ABSOLUTE ORIENTATION OF INDIAN GEODETIC SYSTEM BY GRAVIMETRIC METHOD

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CIVIL ENGINEERING

By
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CERTIFICATE

Certified that the thesis entitled 'ABSOLUTE ORIENTATION OF INDIAN GEODETIC SYSTEM BY GRAVIMETRIC METHOD' which is being submitted by Sri PRADIP K. RAY in fulfilment of the requirements for the award of the Degree of DOCTOR OF PHILOSOPHY IN CIVIL ENGINEERING of the University of Roorkee, is a record of the student's own work carried out by him under our supervision and guidance. The matter embodied in this thesis has not been submitted for the award of any other Degree or Diploma.

This is further to certify that the candidate has worked for a period of three years from December, 1975 to December, 1978 for preparing this thesis at this University.

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A GUIDE TO NOTATIONS

COMMONLY USED SYMBOLS :

a = Semimajor axis of the reference spheroid

b = Semiminor axis of the reference spheroid

e = Eccentricity of the spheroid

f = Flattening of the spheroid

U = Spheropotential at a point

U_o = Potential of the reference spheroid

J_2 = Conventional 2nd-degree spheropotential coefficient

γ = Normal gravity at a point

γ_a = γ at equator

γ_h = γ at height h

f_2, f_4 = Coefficients of normal gravity formula

K = Universal gravitational constant

M = Mass of the earth

R = Average Radius of the earth

A, B, C, D, E, F = Moments and products of inertia of the earth

ω = Angular speed of rotation of the earth

G = Average gravity of the earth

V = Earth's attraction potential

V' = Potential of rotation

W = Total geopotential at a point

W_o = Earth's potential at the geoid

g = Gravity at a point

$\bar{C}_{nm}$, $\bar{S}_{nm}$ = Coefficients of spherical harmonic expansion of the geopotential

ϕ = Latitude of a point

λ = Longitude of a point

$\bar{\phi}$ = Geocentric latitude

P_n = Conventional Legendre polynomial on $\bar{\phi}$, of degree n

$\bar{P}_{nm}$ = Normalized associated Legendre function of degree n and order m

$D\phi, D\lambda$ = Width in ϕ and λ -direction, in degrees

σ = Element area, in steradian

T = Disturbing potential

Δg = Gravity anomaly at a point

N = Height of the geoid above the reference spheroid

ξ = Meridional deviation of the vertical

η = Prime vertical deviation of the vertical

ℓ = Distance between two points

r = Distance of a point from geocentre

x, y, z = Cartesian coordinates

h = Height of a point

ψ = Spherical distance between two points

α = Azimuth of a line

$S(\psi)$ = Stokes' function

$V(\psi)$ = Vening Meinesz' function

$U(\psi)$ = Modified Vening Meinesz' function

$\bar{S}(\psi), \bar{U}(\psi)$ = Normalized weighting functions

PREFIX AND SUBSCRIPTS :

$E\{\Delta g\}$ = Expectance of Δg

$\sigma^2\{y\}$ = Variance of y

$\Delta\phi$ = Difference of ϕ between two points

$\delta\lambda$ = Correction to λ

$a_{g\ell}$ = a of global spheroid

$f_{\ell o}$ = f of local spheroid

ϕ_g = ϕ at a gravity point

λ_c = λ at a computation point

ϕ_a = Astronomic ϕ

N_{gr} = Gravimetric N

N_{ag} = Astro-geodetic N

ξ_n = ξ at any station n

η_o = η at the origin of network

N_v = N of void geoid

ξ_p = ξ of partial geoid

η_i = η effect of innermost zone

VECTORS AND MATRICES :

L = Interstation Vector

T = Transformation Matrix

P = Position Vector

U = Undulation Vector

W = Weighting Vector

C = Orientation Correction Vector

X = Shift Vector

E = Spheroid Correction Matrix

D = Vector of change of spheroid dimension

I = Identity Matrix.

T' = Transpose of T

T^{-1} = Inverse of T

e_{ij} = Element of Matrix E on ith row and j th column

d_k = kth element of Vector D

ABBREVIATIONS :

BGI = Bureau Gravimetrique International

DMAAC = Defense Mapping Agency Aerospace Centre

GEM = Goddard Earth Model

GRS67 = Geodetic Reference System 1967.

IGSN71= International Gravity Standardization Net 1971

MFA = Mean Free-air Anomaly

OSU = Ohio State University

SAO = Smithsonian Astrophysical Observatory

SI = Systeme International d'Unites

WGS = World Geodetic System.

ABSTRACT

More than a century ago, Everest derived the dimensions of a spheroid on which the Indian Geodetic System was based. Its orientation at the Kalianpur origin of the Indian triangulation system has been arbitrarily chosen at various times. Local fitting of the spheroid could lead to conflicting claims by neighbouring countries in the definition of their national boundaries. The absolute orientation of the geodetic system is therefore a prerequisite for the readjustment of the Indian triangulation net for use as a global geodetic system. The present work is the first long - awaited attempt to redefine the values at the orientation parameters at the initial point, with reference to the Geodetic Reference System, 1967, by determining their absolute geocentric values.

The classical gravimetric principle has been used as the principal tool to accomplish the task. The well - known Stokes' formula relates the gravity anomalies over the entire surface of the earth to the undulation of the geoid above a geocentric reference spheroid, as a solution of the third boundary-value problem of the earth's gravitational potential. The Vening Meinesz' expressions similarly provide the meridional and the prime vertical

components of the deviations of the vertical. The results of these three global integrations of known gravity anomalies weighted by functions of the spherical distance, are compared to the corresponding astro-geodetic values existing in terms of the local system to arrive at the required correction parameters.

The irregularity of the gravity field over the earth's surface precludes the functional evaluation of the geoidal undulations, necessitating numerical discrete summation. The spherical surface is accordingly partitioned by finite elements with representative mean values of gravity anomalies expressed over them. The grid divisions have been adopted in this work as being well-suited for automatic computations. Furthermore, the nature of the Stokes' and Vening Meinesz' functions suggests that coarser grids may be used in the exterior regions without seriously affecting the accuracy of computation as long as comparatively finer meshes are used in the region of interest. Five-degree Equal-Area-Blocks have been used in the outer region, and further subdivisions of 1°, $0^{\circ}.25$, $0^{\circ}.05$, $0^{\circ}.01$ have been suggested for the interior region.

The first part of the computational work started with evaluations of the contribution of a recent set of five-degree mean free-air gravity anomalies, extending beyond a considerable margin around India. To suit machine evaluations on a digital computer, a number of analytical schemes have been developed in the formulation, such as,

(a) matrix form of interstation separation,

(b) non-dimensional forms of surface area and anomalies,

(c) modification of the Vening Meinesz' function and rearrangement of the functions in algebraic forms.

The geoidal parameters have been evaluated at the five-degree grid corners covering India and presented as an intermediate bye-product of the present investigation which may be useful for further work. The undulation ranges from -13 metres to -22 metres, whereas the deviation components smoothly vary between $\pm 1''$. A bicubic spline interpolation technique was used to compute the values at any desired point.

The next smaller size of mesh used is the one-square degree Meridian-Parallel-Grid type unit. The available data are nearly complete and updated. Gaps in farther areas have been filled up by a simplified procedure, keeping consistency of the average value over a block. For nearby unrepresented units, however, a local covariance interpolation has been used. The weighting functions which increase with decrease in distance, have been further normalized to minimize inaccuracies caused by exploding terms. After developing working formulae for computations from gridded data, the partial geoid parameters have been computed at 1° corners within India using the one-degree mean free-air anomalies covering the interior region. The profiles are seen to be mutually consistent, whereas the slope components vary sharply.

A combination of the void geoid and the partial geoid gives a pictorial representation of the one-degree mean free-air geoid in India. The variation is from -40 metres to -85 metres, with geoidal lows in the Himalayan region and in the Southern peninsula.

The last part of the main objective has been accomplished by completing the numerical algorithm using denser gravity details in the immediate neighbourhood of the computation point, which includes a further modification of the interstation vector to a differential expression. In order to estimate quarter-degree mean anomalies from point observations, simple average and patchwise surface-fitting have been used. For finer mesh sizes, a truncated pyramid window has been proposed. A few existing and suggested techniques for the evaluation of the effect of the innermost zone have also been enumerated. The numerical work consists of using modified terrain-corrected free-air anomalies around the initial point for further precision in the determination. The final results obtained are,

$\delta N_o = -59.0$ metres,

$\delta \xi_o = +0.65$ arcsecond,

$\delta \eta_o = +2.60$ arcseconds.

Although the orientation through the initial point itself provides the most reliable and stable positioning, the formulation of the gravimetric method permits any other astro-geodetic station also to be considered as a computation point. A first-order triagulation station with a commendable distribution of gravity coverage all around, may even act as a supercontrol point. An invariant shift vector has been introduced to further generalize the procedure and four zones at four geographical corners in India have been chosen for test computations. The limitations of availability and measurements of gravity data called for filling up some compartments of surrounding regions by prediction, for which a truncated local covariance interpolation has been used. Despite all the defects and approximation in these stations, the various sets or orientation parameters provide consistent numerical checks. The variations of results between themselves as well as with those obtained at the initial point are of the order of 3 metres in δN_o and 1" in $\delta \xi_o$ or $\delta \eta_o$, which are a little too high for obvious reasons.

An alternative proposition to obtain the absolute orientation parameters from the regional gravity data and the astro-geodetic geoid, forms the subject matter of a subsequent chapter. The inherent errors in the available informations being of fluctuating nature, a practical solution of the orientation problem may be achieved by the logic of minimization of their non-coincidence in a least-squares sense. The matching of undulations seems preferable to the parallelism condition, and the shift vector formulations are further modified to simpler expressions. The existing astro-geodetic geoid has been converted to one corresponding to the GRS 67 spheroid without changing the present orientation. The comparison of its undulations at some points with those of the gravimetric geoid obtained from one-degree mean free-air anomalies are made to frame condition equations and consequent normalization to yield optimal estimates of orientation parameters. The results differ by 1 metre in δN_o , 1" in $\delta \xi_o$ and 1".3 in $\delta \eta_o$ from the gravimetric results at the origin, showing thereby the possibilities of the exercise for further refinement.

With a view to formulating an integrated strategy to tackle the orientation problem, another plausible solution without requiring the use of any gravity data directly, has been tested in this work.

Similar to the astro-gravimetric geoid matching attempted earlier ,the astro-geodetic geoid heights in this case have been compared with those obtained from the satellite-derived geopotential coefficients, on assuming the apparent misfit to be solely due to the local non-geocentric orientation of the former. To obtain a smoothened geoid a 7th-order surface has been fitted using a number of astro-geodetic deviations, and its comparison with the present geoid shows an average discrepancy of 3 to 4 metres, the difference getting progressively increased with the distance from the origin. The other geoid is computed from the recent GEM 10 coefficients. Whilst the results obtained reveal that further work is necessary in this regard to achieve a reliable solution, the present work contributes all necessary formulations including the various recursion relations to optimize computer economy, which will be useful for future researchers.

The concluding part of the thesis summarizes various outputs of the methods adopted in the present work and compares them among themselves as well as with the datum shift values supplied by the satellite research organizations in respect of their adopted ellipsoids. All the sets fall within the reasonable limits of accuracy and the three alternative methods, viz.,

(i) the general astro-geodetic orientation,

(ii) the least-squares coincidence approach, and even

(iii) the astro-satellite matching provided quite useful checks. The linear shift components obtained from the present determinations are,

$$\Delta X = 243 \text{ metres},$$

$$\Delta Y = 733 \text{ metres},$$

$$\Delta Z = 174 \text{ metres}.$$

The corrections to the existing geoidal heights, latitudes and longitudes, have been presented in functional, digital as well as graphical forms. Finally, the various contributions of the study have been enumerated to delineate the scope of future advancement and further studies in this field.

CHAPTER I

INTRODUCTION

1.1 GENERAL

All levelling and astronomical measurements are necessarily made with respect to the level surface of the earth or the equipotential surface, conventionally coinciding with the mean sea level, called the geoid. The geoid is, however, a complicated surface with discontinuities in its curvature, caused by irregular distribution of masses within the earth, and is therefore ill-suited for being used as a reference surface. This difficulty can be circumvented by adopting a regular mathematical model closely approximating the geoid, as reference for mapping and mathematical computations, as long as the geoid itself is also completely defined with respect to this surface.

1.2 DIMENSIONS OF REFERENCE SURFACE

Attempts at determining the shape and size of a reference surface suitable for mapping the earth date back to primitive ages, when it was assumed to be a plane (Encyclopaedia Britannica, 1962). Later, Pythagorus, Aristotle and Eratosthenes argued that the earth was spherical in shape, and still later it was proved to be an ellipsoid (Heiskanen and Vening Meinesz, 1958), although there remained a controversy, even reflected in literature (Sharni, 1973), as to whether it was an oblate or a prolate spheroid. The rotational ellipsoid concept fitted well with the principles of physical geodesy and is supported by satellite observations also. Other suggestions for a reference, such as a triaxial ellipsoid made by Russian scientists, and the pear-shape (Ramanathan, 1978) derived from satellite data are equally unsuitable owing to their lack of simplicity. An ellipsoid of revolution defined by its major axis and flattening (Figure 1.1) is still found to be the most convenient reference surface. Three such surfaces related to the Indian geodetic system are described in Table 1.1

TABLE 1.1

DIMENSIONS OF SOME REFERENCE SURFACES

Name	a		1/f
Everest Spheroid	20922931.8	Indian feet	300.8017
Hayford International Spheroid	6378388	metres	297.0
Geodetic Reference System 1967	6378160	metres	298.24717

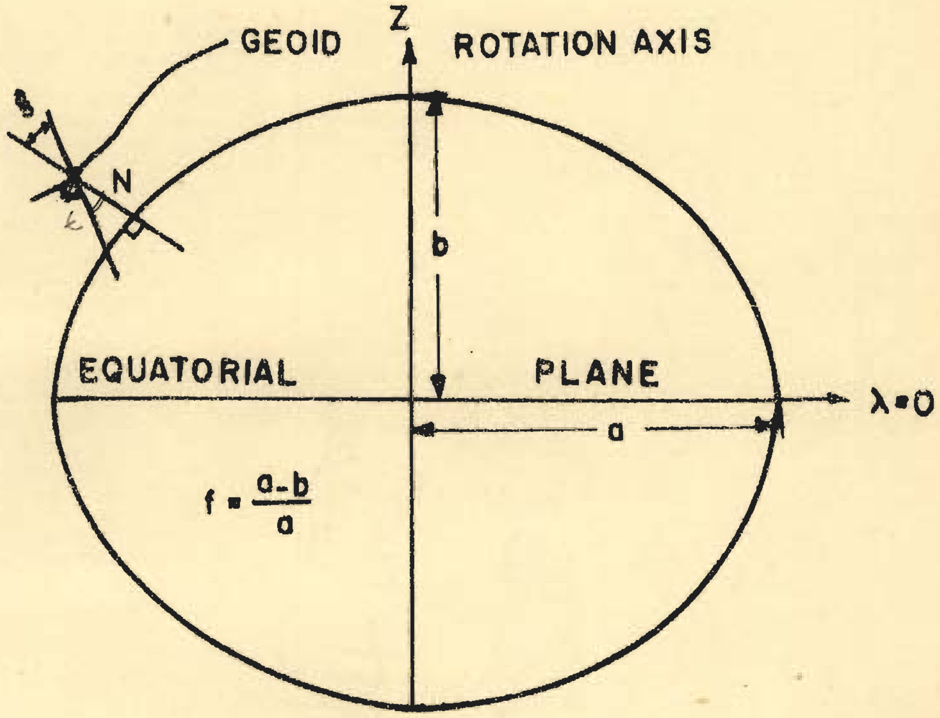


FIGURE 1.1 - REFERENCE SPHEROID AND GEOID

1.3 GEOID DETERMINATION AND ABSOLUTE ORIENTATION OF DATUM

Hirvonen(1934) was the first to compute geoidal undulations gravimetrically, followed by Tanni(1948) with more gravity data. Zhonogolovitch, Heiskanen, Uotila (1959), Talwani are among others who used gravity data for determining the global geoid or a part of it. Wideland (1955) detailed the Swedish part, Honkasalo(1956) the Finland area, and the Russian part was carried out by Molodenskii. Rapp(1974) has given a detailed account of definition and determination of geoid.

The geocentric orientation of their respective triangulation systems have been determined or are being determined by most countries towards providing a globally consistent reference. Rice(1952) oriented the North American system by gravimetrically correcting it at the origin, Meades Ranch. The Australian orientation was similarly carried out by Mather(1970).

1.4 STATUS OF THE INDIAN GEODETIC SYSTEM

In 1840, Everest selected Kalianpur as the initial point of the Indian geodetic system as it was more or less in the centre of the country.in a flat lying area. Figure 1.2 shows its approximate position. The arbitrary values of the geoid parameters were chosen as follows:

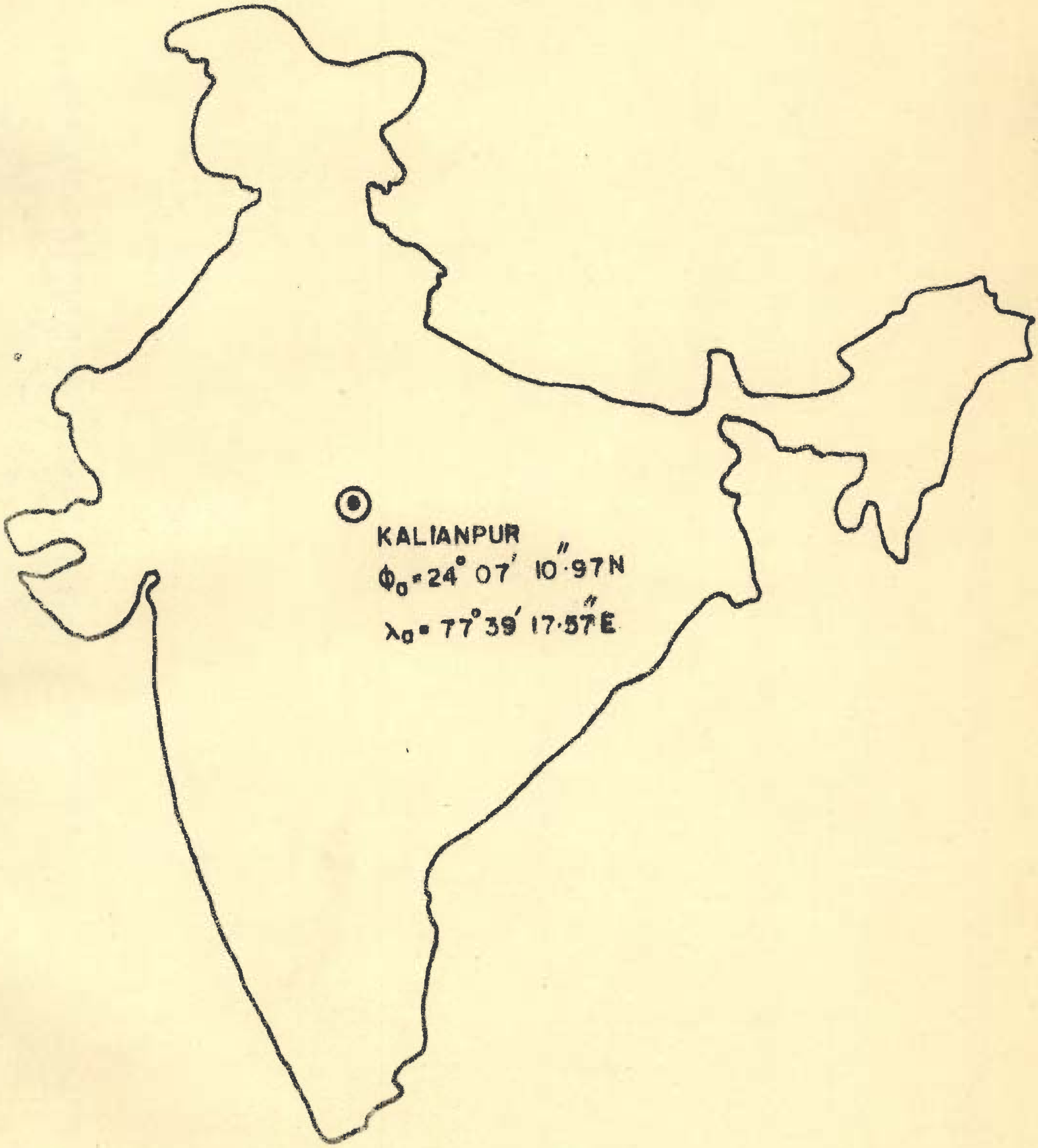


FIGURE 1.2- APPROXIMATE POSITION OF ORIGIN OF INDIAN GEODETIC SYSTEM

$$N_o = 0.0 \text{ m}$$
$$\xi_o = -0''.29 \qquad (1.1)$$
$$\eta_o = 2''.89$$

In 1924, the Hayford spheroid was adopted by the International Union of Geodesy and Geophysics, and in 1927, it was adopted as a reference surface for India. A least-squares solution was carried out to obtain the best fit to the compensated geoid and the resulting values were :

$$N_o = 9.5 \text{ m}$$
$$\xi_o = 2''.42 \qquad (1.2)$$
$$\eta_o = 3''.17$$

Whilst latitudes and longitudes are still expressed in terms of the Everest spheroid, Gulatee(1955) had changed the deviations of the vertical of the astrogeodetic stations in the International spheroid system, reporting a further correction of 3''.16 in the longitude.

1.5 OBJECTIVE OF THE PRESENT WORK

Arbitrary stationing of the reference spheroid by individual countries can lead to discrepancies of over 100 metres or more in the definition of common boundary points and much avoidable confusion in a progressively shrinking world. Such discrepancies could be greatly minimized if the absolute orientation of the spheroid is determined and adopted so as to be globally consistent as far as possible. This provided the basic motivation for undertaking the research work embodied in this thesis, i.e. determination of the absolute orientation values for the Indian geodetic system.

Geocentrically oriented systems being intrinsically absolute and of global character, offer a number of other allied advantages, notably,

(i) in establishing supercontrol points needed for the absolute positioning of artificial satellites (Dixit, 1977),

(ii) in providing more accurate values of the gravitational field for better understanding of mass distribution thereby delineating subscrustal anomalies inside the earth for use in geodynamical as well as exploration studies (Ray and Bhattacharji, 1977).

1.6 METHODS USED AND SCOPE

The present study primarily uses informations contained in gravimetric data to obtain reliable values of orientation parameters. Heterogeneous mass distributions in the earth produce several associated phenomena detectable at the surface, namely,

(i) anomalies in the gravity field,
(ii) undulations of the geoid, and
(iii) deflections of the vertical

All these three variations stemming from the same cause are, naturally, related to each other and each one can be computed from a knowledge of the other. This possibility underlines the basic concept of physical geodesy which is exploited in the gravimetric method discussed in detail in the subsequent chapters.

Starting basically from the gravity data, the present work combines other existing information, particularly astrogeodetic and satellite, with a view to designing an integrated strategy in arriving at an optimal solution within the framework of available resources.

While formulating the physical principles in terms of mathematical expressions and subsequently translating the latter into tractable computational algorithms, various innovations and modifications have necessarily been made for software development that will accomplish acceptable trade-offs between accuracy and cost. However, no attempt has been made to analyze the basic inputs critically nor the claims to their declared accuracies which have been accepted in good faith. Simplifications have been made as and when necessary, keeping in view the specific purpose and order of reliability of the final output, the physical interpretations being considered more important than formal mathematical rigor, without however severely impairing the latter.

CHAPTER II

GRAVIMETRIC PRINCIPLE

2.1 GENERAL

G.G. Stokes, a pioneer in the field of scientific geodesy, provided for the first time (1849) an integral expressing the height of geoid above the reference spheroid, in terms of gravity anomalies over the entire earth. This was extended by Vening Meinesz(1928) to derive the slope components of the geoid, which is directly related to the corrections to be made on the astronomical coordinates, viz., latitude and longitude, to enable one to compute the parameters for the absolute orientation of a geodetic system.

2.2 THE GEOID AND THE SPHEROID

The mean sea level which is the reference surface for astronomical observations and spirit levelling, is a physical surface of constant potential, in equilibrium under the forces of attraction by the underlying masses and those above, including the topographical features and extraterrestrial planetary bodies as well as by the inertial forces arising from the rotation of the earth about its axis.

The potential of gravitational attraction and rotation of the earth at a point P (Figure 2.1) can be written as,

$$W = V + V' = K\int \frac{dm}{\ell} + \frac{1}{2}\omega^2(x^2+y^2) \qquad (2.1)$$

The gradient vector of W, called 'gravity' , is the total force acting on a unit mass at P,

$$[g] = \text{grad } W = [\ \frac{\delta W}{\delta x}, \frac{\delta W}{\delta y}, \frac{\delta W}{\delta z}\ ] \qquad (2.2)$$

A level surface or equipotential surface, is defined as the surface on which

$$W(x,y,z) = \text{constant} = W_n \ , \ n = 0,1,2,... \qquad (2.3)$$

or in other words,

$$[g].[ds] = [\ \frac{\delta W}{\delta x}, \frac{\delta W}{\delta y}, \frac{\delta W}{\delta z}\ ] . [\ dx, dy, dz\ ]^T = 0 \qquad (2.4)$$

where,

[ds] = [dx, dy, dz] is a line element along the level surface.

The vanishing of the dot-product of the vectors implies that the gravity vector is normal to the equipotential surface, termed 'geop' .

Of the whole family of equipotential surfaces described by the above expressions, the particular one for which the constant W_n is equal to W_o and coincides with

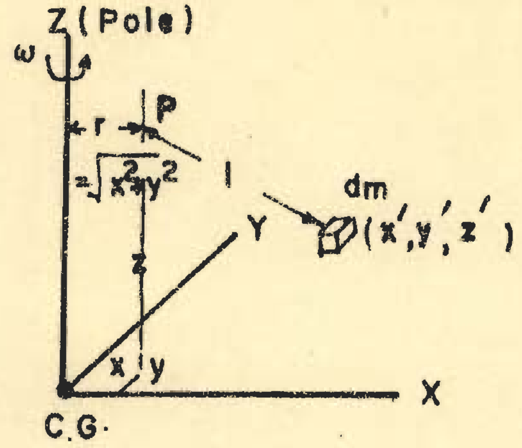


FIGURE 2.1-POTENTIAL AT A POINT

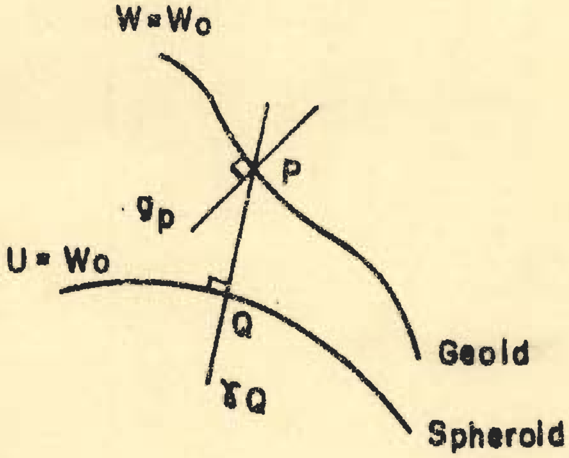


FIGURE 2.2-GEOID AND SPHEROID

the idealized free ocean surface is called the GEOID. The latter being normal to the plumbline, constitutes a reference for all astronomical observations. The surface tangent to the body-bubble in measuring instruments is parallel to the geoid at open sea. On land, however, the curvature of the plumbline must be taken into account while reducing observations down to the geoid.

For mapping and other practical purposes, however, the geoid is not acceptable as a direct reference surface owing to its irregularity which is, in turn, brought about by heterogeneous density distributions inside the earth generally and in the crustal region in particular.

The potential of attraction V(Equation 2.1) may be expressed as follows in terms of spherical harmonic functions,

$$V = K\int \frac{dm}{\ell} = V_o + V_1 + V_2 + V_3 + V_4 + \ldots \qquad (2.5)$$

where,

$$V_o = \frac{KM}{r},$$

represents the major part of the potential and is equal to the attraction potential of a spherical earth with radially symmetric density distributions, the total mass being equal to that of the actual earth.

The magnitude of the first-order term,

$$V_1 = \frac{K}{r^3} (x \int x' \, dm + y \int y' \, dm + z \int z' \, dm) \quad (2.6)$$

depends upon the position of the centre of gravity of the earth with respect to the reference axes. By making the reference system geocentric, V_1 reduces to zero.

Similarly, the second-order term V_2 involves the mass moments of inertia, and may be expressed in terms of the moments and products of inertia about different axes (Figure 2.1) defined as follows,

$$\begin{aligned} A &= \int (y'^2 + z'^2) \, dm \, , \\ B &= \int (x'^2 + z'^2) \, dm \, , \\ C &= \int (x'^2 + y'^2) \, dm \, , \\ D &= \int x' y' \, dm \, , \\ E &= \int y' z' \, dm \, , \\ F &= \int z' x' \, dm \, , \end{aligned} \quad (2.7)$$

It can be shown that V_3, V_4 and other higher order terms can also be interpreted in a similar fashion.

Transforming the geocentric rectangular coordinates to the spherical coordinates and recognizing z axis to be a principal axis, the total geopotential function reduces to

$$W = \frac{KM}{r} + \frac{K}{r^3} \{ \frac{1}{2}(C-\bar{A})(1-3\sin^2\bar{\phi}) + \frac{3}{4}(B-A)\cos^2\bar{\phi}\cos 2\lambda$$

$$+ \frac{3}{2} D \cos^2\bar{\phi} \sin 2\lambda\} + \ldots . \frac{\omega^2 r^2 \cos^2\bar{\phi}}{2} \ldots$$

where, $\bar{A} = \frac{A+B}{2}$, and

$\bar{\phi}$ is the geocentric latitude (2.8)

The surface of the geoid is therefore expressed as

$$W = W_o \text{ , a constant} \qquad (2.9)$$

With the assumption of a symmetrical density distribution about the Z -axis and about the equatorial plane, the shape of the regularized geopotential surface, known as spheroid, becomes

$$U = \frac{KM}{r} \{ 1+ \frac{a^2}{2r^2} J_2(1-3\sin^2\bar{\phi}) + \frac{1}{2} \frac{mr^3}{a^3(1-f)} \cos^2\bar{\phi}$$

$$+ \frac{a^4}{r^4} p(\sin^4\bar{\phi} - \frac{6}{7} \sin^2\bar{\phi} + \frac{3}{35})\} + \ldots . = W_o, \qquad (2.10)$$

where,

$$J_2 = \frac{C-\bar{A}}{Ma^2} = \frac{2}{3} (f - \frac{1}{2} m - \frac{1}{2} f^2 + \frac{1}{7} fm)$$

$$m = \frac{\omega^2 a^3(1-f)}{KM} ,$$

$$p = \frac{7}{2} f^2 - \frac{5}{2} fm$$

The geometry of the fourth-order spheroid surface very nearly corresponds to a rotational ellipsoid with semimajor axis a and flattening f. The residual deviations of the geoid or its gravity field are small enough to be considered linear.

The gradient of this equipotential surface would, in turn, yield a reference for the total gravitational field, which is called the 'normal gravity',

$$[\gamma] = \text{grad } U \tag{2.11}$$

The closed formulae, neglecting terms of order higher than the square of flattening, are as follows:

$$KM = ab\gamma_a\left(1+ \frac{3m}{2} + \frac{3}{7} fm + \frac{9}{4} m^2\right) \tag{2.12}$$

$$U_o = a\gamma_a\left(1- \frac{2}{3}f + \frac{11}{6}m - \frac{1}{5} f^2 - \frac{4}{7} fm + \frac{11}{4}m^2\right)$$

$$\gamma = \gamma_a(1 + f_2 \sin^2\phi + f_4 \sin^4\phi)$$

where,

$$f_2 = - f + \frac{5}{2} m + \frac{1}{2} f^2 - \frac{26}{7}fm + \frac{15}{4}m^2 ,$$

$$f_4 = - \frac{f^2}{2} + \frac{5}{2} fm ,$$

$$f = \frac{a-b}{a} , \text{ and}$$

$$m = \frac{\omega^2 a}{\gamma_a}$$

A careful scrutiny reveals that the expressions may be evaluated from four quantities namely, a, f, γ_a, ω

The change of γ with elevation h is given by,

$$\gamma_h = \gamma - \frac{2\gamma_a}{a}\{1+f+m+(-3f + \frac{5}{2}m)\sin^2\phi\}h + \frac{3\gamma_a}{a^2}h^2 \qquad (2.13)$$

For the International spheroid, the values adopted are (Heiskanen and Moritz, 1967) as follows:

$$\begin{aligned} a &= 6378388 \text{ metres} \\ f &= 1/297.0 \\ \gamma_a &= 978.049 \text{ gals} \\ \omega &= 0.72921151 \times 10^{-4} \text{ rad/sec} \end{aligned} \qquad (2.14)$$

From the above, the following values are obtained

$$\begin{aligned} f_2 &= 0.0052648, \\ f_4 &= 0.0000236, \\ J_2 &= 0.0010920, \\ m &= 0.00344986, \\ U_o &= 6263978.7 \text{ Kgal metre} \\ KM &= 3.9863290 \times 10^{14} \text{ m}^3 \text{ sec}^{-2}, \text{ and} \\ \gamma_h &= \gamma - (0.30877 - 0.00045\sin^2\phi)h + 0.000072h^2 \end{aligned} \qquad (2.15)$$

where h is in metre and γ in milligal.

The recent internationally agreed reference spheroid is the Geodetic Reference System, 1967 (International Union of Geodesy and Geophysics, 1967) with the following basic elements,

$$a = 6378160 \text{ m}$$

$$J_2 = 0.0010827$$

$$KM = 3.98603 \times 10^{14} \text{ m}^3 \text{ sec}^{-2} \qquad (2.16)$$

$$\omega = 7.2921151467 \times 10^{-5} \text{ rad/sec}$$

The other values, as derived from these, are (Williamson and Gaposchkin, 1975) as follows:

$$f = 1/298.247167427$$

$$\gamma = 978031.85(1+5.278895 \times 10^{-3} \sin^2\phi + 2.3462 \times 10^{-5} \sin^4\phi) \text{ mgal} \qquad (2.17)$$

2.3 DISTURBING POTENTIAL AND GRAVITY ANOMALY

As discussed earlier, the major part of the geopotential W may be represented by a smooth normal part, i.e., the spheropotential U, and the residual anomalous component is termed as the disturbing potential T.

Referring to Figure 2.2, the geoid is defined by $W = W_o$, whereas the reference spheroid by $U = W_o$. Bruns' formula relates the separation of the two surfaces by,

$$T = W_p - U_p = W_p - \left\{ U_q + \left(\frac{\delta U}{\delta h} \right)_q N \right\}$$

$$= W_o - W_o + \gamma N = \gamma N$$

or,
$$N = T/\gamma \qquad (2.18)$$

The gravity anomaly at a point is conveniently defined as the actual gravity value reduced at a corresponding point on the geoid minus the normal gravity at the corresponding spheroid point. Referring again to Figure 2.2 and assuming, for the present, that the two normals are almost coincident,

$$\Delta g = g_p - \gamma_q$$

$$= -\left(\frac{\delta W}{\delta h} \right)_p - \left\{ -\left(\frac{\delta U}{\delta h} \right)_p - \left(\frac{\delta \gamma}{\delta h} \right)_p N \right\}$$

$$= -\left(\frac{\delta T}{\delta h} \right) + \left(\frac{\delta \gamma}{\delta h} \right) N \qquad (2.19)$$

$$= \frac{T}{\gamma} \frac{\delta \gamma}{\delta h} - \frac{\delta T}{\delta h}$$ ~~(2.19)~~

A spherical approximation, applied in respect of all the anomalous components, leads to the relations,

$$\gamma \approx \frac{KM}{r^2}$$

$$\frac{\delta \gamma}{\delta h} \approx \frac{\delta \gamma}{\delta r}$$

$$\frac{1}{\gamma} \cdot \frac{\delta \gamma}{\delta h} \approx - \frac{2}{r}$$

$$\Delta g = - \frac{\delta T}{\delta r} - \frac{2T}{r} \qquad (2.20)$$

On the geoid,

$$r = R = \sqrt[3]{a^2 b} = a(1-f)^{1/3} \qquad (2.21)$$

2.4 STOKES' AND VENING MEINESZ' FORMULAE

The rotation term, contained both in the geopotential W and in the spheopotential U, vanishes in the expression for the disturbing potential, making T a harmonic function outside the earth, obeying the Laplace's equation,

$$\Delta T = \frac{\delta^2 T}{\delta x^2} + \frac{\delta^2 T}{\delta y^2} + \frac{\delta^2 T}{\delta z^2} = 0 \qquad (2.22)$$

the atmosphere being assumed to have negligible mass compared with that of the earth. The physical geodesy problem, i.e., to determine the undulation of the geoid from the known gravity anomalies, then reduces simply to the third exterior boundary-value problem of potential theory in which the function T to be determined is harmonic outside a surface s, while a linear combination of T and its normal derivative $\frac{\delta T}{\delta r}$ is specified everywhere on s. The boundary function known on the geoid surface is the gravity anomaly,

$$\Delta g = (-\frac{2}{R})T + (-1)\ \frac{\delta T}{\delta r} \tag{2.23}$$

This classical problem was solved by Stokes(1849) presumably as a mathematical exercise. However, he did not use the spherical harmonic expansion but a closed expression of the following form (Figure 2.3),

$$N = \frac{R}{4\pi G} \oint \Delta g\ S(\psi) d\sigma \tag{2.24}$$

where,

N = undulation of the geoid with respect to the reference spheroid at any point,

R = average radius of the earth,

G = average gravity of the earth,

$d\sigma$ = elemental surface area on the earth,

Δg = gravity anomaly on the element $d\sigma$

$S(\psi)$ = Stokes' function

$$= \frac{1}{\sin\frac{\psi}{2}} - 6\sin\frac{\psi}{2} + 1 - 5\cos\psi$$

$$- 3\cos\psi\ \ln\left(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2}\right),$$

and, ψ = spherical distance of the gravity element from the computation point.

In determining the orientation parameters, the other gravimetric quantity needed to position the mapping surface is the first horizontal derivative of N. Its

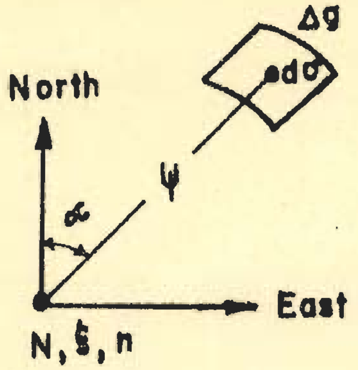


FIGURE 2.3 - GRAVITY ELEMENT AND UNDULATION POINT.

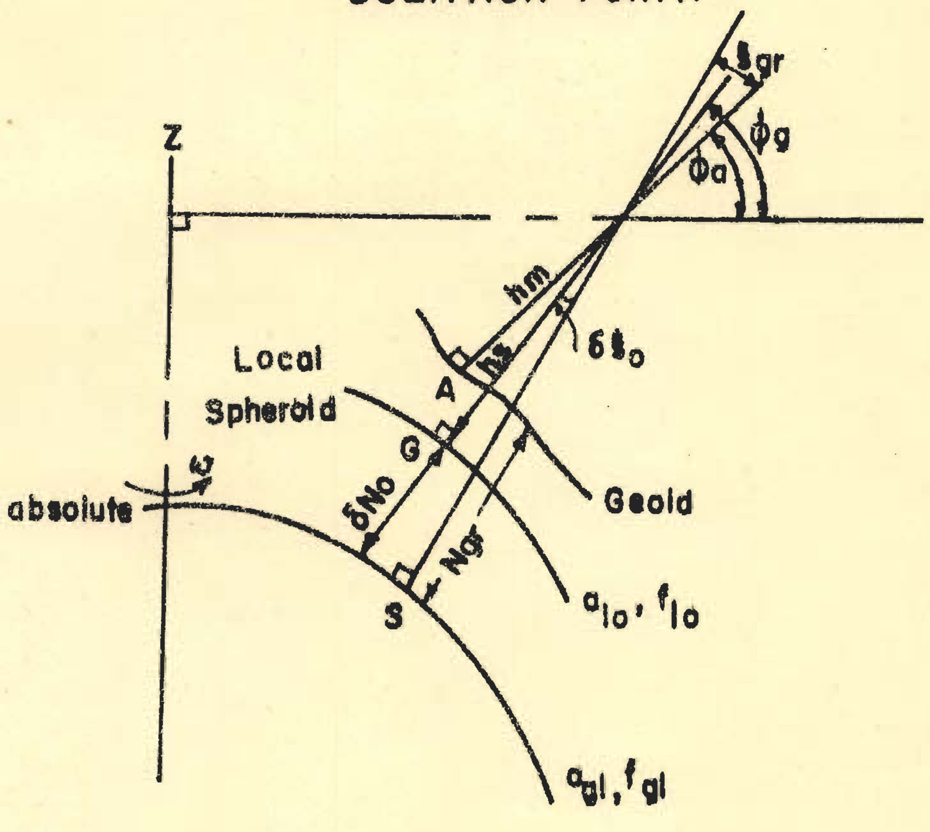


FIGURE 2.4 - ORIENTATION PARAMETERS.

two components, namely, the meridional and the prime vertical deviations of the vertical, are expressed as,

$$\xi = -\frac{\delta N}{R\delta\phi}, \qquad (2.25)$$

$$\eta = -\frac{\delta N}{R\cos\phi\,\delta\lambda},$$

where,

ϕ = geodetic latitude of the point,

δN = elemental increase in geoid undulation (positive above spheroid),

$\delta\phi$ = elemental increase in latitude ϕ (positive northward),

$\delta\lambda$ = elemental increase in longitude λ(positive eastward).

The minus sign comes from the sign convention adopted.

Differentiating Expression 2.24 with respect to ϕ and λ, the expressions (Vening Meinesz, 1928) are obtained as follows

$$\xi = \frac{1}{4\pi G}\oint \Delta g \ V(\psi)\cos\alpha \ d\sigma$$

$$\eta = \frac{1}{4\pi G}\oint \Delta g \ V(\psi)\sin\alpha \ d\sigma \qquad (2.26)$$

where,

ξ = the meridional (North-South) component of the deviation of the vertical,

η = the prime-vertical (East-West) component of the deviation,

α = azimuth of the arc of the great circle joining the deviation point to the gravity element reckoned in the clockwise direction from the North (Figure 2.3),

$$V(\psi) = \frac{dS(\psi)}{d\psi} = \text{Vening Meinesz' function}$$

$$= -\frac{\cos\psi/2}{2\sin^2\psi/2} + 8\sin\psi - 6\cos\psi/2 - \frac{3(1-\sin\psi/2)}{\sin\psi} + 3\sin\psi \ ln(\sin\psi/2+\sin^2\psi/2)$$

Other symbols have been explained earlier.

2.5 ORIENTATION OF NETWORK

A regional geodetic datum may be said to be defined by seven parameters (Ewing and Mitchell, 1970);

(a) lengths of two axes of the rotational ellipsoid used,

(b) two conditions related to geodetic azimuth and parallelism of minor axis,

(c) three parameters to assign the magnitudes of the tilt and the separation between the geoid and the reference ellipsoid at a point on the earth's surface, usually the origin of the geodetic system.

Whilst the first two constants are specified by adopting suitable dimensions of the reference spheroid, the next two are satisfied by astronomic reference and check on Laplace azimuth (Clark, 1968) during measurements.

For the last three unknowns, usually the origin is assigned some specific values of geodetic latitude ϕ_g, longitude λ_g, and altitude h_s above the mapping reference i.e., the local spheroid used. Either the astronomical coordinates ϕ_a, λ_a themselves and the height above the mean sea level h_m are used directly, implying tangency of both the geoid and the spheroid at the point g or some other reasonable criterion, e.g., local least-squares fitting, regional averages etc., is used. The astrogeodetic deviations of the vertical and the relative geoid-spheroid height at the point are then (Figure 2.4)

$$\begin{aligned} \xi_{ag} &= \phi_a - \phi_g \\ \eta_{ag} &= (\lambda_a - \lambda_g) \cos\phi_g \qquad (2.27) \\ N_{ag} &= h_s - h_m \end{aligned}$$

The gravimetric principle of physical geodesy, outlined in the earlier paragraphs, relates the separation of the geoid and a geocentric reference spheroid, including the deviations of the former with respect to the latter, to the gravity anomalies over the entire surface of the earth, involving three global integrations to obtain absolute values of N_{gr}, ξ_{gr}, η_{gr} at any initial point of the national triangulation network. Dimensions of the globally accepted spheroid may differ from the locally used one, as in the case of India.

The corrections to be made to reduce the network to absolute terms, are then, (Figure 2.4)

$$
\begin{aligned}
\delta N_o &= N_{gr} - N_{ag} \\
\delta \xi_o &= \xi_{gr} - \xi_{ag} \\
\delta \eta_o &= \eta_{gr} - \eta_{ag} \\
\delta a &= a_{g\ell} - a_{\ell o} \\
\delta f &= f_{g\ell} - f_{\ell o} ,
\end{aligned}
\qquad (2.28)
$$

where $a_{g\ell}$, $f_{g\ell}$ refer to the new spheroid and $a_{\ell o}$, $f_{\ell o}$ refer to the existing reference spheroid.

Once the absolute orientation parameters defined above are determined, the corrections needed to the existing geographical latitudes, longitudes and spheroidal heights at any point connected to the network are computable from the following datum shift formulae (Vening Meinesz, 1950):

$$-\delta\phi = (\cos\phi_0\cos\phi + \sin\phi_0\sin\phi\cos\Delta\lambda)\,\delta\xi_0$$
$$-(\sin\phi\sin\Delta\lambda)\,\delta\eta_0$$
$$-[(\sin\phi_0\cos\phi-\cos\phi_0\sin\phi\cos\Delta\lambda)\cdot$$
$$\cdot\left(\frac{\delta N_0+\delta a}{R} + \sin^2\phi_0\,\delta f\right)]$$
$$-2\cos\phi(\sin\phi-\sin\phi_0)\,\delta f\ ,$$

$$-\delta\lambda\cos\phi = (\sin\phi_0\sin\Delta\lambda)\,\delta\xi_0 + (\cos\Delta\lambda)\,\delta\eta_0$$
$$+(\cos\phi_0\sin\Delta\lambda)\left(\frac{\delta N_0+\delta a}{R} + \sin^2\phi_0\,\delta f\right), \qquad (2.29)$$

$$\delta h/R = -(\cos\phi_0\sin\phi-\sin\phi_0\cos\phi\cos\Delta\lambda)\,\delta\xi_0$$
$$-(\cos\phi\ \sin\Delta\lambda)\,\delta\eta_0$$
$$+(\sin\phi_0\sin\phi+\cos\phi_0\cos\phi\cos\Delta\lambda)\cdot$$
$$\cdot\left(\frac{\delta N_0+\delta a}{R} + \sin^2\phi_0\,\delta f\right)$$
$$-\frac{\delta a}{R} + (\sin^2\phi-2\sin\phi_0\sin\phi)\,\delta f,$$

where, $\delta\phi$, $\delta\lambda$, δh are the corrections at (ϕ,λ), ϕ_0, λ_0 are the coordinates of the initial point, and

$$\Delta\lambda = \lambda - \lambda_o$$

2.6 SUMMARY

The preceeding paragraphs describe in brief the basic concepts of the gravimetric principle of absolute orientation of a national geodetic system. Starting from the idea of the commonly used equipotential surface, and its division into a regularized and an irregular part, the practical computation formulae needed for a geometrical description of the geoid are discussed, culminating to the expressions for conversions of local geographical latitude, longitude and geometrical altitude of any station to their absolute values.

Various assumptions, explanations and modifications made for computing the integrals numerically are discussed in detail in the subsequent chapters.

CHAPTER III

DETERMINATION OF THE EFFECT OF OUTER REGION ON THE GEOIDAL PARAMETERS

3.1 GENERAL

Theoretically one needs a complete knowledge of the earth's gravity field at every point of its surface in order to compute the departure of the geoid from a reference spheroid through Stokes' and Vening Meinesz' integrals. However, since the gravity field of the earth is irregular, and cannot be expressed in a mathematical form, the integrals can only be evaluated numerically by considering them as sums in finite intervals. The accuracy of computation depends on the size of meshes chosen for the summation as well as on other factors related to the weighting function and the input variance involved.

However, the near-linearity in the variations of the Stokes' function at large radial distances towards the antipode of a computation point and its progressively decreasing magnitude suggests that at larger distance from a point in question, one could choose comparatively larger sized finite elements without sacrificing the precision significantly. This is also true for the slope components as the linearity of the Vening Meinesz' function used for their computation is even more pronounced at larger distances.

In order to carry out the computations, every mesh is characterized by a single value of the gravity anomaly equal to the mean value of the anomaly over it. As explained above, the size of the meshes could be chosen to be fairly large in the region of integration which is far from the point at which computations are made, whilst it has to be smaller for the region of integration immediately surrounding this point.

In the present work, the total domain of integration of the Stokes' and Vening Meinesz' integrals has been divided into a far exterior region made up of large mesh intervals and a near interior region wherein the meshes are of smaller dimensions. Finally in the immediate neighbourhood of the points in question, the meshes are further shortened for gaining higher accuracy. This is done merely to compute once for all the contribution to the geoidal parameters arising from the exterior region which could subsequently be simply superimposed over that arising from the near region and from the immediate neighbourhood, to obtain the total values of the Stokes' and Vening Meinesz' integrals.

The contribution of the exterior region to the geoidal undulations will be referred to as the 'void geoid'. It has been calculated from the recently available mean free-air gravity anomalies published by the Ohio State University (Rapp, 1977).

The computation of the void geoid and various innovations made to accomplish it numerically, forms the subject matters of this chapter, whereas that corresponding to the contribution of the interior region and the immediate neighbourhood are discussed in Chapter IV and V respectively.

3.2 INTERSTATION VECTOR

Evaluations of the Stokes' and Vening Meinesz' functions primarily need the values of ψ and α, the spherical distance and the azimuth of the line joining the computation station and the gravity station. The following relations follow from spherical trigonometry:

$$\cos\psi = \sin\phi_g \sin\phi_c + \cos\phi_g \cos\phi_c \cos(\lambda_g - \lambda_c),$$

$$\cos\alpha = \frac{\sin\phi_g \cos\phi_c - \cos\phi_g \sin\phi_c \cos(\lambda_g - \lambda_c)}{\sin\psi}, \qquad (3.1)$$

$$\sin\alpha = \frac{\cos\phi_g \sin(\lambda_g - \lambda_c)}{\sin\psi}$$

where

ϕ_c, λ_c = latitude and longitude of the station where N, ξ, η are needed

ϕ_g, λ_g = latitude and longitude of the gravity stations, or of the geometrical centre of the gravity element.

The customary formulations mentioned above are not well-suited for automatic computations in the form as they are. A better analytical expression is the matrix form in which the above equations can be written as follows:

$$\begin{bmatrix} \cos\psi \\ -\sin\psi\cos\alpha \\ -\sin\psi\sin\alpha \end{bmatrix} = \begin{bmatrix} -\sin\phi_c & \cos\phi_c\cos\lambda_c & \cos\phi_c\sin\lambda_c \\ \cos\phi_c & \sin\phi_c\cos\lambda_c & \sin\phi_c\sin\lambda_c \\ 0 & \sin\lambda_c & -\cos\lambda_c \end{bmatrix} \begin{bmatrix} -\sin\phi_g \\ \cos\phi_g\cos\lambda_g \\ \cos\phi_g\sin\lambda_g \end{bmatrix} \tag{3.2}$$

The matrix notation, arrived at, offers several advantages such as,

(a) the trigonometric elements related to the two stations are now separated,

(b) the transformation matrix elements are to be computed only once for a computation station and stored for further use,

(c) the transformation matrix is recognized to be orthogonal, and this unique property may be exploited any time with advantage, knowing that its inverse is equal to its transpose.

3.3 THE AREAL ELEMENT

The mean gravity anomalies are usually expressed over a trapezoidal area bounded by two meridians and two parallels. The area in square degrees is usually (Figure 3.1) approximated by

$$a \approx D\phi .\ D\lambda .\ \cos\phi_g \tag{3.3}$$

When the block-size is large, this approximation gives an appreciable error. The true value is,

$$a = \int_{\lambda_g - D\lambda/2}^{\lambda_g + D\lambda/2} \int_{\phi_g - D\phi/2}^{\phi_g + D\phi/2} \cos\phi \, d\phi \, d\lambda$$

$$= \{\sin(\phi_g + D\phi/2) - \sin(\phi_g - D\phi/2)\} D\lambda \tag{3.4}$$

$$= 2.\cos\phi_g .\sin \frac{D\phi}{2} .\ D\lambda$$

The divisor 4π in original Stokes' and Vening Meinesz' integrals may conveniently be absorbed here,

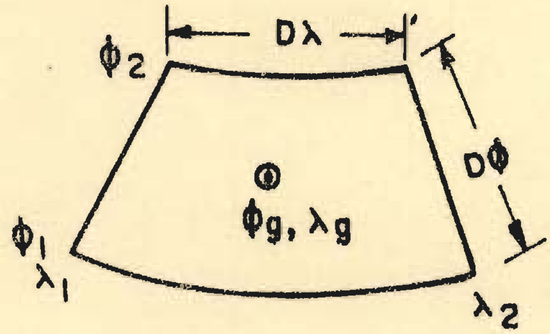
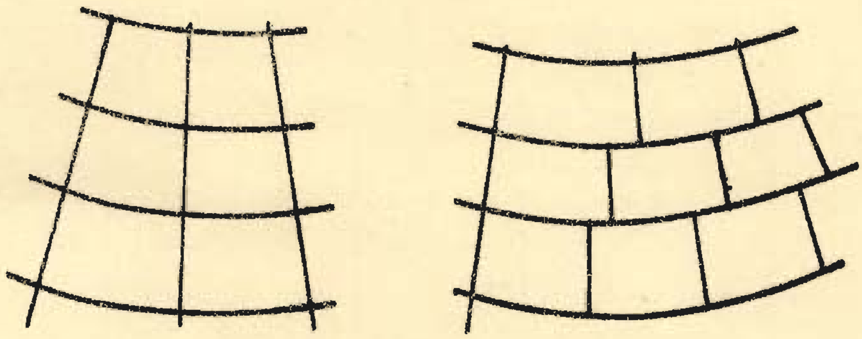


FIGURE 3.1 - SHAPE OF A GRAVITY ELEMENT.



Meridian parallel

Equal area

FIGURE 3·2 - FIVE DEGREE BLOCK TYPES.

to express the area in terms of the solid angle subtended by the quad at the centre of the earth. The customary unit of angle being in degrees,

$$\sigma = a/4\pi$$

$$= 2.\cos\phi_g . \sin\frac{D\phi}{2} . \frac{D\lambda.\pi}{180} . \frac{1}{4\pi} \qquad (3.5)$$

$$= \frac{D\lambda}{360} . \cos\phi_g . \sin\frac{D\phi}{2}$$

where the solid angle σ is now in steradians and $D\lambda$ in degrees,

$D\phi$ = latitude interval,

ϕ_g = latitude of the centre of the block.

The expression has at least two advantages,

(a) the irrational factor π vanishes from the computation, except in two trigonometric terms, which should better be evaluated in double precision,

(b) for any particular belt in the Equal-Area-Block system, every quantity except $D\lambda$ remains invariant.

3.4 SIZE OF AN ELEMENT

The global unit of a surface element for mean anomaly is an element of $5^\circ \times 5^\circ$ size. Two categories of mesh divisions (Figure 3.2) have been used in practice,

(a) Meridian-Parallel Grid : The entire earth's surface may be divided by meridian lines at 5 -degree intervals, and parallels of longitudes at 5 -degree intervals. Due to latitude-longitude asymmetry, the area of a block will vary according to its position. In the equatorial region it will be nearly 25 square-degree, but in the polar region it could be as small as 1 square-degree. The total number of blocks thus chosen will be 2592.

(b) Equal-Area Block : Another way of dividing the earth's surface into elements, are to frame blocks of approximately 25 square-degree surface area on the sphere. The latitude interval may be kept as 5 degrees, but the width in longitude direction changes from 5 degrees in the equatorial region to as large as 120 degrees in the polar region. The total number of blocks thus becomes 1654.

3.5 THE WEIGHTING FUNCTIONS

Another improvement has been made by expressing the Stokes' and Vening Meinesz' expression in algebraic arguments instead of trigonometric quantities, as shown below:

(a) The Stokes' function contains the $\cos\psi$ and $\sin\frac{\psi}{2}$ terms only, of which $\cos\psi$ is directly obtained as the first element of the interstation vector. The other may be obtained by either of the two expressions,

$$\sin\frac{\psi}{2} = \sqrt{\frac{1-\cos\psi}{2}} \;,$$

$$\sin\frac{\psi}{2} = \sqrt{\frac{\sin^2\psi}{2(1+\cos\psi)}} \;, \qquad (3.6)$$

So far as the computational precision is concerned, the first expression clearly becomes inaccurate as $\cos\psi$ tends to 1, whereas the second expression may cause overflow when $\cos\psi$ approaches -1. Noting also that,

$$\sin^2\psi = (-\sin\psi\cos\alpha)^2 + (-\sin\psi\sin\alpha)^2 \;, \qquad (3.7)$$

the Stokes' function can be finally rewritten as,

$$S(\psi) = \frac{1}{x} - 6x + 1 - \ell_1(5+3\ell n(x(1+x))) \qquad (3.8)$$

where,

$$x = \sqrt{0.5(1-\ell_1)} \qquad \text{when } \ell_1 \leq 0$$

$$= \sqrt{0.5(\ell_2^2 + \ell_3^2)/(1+\ell_1)} \qquad \text{when } \ell_1 > 0,$$

in which,

$$\begin{bmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{bmatrix} = \begin{bmatrix} \cos\psi \\ -\sin\psi\cos\alpha \\ -\sin\psi\sin\alpha \end{bmatrix} \;, \qquad (3.9)$$

are the elements of the interstation vector L, given by

$$L = T.P,$$

T being the transformation matrix, and

P, the position vector of the centre of the mesh, detailed in Expression 3.2

(b) Two more trigonometric terms, viz., $\sin\psi$ and $\cos\frac{\psi}{2}$ occur in the Vening Meinesz' function. Moreover, in the final integrals, $\cos\alpha$ and $\sin\alpha$ are to be multiplied which are not explicitly available so far except as the products $-\sin\psi\cos\alpha$ and $-\sin\psi\sin\alpha$.

Dividing the original function by $-\sin\psi$ and rearranging, the modified form becomes,

$$U(\psi) = \frac{V(\psi)}{-\sin\psi} \tag{3.10}$$

$$=\left(\frac{1}{4x^2} + 3 + \frac{3}{4x(1+x)}\right)/x - (8+3\ln(x(1+x)))$$

This expression is fortunately in terms of x only, and in the integrals the α term vanishes as,

$$\begin{aligned} V(\psi)\cos\alpha &= U(\psi)\ell_2 , \\ V(\psi)\sin\alpha &= U(\psi)\ell_3 \end{aligned} \tag{3.11}$$

Figure 3.3 shows the graphical representation of the functions $S(\psi)$, $V(\psi)$ and $U(\psi)$ with $\cos\psi$ as the argument.

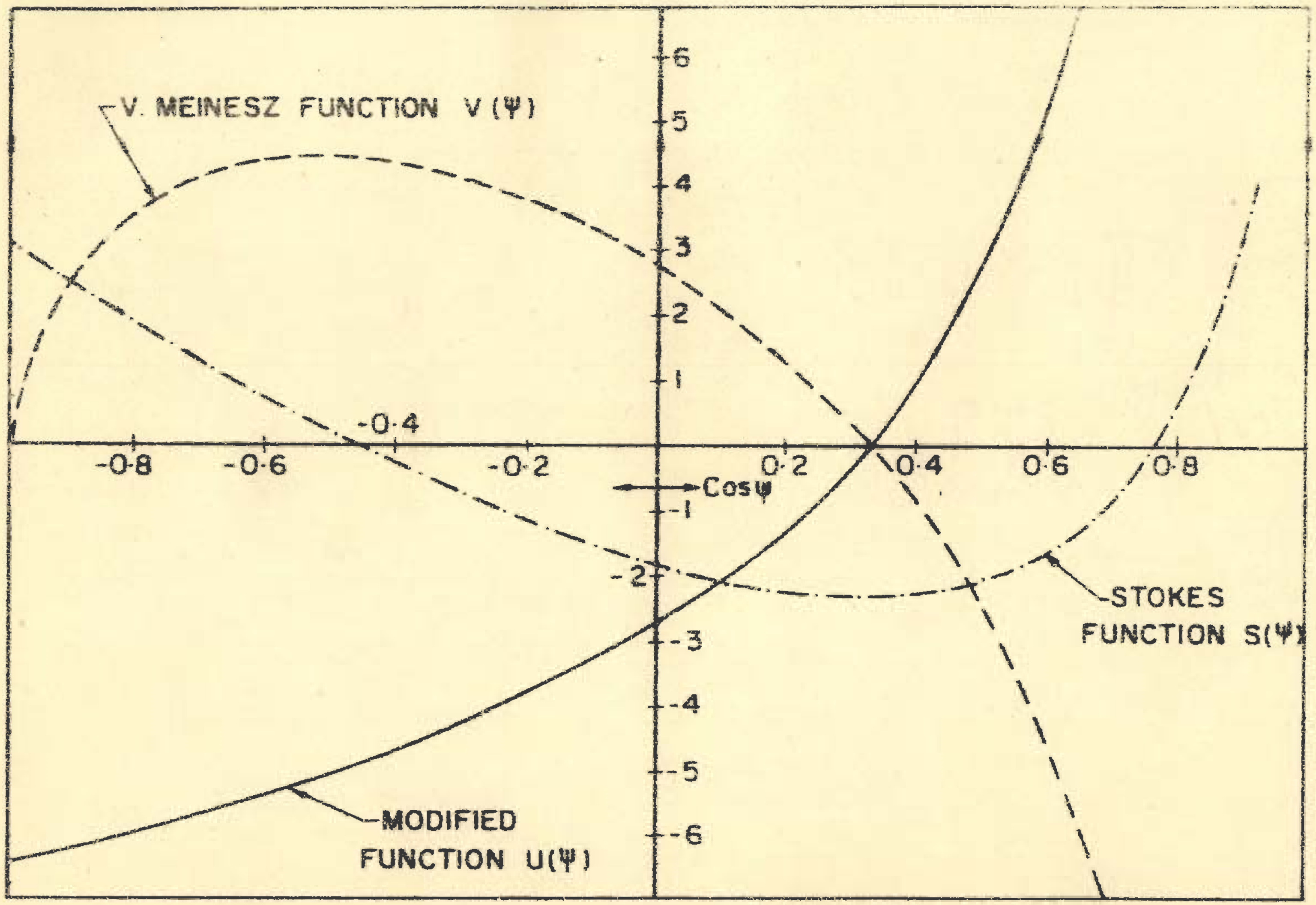


FIGURE 3.3 ORIGINAL AND MODIFIED WEIGHTING FUNCTIONS WITH Cosψ AS ARGUMENT

3.6 GLOBAL AVERAGE VALUES OF GRAVITY AND THE EARTH'S RADIUS

As a result of spherical approximation, two averaged quantities appear in the Stokes' and Vening Meinesz' integrals. These are defined as follows:

(a) G is defined as the average global value of gravity, usually taken as 979.8 gals. To render it more precise, the following derivation is used in the present study:

$$G = \frac{\int_{\lambda=0}^{2\pi} \int_{-\pi/2}^{\pi/2} \gamma \cos\phi \, d\phi \, d\lambda}{\int_{\lambda=0}^{2\pi} \int_{-\pi/2}^{\pi/2} \cos\phi \, d\phi \, d\lambda} \qquad (3.12)$$

where,

$$\gamma = \gamma_a (1 + f_2 \sin^2\phi + f_4 \sin^4\phi)$$

whence,

$$G = \gamma_a \left(1 + \frac{f_2}{3} + \frac{f_4}{5}\right) , \qquad (3.13)$$

which is a general expression for a given spheroid.

In particular, for the Geodetic Reference System 1967, the value of G turns out to be 979757.41 milligals.

(b) R, the equivalent radius of the earth, is the radius of the sphere having the same volume as that of the reference spheroid. Thus,

$$R = \sqrt[3]{a^2 b} = a(1-f)^{1/3} \qquad (3.14)$$

It becomes 6371023.4 metres in respect of the GRS67 spheroid.

3.7 NONDIMENSIONAL FORMS

Input anomalies if divided by G and multiplied by σ ,yield a dimensionless factor,

$$\beta = \frac{\Delta g \, \sigma}{G} \qquad (3.15)$$

Similarly, the geoid-spheroid separation may be divided by the equivalent radius of the earth to render it non-dimensional

$$\begin{aligned} u_1 &= N/R , \\ \text{whereas } u_2 &= \xi, \\ \text{and } u_3 &= \eta \end{aligned} \qquad (3.16)$$

are already non-dimensional being in radians.

3.8 FORMULATION OF BLOCK CONTRIBUTION

The numerical integration procedure may now be expressed as,

$$U = \begin{bmatrix} N/R \\ \xi \\ \eta \end{bmatrix} = \Sigma \, \Delta U , \qquad (3.17)$$

where the undulation vector for each integration element is,

$$\Delta U = \begin{bmatrix} \Delta u_1 \\ \Delta u_2 \\ \Delta u_3 \end{bmatrix} = W.\,\beta,$$

the weighting vector being,

$$W = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} S(\psi) \\ U(\psi)\ell_2 \\ U(\psi)\ell_3 \end{bmatrix}$$

3.9 DATA DESCRIPTION

For the present part of study, three different sources were explored for obtaining the $5^o \times 5^o$ mean free-air gravity anomalies.

(a) One set is that published by the Bureau Gravimetrique International (Coron, 1972), hereafter designated as BGI data. The mesh is on the Meridian-Parallel-Grid system and anomalies are related to the 1930 formula for normal gravity corresponding to the International Spheroid, and the old Potsdam value of gravity.

(b) The second set designated as SAO, is an incomplete Equal-Area-Block set (Williamson and Gaposchkin. 1975) compiled by the Smithsonian Astrophysical Observatory.

The SAO set, consisting of 1452 blocks is referred to the IGSN 71 network of gravity and the 1967 formula for normal gravity.

(c) The third set, i.e. the OSU data set recently published by the Ohio State University (Rapp, 1977) is a complete set of Equal-Area-Block mean free-air anomalies derived on the basis of the IGSN 71 network and the GRS 67 formula.

Before any comparison is made regarding their suitability and reliability, these data sets must be reduced to a common standard, i.e. to the same network and spheroid. Appendix A describes the corrections applied to the BGI set to reduce it to the GRS 67, IGSN 71 base. From the values in and around the Indian continent in the SAO and OSU sets, mean values for 5° gridded divisions are estimated proportionately, in order to convert the blocks to Meridian-Parallel-Grid system. Table 3.1 contains the descriptions of the sample values taken.

From the sample statistics shown in Table 3.1, the primary observation reveals that the fluctuation is higher in the BGI set than that in the other sets, whilst the mean values are nearly same in all cases. The recent data are the results of incorporating greater

TABLE 3.1

STATISTICS OF 5-DEGREE DATA SET SAMPLES

Area Covered = 5° to 40°N, 65° to 100°E

Number of blocks = 49

Unit = mgal

Set	Maximum Δg	Minimum Δg	Mean	Standard Deviation
SAO	33.2	-68.8	-11.1	19.0
BGI	49.2	-67.1	-12.9	25.1
OSU	28.2	-56.5	-11.3	16.6

number of observations and hence likely to produce more averaged-out representative block-mean values, thereby reducing the fluctuations considerably.

Another comparison of the differences of mean anomaly values of the same area obtained from the three sets, is presented in Table 3.2

The same conclusion emerges from Table 3.2 also, where the results for 'BGI minus OSU' and 'BGI minus SAO' are more or less equivalent, the 'OSU minus SAO' values show a marked improvement.

TABLE 3.2

COMPARISON OF SAMPLE SETS OF ANOMALIES

Sets compared	Difference, without regard to sign			Mean Diffe-rence	RMS Diffe-rence
	Maximum	Minimum	Average		
BGI-OSU	39.9	0.1	9.4	-1.7	13.3
OSU-SAO	17.4	0.1	5.3	-0.2	7.4
BGI-SAO	42.8	0.1	8.4	-1.9	13.2

3.10 ON SUITABILITY OF USING THE DATA

Regarding the economy of using the mean anomalies, as input data, the objective of the excercise is the prime determinant. For example, if a generalized geoid shape is needed for the Indian region, the Meridian-Parallel-Grid type is convenient for computation by numerical integration. The computation points in this case may be chosen to coincide with the grid corners for which the coefficients to be multiplied by Δg are to be evaluated only once for the entire latitude belt and for only one side of the meridian, as has been done by Tanni(1948). Moreover, the minimum distance from a computation point to a gravity point, i.e. to the centre of the block, is more or less fixed being equal to 3.5 degrees. This uniformity makes the inaccuracy due to numerical integration uniform and less pronounced for all computation points.

Using the other type of division, namely the Equal-Area-Block, the minimum distance may be as small as 2.5 degrees when a computation point falls on the same meridian as the centre of the upper or lower block, causing the inaccuracy in N to be higher for that point, as the Stokes' function changes sharply with decreasing distance. Therefore, for a general mapping of the geoid from five-degree corner values of N, the Meridian-Parallel-Grid type is better suited. The result of a test run with BGI data is presented in Appendix B, and with another preliminary attempt on absolute orientation on International Spheroid is reported in Appendix C.

All the advantages mentioned would, however, be infructuous if the computation point is a general astrogeodetic station like the origin of a local geodetic system. The Equal-Area-Block obviously affects economy in the computations, the elements being about 35 percent less in number. Moreover, the surface area being nearly the same for all, the weightage of input values are similar to each other. Hence for gravimetric computations for a deviation station, the Equal-Area-Block system would clearly prove to be a better choice.

For the present investigation, therefore, the updated OSU data have been preferred. Since the International Spheroid has been superseded by the universally accepted GRS 67 Spheroid, the absolute orientation parameters determined have been utilized to reduce the published geographical coordinates in terms of the Everest Spheroid to the GRS 67 system.

3.11 INNER LIMIT OF THE EXTERIOR REGION

The effect of the exterior region, from a certain distance onwards right up to the antipode, varies smoothly and is therefore interpolable. The desired accuracy of the determination and the availability of detailed data in the interior region, are the two basic factors which define the inner limit. An aperture of 20° is a generally accepted recommendation for N, whereas 15° may be sufficient for ξ or η. In the present case, a clear margin of 15° beyond the borders of the country was to be chosen, as outside this limit, sufficient coverage of $1^\circ \times 1^\circ$ mean anomalies was not available.

However, owing to lack of continuity of meridians in the Equal-Area-Block type division, the inner bounds of the gravity cannot be defined by distinctly delimiting longitudes. The area for which the gravity anomalies were not considered for evaluating the void geoid are detailed in Table 3.3 and depicted in Figure 3.4

TABLE 3.3

LIMITS OF THE INNER ZONE NOT CONSIDERED IN THE DETERMINATION OF THE VOID GEOID

Latitude Limit of belt		Corresponding Longitude Limit	
Northern	Southern	Western	Eastern
55°N	50°N	57°E	115°E
50°N	45°N	51°E	110°E
45°N	40°N	54°E	115°E
40°N	35°N	51°E	114°E
35°N	30°N	53°E	112°E
30°N	25°N	51°E	113°E
25°N	20°N	54°E	113°E
20°N	15°N	52°E	115°E
15°N	10°N	51°E	113°E
10°N	5°N	51°E	112°E
5°N	0°	50°E	115°E

contd...

Table 3.3 continued

0°	5°S	50°E	115°E
5° S	10°S	51°E	112°E

Total number of blocks = 141

3.12 RESULTS OF COMPUTATION

After developing the computer programmes for determining the void geoid parameters at any general point, 64 grid corners at 5° intervals covering the Indian subcontinent (Figure 3.4), were selected so as to give sufficient interpolable informations. Uotila (1959) also recommended a similar grid with parabolic interpolation. The inner limit, being fixed and independent of the position of the computation point, might ofcourse give rise to nonuniform precision at various corners but values at an interior station in the Indian region would not be sufficiently affected by this.

For evaluating the series of undulation and tilt components at all points of the 8 x 8 grid, the gravity data including positions and sizes of the 1513 blocks must be read from cards over and over again, or to be stored in the computer core as subscripted

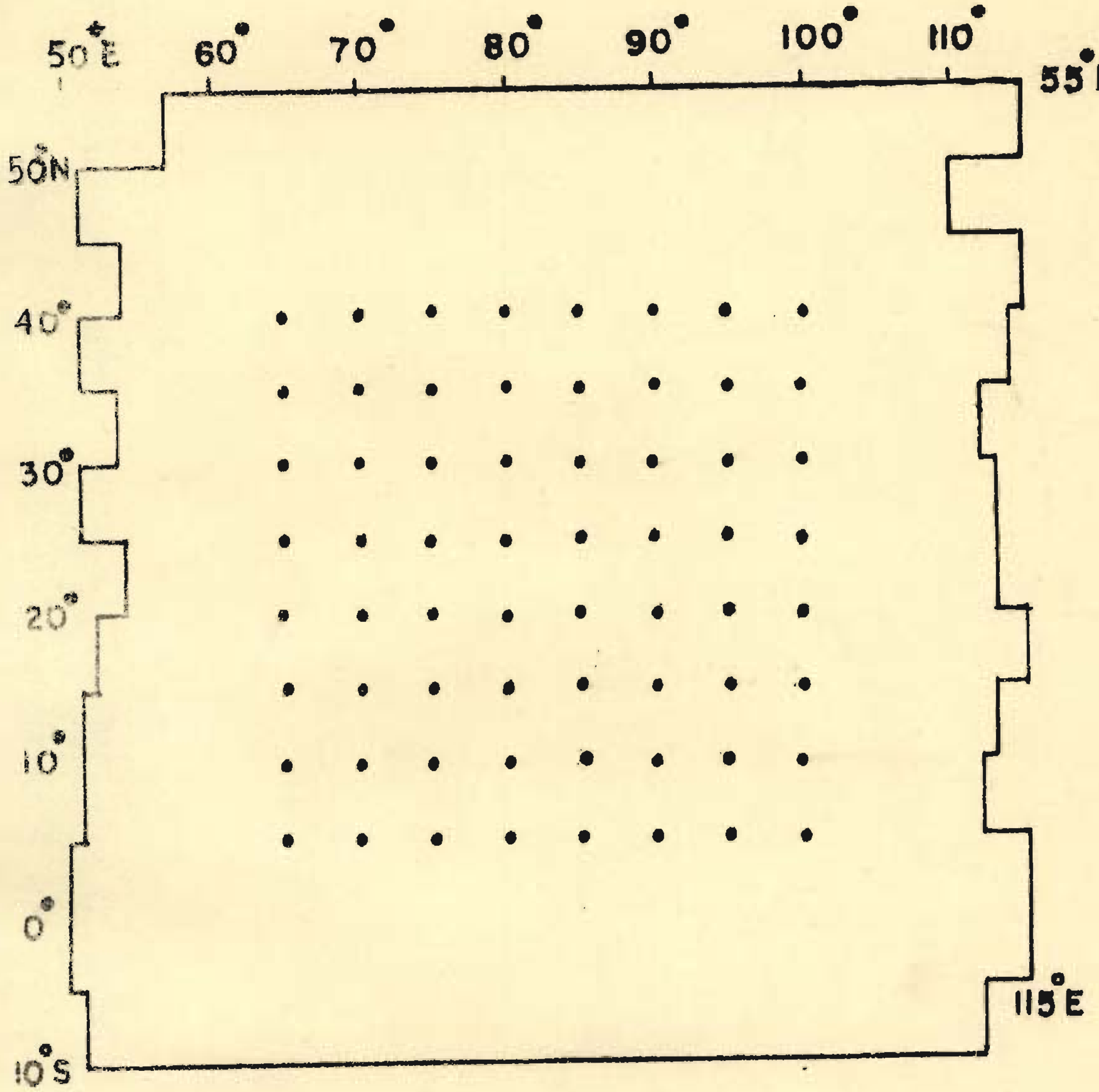


FIGURE 3.4 - INNER LIMIT OF GRAVITY ZONE AND COMPUTATION POINTS OF VOID GEOID.

variables, thereby requiring a large memory and search time. This was avoided by arranging the cards to be first read by the machine, evaluating the elements of the vector P and of β for each block, and then storing these on a magnetic tape as non-subscripted serial array in binary code. In the next phase, the tape was mounted and the usual REWIND statement was used after evaluating N_V, ξ_V, η_V at every grid corner. The total CPU time needed for both the phases was about 5 minutes on IBM 370 computer.

Table 3.4 shows the final results for N_V, the contributions to the void geoid, and the values ξ_V, η_V of the slope components are given in Table 3.5 and 3.6 respectively.

3.13 INTERPOLATION FROM CORNER VALUES

Interpolation of the N_V, ξ_V, η_V values at intermediate points is in this case rather simple as the grid values change smoothly. Even linear interpolation could, therefore, serve the purpose with no significant loss of accuracy, but a two-dimensional cubic spline interpolation has been preferred and developed. The spline-fitting technique is generally recommended to be a very powerful tool for its minimum norm property (Ahlberg et al.,1967) and an odd-order function has been considered to be the

TABLE 3.4

UNDULATIONS OF THE VOID GEOID IN AND AROUND INDIAN REGION

λ	65°	70°	75°	80°	85°	90°	95°	100°
φ	Values are in metres. Negative sign indicates height below GRS 67							
40°	- 9.08	-11.36	-12.87	-13.69	-13.82	-13.29	-12.11	-10.32
35°	-10.34	-12.42	-13.66	-14.15	-13.92	-12.99	-11.39	- 9.20
30°	-12.25	-13.97	-14.91	-15.09	-14.51	-13.19	-11.14	- 8.42
25°	-14.46	-15.79	-16.45	-16.35	-15.46	-13.76	-11.22	- 7.87
20°	-16.61	-17.71	-18.15	-17.85	-16.72	-14.68	-11.67	- 7.63
15°	-18.63	-19.62	-19.97	-19.56	-18.28	-16.01	-12.60	- 7.92
10°	-20.55	-21.49	-21.85	-21.49	-20.21	-17.83	-14.17	- 9.00
5°	-22.26	-23.24	-23.80	-23.68	-22.60	-20.31	-16.59	-11.16

TABLE 3.5

MERIDIONAL DEVIATIONS OF THE VERTICAL OF THE VOID GEOID IN AND AROUND INDIA

λ	65°	70°	75°	80°	85°	90°	95°	100°
ϕ	Values are in arcseconds and referred to GRS 67							
40°	-0.322	-0.279	-0.183	-0.060	0.080	0.231	0.386	0.526
35°	-0.605	-0.498	-0.389	-0.271	-0.140	0.006	0.168	0.334
30°	-0.790	-0.639	-0.525	-0.414	-0.291	-0.146	0.030	0.249
25°	-0.823	-0.702	-0.608	-0.516	-0.410	-0.275	-0.094	0.158
20°	-0.772	-0.713	-0.656	-0.597	-0.521	-0.412	-0.248	0.005
15°	-0.730	-0.704	-0.687	-0.673	-0.642	-0.577	-0.453	-0.237
10°	-0.683	-0.676	-0.711	-0.758	-0.792	-0.787	-0.725	-0.584
5°	-0.573	-0.616	-0.736	-0.877	-0.998	-1.071	-1.083	-1.029

TABLE 3.6

PRIME VERTICAL DEVIATIONS OF THE VERTICAL OF THE VOID GEOID IN AND AROUND INDIA

λ	65°	70°	75°	80°	85°	90°	95°	100°
φ	Values are in arcseconds and referred to GRS 67							
40°	1.298	0.912	0.562	0.228	-0.098	-0.415	-0.721	-1.011
35°	1.154	0.744	0.399	0.057	-0.265	-0.576	-0.866	-1.110
30°	0.907	0.567	0.239	-0.084	-0.408	-0.726	-1.028	-1.285
25°	0.671	0.413	0.117	-0.198	-0.529	-0.868	-1.209	-1.532
20°	0.541	0.312	0.034	-0.278	-0.621	-0.992	-1.388	-1.808
15°	0.491	0.263	-0.004	-0.316	-0.675	-1.082	-1.541	-2.070
10°	0.447	0.255	0.011	-0.299	-0.677	-1.124	-1.646	-2.272
5°	0.411	0.303	0.100	-0.207	-0.612	-1.105	-1.686	-2.383

best. For the present study, the third-order function having continuity up to the second derivative has been extended both along the meridians and the parallels, generalizing it as a 'bicubic' spline surface. This may further prove useful in improving digital models for terrain contouring which is usually accomplished by polynomial DTM technique (Ghosh and Ayeni, 1977). The formulation of the spline-surface and a few useful tables are presented in Appendix D which may be found useful by future researchers in this field.

The parameters of geoid-spheroid departure are, thereafter, interpolated at every 1° corner around the Indian boundary, and are depicted in Figures 3.5, 3.6 and 3.7, in contoured forms. It is important to mention here that the pictorial presentations are not based on any conventional projection system. Equidistant and perpendicular lines are drawn to indicate the meridians and parallels at equal intervals. The national boundaries are also purely approximate, just to indicate a general shape only.

3.14 SUMMARY AND DISCUSSION

As a first step towards determining the parameters of the Indian geoid gravimetrically from the GRS67 spheroid, the effect of the outer region on the undulations of the geoid has been computed and presented in this chapter. This is subsequently added to the contribution of the interior region discussed in succeeding chapters to obtain the parameters of Absolute Orientation.

The conventional formulations have been modified to suit automatic computation on a digital computer with high efficiency. The interstation separation has been formed in a matrix form and the area of the integration element has been expressed in terms of the solid angle subtended by it at the centre of the earth as these are found to be more suitable for computations involving gridded data. The Stokes' function and the modified version of the Vening Meinesz' function have been rearranged in algebraic form, to avoid repetitive computations of trigonometric quantities. Further, the average value of G has been improved. Finally the contribution formula has been expressed involving dimensionless quantities.

After formal comparison of the qualities of the existing data sources for their suitability of use, the OSU data, being most uptodate and also economical for for computer use, has been chosen for the analysis presented in this chapter. The data set, extending beyond a margin of about 15 degrees from the boundaries of the

Indian subcontinent, has been used to compute its contribution to the deviation parameters N_v, ξ_v, η_v at 64 grid corners covering the Indian region. Spline fitting has been used to interpolate values at intermediate points, and the final results are presented in tabular and contoured forms.

Whilst the contribution of the exterior region has been used here as a part of the total objective of this work, i.e., to obtain the parameters of absolute orientation of the Everest Spheroid at the origin of the Indian geodetic system, these results thus made available in a digital form can be used in future to further refine the total correction as and when more close and complete data accrue in respect of the local region and in the immediate neighbourhood of a point in question, simply by superposition.

The shape of the 'void geoid' is found to compare well with that of the satellite-derived geoid in the Indian region (Gaposchkin, 1973), particularly in respect of the following:

(a) the geoidal low in the Southern part of the peninsula,

(b) the downward trend in the North-South profile,

(c) flatness of the geoid in the East-West direction.

The absolute values of the void geoid heights are much smaller, obviously due to the absence of anomalies in the near region later found to be mostly negative. However, these are perfectly consistent with the RMS influence of the region beyond the average spherical radius of 18° (Heiskanen and Moritz, 1967); being ± 18 metres in N and $\pm 1''.8$ in ξ or η, obtained statistically from the degree variances of gravity anomalies (Kaula, 1959).

The choice of the inner limit is further confirmed from Rapp(1974), who presented the standard deviation of undulation assuming gravity coverage beyond a certain limit ψ_o. Whilst a sharp decline is seen from $\psi_o = 0^{\circ}$ to $\psi_o = 20^{\circ}$, the standard deviation remains practically constant further up to $\psi_o = 80^{\circ}$, indicating no practical gain of accuracy by extending the limit beyond 20°.

The computation points for the void geoid lying well within the Indian region thus satisfy all necessary conditions for obtaining the desired accuracy, despite the asymetry of the region of influence with respect to the computation point.

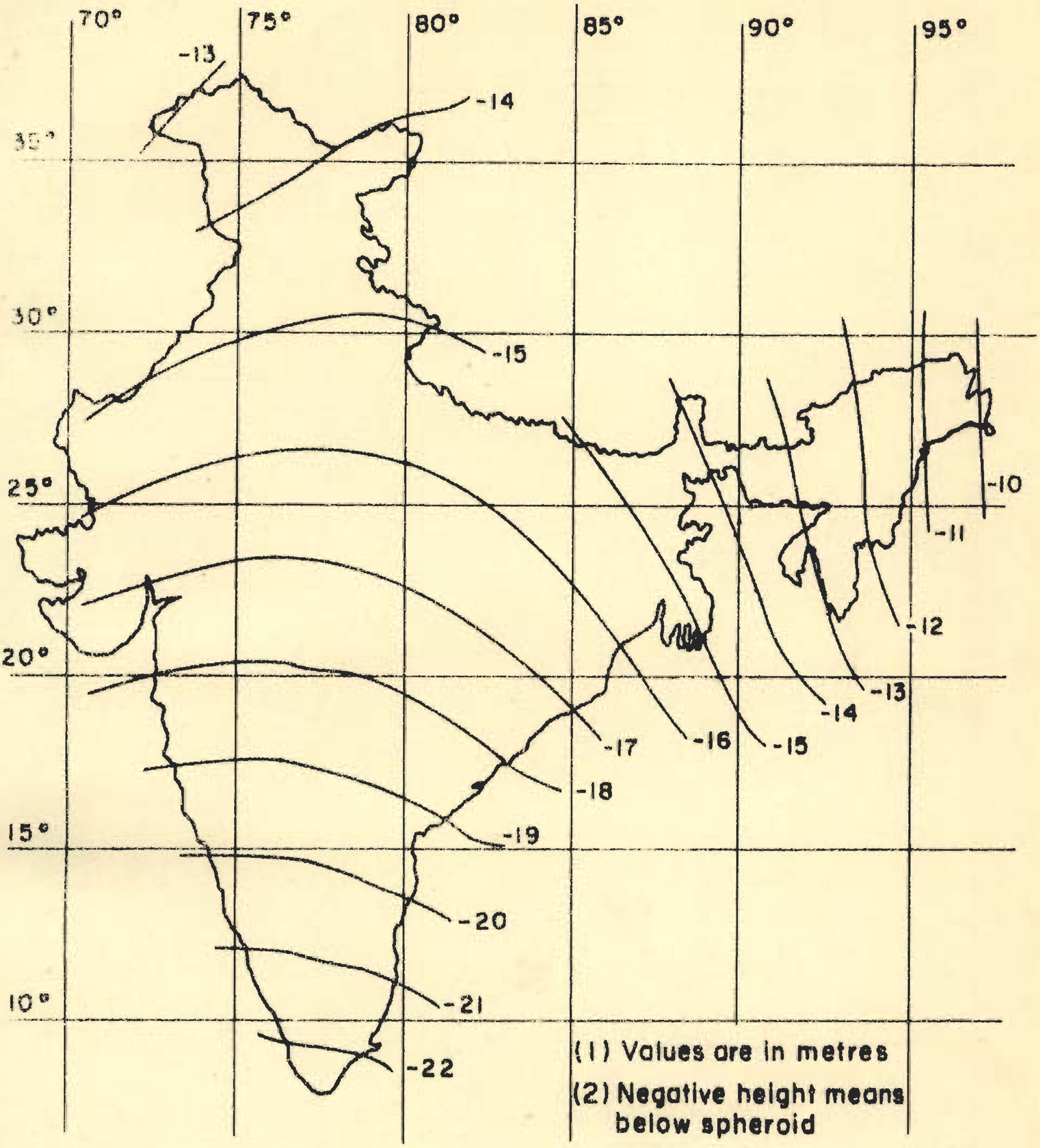


FIGURE 3·5 UNDULATIONS OF THE VOID GEOID IN INDIA

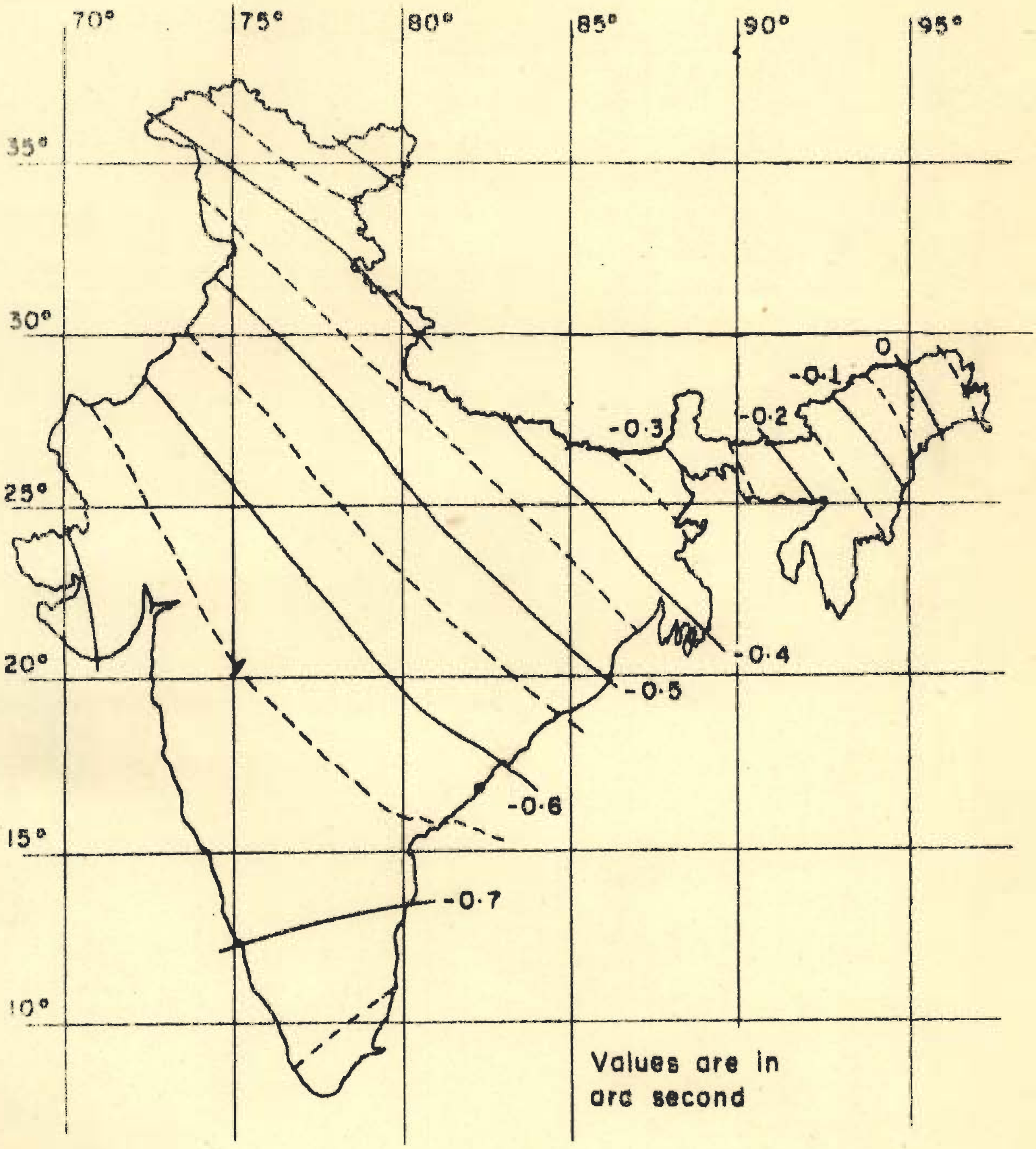


FIGURE 3·6 MERIDIONAL DEVIATIONS OF THE VERTICAL
OF THE VOID GEOID

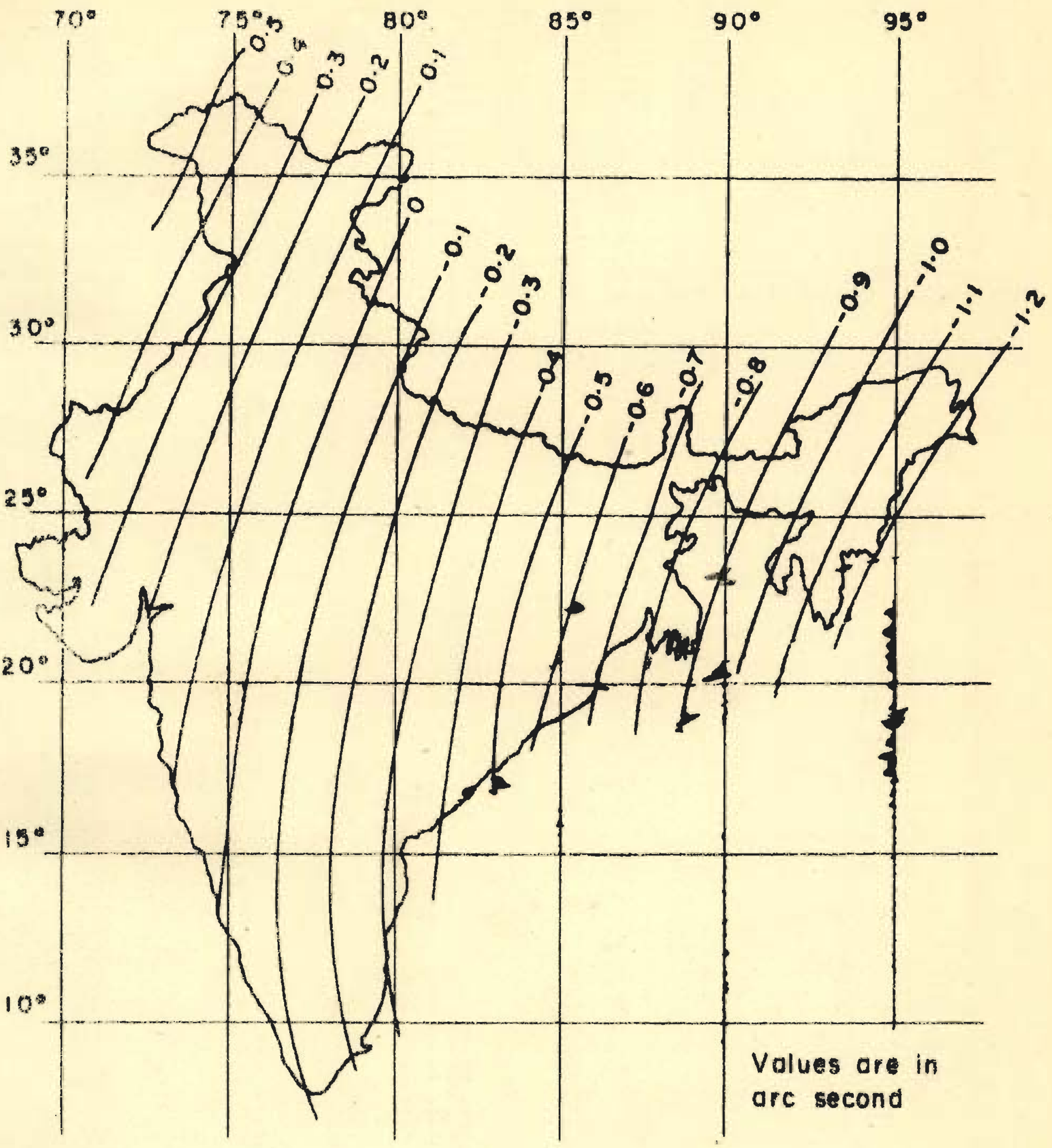


FIGURE 3·7 PRIME VERTICAL DEVIATIONS OF THE VERTICAL OF THE VOID GEOID

CHAPTER IV

EFFECT OF THE INTERIOR ONE DEGREE ANOMALY REGION ON THE GEOIDAL PARAMETERS

4.1 GENERAL

The determination of the geoidal parameters using a basic global coverage of gravity anomalies over five-degree elements, has been attempted in the previous chapter. For computing the effect of the interior region with acceptable precision, however, the size of the integration element has to be considerably smaller.

The next smaller size of areal element for the latter is chosen to be one-square degree Meridian-Parallel-grid. The outer bounds of the interior region were chosen to extend up to a distance of about 15° from the boundaries of the Indian region, coinciding with the inner limit of the exterior region considered for the void geoid and described in Table 3.3.

This chapter embodies the formulation of the problem of computing the effects of the interior region and results obtained at 1° grid corners, using the mean free-air gravity anomalies mostly from the DMAAC data (Defense Mapping Agency Aerospace Centre, 1973), and termed as 'partial geoid'. The superposition of these partial geoid parameters on the 'void geoid' values gives a smoothened set for further use in determining the orientation parameters in absolute terms.

4.2 ASSUMPTIONS MADE IN THE INTEGRALS

(a) The first and foremost source of inaccuracy in the numerical integration arises from the large size of the integral element. In the discrete summation analogue of the integration continuum, two variables are approximated:

i) each of the weighting functions which are actually continuously varying functions of position over an areal element, is replaced by a single value corresponding to the distance of the centre of the areal element with the computation station as pole, for the entire element,

ii) the variations in gravity anomalies within the element have similarly replaced by a single mean value.

(b) Furthermore, the Stokesian integral basically assumes that the disturbing potential T is a harmonic function. The solution will therefore be perfect only if no masses lie outside the geoid, an assumption which is rarely fulfilled over land. This assumption, given effect by free-air reduction, adds another source of inaccuracy as it yields the cogeoid instead of the geoid.

(c) The errors of measurement in the determination of gravity and altitude values combined with modelling errors in averaging also add to the inaccuracies of the final results which therefore tend to represent only estimates with large variance.

(d) Other sources of errors arising from imperfect assumptions, e.g.,

i) equal potential for the geoid and the spheroid,

ii) their possessing equal masses and volumes, thereby meaning a negligible zero order term,

iii) coincidence of the geoid with the mean sea level,

as well as other smaller order discrepancies, though theoretically quite important are considered to be negligible enough to be ignored in the present determination.

4.3 ERRORS OWING TO AVERAGING OF THE WEIGHTING FUNCTIONS

An estimate of the errors caused by replacing the continuous function $S(\psi)$ and $V(\psi)$ by their mean values over an element can be made by considering a suitably oriented nearmost block. Since the block containing the computation point will have $\psi = 0$ where the functions are discontinuous, one bounded by $\psi = 5^{\circ}$ to $\psi = 10^{\circ}$ and $\alpha = -2.^{\circ}5$ to $\alpha = 2.^{\circ}5$ may be chosen for the error estimation.

$$\text{Error in } S(\psi) = \left| 1 - \frac{S(7.5)}{\int_5^{10} S(\psi)\sin\psi d\psi / \int_5^{10} \sin\psi d\psi} \right| \approx 0.4\%$$

$$\text{Error in } V(\psi) = \left| 1 - \frac{V(7.5)}{\int_5^{10} V(\psi)\sin\psi d\psi / \int_5^{10} \sin\psi d\psi} \right| \approx 2.8\ \%$$

In the direction of α, the error will be of the order of

$$\left| 1 - \frac{\sin 2^\circ.5}{\pi \times 2.5/180} \right| ,$$

which is about 0.03 percent only.

The Stokesian solution is based on the spherical approximation which implies that quantities of the order of flattening, i.e., 1/298 or approximately 0.3 percent, can be neglected. Therefore, the averaging of functions does affect the accuracy at $\psi = 10^\circ$ or nearer. The effect of the element referred to above being nearly 0.08 metre/milligal in N and 0.02 arcsecond/milligal in the deviation θ, the absolute orders of error for an element over which the anomaly may be as high as 200 milligals, will be 6 centimetre and $0''.1$ respectively.

4.4 ERRORS DUE TO AVERAGING OF THE GRAVITY ANOMALIES

The gravity anomalies are usually of random type with high fluctuations specially when the topography is rugged or the crustal density contrast is sharp. The error estimates will therefore be different at different places and only a sample was considered from the real data in Indian region.

The mean values may be estimated either as a simple average

$$\overline{\Delta g} = \frac{\sum_{i=1}^{n} \Delta g_i}{n} \qquad (4.1)$$

or, the average after weighting by the corresponding area covered, i.e.

$$\overline{\Delta g}^* = \frac{\Sigma(\cos\phi_g\ \Delta g)_i}{\Sigma(\cos\phi_g)_i} \qquad (4.2)$$

The two means were calculated and compared to the value given by Rapp(1977). Table 4.1 shows the sample statistics.

TABLE 4.1

SAMPLE STATISTICS OF MFA ANOMALIES

Zone covered	25 to 30°N, 79 to 84°E
Range	-163 to + 61
$\overline{\Delta g}$	-72.04
$\overline{\Delta g}^*$	-72.05
Maxm. difference	133
RMS difference	52.9
5° Mean	-68.5 ± 3.1
unit	milligal

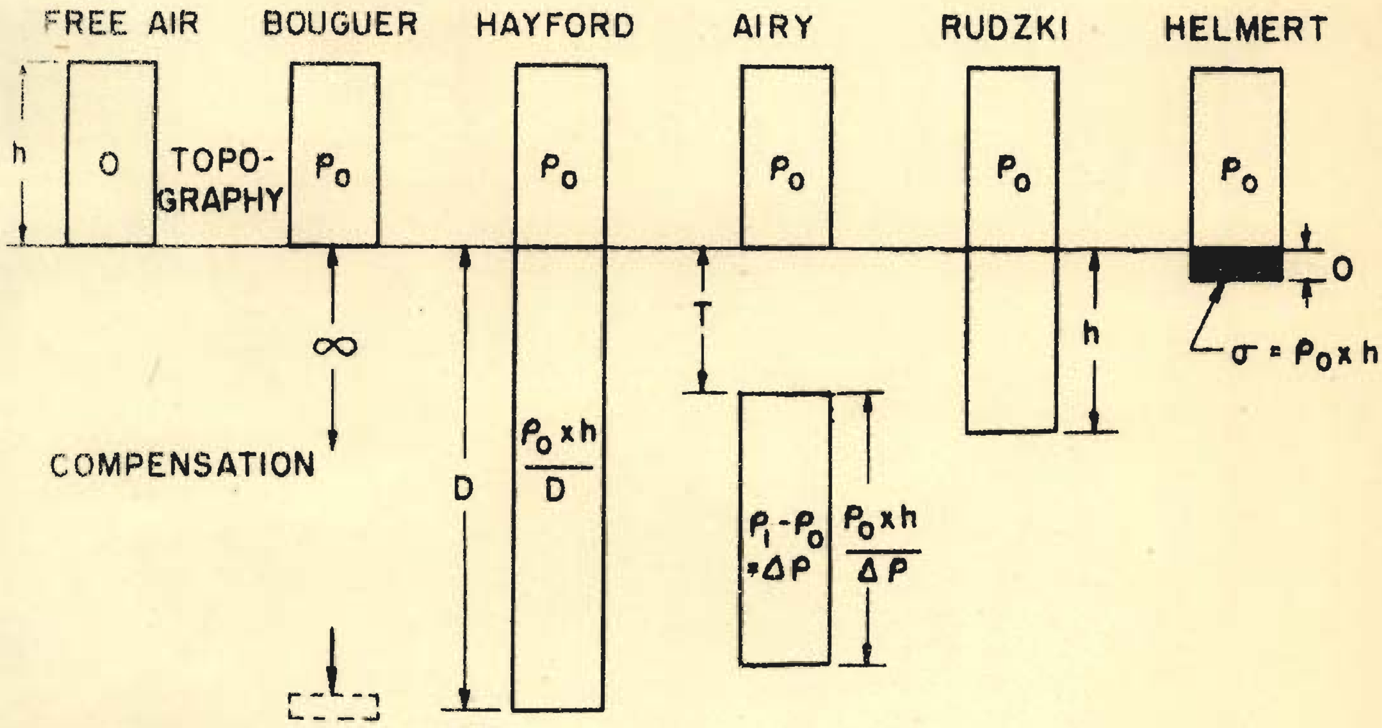
The standard deviation in the set is too high; the region is in the lower Himalayan area and is thus an extreme case of fluctuating values, the deviation being an upper bound.

The three mean values are consistent among themselves as they do not significantly differ from one another.

4.5 REDUCTIONS OF ANOMALIES

The loss of precision for want of suitable reduction methods had been discussed at length by geodesists. Theoretically, the correct thing to do is to apply the condensation correction (Helmert, 1884). However, from practical considerations, the free-air reduction proves to be the optimum. Isostatic anomalies have also been recommended as they are geodynamically significant and interpolable, and the indirect effects involved are moderate (Heiskanen and Moritz, 1967). Mather (1970) computed the Australian geoid on the basis of regional gravity data, where he recommended the method of free-air reduction. As the errors arising from the use of free-air anomalies are comparable to the modern accuracy of measurements, it may be safely used for one-degree mean anomaly data, where the standard error of the mean itself is 5 milligal or more.

Figure 4.1 is a schematic diagram of various reduction methods showing their effects on gravity and geoid-



	Free Air	Bouguer	Hayford	Airy	Rudzki	Helmert
δg	20	150	30	20	?	20 m gal
δN	X	400	10	10	0	2 metre

δg - Change in gravity

δN - Geoid-cogeoid separation

FIG. 4·1 VARIOUS GRAVITY REDUCTION METHODS cogeoid separations. The comparisons clearly establish the facts that simple free-air anomalies may be considered as very nearly equivalent to condensed free-air anomalies, and the free-air cogeoid coinciding with the actual geoid to the same degree of approximation.

4.6 NORMALIZATION OF WEIGHTING FUNCTIONS

The Vening Meinesz' function has already been modified and reported in section 3.5 of chapter III. As the spherical distance involved in the present case is less than 90°, the following formulation for x is adopted

$$x = \sin(\psi/2) = \sqrt{0.5(\ell_2^2 + \ell_3^2)/(1+\ell_1)} \qquad (4.3)$$

As the value of x is now small, evaluation of the function $S(\psi)$ and $U(\psi)$ should be carefully arranged to avoid loss of precision due to computer overflow and truncation. Double precision may well be used in this phase of computation, even though it will involve greater computer time.

A clear review of the computation logic will be in order to illuminate the strategies adopted for effecting economy. Firstly, the behaviour of various terms that comprise the functions $S(\psi)$ and $U(\psi)$ is visualized from their values presented in Table 4.2. After gaining an idea of the order of components, they are rearranged in a sequence of increasing magnitude. This is done to ensure that the larger magnitudes do not render small additions negligible as would happen if the position of the first significant digits of the latter terms are at a place lower than the precision level. (Rajaraman, 1978).

TABLE 4.2

VALUES OF VARIOUS TERMS IN THE WEIGHTING FUNCTIONS

x	$\frac{1}{x}$	$6x$	1	$-\lambda_1 p$	$\frac{1}{4x^3}$	$\frac{3}{x}$	$\frac{3}{4x^2 q}$	$3+p$
0.12	8.3	0.7	1.0	1.0	145	25	47	2
0.11	9.1	0.7	1.0	1.3	188	27	56	2
0.10	10.0	0.6	1.0	1.6	250	30	68	1
0.09	11.1	0.5	1.0	1.9	343	33	85	1
0.08	12.5	0.5	1.0	2.3	488	38	109	1
0.07	14.3	0.4	1.0	2.7	558	43	143	0
0.06	16.7	0.3	1.0	3.2	1156	50	197	0
0.05	20.0	0.3	1.0	3.8	2000	60	285	-1

$q = 1+x$, $p = 5 + 3\,\lambda_n(xq)$

It is further observed that the terms with $1/x$ and $1/x^3$ are the exploding ones. These factors are therefore used as the normalising factors evolving thereby two new normalized expressions wherein the awkward infinity does not occur at all. The proposed functions are, rearranged in increasing order,

$$\bar{S}(\psi) = x\, S(\psi)$$

$$= x(-6x + 1 - \lambda_1 p) + 1 \qquad (4.4)$$

and

$$\bar{U}(\psi) = x^3\, U(\psi)$$

$$= -x(x(x(3+p)-3) - 0.75/q) + 0.25$$

where,

$$x = \sin\psi/2$$

$$\lambda_1 = 1 - 2x^2$$

$$q = x + 1$$

$$p = 5 + 3\, \lambda_n(xq)$$

In these forms, only one type of paranthesis has been used to make these suitable for changing to Fortran statements.

Methematically the term p becomes infinity when x is exactly zero, but for all practical purposes the minimum value of x is 1.37×10^{-13} corresponding to a distance of 1 millimetre for which p is only equal to -64 . However, as is well-known, a positional accuracy of 1 millimetre is only of rigorous theoretical importance.

Figure 4.2 shows the graphs of the normalized functions $\bar{S}(\psi)$ and $\bar{U}(\psi)$ with x as argument.

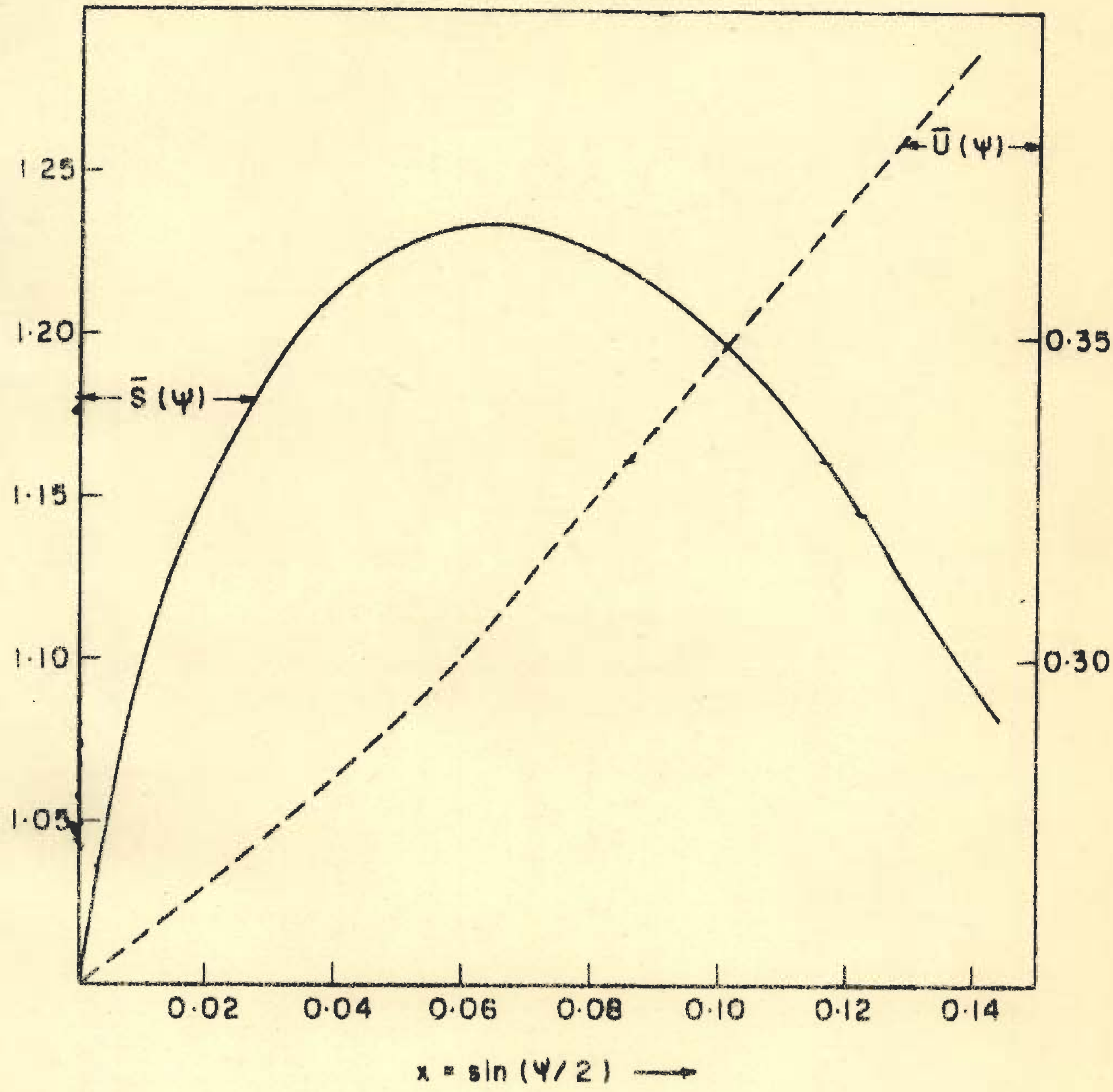


FIG. 4.2 NORMALIZED WEIGHTING FUNCTIONS WITH SIN (ψ/2) AS ARGUMENT

4.7 FORMULATION FOR NUMERICAL INTEGRATION

With the normalized functions, the numerical summation formulation is also refined as follows:

$$\Delta U = \begin{bmatrix} \Delta u_1 \\ \Delta u_2 \\ \Delta u_3 \end{bmatrix} = \bar{W} . \bar{\beta}$$

where,

$$\bar{W} = \begin{bmatrix} \bar{w}_1 \\ \bar{w}_2 \\ \bar{w}_3 \end{bmatrix} = \begin{bmatrix} \bar{S}(\psi)x^2 \\ \bar{U}(\psi)\ell_2 \\ \bar{U}(\psi)\ell_3 \end{bmatrix} \quad (4.5)$$

and $\bar{\beta} = \frac{\Delta g\ \sigma}{Gx^3}$.

The symbols used are self-explanatory, and have been defined earlier.

4.8 ONE-DEGREE DATA SET

Two basic sources of data were explored for one-degree mean free-air anomalies:

(a) A major global coverage, by the Defense Mapping Agency Aerospace Centre (DMAAC, 1973), of $1^\circ \times 1^\circ$ Meridian-Parallel-Grid type division. The DMAAC set is in the IGSN71, GRS67 system.

standard error of the mean determination which is of the order of ±12 milligal and is seldom less than 5 milligals.

(b) The mean free-air anomalies should be converted to condensation-corrected anomalies. Evan a simple terrain correction will yield a modified free-air anomaly set suitable for the present use. The evaluation, however, requires a detailed knowledge of the topography of the region, involving an enormous task. The indirect effects of condensation on N, ξ, η through the change of potential must also be simultaneously taken into account, thus, making the task quite complicated. The resulting change will be about 2 milligals in a point value in the most rugged part. The ruggedness again makes this change fluctuating between positive and negative extremes, least affecting the mean anomaly in an $1^\circ \times 1^\circ$ areal element.

4.9 FILLING OF ELEMENTS HAVING NO DATA

Even after the sets are combined in the manner detailed earlier, there remain several gaps in the region which must be suitably filled up before proceeding to process the anomalies. Luckily enough, the gaps in known values of anomalies correspond to regions that lie at great distances from the origin at Kalianpur, the minimum distance being nearly 1000 kilometres. Fig.4.3 shows the positions of such gaps that lie within approximately 20°

(b) The other data set is from the Lamont Geological Observatory (Kahle and Talwani, 1973). The grid shape is same as that of the DMAAC set, but the values are in the old Potsdam system and referred to the 1930 formula for normal gravity.

The DMAAC data set was first compiled for the region covered by the following boundaries,

$$\phi = 12^\circ S \text{ to } 58^\circ N,$$

$$\lambda = 48^\circ E \text{ to } 118^\circ E,$$

leaving a clear margin of 20° beyond the boundaries of the Indian subcontinent.

For filling up the gaps, the second set was then used after converting the required anomalies to the same system, as outlined in Appendix A. Thereafter updating of some values were done through personal communication with scientists in India (Chugh, 1977) and abroad (Decker,1978).

Two points may be recalled in this context:

(a) The Potsdam correction should not be directly applied for local determinations. Another correction of about 1 milligal comes from the revised IGSN71 network in respect of the Indian region. However, the corrections are needed for a few elements only, mostly lying outside the Indian continent and the oceanic regions not connected by precise gravity networks. Moreover, the amount is less than the

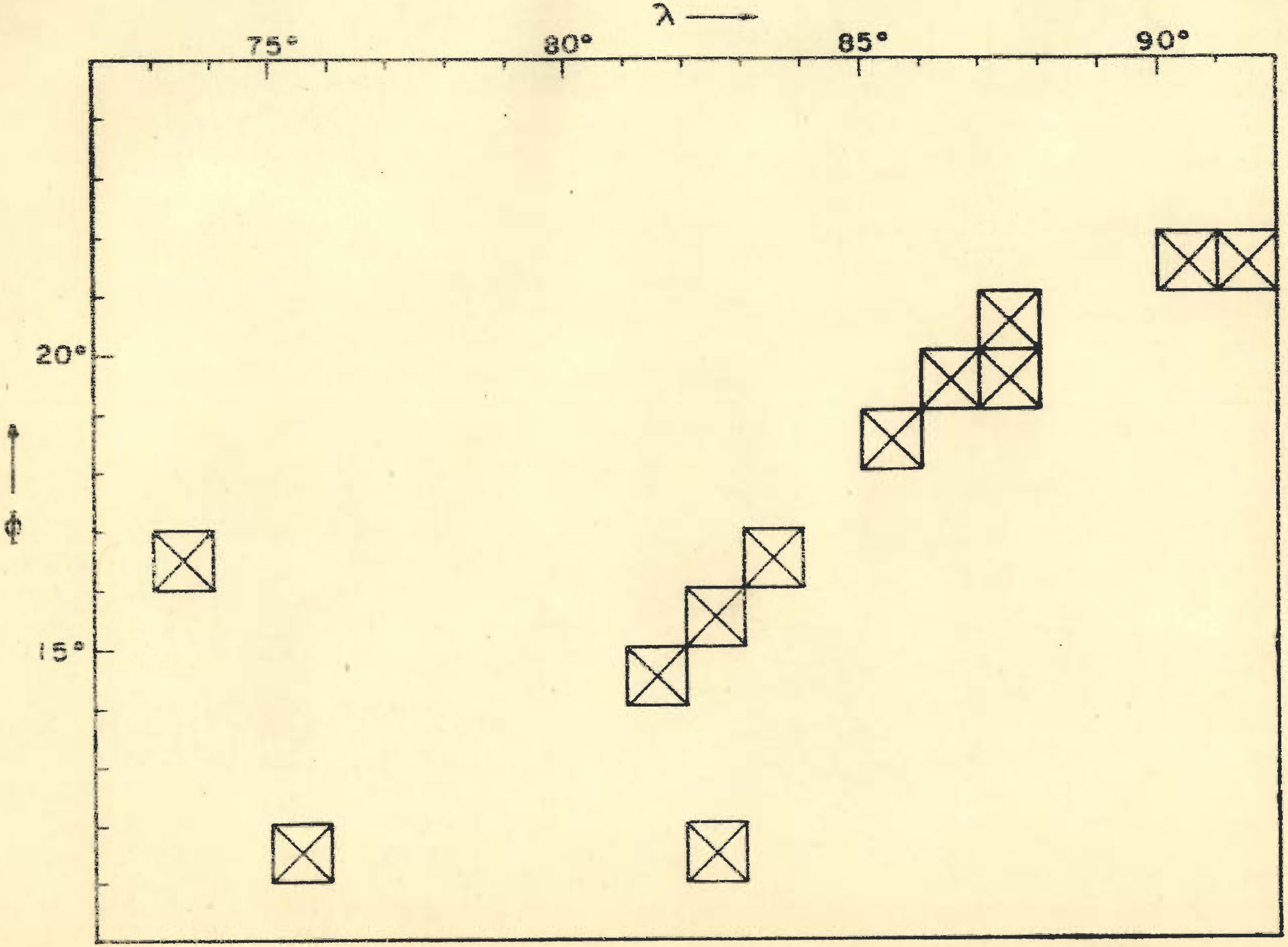


FIG. 4-3 SOME ONE-DEGREE UNITS WITH NO SOURCE DATA

radius from Kalianpur.

Most of the remaining gaps are found to be beyond the 53°E meridian in the West and beyond 100°E in the East. The situation towards the Northern and Southern sides is comparatively more favourable.

For interpolating the values of gravity anomalies in these distant unrepresented areas, a simple procedure detailed below has been adopted, as its precision does not significantly affect the final determination.

For any one-square-degree unit, or units, the corresponding twenty-five square-degree Equal-Area-Block is selected according to Rapp (1977), and the predicted value, or values Δg_p may be determined by,

$$\Delta g_p = \frac{n\Delta g_b - \sum_{i-1}^{n-k} (\Delta g_u)_i}{k} \tag{4.6}$$

where,

n = total number of units in the block,

Δg_b = block mean gravity anomaly,

k = number of unsurveyed units in the block,

$(\Delta g_u)_i$ = the mean anomaly in the i th unit.

Hereafter, the term 'unit' will be used to denote a $1^{\circ} \times 1^{\circ}$ element and 'block' to denote a $5^{\circ} \times 5^{\circ}$ element in the Equal-Area-Block system.

The prediction equation 4.6 implies assigning a value of the anomaly to a unit, or same values to each of the unsurveyed units in the block, such that the simple average of all the units remain the same as the mean anomaly of the block.

For the nearer units, however, a better representation is necessary.

4.10 PREDICTION OF MEAN ANOMALIES

Free-air anomalies are not interpolable and hence any prediction will be rather arbitrary in a strict deterministic sense. A zero anomaly in the unit is the simplest assumption to start with. But a zero value in the midst of high values of the same sign all around would be an improbability.

Another simple way would be to take the mean of the neighbouring units. The probability of such a value being truly representative of the unit will be more than that of null representation. In the absence of any other evidence, therefore, the arithmatic mean is the expectance with least standard deviation for unweighted array and is easy to compute as well.

Another scheme would be to use a weighted mean but theoretically this also presents a problem, as a mathematical model is fitted to non-interpolable data. Reasoning may differ in assigning the weights.

The statistical covariance prediction (Rapp,1964) however appears the most reasonable, being a combination of the classical probabilistic concepts (Kaula, 1959). Based on least-square-error theory, the principle is equally suitable for regular and systematic data, as well as those of random nature. The theory behind the covariance prediction is briefly discussed in Appendix E for the sake of completeness of the present work. The computer-oriented formulations with a few modifications proposed, are also presented therein.

4.11 VARIOUS COVARIANCE FUNCTIONS

The statistical correlation of gravity anomalies are completely characterized by the covariance function, a function relating interdependency of values with their mutual distance. Starting with the value of variance at zero distance, their typical decreasing nature highlights the randomness of the values. Examples from some publications are shown in Figure 4.4, wherein the ordinates have been standardized by dividing the original covariances by the respective variances, in order to render them comparable.

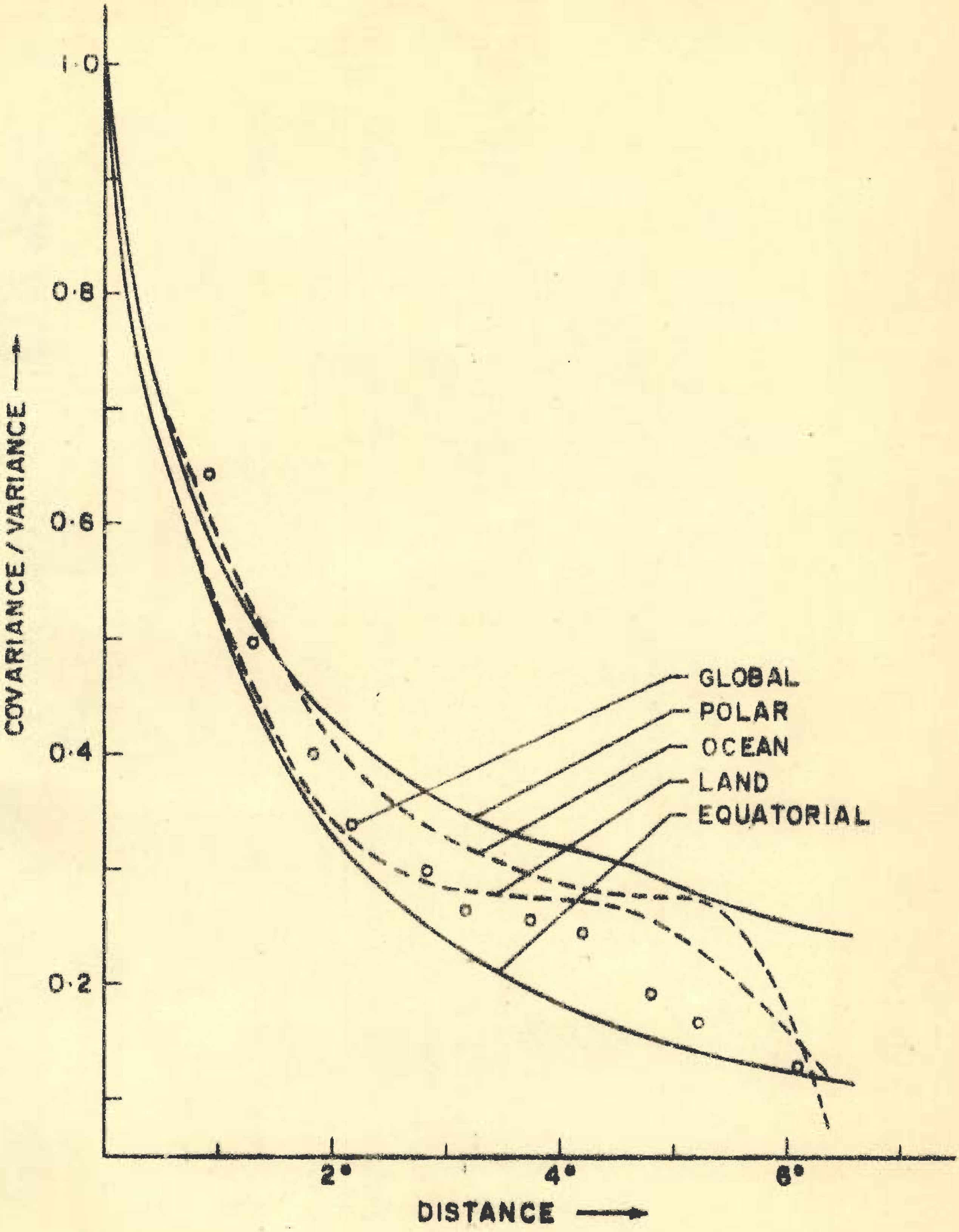


FIG. 4·4 COVARIANCE FUNCTIONS OF VARIOUS REGIONS

A linear estimate that depends only on the covariance, can be derived when stationarity and isotropy are assumed (Kaula, 1967). Stationarity of data means that statistical properties of the data remain unchanged with position whereas isotropy implies that its properties are independent of azimuth or direction.

There is some evidence that gravity data are not stationary (Gaposchkin, 1973) as also discernible from Figure 4.4. The statistical properties would most probably vary over land areas from those over oceanic regions or in equatorial regions from those over polar regions. The gravity anomalies, as computed by subtracting the normal gravity from the observed value after the free - air reduction, are functionally dependent on the latitude ϕ and the topographic height h. The background of non-stationarity is thus evident from this; continents with wider topographic variation and near-surface density contrasts, would be characterized by more random anomalies as compared with those over oceanic areas where no mass lies above the geoid, and deeper heterogenities produce rather smooth anomalies. Furthermore, the flattening of the earth causes a systematic variation in gravity anomalies from equator to pole.

It was therefore felt that a 'local' covariance estimation will yield better results than a global one, which primarily means that the covariance obtained is in respect of a shorter distance.

To illustrate the short-distance property numerically, covariograms around a few unsurveyed units have been presented in Figure 4.5. After some distance, the curves swing towards the negative side.

A number of functional forms have been tried by various authors. For example,

Hirvonen (1963) suggested

$$C(r) = C_o/(1+a^2r^2) \tag{4.7}$$

Kaula proposed

$$C(r) = C_o\ e^{-ar} \tag{4.8}$$

Rapp (1964) recommended

$$C(r) = C_o + C_1r + C_2r^2 + \ldots \tag{4.9}$$

The polynomial presentation appears to be the best, as neither the hyperbolic equation, nor the exponential expression accounts for the negative value at increased distance.

However, so far as the present problem is concerned none of these need be applied at this stage, as the gravity centres are uniformly gridded, except for the inappreciable change due to the parallels not being great circle. This effect can safely be neglected because,

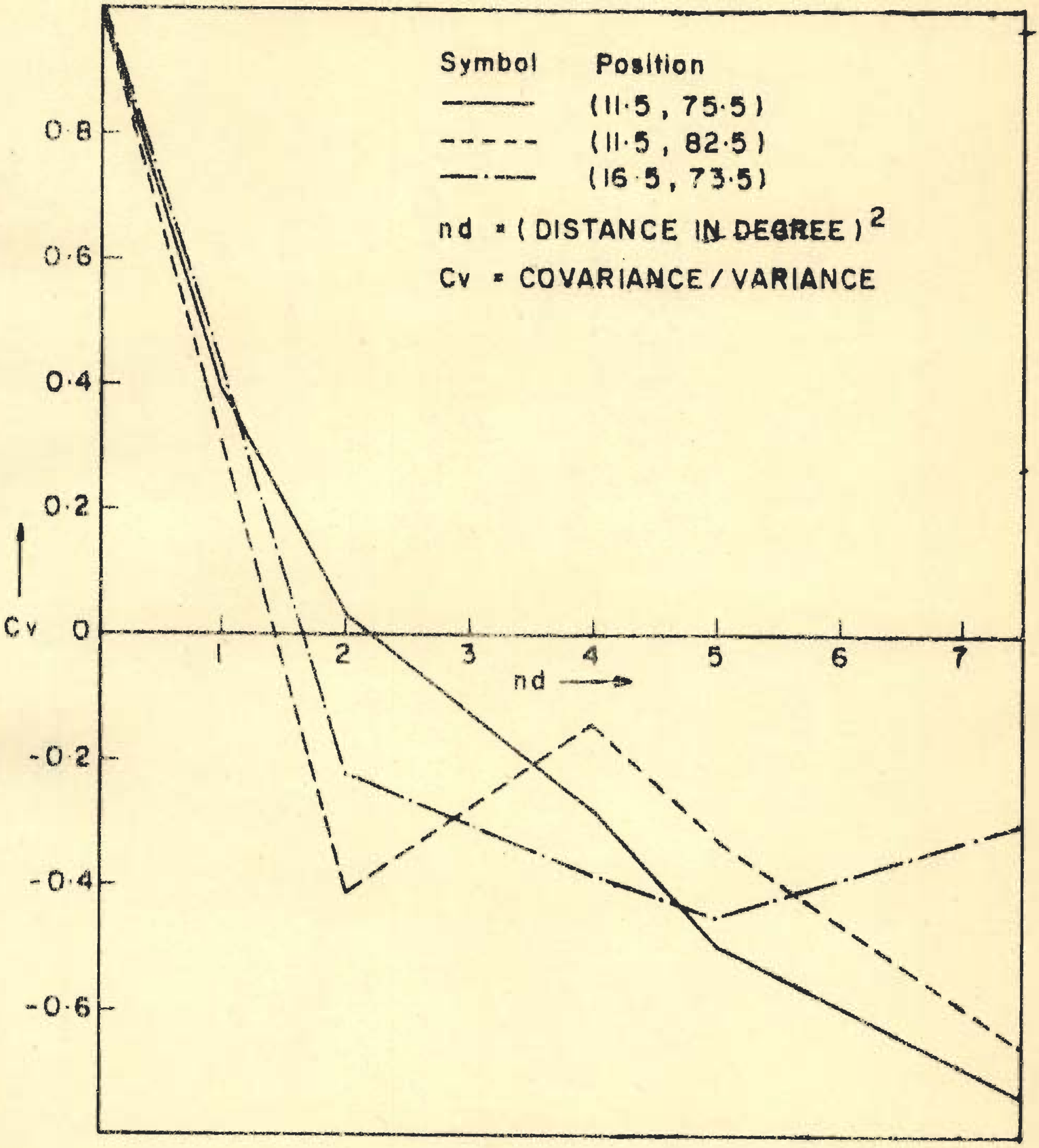


FIG. 4·5 SOME LOCAL COVARIOGRAMS OF UNIT ANOMALIES

(a) the Indian region fortunately lies in the equatorial region,

(b) for the local covariance function, variation from the average grid distance is not appreciable.

The predicted values of the anomalies in the units not represented by measured values, are tabulated in Table 4.3.

TABLE 4.3

UNIT MEAN ANOMALIES ESTIMATED BY COVARIANCE PREDICTION

Latitude	Limit	Longitude	Limit	Anomaly
10°N	11°N	75°E	76°E	-31 mgal
10°N	11°N	82°E	83°E	-61
14°N	15°N	81°E	82°E	-28
15°N	16°N	82°E	83°E	-17
16°N	17°N	73°E	74°E	-29
16°N	17°N	83°E	84°E	-24
18°N	19°N	85°E	86°E	-47
19°N	20°N	86°E	87°E	-29
19°N	20°N	87°E	88°E	-16
20°N	21°N	87°E	88°E	-15
21°N	22°N	90°E	91°E	-27
21°N	22°N	91°E	92°E	+4

4.12 RESULTS OF COMPUTATION

Using the numerical summation formulation with the normalized function, described in section 4.7 of chapter IV, two computer programs have been developed, as follows,

(a) computation of geoidal parameters at any general point,

(b) computation at 1° unit corners only.

The first one is a general program used later with further subdivision of areal elements in the immediate neighbourhood. Otherwise, with 1° unit as the smallest division, a general point may give rise to unrepresentatively high value due to the effect of the unit containing it.

The second program similar to the test program described in Appendix B, is with an aptly simplified algorithm, by considering the effect beltwise and taking advantage of the symmetry of $\bar{w}_1$, $\bar{w}_2$ and the antisymmetry of $\bar{w}_3$, about a meridian. The local effects due to the variations in gravity anomalies within the units, are uniformly truncated during their evaluations at the grid corners, leaving the systematic part behind.

The outer region was defined by the boundaries detailed in Table 3.3. Some logical statement have been incorporated in the computer program to take care of the boundary, by skipping calculations wherever a boundary is crossed on the either side.

The grid undulations N_p as well as the deviation components ξ_p, η_p are evaluated at grid corners well within the Indian geographical boundary, and given in Tables 4.4, 4.5 and 4.6.

Figures 4.6, 4.7, 4.8 present two profiles of each of N_p, ξ_p, η_p one along a meridian and another along a parallel.

4.13 SUMMARY AND DISCUSSION

The effect of the exterior region on the geoidal parameters having been discussed and computed in the last chapter, the contribution of the interior region has been computed with higher precision using a more detailed gravity anomaly set mostly furnished by DMAAC.

After preliminary discussions on the assumptions made in the numerical analogue of the Stokes' and Vening Meinesz' integrals, the errors owing to averaging the components of the integrals, namely the weighting functions and the gravity anomalies, have been estimated alongwith a sample check of consistency of the one - degree data with the OSU data. These are found to be quite satisfactory for use without the necessity of any reductions, such as the condensation reduction etc., for achieving an order of precision compatible with that of the source data.

The numerical algorithm has been further refined in this study by normalization of the Stokes' and modified Vening Meinesz' functions to achieve better computation efficiency.

The compilation of the data set has been done from various sources, after standardizing all into the same system. Gaps have been filled up by using a simple formula for the consistency of data sets. Gravity anomalies of the nearer unsurveyed units are estimated by a local covariance prediction technique. The geoidal parameters at 1° grid corners within the Indian subcontinents have been computed from the data set extending up to the inner limit of the exterior region, and the computed 'partial geoid' are presented in the tabular form, with a few profiles for the graphical representation.

The profiles are found to be mutually consistent. The maxima or minima in N_p tallies with the corresponding zero slope in the appropriate direction, thus providing a numerical check. Whilst the undulations are smoothly varying, the ξ_p, η_p values show occasional sharp changes at places where the changes of the mean anomalies between the adjacent units are also seen to be sharp, thus highlighting the fact that the two deflections components are much affected by local changes.

The partial geoid obtained, however, represents a general trend only, and for obtaining the required higher precision for the absolute orientation of the geodetic system, finer subdivisions of the gravity field are essential. These intermediate results may directly be taken for the purpose without repeating the entire process over again. The steps are,

i) covering an area, bounded by two meridians and parallels of whole number of degrees around any astrogeodetic station, by a dense gravity survey,

ii) computing N_v, ξ_v, η_v from the 'void geoid' parameters at the station, by suitable interpolations ,

iii) computing the effects of the 1^o mean anomalies of the newly covered zone, at the four 1^o corners around the station ,

iv) removing these effects from the corresponding N_p, ξ_p, η_p values and interpolating the net values at the astrogeodetic station ,

v) superimposing the void geoid parameters computed in step (i) and the effects of the detailed gravity region, starting from $0.^o25$ mesh size to finer grids.

The gravimetric measurements may be planned judiciously to obtain representative mean values with optimum number of observations. To match with the precision in the 'void geoid' , a coverage of 3^{o} to 4^{o} around the station, is recommended.

Another use of the 'partial geoid' parameters for obtaining the absolute orientation parameters, will be discussed in chapter VI, where an alternative approach has been proposed.

The geophysical importance of the regional geoid obtained during the course of this investigation, need not however, be overemphasised. Its special significance (Woollard and Khan, 1970) to delineation of mass anomalies and to the resulting geoid prospecting (Ray and Bhattacharji, 1977) is considerable among other things and the results provided here furnish a major portion of total computative effort involved in such problems. Accordingly, a map of the regional free-air geoid in India has been produced in Figure 4.9, which may be found useful by geophysicists generally and by future researchers in geodesy.

TABLE 4.4

UNDULATIONS OF THE PARTIAL GEOID IN INDIAN REGION

ϕ	λ From	To	N values in metres negative sign indicates 'below GRS 67 spheroid'							
35	75	79	-28.8	-27.2	-26.1	-23.8	-21.4			
34	75	79	-30.8	-27.4	-26.4	-25.6	-24.5			
33	75	79	-36.0	-30.0	-24.4	-23.8	-25.3			
32	75	79	-39.4	-35.8	-29.0	-25.3	-26.3			
31	75	80	-38.9	-39.4	-36.3	-31.4	-28.1	-27.4		
30	74	80	-37.3	-38.4	-39.9	-40.2	-39.1	-37.2	-36.7	
29	73	80	-36.9	-37.2	-38.2	-40.0	-42.7	-45.7	-47.6	-48.4
28	71		-34.9	-35.4	-35.6	-36.5	-37.8	-40.0	-43.4	-47.6
		82	-51.8	-53.9	-53.0	-50.1				
28	92	95	-26.6	-28.5	-32.7	-34.9				
27	70		-32.6	-33.4	-33.8	-34.6	-35.8	-37.6	-40.2	-43.5
			-47.2	-51.1	-53.8	-55.1	-55.7	-54.7	-52.8	-50.5
			-45.4	-39.5	-37.4	-36.8	-35.9	-35.4	-34.7	-35.6
		95	-36.9	-36.7						
26	71		-32.6	-33.4	-34.5	-35.6	-37.8	-40.7	-43.1	-45.6
			-48.5	-51.1	-53.7	-56.1	-57.1	-56.5	-54.5	-51.9
			-49.1	-46.5	-44.6	-43.0	-40.8	-39.6	-39.6	-38.1
		95	-36.6							

contd.

Table 4.4 contd.

φ	λ From	λ To	N values in meters negative sign indicates 'below GRS 67 spheroid'
25	71		-33.0 -34.0 -34.9 -36.1 -38.5 -40.8 -42.4 -44.2
			-46.5 -48.4 -50.5 -52.5 -53.4 -52.9 -50.9 -48.9
		88	-47.6 -46.8
25	93	94	-40.6 -38.9
24	71		-34.8 -35.8 -36.7 -38.0 -39.6 -40.9 -42.1 -43.6
			-45.3 -46.3 -47.2 -48.3 -48.6 -48.3 -47.3 -46.1
		88	-45.9 -46.5
23	71		-37.4 -38.6 -39.8 -40.6 -41.4 -42.4 -43.5 -44.3
			-44.7 -45.1 -46.0 -47.2 -48.0 -48.4 -48.5 -48.3
		88	-47.7 -47.3
22	70		-38.3 -39.5 -40.8 -42.0 -42.9 -43.6 -44.7 -45.5
			-45.6 -45.5 -46.0 -47.1 -48.6 -50.1 -51.4 -52.0
		89	-51.6 -50.1 -48.1 -46.9
21	71		-42.6 -43.3 -43.9 -44.8 -45.8 -46.6 -47.2 -47.5
			-48.0 -48.5 -49.4 -51.0 -53.0 -54.4 -54.6 -53.5
		87	-51.5
20	73		-46.6 -47.2 -48.0 -48.8 -49.4 -49.9 -50.6 -51.0
		86	-51.3 -53.1 -55.3 -56.2 -55.8 -54.8
19	73		-49.6 -49.7 -50.3 -51.0 -51.6 -52.1 -52.9 -53.2
		84	-53.5 -55.1 -56.9 -57.9

contd.

Table 4.4 contd.

ϕ	λ From	λ To	N values in metre negative sign indicates 'below GRS67 spheroid'							
18	73		-52.0	-51.8	-52.7	-53.6	-54.2	-54.5	-55.1	-55.2
		83	-55.8	-56.5	-57.0					
17	74		-54.8	-55.5	-56.7	-57.3	-57.5	-57.8	-57.4	-57.3
		82	-57.4							
16	74	81	-58.7	-58.5	-59.2	-60.0	-60.6	-60.6	-59.2	-58.3
15	75	80	-61.2	-60.8	-61.3	-62.2	-61.8	-60.8		
14	75	80	-63.6	-62.1	-61.1	-61.3	-61.2	-62.4		
13	75	80	-65.0	-62.8	-61.2	-61.0	-60.9	-63.0		
12	76	79	-63.9	-62.6	-62.0	-61.4				
11	76	79	-65.6	-63.0	-62.0	-61.9				
10	77	79	-63.0	-62.4	-63.8					

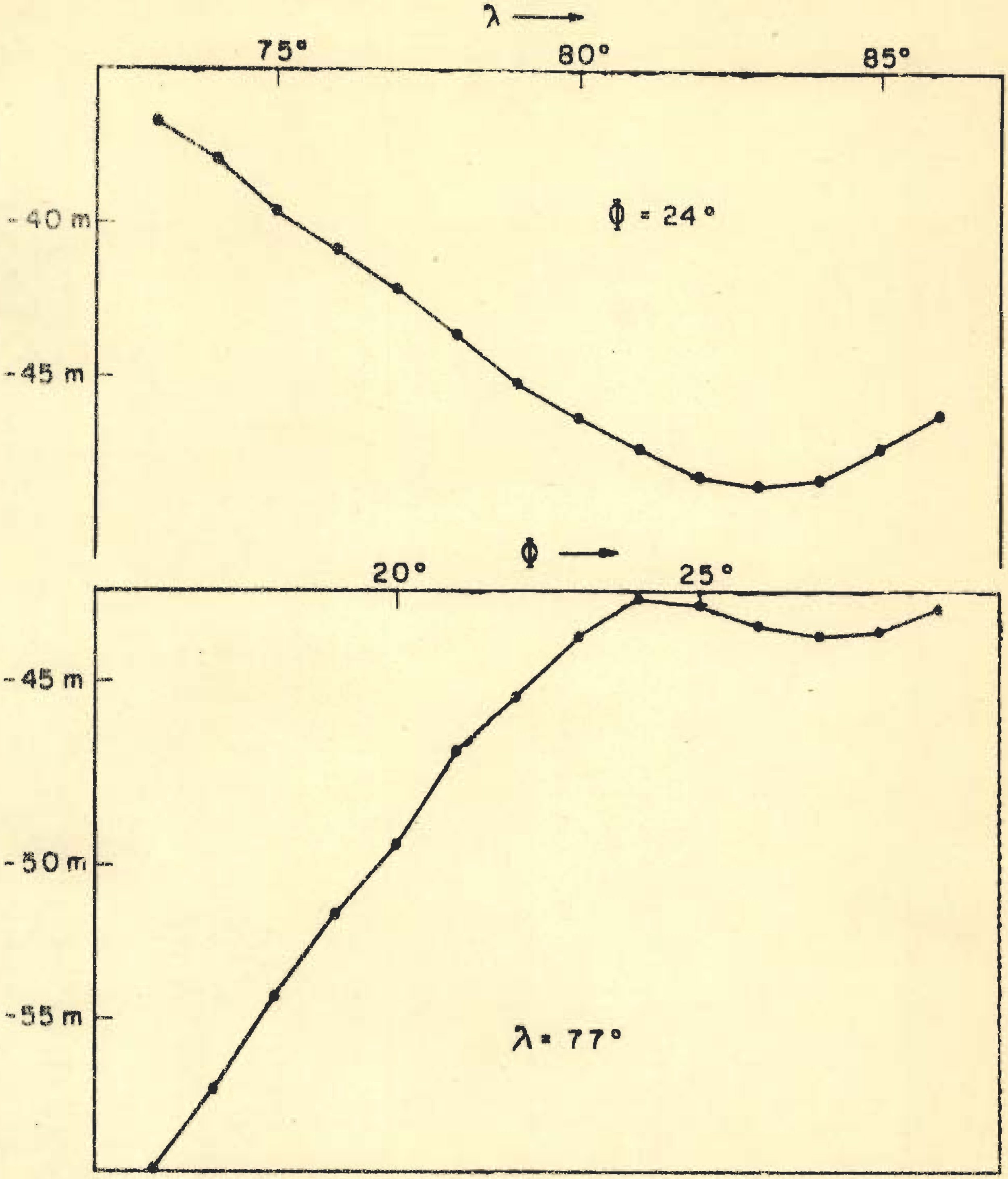


FIG. 4·6 PROFILES OF THE UNDULATIONS OF THE PART GEOID

TABLE 4.5

MERIDIONAL DEVIATIONS OF THE VERTICAL OF THE PARTIAL GEOID IN INDIA

ϕ	λ From	to	ξ in arcseconds							
34	76	78	0.43	6.02	-0.42					
33	76	78	-11.23	5.69	10.06					
32	76	78	-14.09	-24.84	-14.41					
31	76	78	0.45	-7.12	-10.81					
30	75	79	0.69	0.56	-7.53	-20.87	-29.81			
29	75	79	0.87	0.68	-1.51	-5.70	-13.36			
28	74	80	2.53	1.13	-0.40	-0.41	-0.82	-2.90	-3.77	
27	73		1.43	0.49	-0.06	-0.43	0.31	2.92	6.58	6.23
		81	2.14							
26	72		-1.04	-1.08	0.50	-0.73	-1.84	1.29	4.09	4.87
		83	5.79	4.63	2.17	0.62				
25	72		-1.34	-0.00	-2.12	-1.93	1.57	1.92	2.05	3.46
		87	5.40	8.84	12.76	15.22	15.02	13.24	12.26	8.95
24	72		-5.63	-6.94	-5.30	-2.43	-1.11	-0.23	0.28	1.57
		87	3.17	4.63	5.24	5.45	4.58	2.16	-0.35	-0.69
23	72		-4.88	-5.56	-5.12	-4.17	-4.84	-5.59	-2.99	1.31
		87	1.89	0.83	-0.68	-2.76	-4.64	-7.02	-8.61	-6.79

contd...

Table 4.5 continued

ϕ	λ From	To	ξ in arcseconds							
22	75		-4.49	-3.83	-2.05	-1.87	-3.94	-4.93	-5.22	-4.74
		87	-5.73	-7.56	-2.08	-4.73	-3.02			
21	75		-3.89	-3.54	-4.07	-5.28	-5.75	-4.82	-3.48	-4.41
		85	-5.83	-4.51	-3.10					
20	75		-4.14	-4.45	-4.07	-3.92	-4.59	-4.87	-3.70	-3.70
		83	-3.20							
19	75	82	-4.25	-3.84	-4.25	-4.25	-3.88	-3.55	-4.78	-3.73
18	75	81	-4.92	-6.15	-5.63	-4.74	-4.49	-3.78	-4.20	
17	75	80	-5.66	-5.64	-6.25	-6.55	-6.14	-4.63		
16	76	79	-3.33	-4.18	-5.93	-4.57				
15	76	79	-2.49	-0.51	-0.24	0.38				
14	76	79	-2.10	2.04	4.66	3.07				
13	76	79	-0.52	-1.96	-2.54	-1.44				

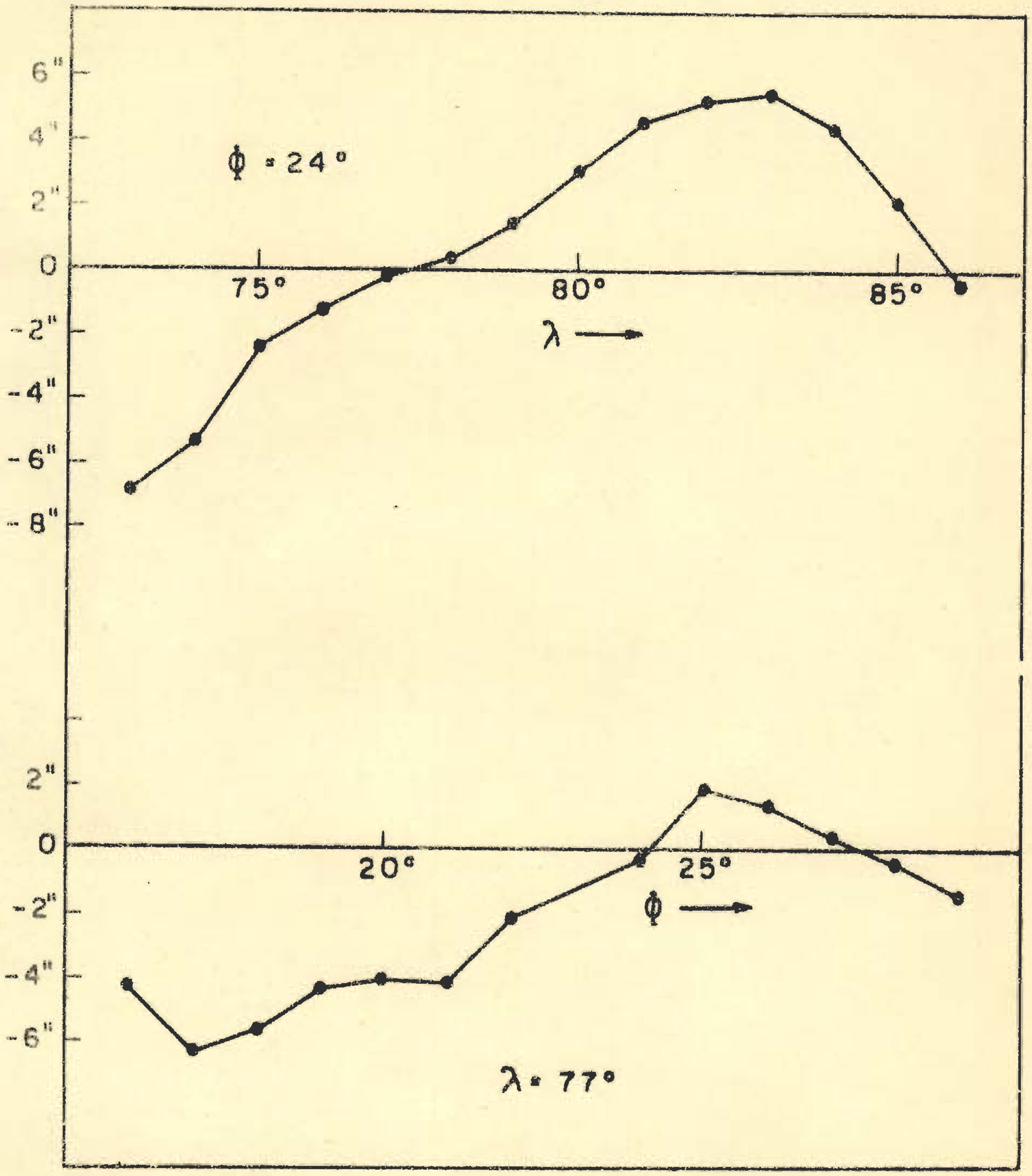


FIG. 4·7 PROFILES OF THE MERIDIONAL DEVIATIONS OF THE VERTICAL OF THE PART GEOID

TABLE 4.6

PRIME VERTICAL DEVIATIONS OF THE VERTICAL OF THE PARTIAL GEOID IN INDIA

ϕ	λ From	To	η in arcseconds							
34	76	78	-4.66	0.79	-4.11					
33	76	78	-19.15	-7.05	4.94					
32	76	78	-15.93	-15.96	-1.25					
31	76	78	-1.55	-12.57	-10.71					
30	75	79	3.40	3.23	-1.83	-3.22	-5.61			
29	75	79	2.11	5.67	6.89	6.84	2.10			
28	74	80	2.71	2.83	6.89	8.08	10.67	8.18	1.60	
27	73		2.10	3.08	4.50	7.01	7.21	9.17	7.88	3.67
		81	2.62							
26	72		2.58	2.12	2.57	6.64	6.06	4.08	6.45	5.87
		83	5.20	6.54	4.34	0.91				
25	72		2.45	1.01	4.15	5.88	4.28	2.13	5.20	4.67
		87	3.29	5.76	3.24	0.80	-3.19	-5.44	-3.23	-1.61
24	72		1.78	1.63	3.71	3.00	2.13	2.68	3.82	3.29
		87	0.71	2.95	1.01	0.11	-1.42	-2.85	-1.91	1.32
23	72		2.42	2.79	0.31	2.52	1.78	2.64	0.44	0.91
		87	0.57	2.70	2.37	0.87	0.86	-0.12	-0.45	-1.64

contd.

Table 4.6 contd.

ϕ	λ From	To	η in arcseconds							
22	75		2.28	1.97	1.22	-1.05	0.15	1.74	3.01	3.33
		87	3.08	2.51	0.18	-1.70	-4.26			
21	75		1.87	1.38	0.62	0.71	0.99	1.07	2.72	4.05
		85	4.60	1.44	-0.79					
20	75		1.97	1.19	1.06	1.34	1.37	-0.06	1.41	6.13
		83	3.22							
19	75	82	1.98	0.96	1.27	0.98	1.99	-0.66		
18	75	81	2.57	1.22	0.87	0.46	1.77	-1.33		
17	75	80	3.44	1.57	0.56	0.08	1.39	-3.18		
16	76	79	1.63	1.95	0.64	-0.95				
15	76	79	0.12	2.36	1.54	-3.60				
14	76	79	-3.55	-0.35	1.09	-1.68				
13	76	79	-4.70	-1.89	0.71	-0.65				

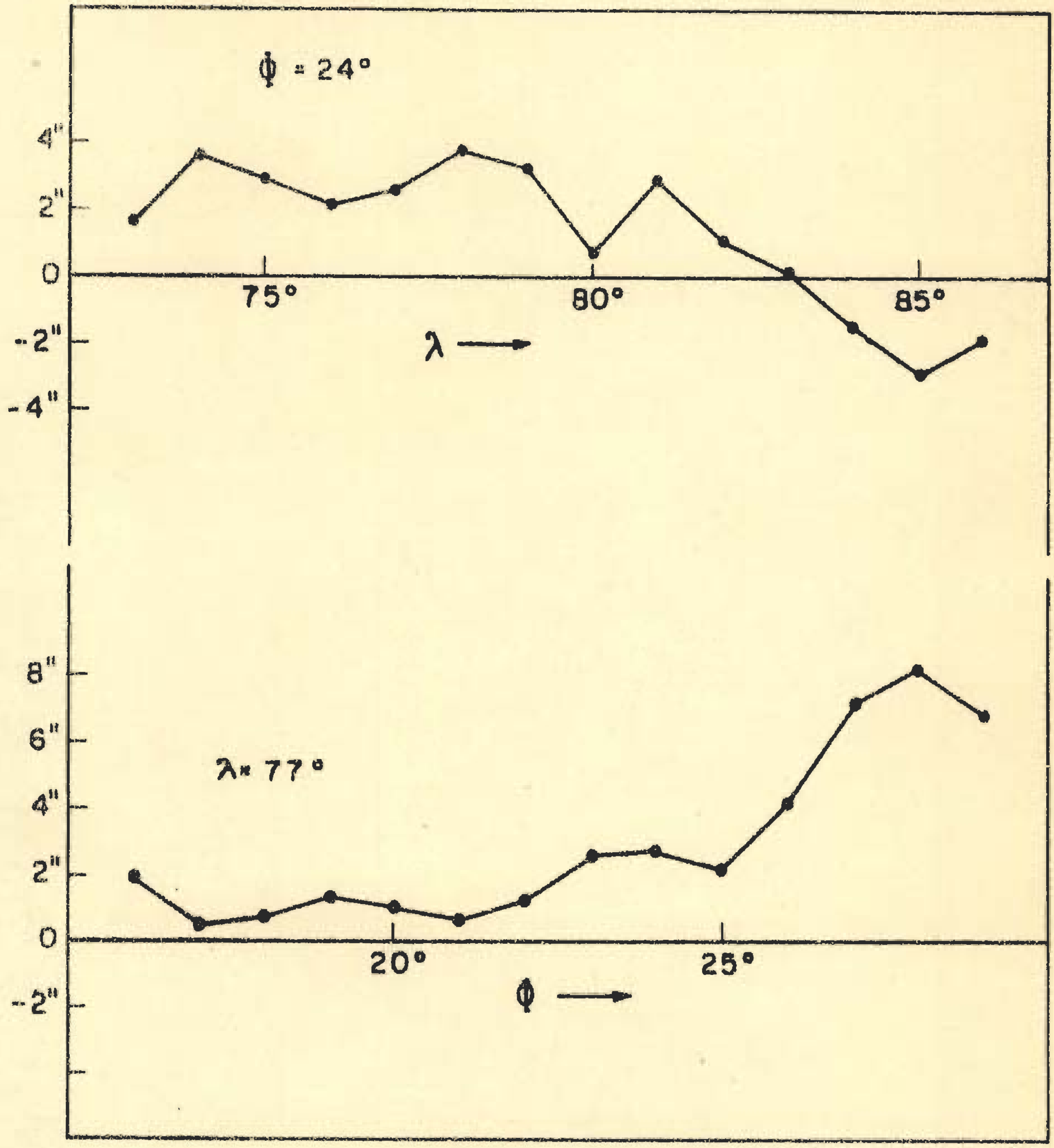


FIG. 4·8 PROFILES OF THE PRIME VERTICAL DEVIATIONS OF THE VERTICAL OF THE PART GEOID

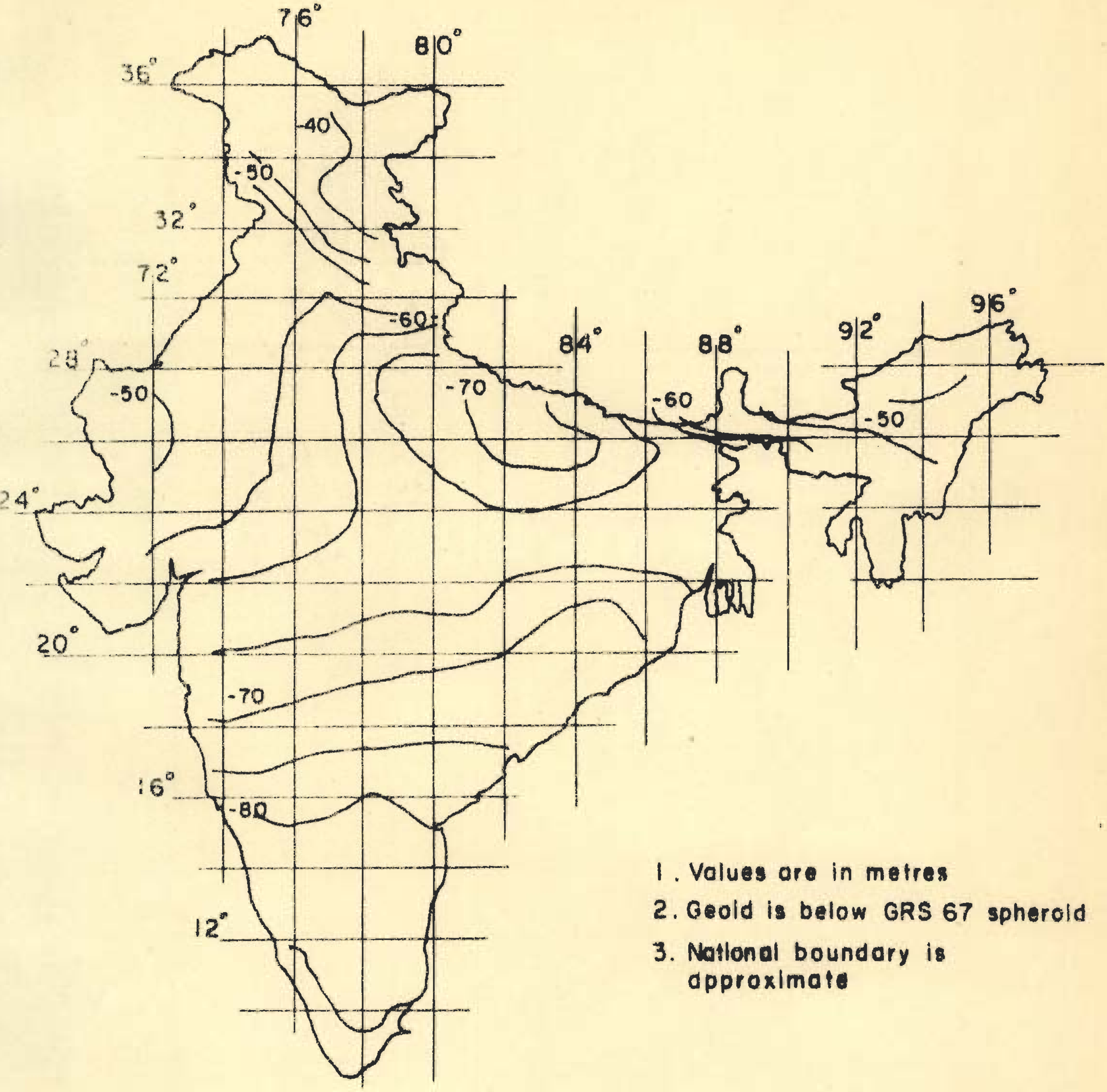


FIG. 4·9 ONE DEGREE MEAN FREE-AIR GEOID IN INDIA

CHAPTER V

GRAVIMETRIC ORIENTATION OF GEODETIC SYSTEM THROUGH GENERAL ASTRO-GEODETIC STATIONS

5.1 GENERAL

The reduction of a local geodetic network to an absolute reference system requires the definition of five orientation parameters. These are the parameters a and f of the reference spheroid, and the absolute geoidal parameters N_o, ξ_o, η_o at the initial point of the triangulation or trilateration network, which together determine the World Geodetic System .

The transformation may be done by determining the geoidal parameters directly at the initial point, using the global broad gravity coverage in the exterior region and dense gravity network in the interior region around. Any astrogeodetic station could also be chosen for the purpose, provided adequate gravity data is available in the interior region around the computation station.

The present chapter deals with formulations related to a finer detailing of the Stokesian integration with specific numerical works carried out for various stations in India, with a view to obtaining gravimetrically the absolute orientation parameters more precisely.

The problem in the present context is of course an inverse one. However, the equations being generalized, a simple interchange of the subscripts o and n serves the purpose.

5.3 NUMERICAL EXAMPLES

(a) Determination of the undulation vector at the origin at Kalianpur from the grid values obtained in the earlier chapters is a matter of routine interpolation. Simple linear interpolations from the nearest four grid corners yield the following values:

$$N_{gr} = N_v + N_p = -59.9 \text{ metres},$$

$$\xi_{gr} = \xi_v + \xi_p = -\ 0''.24 \qquad (5.2)$$

$$\eta_{gr} = \eta_v + \eta_p = +\ 3''.44$$

The formulae 5.1 with interchanged subscripts are reduced to simple expressions, by using $\phi_n = \phi_o = 24.119794$ degrees, $\lambda_n = \lambda_o = 77.654880$ degrees, as

$$\delta N_o = \delta N_n = -59.9 - 0.0 = -59.9 \text{ metres}$$

$$\delta \xi_o = \delta \xi_n = -0.24 - (-0.29) = +\ 0''.05 \qquad (5.3)$$

$$\delta \eta_o = \delta \eta_n = +3.44 - 2.89 = +\ 0''.55$$

(b) Two more stations, one in the North and another in the East are taken as test cases. The astrogeodetic deviations of the vertical being obtained directly in terms

5.2 THE ORIENTATION FORMULA

In order to obtain corrections to be applied to astrogeodetic values of N, ξ, η at a point from those known at the origin, the following datum shift formulae (Vening Meinesz, 1950) could be used directly:

$$\begin{aligned}\frac{\delta N_n}{R} = & (\sin\phi_o \sin\phi_n + \cos\phi_o \cos\phi_n \cos\Delta\lambda)\,\delta\varepsilon_o \\ & -(\cos\phi_o \sin\phi_n - \sin\phi_o \cos\phi_n \cos\Delta\lambda)\,\delta\xi_o \\ & -(\cos\phi_n \sin\Delta\lambda)\,\delta\eta_o - \delta a/R \\ & +(\sin^2\phi_n - 2\sin\phi_o \sin\phi_n)\,\delta f \end{aligned} \tag{5.1}$$

$$\begin{aligned}\delta\xi_n = & -(\sin\phi_o \cos\phi_n - \cos\phi_o \sin\phi_n \cos\Delta\lambda)\,\delta\varepsilon_o \\ & +(\cos\phi_o \cos\phi_n + \sin\phi_o \sin\phi_n \cos\Delta\lambda)\,\delta\xi_o \\ & -(\sin\phi_n \sin\Delta\lambda)\,\delta\eta_o - 2\cos\phi_n(\sin\phi_n - \sin\phi_o)\,\delta f\end{aligned}$$

$$\begin{aligned}\delta\eta_n = & +(\cos\phi_o \sin\Delta\lambda)\,\delta\varepsilon_o + (\sin\phi_o \sin\Delta\lambda)\,\delta\xi_o \\ & +(\cos\Delta\lambda)\,\delta\eta_o\end{aligned}$$

where, ϕ_o, λ_o = coordinates of the origin

ϕ_n, λ_n = coordinates of any station n

$$\delta\varepsilon_o = \frac{\delta N_o}{R} + \frac{\delta a}{R} + \sin^2\phi_o\,\delta f$$

$$\Delta\lambda = \lambda_n - \lambda_o ,$$

other symbols have been defined earlier.

of the Everest spheroid (Gulatee, 1955), the geoidal heights were estimated from a geoidal map (Bhattacharji,1973) where the N_{ag} values are in terms of the International Spheroid with the revised orientation (Expression 1.2). To reduce the heights to the original Everest system again, the datum shift relations would be,

$$\delta N_o = 0.0 - 9.5 \text{ metre}$$

$$\delta \xi_o = -0.29 - 2.42 \text{ arcseconds}$$

$$\delta \eta_o = 2.89 - 3.17 \text{ arcseconds} \qquad (5.4)$$

$$\delta a = 6377299 - 6378388 \text{ metres}$$

$$\delta f = (1/300.8017) - (1/297.0)$$

It is to be noted here that the metric equivalent of the semimajor axis of the Everest spheroid is the result of a conversion from ' Indian feet ' to metres (Bhattacharji, 1961).

The coordinates of the stations and astrogeodetic quantities and the interpolated gravimetric values there are given in Table 5.1. The geoidal parameters are expressed in a non-dimensional form for ease of computation. The change of dimensions of the spheroids, namely the Everest and GRS67, are

$$\delta a/R = 135.14 \ \mu\text{rad}$$

$$\delta f = 28.47 \ \mu\text{rad} \qquad (5.5)$$

TABLE 5.1

GEOIDAL PARAMETERS OF NORTH AND EAST STATIONS

position	code	φ_c	λ_c	$\frac{N_{ag}}{R}$	ξ_{ag}	η_{ag}	$\frac{N_{gr}}{R}$	ξ_{gr}	η_{gr}
North	1	28.734581	77.648975	0.36	-23.75	31.05	-9.54	-14.38	41.58
East	2	23.395269	86.986980	2.23	0.98	-34.42	-9.72	-28.15	-4.26
unit		degree		ppm or microradian					

Table 5.2 presents the three sets of orientation corrections at the origin, obtained from three astrogeodetic stations. Large discrepancies in these results are visualized. For, it is quite obvious that the orientation

TABLE 5.2

EXAMPLE OF ORIENTATION PARAMETERS THROUGH VARIOUS ASTROGEODETIC STATIONS

Point Code	δN_o (metre)	$\delta \xi_o$ (")	$\delta \eta_o$ (")
0	-59.9	+0.05	+0.55
1	-60.1	+0.50	+2.18
2	3.6	-5.25	+2.52

parameters obtained from a single point evaluation with basic input upto one-degree mean will not be reliable. It may be recalled here that astro-geodetic deviations derived from actual measurement possesses a complete spectrum of energy, even upto the shortest wavelength components, whereas the gravimetric values obtained through averaged gravity anomalies are effectively bandlimited.

A comparison of the sets suggest that attempts to determine reliable values of absolute orientation parameters are liable to be futile until both the astrogeodetic and gravimetric sets are equally weighted using either of the following methods:

(a) increasing the accuracy of numerical integration using a more detailed set of gravity anomalies at least in the immediate neighbourhood of the point, say upto 3^{o} radius,

(b) averaging the astrogeodetic values to correspond with the automatically smoothed gravimetric values by filtering out the short wavelength components.

The first of these two alternatives has been attempted and forms the subject matter of this chapter, whilst the second has been tested in the next chapter.

5.4 EFFECT OF ERRORS IN GRAVITY DATA

The success of absolute orientation depends on the quality of gravity data, topographic information and the accuracy of numerical procedures. The physical inhomogenity of the earth causing the warping of the geoid, is fully reflected in the gravity field and in the astronomic results. ~~The precision of gravity anomalies should therefore~~ be of the order comparable to similar 1st-order astro - observations.

The basic sources of errors are as follows:

i) instrumental and observational errors, including drift and tidal effects,

ii) errors in measurements of heights,

iii) errors in station positions,

iv) errors in absolute gravity values derived from the reference base station,

v) inaccuracies in reductions of gravity values to geoid level.

The gravimeters are generally capable of reading gravity values precisely up to 0.5 milligal with usual human skill, Lacoste Romberg gravimeters being more stable and precise than the Worden instrument. Drift errors may be checked by occasional corrections assuming linear variations whereas tidal effects may go up to 0.2 milligal at times.

Inaccuracies in height estimations probably contribute the most. For example, an error of 1 metre, which is quite common, produces an error of 0.3 milligal in the reduced free-air anomaly.

Inaccuracies in the actual coordinates of a station also introduce errors by affecting the normal value of gravity calculated for the given latitude. An error of 0.01 degree in latitude determination can be commonly found to occur. An estimate of discrepancies arising from this source, in the Indian region, can be made from the differential of γ with respect to ϕ, which is found to be 0.5 milligal to 0.9 milligal.

The absolute value of gravity at the reference base station may be in error due to standardization of network. The error being constant will have an effect on N but not on ξ and η.

A condensation correction needs to be applied for the presence of the protruding topographic masses above the geoid. As the free-air anomalies very nearly correspond to the condensation values, error in a flat terrain would normally be inappreciable but could be about 1 milligal in the rugged terrain (Rice,1952). The geoid-cogeoid separation may amount to as much as 3 metres in terrains of large topographic variation and an error of 0.5 metre is usually anticipated.

The total estimate of errors from various sources in the input data thus turns out to be about 1 milligal in the interior region on the average.

Finally, errors also stem from mathematical computations and computer truncations. The mean value of anomalies beyond radial distances of 3° to 4° from the computation points, are estimated to have rms deviation of as much as 5 milligals. Truncation errors during computations are, however mostly eliminated, if the recommended subdivision of meshes are adhered to.

Fortunately, the gravity defects do not so much contribute to geoidal undulations as they essentially amount to summations and are in the nature of being self-compensating.

For a non-recursive digital filter, as the present problem is, where the process may be formulated as

$$E\{y\} = a_1x_1 + a_2x_2 + \ldots\ldots\ldots + a_nx_n \tag{5.6}$$

The estimated variance in y is given by

$$\sigma^2\{y\} = (\Sigma a_n^2)\ \sigma^2\{x\}, \tag{5.7}$$

assuming no correlation between the x values.

Using circle-ring zones as elements, when all factors are reduced to $0''.001$, the variance in the deviation of the vertical, for an rms error of 3 milligal up to 50th zone, turns out to be

$$\sigma^2\{\theta\} = 50(\Sigma(0.001\cos\alpha)^2)(3)^2$$

or, error in $\theta \approx 0''.05$ (5.8)

5.5 ON INTEGRATION TECHNIQUES

The surface integrals in the Stokes' and Vening

Meinesz' formulae, are replaced by discrete summation of contributions arising from compartments of finite size obtained by suitably subdividing the surface of the earth. In the template method which is a graphical procedure, a transparent sheet marked with concentric circle and radial lines is placed on the gravity map, its centre coinciding with the computation point on the map. Another technique uses rectangular compartments formed by the grid lines of geographical coordinates ϕ and λ.

The simplicity and flexibility of the template method made it universally acceptable for a long time. Rice(1952) used it to compute the deflections of the vertical at a number of astrogeodetic stations, whilst Uotila (1959) used it to investigate the shape of the earth. The integrals $S(\psi)$ and $V(\psi)$ are computed over each element, thus avoiding the inaccuracy introduced by averaging the functions, as pointed out in Section 4.3. This advantage is however lost because of the inaccuracies that creep in during the estimation of the mean gravity anomaly obtained through the following steps:

(a) plotting of gravity stations on a map and of the gravity anomaly values,

(b) contouring the iso-anomaly lines by inspection or using a graphical method,

(c) estimating the mean simply by experience or through some qualitative methods.

All the above steps are extremely laborious, greatly susceptible to personal skill and wanting in accuracy as compared with that of the weighting functions, which severely limit its application despite the apparent advantages.

Moreover, the gravity anomalies, specially the free-air, are not generally interpolable except in regions with flat topography and homogeneous crustal structure.

The rectangular elements, on the other hand, enable the use of a general digital technique matching modern needs and amenable to modern tools. The mean anomalies can be evaluated using the same principle analogous to the plotting, contouring and estimating sequence, without any further loss of accuracy, and once evaluated, they may be stored on magnetic tapes for varied future uses such as gravity explorations. Smaller sized compartments also reduce the errors of centering the weighting functions. Fischer (1966) has discussed these advantages at length and computed factors for gravimetric interpolation using an electronic computer.

5.6 COMPARTMENT SIZE AND INNER LIMITS

Following the recommendations of the General Conference on Weights and Measures in 1960 (Ramaswamy and Rao, 1971) to use the SI units - "Systeme International d'Unites" -also accepted by India, the unit for plane angle will be radians (rad) and for solid angle steradians(sr). The multiples kilo, mega and submultiples milli, micro will also be used with their usually acceptable meanings. The division of a circle into 360°, a nondecimal division, is also in vogue, though some countries are using grade systems.

Global maps are made using the degree grids whilst local maps use minutes or linear grids. Gravity maps are often made using a $15'$ gridded interval which permit an easy adoption of decimals being equal to $0^\circ.25$. Use of sub-multiple arcseconds is however generally restricted only to very small order terms like the difference in astronomical and geodetic latitudes. Decimal degree compartments are naturally best suited for grid divisions and for practical measurements whereas radians may be used in computer applications, with the recommended submultiple micro(μ deg, μ rad) used to avoid the repetitive exponent notation.

Mather (1970) used $5^\circ, 1^\circ, \frac{1}{2}^\circ$ and 0.1° grids, but the intermediate subdivisions of 1° to $\frac{1}{2}^\circ$ does not seem to have much advantage. Instead, a division sequence like $5^\circ, 1^\circ, 0^\circ.25, 0^\circ.05, 0^\circ.01$ provides a more balanced scheme, the succeeding divisions being always in fractions of 1/5th or 1/4th of the earlier size. Further, the grid sizes of 0.25 and 0.05 degree are equal to whole number in minutes, being 15' and 3' respectively.

The inner limits of various compartments to be used should be chosen such as to produce comparable effcts on the geoidal parameters. For the same anomaly, the product of the weighting function corresponding to the limiting ψ and the area of the compartment used should remain the same, i.e., for computing undulations

$$S(\psi_1)\,\sigma_1 = S(\psi_2)\,\sigma_2 \qquad (5.9)$$

and for the deviation of the vertical,

$$V(\psi_1)\,\sigma_1 = V(\psi_2)\,\sigma_2 \qquad (5.10)$$

where ψ_1, ψ_2 are the inner limits of compartments and σ_1, σ_2 are the areas of respective compartments.

Starting from the values of 15° to 20° for a $5^\circ \times 5^\circ$ block as used earlier, the inner limits for different compartments covering the above recommendations are given in Table 5.3.

TABLE 5.3

RECOMMENDED INNER LIMITS OF VARIOUS MESH SIZES

Compartment size	Inner limit between
5°	15° to 20°
1°	3° to 4°
$0\overset{\circ}{.}25$	$0\overset{\circ}{.}75$ to 1°
$0\overset{\circ}{.}05$	$0\overset{\circ}{.}15$ to $0\overset{\circ}{.}2$
$0\overset{\circ}{.}01$	$0\overset{\circ}{.}03$ to $0\overset{\circ}{.}04$

5.7 FORMULATION FOR LOCAL COORDINATES

The interstation vector L earlier obtained from the unit position vector P through the transformation matrix T cannot be used as such for evaluation in the immediate neighbourhood owing to the following disadvantages:

(a) the first element $\cos\psi$ becomes very nearly equal to 1 and an exact value may not therefore be obtained owing to truncation errors,

(b) the latitudes ϕ_c and ϕ_g being very near to each other, the negative terms in the second element ℓ_2 differ from the positive terms by a very small amount discernible only in the third or fourth place of decimal; the relative error in the deduction process therefore becomes quite high,

(c) a similar effect is found in the third element ℓ_3 as the longitude difference is also small.

The determinations should therefore be based on 'differences' between the coordinates rather than on their individual values.

Accordingly, the basic expressions in Equations 3.1 can be rearranged in a difference relation form. Introducing the trigonometric versed sine notation, i.e.,

$$\text{vers}\,\theta = 1-\cos\theta \;,$$

the resulting modifications are,

$$\ell_1 = \cos\psi = \cos\Delta\phi - \cos\phi_g\cos\phi_c\text{vers}\Delta\lambda, \qquad (5.11)$$

$$\ell_2 = -\sin\psi\cos\alpha = -(\sin\Delta\phi+\cos\phi_g\sin\phi_c\text{vers}\Delta\lambda),$$

$$\ell_3 = -\sin\psi\sin\alpha = -\cos\phi_g\sin\Delta\lambda$$

where, $\Delta\phi = \phi_g - \phi_c$

$$\Delta\lambda = \lambda_g - \lambda_c$$

$$\text{vers}\Delta\lambda = 1-\cos\Delta\lambda$$

ϕ_g, λ_g and ϕ_c, λ_c are respectively the coordinates of the gravity station and the computation station.

5.8 EFFECT OF THE INNERMOST ZONE

If representative mean values are available, numerical summation may be extended up to compartment sizes of 0.01 degree square for a properly planned gravimetric net, except for the compartment containing the computation point. However, an actual situation may not always be so favourable and the innermost zone may range to a few kilometers.

Whilst the computation formulae for determining this effect using the template method is available, some approximate method must be resorted to for data available in a grid form. Techniques for accomplishing this are enumerated below including others proposed anew:

(1) the innermost zone may be defined by a staggered boundary and split up into a mean circular area and a series of positive and negative parts whose effects are approximated and added/subtracted as the case may be (Figure 5.1a),

(2) the zone may be divided to four rectangles surrounding the computation point (Figure 5.1b) and the effects of ξ,η may be computed by suitable formulation (Fischer, 1966). N-effects are however not yet available.

(3) for directly processing the point gravity anomalies, a triangular division scheme may be proposed. A

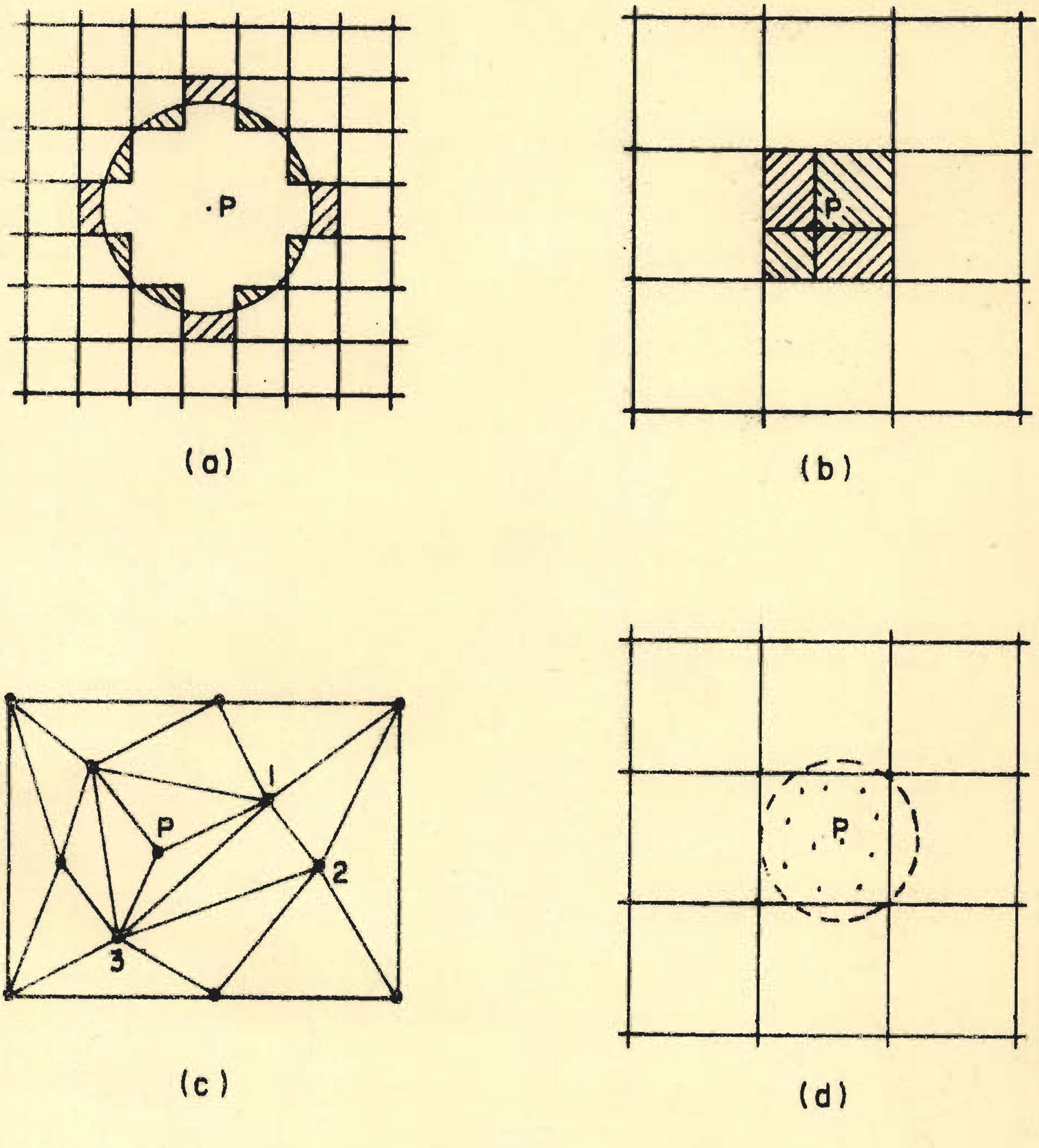


FIG. 5·1 VARIOUS PROPOSALS FOR EVALUATION OF THE EFFECTS OF INNERMOST ZONE chain of triangles is formed (Figure 5.1c) depending upon the distribution of gravity data. The following simplifications are made for any element,

$$\bar{S}(\psi) \approx 1,$$
$$\bar{U}(\psi) \approx 0.25 \qquad (5.12)$$
$$\ell_1 \approx 1$$
$$\ell_2 \approx -\Delta\phi = -(\phi_g - \phi_c) \text{ radian}$$
$$\ell_3 \approx -\cos\phi_c(\lambda_g - \lambda_c) \text{ radian}$$
$$x \approx \frac{1}{2}\sqrt{\ell_2^2 + \ell_3^2}$$
$$\sigma \approx \frac{\cos\phi_c\{\phi_1(\lambda_2-\lambda_3) + \phi_2(\lambda_3-\lambda_1) + \phi_3(\lambda_1-\lambda_2)\}}{8\pi}$$
$$\bar{\Delta}g \approx \frac{\Delta g_1 + \Delta g_2 + g_3}{3}$$
$$\phi_g \approx \frac{\phi_1 + \phi_2 + \phi_3}{3}$$
$$\lambda_g \approx \frac{\lambda_1 + \lambda_2 + \lambda_3}{3}$$

(4) another practical approach is to replace the innermost rectangular zone by a circle of the same area (Figure 5.1d) and apply the original formulae (Heiskanen and Moritz, 1967):

$$N_i = \frac{s_o}{G} \Delta g_p$$

$$\xi_i = -\frac{s_o}{2G} g_x \qquad (5.13)$$

$$\eta_i = -\frac{s_o}{2G} g_y$$

In the proposed modification, then,

$$\pi s_o^{\,2} \approx R^2 \times 2 \times \sin\frac{D\phi}{2} \times \frac{\pi \times D\lambda}{180} \times \cos\phi_c$$

$$g_x = \frac{(\Delta g_N - \Delta g_S)180}{\pi R D\phi} \qquad (5.14)$$

$$g_y = \frac{(\Delta g_E - \Delta g_W)180}{\pi R D\lambda \cos\phi_c}$$

finally reducing to

$$\frac{N_i}{R} = 2\sqrt{\sigma}\ \frac{\Delta g_p}{G} \qquad (5.15)$$

$$\xi_i = -\sqrt{\sigma}\ \frac{180}{\pi}\ \bar{g}_\phi$$

$$\eta_i = -\sqrt{\sigma}\ \frac{180}{\pi}\ \bar{g}_\lambda \sec\phi_c$$

where, $\sigma = \frac{D\lambda}{360} \cos\phi_c \sin\frac{D\phi}{2}$

$\bar{g}_\phi$ = slope of $\frac{\Delta g}{G}$ in ϕ direction, per degree

$\bar{g}_\lambda$ = slope of $\frac{\Delta g}{G}$ in λ direction, per degree

The value Δg_p and the slopes may be obtained by a simple surface fitting of the form,

$$\frac{\Delta g}{G} = A + B\phi + C\lambda + D\phi^2 + E\phi\lambda + F\lambda^2 \qquad (5.16)$$

where ϕ,λ are expressed in degrees;

then,

$$\frac{\Delta g_p}{G} = A + B\phi_c + C\lambda_c + D\phi_c^{\,2} + E\phi_c\lambda_c + F\lambda_c^{\,2}$$

$$\bar{g}_\phi = B + 2D\phi_c + E\lambda_c \qquad (5.17)$$

$$\bar{g}_\lambda = C + E\phi_c + 2F\lambda_c$$

A minimum of six observations should be available in the innermost zone; otherwise, a few outside points should be taken for the matching.

The first method and the third proposal are not fully automatic, owing to difficulties in defining the zigzag boundary and forming the triangles respectively. The second method is the optimum one, but the fourth proposal has been used in the present work, being readily acceptable to computer programming, although it is liable to be inaccurate if the inner zone deviates from a square shape.

5.9 DATUM SHIFT RELATIONS

The conventional datum shift formulation of Equations 5.1 which stems from first principles is unfortunately inconvenient for automatic computation where repetitive use is made of similar quantities. A matrix formulation is therefore adopted.

First the equations are split into two components: the first involving terms related to the corrections in the deviation components and of the undulation, and the second involving changes in the dimensions of the reference ellipsoid. Then, denoting the correction vector at any point as:

$$C = \begin{bmatrix} \delta N/R \\ \delta \xi \\ \delta \eta \end{bmatrix} = C_u + C_e \, , \qquad (5.18)$$

The resulting expressions, after a long sequence of mathematical manipulations involving trigonometric identities not elaborated here in order to retain continuity of purpose, are finally obtained as follows:

$$C_u = T \, T_o' \, C_o$$

and

$$C_e = (T \, B_o + E) D \qquad (5.19)$$

where, the matrices are

$$T = \begin{bmatrix} -\sin\phi & \cos\phi\cos\lambda & \cos\phi\sin\lambda \\ \cos\phi & \sin\phi\cos\lambda & \sin\phi\sin\lambda \\ 0 & \sin\lambda & -\cos\lambda \end{bmatrix}$$

$$B_o = \begin{bmatrix} -\sin\phi_o & 2\sin\phi_o - \sin^3\phi_o \\ \cos\phi_o\cos\lambda_o & \sin^2\phi_o\cos\phi_o\cos\lambda_o \\ \cos\phi_o\sin\lambda_o & \sin^2\phi_o\cos\phi_o\sin\lambda_o \end{bmatrix}$$

$$T_o = \begin{bmatrix} -\sin\phi_o & \cos\phi_o\cos\lambda_o & \cos\phi_o\sin\lambda_o \\ \cos\phi_o & \sin\phi_o\cos\lambda_o & \sin\phi_o\sin\lambda_o \\ 0 & \sin\lambda_o & -\cos\lambda_o \end{bmatrix}$$

$$E = \begin{bmatrix} -1 & \sin^2\phi \\ 0 & -\sin 2\phi \\ 0 & 0 \end{bmatrix}$$

and, the vectors are

$$C_o = \begin{bmatrix} \delta N_o/R \\ \delta\xi_o \\ \delta\eta_o \end{bmatrix} , \quad D = \begin{bmatrix} \delta a/R \\ \delta f \end{bmatrix}$$

It can be further shown that,

$$T_o' E_o = -B_o$$

substitution of which in Expression (5.18) yields

$$C = T\,T_o'\,C_o + (T(-T_o'\,E_o)+E)D$$

or $$C = TX + ED \qquad (5.20)$$

where, $$X = T_o'\,(C_o-E_oD)$$

The newly evolved vector X may be termed as the shift vector. The superscript ' indicating the matrix transpose as usual, whilst the subscript $_o$ refers to the origin. The square matrices T and T_o are seen to be orthogonal and are identified with the transformation matrix introduced in Chapter III.

5.10 THE INVARIANT SHIFT VECTOR

The transformation matrix is recalled to be an orthogonal matrix satisfying the identity,

$$T\,T' = T'\,T = I, \qquad (5.21)$$

where I indicates a unit matrix.

The use of this relationship leads from Equation 5.20 to

$$T'C = T'TX + T'ED = IX + T'ED$$

or

$$X = T'(C - ED)$$

$$= T_n'\,(C_n - E_nD) = T_o'\,(C_o - E_oD), \qquad (5.22)$$

which is a generalized expression introducing X as an invariant vector, an outcome of astro-geo-gravimetric corrections at a general station n. The final quantities sought for the definition of the datum in absolute terms, are then the elements of this shift vector X in relation to the geocentre .

5.11 CHOICE OF STATIONS

A single solution of the orientation vector obtained at the initial point, though theoretically adequate, may be seriously affected by the inadequacies or inaccuracies of the gravity field determinations. On the other hand, control stations away from the network will have less reliability on the geodetic parameters due to the systematic errors in measurements and computations.

The Indian subcontinent has some peculiar features which must be taken into account for the selection of computation stations through which shift vectors have to be determined. It has a diamond shaped boundary surrounded by an ocean on two sides and its topography ranges from medium altitudes to the steepest peaks in the Himalayan region. The choice of the initial point naturally falls over a centrally located flat region. The triangulation net extends from this control point to all four directions along meridians and parallels, with necessary extensions of survey tributaries to cover the vast land.

The astronomical stations also conform to this pattern, having two main orthogonal profiles.

The astrogeodetic stations are chosen to be at the ends of the profiles, but not too near the oceanic zone nor to the rugged regions. This is done to ensure reliability of their determinations and similarity in the order of their systematic errors. Five computation points were selected keeping in view the scope of investigations and the availability of data.

Table 5.4 describes the relevant basic details and Figure 5.2 indicates their approximate positions. The astrogeodetic N values had to be reduced from the International Spheroid system to the Everest Spheroid system with the orientation components given in Expression 5.4.

TABLE 5.4

ASTROGEODETIC QUANTITIES OF THE COMPUTATION STATIONS

Position	Code	ϕ_{pe}	λ_{pe}	N_{ag}/R	ξ_{ag}	η_{ag}	
Central	0	24.119794	77.654880	+0.00	-1.41	+14.01	pe = published Everest coordinates
North	1	28.734581	77.648975	+0.36	-23.75	+31.05	
East	2	23.395269	86.986980	+2.23	+ 0.98	-34.42	
South	3	17.400630	78.556258	-1.62	-24.19	-5.67	
West	4	24.258153	72.184680	+1.29	-39.75	-1.94	
	unit	degrees		ppm or micro rad.			

FIG. 5·2 APPROXIMATE POSITIONS OF COMPUTATION POINTS

5.12 THE DATA SET

The data set gleaned from various sources in the country and supplemented by actual measurements especially made for this purpose can be classified as follows with respect to their information content:

(i) data set comprising of coordinates and gravity anomalies in respect of some points,

(ii) data set comprising of coordinates, heights, gravity values and anomalies in respect of some points,

(iii) data set comprising of station distribution maps, heights and relative gravity values referred to various arbitrary gravity datum.

The available data mainly consisted of gravity values measured by Worden gravimeters supported with altimeter heights or those measured by Lacoste Romberg gravimeters and supported by levelling heights. Additional field work was undertaken with a view to filling up broad gaps in the gravity anomaly data as well as to checking up and standardizing existing data. The gravimeters used for this purpose were (i) the Worden gravimeter (geophysical model) and (ii) the La-coste Romberg gravimeter. Heights were estimated from topographic maps with occasional checks from nearby Bench Marks. Usual drift corrections were made assuming linearity over time.

Compilation of the above data sets into a common format, for storage on computer cards needed various modification such as those given below:

(a) digitization of station position, from plane table sheets or printed maps, to decimal degrees of latitude and longitude,

(b) reduction of all latitudes and longitudes in decimal degree up to the nearest 0.0001 degree,

(c) conversion of heights from feet to metres upto the nearest 0.1 metre,

(d) updating of the relative gravity values to absolute ones by comparison with standard stations,

(e) reduction of all gravity values to IGSN 71 system, by applying the usual Potsdam correction wherever needed,

(f) computation of anomalies by comparing IGSN 71 gravity values with γ_{67}, duly reduced for the appropriate free-air effect.

Whenever gravity values as well as gravity anomalies, were both available, the basic gravity values were converted to free-air anomalies and compared with the corresponding anomalies duly corrected (vide Appendix A). In case where discrepancies were found to exceed 1 to 2 milligals, the source records and punched cards were scrutinized for detection of any human errors and rejected if the differences still persisted. In cases where the discrepancies were within the limit, the derived anomalies were retained for further use.

Table 5.5 shows the area covered by each zone and the total number of gravity stations finally used.

TABLE 5.5

GRAVITY STATION DISTRIBUTION IN VARIOUS ZONES

Zone	Latitude Limits		Longitude Limits		No. of stations	Remarks
Central	23.00	25.00	76.00	79.00	1224	Well-distributed, modified free-air
North	28.00	30.00	76.00	79.00	188	No dense net
East	23.00	24.00	86.00	88.00	392	
South	17.25	17.75	78.25	78.75	135	Limited zone
West	24.00	24.50	72.00	72.50	833	Mostly confined to a very limited area only

5.13 EVALUATION OF THE MEAN ANOMALIES

The mean value over a compartment obtained from the point anomalies computed from field observations of gravity and altitude data, are bound to be functions of the density and distribution of points. In case large coverage is available, a simple arithmetic mean as given below will be adequate:

$$E\{\Delta g_m\} = \frac{\sum_{i=1}^{n} \Delta g_i}{n} \tag{5.23}$$

The major consideration for the adoption of a simple mean is the sample size n. Mather (1970) suggested a lower bound equal to 0.5 N where N is the number of stations in a fully represented compartment. For example, in a 0.25 degree compartment usually a 5 KM x 5 KM station grid is usually sufficient if the region is more or less flat, in which case N will equal 25.

In case the sample size is not satisfactory, some predicted values Δg_j could be generated using correlation techniques, and the mean obtained as

$$E\{\Delta g_m\} = \frac{\sum_{i=1}^{n} \Delta g_i + \sum_{j=n+1}^{N} E\{\Delta g_j\}}{N} \tag{5.24}$$

Prediction technique may be broadly classified into two ,

(i) heterogenous correlation

(ii) autocovariance prediction.

The first of these require a detailed knowledge of the topography with heights measured accurately. For, an error of 1 metre in assessing the heights **will lead** to inaccuracy of 0.3 milligal which will, in turn, affect the correlation factor and the final prediction. The other type (Rapp, 1964) which is based on the interdependance of gravity anomalies with distance will also cause similar cumulative errors due to the inaccuracies of observation. In both these cases, therefore, the reliability of the basic input data is of prime importance. They may therefore be used only if noise is small as compared with the signal.

Another way is to use a polynomial expansion of the anomalies which is quite analogous to the graphical estimation technique (Nagy, 1973). Expanding the anomalies as a function of position,

$$\Delta g(x,y) = A_0 + A_1 x + A_2 y + A_3 x^2 + A_4 xy + A_5 y^2 , \tag{5.25}$$

one obtains for the estimated mean,

$$\overline{\Delta g}_m = \frac{\iint \Delta g \, dx \, dy}{\iint dx \, dy} , \tag{5.26}$$

where the integration is carried out over the compartment.

With the restricted data source and the nature of their variances, the covariance and correlation prediction techniques were ruled out. Wherever the number of stations within a 0.°25 compartment exceeds ten, the arithmetic average was used, whilst in cases where the number was nearer ten, a surface fitting up to the third order was used with the following modifications in Nagy's procedure:

$$\text{(i)} \qquad x_1 = \frac{\phi_i - \phi_m}{D\phi}$$

$$\text{(ii)} \qquad y_i = \frac{\lambda_i - \lambda_m}{D\lambda} \qquad (5.27)$$

$$\text{(iii)} \qquad \Delta g^* = \Sigma \Delta g_i / n$$

$$\text{(iv)} \qquad f_i = \Delta g_i - \Delta g^* = A_o + A_1 x_i + A_2 y_i + \ldots.$$

$$\text{(v)} \qquad \overline{\Delta g_m} = \Delta g^* + \frac{\iint f_i \, dx \, dy}{\int_{-1/2}^{1/2} \int_{-1/2}^{1/2} dx \, dy}$$

where, ϕ_m, λ_m are the coordinates of the centre of the compartment.

This standardization renders the design matrix well-conditioned as the off-diagonal elements are reduced to very nearly zero, specially so when the distribution is of a symmetrical type.

In the case of 0.05 degree square elements however, the situation is usually quite different. Some elements have only one or two gravity stations whereas some others are completely devoid of any data. A compromise scheme had therefore to be sought between the accuracy desired and the economy of computations, the various steps of which are described below :

(a) when the number of observations in an element was more than two, a simple average was taken, the standard deviation being usually within 1 milligal,

(b) when number of observations was two or less, the adjacent elements were also considered and a weighted mean was obtained giving full weight to the values in the particular element and a linearly reducing weightage in the adjacent ones. Figure 5.3 represents this decaying type weighting scheme graphically. This window was used to utilize surrounding values as and when needed, to circumvent defects associated with sudden truncation, and it is simple so far as computer logic is concerned,

(c) when no observations existed in an element but three or more points existed in the adjoining elements, the same weightage scheme was applied thus using the outside values for interpolation,

(d) when none of the above mentioned conditions were satisfied the element mean was not computed, and predicted later on.

The remainder of the excercise was of a statistical nature. The covariance function was evaluated from the mean values for all the compartments. For the interpolation of missing values, neighbouring values were collected up to a distance which corresponded to that

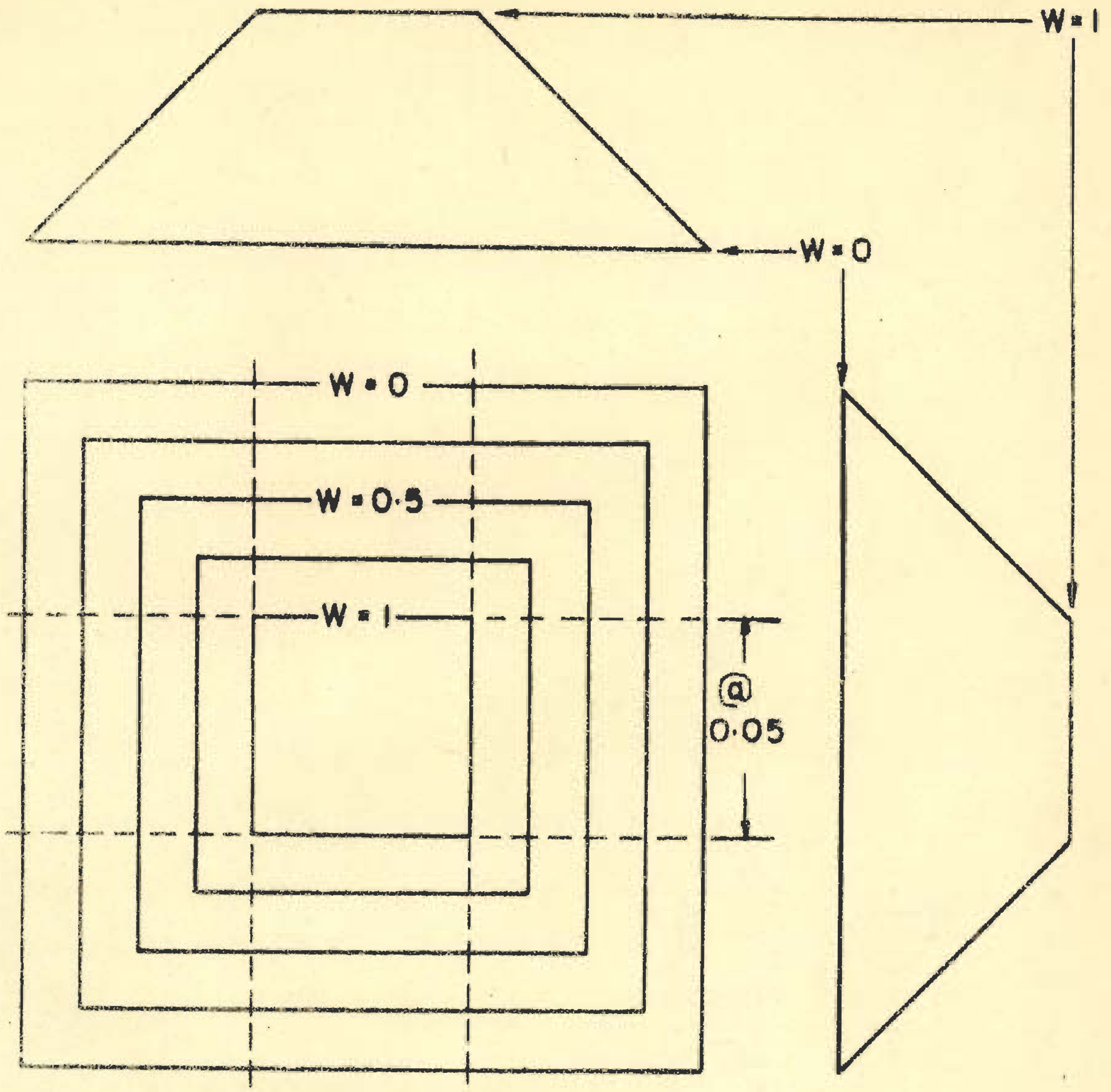


FIG. 5.3 WEIGHTING SCHEME FOR MEAN ANOMALIES
OVER 0.05 DEGREE ELEMENTS

at which the covariance function crossed over to the negative side, except when the number of such source values become less than three in which case the influence zone was extended to at least three sources.

Figure 5.4 explains the coverage made for various compartment sizes. Figure 5.5 and 5.6 show two typical covariograms. Table 5.6 presents some 0.25 degree mean anomalies obtained in the various zones considered.

5.14 RESULTS OF COMPUTATION

A Fortran IV computer programme has been developed to compute the orientation vector at the initial point. This is accomplished by first calculating the gravimetric geoidal parameters at any general astrogeodetic station using gravity mean values starting from five-degree blocks in the exterior region to one-degree unit and smaller compartments in the inner zone, thereafter the invariant shift vector as well as the absolute orientation parameters at the initial point. Various parts of the programme have been discussed in Appendix F.

The final gravimetric values of the geoidal undulations and deviation components are presented in Table 5.7.

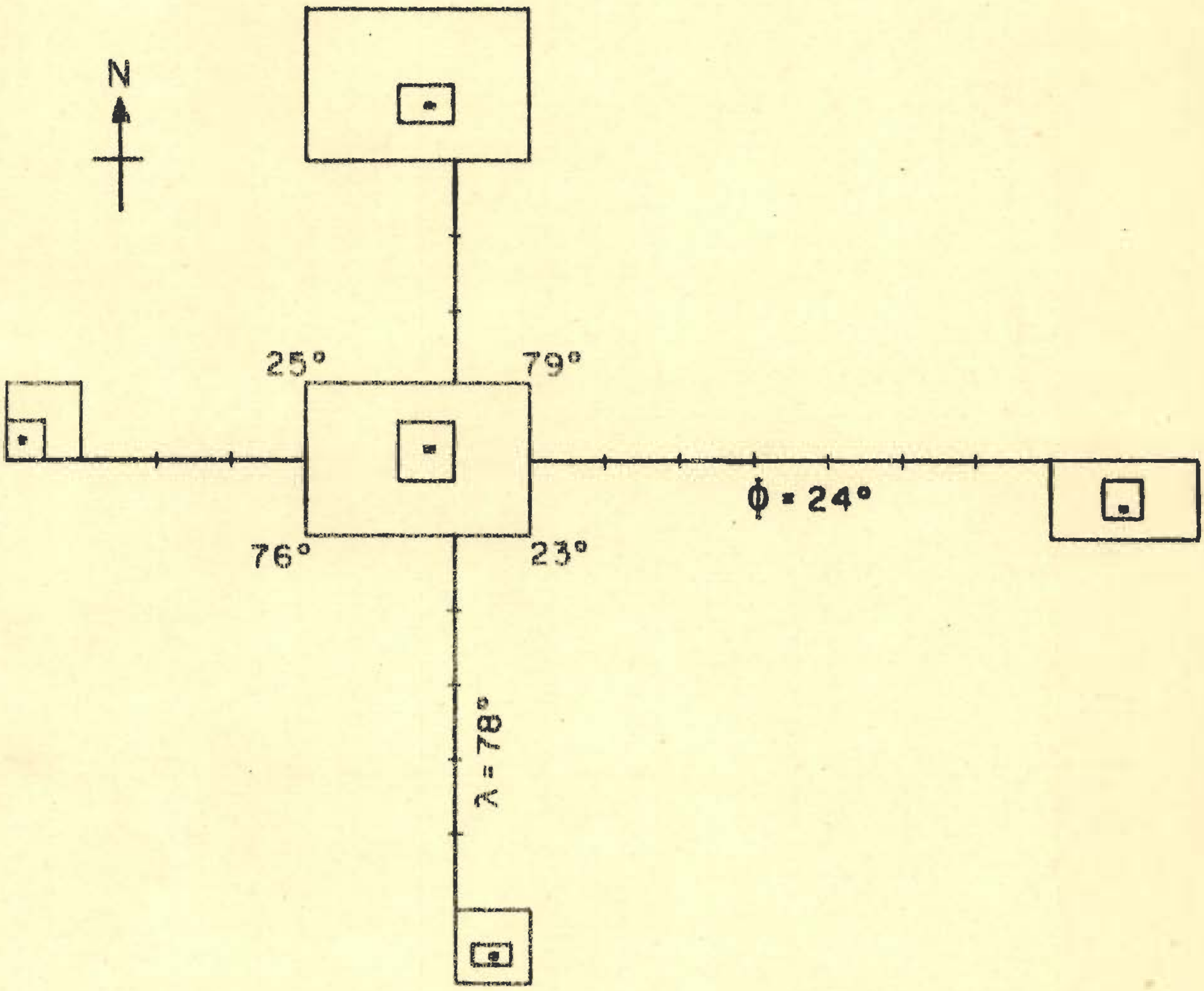


FIG. 5·4 LIMITS OF VARIOUS ELEMENT MESHES AROUND THE COMPUTATION POINTS

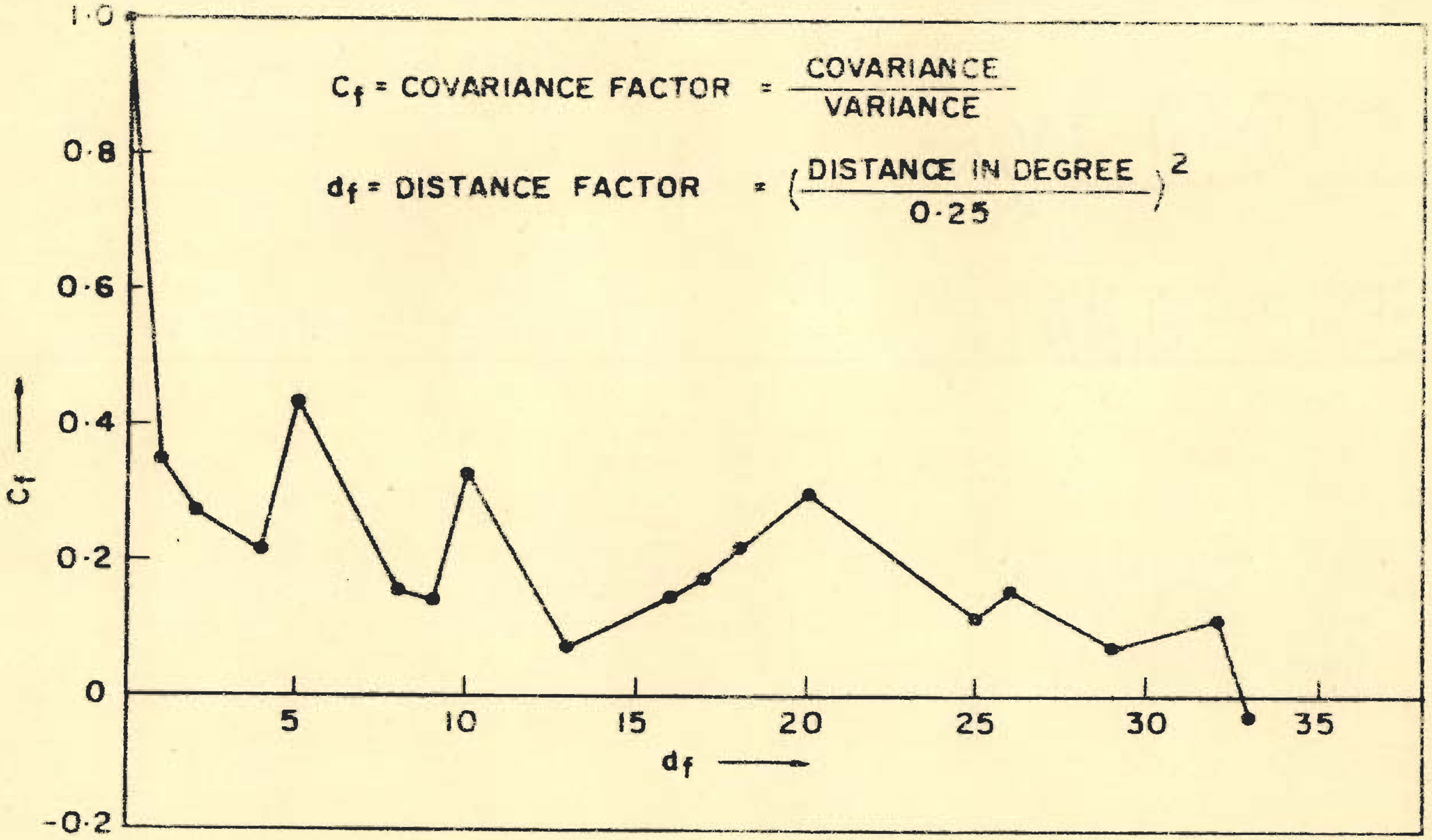


FIG. 5·5 COVARIOGRAM OF 0·25 DEGREE MEAN ANOMALIES IN NORTH ZONE

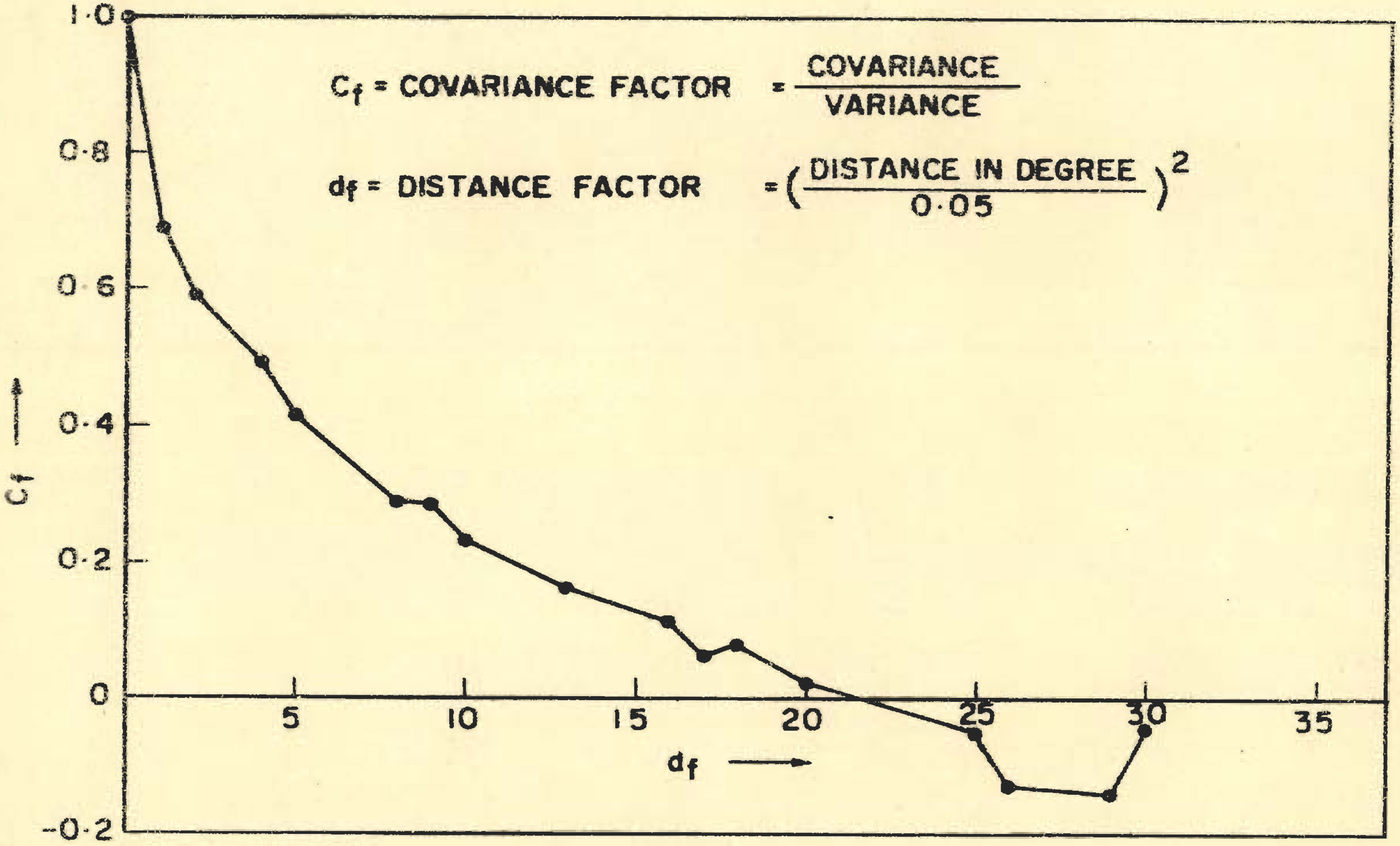


FIG. 5·6 COVARIOGRAM OF 0·05 DEGREE MEAN ANOMALIES IN WEST ZONE

TABLE 5.6

SOME QUARTER-DEGREE MEAN GRAVITY ANOMALIES WITHIN INDIAN REGION

Boundaries of Latitude		Boundaries of Longitude		Anomalies in milligal							
30.00	29.75	76.00	78.00	-26	-29	-49	-57	-69	-81	-93	-111
29.75	29.50	76.00	78.00	-23	-25	-34	-43	-51	-60	-69	-83
29.50	29.25	76.00	78.00	-20	-18	-25	-34	-43	-52	-69	-76
29.25	29.00	76.00	78.00	-14	-13	-21	-27	-29	-37	-44	-61
29.00	28.75	76.00	78.00	-10	-7	-20	-24	-27	-31	-34	-46
28.75	28.50	76.00	78.00	-8	-8	-18	-30	-22	-44	-35	-31
24.50	24.25	72.00	72.50	30	36						
24.25	24.00	72.00	72.50	11	40						
25.00	24.75	76.75	78.50	8	8	9	4	-4	-10	-17	
24.75	24.50	76.75	78.50	1	6	11	2	-5	-2	0	
24.50	24.25	76.75	78.50	3	9	20	16	7	4	8	
24.25	24.00	76.75	78.50	20	11	24	19	15	12	16	
24.00	23.75	76.75	78.50	30	21	21	15	6	8	14	
23.75	23.50	76.75	78.50	32	25	17	10	4	10	18	
23.50	23.25	76.75	78.50	22	25	17	4	6	10	7	
24.00	23.75	86.00	88.00	-1	-2	4	6	26	37	23	-19
23.75	23.50	86.00	88.00	5	-7	15	12	7	16	15	-31
23.50	23.25	86.00	88.00	11	13	14	13	20	12	2	-31
23.25	23.00	86.00	88.00	17	16	12	13	12	3	-13	-35
17.75	17.50	78.25	78.75	-6	-4						
17.50	17.25	78.25	78.75	-8	-12						
17.25	17.00	78.25	78.75	-2	-13						

TABLE 5.7

GRAVIMETRIC GEOID PARAMETERS AT VARIOUS COMPUTATION STATIONS

Code	0	1	2	3	4	unit
N_{gr}	-59.00	-61.86	-62.05	-75.94	-51.86	metre
ξ_{gr}	+ 0.36	- 3.32	- 2.51	- 5.51	- 7.71	arcsec
η_{gr}	+ 5.49	+ 8.26	- 1.14	+ 1.47	- 1.12	arcsec

The elements of the correction vector at any station are computed using the relation.

$$C = \begin{bmatrix} N_{gr}/R \\ \xi_{gr} \\ \eta_{gr} \end{bmatrix} - \begin{bmatrix} N_{ag}/R \\ \xi_{ag} \\ \eta_{ag} \end{bmatrix} \tag{5.28}$$

Table 5.8 shows the intermediate C vector, the corresponding X vector and finally the orientation parameters C_o, obtained from various computation points.

5.15 SUMMARY AND DISCUSSION

Gravimetric determination of the geoidal undulation and the deviations of the vertical at any point, using finer mesh sizes in the immediate neighbourhood,

TABLE 5.8

ABSOLUTE ORIENTATION PARAMETERS THROUGH VARIOUS ASTRO-GEODETIC STATIONS

Point code	0	1	2	3	4	Unit
c_1	-9.26	-10.07	-11.97	-10.30	-9.43	ppm
c_2	3.15	7.63	-13.17	-2.51	2.35	micro rad.
c_3	12.61	9.01	28.87	12.79	-3.51	micro rad.
x_1	-27.24	-29.23	-40.17	-23.46	-28.08	ppm
x_2	38.09	34.28	34.71	36.50	33.36	ppm
x_3	115.04	114.42	110.26	115.86	115.28	ppm
δN_o	-59.00	-62.05	-56.67	-66.14	-61.31	metre
$\delta \xi_o$	+0.65	+ 0.15	-2.24	+1.40	+0.43	arcsec
$\delta \eta_o$	+2.60	+1.86	+2.13	+2.25	+1.64	arcsec

has been discussed in this chapter. If the point happens to be occupied by an astrogeodetic station also, the corrections required to be applied to the astro-geodetic parameters, may be computed. The corresponding corrections at the initial point can then be calculated using the orientation formula suitably rearranged. A few numerical examples have been worked out using the results obtained in the previous chapters. Various sources of errors in the gravity data and their effect on the determination of undulation of the geoid and deviations of the vertical, are briefly discussed. The grid method has been preferred to the graphical technique, and decimal degree compartments used in order to conform with the SI system of units. Consistent inner limits for the size of various compartments have also been discussed.

The transformation of conventional geographical coordinates of any gravity station to local coordinates using the computation station as the pole, is formulated in a differential form. A few proposals for evaluating the effect of the innermost zone have also been included. The datum shift relation is formulated in a generalized form starting with the introduction of a shift vector whose elements are independent of the coordinates of the computation station as well as those of the initial point.

A brief discussion concerning the selection of the astro-geodetic station, is included and five stations have been chosen on the Indian continent along two profiles, intersecting at the origin. The sources and qualities of the data set used in this investigation are of diverse forms, having been collected for different purposes, e.g., explorations etc. which were further supplemented by additional field measurements made. For converting the distributed point gravity anomalies to representative mean values, various practical methods including a sort of weighted average have been chosen. However, in respect of compartments where no mean values were available, the covariance interpolation was used. Finally the results of computation have been presented as sets of the shift vector as well as in the conventional form by δN_o, $\delta \xi_o$, $\delta \eta_o$ values.

The results from various computation stations show that the values of δN_o are more or less consistent among themselves whereas those of the slope components are found to be highly discrepant. A standard deviation of 3 metre in δN_o may be comparable to the similar closure error in the astro-geodetic geoid. In $\delta \xi_o$, the standard deviation works out to be $1''.2$, which is rather high. In $\delta\eta_o$, the deviation is $0''.3$ around the mean, but the values from all other stations are lower than that obtained from the initial point.

Out of all the results, the values obtained from the initial point is obviously the best estimate. There is no systematic error as the network starts from the centrally placed initial point itself. The astronomic determinations are also perfect to the extent possible. The gravity data distribution compared to that at any other station, is also by far the best, leading to as accurate gravimetric results as practicable . The circle-ring method was also applied up to about 500 KM radius and the results (Bhattacharji and Ray,1978) are in satisfactory agreement with those obtained from the grid method applied here. The anomalies used are modified free-air (Bhattacharji, 1971), i.e., corrected for regional terrain effect, and thus provide better simulation of the Stokesian boundary-value problem.

The other determinations are not so satisfactory, basically due to want of necessary density of gravity data around them. The local determinations are more susceptible to altitude and standardization errors. For example, the data in the Western zone has been updated by comparison with a single pendulum station. Moreover, a North-South flowing river separated the two sets of observations. There being no common station between these two groups, the anomalies were matched along the two sides of the river in an arbitrary manner, which could be considerably in error due to possible sharp changes in density along the river-bed separating the two sets. A somewhat similar situation prevailed in the East station also. For the inadequacy of gravity data in the North zone, the mean anomalies over 0.05 degree compartments had to be taken as that over 0.25 degree compartment. In no case was the inner limit suggested in Table 5.3 achieved.

Even if the gravimetric determinations are made accurately, the results from these stations could still differ from the real values because of the unknown systematic errors in the triangulation network. Whilst the gravimetric and astronomic determinations could be of comparable accuracy, the geodetic coordinates have to be burdened with errors propagated along the cantilever extension. The South station is a temporarily occupied one and not a triangulation station. The West point is the only Laplace station used in this study where a discrepancy of 1'' was found (Gulatee, 1955).

For obtaining a better revised set from a general astro-geodetic station, the following steps are recommended:

(i) computation points be chosen to coincide with Laplace stations,

(ii) gravity values used as reference should be standardized,

(iii) gravity data in the inner zone say $3^{\circ} \times 3^{\circ}$ be measured accurately alongwith precise altitude values of the station, Notwithstanding these shortcomings, the present determinations from various computation points have indicated a first-hand check on the numerical computations, which can be utilized for systematic planning in future.

Computations from several stations have earlier been recommended by geodesist for obtaining a reliable estimate. Rice (1952) used 16 stations and Mather (1970) chosen 38 stations, with suitable gravity coverages in the immediate neighbourhoods. An attempt to incorporate a somewhat similar logic has been subsequently discussed in the following chapter.

CHAPTER VI

A LEAST-SQUARES COINCIDENCE APPROACH TO ABSOLUTE ORIENTATION

6.1 GENERAL

Theoretically, a complete knowledge of the gravity anomaly field over the entire earth is a prerequisite for obtaining the absolute orientation of a local geodetic network using the classical gravimetric method. This being an unattainable condition, precise determinations of the absolute geoid and its orientation parameters basically constitute an ill-posed problem in geodesy, calling for careful processing and interpretation of all available data. The techniques and procedures designed to accomplish this task as well as the results obtained in respect of the Indian geodetic system have been discussed in earlier chapters.

The basic guidelines for selecting an appropriate computation station for the determination of the absolute orientation vector are as follows :

(i) the point should be a first-order astro-geodetic station, and

(ii) that it should be surrounded by a reasonably dense gravity network.

The numerical examples cited in Section 5.3 as well as the final results obtained from the various computation stations described later, highlighted the defects arising from the limitations and inaccuracies of astrogeodetic and gravimetric data around a station. However, this could be circumvented if, a number of astrogeodetic stations and regional gravimetric data are available. The use of this proposition forms the subject matter of the present chapter, illustrated with numerical examples to obtain a set of orientation parameters for the Indian geodetic system.

6.2 SOURCES OF ERROR IN ASTRO-GEODETIC DATA

Although the gravimetric orientation through an astro-geodetic station appears perfect theoretically, the solution obtained is never absolutely correct in a mathematical sense, but can only be regarded as an estimate whose reliability depends upon those of the input data and of the various linkages of the overall numerical procedure. Sources of error in gravimetric data have already been discussed in Chapter V. Those in astro-geodetic data can be broadly classified into two categories, viz.,(a) errors associated with astronomic observation and reduction processes and (b) errors in geodetic determination of coordinates.

The basic sources of errors in astronomic determinations as follows :

(i) instrumental and observational errors,

(ii) effect of polar migration,

(iii) use of various star catalogues,

(iv) reduction of observations to the geoid.

Depending upon the instruments used, the observational procedures followed and the human skill deployed, errors in measurements may range from 0".01 for zero-order determination up to even a few seconds. Whereas instruments are liable to produce cumulative errors, various astrofix method are designed to compensate for or minimize them in order to achieve a final reliable result. For the particular purpose of orientation, the astro-station should be of first order, to a precision of 0".05 or higher. Whilst latitudes may be measured with greater accuracy, the longitudes are liable to further errors due to an additional measurement of time, which fortunately has been considerably improved in recent years following the use of wireless signals.

The inaccuracy arising from the slight wandering of the terrestrial pole affects the basic geocentric coordinate system as a whole. This variation is quite significant, and appropriate corrections need be applied to astronomic coordinates including the azimuth.

Another important celestial feature shows up in the star almanac published at various times. Various catalogues have been used for the calculation of astronomic positions, and systematic variations are found to occur between them. As the Indian net includes some century old observations, errors of $0\overset{''}{.}2$ or more may be expected if they are all reduced to the present FK_4 system.

For the purpose of computing geoidal coordinates, observed values must be reduced to the hypothetical mean sea-level below the station. The shape and density distribution of the surrounding topography controls the deviation of the slope of the geop on the earth's surface from that of the geoid below. The normal amount of error, in the Indian zone, is of the order of $0\overset{''}{.}15$ per kilometre of elevation. Though the correction is customarily made, it is computed on the assumption of a 'regularized earth' possessing rotational symmetry and hence applied only to latitudes. Any asymmetry in terrain, however, will have its effect on longitude also, which may be estimated from the gravity data around the station.

The great triangulation survey conducted by the Survey of India was a major step towards producing consistency in the national geodetic grid. However, for such a large subcontinent as India, propagation of inaccuracies due to centilever extension assumes considerable significance. The major inhomogenities in the Indian triangulation system are as follows:

(i) a scale-error due to adoption of various foot - metre ratios at various times; this aspect has been investigated by Bhattacharji (1961),

(ii) lack of azimuth control due to inadequancy of Laplace stations as reported by Gulatee(1955).

Two other errors that generally creep in a geodetic network computation are discussed below:

(i) Reduction of a base-line and other observations to the corresponding ellipsoid of computation needs an a-priori knowledge of the geoid-spheroid separation below the base-line. But these heights are only known after all the astro-geodetic deviations are made available. Thus a recomputation is necessary to correct the whole net after constructing the preliminary geoidal chart.

(ii) Adjustment of closure errors are based on some theoretical assumptions, e.g. equal shift, linear propagation, or least-squares-error. The departures from reality of these assumptions, give rise to inaccuracies at places away from check bases.

These limitations underline the need for a careful consideration of various factors involved if a general astro-geodetic orientation is to be attempted. For a more reliable and stable orientation, it is always preferable to choose the initial point, where repeated astronomic observations and dense network of gravity station may be made available.

6.3 THE LEAST-SQUARES APPROACH

From the above discussions of the various errors and their estimates, it should be clear that an orientation point should be so chosen as to be a supercontrol point at which all measurements of the astronomic, geodetic and gravimetric quantities and their computations are made with utmost care, calling for high skill, best quality instruments and comparable software. However, for a country like India, time and economy are as much of the essence and whilst precision and refinement should be continually improved, a practical solution must be found in the context of available informations with all their inherent errors.

As the errors are largely of a random nature, a reliable assessment can be made by assuming that they constitute a gaussian distribution. Accordingly, one can use the logic of least-squares for minimizing their effect provided that the sample size is well distributed. There are a number of astro-geodetic stations and gravimetric geoidal values determined from mean anomalies over one-degree units. The orientation parameters may thus be selected by framing normal equations so that the sum of the discrepancies between astro-geodetic and gravimetric values is constrained to be a minimum.

Either of two conditions stated below may be fulfilled :

(i) condition of coincidence,

(ii) condition of parallelism.

The coincidence condition assumes that the sum of the squares of the differences between $N_{g\gamma}$ and N_{ag} at suitable points will be least, physically meaning thereby that the two surfaces are in an average sense coincident to each other.

The other condition requires that the two surfaces to be as closely parallel to each other as possible, rather, the total non-parallelism over the region, numerically represented by the sum of the departures of their slope components at various points be constrained to a minimum. The slope vector is conventionally expressed in terms of its components ξ and η in two orthogonal directions. The parallelism condition should thus be obtained by constraining the quantity $\Sigma((\delta\xi)^2 + (\delta\eta)^2)$ to be a minimum.

6.4 FORMULATIONS FOR LEAST-SQUARES ORIENTATION

The general datum shift relations for any station n may be written as follows :

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{n,o} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}_o = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}_n \qquad (6.1)$$

The elements a_{ij} are complex trigonometrical functions of ϕ_n, λ_n, ϕ_o, λ_o and can be obtained by rearranging the orientation formulae (Equations 5.1). The elements of C are the conventional parameters $\delta N_o/R$, $\delta \xi_o$, $\delta \eta_o$ respectively. Elements b_i are the differences between the gravimetric and astrogeodetic geoidal parameters at station n combined with the effect of change in the dimensions of the spheroids.

Using the first row of the matrix B for various stations, coincidence condition equations can be obtained as follows:

$$a_{11(u)} c_{1(o)} + a_{12(u)} c_{2(o)} + a_{13(u)} c_{3(o)} = b_{1(u)},$$

$$u = 1,2,\ldots,U \qquad (6.2)$$

where, U is the total number of undulation stations. If $U > 3$, a normalization will be needed to satisfy the least-squares fitting.

Similarly, for the parallelism condition,

$$a_{21(m)}c_{1(o)} + a_{22(m)}c_{2(o)} + a_{23(m)}c_{3(o)} = b_{2(m)},$$

$$m = 1,2,\ldots,M \qquad (6.3)$$

and

$$a_{31(p)}c_{1(o)} + a_{32(p)}c_{2(o)} + a_{33(p)}c_{3(o)} = b_{3(p)},$$

$$p = 1,2,\ldots,P \qquad ,$$

are the condition equations where m denotes a meridional deviation station and p denotes a prime vertical deviation station, and other symbols have their usual meanings. A station may be common to both groups. If $(M+P) > 3$, normalization will be required to obtain the design matrix. This indicates that while coincidence matching will require at least three computation stations, two common deviation stations would be sufficient for matching the condition of parallelism.

6.5 COMPUTATION WITH SELECTED STATIONS

The initial point together with four astro-geodetic stations were considered in Chapter V for orienting the system by gravimetric method. The same stations may be used for the proposed least-squares matching using the coincidence and parallelism conditions.

First, the contributions of the void geoid parameters are computed by linear interpolation from the four nearmost corner values of N_v, ξ_v, η_v. For the partial geoid parameters, however, a curvilinear interpolation technique has been preferred to take care of the regional changes. The procedure adopted is as follows:

(i) 16 surrounding grid corners around the computation station (ϕ,λ) are considered such that,

$$\phi_{-1} < \phi_o < \phi < \phi_1 < \phi_2 \qquad \text{and} \qquad \lambda_{-1} < \lambda_o < \lambda < \lambda_1 < \lambda_2 \tag{6.4}$$

(ii) The Lagrange polynomial interpolation yields four intermediate values in the ϕ -direction, as

$$H_i = \frac{-p(1-p)(2-p)}{6} Z_{i,-1} + \frac{(1+p)(1-p)(2-p)}{2} Z_{i,o} + \frac{(1+p)p(2-p)}{2} Z_{i,1} - \frac{(1+p)p(1-p)}{6} Z_{i,2} \, , \qquad i = -1,0,1,2 \tag{6.5}$$

where, $p = (\phi-\phi_o)/(\phi_1-\phi_o)$

and $Z_{i,j}$ are the data points at grid corners.

(iii) From these again, the interpolated value of Z is found by the same principle in the λ - direction , as

$$Z = \frac{-q(1-q)(2-q)}{6} H_{-1} + \frac{(1+q)(1-q)(2-q)}{2} H_o$$

$$+ \frac{(1+q)q(2-q)}{2} H_1 - \frac{(1+q)q(1-q)}{6} H_2 \qquad (6.6)$$

where, $q = (\lambda-\lambda_o)/(\lambda_1-\lambda_o)$

The final values of N, ξ, η are given in Table 6.1.

TABLE 6.1

GRAVIMETRIC PARAMETERS OF COMPUTATION STATIONS INTERPCLATED FROM FIVE-DEGREE AND ONE-DEGREE CORNER VALUES

Point code	N_{gr} metre	ξ_{gr} arcsecond	η_{gr} arcsecond
0	-59.8	-0.20	3.53
1	-60.8	-2.97	8.58
2	-61.9	-5.81	-0.88
3	-75.4	-6.42	1.04
4	-51.8	-5.44	1.99

Values are referred to GRS 67

A comparison of these values with those presented in Table 5.7, chapter V, reveals that informations of the order of 1 metre in N and 2" to 3" in ξ or η may be hidden in the local gravity details .

The solution for coincidence matching and parallelism matching involved normalization of matrices of order 5 x 3 and 10 x 3 respectively.

The results are presented in Table 6.2 in conventional form as well as in terms of the shift vector elements.

TABLE 6.2

ORIENTATION PARAMETERS OBTAINED BY IMPOSING LEAST - SQUARES CONDITIONS AT SELECTED STATIONS

Condition	Coincidence	Parallelism
δN_o metre	-60.34	-854.48
$\delta \xi_o''$	-0.02	-0.76
$\delta \eta_o''$	1.43	2.85
x_1 ppm	-31.58	16.09
x_2 ppm	32.94	15.02
x_3 ppm	117.99	3.95

6.6 PARALLELISM VERSUS COINCIDENCE

It is immediately apparent from the results obtained by the above exercise, that a complete solution of the orientation problem is not possible by the method of least-squares parallelism; the value of δN_o is improbable.

In the two-fold process of determining ξ_{gr} and η_{gr} at the astrostation, the original information suffers considerably owing to the following reasons:

(i) In approximating the integration by numerical summation, inaccuracies introduced is greatest from the nearest gravity unit, with maximum effect on the deviations of the vertical. In the absence of adequate gravity detail and finer mesh sizes, the grid corner values thus neither represent the local fluctuations, nor do they have any reason to be considered as zonal values.

(ii) Similarly, any interpolation of ξ_{gr} and η_{gr} from the sharply changing corner values is equally arbitrary.

An alternative scheme of matching the slopes at grid corners does not seem to yield reliable result either for the following reasons:

(i) Astro-geodetic stations are not necessarily located at grid corners thereby rendering interpolated values of ξ_{ag} and η_{ag} at the corners from limited distributed data, also arbitrary.

(ii) The high density of astro-geodetic stations required for a representative interpolation is not practicable.

The differences in gravimetric and astro-geodetic values, in either case mentioned above, contain some significant signal data and should not be assumed to be only the uncorrelated random noise due to various errors.

On the other hand, informations regarding undulations are obtained in a similar way for both cases: gravimetrically, by integration of the amplitudes of the gravity vector and astro-geodetically by integrating the phase of the vector. These N_{gr} and N_{ag} values are therefore of comparable order. The smoothness of the geoid heights in both cases makes them interpolable also. The average coincidence in a way also brings about the parallelism closer as a little change of ξ_o, η_o will affect the N heights systematically.

From the overall considerations, therefore the coincidence matching at grid corners is suggested as a more optimal choice.

6.7 SHIFT VECTOR FORMULATION FOR COINCIDENCE MATCHING

The extension of the least-squares technique to matching of geoid heights at the corner points only, may now be framed as a general procedure.

Corrections due to the change of spheroid without affecting the N_o, ξ_o, η_o values may be done first, by the procedure detailed below.

Rewriting the transformation relations as given in Section 5.9,

$$C = TX + ED$$

$$\text{where,} \quad X = T_o'(C_o - E_oD) \tag{6.7}$$

$$= \bar{X} + \mathring{X}$$

For no change in geoid parameters at the initial point,

$$\delta N_o = \delta \xi_o = \delta \eta_o = 0,$$

hence C_o being a null vector,

$$\bar{X} = T_o'C_o = 0$$

$$\mathring{X} = -T_o'E_oD, \tag{6.8}$$

$$\text{and} \quad \dot{C} = T\dot{X} + ED$$

For computing only the undulations, the first element of $\dot{C}$ will be needed, whose expression turns out to be,

$$\frac{\delta \dot{N}}{R} = -(\sin\phi)\dot{x}_1 + (\cos\phi\cos\lambda)\dot{x}_2 + (\cos\phi\sin\lambda)\dot{x}_3 - d_1 + d_2\sin^2\phi \tag{6.9}$$

In the present case, application of this correction to the Everest geoid will form an 'Arbitrary GRS67 Geoid' which may now be compared with the 'Gravimetric GRS67 Geoid'.

The D vector being now null, the relation is simply given by,

$$\bar{C} = T\,\bar{X} \tag{6.10}$$

of which only the first element is to be used here, yielding,

$$-(\sin\phi)\bar{x}_1 + (\cos\phi\cos\lambda)\bar{x}_2 + (\cos\phi\sin\lambda)\bar{x}_3 = \frac{\delta\bar{N}}{R} \qquad (6.11)$$

where, $\delta\bar{N}$ is the gravimetric value minus the arbitrary GRS67 value.

By solving the above sets of equations for $\bar{X}$ after usual normalization, one obtains the conventional correction at the origin as,

$$C_o = T_o\,\bar{X} \qquad (6.12)$$

Also, the shift vector X may be directly obtained by,

$$X = \bar{X} + \dot{X}$$

$$= \bar{X} - T_o'E_oD \qquad (6.13)$$

The whole procedure has several advantages, notably,

(i) the intermediate bye-product, i.e., the 'arbitrary GRS 67 geoid' may be used in future whenever revised corrections C_o are available,

(ii) the complex trigonometrical relations are eliminated,

(iii) for geographical latitude-longitude grids, the same coefficients will occur for a number of times and subscripted variables may be used to effect an efficient computer programme.

6.8 RESULTS OF COMPUTATION

The grid corners considered for the least-squares coincidence matching for orientation, were arranged in three sets as follows (Figure 6.1)

(a) Set A : 15 corners along two orthogonal profiles,

(b) Set B : 8 points near boundaries of Indian region,

(c) Set C : 23 points, combining set A and set B.

The astro-geodetic geoidal heights were obtained from a contour map (Bhattacharji ,1973) referred to the International Spheroid. The values are converted to those on the Everest Spheroid using the correction parameters given in Expressions 5.4 of chapter V. The arbitrary GRS67 geoid heights are obtained thereafter, using the relation expressed in Equation 6.9.

The gravimetric GRS 67 geoid heights were already obtained superimposing the partial geoid heights computed in chapter IV, on the void geoid heights calculated at 1° corners by cubic spline interpolation from the 5° corner values computed in chapter III. Two profiles of both types of geoid, the arbitrary GRS 67 (astro-geodetic) and the gravimetric GRS 67, are depicted in Figure 6.2.

The solution of Equation 6.11 yields the $\bar{X}$ vector from which the orientation vector C_o and the shift vector X are computed using Equations 6.12 and 6.13 respectively.

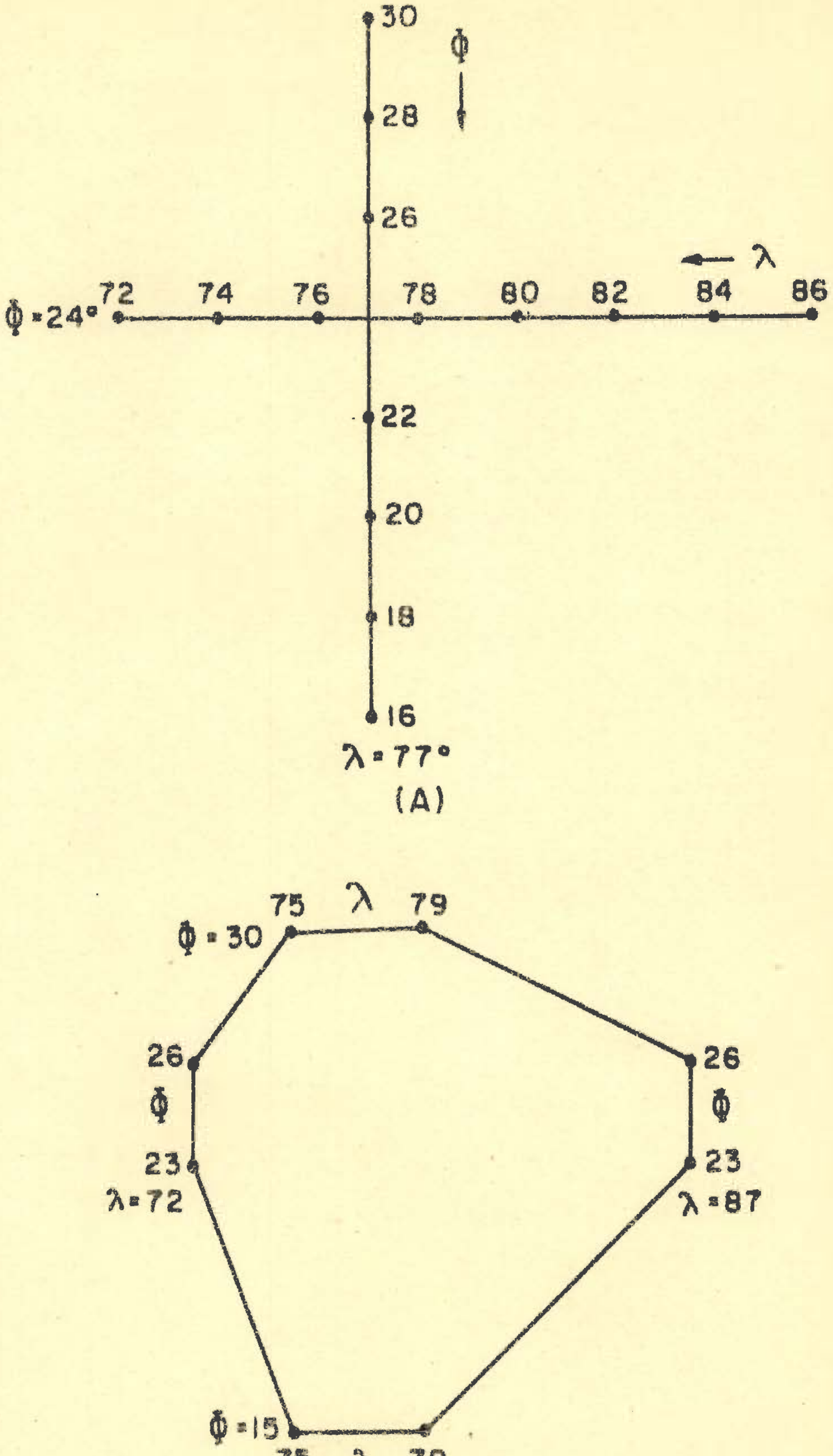


FIG. 6·1 POSITIONS OF POINTS CONSIDERED FOR LEAST-SQUARES COINCIDENCE MATCHING

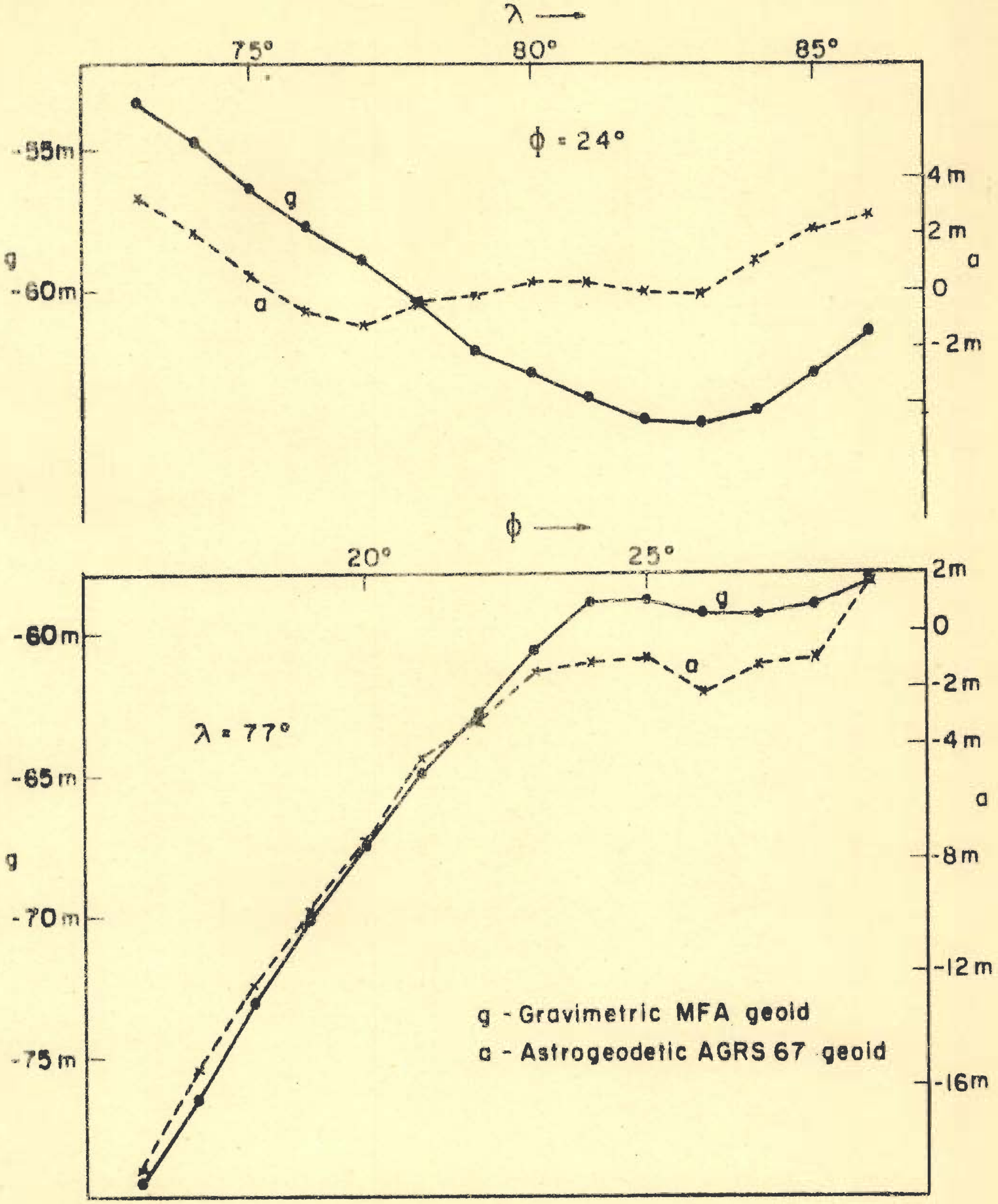


FIG. 6·2 PROFILES OF MEAN FREE AIR GEOID AND ARBITRARY GRS 67 GEOID

The final results obtained from the three sets are tabulated in Table 6.3. The standard errors of various solutions are also indicated therein to illustrate how well they coincide with each other.

TABLE 6.3

SHIFT VECTOR AND ORIENTATION PARAMETERS OBTAINED BY LEAST-SQUARES COINCIDENCE

Set Points compared	A 15	B 8	C 23	unit
x_1	-32.03	-30.92	-31.37	ppm
x_2	32.81	30.36	31.38	ppm
x_3	113.85	114.91	114.46	ppm
Standard error	1.0	2.7	1.7	metre
δN_c	-59.8	-59.8	-59.9	metre
$\delta \xi_o$	-0.45	-0.19	-0.30	arcsec
$\delta \eta_o$	1.59	1.05	1.27	arcsec

Two observations may be made from the results obtained from various sets :

(i) the solutions do not differ from each other significantly thus satisfying the internal consistency of the method.,

(ii) the standard error is minimum for set A, indicating that the result of including greater number of stations may not be always advisable.

6.9 SUMMARY AND DISCUSSION

This chapter begins with discussion of the various sources of inaccuracies involved in the absolute orientation of a geodetic system through a general astro-geodetic station. The sources of error in the astronomic and geodetic measurements and computations, alongwith estimates of errors, are briefly enumerated. Thereafter assuming a gaussian distribution of those errors, a least-squares approach to the problem is followed.

Following the formulations of the least-squares matching of

(i) geoid undulations, and

(ii) slopes of the astrogeodetic and gravimetric geoids,

both the methods have been applied to compute the orientation parameters, using some selected astrogeodetic stations. After a comparative study, finally the coincidence approach has been adopted in a general way. Computer-oriented formulae are developed and three consistent sets of orientation parameters are obtained, which may be compared with the sets obtained through alternative methods.

The virtue of a least-square solution proposed and tried in this study should not however be overstressed, as it only partially circumvents the want of precisely determined gravimetric and astro-geodetic data. For, a large number of observations alone may not produce reliable results in all cases as the variance may become even greater than the original looseness of the set of observations, and an additional station may be an outlier unduly affecting the system as a whole.

Furthermore, the mathematical expression contains another term N_o which has not been taken into account, as it does not appear in the Stokesian geoid height, while the astro-geodetic geoid is not considered to be precise enough to yield this value accurately. The term may be visualized as absorbed in other coefficients or neglected altogether, similar to the term representing the geoid-cogeoid separation.

The present determination must therefore be regarded only a set of secondary check values, whilst a truly representative picture must await more precise knowledge of the gravity field and its accurate reduction for purposees of orientation. Revision of the astro-geodetic geoid with greater detail and sharpening of the tools for treating and handling such data in view of future refinements would also render this exercise more realistic.

CHAPTER VII

ABSOLUTE ORIENTATION BY COMPARISON OF ASTRO-GEODETIC AND SATELLITE-DERIVED GEOIDS

7.1 GENERAL

Although the classical gravimetric method is the main approach followed in this work for determining the absolute orientation parameters of the Indian Geodetic System, an attempt has also been made to determine these using the astro-geodetic deviations of the vertical and the satellite-derived geopotential coefficients.

The astro-geodetic geoid based on the measured deviations of the vertical gives a reasonably detailed picture of the mean sea level surface, its fine structure depending upon the distribution of the deviation stations and the method of integration used. However, this geoid suffers from a major defect in that it is only relative, the extent of non-geocentricity depending upon the arbitrariness of the orientation parameters chosen for the initial station.

Recent satellite data have helped define a generalised shape of the geoid expressed in terms of a limited number of geopotential coefficients. This surface, whilst being unrepresentative of local features, is positioned and oriented in space in an absolute sense.

The satellite-derived geoid may be compared with the astro-geodetic geoid in a least-squares sense. Before this is done however, the latter must be rendered comparable to the former and both these reduced to a common spheroid. The three factors needed to accomplish this are the elements of the shift vector introduced earlier in this work, or the corresponding conventional quantities, viz., the radial translational component and the two tilt components, which incidentally are the parameters sought for absolute orientation.

7.2 THE INDIAN ASTRO-GEODETIC GEOID

The Indian astro-geodetic geoid (Bhattacharji,1973) was obtained using the well-known Helmert's integration (Helmert,1880). Accordingly, a series of loops of geoidal profiles are determined by direct integration of the available deviations of the vertical at appropriate intervals, in the form

$$N_i = N_{i-1} - \int_{i-1}^{i} (\xi \cos\alpha + \eta \sin\alpha)\, ds \qquad (7.1)$$

where, N_i is the geoidal height of the ith station,

ξ, η are respectively the deviation components in the meridional and the prime vertical planes, and

α is the azimuth of the profile connecting the two stations, the (i-1)th and the ith.

However, owing to the ideal assumptions implicit in the mathematical formulation and defects in the data, the loop closures are found to be in appreciable error which are subsequently adjusted usually by some sort of linear distribution of errors along the loops. Thereafter, the geoidal heights are linearly interpolated at points inside the loops and a map of geoidal height contours is drawn with reference to the adopted spheroid.

A variation of this method is to interpolate deviations of the vertical at geographic grid intersections and compute the geoidal profiles along this grid lines, finally adjusting the heights so that the closure errors along the loops reduce to zero.

7.3 DEFECTS IN HELMERT'S INTEGRATION METHOD

The evaluation of the geoid by Helmert's method has amongst others, the following limitations:

(a) both components of the deviation, i.e., in respect of ξ as well as η, must be available at a station; meridional deviation values alone are not usable unless the station perfectly lie along a North-South profile,

(b) station interval should be small so that variations between adjacent stations may be assumed to be linear,

(c) gaps between profiles are not represented with equal weightage, being oversmoothened compared to the profiles.

The alternative method, using interpolation at grid corners, does not provide a reliable geoid either, as the observed deviations of the vertical are truncated and reasonable interpolations are not possible unless the station distribution is dense enough around the grids.

In the case of the Indian geoid, the defects are quite pronounced as there exist only about 500 stations in this vast subcontinent at which values of both deviation components are available, and these too, not uniformly distributed. Only a few dominant profiles can thus be obtained providing about 4 or 5 loops of fairly large size with larger data gaps within them.

7.4 THE SURFACE-FITTING TECHNIQUE

Vaniceck and Merry (1973) have proposed a polynomial surface-fitting technique in three-dimensions amenable to automatic machine computation that may replace Helmert's method. This technique has been used here to compute geoid heights in the Indian region which are, in turn, compared with existing values.

The equation of the surface used in the present work is

$$u_1 = \frac{N}{R} = \sum_{n=0}^{n\,max} \sum_{m=0}^{n} C_{nm} X^{n-m} Y^m \qquad (7.2)$$

where,

$X = \phi$

$Y = \lambda \cos\phi$,

and the latitudes and longitudes are expressed in radians.

The original scheme has been hereby slightly modified

(a) to enable a better computer-oriented arrangement of the power series,

(b) to make all variables dimensionless,

(c) to eliminate the coordinates of the initial point from the expressions of X and Y, so as to avoid underflow in computations which could have occured if as initially

$X = (\phi-\phi_o)R$

$Y = (\lambda-\lambda_o)R\cos\phi$,

specially when a station shares the same meridian or parallel with the initial point.

Another advantage of the modified expressions of X and Y, specially applicable to India because of its location in the North-Eastern part of the globe, stems from the positive values of both X and Y thereby avoiding negative bases in exponentiation.

Recalling that

$$\delta X = \delta\phi \text{ , and}$$

$$\delta Y = \delta\lambda \cos\phi,$$

and differentiating expression 7.2 with respect to ϕ and λ,

$$\frac{\delta u_1}{\delta X} = -\xi = \Sigma\Sigma C_{nm}(n-m)X^{n-m-1}Y^m ,$$

$$\frac{\delta u_1}{\delta Y} = -\eta = \Sigma\Sigma C_{nm}\, X^{n-m}(m)Y^{m-1} \tag{7.3}$$

Expressions 7.3 provide the conditions for obtaining the unknowns C_{nm} by framing the equations, in a matrix form, as

$$AC = B \tag{7.4}$$

where, A is the design matrix, whose elements are functions of position of the deviation station,

C is the coefficient vector, to be determined from the astrogeodetic data,

B is the deviation vector, comprising the components of the deviations of the vertical in radians with reversed sign.

Each measurement of either ξ or η will provide one equation which offers an added advantage over Helmert's method. Both components of deviation at a station provide two equations.

The solution, after usual normalization, is

$$C = (A'A)^{-1} (A'B) \qquad (7.5)$$

It is noted that the coefficient C_{oo} cannot be determined by the least-squares solution, as it does not appear in Equations 7.3. For evaluating C_{oo}, the known geoid height N_o at the initial point will be needed, whence

$$C_{oo} = \frac{N_o}{R} - \sum_{n=1}^{n\,max} \sum_{m=0}^{n} C_{nm} X_o^{n-m} Y_o^m \qquad (7.6)$$

where,

$$X_o = \phi_o ,$$

$$Y_o = \lambda_o \cos\phi_o .$$

The coefficients now being all known, the geoid height at any point may be computed by evaluating expressions 7.2.

7.5 OPTIMUM ORDER OF FITTED SURFACE

A high-order surface might at first appear attractive as it contains larger number of coefficients, but while computing intermediate values of the geoid heights, sudden ripples may occur due to involvement of shortwave components that may have high amplitude.

On the other hand, a low-order surface will be too smooth with a high standard error. The criteria of a logical choice are well-discussed by statisticians, and the unbiased variance may act as a quantitative measure, controlled by the degree of freedom of the equation-unknown compatibility.

The unbiased variance is defined as

$$v = \frac{B'B - C(A'B)}{n_{un} - n_{eq}} \tag{7.7}$$

where,

n_{un} = number of unknowns related to the order of the surface,

n_{eq} = number of equations equal to the number of deviation components used.

The product $B'B$ is computed once for all, while reading the data cards, as actually,

$$B'B = \Sigma\ (\xi^2 + \eta^2) \tag{7.8}$$

The number of unknowns excluding C_{oo}, depends on the order of the surface, n_{ord}, by the relation

$$n_{un} = n_{ord}\ (n_{ord} + 3)/2\ , \tag{7.9}$$

the value of n_{ord} being 1 for a plane surface.

As the number of unknowns cannot exceed the number of equations, the highest order of surface that can be incorporated, turns out to be (Equation 7.9),

$$n_{ord} = \{(\sqrt{9 + 8n_{eq}} - 3)/2\} \qquad (7.10)$$

where the parantheses { } signifies that n_{ord} is the integer value of the expression enclosed.

Beginning thus, with a higher-order surface dictated by Equation 7.10 for calculating variances, the order can be progressively decreased and variances compared, and the order corresponding to the minimum v adopted for a final fitting.

7.6 INVERSION OF THE NORMALIZED MATRICES

The technique for obtaining the optimum order of surface, as outlined above, involves the inversion of large matrices quite a few times, thereby requiring long computer time and large storage. This difficulty has been circumvented in the following manner:

(a) First, normalization of the original equations is carried out up to the highest order and by framing $A'A$ and $A'B$ directly, instead of A. The data need not be stored in this procedure. The normalized matrix being symmetric, only the upper triangular part need be stored in a single-subscripted array.

(b) The least-squares matrix is proved to be a positive definite one and hence inversion by factorization is much more convenient. The factorization into triangular matrices is done by Cholesky's square root method, using the following recurrence relation,

$$A'A = D = R'R \tag{7.11}$$

where,

$$r_{jk} = \frac{d_{jk} - \sum_{i=1}^{j-1} r_{ij}\, r_{ik}}{r_{jj}}, \quad j \leq k$$

The elements r are stored in the same location as d to minimize core space requirements.

(c) Inversion of R is now done using another recurrence relation, i.e.,

$$\bar{r}_{ik} = \frac{-\sum_{j=i+1}^{k} r_{ij}\, \bar{r}_{jk}}{r_{ii}}, \quad i < k \tag{7.12}$$

and $\bar{r}_{ii} = \frac{1}{r_{ii}}$,

$\bar{r}$ denoting the elements of R^{-1}

It is found that truncation of the original normalized matrix D down to any size and subsequent inversion of the factorized triangular matrix leads to exactly the same elements, as those obtained from the direct truncation of the matrix R^{-1} to the same size. Therefore, this computation need be executed only once with the largest sized matrix. While reducing the order of surface, the elements $\bar{r}$ of the shortsized matrix, truncated according to the n_{un} value, need be used.

(d) The coefficients C_{nm} and the variance v for any order of surface may be directly generated by,

$$C = (R^{-1})(R^{-1})' (A' B) \qquad (7.13)$$

and the expression 7.7,

where the number of elements will be truncated accordingly.

7.7 RESULTS OBTAINED BY SURFACE-FITTING

For the sake of comparison with the published geoid map, the deviations of the vertical reduced to the International Spheroid were used. A preliminary study (Ketaurai, 1978) on testing of polynomial surfaces by fitting them piecewise within short blocks of $4^{\circ} \times 4^{\circ}$ size proved unsatisfactory due to insufficient data and errors in extrapolation . A single surface was therefore fitted for the whole Indian region.

366 stations giving both the deviation components, and 387 stations giving only the meridional component were considered, rejecting a number of astrogeodetic stations to avoid clustering of data points. The order of the polynomial surface was reduced from 10 to 3, and the seventh-order surface was found to be optimum.

Table 7.1 presents the standard errors $\sqrt{v}$ in respect of various surfaces. The highest order surface could not be adopted for want of adequate storage and time on the available computer IBM 1620. Also, it was felt that although a detailed geoid would necessitate a higher order surface, the present objective was to obtain only smoothened values, similar to the smooth geoid derived from satellite data.

TABLE 7.1

STANDARD ERRORS OF GEOID SURFACES OF VARIOUS ORDERS

Order of Surface	10	9	8	7	6	5	4	3
Number of Coefficients	65	54	44	35	27	20	14	9
Standard error (arcsecond)	13.9	11.8	11.9	7.9	10.1	10.9	12.1	12.8

Figure 7.1 shows two profiles of the geoid thus obtained alongwith the existing profiles for purposes of comparison. Sample statistics of height comparison of a few points are presented in Table 7.2.

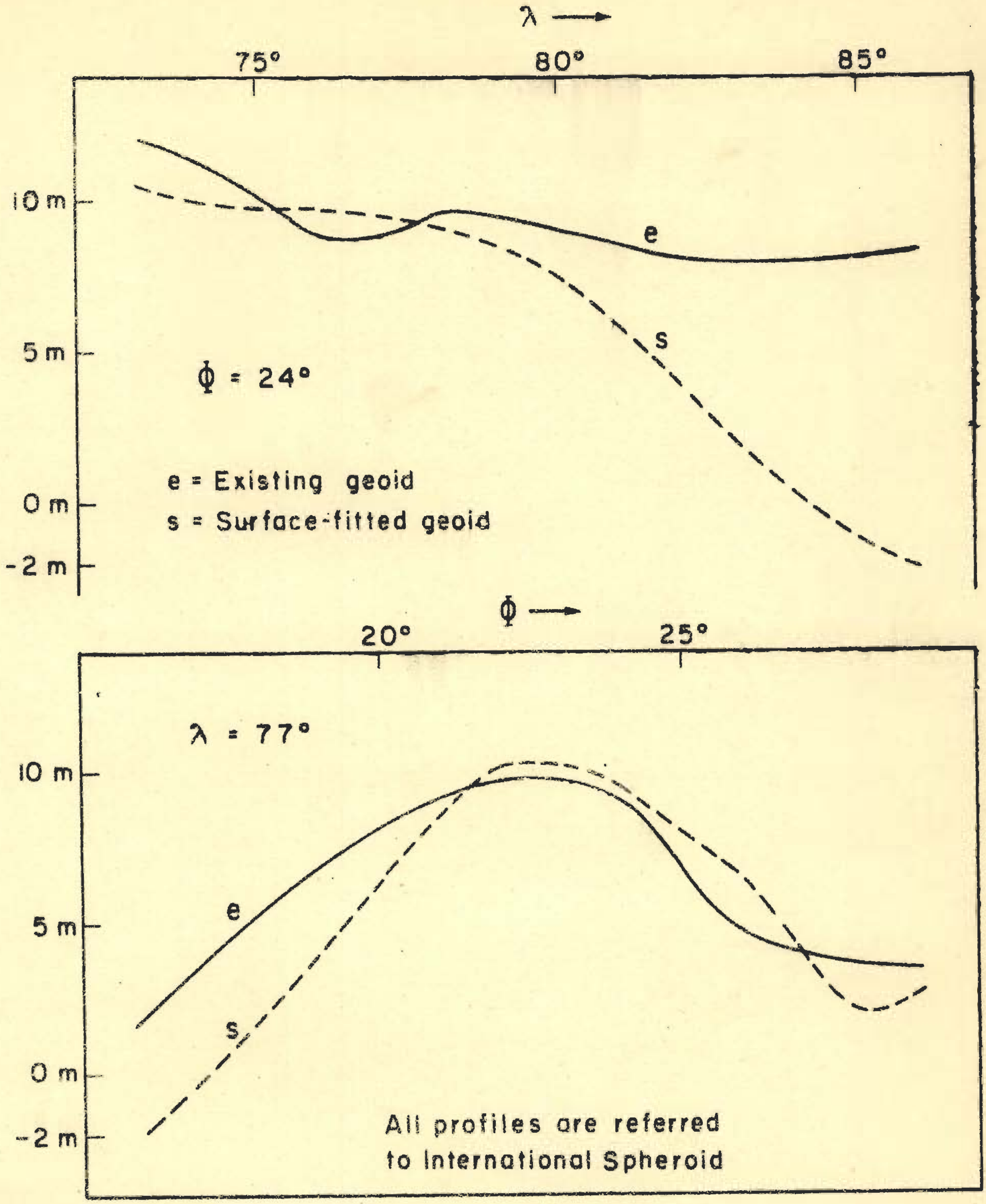


FIG. 7·1 PROFILES OF PRESENT ASTRO-GEODETIC GEOID AND SURFACE-FITTED GEOID

TABLE 7.2

SAMPLE STATISTICS OF COMPARISON OF ASTRO-GEODETIC GEOIDS

	Existing Geoid	Surface-fitted Geoid	Number of points compared = 23
Maximum Height	12.9 metre	12.1 metre	Maximum Difference = 10.4 m
Minimum Height	-2.0 metre	-2.1 metre	Minimum Difference = 0.1 m
Average Height	6.8 metre	5.3 metre	RMS Difference = 3.5 m

7.8 SATELLITE-DERIVED GEOPOTENTIAL COEFFICIENTS

The disturbing potential of the earth being a harmonic function, it can be expressed as a series of spherical harmonic terms. From Bruns' formula, the geoid undulation function finally takes the form,

$$u_1 = N/R$$

$$= (1 - \sum_{n=2}^{\infty} \sum_{m=0}^{n} (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm})$$

$$-(1 - \sum_{n=1}^{\infty} \bar{J}_{2n} \bar{P}_{2n}). \qquad (7.14)$$

The quantities $\bar{C}_{nm}$, $\bar{S}_{nm}$ are the geopotential coefficients computed by analyzing the satellite orbits, and the coefficients $\bar{\bar{J}}_{2n}$ are related to the reference spheroid adopted. The summation involves the second-order and higher terms only, as the zero and first-order terms vanish owing to the equality of mass, the geocentricity and coaxiality of the system with that of the actual earth. The geopotential coefficients yielded by satellites are being continuously refined from time to time to progressively higher degrees and orders as new satellites provide additional informations.

7.9 RECURSION OF THE LONGITUDE AND LATITUDE TERMS

(a) For computation of the longitude terms, the following recursion relations have been used in the present work:

$$\begin{aligned} \sin m\lambda &= \sin(m-1)\lambda\, C_\lambda + \cos(m-1)\lambda\, S_\lambda, \\ \cos m\lambda &= \cos(m-1)\lambda\, C_\lambda - \sin(m-1)\lambda\, S_\lambda \end{aligned} \qquad (7.15)$$

with starting elements,

$$C_\lambda = \cos\lambda,$$

$$S_\lambda = \sin\lambda$$

This simple recursion avoids the repetitive use of the irrational factor π, and the accuracy is not affected if C_λ, S_λ are first computed in double precision.

(b) The latitude term $\bar{P}_{nm}$ in the geopotential series is the fully normalized Associated Legendre function of degree n and order m, the general expression being,

$$\bar{P}_{nm} = \sqrt{\frac{2(2n+1)\,\underline{\lfloor n-m}}{\underline{\lfloor n+m}}} \cdot \frac{(c)^m}{(2)^n} \cdot \sum_{k=o}^{j} \frac{(-1)^k \,\underline{\lfloor 2n-2k}\,(s)^{n-m-2k}}{\underline{\lfloor k}\;\underline{\lfloor n-k}\;\underline{\lfloor n-m-2k}}$$

where, (7.16)

$m \neq 0$

$c = \cos\bar{\phi}$

$s = \sin\bar{\phi}$

and $j = \{ \frac{n-m}{2} \}$,

Here $\bar{\phi}$ is the geocentric latitude of a point, and the paranthesis { } signifies the largest integer value of the quantity within it.

When m = 0, the zonal harmonics are the Legendre's polynomials,

$$\bar{P}_n = \sqrt{2n+1}\; P_n$$

where,

$$P_n = \frac{1}{(2)^n} \sum_{k=o}^{j} \frac{(-1)^k \,\underline{\lfloor 2n-2k}\,(s)^{n-2k}}{\underline{\lfloor k}\;\underline{\lfloor n-k}\;\underline{\lfloor n-2k}} \qquad (7.17)$$

In the spheropotential series, only the even-order polynomials possessing equatorial symmetry occur.

(c) The expressions in the Legendre function contains long series and factorials. A continuous series summation on a computer is likely to affect the precision of the final output due to round-off errors. Further, the factorial being a long product of numbers, gives rise to overflow. Recursion formulae are therefore resorted to for evaluation of these polynomials and functions.

(i) For the zonal harmonics, the following formula is used,

$$P_n = P_{n,o} = \frac{(2n-1)sP_{n-1} - (n-1)P_{n-2}}{n}, \qquad (7.18)$$

and $\bar{P}_n = \sqrt{2n+1}\, P_n$,

taking initially.

$$P_1 = s$$
$$P_2 = 0.5(3s^2-1)$$

(ii) In the case of Associated Legendre functions, however, the normalizing factor under the radical sign should not be isolated as the conventional P_{nm} values are highly divergent for higher degrees and orders. Two types of recursions may be used,

(1) order-wise recurrence, keeping the degree n unchanged [redacted] (Gaposchkin,1973),

(2) degree-wise recurrence, keeping the order m unchanged (Hopkins,1973).

Considering the efficiencies of the computer as regards the storage and machine time, the following recursions were found to be optimum:

(1) diagonal recursion:

$$\bar{P}_{n,n} = c\sqrt{1 + \frac{1}{2n}}\ \bar{P}_{n-1,n-1} \qquad (7.19)$$

(2) prediagonal recursion:

$$\bar{P}_{n,n-1} = t\sqrt{2n}\ \bar{P}_{n,n} \qquad (7.20)$$

(3) row recursion:

$$\bar{P}_{n,m-2} = \frac{(2m-2)t\ \bar{P}_{n,m-1} - \sqrt{(n-m+1)(n+m)}\ \bar{P}_{n,m}}{\sqrt{(n-m+2)(n+m-1)}} \qquad (7.21)$$

where, $t = \tan\bar{\phi} = s/c$,

The relation between the geocentric latitude and the geodetic latitude is,

$$\tan\bar{\phi} = (1-f)^2 \tan\phi \qquad (7.22)$$

7.10 RESULT OBTAINED USING SATELLITE DATA

The recursion formulae for harmonics are first tested numerically. As a test example, the attraction part of the normal gravity was expanded in harmonic series (Ray, 1978a) evaluated in terms of GRS 67 adding the rotation term and compared with the normal formula for γ_{67}. The results are reported in Appendix G.

Thereafter the GEM 10 satellite coefficients (Lerch, 1978) completed upto the 22nd degree and order with a number of additional terms upto 30th degree, were made use of. For the spheropotential coefficients, the normalized values were generated from conventional J_2 value using the relation

$$\bar{J}_{2n} = \frac{(-1)^n . 3 . (e^2)^n \, (1 - n + 5nJ_2/e^2)}{(2n+1)(2n+3)\sqrt{4n+1}} \qquad (7.23)$$

where, $e^2 = f(2-f)$

Two profiles of the GEM 10 geoid obtained are shown in Fig 7.2. Similar profiles are also drawn for a geoid computed by the Smithsonian Astrophysical Observatory (Gaposchkin, 1973) termed as the SAO III geoid.

While the general trend of the geoids compare well, the other differences observed between the two are most likely to be for the following three reasons,

(a) difference of equatorial radius of the reference ellipsoids,

(b) difference of the flattening values,

(c) addition and updating of geopotential coefficients.

Table 7.3 indicates a sample statistics of the satellite-derived geoid and the gravimetric geoid obtained from mean free-air anomalies over one-degree areal elements. A rms difference of 8 metres is readily detectable, the gravimetric geoid heights being numerically greater than those of the satellite geoid. Information in respect of

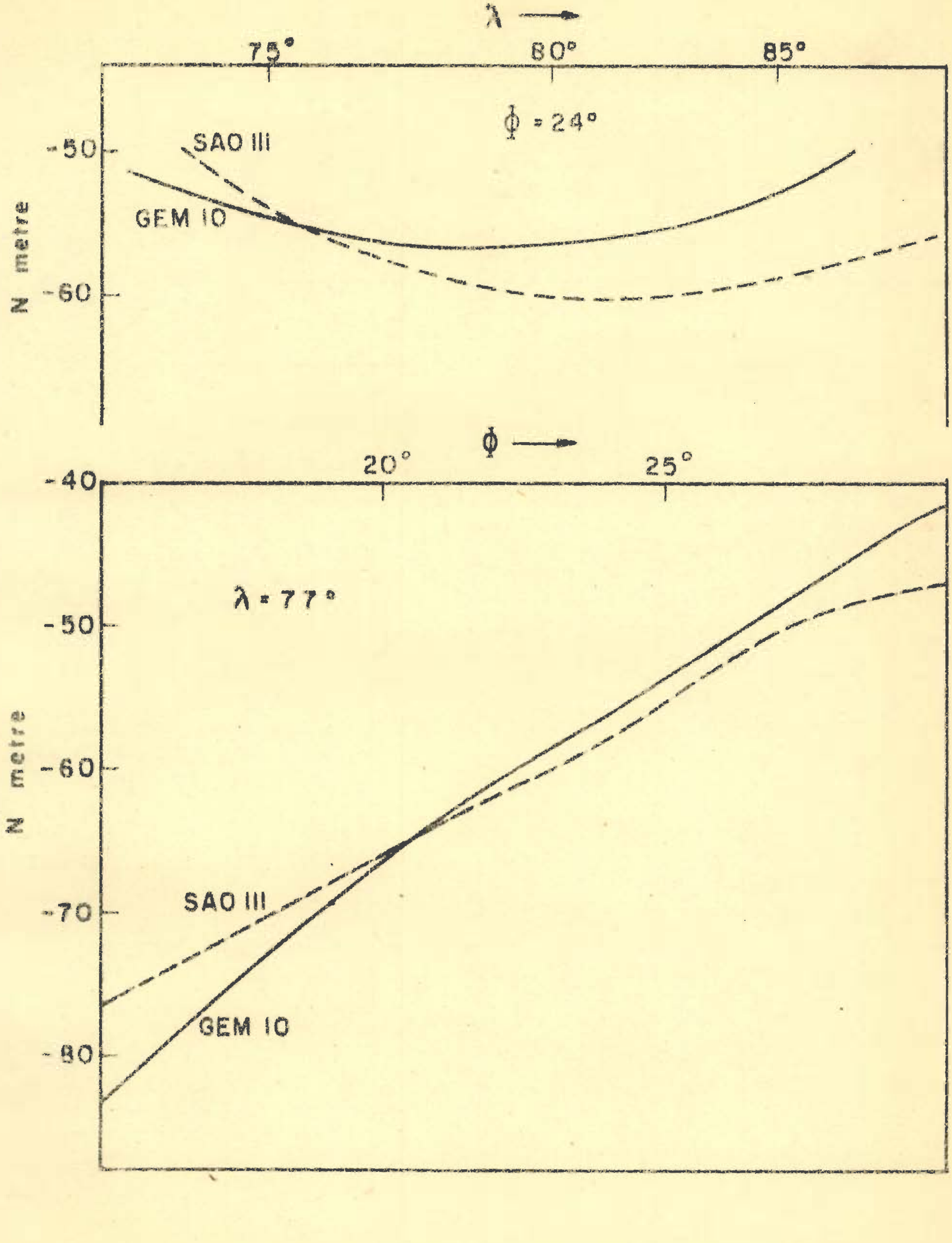


FIG. 7·2 PROFILES OF SAO III AND GEM 10 GEOIDS an average height of about 8 metres still therefore seems to be hidden in the higher order coefficients yet to be evaluated from surface gravity and near-earth satellites.

TABLE 7.3

SAMPLE STATISTICS OF COMPARISON OF SATELLITE - DERIVED AND GRAVIMETRIC GEOID

	Satellite-derived	Gravimetric	Number of points compared = 23
Maximum value	-83.13 metre	-81.48 metre	Maximum difference = 18.38 m
Minimum value	-39.78 metre	-49.17 metre	Minimum difference = 0.41 m
Mean	-56.59 metre	-62.35 metre	RMS difference = 8.07 m

The maximum difference being double this amount, and the minimum being nearer to zero, the residual geoid information is likely to cover a quarter of a wave in the profile span of 15°, corresponding to the 6th degree zonal harmonic.

However, since detailed investigations in this regard were beyond the scope of this work , only some broad comments may be made, notably,

(i) higher degree geopotential coefficients will be required to make the satellite-derived geoid comparable to the gravimetric geoid,

(ii) for purposes of orientation through comparison, the astro-geodetic geoid should be truncated to be equivalent to the satellite-derived geoid.

7.11 ORIENTATION PARAMETER BY ASTRO-SATELLITE MATCHING

The astro-geodetic geoid obtained upon the seventh-order surface fitting is in terms of the International Spheroid. It must be reduced to the 'Arbitrary GRS 67 geoid' (defined in chapter VI) before comparison. A direct conversion was effected by applying the following corrections :

(i) the orientation corrections were applied to refer the geoid to the orientation of Everest Spheroid at the origin from that of the International Spheroid, i.e.

$$\begin{aligned} \delta N_o &= 0.0 - 9.5 \text{ m} \\ \delta \xi_o &= -0''29 - 2''.42 \qquad (7.24) \\ \delta \eta_o &= 2''.89 - 3''.17 \end{aligned}$$

(ii) the dimensions of the reference surface were changed from the International Spheroid to the GRS 67 spheroid, using the relations ;

$$\begin{aligned} \delta a &= 6378160 - 6378388 \text{ m}, \\ \delta f &= 1/298.24717 - 1/297.0 \end{aligned} \qquad (7.25)$$

The same points used for least-squares coincidence matching (section 6.8) were considered in the present case also. (Figure 6.1). Equation 6.11 of chapter VI was also used, as

$$-(\sin\phi)\,\bar{x}_1 + (\cos\phi\cos\lambda)\,\bar{x}_2 + (\cos\phi\sin\lambda)\,\bar{x}_3 = \frac{N_{sat} - N_{ag}}{R} \qquad (7.26)$$

where, N_{sat} and N_{ag} are the heights of the satellite - derived and astro-geodetic geoids respectively. The final results are tabulated in Table 7.4 from which it can be seen that the solutions in this case are also mutually consistent as those obtained in the previous chapter with minimum standard error appearing in the set A.

TABLE 7.4

SHIFT VECTOR AND ORIENTATION PARAMETERS OBTAINED USING ASTRO-SATELLITE GEOID MATCHING

Set Points Compared	A 15	B 8	C 23
$\bar{x}_1$ ppm	-5.18	-5.90	-5.61
$\bar{x}_2$ ppm	-7.99	-6.56	-7.12
$\bar{x}_3$ ppm	-9.83	-10.75	-10.31
Standard error m	3.1	3.4	3.3
δN_o m	-52.3	-53.8	-52.8
$\delta \xi_o$ m	-1.93	-2.11	-2.03
$\delta \eta_o$	-1.18	-0.85	-0.98

7.12 SUMMARY AND DISCUSSION

Beginning with a brief description of the principles behind the astro-geodetic and satellite-derived geoid-matching for absolute orientation, the present status of the Indian astrogeodetic geoid has been discussed. In view of the practical limitations of Helmert's integration method, the surface-fitting technique has been used. Criteria for fixing an optimum order of the surface has been discussed and the unbiased variance has been suggested as the determining factor. The solution algorithm of the high-order polynomial surface-fitting was obtained by inversion of the normal matrix which has been aptly simplified for efficient evaluation on a mediumsized computer. The resulting values were then compared with existing values.

The next part describes the harmonic series expansion of the geoidal undulation using geopotential coefficients. Recursion relations have been judiciously chosen to economize automatic computations. Finally, the geoid - spheroid separation with GEM 10 coefficients were computed at a few points and compared with another satellite-derived geoid, SAO III, and the one-degree mean free-air geoid obtained earlier in this work.

After reducing the surface-fitted astro-geodetic geoid to the GRS 67 system keeping its orientation the same as that of the original Everest Spheroid, it was then compared with the GEM 10 geoid to arrive at values of the orientation parameters. Three sets of points were chosen for the purpose, the first consisting of a couple of intersecting profiles, the second consisting of a few widely spread points and the third, a combination of these two.

The surface-fitting technique could prove better than Helmert's method, with its three-dimensional approach and fully automated formulation. Further, a number of low-order geoidal surfaces constrained by the conditions of continuity are expected to be more representative than a single surface of very high order. This will avoid systematic errors as distances from the initial point increase. A procedure equivalent to forming loops in the Helmert's method may well be adopted with a provision for distributing the closure error, the irregular lines being replaced by regular grids.

The comparison of the satellite-derived geoid with the gravimetric one showed some systematic differences, probably due to missing terms in the harmonic series. Assuming that a change in δN_o will produce equal changes everywhere and that 1" of extra deviation of the vertical at the origin will produce an undulation of 0.5 metre per 1^o of horizontal extent in the appropriate direction, it is estimated that an additional undulation of 4 metre in

N_o and deviations of about 1" each in ξ_o and η_o are necessary to minimize the discrepancy.

The gravimetric method still being theoretically the best for obtaining the parameters of absolute orientation, and significant gaps still existing in the information content of the satellite-derived geoids, therefore, further data would have to be awaited before attempting to obtain a precise set of orientation parameters by matching the astro-geodetic and satellite derived geoids.

CHAPTER VIII

ANALYSIS OF RESULTS AND CONCLUSION

8.1 GENERAL

As explained earlier, the basic objective of the present work was to obtain the parameters of absolute orientation of the Indian Geodetic System. This was a long-awaited job required to relate the local system to an internationally accepted global framework. Originally the orientation parameters of the initial point were chosen on a relative basis in terms of the Everest Spheroid. Later, these were expressed in terms of the International Spheroid by Least-squares fitting but purely on local considerations. It has therefore been felt that the time is ripe for computing the orientation vector at the initial point of the Indian Geodetic System in terms of the GRS 67 spheroid which is gradually replacing all local reference systems.

The main basis of this exercise attempted here is gravimetric. For, despite the advent of sophisticated techniques such as satellite altimetry, gravimetric methods still continue to be the most reliable. Nor is the use of satellite stations (Chatterjee, 1973) quite satisfactory in the Indian context, there being as yet only a single camera station in existence in the entire country, and other techniques such as laser and doppler tracking are yet to be adopted for efficient space triangulation for want of adequate density of tracking stations (Dixit,1976)

The requisite computer software for this exercise has been carefully developed to match future refinements in data acquisition and improvement in data distribution. These formulations have been couched in a simple form exploiting various mathematical manipulations to conserve computer time and memory storage.

The results of this exercise comprise a set of parameters of absolute orientation at the initial point of the Indian geodetic System alongwith a pictorial representation of the Indian geoid which appeared as a bye-product of this investigation.

8.2 SUMMARY OF VARIOUS RESULTS

In the preceeding chapters, three different methods were discussed and used for obtaining sets of orientation parameters. These were:

(a) gravimetric determination of the geoid-spheroid separation and deviations of the plumb-line at astro-geodetic stations,

(b) a least-squares solution using undulations of the regional gravimetric geoid and those of the astro-geodetic geoid, and

(c) a similar comparison of a smoothened astro - geodetic geoid with the regional satellite- derived geoid over the Indian region.

As already stated, the first of these constitutes the central idea behind the exercise for orientation, which can further be classified into two categories :

(i) gravimetric orientation at the initial point itself,

(ii) orientation through any other astro-geodetic station.

The latter two were also used as plausible alternatives where want of adequate gravity data and comparable astro-geodetic accuracy precluded a serious consideration of the former.

Table 8.1 summarizes the final results obtained from these three methods in respect of the following parameters,

(i) the invariant shift vector,

(ii) the conventional correction parameters at the origin, namely δN_o, $\delta\xi_o$, $\delta\eta_o$

Whilst the elements of the invariant shift vector have been expressed in non-dimensional units, as parts per million, the orientation parameters have been tabulated in

TABLE 8.1

SHIFT VECTOR COMPONENTS AND ORIENTATION PARAMETERS OBTAINED BY VARIOUS METHODS

Method code	Station code	x_1 ppm	x_2 ppm	x_3 ppm	δN_o metres	$\delta \xi_o$ arcsec	$\delta \eta_o$ arcsec
a	0	-27.24	38.09	115.04	-59.0	+0.65	+2.60
b	1	-29.23	34.28	114.42	-62.1	+0.15	+1.86
b	2	-40.17	34.71	110.26	-56.7	-2.24	+2.13
b	3	-23.46	36.50	115.86	-66.1	+1.40	+2.25
b	4	-28.08	33.36	115.28	-61.3	+0.43	+1.64
c	23	-31.37	31.38	114.46	-59.9	-0.30	+1.27
d	23	-39.5	20.2	114.42	-52.8	-2.03	-0.98

Method Codes :

a : Gravimetric computation at the initial point

b : Gravimetric computation at any other astro-geodetic station

c : Least-squares coincidence solution

d : Astro-satellite geoid-matching

Station Codes :

0 : The initial point Kalianpur

1 : The Northern astro-geodetic station

2 : The Eastern astro-geodetic station

3 : The Southern astro-geodetic station

4 : The Western astro-geodetic station

23 : All 23 points of set C (Figure 6.1)

conventional units. The deviations of the vertical have been expressed as customary in arcsecond units for ease in analysis and interpretation, although the micro degree unit seems to be consistent with the SI units and the decimal degree divisions discussed earlier.

For the two least-squares conjunctive methods, only the solutions corresponding to the set C, taking 23 points into account, have been cited here. The standard error is not the least for this set but the results are nearer the average of those obtained for all sets.

8.3 ANALYSIS OF VARIOUS RESULTS

Out of the three methods, the classical gravimetric method is still by far the most preferable and yields reliable results provided a certain precision in respect of the input data are duly maintained. The determination based on the initial point is most reliable, as the quality and distribution of data conform to acceptable standards. Moreover, the anomalies being 'modified' by terrain corrections, the theoretical requirement of obtaining the geoid rather than the co-geoid is fulfilled, and the 'code a' results are therefore straightway taken to be the best set of geocentric orientation parameters obtained from the present exercise.

'Code b' results are obtained using the same principle, but applied to various astro-geodetic stations. The values are found to differ from each other as well as from the 'code a' results. The defects that contribute to the discrepancies are already discussed in Section 5.15. The δN_o value from station 3, and $\delta \xi_o$ value obtained from station 2 are very much discrepant whilst all other results are in mutually good agreement with each other. The extrapolation of anomalies over the smaller areal elements was not attempted as the truncated covariance prediction (neglecting negative covariance) involving only a few source values at one side, was suspected to produce erroneous results. The inner limits were inevitably restricted, thereby affecting the approximation of numerical summation, which was, in turn, reflected in the results obtained.

The mean value of δN_o, excluding station 3, and that of $\delta \xi_o$, excluding station 2 provide a satisfactory check on 'code a' values. The mean $\delta \eta_o$ is however about $0''.5$ smaller. Whilst absence of adequate information precludes exploration of various causes, increase of data is likely to yield more encouraging results in the future. Preliminary results from these stations were of prime importance, but the speed of execution was a restrictive factor.

The 'code c' results show marked variations from 'code a' values in respect of the slope components, whereas the geoid-spheroid separation values are comparable. The astro-geodetic geoid heights were estimated from a smoothened map, which has possibly given rise to smaller values of $\delta\xi_o$, $\delta\eta_o$. The estimates in this case mainly depend upon the following:

(i) fineness of discretization of the Stokes' integral especially in the immediate neighbourhood of the computation point where the mesh size should be smaller, even if the 1^o mean anomaly values are only used in the smaller elements,

(ii) distribution of the astro-geodetic stations leading to a small loop closure.

The orientation parameters obtained from the set 'code d' are significantly different from those obtained from the set 'code a'. A discrepancy of over 6 m in δN_o and about 3'' each in $\delta\xi_o$, $\delta\eta_o$ indicate that the principle in the present state is not very encouraging for the purpose of absolute orientation, the main reasons being:

(i) truncation of the geopotential function,

(ii) model errors in the surface-fitting technique,

(iii) inclusion of comparatively fewer points for matching.

Marsh and Vincent (1974) reported discrepancies of over 5 metres between the gravimetric and the satellite - derived geoid, and tilts of more than 1.5 arcseconds between them. Further investigation is therefore necessary in respect of this method, before using it for purpose of absolute orientation.

8.4 COMPARISON OF LINEAR SHIFTS OF SPHEROID CENTRES

A little reflection shows that the shift vector is analogous to the linear shift components in rectangular Cartesian coordinates used by space research organizations, the relation being

$$x_1 = -\Delta Z/R$$
$$x_2 = \Delta X/R \qquad (8.1)$$
$$x_3 = \Delta Y/R$$

The transformation constants to reduce the Indian Everest System to the various World Geodetic Systems, have differed according to different dimensions adopted for the matched spheroids, e.g., the Modified Mercury, the SAO III, the WGS 72 etc.(Ramanathan et al.1976). Theoretically, these values should not differ if all spheroids were absolutely geocentric. However, these are within the limits of precision by satellites, even when only a single station is available for space triangulation in the whole country. The ranges and the mean values in respect of a few recent geodetic systems which nearly conform to the GRS 67 in dimensions, are presented in Table 8.2

TABLE 8.2

SHIFT COMPONENTS DERIVED FROM ARTIFICIAL SATELLITES

	Minimum	Maximum	Mean
x_1 ppm	-40.5	-44.9	-44.0
x_2 ppm	30.5	36.6	33.6
x_3 ppm	106.4	108.8	107.6

For a comparative analysis of various results, the linear scalar magnitude of the shift vector,

$$r = R|x| = R\sqrt{x_1^2 + x_2^2 + x_3^2} \qquad (8.2)$$

has been calculated for each set, and presented in Table 8.3.

A striking feature revealed by this exercise is that the astro- satellite fitting practically yields the same values of r as obtained by other methods despite large differences in individual elements. The hypothetical centre of the satellite-derived geoid is thus found to be displaced from that of the local spheroid by the same amount as the latter from the earth's centre. But, a systematic tilt error appears to creep in the satellite-derivedgeoid, shifting the centre along an arc.

The comparative values of r also quantitatively indicate the order of reliability of the various methods. Whilst the gravimetric determination at the initial point is the most accurate, other methods including even the astro-satellite geoid-matching, can also be considered to be reasonably accurate yielding results better than atleast the existing satellite solution whose positional inaccuracy is about 20 metres.

TABLE 8.3

LINEAR SHIFT OF CENTRE OBTAINED BY VARIOUS METHODS

Method code	Station code	x_1 ppm	x_2 ppm	x_3 ppm	r metre	Remarks
a	0	-27.24	38.09	115.04	791	Code e =
b	Mean of 1, 2,3,4	-30.24	34.71	113.96	783	results from
c	23	-31.37	31.38	114.46	782	artificial
d	23	-39.5	20.2	114.42	781	satellites
e	mean	-44.0	33.6	107.6	771	

8.5 ESTIMATE OF ACCURACY

The accuracy of the computed gravimetric quantities can be estimated on the basis of the quality and distribution of gravity data actually used. Many studies of this nature have been made and nearly as many solutions advanced.

The results of these analyses point out to errors of ± 5m to ± 28m in the geoid height and of $\pm 0''.2$ to $\pm 1''.6$ in the deviation components (Szabo,1962).

Hirvonen (1956) estimated the errors to be as high as 10 metres in N and $0''.85$ in ξ or η. The high values of these overall estimates were mainly due to the lack of data in the Southern hemisphere. Kaula (1957) statistically computed the inaccuracies to be of the order of 5 metres in N and $1''$ in the deviation, assuming gravity data to be available in the surrounding zone only. Henrikson and Nash (1970) cited an error of $0''.55$ in ξ or η for an overall inaccuracy of 5 mgal in the gravity data. Obenson (1973) presented formulations suitable for machine - evaluation and sample calculations, the resulting errors being 4.5 metres and 0.6 arcsecond in undulation and deviation respectively.

However, all these estimates are of global nature and a little worse as compared with local determinations on land derived from closely distributed data. Rice(1952) reported on error of $0''.1$ in ξ or η with the circle-ring method upto zone 51. A lesser amount was estimated in Section 5.4 of chapter V in this thesis. Mather (1970) while orienting the Australian Geodetic Datum, assessed the error to be $0''.2$ in ξ and η and 0.2 metre in N, excluding the zero-degree term which was computed as,

$$N_o = \frac{-RE\{\Delta g\}}{G} \tag{8.3}$$

where, $E\{\Delta g\}$ is the global mean of gravity anomalies. The numerical value of N_o for GRS 67 is -2.8 metres.

The gravimetric results computed at the initial point itself constitute the best set of geocentric orientation parameters. The astronomic observations here contain practically no errors. The origin too is obviously free of geodetic errors. The gravity data for the exterior region, up to one-degree anomalies are most recent and updated up to the year 1978, the inaccuracies diminishing progressively. The surrounding region is also covered with well-distributed data corrected for the terrain- effect, thus providing nearly perfect simulation of the Stokesian model. The existing flat topography around the initial point also renders the geoid-cogeoid departure negligible. The comparison with the circle-ring method provides a further check on the numerical algorithm, and the overall error is not likely to exceed 0".2.

The difference between the mean results obtained from the set 'code b' and those from the set 'code a' are of the order of 2.5m in δN_o, 0".7 in $\delta \xi_o$, 0."6 in $\delta \eta_o$. These bounds, however, include all the errors explained earlier and should not be taken to underestimate the standard of accuracy actually achieved in the final determination. Similarly, the 'code c' and 'code d' results are not to be considered for error estimation as they belong to the different data sources and rest on methodologies which need to be further investigated.

In δN_0, the zero-order term has been automatically excluded from the gravimetric determination. A major part of the discrepancies of various results in respect of this parameter is attributed to the incorrect assessment of astro-geodetic geoid heights which at places may be out by 1 metre or more.

At the initial point, therefore, an estimate of 0.5 metre inaccuracy in δN_0 and 0."4 each in $\delta \xi_0$ and $\delta \eta_0$ may be safely regarded as constituting the upper limits.

8.6 CORRECTIONS TO ASTRO-GEODETIC GEOID AND BASE LINES

The astro-geodetic geoid height at a place needs a correction δN for conversion to geocentric geoid height. The numerical formula for the recommended shift vector and the change of dimensions of the spheroids being given by

$$\delta N = (173.5 + 181.4 \sin\phi)\sin\phi + (242.7\cos\lambda + 732.9\sin\lambda)\cos\phi - 861.0 \text{ metres} \quad (8.4)$$

The same formula yields the correction to be applied to the geometrical or spheroidal height from local to absolute terms.

Table 8.4 presents the correction δN at 5° grid corners covering the Indian region, and Figure 8.1 depicts the the same in contoured form.

Consequent upon absolute orientation and change of spheroid, base lines of the triangulation network also need some corrections. Instead of reducing the base-line to the local spheroid, a further reduction should be made before changing over to another geodetic system, this correction being

$$\delta b = - \delta N/R \tag{8.5}$$

with the sign convention used so far, and δb being positive for addition. Accordingly, the correction formula becomes

$$\begin{aligned} \delta b = & -(27.24 + 28.47 \sin\phi)\sin\phi \\ & -(38.09\cos\lambda + 115.04\sin\lambda)\cos\phi \\ & + 135.14 \quad \text{parts per million} \end{aligned} \tag{8.6}$$

The numerical value of this correction ranges from about 10 mm per km of base length in central India, to as high as 30 mm per kilometre on the South-Eastern side, i.e., in the Burma region.

TABLE 8 . 4

CORRECTIONS FOR CONVERSION OF ASTRO-GEODETIC GEOID HEIGHTS ON THE EVEREST SPHEROID TO GEOCENTRIC GEOID HEIGHTS ON GRS 67

λ	65°E	70°	75°	80°	85°	90°	95°	100°
φ	Values are in metres							
40°	-87.1	-83.4	-84.1	-89.3	-99.0	-113.1	-131.4	-153.9
35°	-73.7	-69.7	-70.5	-76.1	-86.4	-101.5	-121.1	-145.1
30°	-64.8	-60.7	-61.4	-67.3	-78.3	-94.2	-114.9	-140.3
25°	-60.3	-55.9	-56.8	-62.9	-74.4	-91.0	-112.7	-139.3
20°	-59.9	-55.3	-56.2	-62.6	-74.5	-91.7	-114.2	-141.8
15°	-63.3	-58.5	-59.5	-66.1	-78.3	-96.0	-119.1	-147.5
10°	-70.2	-65.4	-66.4	-73.1	-85.5	-103.6	-127.2	-156.1
5°N	-80.6	-75.7	-76.7	-83.5	-96.1	-114.4	-138.2	-167.5

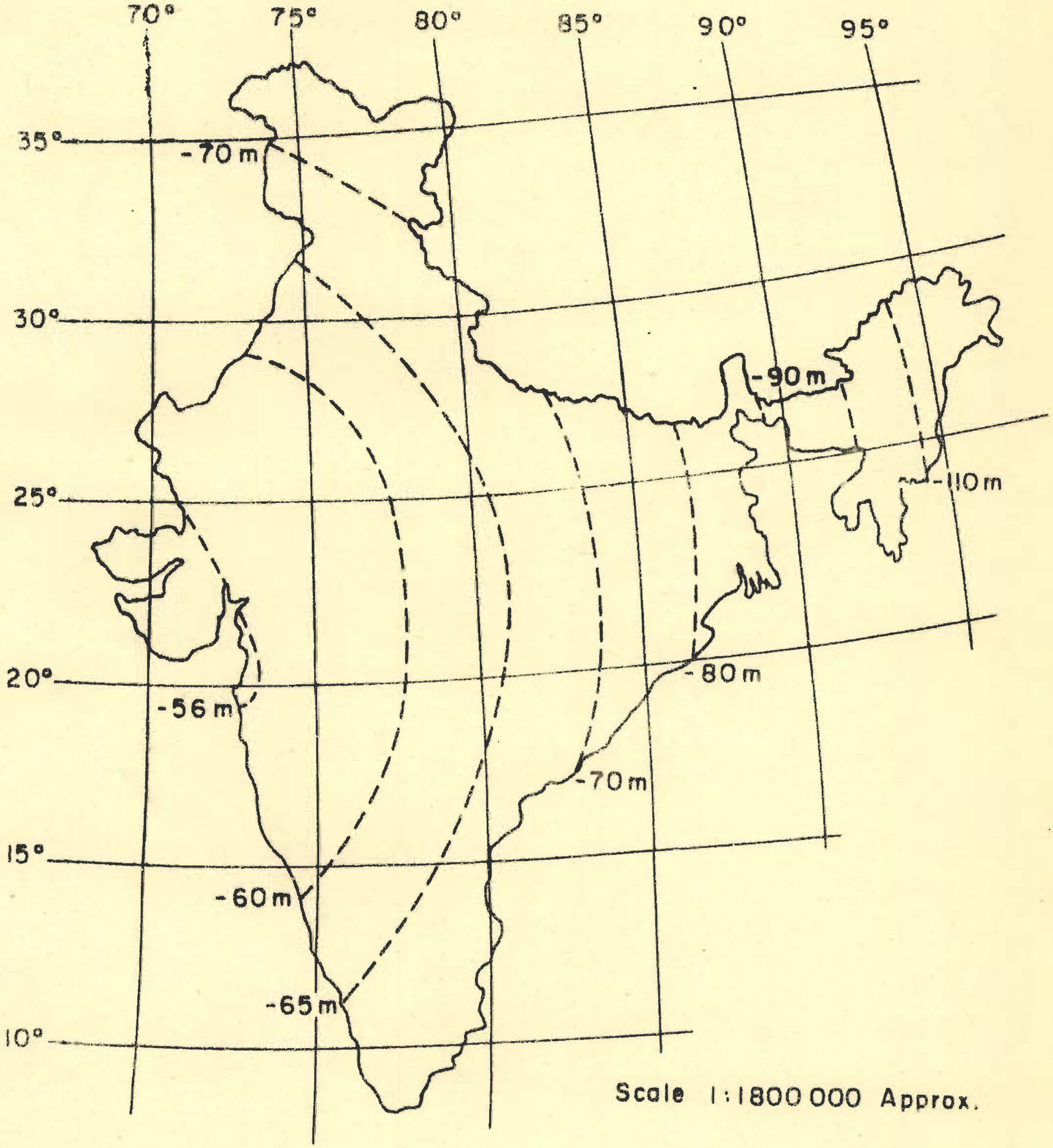


FIG. 8·1 CORRECTIONS ON EXISTING GEOID HEIGHTS, FROM EVEREST SYSTEM TO ABSOLUTE GRS 67

8.7 CORRECTIONS TO EXISTING LATITUDES AND LONGITUDES

The adoption of absolute orientation values will affect all published geodetic latitudes and longitudes. Corrections for these may be obtained directly from the formulae given in Expression 2.29, or from the matrix relationships formulated in this work and the relations

$$\delta\phi = -\delta\xi \qquad (8.7)$$

$$\delta\lambda = -\delta\eta\sec\phi$$

The correction needed in respect of latitudes, to convert from local Everest system to the absolute GRS 67 system, turns out to be equal to :

$$\delta\phi'' = (5.62 + 11.74\sin\phi)\cos\phi - (7.86\cos\lambda + 23.73\sin\lambda)\sin\phi \qquad (8.8)$$

Table 8.5 tabulates the amount of this correction at 5^{o} grid corners. The correction contours are drawn in Figure 8.2.

The published longitudes are reported (Gulatee, 1955) to be already in error by an amount equal to $3''.16$. When this correction is included, the final expression for relevant longitude corrections becomes,

$$\delta\lambda'' = (23.73\cos\lambda - 7.86\sin\lambda)\sec\phi - 3.16 \qquad (8.9)$$

The values at grid corners are tabulated in Table 8.6, TABLE - 8.5

CORRECTIONS TO PUBLISHED LATITUDES IN INDIA FOR CONVERSION TO ABSOLUTE GRS 67 SYSTEM

λ / ϕ	65°	70°	75°	80°	85°	90°	95°	100°
	Values are in arcseconds							
40°	-5.87	-5.98	-5.96	-5.81	-5.55	-5.17	-4.67	-4.06
35°	-4.12	-4.21	-4.19	-4.07	-3.83	-3.49	-3.05	-2.50
30°	-2.46	-2.54	-2.53	-2.42	-2.21	-1.91	-1.53	-1.05
25°	-0.90	-0.97	-0.96	-0.86	-0.69	-0.44	-0.11	+0.29
20°	+0.56	+0.51	+0.52	+0.59	+0.73	+0.94	+1.20	+1.53
15°	+1.94	+1.90	+1.90	+1.96	+2.07	+2.22	+2.42	+2.67
10°	+3.23	+3.20	+3.21	+3.25	+3.32	+3.42	+3.56	+3.72
5°	+4.45	+4.44	+4.42	+4.46	+4.50	+4.55	+4.62	+4.70

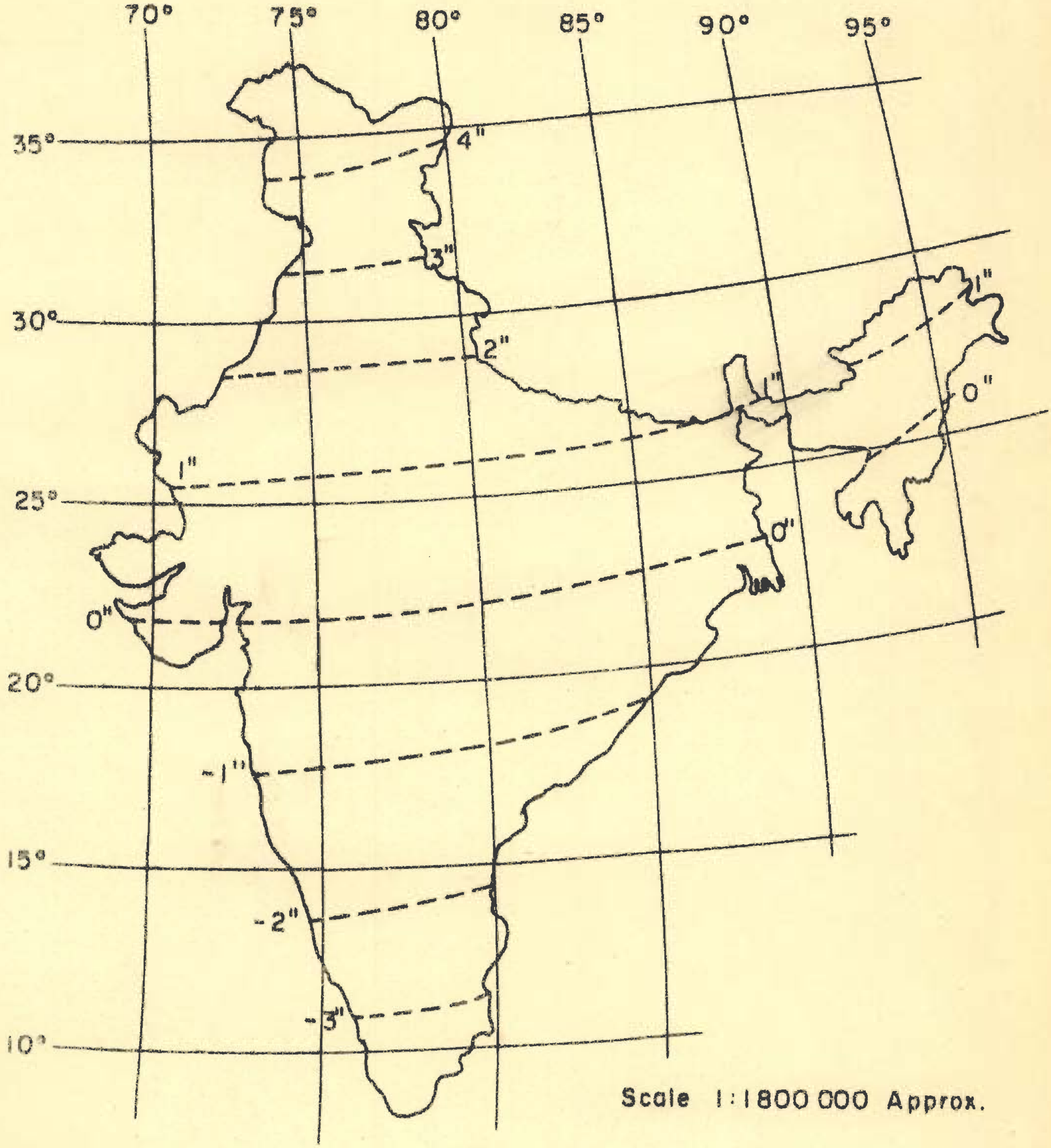


FIG. 8·2 CORRECTIONS ON EXISTING LATITUDES, FROM EVEREST SYSTEM TO ABSOLUTE GRS 67 and the graphical representation is given in Figure 8.3.

The changes in longitude are found to be higher on the Eastern side. A total correction of about 12" is required along the North-East boundary adjoining China, which is equivalent to a linear discrepancy of about 300 metres.

8.8 CONTRIBUTION OF THE WORK PRESENTED IN THE THESIS

The present investigation which constitutes the first integrated effort made in this country for obtaining the absolute orientation of the Indian Geodetic System leads to a set of orientation parameters and corresponding relations to reduce the existing geodetic latitudes and longitudes in India to the absolutely oriented GRS 67 spheroid, so as to relate it to the World Geodetic System.

The work being restricted by available data set, most of the results can only be regarded as being tentative. Leaving aside the gravimetric determination at the initial point, the field data was far below the required standards and had to be accepted as given. Furthermore, non - availability of instruments and data precluded any systematic and elaborate planning of data collection schemes. Notwithstanding this, the computer-oriented formulations of classical expressions and development of a number of

TABLE 8.6

CORRECTIONS TO PUBLISHED LONGITUDES IN INDIA FOR CONVERSION TO ABSOLUTE GRS 67 SYSTEM

λ	65°	70°	75°	80°	85°	90°	95°	100°
ϕ	values are in arcseconds							
40°	0.63	-2.21	-5.05	-7.89	-10.68	-13.42	-16.08	-18.64
35°	0.39	-2.27	-4.93	-7.58	-10.19	-12.76	-15.24	-17.64
30°	0.19	-2.32	-4.83	-7.34	-9.81	-12.24	-14.59	-16.86
25°	0.05	-2.35	-4.76	-7.15	-9.52	-11.83	-14.08	-16.25
20°	-0.07	-2.38	-4.70	-7.01	-9.29	-11.52	-13.69	-15.78
15°	-0.15	-2.40	-4.66	-6.91	-9.13	-11.30	-13.41	-15.44
10°	-0.21	-2.42	-4.63	-6.84	-9.01	-11.14	-13.21	-15.20
5°	-0.24	-2.43	-4.62	-6.79	-8.94	-11.05	-13.10	-15.07

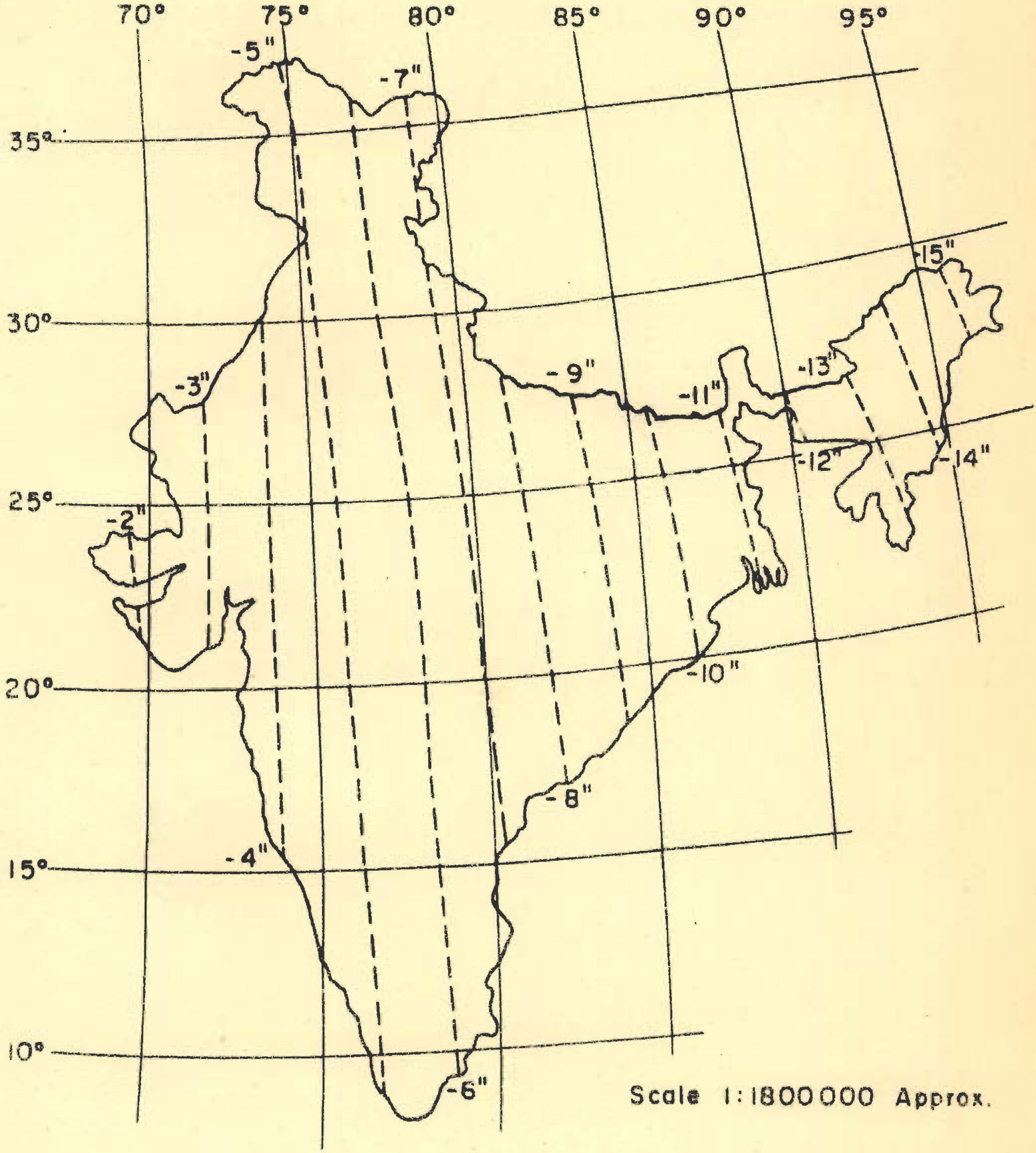


FIG. 8·3 CORRECTIONS ON EXISTING LONGITUDES, FROM EVEREST SYSTEM TO ABSOLUTE GRS 67 subprogrammes were carried out with the maximum refinement possible. Further accuracy in the determination of these parameters can thus be attempted in a routine way as and when more accurate data including denser gravity material becomes available in the future.

The various contributions made in the work presented here, apart from the accomplishment of the main objective, that is, obtaining the parameters of absolute orientation for the Indian geodetic system, may be summarized as follows :

(a) the contributions to geoidal undulations arising from a global coverage of gravity, extending from a certain regional limit, have been provided for the first time (chapter III). These results constitute a permanent asset to future work in the following directions:

(i) refining the orientation parameters from time to time as better data becomes available,

(ii) interpolation in astro-gravimetric levelling,

(iii) construction of detailed geoid for the Indian region,

(b) complete span of the numerical algorithm developed for computing the absolute orientation of the Indian Geodetic System through any given astro-geodetic station, has been detailed. The accuracy of the determination however depends upon the reliability of the relevant astronomic, geodetic, gravimetric and topographic measurements,

(c) whilst absolute orientation is a task normally performed at long intervals of time, another important purpose might occasionally be served by utilizing the computed gravimetric geoidal quantities at any point combined with both astronomic observations and spirit levelling, to arrive at the geocentric coordinates without any geodetic measurements. This astrogravimetric station may act as a control station for satellites and georeceiver experiments with fair accuracy,

(d) the intermediate values of the partial geoidal parameters at 1° grid corners, presented in chapter IV, may be taken directly to compute N_{gr}, ξ_{gr}, η_{gr} at any given point in the Indian region, as and when necessary dense gravity data around the point are made available,

(e) a broad shape of the 1° mean free-air geoid in the Indian region has been obtained and presented which should provide a useful starting point for further studies in this field,

(f) a consistent series of mesh sizes for integration according to the decimal degree system, has been recommended and the inner limits of the meshes are suggested (Table 5.3) to maintain the high precision of gravimetric determinations,

(g) a few one-degree (Table 4.3) and quarter-degree (Table 5.6) representative mean gravity anomalies around and within the Indian region has been contributed for storing and further use in geodetic and geophysical studies,

(h) a simple procedure for obtaining parameters of absolute orientation, by comparison of astro-geodetic and gravimetric geoids, has been attempted. This appears to be full of interesting possibilities for further explorations,

(i) an alternative suggestion, namely the comparison of satellite-derived geoid and surface-fitted astro-geodetic geoid, has also been discussed.

8.9 SUGGESTIONS FOR FURTHER WORK

Many difficulties and limitations were encountered during this investigation, which on reflection yield the following suggestions that may be found useful by future researchers in this area.

(a) The number and distribution of point gravity values in India are totally inadequate for integrated studies such as that attempted here. A centralized compiled data bank at the national level, with reliable topography-cum-gravity data file, in unified IGSN 71 system therefore appears quite essential and should prove valuable to diverse types of studies based on the analyses of gravimetric data.

(b) The distribution of astro-geodetic stations are also quite unsatisfactory for a vast subcontinent such as India and calls for filling up of gaps in order to provide a better representation. A best endeavour would be to provide all latitude stations with additional values of η as well, which will make the average station density equal to 1 in 3000 square kilometres.

(c) Evaluation and prediction of mean anomaly values was carried out in the present work keeping in mind mainly the immediate requirements as well as the limits of available facilities. Not much physical insight could therefore be achieved to establish the theoretical validity of some of the procedures used. Various deterministic approaches and probabilistic concepts (such as, correlation, collocation etc.) may prove more helpful in achieving these goals, with special reference to India as a whole or to some region of interest within it.

(d) Evaluation of the effect of the innermost rectangular compartment, needs to be further examined, to make the procedure complete and error-free,

(e) Four strategically located super-control points may be established at the first-order Laplace stations of primary triangulation at four geographic corners. Gravimetric determination of the geoidal undulations at these points will provide the non-Stokesian, i.e., the zero-order term N_o which controls the scale-error, simultaneously correcting the systematic errors in triangulation.

8.10 SUMMARY AND CONCLUSION

This chapter, the concluding part of the thesis, first recalls the basic objectives of and the motivation for undertaking the work as well as other related outputs. After compiling the shift vector components and correction parameters obtained by using three different principles, they were analyzed to yield finally a set of geocentric orientation parameters. These were again formally compared with datum shift components provided from the orbital analysis of satellites. After providing an estimate of accuracy of the recommended values, necessary formulations have been presented for correction of astro-geodetic geoid heights, lengths of base-lines, published latitudes and longitudes to convert these from the present Everest system to the absolute GRS 67 system. The main contributions of the investigation have thereafter been enumerated with suggestions for further studies and refinement.

The final results answering the basic objective of the work are the following set of required correction parameters of absolute orientation, obtained in respect of the Kalianpur origin, to convert the present Indian Geodetic System based on the Everest Spheroid to the geocentric Geodetic Reference System 1967:

$$\delta N_o = -59.0 \pm 0.5 \text{ metres}$$

$$\delta \xi_o = +0.65 \pm 0.4 \text{ arcseconds}$$

$$\delta \eta_o = +2.60 \pm 0.4 \text{ arcseconds}$$

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APPENDIX - A

CONVERSION OF ANOMALIES TO IGSN 71, GRS 67 SYSTEM

The International Spheroid is defined by the following basic elements (Heiskanen and Moritz, 1967):

$$GM = 3.986329E{+}14 m^3 \sec^{-2}$$

$$\omega = 7.2921151E{-}05 \text{ rad } \sec^{-1}$$

$$a = 6.378388E{+}06 m ,$$

$$f = 1/297.0$$

The corresponding normal gravity formula is,

$$\gamma_{30} = 978049.0(1+0.0052884 \sin^2\phi - 5.9 \times 10^{-6} \sin^2 2\phi)$$

$$= 978049.0(1+5.2648E{-}03 \sin^2\phi + 2.36E{-}05 \sin^4\phi) \text{ milligal} \quad (A.1)$$

The Geodetic Reference System, 1967 (International Union of Geodesy and Geophysics, 1967) is defined by

$$GM = 3.98603E + 14 m^3 \sec^{-2}$$

$$\omega = 7.2921151467E{-}05 \text{ rad } \sec^{-1}$$

$$a = 6.378160E{+}06 \text{ m},$$

$$J_2 = 1.0827E{-}03,$$

from which, flattening is determined to be

$$f = 1/298.247167427,$$

and the corresponding reference gravity in milligal to be

$$\gamma_{67}$$

$$= 978031.85(1+5.278895E\text{-}03 \sin^2\phi + 2.3462E\text{-}05 \sin^4\phi)$$
$$\pm 0.004 \text{ milligal} \qquad (A.2)$$

The International Gravity Standardization Network, 1971 (Morelli and Gantar, 1974) shows a correction of -14.01 milligal at the Potsdam base. In general, therefore, each gravity value referred to the old system should be corrected by

$$g_{new} = g_{old} - 14.01 \qquad (A.3)$$

The correction in the free-air gravity anomaly, neglecting the small change in the vertical gravity gradient, is now given by

$$\Delta = (\Delta g)_{new} - (\Delta g)_{old} = (g_{new} - \gamma_{67}) - (g_{old} - \gamma_{30})$$

From expressions (A.1),(A.2),(A.3), the correction is,

$$\Delta = (g_{old} - 14.01 - 978031.85 - 5162.9274x - 22.9466x^2)$$
$$- (g_{old} - 978049.0 - 5149.232x - 23.082x^2)$$

$$= (0.1354x - 13.6954)x + 3.1400 \text{ milligal} \qquad (A.4)$$

where $x = \sin^2\phi$

APPENDIX - B

INDIAN GEOID FROM BGI DATA

The BGI values (Coron, 1972) are expressed in the Meridian-Parallel-Grid system, and in terms of the old Potsdam value of absolute gravity and the 1930 international formula for normal gravity. Some blocks were assigned more than one values by different sources in which cases average values were adopted. Finally, the anomalies were reduced to IGSN 71, GRS 67 values through a conversion detailed in Appendix - A, and stored on punched cards, after rounding off to nearest 0.1 milligal.

A general computer program in Fortran language was developed to be used for determination of N, ξ, η at grid corners. In the present case, only N values were evaluated at a few corners covering the Indian region. The programme was later on used to compute the partial geoid parameters described in chapter IV, after slight modification needed for the zigzag outer limits in the one-degree data. The flow diagram is shown in chart B.1.

Table B.1 presents the corner values of N in metres, whereas Figure B.1 shows the N-contours in the Indian region.

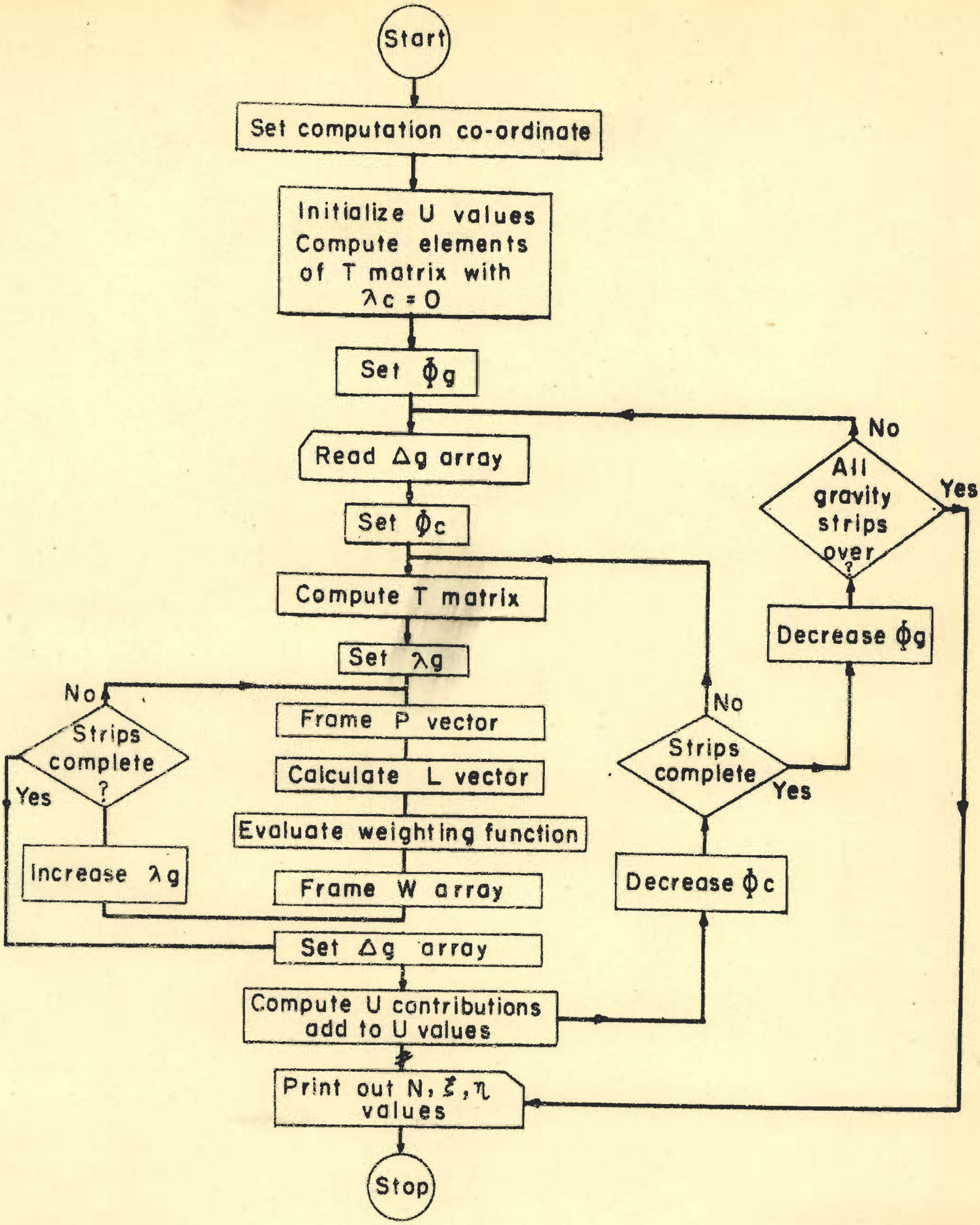


CHART B-1 FLOW DIAGRAM FOR UNDULATIONS FROM MPG TYPE DATA

TABLE B.1

VALUES OF N, IN INDIAN PART, FROM BGI ANOMALIES

λ	65°	70°	75°	80°	85°	90°	95°	100°
ϕ	Values are in metre, referred to GRS67. Geoid is below the Spheroid							
40°	-21.98	-26.81	-28.66	-30.69	-31.63	-30.17	-25.67	-21.22
35°	-12.84	-22.13	-26.60	-25.02	-24.30	-23.12	-20.84	-18.94
30°	-9.81	-22.76	-31.05	-33.12	-32.13	-29.22	-27.40	-24.04
25°	-21.45	-28.55	-35.05	-41.43	-41.41	-38.95	-35.38	-28.08
20°	-34.82	-40.82	-44.70	-45.21	-44.97	-43.93	-36.57	-26.20
15°	-46.66	-56.24	-60.40	-59.14	-55.58	-47.95	-34.44	-20.60
10°	-55.09	-68.55	-75.12	-75.75	-67.68	-51.34	-31.81	-12.67
5°	-57.78	-73.83	-84.55	-85.26	-73.12	-53.05	-29.50	-5.58

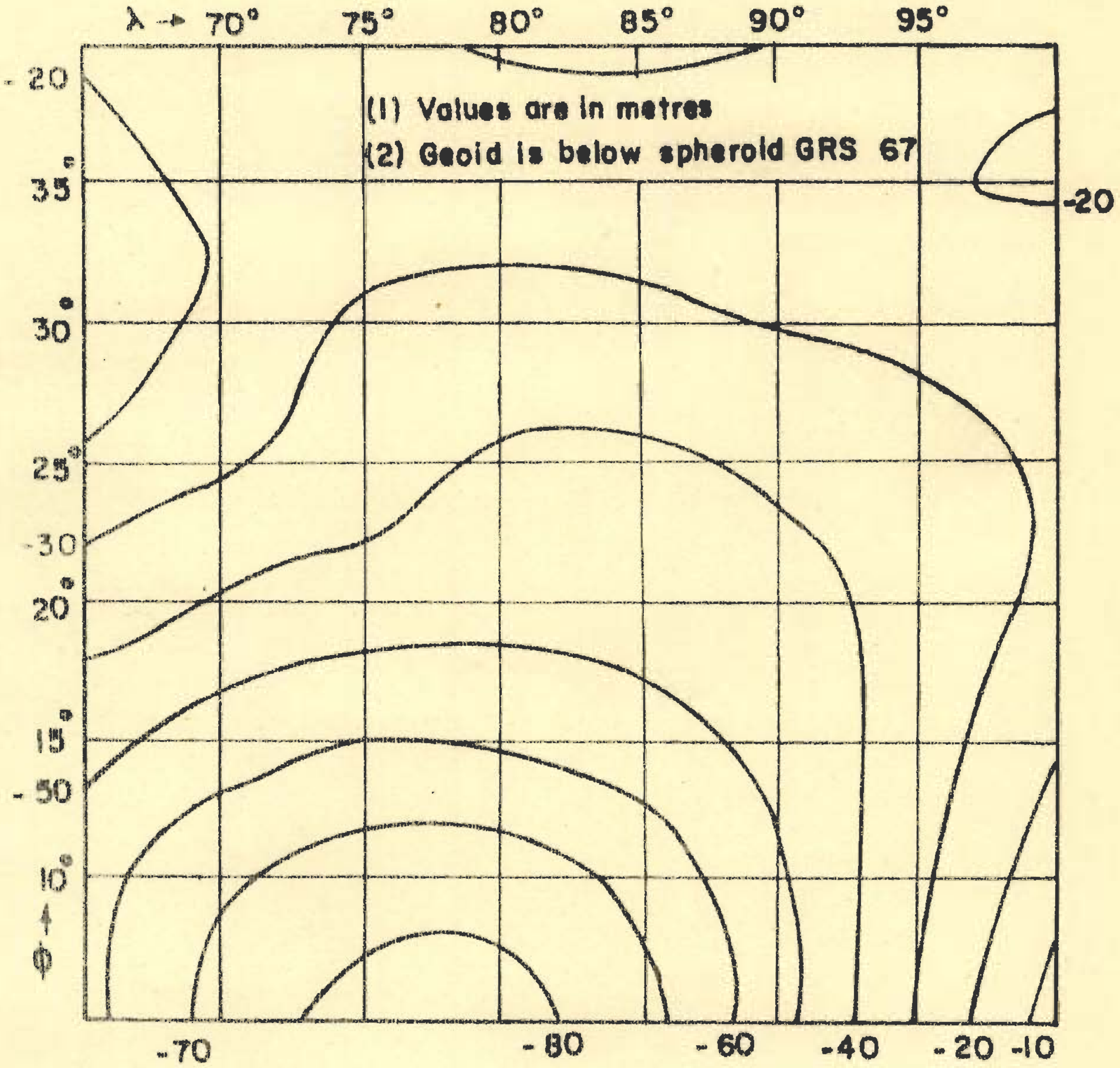


FIGURE B.1 - INDIAN GEOID FROM BGI DATA

APPENDIX - C

BROAD GRAVIMETRIC GEOID IN INDIAN CONTINENT AND PRIMARY ORIENTATION OF NETWORK

(A paper presented in the Indian Science Congress Session, January 1978)

Abstract

Average free-air gravity anomalies of five degree blocks over the entire earth are used to determine the geoid-spheroid separations at five degree corners in Indian part. Numerical integration of the well-known Stokes' formula is done by electronic computer programming. The obtained geoid map, equivalent to the broad wavelength geoid in satellite geodesy term, is then used to compute the primary orientation-vector at the Indian origin of triangulation network assuming linear interpolation along meridians and parallels. Consequently, corrections in the existing latitudes and longitudes are calculated and presented in contoured form.

Introduction

More than a century ago, Everest derived the dimensions of the Everest spheroid on which the Indian geodetic system is based[1]. Its orientation at the datum has been done at various times in arbitrary manner. After the

International Spheroid was adopted by IUGG, the Indian geoid was locally fitted to it by a least square solution, which being again more or less arbitrary, is unfit for use as a World Geodetic System.

The geocentric system refers to the spheroid as a physical surface with its origin at the centre of gravity of the earth and Z -axis as the mean axis of its rotation. The absolute orientation of a geodetic system essentially means defining the reference surface dimensions (a,f) and its deviation components including separation from geoid surface at the control point.

To accomplish the object, one must know the gravity values on the entire earth surface and apply gravimetric principles for computing the gravity anomalies, properly reducing these to geoid as a prerequisite for the boundary value problem, and use numerical integration for Stokes' formula.

Gravity Anomalies

The mean free-air gravity values at $5^{\circ} \times 5^{\circ}$ blocks are taken from Bulletin D'information[2], referring to 1930 international formula with base elements :

Flattening = 1/297 ; Potsdam value = 981274 mgal

$$\gamma = 9780490(1+0.0052884 \sin^2\phi - 0.0000059 \sin^2 2\phi)\text{mgal}$$

The single value for a block has been directly taken, and two values for the same box have been averaged, as serving the present purpose.

Helmert's condensation method for reduction gravity has been recommended as most suitable for geoid determination[3]. However, the mean free-air values correspond very nearly to the condensation reduction and resulting undulation of cogeoid differes by only a few meters, acceptable for primary adjustment.

Basic Formula and Modification

The undulation of geoid from a reference spheroid is obtained by,

$$N = \frac{R}{4\pi G} \int f(\psi) \Delta g_q \, dq \text{ , integrated over whole earth,}$$

where, R = Mean value for earth's radius,

G = Mean value of gravity,

dq = Elemental surface area of the gravity element,

ψ = Angular distance of gravity element from computation point,

Δg_q = Average gravity anomaly in the surface element

and $f(\psi)$ = Stokes' function

$$= 1 - 6\sin\frac{\psi}{2} + \operatorname{cosec}\frac{\psi}{2}$$

$$- \cos\psi\left[5+3\,\mathrm{Ln.}\left(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2}\right)\right]$$

The square division modification changes this to a summation formula. Also if the corner points of the blocks are taken as computation points, then same $f(\psi)$will be repeated at the points along a parallel and hence the evaluation of a set of factors are sufficient for all corners at the same latitude. Tanni[4] did this by Desk Calculator, and that posed no difficulty because of lack of gravity observations at that time. It is worthwhile to mention here that he took an average of 1.5 hrs. for one computation of N, and the present computer programme took only 2 minutes for all the 81 corners, in IBM 360. The resulting geoid contour is presented in Fig.1.

Orientation Vector and Adjustment

The geoid spheroid separation N at Kalianpur, the initial points of Indian triangulation network, now comes to be -53.4 m approximately. The meridional and prime vertical components of the deviation of vertical are also assessed by applying finite-difference slope at corners and linear interpolation along meridians and parallels. Table 1 shows the values obtained, and the existing arbitrary vector.

The effect of this datum shift on latitudes and longitudes are computed by the transformation formulae[5] derived by Vening Meinesz and rearranged for the present

FIG 1

GRAVIMETRIC PRIMARY GEOID IN INDIA

VALUES ARE IN METRE
EVERY WHERE GEOID IS BELOW SPHEROID
(INTERNATIONAL 1930)

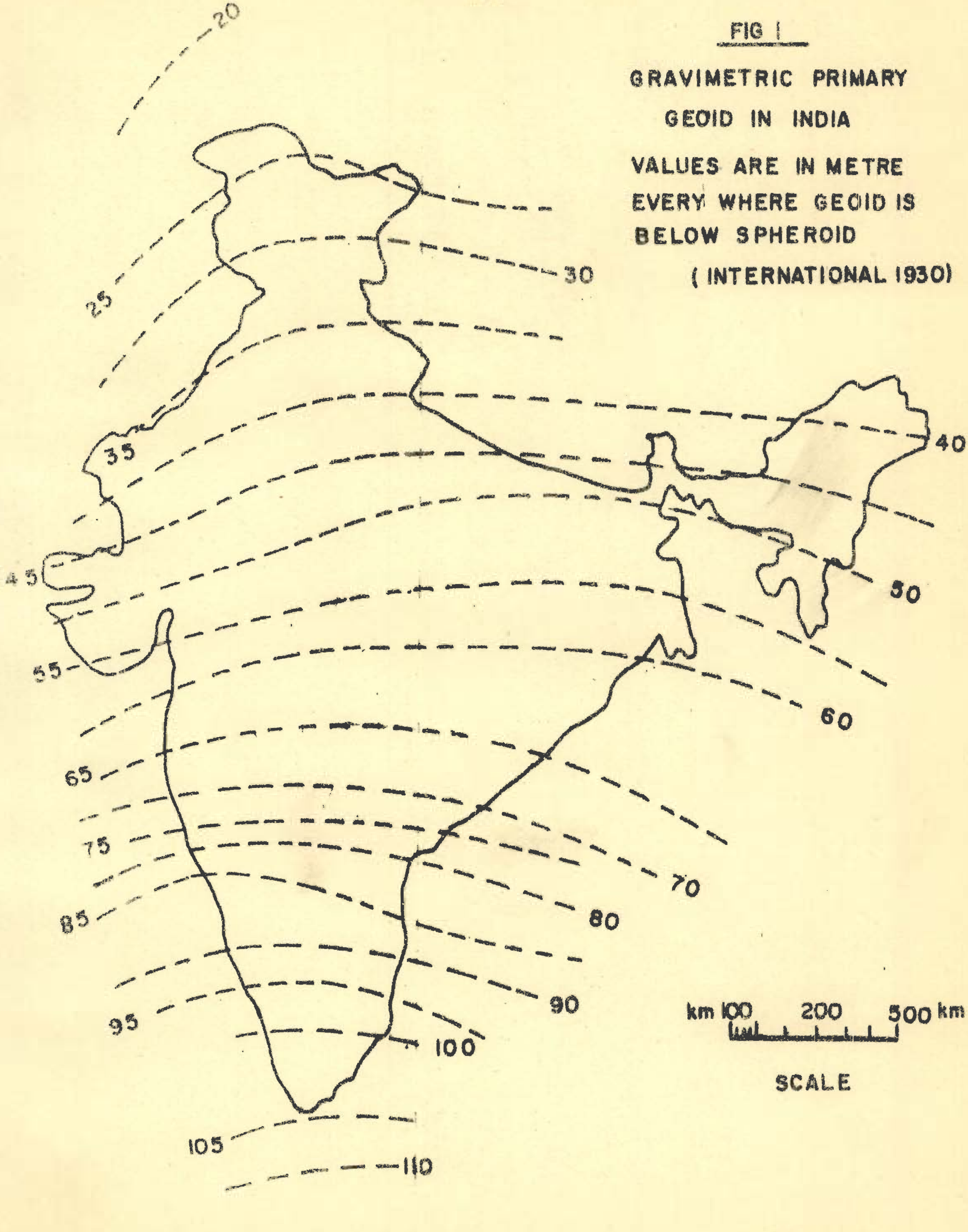


TABLE - 1

Orientation Components of Indian Geodetic Datum

Existing		Primary Gravimetric Computation
+9.5 m	N	- 53.4 m
-2''.42	ξ	- 4''.36
-3''.17	η	+ 0''.25

Both are referred to Hayford International Ellipsoid,
a = 6378388 m ; f = 1/297.0(1930)

purpose. The resulting corrections are shown, in contour form, in Fig.2. This adjustment will thus make the Indian geodetic system primarily converted to the geocentric International Spheriod System.

Conclusion

The broad gravimetric geoid, obtained in the present work by using numerical integration of Stokes' formula on mean $5^{\circ} \times 5^{\circ}$ free air gravity values, is rather a trend-surface and for more detailed evaluation, as needed for accurate absolute orientation, the following points are amongst those to be considered carefully,

i) The free air values, specially in the continental parts and around the computation points should be

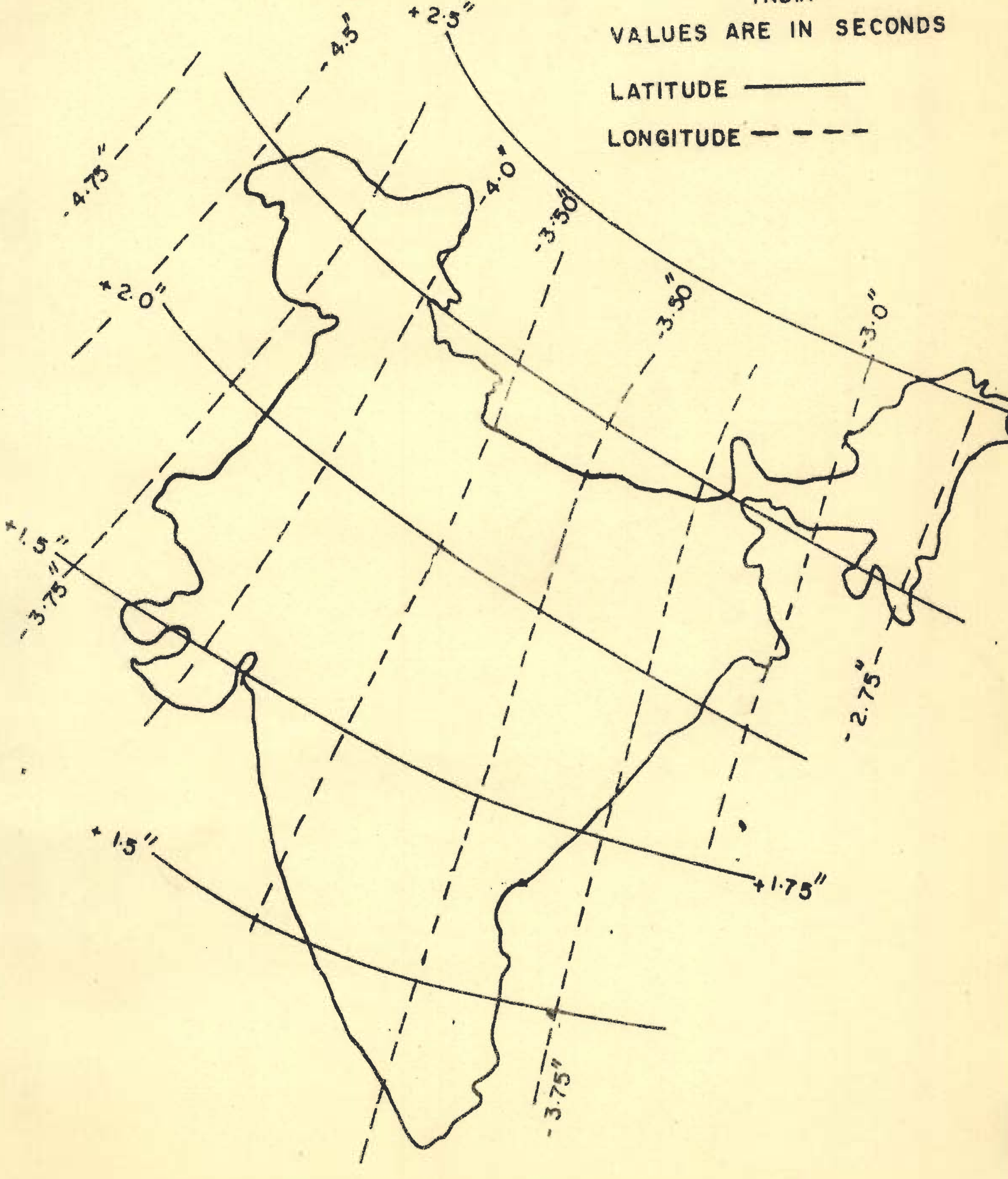
FIG. 2

CORRECTION IN EXISTING LATITUDES AND LONGITUDE IN INDIA

VALUES ARE IN SECONDS

LATITUDE ————

LONGITUDE - - - -



reduced by Helmert's condensation; and finally the indirect effect must be accounted for to arrive at the actual geoid.

ii) Closer network is needed near the computation points, because the Stokes' function diverges as the spherical distance decreases.

iii) More anomaly values are required for $\psi = 20^{\circ}$ or less, to have a correct representation of the gravity changes and proper integration procedure.

iv) The deviation-components of the vertical can better be evaluated by differentiating the Stokes' function, as proposed by Vening Meinesz, and applying that directly as summation weightage.

From satellite-derived geopotential coefficients featuring the broad-wavelength properties, a comparable geocentric geoid may be arrived at. The study of geoid by gravimetry and satellite observation are of prime importance, not only for surveying and mapping purposes like (a) correct base line reduction, (b) proper defining of control points for triangulation, including national boundaries, (c) Super control points for satellite geodesy and long-range hydrographic surveying, but also for researches in geophysics so far as assessing invisible mass anomalies in Indian part, for possible interpretation of natural reasources, are concerned.

Acknowledgement

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APPENDIX- D

TWO-DIMENSIONAL CUBIC SPLINE INTERPOLATION

Let the discrete values of a variable at grid corners be defined by the usual matrix convention,

$$H = \begin{bmatrix} h_{11} & h_{12} & - \\ h_{21} & - & - \\ - & - & h_{KK} \end{bmatrix} \qquad (D.1)$$

where h_{mn} indicates the value at the corner (ϕ_m, λ_n) (Figure D.1)

A cubic curve passing through the points $(\phi_m, \lambda_n, h_{mn})$ and $(\phi_{m+1}, \lambda_n, h_{m+1,n})$ with curvatures $-6p_{m,n}$ and $-6p_{m+1,n}$ at the respective ends may be expressed in Hermitian form as,

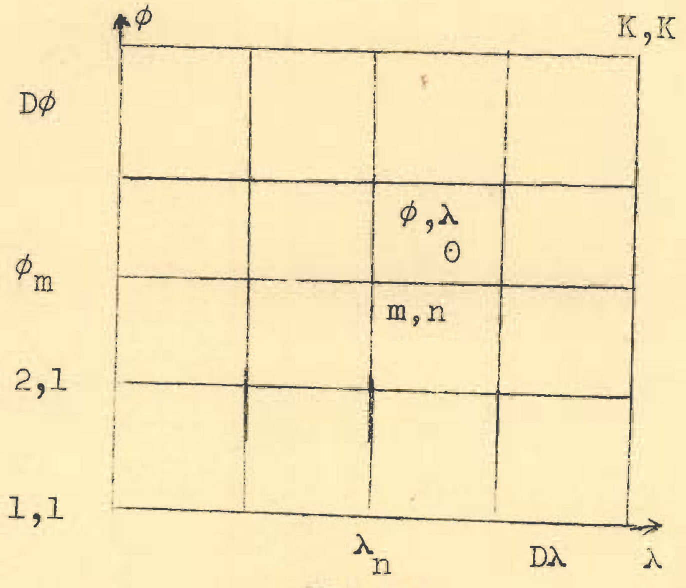


FIG. D.1

$$Z(\phi, \lambda_n) = (h_{m,n})\, u + (h_{m+1,n})x + (p_{m,n})ua$$
$$+ (p_{m+1,n})\, xb$$

where, $\phi_m \leq \phi \leq \phi_{m+1}$,

$$x = (\phi - \phi_m)/D\phi$$
$$u = 1 - x \qquad \text{(D.2)}$$
$$a = 1 - u^2$$
$$b = 1 - x^2$$

Assuming similar cubic variation between $Z(\phi,\lambda_n)$ and $Z(\phi,\lambda_{n+1})$ in the other direction, the general expession becomes,

$$Z(\phi,\lambda) = Z(\phi,\lambda_n)v + Z(\phi,\lambda_{n+1})y + q(\phi,\lambda_n)vc$$
$$+ q(\phi,\lambda_{n+1})yd$$

where, $\lambda_n \leq \lambda \leq \lambda_{n+1}$

$$y = \lambda - \lambda_n$$
$$v = 1 - y \qquad \text{(D.3)}$$
$$c = 1 - v^2$$
$$d = 1 - y^2$$

and the curvatures at the ends are $-6q(\phi,\lambda_n)$ and $-6q(\phi,\lambda_{n+1})$.

The second derivative of expression D.2 will show linearity in p. Assuming a similar linearity in q along ϕ in the form,

$$q(\phi,\lambda_n) = (q_{m,n})u + (q_{m+1,n})x$$

$$\text{and} \quad q(\phi,\lambda_{n+1}) = (q_{m,n+1})u + (q_{m+1,n})x, \tag{D.4}$$

the complete expansion of expression D.3 becomes

$$\begin{aligned} Z(\phi,\lambda) = {} & (h_{m,n})uv + (h_{m,n+1})uy + (h_{m+1,n})xv \\ & +(h_{m+1,n+1})xy + (p_{m,n})uva + \\ & +(p_{m,n+1})uya + (p_{m+1,n})xvb + \\ & +(p_{m+1,n+1})xyb + (q_{m,n})uvc + \\ & +(q_{m,n+1})uyd + (q_{m+1,n})xvc + \\ & +(q_{m+1,n+1})xyd \end{aligned} \tag{D.5}$$

The single value assigned to each of p and q at each corner ensures continuity of curvature of the whole surface. Conditions of slope continuity at both directions at a general grid point ,m,n are achieved by,

$$p_{m-1,n} + 4p_{m,n} + p_{m+1,n} = h_{m-1,n} - 2h_{m,n} + h_{m+1,n}$$

$$q_{m,n-1} + 4q_{m,n} + q_{m,n+1} = h_{m,n-1} - 2h_{m,n} + h_{m,n+1} \tag{D.6}$$

Imposing the end conditions that the surface becomes planar just beyond the boundaries, i.e.,

$$
\begin{aligned}
p_{1,n} &= 0, \\
p_{k,n} &= 0, \\
q_{m,1} &= 0, \\
\text{and} \quad q_{m,k} &= 0
\end{aligned}
\qquad (D.7)
$$

the relations become

$$
T \begin{bmatrix} p_{21} & p_{22} & - \\ - & - & - \\ - & - & p_{k-1,k} \end{bmatrix} = CH ,
$$

$$
T \begin{bmatrix} q_{12} & q_{22} & - \\ q_{13} & - & - \\ - & - & q_{k,k-1} \end{bmatrix} = CH' \qquad (D.8)
$$

where T is a tridiagonal matrix of size (k-2) x (k-2) of the form,

$$
T = \begin{bmatrix} 4 & 1 & & & \\ 1 & 4 & 1 & & \\ & - & - & - & \\ & & 1 & 4 & \end{bmatrix} \qquad (D.9)
$$

$$
\text{and} \quad C = \begin{bmatrix} 1 & -2 & 1 & & \\ & 1 & -2 & 1 & \\ & & - & - & - \\ & & 1 & -2 & 1 \end{bmatrix} \qquad (D.10)
$$

By operating $T^{-1}C$ and adding two rows, of all zeros, at top and bottom (to satisfy the end conditions), a centrosymmetric matrix S is formed. The final solution is then

$$\begin{aligned} P &= S\,H \\ Q' &= S\,H' \end{aligned} \qquad \text{(D.11)}$$

After getting numerical values of P and Q by Equations D.11 from input H and corresponding matrix S, interpolation at any ϕ,λ may be done by Expression D.5.

S matrix for k = 4,5,6,7, and 8 are given below:

$$S_{4x4} = \frac{1}{15}\begin{bmatrix} 0 & 0 & 0 & 0 \\ -4 & 9 & -6 & 1 \\ 1 & -6 & 9 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$S_{5x5} = \frac{1}{56}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -15 & 34 & -24 & 6 & -1 \\ 4 & -24 & 40 & -24 & 4 \\ -1 & 6 & -24 & 34 & -15 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$S_{6x6} = \frac{1}{209}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -56 & 127 & -90 & 24 & -6 & 1 \\ 15 & -90 & 151 & -96 & 24 & -4 \\ -4 & 24 & -96 & 151 & -90 & 15 \\ 1 & -6 & 24 & -90 & 127 & -56 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$S_{7x7} = \frac{1}{780}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -209 & 474 & -336 & 90 & -24 & 6 & -1 \\ 56 & -336 & 564 & -360 & 96 & -24 & 4 \\ -15 & 90 & -360 & 570 & -360 & 90 & -15 \\ 4 & -24 & 96 & -360 & 564 & -336 & 56 \\ -1 & 6 & -24 & 90 & -336 & 474 & -209 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$S_{8x8} = \frac{1}{2911}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -780 & 1769 & -1254 & 336 & -90 & 24 & -6 & 1 \\ 209 & -1254 & 2105 & -1344 & 360 & -96 & 24 & -4 \\ -56 & 336 & -1344 & 2129 & -1350 & 360 & -90 & 15 \\ 15 & -90 & 360 & -1350 & 2129 & -1344 & 336 & -56 \\ -4 & 24 & -96 & 360 & -1344 & 2105 & -1254 & 209 \\ 1 & -6 & 24 & -90 & 336 & -1254 & 1769 & -780 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

APPENDIX - E

PREDICTION OF GRAVITY ANOMALIES USING COVARIANCE FUNCTION

PRINCIPLE

Let h_p be a value to be predicted, and linearly dependent on the known values $h_1, h_2, h_3, \ldots, h_n$ which are, in turn, mutually correlated according to their separation distances. The relation may be expressed as

$$h_p = \sum_{i=1}^{n} a_i h_i \qquad (E.1)$$

If the expectance of the values are to be consistent before and after inclusion of the predicted h_p, then,

$$E\{h_i\} = E\{h_p\}$$

$$= \Sigma\, a_i\, E\{h_i\} = E\{h_i\} \Sigma a_i \qquad (E.2)$$

If $\Sigma a_i \neq 1$ as a general case, then $E\{h_i\}$ must be zero, indicating thereby that h values should initially be <u>centered</u> such that their mean is zero.

The mutual correlation are expressed by the <u>covariance</u> obtained by forming the average product of pairs of of values h_i, h_j constrained by a distance s_{ij}, i.e.

$$c(s_{ij}) = E\{h_i h_j\}, \qquad (E.3)$$

a special case for s = 0 being the variance,

$$c(s_{ii}) = c(0) = v = E\{h_i^2\} = \frac{\Sigma h_i^2}{n} \tag{E.4}$$

The error of prediction may be denoted as,

$$e = h_t - h_p \tag{E.5}$$

where h_t is the true value of h_p.

According to least-squares principle, the relating coefficients a_i are to be chosen in such a way that,

$$\frac{\delta E\{e^2\}}{\delta a_i} = 0 , \quad i = 1,2,\ldots,n \tag{E.6}$$

where,

$$E\{e^2\} = E\{(h_t - \Sigma a_i h_i)^2\}$$

$$= E\{h_t^2\} - 2\Sigma a_i \; E\{h_t h_i\} + \Sigma\Sigma a_i a_j \; E\{h_i h_j\}$$

Substitution of (E.3) in the differential relation (E.6) leads to,

$$0 = 0 - 2c(s_{ti}) + 2\Sigma a_j c(s_{ij}) \; , \; i = 1,2,\ldots,n$$

or, the set of simultaneous equations,

$$\begin{aligned}
a_1 \, c(s_{11}) + a_2 \, c(s_{12}) + \quad & a_n \, c(s_{1n}) = c(s_{t1}), \\
a_1 \, c(s_{21}) + a_2 \, c(s_{22}) + \quad & a_n \, c(s_{2n}) = c(s_{t2}), \\
\vdots \qquad\qquad \vdots \qquad\quad & \vdots \qquad\qquad \vdots \\
a_1 \, c(s_{n1}) + a_2 \, c(s_{n2}) + \quad & a_n \, c(s_{nn}) = c(s_{tn})
\end{aligned} \tag{E.7}$$

The linear equations may be solved if the statistical behaviour of the values are determined first through the covariance function. On getting the coefficient a_i, then, the value h_p may be predicted from the Expression E . 1.

MODIFICATION

In the present work, a few modifications of the basic principle have been made for computer-oriented formulation, which are described below:

(a) to discretize the function, a digitization gap needs to be assigned. The shortest distance between the known points may be used. In gridded data, the gap is simply,

$$d = D\phi = D\lambda \qquad (E.8),$$

(b) to compute and store the covariances as subscripted variable, an equivalent integer subscript is formed as,

$$L_{ij} = \left\{ \frac{(x_i - x_j)^2 + (y_i - y_j)^2}{d^2} + 0.5 \right\} \qquad (E.9)$$

where { } indicates the integer part within it. This avoids unnecessary evaluation of square roots,

(c) as already explained, the original gravity anomalies are centred and h_p is obtained by,

$$\Delta g_p = \Sigma a_i h_i + \overline{\Delta g}$$

where, $\overline{\Delta g} = \frac{\Sigma \Delta g_i}{n}$ (E.10)

and $h_i = \Delta g_i - \overline{\Delta g}$,

(d) the original covariances are standardized by dividing them by the variance; this makes all the diagonal elements of the Equation E-7 as unity. In the program, a counter array has also been provided to count the number of pair of points. The subroutine COVAR developed in Fortran II language suitable for IBM 1620 is given as follows to illustrate the algorithm for computing the standardized covariances.

```
      SUBROUTINE COVAR (X,Y,DG,NP,G,M,C,GM )
      DIMENSION X(100), Y(100), DG(100), C(200), P(200)
C     X,Y,DG = POSITION COORDINATES AND ANOMALIES
C     NP = NUMBER OF DATA POINTS, G = DIGITIZATION GAP
C     M = MAXM. DIMENSION OF C = (LARGEST DISTRANCE/G)**2+1
C    C =STANDARDIZED COVARIANCE,GM= MEAN OF INPUT ANOMALIES
C     P = COUNTING ARRAY
C     INPUT = X,Y,DG,NP,G,M, OUTPUT = C, GM
      DO 41 N = 1,M
      C(N) = 0.0
```

```
      H = NP
      H = 1./H
      GM = 0.0
      V = 0.0
      DO 42 J = 1,NP
      D = DG(J)
      GM = GM+D
42    V = V + D*D
      GM = GM*H
      V = V*H - GM*GM
      H = 1./(G*G)
      DO 43 N = 1,NP
      XN = X(N)
      YN = Y(N)
      ZN = DG(N)-GM
      I = N + 1
      DO 43 J = I,NP
      L = ((XN - X(J))**2 + (YN-Y(J))**2)*H + 0.5
      C(L) = C(L) + ZN*(DG(J)-GM)
43    P(L) = P(L) + 1.0
      DO 45 N = 1,M
      H = P(L)
      IF(H)99,45,44
44    C(N) = C(N)/(H*V)
45    CONTINUE
99    RETURN
      END
```

APPENDIX - F

DESCRIPTION OF COMPUTER PROGRAMMES

F.1 INTRODUCTION

The primary objective of getting an updated definition of the correction parameters at the Indian origin to make the reference frame absolute as well as to convert the existing ill-fitting spheroid to an internationally accepted one , has been achieved through this investigation.

But this is not the final word; the attempts and the findings are constrained by various limitations, specially those of availability and reliability of usable data, of proper consciousness of mathematical and statistical models for processing the same and last but not the least, by restrictions on time, skill, software and finance.

However, the exercise may open up further avenues of research and development in the methodology of gravimetric orientation in India and looking for a practically complete data bank in near future, the comparable efficient computational link hereby will make the revision a routine job.

The universally recognized FORTRAN language has been used in the computations for this study. The FORGO version were needed to run intermediate routines, in IBM 1620 with card output, for checking and converting raw data in formatted cards. An IBM 360 computer, and later on another IBM 370 recently installed could be made available. Peripherals were not exploited fully, due to lack of proper expertise.

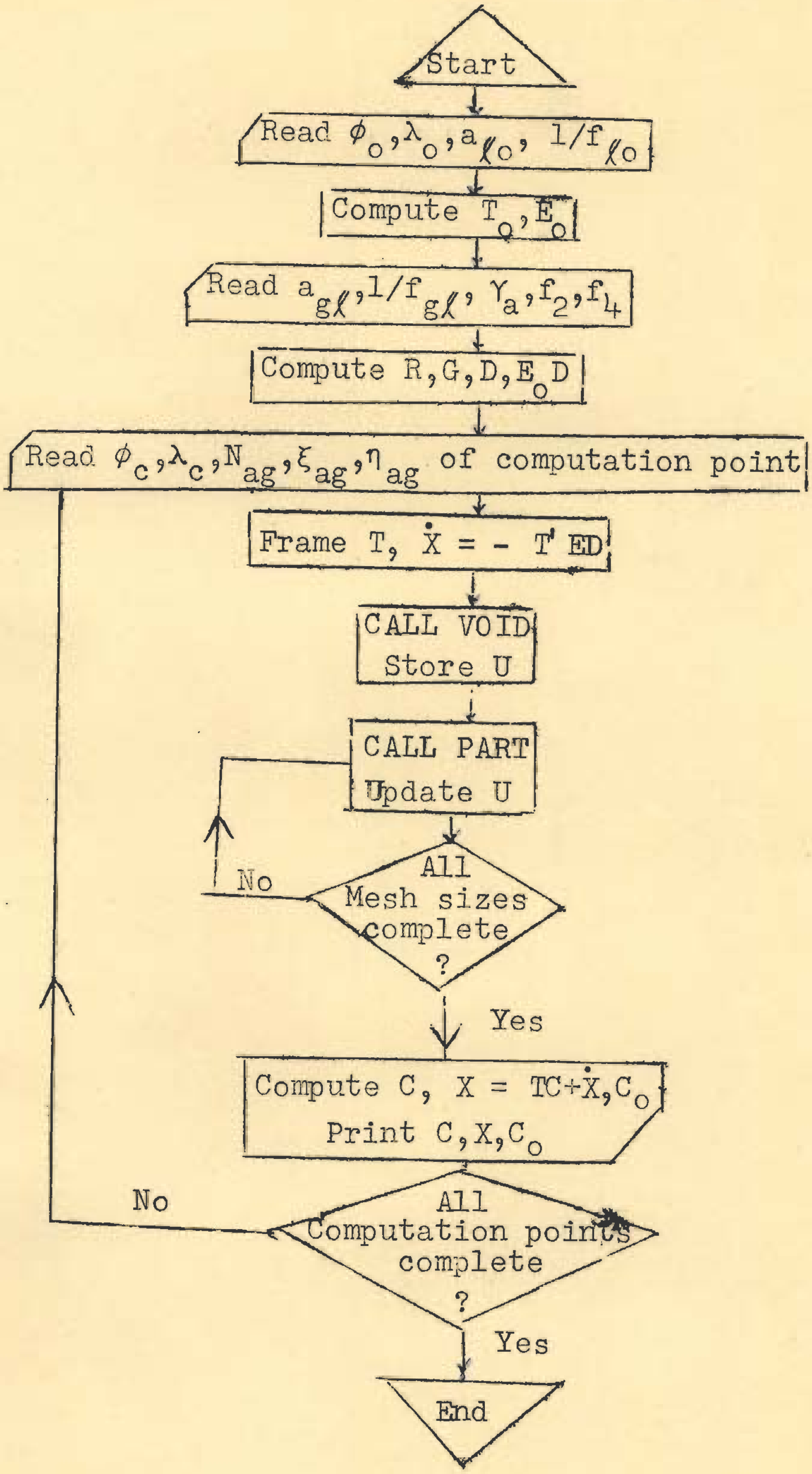
F.2 MAIN PROGRAMMES

The basic programme ORIENT computes the orientation parameters and shift vector, using mean gravity anomalies around any general astro-geodetic station, which may be the initial point itself also, through the subprogrammes VOID and PART.

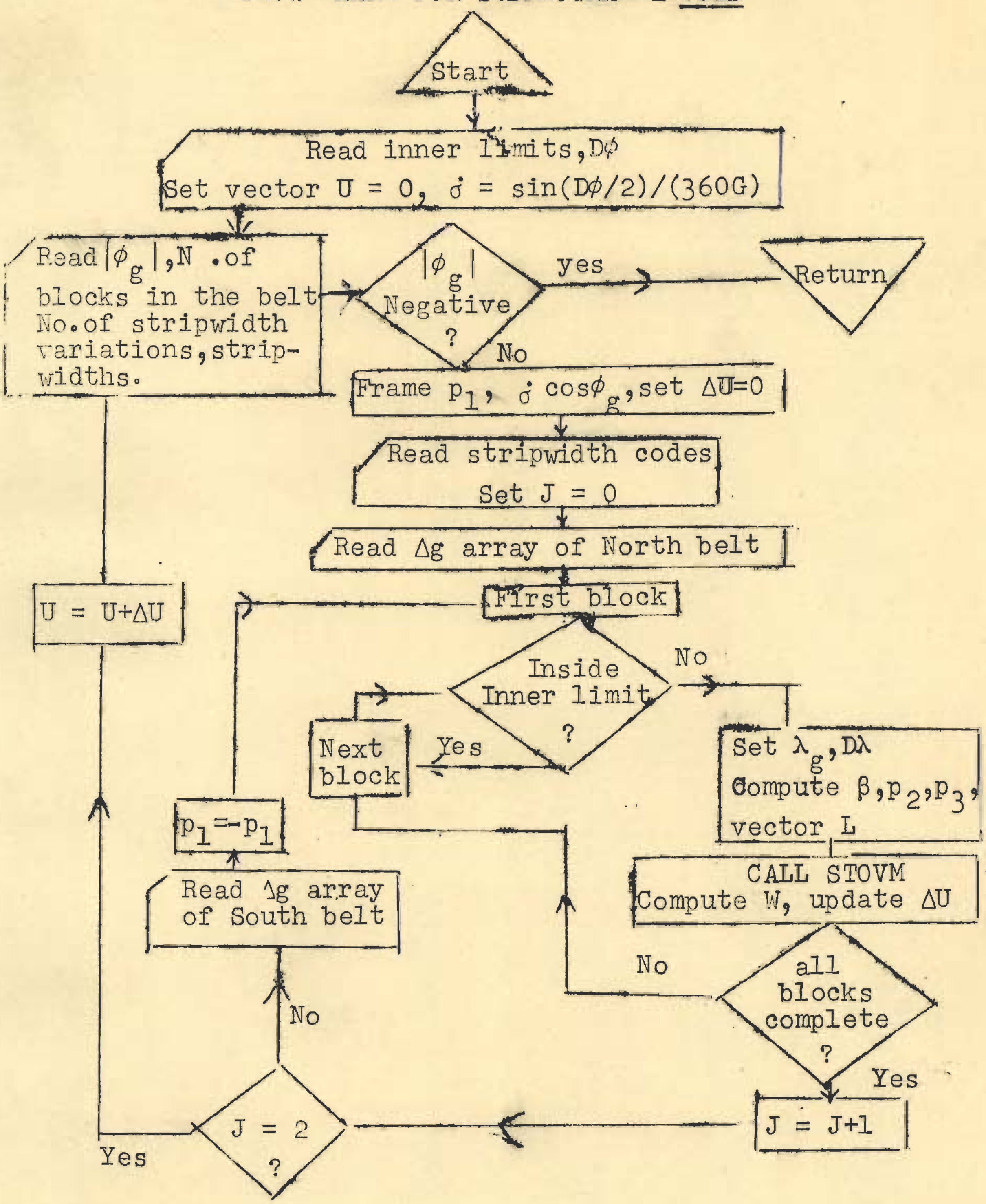
The subprogramme VOID uses Five-degree Equal-Area-Block mean values in the exterior region to compute N_V/R, ξ_V, η_V parameters. The PART subprogramme similarly computes the contribution of the interior region from the gridded mean values of One -degree units and finer compartments, down to the innermost zone (through the subroutine ZONIN).

The flow charts of the programmes ORIENT, VOID and PART are shown in the following pages, with an example of data cards for the VOID subroutine. Other utility subroutines COSIN, DCOSIN, COVERS, STOVM, STOVN, ZONIN and SURFIT are also briefly described thereafter.

FLOW CHART FOR MAIN PROGRAM ORIENT



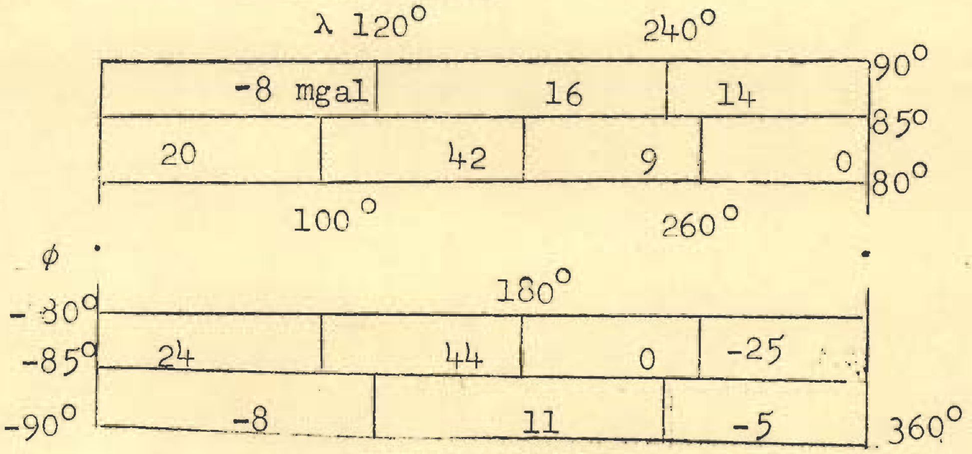
FLOW CHART FOR SUBPROGRAMME VOID



EXAMPLE OF INPUT DATA CARDS FOR BLOCK MEAN ANOMALIES

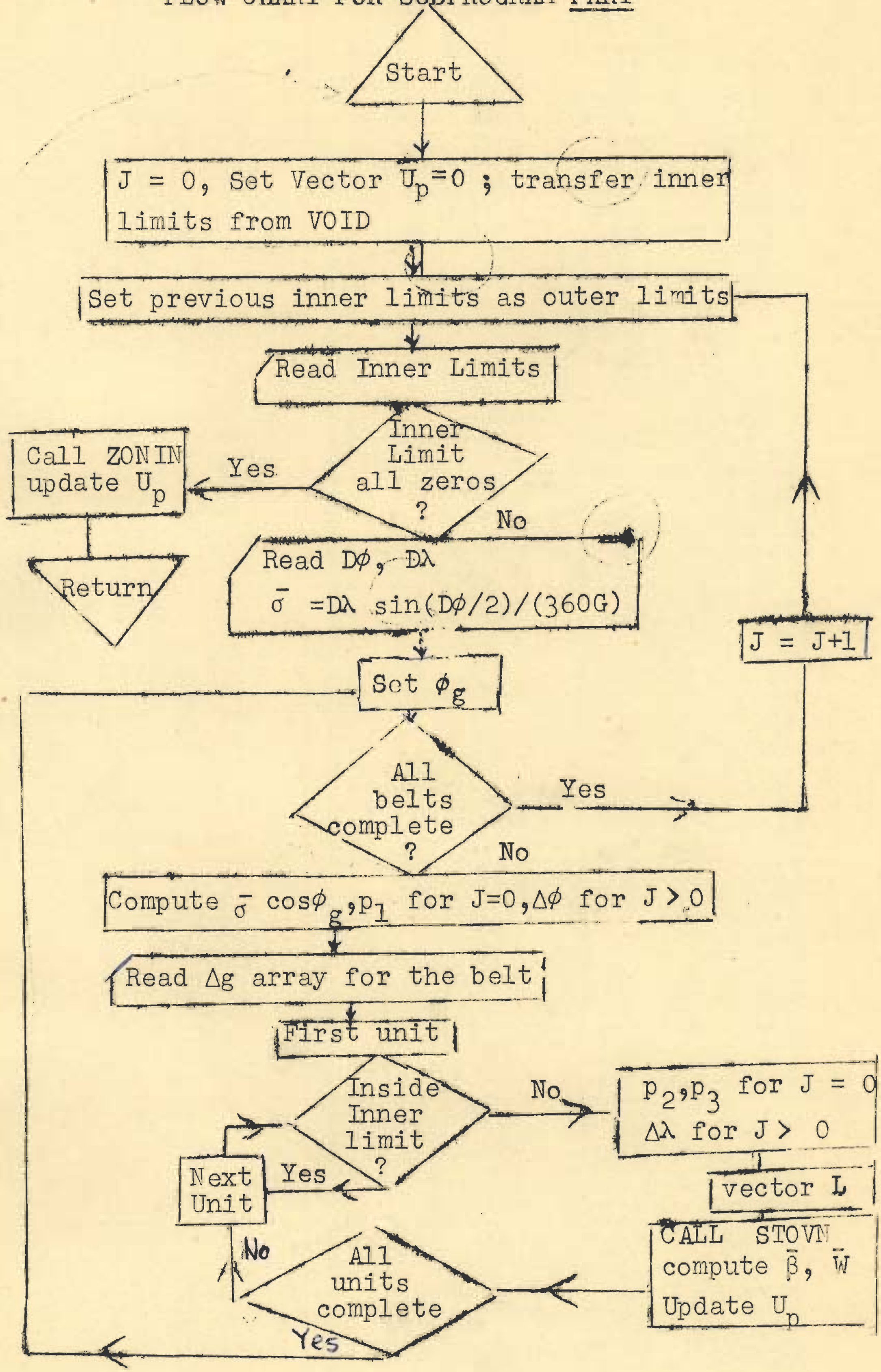
The anomalies are inputted beltwise, from polar to equatorial region, i.e. changing $|\phi_g|$ from 87.5 to 2°.5, with finally a negative buffer. The advantages of this arrangements are : (i) the stripcoding requires lesser number of data cards, (ii) the pairwise input avoids repetitive computation, (iii) India being in the equatorial part, the contributions progressively increase and the beltwise sum minimizes truncation error.

An example of data distribution and corresponding card format is shown below:



						FORMAT
1st Card :	87.5	3	1	120.		(F5.1,2I5,13F5.1)
2nd Card :	111					(72I1)
3rd Card :	-8.	16.	14.			(16F5.1)
4th Card :	-8.	11.	-5.			(16F5.1)
5th Card :	82.5	4	2	100.	80.	
6th Card :	1221					
7th Card :	20.	42.	9.	0.		
8th Card :	24.	44.	0.	-25.		

FLOW CHART FOR SUBPROGRAM <u>PART</u>



Subroutine COSIN : It takes any angle in decimal degrees as input, changes it to radian, and returns the cosine and sine values.

Subroutine DCOSIN : The double precision version of COSIN

Subroutine COVERS : Similar to DCOSIN, it also computes the versin of the angle. In the PART programme, this is used instead of DCOSIN when $J \neq 0$.

Subroutine STOVM : With elements of vector L as input, it checks the sign of ℓ_1, correspondingly computes x, $S(\psi)$ and $U(\psi)$, the modified Vening Meinesz' function.

Subroutine STOVN : It computes x, and the normalized weighting functions $\bar{S}(\psi)$ and $\bar{U}(\psi)$.

Subroutine ZONIN : Taking the point value and the slopes of $\Delta g/G$ at the computation point, this subprogramme calculates the effects N_i/R, ξ_i, η_i of the innermost zone of $D\phi$, $D\lambda$. For processing the gravity anomalies at data points, a subroutine SURFIT is called.

Subroutine SURFIT : From the given point values distributed in a rectangular compartment, this subroutine fits a corresponding polynomial surface, computes the mean over the compartment and also the predicted value and slopes of the surface at any desired point. For zero-order surface, the arithmetic average is obtained.

F.3 OTHER PROGRAMMES

During the course of investigation, many subprogrammes were made for various intermediate computations. Only a few salient routines have been indicated as follows:

SPLINE : This programme reads the grid size and corner values, selects the S matrix (Appendix - D) and computes the elements of P and Q matrices. For interpolating the value at any desired point or points, it selects the corresponding m and n, and computes Z using Equation D.5. The elements of S matrix from 4 x 4 to 8 x 8 size are built in the programme.

COVAR : The standardized covariances (Appendix E) are calculated as subscripted variables. The algorithm and the Fortran II version of the subroutine is given in Appendix E.

PREDIC : Using the covariance function evaluated in COVAR, this programme predicts the anomaly at any desired point. Two options have been provided as follows :

(i) all the data points are used in the prediction,

(ii) the points up to the distance within which the covariance function is positive, are used; if the number of such points is less than three, the influence zone is extended to cover three points.

NOSPIN : The COVAR program actually evaluates some non-uniformly spaced values, as can be seen from the various covariograms shown in chapter V. This program generates all other values by using a general non-uniform cubic spline interpolation. In the present work, however, the subroutine was not used, as intermediate covariances were not needed owing to the gridded data.

GMEAN : The mean anomaly over a compartment from the point anomalies are evaluated through this programme. Depending upon the compartment size and number of stations within it, various options are coded, e.g.,

(i) usual arithmetic average

(ii) surface-fitted mean, using SURFIT

(iii) weighted mean, using adjacent compartments, by calling WTMEAN.

WTMEAN : The truncated pyramid window, as detailed in chapter V, is used to get the average value. The various options are already described in the text.

GEOFIT : The surface-fitting technique, described in chapter VII, to get the geoid height from the astro-geodetic deviations of the vertical has been translated to Fortran statement in this programme. The various parts are :

(i) framing of normalized matrix,

(ii) inversion of the factorized upper triangular matrix,

(iii) selecting the optimum order of surface,

(iv) computing C_{oo}, and then geoid height at any desired point.

SATELA : This programme computes the geoid height at any point, from the inputted geopotential coefficients and spheroidal parameters, namely J_2,f,a. The parts of the programme are as follows :

(i) $\bar{J}_{2n}$ from J_2 value,

(ii) recursion of zonal harmonics,

(iii) recursion of $\bar{P}_{nm}(\bar{\phi})$,

(iv) recursion of longitude terms,

(v) summation of the spherical harmonic series.

A part of the programme, as a test routine, has been presented in Appendix G.

APPENDIX- G

NORMAL GRAVITY BY SPHERICAL HARMONICS

(A paper published in INDIAN SURVEYOR, January,1973)

Abstract

Spherical harmonic expansion of the theoretical gravity on the reference spheroid has been formulated, considering up to second order of eccentricity and cross radial derivative of the sphero-potential. A computer subroutine for evaluating the normalized Legendre polynomial and its differential is presented.

Computations at some latitudes in the Indian part are tabulated and compared with the 1967 normal gravity formula. Finally the results are discussed in the light of satellite geodesy.

Introduction

Geometrical geodesy deals solely with geodetic triangulation and astronomical observations, to give the general shape of the earth and coordinates of points on its surface in terms of latitude, longitude and height. Physical geodesy uses gravimetric method to orient the reference surface in geocentric position by defining the geoid-spheroid separation and deflections of vertical at one or more triangulation stations.

The gravimetric computation requires the normal gravity caused by a regularized earth having symmetry about its axis of rotation and the equatorial plane. Present trends in satellite geodesy needs evaluation of geoid parameters by spherical harmonics.

Expansion of normal gravity in harmonics is one of the requirements for determining gravity anomalies on the geoid surface.

Formulation

The normal gravity has two parts, the major being 'gravitational' due to the attraction of earth's masses, and the other 'rotational' due to the spin of earth about its own axis. The second part is expressable, in closed form and depends upon the rotational rate ω and ellipsoidal geometry (see Fig.1 for symbols), and takes the following simple form

$$y_\omega = \frac{-\omega^2 a \cos^2\phi}{\sqrt{1 - f(2-f)\sin^2\phi}} \tag{1}$$

The other part is the radial derivative of gravitational potential V, as :

$$y_g = \left(\frac{\partial V}{\partial r}\right)^2 + \left(\frac{\partial V}{r\partial \phi}\right)^2 \quad (2)$$

In spherical harmonic expansion, the normal gravitational potential of the reference spheroid is the series (Heiskanen and Moritz, 1967),

$$V = \frac{GM}{r}\left(1 + \sum_{n=1,2}^{\infty} (a/r)^{2n}\, \bar{J}_{2n}\, \bar{P}_{2n}\right) \quad (3)$$

where, $\bar{J}_{2n}$ and $\bar{P}_{2n}$ are respectively the spheropotential coefficients and the Legendre's polynomial of even degree 2n, in fully normalized form.

From (2) and (3) the expression for y_g are obtained as,

$$y_g = + \frac{GM}{r^2}\left\{(1+\Sigma Q_n)^2 + (\Sigma R_n)^2\right\} \quad (4)$$

where $Q_n = (a/r)^{2n} \times (2n+1) \times \bar{J}_{2n}\, \bar{P}_{2n}$

and $R_n = (a/r)^{2n} \times \bar{J}_{2n} \times \bar{P}_{2n}'$

$$\bar{P} = \frac{\partial}{\partial \phi}(\bar{P}_{2n})$$

Binomial expansion of the term under square root and neglecting terms of order more than $(Q_n)^2$ and $(R_n)^2$, finally yields,

$$y_g = \frac{GM}{a^2} \times (q/r)^2 \times \left\{1 + Q_n + \frac{1}{2}(\Sigma R_n)^2\right\} \quad (5)$$

Ellipsoidal Term

The ellipsoid may be expressed in terms of radius vector r and geocentric latitude $\bar{\phi}$, where,

$$\tan\bar{\phi} = (1-f)^2 \tan\phi \qquad (6)$$

Also, we get by series expansion and ignoring higher order terms,

$$(a/r)^2 = 1 + f(2 + 3f) \sin^2\bar{\phi} \qquad (7)$$

Zonal Harmonics

The sphrical harmonics, in fully normalized form, are required for even degree only. A recursive relation for conventional zonal polynomials facilitating computer calculation (Heiskanen and Moritz 1967),

$$P_n = \frac{1}{n}[(2n-1)s\ P_{n-1} - (n-1)P_{n-2}] \qquad (8)$$

with $P_o = 1$, $P_1 = s = \sin\bar{\phi}$

finally, $\bar{P}_n = \sqrt{2n+1} \cdot P_n$

A similar relation for the derivative has been found out, as follows,

$$P'_n = \frac{1}{n-1}[(2n-1)s\ P'_{n-1} - (n)P'_{n-2}] \qquad (9)$$

with $\bar{P}'_o = 0$, $P'_1 = c = \cos\bar{\phi}$

and $\bar{P}'_n = \sqrt{2n+1} \cdot P'_n$

A fortran subroutine has also been developed by the author to calculate the harmonics by recurrence (Appendix (1)). The routine is optimized for computer time, memory storage and accuracy, by avoiding recall of subscripted array, reusing the same space (U, V, PMID, PTOP, PBOT), and noting that n, n-1, 2n-1 are appearing in arithmetic progression.

Fig. 2 shows the values of $\bar{P}_2$, $\bar{P}'_2$, $\bar{P}_4$, $\bar{P}'_4$ for ϕ upto 40°.

Reference Spheroid

In the present article, the spheroid used is Reference Spheroid, 1967, as defined by the following constants (Williamson and Gaposchkin, 1975),

$GM = 3.98603E + 20$ cc/sec/sec

$\omega = 7.2921151E - 05$ rad/sec

$a = 6.378160E + 08$ cm

$J_2 = 0.0010827$

$f = 1/298.24717$

The reference normal gravity is expressed as,

$$y_s = 978031.85(1+5.30286E-03 \sin^2\phi -5.32E06 \sin^2 2\phi) \quad (10)$$

where y_s is in milligal and ϕ is the geographic latitude. The spheropotential coefficients can be derived from J_2 by the relation,

$$\bar{J}_{2n} = (-1)^n \frac{3(e^2)^n}{(2n+1)(2n+3)} \left(1-n+5n \frac{J_2}{e^2}\right)/\sqrt{4n+1}$$

where $e^2 = f(2-f)$

The first three coefficients calculated are,

$\bar{J}_2$ = -4.8419817E - 04,

$\bar{J}_4$ = +7.9042088E - 07, and

$\bar{J}_6$ = -1.6877169E - 09.

This shows that the series is highly convergent, and for mgal accuracy, these three terms are sufficient.

Results and Discussions

The different parts of the computations of normal gravity by spherical harmonics and comparison with the reference formula are shown in Table - 1, for nine geographic latitudes relating to Indian part.

The excellent agreement is evident. The ellipsoidal term, and the rotation term constitutes approximately upto 0.3 percent and 0.4 per cent of the total gravity. The harmonic part is about 0.2 percent, mostly shared by the first harmonic. The tangential component R_n is limited to 1 to 2 mgal only.

For the purpose of computing gravity anomalies, or determining geoid vector from satellite derived geopotential coefficients, thus, three terms of Q_n are sufficient, giving accuracy of the mgal order. The crossradial term also can be safely neglected. The rotational term is usually deducted by the space flight centres, (as done by Smithsonian Astrophysical Observatory) and the disturbing potential is obtained simply by deducting the harmonic part of V.

However, present age of fast digital computers with high precision makes us free from these simplifying assumptions, and direct formulation is recommended keeping in view the truncation inaccuracies only.

Acknowledgement

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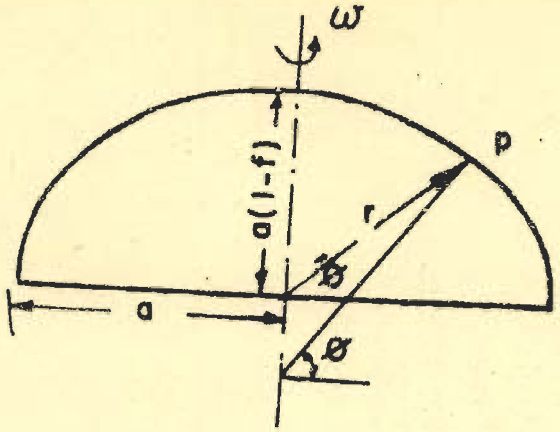

FIG. 1 ELLIPSOID OF REVOLUTION

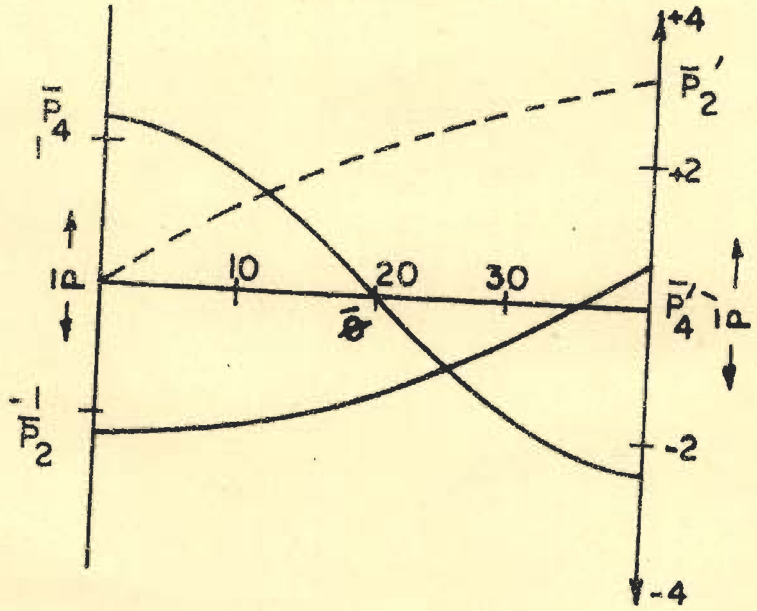

FIG. 2 NORMALIZED HARMONICS

Table - 1

Normal gravity through spherical harmonics and by standard formula
(rounded to 0.1 mgal)

ϕ	$y_a = GM/a^2$	$y_e = y_a(a/r)^2$	y_h harmonic	y_ω rotation	$y=y_g+y_\omega$	y_s standard
0	979827.8	0.0	1595.7	-3391.6	978031.9	978031.8
5	-do-	49.3	1560.0	-3365.9	978071.2	978071.0
10	-do-	196.5	1453.1	-3289.7	978187.7	978187.6
15	-do-	436.9	1278.7	-3165.1	978378.3	978377.8
20	-do	763.4	1041.6	-2996.1	978636.7	978636.2
25	-do-	1166.5	748.8	-2878.5	978954.6	978954.8
30	-do-	1634.3	408.8	-2545.8	979325.1	979324.2
35	-do-	2153.0	31.7	-2278.3	979734.2	979733.1
40	-do-	2707.0	-371.4	-1993.1	980170.3	980169.2

APPENDIX - (1)

```
      SUBROUTINE ZONAL (S,C, NDG, PDS, PDB)
      DIMENSION  PDS(NDG), PDB(NDG)
      PTOP = 1.
      PMID  = S
      U = 3.
      V = 1.
      PDS(1) = S*SQRT(3.)
      DO 104 N = 2, NDG
      PBOT = U*S*PMID - V*PTOP
      V = V + 1.
      PBOT = PBOT/V
      U = U + 2.
      PTOP = PMID
      PMID = PBOT
  104PDS(N) = PBOT*SQRT(U)
      PTOP = 0. 0
      PMID = C
      U = 3.
      V = 1.
      PDB(1) = C*SQRT(3.)
      DO204N = 2,NDG
      PBOT = U*S*PMID-(V+1.)*PTOP
      PBOT = PBOT/V
      V = V + 1.
      U = U + 2.
      PTOP = PMID
      PMID = PBOT
  204PDB(N) = PBOT*SQRT(U)
      RETURN
      END
```

VITA

Born on 5th December, 1943 in West Bengal, Sri Pradip K. Ray graduated in Civil Engineering from Bengal Engineering College in 1965. He obtained his Master's degree in Civil Engineering from Calcutta University in 1967 and was recipient of the University Gold Medal as the topper.

He joined Bengal Engineering College in 1965 as a Senior Fellow under the Teachers' Training programme and subsequently has been serving as a Lecturer in Civil Engineering in the same college till date. He was deputed to the Advanced Survey and Photogrammetry Section in the Civil Engineering Department of the University of Roorkee in 1975 under the Quality Improvement Programme for his doctoral study in the field of geodesy.

Sri Ray is a member of the Institution of Engineers (India), the Indian Society of Earth Sciences, the Indian Science Congress Association and a student member of the Institution of Surveyors(India). He has in his credit 7 technical papers (4 individual, 3 co-authored) published and presented in India and abroad, two of which have been appended herein.