# DYNAMIC STABILITY ANALYSIS OF MULTIMACHINE SYSTEM WITH PARTICULAR REFERENCE TO BHAKRA NANGAL FERTILIZER FACTORY SYSTEM

A DISSERTATION
submitted in partial fulfilment of
the requirements for the award of the degree
of
MASTER OF ENGINEERING
in
WATER RESOURCES DEVELOPMENT



*By*
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# CERTIFICATE

Certified that the dissertation entitled "DYNAMIC STABILITY ANALYSIS OF MULTIMACHINE SYSTEM WITH PARTICULAR REFERENCE TO BHAKRA NANGAL FERTILIZER FACTORY SYSTEM" which is being submitted by Shri Ravinder Nath in partial fulfilment for the award of the degree of Master of Engineering (Water Resources Development) of University of Roorkee, is a record of students' own work carried out by him under my guidance and supervision. The matter embodied in this dissertation has not been submitted for the award of any other Degree or Diploma.

This is further to certify that he has worked for a period of six months from 1.9.75 to 31.3.76 for preparing this dissertation for Master of Engineering at this University.

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## SYNOPSIS

A review of the existing methods for analysing the dynamic and transient stability of power systems has been made. The mathematical models of the various components of the system required for analysis are outlined.

The dynamic stability of Bhakra-Nangal fertilizer factory system when it is run isolated from the system using state space method is analysed. Field observation indicated that the system is dynamically unstable and is also confirmed from the analysis presented here. To stabilize the system feed back signals in the AVR based on first and second derivatives of rotor angle are tried and their results presented. Higher order derivatives of rotor angle are suggested.

# ACKNOWLEDGEMENT

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CONTENTS



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# LIST OF PRINCIPAL SYMBOLS

All the quantities are expressed in per unit on a common base unless other units are given.

| | |
|---|---|
| $e_d$, $e_q$ | terminal voltages on direct and quadrature axes of machine |
| $e_t$, $v_t$ | magnitude of terminal voltage of machine |
| $i_d$, $i_q$ | direct and quadrature axis currents on machine |
| E | excitation voltage or open circuit voltage of machine |
| $\emptyset_{fd}$, $\emptyset_d$, $\emptyset_{kd}$ $\emptyset_q$, $\emptyset_{kq}$ | internal flux linkages of synchronous machine field, d-axis armature, d-axis amortisseur, q-axis armature and q-axis amortisseur winding |
| $i_{fd}$, $i_{kd}$, $i_{kq}$ | rotor circuit currents of synchronous machine |
| $x_{afd}$, $x_{ffd}$, $x_{akd}$, $x_{kkd}$, $x_{akq}$, $x_{fkd}$ | coupling and self reactances of synchronous machine |
| r | armature resistance of synchronous machine |
| $r_{fd}$, $r_{kd}$, $r_{kq}$ | rotor circuit resistances of synchronous machine |
| $\omega_o$ | rated angular frequency, electrical radians per second = $2\pi f$ |
| $\omega$ | instantaneous angular frequency of machine rotor |
| $n = \frac{\omega - \omega_o}{\omega_o}$ | per unit speed deviation of machine rotor |
| $\delta$ | rotor angle of machine in electrical radians |
| $i_D$, $i_Q$ | network terminal currents expressed with respect to network reference axes |

| | |
|---|---|
| $e_D, e_Q$ | network terminal voltages expressed with respect to network reference axes |
| $G_{ij}$ | real component of a network self-or mutual admittance |
| $b_{ij}$ | imaginary component of a network self or mutual admittance |
| $x_d$ | direct axis synchronous reactance |
| $x_q$ | quadrature axis synchronous reactance |
| $x'_d$ | direct axis transient reactance |
| $x''_d$ | direct axis subtransient reactance |
| $x''_q$ | quadrature axis subtransient reactance |
| $T'_{do}$ | direct axis open circuit transient time constant S |
| $T''_{do}$ | direct axis open circuit subtransient time constant S |
| $T''_{qo}$ | quadrature axis open circuit subtransient time constant S |
| $T_D$ | damper winding time constant |
| H | inertia constant KWs/KVA |
| M | inertia constant |
| $T_i$ | initial torque input to rotor |
| $T_g$ | air gap torque of synchronous machine |
| $T_a$ | armature time constant |
| $T_m = 2H$ | inertia time constant |
| $K_d, D$ | damping coefficient |
| p, s, d/dt | differential operators |
| $\Delta$ | incremental operator |

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$$[\delta] = [\delta_1, \delta_2, \ldots\ldots \delta_n]$$

$$[n] = [n_1, n_2, \ldots\ldots n_n]$$

$$[e_1] = [e_{t1}, e_{t2}, \ldots\ldots, n_{tn}]$$

$$[E] = [E_1, E_2, \ldots, E_n]$$

$$[T_g] = [T_{g1}, T_{g2}, \ldots\ldots, T_{gn}]$$

$$[i_N] = [i_{D1}, i_{Q1}, \ldots, i_{Dn}, i_{Qn}]$$

$$[e_N] = [e_{D1}, e_{D2}, \ldots\ldots, e_{Dn}, e_{Qn}]$$

$$[i_m] = [i_{d1}, i_{q1}, \ldots\ldots, i_{dn} i_{qn}]$$

$$[e_m] = [e_{d1}, e_{q1}, \ldots\ldots, e_{dn} e_{qn}]$$

$$[\phi_m] = [\phi_{d1}, \phi_{q1}, \ldots\ldots, \phi_{dn} \phi_{qn}]$$

$$[i] = [i_{fd1}, i_{d1}, i_{kd1}, i_{q1}, i_{kq1}, i_{fd2}, \ldots, i_{qn}, i_{kqn}]$$

$$[\phi] = [\phi_{fd1}, \phi_{d1}, \phi_{kd1}, \phi_{q1}, \phi_{kq1}, \phi_{fd2}, \ldots\ldots, \phi_{qn}, \phi_{kqn}]$$

$$[\Delta\phi_m, \Delta\delta, n] = [\Delta\phi_{d1}, \Delta\phi_{q1}, \Delta\phi_{dn}, \Delta\phi_{qn}, \Delta\delta_1, \ldots, \Delta\delta_n, \Delta\delta_{n1} \ldots, n_n]$$

# CHAPTER I

## REVIEW OF METHODS APPLIED TO POWER SYSTEM STABILITY

### 1. INTRODUCTION

With the growth of power systems to their present size with large capacity generators and long distance transmission lines, stability studies have become an essential part of power system planning. As transmission distances are extended, load centres tend to be widely separated and partially supplied by remote generation with large angular displacements between remote generators and those near load centres. The shift in load between generators is a nonlinear function of the difference in rotor angles, and above a certain angle difference, the incremental load shift due to incremental angle change reverses, and the forces which tended to reduce speed differences become forces tending to increase speed differences, leading to loss of synchronism phenomena. Power system stability is primarily concerned with variations in speed, rotor positions and generator loads. One of the aspects of stability study is to determine the stability regions and to improve these by suitable means.

### Concept of Stability

A system is defined to be stable when subjected to bounded disturbance it produces a bounded response. 'Bounded' means 'of less than some finite magnitude for all finite intervals of time'. Clearly if a system is subjected to an unbounded disturbance and produces an unbounded response, nothing can be said about its stability. But if it is subjected to a bounded disturbance and produces an unbounded response, it is by definition unstable.

In the absence of input excitation any time invariant physical system can be represented mathematically as a set of simultaneous differential equations of the form

$$\frac{d}{dt} X_{(t)} = f(X) \tag{1.1}$$

where X represents the system state variable vector. The function f(X) may be linear or nonlinear.

In physically realizable, linear, time invariant systems, there is only one equilibrium state, which may be made to be origin of the state space coordinate system. If this equilibrium state is stable, the entire state space represents stable region of system response. It may or may not be true in the case of nonlinear systems, i.e., a nonlinear system may have both stable and unstable equilibrium states and both stable and unstable regions of response.

If the system of equations (1.1) is perturbed slightly from its equilibrium state $X_e$ and all subsequent motions remain in a correspondingly small neighbourhood of the equilibrium state, then the system is said to be stable. Mathematically the definition can be stated as:

The response of the system of equations (1.1) is stable if, for any given arbitrary small real positive number $\epsilon$, there can be found another positive number $\delta(\epsilon)$ such that if

$$0 \leqslant X(0) \leqslant \delta$$

then the system solutions are defined for all time by the relation

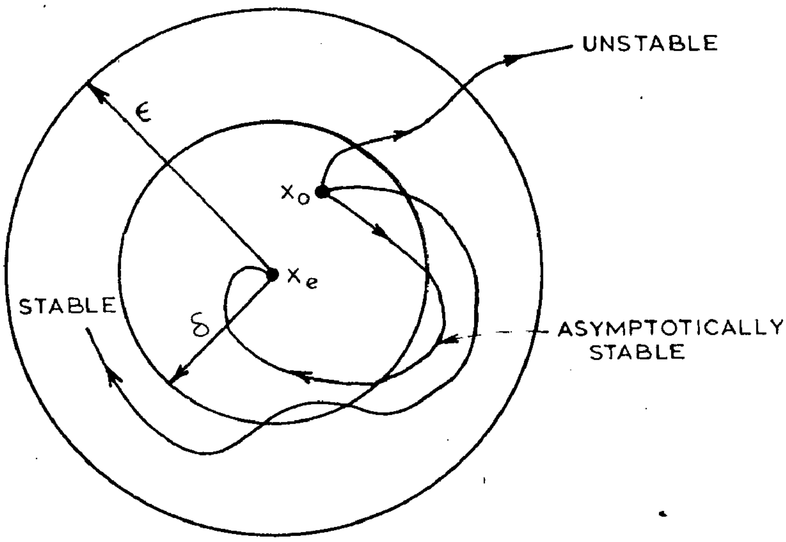
$$0 \leqslant X(t) \leqslant \epsilon$$

Within the n-dimensional state space, the region $S(\epsilon)$ is said to be stable if for any $S(\delta)$, a transient starting in $S(\delta)$_ does not leave $S(\epsilon)$ as shown in Figure 1.1. If the system response is stable as defined above and, in addition if every motion originating sufficiently near $X_e$ converges to $X_e$ as time approaches infinity, then the system is said to be asymptotically stable. These concepts are illustrated in Figure 1.1.

## 2. STEADY STATE DYNAMIC AND TRANSIENT STABILITY

In power system stability studies, the terms steady state, dynamic and transient stability are widely used to distinguish between three kinds of studies.

Steady State Stability [1] is the stability of a system under conditions of gradual or relatively slow changes in load. The load is assumed to be applied at a rate which is slow when compared either with the natural frequency of oscillation of major parts of the




$X_o$ - REPRESENTS THE INITIAL STATE

$X_e$ - REPRESENTS THE EQUILIBRIUM STATE

FIG. 1.1 - DEFINITION OF STABILITY

system or with the rate of change of field flux in the rotating machine in response to change in loading. In steady state stability it is assumed that the control of excitation is very slow and such as to correct the voltage change after each small load change has occurred . As the effect of this is negligible because of assumed slow response, it is a stability limit for an infinitesimal change with constant field current.

Dynamic Stability . If the change in excitation is assumed to take place with or immediately following the change in load, the stability limit under such conditions is termed as dynamic stability. The effect of voltage regulators and governors is important in dynamic stability studies Dynamic stability analysis is concentrated on the problems associated with undamped or poorly damped oscillations of small amplitude. If the oscillations resulting from any initial change diminish with time, the system is said to be dynamically stable. If, on the other hand, the oscillations increase with time, the system is dynamically unstable.

Oscillations may occur between one machine or plant and the rest of the system or between large machine groups. Spontaneous oscillations are initiated by minor disturbances such as variations in load inherent in normal operation. Due to the effect of nonlinearities such oscillations may be limited at some magnitude short of loss of synchronism and system break up.

The only effective way to deal with spontaneous oscillations is to alter the inherent system characteristics which cause them.

Dynamic stability studies cover real time intervals, usually 5 to 10 seconds and occasionally upto 30 seconds. Spontaneous oscillations do not initially involve nonlinear behaviour in any way, and therefore linearized mathematical models are used to study the problem.

Transient Stability analysis is primarily concerned with large and sudden disturbances such as they occur with the effects of transmission line faults on generator synchronoism or sudden changes in load. During a fault, electrical power from nearby generators is reduced drastically, while power from machines somewhat removed from the fault may be scarcely changed. The resultant differences in acceleration produce speed differences over the time interval of the fault, and it is important to clear the fault quickly to limit these speed differences and the associated changes in angle differences. Clearing the fault removes one or more transmission lines from service and at least temporarily weakens the system. This change in transmission system also implies that the generator angle differences which existed prior to the fault no longer represent equilibrium conditions . If speed differences and accumulated angle differences at the time of fault clearing are sufficiently bounded and if transfer impedences between generators subsequent to the fault clearing are sufficiently low, the accelerated machines will pick up load due to their advanced angular positions, slow down and eventually a new synchronous equilibrium will be established. Loss of synchronism if it occurs is usually evident within one second for the first swing stability, and within ten seconds for multi swing stability. Therefore, transient stability studies are limited to short time intervals. They are most often used to determine the stability of a single unit or plant during the initial period of high stress immediately following a nearby fault. Such studies may require the representation of a large system. The effects of voltage regulators and governors are usually limited and often neglected for generators remote from the fault.

## 3. REVIEW

Power system stability has been the subject of intensive study in the US and Canada since 1920 when the first large hydroelectric installations were being developed Long lines and relatively slow circuit breakers and relays made the system stability a serious problem. The nonlinear behaviour of the alternator was known for a long time. It led to the development of two axis theory by Park in 1929. Since then many investigators have shown that voltage regulators can improve the alternator stability in many ways. Crary [2] and Kimbark [3] in their discussions of the factors affecting stability have emphasised the importance of automatic voltage regulators in improving stability limit.

Methods suited to stability analysis are:

1. Dynamic stability - (a) Frequency response methods

   (b) State space methods

2. Transient stability (a) Point by point solution of differential equations

   (b) Liapunov's second method.

Dynamic Stability

The classical approach to the study of dynamic stability problems has been through frequency response methods. It was concordia [4] who discussed the subject at length. Starting from the basic machine equations, he applied the Routh's criterion to the equation of motion for ascertaining the stability of a system under the effect of voltage regulators and secondly, to obtain the steady state limit for a synchronous machine connected through a tie line to an infinite bus and also to the case of two machines with a voltage regulator responsive to the common bus voltage magnitude, by varying the regulator amplification factor, machine field time constant, regulator and exciter time constant. The results obtained by Concordia showed that with properly designed voltage regulator, the steady state limit can be increased to as much as 1.6 times its value without regulator and that the system remains stable for a value of the load anglo δ as high as $115^{\circ}$.

Messerle and Bruck [5] in 1956 studied the effect of voltage and angle regulation on the steady sta te stability limit of a single machine connected to an infinite bus and extended the work to allow for the control of the prime mover torque by means of governors. For analysis, the authors represented synch-r nous machine with Park's equations to ddfine transfer function for the machine. The transfer function so obtained was used together with the Nyquist criterion to give the results in the form of stability contour diagrams. The advantage of this approach is that the results are obtained in a general form and only the axis need to be shifted for finding the effect of changing controller gains.They concluded that the gain margin increases considerably by using a stabilizer, while increase in controller gain reduces tho gain margin.

The authors also claimed that instability of the alternator with feedback usually shows up at the dynamic limit in the form of self excited oscillations in feedback systems as opposed to the steady state case where the intability occurs with slow falling out of synchronism with continuously increasing load angle.

Alderød and Shackshaft [6] applied the Nyquist criterion for the predetermination of synchronous machine stability with and without voltage regulator and connected to an infinite bus treating it as a closed loop system. The effect of main regulator loop parameters such as gain, exciter and main field time constant etc. on the stability of the system were examined and curves obtained to that effect. They concluded that while the steady state limit increased considerably by the use of voltage regulator, the transient stability limit remained practically unaffected. They also considered the saturation type of non linearity and found that its effect was to make the system more stable at higher gains resulting in reduced self excited oscillation.

Jacovides and Adkins [17] made a detailed study of the effects of voltage regulator on the stability limit. The stability was analysed by the Nyquist method. They considered the different types of voltage regulators and concluded that with increased voltage regulator gains the system was stable having operation in the dynamic zone, but after a certain value, further increase in the gain made the system unstable at an angle $\delta$ less than $90^{o}$. They also noted that the effect of resistance and damper winding was to make the system more stable.

Stapleson [8] applied the root locus technique to study the stability and dynamic response of a synchronous machine by plotting a family of loci which indicated on one diagram the damping factor and oscillation frequency of each term in the time response for all values of two parameters such as regulator gain and exciter time constant. Though the system equations were based on the small perturbation theory and valid only near the chosen operating point, the slow variations of poles and zeros with the operating conditions led them to conclude that the results deduced for a specific operating point would be valid over a considerable range.

Ewart and DeMello [9] implemented a digital computer program for plotting the Nyquist diagram and compute the dynamic stability limit for a single machine connected to an infinite bus through a transmission line. They evaluated the effects of generator, excitation system and transmission line parameters on the dynamic stability limit. They found that the increase in machine inertia decreases the dynamic stability limit in contrast to the opposite effect on transient stability in the case studied. They also found that in the overexcited region poorly damped oscillations may be encountered at load levels considerably lower than the absolute stability limit.

Recently, the approach to the study of the stability problems has been towards the utilization of the state space techniques for describing the system behaviour. The state of a system changes with respect to some independent variable which is usually time. State variables are those set of variables (a minimum set) which describe the present state of the system, and which also allow one to use the past history and the present state to determine the future state. The variables in such a set are called state variables. The state variables in a particular system do not form a unique set , but rather that several arbitrarily chosen sets can be found. The state variables are usually chosen based on the following considerations:

(i) the ability to measure all the states.

(ii) the ability to specify a more meaningful performance index.

If a set of state variables is properly chosen, it contains sufficient information to describe the transient behaviour of the linear system being studied.

Laughton [10] investigated the dynamic stability using state space approach. He starts with the description of the performance of a single machine without excitation and prime mover control by general nonlinear equations and linearizes them using small perturbation technique. Relations between system variables are expressed by operating matrix, and using the matrix reduction method, all variables which are not of interest are eliminated to yield relations of the form

$$\dot{X} = AX + Du$$

$$Y = BX$$

where A,B and D are matrices of constant coefficients, X is the state vector, Y is the output vector. The paper extends the representation to include the voltage regulator and applies eigenvalue analysis to determine the stability limits. Finally, he has also considered the case of dynamic stability in multimachine system.

Undrill [11] has also studied the dynamic stability of a multimachine system including the effects of voltage regulators and governors employing the eigen values analysis. The state variables chosen are the flux linkages of the direct and quadrature axis armature and rotor circuits.

Transient Stability

Transient stability analysis is undertaken to determine the response of a power system to large and sudden disturbances. Obtaining transient response of a power system essentially involves solution of non linear and linear equations.

The system of equations describing the power system may be divided into three categories:

(1) Differential equations of the form

$D(X,V,I) - \dot{X} = 0$

where

X is the vector of state variables

V is the vector of bus voltages

I is the vector of bus currents.

These are the equations which describe the time dependence of the prime movers/.

(2) Non-linear algebraic equations of the form

$N(X,V,I) = 0$

At each generator there is a pair of equations which connect the flux linkages vector with the terminal voltage.

(3) Linear algebraic equations of the form

$L(I,V) = 0$

Much of the effort involved in transient stability analysis is in developing programs for the solution of above equations.

Miles [12] studied the transient stability of multimachine system allowing for the representation of saliency, variable flux and damping and the effects of voltage regulators and governors on the analogue as well as digital computer. He concluded that the transient stability studies with normal network analyser assumptions give substantially correct results for system predominantly composed of round rotor machines equipped with excitation systems of moderate response.

Brown, Happ, Person and Young [13] developed matrix computational methods using impedance matrix for solving power system transient stability problems with the inclusion of transient saliency, variable impedance type of loads, voltage regulators and governors. They also reported good convergence characteristics for the impedence matrix method and a significant increase in speed of solution as compared to the nodal iterative method.

Olive [14] developed a program which can be applied for the calculation of transient stability of multi-machine systems (upto 100 machines). He represented the synchronous machine on the d-and q-axis including saturation. Governors and voltage regulators can also be represented to a close approximateion. Non-linear loads can also be represented. Iterative procedure has been outlined for the solution of the network, synchronous machine equations and non-linear loads. According to the author, the iterative procedure is remarkably effective and has not failed even once when the stability was lost.

Dinsley and Kennedy [15] investigated the effect of using an operating signal derived from rotor acceleration for the control of input power as opposed to the conventional velocity feedback on the transient stability of the power system. A governor actuated from the compound of velocity and acceleration signal is described and its effect on the transeint stability is studied. The effect on stability of varying some of the parameters of the system, the machine and governor are described. The paper concludes with a brief study of the effects of various governors on the transient stability of a synchronous generator connected to a large system by a single faulted transmission line that is fitted with auto-reclosing circuit breakers.

Talukdar [16] investigated the multistep integration algorithms suitable for transient stability studies by combining conventional, implicit multistep formulae with new iterative procedures. He concluded that multistep algorithms require significantly less computing time.

Fuller, Hirsch and Lambie [17] have developed a new transient stability algorithm which employs automatic variable step size, automatic variable order of integration and an implicit integration algorithm. The differential equations representing the generation, and algebraic equations representing the network are solved simultaneously thus minimising the interface error due to the time skew in sequential solution of the differential equat ons and the algebraic equations in each fixed small time step. The scheme outlined in the paper is significantly faster and advantageous for long time spans.

State space methods have been applied in recent years to study the transient stability problems. The second method of Liapunov is based on the concept of energy and the relation of stored energy and system stability. Fundamental to the Liapunov's method is the idea that for a stable system the stored energy will decay with time. Since the system is characterized by the state variables which represent the energy state of the system, the stability can be determined by examining a function of the state variables without an explicit solution of the system differential equations. The stored energy in a homogeneous (undriven) stable linear system can be shown to be a non-increasing function of time. If it can be proved that a positive definite energy-like function of system state, generally referred to as Liapunov function or a V function, is a monotonically decreasing time function, that system is assuredly stable, whether linear or nonlinear. Mathematically, a system is asymptotically stable in some region of the state space if, in that region

$$V(X) > 0 \qquad \text{for } X \neq X_e$$

$$\frac{dV}{dt} = \dot{V}(X) < 0 \qquad \text{for } X \neq X_e$$

$$V(X) = 0 \qquad \text{for } X \neq X_e$$

$$V(x) \to \infty \qquad \text{for } \|X\| \to \infty$$

where $X_e$ is the equilibrium state and $\|X\| = (X^T X)^{1/2}$ is the Euclidean length of the vector from the origin.

The Liapunov function may be thought of as a measure of the distance from the equilibrium state. The chief merit of the Liapunov's second method lies in the fact that the system equations need not be solved for determining the regions of stability. However, finding the desired V function is not in general a simple task and failure to find V does not indicate instability, but if atleast one V function can be found, the system is proved stable. Much of the effort has, therefore been directed towards finding V functions. Gless [18], El Abiad and Nagappan [19] and many others [20-25] have successfully applied Liapunov's second method to analyse power system stability problems.

4. STATEMENT OF THE PROBLEM

The problem studied is stated briefly as below:

Three units of Bhakra power house, 100 MVA each are supplying power to Nangal fertilizer factory whose load is 180 MVA . The electrolysis plant constitutes the major load. In addition, there are a number of synchronous motors whose aggregate capacity is 19 MW. The motors are not equipped with voltage regulators. It is known that above system is dynamically unstable when it is operated isolated from the rest of the system. Therefore methods to stabilize such a system are required to be suggested.

# CHAPTER II

# MATHEMATICAL MODELLING OF POWER SYSTEM COMPONENTS

## 1. INTRODUCTION

For analysing a complex physical system on a digital computer, a mathematical model which charactorizes the system behaviour is essential. A set of equations which accurately relate the input and output quantities constitutes a mathematical model. Topologically mathematical model may be represented by a flow graph or a block diagram.

A continuous physical system such as a power system is described in terms of physical laws governing its behaviour by decomposing it intoa schematic representation of individual elements, say as in Fig. (2.1) The term continuous is used to indicate that the system variables (angular velocity,voltage etc) are functions of a continuous independent variable,time. The application of Kirchhoff's laws to lumped parameter circuits and Newtons second law of motion to the dynamical part of the system give a set of equations which constitute a mathematical model of the power system.

A mathematical model may be formulated in different formats. The first format is the classical

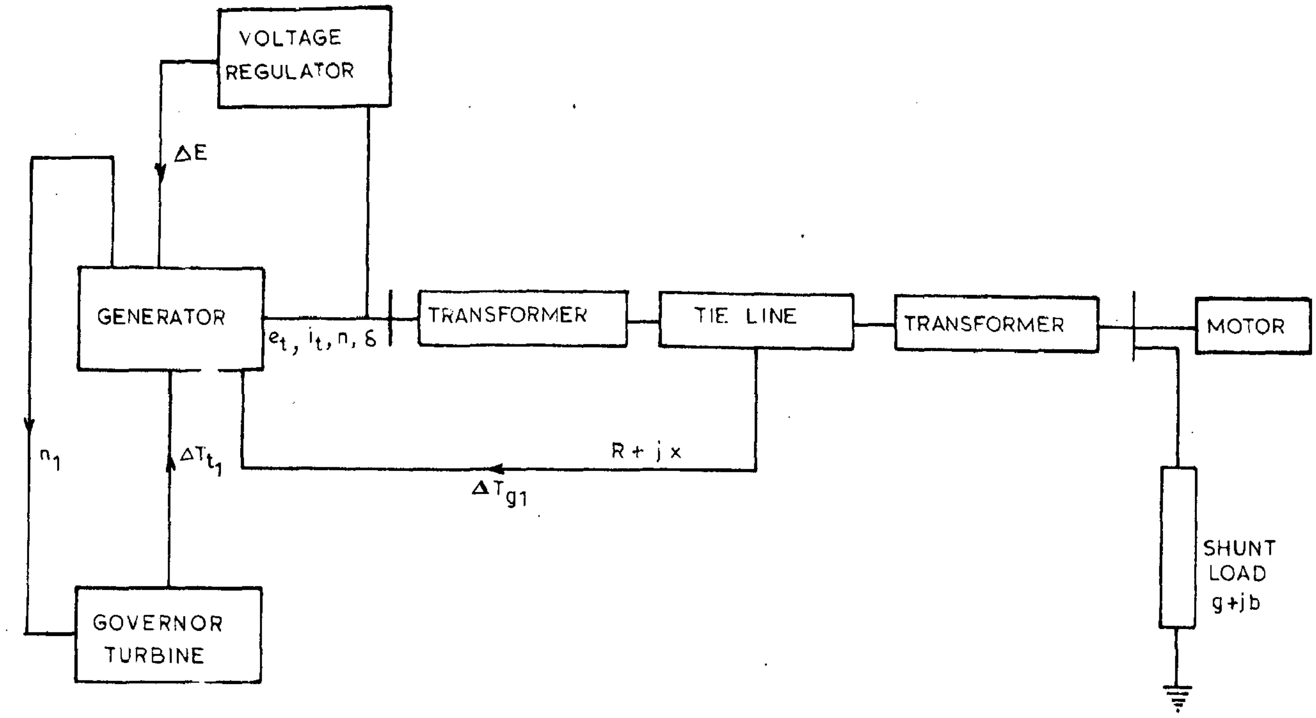



FIG. 2.1

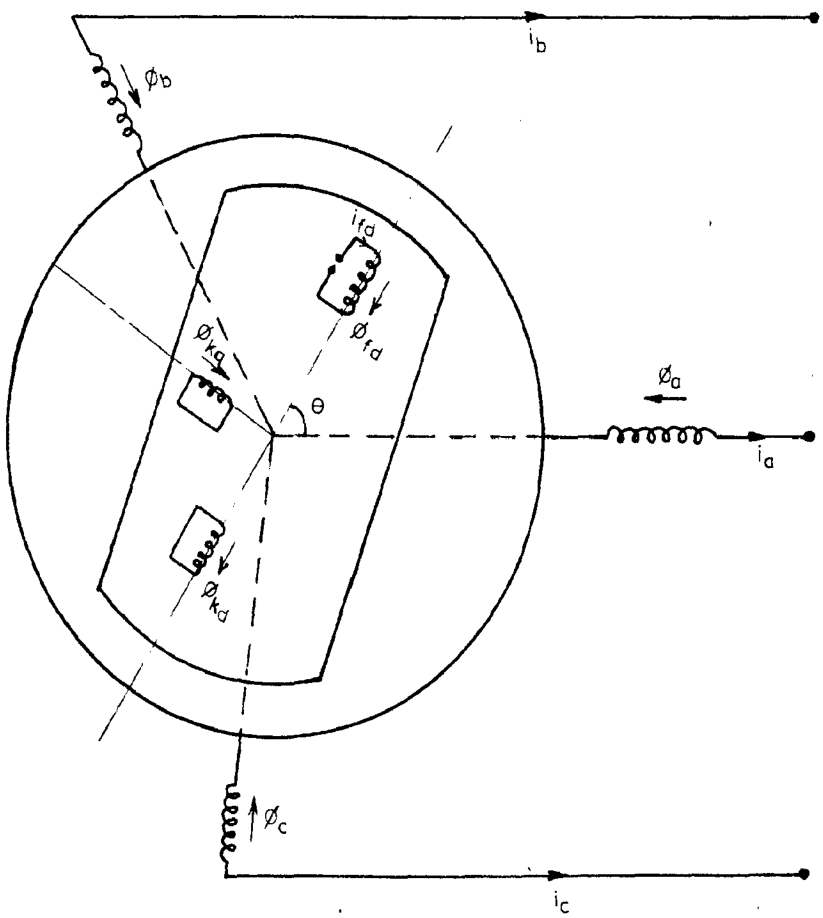



FIG. 2.2 _ SCHEMATIC LAYOUT OF WINDINGS OF A SYNCHRONOUS MACHINE

formulation in which a mathematical model consists of a set of differential equations. The second format is topologically oriented in that it represents a system by a block diagram. This is the classical frequency domain representation. The vector - matrix representation is the third format which has been widely employed in modern system analysis.

The purpose of this chapter is to review and examine the various mathematical descriptions of the salient pole synchronous machines and other components of the control and power system such as voltage regulators hydraulic turbine and speed governing system, transmission lines and loads.

## 2. SYNCHRONOUS MACHINES

The synchronous machine constitutes an important part of the power system as the transient and dynamic behaviour of the system is largely determined by the machine characteristics and its controls. For multimachine dynamic analysis on a digital computer, it is desirable that the mathematical model be such that

(i) known mathematical techniques may be applied for the analysis and optimization of the system.

(ii) the model either uses the data supplied by the machine manufacturers or the model may be such that its significant parameters may be easily determined by tests or calculated.

All the mathematical models in use in varying degrees of detail are derived from the Park's equations of the machine having a field winding and one damper circuit in each axis as shown in Fig. 2.2 by applying the d-q-O transformation to the equations of the machine in phase variables. The equations of this transformation are [26]

$$K_d = \frac{2}{3} [K_a \cos\theta + K_b \cos(\theta - 120) + K_c \cos(\theta - 240)] \quad (2.1)$$

$$K_q = -\frac{2}{3} [K_a \sin\theta + K_b \sin(\theta - 120) + K_c \sin(\theta + 120)] \quad (2.2)$$

$$K_o = \frac{1}{3} [K_a + K_b + K_c] \quad (2.3)$$

The symbol K is replaced by i, $\phi$ or e to give the current, flux linkage or voltage transform respectively. This transformation (equation 1 and 2) resolves the stator quantities into components along the direct and quadrature axes respectively. Equation 3 relates to zero sequence effects in the generator.

When Park's transformation is applied to the equations of the machine in phase variables, the following equations are obtained [26]

Direct axis flux linkages

$$\phi_{fd} = x_{ffd} i_{fd} + x_{fkd} i_{kd} - x_{afd} i_d \quad (2.4)$$

$$\phi_d = x_{afd} i_{fd} + x_{akd} i_{kd} - x_d i_d \quad (2.5)$$

$$\phi_{kd} = x_{fkd} i_{fd} + x_{kkd} i_{kd} - x_{akd} i_d \qquad (2.6)$$

Quadrature axis flux linkages

$$\phi_q = x_{akq} i_{kq} - x_q i_q \qquad (2.7)$$

$$\phi_{kq} = x_{kkq} i_{kq} - x_{akq} i_q \qquad (2.8)$$

Direct axis voltages

$$e_{fd} = \frac{1}{\omega_o} p \, \phi_{fd} + r_{fd} i_{fd} \qquad (2.9)$$

$$e_d = \frac{1}{\omega_o} p\phi_d - r \, i_d - \frac{\omega}{\omega_o} \phi_q \qquad (2.10)$$

$$0 = \frac{1}{\omega_o} p \, \phi_{kd} + r_{kd} \, i_{kd} \qquad (2.11)$$

Quadrature axis voltages

$$e_q = \frac{1}{\omega_o} p \, \phi_q - r \, i_q + \frac{\omega}{\omega_o} \phi_d \qquad (2.12)$$

$$0 = \frac{1}{\omega_o} p \, \phi_{kq} + r_{kq} \, i_{kq} \qquad (2.13)$$

The equations are in per unit form. In per unit system each voltage, flux, current and impedance is expressed as the ratio of its actual value to a selected base value. One per unit field voltage gives one per unit open circuit voltage on the air gap line and one per unit torque is based on rated power at synchronous speed.

The following assumptions have been made in deriving the above equations.

1. The distributed windings of the stator are considered as concentrated winding spatially distributed $\frac{2\pi}{3}$ radians apart.

2. The mmf produced by each stator and rotor winding is sinusoidally distributed in the air gap.
3. Hysteresis and eddy current losses are neglected.
4. The sign convention is such that the generator action is considered positive and positive field flux is that which induces a positive voltage in the stator windings.

The follow ng equations are necessary to complete the description of the synchronous machine.

Generator terminal voltage

$$e_t^2 = e_d^2 + e_q^2 \tag{2.14}$$

Mechanical equations

$$T_g = \phi_d i_q - \phi_q i_d \tag{2.15}$$

$$T_i = Mp^2\delta + T_g + K_d p\delta + \Delta T \tag{2.16}$$

3. SYNCHRONOUS MACHINE MODELS

The models of the synchronous machine and the simplifications made in formulating them from the basic five winding Park's equations are indicated. The resulting equations are stated in the form which are easily solvable by numerical calculation. The models are presented starting with the least complex and finishing with the detailed one

Model 1 [28]

The classical synchronous machine model uses a constant voltage magnitude behind transient reactance.

It is the simplest of the machine model. The following simplifications have been made.

(a) Transformer voltages in the stator equations are neglected.

(b) Speed is assumed constant except in the equations of motion.

(c) Damper windings are neglected.

(d) Saturation is neglected.

(e) Main flux linkages are assumed to be constant.

(f) Transient saliency is neglected by approximating $x_q$ to $x'_d$.

The machine equations are

$$\begin{bmatrix} e_d \\ e_q \end{bmatrix} = \begin{bmatrix} -r & x'_d \\ -x'_d & r \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ e'_q \end{bmatrix}$$

$$T_g = e_q i_q + e_d i_d$$

$$Mp^2\delta = T_i - T_g$$

Model 2 [28]

Transient saliency is taken into account. The field flux linkages remain constant and all other simplifying assumptions as in case of model 1 still hold. The machine equations become

$$\begin{bmatrix} e_d \\ e_q \end{bmatrix} = \begin{bmatrix} -r & x_q \\ -x'_d & -r \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ e'_q \end{bmatrix}$$

$$T_g = e_q i_q + e_d i_d$$

$$Mp^2\delta = T_i - T_g$$

$e_q'$ is calculated from initial conditions and is assumed constant.

Model3 [28]

The field flux linkages are assumed variables, but all other assumptions still hold.

$$\frac{de_q'}{dt} = \frac{1}{T_{do}'}[\, e_{fd} - e_q' - i_d(x_d - x_d')]$$

$$\begin{bmatrix} e_d \\ e_q \end{bmatrix} = \begin{bmatrix} -r & x_q \\ -x_d' & -r \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ e_q' \end{bmatrix}$$

$$T_g = [\, e_q i_q + e_d i_d \,]$$

$$Mp^2\delta = T_i - T_g$$

Model 4 [29]

The synchronous machine equations are represented by Park's voltage equations referred to the direct and quadrature axes on the machine rotor position.

$$\begin{bmatrix} pe_d' \\ pe_q' \end{bmatrix} = \begin{bmatrix} K_1 & \\ & K_2 \end{bmatrix} \begin{bmatrix} e_d' \\ e_q' \end{bmatrix} + \begin{bmatrix} K_3 & \\ & K_4 \end{bmatrix} \begin{bmatrix} e_d \\ e_q \end{bmatrix} + \begin{bmatrix} 0 \\ K_5 \end{bmatrix} e_{fd}$$

$$\begin{bmatrix} e_d \\ e_q \end{bmatrix} = \begin{bmatrix} e_d' \\ e_q' \end{bmatrix} - \begin{bmatrix} r & -x_q' \\ x_d' & r \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

where

$$K_1 = -\frac{x_q}{T'_{qo}\ x'_q}$$

$$K_2 = -\frac{x_d}{T'_{do}\ x'_d}$$

$$K_3 = \frac{x_q - x'_q}{T'_{qo}\ x'_q}$$

$$K_4 = \frac{x_d - x'_d}{T'_{do}\ x'_d}$$

$$K_5 = \frac{1}{T'_{do}}$$

$$T_g = e_q\, i_q + e_d\, i_d$$

$$Mp^2\delta = T_i - T_g$$

Model 5 [28]

It is the simplest of the representations in which damper windings are included. The main flux linkages are assumed variable but the transformer voltages in the stator equations are neglected. Saturation is also neglected. The equations are;

$$\begin{bmatrix} pe'_q \\ pe''_q \\ pe''_d \end{bmatrix} = \begin{bmatrix} -\frac{1}{T'_{do}} & 0 & 0 \\ \left(-\frac{1}{T'_{do}} + \frac{1}{T''_{do}}\right) & -\frac{1}{T''_{do}} & 0 \\ 0 & 0 & \frac{1}{T''_{qo}} \end{bmatrix} \begin{bmatrix} e''_q \\ e''_q \\ e''_d \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & -\left(\frac{(x_d - x'_d)}{T'_{do}}\right) \\ 0 & -\left(\frac{x_d - x'_d}{T'_{do}} + \frac{x'_d - x''_d}{T''_{do}}\right) \\ \frac{x_q - x''_q}{T''_{qo}} & 0 \end{bmatrix} \begin{bmatrix} i_q \\ i_d \end{bmatrix} + \begin{bmatrix} \frac{G'}{T'_{do}} \\ \frac{G'}{T'_{do}} - \frac{G''}{T''_{do}} \\ 0 \end{bmatrix} e_{fd}$$

where

$$G' = \frac{T'_{do} - T_D}{T'_{do} - T''_{do}} \approx 1$$

$$G'' = \frac{T''_{do} - T_D}{T'_{do} - T''_{do}} \approx 0$$

$$\begin{bmatrix} e_q \\ e_d \end{bmatrix} = - \begin{bmatrix} +r & x''_d \\ -x''_q & r \end{bmatrix} \begin{bmatrix} i_q \\ i_d \end{bmatrix} + \begin{bmatrix} e''_q \\ e''_d \end{bmatrix}$$

$$T_e = e''_q\, i_q + e''_d\, i_d + i_d i_q\, (x''_q - x''_d)$$

$$Mp^2\delta = T_1 - T_g$$

4. INCREMENTAL MODELS

The most commonly used models were derived by Concordia [4], Heffron and Philips [30], Laughton [10] and Undrill [11].

5. EXCITATION SYSTEMS [31]

After considering various types of excitation systems in use, IEEE Committee has defined four excitation system types to be used in computer representation.

Type1 continuously acting regulator and excitor

Type 2 rotating rectifier system

Type 3 static with terminal potential and current supplies

Type 4 non continuously acting.

In this chapter only type 1 is discussed. Fig. 2.3 shows the significant transfer function which are included for representation in computer studies. The first transfer function is a simple time constant $T_R$ representing the regulator input filtering. $T_R$ is generally very small and is considered as zero. The first summing point compares the regulator reference with the output of the input filter to determine the voltage error input to the regulator amplifier. The second summing point combines voltage error input with the excitation major

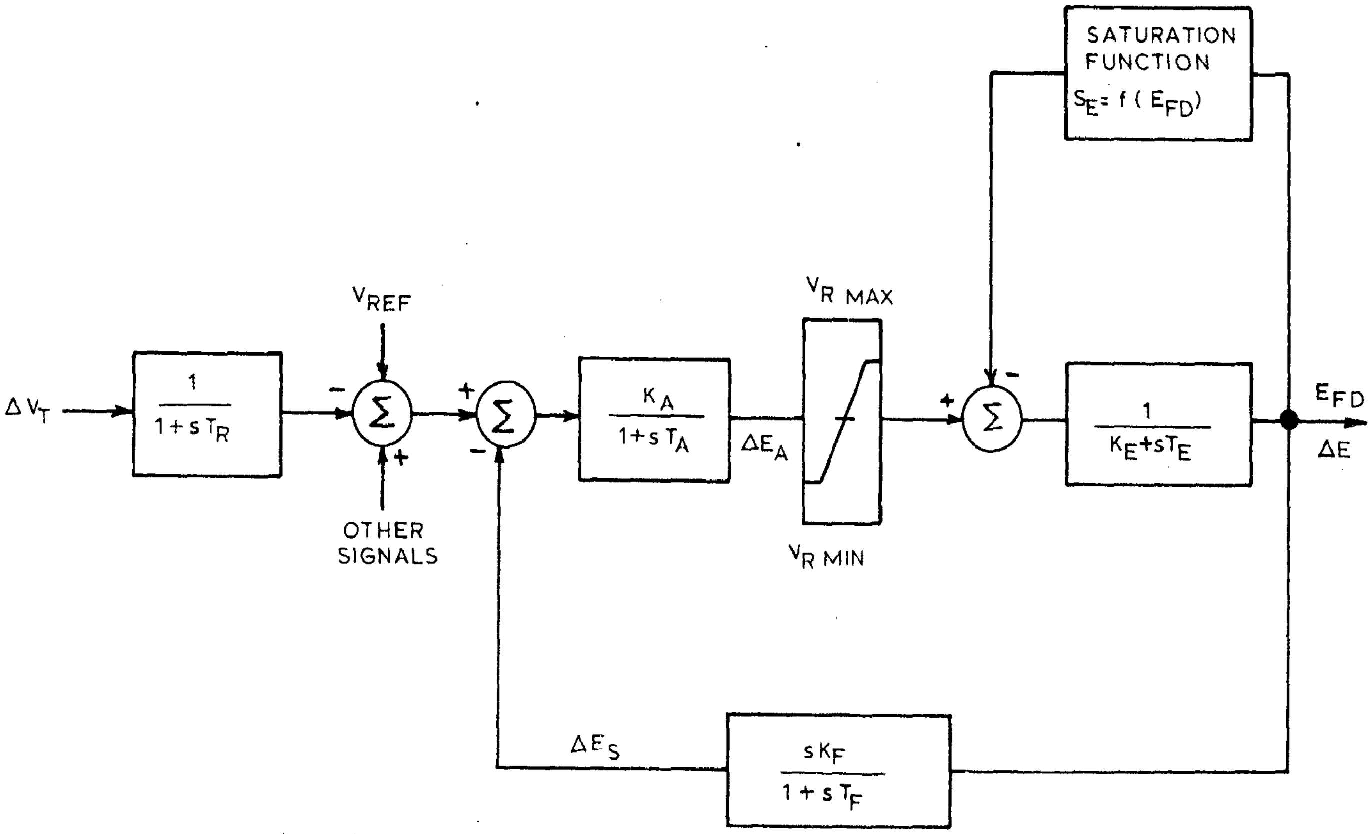



FIG. 2.3 _ TYPE I EXCITATION SYSTEM REPRASENATION, CONTINUOUSLY ACTING REGULATOR AND EXCITER

damping loop signal. The main regulator transfer function is represented as a gain $K_A$ and a time constant $T_A$. Following this , the maximum and minimum limits of the regulator are imposed so that large input error signals cannot produce a regulator output which exceeds practical limits. The next summing point subtracts a signal which represents the saturation function $S_E$ of the exciter The resultant is applied to the exciter transfer function

$$\frac{1}{K_E + ST_E}$$

The excitation system equations may be written in the following general form

$$[\dot{X}] = [A][X] + [B][U]$$

where

[A] is the system matrix

[X] is the state vector

[B] is the control matrix

[U] is the control vector

For type1 excitation system shown in Fig. 2.3 the equations are

$$\begin{bmatrix} p\Delta E \\ p\Delta E_s \\ p\Delta E_A \end{bmatrix} = \begin{bmatrix} -\frac{(K_E+S_E)}{T_E} & 0 & \frac{1}{T_E} \\ -\frac{K_F(K_E+S_E)}{T_E T_F} & -\frac{1}{T_F} & \frac{K_F}{T_E T_F} \\ 0 & -\frac{K_A}{T_A} & -\frac{1}{T_A} \end{bmatrix} \begin{bmatrix} \Delta E \\ \Delta E_s \\ \Delta E_A \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{K_A}{T_A} \end{bmatrix} \Delta e_t$$

where $\Delta E$ , $\Delta E_s$ and $\Delta E_A$ are defined in the figure.

For dynamic stability studies (small perturbation analysis) the saturation function can be neglected. However, in transient stability studies, because of large changes in $\Delta E$, saturation function $S_E$ which represents the saturation in the main exciter cannot be regarded as a constant and the above equation become nonlinear. The limits $V_{R_{max}}$ and $V_{R_{min}}$ have an important effect on the transient behaviour of the machine, and computer representation must be incorporated for these limits.

Typical constants for type 1 excitation system are as follows:

| | | |
|---|---|---|
| $T_R$ | Regulator input filter time constant | 0.0 to 0.06 |
| $K_A$ | Regulator gain | 25 to 50 |
| $T_A$ | Regulator amplifier time constant | 0.06 to 0.20 |
| $V_{R_{max}}$ | Maximum value of $V_R$ | 1.0 |
| $V_{R_{min}}$ | Minimum value of $V_R$ | -1.0 |
| $K_F$ | Regulator stabilizing circuit gain | 0.01 to 0.08 |
| $T_F$ | Regulator stabilizing circuit time constant | 0.35 to 1.0 |
| $K_E$ | Exciter constant related to self-excited field | 0.05 |
| $T_E$ | Exciter time constant | 0.5 |
| $S_{E_{max}}$ | Saturation function (max.) | 0.267 |

$S_{E\ 0.75\ max.}$ Saturation function (0.75 max) 0.074

6. HYDRAULIC TURBINES AND SPEED GOVERNING SYSTEMS

Turbine speed-governing systems play an important role in the behaviour of isolated systems which have eithe r excess load or generation in significant amounts. They also influence damping in dynamic stability studies and have some effect on transient stability behaviour.

The representation of hydraulic turbine and speed governing system for power system stability studies has been investigated by Undrill and Woodward [32], Ramey and Skooglund [33] and Young [34]. IEEE Committee report [35] has defined basic models for speed governing systems and turbines for power system stability studies. The model suggested by the Committee for electrohydraulic governor and mechanical governor is the same as the dynamic performance of the electrohydraulic governor is necessarily adjusted to be essentially the same as that for the mechanical governor in an interconnected system.

The transient characteristics of hydraulic turbine are determined by the dynamics of water flow in the penstock. The conversion of flow and head by the turbine involves only non-dynamic relationships. The precise models of water pressure and flow in penstock which take into consideration the travelling wave phenomena are not usually used for power system stability studies.

The representation of hydraulic turbine and speed governing system for computer simulation is shown in Figure 2.4. Neglecting the turbine and penstock representation, the governing system equations may be written in the following general form

$$[\dot{X}] = [A]\,[X] + [B]\,[U]$$

For the governing system shown in the figure the equations are

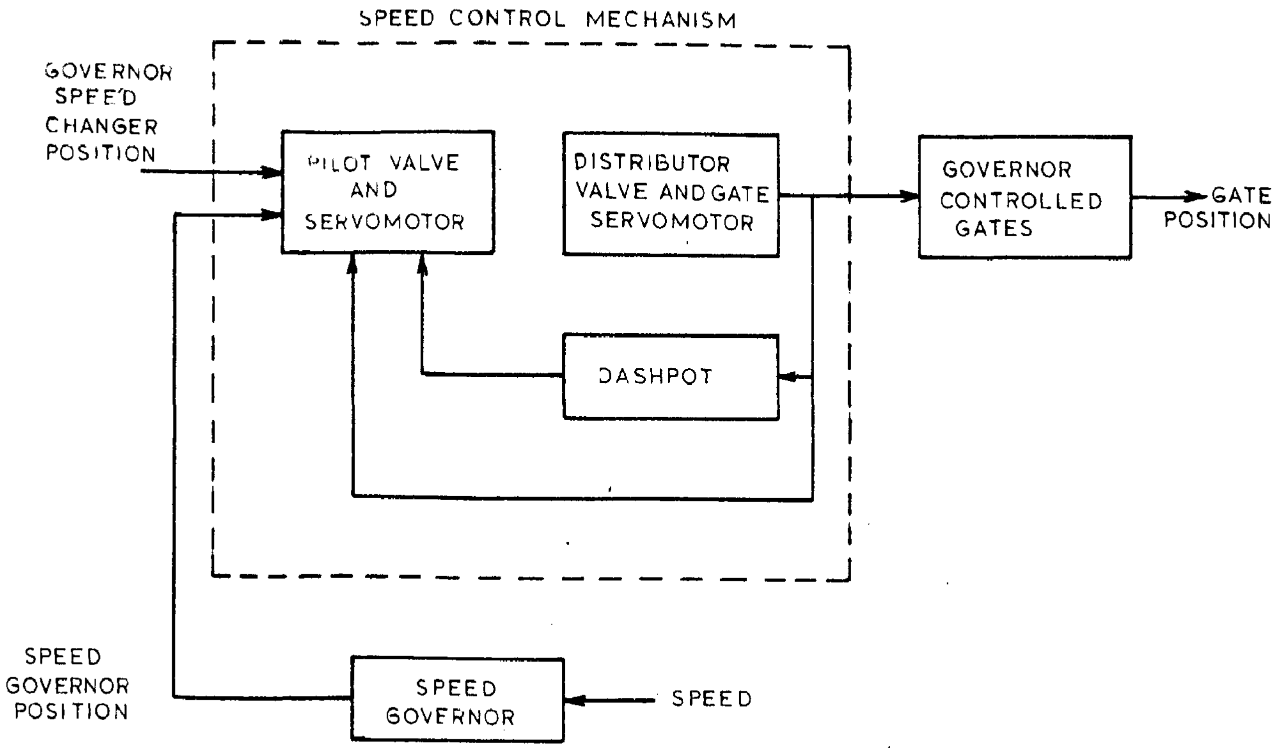
$$\begin{bmatrix} pc \\ pg \\ pg_{fb} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_P} & -\frac{\sigma}{T_P} & -\frac{1}{T_P} \\ \frac{1}{T_G} & 0 & 0 \\ \frac{\delta}{T_G} & 0 & -\frac{1}{T_R} \end{bmatrix} \begin{bmatrix} c \\ g \\ g_{fb} \end{bmatrix} + \begin{bmatrix} \frac{1}{T_P} \\ 0 \\ 0 \end{bmatrix} [n]$$

The effect of water column can be represented by the differential equation derived by $H_o$vey [32] that is

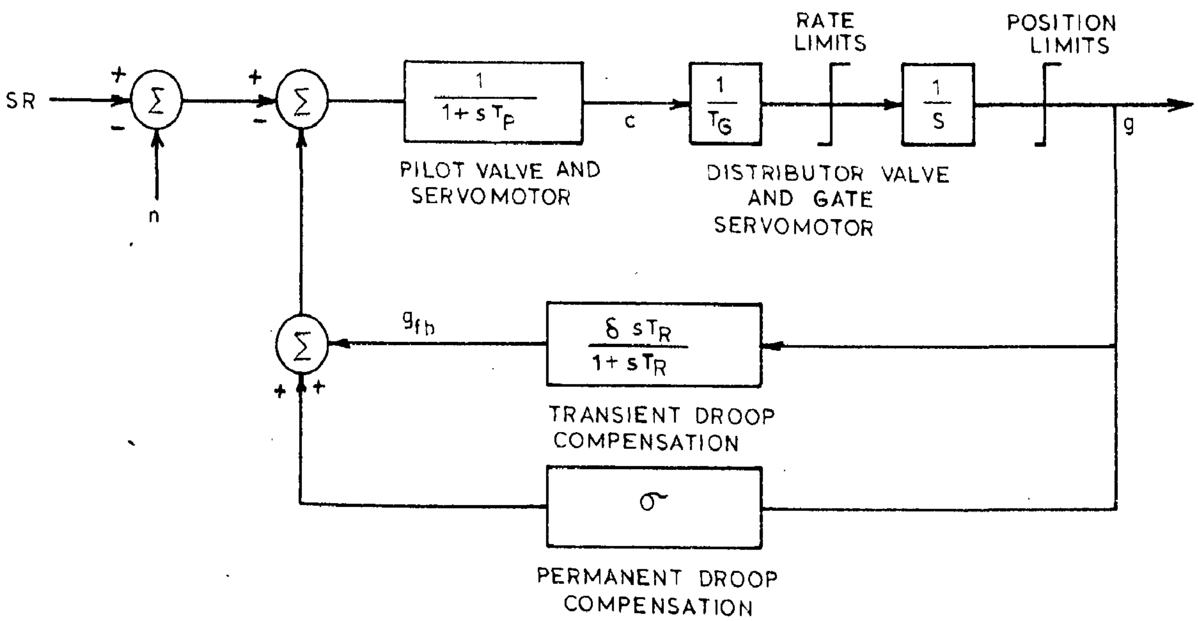
$$\frac{dh}{dt} = -2\,\frac{dg}{dt} - \frac{2}{T_W}\,h$$

Then the equations for the governing system become

$$\begin{bmatrix} pc \\ pg \\ pg_{fb} \\ ph \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_P} & -\frac{\sigma}{T_P} & -\frac{1}{T_P} & 0 \\ \frac{1}{T_G} & 0 & 0 & 0 \\ \frac{\delta}{T_G} & 0 & -\frac{1}{T_R} & 0 \\ -\frac{2}{T_G} & 0 & 0 & -\frac{2}{T_W} \end{bmatrix} \begin{bmatrix} c \\ g \\ g_{fb} \\ h \end{bmatrix} + \begin{bmatrix} \frac{1}{T_P} \\ 0 \\ 0 \\ 0 \end{bmatrix} [n]$$




a _ FUNCTIONAL BLOCK DIAGRAM




b _ APPROXIMATE NONLINEAR MODEL

FIG. 2.4 _ MECHANICAL HYDRAULIC SPEED-GOVERNING SYSTEM FOR HYDROTURBINES

where

$h$ = per unit head deviation at scrol casing

$= \frac{\Delta H}{H}$

$g$ = gate position deviation

$T_W$ = water column time constant in seconds

$H$ = rated head

$n$ = speed deviation in per unit

$T_W$, the water column time constant or the water starting time is associated with the acceleration time for water in the penstock between the turbine inlet and the forebay and can be calculated from the equation

$$T_W = \frac{LV}{Hg}$$

where

$L$ = length of penstock in feet

$V$ = velocity of water in feet per second

$H$ = head in feet

$g$ = acceleration due to gravity in feet per second per second

Typical values of the parameters for speed governing system for hydro-turbines are given in Table 1.

TABLE 1

| Parameter | Typical Value | Range |
|---|---|---|
| Dashpot time constant $T_R$ | 5.0 | 2.5 - 25.0 |
| Gate servomotor time constant $T_G$ | 0.2 | 0.2 - 0.4 |

| | | |
|---|---|---|
| Pilot value time constant $T_p$ | 0.04 | 0.03 - 0.05 |
| Transient speed drop coefficient $\delta$ | 0.3 | 0.2 - 1.0 |
| Permanent speed droop coefficient $\sigma$ | 0.05 | 0.03 - 0.06 |

7. NETWORK

Equations defining the voltage/ current relationship in a network, relating among them the voltages and currents at the machine terminals through the network impedences can be written down easily to satisfy Kirchhoff's laws. There are two main types of equations, namely, mesh current and nodal voltage. The mesh current equations can generally be represented by the matrix equation

$$[V] = [Z][I]$$

Similarly, nodal voltage equations can be represented by the matrix equation

$$[I] = [Y]\,[V] \qquad (2.17)$$

Nodal voltage equations have distinct advantage over the mesh current equations in that the number of equations, particularly in a large power system, is always less and the solution gives the required voltage directly.

The constant coefficients of (2.17) are the functions of branch impedences. These are computed at the rated frequency which introduces error, since the frequency at the different network points is generally variable around the rated value. However, in dynamic stability studies, maximum frequency variations are usually small and the simplification is justified.

8. LOADS [36]

In this section the structural modelling of loads as a function of voltage and frequency is considered. It is quite different from the dynamic modelling of loads as a function of time.

Both the active and reactive components of a load are important in power system calculations. The variations of active load affect directly the generator swings and tie line loadings, while the reactive load variations affect the bus voltages and thus indirectly the synchronizing power and apparant impedences of the lines.

An accurate representation of electric loads in a power system is of interest for the purposes of more detailed system-behaviour studies by simulation. In dealing with the problem of load modelling, the major difficulty encountered relates to the nature of system loads in general, their variety and changing composition. The exact load characteristics are rarely known for a particular bus bar and time.

The load at any bus can be represented as a function of voltage at that bus, V and the system frequency $\omega$. The basic load model chosen can be described by the following equation

$$\begin{bmatrix} \Delta P \\ \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \frac{dP}{dV} & \frac{dP}{d\omega} \\ \\ \frac{dQ}{dV} & \frac{dQ}{d\omega} \end{bmatrix} \begin{bmatrix} \Delta V \\ \\ \Delta\omega \end{bmatrix} \qquad (2.18)$$

where $\Delta P$ and $\Delta Q$ are changes in the active and reactive load demand due to changes in voltage and frequency. This model results by assuming that the active and reactive components of load are differentiable functions of voltage magnitude and frequency, and, using Taylor's expansion of those functions, by omitting higher order terms. The model is therefore applicable primarily for moderate changes in voltage V and frequency $\omega$.

The above equation applies to any load situation where the concept of average power applies. For a particular, ordinary type of load, the derivatives in the equation (2.18) known as the characteristic coefficients may be found from theoretical considerations or experimentally [2] .

However, in power system load flow and stability calculations it is customary to assume that the load at the bus is independent of the voltage at that bus and the system frequency. Thus the usual way of representing loads as constant impedences fits very well into equation (2.17) and avoids analytical complications. This load representation is only approximate and in practical calculations may produce optimistic results, especially in the cases where relatively large voltage variations

# CHAPTER III

## DYNAMIC STABILITY CALCULATIONS

### 1. INTRODUCTION

The dynamic analysis of power system requires that the state of the system at any instant be described by a vector y in an n-dimensional state space and the dynamic response of the system by a set of differential equations of linear form

$$[\dot{y}] = [A]\,[y] \qquad (3.1)$$

where $[A]$ is the system matrix

The basic system studied is shown in Figure 3.1 which is a reduced model combining all synchronous machines at sending and receiving ends into equivalent machines. Thus it becomes a two equivalent machine system coupled by a short transmission line. The equations are written for two machine system and these can be easily extended to the n machine case. Where equations are written in the symbolic form. The dimensions are given for the n-machine case. The synchronous machine is described by a set of Park's equation. The set of state variables chosen to describe the synchronous machine is the flux linkages of the direct and quadrature axis armature and rotor circuits, This set of variables has an advantage over a set of

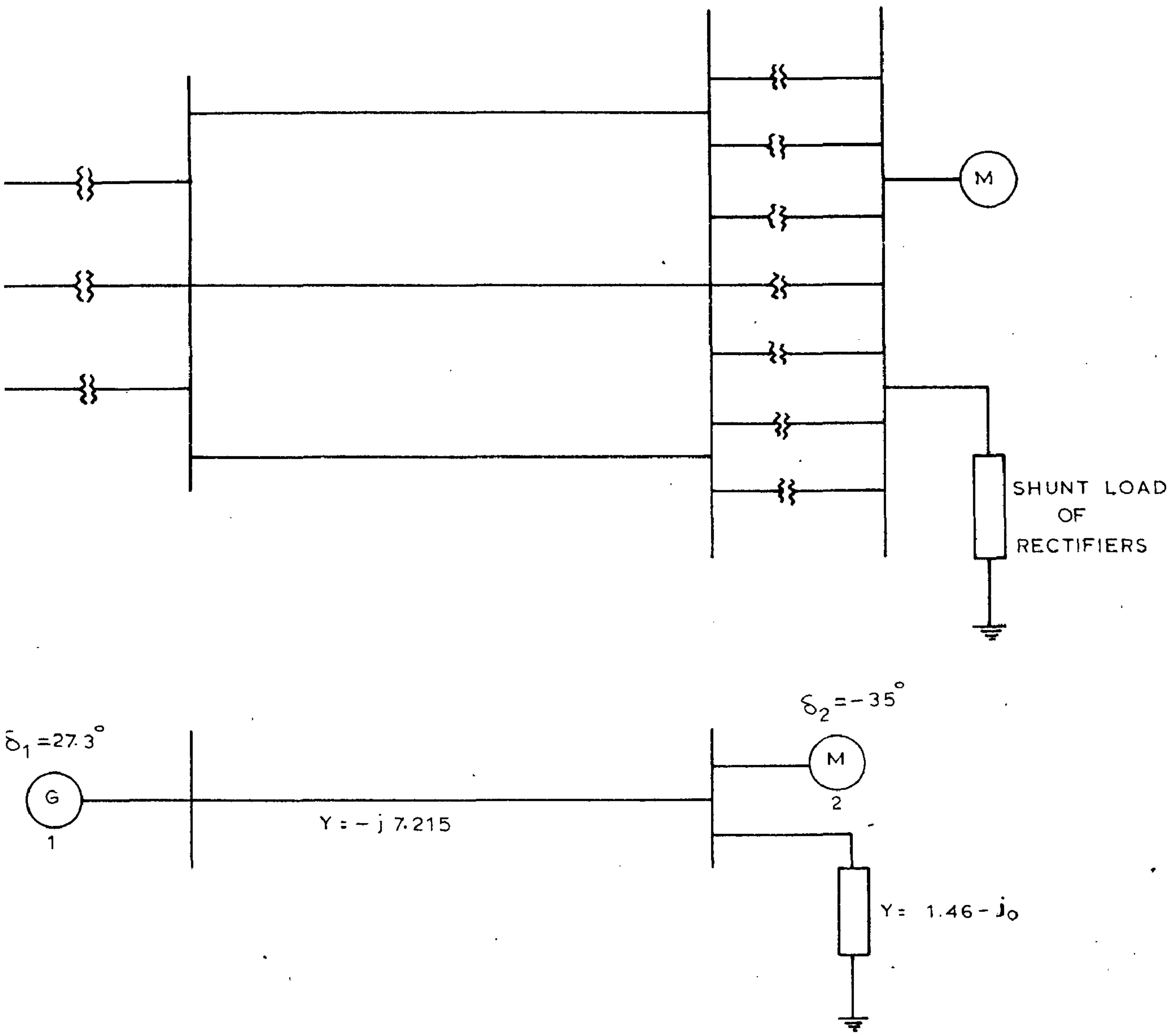



FIG. 3.1 _ ACTUAL AND EQUIVALENT NETWORKS

current and voltages because it is not necessary to compute new initial values after each change in system parameters because of the requirement (Lenz's Law) that the flux linkages in the machine remain constant through any instant($t \Rightarrow t+$) at which such a change occurs. The network is assumed to be described by the nodal admittance matrix equation

$$[I] = [Y][V] \qquad (3.2)$$

where all non-synchronous loads are represented by constant admittances. The excitation system for the generator is assumed to be of type 1, and the prime mover is controlled by a conventional dashpot type governor. The differential equations for the excitation system and the governing system are given in Chapter I.

## 2. SYNCHRONOUS MACHINE EQUATIONS [26]

Park's equations for the synchronous machine are

$$\phi_{fd} = x_{ffd}\, i_{fd} + x_{fkd} i_{kd} - x_{afd} i_d \qquad (3.3)$$

$$\phi_d = x_{afd} i_{fd} + x_{akd} i_{kd} - x_d i_d \qquad (3.4)$$

$$\phi_{kd} = x_{fkd}\, i_{fd} + x_{kkd} i_{kd} - x_{akd} i_d \qquad (3.5)$$

$$\phi_q = x_{akq}\, i_{kq} - x_q i_q \qquad (3.6)$$

$$\phi_{kq} = x_{kkq}\, i_{kq} - x_{akq} i_q \qquad (3.7)$$

$$e_{fd} = \frac{1}{\omega_o}\, p\, \phi_{fd} + r_{fd}\, i_{fd} \qquad (3.8)$$

$$e_d = \frac{1}{\omega_o}\, p\, \phi_d - r i_d - \frac{\omega}{\omega_o}\, \phi_q \qquad (3.9)$$

$$0 = \frac{1}{\omega_o}\, p\, \phi_{kd} + r_{kd}\, i_{kd} \qquad (3.10)$$

$$e_q = \frac{1}{\omega_o} p\phi_q - ri_q + \frac{\omega}{\omega_o} \phi_d \qquad (3.11)$$

$$0 = \frac{1}{\omega_o} p\, \phi_{kq} + r_{kq}\, i_{kq} \qquad (3.12)$$

$$T_g = \phi_d\, i_q - \phi_q\, i_d \qquad (3.13)$$

$$e_t^2 = e_d^2 + e_q^2 \qquad (3.14)$$

For small perturbations the above equations can be linearized by using Taylor series expansion about an operating point.

3. APPROACH TO THE CONSTRUCTION OF [A] MATRIX

The first stage in the construction of [A] is the selection of the frame of reference for the electrical quantities. The equations for each machine are expressed with reference to pairs of axes (d, q) which rotate in synchronism with the rotors of the machines; but the equations of the transmission network which fixes the relationship between the internal quantities of the machines refer to the axes (D, Q) which rotate at the angular frequency of the network current. In steady state all these axes will rotate at the same speed with angular displacements defined in Figure (3.2), but in transient conditions the angles $\delta_1$ will vary as the machine speeds vary. Therefore it is necessary to obtain a relationship between the deviations of the variables $i_m$, $e_m$ and the variables $\delta$ from their steady state equilibrium values. Section 7 gives the derivation of this relationship from (3.23) . The equation is

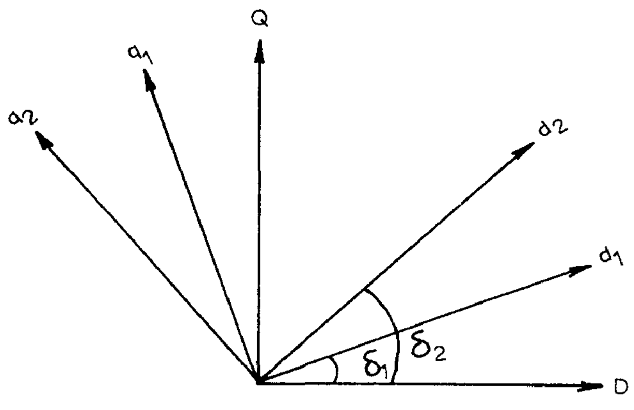



FIG. 3.2 _ ANGULAR RELATIONSHIP BETWEEN NETWORK AND SYNCHRONOUS MACHINE REFERENCE AXES

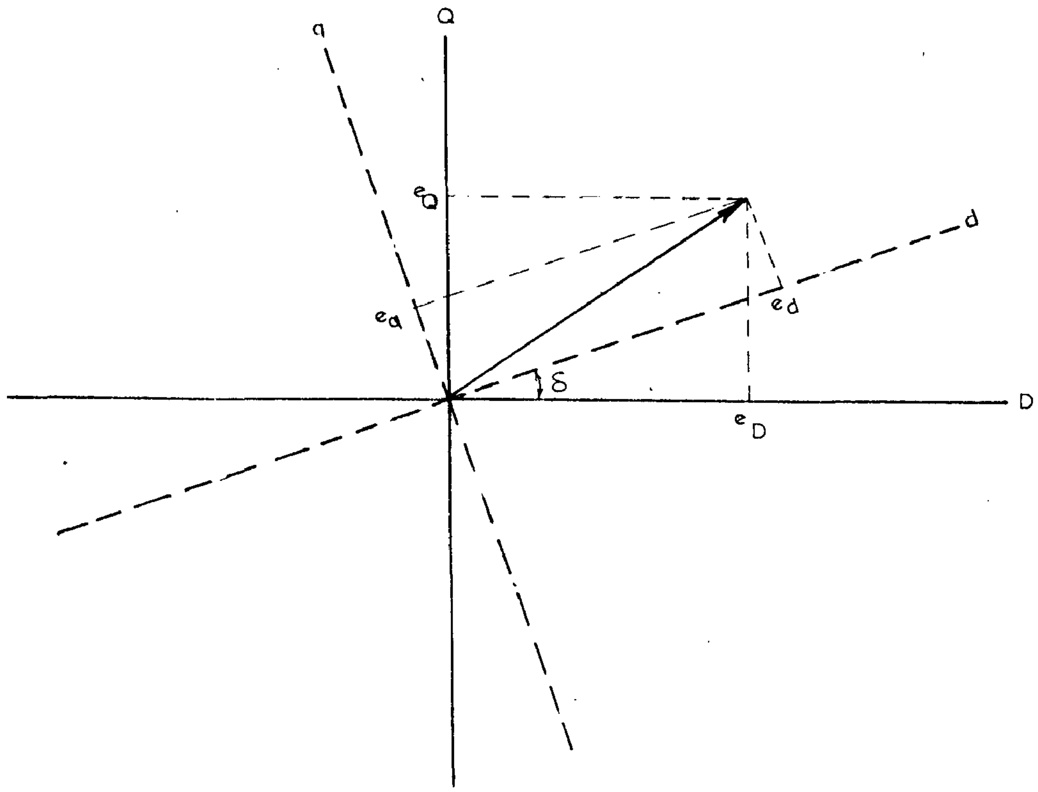



FIG. 3.3

$[\Delta i_m] = [Y_m][\Delta e_m] + [K][\Delta\delta]$

where the matrices $[Y_m]$ and $[K]$ as given by (27) and (28) are functions of the steady state values of $[i_m]$, $[e_m]$, and $[\delta]$ .

The second stage in the construction process involves the elimination of the variables $[\Delta i_m]$ and $[\Delta e_m]$ from the linearized synchronous machine equations to give a set of differential equations describing the performance of the rotor and stator circuits in terms of state variables only. The set of differential equations in matrix form,

$[\Delta\phi] = [X][P]\,[p\Delta\phi] + [X]\;[Q]\;[x]$

which can be written in the form

$$[H]\,[p\Delta\phi] = [F]\,[x] \qquad \text{(a)}$$

where

$$[H] = [X][P]$$

$$[F] = [[1,0] - [X][Q]]$$

and the matrices $[X]$ , $[P]$ and $[Q]$ are derived in section 8.

The vector $[p\Delta\phi]$ contains 5 n variables, the vector $[x]$ contains 8 n variables, and (a) contains 5 n independent differential equations. The 3n additional differential equations needed to give one independent equation per state variable and to pE, pδ and pm terms into the derivative vector are the exciter, rotor speed and rotor angle expressions. These expressions are derived in (3.44), (3.52) and (3.53). Equation (a) when augmented with equations (3.44) , (3.52) and (3.53) gives a set of 8 n differential equations in the 8 n state variables which constitute the vector [x]:

$$[H'] [\dot{x}] = [F'] [x] \qquad (b)$$

The equations of the additional elements in the voltage regulator and governor can now be added subject to the condition that one, and only one, state variable is added to each additional equations to give 13 n equations in the final form

$$[H_m] [\dot{y}] = [F_m] [y] \qquad (c)$$

The above equations yields the required form (3.1) directly since

$$[\dot{y}] = [H_m]^{-1} [F_m] [y]$$

and

$$[A] = [H_m]^{-1} [F_m]$$

The effect of damper winding is small and can be neglected and/ or accounted as extra damping term for the machine by dropping the terms $\Delta\phi_{kd1}$ , $\Delta\phi_{kq1}$ , $\Delta\phi_{kd2}$ and $\Delta\phi_{kq2}$ from the state vector and the corresponding rows and columns from the matrices [H'] and [F'] or $[H_m]$ and $[F_m]$ before performing inversion. This leaves matrices [H'] and [F'] of size 6n x 6n.

4. OBTAINING INCREMENTAL EQUATIONS

Equation (3.3) - (3.7) in the linearized form can be written as

$$\begin{bmatrix} \Delta\phi_{fd} \\ \Delta\phi_d \\ \Delta\phi_{kd} \\ \Delta\phi_q \\ \Delta\phi_{kq} \end{bmatrix} = \begin{bmatrix} x_{ffd} & -x_{afd} & x_{fkd} & & \\ x_{afd} & -x_d & x_{akd} & & \\ x_{fkd} & -x_{akd} & x_{kkd} & & \\ & & & -x_q & x_{akq} \\ & & & -x_{akq} & x_{kkq} \end{bmatrix} \begin{bmatrix} \Delta i_{fd} \\ \Delta i_d \\ \Delta i_{kd} \\ \Delta i_q \\ \Delta i_{kq} \end{bmatrix}$$

Equation (3.8) can be written for small perturbations as

$$\Delta i_{fd} = \frac{\Delta e_{fd}}{r_{fd}} - \frac{p\Delta\phi_{fd}}{\omega_o r_{fd}}$$

But by definition [27]

$$E = x_{afd} \frac{e_{fd}}{r_{fd}}$$

and the field circuit equation becomes

$$\Delta i_{fd} = \frac{\Delta E}{x_{afd}} - \frac{p\Delta\phi_{fd}}{\omega_o r_{fd}}$$

$$\text{or } \frac{\Delta E}{x_{afd}} = \frac{1}{\omega_o r_{fd}} p\Delta\phi_{fd} + \Delta i_{fd}$$

Similarly for small perturbations about an operating point equation (3.10) and (3.12) become

$$0 = \frac{1}{\omega_o r_{kd}} p\Delta\phi_{kd} + \Delta i_{kd}$$

$$0 = \frac{1}{\omega_o r_{kq}} p\Delta\phi_{kq} + \Delta i_{kq}$$

Equations (3.9) , (3.11), (3.13) and (3.14) linearize to

$$\Delta e_d = \frac{1}{\omega_o} p \Delta \phi_d - r \Delta i_d - \frac{\phi_q}{\omega_o} \Delta\omega - \Delta\phi_q$$

$$\Delta e_q = \frac{1}{\omega_o} p\Delta\phi_q - r \Delta i_q + \frac{\phi_d}{\omega_o} \Delta\omega + \Delta\phi_d$$

$$\Delta T_g = \phi_d \Delta i_q + i_q \Delta\phi_d - \phi_q \Delta i_d - i_d \Delta\phi_q$$

$$\Delta e_t = \frac{e_d}{e_t} \Delta e_d + \frac{e_q}{e_t} \Delta e_q$$

Summarizing, the linearized equations of the synchronous machine are

$$\begin{bmatrix} \Delta\phi_{fd} \\ \Delta\phi_d \\ \Delta\phi_{kd} \\ \Delta\phi_q \\ \Delta\phi_{kq} \end{bmatrix} = \begin{bmatrix} x_{ffd} & -x_{afd} & x_{fkd} & & \\ x_{afd} & -x_d & x_{akd} & & \\ x_{fkd} & -x_{akd} & x_{kkd} & & \\ & & & -x_q & x_{akq} \\ & & & -x_{akq} & x_{kkq} \end{bmatrix} \begin{bmatrix} \Delta i_{fd} \\ \Delta i_d \\ \Delta i_{kd} \\ \Delta i_q \\ \Delta i_{kq} \end{bmatrix} \quad (3.15)$$

$$\frac{\Delta E}{x_{afd}} = \frac{1}{\omega_o r_{fd}} p\Delta\phi_{fd} + \Delta i_{fd} \quad (3.16)$$

$$0 = \frac{1}{\omega_o r_{kd}} p\Delta\phi_{kd} + \Delta i_{kd} \quad (3.17)$$

$$0 = \frac{1}{\omega_o r_{kq}} p\Delta\phi_{kq} + \Delta i_{kq} \quad (3.18)$$

$$\Delta ed = \frac{1}{\omega_o} p \Delta\phi_d - r\Delta i_d - \frac{\phi_q}{\omega_o} \Delta\omega - \Delta\phi_q \quad (3.19)$$

$$\Delta e_q = \frac{1}{\omega_o} p\Delta\phi_q - r\,\Delta i_q + \frac{\phi_d}{\omega_o}\Delta\omega + \Delta\phi_d \qquad (3.20)$$

$$\Delta T_g = \phi_d \Delta i_q + i_q \Delta\phi_d - \phi_q \Delta i_d - i_d \Delta\phi_q \qquad (3.21)$$

$$\Delta e_t = \frac{e_d}{e_t}\Delta e_d + \frac{e_q}{e_t}\Delta e_q \qquad (3.22)$$

5. NETWORK

The network can be represented by an admittance matrix whose coefficients are the driving point and transfer admittances of only those nodes to which machines are connected For a network having two nodes to which machines are connected, the matrix equation relating node currents to node voltages in the (D,Q) reference frame is

$$\begin{bmatrix} i_{D1} \\ i_{Q1} \\ i_{D2} \\ i_{Q2} \end{bmatrix} = \begin{bmatrix} g_{11} & -b_{11} & g_{12} & -b_{12} \\ b_{11} & g_{11} & b_{12} & g_{12} \\ g_{21} & -b_{21} & g_{22} & -b_{22} \\ b_{21} & g_{21} & b_{22} & g_{22} \end{bmatrix} \begin{bmatrix} e_{D1} \\ e_{Q1} \\ e_{D2} \\ e_{Q2} \end{bmatrix} . \qquad (3.23)$$

which is in symbolic form

$$[i_N] = [Y_N]\,[e_N]$$

The above equations is a result of expanding a set of two simultaneous complete equations into a set of four real equations. The D,Q axis are common to all the nodes, so that suffixes D and Q are associa-ted with real and imaginary parts respectively of the complex quantities.

## 6. CONNECTION OF MACHINES AND NETWORK

In the previous sections two sets of equations have been given, one set representing the unconnected machine and the other network in terms of the nodes to which the machines are to be connected. The connection implies a relationship between the machine quantities (rotor based voltages) and the network based voltages. The relationship must take into account the displacement $\delta$ of the reference axis d, q for each machine with respect to the reference axis, D, Q for the network. The components of the network voltages in the D, Q reference frame are related to the terminal voltage of the machine in the d, q reference frame by the matrix equation Fig. (3.3)

$$\begin{bmatrix} e_D \\ e_Q \end{bmatrix} = \begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} e_d \\ e_q \end{bmatrix}$$

## 7. TRANSFORMATION OF THE TRANSMISSION NETWORK EQUATIONS

The transformation relating the network based voltages to the rotor based voltages for the system is

$$\begin{bmatrix} e_{D1} \\ e_{Q1} \\ e_{D2} \\ e_{Q2} \end{bmatrix} = \begin{bmatrix} \cos\delta_1 & -\sin\delta_1 & & \\ \sin\delta_1 & \cos\delta_1 & & \\ & & \cos\delta_2 & -\sin\delta_2 \\ & & \sin\delta_2 & \cos\delta_2 \end{bmatrix} \begin{bmatrix} e_{d1} \\ e_{q1} \\ e_{d2} \\ e_{q2} \end{bmatrix}$$

In symbolic form

$$[e_N] = [T][e_m] \qquad (3.24)$$

For small perturbation around the operating point the above equation becomes

$$\begin{bmatrix} \Delta e_{D1} \\ \Delta e_{Q1} \\ \Delta e_{D2} \\ \Delta e_{Q2} \end{bmatrix} = [T] \begin{bmatrix} \Delta e_{d1} \\ \Delta e_{q1} \\ \Delta e_{d2} \\ \Delta e_{q2} \end{bmatrix}$$

$$+ \begin{bmatrix} (-e_{d1}\sin\delta_1 - e_{q1}\cos\delta_1) & \\ (e_{d1}\cos\delta_1 - e_{q1}\sin\delta_1) & \\ & (-e_{d2}\sin\delta_2 - e_{q2}\cos\delta_2) \\ & (e_{d2}\cos\delta_2 - e_{q2}\sin\delta_2) \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \Delta\delta_2 \end{bmatrix}$$

which is, in symbolic form

$$[\Delta e_N] = [T] [\Delta e_m] + [I] [\Delta\delta] \qquad (3.25)$$

The power invariance theorem of Kron [40] requires

$$[i_m] = [T]^t [i_N]$$

which for small perturbations can be written as

$$[\Delta i_m] = [T]^t [\Delta i_N] + [J][\Delta\delta] \qquad (3.26)$$

$$[J] = \begin{bmatrix} i_{q1} & 0 \\ -i_{d1} & 0 \\ 0 & i_{q2} \\ 0 & -i_{d2} \end{bmatrix}$$

The values of $[i_m]$ , $[e_m]$ and $[\delta]$ used in constructing the matrices [T], [I] and [J] are the steady state operating point values which are deduced from the load flow data.

For small perturbations about an operating point (3.23) is

$$[\Delta i_N] = [Y_N] [\Delta e_N]$$

whence, from (3.25), and (3.26)

$$[\Delta i_m] = [Y_m] [\Delta e_m] + [K] [\Delta\delta] \qquad \text{(a)}$$

where

$$[Y_m] = [T]^t [Y_N] [T] \qquad (3.27)$$

$$[K] = [T]^t [Y_N] [I] + [J] \qquad (3.28)$$

The equation (a) is the expression of the constraint imposed by the transmission network on the performance of each synchronous machine. It involves only quantities which are referred to the internal reference axes of the machines, and hence no further transformations are needed for the machine internal variables to be taken as a set of numbers defining a vector in state space.

## 8. EXPRESSION OF SYSTEM EQUATIONS IN TERMS OF STATE VARIABLES

The second stage of the construction process involves the elimination of the variables $[\Delta i_m]$ and $[\Delta e_m]$ from the linearized synchronous machine equations to give a set of differential equations describing the performance of the rotor and stator circuits in terms of state variables only.

The perturbation forms of the Park terminal voltage equations (3.19) and (3.20) for each machine can be expressed collectively as

$$\begin{bmatrix} \Delta e_{d1} \\ \Delta e_{q1} \\ \Delta e_{d2} \\ \Delta e_{q2} \end{bmatrix} = \frac{1}{\omega_o} \begin{bmatrix} p\Delta\phi_{d1} \\ p\Delta\phi_{q1} \\ p\Delta\phi_{d2} \\ p\Delta\phi_{q2} \end{bmatrix} + \begin{bmatrix} -\phi_{q1} & \\ \phi_{d1} & \\ & -\phi_{q2} \\ & \phi_{d2} \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$- \begin{bmatrix} r_1 & & & \\ & r_1 & & \\ & & r_2 & \\ & & & r_2 \end{bmatrix} \begin{bmatrix} \Delta i_{d1} \\ \Delta i_{q1} \\ \Delta i_{d2} \\ \Delta i_{q2} \end{bmatrix} + \begin{bmatrix} -\Delta\phi_{q1} \\ \Delta\phi_{d1} \\ -\Delta_{q2} \\ \Delta\phi_{d2} \end{bmatrix}$$

which may be written in symbolic form as

$$[\Delta e_m] = \frac{1}{\omega_o} [p\Delta\phi_m] + \begin{bmatrix} -\Delta\phi_{q_1} \\ \Delta\phi_{d1} \\ -\Delta\phi_{q2} \\ \Delta\phi_{d2} \end{bmatrix} + [PDQ]\,[n] - [r][\Delta i_m] \quad (3.29)$$

Combining a and 2a gives

$$[\Delta i_m] = [C][p\Delta\phi_m] + \omega_o[C]\begin{bmatrix} -\Delta\phi_{q1} \\ \Delta\phi_{d1} \\ -\Delta\phi_{q2} \\ \Delta\phi_{d2} \end{bmatrix} + \omega_o[CP][n] + [KC]\,[\Delta\delta] \qquad (3.30)$$

where

$$[C] = \frac{1}{\omega_o}\,[[1] + [Y_m]\,[r]]^{-1}\,[Y_m] \qquad (3.31)$$

$$[KC] = [[1] + [Y_m][r]]^{-1}\,[K] \qquad (3.32)$$

$$[CP] = [C]\,[PDQ] \qquad (3.33)$$

Equation (30) may now be rearranged by simple row and column operations to take the form

$$\begin{bmatrix} \Delta i_{d1} \\ \Delta i_{q1} \\ \Delta i_{d2} \\ \Delta i_{q2} \end{bmatrix} = [C][p\Delta\phi_m] + \begin{bmatrix} \omega_oC_{12} & -\omega_oC_{11} & \omega_oC_{14} & -\omega_oC_{13} & KC_{11} & KC_{12} & \omega_oCP_{11} & \omega_oCP_{12} \\ \omega_oC_{22} & -\omega_oC_{21} & \omega_oC_{24} & -\omega_oC_{23} & KC_{21} & KC_{22} & \omega_oCP_{21} & \omega_oCP_{22} \\ \omega_oC_{32} & -\omega_oC_{31} & \omega_oC_{34} & -\omega_oC_{33} & KC_{31} & KC_{32} & \omega_oCP_{31} & \omega_oCP_{32} \\ \omega_oC_{42} & -\omega_oC_{41} & \omega_oC_{44} & -\omega_oC_{43} & KC_{41} & KC_{42} & \omega_oCP_{41} & \omega_oCP_{42} \end{bmatrix}$$

$$\left[\Delta\phi_{d1}, \Delta\phi_{q1}, \Delta\phi_{d2}, \Delta\phi_{q2}, \Delta\delta_1, \Delta\delta_2, n_1, n_2\right]^t$$

which is written symbolically as

$$[\Delta i_m] = [C][p\Delta\phi_m] + [U]\,[\Delta\phi_m, \Delta\delta, n]^t \qquad (3.34)$$

Equation (3.34) can now be expanded to include the rotor circuit equations (3.16), (3.17), (3.18). This operation involves the expanding of the vectors $[\Delta i_m]$, $[p\Delta\phi_m]$, and $[\Delta\phi_m, \Delta\delta, n]$ to the vectors $[\Delta i]$, $[p\,\Delta\,\phi]$, and $[x]$; and corresponding expansion of the matrices [C] and [U] to give the following matrix equation.

1 1 1 = 2 2 2

$$
\begin{bmatrix}
-\frac{1}{\omega_o r_{fd1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & C_{11} & 0 & C_{12} & 0 & 0 & C_{13} & 0 & C_{14} & 0 \\
0 & 0 & -\frac{1}{\omega_o r_{kd1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & C_{21} & 0 & C_{22} & 0 & 0 & C_{23} & 0 & C_{24} & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{\omega_o r_{kq1}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{\omega_o r_{fd2}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & C_{33} & 0 & C_{34} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\omega_o r_{kd2}} & 0 & 0 \\
0 & C_{41} & 0 & C_{42} & 0 & 0 & C_{43} & 0 & C_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\omega_o r_{kq2}}
\end{bmatrix}
$$

$$p\Delta\phi_{fd1}, p\Delta\phi_{d1}, p\Delta\phi_{kd1}, p\Delta\phi_{q1}\ p\Delta\phi_{kq1}, p\Delta\phi_{fd2}, p\Delta\phi_{d2}, p\Delta\phi_{kd2}, p\Delta\phi_{q2}, p\Delta\phi_{kq2}\ ]^t$$

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{x_{afd1}} & 0 & 0 & 0 & 0 & 0 \\
0 & \omega_o C_{12} & 0 & -\omega_o C_{11} & 0 & 0 & \omega_o C_{14} & 0 & -\omega_o C_{13} & 0 & 0 & 0 & KC_{11} & KC_{12} & \omega_o CP_{11} & \omega_o CP_{12} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \omega_o C_{22} & 0 & -\omega_o C_{21} & 0 & 0 & \omega_o C_{24} & 0 & -\omega_o C_{23} & 0 & 0 & 0 & KC_{21} & KC_{22} & \omega_o CP_{21} & \omega_o CP_{22} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{x_{afd2}} & 0 & 0 & 0 & 0 \\
0 & \omega_o C_{32} & 0 & -\omega_o C_{31} & 0 & 0 & \omega_o C_{34} & 0 & -\omega_o C_{33} & 0 & 0 & 0 & KC_{31} & KC_{32} & \omega_o CP_{31} & \omega_o CP_{32} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \omega_o C_{42} & 0 & -\omega_o C_{41} & 0 & 0 & \omega_o C_{44} & 0 & -\omega_o C_{43} & 0 & 0 & 0 & KC_{41} & KC_{42} & \omega_o CP_{41} & \omega_o CP_{42} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$[\Delta\phi_{fd1}, \Delta\phi_{d1}, \Delta\phi_{kd1}, \Delta\phi_{q1}, \Delta\phi_{kq1}, \Delta\phi_{fd2}, \Delta\phi_{d2}, \Delta\phi_{kd2}, \Delta\phi_{q2}, \Delta\phi_{kq2}, \Delta E_1, \Delta E_2, \Delta\delta_1, \Delta\delta_2, n_1, n_2]^t$$

This equation is written symbolically as

$$[\Delta i] = [P]\,[p\,\Delta\,\phi\,] + [Q]\,[x] \qquad (3.35)$$

The elimination of vector $[\Delta i]$ from this equation leaves a set of differential equations involving only variables contained in the required state vector $[x]$ and in $[px]$.

The generator flux linkage equations (3.15) for each machine may be collated to form the matrix equation

$$[\Delta\phi] = [x]\,[\Delta i] \qquad (3.36)$$

where the matrix $[X]$ is a 10x10 square matrix in which each 5x5 submatrix on its diagonal is the matrix of (3.15) for one machine. Combining (3.35) and (3.36) then gives directly

$$[H]\,[p\,\Delta\,\phi] = [F]\,[x] \qquad \text{(c)}$$

where $[H] = [X]\,[P]$

$[F] = [1,0] - [X][Q]$

Hene the matrix [1,0] is the 5nx5n unit matrix augmented with 3n additional zero columns to allow it to be added to the 5n x 8 n matrix [X][Q]

## 9. EXCITATION SYSTEM EQUATIONS

The expressions for the terminal voltage deviations, required in the voltage regulator equations is obtained by rearranging (3.29) in the form

$$[\Delta e_m] = \frac{1}{\omega_o}[p\,\Delta\,\phi_m] + \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & -\phi_{q1} & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & \phi_{d1} & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & -\phi_{q_2} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & \phi_{d_2} \end{bmatrix} \begin{bmatrix} \Delta\phi_{d1} \\ \Delta\phi_{q1} \\ \Delta\phi_{d2} \\ \Delta\phi_{q2} \\ \Delta\delta_1 \\ \Delta\delta_2 \\ n_1 \\ n_2 \end{bmatrix}$$

$$- [r]\,[\Delta i_m]$$

which is written symbolically as

$$[\Delta e_m] = \frac{1}{\omega_o}[p\,\Delta\,\phi_m] + [V]\,[\,\Delta\phi_m, \Delta\delta,\, n\,]^t - [r]\,[\Delta i_m] \qquad (3.37)$$

Rearranging (3.37) and using (3.34) gives

$$[\Delta e_m] = [VP]\,[p\,\Delta\,\phi_m] + [VS][\Delta\phi_m, \Delta\delta, n\,]^t \qquad (3.38)$$

where

$$[VP] = [\frac{1}{\omega_o}[1] - [R][C]] \qquad (3.39)$$

$$[VS] = [[V] - [R][U]] \qquad (3.40)$$

The expressions for the absolute changes in the terminal voltages of the machines is

$$\begin{bmatrix} \Delta e_{t1} \\ \\ \Delta e_{t2} \end{bmatrix} = \begin{bmatrix} \frac{e_{d1}}{e_{t1}} & \frac{e_{q1}}{e_{t1}} & 0 & 0 \\ \\ 0 & 0 & \frac{ed_2}{e_{t2}} & \frac{e_{q2}}{e_{t2}} \end{bmatrix} \begin{bmatrix} \Delta e_{d1} \\ \Delta e_{q1} \\ \Delta e_{d2} \\ \Delta e_{q2} \end{bmatrix}$$

In symbolic form

$$[\Delta e_t] = [EV][\Delta e_m] \qquad (3.41)$$

which when combined with (3.38) gives

$$[\Delta e_t] = [EV][VP][p\Delta\phi_m] + [EV][VS][\Delta\phi_m, \Delta\delta, n]^t \qquad (3.42)$$

The voltage regulator equation for type one representation (neglecting saturation) is

$$\begin{bmatrix} p\Delta E \\ \\ p\Delta E_S \\ \\ p\Delta E_A \end{bmatrix} = \begin{bmatrix} -\frac{K_E}{T_E} & 0 & \frac{1}{T_E} \\ -\frac{K_E K_F}{T_E T_F} & -\frac{1}{T_F} & \frac{K_E}{T_E T_F} \\ 0 & -\frac{K_A}{T_A} & -\frac{1}{T_A} \end{bmatrix} \begin{bmatrix} \Delta E \\ \\ \Delta E_S \\ \\ \Delta E_A \end{bmatrix} + \begin{bmatrix} 0 \\ \\ 0 \\ \\ \frac{K_A}{T_A} \end{bmatrix} [e_t]$$

which in symbolic form is written as

$$[p\,\Delta EF] = [VR][\Delta EF] + [FF][\Delta e_t] \qquad (3.43)$$

Substituting $[\Delta e_t]$ from (3.42) a in (3.43) gives

$$[p\Delta EF] = [VR][\Delta EF] + [FF][\,[EV][VP][p\Delta\phi_m] + [EV][VS][\Delta\phi_m, \Delta\delta, n]^t] \qquad (3.44)$$

$$= [VR][\Delta EF] + [FF][[EP][p\Delta\phi_m] + [ES][\Delta\phi_m, \Delta\delta, n]^t] \qquad (3.45)$$

where

$$[EP] = [EV][VP]$$

$$[ES] = [EV][VS]$$

Writing (3.44) in the expanded form

$$\begin{bmatrix} p\Delta E_1 \\ p\Delta E_{S1} \\ p\Delta E_{A1} \\ p\Delta E_2 \\ p\Delta E_{S2} \\ p\Delta E_{A2} \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{A1}}{T_{A1}} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{K_{A2}}{T_{A2}} \end{bmatrix} \begin{bmatrix} EP_{11} & EP_{12} & EP_{13} & EP_{14} \\ EP_{21} & EP_{22} & EP_{23} & EP_{24} \end{bmatrix} \begin{bmatrix} p\Delta\phi_{d1} \\ p\Delta\phi_{q1} \\ p\Delta\phi_{d2} \\ p\Delta\phi_{d2} \\ p\Delta\phi_{q2} \end{bmatrix}$$

$$
= \begin{bmatrix}
-\frac{K_{E1}}{T_{E1}} & 0 & \frac{1}{T_{E1}} & & & \\
-\frac{K_{E1}K_{F1}}{T_{E1}T_{F1}} & -\frac{1}{T_{F1}} & \frac{K_{F1}}{T_{E1}T_{F1}} & & & \\
0 & -\frac{K_{A1}}{T_{A1}} & -\frac{1}{T_{A1}} & & & \\
 & & & -\frac{K_{E2}}{T_{E2}} & 0 & \frac{1}{T_{E2}} \\
 & & & -\frac{K_{E2}K_{F2}}{T_{E2}T_{F2}} & \frac{-1}{T_{F2}} & \frac{K_{F2}}{T_{E2}T_{F2}} \\
 & & & 0 & -\frac{K_{A2}}{T_{A2}} & \frac{-1}{T_{A2}}
\end{bmatrix}
\begin{bmatrix} \Delta E_1 \\ \Delta E_{S1} \\ \Delta E_{A1} \\ \Delta E_2 \\ \Delta E_{S2} \\ \Delta E_{A2} \end{bmatrix} +
$$

$$
\begin{bmatrix}
0 & 0 \\
0 & 0 \\
\frac{K_{A1}}{T_{A1}} & 0 \\
0 & 0 \\
0 & 0 \\
0 & \frac{K_{A2}}{T_{A2}}
\end{bmatrix}
\begin{bmatrix}
ES_{11} & ES_{12} & ES_{13} & ES_{14} & ES_{15} & ES_{16} & ES_{17} & ES_{18} \\
ES_{21} & ES_{22} & ES_{23} & ES_{24} & ES_{25} & ES_{26} & ES_{27} & ES_{28}
\end{bmatrix}
\begin{bmatrix} \Delta\phi_{d1} \\ \Delta\phi_{q1} \\ \Delta\phi_{d2} \\ \Delta\phi_{q2} \\ \Delta\delta_1 \\ \Delta\delta_2 \\ n_1 \\ n_2 \end{bmatrix}
\tag{3.46}
$$

## 10. ROTATION SYSTEM AND GOVERNING SYSTEM EQUATIONS

The expression for the incremental air gap torques of the machines is

$$\begin{bmatrix} \Delta T_{g1} \\ \Delta T_{g2} \end{bmatrix} = \begin{bmatrix} -\phi_{q1} & \phi_{d1} & 0 & 0 \\ 0 & 0 & -\phi_{q2} & \phi_{d2} \end{bmatrix} \begin{bmatrix} \Delta i_{d1} \\ \Delta i_{q1} \\ \Delta i_{d2} \\ \Delta i_{q2} \end{bmatrix} + \begin{bmatrix} i_{q1} & -i_{d1} & 0 & 0 \\ 0 & 0 & i_{q2} & -i_{d2} \end{bmatrix} \begin{bmatrix} \Delta\phi_{d1} \\ \Delta\phi_{q1} \\ \Delta\phi_{d2} \\ \Delta\phi_{q2} \end{bmatrix}$$

Symbolically

$$[\Delta T_g] = [ST][\Delta i_m] + [IT][\Delta\phi_m] \qquad (3.47)$$

Incorporating (3.34) to eliminate $[\Delta i_m]$ yields

$$[\Delta T_g] = [ST][C][p\Delta\phi_m] + [ST][U][\Delta\phi_m, \Delta\delta, n]^t + [IT][\Delta\phi_m]$$

$$= [ST][C][p\Delta\phi_m] + [[STU] + [IT][1,0]\,][\Delta\phi_m, \Delta\delta, n]^t$$

$$= [STC][p\Delta\phi_m] + [STT]\,[\Delta\phi_m, \Delta\delta, n]^t \qquad (3.48)$$

where,

$$[STC] \quad [ST][C] \qquad (3.49)$$

$$[STT] = [STU] + [IT][1,0] \qquad (3.50)$$

Here the matrix [1,0] is 2n x2n unit matrix augmented with 2n additional zero columns to allow it to be added to the nx4n matrix [STU].

The acceleration equations of the machines are

$$\begin{bmatrix} T_{m1} & \\ & T_{m2} \end{bmatrix} \begin{bmatrix} p_{n1} \\ p_{n2} \end{bmatrix} = -\begin{bmatrix} T_{g1} \\ T_{g2} \end{bmatrix} - \begin{bmatrix} D_1 & \\ & D_2 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Symbolically

$$[T_m][pn] = -[\Delta T_g] - [D][n] \tag{3.51}$$

which after substituting (3.48) becomes

$$[T_m][pn] + [STC][p\Delta\phi_m] = -[STT][\Delta\phi_m, \Delta\delta, n]^t - [D][n] \tag{3.52}$$

In the expanded form

$$\begin{bmatrix} T_{m1} & \\ & T_{m2} \end{bmatrix} \begin{bmatrix} p_{n1} \\ p_{n2} \end{bmatrix} + \begin{bmatrix} STC_{11} & STC_{12} & STC_{13} & STC_{14} \\ STC_{21} & STC_{22} & STC_{23} & STC_{24} \end{bmatrix} \begin{bmatrix} p\Delta\phi_{d1} \\ p\Delta\phi_{q1} \\ p\Delta\phi_{d2} \\ p\Delta\phi_{q2} \end{bmatrix}$$

$$= \begin{bmatrix} STT_{11} & STT_{12} & STT_{13} & STT_{14} & STT_{15} & STT_{16} & STT_{17} & STT_{18} \\ STT_{21} & STT_{22} & STT_{23} & STT_{24} & STT_{25} & STT_{26} & STT_{27} & STT_{28} \end{bmatrix} \begin{bmatrix} \Delta\phi_{d1} \\ \Delta\phi_{q1} \\ \Delta\phi_{d2} \\ \Delta\phi_{q2} \\ \Delta\delta_1 \\ \Delta\delta_2 \\ n_1 \\ n_2 \end{bmatrix}$$

$$- \begin{bmatrix} D_1 & \\ & D_2 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

In angular velocity relationship may be introduced in the form

$$\begin{bmatrix} p\Delta\delta_1 \\ p\Delta\delta_2 \end{bmatrix} = \begin{bmatrix} \omega_o & \\ & \omega_o \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Symbolically

$$[p\Delta\delta] = [\omega_o] \ [n] \qquad (3.53)$$

which incorporates the assumption that the network reference axes rotate at constant speed.

The governing system equations are

$$\begin{bmatrix} pc \\ pg \\ pg_{fb} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_p} & -\frac{\sigma}{T_p} & -\frac{1}{T_P} \\ \frac{1}{T_G} & 0 & 0 \\ \frac{\delta}{T_G} & 0 & \frac{1}{T_R} \end{bmatrix} \begin{bmatrix} c \\ g \\ g_{fb} \end{bmatrix} + \begin{bmatrix} \frac{1}{T_P} \\ 0 \\ 0 \end{bmatrix} [n] \qquad (3.54)$$

11. SELECTION OF ANGULAR REFERENCE

Equation (c) contains a set of equations (3.53) which imply that the network reference axes (D, Q) rotate at constant speed. However, this is only the case for steady state equation where all the synchronous machines rotate at the same speed; and during transients the actual instantaneous speed of the (D, Q) axes is unknown even though the instantaneous speed of each machine is known. Therefore it is necessary to make some assumption with regard to the behaviour of the network reference axes.

The assumption that (D, Q) rotate at a constant speed of $\omega_o$ is not valid since this would force the whole system to remain in synchronism at a speed of $\omega_o$ while the physical situation indicates that the system remains in synchronism at some frequency which is determined by the collective permanent droop action of all its governors and the speed-torque characteristics of all its loads.

An alternative and reasonable assumption is that the network frequency is always identical to that of one arbitrarily chosen machine so that the axes (D, Q) rotate in synchronism with the axes $(d_r, q_r)$ of that machine. This implies that the rotor angle deviation $\Delta\delta_r$ of the rth machine is always zero and that (3.53) must be modified to

$$\begin{bmatrix} p\Delta\delta_1 \\ p\Delta\delta_2 \\ \vdots \\ p\Delta\delta_{r-1} \\ p\Delta\delta_{r+1} \\ \vdots \\ p\Delta\delta_n \end{bmatrix} = \begin{bmatrix} \omega_o & & & -\omega_o & & \\ & \omega_o & & -\omega_o & & \\ & & \omega_o & -\omega_o & & \\ & & & -\omega_o & \omega_o & \\ & & & & & \\ & & & -\omega_o & & \omega_o \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ \\ n_{r-1} \\ n_r \\ n_{r+1} \\ \vdots \\ n_n \end{bmatrix}$$

The resulting change in equation (c) can be affected conveniently by

(a) deleting $\Delta\delta_r$ and $\delta$ $p\Delta\delta_r$ from the vector [y] and $[\dot{y}]$

(b) deleting the appropriate rows and columns corresponding to $\Delta\delta_r$ and $p\Delta\delta_r$

(c) placing $(-\omega_o)$ in the rows of the new right hand side of the matrix corresponding to the remaining angle deviations and the column corresponding to $n_r$.

This change leaves equation (c) of order 13n -1 with all rotor angles referred to the rth machine. The largest machine or the machine which is likely to have the most influence on the frequency of the network current is usually chosen as the reference machine.

## 12. EIGENVALUES AND THEIR SIGNIFICANCE

The stability of an equilibrium point in the state space of a free, linear, stationary system depends solely on the roots of the matrix differential equation

$$[\dot{X}] = [A][X] \qquad (3.55)$$

The roots of the matrix differential equation are called the eigenvalues of the system. Other terms used interchangeably with the eigenvalues are proper values, natural modes, free frequencies, characteristic roots, and characteristic values.

Using the Laplace operator to replace the derivative, the equation becomes with zero initial conditions

$$(sI - A)\, X(s) = 0$$

Since the state vector X is not zero under all conditions then the determinant of the term in the brackets must be zero, i.e.,

$$|sI - A| = 0 \qquad (3.56)$$

The polynomial in s resulting from the expansion of the doterminant

$$a_0 s^n + a_1 s^{n-1} + \ldots\ldots + a_{n-1} s + a_n = 0$$

is the common denominator polynomial (characteristic equation) of all transfer functions between an input vector and an output vector.

A linear, stationary, free system is asymptotically stable at an equilibrium state X = 0 if and only if all roots of equation (3.56) are located in the left half of the complex S-plane. It is precisely this condition that the Liapunov theorems are checking [37]. However in those cases where the characteristic equation is savailable ór easily derived, there is an alternative to the Liapunov approach which involves less work and is more straight - forward.

There are two such alternatives, both of which involve obtaining information about the asymptotic stability of a system from its characteristic polynomial without solving for the polynomial roots. One method is that of Routh and another, by Hurwitz. Both furnish much the same information and involve similar amounts of necessary computation.

In the program developed the characteristic equation is never formed as such. Instead, the eigenvalues of the matrix A of the first order differential equations that describe the system are found by QR transform method [38]. These are identical with the roots of the characteristics equation.

If the coefficients of the differential equation are all real, the roots will either be real or appear as conjugate pairs of complex number. A real root $\alpha$, would give rise to a term in the solution of the form $Ce^{\alpha t}$. A conjugate pair $\alpha \pm j\beta$, would give a term of the form $Ce^{\alpha t} \sin(\beta t + \theta)$. In both cases, the C and $\theta$ are arbitrary constants determined by initial conditions. In the solution, the arbitrary constants contain the information on the initial conditions, while the roots contain the information on the system represented by differential equations. The eigenvalues give the response characteristics of a system's modes.

If all of the roots are plotted on a complex plane, their location gives much information. Any of the roots which lie in the right half of the plane will have positive real parts. They represent terms in the solution which will grow with time, and thus they indicate an unstable system. Roots which lie in the left half plane will represent terms that die out. The farther they are to the left, the larger the value of $\alpha$ and faster the term will disappear. To improve the transient response it is necessary to move the roots to the left, thus increasing damping. The distance of the complex roots from the real axis indicates the frequency of the oscillatory terms in the transient response. Those roots near the real axis will have lower frequency, while those further away will have a higher frequency.

The significance of the root location in the s-plane is illustrated in Fig. 3.4.

13. SYSTEM ANALYSIS

DATA

Machines. (Machine data in p.u. referred to machine rated voltage and apparent power)

1. Generators - 3 nos.

| | |
|---|---|
| VA rating | 100 MVA |
| Cos $\phi$ | 0.9 |
| H | 4.04 KWs/KVA |
| $x_d$ | 0.913 |
| $x'_d$ | 0.30 |
| $x_q$ | 0.56 |
| r | 0.004 |
| $T'_{do}$ | 7.66 s |
| $x_{afd}$ | 1.0 |

2. Motor (Equivalent) 1 No.( Representative data has been assumed)

| | |
|---|---|
| VA rating | 19 MVA |
| cos $\phi$ | 1.0 |

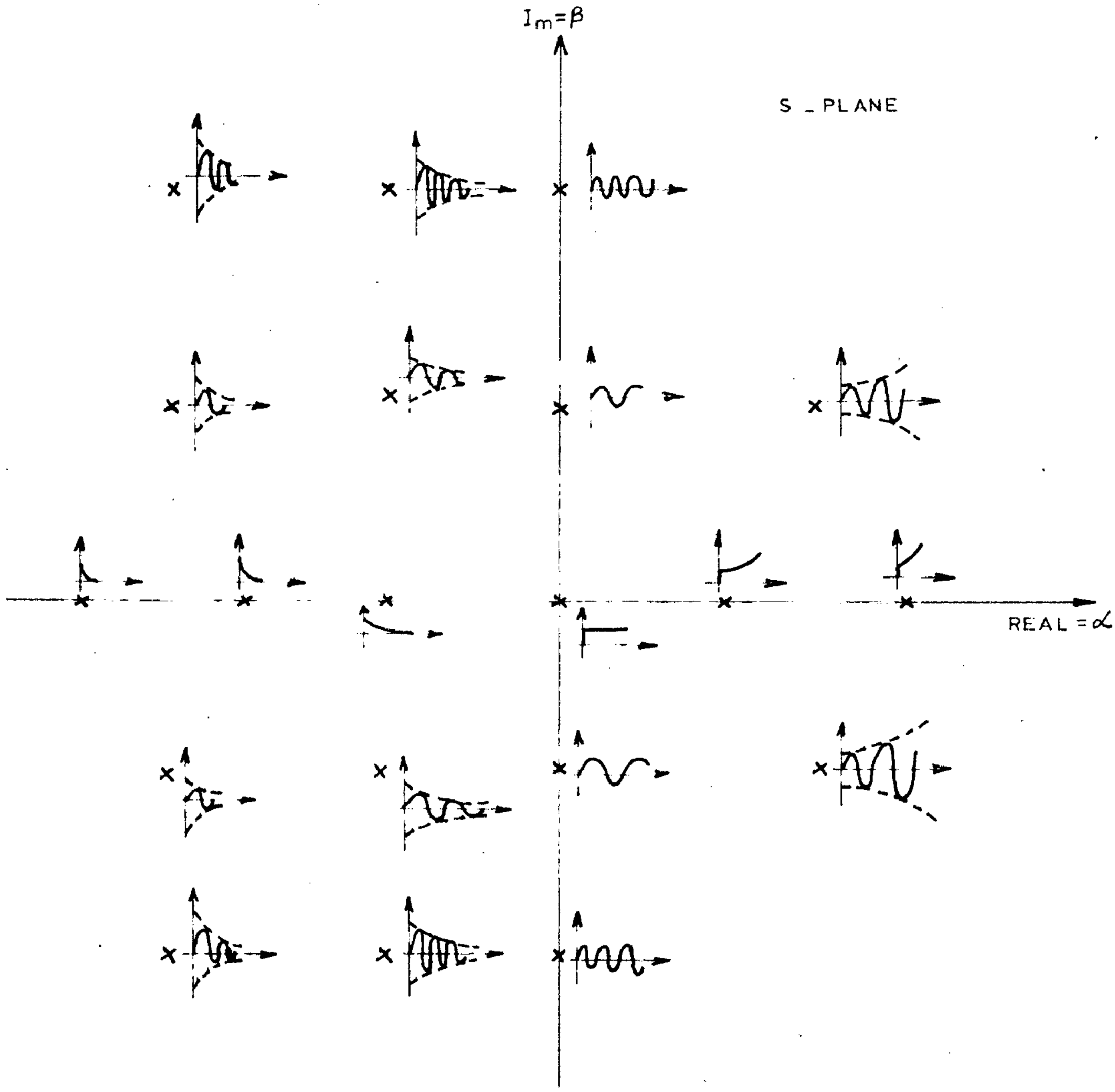



FIG.3.4 _ SIGNIFICANCE OF ROOT LOCATION IN S - PLANE

| | |
|---|---|
| H | 2.0 KWs/KVA |
| $x_d$ | 1.25 |
| $x'_d$ | 0.34 |
| $x_q$ | 0.7 |
| r | 0.002 |
| $T'_{do}$ | 3.18 s |
| $x_{afd}$ | 1.0 |

3. Transformers (Sending end) 3 Nos. (Representative data has been assumed).

| | |
|---|---|
| VA rating | 100 |
| Voltage ratio | 11/66 KV |
| Reactance x | 0.15 |

4. Transformers (Receiving end)

| | |
|---|---|
| VA rating (7 nos) | 35 MVA |
| Voltage ratio | 66/11 KV |
| Reactance x | 0.1145 |
| VA rating (1 No.) | 6.5 MVA |
| Reactance x | 0.0783 |

5. Transmission line

| | |
|---|---|
| No. of circuits | 3 |
| Transmission voltage | 66 KV |
| Length of line | 6 miles |
| Reactance per phase per mile | 0.587 Ω |
| Total series reactance per circuit | 0.587 x 6 = 3.522 |

| | |
|---|---|
| Base MVA | 100 |
| Base impedence of line | $\frac{(KV_{L-L})^2}{\text{base MVA}}$ |
| | $= \frac{(66)^2}{100} = 43.58$ |
| P.U reactance | $= \frac{3.522}{43.5}$ |
| | $= 0.081$ |

6. Load

| | |
|---|---|
| Total load (including motor load) | 180 MVA |
| Shunt admittance | $1.46 - j_o$ |

The actual network and equivalent network configuration are shown in Fig. (3.1)

7. Automatic Voltage Regulator

In the absence of suitable data, the following have been assumed as typical values:

| | | |
|---|---|---|
| Regulator amplifier time constant | $T_A$ | 0.1 |
| Regulator stabilizing circuit time constant | $T_F$ | 0.6 |
| Rogulator stabilizing circuit gain | $K_F$ | 0.05 |
| Exciter constant related to self excited field | $K_E'$ | - 0.05 |
| Exciter time constant | $T_E$ | 0.5 |

Rogulator gain $K_A$ made variable from 10 to 40

8. Turbine Governor

In the absence of suitable data, the following have been assumed as typical values.

| | | |
|---|---|---|
| Dashpot time constant | $T_R$ | 5.0 |

Gate servomotor time constant $T_G$ 0.2

Pilot value time constant $T_P$ 0.04

Transient speed droop coefficient $\delta$ 0.3

Permanent speed droop coefficient $\sigma$ made variable from .02-.0

By Definition [ 27]

$$x'_d = \left[ x_d - \frac{x^2_{afd}}{x_{ffd}} \right]$$

$$X_{ffd} = \frac{x^2_{afd}}{x_d - x'_d}$$

For the generator

$$x_{ffd} = \frac{1.0}{0.913 - 0.30} = 1.63$$

For the motor

$$x_{ffd} = \frac{1}{1.25 - .34} = 1.1$$

By definition

$$T'_{do} = \frac{x_{ffd}}{r_{fd}} \qquad (\, T'_{do} \text{ is in radians})$$

$$r_{fd} = \frac{x_{ffd}}{T'_{do}}$$

For the generator

$$r_{fd} = \frac{1.63}{2410} = 0.000678$$

For the motor

$$r_{fd} = \frac{1.1}{1000} = .0011$$

Steady state operating Point Values

For the motor

$e_{t2} = 1.0 \angle 0$

$\delta_2 = -35.0^\circ$

$i_{d2} = 0.1089$

$i_{q2} = -0.1556$

$e_{d2} = -0.5736$

$e_{q2} = 0.8192$

$\phi_{d2} = e_{q2} = 0.8192$

$\phi_{q2} = -e_{d2} = 0.5736$

For the generator

$e_{t1} = 1.0258 \angle 12.9^\circ$

$\delta_1 = 27.3^\circ$

$i_{d1} = 1.1533$

$i_{q1} = 1.3647$

$e_{d1} = 0.2920$

$e_{q1} = 0.9935$

$\phi_{d1} = e_{q1} = 0.9935$

$\phi_{q1} = -e_{d1} = -0.2920$

14. DIGITAL COMPUTER IMPLEMENTATION

After forming complete $[H_m]$ and $[F_m]$ matrices, the generator is selected as the reference machine and the variable $\Delta\delta_1$ and its derivative $p\,\Delta\delta_1$, are dropped. After forming [A], the program given in Appendix I is used to compute the eigenvalues. The program given in Appendix II used library subroutines HSBG and ATEIG [39] to compute eigen values of the system matrix A. The eigen values for various values of regulator gain $K_A$, damping D and permanent speed droop $\sigma$ are given below:

(1) $K_A = 10$ , $D = 4$ , $\sigma = 0.06$

1. -1743 + j 394.11
2. -1743 - j 394.11
3. -4.52 +j 330.37
4. -4.52 - j 330.37
5. -23.03
6. -2.88 + j 11.32
7. -2.88 - j 11.32
8. -6.015 + j1.596
9. -6.015 - j1.596
10. -2.137
11. +1.126
12. -0.8707
13. -0.6470
14. -0.0306
15. -0.0061

(2) $K_A = 25$ , $D = 4$ , o = 0.06

1. -1743 + j 394.11
2. -1743 - j 394.11
3. -4.52 + j 330.37
4. -4.52 - j 330.37

5. -23.03

6. -2.88 + j 11.32

7. -2.88 - j 11.32

8. -6.18 + j 5.49

9. -6.18 - j 5.49

10. -2.137

11. +1.519

12. -0.9378

13. -0.6323

14. -0.0307

15. -0.0049

(3) $K_A = 40$ ; $D = 4$ , $\sigma = 0.06$

1. -1743 + j 394.11

2. -1743 - j 394.11

3. -4.52 + j 330.38

4. -4.52 - j 330.38

5. -23.03

6. -2.88 +j 11.32

7. -2.88 -j 11.32

8. -6.27 +j 7.52

9. -6.27 -j 7.52

10. -2.137

11. +1.712

12. -0.9544

13. -0.6293

14. -0.0307

15. -0.0046

(4) $K_A = 40$ ; $D = 4$ ; $\sigma = 0.02$

1. $-1743 + j\,394.11$
2. $-1743 - j\,394.11$
3. $-4.52 + j\,330.37$
4. $-4.52 - j\,330.37$
5. $-23.26$
6. $-2.88 + j11.32$
7. $-2.88 - j\,11.32$
8. $-6.27 + j\,7.52$
9. $-6.27 - j\,7.52$
10. $-1.922$
11. $+1.712$
12. $-0.9544$
13. $-0.6292$
14. $-0.0116$
15. $-0.0045$

(5) $K_A = 25$ ; $D = 4$ ; $\sigma = 0.03$

1. $-1743 + j\,394.11$
2. $-1743 - j\,394.11$
3. $-4.52 + j\,330.37$
4. $-4.52 - j\,330.37$
5. $-23.20$
6. $-2.88 + j\,11.32$
7. $-2.88 - j\,11.32$
8. $-6.18 + j\,5.49$
9. $-6.18 - j\,5.49$
10. $-1.975$

11. +1.519

1;2. - .0379

13. -0.6323

14. -0.0166

15. -0.0049

(6) $K_A$ = 10.0 ; D = 12.00 $\sigma$ = 0.06

1. -1743 + j394.11

2. -1743 - j 394.11

3. -4.52 + j 330.37

4. - 4.52 - j 330.37

5. -23.03

6. -8.29 + j 8.81

7. -8.29 - j 8.81

8. -6.017 + j 1.606

9. -6.017 - j 1.6068716

10. -2.137

11. +1.097

12. -0.8413

13. -0.4047

14. -0.2883

15. -0.0298

## 15. INTERPRETATION OF RESULTS AND CONCLUSION

The eigenvalues give complete information on the dynamic stability of the system. These correspond to the natural modes of response. The real part $\alpha$ of each eigen value gives a measure of the decrement of the oscillation of a mode; it is the reciprocal of the time constant of the decay of oscillation. The imaginary part $\beta$ indicates the natural angular frequency of the mode concerned.

The necessary and sufficient condition for dynamic stability of the system is that all real parts of the eigen values should be negative. Negative real parts indicate positive decrements and thus positive system damping.

The system studied is evidently unstable, because of the existence of an eigen value (No.11) having positive real part. The last two eigenvalues lie close to the origin and may move to the right half of s-plane with parameter variations. As the regulator gain is increased from 10 to 40, the eigen value (No.11) moves further to the right; the natural angular frequency of the modes corresponding to the eigen values (No. 8 and 9) increases, leaving the damping term practically unaffected, thus making the system more oscillatory and less damped.

The results obtained corroborate with the large severe oscillations observed on the actual system.

Stabilization of system

The dynamic stability limit can be improved by the use of other input signals to the voltage regulators in addition to the terminal voltage. The signals are chosen to provide a positive damping of the power system oscillations to improve generator stability and damp tie line oscillations. Some of these signals are: rotor speed, rotor acceleration, accelerating power, frequency and rate of change of voltage. When used, they are added as shown in Fig. 3.5. Usually the stabilizing signal is inserted through a transfer function providing gain adjustment and lead-lag compensation for phase shifting. Usually a combination of proportional and first derivative signals is not sufficient and a second or higher derivatives are also required [31],[42]. Therefore a combination $\Delta\dot{\delta}$ and $\Delta\ddot{\delta}$ signals derived from rotor angle were tried to stabilize the system. Time constants of the input filter circuit which are very small have been neglected to simplify the calculations. The values of $K_A$, D and $\sigma$ are the same for all the sets, namely

$$K_A = 40 \ , \ D = 4 \ , \ \sigma = 0.06$$

The eigen values of the system matrix for different values of $K_1$ and $K_2$ are given below.

| Sl. No. | $K_1 = 0.2, K_2 = 5$ | $K_1 = 1, K_2 = 5$ | $K_1 = 5, K_2 = 5$ |
|---|---|---|---|
| 1. | -1742 + j 388 | -1745 + j 389 | -1742 + j 390 |
| 2. | -1742 - j 388 | -1745 - j 389 | -1742 - j 390 |
| 3. | -5.72 + j 330 | -3.57 + j 331 | -5.41 + j 331 |
| 4. | -5.72 - j 330 | -3.57 - j 331 | -5.41 - j 331 |
| 5. | -28.96 | -27.88 | -28.08 |
| 6. | 9.43 + j 22.65 | +9.70 + j 22.97 | +9.89 + j 22.78 |
| 7. | 9.43 - j 22.65 | +9.70 - j 22.97 | +9.89 - j 22.78 |
| 8. | -23.03 | -23.03 | -23.03 |
| 9. | -2.84 + j 11.30 | -2.77 + j 11.37 | -2.80 + j 11.35 |
| 10. | -2.84 - j 11.30 | -2.77 - j 11.37 | -2.80 - j 11.35 |

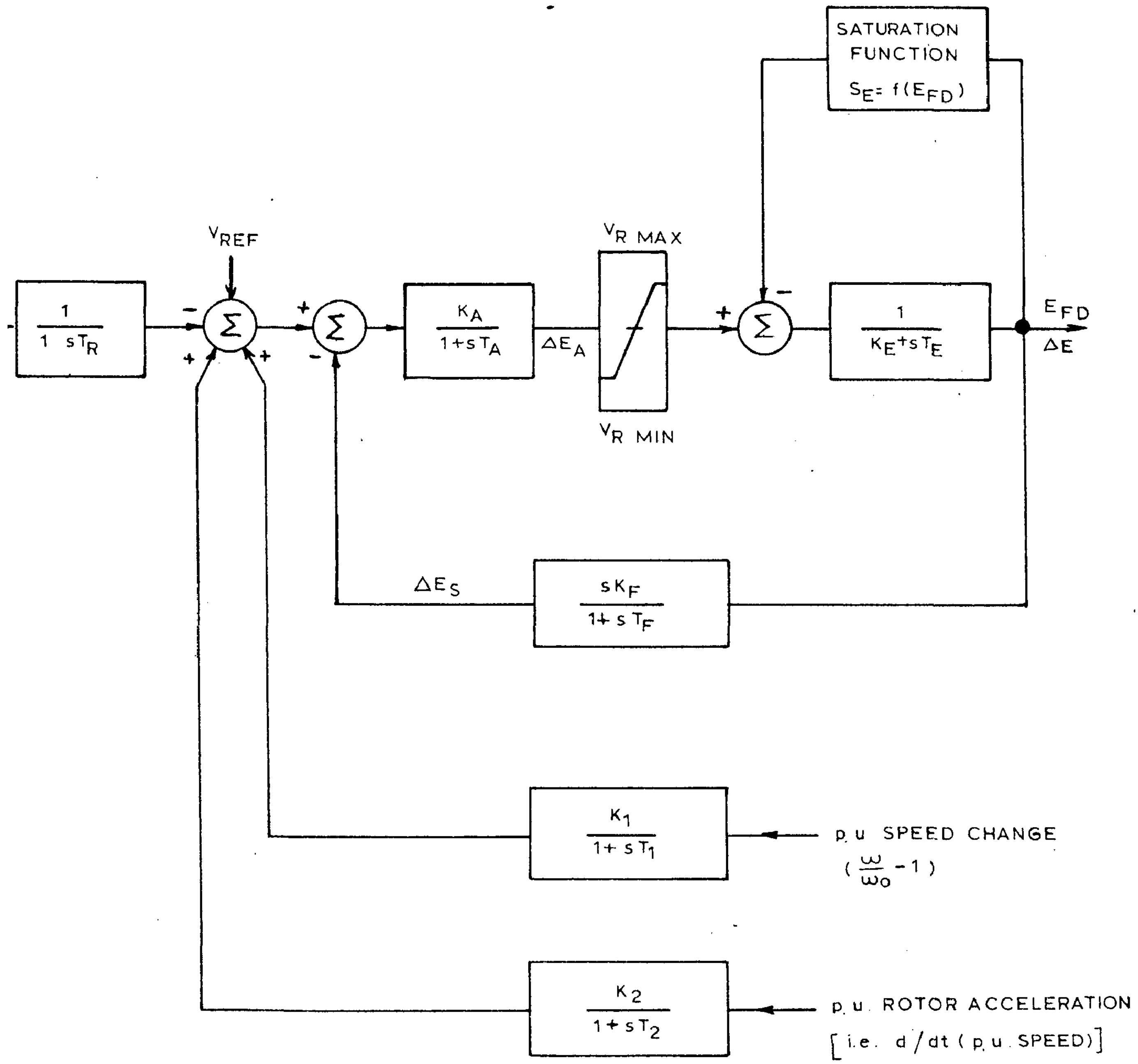
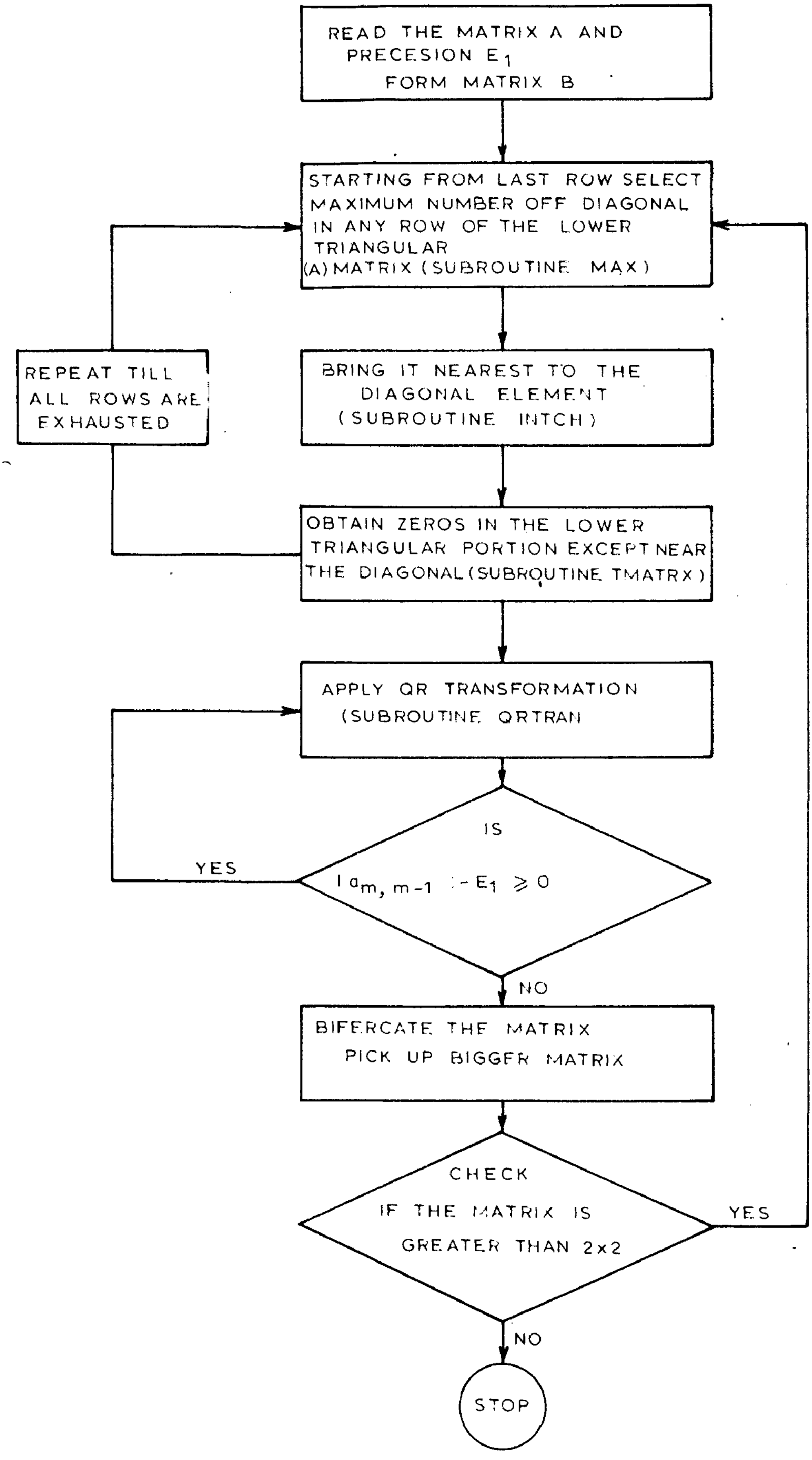



FIG. 2.5 _ TYPE 1 EXCITATION SYSTEM WITH $\dot{\delta}$ AND $\ddot{\delta}$ FEED BACK SIGNALS

| | | |
|---|---|---|
| 11. -2.137 | -2.137 | -2.137 |
| 12. -1.676 | -1.681 | -1.699 |
| 13. -0.714 | -0.7116 | -0.9988 |
| 14. -0.0400 | -0.2009 | -0.7901 |
| 15. -0.0306 | -0.0303 | -0.0304 |

The results indicate that the system is still unstable.

For different values of $K_1$ and $K_2$ tried, the system is unstable. The values of $K_2$ was not varied. Possibly the system can be made stable if the value of gain $K_2$ is varied from 0 to 5. Still higher derivatives of $\Delta\delta$ could also be tried to achieve stability [41].




FLOW CHART FOR COMPUTATION OF EIGENVALUES

NDIX-1 *EIGENVALUES BY QR TRANSFORM METHOD*

```
EIGENVALUES BY QR TRANSFORM METHOD
DIMENSION A(15,15),B(15,15),C1(15),C(15,15),D(15,15),E(15,15)
DIMENSION Q(15,15),Q1(15,15),Q2(15,15),Z(15,15),H1(15,15)
DIMENSION H2(15,15)
COMMON M,N,J1,E1,L,I,J,N1,A,B,C1,C,D,E,Q,Q1,Q2,Z,N3,N4,H1,H2,Y2,N5
READ50,N,E1,MN1
PUNCH 50,N,E1,MN1
IF(MN1)22,20,20
READ51,((A(I,J),J=1,N),I=1,N)
PUNCH 51,((A(I,J),J=1,N),I=1,N)
GOTO23
READ54,((A(I,J),J=1,N),I=1,N)
PUNCH 54,((A(I,J),J=1,N),I=1,N)
DO 52 I=1,N
DO 52 J=1,N
B(I,J)=0.0
DO 53 I=1,N
B(I,I)=1.0
FORMAT(I2,F10.7,I2)
FORMAT(5F14.6)
M=N
N3=M-1
N4=M-2
DO 3 J=1,N3
C1(J)=ABSF(A(M,J))
CALL MAX
CALL INTCH
CALL MULT(A,C,E,N,N,N)
CALL MULT(C,E,Z,N,N,N)
CALL MATEQ(Z,E,N,N)
CALL TMATRX
CALL MULT(E,D,Z,N,N,N)
CALL MATEQ(Z,E,N,N)
DO 6 J=1,N4
D(M-1,J)=(-1.)*D(M-1,J)
CALL MULT(D,E,Z,N,N,N)
CALL MATEQ(Z,E,N,N)
M=M-1
CALL MATEQ(E,A,N,N)
IF(2-M)2,9,9
CONTINUE
FORMAT(5E16.8)
K=0
K=K+1
CALL QRTRAN
CALL MULT(Q,E,Z,N,N,N)
CALL MATEQ(Z,E,N,N)
CALL TRNSPZ(Q,Q1,N,N)
IF(K-1)11,11,12
CALL MULT(Q1,B,Q2,N,N,N)
IF(K-1)14,14,15
CALL MULT(Q2,Q1,Z,N,N,N)
CALL MATEQ(Z,Q2,N,N)
N2=N-1
IF(K-N2)14,16,16
CALL MULT(E,Q2,Z,N,N,N)
CALL MATEQ(Z,E,N,N)
CALL CHECK
```

```
   IF(Y2-E1)17,17,31
   IF(I-N)17,10,17
   IF(N1-N5)75,75,72
   IF(N1-2)76,76,73
   CALL MATEQ(H1,A,N1,N1)
   N=N1
   GOTO21
   IF(N5-2)76,76,78
   CALL MATEQ(H2,A,N5,N5)
   N=N5
   GOTO21
   STOP
   END

   SUBROUTINE CHECK
   DIMENSION A(15,15),B(15,15),C1(15),C(15,15),D(15,15),E(15,15)
   DIMENSION Q(15,15),Q1(15,15),Q2(15,15),Z(15,15),H1(15,15)
   DIMENSION H2(15,15)
   COMMON M,N,J1,E1,K,I,J,N1,A,B,C1,C,D,E,Q,Q1,Q2,Z,N3,N4,H1,H2,Y2,N5
   I=0
)  I=I+1
   IF(I-N)243,251,251
)  I3=I+1
   Y1=E(I3,I)
   Y2=ABSF(Y1)
   IF(Y2-E1)242,242,240
!  DO 252 K=1,I
   DO 252 L=1,I
:  H1(K,L)=E(K,L)
   N1=I+1
   DO 244 K=N1,N
   DO 244 L=N1,N
   K1=K-N1+1
   L1=L-N1+1
)  H2(K1,L1)=E(K,L)
   N1=N1-1
   N5=N-N1
   PUNCH 300
)  FORMAT(10X,9HMATRIX H1)
   PUNCH250,((H1(I,J),J=1,N1),I=1,N1)
   PUNCH 303
)  FORMAT(10X,9HMATRIX H2)
   PUNCH250,((H2(I,J),J=1,N5),I=1,N5)
)  FORMAT(5F16.8)
.  CONTINUE
   RETURN
   END

   SUBROUTINE TMATRX
   DIMENSION A(15,15),B(15,15),C1(15),C(15,15),D(15,15),E(15,15)
   DIMENSION Q(15,15),Q1(15,15),Q2(15,15),Z(15,15),H1(15,15)
   DIMENSION H2(15,15)
   COMMON M,N,J1,E1,K,I,J,N1,A,B,C1,C,D,E,Q,Q1,Q2,Z,N3,N4,H1,H2,Y2,N5
)  DO 161 I=1,N
   DO 161 J=1,N
.  D(I,J)=B(I,J)
   DO 162 J=1,N4
   X6=ABSF(E(M,M-1))
```

```
      IF(X6-0.000001)162,162,163
163   D(M-1,J)=(-1.)*E(M,J)/E(M,M-1)
162   CONTINUE
      RETURN
      END

      SUBROUTINE QRTRAN
      DIMENSION A(15,15),B(15,15),C1(15),C(15,15),D(15,15),E(15,15)
      DIMENSION Q(15,15),Q1(15,15),Q2(15,15),Z(15,15),H1(15,15)
      DIMENSION H2(15,15)
      COMMON M,N,J1,E1,K,I,J,N1,A,B,C1,C,D,E,Q,Q1,Q2,Z,N3,N4,H1,H2,Y2,N5
      DO 201 I=1,N
      DO 201 J=1,N
201   Q(I,J)=B(I,J)
200   J=K
      G3=(1.)/SQRTF(E(J,J)*E(J,J)+E(J+1,J)*E(J+1,J))
      G1=E(J,J)*G3
      G2=E(J+1,J)*G3
      Q(J,J)=G1
      Q(J+1,J+1)=G1
      Q(J+1,J)=(-1.)*G2
      Q(J,J+1)=G2
      RETURN
      END

      SUBROUTINE MAX
      DIMENSION A(15,15),B(15,15),C1(15),C(15,15),D(15,15),E(15,15)
      DIMENSION Q(15,15),Q1(15,15),Q2(15,15),Z(15,15),H1(15,15)
      DIMENSION H2(15,15)
      COMMON M,N,J1,E1,K,I,J,N1,A,B,C1,C,D,E,Q,Q1,Q2,Z,N3,N4,H1,H2,Y2,N5
100   X=C1(1)
      DO 101 J=1,N4
      IF(X-C1(J+1))102,102,101
102   X=C1(J+1)
101   CONTINUE
      J=0
103   J=J+1
      IF(X-C1(J))104,104,103
104   J1=J
      C1(J1)=X
      RETURN
      END

      SUBROUTINE INTCH
      DIMENSION A(15,15),B(15,15),C1(15),C(15,15),D(15,15),E(15,15)
      DIMENSION Q(15,15),Q1(15,15),Q2(15,15),Z(15,15),H1(15,15)
      DIMENSION H2(15,15)
      COMMON M,N,J1,E1,K,I,J,N1,A,B,C1,C,D,E,Q,Q1,Q2,Z,N3,N4,H1,H2,Y2,N5
120   DO 121 I=1,N
      DO 121 J=1,N
121   C(I,J)=B(I,J)
      I1=M-1
      I2=J1
122   C(I1,I1)=0.0
      C(I2,I2)=0.0
      C(I2,I1)=1.0
      C(I1,I2)=1.0
      RETURN
      END
```

APPENDIX-1 *EIGENVALUES BY QR TRANSFORM METHOD*

```
      SUBROUTINE MULT(A,B,C,N,M,L)
      DIMENSION A(15,15),B(15,15),C(15,15)
      DO41I=1,N
      DO41J=1,L
      SUM=0.
      DO42K=1,M
 42   SUM=SUM+A(I,K)*B(K,J)
 41   C(I,J)=SUM
      RETURN
      END

      SUBROUTINE MATEQ(A,B,M,N)
      DIMENSION A(15,15),B(15,15)
      DO 11 I=1,M
      DO 11 J=1,N
 11   B(I,J)=A(I,J)
      RETURN
      END

      SUBROUTINE TRNSPZ(A,B,M,N)
      DIMENSION A(15,15),B(15,15)
      DO 14 I=1,M
      DO 14 J=1,N
 14   B(J,I)=A(I,J)
      RETURN
      END

C     SOLUTION OF QUADRATIC EQUATION
1     READ 2,A11,A12,A21,A22
2     FORMAT(4E16.8)
      X=(0.5)*(A11+A22)
      Y1=(-0.25)*(A11-A22)*(A11-A22)-A12*A21
      IF(Y1)5,4,4
4     Y=SQRTF(Y1)
      GOTO6
5     Y1=-Y1
      Y=SQRTF(Y1)
      R1=X+Y
      R2=X-Y
      Z=0.0
      PUNCH 3,R1,Z
      PUNCH 3,R2,Z
      GOTO7
6     PUNCH 3,X,Y
      Y=-Y
      PUNCH 3,X,Y
3     FORMAT(F14.8,F14.8)
7     GOTO1
      END
```

APPENDIX-2 *EIGENVALUES BY QR TRANSFORM METHOD*

```
C     EIGENVALUES BY QR TRANSFORM METHOD
      DIMENSIONA(15,15),RR(15),RI(15),IANA(15),PRR(2),PRI(2)
      READ 21,N,IA
 21   FORMAT(2I5)
      READ 22,((A(I,J),J=1,N),I=1,N)
      PRINT 22,((A(I,J),J=1,N),I=1,N)
 22   FORMAT(5E16.8)
      CALL HSBG(N,A,IA)
      PRINT 23
 22   FORMAT(10X,MATRIX A IN HESSENBERG FORM)
      PRINT 24,((A(I,J),J=1,N),I=1,N)
 24   FORMAT(5E16.8)
      CALL ATEIG(N,A,RR,RI,IANA,IA)
      PRINT 25
 25   FORMAT(10X,EIGENVALUES OF A MATRIX)
      PRINT 26
 26   FORMAT(10X,RR,20X,RI)
      PRINT 27,(RR(I),RI(I),I=1,N)
 27   FORMAT(2E16.8)
      END
```

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