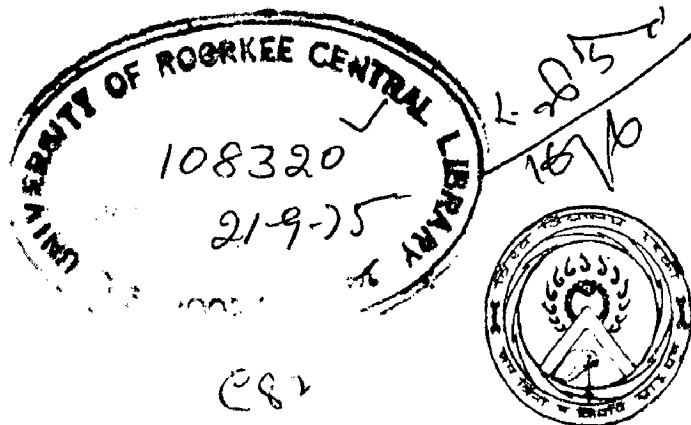# DESIGN OF HYDRAULIC STRUCTURES

ON

NON-HOMOGENEOUS FOUNDATIONS

A DISSERTATION
submitted in partial fulfilment of the
requirements for the Degree
of
MASTER OF ENGINEERING
in
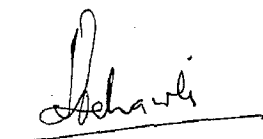WATER RESOURCES DEVELOPMENT

By

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ROORKEE U.P. (INDIA)
1975

C E R T I F I C A T E

Certified that the dissertation entitled " Design
of Hydraulic Structures on Non-homogeneous Foundations"
which is being submitted by Sri H.C.Hemantha Kumar in
partial fulfilment of the requirements for award of the
Degree of Master of Engineering in Water Resources
Development of University of Roorkee is a record of the
Candidate's own work carried out by him under my supervision
and guidance. The matter embodied in this dissertation has
not been submitted for the award of any other degree or diploma.

This is further to certify that Sri H.C.Hemantha Kumar
has worked for a period of 8 months since October, 1974 to
May, 1975 staying at Roorkee, in the preparation of this
dissertation.

Roorkee
May, 29,1975.

(A.S. CHAWLA)
Research Officer
U.P.Irrigation Research Institu
Roorkee.

ACKNOWLEDGEMENTS

C O N T E N T S

Contents (Contd..)

NOTATIONS

a $\qquad = \tanh^2 \dfrac{\pi b}{2T}$ $\quad = $ A constant in ζ plane

b $\qquad =$ Width (or half width) of floor

b_1 and $b_2 =$ Widths of floor at upstream and downstream of cut off.

b_1' and $b_2' =$ Widths of floor at upstream and downstream of cutoff in transformed plane.

b' $\qquad =$ Distance between two cut offs.

C $\qquad =$ Correction factor for mutual interference of piles

d $\qquad =$ Depth of cut off

d_1 $\qquad =$ Depth of cut off measured along its upstream face

d_2 $\qquad =$ Downstream scour depth

d' $\qquad =$ Depth of cut off in transformed plane

G_E $\qquad =$ Exit gradient

g $\qquad =$ Gravitational constant

H $\qquad =$ Difference in (Head) upstream and downstream water levels.

h $\qquad =$ The head at any point

$h_0, h_1, h_2,$ \
h_3, h_4 $\quad =$ Heads at nodes 0,1,2,3,4

$\{\text{grad } H\}$ \
$\{\text{grad } H'\}$ $= $ Head gradient (vector) in original and changed (new) principal axes of anisotropy respecatively

K and K' $=$ Complete elliptic integrals

$[K]$ and $[K']$ = Permeability matrices in original and new principal axes of anisotropy respec-tively

$\left.\begin{array}{c} K_{xx}, K_{yy} \\ k_{x1}, k_{y1} \\ k_x, k_y \end{array}\right]$ = Seepage coefficients in the directions of principal axes of anisotropy

$K_{x'x'}, K_{y'y'}$ = Seepage coefficients in new principal axes of anisotropy

k = Seepage coefficient

k_1, k_2 = Coefficients of permeability in top and bottom layers respectively.

L_1 and L_2 = Lengths of floor in semi-infinite plane

M = T/π = Transformation parameter

M' = $\dfrac{k_1 H}{2T}$ = Transformation parameter

m = d/b = Ratio of depth of central cut off to half width of floor.

m' and m = Modulii

N = Constant of Integration.

n = T/b = Ratio of depth of pervious medium to half widt of floor.

P = Argument of $Sn^{-1}(P,m)$

p = Pressure at any point on seepage fluid

q = Discharge (seepage) per unit length.

$Sn^{-1}(P,m)$ = Elliptic integral of I kind

$Sn(U,m) = P$ = Elliptic function

s	= Length of seepage segment
T	= Depth of pervious medium
T_1, T_2	= Depths of upper and lower stratum (pervious) respectively.
t	= Auxiliary semi-infinite plane.
U $(= Sn^{-1}(P,m))$	= Inverse of Elliptic function $Sn(U,m)=P$.
u and v	= Components of fictitious dimensionless velocity
v_x, v_y, v_z	= Seepage velocities in principal directions of anisotropy.
$\{v\}$, $\{v'\}$	= Velocity vectors in original and new principal directions (axes) of anisotropy.
$w = (\phi + i\psi)$ $w' = (\phi' + i\psi')$	= Rectangular flow fields.
$w_1 = (\phi_1 + i\psi_1)$ $w_2 = (\phi_2 + i\psi_2)$	= Complex potentials in upper and lower stratum respectively
X,Y	= Horizontal and vertical coordinate axes
x,y,z	= Spatial coordinates
$\bar{x}, \bar{y}, \bar{z}$	= Transformed coordinate axes in anisotropic medium.
x',y',z'	= New axes of principal directions of anisotropy
x_1, y_1 or μ, ν	= Axes along principal directions of anisotropy
z	= Complex potential plane
α	= Angle between μ axis (principal axis of anisotropy) and horizontal.
α_1, α_2	= Transformation parameters.

β = Angle between transformed cut off with μ axis.

Υ = Angle between downstream floor and cut off in fictitious μ - μ ν plane

δ = Angle between transformed straight line (floor line) to μ axis.

σ_1 and σ_2 = Transformation parameters

ϕ = Velocity potential (= khϕ)

ϕ_1, ϕ_2 = Velocity potentials in upper and lower stratum respec-tively,

ϕ_x , ϕ_D , ϕ_B = Potentials under the floor and tip of cut off.

Ψ , Ψ_1, Ψ_2 = Stream functions

$\epsilon = \frac{1}{\pi} \tan^{-1} \sqrt{k_2/k_1}$ = Parameter depending on permeability coefficients of upper and lower stratum.

$\rho = \sqrt{k_1/k_2}$ = Square root of ratio of permeability coefficients of upper and lower stratums

μ = - log m' = Negative logarithm of complementary modulus.

$\lambda = \nu^2 = k_{y1} / k_{x1}$ = Ratio of permeability coefficients of in principal directions of anisotropy

ζ = Auxiliary semi infinite plane.

π = Constant.

S Y N O P S I S

Approximate or closed - form solutions are available for the problem of seepage below the hydraulic structures founded on homogeneous and isotropic medium with various boundary conditions. But mostly the foundation met with below hydraulic structure is rarely homogeneous and isotropic. Anisotropy , stratification and non homogeneity to some degree or the other are common characters of foundation below hydraulic structures . The occurrence of pervious foundations of finite depth is also a common feature.

In this dissertation an attempt has been made to review critically the effect of (1) finite depth of pervious medium (2) stratification (3) Anisotropy and (4) Non-homogeneity on the uplift pressures below the structure, exit gradient and seepage discharges.

The effect of finite depth of pervious medium on uplift pressures (1) below the floor (2) at junction of floor and cut off and (3) at tip of cut off has been reviewed. Also the effect of downstream scour depth on uplift pressures and exit gradient in the case of an apron (floor) with downstream end cut off has been studied .

In the case of structure on two strata of equal thickness and of different permeabilities, general formulae are given based on solution obtained by Polubarinova-Kochina, to calculate uplift pressures below the floor and also to find seepage discharge.

An attempt has also been made to study the effect of anisotropy of foundation soil (with principal directions of permeability not coinciding with coordiate axis of physical plane) on the pressures (1) at junction of cut off and floor and (2) at tip of cutoff, in the case of a floor with a downstream end cut off. A vertical cutoff in an anisotropic medium, will , after suitable transformation become an inclined one in the transformed plane. The effect of both finite depth of pervious medium and the depth of inclined cut off on pressures at tip of cut off and exit gradient, in the case of a stepped floor, has been briefly reviewed.

Finally,the versatility and wide range of applicability of finite element method for solution of seepage in Non-homogeneous foundations has been reviewed.

1.1 HISTORICAL BACKGROUND

Design of Hydraulic structure requires testing their
stability against forces due to surface flow and against forces
caused by seepage water. The water seeps under the foundations
of such structures and exerts pressures on the bottom and tends
to wash away the soil under it. The structure has to be made
heavy enough to withstand the uplift pressures and the value
of the exit gradient of the first streamline has to be kept
within safe value by providing end cutoff.

The application of conformal mapping to the solution of
confined flow below the foundations of hydraulic structures was
for the first time indicated by Pavlovsky (14) and Khosla(7).
Based on conformal transformation, general solutions for the
various boundaries have been given by Muskat (12), Harr (6)
and Pollubarinova-Kochina (15) and Arvin and Numerov (3). These
solutions are applicable for the seepage below hydraulic struc-
tures founded on homogeneous amd isotropic permeable soil with
various boundary conditions. In practice, the homogeneous and
isotropic soil extending upto infinite depth is seldom encountered.
The homogeneous permeable soil may be underlain by a homogeneous
stratum of hydraulic conductivity higher or smaller than that
of upper stratum. On the other hand, the hydraulic structure
may be underlain by homogeneous but anisotropic stratum extend-
ing upto finite or infinite depth.

The solutions of seepage below hydraulic structures founded on homogeneous isotropic soil extending upto finite depth and underlain by impermeable or highly permeable layer were obtained by Pavlovskii (14) . Kulandaiswamy (8) and Muthukumaran (13) have prepared design charts for determining exit gradient and uplift pressures below the foundation of the structure founded on soil underlain by an impervious layer.

The solution of the seepage below structures with a flat bottom founded on two layers of equal thickness is given by polubarinova - Kochina (15) and that for two layers of unequal thickness has been obtained by Lenau C.W. (9). The uplift pressures below flat floor with end cut off founded on two layers, upper layer being of higher permeability has been studied by Alamsingh and Punmia (1). The effect of higher permeability of lower layer has been determined by Sharma et. al. (22). They have combined analytical and experimental results to provide design curves for determining exit gradient and uplift pressures, covering the complete range of the values of k_2/k_1 from 0 to infinity.

The effect of anisotropic media on the uplift pressures has been studied experimentally by P.V.Rao (19), Alamsingh and Punmia (2) and by Punmia and Patwa (18).

In the present dissertation an attempt has been made to study the effect of non homogeneity of the foundation on the stability of the structures. The cases with the following boundary conditions have been reviewed critically.

(1) Hydraulic structures founded on permeable soil of finite depth underlain by impermeable or very pervious layer (Chapter II)

(2) Structures founded on stratified foundations (Chapter III)

(3) Structures founded on homogeneous but anisotropic soil (Chapter IV)

(4) Application of Numerical methods for the solution of problem of seepage below hydraulic structures founded on nonhomogeneous soil (Chapter V).

C H A P T E R   II

PERVIOUS   FOUNDATION   OF   FINITE DEPTH

2.1   GENERAL

The solution of seepage below hydraulic structure founded on finite depth of permeable soil could be obtained by conformal mapping, approximate methods such as method of fragment, or by experimental methods. Pavlovskii (14) was the first to treat the problem of seepage as a problem of mathematical physics and used the method of conformal transformation for its solution. He obtained the solution of seepage below a flat apron with a central cut off on limited depth of pervious foundation. Muskat (12) analysed the case of flat floor with sheet piling anywhere on finite depth of permeable soil. Pollubarinova. Kochina(15) gave analytical solutions by the method of conformal mapping for seepage below flat bottomed structures with single and two cut offs founded on permeable foundation of finite depth. Chugaev (4) while finding a method of determining the underground profile of a structure which is safe in regard to percolation introduced a method called the "method of resistive coefficients" (method of fragments) where in the underground profile was divided into seperate elements resistive coefficient for each separate element determined, and hydraulic head distributed in proportion to these

coefficients. The uplift diagram could thus be drawn. The method is suitable for any complicated weir profile with number of cut-offs on pervious foundations of finite depth.

2.2. STEPPED APRON WITH A CUT OFF

The general case of a stepped apron with a cut off founded on pervious foundation of finite depth has been dealt by Sri MuthuKumaran (13). He has given the design procedure and provided charts for determining the exit gradients and uplift pressures at salient points of weir profile.

The weir profile is shown in Fig. 2.1. An impervious floor AECB of length $b_1 + b_2$ located on a permeable soil of stratum depth T has been considered. The floor has a step of depth, d_2 at E and a sheet pile of depth $(d_1 - d_2)$ at the step. The downstream pervious bed is at the same level as the down stream impervious bed. The differential equation govering seepage phenomenon is

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad \ldots (2.1)$$

where ϕ = velocity potential equivalent to 'kh',

h = the head, k = the coefficient of percolation, and x and y are spatial coordinates.

The foundation profile AEDCB forms the inner boundary of flow and represents streamline $\psi = 0$, in which ψ = stream

(a)

Z - PLANE

(b)   p - PLANE

(c)

ω' - PLANE

FIG. 2·1

function. The lower impervious boundary FHG represents
the streamline ψ = q , where q = the total discharge
per unit width percolating through the foundation. Along
the boundary AF at the upstream bed level potential ϕ=kH
i.e., the total head, and at the downstream bed, BG poten-
tial function, ϕ = 0 .

Both the profiles of the structure in the z plane
and the rectangular flow field of the w plane can be trans-
formed on to the lower half of the same semi-infinite p-
plane. Thus

$$z = f(p) \qquad\qquad ...(2.2)$$

and $\qquad$ w = F(p) $\qquad\qquad$ ...(2.3)

are obtained in which z = x+iy , representing physical
plane, p = s+it, representing an auxiliary semi-infinite
plane, and w = ϕ + iψ , representing the rectangular flow
field. For convenience, a new variable w$'$ = ϕ' + iψ'
has been introduced which is defined as

$$\phi' = \frac{\phi}{kH} K \qquad\qquad ...(2.4)$$

$$\psi' = \frac{\psi}{q} K' \qquad\qquad ...(2.5)$$

where K and K$'$ are complete elliptic integrals of the
first kind with modulus 'm' and m$'$ respectively and m$'$
= 1-m . Combining equations 2.2 and 2.3 and replacing
w by w$'$, we obtain

$$z = f(p) = fF^{-1}(w') \qquad\qquad ...(2.6)$$

$$\text{and} \quad w' = F(p) = Ff^{-1}(z) \qquad\qquad ...(2.7)$$

The relationship between z and p fields is given by

$$z = \frac{2}{\pi} (T-d_2) \tanh^{-1} \sqrt{\frac{(\sigma_2 +1)(p-1)}{(\sigma_2 -1)(p+1)}}$$

$$- \frac{2T}{\pi} \tanh^{-1} \sqrt{\frac{(\sigma_1 -1)(p-1)}{(\sigma_1 +1)(p+1)}} + i d_2 \quad \ldots(2.8)$$

The transformation constants are given by

$$\sigma_i = \sec \alpha_i , \quad (i=1,2) \qquad \ldots(2.9)$$

$$\frac{\sin \alpha_1}{\sin \alpha_2} = 1 - \frac{d_2}{T} \qquad \ldots(2.10)$$

$$\alpha_2 = \frac{\pi (1 - \frac{d_1}{T}) - \alpha_1}{(1 - \frac{d_2}{T})} \qquad \ldots(2.11)$$

$$b_1 = \frac{2T}{\pi} \tanh^{-1} \sqrt{\frac{(\sigma_1 +1)(\beta_1 -1)}{(\sigma_1 -1)(\beta_1 +1)}}$$

$$- \frac{2}{\pi} (T-d_2) \tanh^{-1} \sqrt{\frac{(\sigma_2 -1)(\beta_1 -1)}{(\sigma_2 +1)(\beta_1 +1)}} \quad \ldots(2.12)$$

$$b_2 = \frac{2}{\pi}(T-d_2) \tanh^{-1} \sqrt{\frac{(\sigma_2 +1)(\beta_2 -1)}{(\sigma_2 -1)(\beta_2 +1)}}$$

$$- \frac{2T}{\pi} \tanh^{-1} \sqrt{\frac{(\sigma_1 -1)(\beta_2 -1)}{(\sigma_1 +1)(\beta_2 +1)}} \quad \ldots(2.13)$$

where β_1, β_2, σ_1 and σ_2 are transformation parameters. The relation between w' and p fields after transformatin is

$$w' = Sn^{-1} (P,m) \qquad \qquad \ldots (2.14)$$

where $Sn^{-1}(P,m)$ is elliptic integral of first kind with P as argument and m as modulus.

$$m = \frac{(\beta_1 + \beta_2)(\alpha_1 + \alpha_2)}{(\beta_1 + \alpha_2)(\beta_2 + \alpha_1)} \qquad \ldots (2.15)$$

$$P = \sqrt{\frac{(\alpha_2 + \beta_1)(\beta_2 - p)}{(\alpha_2 - p)(\beta_1 + \beta_2)}} \qquad \ldots (2.16)$$

Potentials underside of floor and sides of sheet pile as a ratio of total differential head are given by

$$\phi = \frac{1}{K} Sn^{-1}(P,m) \qquad \qquad \ldots (2.17)$$

2.2.1 Calculation of uplift pressures

It is not convenient to use equation (2.17) when the width of weir is comparable to depth of pervious foundation. Under such circumstances m and P are close to unity, the elliptic integrals in equation (2.17) vary rapidly and the values of m and P have to be computed to many significant places to get accurate results. It is therefore, necessary to rewrite the solution in some other form and express the elliptic integrals in terms of some other parameters.

From equations (2.15) and (2.16)

$$m' = 1 - m = \frac{(\alpha_1 - \beta_1)(\alpha_2 - \beta_2)}{(\alpha_2 + \beta_1)(\alpha_1 + \beta_2)} \qquad \ldots (2.18)$$

$$1 - mP^2 = \frac{(\alpha_2 - \beta_2)(\alpha_4 + p)}{(\alpha_1 + \beta_2)(\alpha_2 - p)} \qquad \ldots (2.19)$$

Equation (2.17) is modified as

$$\phi = \frac{1}{K} dn^{-1}\left[\sqrt{(1-mP^2)}, \ m\right] \qquad \ldots (2.20)$$

or $\quad dn^2(\phi K, m) = 1 - mP^2$

where $dn(\phi K, m)$ is an elliptic function given by

$dn^2(U,m) = 1 - mSn^2(U,m)$, $Sn(U,m) = P$, is an elliptic

function whose inverse is the elliptic integral of the

first kind , $\quad U = Sn^{-1}(P,m)$

The elliptic function dn, can be evaluated by

expanding it in an infinite series of functions of its

argument and m' . The complete integral K can also be

computed for given m' . Thus ϕ can be computed for given

values of $(1-mP^2)$ and m' . The latter quantities can be

computed from known values of d_1, d_2, T, b_1 and b_2, by using

equations (2.18) , (2.19) and (2.8) to (2.13).

2.2.2 Exit Gradient

In addition to uplift pressures it is also important

to find the hydraulic gradient at the downstream end of

the percolation trajectory.

Gradient G at any point is given by

$$G = \frac{dh}{ds} \qquad \qquad \ldots(2.21)$$

where h = head at any point along the floor or cut off

$$= \frac{H}{K} \phi'$$

s = distance along the streamline passing throught the point.

Equation (2.21) can be rewritten as

$$G = \frac{H}{K} \frac{d\phi'}{ds} \qquad \qquad \ldots(2.22)$$

At the exit point B where $w' = \phi'$, exit gradient G_E is given by

$$G_E = \frac{H}{K} \left| \frac{dw'}{dz} \right|_{z = b_2 + id_2} \qquad \ldots(2.23)$$

$$= \text{infinity, if } b_2 > 0 \ .$$

The quantity of seepage is given by

$$q = kHK'/K \qquad \qquad \ldots(2.24)$$

where K' is the complete elliptic integral with modulus m'.

2.2.3 Charts

The potentials at salient points in the underprofile of the weir can be determined by the following steps

(1) α_1 and α_2 are computed using equations (2.9) to (2.11) by trial and error.

(2) β_1 and β_2 are obtained from equations (2.12) and (2.13) by trial and error.

(3) $(1-mP^2)$ is calculated from equation (2.19)

(Note:- p takes values -1,0,1 at E,D,C respec-tively)

(4) m' is determined from equation (2.18)

(5) Using equation (2.20) and steps (3) and (4) ϕ is computed.

The values of σ_1 and β_i (i=1,2) in the equations for potentials are close to unity and hence $\sigma_1 - \beta_1$ is very small. But it cannot be neglected, since small variation in this leads to large variations in potential distribution under the floor. Therefore charts have been provided for facilitating the computations of σ_1 and ($\sigma_1 - \beta_1$) for i = 1,2. For finding σ_1 two charts (Fig.2.2 and 2.3) are provided where σ_1 can be read for given values of d_1/T and d_2/T . Once the value of σ_1 is known σ_2 can be worked out using equations (2.10) and (2.11) . For finding ($\sigma_i - \beta_i$) for i = 1,2, Using Figs. (2.4) and (2.5) the following trial and error procedure is adopted. First a suitable value for ($\sigma_1 - \beta_1$) is assumed, based on experience. Using this value and known values of σ_1 , R_I and R_{II} are read from Figs. (2.4) and (2.5) respectively. Then from equation

$$b_1 \times \frac{\pi}{2T} = R_I - (1 - \frac{d_2}{T}) R_{II} \qquad ...(2.25)$$

b_1/T is computed and compared with known value of b_1/T. If b_1/T is greater than the computed value, then ($\sigma_1 - \beta_1$) must be reduced and vice versa.
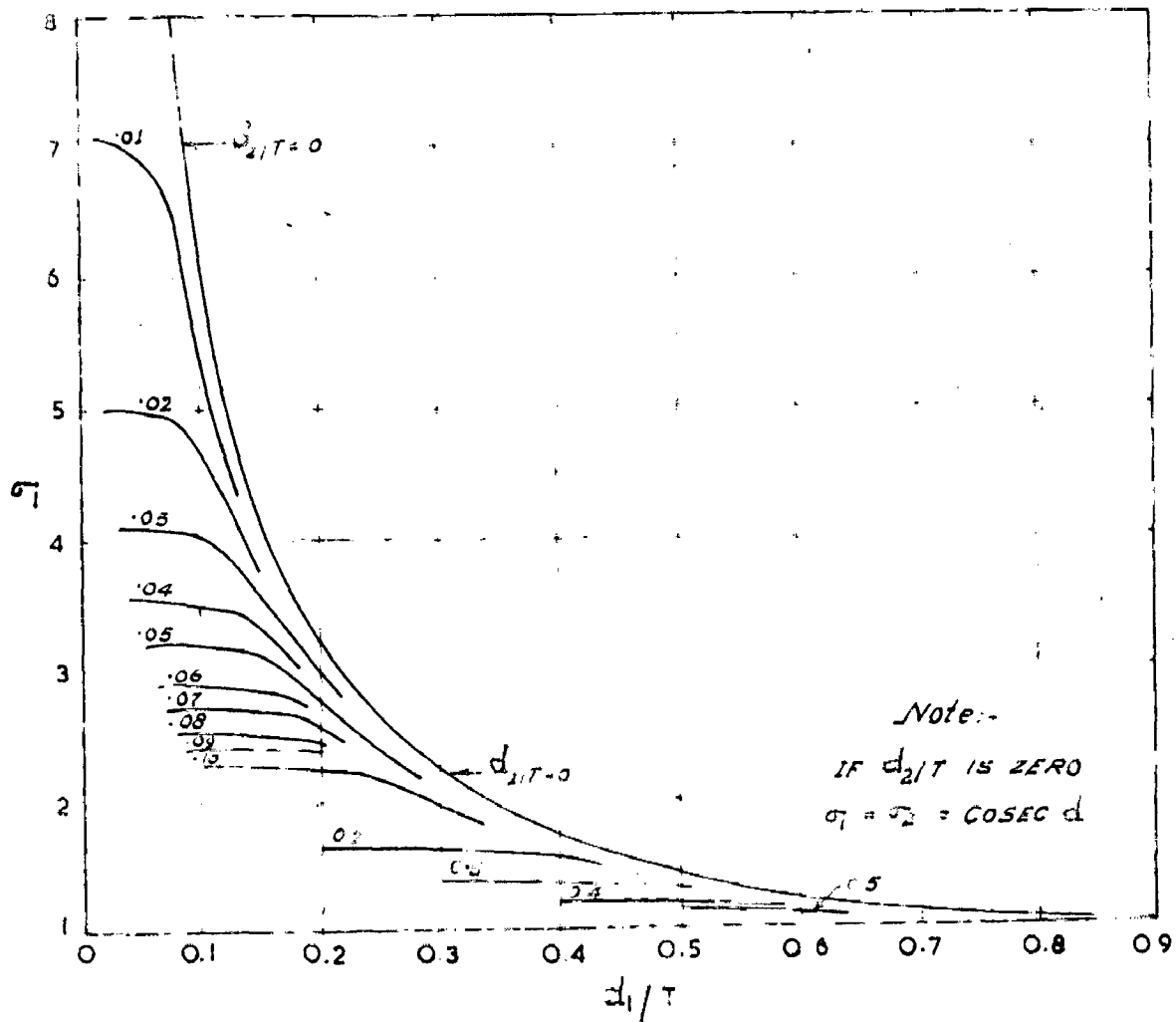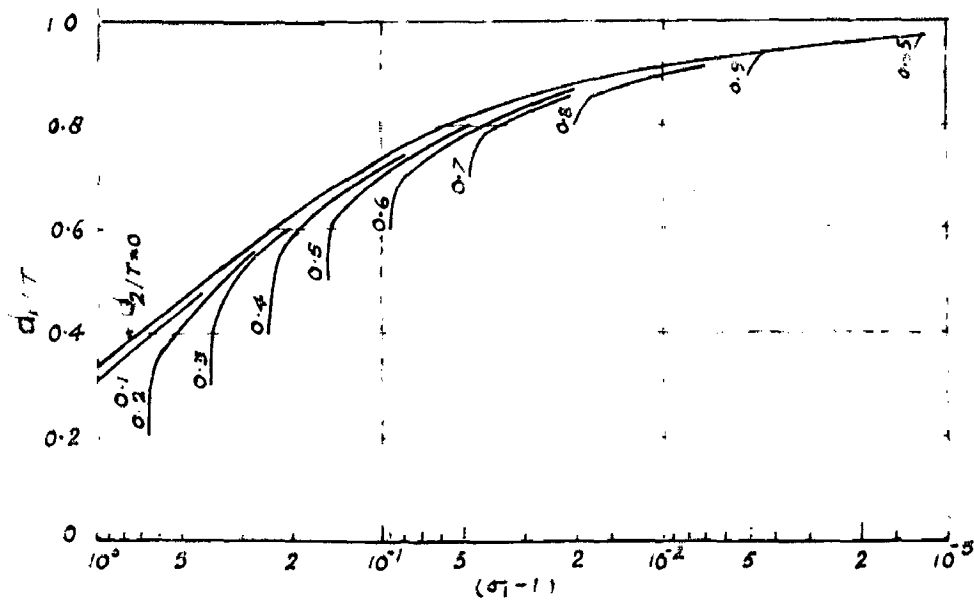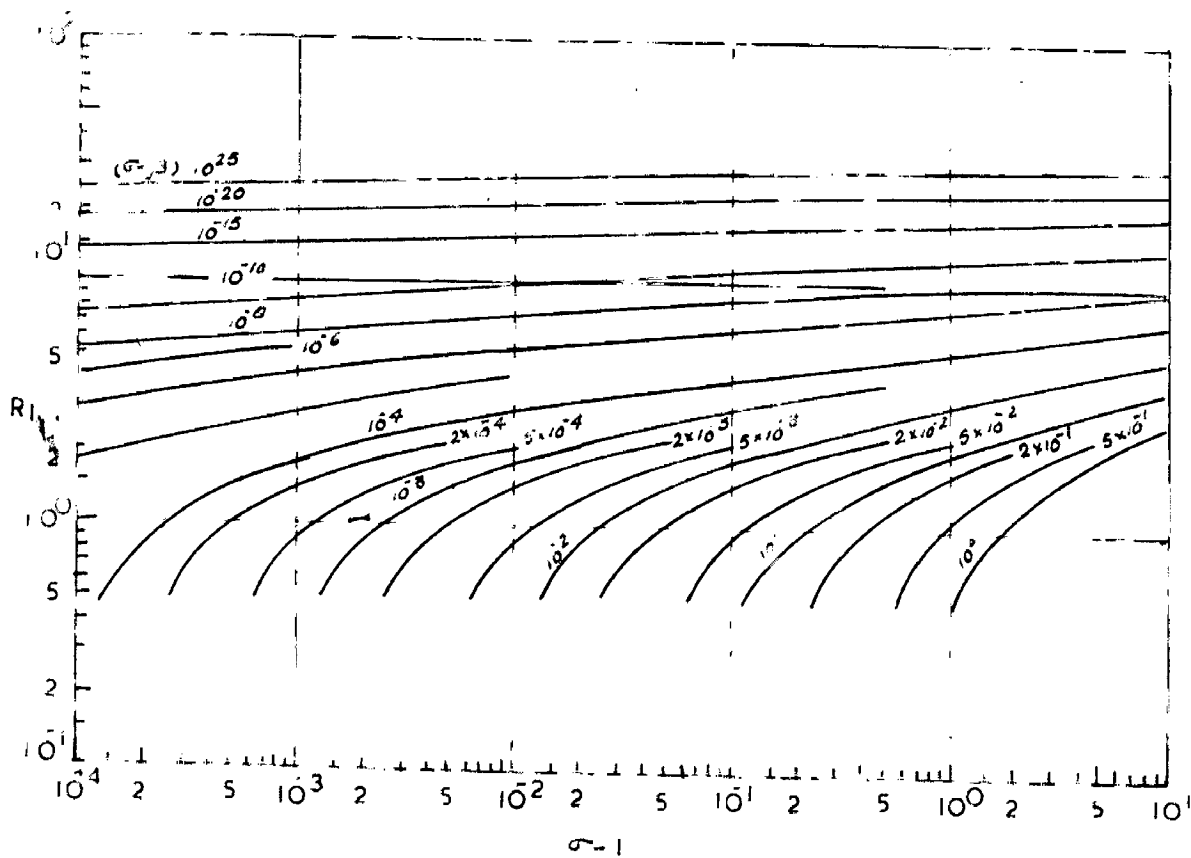
FIG 2·2 CHART FOR σ_1.
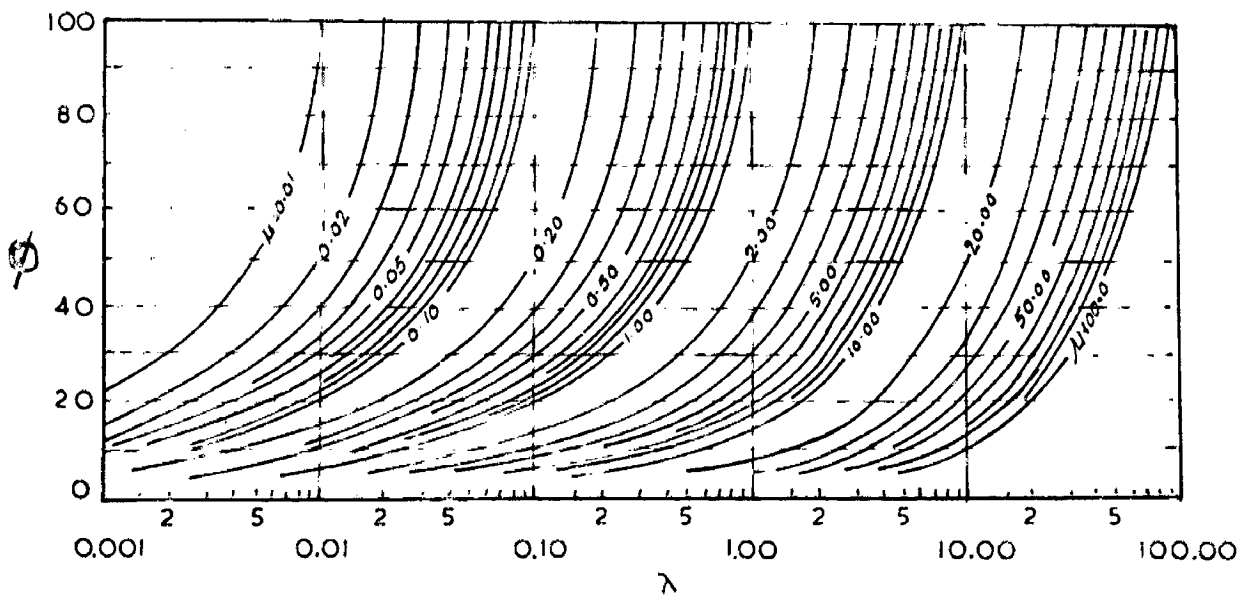

FIG 2·3 CHART FOR (σ_1-1)

FIG 2·4 CHART FOR R₁

FIG 2·6 CHART FOR POTENTIAL

A Similar procedure and the use of equation,

$$\frac{b_2 \pi}{2T} = (1 - \frac{d_2}{T}) R_I - R_{II} \qquad \qquad ...(2.26)$$

will enable, determination of the value of $(\bar{\sigma}_2 - \beta_2)$

Since $(\sigma_1 - \beta_1)$ is small in practical cases $\wedge$ since

R_{II} does not contain $(\sigma_1 - \beta_1)$, R_{II} can be read from Fig. (2.5) , assuming $\sigma_1 = \beta_1$ and R_I is then worked out by equation (2.25) . Therefore in one single trial $(\sigma_1 - \beta_1)$ can be read from figure (2.4) as R_I and σ_1 are known.

Figure (2.6) shows the chart for reading m' and $(1-mp^2)$. The following substitution is used since the values of m' and $(1-mP^2)$ are generally small.

$$\mu = - \log m' \qquad \qquad ...(2.27)$$

$$\lambda = - \log \left[1-mP^2 \right] \qquad \qquad ...(2.28)$$

All the charts prepared, cover the following ranges.

(1) T/b varying from negligible values to about 2.5

(2) T/d_1 varying from negligible values to about 5.

(3) b/d_1 varying from about 0.5 to about 30.

The author observed by trial computations that when T/b = 2.5 and $T/d_1 = 5$, the potentials at salient points were approximately equal to the corresponding values for infinite depth of pervious foundations.
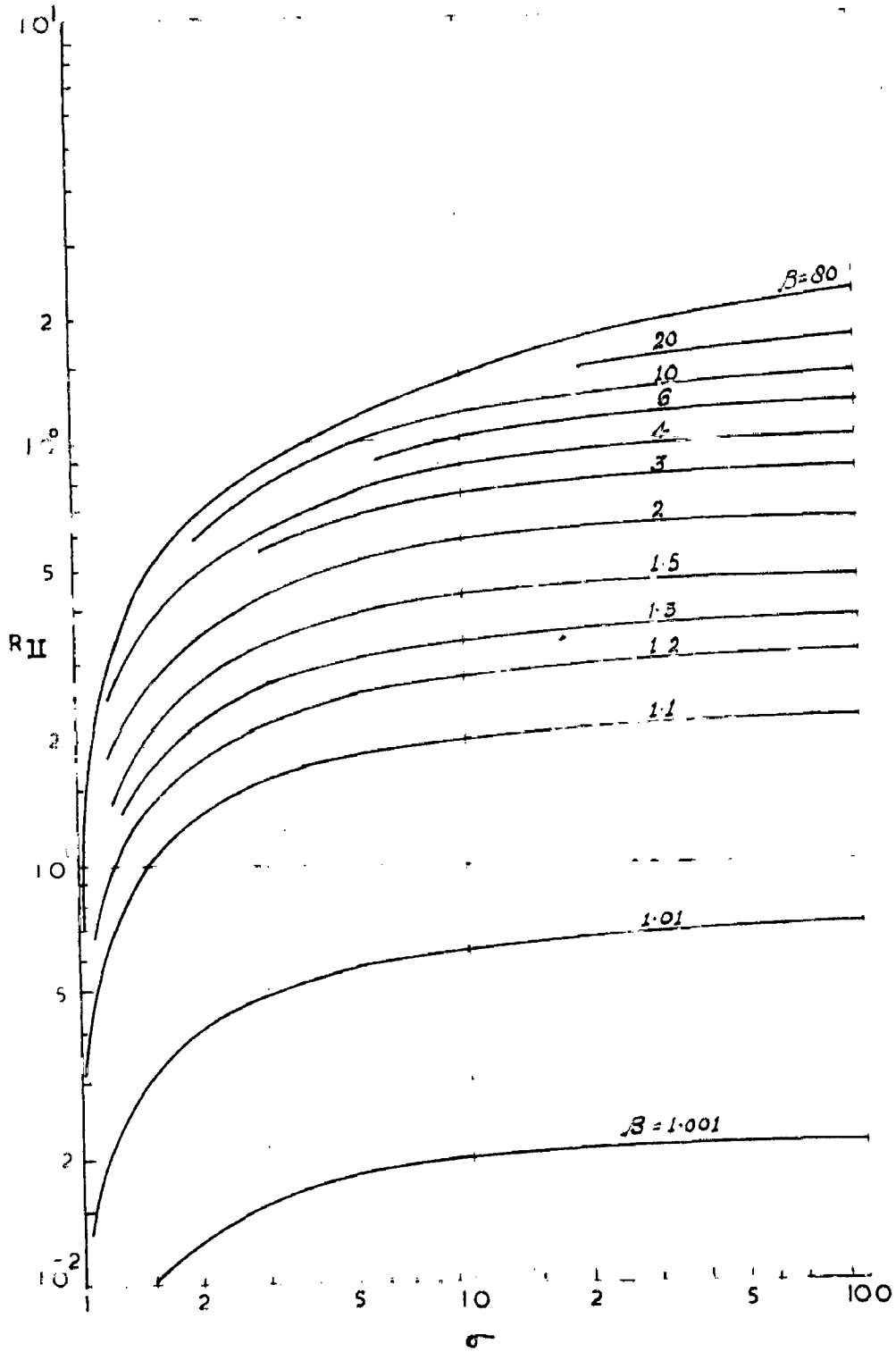
FIG. 25 CHART FOR R_{II}.

2.2.3 Mutual Interference of Piles

By improving the formula given by Khosla, the author gave after experimental verification, formula for mutual interference of piles for finite depth of pervious foundation, as follows:

$$C = 19 \sqrt{\frac{\sin \frac{\pi d_1}{2T}}{\sinh \frac{\pi b'}{2T}}} \cdot \frac{\sin \frac{\pi d_1}{2T} + \sin \frac{\pi d_2}{2T}}{\tanh \frac{\pi b}{2T}} \qquad \ldots (2.29)$$

Where C = Correction at the corner where sheet pile of depth d_2 meets the floor, due to the presence of sheet pile of depth d_1.

b' = distance between piles

b = Width of floor.

It may be seen that equation (2.29) reduces to equation given by Khosla when T is large. The correction in equation (2.29) is additive if pile of depth d_1 is on the down-stream side of pile of depth d_2 and subtractive if otherwise. The equation is valid for the following conditions-

(1)   The intermediate pile is longer or equal to outer pile.

(2)   The distance between the piles is more than twice the length of the outer pile.

(3)   The depth of the pervious stratum below the tip of the pile (t) is greater than 0.2b for b/T varying from negligible values to 2. When b/T

equals 12, the minimum required value of t/b is

negligible. For values of b/T between 2 and 12

minimum required value of t/b varies linearly

from 0.2 to 0.

This equation was verified using both experimental and theoretical results and found to be valid.

2.2.4 Safety Against Piping Failures

Piping failures are caused by seepage erosion or heaving. The seepage erosion, in potential theory is analysed on the basis of the exit gradient at the point on the pervious region nearest to the downstream end of the structure. Khosla, who has derived exit gradient for the general case and other simple weirs on infinite depth foundation (pervious) has shown that the result of the case of a floor with a sheet pile at end is adequate for practical purposes.

2.3 FLAT APRON (WITHOUT A STEP) WITH AN INTERMEDIATE CUT OFF

This cas is a particular form of general case discussed in previous section where a floor with a step and an intermediate cut off was considered. By putting $d_2 = 0$ and $d_1 = d$, we get the present case. The solution for this case was obtained by applying Schwarz-Christoffel transformations successively twice, on the same principles

adopted in the previous case. Charts for finding potentials at salient points and for finding exit gradients were provided by V.C. Kulandaiswamy and Muthu Kumaran(8).

The uplift pressures and exit gradient are given by

$$\phi = \frac{1}{K} \, dn^{-1} \left[\sqrt{\frac{1-\beta_2}{1+\beta_2} \cdot \frac{1-p}{1+p}} \quad , \, m \right] \qquad \ldots(2.39)$$

$$G_E = \frac{H}{K \sqrt{m \, \sin \frac{\pi d}{2T}}} \quad , \qquad \ldots(2.31)$$

where,

$$m = \frac{2(\beta_1 + \beta_2)}{(1+\beta_1)(1+\beta_2)} \qquad \ldots(2.32)$$

The equation (2.31) for exit gradient applies only when the cut off is at the end of floor, while the exit gradient for an intermediate cut off is infinity. As the quantities to be used as variables in the chart namely m' and $(1-mp^2)$ are generally small, their negative logarithms have been used i.e.,

$$\mu = -\log m' \qquad \ldots(2.33)$$

$$\lambda = -\log \left[\frac{1-\beta_2}{1+\beta_2} \, \frac{1-p}{1+p} \right] \qquad \ldots(2.34)$$

where, $m' = 1-m$ and

where $p = -\alpha = -\sin \frac{\pi d}{2T}$ for point E (Refer Fig.2.1a)

$p = 0$ for point D and $p = \alpha = \sin\dfrac{\pi d}{2T}$ for point C.

The charts for potentials (Fig. 2.7) are prepared using μ and λ as variables. The chart for the exit gradient (Fig. 2.8) is computed using μ and d/T as variables. The values of μ and λ can be computed using the available data namely b_1, b_2 , d and T.

The use of charts to find(1) potentials at points E,D and C (Fig. 2.1a) and(2) exit gradient is explained in the following steps.

(i) The values of $\dfrac{1 - \beta_1}{1 + \beta_1}$ and $\dfrac{1 - \beta_2}{1 + \beta_2}$ are computed from the equation,

$$\sqrt{\frac{1 - \beta_1}{1 + \beta_1}} = \frac{\cos\dfrac{\pi d}{2T}\ \operatorname{sech}\dfrac{\pi b_1}{2T}}{1 + \sqrt{1 - \cos^2\dfrac{\pi d}{2T}\operatorname{sech}^2\dfrac{\pi b_1}{2T}}} \qquad \ldots (2.35)$$

(ii) $m'(= 1-m)$ is computed from

$$m' = \frac{1 - \beta_1}{1 + \beta_1} \times \frac{1 - \beta_2}{1 + \beta_2} \qquad \ldots (2.36)$$

(iii) μ is computed from equation (2.33).

(iv) $\dfrac{1 - \sin\dfrac{\pi d}{2T}}{1 + \sin\dfrac{\pi d}{2T}}$ is computed and using equation (2.34) λ is determined.
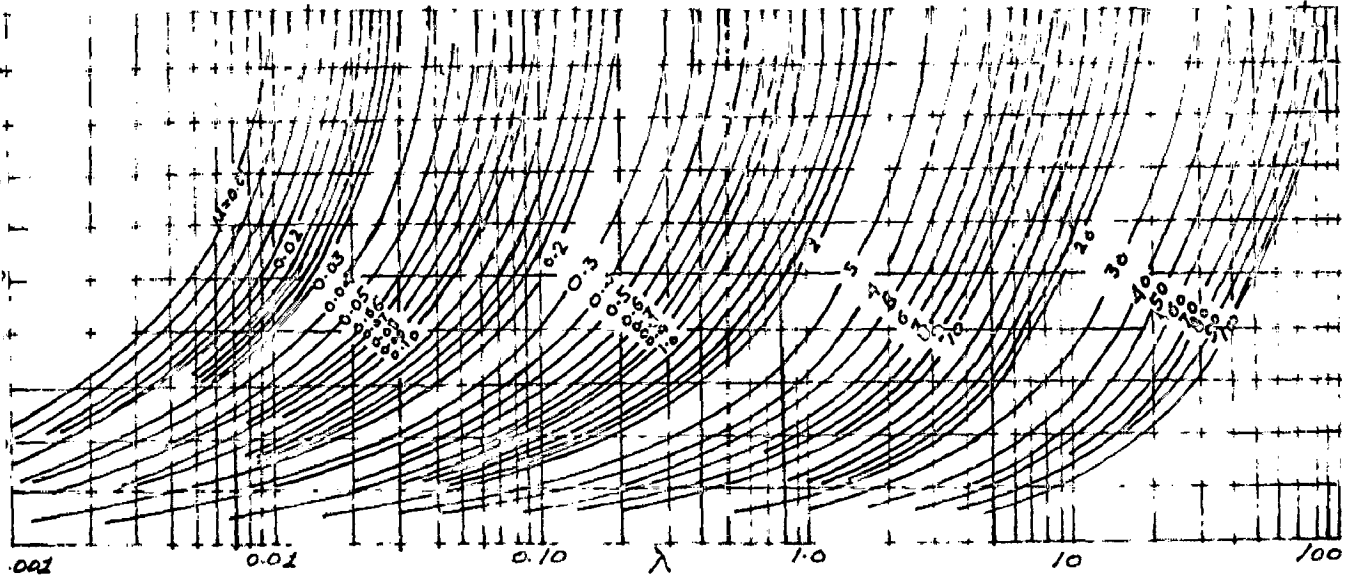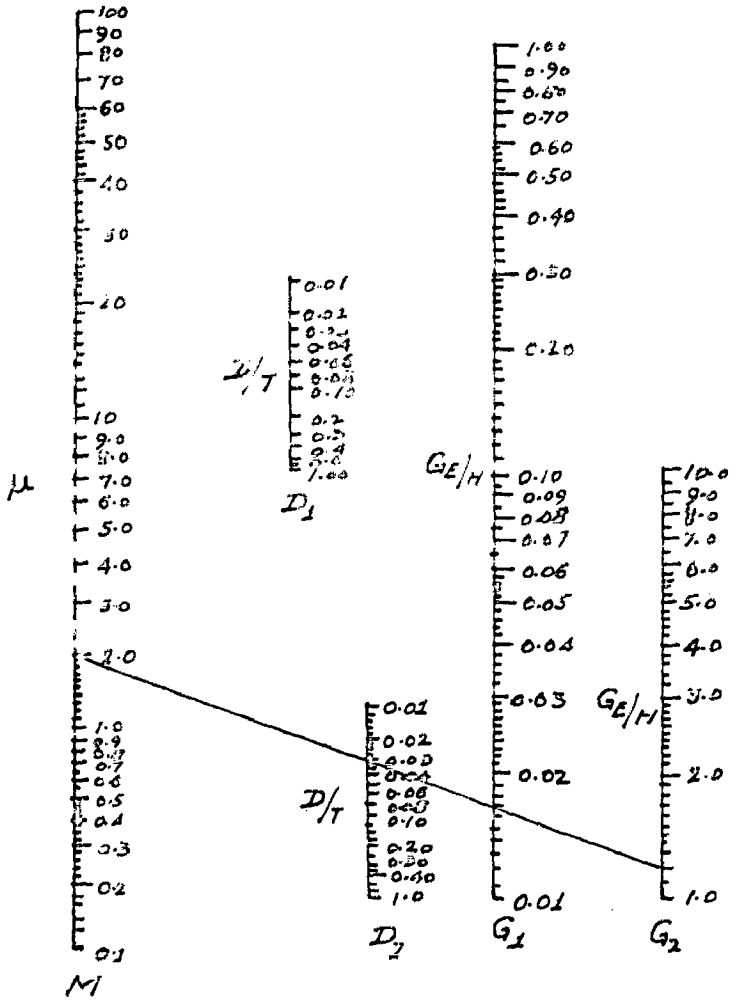
FIG. 2·7 CHART FOR POTENTIALS.

FIG. 2·8 CHART FOR EXIT GRADIENT.

(v)   With the values of μ and λ , the value of ϕ
        is read from Fig. (2.7)

(vi)   From Fig. (2.8) with the known value of μ on line
        M proceed through the d/T value in line
        D_1 or D_2 straight to the corresponding line G
        i.e., from D_1 to G_1 or D_2 to G_2 and read the
        value of G_E / H   i.e., $\dfrac{1}{K\sqrt{m\sin\dfrac{\pi d}{2T}}}$.

(vii)   The exit gradient is computed using equation(2.31)

        The formula for correction for mutual interference
of piles (in case of weirs with more than one pile) and its
limitations, is the same as given in previous section.
The validity of this formula was checked by the authors
by conducting experimental studies and found that the
variation was within 5%.

        The authors observed substantial variation in
both potential at salient points and exit gradients (G_E
increased) when the depth of pervious stratum below tip of
sheet pile was increased from 0.20 to infinity, while
studying a typical weir profile with two piles.

2.4 EFFECT OF IMPERMEABLE LAYER ON UPLIFT PRESSURES

        Based on the results of conformal mapping
Pavlousky (14) studied the effect of lower impermeable
layer on the uplift pressures. He carried out calcula-
tions for various combinations of the different parameters

to determine uplift pressures in the downstream portion
of floor with central cut off. The parameters affecting
the pressures are $n = T/B$ and $m = d/b$ where b = half
width of floor, T = depth of impermeable boundary and
d = depth of central cutoff. Following values of these
parameters were considered .

$$n = 10.0 \text{ and } 1.0$$
$$d/T = 0.3, 0.5 \text{ and } 0.7$$
$$m = 3,5 \text{ and } 7 \text{ for } n = 10.0$$
$$= 0.3 , 0.5 \text{ and} 0.7 \text{ for } n=1.0$$

In addition the extreme case of $n= \infty$ i.e., the depth of
permeable stratum extending upto infinite depth was also
considered.

The effect of m and n on the uplift pressures
was studied at two selected points i.e., the point A, imme-
diately downstream of the piling and a point B, at the
middistance between the piling and the downstream edge of
the floor. The uplift pressures are given in Table 2.1
and plotted in Fig. 2.9. It is seen that-

(a)   The effect of impermeable layer is negligible
for $n > 5$ and $m \leq 0.1$. The difference in th-e
uplift pressure calculated for $n = \infty$ and for $n \geq 5$ is
less than 1% when the value of $,m \leq 0.1$.

(b)   The difference in uplift pressures calculated for $n = \infty$ and $n \geqslant 3$ is less 10% when the value of $m \leqslant 0.4$.

Pavlovs ky investigated another problem of what depth should be given to the piling to make it reasonably effective. He gave two graphs drawn from theoretical calculations (Fig. 2.10) to show efficiency of piling of various depths. The efficiency (h_{eff}) was presented as the ratio of head lost in between points namely upstream junction point of pile and floor and downstream junction point, expressed as a percentage of total head. The interesting point in the graph is that line for $T/b = 2$ for case of central pile is a straight line. The curves on one side of it are concave while those on otherside are convex. So if $T/b = 2$ is taken as critical value it follows, from diagram that for $T/b < 2$ specific efficiency rises with increase in depth of pile while for $T/b > 2$ it drops with increase in depth of pile.

Thus whilst for infra critical values of T/b the piling should be as deep as technically possible, no material advantage is gained in the case of hyper critical values of T/b , by increasing the piling beyond a certain limit.
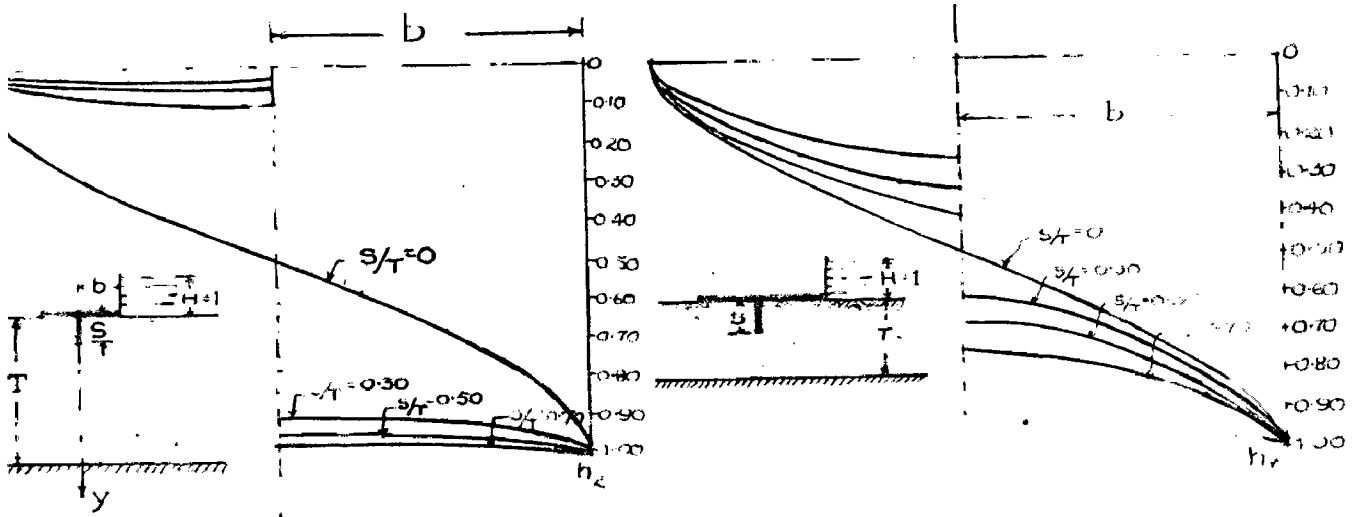
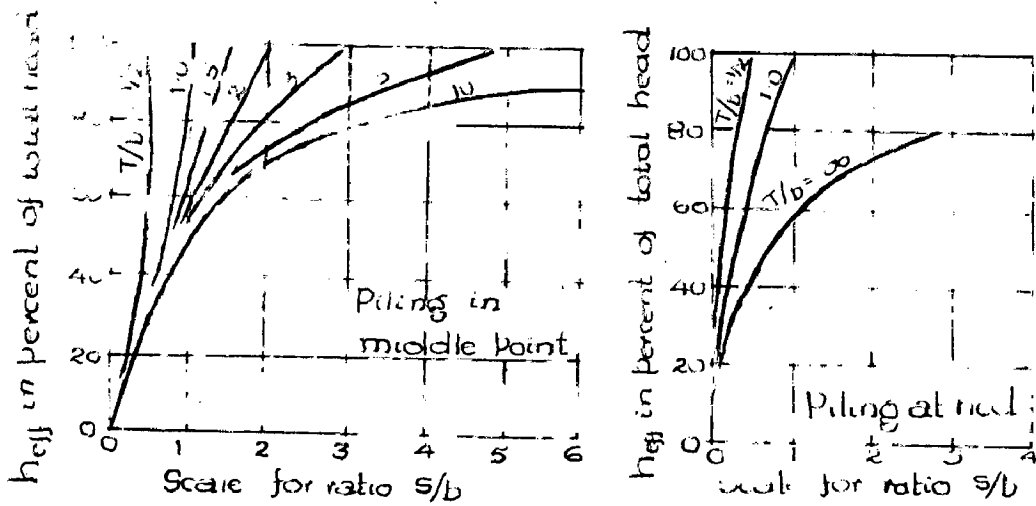thrust Curves for T/b = 10

Upthrust curves for T/b = 1·0

FIG :- 2, 9

FIG :- 2·10

TABLE 2.1

(a) for T/b = 10.0

Point	m = 3			m = 5			m = 7		
	n=10	n=∞	Bligh	n=10	n=∞	Bligh	n=10	n=∞	Bligh
A	0.099	0.101	0.125	0.060	0.063	0.083	0.039	0.045	0.062
B	0.085	0.088	0.061	0.048	0.054	0.042	0.031	0.039	0.031

TABLE 2.1

(b) for T/b = 1.0

Point	m-0.3			m=0.5			m=0.7		
	n=1.0	n=∞	Bligh	n=1.0	n=∞	Bligh	n=1.0	n=∞	Bligh
A	0.394	0.406	0&385	0.328	0.353	0.333	0.256	0.306	0.295
B	0.288	0.310	0.192	0.249	0.282	0.167	0.198	0.251	0.147

2.5 FLAT APRON WITH DOWNSTREAM CUTOFF UNDER SCOURED CONDITIONS

2.5.1 Uplift Pressures

In the previous sections, determination of uplift pressures and exit gradients below apron with an intermediate cut off, either with or without a step, was discussed. Also the formula and chart for finding the exit gradient for the case of a floor with an end cut off were given. But when the apron is under scoured conditions, i.e., when

there is difference in levels between upstream and downstream of an apron (with a downstream end cut off), the pressure below it and the exit gradients differ from the case of apron having equal upstream and downstream elevations. One such case was investigated experimentally by B.C. Punmia et. al. (16) by electrical analogy method.

In the experimental study the authors studied the influence of parameters like b/T, d_1/T, and d_2/T on pressures at (1) below the apron (2) at junction of floor and downstream end cutoff (3) at tip of cut off, where d_2 is the downstream scour depth and other symbols have the usual notation. The sequence of their study was as follows . (a) the values of b/T were kept equal to 1/2, 1, 2 and 4 (b) For each b/T the value of d_1/T was taken as 0.2, 0.4, 0.6 and 0.8 , (c) For each b/T and d_1/T the value of d_2/T was taken as 1/8 , 2/8 , 3/8 and 4/8.

The influence of the ratios of various parameters namely the width of apron, the depth of porous media, depth of cut off downstream scour depth on the pressures (i) below the apron (ϕ_x) (ii) at junction of apron and cut off(ϕ_e) (iii) at tip of cut off (ϕ_D) are shown in Figures (2.11) to (2.18)

From the results of the experiments the following conclusions have been drawn. (1) for a particular width of apron and depth of media, the pressures below apron increase
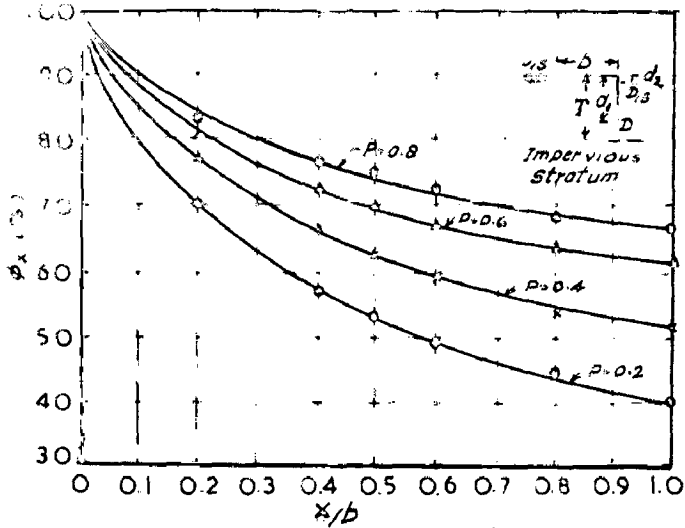
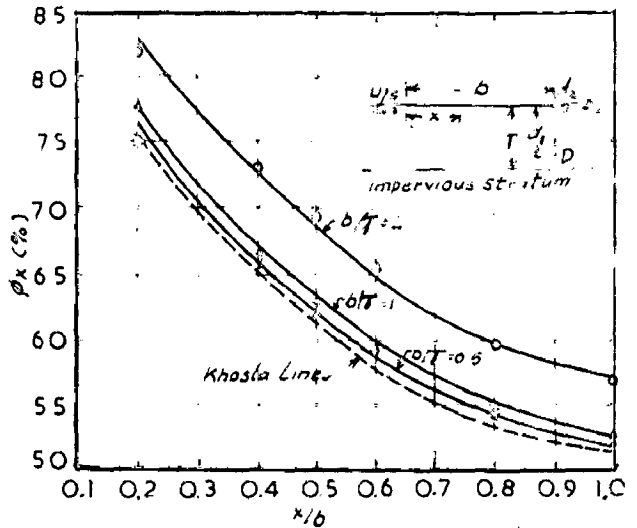FIG.2·11 VARIATION OF ϕ_x WITH $\frac{d_1}{T}$, FOR $\frac{b}{T} = 1$ AND $\frac{d_2}{d_1} = \frac{1}{8}$

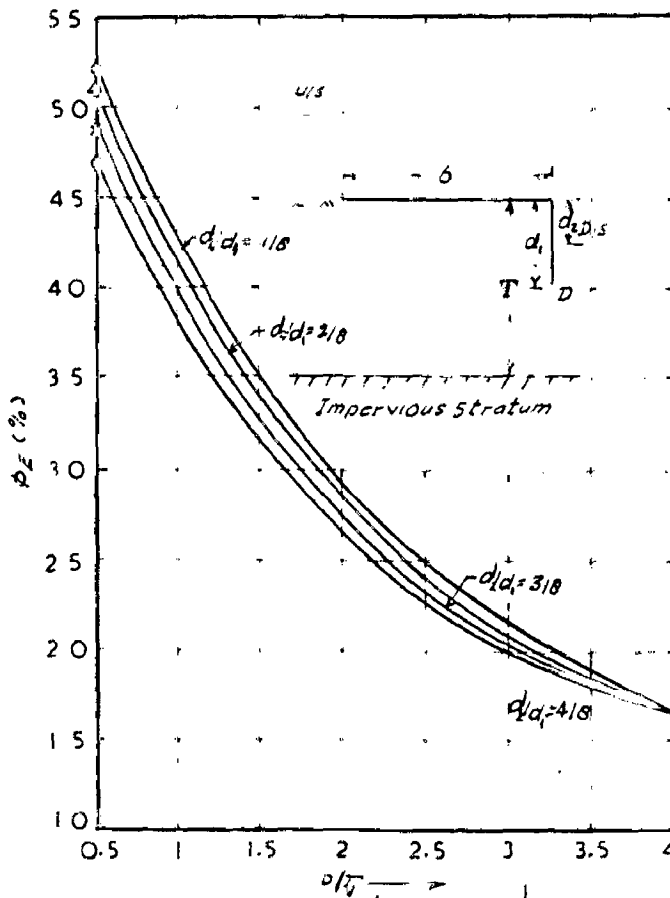FIG.2·12 VARIATION OF ϕ_x WITH $\frac{b}{T}$, FOR $\frac{b}{d_1} = 2.5$ AND $\frac{d_2}{d_1} = \frac{1}{8}$

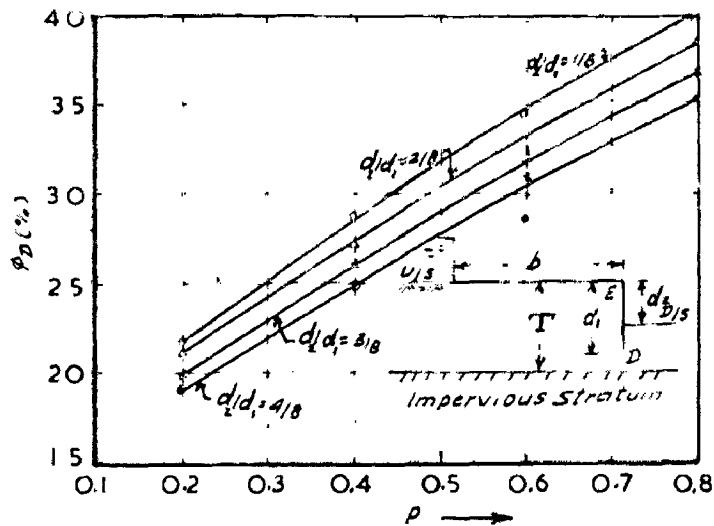FIG.2·13 VARIATION OF ϕ_E WITH $\frac{b}{T}$ AND $\frac{d_2}{d_1}$, FOR $\frac{d_1}{T} = 0.2$

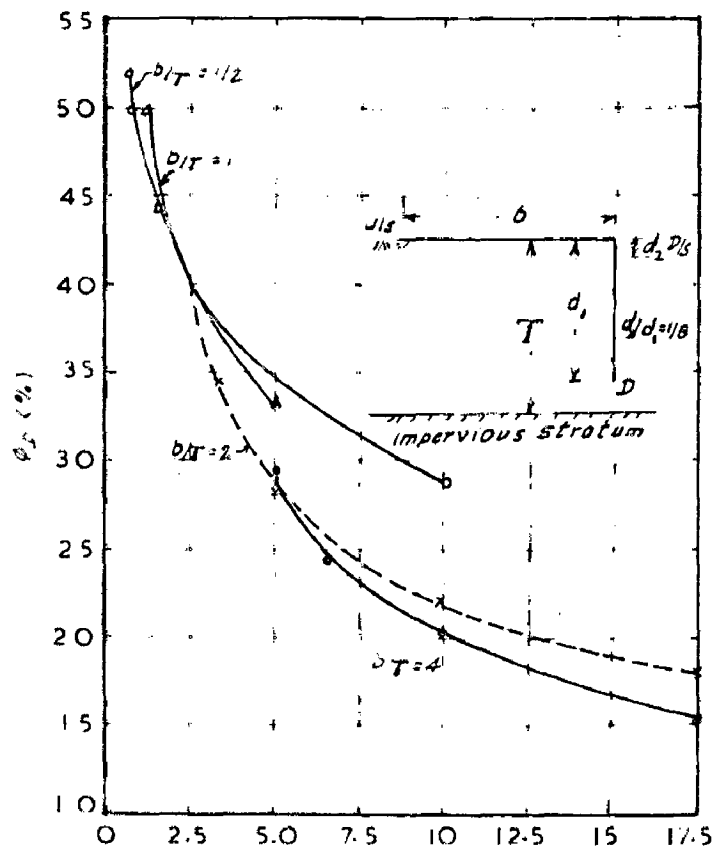FIG 2·14 VARIATION OF ϕ_D WITH $\frac{d_1}{T}$ AND $\frac{d_2}{d_1}$ FOR $\frac{b}{T} = 2$

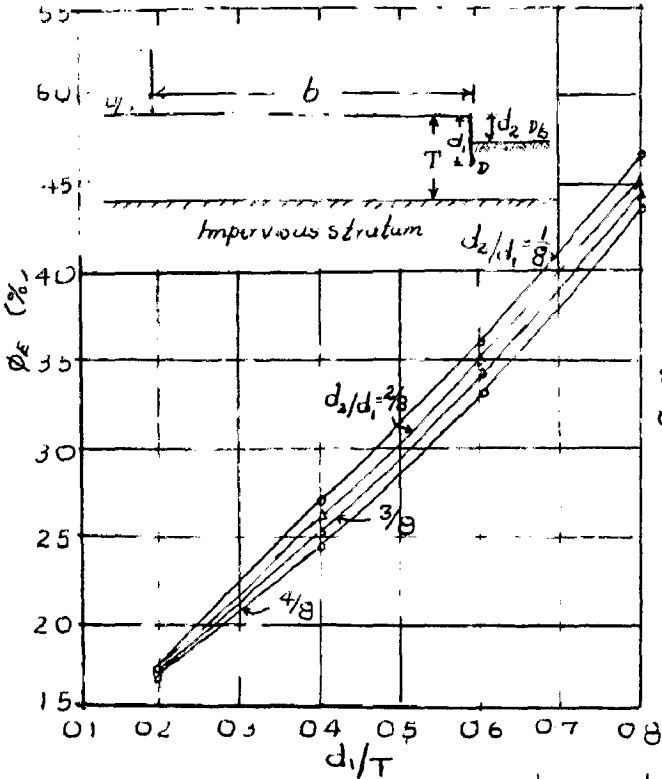FIG 2·15 VARIATION OF ϕ_D WITH $\frac{b}{d_1}$ AND $\frac{b}{T}$ FOR $\frac{d_2}{d_1} = \frac{1}{8}$

FIG 2·16 VARIATION OF ϕ_E WITH $\dfrac{d_1}{T}$ AND $\dfrac{d_2}{d_1}$ FOR $\dfrac{b}{T} = 4$
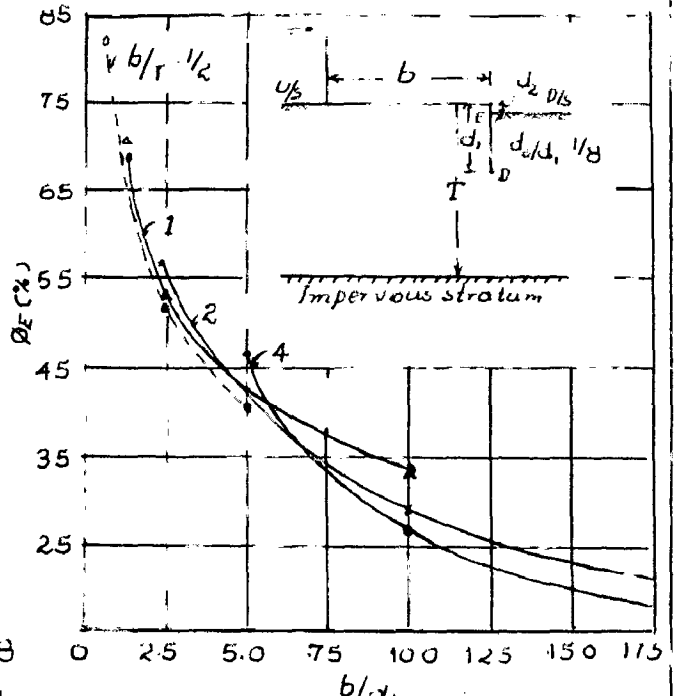
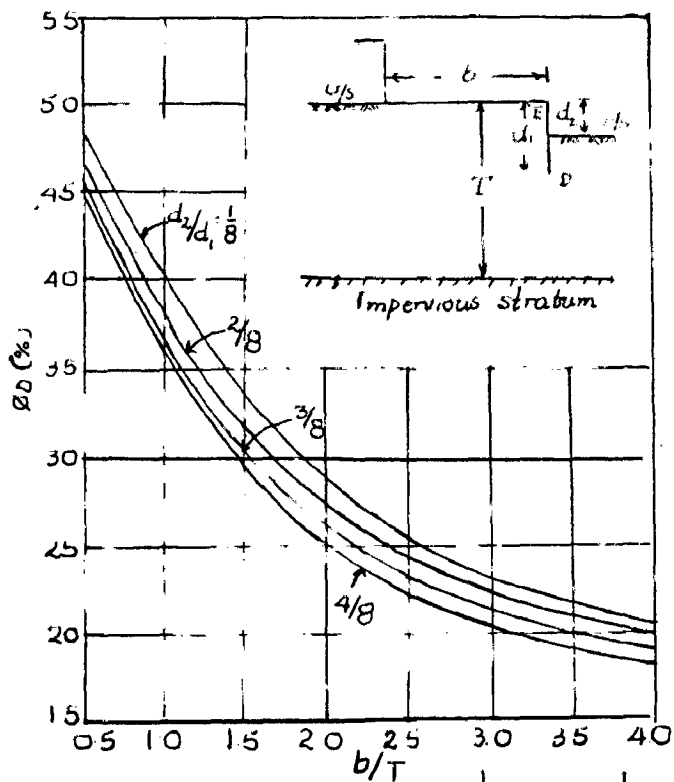FIG 2·17 VARIATION OF ϕ_E WITH $\dfrac{b}{d_1}$ AND $\dfrac{b}{T}$ FOR $\dfrac{d_2}{d_1} = \dfrac{1}{8}$

FIG 2·18 VARIATION OF ϕ_D WITH $\dfrac{b}{T}$ AND $\dfrac{d_2}{d_1}$ FOR $\dfrac{d_1}{T} = 0.4$

as depth of cut off increases. (2) The width of floor, the depth of cut off and the scour depth all being fixed, the pressures at any point below floor increase as the impervious layer comes nearer the floor (3) The pressure at junction point (ϕ_E) and that at tip of cut off (ϕ_D) increase when width of floor decreases (for constant depths of porous media, scour and cut off). But all the other parameters being fixed the pressures ϕ_E and ϕ_D slightly increase with decrease in scour depth (4) Pressures (ϕ_D) and (ϕ_E) increas rapidly with decrease in ratio of width of floor to depth of cut off. (5) ϕ_D and ϕ_E increase rapidly with increase in penetration ratio .

It can therefore be concluded that (i) the depth of the downstream cut off should be kept minimum required for safe value of exit gradient and also from scour considerations.

(ii) The width of floor downstream of the gate line should be kept minimum subject to the requirement from surface flow considerations, (iii) the depth of scour on the downstream, effec ts slightly reduction in uplift pressures, but at the same time would increase the value of exit gradient.

2.5.2 Exit Gradient

The effect of scour downstream of a structure can be studied from the expression for exit gradient at the end of floor with a step and downstream cut off. The

expression obtained by Muthukumaran is

$$G_E \frac{T}{H} = \frac{\pi \, \sigma_2 \, (1+\sigma_1) \sqrt{2 \, (\beta_1 + \sigma_2)}}{2K \, (1 - \frac{d_2}{T})(\sigma_1 + \sigma_2) \sqrt{(1+\beta_1)(1+\sigma_2)}}$$

$$...(2.37)$$

where the symbols have the notations as explained in earlier section. The values of $G_E \frac{T}{H}$ have been evaluated for $b/T = 1.0$ and for following values of d_1/T and d_2/T.

1) $\frac{d_1}{T} = 0.2$ $\frac{d_2}{T} = 0, 0.05, 0.1, 0.15$

2) $\frac{d_1}{T} = 0.4$ $\frac{d_2}{T} = 0, 0.1, 0.2, 0.30$

3) $\frac{d_1}{T} = 0.6$ $\frac{d_2}{T} = 0, 0.3, 0.4, 0.50$

4) $\frac{d_1}{T} = 0.8$ $\frac{d_2}{T} = 0, 0.5, 0.6, 0.70$

The values of σ_1, σ_2 and β_1 were found using the charts given by Muthukumaran (13) the use of which was explained earlier.

The values of $G_E \frac{T}{H}$ are plotted against $\frac{d_2}{T}$ as abcissa for all the four values of d_1/T in Fig. 2.19.

A perusal of the figure indicates that the value of $G_E \frac{T}{H}$ increases with increase in the value of $\frac{d_1}{T}$ and $\frac{d_2}{T}$. However this rate of increase gradually increases as the value of d_2/T increases for a particular value
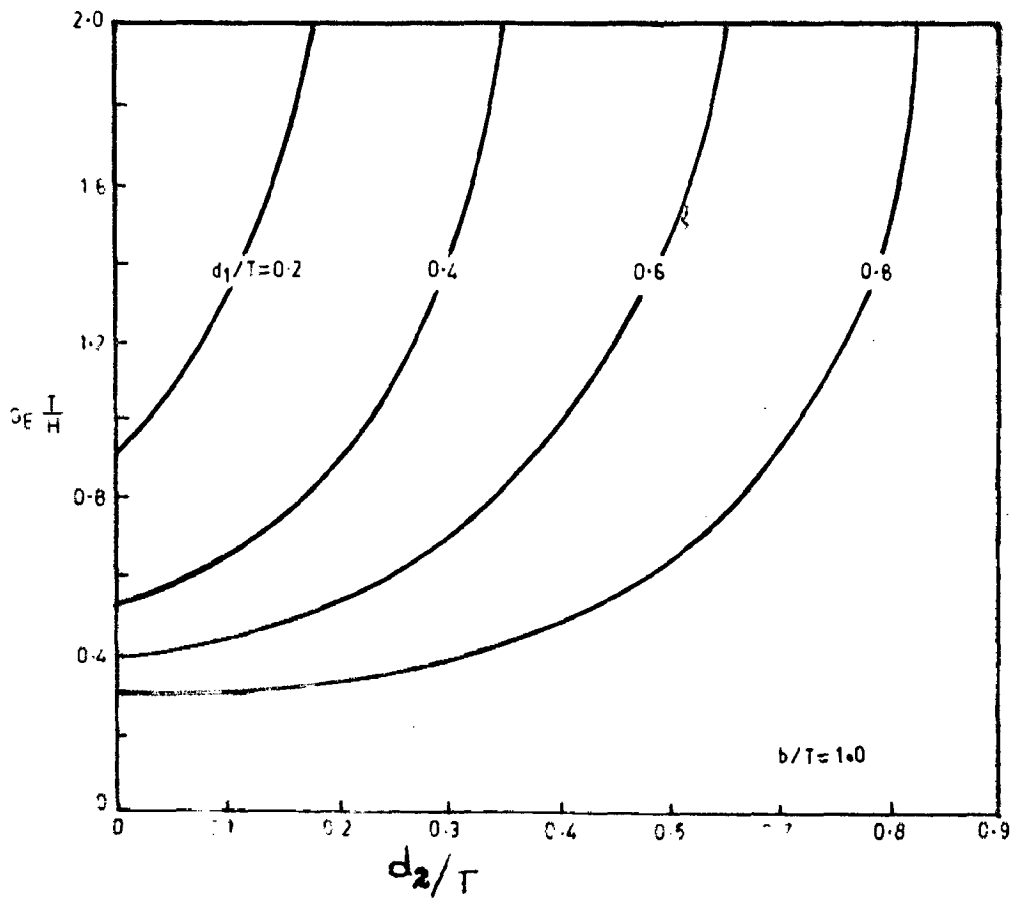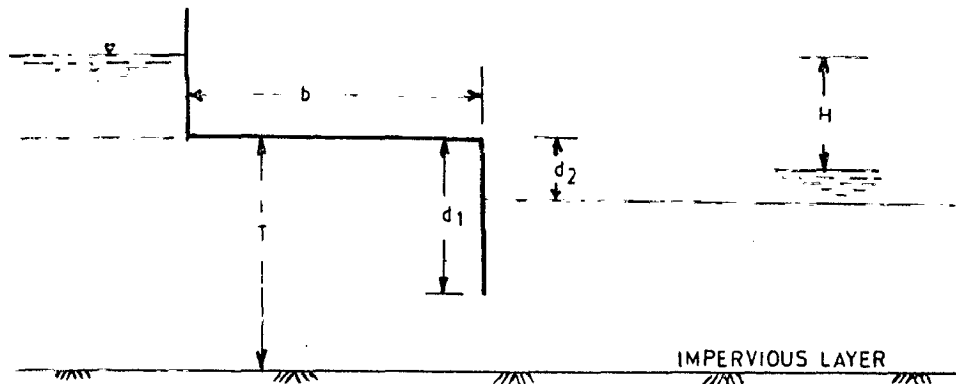
FIG. 2·19—EFFECT OF SCOUR ON EXIT GRADIENT.

of d_1/T . The rate of increase in the value of $G_E \frac{T}{H}$

increases with decrease in the value of d_1/T. For example

for $d_1/T = 0.8$ the value of $G_E \frac{T}{H}$ increases from 0.32 to 0.48

with increase in the value of d_2/T from 0.2 to 0.4. But for

$d_1/T = 0.6$ the value of $G_E \frac{T}{H}$ increases from 0.55 to 1.00

with increase in the value of d_2/T from 0.2 to 0.4.

Since the value of G_E especially for deeper cut off

i.e. $d_1/T > 0.4$, increases only gradually , limited scour

downstream of the end cutoff without step may not result in

significant increase in the value of exit gradient. However,

excessive scour reduces the effectiveness of cut off in

controlling the value of exit gradient. It is therefore seen

that with downstream scour extending over large area, uplift

pressure below the floor decreases whereas the value of G_E

increases. This information would alsohelp in determining the

effect of the depth of filter on the downstream side of a

structure.

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┌─────────────────────────────────────────────────┐
│                                                   │
│           C H A P T E R    III                    │
│                                                   │
│         STRATIFIED    FOUNDATIONS                 │
│                                                   │
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3.0   An exact solution has been obtained by Polubarinova-
Kochina (15) for seepage below flat bottom weir or a cut off
founded on soil consisting of two layers of equal thickness.
Lenau (9) has obtained solution of flow below a flat bottomed
structure founded on two strata of unequal thickness using
perturbation technique. The effect of stratification on the
seepage and uplift pressure below hydraulic structures with
a flat floor only was also studied by Luthra (11) Gurudasram
and Awade (5) on electrical analogy model.

        Alamsingh and Punmia (1) conducted electrical analogy
experiments to determine uplift pressures at key points of a
floor with cut off at its downstream end founded on soil
consisting of two layers of equal thickness. In these exper-
iments the permeability of upper layer was taken to be higher
than that of lower layer. Sharma et al (22) combined theore-
tical and model studies to determine the exit gradient and
uplift pressures below a floor with an end cut off founded
on two layered soil with permeability of upperlayer being
smaller than that of underlying layer. The ratio of the perm-
eabilities of lower and upper layers have been varied between
zero and infinity. The solutions obtained for various
boundary conditions are given below.

3.1 FLOOR ON TWO STRATA OF EQUAL THICKNESS

Consider a flat floor AB(Fig.3.1) of length 2b resting on a stratum of depth T with coefficient of permeability k_1. This stratum is resting on another stratum of thickness T with coefficient of permeability k_2 underlain by an impervious layer. Since the structure is symmetrical about its centre line, only the right half is considered. Along the upstream bed AG_1 . $\phi = -k_1H$ and along downstream bed BF_1 , $\phi = 0$. Due to the symmetry CDE is an equipotential surface where $\phi = \dfrac{k_1H}{2}$ or $\dfrac{k_2H}{2}$. Along the lower impermeable boundary EF_2, $\psi = 0$. The base of the structure CB is an impermeable boundary and may be taken as the stream line $\psi = q$.

$w_1 = \phi_1 + i\psi_1$ and $w_2 = \phi_2 + i\psi_2$ represent the complex potentials in the upper and lower stratum. The boundary condition along the various surfaces are expressed in terms of w_1 and w_2 as follows.

BF_1 $\quad \phi_1 = k_1H$ or $Im(iw_1) = k_1H$

CB $\quad \psi = q$ or $Im(w_1) = q$

CD $\quad \phi_1 = \dfrac{k_1H}{2}$ or $Im(iw_1) = \dfrac{k_1H}{2}$

DE $\quad \phi_2 = \dfrac{k_2H}{2}$ or $Im(iw_2) = \dfrac{k_2H}{2}$
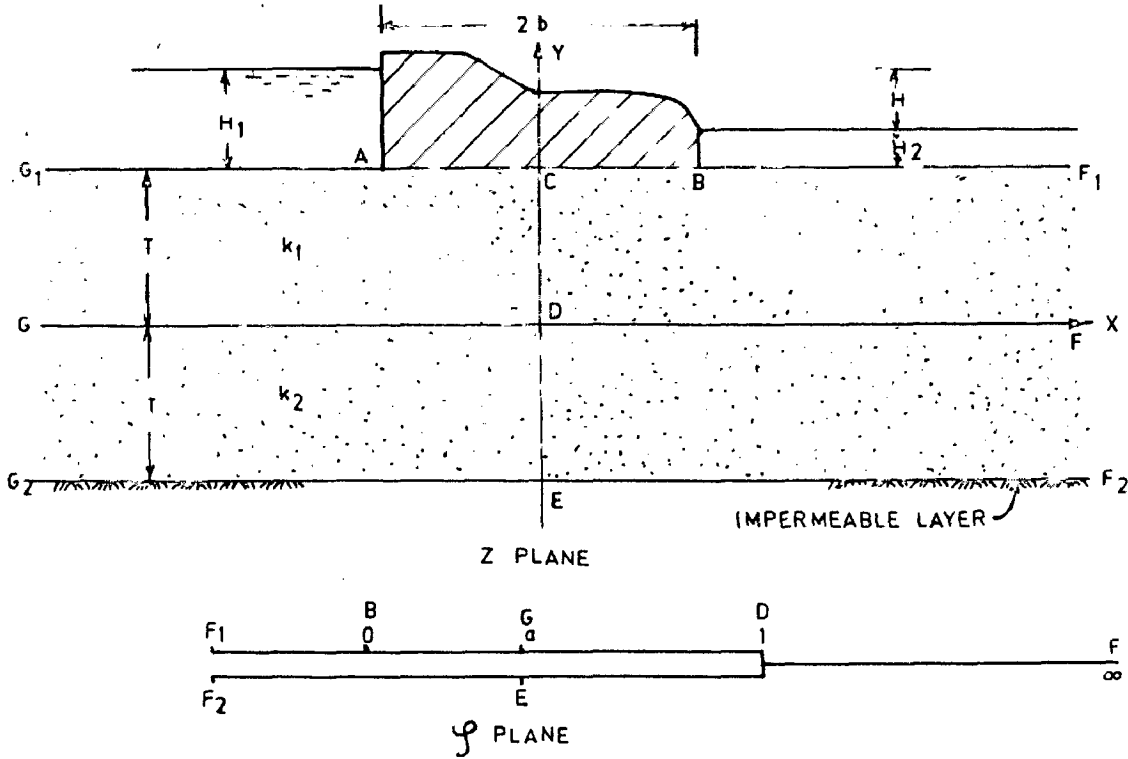
Z PLANE

$\mathcal{G}$ PLANE

FIG.3·1-TRANSFORMATION LAYOUT.
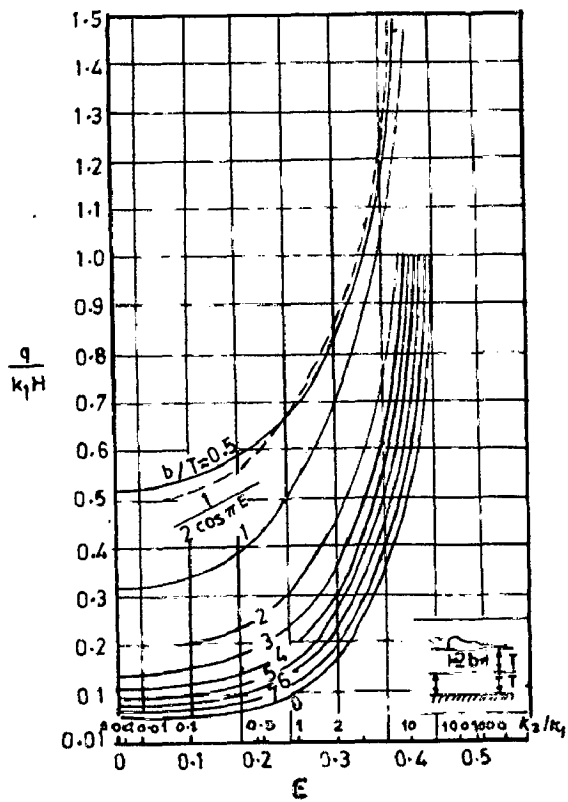
FIG·3·2-VARIATION OF SEEPAGE DISCHARGE WITH k_2/k_1 AND $2b$.

EF_2 $\Psi_2 = 0$ or $Im\ (w_2) = 0$

DF $\dfrac{\phi_1}{k_1} = \dfrac{\phi_2}{k_2}$ or $Im(\dfrac{iw_1}{k_1} - \dfrac{iw_2}{k_2}) = 0$

$$\Psi_1 = \Psi_2 \quad \text{or} \quad Im(w_1 - w_2) = 0$$

The region F_1BCDF and $FDEF_2$ are mapped into the upper half and lower half of the complex ζ plane. In mapping the region of upper stratum to the upper portion of the ζ-plane, the points B,D and F are placed at 0.1 and ∞ along the real axis and point C lies at 'a'. The point E of the lower stratum is placed at $\zeta = a$.

The Schwartz - Christoffel, transformation equation that maps z- plane into ζ plane is

$$\frac{dz}{d\zeta} = M\ (\zeta - a)^{-1/2}(\zeta - 1)^{-1/2} \qquad \text{....(3.1)}$$

or $$z = M \int \frac{d\zeta}{\sqrt{(\zeta - a)\ (\zeta - 1)}} + N$$

$$= M\ \cosh^{-1} \left[\frac{\zeta - \frac{1}{2}(1+a)}{\frac{1}{2}(1-a)} \right] + N \qquad \text{....(3.2)}$$

At point D , $\zeta = 1$, and $z = 0$. Therefore $N = 0$. At point C, $\zeta = a$ and $z = iT$, Therefore

$$iT = M\ \cosh^{-1}\ \ -1$$

$$= Mi\ \pi$$

or $M = T/\pi$

Eq. 3.2 reduces to

$$z = \frac{T}{\pi} \cosh^{-1} \left[\frac{\zeta - \frac{1}{2} (1+a)}{\frac{1}{2} (1-a)} \right] \qquad \ldots (3.3)$$

or $\zeta = \frac{1}{2} (1+a) + \frac{1}{2} (1-a) \cosh \frac{\pi z}{T} \qquad \ldots (3.4)$

At point B, $z = b+iT$ and $\zeta = 0$, therefore from Eq. 3.4

$$a = \tanh^2 \frac{\pi b}{2T} \qquad \ldots (3.5)$$

The potential functions w_1 and w_2 for the upper and lower layers are given by the following integrals

$$w_1 = \frac{M'i}{2} \int \frac{(\sqrt{1-\zeta} + \sqrt{-\zeta})^{2\varepsilon} + (\sqrt{1-\zeta} - \sqrt{-\zeta})^{2\varepsilon}}{\sqrt{\zeta (1-\zeta)(\zeta-a)}} d\zeta + N' \qquad \ldots (3.6)$$

and

$$w_2 = -\frac{M'}{2} \tan \varepsilon\pi \int \frac{(\sqrt{1-\zeta} + \sqrt{-\zeta})^{2\varepsilon} - (\sqrt{1-\zeta} - \sqrt{-\zeta})^{2\varepsilon}}{\sqrt{\zeta (1-\zeta)(\zeta-a)}} d\zeta + N' \qquad \ldots (3.7)$$

where $\varepsilon = \frac{1}{\pi} \tan^{-1} \sqrt{\frac{k_2}{k_1}} \qquad \ldots (3.8)$

Uplift Pressures

Substituting $\zeta = \sin^2 \alpha$, Eq. 3.6 reduces to

$$w_1 = \frac{M'}{2} \int \frac{(\cos\alpha + i\sin\alpha)^{2\varepsilon} + (\cos\alpha - i\sin\alpha)^{2\varepsilon}}{\sqrt{a - \sin^2\alpha}} d\alpha + N' \qquad \ldots (3.9)$$

$$= M' \int \frac{\cos (2\varepsilon\alpha)}{\sqrt{a - \sin^2\alpha}} d\alpha + N' \qquad \ldots (3.10)$$

At point B , $\zeta = 0$ and $w = 0$, therefore $N' = 0$

At point C , $\zeta = a$ and $w = \dfrac{k_1 H}{2}$, therefore

$$\frac{k_1 H}{2} = M' \int_0^{\sin^{-1}\sqrt{a}} \frac{\cos(2\varepsilon\alpha)}{\sqrt{a - \sin^2\alpha}} \, d\alpha \qquad \ldots (3.11)$$

$$= M' \, J$$

Eq. 3.9 therefore reduces to

$$w_1 = \frac{k_1 H}{2J} \int_0^{\sin^{-1}\sqrt{\zeta}} \frac{\cos(2\varepsilon\alpha)}{\sqrt{a - \sin^2\alpha}} \, d\alpha \qquad \ldots (3.12)$$

Direct integration of Eq. 3.12 is not possible

It can however be integrated after expanding into an infinite series.

Seepage Discharge

The seepage discharge can be determined by integrating Eq. 3.6 between limits $\zeta = -\infty$ and $\zeta = 0$. The seepage flow can also be determined by adding the seepage discharge passing through upper layer at section CD and the seepage discharge passing through the lower layer at section DE. The flow rate through the upper layer can be obtained by integrating Eq. 3.6 between the limits $\zeta = a$ and $\zeta = 1$ and the flow rate through the lower layer can be obtained by integrating Eq. 3.7 between the limits $\zeta = a$ and $\zeta = 1$. The total flow rate is given by

$$\frac{q}{k_1 H} = \frac{1}{2J \cos \varepsilon \pi} \int_0^{\sin^{-1} k'} \frac{\cos (2\alpha)}{\sqrt{k'^2 - \sin^2 \alpha}} \, d\alpha \qquad \ldots(3.13)$$

where q = seepage discharge per unit length of the structure. The values of $q/k_1 H$ have been plotted against values of ε and k_2/k_1 for various values of b/T ranging between 0.5 and 8.0 (Fig.3.2). A perusal of the figure indicates that the seepage discharge increases with increase in the value of k_2/k_1. Also the seepage discharge reduces with the increase in the length of floor. This is also evident from Fig.3.2 . It is seen that the discharge reduces rapidly for initial increase in the value of b/T and the rate of reduction reduces with increase in the value of b/T.

3.2   CUT OFF IN TWO STRATA OF EQUAL DEPTH

Pollubarinova - Kochina(15) has als obtained expression for seepage discharge below a cutoff penetrating to a depth 'd' in two layers of equal thickness T with coefficient of permeability being k_1 and k_2.

For d < T

$$\frac{q}{k_1 H} = \frac{1}{J_1 + J_2} \left[\frac{(\cos \frac{\pi d}{2T})^{2\varepsilon}}{\cos \varepsilon \pi} J_3 + \frac{J_1 - J_2}{2} \tan \varepsilon \pi \right] \qquad \ldots(3.14)$$

For d > T

$$\frac{q}{k_1 H} = \frac{J_1 + J_2}{2J_0} \tan \pi \varepsilon \qquad \ldots(3.15)$$

where

$$J_1 = \int_0^{\pi/2} \frac{\left(\sqrt{1-o^2 \sin^2\phi} + o\cos\phi\right)^{2\varepsilon}}{\sqrt{1-o^2 \sin^2\phi}} \, d\phi \qquad \ldots(3.16)$$

$$J_2 = \int_0^{\pi/2} \frac{\left(\sqrt{1-o^2 \sin^2\phi} - o\cos\phi\right)^{2\varepsilon}}{\sqrt{1-o^2 \sin^2\phi}} \, d\phi \qquad \ldots(3.17)$$

$$J_3 = \int_0^{\pi/2} \frac{\cos 2\varepsilon\phi \, d\phi}{\sqrt{1-o'^2 \sin^2\phi}} \qquad \ldots(3.18)$$

$$J_o = J_1 - J_2 + \frac{2 o'^{2\varepsilon}}{\sin \varepsilon\pi} J_3 \qquad \ldots(3.19)$$

$$\phi = \sin^{-1}\sqrt{\frac{\varsigma}{a}} \qquad \ldots(3.20)$$

$$o = \sin\frac{\pi d}{2T} \quad \text{and} \quad o' = \cos\frac{\pi d}{2T} \qquad \ldots(3.21)$$

$$a = -\tan^2\frac{\pi d}{2T} \qquad \ldots(3.22)$$

for $d = T$

$$\frac{q}{k_1 H} = \frac{1}{2}\tan \varepsilon\pi = \frac{1}{2}\sqrt{\frac{k_2}{k_1}} \qquad \ldots(3.23)$$

The values of $G_E \frac{T}{H}$ have been plotted, in Fig.3.3 against the values of d/T and k_2 / k_1 . A perusal of the figure indicates that the value of $G_E \frac{T}{H}$ decreases rapidly with increase in value of d/T = 3/4 . The reduction in the value of $G_E \frac{T}{H}$ with increase in the value of d/T beyond 3/4

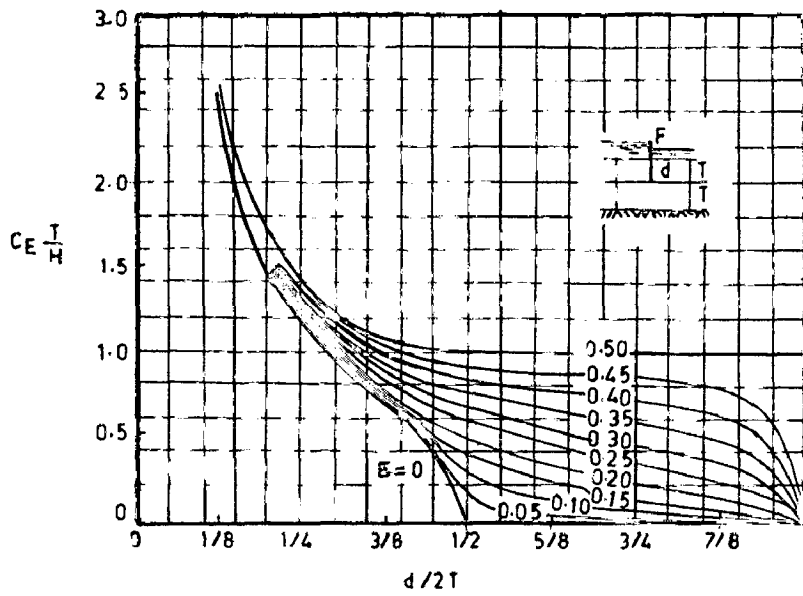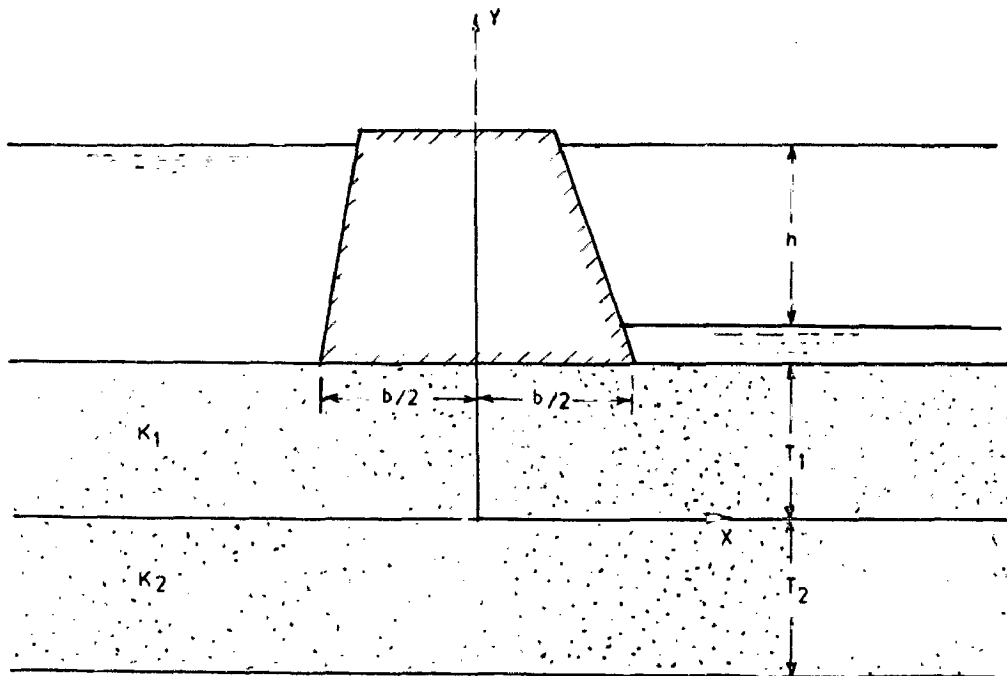FIG.3·3-VARIATION OF EXIT GRADIENT WITH ε & d .

FIG.3·4-FLOOR ON TWO STRATA OF UNEQUAL THICKNESS.

is very small. The ratio of the permeability of lower layer

to that of upper layer has very little effect on the value

of $G_E \frac{T}{H}$ for d/T less than $3/4$.

3.3 FLOOR ON TWO STRATA ON UNEQUAL THICKNESS

Lenau (9) has obtained the solution of seepage

below a flat floor founded on two strata of unequal thick-

ness using "perturbation technique" Fig. (3.4), He has

obtained expressions for the seepage discharge only and has

not derived any relation for uplift pressures. Expression

for discharge is

$$\frac{q}{k_1 H} = \frac{(1-p)^{2p} \sqrt{\gamma}/\pi}{2p \sqrt{\gamma}} + \frac{1}{\pi} \log \left(\frac{1+p}{2p}\right) + O(\rho) \quad ...(3.24)$$

where

$$p = \tanh \left(\frac{\pi b}{8T_1}\right) \qquad ...(3.25)$$

$$\gamma = \frac{T_1}{T_2}$$

$$\rho = \sqrt{k_1/k_2}$$

Eq. 3.24 can give relatively accurate values for smaller

values of ρ and $\xi \sqrt{\gamma}$. The values of $q/k_1 H$ have been

plotted against b/T_1 for different values of $\sqrt{k_1 T_1/k_2 T_2}$

in Fig. 3.5. A perusal of the figure indicates that the

length of the floor becomes unimportant as the value of

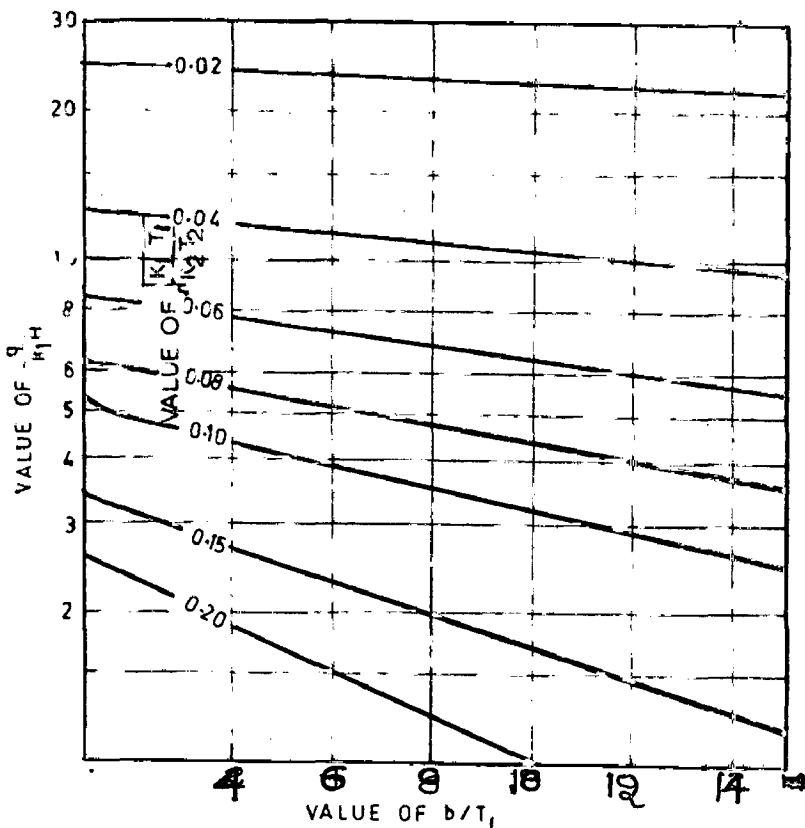$\sqrt{k_1 T_1/k_2 T_2}$ approaches zero and the value of $q/k_1 H$
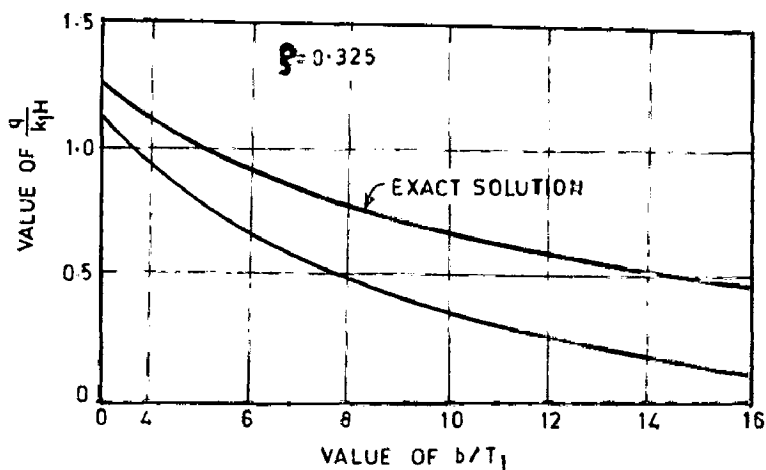
FIG.3·5—q/k_1H V/S b/T_1

FIG.3·6—COMPARISON BETWEEN Eq3·24 AND EXACT SOLUTION GIVEN BY POLUBARINOVA KOCHINA.

approaches the value of $\dfrac{1}{2}\sqrt{\dfrac{k_2 T_2}{k_1 T_1}}$. A similar tendency

is seen from the results of the solution given by Polubarinova-Kochina for floor founded on two layers of equal thickness.

A comparison between the results obtained from Eq.3.24 and exact solution given by Polubarinava-Kochina (15) is illustrated in Fig. 3.6 (for $\varsigma = 0.325$) . The Eq. 3.24 is seen to under-estimate seepage discharge $q/k_1 H$, the accuracy decreasing with increase in b/T_1 . For example when $b/T_1 = 3$ the absolute and relative errors are 0.15 and 15% respectively, and when $b/T_1 = 16$ the absolute and relative errors are 0.34 and several hundred percent. The Eq. 3.24 can be expected (From Fig.3.6) to underestimate the total seepage to a maximum of 1.0 ς if $b/T_1 \leqslant 16$ and $\Upsilon \leqslant 1$. For $\Upsilon > 1$ ($T_1/T_2 > 1$) the error will be somewhat larger but probably no more than $\varsigma\sqrt{\Upsilon}$.

Lenau (10) has also obtained an approximate solution of seepage below the foundation of a structure without cutoff resting on four layers of equal thicknesses. The approximate solution is accurate for very small values of b/T . However as b/T approaches zero the error approaches infinity. Hence the solution is recommended to be used only when $b/T \geqslant 0.2$.

3.4 FLOOR WITH END CUT OFF ON TWO STRATA

Alamsingh et. al (1) has studied on electrical analogy model the effect of stratified foundation on the uplift pressures below hydraulic structures with downstream end cut off . The permeability of upper layer was taken to be more than the lower layer. Sharma et. al (22) combined the theoretical and model studies to determine the exit gradient and uplift pressures below foundations of structures founded on two layers, with the permeability of upper layer being smaller than that of the underlying layer.

Alamsingh et al conducted experiments to determine uplift pressures only for k_1/k_2 ratio as 1,4, 20 and 50, b/T ratio equal to 1,2,4 and 8 and d/T ratio as 0.25, 0.5 0.75 and practically 1.0 . The results indicate that uplift pressures increase with the increase in the permeability ratio k_1/k_2 and penetration ratio d/T (Fig. 3.7 to 3.10). As expected the pressures at the corresponding points decrease with increase in the length of floor. The authors also found that for a given b/T ratio the uplift pressures at D and E decr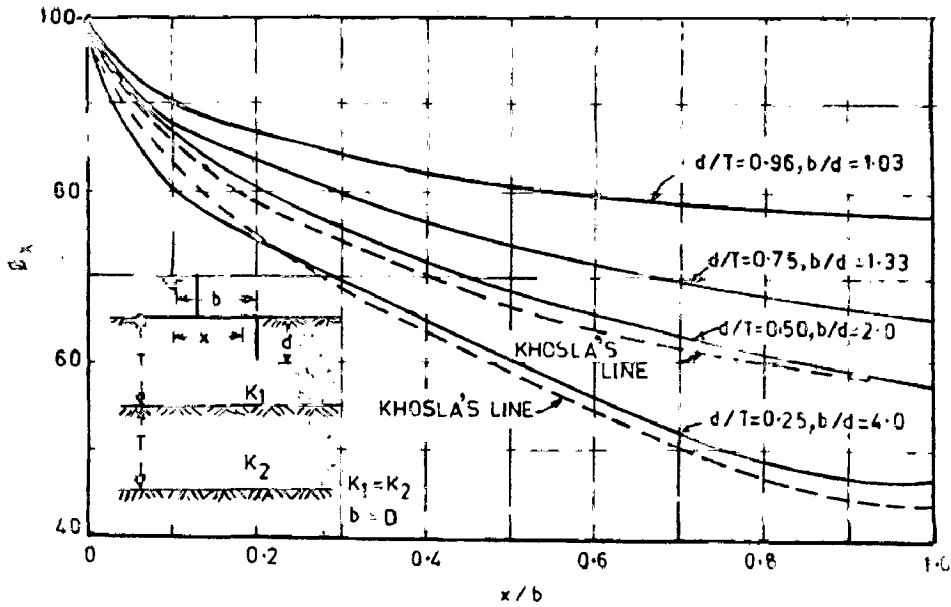ease with increase inthe value of $\sqrt{k_1/k_2}$ for d/T $\leqslant$ 0.5 and increase with increase in the value of $\sqrt{k_1/k_2}$ for d/T $\geqslant$ 0.75 (Fig.3.11 and 3.12)

FIG.3·7-VARIATION OF $\emptyset_x$ FOR VARIOUS VALUES OF d/T & b/d.
b/T=1 AND K_1/K_2=1 FIXED.

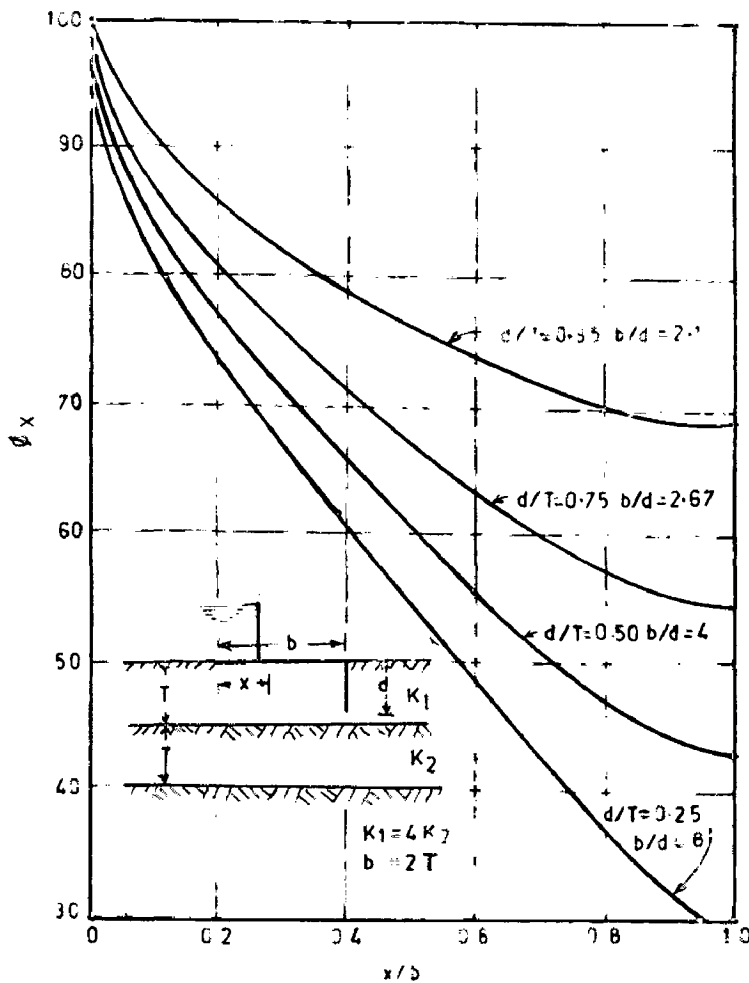
FIG.3·8-VARIATION OF $\emptyset_x$ FOR VARIOUS VALUES OF d/T & b/d,
b/T=2 AND K_1/K_2=4 FIXED.

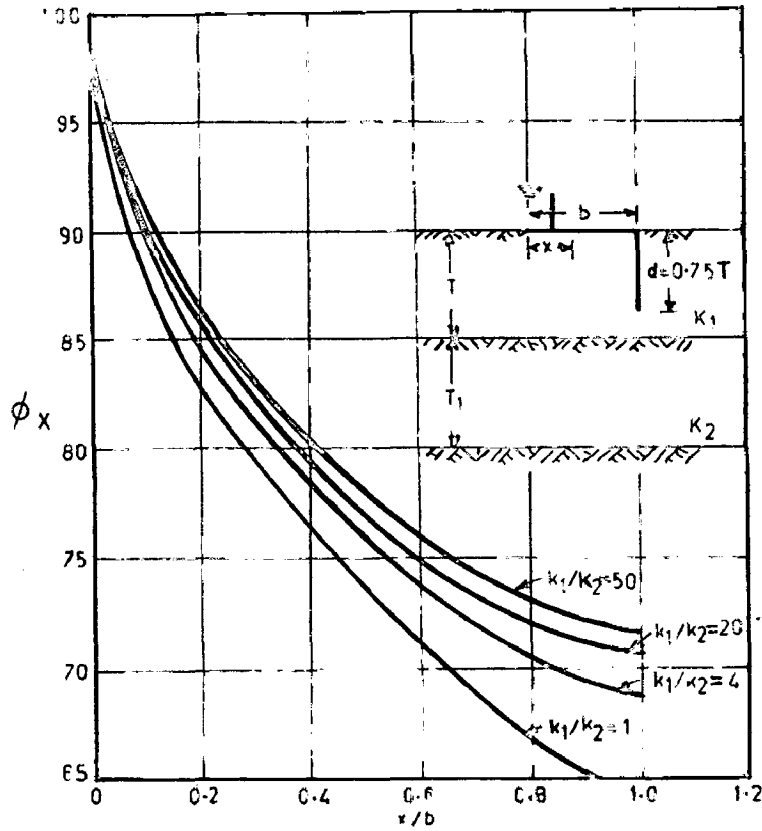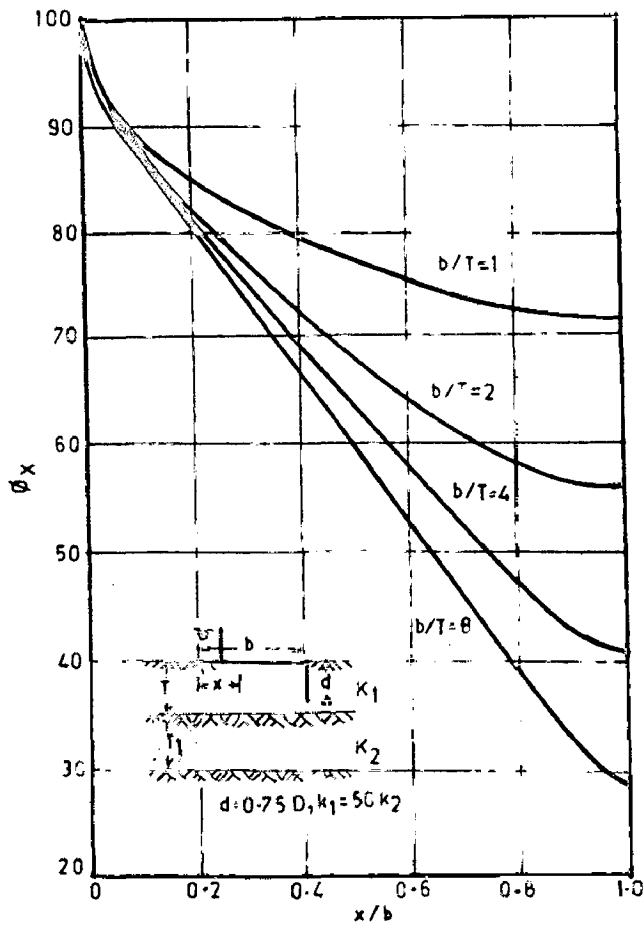FIG.3.9—VARIATION OF ϕ_x FOR VARIOUS VALUES OF K_1/K_2 b/T=1, d/T=0.75 AND b/1=1.33 FIXED.

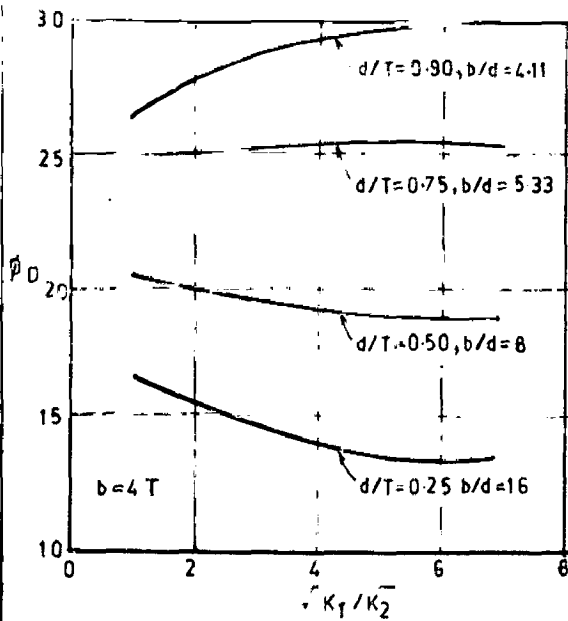FIG.3.10—VARIATION OF ϕ_x FOR VARIOUS VALUES OF b/T, K_1/K_2=50 d/T=0.75 FIXED.

30

d/T=0.90, b/d=4.11

25

d/T=0.75, b/d=5.33

ϕ_D 20

d/T=0.50, b/d=8

15

b=4T

d/T=0.25 b/d=16

10

0   2   4   6   8   10

$\sqrt{K_1/K_2}$

FIG. 3.11-INFLUENCE OF K_1/K_2 d/T ON ϕ_D FOR b/T=4.

25

d/T=0.85, b/d=9.33

20

d/T=0.75, b/d=10.67

15

d/T=0.50, b/d=18

10

d/T=0.25, b/d=32

b=8T

5

0   2   4   6   8   10
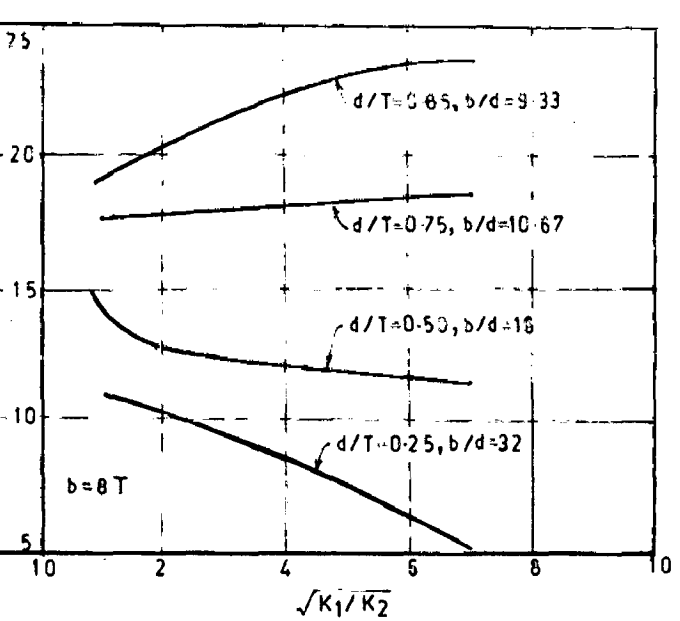
$\sqrt{K_1/K_2}$

FIG. 3.11(a)-INFLUENCE OF K_1/K_2, d/T, b/d ON ϕ_D FOR b/T=8.

85

b=4T

75

E K_1

T K_2

65

ϕ_E

d/T=0.9, b/d=4.11

55

d/T=0.75, b/d=5.33

45

d/T=0.50, b/d=8

35

d/T=0.25, b/d=16

25

15

0   2   4   6   8
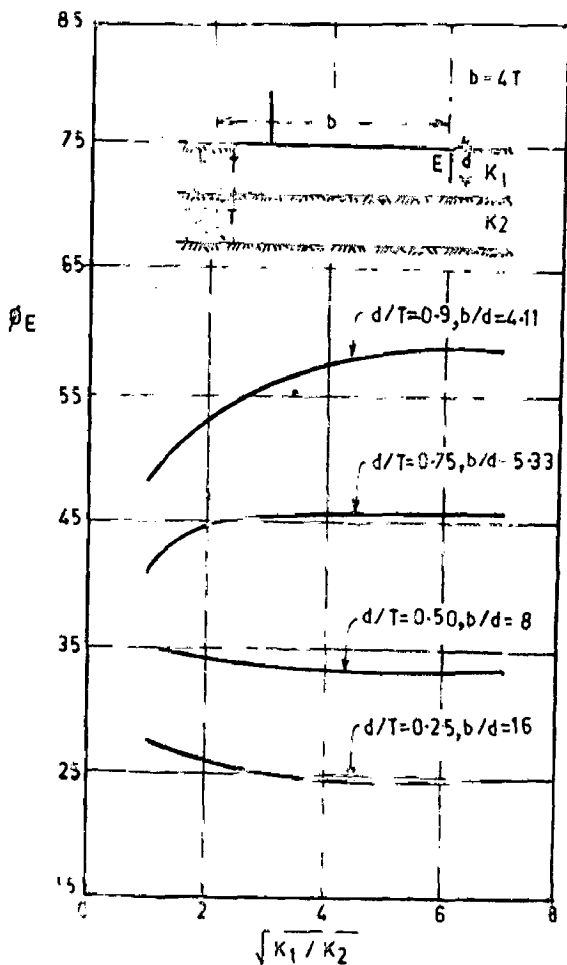
$\sqrt{K_1/K_2}$

FIG. 3.12-INFLUENCE OF K_1/K_2, d/T, b/d ON ϕ_E FOR b/T=4 FIXED.

60

b=8T

50

E K_1

K_2

40

d/T=0.85, b/d=9.33

30

d/T=0.75, b/d=10.67

20

d/T=0.50, b/d=16

10

d/T=0.25, b/d=32

0

0   2   4   6   8   10

$\sqrt{K_1/K_2}$

FIG. 3.12(a)-INFLUENCE OF K_1/K_2, d/T, b/d ON ϕ_E FOR b/T=8 FIXED.

Sharma et al have calculated exit gradient and uplift pressures below floor with end cut off from closed form solutions for $k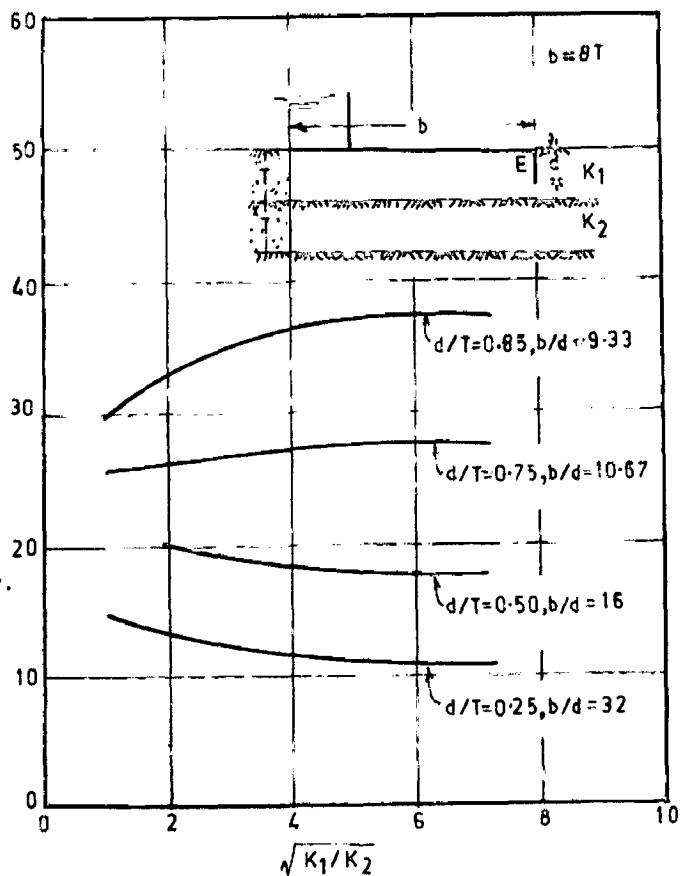_2/k_1 = 0$, 1 and ∞ and employed electrical analogy model for $k_2/k_1 = 0,1,10$ and 100. The theoretical and model test results for $k_2/k_1 = 0$ and 1 were compared and found almost identical. The uplift pressures at the key points i.e., at the junction of the floor and cutoff, E and at the tip of the cut off, D for $d/T_1 = 0.25,0.5$ are shown in Figs. 3.13 to 3.16 . The values of $G_E \frac{T}{H}$ for various values of b/T_1 and d/T_1 are also shown in Fig. 3.17 . Results of the studies indicate that

(i)   With the increase in the value of k_2/k_1 , the uplift pressures increase in the portion of the floor where pressures are less than 50 per cent and decrease in the portion where pressures are more than 50 per cent (Fig. 3.18).

(ii)   As indicated by the studies conducted by Alamsingh the uplift pressures increase with increase in the penetration ratio and decrease at the corresponding point with increase in length of the floor (Fig.3.20). However the pressure s below floor are not affected by increase in length of the floor for large value of k_2/k_1 i.e.,$\geqslant 100$.

(iii)   Thickness of lower stratum has insignificant effect on the uplift pressures (Fig.3.19).

(iv)   The value of exit gradient increases with increase in the permeability of lower layer (Fig.3.17). Increase in the length of the floor has practically insignificant
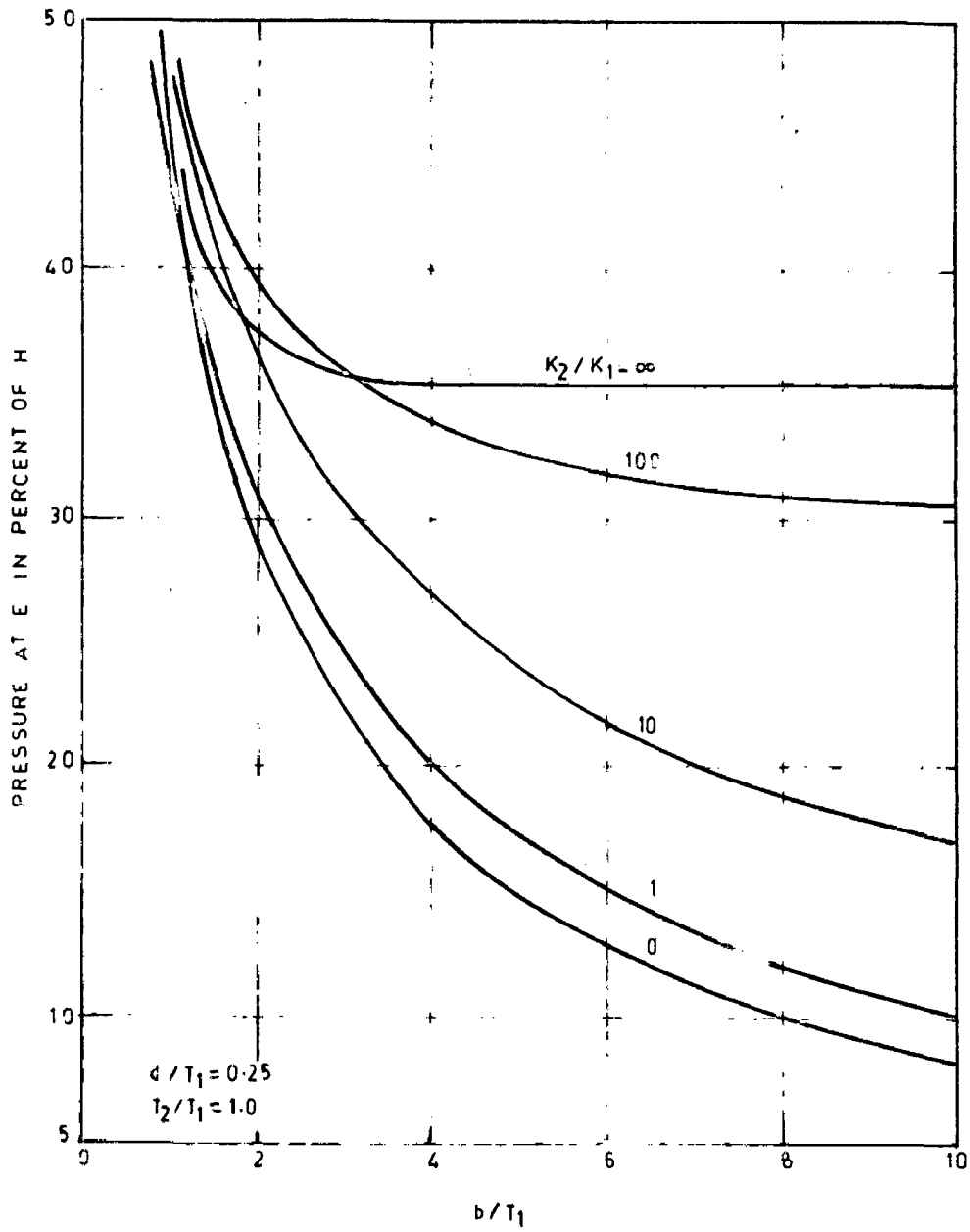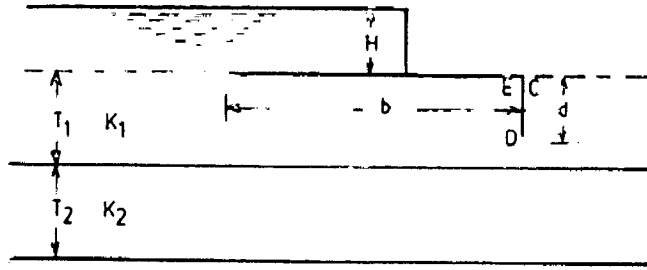
FIG 3·13—UPLIFT PRESSURES ON FLOOR.
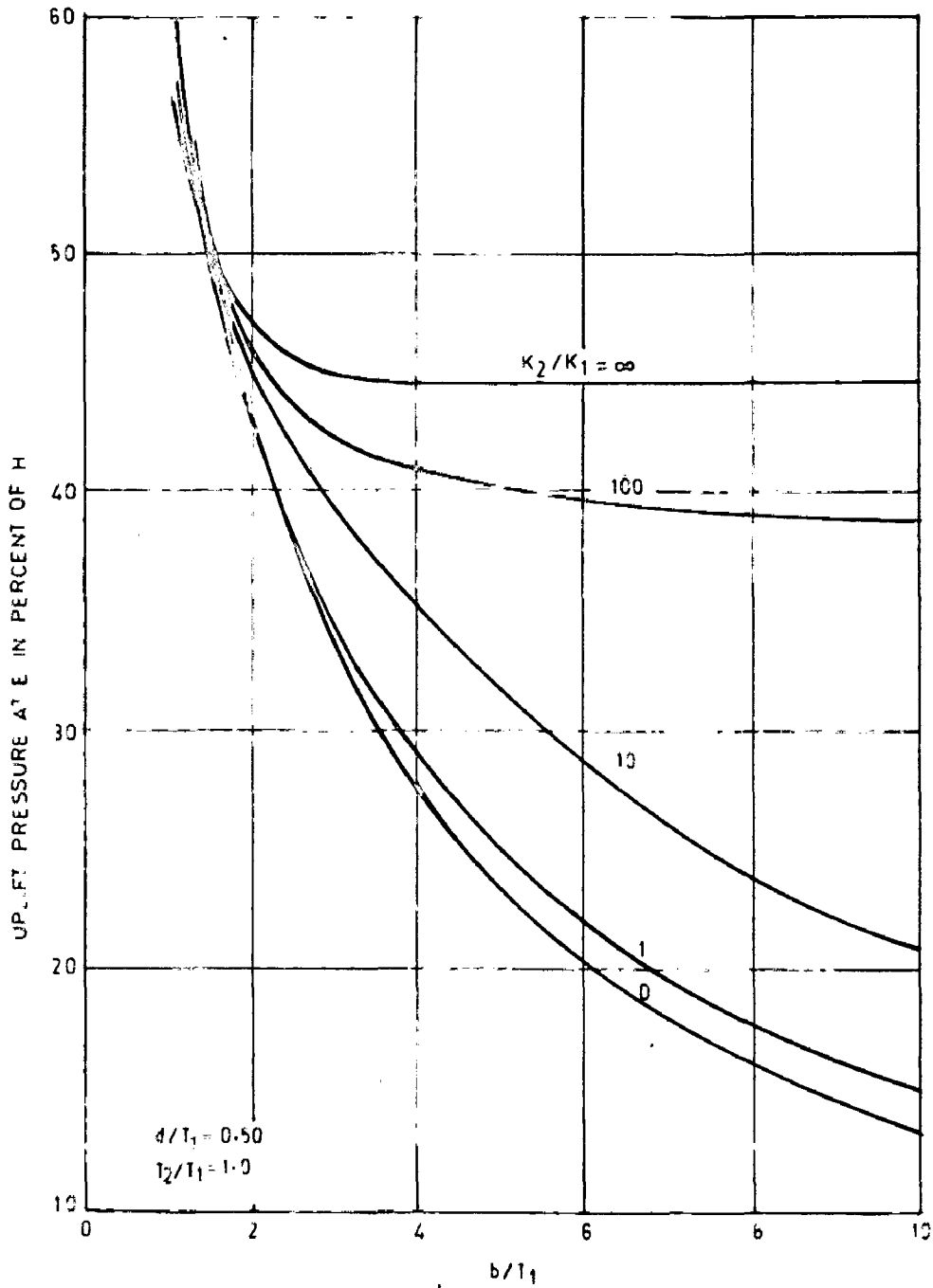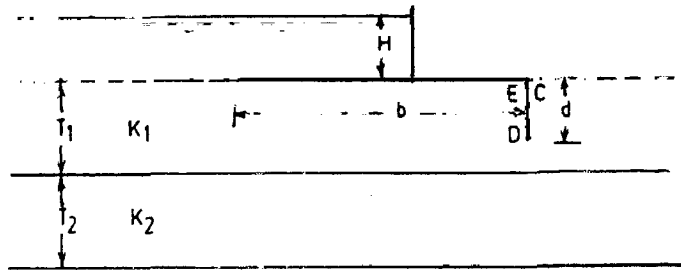
FIG 3·14-UPLIFT PRESSURES ON FLOOR

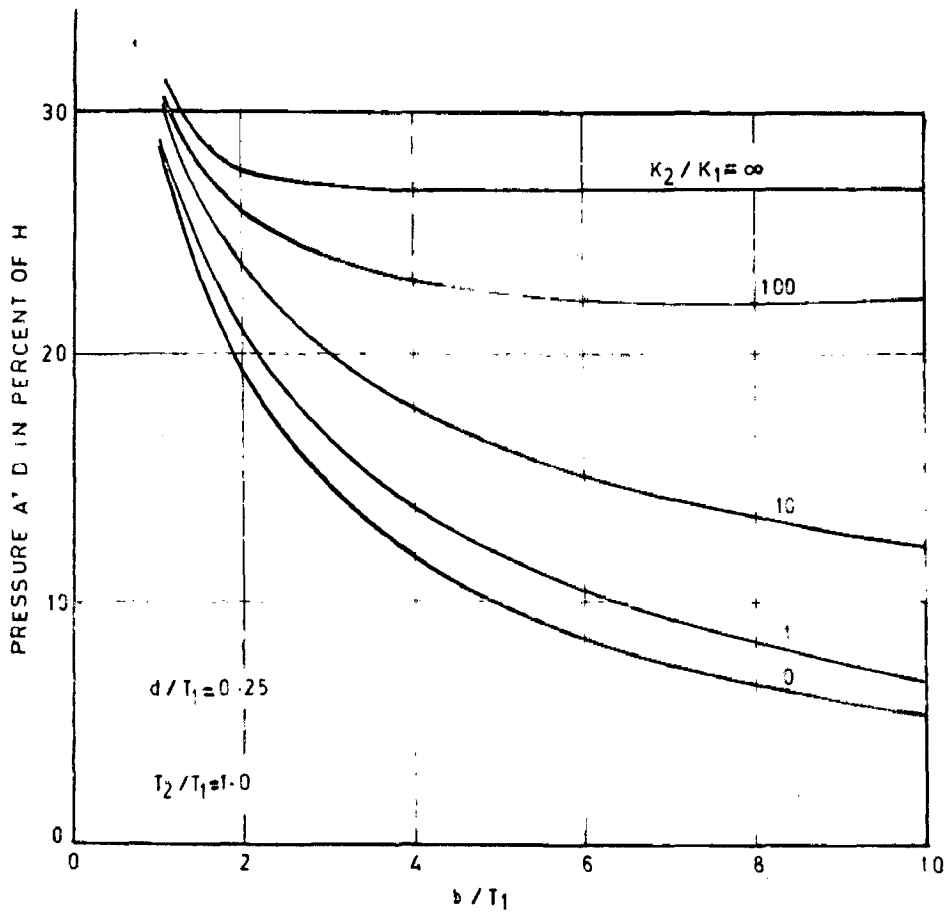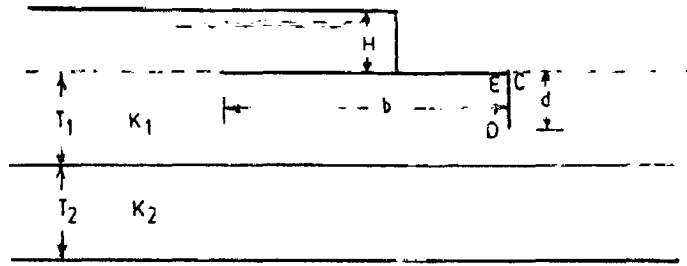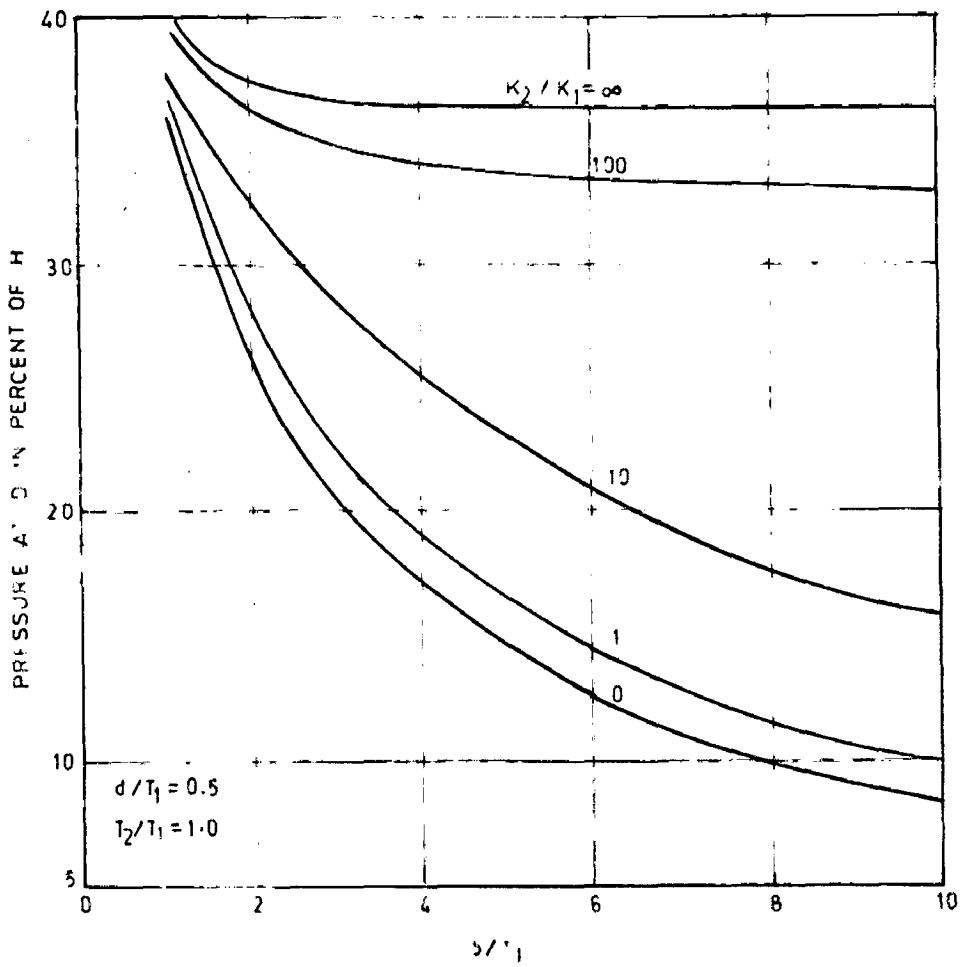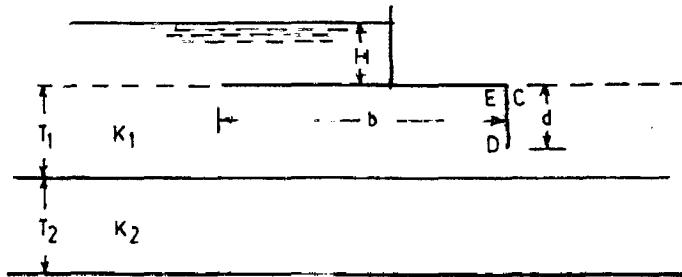FIG.3·15-UPLIFT PRESSURES AT BOTTOM OF CUTOFF.

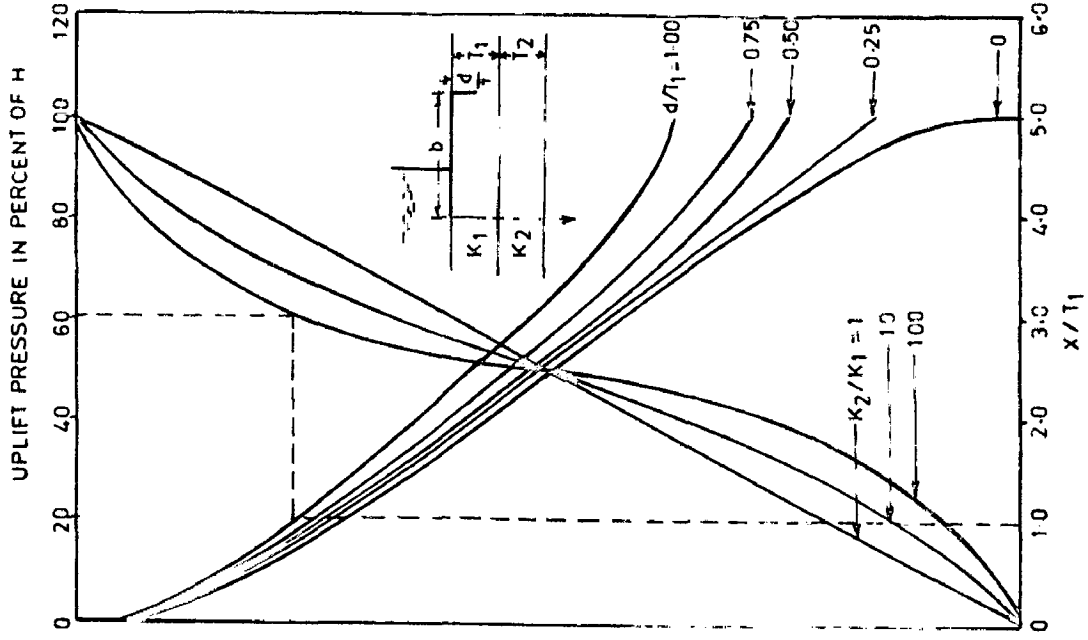FIG 3·16—UPLIFT PRESSURES AT BOTTOM OF CUTOFF.

FIG. 3·18-UPLIFT PRESSURE BELOW FLCOR
$T_2 / T_1 = 1·0$.
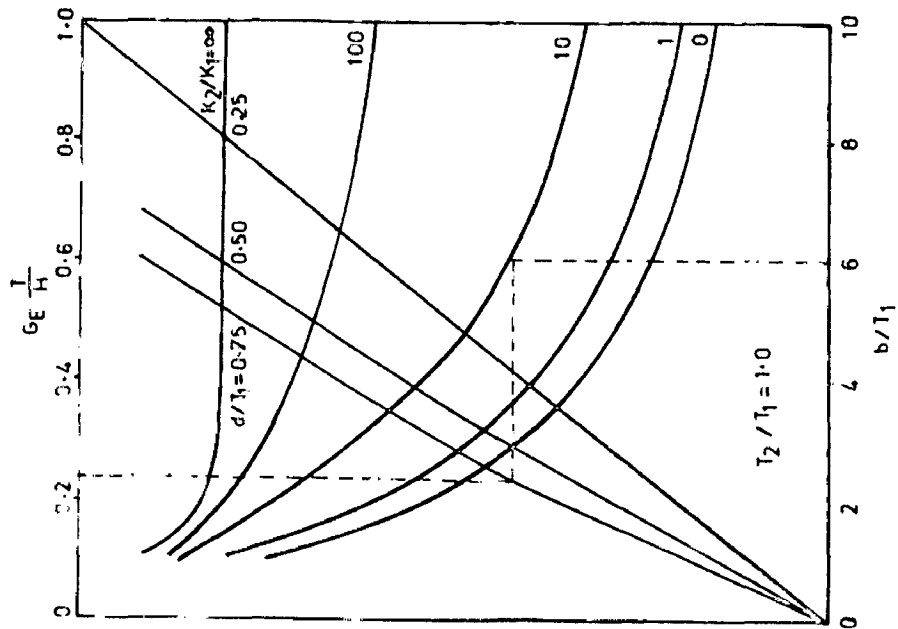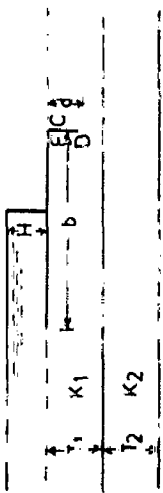
FIG. 3·17-EXIT GRADIENT AT C .
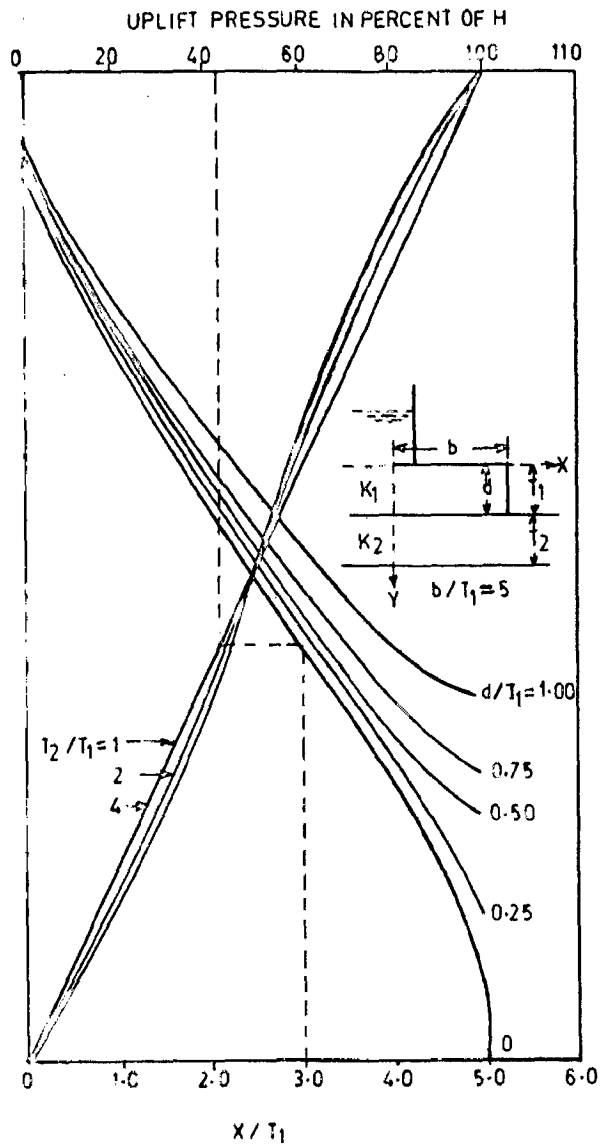
FIG. 3·19-UPLIFT PRESSURES BELOW FLOOR $K_2/K_1=1$.
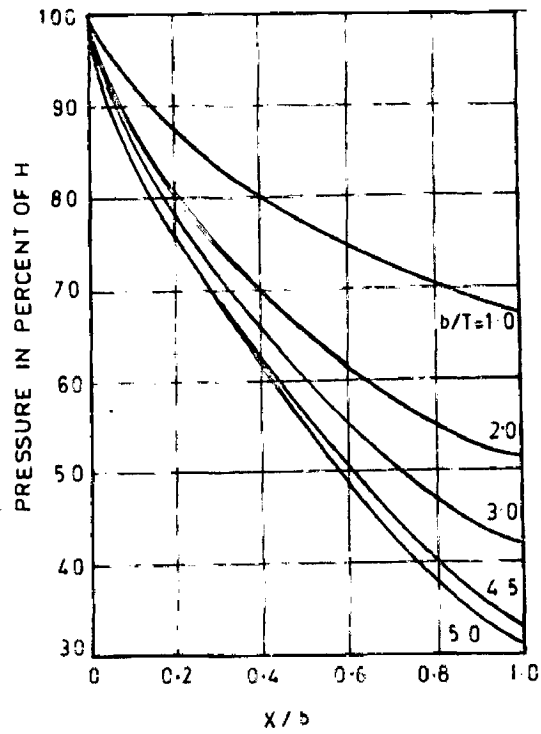
FIG.3·20-UPLIFT PRESSURES BELOW FLOOR $K_2/K_1=1$, $d/T_1=0.75, T_2/T_1=1$
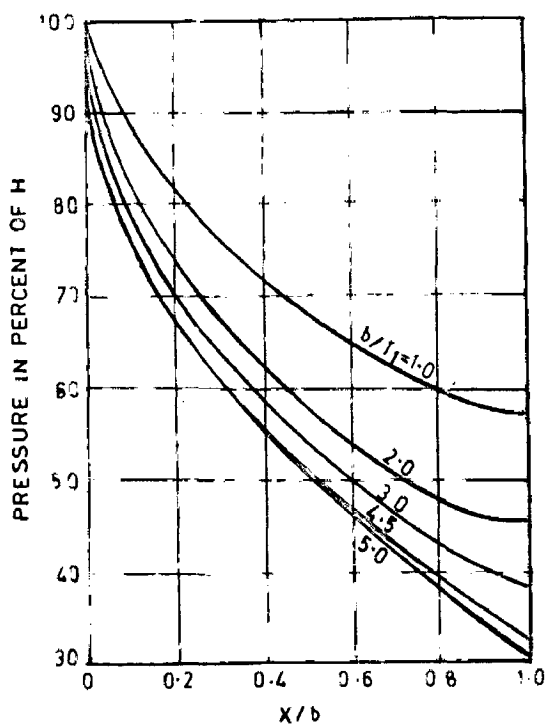
FIG.3·20(a)-UPLIFT PRESSURE BELOW APRON $K_2/K_1=10$, $d/T_1=0.50, T_2/T_1=1$

effect on the value of exit gradient for k_2/k_1 > 10.

(v)   Increase in the depth of cut off has little effect in reducing the value of exit gradient when the permeability of the lower stratum is higher than the upper stratum.

The study conducted by Alamsingh et. al gives galues of uplift pressures at D and E for k_2/k_1 =1.0, 0.25, 0.05 and 0.02 . The experimental values for k_2/k_1 =1.0,0.25, 0.05 and 0.02 should fall along those for k_2/k_1 = 1.0 and between those for k_2/k_1 = 1.0 and k_2/k_1 = 0 obtained theoretically by Sharma et. al. In order to compare these experimental results with those obtained by Sharma at al theoretically for k_2/k_1 = 0 and 1.0 these were plotted on Fig. 3.21. However a perusal of this figure indicates that the pressures obtained experimentally by Alamsingh et al are higher than those calculated theoretically.

3.5 FLOOR WITH INTERMEDIATE CUT OFF ON TWO STRATA (OF EQUAL THICKNESS)

Punmia et al(17) conducted electrical analogy model studies to determine the uplift pressures below hydraulic structures with intermediate cut off founded on two layered media of equal thickness. The permeability of the upper layer was more than that of lower layer. Model tests were conducted with various combinations of b/T, d/T , b_1/b and k_1/k_2 where b_1 is the distance of cut off from upstream end of floor. The results of the studies indicate that -
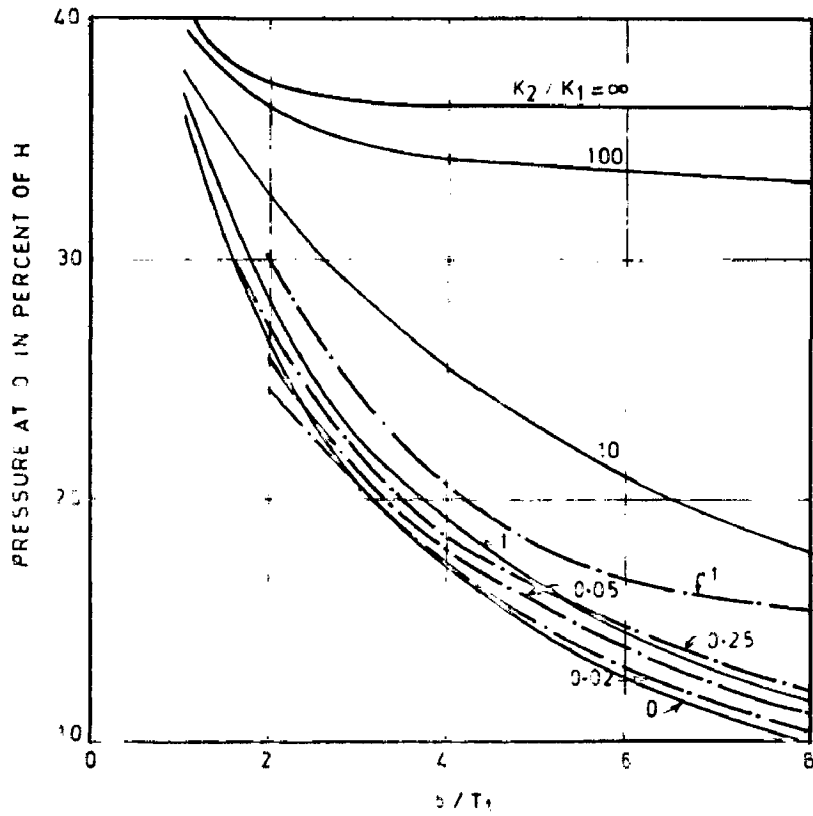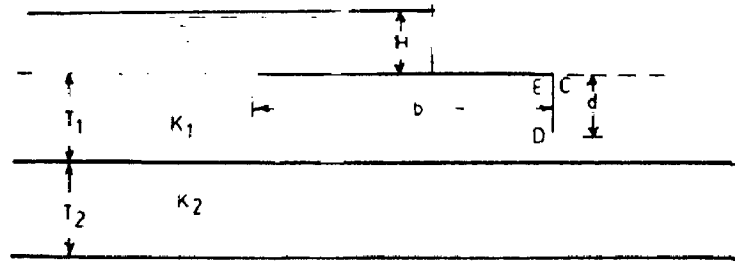
FIG.3·21-UPLIFT PRESSURES AT BOTTOM OF CUTOFF.

(i)   With increase in value of k_1/k_2 the pressures on
        the floor on upstream side of cut off increase while the
        pressures decrease on down stream side.

(ii)   While the other parameters are not altered, the pressure
        at points on the upstream side of cut off increases
        with increase in depth of cut off or penetration ratio
        and the reverse is true for points on downstream of cutoff.

(iii)   With the other parameters being kept constant, the pressure
        at points on upstream side of cut off increases as the width
        of apron decreases and it decreases with decrease in width
        of apron on downstream side.

(iv)   The pressure on the floor on upstream side of cut-off
        decreases as the distance of cut off from upstream floor
        end increases.

(v)   Pressure at key points namely junction of floors
        cutoff (ϕ_E) and tip of cut off decreases as width of apron
        (floor) increases (keeping other parameters unchanged)

(vi)   Both ϕ_E and ϕ_D increase with increase in depth of cut off.

(vii) ϕ_E and ϕ_D increase with increase in k_1/k_2 while the
        other ratios are kept constant.

3.6   TWO LAYERED MEDIA (FOUNDATION) WITH A CUT OFF AT
        UPSTREAM END

        A study of two dimensional seepage under a flat
bottomed structure with a cut off at upstream and resting

on two horizontal layers of different permeabilities and equal thicknesses (Fig.3.22) using a simple electrical analog was made by Stefan (23) .

By current measurements and by using the well known analog

$$q / k_1 H = i / \sigma_1 v$$

where

q = total seepage discharge per unit length.

k_1 = permeability coefficient of upper layer.

H = Difference in pressure head

$i.$ = current

σ_1 = conductivity of upper layer

v = voltage applied between electrodes.

the dimensionless flow rate is $q/ k_1 H$ was obtained and the results were plotted by the author (Fig.3.23) as a function of permeability ratio k_2/k_1 , given by a parameter ε defined by

$$\tan \pi \varepsilon = \sqrt{k_2/k_1}$$

The lengths of the flat bottomed structure and sheet pile cut off (each relative to thickness of layers) and also the permeability ratio were varied.

The analytical solutions of pollubarinova-Kochina, which are special cases of boundary geometries, investigated using electrical analogy were also reproduced for completeness and reference.
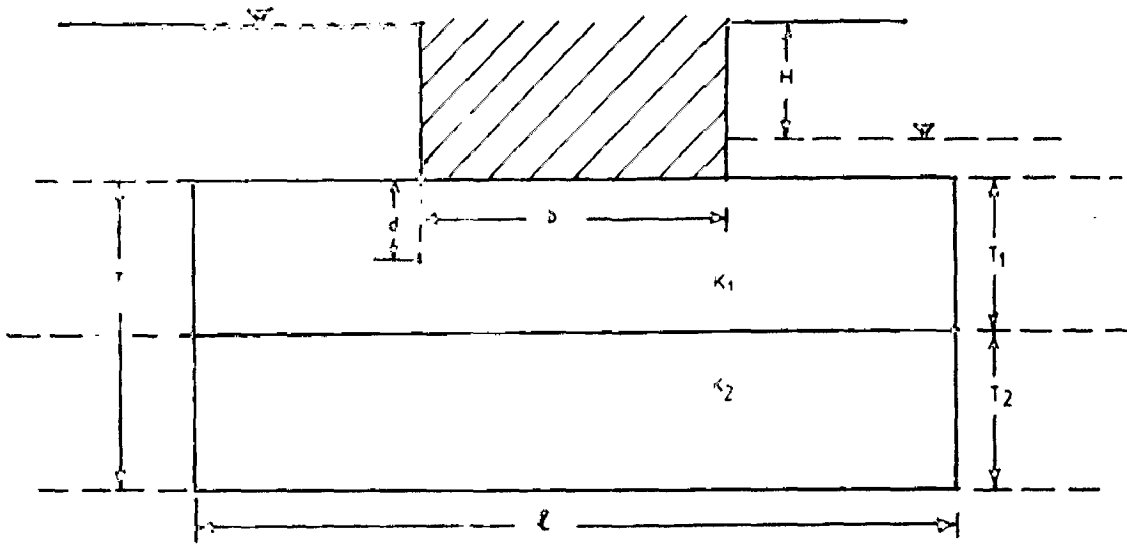
FIG.3·22–FLOOR ON TWO LAYERED MEDIA WITH UPSTREAM
END CUTOFF
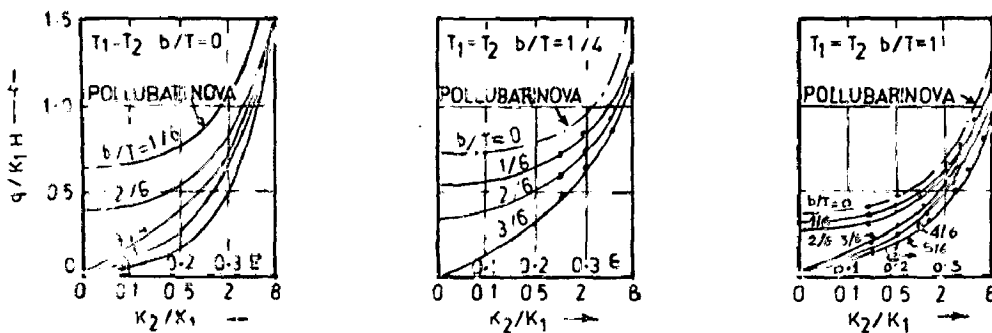
FIG.3·23–DIMENSIONLESS FLOW RATE THROUGH FOUNDATION
AS FUNCTION OF LENGTH OF FLAT STRUCTURE
DEPTH OF CUTOFF WALL AND PERMEABILITY RATIO
OF TWO PERVIOUS LAYERS.

The author condluded praising the technique of
investigation as very simple and easy. It may be helpful
for those not experienced in seepage flow studies. The
results of the study presented provide a basis for evalua-
ting seepage flow through two layers of different permea-
bilities with upstream cut off.

3.7. TWO LAYERED MEDIA (FOUNDATION) WITH TWO CUT OFFS

The effect of foundation stratification on seepage
under a hydraulic structure, on the underground contours
and foundations of the spillways dams of the volga hydro
electric stations named after V.I. Lenin was studied by
Ronzhin (20). The foundation of the first dam was composed
of three distinct layers and the second dam foundation of
two layers. The underground contours comprised an upstream
apron, two sheet piling walls and the spillway foundation
slab.

The two layered foundation was tested by the elec-
trical analogy method at different ratios of permeability
coefficients of individual layers and at various permeabili-
ties of sheet pile walls, but without considering the per-
meability of the upstream apron and that of concrete of
body of dam. In the laboratory models, the upstream apron
sheet piling and sheet piling under the dam were cut to the
full depth of the first layer of the foundation like in the
prototype. The test results showed that the diagram of

It is noteworthy that experimental curves fall well below theoretical ones in the range of high permeability ratios, the reason being limited length of model. The author indicated that errors in flow rate measurements were caused by limited length of anologypaper.

A second point of investigation as observed by the author related to the pressure distribution along cutoff wall and base of structure. Local differential pressure heads with reference to downstream water level for various lengths of cut off and for various permeability ratios were shown . The author pointed out the permeability had no dramatic effect on pressures. However the pressure under the horizontal base of the structure increased with increase in permeability of lower layer.

Terming the ratio of difference in pressure head (at a point below the floor) to the distance of the point from downstream edge of the floor as 'average exit gradient', the author studied. its variation for different ratios of width of floor to depth of media, and obtained finite values of exit gradient . But the author's conception of a finite value of exit gradient (at the downstream edge of floor) is wrong, when there is no cut off at downstream edge of floor. In such a case the exit gradient will be infinite under any conditions.

The author concluded praising the technique of
investigation as very simple and easy. It may be helpful
for those not experienced in seepage flow studies. The
results of the study presented provide a basis for evalua-
ting seepage flow through two layers of different permea-
bilities with upstream cut off.

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trical analogy method at different ratios of permeability
coefficients of individual layers and at various permeabili-
ties of sheet pile walls, but without considering the per-
meability of the upstream apron and that of concrete of
body of dam. In the laboratory models, the upstream apron
sheet piling and sheet piling under the dam were cut to the
full depth of the first layer of the foundation like in the
prototype. The test results showed that the diagram of

seepage uplift pressure acting on the underground contour changed with the increase in relative permeability of the lower layer in comparison with the permeability of the upper layer. Head losses on the sheet pilings decreased while those on the upstream apron increased. The same phenomenon took place by increasing the permeability of sheet pile, but the character of seepage pattern change was different. There was prac-tically no change in the seepage pattern in the dam foundation even after increasing the per= meabilities of sheet pile and lower soil layer from 0 to 0.002 . But only after further increasing the perme-abilities of sheet pile under the dam or that of lower

soil layer, a considerable change of seepage under dam occurred.

Ranzhin also studied the effect of the founda-tion consisting of three layers of different permeabilities. In the tests the permeabilities of upper and lower layers were assumed to be equal, with the middle layer having different permeabilities.

The elements of the dam underground were supposed to be impermeable in the tests. The first and the second layers were modelled to be traversed by the upstream apron cut off and cutoff under the dam. Under such conditions the

seepage uplift pressures on the upstream apron increased
and that on the bas of the damspillway decreased
with the increase of the middle layer permeability.
However, the seepage uplift pressure on the upstream apron
was found to be increasing with the increase in the rela-
tive permeability of the (soil foundation) middle layer
from 0.05 to 1.

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┌─────────────────────────────────────────┐
│                                         │
│         C H A P T E R   -  IV           │
│                                         │
│         ANISOTROPIC   FOUNDATION        │
│                                         │
│                                         │
└─────────────────────────────────────────┘
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4.0   GENERAL

Pervious soils may be subdivided into isotropic
and anisotropic. In isotropic soils, the coefficient of
permeability at any point is independent of direction of
seepage velocity. Isotrop ic soils are further subdivided
into homogeneous and non homogeneous soils. The permeability
coefficent of homogeneous soils is independent of the coor-
dinates of the seepage region and is constant   throughout.
In inhomogeneous soils the permeability coefficient depends
on the coordinates of the seepage region. In anisotropic
soils, the permeability coefficient is dependent on the
direction of seepage velocity. The soil is said to be homo-
geneous and anisotropic if the permeability coefficient is
dependent on the direction of velocity and if this direct-
ional dependence is the same at all points of flow region.
In such a case the permeability coefficient is independent
of coordinates of seepage region.

In nature, it is very difficult to get isotropic
soils. Most soils are anisotropic to some degree and hence
the isotropy of a soil is an ideal case and true for theore-
tical considerations only. One of the marked properties

found from the soil samples obtained from river bed is "transverse anisotropy" wherein the permeability coefficient parallel to bedding planes is different from that normal to it. The average permeability of a natural deposit of silt in the horizontal direction has been shown to be equal to 2 to 10 times that in the vertical direction (24). The ratio becomes larger than 10 for distinctly stratified soils. The ratio (k_x/k_y) according to Justin may be 4 to 20 and in eceptional cases of horizontal seems, may even go upto 20 to 50. Generally, in natural deposits of homogeneous nature the permeability coefficient in horizontal direction is greater than that in vertical direction with one exception of Loess, where the opposite true.

If the direction of maximum permeability in the case of 'inclined anisotropy' caused by geological formations of bed, is inclined towards the river bed then it is of serious consequence to the stability of structure and cannot be overlooked.

The manner in which the permeability coefficient may vary at some point (say 'A') in an anisotropic soil is shown schematically in Fig. 4.1.
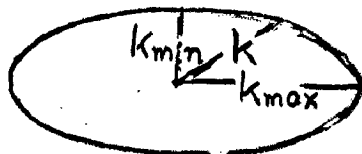

Fig. 4.1

The magnitude k of the radius vector is proportional to the value of the permeability coefficient and its direction coincides with that of the seepage velocity. There are two specific directions in which the permeability coefficient attains a least value of k_{min} and greatest value of k_{max}. as shown in Fig.4.1 These two directions together with a third one perpendicular to them are referred to as "principal directions of anisotropy of soil". A soil often proves to be anisotropic because it is made up of alternating layers having different permeabilities. If, in an anisotropic soil, the principal directions of anisotropy and the values of K_{max} and k_{min} are independent of the coordinates of the seepage region, then the soil si called "homogeneous anisotropic". On the other hand, if either the principal directions of anisotropy, or the values of k_{max} and k_{min}, or both characterisitics, are functions of the coordinates, then the soil is said to be 'inhomogeneous anisotropic".

In case the principal directions of anisotropic soil coincide with the coordinate axes of the physical plane ie the principal directions of permeability are horizontal or vertical, then seepage below a hydraulic structure can be analysed easily. The shape of the actual flow domain is similar to the shape of the transformed, fictitious, isotropic system. Let k_x, k_y and k_z be the principal values of coefficient of permeability in the

direction of coordinate axes. According to Darcy's law

$$v_x = - k_x \frac{\partial h}{\partial x}$$

$$v_y = - k_y \frac{\partial h}{\partial y} \qquad \qquad ...(4.1)$$

$$v_z = - k_z \frac{\partial h}{\partial z}$$

From continuity equation,

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \qquad \qquad ...(4.2)$$

From equations (4.1) and (4.2) we get,

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \qquad ...(4.3)$$

This equation (4.3) can be transformed by substituting

$$\bar{x} = \frac{x}{\sqrt{k_x}} \quad , \quad \bar{y} = \frac{y}{\sqrt{k_y}} \quad , \quad \bar{z} = \frac{z}{\sqrt{k_z}} \qquad ...(4.4)$$

The transformed differential equation is

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad ...(4.5)$$

which is the Laplace's equation.

Thus the effect of anisotropy of the medium is obtained by transforming the coordinates system and solving Laplace's equation in the transformed plane.

The equivalent isotropic permeability of the transformed system is given by $\sqrt[3]{k_x\,k_y\,k_z}$ for three dimensional flor or $\sqrt{k_x\,k_y}$ for two dimensional flow.

4.1 EQUATIONS OF MOTION IN HOMOGENEOUS ANISOTROPIC MEDIUM

When the principal directions of coefficient of permeabilities do not coincide with Horizontal or vertical directions the simple transformation described above is not applicable. In such a case the shape of the boundaries is also deformmed.

For isotropic soil

$$V = -K \text{ grad } h$$

where K = seepage coefficient

$$h = \frac{p}{\rho g} + y + \text{constant}$$

...(4.6)

For homogeneous anisotropic soil, a symmetric seepage tensor may be written in the form

$$\begin{vmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{vmatrix}$$

Taking principal axes of this tensor along x_1 y_1 axes of coordinate system, the tens r is

$$\begin{vmatrix} k_{x1} & 0 \\ 0 & k_{y1} \end{vmatrix}$$

where k_{x1} and k_{y1} are seepage coefficients in the direction of principal axes.

The equation of motion in x_1, y_1 coordinate can be written as

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial x_1} - \frac{g}{k_{x1}} u_1 - g \sin\alpha$$

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial y_1} - \frac{g}{k_{y1}} v_1 - g \cos\alpha$$

...(4.7)

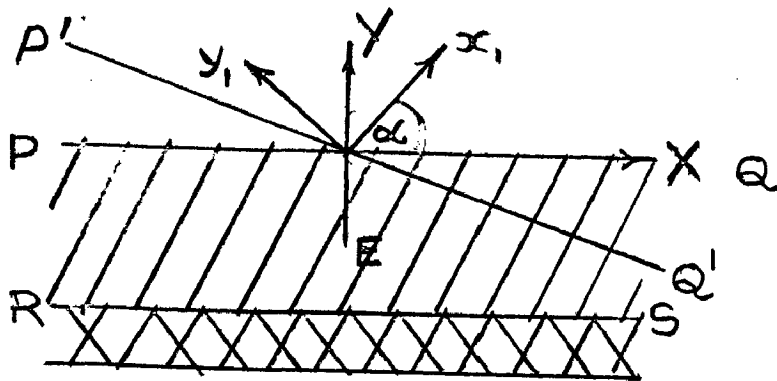where α is the angle measured from horizontal axis.

Figure 4.2.

Putting

$$\phi (x_1, y_1) = \frac{-p}{\rho g} - x_1 \sin\alpha - y_1 \cos\alpha \qquad ...(48)$$

where

$$u_1 = - k_{x1} \frac{\partial \phi}{\partial x_1} \qquad \qquad \ldots (4.9)$$

$$v_1 = - k_{y1} \frac{\partial \phi}{\partial y_1} \qquad \qquad \ldots (4.10)$$

ϕ may be called as the 'reduced head'.

A function ψ (x_1, y_1) , by means of equalities

$$u_1 = \sqrt{k_{x1} \, k_{y1}} \; \frac{\partial \psi}{\partial y_1} \quad , \quad v_1 = - \sqrt{k_{x1} \, k_{y1}} \; \frac{\partial \psi}{\partial x_1}$$

$$\ldots (4.11)$$

is herein introduced. Continuity equation gives

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial v_1}{\partial y_1} = 0 \qquad \qquad \ldots (4.12)$$

from which it follows that ϕ satisfies Laplace's equation

$$k_{x1} \frac{\partial^2 \phi}{\partial x_1^2} + k_{y1} \frac{\partial^2 \phi}{\partial y_1^2} = 0 \qquad \ldots (4.13)$$

With further transformation, i.e. $x = x_1, y_1 = \nu y$, $\nu^2 = k_{y1} : k_{x1}$,

ϕ satisfies Laplace's equation in the new x,y coordinates

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad \qquad \ldots (4.14)$$

The combination $\phi + i\psi$ is a function of complex variable
$z = x + iy$.

The rate of flow through the contour bounded by the
arc AB is expressed by means of stream function as

$$Q = \sqrt{k_{x1} \, k_{y1}} \quad (\Psi_A - \Psi_B) \qquad \dots(4.15)$$

The three coordinate systems that were previously introduced are linked by the equations

$$X = x_1 \cos \alpha - y_1 \sin \alpha = x \cos\alpha - \nu y \sin\alpha$$
$$\dots(4.16)$$
$$Y = x_1 \sin\alpha + y_1 \cos\alpha = x \sin \alpha + \nu y \cos\alpha$$

$$x_1 = x = X \cos \alpha + Y \sin \alpha$$
$$\dots(4.17)$$

$$y_1 = \nu y = -X \sin\alpha + Y \cos \alpha$$

(here the notation $\nu = \sqrt{k_{y1}} : \sqrt{k_{x1}}$ has been introduced)

In particular, the equations of the horizontal lines pQ, RS and of the vertical line MN in the x,y coordinate system are

$$y = - \frac{1}{\nu} x \; \text{tg} \; \alpha$$

$$x \sin\alpha + \nu y \cos \alpha + h = 0 \qquad \dots(4.18)$$

$$y = \frac{1}{\nu} x \; \text{ctg} \; \alpha$$

In Fig. 4.2 p'Q' is the position of PQ in the x,y system.

In the x,y coordinate system, some fictitious flow in the region from that in X,Y plane through the transformation (Eq. 4.16) may be considered. The following conditions on the boundaries of flow region in x,y plane must be satisfied.

(1)   On the boundary of water reservoir

$$\phi(x,y) = \text{constant}$$

On $x_1 \ y_1$ plane, $\phi(x_1 y_1) = \text{constant}$

(2)   On impervious boundary

$$\psi(x,y) = \text{constant}$$

(3)   On the free surface (when the pressure is constant)

$$\phi + x \sin \alpha + vy \cos \alpha = \text{constant} \qquad \qquad ...(4.19)$$

$$\psi(x,y) = \text{constant}$$

Differentiating equation(4.19) along S on the free surface (in x,y plane) and multiplying by $\partial\phi / \partial S$

$$\left(\frac{\partial\phi}{\partial s} \right)^2 + \sin \alpha \ \frac{\partial\phi}{\partial s} \ \frac{dx}{ds} + v \cos \alpha \ \frac{\partial\phi}{\partial s} \ \frac{dy}{ds} = 0 \quad ...(4.20)$$

If $\quad u = \dfrac{\partial\phi}{\partial s} \ \dfrac{dx}{ds} \ , \quad v = \dfrac{\partial\phi}{\partial s} \dfrac{dy}{ds}$

$$u^2 + v^2 + u \sin\alpha + v \ u \cos \alpha = 0 \qquad \qquad ...(4.21)$$

which is the equation of a circle, in the plane of u,v (u, and v are the components of fictitious dimensionless velocity).

This circle passes through the origin of coordinates and has its center in the point $(-1/2 \sin \alpha \ , \ -v/2 \cos\alpha)$.

(4)   The equation 4.19 holds good along seepage surface. Assuming that seepage segment to be a straight line,making an angle β with the x axis and differentiating equation

(4.19) with respect to 'S'

$$\frac{\partial \phi}{\partial s} + \sin \alpha \ \frac{dx}{ds} + v \cos \alpha \ \frac{dy}{ds} = 0$$

$$\frac{\partial \phi}{\partial s} = \frac{\partial \phi}{\partial x} \frac{dx}{ds} + \frac{\partial \phi}{\partial y} \frac{dy}{ds}$$

$$= \frac{\partial \phi}{\partial x} \cos \beta + \frac{\partial \phi}{\partial y} \sin \beta$$

$$= u \cos \beta + v \sin \beta$$

Hence $u \cos \beta + v \sin \beta + \sin \alpha \cos \beta + v \cos \alpha \sin \beta = 0$

$$\dots (4.22)$$

which is the equation of a straight line .

This straight line passes through the point ($-\sin \alpha$, $v \cos \alpha$)
and is perpendicular to the seepage surface.

Particular Case of Anisotropy: ($\alpha = 0$)

In this case for $v < 1$ there will be horizontal
stratification and for $v > 1$ vertical stratification.

Eq. 4.16 becomes $X = x_1 = x$

$$Y = y_1 = vy$$

The directions of the coordinate system coincide, but in
the region of fictitious flow (x,y), this change in
scale along the y axis holds good ie the dam foundation
preserves its length; but the vertical cut off is shortened

for horizontal stratification and stretched for vertical.

The relationship between the reduced flow rate and the

pressure according to Eq. 4.8 is

$$\phi \ (x_1, \ y_1) \ = \ \frac{p}{\rho \ g} \ - \ y_1 \qquad\qquad ...(4.23)$$

4.2 FLOOR WITH A CUT OFF

Consider an impervious floor AB of length $b_1 + b_2 = b$ founded on homogeneous anisotropic permeable soil extending upto infinite depth (Fig.4.3). The floor has a cut off CD of depth d at a distance b_1 from the upstream end. On the upstream and downstream of the floor is pervious bed extending upto infinity. Along the upstream bed AF, $\phi = -kH$ and along downstream bed BG , $\phi = 0$. The foundation profile AEDCB forms the inner boundary and can therefore be taken as a stream line $\psi = 0$.

Let $\lambda (= (k_2/k_1))$ be the coefficient of anisotropy and α be the angle between x-axis and the principal seepage axis μ . The relation between the physical z- plane and the μ , v plane (where μ and v are the directions of principal permeabilities) is given by

$$\mu = x\cos\alpha + y \sin \alpha$$
$$v = \frac{1}{\sqrt{\lambda}} (-x \sin\alpha + y \cos \alpha)$$
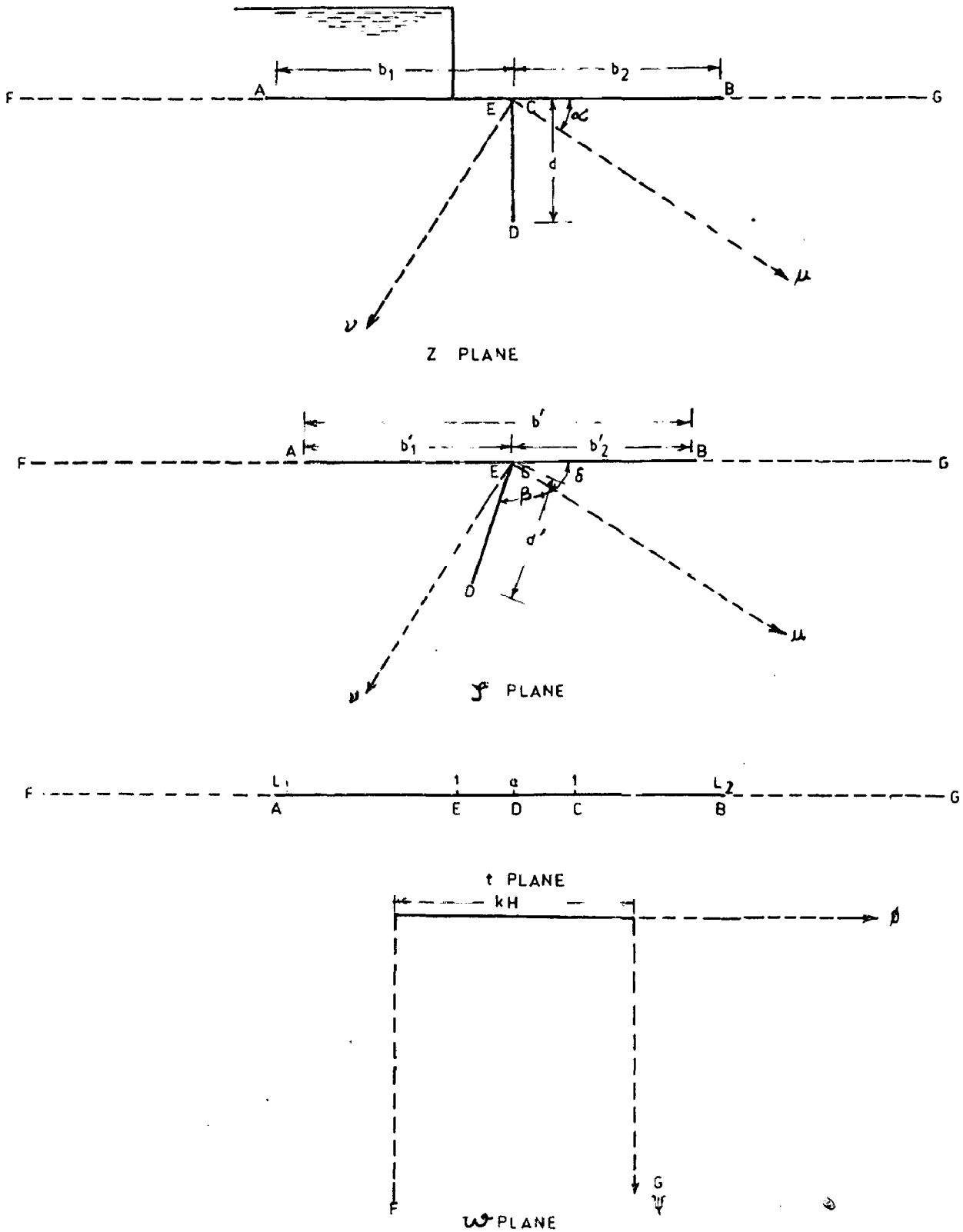
$$...(4.24)$$

FIG. 4·3 − TRANSFORMATION LAYOUT.

The straight line $y = 0$ (floor profile) becomes the straight line

$$\frac{v}{\mu} = - \frac{\tan \alpha}{\sqrt{\lambda}} \qquad \ldots(4.25)$$

Let δ be the angle between the new straight line with the μ axis

$$\tan \delta = - \frac{\tan \alpha}{\sqrt{\lambda}} \qquad \ldots(4.26)$$

In the same way the equation of cut off ($x=0$) is transformed on to the straight line

$$\frac{v}{\mu} = \frac{\cot \alpha}{\sqrt{\lambda}} \qquad \ldots(4.27)$$

If the angle of the transformed cut-off with μ axis is β, it is given by

$$\tan \beta = \frac{\cot \alpha}{\sqrt{\lambda}} \qquad \ldots(4.28)$$

From the Eqs 4.24 we can find the lengths of upstream and downstream aprons in $\mu - v$ plane

$$b_1'^2 = b_1^2 \left(\cos^2\alpha + \frac{\sin^2\alpha}{\lambda} \right) \qquad \ldots(4.29)$$

$$b_2'^2 = b_2^2 \left(\cos^2\alpha + \frac{\sin^2\alpha}{\lambda} \right) \qquad \ldots(4.30)$$

The depth of cut off is given by ,

$$d'^2 = d^2 (\sin^2\alpha + \frac{\cos^2\alpha}{\lambda}) \qquad \ldots(4.31)$$

The relation between the ratio of b/d and b'/d' have been plotted against various values of λ and α in Fig. 4.4.

The angle , $\pi\gamma$ between the downstream floor or bed and the cut off in fictitious $\mu-\nu$ plane is given by

$$\pi\gamma = \tan^{-1} \left[\frac{2\lambda}{(\lambda-1)\sin2\alpha} \right] \qquad \ldots(4.32)$$

The value of $\pi\gamma$ have been plotted against the various values of λ and α in Fig. 4.5 . Transformation of ξ plane into t-plane is given by

$$\xi = A \int (1+t)^{-\gamma} (t-a) (1-t)^{\gamma-1} dt \qquad \ldots(4.33)$$

Integration of this equation yields

$$\xi = - A(1+t)^{1-\gamma} (1-t)^\gamma \qquad \ldots(4.34)$$

and $a = 1 - 2\gamma$ $\qquad \ldots(4.35)$

From the boundary conditions, we find

$$A = d' \frac{e^{-\pi\gamma i}}{(1+a)^{1-\gamma} (1-a)^\gamma} \qquad \ldots(4.36)$$
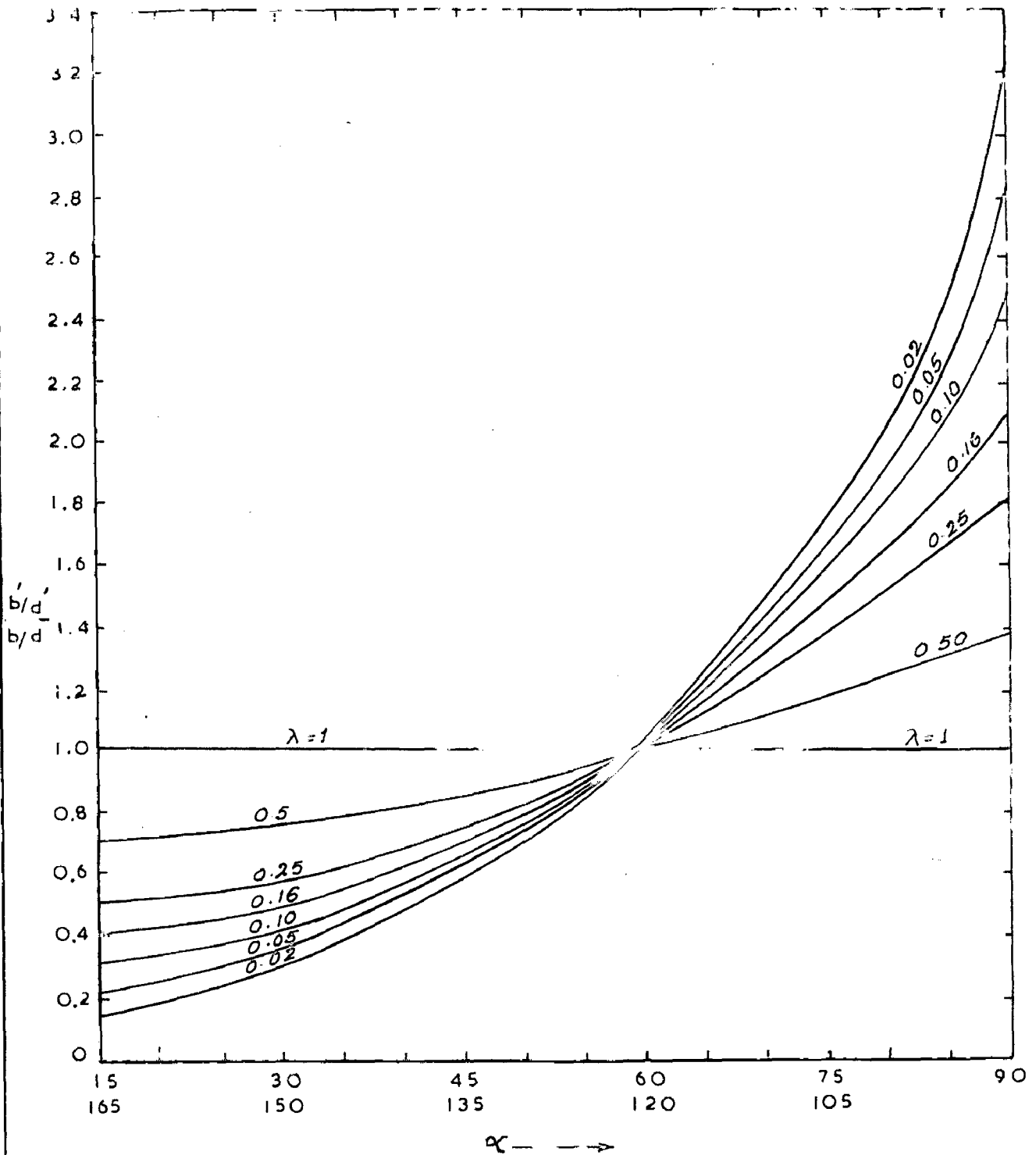
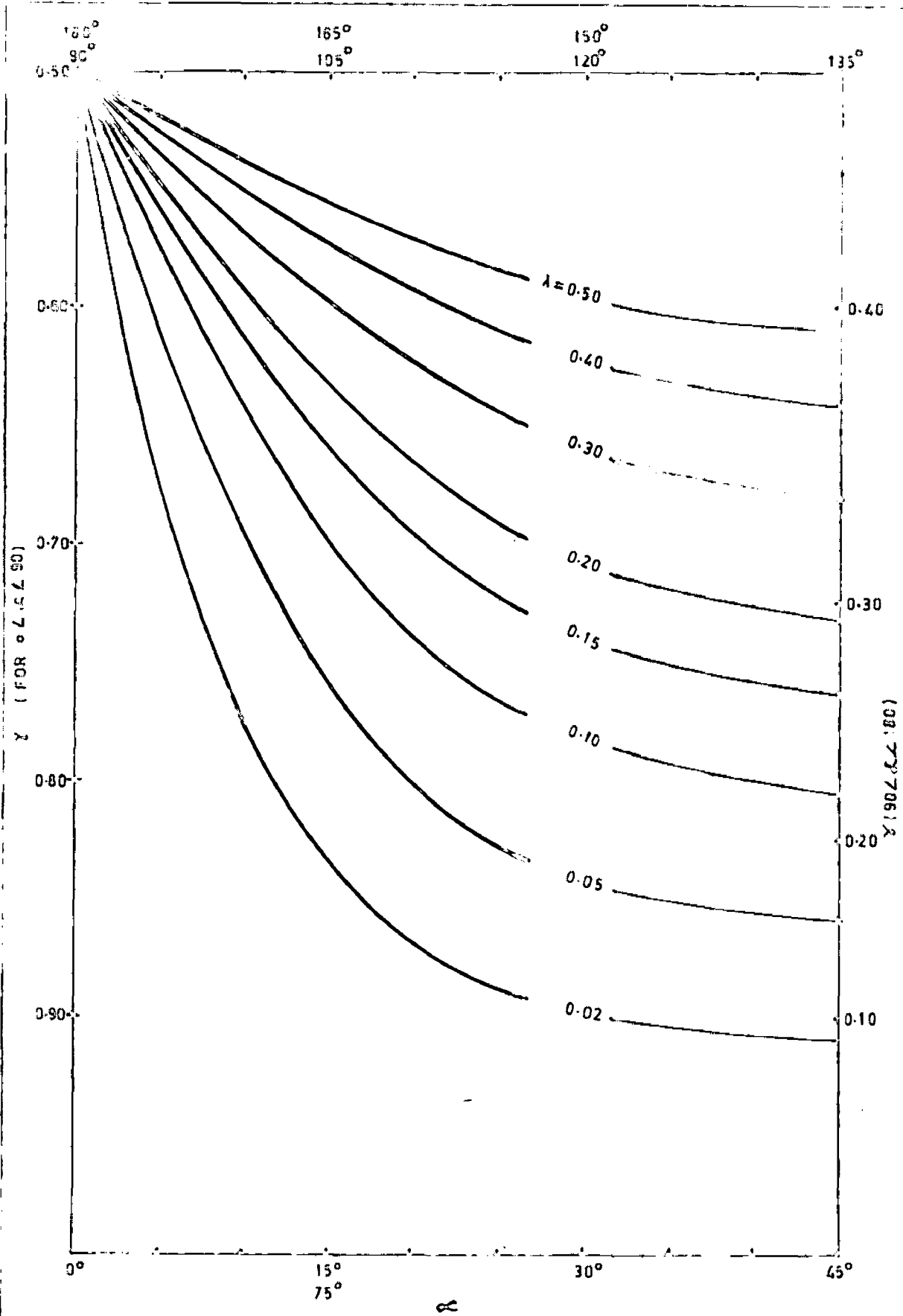The transformation Eq. (4.34) reduces to

FIG 4 4 _ VALUES OF b/d / b/d FOR DIFFERENT λ AND α

FIG. 4·5-VALUES OF r FOR VARIOUS VALUES OF ∝.
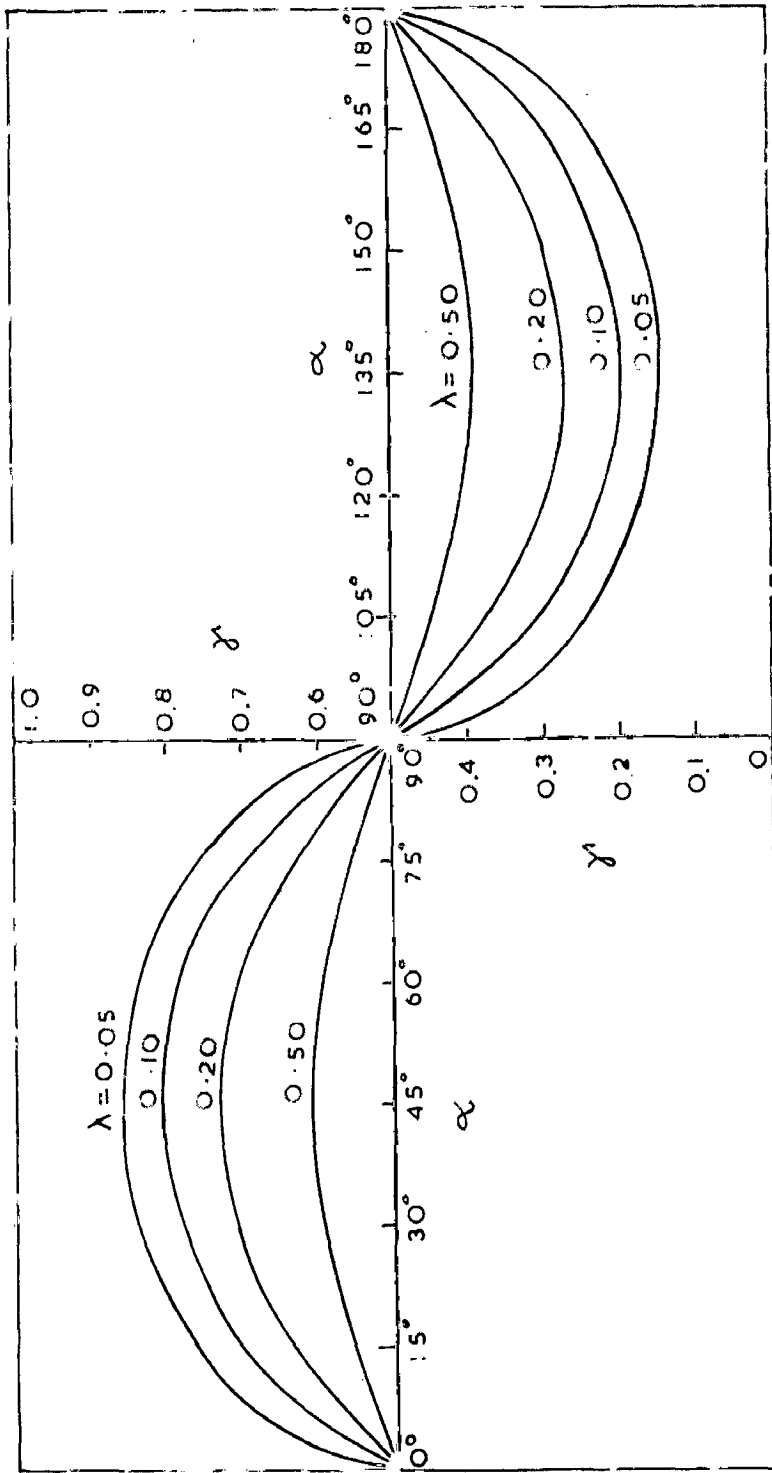
FIG 4.5 (a)   VALUES OF γ FOR α FROM 0° TO 180°

$$\zeta = d' \left(\frac{1+t}{1+a}\right)^{1-\gamma} \left(\frac{t-1}{1-a}\right)^{\gamma} \qquad \ldots(4.37)$$

This equation can be used to determine the values of L_1 and L_2 for the known values of b_1' and b_2' .

The transformation of w- plane onto the lower half of the t plane is given by

$$w = \frac{kH}{\pi} \sin^{-1} \left[\frac{2t + L_1 - L_2}{L_1 + L_2}\right] - \frac{kH}{2} \ldots(4.38)$$

The uplift pressures at D and E below a horizontal structure with an end cut off have been determined and plotted in Fig. 4.6 , 4.7 . These uplift pressures have been plotted against b'/d' values for various values of γ .

In order to determine the effect of the principal directions of anisotropy on uplift pressures at points D and E below floor with an end cut off, curves for $b/d = 5$, 10 and 15 and $\lambda = 0.5$ have been plotted in Fig.4.8 for various values of α . Uplift pressures at thes points for isotropic medium have also been plotted in this figure. A perusal of this figure indicates that value of ϕ_D is minimum for value of α around 120° and maximum at α less than about 30° . The value of α for maximum values of ϕ_D varies with the value of b/d . A perusal of Fig.4.9

FIG.4·6–UPLIFT PRESSURE AT 'E'

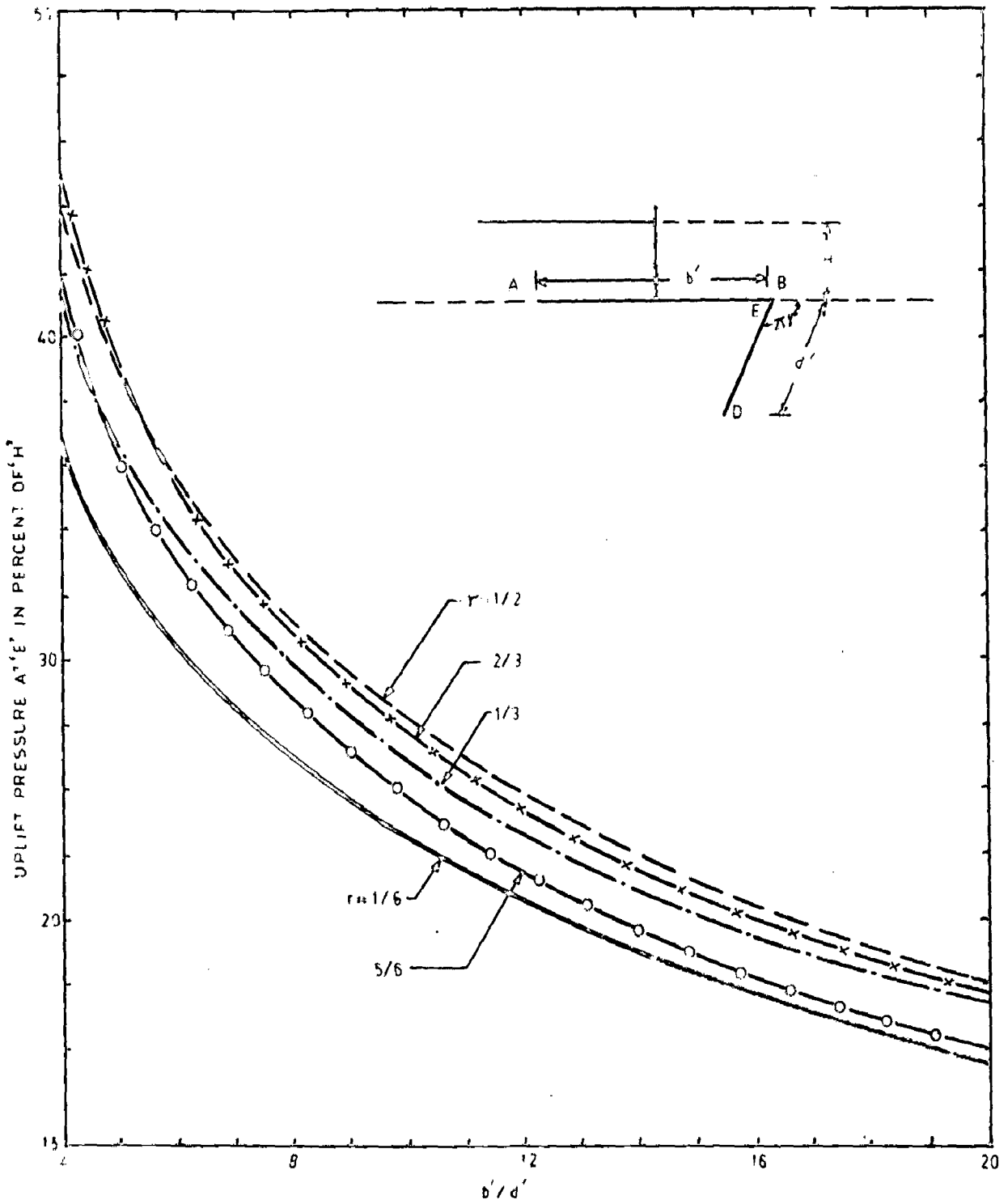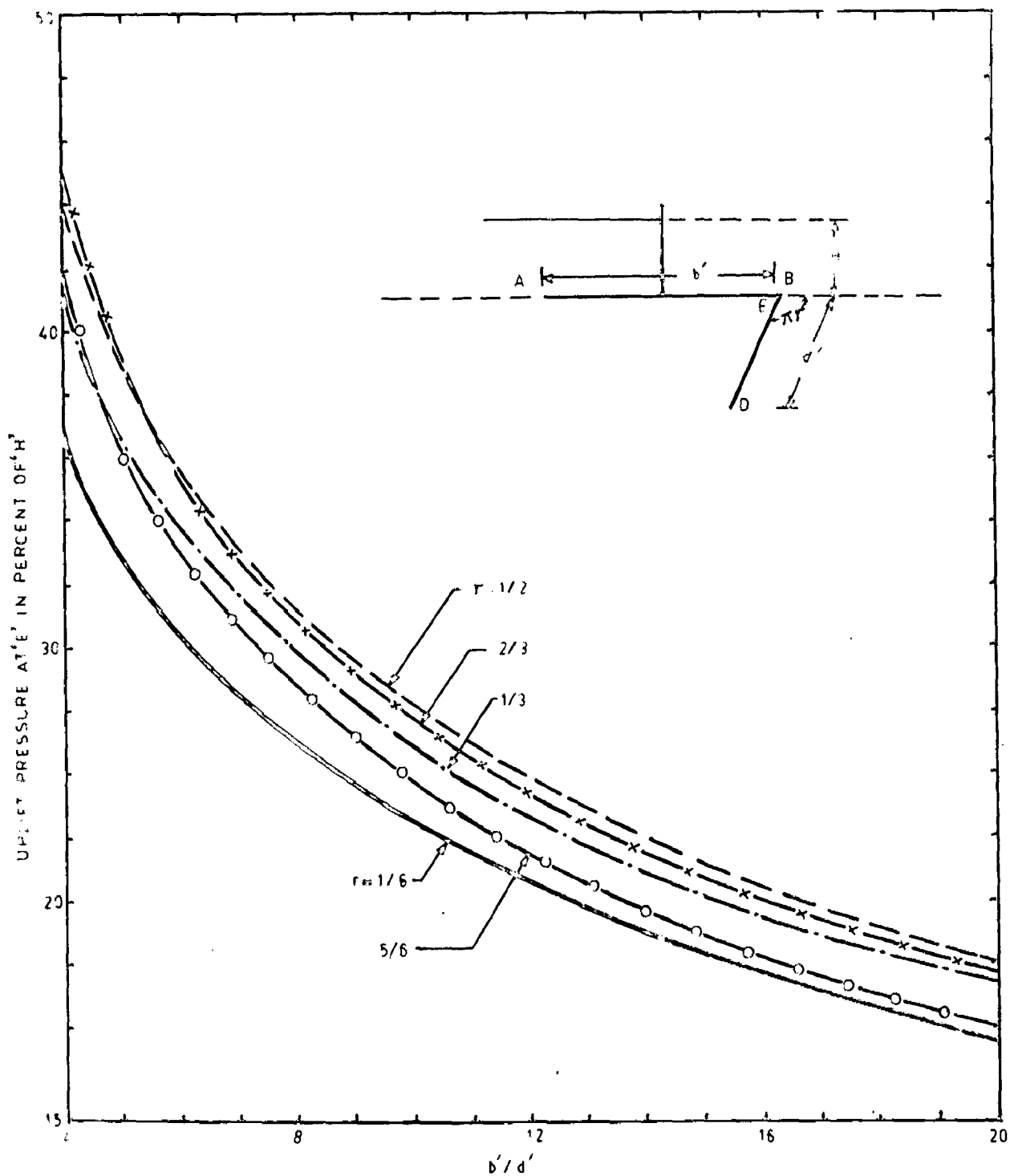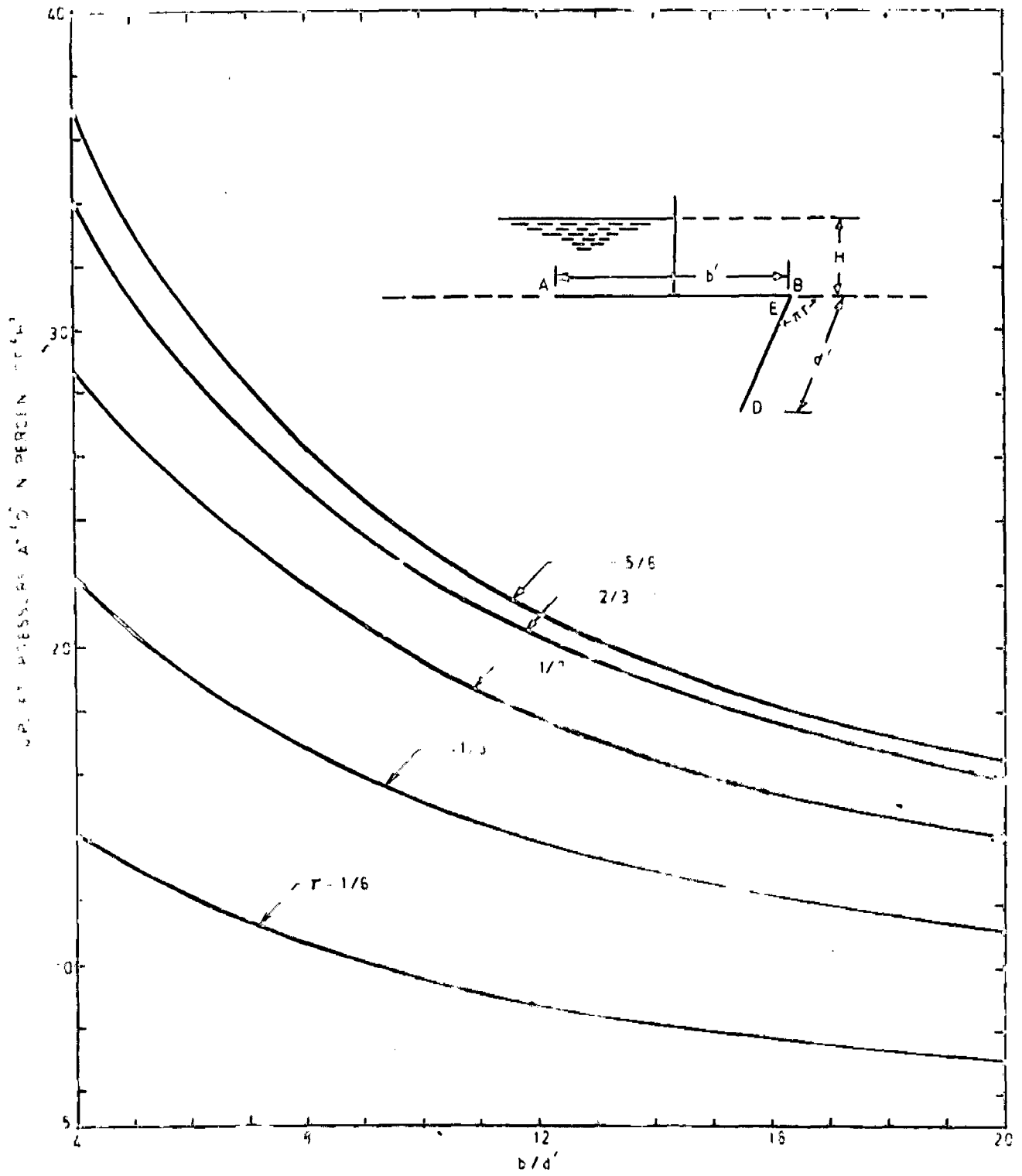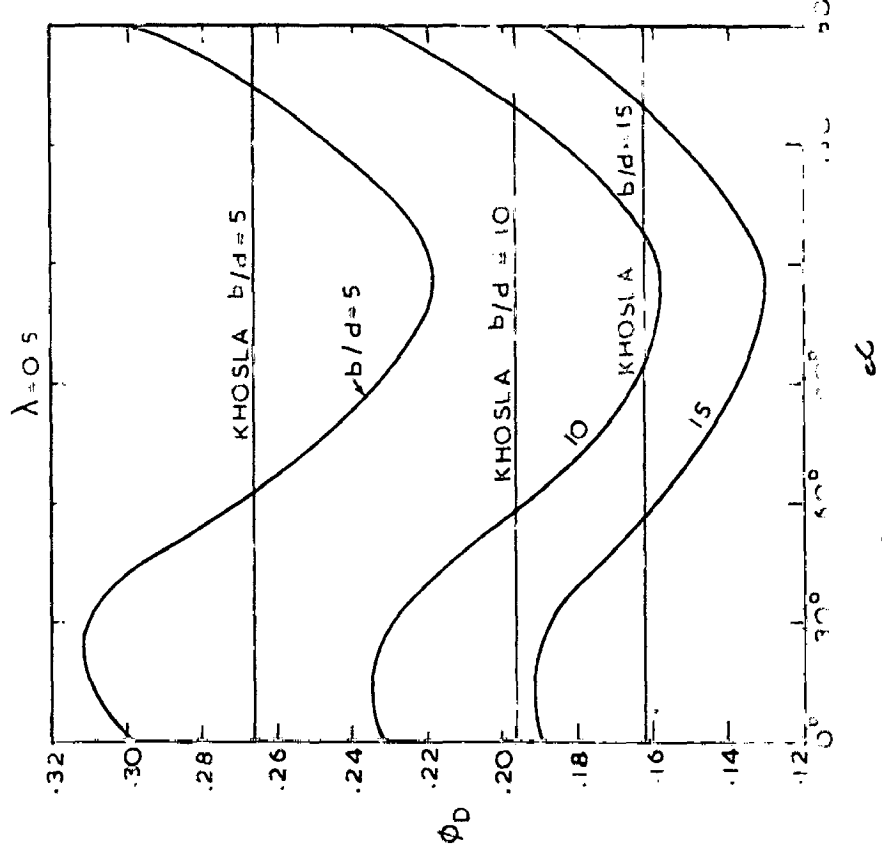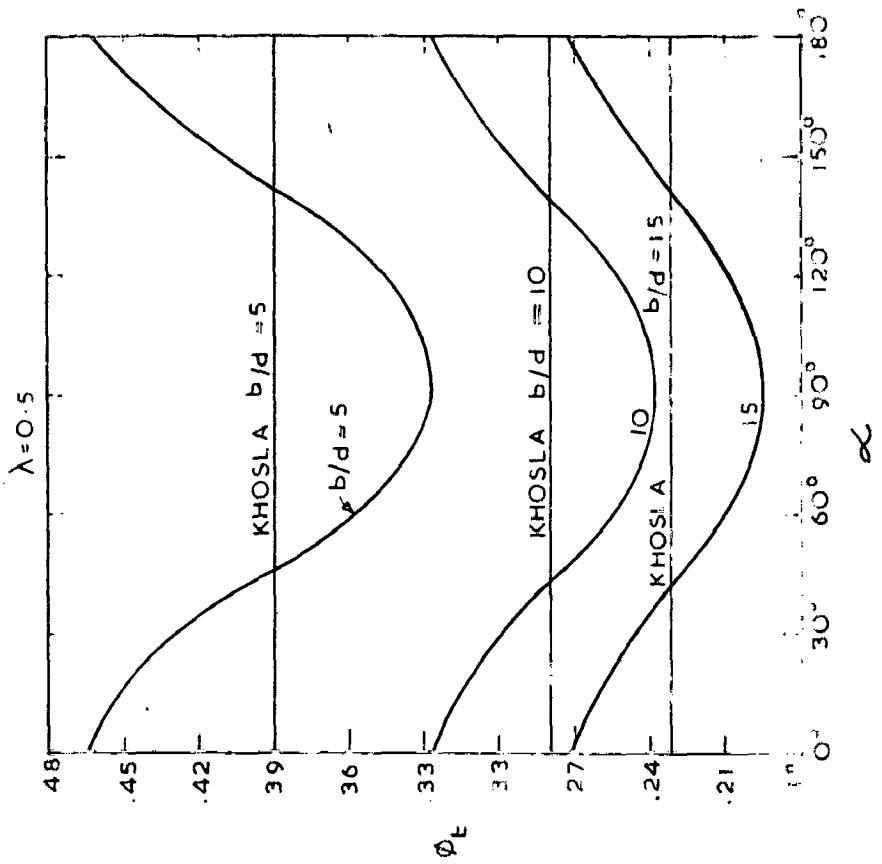FIG.4·6-UPLIFT PRESSURE AT 'E'

FIG·4·7 - JOINT PRESSURE (γ 'D)

FIG.4·8_VALUES OF ϕ_D AND ϕ_t FOR DIFFERENT VALUES OF b/d AND $\propto (\lambda=0.5)$
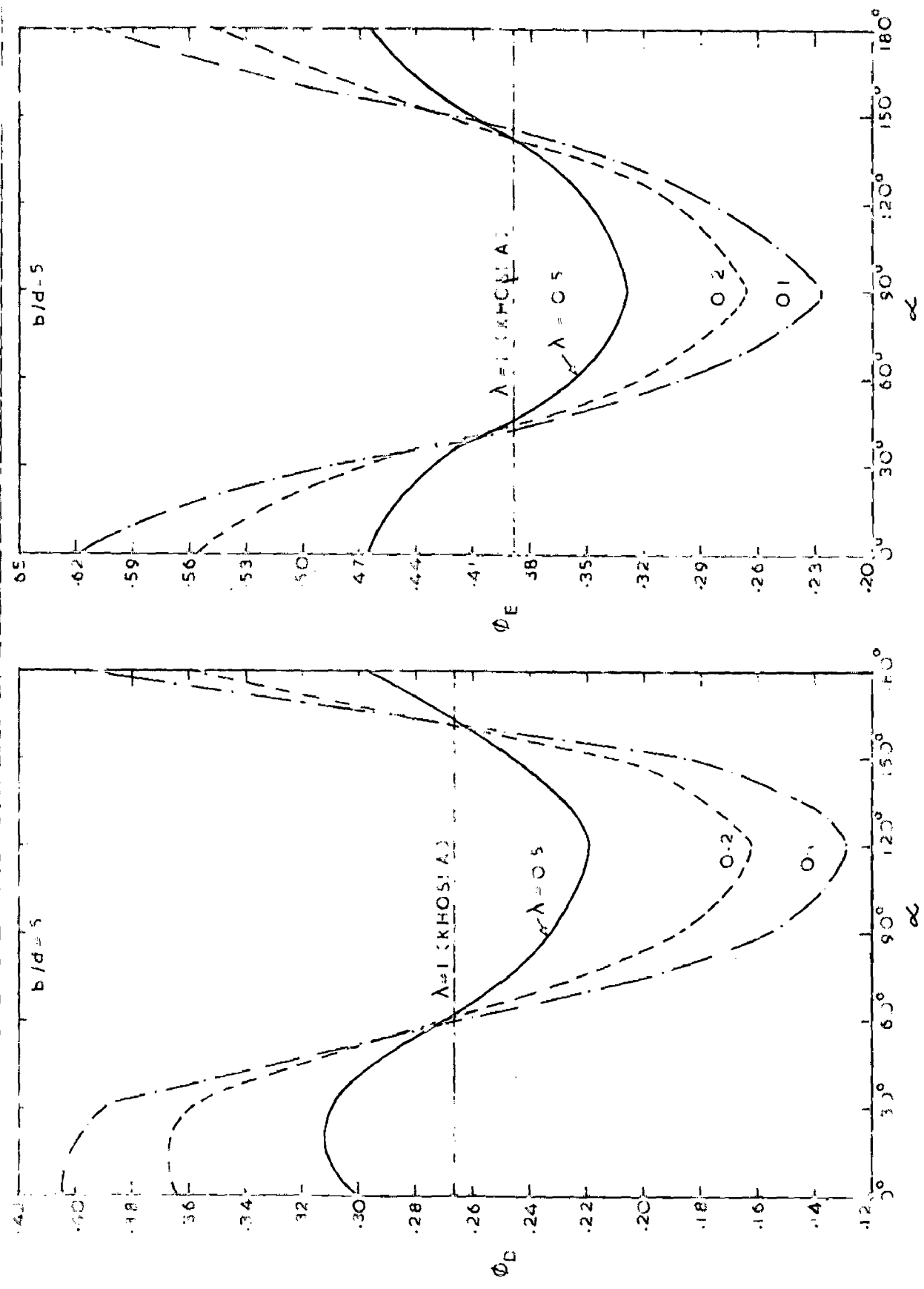
FIG. 4.9—VALUES OF ϕ_D AND ϕ_E FOR DIFFERENT VALUES OF λ AND α

in which values of ϕ_D have been plotted against α for

values of $\lambda = 0.5$, 0.2 and 0.1 and $b/d = 5$ indicates

that the values of ϕ_D are maximum at $\alpha = 0$, for $\lambda = 0.1$,

at α equal to about 15° for $\lambda = 0.2$ and α equal to about

20° for $\lambda = 0.5$. A perusal of these figures also indicates

that for value of α ranging between 60° and about 160°

the values of ϕ_D are less than those for isotropic con-

ditions. For values of $\alpha < 60^\circ$ and greater than 160° the

values of ϕ_D are larger than those for isotropic conditions.

The values of ϕ_E have also been plotted in these figures

for various values of α , λ and b/d. It is seen that the

values of ϕ_E are minimum for $\alpha = 90^\circ$ and maximum for

$\alpha = 0^\circ$. The values of ϕ_E for anisotropic soil are less

than those for isotropic soil for the values of α ranging

between about 45° and 140° . The minimum value of ϕ_E

and ϕ_D decrease with decrease in the value of λ and

their maximum values increase with increase in the value

of λ .

The values of ϕ_D and ϕ_E have also been plotted

in Fig. 4.10 against values of λ , for $\alpha = 45^\circ$ and values

of $b/d = 5$, 10, 15 and 20. A perusal of this figure indi-

cates that values of ϕ_D increase with decrease in the

value of λ from 1.0 to about 0.1 and then decrease with

further decrease in the value of λ . The value of λ for

maximum value of ϕ_D is not equal to 0.1, for all values

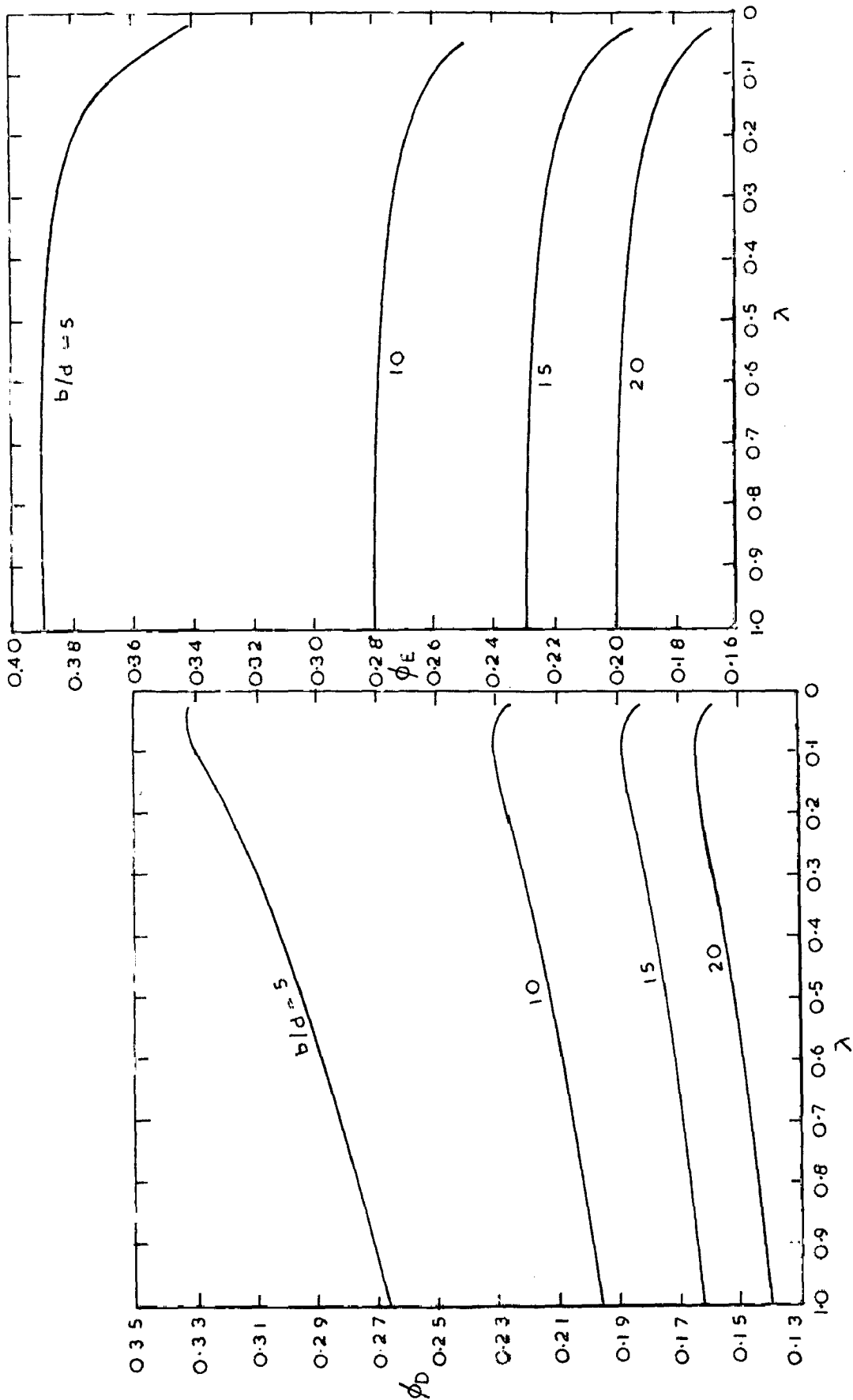of b/d . It increases with increase in the value of b/d.

The values of

FIG. 4.10 VALUES OF ϕ_D AND ϕ_E FOR DIFFERENT VALUES OF b/d AND λ
($\alpha = 45°$)

in the values of λ and the decrease is rapid after $\lambda = 0.2$
The effect of λ on the values of ϕ_E and ϕ_D for b/d =5
and values of $\alpha = 0^O$, 30° , 45°, 90° and 135° can be
studied by the curves plotted in Fig. 4.11. A perusal of
this figure indica tes that values of ϕ_D increase with
decrease in the value of λ for the values of α less than
60° and greater than 160° and decrese with the decrease
in the value of λ for the range $60^O < \alpha < 160^O$. Similarly
the values of ϕ_E increase with decrease in the value of λ
for α less than 45° and greater than 140° , and decrease
with decrease in the value of λ for $45^O < \alpha < 140^O$. The

pressures at E for anisotropic conditions would therefore
be more than those for isotropic condition when α is less
than 45° and greater than 140°, and the pressures at D
for anisotropic conditions would be more than those for
isotropic conditions when α is less than 60° and greater
than 160°.

4.3 FLOOR WITH A CUT OFF FOUNDED ON ANISOTROPIC SOIL
OF FINITE DEPTH

As indicated in para 4.1 and 4.2 the physical
plane with inclined anisotropic soil can be transformed into
a fictitious plane in which, the vertical cut off is
transformed into an inclined cut off. Solution for inclined
cut off with a step and apron on both sides founded on
finite depth of permeable medium has been obtained by
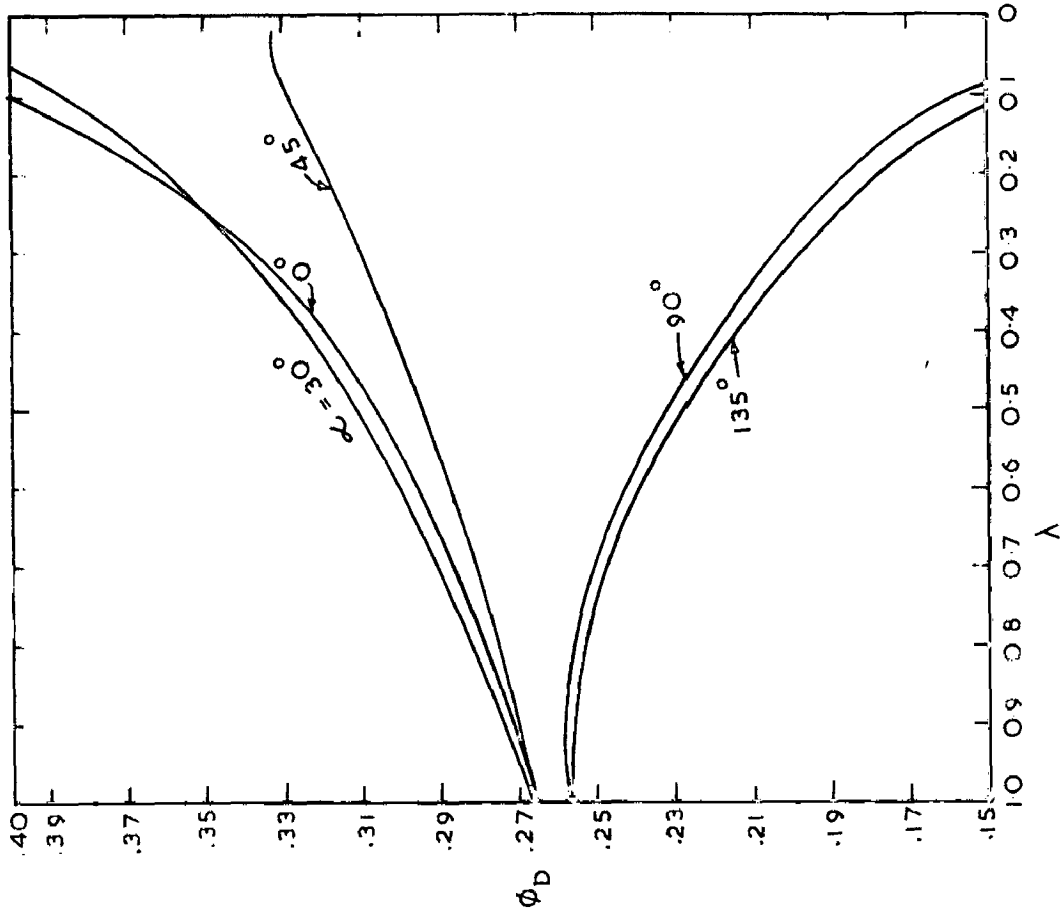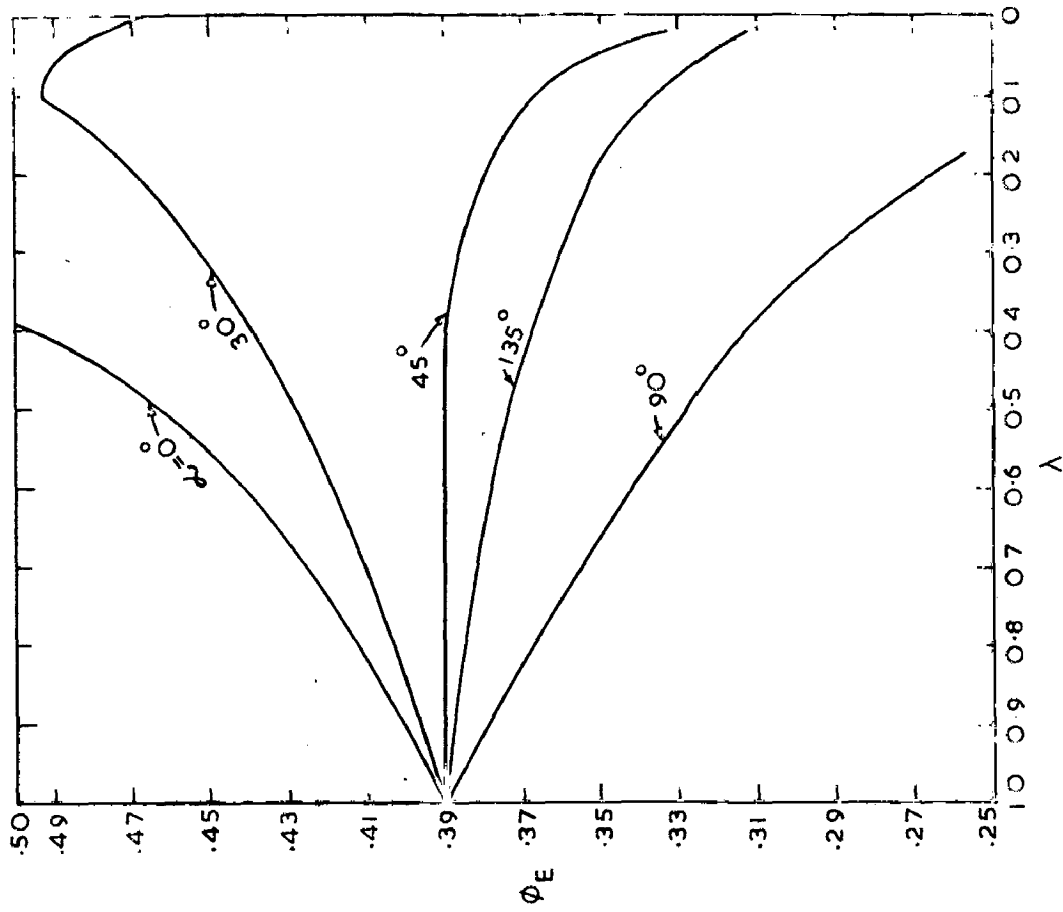
FIG 4.11 - VALUES OF ϕ_D AND ϕ_E FOR DIFFERENT VALUES OF α AND λ

Sivareddy et.al.(21). The transformation layout is shown in Fig. 4.12.

The relation between the physical plane and the fictitious z plane is given by Eqs 4.16 and 4.17 . Transformation of the z plane into the intermediate t-plane is given by

$$z = M \int_{0}^{t} \frac{(t-\beta)\,dt}{t(t-\alpha)^{1-\gamma}(t-1)^{\gamma}} + Z_E \qquad \ldots (4.39)$$

where values of M, α and β can be evaluated from the following equations for known values of T, Z_D and Z_E.

$$M = -\frac{T\,\alpha^{1-\gamma}}{\beta\pi} \qquad \ldots (4.40)$$

$$\frac{Z_D - Z_E}{T} = \frac{1}{\pi(-1)^{\gamma}} \left[B_1 - B_2 \frac{\alpha^{1-\gamma}}{\beta} \right] \qquad \ldots (4.41)$$

$$\frac{Z_E}{T} = \frac{(\beta - \alpha^{1-\gamma})\,B(\gamma, 1-\gamma)}{(-1)^{\gamma}\,\beta\,\pi} \qquad \ldots (4.42)$$

where $B_1 = B_{(\beta-\alpha)/(1-\alpha)}(\gamma, 1-\gamma)$ is incomplete beta function

$B_2 = B_{(\beta-\alpha)/[\beta(1-\alpha)]}(\gamma, 1-\gamma)$ is incomplete beta function

$B(\gamma, 1-\gamma)$ is complete beta function

α and β are transformation parameters.
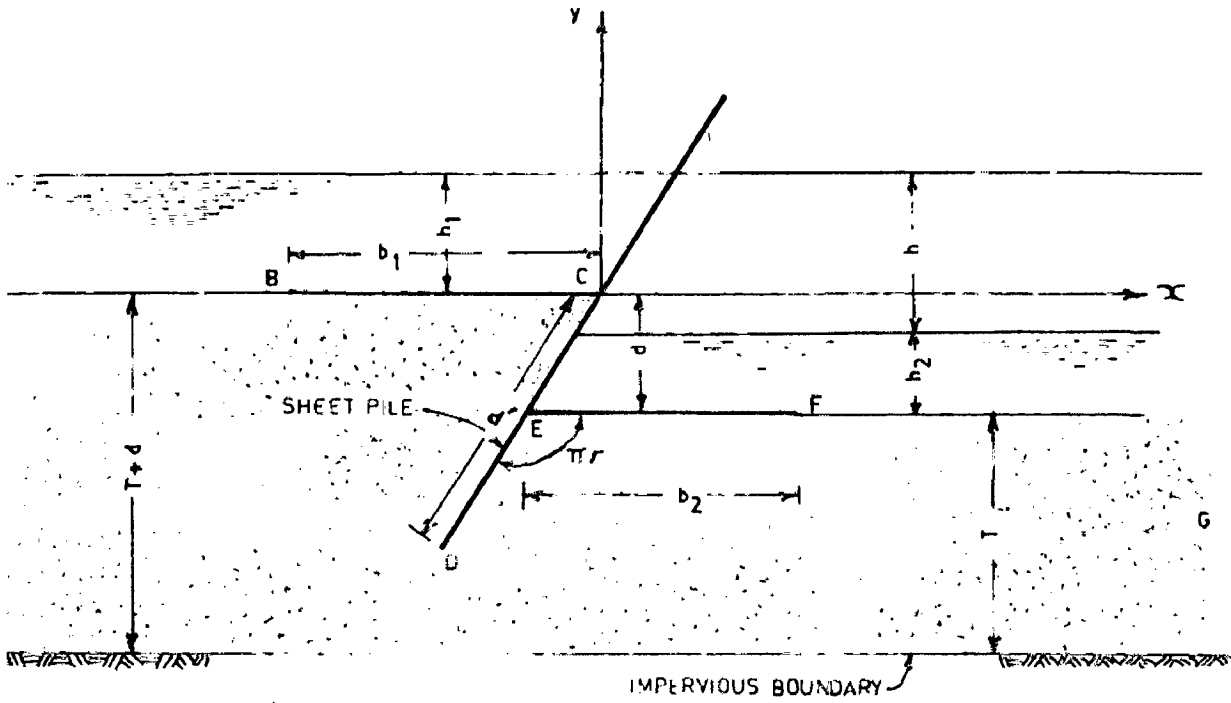
(a) Z PLANE
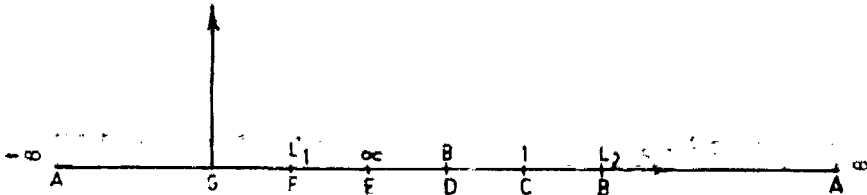
(b) t PLANE
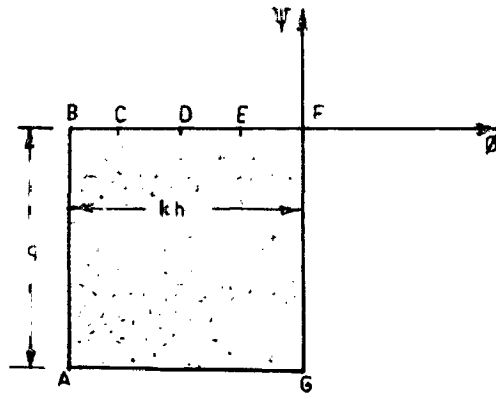
(c) W PLANE

FIG.4.12-DIAGRAM OF PHYSICAL PROBLEM AND CONFORMAL MAPPING PLANES.

Eq 4.39 can then be used to evaluate the values of transformation parameters L_1 and L_2 for known values of b_1 and b_2.

Transformation of w-plane into t-plane is given by

$$w = M' \int_{-\alpha}^{t} \frac{dt}{\sqrt{t(L_1-t)(L_2-t)}} + N' \qquad \ldots(4.43)$$

where $M' = \dfrac{kH}{K} \sqrt{\dfrac{L_2}{4}}$ $\qquad \ldots(4.44)$

$N' = - (kH + iq)$ $\qquad \ldots(4.45)$

where K = complete elliptic function with modulus

$$= \sqrt{\frac{L_2-L_1}{L_2}}$$

q = Seepage discharge per unit length of the structure

$$= \frac{kH}{K} K'$$

K' = complete elliptic function with modulus $= \sqrt{\dfrac{L_1}{L_2}}$

Potential at any point along the floor is given by

$$\frac{\phi}{kH} = \frac{F(\theta, m)}{K} \qquad \ldots(4.46)$$

where $F(\theta, m)$ = incomplete elliptic function

$$\theta = \sin^{-1} \sqrt{\frac{L_2(t-L_1)}{t(L_2 - L_1)}} \qquad \ldots(4.47)$$

$$m = \sqrt{\frac{L_2 - L_1}{L_2}} \qquad \qquad \dots(4.48)$$

Exit gradient at any point along the down-stream bed is given by

$$G_E \frac{T}{H} = \frac{\beta \pi \sqrt{L_2 t}}{2K (t-\beta)} \cdot \frac{\alpha^{\gamma-1}(t-\alpha)^{1-\gamma}(t-1)^{\gamma}}{\sqrt{t (L_1-t)(L_2-t)}} \qquad \dots(4.49)$$

Seepage discharge and pressure at the tip of the cut off have been obtained for a cut off without a step and apron. The uplift pressures at D are shown in Fig. 4.13, and the seepage discharge in Fig. 4.14. Exit gradient at the end of the cut off are also plotted in Fig. 4.15, for various values of γ. A perusal of the figure indicates that the value of exit gradient just downstream of the cut off is zero for $\gamma < 0.5$ and is infinite for $\gamma > 0.5$. For values of $\gamma < 0.5$ the value of exit gradient first increases, reaches a maximum value and decreases as distance from the cut off increases. For $\gamma > 0.5$ the value of exit gradient decreases with increase in distance from the cut off.

A perusal of Fig. 4.13 indicates that the value of ϕ_D/kH is a linear function of d'/T in the range, these values have been plotted.

4.4 EXPERIMENTAL STUDIES

In the previous sections cases of anisotropy where its principal directions do not coincide with coordinate axes
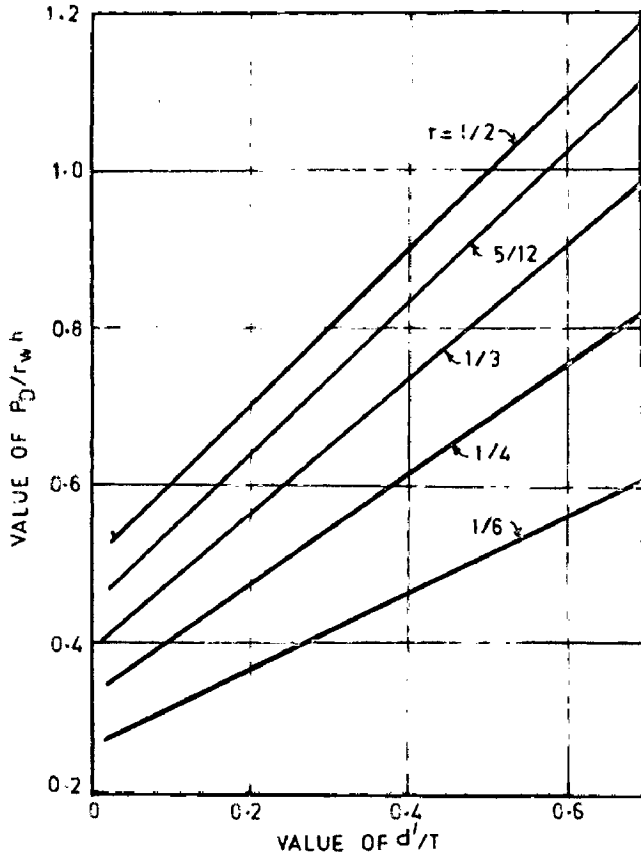
FIG.4·13-PRESSURE AT TIP OF SHEET PILE V/S LENGTH OF SHEET PILE FOR T=10
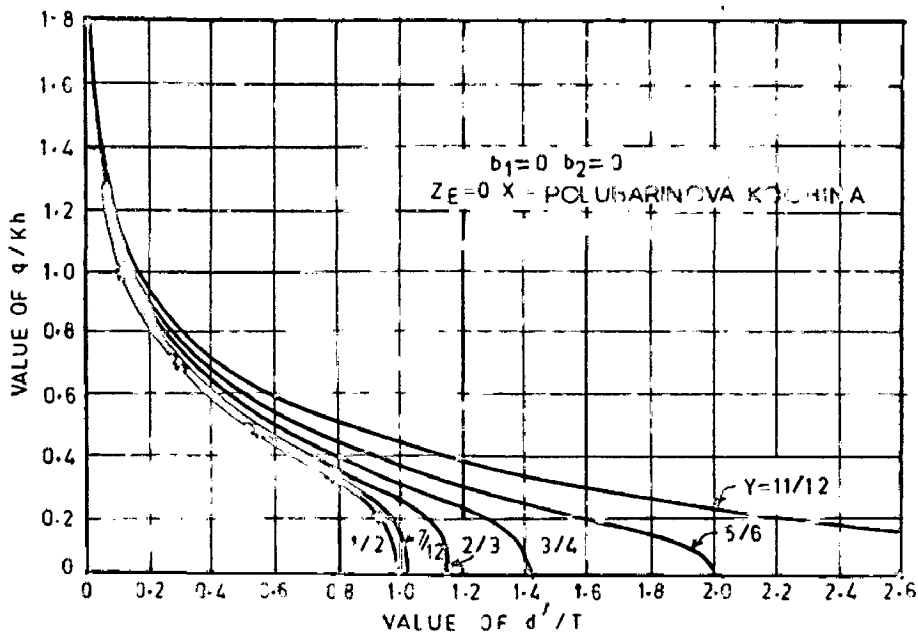
FIG.4·14-EFFECT OF LENGTH OF SHEET PILE ON QUANTITY SEEPAGE.
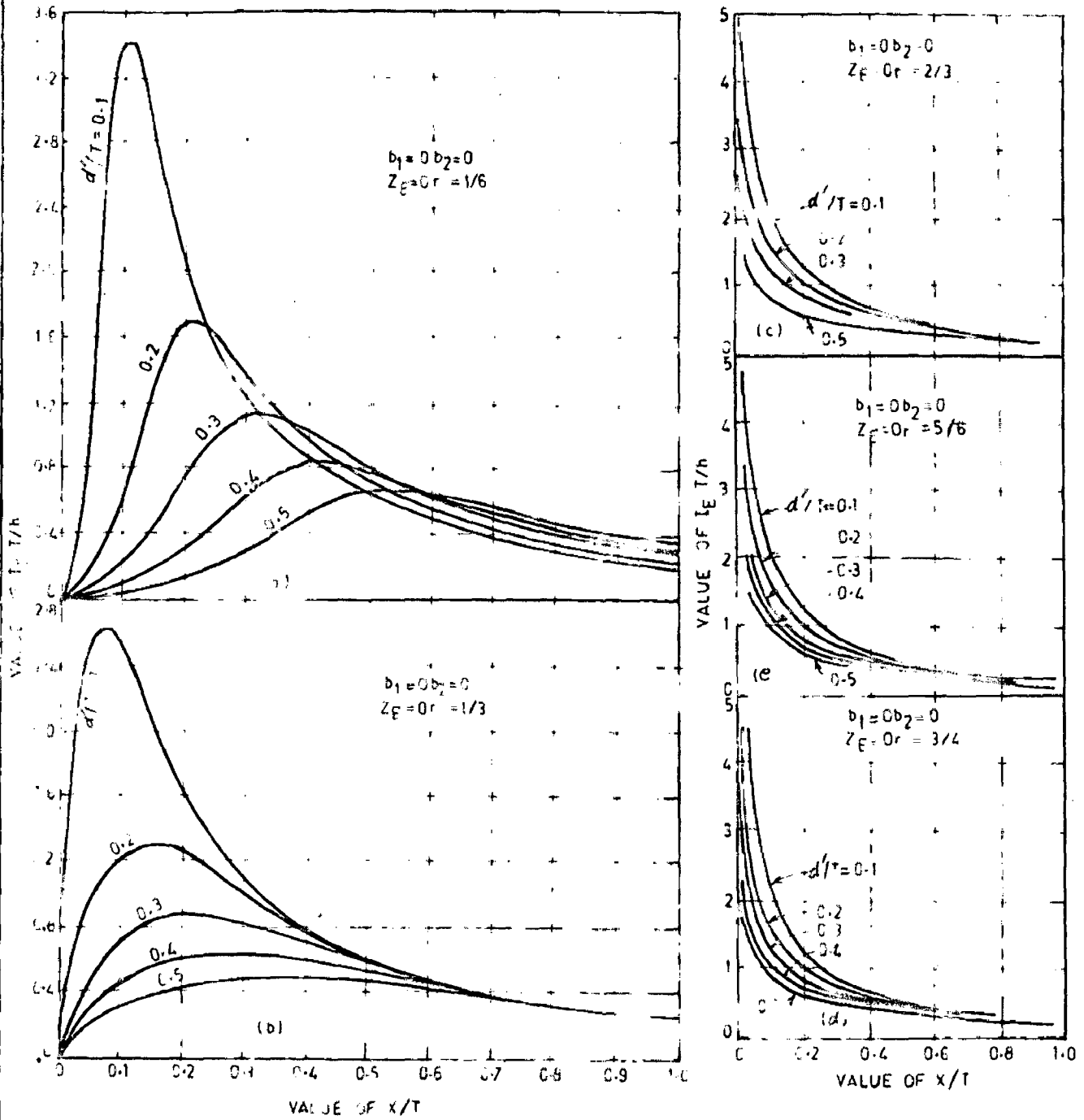
FIG. 4·15—EXIT GRADIENT FOR SHEET PILE OF INCLINATION
(a) π/6. (b) π/3. (c) 2π/3. (d) 3π/4. (e) 5π/6.

of physical plane were considered. The case of anisotropy with its principal directions parallel to the coordinate axes is simple to deal with . However earlier some experimental studies were conducted for this case based on electrical analogy method.

In the case of a depressed floor on infinite depth, P.V. Rao (19) opined that the uplift pressures increased in upstream half of floor length and decreased in downstream half with increase in anisotropic ratio. The maximum deviation in pressure was 8.5% from that of isotropic case, for a value of n (= K_x/ K_y) equal to 15.

The exit gradient was found to increase with 'n' value and this increas was found to be 50% of the pressures for isotropic case for n = 15. In the case of a flat floor with a cut off at either end, the author observed that maximum deviation of pressures from that of isotropic case to be $\pm$ 28% for n = 15.

The uplift pressure distribution below a floor on finite depth of anisotropic foundation for two cases viz. 1) downstream end cut off (2) intermediate cut off, were studied experimentally by Alamsingh et al (2) and Punmia. et.al (18) respec tively. The conclusions of the study were-

(1) Pressure at any point under floor increased with increase in d/T ratio for a fixed b/T ratio (Fig.4.16)

(2)   For a given d/T ratio pressure at any point was

larger for lesser values of b/T and decreased

with increas in b/T ratio (Fig. 4.17).

(3)   The pressure at any point increased sharply with

increas in anisotropic ratio (Fig. 4.18) . The

variation of pressure under floor was smaller for

greater anisotropic ratios.

(4)   For given T/b and d/T ratios, p_E and p_D

increased with increase in anisotropy (Fig.4.19).

Effect of anisotropy was more pronounced for lower

value of T/b.

(5)   Both p_E and p_D decreased with increase in b/d

ratio (Fig. 4.20 and 4.21).
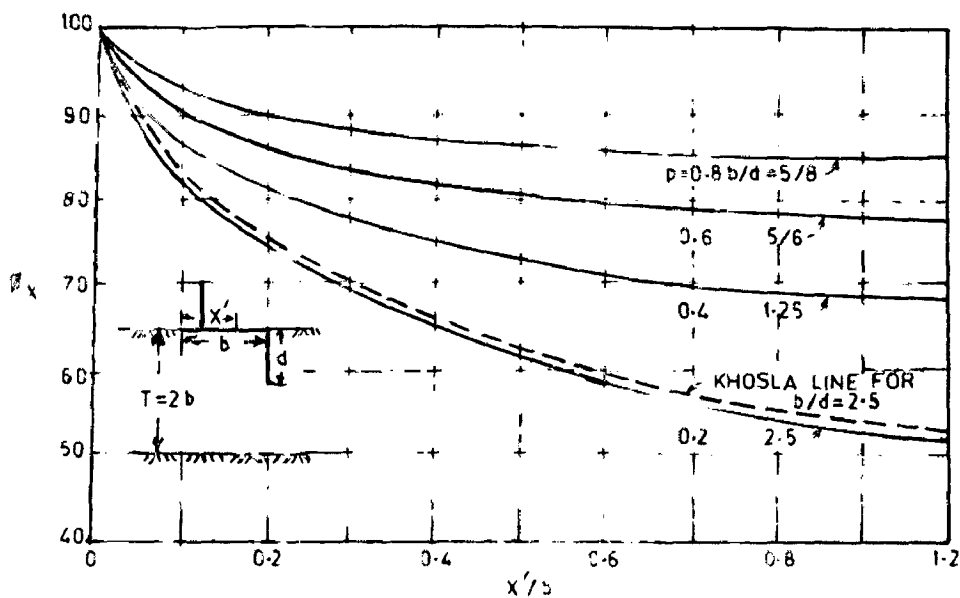
FIG·4·16–VARIATION OF Øx FOR VARIOUS VALUES OF
d/T AND b/d; b/T= 1/2 (FIXED)
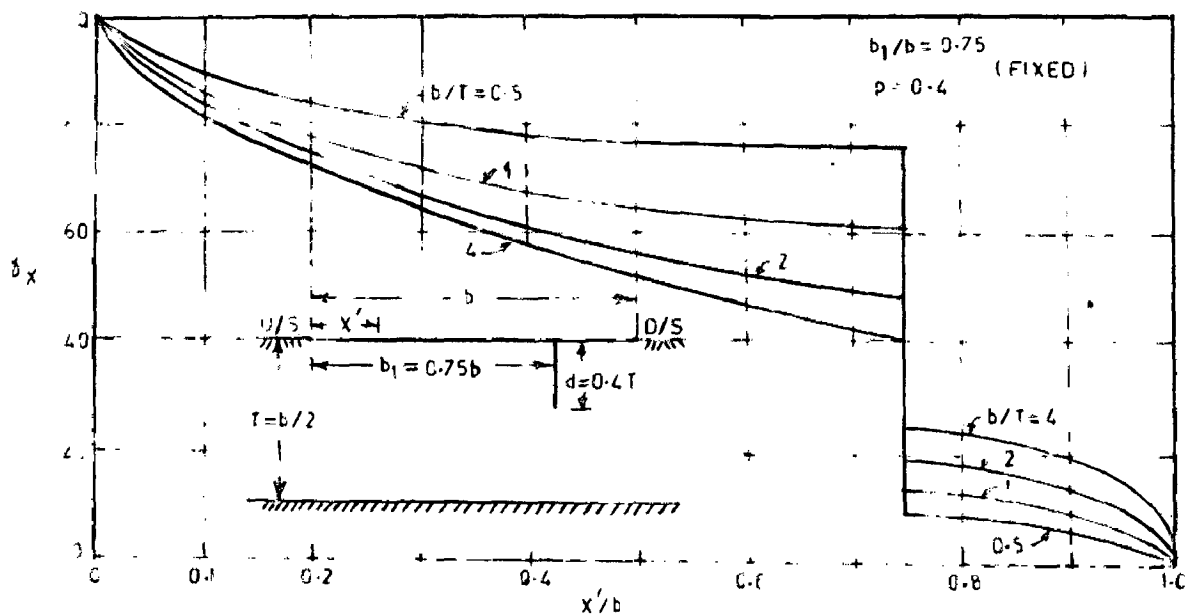
FIG·4·17–EFFECT OF b/T ON PRESSURE DISTRIBUTION·
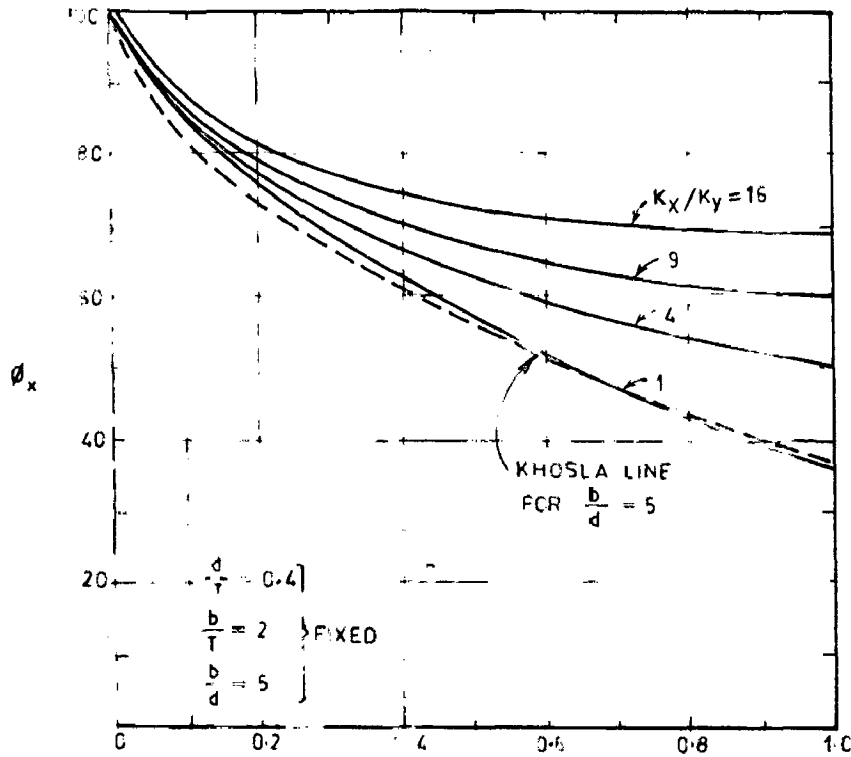
FIG. 4·18-EFFECT OF ANISOTROPY ON PRESSURE DISTRIBUTION
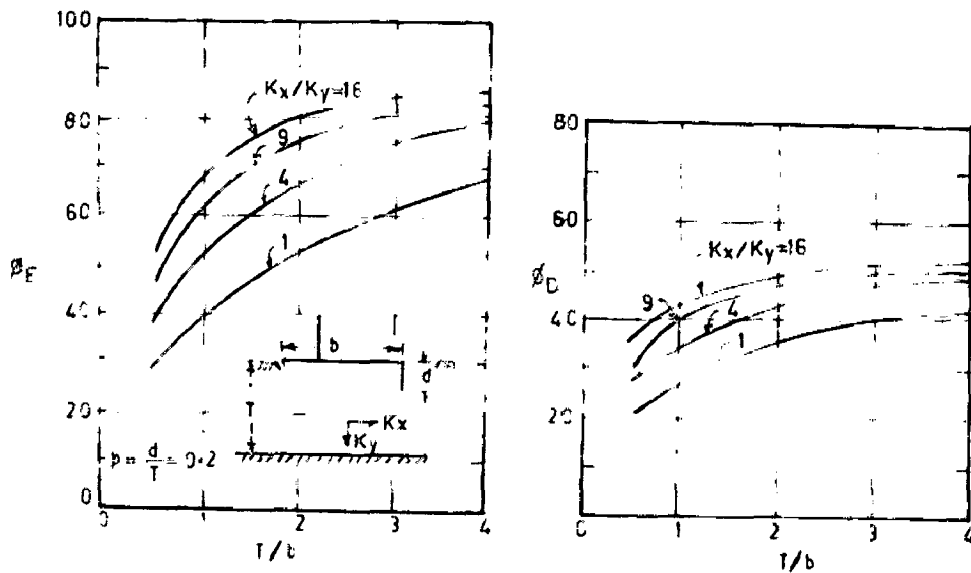
FIG·4·19-INFLUENCE OF T/b RATIO ON ϕ_E & O_D (d/T=0·2)
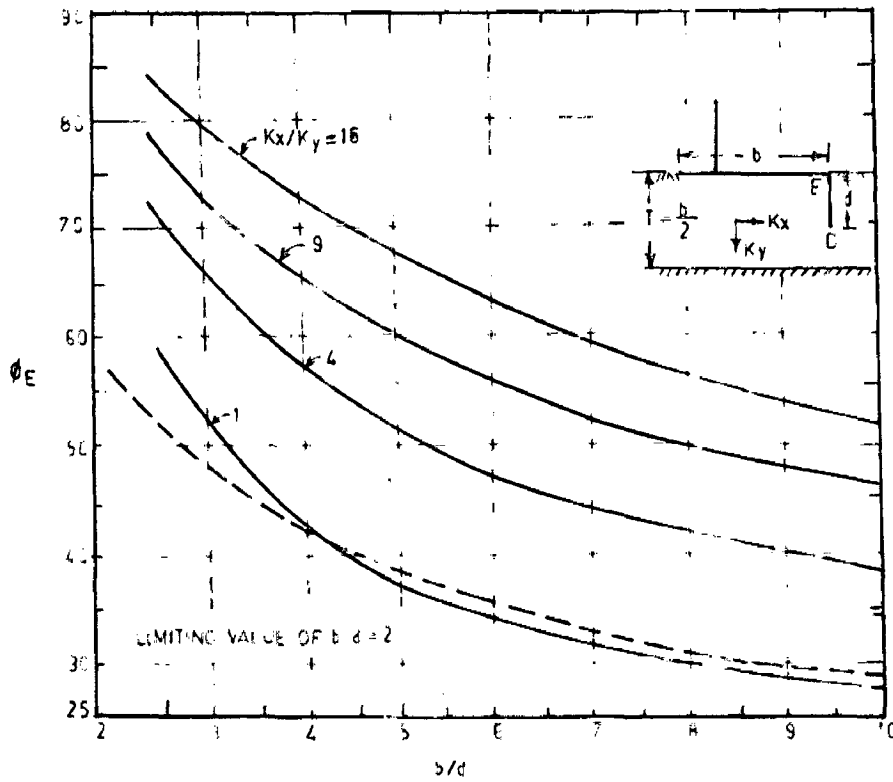
FIG. 20-VARIATION OF ϕ_E WITH b/d AND ANISOTROPY (b/T=2)
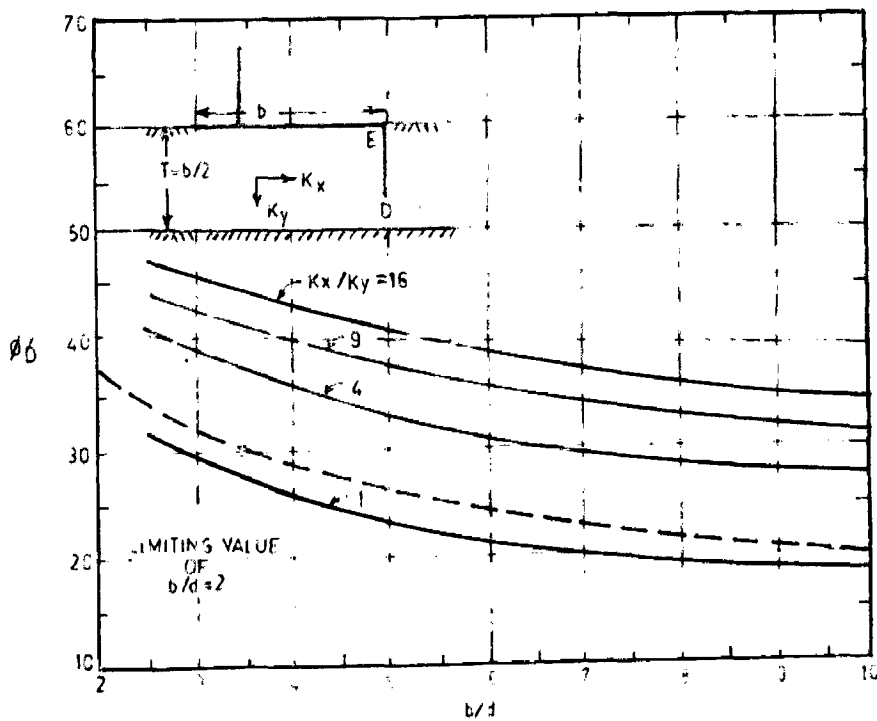
FIG. 21-VARIATION OF ϕ_D WITH b/d & ANISOTROPY (b/T=2)

5.1 GENERAL

The solution of seepage below hydraulic structures
founded on homogeneous anisotropic medium can be obtained
by the application of transformation that reduces the prob-
lem to that of a homogeneous isotropic medium (See Chapter
IV). This method is however is not applicable when the
subsoil comprises of several zones of different permeabili-
ties or values of the permeability or its principal direc-
tions vary from point to point. Numerical methods could
be applied to obtain an approximate solution for
such problems.

5.2 SOLUTION OF ANISOTROPIC SEEPAGE BY FINITE DIFFERENCE METHOD

5.2.1 The partial differential equation governing the
steady state, two-dimensional flow in a homogeneous and
isotropic medium is given by Eq.2·1

The medium is subdivided into squares of equal
area, $\partial x \partial y$. The sides of the squares, ∂x, and ∂y, are
equal and are of finite length Δx , Δy respectively. The

square grid is shown in Fig. 5.1 . The intersections of grid lines are called nodes. The infinitesmal $\Delta x \Delta y$ is approximated by a_g^2 in which a_g is the width of grid interval. The area a_g^2 , is small compared with the area of the medium.

The second differentials of head at node '0' can be approximated by

$$\frac{\partial^2 h}{\partial x^2} = \frac{h_2 + h_4 - 2h_0}{a_g^2} \qquad \qquad \text{...(5.1)}$$

$$\frac{\partial^2 h}{\partial y^2} = \frac{h_1 + h_3 - 2h_0}{a_g^2} \qquad \qquad \text{...(5.2)}$$

Substitution of Eqs 5.1 and 5.2 in Eq.2·1 results in

$$\frac{h_1 + h_2 + h_3 + h_4 - 4h_0}{a_g^2} = 0 \qquad \qquad \text{...(5.3)}$$

or $\quad h_1 + h_2 + h_3 + h_4 - 4h_0 = 0 \qquad \qquad \text{...(5.4)}$

where h_0, h_1, h_2, h_3, and h_4 are the heads at nodes 0,1,2,3,4 respectively.

The heads at every node except those at the up-stream and downstream boundaries are guessed initially, and then by the repeated use of equation (5.4) are adjusted towards their correct values.

Difficulties often arise from the presence of fractional spaces at boundaries of the mesh. These difficulties can be over come by the use of triangular meshes. Suitable arrangement of the triangular meshes ensures that there are no fractional spaces at the boundaries.

G.R. Tomlin (25) has analysed seepage through soils that are zoned into areas of different permeability using the concept of triangular mesh and with the aid of a computer.

5.2.2 Outline of the Method

The soil in each zone is assumed to be homogeneous. In general each zone can be anisotropic with the principal axes of permeability inclined at any angle. In applying the method, straight lines are substituted for curved zone boundaries and 'infinite' boundaries are reduced to some arbitrary finite length, so that the shape of each zone becomes a polygon.

The area within each zone is then divided into triangles by connecting selected pairs of vertices. Within each triangle, a fine-size triangular mesh is constructed with its axes parallel to the side of the triangle. The head at any node in this system of meshes is expressed in terms of heads at surrounding nodes. By the theory of finite differences, equations for the heads at various

types of nodes have been derived, and thes equations are solved by iteration to obtain the values of head at every node. A standard computer programme has been used for the purpose.

5.2.3 Application

The method was tested using several problems which, by virtue of their regular boundary geometry, could be solved accurately by other methods. A close agreement was observed by the author between the solutions by the two methods.

The numerical method of analysis using a triangular mesh system, though mathematically approximate is extremely versatile. Anisotropic zoned soil sections with irregular boundary conditions can be as easily analysed by this method as a simple homogeneous section. Solutions of any desired accuracy can be obtained by increasing the number of iterations and by decreasing the mesh size. An obvious advantage in programming the method for a computer is that solutions can be obtained very quickly.

But the main disadvantage of the method is that it cannot be applied for the solution of seepage through non homogeneous and anisotropic soils. The method has thus limited applicability in the solution of field problems.
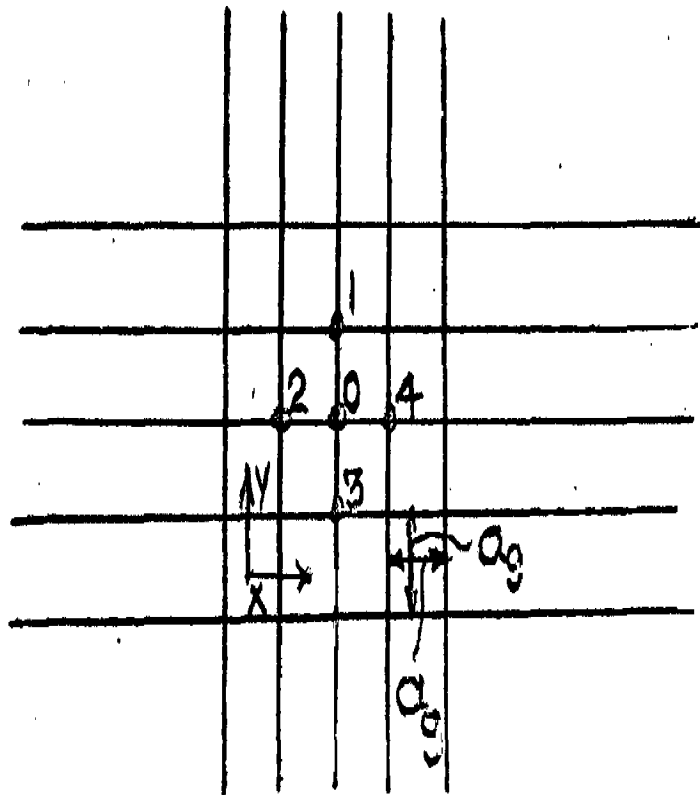
FIG.5.1 Finite difference grid

In contrast to this, finite element method is now-a-days very widely used in the solution of seepage through soils which are both anisotropic and nonhomogeneous. This method is very versatile in the sense it can be applied to all types of foundations (soils) irrespective of their charac ter, and solutions can easily be got with the aid of a standard computer programme.

5.3 SOLUTION OF ANISOTROPIC SEEPAGE BY FINITE ELEMENT METHOD

A general numerical method of solution of two dimensional seepage problems in anisotropic media, particularly well suited for digital computation has been given by O.C. Zienkiewicz (26) This method is based on well known finite element method.

Ferrandon introduced the concept of a 3x3 permeability matrix and showed that it is always symmetrical. Hence it is evident that in any anisotropic material three principal axes exist, in the direction of which the seepage velocity is always colinear with pressure gradient. These directions are known for the material at a particular point and the permeability can be described fully by three values of the permeability coefficients only.

5.3.1 Theory of Flow in Anisotropic Media

The well known Darcy's law of seepage can be generalized to a two dimensional situation If

the velocities along three orthogonal axes x,y and z can
be designated by a column vector $\{v\}$

and head gradients along these axes by another vector
$-\{grad\ H\}$ then by Darcy's law

$$\{v\} = -[K]\ \{grad\ H\} \qquad ...(5.1)$$

where k is a 3x3 matrix of nine coefficients and since,
it is symmetrical only six coefficients are needed to
define it.

If the direction of the axes is changed to x',y'
and z' then it can be shown (Appendix I) that the velocity
vector in the new system is given by

$$v' = -[K']\ \{grad\ H'\} \qquad ...(5.2)$$

where $[K'] = [L][K]\ [L^{-1}]$ $\qquad ...(5.3)$

where $[L]$ is a transformation matrix of direction cosines.
$[K']$ can be reduced to a diagonal matrix , by suitably
choosing the orthogonal directions, procided $[K]$ is symme-
trical. These directions are known as principal axes
of the porous material and the particularly simple relation-
ship (Eq. 6 Appendix 1) in the direction of these axes are
obviously worth noting. The above trans formation is
identical to that used in

computing stress components where the existence of principal axes is well known.

5.3.2 Two Dimensional Seepage in Anisotropic medium

The continuity equation when the x and y directions coincide (locally) with principal axes of material is

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \qquad \qquad ...(5.4)$$

Substituting Eq. 8 (of appendix 1) in above equation,

$$\frac{\partial}{\partial x}\left(K_{xx} \frac{\partial H}{\partial x} \right) + \frac{\partial}{\partial y}\left(K_{yy} \frac{\partial H}{\partial y}\right) = 0 \qquad ...(5.5)$$

which is valid for both homogeneous and nonhomogeneous situations. The solution of Eq. 5.5 is equivalent to finding a function H, which minimizes the following integral taken over the whole region of solution (subject to specified boundary conditions).

$$E = \iint \left[K_{xx}\left(\frac{\partial H}{\partial x} \right)^2 + K_{yy} \left(\frac{\partial H}{\partial y} \right)^2 \right] dx\, dy \quad ...(5.6)$$

If the Euler conditions of minimization are applied to Eq. 5.6 then Eq. 5.5 will be obtained directly. This is valid whether K_{xx} and K_{yy} are constant or variable with the x and y coordinates.

The solution of this type of problem by finite element method is explained in Appendix II.

The procedure is identical with that of a structural analysis and standard computer programs have been used for both assembly and solution of examples. The program first converts the coordinates of the nodal points of each element to its appropriate principal direction and then computes 'stiffness matrices' (S). The assembly and solution follows a standard routine common to many stiffness analysis problems. The data necessary for computation programmed for a digital computer are -

(i)   Coordinates of nodal points chosen in some common coordinate systems.

(ii)   Nodal numbers specifying the elementary triangles along with K_{xx} and K_{yy} , and

(iii)   A direction angle specifying the direction of the principal axes with reference to common coordinates.

5.3.3 Application

Two examples were chosen by Zienkiewicz(26) to illustrate the accuracy and wide range of applicability of the method, respectively. The first was the study of head distribution under an inclined layer of sheet piling on a stratified anisotropic foundation, for which analytical s lution was known.

The equipotentials computed on the basis of exact solution and the number of values of heads computed at the nodes of triangles of finite element triangulation, on comparison were found to be in close agreement.

The second example is illustrated in Fig. 5.2
and 5.3 respectively, which is designed exclusively to
illustrate the versatility of the computer programme
and deals with the flow through a foundation of perhaps
unusual complexity. But it should be noted that once
the data pertaining to the characteristics of each
element were specified no additional difficulty is
presented by the computation.
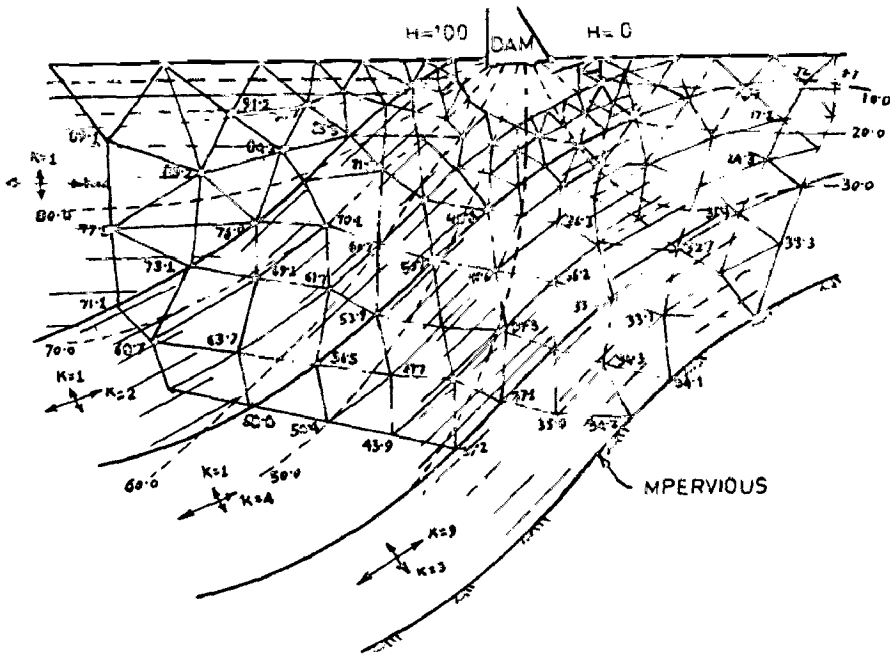
5.2- FLOW UNDER A DAM THROUGH A HIGHLY NONHOMO
GENEOUS ANISTROPIC AND CONTORTED FOUNDATION.
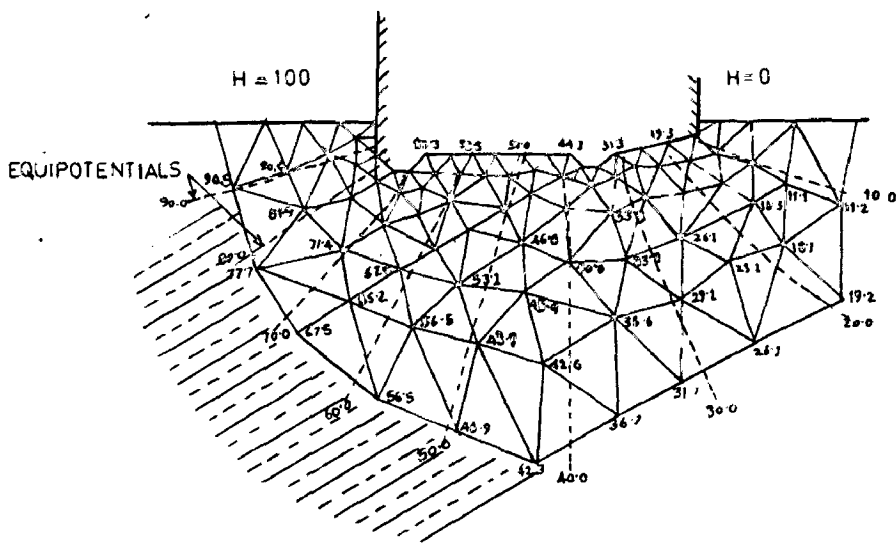
FIG.5.3-DETAIL OF PROBLEM ILLUSTRATED.

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┌─────────────────────────────────────┐
│                                     │
│        C H A P T E R   VI           │
│                                     │
│        C O N C L U S I O N S        │
│                                     │
└─────────────────────────────────────┘
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6.1   Closed-form or approximate solutions are available from the problem of seepage below the hydraulic structures founded on homogeneous and isotropic medium with various boundary conditions. But in practice the foundation material below the hydraulic structure is mostly non-homogeneous and anisotropic. The effect of finite depth, stratification, and anisotropic on the uplift press ures and finite element method for the solution of seepage below hydraulic structure founded on anisotropic and non-homogeneous mediums have been discussed. Following conclusions are drawn from these studies:

1. Design curves are given for determination of exit gradient and uplift pressures at key points below a flat floor with an end cut offfounded on finite depth of permeable medium. The effect of impermeable layer is negligible for $T/b \geqslant 5$ and $d/b \leqslant 0.1$.

2. With the increasing scour downstream of the end cut off founded on finite depth of permeable soil, the uplift pressures below the floor decrease but the value of exit gradient increases . The extent of increase in the value of G_E can be determined from Fig. 2.19. This information can also be used to determine the effect of the depth of the downstream filter on the exit gradient.

3.　　In case of hydraulic structures founded on stratified soils comprising two layers and $k_2/k_1 > 1$, the uplift pressures increas in the portion where pressures are less than 50% and decrease in the portion where pressures are more than 50%. When k_2/k_1 is more than 1, the effect of the thickness of lower stratum on the uplift pressures is negligible. Pressures below the floor with $k_2/k_1 > 10$ are not affected by b/d i.e. depth of cut off The value of exit gradient increases with increase in the permeability of lower stratum. Increase in the length of the floor or depth of the downstream cut off , has practically no effect on the value of exit gradient for $k_2/k_1 > 10$.

4.　　Solutions based on conformal mapping has been obtained for determination of uplift pressures below floor with an assymetric cut off founded on anisotropic soil with principal direction of permeability inclined to the coordinate axis of the physical plane. The results have been given in the form of design curves, to determine pressures at the junction of floor and end cut off(E)and at the tip of end cutoff(D)

　　　It is seen that uplift pressures at E are maximum for anisotropic medium with $\alpha = 0$ and decrease with decrease in the value of λ . The

pressures at E are minimum for $\alpha = 90^{O}$. The
pressures at the point E with anisotropic condi-
tions are more than those for isotropic condit-
ions for thevalue of α less than 45° and greater
than 140°. The pressure at the point D with ani-
sotropic conditions are less than those for isotro-
pic condition for the value of α between 60° and 160°.

The pressures at D increase with decrease in the value
of λ for $\alpha < 60^{O}$ and $\alpha > 160^{O}$ and decrease
with decrease in the value of λ for $60^{O} < \alpha < 160^{O}$.

5. Finite element method can be applied for obtaining
the uplift pressure below structures founded on
anisotropic and non homogeneous foundations. With
the availablity of the fast digital computer
this method has wide applicability and can be
used to obtain results to desired accuracy.

If the velocities in the direction of three or-
thogonal axes, x,y and z are designated   by a vector v   or

$$\{v\} = \left\{ \begin{array}{c} v_x \\ v_y \\ v_z \end{array} \right\} \qquad \ldots(1)$$

and if the head gradient is defined similarly by its

three components,

$$-\{grad\ H\} = \left\{ \begin{array}{c} -\dfrac{\partial H}{\partial x} \\ -\dfrac{\partial H}{\partial y} \\ -\dfrac{\partial H}{\partial z} \end{array} \right\} \qquad \ldots(2)$$

then the most general linear relationship that can exist

between the two quantities is of the form (by Darcy's law)

$$\{v\} = -[K]\ \{grad\ H\} \qquad \ldots(3)$$

in which [K] is a 3 x 3 matrix defined by nine numerical
coefficients.

It can   , however be shown that matrix K must be
symmetrical to satisfy conservation of energy and therefore

it appears that only six coefficients are necessary to define it.

If the direction of the axes is changed to x', y', z' then the velocity vector in the direction of the new axes can be found as

$$\{v'\} = \begin{Bmatrix} v'_x \\ v'_y \\ v'_z \end{Bmatrix} = [L]\ \{v\} \qquad \ldots(4)$$

where L is a transformation matrix of direction cosines. Similarly the new vector of the head gradient

$$-\{ \text{grad } H' \} = \begin{Bmatrix} -\dfrac{\partial H}{\partial x'} \\ -\dfrac{\partial H}{\partial y'} \\ -\dfrac{\partial H}{\partial z'} \end{Bmatrix} = -[L]\ \{ \text{grad } H \} \quad \ldots(5)$$

Combining equations 3,4 and 5 yields

$$v' = -[K']\ \{ \text{grad } H' \} \qquad \ldots\ldots(6)$$

in which the new permeability matrix is

$$[K'] = [L][K][L^{-1}] \qquad \ldots(7)$$

With this type of transformation of K is symmetrical it is always possible to find three orthogonal directions

for which $\begin{bmatrix} K' \end{bmatrix}$ reduces to a diagonal matrix giving

$$\{v'\} = - \begin{bmatrix} k_{x'x'} & 0 & 0 \\ 0 & k_{y'y'} & 0 \\ 0 & 0 & k_{z'z'} \end{bmatrix} \{grad\ H'\}$$

$$\dots(8)$$

The continuity equation, when the x and y direc-
tions coincide (locally) with principal axes of material
is

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \qquad \ldots(1)$$

Substituting Eq. (8) (of appendix 1) in above equation,

$$\frac{\partial}{\partial x}\left(K_{xx} \frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy} \frac{\partial H}{\partial y}\right) = 0 \qquad \ldots(2)$$

which is valid for both homogeneous and non homogeneous
situation. The solution of Eq. 2 is equivalent to finding
a function H , which minimizes the following integral
taken over the whole region of solution (subject to speci-
fied boundary conditions).

$$E = \frac{1}{2} \iint \left\{ K_{xx} \left(\frac{\partial H}{\partial x}\right)^2 + K_{yy} \left(\frac{\partial H}{\partial y}\right)^2 \right\} dx\ dy \qquad \ldots(3)$$

If the Euler conditions of minimization are applied to
Eq. 3, then Eq. 2 will be obtained directly. This is
valid whether K_{xx} and K_{yy} are constant or variable with
the x and y coordinates.

The solution of this type of problem by finite
element method is explained as follows.

The seepage region is divided into arbitrary
elementary triangular areas, or finite elements as shows
in Fig. 1. If the unknown values of the function H
at the nodes of the triangles, define the function thr-
oughout the entire region completely and uniquely, then
differentiating the function E with respec t to each of
these nodal values and equating each of these differentials
to zero with result in a series of simultaneous equations.
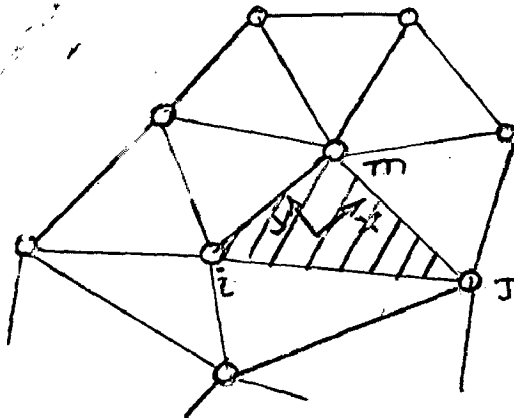
Fig.1 Division of Region into finite elements

From these the final, approximate solution can be obta-
ined by feeding in the appropriate boundary values. If
the nodal values define the function that is sought only
in the element adjacent to a particular node, while ful-
filling continuity requirements along lines separating
the elements then each equation will contain only the
contributions of these adjacent elements.

Considering a typical triangle i, j, m with a
locally defined set of coordinate axes (coinciding with

the principal axes of the material) the simplest defi-
nition of the function H within the element will be obta-
ined by taking a linear function.

$$H = A + Bx + Cy \qquad \qquad \ldots(4)$$

and evaluating the three constants in terms of coordinates
of the nodes and the corresponding nodal values of the
function. Performing the necessary algebra (in matrix
language) yields

$$H = \left[(a_i + b_i x + c_i y) , (a_j + b_j x + c_j y), (a_m + b_m x + c_m y)\right] \begin{Bmatrix} H_i \\ H_j \\ H_m \end{Bmatrix} \qquad \ldots(5)$$

or simply $H = \begin{bmatrix} N_i, & N_j, & N_m \end{bmatrix} \begin{Bmatrix} H^e \end{Bmatrix}$ $\qquad \ldots(6)$

where H^e stands for the values of the function character-
istic of the element considered. The coefficients are
defined as

$$a_i = \frac{x_m y_j - x_j y_m}{2\Delta}$$

$$b_i = \frac{(y_m - y_j)}{2\Delta} \qquad \ldots(7)$$

$$c_i = \frac{(x_j - x_m)}{2\Delta}$$

with others following a cyclic, anticlockwise, order in
i,j,m. In Eq. 7 , Δ is the area of the triangle i,j,m.

The minimum of the functional E is obtained by first obtaining the contributions to the appropriate differentials for each element seperately and then assembling the results for the whole region.

Thus terming E^c as the contribution of the element i, j,m yields

$$\frac{\partial E^c}{\partial H_i} = \iint \left[K_{xx} \frac{\partial H}{\partial x} \frac{\partial}{\partial H_i} \left(\frac{\partial H}{\partial x} \right) + K_{yy} \frac{\partial H}{\partial y} \frac{\partial}{\partial H_i} \left(\frac{\partial H}{\partial y} \right) \right] dxdy$$

$$\ldots(8)$$

With the integration limited to the area of the triangle i,j,m. This on substitution of Eq. 6 becomes

$$\frac{\partial E^c}{\partial H_i} = \iint K_{xx} \left[\frac{\partial N_i}{\partial x}, \frac{\partial N_j}{\partial x}, \frac{\partial N_m}{\partial x} \right] \left\{ H^o \right\} \left(\frac{\partial N_i}{\partial x} \right) dx \, dy$$

$$+ \iint K_{yy} \left[\frac{\partial N_i}{\partial y}, \frac{\partial N_j}{\partial y}, \frac{\partial N_m}{\partial y} \right] \left\{ H^e \right\} \left(\frac{\partial N_i}{\partial y} \right) dxdy$$

$$\ldots(9)$$

Recognising that the element contributes only to the differentials with respect to the values at its three nodal points, assuming the permeability coefficients K_{xx} and K_{yy} to be constant within the element, then substitution of Eq. 7 yields simply,

$$\left\{ \frac{\partial E^e}{\partial H^e} \right\} = \left\{ \begin{array}{c} \dfrac{\partial E^e}{\partial H_i} \\[2mm] \dfrac{\partial E^e}{\partial H} \\[2mm] \dfrac{\partial E^e}{\partial H_m} \end{array} \right\} = \left[S_{ijm} \right] \left\{ \begin{array}{c} H_i \\[2mm] H_j \\[2mm] H_m \end{array} \right\} = \left[S_{ijm} \right] \left\{ H^e \right\} \qquad ,..(10)$$

in which the [S] matrix has coefficients of the type

$$S_{ij} = \left\{ K_{xx}(y_m - y_i)(y_i - y_m) + K_{yy}(x_m - x_j)(x_i - x_m) \right\} \ / \ 4\Delta \qquad ...(11)$$

or explicitly

$$\left[S_{ijm} \right] = \begin{bmatrix} K_{xx}(y_m - y_j)(y_m - y_j) & K_{xx}(y_m - y_j)(y_i - y_m) & K_{xx}(y_m - y_j)(y_j - y_i) \\ + & + & + \\ k_{yy}(x_m - x_j)(x_m - x_j) & k_{yy}(x_m - x_j)(x_i - x_m) & K_{yy}(x_m - x_j)(x_j - x_i) \\[4mm] & k_{xx}(y_i - y_m)(y_i - y_m) & K_{xx}(y_i - y_m)(y_j - y_i) \\ & + & + \\ & k_{yy}(x_i - x_m)(x_i - x_m) & K_{yy}(x_i - x_m)(x_j - x_i) \\[4mm] \text{Symmetrical.} & & K_{xx}(y_j - y_i)(y_j - y_i) \\ & & + \\ & & K_{yy}(x_j - x_i)(x_j - x_i) \end{bmatrix}$$

$$...(12)$$

To assemble a typical equation for a differential of E with respect to any nodal value for the whole region, the contributions of elements adjacent to the node only are non zero and the assembly will clearly according to the pattern

$$\frac{\partial E}{\partial H_i} = \sum \frac{\partial E^e}{\partial H_i} = 0 \qquad \qquad ...(13)$$

$$\text{or} \quad \sum H_n \sum s^e_{in} = 0 \qquad \qquad ...(13b)$$

The procedure is identical with that of structural analysis in which S matrix is a stiffness matrix of a structural component and Eq 13b represents the equation of equlibrium obtained on assembly of such elements. Standard computer programs have been used for both assembly and solution of examples.

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