AGGRADATION OF STREAMS DUE TO INCREASE IN SEDIMENT LOAD

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C E R T I F I C A T E

Certified that the thesis entitled 'AGGRADATION OF STREAMS DUE TO INCREASE IN SEDIMENT LOAD' which is being submitted by Mr. J.P. SONI in fulfilment of the requirements for the award of the Degree of Doctor of Philosophy in Civil Engineering of the University of Roorkee is a record of the student's own work carried out by him under our supervision and guidance. The matter embodied in this thesis has not been submitted for the award of any other degree.

This is to further certify that he has worked for a period of 3 years from August 1972 to August 1975 for preparing this thesis.

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ABSTRACT

Aggradation occurs when the equilibrium of an alluvial stream is disturbed in such a manner that either the sediment carrying capacity of the stream is reduced or the rate of supply of sediment is increased over and above the carrying capacity of the stream. Aggradation is thus found to occur in many situations. The problem of aggradation due to supply of sediment, in excess of what the channel can carry, has been investigated in the present study. The supply of additional sediment is assumed to be continuous and at a constant rate.

The primary objective of the study is to provide a computational procedure for prediction of transient bed profiles on the basis of laboratory experiments. The analysis of experimental data provided also for the first time a clear understanding of resistance to flow and sediment transport in alluvial channels under non-uniform flow condition.

The time dependent variations of a river bed due to natural and/or human interference can be described by equations of motion for flow and equations of continuity for water and sediment. For large scale river morphological processes such as aggradation and degradation analytical models based on these equations have been presented by

some investigators. The parabolic model proposed by de Vries, viz.,

$$\frac{\partial Z}{\partial t} - K \frac{\partial^2 Z}{\partial x^2} = 0$$

has been solved for the boundary conditions of the present problem and following expressions obtained :

$$Z = Z_o \left(1 - \mathrm{erf} \frac{x}{2\sqrt{Kt}} \right) \qquad \ldots \quad \text{(i)}$$

$$\frac{Z_o}{\sqrt{Kt}} = 0.885 \frac{\Delta G}{K(1-\lambda)} \qquad \ldots \quad \text{(ii)}$$

$$l = 3.66 \sqrt{Kt} \qquad \ldots \quad \text{(iii)}$$

in which Z is the aggradation depth at time t at any distance x from the section of sediment injection; Z_o is the maximum depth of deposition at $x = 0$; ΔG is the sediment load at $x = 0$ in excess of the equilibrium sediment transport rate; K is the aggradation coefficient; l is the length of aggradation; and λ is the porosity of the sand mass.

A tilting recirculatory flume of rectangular cross-section 20 cm wide and 30 m long was used for experimental investigation of the problem. The sediment forming the bed and the injected material was natural sand with a median diameter of 0.32mm and a geometric standard deviation of 1.30. After the establishment of uniform flow for a given discharge and slope, the sediment supply rate at the upstream end of the flume was increased to a known value by continuously feeding excess sediment at the upstream end of the flume. The bed and water surface profiles downstream of the section of sediment injection were recorded at intervals. The added sediment load was varied from 0.30 G_e to 4.0 G_e, where G_e is the equilibrium sediment transport rate.

The theoretical expression $Z = Z_o(1- \text{erf}\, x/2\sqrt{Kt})$ i.e. Eq.(i) has been arrived at after many simplifying assumptions and it is to be expected that results from this shall not fit the experimental data directly. On comparison with the experimental data it has been found that the form of the equation is correct, but the value of the aggradation coefficient, K, enabling fit of this equation to the experimental data is different from the theoretical value, $K_o = \frac{1}{3} \frac{b\, G_e}{S_o(1-\lambda)}$ (Here b is the exponent in sediment transport law of the form $G = a\, U^b$ and S_o is the bed slope). This modified value of K (enabling fit of Eq(i) to the experimental data) has been empirically related to the theoretical value, K_o, and to the relative rate of overloading $\Delta G/G_e$. As such this relation alongwith eq.(i) enables prediction of transient bed profiles, when the value of Z_o has been computed from Eq.(ii) with the modified value of K.

The extent of aggradation can be known from the Eq.(iii) using again the modified value of K.

The analysis of non-uniform flow data obtained from aggradation runs has revealed the following facts: (i) The sediment transport law valid for uniform flow conditions cannot be applied directly to non-uniform flow conditions obtained in aggrading streams. (ii) The concept of lag of sediment transport proposed by Kennedy is only partially supported by the present data. (iii) Resistance law under non-uniform flow conditions is seen to be the same as for uniform flow provided the local friction slope is used in place of S_o in the former case.

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LIST OF SYMBOLS

Symbol	Meaning	Dimension
a	Coefficient in sediment transport law	$L^{2-b}\ T^{b-1}$
B	Width of the channel	L
b	Exponent in sediment transport law	Dimensionless
C	Chezy's coefficient	$L^{1/2}\ T^{-1}$
C_o	Coefficient in sediment transport law	$L^{-1}\ T^{2}$
c_1, c_2	Celerities at the water surface	$L\ T^{-1}$
c_3	Celerity for the propagation of disturbance at the bed	$L\ T^{-1}$
c_4	Constant	-
d_1	Constant	-
d	Median size of sediment, d_{50}	L
Fr	Froude number of the Flow	Dimensionless
f	Darcy-Weisbach resistance coefficient	Dimensionless
G	Total sediment transport rate at a cross-section in absolute volume per unit width of stream per second	$L^2\ T^{-1}$
G_e	Average equilibrium sediment transport rate by volume per unit width per second	$L^2\ T^{-1}$
g	Gravitation acceleration	$L\ T^{-2}$
ΔG	Sediment injection rate in absolute volume per unit width per second	$L^2\ T^{-1}$
h	Depth of flow	L
h_o	Depth of uniform flow	L

Symbol	Meaning	Dimension
h_c	Critical depth of flow	L
i	Index for distance	-
j	Index for time	-
K	Aggradation coefficient	$L^2 T^{-1}$
K_o	Theoretical value of aggradation coefficient, $= \frac{1}{3} \frac{b G_e}{S_o(1-\lambda)}$	$L^2 T^{-1}$
K_1, K_2	Coefficients	Dimensionless
k_3, k_4, k_5, k_6	Constants	Dimensionless
k_v	Velocity coefficient	Dimensionless
k_{vp}	Velocity coefficient for plane bed	
k_t	Coefficient in sediment transport law	Dimensionless
L	Length of dunes and antidunes	L
l	Length of aggradation profile	L
m	Exponent in sediment transport law	Dimensionless
n	Manning's roughness Coefficient	$L^{-1/3}$ T
q	Discharge per unit width	$L^2 T^{-1}$
R	Hydraulic radius of the channel	L
R_b	Hydraulic radius corresponding to bed	L
R_{w_1}	Hydraulic radius corresponding to glass side walls	L

Symbol	Meaning	Dimension
R_{W_2}	Hydraulic radius corresponding to painted steel-sheet side wall	L
S_o	Bed Slope	Dimensionless
S_{oi}	Initial bed slope	Dimensionless
S_f	Friction slope	Dimensionless
U	Average velocityof flow at a section	$L\ T^{-1}$
U_*	Shear velocity $(= \tau_o/\rho)$	$L\ T^{-1}$
U_{*c}	Critical shear velocity	$L\ T^{-1}$
T	Time	T
t	Time	T
t_e	Equilibrium time	T
$\forall_o$	Initial reservoir volume per unit width	L^2
$\forall_t$	Reservoir volume per unit width at time t	L^2
x	Distance	L
Z	Depth of Deposition	L
Z_o	Maximum depth of deposition at any time and at $x = 0$	L
γ	Unit weight of water	$M\ L^{-2}\ T^{-2}$
γ_s	Unit weight of sediment	$M\ L^{-2}T^{-2}$
ρ	Mass density of water	$M\ L^{-3}$
ρ_s	Mass density of sediment	$M\ L^{-3}$
η	Dimensionless variable $= \dfrac{x}{2\sqrt{Kt}}$	-
$\Delta\gamma_s$	Difference of unit weights of sediment and water $= \gamma_s - \gamma$	$M\ L^{-2}T^{-2}$

Symbol	Meaning	Dimension
ν	Kinematic viscosity of water	$L^2 \; T^{-1}$
ϕ_T	Dimensionless total load sediment transport parameter $= \left(\frac{\gamma}{\gamma_s - \gamma}\right)^{1/2} \left(\frac{1}{gd^3}\right)^{1/2} \cdot G$	Dimensionless
ϕ_1, ϕ_2	Dimensionless celerities at the water surface; c_1/U and c_2/U respectively	Dimensionless
ϕ_3	Dimensionless celerity of the bed $= c_3/U$	''
λ	Porosity of Sand mass	''
δ	Lag distance in Kennedy's analysis	L
δ'	Lag distance in Engelund's analysis	L
δ_m	Lag distance for section x = 0	L
α	Constant	-
β	Constant	-
τ_o	Average bed shear stress	$M \; L^{-1}T^{-2}$
τ_c	Critical shear stress	''
τ_s'	Average grain shear stress computed from Manning-Strickler relation.	''
τ_t	Effective shear stress for total load transport	''
τ_{*t}	Dimensionless effective shear stress for total load transport	Dimensionless
τ_*	Dimensionless shear stress $= \tau_o/(\gamma_s - \gamma)d$	''
τ_{*s}'	Dimensionless average grain shear stress computed from Manning-Strickler relation.	''

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CHAPTER-I

INTRODUCTION

1.1 PRELIMINARY REMARKS

An insight into the dynamic behaviour of streams is essential in the planning of river basin development. Many rivers are so delicately balanced that any artificial or natural change of a permanent nature in the flow of water and/or sediment may affect the entire system. The rate at which the river will adjust its regime to man-made river works or to natural causes may be very slow or rapid depending on its nature. Such natural adjustment of the river to the imposed changes may prove detrimental to the proper functioning of the engineering works located on or along the river. It is, therefore, important to have methods for quantitative prediction of the nature of the adjustment.

An alluvial stream flowing under equilibrium conditions has been called a graded stream, a balanced stream or a regimen stream. Mackin (23) 1948, has defined a graded stream as one " in which over a period of years, slope is delicately adjusted to provide with available discharge and with prevailing characteristics, just the velocity required for the transportation of the load supplied from drainage basin" . No natural stream, however, is truly in equilibrium. The discharge of a natural stream varies continuously with time, with a ratio of high discharge to low discharge ranging from unity to several hundred or more. The temperature, the rate of sediment supply by tributaries and even the character of the sediment also vary. The result of this continuous change is that the stream under consideration cannot be in true equilibrium. It appears, therefore, that the graded condition is more a matter of degree of balance than that of absolute balance. However, many rivers approach this balance reasonably closely from an engineering point of view in which relatively short periods (compared to the geological cycle) and mostly short stretches of the river are considered.

A change in any of the controlling factors, namely, slope, sediment size, water and sediment discharge, will necessitate changes in one or more of the other factors to restore the equilibrium. The above situation can be explained qualitatively by the well known balance analogy of Lane (22) 1955, where two controlling variables-- sediment discharge and diameter of sediment-- are on one pan and the other two controlling variables- water discharge and slope--are on the other pan. Thus if the sediment load entering a stream in equilibrium is increased keeping sediment size the same, equilibrium can again be restored only if the water discharge and/or slope are increased sufficiently.

The changes in discharge, slope, sediment load and sediment size can be due to natural reasons-such as occurrence of land slides, addition of sediment load by tributaries in excess of what the stream can carry, etc. It can also be due to artificial means such as construction of a dam across a river, provision of silt exclusion works, withdrawal of clear water, etc. A stream disturbed in this way tends to approach a new equilibrium condition through a slow process of deposition or widespread lowering of the stream bed. If the sediment carrying capacity of the stream is decreased, the excess sediment load will be dropped on the bed causing the bed slope to increase with the passage of time; this phenomenon is known as aggradation. On the other hand, increase in the sediment carrying capacity will enable the flow to pick up additional sediment from the bed and banks, and a progressive lowering of the stream bed will result; if banks are erodible the stream width may increase. This phenomenon is called degradation.

Both aggradation and degradation are large scale time-dependent morphological processes of a river and take very long time to reach near equilibrium conditions. Obviously such phenomena create non-uniform, and unsteady flow conditions in a channel.

The problem of determination of resistance to flow, sediment transport and prediction of bed forms in a stream in equilibrium (uniform flow conditions being implied) is far from solved even today. The problem is further complicated when non-uniform and unsteady conditions are created as is the case when this equilibrium is disturbed in a manner explained in the preceding paragraphs.

1.2 AGGRADATION PROCESS

When artificial or natural changes disturb the equilibrium of a stream in such a manner that either the sediment carrying capacity of the stream is reduced or the incoming sediment load is greater than the carrying capacity of the stream, aggradation takes place. Thus aggradation is found to occur under the following conditions:

(i) Upstream of a dam :- Due to backwater formation the sediment carrying capacity of the stream is greatly reduced and the excess sediment is deposited in the reservoir.

(ii) When relatively clear water is withdrawn from an alluvial stream in equilibrium for irrigation or water supply purposes aggradation occurs downstream of the point of withdrawal.

(iii) Downstream of branching streams where the division of discharge leads to a decrease in the carrying capacity of the stream.

(iv) When the tributary brings in more sediment load than what the main stream can carry.

(v) When sediment load is increased in an alluvial stream by series of **land** slides or dumping of mining debris. Due to series of land slides in the Alaknanda valley, in the year 1970, the Ganga canal (36) 1974, in Uttar Pradesh, India, was virtually silted up in its upstream reach; the canal had to be closed and silt clearance done. Another example is the Mu Kwa river in Formosa (22) 1955, the bed of which was raised about 12.0m in 12 years due to addition of sediment to the river from land slides. A two storey hydroelectric power house along the side of the river was completely buried.

Lane (22) 1955, reports that the bed of the Yuba river in northern California, U.S.A. rose about 6.0m due to increase in the sediment load resulting from the dumping into the streams of large quantities of gravel wasted in the hydraulic mining of gold. A similar case is that of Serendah river in Malaya (22) 1955, where the river rose about 6.5m in the years 1922-23 due to addition of sediment load from the hydraulic mining of tin.

Aggradation of stream channels and natural or artificial waterways has the effect of increasing flood stage for a given discharge. Consequently the area inundated and the extent of flood water damages go on increasing although the discharge remains substantially the same. Aggradation causes poor drainage of irrigable lands and excessive wastage of water through consumptive use of uncontrolled vegetation. It also increases the ground water levels in areas surrounding the stream.

1.3 MATHEMATICAL MODELS FOR STUDY OF AGGRADATION

Time dependent variations of river bed due to artificial or natural causes can be described by combining equations of motion, equations of continuity for water and sediment, the resistance law and the sediment transport law. Theoretically these equations can be solved with prescribed boundary conditions of the problem to determine the longitudinal bed profile of the river at various times. However, such a system of equations being nonlinear it is not possible to solve them analytically without making simplifying assumptions. Models based on such simplifying assumptions have been proposed by a few investigators (1,8,10,31,35,39). However, in general, it is necessary to check the results from such models against a known situation, i.e. against a set of observations. A sufficient degree of agreement between analytical predictions and observations may exist but complete identity cannot generally be obtained due to the complex nature of the phenomenon. One way of improving the analytical predictions is to adjust the values of those parameters in the model about which certain simplifying assumptions have been made; the adjustments have to be dependent on the observed results.

1.4 NONUNIFORM FLOW IN OPEN CHANNELS

Various investigators (14,29) have shown that the friction factor for uniform flow in alluvial channels can vary widely because of changes in bed form with varying flow conditions. On the basis of both field and laboratory data it has been found that friction factor of a sand bed stream often changes by a factor of four to five, times and in exceptional cases by a factor of even ten over the range of discharges commonly occuring during a year. This property of streams is illustrated by Alam and Kennedy(2)1969, for the Rio-Grande river in New Mexico.

Friction factor predictors for alluvial channels as of today are by no means as satisfactory as their counterparts for rigid bed channel flow. But one may hope to predict the mean velocity to an accuracy of ± 30 per cent under most circumstances from some of the existing methods.

Regarding sediment discharge relationships, it has been found (30) 1971, that the probable error in sediment discharge calculations even under the most favourable circumstances can be as large as 50 per cent to 100 per cent. When calculations are based on average values of slopes, bed material characteristics, temperature and estimated flow depth and velocity, larger errors can be expected.

While there is an abundance of data on sediment transport and channel resistance under steady uniform flow conditions, there is almost total lack of data under nonuniform/unsteady conditions; as such no check has been made so far as to whether the resistance and sediment transport laws derived under uniform flow conditions can be used under unsteady, nonuniform flow conditions. Kennedy's(19,20) 1963,1969, concept of lag distance—that sediment transport lags local changes in velocity near the bed—indicates that the sediment transport relation is affected by the non-uniform flow conditions. However, no experiments have been carried out to find the magnitude of lag distance and its variation with other pertinent variables. So far as the resistance law is concerned, nothing is known about its applicability for nonuniform flow conditions. It may be emphasized that nonuniform flow and unsteady conditionsare associated with the phenomenon of aggradation.

In view of the above, there is a great need for collecting experimental data on aggradation. Such experiments shall provide basic data on sediment transport and resistance to flow under nonuniform flow conditions. Such information would also help in evolving a procedure for prediction of aggradation, either using an empirical procedure or by modifying the results from a mathematical model. It is to be pointed out that there is almost a total lack of data on aggradation in streams arising out of sudden addition of sediment load. Bhamidipaty and Shen(3) 1971, have collected a limited amount of data on aggradation due to excess of sediment load but under conditions of incipient motion prior to sediment addition. Natural stream beds are generally mobile and as such their study would not be directly applicable to such streams.

1.5 SCOPE OF STUDY

The aggradation caused in an alluvial stream in equilibrium due to increase in sediment supply rate has been studied in this investigation. The stream bed both upstream and downstream of the section of increased supply of sediment shall aggrade; the aggradation in the downstream reach only has been considered here. The primary objective has been to predict analytically the transient bed and water surface profiles under such conditions. Assuming a downstream control section, one can see that after a long time, the aggradation downstream of the section of sediment addition ceases once hydraulic conditions are compatible with the increased load.

Laboratory experiments have also been conducted. The experimental data have been used to support and modify the results obtained from the analytical model. These data have been utilized to study resistance and sediment transport in nonuniform flow conditions.

1.6 LIMITATIONS OF THE STUDY

The following limitations have been imposed on the study :

1. Aggradation downstream of the section of increased sediment supply only has been investigated.
2. Graded sand with median size equal to 0.32 mm and geometric standard deviation equal to 1.30 was used as bed material of the flume as well as for injected material.
3. Water discharge was kept constant for a given slope.
4. Channel width remained the same along the length since flume of constant width was used.

CHAPTER-II

REVIEW OF LITERATURE

2.1 PRELIMINARY REMARKS

Basically there are two approaches to the solution of aggradation problems — analytical and experimental. The phenomenon of aggradation can be described as a problem of propagation of bed profile in a continuous medium. The system of equations governing the phenomenon is nonlinear and it is not possible to solve these equations analytically without making simplifying assumptions. Experimental investigations on the phenomenon of aggradation are few and as such not sufficient information is available on the basic mechanism of the phenomenon.

One of the objects of the present study was to check the applicability of uniform flow sediment transport and resistance laws to non-uniform flow. A number of such relationships are available for uniform flow. A few of these intended to be used in later analysis are reviewed here.

The literature reviewed here has been classified as under :

(i) Analytical models;

(ii) Experimental work on aggradation;

(iii) Resistance laws in uniform flows;

(iv) Sediment transport laws in uniform flows; and

(v) Sediment transport under non-uniform flow condition.

2.2 ANALYTICAL MODELS FOR PREDICTION OF AGGRADATION

For large scale river morphological processes, which are time dependent, mathematical models have been presented by various investigators. Some of these investigators(16,21,27,31) consider variations in channel width, while most others consider channels of constant width and deal only with the variations of pertinent variables along the length as well as with time. The results of such one dimensional analyses have been reviewed in the following sections.

Goncharov (13) 1964, has dealt with one dimensional theory of channel deformation indicating that a theoretical analysis of each type of channel processes—aggradation, degradation—requires the development of special solutions. He has illustrated the applicability of the one dimensional theory to river flood problems and aggradation upstream of a reservoir.

Culling (8) 1960, has developed a mathematical theory of erosion along lines similar to that of the classical Fourier theory of heat flow in solids. Assuming the material flow to take place at a rate proportional to the surface gradient and applying the usual conservation principle he obtained :

$$\frac{1}{K}\frac{\partial Z}{\partial t} = \frac{\partial^2 Z}{\partial x^2} \qquad \ldots \qquad (2.1)$$

in which K may be called the aggradation (or erosion) coefficient; Z is the bed elevation; x is the distance and t is the time. Since the basic hydro-dynamic equations were not used in the derivation of Eq. 2.1, no expression for K was obtained. This mathematical model could be used for predicting transient phases of aggradation provided K is a determinable quantity.

Tsuchiya and Ishizaki (35) 1969, have developed the basic differential equation, viz., Eq. 2.1 and have given an expression for evaluating the value of aggradation coefficient. In their mathematical frame work they used the equation of fluid continuity, i.e., $q = U\,h$, resistance equation for uniform flow, bed load equation of Sato, Kikkawa and Ashida (28) 1958, and the continuity equation for sediment moving as bed load. In arriving at the model represented by Eq. 2.1, the river bed slope, $\partial Z/\partial x$, has been used in place of the energy slope. The model was used to predict aggradation upstream of the Hongu Dam (located on the Joganji river, Japan). A good agreement was found between the predicted values of deposition depths and the data collected for a period of fifteen years (1939-1954).

De Vries (39) 1973, has developed the mathematical models by making use of the dynamic equations of motion and equations of continuity both for water and sediment. The dynamic equation of water and continuity equation for flow as quoted by Chow (5) 1959, have been combined with Chezy's equation to give (see Fig.2.1):

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + g\frac{\partial h}{\partial x} + g\frac{\partial Z}{\partial x} = -\frac{g\,U|U|}{C^2 R} \quad \ldots(2.2)$$

$$\frac{\partial h}{\partial t} + U\frac{\partial h}{\partial x} + h\frac{\partial U}{\partial x} = 0\,, \quad \ldots(2.3)$$

in which h is the depth of flow; U is the average velocity of flow at a section; C is the Chezy's coefficient; R is the hydraulic radius; and g is the gravitational acceleration. The actual phenomenon is of three dimensional nature due to variation of depth and velocity across the section and also due to secondary currents; but only one dimensional treatment is implied in Eqs. 2.2 and 2.3.

The continuity equation for sediment for alluvial stream of constant width is :

$$\frac{\partial Z}{\partial t} + \frac{1}{1-\lambda}\frac{\partial G}{\partial x} = 0 \quad \ldots \quad (2.4)$$

in which G is the total load carried by the channel in absolute volume per unit time per unit width of the stream and λ is the porosity of the sand mass. This equation can be used if the amount of sediment carried in suspension does not change much with x and t. In fact this equation holds truly for bed load only and strictly speaking therefore its applicability is limited to streams with coarse sediment. However, it has been applied to streams carrying medium sands.

He has used a sediment transport law of the form :

$$G = a\,U^b. \qquad \ldots \qquad (2.5)$$

It has been shown later in this Chapter that it is an approximation of Colby's total load relation and may be deemed to be satisfactory over a limited range of depth. De Vries preferred to use the law of the type given by Eq. 2.5 because of its simplicity and also because of his contention that the relations involving shear stress are by no means more accurate. The sediment transport law during unsteady/non-uniform condition is assumed to follow the same relationship as for steady uniform flow, i.e., no inertia effects are considered.

The set of differential equations governing the phenomenon, viz., Eqs. 2.2 to 2.5 are nonlinear partial differential equations and as such are difficult to handle.

In an attempt to simplify the analysis he examined mathematically the possibility whether water movement could be treated as steady when the study of bed movement is of primary interest. It has been shown by him that when Fr is not close to unity, which is so in a majority of practical cases, the celerity c_3, the one for the propagation of disturbance at the bed is very very small compared to c_1 and c_2 — the celerities at the water surface, see Fig.2.2. The term ϕ in Fig.2.2 is a relative celerity = c/U ; ψ is a transport parameter = $h^{-1} \frac{dG}{dU}$; and Fr is the Froude number of flow = $U/\sqrt{gh}$. This means that the bed movement can be studied by assuming $|\phi_{1,2}| \rightarrow \infty$ and hence the water movement can be considered as quasi-steady, i.e., the terms $\partial U/\partial t$ and $\partial h/\partial t$ can be neglected in comparison to other terms in Eqs. 2.2 and 2.3.

Further considering wide rectangular channel (so that R = h) and water movement to be steady and uniform during transient stages, Eq. 2.2 reduces to :

$$\frac{\partial Z}{\partial x} = - \frac{U^3}{C^2 q} \qquad \ldots \qquad (2.7)$$

or

$$\frac{\partial^2 Z}{\partial x^2} = - 3 \frac{U^2}{C^2 q} \frac{\partial U}{\partial x} \qquad \ldots \qquad (2.8)$$

Combining Eqs. 2.4, 2.5 and 2.8, he obtained his parabolic model,viz.

$$\frac{\partial Z}{\partial t} - K \frac{\partial^2 Z}{\partial x^2} = 0 \qquad \ldots \qquad (2.9)$$

where $K = \frac{1}{3} \frac{C^2 q \, dG/dU}{U^2(1-\lambda)}$ ... (2.10)

or $K = \frac{1}{3} U \frac{dG/dU}{(1-\lambda)S_o} \left(\frac{U_o}{U}\right)^3$ ... (2.11)

in which the subscript 0 refers to the original uniform flow situation.

After linearization ($U \simeq U_o$)

$$K = \frac{1}{3} \frac{U_o(dG/dU)}{S_o(1-\lambda)} \qquad \ldots \qquad (2.12)$$

If sediment transport law of the form $G = a U^b$ with constant values of a and b is assumed, then

$$K = \frac{1}{3} \frac{b\,G}{S_o(1-\lambda)} \qquad \ldots \qquad (2.13)$$

This theoretical value of K is designated as K_o subsequently. The parabolic model represented by Eq. 2.9 can be solved analytically for a number of aggradation problems. De Vries points out that the parabolic model has restricted applicability and in certain cases this model might have to be replaced by the more complicated hyperbolic model. The hyperbolic model is characterized by the following differential equation :

$$\frac{\partial Z}{\partial t} - K \frac{\partial^2 Z}{\partial x^2} - \frac{K}{c_3} \frac{\partial^2 Z}{\partial x \, \partial t} = 0 \quad \ldots \qquad (2.14)$$

in which c_3 denotes the celerity of a small disturbance at the bed; the expression for K after linearisation again reduces to Eq.2.13. In the derivation of the above hyperbolic model assumption of uniform flow during the entire process is not made in advance.

His description of aggradation phenomenon is qualitative only and no results have been obtained to show growth of aggradation pattern either upstream of a dam or in streams. However, he showed that for steep slopes and for relatively small depths of flow, the results from the simpler parabolic model are in good agreement with those from the hyperbolic model.

Adachi and Nakatoh (1) 1969, made use of the dynamic equation of fluid motion, viz., Eq. 2.2, equation of fluid continuity,

$$q = U h , \quad \ldots \qquad (2.15)$$

equation of resistance to flow,

$$U_* = \sqrt{f/8}\, U , \quad \ldots \qquad (2.16)$$

and equation of sediment discharge,

$$G = C_o U_*^3 . \quad \ldots \quad (2.17)$$

Here,

U_* is the shear velocity $(= \tau_o/\rho)$;

τ_o is the average shear stress;

ρ is the mass density of water;

f, is the Darcy-Weisbach coefficient under uniform flow; and

C_o is a constant.

Neglecting the temporal and spatial changes in the water depth and velocity and combining the above equations with the equation of continuity for sediment moving as bed load, they obtained a second order differential equation of heat conduction type, viz.,

$$\frac{\partial Z}{\partial t} = K \frac{\partial^2 Z}{\partial x^2} , \quad \ldots \quad (2.18)$$

where $K = C_o \, g \, q \, \sqrt{f/8}$ ... (2.19)

Note that the same differential equation with the different value of the constant was obtained by de Vries. If the discharge of water remains constant and f is assumed to remain constant at the value under uniform flow, Eq. 2.18 can be solved analytically for a variety of aggradation and degradation problems.

On the lines of Tinney's analysis (34) 1962, for prediction of aggradation, Garde(10)1965, has formulated a model for prediction of aggradation upstream of a reservoir. He has used the gradually varied flow equation, resistance equation developed by Garde and RangaRuju(11)1966, and total load equation of Garde and Dattatri(12)1963, in this analysis.

The resistance equation can be written as :

$$U = k_v \left(\frac{h}{d} \right)^{1/6} \sqrt{g\, h\, S_f} \qquad \ldots \qquad (2.20)$$

in which k_v is a velocity coefficient, which has different values depending upon the regime of flow; for ripples and dunes $k_v = 3.2$; and d is the diameter of the sediment.

The total load equation can be expressed as

$$\frac{G}{U_* \, d} = k_t \left(\frac{\tau_o - \tau_c}{[\gamma_s - \gamma] d} \right)^m \qquad \ldots \qquad (2.21)$$

in which k_t is a coefficient; m is an exponents; τ_o is the critical shear stress; and γ_s and γ are the unit weights of sediment and water respectively.

Making use of Eqs. 2.20 and 2.21 and neglecting higher powers of U_{*c}/U, he arived at the following expression for $\partial G/\partial x$:

$$\frac{\partial G}{\partial x} = \left\{ \frac{7}{6} k_t \left(\frac{q}{k_v} \right)^{2m+1} d^{\frac{7-4m}{6}} \left[\frac{m(2m-1)}{h_c^{7/3}} \left(\frac{k_v}{k_{vp}} \right)^2 h^{-\frac{14m-1}{6}} - (2m+1) h^{-\frac{14m+13}{6}} \right] \right\} \frac{\partial h}{\partial x} , \qquad (2.22)$$

in which k_{vp} is the value of velocity coefficient for plane bed without motion (the suggested value being 7.66) and h_c is the depth of flow at the incipient condition for a given discharge and sediment size. Since all the terms in the parenthesis on the right hand side of Eq.2.22 except h, are constants for a given flow situation, Eq.2.22 could be written as :

$$\frac{\partial G}{\partial x} = F(h) \frac{\partial h}{\partial x} \qquad \ldots \qquad (2.23)$$

The continuity equation for sediment, viz., Eq 2.4 then becomes

$$\frac{\partial Z}{\partial t} + F(h) \frac{\partial h}{\partial x} = 0 \qquad \ldots \qquad (2.24)$$

The scheme of computation consists of determination of h and $\partial h/\partial x$ at various sections by integrating the varied flow equation, and calculation of $\partial G/\partial x$ from Eq. 2.22. Substituting the value of $\partial G/\partial x$ in the continuity equation for sediment, viz., Eq. 2.4 the value of $\partial Z/\partial t$ is obtained. Assuming a suitable time interval Δt, ΔZ can be obtained by

$$\Delta Z = \left(\frac{\partial Z}{\partial t} \right) . \Delta t \qquad \ldots \qquad (2.25)$$

The aggradation profile after Δt is obtained by determination of ΔZ at various sections. Calculationsfor the next time interval can be carried out starting with the above aggraded bed.

Because of the neglect of higher powers of U_{*c}/U in arriving at Eq.2.22, the formulation is not expected to give reliable results for sections close to the dam where U_{*c}/U is approximately one. This formulation cannot, however, be used for the problem of aggradation in streams due to excess sediment load since no back water curve exists initially in such cases.

Swamee (33) 1974, has presented a numerical method for predicting aggradation upstream of a dam for a constant discharge flowing through the stream. The method is based on the solution of set of equations represented by Eq. 2.24 and the following equation :

$$\frac{\partial h}{\partial x} = \frac{So_i - \partial Z/\partial x - S_f}{1 - F_r^2} , \quad ... \qquad (2.26)$$

in which So_i is the initial slope of the stream bed. The first deposition profile after time Δt is obtained using Eq.2.24 after substituting the values of $\partial h/\partial x$ from Eq.2.26 with $\partial Z/\partial x = 0$. The subsequent profile is predicted using the relation :

$$Z^{(o)}_{i,j+1} = Z_{i,j-1} + 2\Delta t\left(\frac{\partial Z}{\partial t}\right)_{i,j} \quad ... (2.27)$$

for various i, values. Knowing this profile the bed slope as well as $\partial Z/\partial x$ are computed at each section. Now a new water surface profile is obtained and then $(\partial Z/\partial t)$ values at various sections are computed. A new bed profile can be computed as :

$$Z^{1}_{i,j+1} = Z^{(o)}_{i,j-1} + \frac{\Delta t}{2}\left[\left(\frac{\partial Z}{\partial t}\right)_{i,j-1} + \left(\frac{\partial Z}{\partial t}\right)^{(o)}_{i,j+1}\right] \quad (2.28)$$

This iteration is terminated when desired accuracy is achieved. The high frequency undulations on the bed thus obtained are smoothened using truncated Fourier sine series. Such a smoothening is necessary for obtaining a stable solution over a large period of time. The numerical algorithm was found to be quite stable and results show good agreement with experimentally observed bed profiles.

2.3 EXPERIMENTS ON AGGRADATION

Compared to the analytical investigations of the phenomenon, very little experimental work on aggradation has been carried out. Aggradation has been studied in the laboratory under idealised conditions of a prismatic channel carrying a constant discharge and sediment load— assumptions which are also common to many analytical investigations. As such a comparison between theoretical and experimental results is possible.

Sugio (32) 1962, observed aggradation upstream of a weir in a rectangular laboratory flume. He has analysed these data (covering a range of $h_c/(\gamma_s/\gamma - 1) = 13$ to 25 and S_o from 1/100 to 1/267) on the basis of dimensional analysis to evolve empirical relations for the deposition and depth of water at the moving front of aggradation.

Bhimadipaty and Shen(3),1971, have conducted experiments on aggradation caused by addition of sediment load to a stream under conditions of incipient motion. The increased sediment load creates a moving aggradation front. The down-stream reach of the front at any time remains unaffected by the sediment addition. The typical water surface and bed profiles as recorded by them are shown in Fig. 2.3 They have found that the bed level at a section rises as a logarithmic function of time, viz.,

$$Z = c_4 \ln t + d_1 \qquad \ldots \qquad (2.29)$$

in which c_4 and d_1 are constants. This equation does not give zero deposition at t = 0 and also no attempt has been made by the authors to evaluate the empirical constants c_4 and d_1.

Swamee(33) 1974, studied the problem of aggradation upstream of a dam in a 20cm wide, 30 m long laboratory flume. He found that the phenomenon of aggradation upstream of a dam is a phenomenon of moving front. Upstream of this front he found major peaks followed by minor peaks. As time passes these peaks travel in the downstream direction.

At the time the major peak reaches the dam its height is nearly equal to the dam height and by this time a major part of the reservoir is filled up. The front causes the reduction in the reservoir capacity, for which he obtained an empirical equation of the form :

$$\frac{V_t}{V_o} = 1 - \frac{(t/t_e)^{k_6}}{\left[k_3 + (t/t_e)^{k_4}\right]^{k_5}} \qquad \ldots \qquad (2.30)$$

which gives the reservoir volume V_t for time t; V_o being the initial volume. Here t is the equilibrium time and k_3, k_4, k_5 and k_6 are constants. Empirical equation have also been obtained by him for determination of the height of the front as a function of its distance from the dam and also as a function of the time.

2.4 RESISTANCE LAWS UNDER UNIFORM FLOW CONDITIONS

The Task Committee on sediment transport mechanics (29) 1971, has given an excellent review of a majority of the resistance relations. As such literature pertaining to these relations has not been reviewed here. Only the resistance law proposed by RangaRaju (25)1970, is reviewed briefly since it appeared the most convenient to use in this investigation apart from being reasonably accurate. Garde and RangaRaju (11) 1966, had proposed a resistance relationship involving the parameters $U/\sqrt{(\Delta\gamma_s/\rho)\,R}$, R/d and $S_f/(\Delta\gamma_s/\gamma)$. Later, with the availability of a large amount of additional data, RangaRuju(25) 1970, modified the above resistance law by considering the effect of parameter $g^{1/2}d^{3/2}/\nu$ on the resistance relationship. The resistance relationship obtained by him for all regimes involving sediment motion is shown in Fig. 2.4. This relationship is between the parameters $K_1 . U/\sqrt{(\Delta\gamma_s/\rho)\,R}$ and $K_2(\frac{R}{d})^{1/3} \cdot \frac{S_f}{(\Delta\gamma_s/\gamma)}$ in which K_1 and K_2 are empirical coefficients related to the sediment size as shown in Fig.2.5. On Fig.2.4, the scatter of data drawn from nearly seventy sources is such that an error of less than $\pm$ 30 percent is indicated in the predicted mean velocity.

2.5 SEDIMENT TRANSPORT UNDER UNIFORM FLOW CONDITIONS

Many relations for calculating sediment discharge have appeared in literature. The hydraulic engineer has to select one or more of these for solving his particular problem. The selection is not straightforward since the results of different formulae often differ appreciably and it is not possible to find out which one gives the most realistic results for the case under investigation. After close examination of the various formulae, two total load relations, one based on mean velocity of flow and other based on shear, have been selected for use in the present analysis on the basis of their simplicity and accuracy.

2.5.1 Colby's Sediment Discharge Formula

Initially, Colby(6,7) 1964, gave a relationship for the uncorrected discharge of sand in terms of mean velocity for six median sizes of bed materials, four depths of flow and water temperature of 60°F(15.5°C). The relationship in functional form is :

$$G = F\ (U,\ h,\ d_{50})\ , \qquad \ldots \qquad (2.31)$$

and consisted of sets of curves for water depths of 0.1 ft., 1 ft., 10 ft., and 100 ft. and sediment sizes of 0.10mm, 0.20mm, 0.30mm, 0.40mm, 0.60mm and 0.80mm. For getting the true sediment discharge he recommended that certain corrections be applied to the uncorrected sediment discharge to account for the effect of temperature, presence of fine suspended sediment and changes in sediment size.

Later Colby developed relationships, in the form of curves, between the observed sediment discharge and mean velocity for 5 sand bed streams at average temperature of 60°F(15.5°C). These curves on logarithmic paper have little curvature especially over the narrow ranges of discharge for which the data are generally available and can be fitted satisfactorily by power relations of the form,

$$G = a\,U^b \,, \qquad \ldots \qquad (2.5)$$

where a and b are functions of depth and sediment size. In the evaluation of various sediment discharge formulae by the Task Committee (30) 1971, it has been established that Colby's relationship gives good agreement with the observed sediment discharges.

For a given sediment size and a relatively narrow range of depth 'a' and 'b' in Eq.2.5 would be approximately constant. This is supported by the laboratory data collected by Kennedy (18) 1961. It may be emphasised that Kennedy (19,20) 1963,1969, and de Vries (39) 1973, use Eq. 2.5 in their analyses.

2.5.2 Vittal-Raju-Garde Sediment Discharge Formula:

Relations for rates of total load and bed load transport in alluvial streams have been developed by Vittal-Raju-Garde(38) 1973, using the concept of effective shear stress. For plane bed they showed that the parameter ϕ_T defined as $G\left(\frac{\gamma}{\gamma_s - \gamma}\right)^{1/2}\left(\frac{1}{gd^3}\right)^{1/2}$ is uniquely related to τ_*, see Fig. 2.6. Here $\tau_* = \tau_o/(\gamma_s - \gamma)d$. The effective shear stress for total load τ_t in case of undulated beds has been defined as the shear stress which would give the same rate of total load transport (for a given value of d, γ_s and γ) on a plane bed as the observed rate of total load transport on the undulated bed. Obviously τ_t for a plane bed is equal to the average shear stress on the bed. For ripple and dune beds it has been shown by the authors that τ_t is greater than grain shear stress τ_s' computed by Manning-Strickler equation, the difference being large when the suspended load is high, see Fig. 2.7. Here $\tau_{*t} = \tau_t/(\gamma_s - \gamma)d$ and $\tau_{*s}' = \tau_s'/(\gamma_s - \gamma)d$.

Thus one can use Figs. 2.6 and 2.7 to compute the total load transport in case of ripple and dune beds. The scatter of data is small in comparison to that obtained normally on sediment discharge plots.

2.6 SEDIMENT TRANSPORT UNDER NON-UNIFORM FLOW CONDITIONS

The available sediment transport formulae are all based on uniform flow data. It is not known clearly how far these are applicable to non-uniform flow conditions as obtained in an aggrading stream. But a clue to the possible effects of such non-uniformity can be had by referring to the works of Kennedy and others(15,17,26), who looked at the flow past a dune as non- uniform flow in the microscopic sense with a continual decrease in depth from trough to the crest of the dune.

Kennedy (19,20) 1963,1969, has introduced a physical quantity 'δ' into the sediment discharge formula (in which transport rate is proportional to a power of fluid velocity at the level of the bed) and using the potential theory investigated the stability of a small disturbance at the fluid-bed interface. He defined the quantity, δ, as the distance by which the local sediment transport rate lags the local velocity at the mean level of the bed. In other words because of inertia, the sediment transport rate at a section responds relatively slowly to the changes in hydraulic conditions. Thus on the up-stream face of a dune, the transport rate at a cross-section would be smaller than that given by the equation $G = a\,U^b$ after substituting the local velocity for U. The reverse would obviously be the case on the down-stream face. Kennedy, however, did not give any relation for computing δ. Kennedy's concept of lag distance has often been used in subsequent treatments(15,17,26), and various qualitative reasons have been given for existance of δ.

Engelund(9).1966, considered a sand bed with sinusoidal variation of bed shear (an idealised case) and established that sediment transport is delayed in relation to the bed shear. This lag distance based on shear stress may be designated as δ'. The lag distance, δ', was determined by the structure of turbulent flow, He related $2\pi\delta'/L$

to the Froude number of flow as shown in Fig. 2.8. Here L is the length of dunes and antidunes.

Parker (24) 1975, has gone a step further and argued that if the lag in fact exists, it should be obtainable from the very analysis that indicates instability. He presented an instability analysis of flat bed leading to antidunes and obtained an expression for the lag distance. He showed that the lag distance is a function of bed form wave number; ratio of sediment transport effects to frictional effects; and the ratio of sediment transport velocity to mean fluid velocity.

Thus it is seen that whether sediment transport rate is expressed in terms of velocity or shear lag exists. But little is known about the magnitude of this lag distance. The idea of lag distance has been used primarily in connection with the instability of flat beds leading to the formation of micro-morphological processes such as dunes, antidunes, etc. There is a need, therefore, to explore the applicability of this concept to unsteady and non-uniform flows occurring in case of macro-morphological processes like that of aggradation due to excess sediment load.

2.7 CONCLUDING REMARKS

Review of literature indicates that matehmatical framework for the problem of aggradation and degradation has been well established. Some work has been done regarding the quantitative prediction for the problem of aggradation upstream of a dam but no such work exists for aggradation in a stream due to overloading. Compared to the analytical work very little work has been done on the experimental side. In fact aggradation due to excess sediment load when the sediment on the bed is in motion has not been investigated experimentally at all.

Little information on the resistance of alluvial streams under non-uniform flow is available. Although no studies of sediment transport under non-uniform flow have been conducted, there is evidence to suggest that sediment transport laws developed for uniform flows cannot be applied directly to non-uniform flows.

CHAPTER-III

THEORETICAL ANALYSIS

3.1 PRELIMINARY REMARKS

The natural process of obtaining equilibrium between sediment supply and sediment transport is complicated by various factors such as variability of discharge in a river, variations in total run off from year to year in different parts of the drainage basin and periodic changes in the volume of sediment transport, with the actual mode of transport differing from one stage to another. It is, therefore, difficult to attempt a complete representation of the aggradation due to overloading as it exists in nature. It is, therefore, necessary to choose the simplest model which clearly incorporates the essential features of aggradation. Given precise setting and calibration the simplified model shall respond to a given situation just like the natural phenomenon of aggradation.

As mentioned in Chapter II, the following differential equation for river bed variation has been obtained by a number of investigators :

$$\frac{\partial Z}{\partial t} - K \frac{\partial^2 Z}{\partial x^2} = 0 . \qquad \ldots \qquad (2.9)$$

The value of K varies from investigator to investigator depending upon the simplifying assumptions introduced.

The parabolic model suggested by de Vries(39) 1973, is based on the equations of continuity of water and sediment, equation of motion for water and a sediment transport law of the form $G = a\,U^b$ and the theoretical value of K is given by

$$K_o = \frac{1}{3} \; \frac{b\,G_e}{S_o(1-\lambda)} .$$

The merits of de Vries' work have been outlined in Chapter II. The model proposed by de Vries was chosen for detailed study and his equation has been solved in this Chapter for the boundary conditions of the problem.

Before presenting the theoretical analysis, the salient features of the problem which have a direct bearing on the analysis shall be explained.

3.2 SUDDEN DISCONTINUITY IN TRANSPORT RATE

Suppose that the sediment load at a section is suddenly increased without changing the discharge and the sediment size. This represents a case of sudden discontinuity in the variation of sediment transport rate along the length. In case of aggradation through back water or degradation through draw down, such a discontinuity does not exist; the transporting capacity is gradually increased or decreased from the uniform flow value to some specified value over the length of back-water curve or draw-down curve.

Due to sudden addition of sediment load, the equilibrium is disturbed. To reestablish equilibrium the slope of the bed must increase. Ordinarily, after such a change in flow condition, a new equilibrium tends to be established in the following manner: When the rate of sediment supply is increased the stream between sections O and A (see Fig.3.1) cannot carry the increased load of sediment and some of it is deposited on the bed downstream of O causing the bed to aggrade to O_1. Deposition continues with the addition of sediment load and the bed level rises to O_2 and the aggradation extends further downstream. If the sediment addition continues for a long time, a new equilibrium grade O_3A would ultimately be established such that the new slope with the prevailing water discharge is able to transport the increased sediment supply rate.

The above mentioned changes would also cause a change of slope upstream of the secion O. The rise of the bed level to O_1 would also cause a decrease in the slope upstream of the section O. The stream cannot transport the sediment brought down on this decreased slope and some of it would be deposited upstream of O. With the continued aggradation of stream bed downstream of O, the bed level of the stream upstream would also rise approaching a final equilibrium grade O_3B_3 parallel to original grade OB.

In the theoretical treatment in this Chapter and subsequent analysis only aggradation downstream of the section of sediment injection is considered.

3.3 AGGRADATION DUE TO OVERLOADING

Equation 2.9 shall be solved for boundary conditions of the problem of aggradation due to sudden addition of sediment load to determine the transient bed profiles of the stream. Injection of sediment load is assumed to be taking place continuously and at a constant rate. The discharge of the water and sediment size are assumed to be constant. It is further assumed that the total load transport may be used in the equation of continuity for sediment implying little change in the amount of suspended load from section to section and with time. The width of the stream is taken to be constant.

3.3.1 Mathematical Model

The mathematical model governing the process of river bed variation under simplified assumptions has been given by de Vries (39) 1973, as

$$\frac{\partial Z}{\partial t} - K \frac{\partial^2 Z}{\partial x^2} = 0 \ . \quad \ldots \qquad (2.9)$$

The theoretical value of K designated as K_o is given by

$$K_o = \frac{1}{3} \frac{b\, G_e}{S_o(1-\lambda)} \ . \quad \ldots \qquad (2.13)$$

In the case of overloading at a constant rate at a section $x = 0$, the value of deposition depth at this section shall always be greater than that for $x > 0$. The magnitude of this depth shall go on increasing with increase in time. Prior to injection Z is zero every where (See Fig 3.2). At large distances for $t > 0$, the deposition depths can be taken as zero. Hence the boundary conditions for the present problem are :

$$\left.\begin{array}{ll} Z\,(x,0) = 0 & \\ Z\,(0,t) = Z_o(t) & \text{for } t > 0 \\ Z\,(x,t) = 0 & \text{for } x \to \infty, \ t \geq 0, \end{array}\right\} \quad \ldots \quad (3.1)$$

in which $Z_o(t)$ represents the deposition depth at $x = 0$ as function of time. Assuming further that K is a constant, and taking Laplace transform one gets

$$s\,\bar{Z} - K \frac{d^2\bar{Z}}{dx^2} = 0, \qquad \ldots \quad (3,2)$$

in which bar over the symbol denotes Laplace transform of a quantity and s denotes the transformed time variable. If $\bar{Z}_o$ represents Laplace transform of $Z_o(t)$, the solution of Eq. 3.2 is obtained as (4) 1963,

$$\bar{Z} = Z(x,s) = \bar{Z}_o\, e^{-\sqrt{s/K}\,x}. \qquad \ldots \quad (3,3)$$

Knowing that inverse Laplace transform of $e^{-\sqrt{s/K}\,x}$ is equal to $\frac{x}{\sqrt{2\pi K t^3}}\, e^{-x^2/4Kt}$ and making use of convolution theorm, one obtains (4) 1963,

$$Z(x,t) = \int_0^t \left(\frac{x}{2\sqrt{\pi K}}\, Z^{-3/2}\, e^{-x^2/4KZ} \right) Z_o(t-Z)\,dZ. \quad \ldots \quad (3,4)$$

Letting $u^2 = x^2/4\,KZ$, Eq 3.4 reduces to

$$Z(x,t) = \frac{2}{\sqrt{\pi}} \int_{x/2\sqrt{Kt}}^{\infty} e^{-u^2} Z_o\left(t - \frac{x^2}{4Ku^2}\right) du, \quad \ldots \quad (3.5)$$

In case when $Z_o(0,t) = Z_o$, a constant value, which is representative of a situation very close to the attainment of final equilibrium, i.e., when $t \to \infty$, the solution of Eq.3.2 may be obtained as :

$$Z(x,t) = Z_o(1 - \text{erf}\,\eta) \quad \ldots \quad (3.6)$$

$$= Z_o(\,\text{erfc}\,\eta\,), \quad \ldots \quad (3.7)$$

where $\eta = x/2\sqrt{Kt}$ and erf η and erfc η are defined as

$$\text{erf}\,\eta = \frac{2}{\sqrt{\pi}} \int_{0}^{\eta} e^{-u^2}\,du \quad \text{and,}$$

$$\text{erfc}\,\eta = 1 - \text{erf}\,\eta. \quad \ldots \quad (3.8)$$

The solution obtained in Eq.3.7 is the same as obtained by de Vries in the case of degradation phenomenon downstream of a dam.

The variation of Z with t at x = 0 is not known in the present problem and the explicit solution of the integral Eq. 3.5 is not possible. As such the solution for the transient bed profiles is not forthcoming directly from Eq. 3.5. Purely theoretical considerations would not lead to the determination of $Z_o(t)$ explicitly unless the transient bed profile itself is known. Hence, the information regarding the variation has to come from experiments. The deposition depth at x = 0 i.e., Z_o shall be a function of many variables such as time and fluid, flow, and sediment characteristics. The determination of the correct form of such a functional relationship involving empirical coefficients may not be easy. An alternative to the above scheme which shall avoid determination of the explicit form of $Z_o(t)$ is proposed below and shall be tested against experimental information.

Assuming that the solution obtained in Eq.3.6 be also applicable for transient stages meaning that Z_o is now the value of deposition at x = 0 for any transient bed profile, the value of the deposition at any distance and time can then be computed from Eq. 3.6. It may be pointed out that many simplifications were made in arriving at Eq. 3.6. These simplifications are :(i) Simple form of sediment transport law of the form $G = a U^b$ is used in which the effect of bed forms is not being explicitly accounted for. (ii) The use of sediment transport law under uniform flow conditions for non-uniform flow conditions. (iii) The assumption regarding linearisation made by de Vries. In view of the above it is expected that the value of K enabling prediction of transient profiles in agreement with the measured profiles may be different from the theoretical value of K equal to K_o given by Eq. 2.13.

3.4 LENGTH OF AGGRADATION

It is obvious from Eq. 3.6 that at a given time the aggradation depth Z, downstream of the section of sediment injection decreases asymptotically in the longitudinal direction. However, the region of asymptotic change in Z is taken to end at a section beyond which the change in Z does not exceed some prescribed value. In that case the length of aggradation, l, shall be the distance between the section of sediment injection and the section where the aggradation is assumed to end.

Assuming aggradation ends where $Z/Z_o = 0.01$ a usual assumption made in problems having asymptotic behaviour, e.g., as in a boundary layer theory - one gets from Eq. 3.6

$$\frac{Z}{Z_o} = 0.01 = 1 - \operatorname{erf} \frac{1}{2\sqrt{Kt}}$$

$$\left.\begin{aligned} \text{or} \quad & \frac{1}{2\sqrt{Kt}} = 1.83 \\ \text{or} \quad & l = 3.66\sqrt{Kt} \end{aligned}\right\} \quad \ldots \quad (3.9)$$

If the limit for aggradation is set at $Z/Z_o = 0.02$, one gets

$$\left.\begin{aligned} \frac{l}{2\sqrt{Kt}} &\doteq 1.65 \\ \text{or } l &= 3.30\sqrt{Kt} \end{aligned}\right\} \quad \ldots \quad (3.10)$$

3.5 DETERMINATION OF Z_o

In order that the transient bed profiles be determined by Eq.3.6, the magnitude of Z_o must be known. For the given rate of sediment injection ΔG and the time t during which this is taking place, the magnitude of Z_o can be calculated by equating the volume of sediment coming in during time t to the area under the corresponding transient bed profile,(see Fig.3.2):

$$\Delta G \cdot t = \int_0^l (Z\, dx)(1-\lambda) \quad \ldots \quad (3.11)$$

Combining Eqs.(3.6) and (3.11), one gets

$$Z_o = \left(\frac{\Delta G \cdot t}{(1-\lambda)}\right) \Big/ \int_0^l \left[1 - \text{erf}\left(\frac{x}{2\sqrt{Kt}}\right)\right] dx, \quad (3.12)$$

which yields,

$$Z_o = \frac{\Delta G \cdot t}{(1-\lambda)} \Big/ l\left\{1 - \frac{2}{\sqrt{\pi}}\left[\frac{1}{4}\left(\frac{l}{\sqrt{Kt}}\right) - \frac{1}{96}\left(\frac{l}{\sqrt{Kt}}\right)^3 + \frac{1}{1920}\left(\frac{l}{\sqrt{Kt}}\right)^5 - \frac{1}{4.30\text{x}10^4}\left(\frac{l}{\sqrt{Kt}}\right)^7 + \frac{1}{11\text{x}10^5}\left(\frac{l}{\sqrt{Kt}}\right)^9 - \frac{1}{32.44\text{x}10^6}\left(\frac{l}{\sqrt{Kt}}\right)^{11} + \ldots\right]\right\}$$

$$\ldots \quad (3.13)$$

Taking the length of aggradation profile given by Eq.3.9, the expression within the parenthesis on the right hand side of Eq.3.13 works out to be 0.307. Equation 3.13, therefore, reduces to :

$$Z_o = \frac{\Delta G \quad t}{1.13(1-\lambda)\sqrt{Kt}} \quad \ldots \qquad (3.14)$$

or $$\frac{Z_o}{\sqrt{Kt}} = 0.885 \frac{\Delta G}{K(1-\lambda)} \quad \ldots \qquad (3.15)$$

3.6 TIME SCALE

Equation 3.6 shows that the solution for the problem of aggradation under study involves only the single dimensionless parameter $x/2\sqrt{Kt}$; the time scale of aggradation process can, therefore, be estimated from this equation. Suppose at a section x, α fraction of final aggradation has taken place, then the time t_α in which this would have taken place can be computed from this equation,

$$\alpha = 1 - \text{erf} \frac{x}{2\sqrt{Kt_\alpha}} \quad \ldots \qquad (3.16)$$

or $$\text{erf} \frac{x}{2\sqrt{Kt_\alpha}} = 1 - \alpha \quad \ldots \qquad (3.17)$$

From the table of error functions, the value of

$x/2\sqrt{Kt_\alpha}$ can be determined say β, then

$$\frac{x}{2\sqrt{Kt_\alpha}} = \beta$$

or $$t_\alpha = \frac{x^2}{4\beta K} \qquad \ldots \qquad (3.18)$$

From Eq. 3.18 it follows that time required for any section to reach a given deposition depth is proportional to the square of its distance from the section of sediment injection and varies inversely as the aggradation coefficient. For $\alpha = 0.50$, $\mathrm{erf}(x/2\sqrt{Kt}) = 0.5$ and from tables of error function $\beta = 0.477$. Thus

$$t_{0.50} = \frac{x^2}{0.90K} \qquad \ldots \qquad (3.19)$$

3.7 CONCLUDING REMARKS

Transient bed profiles at different times can be completely determined by Eq.3.6, while the length and maximum depth of deposition are given by Eqs.3.9 and 3.15. These equations have been obtained on the basis of several simplifying assumptions such as constant width of channel, steady water movement during transient stages, use of resistance and sediment transport law developed for uniform flow under non uniform conditions occurring during aggradation.

The theoretical value of aggradation coefficient, i.e., K_o is based on the value of b derived from uniform flow data. As already mentioned there is evidence to indicate that the sediment transport law under nonuniform flow conditions may be different from that for uniform flow implying a possible change in the value of b. Also in the prediction analysis, solution obtained for constant boundary condition, i.e., $Z = Z_o$ is being applied for the case of $Z = Z_o(t)$. These two assumptions are likely to affect the results appreciably in a real situation.

It is therefore, logical to expect that the derived equations would need some corrections such as modification in the value of K. Such a modification in the value of K can only be done on the basis of known set of data. Experimental data collected by the author are used to find an empirical predictor for such a modified value of K. Hence once the modified values of K is known Eq. 3.6 can be used to predict the transient bed profiles.

CHAPTER IV

EXPERIMENTAL INVESTIGATION

4.1 PRELIMINARY REMARKS

The mathematical analysis developed in the previous Chapter for predicting aggradation profiles is based on many simplifying assumptions. As such there is a need for comparison of the analytical results with those observed either in the laboratory or in the field. No laboratory or field data are available for the problem of aggradation in streams arising due to increase in the rate of sediment supply over and above the equilibrium sediment transport rate.Experiments were,therefore, carried out in a laboratory flume to provide information on bed and water surface profiles under such transient conditions. Since aggradation creates non-uniform flow conditions, the data collected also provide an opportunity to study sediment transport and resistance laws under unsteady and non-uniform flow conditions. The salient features of the experimental set up and procedure are described in the following sections.

4.2 EXPERIMENTAL SET-UP

The experiments were conducted in a 20cm wide, 50cm deep and 30m long recirculatory tilting flume located in the hydraulicslaboratory of University of Roorkee,Roorkee.

The flume was provided with glass side wall on one side and a painted steel-sheet wall on the other side. The presence of different materials on two sides necessitates a modification to the side wall correction procedure valid for a flume with the same material on both sides. Such a modified method was devised and is presented in Appendix-I. However, the difference in the nature of the material is not such as to cause unsymmetrical flow conditions; symmetry about the centre line would in no case be obtained even in flumes with same material on both walls because of the three dimensionality of bed undulations.

The recirculatory system consisted of a rectangular tank having sloping bottom to collect the sediment laden flow from the downstream end of the flume (Fig.4.1). A 25-H.P. pump was connected with the tank and a supply pipe for maintaining the recirculation. A 10cm-dia supply pipe line was connected to the upstream end of the flume. An orifice meter of 7.5cm diameter was installed and calibrated in the supply line for the measurement of discharge, and the discharge was controlled by a valve. The maximum discharge that could be carried in the flume was 15 l/sec. A floating wooden wave suppressor provided at the entrance of the flume for damping the disturbances at the free surface. Rails made from metallic tube were provided on the top of side walls. A movable carriage with a pointer gauge having a least count of 0.01 cm. was mounted on a carriage which could move on the rails. This was used for recording water surface and bed elevations. An adjustable gate at the downstream end of the flume was used to control the depth of flow in the flume.

The sand was filled in the flume upto a depth of 15 cm. and levelled parallel to the rails. The sand forming the bed and injected material had a median sieve diameter of 0.32 mm and a geometric standard deviation of 1.30. The grain size distribution curve of the sand used is shown in Fig.4.2. The specific gravity of the sand was 2.65.

4.3 EXPERIMENTAL PROCEDURE

The experiments conducted can be grouped under two categories — uniform flow experiments and aggradation experiments. Uniform flow experiments were conducted for determining the relation for sediment transport and resistance to flow under such conditions for the sediment used in the study. Subsequently experiments were also conducted in which the sediment was injected at the upstream end and the aggradation downstream was studied. Detailed measurements of the bed and water surface profiles at various times were taken; these were useful in the study of sediment transport and resistance to flow under non-uniform flow conditions, apart from providing the basic data on aggradation.

The temperature of water was recorded at the begining and end of each run and since the maximum difference between these two was 1°C, the average of these was taken as the average temperature for the run. A continuous watch was kept on the discharge and it was ensured that it remained almost constant.

4.3.1 Uniform Flow Experiments

A series of 24 uniform flow runs were conducted during this study. In these experiments the flume was filled with sediment as mentioned earlier and then was given the desired slope.

The recirculatory system was then filled with water and the pump started. The valve was slowly adjusted to give the specified discharge and uniform flow was obtained by adjusting the tail gate at the down stream end of the flume and allowing the bed to adjust. Because of the effect of entrance and exit conditions on the flow, about 3 m length of the flume at the upstream and downstream ends was not considered in assessing the uniformity of the flow. Uniform flow condition was considered to have been attained when the measured bed and water surface profiles were parallel to each other. About 4-6 hours were required to establish such a condition. After the uniform flow was established the bed and water surface profiles were recorded at ten sections 2.50 m apart and thus the average depth of flow determined.

The concentration of total sediment load was measured at the downstream end of the flume. The individual sample was obtained by a sampler (see detail 'A' in Fig.4.1), which discharged the sand water mixture into a collector. The sampler collects the total load as it is made to traverse across the width of the flume at a uniform speed. Minimum of 8-10 samples were collected at an interval of 5 minutes. The contents of the collector for each of these samples were filtered through a thick filter paper. The material was dried in an oven, then weighed on a chemical balance and concentration in gms/litre for each sample determined. From these data average sediment concentration was determined for each run. The basic data collected on uniform flow have been summarised in Table 1 of Appendix-II.

4.3.2 Aggradation Studies

In order to study aggradation due to overloading uniform flow conditions were first established for a prescribed discharge and slope as described earlier. The equilibrium concentration was also obtained. To simulate the conditions of aggradation taking place in a stream when the rate of sediment supply is increased over the equilibrium sediment transport rate, additional sediment was dropped manually at a desired constant rate into this recirculatory system at the upstream end of the flume. The section of sediment injection was located near the upstream end of the flume but far enough from the entrance to be unaffected by entrance disturbances. The excess sediment load was progressively deposited in the flume.

It was estimated that only about 5-10 per cent of the injected material was deposited upstream of the section of sediment addition. Since the study was concerned only with the aggradation downstream of the section of sediment addition, the amount of sediment deposited downstream was used in the computation of amount of additional load. It may be noted that in view of the very small amount of deposition upstream, the amount of material actually added is practically equal to the material deposited downstream.

The bed and water surface profiles were measured at various times at eleven sections along the length of the flume downstream of the section of sediment injection. The profiles were generally taken at intervals varying from 10-20 minutes. This time interval was decided on the basis of rates of deposition of sediment, the velocity of propagation of aggradation front and the time required to measure one set of transient profiles so that a few profiles could be recorded before the injected sediment became a part of recirculatory system. The runs were continued till the front reached close to the end of the flume. During the progress of aggradation, the sediment samples were taken at the downstream end of the flume to ensure that the injected sediment had not reached the downstream end of the flume but all of it has been deposited in the flume. Such a check was carried out by studying the variation of sediment transport rate, G, with time, t, at the end of the flume before and after sediment injection. Two such typical plots are shown in Fig. 4.3. These curves show that, on an average, the sediment transport rate at the end of the flume after sediment injection is equal to that prior to the addition of excess sediment. This was true as long as the aggradation front had not reached the downstream end of the flume. The bed and water surface measurements were always stopped well before a part of the added sediment reached the downstream end of the flume.

The area under each transient bed profile was determined and ΔG thus calculated during the time interval. The average of the values so determined for all transient profiles of a run was taken as ΔG for that run. The maximum difference between the individual ΔG values during a run was 15 per cent.

The aggradation experiments were conducted using two discharges- 4 l/s and 7 l/s -- and range of slopes from 0.00212 to 0.00652 under different rates of sediment addition. The rate of sediment injection was varied from 0.30 G_e to 4.00 G_e.

A total of eleven runs were conducted to obtain thirty transient profiles of the bed and water surfaces. A summary of the data collected during such aggradation runs is given in Table 2 of Appendix II. Here run 3.50-U-4 means that the run U-4 listed in Table 1 of Appendix II has been used for aggradation studies and rate of overloading $\Delta G/G_e$ is equal to 3.50; and so is the meaning of other runs listed in the Table 2.

CHAPTER-V

ANALYSIS OF DATA

5.1 PRELIMINARY REMARKS

The experimental data obtained under the present investigation pertain to uniform as well as non-uniform flow conditions. The uniform flow data have been analysed first to establish basic relationships for sediment transport and resistance to flow that are valid for the sediment used in the study. The non-uniform flow data have been utilised for studying the characteristics of flow during aggradation and establishing sediment transport and resistance laws under such condtions. Finally a comparison has been made between the experimentally observed aggradation profiles and profiles obtained from mathematical analysis. As a result, a modified value of K has been proposed for use in the mathematical relations. This modified value of K can be predicted from the empirical relation evolved by the analysis of data. Calculations of transient bed and water surface profiles with the help of the proposed relations have been illustrated taking the case of Colorado river, U.S.A.

5.2 VERIFICATION OF RELATIONS FOR UNIFORM FLOW

Some of the existing uniform flow relations for resistance to flow and sediment transport were tested using the data from uniform flow conditions. Agreement with the well established relations would indicate that the data are reasonably accurate. Where only the form of the relationship is well established, plotting of these data in such form would help in determination of the constants required in the later analysis. Throughout the analysis presented in this Chapter the hydraulic radius with respect to the bed was used in place of R and its value found by the method explained in Appendix I.

5.2.1 Sediment Transport Law in Terms of Velocity

A simple sediment transport law of the form

$$G = a\,U^b \qquad \text{... (2.5)}$$

has been used by some investigators earlier (6,19,20) and was used in the derivation presented in Chapter III. This equation involves a coefficient 'a' and an exponent 'b' which must be determined experimentally. The data from the uniform flow experiments can be utilised to determine the values of a and b. The experimental data are plotted on a log-log paper as G versus U in Fig. 5.1. The relatively small scatter of data permits fitting a straight line through them justifying the form of Eq. 2.5. The best fit line drawn through these points by visual inspection yielded values of a and b as 1.45×10^{-3} and 5.0 respectively

Eq. 2.5 then beomes

$$G = 1.45 \times 10^{-3} U^{5.0} , \quad \ldots \quad (5.1)$$

in which G is in m^3/sec-m and U is in m/sec, and the above equation is used in later analysis. The values of a obtained by Kennedy (18) 1961, for sands with sizes 0.549mm and 0.233mm are 0.17×10^{-3} and 0.35×10^{-3} respectively; the corresponding values of b are 3.4 and 6.2 respectively.

5.2.2 Vittal-Raju-Garde Sediment Transport Law.

Figure 5.2 shows Vittal-Raju-Garde's curve of dimensionless effective shear stress for total load, τ_{*t}, versus dimensionless total load transport rate, ϕ_T, for ripple and dune beds. These parameters were computed for the sediment transport data & are shown plotted in Fig.5.2. Although the data fall within the scatter zone of the original graph, they plot consistently lower, for which no explanation could be found. Nevertheless a mean line has been fitted through the present data purely for the purpose of subsequent comparison with sediment transport data under non-uniform flow conditions.

5.2.3 Ranga Raju Resistance Law

Ranga Raju's resistance relationship in terms of parameters $K_1 . \frac{U}{\sqrt{(\Delta \gamma_s / \rho) R}}$ and $K_2 (\frac{R}{d})^{1/3} (\frac{S_f}{\Delta \gamma_s / \gamma})$

is shown in Fig.5.3. The above parameters were computed for the uniform flow data and these data are shown plotted in the same figure. The data plot well within the band of scatter in the original plot indicating a maximum error of ± 30 per cent in velocity. Uniform flow data may, therefore, be taken to be represented by this resistance relationship. In fact the above resistance relationship and the two sediment transport laws discussed earlier form the basis of subsequent comparison with the experimental data for non-uniform flow conditions.

5.3 CHARACTERISTICS OF FLOW UNDER AGGRADING CONDITIONS

Aggradation of the stream bed takes place due to injection of additional sediment at the upstream end of the flume; this leads to the existence of non-uniform flow conditions. The several intersting characteristics of this phenomenon are discussed below.

5.3.1 Transient Bed and Water Surface Profiles

A typical plot of transient bed and water surface profiles downstream of the section of sediment addition is shown in Fig.5.4. This figure depicts the actual data points and the mean bed and water surface profiles. Averaging of these profiles was required because of the presence of ripples and dunes on the bed. It was seen that the scatter of data around the averaged bed profiles was generally smaller in the aggraded reach than in the region of uniform flow indicating smaller undulations in the aggraded reach. This implies a smaller roughness coefficient in the aggraded reach which was confirmed by calculationsdiscussed later. Figure 5.5 shows the averaged transient bed and water surface profiles for the rates of overloading, $\Delta G/G_e$, equal to 4.0 and 3.50 respectively. The deposition depth, Z, and depth of flow at various sections and for various times during aggradation (listed in Appendix-II) are taken from such averaged profiles.

The shape of the transient bed profile is concave upwards and the ordinate of the curve is monotonically decreasing with increase in distance. However, aggradation was taken to end at a section beyond which differences in Z between two consecutive sections was so small that it could not be measured. This defined the location of aggradation front and hence the length of aggradation at various times. Water surface profiles at small time intervals after sediment addition show a considerable change in curvature indicating probably that upto a certain time the deposit of the injected sediment acts like a local hump on the bed.

5.3.2 Variation of Manning's Roughness Coefficient

From the observed transient bed and water surface profiles during aggradation the mean velocity, U, at various sectionscan be computed. The position of the total energy line can then be determined as $U^2/2g$ above the water surface, after making an assumption that energy correction factor is unity. Selecting a step length Δx, the local friction slope, S_f, at various points on the energy line say ABCDE with end points A and E can be calculated using central difference formula for interior points, i.e., B, C and D; and right hand and left hand difference formulae for points A and E respectively. For example the values of the local values of friction slope at C, A and E would be :

$$(S_f)_C = \frac{Y_B - Y_D}{2\Delta x},$$

$$(S_f)_A = \frac{Y_A - Y_B}{\Delta x}, \qquad \ldots \qquad (5.2)$$

$$(S_f)_E = \frac{Y_D - Y_E}{\Delta x},$$

in which Y with subscripts A, B, C .... represent the ordinates of energy line at points A,B..etc. The value of the local hydraulic radius corresponding to the bed is computed by making use of the local mean velocity and the local friction slope using the procedure explained in

Appendix-I. After the computation of local values of U, R and S_f the Mannings roughness coefficient, n can be determined with the use of the following equation

$$U = \frac{1}{n} R^{2/3} S_f^{1/2} \qquad \ldots \qquad (5.3)$$

The variation of n in the longitudinal direction under transient conditions for given rates of overloading was studied. Three typical plots showing variation of n with x for values of $\Delta G/G_e$ equal to 1.0, 3.0 and 4.0 are shown in Fig. 5.6. In general the value of n in the aggraded reach is smaller than that under equilibrium condition; the maximum reduction in the value of n is about 30 per cent. The decrease in value of n is consistent with the observation that during aggradation the dunes are flattened out. Further the variation in 'n' from equilibrium value seems to be small for runs with small values of $\Delta G/G_e$. But there is no systematic variation in the value of n with sediment injection rates. At any particular time after sediment addition roughness coefficient, n, tends to decrease with increase in x initially and then again approaches the equilibrium value.

5.3.3 Variation of Sediment Transport Rate Along Aggraded Reach.

The sediment transport rate at various sections downstream of the section of sediment injection for $t = t_1, t_2 \ldots$ etc., can be determined from the equation of continuity for sediment, viz. Eq. 2.4. Reducing Eq. 2.4 into finite difference form and applying it to two consecutive sections A and B, Δx apart see Fig. 5.7(a), one obtains:

$$V = \left[\int G.dt \right]_A - \left[\int G.dt \right]_B \qquad (5.4)$$

in which V is the absolute volume under the transient bed profile $= \Delta Z \cdot \Delta x \; (1 - \lambda)$. Customarily used value of 0.40 was taken for λ. The manner of variation of G with respect to time at both the sections being unknown, the above equation cannot be solved directly. At the section x = 0, the sediment injection is taking place continuously at a constant rate. The sediment transport rate at this section is thus constant and is equal to $G_e + \Delta G$. So if the sections of sediment injection is chosen as one of the sections, then only one unknown is left in Eq. 5.4. Even then the direct solution of Eq. 5.4 is not possible because of lack of knowledge of variation of G with respect to time. However, starting from the section of sediment injection, Eq. 5.4 can be solved by the following graphical trial and error procedure.

On a G-t plot, (See Fig.5.7) draw a horizontal line H-I at the known specified transport rate equal to $G_e + \Delta G$. Assume a trial G-t curve O-P-Q-R for a particular section say A. The area between the assumed curve and line H-I is calculated and compared with the area under the transient bed profile for the same time interval and between the sections O and A. In other words the two shaded areas must give the same volume after taking into account the porosity of the sand mass. In case of difference the assumed G-t curve is adjusted in such a manner that the areas become equal. The point P on the G-t curve represents the sediment transport rate at the point(i,j+1) on the transient bed profile. The trial G-t curve O_1-M-N-S is assumed for the section B, the variation of the sediment transport rate at section A now being known. The area between the curves O-P-Q-R and O_1-M-N-S between time t = 0 and $t = t_1$ is then comapared with the area under the transient bed profile between section A and B. In case of difference the trial G-t curve namely O_1-M-N-S is adjusted till the areas are balanced. The point M on the G-t curve now represents the sediment transport rate at the point (i+1,j+1) on the transient bed profile. Proceed in a similar fashion for all the sections till the end of transient bed profile for $t = t_1$.

The sediment transport rate for the second transient bed profile for $t = t_2$, i.e., for points (i,j+2), (i+1,J+2),(i+2,J+2).. etc.,is determined by extending and adjusting the previously assumed G-t curves in such a fashion that the requirements of continuity are satisfied. The transient bed profile for $t = t_1$ serves as a base for working out sediment transport rates for the transient bed profile $t = t_2$. The sediment transport rates at various locations for all the subsequent profiles are worked out in a similar fashion.

During the process of adjustment by trial and error, it was ensured that the calculated G-t curves are smooth and that there is a general similarity in shape of these curves at various x values. Apart from the possible errors in averaging, these computed valuesmay be prone to some error because the above procedure is based on the presumption that suspended load varies little with x and t. While it was observed that most of the added sediment settled to the bed in a short length and then moved as bed load, variation in suspended load along the length (and with time) cannot be ruled out. Since this variation is likely to be more significant close to the section of sediment injection, computed values of G at small x values are likely to be less accurate than those at large distances. Further some inaccuracy may also creep in due to error in measurements towards the tail end of transient bed profile where very small deposition depths are involved. Nevertheless, these computations provide useful information on the variation of sediment transport rate along the length of the channel.

Figure 5.8 shows typical curves of G versus t with distance x as the third parameter. It is seen from these plots that the sediment transport at a particular section increases very fast from the equilibrium value once the aggradation front reaches that section and then gradually increases and approaches the increased sediment transport rate $G_e + \Delta G$ asymptotically.

5.4 SEDIMENT TRANSPORT LAW UNDER NON-UNIFORM FLOW CONDITIONS

As discussed in Chapter II data on sediment transport under non-uniform flow conditions are practically non-existent. The values of G under such conditions computed in section 5.3.3 provide an excellent opportunity of examining the applicability of the relations developed for uniform flow to such non-uniform flow conditions as are obtained under aggradation. The relations of Vittal, et al. and the one discussed earlier in this Chapter are used for this purpose.

5.4.1 Comparison with Vittal, et al. Relation

The parameter ϕ_T was calculated by making use of the computed sediment transport rates at various sections along the length of the transient profiles at various times. The values of the dimensionless grain shear stress τ'_{*s} were computed by making use of the corresponding local values of U, R_h and S_f. The values of dimensionless effective shear stress τ_{*t} were read against these τ'_{*s} values from Fig.2.7. The parameter ϕ_T and τ_{*t} obtained for two aggradation runs 1.8-U-3 and 4.0-U-1 are shown plotted in Fig.5.9, along with the Vittal,et al.curve and mean curve drawn through the uniform flow data of the present study. The mean curves are drawn through these points and shown dotted. The arrows on these two mean curves show that with increasing distance from the section of sediment injection the data fall closer to the curve for uniform flow and eventually merge with the uniform flow data of the present study as indeed they should. It can, therefore, be concluded that in case of pronounced non-uniformity the relation between the shear and sediment transport is different from the relationship valid under uniform flow conditions. The data of all the aggradation runs is shown plotted in Fig. 5.10. It is interesting to note from this figure that although majority of the data points fall above the curve for uniform flow, there are still a fair number of points falling below the curve. Such dispersion of the data is clearly indicative of the complexity of the problem of sediment transport under non-uniform flow conditions.

The relation between the sediment transport rate and the mean velocity was derived as under uniform flow conditions. The local sediment transport rates computed at various sections along the length of transient bed profiles for various times are plotted against the local mean velocity in Fig.5.11, on which is also shown Eq. 5.1. From this plot it can be seen that relationship between sediment transport rate and velocity under non-uniform flow conditions is different from the G-U relationship valid for uniform flow conditions. In general the transport rate is smaller than that under uniform flow conditions for a particular mean velocity.

5.4.2 Study of Variation of δ

It was concluded in the previous section that under non-uniform flow (and unsteady) conditions the sediment transport relations for uniform flow are not directly applicable. Based on such a speculation, Kennedy introduced the concept of lag distance, δ, for which a physical explanation has been given but no quantitative estimates of this parameter are available. The data on aggradation can be used, as done below to find δ and study its variation.

The depth of flow at various sections in the longitudinal direction was determined from the observed transient bed and water surface profiles . Knowing the discharge of flow, the mean velocity at these sections was computed. The velocity corresponding to the local sediment transport rate was computed from Eq.5.1. The section where this mean velocity occurs on the aggraded profile was then determined. The distance of this section from the section under consideration (for which G was known) is obviously the lag distance, δ, (See Fig. 5.12). The lag distance was considered positive when the sediment transport was related to the mean velocity at a section downstream of the section under consideration and negative when related to the mean velocity at a section upstream of the section under consideration. The δ-values computed in this manner are tabulated in Appendix II.

Figure 5.13 shows the typical variation of G computed with x; G being computed in two ways. In the first case G is computed from Eq.5.1 and in the other case it is computed from the continuity relationship as explained in section 5.3.3. It can be seen that δ shall be positive throughout the length of aggradation profile in run 1.8-U-3 because the curve representing G computed from Eq.5.1 is lying throughout above the other curve; whereas it shall be positive over a part of the length and negative over the remaining length of the profile in the second run.

The typical variation of the lag distance, δ, with x along the length of transient bed profiles is shown in Fig.5.14 for the runs 3.5-U-4 and 0.90-U-9. Extrapolating such curves on to the section x = 0, the maximum values of the lag distance, δ_m, were determined for all runs except one; and are listed in Appendix II. In run 0.30-U-11 it was not possible to get values of δ, because the required mean velocity was not available in the length over which measurements were made. Figure 5.15 depicts the nondimensional plot of δ/δ_m versus x/l for the runs listed in Appendix II. The data disperse greatly indicating no unique relation between these parameters. It is further seen that for about 20 per cent of the data, δ values are negative. An attempt was made to relate δ_m with the relevant flow parameters. A plot of δ_m/l versus Froude number of uniform flow (not shown) showed poor correlation. Figure 5.16 shows the variation of δ_m/R with the nondimensional time Ut/R. The plot indicates a general increase in δ_m/R with increase in Ut/R, but the scatter of data indicates that there is no exact relation between these parameters.

In his stability analysis of plane bed leading to micro-morphological features-ripples, dunes, anti-dunes— Kennedy (19,20) 1963,1969, hypothesised that sediment transport rate lags the local changes in velocity at the mean level of the bed. In such a case, therefore, sediment transport rate at a section is always related to the velocity upstream of this section, and hence as per the author's definition δ shall always be negative. The analysis based on non-uniform flow data has shown that δ can either be positive or negative. The above analysis, therefore, does not fully support Kennedy's hypothesis. Some explanation can be offered for the discrepancy between the present results and Kennedy's hypothesis.

(i) Due to sudden injection of sediment, unsteadiness created in the flow is severe near the section of sediment injection. There occurs a sudden discontinuity in the sediment transport rate at this section. No such discontinuity occurs in Kennedy's analysis of instability.

(ii) It is emphasised that the G-values used in the analysis are computed values and not measured ones. G was computed considering the variation of suspended load with x and t to be negligible. If suspended load content in a certain reach is assumed to increase in a particular period because of sediment injection, the actual value of G would be smaller than those listed in Appendix-II.

(iii) Kennedy dealt with a micro-morphological process-ripples, dunes,etc.- whereas in the present problem a macromorphological process like aggradation is involved.

(iv) Due to rise in bed level at x = 0 caused by sediment addition, aggradation shall also take place upstream of this section resulting in lower velocities. Since no data were collected upstream of this section it was not possible to check whether the required velocity in the first few metres of the aggraded reach were really available upstream also.

(v) Kennedy's potential flow model is based on the velocity close to the bed. In the foregoing computations the mean velocity of flow at the section has been used because the velocity distribution in the vertical was not known and could not be predicted.

Figures 5.15 and 5.16 were prepared as a logical step following the computation of δ. It is emphasised, however, that not much significance can be attached to the positive value of δ and any significant conclusions regarding Figs.5.15 and 5.16 must await a better understanding of the mechanics of sediment transport in non-uniform flows.

Summing up on the subject of sediment transport relation under non-uniform flow conditions, it has been shown that existing relations for sediment transport cannot be used directly under such conditions. The present analysis establishes for the first time that lag distance, δ, does exist and is a real physical entity and provides some ideas regarding its order of magnitude in aggrading flows. However, it has not been possible to prove the hypothesis of Kennedy that sediment transport at a section is always related to velocity upstream of the section. The possible reasons for such discrepancy between actual data and Kennedy's hypothesis have been clearly mentioned. But it is quite possible that the concept of δ introduced by Kennedy to discuss the micro-morphological process of bed formation under uniform flow may not be directly valid for macro-morphological process of wide spread aggradation.

5.5 RESISTANCE RELATIONSHIP FOR NON UNIFORM FLOW

In rigid boundary channels, resistance law valid for uniform flow is seen to yield reasonably reliable results under gradually varied flow conditions if local friction slope is used in place of the bed slope. The uniform flow data of the present study on sediment transporting flows were found to agree with resistance relationship proposed by Ranga Raju (25) 1970, As such the non-uniform flow data were also plotted on this figure using S_f in place of S_o to check the applicability of the above relationship for non-uniform flows, see Fig. 5.17. The data plotted on this figure correspond to various sections along the length of the aggraded reach and at different times after sediment injection.

It is seen that all the data points, barring a few fall around the proposed curve and well within the scatter zone of the original plot. Detailed scrutiny of the data which fall quite far from the resistance curve showed that most of these pertain to a single run 3.5-U-4. This particular run is also the one which shows a departure from the trend of other data in the plot of K/K_o versus $\Delta G/G_e$ discussed later (Fig.5.24). This aggradation run was taken on a flat bed slope covered with large undulations and might have led to inaccurate measurements at some locations. But it seems logical to expect that in view of generally good agreement of the rest of the data with the resistance curve, the relation may be used for gradually varied flow (as in case of aggradation) with confidence after replacing the bed slope by the local friction slope. In view of above, one is inclined to conclude that any resistance relationship valid for a given river under uniform flow conditions can be used on the same river under non-uniform flow conditions provided local friction slope is used in place of bed slope.

5.6 PREDICTION OF TRANSIENT BED PROFILE

The differential equation for the solution of transient bed profiles due to overloading, namely Eq.2.9 has been used by de Vries and Adachi and Nakatoh. They, however, propose different theoretical expressions for the aggradation coefficient in this equation. A comparison between the actual profiles and those predicted from these models is presented below.

5.6.1 Adachi and Nakatoh Model

The theoretical expression for K proposed by Adachi and Nakatoh is :

$$K = C_o \, g \, q \sqrt{f/8} \qquad \ldots \qquad (2.19)$$

in which C_o is a constant in the sediment transport law ; $G = C_o U_*^3$, and f is the Darcy-Weisbach friction coefficient considered as constant for a particular run and is given by

$$\sqrt{\frac{f}{8}} = \frac{U_*}{U} \qquad \ldots \qquad (2.16)$$

The uniform flow data computed in terms of parameter

U_* ($= \sqrt{g R S_f}$) and G are shown plotted in Fig.5.18. A best fit line with a slope of 1:3 (H:V) is drawn through the plotted points and value of C_o determined as 0.372. The sediment transport law is, therefore,

$$G = 0.372\ U_*^3 \qquad \ldots \qquad (5.5)$$

It is seen from Fig.5.18 that Eq. 5.5 is not the best fit line to the data as low and high sediment discharge data fall away from this line. But there is no freedom in the choice of exponent of U_*, the sediment transport relation $G = C_o U_*^3$ being an essential part of his analysis. Adachi and Nakatoh obtained simple diffusion model only with the exponent of U_* as 3. If any other value of exponent is used the model shall become complicated. As a consequence one is compelled to use Eq. 2.17 with C_o = 0.372 even though the equation may not be truly representative of the data.

As shall be shown later in section 5.6.5, K showed poor correlation with $\Delta G/G_e$ when theoretical value of aggradation coefficient given by Adachi and Nakatoh was used for nondimensionalising K,(see Fig.5.25).

With the sediment transport law of the type $G = a\ U^b$, there is complete freedom in the choice of the exponent b. For any given situation, by passing the best fit line on a G-U plot on log-log paper, value of exponent b (and of a) can be determined. It means that the sediment transport law most representative of the situation is obtained by this procedure. It has been already shown that the relation $G = 1.45 \times 10^{-3}\ U^{5.0}$ (Eq.5.1) fits the data of uniform flow very well.

In the light of the above, de Vries model which makes use of the sediment transport law of the type $G = a\ U^b$ shall be used in preference to that of Adachi and Nakatoh in the prediction of transient bed profiles.

5.6.2 De Vries Model

The data concerning aggradation downstream of the section of sediment injection was used to check Eq.3.6. Letting K_o be the value of aggradation coefficient computed from Eq. 2.13 with value of $G = G_e$, the experimental data for all the transient bed profiles were plotted on a plot of Z/Z_o and $x/2\sqrt{K_o t}$ (not shown here). It was seen that the form of the theoretical equation is basically correct but the value of aggradation coefficient K, enabling fit of Eq. 3.6 to the experimental data was different from the theoretical value K_o. To illustrate the above point and for clarity the data from only three such aggradation runs are shown in Fig. 5.19 along with the theoretical curve, i.e., Eq. 3.6.

A significant feature of this figure is that different profiles for various times during any run fall on a single curve when Z/Z_o and $x/2\sqrt{K_o t}$ are used as the abscissa and ordinate respectively. It may be recalled that in Chapter III the scaling parameter for Z was changed from theoretically obtained $Z_o(t)$ to Z_o. That the data at various times are brought together on a single curve by the use of Z_o as the scaling parameter indicates the validity of the approximation introduced in this connection in Chapter III.

The parameter $Z_o/\sqrt{K_o t}$ is plotted against $\Delta G/K_o(1-\lambda)$ in Fig. 5.20 in which Eq.3.15 is also plotted as a solid line. One aggradation run is represented by a single point on this figure as an average of the $Z_o/\sqrt{K_o t}$ values obtained for the individual transient bed profiles of the run. This was done since the variation of the individual values from the average was not large. The data are seen to plot consistently above the line corresponding to Eq. 3.15.

In deriving the theoretical expression for K, i.e., Eq. 2.13, two major simplifications were made. Firstly the linearization $U \approx U_o$ was made where subscript O refers to the origonal uniform flow situation. Secondly the sediment transport law valid for uniform flow conditions was used for the aggrading case as well. But it has been shown that this law is not applicable during the aggradation process. As such there is every reason to expect the value of K to be different from the theoretical value K_o. The value of K which provides agreement between theory and the experimental data was determined as described below.

From Fig 5.20, the slopes m_1, m_2, ... of the experimental curve such as OA were determined. From this line one obtains the expression

$$\frac{Z_o}{\sqrt{K_o t}} = m \frac{\Delta G}{K_o(1-\lambda)} \quad \ldots \quad (5.6)$$

in which m represents the slope of experimental curve and hence assumes different values for each run. Theoretical relationship between the parameters is,

$$\frac{Z_o}{\sqrt{K t}} = 0.885 \frac{\Delta G}{K(1-\lambda)} \quad \ldots \quad (3.15)$$

Dividing Eq. 3.15 by Eq 5.6 and simplifying one gets,

$$\frac{K}{K_o} = \left(\frac{0.885}{m}\right)^2 \quad \ldots \quad (5.7)$$

The value of K was obtained from Eq. 5.7 for all the runs.

5.6.3 Dimensionless Plots with Modified Aggradation Coefficient

Figure 5.21 shows a plot of $Z_o/\sqrt{Kt}$ versus

$\Delta G/[K(1-\lambda)]$ based upon values of K computed as explained above. As can be expected the data fall around the line given by Eq. 3.15 and the scatter of data about the mean line appears to be within acceptable limits. The bed profile in dimensionless form is shown for all the runs in Fig. 5.22; it is based upon the above values of K. The data scatter on either side of the theoretical relation given by Eq.3.6. In general the scatter of data is small at small values of $x/2\sqrt{Kt}$ and relatively larger for higher values of $x/2\sqrt{Kt}$. This is partly due to the possible errors in measurement of small depths of deposition near the end of the aggradation profile. The aggradation depths will be large at sections close to the section of sediment injection and decrease rapidly in the downstream direction. Thus the relatively small scatter at small $x/2\sqrt{Kt}$ values ensures accurate prediction of the large values of Z close to the section of sediment injection. The small depths of aggradation at large values of $x/2\sqrt{Kt}$ are not predicted with the same degree of accuracy. But for most practical purposes this does not assume much importance.

5.6.4 Length of Aggradation Profile

With the modified value of aggradation coefficient,K, the experimental data in terms of l versus $\sqrt{Kt}$ are shown plotted in Fig. 5.23 alongwith the theoretical relationships Eqs. 3.9 and 3.10. It is seen that the experimental data lie closer to the line corresponding to Eq. 3.10 than Eq. 3.9. This implies that the length of transient bed profiles determined from experimental plots (see Fig. 5.5) are from the section of sediment injection to a section where the depth of deposition is about 2 percent of the deposition at the section of sediment injection. Obviously values of Z with Z/Z_o less than 0.02 were too small for measurement.

5.6.5 Variation of K/K_o With Increased Sediment Load $\Delta G/G_e$.

In the previous two sections, good agreement has been shown between experimental data and theory based on the modified value of K. Therefore, the theoretical relations can be used for prediction of transient bed profiles provided K can be related to the known sediment, fluid and flow characteristics.

The injection of additional sediment at a section in the stream is responsible for aggradation resulting in non-uniform flow conditions leading to difference between the actual and theoretical values of the aggradation coefficient. The aggradation coefficient K may, therefore, be expected to vary with the increased sediment load, apart from the theoretical value of K, viz., K_o. From dimensional considerations one can expect K/K_o to be a function of $\Delta G/G_e$. A plot between these parameters based on K_o values obtained from de Vries model and Adachi and Nakatoh models is shown in Figs. 5.24 and 5.25. It is obvious from these plots that Fig.5.24, shows better correlation between K/K_o and $\Delta G/G_e$ as compared to Fig.5.25. Hence de Vries model is adopted for use in the prediction analysis.

It is seen from Fig. 5.24 that the value of K/K_o increases with increase in the values of $\Delta G/G_e$. Despite some scatter a mean curve is fitted through the plotted points. This plot would doubtless require modification or inclusion of a third parameter as more data on aggradation particularly with different sediment sizes become available. Until such modifications or refinements are done, Fig.5.24 may be used for estimation of the correct aggradation coefficient required for prediction of transient bed profiles. One can determine K from this figure and then use Eqs. 3.15, 3.6 and 3.9 to determine the maximum depth of deposition, the profile and the length of aggradation.

In view of the uncertainties regarding Fig.5.24, one may profitably use prototype measurements over a limited period (if available) to predict successive bed profiles. The value of K can be obtained from Eq. 3.15 if one knows Z_o at a given time t for a given value of ΔG. Since the value of K remains independent of time, this value of K can be used alongwith Eq. 3.6 to predict the bed profiles at other times.

5.7 COMPUTATION OF WATER SURFACE PROFILE

Assuming uniform flow conditions to exist at the end of the aggraded reach and making use of energy equation at two sections of the aggraded reach, Δx apart, the following expression can be written for a wide rectangular channel (see Fig. 5.27) :

$$h_1 + \frac{U_1^2}{2g} + Z_1 + \frac{1}{2}(S_{f1} + S_{f2})\Delta x = h_2 + \frac{U_2^2}{2g} + Z_2 + S_o \Delta x \quad \ldots \quad (5.8)$$

For constant discharge, $q = Uh$, the above equation can be rewritten in the form

$$h_1 + \frac{q^2}{2gh_1^2} + \frac{1}{2} S_{f1} \cdot \Delta x - (S_o \Delta x + Z_2 - Z_1) = h_2 + \frac{q^2}{2gh_2^2} - \frac{1}{2} S_{f2} \cdot \Delta x \quad \ldots \quad (5.9)$$

Equation 5.9 can be solved for h_2 by trial and error making use of the resistance relationship of Ranga Raju which has been shown to be applicable under non-uniform flow conditions if friction slope is used in place of S_o. The depth of flow is substituted in place of hydraulic radius for wide rectangular channel.

The steps for the computation of water surface profiles are :

(i) Start from the downstream end of the transient bed profile, where $h_1 = h_o$, the uniform flow depth. Decide the step length and evaluate the left hand side of the Eq.5.9, computations being performed in upstream direction.

(ii) Assume a trial value of h_2 and calculate S_{f_2} from Eq. 5.9.

(iii) Compute the value of parameter $K_1 \frac{U}{\sqrt{\Delta\gamma_s h/\rho}}$ and read the value of the parameter, $K_2(h/d)^{1/3}\left(\frac{S_f}{\Delta\gamma_s/\gamma}\right)$ from Fig.2.4 and hence determine the value of friction slope S_f.

(iv) Compare the value of S_{f_2} against the value determined in step (iii). If these values are different, assume another value of h_2 and repeat the steps (ii) and (iii) till these tally.

(v) Repeat steps (ii) to (iv) for the next step length and work out the water surface profile over the whole of the aggraded reach.

5.8 PREDICTION OF TRANSIENT BED AND WATER SURFACE PROFILES FOR COLORADO RIVER-- A CASE STUDY

The bed material size of the sediment of the Colorado River at Taylor Ferry, U.S.A is 0.32 mm, the same as in the present laboratory investigation. With the effect of size of sediment eliminated, it was, therefore, decided to employ the proposed method of computation for predicting transient bed and water surface profiles for a large stream like the Colorado. In the absence of any prototype data to compare with, the only check can be whether the results are realistic. The rate of overloading $\Delta G/G_e$, was assumed to be equal to unity and the aim was to predict the bed and water surface profiles at the end of 6 months. The following are the known hydraulic and sediment parameters of the river(37)1961

Discharge, q	=	1.38 m^3/sec-m
Flow depth, h	=	1.80 m
Equilibrium sediment transport rate, G_e.	=	$115x16^{-6} m^3$/sec-m
Bed slope, S_o	=	0.000217
Median size of the bed material, d	=	0.32 mm

From the known sediment transport rates and corresponding mean velocities sediment transport ...

law in the form $G = a\ U^b$ was determined for the river reach under consideration as shown in Fig.5.26. The value of the exponent b was found to be 7.3. The values of K_o and K were found to be 2.07 and 0.414 respectively. Following the steps of computation detailed in sections 5.6.5 and 5.7 the bed and water surface profiles at the end of 6 months are computed and shown in Fig. 5.27. The maximum aggradation at section x = 0, i.e., Z_o is found to be 1.01-m decreasing to 0.01 in a distance of 9.42 kms; the bed wave will travel to 6.67km, 9.42km and 13.40 km at the end of three months, 6 months and one years respectively; these values may be considered realistic. It can be seen that the computed water surface maintains the essential characteristics of the transient water surface profiles observed in the laboratory

5.9 CONCLUDING REMARKS

The elements of a transient bed profile, i.e., maximum depth of deposition, deposition at any distance, and length of aggraded profile can be worked out with the help of relationsdeveloped for these but using a modified value of K which can be determined using Fig.5.24 and Eq.2.13. It has been found that sediment transport law under uniform flow conditions cannot be applied directly to non-uniform flow conditions obtained in aggrading streams.

Kennedy's concept of lag distande, δ, has not been fully supported by the present data. However, the resistance relationship developed by Ranga Raju for uniform flow was seen to be valid under non-uniform flow conditions if the local friction slope, S_f, is used in place of S_o.

CHAPTER-VI

CONCLUSIONS

The main objective of the present study was to formulate an analytical computational procedure for predicting transient bed profiles in case of aggradation due to continuous overloading at a constant rate. It was also intended that this procedure should be supported by data from laboratory experiments. Theoretical relations for the transient bed profile based on the parabolic model proposed by de Vries have been modified on the basis of these data. The aggradation experiments conducted during this study provide data for non-uniform flow conditions as a by-product. The non-uniform flow data have been utilised in studying the characteristics of flow under aggradation and studying for the first time in detail the sediment transport and resistance laws under such conditions.

The main conclusions drawn from the analytical and experimental investigation are summarized below:

(1) The maximum aggradation in a stream, Z_o, occurs at the section of sediment injection and for a given time, t, and rate of injection,

ΔG, Z_o is given by Eq.3.15 namely

$$\frac{Z_o}{\sqrt{Kt}} = 0.885 \frac{\Delta G}{K(1-\lambda)}$$

(2) Downstream of the section of sediment addition the extent of aggradation decreases monotonically in the down stream direction and approaches a value of zero asymptotically. The value of aggradation at any section and at any time t can be computed from Eq. 3.6, namely

$$Z = Z_o \left(1 - \operatorname{erf} \frac{x}{2\sqrt{Kt}} \right)$$

(3) The region of asymptotic change in Z is assumed to end at a section beyond which the change in aggradation depth does not exceed some small prescribed value taken as 1 per cent of the maximum depth Z_o. This defines the length, l, of transient bed profile for time t, and is given by the following equation:

$$l = 3.66 \sqrt{Kt}$$

(4) The aggradation coefficient K appearing in Eqs. 3.6, 3.9 and 3.15 expressed as K/K_o is related to the ratio of the excess load to the equilibrium load, $\Delta G/G_e$. However, more data

are needed in support of this relationship shown in Fig.5.24.

(5) In view of uncertainities regarding Fig 5.24, the value of K may be obtained from Eq.3.15 if one knows Z_o at a given time, t, and for given value of ΔG from prototype observations. Since the value of K remains independent of time, this value of K can be used in Eq. 3.6 to predict the bed profiles at other times.

(6) The sediment transport rate at a section on the aggrading reach initially increases very fast from the equilibrium value but it increases gradually later and approaches the increased sediment transport rate asymptotically.

(7) The value of Mannings roughness coefficient on the aggraded reach is generally smaller than the value of uniform flow; the decrease may be to the extent of 20-30 per cent. However, there is no systematic variation of n with sediment injection rate and such other parameters.

(8) The sediment transport formulae for steady uniform flows cannot be directly applied to the unsteady non-uniform flow conditions obtained in aggrading streams.

(9) The sediment transport rate at a particular section is not related to the mean velocity of flow at this section by the same relationship as for uniform flow. However, the local sediment transport rate at a section is related by this relation to the local mean velocity upstream or downstream of this section. Kennedy's hypothesis is that there would be a lag 'δ' between sediment transport and mean velocity (implying relation of local transport rate to the velocity upstream of the section) and the present data do not thus support the hypothesis fully.

(10) The resistance law under non-uniform flow conditions is seen to be the same as for uniform flow provided the local friction slope is used instead of S_o in the former case.

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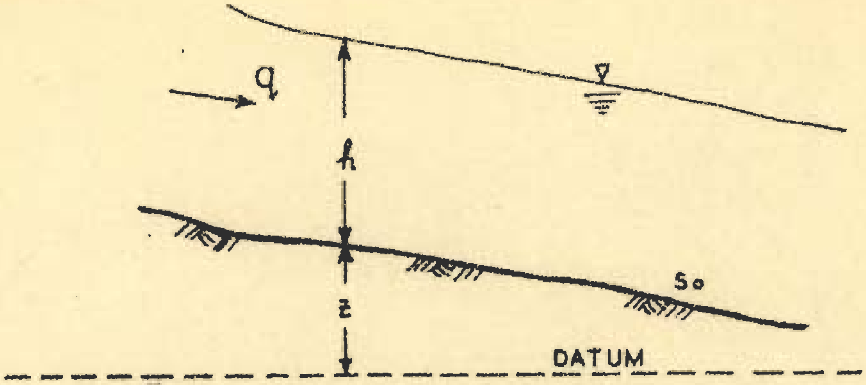


FIG. 2·1 – DEFINITION SKETCH

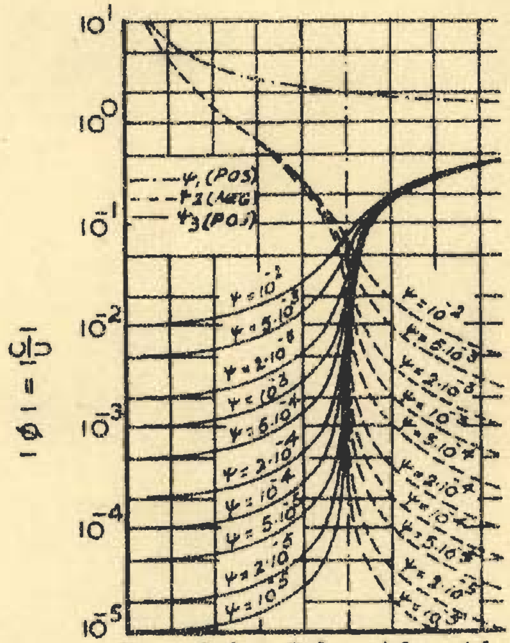
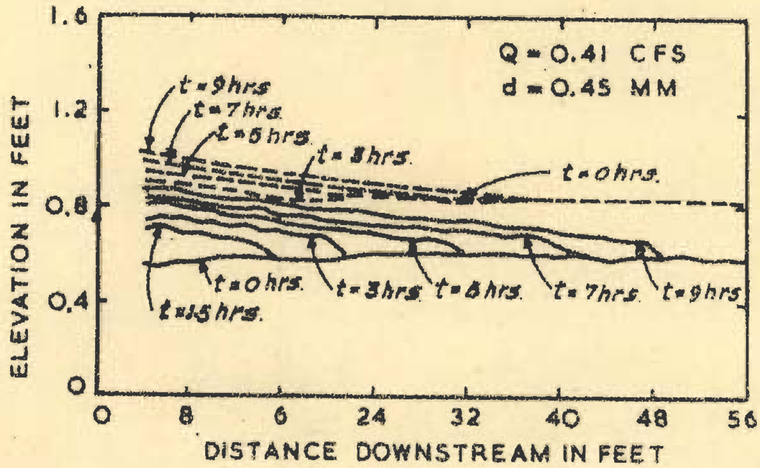


FIG. 2·2 - DIAGRAM SHOWING RELATIVE CELERITIES IN UNSTEADY ALLUVIAL CHANNEL FLOW (REF. 39)



FLUME SLOPE = 0.000484

------ W.S. PROFILE ——— BED PROFILE

FIG. 2·3 - TYPICAL BED AND WATER SURFACE PROFILES DURING AGGRADATION (REF. 3)

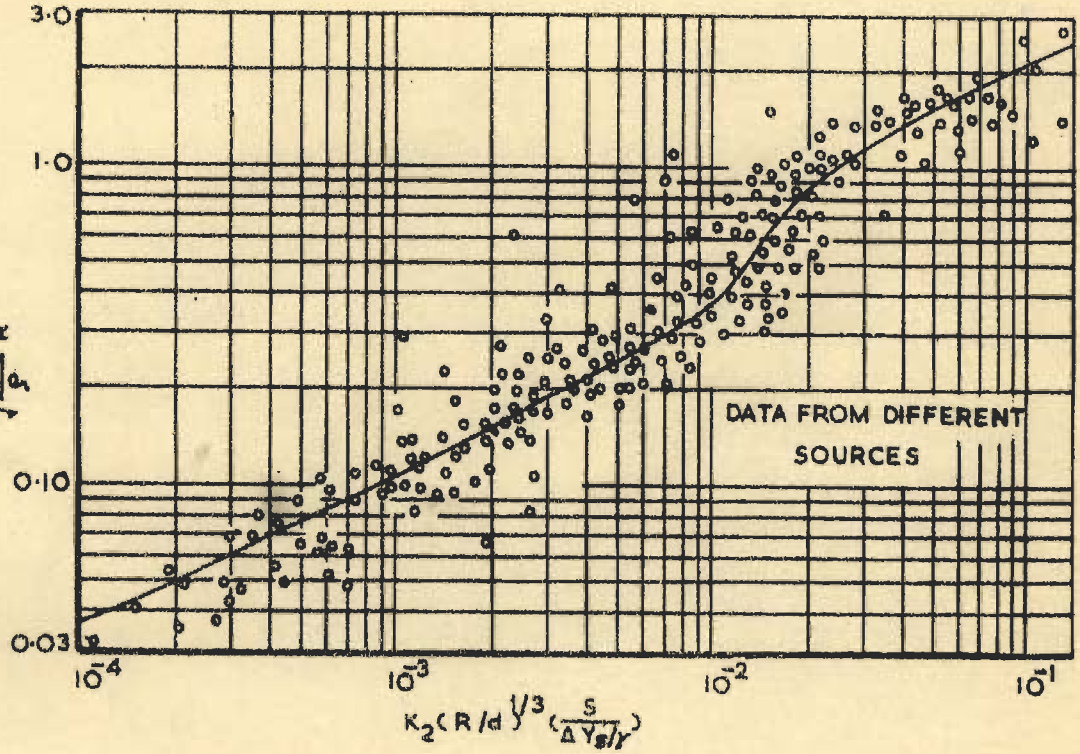


FIG 2·4 - RANGA RAJU RESISTANCE RELATIONSHIP FOR STEADY UNIFORM FLOW IN ALLUVIAL CHANNELS (REF. 25)

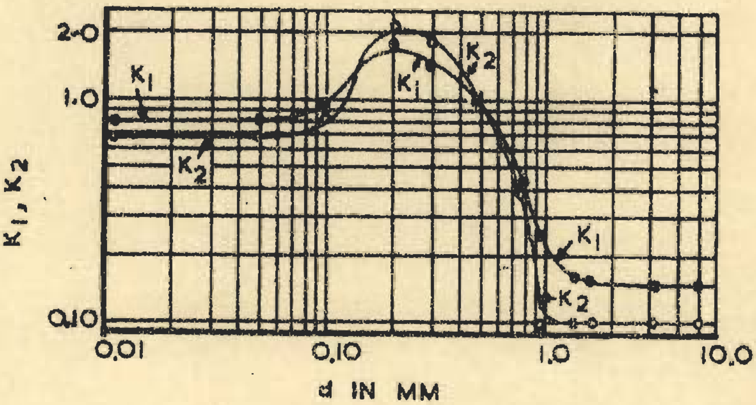


FIG. 2·5 - VARIATION OF K_1 AND K_2 WITH SEDIMENT SIZE (REF. 25)

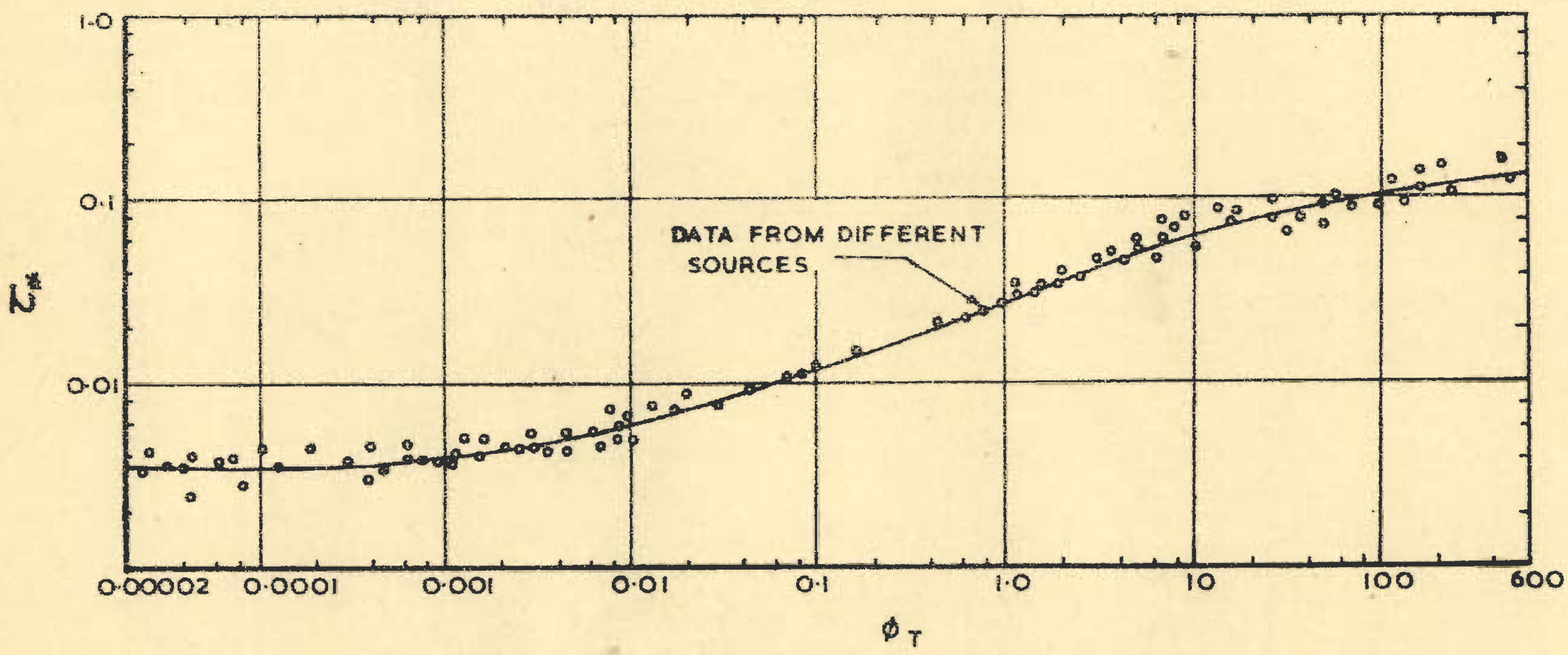


FIG. 2·6–VITTAL, ET AL. TOTAL LOAD RELATION FOR PLANE BED DATA (REF.38)

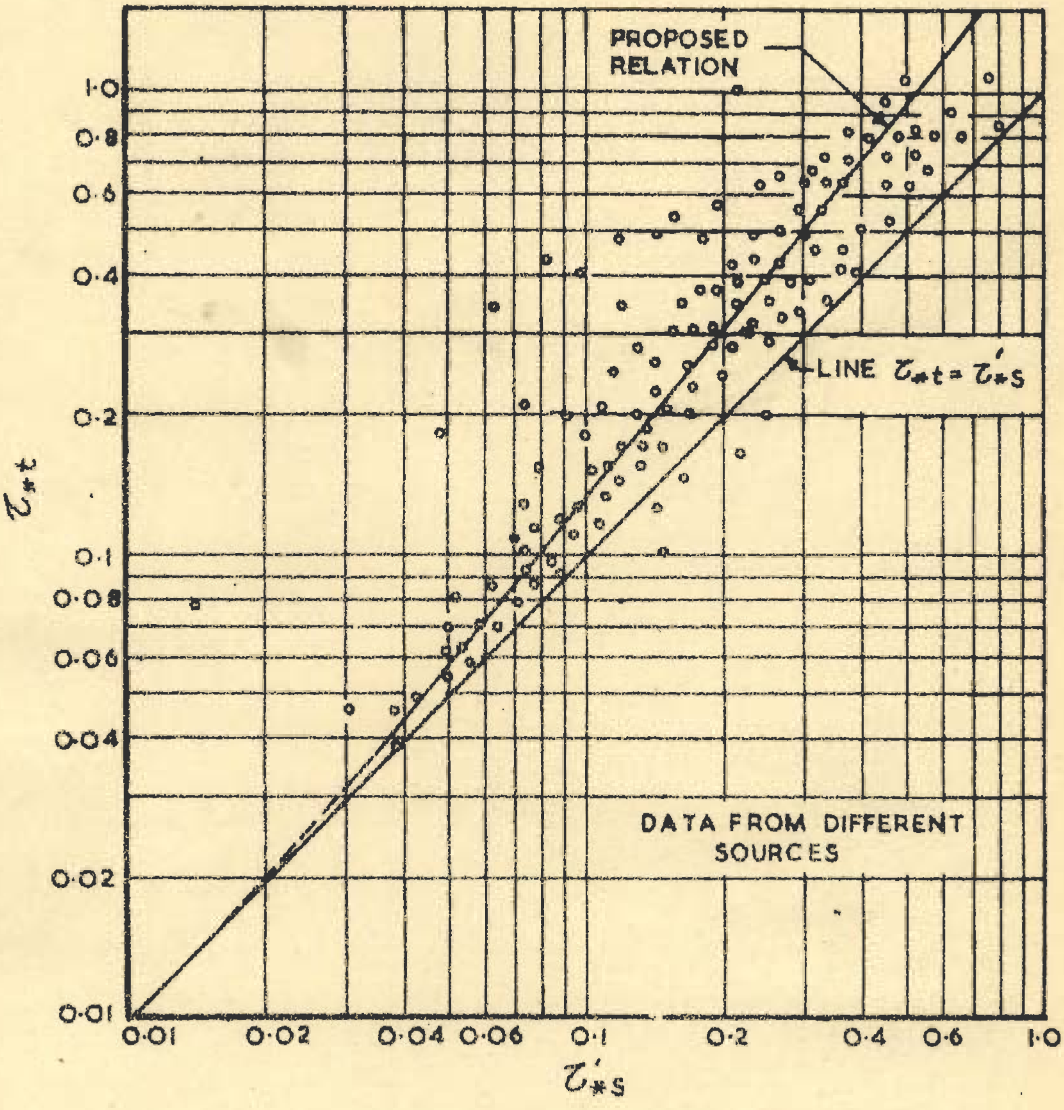


FIG. 2·7 – VARIATION OF τ_{*t} WITH τ'_{*s} FOR RIPPLE AND DUNE BED DATA (REF. 38)

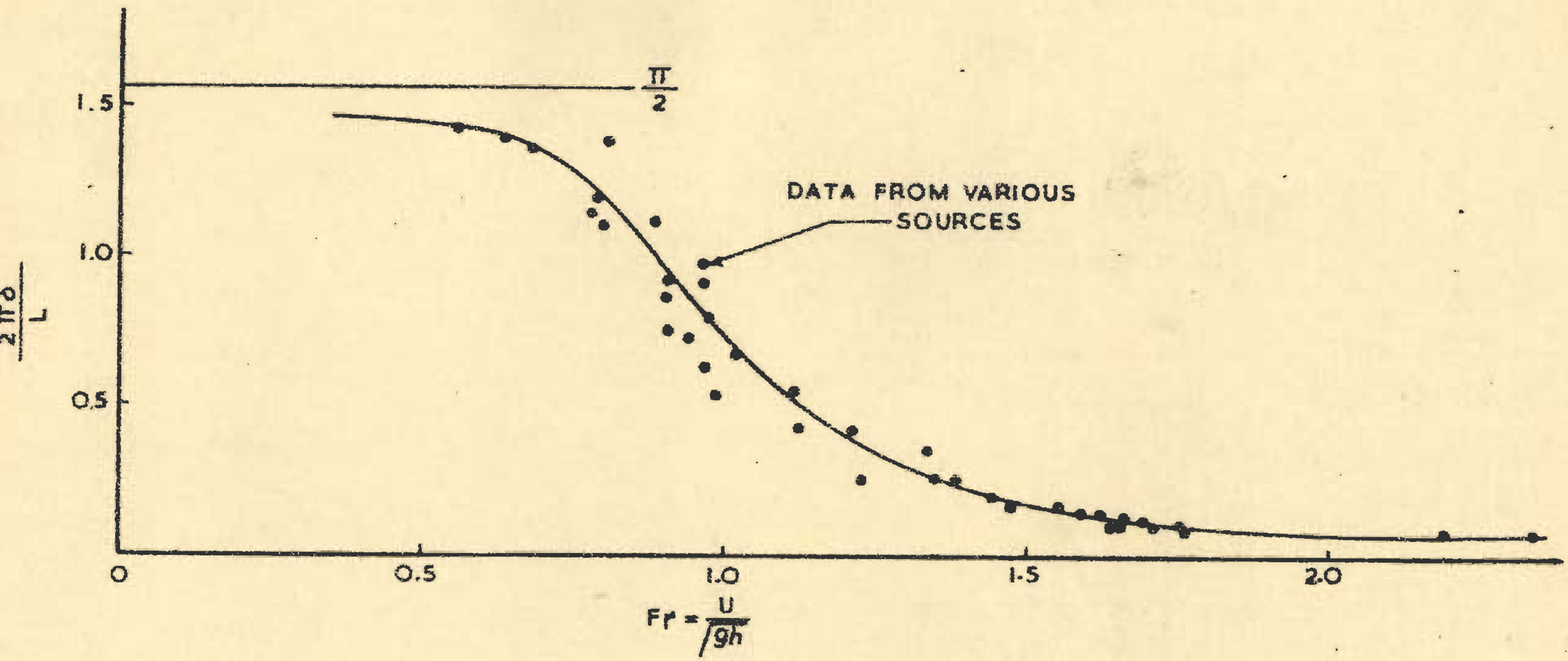


FIG. 2·8–ENGELUND'S RELATIONSHIP FOR LAG DISTANCE δ' (REF. 9)

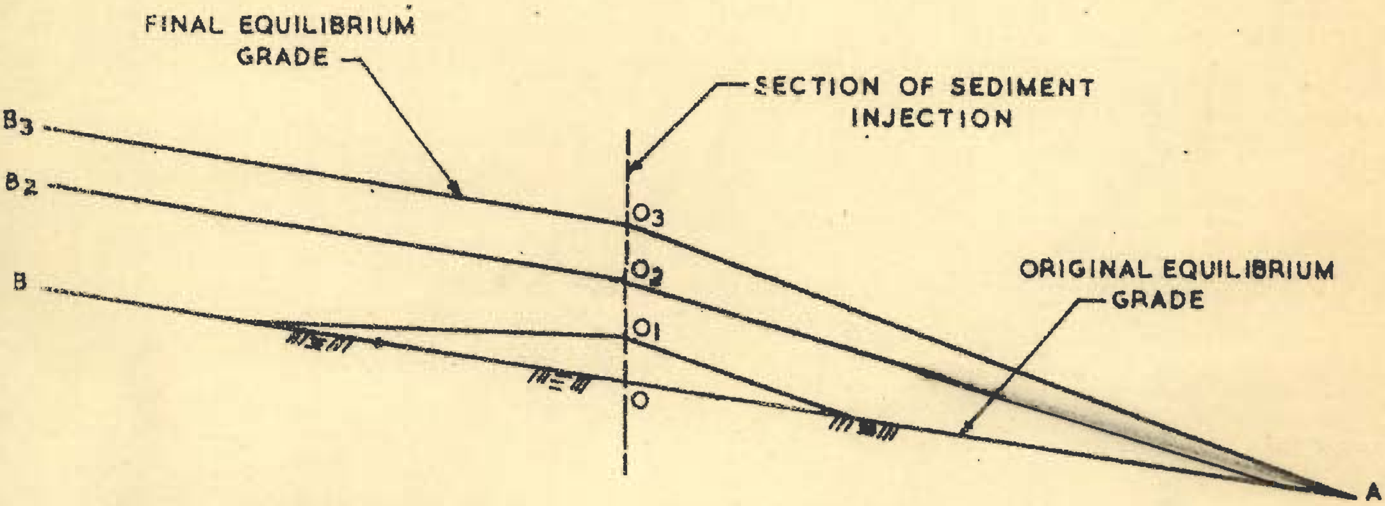


FIG. 3·1 _ SCHEMATIC DIAGRAM OF AGGRADATION DUE TO OVERLOADING

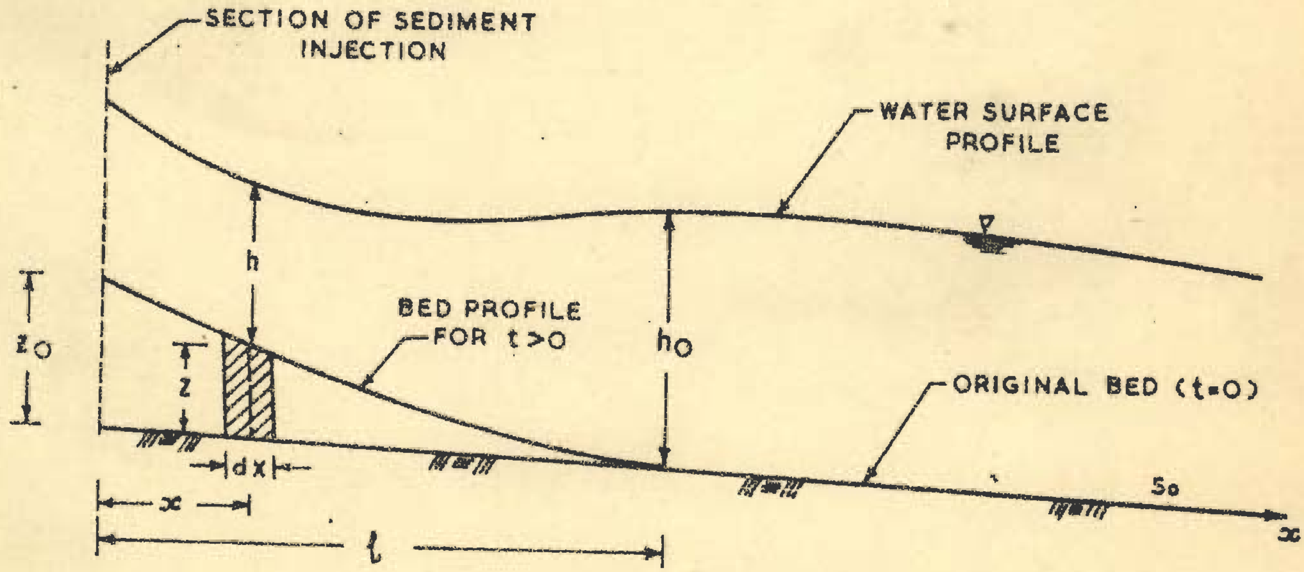


FIG. 3·2_DEFINITION SKETCH

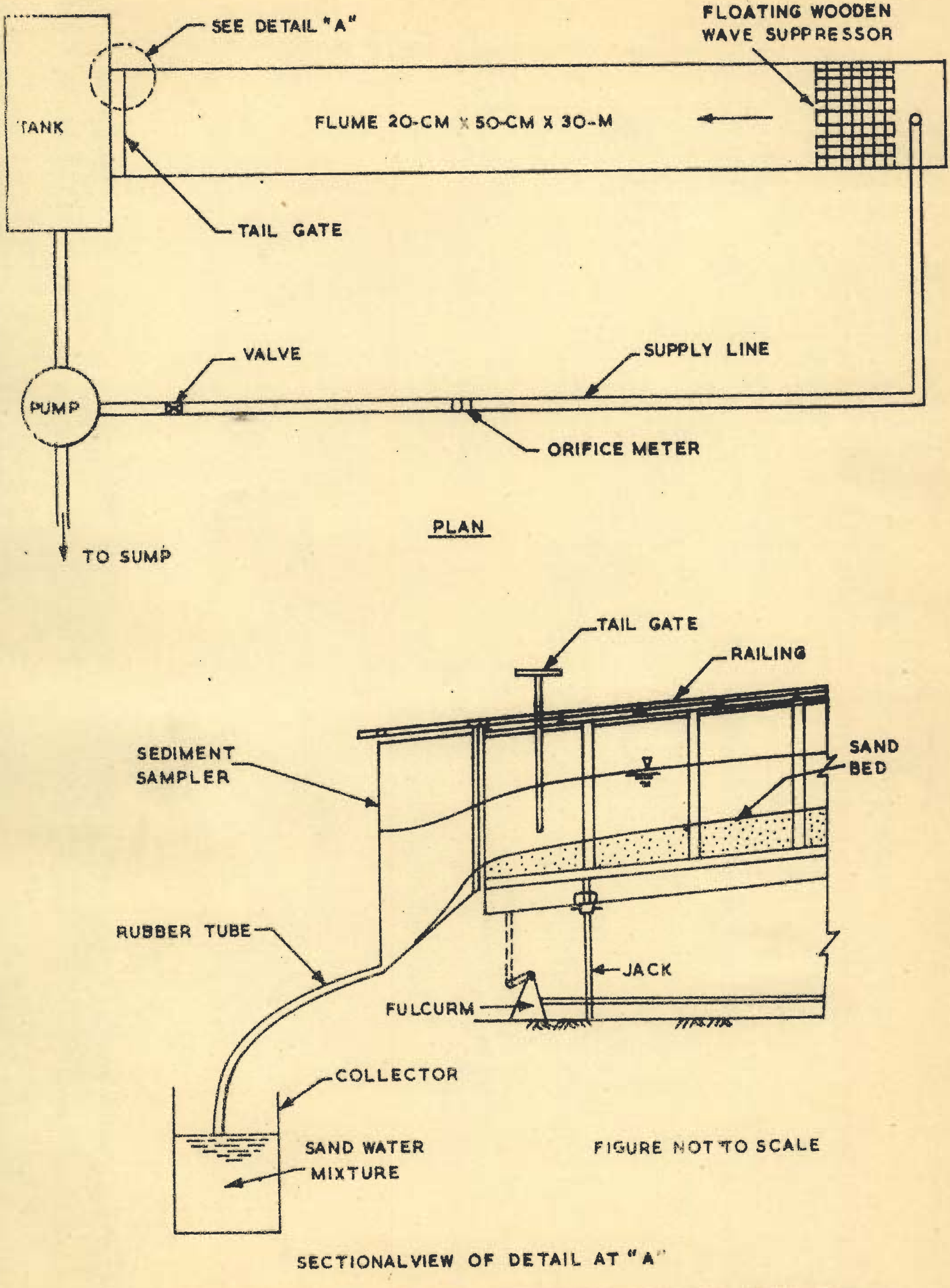


FIG. 4·1 - SCHEMATIC DIAGRAM OF THE EXPERIMENTAL SET UP

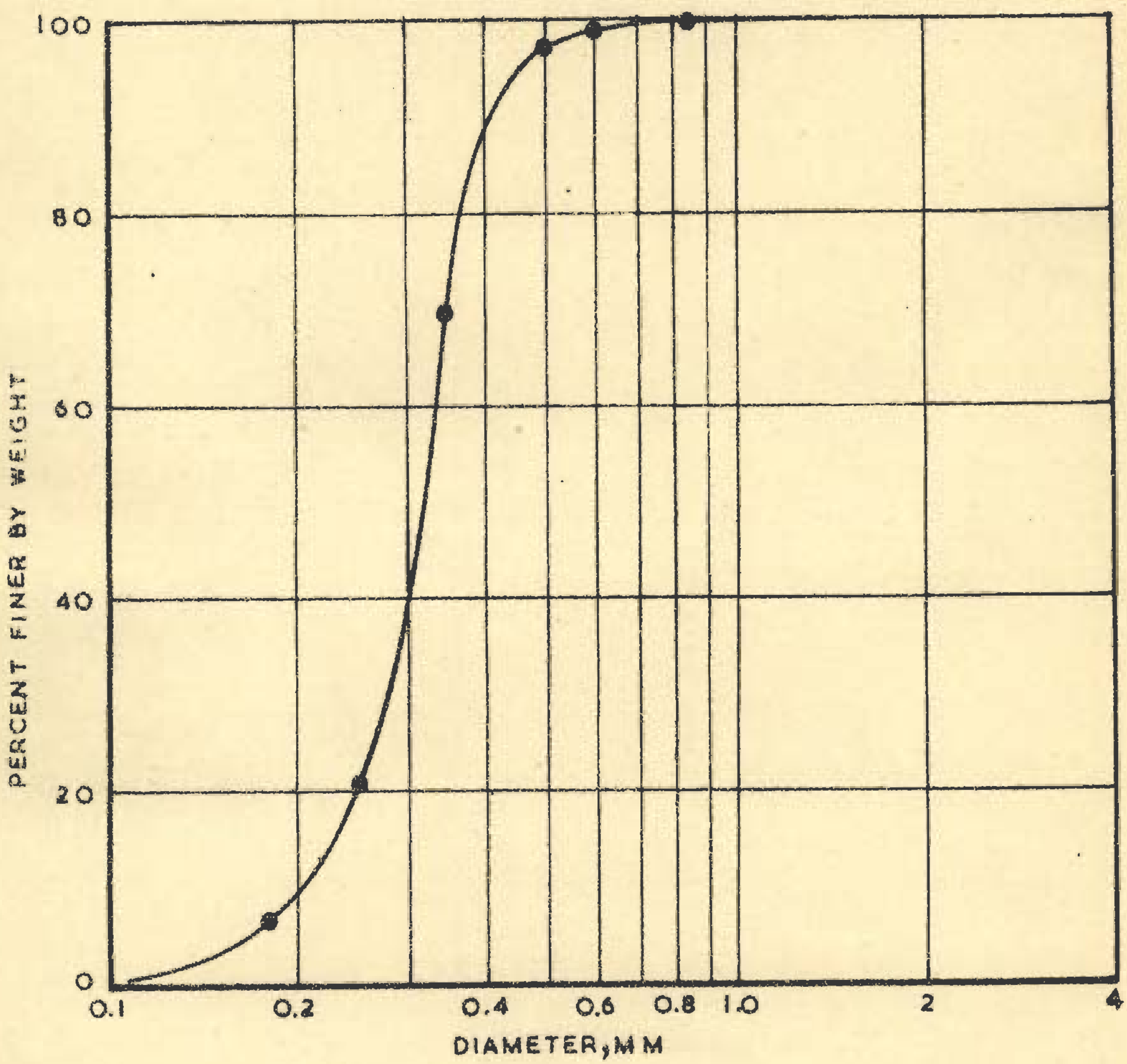


FIG 4·2_GRAIN SIZE DISTRIBUTION OF SAND USED FOR BED MATERIAL AS WELL AS FOR INJECTION.

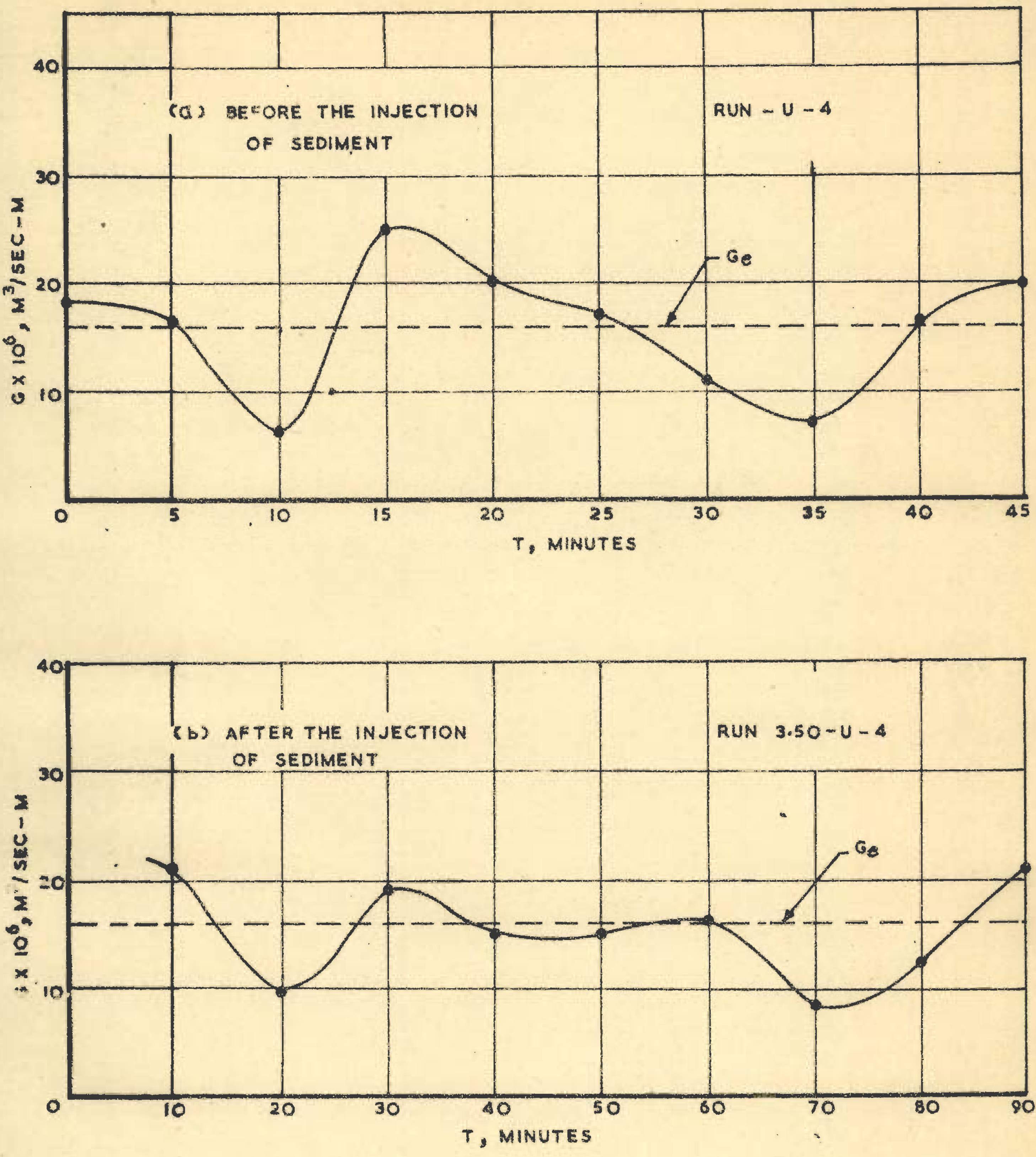
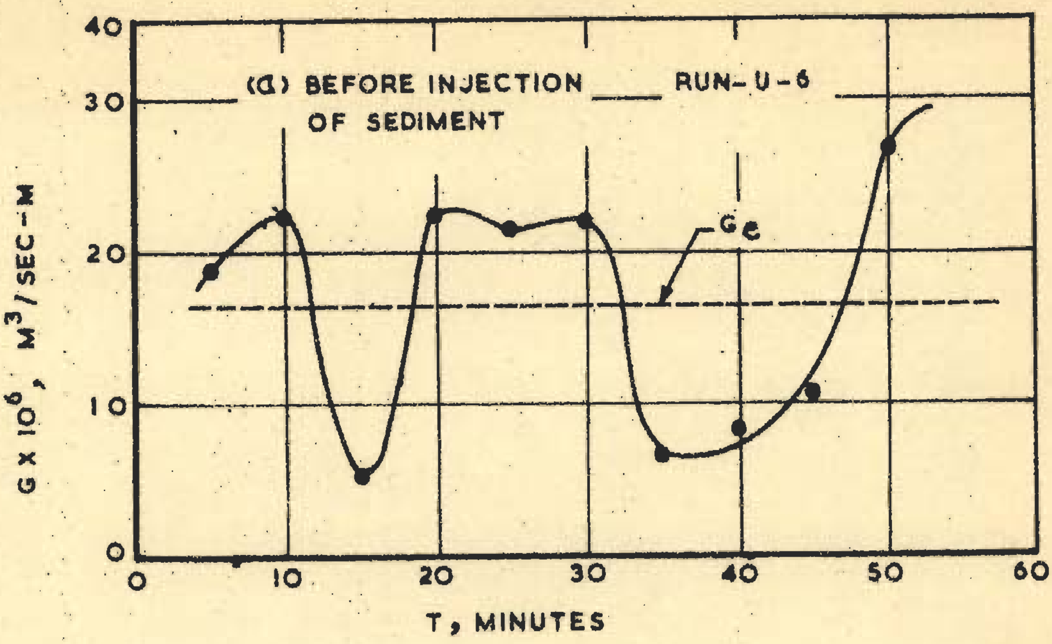


FIG. 4.3 - VARIATION OF G WITH T AT THE END OF FLUME



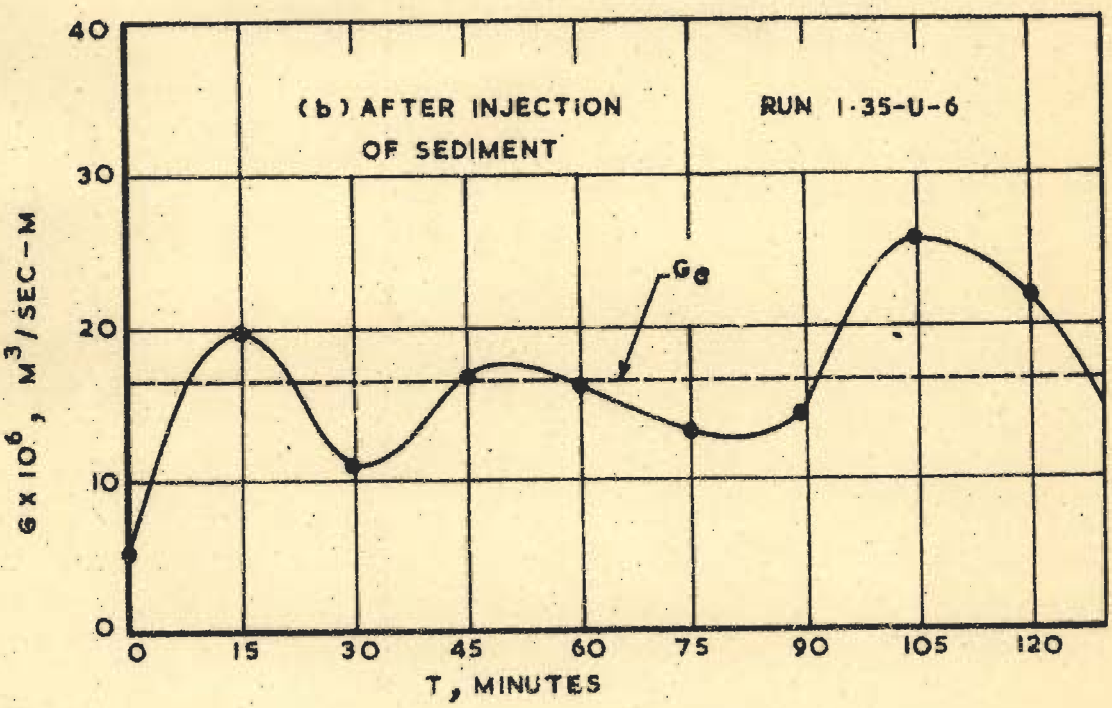


FIG. 4·3–VARIATION OF G WITH T AT THE END OF FLUME (CONTINUED)

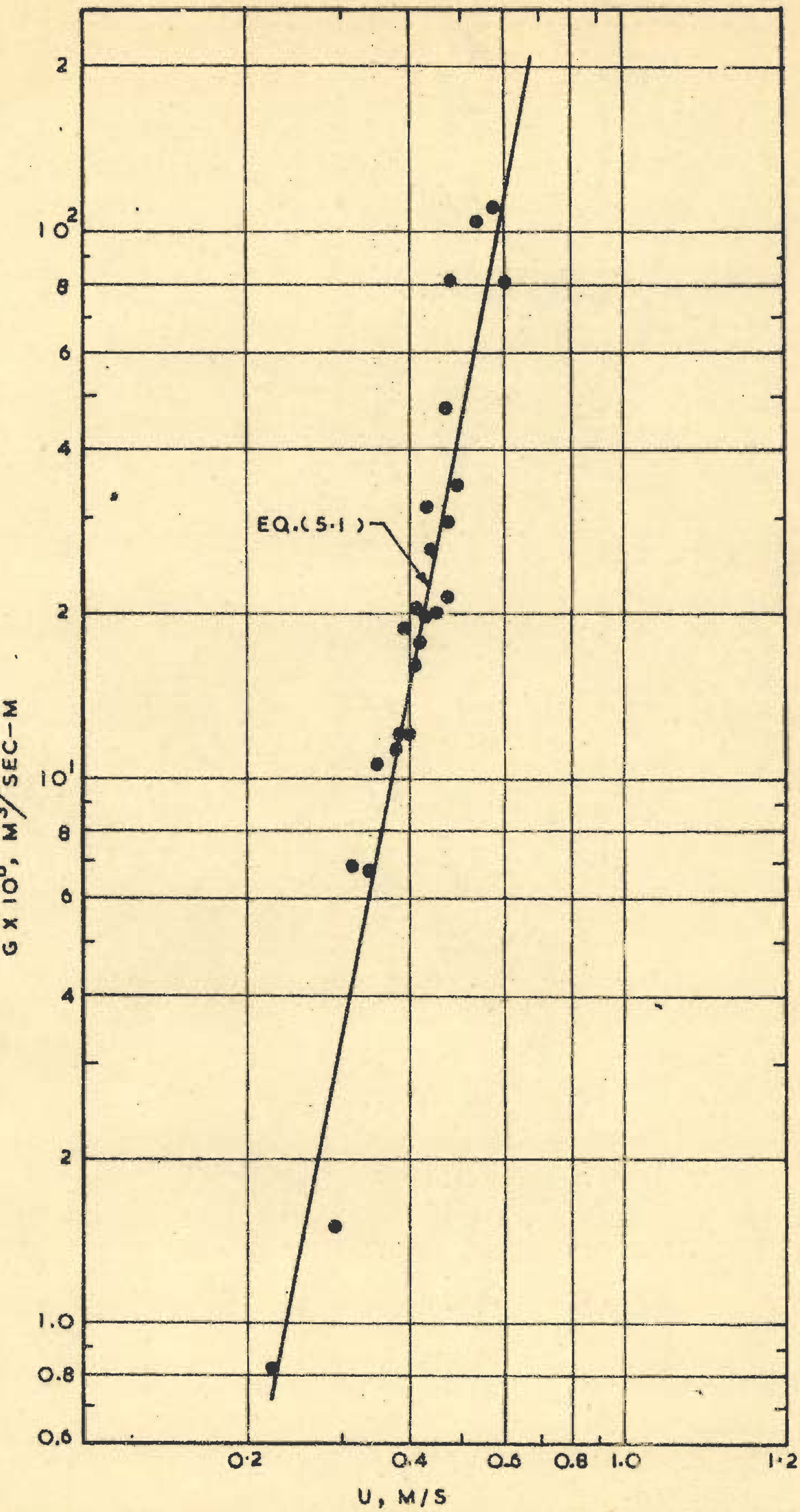


FIG. 5·1_SEDIMENT TRANSPORT LAW FOR UNIFORM FLOW DATA

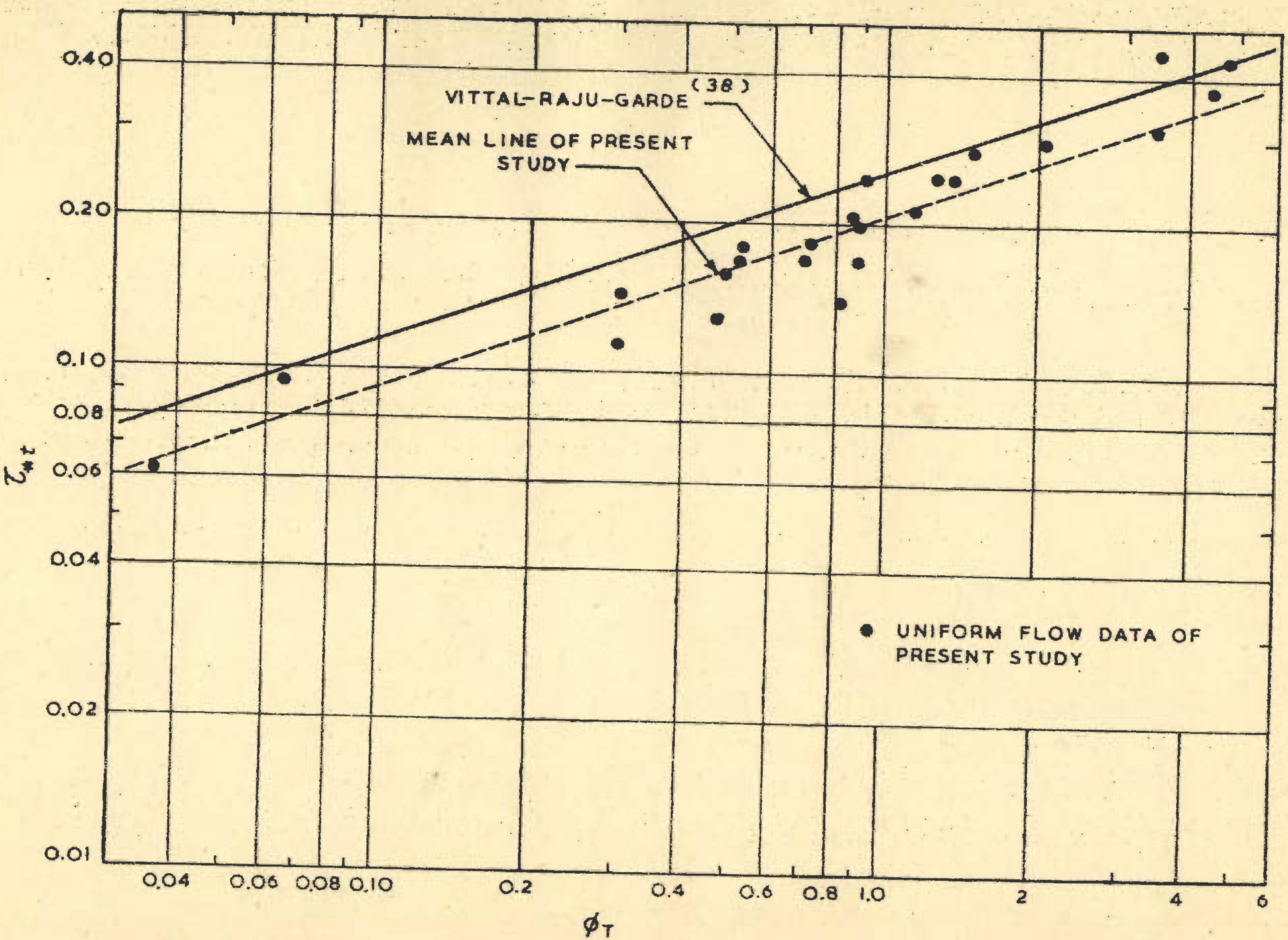


FIG. 5·2 – VARIATION OF ϕ_T WITH τ_{*t} FOR RIPPLE AND DUNE BEDS.

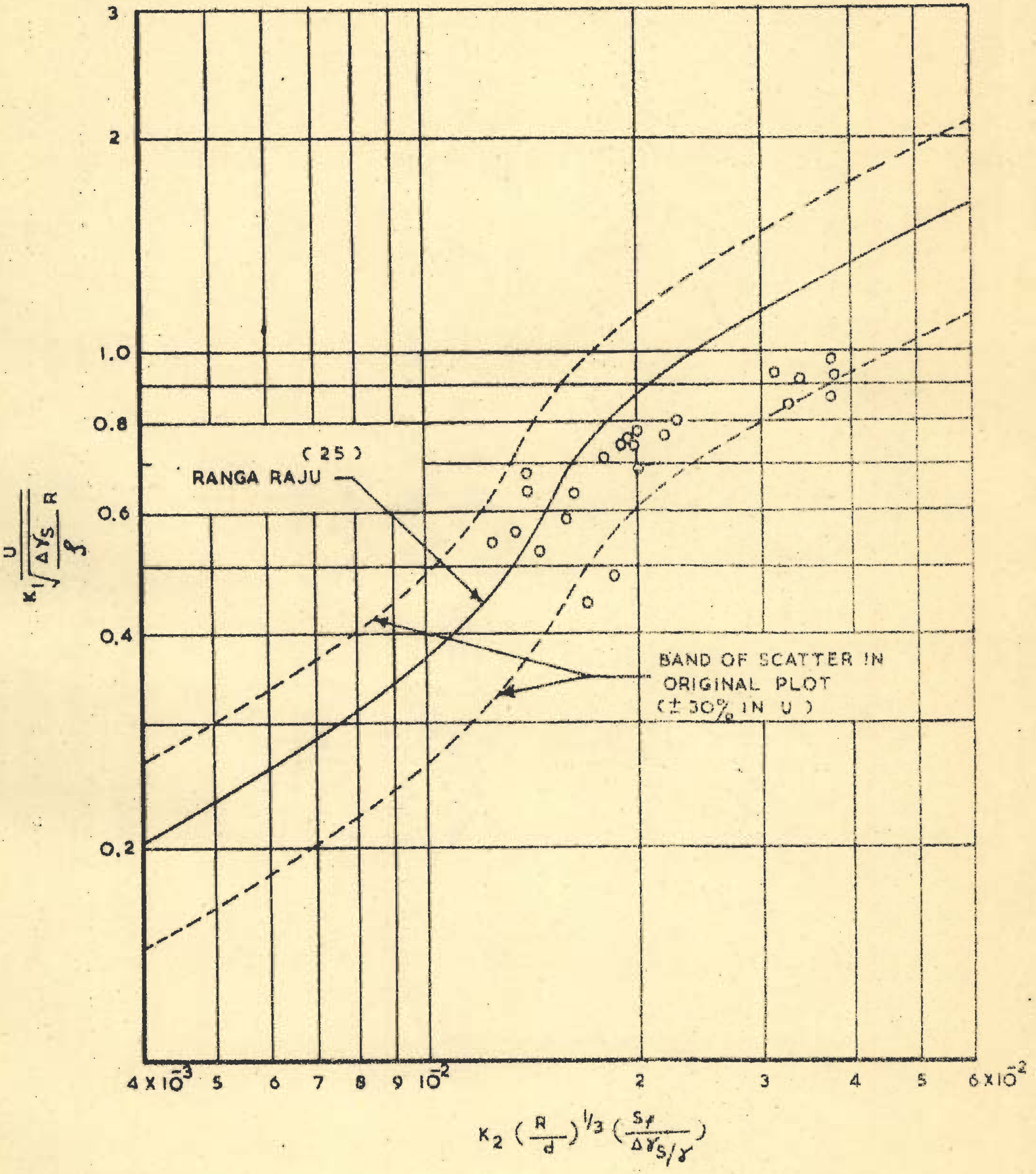


FIG. 5·3 – AUTHOR'S UNIFORM FLOW DATA PLOTTED ON RANGA RAJU RESISTANCE RELATION

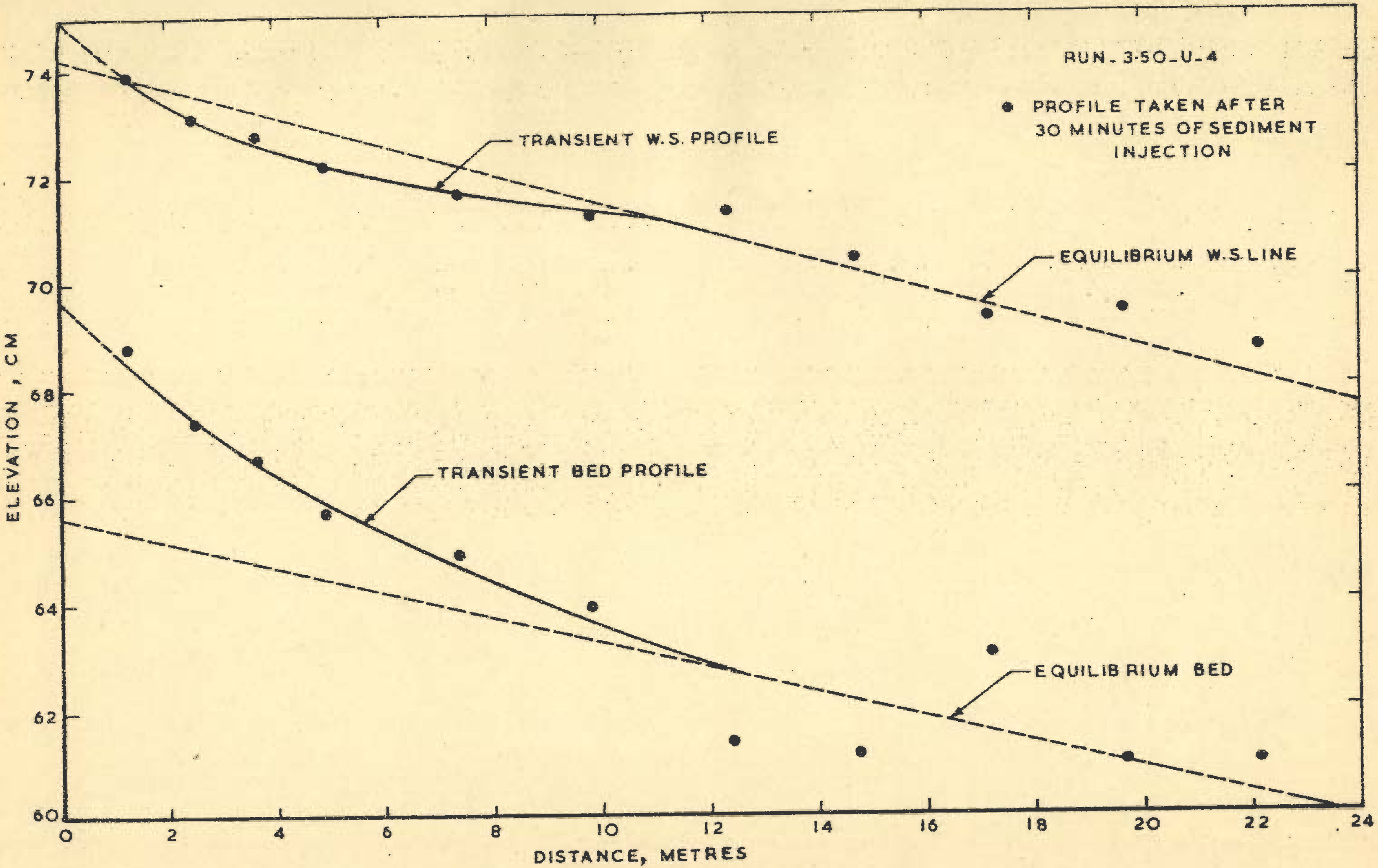


FIG. 5·4_TYPICAL AVERAGING OF TRANSIENT BED AND WATER SURFACE PROFILES

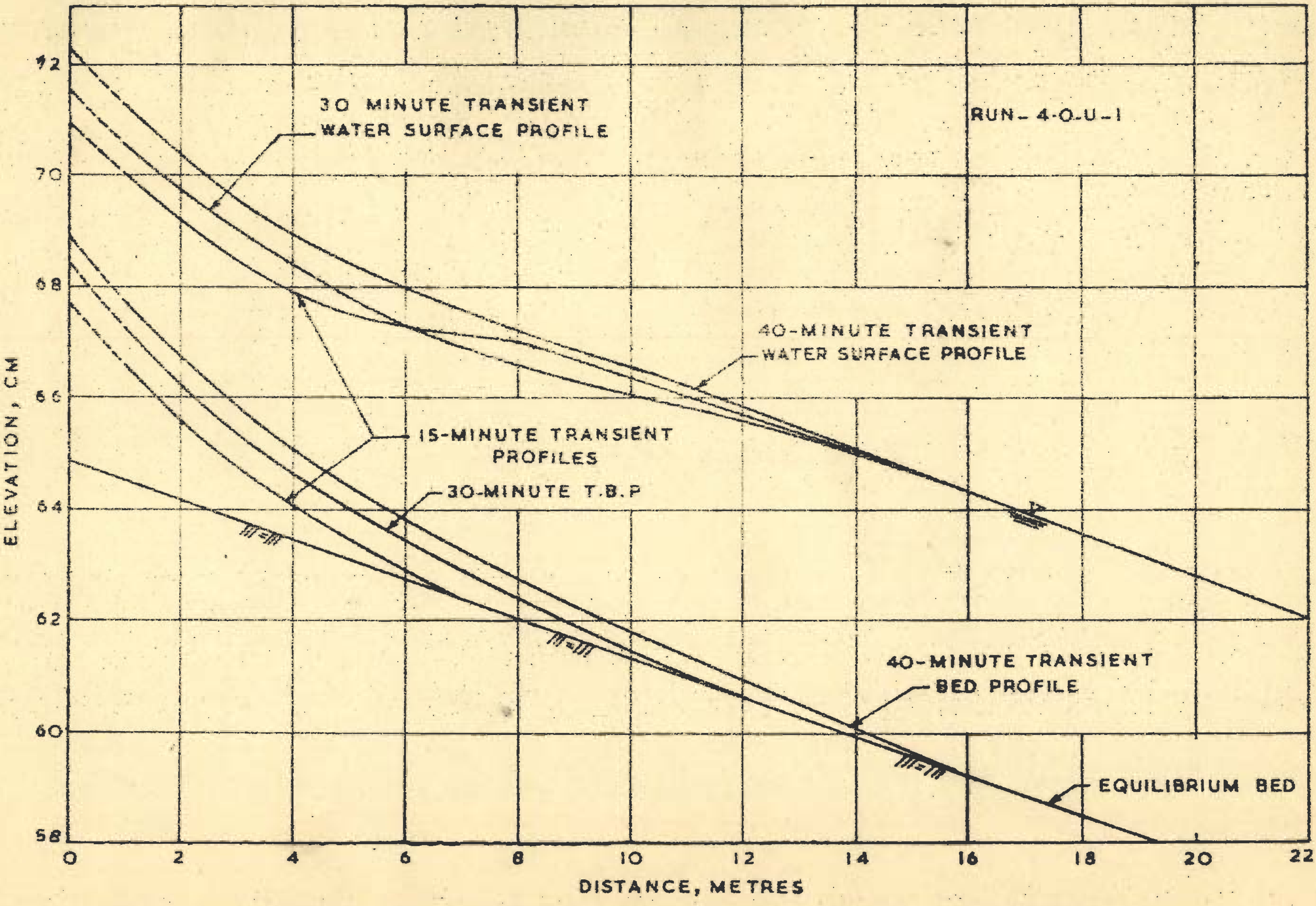


FIG. 5·5_TRANSIENT PROFILES AT VARIOUS TIMES AFTER SEDIMENT INJECTION

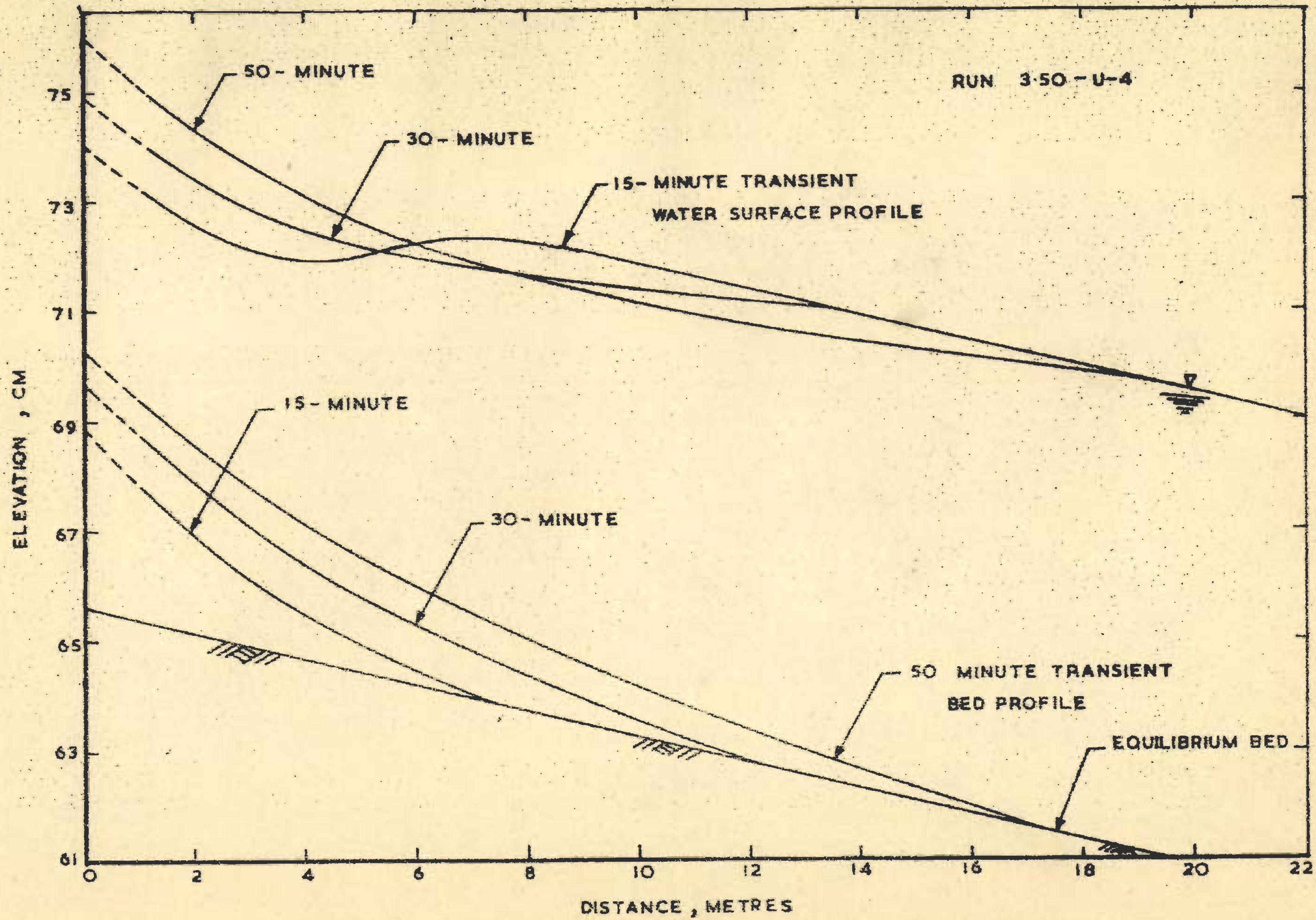


FIG. 5·5 – TRANSIENT PROFILES AT VARIOUS TIMES AFTER SEDIMENT INJECTION (CONTINUED)

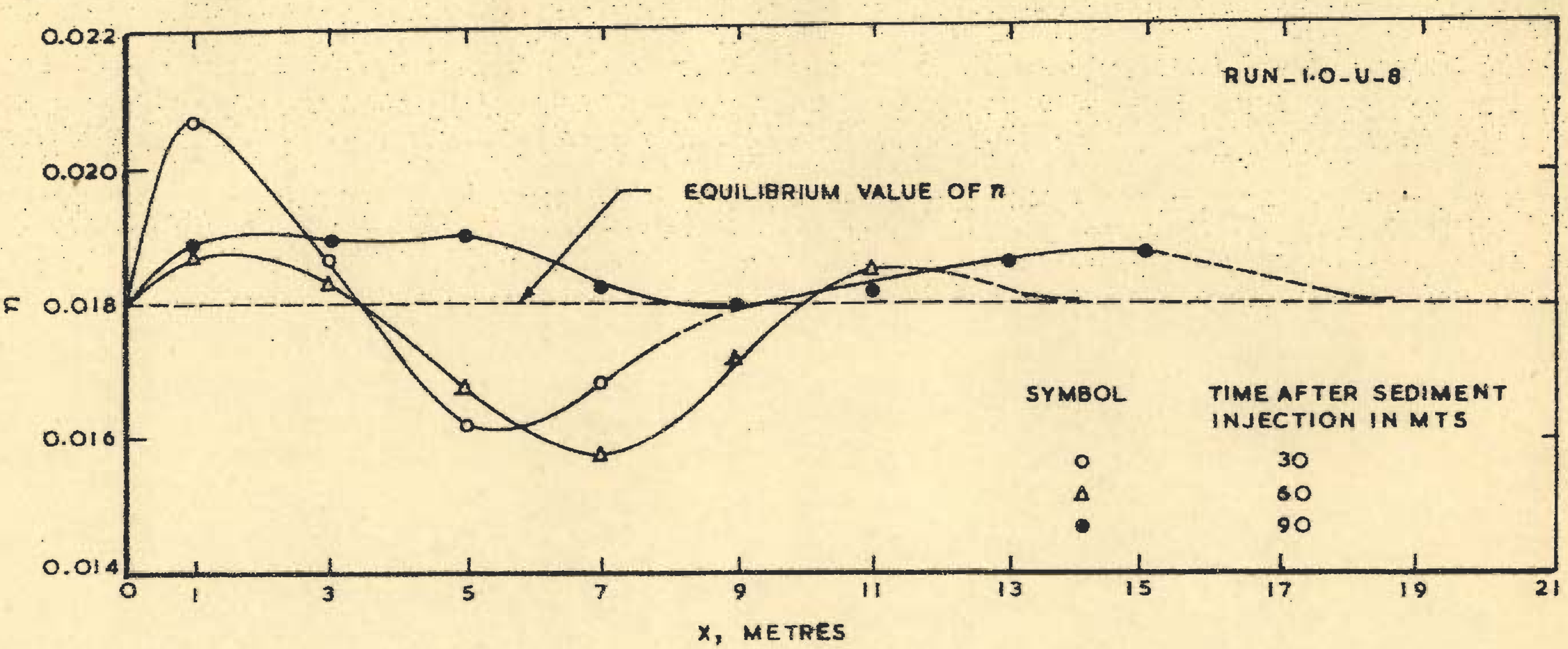


FIG. 5·6 _ VARIATION OF MANNING'S ROUGHNESS COEFFICIENT WITH DISTANCE AND TIME

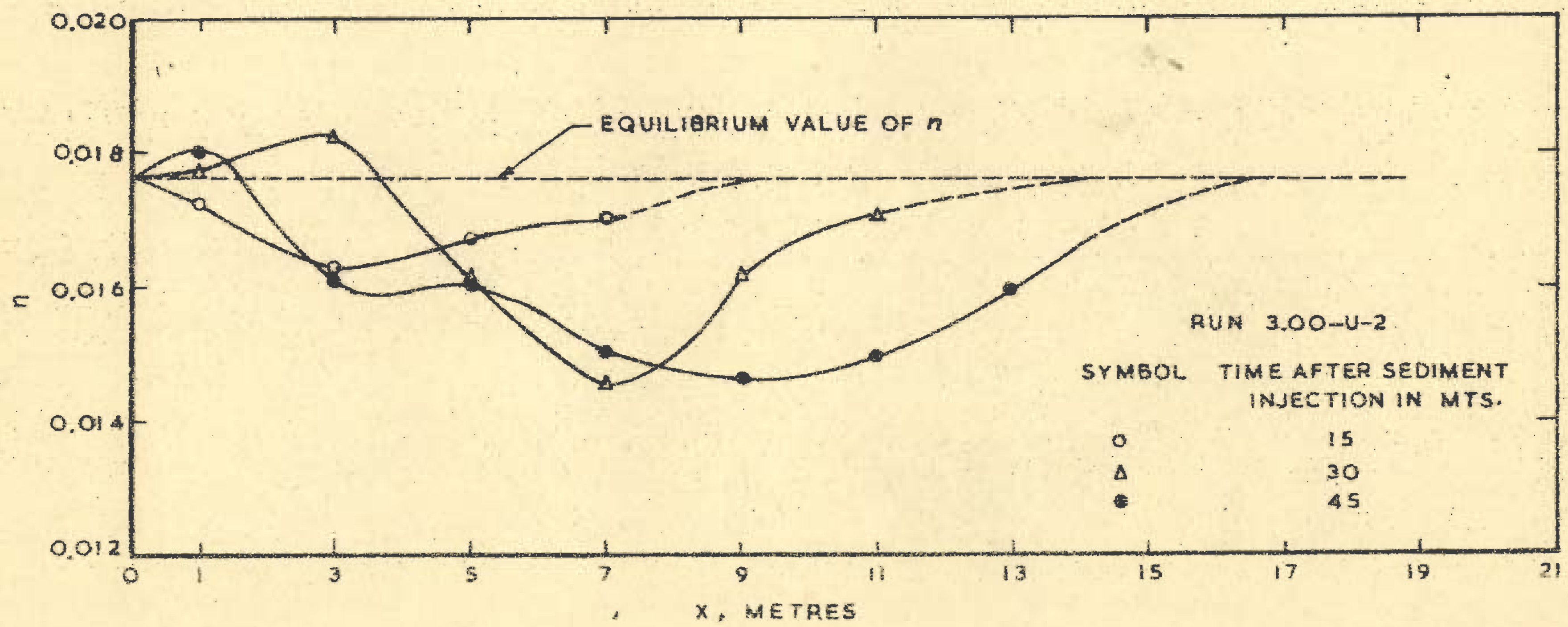


FIG. 5·6 _ VARIATION OF MANNING'S ROUGHNESS COEFFICIENT WITH DISTANCE AND TIME (CONTINUED)

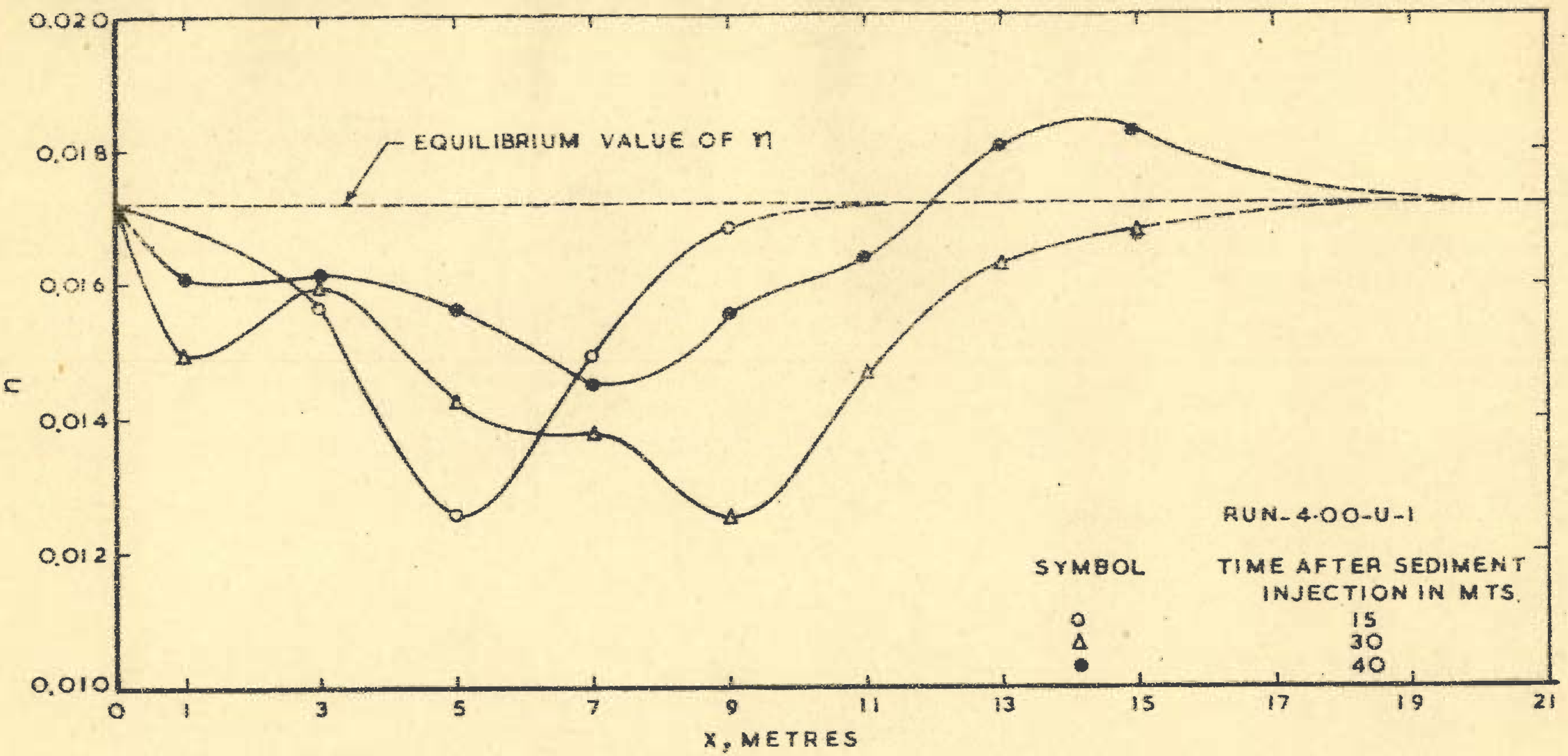
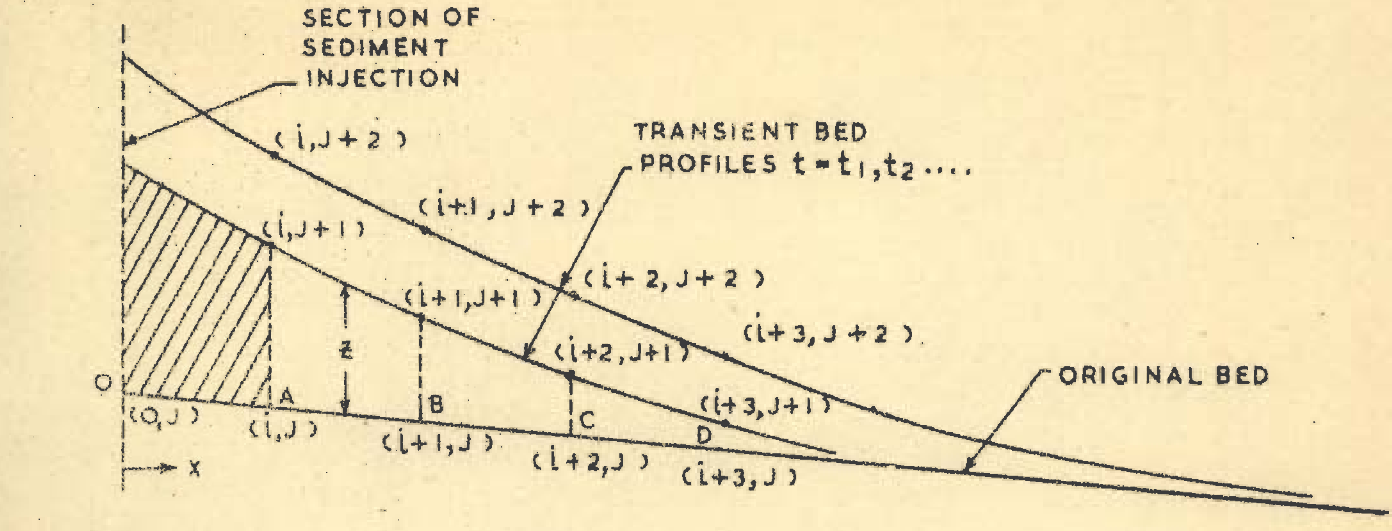
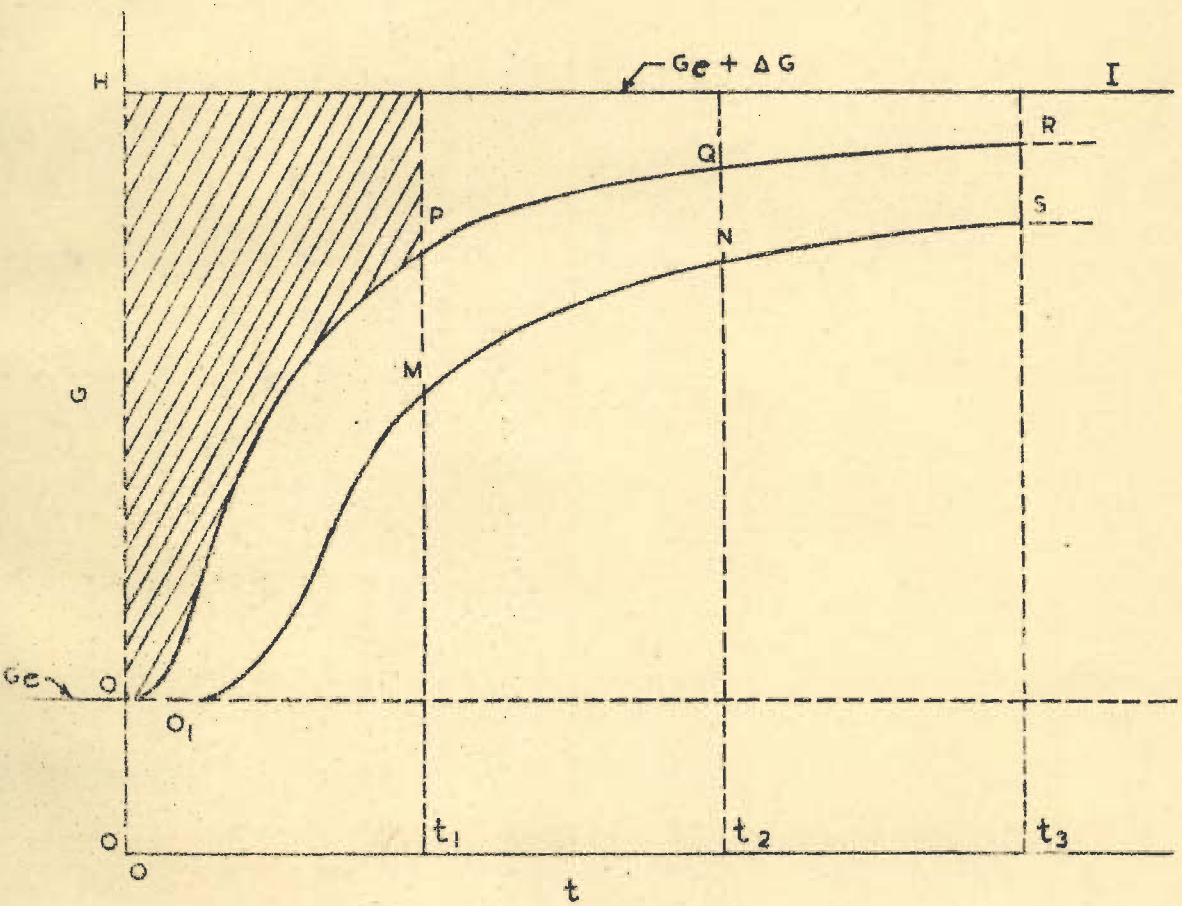


FIG. 5·6 _ VARIATION OF MANNING'S ROUGHNESS COEFFICIENT WITH DISTANCE AND TIME (CONTINUED)



(a)



(b)

FIG. 5·7 – DEFINITION SKETCH FOR COMPUTING SEDIMENT TRANSPORT RATE ALONG TRANSIENT BED PROFILES

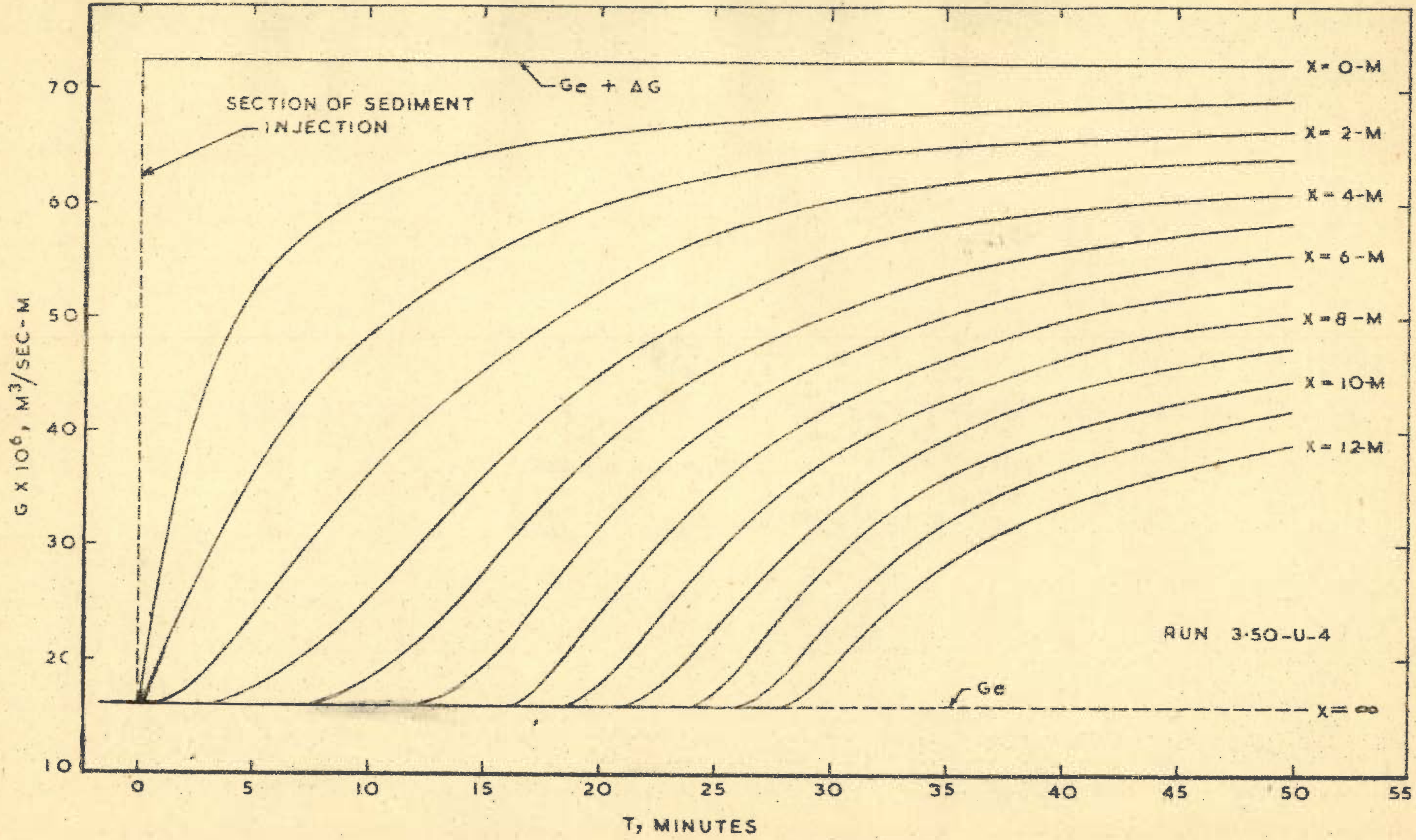


FIG. 5·8_VARIATION OF SEDIMENT TRANSPORT RATE WITH TIME AND DISTANCE

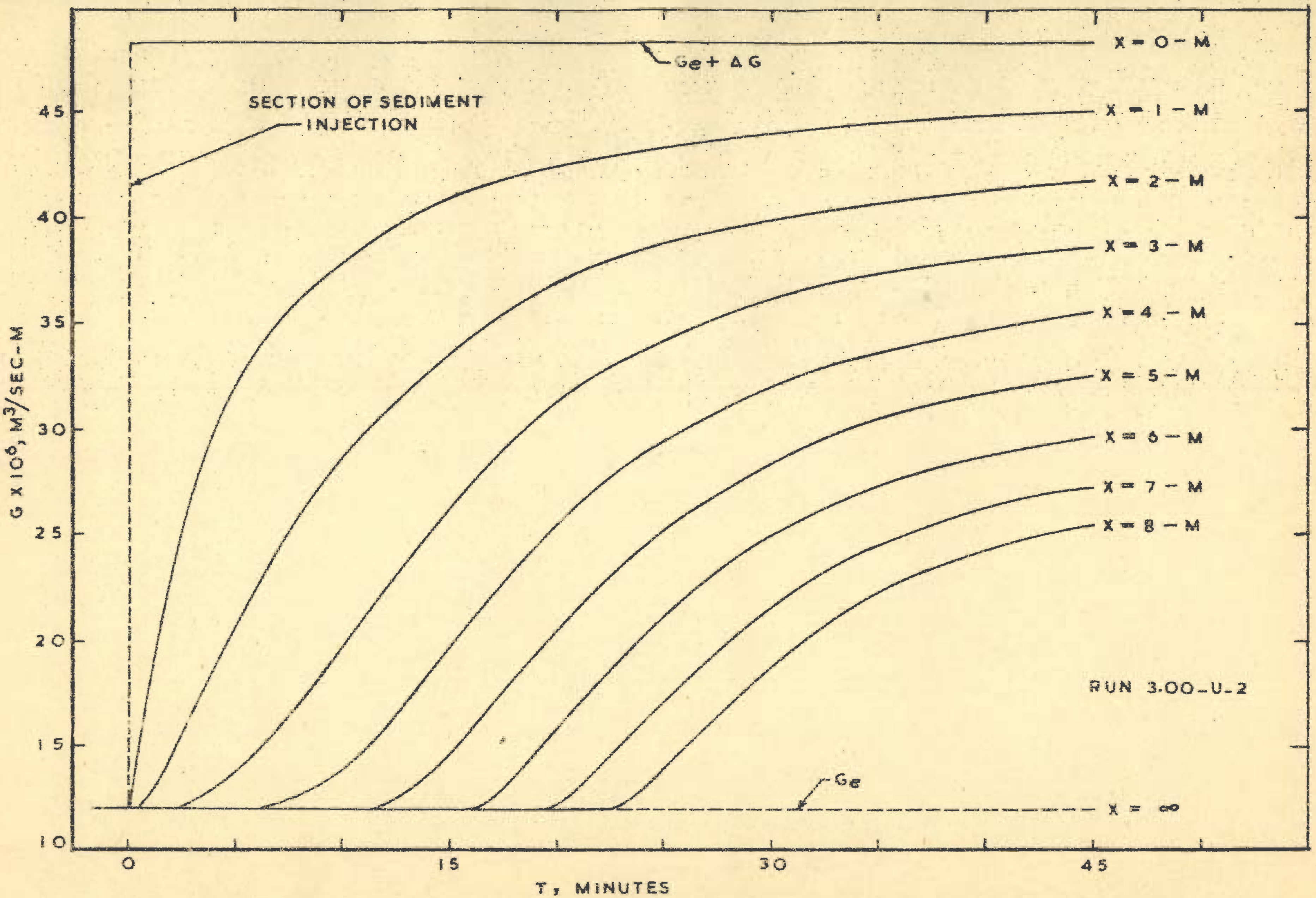


FIG. 5·8 – VARIATION OF SEDIMENT TRANSPORT RATE WITH TIME AND DISTANCE (CONTINUED)

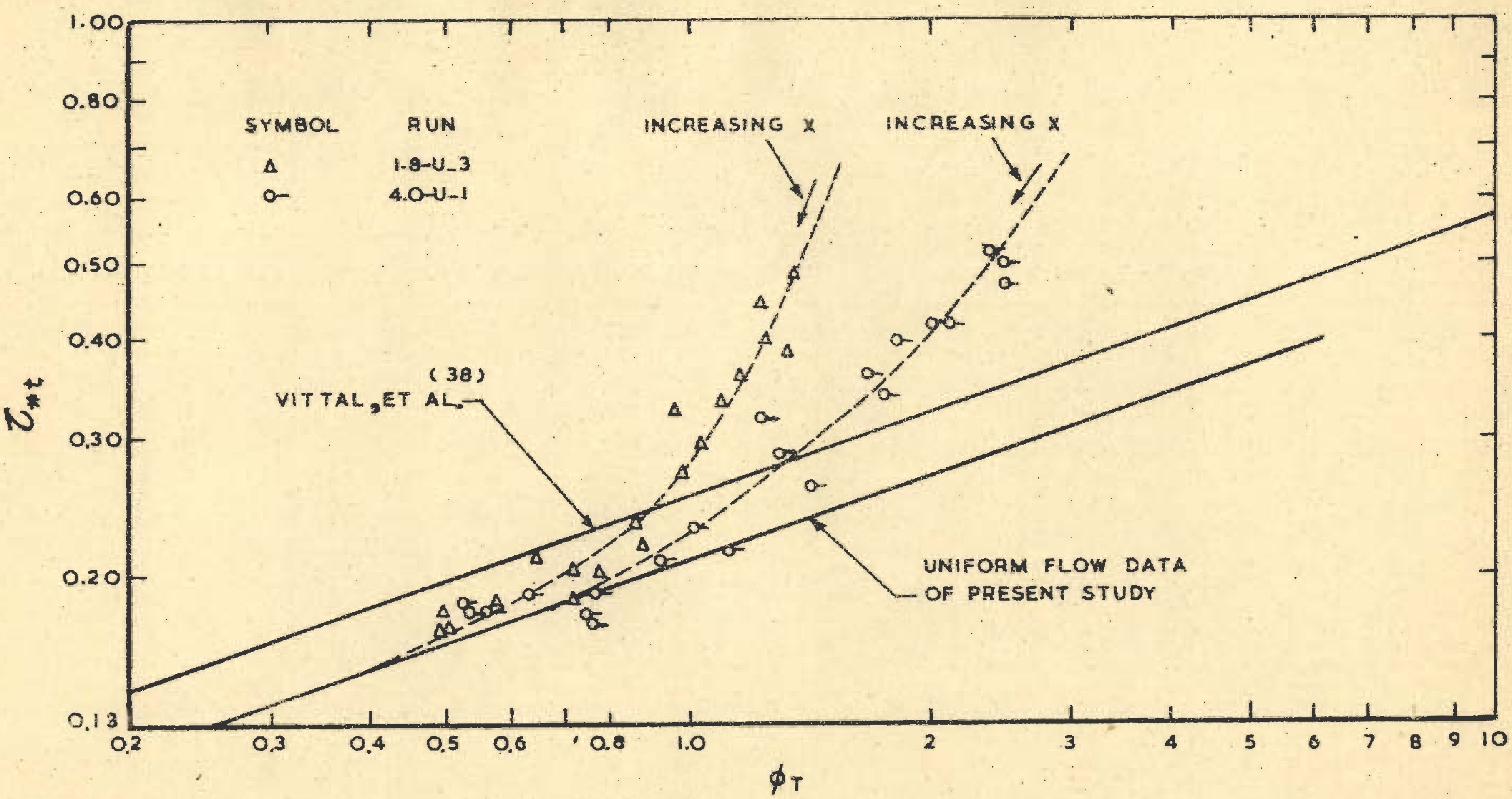


FIG. 5-9 _ EFFECT OF NON-UNIFORMITY OF FLOW ON SEDIMENT TRANSPORT RELATION BASED ON SHEAR

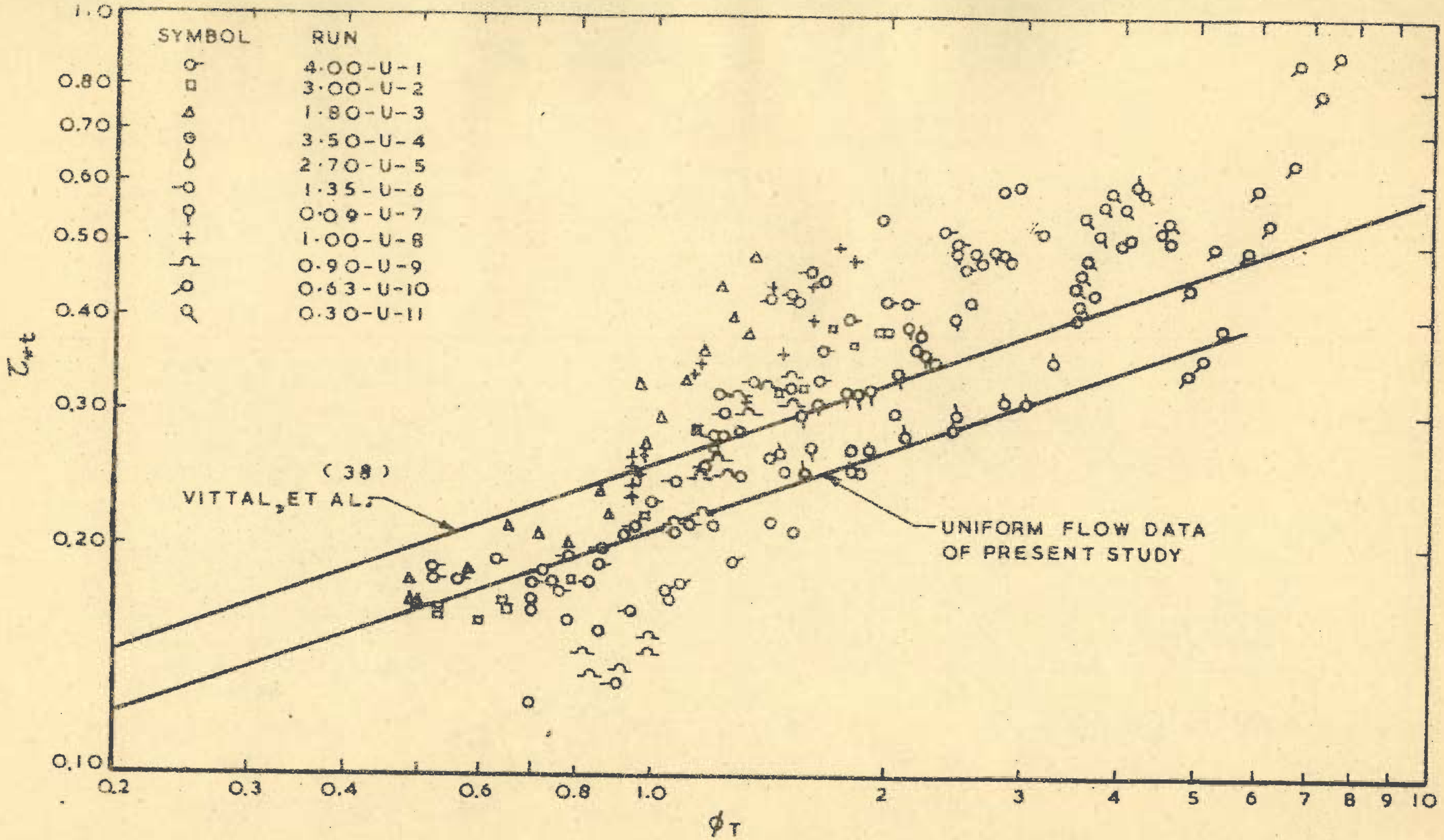


FIG. 5·10 _ VARIATION OF ϕ_T WITH τ_{*t} FOR NON UNIFORM FLOW DATA

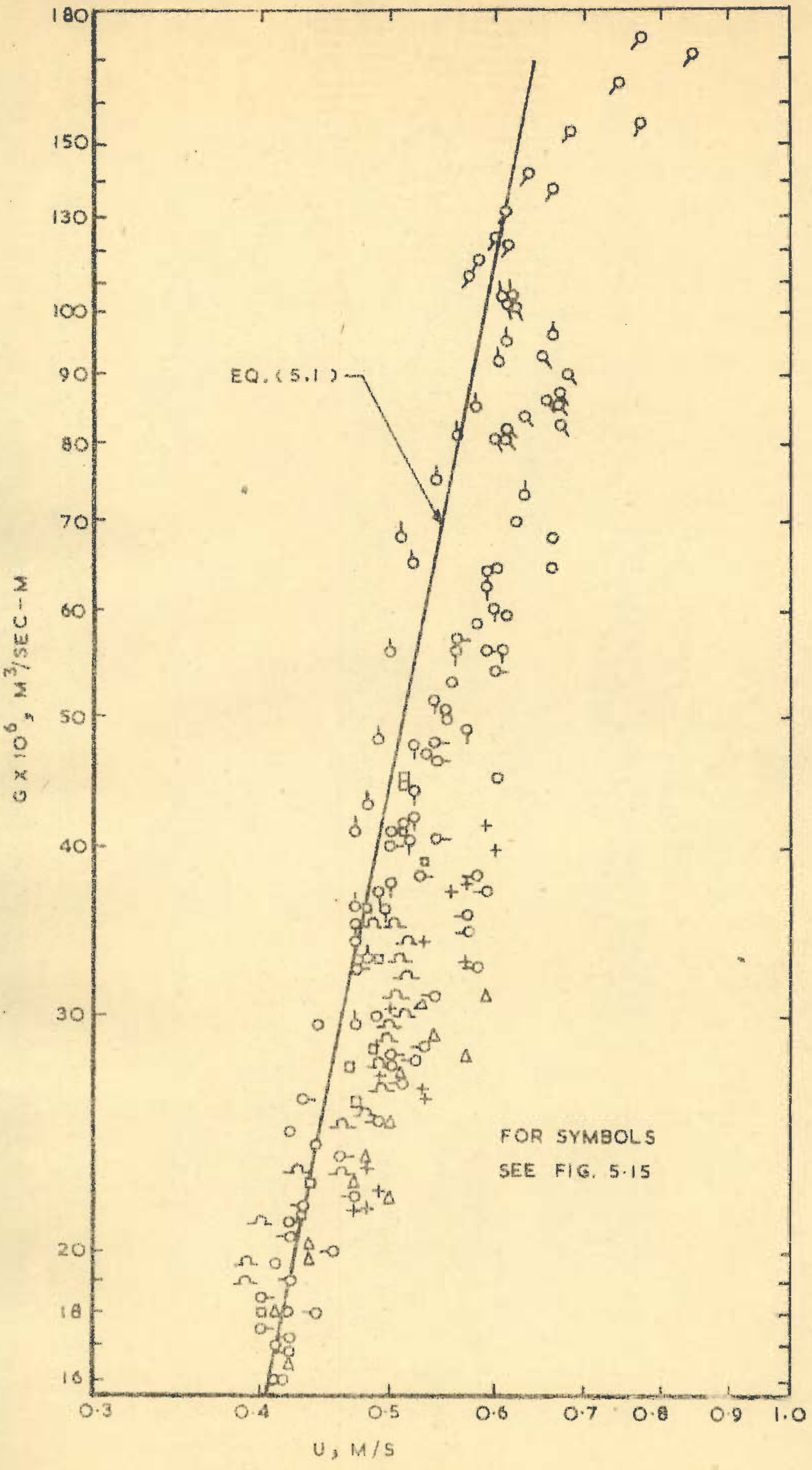


FIG. 5·11_VARIATION OF G WITH U FOR NON UNIFORM FLOW

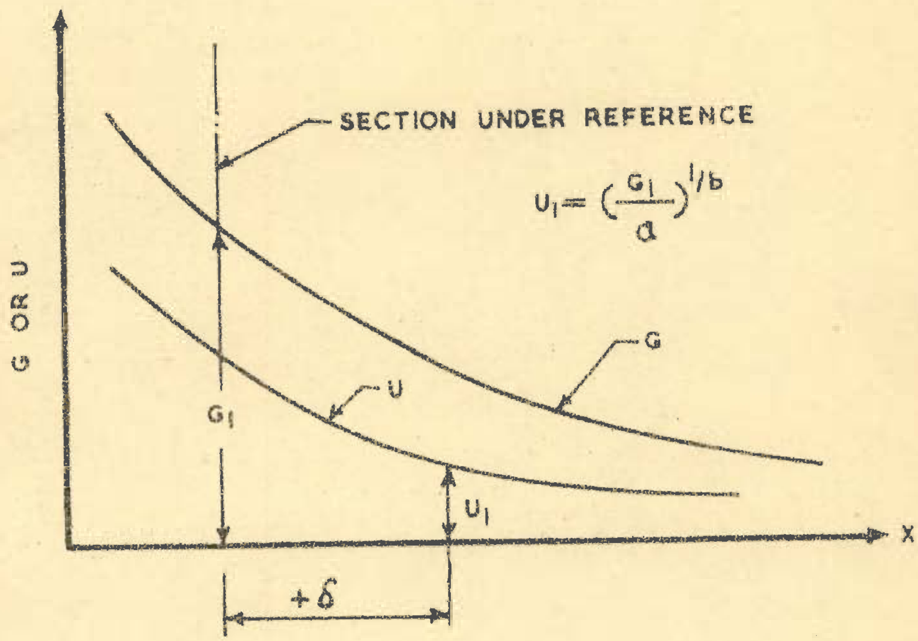


FIG. 5·12 – DEFINITION SKETCH FOR DETERMINATION OF δ

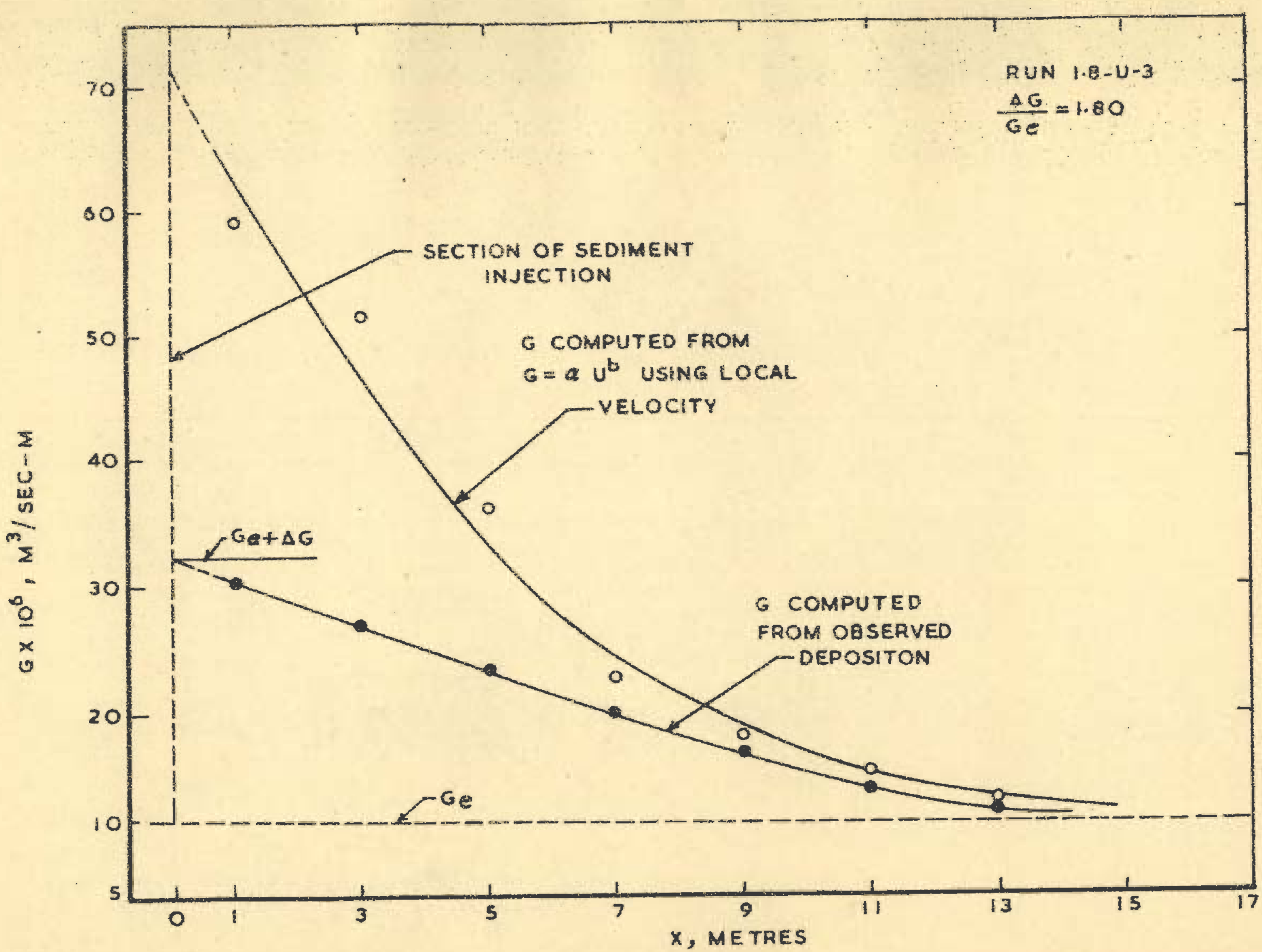


FIG. 5·13_FIGURE SHOWING EXISTENCE OF LAG DISTANCE 'δ' IN AGGRADING FLOWS

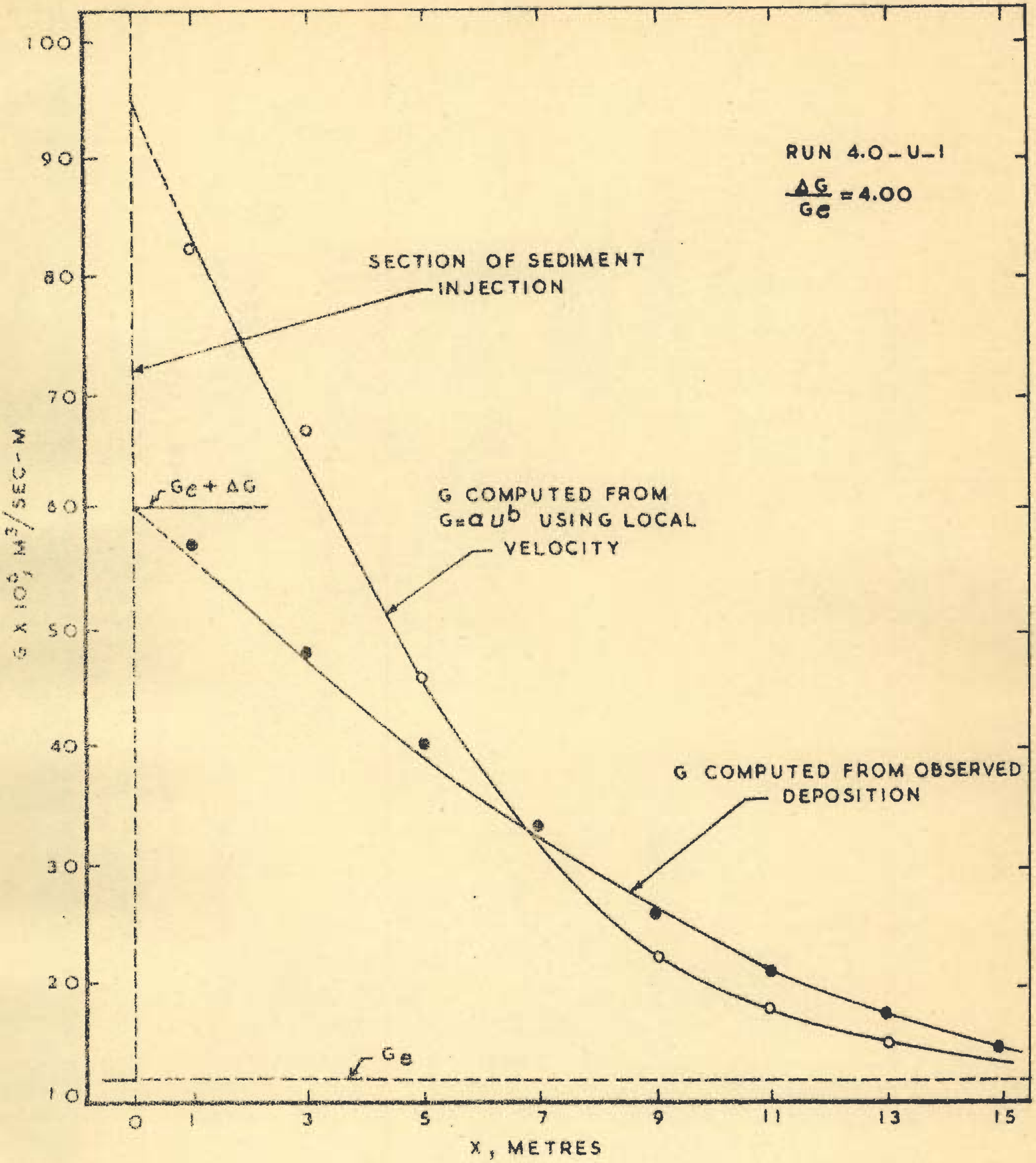


FIG. 5.13. FIGURE SHOWING EXISTENCE OF LAG DISTANCE 'δ' IN AGGRADING FLOWS (CONTINUED)

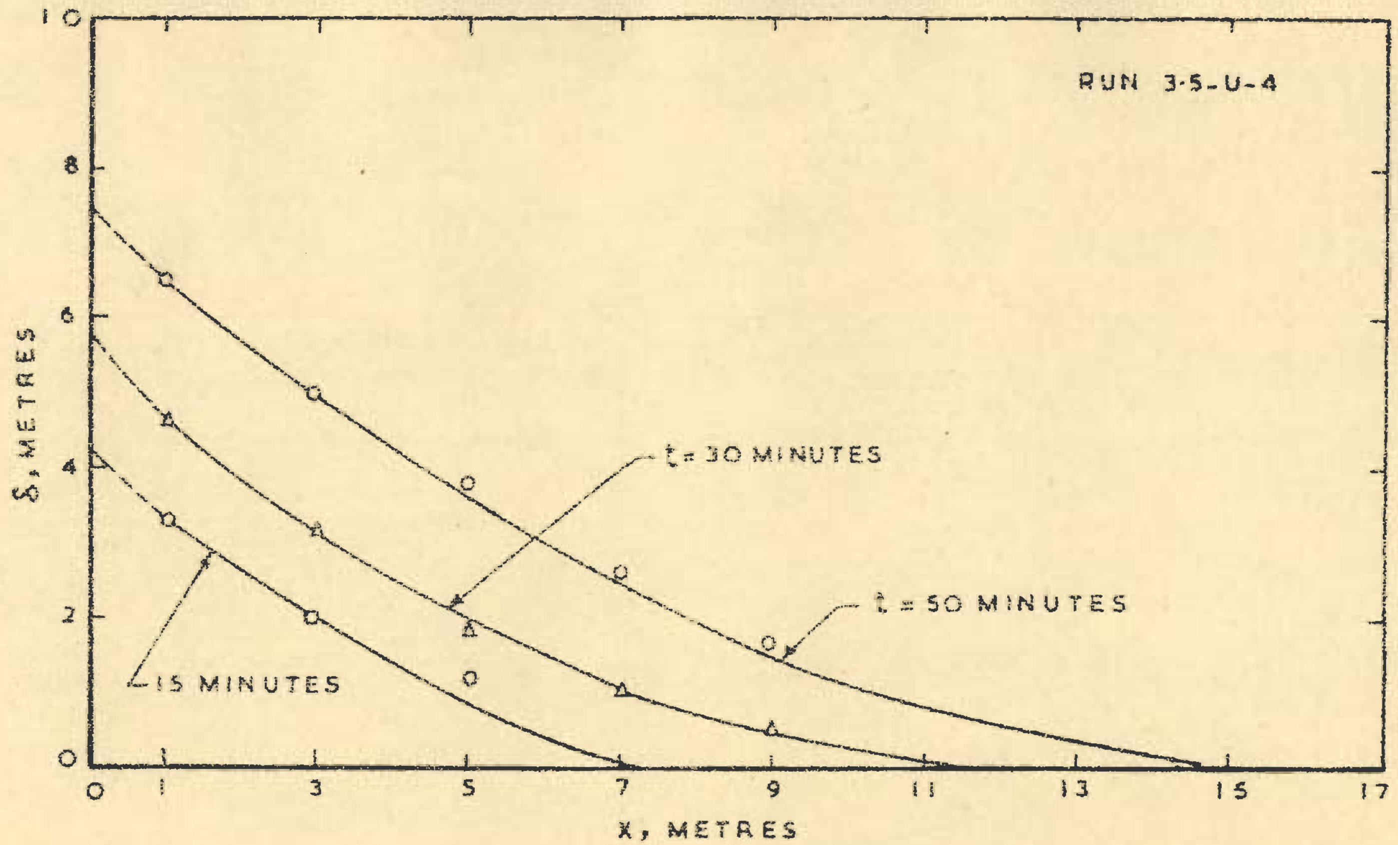


FIG. 5·14_VARIATION OF δ WITH x ALONG TRANSIENT BED PROFILES

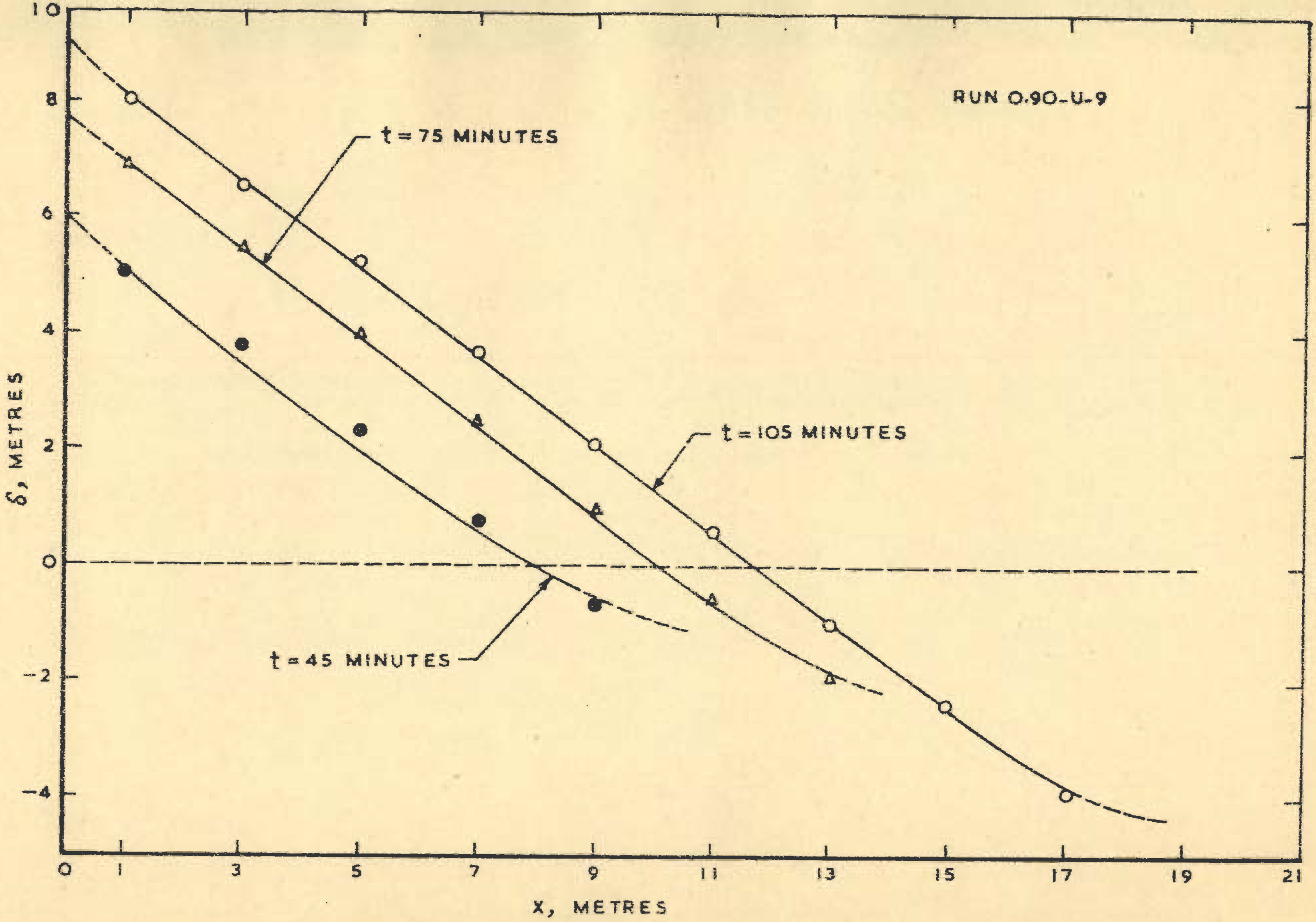


FIG. 5·14_VARIATION OF δ WITH x ALONG TRANSIENT BED PROFILES
(CONTINUED)

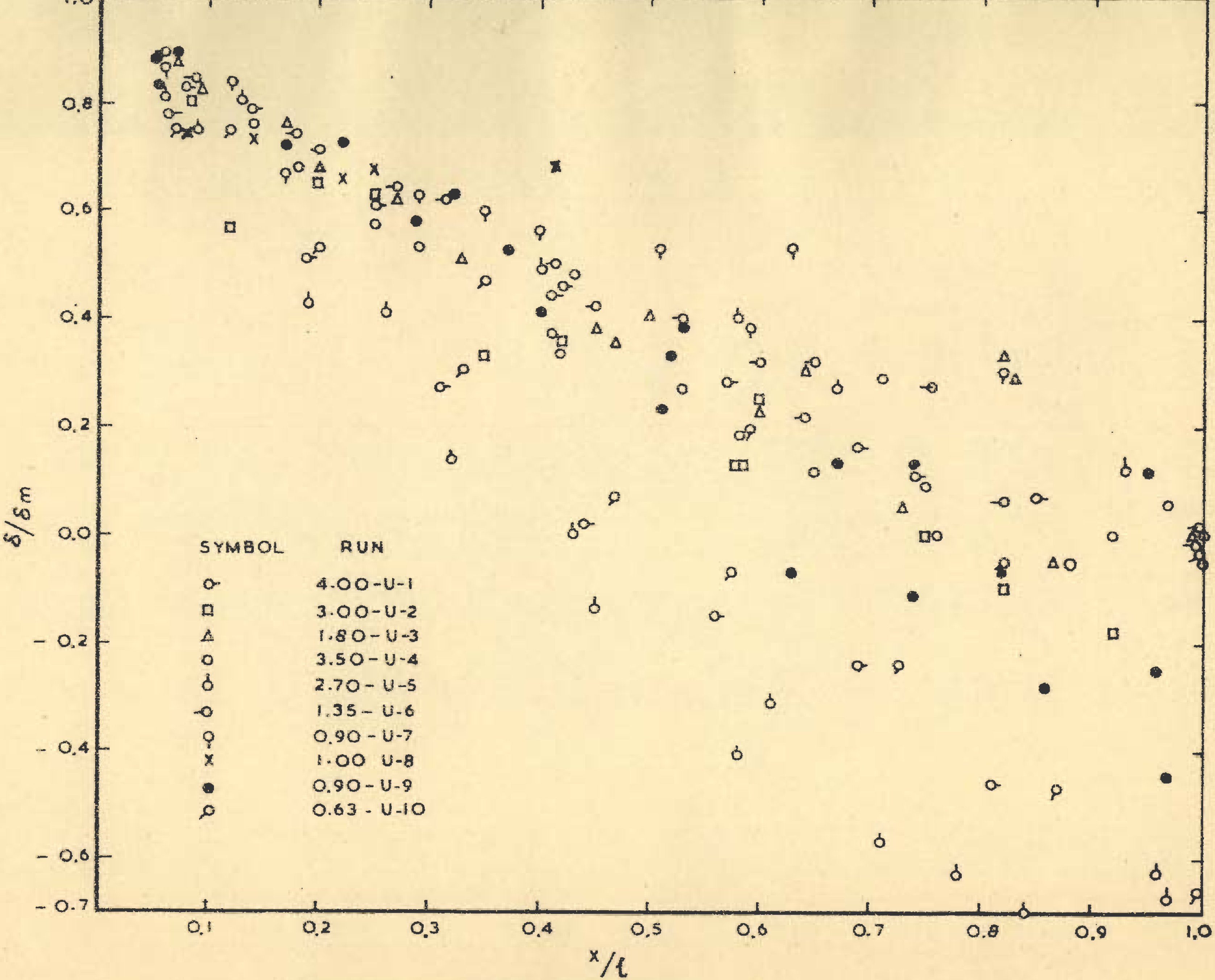


FIG. 5·15_VARIATION OF $\delta/\delta m$ WITH x/l

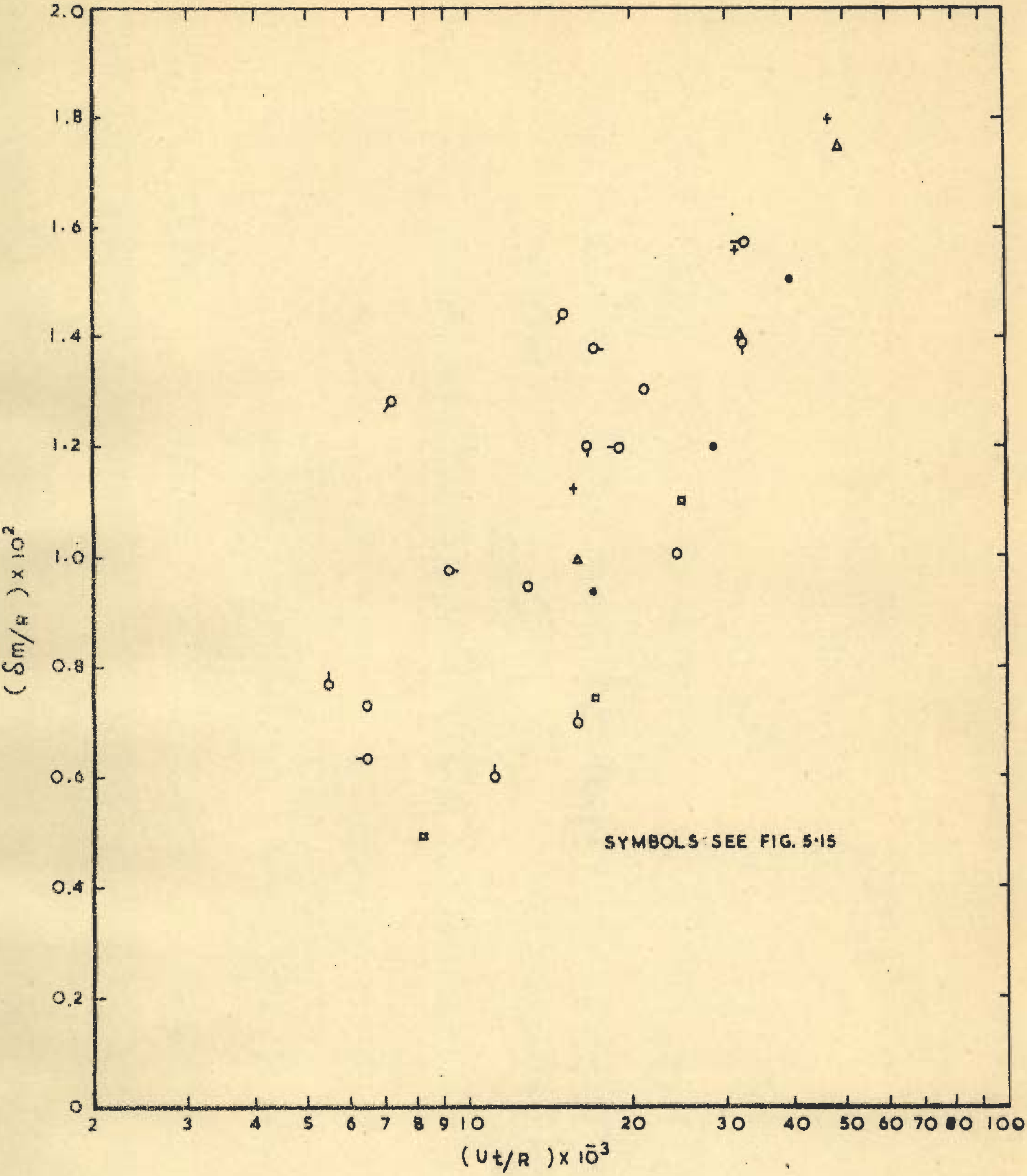


FIG. 5·16_VARIATION OF δm/R WITH Ut/R

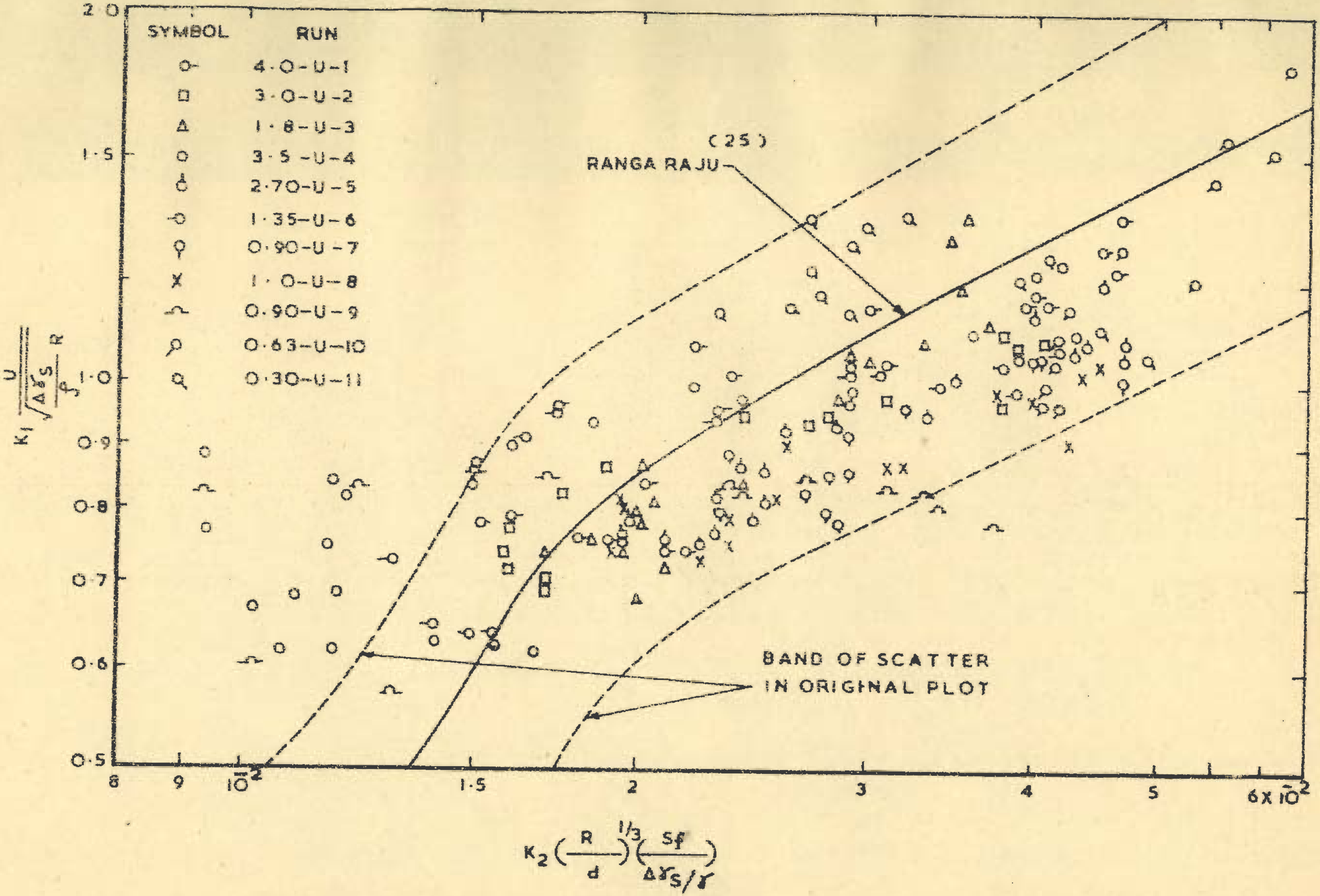


FIG. 5·17-RESISTANCE LAW FOR NON UNIFORM FLOW DATA

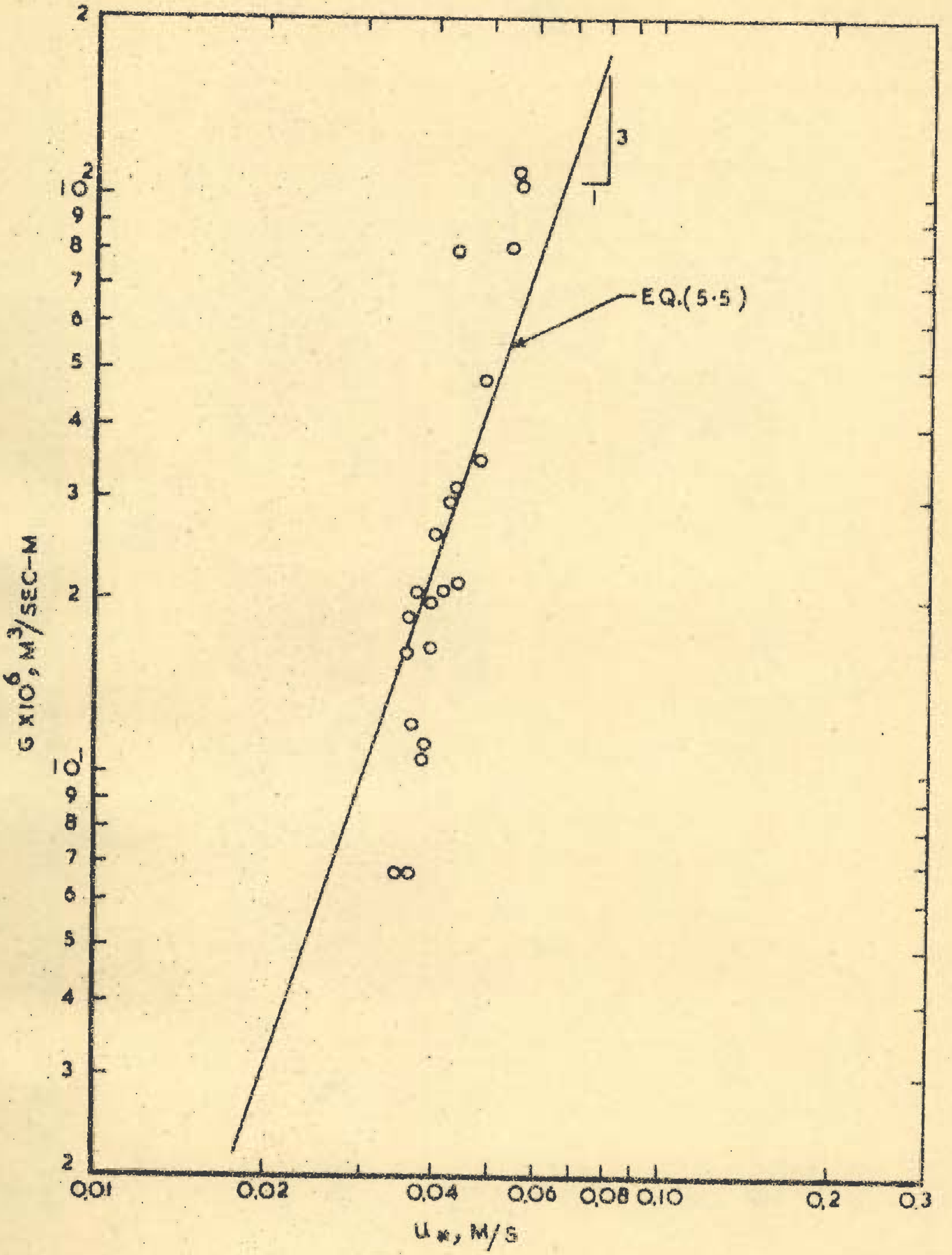


FIG. 5·18_VARIATION OF G WITH U_* FOR UNIFORM FLOW DATA

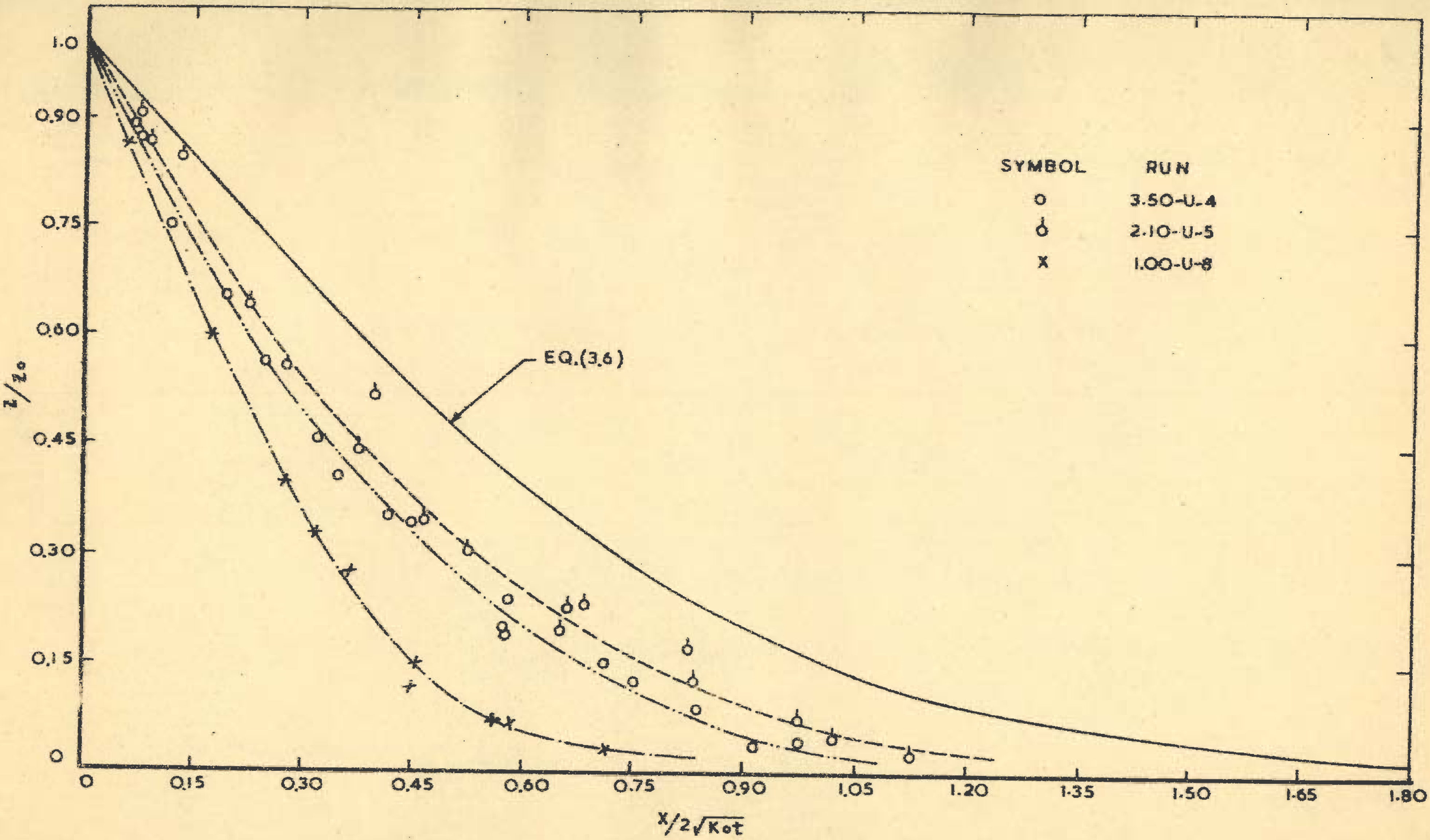


FIG. 5·19 _ VARIATION OF z/z_o WITH $x/2\sqrt{K_o t}$

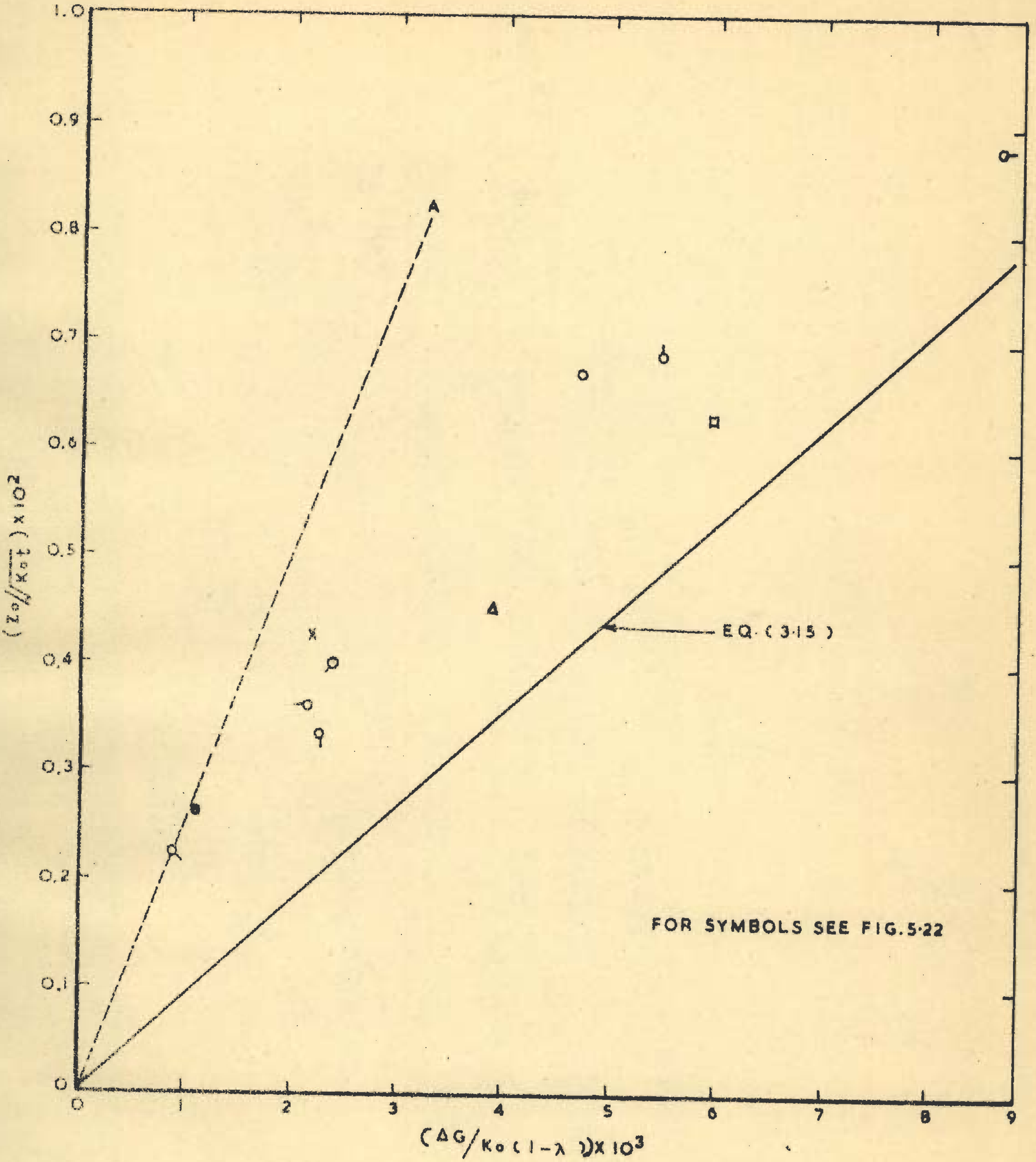


FIG. 5.20_VARIATION OF $Z_0/\sqrt{K_0T}$ WITH $\Delta G/K_0(1-\lambda)$

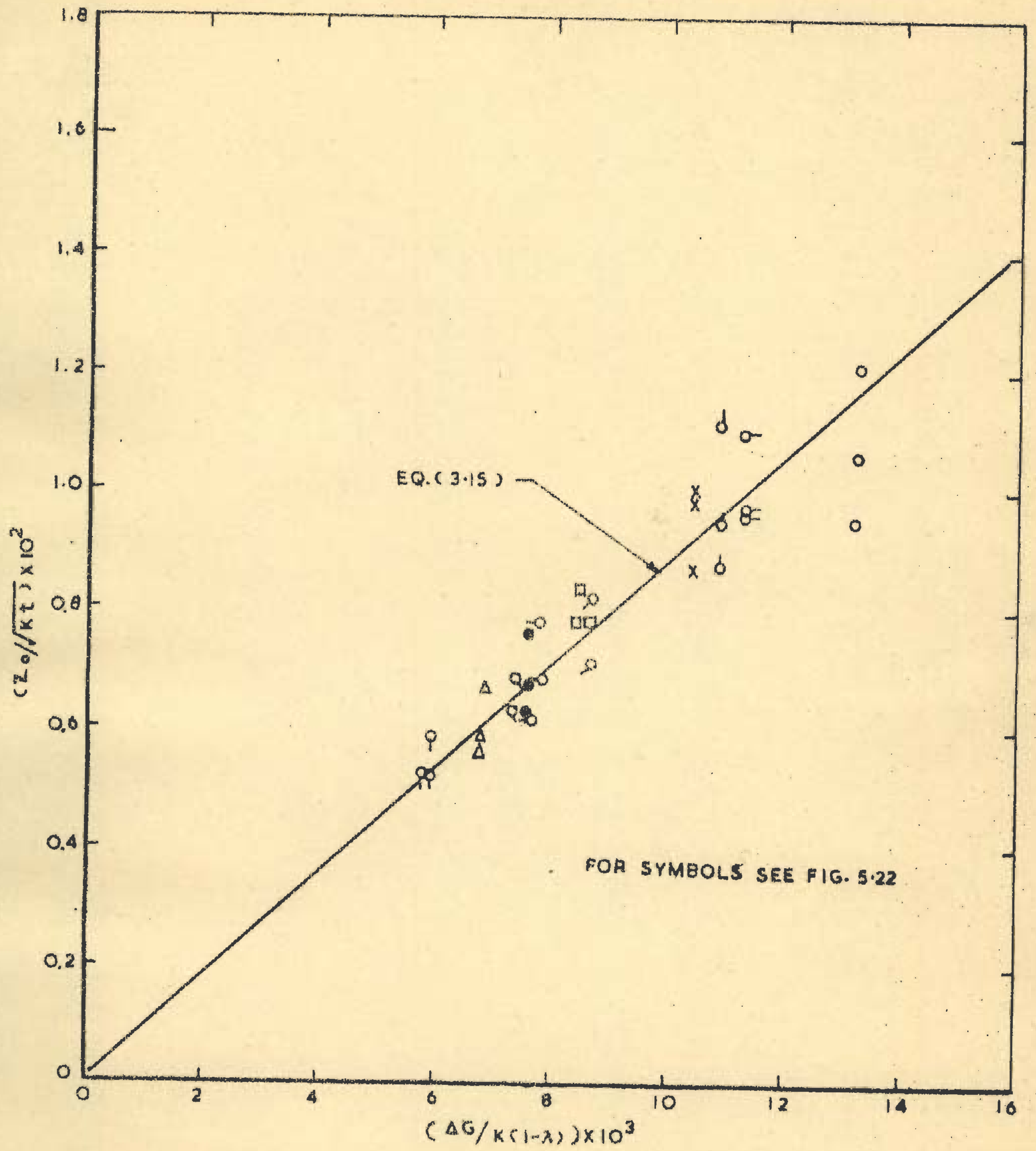


FIG. 5·21 _ VARIATION OF $Zo/\sqrt{Kt}$ WITH $\Delta G/K(1-\lambda)$

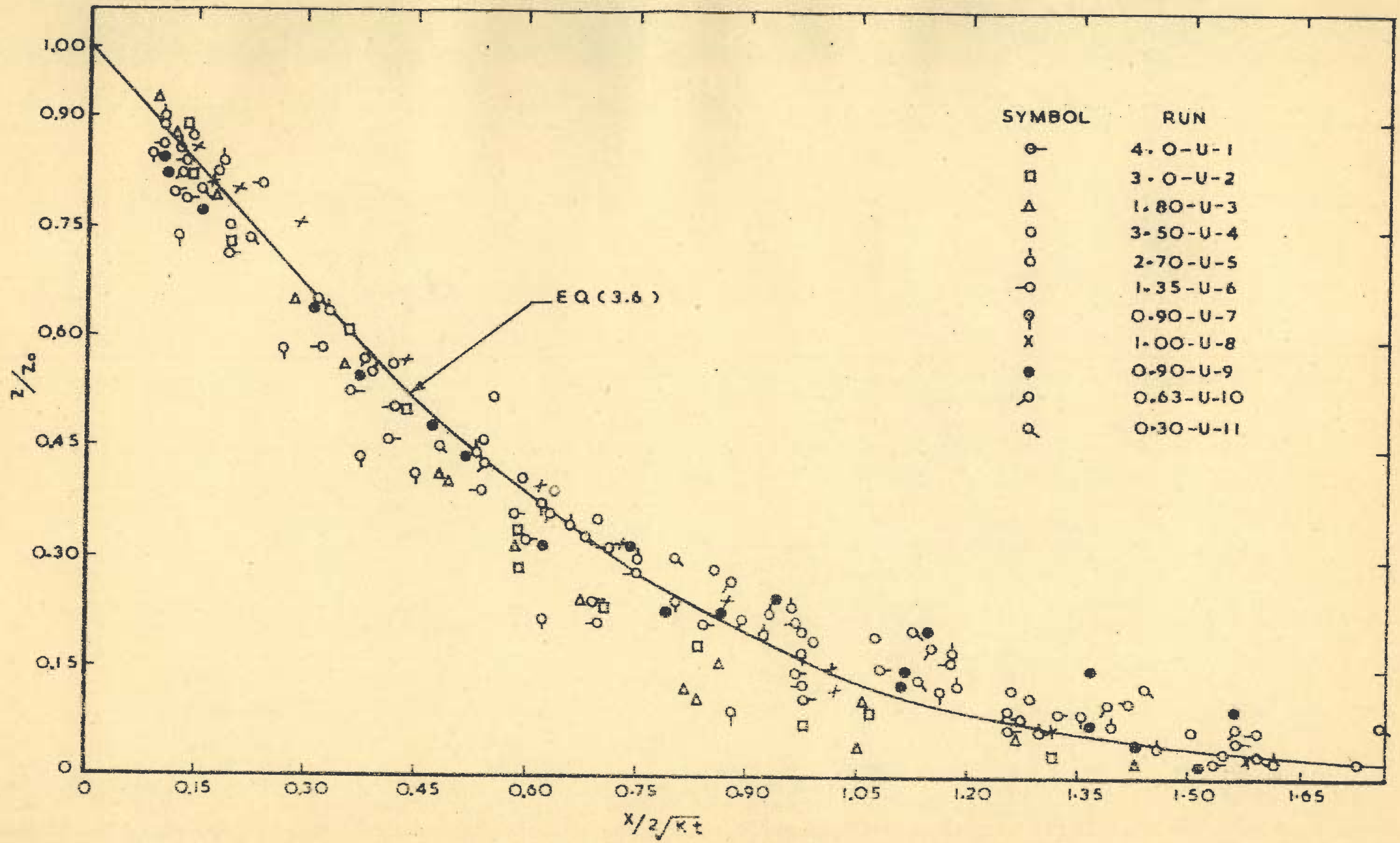


FIG. 5·22-DIMENSIONLESS PLOT OF TRANSIENT BED PROFILES

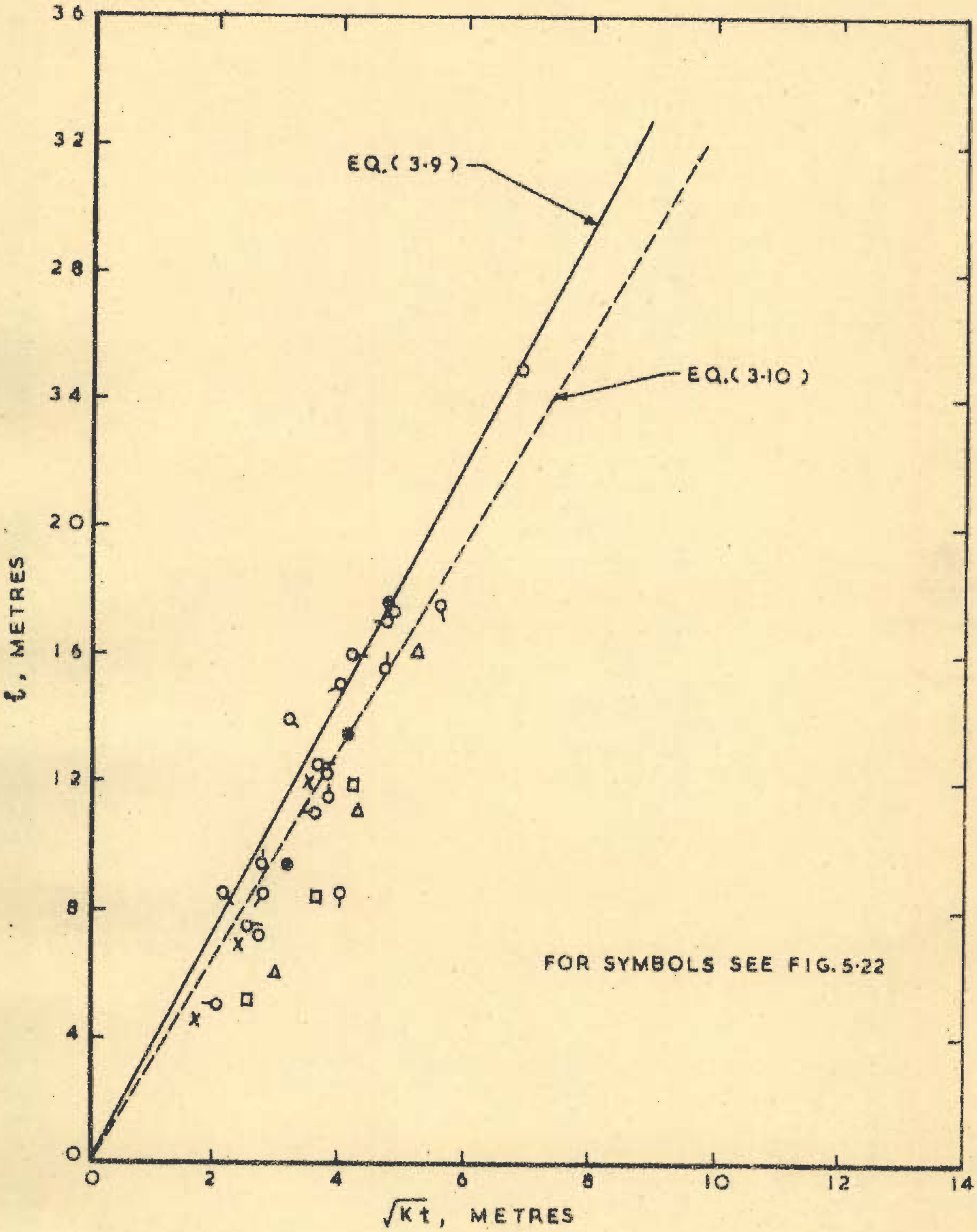


FIG. 5·23 LENGTH OF AGGRADED REACH

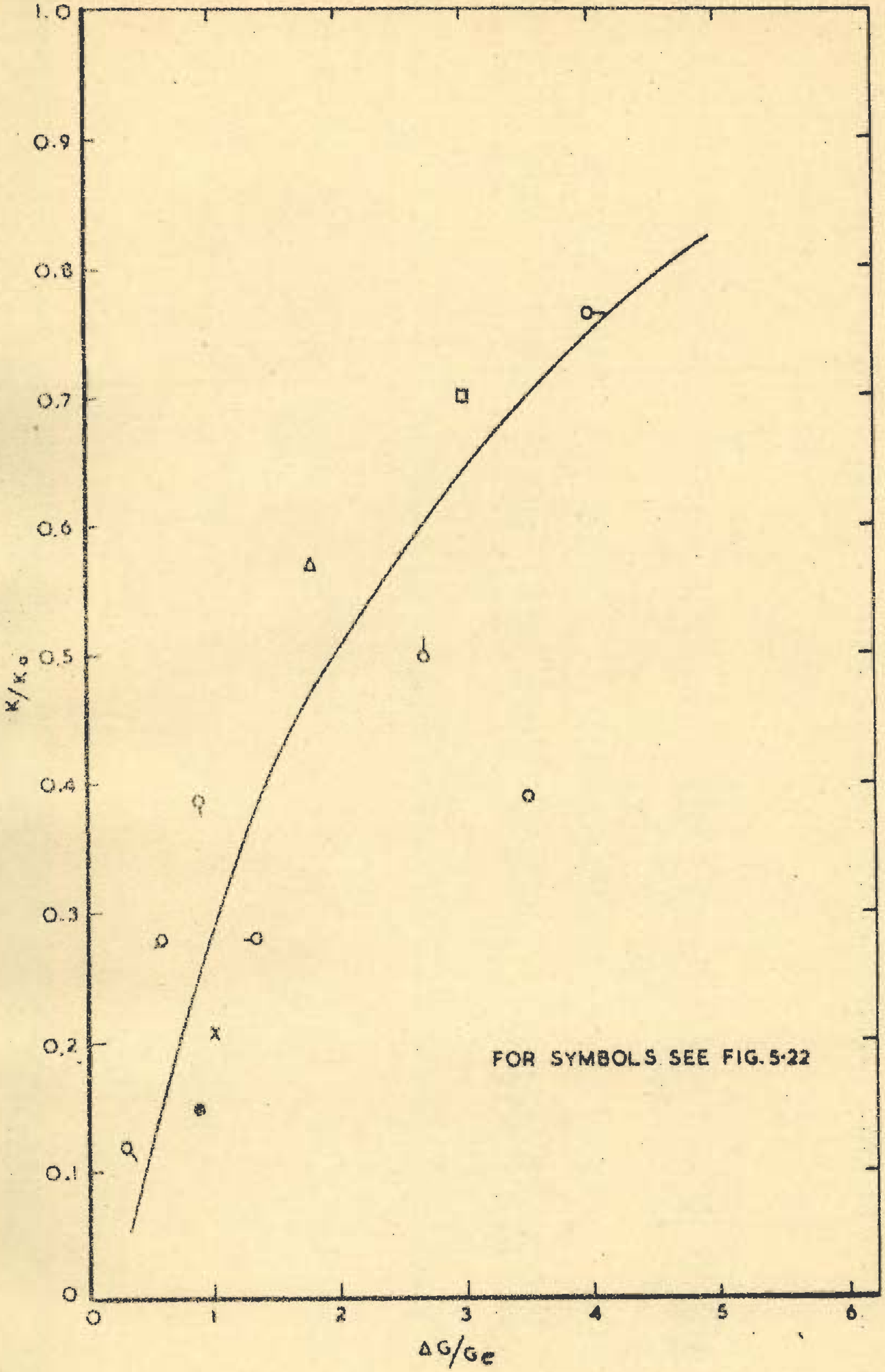


FIG. 5·24 _ VARIATION OF K/K_o WITH $\Delta G/G_e$ (DE VRIES MODEL)

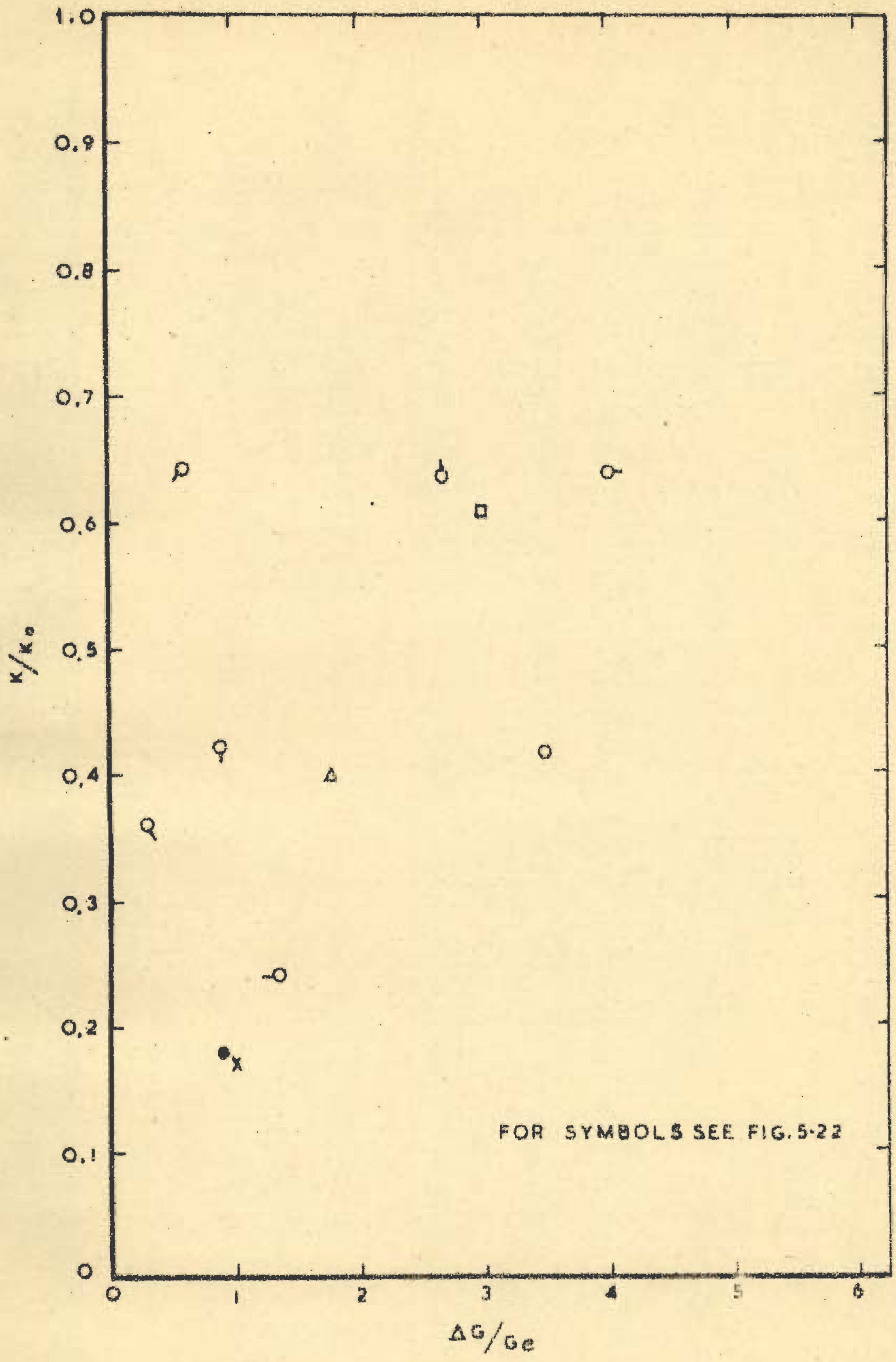


FIG. 5·25 VARIATION OF K / Ko WITH ΔG / Ge
(ADACHI AND NAKATOH MODEL)

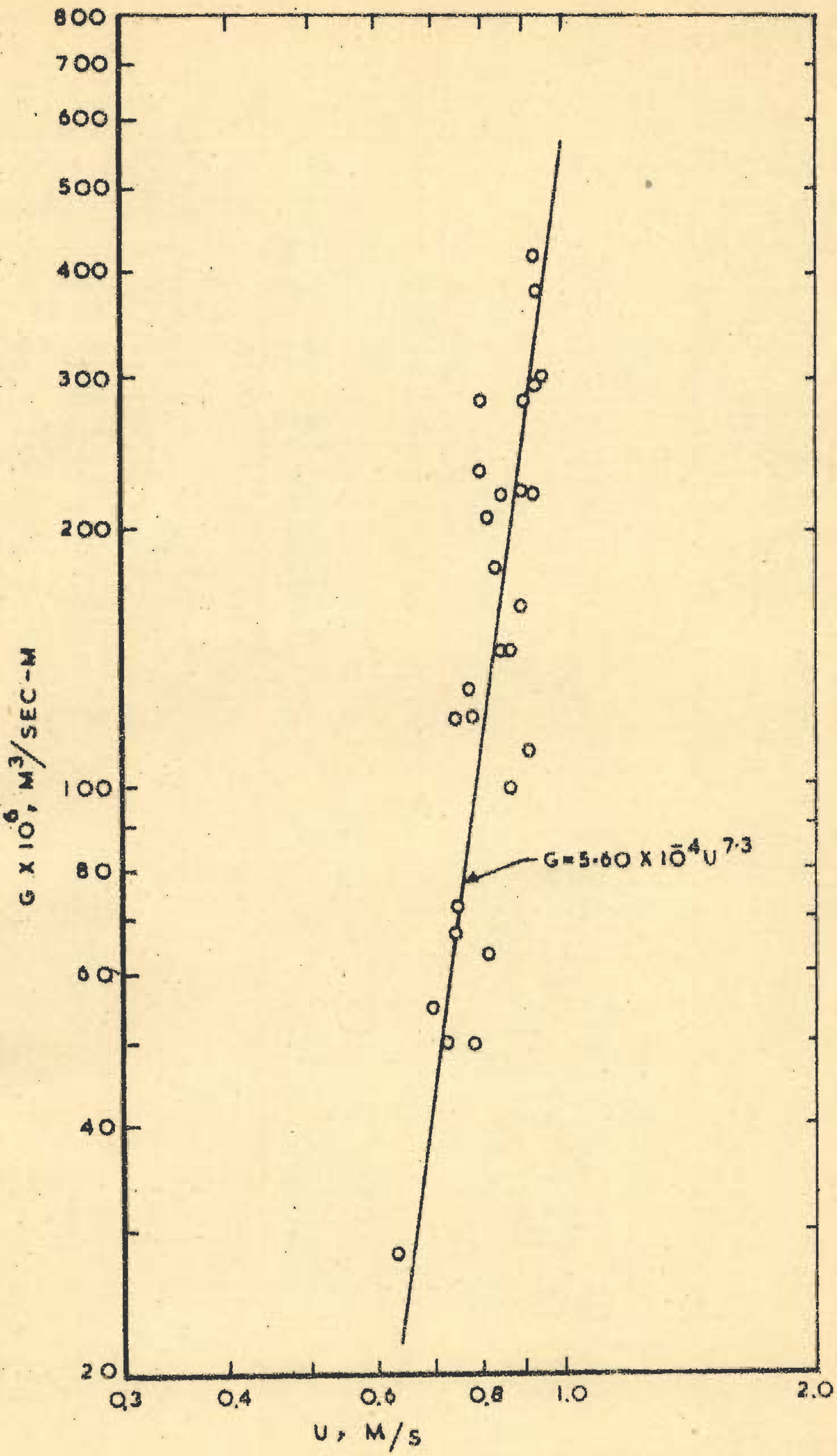


FIG. 5·26_SEDIMENT TRANSPORT LAW FOR THE COLORADO RIVER

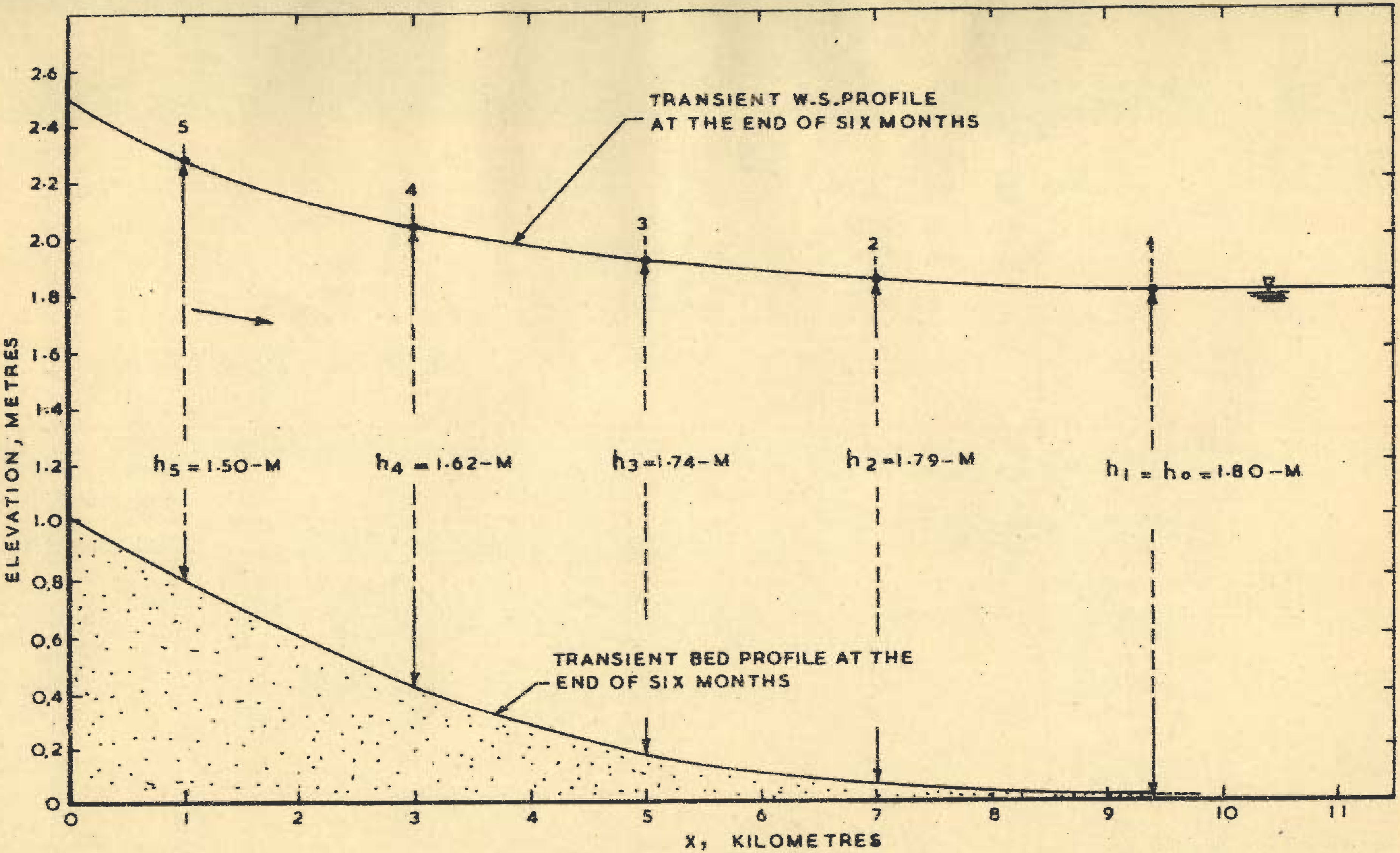


FIG. 5.27_COMPUTED TRANSIENT BED AND WATER SURFACE PROFILES FOR COLORADO RIVER AT TAYLOR-FERRY.

APPENDICES

APPENDIX-I

PROCEDURE FOR APPLYING SIDE WALL CORRECTION

The side walls of the flume were made of different materials— one side being of glass and other of painted steel sheet. There is no standard procedure for applying side wall correction to get the hydraulic radius corresponding to the bed in such a case. The following procedure was, therefore, devised :

Let the total area of cross-section, A, be divided into three parts— A_{w_1}, A_{w_2}, and A_b— representing the areas corresponding to glass and steel wall and the bed respectively.

$$\left.\begin{aligned} \text{i.e.} \quad A &= A_b + A_{w_1} + A_{w_2} \\ \text{or} \quad Bh &= BR_b + h\,R_{w_1} + h\,R_{w_2} \end{aligned}\right\} \quad \ldots \text{(A.1)}$$

in which R_b, R_{w_1} and R_{w_2} are the hydraulic radii corresponding to the bed and glass and steel walls respectively. For any given run at any given section the values of the friction slope, S_f, and the mean velocity, U, are known. The values of R_{w_1} and R_{w_2} can be found from Manning's formula as :

$$R_{w_1} = \left(\frac{U n_{w_1}}{S_f^{1/2}} \right)^{3/2} \quad \ldots \quad \text{(A-2)}$$

and $$R_{w_2} = \left(\frac{U n_{w_2}}{S_f^{1/2}} \right)^{3/2} \quad \ldots \quad \text{(A-3)}$$

in which n_{w_1} and n_{w_2} represent the Manning's roughness coefficient for glass and steel side walls respectively. Substituting Eqs. A-2 and A-3 in Eq.A-1 and simplifying:

$$\frac{R_b}{h} = 1 - \frac{U^{1.5}}{S_f^{0.75}} \left[\frac{n_{w_1}^{3/2} + n_{w_2}^{3/2}}{B} \right] \quad \ldots \text{(A-4)}$$

In the present case, the width of the flume, B, is equal to 0.20m and values of n_{w_1} and n_{w_2} read from pages 110-111 of 'Open Channel Hydraulics' by V.T. Chow are 0.010 and 0.013 respectively. With the substitution of these numerical values, Eq. A-4 reduces to :

$$\frac{R_b}{h} = 1 - 0.0125 \frac{U^{1.5}}{S_f^{0.75}} \quad \ldots \quad \text{(A-5)}$$

Here U is in metres/sec and R_b and h are in metres. Eq. A-5 was used in the calculation of R_b.

APPENDIX-I

PROCEDURE FOR APPLYING SIDE WALL CORRECTION

The side walls of the flume were made of different materials— one side being of glass and other of painted steel sheet. There is no standard procedure for applying side wall correction to get the hydraulic radius corresponding to the bed in such a case. The following procedure was, therefore, devised :

Let the total area of cross-section, A, be divided into three parts— A_{w_1}, A_{w_2}, and A_b— representing the areas corresponding to glass and steel wall and the bed respectively.

$$\left.\begin{aligned} \text{i.e.} \quad A &= A_b + A_{w_1} + A_{w_2} \\ \text{or} \quad Bh &= BR_b + h\,R_{w_1} + h\,R_{w_2} \end{aligned}\right\} \quad \ldots (A.1)$$

in which R_b, R_{w_1} and R_{w_2} are the hydraulic radii corresponding to the bed and glass and steel walls respectively. For any given run at any given section the values of the friction slope, S_f, and the mean velocity, U, are known. The values of R_{w_1} and R_{w_2} can be found from Manning's formula as :

$$R_{w_1} = \left(\frac{U n_{w_1}}{S_f^{1/2}} \right)^{3/2} \quad \ldots \quad (A-2)$$

APPENDIX-II

TABLE-1

SUMMARY OF UNIFORM FLOW DATA

S.No.	RUN	$q \times 10^3$ m^3/sec.m	$h \times 10^2$ m	$S_f \times 10^3$	U * m/sec.	G $\times 10^6$ m^3/sec.m	Temp. ^oC	Remarks
1	U-1	20.00	5.00	3.56	0.400	12.10	29.00	
2	U-2	20.00	5.20	3.30	0.385	12.10	27.50	
3	U-3	20.00	5.30	3.60	0.378	11.30	28.00	
4	U-4	35.50	8.60	2.25	0.413	16.10	30.50	RUNS U-1 to U-11 have been used for aggradation studies
5	U-5	35.50	7.5o	3.38	0.413	29.50	29.00	
6	U-6	35.50	8.50	2.63	0.417	16.60	30.00	
7	U-7	35.50	7.20	4.27	0.493	34.40	30.00	
8	U-8	35.50	7.50	3.63	0.473	21.40	27.50	
9	U-9	35.50	9.20	2.12	0.386	18.80	28.50	
10	U-10	35.50	6.20	6.52	0.572	111.00	29.50	
11	U-11	35.50	5.80	4.82	0.612	80.50	28.00	
12	U-12	20.00	7.00	2.74	0.286	1.50	29.75	
13	U-13	25.00	7.40	2.07	0.337	6.80	29.50	
14	U-14	30.00	8.50	2.35	0.353	10.66	28.00	
15	U-15	40.00	9.85	2.50	0.407	20.50	31.50	
16	U-16	45.00	10.00	2.25	0.450	20.40	28.00	
17	U-17	7.00	3.20	3.72	0.219	0.82	28.50	
18	U-18	25.00	5.85	3.50	0.427	19.85	29.00	
19	U-19	30.00	6.80	3.30	0.441	26.10	29.50	
20	U-20	7.00	2.20	6.58	0.318	6.88	27.00	
21	U-21	15.00	3.50	6.50	0.429	31.40	29.00	
22	U-22	20.00	4.30	6.70	0.465	47.60	28.00	
23	U-23	25.00	5.20	7.00	0.481	80.50	28.00	
24	U-24	30.00	5.70	6.90	0.526	104.30	27.50	

* U values are computed ones.

TABLE-2

AGGRADATION STUDIES—BASIC AND COMPUTED PARAMETERS:RUN 4.0-U-1*

TEMPERATURE = 28.0°C $K = 0.72 \times 10^{-2}$

DISTANCES	1	3	5	7	9	11	13	15	17	19
	T = 15 MINUTES				z_o = 0.028			L= 7.2	$G_* = 13.4 \times 10^{-6}$	
$z \times 10^2$	2.00	1.00	0.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$h \times 10^2$	3.35	3.70	4.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00
$G \times 10^6$	54.00	41.00	28.00	17.50	12.10	12.10	12.10	12.10	12.10	12.10
$n \times 10^2$	1.42	1.57	1.26	1.40	1.68	1.72	1.72	1.72	1.72	1.72
	T = 30 MINUTES				z_o = 0.035			L = 12.2	$G_* = 14.0 \times 10^{-6}$	
$z \times 10^2$	2.75	1.60	0.85	0.50	0.25	0.10	0.00	0.00	0.00	0.00
$h \times 10^2$	3.40	3.70	3.80	4.10	4.35	4.90	5.00	5.00	5.00	5.00
$G \times 10^6$	56.00	46.50	38.00	30.00	23.50	17.00	13.00	12.10	12.10	12.10
$n \times 10^2$	1.49	1.60	1.42	1.42	1.38	1.25	1.47	1.63	1.68	1.72
	T = 40 MINUTES				z_o = 0.040			L = 16.0	$G_* = 14.3 \times 10^{-6}$	
$z \times 10^2$	3.20	2.10	1.30	0.85	0.60	0.35	0.20	0.10	0.00	0.00
$h \times 10^2$	3.55	3.70	4.00	4.25	4.65	4.80	5.00	5.00	5.00	5.00
$G \times 10^6$	57.00	48.00	40.00	32.50	26.00	21.00	17.50	14.50	13.00	12.10
$n \times 10^2$	1.62	1.61	1.56	1.45	1.55	1.64	1.81	1.83	1.72	1.72

* In all these Tables G is in M^3/Sec-M; the depth of deposition, z; the maximum depth of deposition at x = 0, z_o; the length of aggradation profile, L; and the depth of flow, h, are in metres. G_* is the total sediment transport rate at the end of flume in M^3/sec-M.

TABLE-2 (Continued)

AGGRADATION STUDIES-BASIC AND COMPUTED PARAMETERS:RUN 3.0-U-2

TEMPERATURE 27.5°C $K = 0.70 \times 10^{-2}$

DISTANCES	1	3	5	7	9	11	13	15	17	19
		T = 15 MINUTES				z_o = 0.021			L = 5.2	$G_* = 12.0 \times 10^{-6}$
$z \times 10^2$	1.50	0.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$h \times 10^2$	3.90	4.25	5.20	5.20	5.20	5.20	5.20	5.20	5.20	5.20
$G \times 10^6$	41.00	26.00	15.00	12.10	12.10	12.10	12.10	12.10	12.10	12.10
$n \times 10^2$	1.72	1.63	1.68	1.70	1.76	1.76	1.76	1.76	1.76	1.76
		T = 30 MINUTES				z_o = 0.028			L = 8.5	$G_* = 18.6 \times 10^{-6}$
$z \times 10^2$	2.30	1.40	0.65	0.20	0.00	0.00	0.00	0.00	0.00	0.00
$h \times 10^2$	3.90	4.10	4.30	4.70	5.20	5.20	5.20	5.20	5.20	5.20
$G \times 10^6$	44.50	36.00	28.50	21.00	14.75	12.10	12.10	12.10	12.10	12.10
$n \times 10^2$	1.77	1.82	1.62	1.45	1.63	1.70	1.76	1.76	1.76	1.76
		T = 45 MINUTES				z_o = 0.033			L = 12.0	$G_* = 12.2 \times 10^{-6}$
$z \times 10^2$	2.90	2.00	1.10	0.60	0.30	0.10	0.00	0.00	0.00	0.00
$h \times 10^2$	3.90	3.80	4.10	4.30	4.60	4.95	5.20	5.20	5.20	5.20
$G \times 10^6$	45.25	38.75	33.00	27.50	22.50	18.00	14.00	12.50	12.10	12.10
$n \times 10^2$	1.77	1.61	1.62	1.50	1.47	1.50	1.60	1.76	1.76	1.76

TABLE-2(Continued)

AGGRADATION STUDIES-BASIC AND COMPUTED PARAMETERS:RUN 1.8-U-3

TEMPERATURE 28.0°C $K = 0.50 \times 10^{-2}$

DISTANCES	1	3	5	7	9	11	13	15	17	19
		T = 30 MINUTES				z_o = 0.020			L = 6.0	$G_* = 11.6 \times 10^{-2}$
$z \times 10^2$	1.60	0.80	0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$h \times 10^2$	3.70	4.00	4.80	5.25	5.15	5.30	5.30	5.30	5.30	5.30
$G \times 10^6$	29.00	22.00	15.00	11.30	11.30	11.30	11.30	11.30	11.30	11.30
$n \times 10^2$	1.47	1.50	1.65	1.86	1.89	1.92	1.92	1.92	1.92	1.92
		T = 60 MINUTES				z_o = 0.025			L =11.0	$G_* = 11.6 \times 10^{-2}$
$z \times 10^2$	2.20	1.40	0.80	0.30	0.10	0.00	0.00	0.00	0.00	0.00
$h \times 10^2$	3.80	3.90	4.20	4.60	4.80	5.00	5.10	5.20	5.30	5.30
$G \times 10^6$	30.50	26.75	23.50	19.75	16.50	13.25	11.50	11.30	11.30	11.30
$n \times 10^2$	1.58	1.56	1.61	1.69	1.62	1.64	1.64	1.70	1.92	1.92
		T = 90 MINUTES				z_o = 0.029			L = 16.0	$G_* = 13.4 \times 10^{-2}$
$z \times 10^2$	2.70	1.90	1.20	0.70	0.45	0.30	0.15	0.05	0.00	0.00
$h \times 10^2$	3.40	3.50	4.00	4.30	4.60	4.90	5.05	5.15	5.30	5.30
$G \times 10^6$	31.00	28.00	25.25	22,50	20.25	18.00	16.50	14.50	13.00	12.00
$n \times 10^2$	1.29	1.33	1.54	1.50	1.53	1.66	1.74	1.80	1.90	1.92

TABLE-2 (Continued)

AGGRADATION STUDIES--BASIC AND COMPUTED PARAMETERS:RUN 3.5-U-4

TEMPERATURE 29.0°C $K = 0.78 \times 10^{-2}$

DISTANCES	1	3	5	7	9	11	13	15	17	19
			T = 15 MINUTES			z_o = 0.032			L = 7.2	$G_* = 21.0 \times 10^{-6}$
$z \times 10^2$	2.40	1.30	0.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$h \times 10^2$	5.40	5.90	7.05	8.55	8.60	8.60	8.60	8.60	8.60	8.60
$G \times 10^6$	64.50	45.00	27.50	16.10	16.10	16.10	16.10	16.10	16.10	16.10
$n \times 10^2$	1.49	1.43	0.67	0.59	1.73	1.85	1.72	1.72	1.72	1.72
			T = 30 MINUTES			z_o = 0.040			L = 12.2	$G_* = 18.5 \times 10^{-6}$
$z \times 10^2$	3.50	2.25	1.40	0.80	0.50	0.15	0.00	0.00	0.00	0.00
$h \times 10^2$	5.35	5.80	6.40	7.00	7.50	8.10	8.55	8.70	8.60	8.60
$G \times 10^6$	68.00	59.50	50.50	41.50	32.50	24.00	18.00	16.10	16.00	16.10
$n \times 10^2$	1.52	1.59	1.45	1.29	1.18	1.16	1.36	1.77	1.93	1.72
			T = 50 MINUTES			z_o =0.046			L = 17.2	$G_* = 15.0 \times 10^{-6}$
$z \times 10^2$	3.90	3.00	2.10	1.60	1.10	0.70	0.40	0.20	0.00	0.00
$h \times 10^2$	5.70	5.90	6.10	6.40	6.75	7.15	7.60	8.00	8.40	8.60
$G \times 10^6$	70.00	64.50	58.50	53.00	47.00	41.00	35.00	29.50	24.50	19.50
$n \times 10^2$	1.72	1.72	1.52	1.37	1.34	1.28	1.24	1.33	1.39	1.38

TABLE-2 (Continued)

AGGRADATION STUDIES--BASIC AND COMPUTED PARAMETERS:RUN 2.7-U-5

TEMPERATURE 29°C   K = 1.21 x 10⁻²

DISTANCES	1	3	5	7	9	11	13	15	17	19
		T = 10 MINUTES				z_o = 0.030			L = 7.5	$G_* = 37.0 \times 10^{-6}$
$z \times 10^2$	2.50	1.50	0.60	0.10	0.00	0.00	0.00	0.00	0.00	0.00
$h \times 10^2$	5.40	5.65	6.45	7.35	7.50	7.50	7.50	7.50	7.50	7.50
$G \times 10^6$	96.00	73.00	50.00	33.50	29.50	29.50	29.50	29.50	29.50	29.50
$n \times 10^2$	1.62	1.63	1.64	1.65	1.67	1.74	1.74	1.74	1.74	1.74
		T = 20 MINUTES				z_o = 0.041			L = 11.50	$G_* = 32.0 \times 10^{-6}$
$z \times 10^2$	3.10	2.00	1.25	0.70	0.45	0.15	0.00	0.00	0.00	0.00
$h \times 10^2$	5.80	5.90	6.35	6.90	7.10	7.40	7.50	7.50	7.50	7.50
$G \times 10^6$	103.00	92.00	81.00	68.00	56.00	43.00	34.00	30.00	29.50	29.50
$n \times 10^2$	1.88	1.82	1.76	1.71	1.71	1.78	1.73	1.74	1.74	1.74
		T = 30 MINUTES				z_o = 0.041			L = 15.50	$G_* = 26.0 \times 10^{-6}$
$z \times 10^2$	3.70	2.60	1.80	1.25	0.95	0.70	0.30	0.10	0.00	0.00
$h \times 10^2$	5.90	5.80	6.15	6.55	6.85	7.10	7.25	7.50	7.50	7.50
$G \times 10^6$	105.00	95.00	85.00	75.00	65.00	56.00	48.00	41.00	36.00	32.00
$n \times 10^2$	1.92	1.79	1.73	1.61	1.66	1.80	1.83	1.83	1.76	1.74

TABLE-2 (Continued)

AGGRADATION STUDIES--BASIC AND COMPUTED PARAMETERS:RUN 1.35-U-6

TEMPERATURE 29°C $K = 0.485 \times 10^{-2}$

DISTANCES	1	3	5	7	9	11	13	15	17	19
		T = 15 MINUTES				z_o = 0.016			L = 5.0	$G_* = 20.0 \times 10^{-6}$
$z \times 10^2$	1.30	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$h \times 10^2$	6.20	6.90	8.50	8.50	8.50	8.50	8.50	8.50	8.50	8.50
$G \times 10^6$	35.50	26.75	20.50	16.75	16.40	16.40	16.40	16.40	16.40	16.40
$n \times 10^2$	1.55	1.02	0.68	1.80	1.87	1.87	1.87	1.87	1.87	1.87
		T = 45 MINUTES				z_o = 0.024			L = 11.0	$G_* = 16.6 \times 10^{-6}$
$z \times 10^2$	2.10	1.30	0.60	0.40	0.20	0.00	0.00	0.00	0.00	0.00
$h \times 10^2$	6.00	6.15	6.70	7.30	8.20	8.50	8.50	8.50	8.50	8.50
$G \times 10^6$	37.00	32.50	28.50	24.75	21.50	19.00	17.00	16.40	16.40	16.40
$n \times 10^2$	1.74	1.53	1.27	0.59	0.95	1.76	1.80	1.87	1.87	1.87
		T = 75 MINUTES				z_o = 0.029			L = 17.0	$G_* = 13.0 \times 10^{-6}$
$z \times 10^2$	2.50	1.70	1.10	0.80	0.60	0.45	0.30	0.20	0.00	0.00
$h \times 10^2$	6.10	6.20	6.60	6.90	7.30	7.60	7.80	8.00	8.50	8.50
$G \times 10^6$	38.00	34.50	31.00	27.75	24.75	22.00	19.75	18.00	17.00	16.40
$n \times 10^2$	1.81	1.71	1.50	1.29	1.33	1.24	1.27	1.42	1.69	1.80

TABLE-2 (Continued)

AGGRADATION STUDIES--BASIC AND COMPUTED PARAMETERS: RUN 0.9-U-7

TEMPERATURE = 27.5°C $K = 0.877 \times 10^{-2}$

DISTANCES	1	3	5	7	9	11	13	15	17	19
		T = 30 MINUTES				z_o = 0.023			L = 8.5	$G_* = 22.2 \times 10^{-6}$
$z \times 10^2$	1.70	1.00	0.50	0.20	0.00	0.00	0.00	0.00	0.00	0.00
$h \times 10^2$	6.00	5.90	6.25	6.80	7.25	7.20	7.20	7.20	7.20	7.20
$G \times 10^6$	62.50	56.00	49.00	42.00	37.00	34.40	34.40	34.40	34.40	34.40
$n \times 10^2$	1.99	1.75	1.58	1.60	1.75	1.88	1.88	1.88	1.88	1.88
		T = 60 MINUTES				z_o = 0.029			L = 17.5	$G_* = 23.8 \times 10^{-6}$
$z \times 10^2$	2.50	1.70	1.20	0.90	0.70	0.50	0.35	0.25	0.10	0.00
$h \times 10^2$	6.00	5.90	6.30	6.55	6.80	6.85	7.00	7.10	7.20	7.20
$G \times 10^6$	64.00	60.00	56.00	51.50	47.50	44.00	40.50	37.50	36.00	34.40
$n \times 10^2$	1.99	1.76	1.71	1.73	1.83	1.79	1.85	1.92	1.98	1.88

TABLE-2 (Continued)

AGGRADATION STUDIES-- BASIC AND COMPUTED PARAMETERS:RUN 1.0-U-8

TEMPERATURE = 29.0°C $k = 0.35 \times 10^{-2}$

DISTANCES	1	3	5	7	9	11	13	15	17	19
		T = 15 MINUTES				z_o = 0.017			L = 4.5	$G_* = 19.8 \times 10^{-6}$
$z \times 10^2$	1.30	0.40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$h \times 10^2$	6.40	6.70	7.35	7.60	7.50	7.50	7.50	7.50	7.50	7.50
$G \times 10^6$	37.00	26.50	21.50	21.50	21.50	21.50	21.50	21.50	21.50	21.50
$n \times 10^2$	2.07	1.86	1.62	1.68	1.80	1.80	1.80	1.80	1.80	1.80
		T = 30 MINUTES				z_o = 0.025			L = 7.0	$G_* = 26.4 \times 10^{-6}$
$z \times 10^2$	2.00	1.00	0.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$h \times 10^2$	5.90	6.20	6.70	7.25	7.60	7.55	7.50	7.50	7.50	7.50
$G \times 10^6$	39.75	32.50	26.00	22.25	21.50	21.50	21.50	21.50	21.50	21.50
$n \times 10^2$	1.87	1.83	1.66	1.57	1.72	1.85	1.80	1.80	1.80	1.80
		T = 60 MINUTES				z_o = 0.030			L = 12.0	$G_* = 28.0 \times 10^{-6}$
$z \times 10^2$	2.60	1.70	1.00	0.45	0.20	0.10	0.00	0.00	0.00	0.00
$h \times 10^2$	6.00	6.20	6.70	7.10	7.30	7.45	7.60	7.50	7.50	7.50
$G \times 10^6$	41.50	37.50	34.00	30.00	26.75	23.25	22.00	21.50	21.50	21.50
$n \times 10^2$	1.89	1.89	1.90	1.82	1.80	1.81	1.87	1.88	1.80	1.80

TABLE-2 (Continued)

AGGRADATION STUDIES--- BASIC AND COMPUTED PARAMETERS:RUN 0.90-U-9

TEMPERATURE = 29.0°C $K = 0.37 \times 10^{-2}$

DISTANCES	1	3	5	7	9	11	13	15	17	19
		T = 45 MINUTES				z_o = 0.019			L = 9.5	$G_* = 14.5 \times 10^{-6}$
$z \times 10^2$	1.85	1.10	0.55	0.30	0.10	0.00	0.00	0.00	0.00	0.00
$h \times 10^2$	6.90	6.90	7.20	7.70	8.65	9.20	9.20	9.20	9.20	9.20
$G \times 10^6$	34.25	30.25	26.50	23.00	20.25	19.00	18.80	18.80	18.80	18.80
$n \times 10^2$	2.02	1.77	1.32	2.08	1.88	1.88	1.88	1.88	1.88	1.88
		T = 75 MINUTES				z_o = 0.027			L = 13.5	$G_* = 12.0 \times 10^{-6}$
$z \times 10^2$	2.20	1.50	0.90	0.60	0.40	0.20	0.05	0.00	0.00	0.00
$h \times 10^2$	7.10	6.90	7.05	7.15	7.40	7.90	8.60	9.00	9.20	9.20
$G \times 10^6$	34.75	32.00	29.50	27.50	25.00	22.00	20.75	19.50	18.80	18.80
$n \times 10^2$	2.28	1.95	1.62	1.34	0.91	1.18	1.32	1.62	1.88	1.88
		T = 105 MINUTES				z_o = 0.030			L = 17.50	$G_* = 20.0 \times 10^{-6}$
$z \times 10^2$	2.60	1.90	1.30	0.95	0.75	0.60	0.40	0.20	0.05	0.00
$h \times 10^2$	7.30	7.00	7.05	7.25	7.50	7.80	8.35	8.80	9.15	9.20
$G \times 10^6$	35.00	33.00	30.75	28.75	26.75	24.75	23.00	21.25	19.50	19.00
$n \times 10^2$	2.47	2.10	1.69	1.42	1.22	1.07	1.01	1.12	1.57	1.80

TABLE-2 (Continued)

AGGRADATION STUDIES--BASIC AND COMPUTED PARAMETERS:RUN 0.63-U-10

TEMPERATURE = 26,5°C $k = 1.32 \times 10^{-2}$

DISTANCES	1	3	5	7	9	11	13	15	17	19
		T = 10 MINUTES				z_o = 0.023			L = 8.5	$G_* = 93.0 \times 10^{-6}$
$z \times 10^2$	1.80	1.00	0.50	0.20	0.00	0.00	0.00	0.00	0.00	0.00
$h \times 10^2$	4.20	4.60	5.40	5.85	6.20	6.20	6.20	6.20	6.20	6.20
$G \times 10^6$	171.00	155.00	138.00	122.50	112.00	111.00	111.00	111.00	111.00	111.00
$n \times 10^2$	1.29	1.41	1.64	1.76	1.92	186	1.86	1.86	1.86	1.86
		T = 20 MINUTES				z_o = 0.028			L = 15.0	$G_* = 106.0 \times 10^{-6}$
$z \times 10^2$	2.30	1.60	1.00	0.75	0.50	0.30	0.20	0.00	0.00	0.00
$h \times 10^2$	4.60	4.80	5.20	5.60	5.80	5.90	6.10	6.20	6.20	6.20
$G \times 10^6$	174.50	164.00	153.00	142.00	132.50	124.00	117.00	112.50	111.00	111.00
$n \times 10^2$	1.49	1.49	1.54	1.66	1.71	1.77	1.88	1.96	1.86	1.86

TABLE-2 (Continued)

AGGRADATION STUDIES-- BASIC AND COMPUTED PARAMETERS: RUN 0.30-U-11

TEMPERATURE = 29.5°C   K = 0.542 x 10^{-2}

DISTNACES	1	3	5	7	9	11	13	15	17	19
		T = 15 MINUTES				z_o = 0.015			L = 8.5	G_* = 92.5 x 10^{-6}
$z \times 10^2$	1.10	0.50	0.20	0.05	0.00	0.00	0.00	0.00	0.00	0.00
$h \times 10^2$	5.70	5.45	5.30	5.30	5.80	5.90	5.80	5.80	5.80	5.80
$G \times 10^6$	101.00	93.00	87.00	82.50	80.50	80.50	80.50	80.50	80.50	80.50
$n \times 10^2$	1.78	1.51	1.20	1.12	1.30	1.23	1.30	1.30	1.30	1.30
		T = 30 MINUTES				z_o = 0.020			L = 14.0	G_* = 70 x 10^{-6}
$z \times 10^2$	1.60	0.90	0.60	0.40	0.25	0.15	0.05	0.00	0.00	0.00
$h \times 10^2$	5.80	5.30	5.20	5.40	5.60	5.70	5.80	5.80	5.80	5.80
$G \times 10^6$	102.50	95.50	90.00	86.00	83.50	81.50	80.50	80.50	80.50	80.50
$n \times 10^2$	1.92	1.48	1.23	1.23	1.24	1.30	1.39	1.30	1.30	1.30

TABLE-3

AGGRADATION STUDIES—COMPUTED VALUES OF 'δ'

DISTANCES / TIME	0**	1	3	5	7	9	11	13	15	17	19
RUN NO. 4.0- U - 1											
15 MIN.	3.80	3.00	1.75	0.60	0.25	0.00	–	–	–	–	–
30 MIN.	5.40	4.40	3.30	2.40	1.50	0.60	0.40	0.0	–	–	–
40 MIN.	4.10	3.20	2.10	1.10	0.10	-0.60	-1.00	-1.90	–*	–*	–
RUN NO. 3.0- U- 2											
15 MIN.	2.00	1.30	0.50	0.00	–	–	–	–	–	–	–
30 MIN.	3.00	1.70	1.00	0.40	-0.30	-1.20	–	–	–	–	–
45 MIN.	4.50	3.70	2.80	1.60	0.60	0.00	-0.80	–	–	–	–
RUN NO. 1.8- U- 3											
30 MIN.	4.20	3.20	1.70	1.20	–*	–	–	–	–	–	–
60 MIN.	6.00	4.90	3.70	2.30	1.80	2.00	3.50	–	–	–	–
90 MIN.	7.40	6.50	5.00	3.80	2.60	1.70	0.40	-0.40	0.00	–	–

* Corresponding velocity was not available anywhere in the flow.

** Lag distance values at distance 0-m are extrapolated values.

TABLE 3 (Continued)

AGRADATION STUDIES--COMPUTED VALUES OF 'δ'

DISTANCES / TIME	0**	1	3	5	7	9	11	13	15	17	19
RUN NO. 3.5-U-4											
15 MINUTES	4.20	3.20	2.00	1.20	-*	-	-	-	-	-	-
30 MIN.	5.40	4.50	3.10	1.80	1.00	0.50	0.00	0.00	-	-	-
50 MIN.	7.50	6.60	5.10	4.00	2.75	2.00	0.85	0.00	-0.50	-0.50	-
RUN NO. 2.70-U-5											
10 MIN.	4.10	3.30	2.00	1.10	0.50	-*	-	-	-	-	-
20 MIN.	3.20	2.40	1.30	0.00	-1.00	-2.00	-2.00				
30 MIN.	3.70	3.00	1.60	0.50	-0.50	-1.50	-2.10	-2.60	-2.50	-	-
RUN NO.1.35-U-6											
15 MIN.	3.80	2.70	1.20	0.00	-	-	-	-	-	-	-
45 MIN.	7.20	6.00	4.60	3.00	1.60	0.40	0.00	-	-	-	-
75 MIN.	9.40	8.30	7.00	5.80	4.70	3.80	3.00	2.50	-*	-*	-

TABLE 3 (Continued)

AGGRADATION STUDIES— COMPUTED VALUES OF 'δ'

DISTANCES / TIME	0**	1	3	5	7	9	11	13	15	17	19
RUN NO. 0.9-U-7											
30 MIN.	6.50	5.50	3.90	2.50	2.00	-*	-	-	-	-	-
60 MIN.	7.50	6.50	5.00	4.70	4.20	4.00	4.00	-*	-*	-*	-
RUN NO. 1.0-U-8											
15 MIN.	6.40	4.20	-*	-*	-	-	-	-	-	-	-
30 MIN.	8.40	6.20	-*	-*	-*	-	-	-	-	-	-
60 MIN.	9.60	7.30	6.5	6.5	-*	-*	-*	-*	-	-	-
RUN NO. 0.90-U-9											
45 MIN.	6.00	5.00	3.80	2.30	0.80	-0.70	-	-	-	-	-
75 MIN.	7.60	6.80	5.50	4.00	2.50	1.00	-0.5	-2.00	-	-	-
105 MIN.	9.00	8.00	6.50	5.20	3.70	2.10	0.60	-1.00	-2.50	-4.00	-
RUN NO. 0.63-U-10											
10 MIN.	6.00	4.50	2.80	1.20	-0.30	0.00	-	-	-	-	-
20 MIN.	6.80	5.10	3.50	2.00	0.50	-0.50	-1.60	-3.20	-4.50	-	-