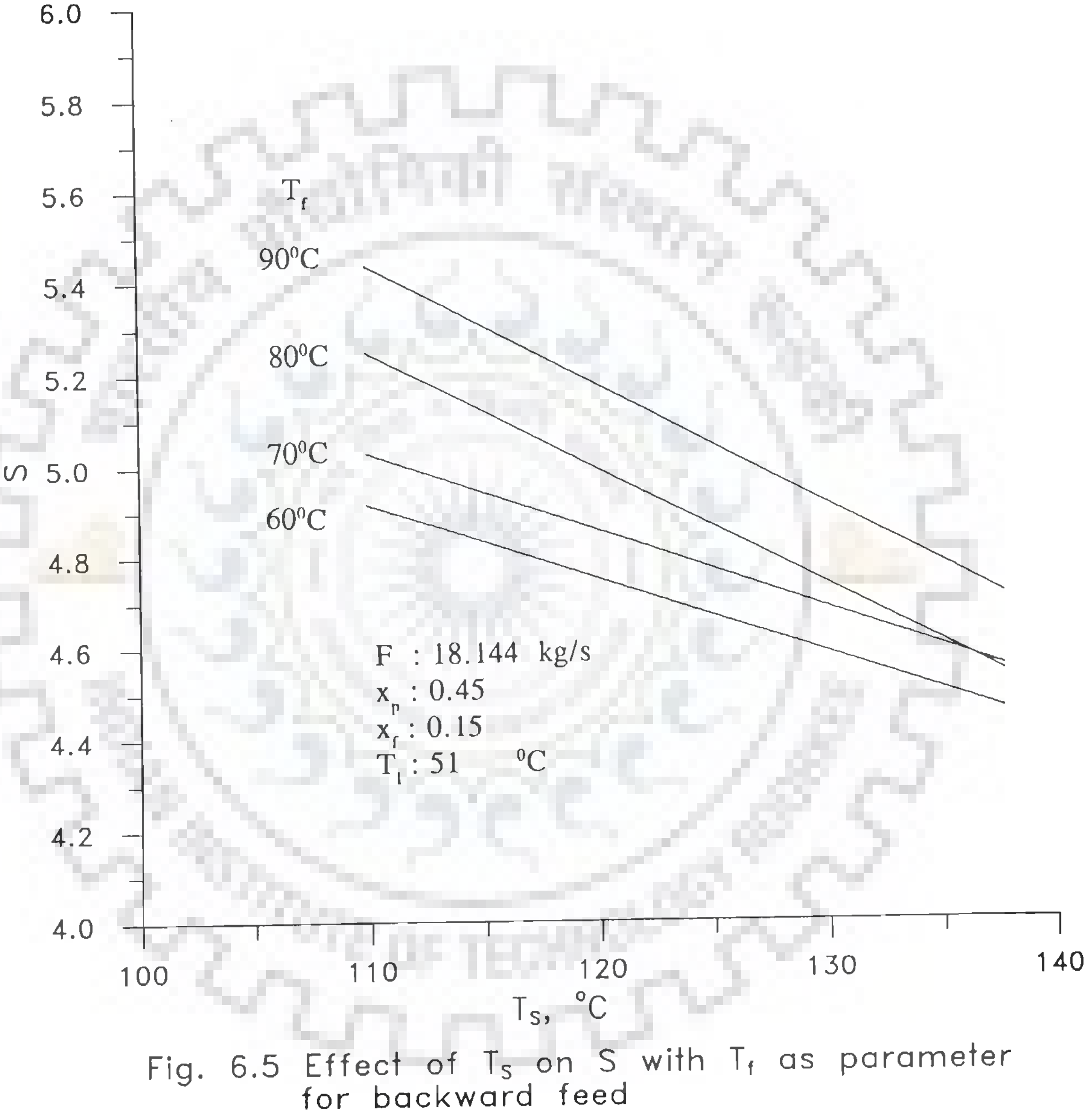

Fig. 6.5 Effect of T_S on S with T_f as parameter for backward feed

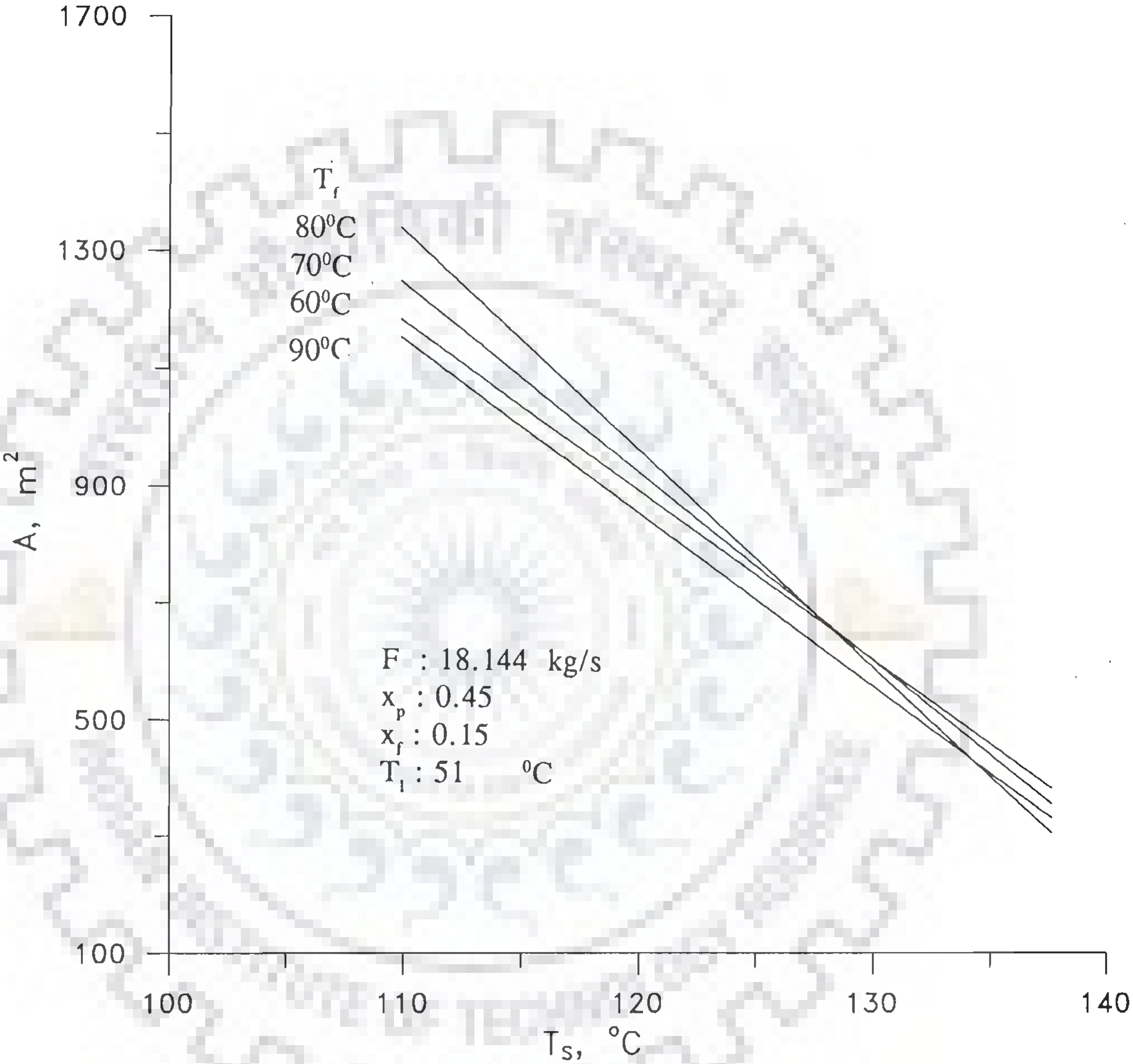

Fig. 6.6 Effect of T_S on A with T_f as parameter for backward feed

Figures 6.7, 6.8 and 6.9 are embodiment of the interactions of T_s and x_f on SC, S and A, respectively. From Figure 6.7, it is noted that SC increases with T_s but exhibits a reverse trend with respect to x_f. Evidently, for the specified parameters, shown on the plot, the evaporation capacity goes down with increasing values of x_f, thereby necessitating lower steam consumption. Figure 6.8 demonstrates the negative sensitivity coefficient for both the input variables T_s and x_f, as can be seen from Table 6.4 (S_{Ts-S} = - 0.58211, and S_{xf-S} = -0.19883).

Figure 6.9 reveals that A decreases very steeply with rise in T_s. However, the effect of x_f on A is not appreciable. This is reflected from the values S_{Ts-A} (=-1.83402) and S_{xf-A} (=-0.30023).

Figures 6.10 through 6.12 are intended to show the effect of T_s on SC, S and A, with F as parameter. The value of SC increases somewhat slowly with T_s, but quite appreciably with F. It is quite difficult in Figure 6.11 to locate precisely four different curves corresponding to the different values of F, indicating that S does not change with F.

The curves in Figure 6.12 show that area is a strong function of both T_s and F. The values of sensitivity coefficients S_{Ts-A} (=-1.83402) and S_{F-A} (= 1.000) reinforce the above observations. However, the curves diverge with decreasing values of T_s.

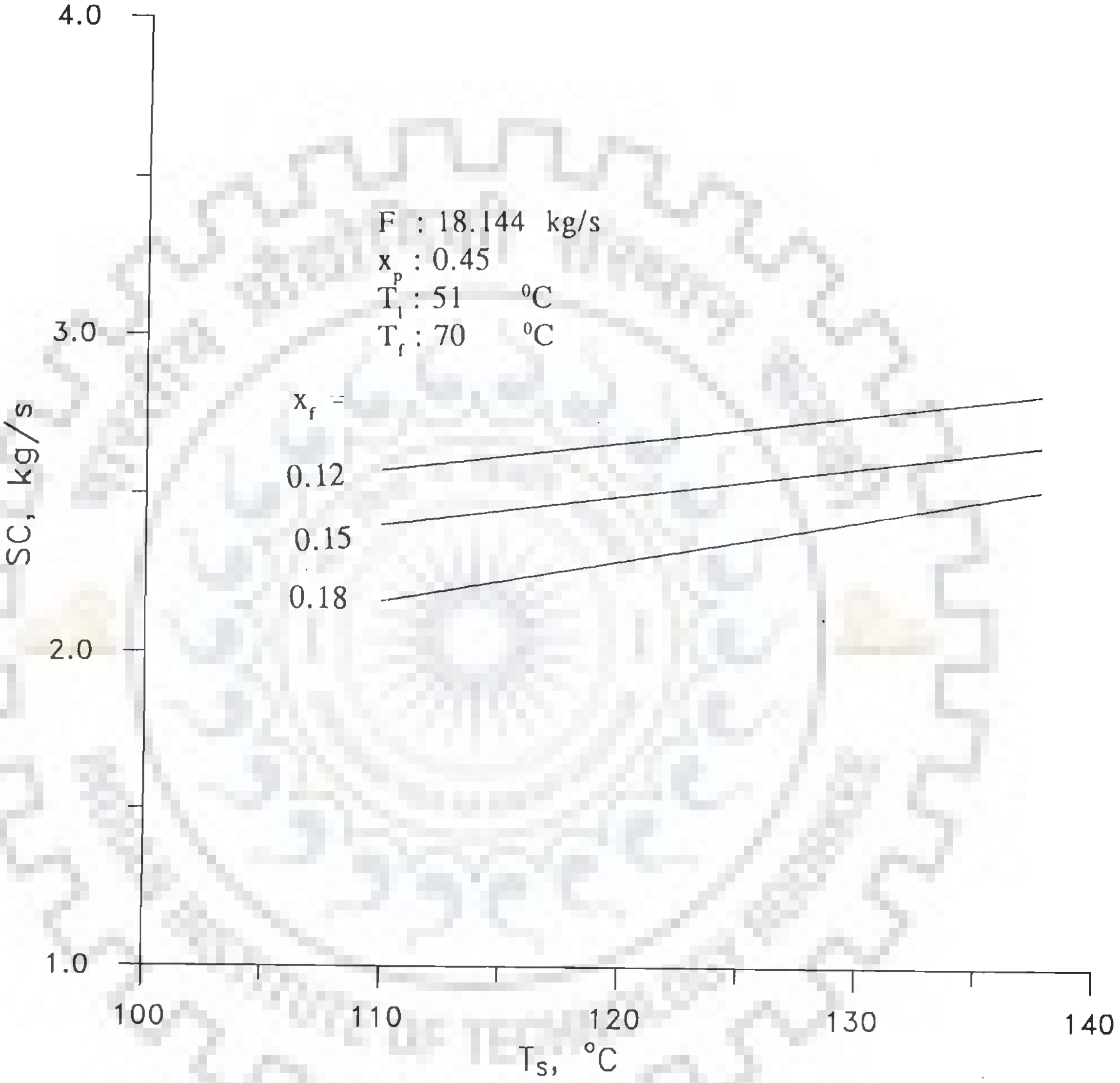

Fig. 6.7 Effect of T_S on SC with x_f as parameter for backward feed

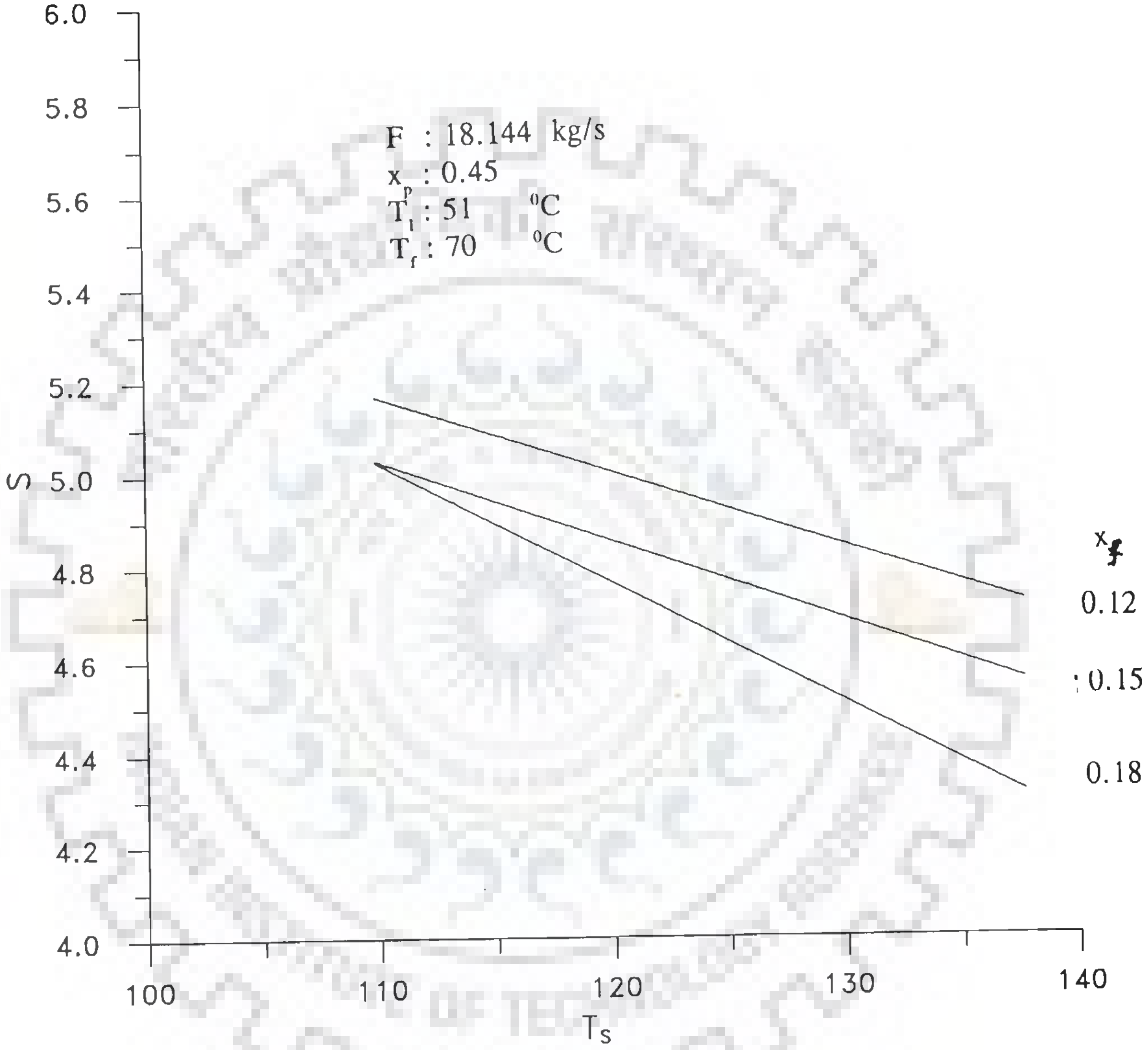

Fig. 6.8 Effect of T_S on S with x_f as parameter for backward feed

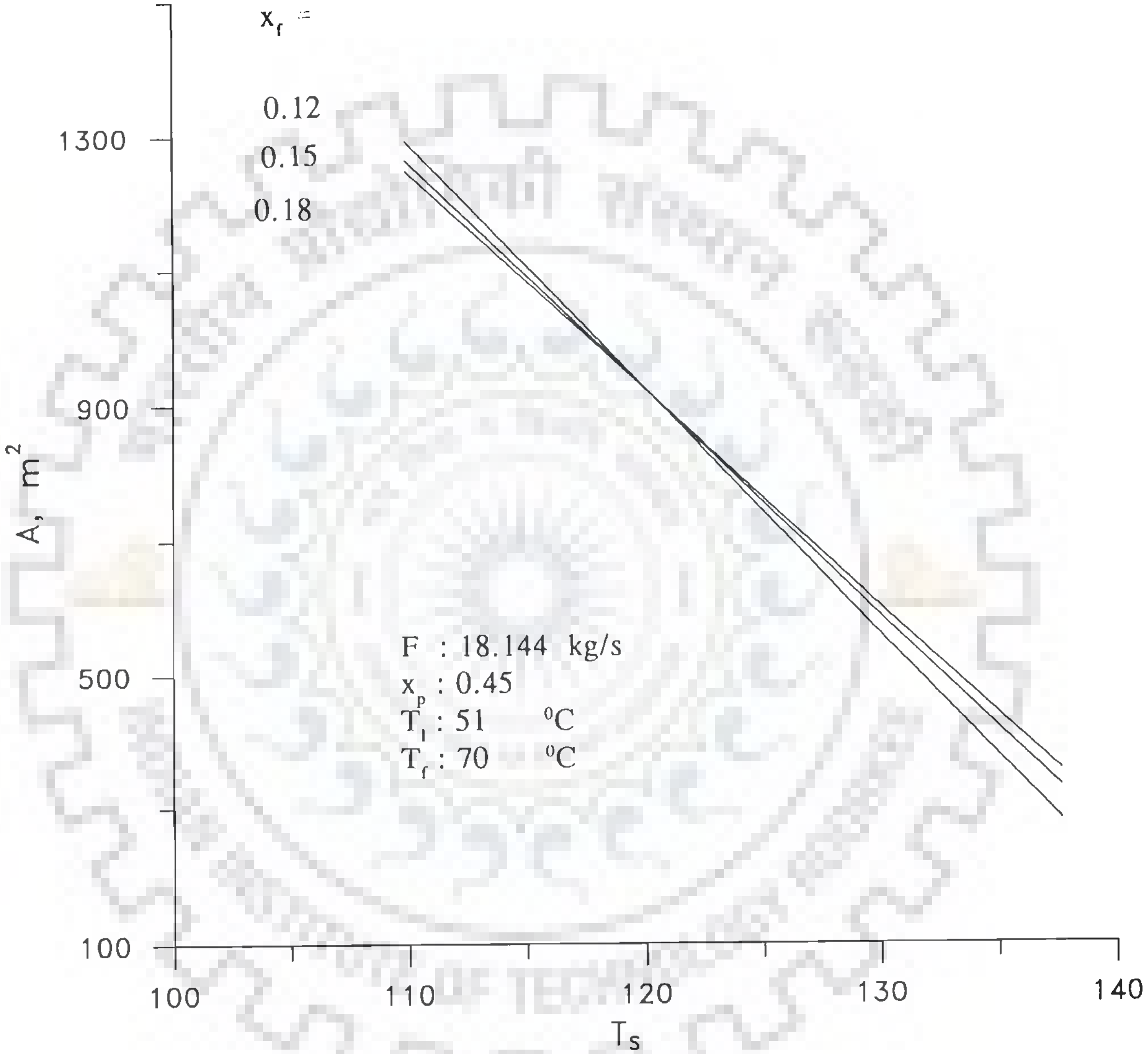

Fig. 6.9 Effect of T_s on A with x_f as parameter for backward feed

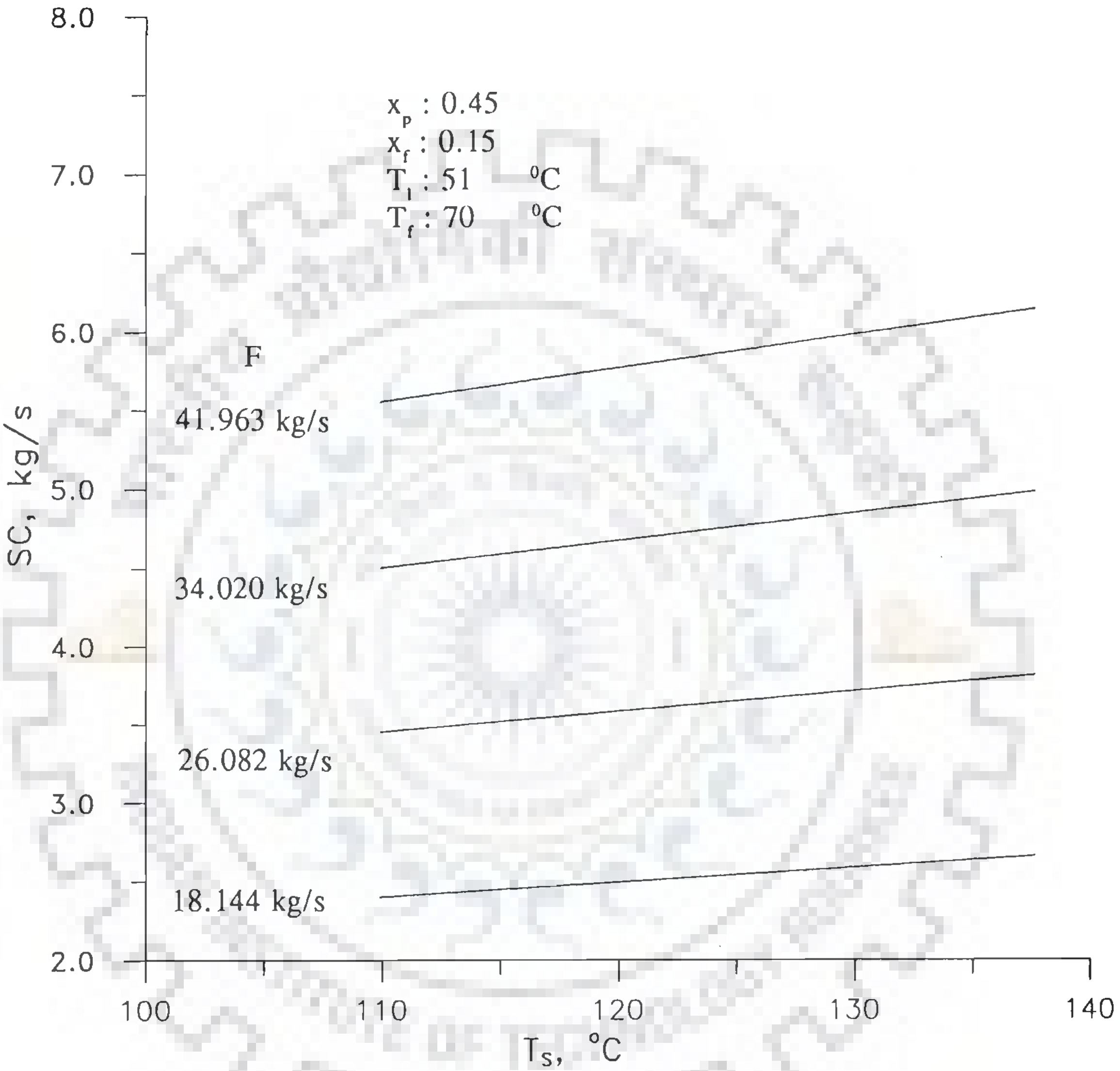

Fig. 6.10 Effect of T_S on SC with F as parameter for backward feed

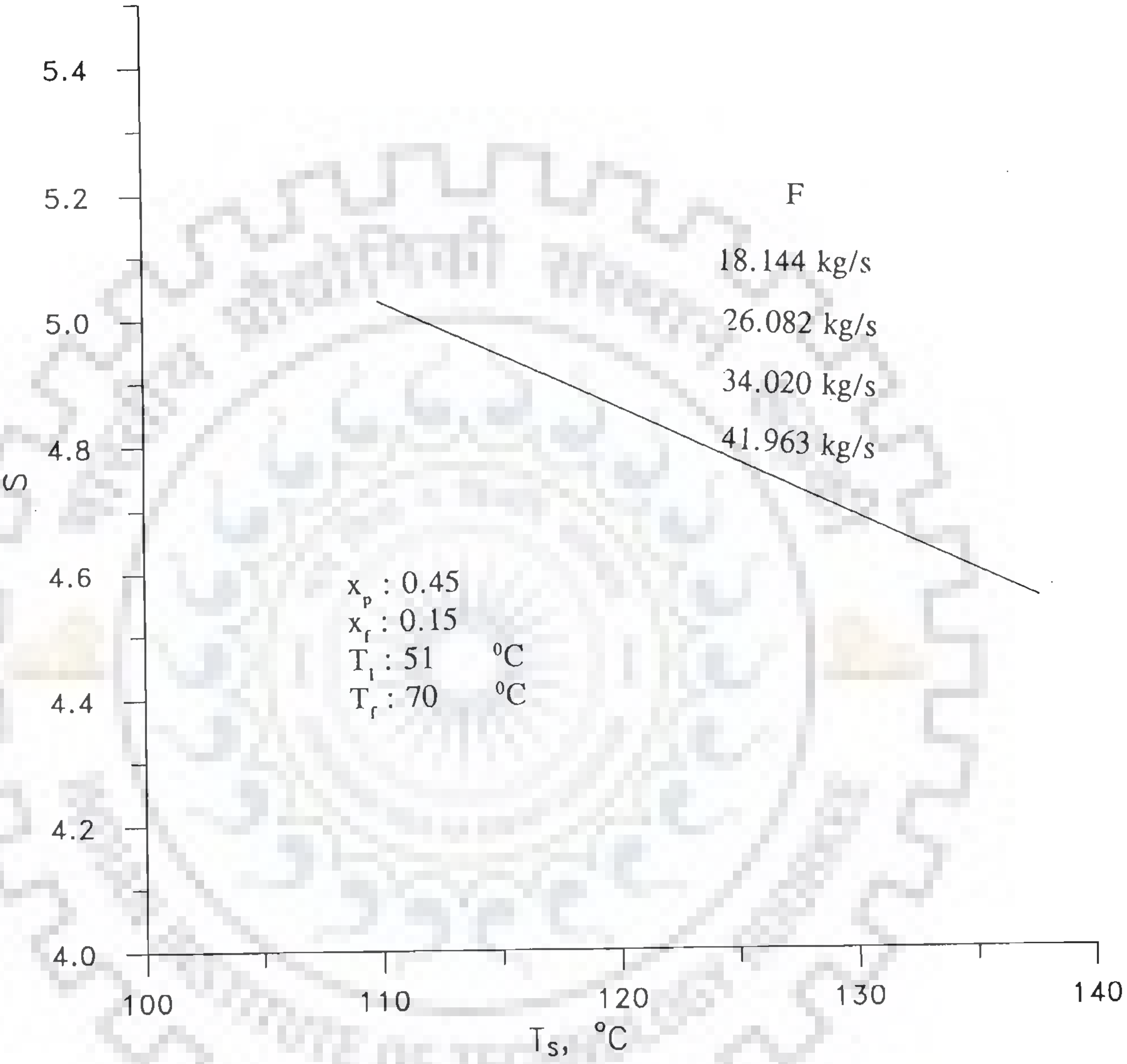

Fig. 6.11 Effect of T_S on S with F as parameter for backward feed

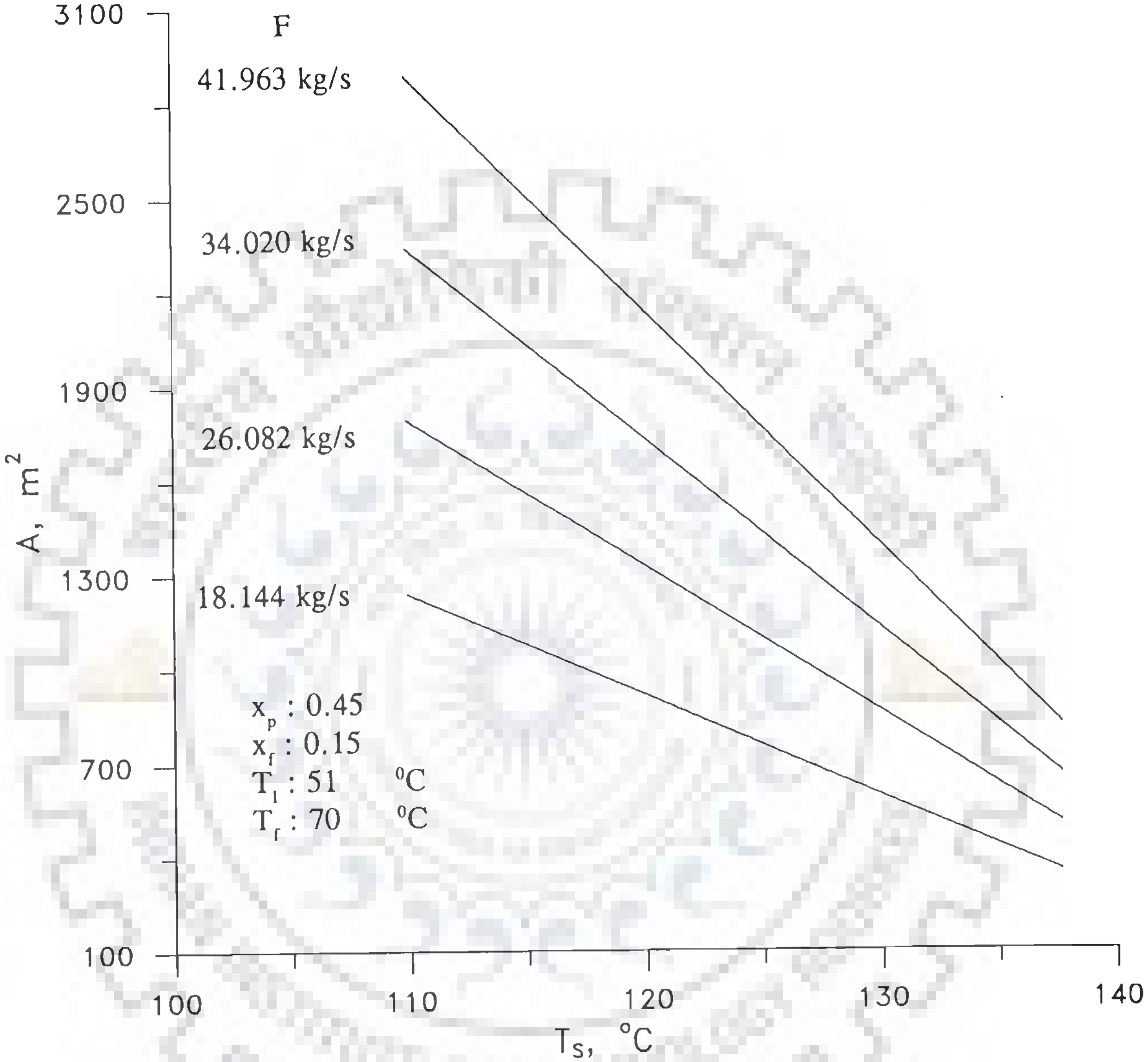

Fig. 6.12 Effect of T_S on A with F as parameter for backward feed

6.7 SEXTUPLE EFFECT EVAPORATOR EMPLOYING BACKWARD FEED WITH AND WITHOUT SPLITS OF FEED AND STEAM

First of all, it is important to mention that the computed values of steam economy in this and subsequent sections for different liquor flow sequences analysed will be slightly lower than that expected in actual plant practice. This is primarily because of the fact that contributions of the product-and condensate-flashes have not been accounted for in this study. However, it is expected that this will not materially affect the results and conclusions drawn there-on. In fact, the contribution of flashes is expected to be identical for all the liquor flow sequences considered and hence their absence in a comparative study, such as the present investigation, shall not make any difference.

In this study, the following liquor flow sequences for the sextuple effect evaporator have been considered. They have been marked as sequences **a**, **b** and **c** in Table 3.2 of Chapter 3. However, they are reproduced below for ready reference :

(i) Backward feed without any split - sequence **a**

(ii) Backward feed with equal split of feed among the fifth and sixth effects - sequence **b**, and

(iii) Backward feed with equal split of feed among the fifth and sixth effects and the split of steam in the two bodies of the first effect - sequence **c**

To appreciate the effects of input parameters on the output parameters, the following sections have been included.

6.7.1 Effect of Liquor Feed Temperature

Figure 6.13 has been drawn to represent the effect of liquor feed temperature, T_f on steam consumption, SC, for all the sequences **a**, **b** and **c**. From the plot, it is noted that for values of T_f, up to about 58°C, steam consumption is the lowest corresponding to sequence **a**, while beyond this value of T_f, sequence **c** achieves the lowest values of steam consumption. An implication of this would be that liquor flow sequence **c** is the best of the sequences considered with regard to steam consumption.

Figure 6.14 has been drawn to represent the effect of feed temperature, T_f on steam economy, S for the above mentioned liquor sequences.

From this plot, the following note-worthy facts are observed:

(i) S increases with rise in T_f for all the three sequences. However, the trend of change differs with the sequence.

(ii) Value of S is the highest for sequence **a** followed by sequence **c** and sequence **b** in descending order for feed temperatures below 58°C. However, as T_f increases the best economy is for sequence **c**. Beyond a T_f of about 68°C, the next best is for sequence **b**, while the sequence **a** has the lowest value of S. It

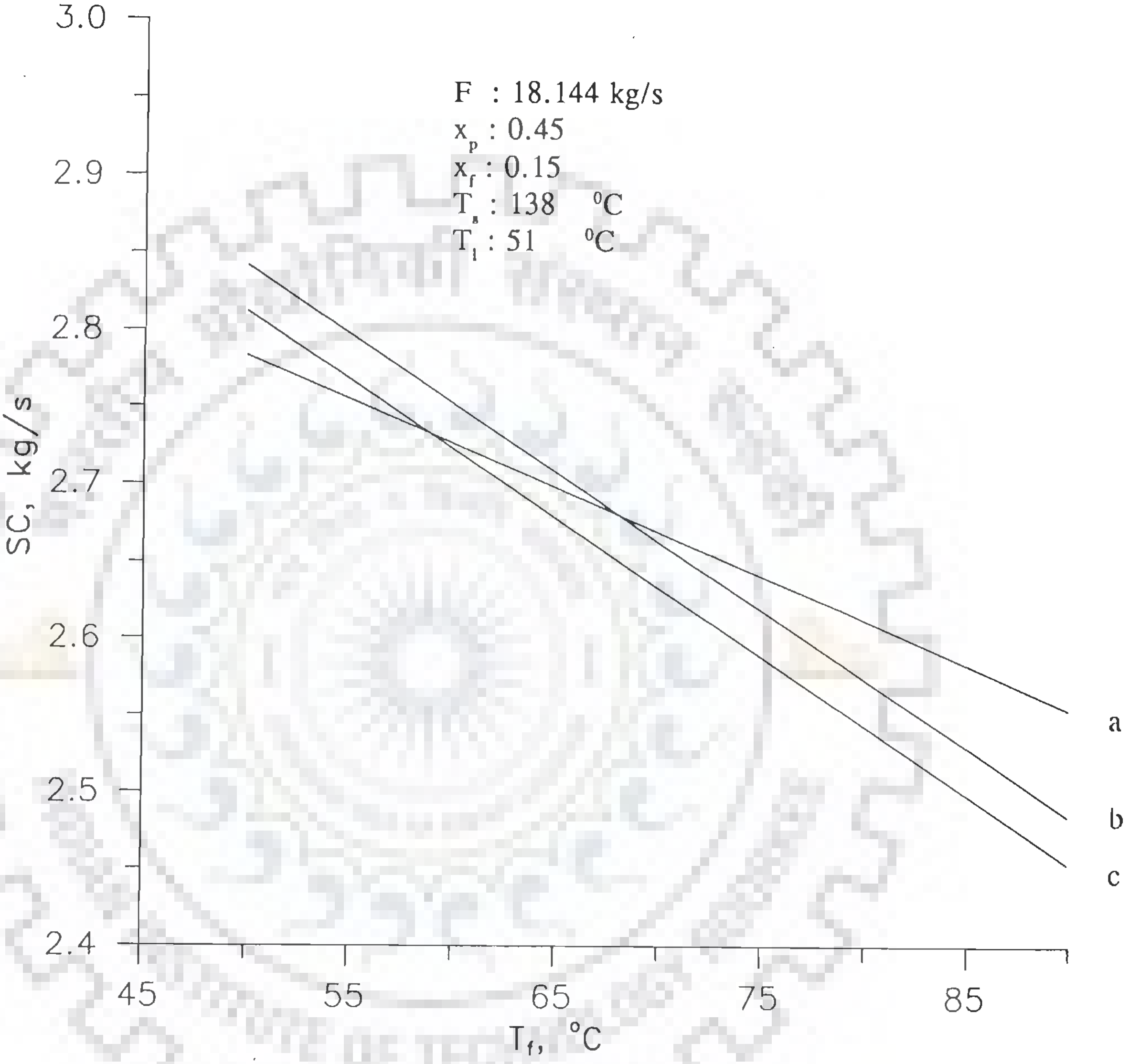

Fig. 6.13 Effect of T_f on SC for sequences a, b & c

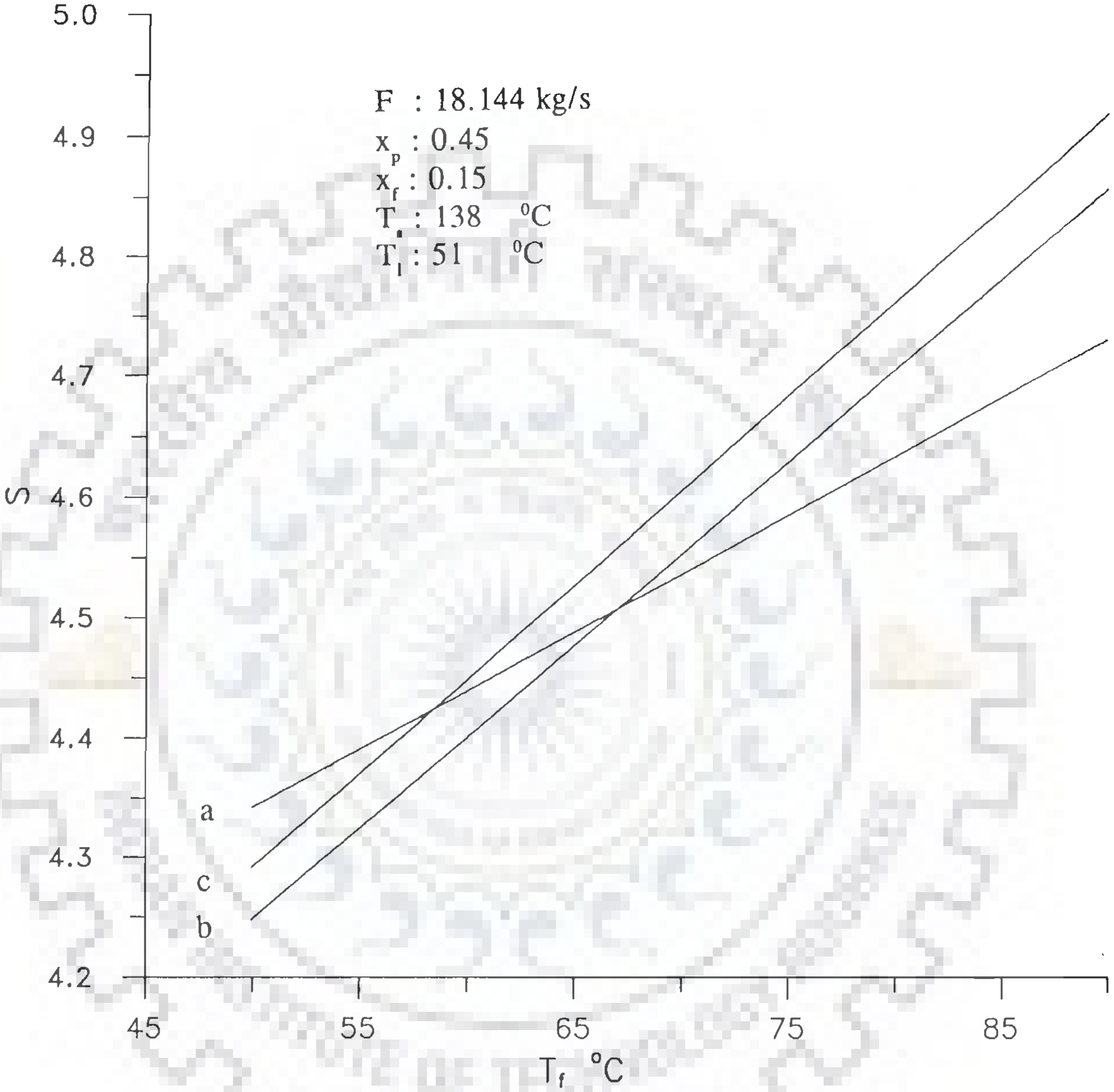

Fig. 6.14 Effect of T_f on S for sequences a, b & c

is therefore concluded that to achieve the best steam economy, the feed and steam both should be split.

Figure 6.15 is a similar plot which demonstrates the variation of heating area A, due to changes in T_f. From the plot it is clearly brought out that the heating area of each effect is the minimum for sequence **c** over the entire range of T_f investigated.

From the plots of Figures 6.13 through 6.15 it can be concluded that for the feed temperatures of about 58^0C or more, normally encountered in the Indian pulp and paper mills, the backward feed sequence with split of feed and steam corresponds to the lowest steam consumption, along with the highest steam economy and the lowest heating area.

6.7.2 Effect of Liquor Feed Concentration

Figures 6.16 through 6.18 depict the response to changes in liquor feed concentration. It is clearly seen that steam consumption and steam economy decrease continuously with an increase in the value of x_f for all the three flow sequences. Further, the value of SC is slightly higher and that of S only marginally smaller for the sequence **c**, when compared with the sequences **a** and **b**, over the entire range of x_f. The magnitude of A decreases quite feverishly with increasing values of x_f for the sequences **a** and **c**, while it is comparatively steeper for the sequence **b**. It is also noted that A is the minimum for sequence **c** for values of x_f upto 0.16.

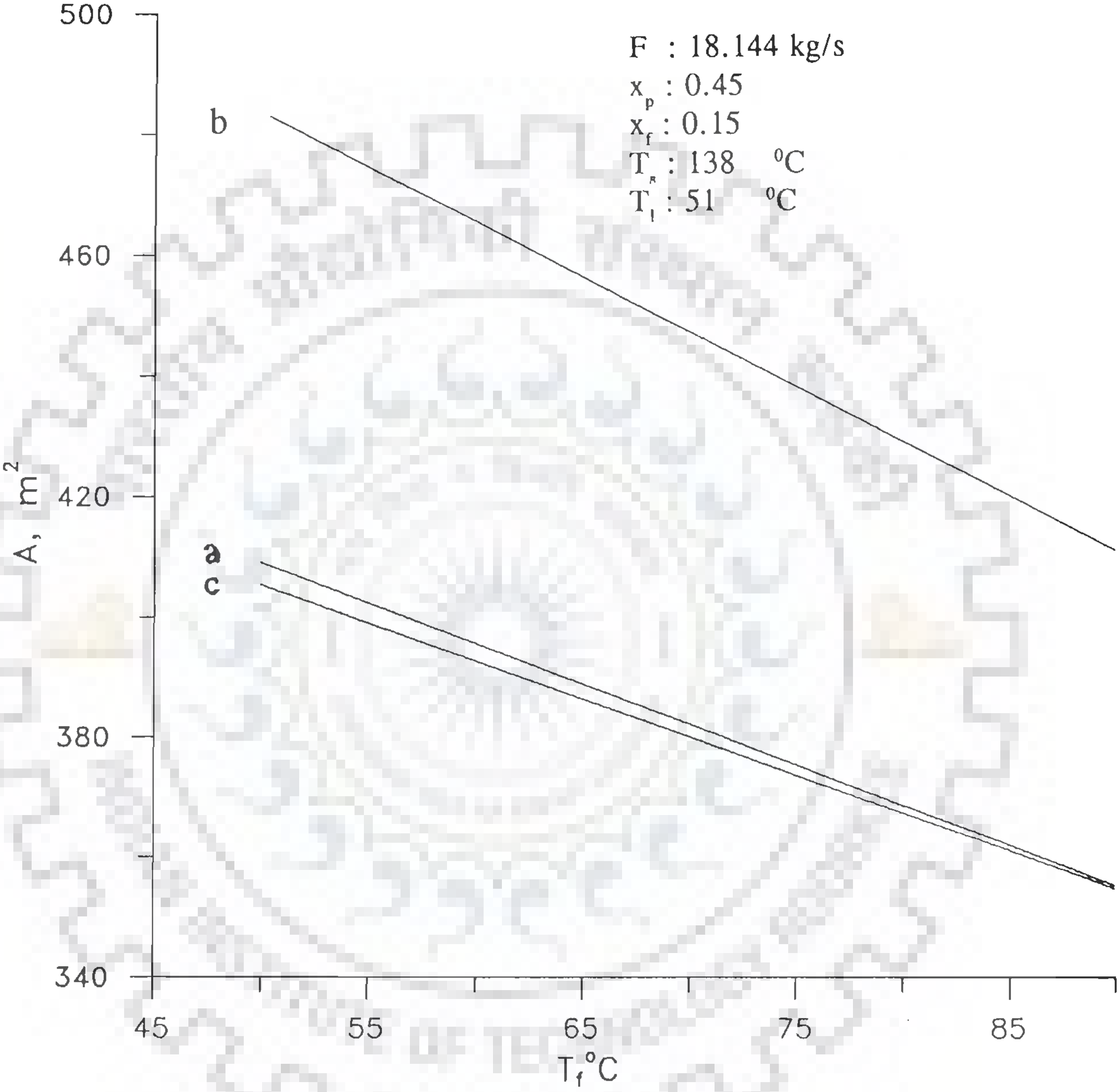

Fig. 6.15 Effect of T_f on A for sequences a,b & c

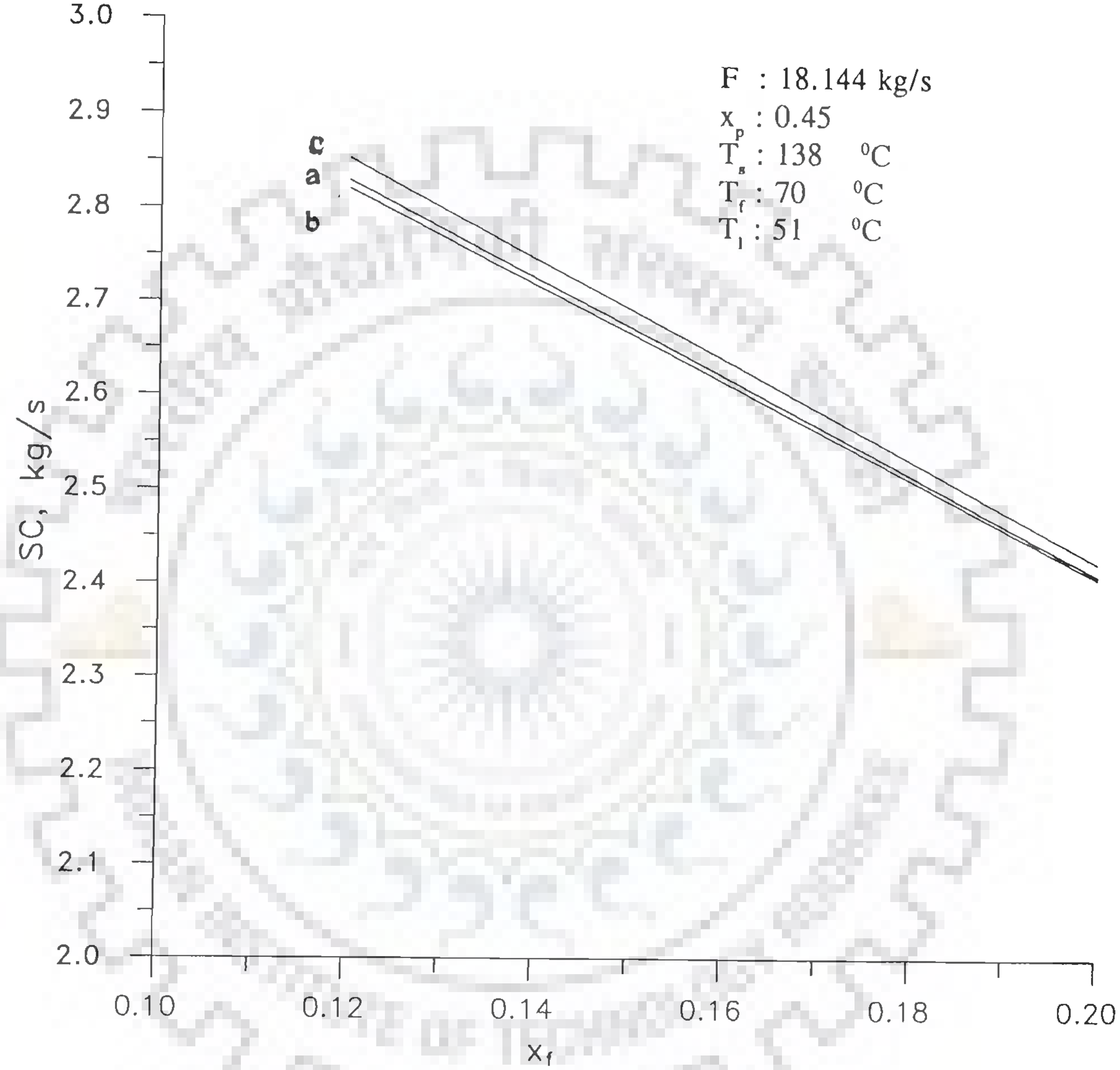

Fig. 6.16 Effect of x_f on SC for sequences a, b & c

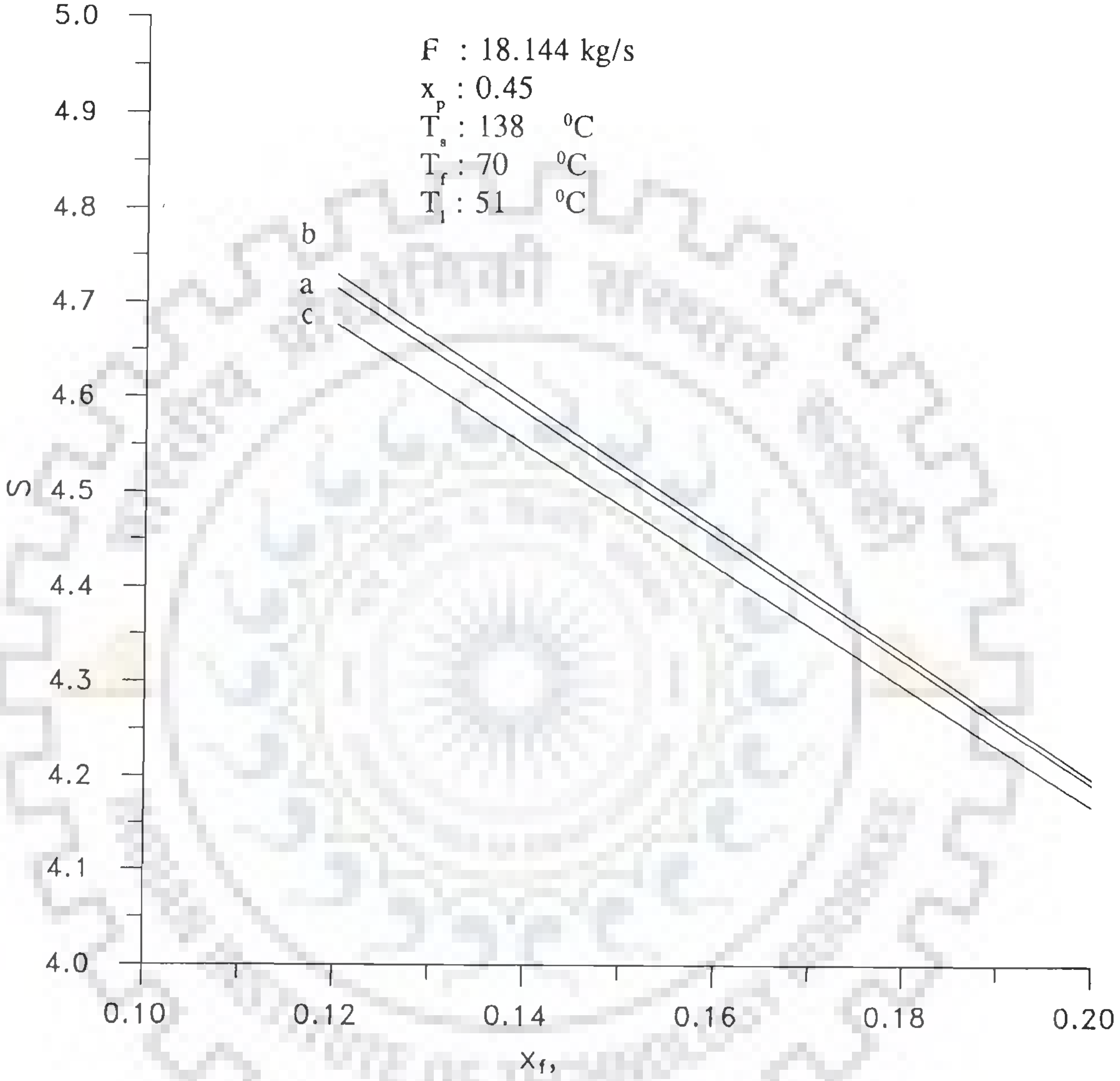

Fig. 6.17 Effect of x_f on S for sequences a, b & c

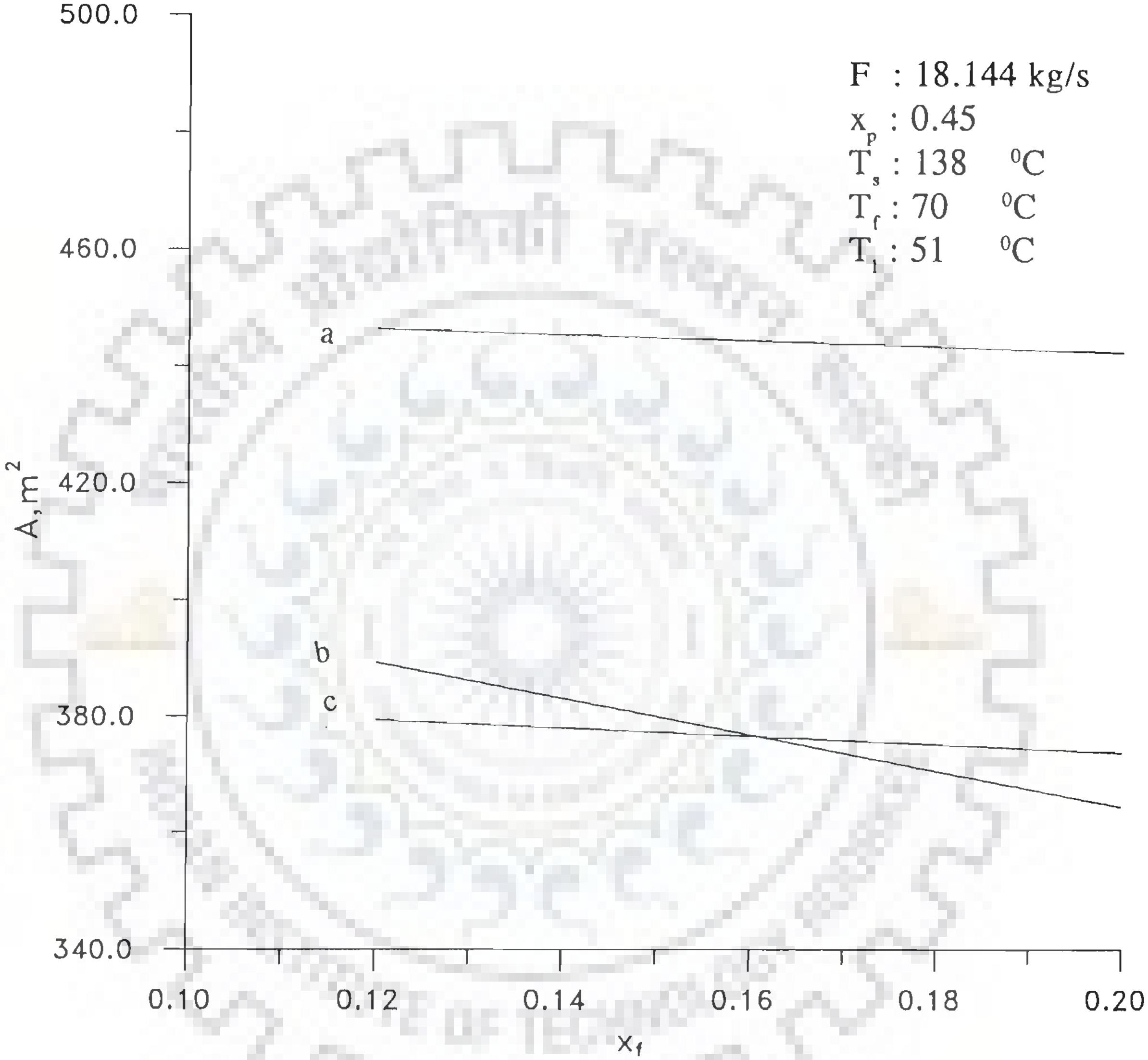

Fig. 6.18 Effect of x_f on A for sequences a,b & c

It is thus inferred that for the feed concentrations normally employed in the Indian mills, none of the three cases shows any distinct advantage over the other considered in this analysis, as far as SC and S are concerned.

6.7.3 Effect of Steam Temperature

Dependence of the SC, S, and A on T_s is depicted in Figures 6.19 through 6.21. The steam consumption increases and the steam economy decreases with increasing values of T_s for all the three sequences. It is seen that S attains the highest value for the sequence **c** for all values of T_s above 120°C. Figure 6.21 points out that the requirement of the heat transfer area is the lowest for the sequence **c** over the entire range of T_s explored.

Therefore, these plots evince out the fact that for the steam temperatures normally employed in the mills, the backward feed along with the split of feed and steam is the best arrangement out of the three flow sequences considered.

6.7.4 Effect of Last Effect Temperature

The influence of variations in T_1 on SC, S, and A is shown in Figures 6.22 through 6.24. As is seen, SC decreases while S rises with an increase in the value of T_1. Once again a similar phenomenon is observed in that the values of SC and A are the lowest and that of S the highest for the sequence **c**. In fact, at the standard value of T_1 equal to 51°C, the provision for the splits of feed and steam to an otherwise simple backward feed

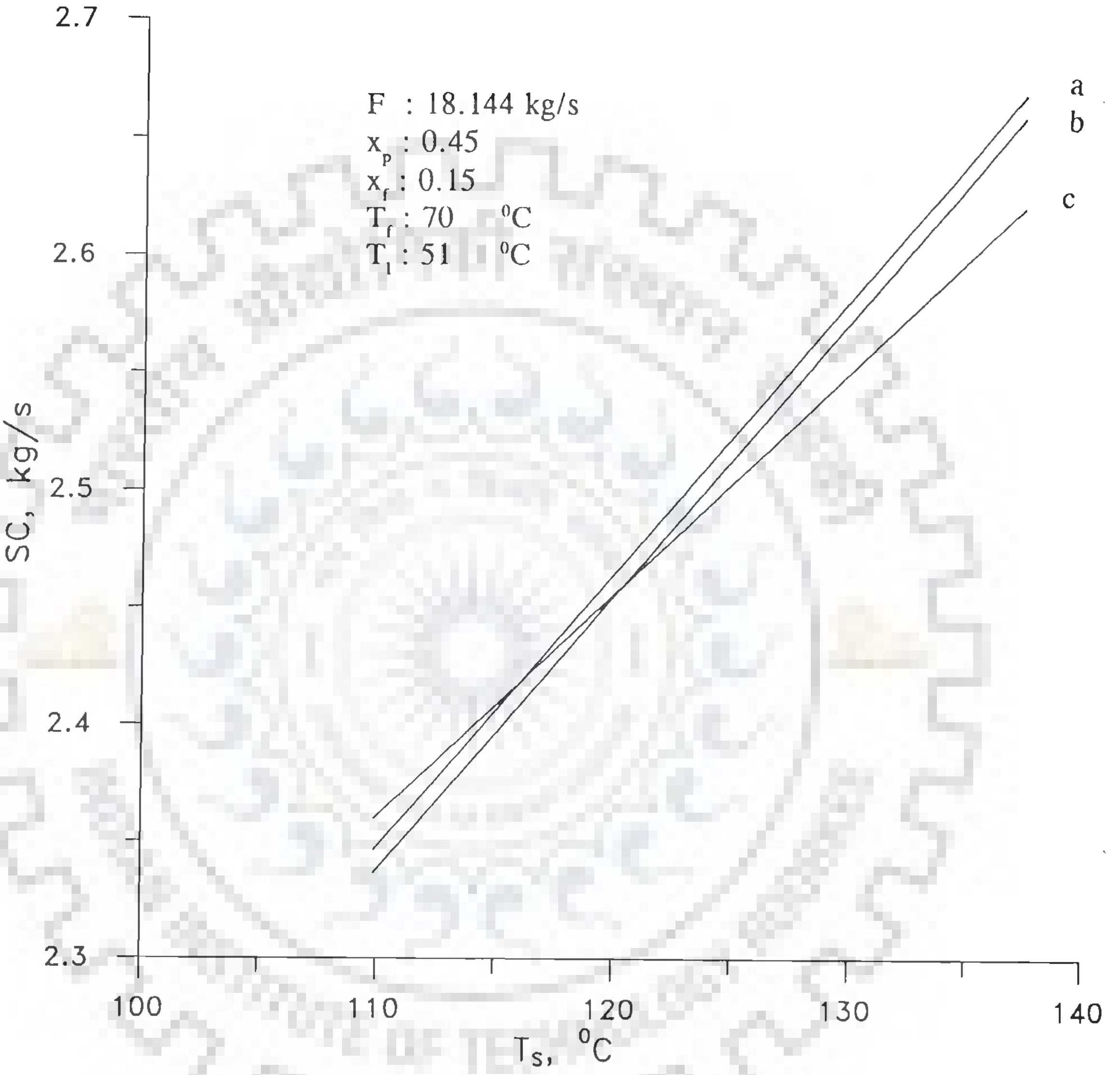

Fig. 6.19 Effect of T_s on SC for sequences a,b & c

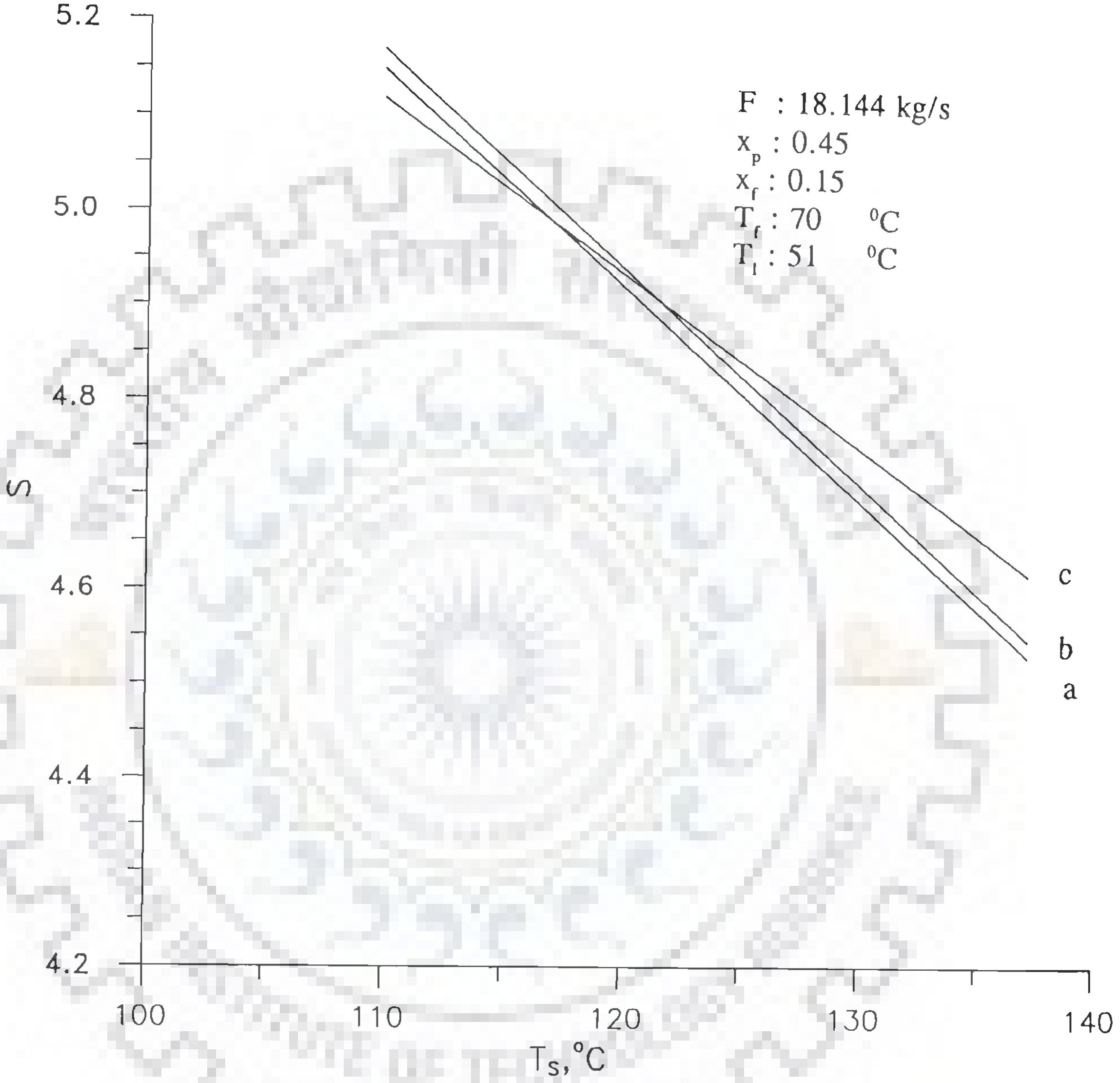

Fig. 6.20 Effect of T_S on S for sequences a,b & c

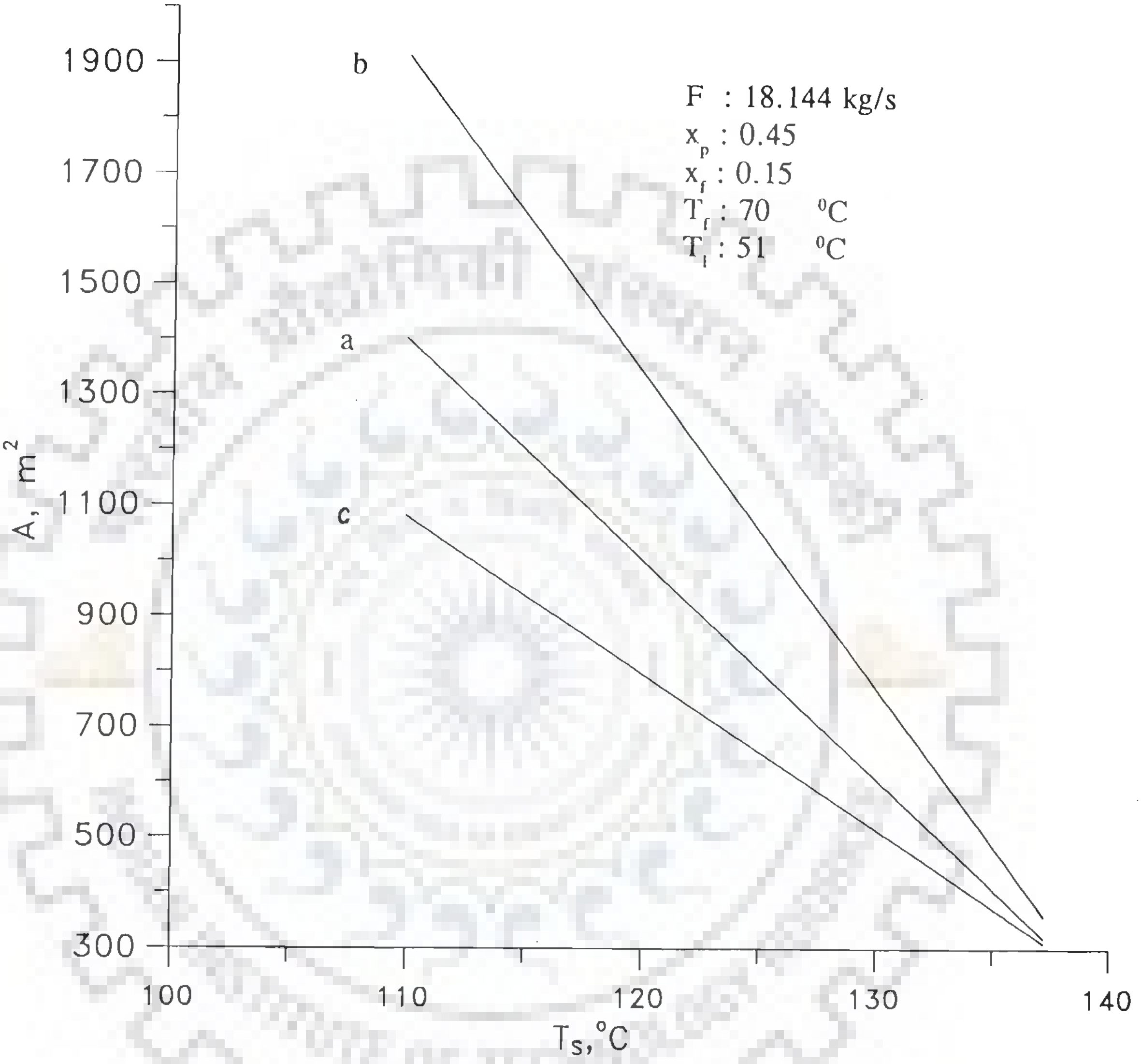

Fig. 6.21 Effect of T_s on A for sequences a,b & c

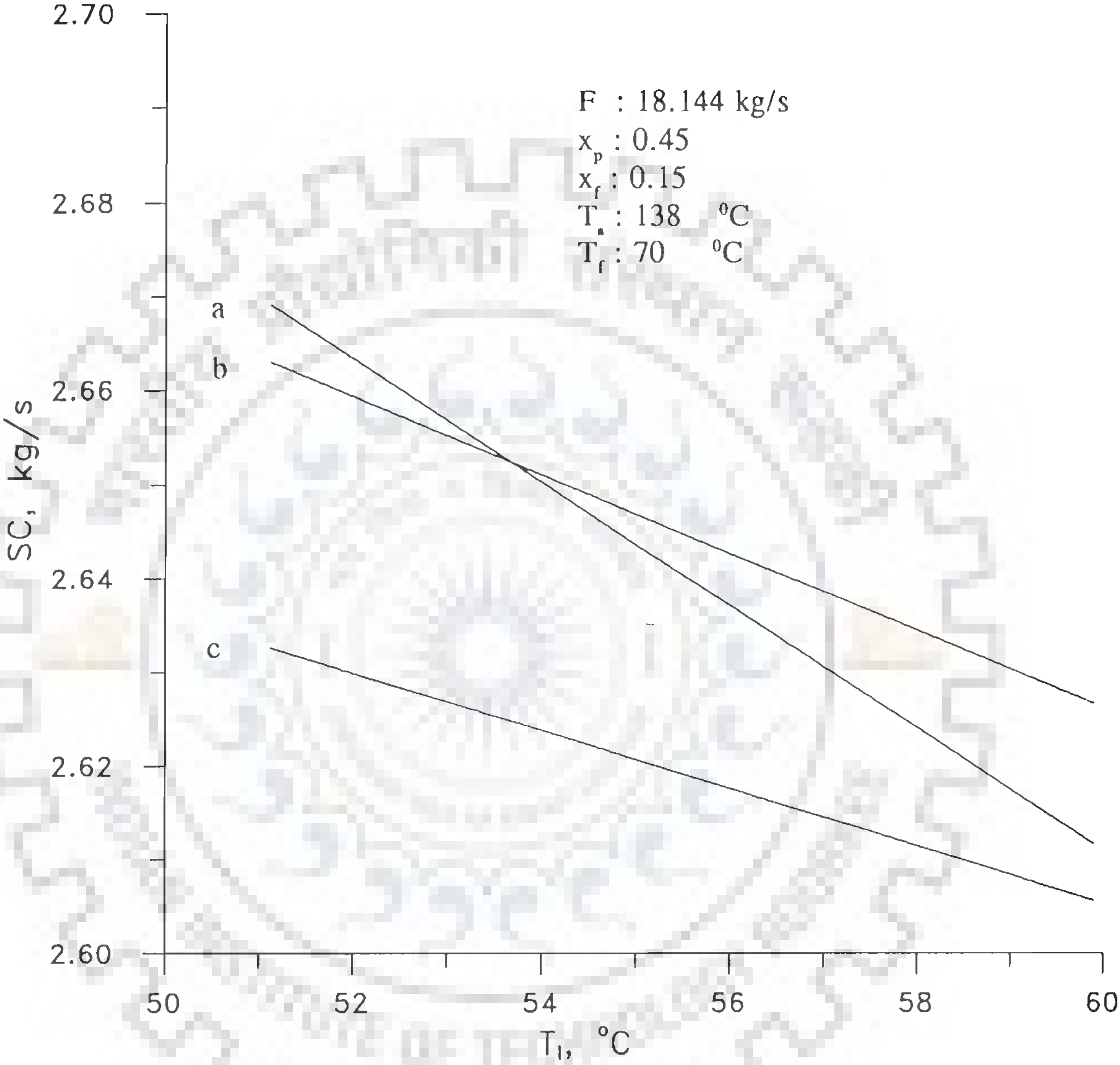

Fig. 6.22 Effect of T_l on SC for sequences a,b & c

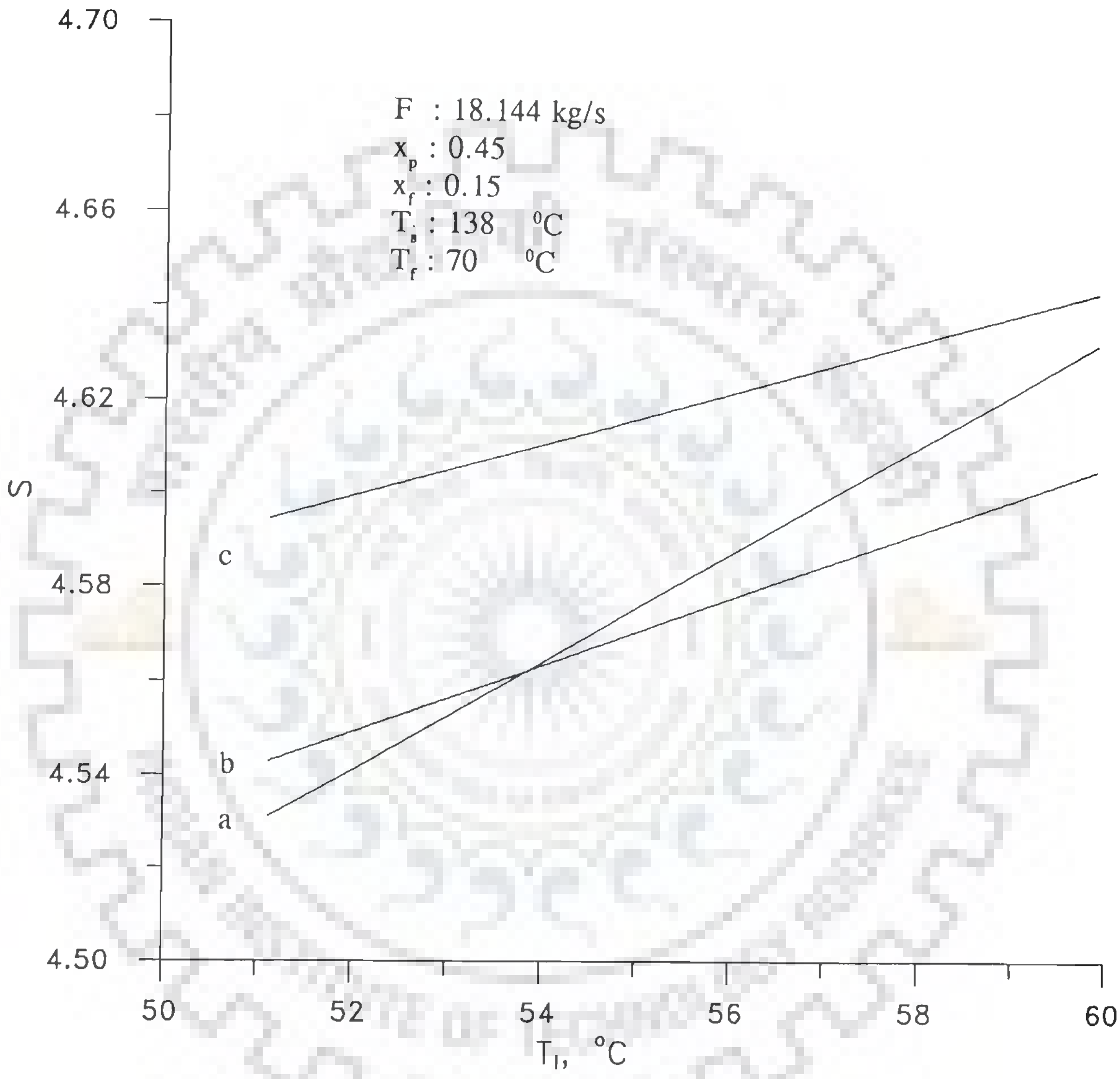

Fig. 6.23 Effect of T_l on S for sequences a,b & c

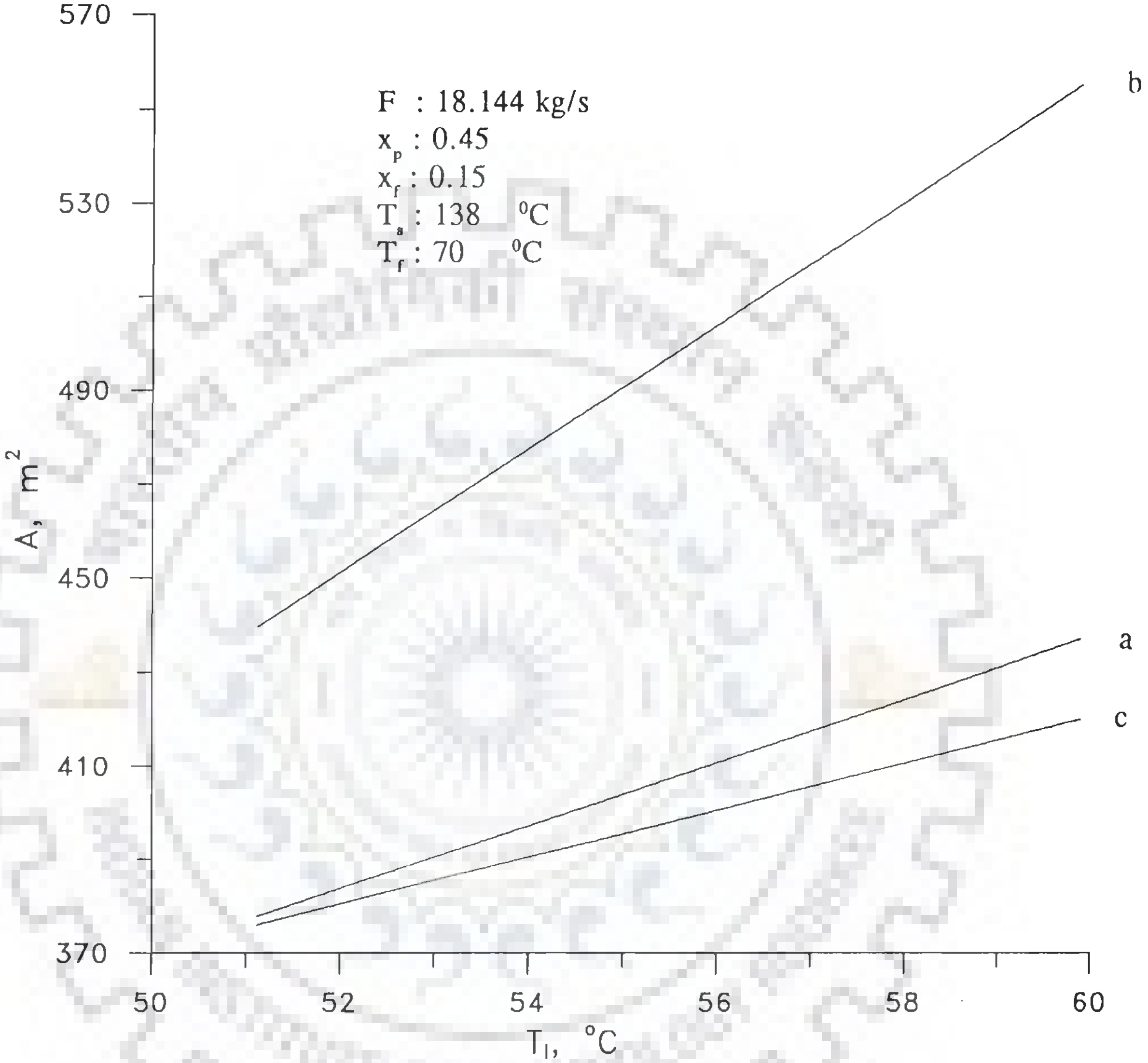

Fig. 6.24 Effect of T_l on A for sequences a,b & c

raises the steam economy by 1.40 per cent which is a small improvement. However, the decrease in the heating area is only marginal.

6.7.5 Effect of Liquor Feed Rate

Figures 6.25 through 6.27 represent the plots for the effect of liquor feed rate F on SC, S and A. Figure 6.26 exhibits that S is absolutely immune to changes in F although SC and A increase linearly with rise in values of F, as shown in Figures 6.25 and 6.27, respectively. The heating surface area requirement is the minimum for sequence **c**, but the steam economy is the lowest for this sequence, though the difference with other cases is only marginal.

The above discussions on the three sequences **a**, **b** and **c** carried out in this section bring out the fact with no uncertain clarity that the sequence **c** which employs the splits of feed and steam, turns out to be the best.

6.8 SEXTUPLE EFFECT EVAPORATOR EMPLOYING VARIOUS MIXED FEED SEQUENCES

The various liquor flow sequences of mixed feed considered for the present investigation from Table 3.2 are reproduced below.

	Liquor flow sequence	
(i)	**5** - 6 - 4 - 3 - 2 - 1	sequence **d**
(ii)	4 - 5 - 6 - 3 - 2 - 1	sequence **e**

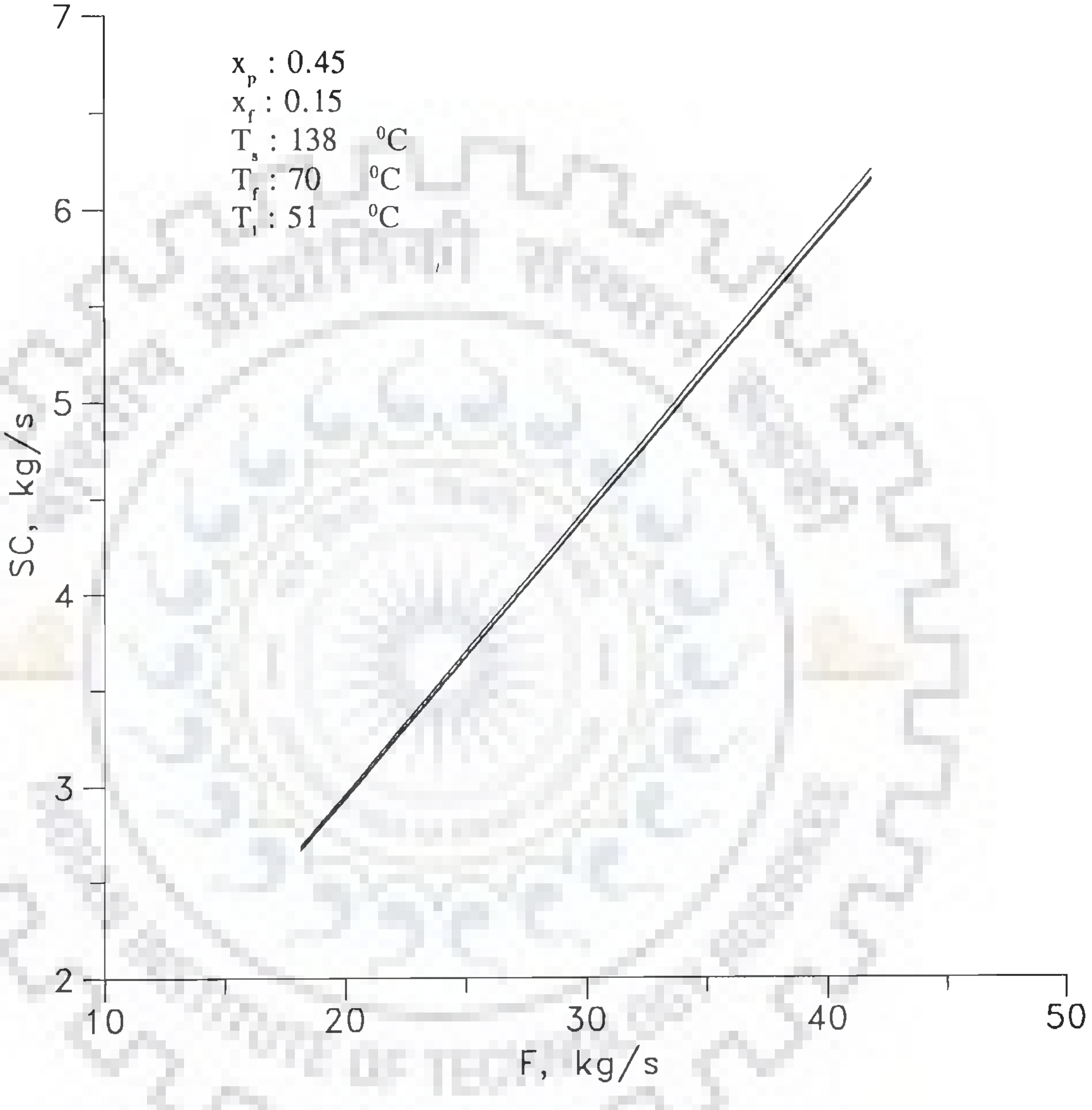

Fig. 6.25 Effect of F on SC for sequences a,b & c

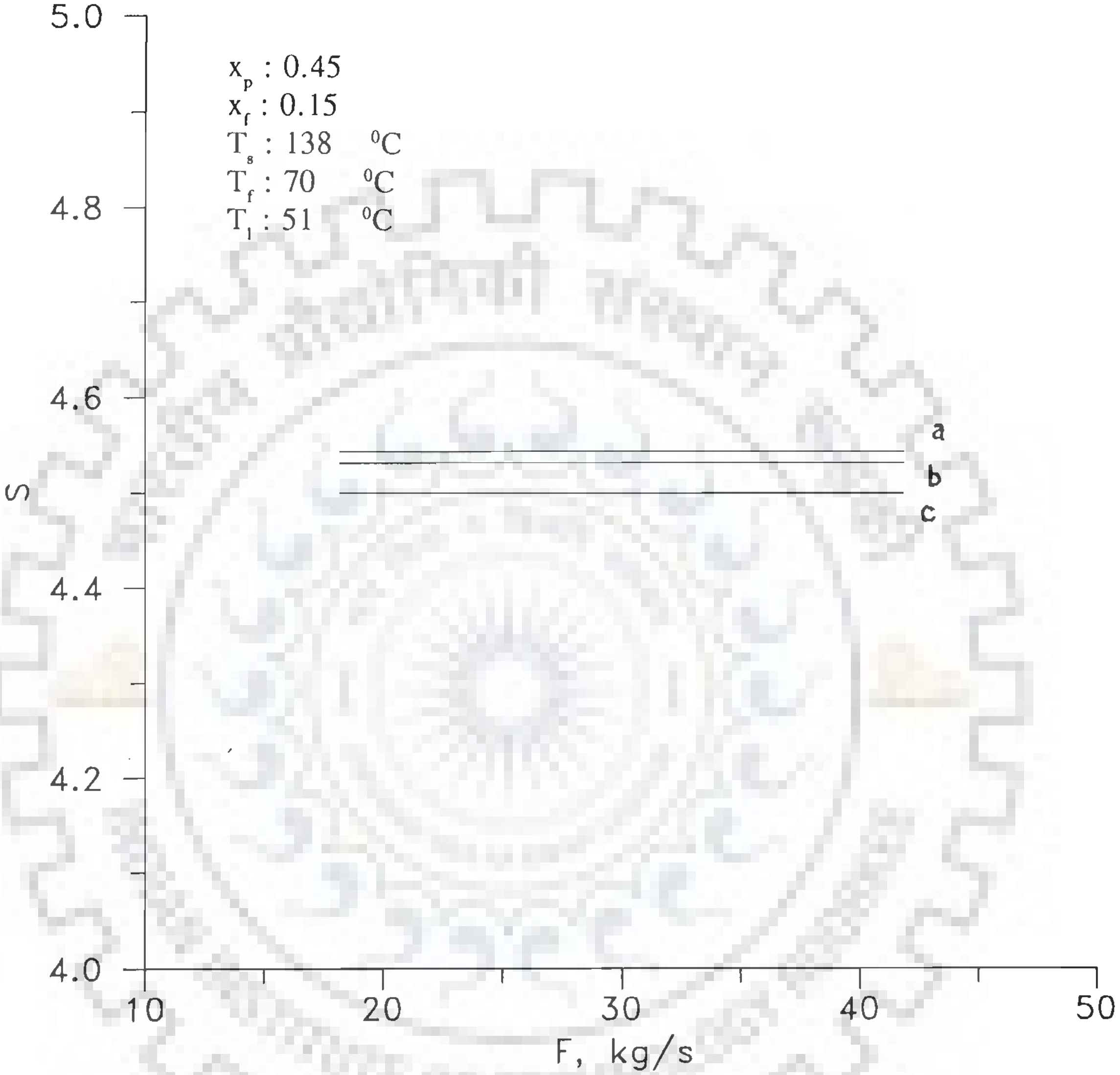

Fig. 6.26 Effect of F on S for sequences a,b & c

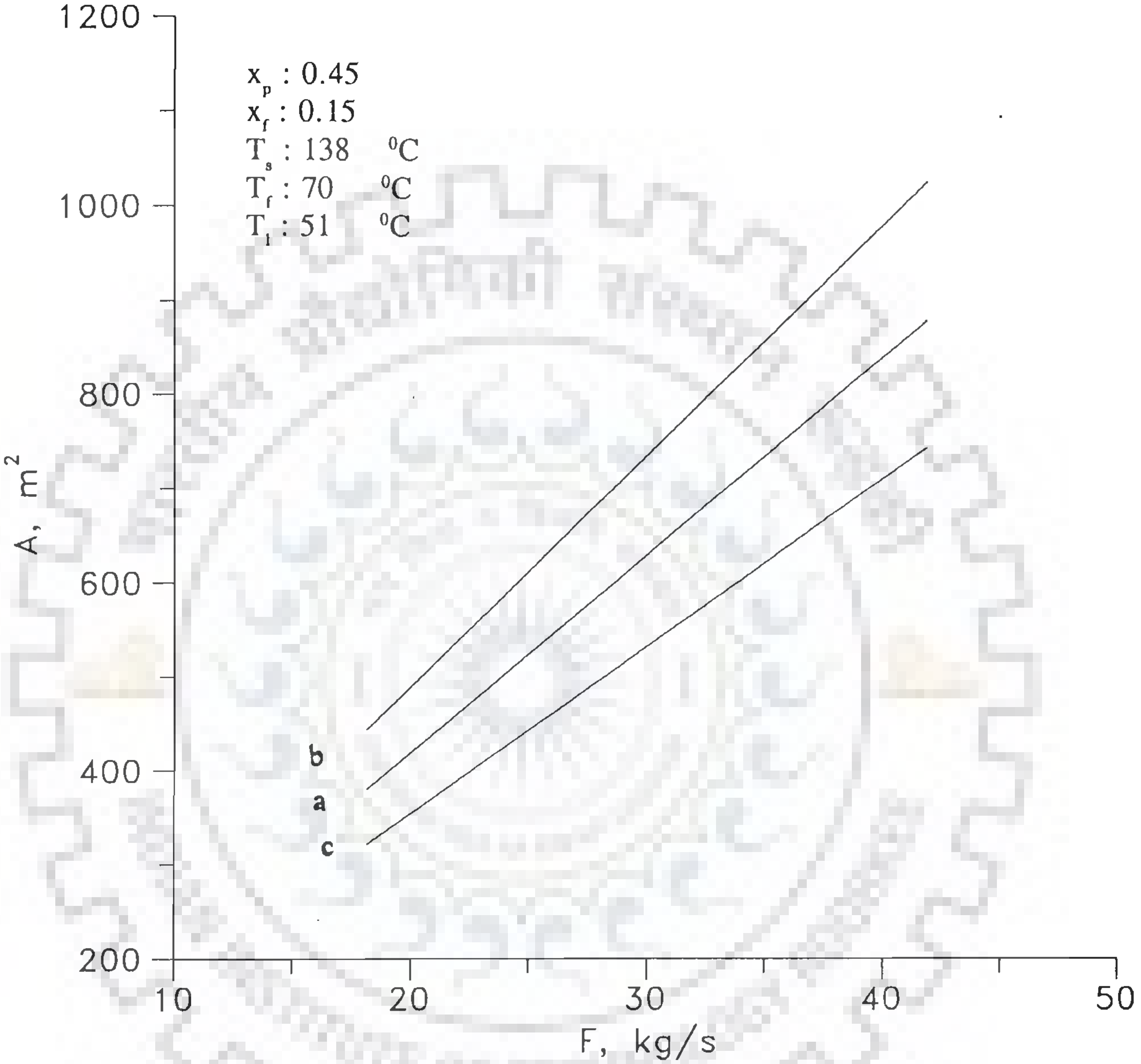

Fig. 6.27 Effect of F on A for sequences a,b & c

(iii) 3 -> 4 -> 5 -> 6 -> 2 -> 1 sequence, **f**

(iv) 2 -> 3 -> 4 -> 5 -> 6 -> 1 sequence, **g**

These sequences are represented by letters **d**, **e**, **f** and **g**, respectively in the figures drawn in the subsequent sections.

6.8.1 Effect of Liquor Feed Temperature

Figure 6.28 has been plotted to show the dependence of steam consumption, SC on liquor feed temperature, T_f for the four liquor flow sequences mentioned above. As would be expected, the steam consumption decreases continuously with rise in the value of T_f for all the sequences considered. It is also noted that for the entire range of T_f explored, the value of SC is the lowest for sequence **d** implying that this sequence is the best amongst all.

Figure 6.29 is drawn to demonstrate the effect of T_f on the steam economy , S of the evaporator. It is seen that S increases with a rise in T_f and is the largest for the flow sequence **d**.

To determine the effect of T_f on the requirement of heating surface area, A, Figure 6.30 is drawn. It is found that the heating surface required decreases with increasing value of T_f for all the four cases. An important observation from this plot is that the heating area needed for the sequence **d** is the lowest upto a feed temperature of about 77°C. Beyond this, it is marginally greater than those for sequences **e** and **f**.

The foregoing discussions lead to the conclusion that for a

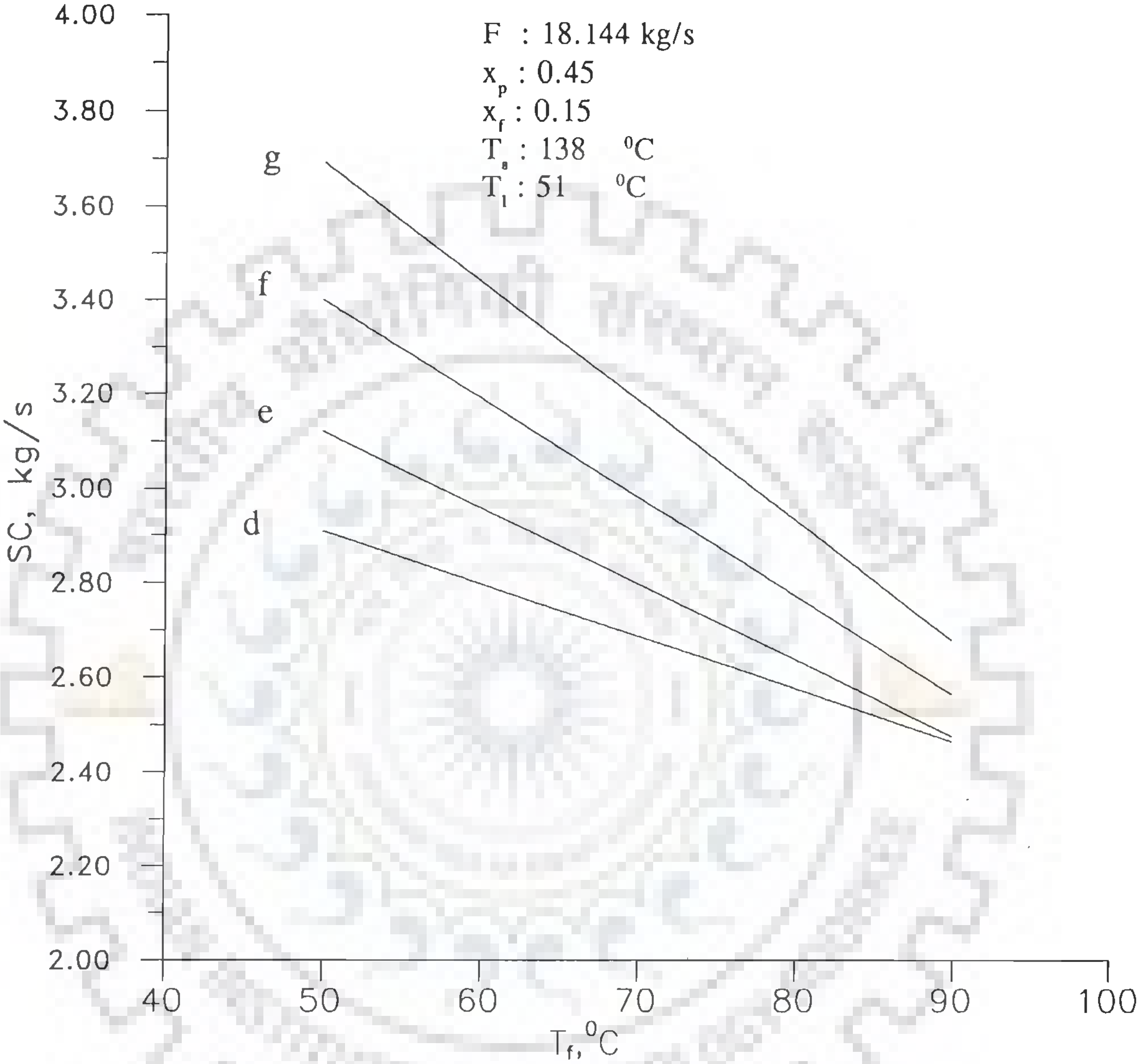

Fig. 6.28 Effect of T_f on SC for sequences d,e, f & g

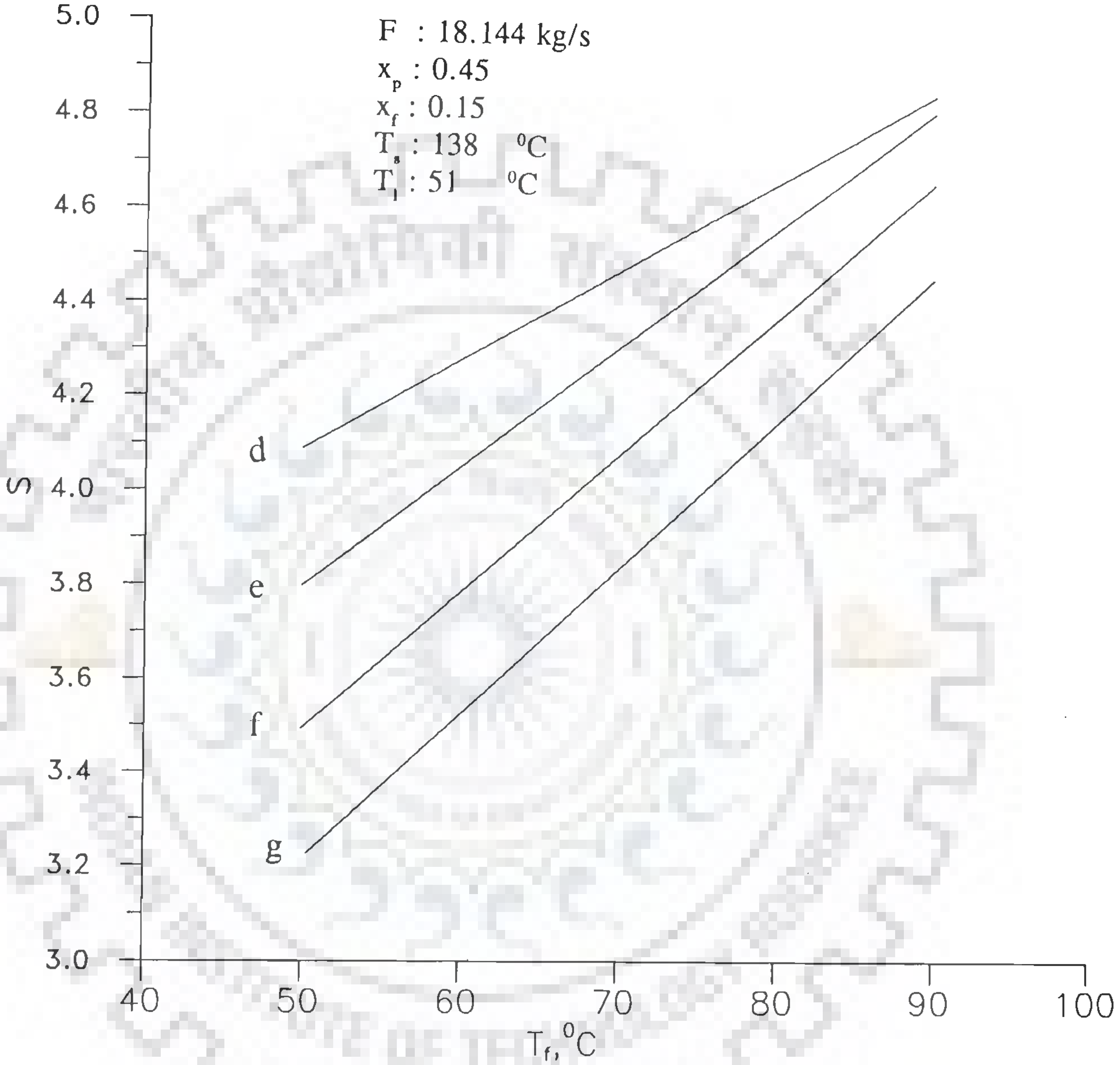

Fig. 6.29 Effect of T_f on S for sequences d,e, f & g

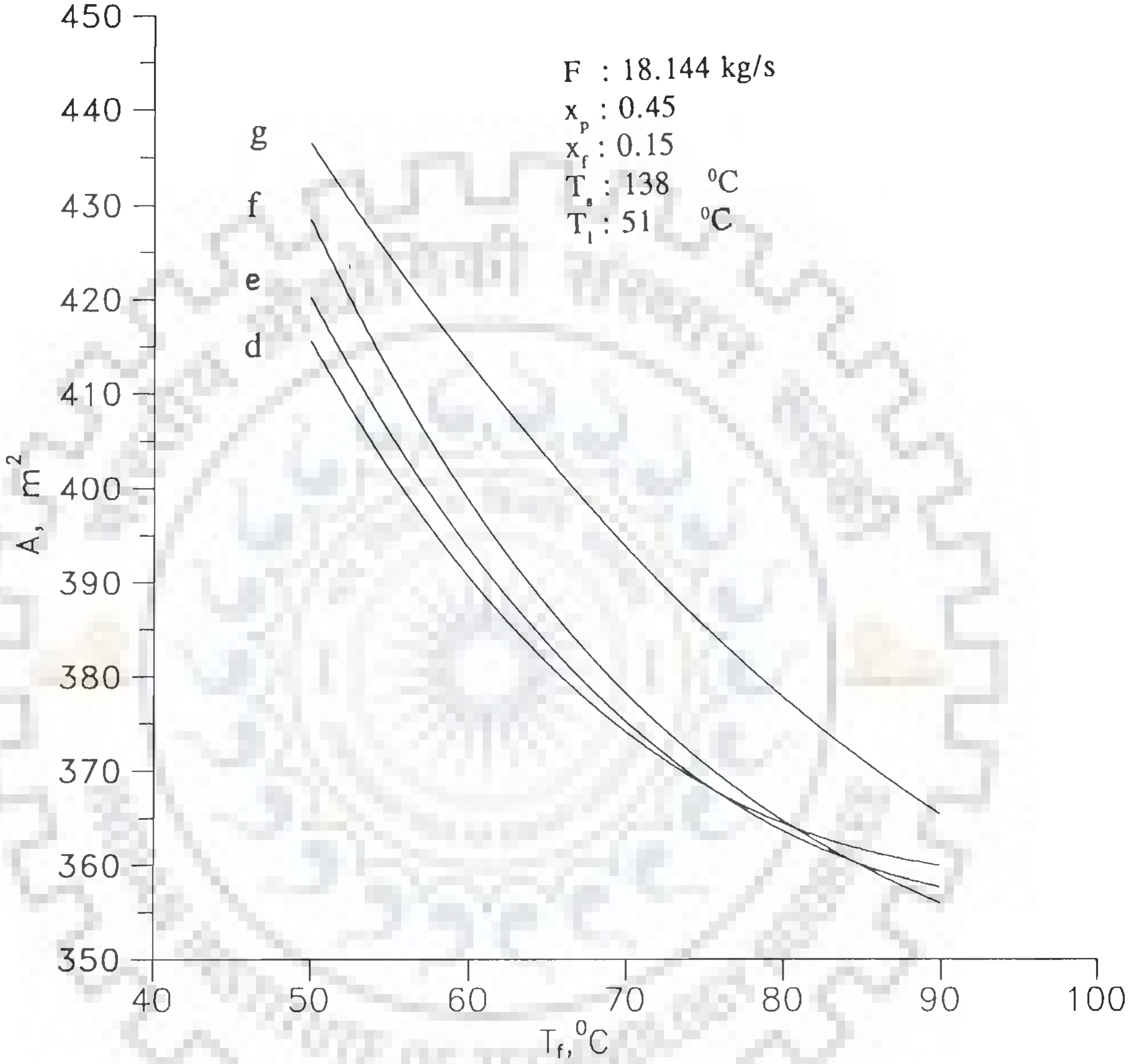

Fig. 6.30 Effect of T_f on A for sequences d,e, f & g

majority of the industrial practices, as far as the feed temperature is concerned, the sequence **d** is the best of the four sequences considered.

6.8.2 Effect of Last Effect Temperature

Figure 6.31 is a plot between SC and T_1. It shows vividly that the dependence of SC on T_1 is by and large acutely small for a given liquor flow sequence. Out of all the sequences considered, it is noted that sequence **d** has the lowest value of SC for the entire range of T_1. A similar observation is made from Figure 6.32 about the dependence of S on T_1. However, S is the highest for the sequence **d**. Figure 6.33 shows a rising trend for A with T_1. Furthermore, A is the lowest for sequence **d**.

6.8.3 Effect of Steam Temperature

In Section 6.5, it has been pointed out that the three output parameters, SC S, and A are quite sensitive to variations in T_S. Figure 6.34 shows a rising trend for SC with rise in T_S, while the trend for S in Figure 6.35 is seen to be just the opposite. This is quite expected and has been already discussed in Section 6.6. The important aspect of these figures is that sequence **d** has the lowest SC.

The heating surface required decreases steeply with increasing values of T_S. This trend can be seen so vividly in Figure 6.36. However, beyond a T_S value of around 132°C, the plots for the sequences considered almost overlap, thereby demonstrating that there is little to choose from amongst them,

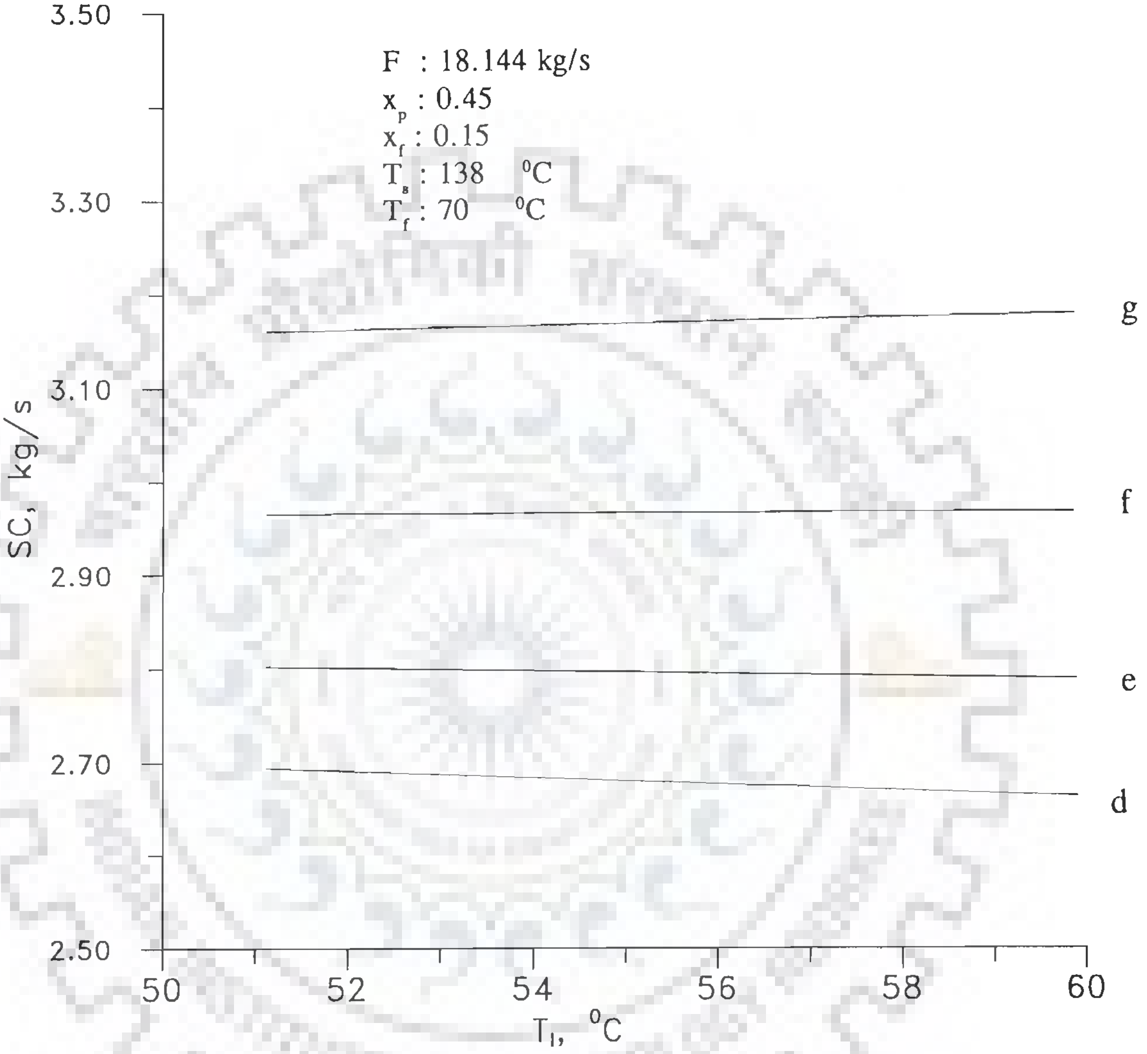

Fig. 6.31 Effect of T_l on SC for sequences d,e, f & g

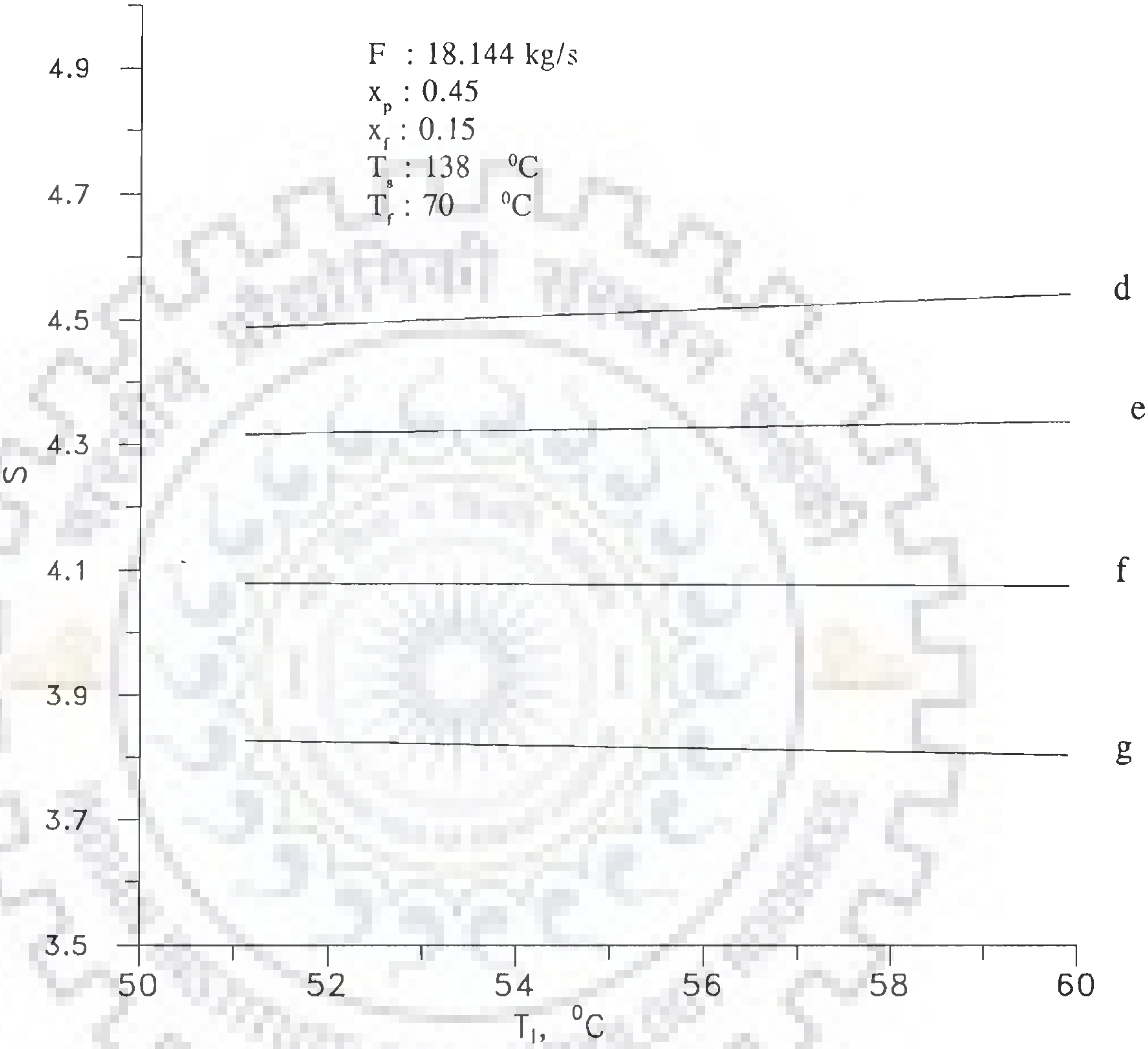

Fig. 6.32 Effect of T_l on S for sequences d,e, f & g

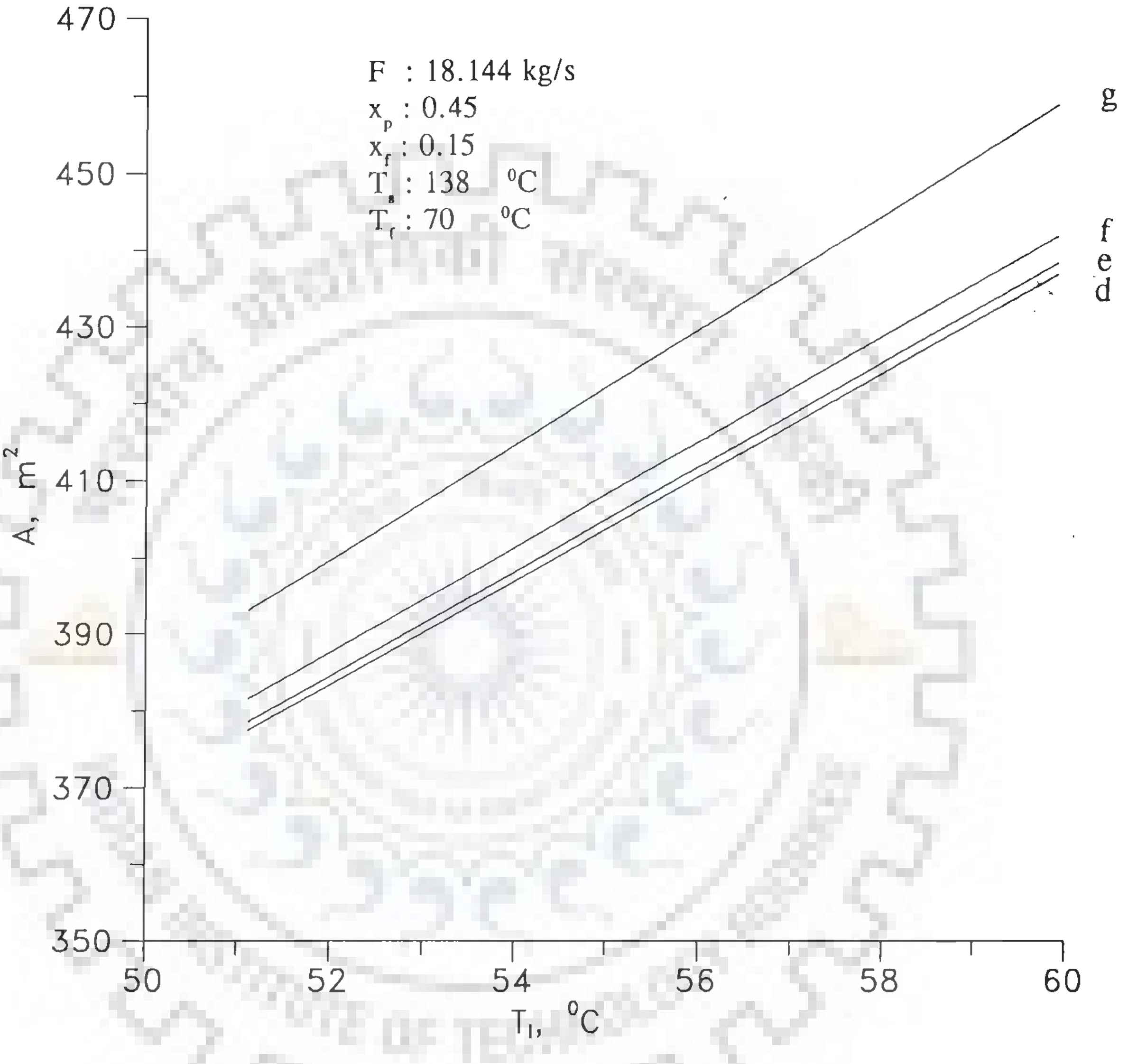

Fig. 6.33 Effect of T_1 on A for sequences d,e, f & g

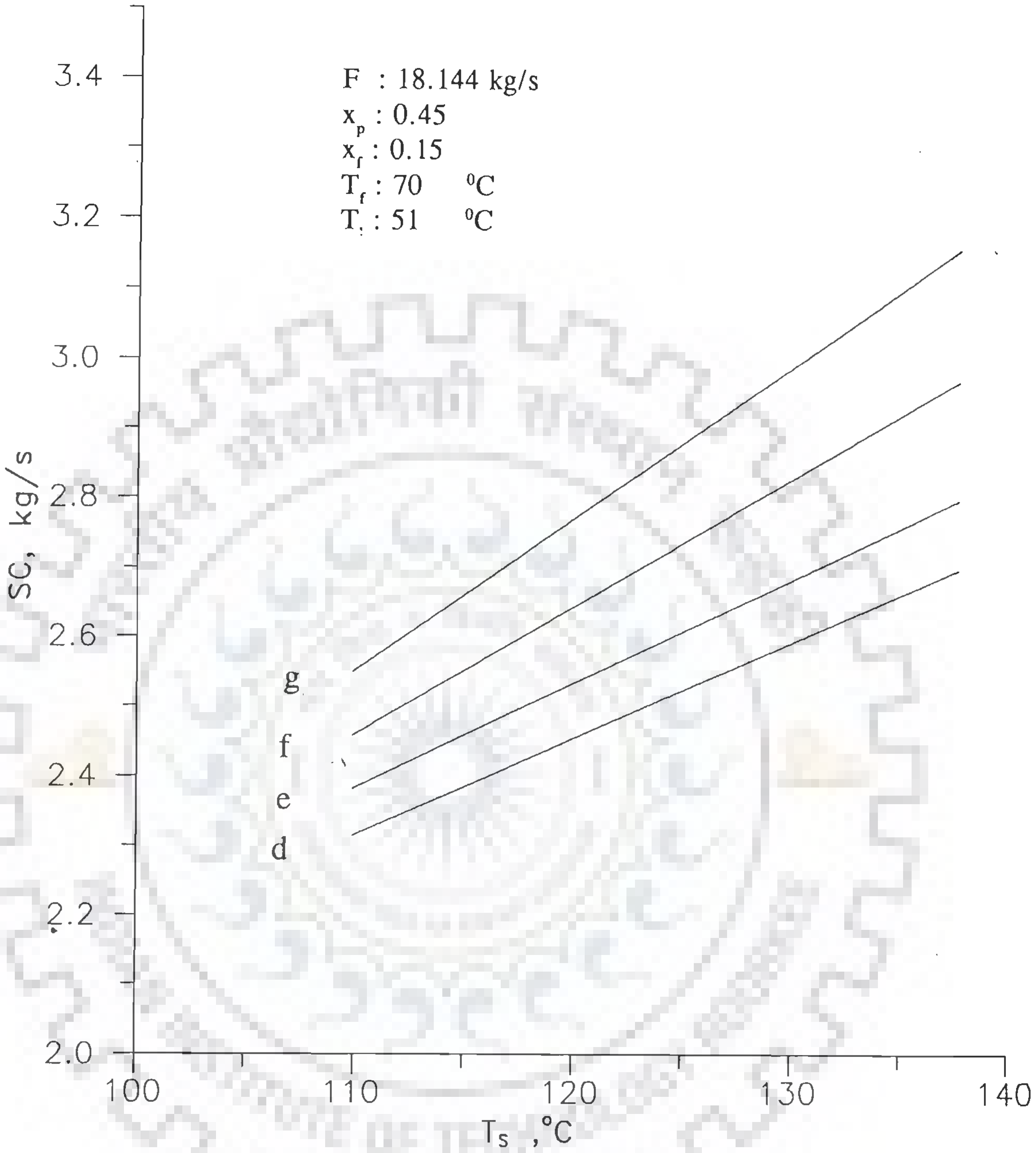

Fig. 6.34 Effect of T_s on SC for sequences d,e, f & g

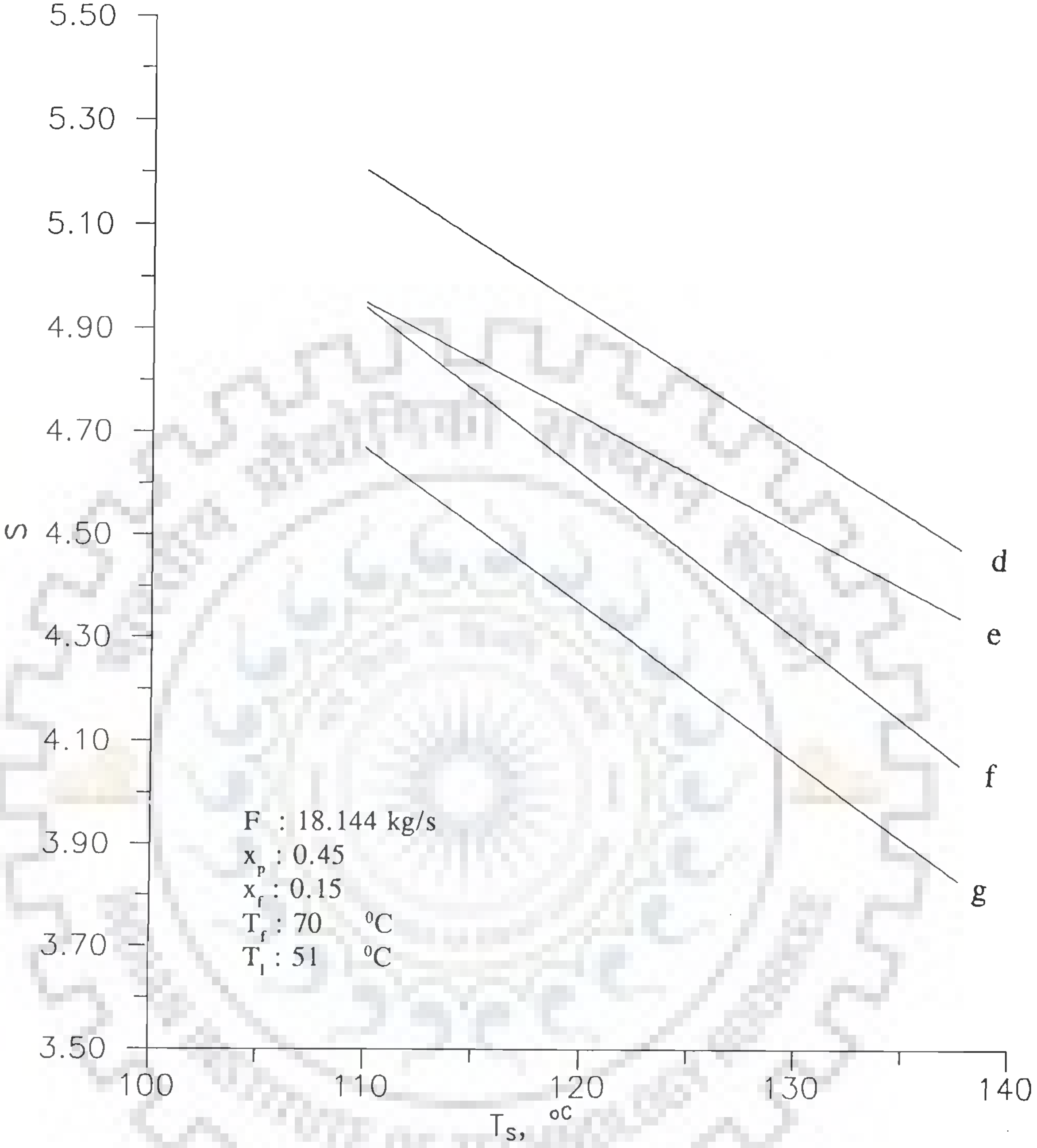

Fig. 6.35 Effect of T_S on S for sequences d,e, f & g

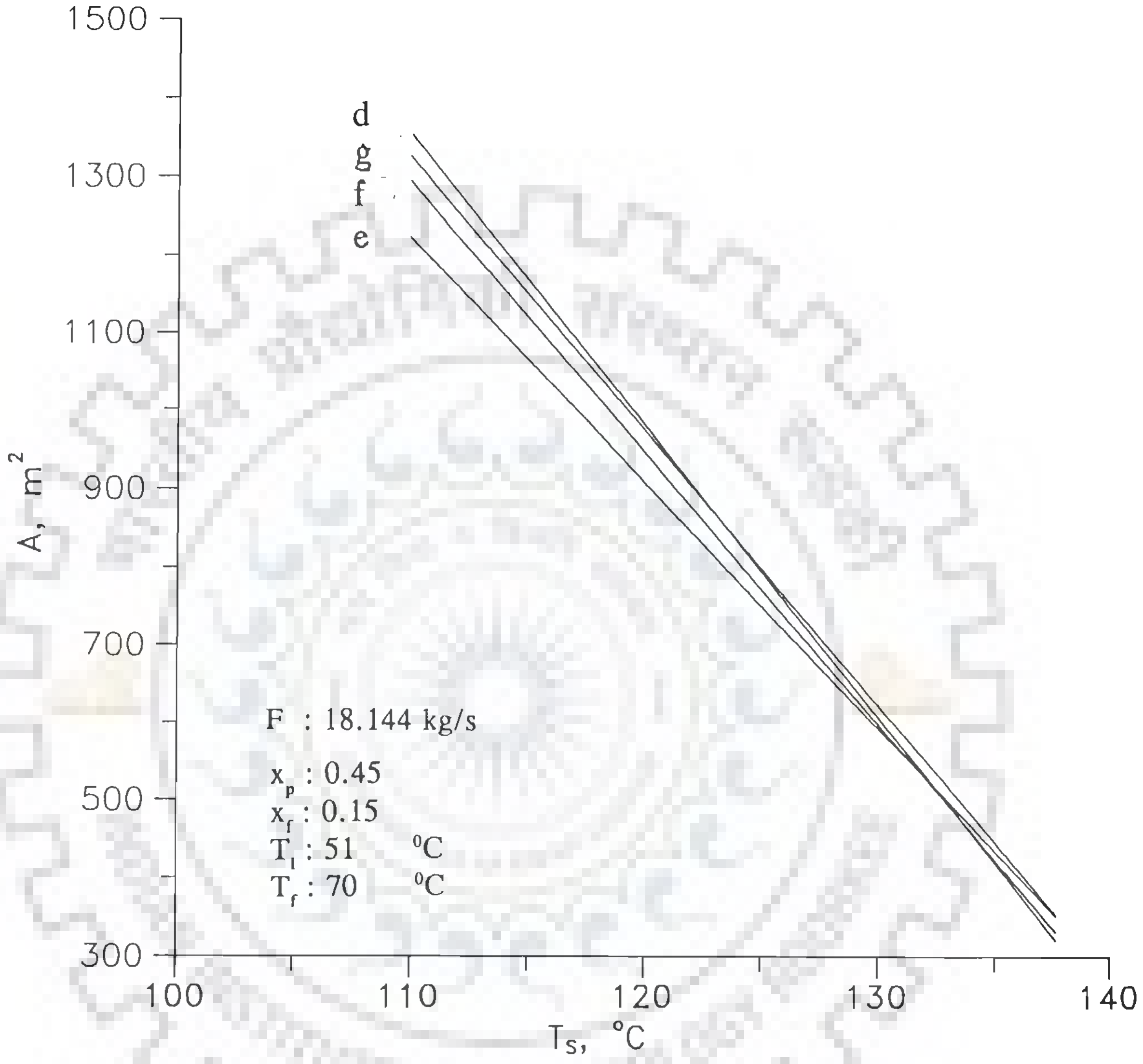

Fig. 6.36 Effect of T_S on A for sequences d,e, f & g

although they correspond to lower values of surface area when compared with the sequence **g**.

It is important to recall that the distinct large changes in SC, S and A with T_S as observed in Figures 6.34 through 6.36 are expected as they are in accordance with the values of sensitivity coefficients in Table 6.4.

Therefore, sequence **d** is the best of the four as far as the choice based on T_S is made.

6.8.4 Effect of Liquor Feed Concentration

A manifestation of the role of liquor feed concentration, x_f on the three output parameters, SC, S, and A is represented in Figures 6.37, 6.38 and 6.39. It is interesting to note that all the three parameters have a declining trend with increase in liquor feed concentration values. Once again the sequence **d** has the lowest steam consumption, SC and the heating surface area requirement, A and also the maximum steam economy, S, proving itself to be the best. For an increase in the value of liquor feed concentration from 0.12 to 0.16, the area decreases by 3.6 percent in this case.

6.9 COMPARISON BETWEEN SEQUENCES c AND d

As pointed out earlier, in Sections 6.7 and 6.8, the sequence **c** is the best among the three alternatives **a**, **b** and **c** employing backward feed, whereas sequence **d** has emerged to be the most acceptable among the various mixed feed sequences

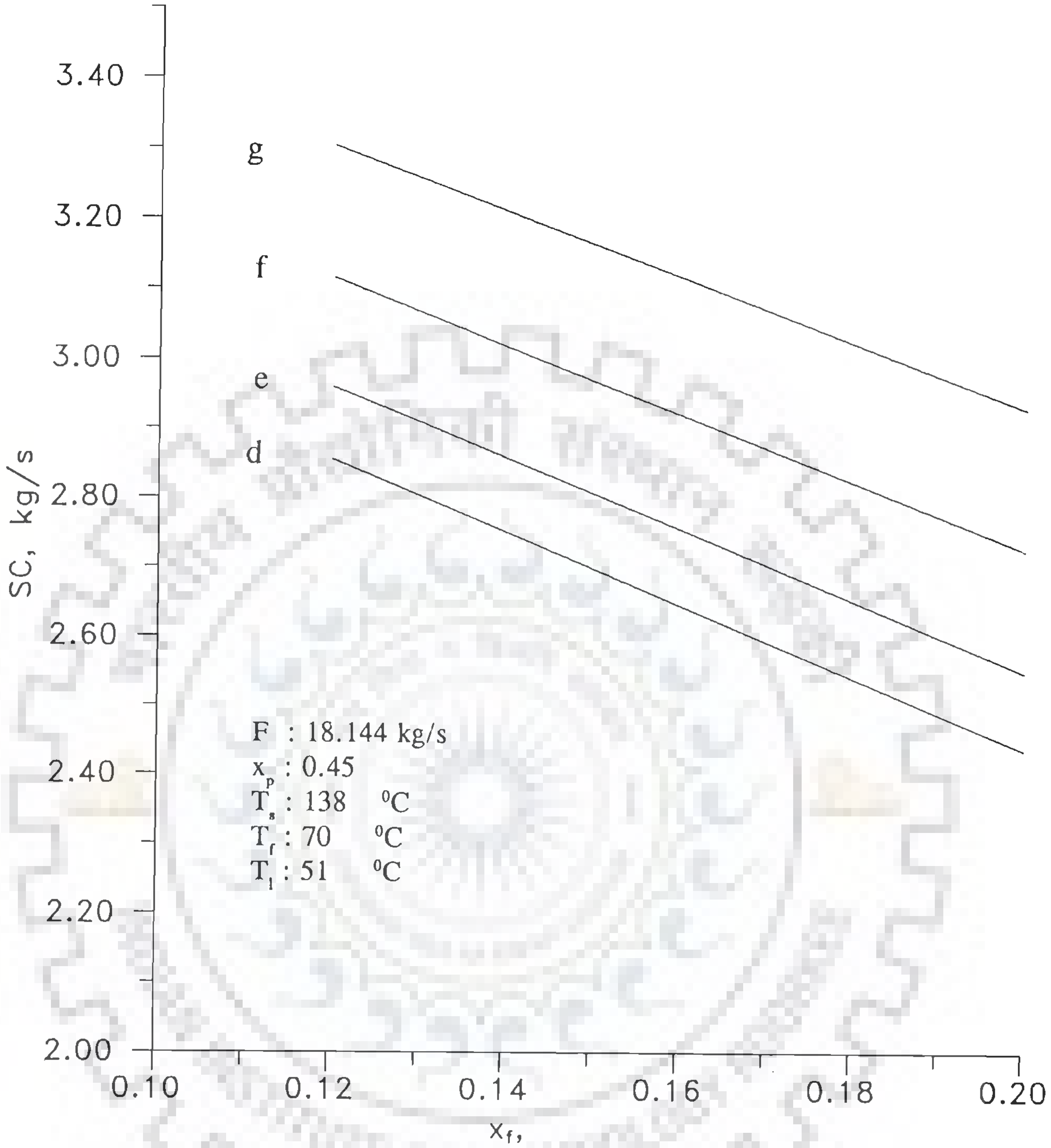

Fig. 6.37 Effect of x_f on SC for sequences d,e, f & g

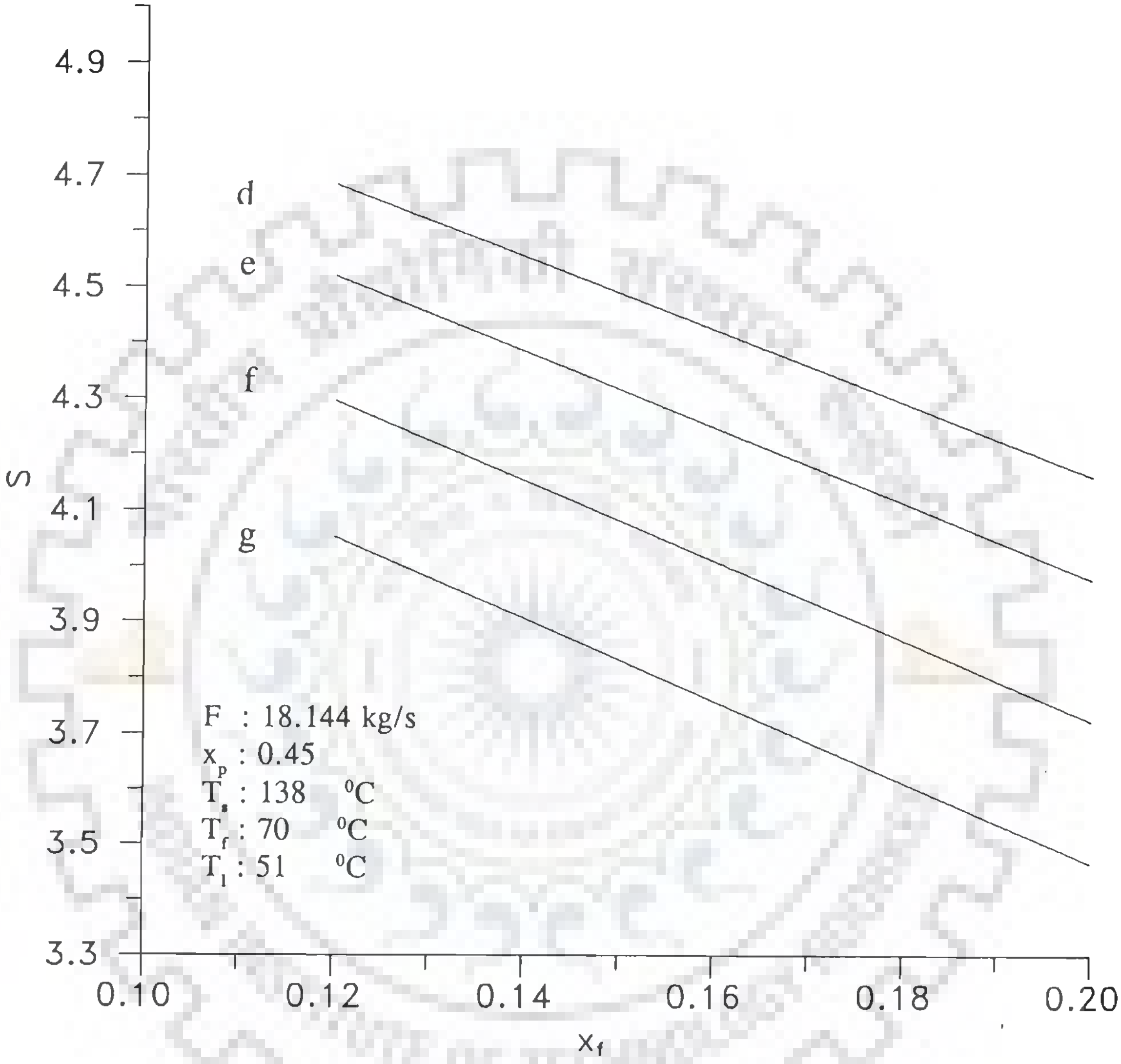

Fig. 6.38 Effect of x_f on S for sequences d,e, f & g

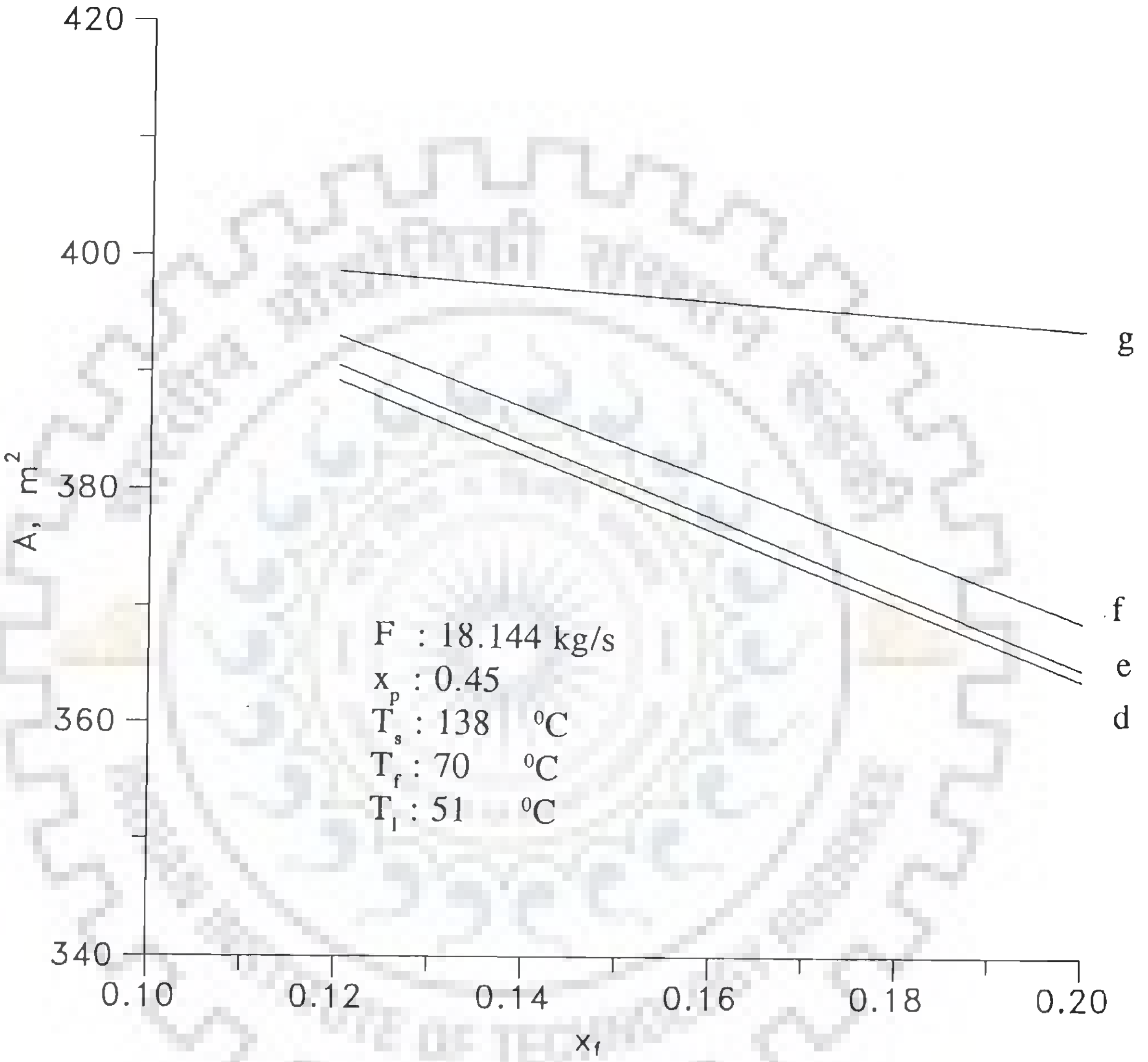

Fig. 6.39 Effect of x_f on A for sequences d, e, f and g

studied. It will now be worthwhile to examine these two sequences so as to determine the better one among these.

The following analysis is aimed at achieving this objective:

6.9.1 Effect of Steam Temperature

Figures 6.40, 6.41 and 6.42 have been drawn to show the effect of T_s on SC, S, and A.

As is evident from Figure 6.40, the steam consumption is higher for sequence **c** upto a T_s of about 132°C as compared to sequence **d**. Beyond this temperature the trend is reversed although marginally. However, Figure 6.41 depicts an opposite trend in the case of steam economy as expected. From the point of view of requirement of heating surface, Figure 6.42 clearly reflects the superiority of sequence **c** though the benefit narrows down with increasing values of T_s.

As the steam temperature employed in conventional mill practice is generally above 130°C, it follows that sequence **c** has an edge over sequence **d**.

6.9.2 Effect of Liquor Feed Temperature

Figures 6.43, 6.44 and 6.45 show a comparison of the effect of T_f on SC, S, and A.

Figure 6.44 reflects the marginal superiority of sequence **c** over sequence **d** for T_f values upto 73°C. However, it is seen from Figure 6.43 that steam consumption is higher for values of

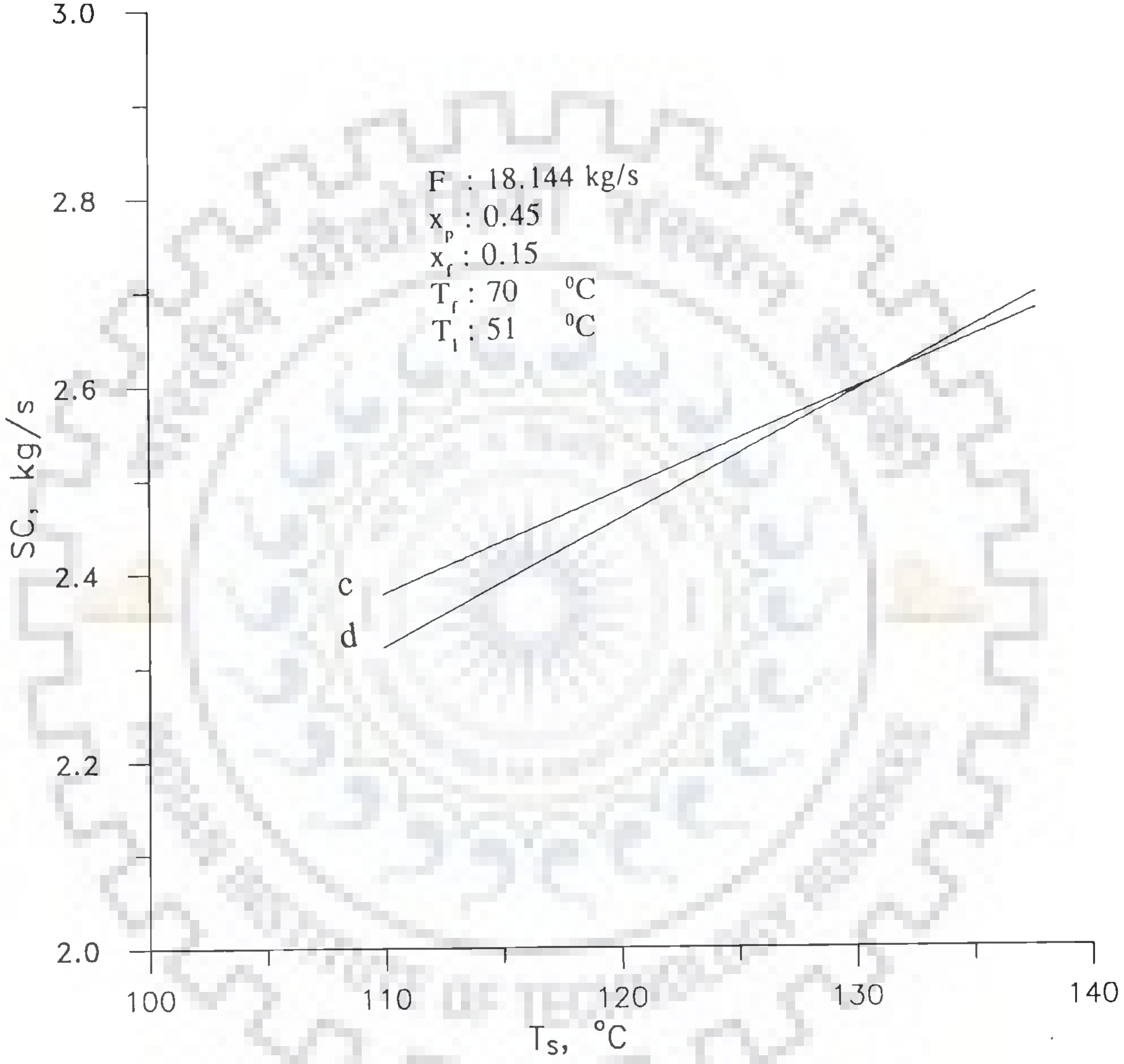

Fig. 6.40 Effect of T_s on SC for sequences c and d

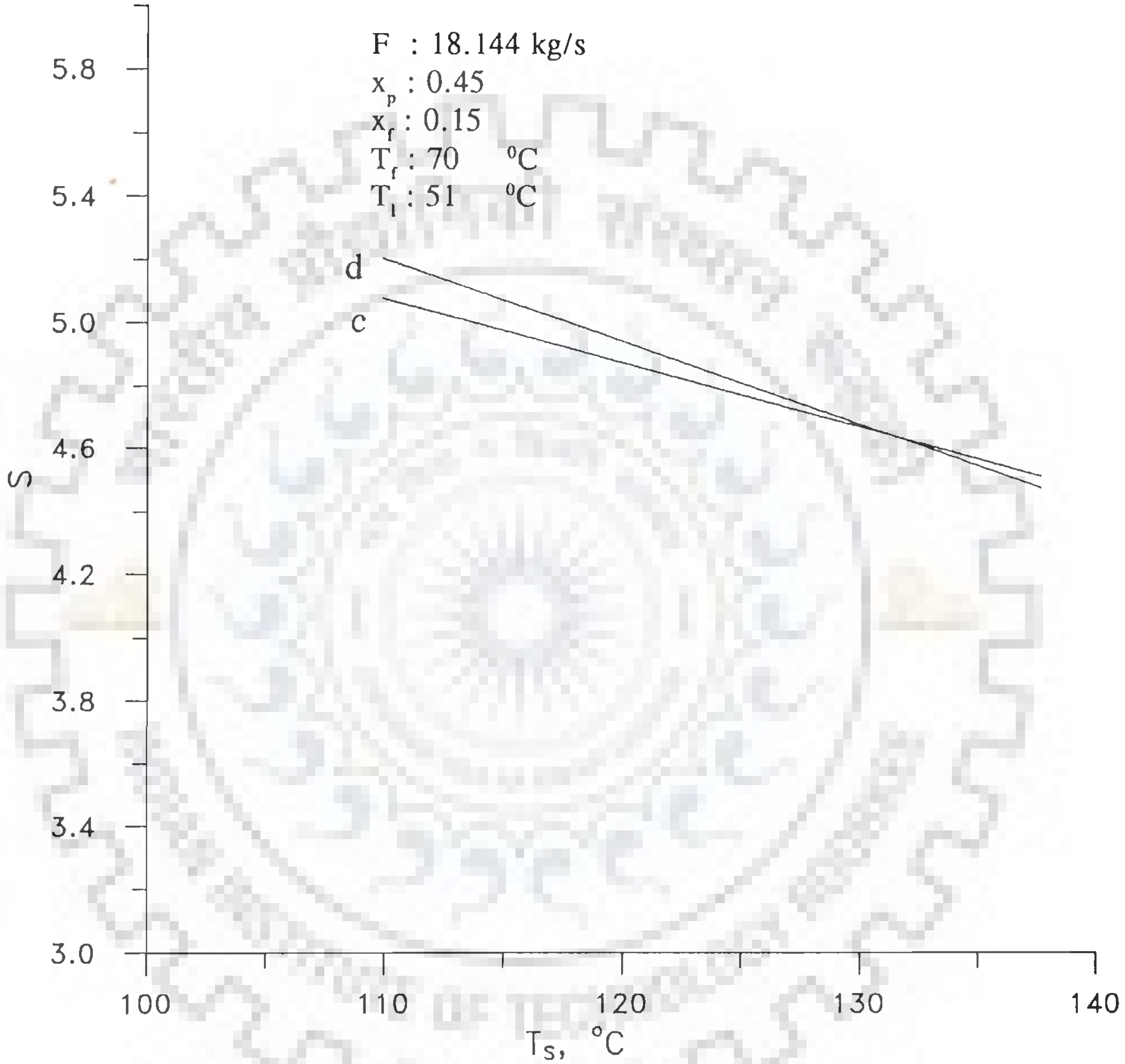

Fig. 6.41 Effect of T_S on S for sequences c and d

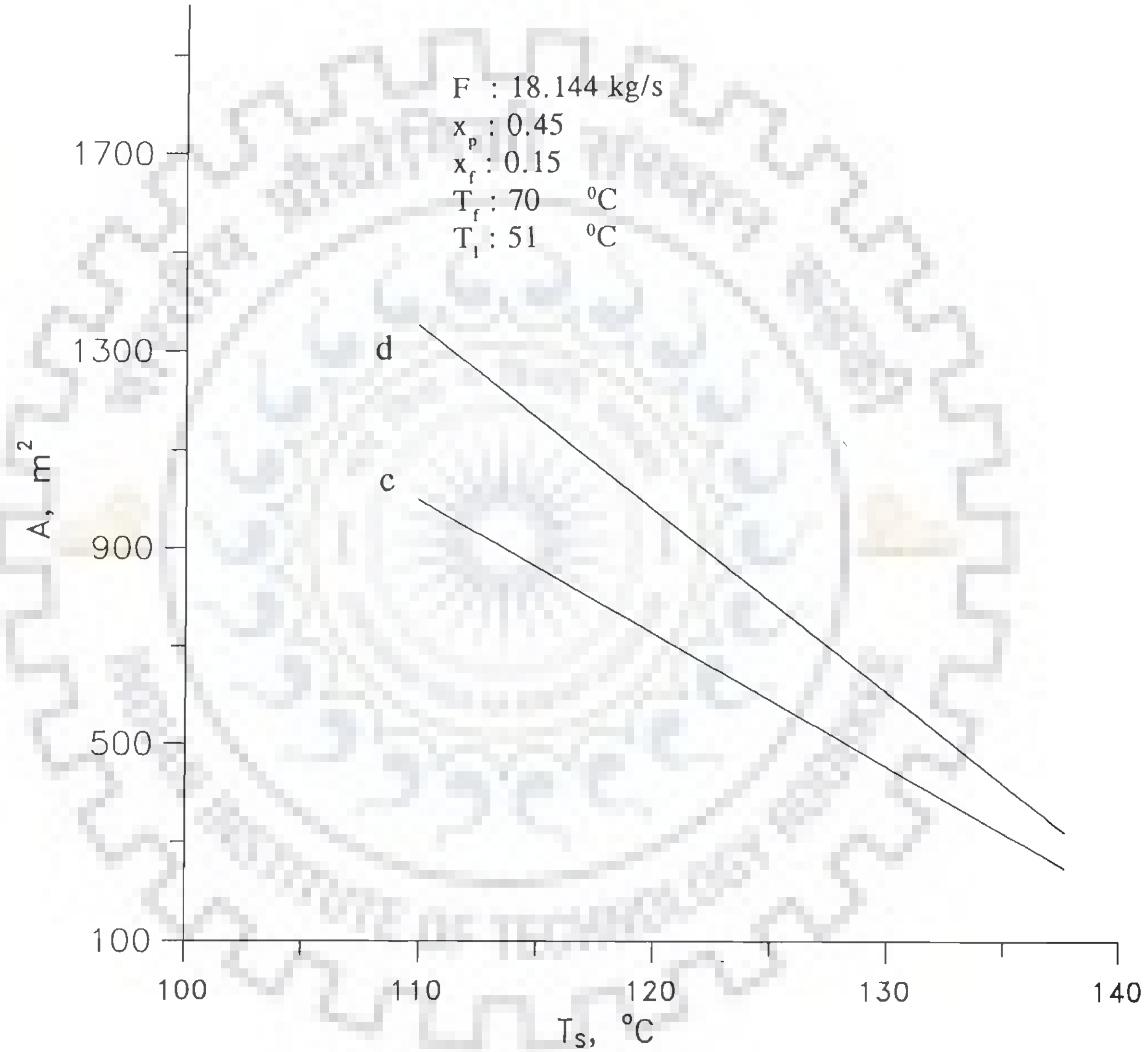

Fig. 6.42 Effect of T_S on A for sequences c and d

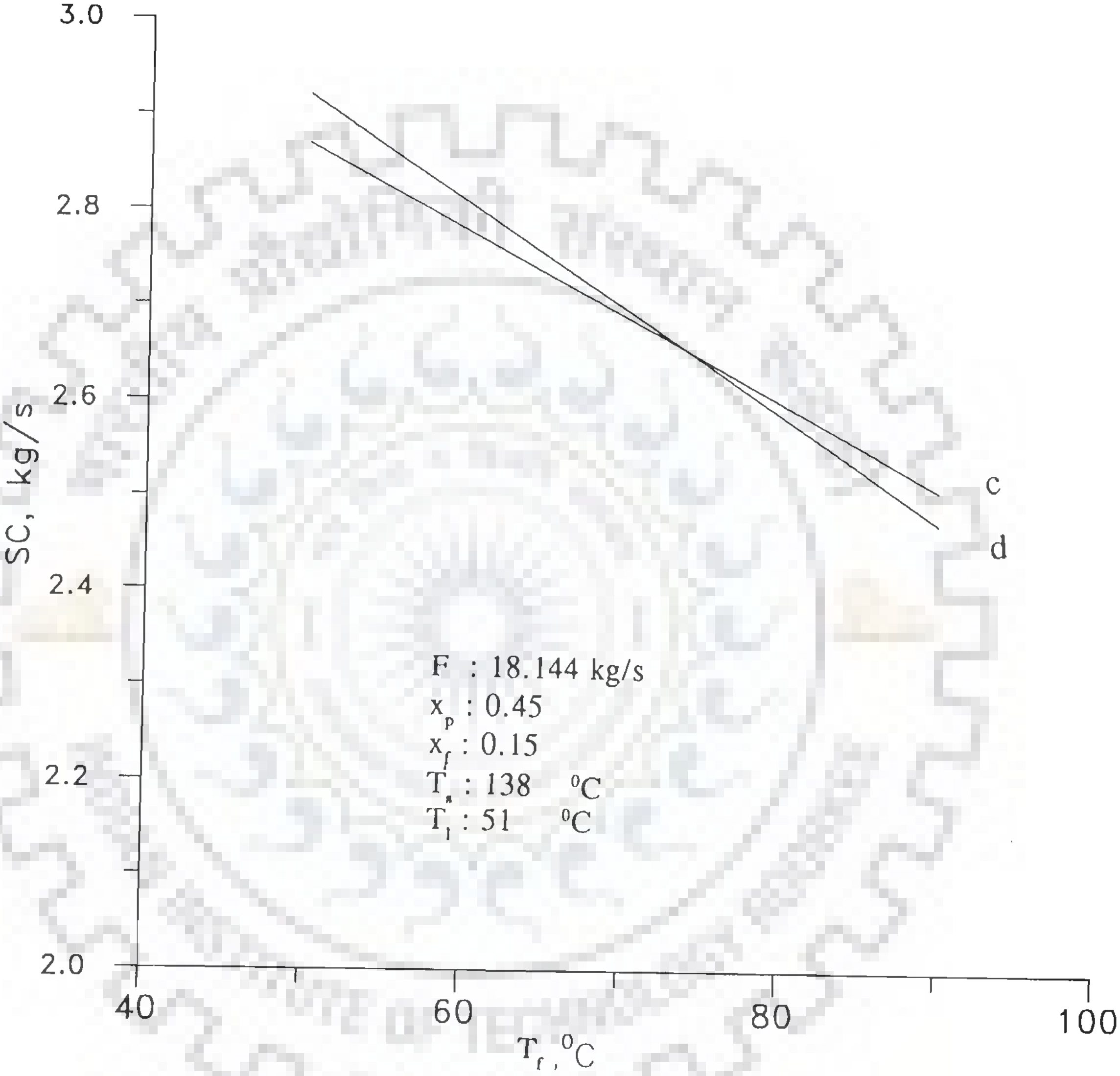

Fig. 6.43 Effect of T_f on SC for sequences c and d

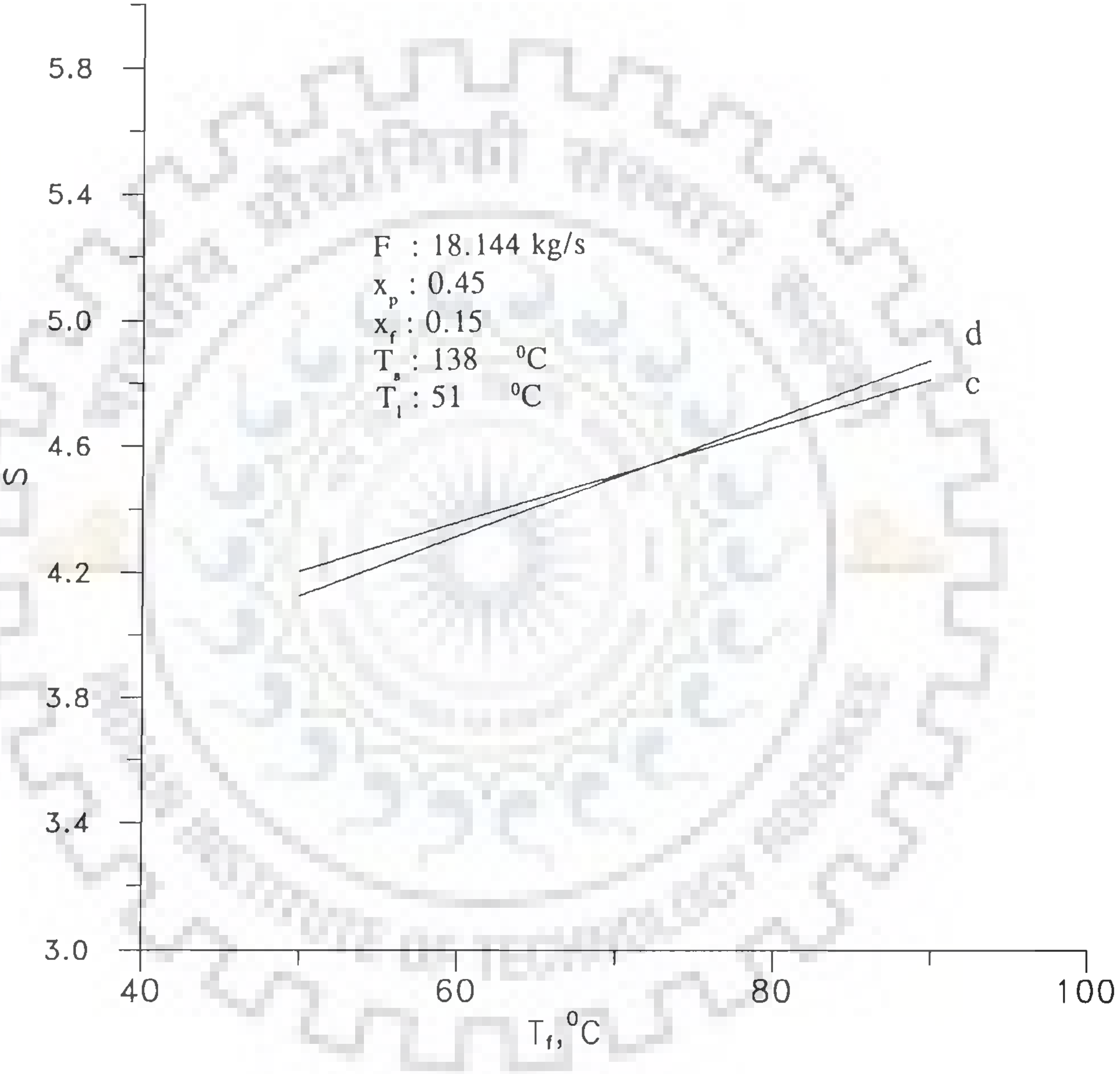

Fig. 6.44 Effect of T_f on S for sequences c and d

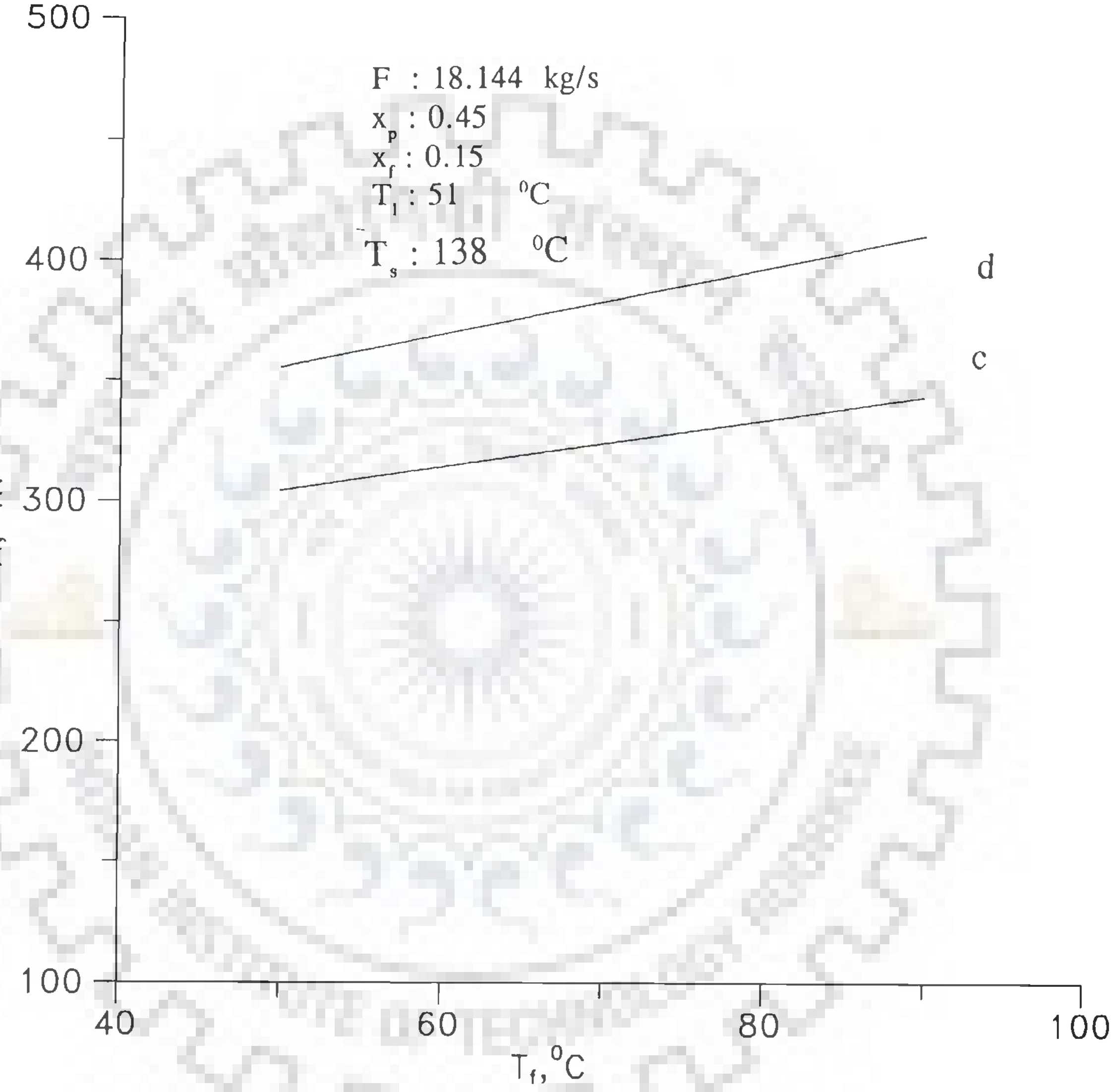

Fig. 6.45 Effect of T_f on A for sequences c and d

T_f greater than 73°C for this sequence. Of course, the area requirements are always lower for sequence c for the entire range of T_f investigated, as is evident from Figure 6.45.

6.9.3 Effect of Last Effect Temperature

Influence of T_1 on the three parameters under study is depicted in Figures 6.46, 6.47 and 6.48.

Sequence **c** is clearly the better choice among the two from the viewpoint of all of the three parameters studied over the entire range of T_1 traversed in this study. However, the benefits regarding S and SC are only marginal.

6.9.4 Effect of Liquor Feed Concentration

The dependence of the three output parameters on x_f is represented in Figures 6.49, 6.50 and 6.51.

As expected, sequence **c** is better than sequence **d** over the entire range of x_f investigated, although the benefits in terms of S and SC are not significant.

6.9.5 Effect of Liquor Feed Rate

Figures 6.52, 6.53 and 6.54 represent, respectively, the variation of SC, S and A as a result of changes in F.

Figure 6.52 shows that the plots for the two sequences **c** and **d** are almost overlapping for the entire range of F investigated, meaning thereby that the steam consumptions in the two cases are

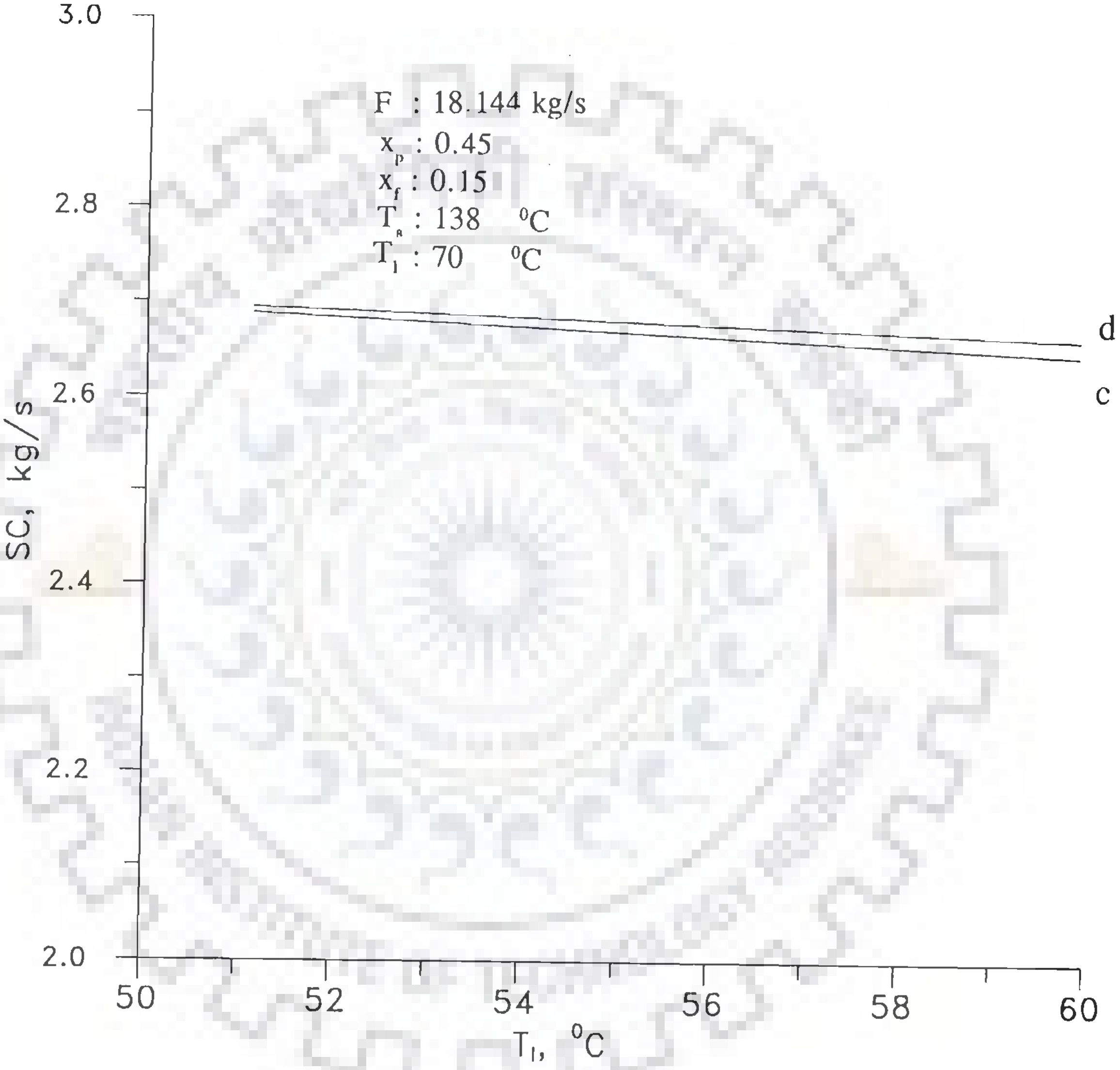

Fig. 6.46 Effect of T_l on SC for sequences c and d

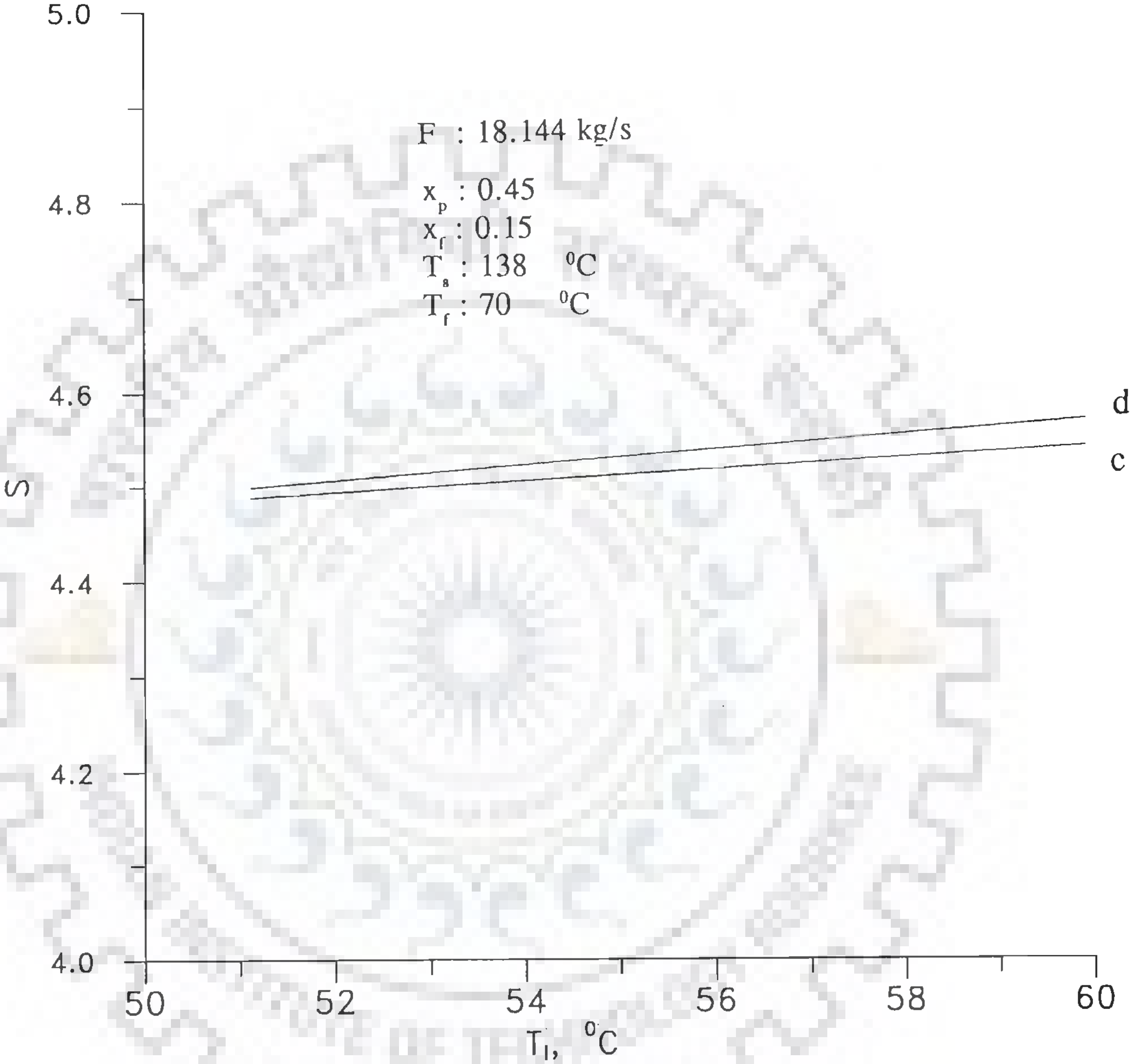

Fig. 6.47 Effect of T_l on S for sequences c and d

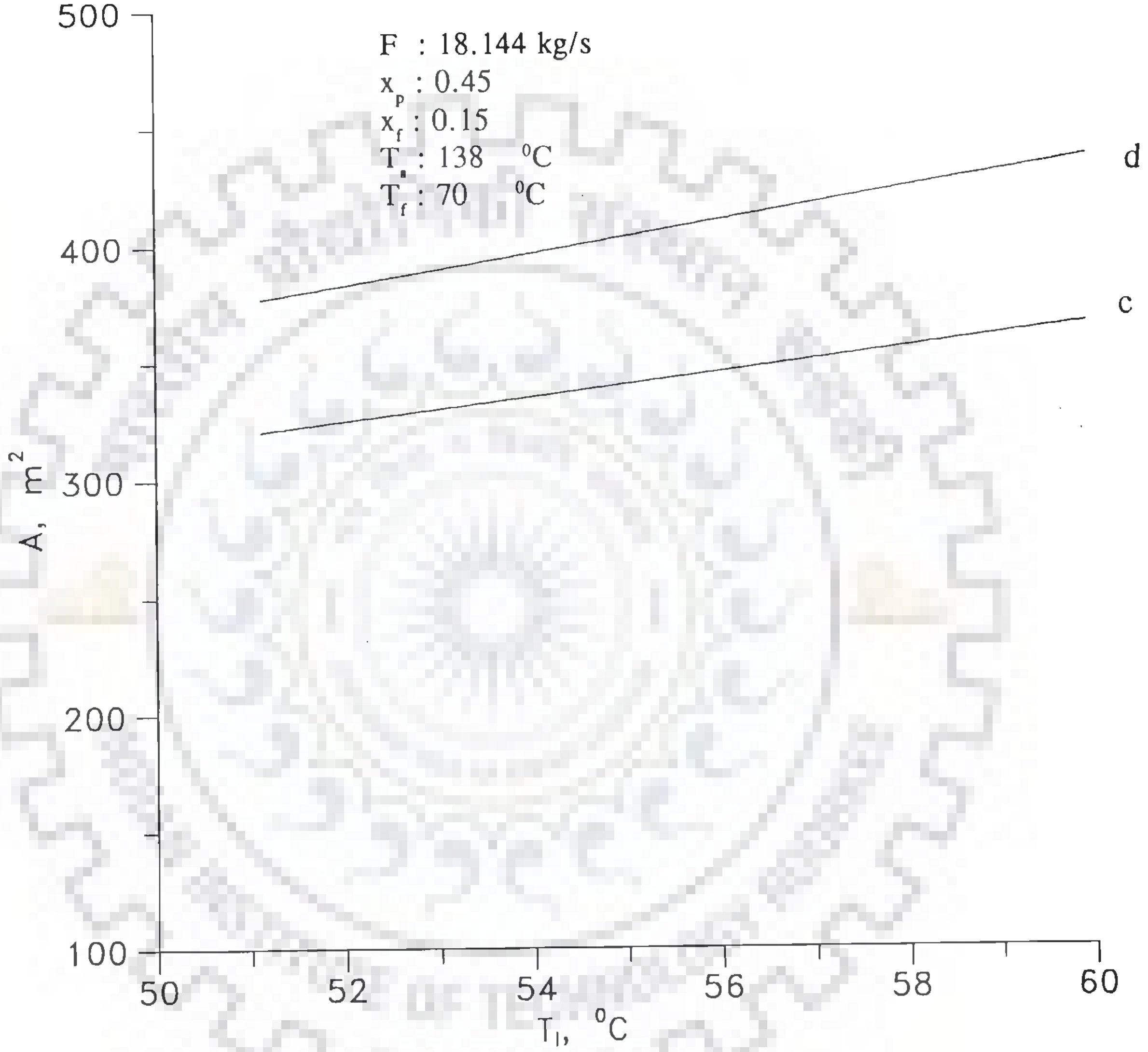

Fig. 6.48 Effect of T_l on A for sequences c and d

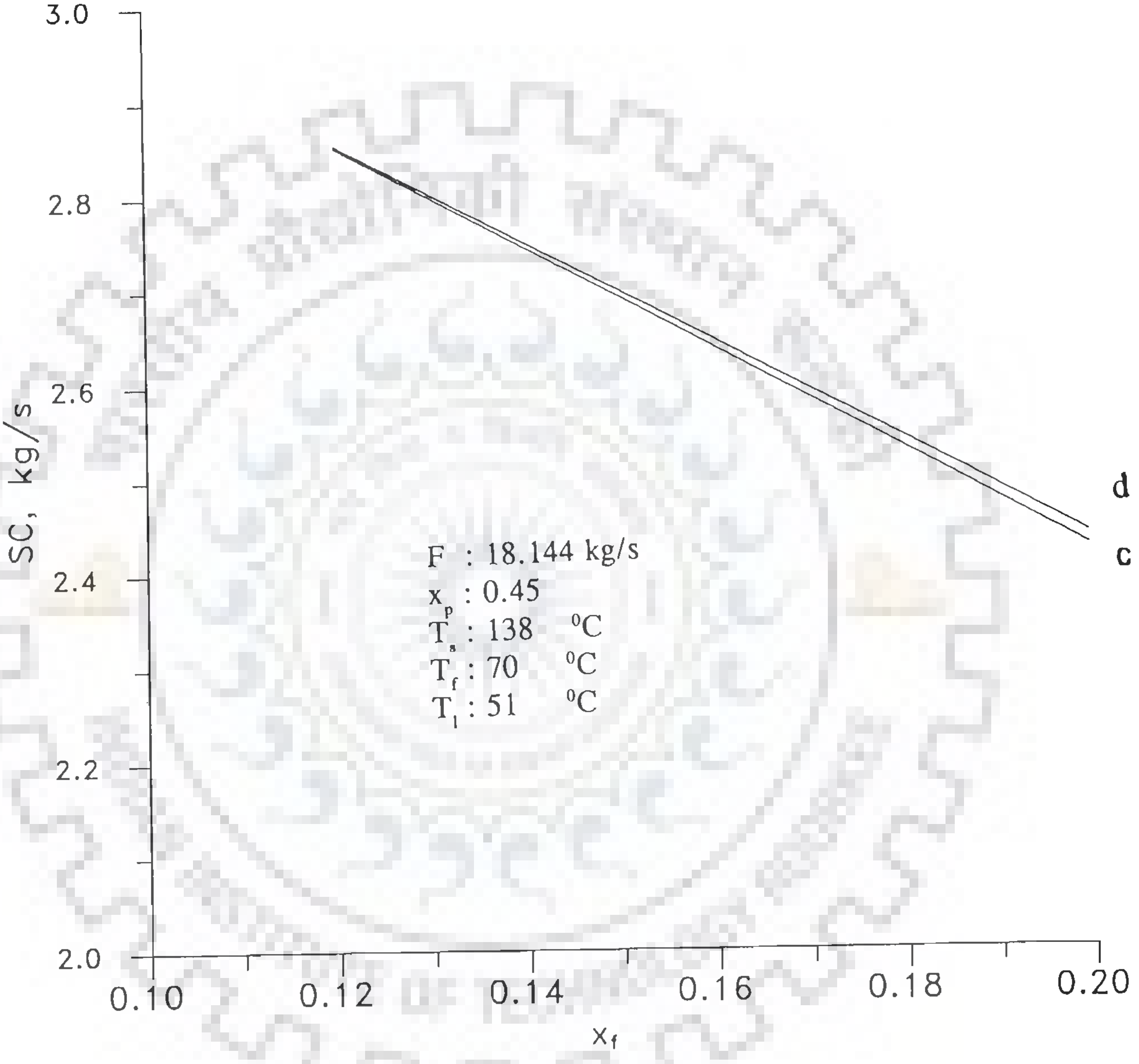

Fig. 6.49 Effect of x_f on SC for sequences c and d

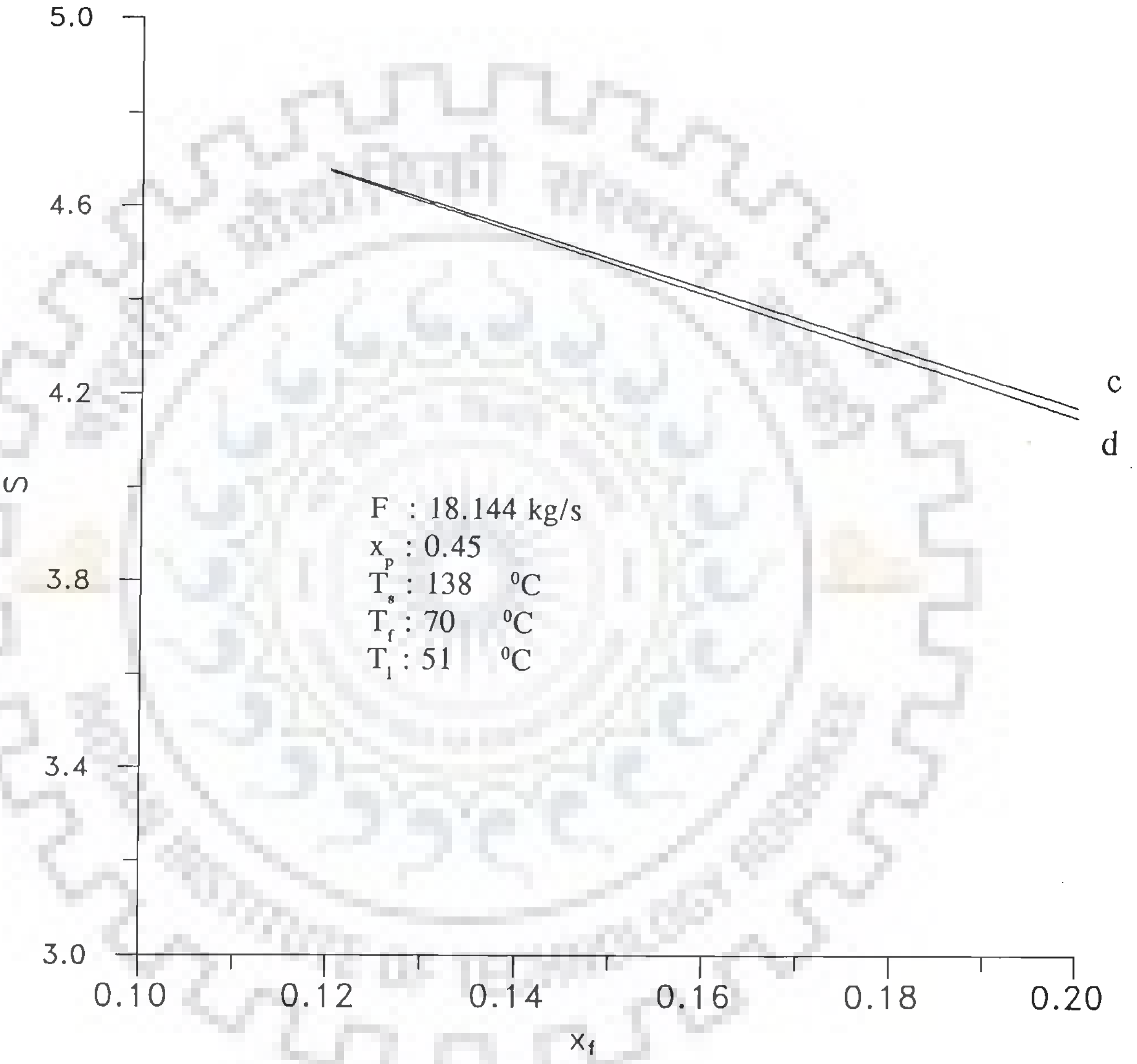

Fig. 6.50 Effect of x_f on S for sequences c and d

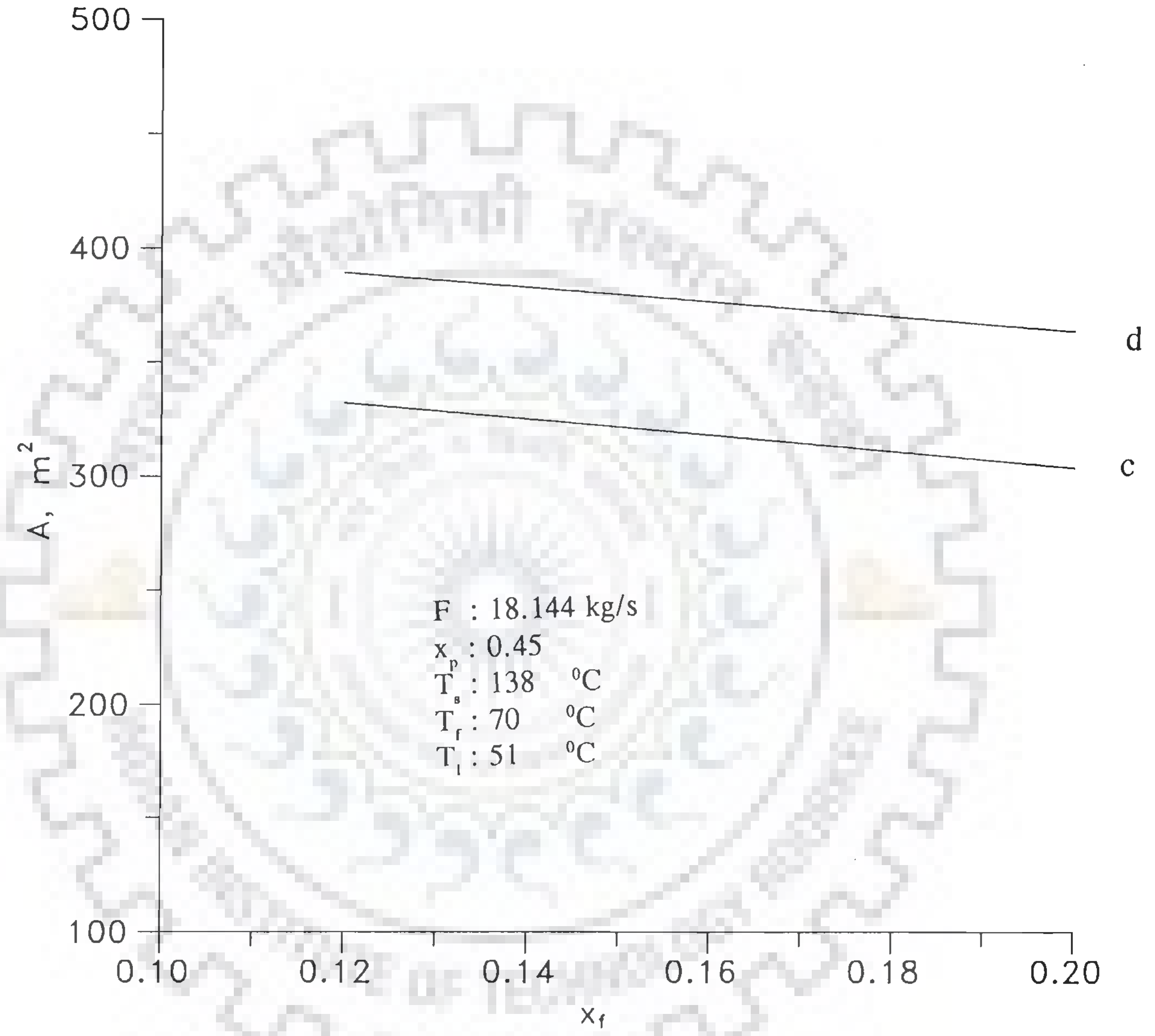

Fig. 6.51 Effect of x_f on A for sequences c and d

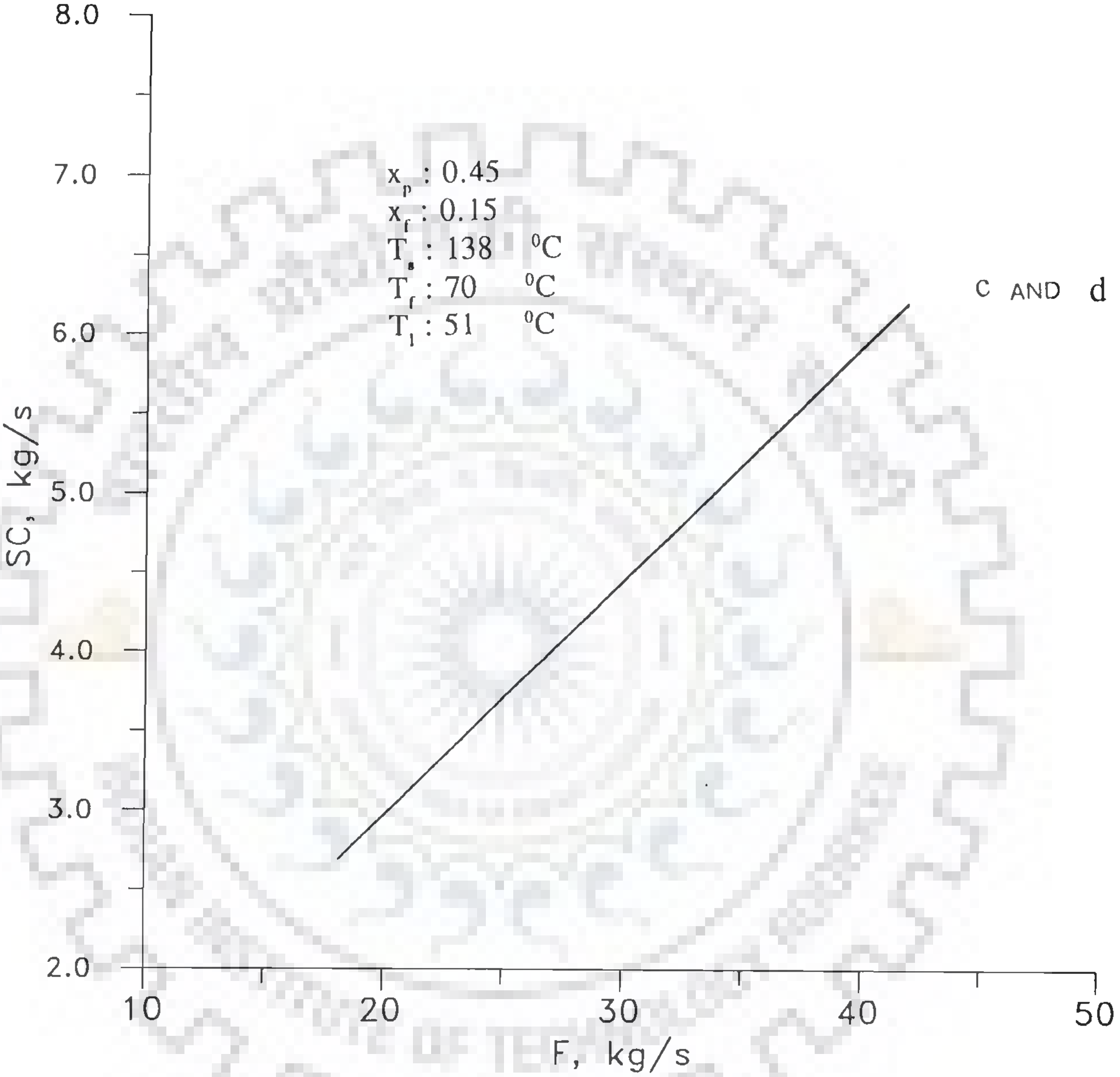

Fig. 6.52 Effect of F on SC for sequences c and d

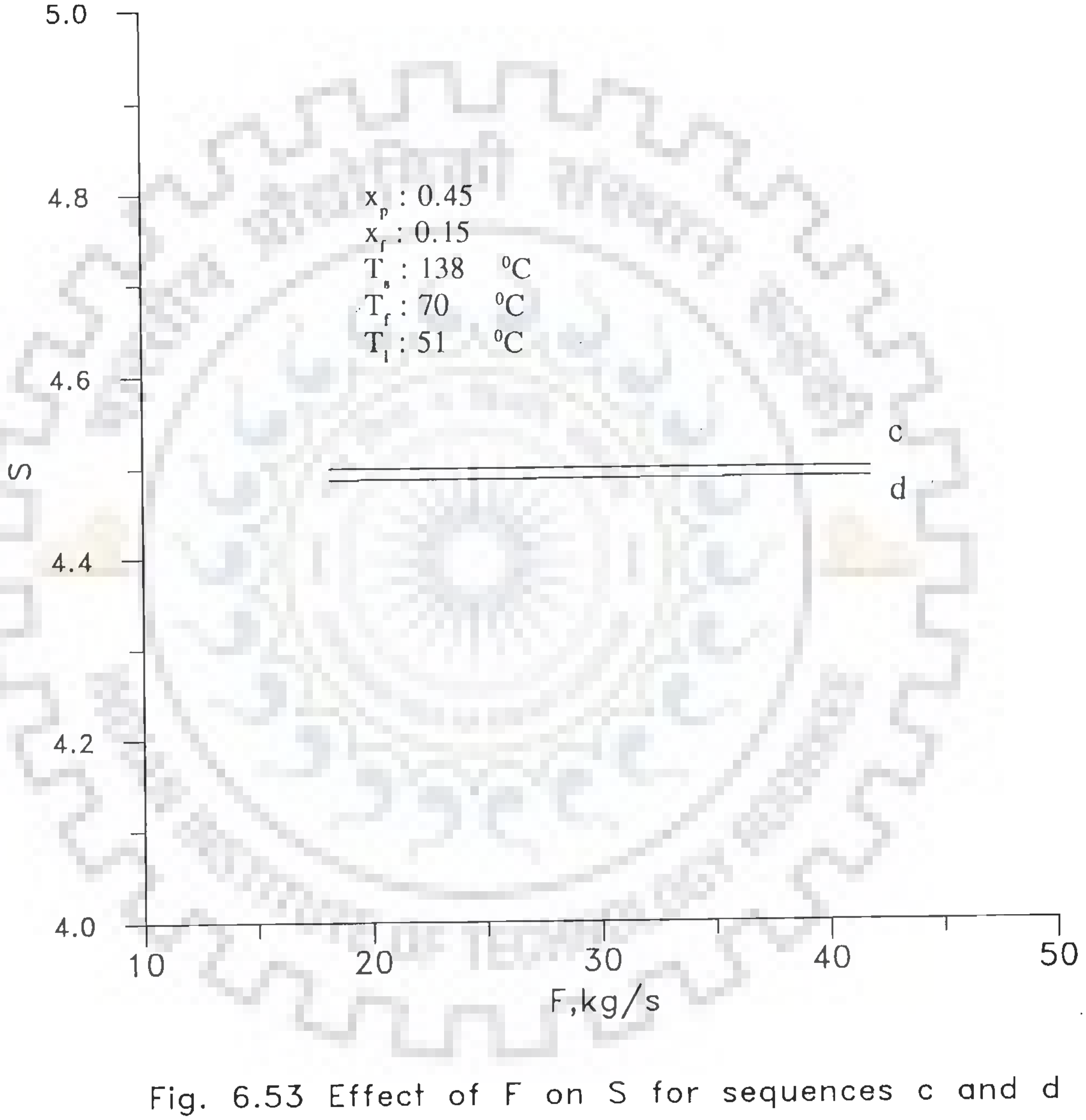

Fig. 6.53 Effect of F on S for sequences c and d

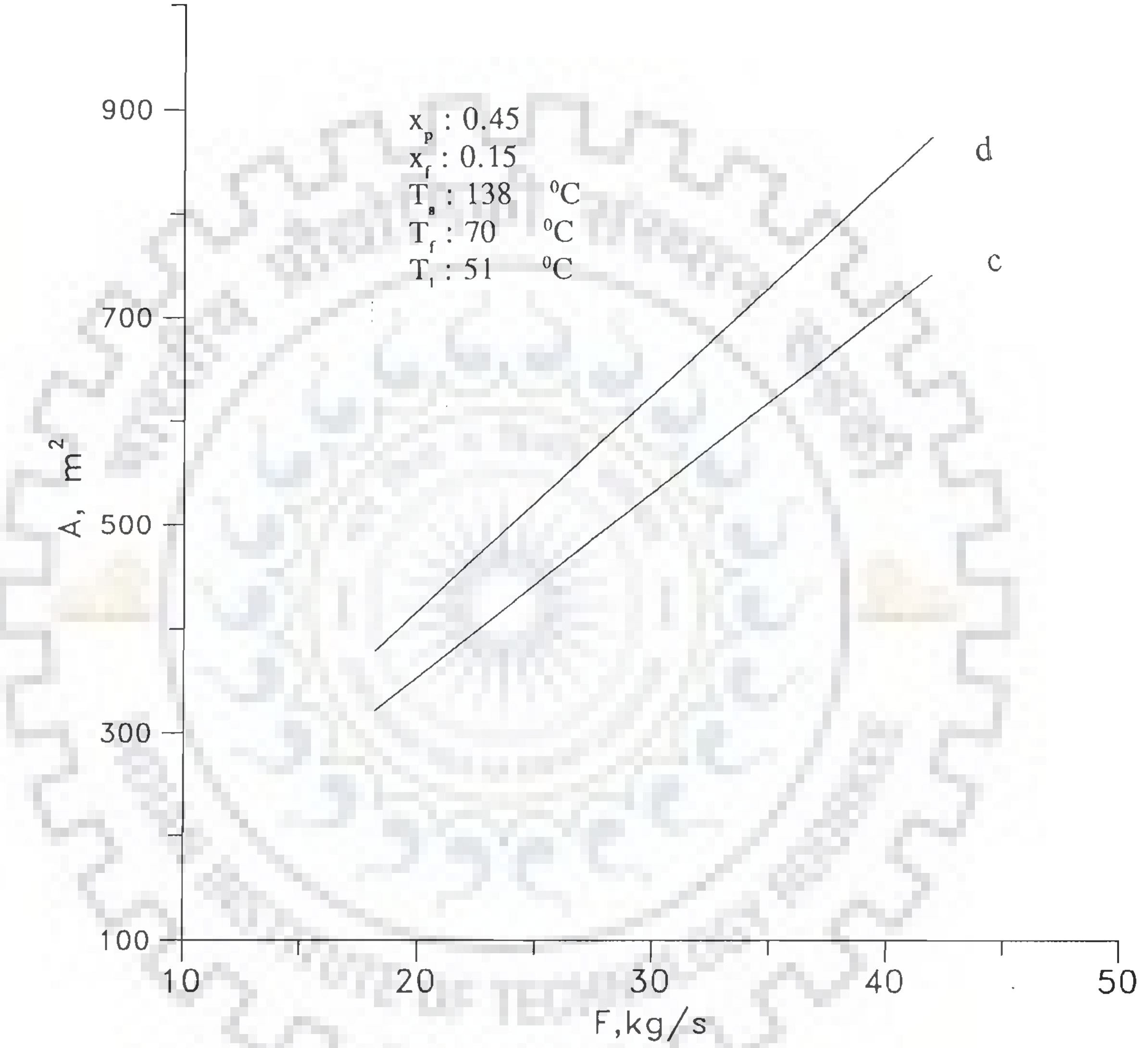

Fig. 6.54 Effect of F on A for sequences c and d

almost identical. The minute difference among them is better represented in Figure 6.53, representing S vs F for these sequences.

As is clear from Figure 6.54, the heating surface required for sequence **c** is lower than that for sequence **d** over the entire range of F investigated.

From all the foregoing discussions in this Section, it is, therefore, inferred that flow sequence **c** scores over sequence **d**, though not by an appreciable margin with regard to steam consumption, SC and steam economy, S. However, an appreciable benefit for this flow sequence **c** is noted as far as heating surface area is concerned.

6.10 STUDIES ON SCANDINAVIAN SEQUENCE

Scandinavian sequence (3-4-5-1A-1B-2) is probably employed with the thought that the liquor with the highest concentration should not be exposed to the highest temperature, as this aggravates the problem of scaling inside the evaporator tubes. As a result, the overall heat transfer coefficient declines bringing down the steam-economy and making the system more inefficient. It has therefore been considered proper to analyse this sequence with regard to the three output parameters SC, S, and A as functions of the five input parameters used for other similar studies.

6.10.1 Comparison Between Results for Sequence c and Quintuple Effect Evaporator Using Scandinavian Sequence

In Section 6.9, it has been concluded that the sextuple effect evaporator employing backward feed along with the splits of both feed and steam, i.e., sequence **c**, is better than the one employing a mixed feed sequence 5-6-4-3-2-1. It is now worthwhile to make a comparison between the sequence **c** and the Scandinavian sequence.

For this purpose, various quantities from the respective models have been calculated and are compared in Tables 6.6 through 6.10.

TABLE 6.6

EFFECT OF T_s ON THE OUTPUT PARAMETERS

($F = 18.144$ kg/s, $x_f = 0.15$, $x_p = 0.45$
$T_1 = 51$ °C, $T_f = 70$ °C)

T_s	Steam Consumption		Steam Economy		Area	
	c*	Scan*	**c**	Scan	**c**	Scan
120	2.4693	3.1311	4.899	3.863	633.7	495.2
130	2.5897	3.2656	4.671	3.704	401.6	429.8
138	2.6884	3.4015	4.499	3.556	321.7	361.6

Note : * **c** - refer Table 3.2
* Scan - Scandinavian sequence

TABLE 6.7

EFFECT OF T_f ON THE OUTPUT PARAMETERS

(F = 18.144 kg/s, x_f = 0.15, x_p = 0.45
T_1 = 51 ^oC, T_s = 138^oC)

T_f	Steam Consumption		Steam Economy		Area	
	c	Scan	c	Scan	c	Scan
50	2.8696	3.7830	4.215	3.198	345.8	393.8
60	2.7790	3.5901	4.353	3.369	332.3	376.9
70	2.6884	3.4015	4.499	3.556	321.7	361.6
80	2.5978	3.2145	4.656	3.763	313.2	348.6
90	2.5074	3.0308	4.824	3.991	306.3	335.7

TABLE 6.8

EFFECT OF T_1 ON THE OUTPUT PARAMETERS

(F = 18.144 kg/s, x_f = 0.15, x_p = 0.45
T_f = 70 ^oC, T_s =138^oC)

T_1	Steam Consumption		Steam Economy		Area	
	c	Scan	c	Scan	c	Scan
51	2.6884	3.4015	4.499	3.556	321.7	361.6
55.5	2.6655	3.3798	4.538	3.579	342.7	383.0
60.0	2.6464	3.3708	4.571	3.588	367.5	406.7

TABLE 6.9

EFFECT OF x_f ON THE OUTPUT PARAMETERS

($F = 18.144$ kg/s, $x_p = 0.45$, $T_1 = 51^{o}C$,
$T_f = 70\ ^{o}C$, $T_s = 138\ ^{o}C$)

x_f	Steam Consumption		Steam Economy		Area	
	c	Scan	c	Scan	c	Scan
0.12	2.8549	3.6139	4.661	3.682	331.9	309.1
0.14	2.7430	3.4708	4.557	3.601	325.2	330.4
0.15	2.6884	3.4009	4.499	3.557	321.7	361.8
0.18	2.5286	3.1965	4.305	3.406	310.8	372.1
0.20	2.4252	3.0644	4.156	3.289	303.7	392.2

TABLE 6.10

EFFECT OF F ON THE OUTPUT PARAMETERS

($x_p = 0.45$, $x_f = 0.15$, $T_1 = 51^{o}C$,
$T_f = 70\ ^{o}C$, $T_s = 138\ ^{o}C$)

F	Steam Consumption		Steam Economy		Area	
	c	Scan	c	Scan	c	Scan
18.144	2.6884	3.4010	4.499	3.557	321.7	361.8
26.082	3.8646	4.8897	4.499	3.556	462.4	519.8
34.02	5.0407	6.3766	4.499	3.557	603.2	678.5
41.963	6.2176	7.8669	4.499	3.556	744.0	836.3

It is obvious from Table 6.6 that the sequence **c** is distinctly better. The values for the area, A cross-over at a value of T_s equal to $130^{o}C$, thereby indicating that sequence **c** is better beyond this temperature which also represents the normal practice in mills.

As regards the effect of liquor feed temperature T_f, it is noted from Table 6.7 in no uncertain terms that the sequence **c** is definitely better.

Table 6.8 reveals that with regard to the last effect temperature T_1, sequence **c** once again turns out to be a better choice.

Accounting for the effect of feed concentration, Table 6.9 points out towards sequence **c** as the better competitor for $x_f \geq 0.14$. A similar conclusion is drawn from Table 6.10 with respect to the role of F.

A re-examination of Tables 6.6 through 6.10 reveals that for the standard set of values of the input parameters, viz., F=18.144 kg/s, T_s=138°C, T_f=70°C, T_1=51°C, x_f=0.15 and x_p=0.45 the steam economy of sequence **c** is higher than that of Scandinavian sequence by approximately 29%, while the steam consumption is lower by approximately 24%. However, the requirement of heating surface area is only marginally lower by just 4%.

Therefore, the obvious conclusion that emerges out of this study is that the sextuple effect evaporator employing backward feed with equal splits of the feed among the fifth and sixth effects and the split of steam in the two bodies of the first effect is the best out of all the sequences, mentioned in Table 3.2.

CHAPTER - 7

CONCLUSIONS AND RECOMMENDATIONS

In this Chapter, various conclusions drawn from the present work and also the recommendations for future work are summarized.

7.1 CONCLUSIONS

Based on the results of the present investigation, several note-worthy conclusions can be drawn for their implementation in pulp and paper mills for an economic and efficient operation of the black liquor multiple effect evaporators. However, this will be subject to the condition that a given input parameter varies over its specified range of operation, as presented in Table 3.3. Also the conclusions are applicable to black liquors whose density, viscosity, specific heat, thermal conductivity and boiling point rise are respectively calculated from Eqs. (2.1), (2.8), (2.13), (2.15) and (2.17) and the overall heat transfer coefficient using Gudmundson's model, described in Chapter 2. The salient conclusions are given below :

1. Considering the total annual cost of multiple effect evaporators for concentrating the black liquor, it is recommended that sextuple effect evaporators should be adopted in mills, as they are the most economical.

2. The sensitivity analysis has succeeded in pin- pointing that perturbations in live steam temperature, T_S, are likely to affect the output parameters the most. Hence,

it seems desirable to control the perturbations in this parameter more carefully to ensure steady and trouble- free operation of evaporators.

3. It is also established that for the backward feed sequence of the sextuple effect evaporators, it is necessary to split the feed evenly among 5th and 6th effects and the steam in the 1st effect to have the best steam energy economy, along with the requirement of the smallest heating surface area if the liquor feed concentration lies in a range from 0.12 to 0.16, its temperature is greater than $58^{\circ}C$ and saturation temperature of the heating steam exceeds $120^{\circ}C$, irrespective of temperature of the last effect. This liquor flow sequence is represented by
5
 >4- 3- 2- 1B- 1A.
6

4. Another conclusion, which merits its inclusion is that for the mixed feed sequences it is advantageous to employ a sequence in which liquor feed enters into 5th effect, represented by (5- 6- 4- 3- 2- 1). This, in fact, corresponds to the best steam economy coupled with the lowest requirement of heating surface area, except for a marginal increase over the area requirements of other mixed feed sequences for feed temperature exceeding $77^{\circ}C$.

5. Yet another conclusion worth-noting is that the liquor flow sequence $\begin{smallmatrix}5\\6\end{smallmatrix}{>}4-\ 3-\ 2-\ 1B-\ 1A$ has an advantage over the sequence 5- 6- 4- 3- 2- 1 with regard to the requirements of heating surface area. However, this superiority is not so appreciable as far as steam consumption and steam economy are concerned.

6. It is found that the liquor flow sequence $\begin{smallmatrix}5\\6\end{smallmatrix}{>}4-\ 3-\ 2-\ 1B-\ 1A$ for the concentration of weak kraft black liquor scores over the Scandinavian sequence 3- 4- 5- 1A- 1B- 2.

7. It is also to be noted that the close agreement between the quantities predicted by Eqs. (5.81 through 5.94) and the reported data of McDonald suggests that these equations provide accurate values of the various parameters. Hence, the sets of equations, developed in the present work for the various flow sequences are recommended for computation of the boiling point rise, concentration and temperature in each effect, steam consumption, steam economy and heating area requirements of multiple effect evaporators, with high confidence levels.

8. It may be emphasized that the equations in Chapter 5 have been developed with no constraints of values of physical properties, and hence are of general applicability. They can, therefore, be employed for any black liquor based on different raw materials, say

bagasse, straw, bamboo, etc., provided one uses suitable correlations for physico-thermal properties.

7.2 RECOMMENDATIONS

Following recommendations are made for extension of the present work :

1. It would be more meaningful if a similar analysis of multiple effect evaporators is carried out taking into account the effects of, (a) heat dissipation from evaporator to the surroundings, and (b) fouling in evaporator tubes, while employing the heat transfer rate equation in the models.

2. For further improvement in energy conservation measures, a critical quantitative analysis should be made to study the effect of variations of the input parameters on the output parameters studied in this work by incorporating condensate flash steam and also heat exchange among different liquor and feed streams.

3. An IBM PC based software package should be developed incorporating the above recommendations to help operators to take care of any possible changes in characteristics of weak kraft black liquor or perturbations in input operating parameters. This would enable them to take on the spot decisions and run the multiple effect evaporator house at optimum conditions.

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APPENDIX - A

COMPUTER PROGRAM FOR A SEXTUPLE EFFECT EVAPORATOR WITH BACKWARD FEED

A.1 PROGRAM DESCRIPTION

The computer program given in this Appendix is written in FORTRAN-77 to design/analyse a sextuple effect evaporator employing backward feed sequence. The parameters used in different subroutines and functions are annotate within the subroutine itself. The work of main program along with the different SUBROUTINES and FUNCTIONS are described below :

A.2 MAIN PROGRAM: It takes steam temperature, T_s, feed temperature, T_f, last effect temperature, T_1, and, feed concentration, x_f, data in the indicated sequence from data file TNS.DAT. The other required parameters are supplied through DATA STATEMENT in the main program. It calls SUBROUTINE NLSYST, HTC and TABLE to perform various tasks.

A.3 SUBROUTINES

A.3.1 NLSYST : This subroutine solves a system of non-linear equations by Newton-Raphson's method. The partial derivatives of the functions are estimated by difference quotients when a variable is perturbed by an amount equal to DELTA (DELTA is added). This is done for each variable in each function. Increments to improve the estimates for **X**-values are computed from a system of equations using subroutine ELIM.

A.3.2 ELIM : This subroutine solves a set of linear equations and gives an LU decomposition of the coefficient matrix. The Gauss elimination method is used, with partial pivoting. Multiple right hand sided are permitted, they should be supplied as columns that augment the coefficient matrix.

A.3.3 HTC : This SUBROUTINE is used to calculate overall heat transfer coefficient of a multiple effect evaporator using Gudmundson's model.

A.3.4 TABLE : This SUBROUTINE tabulates the results of the program.

A.3.5 FCN : SUBROUTINE FCN is used for generation of mass-and energy-balance equations for multiple effect evaporator employing backward feed.

A.4 FUNCTIONS

The physical properties of black liquor and steam are supplied to the SUBROUTINES using FUNCTIONS as are described below :

A.4.1 **PL1** : Supplies latent heat of condensation of saturated steam at given temperature.

A.4.2 **BPR1** : Returns boiling point rise of black liquor at given concentration

A.4.3 **PRS** : Returns saturation pressure of steam at given saturation temperature.

A.4.4 **AMU1** : Supplies viscosity of black liquor at given concentration and temperature.

A.4.5 **CP2** : Supplies specific heat of black liquor at given concentration of black liquor.

A.4.6 **RHO1** : Returns density of black liquor at given concentration.

A.4.7 **THERC1** : Supplies thermal conductivity of black liquor at given concentration and temperature.

```
C      MAIN PROGRAMME FOR DESIGN/ANALYSIS OF A SEXTUPLE EFFECT
C      EVAPORATOR EMPLOYING BACKWARD FEED SEQUENCE
C
      REAL X(12),F(12),DELTA,XTOL,FTOL,AX(12),AHU(6),HHU(12)
      REAL T(6),BPR(6+1),PL(6),CP(6+1),C(6),HU(6),PR(6)
      REAL FLI(7),DELT(7),EVAP(7),AUP(7),ALP(7),BPRP(7),TLP(7)
      REAL CO,CF,TO,TF,TN,CPV,FR
      INTEGER N,MAXIT,I
      COMMON/B1/CO,CF,TO,TF,TN,CPV,FR,PLO
      COMMON/B2/HU,BPR,PL,CP,C,T,PR
      EXTERNAL FCN
      DATA I,N,NE,MAXIT,DELTA/0,12,6,20,0.01/
      DATA X,XTOL,FTOL,FR/0.51,0.61,0.7,0.4,0.4,0.9,0.8,0.7,0.6,0.5,
     10.2,1.13,0.0000001,0.0000001,144000/
      DO 99 K=1,12
      AX(K)=X(K)
  99  CONTINUE
      DATA CF,CPV/0.45,0.5/
      DATA HU/262,295,252,251,221,221/
      DO 909 K1=1,6
      AHU(K1)=HU(K1)
 909  CONTINUE
      OPEN(UNIT=1,FILE='TNS.DAT')
      DO 21 LP1=1,1
      READ(1,*)TO,TF,TN,CO
      TO=TO*9/5+32
      TF=TF*9/5+32
      TN=TN*9/5+32
      N1=NE-1
      DO 20 IKL=1,300
      CALL NLSYST(FCN,N,NE,MAXIT,X,F,DELTA,XTOL,FTOL,I)
C
C      DETERMINATION OF OVERALL HEAT TRANSFER COEFFICIENT
C
      DO 111 J=1,N1
      ALP(J+1)=X(J)
      AUP(J)=X(J+N1)
111   CONTINUE
      AUP(NE)=TN/TO
      AUP(NE+1)=TF/TO
      ALP(1)=CO/CF
      DO 112 I=1,NE
      AUP(I)=AUP(I)*TO
      ALP(I)=ALP(I)*FR
      BPRP(I)=BPR(I)*TO
      TLP(I)=BPRP(I)+AUP(I)
112   CONTINUE
      DO 113 M=1,N1
      FLI(M)=ALP(M+1)
      DELT(M+1)=AUP(M)-TLP(M+1)
      EVAP(M)=ALP(M+1)-ALP(M)
113   CONTINUE
```

```
       FLI(NE)=FR
       DELT(1)=TO-TLP(1)
       EVAP(NE)=FR-ALP(NE)
       ST2=X(N)*(FR/50)
       TLP(NE+1)=TF
       C(7)=CO
       DO 33 KI=1,6
       HHU(KI)=HU(KI)
33     CONTINUE
       DO 114 IJ=1,NE
       IF(IJ.LT.NE)THEN
       CALL HTC(C(IJ),TLP(IJ),TLP(IJ+1),OHTC,FLI(IJ),EVAP(IJ),PL(IJ)
     1,ST2,IKL,IJ,C(IJ+1))
        ELSE
       CALL HTC(C(IJ),TLP(IJ),TLP(IJ+1),OHTC,FLI(IJ),EVAP(IJ),PL(IJ)
     1,ST2,IKL,IJ,CO)
        ENDIF
        IF(IKL.LE.200)THEN
        FF1=0.05
        ELSE
        FF1=0.01
        ENDIF
        HU(IJ)=OHTC+FF1*(OHTC-HHU(IJ))
C       WRITE(*,*)'IJ=',IJ
114    CONTINUE
C       WRITE(*,*)'IKL=',IKL
C       WRITE(*,*)'X',(X(KL),KL=1,12)
C       WRITE(*,*)'F',(F(KL),KL=1,12)
20     CONTINUE
       CALL TABLE(X,N,NE)
        DO 991 K=1,12
        X(K)=AX(K)
 991    CONTINUE
        DO 908 K1=1,6
        HU(K1)=AHU(K1)
 908    CONTINUE
21     CONTINUE
       STOP
       END
C     ----------------------------------------------------------------
       SUBROUTINE FCN(X,F,N,NE)
C      -----------------------------------------------------------
C      SUBROUTINE FCN FOR AUTOMATIC GENERATION OF MASS AND ENERGY
C      BALANCE EQUATIONS FOR MULTI EFFECT EVAPORATOR SYSTEMS
C      CO:     INITIAL CONCENTRATION OF FEED
C      CF:     FINAL CONCENTRATION OF PRODUCT
C      TO:     STEAM TEMPERATURE
C      TF:     FEED TEMPERATURE
C      TN:     TEMPERATURE OF Nth BODY
C      F :     FEED FLOW RATE
C      NE:     NO OF EFFECTS
C      AU:     SCALED TEMPERATURES OF EFFECTS
C      AL:     SCALED FLOW RATE OF EFFECTS
C      T :     TEMPERATURE OF DIFFERENT EFFECTS
```

```
C      BPR     BOILING POINT RISE( SCALED) IN DIFFEREENT EFFECTS
C      PL:     LATENT HEAT OF VAPORIZATION IN DIFFEREENT EFFECTS
C      CP:     SPECIFIC HEAT OF DIFFERENT EFFECTS
C      C :     CONCENTRATION OF LIQUOR IN DIFFERENT EFFECTS
C
C
C
C      -------------------------------------------------------------
       REAL X(12),F(12),T(6),BPR(6+1),PL(6),CP(6+1),C(6),AL(6+1)
       REAL AU(6+1),HU(6),PR(6)
       REAL CO,CF,TO,TF,TN,CPV,FR
       COMMON/B1/CO,CF,TO,TF,TN,CPV,FR,PLO
       COMMON/B2/HU,BPR,PL,CP,C,T,PR
       N1=NE-1
       AU(NE+1)=TF/TO
       AL(NE+1)=1.0
       AU(NE)=TN/TO
       T(NE)=(AU(NE)*TO-32)*0.5555
       TF1=(TF-32)*0.5555
       TO1=(TO-32)*0.5555
       CP(NE+1)=CP2(CO)/4187
       PLO=PL1(TO1)
       AL(1)=CO/CF
       BPR(NE+1)=0.0
C      WRITE(*,*)'CHECK',PLO,AL(1),TF1,TO1
       DO 30 KP=1,N1
       AL(KP+1)=X(KP)
       AU(KP)=X(KP+N1)
       T(KP)=(AU(KP)*TO-32)*0.5555
  30   CONTINUE
       DO 40 K2=1,NE
       C(K2)=CO/AL(K2)
       BPR(K2)=BPR1(C(K2))/TO
       CP(K2)=CP2(C(K2))/4187
       PL(K2)=PL1(T(K2))
       PR(K2)=PRS(T(K2))
C      WRITE(*,*)'PHY',C(K2),BPR(K2),CP(K2),PL(K2),T(K2),PR(K2)
  40   CONTINUE
C
C
C
C      ----------------------------------------------------------------
C      DEVELOPMENT OF EQUATIONS FOR MULTI EFFECT EVAPORATOR SYSTEM
C      FROM EFFECT NO. 1 TO EFFECT NO. NE
C
C      FOR FIRST EFFECT
C
       F(1)=TO*AL(2)*(CP(2)*(AU(2)+BPR(2))-CP(1)*(AU(1)+BPR(1)))/PLO+
     1X(N-1)-(AL(2)-AL(1))*(PL(1)+CPV*BPR(1)*TO)/PLO
       F(2)=HU(1)*TO*X(N)*(1-(AU(1)+BPR(1)))/(50*PLO)-X(N-1)
C
C      FOR SECOND TO LAST EFFECT(FIXED BY THE VALUE OF NE)
C
```

```
       KL=3
       DO 20 J=2,NE
       F(KL)=TO*AL(J+1)*(CP(J+1)*(AU(J+1)+BPR(J+1))-CP(J)*(AU(J)+
     1BPR(J)))/PLO+(AL(J)-AL(J-1))*(PL(J-1)+CPV*BPR(J-1)*TO)/PLO
     1-(AL(J+1)-AL(J))*(PL(J)+CPV*BPR(J)*TO)/PLO
       F(KL+1)=HU(J)*TO*X(N)*(AU(J-1)-(AU(J)+BPR(J)))/(50*PLO)
     1-(AL(J)-AL(J-1))*(PL(J-1)+CPV*BPR(J-1)*TO)/PLO
       KL=KL+2
  20   CONTINUE
C
C      -------------------------------------------------------------
C      EQUATION FOR PHYSICO-THERMAL PROPERTIES ARE INCLUDED INTERMS
C      OF FUNCTIONS PL1(T),CP1(CC,T),BPR1(CC)
      RETURN
      END
      FUNCTION PL1(T)
      PL1=(2503.22-2.4245*T+0.00119125*T**2-1.57346E-05*T**3)/2.3249
      RETURN
      END
      FUNCTION CP1(CC,T)
      CP1=0.990-4.44E-05*T-3.67E-04*CC*T
      RETURN
      END
      FUNCTION BPR1(CC)
      BPR1=41.4*(CC+0.1)**2
      RETURN
      END
      FUNCTION PRS(T)
      T1=T*1.8+32
      PRS=3.73812-0.108896*T1+0.0012806*T1**2-6.69111E-06*T1**3
     1+1.99203E-08*T1**4
      RETURN
      END

      SUBROUTINE TABLE(X,N,NE)
C
C      THIS SUBROUTINE TABULATES THE RESULTS IN TABULAR FORM
C
       REAL CO,CF,TO,TF,TN,CPV,FR,PLO
       REAL X(14),T(6),BPR(6+1),PL(6),CP(6+1),C(6),AL(7),AU(7),HU(6)
       REAL PR(6),TL(6),DELT(6),FLI(6),EVAP(6)
       INTEGER N,I,KK(6)
       COMMON/B1/CO,CF,TO,TF,TN,CPV,FR,PLO
       COMMON/B2/HU,BPR,PL,CP,C,T,PR
       WRITE(*,90)
       N1=NE-1
       DO 5 K=1,6
       KK(K)=K
5      CONTINUE
       DO 10 J=1,N1
       AL(J+1)=X(J)
       AU(J)=X(J+N1)
10     CONTINUE
       PLO=PLO*2.323
```

```
      AU(NE)=TN/TO
      AU(NE+1)=TF/TO
      AL(1)=CO/CF
      DO 20 I=1,NE
      AU(I)=(AU(I)*TO-32)*0.5555
      AL(I)=AL(I)*FR*1.260E-04
      BPR(I)=BPR(I)*TO/1.8
      TL(I)= BPR(I)+AU(I)
      PR(I)=PR(I)*6.895
      PL(I)=PL(I)*2.326
      HU(I)=HU(I)*5.678
      CP(I)=CP(I)*4.187
20    CONTINUE
      DO 30 M=1,N1
      FLI(M)=AL(M+1)
      DELT(M+1)=AU(M)-TL(M+1)
      EVAP(M)=AL(M+1)-AL(M)
30    CONTINUE
      FR1=FR*1.260E-04
      FLI(NE)=FR1
      DELT(1)=(TO-32)/1.8 -TL(1)
      EVAP(NE)=FR1-AL(NE)
      ST1=X(N-1)*FR1
      ST2=X(N)*(FR/50)*9.290E-02
      SE=(FR1*(1-CO)-AL(1)*(1-C(1)))/ST1
      TO=(TO-32)*0.555
      TF=(TF-32)*0.555
      TN=(TN-32)*0.555
      WRITE(*,190)FR1,CO,CF
      WRITE(*,210)TO,TF,TN
      WRITE(*,180)
      WRITE(*,100)(KK(L),L=1,6)
      WRITE(*,180)
      WRITE(*,80)(PR(I),I=1,6)
      WRITE(*,110)(AU(I),I=1,6)
      WRITE(*,120)(BPR(I),I=1,6)
      WRITE(*,125)(TL(I),I=1,6)
      WRITE(*,150)(HU(I),I=1,6)
      WRITE(*,155)(DELT(I),I=1,6)
      WRITE(*,157)(FLI(I),I=1,6)
      WRITE(*,160)(AL(I),I=1,6)
      WRITE(*,159)(EVAP(I),I=1,6)
      WRITE(*,170)(C(I),I=1,6)
      WRITE(*,180)
      WRITE(*,200)ST1,ST2
      WRITE(*,220)SE
80    FORMAT(2X,'PRESS  ',2X,7(F8.3,2X))
125   FORMAT(2X,'L.TEMP ',2X,7(F6.2,4X))
155   FORMAT(2X,'DELT   ',2X,7(F6.2,4X))
157   FORMAT(2X,'L/I    ',2X,7(F7.3,3X))
159   FORMAT(2X,'EVAP   ',2X,7(F7.3,3X))
90    FORMAT(/2X,'RESULTS FOR MULTI EFFECT EVAPORATOR SYSTEMS'
     1/2X,'(BACKWARD FEED )'/)
100   FORMAT(2X,'EFFECTS',2X,7(I2,8X))
```

```
110     FORMAT(2X,'S.TEMP ',2X,7(F6.2,4X))
120     FORMAT(2X,'BPR    ',2X,7(F6.2,4X))
150     FORMAT(2X,'H.T.C. ',2X,7(F8.1,2X))
160     FORMAT(2X,'L/O    ',2X,7(F7.3,3X))
170     FORMAT(2X,'CONC.  ',2X,7(F7.5,3X))
180     FORMAT(2X,76('-'))
190     FORMAT(2X,'FEED RATE= ',2X,F10.4,2X,'FEED CONC.=',F7.5,2X,
     1'PRODUCT CONC.=',F7.5)
200     FORMAT(2X,'STEAM CONSUMPTION = ',2X,F10.4,2X,'  AREA=',F10.3)
210     FORMAT(2X,'STEAM TEMP.= ',F6.2,'   FEED TEMP.= ',F6.2,'  LAST'
     1 ' EFFECT TEMP.= ',F6.2/)
220     FORMAT(2X,'STEAM ECONOMY = ',F8.3)
        RETURN
        END

        SUBROUTINE HTC(CC,T,TI,OHTC,AL,EVP,PLK,ARA1,I3,I4,CC1)
        REAL CC,CC1,T,TI,OHTC,AL,EVP,PLK
C
C       DETERMINATION OF OVERALL HEAT TRANSFER COEFFICIENT FOR LONG TUB
C       VERTICAL EVAPORATORS. THE FUNCTIONAL RELATIONSHIP OF OVERALL
C       HEAT TRANSFER COEFFICIENT WITH OTHER OPERATING PARAMETERS ARE
C       GIVEN BELOW:
C              p2
C       U=p1.CK   .(p3.Q+p4.Q**2+p5.Q**3)
C
C       CK = CAMU*CT*CF
C
C       WHERE p1 TO p5 ARE CONSTANTS.CAMU ACCOUNTS FOR THE CHANGES IN
C       VISCOSITY, CT IS THE CORRECTION FACTOR FOR BOILING POINT AND CF
C       IS THE COMBINED CORRECTION FACTOR FOR LIQUID FEED RATE IN
C       COMBINATION WITH LIQUID FEED TEMPERATURE AND HEAT FLUX.
C
C       GEOMETRICAL PARAMETERS:
C
C       LENGTH OF TUBE                  : 8.5 m
C       DIAMETER OF TUBE                : 51/48 mm
C       MATERIAL OF CONSTRUCTION        : STAINLESS STEEL SIS 2343
C       THERMAL CONDUCTIVITY            : 14.6 W/mK ( 70-130 DEG.C)
C
C       DESCRIPTION OF VARIABLES:
C
C       TB       : BOILING POINT POINT OF LIQUOR (TSAT+BPR) DEG.C
C       Q        : HEAT FLUX, W/sq.m
C       FD       : FEED RATE PER UNIT AREA, kg/(s.sq.m)
C       TSUB     : SUBCOOLING OF LIQUID FEED ( TMAX-TIN) DEG.C
C       TSUP     : SUPERHETING OF LIQUOR FEED ( TIN-TMAX) DEG.C
C       EVP      : EVAPORATION RATE FROM A BODY
C       PLK      : LATENT HEAT OF VAPORIZATION
C       AL       : LIQUOR FLOW RATE
C
C
C
C       CONVERSION OF UNITS
        TIN=(TI-32)*0.555
```

```
      T1=(T-32)*0.5555
      ALF=AL*1.260E-04
      EVA=EVP*1.260E-04
      PLL=PLK*2326
      ARA=ARA1*9.290E-02
C
C     T1,TIN       : DEG.C
C     ALF,EVA      : kg/s
C     PLL          : J/kg
C     ARA          : sq.m
C
C     DETERMINATION OF CORRECTION FACTOR CK (=CAMU*CT*CF)
C
C     DETERMINATION OF PHYSICAL PROPERTIES
C
C      A1=1.0E-03*AMU1(CC,T1)
C      A2=1.0E-03*AMU1(CC1,TIN)
C      AMU=(A1+A2)/2.0
       AMU=1.0E-03*AMU1(CC,T1)
      CP=CP2(CC)
      RHO=RHO1(CC,T1)
      THERC=THERC1(CC,T1)
C     WRITE(*,*)'T1,TIN,EVA,ARA',T1,TIN,EVA,ARA
C
      L1=0
      TB=T1
C
C     DETERMINATION OF FD VALUE AND HEAT FLUX
      ANT=ARA/(8.5*3.14*0.051)
      CSANT=3.14*(0.048/2)**2*ANT
      FD=ALF/CSANT
      Q=EVA*PLL/ARA
C     WRITE(*,*)'Q,EVA,PLL,ARA',Q,EVA,PLL,ARA
C     DETERMINATION OF CORRECTION FACTOR, CAMU
      IF(AMU.GT.0.AND.AMU.LE.15)THEN
      CAMU=1.0-0.07858*AMU+0.002735*AMU**2-0.1092E-05*AMU**3
      ENDIF
      IF(AMU.GT.15)CAMU=1.576-0.095*LOG10(AMU)
C
C     DETERMINATION OF CORRECTION FACTOR, CT
C
      CT=-1.02+0.0467*TB-0.364E-03*TB**2+0.946E-06*TB**3
C
C     DETERMINATION OF FACTOR, CF
C
      TMAX=15 0+(0.12-0.001*TB)*FD+(3.0-0.025*TB)*LOG10(AMU)-0.12*TB
      TSUB=TMAX-TIN
      IF(TSUB.LT.0)L1=1
      IF(TSUB.LT.0)TSUB=0.0
      IF(AMU.LT.1.0)AMUC=1.0
      IF(AMU.GE.1.0)AMUC=AMU
C     WRITE(*,*)'TSUB,TMAX,Q',TSUB,TMAX,Q
C
C     CALCULATION OF HEAT FLUX BOUNDS
```

```
C
      BOUND1=19500+2000*LOG10(AMUC)-95.0*TB
      BOUND2=4300+100*LOG10(AMUC)-14.0*TB
C     WRITE(*,*)'BOUND1,BOUND2',BOUND1,BOUND2
C
      IF(TSUB.EQ.0)THEN
      Z=1.0
      GO TO 33
      ENDIF
      Z=1.0+0.01*(TANH((Q-BOUND1)/21800))*TSUB**0.40
33    CONTINUE

      CF=Z+(-0.0515+0.0505*Z)*FD
      CK=CAMU*CT*CF
C     WRITE(*,*)'Z,CAMU,CT,CF,CK',Z,CAMU,CT,CF,CK
C
C     DETERMINATION OF ADDED HEAT FLUX DUE TO SUPER HEATING OF LIQUID
C     FEED, DELQ
C
      IF(L1.EQ.1)GO TO 10
      GO TO 20
10    CONTINUE
      TSUP=TIN-TMAX
      DELQ=TSUP*CP*FD*0.00181*0.61/1.321
20    CONTINUE
C
C    DETERMINATION OF OVERALL HEAT TRANSFER COEFFICIENT IF THE VALUE O
C    Q LIES BETWEEN 0 TO 20000 W/sq.m
C
C     WRITE(*,*)'Q,DELQ',Q,DELQ
      Q1=Q+DELQ
      IF(Q1.GT.20000)Q1=20000
      IF(Q1.GT.0.0.AND.Q1.LE.20000)THEN
      OHTC=0.981*(CK**1.0)*(0.2334*Q1-0.1006E-04*Q1**2+0.1362E-
     109*Q1**3)
      ENDIF
      OHTC=OHTC/5.678
C     WRITE(*,*)'TSUP,Q1,OHTC,DELQ',TSUP,Q1,OHTC,DELQ
C
C    AT EXTREMELY LOW HEAT FLUXES GIVEN BY Q < BOUND2 THE METHODOLOGY
C    FOR DETERMINATION OF OVERALL HEAT TRANSFER COEFFICIENT DIFFERS
      IF(Q1.LE.BOUND2.AND.I3.EQ.100)THEN
      WRITE(*,30)Q1,BOUND2,I4
      ENDIF
      IF(Q1.GT.BOUND2)GO TO 40
      IF(Q1.LE.BOUND2)THEN
C     WRITE(*,30)BOUND2
      ENDIF
      VEL=FD/RHO
      REN=0.048*VEL*RHO/AMU
      PRN=CP*AMU/THERC
      H1=0.023*REN**0.8*PRN**0.4*THERC/0.048
      Q2=BOUND2
      HTCB=0.981*(CK**1.0)*(0.2334*Q2-0.1006E-04*Q2**2+0.1362E-
```

```
109*Q2**3)
    CSUB=1.0+0.04*TSUB*((BOUND2-Q1)/BOUND2)**2
    OHTC=(H1+(HTCB-H1)*Q1/BOUND2)*CSUB
    WRITE(*,*)'REN,PRN,THERC,CP,RHO,AMU',REN,PRN,THERC,CP,RHO,AMU
    CONTINUE
    CONTINUE
    FORMAT(2X,'Q= ',F8.1,'< BOUND2=',F8.1,'  BODY NO.=',I3)
    RETURN
    END

    FUNCTION AMU1(CC,T)
    UNIT OF AMU, kg/sm *1.0E03      ( i.e. cp)
    T, TEMPERATURE OF LIQUOR, DEG.C
    C1, TOTAL SOILD CONTENT IN LIQUOR, %

    C1=CC*100
    A=0.4717-0.02472*T+0.7059E-05*T**2
    B=0.06973-0.5452E-03*T+0.1656E-05*T**2
    C=0.002046+0.3183E-04*T-0.9761E-07*T**2
    D=0.5793E-04-0.6129E-06*T+0.1837E-08*T**2
    E=A+B*C1+C*C1**2+D*C1**3
    AMU1=EXP(E)
    RETURN
    END

    FUNCTION CP2(CC)
    UNIT OF CP OF BLACK LIQUOR, Ws/kgK
    T, TEMPERATURE OF LIQUOR, DEG.C
    C1, TOTAL SOILD CONTENT IN LIQUOR, %
    C1=CC*100
    CP2=(1.0-0.0054*C1)*4187
    RETURN
    END

    FUNCTION  RHO1(CC,T)
    UNIT OF RHO OF BLACK LIQUOR, kg/cu.m
    T, TEMPERATURE OF LIQUOR, DEG.C
    C1, TOTAL SOILD CONTENT IN LIQUOR, %
    C1=CC*100
    RHO1=1007-0.495*T+6.0*C1
    RETURN
    END

    FUNCTION THERC1(CC,T)
    UNIT OF THERMALCONDUCTIVITY OF BLACK LIQUOR, W/mK
    T, TEMPERATURE OF LIQUOR, DEG.C
    CC, TOTAL SOILD CONTENT IN LIQUOR, MASS FRACTION
    THERC1=0.504-0.282*CC+1.35E-03*T
    THERC1=THERC1*1.163
    RETURN
    END
```

```
       SUBROUTINE NLSYST(FCN,N,NE,MAXIT,X,F,DELTA,XTOL,FTOL,I)
C      -----------------------------------------------------------------
C       SUBROUTINE NLSYST :
C       THIS SUBROUTINE SOLVES  A SYSTEM  OF NON-LINEAR  EQUATIONS  BY
C       NEWTON'S METHOD. THE PARTIAL DERIVATIVES OF THE FUNCTIONS  ARE
C       ESTIMATED BY DIFFERENCE QUOTIENTS WHEN A VARIABLE IS PERTURBED
C       BY AN AMOUNT EQUAL TO DELTA( DELTA IS ADDED). THIS IS DONE FOR
C       EACH VARIABLE IN EACH FUNCTION.   INCREMENTS  TO  IMPROVE  THE
C       ESTIMATES FOR THE X-VALUES ARE COMPUTED FROM A SYSTEM OF EQUATI
C       USING SUBROUTINE ELIM.
C      -----------------------------------------------------------------
C      PARAMETERS ARE:
C      FNC            : SUBROUTINE THAT COMPUTES VALUES OF THE FUNCTIONS
C                       MUST BE DECLARED EXTERNAL IN THE CALLING PROGRAM
C      N              : THE NUMBER OF EQUATIONS
C      MAXIT          : LIMIT TO THE NUMBER OF ITERATIONS THAT WILL BE U
C      X              : ARRAY TO HOLD THE X VALUES. INITIALLY THIS ARRAY
C                       HOLDS THE THE INITIAL GUESSES. IT RETURNS THE FI
C      F              : AN ARRAY THAT HOLDS THE VALUES OF THE FUNCTION
C      DELTA          : A SMALL VALUE USED TO PERTURB THE X VALUES SO PA
C                       DERIVATIVES CAN BE COMPUTEDBY DIFFERENCE QUOTIEN
C      XTOL           : TOLERANCE VALUE FOR CHANGE IN X VALUES TO STOP I
C                       WHEN THE LARGEST CHANGE IN ANY X MEETS XTOL, THE
C                       TERMINATES
C      FTOL           : TOLERANCE VALUE ON F TO TERMINATE. wHEN THE LARG
C                       VALUE IS LESS THAN FTOL, SUBROUTINE TERMINATE.
C      I              : RETURNS VALUES TO INDICATE HOEW THE SUBROUTINE T
C      I=1            : XTOL WAS MET
C      I=2            : FTOL WAS MET
C      I=-1           : MAXIT EXCEEDED BUT TOLERANCES NOT MET
C      I=-2           : VERY SMALL PIVOT ENCOUNTERED IN GAUSSIAN ELIMINA
C                       STEP-NO RESULTS OBTAINED
C      I=-3           : INCORRECT VALUE OF N WAS SUPPLIED- N MUST BE BET
C                       2 AND 10
C     ------------------------------------------------------------------
       REAL X(N),F(N),DELTA,XTOL,FTOL
       INTEGER N,MAXIT,I
       REAL A(10,13),XSAVE(10),FSAVE(10)
       INTEGER NP,IT,IVBL,ITEST,IFCN,IROW,JCOL
C     ------------------------------------------------------------------
C      CHECK VALIDITY OF VALUES OF N
C
       IF(N.LT.2.OR.N.GT.12)THEN
       I=-3
       WRITE(*,1004) N
       RETURN
       END IF
C     ------------------------------------------------------------------
C      BEGIN ITERATIONS-SAVE X VALUES, THEN GET F VALUES
C
       NP=N+1
       DO 100 IT=1,MAXIT
       DO 10 IVBL=1,N
       XSAVE(IVBL)=X(IVBL)
```

```
  10   CONTINUE
       CALL FCN(X,F,N,NE)
C     ----------------------------------------------------------------
C     TEST F  VALUES AND SAVE THEM
C
       ITEST=0
       DO 20 IFCN=1,N
       IF(ABS(F(IFCN)).GT.FTOL)ITEST=ITEST+1
       FSAVE(IFCN)=F(IFCN)
 20    CONTINUE
       IF(I.EQ.0)THEN
       WRITE(*,1000)IT,X
       WRITE(*,1001)F
       END IF
C
C     ----------------------------------------------------------------
C     SEE IF FTOL IS MET. IF NOT, CONTINUE. IF SO, SET I=2 AND RETURN
       IF(ITEST.EQ.0)THEN
       I=2
       RETURN
       END IF
C    -----------------------------------------------------------------
C     THIS DOUBLE LOOP COMPUTES THE PARTIAL DERIVATIVES OF EACH FUNCTI
C     FOR EACH VARIABLE AND STORES THEM IN A COEFFICIENT ARRAY.
       DO 50 JCOL=1,N
       X(JCOL)=XSAVE(JCOL)+DELTA
       CALL FCN(X,F,N,NE)
       DO 40 IROW=1,N
       A(IROW,JCOL)=(F(IROW)-FSAVE(IROW))/DELTA
 40    CONTINUE
C
C      RESET X VALUES FOR NEXT COLUMN OF PARTIALS
C
       X(JCOL)=XSAVE(JCOL)
 50    CONTINUE
C      ---------------------------------------------------------------
C      NOW WE PUT NEGATIVE OF F VALUES AS RIGHT AND SIDES AND CALL ELI
       DO 60 IROW=1,N
       A(IROW,NP)=-FSAVE(IROW)
 60    CONTINUE
       CALL ELIM(A,N,NP,12)
C
C      ---------------------------------------------------------------
C      BE SURE THAT THE COEFFICIENT MATRIX IS NOT TOO ILL CONDITIONED
       DO 70 IROW=1,N
       IF(ABS(A(IROW,IROW)).LE.1.0E-6)THEN
       I=-2
       WRITE(*,1003)
       RETURN
       END IF
 70    CONTINUE
C
C      ---------------------------------------------------------------
C      APPLY THE CORRECTIONS TO THE X VALUES, ALSO SEE IF XTOL IS MET
```

```
       ITEST=0
       DO 80 IVBL=1,N
       X(IVBL)=XSAVE(IVBL)+A(IVBL,NP)
       IF(ABS(A(IVBL,NP)).GT.XTOL)ITEST=ITEST+1
 80    CONTINUE
C
C      -------------------------------------------------------------
C      IF XTOL IS MET, PRINT LAST VALUES AND RETURN , ELSE DO ANOTHER
C      ITERATION
       IF(ITEST.EQ.0)THEN
       I=1
       IF(I.EQ.0)WRITE(*,1002)IT,X
       RETURN
       END IF
 100   CONTINUE
C
C      ------------------------------------------------------------
C      WHEN WE HAVE DONE MAXIT ITERATIONS, SET I=-1 AND RETURN
C
       I=-1
       RETURN
1000   FORMAT('  AFTER ITERATION NUMBER',I3,'  X AND F VALUES ARE'
     1/12F6.3)
1001   FORMAT(12F6.3)
1002   FORMAT(' AFTER ITERATION NUMBER',I3,' X VALUES(MEETING',
     1' XTOL) ARE'/12F6.3)
1003   FORMAT('  CANNOT SOLVE SYSTEM. MATRIX NEARLY SINGULAR')
1004   FORMAT('  NUMBER OF EQUATIONS PASSED TO NLSYST IS INVALID.',
     1' MUST BE 1<N<11. VALUE WAS ',I3)
       END

       SUBROUTINE ELIM(AB,N,NP,NDIM)
C      ------------------------------------------------------------
C      SUBROUTINE ELIM:
C     THIS SUBROUTINE SOLVES A SET OF LINEAR EQUATIONS AND GIVES AN
C     LU DECOMPOSITION OF THE COEFFICIENT MATRIX. THE GAUSS ELIMINATIO
C     METHOD IS USED, WITH PARTIAL PIVOTING. MULTIPLE RIGHT HAND SIDED
C     ARE PERMITTED, THEY SHOULD BE SUPPLIED AS COLUMNS THAT AUGMENT T
C     COEFFICIENT MATRIX.
C     -------------------------------------------------------------
C     PARAMETERS ARE:
C     AB          : COEFFICIENT MATRIX AUGMENTED WITH R.H.S. VECTORS
C     N           : NUMBER OF EQUATIONS
C     NP          : TOTAL NUMBER OF COLUMNS IN THE AUGMENTED MATRIX
C     NDIM        : FIRST DIMENSION OF MATRIX AB IN THE CALLING PROGRAM
C
C    THE SOLUTION VECTOR(S) ARE RETURNED IN THE AUGMENTATION COLUMNS
C    OF AB
C    --------------------------------------------------------------
C
       REAL AB(NDIM,NP)
       INTEGER N,NP,NDIM
       REAL SAVE, RATIO, VALUE
       INTEGER NM1,IPVT,IP1,J,NVBL,L,KCOL,JCOL,JROW
```

```
C
C   ----------------------------------------------------------------
C
C       BEGIN    THE REDUCTION
C
        NM1=N-1
        DO 35 I=1,NM1
C
C      FIND THE ROW NUMBER OF THE PIVOT ROW. WE WILL THEN INTERCHANGE
C      ROWS TO PUT THE PIVOT ELEMENT ON THE DIAGONAL.
C
       IPVT=I
       IP1=I+1
       DO 10 J=IP1,N
       IF(ABS(AB(IPVT,I)).LT.ABS(AB(J,I)))IPVT=J
 10    CONTINUE
C
C      -------------------------------------------------------------
C      CHECK FOR A NEAR SINGULAR MATRIX
C
        IF(ABS(AB(IPVT,I)).LT.1.0E-6)THEN
        WRITE(*,100)
        RETURN
        END IF
C
C       NOW INTERCHANGE, EXCEPT IF THE PIVOT ELEMENT IS ALREADY ON THE
C       DIAGONAL, DON'T NEED TO
C
        IF(IPVT.NE.I)THEN
        DO 20 JCOL=1,NP
        SAVE=AB(I,JCOL)
        AB(I,JCOL)=AB(IPVT,JCOL)
        AB(IPVT,JCOL)=SAVE
 20     CONTINUE
        END IF
C
C       ------------------------------------------------------------
C
C       NOW REDUCE ALL ELEMENTS BELOW THE DIAGONAL IN THE I-TH ROW.
C       CHECK FIRST TO SEE IF A ZERO ALREADY PRESENT. IF SO, CAN SKIP
C       REDUCTION ON THAT ROW.
C
        DO 32 JROW=IP1,N
        IF(AB(JROW,I).EQ.0)GO TO 32
        RATIO = AB(JROW,I)/AB(I,I)
        AB(JROW,I)=RATIO
        DO 30 KCOL=IP1,NP
        AB(JROW,KCOL)=AB(JROW,KCOL)-RATIO*AB(I,KCOL)
 30     CONTINUE
 32     CONTINUE
 35     CONTINUE
C
C      -------------------------------------------------------------
C       WE STILL NEED TO CHECK AB(N,N) FOR SIZE
```

```
      IF(ABS(AB(N,N)).LT.1.0E-6)THEN
      WRITE(*,100)
      RETURN
      END IF
C
C     ------------------------------------------------------------------
C     NOW WE BACK SUBSTITUTE
C
      NP1=N+1
      DO 50 KCOL=NP1,NP
      AB(N,KCOL)=AB(N,KCOL)/AB(N,N)
      DO 45  J=2,N
      NVBL=NP1-J
      L=NVBL+1
      VALUE=AB(NVBL,KCOL)
      DO 40 K=L,N
      VALUE=VALUE-AB(NVBL,K)*AB(K,KCOL)
 40   CONTINUE
      AB(NVBL,KCOL)=VALUE/AB(NVBL,NVBL)
 45   CONTINUE
 50   CONTINUE
      RETURN
C
 100  FORMAT(/'SOLUTION NOT FEASIBLE. A NEAR ZERO PIVOT',
     1 ' WAS ENCOUNTERED.')
      END
```

SAMPLE DATA AND RESULTS

```
SAMPLE DATA FILE(TNS.DAT)

138,70,51,0.15


SAMPLE RESULTS

 RESULTS FOR MULTI EFFECT EVAPORATOR SYSTEMS
 (BACKWARD FEED)

 FEED RATE=    18.1440   FEED CONC.=.15    PRODUCT CONC.=   .45
 STEAM TEMP.= 138        FEED TEMP.=  70   LAST EFFECT TEMP.=51
 ---------------------------------------------------------------------
 EFFECTS    1         2         3         4         5         6
 ---------------------------------------------------------------------
 PRESS      168.614   106.860    67.814    41.491    23.205    12.979
 S.TEMP    114.93    101.50     89.12     76.74     63.34     51.00
 BPR         6.96      4.01      2.81      2.20      1.86      1.65
 L.TEMP    121.89    105.51     91.93     78.94     65.20     52.65
 H.T.C.       951.2    1568.9    1451.4    1237.5     935.6     846.9
 DELT       16.01      9.42      9.57     10.18     11.54     10.69
 L/I         8.564    10.901    12.990    14.757    16.197    18.144
 L/O         6.048     8.564    10.901    12.990    14.757    16.197
 EVAP        2.516     2.337     2.089     1.768     1.440     1.947
 CONC.      .45000    .31780    .24968    .20952    .18443    .16803
 ---------------------------------------------------------------------
 STEAM CONSUMPTION = 2.6697    AREA=    380.278

 STEAM ECONOMY =     4.531
Stop - Program terminated.
```

Note: *Units used for computation work and results obtained are indicated in the nomenclature*