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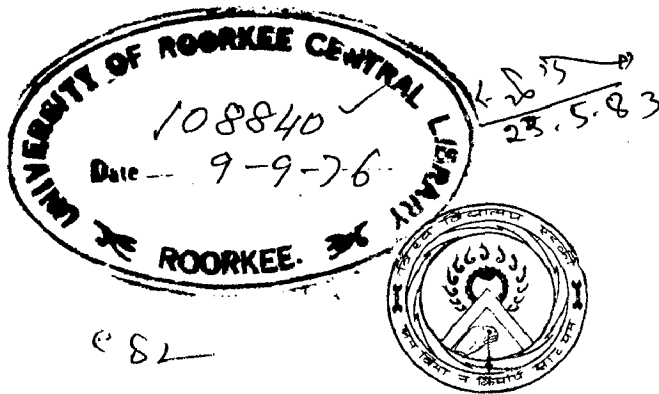
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OPTIMAL SCHEDULING OF HYDRO-THERMAL SYSTEMS

A Dissertation
submitted in partial fulfilment
of the requirements for the Degree of
MASTER OF ENGINEERING
in
WATER RESOURCES DEVELOPMENT

By

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C E R T I F I C A T E

Certified that the dissertation entitled "OPTIMAL SCHEDULING OF HYDRO THERMAL SYSTEMS" which is being submitted by Sri M. L. Gupta in partial fulfilment for the award of Degree of Master of Engineering in Water Resources Development of University of Roorkee is a record of candidate's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma. This is to certify that he worked for a period of more than 9 months from October 1972 to June 1974 for preparing dissertation for Master of Engineering Degree of the University.



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A C K N O W L E D G E M E N T S

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_S_Y_N_O_P_S_I_S_

In this dissertation on Optimal Scheduling of Hydro-Thermal Systems, various aspects of scheduling i.e. allocating both active and reactive power to hydro and thermal units in a combined system have been discussed so as to achieve maximum availability of generating capacity, obtain energy at minimum cost and utilise the resources most effectively.

Optimal scheduling of power in hydro thermal systems is more involved as compared to optimal operation of an all thermal system. Optimal operation of a thermal system is a static optimisation problem i.e. the objective is to minimise instantaneous fuel cost which is function of type, size and loading of the thermal unit subject to the equality constraints of the power system (Demand and losses = Generation) as well as inequality constraints imposed by the equipment ratings. In case of operation of hydro thermal systems, the problem is of dynamic optimisation i.e. one has to take into account the water usage policy over a period for optimum benefits. Based on operating period, the problem could be classified as :

- i) Long range scheduling for plant maintenance and short time availability of reservoir. The time base is year/month in this case.
- ii) Short term scheduling for unit commitment and unit hourly schedules over a

Hydro stations in Himalayan Regions of Northern India are characterised by three distinct periods in a year viz

- i) Period of minimum head in the storage reservoirs, during this period generating capability of the hydro stations is minimum and maximum number of hydro units should be available during this period for generation.
- ii) Maximum water period (Monsoons) when heads are high, ample water is available and maximum hydro-generation may be desirable even at lesser hydro plant efficiency.
- iii) Water shortage months, during this period even though the heads are high enough water may not be available and hydro units may run as peaking units only. This is the period of hydro unit maintenance.

Based upon above considerations and also on type of hydro power plant whether run of river scheme or storage plants, quantity of water to be used over a day or week is decided.

Depending on water availability over a day/week, hourly generation schedules of hydro & thermal stations are to be worked out (short term scheduling) as to achieve optimum benefits i.e. minimum overall thermal cost over the period. The various optimisation techniques

that have been developed and deployed for problem of this nature are :

- i) Calculus of variations,
- ii) Dynamic programming
- iii) Discrete maximum principle

These methods are discussed in this dissertation.

A computer programme for optimal scheduling over a day of one hydro (constant head) and one thermal system based on incremental dynamic programming method has been developed and tested on a sample system. The merits of dynamic programming method over other methods have been enumerated and its extension to include more hydro stations have been detailed. Short term scheduling methods which account for change of head by way of change in reservoir and tail water level and efficiency etc. with discharge have also been described.

The long range scheduling has been viewed in the light of determining appropriate generating reserve based on probability methods and scheduling maintenance of units based on constant risk principle. This results in maximum capacity benefits, optimum utilisation of hydro and thermal capacities and energy benefits and maintenance of appropriate reliability standards over long periods. On this account, various probability methods of determining generating reserves and method of working maintenance schedules on constant risk principle have been explained.

Optimum operation of power system involves optimising both active and reactive power. Optimum scheduling of power plants for active power is obtained by operating power plants on the principles enumerated above whereas optimisation of reactive power involves its allocation to various generating units subject to their MVAR capacity constraints and also its optimum generation in the system by use of capacitors/reactors so as to minimise losses in the system and also to maintain voltage profiles within National Permissible Standards. The desired allocation of reactive power may be achieved by excitation control of the generators, changing taps on the transformers and underexciting or overexciting synchronous condensers. This aspects could be looked after only by load flow studies as such a brief account of load flow methods have been given and a computer flow diagram for load flow has been included so that this aspect could be achieved.

CHAPTER - 1

INTRODUCTION

1. IMPORTANCE OF OPTIMAL SCHEDULING

1.1 General - Optimum economy in operation of a power system assumes greater importance with the size of the system especially where power systems are expanding at such a rapid rate as ours (doubling every 5 - 7 years). With development of efficient modern power station equipment and competition in electrical industry, maximum economy in system operation is a growing challenge to Operation Engineer. An electric utility system in the United States has achieved economy to the extent of about Rs.4,00,000/1000 MW of installed capacity by resorting to optimal operation technique⁽¹⁾. An optimal operation of power systems in Kerala for the year 1971 was estimated to result in saving of Rs.4.75 lakhs⁽²⁾.

1.2 Development of Hydro-Thermal System in India - Electrical system in India is in fast developing State. The total installed capacity which stood at 2.3 million KW at beginning of first five year plan has now touched the figure of 17 million KW and by the end of fifth plan it is expected to reach 52 million KW. Per capita consumption rose from 18 Kwh in the year 1949 to 95 Kwh at the end of 4th plan periods and it is expected to touch 250 kwh by the end of the decade.

(1) Optimum Economy Scheduling and Allocation of Real Power in a Power System-By S.Ray Basak-Symposium on Load Despatch Techniques and Integrated Operation of Power System - 42nd Annual Board Session of C.B.I.P. - Dec. 1969.

(2) Economic Operation - Kerala Power System. - by V.Anantha

Power plants at present consist of hydro, thermal and nuclear generating stations. This is because it has been found that electrical energy at minimum overall cost could be derived if both hydro and thermal power plants are developed hand in hand. The hydel and thermal power plants have many contrasting features and the best results are obtained when both these are harmoniously blended in power expansion programmes. Table I is an illustrative statement to highlight the comparative advantages of hydel and thermal power plants. The hydel and thermal power plants are no longer construed as competitors but as essential complements to each other. This is supported by examples of countries like England and Japan having predominant thermal power with meagre water resources seriously exploring the possibilities of large pumped storage schemes for affecting reduction of the overall cost of electrical energy on one hand and on the other hand countries like Norway and Sweden blessed with abundant hydel resources are implementing installation of large nuclear powered and oil fired thermal plants for effective utilisation of water resources. In country like ours blessed with both hydro and thermal resources a mixed pattern of hydel and thermal generation is necessary in the interest of national conservation of fuel energy and effective utilisation of water resources. The relative amount of hydel and thermal power installed would have to be arrived at by examining several alternative expansion schemes and then evaluating the financial implications as saving over a base case.

TABLE - 1

COMPARATIVE MERITS OF HYDEL AND THERMAL POWER PLANTS

Sl.No.	Description	Hydel	Thermal
1.	Initial Capital cost	High	Low
2.	Time for implementation	Long	Short
3.	Flexibility in siting	No flexibility	Flexible
4.	Annual Establishment expenses	Low	High
5.	Annual Maintenance expenses	Low	High
6.	Fuel expenses	Nil	Significant
7.	Life expectancy	Long	Short
8.	Plant shutdown for maintenance	Short	Long
9.	Forced outage rate	Low	High
10.	Power availability as needed	Limited by availability of water	No limitation
11.	Spinning reserve	More suitable for providing spinning reserve	Capability for providing spinning reserve is limited
12.	Quick start up and shut down for stand by reserve	Starting up time is very low	Starting up time is long

In such a vast and developing hydro thermal system as ours and in view of tight financial position even a small saving in operation of the system would further the power development programme.

2. OBJECTIVES OF OPTIMAL OPERATION

2.1 General - The operation of power system has to be such as to maintain equality between active and reactive power generated and their demand at consumer and plus losses incurred during transfer of power. In addition, to maintain continuity of supply appropriate reserves should be provided in the system to account for any forced outage of the equipment and other unforeseen eventualities.

2.2 Real Power Balance - Difference in power generated and power required is reflected in the frequency of the system. Maintenance of frequency at as constant a value as possible depending upon load and frequency control equipment provided is of great significance for the healthy operation of power system for following reasons :

- i) A.C.motors which often contribute to the bulk of the load run at speeds that are directly related to frequency.
- ii) Synchronous driven electric clocks depend on the frequency error and its integral for its accuracy.
- iii) The operation of the system will be smoother, efficient and better controlled if the frequency error is kept within limits.

Under normal operating conditions, the system generators should run synchronously and generate together power that at each moment is being drawn by all loads plus the real transmission losses. The ideal way to operate the system would thus be to set the water gates and steam valves of the various generators at values that would exactly correspond to the load demand. Unfortunately the reality is not so accommodating, since the fluctuation in load demand are entirely random in nature and a mismatch in active power balance is always present causing frequency fluctuations. The rate of these fluctuations depends on total moment of inertia of the running equipment.

Adoption of proper methods of load frequency control with due regard to the factors like characteristics of the generating units, loads, contractual obligations in energy supplies, etc., thus assumes greater importance in the efficient operation of power system. The various methods in the case of interconnected power system are flat frequency control, flat tie line control and tie line load bias control.

2.3 Reactive Power Balance - The balance of reactive power generation in a system could be obtained by varying excitation of the generators, use of shunt capacitors, synchronous condensers, shunt reactors, tap changing transformers etc. The point of generation of reactive power has an important say in the system losses and hence on the economics of the system.

The voltage profile in a power system largely depend upon the balance between produced and consumed reactive power, whenever the magnitude of a particular bus voltage undergoes variations this means that reactive power balance is not kept at the bus in question.

Further practically all equipment used in a power system are designed to operate at a certain voltage level, the rated voltage. If the system voltage deviates from this value, the performance of the device suffers and its life expectancy drops. Thus motivations for controlling the voltage levels on a power system are strong. There is, however, no excessive rigidity in the requirements for voltage constancy and nation wide standards allow a tolerable fluctuations in voltage levels of $\pm 12\frac{1}{2}\%$ at transmission level and $\pm 5\%$ at distribution level. A much reduced limit would however be desirable.

2.4 Scheduled Maintenance and Reserve Capacity - In order to ensure uninterrupted supply of power to the consumers, a power system has to maintain some reserve capacity for meeting the sudden forced outage of a generating equipment and meet unforeseen load demands. Part of the reserve have to be kept spinning and is called spinning reserve.

The amount of reserve required for meeting forced outage is dependent upon the number and size of units in a power system and the expected frequency of the outage of each generating unit called the outage rate. The outage rate even for the same size unit may vary from system to

system. Each system will, therefore, have to evaluate performance of its generating equipment from its own operating experience. An attempt to maintain record in this direction is required in the country.

Depending upon the outage rates and average duration of an outage of the generating units in a system, the expected frequency of simultaneous forced outage of various magnitude is determined with the help of probability studies. The required reserve capacity for forced outages for a desired standard of reliability, such as for simultaneous outages expected once in ten years (which for example, is followed in the U.S.A.) is thus calculated. Of course the once in ten year criterion may not be considered realistic in the context of our present economy in India and a lower level of reliability may be acceptable for the sake of rapid development.

While the quantum of reserve requirements in a system is generally governed by the above criteria, the manner of keeping the spinning reserve depends mainly on the type of generator available in the system. When the system consists mainly of thermal sets, the tendency should be to distribute the reserve over a large number of sets in the system. However when the system also includes hydro electric power stations, the cold reserve capacity is provided mainly by the storage hydro-electric stations. The power variations to which the thermal sets especially nuclear power plants could be subjected is usually only a fraction of their maximum capacity. In the case of hydro-electric sets large

amount of spinning reserve capacity could be conveniently provided by operating few of the units as motoring unit.

2.5 Co-Ordinated Operation of Hydro-Thermal Units - Various points which merit special attention with respect to co-ordinated operation of hydro-thermal units are as follows:-

- 1) Adequate transmission system for planned and forced outages and co-ordination of maintenance schedules.
- ii) Well planned emergency operation procedure.
- iii) A well co-ordinated protective scheme with no chances for cascade tripping.

Accordingly, the optimum operation of Hydro-Thermal Power System ensures operation of the various units at their optimum efficiency and has also to maintain appropriate reliability, frequency and voltage standards as discussed above so as to entail minimum fuel cost and optimum utilisation of water over the operating period.

2.6 Contents of Dissertation - The subject optimum scheduling of hydro-thermal system is proposed to be discussed under the following headings:

- i. Scheduled Planned Maintenance and Evaluation of Reserve for Optimising Capacity Benefits.
- ii. Techniques and methods for optimising scheduling in Hydro-Thermal Systems.

- iii) Scheduling for Reactive Power and Voltage Control in the System.
- iv) Case Study of Optimal Scheduling of a Sample system. Computer Programme - results and discussions.
- v) Application techniques
- vi) Conclusions.

Under the first heading principles of probability methods and methods such as loss of load method, loss of energy and frequency duration method of finding generation reserve have been discussed.

Hydro-Thermal Scheduling Methods, include calculus of variations, dynamic programming method and method of discrete maximum principles. Method of dynamic programming has been preferred for the reasons listed in this Chapter.

Various load flow methods are discussed in Chapter Four of the dissertation and a flow programme for making a computer programme has been given so that reactive power requirements of the system are scheduled for minimum losses and adequate voltage control in the system is achieved.

The method of optimal hydro-thermal scheduling based on incremental dynamic programming has been illustrated by taking a sample study of one system comprising one hydro (constant head) and one thermal system. The computer programme has been developed and successfully tested on the sample case. The results are also discussed in this Chapter.

The Chapter on Application Techniques contains a brief account of method used in converting these operating schedules into practice. A brief account of load despatch techniques is given in this Chapter.

CHAPTER - 2

2. SCHEDULING PLANNED MAINTENANCE AND EVALUATION OF RESERVES FOR OPTIMISING CAPACITY BENEFITS

2.1 General :

The maximum planned capacity in a system must be equal to the sum of the peak load, scheduled maintenance requirement and the capacity required for forced outages. Capacity benefits in the form of increased peak load supplies can be attained by properly scheduling planned maintenance of hydro & thermal unit in the systems and optimum evaluation of capacity required for forced outages keeping in view the reliability standard required.

The above problem could thus be sub-divided into two parts viz

- (1) Determination of Generation reserve for forced outages.
- (ii) Determination of planned maintenance Schedule.

2.2 Application of Probability Methods :

Application of probability methods have been recommended by A.I.E.E. Committee for evaluation of reserve capacity in the system⁽¹⁾. Several criteria and methods of computations have been developed over the years so as to bring into focus several criteria of reliability of a system. In the analysis of evaluation of system reserve

(1) A.I.E.E. Committee Report on Generating Capacity Problem, A.I.E.E. Trans. PA&S Vol.79-February 1961.

problems, the simplest criteria of reliability is the computed probability that the outage of generating capacity would exceed the reserve available at the time of peak load. However, analysis of generation reserve problem is complicated by the effects on reliability of daily & seasonal load shapes, of maintenance outages, of interconnection, of uncertainties in load forecasts and of river flow and storage when hydro - electric capacity is involved.

Depending upon measure of reliability three methods of generation reserve evaluation are as follows:-

- (i) Loss of load method
- (ii) Loss of energy method
- (iii) Frequency duration method

Application of these methods with special reference to the evaluation of the effect on system capacity requirements, of maintenance scheduling has been discussed.

Before describing the methods the following terms usually encountered in probability studies of the determination^{of}/generation reserve are defined.

2.2.1 Definitions: -

(a) Forced Outages: - A forced outage is the one which results from the failure of the turbine, generator or any of their auxiliaries or pertinent structures extending from the water passage to the generator loads that requires that the unit be taken out of service.

(b) Forced Outage Rate - t/T , where t is duration of forced or emergency outage in days and T is the total time during which equipment has been exposed.

(c) Duration - Duration of the outage extends from the time the unit was removed from service until the repair crew clears the unit for operation.

(d) Minimum Duration - Minimum duration of forced outage is the minimum time required taking recourse to overtime, shift work etc. to complete the necessary repairs to make the unit available for service.

Average duration of outage (d)

$$d = \frac{\text{Total forced outage time for outages on record}}{\text{Total number of forced outages}}$$

Average Interval of outages (T)

$$= \frac{\text{Exposed time of the unit on record}}{\text{Number of forced outages}}$$

Frequency of Outages = $(1/T)$, reciprocal of interval

(e) Time of Operation - Time of operation is the total time the units was synchronised with the system, whether or not it was carrying load. Also, on any calendar day that a unit operated, but was shut down for a period of 24 hours or less due to light load conditions, the unit is assumed to have operated continuously over the 24 hours because the risk of outage due to starting and stopping is assumed to be a great as if it had remained in service.

(f) Uniform System - A system where all machines are equal, in unit size outage rate and average duration of outages.

2.2² Loss of Load Method - In this method a probability table of capacity outage of the various combination of the machine based on the formulae given in Appendix entitled "Mathematical Methods of Reserve Studies", is constructed. The combinations are generally restricted to the capacity outage of the 4 machines at the most to avoid excessive calculation. From this table a cumulative capacity outage table for the system i.e. a table showing what probability exists of losing capacity equal to or exceeding various amounts, is constructed.

The cumulative probability table arrived at does not indicate what probability exist of a system loss of load but confines itself to loss of generation only. To know what probability exists of a system loss of load, a knowledge of system load characteristics is required. A load duration curve for a typical system over a year is shown in Figure 2.1. The steps involved to find out loss of load probability for the year are given in flow diagram shown in Figure 2.2.

Maintenance should be considered as a reduction of capacity. However, without significant error it may be considered as an increase of load. The inclusion of maintenance alters the peak load variation curve. The revised load duration curve becomes as shown in Figure 2.1(a)

Maintenance is to be included in such a manner as to minimise the ordinates of daily peak load variation curve. However a maintenance pattern once followed is not considered to change even though a maintenance pattern that was an optimum one at a high load level might not be optimum at low load level.

After the maintenance has been incorporated in daily peak load variation curve, computer works out the loss of load probability associated with peak hourly load selected. The flow diagram is given in Figure 2.2.

Next step is to select various values of peak hourly integrated load and to obtain the associated loss of load probability for each so that the loss of load probability be plotted as function of system load as shown in Figure 2.3.

The time required to calculate a table of capacity outage probability depends upon numbers and ratings of the units comprising the system. A system composed of identical units will yield a much shorter outage probability table than one having same number of units but of different ratings.

The time required to obtain a set of points describing loss of load probability as a function of load depends upon number of points taken. By increasing the value of load increments, few points will be taken and calculation time reduced for calculation of loss of load probability for given load.

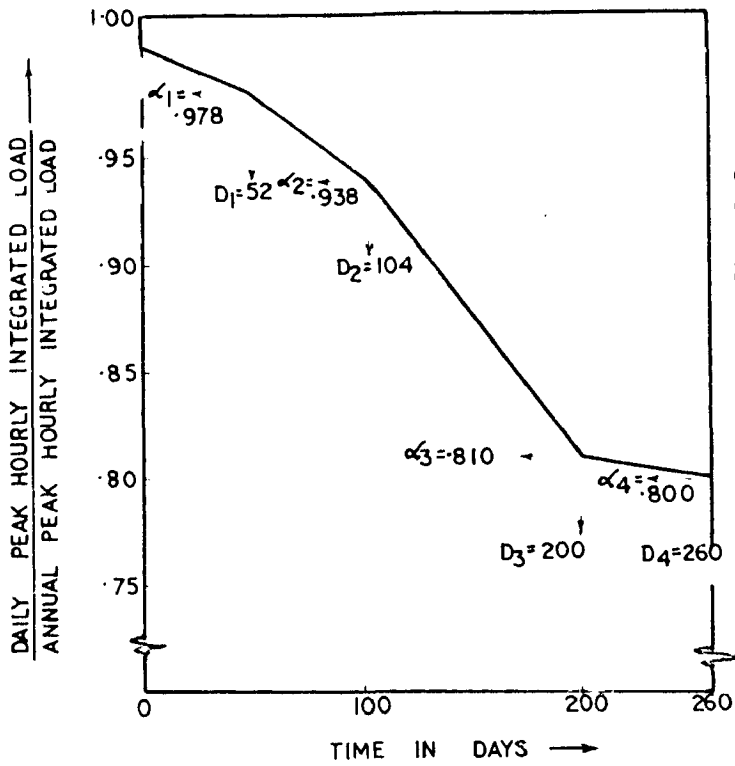


Fig. 2-1 DAILY PEAK LOAD VARIATION CURVE IN PER UNIT OF PEAK ANNUAL LOAD

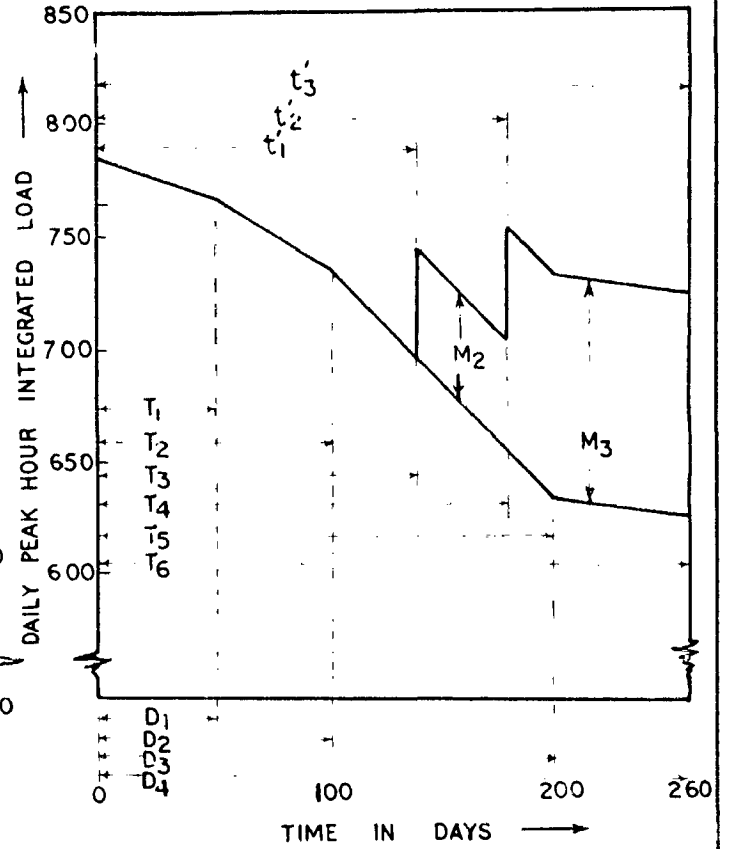


Fig. 2-1a. DAILY PEAK LOAD VARIATION CURVE FOR ANNUAL PEAK LOAD OF 785 MW WITH MAINTENANCE CONSIDERED

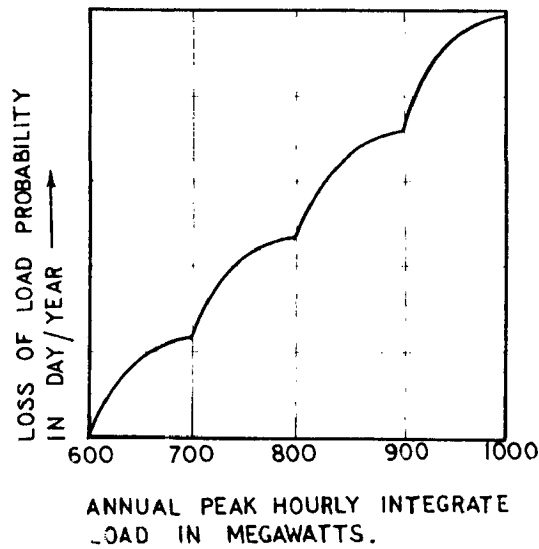


FIG 2-3 LOSS OF LOAD PROBABILITY AS FUNCTION OF ANNUAL PEAK HOURLY INTEGRATED LOAD

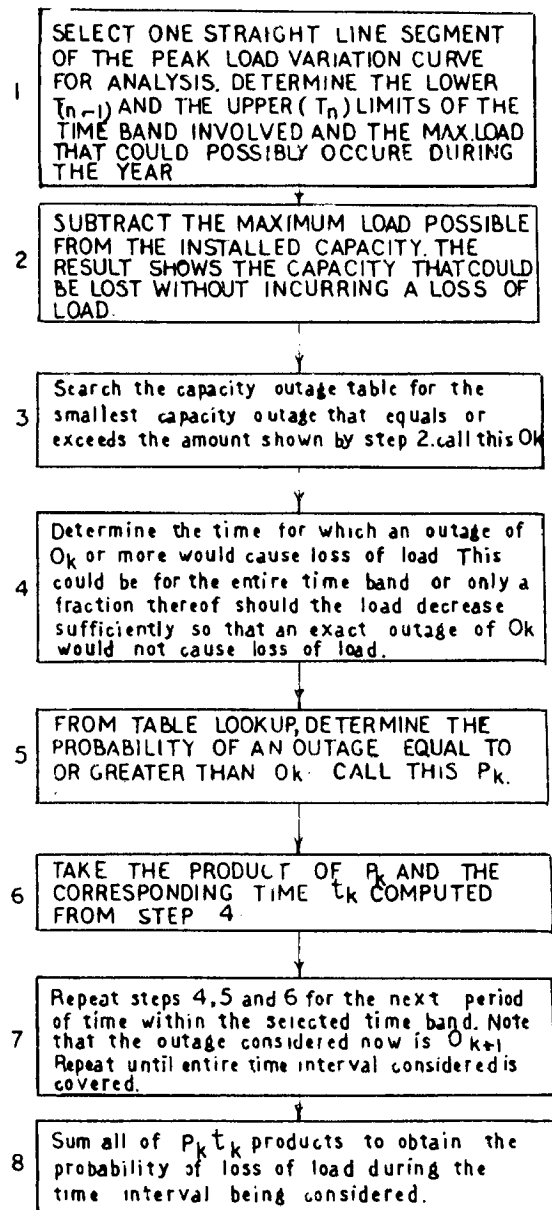


FIG. 2-2 DETERMINATION OF LOSS-OF-LOAD PROBABILITY

An acceptable probability of load in excess of available capacity may be taken as less than 1 day per year.

2.2.3. Loss of Energy Method - The first step is to compute capacity outage vs probability curve for the system. To find out loss of energy during each month, recourse is made to load duration curve of each month (See Figure 2.4).

From the Figure 2.4, we conclude that for outage of capacity less than OK_1 , there will be no loss of energy. For capacity outage exceeding OK_1 , the energy loss corresponding to say OK_1 is given by area A - in Figure 2.4. Similarly energy loss B corresponds to outage OK_2 and so on.

Depending upon probability of forced outage and capacity outage and the peaking energy terms (Area A, B and so on) the expected energy loss (ΣA) for the entire month is worked out for the load duration curve in question. The calculations are made for each month of the year taking into account the representative load duration curve for each month and anticipated reduction in net dependable capacity caused by equipment overhauls and factors such as reduced operating heads resulting from drawdown at hydro-electric plants.

By finding the expected energy loss during the entire year, the index of reliability defined below could be found for the year.

Expected index of reliability = (1-expected energy loss) /
System load ^{hrs} by available for the
month.

The index of reliability for capacity outage of OK should not be less than long time historical index. But the calculations must be repeated with other values of reserve if the calculated index of reliability is unsatisfactory. A curve similar to the curve shown in Figure 2.5 is plotted and necessary reserve corresponding to desired index of reliability found out.

2.2.4. Frequency - Duration Method - Expectations of forced outages are calculated by probability methods which give results in terms of frequencies, intervals and average durations. This method gives more significant results than the usual probability calculations which give results in terms of ratio. A probability of .01 indicates an outage for 1% of time but it does not state for example whether the outage occurs once in 100 days with a duration of 1 day or once in 500 days with duration of 5 days, the later being a more serious.

Excessive calculations are required for a completely diversified systems and it is not practicable to handle a complex diversified power system of modern type even with the aid of digital computer. For practical purposes it is thus found convenient and sufficiently accurate to subdivide the system into subgroups of units which could be considered reasonably uniform Sub-groups shall be so selected that the

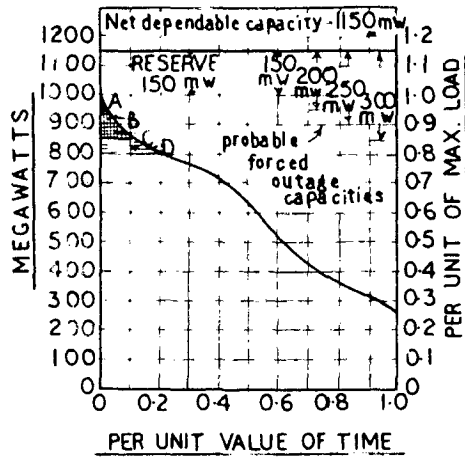


FIG. 2.4 LOAD DURATION CURVE

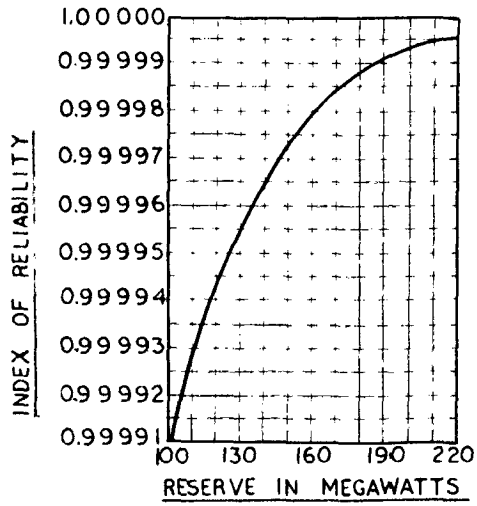


FIG. 2.5 RELIABILITY VERSUS RESERVE

deviation of individual sizes from the average within the group, as measured by the standard deviations will give results within the desired accuracy. The standard deviation is a measure of the discrepancy in the magnitude of multiple outage introduced by using multiples of average values for the group instead of the sum of their individual values in their possible combination.

Standard deviation is defined as

$$= \sqrt{\frac{\sum f (x - \bar{x})^2}{\sum f}}$$

Where \bar{x} = arithmetic mean, f frequency of occurrence and x are the individual values.

or
$$\sigma = \sqrt{\frac{\sum (x_n)^2}{n} - (\bar{x})^2}$$

Where n is the number of terms, x_n the individual value of each term.

For calculating the values in each sub-group the Appendix A entitled "Mathematical ^{methods} ~~calculation~~ ^{determination} in reserve/may be seen.

After the values for each sub-group have been calculated the results are ^{meshed} ~~marked~~ mathematically with each other to give the values for the composite system (See Appendix A)

These values of probability, average duration, interval between outages vs capacity outages are then plotted and depending upon the interval acceptable as basis of reliability i.e. either once in five ^{or ten} years, (^{1/10 years} a normally taken value), the duration and outages are found out and as

such the required reserve.

2.2.4.1. Effect of monthly Peaking Factor:

The figure number 2.6 depicts the variation of daily peak load within each month over several years for a typical system. The abscissa shows the difference between monthly peak and daily peak and the ordinate shows the probability that these load differences will be equal to or greater than the amount shown. The curve shows that for the year under consideration in 90% of the cases, it may be expected that the load will be at least 65 MW below the monthly peak. As a matter of judgement, the provision for the 10% cases where the load might be nearer to the peak is not justified. Therefore, the peaking factor may be subtracted from the reserve requirements worked out. If a higher monthly peaking factor was taken, not only the necessity of voltage reduction would arise more frequently but also the magnitude of reduction would be greater.

2.2.5. Comparison of 3 Methods - Relative merits of method 1, 2 & 3 lies in physical insight the results give and flexibility to changes in system conditions. Whereas methods 1 & 2 easily accounts for the system load variation method 3 gives more insight into the actual outage duration and interval of outage but it has no relation to system load curves. In each one of them the reliability is measured by a different index.

2.3 Maintenance Schedules Determination for Maximum Capacity :
Benefits:

Planned maintenance schedules are to be worked out so that the maximum capacity benefits are achieved and also maximum use is made of available hydro resources. The constant risk principle is followed to work out optimum maintenance schedules. It states that sum of monthly peak load and overhauling requirements and limitations (due to warmer condenser circulating water) is constant throughout the year. This scheme has the effect of providing essentially the same amount of capacity to take care of forced outages on any monthly peak day through^{out} the year. In contrast excess reserve will be available during most of the year if the reserve were planned with relation to only the maximum demand of the year.

Because ability to meet emergencies is essentially constant throughout the year with reference to 12 monthly peaks, this scheme makes the most efficient use of reserve capacity. The result therefore is minimum required total capacity for a given degree of service reliability.

The development of the constant risk principle has been a consequence of change in yearly load curve.

In practice the idea of constant risk principle can not be fully achieved for the following :

It is practically impossible to match the overhauling schedule with the load curve so as to achieve a perfect constant total of monthly peaks, overhaulings and limitations. This problem increases as large units with overhauling period of longer duration are added to the system. For this reason the sum of megawatt month figure for the required overhauling and maintenance of the various pieces of equipment is increased by a starting allowance. A stacking allowance of 15% applied to the calculated overhauling requirements is considered adequate and this percentage is generally used in calculating planned reserve requirement.

Second the equipment exposed to forced outages is not constant throughout the year because the equipment in operation varies on account of overhauling and limitations. As a consequence expected forced outages of a given frequency are not constant throughout the year, and therefore risk is not strictly constant even if the margin in capacity provided for forced outages is constant. Calculations shows that allowing for the variation in exposure results in about 5% in maximum derivations in magnitude of forced outages from the yearly average.

The constant risk principle is illustrated in Figure 2.7

In case during any month sum of monthly peak load, ^{limitations} ~~line tables~~, with no overhauling exceeds, the sum of monthly peak load and overhauling and limitation for the other months, the required capacity can be so evaluated that a

a somewhat smaller reserve in the month is compensated by a somewhat longer but constant reserve in the remaining eleven months. In this way the ability to meet forced outage expected upto any standard of reliability would still be maintained. However a caution is to be exercised and in no case the reserve margin for the month under consideration should be less than required to take care of outage of frequent sequence once in 2 or 3 years if the reliability standard followed for other months is once in five years.

In case the generation capabilities of the system are varying throughout the year, the principle of scheduling is to arrange scheduling such that water is utilised fully. In a developing system as ours, the load is restricted by the available capacity in the most critical period. For northern region this period is May/June when the reservoir is depleted, head is minimum. For maximum utilisation during this period no units should be on maintenance (neither hydro nor thermal). Because of reduced capability of hydro units at low head maximum number of hydro units should be available for same generation. Maintenance of thermal units will cause curtailment of capacity. From July to September i.e. during the period when reservoir is filling and water inflow is maximum again no hydro units are put on operation so that maximum number of units could be run to utilise the monsoon inflow & avoid spilling. This is ideal time for maintenance of thermal units.

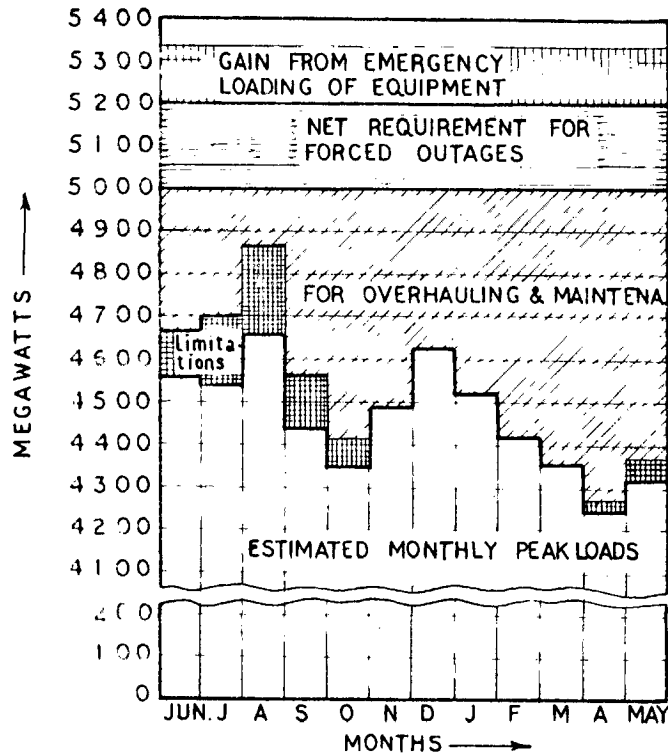


FIG.2-7 CONSTANT RISK PRINCIPLE - ILLUSTRATION

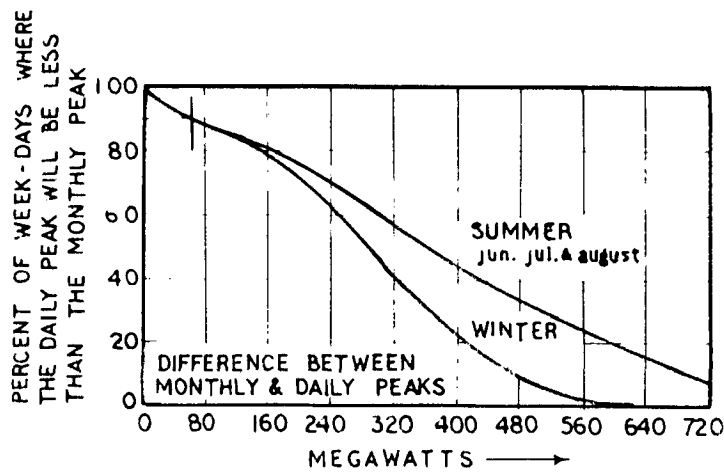


FIG.2-6 MONTHLY PEAKING FACTOR

CHAPTER - 3

3. TECHNIQUES AND METHODS FOR OPTIMISING
SCHEDULING IN HYDRO THERMAL SYSTEMS

3.1 General

The essential difference between the most economical operation of an all thermal system and a hydro-thermal system is that the most economical loading of the units in all thermal system depends only on the conditions that exist from instant to instant whereas, the most economical loading of the units of a combined hydro and thermal system depends on the conditions that exist over the whole of an operating period. If instant to instant economic criteria is followed for hydro-thermal systems too, it will be found that in the initial stages the whole of the load demand is to be met from hydro plants only. This is because incremental cost of hydro generator is negligible as compared to thermal cost. For the later part of operating period, the whole of the load demand would have to be met from thermal sources only as the water quantity would have been exhausted in the initial stages only if instant to instant economic criteria is followed for hydro thermal operation. Such an operation with limited quantity of water available over an operating period may be far from optimum. The problem of optimal hydro thermal scheduling is thus of dynamic optimisation i.e. one has to take in account the water usage policy over the operating period for optimum benefits. It is therefore necessary that the representation of the operation of a hydro thermal system must, in effect,

From October to February, the hydro reservoirs are full i.e. head is high but water inflow starts reducing. At this time the water should be preserved for maximum use in future as such this is ideal period for maintenance of hydro units. The reserve generating capacity is worked out by either of the method described and accordingly maintenance schedules are worked out.

be able to "remember" all the pertinent conditions that are expected to occur over the operating period and then to relate them to each other so as to result in the most economical operation.

3.2 Pertinent Factors:

3.2.1 For an all thermal system knowledge of the following is required for the determination of the optimum operation:

1. Incremental thermal production cost as a function of thermal generator, (Fig.3.1).
2. Incremental transmission loss characteristics.
3. Expected future load for appropriate short time intervals, preferably at least by hours over the operating period under study.
4. Units between which load is to be allocated.

3.2.2 For hydro-thermal systems, additional knowledge of following is required for determination of optimum operation.

1. Expected future pond inflows for the same short time intervals as the system load, also over the operating period under study.
2. Hydro plant characteristics, which may comprise knowledge of
 - a) generation of the units as a function of discharge and the head. This can include effect of intake losses when each unit is supplied by its own intake (Fig.3.2 & Fig.3.3).

- b) Pond elevation as a function of storage which later is a function of pond inflow, plant discharge and time, the evaporation losses and other losses in the reservoir area are neglected. (Figure 3.4).
- c) intake losses when two or more hydro units are supplied by a common intake.
- d) Tail race elevation as a function of the plant discharge (Figure 3.5).

These various characteristics can extend over quite a wide range from fairly flat to fairly steep system incremental thermal production cost curves, average hydro equivalent curves and tail race and pond elevation characteristics curves with rather high and steep and rather low and flat incremental hydro equivalent curves.

3.3 Type of Hydro-Plants and Their Comparison with Thermal Plant Characteristics:

Hydro plants may consist of a number of types of power plants as given below having different operational characteristics:

- 1) Run of river plants without pondage
- 2) Run of river plants with pondage
- 3) Reservoir plants or storage plants
- 4) Pumped storage hydro plants.

Water storage is stock pile of energy & similar to thermal plant coal pile with the essential difference that its stock is replenished only at specific interval and this

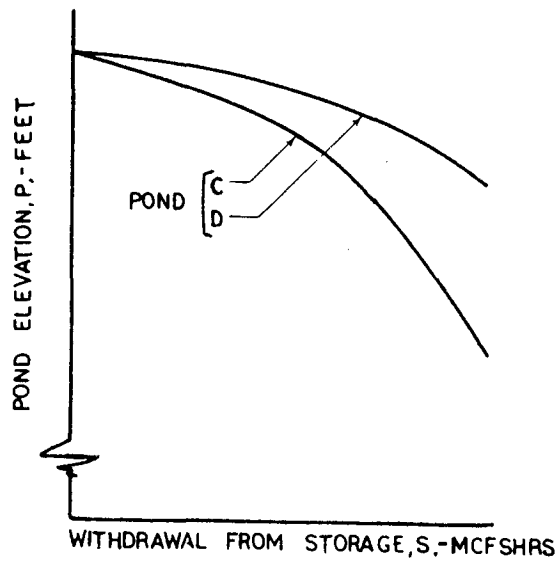


FIG. 3-4 TYPICAL PONDAGE CURVES OF HYDROELECTRIC GENERATING PLANTS

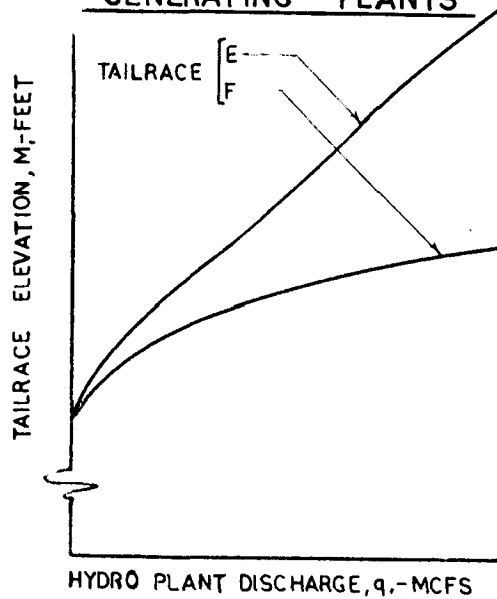


FIG. 3-5 TYPICAL TAILRACE CURVES OF HYDROELECTRIC GENERATING PLANTS

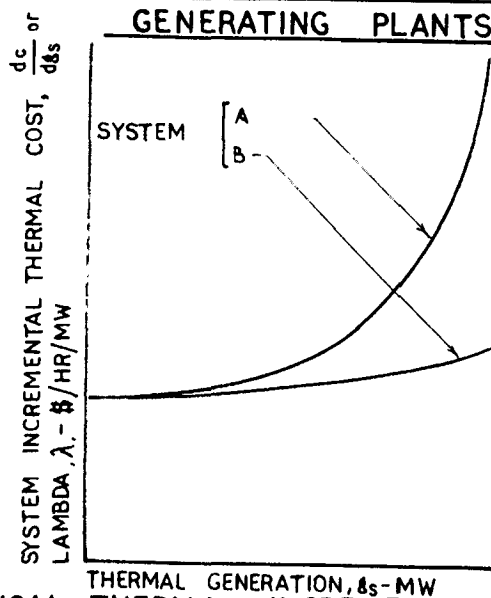


FIG. 3-1 TYPICAL THERMAL INCREMENTAL COST CURVES OF ELECTRIC POWER GENERATING SYSTEMS

depends upon the weather conditions. Once used water resources can not be replaced until the next natural period, but if not used, there natural replacement is necessarily wasted. This gives rise to a judicious estimation of the water energy based on probability approach and their use for optimum benefits accordingly.

3.3.1 Run of River Plants Without Pondage - Run of river plants produce electric power at almost constant production cost. But the river discharges may vary from season to season. In flood seasons there may be excess of water available for generation. In such plants the maximum possible energy must be generated by utilising the water available to the maximum extent possible without wasting it. This arrangement relieves the generation of power, to the extent of run of river plant maximum capacity in other plants where water may be easily conserved/fuel saved and better utilised in future.

3.3.2 Run Of River Plants with Pondage - In these plants pondage to take over load variation during any day in the week is provided. The plants are required to be scheduled in accordance with short term optimum methods described in this Chapter.

These hydro plants could also be used for peaking purposes i.e. the daily inflow is most economically used over the daily peak load period. This means that hydro plants discharges over such an operating period is much larger than the inflow so that a loss of pond elevation and consequently a loss of head will always result as compared with maintaining

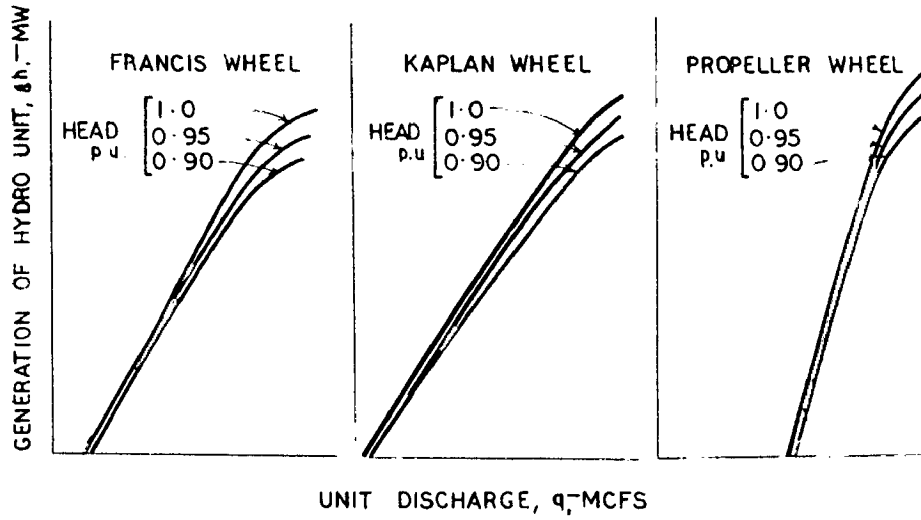


FIG. 3-2 POWER DISCHARGE CURVES OF HYDROELECTRIC GENERATING UNITS

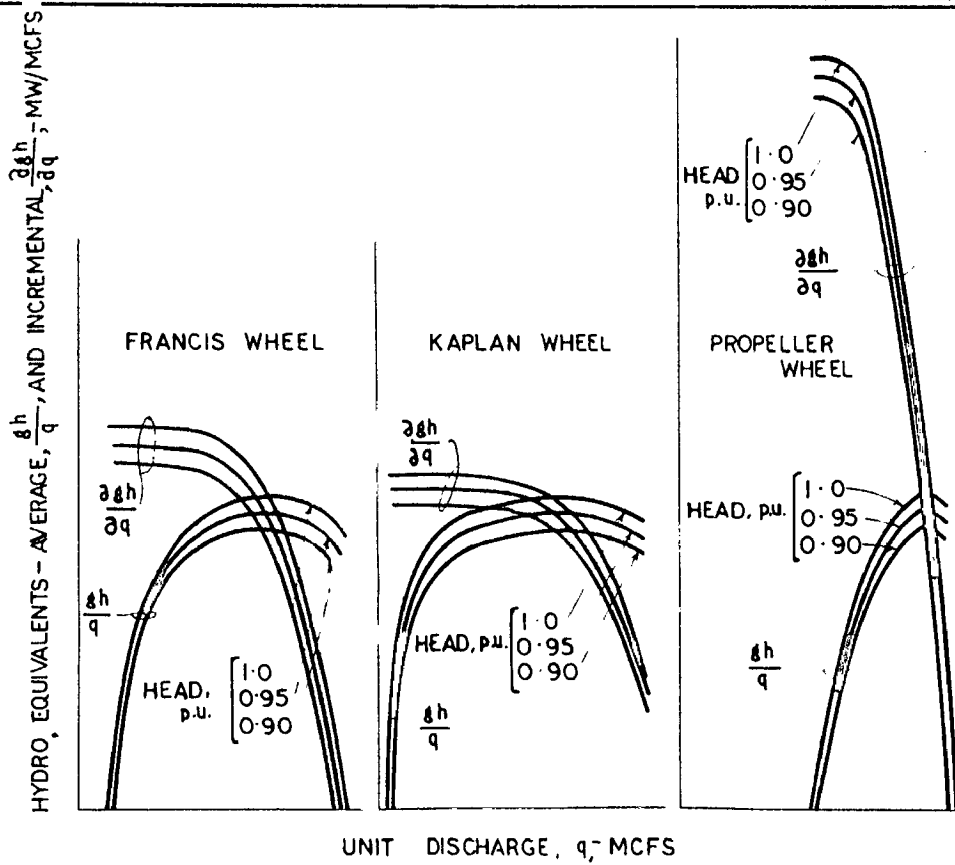


FIG. 3-3 AVERAGE AND INCREMENTAL HYDRO EQUIVALENT CURVES OF HYDROELECTRIC GENERATING UNITS

a full pond. If we assume which generally is the case that the discharges of the hydro units at the maximum efficiency loadings taking into account the variation of such discharges with head is commensurate with the plant total discharge, if we should now load the hydro units for maximum efficiency the head would be lower all day than if they were loaded at less than for maximum efficiency during the earlier part of the day and then gradually increased to loadings at maximum efficiency and then beyond towards the end of the day. Some degree of this latter type of operation results in optimum co-ordination.

3.3.2 Storage Plants - The hydel stations where water is drawn from a storage reservoirs, the regulation of water for generation of power should be as economical as possible. In other words, the generating units must be loaded to their maximum efficiency for longer duration of a day if water conditions permit. Further the load allocation on the plants should be done keeping the availability of water in different season in view. During flood seasons (Monsoon) the plants may be operated at reduced efficiency if necessary even though the incremental water rate of flow is high in order to utilise the excess water which is likely to go waste. For facilitating this, the drawal of water from the reservoir is increased in advance depending upon the probable water inflow estimated by past experience and by taking precipitation and surface water flow in the catchment area of the reservoir into consideration. The pond shortage likely due to advance withdrawal of excess of water is replaced with excess river flow which would otherwise have been spilled over.

Storage plants are also ideally suited for working on peaking units provided sufficient balancing storage can be provided downstream of the power plant to even out irrigation releases in case of multi-purposes schemes.

3.3.4 Pumped Storage Plants - They are primarily meant for taking short term and unusual peak load on a power system and for serving as an instantly available reserve capacity for the system. The basic principle of their operation is to store the surplus Electrical Energy either hydro & thermal generated by a power plant or available in a system in off peak periods in the form of hydraulic potential energy with the aim to regain it in periods when the peak demand on the system exceeds the total capacity of generating plants. There is no pumped storage plant at present working in India.

3.4 Advantages of Hydro-Thermal Systems :

Power plants at present consist of hydro & thermal systems. This is because it has been found that electrical energy at minimum overall cost could be derived if both hydro and thermal power plants are developed hand in hand. This also results in effective utilisation of water resources. Thermal power plants have the advantage of less initial cost, less time for implementation, flexibility in siting and better availability as compared to the hydro plants. Hydro Power plants are quick to start and shut down, are better in respect of spinning reserves and are ideally suited for providing reserve and peaking capacity. Further annual maintenance expenses, fuel expenses are low. The units are suitable for pumped storage operation, stable operation, are more flexible in load sharing and annual establishment

expenses are low. Hydro plants have less forced out age rates and more life expectancy. Because of these contrasting properties of hydro & thermal systems, the best results are obtained when both these power plants are harmoniously blended in power systems.

3.5 Optimum Scheduling of Hydro - Thermal Systems :

Factors that are to be considered in the operation of a system with both thermal and hydro units depend largely on the degree of storage development and the proportion of the two types of supply.

The large and efficient thermal stations are best utilised for base load and hydro plants with their favourable characteristics are best utilised in meeting the peak loads.

The spinning reserve requirements of the system are most advantageously supplied by the hydro plants.

When stream flows are high some of the hydro plants may be operated continuously saving coal in thermal generation. Even if peak demand thermal is necessary to supplement, it should be done such that variation on the thermal energy are minimum. Conversely, when stream flow is less, thermal stations are worked longer to store power in the reservoir to maintain higher heads and provide higher peaking capacity. It is in this connection that the economic utilisation of pumped storage scheme is best utilised. On these considerations listed here, the quantity of water to be utilised over a

day/week is fixed. Before allocating the generation among various units, it must be decided as to which are the units to be operated in a power plant.

3.5.1 Unit Commitment - Unit participation programme helps the load despatcher to decide which machines shall be operated and when a machine or machines shall be connected ^{or} ~~dis~~ - connected from the system. The principal consideration in deciding number of units to be operated in power plants ^{is} that during any hour sufficient units must be run to carry the load and provide the spinning reserve. Whereas shutting down an inefficient unit at minimum load means that the load could be transferred to more efficient unit, the decision to shut down a unit depends upon number of factors including the following :-

- i) The number of hours the unit can be shut down before it is required again.
- ii) The cost of starting up the unit
- iii) The relative efficiency of the unit to shut down compared to the efficiency of the unit left running.

The unit commitment programme is decided by model study of the system by computing the total production cost by calculating the fuel cost, the cost of starting up and the cost of shut down if any, for various combinations of m/c to minimise the total production cost.

3.6 Short Range Scheduling Methods of Hydro Thermal System :

The various optimisation techniques that have been successfully employed for solving the short range optimal scheduling hydro thermal problem i.e. dividing the total generation required to meet system load between hydro & thermal system so that minimum thermal cost is obtained over the operating period and constraints on equipment rating are met with and quantity of water is utilised fully for hydro resources, are

- i) Calculus of variations
- ii) Discrete Maximum principle
- iii) Dynamic programming

In calculus of variation methods, performance differential equation for minimum cost are written both for hydro sub-system and thermal sub-system. These equations are solved for optimum scheduling by making use of physical system equality constraints (i.e. generation and losses must be equal to system load during any hour) and also that total quantity of water used equals the quantity of water available for use over the day. This method becomes extremely complicated when constraints on system variables (like minimum and maximum value of hydro/thermal generation) are imposed, the fundamental assumption of this method being that independent and arbitrary variations can be made in system variables.

Dynamic programming method is quite versatile in solving problem of this nature and over ³³cause the difficulties of method of calculus of variations listed above. It operates the system in every possible combination of hydro and thermal

generations from hour to hour and then selects the combination of hour to hour scheduling that results in minimum thermal cost over the operating period. It however suffers from the disadvantage that a very extensive search has to be carried out which results in more time and also requirements of memory increases considerably with the dimensions of state variables.

Much attention has recently been given to the method of Pontrygin's maximum principle. This is applicable for continuous systems described by first order differential equations. A finite number of discontinuities and constraints on system variables can be easily handled.

The above 3 methods are described in detail below.

3.6.1. Short Range Optimum Scheduling of a Combined Thermal Hydro Electric Power System - Variational Techniques

The optimum scheduling problem with transmission losses rigorously considered is defined as follows:

The objective is to minimise the total fuel cost over the given operating period of time while using desired amount of water from the hydro plants

i.e.

$$\sum_n \int_0^T F (PS_n) dt = \text{minimum} \quad \dots(1)$$

where $F(PS_n)$ = Production cost per unit time of thermal plant n as a function of its output PS_n in MW
 $n = 1, 2, 3, \dots, \alpha$, α denoted total no. of thermal plants in the system.

PS_n = Steam generation of nth unit in M.W.

PH_j = Hydro " jth "

$$\sum_{n=1}^{\alpha} PS_n + \sum_{j=1}^{\beta} PH_j - P_L = P_R \quad \dots\dots(2)$$

$J = 1, 2, \dots, \beta$ the total number of hydro plants in the system, P_L is system losses & P_R the load demand in MW.

A specific amount of water is to be used over the period at each plant.

$$\int_0^T q_j dt = k, \text{ a constant} \quad \dots\dots(3)$$

q_j = discharge of jth unit

If PH_j is considered function of discharge alone (i.e. constant head).

$$q_j = q_j (PH_j) \quad \dots\dots(4)$$

The co-ordination equation for optimum scheduling of hydro thermal systems are given by Equation (5) & (6)

$$\frac{dF_n}{dPS_n} + \lambda \frac{\partial L_T}{\partial PS_n} = \lambda, \quad n = 1, 2, \dots, \alpha \quad (5)$$

$$r_j \frac{dq_j}{dH_j} + \lambda \frac{\partial L_T}{\partial PH_j} = \lambda \quad \dots\dots(6)$$

Where $J = \alpha + 1, \dots, (\alpha + \beta)$

where

$\frac{dF_n}{dPS_n}$ = incremental fuel cost of thermal plant n, in Rs. per MW hours.

$\frac{\partial L_T}{\partial PS_n}$ = Incremental transmission loss of thermal plant n.

$\frac{dq_j}{dPH_j}$ = Incremental water rate at hydro plant j, in cft/Mwh.

$\frac{\partial L_T}{\partial PH_j}$ = incremental transmission loss of hydro plant j

λ = Incremental cost of received power in Rs/Mwh

r_j = Constant which effectively convert an incremental water rates into incremental plant costs.

α = Number of thermal plants

β = Number of Hydro Plant

r_j is also Lagrangian multiplier which effectively converts the incremental water rate into equivalent plant cost r_j is chosen so that desired amount of water is used.

If the $\frac{dF_n}{dPS_n}$ & $\frac{dq_j}{dPH_j}$ terms in equation (5) & (6) are

assumed linear functions of output power they can be written in the form

$$\frac{dF_n}{dPS_n} = F_{mn} PS_n + f_n \quad \dots\dots(7)$$

$$\frac{dq_j}{dPH_j} = q_{jj} PH_j + q_j \quad \dots\dots(8)$$

Transmission losses could be expressed by equation

$$L_T = \sum_m \sum_n P_m B_{mn} P_n$$

Where P_n are the plant loadings & B_{mn} loss formula coefficients thus (5) & (6) becomes

$$F_{mn} PS_n + f_n + 2\lambda \left(\sum_m B_{nm} PS_m + \sum_j B_{nj} PH_j \right) = \lambda \quad \dots\dots(9)$$

$$r_j q_{jj} PH_j + r_j q_j + 2\lambda \left(\sum_k B_{jk} PH_k + \sum_m B_{jm} PS_m \right) = \lambda \quad \dots\dots(10)$$

The eqns are solved for different values of λ & r_j such that the total generation - losses = desired load and that quantity of water used in hydro plant per the period is the desired quantity.

Flow chart for solution of equation is given in Figure 3.6

Effect of Variation of Head:

For variable head problem the water conversion coefficient becomes a function of time instead of a constant as in the fixed head derivation. The calculations and economic schedule for this case of variational head hydro electric problem requires solution of non-linear differential equation.

Given below is the Ricard equation for a loss less system considering effect of head variation⁽¹⁾.

$$\lambda \left(\frac{\partial q}{\partial g} \right)_n = r = r_0 \exp. \left(\int_0^t \frac{\partial g}{\partial H} \frac{dt}{A} \right) \dots\dots(11)$$

Where λ = system incremental thermal cost at time t in Rs./h/mw

gh = generation of hydro plant at time t in Rs/h/MW

q = Hydro plant discharge at time t in thousand ft^3/S

r = water Conversion Coefficient at time t in Rs/hr/ thousand ft^3/S

r_0 = Base water conversion coefficient in Rs/hr/thousand ft^3/S .

H = Head on the hydro plant at time " t " in ft.

A = Area of pond at time t in sq.ft.

⁽¹⁾ Inside Hydro-Thermal Co-Ordination, C.W. Watchorn-IEEE transaction PA & S, Vol.86 - January 1967, pp.106.

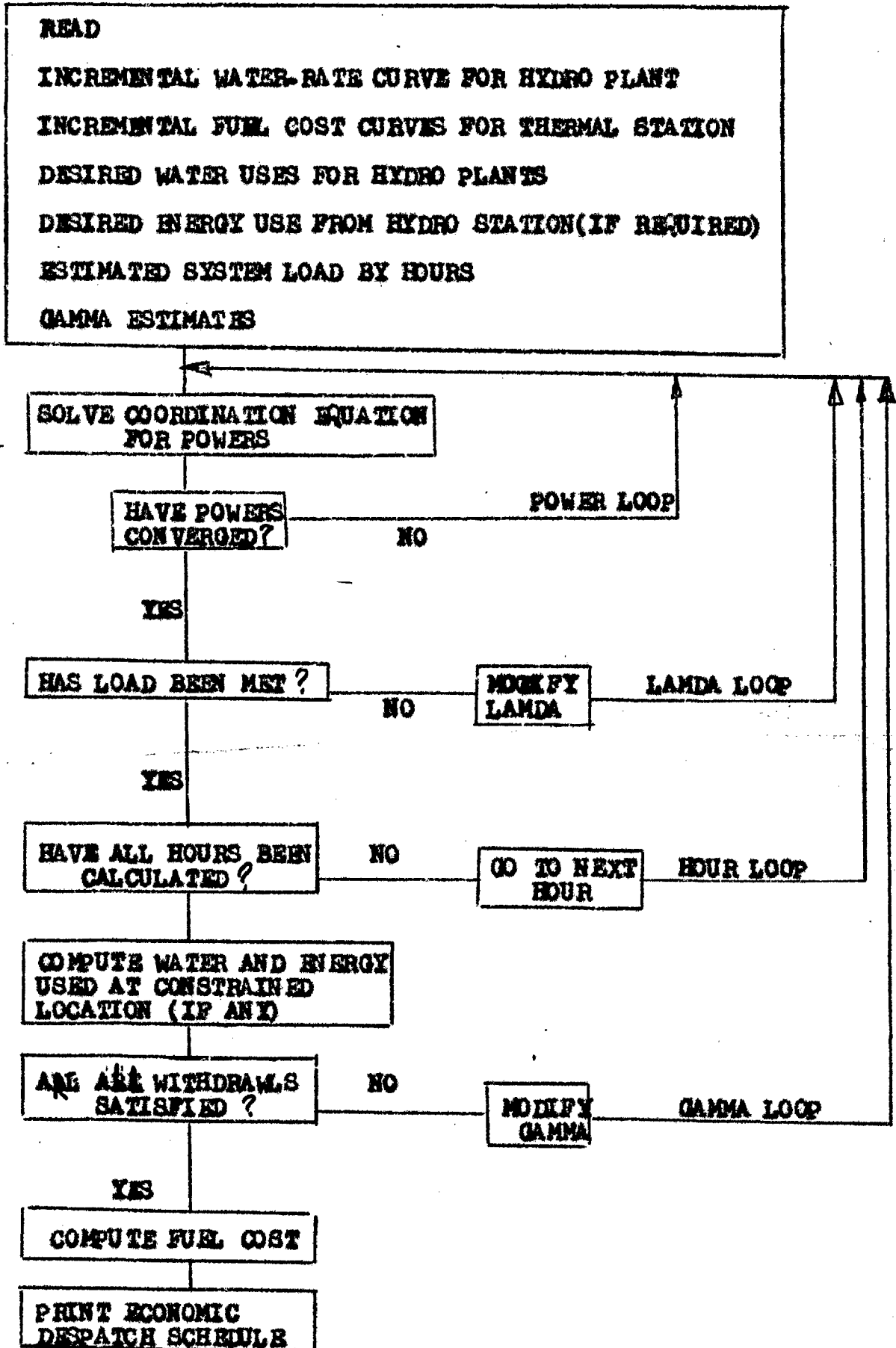


FIGURE - 3.6 - FLOW CHART FOR HYDRO THERMAL SCHEDULING

The partial derivative $\left(\frac{\partial gh}{\partial q}\right)_n$, the nominal incremental hydro equivalent (before adjustment for the effect of incremental head changes with time as a result of withdrawals from the storage) is substituted for partial derivative $\frac{\partial gh}{\partial q}$, given in the statement of Ricard Equations. This substitution is made so as to allow for the effect of changes in tail race and intake losses in addition to the effect of changes in the unit characteristics with changes in plant discharges. This partial derivative then accounts for all the effects of changes in plant discharges except that of change in pond elevation. The RHS of equation (11) is then intended to account for the effect of the incremental changes in head with changes in storage. The partial $\frac{\partial g}{\partial H}$ in the integrand of the exponential depends on the characteristics of the units in the hydro plant and is always negative. Its value at time "t" depends on the hydro plant loading at that time. The value of the area of the Pond A, also at time "t" is always positive and depends on the elevation of pond at that time being smaller for lower pond elevation and longer for the higher.

This means that the integrand is always negative with the result that the integral exponent is always negative except for $t = 0$, when it equals zero & $r = r_0$. The exponent then, except when the hydro plant is shut down becomes a larger negative number with time. Its rate of change is governed by the unit and pond characteristics and the loading of the plant. It increases faster with heavier plant loadings and the smaller pond areas that results from lower pond elevation,

when the plant discharges are larger than the inflow and is constant when the plant is shut down. This then means that r is always smaller than r_0 during an operating period always becoming smaller with time, which is shown by decreasing value of r between t_0 and t_e in the lower portion of Fig. 3.7. This requirement of decreasing value of r is satisfied with initial values of both λ and $(\partial gh / \partial q)_n$ being on longer side and then both decreasing with time. Figure 3.1 shows that this condition for λ is met initially with larger value of thermal generation and then with decreasing values with time. Figure 3.3 shows indirectly that the conditions for $(\partial gh / \partial q)_n$ is met initially with smaller values of unit discharge and consequently smaller value of hydro generation as seen from Figure 3.2 and then with increasing value of hydro generation with time. This indirect relationship between $\partial gh / \partial q$ of Figure 3.3 and $(\partial gh / \partial q)_n$ is because figure 3.3 is shown for constant head on the units whereas $(\partial gh / \partial q)_n$ is for the plant which is subject to head changes with changes in plant discharge that must also be taken into account. They are related by the following:

$$\left(\frac{\partial gh}{\partial q}\right)_n = \frac{\partial gh}{\partial q} - \frac{\partial gh}{\partial H} \left(\frac{dI}{dq} + \frac{dM}{dq}\right) \dots\dots(12)$$

where I is intake losses at time " t " in ft., M is tail race elevation at time " t " in feet.

A novel feature of the Ricard equation is that because of its continually providing values of r over the whole of applicable operating period, it determines a single path along which the operation must proceed. This operation

will always be optimum if the amount of water ~~is~~ used is equal to that which is to be used in that operating period. In the event of required and desired water uses are not equal, new values of r_0 are tried until they are.

The applicability of Ricard's equation is limited to the co-ordination of hydro thermal system for a single day at a time at the most because of its continually decreasing value of r . This is because r is discontinuous and takes on a sudden increase in value at the time the operation for one day ends and that for the next begins which may be defined as the time, usually in the morning when the pond is at its maximum elevations. This characteristic is illustrated in Figure 3-7. This characteristics present no difficulty so long as the optimum operation requires that pond be refilled each day. However if the optimum operation requires partial refill during the day or from day to day the same difficulty of discontinuous r is experienced. This is shown in Figure 3-8

Dynamic programming method and other conventional methods have been found satisfactory to avoid the above difficulties of the Ricard equation but they all lack a way of knowing how far we are from the optimum operation with the use of any of these methods.

A modification to overcome the above predicament is the properties that true incremental water value over any valid operating period is constant. Its derivation is based on the concept that true incremental hydro equivalent dgh/dq , the total derivative of the generation of a hydro

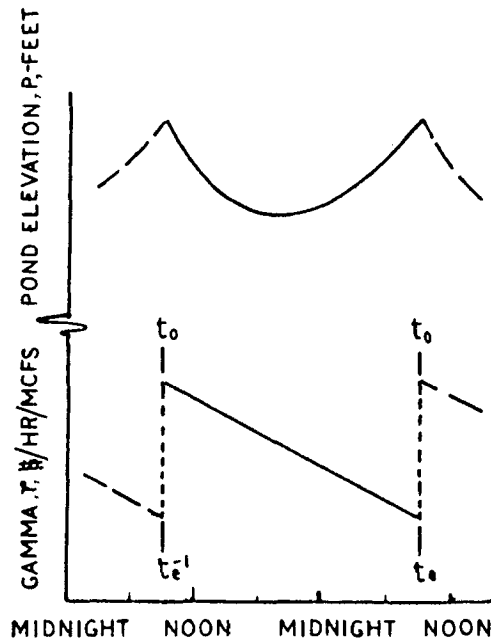


FIG. 3-7 VARIATIONS IN GAMMA AND POND ELEVATION AS FUNCTIONS OF TIME FOR OPTIMIZED HYDROTHERMAL COORDINATION WITH DAILY REFILL OF THE POND AND NO PARTIAL REFILL DURING THE DAY

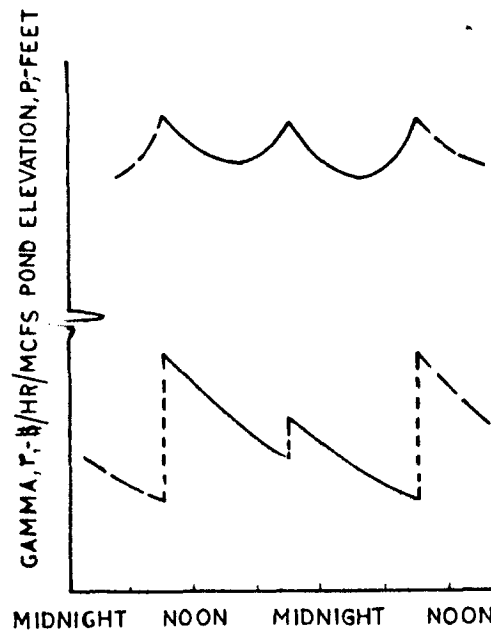


FIG. 3-8 SAME AS FIG. 3-7 EXCEPT WITH PARTIAL REFILL DURING THE DAY

plant with respect to discharge, should include the effect of incremental changes in head with time as a result of changes in pondage with changes in plant discharge.

This requires that expression

$$\frac{\partial gh}{\partial H} \frac{dP}{dP} = (q - r) \frac{\partial gh}{\partial H} \frac{dq}{ds} / q \dots\dots(13)$$

be added to nominal hydro equivalent $(\partial gh / \partial q)_n$ in order to obtain dgh/dq , the time hydro equivalent

$$\frac{dgh}{dq} = \frac{\partial gh}{\partial q} + \frac{\partial gh}{\partial H} \left\{ \frac{(q-r) \frac{dP}{dS}}{q} - \frac{dI}{dq} - \frac{dM}{dq} \right\} \dots\dots(14)$$

for a convention hydro development &

$$\frac{dgh}{dq} = \frac{\partial gh}{\partial q} + \frac{\partial gh}{\partial H} \times \left(\frac{q \frac{d}{ds}(P-M) - r_1 \frac{dP}{ds} - r_2^2 \frac{dM}{ds}}{q} \right) \dots\dots(15)$$

For a pure pump storage project where

P = Pond or upper reservoir elevation at time t in ft.

S = Withdrawals from storage of the pond or upper reservoir & the addition to the storage of the lower reservoir in thousand ft³/S/hours at time t as compared with zero withdrawal for a full pond or upper reservoir and zero addition for an empty lower reservoir.

r = rate of pond inflow in thousand ft³/S at time t with sub-script 1 & 2 designating respectively the upper and lower reservoirs for a pump storage project.

Then with e = e (gs)(16)

$g_1 = L(t) - gh(q, H)$ (17)

$q = q(t)$ (18)

we have $\frac{dc}{dq} = \frac{dc}{dgs} \cdot \frac{dgs}{dq} = \lambda \left(\frac{L}{q} - \frac{dqh}{dq} \right) = N \dots (19)$

where c is cost of system steam generation in Rs/hr.

L = load at time t in Mw

N = True incremental water value in Rs/h/thousand ft^3/S

Which have been postulated as being constant over any valid operating period.

3.6.2 Discrete Maximum Principle :

As an introduction to this method the economic operation of an extremely simplified model system which may be called the fundamental system is considered. A general multi reservoir system can be decomposed into a combination of such fundamental system by recursive relaxation approximation technique and analysis of power system can then be carried out.

The fundamental system consists of two plants, one hydro with reservoir and one thermal, jointly supplying a load located between two stations.

Available water storage is to be utilised in such a way that using given water inflow, the required power demand is satisfied and finally the specified value of storage is obtained and in this process the total fuel cost $\sum_{n=1}^N F^n(G^n)$ is minimised, other quantities i.e. hydro generation p^n and thermal generation G^n , water discharge ϕ_n satisfying the constraints put on them.

The total optimisation time interval is divided into N sub-intervals.

The fuel cost of the thermal plant during any interval is assumed to be :

$$F^n = a_1 G^n + a_2 (G^n)^2 \quad \dots\dots(20)$$

$$\text{Incremental fuel cost } \lambda^n = \frac{dF^n}{dG^n} = a_1 + 2a_2(G^n) \quad \dots\dots(21)$$

The water continuity relation in per unit system at the reservoir is given by

$$X_1^n = X_1^{n-1} + J^n - Q^n \quad \dots\dots(22)$$

$$\text{or } X_1^n = T_1^n (X_1^{n-1}, Q^n)$$

where T_1^n is the transformation operation at the n th interval
The hydro power for conventional hydro plants may be approximated in per unit system as

$$P^n = H \left(1 + C/2 (X_1^{n-1} + X_1^n) \right) (Q^n - q) \quad \dots\dots(23)$$

where H = basic head

C = water head correction factor for change in water storage.

q = non effective discharge

From equations (22) and (23) we get

$$P = H(1+C/2)(2X_1^{n-1} + J^n - Q^n)(Q^n - q) \quad \dots\dots(24)$$

The power balance equation will be

$$P_R^n - G^n - P^n = 0 \quad \dots\dots(25)$$

Introducing an another variable X_2^n representing the partial sum of the total fuel cost from the first to the n^{th} sub-interval, the following relation is obtained:

$$X_2^n = X_2^{n-1} + F^n \quad \dots\dots(26)$$

or $X_2^n = T_2^n (X_2^{n-1}, Q^n)$

where $F^n =$ fuel cost during n^{th} interval

The objective of the problem is to minimise X_2^n or to maximise $-X_2^n$

Applying discrete maximum principle, the Hamiltonian Function can be written as

$$H^n = z_1^n T_1^n + z_2^n T_2^n \quad \dots\dots(27)$$

or $H^n = z_1^n (X_1^{n-1} + j^n - Q^n) + z_2^n (X_2^{n-1} + F^n)$

The adjoint vector is

$$z_1^{n-1} = \frac{\partial H^n}{\partial X_1^{n-1}} = z_1^n + z_2^n \frac{\partial F^n}{\partial X_1^{n-1}} \quad \dots\dots(28)$$

$$z_2^{n-1} = \frac{\partial H^n}{\partial X_2^{n-1}} = z_2^n \quad \dots\dots(29)$$

From the principle of optimality, it can be shown that for even adjoint variable the corresponding state variable of which, is not specified, we have

$$z_2^n = -1 = \text{constant} \quad \dots\dots(30)$$

Substituting equation (30) in equation (28) we get

$$z_1^n = z_1^{n-1} + \frac{\partial F^n}{\partial x_1^{n-1}} \dots\dots(31)$$

$$z_1^n = z_1^{n-1} + \lambda^n \left(\frac{\partial G^n}{\partial p^n} \right) \left(\frac{\partial p^n}{\partial x_1^{n-1}} \right) \dots\dots(32)$$

The optimal reservoir policy can be obtained by the relation
(Q^n is control variable)

$$\frac{\partial H^n}{\partial Q^n} = - z_1^n - \left(\frac{\partial F^n}{\partial G^n} \right) \frac{\partial G^n}{\partial p^n} \frac{\partial F^n}{\partial Q^n} = 0$$

$$z_1^n = - \lambda \frac{\partial G^n}{\partial p^n} \cdot \frac{\partial p^n}{\partial Q^n} \dots\dots(33)$$

Differentiating equation 24 w.r.t. x_1^{n-1} and Q^n and differentiating 5 w.r.t. p^n , we get

$$\frac{\partial p^n}{\partial x_1^{n-1}} = \frac{HC}{H} (Q^n - q)$$

$$\frac{\partial p^n}{\partial Q^n} = H (1 + C/2 (2 x_1^{n-1} + j^n - 2 Q^n + q))$$

$$\frac{\partial G^n}{\partial p^n} = -1$$

Substituting these in equation (32) and (33) we get

$$z_1^n = z_1^{n-1} - \lambda^n HC (Q^n - q) \dots\dots(34)$$

$$z_1^n = \lambda^n H (1 + C/2 (2 x_1^{n-1} + j^n - 2 Q^n + q)) \dots\dots(35)$$

equating 34 & 35, we get $\lambda^n = \frac{z_1^{n-1}}{H(1+C(x_1^{n-1}+j^n-q)/2)} \dots\dots(36)$

Equations 34 & 35 are the two required relations solving these two numerically an optimal reservoir operating policy can be determined.

There would be no serious problem for the care of constraints on control variables, since the optimization is carried out only by the maximization of the Hamiltonian function within the admissible control region. In case either of the values of Q^n and G^n found from equations 3* & 3/ ^{violates constraints} imposed on it, its value is made equal to the bounded value and the other variable is suitably changed to satisfy the load demand. The new values of X^n , G^n , P^n and Q^n so found are then substituted in equation 3* to get the value of adjoint variable in the interval under consideration.

3.6.3. Hydro Thermal Economic Scheduling - By Dynamic Programming :

The size of the hydro thermal scheduling problem depends on number of hydro & thermal stations in the system and time over which the operating hourly schedules are to be found. For example in case of two fixed head hydro station, 2 variable head hydro station and three thermal stations, if operating hourly schedules are to be found over a day, $7 \times 24 = 168$ number of variables are to be found. This is because of the constraints on water usage over the operating period, each hour operation cannot be considered separately. Enlarging the model to include more hydrostations and interconnections will be cause increase the number of variables. Thus from practical view point it is absolutely necessary to have a scheduling method which is computationally efficient.

By using recursive methods of solution such as dynamic programming, scheduling problems can be brought off as 24 optimisation problem each in seven variables instead of one optimisation problem in 168 variables. While this is a tremendous improvement it still leaves a computational problem of unmanageable size.

A further simplification of the problem is achieved by considering the overall system as consisting of two sub-systems - one all thermal and one all hydro. At each hour there is then three dimensional scheduling problem in all thermal systems (i.e. generation is to be allocated to three thermal stations) and a four dimensional scheduling problem in all hydro systems. If the two parts can be scheduled separately and combined to provide a solution to the overall problems a further reduction in magnitude of computation will have been achieved.

The extension of the basic procedure for combining hydro thermal schedules to the case of a system with more than one fixed head hydro stations is also made. The major advantage of this extension is that multi-dimensional problem of scheduling several fixed head hydro stations can be reduced to scheduling each hydro plant separately using one dimensional incremental dynamic programming as described below.

Statement of the Problem :

Day is divided into 24 hourly intervals numbered 1,2,..

24.

- D_1 = Average hourly demand for power during hour 1
 S_1 = Average hourly steam generation " "
 Q_1 = Average hourly discharge through the hydro plant during hour 1.
 H_1 = Hydro generation corresponding to Q_1
 L_1 = Average line losses during hour 1
 $F(S_1)$ = Fuel cost of supplying S_1 units of steam generation during hour 1.

$$H = H(Q) \quad \dots\dots(37)$$

$$L = a S^2 + b H^2 + c SH \quad \dots\dots(38)$$

Cost F is assumed to be an increasing strict convex differentiable function of "S" so that dF/dS is strictly increasing function of S . Strictly convex function is that for which a line joining any two points on the characteristics lies above the characteristics.

It is desired to find S_1, S_2, \dots, S_{24}

$Q_1, 1 = 1, 24$ so that

$F(S_1) + F(S_2) + \dots + F(S_{24})$ is minimum. Where

$$S_1 + H_1 = D_1 + L_1 \quad \dots\dots(39)$$

$$Q_1 + Q_2 + \dots + Q_{24} = V \quad \dots\dots(40)$$

(V = Value of water available for use over the day)

$$0 \leq Q_1 \leq Q_{\max} \quad \dots\dots(41)$$

$$1 = 1, 24$$

$$0 \leq S_{\min} \leq S_1 \leq S_{\max}, \quad 1 = 1, 24 \quad \dots\dots(42)$$

Method of Solution:

There are many ways of dividing system hourly outputs into thermal & hydrogeneration. Different division will result in different total fuel cost over a day. In determining the optimum mode of operating, successive hydro schedules will be calculated such that the corresponding cost of thermal generation decreases monotonically. The procedure will come to an end when successive thermal cost differ by amounts which are sufficiently small that they may be neglected and also that there is no change in state.

The procedure consists in assuming a trial schedule which satisfy equations (38) to (42) and then finding the better schedule so that the thermal cost decreases.

Know Q_2^0 (initial trial schedule)

$H_1 = H(Q_1^0)$ is calculated

Corresponding S_1^1 is calculated from solving equation

$$a S_1^2 + (e H_1^0 - 1) S_1^1 + (D_1 - H_1^0 + b H_1^0) = 0 \dots (43)$$

(obtained from equation 38 & 39).

The constraints (42) on S_1 ensure that two roots of equation are real & positive. The physically correct root is the one satisfying the constraints.

Knowing S_1^1 , $F(S_1^1)$ and $C(S_1^1)$ (DF/dS_1^1) are obtained by substitution. The value of $G(H_1^0, S_1^1)$ defined below are calculated next.

Iteration 1 : next step is to improve the initial division of load between hydro & thermal system from (39) we get

$$\begin{aligned} dS + dH &= dL \\ &= \frac{\partial L}{\partial S} \cdot dS + \frac{\partial L}{\partial H} \cdot dH \end{aligned} \quad \dots\dots(44)$$

$$\text{or} \quad dS = - \frac{\left(1 - \frac{\partial L}{\partial H}\right)}{\left(1 - \frac{\partial L}{\partial S}\right)} dH \quad \dots\dots(45)$$

Let Q_1 be any proposed alternative to Q_1^0 where the change

$$H(Q_1) - H(Q_1^0) = H_1 - H_1^0$$

Small From 45 it follows that corresponding change in thermal output can be approximated by

$$\Delta S_1^1 = - \frac{\left(1 - \frac{\partial L}{\partial H}\right)(H_1^0, S_1^1)}{\left(1 - \frac{\partial L}{\partial S}\right)(H_1^0, S_1^1)} (H_1 - H_1^0) \quad \dots\dots(46)$$

In practice the factor multiplying $(H_1 - H_1^0)$ in equation (46) is always negative so that increase in hydro generation during hour 1 will result in decrease in thermal generation, whereas decrease in hydro generation will result in increasing thermal generation. The change in hydro generation $(H_1 - H_1^0)$ is worth approximately.

$$\frac{C(S_1^1) S_1^1}{1} = G(H_1^0, S_1^1) (H_1 - H_2^0) \quad \dots\dots(47)$$

$$\text{where } G(H_1^0, S_1^1) = + C(S_1^1) \frac{\left(1 - \frac{\partial L}{\partial H}\right)(H_1^0, S_1^1)}{\left(1 - \frac{\partial L}{\partial S}\right)(H_1^0, S_1^1)} \quad \dots\dots(48)$$

and $G(H_1^0, S_1^1)$ are the constants, called incremental worth of power at bus bar of Hydrostations.

If $H_1 < H_1^0$,(47)

is positive whereas if $H_1 > H_1^0$,(47) is negative. Thus the alternate hydro schedule Q_1 should be so chosen so as to minimise $\sum_{i=1}^{24} C(S_1^1) \Delta S_1^1$ or equivalently to maximise

$$\begin{aligned} & \sum_{i=1}^{24} G(H_1^0, S_1^1) (H_1 - H_1^0) \\ & = \sum_{i=1}^{24} G(H_1^0, S_1^1) H_1 - \sum_{i=1}^{24} G(H_1^0, S_1^1) H_1^0 \quad \dots(49) \end{aligned}$$

where the various constraints of equations 39 to 42 are satisfied and only value of Q_1 in the neighbourhood of Q_1^0 is permitted. Since the II term in equation (49) is independent of Q_1^1 , the best neighbouring alternative to Q_1^0 is finding values (Q_1, Q_2, \dots, Q_{24}) which maximise

$$\sum_{i=1}^{24} G(H_1^0, S_1^1) H_1 \quad \dots(50)$$

Expression (50) will be referred to as weighted output of the hydro electric station, where the weights are

$$G(H_1^0, S_1^1) \quad \dots(i = 1 \text{ to } \dots 24)$$

The first part of iteration 1 consist of calculating Q_1^1 which maximise (50) subject to required physical constraint next S_1^2 are calculated finally the values of $G(H_1^1, S_1^2)$ are computed. This ends iteration 1.

The basic property of this method of solution is that the total cost of the thermal schedule corresponding to Q_1^1 must now be equal to or less than the total cost of thermal

schedule corresponding to Q_1^{j-1} .

The value of Q_1 is to be selected in neighbourhood of Q_1^0 and this is illustrated below

V_1 = denotes the volume at the beginning of i^{th} hour

$$V_{i+1} = V_i - Q_i \quad \dots(51)$$

for $i = 1, 2, 3$

for $i = 24, V_{24} = V_{23} - Q_{23} = V_{24}$

and V_1 = Total volume of water available for use during the day. The relation given by (51) is valid if appropriate units of measurements are used e.g. volume in cfs/hr (Cft/sec/hr).

Now schedule Q_1, \dots, Q_{24} could be specified by stating V_1, \dots, V_{23} , since

$$Q_i = V_i - V_{i+1} \quad i = 1, 2, 3$$

Let v be very small increment of volume and consider the states $V_i - v, V_i, V_i + v, i = 1, 24$

at the beginning of interval i.e. $i = 1$, only the states $V_i = V$ is permissible for $i = 2, 23$,

it is possible to go from the permissible state $V_i - v, V_i, V_i + v$ to the permissible state $V_{i+1} - v, V_{i+1}, V_{i+1} + v$ in at most nine possible ways or passages

from $V_1 - v$ to $V_{i+1} - v$
to V_{i+1}
to $V_{i+1} + v$

From V_1 to $V_{i+1} - v$
to V_{i+1}
to $V_{i+1} + v$ (52)

From $V_1 + v$ to $V_{i+1} - v$
to V_{i+1}
to $V_{i+1} + v$

This results in at the most fine discharges

$$Q_1 - 2V$$

$$Q_1 - v$$

$$Q_1$$

$$Q_1 + v$$

$$Q_1 + 2V$$

.....(53)

Of the discharges ⁵³(~~54~~) only those which satisfy the constraints are permissible.

The phrase Q_1 is restricted to some neighbourhood of Q_1^* means that Q_1 may only take to these values of 53 which satisfy 40, 41 & 42 and which in turn results from the passage equation.

The dynamic programme technique consists of the following. For each permissible state at the beginning of hour 24, determine the optimum mode of operation during the hour 24. Use this to evaluate the optimum mode of operation during the last two hours for any permissible state at the beginning of hour 23. Based on these results calculate the optimum mode of operation during the last 3 hours for any permissible state at the beginning of hour 22. Working back the optimum mode of operation is determined successively for the last 4 hours, and continued until the optimum mode of operation for the entire day is obtained.

The calculations of the optimum discharges during the last interval is trivial. For the state $V_{24} - v$, the optimum and only discharge during hour 24 is $Q_{24} - v$, similarly for V_{24} and $V_{24} + v$, the optimum discharges are Q_{24} and $Q_{24} + v$ respectively. The maximum weighted outputs for these modes of operation will be denoted by $R_{24}(V_{24} - v)$, $R_{24}(V_{24})$, and $R_{24}(V_{24} + v)$.

$$\cancel{R_{24}(V_{24} - v)}, \text{ and } \cancel{R_{24}(V_{24} + v)}$$

$$R_{24}(V_{24} - v) = G_{24} H(Q_{24} - v)$$

$$R_{24}(V_{24}) = G_{24} H(Q_{24})$$

$$R_{24}(V_{24} + v) = G_{24} H(Q_{24} + v)$$

.....(54)

Whereas corresponding to the states $V_{23} - v$ at the beginning of hour 23, at most 3 discharges are possible during hour 23, namely Q_{23} , $Q_{23} - v$, $Q_{23} - 2v$. The resulting weighted outputs over the last two hours are

$$G_{23} H(Q_{23}) + R_{24} (V_{24} - v)$$

$$G_{23} H(Q_{23}-v) + R_{24} (V_{24}) \quad \dots(55)$$

$$G_{23} H(Q_{23} - 2v) + R_{24} (V_{24} + v)$$

respectively. The optimum discharge is the one yielding the maximum weighted output ^{here} and denoted by $R_{23}(V_{23} - v)$

Similarly for state V_{23} there are at most 3 discharges possible during hour 23, namely $Q_{23} + v$, Q_{23} , $Q_{23} - v$ for ^{which} the weighted outputs over the last two hours are

$$G_{23} H(Q_{23}+v) + R_{24} (V_{24}-v)$$

$$G_{23} H(Q_{23}) + R_{24} (V_{24}) \quad \dots(56)$$

$$G_{23} H(Q_{23}-v) + R_{24} (V_{24}+v)$$

Again the optimum discharge is one yielding the maximum weighted output denoted by $R_{23}(V_{23})$. A similar calculation yields $R_{23}(V_{23} + v)$. The process is continued till we reach the first hour. Then the operation scheduling for rest of the hours during the day is retraced back from values stated earlier.

3.7 Extension of this Basic Theory of Incremental Dynamic Programming of One Hydro & One Thermal System to include More Hydrostations:

Two fixed head hydrostations and one steam station are considered now. More fixed head hydrostations involves no new considerations and could be similarly considered.

Let Q_{11} & Q_{21} be average hourly discharges of hydro generation corresponding to these discharges.

$F_1(S_1)$ = fuel cost of supplying S_1 units of steam generation
line losses L are represented by

$$L = B_{11}H_1^2 + B_{22}H_2^2 + B_{33}S^2 + 2B_{12}H_1H_2 + 2B_{13}H_1S + 2B_{23}H_2S \quad \dots\dots(57)$$

B_{ij} are loss constants,

It is desired to find Q_{11} & Q_{21} & S_1 so that

$$F_1(S_1) + F_2(S_2) + \dots + F_{24}(S_{24}) = \min \quad \dots\dots(58)$$

$$(S_1 + H_{11}H_{21}) = D_1 + L_1 \quad (i = 1, 24) \quad \dots\dots(59)$$

$$\sum_{i=1}^{24} Q_{11} = V_1 = \text{Volume of water available for the day for first Hydro stations} \quad \dots\dots(60)$$

$$\sum_{i=1}^{24} Q_{21} = V_2 = \text{" " for IIInd " " } \quad \dots\dots(61)$$

$$0 \leq Q_{1,1} \leq Q_{1\max} \quad \dots\dots(62)$$

$$0 \leq Q_{21} \leq Q_{2\max} \quad \dots\dots(63)$$

$$0 \leq S_{\min} \leq S_1 \leq S_{\max} \quad \dots\dots(64)$$

Equation 60 & 61 must be expressed in suitable units

Method of Solution:

To determine the division of load between hydro & thermal systems resulting in minimum fuel cost successive hydro schedules starting from an initial trial schedules are calculated such that corresponding cost of thermal

generation decreases monotonically. This procedure which is iterative in nature comes to an end when successive thermal costs differ by amounts which are sufficiently small that may be neglected.

Let $S_1^*, \dots, S_{24}^*, Q_{11}^*, \dots, Q_{1,24}^*, Q_{21}^*, Q_{2,24}^*$ be any schedule satisfying equations 59 to 64. It is desired to calculate a thermal schedule (S_1, \dots, S_{24}) having daily cost less than or equal to the fuel cost of the schedule S_1^*, S_{24}^* .

If $Q_{1,1}$ & Q_{21} , $1 = 1, 24$ are any proposed alternative to Q_{11}^* & Q_{21}^* such that corresponding changes

$$\Delta H_{11} = (H_{11} - H_{11}^*) \quad \dots(65)$$

$$\Delta H_{21} = (H_{21} - H_{21}^*)$$

are small then resulting change in total fuel cost is approximately.

$$\sum_{i=1}^{24} \frac{d F_1}{d S_1} \Delta S_1 \quad \dots(66)$$

$$\Delta S_1 + \Delta H_{11} + \Delta H_{21} = \Delta L_1 \quad \dots(67)$$

$$= \frac{\partial L_1}{\partial S_1} \Delta S_1 + \frac{\partial L_1}{\partial H_{11}} \Delta H_{11} + \frac{\partial L_1}{\partial H_{21}} \Delta H_{21}$$

Where the partial derivatives are evaluated at $S_1 = S_1^*$ & Q_{11}^* & Q_{21}^* .

$$\text{Solving for } \Delta S_1 = \frac{(1 - \frac{\partial L_1}{\partial H_{11}})}{(1 - \frac{\partial L_1}{\partial S_1})} \Delta H_{11} - \frac{(1 - \frac{\partial L_1}{\partial H_{21}})}{(1 - \frac{\partial L_1}{\partial S_1})} \Delta H_{21} \quad \dots(68)$$

Substituting equation 68 in equation 66, the change in total fuel cost is seen to be negative of the expression.

$$\sum_{i=1}^{24} \frac{dF_1}{dS_1} \frac{(1 - \frac{\partial L_1}{\partial H_{1i}})}{(1 - \frac{\partial L_1}{\partial S_1})} \Delta H_{1i} +$$

$$\sum_{i=1}^{24} \frac{dF_2}{dS_1} \frac{(1 - \frac{\partial L_1}{\partial H_{2i}})}{(1 - \frac{\partial L_1}{\partial S_1})} \Delta H_{2i} \dots (69)$$

$$= \sum_{i=1}^{24} G_1(S_{1i}^*, H_{1i}^*, H_{2i}^*) \Delta H_{1i} + \sum_{i=1}^{24} G_2(S_{1i}^*, H_{1i}^*, H_{2i}^*) \Delta H_{2i}$$

The values of G_1 & G_2 are defined by equation (69)

G_1 & G_2 are incremental worth of power at the bus bar of Hydro station 1 & 2 respectively during hour i of the iteration in question.

In each step of iteration, it is necessary to solve problem of the following form:

Determine discharges (Q_{11}, \dots, Q_{24} & $Q_{21}, \dots, Q_{2,24}$) that maximise

$$\sum_{i=1}^{24} G_{1i} H(Q_{1i}) + \sum_{i=1}^{24} G_{2i} H(Q_{2i})$$

subject to the equality and system constraints on equipment rating.

Because of constraints

$$H_1(Q_{1i}) + H_2(Q_{2i}) - L(S_{1\max}, H_{1i}, H_{2i}) \geq D_i - S_{\max} \dots (70)$$

$$\text{and } H_1(Q_{11}) + H_2(Q_{21}) - L(S_{\min}, H_{11}, H_{21}) \quad D_1 - S_{\min} \quad \dots (71)$$

The problem of maximising the weighted output of the hydro sub-station system is a two dimensional incremental dynamic programming. To reduce the computational involved in maximising the weighted outputs of a hydro sub-system it is desirable to replace the two dimensional problem by two one dimensional problem with some approximation.

Free Joint Variation :

Under certain circumstances, the two dimensional problem reduces to two one dimensional problem without any approximations. Thus for hour i consider all pairs $(Q_{1,i} & Q_{2,i})$ formed by taking Q_{11} & Q_{21} from the given neighbourhood of Q_{11}^* & Q_{21}^* where Q_{11}^* & Q_{21}^* satisfy the constraints (61) to (64). If these pairs also satisfy (69) & (70) then hydro-stations are said to be capable of free joint variation during hour i . If the hydro stations are capable of free first joint variation all over the day, then the maximum weight output of the hydro sub-system could be obtained by maximising the weighted output of each hydro station separately using one dimensional incremental dynamic programme.

The term free refers to the fact that the individual stations are subject to natural end point constraints only so that they can be scheduled separately without introducing any approximation. The term joint is used because both Q_{11} & Q_{21} are defined in terms of the incremental thermal costs dF_1/dS_1 and incremental line losses of the schedule $(S_1^*, \dots, S_{24}^*), (Q_{1,1}^* \dots Q_{1,24}^*), (Q_{21}^*, \dots, Q_{2,24}^*)$

If the hydro-stations can not be considered separately ~~it is possible~~, the possible states of hydro sub-systems consists of 9 pairs (at the most) obtained by taking all the combination of state of station 1 at the beginning of hour 1 with the corresponding states of station 2. Thus there will be 81 passages from the 9 possible states of the hydro sub-system at the beginning of hours 1, to the 9 possible states at the end. These passages will however involve 25 discharges.

Thus, ^{even} to solve one two dimensional problem require considerable computation.

If three fixed head hydro stations had to be scheduled simultaneously, there would be thus $27 \times 27 = 729$ possible passages during hour 1 & $5 \times 5 \times 5$ possible discharges.

Constrained Joint Variation:

To reduce the two dimensional problem which occur if free joint variations is not possible at all hours, to two one dimensional problem, the following approximation will be used.

The order in which the stations are to be scheduled are specified (If hydro stations are capable of free joint variation during this aspect does not matter). Assume that order is specified by previous designation.

Hydro station 1 will then be scheduled by incremental dynamic programming, only the natural end point constraints need be considered at these hours at which free joint variation applied. At the remaining hour induced end point constraints

must be also included e.g. if hydro-stations are not under free joint variations during hour 1, then Q_{11} must satisfy induced end point constraints with Q_{21} set equal to Q_{21}^* .

Hydro stations 2 will be then scheduled, as with station 1 only the natural end point constraints apply at these hour in which the plants are under free joint variation. If the plants are not under free joint variation during hour 1 then Q_{21} , the discharge through hydro station 2 must satisfy the induced end point constraints obtained by setting Q_{11} in equation 69 & 70 equal to the optimum value of Q_{11} just obtained.

The mode of operation obtained using this approximation will be described by saying that the hydro-stations are under constrained joint variation.

Suppose the order in which the plant schedules are to be computed has been specified on the basis of incremental hydro e.g. if $\frac{dH_1(Q_{11}^*)}{dQ_{11}} \geq \frac{dH_2(Q_{21}^*)}{dQ_{21}} \quad i = 1, \dots, 24$

plant 1 schedules will be computed first. In this case constrained joint operation means that incrementally more efficient hydro station is permitted the first choice of departing from the previous optimum schedule. The departure may not exceed the amount allowable with the less efficient station held at its previous optimum value. The less efficient plant is then permitted to depart from its previous optimum discharge, the departure not to exceed the amount allowable with more efficient plant held at its new optimum discharge testing for free joint discharge.

The concept of free joint variation is clearly fundamental in reducing the computational labour involved in solving the scheduling problem. To test whether equation 70 is violated at hour i , it is only necessary to consider the discharges.

$$\text{Max } (Q_1 \text{ min, } Q_{1i}^* - 2 v_1) \quad \dots\dots(72)$$

and

$$\text{Max } (Q_2 \text{ min, } Q_{2i}^* - 2 v_2) \quad \dots\dots(73)$$

If equation ⁷⁰ is satisfied for these discharges, then it is satisfied for remainder of 25 passages involved in passing from 9 possible states of the hydro sub-system at the beginning of hour i , to the 9 possible states at the beginning of hour $i+1$. This follows from the fact that $H_1(Q_{1i})$, $H_2(Q_{2i})$ are increasing functions.

Similarly to test whether express 71 is violated at hour i or not, it is only necessary to consider

$$\text{Min } (Q_1 \text{ max, } Q_{1i} + 2 v_1) \quad \dots\dots(74)$$

and

$$\text{Min } (Q_2 \text{ max, } Q_{2i} + 2 v_2) \quad \dots\dots(75)$$

Cycling Scheduling :

Other approximation which will permit the hydro-station to be scheduled separately can of course be used. This starting from Trial schedules, the stations may be rescheduled in the sequence hydro station 1, thermal station, hydro-station 2, Thermal Station, hydro-station 1, thermal

station, hydro station 2 and so on. In this approximation which will be referred to as cyclic scheduling it is to be understood that if a station is being re-scheduled, the outputs of the remaining two plants are to be kept constant at their previous determined value in the sequence. A hydro station will be dropped from the constraint as soon as additional reduction in fuel cost gained by retaining it becomes less than a pre-assigned value. When all hydro stations have been dropped computation stops.

3.8 Advantage of Incremental Dynamic Programming Over Method of Calculus of Variation:

1. The chief advantage of the method of incremental dynamic programming over method of calculus of variations is more economic schedule in less computing time while taking operating constraints into account.
2. Computing time increases linearly with number of hydro-stations whereas it increases exponentially for solution based on calculus of variation.
3. In calculus of variations it is difficult to include corner points i.e. natural & induced constraints.

CHAPTER - 4

4. SCHEDULING FOR REACTIVE POWER AND VOLTAGE CONTROL

4.1 General

Reactive power component of load supplied from a generator, introduces additional component of transmission losses, scheduling of reactive power for optimal benefits means its allocation to various generating units and circuit elements like shunt capacitors both static and synchronous and shunt reactors etc. For optimum benefits i.e. minimum losses in the system, the location of its sources (other than generating equipment) have to be optimised and its allocation to generating units properly scheduled. Whereas inductive circuits absorb lagging KVAR, a capacitive load, may be thought of supplying a lagging KVAR to the system or absorbing leading KVAR. Reactive power requirements of load, circuit elements have to be supplied by generating units, and capacitors installed in the system in case the requirements are of lagging KVAR, and shunt reactors in case system requires leading KVAR. The unbalance between reactive power demand and generation causes the voltage to vary till the balance is achieved.

4.2 Effect of Reactive Power on System Voltage :

If the reactive power supply denoted by Q_s and demand by Q_d are equal i.e. $Q_s - Q_d = 0$, the system voltages remains stable. In the case where the supply exceeds the reactive demand say due to capacitive effect of cable networks or due to long extra high voltage overhead lines during

the period of light load, we have $Q_s - Q_d = Q_r$, where Q_r represents the surplus reactive power. In this case the system voltage will rise. Reducing Q_r to zero by disconnecting some cables or transmission lines or alternatively by increasing the reactive power demand by connecting shunt reactors or under exciting the synchronous condensers stabilises voltage. Conversely when the demand Q_d increases Q_s as is the case during peak loads or heavy loads, the static capacitors have to be switched on and synchronous condenser are to be over-excited. The capacitors in such a case should be as close to the load centre as possible.

The change in system voltage

$$\Delta V = \frac{RP + XQ}{V} \dots\dots(1)$$

Where ΔV is difference between sending and received end voltage.

R & X transmission circuit resistance & reactance, and P & Q are active & reactive power requirement of load and V is the load end voltage. (1)

4.3 Reactive Power Generation at the Load Centre:

Reactive power demand should preferably be met by generation as close to the load centre as possible. Of course some reactive power has to be generated at the sending end to meet reactive loss of transmission circuit & overall reasons of economy. From equation (1) above we observe that ΔV could be reduced if Q is reduced i.e. if a capacitor is installed at the load end. It is further preferable because this type of compensation i.e. generation of Q at the load end, eliminates the line voltage drop, reduces transmission losses, releases

(1) Electric Power System by B.M.Weedy Book, pp.35

generation, transmission & transformer capacity thus permitting the tie line (between load & generator) to carry a higher position of load. (active power)

4.4 Control of Reactive Power and Voltage :

The allocation of reactive power between its various generators can be achieved by the control of the voltage of the generators, which can be effected by operation of exciter field rheostats or by automatic voltage regulators. Keeping the voltage of a generator constant means maintaining for a fixed output of active power ^a or fixed output re- of/active power. If the reactive power demand is suddenly increased or decreased, the automatic voltage regulator commission adjusts the voltage to the corresponding new reactive power.

4.5 Optimisation of the Allocation of Reactive Power:

4.5.1 General - Although it is preferable to generate reactive power at point of its demand as explained above to the maximum economic extent possible, yet quite a large amount of reactive power will be supplied by generators. The auxiliary reactive power generators (capacitors) are to be located at suitable points on the transmission system and their co-ordinated operation have to be so arranged that the total real power loss is minimum and voltages are kept up within permissible standards at various busses.

4.5.2 Economic Aspects of Reactive Power Generator :

The analysis of relative net cost of installing KVAR capacity in generators, large synchronous condensers, in

switching stations, small synchronous condensers in sub-stations and static capacitors on distribution circuits reveals that⁽²⁾

- i) The static capacitors installation are the most economical because they are close to load.
- ii) The small added cost of providing reactive power capacity in generators is next least expensive even with added system investment and no saving in losses.
- iii) Adding capacity in the form of synchronous condensers wherever located is the most expensive of all due to their higher initial cost.

4.5.3 Methods - The problem under consideration is to fix up the optimum value of reactive powers at the different buses especially in intermeshed network where no limits on the reactive capacity is specified. The capacitors so fixed up must result in a well balanced voltage distribution, small voltage angles (from stability consideration) and the voltage distribution should lead to minimum I^2R losses for economic reasons.

In the minimisation technique K. Zollenkopf⁽⁴⁾ has employed load flow programme.

(2) Sels Hellis K and Seely Theodox "Facility for supply of KW & KVAR" AIEE Transaction, PA&S, Vol. PAS-61, pp. 249, May 1942.

(4) Zollenkopf, K., "Load Flow calculation Using Loss minimisation Techniques", Proceedings of I.E.E. Vol. 115, No. 1, pp. 121-127 January 1968.

The rigorous determination of minimum real power loss dependent on the reactive power input at different buses is very complex. For this reasons it is more convenient to substitute the imaginary components of current ^{entering} ~~entering~~ at the buses in place of reactive powers. Equations for loss minimisation based on this modification yield relatively simple expressions, which can easily be incorporated into iterative process of a load flow calculations. This simplification is based on the assumption that all nodal voltage vectors do not deviate substantially from the real axis.

The nodal voltage equations with respect to the reference bus bar can be expressed as

$$Y (V - V_R) = I \quad \dots\dots(2)$$

Where Y is the nodal admittance matrix

$$\text{Then} \quad V - V_R = Z I \quad \dots\dots(3)$$

$$\text{where} \quad Z = Y^{-1}$$

The total nodal currents I are composed of the following parts -

$$I_g = \text{Generation}$$

$$I_l = \text{Load}$$

$$I_c = \text{Line charging, static capacitor or other reactances to ground.}$$

The real part of the total system loss is given by

$$P_L = R_e (\underline{I}_t^* Z \underline{I} - \Delta \underline{I}_t^* \underline{V}) \quad \dots\dots(4)$$

Where the current vector $\Delta \underline{I} = \underline{I}_c$

Assuming that all impedances to earth such as line charging are pure reactances, the last scalar term in equation (4) may be neglected.

By separating real and imaginary parts, equation (4) then becomes,

$$P_L = \underline{I}_t' \cdot R \underline{I}' + \underline{I}_t'' R \underline{I}'' \quad \dots\dots(5)$$

Where superscripts,

' means real part of a complex quantity

'' means imaginary part of a complex quantity.

and subscripts, t means transpose

$$\underline{I} = \underline{I}_g + \underline{I}_l + \underline{I}_c \quad \dots\dots(6)$$

and R = Real part of Z matrix

According to the assumption, the approximation to minimum loss is to be evaluated dependent on the reactive power generation. This quantity can be found from the partial derivatives of the total system power loss with respect to the nodal current \underline{I}'' , assuming that all other current remain constant.

On taking partial derivative of P_L in equation (5) with respect to \underline{I}'' , the first term becomes zero and so far convenience expanding only the second term of equation (5).

$$\underline{I}'' \cdot R \cdot \underline{I}'' = [I_1'' \ I_2'' \ I_3'' \ \dots \ I_n''] [R] \begin{bmatrix} I_1'' \\ I_2'' \\ I_3'' \\ \vdots \\ I_n'' \end{bmatrix}$$

$$= I_1'' R_1 + I_2'' R_2 + I_3'' R_3 + \dots + I_n'' R_n \begin{bmatrix} I_1'' \\ I_2'' \\ I_3'' \\ \vdots \\ I_n'' \end{bmatrix}$$

$$= I_1'' \sum_{j=1}^n R_{1j} I_j'' + I_2'' \sum_{j=1}^n R_{2j} I_j'' + I_3'' \sum_{j=1}^n R_{3j} I_j'' + \dots + I_n'' \sum_{j=1}^n R_{nj} I_j''$$

On taking now the derivatives with respect to I'' say first with I_1'' , and equating to zero,

$$\frac{\partial P_L}{\partial I_1''} = 2I_1'' R_{11} + \sum_{\substack{j=1 \\ j \neq 1}}^n R_{1j} I_j'' + R_{21} I_2'' + R_{31} I_3'' + \dots + R_{n1} I_n'' = 0$$

$$\text{or, } 2I_1^n R_{11} + \sum_{\substack{j=1 \\ j \neq 1}}^n R_{1j} I_j^n + \sum_{\substack{j=1 \\ j \neq 1}}^n R_{j1} I_j^n = 0$$

$$\text{or } 2I_1^n R_{11} + \sum_{\substack{j=1 \\ j \neq 1}}^n (R_{1j} + R_{j1}) I_j^n = 0$$

$$\therefore I_1^n = - \frac{1}{2 R_{11}} \sum_{\substack{j=1 \\ j \neq 1}}^n (R_{1j} + R_{j1}) I_j^n$$

Similarly,

$$I_2^n = - \frac{1}{2 R_{22}} \sum_{\substack{j=1 \\ j \neq 2}}^n (R_{2j} + R_{j2}) I_j^n$$

and for any bus k,

$$I_k^n = - \frac{1}{2 R_{kk}} \sum_{\substack{j=1 \\ j \neq k}}^n (R_{kj} + R_{jk}) I_j^n \dots\dots(7)$$

Equation (7) gives the value of the imaginary part of the current for minimum loss condition, from which the value of the reactive power can be calculated.

In equation (5) R is the real part of the complex matrix Z., Where $Z = Y^{-1}$,

Then to evaluate R, let,

$$Y = G + j B \dots\dots(8)$$

Also $Z = Y^{-1} = C + j D \dots\dots(9)$

C and D are to be evaluated from equation (8) & (9)

$$Y^{-1} Y = CG - DB + j (DG + CB) = I + jO$$

where I = unit matrix

equating real and imaginary parts

$$CG - DB = I = GC - BD \quad \dots\dots(10)$$

$$\text{and} \quad DG + CB = 0 = GD + BC \quad \dots\dots(11)$$

on Multiplication with G^{-1} in equation (11)

$$G^{-1} GD + G^{-1} BC = 0$$

$$\text{or} \quad D = -G^{-1} BC$$

$$\text{Similarly} \quad C = (G + BG^{-1} B)^{-1}$$

This real part of Z matrix is given by

$$R = C = (G + BG^{-1} B)^{-1} \quad \dots\dots(12)$$

4.5* Incorporation of the Loss Minimization Procedure in a Load Flow Program :

The equations developed above can be incorporated into the iterative process of load flow calculations in the following sequence:-

- (1) Form the nodal admittance matrix, Y.
- (2) Take inversion of Y^{-1} matrix so that $Y^{-1} = Z$
- (3) Calculate real part of Z matrix according to equation (12)
- (4) Estimate initial nodal voltage V (except the swing bus bar whose voltage is completely known $1 + j0$)
- (5) Estimate initial reactive power Q for generator as well as load buses.

- (6) Set all total nodal currents $I = 0$
 (7) Calculate additional injected nodal current.

It representing time charging from

$$\underline{I}_C = (\underline{D}_C' + j \underline{I}_C'') = (V_r + j V_1)(j\omega C)$$

Equating real and imaginary parts

$$I'_C = -V_1 \cdot \omega C$$

$$I''_C = V_r \cdot \omega C$$

- (8) Calculate real component of Generator as well as load currents since active power is specified at all the buses, from the following relation:

At any bus

$$\text{Total Power } S = P + j Q = V \cdot I^*$$

(Where superscript* indicates complex conjugate)

or
$$P + j Q = (V_r + j V_1) (I' - j I'')$$

Equating real and imaginary parts,

$$I' = \frac{P V_r + Q V_1}{V_r^2 + V_1^2}$$

Where P is the specified active power (Generation or load).
 and Q is the reactive power as calculated by the preceding iteration step. For the first iteration step, Q is taken from the estimated initial values.

- (9) Calculate total real components of nodal currents for generator as well as load buses according to equation (6) then

$$I'(\text{Total}) = I' + I'_C$$

- (10) Calculate imaginary components of all currents according to equation (7)

$$I''_k = -\frac{1}{2R_{kk}} \sum_{\substack{j=1 \\ j \neq k}}^n (R_{kj} + R_{jk}) I''_j$$

- (11) Calculate total imaginary component of nodal currents for generator as well as load buses according to equation (6), then

$$I''(\text{Total}) = I''_k + I''_c$$

- (12) Calculate all nodal voltage from equation (3)

$$V - V_R = Z I$$

V = nodal voltage

V_R = Voltage of Reference or swing bus

$$Z = Y^{-1}$$

$$\text{and } I = (I'(\text{Total}) + I''(\text{Total}))$$

i.e. I is composed of generation, load and line charging components.

- (13) Calculate again additional injected nodal currents I_c representing line charging i.e.

$$I'_c = -V_l \cdot wc$$

$$I''_c = V_r \cdot wc$$

- (14) Calculate actual nodal currents due to generator or load from $I_g + I_l = I - I_c$

$$\text{i.e. } I'(\text{net}) = I'(\text{Total}) - I'_c$$

$$\text{and } I''(\text{net}) = I''(\text{Total}) - I''_c$$

- (15) Calculate actual values of Active & Reactive powers according to relation

$$P + jQ = V \cdot I^*$$

$$\text{which yields, } P = V_r I'(\text{net}) + V_i I''(\text{net})$$

$$\text{and } Q = V_i I'(\text{net}) - V_r I''(\text{net})$$

- (16) Compare mismatches in all active powers with specified precision index

$$\text{i.e. } |P(\text{calculated})| - |P(\text{Specified})| \leq \sigma$$

where σ is the precision index.

- (17) Repeat steps ~~in~~ ^{from} (8) to (16) until mismatches in all active power exceeds the specified precision index.

The flow chart showing the sequence of steps to be followed in the determination of the reactive capacitors at different buses is given in Fig.4.1

interval for V_1 and used in the maximisation procedure. The division of discharges to various units so as to result in maximum station efficiency is found by model studies of the system and the input output characteristics so arrived at is called exact curve of the station. A typical exact curve for a hydro station is shown in Figure 5.2. There is a discontinuity in this curve at the moment a new unit is started. The maximum discharge, Q_{max} , is 39500 cfs and the corresponding output is 382.00 MW. 7,11,000 cfs hours of water is initially available for use over the day.

The thermal station selected for this example contains four essentially identical units, each with a maximum net output of 100 MW. The input output equation for the thermal plant determined by fitting a parabola into input out^{put} data obtained from tests on these machines is given below. It was assumed in this study that all thermal units are on the line at all times. Maximum plant efficiency is then obtained by operating all units at equal incremental values which in this case mean that all units must be operated at equal output.

$$F(S) = 373.704 + 9.60644 S + 0.001991 S^2 \text{ (for this sample problem).}$$

Where S is in MW and $F(S)$ in millions of BTU per hour. The station incremental heat rate in million of BTU per MW hours is

$$C(S) = \frac{d F(S)}{d S} = 9.60644 + 0.0039822 S$$

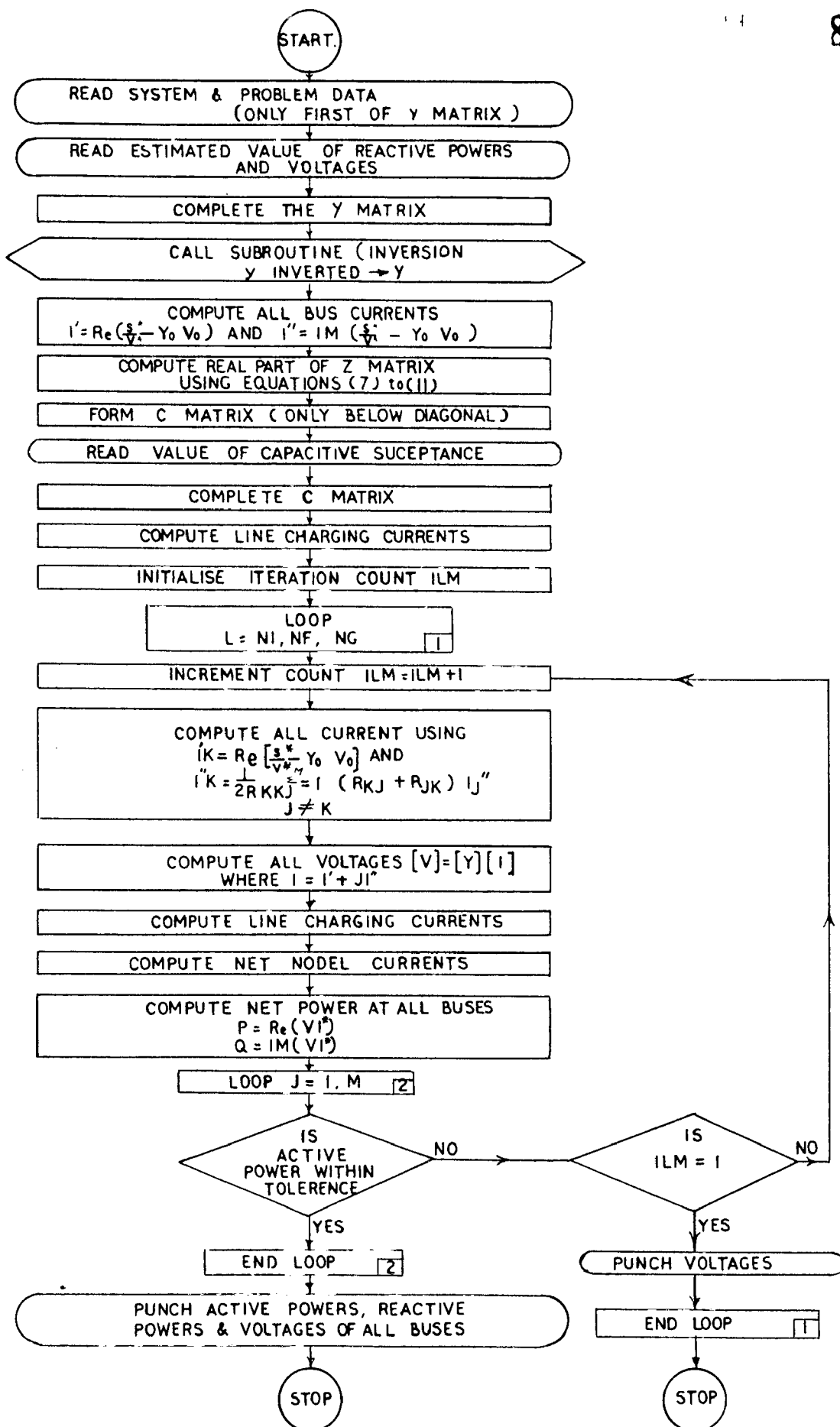


FIG. 4-1 FLOW CHART FOR LOAD FLOW USING LOSS MINIMIZATION TECHNIQUES

CHAPTER - 5

5. CASE STUDY OF OPTIMAL SCHEDULING OF A SAMPLE
SYSTEM, COMPUTER PROGRAMME - RESULTS AND
DISCUSSION

5.1 General:

Detailed description of method of dynamic programming for optimal hydro thermal scheduling is given in Chapter 4. A computer programme in Fortran - II has been developed for optimal solution of one hydro (fixed head) and one thermal sub-system over a day. The programme has been tested on a sample problem taken from the paper entitled "Hydro Thermal Economic Scheduling Part - I ", B. Bernhaltz & L.J. Graham, published in I.E.E.E. Transaction, P A & S December 1960 - pp 921-932" and the results arrived at by the Computer tally with those listed in the reference.

5.2 The Problem & Data :

The hydro station selected for this example contains two groups of four units, units 1, 2, 3 & 4 can be considered identical and constitute one group, while units 5, 6, 7 & 8 can be considered identical and constitute the other group. The most efficient mode of operation of the entire station for discharges exceeding 22000 cfs (only these values are required in this example) and a net operating head of 134 feet is fed as a data to computer. Discharges are tabulated at an interval of 500 cfs since $v = 500$ cfs hours was the tabular

interval for V_1 and used in the maximisation procedure. The division of discharges to various units so as to result in maximum station efficiency is found by model studies of the system and the input output characteristics so arrived at is called exact curve of the station. A typical exact curve for a hydro station is shown in Figure 5.2. There is a discontinuity in this curve at the moment a new unit is started. The maximum discharge, Q_{max} , is 39500 cfs and the corresponding output is 382.00 MW. 7,11,000 cfs hours of water is initially available for use over the day.

The thermal station selected for this example contains four essentially identical units, each with a maximum net output of 100 MW. The input output equation for the thermal plant determined by fitting a parabola into input out^{put} data obtained from tests on these machines is given below. It was assumed in this study that all thermal units are on the line at all times. Maximum plant efficiency is then obtained by operating all units at equal incremental values which in this case mean that all units must be operated at equal output.

$$F(S) = 373.704 + 9.60644 S + 0.001991 S^2 \quad (\text{for this sample problem}).$$

Where S is in MW and $F(S)$ in millions of BTU per hour. The station incremental heat rate in million of BTU per MW hours is

$$C(S) = \frac{d F(S)}{d S} = 9.60644 + 0.0039822 S$$

The maximum station output, S_{\max} is 400 MW and minimum output $S_{\min} = 40$ MW. Cost of fuel was assumed to be 35 cents/million BTU. The generating stations are connected to the load centre by a transmission network. The following is the line loss equation.

$$L = 0.00015 H^2 + 0.00005 S^2 + 0.00002 HS$$

Where L, H & S are in MW.

The load curve for the day is shown in Figure 5.1.

5.3 Development of Computer Programme

A computer programme in Fortran-II has been developed and attached as Appendix A₁. The programme was run on IBM 1620 Computer at S.E.R.C. Roorkee and the execution time taken for the programme was 30 minutes. The compilation time was 12 minutes.

The programme consists of a main programme and five sub-routine sub-programmes (Appendix A-2). The number of commands in main programme is about 160.

The criterion for optimisation was taken as when the successive thermal cost differ by \$10.0. The actual value taken in the reference was \$1.0 and it required 12 iterations to arrive at the final optimised value by taking \$1.0 as the cost difference base. This difference was taken as \$10.0 in the study undertaken because there was limitation

of 35 minutes for one programme execution in Fortran - II on IBM - 1620 at S.E.R.C. Roorkee because of the huge rush. The final optimised cost value (with \$ 10.0 as difference base) was arrived at after 7 iterations and in 30 minutes time.

5.4 Results :

The results are listed under an Appendix A-2. For each iteration the values of hydro generation, the corresponding thermal generation during 24 hours of the day and thermal cost during the day and also the value of maximum weighted output R in million BTUs have been punched. The initial trial scheduling was costing \$ 28215.02 and optimised operation schedules worked out after 7 iterations cost \$ 28049.46. There is thus net saving of \$ 165.56 or Say \$ 166. The optimum cost reduction using \$ 1.0 as the cost difference taken from the reference was \$ 172.0. Thus about 96% cost reduction have been achieved in 7 iterations. The cost reductions constitute about $\frac{166}{28215} \times 100 = 0.59\%$ of the total cost per day which when integrated over a year or so a period becomes considerable. The optimum operation hydro schedule has been shown in Figure 5.1. The schedule of hydro generation shown in Figure 5.1. exhibits during the hour from 12 mid night to 8 a.m. a rather complicated behaviour which arises from the discrete nature of the digital calculations and the complexity of the hydro stations input output curve. The results described above tally with the reference under considerations.

5.5 Effect of Trial Schedule and Tabular Interval on the Results: -

The dynamic programming aims at optimising an initial trial schedule. The computing time required depends upon the initial trial schedule and so is the savings in cost obtained. Large savings in cost with longer computing time is the result of obviously poor trial schedules. In the example under consideration the trial schedule is very near to the optimum schedule as such number of iterations required to arrive at optimum results is considerably less. The final optimum schedule has almost the same total operating cost whatever is the initial trial schedule.

Reduction of tabular interval taken to be 500 cfs increases the computation time in almost same proportion.

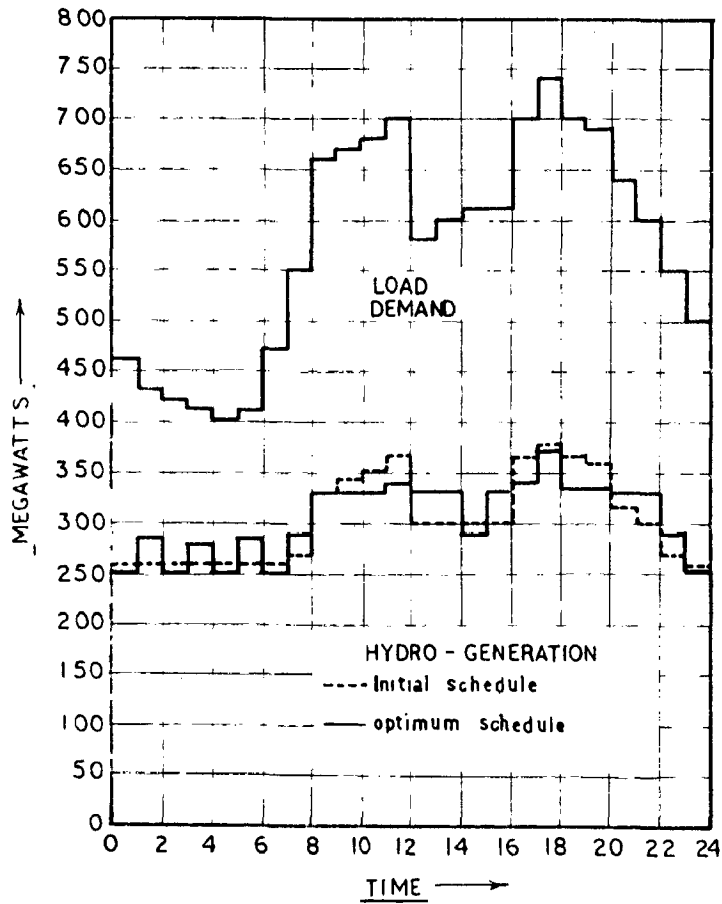


FIG.5.1 OPTIMUM DAILY SCHEDULE OF HYDRO-GENERATION.

CHAPTER - 66. APPLICATION TECHNIQUES6.1 General :

Load Despatchers organisation are required to ensure that the operational planning aspects discussed earlier are translated into practice. They also continuously search for new avenues of savings which only continuous monitoring can reveal.

The type of organisation would depend on the accepted policy in respect of interconnected operation. It is desired to have a separate load despatch organisation for each system which will monitor detailed operation within the system. A central load despatch to act as a clearing house for transactions between systems may also help though may not be indispensable if individual load despatch departments can work with enlightened co-operation. If completely integrated operation between the systems is desired, central load despatch organisation would be valuable.

6.2 Equipment :

The system size and complexity as well as economic considerations influence the methods used to collect and utilise the information required to operate the system. The main equipment in load despatch stations :-

- (a) A mimic board forming a miniature of the power system showing a single line diagram of the system with generating units, transformers, circuit breakers, disconnects,

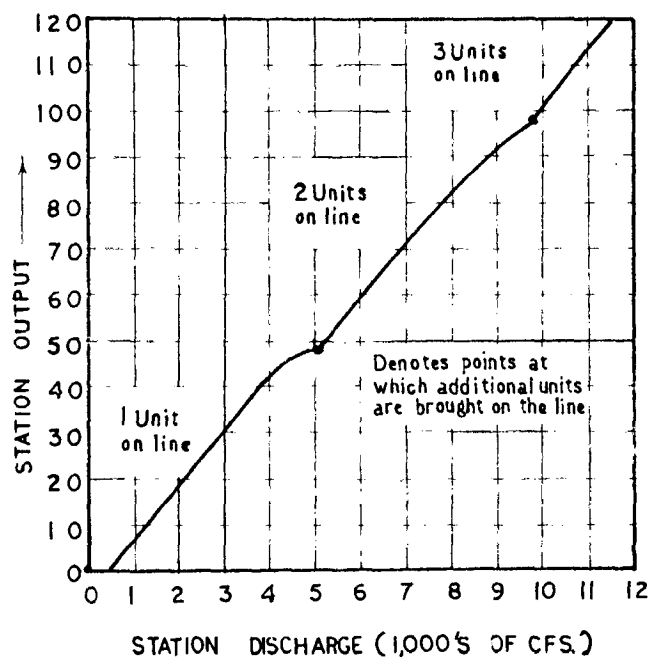


FIG. 5-2 MAXIMUM STATION OUTPUT AS A FUNCTION OF STATION DISCHARGE

CHAPTER - 66. APPLICATION TECHNIQUES6.1 General :

Load Despatchers organisation are required to ensure that the operational planning aspects discussed earlier are translated into practice. They also continuously search for new avenues of savings which only continuous monitoring can reveal.

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6.2 Equipment :

The system size and complexity as well as economic considerations influence the methods used to collect and utilise the information required to operate the system. The main equipment in load despatch stations is :-

- (a) A mimic board forming a miniature of the power system showing a single line diagram of the system with generating units, transformers, circuit breakers, disconnects,

station bus bars, transmission lines etc. with blue and red indications for open and close positions of breakers and semaphores for disconnects.

(b) Indicating and recording meters and instruments for measuring and knowing

- i) power flow both active and reactive from each generating stations , tie lines, key points in important transmission lines.
- ii) Bus voltage at each generating station and sub-station
- iii) System frequency
- iv) Accumulated frequency time error based on standard time.
- v) Tap positions of load ratio control transformers.
- vi) Fuel supply for each plant and Reservoir water level at each hydro electric generating plant having storage.
- vii) Energy supply from hydro & thermal stations and tie lines.
- viii) River flow data in the vicinity of hydro electric station.

In addition system operation must have upto date information of weather data including storm warning & contractual obligation in load and frequency control with neighbouring systems.

(c) The use of telemetering and telecontrol ~~for~~ equipment required for the functions listed above is a matter of economics and depends upon the size of the system.

(d) Communication and control equipment required must be fool proof. Communication is the backbone of load despatching. To guard against interruption, parallel telephone circuits and different mode of communications are provided as follows:-

- i) Automatic telephone exchange connections
- ii) Power line carrier communication
- iii) Microwave communication
- iv) A code call system of communication.

(e) Teleprinters for collection of data, statistics, reports and for administrative purposes.

(f) Supervisory control equipment if employed for control functions such as switching of lines, starting synchronous condensers, hydro plants, change levels in transformer taps etc.

(g) Net work analyser or digital computer used for planning controlling and load despatching functions.

6.3 Types of Load Despatching System :

The load despatch station may be manual or automatic depending upon potential benefits that could be achieved from the integrated operation of the system.

6.3.1. Manual Load Despatching System :

The first step in manual load despatching of a power area is that of predicting daily system load vs time curve of that system. Total generation necessary to supply the system load will include reserve requirements and tie line flows. The base load part of system load is allocated among various generating plants in the area giving due consideration to the types of generating plants and availability of water or fuel, etc. The variable part of system load is allocated to efficient big and new plants and this part of the load is controlled by manual despatching techniques.

As a part of load despatching, the system frequency and net tie line power flows are measured and compared with standard frequency and the net estimated system variable load and their deviation are recorded in the despatching office. Depending upon the magnitude of combined deviation the load allocation between regulating plants or units is worked out on the basis of equal incremental cost of power received at a point on a tie line.

The calculated values are allocated among regulating units of every station by the help of incremental loading slide rule manually. By making use of incremental cost curves in the case of steam plants and incremental rate of water flow or incremental plant cost curve in the case of hydro plant, load is regulated among generating units in each station.

The manual controlling becomes cumbersome if load variation of system is very frequent for the fact that evaluation of incremental fuel cost of incremental water flow, etc., involves lot of time and work. Moreover, by the time a desired scheduling is calculated, system condition may change. In such cases, automatic controlling is preferable.

6.3.2. Automatic Load Despatching Arrangement :

Basically, load despatching arrangement comprises load frequency control and economic loading of generating units on the basis of techniques discussed in Chapter IV.

As in the case of manual despatching, prediction of system load requirements is done and total regulation of generation required, etc., estimated and the economic aspects for allocating generating units, etc., are taken into consideration.

6.3.2.1. Load Frequency Control - Load frequency control system comprises measuring and computing system which has a number of sensing elements to measure the net interchange of power system frequency and time error. The tele-metered signals from the various interchange points are added together and sent to a totalizer (or interchanger). The net interchange from the power totalizer is compared with the schedule by the net interchange measuring circuit. Similarly system frequency is compared with frequency standard and the time error is compared with time error set point. Any resulting error is applied as input to the area requirement computer. The net error signal which is the algebraic sum

of net interchange frequency and time error signals, is then sent to the controller which converts it into proportional and reset actions either combined or separately. The combined signals of all the mentioned error signals becomes the 'area load requirement signal'.

The "area load requirement signal" is then fed into the controller manually for semi-automatic load control and automatically for automatic load control. The controller generates signal in the form proportional and reset controlling signals which are in terms of electrical phase shift from reference supply, i.e. system frequency. The action of the proportional and reset controllers is such that "area requirement" signals cause an integration of electrical degrees of phase shift and the integrated degrees indicate the level of generation under control.

The proportional controlling signal corrects major portion of the requirement quickly and the remainder is slowly by the reset action. The reset action continues to function until the requirement signal is reduced to zero.

The degrees phase shift due to "area signal" from the system frequency may be increased or decreased manually to compensate for line losses. ~~(kx)~~. The supply frequency signal is then fed to frequency dividers, one for each station. Where, it is reduced to 12 to 24 cycles/sec. signal for transmissions to the generating stations in

order to reduce unnecessary interference and use only one channel between despatching office and generating plants.

At station, there is a master station equipment which produces the same angular displacements of the signal received and translates the combined action of proportional and reset controlling and penalty factor effects. Then these signals actuate circuits to allocate the extent of load to be generated by each regulating unit, on the basis of economic principle described earlier in Chapter 4.

The master controller at station actuates circuits for governor motors such that as soon as the desired load regulating signal is received, proportional control and reset control come into operation and the desired generation is attained. When the desired generation is obtained, the control actions by the master controller stop.

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CHAPTER - 7

7. CONCLUSIONS

7.1 Optimum maintenance scheduling described could help in considerable energy/capacity benefits. Detailed probabilistic methods be employed to find out generation reserve for different months of the year. The outage rates and average outage duration record of the power plant units should be properly maintained so that realistic figures could be taken.

7.2 The regional grid should function with arrangement for optimal hydro thermal scheduling so as to derive maximum economic benefits. Various methods of hydro thermal scheduling (short range) are available. Dynamic Programming method of hydro thermal scheduling is recommended for adoption. It is further considered that for this purpose an encyclopedia of operating schedule for various conditions which could be revised from time to time with current change in conditions be made and that an on line computer be used for continuous optimal operation of power systems at the basis of methods outlined in this dissertation.

7.3 Optimum scheduling of reactive power by allocating the same to various sources of its generation i.e. Generators, capacitors etc. may result in substantial minimisation of losses and is also conducive to better voltage regulation in the system.

7.4 A computer programme has been developed for optimal solution of one hydro and one thermal system based on incremental dynamic programming method. This programme has been applied to a sample system to show the saving achieved by resorting to optimal operation.

7.5 A load despatch centre is required to apply and co-ordinate the various techniques of scheduling both active as well as reactive power in the system and also schedule maintenance of the units.

APPENDIX - A

MATHEMATICAL METHODS OF RESERVE STUDIESI. Frequency and Duration of Forced Outages of a Uniform System :

Assume a uniform system of n units of equal size of M mw with a uniform outage rate of p . Then

$$p = t/T \quad \dots\dots(1)$$

where t is the average duration of individual outages in days, and T is the average interval of individual outages in days. The frequency of individual events is

$$F = \frac{1}{T} = \frac{p}{t} \quad \dots\dots(2)$$

The probability of r -fold simultaneous outages is expressed by the binomial formula

$$P_r = \frac{n!}{r!(n-r)!} p^r (1-p)^{n-r} \quad \dots\dots(3)$$

The average duration in days of r -fold simultaneous outages is

$$t_r = \frac{t_p (1-p)}{r+p(n-2r)} = \frac{t (1-p)}{r+p(n-2r)} \quad \dots\dots(4)$$

From equations 3 and 4 the average interval in days of r -fold simultaneous outages is derived by

$$T_r = \frac{t_r}{P_r} \quad \dots\dots(5)$$

and the average frequency by

$$F_r = \frac{P_r}{t_r} \dots\dots(6)$$

The magnitude of the r-fold outage is

$$M_r = r M \dots\dots(7)$$

If the system is actually not uniform with regard to size of units, but consists of units with an average size M and a standard deviation σ , then the magnitude of an r-fold outage becomes $r M$ with a deviation of $\pm \sigma/\sqrt{r}$

II. Derivation of Equation for Duration of r-fold Simultaneous Outages:

The derivation of equation 4 is as follows:

For a system of n machines with an individual outage rate of p , an average outage duration of t days, and an average interval between individual outages of T days, the average duration of r-fold outages is

$$t_r = \frac{t(1-p)}{r+p(n-2r)} \text{ days}$$

Assuming first that the events can change position in time only in units steps, then for each of the T possible positions of an event in the period T , there will be t time units occupied and $(T-t)$ unoccupied. There will be therefore for the T possible positions (Tt) occupied time units and $T(T-t)$ unoccupied time units. For each position of an event there are $t^{(r-1)}$ positions which the $(r-1)$ other simultaneous events can take, so that they overlap partly or completely with this event. Therefore, for the T

possible positions of the event, there are $Tt^{(r-1)}$ or Tt^r time units it can occupy together with the $(r-1)$ other events. That just r and not more than r events coincide, the $(n-r)$ other events, which are not involved in the r -fold simultaneous outage, must take positions not overlapping any part of the interval occupied by the r simultaneous events. In other words, the interval occupied by the r simultaneous events must be unoccupied by the $(n-r)$ other events. This is possible in $(T-t)^{n-r}$ ways. Finally, there are $n!/r!(n-r)!$ possible ways in which r events out of n events may be combined. Therefore, for all possible different positions of n events in the interval T , the total number of time units occupied in simultaneous r -fold occurrences is

$$T(n!/r!(n-r)!) t^r (T-t)^{n-r}$$

Individual r -fold simultaneous events can have durations varying from the minimum overlap of one time unit to complete overlap of t time units. To obtain the average duration of overlap of r events, the total number of $T n!/r!(n-r)!$ $t^r (T-t)^{n-r}$ time units which can be occupied must be divided by the number of possible r -fold individual events. An individual event is defined as a succession of occupied time units not affected by the start of a new event, nor the end of an event, nor the simultaneous end of one event and the start of another event, which, while not changing the number of simultaneous events, would change their identity. This latter requirements is important in calculations where events may involve, for example, outages of equipment of different capacities.

Of the t successive time units of an individual event there are $(t-1)$ units directly preceded by another unit. These may be called for simplicity "connected" units; and there is one unit not preceded by another which may be called the "leading" unit. Since the t successive units of an individual event should be counted only as one event, only the leading unit out of the t units should be counted and the connected units should not be counted. Similarly of the $(T-t)$ unoccupied units, only the leading unit should be counted in determining the number of unoccupied intervals and the $(T-t-1)$ connected units should not be counted.

In determining the number of r -fold simultaneous events the following observations apply. Only a leading unit can create a leading units of a multiple event either by coincidence with other leading units or with connected units. The coincidence of connected units with connected units creates only connected units of multiple events. This applies both to intervals of occupied as well as to intervals of unoccupied units. The leading unit of an interval of unoccupied units starts a change in events as it is equivalent to the end of an interval of occupied units. The coincidence of connected unoccupied units with connected unoccupied units does not cause a change in events.

Now, for the

$$t_r = \frac{t(1-p)}{r + p(n-2r)} \quad \text{days}$$

units occupied in all possible, r-fold simultaneous events in the interval T, there are $T(t-1)^r$ connected occupied units combined with $(T-t-1)^{n-r}$ connected unoccupied units in

$$\frac{n!}{r!(n-r)!}$$

possible combinations. Therefore, there are

$$T = \frac{n!}{r!(n-r)!} (t-1)^r (T-t-1)^{n-r}$$

connected occupied units in all possible cases of r-fold simultaneous events in the interval T. As a consequence there are

$$T \frac{n!}{r!(n-r)!} t^r (T-t)^{n-r} - T \frac{n!}{r!(n-r)!} (t-1)^r (T-t-1)^{n-r}$$

leading occupied units or different individual events in all possible cases of r-fold individual events in the interval T.

Table - Schematic Pattern of Meshing of Two Subgroups

System B	System A				
	A ₀	A ₁	A ₂	A ₃	A ₄
B ₀	A ₀ B ₀	A ₁ B ₀	A ₂ B ₀	A ₃ B ₀	A ₄ B ₀
B ₁	A ₀ B ₁	A ₁ B ₁	A ₂ B ₁	A ₃ B ₁	A ₄ B ₁
B ₂	A ₀ B ₂	A ₁ B ₂	A ₂ B ₂	A ₃ B ₂	A ₄ B ₂
B ₃	A ₀ B ₃	A ₁ B ₃	A ₂ B ₃	A ₃ B ₃	A ₄ B ₃
B ₄	A ₀ B ₄	A ₁ B ₄	A ₂ B ₄	A ₃ B ₄	A ₄ B ₄

The average duration of an r -fold simultaneous event is the ratio of the number of all occupied units to the number of all individual events or

$$t_r = \frac{T \frac{n!}{r!(n-r)!} t^r (T-t)^{n-r}}{T \frac{n!}{r!(n-r)!} t^r (T-t)^{n-r} -$$

$$T \frac{n!}{r!(n-r)!} (t-1)^r (T-t-1)^{n-r}}$$

$$\text{or } t_r = \frac{n!}{t^r (T-t)^{n-r} - (t-1)^r (T-t-1)^{n-r}} t^r (T-t)^{n-r}$$

The restriction that events can change position only by unit steps can be removed by expressing both t and t_r as fractions of T and by letting T increase to infinity. By definition $p = r/T$. Therefore substituting pT for t yields the following equations for the average duration of an r -fold event expressed as a fraction of T :

$$\frac{t_r}{T} = \frac{1}{T} \frac{(pT)^r (T-pT)^{n-r}}{(pT)^r (T-pT)^{n-r} - (pT-1)^r (T-pT-1)^{n-r}}$$

or, after dividing numerator and denominator by T^n

$$\frac{t_r}{T} = \frac{1}{T} \frac{p^r (1-p)^{n-r}}{p^r (1-p)^{n-r} - (p-1/T)^r (1-p-1/T)^{n-r}}$$

Then, by expanding and omitting terms which will vanish if T becomes infinity

$$\lim_{T \rightarrow \infty} \frac{t_r}{T} = \frac{p(1-p)}{r+p(n-2r)}$$

$$\text{or } t_r = \frac{Tp(1-p)}{r+p(n-2r)} = \frac{t(1-p)}{r+p(n-2r)}$$

III. Meshing of Two Subgroups of a Nonuniform System :

Consider a system AB composed of two uniform subgroups A and B. For each of the subgroups the following values are determined for all values of r from 0 to r according to the equations in section I.

M_A and M_B , magnitude of r-fold outage

σ_A and σ_B , standard deviation for r-fold outage

P_A and P_B , probability of r-fold outage

t_A and t_B , average duration of r-fold outage.

Then, the values for the composite system are calculated from the values for the subgroups for all possible combinations of r according to the matrix pattern shown in Table.

The composite values are as follows:

$$M_{AB} = M_A + M_B \quad \dots\dots(8)$$

$$\sigma_{AB} = \sqrt{\sigma_A^2 + \sigma_B^2} \quad \dots\dots(9)$$

$$P_{AB} = P_A P_B \quad \dots\dots(10)$$

$$t_{AB} = \frac{t_A t_B}{t_A + t_B} \quad \dots\dots(11)$$

$$T_{AB} = t_{AB}/P_{AB} \quad \dots\dots(12)$$

$$F_{AB} = 1/T_{AB} \quad \dots\dots(13)$$

The combination of a third subgroup \check{C} with a combined group AB is done in a similar way.

The derivation of equation 11 for t_{AB} is as follows:

Assume t_A and T_A are the duration and interval of an r -fold outage in system A and t_B and T_B are the duration and interval of an s -fold outage in system B. In general T_A and T_B will be different, so that $T_A = fT_B$ or the event in system B will occur once in the interval T_B but f times in the interval T_A . According to equation 12 of reference 1, the combined duration t_{AB} is the x^2 term of the expression of

$$\frac{t_A x + (T_A - t_A) \quad ft_B x + (T_A - ft_B)}{t_A x + (T_A - t_A) \quad ft_B x + (T_A - ft_B) - (t_A - 1)x + (T_A - t_A - 1) \quad (ft_B - f) \quad x + (T_A - ft_B - f)}$$

The x^2 term is

$$t_{AB} = \frac{t_A ft_B}{t_A ft_B - (t_A - 1)(ft_B - f)} = \frac{t_A t_B}{t_A t_B - (t_A - 1)(t_B - 1)}$$

$$= \frac{t_A t_B}{t_A + t_B - 1}$$

Then removing the limitation of unit steps by making the size of the steps very small, or t_A and t_B very large, compared with 1, yields

$$t_{AB} = \frac{t_A t_B}{t_A + t_B}$$

IV. Grouping and Plotting of Results:

A plot of the interval versus magnitude of forced outages for a complex composite system calculated according to the methods described in section III results in an array of points as shown by the crosses in Fig. 6. These results apply to a 5,608 mw system composed of eight subgroups of different sizes and outages rates. The apparent, irregular array of the points is due to the nature of the system. Certain combinations of outages are relatively more likely than other intermediate values. In evaluating these data, it must be recognized that the values are by nature discontinuous. Only certain combinations of magnitudes of outages are possible, and the frequency of intermediate values is zero. Nevertheless, to apply the results to reserve studies, it is necessary to express them by a continuous curve of interval versus magnitude of outages. This curve must represent the summation of the individual frequencies. This can be most readily accomplished by grouping the results in ranges of magnitudes, such as 50 or 100-mw steps and determining the total frequency for each group. In selecting the group width, it must be recognized that a large group width will result in smooth curves, but conservatively high values of expected outages. Grouping in narrow megawatt steps will result in more accurate magnitudes of outage but more irregular curves. A group width equal to the average size of all units of the combined system was found to give most accurate results. The grouped results in Fig. 6 are shown by circles. The grouping was done in 100-mw steps, which is close to the average size of the units of this system.

The curve through the grouped points is slightly beyond the envelope curve of the individual points, as should be expected, because the curve represents the summation of the individual frequencies, and the frequencies of the envelope points have the greatest weight.

The parameters of a group are derived from the individual values of the group by the following equations:

$$F_g = \sum F \quad \dots\dots(14)$$

where F_g is the frequency of the group and F are the frequencies of individual terms in the group.

$$P_g = \sum P \quad \dots\dots(15)$$

where P_g is the probability of the group and P are the individual probabilities;

$$T_g = 1/F_g \quad \dots\dots(16)$$

where T_g is the interval for the group;

$$t_g = P_g T_g \quad \dots\dots(17)$$

where t_g is the average duration for the group;

$$M_g = \frac{\sum MP}{P_g} \quad \dots\dots(18)$$

where M_g is the average magnitude of outage for the group, M are the individual magnitudes, and P the corresponding probabilities, and

$$g = \frac{\sum \sigma P}{P_g} \quad \dots\dots(19)$$

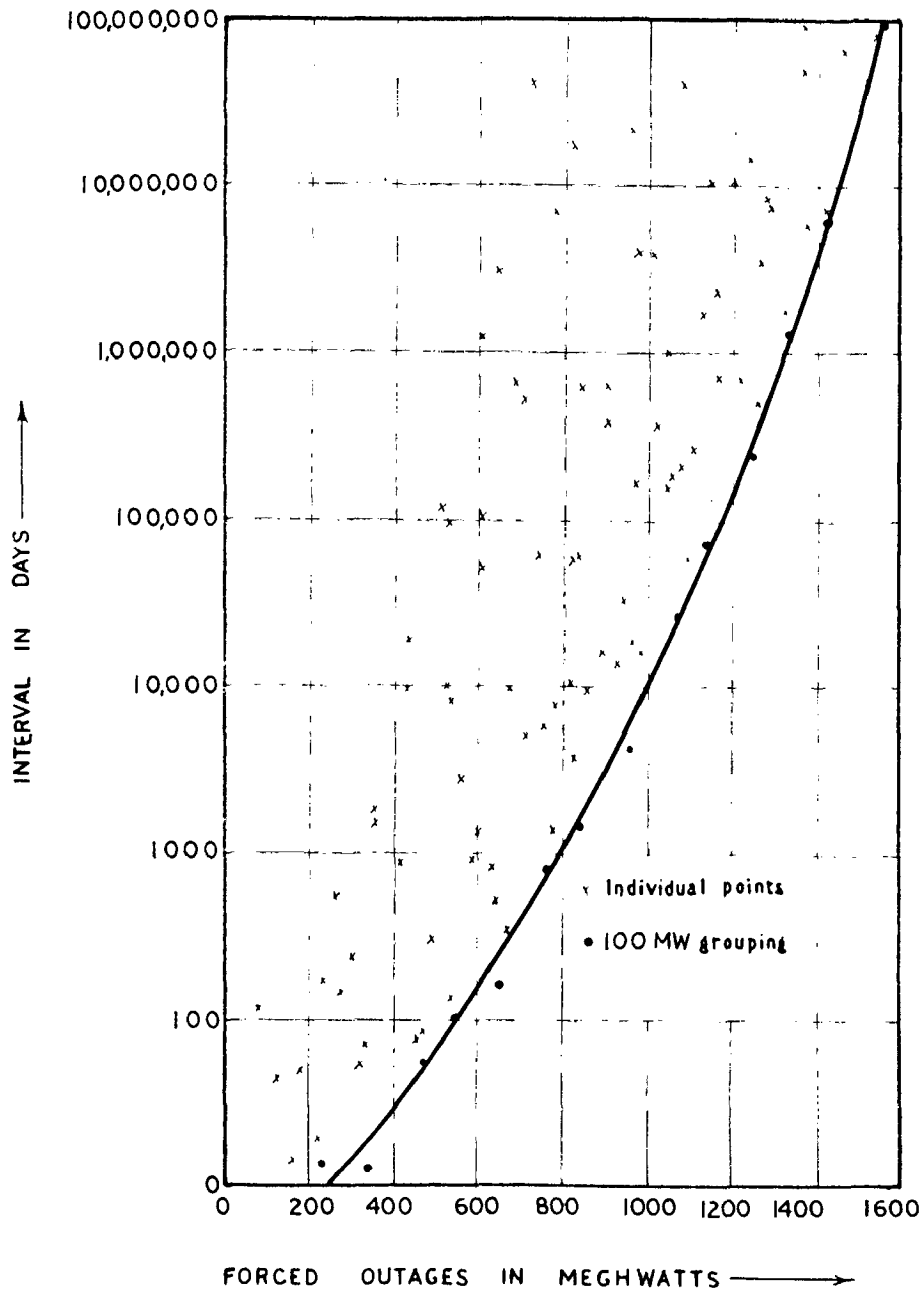


FIG. 6. FREQUENCY OF FORCED OUTAGES FOR A 5608 MW SYSTEM, INDIVIDUAL & GROUP RESULTS.

where σ_g is the standard deviation for the group, and σ is the standard deviation for the individual values.

V. Division of Gains from Interconnection of Two Systems:

NOMENCLATURE

C	= capacity
L	= load
F	= forced outages
OH	= overhauling
J	= limitations
E	= excess of emergency load capability over normal load capability.
I	= interruptible load
T	= help from previously existing ties
PF	= peaking factor
X	= interchange over new interconnection
D	= I + T + PF

Furthermore, subscripts A and B apply to the two interconnected systems respectively, and subscript AB to the combined systems.

Division of Required Capacity Between Two Systems Based on

Equal Interchange:

$$C_{AB} = C_A + C_B$$

$$C_A = L_A + (F_A + OH_A + J_A - E_A) - D_A - X$$

$$C_B = L_B + (F_B + OH_B + J_B - E_B) - D_B - X$$

Within relatively narrow limits, it is permissible to assume that $(F + OH + J - E)$ is proportional to C or equal to YC . Then

$$C_A = L_A + Y_A C_A - D_A - X$$

$$C_B = L_B + Y_B C_B - D_B - X$$

is obtained. Setting $Z = 1 - Y$ yields

$$C_A Z_A = L_A - D_A - X$$

$$C_B Z_B = L_B - D_B - X$$

which results in

$$C_A = \frac{(L_A - D_A) - (L_B - D_B) + C_{AB} Z_B}{Z_A + Z_B}$$

$$C_B = \frac{(L_B - D_B) - (L_A - D_A) + C_{AB} Z_A}{Z_A + Z_B}$$

Interchange Over Tie

The value of X is obtained by substituting the value of C_A in the equation

$$C_A Z_A = L_A - D_A - X$$

or from equation

$$X = \frac{F_A + F_B - F_{AB}}{2}$$

which is derived from the following four equations:

$$C_A = L_A + F_A + OH_A + J_A - E_A - D_A - X$$

$$C_B = L_B + F_B + OH_B + J_B - E_B - D_B - X$$

$$C_{AB} = (L_A + L_B) + F_{AB} + (OH_A + OH_B) + (J_A + J_B) - (E_A + E_B) - (D_A + D_B)$$

$$C_{AB} = C_A + C_B$$

Gain from Interconnection:

The reduction in capacity made possible by the interconnection is determined for each system respectively by

Required capacity before interconnection

$$C = L + CY - D$$

Required capacity after interconnection

$$C' = L + C'Y - D - X$$

Therefore

$$\text{Gain} = C - C' = \frac{X}{1 - Y} = \frac{X}{Z}$$

MAIN COMPUTER PROGRAMME

```

C C OPTIMAL HYDRO-THERMAL SCHEDULING USING NUMERICAL DYNAMIC PROGRAMMING WRITING
  DIMENSION IQ(40),IH(40),IQT(24),IQMIN(24),IHMIN(24),IDEM(24)
  DIMENSION ALV(3,24),R(3,24),S1(24),IHDR(24),AIQOPT(3,24),COST1(2)
C IQ AND IH STAND FOR DISCHARGE VS CORROS HYDRO POWER IN MW
C IDEM FOR LOAD IN MW,IQT FOR TRIAL DISCHARGES
C ALV IS VOLUME MATRIX CORROS TO INCREMENTS ON TRIAL SCHEDULE
C R IS WEIGHTED OUTPUT MATRIX
C S1 IS THERMAL GEN,IHDR HYDRO GEN
C AIQOPT IS OPTIMAL DISCHARGE
C ALVOL IS TOTAL VOLUME OF WATER, INCD-INCREMENT
C AL1,AL2,AL3, ARE LOSS COEFF AL1-S*S, AL2-H*H
C AC1,AC2,AC3, ARE COST CONSTANTS,AC1-S*S,AC2-S
C SMAX,SMIN,IQMAX ARE CONSTRAINTS ON THERMAL/HYDRO GEN
C FRATE COST OF FUEL PER MILLION BTU
C DDIFF STANDS FOR PERMISSIBLE DIFFERENCE OF COST BETWEEN 2 ITER
  READ10,M,INCD,ALVOL
 10  FORMAT(2I5,F10.1)
  READ15,(IQT(I),I=1,24)
  READ15,(IDEM(I),I=1,24)
 15  FORMAT(14I5)
  READ16,AL1,AL2,AL3,AC1,AC2,AC3
 16  FORMAT(3E20.8)
  READ18,SMAX,SMIN,IQMAX,FRATE
 18  FORMAT(2F10.2,I5,F10.4)
  READ15,(IQ(I),IH(I),I=1,M)
  READ17,DDIFF
 17  FORMAT(F10.1)
C CALCULATION OF MINIMUM HOURLY DISCHARGES
  DO 23 I=1,24
  LD=IDEM(I)
  CALL DISCH(LD,SMAX,AL1,AL2,AL3,INMIN)
  DO 20 J=1,M
  IF(INMIN-IH(J)) 19,19,20
 20  CONTINUE
 19  NN=J
  IHMIN(I)=IH(NN)
 23  IQMIN(I)=IQ(NN)
  PUNCH 24
 24  FORMAT(5X6H IQMIN5X6H IHMIN5X5H IDEM)
  PUNCH 21,(IQMIN(I),IHMIN(I),IDEM(I),I=1,24)
 21  FORMAT(6X,I5,6X,I5,6X,I5)
C GENERATION OF ALV(3,24) MATRIX CORROS TO TRIAL DISCHARGES
  ITER=0
 28  ALV(1,24)=IQT(24)+INCD
  ALV(2,24)=IQT(24)
  ALV(3,24)=IQT(24)-INCD
  ALV(1,1)=ALVOL
  ALV(2,1)=ALVOL
  ALV(3,1)=ALVOL
  DO 22 I=3,24
  IX=26-I
  AIQT=IQT(IX)
  DO 22 J=1,3
 22  ALV(J,IX)=ALV(J,IX+1)+AIQT
C EVALUATION OF HYDRO GEN CORROS TO TRIAL DISCHARGES
  DO 25 I=1,24

```

```

COST=0.
DO30I=1,24
LD=IDEM(I)
LHDR1=IHDR(I)
CALL THERM(LD,AL1,AL2,AL3,LHDR1,S11)
COST=COST+FRATE*(AC1*S11*S11+AC2*S11+AC3)
30 S1(I)=S11
COST1(1)=COST
PUNCH 33
33 FORMAT(6X5H IHDR8X2HS1)
PUNCH31,(IHDR(I),S1(I),I=1,24)
31 FORMAT(6X,15,5X,F10.2)
PUNCH32,COST1(1),ITER
32 FORMAT(19HTOTAL THERMAL COST=F10.2,2X,15HAFTER ITERATIONI3)
C CALCULATION OF MAXIMUM WOUTPUTSAND OPTIMALDISCHARGES
DIMENSIONLQ2(3),R2(3),RI(3),AVOPT(24),ROPT(24)
C CALCULATION OF R FOR24TH HOUR
LD=IDEM(24)
DO35J=1,3
LAQ=ALV(J,24)
AIQOPT(J,24)=ALV(J,24)
CALL POWER(LAQ,IQ,IH,LAH,M)
CALL THERM(LD,AL1,AL2,AL3,LAH,S1A)
CALLGQ(AL1,AL2,AL3,AC1,AC2,LAH,S1A,R1)
35 R(J,24)=R1
DO70I=3,24
NX=26-I
DO70J=1,3
DO65IJ=1,3
LQ2(IJ)=ALV(J,NX)-ALV(IJ,NX+1)
LLD=IDEM(NX)
IF(LQ2(IJ)-IQMIN(NX))40,50,50
50 IF(LQ2(IJ)-IQMAX)55,55,40
40 R2(IJ)=0.
GO TO 56
55 LLQ2=LQ2(IJ)
CALLPOWER(LLQ2,IQ,IH,LFF,M)
CALLTHERM(LLD,AL1,AL2,AL3,LFF,S1B)
CALLGQ(AL1,AL2,AL3,AC1,AC2,LFF,S1B,R11)
R2(IJ)=R11
56 RI(IJ)=R(IJ,NX+1)+R2(IJ)
65 CONTINUE
R1=RI(1)
R21=RI(2)
R31=RI(3)
CALLGMAXI(R1,R21,R31,RMAX,IIX)
R(J,NX)=RMAX
70 AIQOPT(J,NX)=LQ2(IIX)
C CALCULATION OFR FOR 1ST HOUR
LD=IDEM(1)
DO75J=1,3
LQ2(J)=ALV(1,1)-ALV(J,2)
IF(LQ2(J)-IQMIN(1))76,77,77
77 IF(LQ2(J)-IQMAX)78,78,76
76 R2(J)=0.
GO TO 79
78 LIQ2=LQ2(J)
CALLPOWER(LIQ2,IQ,IH,LBH,M)
CALLTHERM(LD,AL1,AL2,AL3,LBH,S1C)
CALLGQ(AL1,AL2,AL3,AC1,AC2,LBH,S1C,R1)
R2(J)=R1
79 R(J,1)=R2(J)+R(J,2)
AIQOPT(J,1)=LQ2(J)
75 CONTINUE

```


SUBROUTINES OF MAIN PROGRAMME

```

SUBROUTINE GMAXI(G1,G2,G3,GMAX,IX)
  IF(G1-G2)5,5,8
5  IF(G2-G3)6,6,10
6  GMAX=G3
  IX=3
  RETURN
8  IF(G1-G3)6,7,7
7  GMAX=G1
  IX=1
  RETURN
10 GMAX=G2
  IX=2
  RETURN
  END

**
C  SUBROUTINE GQ(AL1,AL2,AL3,AC1,AC2,LFH,S1,R1)
  AC1COST CONS RELATING TO S*S,AC2-S
  DF=2.*S1*AC1+AC2
  ALFH=LFH
  DLH=2.*AL2*ALFH+AL3*S1
  DLS=2.*AL1*S1+AL3*ALFH
  R1=DF*(1.-DLH)*ALFH/(1.-DLS)
  RETURN
  END
  SUBROUTINE THERM(LD,AL1,AL2,AL3,LFH,S1)
  ALD=LD
  ALFH=LFH
  B=AL3*ALFH-1.
  C=AL2*ALFH*ALFH+ALD-ALFH
  S1=(-B-SQRTF(B*B-4.*AL1*C))/(2.*AL1)
  RETURN
  END
  SUBROUTINE POWER(LAQ,IQ,LH,LFH,M)
  DIMENSIONIQ(40),LH(40)
  DO4I=1,M
  IF(LAQ-IQ(I))3,3,4
4  CONTINUE
3  J=I
  LFH=LH(J)
  RETURN
  END
  SUBROUTINE DISCH(LD,SMAX,AL1,AL2,AL3,IHMIN)
  ALD=LD
  B=AL3*SMAX-1.
  C=AL1*SMAX*SMAX+ALD-SMAX
  IHMIN=(-B-SQRTF(B*B-4.*AL2*C))/(2.*AL2)
  IHMIN=IHMIN
  RETURN
  END

```

RESULTS

IQMIN	IHMIN	IDEM
22500	232	460
22500	232	430
22500	232	420
22500	232	410
22500	232	400
22500	232	410
22500	232	470
22500	232	550
27500	284	660
28500	294	670
29500	304	680
32000	330	700
22500	232	580
22500	232	600
22500	232	610
22500	232	610
32000	330	700
37500	372	740
32000	330	700
30500	315	690
25500	263	640
22500	232	600
22500	232	550
22500	232	500
IHDR	S1	
258	215.41	
258	184.64	
258	174.40	
258	164.17	
258	153.96	
258	164.17	
258	225.69	
268	298.84	
330	354.97	
343	353.31	
352	355.40	
366	363.35	
298	301.66	
298	322.44	
298	332.84	
298	332.84	
366	363.35	
377	395.10	
366	363.35	
359	359.36	
315	348.13	
298	322.44	
268	298.84	
258	256.60	

TOTAL THERMAL COST= 28215.02 AFTER ITERATION 0

ROPT
 73527.71
 70940.76
 68429.64
 65930.48
 63443.25
 61006.81
 58558.67
 55999.37
 53169.76
 49776.13
 46265.84
 42670.36
 38988.11
 35918.26
 32899.54
 29786.45
 26673.35
 22938.36
 19065.66
 15353.20
 11687.93
 8403.63
 5384.91
 2595.78

IHDR

S1

263	210.70
258	184.64
258	174.40
258	164.17
253	158.66
253	168.88
258	225.69
279	288.44
330	354.97
343	353.31
352	355.40
359	369.82
304	296.04
294	326.20
304	327.20
304	327.20
366	363.35
375	396.94
363	366.12
359	359.36
320	343.45
294	326.20
274	293.16
258	256.60

TOTAL THERMAL COST= 28160.24 AFTER ITERATION 1

ROPT
73602.93
70976.56
68465.44
65966.28
63518.14
61120.91
58672.77
56073.52
53203.77
49810.14
46299.84
42704.37
39075.51
35920.14
32901.43
29868.65
26668.76
22956.30
19106.22
15393.76
11728.49
8444.19
5425.47
2636.34

IHDR

S1

268	205.99
258	184.64
258	174.40
253	168.88
248	163.37
253	168.88
263	220.97
284	283.73
330	354.97
343	353.31
352	355.40
352	376.31
315	285.76
294	326.20
294	336.61
315	316.90
363	366.12
372	399.72
363	366.12
359	359.36
320	343.45
294	326.20
274	293.16
263	251.87

TOTAL THERMAL COST= 28128.99 AFTER ITERATION 2

ROPT
 73664.20
 70990.96
 68479.84
 66019.97
 63571.83
 61174.61
 58765.88
 56079.91
 53210.15
 49816.52
 46306.23
 42741.25
 39143.18
 35987.82
 32969.10
 29856.01
 26656.12
 22943.66
 19093.58
 15381.12
 11768.97
 8484.67
 5465.95
 2636.34

IHDR

S1

274	200.36
258	184.64
253	179.11
253	168.88
248	163.37
248	173.59
274	210.63
284	283.73
330	354.97
343	353.31
348	359.11
348	380.03
315	285.76
294	326.20
304	327.20
315	316.90
363	366.12
372	399.72
363	366.12
352	365.85
320	343.45
294	326.20
279	288.44
263	251.87

TOTAL THERMAL COST= 28104.61 AFTER ITERATION 3

ROPT

73701.42
 71028.18
 68517.06
 66057.19
 63648.47
 61251.24
 58842.51
 56156.54
 53207.50
 49813.87
 46334.40
 42807.83
 39209.76
 36054.40
 33035.68
 29835.79
 26635.91
 22953.66
 19103.58
 15421.34
 11809.19
 8524.89
 5506.17
 2676.56

IHDR

S1

274	200.36
258	184.64
253	179.11
248	173.59
248	163.37
248	173.59
274	210.63
294	274.34
330	354.97
339	357.03
343	363.75
348	380.03
315	285.76
294	326.20
315	316.90
315	316.90
359	369.82
372	399.72
359	369.82
352	365.85
320	343.45
294	326.20
279	288.44
268	247.15

TOTAL THERMAL COST= 28090.54 AFTER ITERATION 4

ROPT

73745.42
 71072.18
 68561.06
 66101.19
 63653.05
 61255.83
 58847.10
 56161.13
 53212.09
 49818.46
 46338.98
 42843.38
 39284.12
 36128.75
 33030.11
 29830.23
 26630.34
 23001.48
 19151.41
 15469.16
 11857.01
 8572.71
 5553.99
 2724.38

IHDR

S1

274	200.36
258	184.64
253	179.11
253	168.88
248	163.37
248	173.59
274	210.63
294	274.34
330	354.97
339	357.03
339	367.48
343	384.68
315	285.76
304	316.80
315	316.90
315	316.90
352	376.31
372	399.72
359	369.82
352	365.85
320	343.45
294	326.20
279	288.44
274	241.50

TOTAL THERMAL COST= 28074.58 AFTER ITERATION 5

ROPT
 73787.87
 71114.63
 68603.51
 66143.64
 63695.50
 61298.27
 58850.13
 56164.16
 53215.12
 49821.49
 46380.83
 42885.23
 39357.24
 36201.88
 33016.87
 29816.98
 26617.10
 22988.24
 19138.16
 15509.30
 11897.16
 8612.86
 5594.14
 2724.38

IHDR

S1

274	200.36
258	184.64
253	179.11
253	168.88
248	163.37
253	168.88
274	210.63
294	274.34
330	354.97
334	361.69
339	367.48
339	388.41
315	285.76
315	306.51
315	316.90
315	316.90
352	376.31
372	399.72
352	376.31
352	365.85
320	343.45
294	326.20
284	283.73
274	241.50

TOTAL THERMAL COST= 28058.55 AFTER ITERATION 6

IQT	IHDR	ROPT	S1
26500	274	73813.14	200.36
25000	258	71139.90	184.64
25000	258	68628.78	174.40
24500	253	66129.63	168.88
24000	248	63681.49	163.37
24500	253	61284.26	168.88
26500	274	58836.12	210.63
28500	294	56150.14	274.34
32000	330	53201.11	354.97
32000	330	49807.48	365.42
33000	339	46398.11	367.48
33000	339	42902.51	388.41
30500	315	39374.52	285.76
30500	315	36219.16	306.51
31000	320	33034.16	312.23
30500	315	29795.35	316.90
34500	352	26595.46	376.31
37500	372	22966.60	399.72
34000	348	19116.53	380.03
34000	348	15518.46	369.56
31000	320	11936.96	343.45
28500	294	8652.66	326.20
28000	289	5633.94	279.03
26500	274	2724.38	241.50
TOTAL OPTIMISED COST=		28049.46	AFTER ITERATIONS
			7

DATA

35 500711000.0
 2500025000250002500025000250002500026000320003350034500365002900029000
 29000290003650038500365003550030500290002600025000
 460 430 420 410 400 410 470 550 660 670 680 700 580 600
 610 610 700 740 700 690 640 600 550 500
 .00005 .00015 .00002
 .0019911 9.60644 373.704
 400.00 40.0 39500 .3500
 22500 23223000 23823500 24324000 24824500 25325000 25825500 263
 26000 26826500 27427000 27927500 28428000 28928500 29429000 298
 29500 30430000 30930500 31531000 32031500 32532000 33032500 334
 33000 33933500 34334000 34834500 35235000 35535500 35936000 363
 36500 36637000 36937500 37238000 37538500 37739000 38039500 382
 10.0

B_I_B_L_I_O_G_R_A_P_H_YA Note on Bibliography:

A detailed bibliography on the "Optimum Operation of Power Systems 1919 - 1959" has been given in AIEE Transactions on Power Apparatus and Systems February 63 pp.864-871. This bibliography contains 436 authorized papers, books and other material in the field of optimum operation of Power System and related areas.

This bibliography has been supplemented upto 1969 by Shri A.L. Puttaswamy in his M.E.(WRD) dissertation entitled "Optimum Scheduling of Power in Power Systems" University of Roorkee, Roorkee, 1973. He has added 131 references on the topic.

The references on the topic taken from IEEE Transaction on Power System & Apparatus upto 73 have been listed here.

A detailed bibliography on application of probability method has been compiled by R. Billington and published under the above caption in IEEE Transactions on Power Systems and Apparatus - March/April 1972 - pp.649. This bibliography is divided under six sub-parts as listed below:

1. Static Generating Capacity reliability Evaluation - 103 references on this aspect have been listed and contain papers on the above subject from 1933 onwards to 1971.
2. Spinning Generating Capacity reliability evaluation - 9 papers have been grouped under this sub-heading.
3. Bibliography on equipment Outage rate - 22 references have been given on this topic.
4. Bibliography on transmission system reliability evaluation - 29 references have been given under this caption.
5. Bibliography on general considerations - 49 references have been compiled under this sub-heading.
6. Bibliography on Theory - 36 references have been given under this sub-heading.

The papers on the topic after 1970 published in IEEE Transactions PA&S upto 1973 have been brought up in the bibliography of dissertation.

{1} Optimal Scheduling

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12. H.Nicholson & M.J.H.Sierling, "Optimum Dispatch of Active and Reactive Power by Quadratic Programming", I.E.E.E., P A & S, March/April 1973, pp.644.
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