ENERGY CONSERVATION IN MULTIPLE EFFECT EVAPORATORS

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By

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To

my reverend

PARENTS

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I hereby certify that the work which is being presented in the thesis entitled "*ENERGY CONSERVATION IN MULTIPLE EFFECT EVAPORATORS* " in fulfilment of the requirements for the award of the Degree of **DOCTOR OF PHILOSOPHY** and submitted in the *Department of Chemical Engineering* of the University is an authentic record of my own work carried out during a period from September, 1983 to October 1992 under the supervision of Professor *S. C. Gupta.*

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other university.

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ABSTRACT

The present investigation pertains to a theoretical study of energy conservation in multiple effect evaporator by simulating the variables of an evaporator for an improvement in steam economy. Basically, it deals with the development of mathematical models for the evaporation of aqueous solutions of sugar, black- liquor, and caustic soda in quadruple-, quintuple-, and triple- effect evaporator, respectively under forward, backward, and mixed feed arrangements and their solutions to determine steam economy. It also includes the parametric effect of operating variables on the steam economy and end-product concentration so as to obtain the condition for improved steam economy of the evaporator. Finally, it describes a procedure to counteract a change in end-product concentration caused by any upset in operating variables in such a way that steam economy of the evaporator does not suffer at all.

Using the equations of solute material balance, overall material balance, energy balance, heat transfer rate, and boiling point rise in individual effects of an N-effect evaporator, pertinent models of 4N nonlinear simultaneous algebraic equations have been developed for quadruple, quintuple-, and triple- effect evaporators employing aqueous solutions of sugar, black-liquor, and caustic soda, respectively with forward, backward, and mixed feed arrangements. The equations have been linearised by Newton-Raphson method and then solved by L-U decomposition method to determine the values of unknown variables and thereby steam economy and the end-product concentration. A set of initial guess value of unknown variables has been used to initiate the iterative computation. The value of an unknown variable has been taken to be converged when the deviation between two successive values of the variable is found to be of the order of one tenth of a micro unit. The model has predicted pertinent quantities of steam consumption; saturation temperature of vapour of each effect; concentration, flow rate, and temperature of the aqueous solution in each effect for the known values of operating variables viz; feed temperature, feed concentration, feed rate, pressure in the last effect, and steam pressure for a given solution and feed arrangement. The validity of the model for its applicability to industrial situations has been examined by comparing the predicted values of solute concentration, saturation temperature of vapour, and the liquid temperature in individual effects of the multiple effect evaporator against the plant values for the known values of the operating variables being used in Indian mills. The maximum deviation between the predictions and the plant values has been found to be of the order of ±10%.

Parametric effect of operating variables on steam economy has been studied for the multiple effect evaporators using various solutions and feed arrangements. As a result of it, the range of operating variables for the highest steam economy of the evaporator has been

determined. These values, undoubtedly, are likely to revamp the performance of existing evaporators and also help in the design of energy-efficient multiple effect evaporators.

Application of multiple linear regression analysis to the values of steam economy and operating variables has resulted in the development of various correlations for aqueous solutions of sugar, black-liquor, and caustic soda with forward, backward, and mixed feed arrangements. The maximum deviation for each of the correlation has been of the order of $\pm 5.5\%$. The correlations are of the following general form:

$$E = C\tau_f^{a} X_f^{b} F^{c} T_l^{d} T_s^{e}$$

Where values of the constant, C and the exponents, a, b, c, d, and e depend upon the aqueous solution to be concentrated and the feed arrangement used in evaporator.

Present analysis has also been extended to investigate the parametric effect of operating variables on the end-product concentration of evaporators so that the results may be of direct relevance to maintain end-product concentration at a specified level as might be necessary due to process constraints. Based on it, end-product concentration has been found to vary directly with feed temperature and steam pressure and inversely with feed rate and pressure in last effect of the evaporator. Effect of feed concentration on the end-product concentration has differed from solution to solution.

Using multiple linear regression analysis the end-product concentration of the evaporator has been correlated with operating variables for aqueous solutions of sugar, black-liquor, and caustic-soda with various feed arrangements. The general form of the correlation is as follows:

$$X_p = K\tau_f^{p} X_f^{q} F^{r} T_l^{s} T_s^{t}$$

Where the values of constant, K and the exponents p, q, r, s, and t vary with feed arrangement and the aqueous solution used in evaporator. The maximum deviation of a correlation from its mean value has been of the order of $\pm 5.26\%$.

This investigation has also attempted to evolve a procedure to meet the situation of the change in end-product concentration that might arise due to unforeseen variation in one or more operating variables and thereby the steam economy of the evaporator undergoes a change. Based on the generalized correlation of end-product concentration, the following equation has been obtained to determine the corresponding change in the operating variables so that end-product concentration does not alter.

$$\begin{aligned}&[p(\Delta\tau_f/\tau_f) + q(\Delta X_f/X_f) + r(\Delta F/F) + s(\Delta T_l/T_l) + t(\Delta T_s/T_s)]\\ &= pq(\Delta\tau_f/\tau_f)(\Delta X_f/X_f) - rs(\Delta F/F)(\Delta T_l/T_l) - st(\Delta T_l/T_l)(\Delta T_s/T_s) - rt(\Delta F/F)(\Delta T_s/T_s)\end{aligned}$$

This, obviously, leads to many options for the readjustment of operating variable. Each is likely to provide a different value of steam economy. Therefore, each option must be evaluated for its impact on steam economy and thereby the most appropriate one which yields the highest steam economy must be selected. Following relationship has been developed for the calculation of deviation in steam economy of the evaporator with readjusted changes in values of operating variables:

$$
\begin{aligned}
[\Delta E/E] = {} & a(\Delta\tau_f/\tau_f) + b(\Delta X_f/X_f) + c(\Delta F/F) + d(\Delta T_l/T_l) + e(\Delta T_s/T_s) \\
& + a(\Delta\tau_f/\tau_f)\,\{b(\Delta X_f/X_f) + c(\Delta F/F) + d(\Delta T_l/T_l) + e(\Delta T_s/T_s)\} \\
& + b(\Delta X_f/X_f)\,\{c(\Delta F/F) + d(\Delta T_l/T_l) + e(\Delta T_s/T_s)\} \\
& + c(\Delta F/F)\,\{d(\Delta T_l/T_l) + e(\Delta T_s/T_s)\} \\
& + de(\Delta T_l/T_l)(\Delta T_s/T_s)
\end{aligned}
$$

Above equations provide a useful procedure to determine the necessary changes the operating variables needed to nullify the variation in end-product concentration caused by any upset in other variables of the evaporator. Besides, the resultant variation in the steam economy can also be determined, and then the operating variables can be readjusted to provide the highest steam economy. This will reduce the steam consumption to the evaporator under consideration.

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NOMENCLATURE

A	Area of heating surface	m^2
C	Heat capacity	kJ/kg.°C
D	Number of working days	
E	Steam economy	kg/kg
F	Feed rate	ton/hr
g	Function defined in Eq(4.1)	
H	Enthalpy of vapour	kJ/kg
h	Enthalpy of aqueous solution	kJ/kg
I	Cost	Rs.
L	Flow rate of aqueous solution	ton/hr
N	Number of effects	
P	Pressure	kPa
R	Number of working days in one cycle	
T	Saturation temperature of water vapour/water	°C
U	Overall heat transfer coefficient	kJ/hr.m^2.°C
V	Flow rate of vapour/steam	ton/hr
X	Concentration of aqueous solution	°Bx or °Tw or %
Z	Percentage of total capital as deprication	
τ	Temperature of aqueous solution	°C
λ	Latent heat of vaporization	kJ/kg
ϵ	Boiling point rise (= τ - T)	°C
θ	Operating time	hr

Subscripts

b	Single effect
c	Cleaning
f	Feed
M	Maintenance
o	Operational
P	End-product
s	Steam
t	Total

Subroutines

START	The selection of initial sets
MBL	Material balance
EBL	Energy balance
BPREQ	Boiling point of aqueous solution
RATE	Heat transfer rate
GAUSS	Solution of simultaneous equations
FINAL	Final values of unknown variables

CHAPTER-1

INTRODUCTION

Evaporation is widely employed in chemical-, petrochemical-, food-, refrigeration, power plant-, and other allied-industries to concentrate dilute aqueous solutions to the desired concentration so as to make end-product suitable for further processing and marketable. It is an energy-intensive operation as multitudes of thermal energy in the form of steam are used in it. In recent years, a radical increase in energy cost in relation to the capital equipment cost has caused a dramatic increase in the operating expenses of the evaporators. This trend is quite likely to continue in future too, due to ever-increasing rapid depletion in the reserves of fossil fuels and also their continuous consumption at alarming rates. Thus, energy plays a dominating role in the economic design of evaporators. This emphasises the need of energy conservation in evaporators. One of the basic factors contributing to it is steam economy as it represents a measure of the quantity of water evaporated per unit of steam consumption. As a matter of fact, high values of steam economy are desirable in multiple effect evaporators so that steam consumption is kept at the lowest level for a given evaporation of water.

Steam economy of a multiple effect evaporator having a specified feed arrangement depends on the number of effects; temperature, flow rate, and concentration of the feed; pressure in individual effects; steam pressure and the physico-thermal properties of the solution. Obviously, a change in any one of these variables can affect the steam economy of the evaporator. Therefore, values of these variables are to be determined so that evaporators with a given feed arrangement can operate for increased steam economy. This calls for a detailed investigation to study the parametric effect of above variables on steam economy of an evaporator for a given feed arrangement. This will also help plant engineers to revamp their existing evaporators by adjusting values of operating variables to attain improved steam economy.

It is important to mention that in some of the cases a plant engineer does not have much flexibility to change the value of some of the above variables due to process constraints. For example, concentration of the end-product in a sugar solution, multiple effect evaporator is kept at the level of 60 ± 5 °Bx, otherwise seeding of sugar crystals will take place in the evaporator itself. Similarly, in the evaporation of caustic-soda solution, concentration of the end-product is limited to a maximum of 50%, as beyond this concentration the freezing point of the caustic-soda solution starts rising steeply. However, in India and other tropical countries, the end-product concentration of caustic soda is maintained at 47.5% because the ambient temperature in these countries rarely falls below the freezing point of 47.5% caustic soda solution. There is another situation with the captive caustic-soda plants where relatively higher sodium chloride contents in a caustic soda solution can be tolerated and the end-product concentration is restricted to only 30-35%. Thus, end-product concentration of a solution may be fixed by the process technology, economics, and other factors. For such systems, values of operating variables have to be determined which can yield the best possible steam economy of the evaporator. This necessitates the knowledge of the parametric effect of operating variables, namely; feed temperature, feed concentration, feed rate, pressure in the last effect, and steam pressure on the concentration of end-product of an evaporator with various feed arrangements. Such a study will help in revamping the values of operating variables of an evaporator to achieve evaporation with increased steam economy.

Another important factor of interest is that in industries, values of operating variables undergo changes due to process alterations preceding and / or following the evaporators. For instance; in caustic soda plants, low-amperage cells like Vorce, Allen Moore, etc. have now been replaced by high-amperage diaphragm cells offering many additional advantages which include clean and pollution free simple operation and also higher cell liquor concentration. Now, a given evaporator with a feed of increased cell liquor concentration will consequently have concentration of the end-product different than that with liquor concentration from low-amperage cells. The steam economy will also be affected. Hence, each change must be evaluated not only to determine their impact on steam economy but also to maintain a predetermined end-product concentration. This, obviously, is possible by the readjustment of the value of the remaining variables to nullify any changes in end-product concentration. This, of course, shall be of immediate utility to plant engineers to tackle the day to day problems of evaporator arising out from changes in operating variables, if any.

Keeping the above in view, present investigation on energy conservation in a multiple effect evaporator with a given feed arrangement has been planned with the following objectives:

1. To develop a mathematical model for the simulation of multiple effect evaporators for aqueous solutions of sugar, black-liquor and caustic soda and to examine its applicability to the existing industrial evaporators.

2. To determine steam economy of a multiple effect evaporator for sugar-, black-liquor-, and caustic soda solution with the changes in operating variables: feed temperature, feed concentration, feed rate, pressure in the last effect, and steam pressure; and thus to determine values of operating variables for which an evaporator with a given feed arrangement should operate at increased steam economy.

Also to develop appropriate correlations of steam economy of evaporators for the solutions of sugar, black-liquor, and caustic soda with different feed arrangements.

3. To study the parametric effect of operating variables on the concentration of end-product of a multiple effect evaporator of sugar, black-liquor, and caustic soda solutions and thereby to establish suitable correlations of end-product concentration from evaporators handling various aqueous solutions under different feed arrangements with the above cited operating variables.

4. To develop a procedure for plant engineers to counteract the situation arising out of change in one or more operating variables of an evaporator by manipulating the values of the remaining operating variables so that concentration of the end-product is maintained at a predetermined- value and the steam economy of the evaporator is improved.

CHAPTER-2

LITERATURE REVIEW

Evaporation has been the subject of active research from its inception. It has been basically with the twin objective of conserving energy & material and upgrading the quality of concentrated product leaving evaporators. In ancient times, the evaporation was confined to direct fired open pans for producing salt. The advent of closed vessels for the evaporation of aqueous solutions represents a marked development in the history of evaporators as it allowed considerable savings in the consumption of fuels over the conventional open pan evaporators. Notable among them were jacketed- and kettle- evaporators. Soon they were replaced by tubular one as they offered efficient operation and less consumption of energy. However, they had the drawback of loss of high temperature vapour produced by the evaporation of valuable liquid. Therefore,soon multiple effect evaporators which permit the use of vapour of an effect as heating media in successive effects came into existence. Since then they have been extensively used in industries to concentrate various aqueous solutions. This type of system is advantageous as it offers high steam economy, improved quality of product, and increased turn-over. Since then a number of improvements in the design, construction and operation of multi-effect evaporators has taken place [A1,L1,S4]. These include the modifications in the type of evaporators,type of liquid circulation, feed arrangement, analysis and simulation and the use of energy conservation measures such as condensate handling, heat pump, heat pipe, vapour bleeding, splitting of feed, etc. Following Paragraphs discuss some of the important investigations pertaining to analysis and design of multiple effect evaporators and the energy conservation measures incorporated in them:

Analysis of multiple effect evaporators is of paramount importance to describe the mechanism and operation of evaporators. Perhaps, Badger & McCabe [B1& M2] were the first to carry out a systematic analysis of evaporators and to recommend a design method. It includes the iterative step by step procedure in which evaporation rate and heating surface area for each effect are calculated by means of enthalpy balance and heat transfer rate equations.In this method, values of temperature for first and second effects of a triple effect evaporator are assumed.The calculations are repeated until equal surface area in each effect is obtained. This procedure has been widely employed in industries for the design of evaporators.

Bonilla [B2] has recommended a simplified method for the calculation of minimum total area for multiple effect evaporators. Based on the assumption of equal heating surface area in each effect, he has used the following equation to calculate temperature gradient in an effect which enables the determination of minimum total area:

$$\Delta T_i = \frac{\sum_{j=1}^{N} \Delta T_j}{\sum_{j=1}^{N} q_j/U_j} (q_i/U_i) \qquad \ldots(2.1)$$

Eq(2.1) is applied recursively until the area in each of the effects is found to be the same.

In another investigation, Bonilla [B3] developed the following expression similar to that of Eq(2.1) for the determination of temperature drop in an effect. Further he differentiated the total area with respect to the temperature drop in each effect assuming that boiling point rise is negligible and heat transfer coefficient and heat transfer rate do not vary with temperature.

$$\Delta T_i = \frac{\sum_{j=1}^{N} \Delta T_j}{\sum_{j=1}^{N} (q_j/U_j)^{0.5}} (q_i/U_i)^{0.5} \qquad \ldots(2.2)$$

The design procedure in this investigation is similar to that for the equal area case,except that the successive values of the temperature drop in ith effect are calculated by the use of Eq(2.2) and the calculations are considered to be completed when the value of $A_i/\ \Delta T_i$ is found to be same in each effect.

Above procedures do not offer general solution because of several assumptions which do not hold true in industrial evaporators. Therefore, Itahara & Stiel [I1&I2] used dynamic programming to determine the optimum temperature distribution for the minimum area in multiple effect evaporators. They have considered linear variation of overall heat transfer coefficient with temperature in an effect and developed the objective function of ΔT for the last and the ith effect of an evaporator. Using Fibonacci search method,they have determined the optimum temperature distribution for minimum area in multiple effect evaporator. They have also compared their results with those obtained by the use of Bonilla's procedure and found that Bonilla's criterion [B2 &B3] does not give the exact solution of the problem.Further, it has also been pointed out that the saving in area over the equal area design method increases with the number of effects in the evaporator and depends on the type of evaporator and the inlet and outlet conditions.

Coates [C2] has recommended a simplified method for the calculation of evaporator capacity and steam consumption in a multiple effect evaporator.

In 1950, Meisler [K1] carried out the design of a quadruple effect evaporator for the evaporation of aqueous sugar solution on the basis of equal heating surface area in each effect. His method is a trial and error one requiring the calculation of heating surface area of various effects based on equal pressure drop in each effect.This method has been successfully employed for multiple effect evaporators of other industries too. However, later on the assumption of equal pressure drop in each effect of the evaporator was abandoned in favour of equal temperature drop in various effects. This approach could save a considerable time. Since in most of the cases boiling point elevation, heat of dilution and the change in specific heat of the solution are unknown, therefore the above method was modified by including enthalpy- concentration diagram. However, it involved the assumption of equal evaporation in each effect in place of equal temperature drop.

In another significant attempt Stewart & Beveridge [S2] used the computers for the analysis and steady state cascade simulation of multiple effect evaporators.

Evaporator modeling has been the subject of active research for many investigators. Prominent investigations have been of Harper & Tsao [H1], Newell & Fischer [N2], Holland [H3],Burdett & Holland [B5], Radovic et al. [R1] and Newell [N1]. The models developed by almost all these researchers have been for simulation purposes.

Gas Symmes Coates [G1] has developed a computer program for the evaluation of performance of a multiple effect evaporator for black- liquor solution. It also pinpoints the sources of trouble in the operation. The results of this study reveals short range scaling (the type of scale that can be removed by water boil-out) in the first effect whereas the scaling of a less serious extent in second and third effects of the evaporator. This investigation is useful to find out the source of trouble and the procedure for the treatment of scale.

Burdett & Holland [B5] solved the problem of design of unsteady state multiple effect evaporator by simulating all the pertinent variables .They used 17 effects in the evaporator system. Their approach includes the simultaneous solution of nonlinear algebraic equations of material balance, energy balance, heat transfer rate and phase equilibria of the aqueous solutions by the use of Newton- Raphson method.They have suggested the application of a generalized scaling procedure to reduce the magnitude of quantities appearing in the equations so as to achieve fast convergence while using Newton- Raphson method. This approach is of great significance as it permits the solution of complex mathematical models of multiple effect evaporators and thereby optimization of the system by dynamic programming either by finding the minimum area or by the minimum annualized operating cost.

Holland [H3] developed a rigorous mathematical model of steady state multiple effect evaporator for the aqueous solutions with forward and backward feed arrangements. The model consists of basic equations of material- and energy- balance, heat transfer rate and phase equilibria of the solution in each effect. The model has been successfully applied to two different situations-design of a new evaporator system and the analysis of an existing evaporator. In both the cases overall heat transfer coefficient and heat capacity of the aqueous solution in each effect have been assumed to be independent of temperature and concentration. The author has considered area of each effect to be equal while applying it for the design of a new evaporator system, whereas no such assumption has been made for the analysis of an existing system. He has also used the Newton- Raphson method and the scaling procedure of [B5] to solve the model. This method enabled rigorous analysis and also the optimisation of evaporators. The problem of evaporation of caustic soda in triple effect evaporator has been solved by the above method.

Based on the data of boiling point of caustic soda solution, following equation of boiling point rise has been developed for the use in the model [H3]:

$$\epsilon = 271.3627x^2 + x(0.142T - 9.42) \qquad \text{...(2.3)}$$

Radovic et al. [R1], in a similar attempt, developed a mathematical model of a quintuple effect sugar evaporator for its design and analysis. The model includes vapour bleeding in the system, variation of heat capacity and overall heat transfer coefficient with temperature and concentration; and also enthalpy and latent heat of vaporization as a function of temperature. Their model is quite similar to that of Burdett & Holland [B5] and Holland [H3]. For overall heat transfer coefficient they have employed the following equations of various investigators:

Baloh Equation

$$U_i = 1884/\,[(X_{i-1})^2 + (X_i)^2 + 0.08] \qquad \text{...(2.4)}$$

Schwedenformel Equation

$$U_i = 18.083\,(\tau_i/X_i) \qquad \text{...(2.5)}$$

Speyerer Equation

$$U_i = 16.744\,(\tau_i/X_i) \qquad \text{...(2.6)}$$

Hopstock Equation

$$U_i = 2.512\,(\tau_i/X_i)(3.5 + 0.04\tau_i) \qquad \text{...(2.7)}$$

Newton-Raphson method has been used by the authors to solve the model. The model has been employed to design the evaporators using operating variables and Eqs(2.4 -- 2.7) for the calculation of overall heat transfer coefficient. As a result, bleed steam has been found to affect the solute concentration, steam consumption and heat transfer area of each effect significantly. Further, they have also determined optimal value of vapour bleed for quintuple effect sugar evaporator under forward feed arrangement. The solute concentration has been found to agree well with the plant value of an existing evaporator but heat transfer surface area to depend upon the equation of heat transfer coefficient employed in the model.This analysis has shown that the Baloh equation provides lower values as compared to those from the Speyerer equation.The method is quite useful in the evaluation of evaporator performance and in choosing the distribution of vapour bleed in various effects.

Ghosh [G2] has discussed some of the aspects of energy conservation in multiple effect sugar evaporators on the basis of Rallieux's principle. These include the merit of greater number of effects, extensive vapour bleeding and close location of bled body to the last effect. According to him, these measures are useful in an open system where last effect vapour discharge to condenser but not in a closed system in which all the vapour streams are returned for complete reuse.

In another paper, Ghosh [G3] has reported some of the design and operational aspects of sugar evaporators. He has suggested the following equation for the calculation of optimum number of effects in a multiple effect evaporator:

$$N = [\{D\ I_s\ V_1)/(I_b\ Z)\}+\{(I_M/100)(V_1/V_T)\}+\{K\ I_c\ D/R\}] \qquad ...(2.8)$$

Eq (2.8) has been obtained by the Rillieux's principle. This equation has further been reduced to the following form by omitting the term $\{(I_M/100)(V_1/V_T)\}$ which represents the maintenance cost, being of comparatively smaller magnitude than that of other values:

$$N = \{D\ I_s\ V_1/(I_b\ Z)\} +\{ K\ I_c\ (D/R)\} \qquad ...(2.9)$$

Both these equations do not consider the effect of boiling point rise of the solution.

Ghosh & Ray [G6,R4 &R5] undertook an investigation on energy conservation in sugar multiple effect evaporators. On the basis of simple calculations, they have shown that heating a stream by means of bleed steam from an evaporator not only reduces the amount of live steam needed to heat the stream but also brings down the losses from the evaporator. Thus bleeding contributes to improve the steam economy of the evaporator.

Reinhold & Connelly [R2] carried out a systematic analysis of multiple effect evaporators to determine the optimum number of effects.Their analysis is based on the total cost equation which has been obtained by the summation of fixed- and operating- cost due to steam. It is as follows:

$$V = (C_2/A)N^{0.75} + \{(1 - S_2)hWC_2\}/\{S_1(1 - S_2^N)\} + V_o \qquad ...(2.10)$$

The operating cost due to other items such as labour, cooling water, power and maintenance has not been included in Eq(2.10) as it does not vary significantly with the number of effects. Eq (2.10) has been solved by the application of calculus of finite differences to determine the optimum number of effects in an evaporator.The resulting expression is in the form of a cost factor,P (= Ahw C_2/C_1) as a function of optimum number of effects.It is as follows:

$$P = \frac{[(N + 1)^{0.75} - N^{0.75}]s_1(1 - s_2^N)(1 - s_2^{N+1})}{(1 - s_2)^2 s_2^N} \qquad ...(2.11)$$

where;

A = payout time in years
c_1 = cost of a single effect including auxiliaries
c_2 = cost of steam per unit mass
h = operating time, hr/year
N = number of effect
s = steam economy
s_1 = steam utilization in a single effect
s_2 = steam utilization in additional effects
V = total annual cost per year
W = evaporation rate, mass per unit time

The value of cost factor can be easily calculated by the process and economic data. and then the corresponding number of effects are obtained from the tables which have been prepared from Eq (2.11).

Veermani [V1] simulated long tube vertical evaporators for black liquor used in pulp & paper industry by considering various heat transfer mechanisms- natural convection, nucleate boiling (bubble flow and part of slug flow regime), forced convective boiling (remainder of slug flow and annular flow regime) in the tube. The equations of continuity, momentum, energy balance and frictional pressure drop alongwith heat transfer correlation for pertinent regime have determined the changes in liquid temperature, pressure, pressure gradient, heat flux, void fraction and quality in various parts of the tube. These data helped in the prediction of nonboiling- and boiling- section length ,evaporation rate, heat transfer rate and the heat transfer coefficient along the length of a tube. The accuracy of the simulation program has been compared with the experimental data of Brooks & Badger [B4] for the evaporation of water in a long tube vertical evaporator. As a result, this programme has been found to predict the experimental conditions [B5] within an accuracy of about 15% .

In another research investigation, following empirical correlation for the specific heat of black-liquor solution has been recommended by Veeramani[V2]:

$$C = 7.53\text{x}10^{-3} (\tau - T_r)X - 2.25383X + 4.182 \qquad ...(2.12)$$

Eq (2.12) has been employed by the author [V1] for the simulation of black liquor evaporators.

Saranathan [S1] carried out a detailed study of caustic soda evaporator house. Based on the performance of evaporator plant in Mettur chemical & Industrial Corporation Limited, Mettur Dam, India; he has discussed various factors related to the operation of a triple effect mixed feed evaporator operating with caustic soda cell liquor from Hooker S- type cells. According to him, the concentration and temperature of cell liquor, and level of vacuum affects the steam consumption of the evaporator. A mere decrease of the cell liquor concentration from 10% to 9% increases the steam consumption by about 530 kg/ton of caustic soda. Similarly a drop in the temperature of cell liquor leads to higher steam consumption. Reduction in vacuum brings down the capacity of the plant and increases the steam consumption. He has also suggested steps to maintain proper concentration and temperature of the cell liquor and vacuum in the system. The author has also discussed the role that caustic losses and the recovery and recycling of salt play in the economics of a caustic soda plant. Effective treatment procedure has also been recommended.

Gupta et al. [G8] has discussed evaporation of cell liquor in caustic soda manufacture using diaphragm cell. They have reviewed various factors affecting the selection of end-product concentration, tolerable salt content, type of evaporation, and the economy of operation. They have reported that the design of cell liquor evaporation system must invariably consider the end use of caustic-lye so that salt content in caustic-lye and the loss of caustic soda may be decided. Further, the effect of cell liquor concentration on the plant design capacity has been found to be more pronounced than that of end product concentration . For example, by increasing cell liquor concentration from 9 to 12% the evaporator capacity reduces by 35%, whereas a decrease in end product concentration from 50 to 47.5% results a mere reduction of 3% in evaporator capacity. The selection of end-product concentration is governed by freezing point curve. That is why 50% caustic soda concentration has been the highest limit for caustic-lye. The paper has also discussed various factors responsible for steam economy in a multiple effect evaporator and the means of minimizing the caustic losses.

In an article, Ghosh [G4 & G5] has discussed some of the design and operational aspects related to sugar evaporators. He has given an account of various heating surface materials and the effect of tube pitch arrangement and noncondensables on evaporator capacity and heat transfer rate. The author has also suggested the preheating of the feed, change in tube geometry, and the recirculation of the aqueous solution as some of the potential aspects to improve the heat transfer rate in evaporators. The emerging velocity of liquid droplets in individual effects has been found to increase with the decrease in the number of effects in the evaporator. However, the effect of velocity becomes quite prominent when the number of effects in an evaporator are changed from four to three. Therefore, the last body of the triple effect evaporator has been found to be more prone to entrainment than other effects of the quadruple effect. Accordingly, the design features of the last body regarding the vapour space dimensions should be considered.

Suri et al. [S4] have discussed various considerations and factors responsible for the efficient evaporation of black-liquor solution of bamboo wood used in pulp & paper industries. They pointed out that the characteristics of solids present in the black-liquor and the scale formation in multiple effect evaporators control the rate of heat transfer and thereby the evaporation rate. Further, they have discussed some of the important remedial measures. As suggested by them the measures for removing the scale formation are disilification; weak liquor boiling; addition of residual alkali in black-liquor; minimizing sulphate, carbonate and fibre in black-liquor; adoption of cleaning schedules and the use of stainless steel tubes instead of carbon steel tubes.

Ray [R3] carried out a theoretical study on short tube multiple effect evaporator for the concentration of sugar solution from 15°Bx to 60°Bx. He has developed correlations for physico-thermal properties of the solution as a function of concentration and temperature of sugar juice and also the recirculation ratio. Based on the correlations of properties, he has succeeded to obtain the following equations which permit the calculation of boiling point rise due to dissolved solids, ϵ_d:

$$\epsilon_d = 1.7895\{X/(100-X)\}\ \{0.0162T^2/\lambda\}\ \{0.01733X + 0.4\} \qquad \ldots(2.13)$$

For the case of no recirculation, X is equal to feed inlet concentration, whereas for the evaporators with recirculation, the value of X should be equal to X_m which is defined as:

$$X_m = (X_i + X_o)/(r + 1) \qquad \ldots(2.14)$$

where X_i, X_o and r refer to concentration of the solution at the inlet, and outlet of the tube and recirculation ratio, respectively. Eq(2.13) has compared quite closely with the experimental data of Hugot [H4] which is widely used in the design of multiple effect evaporators. The author has attempted equations for cumulative boiling point rise due to pressure drop from the knowledge of static- and dynamic- head. Besides, correlations for the pressure drop due to static head, due to friction and due to momentum change in the evaporator body have also been proposed. In addition to the above, it has also suggested an analysis of the time elapsed for sugar evaporator with forward feed to reach steady state equilibrium conditions.

The potential of evaporators for upgrading their performance, and guidelines for formulating an upgrading program have been discussed in Technology Application Manual [T1]. Correction of operating variables ,heat recovery from waste hot outgoing streams, vapour recompression , addition of effects , and application of new technology are some of the areas to upgrade the capacity and steam economy of existing multiple effect evaporators. The report has also discussed a method of evaluation for existing evaporators and determination of an appropriate upgradation scheme.

Executive briefing Report [T2] has discussed the scope of multiple effect evaporators for reducing energy costs and various options for upgrading their capacity and steam economy. The options have been categorized in three major groups viz; low-investment method, moderate-investment method, and large capital investment method. The economics for each of these options depends largely on the existing plant situation. For example, if the heat in the condensate is largely wasted, then the installation of flash tank is advantageous. On the other hand, if the bulk of the energy in condensate is recovered at the boiler, which is usually the case, flash tank may not be an economical choice. The low-investment schemes include the correction of venting rates, air leakage, fouling,optimum pressure profile, water leakage, separation efficiency, and radiation and convection losses. The moderate-investment schemes discussed in this report includes the improved heat recovery, condensate and product flashing, and instrumentation and control. The large capital investment schemes calls for the vapour recompression, and installing additional effects in the evaporator

Siota [S3] applied the concept of heat exchanger network synthesis to evaporator systems. He demonstrated that significant energy savings can occur from the use of heat exchangers and flash tanks between the effects. However, he neither included process streams in the heat recovery scheme nor predicted a lower limit on the utility requirement.

Other investigators [C5, F1, F2, H2, M1, and M2] have also recommended similar measures for energy conservation in multiple effect evaporators.

Nishitani & Kunugita [N3] have discussed the optimization of the flow patterns for a multiple effect evaporator system. They have employed equations of solute balance, overall material balance, energy balance, heat transfer rate , boiling point rise and the relevant enthalpy equations of water vapour, liquid streams and latent heat of vaporisation to obtain a set of system equations for N-effect evaporator systems. Since the equations are linear with respect to the flow rates of liquid, vapour and steam in the evaporator, the set is represented in the form of an occurrence matrix. Based on this, a program for the design has been developed for various flow patterns of an arbitrary number of effects. The program determines the optimal flow pattern of the evaporator system.

Hillenbrand & Westerberg [H2] have developed a mathematical model to compute the utility consumption for multiple effect evaporator systems for which the temperature levels and liquid flow pattern are specified. The modified grand composite curve which is a plot between temperature and the maximum amount of process heat which can be passed through the system in a single effect has been developed. The model and the modified grand composite curve permits to discover the approximate best temperature at which a single effect evaporator uses the minimum utilities. With this effect operating at the best temperature , the paper also shows how to place a second effect and so on. Thus multiple effect evaporator is synthesised for minimum consumption of utilities .

Westerberg and Hillenbrand [W1] have suggested a graphical method to compute the extra amount of utility required for a multiple effect evaporator system caused by the change in liquid flow pattern . It is caused by the formation of heat shunts for both the product and the condensate produced by the system where these streams follow complex temperature path through the system. By this method it is possible to evaluate the heat shunts for any given path and then to identify its impact and thereby the estimation of extra utility consumption without a detailed simulation of the process. All the flow paths can be compared to obtain the one which offers minimum utility consumption. In this method they have made the assumptions of constant boiling point elevation and negligible heat of mixing.

CHAPTER-3

MATHEMATICAL MODEL OF MULTIPLE EFFECT EVAPORATORS

Multiple effect evaporators involve a large number of state- and design-variables. A change in any variable can upset the operation of the evaporator. To achieve the goel of energy conservation in multiple effect evaporators, it is necessary to know how does steam economy alter with changes in operating variables for a given end-product concentration. To quantise the changes in steam economy, a functional relationship correlating it with variables should be developed. For this, it is necessary to identify all the variables which affect the steam economy of a multiple effect evaporator.

3.1 VARIABLES OF A MULTIPLE EFFECT EVAPORATOR

In an evaporator, the variables can be classified as geometrical-operating, and self-balancing-variables. As regards the geometrical variable, it is the area of heat transfer surface in each effect of an evaporator. Hence, N-effect evaporator will have N number of geometrical variables.

From industrial practices, we know that there are some operating variables which plant engineer can change them independently to annul any inbalance in the operation of an evaporator. They include: feed temperature, feed concentration, feed flow rate, steam temperature (pressure), and saturation temperature (pressure) in the last effect. Feed arrangement (forward/backward/mixed) is also one of the operating variable. Thus, total number of operating variables are six.

As regards the vapour and liquid streams from each effect of a multiple effect evaporator, they can not be changed independently by a plant engineer. Therefore, they are self-balancing streams. The variables associated with these streams are: flow rate, and temperature of vapour streams; flow rate, temperature, and concentration of liquid streams; and saturation temperature (pressure) of each effect. However, temperature of vapour stream equals to the temperature of liquid stream. In this way, for N-effect evaporator the number of self- balancing variables becomes 5N. It is important to point out here that the saturation temperature (pressure) of the last effect, has already been accounted as an operating variable for the reason explained therein. Therefore, it can not be considered as a self-balancing variable. Flow rate of steam to the first effect is another self-balancing variable whose value is usually not altered. Thus the total number of net self-balancing variables for N-effect evaporator, becomes 5N [=5N -1 + 1].

The summation of geometrical-, operating-, and self- balancing-variables gives the total number of variables in an evaporator. They are equal to 6N+6 [=N+6+5N].

3.2 MATHEMATICAL MODEL

A mathematical model of a multiple effect evaporator is a relationship amongst the geometrical-, operating-, and self-balancing- variables. This can be obtained from the equations of material balance, energy balance, heat transfer rate, and boiling point rise.

For the simplicity of the mathematical model, following assumptions have been made in this analysis:

1. The vapours entering into steam chest of respective effects are at their saturation temperature.
2. There is no subcooling of the condensate from different steam chests.
3. Condensation of vapour in steam chest occurs at constant pressure.
4. There is no carry-over of liquid droplets with vapours leaving the respective effects.
5. There is no heat dissipation to surroundings.
6. Heat transfer surface does not undergo fouling.

In the following Sections, mathematical models have been developed for the evaporation of aqueous sugar-, black-liquor-, and caustic soda- solution under various feed arrangements.

3.2.1 FORWARD FEED

Figure 3.1 is a typical flow diagram of a multiple effect evaporator for forward feed arrangement.

The equations of material balance, energy balance, heat transfer rate, and boiling

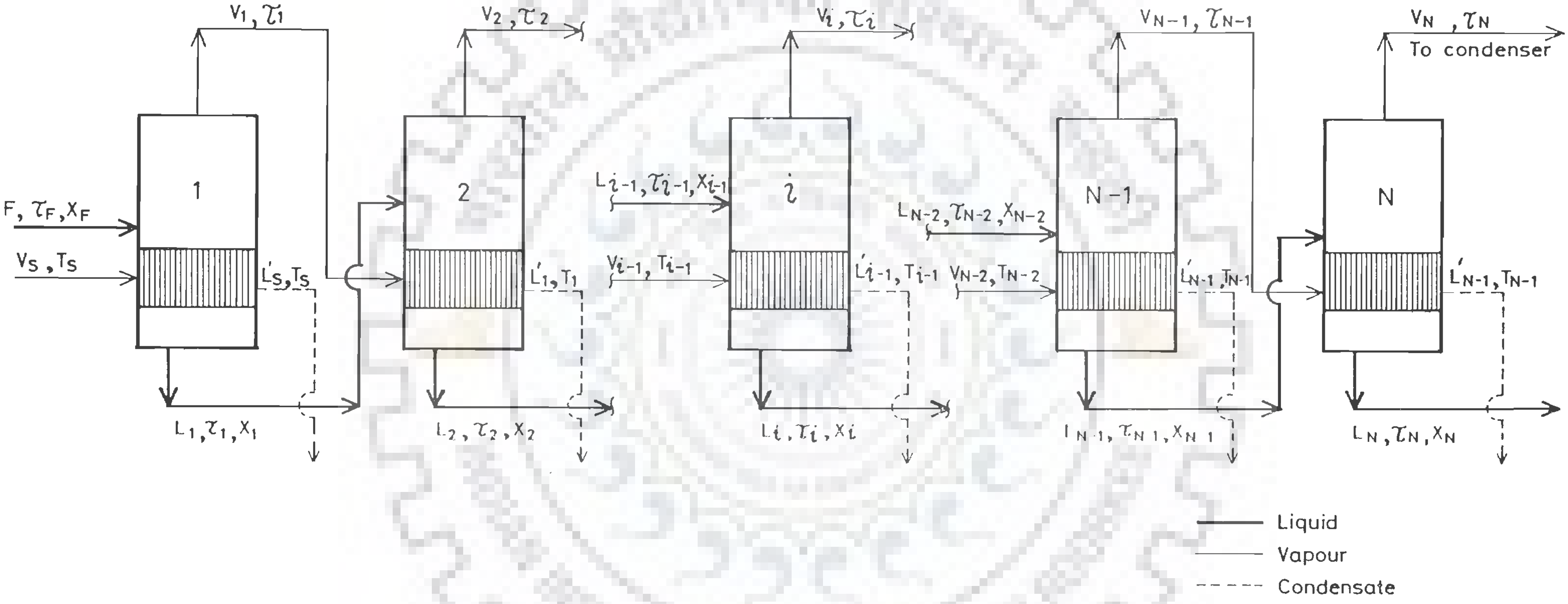

Fig.3.1 Flow diagram of a multiple effect evaporator for a forward feed arrangement

point rise about the ***i*** th effect of a multiple effect evaporator are as follows:

SOLUTE MATERIAL BALANCE

$$F.X_f = L_i.X_i \qquad \ldots(3.1a)$$

OVERALL MATERIAL BALANCE

$$L_{i-1} = V_i + L_i \qquad \ldots(3.1b)$$

ENERGY BALANCE

$$L_{i-1}.h(\tau_{i-1},X_{i-1}) + V_{i-1}.H(P_{i-1}) = L_i.h(\tau_i,X_i) + V_i.\ H(P_i,\tau_i) + L'_{i-1}.h(P_{i-1})$$

Since $L'_{i-1} = V_{i-1}$

$$L_{i-1}.h(\tau_{i-1},X_{i-1}) + V_{i-1}.\ \lambda(P_{i-1}) = L_i.h(\tau_i,X_i) + V_i.H(P_i,\tau_i) \qquad \ldots(3.1c)$$

HEAT TRANSFER RATE

$$V_{i-1}.H(P_{i-1}) - L'_{i-1}.h(P_{i-1}) = U_i.\ A_i.(T_{i-1} - \tau_i)$$

Since $L'_{i-1} = V_{i-1}$

$$V_{i-1}.\ \lambda(P_{i-1}) = U_i.A_i(T_{i-1} - \tau_i) \qquad \ldots(3.1d)$$

BOILING POINT RISE

The equation, Eq(A.6) for boiling point rise of various aqueous salt solutions has been developed in Appendix - A. It is reproduced here as follows:

$$\tau_i = T_i + (\gamma + nT_i)X_i + \alpha X_i^2 + \beta X_i^3 \qquad \ldots(A.6)$$

Substitution of Eq(3.1b) into Eqs(3.1c, & 3.1d) eliminates the term of vapour flow rate and thereby following equations are obtained:

$$L_{i-1}.h(\tau_{i-1},X_{i-1}) + (L_{i-2}-L_{i-1})\,\lambda(P_{i-1}) = L_i h(\tau_i,X_i) + (L_{i-1}-L_i).H(P_i,\tau_i) \qquad \ldots(3.1e)$$

$$(L_{i-2} - L_{i-1})\,\lambda(P_{i-1}) = U_i.A_i.(T_{i-1} - \tau_i) \qquad \ldots(3.1f)$$

Equations (3.1a, 3.1e, 3.1f, and A.6) represent the model for forward feed arrangement in an N-effect evaporator. However, it can not be used in its present form as it contains quantities like enthalpy of liquid, h ; enthalpy of vapour, H ; latent heat of vaporization of steam, λ ; and overall heat transfer coefficient, U whose direct measurement is difficult. The functional relationships of these quantities with temperature and concentration of the liquid streams and the pressure of steam fed to the evaporator have been given by various equations in Appendices B and C. Therefore, Eq(B.5) for H, and Eq(B.6) for λ alongwith pertinent equations for h, and U of a given aqueous solution are substituted into Eqs(3.1e & 3.1f). The resultant equations alongwith Eqs(3.1a & A.6) constitute the model for the evaporation of a given aqueous solution in an N-effect evaporator having forward feed arrangement. The models for aqueous sugar-, aqueous black-liquor, and aqueous caustic soda-solution are as follows:

a. Sugar Solution

$$F.X_f = L_i.X_i \qquad \ldots(3.1S1)$$

$$\begin{aligned}
&L_{i-1}\,(4.182 - 2.2403X_{i-1})(\tau_{i-1} - T_r) + (L_{i-2} - L_{i-1})[-80.345T_{i-1} \\
&- 21035.87/T_{i-1} + 2049.123T_{i-1}^{1/2} - 4213.519\ln T_{i-1} + 0.0918T_{i-1}^{2} \\
&- 1.04\times10^{-4}T_{i-1}^{3} + 8597.953] \\
&= L_i(4.182 - 2.2403X_i)(\tau_i - T_r) + (L_{i-1} - L_i)[4.154(T_i - T_r) \\
&+ 2.0125\times10^{-4}(T_i^2 - T_r^2) + 1.62(\tau_i - T_i) + 2.0285\times10^{-4}(\tau_i^2 - T_i^2) \\
&- 0.3747\times10^{-7}(\tau_i^3 - T_i^3) - 80.345T_i - 21035.87/T_i + 2049.123T_i^{1/2} \\
&- 4213.519\ln T_i + 0.0918T_i^2 - 1.04\times10^{-4}T_i^3 + 8597.953] \qquad \ldots(3.1S2)
\end{aligned}$$

$$\begin{aligned}
&(L_{i-2} - L_{i-1})[-80.345T_{i-1} - 21035.87/T_{i-1} + 2049.123T_{i-1}^{1/2} \\
&- 4213.519\ln T_{i-1} + 0.0918T_{i-1}^{2} - 1.04\times10^{-4}T_{i-1}^{3} + 8597.953] \\
&= 18.083(\tau_i/X_i)\,A_i\,(T_{i-1} - \tau_i) \qquad \ldots(3.1S3)
\end{aligned}$$

$$\tau_i = T_i + 7.20X_i - 11.50X_i^2 + 29.50X_i^3 \qquad \ldots(3.1S4)$$

b. Black-Liquor Solution

$$F.X_f = L_i.X_i \qquad \ldots(3.1B1)$$

$$\begin{aligned}
&L_{i-1}[7.53\text{x}10^{-3}(\tau_{i-1}^2 - T_r^2)X_{i-1} - 2.25383(\tau_{i-1} - T_r)X_{i-1} + 4.182(\tau_{i-1} - T_r)] \\
&+ (L_{i-2} - L_{i-1})[-80.345T_{i-1} - 21035.87/T_{i-1} + 2049.123T_{i-1}^{1/2} \\
&- 4213.519\ln T_{i-1} + 0.0918T^2_{i-1} - 1.04\text{x}10^{-4}T_{i-1}^3 + 8597.953] \\
&= L_i[7.53\text{x}10^{-3}(\tau_i^2 - T_r^2)X_i - 2.25383(\tau_i - T_r)X_i + 4.182(\tau_i - T_r)] \\
&+ (L_{i-1} - L_i)[4.154(T_i - T_r) + 2.0125\text{x}10^{-4}(T_i^2 - T_r^2) + 1.62(\tau_i - T_i) \\
&+ 2.0285\text{x}10^{-4}(\tau_i^2 - T_i^2) - 0.3747\text{x}10^{-7}(\tau_i^3 - T_i^3) - 80.345T_i \\
&- 21035.87/T_i + 2049.123T_i^{1/2} - 4213.519\ln T_i + 0.0918T_i^2 \\
&- 1.04\text{x}10^{-4}T_i^3 + 8597.953] \qquad \ldots(3.1B2)
\end{aligned}$$

$$\begin{aligned}
&(L_{i-2} - L_{i-1})[-80.345T_{i-1} - 21035.87/T_{i-1} + 2049.123T_{i-1}^{1/2} \\
&- 4213.519\ln T_{i-1} + 0.0918T_{i-1}^2 - 1.04\text{x}10^{-4}T_{i-1}^3 + 8597.953] \\
&= [13.392(\tau_{i-1} + \tau_i) - 3960.0(X_{i-1} + X_i) + 4800.0]A_i(T_{i-1} - \tau_i) \qquad \ldots(3.1B3)
\end{aligned}$$

$$\tau_i = T_i - 3.55X_i + 84.0X_i^2 - 107.5X_i^3 \qquad \ldots(3.1B4)$$

c. Caustic Soda Solution

$$F.X_f = L_i.X_i \qquad \ldots(3.1C1)$$

$$\begin{aligned}
&L_{i-1}[62.015 + 3.884\tau_{i-1} - 887.125X_{i-1} + 2316.504X_{i-1}\exp(-1.15\text{x}10^{-3}\tau_{i-1}^2)] \\
&+ (L_{i-2} - L_{i-1})[-80.345T_{i-1} - 21035.87/T_{i-1} + 2049.123T_{i-1}^{1/2} \\
&- 4213.519\ln T_{i-1} + 0.0918T_{i-1}^2 - 1.04\text{x}10^{-4}T_{i-1}^3 + 8597.953] \\
&= L_i[62.015 + 3.884\tau_i - 887.125X_i + 2316.504X_i\exp(-1.15\text{x}10^{-3}\tau_i^2)] \\
&+ (L_{i-1} - L_i)[4.154(T_i - T_r) + 2.0125\text{x}10^{-4}(T_i^2 - T_r^2) + 1.62(\tau_i - T_i) \\
&+ 2.0285\text{x}10^{-4}(\tau_i^2 - T_i^2) - 0.3747\text{x}10^{-7}(\tau_i^3 - T_i^3) - 80.345T_i \\
&- 21035.87/T_i + 2049.123T_i^{1/2} - 4213.519\ln T_i + 0.0918T_i^2 \\
&- 1.04\text{x}10^{-4}T_i^3 + 8597.953] \qquad \ldots(3.1C2)
\end{aligned}$$

$$(L_{i-2} - L_{i-1})[-80.345T_{i-1} - 21035.87/T_{i-1} + 2049.123T_{i-1}^{1/2}$$
$$- 4213.519\ln T_{i-1} + 0.0918T_{i-1}^{2} - 1.04\mathrm{x}10^{-4}T_{i-1}^{3} + 8597.953]$$
$$= 977.66[\tau_i/X_i]^{0.2823}\ A_i\ (T_{i-1} - \tau_i) \qquad \ldots(3.1C3)$$

$$\tau_i = T_i + (0.142T_i - 9.42).X_i + 271.363X_i^2 \qquad \ldots(3.1C4)$$

The above models, Eqs(3.1S1 -- 3.1S4), Eqs(3.1B1--3.1B4), and Eqs (3.1C1 -- 3.1C4) for sugar-, black-liquor-, and caustic soda- solution respectively, can be used to determine the values of self-balancing variables from the knowledge of operating- and geometrical-variables in a forward feed multiple effect evaporator.

3.2.2 BACKWARD FEED

Figure 3.2 depicts a typical flow diagram representing a multiple effect evaporator using backward feed arrangement.

For this arrangement, equations of material balance, energy balance, heat transfer rate, and boiling point rise about the ***i*** th effect are as follows:

SOLUTE MATERIAL BALANCE

$$F.X_f = L_i.X_i \qquad \ldots(3.2a)$$

OVERALL MATERIAL BALANCE

$$L_{i+1} = V_i + L_i \qquad \ldots(3.2b)$$

ENERGY BALANCE

$$L_{i+1}.h(\tau_{i+1},X_{i+1}) + V_{i-1}.H(P_{i-1})$$
$$= L_i.h(\tau_i,X_i) + V_i.H(P_i,\tau_i) + L'_{i-1}.h(P_{i-1})$$

Since $L'_{i-1} = V_{i-1}$

$$L_{i+1}.h(\tau_{i+1},X_{i+1}) + V_{i-1}.\ \lambda(P_{i-1})$$
$$= L_i.h(\tau_i,X_i) + V_i.H(P_i,\tau_i) \qquad \ldots(3.2c)$$

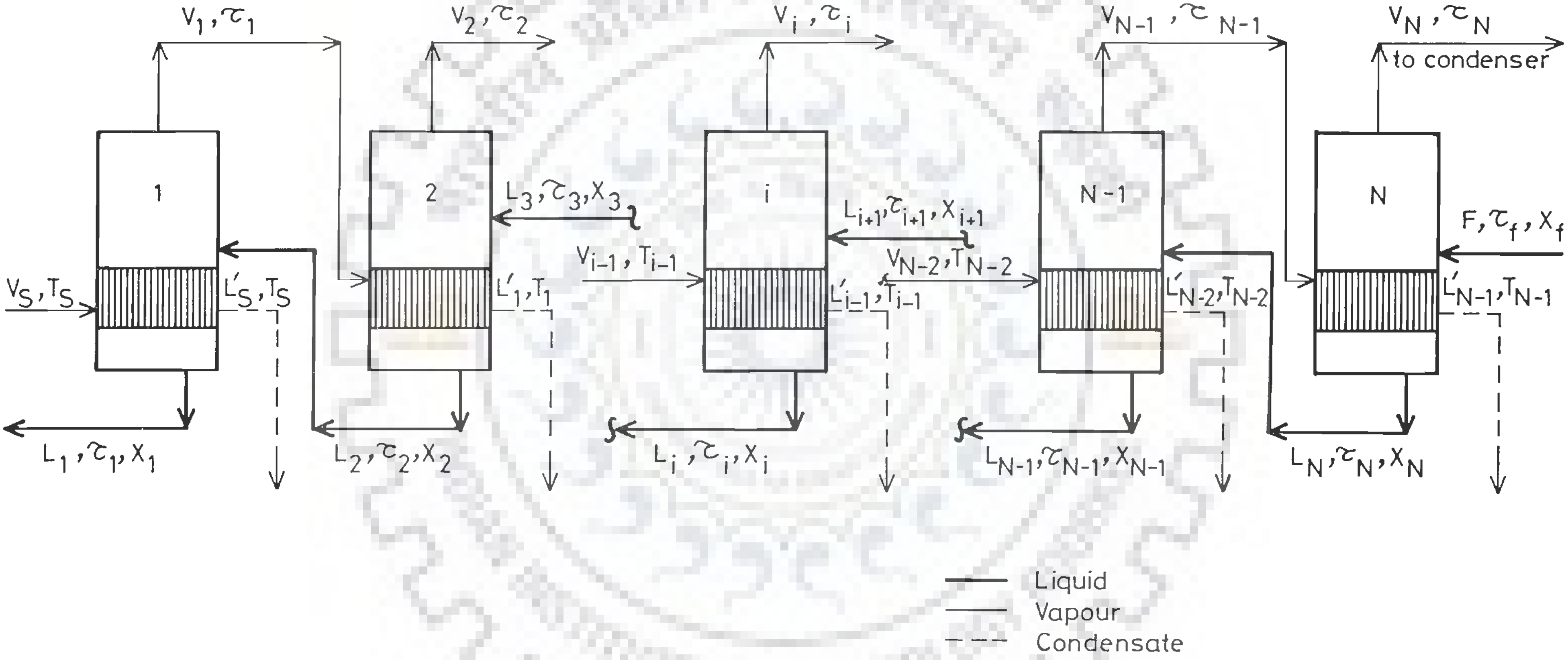

Fig. 3·2 Flow diagram of a multiple effect evaporator for a backward feed arrangement

HEAT TRANSFER RATE

$$V_{i-1}.H(P_{i-1}) - L'_{i-1}.h(P_{i-1}) = U_i.A_i.(T_{i-1} - \tau_i)$$

Since $L'_{i-1} = V_{i-1}$

$$V_{i-1}.\ \lambda\ (P_{i-1}) = U_i.\ A_i.(T_{i-1} - \tau_i) \qquad \ldots(3.2d)$$

BOILING POINT RISE

$$\tau_i = T_i + (\gamma + nT_i)X_i + \alpha X_i^2 + \beta X_i^3 \qquad \ldots(A.6)$$

Substitution of Eq(3.2b) into Eqs(3.2c & 3.2d) provides the following equations:

$$\begin{aligned} &L_{i+1}.h(\tau_{i+1},X_{i+1}) + (L_i - L_{i-1})\,\lambda(P_{i-1}) \\ &= L_i.h(\tau_i,X_i) + (L_{i+1} - L_i).H(P_i,\tau_i) \end{aligned} \qquad \ldots(3.2e)$$

$$(L_i - L_{i-1})\,\lambda\,(P_{i-1}) = U_i.A_i.(T_{i-1} - \tau_i) \qquad \ldots(3.2f)$$

Eqs(3.2a, 3.2e, 3.2f, and A.6) represent a model for the evaporation of aqueous solution in an N-effect evaporator having backward feed arrangement. The terms h, H,λ, and U contained in Eqs(3.2e & 3.2f) are substituted by their respective expressions as given in Appendices B and C. The resulting equations alongwith (3.2a & A.6) describe the model for a given aqueous solution under backward feed arrangement in an N-effect evaporator. They are as follows:

a. Sugar Solution

$$F.X_f = L_i.X_i \qquad \ldots(3.2S1)$$

$$\begin{aligned} &L_{i+1}[(4.182 - 2.2403X_{i+1})(\tau_{i+1} - T_r)] \\ &+ (L_i - L_{i-1})[-80.345T_{i-1} - 21035.87/T_{i-1} + 2049.123T_{i-1}^{1/2} \\ &- 4213.519\ln T_{i-1} + 0.0918T^2_{i-1} - 1.04x10^{-4}T^3_{i-1} + 8597.953] \\ &= L_i[(4.182 - 2.2403X_i)(\tau_i - T_r)] \\ &+ (L_{i+1} - L_i)[4.154(T_i - T_r) + 2.0125x10^{-4}(T_i^2 - T_r^2) \\ &+ 1.62(\tau_i - T_r) + 2.0285x10^{-4}(\tau_i^2 - T_1^2) \\ &- 0.3747x10^{-7}(\tau_i^3 - T_r^3) - 80.345T_i - 21035.87/T_i \\ &+ 2049.123T_i^{1/2} - 4213.519\ln T_i + 0.0918T_i^2 \\ &- 1.04x10^{-4}T_i^3 + 8597.953] \end{aligned} \qquad \ldots(3.2S2)$$

$$(L_i - L_{i-1})[-80.345T_{i-1} - 21035.87/T_{i-1} + 2049.123T_{i-1}^{1/2}$$

$$- 4213.519\ln T_{i-1} + 0.0918T_{i-1}^{2} - 1.04\times10^{-4}T_{i-1}^{3} + 8597.953]$$

$$= 18.083\ (\tau_i/X_i)\ A_i\ (T_{i-1} - \tau_i) \qquad \ldots(3.2S3)$$

$$\tau_i = T_i + 7.20\ X_i - 11.50\ X_i^2 + 29.50\ X_i^3 \qquad \ldots(3.2S4)$$

b. *Black-Liquor Solution*

$$F.X_f = L_i.X_i \qquad \ldots(3.2B1)$$

$$L_{i+1}[7.53\times10^{-3}(\tau_{i+1}^{2} - T_r^2)X_{i+1} - 2.25383(\tau_{i+1} - T_r)X_{i+1}$$

$$+ 4.182(\tau_{i+1} - T_r)]$$

$$+ (L_i - L_{i-1})[-80.345T_{i-1} - 21035.87/T_{i-1} + 2049.123T_{i-1}^{1/2}$$

$$- 4213.519\ln T_{i-1} + 0.0918T_{i-1}^{2} - 1.04\times10^{-4}T_{i-1}^{3} + 8597.953]$$

$$= L_i[7.53\times10^{-3}(\tau_i^2 - T_r^2)X_i - 2.25383(\tau_i - T_r)X_i + 4.182(\tau_i - T_r)]$$

$$+ (L_{i+1} - L_i)[4.154(T_i - T_r) + 2.0125\times10^{-4}(T_i^2 - T_r^2)$$

$$+ 1.62(\tau_i - T_i) + 2.0285\times10^{-4}(\tau_i^2 - T_i^2)$$

$$- 0.3747\times10^{-7}(\tau_i^3 - T_i^3) - 80.345T_i - 21035.87/T_i$$

$$+ 2049.123T_i^{1/2} - 4213.519\ln T_i$$

$$+ 0.0918T_i^2 - 1.04\times10^{-4}T_i^3 + 8597.953] \qquad \ldots(3.2B2)$$

$$(L_i - L_{i-1})[-80.345T_{i-1} - 21035.87/T_{i-1} + 2049.123T_{i-1}^{1/2}$$

$$- 4213.519\ln T_{i-1} + 0.0918T_{i-1}^{2} - 1.04\times10^{-4}T_{i-1}^{3} + 8597.953]$$

$$= [13.392(\tau_{i+1}+\tau_i) - 3960.0(X_{i+1} + X_i)$$

$$+ 4800.0]\ A_i\ (T_{i-1} - \tau_i) \qquad \ldots(3.2B3)$$

$$\tau_i = T_i - 3.55X_i + 84.0X_i^2 - 107.5X_i^3 \qquad \ldots(3.2B4)$$

c. *Caustic Soda Solution*

$$F.X_f = L_i.X_i \qquad \ldots(3.2C1)$$

$$\begin{aligned}
&L_{i+1}[62.015+3.884\tau_{i+1}-887.125X_{i+1}+2316.504X_{i+1}\exp(-1.15x10^{-3}\tau_{i+1}^{2})]\\
&+ (L_i - L_{i-1})[-80.345T_{i-1} - 21035.87/T_{i-1} + 2049.123T_{i-1}^{1/2}\\
&- 4213.519\ln T_{i-1} +0.0918T_{i-1}^{2} - 1.04x10^{-4}T_{i-1}^{3} + 8597.953]\\
&= L_i[62.015+3.884\tau_i -887.125X_i+2316.504X_i\exp(-1.15x10^{-3}\tau_i^{2})]\\
&+ (L_{i+1}-L_i)[4.154\ (T_i-T_r) +2.0125x10^{-4}(T_i^2-T_r^2) +1.62(\tau_i -T_i)\\
&+ 2.0285x10^{-4}(\tau_i^2 -T_i^2) -0.3747x10^{-7}(\tau_i^3 -T_i^3) -80.345T_i\\
&- 21035.87/T_i +2049.123T_i^{1/2} -4213.519\ln T_i +0.0918T_i^2\\
&- 1.04x10^{-4}T_i^3 + 8597.953] \qquad \ldots(3.2C2)
\end{aligned}$$

$$\begin{aligned}
&(L_i - L_{i-1})[-80.345T_{i-1} -21035.87/T_{i-1} +2049.123T_{i-1}^{1/2}\\
&- 4213.519\ln T_{i-1} + 0.0918T_{i-1}^{2} - 1.04x10^{-4}T_{i-1}^{3} + 8597.953]\\
&= 977.66[\tau_i/X_i]^{0.2823} A_i (T_{i-1} - \tau_i) \qquad \ldots(3.2C3)
\end{aligned}$$

$$\tau_i = T_i + (0.142T_i - 9.42).X_i + 271.363X_i^2 \qquad \ldots(3.2C4)$$

Self-balancing variables of a sugar evaporator can be calculated from equations, Eqs(3.2S1, 3.2S2, 3.2S3, and 3.2S4) whereas for black-liquor-, and caustic soda- evaporators their respective equations, Eqs(3.2B1, 3.2B2, 3.2B3, and 3.2B4), and Eqs(3.2C1, 3.2C2, 3.2C3, and 3.2C4) are to be used.

3.2.3 MIXED FEED

Figure 3.3 is a typical flow diagram of a multiple effect evaporator with mixed feed arrangement.

As seen from this Figure, it is a combination of forward and backward feed arrangements. In it, feed can enter in any one of the effects between first- and last-effect of the evaporator. The effects which follow the feed-introduction effect behave as forward, whereas the remaining effects as backward. Thus, the position of feed-introduction

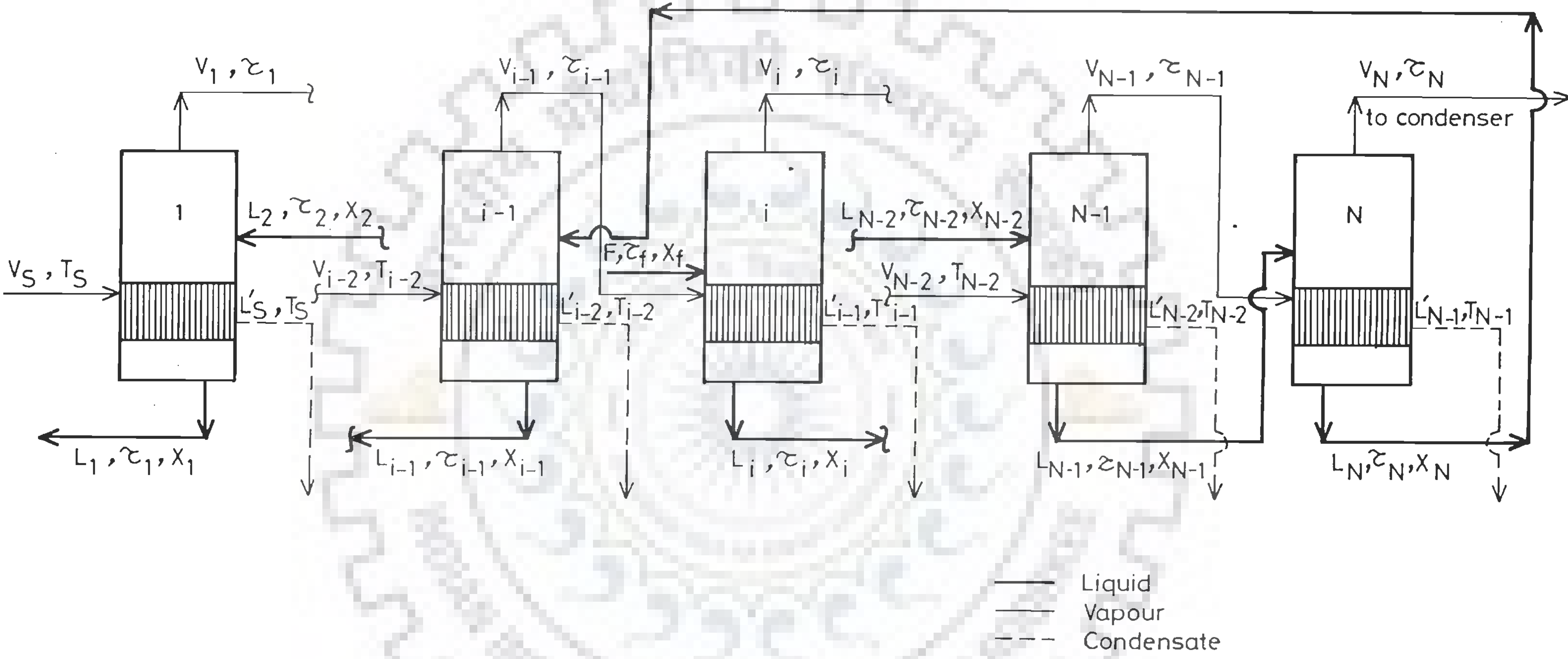

Fig. 3·3 Flow diagram of a multiple effect evaporator for a mixed feed arrangement effect influences the working of a given effect in the evaporator. Therefore, in the present analysis the liquid stream entering to the ith effect has been taken as L_k.

The equations of material balance, energy balance, heat transfer rate, and boiling point rise about the *i* th effect are as follows:

SOLUTE MATERIAL BALANCE

$$F.X_f = L_i.X_i \qquad ...(3.3a)$$

OVERALL MATERIAL BALANCE

$$L_k = V_i + L_i \qquad ...(3.3b)$$

ENERGY BALANCE

$$L_k.h(\tau_k,X_k) + V_{i-1}H(P_{i-1}) = L_i.h(\tau_i,X_i) + V_i.H(P_i,\tau_i) + L'_{i-1}.h(P_{i-1})$$

Since $L'_{i-1} = V_{i-1}$

$$L_k.h(\tau_k,X_k) + V_{i-1}.\ \lambda(P_{i-1}) = L_i.h(\tau_i,X_i) + V_i.H(P_i,\tau_i) \qquad ...(3.3c)$$

HEAT TRANSFER RATE

$$V_{i-1}.H(P_{i-1},T_{i-1}) - L'_{i-1}.h(P_{i-1}) = U_i.A_i.[T_{i-1} - \tau_i]$$

Since $L'_{i-1} = V_{i-1}$

$$V_{i-1}.\ \lambda(P_{i-1}) = U_i.A_i.[T_{i-1} - \tau_i] \qquad ...(3.3d)$$

BOILING POINT RISE

$$\tau_i = T_i + (\gamma + nT_i)X_i + \alpha X_i^2 + \beta X_i^3 \qquad ...(A.6)$$

Substitution of Eq(3.3b) into Eqs(3.3c, & 3.3d) leads to the following respective Eqs(3.3e & 3.3f):

$$L_k.h(\tau_k,X_k) + (L_{k-1} - L_{i-1})\ (P_{i-1}) = L_i.h(\tau_i,X_i) + (L_k - L_i)H(P_i,\tau_i) \qquad ...(3.3e)$$

$$(L_{k-i} - L_{i-1}).\ (P_{i-1}) = U_i.A_i.[T_{i-1} - \tau_i] \qquad ...(3.3f)$$

It is important to mention here that when ith effect corresponds to the feed-introduction effect, $L_k = F$. For the case when ith effect is located after the feed-introduction effect, $L_k = L_{i-1}$; Whereas $L_k = L_N$ when ith effect is just before the feed-introduction effect; and for all other situations, $L_k = L_{i+1}$.

Following the procedure as employed for forward-, and backward- feeds, Eqs(3.3e, & 3.3f) have been transformed into respective equations for aqueous sugar, aqueous black-liquor-, and aqueous caustic soda- solution by substituting the respective values of h, H, λ, and U from Appendices B and C. The respective models for sugar, black-liquor-, and caustic soda-solution are as follows:

a. Sugar Solution

$$F.X_f = L_i.X_i \qquad \ldots(3.3S1)$$

$$\begin{aligned}
&L_k(4.182 - 2.2403X_k)(\tau_k - T_r) \\
&+ (L_{k-1} - L_{i-1})[-80.345T_{i-1} - 21035.87/T_{i-1} + 2049.123T_{i-1}^{1/2} \\
&- 4213.519\ln T_{i-1} + 0.0918T_{i-1}^2 - 1.04\times10^{-4}T_{i-1}^3 + 8597.953] \\
&= L_i(4.182 - 2.2403X_i)(\tau_i - T_r) + (L_k - L_i)[4.154(T_i - T_r) \\
&+ 2.0125\times10^{-4}(T_i^2 - T_r^2) + 1.62(\tau_i - T_i) + 2.0285\times10^{-4}(\tau_i^2 - T_i^2) \\
&- 0.3747\times10^{-7}(\tau_i^3 - T_i^3) - 80.345T_i - 21035.87/T_i + 2049.123T_i^{1/2} \\
&- 4213.519\ln T_i + 0.0918T_i^2 - 1.04\times10^{-4}T_i^3 + 8597.953] \qquad \ldots(3.3S2)
\end{aligned}$$

$$\begin{aligned}
&(L_{k-1} - L_{i-1})[-80.345T_{i-1} - 21035.87/T_{i-1} + 2049.123T_{i-1}^{1/2} \\
&- 4213.519\ln T_{i-1} + 0.0918T_{i-1}^2 - 1.04\times10^{-4}T_{i-1}^3 + 8597.953] \\
&= 18.083(\tau_i/X_i)\, A_i\, (T_{i-1} - \tau_i) \qquad \ldots(3.3S3)
\end{aligned}$$

$$\tau_i = T_i + 7.20X_i - 11.50X_i^2 + 29.50X_i^3 \qquad \ldots(3.3S4)$$

b. Black-Liquor Solution

$$F.X_f = L_i.X_i \qquad \ldots(3.3B1)$$

$$\begin{aligned}
&L_k[7.53\times10^{-3}(\tau_k^2 - T_r^2)X_k - 2.25383(\tau_k - T_r)X_k + 4.182(\tau_k - T_r)] \\
&+ (L_{k-1} - L_{i-1})[-80.345T_{i-1} - 21035.87/T_{i-1} + 2049.123T_{i-1}^{1/2} \\
&- 4213.519\ln T_{i-1} + 0.0918T_{i-1}^2 - 1.04\times10^{-4}T_{i-1}^3 + 8597.953] \\
&= L_i[7.53\times10^{-3}(\tau_i^2 - T_r^2)X_i - 2.25383(\tau_i - T_r)X_i + 4.182(\tau_i - T_r)] \\
&+ (L_k - L_i)[4.154(T_i - T_r) + 2.0125\times10^{-4}(T_i^2 - T_r^2) \\
&+ 1.62(\tau_i - T_i) + 2.0285\times10^{-4}(\tau_i^2 - T_i^2) - 0.3747\times10^{-7}(\tau_i^3 - T_i^3) \\
&- 80.345T_i - 21035.87/T_i + 2049.123T_i^{1/2} - 4213.519\ln T_i \\
&+ 0.0918T_i^2 - 1.04\times10^{-4}T_i^3 + 8597.953] \qquad \ldots(3.3B2)
\end{aligned}$$

$$(L_{k-1} - L_{i-1})[-80.345T_{i-1} - 21035.87/T_{i-1} + 2049.123T_{i-1}^{1/2}$$
$$- 4213.519\ln T_{i-1} + 0.0918T_{i-1}^{2} - 1.04x10^{-4}T_{i-1}^{3} + 8597.953]$$
$$= [13.392(\tau_k + \tau_i) - 3960.0(X_k + X_i)$$
$$+ 4800.0]\, A_i\, (T_{i-1} - \tau_i) \qquad \text{...(3.3B3)}$$

$$\tau_i = T_i - 3.55X_i + 84.0X_i^2 - 107.5X_i^3 \qquad \text{...(3.3B4)}$$

c. Caustic Soda Solution

$$F.X_f = L_i.X_i \qquad \text{...(3.3C1)}$$

$$L_k[62.015 + 3.884\tau_k - 887.125X_k$$
$$+ 2316.504X_k\exp(-1.15x10^{-3}\tau_k^2)]$$
$$+ (L_{k-1} - L_{i-1})[-80.345T_{i-1} - 21035.87/T_{i-1} + 2049.123T_{i-1}^{1/2}$$
$$- 4213.519\ln T_{i-1} + 0.0918T_{i-1}^{2} - 1.04x10^{-4}T_{i-1}^{3} + 8597.953]$$
$$= L_i[62.015 + 3.884\tau_i - 887.125X_i$$
$$+ 2316.504X_i\exp(-1.15x10^{-3}\tau_i^2)]$$
$$+ (L_k - L_i)[4.154(T_i - T_r) + 2.0125x10^{-4}(T_i^2 - T_r^2)$$
$$+ 1.62(\tau_i - T_i) + 2.0285x10^{-4}(\tau_i^2 - T_i^2)$$
$$- 0.3747x10^{-7}(\tau_i^3 - T_i^3) - 80.345T_i - 21035.87/T_i$$
$$+ 2049.123T_i^{1/2} - 4213.519\ln T_i + 0.0918T_i^2$$
$$- 1.04x10^{-4}T_i^3 + 8597.953] \qquad \text{...(3.3C2)}$$

$$(L_{k-1} - l_{i-1})[-80.345T_{i-1} - 21035.87/T_{i-1} + 2049.123T_{i-1}^{1/2}$$
$$- 4213.519\ln T_{i-1} + 0.0918T_{i-1}^{2} - 1.04x10^{-4}T_{i-1}^{3} + 8597.953]$$
$$= 977.66[\tau_i/X_i]^{0.2823}\, A_i\, (T_{i-1} - \tau_i) \qquad \text{...(3.3C3)}$$

$$\tau_i = T_i + (0.142T_i - 9.42).X_i + 271.363X_i^2 \qquad \text{...(3.3C4)}$$

Models represented by Eqs (3.3S1--3.3S4), Eqs(3.3B1--3.3B4), and Eqs(3.3C1--3.3C4) are to be employed for the calculations of self-balancing variables of sugar-, black-liquor, and caustic soda- evaporators, respectively from the knowledge of operating- and geometrical-variables in an N-effect mixed feed evaporator.

STEAM ECONOMY

The above models, developed in Sections 3.2.1 through 3.2.3 have been solved in the following Chapter for the determination of temperature, concentration, and quantities of various streams entering and leaving each effect of the evaporator. This, in turn, will lead to the calculation of steam economy which is a barometer of energy conservation in an evaporator.

CHAPTER - 4

SOLUTION OF THE MATHEMATICAL MODEL

This Chapter presents the solution of the mathematical models of multiple effect evaporators for forward, backward-, and mixed- feed arrangements dealing with sugar, black- liquor, and caustic soda solutions.

4.1 METHOD OF SOLUTION

The models, developed in Chapter-3, for the calculation of self-balancing variables have simultaneous nonlinear algebraic equations. To solve them, the Newton-Raphson method [C4] has been employed. In fact, it linearises the nonlinear terms by means of Taylor series expansion. The resulting expressions, in turn, are transformed into a matrix equation which is as follows:

$$J_p \ \Delta y_p = - g_p \qquad \ldots(4.1)$$

Where subscript, p (= 1, 2, 3, . . ., 4N) refers to the number of unknown variables.

$$J_p = \begin{bmatrix} \dfrac{\partial g_1}{\partial y_1} & \dfrac{\partial g_1}{\partial y_2} & \cdots & \dfrac{\partial g_1}{\partial y_{4N}} \\[2ex] \dfrac{\partial g_2}{\partial y_1} & \dfrac{\partial g_2}{\partial y_2} & \cdots & \dfrac{\partial g_2}{\partial y_{4N}} \\ \cdot & \cdot \quad \cdot \quad \cdot & & \cdot \\ \cdot & \cdot \quad \cdot \quad \cdot & & \cdot \\ \cdot & \cdot \quad \cdot \quad \cdot & & \cdot \\ \dfrac{\partial g_{4N}}{\partial y_1} & \dfrac{\partial g_{4N}}{\partial y_2} & \cdots & \dfrac{\partial g_{4N}}{\partial y_{4N}} \end{bmatrix}$$

$$\Delta y_p = [\Delta y_1 \ \Delta y_2 \ . \ . \ . \ \Delta y_{4N}]$$

and

$$g_p = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ \cdot \\ \cdot \\ g_{4N} \end{bmatrix}$$

Here Δy_p refers to the value of increment in the variable y_p. Matrix J_p is known as Jacobian.

For the convergence of the Newton-Raphson method, the variables contained in the models have been scaled down to obtain their dimensionless forms. Accordingly, following dimensionless variables have been used:

(1) Dimensionless flow rate of aqueous solution
= flow rate of aqueous solution in individual effect / feed rate
$l = L / F$

(2) Dimensionless steam flow rate
= steam flow rate / feed rate
$v_s = V_s / F$

(3) Dimensionless temperature of aqueous solution
= temperature of aqueous solution in individual effect/steam temperature
$u = \tau / T_s$

(4) Dimensionless saturation temperature of vapour
= saturation temperature of vapour in individual effect / steam temperature
$w = T / T_s$

Using above dimensionless variables, the functional relationships for each effect of a multiple effect evaporator are determined for sugar, black-liquor, and caustic-soda evaporators having a given feed arrangement. Accordingly, four functional relationships corresponding to each effect of an N-effect evaporator are obtained. Hence total number of functional relationships for N-effect evaporator becomes 4N. These functional relationships, so obtained, form a matrix, g_p. To obtain the elements of Jacobian J, partial differentiation of each functional relationship with respect to self-balancing variables is carried out.

Substitution of g_p and J in Eq(4.1) provides a set of 4N simultaneous linear equations. Now L - U decomposition method is used to solve the set of equations and thus self-balancing variables are computed. The flow chart of the computational procedure used for the solution of a model is shown in Figure 4.1.

To initiate the computations, it is necessary to calculate initial guess values of self-balancing variables. Following Section describes the methodology for the selection of initial guess values of self-balancing variables:

INITIAL GUESS VALUES

Based on the assumption of equal evaporation in each effect of a multiple effect evaporator, preliminary calculations of all the self-balancing variables are carried out. These values serve as initial guess values to initiate the iterative computations.

It is important to point out that initial guess values of self-balancing variables alter with the change of any operating variables for a given feed arrangement and specified geometrical variables. Therefore, a new set of initial guess values of self-balancing variables should be computed for each set of operating variables of sugar, black-liquor, and caustic soda evaporators for a known feed arrangement and heat transfer area of individual effects. Table 4.1 provides a typical set of initial guess values of self-balancing variables for the analysis of a quadruple effect sugar evaporator having forward feed arrangement.

Table 4.1 A set of initial guess values for a quadruple effect sugar forward feed evaporator

Effects	Flow rate of steam, ton/hr	Flow rate of liquid ton/hr	Concentration, °Bx	Temperature of liquid, °C	Saturation temperature of vapour, °C
I	13.334	58.333	21.60	104.91	103.59
II	--	46.667	27.00	97.56	95.87
III	--	35.000	36.00	86.74	84.26
IV	--	23.333	54.00	66.04	--

This set of initial guess values is for the operating- and geometrical- variables, given in Table 4.2. Calculations are performed on a DEC-2050 computer. Value of a self-balancing variable is taken to be converged when a deviation between two successive values of a variable is found to be of the order of one tenth of a micro unit.

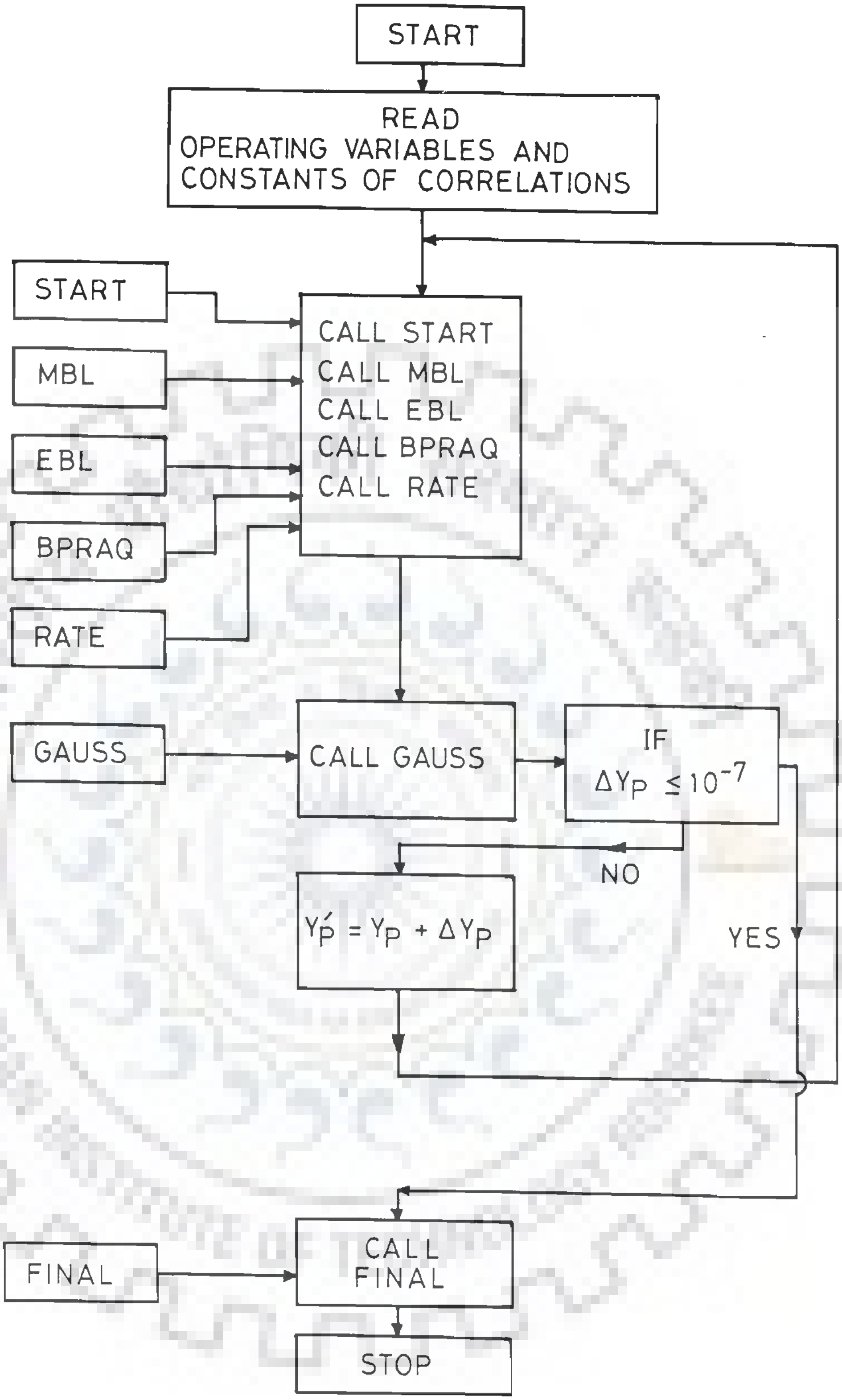

FIG. 4.1 FLOW CHART OF COMPUTATION PROCEDURE

Table 4.2 Operating- and geometrical- variables for a quadruple effect forward feed sugar evaporator

Feed rate, F	=	70.263 ton/hr
Feed concentration, X_f	=	18.00°Bx
Feed temperature, τ_f	=	100.00°C
Steam temperature T_0	=	110.00°C
Saturation temperature of last effect, T_4	=	55.00°C
Heating surface area of each effect	=	665.00 m^2

Following the above computational procedure, solutions were obtained for sugar quadruple effect evaporator, black-liquor quintuple effect evaporator, and caustic soda triple effect evaporator having forward, backward, and mixed feeds.

Appendix D describes various functional relationships, g_p of a quadruple effect sugar evaporator having forward feed arrangement. These relationships are obtained by substituting the dimensionless variables into equations, Eqs (3.1S1, 3.1S2, 3.1S3, and 3.1S4) for individual effects of a quadruple effect sugar evaporator. This Appendix also contains the elements of Jacobian.

Similar functional relationships and elements of Jacobian are derived for quadruple effect sugar evaporator having backward-, and mixed- feed arrangements. Likewise, similar relationships and Jacobian elements are obtained for a quintuple effect black-liquor evaporator having forward, backward, and mixed feeds; and also for a triple effect caustic-soda evaporator having different feed arrangements. However, they have not been included in this thesis due to their similarity with those reported in Appendix-D.

The solution of model provides the values of self-balancing variables, viz; flow rate of steam; flow rate, temperature, and concentration of liquid streams; and saturation temperature of vapour streams from each effect for a given set of operating- and geometrical- variables in an N-effect evaporator. It can also determine a change in the value of a self-balancing variable if any of the operating variables undergoes a change from its pre-set value. Thus, it provides a method to evaluate steam economy of an evaporator.

4.2 LIMITATIONS OF THE MODEL

Above models have been developed on the basis of assumptions listed in Chapter-3. It also does not cover the situations where splitting of streams, flashing, vapor bleeding and other energy conservation measures in an evaporator occur. Therefore, it is recommended that before undertaking the use of models, the case must be examined for any deviation from the above constraints. If any deviation is found to exist the model should be suitably modified.

CHAPTER-5

TESTING OF THE MODEL

This Chapter examines the validity of the mathematical models, Eqs(3.1S1 -- 3.1S4, 3.2S1 -- 3.2S4, 3.3S1 -- 3.3S4) for sugar; Eqs(3.1B1 -- 3.1B4, 3.2B1 -- 3.2B4, 3.3B1 -- 3.3B4) for black-liquor; and Eqs(3.1C1 -- 3.1C4, 3.2C1 -- 3.2C4, 3.3C1 -- 3.3C4) for caustic-soda to the plant values of existing evaporators employed in respective industries.

As regards sugar evaporator, the geometrical and operating variables of a typical Indian sugar industry have been employed. These variables are given in Table 5.1. This industry employs quadruple effect forward feed evaporators.

Table 5.1 **Typical industrial data for a sugar quadruple effect forward feed evaporator**

Variables	Feed	Steam	Liquid/Vapour from effects I	II	III	IV
Flow rate, ton/hr	70.263*	--	63.079	46.669	34.182	24.563
Concentration, °Bx	18.00*	--	20.05	27.10	37.00	51.50
Temperature, °C	100.00*	110.00*	103.60	93.90	82.9	57.80
Saturation temperature, °C	--	--	103.00	92.90	80.90	55.50*
Heating surface area, m^2	--	--	696.77**	557.42**	557.42**	557.42**

* *Operating variables*

** *Geometrical variables*

For typical Indian black-liquor evaporator, geometrical-, and operating - variables are from a typical paper industry using quintuple effect evaporator having mixed feed arrangement. In this system feed enters in the third effect. The variables are described in Table 5.2.

Table 5.2 Typical industrial data for a black-liquor quintuple effect mixed feed evaporator

Variables	Feed	Steam	Liquid/Vapour from effects				
			I	II	III	IV	V
Flow rate, ton/hr	70.583*	--	32.350	40.864	59.724	51.760	43.134
Concentration, °Tw	22.00*	--	48.00	38.00	26.00	30.00	36.00
Temperature, °C	90.00*	135.00*	117.70	98.60	91.10	79.50	60.60
Saturation temperature, °C	--	--	111.90	93.70	88.20	75.90	56.00*
Heating surface area, m^2	--	--	371.43**	371.43**	371.43**	405.09**	405.09**

* *Operating variables*
** *Geometrical variables*

The typical geometrical-, and operating- variables for caustic-soda evaporator have been taken from an Indian chemical industry as given in Table 5.3. These values are for a triple effect evaporator having mixed feed arrangement.

Using the above values of operating-, and geometrical- variables in respective mathematical models, solutions were obtained following the procedure as described in Section 4.1 of Chapter-4 to determine the predicted values of the self-balancing variables for the evaporator of aqueous solutions of sugar, black-liquor, and caustic soda. The so-called values of self-balancing variables are compared with plant values of self-balancing variables listed in respective Tables 5.1 through 5.3. The self-balancing variables include: liquid concentration, liquid temperature, liquid flow rate, and saturation temperature of the vapour from individual effects of the evaporator.

5.1 COMPARISON BETWEEN PLANT- AND PREDICTED- SOLUTE CONCENTRATION OF AQUEOUS SOLUTION FROM INDIVIDUAL EFFECTS OF AN EVAPORATOR

Figure 5.1 represents a plot between plant- and predicted- solute concentration for sugar, black-liquor, and caustic soda solutions from each effect of an evaporator. From this plot it is evidently noted that all the data points lie around a 45° straight line with a maximum scatter of ±10%. This means that the predictions due to models are in excellent agreement with those from the plant values. However, at higher concentrations, predictions from the model seems to be larger than plant values.

Table 5.3 Typical industrial data for a caustic-soda triple effect mixed feed evaporator

Variables	Feed	Steam	Liquid/Vapour from effects		
			I	II	III
Flow rate, ton/hr	13.091*	--	2.846	8.727	5.951
Concentration, %	10.00*	--	46.00	15.00	22.00
Temperature, °C	95.00*	155.00*	145.70	90.00	58.00
Saturation temperature, °C	--	--	100.00	88.00	42.00*
Heating surface area, m^2	--	--	85.340**	77.450**	69.740**

* *Operating variables*

** *Geometrical variables*

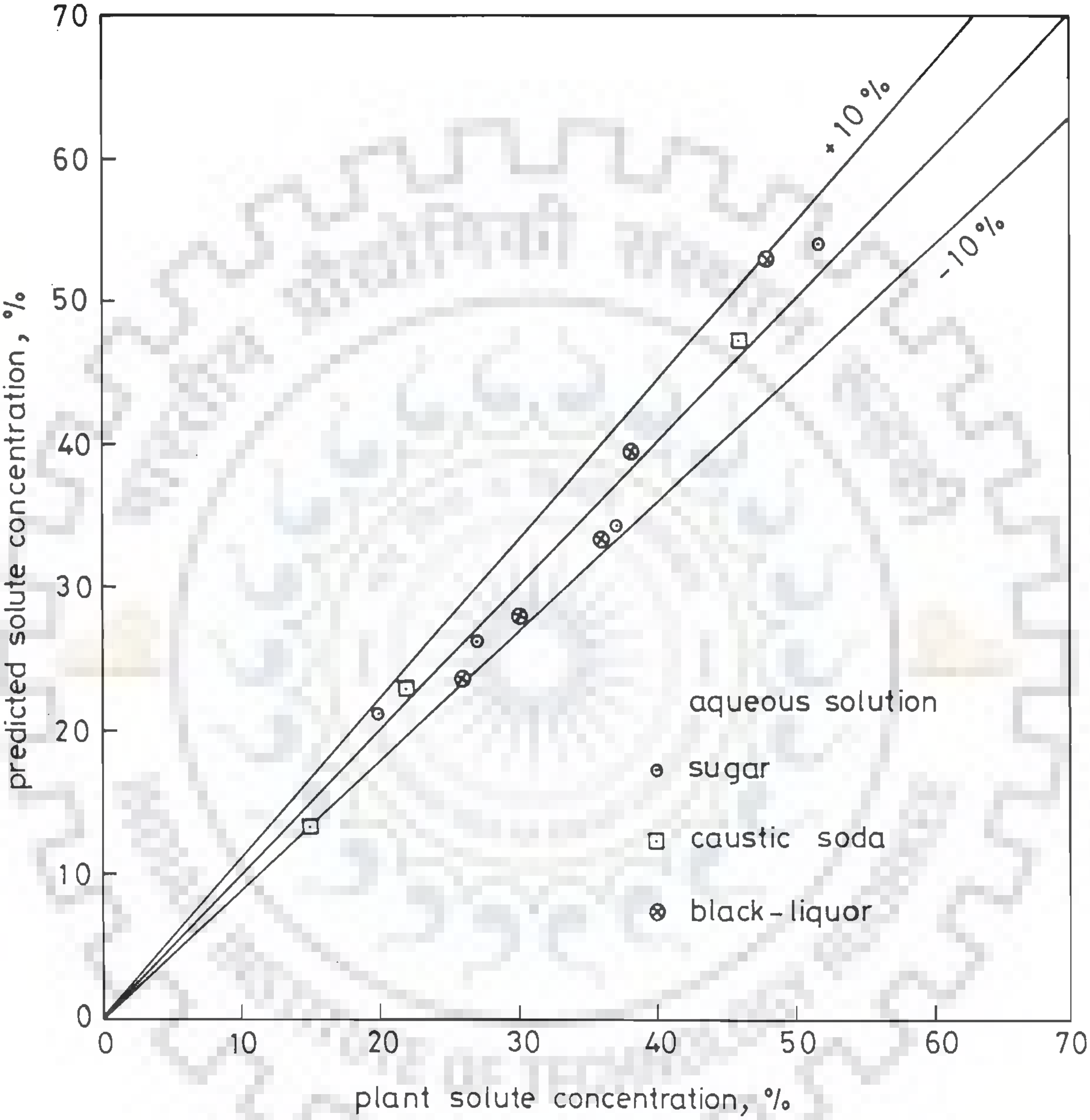

Fig. 5.1 Comparison between plant and predicted solute concentration of various aqueous solutions

5.2 COMPARISON BETWEEN PLANT- AND PREDICTED- TEMPERATURES OF LIQUID FROM INDIVIDUAL EFFECTS OF AN EVAPORATOR

Figure 5.2 is a typical plot between plant- and predicted- temperature of liquid from each effect of sugar, black-liquor, and caustic soda evaporators. This plot reveals that the predicted values compare excellently with the plant values having a maximum deviation of ±10%. The features of this plot resembles with those of Figure 5.1.

5.3 COMPARISON BETWEEN PLANT- AND PREDICTED- FLOW RATES OF LIQUID STREAM FROM INDIVIDUAL EFFECTS OF AN EVAPORATOR

Figure 5.3 shows a comparison between the plant- and predicted- flow rates of various aqueous salt solutions from each effect of the evaporator. This plot has essentially the same characteristic features as that of the preceding Figure, i.e. the predictions due to model agree excellently with the plant values with a maximum deviation of ±10%.

5.4 COMPARISON BETWEEN PLANT- AND PREDICTED- SATURATION TEMPERATURE OF VAPOUR FROM INDIVIDUAL EFFECTS OF AN EVAPORATOR

Figure 5.4 represents a typical plot between plant- and predicted- saturation temperature of vapour leaving each effect of an evaporator for sugar, black- liquor, and caustic soda solutions. This plot indicates that the data of black- liquor lie on a 45° straight line implying an excellent agreement between the predictions and plant values. However, the predicted temperatures of vapour for aqueous sugar solution are somewhat larger than the plant values with a maximum deviation of +6%. Further, in the case of caustic soda solution the predictions are lower than the plant values. However, the maximum deviation between the two values is not significantly large.

From the above, it is inferred that the present models have succeeded to determine the performance of an N-effect evaporator for the concentration of aqueous salt solutions with a reasonable accuracy. Hence, they can be employed to compute the values of self-balancing variables for a given set of operating- and geometrical- variables. This, in turn, will enable the evaluation of steam economy and end-product concentration of sugar-, black-liquor-, and caustic soda- evaporators. Besides, it will also facilitate the determination of the values of operating variables which corresponds to improved steam economy of the evaporator in question and thereby a substantial reduction in the consumption of steam.

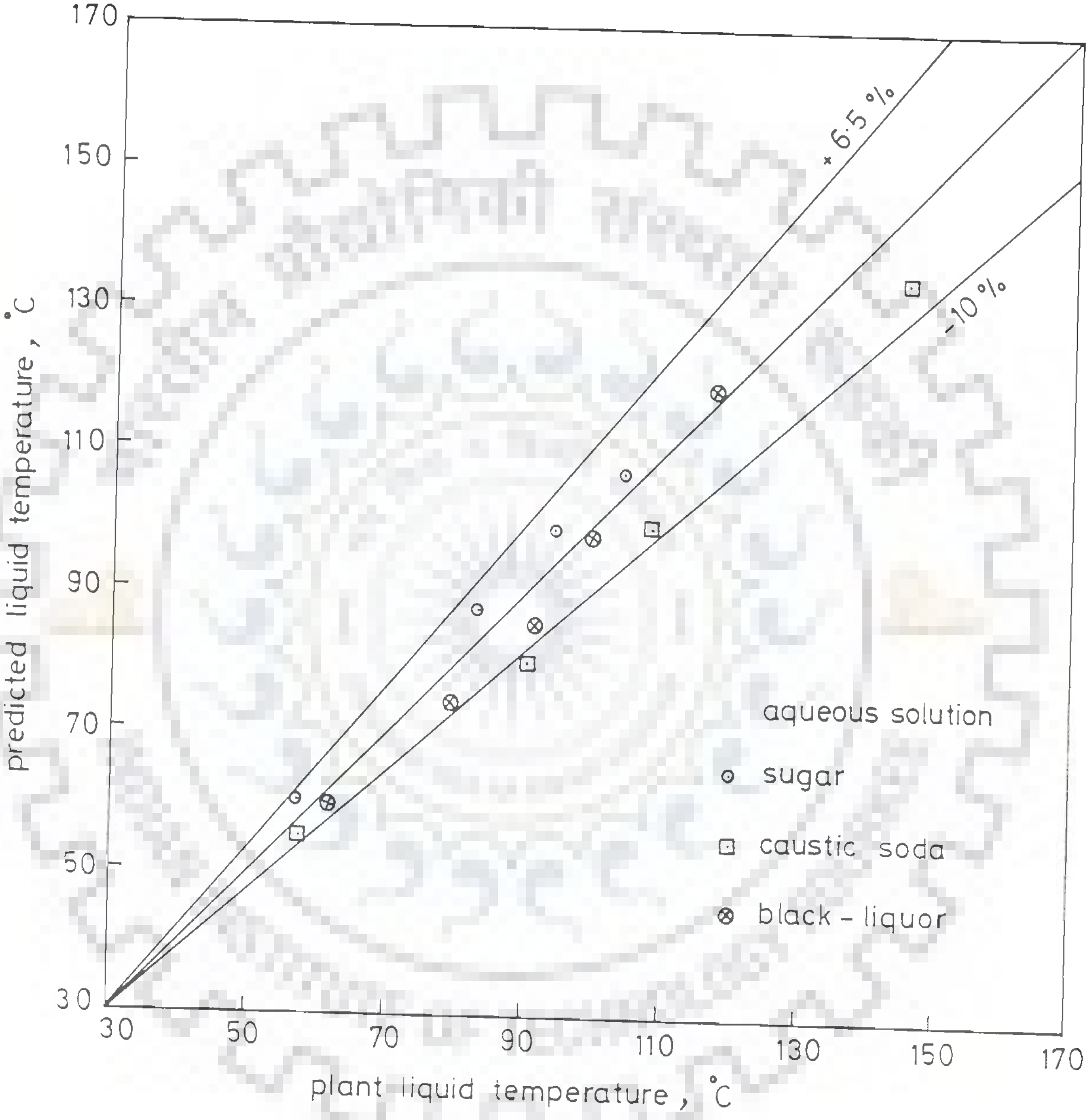

Fig. 5.2 Comparison between plant and predicted temperatures of various aqueous solutions

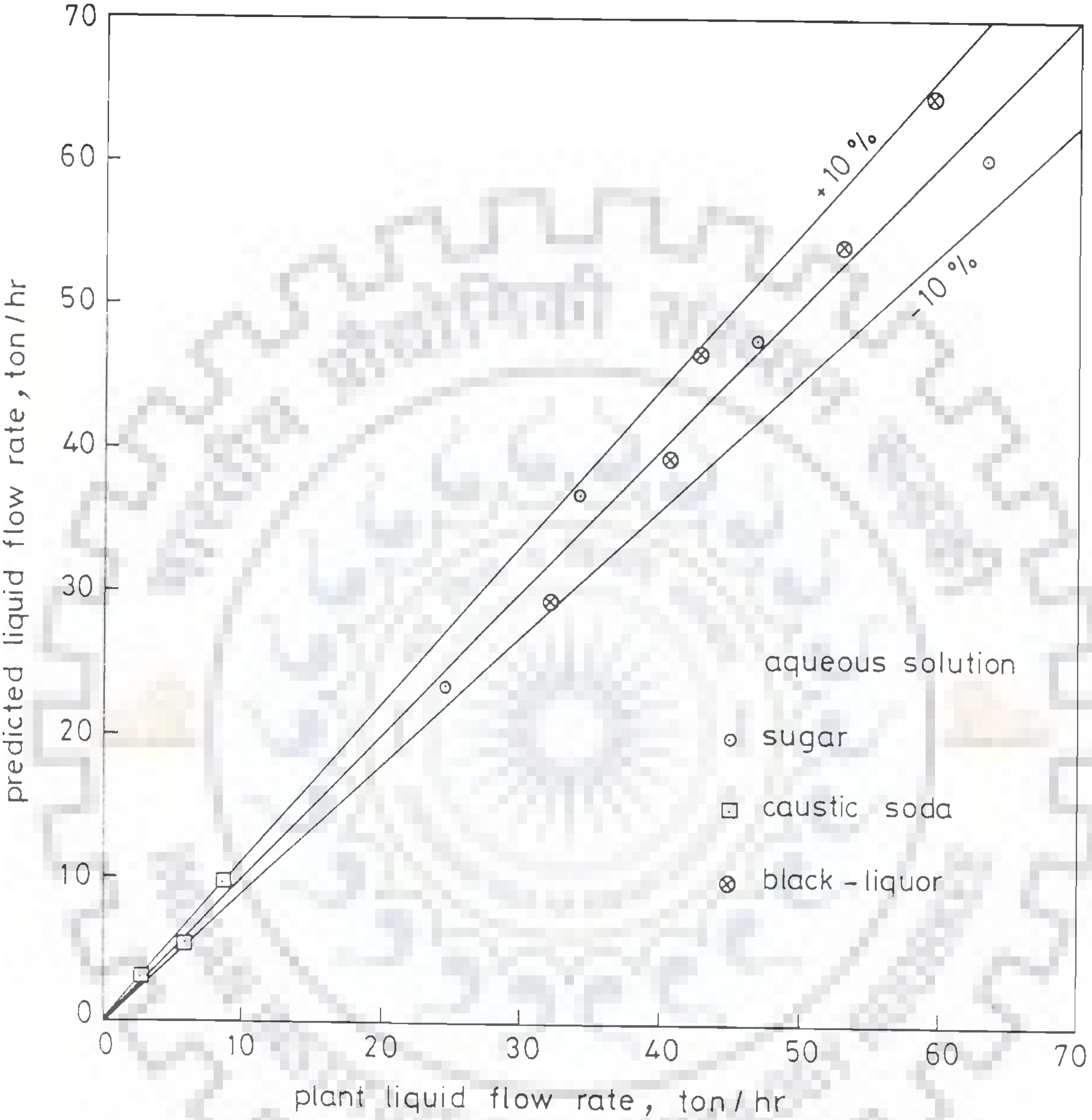

Fig. 5.3 Comparison between plant and predicted liquid flow rates of various aqueous solutions

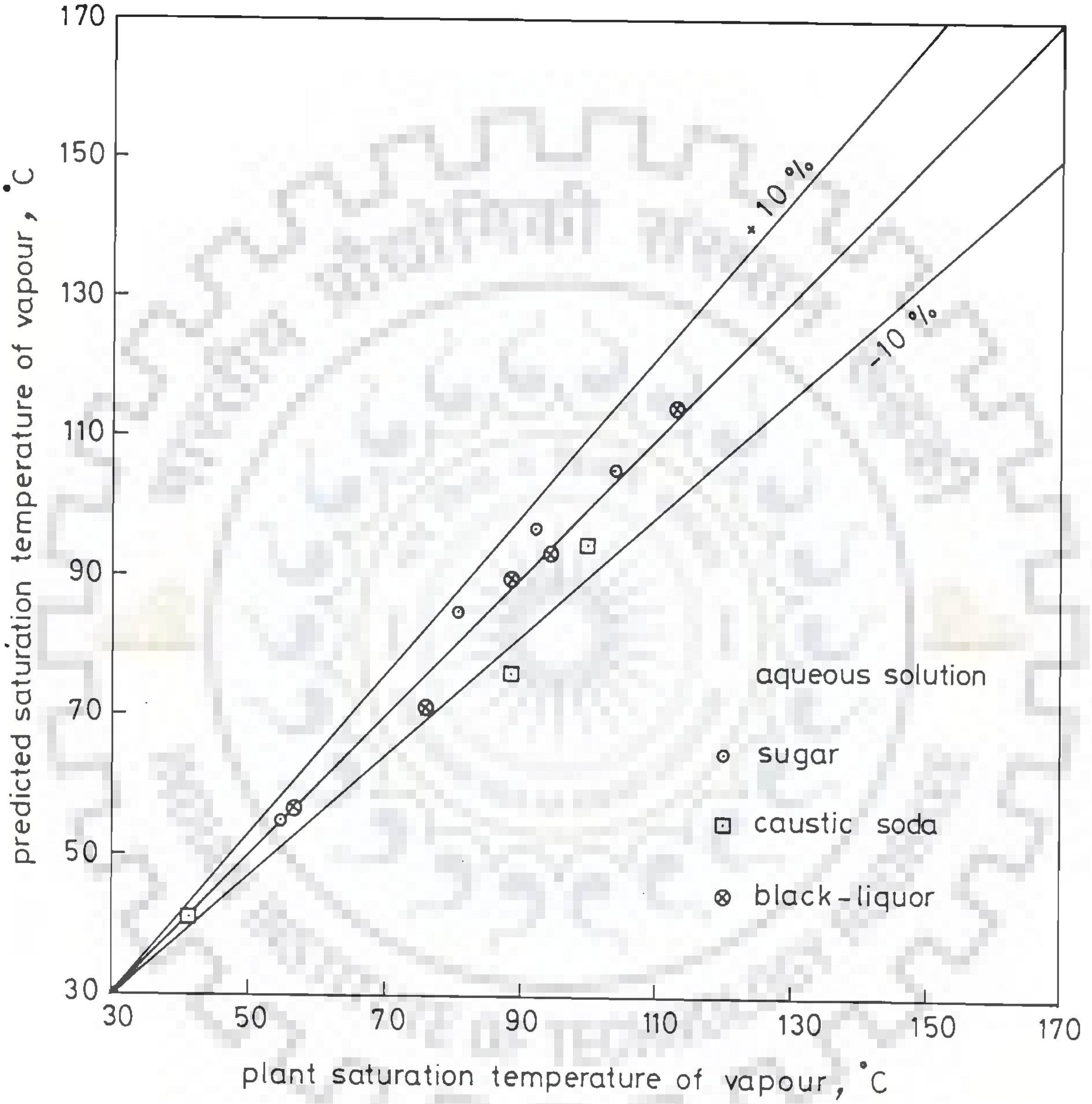

Fig. 5.4 Comparison between plant and predicted saturation temperatures of vapour of various aqueous solutions

CHAPTER-6

RESULTS AND DISCUSSION

This Chapter describes the effect of operating variables on steam economy and also end-product concentration of various aqueous solutions in a multiple effect evaporator. It also discusses a procedure to meet the situation that might arise due to a change in any of the operating variables so that end-product concentration is maintained at a predetermined value without adversely affecting the performance of the evaporator. This, in a way, ensures uninterrupted evaporation of the aqueous solution with the highest steam economy and thereby the conservation of energy in the evaporator. It is important to mention that the values of operating variable are selected in such a way that they have industrial relevance. In following Section the ranges of operating variables are mentioned:

6.1 RANGE OF OPERATING VARIABLES

The upper- and lower- limits of operating variables of evaporators for a given aqueous solution are decided to meet the technological requirements of the industry. The values of operating variables for the aqueous solutions of sugar, black-liquor, and caustic soda are given in Table 6.1. Quadruple, quintuple, and triple effect evaporators have been used for the evaporation of sugar, black-liquor, and caustic soda solutions, respectively. The heating surface area of each effect of sugar-, black-liquor-, and caustic soda- evaporator has been 665, 350, and 95 m^2, respectively. Besides the operating variables of Table 6.1, feed temperature of 95°C and pressure in the last effect of 16.577 kPa have been used to show the effect of other operating variables on steam economy and end- product concentration of caustic soda and black-liquor evaporator, respectively.

Steam economy of the evaporator is calculated by dividing the summation of vapour formed in individual effects with the steam consumption. The values of steam economy and the concentration of the end-product from multiple effect evaporators for the concentration of aqueous sugar, black-liquor, and caustic soda solutions under forward-, backward-, and mixed-feed arrangements for various values of the operating variables are given in Appendix-E.

6.2 STEAM ECONOMY OF AN EVAPORATOR

Steam economy is an important quantity of direct relevance to the steam consumption and thereby energy conservation in evaporators. Its value differs from plant to plant depending upon type, and size of the evaporator, physical properties of aqueous solutions, feed arrangement, feed temperature, feed concentration, feed rate, pressure in the last effect, and steam pressure. As a matter of fact, a change in the value of any one of these variables is likely to affect the rate of vapour formation and the steam consumption and thereby steam economy of the

Table 6.1 RANGE OF OPERATING VARIABLES

Variable	Sugar	Black-liquor	Caustic soda
Feed temperature,°C	70	60	60
	80	70	70
	90	80	80
	100	90	90
	110	100	100
		110	110
Feed concentration	12 °Bx	16 $^{\circ}T_w$	8%
	14	18	10
	16	20	12
	18	22	14
	20	24	16
		26	18
Feed rate, ton/hr	60	60	15
	70	65	20
	80	70	25
	90	75	30
	100	80	35
	110	85	40
Steam temperature (pressure), °C (kPa)	105.0 (120.80)	125 (232.0)	155 (543.2)
	107.5 (132.00)	130 (270.0)	160 (618.1)
	110.0 (143.27)	135 (313.1)	165 (700.8)
	112.5 (156.20)	140 (362.0)	170 (792.0)
	115.0 (169.10)	145 (416.0)	175 (892.4)
Saturation temperature (pressure) of last effect, °C (kPa)	45 (9.58)	45 (9.58)	30 (4.24)
	50 (12.34)	50 (12.34)	35 (5.226)
	55 (15.74)	55 (15.74)	40 (7.38)
	60 (19.92)	60 (19.92)	45 (9.58)
	65 (25.01)	65 (25.01)	50 (12.34)
		70 (31.24)	55 (15.74)

evaporator. Following Sub-sections have been devoted to discuss the effect of operating variables on steam economy of a quadruple effect sugar evaporator, quintuple effect black-liquor, and triple effect caustic soda evaporator under forward-, backward-, and mixed-feed arrangements:

6.2.1 VARIATION OF STEAM ECONOMY WITH FEED TEMPERATURE

Figure 6.1 is a typical plot showing the variation of steam economy of a quadruple effect sugar evaporator as a function of feed temperature for forward-, backward-, and mixed-feed arrangement. This plot is for feed concentration, 18 °Bx; feed rate, 70 ton/hr ,pressure in the last effect, 15.74 kPa; and steam pressure, 143.27 kPa.

An inspection of the plot reveals the following salient features:

(i) Steam economy of a quadruple effect sugar evaporator increases continuously with the increase in feed temperature, irrespective of feed arrangement.

(ii) The curves, representing various feed arrangements intersect each other at points a, b, and c corresponding to the feed temperatures of 81.5°C, 83°C, and 85.5°C, respectively, implying that for temperature smaller than 81.5°C the backward feed arrangement offers the highest steam economy, followed by mixed-, and forward- feed arrangement in decreasing order. When feed temperature exceeds 85.5° C, the trend is reversed and the steam economy decreases as the feed arrangement shifts from forward to mixed to backward. However, for feed having temperature between 81.5°C and 83° C, the order of feed arrangement for the decrease of steam economy becomes mixed to backward to forward. Further, for the feed of temperature lying between 83°C and 85.5°C, the steam economy improves as the feed arrangement changes from backward to forward to mixed feed.

(iii) For a feed at 100°C, which is generally practiced in evaporators employed in Indian sugar Mills, the forward feed arrangement offers the steam economy 3.84% and 11.76% higher than those of mixed- and backward- feed arrangement, respectively. It further increases at feed temperature exceeding 100°C. Thus, the use of forward feed arrangement is advantageous and recommended. This, indeed, corroborates the industrial practice of using forward feed for the evaporation of aqueous sugar solution in India.

Possible explanation for the above features are as follows: Evaporation can take place only when the temperature of feed is either equal or greater than saturation temperature. For the feed having temperature less than saturation temperature, the amount of preheat required to raise it to saturation temperature is proportional to $\Delta T (= T_s - \tau_f)$. An increase in the feed temperature, τ_f reduces the value of preheat load and thereby steam economy increases. When feed temperature exceeds the saturation temperature, no preheating is needed. Instead, flashing results in the formation of vapour without any consumption of steam.

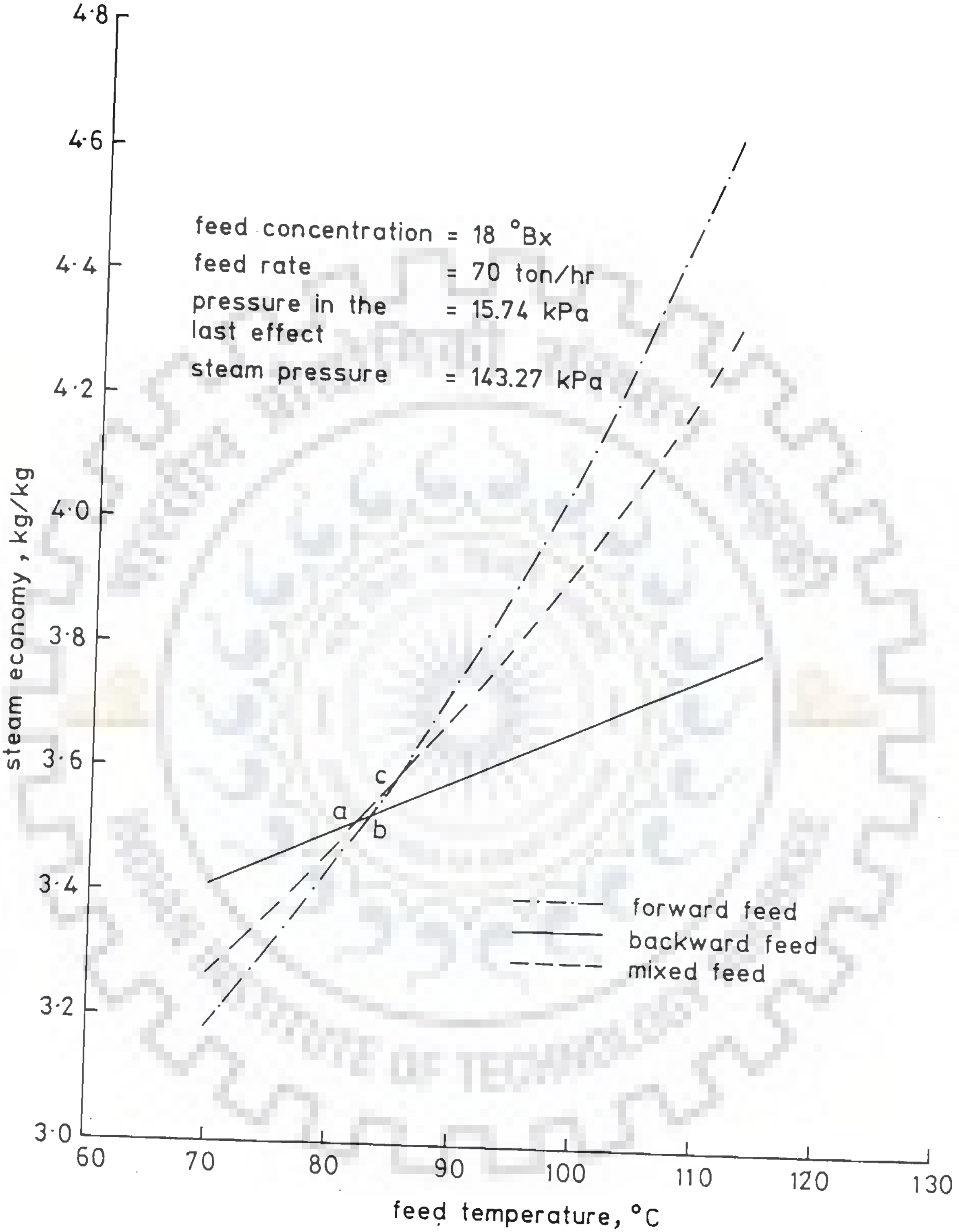

Fig. 6.1 Steam economy versus feed temperature for a quadruple effect sugar evaporator

Raising the temperature of the feed beyond saturation temperature leads to increased flashing and thereby, steam economy of an evaporator is found to be improved. This holds true irrespective of the feed arrangement employed in the evaporator. However, the rate of increase in steam economy depends upon the feed arrangement. In forward feed arrangement, an increase in feed temperature directly contributes to increase the amount of vapour in the first effect which are subsequently used in the following effects to produce the increased amount of vapour. Therefore, steam economy of forward feed evaporator rises sharply with feed temperature. As regards backward feed, increase in steam economy with feed temperature is at a slow pace owing to the fact that vapour of the last effect are not at all being utilized in evaporation process. The rate of increase of steam economy with feed temperature for mixed feed arrangement lies between that of forward-, and backward- feed as it is a combination of the two.

It is important to mention that for a feed of given temperature which is less than the saturation temperature, the heat load of the evaporator decreases continuously as the feed arrangement is shifted from forward to mixed to backward. Therefore, backward feed arrangement provides the highest steam economy to be followed by mixed and forward feed arrangements in decreasing order. Further, it is also seen that decrease in heat load of the evaporator is large when feed arrangement is changed from forward to mixed as compared to that in the change-over from mixed to backward. That is why steam economy of mixed feed evaporator is found to be substantially larger than that of the forward feed evaporator and smaller than that of backward feed evaporator. In other words, the difference between steam economies of backward and mixed feed evaporator is small in comparison to that between mixed- and forward- feed evaporator.

From the above, it is clear that on raising feed temperature the difference amongst the steam economies of the three types of feed evaporators goes on decreasing. It continues upto a temperature of the feed at which steam economy of the backward feed evaporator is equal to that of mixed feed evaporator. This temperature, in the present investigation, corresponds to 81.5°C. Further increase in feed temperature causes the difference between steam economies of backward-, and mixed- feed evaporators to increase. This continues upto a temperature at which the difference between steam economies of forward and backward feed evaporators disappear. However, the steam economy of mixed feed remains to be higher than either of the two arrangements. Such a temperature corresponds to 83° C. Still further increase in the feed temperature leads to widen the difference between steam economies of forward and backward feeds, but that between forward and mixed feeds continues to decrease. Ultimately, at a certain temperature the steam economy of forward feed becomes equal to mixed feed evaporator. This temperature corresponds to 85.5° C. Any increase in feed temperature beyond 85.5° C results to increase the difference amongst the steam economies of all the three evaporators.

Thus, it can be inferred that for the feed of aqueous sugar solution having temperature less than 81.5° C, backward feed provides the highest steam economy, to be followed by mixed and forward feed in decreasing order. For the feed of temperature ranging between 81.5°C to 83°C, the sequence of feed arrangement in which steam economy decreases is mixed to backward to forward; and for feed having temperature from 83°C to 85.5°C, steam economy decreases as feed arrangement is shifted from mixed to forward to backward. Thereafter, any increase in the feed temperature causes the steam economy to decrease when the sequence of feed arrangement is forward to mixed to backward.

Figures 6.2 & 6.3 are the similar plots showing the variation of the steam economy with feed temperature for forward-, backward-, and mixed- feed arrangement for the evaporation of aqueous solutions of black-liquor in quintuple- and caustic soda in triple- effect evaporator, respectively. They have been found to possess essentially the same characteristic behaviour as that observed in the evaporation of aqueous sugar solution except that the temperatures at which the curves representing various feed arrangements intersect each other, are different. This behaviour is an expected one in view of the differing physico-thermal properties of the aqueous solution involved, and the set of operating variables used.

Table 6.2 provides the range of feed temperature for which multiple effect evaporator operates at its highest steam economy for a given aqueous solution and a feed arrangement.

Table 6.2 Range of the feed temperature for the highest steam economy as a function of feed arrangement and aqueous solution

Aqueous solution	Feed temperature,°C		
	Forward feed	Backward feed	Mixed feed
Sugar	> 85.5	< 81.5	81.5 -- 85.5
Black-liquor	> 71.5	< 71.5	--
Caustic soda	> 73.0	< 73.0	--

Table 6.2 can be used to select feed temperature for the highest steam economy of an evaporator having a specified feed arrangement and aqueous solution. Alternatively, feed arrangement can also be selected for the evaporation of an aqueous solution of given temperature so as to operate it with the highest steam economy. This will help to adjust the feed temperature in existing evaporators so that the operation be carried out for the highest steam economy and the energy is conserved.

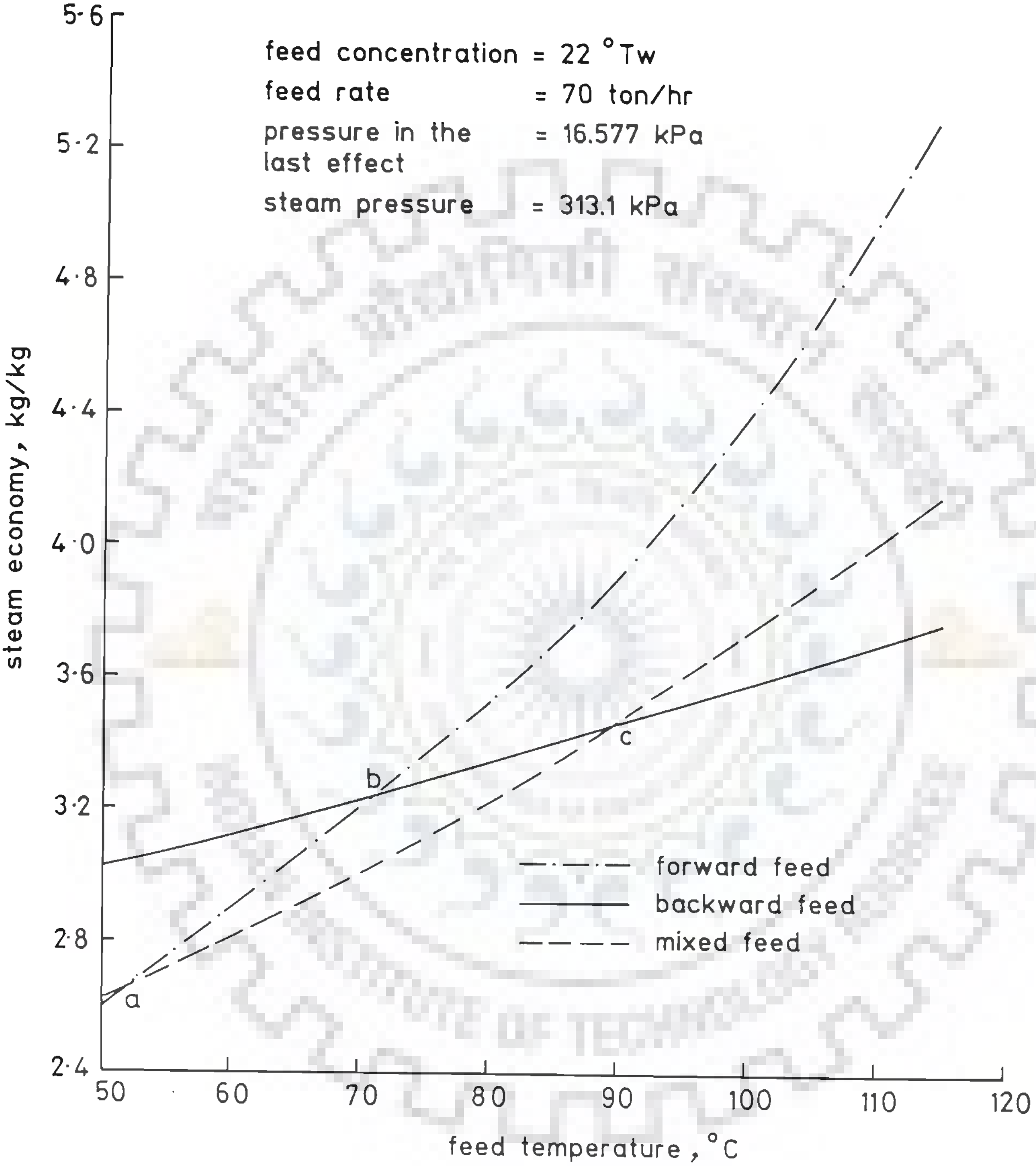

Fig. 6.2 Steam economy versus feed temperature for a quintuple effect black-liquor evaporator

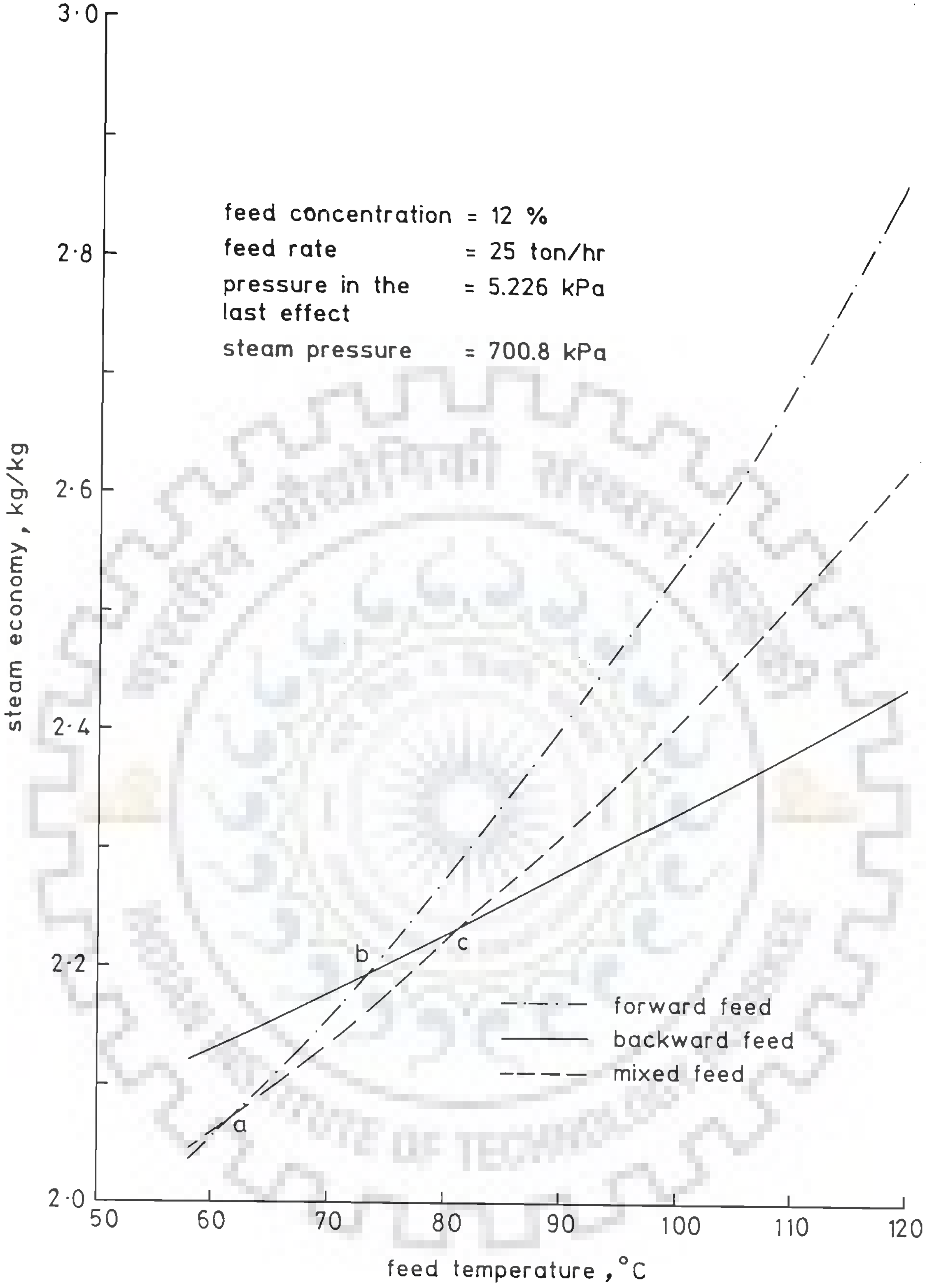

Fig. 6.3 Steam economy versus feed temperature for a triple effect caustic soda evaporator

6.2.2 VARIATION OF STEAM ECONOMY WITH FEED CONCENTRATION

Figure 6.4 represents the variation of steam economy of a quadruple effect sugar evaporator as a function of feed concentration for forward-, backward-, and mixed-feed arrangements. This plot is for feed temperature, 100°C; feed rate, 70 ton/hr; pressure in the last effect, 15.74 kPa; and steam pressure, 143.27 kPa.

An examination of this plot leads to the following features:

(i) An increase in feed concentration decreases the steam economy for mixed-, and backward- feed evaporator, while increases that for forward feed evaporator.

(ii) The curves for forward-, and mixed- feed intersect each other at a point corresponding to a feed concentration of 14.5°Bx. However, backward feed curve lies lower than those for other two feed. This implies that for a feed having a concentration lower than 14.5°Bx, mixed feed arrangement offers the highest steam economy to be followed by forward and backward feeds in decreasing order. But for a feed of concentration higher than 14.5°Bx, the trend is opposite i.e. the steam economy decreases when feed arrangement is shifted from forward to mixed to backward.

The above features are due to the following reasons:

An increase in feed concentration leads to two pronounced effects in the evaporator -- the reduction in water load and in the heat load of the evaporator. Heat load is directly related to heat transfer coefficient and temperature gradient, ΔT. An increase in feed concentration raises the saturation temperature of the solution, and thereby decreases the value of ΔT. It also reduces the value of heat transfer coefficient because of the inverse relationship between the two. Therefore, increase in feed concentration is directly responsible to decrease the heat load as well as the water load in the evaporator. This, in turn, decreases the vapour formation in various effects of the evaporator. In fact, reduction in the heat load is also responsible to less consumption of steam. Thus, the net result of increasing the concentration of feed is the decrease in vapour formation as well as steam consumption in the evaporator. However, the magnitude of vapour formation and steam consumption depends upon the feed arrangement. For forward feed evaporator, steam consumption is directly related to water load in the feed. Hence, decrease in steam consumption is more pronounced as compared to vapour formation. Consequently, steam economy for a forward feed evaporator is found to increase with feed concentration. In backward feed evaporator decrease in water load in the solution of the first effect is not much. So the steam consumption in backward feed evaporator does not decrease with the same rate as vapour formation. As a result of it, steam economy decreases with feed concentration. In mixed feed evaporator also, decrease

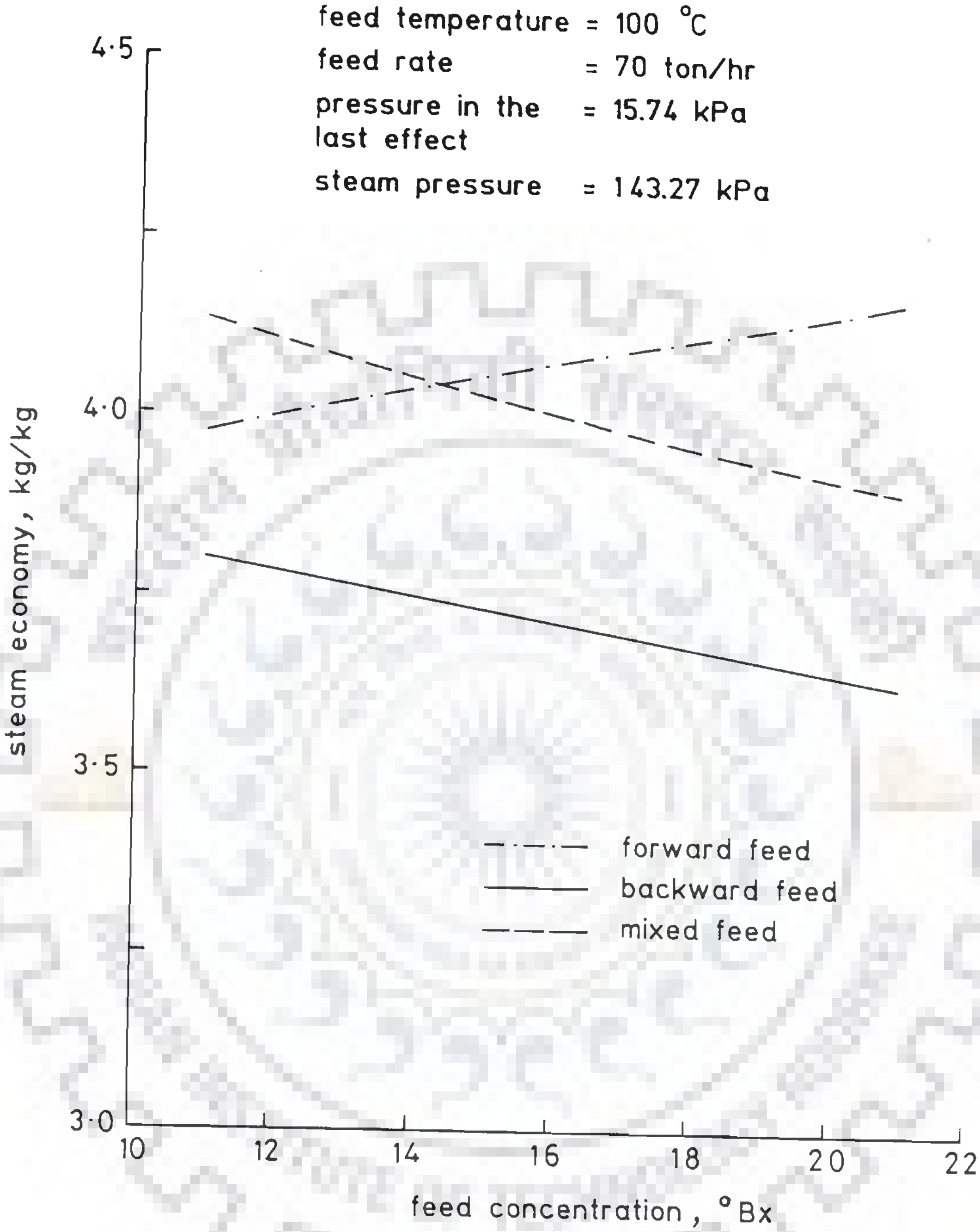

Fig. 6.4 Steam economy versus feed concentration for a quadruple effect sugar evaporator

in steam consumption is not as large as vapour formation. Hence, the steam economy of mixed feed evaporator decreases with increase in feed concentration.

Possible reason for the behaviour that steam economy decreases as feed arrangement is shifted from forward to mixed to backward for a given feed of concentration 18°Bx and temperature more than 85.5°C has already been explained in the previous Section. This feature holds true even at lesser concentration of feed. At a concentration of 14.5°Bx, steam economies of forward and mixed feed evaporators equalizes while that of the backward feed remains lower than either of the two feeds. Any further reduction in the feed concentration causes steam consumption of multiple effect evaporator to be lower than that of forward feed evaporator due to reduced water load in the first effect of the evaporator. As a consequence of it, steam economy is found to decrease when feed arrangement is changed from mixed to forward to backward.

Figure 6.5 represents the variation of steam economy of a quintuple effect black-liquor evaporator as a function of feed concentration for various feeds. This plot refers to the feed temperature, 90°C; feed rate, 70 ton/hr ; pressure in the last effect, 16.577 kPa; and steam pressure, 313.1 kPa. From this plot, the following important points emerge out:

(i) steam economy decreases with feed concentration for all the feed arrangements investigated in the present study.

(ii) The curves for backward- and mixed- feed arrangements intersect each other at a point corresponding to a feed concentration of 24.9°Tw This indicates that for the feed having concentration less than 24.9°Tw the steam economy decreases as feed arrangement is changed from forward to mixed to backward; whereas for highly concentrated feeds, (>24.9°Tw) the steam economy decreases as the feed arrangement is changed from forward to backward to mixed.

Similarly Figure 6.6 shows the variation of steam economy of a triple effect caustic-soda evaporator as a function of the feed concentration for forward-, backward-, and mixed-feed arrangement. This plot is for feed temperature, 95°C; feed rate, 25 ton/hr; pressure in the last effect, 5.226 kPa; and steam pressure, 700.8 kPa. The features of this plot are identical to those obtained in the evaporation of aqueous black-liquor solution. The cut-off concentration in this case corresponds to 8.6%.

Possible reason for these observations is attributed to differing physico-thermal properties of the aqueous solutions, values of operating variables, and the number of effects used in the evaporation of these solutions. As a matter of fact, these aqueous solutions exhibit significant increase in saturation temperature when feed concentration is increased. This, in turn, causes heat load of first effect to be substantially larger than that for forward

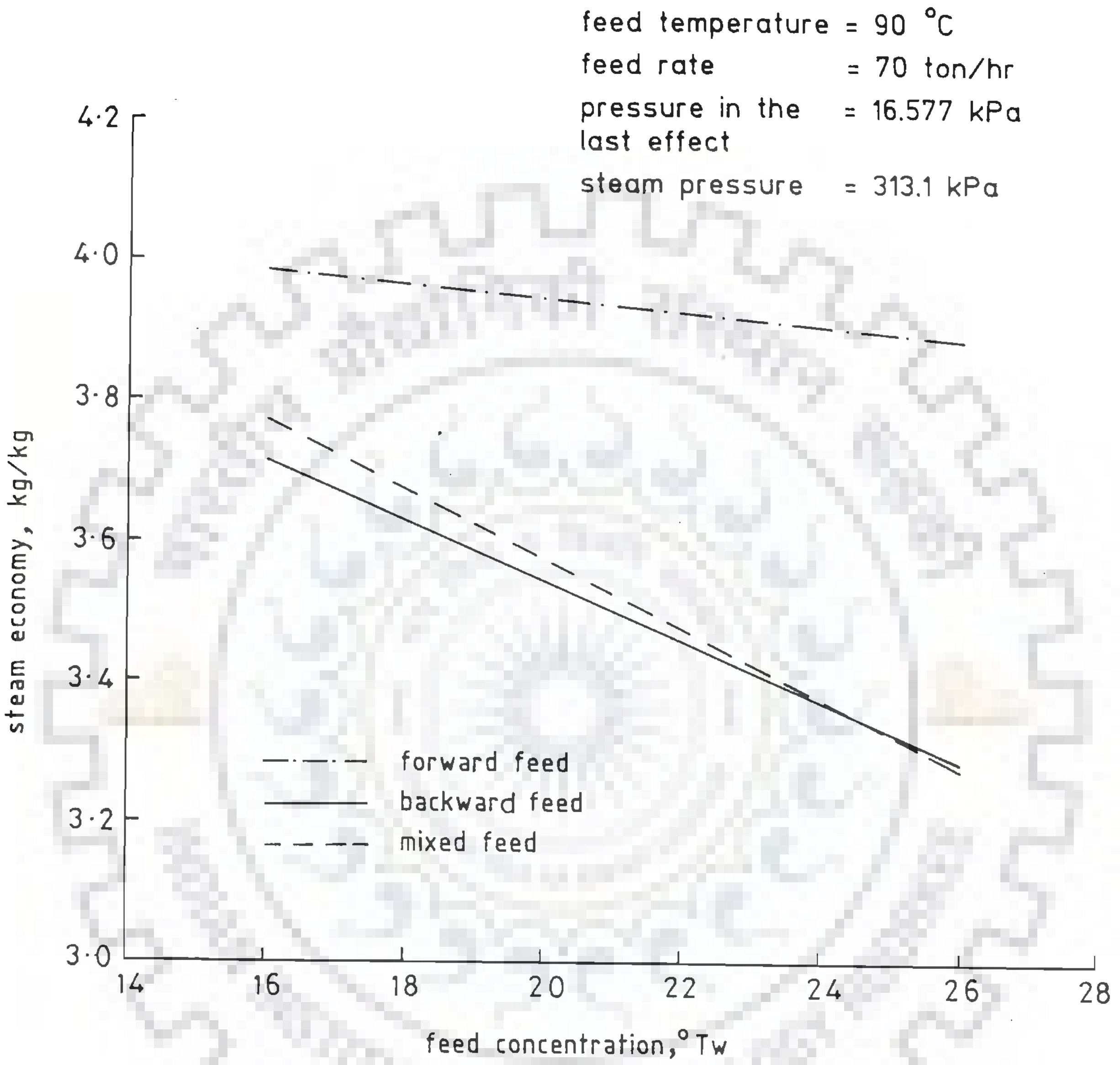

Fig. 6.5 Steam economy versus feed concentration for a quintuple effect black-liquor evaporator

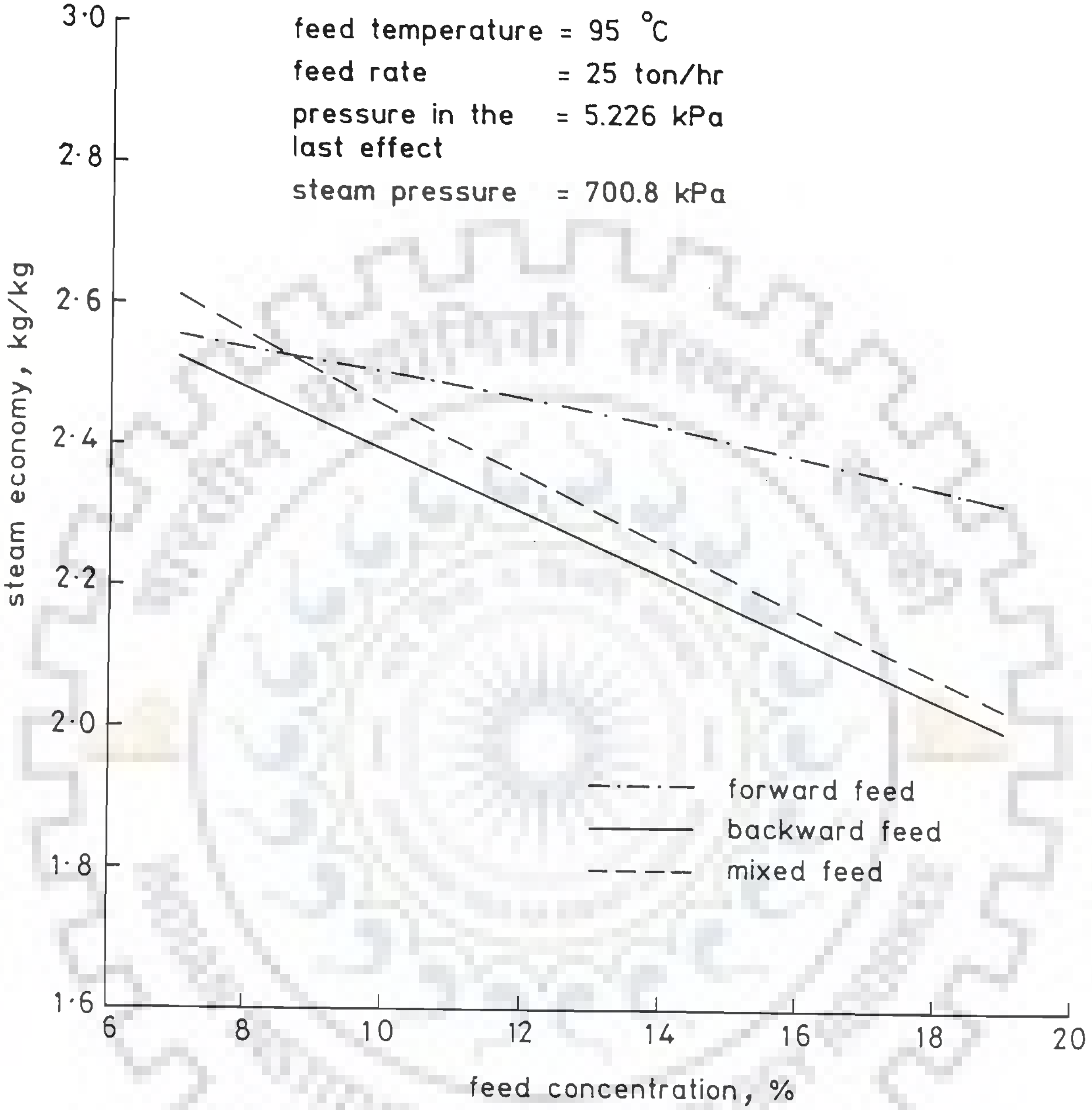

Fig. 6.6 Steam economy versus feed concentration for a triple effect caustic soda evaporator

feed sugar evaporator. That is why steam economies for forward feed black-liquor and caustic soda evaporators are found to decrease with feed concentration.

From the above, it can be concluded that for a given aqueous solution of specified concentration there exists an unique feed arrangement which will offer the highest steam economy. The range of feed concentration that can provide the highest steam economy for a given feed arrangement and aqueous solution are listed in Table 6.3.

Table 6.3 Range of feed concentration for the highest steam economy as a function of feed arrangement and aqueous solution

Aqueous solution	Feed concentration		
	Forward feed	Backward feed	Mixed feed
Sugar	14.5--20.0°Bx	--	12.0--14.5°Bx
Black-liquor	16.0--26.0°Tw	--	--
Caustic soda	8.0-- 8.6%	--	8.6--18.0%

The above Table can be used for the selection of feed arrangement from the knowledge of feed concentration and aqueous solution being concentrated. The compatibility between feed concentration and feed arrangement for a given aqueous solution ensures the evaporation with the highest steam economy. This is important to ensure the minimum consumption of steam in an evaporator.

6.2.3 VARIATION OF STEAM ECONOMY WITH FEED RATE

Figure 6.7 is a typical plot showing the variation of steam economy of a quadruple effect sugar evaporator as a function of the feed rate for forward-, backward-, and mixed- feed. This plot is for feed temperature, 100°C; feed concentration, 18° Bx; pressure in the last effect, 15.74 kPa; and steam pressure; 143.27 kPa.

An inspection of the curves of this plot reveals the following salient features:

(i) An increase in feed rate increases the value of steam economy for forward feed arrangement, and decreases for backward-, and mixed- feed arrangement.

(ii) At a given feed rate, forward feed arrangement offers the highest steam economy to be followed by mixed-, and backward- feed arrangement in decreasing order.

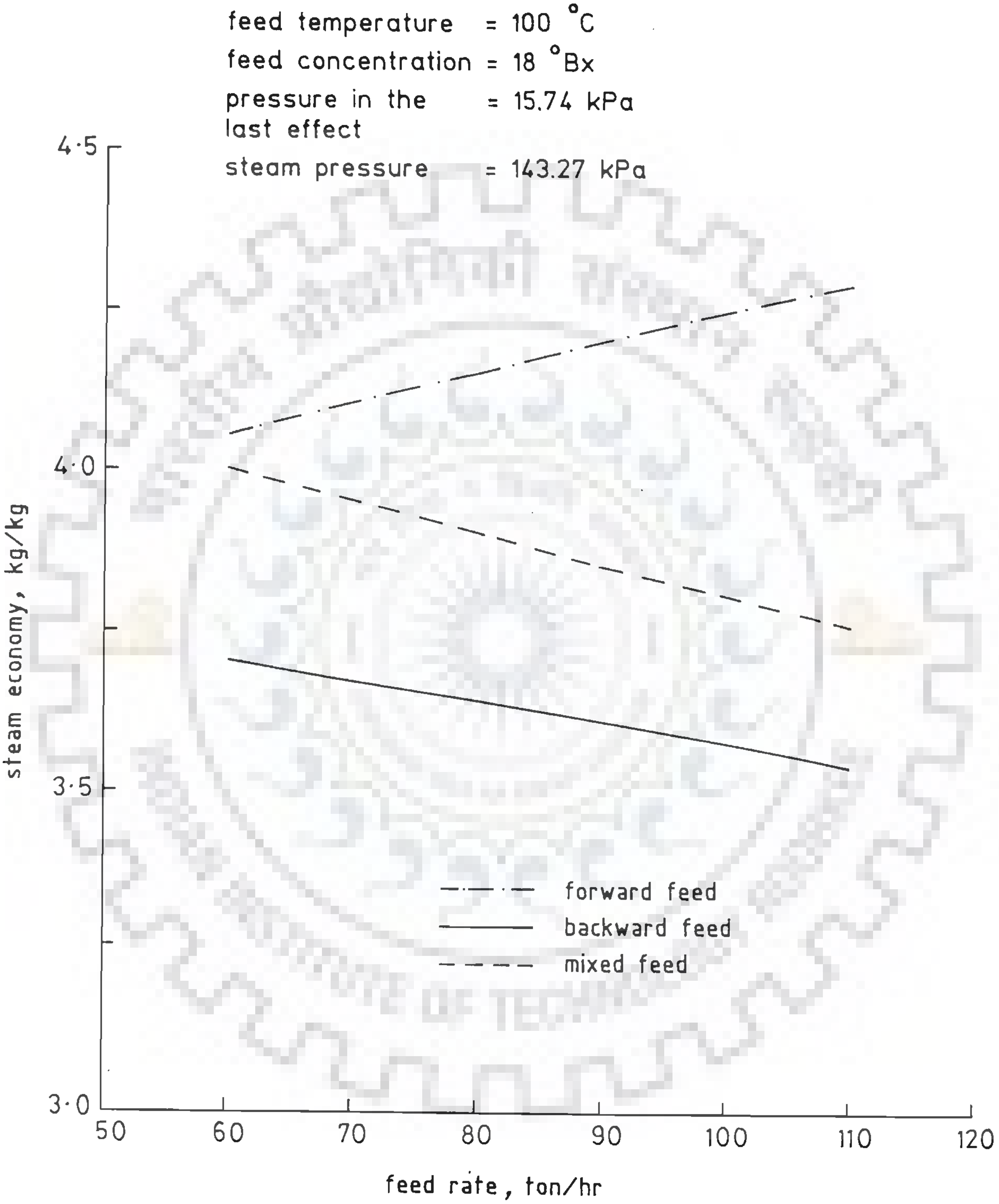

Fig. 6.7 Steam economy versus feed rate for a quadruple effect sugar evaporator

The above observations can be explained as follows:

An increase in feed rate leads to increase the water load in the evaporator which gives rise to more vapour formation. In addition to it, the value of heat transfer coefficient also rises. This, in turn, increases the heat load and thereby steam consumption. However, the variation in both of them with the feed rate is not at the same rate. As a matter of fact, it depends upon the feed arrangement employed in the evaporator. In forward feed evaporator, an increase in feed rate causes heat load of the first effect and flashing in subsequent effects to increase. Since the amount of vapour formation due to flashing is more than the steam required to preheat the solution, therefore the rate of vapour formation is higher than that of steam consumption. Consequently, steam economy of a forward feed evaporator rises with the increase in feed rate. But in the case of backward-, and mixed- feed evaporator, increase in heat load is more than the flashing. Therefore, the rate of steam consumption is higher than that of vapor formation and thereby steam economy for backward-, and mixed- feed evaporator is found to decrease with feed rate.

At a given feed rate, shifting the feed arrangement from forward to backward decreases the amount of vapour formation by flashing. Hence, steam economy of forward feed evaporator is found to be higher than that of backward feed evaporator. Since in mixed feed evaporator, three of the quadruple effect evaporator operate as forward feed, the vapour formation for mixed feed lies between that for forward and backward feed arrangements. Thus, steam economy of mixed feed evaporator is greater than that of backward feed evaporator but lower than that of forward feed evaporator. In other words, steam economy of a quadruple effect sugar evaporator decreases as the feed arrangement is changed from forward to mixed to backward.

A noteworthy point which is of immediate concern is that at a feed rate of 70 ton/hr, as it is usually used in Indian quadruple effect sugar evaporators, the steam economy for forward feed arrangement is 3.84 % and 11.76 % higher than those for mixed-, and backward- feed arrangement, respectively. It is still higher when the feed rate is raised further. Thus, the use of forward feed arrangement for the evaporation of aqueous sugar solution at or above 70 ton/hr feed rate is recommended for the sake of high steam economy and thereby conservation of energy. In fact, this corroborates the widely accepted practice of using forward feed arrangement in sugar mills.

Figure 6.8 represents the effect of feed rate on the steam economy of a quintuple effect black-liquor evaporator for various feed arrangements. The features of this plot are similar to that of Figure 6.7 for the evaporation of aqueous sugar solution. Figure 6.9 shows a plot of steam economy versus feed rate for the evaporation of aqueous caustic soda solution. This plot also has essentially the same characteristics except that the curves representing the forward and mixed feed arrangements intersect each other at a point corresponding to the

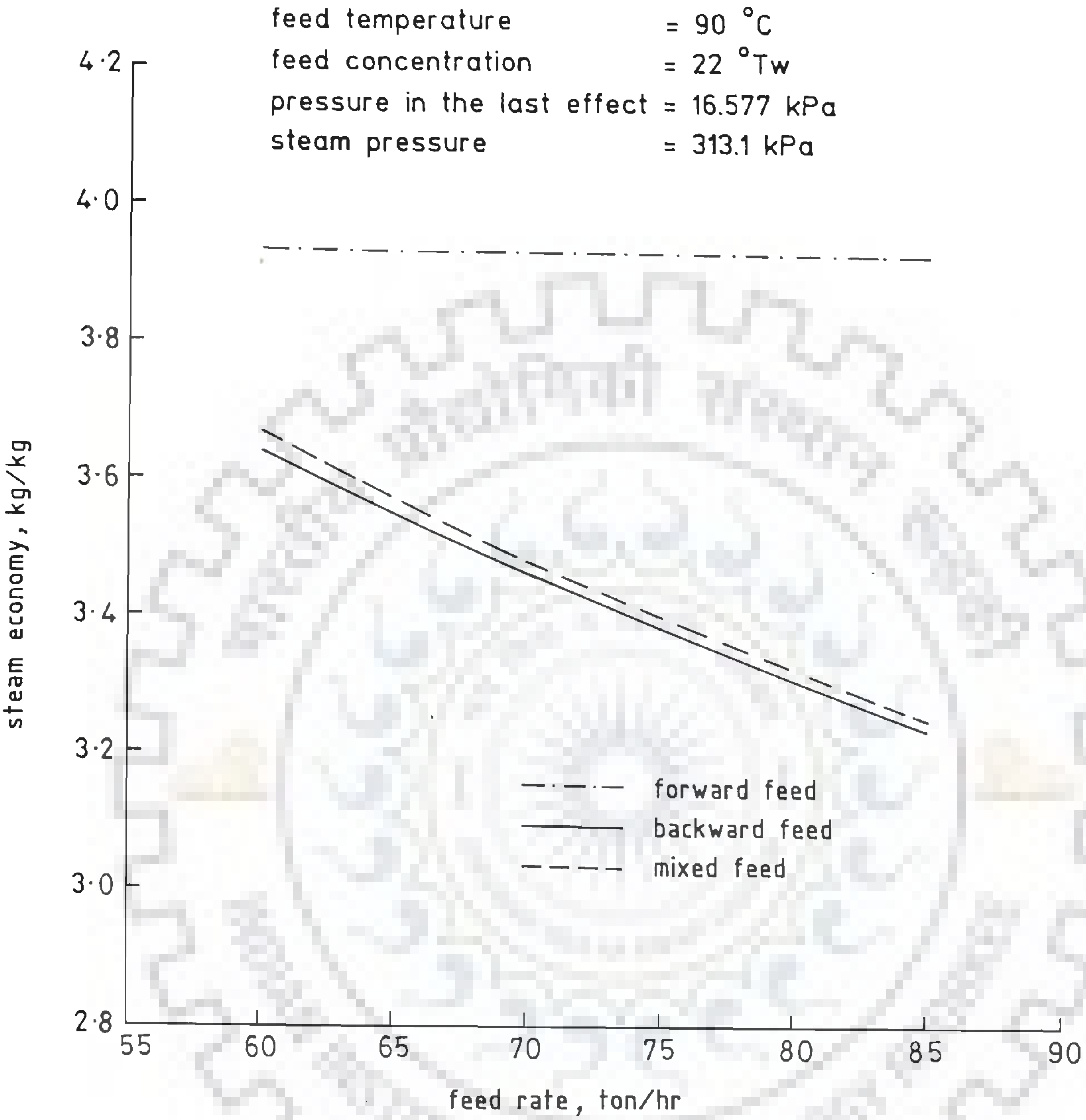

Fig. 6.8 Steam economy versus feed rate for a quintuple effect black-liquor evaporator

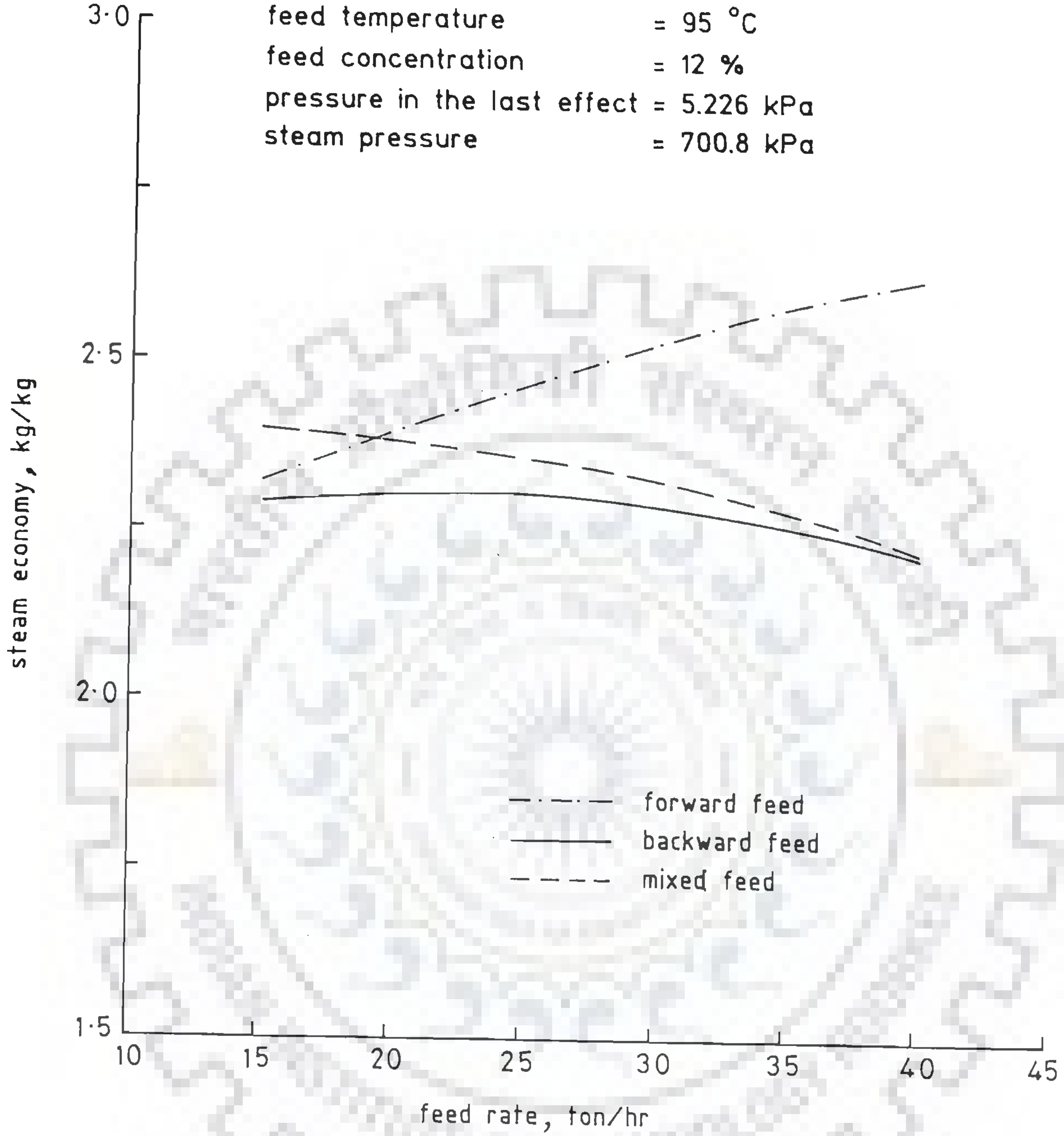

Fig. 6.9 Steam economy versus feed rate for a triple effect caustic soda evaporator

feed rate of 19 ton/hr. This clearly indicates that for the feed rate exceeding 19 ton/hr, the steam economy changes with the feed arrangement in the same way as for the aqueous solutions of sugar and black-liquor, i.e. it decreases from forward- to mixed- to backward- feed arrangement. On the other hand, for the feed rates less than 19 ton/hr, the order of feed arrangement in which the steam economy decreases is from mixed to forward to backward. This is obviously, on account of the differing rate of vapour formation and steam consumption for various feed arrangements. In fact, they vary in such a way that at a feed rate of 19 ton/hr steam economy of mixed feed evaporator becomes equal to that of forward feed evaporator.

This analysis can be extended to determine the feed arrangement which will provide the highest steam economy for the given aqueous solution at a specified feed rate. Table 6.4 gives the range of feed rate of a given aqueous solution for which a particular feed arrangement offers the highest steam economy.

Table 6.4 Range of feed rate for the highest steam economy as a function of feed arrangement and aqueous solution

	Feed rate, ton/hr		
Aqueous solution	Forward feed	Backward feed	Mixed feed
Sugar	60 --110	--	--
Black-liquor	60 --85	--	--
Caustic soda	19 --40	--	15 --19

As is evident from the above Table, steam economy of the evaporator is the highest for forward feed for all the solutions except that for the lower range of feed rate in the case of caustic soda solution.

6.2.4 VARIATION OF STEAM ECONOMY WITH PRESSURE IN THE LAST EFFECT OF AN EVAPORATOR

Figure 6.10 shows a typical plot for the variation of steam economy with pressure in the last effect of a quadruple effect sugar evaporator. This plot is for feed temperature, 100°C; feed concentration, 18° Bx; feed rate, 70 ton/hr; and steam pressure, 143.27 kPa.

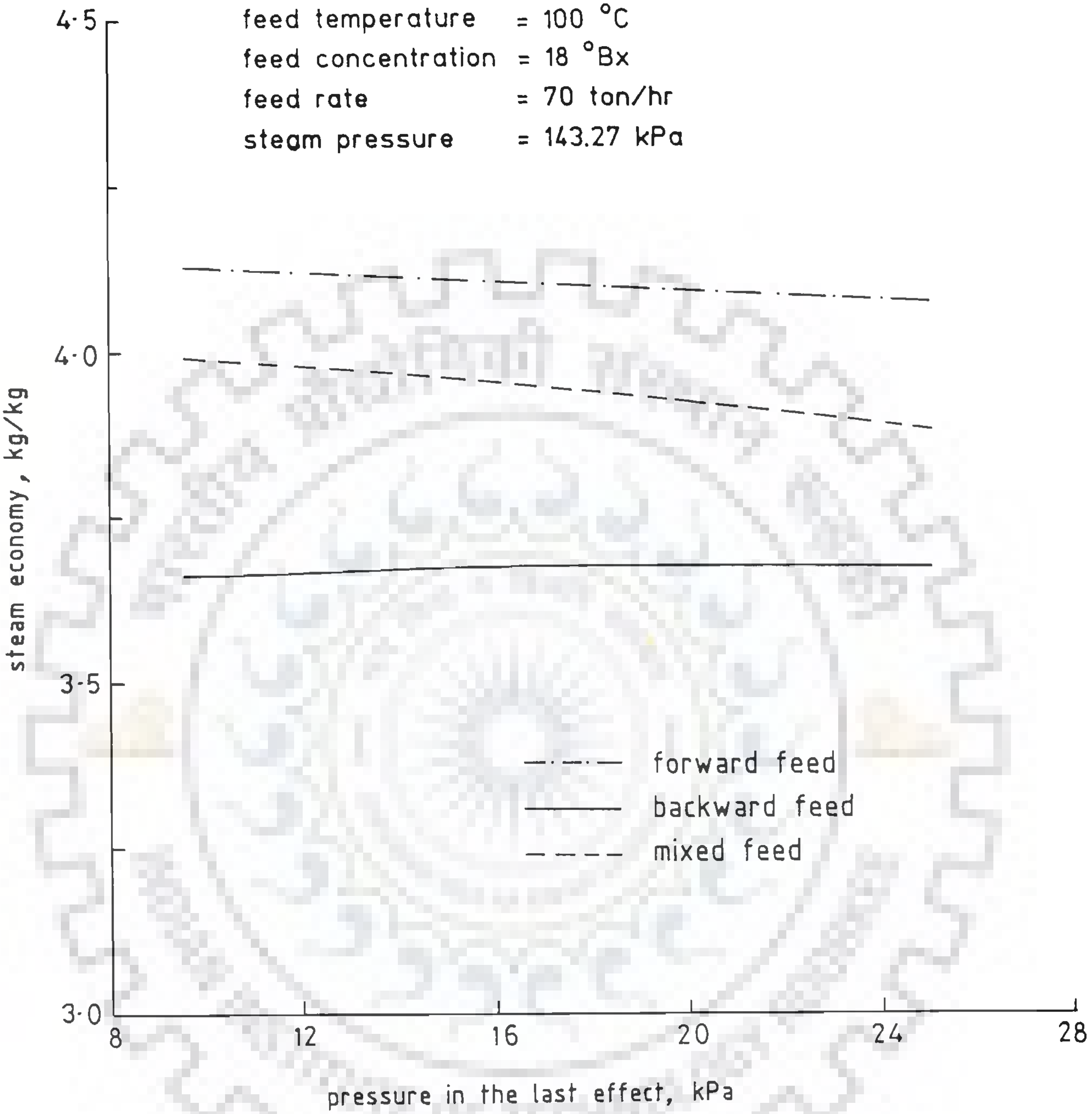

Fig. 6.10 Steam economy versus pressure in the last effect of a quadruple effect sugar evaporator

An examination of this plot brings out the following salient features:

(i) Raising the pressure in the last effect of the evaporator increases the steam economy for backward feed arrangement and decreases for forward- and mixed- feed arrangement.

(ii) At a given pressure in the last effect, forward feed arrangement provides the highest steam economy to be followed by mixed, and backward feed arrangements in decreasing order.

The above observations can be explained as follows:

An increase in pressure of the last effect causes evaporation to proceed at higher pressures in various effects of the evaporator. This, in turn, affects the amount of heat load and flashing depending upon the feed arrangement used in the evaporator. Consequently, both the amount of steam consumption and vapour formation changes. In the case of forward feed arrangement, rate of decrease in steam consumption is more than that of vapour formation for higher pressure in the last effect. This is in view of the fact that change in preheat load is more significant as compared to flashing in second, third, and fourth effect of the quadruple effect evaporator. Hence, steam economy of forward feed evaporator decreases with the pressure in the last effect. In the case of mixed feed arrangement, the same behaviour is obtained. This is an expected trend since most of the effects in the quadruple sugar evaporator operate as forward feed. Contrary to the above, the steam economy of backward feed arrangement increases with the increase in pressure in the last effect. This is attributed to the fact that steam consumption decreases at a faster rate than the vapour formation owing to reduced preheating of the solution for high value of pressure in the last effect. Thus, the steam economy of backward feed evaporator improves when pressure in the last effect is raised.

The observation that at given pressure in the last effect of the evaporator, steam economy decreases when feed arrangement is shifted from forward to mixed to backward is an expected one. This is due to the fact that amount of vapour formation by flashing reduces whereas the preheat load increases when feed arrangement is changed from forward to mixed to backward. This contributes to decrease the steam economy of forward feed evaporator to be followed by mixed-, and backward- feed evaporator in decreasing order.

Figures 6.11 and 6.12 depict the variation of steam economy of black-liquor and caustic soda evaporators with pressure in the last effect of the evaporator, respectively. The plot for caustic soda, shown in Figure 6.12, possesses the same behaviour as found in the evaporation of sugar solution except that the curves of mixed and backward feeds intersect each other at a point corresponding to 14.1 kPa pressure in the last effect. This means that for the last effect pressure ranging from 4.24 kPa to 14.1 kPa, steam economy decreases on shifting the feed arrangement from forward to mixed to backward and for the pressure ranging from 14.1 kPa to 15.74

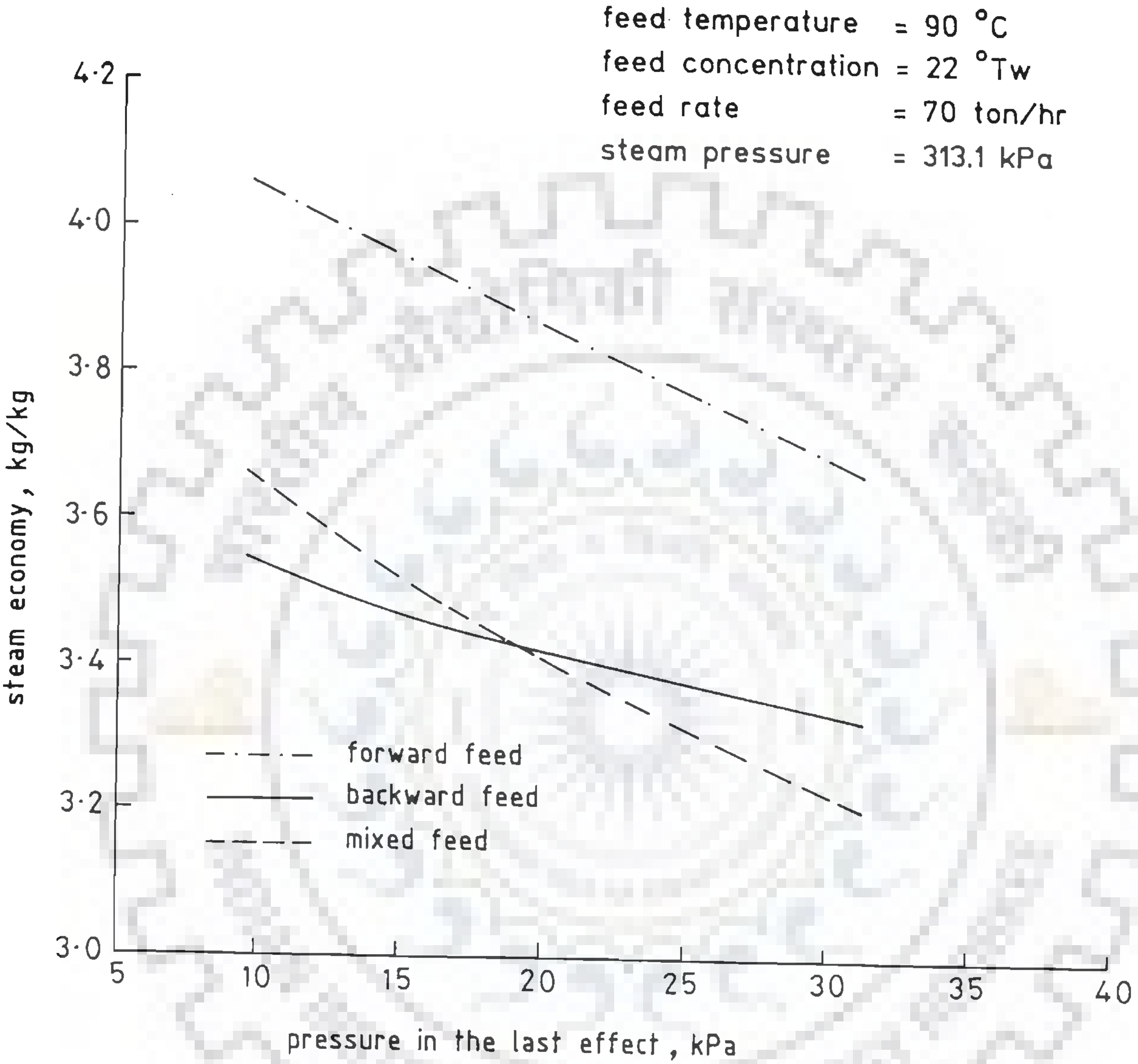

Fig. 6.11 Steam economy versus pressure in the last effect of a quintuple effect black-liquor evaporator

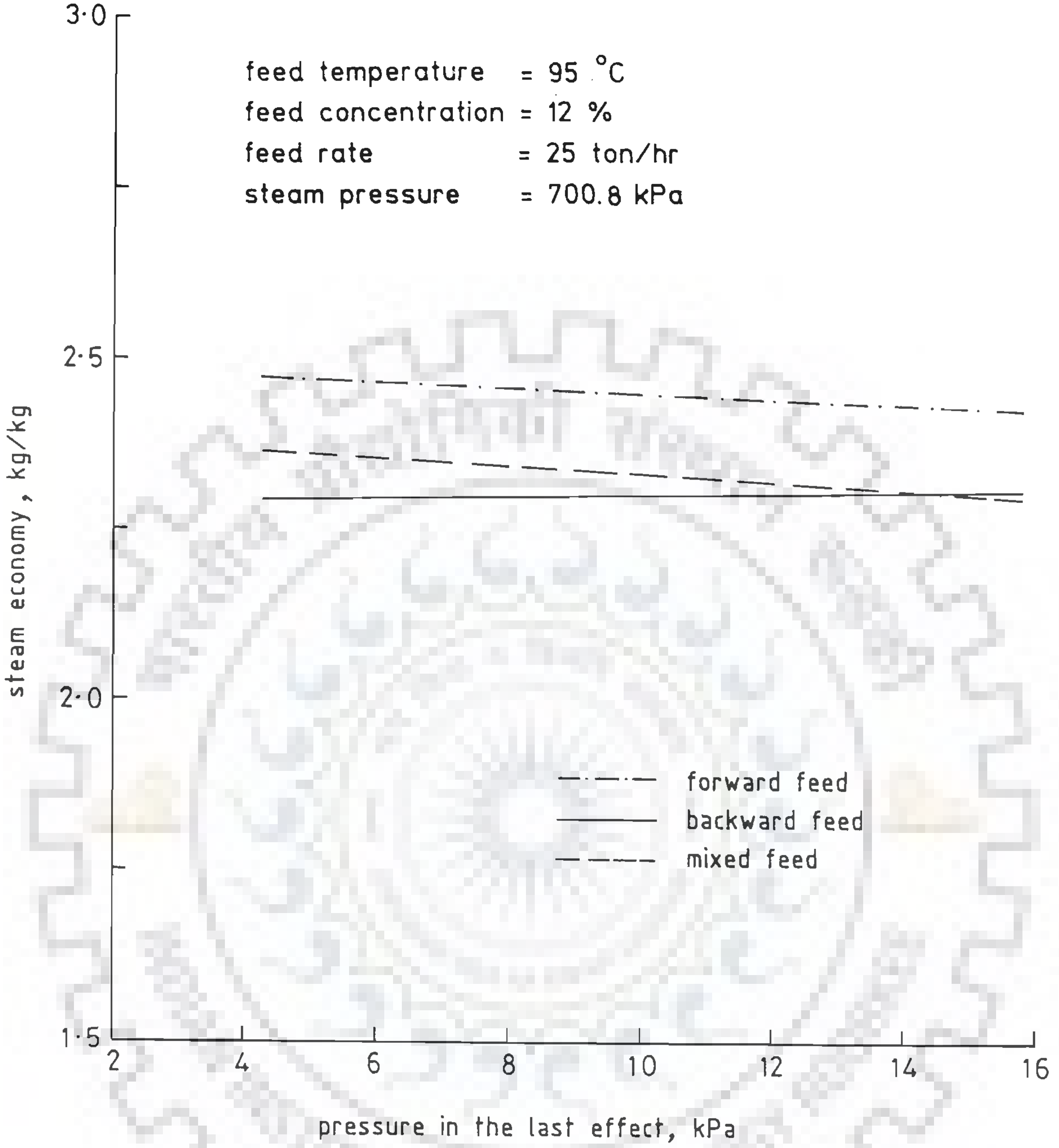

Fig. 6.12 Steam economy versus pressure in the last effect of a triple effect caustic soda evaporator

kPa the sequence of feed arrangement for the decrease of steam economy is from forward to backward to mixed. This behaviour is not at all a surprising one in view of opposing trend of the curves representing mixed-, and backward- feed. Further, it is also seen that for the complete range of last effect pressure, forward feed arrangement provides the highest steam economy out of all the three feed arrangements. Black-liquor solution exhibits different behaviour. In this case, steam economy decreases with pressure in the last effect for all the feed arrangements. Further, the curves for mixed- and backward- feed intersect each other at a point corresponding to 18 kPa pressure in the last effect. The reason for this behaviour is that the rate of decrease in steam economy for mixed feed is greater than that of backward feed on account of differing amount of flashing in these arrangements. Thus, the curves are found to intersect each other. In this way for the last effect pressure ranging from 9.58 kPa to 18 kPa, steam economy decreases from forward to mixed to backward whereas for the pressure ranging from 18 kPa to 31.24 kPa, the steam economy decreases when the sequence of the feed arrangement is from forward to backward to mixed.

Table 6.5 lists the range of pressure in the last effect for a given aqueous solution and feed arrangement that provides the highest steam economy of an evaporator.

Table 6.5 Range of pressure in the last effect for the highest steam economy as a function of feed arrangement and aqueous solution

Aqueous solution	Pressure in the last effect, kPa		
	Forward feed	Backward feed	Mixed feed
Sugar	9.58--25.01	--	--
Black-liquor	9.58--31.24	--	--
Caustic soda	4.24--15.74	--	--

From Table 6.5 it is clear that the steam economy of an evaporator always remains at the highest level for forward feed arrangement irrespective of the aqueous solution and the pressure of the last effect. Therefore, the use of forward feed arrangement for the evaporation of sugar, black-liquor, and caustic soda aqueous solutions in their respective multiple effect evaporators is recommended, unless process technology demands for any change in the above arrangement.

.2.5 VARIATION OF STEAM ECONOMY WITH STEAM PRESSURE

igure 6.13 shows a plot to represent the variation of steam economy of a quadruple effect ugar evaporator with steam pressure for forward-, backward-, and mixed- feed arrangement. he operating variables for this plot are: feed rate, 70 ton/hr; feed concentration, 18°Bx; feed emperature, 100°C; and pressure in the last effect, 15.74 kPa.

he noteworthy features of this plot are as follows:

i) Steam economy of a quadruple effect sugar evaporator decreases with the increase in steam pressure, irrespective of the feed arrangement.

ii) At a given steam pressure, forward feed arrangement offers the highest steam economy to be followed by mixed and backward feed arrangements in decreasing order.

he decrease of steam economy with steam pressure can be arrributed to the following:

Jse of steam at higher pressure in the steam chest of the first effect increases the heat load of the evaporator due to higher temperature gradient, $\Delta T(=T_s - \tau_1)$. This causes amount of apour formation to be larger because the amount of heat utilized in preheating does not ncrease with the same pace as that of heat load. Since steam of high pressure has small latent eat of condensation, large amount of steam is consumed to provide the necessary heat load the evaporator. As a matter of fact, steam consumption increases at a faster rate than that f the vapour formation, hence steam economy of an evaporator is found to be lower when team of elevated pressure is employed. This holds true for all the feed arrangements. Iowever, the rate of variation of steam economy with steam pressure depends upon the feed rrangement used in the evaporator. It is the largest for forward feed evaporator; the smallest or backward feed and in between the two for mixed feed. It is obviously due to differing amount f preheat load in the first effect of the evaporator for various feeds. In fact, in forward feed vhole of the feed is preheated, whereas in backward feed reduced amount of concentrated iquid of the second effect is preheated, and in mixed feed the liquid of the last effect is reheated by steam. Therefore, the rate of decrease of steam economy with steam pressure the highest for forward feed evaporator to be followed by mixed and backward feed vaporators in decreasing order.

he behaviour that at given steam pressure, steam economy of the evaporator decreases when eed arrangement is shifted from forward to mixed to backward is due to reduction in flashing nd increase in preheat load of the evaporator.

he variation of steam economy with steam pressure for black-liquor solution is shown in igure 6.14. This plot has also the similar characteristics as those of Figure 6.13 except that

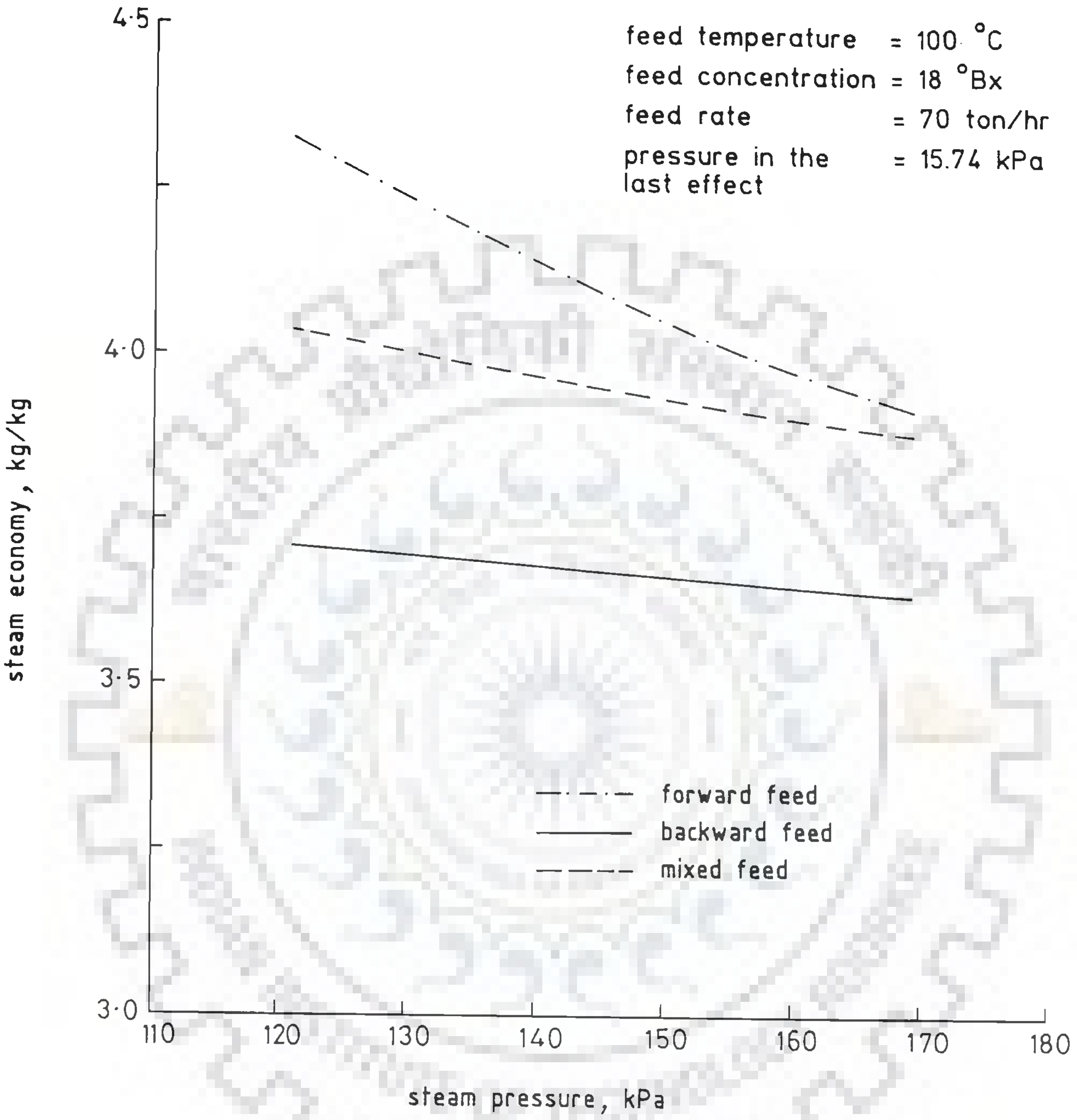

Fig. 6.13 Steam economy versus steam pressure for a quadruple effect sugar evaporator

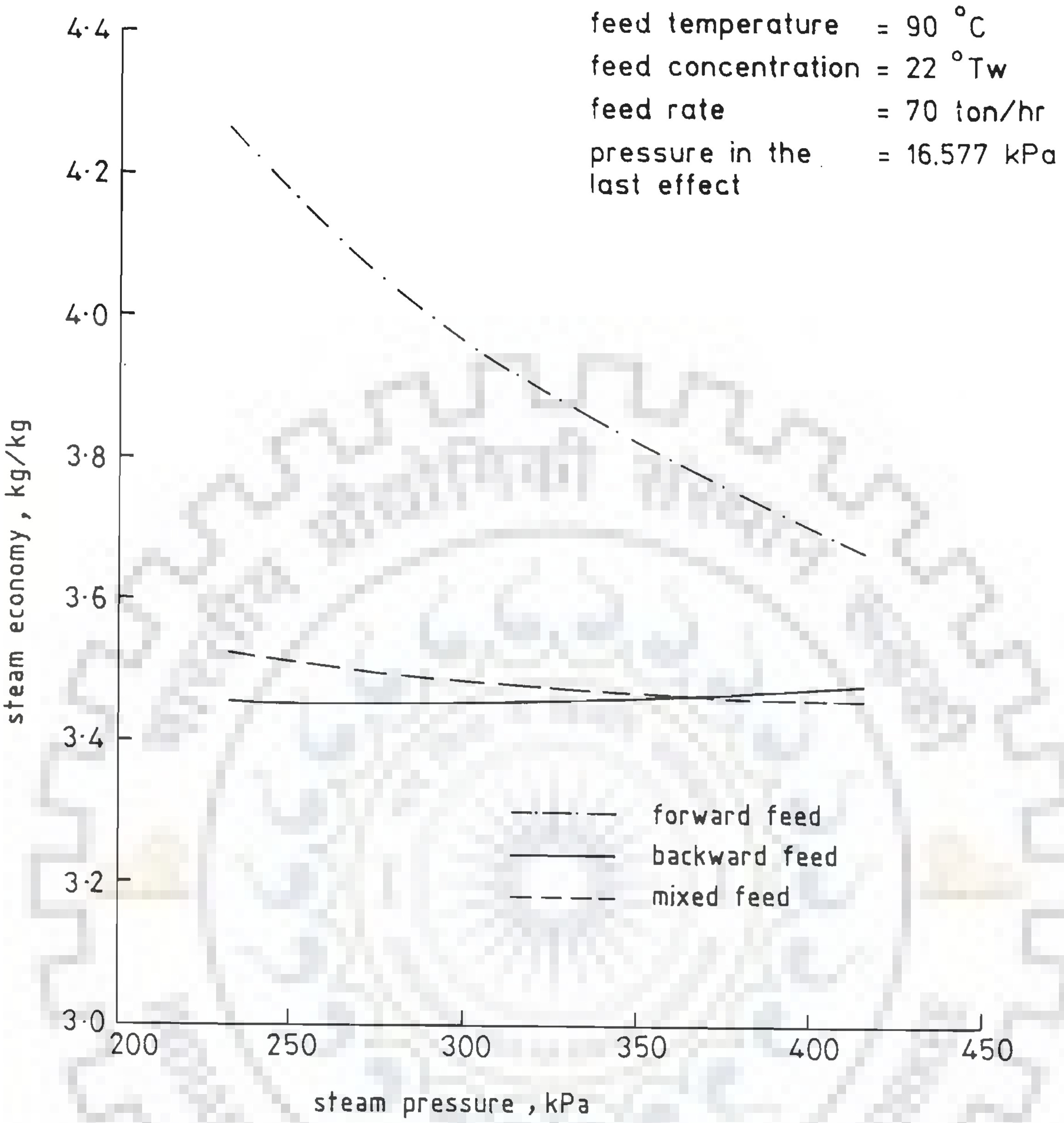

Fig. 6.14 Steam economy versus steam pressure for a quintuple effect black-liquor evaporator

the curves for backward- and mixed- feed intersect each other at a point corresponding to a steam pressure of 362.5 kPa. This implies that for the steam having pressure less than 362.5 kPa, the sequence of feed arrangement for the decrease of steam economy is forward to mixed to backward; whereas for steam pressure exceeding 362.5 kPa, the decrease in steam economy is observed when feed arrangement is shifted from forward to backward to mixed. Possible reason for the above behaviour lies in the fact that the rate of decrease of steam economy with steam pressure for mixed feed evaporator is more than that of backward feed evaporator.

The variation of steam economy for caustic soda evaporator with steam pressure for various feed arrangements is shown in Figure 6.15. The features of this plot are similar to that of Figure 6.13 for aqueous sugar solution.

The range of steam pressure for a given aqueous solution and feed arrangement that can yield the highest steam economy of the evaporator is given in Table 6.6.

Table 6.6 Range of steam pressure for the highest steam economy as a function of feed arrangement and aqueous solution

Aqueous solution	Steam pressure, kPa Forward feed	 Backward feed	 Mixed feed
Sugar	120.8--169.1	--	--
Black-liquor	232.0--416.0	--	--
Caustic soda	543.2--892.4	--	--

Above Table clearly demonstrates that forward feed arrangement can provide the highest steam economy irrespective of the aqueous solution being concentrated and the steam pressure used.

6.2.6 CORRELATION OF STEAM ECONOMY OF AN EVAPORATOR

To understand the quantitative effect of operating variables on steam economy of a multiple effect evaporator for various aqueous solutions under forward-, backward- and mixed-feed arrangement, a correlation of steam economy with operating variables, viz; feed temperature, feed concentration, feed rate, pressure in the last effect (temperature), and steam pressure (saturation temperature) has been developed. For this purpose the computed values of steam economy of Tables E.1 to E.15 of Appendix-E have been correlated with the operating variables by the multiple linear regression analysis. Following are the resultant correlations for sugar quadruple-, black-liquor quintuple-, and caustic soda triple- effect

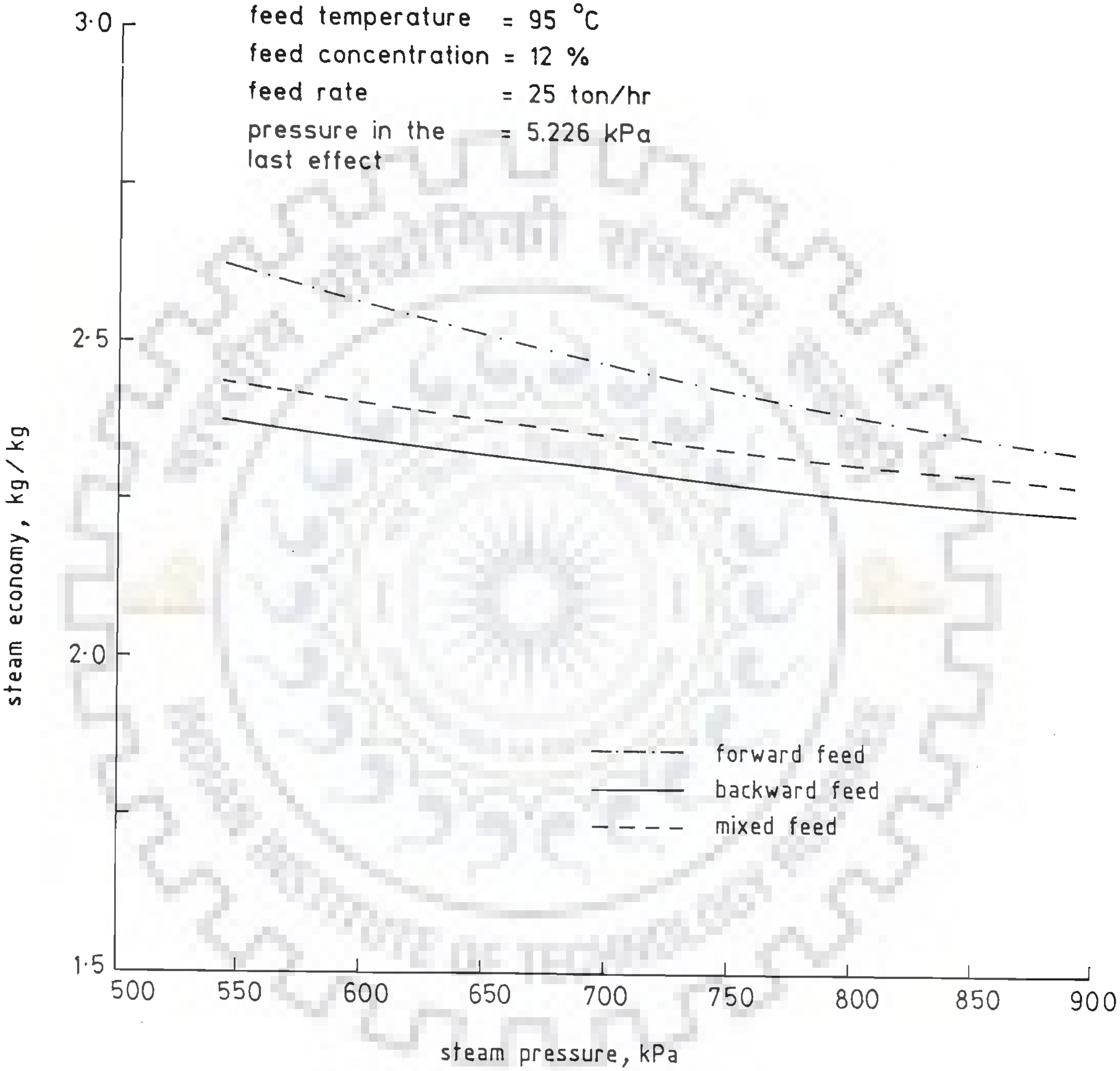

Fig. 6.15 Steam economy versus steam pressure for a triple effect caustic soda evaporator

evaporator for forward, backward, and mixed feed arrangements:

a. Forward feed arrangement

Sugar solution

$$E = 6.964\tau_f^{0.779} X_f^{0.048} F^{0.092} T_l^{-0.036} T_s^{-1.091} \qquad \ldots(6.1)$$

Maximum deviation = ±4.56%

Black-liquor solution

$$E = 32.342\tau_f^{0.885} X_f^{-0.050} F^{-0.219} T_l^{-0.203} T_s^{-1.014} \qquad \ldots(6.2)$$

Maximum deviation = ±4.78%

Caustic soda solution

$$E = 20.86\tau_f^{0.443} X_f^{-0.097} F^{0.127} T_l^{-0.035} T_s^{-0.993} \qquad \ldots(6.3)$$

Maximum deviation = ±4.60%

b. Backward feed arrangement

Sugar solution

$$E = 9.697\tau_f^{0.215} X_f^{-0.074} F^{-0.747} T_l^{0.014} T_s^{-0.206} \qquad \ldots(6.4)$$

Maximum deviation = ±4.95%

Black-liquor solution

$$E = 2.316\tau_f^{0.283} X_f^{-0.137} F^{-0.003} T_l^{-0.159} T_s^{0.046} \qquad \ldots(6.5)$$

Maximum deviation = ±5.15%

Caustic soda solution

$$E = 27.687\tau_f^{0.183} X_f^{-0.236} F^{-0.004} T_l^{-0.034} T_s^{-0.505} \qquad \ldots(6.6)$$

Maximum deviation = ±4.90%

c. Mixed feed arrangement

Sugar solution

$$E = 11.560\tau_f^{0.573}X_f^{-0.096}F^{-0.103}T_l^{-0.073}T_s^{-0.425} \qquad ...(6.7)$$

Maximum deviation = ±5.50%

Black-liquor solution

$$E = 3.511\tau_f^{0.602}X_f^{-0.284}F^{-0.014}T_l^{-0.271}T_s^{-0.120} \qquad ...(6.8)$$

Maximum deviation = ±5.20%

Caustic soda solution

$$E = 40.515\tau_f^{0.326}X_f^{-0.252}F^{-0.075}T_l^{-0.045}T_s^{-0.545} \qquad ...(6.9)$$

Maximum deviation = ±5.35%

These correlations can be used to calculate the steam economy of a multiple effect evaporator for a given aqueous solution from the knowledge of the values of operating variables and the feed arrangement.

Above correlations can be written in the following form to represent a generalized correlation:

$$E = C\,\tau_f^{a}\,X_f^{b}\,F^{c}\,T_l^{d}\,T_s^{e} \qquad ...(6.10)$$

Where the values of constant, C and the exponents a, b, c, d, and e depend upon the aqueous solution and the feed arrangement used. The values of constant C and exponents a, b, c, d, and e are given in Table 6.7.

At this juncture it is important to mention that Eqs(6.1--6.9) hold true only for the range of operating variables given in Table 6.1, the aqueous solutions of sugar, black-liquor, and caustic soda, and the number of effects used in the respective evaporators. Therefore, no attempt should be made to use the above correlations beyond these limitations. In other words, determination of steam economy for the solution and operating variables other than those of this investigation will require the determination of the constant, C and exponents a, b, c, d, and e of Eq(6.10) for the aqueous solution in question.

Table 6.7 Values of constant, C and the exponents a, b, c, d, and e in Eq(6.10) for different aqueous solutions and feed arrangements

Aqueous solution	Constant, C	Values of exponents				
		a	b	c	d	e
			Forward feed arrangement			
Sugar	6.964	0.779	0.048	0.092	-0.036	-1.091
Black-liquor	32.342	0.885	-0.050	-0.219	-0.203	-1.014
Caustic soda	20.860	0.443	-0.097	0.127	-0.035	-0.993
			Backward feed arrangement			
Sugar	9.697	0.215	-0.074	-0.747	0.014	-0.206
Black-liquor	2.316	0.283	-0.137	-0.003	-0.159	0.046
Caustic soda	27.687	0.183	-0.236	-0.004	-0.034	-0.505
			Mixed feed arrangement			
Sugar	11.560	0.573	-0.096	-0.103	-0.073	-0.425
Black-liquor	3.511	0.602	-0.284	-0.014	-0.271	-0.120
Caustic soda	40.515	0.326	-0.252	-0.075	-0.045	-0.545

REMARKS ON STEAM ECONOMY OF AN EVAPORATOR

A scrutiny of the preceding Sections clearly brings out the fact that operating variables, feed arrangement and the solution have a profound effect on the steam economy of an evaporator. It has also established the range of operating variables which can offer the highest steam economy for a given aqueous solution and feed arrangement. Generally, the decision regarding the type of feed arrangement in the evaporator is governed by the nature of the solution, its handling and transportation properties, process technology, economics of the plant, and type and geometry of evaporator. In fact, very little consideration is given to the values of operating variables in the selection of feed arrangement. Due to it, steam economy of the evaporator may not be at the highest level. Such situations should be corrected by modifying the values of operating variables, whereas possible, so that they fall in the ranges specified for the arrangement and the solution under consideration. This, undoubtedly, will reduce the expenditure of steam in the evaporator and thereby will lead to conserve energy.

6.3 END-PRODUCT CONCENTRATION OF AN EVAPORATOR

Concentration of the end-product discharging from an evaporator is of primary interest as it provides the information about the amount of water present in it and thereby its quality so that further processing may be carried out to obtain the marketable product. End-product concentration depends upon the aqueous solution, feed arrangement, number of effects in the evaporator, and the values of operating variables, viz; feed temperature, feed concentration, feed rate, pressure in the last effect, and steam pressure. Many a times, any one of them may undergo a change. As a consequence of it, the end-product concentration may alter. However, it is desired to maintain the concentration at a pre-determined level. This requires the determination of parametric effect of operating variables on the end-product concentration of an evaporator. For this purpose, calculations were performed to determine the values of end-product concentration for the aqueous solutions of sugar, black-liquor, and caustic soda from quadruple-, quintuple-, and triple- effect evaporator, respectively, for the operating variables of Table 6.1. The computed values of end-product concentration are given in Appendix-E. Following Subsections discuss the effect of operating variables on end-product concentration of an evaporator.

6.3.1 VARIATION OF END- PRODUCT CONCENTRATION WITH FEED TEMPERATURE

Figure 6.16 is a plot to represent the variation of end-product concentration with feed temperature for a quadruple effect sugar evaporator for forward-, backward-, and mixed- feed arrangement. This plot has been prepared for feed rate, 70 ton/hr; feed concentration, 18°Bx; pressure in the last effect, 15.74 kPa; and steam pressure, 143.27 kPa.

From this plot the following important features are noted:

(i) An increase in feed temperature increases the end-product concentration of the sugar evaporator irrespective of the feed arrangement. However, the rate of variation differ from arrangement to arrangement.

(ii) At a given feed temperature, forward feed is found to offer the highest end-product concentration to be followed by backward and mixed feed arrangements in decreasing order.

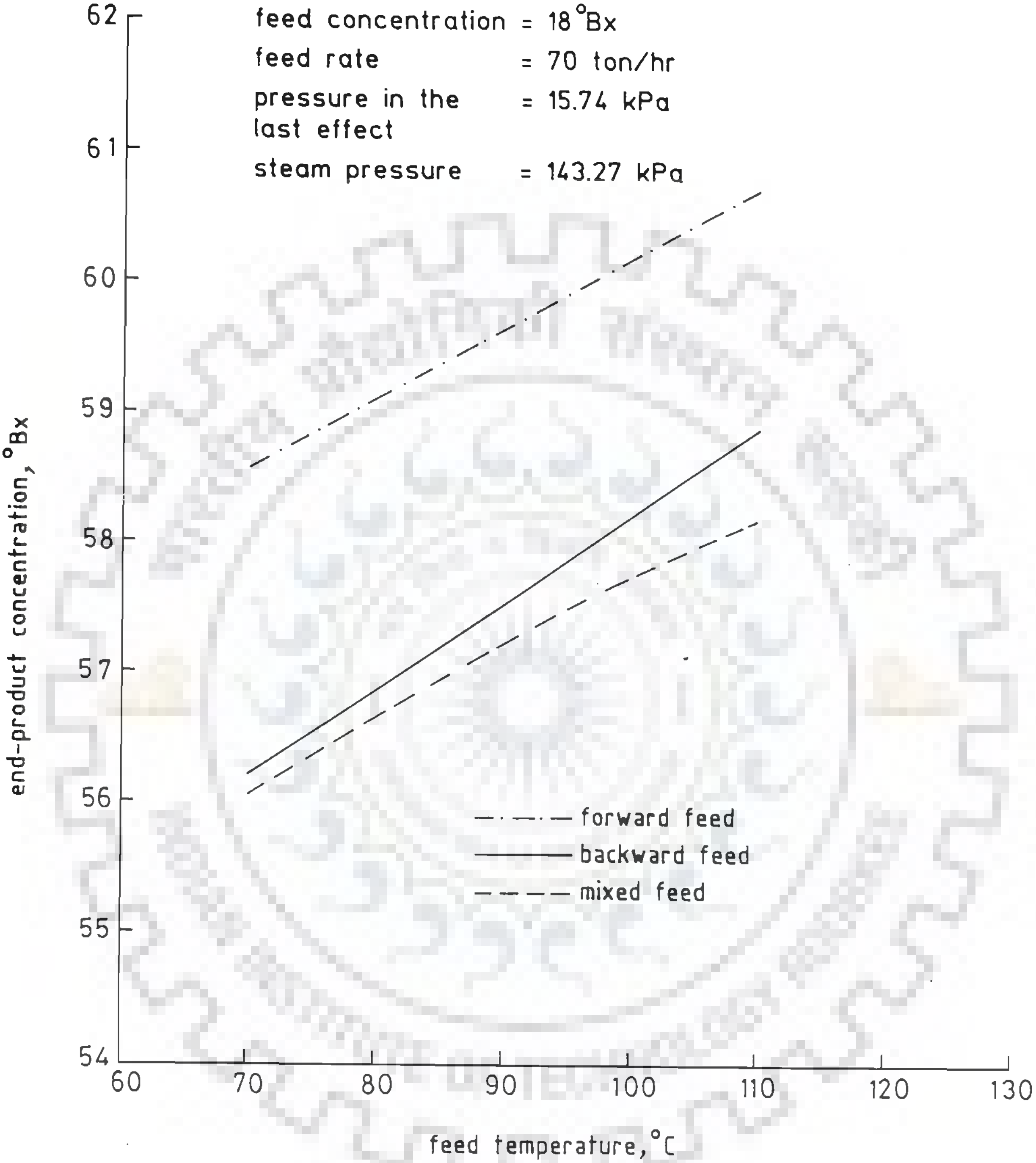

Fig. 6.16 End-product concentration versus feed temperature for a quadruple effect sugar evaporator

These features can be attributed to the following:

As explained in Section 6.2.1, an increase in feed temperature leads to increase the amount of vapour formation in the evaporator for all the feed arrangements. Consequently, the end-product discharging from the evaporator is found to be of higher concentration when feed is at elevated temperature.

At a given feed temperature, the amount of vapour formation decreases when feed arrangement is changed from forward to backward to mixed due to varying amount of flashing and preheating in the evaporator for various feed arrangements. Therefore, forward feed evaporator provides the highest end-product concentration to be followed by backward feed and mixed feed evaporator in decreasing order.

Variation of end-product concentration with feed temperature for black-liquor and caustic soda solutions for various feeds have been shown in Figures 6.17 and 6.18, respectively. The operating variables for black-liquor are: feed concentration, 22°Tw; feed rate, 70 ton/hr; pressure in the last effect, 16.577 kPa; and steam pressure, 313.1 kPa whereas for caustic soda they are: feed concentration, 12%; feed rate, 25 ton/hr; pressure in the last effect, 5.226 kPa, and steam pressure, 700.8 kPa. Both the plots have essentially the same features as that of aqueous sugar solution. However, in the case of caustic soda solution, the curves representing mixed and backward feeds intersect at a point corresponding to a feed temperature of 87.5°C. This means that for temperature smaller than 87.5°C, the end-product concentration decreases when feed arrangement shifts from forward to mixed to backward, and for the feed having temperature greater than 87.5°C, the end-product concentration decreases in the same order as has been observed for black-liquor and sugar solutions. The reason for this lies in the fact that more amount of water evaporates in backward- than that of mixed- feed evaporator for a given increase in the temperature of feed. This causes the curve for backward feed to be steeper than that of mixed feed and thus they intersect each other at a feed temperature of 87.5°C where the end-product concentration of mixed feed evaporator is equal to that of backward feed evaporator and beyond this temperature, the concentration of end-product of backward is higher than that of mixed feed.

6.3.2 VARIATION OF END- PRODUCT CONCENTRATION WITH FEED CONCENTRATION

The variation of end-product concentration of a quadruple effect sugar evaporator with feed concentration for various feed arrangements is shown in Figure 6.19. This plot is for the operating variables; feed temperature, 100°C; feed rate, 70 ton/hr; pressure in the last effect, 15.74 kPa; and steam pressure, 143.27 kPa.

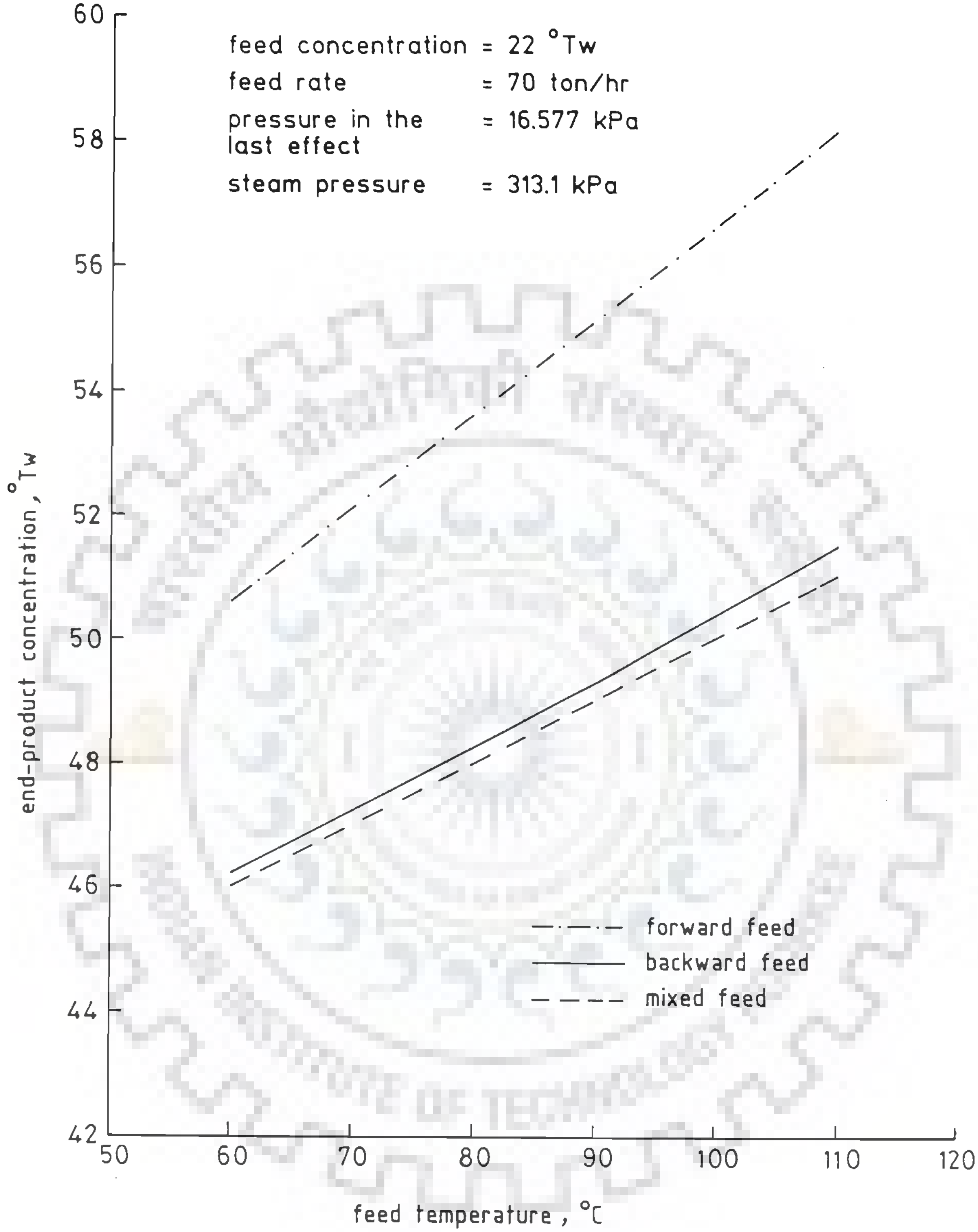

Fig. 6.17 End-product concentration versus feed temperature for a quintuple effect black-liquor evaporator

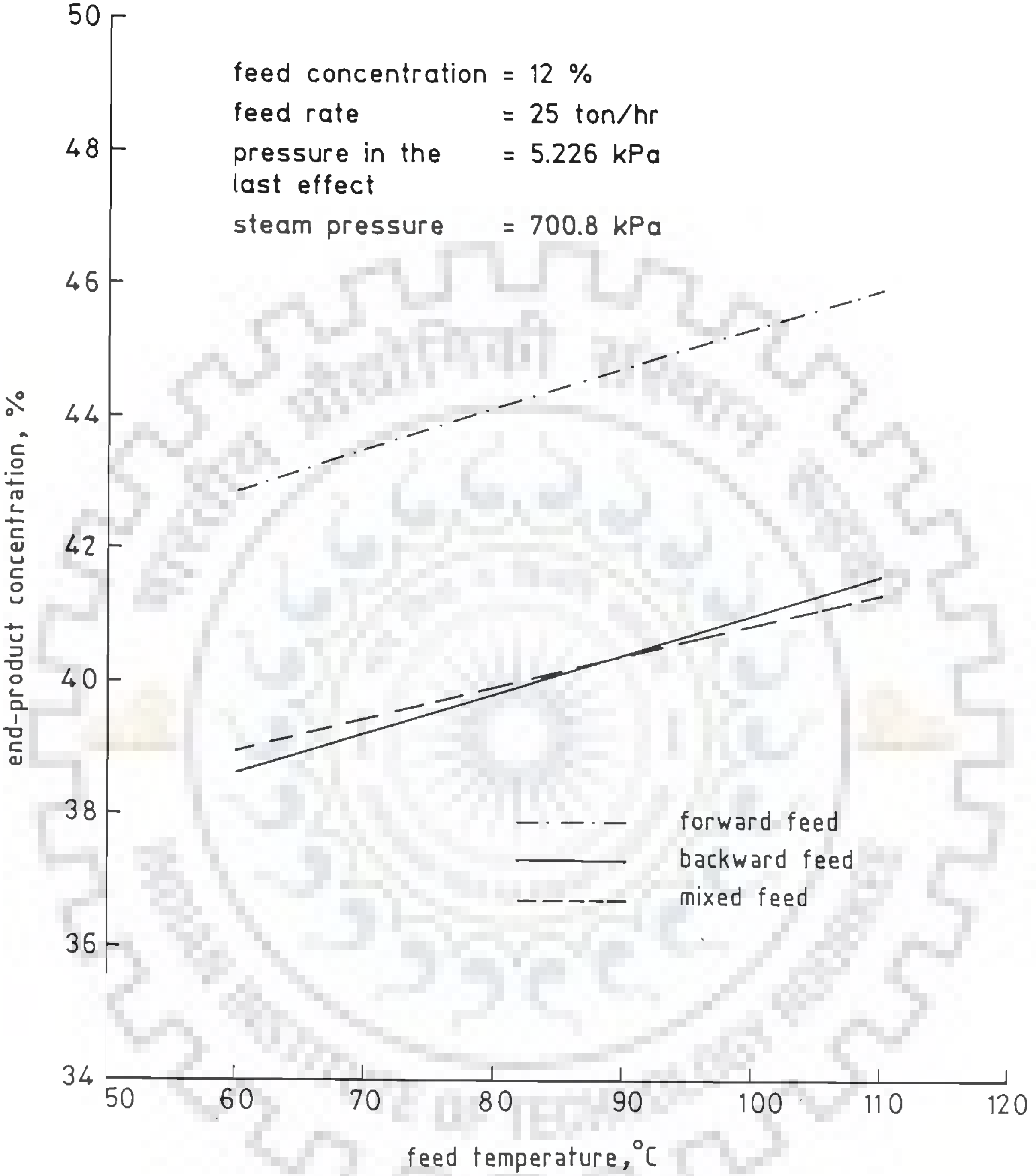

Fig. 6.18 End-product concentration versus feed temperature for a triple effect caustic soda evaporator

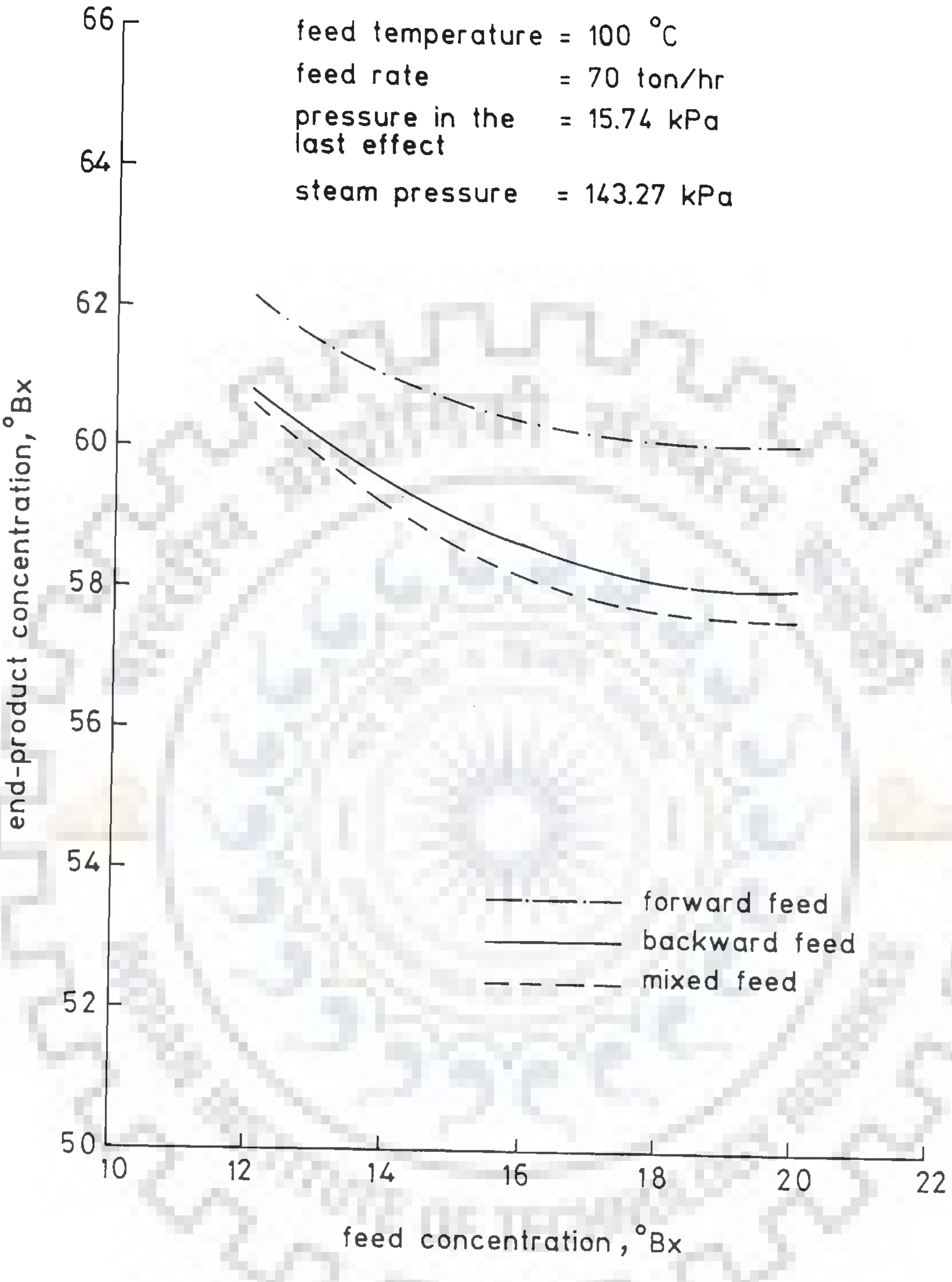

Fig. 6.19 End-product concentration versus feed concentration for a quadruple effect sugar evaporator

From this plot the following important points emerge out:

(i) An increase in feed concentration decreases the end-product concentration for a given feed arrangement.

(ii) At a given feed concentration, the end-product concentration decreases in the order as has been witnessed in the case of feed temperature i.e. from forward- to backward - to mixed- feed.

As explained in Section 6.2.2, an increase in feed concentration leads to reduce the heat load and water load. Since the decrease in former is more than that of latter, the concentration of end-product is found to decline when feed of higher concentration is employed in the evaporator.

Figure 6.20 represents a plot to demonstrate the variation of end-product concentration with feed concentration of black-liquor solution for various feed arrangements. The characteristics of this plot are as follows:

(i) For a given feed arrangement, the concentration of end-product discharging from an evaporator improves with the increase in feed concentration.

(ii) At a given feed concentration, the end-product concentration decreases when feed arrangement is shifted from forward to backward to mixed.

Rise in end-product concentration with increase in feed concentration can be explained by the fact that changes in heat load and water load in the evaporator are accompanied with feed concentration. In fact, heat transfer coefficient and boiling point rise of black-liquor solution are unique functions of feed concentration. With the increase in feed concentration, heat transfer coefficient rises and after attaining a maximum value it decreases. The boiling point of black-liquor solution does not rise uniformly with feed concentration. Both these quantities vary in such a way that the cumulative heat load in the evaporator rises with feed concentration. On the other hand, water load reduces when feed is of higher concentration. The net effect of both these leads to more evaporation of water from the solution. In this way end-product concentration of the solution is found to be improved with the increase in feed concentration.

As can be seen from Figure 6.21, the important feature for the variation of end-product concentration of caustic soda solution with feed concentration is as follows:

An increase in feed concentration decreases the concentration of end-product. This continues upto a particular feed concentration. Any further increase in the

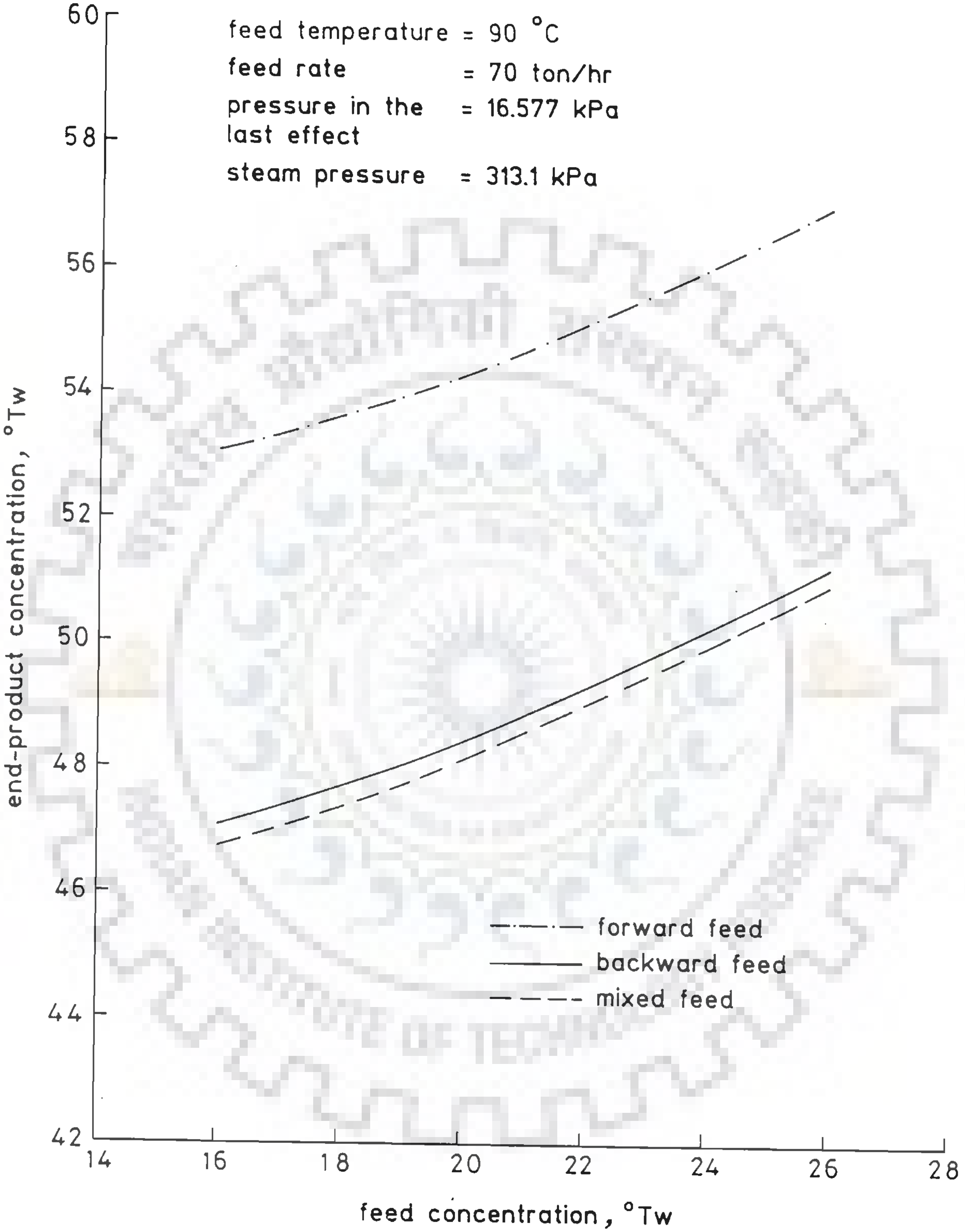

Fig. 6.20 End-product concentration versus feed concentration for a quintuple effect black-liquor evaporator

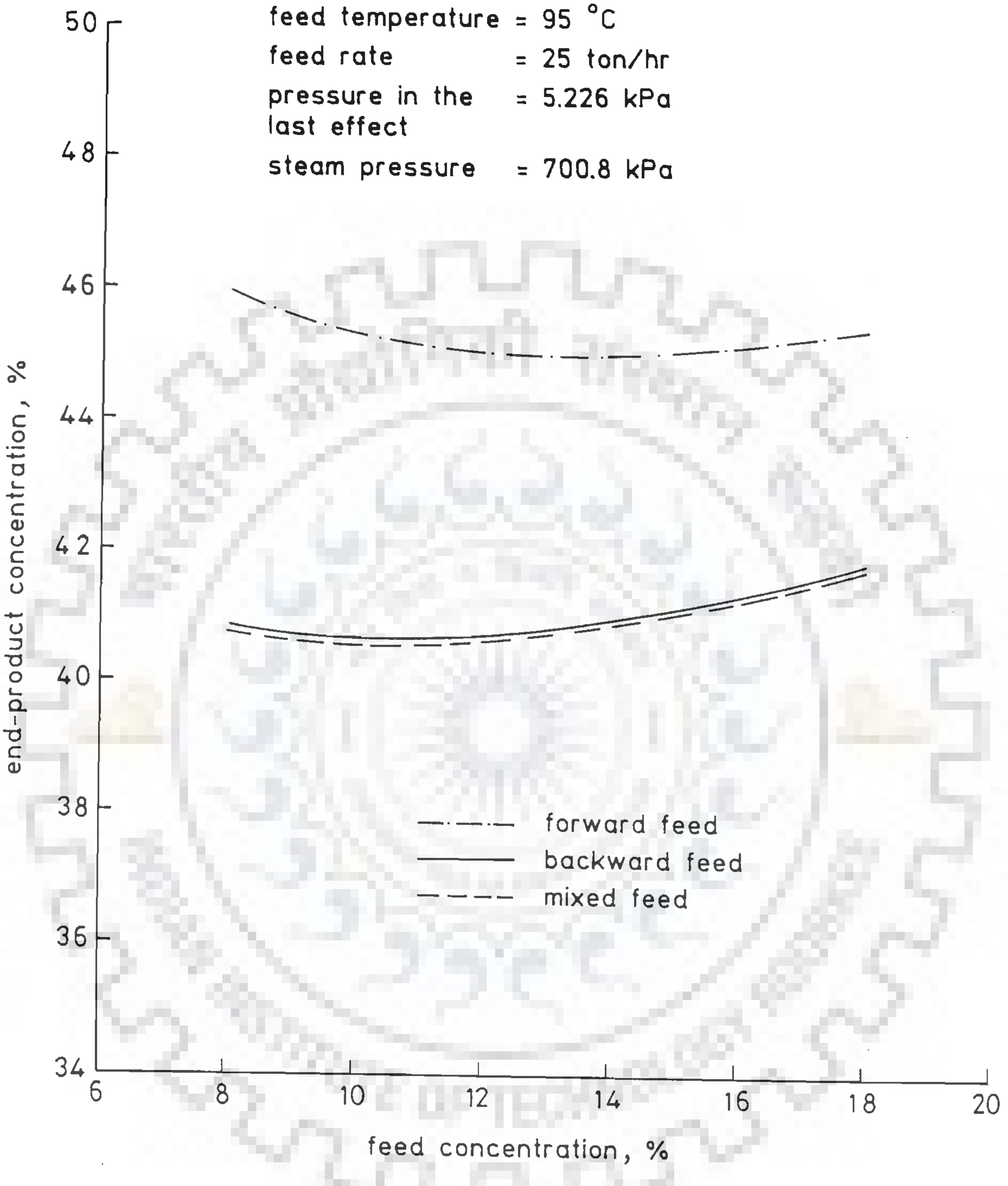

Fig. 6.21 End-product concentration versus feed concentration for a triple effect caustic soda evaporator

concentration of feed causes end-product concentration to increase. This has been found in all the feed arrangements studied in this investigation.

This behaviour is attributed to the decrease in the value of heat transfer coefficient and increase in the boiling point rise of the solution with feed concentration. Consequently, heat load in the evaporator varies with feed concentration. However, the variation depends upon the range of feed concentration. As a matter of fact, for lower range of feed concentration heat load decreases, whereas for higher range of feed concentration it increases. Besides, water load reduces with increase in feed concentration. The resultant effect of these two quantities is that for lower range of feed concentration, the end-product concentration declines whereas for higher range it increases.

6.3.3 VARIATION OF END-PRODUCT CONCENTRATION WITH FEED RATE

Figure 6.22 has been drawn to demonstrate the variation of concentration of end-product of a quadruple effect sugar evaporator with feed rate under forward -, backward -, and mixed-feed arrangement. The value of operating variables for this plot have been: feed temperature, 100°C; feed concentration, 18°Bx; pressure in the last effect, 15.74 kPa; and steam pressure, 143.27 kPa.

The following noteworthy points emerge out from this plot:

(i) An increase in feed rate decreases the concentration of the end-product from the evaporator, irrespective of feed arrangement employed.

(ii) At a given feed rate, forward feed arrangement provides the highest end-product concentration to be followed by backward-, and mixed- feed arrangement in decreasing order.

Black-liquor and caustic soda solutions also exhibit the same behaviour as can be seen from Figures 6.23 and 6.24, respectively.

Decrease in the concentration of end-product with the feed rate is attributed to the fact that both the heat load and water load are affected by it. However, increase in the amount of vapour formation is not as much as in water load because of significant increase in the amount of heat required to preheat the solution. Consequently, the amount of water in end-product increases and thereby the concentration of end-product is found to decrease with the rise in feed rate.

At a given feed rate, the amount of vapour formation decreases when feed arrangement is shifted from forward to backward to mixed owing to varying amounts of flashing and preheating in individual effects of the evaporator for various feed arrangements. Thus,

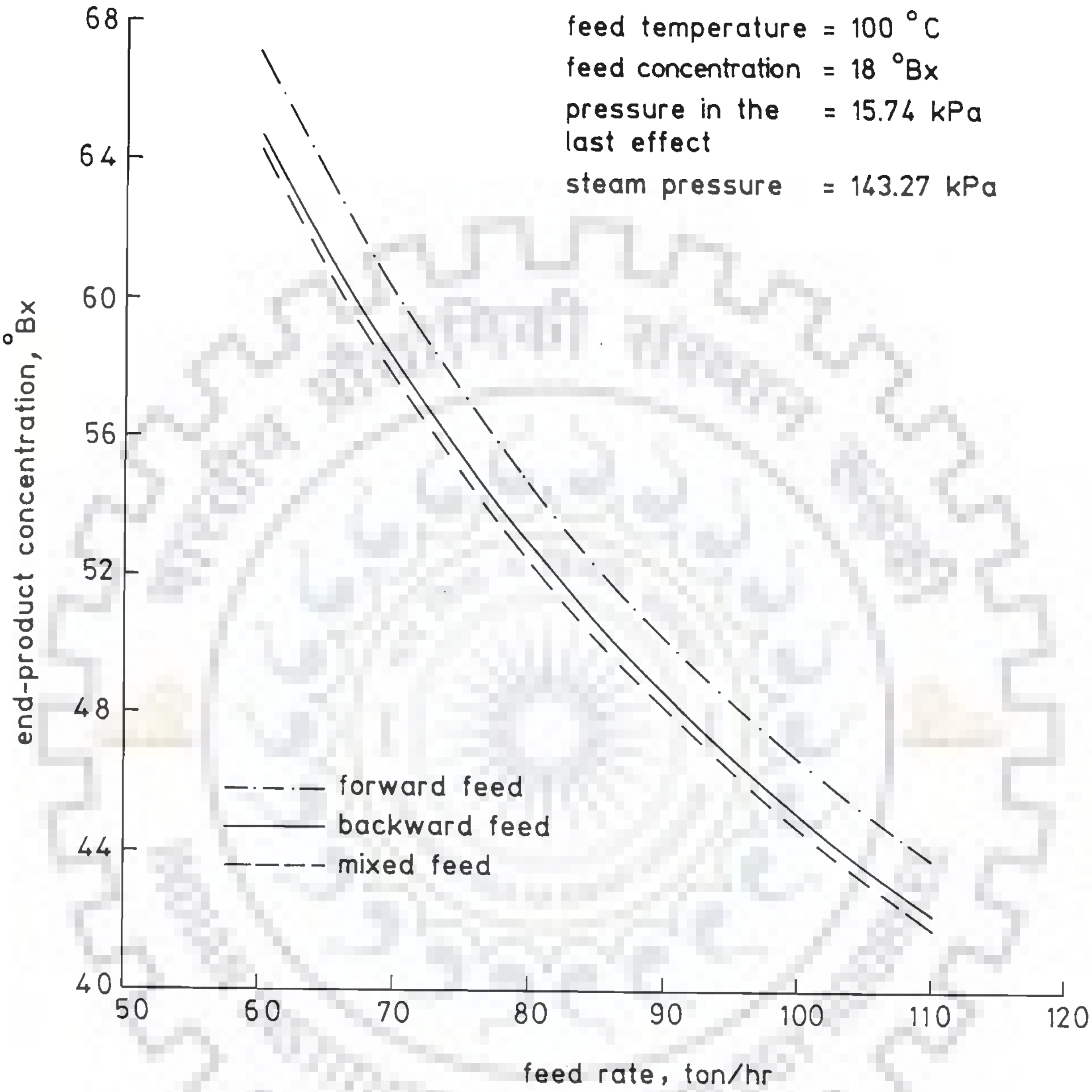

Fig. 6.22 End-product concentration versus feed rate for a quadruple effect sugar evaporator

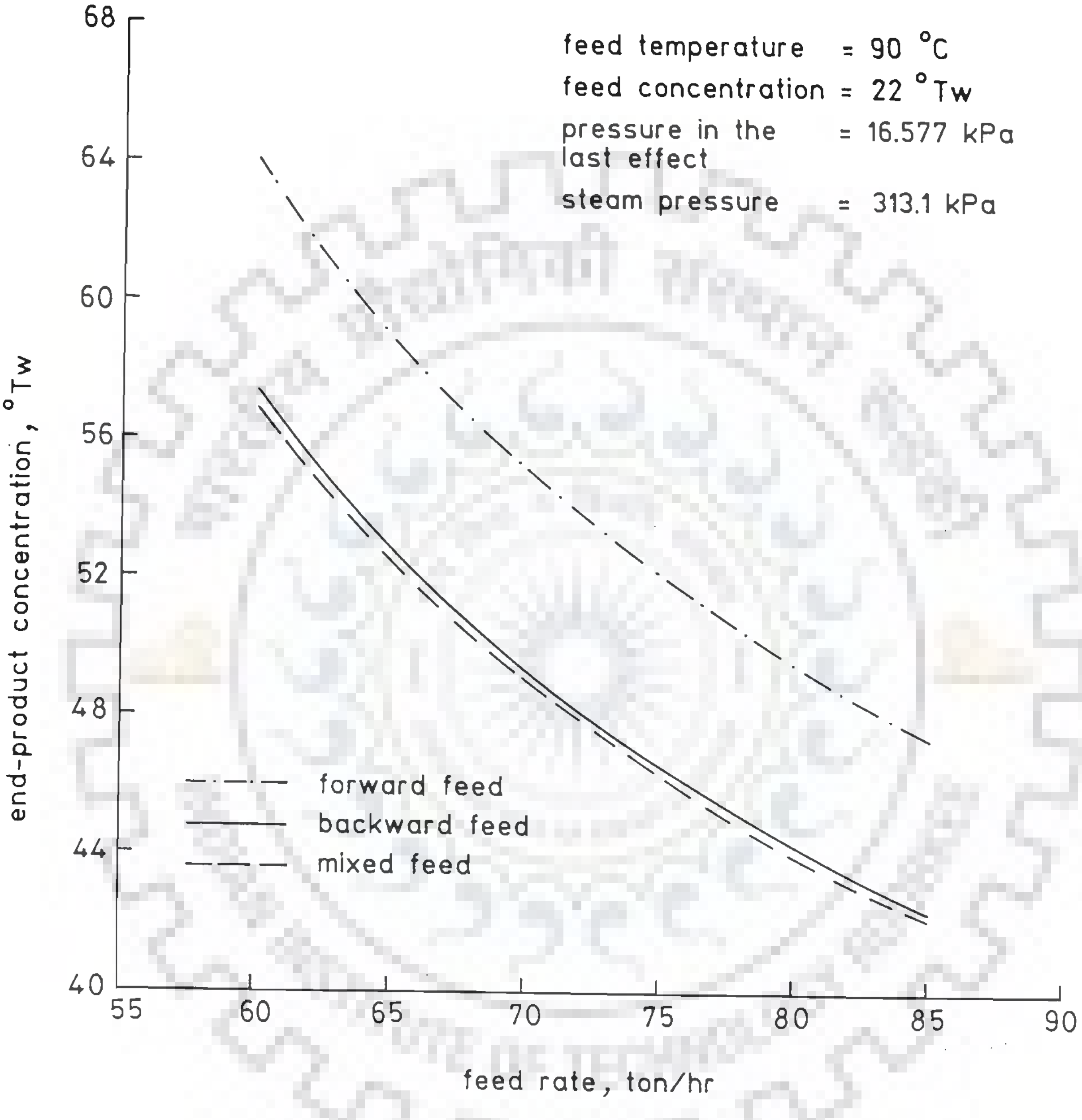

Fig. 6.23 End-product concentration versus feed rate for a quintuple effect black-liquor evaporator

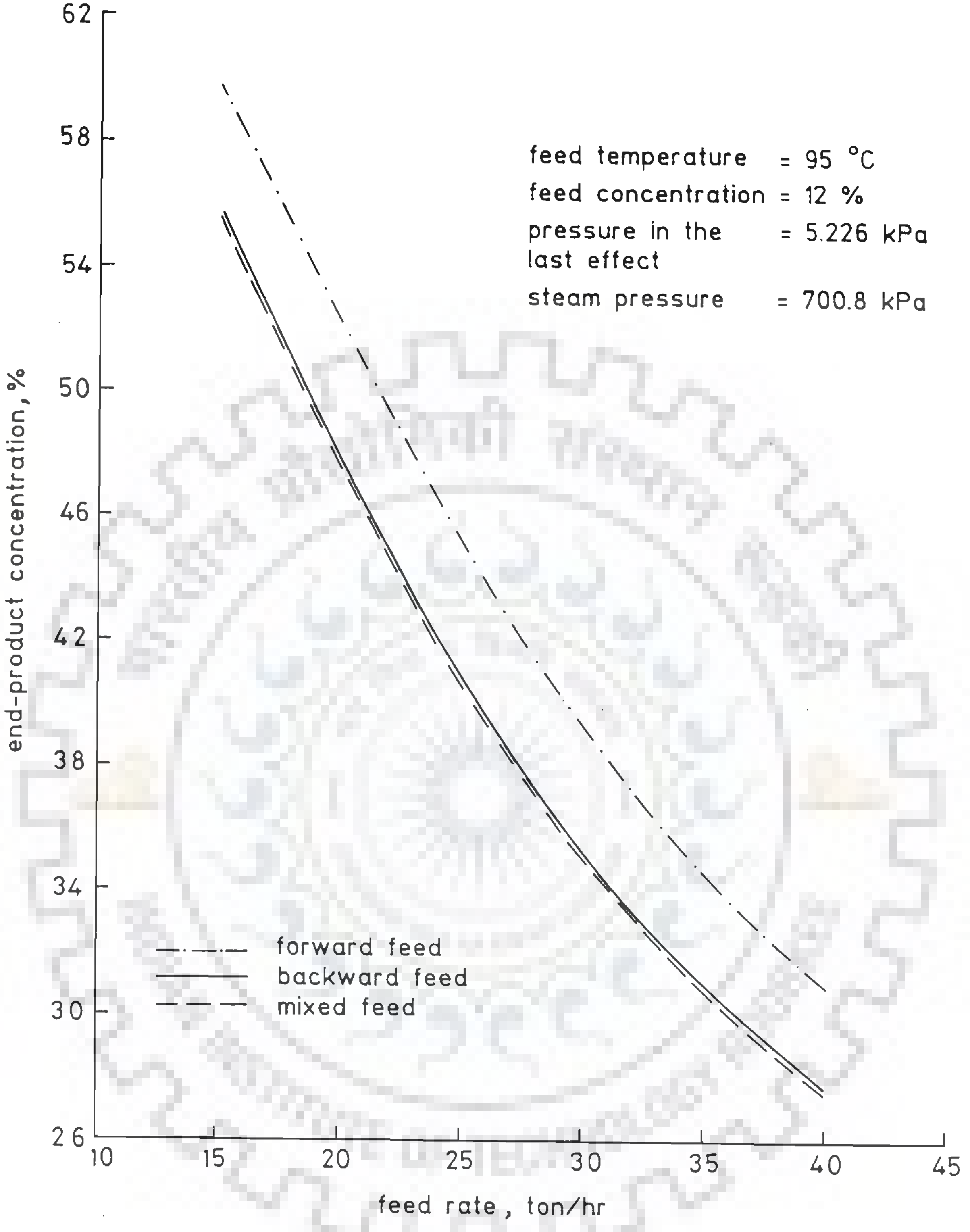

Fig. 6.24 End-product concentration versus feed rate for a triple effect caustic soda evaporator

forward feed evaporator provides the highest end-product concentration to be followed by backward and mixed feeds in decreasing order.

6.3.4 VARIATION OF END-PRODUCT CONCENTRATION WITH PRESSURE IN THE LAST EFFECT OF AN EVAPORATOR

Figure 6.25 is a plot to depict the variation of end-product concentration of a quadruple effect sugar evaporator as a function of pressure in the last effect of an evaporator for forward, backward, and mixed feed arrangements. This plot is for the operating variables; feed temperature, 100°C; feed concentration, 18°Bx; feed rate, 70 ton/hr; and steam pressure, 143.27 kPa.

An inspection of the plot reveals the following salient features:

(i) For a given feed arrangement, the concentration of end-product decreases with the increase of pressure in the last effect of an evaporator.

(ii) At a given pressure in the last effect of an evaporator, concentration of the end-product is the highest for forward feed, the lowest for mixed feed and in between the two for the backward feed arrangement.

As explained in Section 6.2.4, an increase of pressure in the last effect reduces vapour formation in all the effects of the evaporator. Thus the amount of water in the concentrated product is found to be more and thereby the concentration of end-product declines when pressure in the last effect is raised.

The features of black-liquor and caustic soda solution can be noted from Figures 6.26 and 6.27, respectively. They resemble to those of sugar solution, already discussed above.

6.3.5 VARIATION OF END- PRODUCT CONCENTRATION WITH STEAM PRESSURE

To demonstrate the effect of steam pressure on the concentration of end-product discharging from a quadruple effect evaporator for sugar solution, Figure 6.28 has been drawn. This figure contains the curves for forward-, backward-, and mixed- feed arrangement. This plot refers to the operating variables: feed temperature, 100°C; feed concentration, 18°Bx; feed rate, 70 ton/hr; and pressure in the last effect, 15.74 kPa.

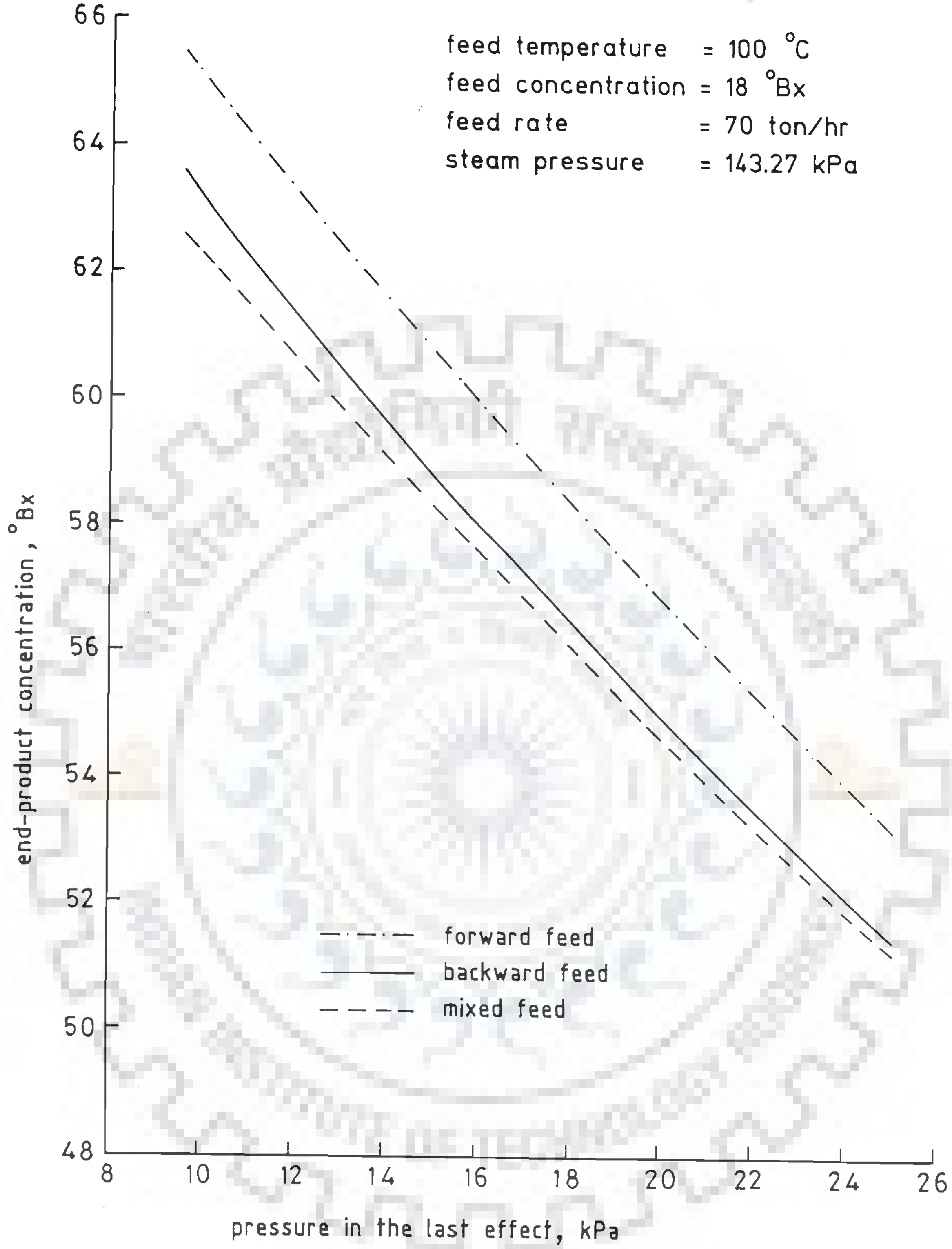

Fig. 6.25 End-product concentration versus pressure in the last effect of a quadruple effect sugar evaporator

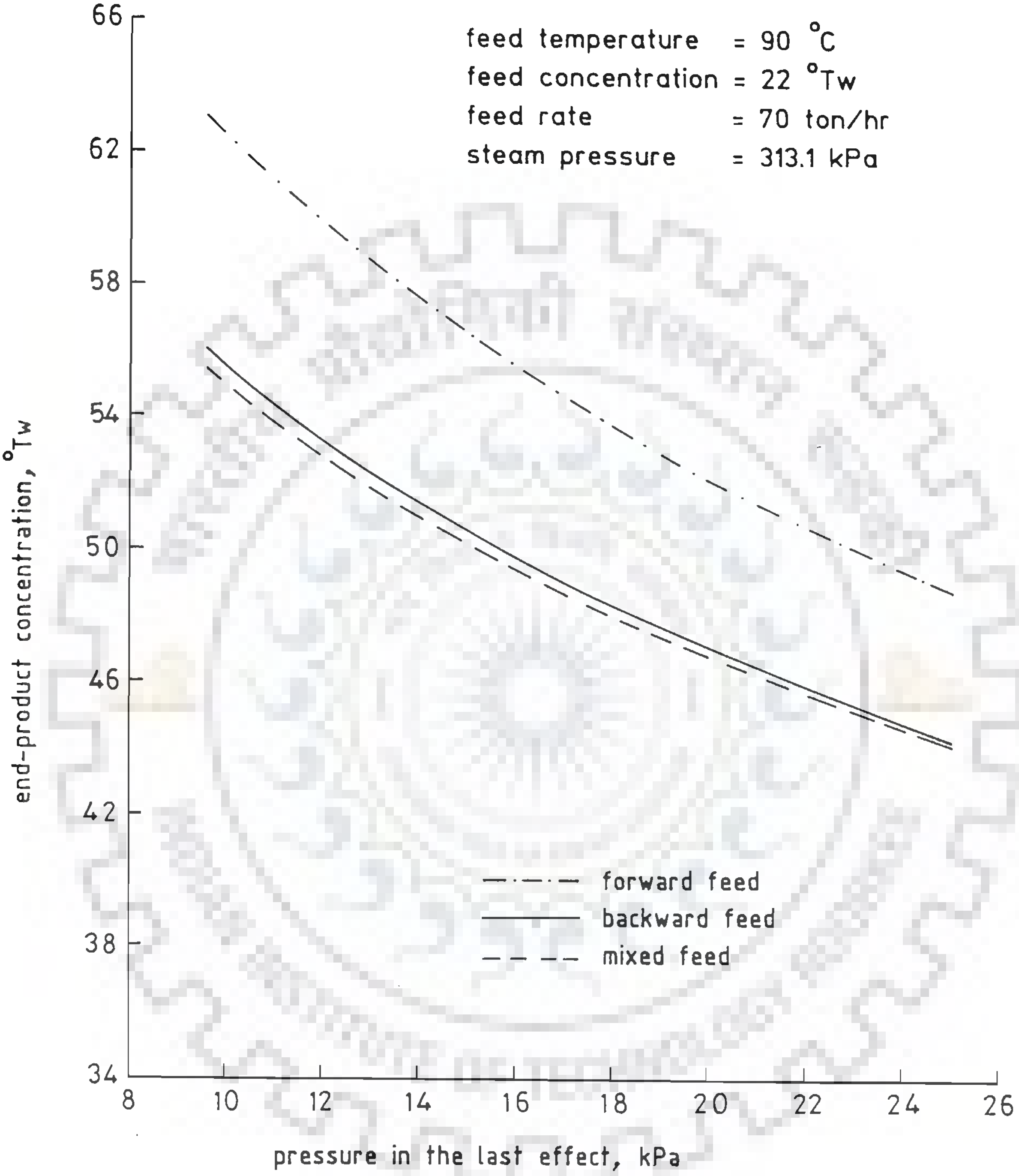

Fig. 6.26 End-product concentration versus pressure in the last effect of a quintuple effect black-liquor evaporator

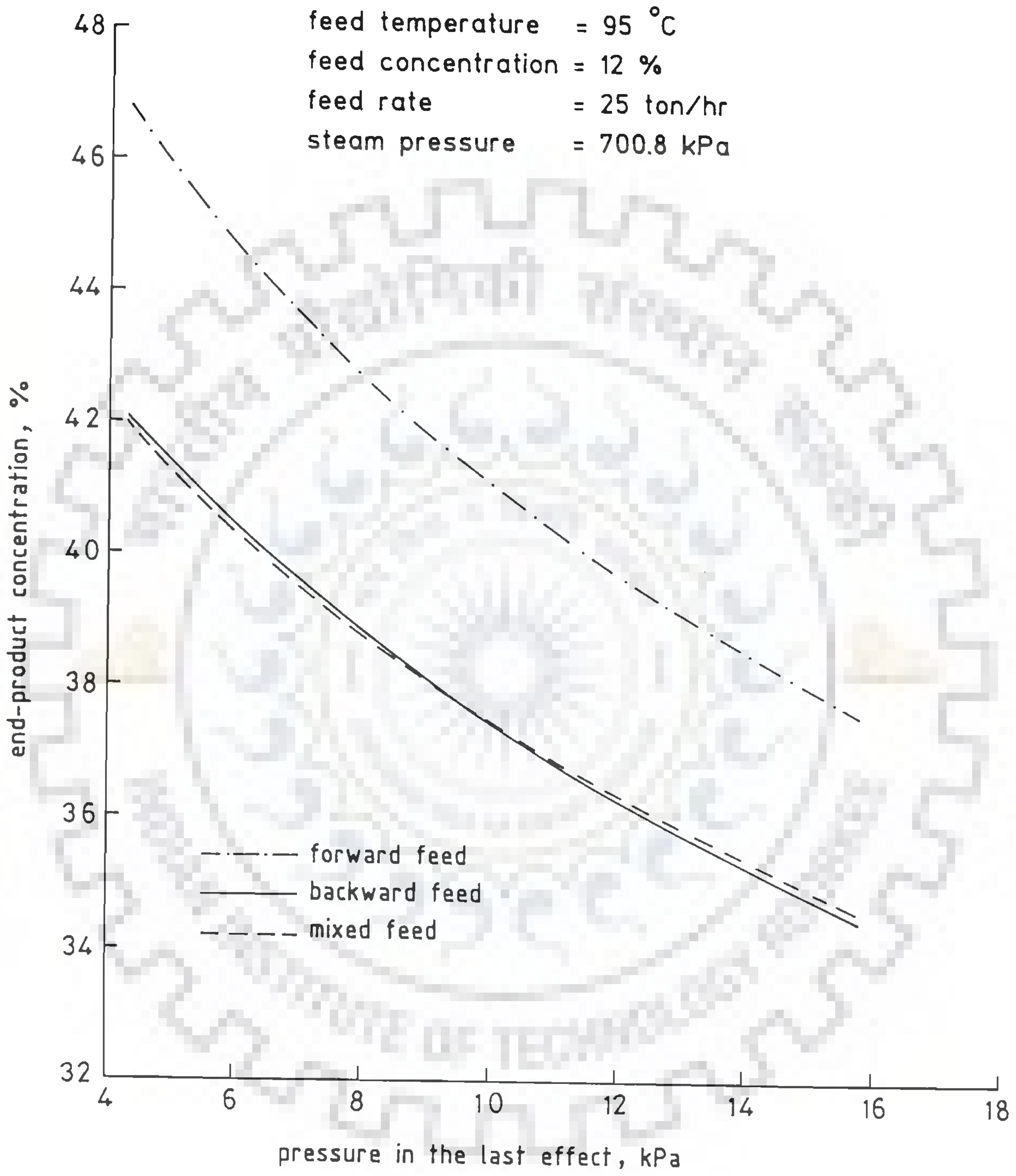

Fig. 6.27 End-product concentration versus pressure in the last effect of a triple effect caustic soda evaporator

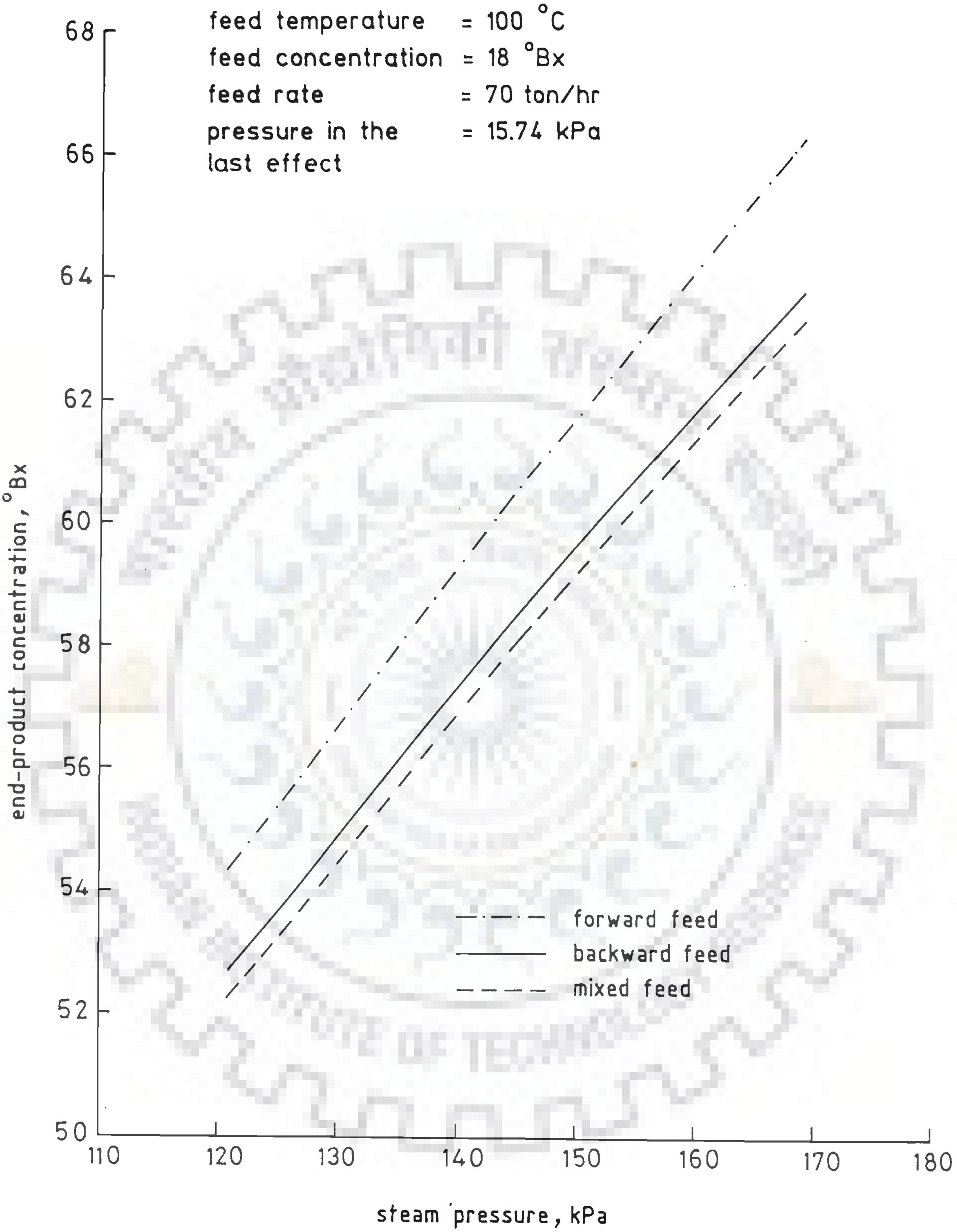

Fig. 6.28 End-product concentration versus steam pressure for a quadruple effect sugar evaporator

From this plot the following main points emerge out:

(i) An increase in steam pressure raises the concentration of the end-product. This holds true for all the feed arrangements of this investigation.

(ii) At a given steam pressure, the sequence of the feed arrangement in which the end-product concentration decreases is from forward to backward to mixed.

Increase in end-product concentration with steam pressure can be explained by the fact that an increase in steam pressure leads to increase the heat load of the evaporator which, in turn, give rise to more vapour formation. This has been explained in detail in Section 6.2.5. Hence, the amount of water evaporated from the solution increases and thus the concentration of end-product is observed to be higher when steam of elevated pressure is used.

As depicted in Figures 6.29 and 6.30 for black-liquor and caustic soda solutions, respectively, both the solutions have the same features as that of sugar solution.

6.3.6 CORRELATION FOR END- PRODUCT CONCENTRATION OF AN EVAPORATOR

In order to demonstrate the quantitative effect of operating variables on end-product concentration, various correlations of end-product concentration have been developed. These pertain to end-product concentration of aqueous solutions of sugar, black-liquor, and caustic soda evaporating in quadruple-, quintuple-, and triple- effect evaporator, respectively, with forward, backward, and mixed feeds. Using the data of Appendix-E, following correlations of end-product concentrations have been obtained by the use of multiple linear regression analysis. The maximum deviation from its mean value has also been indicated for each correlation.

a. Forward feed arrangement

Sugar solution

$$X_p = 40.038\tau_f^{0.083} X_f^{-0.061} F^{-0.704} T_1^{-0.565} T_s^{2.195} \qquad \ldots(6.11)$$

Maximum deviation = $\pm 3.59\%$

Black-liquor solution

$$X_p = 46.960\tau_f^{0.231} X_f^{0.149} F^{-0.861} T_1^{-0.745} T_s^{2.298} \qquad \ldots(6.12)$$

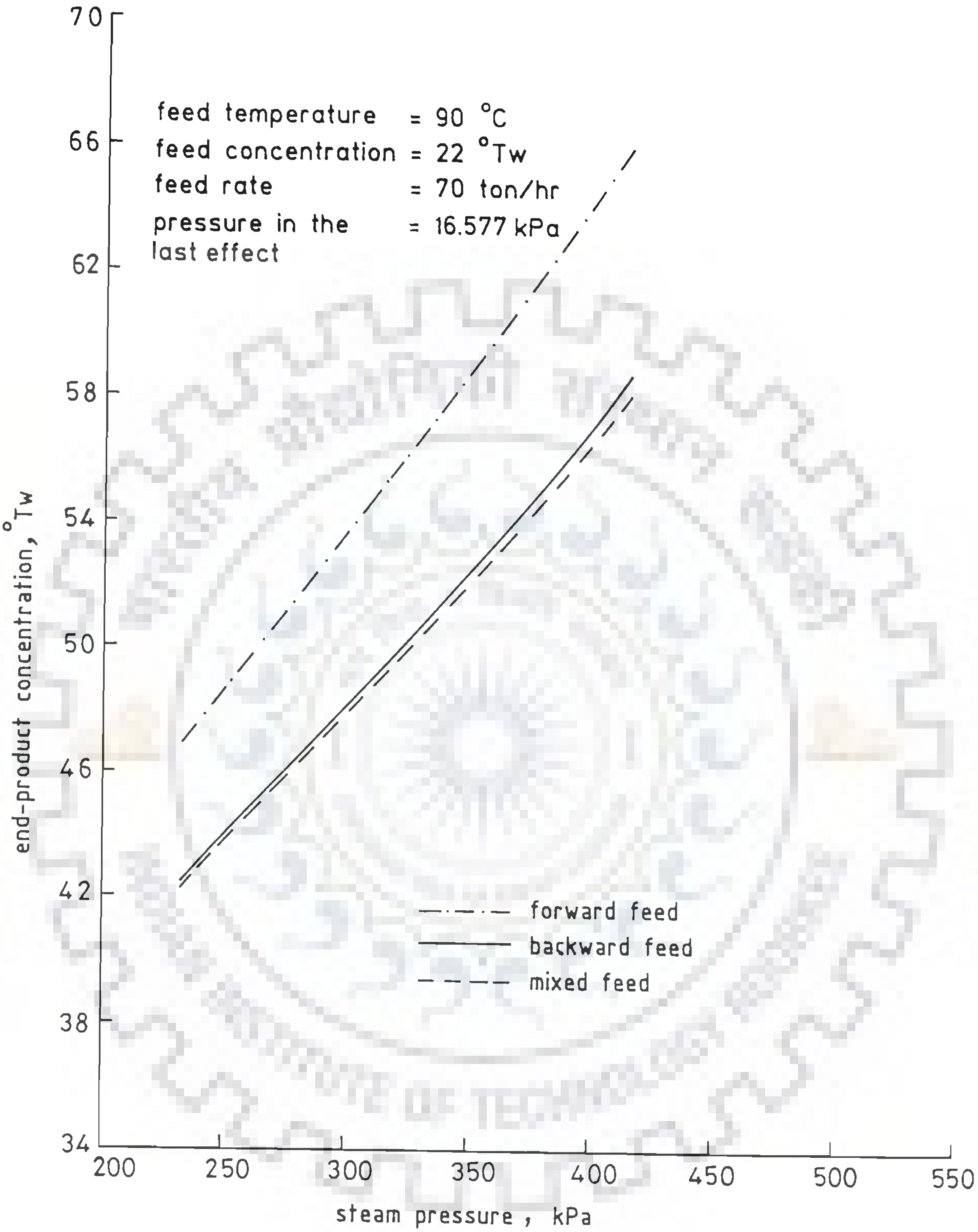

Fig. 6.29 End-product concentration versus steam pressure for a quintuple effect black-liquor evaporator

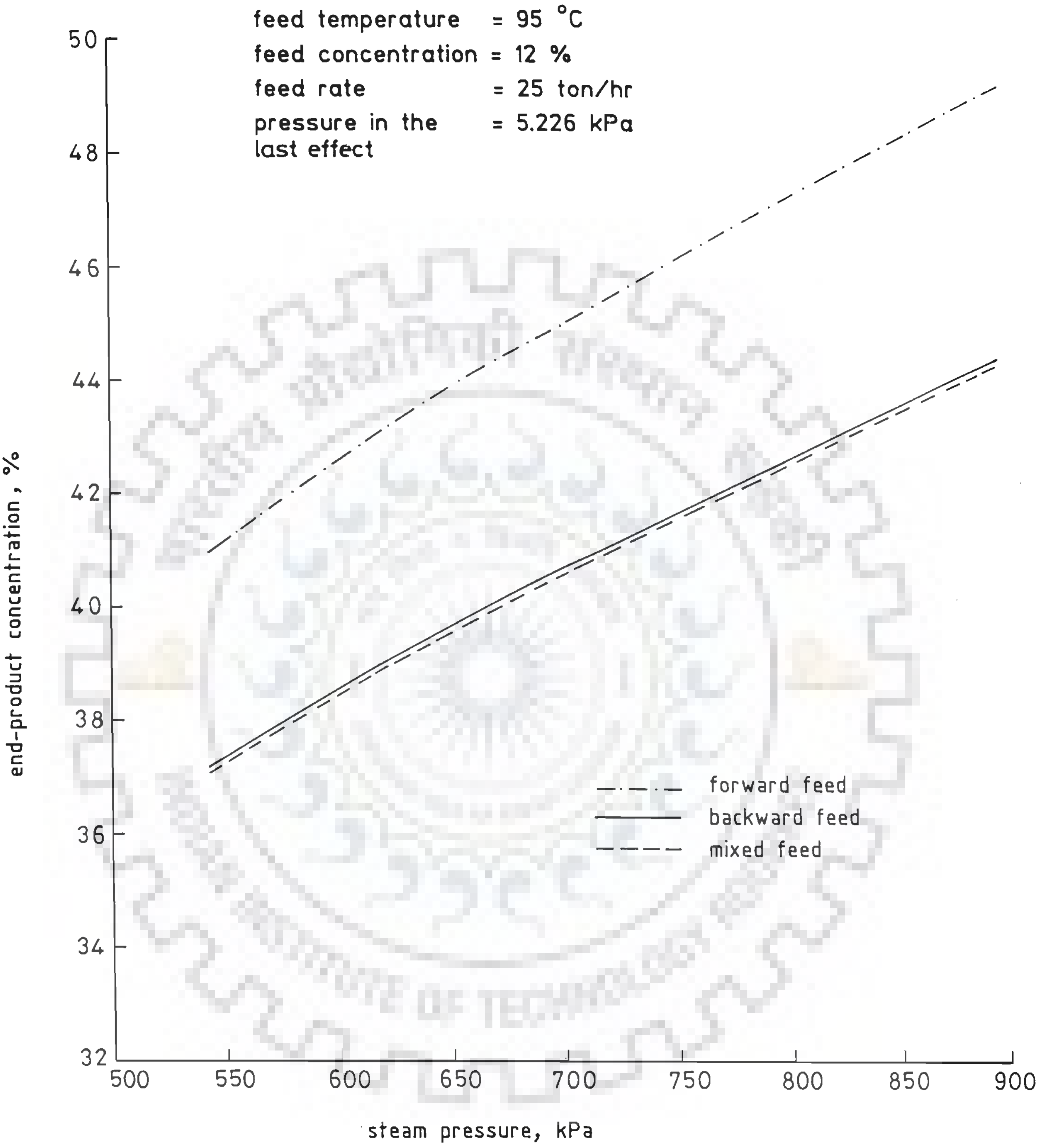

Fig. 6.30 End-product concentration versus steam pressure for a triple effect caustic soda evaporator

Maximum deviation = $\pm 4.46\%$

Caustic soda solution

$$X_p = 9.346\tau_f^{0.113}X_f^{-0.015}F^{-0.524}T_1^{-0.364}T_s^{1.506} \quad \ldots(6.13)$$

Maximum deviation = $\pm 3.75\%$

b. Backward feed arrangement

Sugar solution

$$X_p = 58.810\tau_f^{0.101}X_f^{-0.094}F^{-0.706}T_1^{-0.571}T_s^{2.118} \quad \ldots(6.14)$$

Maximum deviation = $\pm 4.55\%$

Black-liquor solution

$$X_p = 104.840\tau_f^{0.180}X_f^{0.175}F^{-0.864}T_1^{-0.679}T_s^{2.095} \quad \ldots(6.15)$$

Maximum deviation = $\pm 4.95\%$

Caustic soda solution

$$X_p = 67.880\tau_f^{0.097}X_f^{0.029}F^{-0.726}T_1^{-0.330}T_s^{1.469} \quad \ldots(6.16)$$

Maximum deviation = $\pm 4.20\%$

c. Mixed feed arrangement

Sugar solution

$$X_p = 107.650\tau_f^{0.082}X_f^{-0.101}F^{-0.705}T_1^{-0.539}T_s^{1.982} \quad \ldots(6.17)$$

Maximum deviation = $\pm 5.26\%$

Black-liquor solution

$$X_p = 69.270\tau_f^{0.188}X_f^{0.175}F^{-0.850}T_1^{-0.652}T_s^{2.115} \quad \ldots(6.18)$$

Maximum deviation = ±4.34%

Caustic soda solution

$$X_p = 61.435\tau_f^{0.098}X_f^{0.031}F^{-0.725}T_l^{-0.321}T_s^{1.478} \qquad ...(6.19)$$

Maximum deviation = ±5.10%

Eqs(6.11 to 6.19) can be written in the following form to represent a generalized correlation of end-product concentration with operating variables, namely; feed temperature, feed concentration, feed rate, pressure in the last effect, and steam pressure:

$$X_p = K\,\tau_f^{p}\,X_f^{q}\,F^{r}\,T_l^{s}\,T_s^{t} \qquad ...(6.20)$$

Where the values of constant, K and the exponents p, q, r, s, and t depend upon the aqueous solution, feed arrangement, number of effects in the evaporator, and the values of operating variables. The values of constant K and exponents p, q, r, s, and t are given in Table 6.8.

Table 6.8 Values of constant, K and the exponents p, q, r, s, and t in Eq(6.20) for different aqueous solutions and feed arrangements

Aqueous solution	Constant, K	Value of exponents				
		p	q	r	s	t
			Forward feed arrangement			
Sugar	40.038	0.083	-0.061	-0.704	-0.565	2.195
Black-liquor	46.960	0.231	0.149	-0.861	-0.745	2.298
Caustic soda	9.346	0.113	-0.015	-0.524	-0.364	1.506
			Backward feed arrangement			
Sugar	58.810	0.101	-0.094	-0.706	-0.571	2.118
Black-liquor	104.840	0.180	0.175	-0.864	-0.679	2.095
Caustic soda	67.880	0.097	0.029	-0.726	-0.330	1.469
			Mixed feed arrangement			
Sugar	107.650	0.082	-0.101	-0.705	-0.539	1.982
Black-liquor	69.270	0.188	0.175	-0.850	-0.652	2.115
Caustic soda	61.435	0.098	0.031	-0.725	-0.321	1.478

Eq(6.20) is a simple and convenient equation to determine the concentration of end-product from evaporators from the knowledge of the values of the operating variables, viz; feed temperature, feed concentration, feed rate, pressure in the last effect, and steam pressure and the values of the constant, K and exponents p, q, r, s, and t for a given aqueous solution and the feed arrangement. However, it should be borne in mind that Eq(6.20) is applicable to only those values of the operating variables which lie within the range used in this investigation. An extrapolation of it will lead to erroneous determination of end-product concentration. Hence, sufficient care must be taken while using Eq(6.20) as regards to the values of the operating variables, feed arrangement and the solution to be evaporated in a multiple effect evaporator.

6.4 A PROCEDURE TO COUNTERACT ANY CHANGE IN END-PRODUCT CONCENTRATION

The forgoing Section has clearly demonstrated that a change in the value of one or more operating variables affects end-product concentration considerably. Usually operating variables undergo changes due to unforeseen reasons beyond the control of plant engineers and consequently the concentration of end-product suffers a change. This is undesirable and uneconomical as it upsets the evaporator plant capacity and steam consumption. Moreover, in many situations maintenance of the concentration of end-product at a predetermined level is necessary to meet the technological requirements of the process. This calls for the readjustment of remaining variables in such a way that the change in end-product concentration is nullified without adversely affecting the steam economy of the evaporator. This is essential to ensure the minimum expenditure of steam in the evaporator. Following paragraphs have been devoted to describe a procedure for the above:

Let the initial values of operating variables namely; feed temperature, feed concentration, feed rate, pressure (temperature) in the last effect, and steam pressure (saturation temperature) in an evaporator for a given aqueous solution and feed arrangement be τ_f, X_f, F, T_l, and T_s, respectively. The end-product concentration of the evaporator, X_p is given by the Eq(6.20), which is rewritten below:

$$X_p = K\,\tau_f^{\,p}\,X_f^{\,q}\,F^r\,T_l^{\,s}\,T_s^{\,t} \qquad \ldots(6.20)$$

The new values of operating variables are τ'_f, X'_f, F', T'_l, and T'_s. In other words

$$\tau'_f = \tau_f + \Delta\,\tau_f,$$
$$X'_f = X_f + \Delta\,X_f,$$
$$F' = F + \Delta\,F,$$
$$T'_l = T_l + \Delta\,T_l, \text{ and}$$
$$T'_s = T_s + \Delta\,T_s.$$

Where $\Delta\tau_f$, ΔX_f, ΔF, ΔT_l, and ΔT_s represent changes in the values of τ_f, X_f, F, T_l, and T_s, respectively.

The modified value of end-product concentration, X'_p is related to the new values of operating variables by the following equation:

$$X'_p = K\,\tau'^{\,p}_f\,X'^{\,q}_f\,F'^{\,r}\,T'^{\,s}_l\,T'^{\,t}_s \qquad \text{or}$$

$$X_p' = K(\tau_f + \Delta\tau_f)^p.(X_f + \Delta X_f)^q.(F + \Delta F)^r.(T_l + \Delta T_l)^s.(T_s + \Delta T_s)^t \qquad \ldots(6.21)$$

Each term of the right hand side of Eq(6.21) can be written in the following from by the application of Taylor series expansion:

$$(\tau_f + \Delta\tau_f)^p = \tau_f^p[1 + p(\Delta\tau_f/\tau_f) + \{p(p-1)/2\}(\Delta\tau_f/\tau_f)^2 + \;.\;.\; + (\Delta\tau_f/\tau_f)^p] \qquad \ldots(6.22a)$$

$$(X_f + \Delta X_f)^q = X_f^q[1 + q(\Delta X_f/X_f) + \{q(q-1)/2\}(\Delta X_f/X_f)^2 + \;.\;.\; + (\Delta X_f/X_f)^q] \qquad \ldots(6.22b)$$

$$(F + \Delta F)^r = F^r[1 + r(\Delta F/F) + \{r(r-1)/2\}(\Delta F/F)^2 + \;.\;.\; + (\Delta F/F)^r] \qquad \ldots(6.22c)$$

$$(T_l + \Delta T_l)^s = T_l^s[1 + s(\Delta T_l/T_l) + \{s(s-1)/2\}(\Delta T_l/T_l)^2 + \;.\;.\; + (\Delta T_l/T_l)^s] \qquad \ldots(6.22d)$$

$$(T_s + \Delta T_s)^t = T_s^t[1 + t(\Delta T_s/T_s) + \{t(t-1)/2\}(\Delta T_s/T_s)^2 + \;.\;.\; + (\Delta T_s/T_s)^t] \qquad \ldots(6.22e)$$

$(\Delta\tau_f/\tau_f)$, $(\Delta X_f/X_f)$, $(\Delta F/F)$, $(\Delta T_l/T_l)$, and $(\Delta T_s/T_s)$ denote fractional change in the value of feed temperature, feed concentration, feed rate, temperature (pressure) in the last effect, and steam saturation temperature (pressure), respectively. The magnitude of these changes is usually small. Therefore, the magnitude of second and other higher order terms of Eqs(6.22a--6.22e) can be neglected without any significant loss of accuracy. Incorporation of this into Eqs(6.22a -- 6.22e) leads to the following equations:

$$(\tau_f + \Delta\tau_f)^p = \tau_f^p\,[1 + p(\Delta\tau_f/\tau_f)] \qquad \ldots(6.23a)$$

$$(X_f + \Delta X_f)^q = X_f^q\,[1 + q(\Delta X_f/X_f)] \qquad \ldots(6.23b)$$

$$(F + \Delta F)^r = F^r\,[1 + r\,(\Delta F/F)] \qquad \ldots(6.23c)$$

$$(T_l + \Delta T_l)^s = T_l^s\,[1 + s\,(\Delta T_l/T_l)] \qquad \ldots(6.23d)$$

$$(T_s + \Delta T_s)^t = T_s^t\,[1 + t\,(\Delta T_s/T_s)] \qquad \ldots(6.23e)$$

Substitution of Eqs(6.23a--6.23e) in Eq (6.21) provides the following expression:

$$X'_p = K\,\tau_f^p\,X_f^q\,F^r\,T_l^s\,T_s^t[\{1+p\,(\Delta\tau_f/\tau_f)\}\{1+q\,(\Delta X_f/X_f)\}\{1+r\,(\Delta F/F)\}$$
$$\{1+s\,(\Delta T_l/T_l)\}\{1+t\,(\Delta T_s/T_s)\}] \qquad \ldots(6.24)$$

To satisfy the condition of no change in the concentration of end-product , the value of concentration obtained from Eq(6.24) is equated to that determined by Eq (6.20). This gives the following equation:

$$K\,\tau_f^p\,X_f^q\,F^r\,T_l^s\,T_s^t = K\,\tau_f^p\,X_f^q\,F^r\,T_l^s\,T_s^t[\{1+p\,(\Delta\tau_f/\tau_f)\}\{1+q\,(\Delta X_f/X_f)\}$$
$$\{1+r\,(\Delta F/F)\}\{1+s\,(\Delta T_l/T_l)\}\,\{1+t\,(\Delta T_s/T_s)\}]$$

or

$$1 = \{1+p\,(\Delta\tau_f/\tau_f)\}\{1+q\,(\Delta X_f/X_f)\}\{1+r(\Delta F/F)\}\{1+s\,(\Delta T_l/T_l)\}\{1+t(\Delta T_s/T_s)\}$$

or

$$\{1+p(\Delta\tau_f/\tau_f)\}^{-1}\{1+q(\Delta X_f/X_f)\}^{-1} = \{1+r(\Delta F/F)\}\{1+s(\Delta T_l/T_l)\}\{1+t\,(\Delta T_s/T_s)\} \qquad \ldots(6.25)$$

Use of binomial expansion to Eq(6.25) results the following:

$$\{1-p(\Delta\tau_f/\tau_f)\}\{1-q(\Delta X_f/X_f)\} = \{1+r(\Delta F/F)\}\{1+s(\Delta T_l/T_l)\}\{1+t(\Delta T_s/T_s)\}$$

or

$$[1-q(\Delta X_f/X_f) - p(\Delta\tau_f/\tau_f)+pq(\Delta\tau_f/\tau_f)(\Delta X_f/X_f)] = 1+r(\Delta F/F)+s(\Delta T_l/T_l)+rs(\Delta F/F)(\Delta T_l/T_l)$$
$$+t(\Delta T_s/T_s)+rt(\Delta F/F)(\Delta T_s/T_s)$$
$$+\,st(\Delta T_l/T_l)(\Delta T_s/T_s)$$
$$+\,rst(\Delta F/F)(\Delta T_l/T_l)(\Delta T_s/T_s) \qquad \ldots(6.26)$$

The last term of right hand side of Eq(6.26) is of insignificant magnitude as it represents a multiplication of three fractional quantities; $(\Delta F/F)$,$(\Delta T_l/T_1)$ and $(\Delta T_S/T_S)$. Hence its omission is not liable to affect Eq(6.26) significantly. This reduces Eq(6.26) to the following form:

$$[p(\Delta\tau_f/\tau_f) + q(\Delta X_f/X_f) + r(\Delta F/F)+s(\Delta T_l/T_l)+ t(\Delta T_s/T_s)]$$
$$= pq(\Delta\tau_f/\tau_f)(\Delta X_f/X_f) - rs(\Delta F/F)(\Delta T_l/T_l) - st(\Delta T_l/T_l)(\Delta T_s/T_s) - rt(\Delta F/F)\,(\Delta T_s/T_s) \qquad \ldots(6.27)$$

Eq(6.27) is the resultant equation that can be used to determine change in the value of one or more operating variables that might be necessary to counteract an alteration in the end-product concentration caused by upset in some of operating variables.

To demonstrate the usefulness of Eq(6.27), a plot between $(\Delta X_f/X_f)$ and $(\Delta\tau_f/\tau_f)$ has been made with $(\Delta F/F)$ as a parameter for the evaporation of aqueous sugar solution in quadruple effect forward feed evaporator. It is based on the assumption that there is no change in values of T_l and T_s. This is shown in Figure 6.31. Figures 6.32 and 6.33 represent the similar plots, but for the parameter $(\Delta T_l/T_l)$ and $(\Delta T_s/T_s)$ respectively.

Using the above plots, the alteration in one or more operating variables can be determined for the given change in the values of other operating variables so that end-product concentration is maintained at a predetermined value.

It is interesting to note that the right hand side of Eq(6.27) becomes zero if the fractional changes in operating variables are of small magnitude. The equation so obtained, in fact, is the result of total differentiation of Eq(6.20) under the condition of no change in end-product concentration. Thus, Eq(6.27) is a general expression for deviation in the values of operating variables so that the concentration of end-product of a multiple effect evaporator does not alter.

Eq(6.27) does not provide any information about the change in steam economy of the evaporator that might occur due to modified values of the operating variables. Further, it does not specify the variable whose value should be altered so that steam economy of the evaporator does not suffer.

From the preceding Figures, changes in the values of operating variables can be determined. As mentioned above, Eq(6.27) does not give any information about the alteration in steam economy that arises due to change in the values of operating variables. Therefore, it is essential that Eq(6.10) for steam economy be treated to obtain the deviation in steam economy as a function of the changes in the operating variables. For this purpose, Eq (6.10) is treated in the manner analogous to Eq (6.20) to give the following expression of steam economy, E':

$$E' = C\,\tau_f^a X_f^b F^c T_l^d T_s^e \{1 + a(\Delta\tau_f/\tau_f)\}\{1 + b(\Delta X_f/X_f)\}\{1 + c(\Delta F/F)\}$$
$$\{1 + d(\Delta T_l/T_l)\}\{1 + e(\Delta T_s/T_s)\} \qquad \ldots(6.28)$$

$E' = E + \Delta E$, where ΔE represents a change in steam economy. This provides the following:

$$[(E + \Delta E)/E] = [\{1 + a(\Delta\tau_f/\tau_f)\}\{1 + b(\Delta X_f/X_f)\}\{1 + c(\Delta F/F)\}$$
$$\{1 + d(\Delta T_l/T_l)\}\{1 + e(\Delta T_s/T_s)\}] \qquad \ldots(6.29)$$

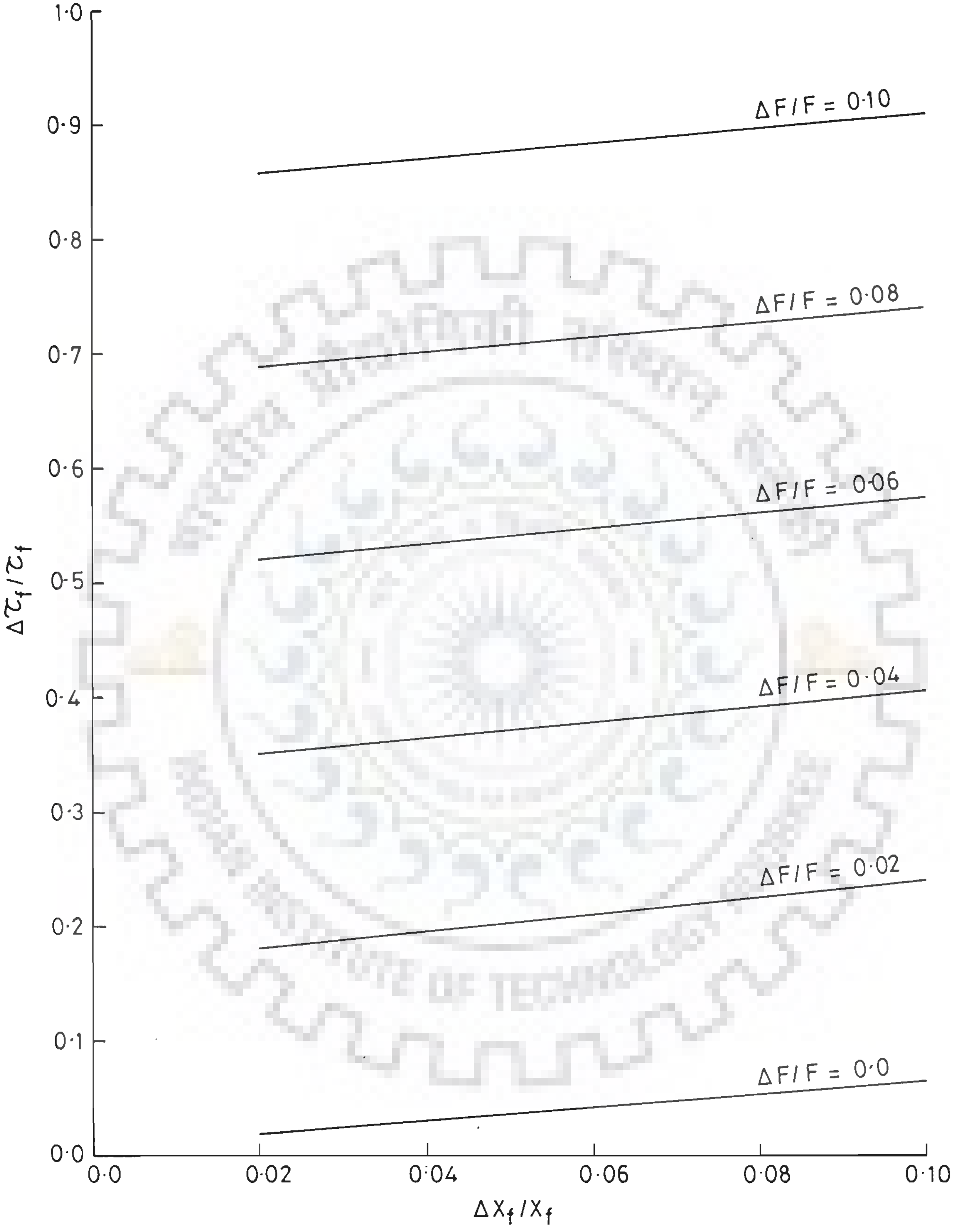

Fig. 6.31 $\Delta\tau_f/\tau_f$ versus $\Delta X_f/X_f$ with ΔF/F as a parameter for sugar solution

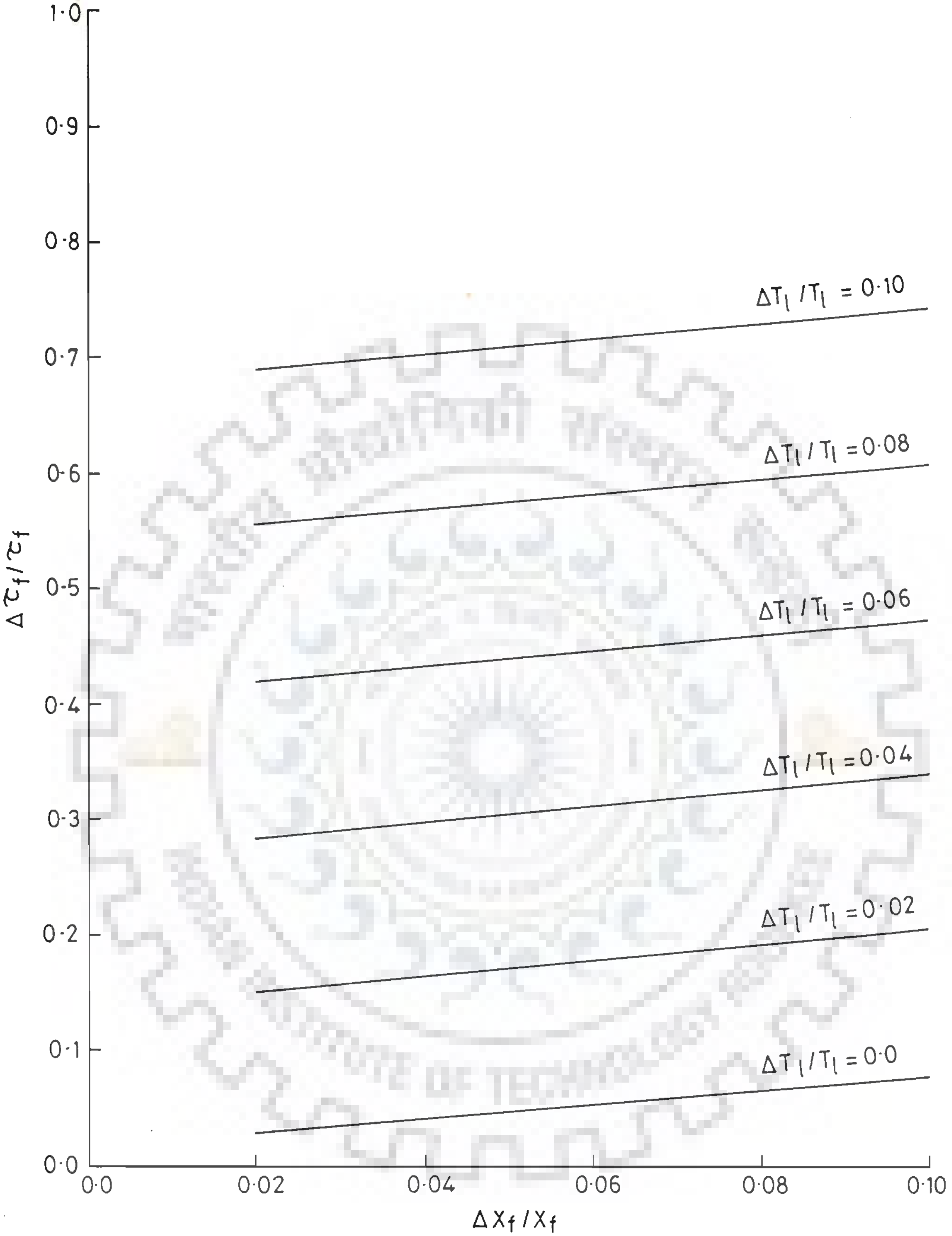

Fig. 6.32 $\Delta\tau_f/\tau_f$ versus $\Delta X_f/X_f$ with $\Delta T_l/T_l$ as a parameter for sugar solution

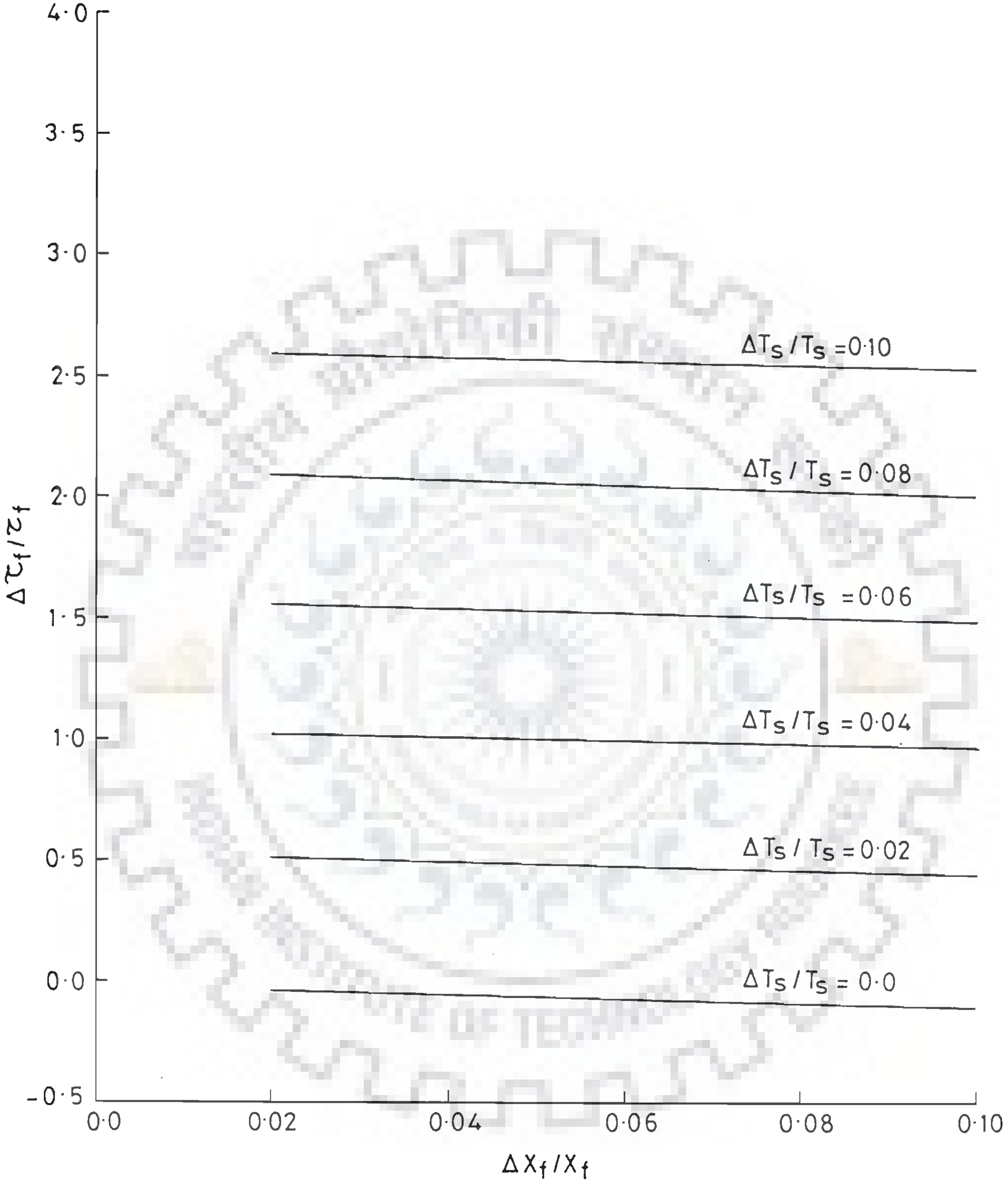

Fig. 6.33 $\Delta\tau_f/\tau_f$ versus $\Delta X_f/X_f$ with $\Delta T_s/T_s$ as a parameter for sugar solution

on simplification:

$$\begin{aligned}[\Delta E/E] = {} & a(\Delta\tau_f/\tau_f) + b(\Delta X_f/X_f) + c(\Delta F/F) + d(\Delta T_l/T_l) + e(\Delta T_s/T_s) \\ & + a(\Delta\tau_f/\tau_f)\,\{b(\Delta X_f/X_f) + c(\Delta F/F) + d(\Delta T_l/T_l) + e(\Delta T_s/T_s)\} \\ & + b(\Delta X_f/X_f)\,\{c(\Delta F/F) + d(\Delta T_l/T_l) + e(\Delta T_s/T_s)\} \\ & + c(\Delta F/F)\,\{d(\Delta T_l/T_l) + e(\Delta T_s/T_s)\} \\ & + de(\Delta T_l/T_l)(\Delta T_s/T_s) \qquad \ldots(6.30)\end{aligned}$$

Eq(6.30) can be used to calculate changes in the value of steam economy due to changes of operating variables.For the sake of convenience, Eq(6.30) has been plotted in Figures 6.34 to 6.37. Figure 6.34 is a plot of ($\Delta E/E$) versus ($\Delta X_f/X_f$) with ($\Delta\tau_f/\tau_f$) as a parameter for the condition of no change in the values of remaining operating variables. Similarly, Figures 6.35 to 6.37 represent the identical plots but with ($\Delta F/F$), ($\Delta T_l/T_l$), and ($\Delta T_s/T_s$) as parameter, respectively. In the preparation of these plots, the remaining operating variables have been assumed to be invariant. Thus, the deviation in steam economy can be obtained from the knowledge of the changes in operating variables.

From the above plots, deviation in steam economy can be determined from the knowledge of changes in operating variables for the evaporation of a given aqueous sugar solution in a quadruple effect forward feed evaporator.

Eq(6.30) can be reduced to a simple form when changes in the values of operating variables are considered to be of small magnitude. This, in-deed, is the result of total differentiation of Eq(6.10). Thus, Eq(6.30) is a generalized expression to represent the deviation in steam economy of a multiple effect evaporator due to changes in operating variables.

The plots of Figures 6.31 to 6.33 together with those of Figures 6.34 to 6.37 provide a procedure to determine changes in operating variables required to counteract any deviation in the concentration of end-product from a predetermined value caused by the unforeseen alterations in some of the operating variables and subsequently to obtain the impact of these changes on steam economy of the evaporator. At this juncture it is worthwhile to state that there may exist many options of the operating variables which can annul the change in end-product concentration. However, each option is likely to yield different value of steam economy. In fact, some of them may even lower the steam economy of the evaporator and thus jeopardize the functioning. Such situations are unwarranted from the point of view of energy consumption in evaporator. This necessitates the judicious selection of operating variables so as to achieve the improved steam economy of the evaporator with modified values of operating variables. This can be carried out by the use of preceding figures as illustrated below:

In a quadruple effect sugar evaporator, a fractional increase in feed concentration of 0.02

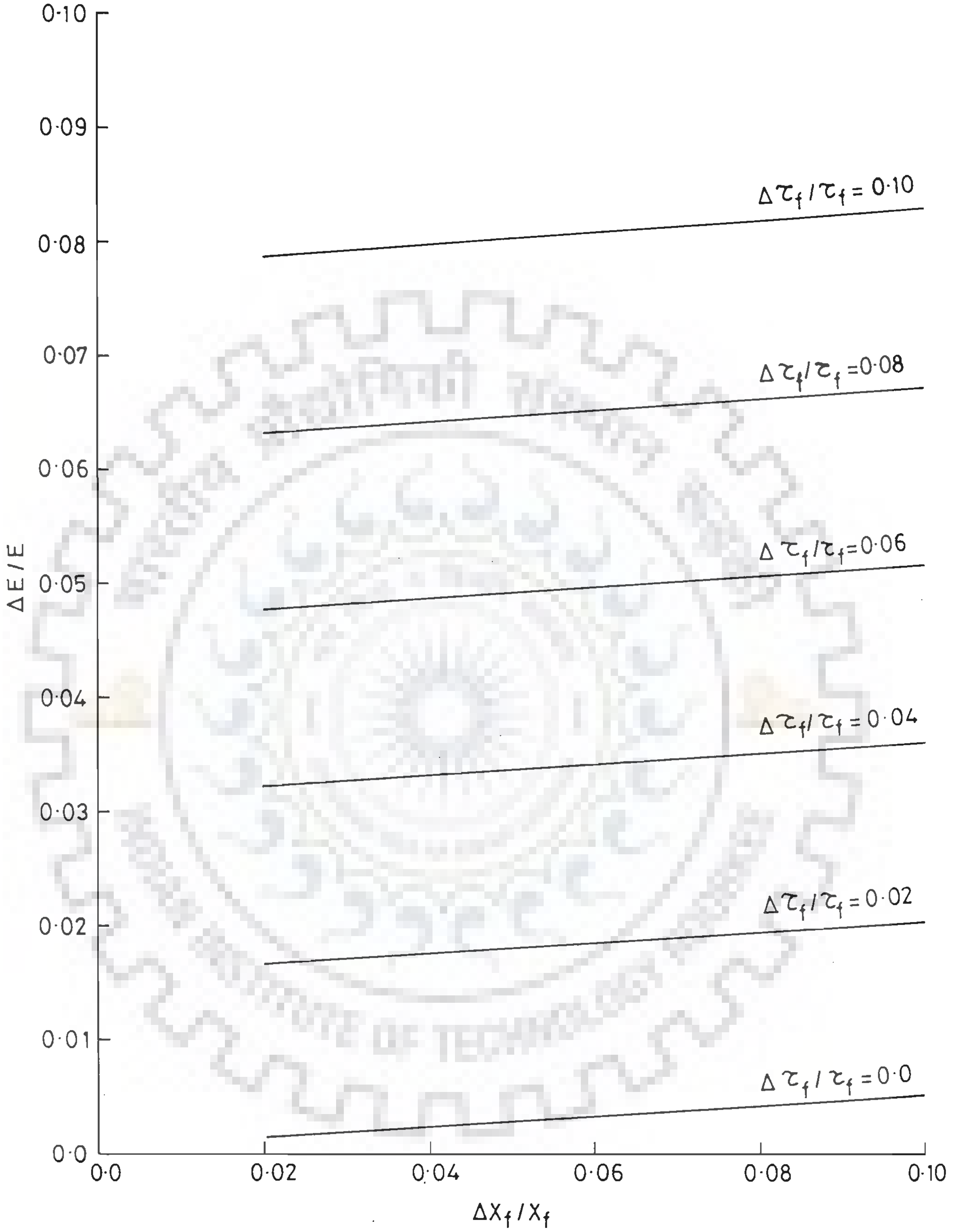

Fig. 6.34 $\Delta E/E$ versus $\Delta X_f/X_f$ with $\Delta \tau_f/\tau_f$ as parameter for sugar solution

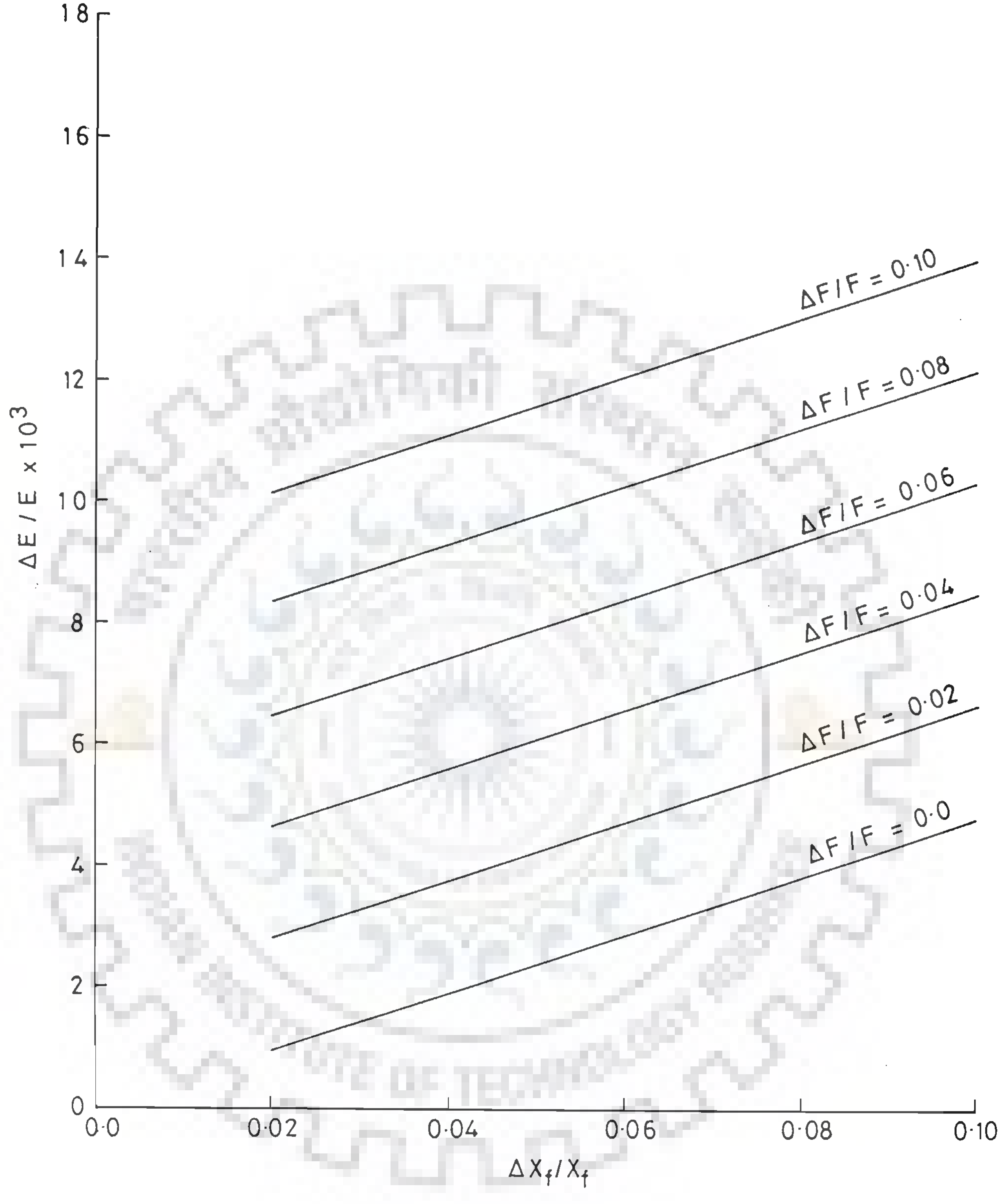

Fig. 6.35 $\Delta E/E$ versus $\Delta X_f/X_f$ with $\Delta F/F$ as a parameter for sugar solution

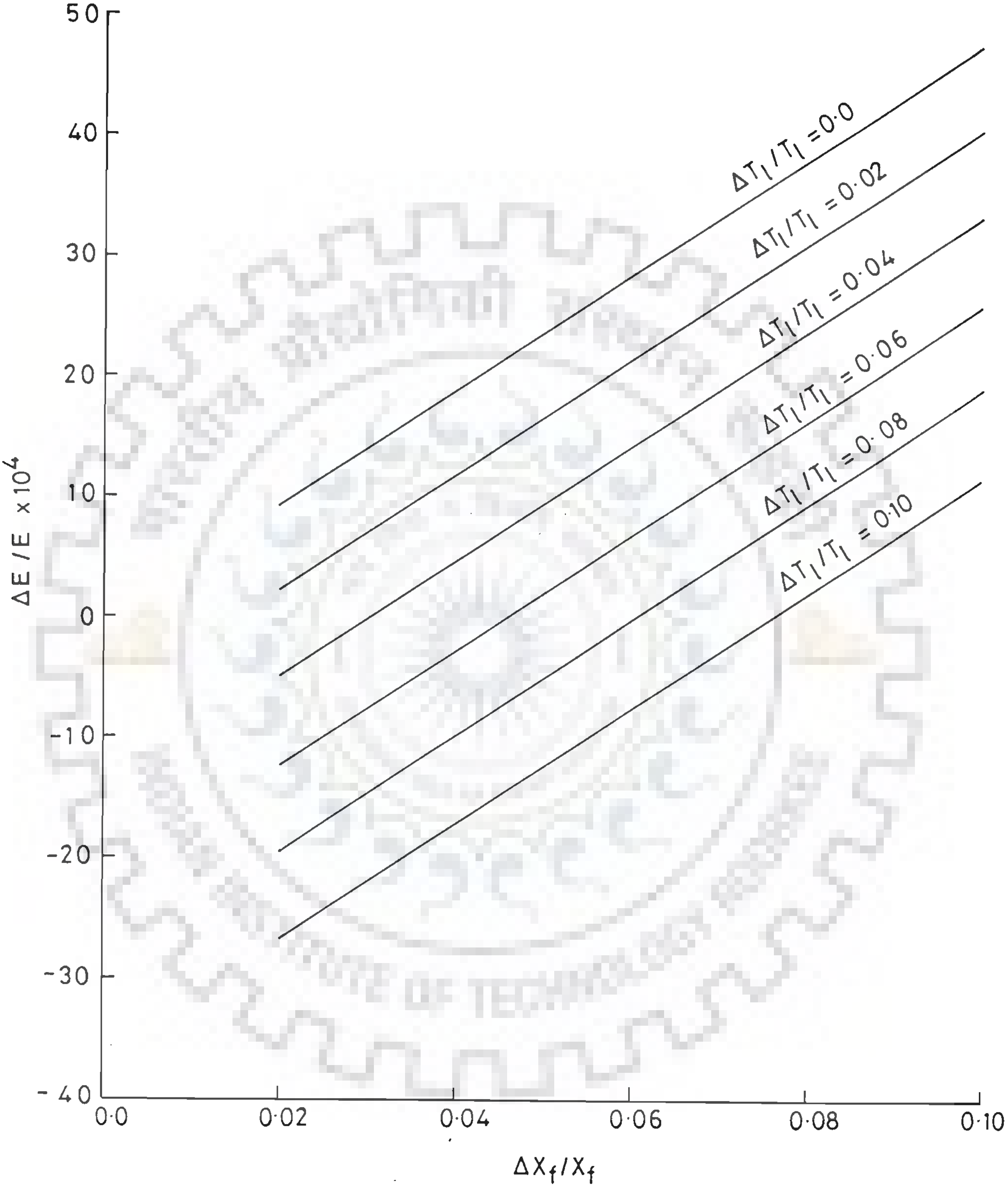

Fig. 6.36 $\Delta E/E$ versus $\Delta X_f/X_f$ with $\Delta T_l/T_l$ as a parameter for sugar solution

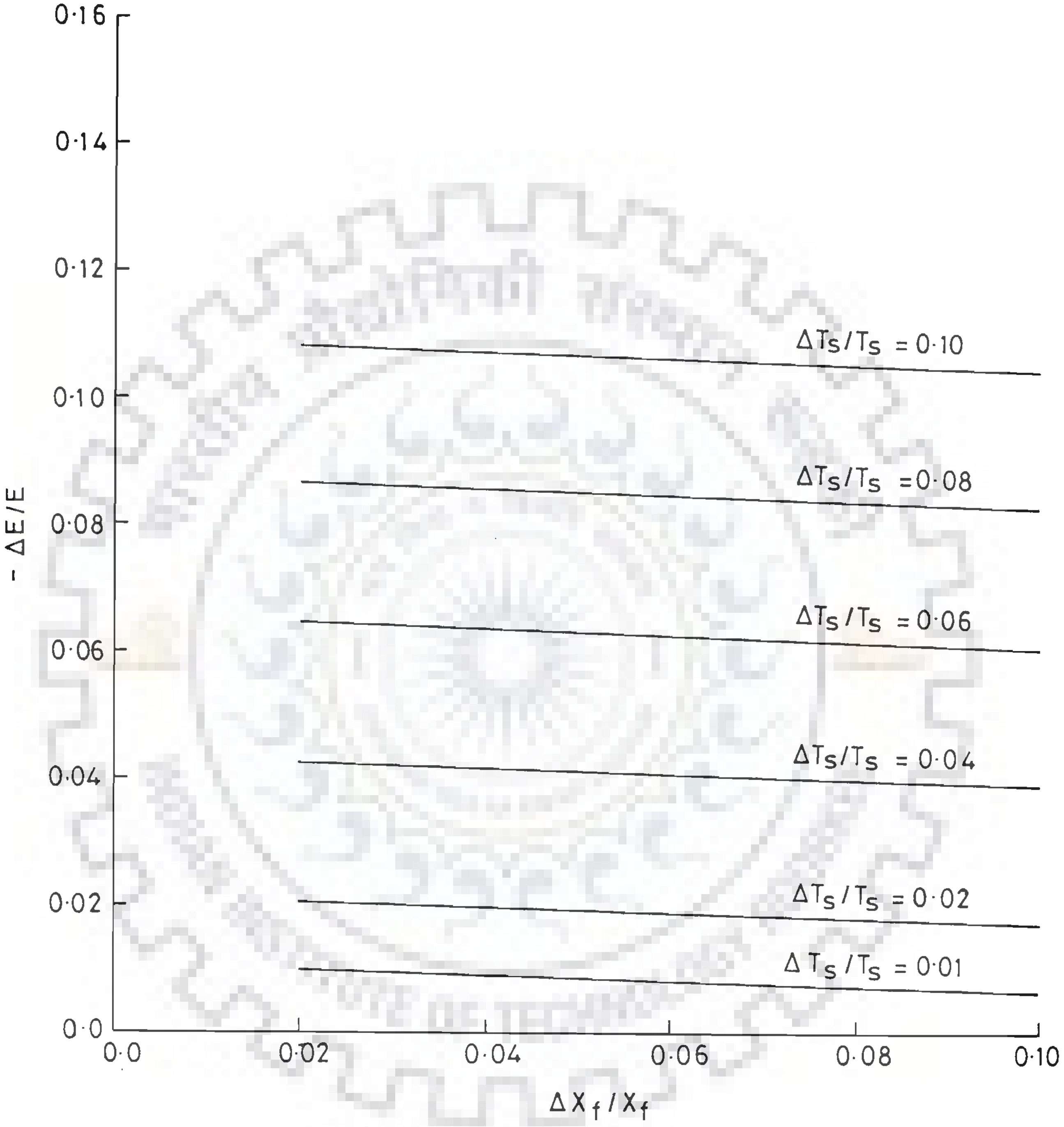

Fig. 6.37 $\Delta E/E$ versus $\Delta X_f/X_f$ with $\Delta T_S/T_S$ as a parameter for sugar solution

requires readjustment of any of the remaining variables to maintain the condition of no change in end-product concentration. The corresponding fractional change may be any one out of the following: 0.02 in feed temperature, 0.004 in saturation temperature of steam, -0.007 in the temperature of the last effect, and -0.0025 in feed rate. These values have been obtained from Figures 6.31 to 6.33. However, each one of them is likely to provide different value of steam economy. As can be seen from Figures 6.34 to 6.37, fractional change in feed temperature, saturation temperature of steam, and feed rate causes a fractional increase in steam economy of 0.0165, 0.004, and 0.0008, respectively, whereas that in the temperature of last effect brings a fractional decrease of 0.0036. Obviously the increase of feed temperature will provide the most profitable evaporation.

REMARKS ON THE PROCEDURE

The above mentioned procedure enables one to determine the necessary changes in operating variables needed to maintain the end-product concentration at a predetermined level due to any upset in other variables of the evaporator. It also facilitates the calculations of resultant deviation in steam economy and thereby the selection of the variable which yields the highest steam economy of the evaporator in question.

This procedure is based on Eqs(6.10 and 6.20) which have been obtained for the range of operating variables of this investigation. Hence, adequate care must be exercised in using the procedure for the aqueous solutions, feed arrangement, and range of operating variables other than those of Table 6.1. Such situations demand the determination of exponents of Eqs(6.10 and 6.20) before undertaking the use of this procedure.

CHAPTER-7

CONCLUSIONS AND RECOMMENDATION

As a result of the present investigation, some of the important conclusions pertaining to the evaporation of the aqueous solutions, namely; sugar, black-liquor and caustic soda in quadruple-, quintuple-, and triple- effect evaporator, respectively with forward, backward, and mixed feed arrangements are listed below:

1. A model of non-linear simultaneous algebraic equations has been developed to describe the performance of a multiple effect evaporator with various feed arrangements for operating variables; feed temperature, feed concentration, feed rate, pressure (temperature) in the last effect, and steam pressure (saturation temperature). The resulting model has been solved by the application of Newton-Raphson method and the L-U decomposition method to predict pertinent quantities for each set of operating variables. The model has successfully predicted the solute concentration, liquid flow rates, and the vapour- and the liquid- temperature which are in agreement with the plant values of Indian mills.

2. Steam economy of a quadruple effect sugar evaporator for forward feed has been found to increase with the increase in feed temperature, feed concentration, and feed rate and to decrease with the increase in the pressure of last effect, and steam pressure. In backward feed an improvement in steam economy has been observed with the rise of feed temperature, and the pressure of last effect; but it has suffered with the increase in feed concentration, feed rate, and steam pressure. As regards the mixed feed, steam economy has decreased with the increase in feed concentration, feed rate, pressure in the last effect, and steam pressure; and it has increased with the rise in the temperature of the feed. The evaporation of black-liquor and the caustic soda solutions in their respective multiple effect evaporators have shown different trends due to differing physico-thermal properties of the aqueous solutions involved and the values of the operating variables.

   Comparison of steam economy of the evaporator with different feed arrangements has resulted the range of operating variables which can yield the highest steam economy of the evaporator as a function of feed arrangement and aqueous solution. However, it has been found that for high values of operating variables forward feed provides the highest steam economy to be followed by either backward or mixed feed depending upon the value of the operating variables.

3. Using multiple linear regression analysis, various multivariable correlations of steam economy of forward, backward, and mixed feed evaporators for aqueous solutions of sugar, black-liquor, and caustic soda have been developed. They are of the following form:

$$E = C\,\tau_f^{a}\,X_f^{b}\,F^{c}\,T_l^{d}\,T_s^{e}$$

where values of the constant,C and the exponents, a, b, c, d, and e depend upon the aqueous solution to be concentrated and the feed arrangement to be used in evaporator.

4. Parametric effect of feed temperature, feed concentration, feed rate, pressure in the last effect, and steam pressure on the concentration of end-product of evaporator with various feed arrangements for sugar, black-liquor, and caustic soda solutions has been studied. As a result of it, end-product concentration has been found to vary directly with feed temperature, and steam pressure and to vary inversely with the feed rate and pressure in the last effect for all the aqueous solutions of this investigation. However, the variation of end-product concentration with feed concentration has differed from solution to solution.

   For the entire range of operating variables, the concentration of the end-product has been found to be the highest for forward feed to be followed by backward and mixed feeds in decreasing order.

5. Various correlations of end-product concentration of the evaporators for various aqueous solutions have been developed by the use of multiple linear regression analysis. The generalized form of the correlations is as follows:

$$X_p = K\,\tau_f^{p}\,X_f^{q}\,F^{r}\,T_l^{s}\,T_s^{t}$$

where the values of constant,K and the exponents, p, q, r, s, and t vary with the feed arrangement and the solution used in the evaporator.

6. This investigation has also succeeded to obtain the following equation to determine the corresponding change in the value of operating variables which are called for so that the concentration of the end-product from multiple effect evaporator does not undergo any alteration due to unforeseen variation in any of the operating variables:

$$[p(\Delta\tau_f/\tau_f) + q(\Delta X_f/X_f) + r(\Delta F/F)+s(\Delta T_l/T_l)+ t(\Delta T_s/T_s)]$$
$$= pq(\Delta\tau_f/\tau_f) (\Delta X_f/X_f) - rs(\Delta F/F)(\Delta T_l/T_l) - st(\Delta T_l/T_l)(\Delta T_s/T_s)- rt(\Delta F/F) (\Delta T_s/T_s)$$

The corresponding change in steam economy of the evaporator, due to modification of any of the operating variables, has been correlated by the following relationship:

$$\begin{aligned}[\Delta E/E] = {} & a(\Delta\tau_f/\tau_f) + b(\Delta X_f/X_f) + c(\Delta F/F) + d(\Delta T_l/T_l) + e(\Delta T_s/T_s) \\ & + a(\Delta\tau_f/\tau_f) \{b(\Delta X_f/X_f) + c(\Delta F/F) + d(\Delta T_l/T_l) + e(\Delta T_s/T_s)\} \\ & + b(\Delta X_f/X_f) \{c(\Delta F/F) + d(\Delta T_l/T_l) + e(\Delta T_s/T_s)\} \\ & + c(\Delta F/F) \{d(\Delta T_l/T_l) + e(\Delta T_s/T_s)\} \\ & + de(\Delta T_l/T_l)(\Delta T_s/T_s)\end{aligned}$$

Above equations provide an useful procedure to the plant engineers to determine the necessary change in the value of operating variables needed to counteract the change in end-product concentration caused by any upset in the value of other variables of the evaporator. It also enables the determination of the deviation in the economy and thereby the selection of the most appropriate operating variable so that evaporator has the highest steam economy. Undoubtedly, this curtails any possibility of reduction in steam economy of the evaporator and thus ensures the minimum consumption of steam in the evaporator.

Based on the results of the present investigation, following is recommended for future work:

1. The model developed in chapter-3 for multiple effect evaporators using various aqueous solutions does not include the effect of foaming, entrainment, liquid head, scaling, leakage, etc. on evaporation. Rather, it is based on the correlation of overall heat transfer coefficient which has been obtained by correlating the plant values. It is desirable that overall heat transfer coefficient be calculated from the knowledge of individual film heat transfer coefficients and the fouling resistance. This requires suitable equations of individual film heat transfer coefficients and of fouling resistance as a function of physico-thermal properties of the solution and operating variables. Incorporation of it will generalize the present model.

2. The present treatment regarding the parametric effect of operating variables on steam economy and end-product concentration has been confined to the values of operating variables mentioned in Table 6.1 and three aqueous solutions, namely; sugar, black-liquor, and caustic soda. It would be interesting if the range of operating variables is inflated and other solutions such as sodium-dichromate, sodium-sulphate, urea, potassium- chloride, saline-water and alike are also included in the

investigation. Such an study will help to generalize the scope of energy conservation through operating variables in evaporator.

3. The procedure to counteract a deviation in end- product concentration due to variation in any of the operating variables requires the readjustment of remaining one or more variables so as to obtain the improved steam economy of the evaporator. This means an expenditure of energy for the readjustment of operating variables. Therefore, it will be desirable if a comparative study between the investment needed to carry out the desired change in the variable and the savings as a result of the improvement in the steam economy of the evaporator is made. Such an analysis will go a long way to decide the economic feasibility of the change in the variable.

4. The present investigation has not considered the inclusion of energy conservation measures such as vapour bleeding, vapour recompression, splitting of feed, condensate flashing, addition of effects, etc. in the evaporator. Therefore, they should be included in the system and then parametric effect of operating variables on steam economy as well as on end-product concentration be investigated. This will help in deciding the suitability of adapting these energy conservation measures in evaporator.

APPENDIX-A

BOILING POINT OF AQUEOUS SOLUTIONS

The boiling point of an aqueous solution depends upon the boiling point of water, the nature of solute, and its concentration dissolved in the solution. Equations of boiling point for the aqueous solutions of sugar, black-liquor, and caustic soda are as follows:

a. Sugar Solution

Hugot [H4] conducted extensive experiments to determine boiling point rise of aqueous sugar solution. These data have been fitted into a third order polynomial by the method of Least Square. The resulting equation is as follows:

$$\epsilon = 7.20X - 11.5X^2 + 29.50X^3 \qquad ...(A.1)$$

The boiling point of aqueous sugar solution, τ is calculated by adding the value of ϵ to the boiling point of water. The resultant equation of boiling point of aqueous sugar solution is as follows:

$$\tau = T + 7.20X - 11.5X^2 + 29.50X^3 \qquad ...(A.2)$$

Eq(A.2) correlates the experimental data of aqueous sugar solution within a maximum deviation of $\pm 2.5\%$.

b. Black-liquor Solution

The empirical correlation of boiling point rise for aqueous black-liquor solution due to Veeramani [V2] is as follows:

$$\epsilon = -3.55X + 84.0X^2 - 107.5X^3 \qquad ...(A.3)$$

Therefore, the boiling point of aqueous black-liquor solution is as follows:

$$\tau = T - 3.55X + 84.0X^2 - 107.5X^3 \qquad ...(A.4)$$

c. Caustic Soda Solution

Following correlation due to Holland [H3] for boiling point rise of aqueous caustic- soda solution has been employed:

$$\epsilon = 271.363\ X^2 + X(0.142\ T - 9.42) \qquad \text{...(2.3)}$$

The boiling point of aqueous caustic-soda solution is as follows:

$$\tau = T + (0.142T - 9.42)X + 271.363X^2 \qquad \text{..(A.5)}$$

Eqs(A.2, A.4, & A.5) can be rewritten in the following general form of equation:

$$\tau = T + (\gamma + nT)X + \alpha X^2 + \beta X^3 \qquad \text{...(A.6)}$$

The values of the constants n, α, β, and γ of Eq(A.6) for the above aqueous solutions are given in Table A.1.

Table A.1 Values of constants n, α, β, and γ in Eq(A.6)

Solution	n	α	β	γ
Sugar [H4]	0.0	-11.50	29.50	7.20
Black- liquor[V2]	0.0	84.00	-107.50	-3.55
Caustic soda[H3]	0.142	271.363	0.0	-9.42

APPENDIX-B

ENTHALPIES OF AQUEOUS SOLUTIONS AND STEAM

This Appendix contains expressions for the enthalpies of aqueous solutions of sugar, black-liquor, and caustic soda and that for steam.

B.1 ENTHALPY OF AQUEOUS SOLUTION

The correlations of enthalpies for the aqueous solutions do not seem to be available in literature. However, data of enthalpies or heat capacities as a function of temperature and concentration are available. Therefore, in case of sugar- and caustic soda- solutions they have been used to develop correlations of enthalpy. For black- liquor solution correlation of heat capacity exists, hence the same has been employed to obtain an expression for the enthalpy of black- liquor.

a. Sugar Solution

Data of heat capacity of the aqueous sugar solution as a function of concentration are available in literature [H4]. They have been correlated by the method of Least Square to obtain the following equation :

$$C = 4.182 - 2.2403X \qquad \ldots(B.1)$$

Eq(B.1) has been compared with the experimental data of Hugot[H4] and Kern[K1] in Figure B.1. As can be seen from this plot, Eq(B.1) has correlated the experimental data excellently within a maximum deviation of ±1.67%. Using Eq(B.1), the following expression relating the enthalpy of aqueous sugar solution with temperature and concentration has been developed:

$$h = (4.182 - 2.2403X)(\tau - T_r) \qquad \ldots(B.2)$$

b. Black-liquor Solution

Veeramani [V 2] has reported a correlation of heat capacity of the black- liquor solution as a function of temperature and concentration. It is reproduced below :

$$C = 7.53 \times 10^{-3} (\tau - T_r)X - 2.25383X + 4.182 \qquad \ldots(2.12)$$

Using the Eq(2.12), the following relationship between the enthalpy of the aqueous

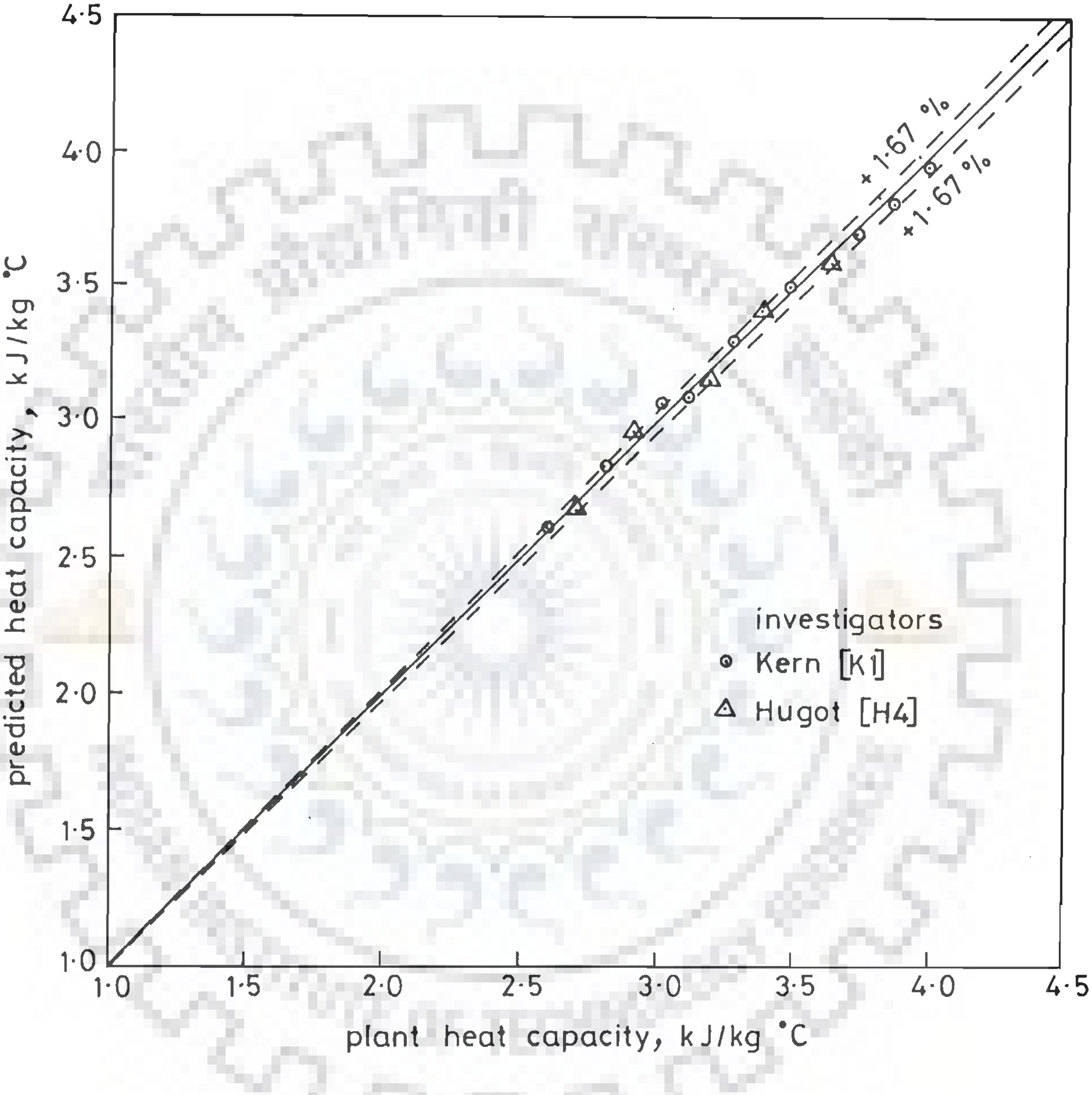

Fig. B·1 A comparison between the plant and predicted heat capacities of aqueous sugar solutions due to Eq (B.1) black- liquor solution and temperature and concentration has been developed:

$$h = 7.53\times10^{-3}(\tau^2 - T_r^2)X - 2.25383X(\tau - T_r) + 4.182(\tau - T_r) \qquad ...(B.3)$$

c. *Caustic Soda Solution*

Goodall [G7] has reported the data of enthalpy of aqueous caustic-soda solution as a function of temperature and concentration. Therefore, these data have been correlated by Least Square method into the following polynomial:

$$h = 62.015 + 3.884\tau - 887.125X + 2316.504X\{\exp(-1.15\times10^{-3}\tau^2)\} \qquad ...(B.4)$$

The enthalpies predicted by Eq(B.4) have the maximum deviation of ± 6.0% from the experimental values.

A.2 ENTHALPY OF STEAM

Enthalpies of steam[P1, V3] were also correlated as a function of temperature by means of Least Square method as follows:

$$\begin{aligned} H = {} & 4.154(T - T_r) + 2.0125\times10^{-4}(T^2 - T_r^2) \\ & + 1.62(\tau - T) + 2.0285\times10^{-4}(\tau^2 - T^2) - 0.3747\times10^{-7}(\tau^3 - T^3) + \lambda \end{aligned} \qquad ...(B.5)$$

and

$$\begin{aligned} \lambda = {} & -80.345T - 21035.87/T + 2049.123T^{1/2} - 4213.519\ln T \\ & + 0.0918T^2 - 1.04\times10^{-4}T^3 + 8597.953 \end{aligned} \qquad ..(B.6)$$

APPENDIX-C

OVERALL HEAT TRANSFER COEFFICIENT

Heat transfer coefficient is a complicated factor having its significance in the design and operation of heat transfer equipment including evaporators. It depends upon the physico- thermal properties of the solution, their fouling characteristics, geometry of the evaporator and variables like concentration, temperature, flow rate of aqueous solution and many other factors. The value of overall heat transfer coefficient can be obtained by evaluating film heat transfer coefficients on both sides of the tube, the resistance due to scale formed on tube and the metallic resistance. But such a value may not represent the true state as it also depends upon time of operation. Therefore, in the present investigation it has not been determined from the knowledge of film heat transfer coefficients. Instead, the industrial data of mean overall heat transfer coefficients for aqueous solutions of sugar, black-liquor, and caustic-soda have been used.

a. Sugar Solution

Several correlations of overall heat transfer coefficient for sugar solution are available in literature [R1]. They are mentioned in Chapter-2. These correlations are empirical in nature. To select a suitable one, it was considered necessary to examine their validity against the industrial data of a typical Indian sugar mill.

Figure C.1 depicts a comparison between the industrial data of overall heat transfer coefficient and those predicted by Eqs (2.4, 2.5, 2.6, and 2.7). From this plot it is clearly seen that the Eq (2.5) due to Schwedenformel, [R1] correlates the plant values excellently within a maximum deviation of -6.7% . However, the predictions due to Eq(2.6) due to Speyerer, [R1] are always lower than the plant values by 11% whereas those due to Hopstock, Eq(2.7),[R1] and Baloh, Eq(2.4)[R1] are higher by 7.8 % and 40% respectively from the plant values. This clearly demonstrates the superiority of the Eq(2.5) due to Schwedenformel over other correlations. Eq(2.5) is reproduced below:

$$U = 18.083(\tau/X) \qquad \ldots(C.1)$$

b. Black-liquor Solution

The following relationship of overall heat transfer coefficient for aqueous black-liquor solution has been developed by correlating the experimental data [K1] using Least Square

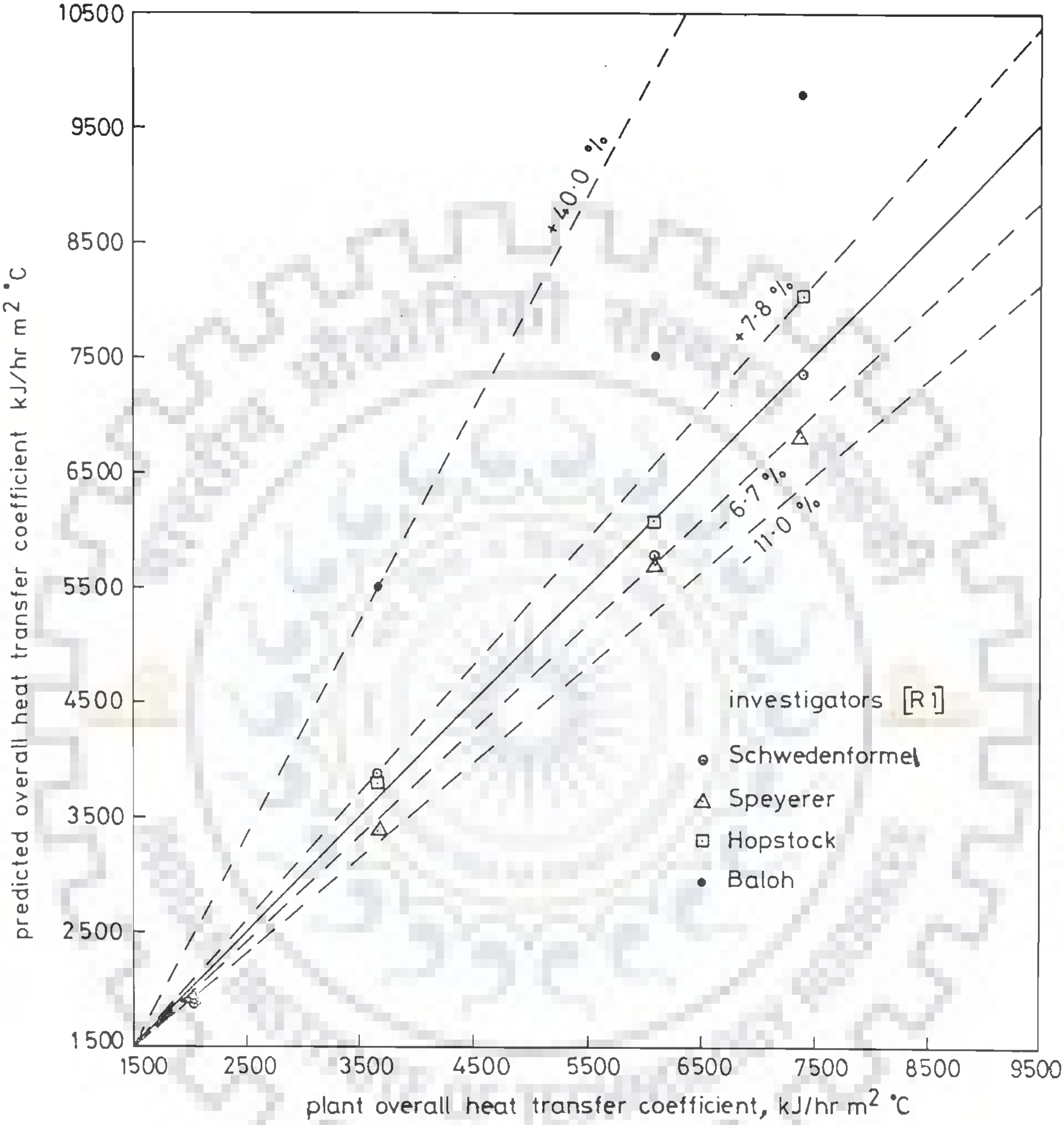

Fig. C·1 Comparision between the plant and predicted overall heat transfer coefficients for aqueous solution of sugar due to Eq (C.2) method:

$$U = 13.392(\tau_{i-1} + \tau_i) - 3960.0(X_{i-1} + X_i) + 4800.0 \qquad \ldots(C.2)$$

Eq(C.2) has a maximum deviation of ±7.62% from the experimental values.

c. Caustic Soda Solution

The experimental data of overall heat transfer coefficients [H3] of aqueous caustic-soda solution have been correlated by the use of Least Square method into the following equation:

$$U = 977.66(\tau/X)^{0.2823} \qquad \ldots(C.3)$$

This equation has the maximum deviation of ±3.53% from the experimental values.

$$\begin{aligned}
g_7 &= l_1[(4.182 - 2.2403X_1)(T_s u_1 - T_r) - (4.182 - 2.2403X_2)((T_s u_2 - T_r)]/\lambda_s \\
&\quad + \{(1-l_1)/\lambda_s\}[-80.345T_s w_1 - 21035.87/(T_s w_1) + 2049.125(T_s w_1)^{1/2} \\
&\quad - 4213.519\ln(T_s w_1) + 0.0918(T_s w_1)^2 - 1.04\text{x}10^{-4}(T_s w_1)^3 + 8597.453] \\
&\quad - \{(l_1 - l_2)/\lambda_s\}[4.154(T_s w_2 - T_r) + 2.0125\ \text{x}10^{-4}(T^2_s w_2^2 - T_r^2) \\
&\quad + 1.62\ T_s(u_2 - w_2) + 2.0285\text{x}10^{-4}\ T_s^2(u_2^2 - w_2^2) - 0.3747\text{x}10^{-7}T_s^3(u_2^3 - w_2^3) \\
&\quad - 80.345T_s w_2 - 21035.87/(T_s w_2) + 2049.125(T_s w_2)^{1/2} - 4213.519\ln(T_s w_2) \\
&\quad + 0.0918(T_s w_2)^2 - 1.04\text{x}10^{-4}(T_s w_2)^3 + 8597.453 \\
&\quad - (4.182 - 2.2403X_2)(T_s u_2 - T_r)] \qquad \ldots(\text{D.7})
\end{aligned}$$

$$\begin{aligned}
g_8 &= 18.083T_s^2 u_2 A_2(w_1 - u_2)/(X_2 F\lambda_s) - \{(1-l_1)/\lambda_s\}[-80.345T_s w_1 \\
&\quad -21035.87/T_s w_1) + 2049.125(T_s w_1)^{1/2} - 4213.519\ln(T_s w_1) \\
&\quad + 0.0918(T_s w_1)^2 - 1.04\text{x}10^{-4}(T_s w_1)^3 + 8597.453] \qquad \ldots(\text{D.8})
\end{aligned}$$

$$g_9 = (x_f - l_3 x_3)/\lambda_s \qquad \ldots(\text{D.9})$$

$$g_{10} = [T_s w_3 + (\gamma + nT_s w_3)X_3 + \alpha X^2_3 + \beta X^3_3 - T_s u_3]/(F.\lambda_s) \qquad \ldots(\text{D.10})$$

$$\begin{aligned}
g_{11} &= l_2[(4.182 - 2.2403X_2)(T_s u_2 - T_r) - (4.182 - 2.2403X_3)((T_s u_3 - T_r)]/\lambda_s \\
&\quad + \{(l_1 - l_2)/\lambda_s\}[-80.345T_s w_2 - 21035.87/(T_s w_2) + 2049.125(T_s w_2)^{1/2} \\
&\quad - 4213.519\ln(T_s w_2) + 0.0918(T_s w_2)^2 - 1.04\text{x}10^{-4}(T_s w_2)^3 + 8597.453] \\
&\quad - \{(l_2 - l_3)/\lambda_s\}[4.154(T_s w_3 - T_r) + 2.0125\ \text{x}10^{-4}(T^2_s w_3^2 - T_r^2) \\
&\quad + 1.62\ T_s(u_3 - w_3) + 2.0285\text{x}10^{-4}\ T_s^2(u_3^2 - w_3^2) - 0.3747\text{x}10^{-7}T_s^3(u_3^3 - w_3^3) \\
&\quad - 80.345T_s w_3 - 21035.87/(T_s w_3) + 2049.125(T_s w_3)^{1/2} - 4213.519\ln(T_s w_3) \\
&\quad + 0.0918(T_s w_3)^2 - 1.04\text{x}10^{-4}(T_s w_3)^3 + 8597.453 \\
&\quad - (4.182 - 2.2403X_3)(T_s u_3 - T_r)] \qquad \ldots(\text{D.11})
\end{aligned}$$

$$\begin{aligned}
g_{12} &= 18.083T_s^2 u_3 A_3(w_2 - u_3)/(X_3 F\lambda_s) - \{(l_1 - l_2)/\lambda_s\}[-80.345T_s w_2 \\
&\quad -21035.87/T_s w_2) + 2049.125(T_s w_2)^{1/2} - 4213.519\ln(T_s w_2) \\
&\quad + 0.0918(T_s w_2)^2 - 1.04\text{x}10^{-4}(T_s w_2)^3 + 8597.453] \qquad \ldots(\text{D.12})
\end{aligned}$$

$$\begin{aligned} g_7 = {} & l_1[(4.182 - 2.2403X_1)(T_s u_1 - T_r) - (4.182 - 2.2403X_2)((T_s u_2 - T_r)]/\lambda_s \\ & + \{(1-l_1)/\lambda_s\}[-80.345T_s w_1 - 21035.87/(T_s w_1) + 2049.125(T_s w_1)^{1/2} \\ & - 4213.519\ln(T_s w_1) + 0.0918(T_s w_1)^2 - 1.04\times10^{-4}(T_s w_1)^3 + 8597.453] \\ & - \{(l_1 - l_2)/\lambda_s\}[4.154(T_s w_2 - T_r) + 2.0125\times10^{-4}(T^2_s w_2^2 - T_r^2) \\ & + 1.62\, T_s(u_2 - w_2) + 2.0285\times10^{-4}\, T_s^2(u_2^2 - w_2^2) - 0.3747\times10^{-7} T_s^3(u_2^3 - w_2^3) \\ & - 80.345T_s w_2 - 21035.87/(T_s w_2) + 2049.125(T_s w_2)^{1/2} - 4213.519\ln(T_s w_2) \\ & + 0.0918(T_s w_2)^2 - 1.04\times10^{-4}(T_s w_2)^3 + 8597.453 \\ & - (4.182 - 2.2403X_2)(T_s u_2 - T_r)] \qquad \ldots(D.7) \end{aligned}$$

$$\begin{aligned} g_8 = {} & 18.083T_s^2 u_2 A_2(w_1 - u_2)/(X_2 F\lambda_s) - \{(1-l_1)/\lambda_s\}[-80.345T_s w_1 \\ & - 21035.87/T_s w_1) + 2049.125(T_s w_1)^{1/2} - 4213.519\ln(T_s w_1) \\ & + 0.0918(T_s w_1)^2 - 1.04\times10^{-4}(T_s w_1)^3 + 8597.453] \qquad \ldots(D.8) \end{aligned}$$

$$g_9 = (x_f - l_3 x_3)/\lambda_s \qquad \ldots(D.9)$$

$$g_{10} = [T_s w_3 + (\gamma + nT_s w_3) X_3 + \alpha X^2_3 + \beta X^3_3 - T_s u_3]/(F.\lambda_s) \qquad \ldots(D.10)$$

$$\begin{aligned} g_{11} = {} & l_2[(4.182 - 2.2403X_2)(T_s u_2 - T_r) - (4.182 - 2.2403X_3)((T_s u_3 - T_r)]/\lambda_s \\ & + \{(l_1 - l_2)/\lambda_s\}[-80.345T_s w_2 - 21035.87/(T_s w_2) + 2049.125(T_s w_2)^{1/2} \\ & - 4213.519\ln(T_s w_2) + 0.0918(T_s w_2)^2 - 1.04\times10^{-4}(T_s w_2)^3 + 8597.453] \\ & - \{(l_2 - l_3)/\lambda_s\}[4.154(T_s w_3 - T_r) + 2.0125\times10^{-4}(T^2_s w_3^2 - T_r^2) \\ & + 1.62\, T_s(u_3 - w_3) + 2.0285\times10^{-4}\, T_s^2(u_3^2 - w_3^2) - 0.3747\times10^{-7} T_s^3(u_3^3 - w_3^3) \\ & - 80.345T_s w_3 - 21035.87/(T_s w_3) + 2049.125(T_s w_3)^{1/2} - 4213.519\ln(T_s w_3) \\ & + 0.0918(T_s w_3)^2 - 1.04\times10^{-4}(T_s w_3)^3 + 8597.453 \\ & - (4.182 - 2.2403X_3)(T_s u_3 - T_r)] \qquad \ldots(D.11) \end{aligned}$$

$$\begin{aligned} g_{12} = {} & 18.083T_s^2 u_3 A_3(w_2 - u_3)/(X_3 F\lambda_s) - \{(l_1 - l_2)/\lambda_s\}[-80.345T_s w_2 \\ & - 21035.87/T_s w_2) + 2049.125(T_s w_2)^{1/2} - 4213.519\ln(T_s w_2) \\ & + 0.0918(T_s w_2)^2 - 1.04\times10^{-4}(T_s w_2)^3 + 8597.453] \qquad \ldots(D.12) \end{aligned}$$

$$g_{13} = (x_f - l_4\ x_4)/\lambda_s \qquad \ldots(D.13)$$

$$g_{14} = [T_s w_4 + (\gamma + nT_s w_4)\ X_4 + \alpha X^2_4 + \beta X^3_4 - T_s\ u_4]/(F.\lambda_s) \qquad \ldots(D.14)$$

$$\begin{aligned} g_{15} =\ & l_3[(4.182 - 2.2403X_3)\ (T_s u_3 - T_r) - (4.182 - 2.2403X_4)((T_s u_4 - T_r)]/\lambda_s \\ & + \{(l_2 - l_3)/\lambda_s\}[-80.345T_s w_3 - 21035.87/(T_s w_3) + 2049.125(T_s w_3)^{1/2} \\ & - 4213.519\ln(T_s w_3) + 0.0918(T_s w_3)^2 - 1.04x10^{-4}(T_s w_3)^3 + 8597.453] \\ & - \{(l_3 - l_4)/\lambda_s\}[4.154(T_s w_4 - T_r) + 2.0125\ x10^{-4}\ (T^2_s\ w_4^{\ 2} - T_r^{\ 2}) \\ & + 1.62\ T_s(u_4 - w_4) + 2.0285x10^{-4}\ T_s^{\ 2}(u_4^{\ 2} - w_4^{\ 2}) - 0.3747x10^{-7}T_s^{\ 3}(u_4^{\ 3} - w_4^{\ 3}) \\ & - 80.345T_s w_4 - 21035.87/(T_s w_4) + 2049.125(T_s w_4)^{1/2} - 4213.519\ln(T_s w_4) \\ & + 0.0918(T_s w_4)^2 - 1.04x10^{-4}(T_s w_4)^3 + 8597.453 \\ & - (4.182 - 2.2403X_4)(T_s u_4 - T_r)] \qquad \ldots(D.15) \end{aligned}$$

$$\begin{aligned} g_{16} =\ & 18.083T_s^{\ 2}u_4 A_4(w_3 - u_4)/(X_4 F\lambda_s) - \{(l_2 - l_3)/\lambda_s\}[-80.345T_s w_3 \\ & -21035.87/T_s w_3) + 2049.125(T_s w_3)^{1/2} - 4213.519\ln(T_s w_3) \\ & + 0.0918(T_s w_3)^2 - 1.04x10^{-4}(T_s w_3)^3 + 8597.453] \qquad \ldots(D.16) \end{aligned}$$

D.2 Jacobian Elements

$$(\partial\ g_1/\partial\ l_1) = -\ X_1/\lambda_s$$

$$(\partial\ g_1/\partial\ X_1) = -\ l_1/\lambda_s$$

$$(\partial\ g_2\ /\partial\ X_1) = [(\gamma + nT_s w_1) + 2\alpha X_1 + 3\beta X^2_1]/(F.\lambda_s)$$

$$(\partial\ g_2\ /\partial\ u_1) = -\ T_s\ /(F.\lambda_s)$$

$$(\partial\ g_2\ /\partial\ w_1) = [\ T_s(1 + nX_1)]/\ (F.\lambda_s)$$

$$(\partial\ g_3\ /\partial\ v_S) = 1.0$$

$$\begin{aligned} (\partial\ g_3\ /\partial\ l_1) =\ & [4.154(T_s w_1 - T_r) + 2.0125\ x10^{-4}\ (T^2_s\ w_1^{\ 2} - T_r^{\ 2}) + 1.62\ T_s\ (u_1 - w_1) \\ & + 2.0285x10^{-4}\ T_s^{\ 2}(u_1^{\ 2} - w_1^{\ 2}) - 0.3747x10^{-7}T_s^{\ 3}(u_1^{\ 3} - w_1^{\ 3}) - 80.345T_s w_1 \\ & -21035.87/(T_s w_1) + 2049.125(T_s w_1)^{1/2} - 4213.519\ln(T_s w_1) + 0.0918(T_s w_1)^2 \\ & -1.04x10^{-4}(T_s w_1)^3 + 8597.453 - (4.182 - 2.2403X_1)(T_s u_1 - T_r)]/\lambda_s \end{aligned}$$

$$(\partial g_3/\partial X_1) = -[2.2403\, l_1(T_s u_1 - T_r)]/\lambda_s$$

$$\begin{aligned}(\partial g_3/\partial u_1) &= \{(4.182-2.2403X_1)T_s\}/\lambda_s - \{(1-l_1)/\lambda_s\}[1.62T_s + 4.057\text{x}10^{-4}\, T_s^2\, u_1 \\ &\quad - 1.1241 \text{ x } 10^{-7}\, T_s^{\,3}\, u_1^{\,2} - (4.182-2.2403X_1)T_s]\end{aligned}$$

$$\begin{aligned}(\partial g_3/\partial w_1) &= [-3.12\text{x}10^{-4}\, T_s^3\, w_1^{\,2} + 0.1835\, T_s^2\, w_1 - 77.811\, T_s \\ &\quad + 21035.85/(T_s\, w_1^{\,2}) + 1024.562\,(T_s/w_1)^{1/2} \\ &\quad - 4213.519/w_1]\,\{(1-l_1)/\lambda_s\}\end{aligned}$$

$$(\partial g_4/\partial v_S) = -1.0$$

$$(\partial g_4/\partial X_1) = -18.083\, T_s^2\, u_1\, A_1\,(1-u_1)/(F\lambda_s\, X_1^{\,2})$$

$$(\partial g_4/\partial u_1) = 18.083\, T_s^2\, A_1\,(1-2u_1)/(F\lambda_s\, X_1)$$

$$(\partial g_5/\partial l_2) = -X_2/\lambda_s$$

$$(\partial g_5/\partial X_2) = -l_2/\lambda_s$$

$$(\partial g_6/\partial X_2) = [(\gamma + nT_s w_2) + 2\alpha X_2 + 3\beta X_2^{\,2}]/(F.\lambda_s)$$

$$(\partial g_6/\partial u_2) = -T_s/(F.\lambda_s)$$

$$(\partial g_6/\partial w_2) = [T_s(1+nX_2)]/(F.\lambda_s)$$

$$\begin{aligned}(\partial g_7/\partial l_1) &= [\{(4.182 - 2.2403X_1)(T_s u_1 - T_r) - (4.182 - 2.2403X_2)((T_s u_2 - T_r)\} \\ &\quad - \{-80.345T_s w_1 - 21035.87/(T_s w_1) + 2049.125(T_s w_1)^{1/2} \\ &\quad - 4213.519\ln(T_s w_1) + 0.0918(T_s w_1)^2 - 1.04\text{x}10^{-4}(T_s w_1)^3 + 8597.453\} \\ &\quad - \{4.154(T_s w_2 - T_r) + 2.0125\text{ x}10^{-4}(T^2_s\, w_2^{\,2} - T_r^{\,2}) + 1.62\, T_s(u_2 - w_2) \\ &\quad + 2.0285\text{x}10^{-4}\, T_s^{\,2}(u_2^{\,2} - w_2^{\,2}) - 0.3747\text{x}10^{-7}T_s^{\,3}(u_2^{\,3} - w_2^{\,3}) - 80.345T_s w_2 \\ &\quad - 21035.87/(T_s w_2) + 2049.125(T_s w_2)^{1/2} - 4213.519\ln(T_s w_2) \\ &\quad + 0.0918(T_s w_2)^2 - 1.04\text{x}10^{-4}(T_s w_2)^3 + 8597.453 \\ &\quad - (4.182-2.2403X_2)(T_s u_2 - T_r)\}]/\lambda_s\end{aligned}$$

APPENDIX - D

FUNCTIONAL RELATIONSHIPS AND JACOBIAN ELEMENTS

Functional relationships and Jacobian elements developed for quadruple effect sugar evaporator under forward feed arrangement are as follows:

D.1 Functional relationships

$$g_1 = (x_f - l_1 x_1)/\lambda_s \qquad \ldots(\mathrm{D.1})$$

$$g_2 = [T_s w_1 + (\gamma + nT_s w_1)X_1 + \alpha X^2_1 + \beta X^3_1 - T_s u_1] / (F.\lambda_s) \qquad \ldots(\mathrm{D.2})$$

$$\begin{aligned} g_3 = {} & [(4.182 - 2.2403 X_f)(\tau_f - T_r) - (4.182 - 2.2403 X_1)((T_s u_1 - T_r)]/\lambda_s \\ & + v_s - \{(1 - l_1)/\lambda_s\}[4.154(T_s w_1 - T_r) + 2.0125 \times 10^{-4} (T^2_s w_1^2 - T_r^2) \\ & + 1.62 T_s (u_1 - w_1) + 2.0285 \times 10^{-4} T_s^2 (u_1^2 - w_1^2) - 0.3747 \times 10^{-7} T_s^3 (u_1^3 - w_1^3) \\ & - 80.345 T_s w_1 - 21035.87/(T_s w_1) + 2049.125 (T_s w_1)^{1/2} - 4213.519 \ln(T_s w_1) \\ & + 0.0918 (T_s w_1)^2 - 1.04 \times 10^{-4} (T_s w_1)^3 + 8597.453 \\ & - (4.182 - 2.2403 X_1)(T_s u_1 - T_r)] \end{aligned} \qquad \ldots(\mathrm{D.3})$$

$$g_4 = 18.083 T_s^2 u_1 A_1 (1 - u_1)/(X_1 F \lambda_s) - v_s \qquad \ldots(\mathrm{D.4})$$

$$g_5 = (x_f - l_2 x_2)/\lambda_s \qquad \ldots(\mathrm{D.5})$$

$$g_6 = [T_s w_2 + (\gamma + nT_s w_2) X_2 + \alpha X^2_2 + \beta X^3_2 - T_s u_2]/(F. \lambda_s) \qquad \ldots(\mathrm{D.6})$$

$$(\partial g_7/\partial X_1) = -[2.2403\, l_1\, (T_s\, u_1 - T_r)]/\lambda_s$$

$$(\partial g_7/\partial u_1) = \{(4.182-2.2403X_1)T_s\, l_1\}/\lambda_s$$

$$\begin{aligned}(\partial g_7/\partial w_1) = {} & [-3.12\text{x}10^{-4}\, T_s^{\,3}\, w_1^{\,2} + 0.1835\; T_s^{\,2}\; w_1 + 80.345\, T_s \\ & + 21035.87/(T_s\, w_1^{\,2}) + 1024.562\, (T_s / w_1)^{1/2} \\ & - 4213.519/w_1\,]\, \{(1-l_1)/\lambda_s\}\end{aligned}$$

$$\begin{aligned}(\partial g_7/\partial l_2) = {} & [4.154(T_s w_2 - T_r) + 2.0125\text{ x}10^{-4}\, (T^2_s\, w_2^{\,2} - T_r^{\,2}) + 1.62\, T_s(u_2 - w_2) \\ & + 2.0285\text{x}10^{-4}\, T_s^{\,2}(u_2^{\,2} - w_2^{\,2}) - 0.3747\text{x}10^{-7} T_s^{\,3}(u_2^{\,3} - w_2^{\,3}) - 80.345 T_s w_2 \\ & - 21035.87/(T_s w_2) + 2049.125(T_s w_2)^{1/2} - 4213.519\ln(T_s w_2) + 0.0918(T_s w_2)^2 \\ & - 1.04\text{x}10^{-4}(T_s w_2)^3 + 8597.453 - (4.182-2.2403X_2)(T_s u_2 - T_r)]/\lambda_s\end{aligned}$$

$$(\partial g_7/\partial X_2) = -[2.2403\, l_2(T_s\, u_2 - T_r)]/\lambda_s$$

$$\begin{aligned}(\partial g_7/\partial u_2) = {} & \{(4.182-2.2403X_2)T_s\}/\lambda_s - \{(l_1 - l_2)/\lambda_s\}[1.62T_s + 4.057 \text{ x } 10^{-4}\, T_s^{\,2}\, u_2 \\ & - 1.1241 \text{ x } 10^{-7}\, T_s^{\,3}\, u_2^{\,2} - (4.182-2.2403X_2)T_s]\end{aligned}$$

$$\begin{aligned}(\partial g_7/\partial w_2) = {} & [-3.12\text{x}10^{-4}\, T_s^{\,3}\, w_2^{\,2} + 0.1835\; T_s^{\,2}\; w_2 - 77.811\, T_s \\ & + 21035.87/(T_s w_2^{\,2}) + 1024.562\, (T_s / w_2)^{1/2} - 4213.519/w_2]\, \{(l_1 - l_2)/\lambda_s\}\end{aligned}$$

$$\begin{aligned}(\partial g_8/\partial l_1) = {} & [-80.345T_s w_1 - 21035.87/(T_s w_1) + 2049.125(T_s w_1)^{1/2} \\ & - 4213.519\ln(T_s w_1) + 0.0918(T_s w_1)^2 - 1.04\text{x}10^{-4}(T_s w_1)^3 + 8597.453]/\lambda_s\end{aligned}$$

$$\begin{aligned}(\partial g_8/\partial w_1) = {} & 18.083\, T_s^{\,2}\; A_2\, u_2 /(F\lambda_s\, X_2) - [-3.12\text{x}10^{-4}\, T_s^{\,3}\, w_1^{\,2} \\ & + 0.1835\; T_s^{\,2}\; w_1 - 80.345\, T_s + 21035.87/(T_s\, w_1^{\,2}) \\ & + 1024.562\, (T_s / w_1)^{1/2} - 4213.519/w_1\,]\, \{(1-l_1)/\lambda_s\}\end{aligned}$$

$$(\partial g_8/\partial X_2) = -18.083\, T_s^{\,2}\; u_2\, A_2\, (1-u_2)/(F\lambda_s\, X_2^{\,2})$$

$$(\partial g_8 / \partial u_2) = 18.083\, T_s^2\; A_2\, (1-2u_2)/(F\lambda_s\, X_2)$$

$$(\partial g_9 / \partial l_3) = -\, X_3 / \lambda_s$$

$$(\partial g_9 / \partial X_3) = -\, l_3/\lambda_s$$

$$(\partial g_{10} / \partial X_3) = [(\gamma + nT_s w_3) + 2\alpha X_3 + 3\beta X_3^2] / (F.\lambda_s)$$

$$(\partial g_{10} / \partial u_3) = -\, T_s / (F.\lambda_s)$$

$$(\partial g_{10} / \partial w_3) = [\, T_s (1 + nX_3)] / (F.\lambda_s)$$

$$\begin{aligned}(\partial g_{11} / \partial l_1) = {} & [-\, 80.345 T_s w_2 - 21035.87/(T_s w_2) + 2049.125 (T_s w_2)^{1/2} \\ & -4213.519 \ln(T_s w_2) + 0.0918 (T_s w_2)^2 - 1.04 \mathrm{x} 10^{-4} (T_s w_2)^3 + 8597.453] / \lambda_s\end{aligned}$$

$$\begin{aligned}(\partial g_{11} / \partial l_2) = {} & [\{(4.182 - 2.2403 X_2)(T_s u_2 - T_r) - (4.182 - 2.2403 X_3)((T_s u_3 - T_r)\} \\ & -\{-80.345 T_s w_2 - 21035.87/(T_s w_2) + 2049.125 (T_s w_2)^{1/2} \\ & -4213.519 \ln(T_s w_2) + 0.0918 (T_s w_2)^2 - 1.04 \mathrm{x} 10^{-4} (T_s w_2)^3 + 8597.453\} \\ & -\{4.154 (T_s w_3 - T_r) + 2.0125 \mathrm{x} 10^{-4} (T^2_s\, w_3^2 - T_r^2) + 1.62\, T_s (u_3 - w_3) \\ & +2.0285 \mathrm{x} 10^{-4}\, T_s^2 (u_3^2 - w_3^2) - 0.3747 \mathrm{x} 10^{-7} T_s^3 (u_3^3 - w_3^3) - 80.345 T_s w_3 \\ & -21035.87/(T_s w_3) + 2049.125 (T_s w_3)^{1/2} - 4213.519 \ln(T_s w_3) + 0.0918 (T_s w_3)^2 \\ & -1.04 \mathrm{x} 10^{-4} (T_s w_3)^3 + 8597.453 - (4.182 - 2.2403 X_3)(T_s u_3 - T_r)\}] / \lambda_s\end{aligned}$$

$$(\partial g_{11} / \partial X_2) = -\, [2.2403\, l_2\, (T_s\, u_2 - T_r)] / \lambda_s$$

$$(\partial g_{11} / \partial u_2) = \{(4.182 - 2.2403 X_2) T_s\, l_2\} / \lambda_s$$

$$\begin{aligned}(\partial g_{11} / \partial w_2) = {} & [-3.12 \mathrm{x} 10^{-4}\, T_s^3\, w_2^2 + 0.1835\;\; T_s^2\, w_2 - 80.345\, T_s \\ & + 21035.87/(T_s\, w_2^2) + 1024.562\, (T_s / w_2)^{1/2} - 4213.519/w_2]\; \{(l_1 - l_2)/\lambda_s\}\end{aligned}$$

$$\begin{aligned}(\partial g_{11}/\partial l_3) \quad &= [4.154(T_s w_3 - T_r) + 2.0125 \times 10^{-4} (T^2_s w_3^2 - T_r^2) + 1.62\, T_s(u_3 - w_3) \\ &+ 2.0285 \times 10^{-4}\, T_s^2(u_3^2 - w_3^2) - 0.3747 \times 10^{-7} T_s^3(u_3^3 - w_3^3) - 80.345 T_s w_3 \\ &- 21035.87/(T_s w_3) + 2049.125(T_s w_3)^{1/2} - 4213.519 \ln(T_s w_3) + 0.0918(T_s w_3)^2 \\ &- 1.04 \times 10^{-4}(T_s w_3)^3 + 8597.453 - (4.182 - 2.2403 X_3)(T_s u_3 - T_r)]/\lambda_s\end{aligned}$$

$$(\partial g_{11}/\partial X_3) \quad = -[2.2403\, l_3(T_s u_3 - T_r)]/\lambda_s$$

$$\begin{aligned}(\partial g_{11}/\partial u_3) \quad &= \{(4.182 - 2.2403 X_3)T_s\}/\lambda_s - \{(l_2 - l_3)/\lambda_s\}[1.62 T_s + 4.057 \times 10^{-4}\, T_s^2 u_3 \\ &- 1.1241 \times 10^{-7}\, T_s^3 u_3^2 - (4.182 - 2.2403 X_3)T_s]\end{aligned}$$

$$\begin{aligned}(\partial g_{11}/\partial w_3) \quad &= [-3.12 \times 10^{-4}\, T_s^3 w_3^2 + 0.1835\, T_s^2 w_3 - 77.811\, T_s + 21035.85/(T_s w_3^2) \\ &+ 1024.562\,(T_s / w_3)^{1/2} - 4213.519/w_3]\, \{(l_2 - l_3)/\lambda_s\}\end{aligned}$$

$$\begin{aligned}(\partial g_{12}/\partial l_1) \quad &= [80.345 T_s w_2 + 21035.87/(T_s w_2) - 2049.125(T_s w_2)^{1/2} + 4213.519 \ln(T_s w_2) \\ &- 0.0918(T_s w_2)^2 + 1.04 \times 10^{-4}(T_s w_2)^3 - 8597.453]/\lambda_s\end{aligned}$$

$$(\partial g_{12}/\partial l_2) \quad = -(\partial g_{12}/\partial l_1)$$

$$\begin{aligned}(\partial g_{12}/\partial w_2) \quad &= 18.083\, T_s^2 A_3 u_3 /(F\lambda_s X_3) - [-3.12 \times 10^{-4}\, T_s^3 w_2^2 + 0.1835\, T_s^2 w_2 \\ &- 80.345\, T_s + 21035.87/(T_s w_2^2) + 1024.562\,(T_s / w_2)^{1/2} \\ &- 4213.519/w_2]\, \{(l_1 - l_2)/\lambda_s\}\end{aligned}$$

$$(\partial g_{12}/\partial X_3) \quad = -18.083\, T_s^2\, u_3 A_3 (1 - u_3)/(F\lambda_s X_3^2)$$

$$(\partial g_{12}/\partial u_3) \quad = 18.083\, T_s^2\, A_3 (1 - 2u_3)/(F\lambda_s X_3)$$

$$(\partial g_{13}/\partial l_4) \quad = -X_4/\lambda_s$$

$$(\partial g_{13}/\partial X_4) \quad = -l_4/\lambda_s$$

$$(\partial g_{14}/\partial X_4) = [(\gamma + nT_s w_4) + 2\alpha X_4 + 3\beta X_4^2]/(F.\lambda_s)$$

$$(\partial g_{14}/\partial u_4) = -\ T_s/(F.\lambda_s)$$

$$\begin{aligned}(\partial g_{15}/\partial l_2) = {} & [-80.345T_s w_3 - 21035.87/(T_s w_3) + 2049.125(T_s w_3)^{1/2} - 4213.519\ln(T_s w_3) \\ & + 0.0918(T_s w_3)^2 - 1.04\text{x}10^{-4}(T_s w_3)^3 + 8597.453]/\lambda_s\end{aligned}$$

$$\begin{aligned}(\partial g_{15}/\partial l_3) = {} & [\{(4.182 - 2.2403X_3)(T_s u_3 - T_r) - (4.182 - 2.2403X_4)((T_s u_4 - T_r)\} \\ & - \{-80.345T_s w_3 - 21035.87/(T_s w_3) + 2049.125(T_s w_3)^{1/2} - 4213.519\ln(T_s w_3) \\ & + 0.0918(T_s w_3)^2 - 1.04\text{x}10^{-4}(T_s w_3)^3 + 8597.453\} - \{4.154(T_s w_4 - T_r) \\ & + 2.0125\text{x}10^{-4}(T^2_s w_4^2 - T_r^2) + 1.62T_s(u_4 - w_4) + 2.0285\text{x}10^{-4}T_s^2(u_4^2 - w_4^2) \\ & - 0.3747\text{ x }10^{-7}T_s^3(u_4^3 - w_4^3) - 80.345T_s w_4 - 21035.87/(T_s w_4) \\ & + 2049.125(T_s w_4)^{1/2} - 4213.519\ln(T_s w_4) + 0.0918(T_s w_4)^2 \\ & - 1.04\text{x}10^{-4}(T_s w_4)^3 + 8597.453 - (4.182 - 2.2403X_4)(T_s u_4 - T_r)\}]/\lambda_s\end{aligned}$$

$$(\partial g_{15}/\partial X_3) = -[2.2403\, l_3(T_s u_3 - T_r)]/\lambda_s$$

$$(\partial g_{15}/\partial u_3) = \{(4.182 - 2.2403X_3)T_s l_3\}/\lambda_s$$

$$\begin{aligned}(\partial g_{15}/\partial w_3) = {} & [-3.12\text{x}10^{-4}T_s^3 w_3^2 + 0.1835T_s^2 w_3 - 80.345T_s + 21035.85/(T_s w_3^2) \\ & + 1024.562(T_s/w_3)^{1/2} - 4213.519/w_3]\{(l_2 - l_3)/\lambda_s\}\end{aligned}$$

$$\begin{aligned}(\partial g_{15}/\partial l_4) = {} & [4.154(T_s w_4 - T_r) + 2.0125\text{x}10^{-4}(T^2_s w_4^2 - T_r^2) + 1.62T_s(u_4 - w_4) \\ & + 2.0285\text{x}10^{-4}T_s^2(u_4^2 - w_4^2) - 0.3747\text{x}10^{-7}T_s^3(u_4^3 - w_4^3) - 80.345T_s w_4 \\ & - 21035.87/(T_s w_4) + 2049.125(T_s w_4)^{1/2} - 4213.519\ln(T_s w_4) + 0.0918(T_s w_4)^2 \\ & - 1.04\text{x}10^{-4}(T_s w_4)^3 + 8597.453 - (4.182 - 2.2403X_4)(T_s u_4 - T_r)]/\lambda_s\end{aligned}$$

$$(\partial g_{15}/\partial X_4) = [2.2403\, l_4(T_s u_4 - T_r)]/\lambda_s$$

$$(\partial g_{15} / \partial u_4) = \{(4.182-2.2403X_4)T_s\} / \lambda_s - \{(l_3-l_4)/\lambda_s\}[1.62T_s + 4.057 \times 10^{-4} T_s^2 u_4 - 1.1241 \times 10^{-7} T_s^3 u_4^2 - (4.182-2.2403X_4)T_s]$$

$$(\partial g_{16} / \partial l_2) = [80.345T_s w_3 + 21035.87/(T_s w_3) - 2049.125(T_s w_3)^{1/2} + 4213.519\ln(T_s w_3) - 0.0918(T_s w_3)^2 + 1.04\times10^{-4}(T_s w_3)^3 - 8597.453]/\lambda_s$$

$$(\partial g_{16} / \partial l_3) = -(\partial g_{16} / \partial l_2)$$

$$(\partial g_{16} / \partial w_3) = 18.083\, T_s^2 A_4 u_4 / (F\lambda_s X_4) - [-3.12\times10^{-4} T_s^3 w_3^2 + 0.1835\, T_s^2 w_3 - 80.345\, T_s + 21035.87/(T_s w_3^2) + 1024.562\, (T_s / w_3)^{1/2} - 4213.519/w_3]\, \{(l_2-l_3)/\lambda_s\}$$

$$(\partial g_{16} / \partial X_4) = -18.083\, T_s^2 u_4 A_4 (1-u_4)/(F\lambda_s X_4^2)$$

$$(\partial g_{16} / \partial u_4) = 18.083\, T_s^2 A_4 (1-2u_4)/(F\lambda_s X_4)$$

All the remaining elements of the Jacobian are zero.

Similar relationships and Jacobian elements have also been developed for quadruple effect sugar evaporator under backward- and mixed- feed arrangement and for quintuple effect black- liquor evaporator and triple effect caustic soda evaporator under forward, backward, and mixed feed arrangements.

APPENDIX-E

VALUES OF STEAM ECONOMY AND END-PRODUCT CONCENTRATION

Table E.1 Computed values of vapour flow rate from each effect, steam consumption, steam economy, and end- product concentration of a sugar quadruple effect evaporator as a function of feed temperature for various feed arrangements

Operating variables:

Feed rate = 70 ton/hr
Feed concentration = 18.00°Bx
Steam pressure (temperature) = 143.27 kPa
Pressure (temperature) in the last effect = 15.74 kPa

Feed temp-erature °C	Vapour flow rate, ton/hr I effect	II effect	III effect	IV effect	Steam consum-ption ton/hr	Steam economy kg/kg	end-product concentra-tion °Bx
			Forward feed arrangement				
70	11.086	11.686	12.373	13.336	15.223	3.185	58.56
80	11.135	11.735	12.422	13.387	14.131	3.445	59.10
90	11.183	11.783	12.469	13.436	13.039	3.748	59.64
100	11.230	11.829	12.516	13.485	11.945	4.107	60.18
110	11.276	11.875	12.561	13.532	10.851	4.538	60.71
			Backward feed arrangement				
70	12.792	11.792	10.786	12.221	13.923	3.418	56.23
80	12.554	11.572	10.584	13.137	13.668	3.500	56.88
90	12.316	11.351	10.381	14.051	13.412	3.586	57.54
100	12.077	11.129	10.178	14.966	13.156	3.675	58.20
110	11.837	10.907	9.974	15.880	12.900	3.767	58.88
			Mixed feed arrangement*				
70	12.375	10.895	11.645	12.612	14.529	3.271	56.07
80	11.650	11.200	11.962	12.953	13.750	3.474	56.67
90	10.920	11.500	12.272	13.288	12.968	3.700	57.23
100	10.187	11.795	12.577	13.616	12.181	3.955	57.73
110	9.448	12.084	12.874	13.936	11.390	4.244	58.18

* *Feed enters into II effect*

Table E.2 Computed values of vapour flow rate of an effect, steam consumption, steam economy, and end-product concentration of a quintuple effect black-liquor evaporator as a function of feed temperature for various feed arrangements

Operating variables:

Feed concentration = 22.00°T_w
Feed rate = 70 ton/hr
Pressure (temperature) in the last effect = 16.577 kPa
Steam pressure (temperature) = 313.10 kPa

Feed temperature, °C	Vapour flow rate, ton/hr I effect	II effect	III effect	IV effect	V effect	Steam consumption, ton/hr	Steam economy, kg/kg	end-product concentration °T_w
Forward feed arrangement								
60	5.768	6.737	7.802	8.969	10.291	13.567	2.916	50.61
70	5.917	6.899	7.975	9.154	10.494	12.638	3.200	52.10
80	6.060	7.054	8.141	9.329	10.687	11.684	3.532	53.61
90	6.196	7.201	8.298	9.495	10.872	10.706	3.929	55.13
100	6.326	7.342	8.448	9.653	11.049	9.705	4.412	56.66
110	6.451	7.476	8.591	9.804	11.219	8.683	5.015	58.21
Backward feed arrangement								
60	9.987	8.366	6.910	5.646	5.770	11.747	3.122	46.22
70	9.845	8.251	6.818	5.598	6.872	11.571	3.231	47.22
80	9.700	8.133	6.724	5.545	7.979	11.390	3.343	48.25
90	9.552	8.013	6.627	5.488	9.091	11.206	3.460	49.32
100	9.401	7.889	6.528	5.426	10.209	11.017	3.581	50.42
110	9.247	7.764	6.426	5.361	11.333	10.825	3.708	51.56
Mixed feed arrangement*								
60	10.919	7.556	4.999	5.975	7.079	12.958	2.819	46.01
70	10.414	7.090	5.501	6.537	7.705	12.345	3.017	47.02
80	9.899	6.615	6.001	7.096	8.325	11.721	3.237	48.03
90	9.373	6.129	6.502	7.653	8.941	11.088	3.481	49.04
100	8.837	5.635	7.003	8.208	9.552	10.446	3.756	50.06
110	8.292	5.132	7.504	8.761	10.159	9.796	4.068	51.08

* *Feed enters into III effect*

Table E.3 Computed values of vapour flow rate of an effect, steam consumption, steam economy, and end-product concentration of a triple effect caustic soda evaporator as a function of feed temprature for various feed arrangements

Operating variables:

Feed concentration = 12.00%
Feed rate = 25 ton/hr
Pressure (temperature) in the last effect = 5.226 kPa
Steam pressure (temperature) = 700.80 kPa

Feed temperature, °C	Vapour flow rate, ton/hr			Steam consumption, ton/hr	Steam economy, kg/kg	end-product concentration %
	I effect	II effect	III effect			
			Forward feed arrangement			
60	5.175	6.126	6.703	8.752	2.057	42.89
70	5.213	6.164	6.727	8.376	2.161	43.51
80	5.250	6.201	6.750	7.998	2.276	44.13
90	5.286	6.238	6.772	7.619	2.401	44.75
100	5.321	6.273	6.793	7.239	2.540	45.37
110	5.356	6.308	6.813	6.858	2.694	45.99
			Backward feed arrangement			
60	6.191	5.054	5.988	8.083	2.132	38.63
70	6.080	4.964	6.307	7.962	2.179	39.23
80	5.969	4.873	6.625	7.840	2.228	39.83
90	5.857	4.780	6.942	7.717	2.278	40.43
100	5.744	4.687	7.257	7.593	2.329	41.03
110	5.630	4.593	7.570	7.468	2.383	41.63
			Mixed feed arrangement			
60	5.818	5.197	6.280	8.401	2.059	38.94
70	5.598	5.350	6.446	8.138	2.137	39.44
80	5.376	5.501	6.609	7.872	2.221	39.93
90	5.154	5.652	6.769	7.605	2.311	40.41
100	4.931	5.801	6.927	7.336	2.407	40.87
110	4.708	5.949	7.082	7.065	2.511	41.32

Table E.4 Computed values of vapour flow rate from each effect, steam consumption, steam economy, and end-product concentration of a sugar quadruple effect evaporator as a function of feed concentration for various feed arrangements

Operating variables:

Feed rate = 70 ton/hr
Feed temperature = 100.00°C
Pressure (temperature) in the last effect = 15.74 kPa
Steam pressure (temperature) = 143.27 kPa

Feed concent- tration, °Bx	Vapour flow rate, ton/hr				Steam consum- ption, ton/hr	Steam econ- omy, kg/kg	end- product concen- tration °Bx
	I effect	II effect	III effect	IV effect			
			Forward feed arrangement				
12	13.280	13.757	14.298	15.140	14.121	3.999	62.11
14	12.571	13.096	13.693	14.582	13.364	4.036	61.03
16	11.886	12.451	13.096	14.027	12.637	4.072	60.42
18	11.230	11.829	12.516	13.485	11.945	4.107	60.18
20	10.604	11.233	11.955	12.958	11.289	4.141	60.18
			Backward feed arrangement				
12	13.807	12.996	12.196	17.204	14.865	3.781	60.89
14	13.230	12.364	11.501	16.433	14.293	3.745	59.50
16	12.650	11.738	10.826	15.686	13.720	3.710	58.64
18	12.077	11.129	10.178	14.966	13.156	3.675	58.20
20	11.517	10.540	9.559	14.277	12.607	3.640	58.08
			Mixed feed arrangement*				
12	11.982	13.989	14.647	15.531	13.671	4.107	60.65
14	11.387	13.224	13.932	14.878	13.184	4.052	59.12
16	10.784	12.492	13.240	14.238	12.683	4.002	58.20
18	10.187	11.795	12.577	13.616	12.181	3.955	57.73
20	9.606	11.134	11.943	13.016	11.688	3.910	57.62

* *Feed enters into II effect*

Table E.5 Computed values of vapour flow rate of an effect, steam consumption, steam economy, and end-product concentration of a quintuple effect black-liquor evaporator as a function of feed concentration for various feed arrangements

Operating variables:

Feed temperature = 90.00°C
Feed rate = 70 ton/hr
Pressure (temperature) in the last effect = 16.577 kPa
Steam pressure temperature) = 313.10 kPa

Feed concentration, $^{\circ}T_w$	Vapour flow rate, ton/hr I effect	II effect	III effect	IV effect	V effect	Steam consumption, ton/hr	Steam economy, kg/kg	end-product concentration $^{\circ}T_w$
Forward feed arrangement								
16	7.774	8.735	9.734	10.767	11.884	12.267	3.986	53.07
18	7.220	8.195	9.229	10.320	11.529	11.720	3.967	53.61
20	6.694	7.684	8.750	9.870	11.192	11.200	3.948	54.30
22	6.196	7.201	8.298	9.495	10.872	10.706	3.929	55.13
24	5.724	6.745	7.871	9.115	10.569	10.238	3.910	56.05
26	5.280	6.316	7.469	8.756	10.284	9.795	3.890	57.07
Backward feed arrangement								
16	10.920	9.472	8.119	6.963	10.738	12.444	3.714	47.09
18	10.439	8.956	7.591	6.442	10.161	12.011	3.629	47.71
20	9.983	8.470	7.094	5.950	9.612	11.598	3.545	48.46
22	9.552	8.013	6.627	5.488	9.091	11.206	3.460	49.32
24	9.144	7.582	6.189	5.054	8.599	10.832	3.376	50.25
26	8.760	7.178	5.778	4.647	8.136	10.479	3.292	51.27
Mixed feed arrangement*								
16	10.648	7.623	8.112	9.240	10.433	12.202	3.775	46.78
18	10.199	7.095	7.546	8.680	9.907	11.811	3.677	47.42
20	9.774	6.598	7.009	8.151	9.410	11.441	3.579	48.18
22	9.373	6.129	6.502	7.653	8.941	11.088	3.481	49.04
24	8.993	5.688	6.024	7.184	8.498	10.754	3.384	49.99
26	8.634	5.275	5.575	6.743	8.082	10.437	3.287	51.00

* *Feed enters into III effect*

Table E.6 Computed values of vapour flow rate of an effect, steam consumption, steam economy, and end-product concentration of a triple effect caustic soda evaporator as a function of feed concentration for various feed arrangements

Operating variables:

Feed temperature = 35.00°C
Feed rate = 25 ton/hr
Pressure (temperature) in the last effect = 5.226 kPa
Steam pressure (temperature) = 700.80 kPa

Feed concentration, %	Vapour flow rate, ton/hr I effect	II effect	III effect	Steam consumption, ton/hr	Steam economy, kg/kg	end-product concentration %
			Forward feed arrangement			
8	6.068	7.038	7.539	8.151	2.533	45.94
10	5.682	6.645	7.155	7.781	2.504	45.32
12	5.304	6.255	6.783	7.429	2.469	45.06
14	4.932	5.873	6.423	7.091	2.430	45.04
16	4.568	5.500	6.077	6.765	2.387	45.17
18	4.211	5.135	5.743	6.447	2.340	45.41
			Backward feed arrangement			
8	6.447	5.557	8.104	8.099	2.483	40.89
10	6.121	5.134	7.593	7.881	2.392	40.65
12	5.800	4.734	7.099	7.655	2.304	40.73
14	5.487	4.355	6.622	7.425	2.217	41.01
16	5.181	3.994	6.159	7.192	2.132	41.39
18	4.883	3.650	5.711	6.955	2.048	41.84
			Mixed feed arrangement			
8	5.713	6.588	7.786	7.865	2.554	40.73
10	5.373	6.148	7.310	7.672	2.454	40.53
12	5.043	5.726	6.848	7.471	2.358	40.64
14	4.724	5.320	6.403	7.264	2.264	40.93
16	4.416	4.927	5.973	7.055	2.171	41.31
18	4.117	4.546	5.559	6.843	2.078	41.76

Table E.7 Computed values of vapour flow rate from each effect, steam consumption, steam economy, and end- product concentration of a sugar quadruple effect evaporator as a function of feed rate for various feed arrangements

Operating variables:

Feed temperature	=	100.00°C
Feed concentration	=	18.00°Bx
Pressure (temperature) in the last effect	=	15.74 kPa
Steam pressure (temperature)	=	143.27 kPa

Feed rate, ton/hr	Vapour flow rate, ton/hr				Steam consumption, ton/hr	Steam economy, kg/kg	end-product concentration °Bx
	I effect	II effect	III effect	IV effect			
			Forward feed arrangement				
60	10.174	10.647	11.182	11.920	10.825	4.057	67.18
70	11.230	11.829	12.516	13.485	11.945	4.107	60.18
80	12.132	12.864	13.711	14.928	12.907	4.156	54.62
90	12.907	13.774	14.789	16.267	13.735	4.203	50.21
100	13.573	14.579	15.769	17.518	14.454	4.251	46.68
110	14.149	15.296	16.664	18.697	15.080	4.298	43.81
			Backward feed arrangement				
60	10.756	9.990	9.231	13.336	11.687	3.706	64.72
70	12.077	11.129	10.178	14.966	13.156	3.675	58.20
80	13.252	12.117	10.969	16.443	14.490	3.642	52.90
90	14.300	12.973	11.627	17.788	15.707	3.609	48.64
100	15.238	13.717	12.172	19.023	16.823	3.575	45.17
110	16.083	14.364	12.622	20.164	17.853	3.542	42.34
			Mixed feed arrangement *				
60	9.170	10.657	11.279	12.089	10.777	4.008	64.27
70	10.187	11.795	12.577	13.616	12.181	3.955	57.73
80	11.038	12.774	13.722	15.004	13.457	3.904	52.44
90	11.746	13.618	14.737	16.274	14.620	3.856	48.18
100	12.333	14.347	15.642	17.443	15.687	3.810	44.74
110	12.817	14.981	16.453	18.525	16.673	3.765	41.93

* *Feed enters into II effect*

Table E.8 Computed values of vapour flow rate of an effect, steam consumption, steam economy, and end-product concentration of a quintuple effect black-liquor evaporator as a function of feed rate for various feed arrangements

Operating variables:

Feed temperature = 90.00°C
Feed concentration = 22.00°T_w
Pressure (temperature) in the last effect = 16.577 kPa
Steam pressure (temperature) = 313.10 kPa

Feed rate, ton/hr	Vapour flow rate, ton/hr I effect	II effect	III effect	IV effect	V effect	Steam consumption, ton/hr	Steam economy, kg/kg	end-product concentration °T_w
			Forward feed arrangement					
60	6.051	6.888	7.784	8.746	9.885	10.006	3.933	63.94
65	6.134	7.056	8.052	9.130	10.379	10.369	3.930	58.98
70	6.196	7.201	8.298	9.495	10.872	10.706	3.929	55.13
75	6.237	7.325	8.523	9.843	11.358	11.019	3.928	52.03
80	6.260	7.429	8.729	10.175	11.836	11.310	3.928	49.48
85	6.267	7.515	8.917	10.490	12.305	11.582	3.928	47.34
			Backward feed arrangement					
60	8.895	7.626	6.456	5.473	8.549	10.170	3.638	57.39
65	9.227	7.824	6.547	5.485	8.825	10.696	3.544	52.78
70	9.552	8.013	6.627	5.488	9.091	11.206	3.460	49.32
75	9.861	8.184	6.691	5.476	9.344	11.696	3.382	46.56
80	10.152	8.337	6.737	5.451	9.583	12.168	3.309	44.29
85	10.425	8.470	6.767	5.411	9.808	12.622	3.239	42.39
			Mixed feed arrangement*					
60	8.707	6.059	6.354	7.313	8.363	10.033	3.668	56.89
65	9.044	6.100	6.434	7.489	8.657	10.569	3.569	52.43
70	9.373	6.129	6.502	7.653	8.941	11.088	3.481	49.04
75	9.685	6.142	6.554	7.800	9.210	11.589	3.399	46.34
80	9.977	6.136	6.591	7.930	9.463	12.069	3.322	44.11
85	10.250	6.113	6.613	8.045	9.702	12.531	3.250	42.24

** Feed enters into III effect*

Table E.9 Computed values of vapour flow rate of an effect, steam consumption, steam economy, and end-product concentration of a triple effect caustic soda evaporator as a function of feed rate for various feed arrangements

Operating variables:

Feed temperature = 95.00°C
Feed concentration = 12.00%
Pressure (temperature) in the last effect = 5.226 kPa
Steam pressure (temperature) = 700.80 kPa

Feed rate, ton/hr	Vapour flow rate, ton/hr I effect	II effect	III effect	Steam consumption, ton/hr	Steam economy, kg/kg	end-product concentration %
			Forward feed arrangement			
15	3.714	4.144	4.121	5.161	2.321	59.63
20	4.611	5.286	5.487	6.415	2.398	52.01
25	5.304	6.255	6.783	7.429	2.469	45.06
30	5.797	7.042	7.973	8.225	2.530	39.19
35	6.125	7.665	9.047	8.844	2.582	34.53
40	6.328	8.156	10.016	9.332	2.625	30.97
			Backward feed arrangement			
15	3.777	3.279	4.718	5.141	2.290	55.83
20	4.868	4.098	5.999	6.495	2.304	47.69
25	5.800	4.734	7.099	7.655	2.304	40.73
30	6.552	5.185	8.018	8.639	2.287	35.15
35	7.135	5.475	8.779	9.478	2.257	30.86
40	7.575	5.636	9.416	10.206	2.217	27.63
			Mixed feed arrangement			
15	3.352	3.966	4.444	4.908	2.397	55.63
20	4.285	4.946	5.722	6.275	2.383	47.56
25	5.043	5.726	6.848	7.471	2.358	40.64
30	5.606	6.316	7.812	8.502	2.321	35.07
35	5.988	6.741	8.628	9.390	2.274	30.79
40	6.224	7.041	9.320	10.165	2.222	27.56

Table E.10 Computed values of vapour flow rate from each effect, steam consumption, steam economy, and end- product concentration of a sugar quadruple effect evaporator as a function of pressure in the last effect for various feed arrangements

Operating variables:

Feed temperature = 100.00°C
Feed concentration = 18.00°Bx
Feed rate = 70 ton/hr
Steam pressure (temperature) = 143.27 kPa

Pressure in the last effect, kPa	Vapour flow rate, ton/hr				Steam consumption, ton/hr	Steam economy, kg/kg	end-product concentration °Bx
	I effect	II effect	III effect	IV effect			
			Forward feed arrangement				
9.58	11.585	12.204	12.919	14.036	12.286	4.130	65.44
12.34	11.434	12.045	12.747	13.790	12.141	4.120	63.06
15.74	11.230	11.829	12.516	13.485	11.945	4.107	60.18
19.92	10.964	11.550	12.216	13.112	11.691	4.092	56.87
25.01	10.623	11.192	11.833	12.657	11.364	4.075	53.18
			Backward feed arrangement				
9.58	12.472	11.395	10.262	16.055	13.705	3.662	63.59
12.34	12.294	11.282	10.247	15.544	13.452	3.670	61.07
15.74	12.077	11.129	10.178	14.966	13.156	3.675	58.20
19.92	11.805	10.921	10.045	14.313	12.804	3.677	54.99
25.01	11.462	10.641	9.836	13.574	12.379	3.677	51.46
			Mixed feed arrangement*				
9.58	10.244	12.224	13.075	14.324	12.493	3.992	62.59
12.34	10.233	12.041	12.859	13.999	12.351	3.978	60.39
15.74	10.187	11.795	12.577	13.616	12.181	3.955	57.73
19.92	10.090	11.478	12.220	13.164	11.966	3.924	54.67
25.01	9.927	11.079	11.776	12.632	11.687	3.886	51.25

** Feed enters into II effect*

Table E.11 Computed values of vapour flow rate of an effect, steam consumption, steam economy, and end-product concentration of a quintuple effect black-liquor evaporator as a function of pressure in the last effect for various feed arrangements

Operating variables:

Feed temperature = 90.00°C
Feed concentration = 22.00° Tw
Feed rate = 70 ton/hr
Steam pressure (temperature) = 313.10 kPa

Pressure in the last effect kPa	Vapour flow rate, ton/hr					Steam consumption, ton/hr	Steam economy, kg/kg	end-product concentration °Tw
	I effect	II effect	III effect	IV effect	V effect			
				Forward feed arrangement				
9.58	6.751	7.815	8.972	10.237	11.784	11.201	4.067	63.02
12.34	6.524	7.564	8.697	9.934	11.399	10.999	4.011	59.51
15.74	6.254	7.266	8.369	9.574	10.964	10.758	3.944	55.86
19.92	5.946	6.925	7.994	9.162	10.484	10.483	3.864	52.23
25.01	5.602	6.543	7.575	8.702	9.960	10.176	3.772	48.71
31.24	5.224	6.124	7.113	8.195	9.393	9.840	3.664	45.36
				Backward feed arrangement				
9.58	10.267	8.602	7.093	5.881	10.649	11.998	3.542	55.99
12.34	9.960	8.351	6.897	5.717	9.963	11.660	3.507	52.90
15.74	9.623	8.072	6.675	5.528	9.240	11.285	3.468	49.90
19.92	9.254	7.763	6.425	5.315	8.481	10.870	3.426	46.43
25.01	8.851	7.423	6.145	5.075	7.685	10.413	3.378	44.23
31.24	8.411	7.049	5.834	4.808	6.853	9.912	3.325	41.57
				Mixed feed arrangement *				
9.58	9.75	6.327	7.377	8.659	10.083	11.524	3.662	55.41
12.34	9.60	6.250	6.997	8.221	9.587	11.347	3.583	52.48
15.74	9.41	6.152	6.587	7.751	9.052	11.135	3.498	49.61
19.92	9.196	6.030	6.149	7.248	8.480	10.885	3.409	46.82
25.01	8.943	5.885	5.679	6.712	7.870	10.592	3.313	44.12
31.24	8.652	5.713	5.178	6.141	7.221	10.255	3.209	41.52

* *Feed enters into III effect*

Table E.12 Computed values of vapour flow rate of an effect, steam consumption, steam economy, and end-product concentration of a triple effect caustic soda evaporator as a function of pressure in the last effect for various feed arrangements

Operating variables:

Feed temperature = 95.00°C
Feed concentration = 12.00%
Feed rate = 25 ton/hr
Steam pressure (temperature) = 700.80 kPa

Pressure in the last effect, kPa	Vapour flow rate, ton/hr			Steam consumption, ton/hr	Steam economy, kg/kg	end-product concentration %
	I effect	II effect	III effect			
			Forward feed arrangement			
4.24	5.405	6.365	6.829	7.518	2.474	46.88
5.226	5.304	6.255	6.783	7.429	2.469	45.06
7.38	5.197	6.140	6.719	7.336	2.461	43.21
9.58	5.084	6.018	6.639	7.238	2.451	41.34
12.34	4.963	5.887	6.544	7.134	2.438	39.45
15.74	4.834	5.746	6.432	7.021	2.423	37.56
			Backward feed arrangement			
4.24	5.854	4.753	7.275	7.788	2.296	42.15
5.226	5.800	4.734	7.099	7.655	2.304	40.73
7.38	5.742	4.707	6.902	7.517	2.308	39.23
9.58	5.679	4.672	6.685	7.372	2.311	37.68
12.24	5.607	4.628	6.450	7.219	2.311	36.08
15.74	5.525	4.573	6.197	7.055	2.310	34.47
			Mixed feed arrangement			
4.24	5.044	5.846	6.968	7.552	2.365	42.02
5.226	5.043	5.726	6.848	7.471	2.358	40.64
7.38	5.037	5.597	6.710	7.386	2.348	39.19
9.58	5.025	5.456	6.553	7.295	2.335	37.67
12.34	5.006	5.304	6.381	7.196	2.319	36.11
15.74	4.977	5.140	6.192	7.089	2.301	34.52

Table E.13 Computed values of vapour flow rate from each effect, steam consumption, steam economy, and end- product concentration of a sugar quadruple effect evaporator as a function of steam pressure for various feed arrangements

Operating variables:

Feed temperature = 100.00°C
Feed concentration = 18.00°Bx
Feed rate = 70 ton/hr
Pressure (temperature) in the last effect = 15.74 kPa

Steam pressure, kPa	Vapour flow rate, ton/hr				Steam consumption, ton/hr	Steam economy, kg/kg	end-product concentration °Bx
	I effect	II effect	III effect	IV effect			
			Forward feed arrangement				
120.8	10.673	11.271	11.958	12.900	10.816	4.327	54.32
132.0	10.961	11.560	12.248	13.205	11.389	4.212	57.21
143.3	11.230	11.829	12.516	13.485	11.945	4.107	60.18
156.2	11.481	12.080	12.764	13.742	12.488	4.009	63.22
169.1	11.716	12.313	12.993	13.977	13.017	3.918	66.32
			Backward feed arrangement				
120.8	11.443	10.548	9.646	14.446	12.422	3.710	52.69
132.0	11.772	10.850	9.923	14.716	12.801	3.692	55.42
143.3	12.077	11.129	10.178	14.966	13.156	3.675	58.20
156.2	12.357	11.386	10.412	15.195	13.491	3.658	61.02
169.1	12.616	11.622	10.628	15.406	13.806	3.641	63.87
			Mixed feed arrangement*				
120.8	9.461	11.296	12.062	13.061	11.365	4.037	52.24
132.0	9.837	11.556	12.330	13.350	11.786	3.994	54.96
143.3	10.187	11.795	12.577	13.616	12.181	3.955	57.73
156.2	10.512	12.014	12.803	13.859	12.553	3.918	60.55
169.1	10.813	12.216	13.010	14.083	12.904	3.884	63.39

* *Feed enters into II effect*

Table E.14 Computed values of vapour flow rate of an effect, steam consumption, steam economy, and end-product concentration of a quintuple effect black-liquor evaporator as a function of steam pressure for various feed arrangements

Operating variables:

Feed temperature = 90.00°C
Feed concentration = 22.00°T_w
Feed rate = 70 ton/hr
Pressure (temperature) in the last effect = 16.577 kPa

Steam pressure kPa	Vapour flow rate, ton/hr I effect	II effect	III effect	IV effect	V effect	Steam consumption, ton/hr	Steam economy, kg/kg	end-product concentration °T_w
				Forward feed arrangement				
232	5.357	6.291	7.321	8.456	9.731	8.710	4.266	46.89
270	5.781	6.752	7.817	8.986	10.309	9.705	4.085	50.74
313	6.196	7.201	8.298	9.495	10.872	10.706	3.929	55.13
362	6.600	7.636	8.761	9.983	11.423	11.711	3.792	60.17
416	6.988	8.053	9.201	10.443	11.969	12.717	3.669	65.97
				Backward feed arrangement				
232	8.272	6.899	5.673	4.668	8.291	9.781	3.456	42.55
270	8.916	7.459	6.153	5.080	8.693	10.501	3.457	45.70
313	9.552	8.013	6.627	5.488	9.091	11.206	3.460	49.32
362	10.187	8.567	7.102	5.895	9.489	11.895	3.467	53.55
416	10.836	9.133	7.587	6.311	9.894	12.571	3.481	58.70
				Mixed feed arrangement*				
232	8.019	5.032	5.763	6.830	8.031	9.555	3.525	42.40
270	8.701	5.583	6.135	7.245	8.490	10.331	3.500	45.50
313	9.373	6.129	6.502	7.653	8.941	11.088	3.481	49.04
362	10.040	6.676	6.867	8.057	9.386	11.829	3.468	53.16
416	10.716	7.234	7.236	8.466	9.835	12.555	3.464	58.09

** Feed enters into III effect*

Table E.15 Computed values of vapour flow rate of an effect, steam consumption, steam economy, and end-product concentration of a triple effect caustic soda evaporator as a function of steam pressure for various feed arrangements

Operating variables:

Feed temperature = 95.00°C
Feed concentration = 12.00%
Feed rate = 25 ton/hr
Pressure (temperature) in the last effect = 5.226 kPa

Steam pressure, kPa	Vapour flow rate, ton/hr			Steam consumption, ton/hr	Steam economy, kg/kg	end-product concentration %
	I effect	II effect	III effect			
Forward feed arrangement						
543.2	5.055	6.004	6.621	6.735	2.625	40.99
618.1	5.183	6.134	6.708	7.083	2.545	43.01
700.8	5.304	6.255	6.783	7.429	2.469	45.06
792.0	5.418	6.369	6.847	7.775	2.396	47.13
892.4	5.526	6.475	6.902	8.122	2.327	49.21
Backward feed arrangement						
543.2	5.521	4.515	6.889	7.129	2.374	37.16
618.1	5.665	4.629	6.999	7.394	2.339	38.93
700.8	5.800	4.734	7.099	7.655	2.304	40.73
792.0	5.926	4.831	7.193	7.913	2.268	42.56
892.4	6.042	4.921	7.279	8.169	2.233	44.41
Mixed feed arrangement						
543.2	4.749	5.521	6.630	6.934	2.437	37.05
618.1	4.901	5.628	6.744	7.205	2.397	38.83
700.8	5.043	5.726	6.848	7.471	2.358	40.64
792.0	5.175	5.818	6.944	7.734	2.319	42.48
892.4	5.298	5.902	7.033	7.993	2.281	44.34

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