MULTI OBJECTIVE ANALYSIS OF WATER RESOURCES PROJECTS THROUGH SYSTEMS APPROACH

—A Review—

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CERTIFICATE

Certified that the dissertation entitled "Multi-Objective Analysis of Water Resources Projects through Systems Approach - A Review", which is being submitted by Shri M.Gopalakrishnan in partial fulfilment for the award of Degree of Master of Engineering in Water Resources Development of the University of Roorkee is a record of the candidate's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma, to the best of my knowledge.

This is to further certify that he has worked for a period of over nine months for the preparation of this dissertation from October 1975 to July 1976.

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Dated 31.7.1976

A C K N O W L E D G E M E N T

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SYNOPSIS

This study is an attempt to consider a multiple objective planning base for water resource projects. It identifies the objectives relevant to water development as National Economic Development, Income redistribution and Environmental Quality: some other objectives identifiable from planning documents have been listed.

Social benefit-cost framework that simultaneously consider 'willingness' to pay of the beneficiaries, opportunity costs and shadow prices has been brought out. Evaluation procedures for regional income distribution are described. Necessity for devising suitable scales for other objectives has been indicated. A combined treatment of social rate of discount and social value of investment has been favoured that also takes into account uncertainty.

The classical approach to multi-objective analysis attempts to evolve the technical transformation curve and social preference curves for locating an optimal choice. While this approach is illustrated the need for systems approach to handle complex problems is emphasized. Following this, the solution strategies for multi-objective optimization have been detailed after a brief survey on models for multi-objective decision making. As multiplicity of objectives introduce a vector in optimization function, maximization (or minimization) of the objective function in the classical sense is difficult. The concept of non-inferiority has been brought

out. Trade-offs that explicitly consider preferences between objectives are obtainable by systems techniques. Solution procedures of this class relevant either for a'top-down methodology' or a 'bottom-up methodology' have been discussed.

For practical application to complex water resources systems, the use of system modelling including multi-objective analysis has been indicated. The study has been concluded with a review on a simple'case study.'

C H A P T E R O

INTRODUCTION

INTRODUCTION

0.0. FUNDAMENTAL OBJECTIVE

Water Resources Development has occupied a prominent position in the development plans of India.[43] Projects to serve this end are major public investment programmes which have a far-reaching economic, social and environmental impacts on implementation. The fundamental goal for planning the water resource projects should therefore be the enhancement of the general welfare of the nation. The aim of this study is to attempt how best this could be achieved in the planning process by an explicit consideration of objectives that reflect the society's welfare: and how best the systems approach could be utilized to analyse the problem thus formulated.

0.1. PROBLEM DEFINITION

0.1.1. Problem 1

0.1.1.1. The water Resource Project formulation concerns presently with the economic efficiency. The Benefit-Cost Analysis which serves to this end relies upon 'consumer sovereignty' and 'competitive market mechanism' to determine prices. The normative economic theory that underlies the Benefit-Cost Analysis stresses upon 'Pareto Optimality' as the ideal that is sought.

0.1.1.2. Equity and other considerations have resulted in an increasing awareness for the need for simultaneous consideration of other objectives that reflect the welfare of society. The problem number 1 of the present study is therefore, identified as "Identification of Objectives".

0.1.2. Problem 2

0.1.2.1. The traditional Benefit-Cost calculations strive for a 'commercial profitability', through the economic efficiency criterion. With the existence of imperfect markets and structural disequilibrium, these calculations do not reflect social gains or losses adequately. The need is apparent for a social benefit-cost calculations aimed at systematizing the complex problem of project planning from the point of view of society or nation.

0.1.2.2. Also criterion for measurement of benefits and costs for the various objectives other than economic efficiency is important so as to have a proper evaluation.

0.1.2.3. Public investment in water resources involves commitments over time whose implications need a careful examination in project evaluation. This brings into consideration the social rate of discount, the opportunity cost of investment and risk/uncertainty.

0.1.2.4. The problem number 2 has therefore been identified as the complexities in the evaluation plans, given the objectives.

0.1.3. Problem 3

Essentially multi-objective planning elevates many of the non-commensurate objectives at par with that of economic efficiency. This necessitates the treatment of an objective function that is a vector in place of conventional scalar objective function.

The modelling, mathematical programming and evaluation of multiple objective vector functions have been identified as problem No.3.

0.2. REPORTING FORMAT

The identification of the relevant objectives for water resources planning, the complexities in evaluation of the projects together with a classical approach to multi-objective problem solution have been taken up in Part I... THE SETTING. Problems Identified as 1 & 2 are dealt with in this part.

Part II ..... ANALYSIS is devoted to deal with problem 3 above. It details multi-objective problems, solution strategies and decision-making methods.

Part III concludes the work with a SUMMARY, and reports FINDINGS and recommendations for further work.

0.2.1. <u>The Arrangement of Chapters</u>

0.2.1.1. Multi objective planning for water resources is a generalization of traditional benefit-cost analysis. However, unlike the traditional benefit-cost calculations which reflect the projects contribution to the national income, in multi-objective analysis, projects are evaluated in terms of their contributions to all socially relevant objectives identified prior to planning. An useful point wherefrom the study could set to roll is, therefore, an inlook into the evaluation processes in water resources planning. The history, a critical review of the traditional benefit-cost analysis and the necessity for multi-objective planning have been taken up in <u>Chapter 1</u>.

0.2.1.2. In Chapter 2 the different objectives are explored. The recent practices in U.S. has been reviewed and a comparison to Indian Planning objectives attempted. Besides the framework for a social cost-benefit analysis to obtain the real social gains/losses to the nation, the appropriate measurement procedures for the income distribution benefits have been attempted. For other non-commensurate objectives like environmental quality, the need for scales of measurement are indicated.

0.2.1.3. Chapter 3 looks into problems in evaluation viz; considerations with respect to social rate of discount for different objectives, the social value of investment. Attention has also been paid to the element of uncertainty/risk.

0.2.1.4. Basic to requirement of solution of multi-objective problem is one of evaluating the transformation surface: this together with 'a priori' knowledge of indifference among objectives yield solution in a simple manner. This classical approach to problem is presented in Chapter 4.

0.2.1.5. The need for a systematic approach in planning to improve the decision-making process is obvious. A survey of the available models in decision-making process is made in Chapter 5; most of the solution techniques for multi-objective problems are found as a combination of these methods to best suit the purpose.

0.2.1.6. Chapter 6 has been devoted to the mathematical modelling of the multi-objective problem.

0.2.1.7. Though the concept of multi-objective analysis is recent, the related solution strategies are varied and plenty even at this early stage. These solution techniques have been categorised in three broad groups as below based on their approach to problem solution and dealt with in Chapter 7.

i) Generating Techniques for multi-objective problem,

ii) Techniques that rely on prior knowledge of preferences,

iii) Interactive solution techniques.

0.2.1.8. The application of the multi-objective planning for a large water resource system with multiple objectives is complex. The aid of the systems modelling and computer based analysis procedures are needed to handle them effectively. This aspect together with a brief review of a case study has been the consideration for Chapter 8.

2.2. In the final Part III, Summary, Findings and Recommendations, a recapitulation of the multiple objective framework has been presented in a nutshell; the findings are listed; and recommendations for further studies indicated.

PART I

THE SETTING

CHAPTER 1

TRADITIONAL BENEFIT-COST ANALYSIS

1.1. BENEFIT-COST ANALYSIS - A HISTORY

Benefit-cost Analysis is an aid to implementation of the strategy of development. It subjects the public investment decisions to quantitative economic analysis. The goals of the Benefit-Cost analysis as well as economic choice, in general, can be stated as maximisation of utility subject to whatever constraints the social, economic and political environment impose.

1.1.2. The initial work on benefit-cost analysis has been traced to the work of Jules Dupuit.[65] However, systematic efforts to apply the technique have been under constant review, notably in U.S. Following considerable works in the field in 1920-1930, the U.S.Flood Control Act 1936 stipulated that feasibility can be interpreted to mean "the benefits, to whomsoever they may accrue, are in excess of estimated costs". Projects were formulated and designed to demonstrate that the above standard was being met.

1.1.3. The criterion to work out the Benefit-Cost analysis to satisfy the objective set forth in the U.S.Flood Control Act 1936 came only in 1950 when an interagency committee attempted to introduce uniformity into standards and criteria. This was known more famously as "Green Book"[119]

1.1.4. The "Green Book" did not have an official sanction. In 1952, the U.S.Budget Bureau issued a circular A-47 which formed the basis for appraisal of projects. This document, however, faced lot of criticism for its overly narrow "accountant's view".[31]

1.1.5. For the meanwhile, there were considerable variations among agencies like U.S.B.R., Army Corps of Engineers and Department of Agriculture, in their practices for evaluation. Which shall govern project selection? Maximization of difference between Benefits and Costs or the ratio of benefits to cost, or rate of return? There was also disparity in working out "project costs"/"benefits"[31]. To review the standards and criteria for river development, a panel of consultants was appointed in U.S. which submitted its report in 1961. A new U.S.interagency committee was subsequently appointed to inves-

1.1.8. The water Resources Council constituted in U.S. under Water Resources Planning Act of 1965 proposed a multi-objective system of 4 objectives and 4 evaluation accounts:[122] viz

National income

Regional development

Environmental quality, and

Social well-being

The final product as approved in 1973 in U.S. provided 2 objectives in plan formulation[123] viz

National economic development, and

Environmental quality

and 4 accounts for recording beneficial and adverse effects[123]

National economic development

Environmental quality

Regional development, and

Social well being

with the proviso that no one objective is to be viewed as inherently superior to another. This was followed by various agencies in U.S. evolving their own guidelines for the evaluation of projects[21] keeping in view the provisions as above in the Federal Register.[123]

1.1.9. It is pertinent now to look on our own methods of analysis. The Indian analysis of feasibility of the projects was on the basis of direct financial return, since the nineteenth century upto independence. The feasibility test stipulated tigate water-resources investment criteria and its recommendations, published as Senate Document No. 97, 87th Cong., came into application in U.S. since 1962.

1.1.6. With an inter-regional symposium on project preparation and evaluation held in Prague in 1965, the United Nations Industrial Development Organisation, Vienna started work in the field of "methodology and practice of rational benefit-cost analysis". The cumulative experience of UNIDO coupled with the recommendations that emerged through the symposium, led to the development of a set of guidelines by UNIDO for use by the developing countries. While the background papers were written in 1965/66, the ultimate outcome has emerged as a Guidelines for project formulation and evaluation in 1972.[118]

1.1.7. Reverting to developments in U.S. itself, multiple objectives came to be identified more and more. The seminal intellectual work was published in the Harvard water program, Design of Water Resource Systems[5] and in Marglin's public investment criteria[90]. Following this a U.S. Federal interagency group proposed in 1969 a set of uniform criteria for bringing multiple objective analysis into the everyday evaluation process of water agencies.[120] The new criteria were also due to the deficiencies pointed out in the benefit-cost analysis by Maass & Major, advocating at the same time a multi-objective evaluation of water resources and other public investments.[6,7,80]

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1.1.9. It is pertinent now to look on our own methods of analysis. The Indian analysis of feasibility of the projects was on the basis of direct financial return, since the nineteenth century upto independence. The feasibility test stipulated a minimum specified return on the sum at charge on full development.[56]. After independence, the thinking underwent revision. Economic Efficiency criterion, similar to approaches in vogue in U.S. and other developed countries was proposed in1964 by the Nijalingappa Committee formed to suggest planning strategy for irrigation projects[44]. The same criterion also was embodied in Planning Commission's instructions "Criteria for Appraising the Feasibility of Irrigation Project"(1965).[115] The relevance of social benefit cost analysis, for our conditions was also not lost sight of.[41] The National Council of Applied Economic Research, New Delhi in its report on "Criteria for fixation of water rates and selection of irrigation projects (1959) recommends as social benefit cost analysis and the following rule for selection:[98]

> " The marginal social benefit of technically possible increment of investment must be equal to the marginal social cost and the ratio between marginal social benefit and marginal social cost must be the same for all investments".

It has been observed therein:

> "Since India has adopted planning as a technique of resource allocation and since planning in India covers a significant part of the economy, estimates of anticipated social benefit and social cost should be accepted as guides for resource allocation for irrigation projects. This is necessary in view of the fact that it has not been possible to evolve refined statistical technique for the purpose. Further, since

planning embraces the private sector as well as the public sector, the separate measurements of secondary benefit is not at all necessary. Each sector of the economy can be viewed in terms of its contribution to national income".

Inspite of the realisation that social costs and benefits should be accounted for in the analysis, our benefit cost analysis is based on economic efficiency criteria (Gadgil Committee) and follows the guidelines of Planning Commission (Research Programme Committee).[45]

Thus our system in project evaluation is based on traditional benefit-cost analysis, the aspects of which will be looked into, in a brief manner, in what follows.

1.2. THE ECONOMIC ASPECTS AND RELATED PROBLEMS IN EVALUATION OF TRADITIONAL BENEFIT-COST ANALYSIS

1.2.1. General

1.2.1.1. Benefit-cost analysis has been universally employed as a powerful tool for project selections besides ranking of alternatives showing its relative preferredness. Benefit-cost analysis is characterised as the collection and organisation of relevant data by some conceptually meaningful criteria.[70] Water Resource Projects are meant to serve multiple purposes recognised since decades.[56] All the contributions to the national output arising out of each of them are evaluated besides a similar exercise for costs. This demands quantification in two important terms:

(i) physical

(ii) economical

The quantifying in economic terms requires a base or a common denominator; money is commonly employed to serve this end.

1.2.1.2. Basic Concept

The problem of efficient resource allocation is solved by the traditional benefit-cost analysis following the 2 basic general concepts:

1) build every project for which benefit exceeds cost*;
2) develop every project to the point where marginal benefits equals marginal costs.*

The solution assumes that

1) Funds/Resources are unlimited
2) Benefits as well as costs are correctly assessable
3) A static equilibrium condition in the economy prevails

1.2.1.3. Problems

There have been considerable problems in the measurement of benefits and costs. Traditonally benefits (costs can be considered as negative benefits) are classified as:

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*Variations in this principle occurs with the constraints of limited resources or institutional abilities to build projects. As (B-C) tilts the selection in favour of huge projects, B/C is considered as a criterion; also rate of return is another practice (as in India) for project selection.

i) Primary (Direct) Benefits

ii) Secondary (Indirect) Benefits

(a) "Stemming from" secondary benefits

(b) "Induced by" secondary benefits

Besides intangible benefits also used to be identified in a qualitative manner and described appropriately.

The tangible benefits, which encompasses direct and indirect benefits, are subject to problems connected with[18]

1) Elasticities of demand and supply

2) Economic fluctuations/changes over time of

(a) preferences, and

(b) technology

3) Functioning and results of the price system itself affected by

(a) aggregate income

(b) income redistribution, and

(c) market form

4) Institutional restrictions that interact on the validity and relevance of values directly yielded by market.[43]

The problems associated with secondary benefits are more complex.

All these led to adoption of conflicting evaluation procedures by different authorities to solve the various problems by practical approximations. However, such semantics as tangibles and intangibles did strenthen confusion and emotional attitudes in matters of evaluation.[43]

1.2.2. Benefit-Cost Evaluation

1.2.2.1The theoretical basis of traditional benefit-cost calculations relies on the market mechanism to establish prices. It was Adam Smith[1] who propounded first, welfare maximization in the market through "invisible hand" of perfect competition. The appropriateness to base the theoretical benefit-cost calculation on the competitive model of market mechanism has been excellently brought out concisely by Otto Eckstein.[31]

1.2.2.2. Assumptions

For an efficient resource allocation the model presupposes that

1) there exists perfect competition

2) factors are: i) completely independent
ii) divisible
iii) perfectly mobile

Also certain other assumptions are made regarding

i) consumers
ii) producers
iii) resources
and iv) factors of production

1.2.2.3. Optimality Condition

The classical analysis yields an equilibrium condition where it would not be possible to make anyone better off without making someone else worse off. This concept is the basis of the traditional cost benefit-analysis following the definition of Pareto Optimality.[99]

1.2.2.4. Paretian Optimality and Compensation Criteria

Though logical and appears unobjectionable, Pareto Optimality glosses over the issue of who pays and who benefits. For this purpose a technique known as Compensation Criterion[58] is utilised to resolve the issue. Any project that fully compensates each individual for any losses[94] and increases the utilities of some individuals (without reducing those of others) emerge as "a preferable one". However, there exists an asymmetry in "willingness to pay as compensation" and "willingness to be paid for". These dilemma are solved by good judgement/ negotiations.[26]

1.2.2.5. Defects and Inadequacies

A few words about Pareto Optimality. For each initial distribution of wealth ,there exists a Pareto Optimal point. This indicates an infinite number of Pareto Optimal points for different distributions of wealth. Besides the Pareto Optimality condition defines only a limited subset of circumstances under which social welfare will improve. Bator's demonstrations on this aspect is lucid.[5]

1.2.2.6. Other Aspects

There thus seems to be nothing sacrosanct about Pareto Optimality. But having made the assumption that the Government seeks Pareto Optimality, one has to look into the inadequacies inherent in the analysis.[93] There are considerable disputable points which can be categorised broadly as

(i) Imperfect prices

(ii) Constraints on resources affecting market price

(iii) Price supported programmes

(iv) Changes in supply and demand conditions

(v) Unemployed resources

(vi) Externalities.

Fresh water, characteristic of utilization is a scarce commodity. However, items that are considered scarce, change with time. Water (of acceptable standard) once considered ample and thus of little value is increasingly becoming scarce with resultant rise in its value. While scarcity is registered in market place, establishing common markets for water is difficult. This leads to the situation in which one finds that some benefits and costs are correctly registered in market by prices, some are incorrectly registered by market prices, some are registered in no markets although simulated market values are computable. For others it is nearly impossible to have an adequate market valuation.[63] Also constraints on resource use is typical. Cases like quota system introduce complexity due to competitive and conflicting nature of users. Similarly the value of marginal unit gets affected by program of subsidies. Anticipated charges in supply and demand condition for items like water, are likely to change considerably in years to come. For projects planned on a longer horizon estimation of costs or gain with due adjustment for change is complex. Unemployed resources and utilisation is another important issue that does not get reflect in B-C-analysis in its traditional form.

1.2.2.7. Externalities

Externalities are due to non-realisation of the condition that factors are independent. This results in

(1) benefits whose value does not occur to the group implementing the project

(2) costs which are not forced back onto the owners or operators of a system.[26]

Externalities, grouped under the headings of

time

collective goods

altruistic and misanthropic considerations and

jurisdictional relationship

or constitutional arrangement

have been treated in an excellent manner by Marglin[88], with due suggestions for accounting them in the analysis.

Though the above problems were recognised, project evaluation has been historically based on optimality condition; evaluation has taken place - mainly in terms of economic benefits and costs, commonly known as "economic efficiency".

1.2.3. <u>Summary Comments</u>

To the extent that the welfare aspects other than the efficiency objective is considered away, the traditional benefit-cost analysis indeed reflects a full approximation of welfare. Eckstein has derived a solution strategy for a model based on welfare theory.[31] Steiner has evolved a different model introducing sectoral impacts(public and private) with budget constraint,[115] besides another model that accounts for the roles of alternative cost in project design

1.3. WELFARE THEORY AND THE NEED FOR EMPLOYING MULTIPLE OBJECTIVES FOR PLANNING

1.3.1. General

Satisfying the Pareto's Optimality, the traditional benefit-cost analysis strives to be economically efficient. Here, national welfare is identified with national income, of course, ignoring the other non-economic dimensions of welfare. However, society prefers redistribution of generated income to lower income groups and regions in order to achieve greater equality.[5] Unless other governmental measures are resorted to such as taxation, subsidy, quota etc. that redistribute income directly from one group to another (which is more difficult and might be unacceptable to a democratic society), incorporating the desired distribution of income as a criterion itself while designing the water resources development programme, even at the expense of potential increase to national income may be desirable.[5]

The traditional benefit-cost analysis also does not take into account many other aspects such as environmental quality etc. that can affect the quality of life; and consequently national welfare.

1.3.2. Welfare Aspect

Welfare as a concept for guidance of planning water resources programme is complex.[131] It is multi-dimensional. It is not restricted to either economic efficiency or resource readjustment or both. Gaffney[37] states

> "Economics, contrary to common usage, begins with the postulate that man is the measure of all things. Direct damage to human health and happiness is more directly 'economic', therefore, than damage to property which is simply an intermediate means to health and happiness. Neither do economists regard 'economic' as a synonym for 'pecuniary'. Rather money is but one of many means to ends as well as a useful measure of value. 'Economic damage' therefore includes damages to human functions and pleasure. The economist tries to weigh these direct effects on people in the same balance with other costs and benefits to the end of making decisions to maximize net social benefits".

It would appear that welfare theory has encompassed a more general concept; maximization of welfare has required the consideration of objectives and criteria in addition to that of economic efficiency.

1.3.3. On Other Objectives

Objectives other than economic efficiency have been identified historically with water resources development. However, lack of clearly defined goals in this field has not facilitated a precise definition of these objectives to guide planning better, including them in the analysis. Nevertheless the awareness to employ multiple objective has been a growing phenomenon.[131]

The choices made by the political process determine significant resource and factor allocation. On implementation of a water resources project, such decisions cause an irreversible social, ecological and other impacts. It is, therefore, desirable that planning is done on a broader basis encompassing objectives and criteria that best approximate social welfare. Besides economic, the following other dimensions of welfare are considered relevant by U.S. Water Resource Council after a recent detailed study.[122]

(i) Environmental quality

(ii) Regional development

and (iii) Social well-being.

1.3.4. Planning for Other Objective

A multi-objective planning process for development of alternative proposals that yield varying levels of achievement in case of different objectives can be schematically shown as in Fig. 1.1.[92]

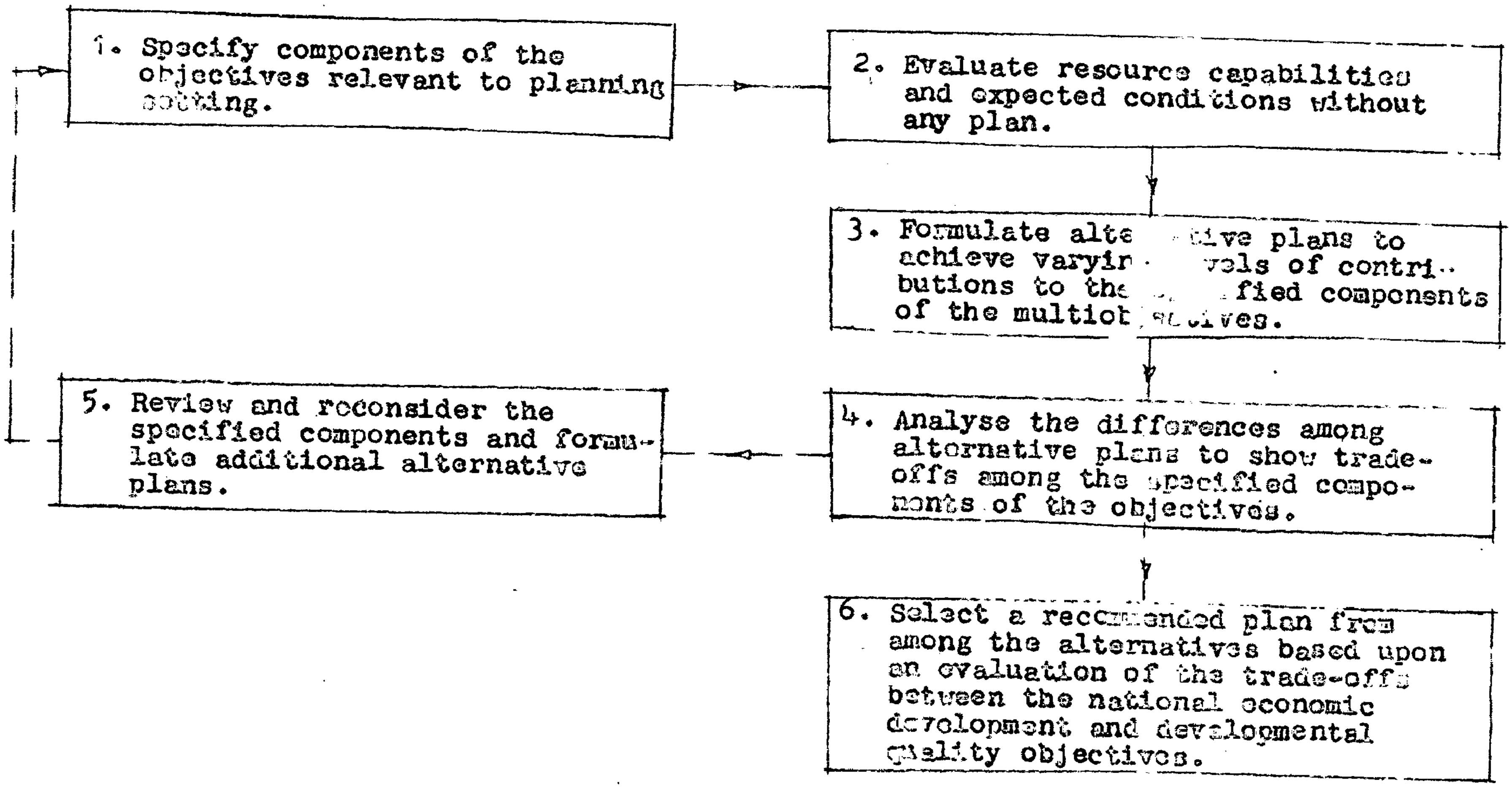


Figure 1.1. Planning Process.

1.3.5. Indian Conditions

Perspective and Five Year (short term) plans determine the broad strategy of planning in India.[46] Besides the national economic development objective, objectives like Income Redistribution, Social Well-being are implicit in planning documents and programmes of action set from time to time.[46,47,48]

Our emphasis on environmental quality, however, is comparably less than that of advanced nations like U.S. Attitudes and values that people hold, however, change with time. Environmental quality is likely to be a major concern in course of time. Projects planned in response to current demands must keep this factor in mind.

1.3.6. Summary Comments

There is need for multi objective analysis for evaluation and selection of the water resource plans using welfare concept. The studies that follow hence hold relevance to regional development and environmental quality objectives, besides the objective of national economic development. The other objectives of relevance are also discussed in para 2.4 that follows. Once set with these objective parameters, the multi-objective analysis aims to integrate them to provide acceptable non-inferior solutions to choose from.

C H A P T E R 2

ON OBJECTIVES FOR PLANNING

2.1. NATIONAL ECONOMIC DEVELOPMENT

2.1.1. General

2.1.1.1. The national economic development (NED) objective is enhanced by increasing the value of nation's output of goods and services and improving national economic efficiency.

2.1.1.2. The GNP comprising of economic goods and services produced for consumption, investment and export purposes indicates the aggregate consumption. The concept of NED is broader than that of national income and is a measure of the impact of governmental investment on the total national output. The gross national product and national income accounts do not give a complete accounting of the value of output of final goods and services resulting from governmental investments because only government expenditures are included.

2.1.2. Effects of the System Design

2.1.2.1. A typical list of contributions to national output by direct increase in productivity from an envisaged water-resources system plan could be[56]

(i) increases in crop yields from irrigation facilities

(ii) hydropower, especially peaking capacity for power systems

(iii) reduced disruption of economic activity due to floods, droughts

(iv) reductions of constraints due to non-availability of water for industrial production, domestic water supply etc.

(v) increased production from the employment of otherwise unemployed or underemployed resources.

(vi) fish and wild life preservation

(vii) pollution control

(viii) recreation

(ix) other external economies

2.1.2.2. Adverse Effects

Achievement of the above beneficial accounts is not devoid of adverse effects. The main adverse effect, from NED angle due to a development plan is simply the economic value that these resources would have had, in their alternative best, feasible, realistic exploitation. They are categorised, broadly,

(i) resources required or displaced to produce final or intermediate goods and services by the plan alternative

and (ii) decrease in outputs due to external diseconomies.

2.1.3. Measurement of Benefits

"With and without analysis" technique forms the basis for measurement of benefits (net after accounting for adverse effects) of the plan alternative.

The output of goods due to the project can be categorised broadly as

1. Marketable output

   (i) consumer goods

   and (ii) producer goods

   depending upon whether the goods/services are directly consumed or employed in the economy for production of other goods/services.

2. Non-marketable output

   represents stream of output that has no market at all.

2.1.3.1. Marketable Output

2.1.3.1.1. The important factor in benefit estimation of the systems is to examine if

(i) the net output, either the goods or services that are contributed, represent an addition to the economy.

(ii) the net output represents only substitution leaving total supply before and after the emergence of system, constant.

In case (i), the output due to the system (which is not available to the economy but for the project) is considered for benefit calculation. If case (ii) is valid, the net benefits created by the system are the newly available resources that have been released from the alternative supply prior to availability of the output of the system.

An illustration for case (ii) will be appropriate. Let the systems output represent increase in food grains due to extensive irrigation facilities, which otherwise was imported, then the net benefit is actually the money saved from imports, in foreign exchange.[118]

As a first step in benefit estimation, the system output should be divided as

(i) additions to supply in the nation's economy as a whole

and (ii) substitution for supply in the nation's economy.

For the second category, again, identification of resources previously used in the alternative source of supply is necessary.

2.1.3.1.2. The net benefit, due to additional supply for the economy can be mathematically expressed as[5]

$$W(A) = E(\underline{Y}) - C(\underline{X}) \quad (2\text{-}1)$$

where $W(A)$ = net social benefits of the systems

$E(\underline{Y})$ = social benefits of the system

& $C(\underline{X})$ = social costs due to the system.

In the expression the vector $\underline{Y}$ represents the variable system outputs $(y_1, y_2 \ldots\ldots y_m)$ assuming m categories of outputs. Similarly vector $\underline{X}$ represents system inputs $(x_1, x_2 \ldots\ldots x_n)$ with n categories of inputs. The implied condition being that the system outputs and inputs are related to each other by production function.

$$f(\underline{X}, \underline{Y}) = 0 \quad (2\text{-}2)$$

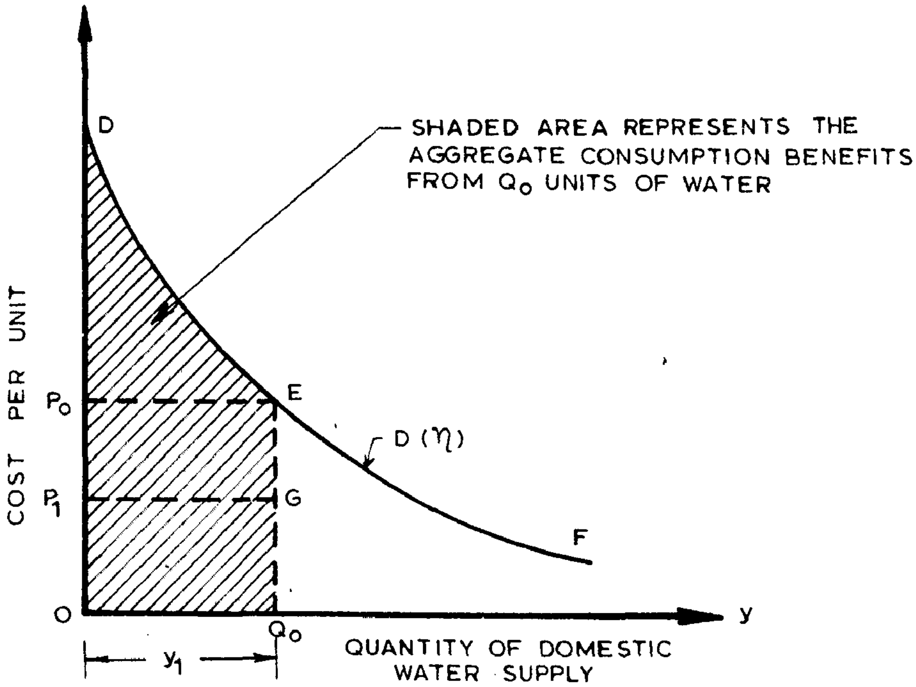
The real problem is then the estimation of $E(\underline{Y})$ or the benefits due to a system output. This is measured by the criterion "willingness to pay". This will be dealt with for the following two categories; separately,

(i) outputs directly consumed by consumers

(ii) outputs yielding producer goods.

2.1.3.1.2.1. Outputs directly consumed by Consumers

The benefits in this case are the increments valued in terms of individuals' willingness to pay. This need not necessarily be what is actually paid for. This is shown in Fig. 2.1. The willingness to pay for any output, say domestic



DF	DEMAND CURVE AT SUCCESSIVE PRICE LEVELS
AREA OQ_0ED	WILLINGNESS TO PAY FOR QUANTITY Q_0
Q_0E	'COMPETITIVE' PRICE AT Q_0 LEVEL OF DEMAND
AREA OQ_0P_0E	COMPETITIVE MARKET PRICE REALISED FROM SALE OF Q_0
P_0ED	CONSUMER'S SURPLUS
Q_0G	SUBSIDIZED PRICE (IF ONE SUCH IS SET)
AREA OQ_0GP_1	ACTUAL PAYMENT IN CASE OF SUBSIDY

FIG. 2.1 – DEMAND FOR DOMESTIC WATER, WILLINGNESS TO PAY, CONSUMER'S SURPLUS, COMPETITIVE MARKET VALUE AND ACTUAL PAYMENT

water, for example, is equal graphically to the area under the aggregate demand curve. The aggregate demand curve is a representation of how much quantum of output the beneficiaries purchase at successive points if the same is sold in free market.* If Qo represents output level for Y_1, $E(Y1)$ is the shaded area under the figure. In other words, if the aggregate demand is a function $y(p)$ of price p, the willingness to pay

$$E(Y) = \int_0^Y D(\eta)\, d\eta \qquad (2\text{-}3)$$

where Y is level of output

and $d\eta$ represents dummy variable of integration.

The overall benefits due to various system outputs m in number then is

$$E(Y) = \sum_{j=1}^{m} \int_0^Y D(\eta_j)\, d\eta_j \qquad (2\text{-}4)$$

The relations among willingness to pay, competitive market value, consumers' surplus and actual payment in case of subsidy can be read from the figure 2.1.

If a system's output is not large relative to the total market, the system will probably not affect the market prices current market prices may then be used to value the output@. This is apparent from Fig.2.2 where (Q2-Q3) represents output.

---

* Thus the assumptions of the competitive model viz constant marginal utilities, profit maximisation of the producer etc. hold good.

@ The following economic assumptions are necessary behind this statement:

1) no rationing or restrictions on the commodity
2) no monopsony exists.

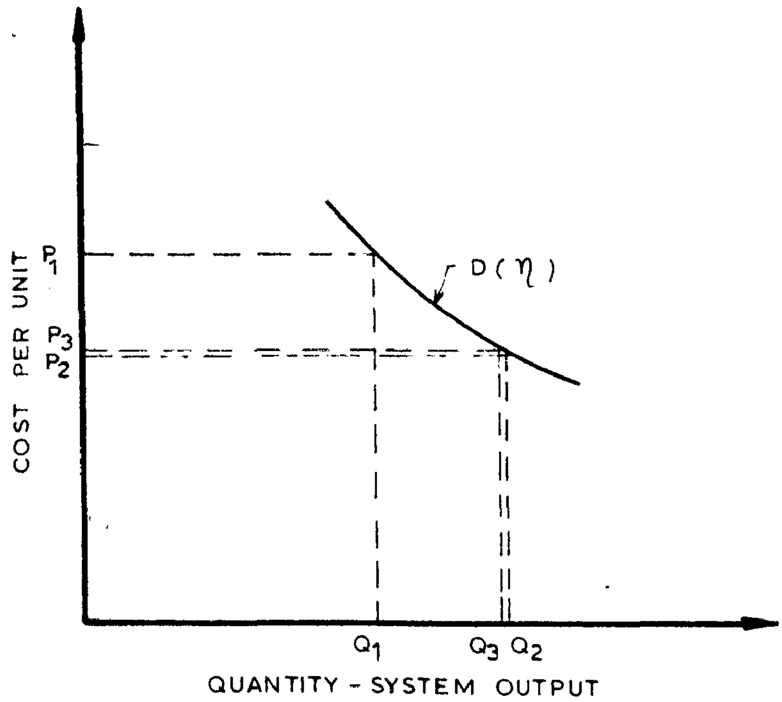


FIG. 2.2 _ IMPACT OF CONSIDERABLE INCREASE IN PRODUCTION

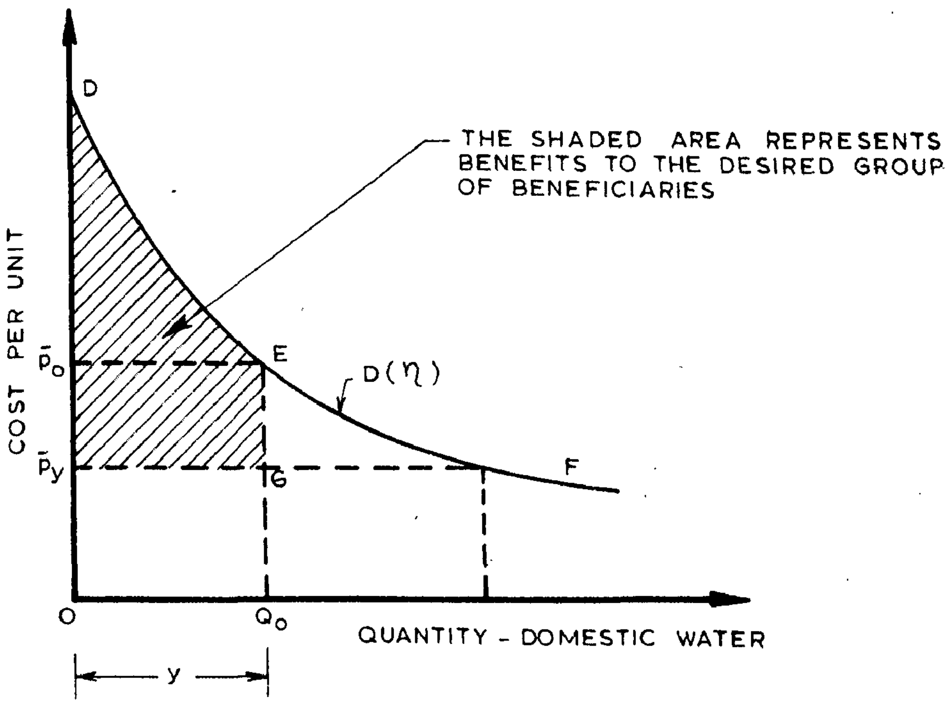


FIG. 2.3 _ REDISTRIBUTED BENEFIT - GRAPHICAL REPRESENTATION

However if a system's output is large relative to the total current production, the appearance of that output on the market will force the price down. The downward trend is apparent from Fig. 2.2. This indicates two possible conditions:[63]

(i) the benefits are less than those that would be calculated if the pre-project price were used to evaluate project output (as in traditional benefit-cost estimations).

& (ii) the producer's gross income may be seriously affected.

2.1.3.1.2.2. Outputs yielding Producer Goods

When the relevant net output of a system is used in the production of other goods and services, such as irrigation water, hydropower etc., the principle of measurement according to consumers' willingness to pay still stands. However, the complexity lies in that the ultimate increase in consumption made possible by the increased availability of the producer goods is also to be considered and this may be many stages of production removed from the system output itself. The full value of system benefits, in other words, mean the immediate purchaser's willingness to pay plus the extra benefits enjoyed further along the line by those people whose willingness to pay for the processed goods exceeds market price.[118]

2.1.3.2. Non-marketable outputs

Markets in the usual sense do not exist for some services resulting from an output of the system. Prime examples are flood protection, recreation on public land and waters, preservation of wilderness or other natural and historic features.

Though the problem is complex, it is necessary and possible to infer, as best as it can be, a reasonable valuation for this kind of outputs also, from observed behaviour of beneficiaries and other reasonable assumptions.

The typical example is the flood control output. To estimate how much annual damage will be averted and how much facilities accrue to new users as a result of project is possible if a reasonable assumption that the present flood plain occupants would be willing to pay any price upto the full amount of the expected damage, is made.[63]

2.1.3.3. Foreign Exchange

When substitution results in savings or earnings of foreign exchange the concept "willingness to pay" for foreign exchange (in terms of local currency) becomes equally valid. This would necessitate adoption of shadow prices if the official rate of exchange is different from the domestic "willingness to pay".[118]

2.1.4. Measurement of Costs

2.1.4.1. A cost is a sacrificed benefit.[90] Thus the distinction between benefit and cost may be considered as simply one of sign. It follows therefore that there need be no analytical distinction between measuring benefits and measuring costs. Hence the philosophy of "willingness to pay" is equally valid for measurement of aggregate consumption cost. This would in fact not only include the immediate "would be purchasers' willingness to pay" but also the excess "willingness to pay" over actual payments for all purchases down the line, as argued in benefit side.

2.1.4.2. Concept of Alternative Cost

The appropriate cost to be considered in analysis is the opportunity cost. This can be defined as the maximum benefits forgone from a feasible, realistic alternative.[116]

2.1.4.3. Cost of Physical Inputs

The net inputs that go into making of the system withdraw from the rest of the economy goods and services, the cost of which shall represent the cost of the project. Two cases are apparent:

(i) The use of various physical inputs for a project results in a decline of the total availability of those inputs to the rest of the economy by exactly equal amount consumed in the project making. The cost shall be computed, then, from demand margin.

(ii) On the other hand, suppose, in response to the demand by the project for these inputs, their supply is correspondingly increased in rest of economy. In this case, there is no change in the total availability of the goods/services used as an input into the system. The net input to the project will then consist of those godds and services whose availability to the rest of the economy is reduced because these are used up in producing inputs for the project. Supply margin shall be the yardstick for costs of inputs that are met by increased supply from other sources.[118]

2.1.4.4. Cost of Labour

The immediate effect of engaging a man's service on a project is to deprive the rest of the economy those services. If the necessary conditions involving competitive model and relatively small change in supply can be assumed to hold, the market price or a wage rate of a particular grade of labour may be taken as an appropriate measure of "willingness to pay". But for countries like India, where labour markets are uncompetitive the use of shadow price is appropriate. The real cost of an unskilled labour is thus,indeed, zero; however, supply of unskilled labour cannot be varied in the short run as it should be considered alongwith the long run demographic trends. The regional dimensions of labour supply and transfer costs, therefore, need to be considered. Regarding skilled labour, increased needs can be taken care of by "human capital formation". The real cost of skilled labour, however, is not zero.[118]

2.1.4.5. Cost of land and Natural Resources used up in the project

For the water resource system design, land as an input is measured in demand margin. When land is used up by the project, that land is denied to the rest of the economy and cannot be substituted from any other source of supply. The appropriate measure of cost of land as an input is the ultimate consumers' "willingness to pay" for the aggregate consumption benefits made possible by use of land.

An example for illustration: Should the land used up in the project has no other potential use, the market clearing price of land is zero, and irrespective of the cost that may be actually paid for it, the land must be measured at zero cost as an input to the project; contrarily, if the land does have an alternative use but the market does not provide an appropriate measure of its value, it is necessary that the cost of land is measured by the net benefits foregone because the land can no longer be devoted to alternative use.

Water and other natural resources need alike treatment. Unlike land, no market exists for water and "willingness to pay" for water uses need to be simulated.

2.1.4.6. Foreign Exchange costs

It could be:

(i) directly imported inputs

(ii) Loss to economy of exportable goods/services by use in project as an input.

(iii) input resources that include foreign exchange.

Fixed quota of imports of a product used up in a project reduces the availability of the product to the rest of the economy. In such cases, the effective net input is not foreign exbhange but the product itself and its cost should therefore be measured in terms of willingness to pay for the product.

As brought in the case of benefit estimation, for the foreign exchange component, appropriate shadow price, should one exist, must be applied as a correction for "willingness to pay".[118]

2.1.5. Social Benefit and Cost Analysis

2.1.5.1. General

One of the objectives in social benefit-cost analysis is to measure as many of the impacts of a project on any economy as completely as possible. It must account for both direct as also indirect benefits and costs.

The indirect benefits and costs is an arbitrary distinction, and refers to project gains/losses that could not wholly be captured and analysed in project. These are due to externalities and quantification and valuation is subject to serious overestimation or under estimation. The limitation, while recognized, could be qualitatively continued to be described as fully as possible so that the decision-maker is aware of the unanalysed factors in the study.

Certain other aspects of importance in the analysis like social rate of discount, uncertainty and risk, saving/investment etc. pertinent very much to analysis are separately taken up in Chapter 3.

2.1.5.2. Analysis

The problem of estimation of social benefit/cost is best solved by mathematical programming. Besides yielding a rapid solution of suffficient accuracy, the programming technique focusses sharp thinking and precise statements. To make this point clear, a model evolved by Haveman & Krutilla is presented in Appendix I. The model, in brief, is solved for a solution in any evaluation of a project as follows:

Step I

For any system that is proposed, the following are assessed:

1) demands that the expenditure imposes on the economy with as fine detailing as possible as waranted by data.
2) examination of prevailing conditions including pattern of unemployment and a comparison.

The former step is solved using empirical analysis based on input-output models that sort out the demands which the public expenditures for water-resources development imposes on the economy1

Step II

In this step, social costs are evaluated after due considerations for unemployed resources and use of pertinent shadow price.

The categories of final contribution of the expenditure are broadly classified under six categories (shown in Appendix I) and thus the contributions of labour, capital land and government are the final product isolated by the occupational or industrial sector of origin within each payment category.

2.1.5.3. Data Requirement

As would be apparent from a perusal of the model (shown in Appendix I), the essential pre-requisite for an application of such procedures is the evolution of an inter-industry relations matrix, after a formal classification of the industries and industry-occupation relations matrix after a similar broad classification of the occupational categories. Inter-alia, other requirements include an assessment of the unemployment and excess capacity so as to adjust nominal or market price to account for the shadow effects.

2.1.5.4. Summary Findings

Thus it is obvious that the social benefit-cost analysis required to evaluate the National Economic Development benefits in the above manner requires an inter-disciplinary effort of the several fields, like engineering, economics, statistics and social sciences.

2.2. INCOME DISTRIBUTION

2.2.1. General

2.2.1.1. Apart from National Economic Development, the concept of relevance of distributional effect in project selection has been the earliest to be identified in the evaluation of water resource projects. Otto Eckstein notes "one of the criteria on which a project must be judged and which benefit-cost analysis disregard altogether is the redistribution of income which a project brings about.[31]"

John Krutilla provides an estimation of distribution both among income classes and geographic regions of a particular river basin projects' costs and benefits; he observes: " although we have concentrated on questions of economic efficiency we cannot ignore the redistributive consequences and the issues which these raise in terms of equity".[70] The objective was examplified lucidly in the masterly premier work of Maass and Marglin (in "Design of Water Resource Systems[5]) and later by Haveman, Krutilla[57], Mckean[93], Weisbord[130] and others. The ways by which distribution effects might be taken into consideration were brought out by Marglin[88] and later by Weisbord[130].

2.2.1.2. "Why must redistribution goals be achieved via individual projects? Why not resort to taxations, transfers and other instruments of national fiscal policy so that the economic efficiency need alone be considered in project evaluation?" Whatever may be said in favour of the above approach of Hick and Kaldor[67], it is apparently unacceptable if pure lumpsum transfers of income are not costless to undertake. If such pure transfer systems do involve costs - administrative or political - any unfavourable income distribution by-product of a government investment project can be costly to undo while any favourable effects will bring savings in the costs of an equivalent pure redistribution effect.[130]

2.2.1.3. For developing countries, the objective of development does stress upon equity in addition to the efficiency. The need for inclusion of this objective for analysis is obvious from various plan objectives and consequent governmental measures taken in India. Prof. V.K.R.V.Rao, proposed a study to National Development Council (1963) and pursued it in the meetings of State Planning Secretaries. The objective was defined as[50]

> "In the context of the need for balanced development of the different parts of the country and extention of the benefits of economic progress to less developed regions, a study on the level of development in different parts of the country and growth thereof becomes important. A knowledge of the interstate and inter-regional differences with reference to various socio-economic indicators is thus necessary to devise appropriate measures for balanced development in successive development periods".

Following this an analysis of differences in improvements made in agriculture and other fields and on levels of consumption and employment among different regions and different sections of population in each state has been made.[50] Standard classifications evolved by National Sample Survey for divisional classification of regions based on appropriate socio-economic footings have been adopted.

2.2.1.4. All these lead to show the importance of inclusion of the distribution objective, in the envisaged multi-objective analysis of projects.

2.2.2. Distributed Benefits and Costs

2.2.2.1. Goals set for redistribution objective are varied. Distribution programme may be aimed to favour relatively ill-placed region, group, sex, income recipients or other considerations. Assuming a broader basis, the regional development aim could be one of the major redistribution programme of a nation.[50] The objective then embraces certain aims and related positive effects like[122]

(i) Increased regional income

(ii) Increased regional employment

(iii) Diversification of the regional economic base

(iv) Enhancement of environmental and social well-being condition of special region concerned, &

(v) Any other specified components of regional development objective.

2.2.2.2. The redistributional benefits to a group are equal to the immediate aggregate consumption benefits it receives minus any offsetting payments made to other groups; the redistributional costs to the group are equal to the immediate aggregate consumption costs it incurs minus any compensating receipts from other groups.

2.2.2.3. Four possible types of benefits in a project with distribution objective arise:[123]

(i) The value of increased outputs of goods and services from the plan to the users residing in the concerned region.

(ii) The value of output to users residing in the region under consideration resulting from external economies

(iii) The value of output resulting from the use of resources in the region under consideration which are otherwise unemployed or under-employed; and

(iv) Additional net income accruing to the region under consideration from the construction or implementation of a plan and from other economic activities induced by operations of the plan.

2.2.2.4. Similarly, due to the adverse effects of a plan upon a particular region, regional development costs arise; they can be broadly[123]

(i) The value of resources contributed from within the region to achieve the output of a plan.

(ii) Payments through taxes, assessments or reimbursement by the region for resources contributed to the plan from outside the region.

(iii) Losses in output resulting from technological external diseconomies to users residing in the region under consideration.

(iv) Loss of assistance payments from sources outside the region to otherwise unemployed or under-employed resources residing in the region under consideration.

and (v) Loss of net income in the region under consideration from other economic by construction or operation of a plan.

2.2.2.5. Whether the net output of the project consists of the particular goods and services it produces, or of goods and services it releases from alternative source of supply, the immediate beneficiaries may be identified as the persons who make use of additional supply. Their "willingness to pay" for it measures corresponding direct aggregate-consumption benefits. To the extent that the immediate beneficiaries must pay for their use of the project net output, their redistribution gains are reduced and those of the groups receiving the payment are increased. Thus depending upon the associated cash transfers, the direct aggregate-consumption benefits of a project may be spread over a number of different groups other than the immediate beneficiary.[118]

The corrollary conditions ascertain redistributed costs.

2.2.2.6. Derivation of Redistributed Benefits

Within the sphere of income redistribution, project that yield maximum redistributed gains ranks superior to the rest of the projects. If annual net income generated from design A is denoted by I(A), gross income by G(A) and system revenues by R(A), the superiority of project A^1 and A^2 for a given year is given by the criterion

$$I(A^1) > I(A^2) \qquad (2.5)$$

or

$$G(A^1) - R(A^1) > G(A^2) - R(A^2) \qquad (2.6)$$

In other words the function

$$I(A) = G(A) - R(A) \qquad (2.7)$$

fulfils requirement for ranking with respect to redistribution of income (for any single year).

The gains due to redistribution objective is apparent from Fig. 2.3. Unlike aggregate consumption objective, the basis to determine net increase in benefits in case of redistribution objective is affected by the pricing policies of Government. As seen from the figure, the shaded area represents the benefit to the desired groups of beneficiaries of y' units of water sold at price $\bar{p}_y$, and this represents the difference between willingness to pay and actual payments ($OQ_oED - OQ_oG\,\bar{P}_Y$) we thus have

$$G(Y) = E(Y) = \int_0^{y} D(\eta)\, d\eta \qquad (2.8)$$

Where $E(y)$ willingness to pay (refer 2-3)

$D(\eta)$ represents the aggregate demand schedule for domestic water in the region to be improvised, denoted by y. *

For the case shown with price set at $\bar{p}_y$ the redistributed benefits = $E(y) - \bar{p}_y\, y'$.

*Basic assumptions of competitive market are implied in the derivation viz; that the marginal utility of income is assumed constant; that there is no external effects and the prices equal marginal costs through out economy and is unaffected by water-resources development.

For a multiple-purpose case, if system outputs are deemed independent and is represented by $\underline{Y} = (y_1 \ldots y_m)$, the annual redistribution benefits are equal to the sum of willingness to pay for all outputs less the actual payments for them.

$$I(\underline{Y}) = \sum_{j=1}^{m} \left[\int_{0}^{y_j} D_j(\eta_j)\, d\eta_j - \bar{p}_{y_j} y_j \right] \qquad (2\text{-}10)$$

The goal of translation of the income distribution objective criterion into design has problems. These will be discussed subsequently.

2.2.3. <u>'Weight' for Redistribution Objective-Identification</u>

The explicit consideration for the redistribution benefit in project evaluation, after arriving at the redistributed benefits/costs, has been first attempted by Marglin.[88] He suggested 3 methods viz

(i) Maximize NED benefits subject to redistribution constraint.

(ii). Maximize an objective function which combines efficiency objective with redistribution objective.

(iii) Maximize redistribution subject to an efficiency constraint.

2.2.3.1. Method (i)

Let $\bar{I}_1 \ldots . \bar{I}_T$ represent the annual income to be achieved for the favoured group in the redistribution objective. For the system design period of T, the design criterion strives to maximize the efficiency objective function

$$\sum_{t=1}^{T} \theta_t \left[E_t(\underline{Y}_t) - M_t(\underline{X}) \right] - K(\underline{X}) \qquad (2\text{-}11)$$

subject to

$$E_1(\underline{Y}_1) - \sum_{j=1}^{m} py_{1j}\, y_{1j} \geqslant \bar{I}_1 \qquad (2\text{-}12)$$

$$\ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots$$

$$E_T(\underline{Y}_T) - \sum_{j=1}^{m} py_{Tj}\, y_{Tj} \geqslant \bar{I}_T \qquad (2\text{-}13)$$

and to the production function

$$f(\underline{X}, \underline{Y}_T) = 0 \qquad (2\text{-}14)$$

where θ_T = discount factor applicable to demand period T

$E_t(\underline{Y}_t)$ = Efficiency benefits in year t represented by expression (2.4) for willingness to pay

$M_t(\underline{X})$ = Operation, maintenance and replacement cost of system, year t

$K(\underline{X})$ = Construction costs.

and $\underline{X}, \underline{Y}$ denote vectors of system inputs and outputs.

By setting the derivatives of the Lagrangian form of this constrained maximisation problem equal to zero and substituing, the following marginal conditions of optimization result:

$$\sum_{t=1}^{T} \theta_t \left[D_{tj}(y_{tj}) \frac{\partial y_{tj}}{\partial x_i} - \frac{\partial M_t}{\partial x_i} \right] - \frac{\partial K}{\partial x_i} = \qquad (2\text{-}15)$$

$$= \sum_{t=1}^{T} \lambda_t \left[D_{tj} (y_{tj}) - \bar{p}_{yt_j} \right] \frac{\partial y_{tj}}{\partial x_i} \qquad (2\text{-}16)$$

$i = 1, \ldots\ldots\ldots n$ (system inputs)

$j = 1, \ldots\ldots\ldots m$ (system outputs)

The left-hand side of the above equation is the marginal net efficiency benefit from an increase of one unit in the size of i_{th} structure:

$$\sum_{t=1}^{T} \theta_t \left[MVP_{t(xi)} - MM_{t(x_i)} \right] - MK_{(xi)} \qquad (2\text{-}17)$$

On the right-hand side we have the multiplier λt multiplying the difference between the gross income derivable by the beneficiaries in each year of period t from a unit increase in the quantity of i_{th} input, $D_{tj}(y_{tj}) \frac{\partial y_{tj}}{\partial x_i}$ and the marginal charge they will actually be called upon to pay in period t. $(\overline{p}_{y_{tj}} \frac{\partial y_{tj}}{\partial xi})$; in simpler notation it is equal to

$$\sum_{t=1}^{T} \lambda_t \; MI_{t(xi)} \qquad (2\text{-}17.a)$$

$i = 1, 2 \ldots\ldots\ldots n.$

The equation thereby yields that for the most efficient design fulfilling the redistribution constraints, the present value of marginal net efficiency benefit of each structure must equal a weighted sum of the marginal net increase in the income of the redistribution beneficiaries over the life of the system.

The notation λ_t is the weight to be identified; it is indicative of the shadow price of redistribution in terms of efficiency. In other words it denotes the efficiency loss per rupee of net income provided for benefit of region. The λ_t plays a significant role in indicating the opportunity cost.

Constraints as indicated in the method cannot, however, be confidently set in the beginning as the sensitivity of λ at the margin may not be initially apparent; for example, "incomes of Nagaland region are to be increased by Rs. one crore" means that this must be accomplished no matter what the costs in efficiency are. Policy makers may like to relax such a decision should the analysis subsequently indicate that the last Rs. 25,00,000 entails an inordinate sacrifice of efficiency benefits to the nation. The converse situation is also valid; should it be possible to have a higher distributional effect at least cost by the proposed water resource system, planning would dictate fuller exploitation of the possibility, at the expense of alternative necessary requisite measures.

2.2.3.2. Method (ii)

This procedure aims at a Grand Efficiency function, which integrates the efficiency objective and redistribution objective in a single objective function. The pre-requisite for this procedure, however, is the advance information of the "weight" for the redistributional objective in relation to the economic efficiency i.e. NED objective. If μ_t^o (the negative of the marginal opportunity cost λt) represents the redistributed rupees worth, a composite objective function in the following form can be maximised for an optimal plan subject to constraints imposed by production function

$$f\ (\underline{X},\underline{Y}) = 0 \qquad (2\text{-}18)$$

$$\sum_{t=1}^{T} \mu_t^o \left[E_t\ (\underline{Y}_t) - \sum_{j=1}^{m} \bar{p}_{y_{tj}}\ y_{tj} \right] \qquad (2\text{-}19)$$

$$+ \sum_{t=1}^{T} \theta_t \left[E_t\ (\underline{Y}_t) - M_t\ (\underline{X}) \right] - K\ (\underline{X})$$

where t = 1,2,......... T particular demand period

T = the number of demand periods in the economic life of the system

$E_t(\underline{Y}_t)$ = Willingness to pay for the output by beneficiary at t_{th} period.

$\underline{Y} = y_1, y_2 \ldots\ldots\ldots y_m$ - output of the system.

$p_{y_{tj}}$ = pre-assigned price for the output y_j in t_{th} period for the level of output corresponding to y_j ; (y = 1,2...j...m)

θ_t = Discount factor applicable to demand period t.

$M_t(\underline{X})$ = Operation, maintenance and replacement costs in tth period with input as per vector $(\underline{X})$

$K(\underline{X})$ = Construction cost for inputs as per vector$(\underline{X})$

where $\underline{X} = 1, \ldots\ldots 2 \ldots\ldots i \ldots\ldots\ldots n$

For an intertemporal evaluation, the expression can be:

$$\mu_1^o \sum_{t=1}^{T} \left(\frac{E_t\,(\underline{Y}_t) - \sum_{j=1}^{m} \bar{p}_{y_{tj}}\, y_{tj}}{(1 + r_I)^t} \right) + \sum_{t=1}^{T} \left(\frac{E_t(\underline{Y}_t) - M_t(\underline{X})}{(1 + r)^t} \right) - K(\underline{X}) \qquad (2\text{-}21)$$

where demand period t represent one year length and r_I = redistribution interest rate, and μ_1^o = efficiency value of redistribution benefits in year one.

2.2.3.3. Method (iii)

This method maximizes redistribution subject to no reduction in the aggregate consumption benefits from a set level.

The design criterion implementing the present objective is maximization of the redistributive objective function:

$$\sum_{t=1}^{T} \theta_t^I \left[E_t\,(\underline{Y}_t) - \sum_{j=1}^{m} \bar{p}_{y_j}\; y_{tj} \right] \qquad (2\text{-}22)$$

subject to the constraint on net efficiency benefit

$$\sum_{t=1}^{T} \theta_t \left[E_t\,(\underline{Y}_t) - M_t(\underline{X}) \right] - K(\underline{X}) \geqslant \begin{matrix}\text{NED}\\ \text{sum benefit}\end{matrix} \qquad (2\text{-}23)$$

Sl. No.	Description of Methodology.	Method I	Method II	Method III
1.	Dimension parameter (set by policy-makers)	Level of distribution to be attained.	Efficiency values of redistribution.	Level of efficiency to be attained and relative redistribution values
2.	Objective function: maximize-	Efficiency.	Weighted sum of redistribution and efficiency.	Redistribution.
3.	Constraints.	Redistribution	None	Efficiency
4.	Results of design revealed in terms of consequence parameters.	Opportunity cost.	Level of redistribution attained.	Opportunity costs and levels of redistribution attained.

Table. 2.1. Statement of comparison of Methodologies for consideration of redistribution objective.[5]

and subject to $f(\underline{X}, \underline{Y}_t) = 0$ (2-24)

The assigned benefits to the nation should be zero, positive or negative as decided by policy makers.

The schema has been summarised by Marglin[88] as shown in Table 2.1. In real world problems, probably almost a mix of 3 methods might be needed, in view of the unknowns in the problem.

2.2.4. Weisbord's Model for Identification of Weights

The real problem of identification of the weight that planners attach for redistribution objective is attempted by Weisbord, from past government decisions. He observes[130]

> "Suppose there is a project 2, which receives priorities over project 1 of equal cost, even though the real benefits from project 1 are greater when a dollar's worth of additional income receives a weight equal to unity regardless of who receives it. If project 2 is preferred nonetheless, then it must produce benefits that would be found to be at least as large as those of project 1 if appropriate differential weights were attached to the income received by each beneficiary."

Based on this for the NED and income redistribution objectives, the following equations for the model are proposed:[1]

---

1 The model in Weisbord's paper has assumed redistribution to several groups; the model presented herein, however, has been simplified so as to consider income redistribution as an objective, only for one group.

$$\text{Project 1}: \quad B_{NED_1} + \lambda_1 \, B_{RI_1} = B_1^{*} \qquad (2\text{-}25)$$

$$\text{Project 2}: \quad B_{NED_2} + \lambda_2 \, B_{RI_2} = B_2^{*} \geq B_1 \qquad (2\text{-}26)$$

if project 2 is chosen instead of project 1.

B_{NED} represents net benefits to economic efficiency objective,

B_{RI} net redistributed benefits and

$\lambda_1 = \lambda_2 = \lambda$ is the weight associated with the objective for income redistribution. B_1 B_2 etc. represent the equivalent net benefits for both the objectives.

The solution for λ is obvious. Where there m groups to be compensated by the redistribution objective and n available project decisions based on choice from valid alternatives, We have 3 cases:

(i) $m > n$ - Derivation of weights are difficult

(ii) $m = n$ - The derivations are just identifiable

(iii) $m < n$ - Thenumber of unknowns, n, if excessive overidentification results. The weights that approximately solve all the equation can be found by, the theory of least squares.

A case illustration and identification of weight is also presented by Weisbord.[130] However, Haveman contends[130] the validity of Weisbord's approach on 2 major grounds:

(i) Past decision of political system may not yield a correct inference of weights since decisions on these are based on a multitude of considerations and not necessarily income distribution, also,

(ii) Reliable derivation is not possible by examining just a few programme decisions.

2.2.4.1. UNIDO Approach

However a similar approach as that of Weisbord's has been proposed by UNIDO in their guidelines for project evaluation. A graphical solution accompanied by lucid explanations for calculating the weight, designated as "switching value for policy-makers", has been presented.[118]

2.2.4.2. The problem may be simpler, if Indian Planning Commission (or CPO as used in UNIDO text) could articulate such weights for an easy evaluation of project by formulators.

2.2.5. <u>Regional Income Multiplier</u>

The ultimate benefits that accrue to the beneficiaries due to redistribution objective requires further adjustment for the effects due to this phenomenon.

The propensity of direct beneficiaries to spend a portion of their earnings within the region creates secondary, tertiary and later rounds of activities. However, the chain of indirect benefits, though in principle, can continue indefinitely diminishes in magnitudes progressively.

If γ represents the marginal proportion of the 'direct' net redistributional benefits R^D, which, when respent, results in additional net benefits to the region, then the value of 'indirect' net redistributional benefits R^I can be expressed as:

$$R^I = \gamma R^D + \gamma (\gamma R^D) + \gamma (\gamma^2 R^D) + \gamma(\gamma^3 R^D) + \ldots\ldots$$

$$= R^D (\gamma + \gamma^2 + \gamma^3 + \ldots\ldots\ldots\ldots) \qquad (2\text{-}28)$$

and the total net redistributional benefits to the region, R^T is given by

$$R^T = R^D + R^I = R^D (1 + \gamma + \gamma^2 + \gamma^3 + \ldots.) =$$

$$= R^D \left(\frac{1}{1-\gamma} \right) \qquad (2\text{-}29)$$

The expression $\left(\frac{1}{1-\gamma}\right)$ is called the "regional income multiplier".

Thus the net benefits to a region, applying "willingness to pay" concept is

$$\left(\frac{1}{1-\gamma} \right) \left[E_t (\underline{y}_t) - \sum_j \bar{p}_{y_{tj}} \; y_{tj} \right] \qquad (2\text{-}30)$$

Irrespective of methodology followed in case of redistribution objective for solution, it has been observed that the multiplier effect improves the system performance in terms of efficiency.

2.2.6. <u>Regional Sectoral Model</u>

The problem of regional analysis accounting for the sectoral impact of public investment is more complex.

A multi-regional model, like the national model (Appendix I) is of great assistance to estimate the regional demands imposed by a final expenditure. Haveman and Krutilla have evolved a regional sectoral model as an extension to national sectoral model, based on a number of assumptions that are necessary to solve the intra-regional impacts due to system input.[57] The type of assumptions, intealia, relate to

(i) Inter-regional distribution of the demands not met by the region in which the project is built.

(ii) The extent of intra-regional preference given to industries within the project region.

(iii) Stability of these patterns over time across regions and across project type.

Basic to arriving at the above factors is the knowledge pertaining to regional trading patterns and inter-regional differences in industry production functions. Data on the above for Indian conditions need to be generated and efforts in this direction will be the premier step leading to adoption of such models in analysis of water resource projects.

2.3. ENVIRONMENTAL QUALITY

2.3.1. Introduction

Environmental quality, as an objective, contemplates the preservation and enhancement of natural values and, in addition, the correction of "misfits".[123] Water resources development plans can have both beneficial and adverse effects on environmental quality. Since one has to deal with "a hard-to-define" loss in quality of environment, the consideration of environmental quality, even limited to water resource projects, is complex.[63]

2.3.2. Indian Context

The relevance of the objective to Indian conditions is open to argument. Costly measures to serve the objective, may be thought of as the luxuries of the affluent and irrelevant to a developing nation on the march of growth.[101] However, there need be no such misgivings. The concern of the government in the preservation and enhancement of environment is more and more apparent from the various measures and schemes under hand.[51] The organisation in charge of Environmental Planning and Coordination in the Department of Science and Technology, Government of India, has, in fact, launched upon several eco-development projects in various parts of the country.[110] The OEPC has also undertaken, in cooperation with UNESCO to coordinate the national "man and Biosphere" programme of ecological research. The programme will study the effects of man's activities on the natural environment.

2.3.4. Problem

Given its relevance, then the problem centres around the measurement of objective as the environmental quality is characterized by its non-market, non-monetary nature.[63] It is not possible to identify, ~~under~~ much less measure, all effects or changes, at present. Only reasoned judgements by an inter-disciplinary team that can express the impacts in some form, indicating inherent limitations, can be of use for decision-maker.[123]

2.3.5. Classification of Environmental Quality Objective

For effective presentation of the effects of a development plan on environment, the following classifications have been adopted by the U.S.Federal Government based on recommendations of Water Resources Council:[122]

2.3.5.1. Environmental Quality Benefits

1. Beneficial effects resulting from the protection, enhancement or creation of open and green space, wild and scenic rivers, lakes, beaches, shores, mountains and wilderness areas, estuaries or other areas of natural beauty.
2. Beneficial results from the preservation or enhancement of especially valuable archeological, historical, biological and geological resources and selected ecological systems.
3. Beneficial effects resulting from the enhancement of selected quality aspects of water, land and air by control of pollution.
4. Benefits resulting from the preservation of freedom of choice to future resource users by actions that minimize or avoid irreversible or inter-reversible effects or conversely the adverse effects resulting from failure to take such actions.

2.3.5.2. Environmental Quality Costs

The adverse environment effects, as a result of a plan, are considered to be the obverse of the beneficial environmental effects described above. The U.S.Government proposes that plan effects of environmental quality objective should be suitably displayed showing separately the beneficial and adverse effects.[123]

2.3.6. Measurement of Environmental Quality Objective

With the recognition of environmental quality as an objective, placed at par with national economic development objective in United States, considerable impetus has come in the evolution of measuring scales for human and ecosystem values; techniques for measurement are growing. Impact analysis, map-overlay planning and resource inventory, land-use zoning methods etc. are some of these techniques under evolution.[107]

As an example as to how such information can be generated, the "quantitative comparison of some aesthetic factors among rivers" studied by Leopold[72], can be cited. Leopold develops what he calls a 'uniqueness index' that could be of utility to planner to compare alternative potential sites. Leopold surveyed a number of streams and recorded 46 characteristics or factors, each assigned with a valuation number of 1 to 5. (Shown in Table 2.2). For the surveyed streams, Leopold assigns values based on the above procedures, and considers that stream 'more unique' with respect to a particular factor, the fewer the number of streams sharing the same value assigned to that factor. For example, with respect to depth at low (table 2.3) stream 2 was the only one assigned value of 3, whereas six out of the 12 streams had values of 2, two had values of 4 and three had values of 5. Stream 2 would thus be assigned a 'uniqueness ratio' with respectto depth at low flow of 1/1 = 1.0, whereas six of the streams would have a uniqueness ratio with respect to ;

Table 2.2

Typical 'Extracts' on Field Evaluation of Aesthetic Factors: Leopold's Evaluation Numbers for Descriptive categories.[72]

Factor No.	Descriptive categories.	Evaluation number of descriptive categories				
		1	2	3	4	5
	Physical Factors					
1	River width at low flow ft.	<3	3 to 10	10 to 30	30 to 100	>100
2	Depth at low flow ft	<.5	.5 to 1	1 to 2	2 to 5	> 5
3	Velocity at low flow ft	<.5	.5 to 1	1 to 2	3 to 5	> 5
	... ...					
	... ...					
6	River pattern	Torrent	Pool & Riffle	Without riffle	Meander	Braided
	... ...					
	Biologic & Water Quality					
15	Water colour	Clear & colourless	...	Green tints	....	Brown
16	Turbidity mg/l	<25	25 to 100	150 to 1000	1000 to 5000	>5000
	... ...					
	Human use and interest					
	....					
39	Land use	Wilderness	Grazed	Lumbering	Forest & mixed recreation.	Urbanized
	... ...					
	... ...					
46	Misfits	None	-...	-...	-...	Many

Table 2.3

Category Assignment for Aesthetic Factors (Extracts from Leopold)[72]

Descriptive Categories.	Category Assigned to Factors at different sites											
	1	2	3	4	5	6	7	8	9	10	11	12
<u>Physical Factors</u>												
1. River width at low flow.	4	4	4	4	5	3	3	4	5	5	4	4
2. Depth at low flow.	2	3	4	2	5	2	2	2	5	5	2	4
3. Velocity at low flow.	5	4	4	3	4	3	1	4	4	4	4	4
....												
....												
6. River Pattern.	2	2	1	2	2	2	4	2	2	2	1	2
....												
....												
<u>Biological & Water Quality</u>												
15. Water colour.	1	3	2	4	3	5	5	3	3	2	1	3
16. Turbidity	1	1	1	1	2	1	3	2	2	2	1	3
....												
....												
<u>Human use and interest</u>												
39. Land use.	4	2	4	4	2	4	2	4	2	1	1	4
....												
....												
46. Misfits	1	2	2	1	1	1	1	2	2	1	1	1

Table 2.4

Extract from Leopold's Uniqueness Ratios for aesthetic factors
(at Hells Canyon Site)[72]

Categories and factor No.	Site No. 1	2	3	4	5	6	7	8	9	10	11	12
<u>Physical</u>												
1	.14	.14	.14	.14	.33	.50	.50	.14	.33	.33	.14	.14
2	.17	1.00	.50	.17	.33	.17	.17	.17	.33	.33	.17	.50
3	1.00	.12	.12	.50	.12	.50	1.00	.12	.12	.12	.12	.12
..	..	..	..	..	..	..	..	..	..	..	..	..
6	.17	.17	.50	.17	.33	.17	.17	.17	.33	.33	1.0	.50
..	..	..	..	..	..	..	..	..	..	..	..	..
..	..	..	..	..	..	..	..	..	..	..	..	..
Sub Total	3.06	3.73	3.53	4.61	3.20	3.75	8.88	3.26	328	3.3	3.43	2.34
<u>Biologic</u>												
15	.50	.20	.50	1.0	.20	.50	.50	.20	.20	.50	.50	.20
16	.17	.17	.17	.17	.25	.17	.50	.25	.25	.25	.17	.50
..	..	..	..	..	..	..	..	..	..	..	..	..
..	..	..	..	..	..	..	..	..	..	..	..	..
Sub Total	2.92	2.66	2.81	5.15	4.41	3.67	9.74	4.24	2.65	3.96	3.06	3.70
<u>Human Interest</u>												
..	..	..	..	..	..	..	..	..	..	..	..	..
39	.17	.25	.17	.17	.25	.17	.25	.17	.25	.50	.50	.17
..	..	..	..	..	..	..	..	..	..	..	..	..
46	.12	.25	.25	.12	.12	.12	.12	.25	.25	.12	.12	.12
Sub Total	5.09	4.61	5.53	4.09	8.48	3.75	4.48	6.28	4.32	7.05	5.46	3.67
Total	11.07	11.00	11.87	13.93	16.09	11.17	28.1	13.78	10.25	14.31	11.95	10.21

Table 2.5

Sites in order of Uniqueness Ratio
(After Leopold)[72]

Rank / Site No	Aesthetic Factors			Total
	Physical	Biologic	Human interest	
1	7	7	5	7
2	4	4	10	5
3	6	5	8	10
4	2	8	3	4
5	3	10	11	8
6	11	12	1	11
7	10	6	2	6
8	9	11	7	3
9	8	1	9	12
10	5	3	4	1
11	1	2	6	2
12	12	9	12	9

that factor of 1/6 = 0.17. All the uniqueness ratio are then summarized as in Table 2.4. Though there can exist different methods to combine the 46 uniqueness ratios of each stream into one index, Leopold chose to add the ratios over subsets of the factors and overall factors (Tables 2.5). The total index provides a ranking of stream in uniqueness. Though there are such trickish questions needing attention like the universe that should considered, method of arrival of uniqueness, overall or subset, and other similar problems in such an analysis, the indices as above are extremely valuable for a solution through systems approach.

2.3.7. <u>Scientific Tools to Study the Problem</u>

The effects of the expected developmental plan on the environmental quality is more precisely studied by impact studies. The categories for impact studies could be in the broad fields like:[60]

Physical

Biological

Economic

Aesthetic

& Social.

The physical portion of the analysis is to measure characteristics of site such as geology, topography, soils, climate and hydrology. The biological impacts can be assessed by classifying the members, locations and diversity of the plant and animal communities established in the area of, or otherwise affected by the development. The economic portion of the study is of relevance for identification of losers and gainers due to project implementation and is of special use where regional income redistribution is not an objective by itself. The aesthetic impacts include the visual and other discernible changes. Social impacts encompass projects influences on urbanisation, flood plain utilisation etc.

Natural resources inventories for a nation is a basic requisite for a proper environmental impact analysis. Geologic biologic, climatologic, cultural and demographic data provide the necessary base for any assessment for studies aimed at balancing water development and environmental quality.[42]

2.3.8. Conclusion

The cost of a clean environment is impossible to establish.[71] But attention in the prevention of deterioration of environmental qualities is worthy as the environmental degradation in the past has been alarming due to development activities. As John Platt [66] puts it

> "In the past we have had science for intellectual pleasure, and science for the control of nature. We have had science faraway. But today the whole human experience may hang on the question of how fast we now press the development of science for survival."

2.4. ON OTHER OBJECTIVES

2.4.1. General

India has chosen planned economic development to promote the welfare of the people. The objective has been declared as "raising living standards and opening up to the people new opportunities for a richer and more varied life." [43]

The Third Five Year Plan embodied "a socialist pattern of society .... as the goal of country's programme for social and economic development," and "the pattern of development and the structure of socio-economic relations should be so planned that they result not only in an appreciable increase in national income and employment but also in greater equality in income and wealth".[41]

The Fourth Plan while endorsing the above aspects of the earlier plans names achievement of self-reliance a major objective, and that... "the country's requirements will be met from within to the maximum possible extent." With this objective stress has been laid on export promotion and import-substitution.[42]

The plan also deals with income and provides that ".... as regards distribution, the perspective (plan) has to provide for a reduction of inequality of income properly not only by income groups but also by urban and rural areas, developed and backward regions of the country and sections within the agricultural communities like the large holders, small holders and agricultural labourers."[19]

Social well-being has been explored in a separate section in the plan documents and aspects like education, health, welfare programmes have been stressed.

2.4.2. Types of Miscellaneous Objectives

The foregoing is an indicator that the objectives deployed in any planning process, given the political system, can vary. The plan documents for our country thus stipulates, as could be screened from the foregoing paras; the following objectives:

(i) Aggregate consumption

(ii) Income-redistribution.

(iii) Employment

(iv) Self-reliance

& (v) Merit wants

Items (i) and (ii) have been covered already in earlier section. A brief coverage of the rest is attempted now.

2.4.2.1. Employment Level

Unemployment can be considered as an evil by the society, as it denies human dignity. Thus the "employment" as an objective is very relevant. Also unemployment and underemployment results in loss of skill and expertise through lack of practice.[118] The impact of this process of "unlearning" is difficult to be quantified; and its impact on future consumption/NED benefits is more complicated.

2.4.2.2. Self Reliance

This avowed objective has been considered in the planning proposals for the nation to reduce dependance on foreign countries.

A particular project may help to achieve the self-reliance while the other increase country's dependance on foreign assistance. Thus the inclusion of such an objective has its validity.

2.4.2.3. Merit Wants

In a backward rural society, people may be reluctant to spend money for education etc. but public policy may be to provide this service despite the expenditure on exchequer. A similar case exists for upliftment through special measures or assistance to certain groups considered lagging behind the rest of society.

2.4.3. Measurement of Objectives

Having recognised such other types of objectives, the problem for the system analyst is one of measurement.[79] This is not insurmountable as one can measure some of the above items as provision of employment, self-reliance or merit wants by suitable scales. The evaluation of scales of performance for systems' attributes are basic in systems methodologies.[79] Thus employment objective can be taken care of by the actual number of employment opportunities provided for by the project; a suitable trade-off can be established. The scale for self-reliance can be the exports earnings and impert saved. With suitable trade-off between NED and these objectives a "best compromise solution" is feasible.

CHAPTER 3

PROBLEMS IN EVALUATION

3.1. SOCIAL RATE OF DISCOUNT

3.1.1. General

3.1.1.1. The social rate of discount is an important design parameter that is required to have a relative valuation of benefits available at different points in time. This may be different for each objective.

3.1.1.2. So as to achieve the purpose of reducing the benefits available over a time stream to comparable value (at present, for example), we resort to

1) Relative weights placed on benefits at different times
2) Introduce constraints on programme performance for the different periods.

In case (2), the weights become implicit in the level of constraints.

3.1.1.3. If w_t represents such a weight conceived in case (1) above, for the t_{th} year hence, then the present value of the benefits accruing in that year would equal

$$w_t B_t \qquad (3.1)$$

and we can evaluate the benefits accruing from a time stream of T years as

$$w_o B_o + w_1 B_1 + \cdots\cdots + w_t B_t \cdots\cdots + w_T B_T \qquad (3.2)$$

For lack of other alternative manner, it may be better to presume the weights decrease in a geometric fashion akin to compound interest. This then yields

$$w_t = \frac{1}{(1 + r)^t} \qquad (3.3)$$

where r denotes the social rate of discount with which we are interested. Let us concern about the NED objective for further analysis on social rate of discount.

3.1.1.4. With an assumed r as above that is constant over time, the contribution due to a project to aggregate consumption (NED benefit, in other words) is

$$\sum_{i=1}^{\infty} \frac{B_i(\underline{X})}{(1+r)^i} - K(\underline{X}) \qquad (3.4)$$

where

$B_i(\underline{x})$= net addition in i_{th} year from project output with input level ($\underline{X}$)

K(X) = cost incurred at zero time for the project inputs (X)

r = the social rate of discount

3.1.2. Social Rate of Discount - Choice

The crux of the problem lies in the choice of r in the above formula.

3.1.2.1. One possible solution for the above is the adoption of "opportunity cost". According to Baumol[61], the capital required for government investments is drawn from the private sector, where it would otherwise be spent on consumers goods or reinvestment. To determine the opportunity cost of this capital, one needs first to examine the rates of return that are appropriate for these alternative uses. Second, it is necessary to weight the proportion of the capital obtained from both sources to obtain a composite opportunity cost.

3.1.2.2. Howe takes a position[63] that no public project should be undertaken that would generate a rate of return less than the rate of return that would have been experienced on private uses of funds, that would be precluded by the financing of the public project.

3.1.2.3. Arrow[4] while dealing at length the social rate of discount observes that selection of a suitable rate involves value judgements.

3.1.2.4. Marglin[30] analyses

two logical rates, as described below

(i) Marginal rate of return in private sector for profits (r^*), and

(ii) Marginal rate of return of consumption provided by private investment.

He has seen that they are not suited for adoption for various reasons. Marglin's observations are that[90]

> "The government cannot divorce investment decisions from decisions about the time pattern of ~~at~~ consumption and therefore cannot escape making a judgement as to the relative value of benefits at different times in the formulation of investment criteria. Nor can the government infer an appropriate role of discount for comparing the present value of the benefits of public investment with the present value of the alternative use of resources from rates of return of either revenues or consumption unless the government is prepared to judge the overall rate of investment in the economy optimal. Otherwise the appropriate rate of discount,... can be inferred only from the inter-temporal consumption preferences that the government holds as proxy for the people."

3.1.2.5. UNIDO Guidelines[118] advocates two approaches viz:

(i) The social rate of discount, as a value judgement to be prescribed by government during different periods for different objectives;

(ii) The social rate of discount as an unknown of project evaluation.

(i) In the first method the social rate of discount is expressed as

$$r = -EG \qquad (3.5)$$

where E = Elasticity of marginal utility with respect to per capita consumption

G = Rate of growth of per capita consumption in percentage terms.

However, the inference of elasticity from a proposed plan for the nation necessitates verification of the realism for projection of G in it and also the optimality. Moreover, it would be necessary to know in some detail the inter-temporal consumption patterns of alternative plans that have been rejected during the planning process. With such information it would be possible to estimate the social marginal productivity of capital and by virtue of optimality of the plan, to use this marginal productivity as a measure of social rate of discount.

(ii) In the second method, the estimation is recommended through a sensitivity analysis of the past projects that are chosen. The "switching value" that can be imputed denotes the social rate of discount; this is the numerical value of the discount rate for which the projects present net value is zero. Algebraically we have, thus

$$\sum_{t=0}^{T} \frac{B_t(\underline{X})}{(1+r)^t} = 0 \qquad (3.6)$$

where B_t denotes the net discounted benefit for t^{th} year and T is the economic life of the project considered.

3.1.2.6. The problem of inter-temporal choice becomes more complicated where objectives are many; the unknown parameters then increase and the "switching over" values derived as in para above could only be related to that particular set of given parameters.

3.1.3. It is interesting to conclude this chapter on social rate of discount with the extracted quotations from de Neufville[27]:

> "The determination of the appropriate discount rate is a technical question. It is more complex to estimate than the strength of steel or the stability of an earth embankment. But it is not therefore more vague........ since the discount is a technical matter, it should be treated with respect."

3.2. SOCIAL VALUE OF INVESTMENT

3.2.1. General

The social value of investment is the net present value of benefit stream that accrues due to a unit of marginal investment.[118] In a simple model with no reinvestment of benefits, we consider only the direct benefits; by applying the appropriate social rate of discount, we convert them to present equivalents. The shadow price on investment is restricted to capital productivity. However, in any realistic analysis, the shadow price of investment must also reflect the consumption produced indirectly by reinvestments.

3.2.2. Shadow Price on Investment

It is seen that the investment outlays in private and public sectors are interlinked in a developing economy as in our country due to limited availability of capital goods; this implies that more the public investment, less the private investment and vice versa.[90]

3.2.2.1. To consider the impact of the above aspect as well as reinvestment, let us consider a simple example:[85]

Let θ represent the component of rupee that has been drawn for the public project (through taxes etc.) from reinvestmemt in private sector; Let $(1 - \theta)$ represent, the portion of a rupee of public investment that displaces private consumption.

The portion that is drawn from private investment i.e. θ, would have generated, applying a marginal internal rate of return in private investment ϱ, an amount equal to $\theta\varrho$, next year. If r is social rate of discount, the present value of $\theta\varrho$ is then equal to $\frac{\theta\varrho}{r}$ (3.7)

The present value of the rupee investment in public project by taxing a rupee in private sector is therefore

$$a = \frac{\theta\varrho}{r} + (1 - \theta) \qquad (3.8)$$

Thus the shadow price 'a'[90] of public investment drawn from both the consumption and investment of private sector is represented by the above equation.

3.2.2.2. Effect on Project Selection

The maximisation of the objective function for aggregate consumption under NED objective (Equation 3.4) can then be modified as

$$\sum_{i=1}^{\infty} \frac{B_i\ (\underline{X})}{(1+r)^i} - a\,K\ (\underline{X}) \qquad (3.4(a))$$

(Notations as in Equation 3.4)

The effect of the shadow price is an increase in the capital costs.

As an example, let us consider a social rate of discount of 5% for a water resources project; let the marginal rate of return on investment in private sector be 15%. If $\theta = 0.4$, then we have

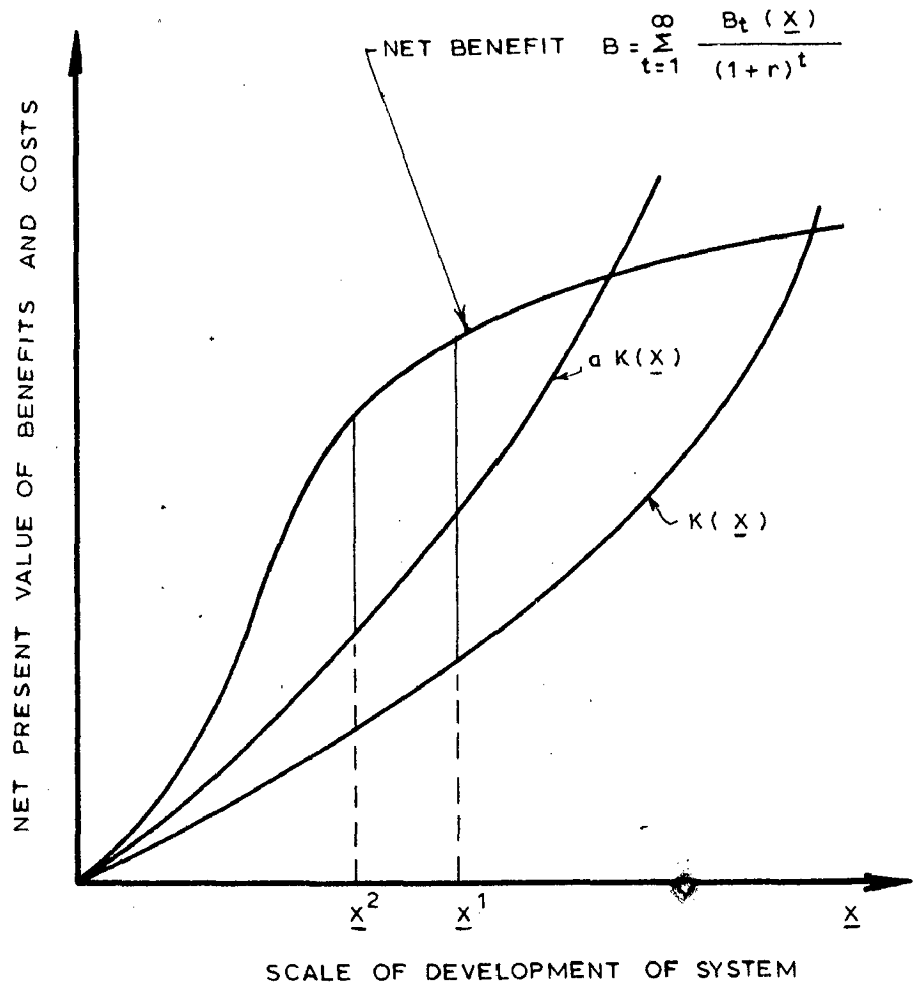
$$\frac{\theta\rho}{r} + (1 - \theta) = \frac{0.4 \times 0.15}{0.05} + 0.6$$

$$= 1.2 + 0.6 = 1.8$$

The shadow price on investment thus becomes 1.8. The effect of such a change on the choice of ($\underline{X}$) (the scale of project) is at once apparent from Fig. 3.1.[85]

3.2.3. Justification for Adoption of Shadow Price on Investment

There could be no doubt that the economic merit of a project is judged in terms of social rate of discount. However, for major, capital intensive durable schemes like water resources projects, direct utilization of ρ , the



$\underline{X}^1$ SCALE OF DEVELOPMENT WITHOUT ADJUSTMENT FOR SOCIAL VALUE OF INVESTMENT

$\underline{X}^2$ SCALE OF DEVELOPMENT DUE TO ACCOUNTING SHADOW PRICE

MAX.(B – C)

FIG. 3.1 _ EFFECT OF APPLICATION OF SHADOW PRICE ON INVESTMENT COSTS marginal rate of return of private investment, as a discount rate is an undue discriminating test. The social rate of discount r could alone lift them as meritorious. However, at the same time, due attention should be paid to consider the shadow price for investment by utilizing factor 'a' to adjust capital investment figures. Marglin finds that any bias due to introducing r for social rate of discounting is thus equalised by such a measure of accounting shadow price for investment.[90]

3.2.4. Synthetic Discount Rate

The social value of investment requires further review and adjustments for:

(i) Utilization of unemployed labour

(ii) Consumption of a portion of output by private sector for capital formation etc.

Marglin's detailed analysis, evolves a synthetic discount rate as a function of social rate of discount, shadow price and economic life.

3.2.5. The UNIDO guidelines adopts rather a similar approach to that of Marglin, as indicated above. It observes that though such a criterion is somewhat more difficult to apply, the difficulty is inescapable, if value judgement and opportunity costs are to play their designed roles.

3.3. RISK AND UNCERTAINTY

3.3.1. General

It is well-known that the planning, design and construction of water-resources systems involve many risks and uncertainties, the importance of which cannot, however, be minimized because of the durable nature of water resource projects.

A risky situation is one in which the probability of outcome is known. An uncertain situation is one in which even this information is totally unavailable. [90]

3.3.2. Types of Uncertainties

The commonly recognized risks/uncertainties can be broadly categorised, as it applies to water resources projects[54] as

(i) the growth of population, agriculture and industries

(ii) the projected cost of labour, material and inflation

(iii) the project benefits/costs

(iv) changes in engineering, science and technology

& (v) assumption to model and inherent uncertainties in modelling with particular reference to hydrology.

3.3.3. <u>Methods for Solving the Problem</u>

The solution strategies, commonly adopted, to deal with uncertainties are:

(i) conventional solutions

(ii) statistical methods

(iii) solution techniques under decision theory.

3.3.3.1. Conventional Solutions

The procedural guidelines for economic analysis such as "Green Book" in U.S.proposed three types of solution categories:[119]

(i) conservatism in estimating benefits and costs

(ii) addition of a premium to discount rate, varying in proportion to lack of confidence in benefit and costs estimation

& (iii) conservatism in estimating economic life of projects.

The criterion indicates in general an aversion to risk and as Marglin rightly points out, reflects pessimism in the approach of government.

Marglin [90] therefore suggests that

(i) Policy-makers must specify their attitude towards fluctuations in benefits and costs.

(ii) Risks and uncertainty should be pooled since this facilitates a greater tolerance of variance and skewness in the programme of individual projects,

3.3.3.2. Statistical Techniques

As Dorfman[29] points out, there are three "time honoured" approaches to the problem of uncertainty:

(i) Certainty equivalents

(ii) Gambler's Indifference Map

& (iii) Risk discounting

(i) Certainty Equivalents

In this concept, it is implied that to every uncertain situation, there corresponds some riskless one that is indifferent to it. Where a choice is to be made amongst a number of uncertain situations, the one with highest certainty equivalent should be selected. This method however, does not provide any rational procedure to evaluate the certainty equivalent.

(ii) Gambler's Indifferent Map

This is a diagram shown in Fig. 3.2, in which a family of expected value-standard deviation that are indifferent to one another is drawn. These hypothetical lines of

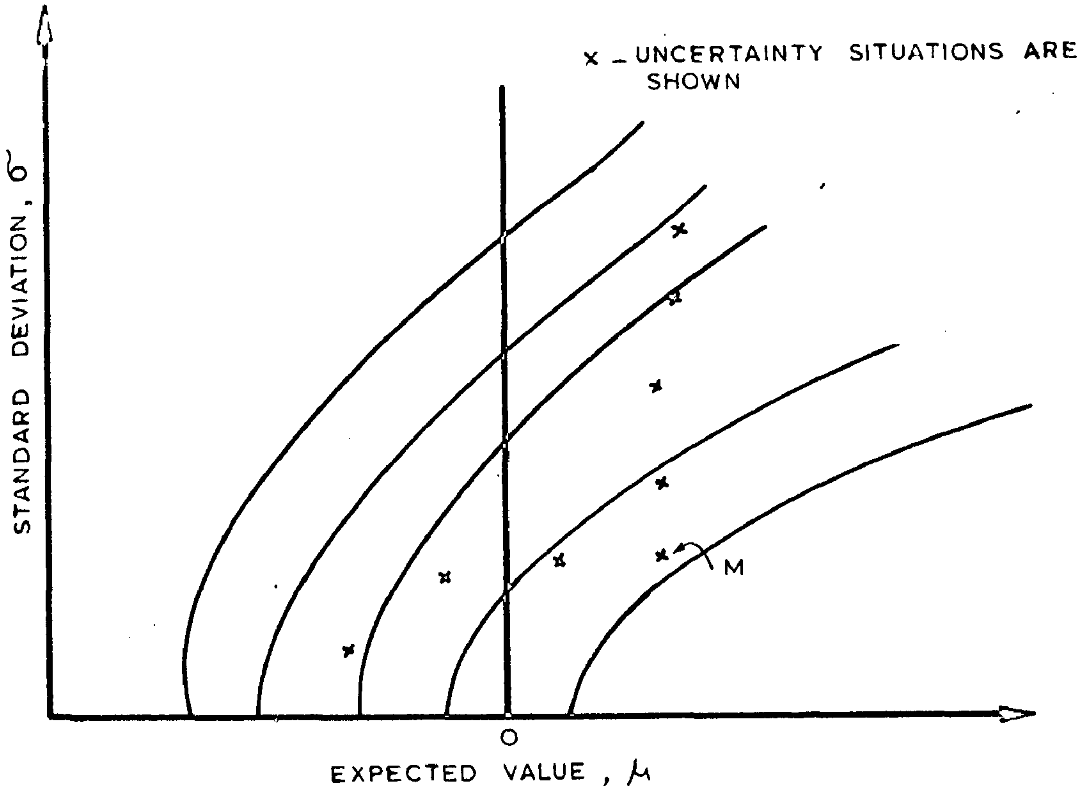


FIG. 3.2 _ GAMBLER'S INDIFFERENCE MAP[5]

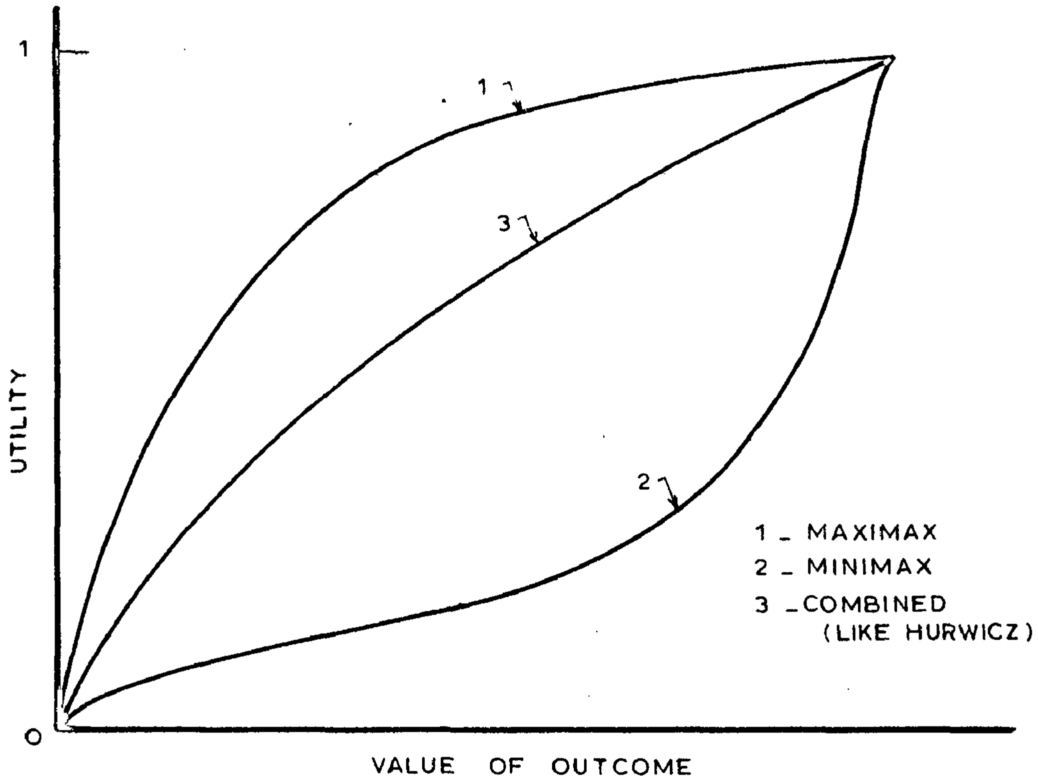


FIG.3.3 _ SHAPE OF UTILITY FUNCTIONS[26]

shape shown indicate the trade-off between expected value and riskness, as we move along it. In fact this diagram is an aid to determine the certainty equivalent required in method (i) above.

For the number of alternatives, we need only calculate the expected value μ and standard deviation σ and show them in the diagram. (Cross points). The maximization of expected value can be achieved by choosing the optimal point such as M in the Fig. 3.2.

The main difficulty, however, lies in establishing the indifference map. Besides, Dorfman[29] has demonstrated fundamental inconsistency that might lead to adoption of an inadmissible alternative.

(iii) Risk Discounting

In this method, the certainty equivalent to a risky venture is computed by multiplying the expected value of the outcome by 'a' factor. It then follows that higher the risk, the smaller the factor is.

This factor is commonly represented by

$$\frac{1}{1 + \sigma c} \tag{3.9}$$

where c is a behavioural constant

and σ is the standard deviation.

c can be shown as the additional requirements to compensate for risks, as below:

Let $B(\underline{X})$ denote the benefits from the system. Then risk discounted net benefits is

$$\frac{B\ (\underline{X})}{1 + \sigma c} \qquad (3.10)$$

where σ and c are as expressed above.

This discounted benefit can be deemed as certainty equivalent. Differentiation of the formula yields $\frac{c}{1 + \sigma c}$; this is the percentage increase in expected net benefits necessary to compensate for a one-unit increase in the standard deviation of the outcome distribution. From this, c is found to be the additional requirement to compensate for risks.[29]

Following the procedure yielded by this method, Howe[63] advocates calculation of benefit and cost streams as per the formula below:

$$B\ (\underline{X}) = E\ [B_o\ (\underline{X})] + \frac{E\ [B_1\ (\underline{X})]}{1 + r + \hat{r}} + \frac{E\ [B_2(\underline{X})]}{(1+r + \hat{r})^2}$$

$$+ \ldots\ldots\ldots\ldots + \frac{E\ [B_t(\underline{X})]}{(1 + r + \hat{r})^t} \qquad (3.11)$$

where,

$E\ [B_t(\underline{X})]$ represents the expected value of benefits in t^{th} year due to systems outputs ($\underline{X}$)

r riskless discount rate (we can take it as social discount rate)

$\hat{r}$ additional discount factor for what is likely to be the compounding level of variability of risk accruing in future periods.

For costs, Howe proposes

$$C(\underline{Y}) = E[(C_o)(\underline{Y})] + \frac{E[C_1(\underline{Y})]}{1 + r - \hat{r}} + \frac{E[C_2(\underline{Y})]}{(1 + r - \hat{r})^2} + \cdots\cdots\cdots + \frac{E[C_t(\underline{Y})]}{(1 + r - \hat{r})^t} \quad (3.12)$$

where,

$E(C)_t(\underline{Y})$ represents the expected value of costs in t^{th} year due to systems inputs: $(\underline{Y})$

r and $\hat{r}$ as above.

3.3.3.3. Solution Techniques from Decision Theory

The decision theory helps to choose a particular solution when a choice is to be made among a number of alternative courses of action.[12] We will consider 3 such criteria as below and extend it to Bayesian approach for dealing with risks/uncertainty.

(i) Maximin and maximax criteria

(ii) Minimax or regret criterion

(iii) Laplace criterion

These approaches are best explained by the solution of an example problem, as below

For the construction of an embankment dam, the initial exploration indicated that hard impervious stratum may lie at ± 50'. The option of the cut-off has been left to the decision maker.

There are three likely possibilities:

(i) adopt steel sheet-pile diaphragm and order it in 40' lengths or 50' lengths (only 2 lengths are available)

(ii) adopt concrete-diaphragm; but as a requirement to this, costly patented techniques are to be resorted to.

The pay-off matrix for the various states, for different alternatives, is as below:

Table 3.1

	State / action	Hard & impervious stratum met at 40'	50'	60'
I	Order 40' sheet-pile length.	100	60	20
II	Order 50' -do-	70	90	35
III	Resort to concrete diaphragm.	30	50	80

Table 3.1. Payoff matrix showing utilities

Let us solve the problem by each of the above criteria which is as follows:

Maximax and Maximin criteria

Maximin rule:

1. Evaluate alternative by minimal return that it guarantees
2. Choose the one with the highest guarantee.

For our example case, we have the minimal payoff for the 3 alternatives as 20, 35 and 30 respectively. The maximum of the minimum pay off being 35, the choice should go in favour of alternative II.

Maximax Criteria

This suggests that the choice of an alternative should be such that it maximizes the maximum possible value of outcomes.

We have for the three alternatives the maximum outcomes as 100, 90, 80 respectively; the maximum of this is 100. The choice should then go in favour of alternative I.

The utility functions implied by maximin and maximax criteria are shown in Fig. 3.3. As seen therefrom, both of them implies extreme utility functions. While the maximin indicates a great deal of risk aversion, the latter i.e. maximax criterion is insensitive to degree of loss below maximum. As de Neufville[26] considers, neither of them stands to logic.

Hurwicz [76] has, therefore attempted to combine the two by using an index of optimism α. In other words he attempts to solve the conflict by tracking a via-media approach.[103] Let us consider a decision maker of 4/10th optimism. We then, have the results of alternatives as per table 3.2. The choice as per Hurwicz goes in favour of alternative II.

Table 3.2

Alternative.	Weight on Optimism		Pessimism		Weighted total of pay-off
I	0.4 (100)	+	0.6(20)	=	52
II	0.4 (90)	+	0.6 (35)	=	57
III	0.4 (80)	+	0.6 (30)	=	50

The results with regard to ranking by the adoption of the 3 criteria, maximin, maximax and Hurwicz are, then, as below (Table 3.3). Considerable variations in choice can be observed.

Table 3.3

Alternative.	Ranking according to Decision rule		
	Maximin	Maximax	Hurwicz
I	2	1	2
II	3	2	1
III	1	3	3

Though Hurwicz's approach is better than the other two, it discards the intermediate values that may be of interest also. Hence none of the approaches by itself appears to be a panacea for the problem involving risk/uncertain situations.

Minimax or Regret Criterion[29]

This method aims at minimizing maximum regret. Savage [76] defines regret of an outcome as equal to the difference between the value of the outcome and the maximum value of the outcome possible for the particular chance of of events that occured. In other words, the risk in an uncertain situation is compared from a table of return by subtracting each return from the highest figure in its column.

For the example problem, we can evolve the regret matrix table as below:

Table 3.4

Alternative.	Regret if impervious stratum is met at		
	40'	50'	60'
I	0	30	60
II	30	0	45
III	70	40	0

It is seen from the above that the maximum regrets for the three cases are 60, 45, 70. The regret is minimized, therefore by choosing alternative II.

However, the regret criterion does not yield absolute solution values, as we co-relate them to other alternatives. If alternatives are dropped or added, the result changes considerably. For example, let us omit the adoption of alternative III from choice range (i.e. concrete diaphragm alternative). We then find that the criterion favours alternative I. Similar exercise by dropping other alternative or inclusion of any fresh alternative also changes picture.[26] There is, besides, no transitivity and hence ranking becomes a problem. Since the fundamental requirement for utility theory on transitivity does not stand fulfilled, the adoption of this method for solution is questionable.

Laplace Criterion

This criterion is very useful where probability chances of outcome are unknown. It assumes, in such a case, equal probabilistic chances. The expected value derived from the first moment of distribution is maximized for a desirable choice.

For our problem example, we have the expected values as below:

Table 3.5

Alternative I	1/3 (100 + 60 + 20)	=	60
Alternative II	1/3 (70 + 90 + 35)	=	65
Alternative III	1/3 (30 + 50 + 80)	=	53

Thus scales tilt in favour of alternative II. Laplace criterion is normally followed due to its intuitive appeal. However, this method is sensitive to description of chance events.[26] If one or other of the chance events are altered the results becomes distorted. For example if we do not anticipate the contingency of impervious strata being met at 40' depth, the expected value is as below:

Table 3.6

Alternative	Depth of impervious strata		Expected value
	50'	60'	
Alternative I 40' sheet pile.	60	20	40
Alternative II 50' sheet pile.	90	35	63
Alternative III C.C. diaphragm.	50	80	65

Thus alternative III emerges as the favourable alternative now.

Bayesian Approach[100]

This approach advocates estimation of probabilities for all chance events. Based on them, the utilities are calculated & choice is made so as to maximize expected utility. The approach also attempts to constantly improve the probability values, in terms of fresh known information. Hence the procedure is valuable for systems that have been committed also.

3.3.4. Summary Comments

Problems linked with risk and uncertainty cannot be tackled ~~looked~~ by precise means. The major difficulty in the real world problem lies in evaluating utilities functions. As such maximization of expected value in monetary terms instead of maximization of expected utilities seems to be a possible recourse; for this purpose it is worth investigating the probabilistic chance values[26] for events.

Attachment of an extra discount to the normal discount factor appears to be also a reasonable practical solution. However, as a consequence the benefits accruing in later years diminish and in projects where benefits accrue only after a sluggish period of initial years (like irrigation projects) this is a great disadvantage.[61,63,118]

When direct measures of risk to be avoided can be defined (as in hydrologic risk quantification) risk can be treated adequately by chance controlled non-decisioned inputs in system. Further research in this field is worthwhile to include risk as an objective in multi-objective optimization and treated adequately.[54]

CHAPTER 4

A GENERAL APPROACH FOR MULTI-OBJECTIVE PROBLEM SOLUTION

4.1. INTRODUCTION

The introduction of multi-objectives in the evaluation of water resources system has complicated the analysis process. So long as a single objective(viz; economic efficiency) dominates over all others and a single point of view (e.g. national income) is considered as the primary, the optimisation can proceed along classical lines using either mathematical decision models or judgement as desired.[54] Secondary objectives and points of view can be taken care of through judgement based constraints. However, when objectives are considered at par , a different analysis stragety is called for. The real world multi-objective, multipurpose project analysis is however complex. The aid of solution techniques based on systems approach are taken up in subsequent chapters 5 - 8. In this Chapter, an attempt is made to illuminate the concept underlying the choice of optimal solution for simple, dual objective cases.

4.2. STEPS IN ANALYSIS

The steps in the multi-objective analysis are fourfold:[84]

(i) The choice of the relevant objectives

(ii) Plan formulation and development of the transformation curves.

(iii) The analysis of preferences of different interest groups indicating trade-offs between objectives.

(iv) The choice of an optimal plan.

Since (i) above has been dealt with in Chapter 2 already, we take (ii) now for consideration.

4.3. TRANSFORMATION CURVES

4.3.1. The transformation curve can be defined as the boundary of the set of feasible combinations of net benefits towards chosen objectives.

4.3.2. For the simple case of dual objectives, this is best illustrated in a two-dimensional graphic plot in which each of the major axes is chosen to represent the net benefits for one of the objectives. It is important that many feasible alternatives for the system under study are evaluated. It may be in an exploratory manner. The net benefits available from each of the projects to the various objectives are utilized to delineate the Transformation curve, as fully as possible. We may, in fact, compare the Transformation curve as analogous to Production Possibility Frontier%

% Mckean gives a pertinent illustration of PPF in his book on Govt.Systems Analysis.93

A wider and broad analysis can extend to all Government Sectors for the derivation of Transformation Curve for a typical National Economic Development - Regional Income (NED - RI) dual objective case.

4.3.3. An example will illustrate the above concept better. Let us consider Fig. 4.1 in which the Transformation Curve for a typical two objective case viz;

(i) National Economic Development (NED)

(ii) Regional Income Distribution (RI)

is shown.

The planning efforts contributing to derive such a curve viz estimation of benefits and costs and derivation of net benefits have been outlined earlier in Chapters 2 and 3. However, it is worth emphasizing again that the income of a region is just not a simple subset of NED income. Regional Income includes transfer and NED income does not.*

4.3.4. For generating requisite data for many feasible alternatives so as to delineate the Transformation Curve appropriately, an input-output model (for national and regional sector) is best made use of.**

4.3.5. To quote another example, Fig. 4.2 may be seen. Cohon & Marks[18] provide in this study of the case project at

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*Details of this are found in Chapter 2.2.

** An input-output model after Haveman & Krutilla is briefed in Chapter 2.1 and described in Appendix I.

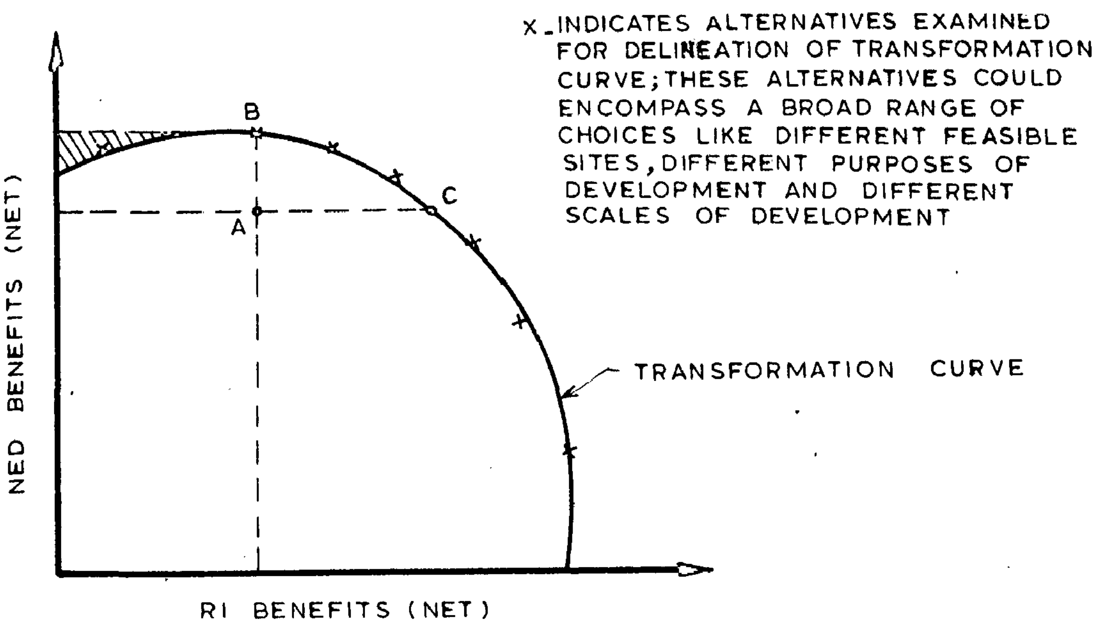

FIG. 4·1_ TRANSFORMATION CURVE FOR DUAL OBJECTIVES (National Econ. Development & Regional Income)

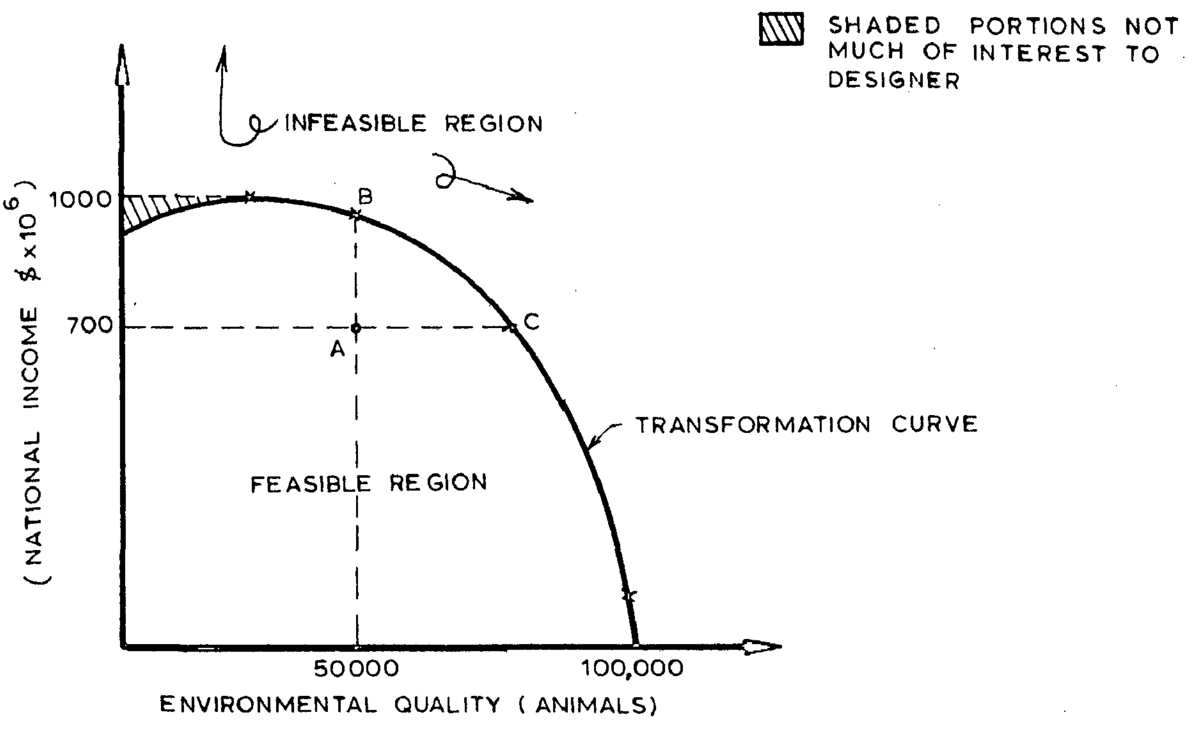

FIG. 4·2_ TRANSFORMATION CURVE FOR THE DUAL OBJECTIVE NED AND EQ.[18]

Alaska where a pipeline is to be built for the transportation of crude oil from the frozen "oil-rich" northern shore to the relatively warmer southern shore. The project involves pumping oil at 140°F which melts frozen tundra. This, therefore, threatens wild-life by preventing net present annual migration.

The planning objective has been defined in this case, by policy-makers, as

(i) Maximization of National Income - in terms of $.

(ii) Preserve environmental quality (EQ) in terms of number of species existing at Alaska.

For this case, therefore, the Transformation Curve represents the boundary of the set of feasible alternatives viz net $ NI benefits and corresponding EQ effects measured in terms of number of species existing at Alaska.

4.3.6. The two cases (Fig. 4.1 and Fig. 4.2) thus demonstrate the aspect of generation of Transformation Curve. When we have examined all the alternatives that are feasible, we need consider only the portion of Transformation Curve that is relevant for decision making.

The characteristics of Transformation Curve is worth a mention. Analogous to Production Possibility Frontier concept, as already indicated, a point outside Transformation curve can be construed as physically infeasible. Similarly all points inside the Transformation Curve are feasible (between origin and Transformation Curve) but not as good as at least one point on the boundary.[84] For example, considering Fig. 4.1, 4.2, point A is feasible but is worse than point B, because national income benefits can be increased by moving from A to B without decreasing RI benefits. Similar arguments hold for the other case.

4.3.7. As "more" of each objective is better than "less" of the same objective (normally), the portion of the boundary of feasible sets that slopes from NW to SE turns out to be relevant portion of maximum interest for decision making.[85] This is so because all other points are dominated by at least one other point lying towards the north-east. Thus it is this portion in which we are confronted with hard choices that requires us to give up benefits towards one objective to achieve more benefits towards another.

4.4. SOCIAL WELFARE CURVES/INDIFFERENCE CURVES

4.4.1. The next requisite in the problem is one of analysis of preferences of different interest groups, also known through societal indifference curves. This leads us to social welfare functions.[58]

4.4.2. The social welfare function is an unique function which expresses the total welfare of the public as a function of all goods and services available.[26] This, in other words, implies that it is the utility function of the society as a whole.[13,105] Though powerful in concept, precise definition of social welfare for any community poses considerable problems. This is due to the fact that the relationship between the welfare of the society and utility of individuals is not known. How a social welfare might depend on the utility of individuals is a matter of debate on ethical and technical grounds. However, there is a concensus that it is highly non-linear, as shown in Fig. 4.3 which is an illustration of the social welfare function in a simple two person community.[26]

4.4.3. The explicit shape of social welfare function depends upon the moral and ethical foundations of society. Two examples of alternative forms of social welfare functions appropriate to different social organisations are sketched in Fig. 4.4 and 4.5. The old zamindari system that desired a greater weight on the utility of zamindars compared to their bonded labour could be somewhat like Fig. 4.4. In fact, the ancient Roman republic explicitly used such weighing in decision-making. The votes of members of certain classes simply counted as an explicit multiple of those of the lower classes. In an individualistic society where each individuals utility counts equally in the determination of social welfare, the curve could be as shown in Fig. 4.5. For such cases, the function for social welfare is essentially linear and obtained by addition of utilities.[26]

4.4.4. For a democratic set up like ours the social welfare function could be construed. While we constitutionally assure equality, equal opportunity and sovereignty, we do subscribe that deprivation of disadvantaged is undesirable and recognize poor groups of population/areas of the country that are under-developed and support special consideration

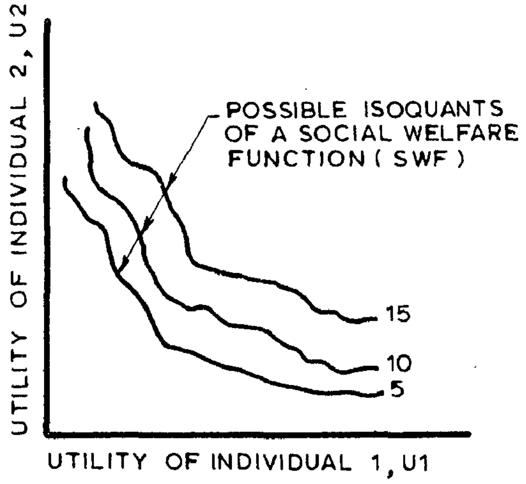

FIG. 4.3 The social welfare function as a function of the utility of individual[26]

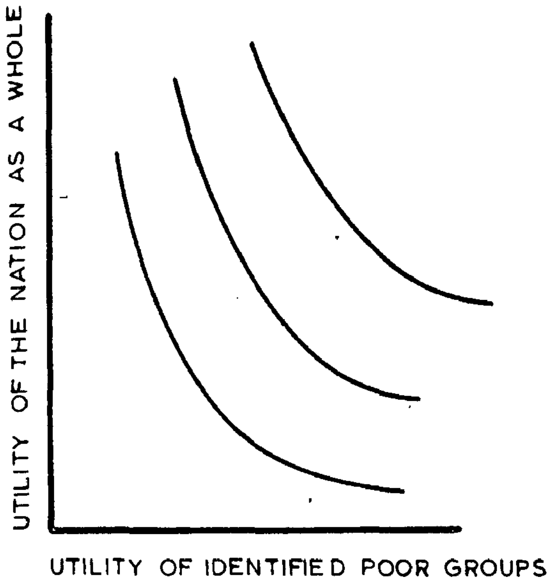

FIG. 4.6 Simplified approximation of the social welfare function for a nation

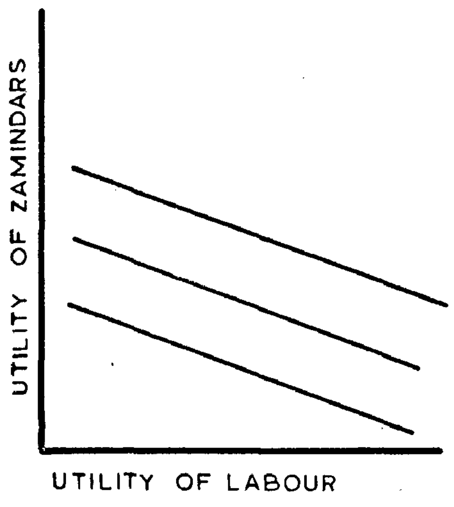

FIG. 4.4

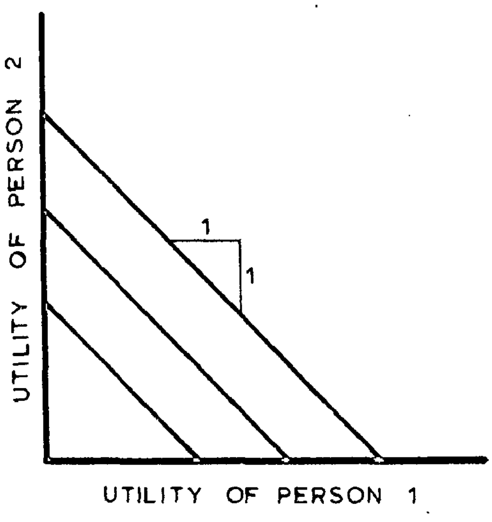

FIG. 4.5

SIMPLIFIED PRESENTATION OF SWF IMPLICIT IN DIFFERENT FORMS OF SOCIETY[26] for their welfare. Specific projects that help achieve this objective are taken up with priority. Thus we can expect that the social welfare function for our country may be somewhat similar to fig. 4.6 which is a notional mix of Fig. 4.4 and Fig. 4.5.

4.4.5. For the determination of an exact form of social welfare function, a logical notion could be some form of questioning put to society or to its electoral representatives. In other words, the political system as a decision making body could assume to reflect the preferences of the society truly. The social welfare function could then be assessed through them for analysis.

Arrow[3] has demonstrated the impossibility of relying upon polls to obtain any useful assessment of social function. To counter this, the system of voting by participants, not only on several issues but also as to how others may vote is useful (value voting). This solves the problems of intransitivity also. Observations on log-rolling also provide one way of understanding the trade-off acceptable to community. Coleman[20] has indicated that even community-wide majority agreement on the ranking of alternatives can thus be obtained. This consensus on community preferences may also help in formulation of social welfare functions.

4.4.6. The final problem in the issue is one of interpersonal comparison of utilities.[13] One cannot compare each person's utility and transform it into some appropriate common denominator and determine the social utilities. To quote an example, one may elect to be relatively poor rather than endure the stress of "climbing the ladder of success" that the other may opt. Their measures of utilities cannot then be combined in any objective way. Thus we recognize that as no analytic way to solve the problem of social welfare functions exists one has to rely on the political process to read the social preferences and reflect them in programme and project designs.

4.4.7. Major[80] has argued that for dual objectives where only societal indifference curves are required, there is no need for direct interpersonal utility comparisons; decisions reached in an informal legislative process can also be used. Maass[6] also presents a strong argument (based empirically on studies by David C.Major and A.Maass) in favour of this, especially when the decision process is crystallized by adequate executive initiative.

4.4.8. Even when preference information is sketchy and uncertain, an analytical approach in ascertaining trade-offs is of great value for the decision-maker. When the actual preference curves are unknown, the analysts can provide to policy makers all pertinent informations on trade-off and sensitivity that the transformation curve embodies, in the choice range of interest (vide para 4.3.6 & 4.3.7). This aspect can be adequately taken care of by the multi-objective solution techniques. (See Chapter 7 & 8).

4.5. THE CHOICE PROCESS

4.5.1. Method of Weights

4.5.1.1. Once the social welfare curves and the implicit societal ordering of the combination of chosen objectives are known, the society's relative marginal social values for, or the trade-offs between chosen objectives are explicit. In figures 4.7 and 4.8 we have designated the family of indifference curves by IC_1, IC_2 etc. Since "more" is better than "less" (normally) we can consider welfare increases as we go up in NE/ direction and can assume $IC_2 \succ IC_1$ and so on.

4.5.1.2. The basic problem in planning then boils down to choosing the best attainable combination of the objectives.

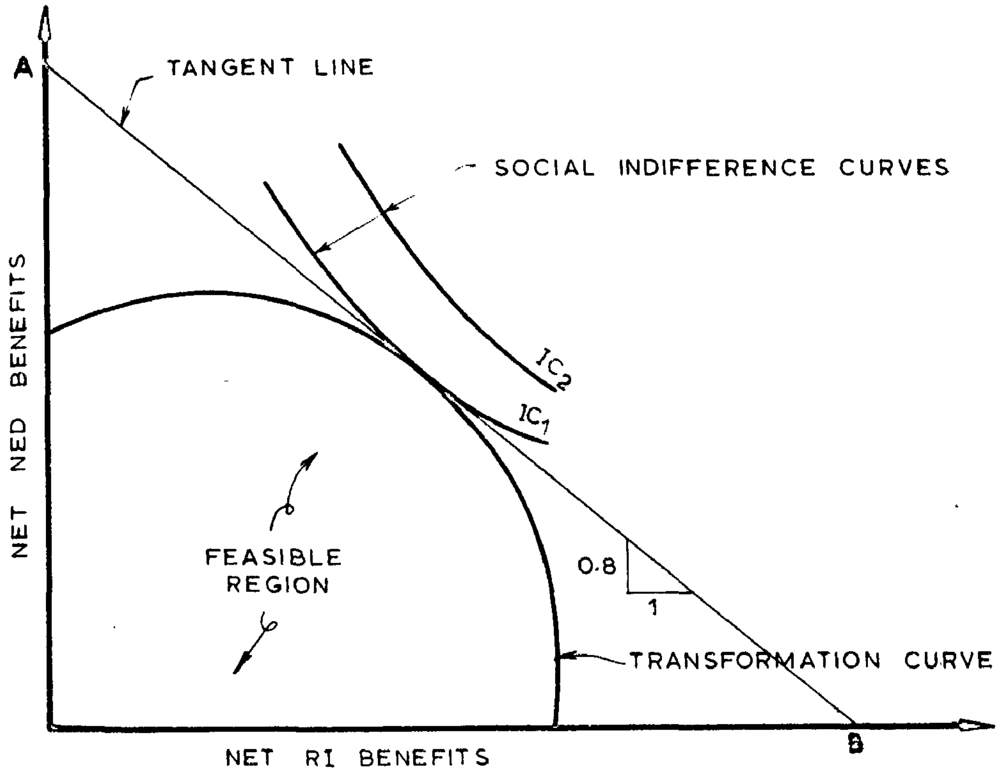

FIG. 4.7_ GRAPHICAL REPRESENTATION OF MULTI-OBJECTIVE ANALYSIS
(NED-RI dual objective case)

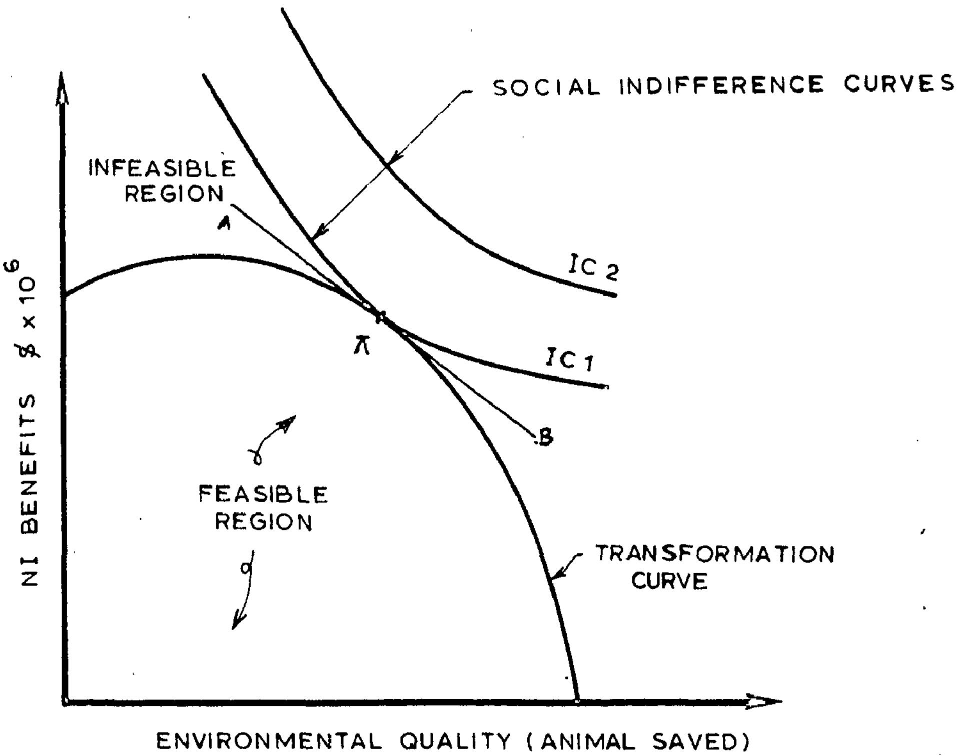

FIG.4.8_ GRAPHICAL REPRESENTATION OF MULTI-OBJECTIVE ANALYSIS[18]
(NI-EQ dual objective case)

This is defined by the point of tangency of transformation curve and the highest attainable indifference curve.[80] (π in Fig. 4.7. ; and Fig. 4.8.). The slope of the tangent line through this point of tangency of the Transformation Curve and the highest attainable Indifference Curve is a measure that yields the relative marginal social values placed on the dual objectives.[85]

4.5.1.3. The line AB in Fig. 4.7 is drawn tangential to both the Transformation Curve and Indifference Curve at optimal point. In this case, thus, the slope of this line represents the relative marginal social value placed on the net NED benefits and net regional income benefits to the specific region. In other words, suppose we assign the value of one to Re. one of the net NED benefits, the price of Re.one of net Regional Income benefits is given by the negative of the slope of tangent line. For the example case in Fig.4.7 this is equal to 0.8. This value thus indicates that by selecting π as the optimal point, society has shown that it is willing to give up, at the margin, Re. one of net NED benefits if Rs. 1.25 of net Regional Income benefits are provided, in return. It is interesting to look into Benefit-Cost comparisons as done in traditional methods, with the foregoing in the background. The maximization function for our project can be expressed as:*

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* The implicit social discount factors and other assumption to derive present value are not repeated.

$$1\{B_n(\underline{X}) - C_n(\underline{Y})\} + 0.8\{(B_r(\underline{X}) - C_r(\underline{Y})\}$$

and form the benefit-cost ratio, for expository interest, for the multi-objective formulation as[85]

$$\frac{1\{B_n(\underline{X})\} + 0.8\{(B_r(\underline{X})\}}{1\{C_n(\underline{Y})\} + 0.8\{C_r(\underline{Y})\}} \qquad (4.2)$$

where

$B_n(\underline{X})$ and $B_r(\underline{X})$ are the gross benefits for NED and Regional Income objective for ($\underline{X}$) outputs

& $C_n(\underline{Y})$ and $C_r(\underline{Y})$ are the gross costs for NED and regional income objectives for ($\underline{Y}$) inputs.

The gross costs and benefits can be derived from planning information.

4.5.1.4. IN OTHER WORDS, A MULTI OBJECTIVE BENEFIT-COST RATIO IS COMPOSED OF ALL THE WEIGHTED BENEFITS AND COSTS FOR ALL OBJECTIVES. By keeping weightage factors for other objectives as zero, the generalized equation for multi-objective analysis reduces to traditional benefit-cost equations. Thus we see that THE TRADITIONAL CASE IS A PARTICULAR SUBSET OF THE ENVISAGED MULTI-OBJECTIVE ANALYSIS.

4.5.1.5. Let us consider Fig. 4.8 of the second illustrative example. In this case, we infer from the property of line AB tangential to both the Transformation Curve and societal indifference curve that society is willing to give up one head of wild life if it gets $ 10,000 in NED benefits, in return, at the margin.

The multi-objective Benefit-Cost ratio for this project, on a similar analogy can be inferred. We must remember that the ratio, in this case, has been based on imputed monetary values for non-monetary objectives, taken at the margin.

4.5.2. Choice of the Weights of Objectives

In case of the examples shown above, the societal Indifference Curve (or social welfare curve-considered analogously in a restricted sense for our convenience) were utilized to derive weights in the margin for the chosen objectives. Alternatively, at national level of planning,* weights could also be set up explicitly, after an explicit declaration by the policy-makers. As this may be rather difficult due to various considerations, it becomes necessary that the analysts attempt to impute the weights from the past choices considered rational. An excellent methodology in this direction is best outlined by Sen, Das Gupta and Marglin in the UNIDO guidelines for Project Evaluation.[118] While initially weights are considered as unknowns, analysts can attempt to infer the switching values favoured by the decision-makers. Instead of requiring an explicit declaration of weights by policy-makers, as indicated above in what can be termed as "TOP DOWN METHODOLOGY", this new methodology, termed "BOTTOM UP METHODOLOGY", considers weights as unknown to begin with.[90,118] The values of weights that make significant difference in the design and operation

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* For example "Planning Commission" in Centre.

of projects are identified from past choices and a set of project variants that are optimal are elaborated in the different ranges of parameters values. These set of variants are submitted in its entirety to the policy-makers for a final assessment on the implications of different choices. The advantages of this "bottom-up Methodology" are claimed to be that[118]

(i) All relevant alternatives are brought to the attention of the decision-makers.

(ii) It focuses choice on the relevant variables by relating political decisions to national parameters.

(iii) It thereby serves to introduce the importance for national parameters to policy-makers, and

(iv) Finally, it forms the basis for a deliberate systematic determination of consistent national parameters in course of time.

4.5.3. Constraint Approach for Solution

In the treatment of the subject so far, we considered the method of setting weights to problem solution. There are other methods available for solution, typically for dual objective case. Marglin has detailed two other methods, which we can call as "constraint methodologies" for problem solution.[5, 90] We have described some aspects of these methods in Chapter 2.2. For the first alternative we maximize the NED benefits setting a constraint that a prescribed net income increase shall accrue to particular region. For the second type we maximize the regional income net benefit function setting constraints on NED benefits. The example applies to a dual NED-RI distribution objective case. A number of objectives other than the above could also be handled in an alike manner.

4.6. PURPOSE AS AN OBJECTIVE IN THE CHOICE PROCESS

We can, in certain cases, find the purpose itself turnigg out to be an objective. To quote an example, let us look into water resources projects, designed for multiple-purposes. Traditionally, the purposes are identified as different combinations of the following:[56]

(i) Generate hydro-electric power.

(ii) Provide irrigation facilities.

(iii) Cater to municipal or industrial needs of water supply.

(iv) Improve flood-control capability of the system.

(v) Improve navigation facilities.

(vi) Provide for recreation facilities/fishing.

(vii) Maintain suitable quality of ground/surface water.

(viii) Provide buffer for groundwater recharge or for drought.

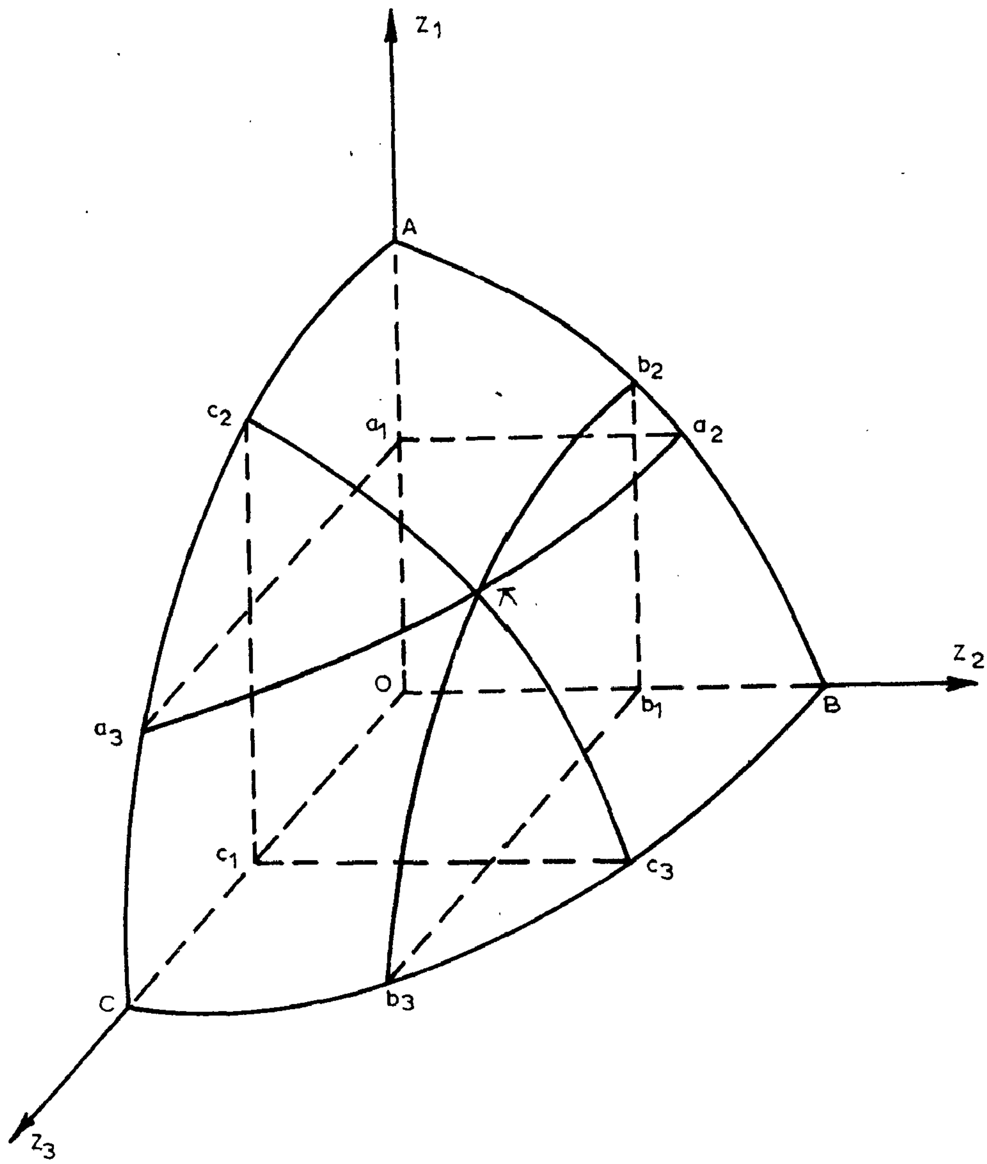
(ix) Improved related land use and prevent damage due to runoff etc.

It could be an aim in planning directives that a constrained maximization of one or several of the objectives be provided. Trade-offs between alternative purposes, now considered as objectives, are then needed to be identified. However, the other procedures outlined for treatment of objectives would remain unaltered. A typical case study in this direction is found in the works of Rao,[104] where the purposes have been identified as objectives for reasons defined therein.

4.7. SUMMARY FINDINGS

4.7.1. When the objectives are more than two, it is difficult to have a two-dimensional graphical representation as has been done in the above treatments. A triple objective case can also be represented in a three-dimensional figure as shown in Fig. 4.9.[59] for an easy visualization. The multi-dimensional transformation curve in this case, is, by analogy to a three-dimensional skin to a space, an envelope or hull to feasible regions of transformation. When the dimensions are more than three, however, a visual conception is difficult. As solutions for multi-dimensional situation are in hyper-planes, there is no graphical representation possible.

4.7.2. The multi-dimensional case requires a more sophisti-



π _ CHOICE POINT

FIG. 4.9 _ GRAPHICAL REPRESENTATION OF 3D TRANSFORMATION SURFACE cated approach. In fact the method of treatment of multiobjective problem, as presented in this chapter has been simple and classical, on the basic assumption that weights can be assigned to the relative importance of each of the objectives. With these weights the attempt has been essentially to reduce the multiple objectives to an equivalent scalar value. The standard optimization of this scale value has been the objective. However, it has been found that there exist a perennial difficulty in the concept of scalarization.[125] In many complex systems, it is neither easy nor desirable to summarise the essentials of a consequence by means of a single numerical value. This has been found so especially in case of environmental quality objective. To quote as an example, for environmental pollution, attribute 1 may reflect the cost of cleaning a polluted environment, attribute 2 may reflect the degree of discomfort, attribute 3 may refer to the cost of damage done and so forth.

The complex systems that are to be analysed with multiple objectives are best handled by systems techniques, mathematical programming models, and computer based solutions.[82] Our aim in the following chapters will be to look into these aspects by studying the various available solution techniques.

PART II

ANALYSIS BY SYSTEMS METHODS

CHAPTER 5

MULTI-OBJECTIVE DECISION-MAKING MODELS

5.1. INTRODUCTION

Decision making, keeping in vision the multi-dimensions of the objectives, is rather complex. However, there are a variety of models, descriptive and prescriptive, to indicate solution strategies. Basically, we can classify them[77] as:

(i) Multiple attribute decision making models, and

(ii) Multiple objective decision making models.

Attributes refer to, commonly, a particular characteristics, aspect, factor, performance parameter, component etc. On the contrary, objectives encompass a broader forum, treating the mean-end relationship. Where there is one to one relationship between attributes and objectives, however, both types merge as one. Two other semantics have come into field recently viz; multiple criteria and multiple dimensional decision situation. We confine our attention in this work to multiple objectives, besides reviewing the multiple attribute solution techniques which are similar.

In what follows therefore, the treatment is aimed basically to develop the modelling process progressively to deal with complicated problems. Most of the solution techniques for multi-objective problems given in Chapter 7 are found to be an effective combination of these different models, which are categorised[77] as hereunder for convenience in review:

(i) Weighting methods

(ii) Sequential elimination methods

(iii) Mathematical programming methods, and

(iv) Spatial proximity methods

5.2. WEIGHTING METHODS

These methods fall under three categories viz;

(i) inferred preferences

(ii) directly assessed preferences - general aggregation.

(iii) directly assessed preferences - specialized aggregation.

The common characteristics of these methods can be found as:

(i) Availability of a set of alternative.

(ii) Their attributes and values

(iii) Intra and inter attribute preferences.

(iv) A well specified objective function.

(v) A rule for the choice of the alternative.

5.2.1. Inferred Preferences of Decision Maker[100]

In this method, the statistical techniques are utilized to study the past choices of DM*, provided there are sufficient repetetive decision situations that can be grouped together. The common methods are:

(i) Linear and Quasi-linear regression analysis

(ii) Analysis of variance.

Quasi-linear and linear regression models have been developed with irrigation water and crop yield as attributes and utilized for approximate studies in screening models.[33]

5.2.2. Directly Assessed Preferences of DM-General Aggregation

Under this, we have the following methods:

1. Simple additive weighting
2. Hierarchical additive weighting.
3. Quasi-additive weighting
4. Trade-offs.

5.2.2.1. Simple Additive Weighting

The requirements to adopt this method are that:

(i) DM assign importance weights for each of the attribute/objective.

(ii) DM reflect his marginal worth assessment within attribute.

(iii) DM also make a numerical scaling of intra-attribute or intra-objective values.

---

* Decision maker is referred to as DM throughout this part.

With the above, it is possible to have a total score for each alternative simply by multiplying the scale rating for each attribute/objective and then summing these products over all attributes/objectives. The alternative with highest score is the choice. However, this method runs the risk of ignoring interaction among attributes/objectives.

5.2.2.2. Hierarchical Additive Weighting[96]

This method recognizes that attributes may simply be means towards higher level objectives. Hence the DM assigns values or preferences to the higher level objectives. He then explores the instrumentality of the other objectives in attaining the higher level objectives. In this way he infers the inter-objective weightings from his direct assessment of higher level objectives.

5.2.2.3. Quasi Additive Weighting

When the utility of expected outcome of a choice of an objective depends on the joint distribution of the other objective and not just on marginal distribution, the above methods, para 5.2.2.2. which considered the independence of utilities for individual objectives is inadequate. Then by obtaining conditional utility functions on the objectives if some of the objectives are utility independant of others, an overall performance of quasi-additive form is obtained.

De Neufville & Keeny[27] have utilized this approach for studying the development of Mexican Airport facilities using multi-plicative utility over attributes.

5.2.2.4. Trade-off

By questioning the DM on the marginal rates at which he would be willing to trade-off one attribute for another, trade-off information is obtained. This helps analyst to solve the problem in a desirable manner.

An interesting solution with trade-off is found in Benjamin Franklin's letter to Joseph Priestley, dating backing to nearly 2 centuries.[34]

5.2.3. Directly Assessed Preferences of DM - A Specialised Aggregation:

Two categories of decision falling under this are Maximin and Maximax procedures,[126] which were detailed in Chapter 3.3.3.3. Maximin adopts a specialized degenerate weighting different for each alternative. It assigns a weight of one to the worst attribute value and a weight of zero to others. This method is suggested only where the over-all performance of the system is determined by the weakest attribute. The maximax approach, in contrast, considers the best attribute value and utilizes a specialized degenerate weighting of 1 assigned to the best attribute and zero to the rest.

5.3. SEQUENTIAL ELIMINATION METHODS

They are of the following types:

1. Alternative versus standard
2. Alternative versus alternative:comparison across attributes.
3. Alternative versus alternative: comparison across alternatives.

5.3.1. Alternative Versus Standard

The DM sets up standards to be applied to the values on certain attributes. In the conjunctive form (characterized by "and") all the standards must be passed in order for the alternative to be acceptable. In the disjunctive form (characterized by "or") only one standard of a specified group must be exceeded for an alternative to be acceptable. The conjunctive constraints are akin to maximin rule; they help filtering alternatives effectively unless standards are at minimal level as they have to pass a number of standards. The disjunctive form, contrarily, permits a larger number of alternatives to emerge, unless standards are set at a maximal level.

5.3.2. Alternative Versus Alternative: Comparison across atrributes

In the disjunctive and conjunctive constraints procedures, one alternative which is a real object of choice is compared to another alternative, which is hypothetical alternative of standards, in order to effect choice. In this method, we compare one real alternative against another real one to see if poorer alternatives can be eliminated. If one alternative has attribute values that are at least as good as those of another alternative for all attributes, and if it has one or more values that are better, then the first alternative is considered to "dominate" the second. In other words "Dominance" rules choice as per this method.

5.3.3. Alternative Versus Alternative by Comparing the Attribute Across Alternatives

In this group, the values of each successive attribute are compared across alternatives. There are two sub-sects:

5.3.3.1. Lexicography

In this method, the attributes are ranked in the order of their importance with their intra-attribute values placed om an ordinal scale.

5.3.3.2. Elimination by Aspects

This is similar to Lexicography but eliminates alternatives which do not satisfy a standard that is set; the process is continued to eliminate all but one. The method also does not order the attributes in an ordinal scale as per importance, but in terms of their discrimination power in a probabilistic mode.

5.4. MATHEMATICAL PROGRAMMING MODELS

The inherent characteristics of this class are:

(i) An infinite or very large set of alternatives which are inferable from a set of description (i.e. constraint specified attribute values).

(ii) A set of technological constraints.

(iii) An objective function, either global or local that is compensatory.

We have, under this class:

(i) Optimisation models

(ii) Goal programming, and

(iii) Interactive multi-criterion models.

5.4.1. Optimisation Models[22]

These may be viewed as a multiple attribute decision method. The variables are attributes, the linear constraints are conjunctive constraints on combination of attributes and there is a linear, compensatory objective function. In this class we have not a small explicit list of alternatives from which to choose, but rather an infinite set of alternatives, implicitly defined by constraints. This procedure is rather a solution for a design problem than that of choice, since the purpose is to design the optimal alternative by putting together the best combination of attribute values (i.e. values of variables).

5.4.2. Goal Programming[16]

In this method, the DM specifies an acceptable or desired levels on single attributes values (i.e. one variable constraints) or on combination of attributes (i.e. multi-variables constraints) and these serve as primary goals. The DM also indicates his preferences for deviations in each direction from these goal levels. The objective then is the minimization of these deviations. As a multi-variable constraint enters as a goal and there are a number of such constraints, it is a multi-objective method of analysis; however, the goal deviations are ultimately combined to form a global objective.

5.4.3. Interactive, Multi-criterion Programming[11,54,97]

This class is recent due to research in mathematical programming. It is apt for multi-objective models since the DM's processing limitations are given due considerations.

Interactive multi-criterion programming does not assume a single global objective. It requires the DM to provide his local trade-offs in the neighbourhood of a feasible alternative. These trade-offs are used in a local objective function for a mathematical programming algorithm to generate optimal (rather non-inferior) solutions. The DM is given opportunities to give his trade-off preferences repeatedly which are input to algorithm until the desired level of satisfaction is attained.

5.5. SPATIAL PROXIMITY METHODS

The procedure has the following characteristics:

(i) A set of identified alternatives (with attribute values)

(ii) A process for obtaining intra-and inter-attribute judgements (or perhaps an aggregated judgement).

(iii) The construction of spatial representation.

& (iv) The identification of ideal configuration and the choice rule based on the proximity of alternatives to these ideal configuration.

5.5.1. Indifference Map

We have dealt with this approach also in Chapter 4.4 already. As stated therein, the alternatives to be considered are located in a graphical/spatial representation, which forms the transformation curve; the preferences of DM is obtained in the form of indifference surfaces which show the combination of attribute values that are equally preferred. In other words the trade-off approach detailed in para 5.2.2.4 above is given an explicit graphical form. Unlike trade-off, however, different marginal rates of substitution, at different values of attribute levels, are possible to be considered in this approach.

5.5.2. Multi-dimensional Scaling with ideal points[117]

Alternatives are represented in this method by points in Euclideon space. The DM's preferences of alternatives are also indicated in space and the distance from this ideal point to various alternatives utilised to rank the alternatives in terms of DM's preference. The relationship between mathematical programming and multi-dimensional scaling is developed by Srinivasan and Shocker[117].

5.5.3. Graphical Overlays[87]

In this method, a number of transparent sheets reflect the desired way to attain a particular objective. The multiple objective aggregation can be done by a visual aggregation of subset of transparencies overlayed on one another until all objectives have been incorporated. This procedure has an explicit hierarchial form.

The Multi-Objective Problem

The classical optimization strategies utilized for water resource systems aim at maximizing economic efficiency subject to a set of constraints. Essentially, this reduces to the model represented by

$$\text{Max } z = \underline{C}^T \underline{X} \qquad (6.1)$$

subject to

$$\underline{AX} \leqslant \underline{B} \qquad (6.2)$$

$$\underline{X} \geqslant 0 \qquad (6.3)$$

where

z is the scalar indicating objective

$\underline{C}$ n x 1 vector (which indicates cost of $\underline{X}$)

$\underline{X}$ n x 1 vector of decision variable

$\underline{A}$ m x n matrix and is known.

$\underline{B}$ m x 1 vector indicates resources available.

Being a general linear programming problem the solution can be easily evaluated utilizing the well-known techniques like simplex method[22] etc.

The multi-objectives in WR systems planning, introduce in the objective function itself, a vector. The general form of a multi-objective problem with p objectives then becomes

$$\text{Max } \underline{Z} = \underline{C}\,\underline{X} \tag{6.4}$$

subject to

$$\underline{A}\underline{X} \lesseqgtr \underline{B} \tag{6.5}$$

$$\underline{X} \geqslant 0 \tag{6.6}$$

where $\underline{Z}$ is p x 1 column vector $(z_1, z_2 \ldots z_p)^T$ (6.7)

$\underline{C}$ is a p x n matrix

$$\begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ c_{p1} & c_{p2} & \cdots & c_{pn} \end{bmatrix} \tag{6.8}$$

$\underline{A}$ $\underline{X}$ $\underline{B}$ are of the same definition as in the case of scalar optimization problem (6.1) to (6.3).

The complexity of solution of multi-objective problem is apparent from the fact that $\underline{Z}$ is now a vector, which cannot be directly maximized or minimized.

In general, in most of our problems n >> p i.e. the decision variables $x_1, x_2 \ldots x_n$ is quite large compared to objectives $z_1.z_2 \ldots z_p$.

Strict inequality holding good for at least some $\underline{X}$.

In the multi-objective problem optimality is replaced by the notion of 'non-inferiority'.

A solution $\underline{X}_1 \in T$ is inferior if there is some solution $\underline{X}_2 \in T$ for which

$$\underline{Z}\ (\underline{X}_2) \geqslant \underline{Z}\ (\underline{X}_1) \qquad (6.14)$$

i.e.

$$\left.\begin{array}{l} z^1\ (\underline{X}_2) \geqslant z^1(\underline{X}_1) \\ z^2\ (\underline{X}_2) \geqslant z^2(\underline{X}_2') \\ \ldots\quad \ldots\quad \ldots\quad \ldots \\ z^p\ (\underline{X}_2) \geqslant z^p(\underline{X}_1) \end{array}\right\} \qquad (6.15)$$

where at least one of the expressions in(6.15) must be satisfied as a strict inequality.

Similarly a solution $\underline{X}_1^*$ is said to be non-inferior if condition (6.13) is obtained.

Thus the solution of multi-objective linear programming is the definition of the set of non-inferior solution.

A few words about non-inferior solution set. These points represent the same as that of 'Pareto Optima' discussed in Chapter 1. The definition of these multi-objective points of efficiency is given by Koopman. To quote[69]

> ' A possible point inthe commodity space is called efficient whenever an increase in one of its coordinates (the net output of one good) can be achieved only at the cost of a decrease in some other coordinate (the net output of another good) '

Let NI be defined as

We have*/

$\underline{X} \in R^n$ - the decision vector

$\underline{Z}: R^n \rightarrow R^p$ - the objective function vector

$\underline{C}: R^n \rightarrow R^p$ - the constraint vector

and $\underline{B} \in R^m$ - the resources vector.

The constraint vector $A\underline{X} \leqslant \underline{B}$ determine a feasible set T of values for the decision vector $\underline{X}$

Or

$$T = \left\{ \underline{X} \;\middle|\; \begin{matrix} \underline{AX} \leqslant \underline{B} \\ \text{and } \underline{X} \geqslant 0 \end{matrix} \right\} \qquad (6.9)$$

Each vector $\underline{X} \in T$ determines a unique value of $\underline{Z}$ and hence there exists a set S of these feasible values

or

$$S = \{ \underline{Z} \mid \underline{X} \in T \} \qquad (6.10)$$

We can thus state the problem as

$$\text{Max } \underline{Z} = \underline{C}\,\underline{X} \text{ subject to } \underline{X} \in T \qquad (6.11)$$

<u>or</u> in its dual form

$$\text{Max } \underline{Z} = \underline{CX} \text{ subject to } \underline{Z} \in S \qquad (6.12)$$

<u>Optimality and Non-inferiority</u>

In single objective linear programming, the goal of the solution is optimality, which is uniquely defined.

The vector $\underline{X}^* \in T$ is optimal when maximizing if

$$\underline{Z}(\underline{X}^*) \geqslant \underline{Z}(\underline{X}) \quad \forall \; \underline{X} \in T \qquad (6.13)$$

*/ The notations for set are generally following the practice of Mangasarian.[86]

Strict inequality holding good for at least some $\underline{X}$.

In the multi-objective problem optimality is replaced by the notion of 'non-inferiority'.

A solution $\underline{X}_1 \in T$ is inferior if there is some solution $\underline{X}_2 \in T$ for which

$$\underline{Z}\ (\underline{X}_2) \geqslant \underline{Z}\ (\underline{X}_1) \qquad (6.14)$$

i.e.

$$\left.\begin{array}{l} z^1\ (\underline{X}_2) \geqslant z^1(\underline{X}_1) \\ z^2\ (\underline{X}_2) \geqslant z^2(\underline{X}_2) \\ \dots\ \dots \quad \dots\ \dots \\ z^p\ (\underline{X}_2) \geqslant z^p(\underline{X}_1) \end{array}\right\} \qquad (6.15)$$

where at least one of the expressions in(6.15) must be satisfied as a strict inequality.

Similarly a solution $\underline{X}_1^*$ is said to be non-inferior if condition (6.13) is obtained.

Thus the solution of multi-objective linear programming is the definition of the set of non-inferior solution.

A few words about non-inferior solution set. These points represent the same as that of 'Pareto Optima' discussed in Chapter 1. The definition of these multi-objective points of efficiency is given by Koopman. To quote[69]

> ' A possible point inthe commodity space is called efficient whenever an increase in one of its coordinates (the net output of one good) can be achieved only at the cost of a decrease in some other coordinate (the net output of another good) '

Let NI be defined as

$$NI = \{\underline{Z}(\underline{X}^*) \mid \underline{X}^* \text{ is a non-inferior solution}\} \quad (6.16)$$

The property $\underline{Z}(\underline{X}^*) \in NI$ (6.17)

is that they lie on the boundary of S. Referring to Fig. 4.1 any interior point like point A inside feasible region will be inferior to at least one boundary point. (point B and C). Not all the transformation curve shown in Fig 4.1 is however noninferior. Portion shaded, for instance, is inferior in this case.

For any two concave functions, the non-inferior set is continuous; for example, every point in the interval of maxima $\underline{X}_1^*$, $\underline{X}_2^*$ of the bicriterion problem shown in figure 6.1 is non-inferior. The non-concave functions may have, however, non-connected, noninferior points.

Social Indifference Curves

We have for each value $\underline{Z} \in R^p$, some benefit that would accrue to society from $z_1, z_2 \ldots . z_p$ units for the various objectives, 1,2.....p. This benefit is called utility and discussed in Chapter 4.4 i.e. ($u: R^p \rightarrow R^1$). The curves traced by utility are the social welfare curves.

The surface of equal utility in R^p ($u(\underline{Z})$ = constant) indicates the society's indifference for the various points on it. Thus the surface of equal utility viz; isopreference surface represents an indifference surface. The points where the social indifference surfaces are tangent to non-inferior set, called indifference band, is of importance to decision maker (DM).

An Example

To best illustrate the various aspects an illustrative example is solved below:

Let us have the objective vector $\underline{Z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$ as

$$\text{Max} \quad z_1 = 2x_1 + x_2$$

$$\text{Max} \quad z_2 = x_1 + 2x_2$$

subject to following constraint equations, i.e

$$\underline{A}\,\underline{X} \leqslant \underline{b}$$

and $-\underline{X} \leqslant 0$

i.e.

$$-2x_1 - 3x_2 + 6 \leqslant 0$$

$$+3x_1 + 2x_2 - 15 \leqslant 0$$

$$x_1 + 3x_2 - 15 \leqslant 0$$

$$-x_1, -x_2 \leqslant 0$$

The feasible region satisfying the above, i.e. the set T is shown in the decision space R^2(Fig.6.2.1). The set of noninferior solutions $\underline{X}^*$ is also shown in the same figure.

The feasible region in decision space $R^2 \rightarrow R^2$ is shown as set S in Figure 6.2.2. The non-inferior set (NI) in function space is shown NI. We find the NI is subset of S in function space and lie in the boundary; i.e NI $\subseteq$ S.

In the absence of preference information none of noninferior solutions can be considered, by analyst, to be

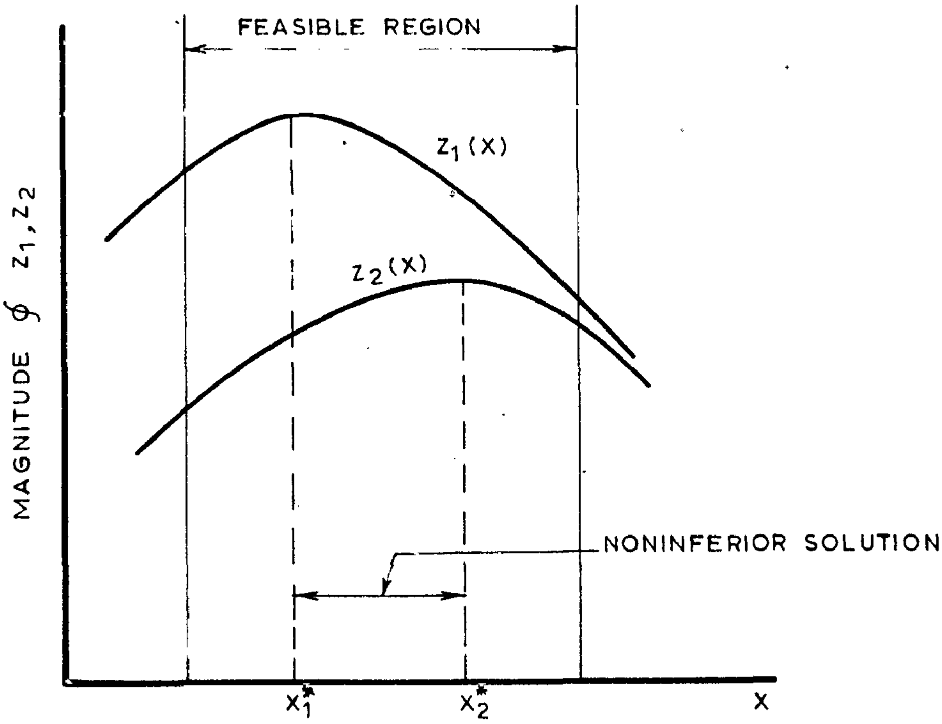


FIG. 6.1 _ NONINFERIOR POINTS-CONCAVE BI-CRITERION PROBLEM

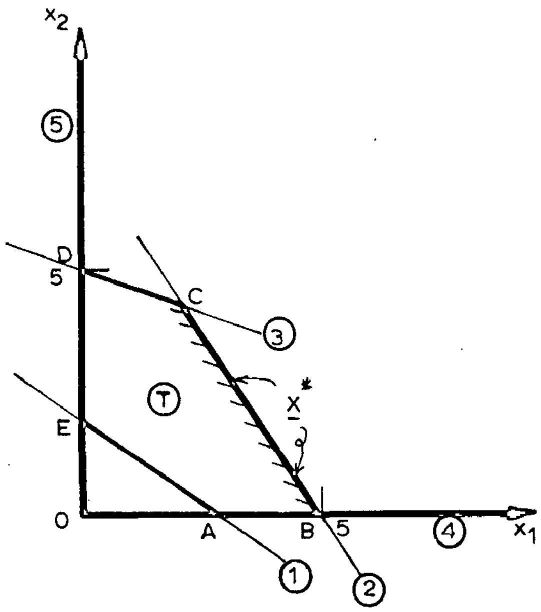


FIG. 6.2.1
DECISION SPACE

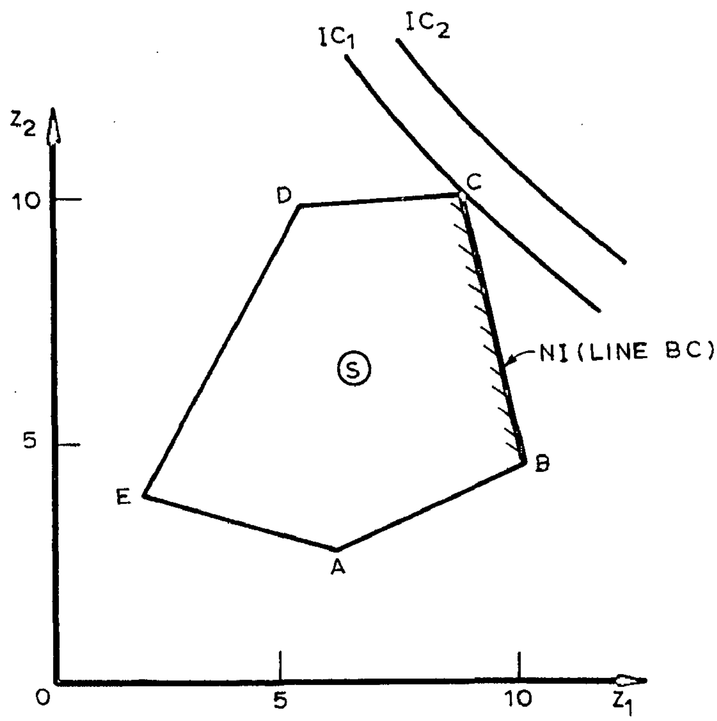


FIG. 6.2.2
TRANSFORMATION IN FUNCTION SPACE

EXAMPLE PROBLEM IN CHAPTER 6

preferable to the other non-inferior solution. However, when preferences, as indicated by indifference surfaces, are known, then, one of the non-inferior solutions can be identified as the 'best compromise solution.' *

For the example considered, suppose the indifference curves are as shown- IC_1, IC_2. It is evident that the choice of C is the 'best compromise solution'. Suppose we consider that z_2 is an objective of regional income distribution not considered as relevant to planning, then, as can be observed, B will be the solution as the indifference curves now parallel z_2 axis. This will be the case with traditional analysis for maximization of economic efficiency represented by say z_1 axis.

* Belenson and Kapur suggested the word 'best compromise solution'. As this is indicative of the significance clearly that the choice is only optimal in terms of value judgement, this word has been preferred in this study.

CHAPTER 7

MULTI OBJECTIVE SOLUTION STRATEGIES

7.1. GENERATING TECHNIQUES

7.1.1. The fundamental requirement of a solution for a multi-objedtive programming problem is the generation of non-inferior set.

The techniques utilized to generate the non-inferior set are:

(i) Weighting methods

(ii) Constraint methods

(iii) Parametric approach

(iv) Functional relation of objectives approach

& (v) Adaptive search methods.

7.1.2. Weighting Methods

Weighting methods were the premier solution techniques available to solve the problem of vector optimization. In this methodology, the approach is to generate the non-inferior set by the transformation of vector function of non-commensurate objective to a scalar valued function. The transformed problem is solved to yield a point $\underline{Z}^{*} \in NI$. By varying the parameters used in transformation, attempt to generate a large number of points is made.

7.1.2.1. Kuhn & Tucker[71] Method

The origination of vector optimization method is attributed to the above authors. They developed the mathematical solution for the problems presenting the necessary conditions.

Restarting our problem;

$$\text{Max } \underline{Z} = \underline{C}\,\underline{X} \qquad (7.1)$$

subject to

$$g_i(\underline{X}) \leqslant 0 \qquad i = 1,2 \ldots m \qquad (7.2)$$

$$-x_j \leqslant 0 \qquad j = 1,2 \ldots n \qquad (7.3)$$

The Langrangian form of the above expression can be denoted as

$$L = \sum_{k=1}^{p} w_k\, z_k(\underline{X}) - \sum_{i=1}^{m} \lambda_i\, g_i(\underline{X}) \qquad (7.4)$$

where $\underline{X} \in T$

Alongwith the presentation of conditions for optimality for scalar optimization problems, Kuhn-Tucker gave in their work, the requisite conditions for non-inferiority of the above problem. If a solution to the vector optimization problem in (7.1) - (7.3) is non-inferior, then there exist[15]

$w_k \geqslant 0$ where $k = 1,2 \ldots \ldots p$, such that

w_k is strictly positive ..

for some values say w_r, where

$r = 1,2,\ldots\ldots p)$ (7.5)

& $\lambda_i \geqslant 0$ where $i = 1,2\ldots\ldots m$ (7.6)

such that

$$\lambda_i g_i(\underline{X}) = 0 \qquad i = 1,2\ldots\ldots m, \qquad (7.7)$$

and $\underline{X} \in T$ (7.8)

$$\text{and} \quad \sum_{k=1}^{p} w_k \nabla z_k(\underline{X}) - \sum_{i=1}^{m} \lambda_i \nabla g_i(\underline{X}) = 0 \qquad (7.8.a)$$

Apart from the above necessary conditions, the sufficient conditions for the NI solutions are that all of $z_k(\underline{X})$ be concave ($k = 1,2\ldots\ldots p$) and $\underline{X}$ be a convex set.[62,91]

7.1.2.2. Further works on weighting techniques transform the multi-dimensional vector optimization problem into a scalar optimization problem. This approach is attributed to several authors viz Zadeh[132], Savir[111], Geoffrion[39,40] and Kapur[68].

The problem

$$\text{Max} \sum_{k=1}^{p} w_k z_k(\underline{X}) \qquad (7.9)$$

subject to $\underline{X} \in T$

can be transformed to a scalar as below:

$$\text{Max} \; w_r z_r(\underline{X}) + \sum_{\substack{k=1 \\ k \neq r}}^{p} w_k z_k(\underline{X}) \qquad (7.10)$$

subject to $\underline{X} \in T$

where $w_k \geqslant 0$ for all k and strictly positive for at least one of the objectives. Conventional optimization

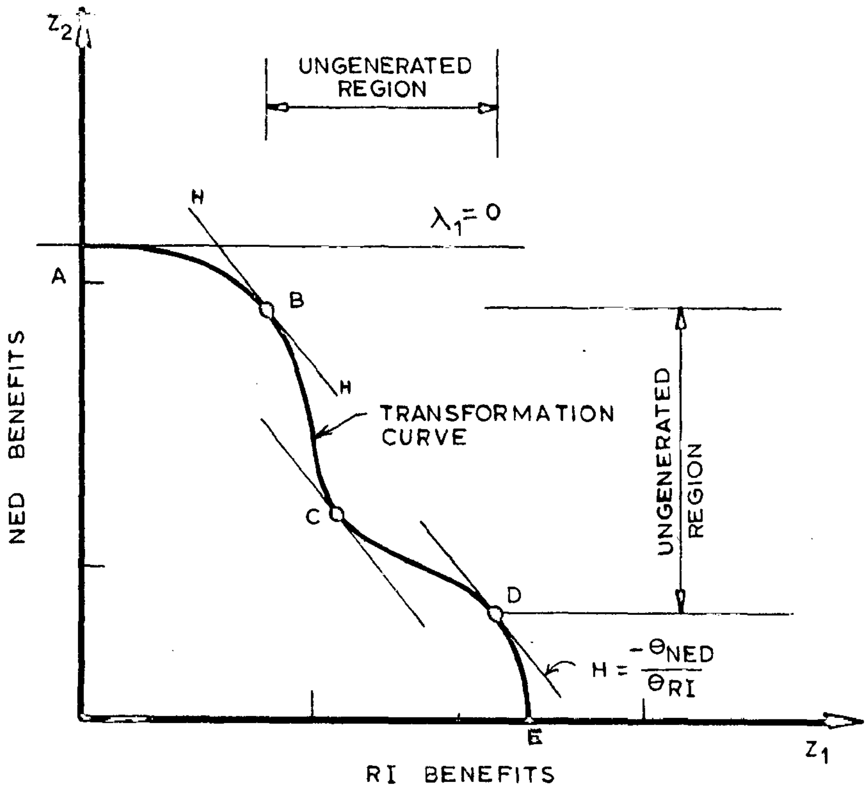


FIG.7.1 _ FEASIBLE SET IN FUNCTION SPACE AND NONINFERIOR SOLUTIONS INDICATING NATURE OF DUALITY GAP

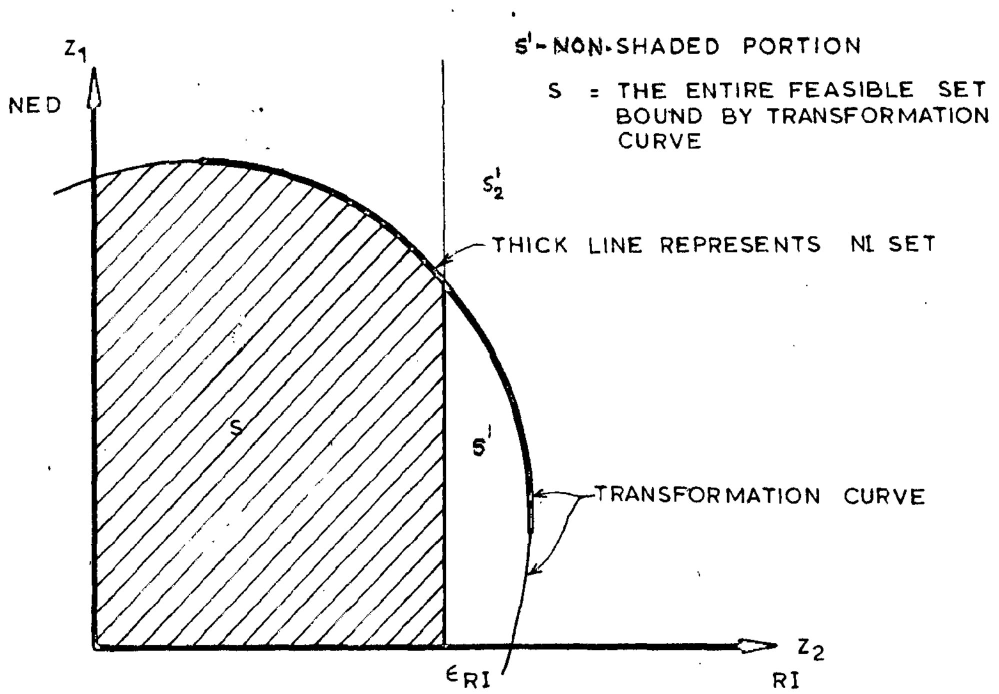


FIG.7.2 _ THE CONSTRAINT APPROACH - A GEOMETRIC REPRESENTATION

can now be adopted as (7.10) is a scalar.

7.1.2.3. Usually one of the w_k, say w_r, can be selected to be equal to 1. This as numeraire, all other objectives can then be weighted i.e. $\frac{w_k}{w_r}$ or λ_k. Example cases are in Chapter 4.

7.1.2.4. Successive variation of weights w_k in the objective function will yield the non-inferior set, as in example (Fig. 4.2) in Chapter 4. The generation of non-inferior solution is difficult with cases where the problems of duality gap arise. This can be seen from figure 7.1, where a hypothetical feasible set for a dual objective case is shown. The weighting methods described in this case, would yield the non-inferior sets for points A to B and D to E shown in the curve in Fig.7.1, but not the solutions for portions between B to D. Thus a basic requirement for the application of weighting methods for generation of non-inferior set is that the NI set must be convex.

The problem as regards to duality gap is best avoided by the constraint method.

7.1.3. Constraint Method

7.1.3.1. An earlier reference of this method has been given in Chapter 2. In this method, we are required to specify a pre-assigned "attainment levels" for each of the various objectives than one, which we wish to maximize. Let the attainment levels thus desired be $\epsilon_1, \epsilon_2, \ldots \epsilon_k \ldots \epsilon$

each objective except one ($k = 1,2\ldots.p$ & $k \neq r$)

The problem is

$$\left.\begin{array}{l} \text{Max } z_r(\underline{X}) \\ \text{subject to } \underline{X} \in T \\ z_k(\underline{X}) \geqslant \epsilon_k \quad \text{for } k = 1,2\ldots\ldots p \\ \qquad\qquad\qquad\qquad k \neq r \end{array}\right\} \quad (7.11)$$

7.1.3.2. Let us examine the utility aspect of the above. It is tantamount to saying that benefit to society from k^{th} objective ($k = 1,2\ldots.p$ & $k \neq r$) is constant as long as the level ϵ_k is not reduced, but indefinitely harmful below this level. Expressing in equations, we have the utility function as

$$u_k \left\{z_k(\underline{X})\right\} = \begin{cases} -\infty; & z_k(\underline{X}) < \epsilon_k \\ \text{constant}; & z_k(\underline{X}) \geqslant \epsilon_k \end{cases} \quad (7.12)$$

7.1.3.3. We can deduce the formulation of the constraint approach from the equation giving Kuhn-Tucker condition,[15] indicated above.

We have, from equation (7.4) above expressed in Lagrangian form

$$L = w_r z_r(\underline{X}) + \sum_{\substack{k=1 \\ k \neq r}}^{p} w_k \, z_k(\underline{X}) - \sum_{i=1}^{m} \lambda_i g_i(\underline{X}) \quad (7.13)$$

therefore

$$dL = w_r \nabla z_r(\underline{X}) + \sum_{\substack{k=1 \\ k \neq r}}^{p} w_k \nabla z_k(\underline{X}) - \sum_{i=1}^{m} \lambda_i \nabla g_i(\underline{X}) = 0 \quad (7.14)$$

Since only relative values of the weights are of significance, the r^{th} objective can be selected as the numeraire so that $w_r = 1$. Then the above equation becomes

$$\nabla z_r(\underline{X}) + \underbrace{\sum_{\substack{k=1\\k\neq r}}^{p} w_k \nabla z_k(\underline{X})} - \sum_{i=1}^{m} \lambda_i \nabla g_i(\underline{X}) = 0 \qquad (7.15)$$

The portion underlined in the above equation(7.15) represents a weighted sum of the gradients (p-1) lower bound constraints, since there is a plus sign before the summation. The above deduces the problem in terms of equation viz

$$\text{Max} \quad z_r(\underline{X}) \quad \text{subject to } \underline{X} \in T \text{ and } z_k(\underline{X}) \geqslant \epsilon_k \quad \text{for } \forall\, k;\ k \neq r \qquad (7.16)$$

Since this is scalar, the problem is solvable by the usual non-linear programming techniques.[62]

7.1.3.4. To generate the entire range of non-inferior solutions, parametric variations of ϵ_k in equation (7.16) is done and computations proceeded with. For this purpose, we can set ϵ_k at zero or at some predetermined value and then increase incrementally until the solution becomes infeasible. At every value of ϵ_k thus assumed, the problem is solved to yield a non-inferior point.

7.1.3.5. We also find that there exists a dual variable (shadow price) λ_k associated with the constraints $z_k \geqslant \epsilon_k$ in equation (7.11). At every solution point of equation, the value of λ_k thus found, when substituted in equation (7.10) of the weighting method discussed in para (7.1.2.2) would yield the same solution.

7.1.3.6. Geometrically the approach of additional constraints reduces the feasible decision space T or equivalently the feasible solution space S.

If $$T_k^r = \left\{ \underline{X} \;\middle|\; z_k(\underline{X}) \geq \epsilon_k;\ \text{for } k = 1,2 \ldots p,\ k \neq r \right\} \quad (7.17)$$

then

$$S_k^r = \left\{ z_k(\underline{X}) \;\middle|\; \underline{X} \in T_k^r;\ \text{for } k = 1,2 \ldots p,\ k \neq r \right\} \quad (7.18)$$

and $$S^r = S \cap S_1^r \cap S_2^r \ldots \cap S_k^r \cap \ldots \cap S_p^r \quad (7.19)$$

where $k \neq r$

and the problem is thus to evaluate

$$\text{Max } z_r(\underline{X}); \quad \text{subject to } \underline{z} \in S^r \quad (7.20)$$

7.1.3.7. In other words, each constraint $z_k(\underline{X}) \geq \epsilon_k$ for (p-1) values of k (except r) define the half space in R^p on the positive side of a hyperplane perpendicular to z_k axis at $z_k(\underline{X}) = \epsilon_k$. The intersection of all of these half spaces with S gives the new feasible space S^r.

For the dual objective case, we can see this in Fig. 7.2, where $S^1 = S \cap S_2^1$ where S^1 is the half plane to the right of $z_2 = \epsilon_{R1}$.

7.1.3.8. The advantages of the constraint methods are:

(i) it is preferable to weighting methods as it does not require the convexity of non-inferio set.

(ii) operational considerations favour a constraint approach rather than the weighting method of approach: this is because the weights vary differently at different levels.

(iii) parameterisation of the constraint coefficients are easier and straight forward than parametrically varying weighted coefficients of the objective function.

However, the problems associated with this procedure are two-fold:

(i) determination of the maximum level for varous objectives

(ii) adoption of the particular order of preference for solution as in (i) above.

7.1.4. Parametric Approach[38]

7.1.4.1. The weighting methods and its dual, constraint methods constitute the parametric approach. It is assumed that

(i) the relative utilities of all the objectives are well established

& (ii) the relative utilities are constant at all levels.

7.1.4.2. The problem is stated as

$$\text{Max} \quad z = \sum_{k=1}^{p} \theta_k z_k(\underline{X}) \quad \text{subject to } \underline{X} \in T \qquad (7.21)$$

where $\theta_k > 0$, $k = 1, 2 \ldots . p$ are the weighting coefficients determined according to relative importance of objectives, and can be denoted by a vector

$$\underline{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ .. \\ \theta_k \\ \theta_p \end{bmatrix} \qquad (7.22)$$

The θ_k can be normalized so as to obtain

$$\sum_{i=1}^{p} \theta_k = 1 \qquad (7.23)$$

7.1.4.3. For a geometric interpretation let us denote set H

$H = \{ \underline{Z} \mid \underline{\theta}^T \underline{Z} = c \}$ where c is a constant.

This will be a hyperplane in R^p with outward normal denoted by $\underline{\theta}$. The maximization of $\underline{\theta}^T \underline{Z}$ can be viewed as moving this hyperplane H with fixed $\underline{\theta}^T$ in a positive direction as far as possible such that $H \cap S$ is non-null. This maximum will normally occur where H is tangent to S.

7.1.4.4. For a dual objective case, this is illustrated in Fig. 7.1. H is a line in this case with slope $-\frac{\theta_{NED}}{\theta_{RI}}$: the maximum of this occurs at point B, whichis the "best compromise solution".

7.1.5. <u>Derivation of Functional Relationship Method</u>

7.1.5.1. The main contribution to this field is due to Reid & Vemuri. They have shown that for a certain class of problems a functional relationship between each objective and a set of weights on all objectives could be derived. When these relationships can be established, the value of any objective can be found by simple calculations. The selection of weights for each of the objectives is postponed until after the performance characteristics of the problem are well understood. The procedure also generates a set of equally viable solutions which are non-inferior as in the other cases "Generating Technique Methodologies".

7.1.5.2. The 3 basic features of the algorithm are as follows[125]

(i) performance of an unconstrained optimization in terms of an auxiliary performance index $\tilde{Z}$ rather than $\underline{Z}$ itself.

(ii) the auxiliary index $\tilde{Z}$ must be a positive polynomial and if so

(iii) the non-inferior solution $\underline{Z}^* = (z_1^*, z_2^* \ldots z_p^*)^T$ can be related to the optimum auxiliary index $\tilde{Z}^*$ via a simple functional form involving weightage coefficients.

The main requirement for the application of this technique is that the required objectives be expressible as a "product of n terms" such as "Cobb-Douglas Type" viz;

$$z_k(\underline{X}) = \prod_{j=1}^{n} (x_j)^{b_{jk}} \quad \ldots\ldots \forall \ k \qquad (7.24)$$

where $x_j > 0$

and b_{jk} is a real number.

This equation has the special property of derivation of the functional relationship between $z_k(\underline{X})$; w_k for all values of k. Reid & Vemuri have preferred the use of Cobb-Douglas function for the functional relationship because of the fundamental duality between these functions and cost functions.[125]

The second step is to define an auxiliary scalar index $\tilde{z}$ as

$$\tilde{z} = \sum_{k=1}^{p} \alpha_k \, z_k(\underline{X}) \quad \text{where} \quad \alpha^k \geqslant 0 \qquad (7.25)$$

$$= \sum_{k=1}^{p} \alpha_k \prod_{j=1}^{n} (x_j)^{b_{jk}}$$

Thus the use of Cobb-Douglas types of functions in equation leads to an auxiliary index that is a posynomial.

We have the necessary condition for maximization of $\tilde{z}$ is

$$\frac{\partial \tilde{z}}{\partial x_r} = \sum_{k=1}^{p} \alpha_k \, b_{rk} (x_r)^{b_{rk}-1} \left[\prod_{\substack{j=1 \\ j \neq r}}^{n} (x_j)^{b_{jk}} \right] = 0$$

$$\text{subject to} \quad x_r \geqslant 0 \qquad (7.26)$$

The solution of the above equation defines the value of x_r that maximizes $\tilde{z}$.

The above equation can be reduced to the following form after denoting the maximum values by an asterisk:

$$\frac{1}{x_r^*} \sum_{k=1}^{p} b_{rk} \, \alpha_k \, z_k^* \, (x^*) = 0 \qquad (7.27)$$

By substituting $x = x^*$ in Equation (7.25) and dividing throughout by $\tilde{z}^*$, we have

$$\sum_{k=1}^{p} w_k = 1 \qquad (7.28)$$

where the performance weights have been defined as

$$w_k = \frac{\alpha_k \, z_k^* \, (x^*)}{\tilde{z}^* \, (x^*)} \qquad (7.29)$$

In view of the relation (7.28), the maximum value of $\tilde{z}^*(x^*)$ may be written as

$$\tilde{z}^* \, (x^*) = \prod_{k=1}^{p} \left[\tilde{z}^*(x^*) \right]^{w_k} = \prod \left[\frac{\alpha_k \, z_k^* \, (x^*)}{w_k} \right]^{w_k}$$

by virtue of equation (7.29).

or
$$\tilde{z}^*(x^*) = \prod_{k=1}^{p} \left(\frac{\alpha_k}{w_k}\right)^{w_k} \qquad (7.31)$$

which is the parametric representation of $z^*(x^*)$ in terms of cost coefficients α_k and the weighting constants for k = 1,2,3....... p.

With the help of equation (7.29) we can now determine the non-inferior set, in terms of α_k, w_k as

$$z_k^*(x^*) = \left(\frac{w_k}{\alpha_k}\right) \prod_{r=1}^{p} \left(\frac{\alpha_r}{w_r}\right)^{w_r} \quad \forall k \qquad (7.32)$$

or
$$z_k^*(x^*) = C\left(\frac{w_k}{\alpha_k}\right) \prod_{r=1}^{p} (\alpha_r)^{w_r} \quad \forall k \qquad (7.33)$$

Thus we now determine the elements of non-inferior set (as the auxiliary performance vector is now not in the above equation). The final equation is found to be a function of 'α', the cost coefficient.

The disadvantages of this method are:

(i) the specific form of representing objectives as a product of n terms of decision variables is difficult for water resource planning problems.

(ii) there is a necessity to differentiate each objective with respect to each variable. For water resource planning problems the objectives could be many and variables are considerably large and differentiating each objective with respect to each variable and subsequent solution of the set of simultaneous equation is computationally infeasible.

(iii) the method limits itself to unconstrained optimization and hence unsuitable for water resources planning.

Thus though the method is powerful to give results for any range of weights by simple computations, the idea is difficult to apply for water resource problem due to the large number of variables involved. A typical example for a small scale problem, adopting this approach has been demonstrated by Vemuri.[125] However, for developing the functional relationship certain heuristic assumptions have been necessary. This points out the need for further works in establishing such functional relationships before the method could be applied to real world problems as in water resource projects.

7.1.6. <u>Adaptive Search Method</u>

7.1.6.1. This method aims at determination of non-inferior values and is due to Beeson & Meisol.[10]

7.1.6.2. The method starts from an initial non-inferior solution to approximation of other non-inferior solutions.

It is initially assumed that $\underline{X}_0$ will be non-inferior in decision space T. The problem is

$$\text{Max} \quad \underline{Z}\,(\underline{X}_k) \qquad \text{subject to} \quad \underline{X} \in T \qquad (7.34)$$

The new iterative solutions are generated by the following recursive formula, the direction of search being determined by the gradients of objective function.

$$\underline{X}_{k+1} = \underline{X}_k - a_k\, J^T\, (\underline{X}_k)\;\; \underline{w}_k + \underline{c}_k \qquad (7.35)$$

where
- a_k controls the step size
- J is the Jacobian matrix of partial derivative of the objectives with respect to decision variables.
- w_k determines direction control
- & $\underline{c}_k$ controls the feasibility of the solution.

The search is restarted a large number of times to have an adequate coverage of the non-inferior set.

Leytham[75] has applied this technique to water resources problems. He has combined the steepest ascent and a modified version of pattern search that use the gradient estimates and incorporated it in a simulation model (Chap.8.2.3) to optimize an example water resource system, following the strategy given in Fig. 7.3.

7.1.7. Marglin[88] is the first to introduce weighting methods. Major proposed the concept of "Grand Benefit-Cost Analysis" extending the approach to water resource planning and project selection.[80] In both their approach to

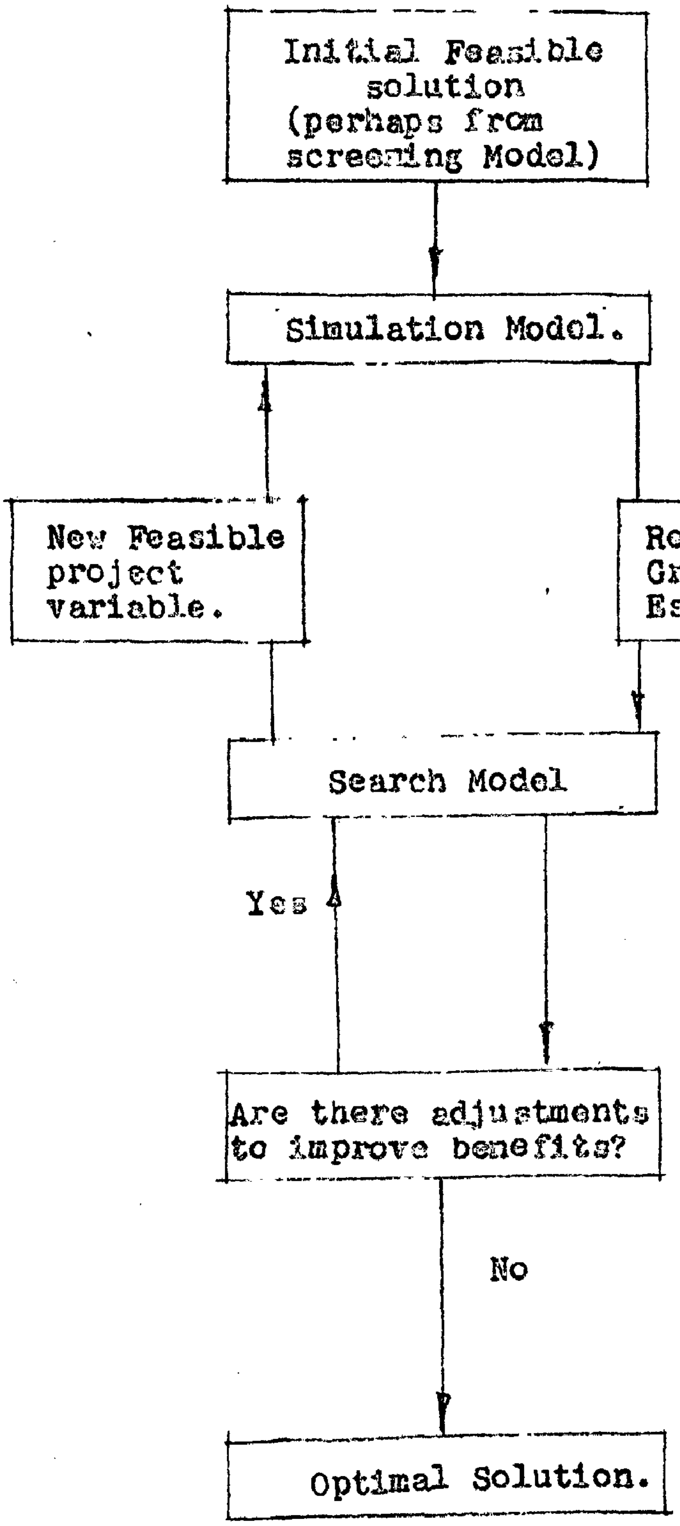


Fig. 7.3. Information flow for the conjunctive use of simulation model and search technique.(73)

the weighting and constraint method, however, a socially optimal single solution is possible as the weightage factors are assumed to be "known". In other words, Marglin and Major methods do not yield the generation of whole range of non-inferior set. The UNIDO's approach to discover the switching values,[118] (Chap.4) is a tacit version to define a constant weight. This comes also, therefore, under the weighting class. Parametric approaches assume constant weighting factors for the objectives. It requires a careful predetermination of "price",using value judgement of each objective, for all possible combination of levels of attainment. This particular aspect, implicit in Major's semantic work[80] faced severe but valid criticism from Freeman and Haveman,[36] and also by Vaut & Willis[124].

The changes in objective levels, either expected or proposed, are trivial from a national view point, unless there are intentions for major changes in the objectives itself. As water resource planning problems are of national nature, the pressumption of constant weight for different objectives, at least in a "partial-equilibrium analysis" may be considered as valid. Thus the methods of weighting techniques or constraint approach provide a broad scope to deal effectively multi-objective analysis problem as shown inChapter 4.

7.2. MULTI OBJECTIVE SOLUTION TECHNIQUES THAT RELY ON PRIOR ARTICULATION OF PREFERENCES

7.2.1. A general feature of the techniques described under this class is that the computational effort for deriving a solution is the least, as they do not assign to themselves the generation of complete non-inferior set as in Section 7.1. Only a part or even a single computation may lead to a solution of the problem. This facility is due to the fact that prior articulation of preferences reduces the work yielding direct solutions.

7.2.2. Lexicography

In Chapter 5 on multi-objective decision-making we had examined this approach in a brief way. Basic to this method is a ranking of objectives in the order of importance by Decision-Maker (DM). The method then obtains the "best compromise solution" by maximizing as many of the objectives as possible simultaneously, starting with the most important objective followed by the other objectives in the hierarchical order.

Assuming the order of objectives are z_1, $z_2 \ldots z_p$, we can state the problem, thus

$$\text{Max} \quad z_1(\underline{X}) \qquad \text{subject to} \quad \underline{X} \in T \tag{7.36}$$

the solutions being obtained being the all possible ones. Let it be denoted by set S_1. Then the next most important objective z_2 ($\underline{X}$) is maximized subject to $\underline{X} \in S_1$ to find solution set S_2

i.e

$$\text{Max} \quad z_2(\underline{X}) \quad \text{s.t} \quad \underline{X} \in S_1 \tag{7.37}$$

The process is repeated until all the p objectives have been considered. If the solution set S_k at the k^{th} iteration has only one element, then this will be the solution to the entire problem and the remaining objectives ranked less important than z_k are ignored.

The main advantage of this approach is that it reflects the logical manner in which solutions for similar problems are solved by us usually. However, there are two major disadvantages with this approach viz;

(i) the solutions are very sensitive to ranking and the ranking needs to be done carefully.

(ii) when the objectives are of equal importance, the arbitrary ranking, if not done properly, would result in inferior solutions being chosen.

Waltz[127] has proposed a variation of this method which accounts for small variations in levels of attainment of the objectives maximized while maximizing the subsequent objective in order of ranking.

7.2.3. Goal Programming

The pre-requisite for this method is that the Decision-Maker sets goals that he would like to attain in each of the objectives. The Goal Programming is aimed at reducing to the minimum the weighted absolute deviations from the set targets for the various objectives.

Let $\hat{\underline{Z}}$ be the vector of goals set for the objectives Then, the problem is formulated as

$$\text{Min} \quad \left\| \underline{Z} - \hat{\underline{Z}} \right\| \qquad (7.38)$$

subject to $\underline{X} \in T$

where $\|\cdot\|$ denotes any norm. Important to observe is that the goal vector $\hat{\underline{Z}}$ need not be in the feasible set S. In fact if the goal vector is inside feasible region then it may yield an inferior solution, as will be seen subsequently.

This method is due to Charmes and Cooper[16] who applied it to linear problems. Using the sum of absolute values of the deviations as the norm they keep the problem linear by defining vectors of slack variables $\underline{d}^+ \geqslant \underline{0}$ and $\underline{d}^- \geqslant \underline{0}$ such that

$$\underline{Z} - \hat{\underline{Z}} = \underline{d}^+ - \underline{d}^- \qquad (7.39)$$

$\underline{d}^+$ represents the vector of $d_1, d_2 \ldots d_p$ where d_k^+ represents the over-attainment of k^{th} objective. Similarly d_k^- represents the under-attainement and $\underline{d}^-$ represents the vector of underattainment values.

The problem then is formulated as

$$\text{Min} \sum_{k=1}^{p} d_k^+ + d_k^- \qquad (7.40)$$

$$\text{subject to} \quad \underline{Z} - \hat{\underline{Z}} = \underline{d}^+ - \underline{d}^- \qquad (7.41)$$

$$\text{and} \quad \underline{X} \in T$$

Goal Programming can also consider the aspect of priorities between objectives, say p_k for k^{th} objectives. If the objective r is identifies as a high-priority objective, implying thereby

$$p_r \ggg p_k \qquad (7.42)$$

the consideration in analysis is simple. The objective function of the Goal Programming for equal importance between objectives vide equation (7.40) above can be reformulated as

$$\text{Min} \sum_{k=1}^{p} p_k \,(d_k^+ + d_k^-) \qquad (7.43)$$

to include preferences for objectives.

Another extension of this method is "mean square approach", and has been adopted by Salukvadze[109]. This assumes that k^{th} component of goal vector $\hat{\underline{Z}}$ will be $\bar{Z}_k$ the maximum value of $Zk(\underline{X})$ subject to $\underline{X} \in T$ and uses a least square norm.

The advantages of Goal Programming are:

(i) it is computationally efficient to other methods

(ii) it is very useful in multi-objective decision making, especially in private sector where the decision maker fully comprehends the system and rationally decides the priorities and targets for the different objectives.

The disadvantages of the method are:

(i) the method demands explicit value judgement from the Decision-maker even prior to evaluating alternatives.

(ii) Public sector problems are complex; for water resource system planning, for instance, it is difficult to set a target without analysis for national objectives and other objectives. In fact the relative levels of social objectives and not the absolute levels, are of importance in these problems. Absolute quantification of targets and priorities with no knowledge of the feasible trade-off embodied in the non-inferior set is indeed a problem.

(iii) it is also necessary that a wide coverage of sensitivity analysis is done to establish the non-inferiority of the solution obtained by Goal-Programming.

The aspect (iii) above is apparent from Fig.7.4. In this dual objective axis, let z_{NED} and z_{RI} be the two axes as shown. Let the set target for each of the objectives, as above, be, z_{NED} and z_{RI}, as shown to scale in the graph. Assuming that the order of assigned priorities are to RI and NED respectively i.e. $p_{RI} >>> p_{NED}$. If $\hat{z}_{RI(1)}$ is the goal set, then the solution yielded will be A. This is due to the fact that the programme minimizes $|\hat{z}_{RI} - z_{RI}|$ to begin with, and at A, this is the least. So long as the goal set for RI objective is $\geq$ to $z_{RI(3)}$ (with goal set for NED at $\hat{z}_{NED}$), the programme will yield a solution in the non-inferior set. But if the goal is set at $\hat{z}_{RI(3)}$ as shown with the other goal at $\hat{z}_{NED}$, we would obtain an inferior solution. Hence the necessity for sensitivity analysis shown in (iii) above.

(iv) This method also suffers from the duality gap problem (para 7.1.2.4).

David[23] has utilized this method for water resource planning considering purposes such as water requirement, Flood protection etc. as objectives and solution ranking done through the "Electre" algorithm (para 7.2.6).

7.2.4. Goal Attainment Method

This method is slightly different from the goal programming method. In this method, the attempt is to obtain the goal vector $\hat{Z}$ in such a way to allow the other components to vary within certain bounds or tolerances.

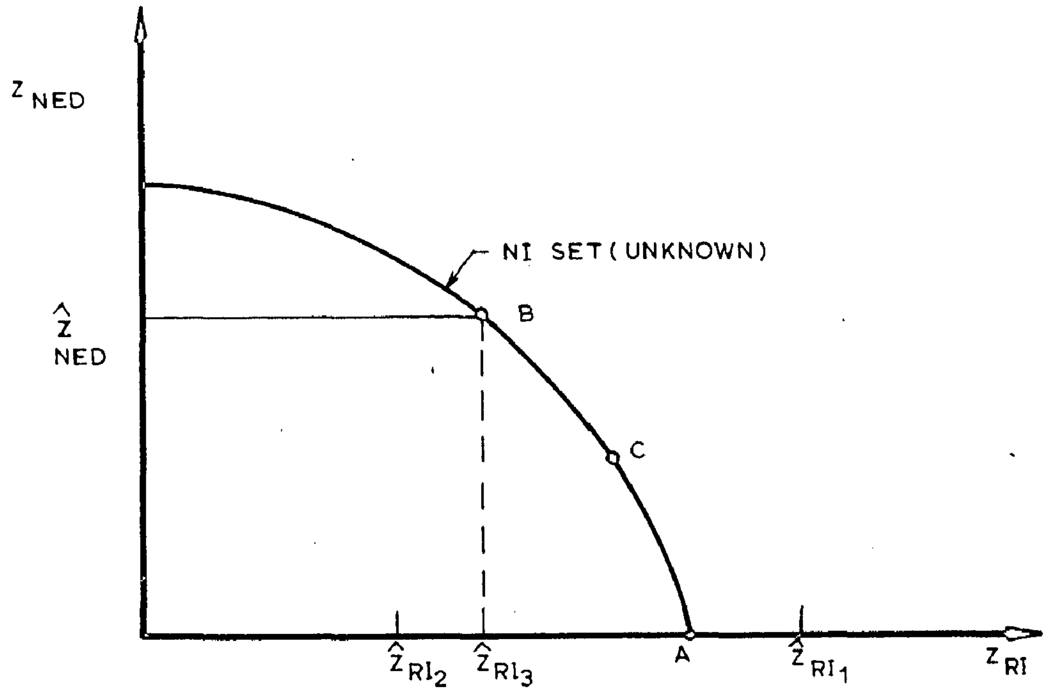


FIG. 7.4 _ FUNCTION SPACE SHOWING NI SOLUTION SET AND THE GOALS $\hat{Z}$ FOR EACH OBJECTIVE

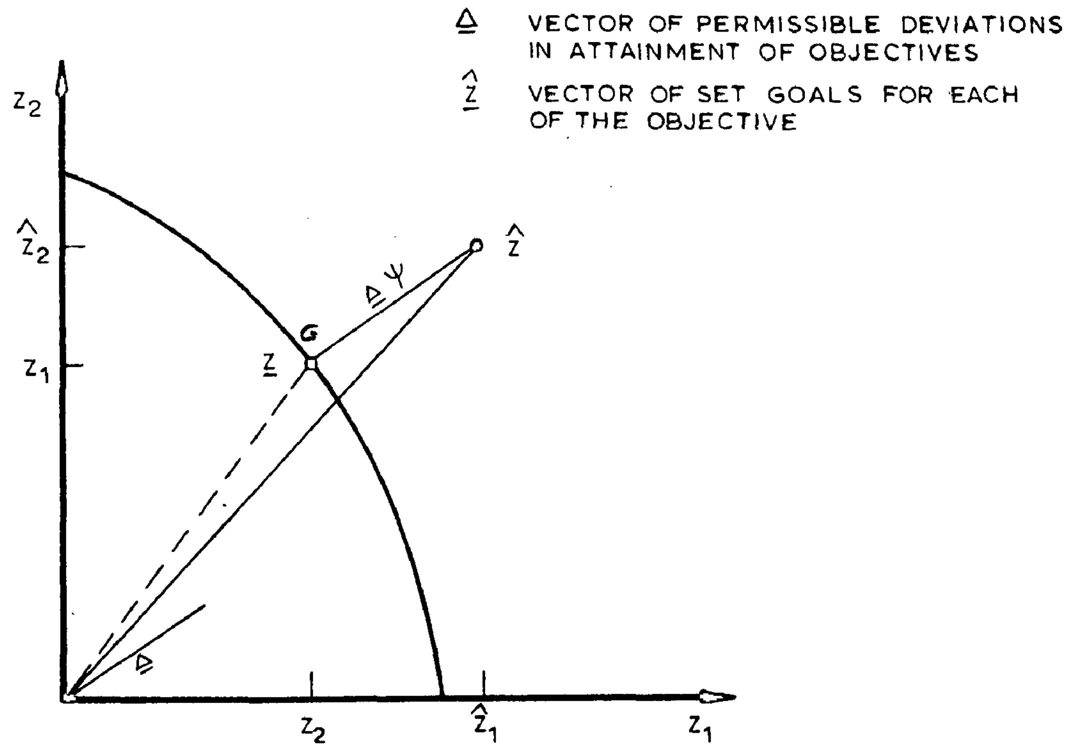


FIG. 7.5 _ GOAL ATTAINMENT METHOD (FUNCTION SPACE REPRESENTATION)

Let $\underline{\Delta}$ be a vector indicating the permissible positive or negative deviation of the desired goals.

Basic to the problem is, then, the determination of

(i) the attainment levels for different objectives or the goal vector $\hat{\underline{Z}}$

(ii) the permissible deviation for each objective or the deviation vector $\underline{\Delta}$

The problem is

$$\text{Min } \psi$$

$$\text{subject to } \underline{Z} + \underline{\Delta}\psi \leq \hat{\underline{Z}} \qquad (7.44)$$

$$\underline{X} \in T$$

$$\& \quad \underline{\Delta} \geq \underline{0} \qquad (7.45)$$

where ψ is a scalar variable of unrestricted sign. Normally $\underline{\Delta}$ is normalized so that $\sum_{k=1}^{p} \delta_k = 1$.

For a dual objective case, the problem is shown in Figure 7.5. $\underline{\Delta}$ and $\hat{\underline{Z}}$ fix the directon of solution vector $\hat{\underline{Z}} - \psi\underline{\Delta} = \underline{Z}$. The minimum of ψ occurs at G for the case $\underline{X} \in T$ & $\underline{Z} \in S$

$$\text{or } \underline{Z} \in NI \subset S. \qquad (7.46)$$

The advantages and disadvantages of this approach are similar to the one stated for Goal Programming. In addition to deciding $\hat{\underline{Z}}$, the Decision Maker is to determine $\underline{\Delta}$ also in this method, prior to having a knowledge of the feasible solutions of NI class.

7.2.5. Utility Functions & Optimal Weights

7.2.5.1. Utility Functions

The concept of utility function was brought out in Chap. 4.4 and also discussed in Chap. 5.5.1 as a tool for decision-making process. As was brought out therein, the 'best compromise solution' for multi-objective problem is definable as the point at which social indifference curve or the contours of equal utility, is tangential to the non-inferior set (or the transformation curve).

A direct derivation of the benefit-cost solution bypassing the generation of the entire NI set in most cases is also available and is due to Geoffrion[39]. In this case, the problem is as follows:

$$\text{Max} \quad U\left[(\underline{Z})\right] \quad \text{subject to } \underline{X} \in T \qquad (7.47)$$

where $U(z_1)$, $u(z_2)$ etc. represented utility due to k^{th} obj $\forall k$ & $U\left[(\underline{Z})\right]$ indicates total utility.

Restrictions as below are necessary in such a case:

(i) the utility function shoud be monotonically increasing & ordinal

(ii) it should be preferably quasi-concave

(iii) the total utility should satisfy the direct additive weighting (para 5.2.2).

7.2.5.2. Optimal Weights

As was brought out elsewhere, the line which passes through the point of tangency of non-inferior set and social indifference curve is a substitute for preferences; the slope of this line is proportional to the ratio of the weights of objectives. Marglin[90], Major,[80], Sen,[118] Das Gupta[118] designated the ratios thus derived as optimal weights. They interpreted this to be the relative value that the society holds for these objectives.

If the optimal weights $\underline{w}^*$ is known, then the "best compromise solution" can be obtained by solving

$$\text{Max} \quad \underline{Z}\,(\underline{X} : \underline{w}^*) \quad \text{subject to } \underline{X} \in T \qquad (7.48)$$

$$\text{or Max} \quad \sum_{k=1}^{p} w_k\, z_k\,(\underline{X}) \quad \text{subject to } \underline{X} \in T \qquad (7.49)$$

This is the same as in equation (7.9) with the difference that the optimal weights are provided by Decision maker to analyst in this case. The computational burden is therefore the least. In the earlier case of Generating Techniques (para 7.1.2.2), the attempt was to generate the whole non-inferior set with varying w_k (weights), since w_k were unknowns in that problem.

7.2.5.3. The advantages of the above method are:

(i) It considers trade-offs explicitly.

(ii) Since the optimal weights or the utility function (social indifference curve) is known in the zone of interest, the need to generate the entire non-inferior set is avoided. Even only one solution is adequate though sensitivity analysis on weights would be desirable.[19]

The disadvantages of this method are:

(i) the explicit quantification of trade-off does not insure optimality of decision when the information based on which decisions are made are insufficient.

(ii) the decision makers is to articulate value judgements (on weights of utility function) and this is to be done without full details of analysis or the results thereof.

(iii) the deviation of optimal weights even though based on past decision is still open to doubt.

7.2.6. Other Approaches

An important approach of recent origin is to generate a partial ordering of non-inferior solution by a method known as "ELECTRE". This method is due to Roy[108]. An out-ranking relationship 'R' has been proposed which is analogous to Decision Maker's preference ordering but without the need for transitivity. The method starts with an available set of non-inferior solution $\underline{X}^*$ and denotes to build a ranking with the Decision-Makers value judgement.[19] The use of "Electre" in water resource planning problem is found in David.[23]

7.3. INTERACTIVE SOLUTION TECHNIQUES

7.3.1. The techniques in Section 7.2 required articulation of prior preferences. It is apparent that this type of value judgement may not be appropriate in public decision making process as it calls for a previous determination of the weights or societal indifference trade-offs between objectives.

In this section we attempt to review the class of solution techniques which are interactive. In other words it means that a progressive articulation of preferences of the DM will be the aim of these solution strategies, and the preferences thus sought are by placing before the DM the results of analysis carried out thus far.

The common algorithm applicable to these solution techniques can be depicted as shown in Fig. 7.6.

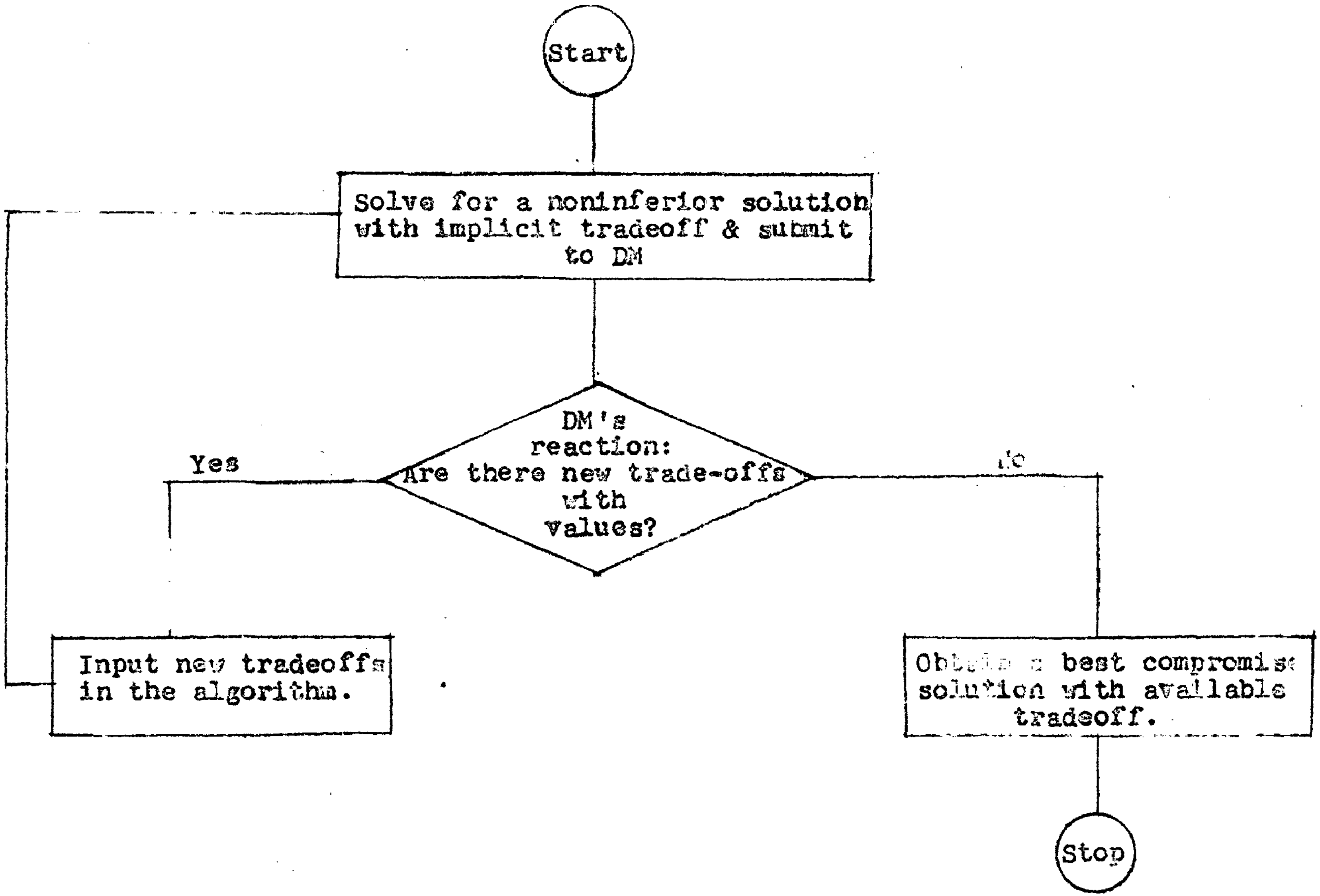


Fig. 7.6 - Common Algorithm of Interactive Solution Techniques.

Though there have been numerous methods available under this class also, we restrict the review to three common methods viz;

(i) Stem Method (1971)

(ii) SEMOPS Method (1973)

& (iii) SWT method (1974)

7.3.2. Step Method "STEM"[11]

7.3.2.1. Basic to the step method is the construction of a 'payoff table' shown in Table 7.1. This is done by solving a problem as below:

$$\text{Max. } z_k(\underline{X}) = \sum_{j=1}^{n} c_j^k \; x_j \qquad (7.50)$$

$$\text{subject to } \quad \underline{X} \in T \quad ; \quad \text{for } \forall k \qquad (7.51)$$

Let the solution to this problem be $\underline{X}^k$.

By definition, $\underline{X}^k$ yields maximum z_k which we will denote as M_k.

Let the values of the other objectives with $\underline{X}^k$ be z_j^k where j = 1,2 .... p and $j \neq k$. i.e.

$$z_j^k = z_j(\underline{X}^k) \quad \text{for } j = 1,2\ldots p \quad j \neq k \qquad (7.52)$$

The values thus obtained for the various objectives are utilized to construct the 'payoff table' shown in Table (7.1) i.e. any row such as k in table gives the values of objectives for the solution $\underline{X}^k$.

Obviously the diagonal cells M_k, $\forall k$ in the payoff table in (Table 7.1) is the ideal solution, since it maximizes

Solution which optimizes kth objective.	Value of kth objective z_1	z_2	...	...	z_k	...	...	z_p
x^1	M_1	z_2^1	...	...	z_k^1	...	...	z_p^1
x^2	z_1^2	M_2	...	...	z_k^2	...	...	z_p^2
...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...
x^k	z_1^k	z_2^k	...	...	M_k	...	...	z_p^k
...	...	...	...	...	...	...	...	...
x^p	z_1^p	z_2^p	...	...	z_k^p	...	...	M_p

Table 7.1. 'Stem' Method pay-off Table.[19]

all the objectives individually.

7.3.2.2. Since the objectives are in conflict (and hence the multi-objective problem) this solution is infeasible. The first step in this method is then to find a non-inferior solution, which is the "nearest"* in the minimax@ sense to the ideal solution $\underline{X}^*$ where $\underline{X}^k$ is the solution that yields M_k, $k = 1,2.....p$.

The problem is then

$$\text{Minimize } D \qquad (7.53)$$

subject to

$$\pi_k \left[M_k - z_k(\underline{X}) \right] - D \leqslant 0$$

$$k = 1,2....p \qquad (7.54)$$

$$\underline{X} \in T^i$$

$$D \geqslant 0 \qquad (7.56)$$

$$\left.\begin{array}{ll} \text{where } T^i = & T \text{ for } i = 1 \\ \text{and} & = T^i \text{ for } i > 1 \end{array}\right\} \qquad (7.57)$$

T^i is the modified form of feasible set in decision space due to progressive reaction and articulations of DM due to solutions presented in $(i - 1)^{th}$ iteration.

7.3.2.3. The D in the above equation (7.53) is the maximum weighted deviation of an objective from the ideal solution which is to be minimized.

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* Para 5.5 "Spatial Proximity methods" in multi-objective decision making models is relevant.

@ Para 3.3.3.3 may be seen for Minimax rule.

The weight π_k is defined as

$$\pi_k = \frac{d_k}{\sum_{k=1}^{p} d_k} \qquad (7.58)$$

where

$$d_k = \frac{M_k - m_k}{M_k} \left[\sum_{j=1}^{p} (c_j^k)^2 \right]^{-1/2} \qquad (7.59)$$

in which m_k is the minimum value of k^{th} objective found by looking to the smallest value cell in the k^{th} column of Table 7.1.

c_j^k is the coefficient for j^{th} decision variable for the k^{th} component of the objective function (vide equation 7.50). In brief π_k thus represents normalized weight on the k^{th} objective which by equation (7.59) depend on the variation of the value of the objective from the 'ideal solution' M_k.

The second step (Fig.7.6) is to elicit the DM's preferences. The effort is to find out which objectives in the solution can be decreased so that "permissible unsatisfactory level in the analysis" can be increased. The problem is repeated (Fig.7.6) by redefining $\underline{X}^i$ at the i^{th} iteration, by incorporating the DM's progressive articulated preferences; this is done by changing decision variables $\underline{X}$ and also the feasible set in decision space T.

The procedure stops at an iteration when DM is satisfied with the attainments or at p^{th} iteration. If the performance is still considered unsatisfactory at the p^{th} iteration then the method concludes that "no-best compromise solution exists.

The advantages of the method are:

(i) DM is able to feel the effect of the preferences and able to modify the decisions progressively.

(ii) The solution finally considered by DM is after an explicit understanding of the analysis and the sensitivities.

The disadvantages of the method can be stated as:

(i) If after p iterations, the DM is unable to compromise, no solution exists as per the method.

(ii) Explicit consideration of trade-offs are not available as the weights do not reflect the value judgements on the part of DM.

The method, though computationally efficient and rational, has not been widely applied in water resource planning problem as the decision vectors are large enough.

7.3.3. Sequential Multi-Objective Problem Solving "SEMOPS" Method

7.3.3.1. This method is due to Monarchi et al[97]. The main feature of this procedure, like others in the category, is to generate information sufficient enough (in a progressive manner) so as to enable the DM to select an alternative "in an aware" situation. The algorithm does not solve the problem by itself but provide adequate information for a decision sequentially; it is as follows.

7.3.3.2. Let the goals for p objectives be $\hat{\underline{Z}}$. The problem is to decide the policy decision vector $\underline{X}$ subject to usual equality and inequality constraints. A constraint, upper and lower bound, to the decision vector, specify the range of the desired solution i.e.

$$0 \leqslant \underline{b}_l \leqslant \underline{X} \leqslant \underline{b}_u < \infty \qquad (7.60)$$

Peculiar to this method are the use of 'criterion functions' and "a surrogate objective function".

The critdrion function, akin to classical objective functions is of the form $\underline{Z} = Z_k(\underline{X})\ \forall k$ and is utilized to judge the achievement of p objectives. The range of k^{th} element of $\underline{Z}$ is $\Gamma\ (z_k)$.

Alongwith the set goals for p objectives $\hat{\underline{Z}}$, an aspiration level $\underline{A}$ is incorporated in the algorithm; $\hat{\underline{Z}}$ is supposed to be an externally stipulated goal level while $\underline{A}$ indicates the attainments DM desires as the solution progresses.

7.3.3.3. The method distinguishes five possible types in which the goals could be set, and suggests an appropriate transformation of them into function 'd' with values of real positive numbers. The five categories and their transformation are below:

(i) at most $\quad d = \frac{z}{A}$

(ii) at least $\quad d = \frac{A}{z}$

(iii) equals $\quad d = \frac{1}{2}\left(\frac{A}{z} + \frac{z}{A}\right)$

(iv) within an interval $d = \left(\frac{A2}{A1+A2}\right)\left(\frac{A_1}{z} + \frac{z}{A_2}\right)$

(v) outside an interval $d = \left(\frac{A1+A2}{A_2}\right)\left(\frac{1}{\frac{A_1}{z} + \frac{z}{A_2}}\right)$

In each instance value of d < 1 implies that the goal is satisfied. The dimensionless indicator of the attainment of goals for p objectives is:

$$\underline{d} = \underline{D}(\underline{X}) \qquad (7.62)$$

The transformations proposed are all nonlinear functions of the criterion function $\{\underline{Z} = \underline{Z}(\underline{X})\}$ except for case (i); the construction of transformation is consistent for an uniform interpretation of p goals.

7.3.3.4. The cyclical optimization is done through a "surrogate objective function" s defined as below

$$s = \sum_{k \in p'} d_k \qquad (7.63)$$

where p' the subset of the set of p goals that make up at any iteration process, and the value of each d_k in s reflect whether the goals have been satisfied; unsatisfied goals have values > 1.

7.3.3.5. Since the transformation d_k is non-linear (besides possible non-linearity of the criterion function z_k), a direct comparison among d_k is difficult; in other words a unit change in d_k has a different meaning within the same goals from one iteration to the next. Minimizing s provides information to DM to help define the next cycle in the search of 'satisfactum'. Thus the method tries to maximize the attainment of each goal set for the objective by minimising s; the definition of s as the "surrogate" objective function is thus implicit.

7.3.3.6. The goals set for objectives not included in set p' in the 'surrogate objective function' s viz (p-p') are added as constraints. While the presence of a goal in s is indicative of the desire of the DM to achieve the corresponding A with the awareness that this might also be not possible, a goal that is entered in the constraint set means that the same must be accomplished.

7.3.3.7. The 3 step algorithm proceeds as follows:

(i) Set up

(ii) Iteration

(iii) Termination

Set up involves transforming the original problem as in para 7.3.3.2 and para 7.3.3.3. above. The iteration step is the core and is the true interactive segment of the algorithm; this involves a cycling between optimization phase and an evaluation phase until a satisfactum is reached. The termination follows.

7.3.3.8. At the beginning i.e., $i = 0$, (where i denotes the iteration) the aspiration levels are equalized to the set goals, $\underline{A} \equiv \underline{\hat{Z}}$. After iterations, at any i^{th} stage, a principle problem is formulated with goals set for certain objectives entered as additional constraints (if so desired). The surrogate objective function is, in such a case denoted by:

$$s_{3,6}^{i} = \sum_{k \in p'} d_k \qquad (7.64)$$

where goals for objectives $3,6 \notin p'$.

7.3.3.9. The solution, which is the optimization phase follows, and the resultant vector in decision space is obtained as

$$\underline{X}_i = (x_1^{\,i},\ x_2^{\,i} \ldots\ldots\ldots x_n^{\,i}) \qquad (7.65)$$

The 'optimality' implied here is in the mathematical sense as DM may not consider this as the 'best compromise solution'.

The corresponding values of the criterion function (objective function) is

$$\underline{Z}_{i,36} = (z_1^{i,36},\ z_2^{i,36} \ldots\ldots\ldots z_p^{i,36}) \qquad (7.66)$$

The dimensionless indicator of attainment for the i^{th} cycle is yielded by

$$\underline{d}_i = (\ d_1^i\ ,\ d_2^i \ldots\ldots\ldots\ d_p^i\ ) \qquad (7.67)$$

7.3.3.10. Any i^{th} cycle contains a principal problem as above and set of auxiliary problems. These auxiliary problems are due to the set of goals for objectives not entered in the principal problem and kept as constraints. Thus within each cycle, p' auxiliary problems are solved with reformulated surrogate s for each case.

7.3.3.11. Monarchi et al have solved an example[97] for a hypothetical pollution problem where each of the goals for the objectives (related to pollution level, cost, social rate of discount etc.) are modelled mathematically so as to connect the variables in decision space. With progressive interactive approach, the DM's preferences are articulated and 'satisficing' solution evolved.

Another demonstration of the use of "Semops" has been by English et al,[32] for the multi-purpose control of a natural lake. Four goals for the lake level control considered in the analysis are:

recreation; flood control; low level control for quality, and irrigation.

Each of the goals has been expressed in its own units; an interactive approach with DM has been followed for "The best compromise solution" in deciding lake level that would optimise the objectives.

7.3.3.12. The advantages of the method are:

(i) It does not attempt to solve the problem but limits itself to generate information, as desired by DM, so that he can choose an alternative.

(ii) The information is generated interactively and hence avoids the advance need of specification of the preference structure.

(iii) The DM can revise the preferences during the course of interaction based on results and can develop a ranking of goals for the objectives in a way that combine subjectivity and objectivity.

7.3.3.13. The disadvantages are:

(i) where the problems are of nature involving a large number of variables, the solution strategy may be difficult to apply.

(ii) the technique does not account for uncertainty.

(iii) a relatively larger burden is placed on the DM; in other words much subjective decision-making is called for.

7.3.4. Surrogate Worth Trade-off Methods (SWT Method)

7.3.4.1. This method is due to Haimes, Hall and Freedmen[53,54]. It recognizes that optimization is more concerned with the relative values of additional increments of the various non-commensurable objectives, at a given value of each objective function, than it is with their absolute values. The Decision-maker in this approach needs to assess only whether an additional quantity of one objective is worth more or less than that which may be lost from another, given the attainment levels of each objective. To help this decision, the SWT method generates trade-off functions which show the relationship between a weight on one objective when another objective is the numeraire and the values of that objective. A set of trade-off functions which can be interpreted as a disaggregated non-inferior set in which objectives are considered in pairs is obtained by the solution strategy.

7.3.4.2. The computational procedure can be briefly reviewed as below:

As a first step, the maximum* value of each of the objective function is evaluated i.e.

$$\text{Max} \quad z_k(\underline{X}) \qquad \text{subject to } \underline{X} \in T \qquad \forall\, k\ (1,2,\ldots.p) \tag{7.68}$$

ignoring the other (p - 1) objectives. We obtain thus $\hat{\underline{Z}}\ (\underline{X})$ which represent the maximum values.

7.3.4.3. The general approach to the problem is to find the maximum value of each objective function subject to a set of constraints. The multi-objective problem is therefore framed utilizing constraint method discussed in Chap.7.1.3.

7.3.4.4. The problem can be stated as

$$\text{Max} \quad z_r(\underline{X}) \tag{7.69}$$

$$\text{Subject to } \underline{X} \in T \quad \text{or} \quad g_j(\underline{X}) \leqslant 0;\ j = 1,2\ldots.m$$

$$\left.\begin{array}{l} \text{and} \quad z_k\ (\underline{X}) \geqslant \epsilon_k \\ \text{where} \quad \epsilon_k = \hat{z}_k(\underline{X}) - \bar{\epsilon}_k \\ \bar{\epsilon}_k > 0 \end{array}\right\} \begin{array}{l} k = 1,2,\ldots\ldots p \\ k \neq r \end{array} \tag{7.70}$$

* Minimum has been obtained in the author's works as minimizing problems are considered therein.

where $\hat{z}_k(\underline{X})$ is obtained from equation (7.68) above and $\bar{\epsilon}_k$ is varied parametrically in the process of constructing trade-off functions.

7.3.4.5. The generalized Lagrangian L to the system is formulated as

$$L = z_r(\underline{X}) + \sum_{j=1}^{m} \mu_j g_j(\underline{X})$$

$$+ \sum_{\substack{k=1 \\ k \neq r}}^{p} \lambda_{rk}(z_k(\underline{X}) - \epsilon_k) \qquad (7.71)$$

where μ_j, $j = 1,2.....m$ and λ_{rk}, $k = 1,2....p$ and $k \neq r$, are generalized Lagrange multiplier.

Let χ denote the set of all x_i, $i = 1,2....n$, that satisfy Kuhn-Tucker conditions in equations (7.72), (7.73)[71,91] that follow.

Let Ω denote the set of all Lagrange multipliers that satisfy those Kuhn-Tucker conditions.

From Kuhn-Tucker conditions, we have, for stationary values of $\underline{X}$, μ_j and λ_{rk},

(k = 1,2....p and j = 1,2.....m)

$$\lambda_{rk}\left[z_k(\underline{X}) - \epsilon_k \right] = 0 \ , \quad k = 1,2....p, \ k \neq r \qquad (7.72)$$

$$\text{and} \quad \lambda_{rk} \geq 0 \ , \quad k = 1,2....p, \ k \neq r \qquad (7.73)$$

Equation (7.72) holds if $\lambda_{rk} = 0$ or $z_k(\underline{X}) - \epsilon_k = 0$ or both. However, if $z_k(\underline{X}) - \epsilon_k > 0$ for any $k = 1,2...p$; $k \neq r$, then the corresponding $\lambda_{rk} = 0$. Thus when the k^{th} objective is not binding, the corresponding Lagrange multiplier (shadow price) is zero. Denoting the set of inactive constraints associated with a specific value of ϵ_k as $I(\epsilon_k)$, we have:

$$I(\epsilon_k) = \left\{ k \;\middle|\; \begin{array}{l} \underline{X} \in \chi \;;\; z_k(\underline{X}) - \epsilon_k > 0 \\ k = 1,2 \ldots \ldots p \\ k \neq r \end{array} \right\} \qquad (7.74)$$

Similarly let the sets of active or binding constraints associated with the specific value of ϵ_k be $A(\epsilon_k)$:

$$A(\epsilon_k) = \left\{ k \;\middle|\; \begin{array}{l} \underline{X} \in \chi \;;\; z_k(\underline{X}) - \epsilon_k = 0 \\ k = 1,2 \ldots\ldots p \\ k \neq r \end{array} \right\} \qquad (7.75)$$

The values of $\lambda_{rk}\,[A(\epsilon_k)]$ $k = 1,2 \ldots\ldots p$, $k \neq r$, are of special interest. They indicate the marginal benefit of the objective function $z_r(\underline{X})$ due to an additional unit of ϵ_k.

We have from equation (7.71)

$$\frac{\partial L}{\partial \epsilon_k} = -\lambda_{rk}\,(\epsilon_k) \qquad \left. \begin{array}{l} k = 1,2\ldots p \\ k \neq r \end{array} \right\} \qquad (7.76)$$

for $\underline{X} \in \chi$; λ_{rk}, $\mu_j \in \Omega$; for all j and k, we have

$$z_r(\underline{X}) = L \qquad (7.77)$$

Thus $$\lambda_{rk}(\epsilon_k) = \frac{-\partial z_r(\underline{X})}{\partial \epsilon_k}, \quad \left. \begin{array}{l} k = 1,2\ldots p \\ k \neq r \end{array} \right\} \qquad (7.78)$$

For all $\lambda_{rk}[A(\epsilon_k)]$, $z_k(\underline{X}) = \epsilon_k$, $k = 1,2\ldots\ldots p$, $k \neq r$, since the constraints are active. Equation (7.78) can therefore be modified as

$$\lambda_{rk} \left[A (\epsilon_k) \right] = \frac{-\partial z_r(\underline{X})}{\partial z_k(\underline{X})} \qquad (7.79)$$

7.3.4.7. From this we find that the trade-off function denoted by the above equation can be obtained for any two non-commensurable objectives e.g. \$ 10000/head of animal saved-para 4.5.1.5. and Fig. 4.8.)

The above equation is valid for all $\lambda_{rk} [A (\epsilon_k)]$ i.e. for active constraints. We can infer from this, that if a Lagrange Multiplier is non-zero, then the particular constraint limits the optimum; also, that zero Lagrange multipliers indicate inferior set of solutions. The set of non-zero Lagrange multipliers represent the set of trade-off ratios between the principal objective function as well as the level of all other objectives, satisfied as equality (binding) constraints. Hence these solutions represent the non-inferior set. Since we need study only the non-inferior solutions, that is active constraints only, we can simplify $\lambda_{rk} \left[A(\epsilon_k) \right] > 0$ as $\lambda_{rk} (\epsilon_k)$.

7.3.4.8. Derivation of Trade-off

The trade-off function between any two objectives r and k is defined as

$$T_{rk}(\underline{X}) = \frac{dz_r(\underline{X})}{dz_k(\underline{X})} \qquad (7.80)$$

where $$dz_r(\underline{X}) = \sum_{k=1}^{p} \frac{\partial z_r(\underline{X})}{\partial x_k} dx_k \qquad (7.81)$$

$$\text{or} \quad T_{rk}(\underline{X}) = \frac{(\nabla_{\underline{X}}\, z_r(\underline{X}),\ d\underline{X})}{(\nabla_{\underline{X}}\, z_k(\underline{X}),\ d\underline{X})} \qquad (7.82)$$

and the functions $T_{rk}(\underline{X})$ have the property that

$$T_{rk}(\underline{X}) = 1 \qquad \text{if} \quad r = k \qquad (7.83)$$

$$T_{rk}(\underline{X}) = \frac{1}{T_{kr}(\underline{X})} \qquad \forall,\ r,\ k.. \qquad (7.84)$$

It is possible to generate these trade-off ratios by calculating Lagrange multipliers λ_{rk} vide equation(7.79) as a function of ϵ_k. This is accomplished by solving the equations given in (7.69) and (7.70) for a certain number of values say L i.e. $\epsilon_k^1, \epsilon_k^2, \epsilon_k^3 \quad .. \quad .. \quad \epsilon_k^L$ where all other ϵ_m $(m = 1,2 \ldots\ldots p ; m \neq r,k)$ are fixed at some level ϵ_m^o. With various values of λ_{rk} obtained for different levels of ϵ_k {or $z_k(\underline{X})$ } (Since the active constraints alone are considered), a regression analysis can be performed to obtain the function $\lambda_{rk}[z_k(\underline{X})]$, as shown in Fig.7.7. However, λ_{rk} is also a function of the values of ϵ_m^o $(m = 1,2 \ldots\ldots p$ and $m \neq r,k)$; a multiple regression analysis may be needed if the first regression is sensitive to the levels of ϵ_m^o..

7.3.4.9. Surrogate Worth Function: (SWF)

When all λ_{rk} $(r,k = 1,2 \ldots\ldots\ldots p)$ have been determined, the functional matrix Δ can be constructed showing trade-off functions. However, since the trade-offs are in non-commensurable units, the authors propose a "surrogate worth function" of the following properties:

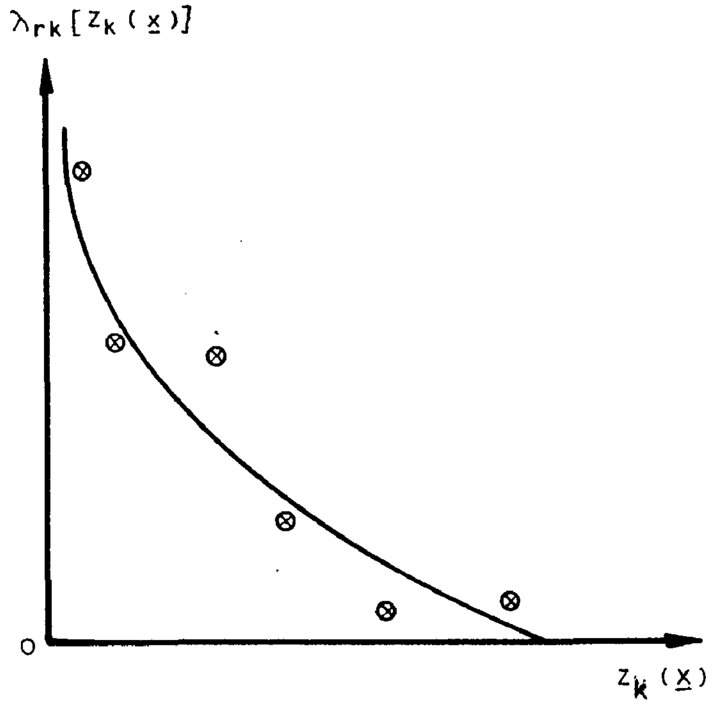

FIG. 7.7 _ REGRESSION FITTING OF $\lambda_{rk}[Z_k(\underline{X})]$ AS A FUNCTION OF $Z_k(\underline{X})$

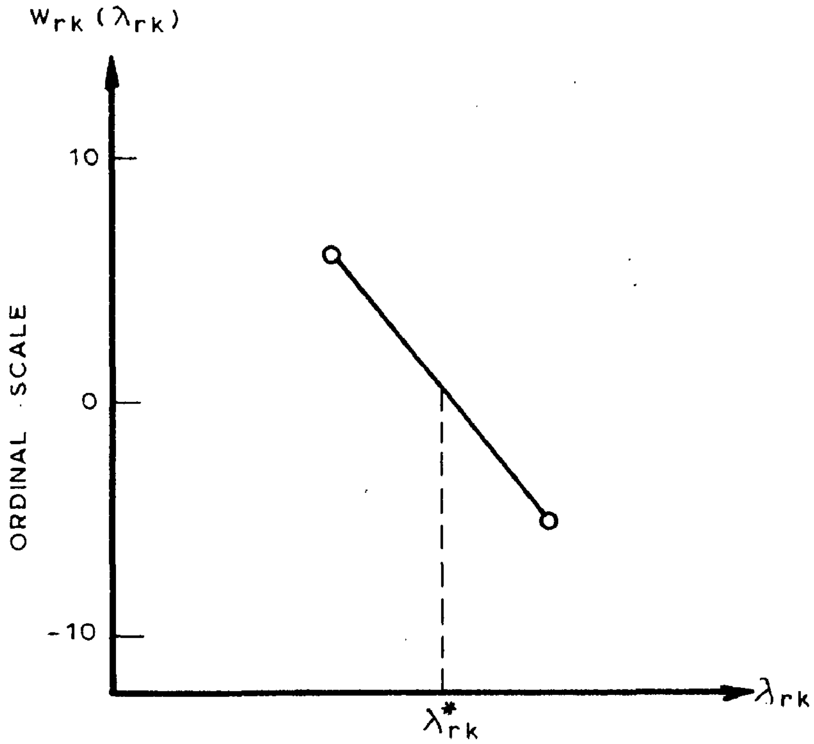

FIG. 7.8 _ DETERMINATION OF INDIFFERENCE BAND λ^*_{rk} AND SWF CONSTRUCTION

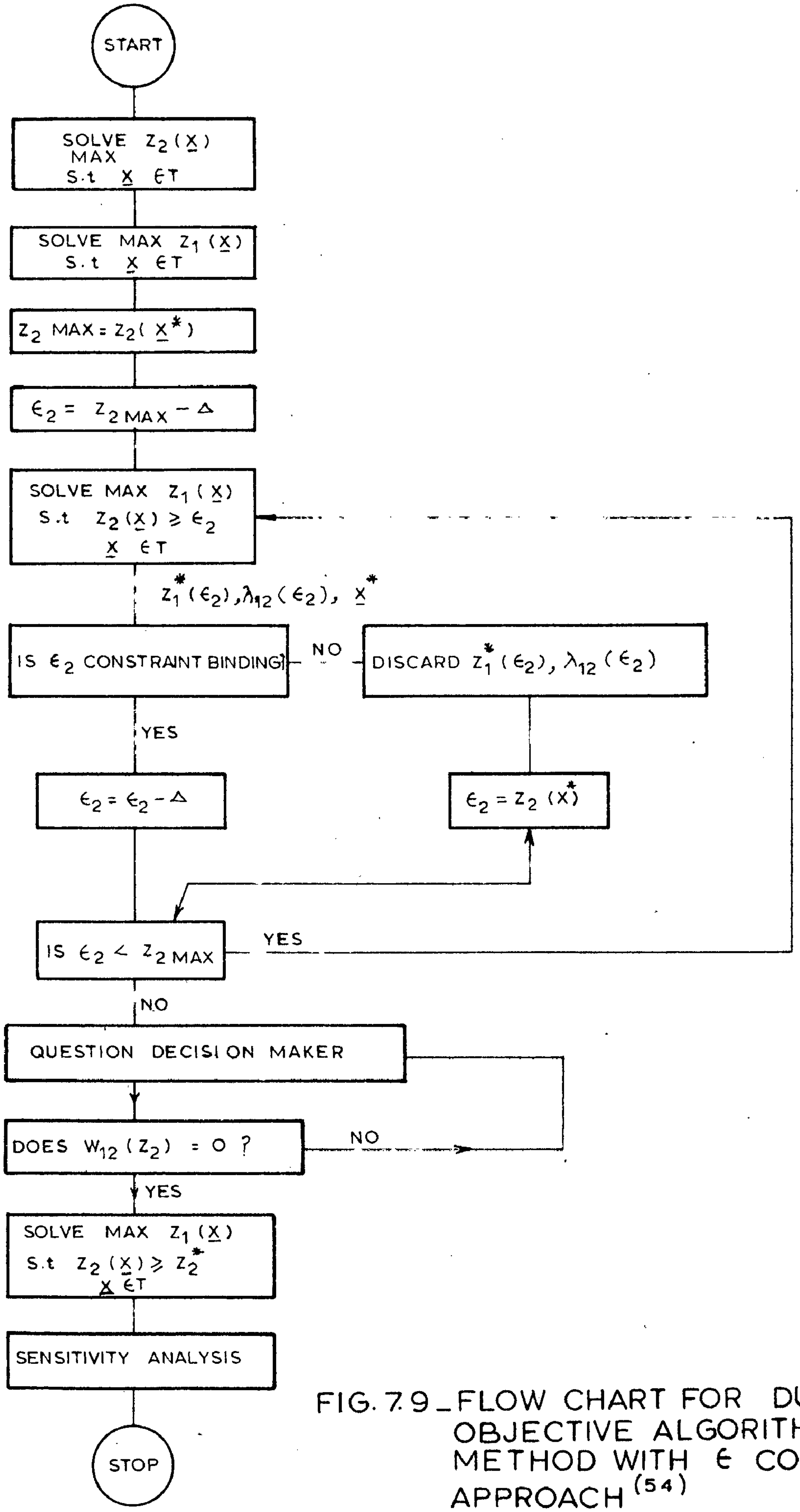

FIG. 7.9 _ FLOW CHART FOR DUAL OBJECTIVE ALGORITHM OF SWT METHOD WITH ϵ CONSTRAINT APPROACH (54)

(i) identification of optimal weight from the DM by progressive articulation.

(ii) implicit DM's comparison of them with slopes of societal indifference curves, and

(iii) specifying a "best-compromise solution" therefrom.

The usage of SWT method has been demonstrated in examples (including certain applications to problems related to water resource systems) for dual and multiple objectives for static and dynamic cases by Haimes et al.[54]

The advantages of this method are:

(i) it is possible to handle non-commensurable objective functions quantitatively in an effective manner.

(ii) the method provides a systematic comparison of the objectives, two at a time. This enables clarity in case of problems of higher dimensions in function space.

(iii) the decisions DM has to make is minimal. To facilitate these decisions, a constant interaction posing relative objectives is made, thereby bringing clarity and meaning to a multiple objective problem.

(iv) for large number of objectives in multi-objective problems ($p > 4$) this method offers computational efficiency.

The disadvantages[19] are:

(i) Trade-offs are generated only between two objectives assuming fixed values for the remaining objectives. The variations of the trade-offs with the attainment levels of the rest of the objectives are captured only in a limited sense.

(ii) For $p < 3$ computations involved are more compared to weighting techniques.

(iii) The method is vulnerable to computational sensitivity to the number of objectives.

CHAPTER 8

A PRACTICAL APPROACH FOR WATER RESOURCE PLANNING PROBLEMS

8.1. INTRODUCTION

8.1.1. Of the various methods discussed in Chapters 7 the following methods consider trade-offs explicitly:

(i)	Weighting methods	in generating methods for non-inferior solutions
(ii)	Constraint methods	
(iii)	The estimation of optimal weights	amongst the class of solution technique that rely on prior articulation of preferences.
& (iv)	The SWT method	In the class of interactive solution technique.

Our treatment of weighting and constraint methods were rather similar. While the SWT method has its own values, for the institutional set up that obtain in our country for dealing water resource planning problems, difficulties are apparent for a constant interaction between analyst and the decision makers. As such, the choice is left between weighting and constraint methods and the estimation of optimal weights. As discussed in para 4.5.2 earlier, the prior articulation of preferences in 'a top-down' methodology has inherent difficulties and drawbacks. For a 'bottom-up' methodolgy as suggested by UNIDO (briefed in para 4.5.2), the generating methods for non-inferior solution, particularly the weighting methods and constraint methods, are most useful. As the constraint methods can yield solution even for non-convex non-inferior set in function space, the method was considered superior to weighting method (para 7.2.4).

8.1.2. Water Resource Systems - A general Statement

8.1.2.1. Water resource systems represent a complex stochastic physical system embedded in an economic, social and institutional frame work that is difficult to model. The decision variables in the system range from deciding whether water resources investment should take place to questions as to where and when facilities such as reservoirs, diversions, power generation, irrigation areas etc. should be developed and to microscale questions of how an individual component should be operated over time. Dorfman[30] has named the concept of modelling these inter-related decisions in the context of physical and non-physical environment as screening model.

For a complex river basin, with many interest groups and many potential development alternatives, the determination of optimal alternatives, has become a difficult problem.

8.1.2.2. With the employment of multi-objectives for planning, a particular alternative, together with necessary assumptions about its physical and economic behaviour needs a careful analysis to assess its contributions to each of the objectives.

8.1.2.3. For this kind of problem, the determination of the net benefit transformation surface representing the non-inferior solution set is the first requirement. This is complex as there could be a large number of feasible alternatives possible for adoption. To analyse these feasible alternatives, it is best that recourse is made to systems analysis through mathematical models.

8.2. MODELS IN SYSTEMS ANALYSIS

Models employed to solve water resources planning problems are varied. Broadly they fall under:

(a) Optimization models

(b) Simulation models.

The optimization models are first considered. Simulation models are taken up in para 8.2.3.

8.2.1. Optimization Models

Models under this category aim at generation of optimal solutions as measured by an explicit objective ranking function. Characteristics of optimization models are:

(i) They are analytic and close to the realistic situations

(ii) They are of closed form.

(iii) They generate as many feasible alternatives as possible with major simplifying assumptions such as deterministic hydrology for every year etc.

Since the intent of the models under this category is to pick out the promising alternatives by screening out those that are not attractive (in achieving the objectives), it can be termed as screening models as envisaged by Dorfman[30]. The realism of such models is dependant upon the adequacy of representation of the various issues that affect the system planning. To limit the computational process, normally such factors as hydrologic stochasticity and temporal considerations are not included in these models. Rather the aim is confined to gather an insight into the planning problem.

8.2.2. Stochastic Screening Models

8.2.2.1. Though the deterministic multi-objective screening models provide a valid representation of the system behaviour on a gross scale, the stochastic nature of events may seriously distort the solutions obtained therefrom. Stochastic-screening models[75] are a new class that attempts an explicit quantification of the important stochastic elements such as:

(i) resource supplies viz hydrologic inflows

(ii) resource demands viz irrigation water requirements

(iii) uncertainty in economic evaluation such as benefit cost estimation, social rate of discount etc.

(iv) other planning uncertainties including those in modelling.

8.2.2.2. Many of the stochastic elements and uncertainties can be viewed as imperfect predictions to be characterized by probability distributions for the different possible events. This requires the consideration of each possible event and the associated probability of occurence.

8.2.2.3. Two classes of models have emerged to deal with stochastic screening models. They are stochastic LP model or Linear Programming under uncertainty and chance constrained programming. Loucks[75], Havan[52] and de Lucia[24] have worked in these fields.

8.2.3. Simulation Models

Simulation models consist of a sequence of mathematical and logical statements describing the design and operation of a system. Such a sequence of statements adjusted to coincide with the characteristics of a basin and together with a series of historical or synthetic streamflows provide a means of simulating the operation of that system in order to predict and analyse its performance.[64] Simulation models are not normally intended to indicate what system designs produce the optimality of an objective function. They produce, instead only a measure of the value of an objective function for a particular system configuration.[55] Investigation of a large number of alternative development possibilities to locate that with the best response would involve revising the model with each system design and would therefore be expensive.

8.2.4. Hierarchy of Models

From the above, it is seen that neither optimization models nor simulation models offer independently, enough scope to identify the "best compromise solution" for a large scale water resource systems design. It appears that the role of systems-analysis-models in the water resources planning process should start with the use of an optimization model to locate good feasible solution (screening models as is commonly termed): this may be followed by an examination of this solution with other similar solutions in a simulation model. Subsequently a sequencing model can be used for basin studies to consider an explicit optimal time scheduling, considering all other relevant factors.[82]

8.3. MULTI OBJECTIVE PROBLEM AND A CASE STUDY

The planning problem in case of multi-objective consideration is

$$\text{Max. } \underline{Z}$$

$$\text{where } \underline{Z} = z_1, z_2 \ldots\ldots\ldots z_p$$

$$\& \; z_1 = \underline{C}_1^T \, \underline{X}$$

$$\text{Subject to } \underline{A}\,\underline{X} \leqslant \underline{b}$$

The solution yields results to enable selection of the decision variables $\underline{X}$ besides $\underline{Z}$.

The decision variables $\underline{X}$, $\underline{b}$ represent generally

(i) project or plan outputs

(ii) allocation of scarce resources as inputs

(iii) policies for project and plan operation.

The constraints $\underline{A}\,\underline{X} \leqslant \underline{b}$ take care of
(i) physical, (ii) technical, (iii) Economic,
(iv) institutional, (v) budgetary
or other constraints that are relevant.

In para 8.1.1. we had seen that for generation of large number of non-inferior solution set the "constraint techniques" is quite useful. The vector maximisation is transformable into a scalar by these methods; as in equation (7.10) as

$$\text{Max. } w_r z_r (\underline{X}) + \sum_{\substack{k=1 \\ k \neq r}}^{p} w_k z_k (\underline{X})$$

subject to constraints on $\underline{X}$ imposed by various considerations.

8.3.1. PRACTICAL EXAMPLE IN MODELLING: A CASE STUDY REVIEW

As an example of mathematical modelling for solution of multiple objective planning problem, Cohon & Marks' example problem[18] solved through a deterministic screening model is reviewed below.

The problem relates to the development of the water resources of a hypothetical river basin shown schematically in Fig. 8.1. The river flows through 4 regions and is the boundary for regions 2 and 3. All the different regions have their own plans for various possible developments: their demand is more than the water available in the river. The flow in the river also exhibits seasonal fluctuations. It is unevenly distributed in a temporal sense differing from the preferred demand for irrigation and power generation during different

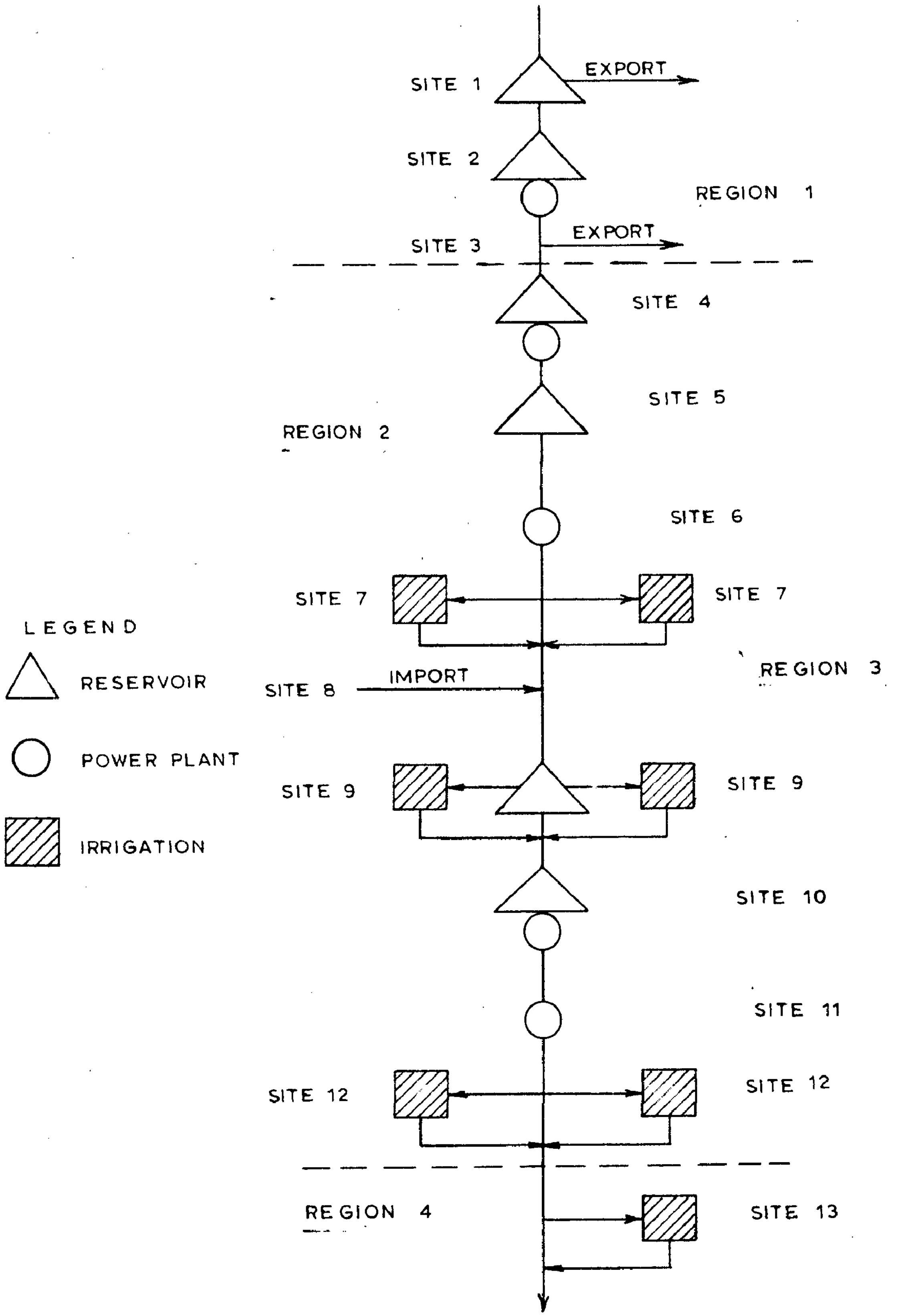

FIG. 8.1 _ THE RIVER BASIN AND DEVELOPMENT ALTERNATIVES[18]

periods. Exports and imports from other river basins and dams have been contemplated to provide storage for inter-temporal allocation.

18.3.2. The objectives considered are:

(i) development objective as reflected by net NI* benefits

(ii) regional concern objective as reflected by the 'just' regional water allocation.

Minimisation of the absolute deviation of regional water use from average regional water use was chosen as a measure of objective(2).

18.3.3. This is represented by the equation

$$\text{Min } D = \sum_{i=1}^{p} W_i - \bar{W} \qquad (18.1)$$

in which D is the deviation in m^3/sec, W_i is the water used in the region in m^3/sec, $\bar{W}$ is the average regional water use in m^3/sec, and p is the number of regions

Just as in the case of other objectives, the allocation objective should be measured by a discounted metric. For the example shown, the authors chose no discounting, however.

18.3.4. The transformation of the absolute value that is non-linear in equation (18.1) above, needs a change, as below to be considered in the linear programming model utilized to solve the problem.

$$\text{Min } D = \sum_{i=1}^{p} (G_i + T_i) \qquad (18.2)$$

Such that $$W_i - \bar{W} = G_i - T_i \qquad \forall i \qquad (18.3)$$

$$G_i, T_i, W_i, \bar{W} \geq 0 \qquad \forall i \qquad (18.4)$$

---

* NI = National Income.

in which G_i and T_i are the deviation of W_i from $\overline{W}$ and only G_i or T_i, not both, can be nonzero for each of the p constraints. For a given deviation $G_i - T_i$, the sum $G_i + T_i$ is minimized when G_i or T_i equals zero.

8.3.5. In a constraint method, the screening model proposed for the case study for the dual objective of (i) national income maximization by increasing the discounted net available benefits and (ii) the water allocation objective is as indicated below:

$$\text{Min} \quad D = \sum_{i=1}^{p} (G_i + T_i)$$

$$W_i - \overline{W} = G_i - T_i$$

$$z_1(\underline{X}) \geq e_1 \qquad \text{Subject to} \quad \underline{X} \in T$$

$$\overline{W} = \frac{1}{p} \sum_{i=1}^{p} W_i \qquad \forall i$$

$$W_i = \sum_{t=1}^{N} \sum_{s \in S_i} (\overline{V}_{st} + \overline{F}_{st}) \qquad \forall i$$

In the above expressions all variables are non-negative, N is the number of seasons, S_i is the set of sites in region i, $\overline{V}_{st}$ is the average diversion for irrigation at site s during season t in m^3/sec. and $\overline{F}_{st}$ is the average inter-basin export at site s during season t in m^3/sec.

8.3.6. As a first step in solution, an unconstrained NI maximization has been done (Max $z_1(\underline{X})$ subject to $\underline{X} \in T$) so as to delineate an upper bound for $z_1(\underline{X})$. Each of the steps that followed consisted of several solutions obtained by parametrically varying z_1 from an initial value of say 0 to z_1(max). The solution procedure and results of the analysis are as

Table 8.1

Solution Procedure and Results for Generating the Non-inferior set

Step No.	$Z = B_1$ pesos x 10^{12}	D_1 m3/sec.	Dual Activity	Computation Cost,$	Point on transformation Curve
1	2.10005	436	--	9	J
2	0	0	0		
	0.5	0	0		
	1.0	0	0		
	1.5	0	0		
	2.0	102.1	-0.00064	20	C
3	1.6	0	0		
	1.7	0	0		
	1.8	0	0		A
	1.9	41.0	-0.00045		B
	2.0	102.1	-0.00064		C
	2.1	397.7	-0.04787	23	I
4	2.05	136.6	-0.00069		D
	2.06	143.6	-0.00070		E
	2.07	150.6	-0.00070		F
	2.08	157.7	-0.00072		G
	2.09	168.0	-0.00114		H
	2.10	397.7	-0.04787	25	I

The total cost equals $ 77.

shown in Table 8.1.

The last column of Table 8.1 labels the noninferior points which are also shown in the Transformation Curve Fig. 8.2.

8.3.7. The important features of the transformation curve can be stated as

(i) Since a smaller deviation in water allocations are preferred by the regions, the plotting of D decreases on its axis from origin.

(ii) The National Income starts at a value of 1.8×10^{12} pisos. This is so because D = 0 for values of NI benefits $< 1.8 \times 10^{12}$. These points, although feasible, are inferior.

(iii) The curve stops at D = 436 corresponding to point J at which $z_1 = z_1$ max. Points to the left of J with D > 436 must have values $z \leq z$ max. for feasibility and therefore are inferior.

8.3.8. Table 8.2 indicates the full details of benefits and other relevant information regarding the various design alternatives. The transformation curve in Fig. 8.2 and the informations in Tables 8.1 and 8.2 provide adequate data for a choice by the DM, though, in general, this requires preference informations such as optimal weights or social indifference curve etc.

8.3.9. For the typical case shown, the decision-making process would prefer a plan in the vicinity of point H. Using a geometric argument, point H would be optimum for a typical idifference curve. From a trade-off analysis also, it is unlikely that a point in segment JH would be selected because of the relatively little NI gains, by movement from H to J. Significant deviation in water allocation disproportionate to gain is apparent. Similarly, when moving from H to A, the water allocation is not greatly improved but a rather large amount of national income is sacrificed.

8.3.10. We thus see that the information generated by the model and contained in the TC is useful to decision-makers in the selection of optimal public investment alternatives, designed keeping in view multiple objectives.

Table 8.2

Design Capacities Corresponding to Points on the Transformation Curve[18]

Site No.	Region No.	Alternative*	Points on Transformation Curve									
			A	B	C	D	E	F	G	H	I	J
1	1	RES	0	0	27.5	77	88	100	112	121	121	121
		EXP	0	0	0	0	0	0	0	0	0	0
2	1	RES	7595	3258	3338	3388	3399	3411	3448	3431	3431	3431
		PP	200	200	200	200	200	200	200	200	200	200
3	1	EXP	84	91	98	103	104	105	106	107	107	107
4	2,3	RES	45	313	225	223	215	207	229	596	805	805
		PP	0	0	0	2	2	2.5	3.8	11.4	12.8	15.6
5	2,3	RES	353	0	0	0	0	0	0	0	0	0
6	2,3	PP	116	116	123	111	108	105	102	93	92	92
7	2,3	IRR	70200	62843	51657	46212	44981	43711	42317	23158	15000	3500
8	2,2	IMP	130	130	130	130	130	130	130	130	130	130
9	2,3	RES	951	1238	1397	1454	1451	1444	1431	1418	1417	1417
		IRR	75100	80277	83880	85964	86694	87507	88485	100000	35210	35000
10	2,3	RES	206	0	0	0	0	0	0	0	0	0
		PP	0	0	0	0	0	0	0	0	0	0
11	2,3	PP	0	0	0	3	3.7	4.6	5	1.8	0	0
12	2,3	IRR	66500	66500	66500	66500	66500	66500	66500	66500	66500	66500
13	4	IRR	133259	129920	129361	127751	126496	125000	122999	126127	166472	173272
Water use and benefit information.												
W_1			251	272	295	309	312	316	319	322	322	322
W_2			251	251	244	241	241	241	241	237	113	99
W_3			251	231	193	173	171	170	169	154	125	114
W_4			251	251	244	241	238	236	233	238	314	327
$\overline{W}$			251	251	241	241	241	241	241	238	218	215
D(m^3/sec)			0	41.0	102.1	136.6	143.6	150.6	157.7	168.0	397.7	436
z (pesos x 10^{12})			1.8	1.9	2.0	2.05	2.06	2.07	2.08	2.09	2.10	2.10005

* RES, reservoir in Hm^2; PP, power plant in Mw, IRR, Irrigation in Has: EXP, export in m^3/sec. IMP. Import in m^3/sec; W_i water use in region i in m^3/sec; and $\overline{W}$ average regional water use in m^3/sec.

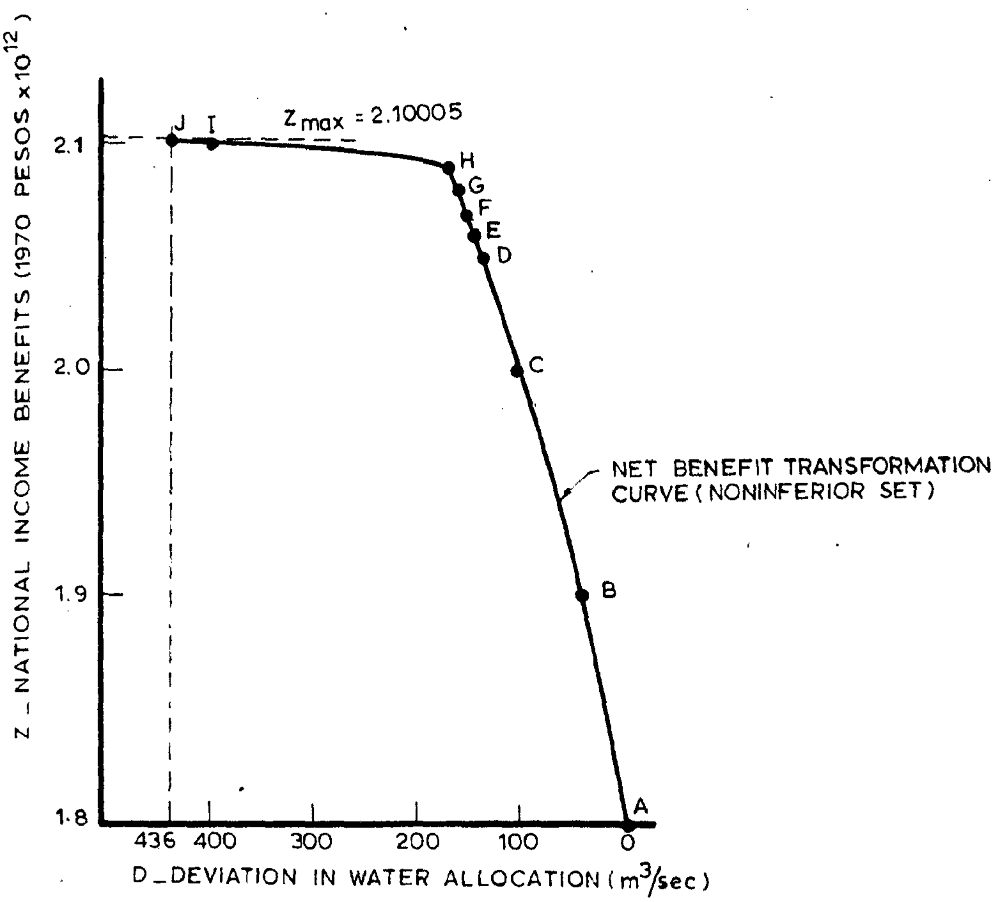

FIG. 8.2 _ THE GENERATED NET BENEFIT TRANSFORMATION CURVE[18]

8.4. SUMMARY COMMENTS

In this chapter, one of the various methods outlined for solving multi-objective problems in Chapter 7 is chosen and through systems approach, the solution strategy available has been briefly pointed out employing mathematical modelling techniques. An application for a case study has been brought out for discussions. The screening models and simulation models, widely employed for solving multi-objective water resource systems analysis have been viewed as complementary. It is recommended that a hierarchy of models viz screening models, simulation models and sequencing models, be utilized for practical problems. A detailed discussion on these models is, however, considered beyond the scope of the present study.

PART III

CONCLUSION

C H A P T E R 9

SUMMARY, FINDINGS AND RECOMMENDATIONS

SUMMARY, FINDINGS AND RECOMMENDATIONS

SUMMARY

At the present state of art, the evaluation of water resources projects are based on the traditional benefit-cost analysis. This procedure is essentially based on the 'Pareto Optimality'. Underlying this is a decentralized optimum achieved through the 'invisible hand' of competitive market mechanism that Adam Smith envisaged.

In reality, the objective functions of water resource investment programmes are complex and economic efficiency is just but one of them. A detailed survey indicates that the relevant objectives of water resources projects could be National Economic Development (NED), the Regional Income Distribution (RI) and the Environmental Quality (EQ). Other objectives relevant to general planning as envisaged in our planning documents are 'self reliance', 'employment opportunity', 'social well-being' etc. An explicit consideration of these objectives in water resources project planning is desirable.

Given the set of objectives, the need for adequate measurement of benefits and costs for each of the objectives is apparent. The GNP and national accounts do not give a complete picture of the value of output to NED. This is also affected by beneficial and adverse externalities, imperfect market conditions and changes in productivity of resource inputs due to investment. The real need therefore lies in social benefit-cost analysis that would reflect the social gains and losses, as fully as it can. Benefits are measured by the concept 'willingness to pay'. Shadow prices are used to measure economic value of production factors where there are disequilibrium between demand and supply. The regional income benefit is not just a subset of national income as it includes transfer. The 'willingness to pay' for the benefits, for example, is the total willingness to pay of the users in the specified region minus any charges for the output imposed on these users. Having found the regional income benefits, a classical solution for a simultaneous treatment of NED and RI objective is possible by formulating a combined 'weighted function' that can be maximized: alternatively maximizing each of the objectives subject to a set of constraint is also feasible.

Environmental Quality as an objective is becoming important in case of water development due to society's changing values, reflected in recent measures of Government. The problem of equating these non-commensurate objectives with those that are commensurate is possible through a recourse to 'systems Approach' after evaluating appropriate scales.

Chapters 1 and 2 deal with the above aspects.

Present and future consumptions are traded off by the choice of an appropriate social rate of discount. The need for a lower discount rate than that of 'opportunity cost' for durable major investment projects like the water resources project is an established factor. Social values of investment are affected by the shadow price of investment. A combined treatment of social rate of discount set rather low and shadow price of investment eliminates offsetting factors in evaluation. Another important element in evaluation complexities is related to risk/uncertainties. Maximizing expected value of return or the adoption of a synthetic discount rate that account for risk is one way to consider them. Risk can also be input as a chance controlled non-decisioned parameter, if possible, in system evaluation. Chapter 3 considers these problems in evaluation.

Given the objectives, by investigating a sufficiently large number of alternative projects, the boundary of the net benefits that accrue to each of the objectives can be established. If adequate information of social welfare functions are available or the indifference in the choice range of alternatives between objectives is known, the optimal choice is indicated by the point of tangency of the indifference surface and the boundary of feasible sets. Problem solution is simple if prior knowledge of weights, as obtainable in a "TOP-DOWN METHODOLOGY", is known. It is possible in such cases to derive a 'grand Benefit-Cost Ratio' for multiple objectives, and show that the traditional benefit-cost ratio is just a subset of the grand benefit-cost ratio envisaged with multiple objectives in perspective. Chapter 4 deals with the above aspects.

Systems Approach is useful when problems to be analysed are complex. When choices are to be made between alternative courses of action, in absence of knowledge about weights, more realistic appraisal of alternative courses of action is necessary by considering trade-offs, as provided by Systems Methods.

Systems Techniques are charactrised by the ability to focus on objectives being sought and alternative ways to achieve them.

Multi-objective solution strategies are interlinked with multi-objective decision-making process. They are framed by the combination of models used for the latter which are of four broad categories: weighting methods, sequential elimination methods, mathematical programming methods and spatial proximity methods. In Chapter 5, a survey of these is given.

The multi-objective problems attempts to maximize a vector in the objective function and is of the general form:

$$\text{Max} \quad \underline{Z} = \underline{C}\,\underline{X} \quad \text{subject to} \quad \underline{A}\,\underline{X} \leqslant \underline{0} \quad \text{and} \quad -X \leqslant \underline{0} \; .$$

The complexity lies in the fact that it is not possible to directly maximize (or minimize) a vector valued objective unlike the uni-objective scalar in case of classical benefit cost analysis. This leads to the concept of non-inferiority instead of optimality as indicated by the transformation hull in the objective space. The mathematical modelling of multi-objective function and a typical example are given in Chapter 6.

The solution of the objective function is the goal of various techniques in the field which fall under 3 categories dependant upon their type of approach, as seen in Chapter 7:

i) Generating Techniques

ii) Techniques that solve the problem based on 'a priori' knowledge of preferences for objectives

& iii) Interactive solution techniques.

The generating techniques attempt to provide all information necessary to obtain the transformation surface from a multi-objective model. This is accomplished without preference from decision-maker (DM). The weighting and constraint methods with parametric variation identify the non-inferior solutions for multi-objective problems. They consider trade-offs explicitly and display in totality all non-inferior set. The functional Relationship method is useful for the restricted class of the control variables. The adaptive search method is yet another technique of this class being experimented to provide a range of alternative project configuration near the optimum for use by DM for simple systems.

Where preferences amongst objectives are known prior to problem solution, results are easily obtainable. Lexicography method attempts to maximize as many objectives as possible simultaneously starting from the most important objective and followed by others in a hierarchical order. The Goal Programming attempts to minimize the weighted absolute deviations from targets of each objectives. A further extension of this category is the Goal Attainment Method that specify the allowable tolerances of the goals attainable. 'Utility Functions and Optimal Weights' method also are of this class which provide direct solution.

Interactive methods of solution for multi-objective problem are seen to be powerful. In these methods, a non-inferior solution is found, the reactions of DM is obtained, the problem is modified and analysed iteratively to attain 'a best compromise solution'. The methods reviewed are Step Method, Sequential Multi-objective Problem Solving Method and Surrogate Worth Trade-off Method. The last one has the advantage of dealing in function space and since trade-offs are made explicit in each stage by this method, it brings clarity and meaningful choice for water resource problems.

The practical problem of analysing water resource systems taking into account multiple objective planning base is best done in a hierarchy of models: optimization, simulation and sequencing. Screening models in optimization class attempt to generate as many feasible alternatives with major simplifying assumptions. Stochastic screening models are recent in this class that can consider uncertainty in performance.of the system. Simulation models refine the results obtained by screening models to get an improved result for objective function. Sequential model is utilized to evaluate an optimal scheduling for construction. Chapter 8 covers these aspects.

A simple example utilizing mathematical modelling for solution of multi-objective problem has also been reviewed in this chapter. The objectives considered in this study done by other authors[18] are:

(i) contributions to national income, and
(ii) regional concern objective as reflected by just regional water allocation.

The information generated by the model to delineate the transformation curve is found to be of value to decide on an optimal choice.

FINDINGS

(i) The present planning practices for water resources project evaluation need a revision to consider the multiple objective as planning base, which are identifiable from Nation's Planning Documents.

(ii) The benefit-cost analysis considering multiple objectives need to be broad based.

(iii) The transformation from traditional Benefit-Cost Analysis and its associated uni-objective planning models to multi-objective analysis is best handled by systems approach. A hierarchy of models for systems design is relevant with multi-objective planning base.

(iv) For a 'bottom-up methodology' the weighting and constraint methods of generating technique for multi-objective planning problem is useful: in 'top-down methodology' indicating weights the solutions by optimal weights are direct.

(v) Where a constant interaction between analyst and Decision-Maker is feasible, the surrogate worth trade-off method that progressively accounts for the articulated preferences is suggestive.

RECOMMENDATIONS FOR FURTHER WORK

I. Evaluation of Projects for Different Objectives

(i) For an adequate implementation of the sectoral model for derivation of social benefits and costs, a large number of primary data are needed viz; input-output coefficients, coefficients describing comparison of occupational requirements within each industry, coefficients relating to value added components to gross outputs for each industry. Works in these fields are of inter-disciplinary nature.

(ii) For accounting for other objectives like environmental quality, there is a lack of adequate scales (for an objective evaluation). Social evaluators are useful to study the impacts on the various components such as income distribution etc.

II. Multi-Objective Analysis of Projects

(iii) In the field of multi-objective analysis, the search method offers promise for further research for a wider application by improving the techniques.

(iv) Methods to establish function relationships between the objectives and decision variables are useful for adopting techniques like Reid and Vemuri Method, described under 'Generating Techniques'.

(v) The 'Surrogate worth Trade-off' method that is outlined has been for deterministic problems. Further studies to account for stochasticity would be useful. Also possibility of treating risk and uncertainty as objectives and applying the methods needs to be explored.

(vi) In the development of interactive techniques the basic assumption is that a single quantitative value for preferences (e.g. Trade-offs) is obtainable from decision-maker. In real world problems (particularly Water Resource Projects involving public decisions) there are a number of decision makers with diverse opinions. This has led to the emergence of a new field 'multi-objective multiple decision making'. Research in this field is required to adequately account for such diverse factors keeping multiple objectives as planning base.

(vii) Finally the most important is application of the multi-objective method of analysis for real world problems viz; water resource systems, which is complex. Basinwise studies that explicitly consider the declared multiple objectives are needed in this regard.

APPENDIX I

NATIONAL SECTORAL MODEL

The Haveman-Krutilla model[57] is a general model that permit decomposition of the rupee expenditure for a final good into its detailed sectoral component and then to allocate these components to the various original contributors of the output in proportion to the value of their contribution. The contributions of labour, capital, land and government to the final product are isolated by the occupational or industrial sector of origin within each payment category.

The model contains Z occupational categories and N industries. Each industry produces a homogeneous output by combining factor inputs with purchased inputs from other industries. In the model, all exchanged commodities and services are measured in physical units and evaluated at base year prices. In the glossary for notation, capital letters represent matrices and lower case letter represent vectors. Matrix and vector dimensions are stated in parentheses

Notational Glossary:

- f - Total cost of project construction
- u - row vector consisting all ones, of appropriate dimensionality.
- y - column vector (NX1) of final demand for materials, equipment and supplies required from each industry as inputs into project construction.
- t_2 - column vector (ZX1) of total on-site labour costs for project construction, by occupational category.

g - Contractor's profit and overhead and other project costs not included in either on-site labour cost or expenditures for materials, equipment and supplies.

x - column vector (NX1) of the gross output level of each industry required by the final demand.

A - Square matrix (NXN) of input-output co-efficients which define the source and quantity of inputs to each industry per rupee worth of output from that industry.

E - Diagonal matrix (NXN) of the total man-year labour requirements per rupee's worth of gross output in each industry.

d - Column vector (NX1) of total man-year labour requirements for each industry required by the final demand.

B - Rectangular matrix (ZXN) containing labour co-efficients which define the volume of man-year occupational requirements in each industry per unit of man-year labour requirements in that industry.

m - Column vector (ZX1) of total man-year labour requirements for each occupational category required by the final demand.

W - Diagonal matrix (ZXZ) of average annual wage and salary income by occupational category.

$\underline{t}'_1$ - Column vector (ZX1) of total off-site labour cost generated by the final demand by occupational category.

$\underline{X}$ - Diagonal matrix (NXN) of the gross output level of each industry required by the final demand; vector x transformed into a diagonal matrix.

$\underline{e}, \underline{r}, \underline{c}, \underline{i}, \underline{p}, \underline{q}$ - Six column vectors (NX1) of ratio of value added components (respectively employee compensation, net interest, capital consumption allowances, indirect business taxes, corporate profits and proprietor and rental income) to gross output by industry.

$\underline{e}_1, \underline{r}_1, \underline{c}_1, \underline{i}_1, \underline{p}_1, \underline{q}_1$ - Six column vectors (NX1) of value added components (respectively, employee compensation, net interest, capital consumption allowances, indirect business taxes, corporate profits and proprietor and rental income) generated by the final demand by industry.

$\underline{C}$ - Rectangular matrix (NX6) defined by $(\underline{e}, \underline{r}, \underline{c}, \underline{i}, \underline{p}, \underline{q})$

$\underline{D}$ - Rectangular matrix (NX6) defined by $(\underline{e}_1, \underline{r}_1, \underline{c}_1, \underline{i}_1, \underline{p}_1, \underline{q}_1)$.

$\underline{t}_1$ - Column vector (ZX1) of the total off-site employee compensation (adjusted) generated by final demand, by occupational category.

$\underline{t}_*$ - Column vector (ZX1) of total labour income generated in each occupational category, by project construction.

The Model:

The total expenditure for project construction is divided into

(a) On-site employee compensation

(b) Final expenditures for material, equipment and supplies.

and (c) Contractors' profit, overhead and other project costs not included in (a) and (b).

$$f = \underline{u}\underline{t}_2 + \underline{u}\underline{y} + g \qquad (1)$$

The gross output of each industry generated by the final demand is equal to the product of the final demand for material, requirement and supplies by industry and the inverse of the inter-industry technical co-efficient matrix.

$$\underline{x} = (\underline{I} - \underline{A})^{-1} \cdot \underline{y} \qquad (2)$$

Total man-year labour requirements by industry ($\underline{j}$) are derived by applying the appropriate labour coefficients($\underline{E}$) to the gross output level ($\underline{x}$) derived above i.e.,

$$\underline{j} = \underline{E} \cdot \underline{x} \qquad (3)$$

These man-year labour requirements are disaggregated into detailed occupational categories by applying the appropriate occupational coefficients to the total industry labour demands.

$$\underline{m} = \underline{B} \cdot \underline{j} \qquad (4)$$

The occupational break-down of generated labour income is obtained by applying average annual occupational wage and salary estimates to the occupational man-year labour requirements

$$\underline{t}'_1 = \underline{W}\cdot\underline{m} \qquad (5)$$

The value of the value-added components generated by the final demand for materials, equipment and supplies, by industry, is the product of the gross industrial outputs and the appropriate ratios of value-added components to gross output.

$$\underline{D} = \underline{X}\cdot\underline{C} \qquad (6)$$

To equate the estimates of total labour income obtained through the occupational man-year procedure (equation5) with the estimates of labour income obtained through the value added component procedure (equation 6), the occupational breakdown obtained in equation 5 is adjusted by the ratio of the total employee compensation figure secured in equation 6 to the total wage and salary income figure generated through equation 5.

$$\underline{t}_1 = \frac{(\underline{u}\cdot \underline{e}_1)}{(\underline{u}\cdot \underline{t}'_1)}\cdot\underline{t}'_1 \qquad (7)$$

The total employee compensation generated by the expenditure for project construction, by occupational category, is the sum of the occupational distribution of off-site and on-site employee compensation.

$$\underline{t}_* = \underline{t}_1 + \underline{t}_2$$

By definitional accounting identities, the value of final expenditure is equal to the sum of the value-added components which enter its production.

$$\underline{uy} = \underline{ue}_1 + \underline{ur}_1 + \underline{uc}_1 + \underline{ui}_1 + \underline{up}_1 + \underline{uq}_1 \qquad (9)$$

$$\underline{uy} = \underline{ut}_1 + \underline{ur}_1 + \underline{uc}_1 + \underline{ui}_1 + \underline{up}_1 + \underline{uq}_1 \qquad (10$$

The sequential pattern of sectoral analysis pursued in the formal model is as follows:

Having secured as basic data the detailed final demand and on-site labour vectors (equation 1), we trace the gross industrial requirements generated by the final material demand through the economy by accounting for inter-industry demands imposed by industries for each other.

Performed in equation 2 are the input out computation and the gross output required from each of the N industries to produce final demand.

Equations 3 and 4 translate these gross industrial outputs into gross industrial man-year labour demands and then decompose these industrial labour requirements into Z occupational categories.

In equations 5, 7, and 8, the off-site labour costs associated with the occupational demands secured in equation 4 are estimated, adjusted and then added to the on-site occupational labour costs to yield occupational breakdown of total labour costs.

In equation 6, the remaining value added components of the bill of final godds by industry are estimated by applying sets of ratio of value-added components to gross output to the data on gross output obtained in equation 2.

The total project cost thus is allocated among the value added components and then each of these components is disaggregated into either occupational or industrial detail.

For the application of the model, a large number of data computation is needed. The requirements are of 2 types:

(1) Data inputs peculiar to the final expenditure which the model is to analyse.

(2) Data inputs intrinsic to the model itself.

Under the former, we have

(1) Detailed final demands for material, equipment and supply inputs (vector $\underline{f}$).

(2) On-site employee compensation payments by occupational (vector $\underline{i}_2$)

Under the latter are:

(1) Input-output coefficients describing the inter-relationship among industries in the economy (matrix $\underline{A}$)

(2) The coefficients describing the composition of occupational requirements within each industry (matrix $\underline{B}$).

(3) The coefficients relating the value-added components to gross output for each industry (Matrix $\underline{C}$).

Haveman & Krutilla have, in their novel presentation analysed 47 projects of different natures in water resources development. Primary expenditure data were generated in a standard industrial classification details. For data intrinsic to the model, the input-output coefficients (82 x 82) evolved for 82 industries in U.S. has been utilised, based on data from the office of Business Economics. The matrix of occupational coefficients by industries had 156 occupations based on data of the Division of Occupational Employment Statistics of the Bureau of Labour Statistics. This illustrates the need for generation of similar statistics for Indian conditions and updating it yearwise for a fruitful and realistic analysis.

The values obtained need further adjustment to account for unemployment and idle capacity. For further details in this respect, the works of Haveman & Krutilla is referred to.[57]

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