SEEPAGE FROM CANAL WITH ASYMMETRIC DRAINAGES

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in
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By

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled **SEEPAGE FROM CANAL WITH ASYMMETRIC DRAINAGES** in fulfilment of the requirement for the award of the Degree of Doctor of Philosophy, submitted in the department of Water Resources Development Training Centre of the University of Roorkee, is an authentic record of my own work carried out during a period from April 1988 to September 1993 under the supervision of Prof. (Dr.) A.S. Chawla.

The matter embodied in this thesis has not been submitted by me for the award of any other degree.

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As for our own faults, it could take a large slate to hold the account of them, but, thank God, we know where to take them, and how to get the better of them. With all our faults, God loves us still if we are trusting in His Son, therefore let us not be down hearted, but hope to live and learn, and do some good service before we die. Though the cart creaks it will get home with its load, and the old horse, broken kneed as he is; will do a sight of work yet. There's no use in lying down and doing nothing, because we cannot do everything as we should like. Faults or no faults, ploughing must be done, and imperfect people must do it too, or there will be no harvest next year; bad ploughman as John may be, the angels won't do his work for him, and so he is off to do it himself. Go along, Violet! Gee woa! Depper!

from C.H. Spurgeon's "John Ploughman's Talks", Baker Book House, Grand Rapids, Michigan.

DEDICATED TO HIM WHO IS PERFECT

ABSTRACT

Canals are widely used in irrigation schemes as a major conveyance system. Most of the canals are unlined and the amount of actual water finally available for irrigation is significantly less than the quantity of water released at the head. One of the major causes of these losses is seepage from canals. Hence, an understanding of the mechanism of the seepage losses from canals leads to improved management of the water resources.

Seepage losses from unlined canals depend on the shape and size of the canal cross section, depth of water in the canal, location of drainages on either side of the canal and the type of subsoil. Several analytical solutions for prediction of seepage loss had been presented for different canal cross sections and, boundary conditions. In the solutions thus far obtained, it is assumed that symmetric seepage flow takes from the canal to the drainages which are shallow or deep. However, in practice canals seldom have symmetric drainages on either side.

Exact solution of the problem of seepage from a canal in homogeneous medium to asymmetric drainage(s) located at finite distance(s) from the canal is presented in this work. Solutions are presented for the following problems :

(i) seepage from a canal with negligible water depth to asymmetrically located drainages at either side of the canal;

(ii) seepage from trapezoidal canal to asymmetrically disposed drainages at either side of the canal and

(iii) seepage from trapezoidal canal to a drainage on one side.

The analytical solution for the determination of the shape of the free surface and the calculation of the seepage quantity through the system was obtained by finding the relationship between the physical plane (z-plane) and the complex potential plane (w-plane). This was done by employing successive transformations through the use of the Zhukovsky function as well as Shwarz-Christoffel and bilinear conformal mapping equations.

The results of seepage discharge for various values of dimensionless physical parameters are prepared as nomographs for practical uses. The case of the symmetric drainages on either side of canal is a particular case of the present study and the results obtained in this study for symmetrical drainages agree with that presented by earlier workers. The computed seepage loss to asymmetric drainages by decomposing the asymmetric flow domain at the centre of the canal and treating each part as a part of the corresponding symmetric cases is found to differ from the seepage loss computed by this method. The difference depends on the degree of variation in the drainage distance and elevation on either side. The present direct and exact solution to asymmetric drainages shows that the drainage which is at the higher level and farther from the canal is receiving less seepage from the canal. As the level of the drainage of the higher level is raised, the seepage to drainage reduces and at certain level the drainage does not receive any seepage water ($h_2 = h_c$). This critical ratio of the levels of the drainages

(h_c/h_1) and the critical location of the drainages have been identified. This critical position of the drainage is found to depend on the canal cross section, depth of water in the canal and distance of the drainage on the other side.

Free surfaces on either side of the canal rise with increase in the bed width and increase in drainage distances. Free surface also rises if drainage on the other side is located at a higher level. The effect of change of the side slope of canal on the free surface is negligible. However, increase in depth of water in the canal significantly raises the free surface.

The results pertaining to the case of drainage on one side of the canal had been compared with that given by Polubarinov-Kochina, 1962. The seepage quantity computed for the case in which the depth of the canal is small compares well with the results given by the above author. In the present work, shape of the canal is considered and the shape of the phreatic lines on both sides of the canal have been plotted to show the effect of the physical dimensions.

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NOTATIONS

The following symbols are used in this thesis.
[In Chapter 2, which deals with review of literature, original notations have been used]

b'	=	bed width of canal in θ-plane;
B_2	=	bed width of canal in z-plane;
$F(\beta,m)$	=	Incomplete elliptic integral of the first kind with amplitude β and modulus m;
h_1,h	=	difference between water levels of canal and the right drainage;
h_2,h	=	difference between water levels of canal and the left drainage;
H	=	water depth of canal;
k	=	coefficient of permeability;
K	=	$F(\pi/2,m)$ = complete elliptic integral of the first kind;
K'	=	$F(\pi/2,m')$ = associated complete elliptic integral of the first kind;
L_1,L	=	distance from canal to right drainage;
L_2,L	=	distance from canal to left drainage;
m	=	modulus of elliptic integral;
m'	=	$\sqrt{(1-m*m)}$; comodulus
M,M',M_2	=	constants;
p	=	atmospheric pressure;
πa	=	canal side slope angle with horizontal in z-plane;
πa_1, πa_2	=	angles with horizontal in θ-plane respectively of the right and left canal side slopes.

q, q_s, q_c	=	volume rate of seepage per unit length of canal;
q'	=	canal seepage discharge component directly emerging on the right hand drainage;
r, s, r', s'	=	cartesian coordinates;
t	=	$r + is$ = parametric plane in Chapter 3, 4, and 5;
t'	=	$r' + is'$ = parametric plane in Appendix C;
w	=	complex potential = $\phi + i\psi$;
z	=	complex variable = $x + iy$;
γ_w	=	specific weight of water;
γ, σ, ρ	=	transformation parameters;
θ	=	$\theta_1 + i\theta_2$, complex variable representing Zhukovsky's function;
$\Pi(\beta, \alpha_1^2, m)$	=	Incomplete elliptic integral of the third kind with amplitude β and parameter α_1^2 and modulus m;
Π_1	=	$\Pi(\pi/2, \alpha_1^2, m) = \Pi(\alpha_1^2, m)$, complete elliptic integral of the third kind;
Π_2'	=	$\Pi(\pi/2, \alpha_2^2, m') = \Pi(\alpha_2^2, m')$, complete elliptic integral of the third kind with parameter α_2^2 and modulus m' ;
Π_3	=	$\Pi(\pi/2, \alpha_3^2, m) = \Pi(\alpha_3^2, m)$, complete elliptic integral of the third kind with parameter α_3^2 and modulus m;
ϕ	=	velocity potential function;
ψ	=	stream function.

CHAPTER 1

INTRODUCTION

Canals continue to be widely used for delivering water for irrigation in most parts of the world. Estimation of seepage from canals and assessment of the water logging problem resulting from the introduction of canals is much required for a rational water resources management. In India, many of the major irrigation canals are unlined and are constructed in alluvial soil. It had been recorded that more than 40 percent of the water from canals are lost through seepage and the menace of water logging had left many fertile lands unsuitable for productive farming [Sharma & Chawla, 1974]. Hence, estimation of seepage from unlined canals and assessment of its impact on the groundwater regime are very important.

Seepage loss from an unlined canal depends on the canal geometry, depth of water in the canal, locations of drainages on either side of the canal and the hydraulic characteristics of the subsoil.

Seepage computation demands understanding of the physics of fluids (water) flow through porous media (soil). Properties of the soil accounts to one of the major factors on which the fluid flow depends. Most of the theoretical analyses of groundwater flow problems assume the porous media to be isotropic and homogeneous with respect to the coefficient of permeability. Most natural and man made soil deposits are anisotropic. Flow through anisotropic porous media is generally analyzed by first transforming the anisotropic actual flow domain to fictitious isotropic flow region by a suitable co-ordinate transformation and applying a method of solution to the transformed section. From the solution of the problem in the transformed region, the solution of the actual problem in the anisotropic region can be obtained [Harr, 1962]. In case directions of the principal coefficient of permeability coincide with horizontal and vertical directions, the shape of flow domain is not altered after transformation. For such type of anisotropic behaviour of the porous media, the existing solution for flow in isotropic media can be made use of in arriving at the flow characteristics concerned with anisotropic porous media [Harr, 1962].

Coupled with the nonuniformity of soil in the horizontal as well as vertical extent, seepage flow systems are also characterized by channels of irregular cross section, changing elevations of the water surface in the channel and of the water table in the soil and other complications [Bouwer, 1969]. The process of erosion, sedimentation, biological action, etc. in the channel as well as the influences of chemical constituents of the water and also the air content in soil on the hydraulic properties of the soil are sources of complications of the seepage flow process. If the water table is sufficiently close to the surface of the soil, the influence of evaporation on the free surface may come into picture. Infiltration from rainfall or from irrigated land also causes a downward flux across the water table.

Numerical and other approximate methods using digital computers and electric analogs can handle many types of soil and boundary conditions. But, the more difficult process than the actual calculation of the seepage, once a realistic representation of the field situation has been developed, is the uncertainty in the field evaluation of pertinent boundary conditions and hydraulic properties of the soil [Bouwer, 1969]. Moreover, the results of the digital or analog methods can be unreliable unless their validity is verified by the exact solutions arrived at by the use of the analytic methods. In addition to that, through the understanding of principles, the closed-form solutions are useful in gaining insight into the physics of fluid flow in porous media and this may lead to the identification of specific sources of uncertainty [Schilfgaarde, 1970]. Hence, the importance of working out a closed-form solutions to the problems of seepage cannot be denied.

Several analytical solutions of seepage problems had been presented for various shape and size of canal cross section and for different boundary conditions. Vedernikov [1936] obtained solutions for the seepage from a channel with triangular, trapezoidal or other shapes to ground water table at infinite depth. Analytical solution had also been obtained for seepage from a canal to an underlying highly pervious layer at finite depth [Harr, 1962; Bouwer, 1965].

Garg and Chawla [1970] presented solution of seepage from a trapezoidal channel to shallow water table with symmetric and horizontal or vertical drainages at finite distance. The channel was assumed to be laid in homogeneous and isotropic medium extending up to infinite depth. Earlier, Vedernikov [1939], had solved the problem of seepage from canal of negligible water depth to symmetrically disposed horizontal drainages. Sharma and

Chawla [1974] obtained analytical solution of seepage from a canal with negligible water depth to the symmetrical drainages on either side of the canal with pervious medium extending up to finite depth. The above solutions assume symmetric flow from the canal to the drainages. However, in practice canals seldom have symmetric drainages on either side.

Exact solution of the problem of seepage from a canal in homogeneous and isotropic medium to asymmetric drainages located at finite distances from the canal is presented in this work. The problems studied in the present work include seepage from trapezoidal canal to drainages located on either side at different levels and distances. As special case of the above, solutions have been obtained for seepage from a channel of negligible water depth to asymmetric drainages and also for the case of a trapezoidal canal with drainage on one side only. The solutions are presented in the following order :

(i) seepage from canal of negligible water depth to asymmetrically located drainages;

(ii) seepage from trapezoidal canal to asymmetrically located drainages; and

(iii) seepage from a trapezoidal canal to a drainage located on one side of the canal.

In the first two cases seepage from the canal emerges into two asymmetric drainages on either side of the canal and the drainages are assumed to be wide. In the third case the seeping water emerges into a wide drainage located on one side of the canal.

In view of the multitude of complexities to which seepage flow systems are subjected in nature, theoretical treatment must begin with simplification of the soil and boundary conditions. This is particularly more so in the case of the mathematical treatment of the problem of seepage analysis. In this study the porous media is assumed to be homogeneous, isotropic, and undeformable. The flow is assumed to be laminar and therefore Darcy's law is applicable.

Conformal mapping still is a useful tool in groundwater mechanics and serves to obtain solutions to simplified versions of complex problem, which may be used to gain insight into the problem prior to using a numerical method which uses the digital computing facilities capable of generally more detailed solutions [Strack, 1989].

In the present study, the analytical solutions are derived using the special mapping techniques of the Schwarz-Christoffel conformal mapping equation, the bilinear or Möbius transformation and the Zhukovsky function. These mapping techniques are briefly discussed in Appendix A.

The thesis is divided into six chapters. Review of literature is given in Chapter 2. Solutions to predict the seepage loss and the shape of the free surface at either side of the canal of negligible water depth are presented in Chapter 3. The derivation of the equations and the analyses in chapter 3 is considered as a logical step towards the solutions arrived at in the subsequent chapter.

In chapter 4, the solutions for trapezoidal canal with drainages at different elevations and distances are given. It may be noted that rectangular and triangular sections can be treated as special cases of the trapezoidal cross-section. Hence, in this chapter the geometry and water depth of the canal are considered in the derivation of the solutions.

In chapter 5, derivation of the solution to the case of seepage from trapezoidal canal to a drainage located only on one side of the canal has been given.

General conclusions reached from the present work have been presented in chapter 6. In this chapter, recommendations for further work have also been suggested but comparison with previous works have been made at different stages of the development of the solutions.

CHAPTER 2

REVIEW OF LITERATURE

Introduction

Many investigators have contributed to the study of seepage from canals. The classic seepage analyses concentrated on the use of analytical methods based on special mapping techniques such as Zhukovsky's function and the hodograph method [Polubarinova-Kochina, 1962 ; Harr, 1962 ; Aravin and Numerov, 1965]. The electric analog method and the graphical approach as well as the numerical methods such as the finite differences and the finite elements had also been adopted in the study of seepage from canals [Bouwer, 1965 ; Jeppson, 1968 ; Bear, 1972 ; Verrujit, 1982 ; etc.]. Though much of the works has been done in the area of steady state conditions, a number of investigators were also drawn to study the case of unsteady state condition as well [Mishra , 1992 ; Bhargava, 1988]. According to Muskat [1946] and Bouwer [1969], an unsteady state can be treated as a succession of steady state conditions. The validity of this assumption has been reasoned out by Muskat in detail [Muskat 1946, pp.621-625]. The review of literature presented in this chapter focuses on the research works which deal with steady state seepage problems. The experimental methods of measurement of seepage (eg. ponding method or inflow outflow method) have been excluded from review. But, the Zhukovsky function, the Schwarz-Christoffel conformal mapping and the bilinear or Möbius transformation which are widely used at different stages of the development of the solutions to the problems tackled in the thesis, a brief discussion on the above techniques has been included but is given separately in Appendix A.

2.1 General Review of Works in Seepage from Canals, Channels or Ditches.

Among those who dealt with steady seepage from canal to ground water located at large depth are Kozeny, Muskat, Vedernikov, Risenkampf, Morel-Seytoux and Jeppson.

As early as 1931, Kozeny [1931] found that the maximum width of the sheet of water seeping down into the porous media from a canal conforming to a special cross sectional shape is equal to (B+2H). Here, B is the width of the canal at the water surface and H is the maximum depth of water in the canal. The free surfaces are bounded by vertical asymptotes. The equation that has been derived by Kozeny which describes the cross section of the canal is,

$$\pm x = - \sqrt{H^2 - y^2} + [(B + 2H)/\pi] \cos^{-1}(y/H) \quad \ldots (2.1)$$

The seepage from such a canal is stated as,

$$q = k (B + 2H) \quad (2.2)$$

where, x and y are cartesian coordinates with origin at the centre of water surface; H is the maximum depth of water in the canal and k is the coefficient of permeability.

One of the limitations in the use of the above results is that it is not applicable to a case were the water table is at shallow depth. It, therefore, follows that that the porous medium must be of very large thickness so that the seeping water can maintain its vertical downward movement. The shape of the canal also should follow the equation given above (Eq.2.1).

Although slight deviation in the shape of the ditch from those given by Eq.2.1 will in themselves cause no serious errors [Muscat, 1941] in the use of Eq.2.2, the assumption of the ground water table being at a great depth below the base of the ditch definitely limits its applicability to only few practical cases where the porous medium is also assumed to extend to infinite depth. In many practical situations, however, the water seeping down from the ditch will reach the normal ground water level at a relatively shallow depth, thus forcing the streamlines to assume a horizontal rather than a vertical trend.

Kozeny also found the seepage from a canal for the case in which the equipotential lines could be considered as segments of circles at large distance from the ditch or canal with radially spreading free surfaces. The shape of the ditch is defined by the equation,

$$\pm x = \sqrt{H^2 - y^2} + [\,(B - 2H\,)/\pi\,]\cos^{-1}(y/H) \qquad \ldots (2.3)$$

The quantity of seepage is given by,

$$q = k\,(B - 2H) \qquad (2.4)$$

Vedernikov [1936] studied seepage from canal of trapezoidal section of which the triangular cross section is a particular case. He applied the inverse transformation to the region of the complex velocity, i.e., in the hodograph plane, to obtain an exact solution for these shapes of canal when ground water is at infinite depth. Solutions for various other simplified flow geometries were also obtained by Risenkampf [1940]. Hammad [1960] solved the same problem by an approximate method for a series of channels in which the shape of the channels have not been conserved. Seepage from a series of triangular channels was found by El Nimr (1963). He used the inverse hodograph method for solution of the problem. Bruch and Street [1967] made an improvement in the above solution. Morel-Seytoux [1964] considered canals of different shapes including cross sections deviating from the standard rectangular, trapezoidal and triangular ones. Morel-Seytoux applied the analytical methods of hodograph techniques and the Schwarz-Christoffel transformations, and the Green-Neumann function to obtain solutions of the seepage problem.

All the above solutions are obtained by assuming the water table to be at infinite depth. The above studies indicate that when the ground water table is at large depth the shape of the canal has small influence on the seepage discharge. Muskat [1946] considered three different shapes of canals. He compared the values of seepage discharge for the three shapes of canal and stated that the extreme variation in seepage, due to the effect of shape of canal or ditch is about 10 percent.

The finite difference method was employed by Jeppson [1968] to solve the problem of seepage from canal to underlying pervious stratum. All of these derivations have limited utilities because the condition of the water table at infinite depth, or because existence of an underlying pervious layer is seldom met in practice.

In the work of Dachler [1933], shallow water table condition has been considered. He derived a procedure in which both model experiment and an approximate analysis were combined for computing the seepage from a trapezoidal channel set in a porous medium of finite depth, to a fully penetrating vertical drain at some distance away from the channel. Dachler determined only one point on the phreatic line and this point is arbitrarily connected to the canal and the drain. He postulated that this point be joined to the canal water level by an arbitrary curve and to the drainage water level by a straight line. This point shifts as the depth of impermeable layer is increased, and in some cases it may even fall beyond the drainage, which is unrealistic [Garg and Chawla, 1970]. Thus, the Dachler approach does not define the seepage line completely. Garg and Chawla [1970] have compared the Dachler method with their exact analytical solution for vertical drainage and have found that the Dachler approach gives much lower phreatic surfaces.

The problem of seepage from an earth dam with horizontal drain on a pervious stratum of infinite depth had also been solved [Polubarinova-Kochina, 1962, pp.247-260]. The drain is located in the body of the dam at some distance away from the upstream face of the dam. This problem may be considered to be equivalent to that of seepage from a channel of infinite bed width into horizontal drainage which is at the same level as the bottom of the canal. Vedernikov had also solved the problem of seepage from a canal having the form of a horizontal segment to horizontal drains. Todd and Bear [1961] made use of the electrical analog method to analyze seepage from leveed rivers into the low lying adjoining lands.

Bouwer [1965] made a detailed study to determine how seepage from canals or streams is affected by the cross sectional shape and the depth of water in the channel, by the position of the ground water table, and by the sub-soil conditions.

Bouwer [1969] reduced the multitude of soil conditions that may be encountered in practice to three basic conditions :

(i) the channel is in uniform soil which is under lain by much more permeable material, designated as condition A;

(ii) the channel is in uniform soil which is under lain by much less permeable material, designated as condition B, and

(iii) the channel is surrounded by a thin, slowly permeable (clogged) layer along its wetted perimeter, designated as condition C.

The condition of seepage to a free draining, permeable layer is a special case of condition A and it is termed as condition A'. Drainages are considered to be symmetrically located at either side of the canal and the drainage distance from the canal centre line is fixed to be $10W_b$, where W_b is the bottom width of the canal. The electrical resistance network analog is used for solution of steady state seepage systems for condition A, A' and B. For condition C, an equation is presented which gives the seepage as a function of the geometry of the channel, the hydraulic impedance of the slowly permeable (clogged) layer, and the pressure condition in the unsaturated underlying material as determined by the unsaturated hydraulic conductivity characteristics of that material. He has considered canals with triangular, trapezoidal and rectangular sections.

The results for condition A, A' and B were expressed in dimensionless graphs showing seepage in relation to the position of the ground water table at different depths to the permeable or impermeable layer, and different water depths in the canals. The canal was taken as trapezoidal with 1H:1V side slope. Bouwer obtained results which agreed with the theoretical values of Vedernikov for underlying pervious stratum, and with the semi-empirical method of Dachler for underlying impervious strata. The conclusion reached by Bouwer are as follows.

(1) The graphs showed that the effect of a permeable or impermeable layer on seepage becomes rather small when this layer lies below the channel bed at depth 5 times the bottom width, W_b, of the channel. This suggests that soil explorations for new canal do not need to go deeper than $5W_b$ below the proposed bottom elevation.

(2) Seepage rates increase with increasing depth to the ground water table, but at a decreasing rate. If the water table at a distance of $10W_b$ from the channel centre is at a depth more than 2.5 times the width of the channel at the water surface, the corresponding seepage is close to that which would occur if the water table is at infinite depth.

Bouwer also summarized the following results of analyses regarding the effect of channel shape on seepage.

For a given surface width W_T of the water surface and water depth H_w in the center of the channel, seepage increases from a triangular to a trapezoidal and from trapezoidal to a rectangular cross section. The magnitude of the increases depends on the soil and water table conditions. For most conditions, this increase is only moderate and less than the corresponding increase in hydraulic discharge capacity of the channel. Therefore, for a certain width and depth of the water, rectangular channels have lower relative water losses due to seepage than trapezoidal or triangular channels. An exception to this rule may be condition A' if the permeable drainage layer is at very small distance below the channel bottom.

According to Bouwer, the seepage from open channels increases with increasing water depth in the channel. For uniform flow, the discharge in the channel also increases with increasing water depth in the channel. For all three soil conditions, however, the rate of increase in seepage was less than the rate of increase in discharge. Therefore, canals with uniform flow and uniform soil conditions along the wetted perimeter become more efficient conveyor of water with increasing water depth in the canal.

Bouwer has furnished a detailed study on seepage from canals but he has furnished seepage discharge for defined channel geometries and for fixed distance from the center line of canal to the drainage, namely 10 W_b. He has used a side slope of 1:1 and the bed width - water depth ratio (W_b/H_w) of 1.33, 2.0 and 4.0. The W_b/H_w ratio of 1.33 and 2 are encountered very infrequently in channels of any consequential size, while the drainage distances generally greatly exceed 10W_b [Garg and Chawla, 1970].

Garg and Chawla [1970] obtained exact solutions of problem of seepage from canals in homogeneous media to drains located symmetrically at finite distance from the canal considering vertical and horizontal drainages. This solution and results are separately discussed in some detail under section 2.2. Sharma and Chawla [1974] obtained analytical solution of seepage from a canal with negligible water depth to the symmetrical drainages on either side of the canal with pervious medium extending up to finite depth.

Mishra and Seth [1988] , using Zhukovsky's function and Schwarz-Christoffel conformal mapping technique, analyzed unconfined seepage from a river of large width for a steady state condition. Seepage quantities occurring through the bed and bank of the river have been estimated separately. The reach transmissivity constant for a river with large width has been determined. The reach transmissivity has been defined as the constant of proportionality between the return flow to river and the difference of potentials at the periphery of the river and in the aquifer in the vicinity of the river [Morel-Seytoux and Daly, 1975]. Morel-Seytoux and Daly [1975] introduced the use of reach transmissivity for solving unsteady state stream-aquifer interaction problem. Mishra and Seth [1988] found that if the distance between the river bank and the observation well is more than half of the saturated thickness of the aquifer below the river bed the reach transmissivity constant is independent of draw down at the observation well. The reach transmissivity constant depends on the depth of water in the river bed and the distance of the observation well from the river bank. The seepage loss from the river at any time is product of the reach transmissivity constant and the difference in water level in the river and at the observation well at the time of observation.

As mentioned earlier, it had been reasoned that an unsteady state condition can be treated as a succession of steady state conditions [Muscat, 1946 and Bouwer, 1969]. Based on the above principle, the reach transmissivity constant, even though it had been derived on the assumption of steady flow condition, had been used for analysis of unsteady state problems [Morel-Seytoux 1975a, 1975b, 1975c, 1975d, 1975e]. Bhargava [1988] had reviewed the works of various investigators who derived the reach transmissivity constant for different canal and aquifer geometry. In his work, Bhargava had solved the unsteady seepage from two parallel canals when the water table is shallow or deep.

In the literature, analytical solution had been reported for the problem of seepage from unlined trapezoidal canal taking into account the general anisotropic behaviour of the porous medium [Reddy and Basu, 1976]. However, since a canal in an anisotropic medium transforms into one with unequal slopes in an equivalent isotropic porous medium, the solution has been given for the problem of seepage flow from an unsymmetrical trapezoidal canal. In practice, the main difficulty will be to find out the actual anisotropic behaviour of the porous medium that occurs in the field.

The literature on steady seepage from canal and its impact on ground water regime has been extensively documented by Muskat [1946], Harr [1962], Polubarinova-Kochina [1962], Aravin and Numerov [1965], Bouwer [1969], Bear [1972], Hálek and Švec [1979], Kovacs [1981], Verrujit [1982], and Huisman and Olsthoorn [1983].

2.2 Seepage from Trapezoidal Channel. with Symmetrically Located Drainages, [Garg and Chawla, 1970].

Vedernikov [1939, vide Polubarinova-kochina, 1962, pp.130 132] had solved the problem of seepage from a canal having negligible water depth to symmetrically disposed drains. Exact solutions of the problem of seepage from trapezoidal canals in homogeneous media to drainages symmetrically located at finite distances from the canal considering vertical and horizontal drainages were presented by Garg and Chawla [1970]. The method of Zhukovsky transformation and Schwarz-Christoffel conformal mapping technique were employed. The resulting integrals were evaluated numerically.

The analysis assumed the shape of the channel to be trapezoidal in the θ-plane. Then the equation of the side slopes in the z-plane was derived. Here the advantage of symmetry was exploited and the analysis was carried out by considering only half of the flow regime. If the drainages were not symmetrically located then the above procedure would have resulted in different side slopes at the left and right of the canal in the z-plane. In the analysis, the influence of the relative distances and levels of the drainages on the seepage flow as well as on the free surface were not considered.

Two basic operations were made in the solution of the problem. In the first operation, the Zhukovsky plane was transformed onto an intermediate semi-infinite plane. In the second operation, the rectangular flow field in the complex potential plane was mapped onto another auxiliary semi-infinite plane. These transformations were made by the use of the Schwarz-Christoffel transformation. The relationship between the two auxiliary planes was obtained using the bilinear transformation.

The seepage discharge from the canal is given by the following equation.

$$q/(kh) = 2K'/K \qquad (2.4)$$

in which, q is the volume rate of seepage per unit length of channel, h is the drop between channel and drain water levels, k is the coefficient of permeability, K and K' are complete elliptic integrals of first kind with modulus m and m'. The values of m and m' are given by the following equations.

$$m = \sqrt{\gamma/(\beta+\gamma)} \qquad (2.5)$$

$$m = \sqrt{1 - m*m} \qquad (2.6)$$

in which β and γ are transformation parameters which are closely related with the bed width of the canal and the drainage distance from water line of the channel respectively.

As direct evaluation of β and γ for any given channel dimensions and drainage distance was not possible [Garg and Chawla, 1970], channel dimensions and drainage distances had been evaluated for various values of β and γ and were plotted. Then, from the figure plotted, knowing the dimensions of the system in the physical plane, the two parameters β and γ were determined. After knowing these, from Eqs.2.5, 2.6 and then from Eq.2.4, the seepage discharge could be obtained. Determination of β and γ was a trial and error process [Garg and Chawla, 1970]. The results were plotted in the form of curves from which the seepage discharge and the phreatic surface profile for various channel geometries could be obtained.

In Chapter 4 , comparison of the results as obtained by Garg and Chawla [1970] and the new method suggested in this thesis is presented.

2.3 Seepage from canal to a collector [Polubarinova-Kochina, 1962]

In the literature review above, it is found that solutions are obtained only for cases where symmetric seepage flow from the canal to the drainages, whether shallow or deep, takes place. In practice canals seldom have symmetric drainages on either side. However, a solution of seepage from a canal with drainage only on one side had been reported [Polubarinova-Kochina, 1962, pp.132-133]. In this work, the shape of the canal as well as the depth of water in the canal were not considered.

The seepage from canal of negligible water depth to a collector on one side, the following equations were obtained.

$$B/h = m^2/m'^2 \ (L/h) + K(m)/K(m') \tag{2.7}$$

$$q/(kh) = K(m)/K(m)' \tag{2.8}$$

in which, B is the bed width of the canal, L is the drainage distance from water line of the canal, K(m) and K(m′) are complete elliptic integrals of the first kind with modulus m and m′ respectively, and $m' = \sqrt{(1-m*m)}$.

In Polubarinova-Kochina's reporting [1969, pp.132-133], the full derivation was not developed and hence the equations giving the shape of the free surfaces were not given. In Appendix C , the full derivation is given. In Chapter 5 of the present work, the solution is extended to cover the case of seepage from trapezoidal canal to drainage on one side.

CHAPTER 3

SEEPAGE FROM CANAL OF NEGLIGIBLE WATER DEPTH TO ASYMMETRIC DRAINAGES

Introduction

As reviewed in Chapter 2, so far the solutions of seepage from canals assume symmetric flow from the canal to the drainages whether shallow or deep. However, in practice canals seldom have symmetric drainages on either side. In this chapter, solution is presented for the case of a canal of negligible water depth with drainages asymmetrically located on either side of the canal.

3.1 Formulation of problem

The water seeping from a canal located on a watershed flows through porous medium and emerges in drainages located on either side. The steady flow through the porous media satisfies two dimensional Laplace's equation (Eq.A.1).

The equation is based on few assumptions. First, the soil below the canal is assumed to be homogeneous and isotropic. Secondly, capillary and surface tension effects are neglected. Thirdly, the flow is assumed to be laminar and therefore follows Darcy's law. In addition, for obtaining solution of this problem, it is assumed that water depth in the canal is small and negligible. Seepage from the canal having a bed width, B_2, emerges into two asymmetric drainages on either side of the

A TECHNICAL NOTE BASED ON THIS CHAPTER HAS BEEN RECOMMENDED BY *ASCE* FOR PUBLICATION IN *Journal of Irrigation and Drainage*.

canal and these drainages are assumed to be wide.

Shape of the free surface of seepage from the canal is curvilinear and is not known a priori. But the curvilinear phreatic line is transformed into a straight line in the Zhukovsky plane. The Zhukovsky function θ has been defined [Section A.3] as :

$$\theta = \theta_1 + i\theta_2$$

$$= z - iw/k \tag{3.1}$$

in which, $z = x + iy$ and $w = \phi + i\psi$; x and y are the spatial coordinates in the z-plane ; ϕ is the velocity potential and ψ is the stream function ; k is coefficient of permeability. Then,

$$\theta_1 = x + \psi/k \tag{3.2}$$

and,

$$\theta_2 = y - \phi/k \tag{3.3}$$

θ_2 is a harmonic function of y and ϕ, and its conjugate function θ_1 is a harmonic function of x and ψ.

It is known [Aravin and Numerov, 1965] from the relation between velocity potential ϕ and the pressure p that :

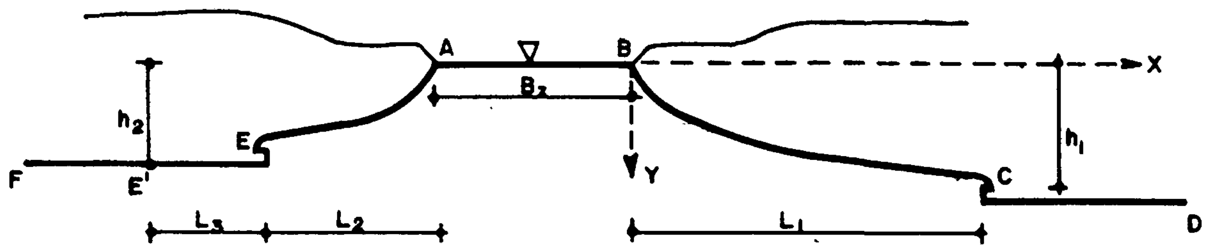
$$p/\gamma_w = y - \phi/k \tag{3.4}$$

in which,

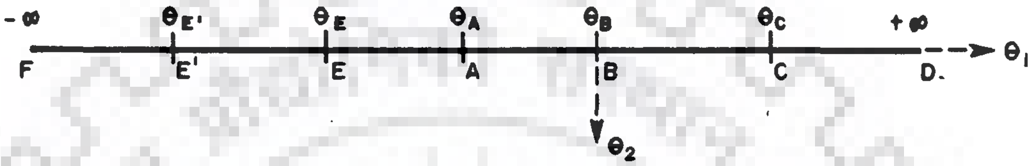
y = the ordinate assumed positive downwards.

γ_w = the specific weight of water;

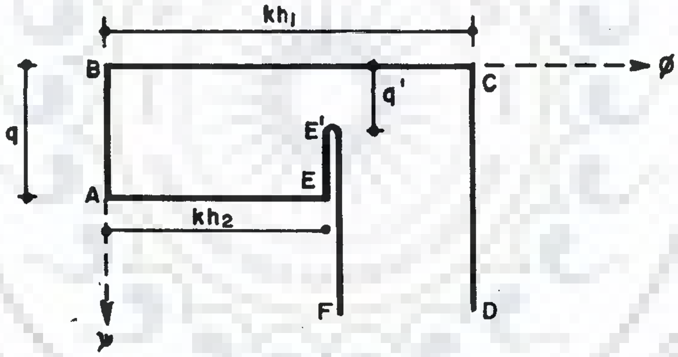
Along the free surface, the pressure is atmospheric, and from Eqs.3.4 and 3.3, $\theta_2 = 0$. Therefore, the free surface is represented by a straight line in the θ-plane. Fig.3.1(a) depicts conditions in the physical z-plane. Fig.3.1(b) shows the



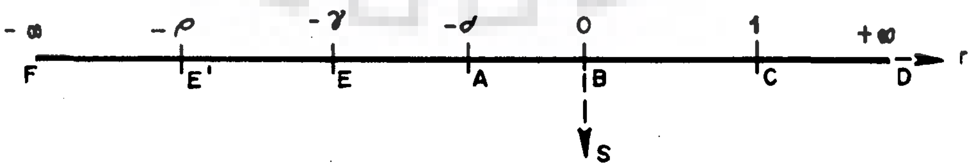
(a) Physical plane (z-plane)



(b) Zhukovsky plane (θ-plane)



(c) Complex potential plane (w-plane)



(d) Auxilliary plane (t-plane)

FIG. 3.1 TRANSFORMATION LAYOUT

boundaries as they look in the θ-plane as a result of Zhukovsky's transformation [Eq.3.1]. In the z-plane, AB is an equipotential line and corresponds to ϕ = constant. This constant is assumed to be zero, and so $\phi = 0$ along this line. Along the stream line and free surface BC, the value of ψ is taken to be zero. Then, $\psi = q$ along the stream line and free surface AE, where q is the seepage discharge per unit length of the canal. Along the drainage CD, which is an equipotential line, ϕ is equal to kh_1, in which h_1 is the difference between water levels of the canal and the right-hand drainage. Along EE'F, the left-hand drainage which is an equipotential line , $\phi = kh_2$, in which h_2 is the difference between the water levels of the canal and the left-hand drainage. Along the left-hand drainage, which is assumed to be at higher level than the right-hand drainage, seepage from the canal emerges between E and E'. Due to difference in the elevations of the left and the right drainages, there will be seepage from the left-hand drainage to the right-hand drainage. This seepage will take place from E'F to portion of the right drainage beyond some distance from Point C. The location of the point E' will depend on the relative values of h_1, h_2, L_1 and L_2, in which L_1 and L_2 are the distances of the drainages on the right and the left side of the canal respectively.

The w-plane [Fig.3.1(c)] and the θ-plane are mapped onto the lower half of an intermediate t-plane where $t = r + is$, and the following relations were obtained :

$$z = f(\theta) = f_1(t) \tag{3.5}$$

$$w = F(t) \qquad (3.6)$$

Combining Eqs.3.5 and 3.6 resulted in the following relationships.

$$z = f_1(t) = f_1[F^{-1}(w)] \qquad (3.7)$$

$$w = F(f_1^{-1}(z)) \qquad (3.8)$$

3.2 Solution of the Problem

3.2.1 First operation.

In this operation the physical plane is transformed onto the θ-plane. The transformation of the physical plane [Fig.3.1(a)] onto the θ-plane [Fig.3.1(b)] was obtained through Eq.3.1. The location of the various points in z-plane and θ-plane are given in Table 3.1.

Table 3.1 LOCATION OF POINTS ON THE FOUR PLANES

POINT	z - plane		w - plane		θ - plane	t-plane
	x	y	ϕ	ψ	$\theta = \theta_1$	t = r
A	$-B_2$	0	0	q	$-B_2+q/k$	$-\sigma$
B	0	0	0	0	0	0
C	L_1	h_1	kh_1	0	L_1	1
D	∞	h_1	kh_1		∞	∞
E	$-B_2-L_2$	h_2	kh_2	q	$-B_2-L_2+q/k$	$-\gamma$
E′	$-B_2-L_2-L_3$	h_2	kh_2	q′	$-B_2-L_2-L_3+q'/k$	$-\rho$
F	$-\infty$	h_2	kh_2		$-\infty$	$-\infty$

In Table 3.1, L_3 is the length of EE′ as shown in Fig.3.1(a) and q′ is the canal seepage component directly emerging on the right-hand drainage.

3.2.2 Second operation.

In this operation the w-plane [Fig.3.1(c)] is transformed onto the lower half of the t-plane [Fig.3.(1d)]. This is done by using the Schwarz-Christoffel transformation. Two arbitrary values t = 0 and t = 1 are assigned respectively to points B and C, and a third point, D is mapped on the t-plane at $t = +\infty$. Point F would be mapped at $t = -\infty$. Mapping of A, E and E' is made on the points $t = -\sigma$, $t = -\gamma$ and $t = -\rho$, respectively, as indicated in Fig.3.1(d). The values of σ, γ and ρ are to be determined. The location of the points in the two planes, i.e., the w-plane and t-plane, are summarized in Table 3.1.

The Schwarz-Christoffel transformation [Eq.A.6] that maps the w-plane onto the lower half of the t-plane is as given below.

$$\int dw = M' \int \frac{(t+\rho)dt}{\sqrt{(1-t)t(t+\sigma)(t+\gamma)}} \tag{3.9}$$

where M' is a complex constant.

At Point B, w = 0 and t = 0. Hence, for the region BC, i.e. [$-\gamma < -\sigma < 0 < t \le 1$], Eq.3.9 takes the following form.

$$\int_0^w dw = M' \int_0^t \frac{(t+\rho)dt}{\sqrt{(1-t)(t-0)(t+\sigma)(t+\gamma)}} \tag{3.10}$$

where, w = ky is the complex potential function value at any point on the free surface BC having the ordinate value of y. Therefore, the integration of the right-hand side [Byrd and

Friedman, 1971, section 256.11 and 256.00] and the left-hand side of Eq.3.10 yielded the following.

$$ky = M\ [\ \sigma \Pi(\beta_1, \alpha_1^2, m) + (\rho-\sigma)F(\beta_1, m)\ ] \tag{3.11}$$

in which, M is a complex constant and

$F(\beta_1, m)$ = Elliptic integral of the first kind,

$\Pi(\beta_1, \alpha_1^2, m)$ = Elliptic integral of the third kind,

$$\alpha_1^2 = 1/(1+\sigma) \tag{3.12}$$

$$m^2 = (\gamma-\sigma)/(\gamma(1+\sigma)) \tag{3.13}$$

$$\beta_1 = \sin^{-1}\sqrt{(1+\sigma)t/(t+\sigma)} \tag{3.14}$$

At point C, $t = 1$, $w = kh_1$ and hence the following was obtained from Eq.3.11 and Eq.3.14.

$$kh_1 = M\ [\ \sigma \Pi_1 + (\rho-\sigma)K\ ] \tag{3.15}$$

in which,

$$\Pi_1 = \Pi(\pi/2, \alpha_1^2, m) \tag{3.16}$$

$$K = F(\pi/2, m) \tag{3.17}$$

Integrating Eq.3.9 in the region AB , i.e in the region [$1 > 0 \geq t > -\sigma > -\gamma$], [applying Byrd and Friedman, 1971, Section 254.10 and 254.00 for the right-hand side integration] and knowing that at Point A, $w = iq$, and $t = -\sigma$ and at Point B, $w = 0$ and $t = 0$ the following relationships were found.

$$\int_{iq}^{0} dw = \frac{M'}{i} \int_{-\sigma}^{0} \frac{(t+\rho)dt}{\sqrt{(1-t)(0-t)(t+\sigma)(t+\gamma)}} \tag{3.18}$$

$$q = M\,[(\gamma-\sigma)\,\Pi_2' + (\rho-\gamma)K']$$ (3.19)

in which,

$$\Pi_2' = \Pi(\pi/2, \alpha_2^2, m')$$ (3.20)

$$K' = F(\pi/2, m')$$ (3.21)

$$m'^2 = 1 - m^2$$ (3.22)

$$\alpha_2^2 = \sigma/\gamma$$ (3.23)

At any point along the free surface AE, ϕ = ky and ψ = q, where y is the ordinate of the point considered. Hence, at Point A, w = iq and from Table 3.1, t = $-\sigma$, and considering the region EA, i.e in the region [$1 > 0 > -\sigma \geq t > -\gamma$], Eq.3.9 takes the following form.

$$\int_{w}^{iq} dw = \frac{M'}{-1} \int_{t}^{-\sigma} \frac{(t+\rho)\,dt}{\sqrt{(1-t)(0-t)(-\sigma-t)(t+\gamma)}}$$ (3.24)

where, w = (ky + iq).

Integrating the right-hand side [Byrd and Friedman, 1971, Section 253.11 and 253.00] as well as the left-hand side of Eq.3.24, the following result was obtained.

$$ky = M\,[\rho F(\beta_2, m) - \sigma\,\Pi(\beta_2, \alpha_8^2, m)]$$ (3.25)

where,

$$\beta_2 = \sin^{-1}\sqrt{[\gamma(t+\sigma)]/[t(\gamma-\sigma)]}$$ (3.26)

$$\alpha_8^2 = (\gamma-\sigma)/\gamma$$ (3.27)

At point E, $w = kh_2 + iq$ and hence from Eq.3.24 and 3.26 the following result was found.

$$kh_2 = M \left[\rho K - \sigma \Pi_8 \right] \tag{3.28}$$

in which,

$$\Pi_8 = \Pi(\pi/2, \alpha_8^2, m) \tag{3.29}$$

At Point E, $w = (kh_2 + iq)$ and $t = -\gamma$ and at Point E', $w = (kh_2 + iq')$ and $t = -\rho$ [refer Table 3.1]. Hence, considering the region E'E , i.e [$1 > 0 > -\sigma > -\gamma > t$], Eq.3.9 was put in the following form after integrating the left-hand side of the equation.

$$-i(q' - q) = \frac{M'}{-i} \int_{-\rho}^{-\gamma} \frac{(t + \rho)\, dt}{\sqrt{(1-t)(0-t)(-\sigma-t)(-\gamma-t)}} \quad . . . (3.30)$$

Integrating the right-hand side of Eq.3.30 [Byrd and Friedman, 1971, Section 251.03 and 251.00] and then rearranging yielded the following equation.

$$q' = q + M \left[(\gamma-\sigma)\, \Pi(\beta_4, \alpha_4^2, m') - (\rho-\sigma) F(\beta_4, m') \right] \quad . . . (3.31)$$

in which,

$$\beta_4 = \sin^{-1}\sqrt{[(1+\sigma)(\rho-\gamma)]/[(1+\gamma)(\rho-\sigma)]} \quad . . . (3.32)$$

$$\alpha_4^2 = (1+\gamma)/(1+\sigma) \tag{3.33}$$

3.2.3 Third operation.

In this last operation, the θ-plane was transformed onto the intermediate semi-infinite t-plane. The above mapping was made by using the bilinear or Möbius transformation [refer Appendix A].

The boundaries of the lower half regions of the θ-plane and t-plane were considered as limiting cases of circles and hence to map the region in θ-plane to t-plane the bilinear transformation was adopted [section A.2]. As any three arbitrary points on the boundary can define a circle, points B, C and D were chosen for this purpose. To apply the bilinear transformation what is commonly called the cross-ratio formula as given in section A.2 was used. The resulting equation is as follows.

$$\frac{(\theta - \theta_B)(\theta_D - \theta_C)}{(\theta - \theta_C)(\theta_D - \theta_B)} = \frac{(t - t_B)(t_D - t_C)}{(t - t_C)(t_D - t_B)} \quad \ldots (3.34)$$

After the appropriate values for points B, C and D [refer Table 3.1] are substituted in Eq.3.34, and rearranging the resulting equation, the following relationship was obtained.

$$t = \theta/L_1 \quad (3.35)$$

At point A, $t = -\sigma$ and $\theta = -(B_2 - q/k)$ [refer Table 3.1]. Substituting these values in Eq.3.35,

$$\sigma = (B_2 - q/k)/L_1 \quad (3.36)$$

Similarly, the corresponding values in t-plane and θ-plane for point E were substituted in Eq.3.35 and the following result was obtained.

$$\gamma = (B_2 + L_2 - q/k)/L_1$$

$$= (B_2 - q/k)/L_1 + L_2/L_1 \qquad (3.37)$$

But, From Eq.3.36, $(B_2 - q/k)/L_1 = \sigma$ and hence substituting this in Eq.3.37,

$$\gamma = \sigma + L_2/L_1 \qquad (3.38)$$

Also, the appropriate values for Point E' from Table 3.1 were substituted in Eq.3.35 and yielded the following relationship.

$$\rho = (B_2 + L_2 + L_3 - q'/k)/L_1 \qquad (3.39)$$

Moreover, rearranging Eq.3.39 yielded,

$$L_3 = \rho L_1 - L_2 - B_2 + q'/k \qquad (3.40)$$

3.3 Dimensionless form of Equations

3.3.1 General Case

Dividing Eq.3.19 and 3.28 by Eq.3.15, the following set of dimensionless quantities were obtained.

$$q/kh_1 = \frac{(\gamma-\sigma)\Pi_2' + (\rho - \gamma)K'}{\sigma\Pi_1 + (\rho - \sigma)K} \qquad (3.41)$$

$$h_2/h_1 = \frac{\rho K - \sigma\Pi_3}{\sigma\Pi_1 + (\rho - \sigma)K} \qquad (3.42)$$

Dividing Eq.3.31 by kh_1 as given by Eq.3.15, the following non-dimensional equation was found.

$$\frac{q'}{kh_1} = \frac{q}{kh_1} + \frac{[\ (\gamma-\sigma)\,\Pi(\beta_4,\alpha_4^2,m') - (\rho-\sigma)F(\beta_4,m')\ ]}{\sigma\,\Pi_1 + (\rho - \sigma)K} \quad \ldots (3.43)$$

Rearranging Eq.3.42, resulted in the following equation as well.

$$\rho = \frac{\sigma\ [\ (h_2/h_1)\ (\Pi_1 - K) + \Pi_3\ ]}{[\ K\ (1 - h_2/h_1)\ ]} \qquad (3.44)$$

Dividing the numerator and denominator of the right-hand side of Eq.3.36 by h_1, the following relationship for the parameter σ was obtained.

$$\sigma = (B_2/h_1 - q/kh_1)/(L_1/h_1) \qquad (3.45)$$

3.3.1 Special Cases

3.3.1.1 Case of symmetrically located drainages

For the case of drainages symmetrically located on either side of the canal, $L_2 = L_1 = L$; $h_2 = h_1 = h$ and $q = q_s$, where q_s is seepage from the canal when the drainages are symmetrically located. So, from Eq.3.38, we have $\gamma = (1+\sigma)$. Hence, substituting this in Eqs.3.12, 3.13, and 3.27, the following equations were obtained.

$$\alpha_1^2 = 1/(1+\sigma) = m \qquad (3.46)$$

$$\alpha_3^2 = 1/(1+\sigma) = m \qquad (3.47)$$

For the case of the parameters as related by Eqs.3.46 and 3.47 the following relationship was made [Byrd and Friedman, 1971, p.227, section 412.01],

$$\Pi_1 = \Pi_2 = [\ \pi + 2(1-m)K\ ]/[\ 4(1-m)\ ] \tag{3.48}$$

Substituting Eq.3.48 in Eq.3.44, the right-hand side of the resulting equation will be free from the elliptic integral of the third kind. For symmetrical case, $h_2/h_1 = 1$ and substituting this in the above result and after some manipulation, it is found that $\rho \rightarrow \infty$. It is to be noted that for $\sigma > 0$, [Fig.3.1 and Eq.3.36], m is different from one [Eq.3.46 and 3.47] and thus refering Byrd and Friedman, 1971, p.11, section 115.05, K is different from infinity. Similarly, Π_2' and K' are different from ∞ and thus knowing that for symmetrical case $(\gamma - \sigma) = 1$, Eq.3.41 after simplification resulted in the following relationship.

$$\frac{q_s}{kh} = K'/K \tag{3.49}$$

Similarly, simplifying the second term on the right-hand side of Eq.3.43 for the case of $\rho \rightarrow \infty$ and substituting the result obtained for q as given by Eq.3.49, the following result was also obtained.

$$q'/kh = K'/K - F(\beta_4, m')/K \tag{3.50}$$

Also, Eq.3.32 reduced to,

$$\beta_4 = \sin^{-1}\sqrt{(1+\sigma)/(2+\sigma)} \tag{3.51}$$

Hence, from Eq.3.51 and applying basic trigonometry,

$$\cot(\beta_4) = 1/\sqrt{1+\sigma} \tag{3.52}$$

Again applying basic trigonometry and using Eq.3.46,

$$\tan^2\beta_4 = 1 + \sigma = 1/m \qquad (3.53)$$

Comparing Eqs.3.46, 3.52 and 3.53, the following relationship was made.

$$\cot(\beta_4) = m \tan(\beta_4) \qquad (3.54)$$

As the condition as given by Eq.3.54 exists, $F(\beta_4,m')$ is equal to 0.5K' [Byrd and Friedman, 1971, p.13, section 117.01]. Substituting this result in Eq.3.50,

$$q'/kh = 0.5K'/K \qquad (3.55)$$

Hence, substituting Eq.3.49 in Eq.3.55, the following relationship for the case of symmetrical drainages was obtained and which obviously is as must be expected.

$$q'/kh = 0.5q_s/kh \qquad (3.56)$$

3.3.2.2 Total seepage from the canal going to one drainage only.

Seepage discharge from the canal emerging in the drainage on the left-hand side decreases as its level rises and correspondingly L_s will reduce. With further rise in the level of the left-hand drainage, entire seepage from canal may appear on the right-hand drainage ($L_s = 0$). In addition, seepage will also take place from left-hand drainage to the right-hand drainage.

Then, for this case in which all the seeping water from the canal goes to one drainage only, say, to the right drainage only, $q' = q$ and $L_s = 0$. Substituting these in Eq.3.39,

$$\rho = (B_2 + L_2 - q/k)/L_1$$

$$= (B_2 - q/k)/L_1 + L_2/L_1 \qquad (3.57)$$

Substituting Eqs.3.36 and 3.37 in Eq.3.57, the following relationship for the case where the total seepage from the canal goes to only one drainage, i.e. the right-hand side drainage was obtained.

$$\rho = \sigma + L_2/L_1$$

$$= \gamma \qquad (3.58)$$

Substituting Eq.3.58 in Eq.3.32, $\beta_4 = 0$, produced [Byrd and Friedman, 1971, p.10, section 111.00] the following result.

$$\Pi(\beta_4, \alpha_4^2, m') = F(\beta_4, m') = 0 \qquad (3.59)$$

Substituting the above relationships, i.e. Eq.3.58 and Eq.3.59 in Eq.3.41, $q = q_c$, where q_c is the total seepage from the canal in the case where only the right drainage gets the seepage water from the canal.

$$q_c/kh_1 = \frac{(L_2/L_1)\Pi_2'}{\sigma\Pi_1 + (L_2/L_1)K} \qquad (3.60)$$

Also, Eq.3.42 became,

$$\frac{h_2}{h_1} = \frac{\gamma K - \sigma\Pi_3}{\sigma\Pi_1 + (\gamma - \sigma)K}$$

$$= \frac{(L_2/L_1)K + \sigma(K - \Pi_3)}{\sigma\Pi_1 + (L_2/L_1)K} \qquad (3.61)$$

The ratio h_2/h_1 as given by Eq.3.61 equals to h_c/h_1, where, h_c is the critical level of the left drainage at which it is not receiving any water from the canal.

But, from Eq.3.12, 3.13 and Eq.3.27, $\alpha_1^2 = m^2/\alpha_3^2$ and hence the following relationship could be obtained [Byrd and Friedman, 1971, p.13, section 117.02(a)].

$$\Pi_3 + \Pi_1 = K + (0.5\pi/\sigma)\sqrt{(1+\sigma)(L_2/L_1 + \sigma)} \quad . . . (3.62)$$

Substituting Eq.3.62 in Eq.3.61, the following equation giving the critical ratio of the elevations of the left and right drainages, i,e (h_c/h_1), at which the total seepage from the canal flows only to the drainage which is at lower elevation was obtained.

$$h_c/h_1 = 1 - \frac{(0.5\pi)\sqrt{(1+\sigma)(L_2/L_1 + \sigma)}}{\sigma\Pi_1 + (L_2/L_1)K} \quad (3.63)$$

3.3.3 The shape of the free surfaces.

3.3.3.1 Free surface BC.

Eq.3.11 was divided by Eq.3.15 to obtain the ordinate of any point on the free surface BC in a non-dimensional form as given below.

$$y/h_1 = \frac{[\sigma\Pi(\beta_1,\alpha_1^2,m) + (\rho - \sigma)\ F(\beta_1,m)\ ]}{\sigma\Pi_1 + (\rho - \sigma)K} \quad (3.64)$$

On the free surface BC, ϕ = ky and ψ = 0, and hence from Eq.3.3, 3.2 and 3.1, the relation $\theta = \theta_1 = x$ was obtained. Substituting this result in Eq.3.35, the following equation was found.

$$t = x/L_1 \tag{3.65}$$

From Eq.3.65, for a given value of x, which is the horizontal projection of any point on the free surface BC , and L_1, the corresponding value of t is obtained. This value of t is then substituted in Eq.3.14, to get the value of β_1 which in turn is needed to compute y/h_1 from Eq.3.64.

3.3.3.2 Free surface AE.

On the free surface AE, ϕ = ky and ψ = q, and hence from Eq.3.3, 3.2 and 3.1 we have $\theta = \theta_1 = x + q/k$. Substituting this in Eq.3.35, the following equation results.

$$t = (x + q/k)/L_1 \tag{3.66}$$

If X_L is the horizontal distance of a point on the free surface AE measured from point A, see Fig.3.1(a), from Eq.3.66 the following relationship could be obtained.

$$\begin{aligned} t &= (-B_2 - X_L + q/k)/L_1 \\ &= -[\, X_L/L_1 + (B_2 - q/k)/L_1 \,] \end{aligned} \tag{3.67}$$

But, from Eq.3.36, $(B_2 - q/k)/L_1 = \sigma$. Substituting this in Eq.3.67, the result below was obtained.

$$t = -(\, X_L/L_1 + \sigma \,) \tag{3.68}$$

Also, dividing Eq.3.25 by Eq.3.28, the ordinate of the point on the free surface AE in a non-dimensional form as given below was found.

$$y/h_2 = \frac{\rho F(\beta_2, m) - \sigma \Pi(\beta_2, \alpha a^2, m)}{\rho K - \sigma \Pi_a} \quad (3.69)$$

Knowing X_L/L_1 and σ, the value of t is computed from Eq.3.68 which in turn is substituted in Eq.3.26 to yield the value of β_2. This value of β_2 is used in Eq.3.69, to find the non-dimensional ordinate (y/h_2) of the point on AE at distance of X_L from Point A.

3.4 Results and Discussions.

From the above equations, it is seen that total seepage from the canal and the seepage discharge components emerging to either of the drainages cannot be explicitly expressed in terms of physical dimensions such as canal bed width, drainage distances and levels of the drainages. Similarly, coordinates of the free surface are also not obtained in explicit form. The physical dimensions and seepage discharge and free surface coordinates are related to intermediate parameters σ, γ and ρ. Therefore, values of these parameters are determined for given values of physical parameters such as B_2/h_1, L_1/h_1, L_2/h_1 and h_2/h_1. Since the expressions relating physical and intermediate parameters also involve values of seepage discharge which in turn depend on the values of the intermediate parameters (σ, γ and ρ), computation of the seepage discharge and coordinates of the free surface is an iterative process [refer flow chart in Fig.B.1]. In the computational steps it was required to evaluate values of elliptic integrals. Subroutines to compute the values of the elliptic integrals of the first, second and third kind were thus developed. In the development of these subroutines, the results were verified with the corresponding values given in the tables of the *Hand book of Elliptic Integrals for Engineers and Scientists* [Byrd and Friedman, 1971] and *A Table of the Incomplete Elliptic Integrals of the Third Kind* [Selfridge & Maxfield, 1958].

The results of calculations for seepage discharge for values of B_2/h_1 = 10, 20 and 30 ; L_1/h_1 = 10, 10^2, 10^3, 10^4 and 10^5; L_2/h_1 = 10, 10^2, 10^3, 10^4, and 10^5 ; and h_2/h_1 = 1, 0.9, 0.8 and 0.7 are tabulated in Tables 3.2 to 3.5. and are plotted in Figs.3.2 to 3.5. Canal seepage discharge components towards the left and right-hand side drainages for the above given various physical dimensions of the flow system are presented in Tables 3.6 to 3.9.

Dimensionless coordinates of the free surface curves on either side of the canal were computed using Eqs.3.64, 3.65, 3.68 and 3.69. The profiles of the free surfaces for various combinations of the physical parameters of the flow system are plotted in Figs.3.6(a), 3.6(b), 3.7, 3.8(a) and 3.8(b) in order to highlight the separate influence of each parameter on the free surface.

The values of a critical depth ratio, h_c/h_1, for B_2/h_1 = 10, 20 and 30 and various values of L_1/h_1 and L_2/h_1 are plotted in Figs.3.9, 3.10 and 3.11 respectively.

Table 3.2 Total Seepage Discharge From Canal To Asymmetric Drainages. [Water depth negligible, $h_2/h_1 = 1.0$]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	q/kh_1				
			$B_2/h_1 = 10$		
10	1.23186	1.01537	0.98090	0.97721	0.97684
10^2	1.01537	0.69823	0.61780	0.60734	0.60626
10^3	0.98090	0.61780	0.46631	0.42856	0.42363
10^4	0.97721	0.60734	0.42856	0.34817	0.32661
10^5	0.97684	0.60626	0.42363	0.32661	0.27758
			$B_2/h_1 = 20$		
10	1.52796	1.22051	1.16197	1.15543	1.15477
10^2	1.22051	0.82230	0.71316	0.69869	0.69718
10^3	1.16197	0.71316	0.52121	0.47440	0.46834
10^4	1.15543	0.69869	0.47440	0.37786	0.35257
10^5	1.15477	0.69718	0.46834	0.35257	0.29605
			$B_2/h_1 = 30$		
10	1.72582	1.35799	1.27773	1.26842	1.26747
10^2	1.35799	0.91166	0.77998	0.76210	0.76023
10^3	1.27773	0.77998	0.55906	0.50555	0.49863
10^4	1.26842	0.76210	0.50555	0.39750	0.36959
10^5	1.26747	0.76023	0.49863	0.36959	0.30795

Table 3.3 Total Seepage Discharge From Canal To Asymmetric Drainages. [Water depth negligible, h_2/h_1 = 0.9]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	q/kh_1				
			B_2/h_1 = 10		
10	1.17253	0.93847	0.89216	0.88389	0.88199
10^2	0.99330	0.66384	0.56886	0.55116	0.54751
10^3	0.97375	0.60567	0.44315	0.39427	0.38418
10^4	-	-	0.42023	0.33083	0.30041
10^5	-	-	-	0.32026	0.26374
			B_2/h_1 = 20		
10	1.45326	1.12950	1.05683	1.04435	1.04167
10^2	1.19132	0.78158	0.65668	0.63388	0.62938
10^3	1.15240	0.69884	0.49525	0.43638	0.42463
10^4	-	-	0.46515	0.35901	0.32424
10^5	-	-	-	0.34570	0.28126
			B_2/h_1 = 30		
10	1.64093	1.25827	1.16258	1.14631	1.14296
10^2	1.32344	0.86642	0.71838	0.69140	0.68621
10^3	1.26627	0.76402	0.53120	0.46501	0.45207
10^4	-	-	0.49566	0.37766	0.33988
10^5	-	-	-	0.36239	0.29257

NOTE :- In the table above and in all other tables, '-' indicates that the location of the left drainage is beyond the critical range.

Table 3.4 Total Seepage Discharge From Canal To Asymmetric Drainages. [Water depth negligible, h_2/h_1 = 0.8]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	q/kh_1				
B_2/h_1 = 10					
10	1.11296	0.86127	0.80303	0.79012	0.78668
10^2	0.97120	0.62939	0.51982	0.49485	0.48863
10^3	-	-	0.41998	0.35994	0.34468
10^4	-	-	-	0.31348	0.27419
10^5	-	-	-	-	0.24989
B_2/h_1 = 20					
10	1.37838	1.03831	0.95143	0.93298	0.92825
10^2	1.16210	0.74082	0.60014	0.56899	0.56148
10^3	-	0.68451	0.46929	0.39833	0.38089
10^4	-	-	-	0.34015	0.29591
10^5	-	-	-	-	0.26647
B_2/h_1 = 30					
10	1.55588	1.15840	1.04724	1.02399	1.01821
10^2	1.28886	0.82115	0.65672	0.62063	0.61211
10^3	-	0.74806	0.50333	0.42446	0.40548
10^4	-	-	-	0.35781	0.31017
10^5	-	-	-	-	0.27718

Table 3.5 Total Seepage Discharge From Canal To Asymmetric Drainages. [Water depth negligible, $h_2/h_1 = 0.7$]

$L_1/h_1 \rightarrow$	10	10^2	10^3	10^4	10^5
L_2/h_1	q/kh_1				
$B_2/h_1 = 10$					
10	1.05315	0.78379	0.71349	0.69591	0.69090
10^2	0.94906	0.59489	0.47068	0.43841	0.42960
10^3	-	-	0.39679	0.32558	0.30513
10^4	-	-	-	-	0.24795
10^5	-	-	-	-	-
$B_2/h_1 = 20$					
10	1.30331	0.94692	0.84579	0.82131	0.81453
10^2	1.13284	0.70002	0.54352	0.50401	0.49348
10^3	-	-	0.44332	0.36026	0.33713
10^4	-	-	-	0.32130	0.26756
10^5	-	-	-	-	-
$B_2/h_1 = 30$					
10	1.47068	1.05839	0.93170	0.90144	0.89324
10^2	1.25424	0.77584	0.59500	0.54979	0.53793
10^3	-	-	0.47544	0.38389	0.35887
10^4	-	-	-	0.33796	0.28045
10^5	-	-	-	-	-

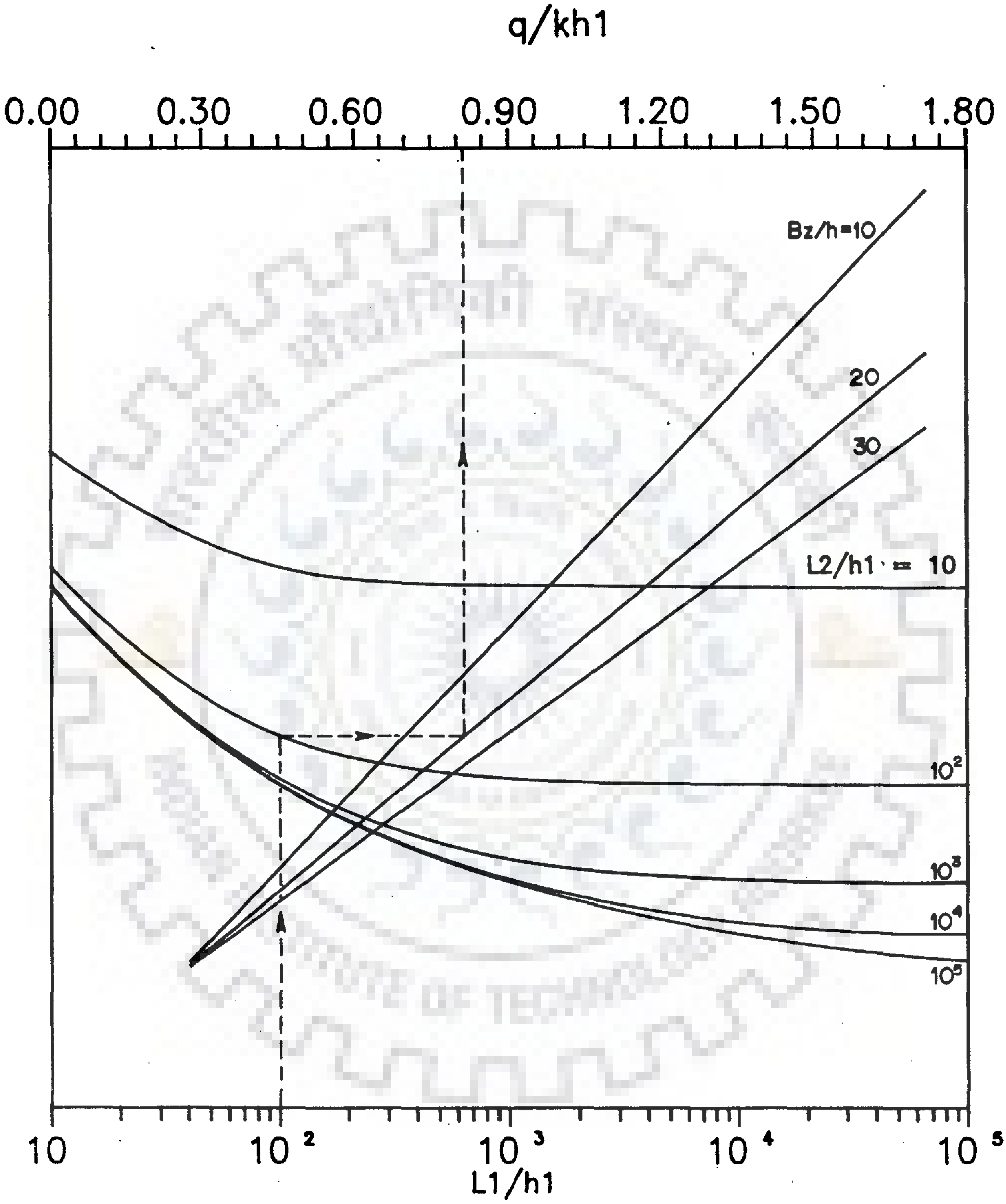


Fig.3.2 TOTAL SEEPAGE DISCHARGE TO ASYMMETRIC DRAINAGES. (h2/h1 = 1.0)

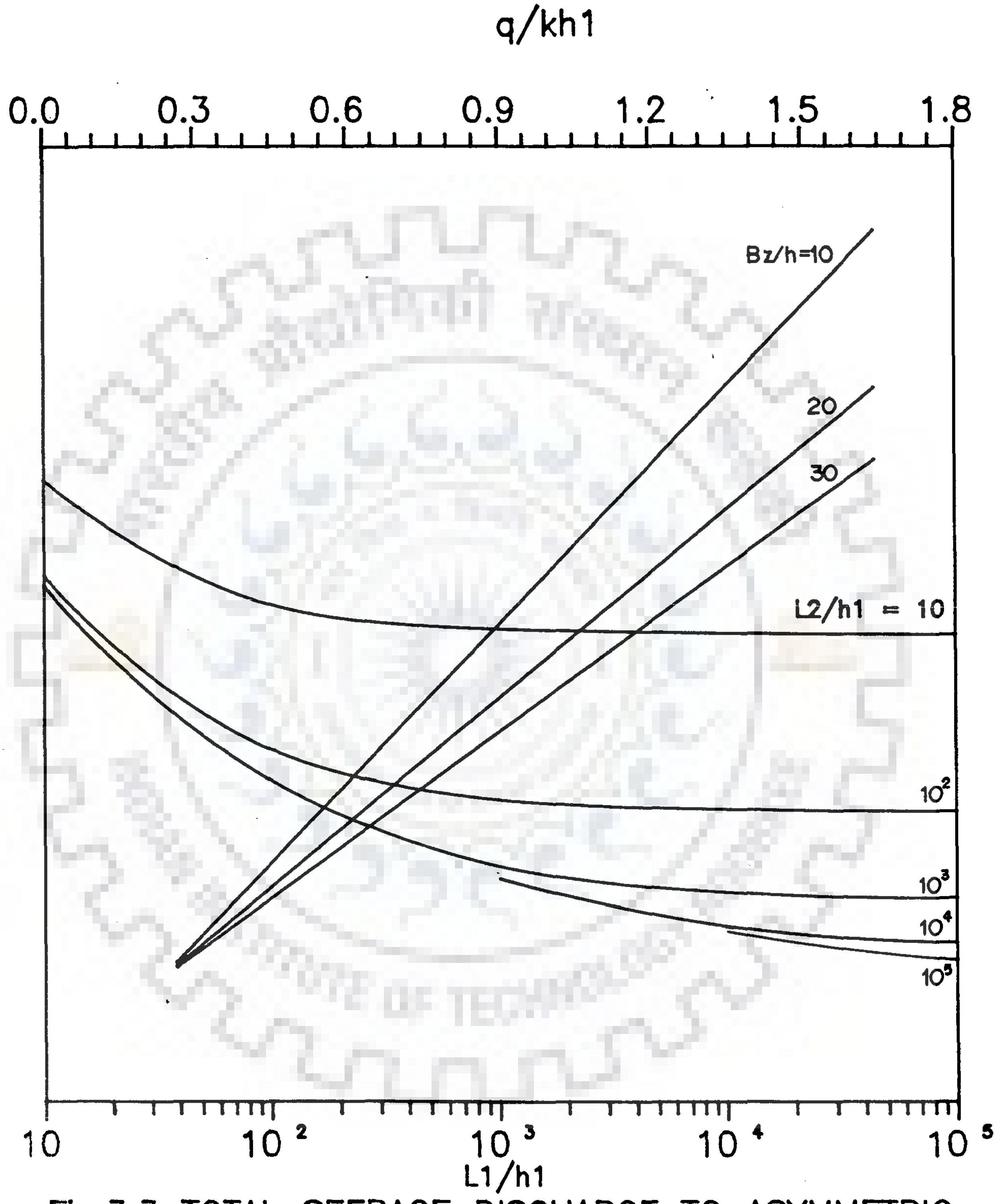


Fig.3.3 TOTAL SEEPAGE DISCHARGE TO ASYMMETRIC DRAINAGES (h2/h1 = 0.90)

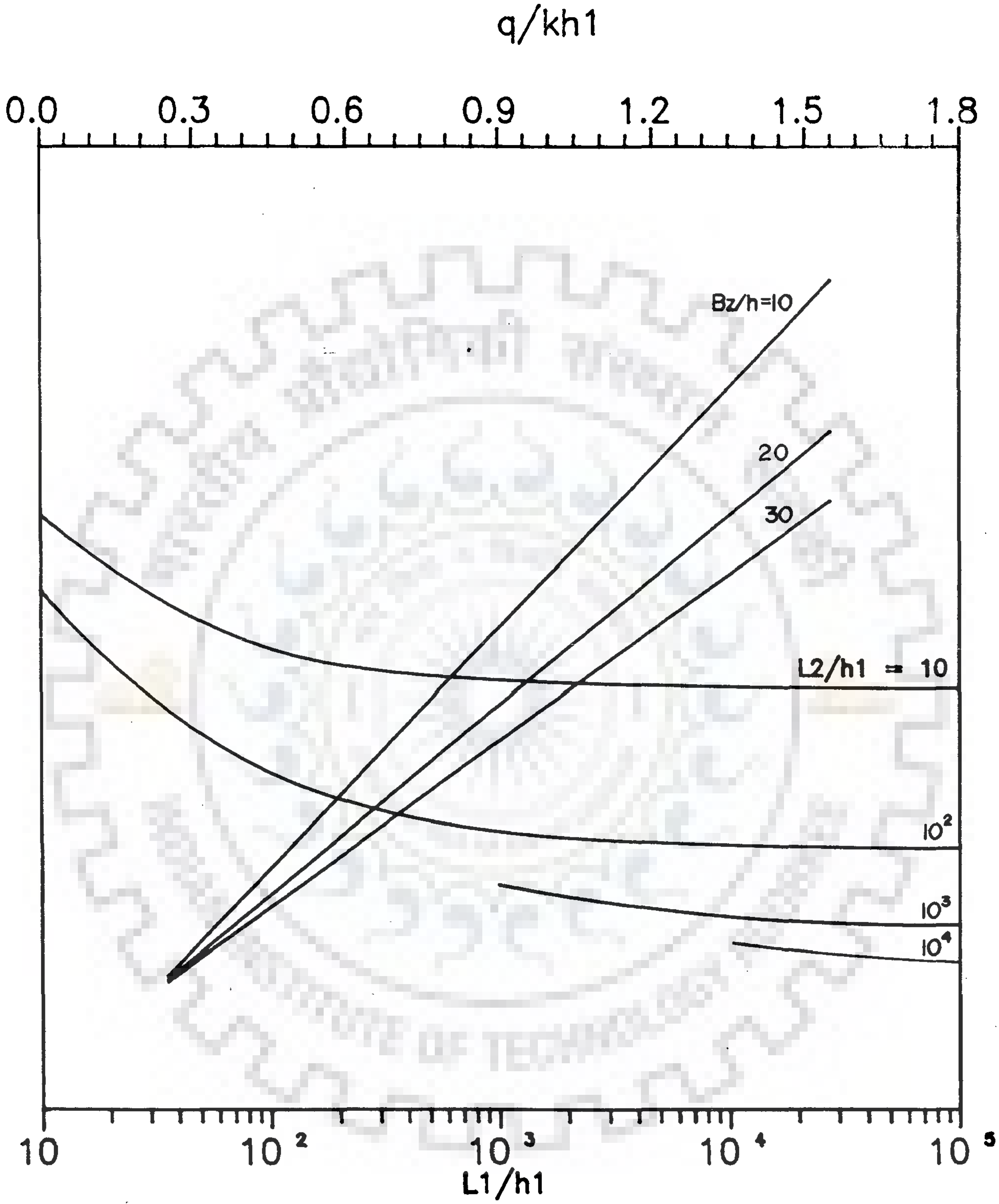


Fig.3.4 TOTAL SEEPAGE DISCHARGE TO ASYMMETRIC DRAINAGES (h2/h1 = 0.80)

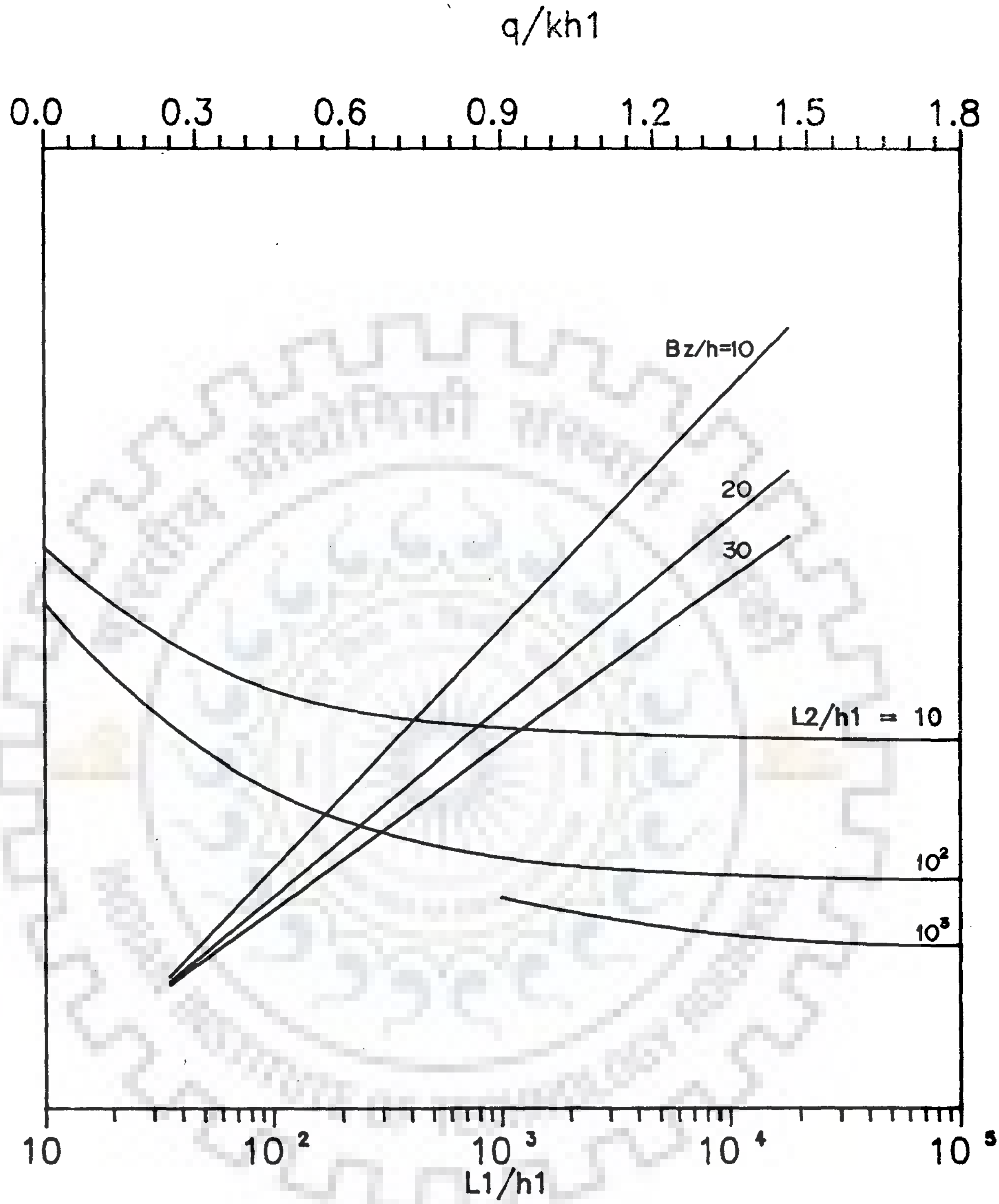


Fig.3.5 TOTAL SEEPAGE DISCHARGE TO ASYMMETRIC DRAINAGES (h2/h1 = 0.70)

Table 3.6 Seepage Discharge Components to the Left and Right Drainages. [Water depth negligible, h_2/h_1 = 1.0]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	Values* in non-dimensional form in terms of kh_1				
			B_2/h_1 = 10		
10	0.61593 0.61593	0.78885 0.22652	0.90764 0.07326	0.95400 0.02321	0.96952 0.00732
10^2	0.22652 0.78885	0.34912 0.34912	0.49508 0.12272	0.56798 0.03936	0.59381 0.01245
10^3	0.07326 0.90764	0.12272 0.49508	0.23316 0.23316	0.34487 0.08369	0.39672 0.02691
10^4	0.02321 0.95400	0.03936 0.56798	0.08369 0.34487	0.17409 0.17409	0.26295 0.06366
10^5	0.00732 0.96952	0.01245 0.59381	0.02691 0.39672	0.06366 0.26295	0.13879 0.13879
			B_2/h_1 = 20		
10	0.76398 0.76398	0.92405 0.29646	1.06494 0.09703	1.12465 0.03078	1.14505 0.00972
10^2	0.29646 0.92405	0.41115 0.41115	0.56887 0.14429	0.65240 0.04629	0.68255 0.01463
10^3	0.09703 1.06494	0.14429 0.56887	0.26060 0.26060	0.38156 0.09284	0.43851 0.02983
10^4	0.03078 1.12465	0.04629 0.65240	0.09284 0.38156	0.18893 0.18893	0.28383 0.06873
10^5	0.00972 1.14505	0.01463 0.68255	0.02983 0.43851	0.06873 0.28383	0.14802 0.14802
			B_2/h_1 = 30		
10	0.86291 0.86291	1.00853 0.34946	1.16200 0.11573	1.23165 0.03677	1.25586 0.01161
10^2	0.34946 1.00853	0.45583 0.45583	0.61948 0.16050	0.71058 0.05152	0.74394 0.01629
10^3	0.11573 1.16200	0.16050 0.61948	0.27953 0.27953	0.40633 0.09922	0.46680 0.03183
10^4	0.03677 1.23165	0.05152 0.71058	0.09922 0.40633	0.19875 0.19875	0.29752 0.07207
10^5	0.01161 1.25586	0.01629 0.74394	0.03183 0.46680	0.07207 0.29752	0.15398 0.15398

* NOTE :- Top values in each row in tables 3.6 to 3.9 are seepage discharge components to the left drainage whereas the corresponding values below them are those to the right drainage.

Table 3.7 Seepage Discharge Components to the Left and Right Drainages. [Water depth negligible, $h_2/h_1 = 0.9$]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	Values in non-dimensional form in terms of kh_1				
		$B_2/h_1 = 10$			
10	0.45325 0.71928	0.61567 0.32280	0.74207 0.15009	0.80689 0.07700	0.84017 0.04182
10^2	0.11063 0.88267	0.21782 0.44602	0.36024 0.20862	0.44780 0.10336	0.49245 0.05506
10^3	0.00231 0.97144	0.03050 0.57517	0.12043 0.32272	0.23242 0.16185	0.30090 0.08328
10^4	-	-	0.00709 0.41314	0.07369 0.25714	0.16446 0.13595
10^5	-	-	-	0.00006 0.32020	0.04749 0.21625
		$B_2/h_1 = 20$			
10	0.58543 0.86783	0.73383 0.39567	0.87756 0.17927	0.95435 0.09000	0.99328 0.04839
10^2	0.16961 1.02171	0.27143 0.51015	0.42360 0.23308	0.52033 0.11355	0.56940 0.05998
10^3	0.01386 1.13854	0.04577 0.65307	0.14291 0.35234	0.26326 0.17312	0.33644 0.08819
10^4	-	-	0.01185 0.45330	0.08513 0.27388	0.18157 0.14267
10^5	-	-	-	0.00112 0.34458	0.05417 0.22709
		$B_2/h_1 = 30$			
10	0.67409 0.96684	0.80821 0.45006	0.96138 0.20120	1.04682 0.09949	1.08986 0.05310
10^2	0.21546 1.10798	0.31049 0.55593	0.46729 0.25109	0.57045 0.12095	0.62271 0.06350
10^3	0.02550 1.24077	0.05786 0.70616	0.15864 0.37256	0.28426 0.18075	0.36057 0.09150
10^4	-	-	0.01544 0.48022	0.09283 0.28483	0.19286 0.14702
10^5	-	-	-	0.00029 0.36021	0.05853 0.23404

Table 3.8 Seepage Discharge Components to the Left and Right Drainages. [Water depth negligible, $h_2/h_1 = 0.8$]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	Values in non-dimensional form in terms of kh_1				
			$B_2/h_1 = 10$		
10	0.33802 0.77494	0.48804 0.37323	0.61605 0.18698	0.68996 0.10016	0.73132 0.05536
10^2	0.04549 0.92571	0.13444 0.49495	0.26874 0.25108	0.36277 0.13208	0.41628 0.07235
10^3	-	-	0.05624 0.36374	0.16143 0.19851	0.23717 0.10751
10^4	-	-	-	0.02274 0.29074	0.10589 0.16830
10^5	-	-	-	-	0.00705 0.24284
			$B_2/h_1 = 20$		
10	0.45439 0.92399	0.59002 0.44829	0.73173 0.21970	0.81687 0.11620	0.86441 0.06384
10^2	0.09209 1.07001	0.17953 0.56129	0.32245 0.27769	0.42476 0.14423	0.48287 0.07861
10^3	-	0.00349 0.68102	0.07356 0.39573	0.18697 0.21136	0.26731 0.11358
10^4	-	-	-	0.03047 0.30968	0.11970 0.17621
10^5	-	-	-	-	0.01068 0.25579
			$B_2/h_1 = 30$		
10	0.53279 1.02309	0.65468 0.50372	0.80342 0.24382	0.89640 0.12759	0.94833 0.06988
10^2	0.13020 1.15866	0.21294 0.60821	0.35969 0.29703	0.46767 0.15296	0.52904 0.08307
10^3	-	0.00959 0.73847	0.08599 0.41734	0.20449 0.21997	0.28785 0.11763
10^4	-	-	-	0.03588 0.32193	0.12886 0.18131
10^5	-	-	-	-	0.01320 0.26398

Table 3.9 Seepage Discharge Components to the Left and Right Drainages. [Water depth negligible, h_2/h_1 = 0.7]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	Values in non-dimensional form in terms of kh_1				
B_2/h_1 = 10					
10	0.24332 0.80983	0.37904 0.40475	0.50404 0.20945	0.58194 0.11397	0.62753 0.06337
10^2	0.00703 0.94203	0.07250 0.52239	0.19414 0.27654	0.28925 0.14916	0.34707 0.08253
10^3	-	-	0.01533 0.38146	0.10611 0.21947	0.18347 0.12166
10^4	-	-	-	-	0.06219 0.18576
10^5	-	-	-	-	-
B_2/h_1 = 20					
10	0.34385 0.95946	0.46560 0.48132	0.60135 0.24444	0.68962 0.13169	0.74158 0.07295
10^2	0.03799 1.09485	0.10877 0.59125	0.23878 0.30474	0.34147 0.16254	0.40391 0.08957
10^3	-	-	0.02680 0.41652	0.12673 0.23353	0.20867 0.12846
10^4	-	-	-	0.0068 0.31962	0.07291 0.19465
10^5	-	-	-	-	-
B_2/h_1 = 30					
10	0.41201 1.05867	0.52092 0.53747	0.66170 0.27000	0.75704 0.14440	0.81346 0.07978
10^2	0.06734 1.18690	0.13638 0.63946	0.26993 0.32507	0.37767 0.17212	0.44333 0.09460
10^3	-	-	0.03557 0.43987	0.14099 0.24290	0.22589 0.13298
10^4	-	-	-	0.00393 0.33403	0.08010 0.20035
10^5	-	-	-	-	-

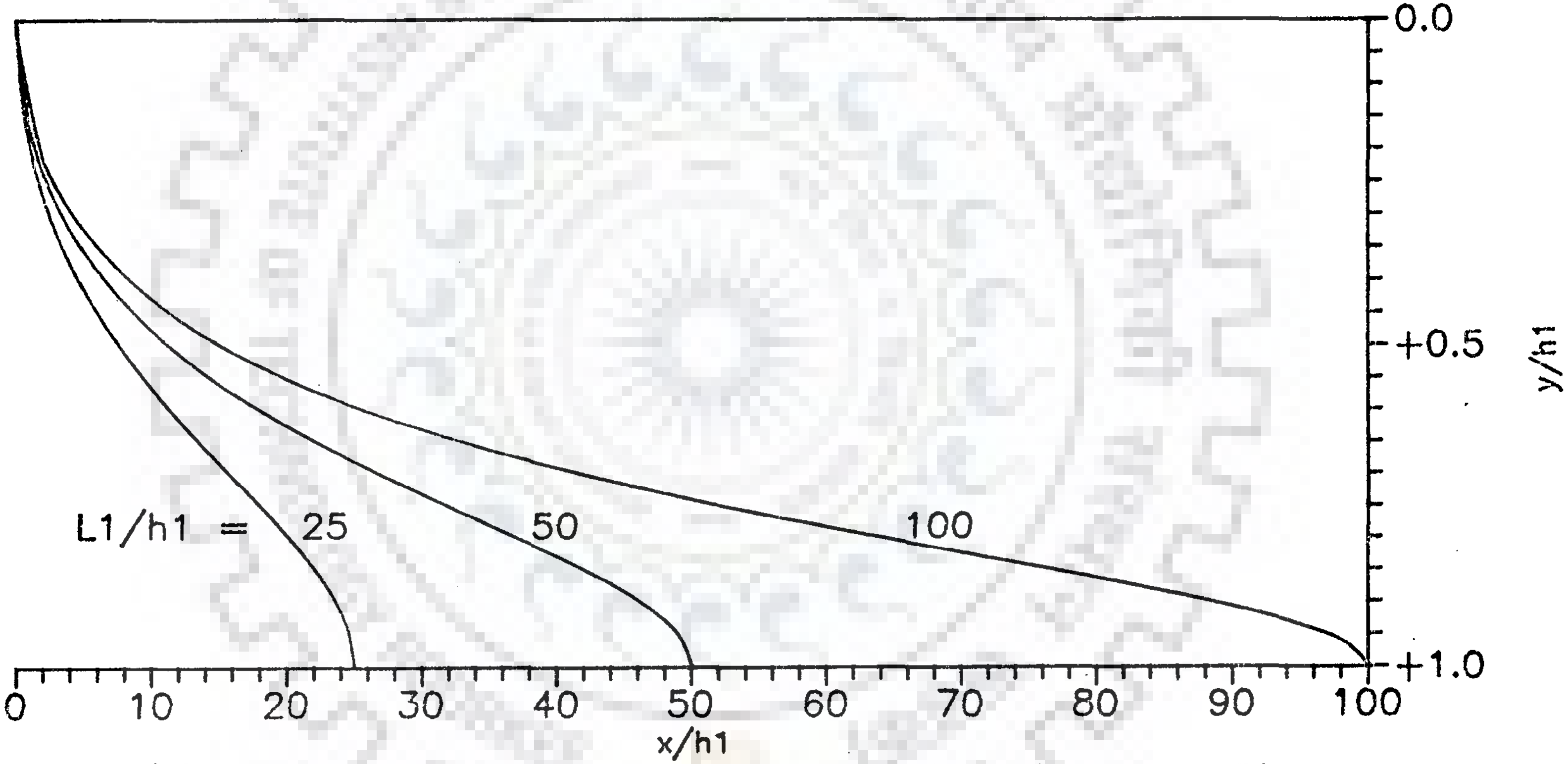


Fig.3.6(a) FREE SURFACE ON RIGHT-HAND SIDE – Effect of Right Drainage Distance. (h2=h1. Bz/h1 = 10. L2/h1 = 50)

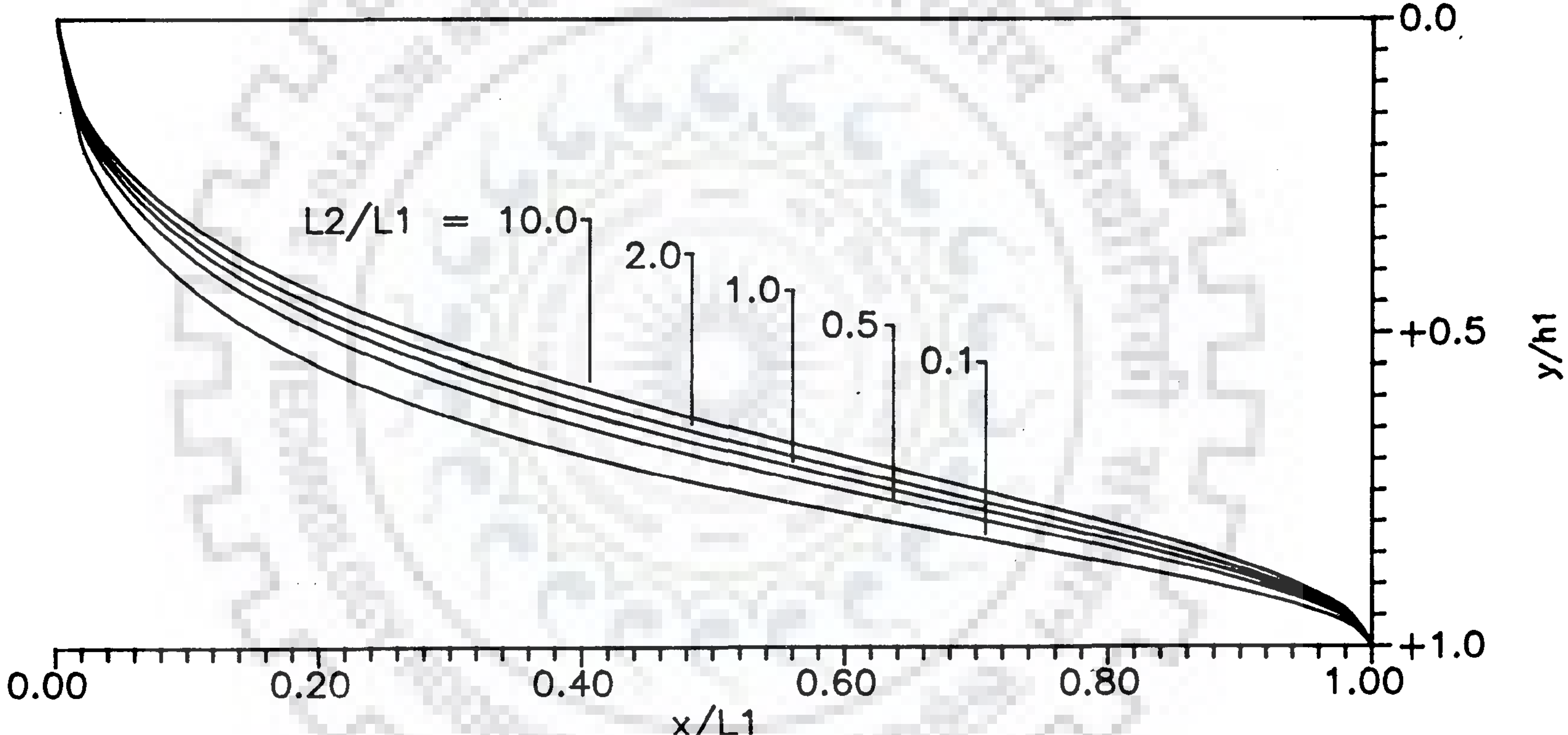


Fig.3.6(b) FREE SURFACE ON RIGHT-HAND SIDE – Effect of Left Drainage Distance.(h2=h1, Bz/h1 = 10, L1/h1 = 50)

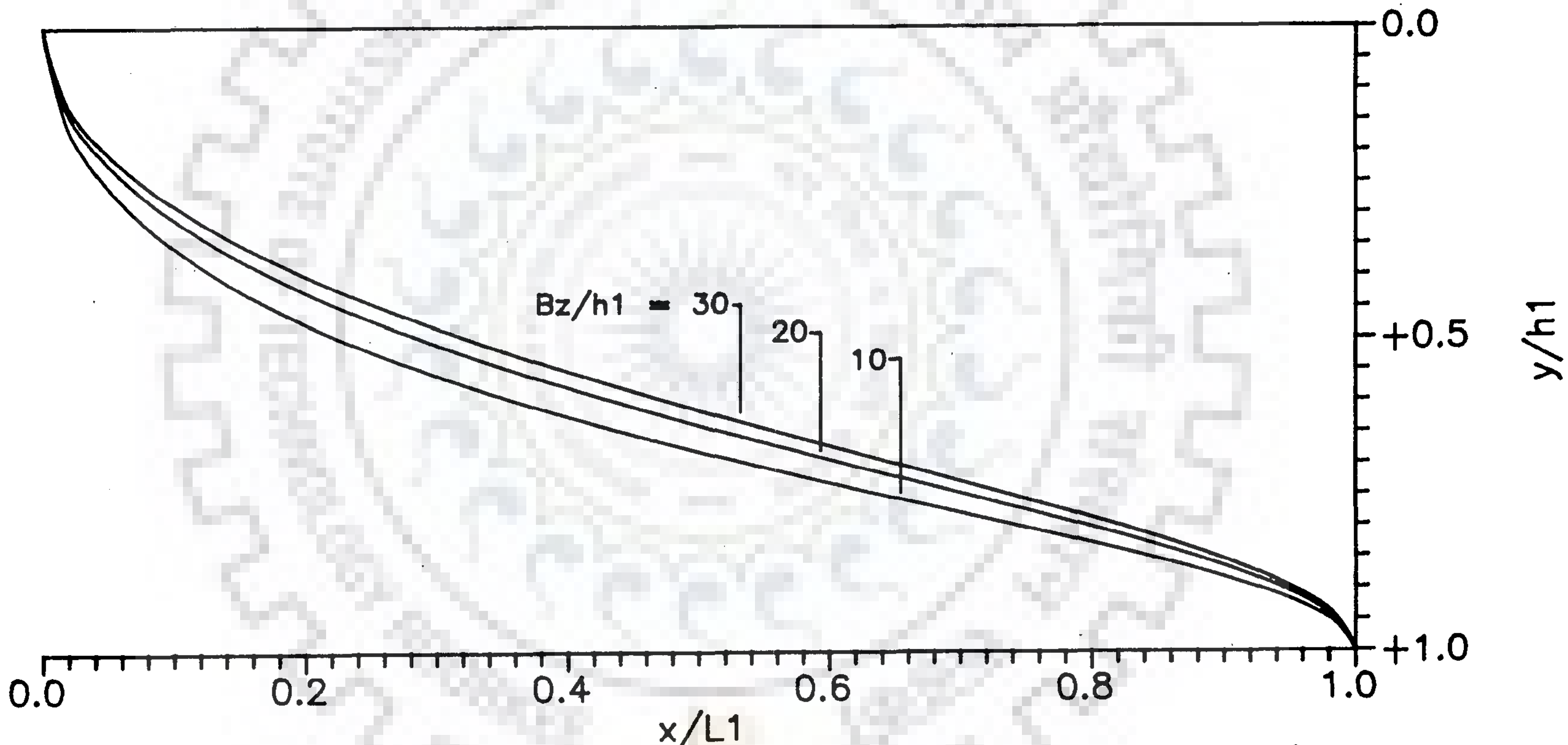


Fig.3.7 FREE SURFACE ON RIGHT-HAND SIDE – Effect of Bed Width
(h2/h1 = 1.0, L1 = L2 = 50h1)

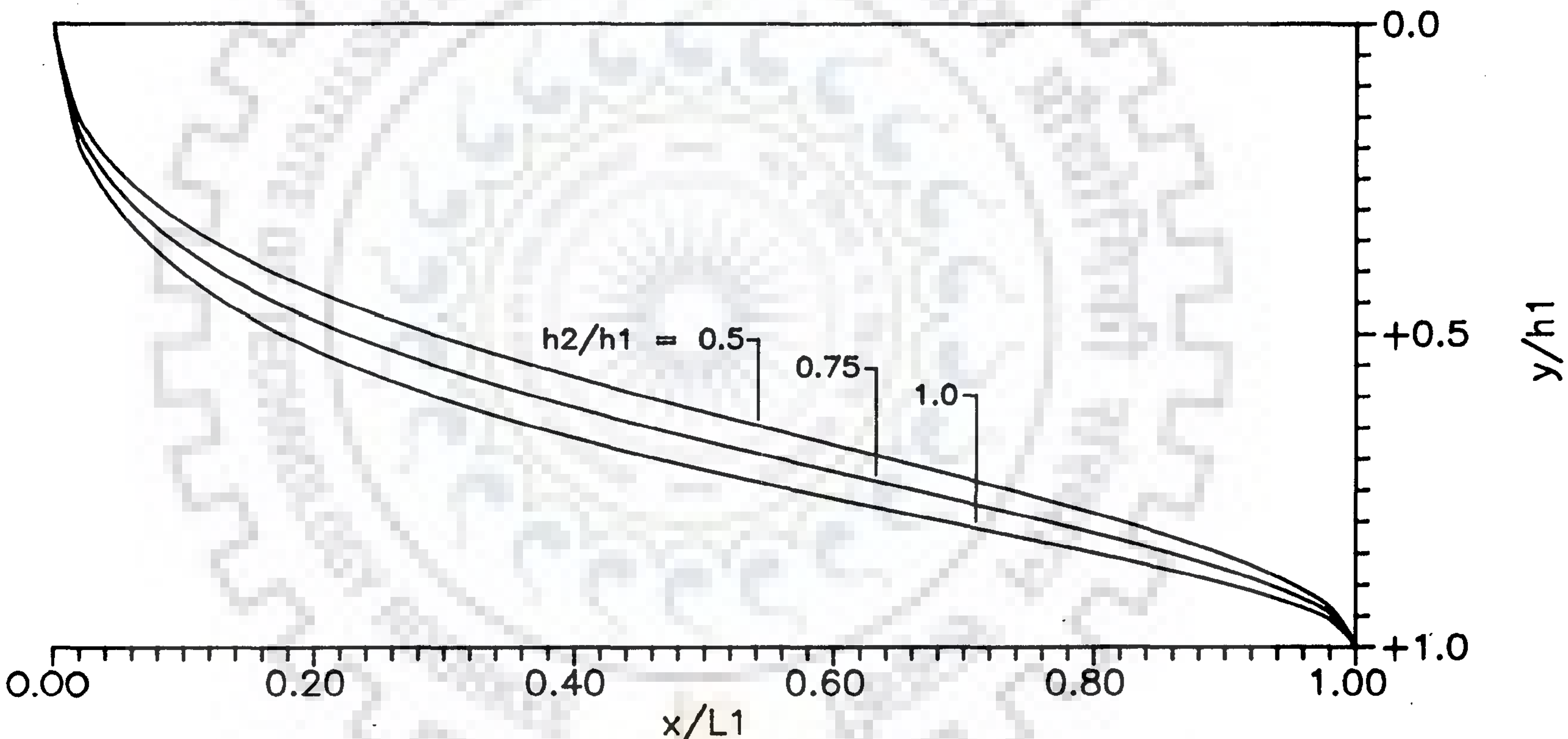


Fig.3.8(a) FREE SURFACE ON RIGHT-HAND SIDE – Effect of Level of the Left Drainage. (Bz/h1 = 10, L1 = L2 = 100h1)

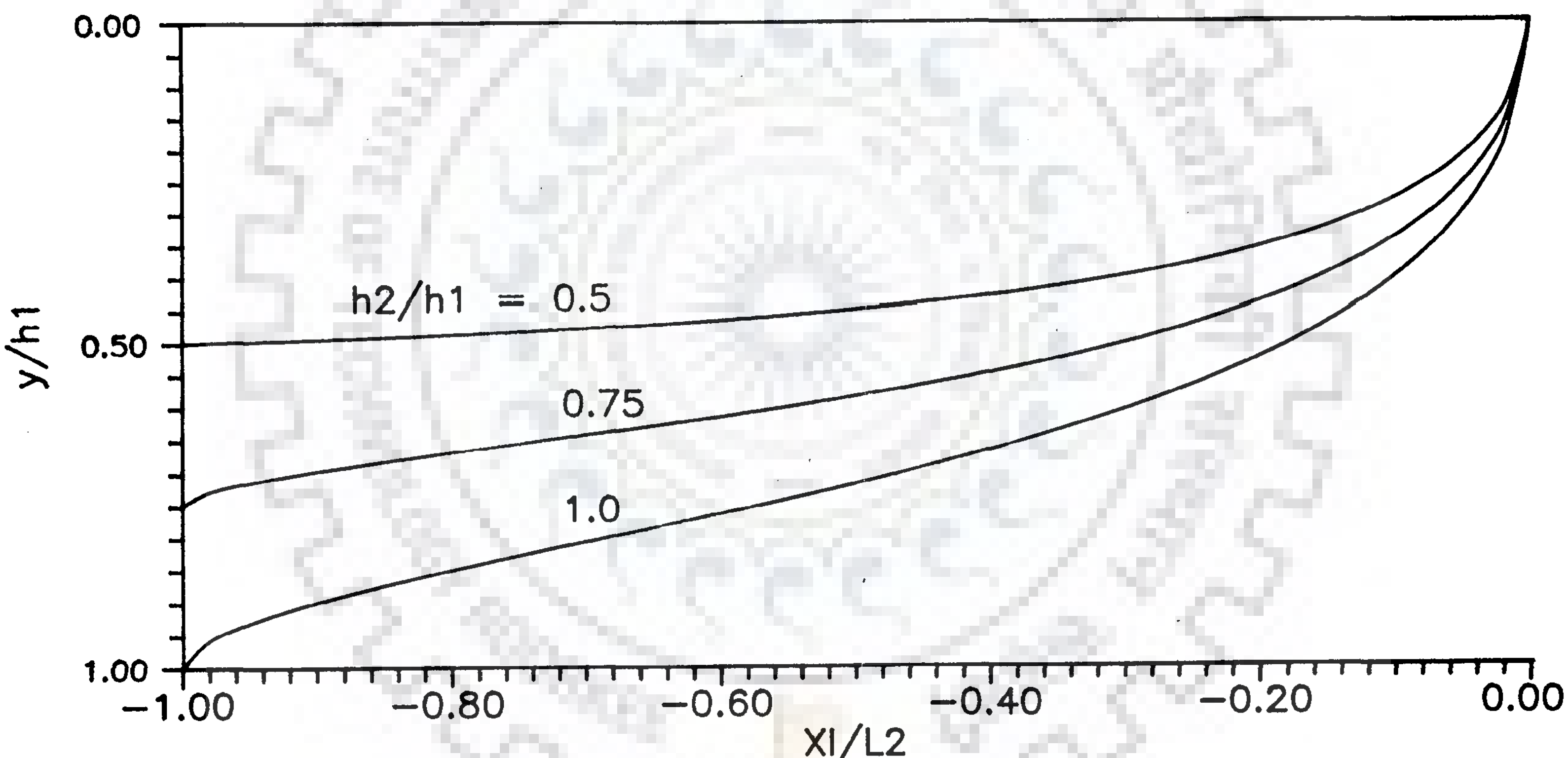


Fig.3.8(b) FREE SURFACE ON LEFT-HAND SIDE – Effect of Level of Left Drainage. (Bz/h1 = 10. L1 = L2 = 100h1)

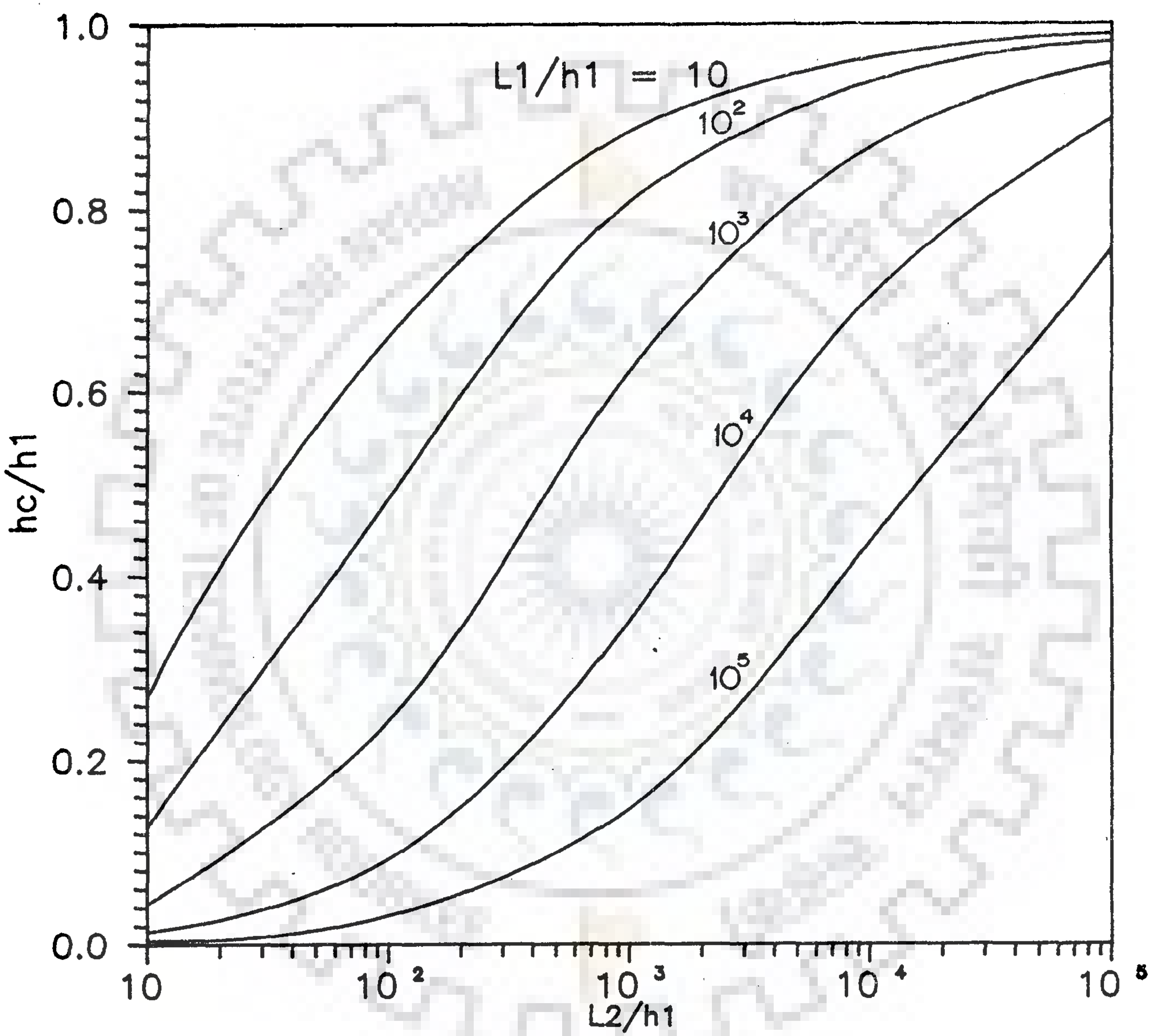


Fig.3.9 CRITICAL LEVEL OF LEFT DRAINAGE BELOW CANAL WATER LEVEL. (H = 0, Bz = 10h1)

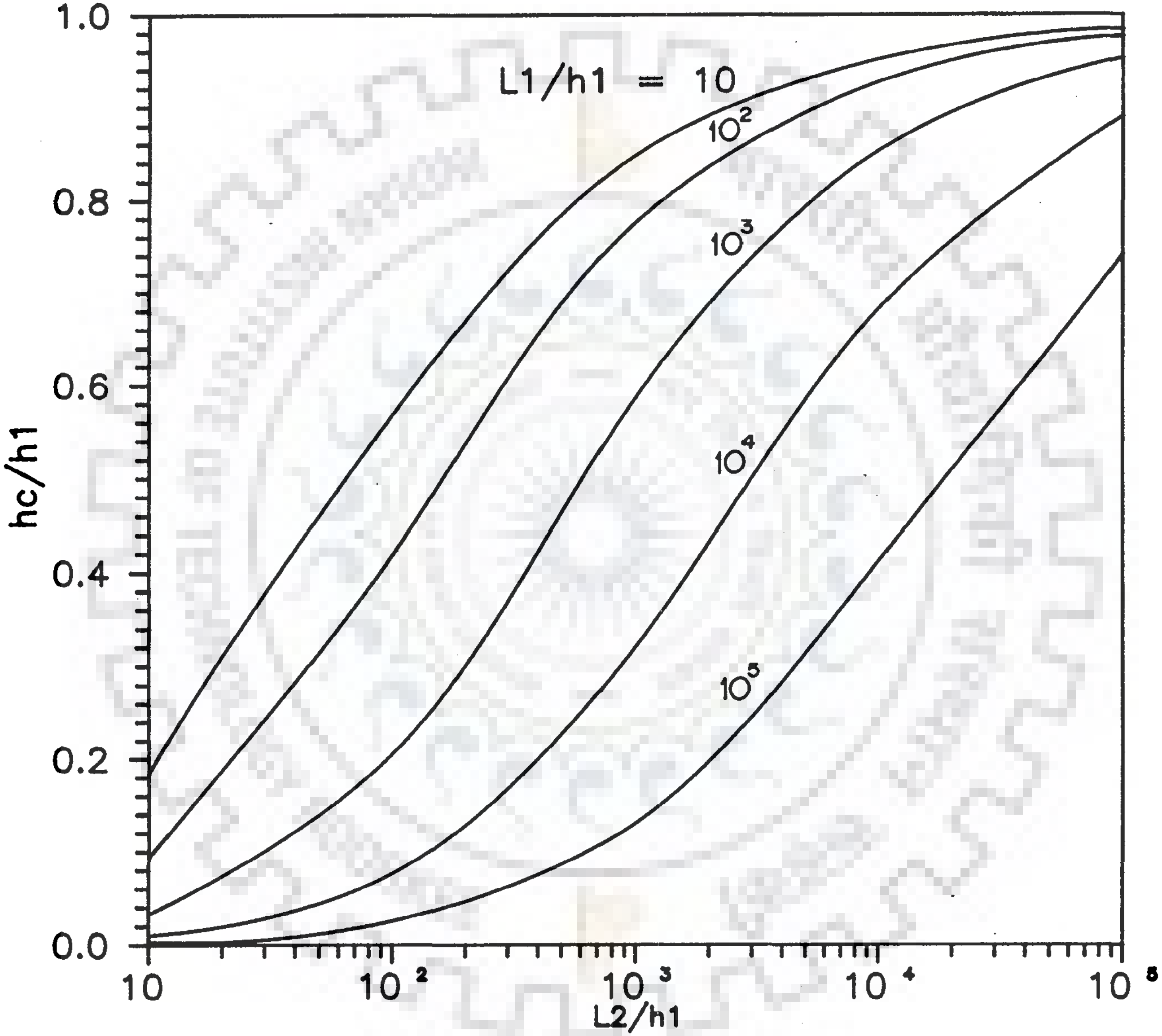


Fig.3.10 CRITICAL LEVEL OF LEFT DRAINAGE BELOW CANAL WATER LEVEL. (H = 0, Bz = 20h1)

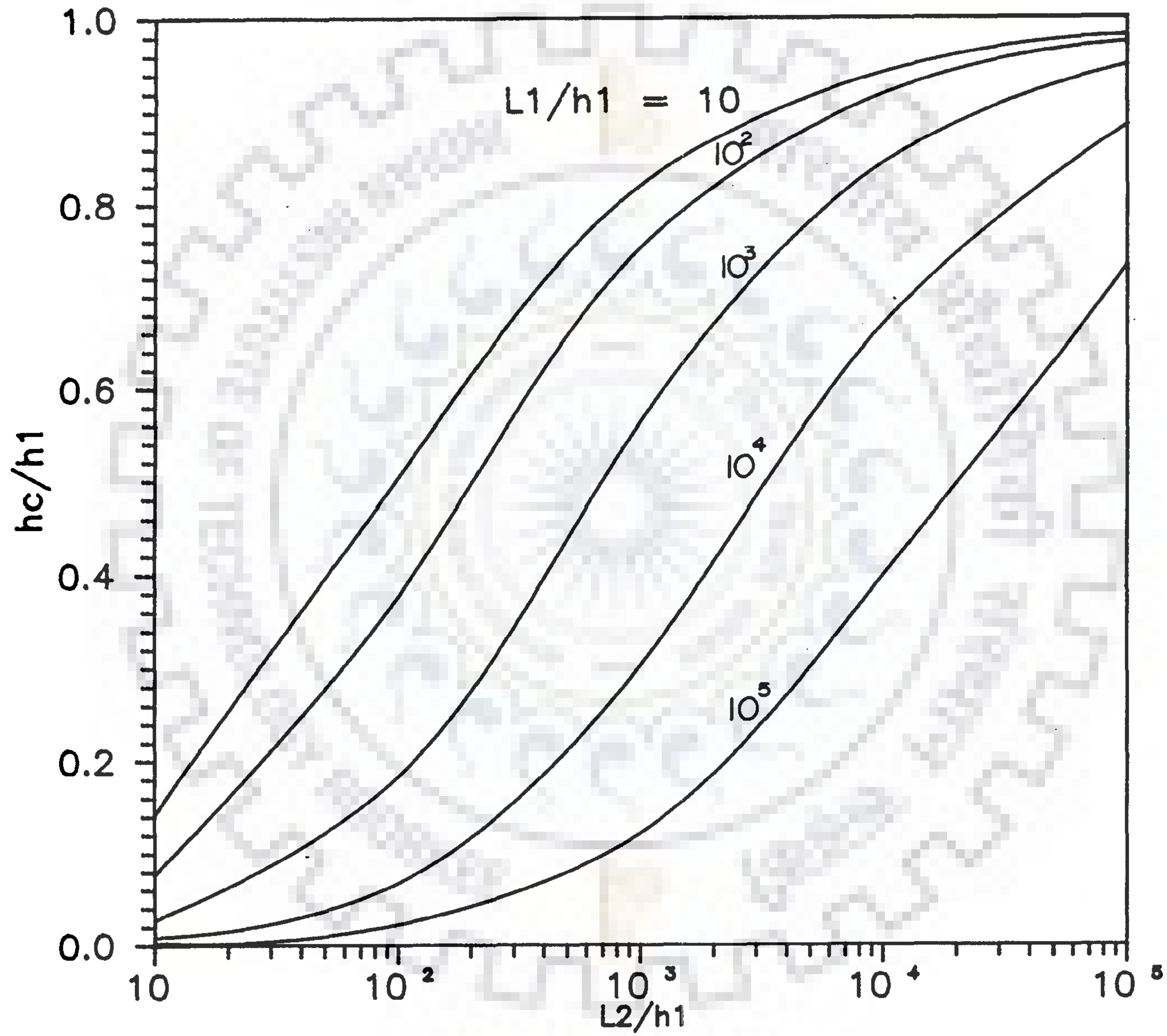


Fig.3.11 CRITICAL LEVEL OF LEFT DRAINAGE BELOW CANAL WATER LEVEL. (H = 0, Bz = 30h1)

A perusal of Figs.3.2 to 3.5 indicates that the seepage discharge decreases with increase in the values of L_1/h_1 and L_2/h_1, i.e as the drainage distance increases, the seepage discharge decreases. It is also seen that the seepage discharge increases with increase in the value of B_2/h_1. However, increase in the seepage discharge due to increase in bed width is not proportional to the increase in bed width. Therefore, the practice of expressing the seepage from canals in terms of their wetted perimeters irrespective of their size is not correct.

From Tables 3.6 to 3.9, it is observed that the components of seepage discharge from the canal towards the left and right-hand side drainages are equal when the drainages are symmetrically located, i.e. when their respective distances and elevations with reference to the canal are equal. But, when the ratios L_2/L_1 and h_2/h_1 are different from one, there is a difference between the amount of seepage discharge received by the left drainage and the right drainage. Moreover, a perusal of the Tables 3.2 to 3.9 further indicates that keeping h_2/h_1 constant, with increase in one of the drainage distances, both the total seepage loss, q/kh_1, and seepage towards the drainage, distance of which is increased, decrease but the seepage component to the other drainage increases. For B_2/h_1 = 10., L_2/h_1 = 100 and h_2/h_1 = 1.0, the total dimensionless seepage discharge, q/kh_1, decreases from 1.01537 to 0.60626 with increase in drainage distance on the right-hand side from L_1/h_1 = 10 to L_1/h_1 = 10^5 [refer Table 3.2]. Referring to Table 3.6, it is seen that out of the above total seepage discharge, the dimensionless seepage discharge towards the right side drainage

(L_1/h_1 = 10) is 0.78885 and that on the left side (L_2/h_1 = 100) is 0.22652. But, when L_1/h_1 is increased to 10^5, the dimensionless seepage component towards the right drainage decreased to 0.01251 whereas that towards the left drainage increased to 0.48116.

However, if one of the drainages, say that on the left, is at higher level than the other drainage, i.e. $h_2/h_1 < 1.$, then as the distance of the drainage which is at higher level is increased (L_2 is increased), the component of seepage discharge received by this drainage decreases and tends to zero. Beyond certain distance, the drainage which is on higher level will be ineffective [refer Tables 3.3 to 3.5].

With respect to the canal, the drainages are located at different finite horizontal distances and vertical levels. In addition to seepage from the canal, there will be seepage taking from the drainage at higher level to the drainage at lower level. As mentioned earlier, the seepage from the canal flows partly to the drainage on the left and partly towards the drainage on the right. The seepage discharge to the drainage which is at lower level and is nearer to the canal is more than that to the other drainage. As the level of the upper drainage is raised (say value of h_2 is reduced), the component of canal seepage to this drainage is reduced. At a certain value of h_2 (say $h_2 = h_c$), the canal seepage to this drainage approaches zero and the entire canal seepage water will emerge in the other drainage. If depth of the higher drainage below canal water level is further reduced ($h_2 < h_c$), this drainage continues to be ineffective [for case of single drainage refer to Chapter 5].

However, if depth of the drainage below the canal water level is lowered such that $h_2 > h_c$, the drainage becomes effective and part of the canal seepage water emerges in this drainage and the remaining canal seepage water emerges in the other drainage. The value of this critical depth (h_c) is a function of B_2/h_1, L_2/h_1 and L_1/h_1. The values of this critical depth (h_c) of the drainage below canal water level for B_2/h_1 = 10, 20 and 30 and different values of L_1/h_1 and L_2/h_1 as plotted in Fig.3.9 to 3.11, indicates that as the value of L_1/h_1 is increased, the value of h_c/h_1 decreases, i.e. the critical level of the drainage at which it becomes effective is higher. On the other hand, with increase in the value of L_2/h_1, the value of h_c/h_1 increases, i.e. the critical level of the drainage at which it becomes effective is lowered. With increase in the value of B_2/h_1, the value of h_c/h_1 decreases, i.e. the critical level of the drainage at which it becomes effective is higher.

So far the seepage discharge for an asymmetric case could be approximately determined by adding the computed seepage discharges towards drainages on either side assuming these to be equivalent to two separate symmetric cases. The values of seepage computed by following the above mentioned procedure are compared with those directly obtained from the solution presented herein (Tables 3.10 to 3.12). It is seen that for the case of the symmetrical layout of the drainages, i.e $L_1 = L_2$ and $h_1 = h_2$, the results found by the above mentioned approximate method and the exact method are identical [refer to Table 3.10 for the case of $h_2/h_1 = 1$ and $L_1 = L_2 = 50.0$]. But in the case of the asymmetric layout of the drainages, the

Table 3.10 Comparison of Seepage Quantitities as Calculated By Equations Obtained from Symetrical and Asymmetrical Considerations. [h_2/h_1 = 1.0 and L_2/h_1 = 50]

Note : seepage quantities are given in terms of kh_1.		FROM SUM OF TWO SEPARATE SYMMETRIC CASES			OBTAINED DIRECT FROM ASYMMETRIC CASE		
		SEEPAGE TO DRAIN ON			SEEPAGE TO DRAIN ON		
L_1/h_1	B_2/h_1	LEFT	RIGHT	TOTAL	LEFT	RIGHT	TOTAL
25.0	10	0.40695	0.48223	0.88918	.36040	.53409	0.89449
	20	0.48881	0.58974	1.07855	.44550	.63795	1.08345
	30	0.54760	0.66543	1.21303	.50793	.70937	1.21730
50.0	10	0.40695	0.40695	0.81390	.40695	.40695	0.81390
	20	0.48881	0.48881	0.97762	.48881	.48881	0.97762
	30	0.54760	0.54760	1.09520	.54760	.54760	1.09520
100.0	10	0.40695	0.34912	0.75607	.45812	.30292	0.76104
	20	0.48881	0.41115	0.89996	.54004	.36516	0.90520
	30	0.54760	0.45583	1.00343	.59719	.41133	1.00852
500.0	10	0.40695	0.25946	0.66641	.56461	.14304	0.70765
	20	0.48881	0.29376	0.78257	.65601	.17315	0.82916
	30	0.54760	0.31778	0.86538	.71763	.19630	0.91393

Table 3.11 Comparison of Seepage Quantitities as Calculated By Equations Obtained from Symetrical and Asymmetrical Considerations. [h_2/h_1 = 0.75 and L_2/h_1 = 50]

Note : seepage quantities are given in terms of kh_1.		FROM SUM OF TWO SEPARATE SYMMETRIC CASES			OBTAINED DIRECTLY FROM ASYMMETRIC CASE		
		SEEPAGE TO DRAIN ON			SEEPAGE TO DRAIN ON		
L_1/h_1	B_2/h_1	LEFT	RIGHT	TOTAL	LEFT	RIGHT	TOTAL
25.0	10	0.30679	0.48223	0.78902	0.10544	0.70086	0.80630
	20	0.36783	0.58974	0.95757	0.16338	0.81021	0.97359
	30	0.41177	0.66543	1.07720	0.20760	0.88403	1.09163
50.0	10	0.30679	0.40695	0.71374	0.13946	0.57455	0.71401
	20	0.36783	0.48881	0.85664	0.19574	0.66111	0.85685
	30	0.41177	0.54760	0.95937	0.23746	0.72208	0.95954
100.0	10	0.30679	0.34912	0.65591	0.17977	0.46853	0.64830
	20	0.36783	0.41115	0.77998	0.23603	0.53552	0.77156
	30	0.41177	0.45583	0.86760	0.27632	0.58408	0.86040
500.0	10	0.30679	0.25946	0.56625	0.27659	0.29169	0.56828
	20	0.36783	0.29376	0.66159	0.33829	0.32818	0.66646
	30	0.41177	0.31778	0.72955	0.38044	0.35518	0.73562

Table 3.12 Comparison of Seepage Quantitities as Calculated By Equations Obtained from Symetrical and Asymmetrical Considerations. [h_2/h_1 = 0.75 and L_1/h_1 = 50]

Note : seepage quantities are given in terms of kh_1.		FROM SUM OF TWO SEPARATE SYMMETRIC CASES			OBTAINED DIRECTLY FROM ASYMMETRIC CASE		
		SEEPAGE TO DRAIN ON			SEEPAGE TO DRAIN ON		
L_2/h_1	B_2/h_1	LEFT	RIGHT	TOTAL	LEFT	RIGHT	TOTAL
25.0	10	0.36417	0.40695	0.77109	0.23247	0.53126	0.76373
	20	0.44423	0.48881	0.93304	0.30588	0.62022	0.92610
	30	0.50072	0.54760	1.04832	0.35746	0.68432	1.04178
50.0	10	0.30679	0.40695	0.71374	0.13946	0.57455	0.71401
	20	0.36783	0.48881	0.85664	0.19574	0.66111	0.85685
	30	0.41177	0.54760	0.95937	0.23746	0.72208	0.95954
100.0	10	0.26286	0.40695	0.66981	0.06851	0.61803	0.68655
	20	0.30914	0.48881	0.79795	0.10824	0.70662	0.81487
	30	0.34255	0.54760	0.89015	0.13939	0.76714	0.90653
500.0	10	0.19502	0.40695	0.60197	-	-	-
	20	0.22062	0.48881	0.70943	0.00182	0.78445	0.78627
	30	0.23859	0.54760	0.78619	0.00931	0.85588	0.86520
1000.0	10	0.17518	0.46950	0.64468	-	-	-
	20	0.19566	0.48881	0.68447	-	-	-
	30	0.20982	0.54760	0.75742	-	-	-

results obtained by the exact and the approximate method differ significantly. The dimensionless total seepage discharge computed by the present solution is more than the value computed by the approximate method where the asymmetric condition is decomposed to two "equivalent" symmetric cases and then half from each of the results for the total seepages in the two symmetric cases are added together to give the amount of the total seepage for the original asymmetric case. For example, from Table 3.10, for the case of h_2/h_1 = 1 and L_1/h_1 = 500 and L_2/h_1 = 50, the total dimensionless seepage (q/kh_1) as calculated approximately from two equivalent symmetric cases is 0.66641, 0.78257, and 0.86538 for B_2/h_1 = 10, 20 and 30 respectively, whereas as calculated by the present direct method, the corresponding results are 0.70765, 0.82916 and 0.91393 respectively. It is observed that the percentage difference between the results as obtained by the two methods decreases with increase in B_2/h_1 and increases with increase in the value of L_1/h_1. Moreover, from Tables 3.10 to 3.12, it is seen that the differences in the results obtained by the two methods are more marked when the corresponding quantity of seepage components received by the left and the right-hand drainages are separately compared. From Table 3.10, for the case of h_1 = h_2, the approximate method to the solution for asymmetric cases underestimates the seepage components received by the nearer drainage whereas it overestimates the seepage components received by the farther drainage.

As the influence of the different elevations of the drainages (for h_2/h_1 different from one) and the distances of the drainages (for L_2/L_1 different from one) is not considered in the development of the solutions for asymmetric case, the results as calculated by this method showed big differences in the calculation of the total seepage discharge or the quantities of seepage components received by the left and the right-hand drainages as compared to those obtained by the solution of asymmetric case. The comparison of results as shown in Table 3.10 for h_2/h_1 = 1 and Tables 3.11 and 3.12 for h_2/h_1 = 0.75 highlights the above mentioned differences. Thus, estimating seepage losses from canal towards asymmetric drainages by decomposing into two symmetric cases is not correct.

Dimensionless coordinates of the free surface on the right-hand side of the canal for B_2/h_1 = 10, L_2/h_1 = 50, h_1 = h_2 and L_1/h_1 = 25, 50 and 100 are plotted in Fig.3.6(a). This is to show the effect of the distance of the right-hand drainage on the right-hand side free surface. The free surface on the right-hand side becomes higher as the value of L_1/h_1 increases, i.e with increase in drainage distance the free surface is raised. Fixing the value of L_1/h_1 = 50 ; h_1 = h_2 ; B_2/h_1 = 10 and varying the left drainage distance, i.e. varying L_2, the free surface on the side of the drainage, distance of which is fixed, is raised [Fig.3.6(b)]. To see the effect of the canal dimension, i.e bed width B_2, the free surface shapes are plotted in Fig.3.7 for h_2 = h_1 ; L_1 = L_2 = $50h_1$ and B_2/h_1 = 10, 20 and 30. All other things kept constant, the free surface rises with increase in the bed width. From Fig.3.7, it is observed that the rise in the free surface as B_2 is increased from $10h_1$ to $20h_1$ is more than the corresponding rise when B_2 is increased from $20h_1$

to $30h_1$. Free surface variation with respect to h_2/h_1 is plotted in Fig.3.8(a) and 3.8(b). Keeping all other things constant, if the level of one of the drainages is varied, it has effect on the free surfaces on either side of the canal. For example, for fixed h_1, and for $B_2 = 10h_1$ and $L_1 = L_2 = 100h_1$, if the left-hand drainage is lowered, i.e. the value of h_2 is increased from $0.5h_1$ to $0.75h_1$ and then to $1.0h_1$, the free surface on the right-hand side is lowered as shown in Fig.3.8(a) whereas the effect of lowering of left-hand drainage on the left-hand side free surface is shown in Fig.3.8(b).

CHAPTER 4

SEEPAGE FROM TRAPEZOIDAL CANAL
TO
ASYMMETRIC DRAINAGES

Introduction

Seepage losses from unlined canals depend on the shape and size of the canal cross section, location of drainages on either side of the canal and the subsoil properties. In Chapter 3, the shape of the canal and the depth of the water in the canal were not considered. In this chapter, exact solution of the problem of seepage from a trapezoidal canal in homogeneous medium to asymmetric drainages located at finite distances is presented. Hence, the shape of the canal and depth of water in the canal have been considered in this analysis.

4.1 Formulation of Problem

To derive the Laplace equation (Eq.A.1) and to make the problem amenable to analytical solution, the few assumptions that had been made in Chapter 3 are applicable in this work too. The assumptions are : (i) the porous medium is homogeneous and isotropic extending up to large depth; (ii) capillary and surface tension effects are negligible; (iii) the flow is laminar and therefore follows Darcy's law, and (iv) the drainages are wide.

Zhukovsky's function which is defined in Eq.A.15 is also applied in this problem. Hence, after applying Eqs.3.1, 3.2, 3.3 and 3.4, the following statements could be made. Along the free surface, the pressure is atmospheric and from Eqs.3.4 and 3.3, $\theta_2 = 0$. The curved free surface in the z-plane [Fig.4.1(a)] is therefore represented by a straight line in the θ-plane.

The θ-plane [Fig.4.1(b)] was mapped onto the lower half of the ζ-plane [Fig.4.1(c)], where $\zeta = \xi + i\eta$, and the w-plane [Fig.4.1(d)] was mapped onto the lower half of the t-plane, where $t = r + is$ as shown in Fig.4.1(e). Table 4.1 summarizes the values of the corresponding points in the different planes. The relationship between the ζ-plane and the t-plane was obtained through the bilinear transformation . Hence,

$$\theta = f_1(\zeta) \qquad (4.1a)$$

in which,

$$\theta = z - iw/k \qquad (4.1b)$$

$$w = f_2(t) \qquad (4.2a)$$

$$t = f_3(\zeta) \qquad (4.2b)$$

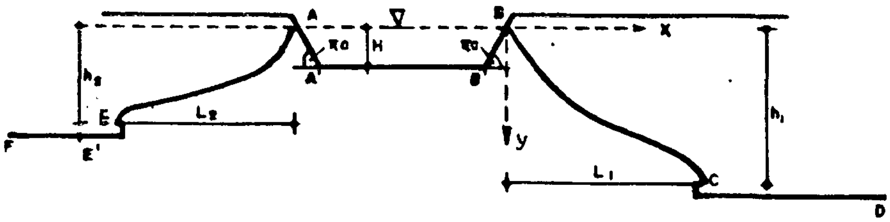
On combining Eqs.4.1(a), 4.1(b), 4.2(b) and 4.2(a), the following relationships were obtained.

$$z = iw/k + f_1(\zeta) \qquad (4.3a)$$

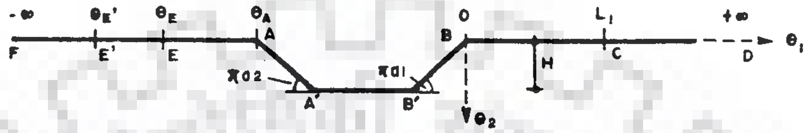
$$w = f_2\,[f_3(\zeta)]$$

$$= f_4(\zeta) \qquad (4.3b)$$

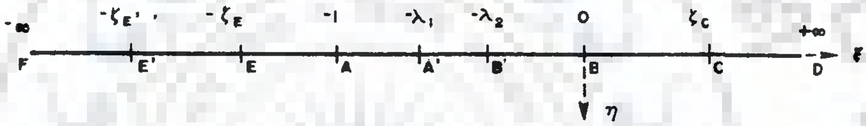
$$z = (i/k)[f_4(\zeta)] + F_1(\zeta) \qquad (4.3c)$$



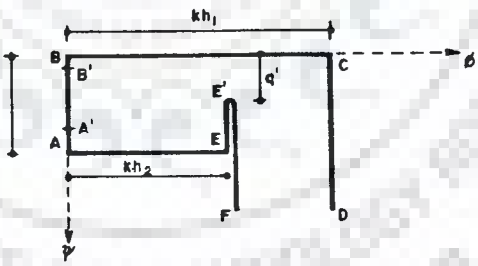
(a) z-plane



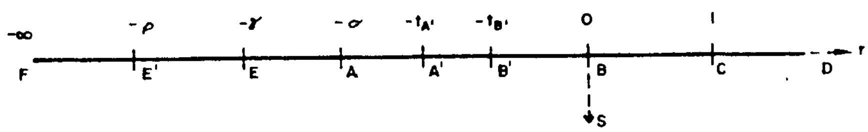
(b) Θ-plane



(c) ζ-plane



(d) w-plane



(e) t-plane

FIG. 4.1 TRANSFORMATION LAYOUT

4.2 Boundary Conditions

In the z-plane [Fig.4.1(a)], AA'B'B is an equipotential line and corresponds to ϕ = constant. This constant is assumed to be zero. Therefore, ϕ is zero along this line. Along the phreatic line BC which is a stream line, the value of ψ is taken to be zero. For the phreatic line AE, ψ has been assigned a value equal to q, where q is the unknown seepage loss per unit length of the canal. Along the drainage CD, which is an equipotential line, ϕ is equal to kh_1, in which h_1 is the difference between water levels of the canal and the right-hand drainage. The left-hand drainage EE'F is an equipotential line , $\phi = kh_2$, in which h_2 is the difference between the water levels of the canal and the left-hand drainage. The part EE' of the left-hand drainage which is at a higher level than the right-hand drainage receives fraction of the seepage from the canal. Due to the difference in the elevations of the left and the right drainages, there will be seepage from the left-hand drainage to the right-hand drainage. This seepage will take place from E'F to some portion of the right drainage. The location of the point E' will depend on the relative values of B_2, H, πa, h_1, h_2, L_1 and L_2, in which L_1 and L_2 are the distances of the drainages on the right and the left side of the canal respectively ; B_2 is bed width, πa is the side slope angle and H is the maximum depth of water in the canal.

4.3 Solution of the Problem

It was found convenient to obtain the solution of the problem through two separate operations by introducing two auxiliary semi-infinite planes. In the first operation, applying the Zhukovsky function [Eq.A.15], the physical plane is transformed onto the θ-plane, which in turn is mapped to an intermediate semi-infinite ζ-plane through the use of the Schwarz-Christoffel conformal mapping [Eq.A.2]. In the second operation, the w-plane [Fig.4.1(d)] is transformed onto the intermediate semi-infinite t-plane using the Schwarz-Christoffel transformation. The relationship between the ζ-plane and the t-plane was obtained using the bilinear transformation.

The equations so obtained were integrated on different regions of the boundary to obtain relationships between physical parameters and seepage losses and the coordinates of the phreatic lines. The improper integrals appearing in the transformation procedure have been converted into proper integrals by method of substitution. The proper integrals are then evaluated using Gaussian quadrature formula [Abramwitz and Stegun, 1970; Davis and Rabinowitz, 1975; Stoer and Bulirsch, 1980].

4.3.1 Mapping of the z - plane onto the θ - plane.

In case section of the canal is trapezoidal in z-plane, the cross section of the canal in θ-plane will not be exactly trapezoidal. However, for this analysis, the cross section of the canal has been assumed to be trapezoidal in θ-plane with side slope angles of πa_1 and πa_2 on the right and left sides, respectively. The corresponding section in the z-plane is approximated by a nearly trapezoidal section in z-plane with side slope angle of πa on either side [Fig.4.1(a)]. The variation in canal section from trapezoidal slope is very small. In practice unlined canals cross section are seldom exactly trapezoidal [Garg and Chawla, 1970 and 1971].

Using the Zhukovsky transformation (A.15) and the geometry as given in Fig.4.1(b), at point B , $\theta = 0 = \theta_B$ and at Point B',

$$\begin{aligned} \theta_{B'} &= z - iw/k \\ &= (x + iy) - i(\phi + i\psi) \\ &= (-H\cot(\pi a) + iH) - i(i\psi_{B'})/k \\ &= (-H\cot(\pi a) + \psi_{B'}/k) + iH \end{aligned} \tag{4.4}$$

Hence, from Eq.4.4 and Fig.4.1(b), in the θ-plane, vertical distance BB' is H and the horizontal distance BB' = [$H\cot(\pi a) - \psi_{B'}/k$]. Hence, from geometry [Fig.4.1(b)],

$$\begin{aligned} \cot(\pi a_1) &= [\, H\cot(\pi a) - \psi_{B'}/k \,]/H \\ &= \cot(\pi a) - \psi_{B'}/kH \end{aligned} \tag{4.5}$$

where, $\psi_{B'}$ is the stream function value at B'.

Similarly, at point A' (using Eq.3.1),

$$\begin{aligned} \theta_{A'} &= [(-H\cot(\pi a) - B_z) + iH] - i(0 + i\psi_{A'})/k \\ &= (-H\cot(\pi a) - B_z + \psi_{A'}/k) + iH \end{aligned} \tag{4.6}$$

where, B_z is the bottom width of the trapezoidal canal in the z-plane. Also at point A,

$$\begin{aligned} \theta_A &= [-2H\cot(\pi a) - B_z\,] - i(0 + iq)/k \\ &= -2H\cot(\pi a) - B_z + q/k \end{aligned} \tag{4.7}$$

Hence, the vertical distance AA' [Fig.4.1(b)] is H and the horizontal distance AA' = [$H\cot(\pi a) + (\psi_{A'} - q)/k$]. Again, from geometry [Fig.4.1(b)],

$$\cot(\pi a_2) = \cot(\pi a) - (q - \psi_{A'})/(kH) \tag{4.8}$$

where, $\psi_{A'}$ is the stream function value at A'.

The values at the different points in the z-plane and θ-plane are summarized in Table 4.1(a).

4.3.2 Mapping of the θ - plane onto the ζ - plane.

Mapping of the θ-plane onto the lower half of ζ-plane through the Schwarz-Christoffel transformation is as follows.

$$\int d\theta = M_2 \int \frac{d\zeta}{(\zeta+1)^{a_2}(\zeta+\lambda_1)^{-a_2}\quad(\zeta+\lambda_2)^{-a_1}\ \zeta^{a_1}}$$

$$= M_2 \int \left[\frac{\zeta+\lambda_2}{\zeta}\right]^{a_1} \left[\frac{\zeta+\lambda_1}{\zeta+1}\right]^{a_2} d\zeta \qquad (4.9)$$

Integrating Eq.4.9 in the region BC, i.e.,in the the region $0 \le \zeta \le \zeta_c$ and $0 \le \theta \le L_1$, where θ = x [refer Eq.A.15],

$$\int_0^x d\theta = M_2 \int_0^{\zeta} \left[\frac{\zeta + \lambda_2}{\zeta}\right]^{a_1} \left[\frac{\zeta + \lambda_1}{\zeta + 1}\right]^{a_2} d\zeta$$

$$x = M_2 \int_0^{\zeta} \left[\frac{\zeta + \lambda_2}{\zeta}\right]^{a_1} \left[\frac{\zeta + \lambda_1}{\zeta + 1}\right]^{a_2} d\zeta \qquad (4.10)$$

where, $0 \le x \le L_1$.

Substituting,

$$\zeta = \sinh^2(u) \qquad (4.11)$$

$$d\zeta = 2\sinh(u)\cosh(u)du \qquad (4.12)$$

from Eq.4.11, the lower limit and the upper limit of the integral of the right-hand side of Eq.4.10 were found to be $u = 0$ and $u = \sinh^{-1}\sqrt{\zeta}$ respectively. Hence, after making the

Table 4.1(a)

Values of Corresponding Points in the Three Planes.
[z, θ and w planes]

POINT	z-plane		θ-plane		w-plane	
	x	y	θ_1	θ_2	ϕ	ψ
A	$-2H\cot(\pi a)-B_2$	0	$-2H\cot(\pi a)-B_2+q/k$ or $-H\cot(\pi a_1)-H\cot(\pi a_2)-b'$	0	0	q
A'	$-H\cot(\pi a)-B_2$	0	$-H\cot(\pi a)-B_2+\psi_{A'}/k$ or $-H\cot(\pi a_1)-b'$	0	0	$\psi_{A'}$
B'	$-H\cot(\pi a)-B_2$	0	$-H\cot(\pi a)+\psi_{B'}/k$ or $-H\cot(\pi a_1)$	0	0	$\psi_{B'}$
B	0	0	0	0	0	0
C	L_1	h_1	L_1	0	kh_1	0
D	∞	h_1	∞	0	kh_1	∞
E	$-2H\cot(\pi a)-B_2-L_2$	h_2	$-2H\cot(\pi a)-B_2-L_2+q/k$	0	kh_2	q
E'	$-2H\cot(\pi a)-B_2-L_2-L_3$	h_2	$-2H\cot(\pi a)-B_2-L_2-L_3+\frac{q'}{k}$	0	kh_2	q'
F	$-\infty$	h_2	$-\infty$	0	kh_2	∞

Table 4.1(b)

Values of Corresponding Points
in the ζ and t-Planes.

POINT	ζ-plane		t-plane	
	ξ	η	r	s
A	-1	0	$-\sigma$	0
A'	$-\lambda_1$	0	$-t_{A'}$	0
B'	$-\lambda_2$	0	$-t_{B'}$	0
B	0	0	0	0
C	ζ_C	0	1	0
D	∞	0	∞	0
E	$-\zeta_E$	0	$-\delta$	0
E'	$-\zeta_{E'}$	0	$-\rho$	0
F	$-\infty$	0	$-\infty$	0

appropriate substitutions [Eq.4.11 and 4.12] and rearranging,

$$\frac{x}{2} = M_2 \int_0^{\sinh^{-1}\sqrt{\zeta}} \left[\frac{\sinh^2 u + \lambda_2}{\sinh^2 u}\right]^{a_1} \left[\frac{\sinh^2 u + \lambda_1}{\sinh^2 + 1}\right]^{a_2} \sinh(u)\cosh(u)\, du \qquad \ldots (4.13)$$

$$= M_2 \int_0^{\sinh^{-1}\sqrt{\zeta}} [\sinh^2 u + \lambda_2]^{a_1} [\sinh(u)]^{1-2a_1} [\sinh^2(u) + \lambda_1]^{a_2} [\cosh(u)]^{1-2a_2} du \qquad \ldots (4.14)$$

At Point C, $x = L_1$ and $\zeta = \zeta_c$. Sustituting these results in Eq.4.14,

$$L_1 = 2(M_2)(I_1) \qquad (4.15a)$$

where,

$$I_1 = \int_0^{\sinh^{-1}\sqrt{\zeta_c}} [\sinh^2 u + \lambda_2]^{a_1} [\sinh(u)]^{1-2a_1} [\sinh^2(u) + \lambda_1]^{a_2} [\cosh(u)]^{1-2a_2}\, du \qquad \ldots (4.15b)$$

Integrating Eq.4.9 in the region B'B, i.e, in the region $-\lambda_2 \le \zeta \le 0$ and $\theta_B' \le \theta \le \theta_B$,

$$\int_{\theta_B'}^{\theta_B} d\theta = M_2 \int_{-\lambda_2}^{0} \left[\frac{\zeta + \lambda_2}{\zeta}\right]^{a_1} \left[\frac{\zeta + \lambda_1}{\zeta + 1}\right]^{a_2} d\zeta$$

$$= M_2 \int_{-\lambda_2}^{0} (-1)^{a_1} \left[\frac{\zeta + \lambda_2}{-\zeta}\right]^{a_1} \left[\frac{\zeta + \lambda_1}{\zeta + 1}\right]^{a_2} d\zeta \quad \ldots (4.16)$$

where, θ_B and θ_B' are the values in the θ-plane at the points B and B' respectively.

From complex number theory [Churchill, 1948],

$$(-1)^{a_1} = [\cos(\pi) - i \sin(\pi)]^{a_1} = e^{-i\pi a_1} \quad (4.17)$$

Also, integrating the left-hand side of Eq.4.16 and applying Eq.4.17,

$$\int_{\theta_B'}^{\theta_B} d\theta = \theta_B - \theta_B'$$

$$= 0 - [-H\cot(\pi a_1) + iH]$$

$$= H[\cos(\pi a_1) - i \sin(\pi a_1)]/\sin(\pi a_1)$$

$$= He^{-i\pi a_1}/\sin(\pi a_1) \quad (4.18)$$

Substituting results of Eq.4.17 and 4.18 in Eq.4.16,

$$\frac{H}{\sin(\pi a_1)} = M_2 \int_{-\lambda_2}^{0} \left[\frac{\zeta + \lambda_2}{-\zeta}\right]^{a_1} \left[\frac{\zeta + \lambda_1}{\zeta + 1}\right]^{a_2} d\zeta \quad \ldots (4.19)$$

Substituting,

$$\zeta = -\lambda_2 \cos^2 u \quad (4.20a)$$

$$d\zeta = 2\lambda_2 \cos(u) \sin(u)\, du \quad (4.20b)$$

in Eq.4.19, and putting the appropriate limits of integration,

$$\frac{H}{2\lambda_2 \sin(\pi a_1)} = M_2 \int_0^{\pi/2} \left[\frac{\sin^2 u}{\cos^2 u}\right]^{a_1} \left[\frac{\lambda_1 - \lambda_2 \cos^2 u}{1 - \lambda_2 \cos^2 u}\right]^{a_2} \cos(u)\sin(u)du$$

$$= M_2 \int_0^{\pi/2} \left[\sin(u)\right]^{1+2a_1} \left[\cos(u)\right]^{1-2a_1} \left[\frac{\lambda_1 - \lambda_2 \cos^2 u}{1 - \lambda_2 \cos^2 u}\right]^{a_2} du \qquad ...(4.21a)$$

$$H = 2\lambda_2 [\sin(\pi a_1)][M_2][I_2] \qquad (4.21b)$$

where,

$$I_2 = \int_0^{\pi/2} \left[\sin(u)\right]^{1+2a_1} \left[\cos(u)\right]^{1-2a_1} \left[\frac{\lambda_1 - \lambda_2 \cos^2 u}{1 - \lambda_2 \cos^2 u}\right]^{a_2} du \qquad ...(4.21c)$$

Similarly, integrating Eq.4.9 in the region AA', i.e., in the region $-1 \le \zeta \le -\lambda_1$ and $\theta_A \le \theta \le \theta_{A'}$, where θ_A and $\theta_{A'}$ are the values in the θ-plane corresponding to the Points A and A', respectively, the following relationships were obtained.

$$H = 2 [\sin(\pi a_2)] [M_2] [I_3] \qquad (4.21d)$$

where,

$$I_3 = \int_0^{\cos^{-1}\sqrt{\lambda_1}} [\cos^2 u - \lambda_2]^{a_1} [\cos(u)]^{1-2a_1} [\cos^2 u - \lambda_1]^{a_2} [\sin(u)]^{1-2a_2} du \qquad ...(4.21e)$$

Integrating Eq.4.9 in the region A'B', i.e., in the region $-\lambda_1 \le \zeta \le -\lambda_2$ and $\theta_{A'} \le \theta \le \theta_{B'}$,

$$\int_{\theta_A'}^{\theta_B'} d\theta = M_2 \int_{-\lambda_1}^{-\lambda_2} \left[\frac{\zeta + \lambda_2}{\zeta}\right]^{a_1} \left[\frac{\zeta + \lambda_1}{\zeta + 1}\right]^{a_2} d\zeta \qquad (4.22)$$

where,

$$\int_{\theta_A'}^{\theta_B'} d\theta = \theta_B' - \theta_A'$$

$$= [-H\cot(\pi a_1) + iH\ ] - [-H\cot(\pi a_1) - b' + iH\ ]$$

$$= b' \qquad (4.23)$$

where, b' is the bottom width of the canal in θ-plane.

Substituting Eq.4.23 in Eq.4.22,

$$b' = M_2 \int_{-\lambda_1}^{-\lambda_2} \left[\frac{\zeta + \lambda_2}{\zeta}\right]^{a_1} \left[\frac{\zeta + \lambda_1}{\zeta + 1}\right]^{a_2} d\zeta$$

$$= [M_2]\ [I_4] \qquad (4.24a)$$

where,

$$I_4 = \int_{-\lambda_1}^{-\lambda_2} \left[\frac{\zeta + \lambda_2}{\zeta}\right]^{a_1} \left[\frac{\zeta + \lambda_1}{\zeta + 1}\right]^{a_2} d\zeta \qquad (4.24b)$$

Subtracting Eq.4.4 from Eq.4.6,

$$\theta_B' - \theta_A' = B_z + (\psi_B' - \psi_A')/k \qquad (4.24c)$$

From Eq.4.23, $(\theta_B' - \theta_A') = b'$, hence Eq.4.24(c) becomes,

$$B_z = b' + (\psi_A' - \psi_B')/k \qquad (4.24d)$$

Substituting Eq.4.24(a) in Eq.4.24(d),

$$B_E = [M_2]\,[I_4] + (\,\psi_A' - \psi_B'\,)/k \qquad (4.24d)$$

Integrating Eq.4.9 in the region EA, i.e, in the region $-\zeta_E \le \zeta \le -1$ and $\theta_E \le \theta \le \theta_A$,

$$\int_{\theta}^{\theta_A} d\theta = M_2 \int_{\zeta}^{-1} \left[\frac{\zeta + \lambda_2}{\zeta}\right]^{a_1} \left[\frac{\zeta + \lambda_1}{\zeta + 1}\right]^{a_2} d\zeta \qquad (4.25)$$

But, from Eq.A.15 it is known that on the free surface AE,

$$\theta = q/k + x$$

$$= q/k + [-2H\cot(\pi a) - B_E - X') \qquad (4.26)$$

where, x is the abscissa of the point on the free surface AE, and X' is the horizontal distance of the point from Point A.

Integrating the left hand side of Eq.4.25 and substituting the results given by Eq.4.7 and 4.26,

$$\int_{\theta_E}^{\theta_A} d\theta = \theta_A - \theta = X' \qquad (4.27)$$

Putting ,

$$\zeta = -\cosh^2 u \qquad (4.28a)$$

$$d\zeta = -2\cosh(u)\,\sinh(u)\,du \qquad (4.28b)$$

Substituting the above, i.e. Eqs.4.27 and 4.28 and putting the appropriate limits of integration in Eq.4.25,

$$\frac{X'}{2} = M_2 \int_0^{\cosh^{-1}\sqrt{-\zeta}} [\cosh^2 u - \lambda_2]^{a_1} [\cosh(u)]^{1-2a_1} [\cosh^2(u) - \lambda_1]^{a_2} [\sinh(u)]^{1-2a_2} du$$

. . .(4.29a)

At Point E, $X' = L_2$ and $\zeta = -\zeta_E$ and hence, substituting these in Eq.4.29(a),

$$L_2 = 2\ [M_2]\ [I_5] \qquad (4.29b)$$

where,

$$I_5 = \int_0^{\cosh^{-1}\sqrt{\zeta_E}} \left[\cosh^2 u - \lambda_2\right]^{a_1} \left[\cosh(u)\right]^{1-2a_1} \left[\cosh^2(u) - \lambda_1\right]^{a_2} \left[\sinh(u)\right]^{1-2a_2} du$$

. . .(4.29c)

Integrating Eq.4.9 in the region E'E, i.e., in the region $-\zeta_{E'} \leq \zeta \leq -\zeta_E$ and $\theta_{E'} \leq \theta \leq \theta_E$, where $\theta_{E'}$ and θ_E are the values in the θ-plane at the points E' and E, respectively,

$$\int_{\theta_{E'}}^{\theta_E} d\theta = M_2 \int_{-\zeta_{E'}}^{-\zeta_E} \left[\frac{\zeta + \lambda_2}{\zeta}\right]^{a_1} \left[\frac{\zeta + \lambda_1}{\zeta + 1}\right]^{a_2} d\zeta \qquad (4.30)$$

But, using Eq.A.15,

$$\theta_E = [-2H\cot(\pi a) - B_2 - L_2 + ih_2] - i(kh_2 + iq)/k$$

. . .(4.31a)

$$\theta_{E'} = [-2H\cot(\pi a) - B_2 - L_2 - L_3 + ih_2] - i(kh_2 + iq')/k$$

. . .(4.31b)

Integrating the left-hand side of Eq.4.30 and substituting Eqs.4.31a and 4.31b,

$$\int_{\theta_E'}^{\theta_E} d\theta = \theta_E - \theta_E'$$

$$= L_s + (q - q')/k \tag{4.32}$$

Substituting Eq.4.32 in Eq.4.30 and rearranging,

$$L_s = [\ M_2\ ]\ [\ I_\sigma\ ] - [\ (q-q')/k\ ] \tag{4.33a}$$

where,

$$I_\sigma = \int_{-\zeta_E'}^{-\zeta_E} \left[\frac{\zeta + \lambda_2}{\zeta}\right]^{a_1} \left[\frac{\zeta + \lambda_1}{\zeta + 1}\right]^{a_2} d\zeta \tag{4.33b}$$

4.3.3 Mapping of the w - plane onto t - plane.

The Schwarz-Christoffel transformation that maps the w-plane onto the lower half of the t-plane is given as below (this is similar to that obtained in Chapter 3 except for suitable modifications wherever necessary).

$$\int dw = M \int \frac{(t+\rho)dt}{\sqrt{(1-t)t(t+\sigma)(t+\gamma)}} \tag{4.34}$$

where, M is a complex constant.

The integration of Eq.4.34 is made between limits as set by the region over which the integration is made. The definite integral of the right-hand side of Eq.4.34 results in elliptic integrals [Byrd and Friedman, 1971] as shown below.

Integrating Eq.4.34, in the region BC, i.e., in the range $1 \geq t > 0 > -\sigma > -\gamma$, [Byrd and Friedman, 1971, section 256.11 and 256.00],

$$\int_{W_B}^{W} dw = M \int_0^t \frac{(t+\rho)dt}{\sqrt{(1-t)t(t+\sigma)(t+\gamma)}}$$

$$= Mg \left[\sigma \Pi(\beta_r, \alpha_1^2, m) + (\rho-\sigma)F(\beta_r, m) \right] \quad \ldots(4.35)$$

where, g is a real constant and

$\Pi(\beta_r, \alpha_1^2, m)$ = elliptic integral of the third kind with parameters β_r, α_1 and modulus m.

$F(\beta_r, m)$ = elliptic integral of the first kind.

where,

$$\beta_r = \sin^{-1}\sqrt{(1+\sigma)t/(t+\sigma)} \quad (4.36)$$

$$m^2 = (\gamma-\sigma)/\{\gamma(1+\sigma)\} \quad (4.37)$$

$$m^2 < \alpha_1^2 = 1/(1+\sigma) < 1 \quad (4.38)$$

The complex potential values of the Point B and any point on the free surface are $W_B = 0$ and $W = ky$ respectively, where y is the ordinate of the point considered on the free surface BC. Integrating the left-hand side of Eq.4.35 and substituting the values of W_B and W,

$$ky = Mg \left[\sigma \Pi(\beta_r, \alpha_1^2, m) + (\rho-\sigma)F(\beta_r, m) \right] \quad \ldots(4.39)$$

At Point C, $y = h_1$ and $t = 1$. Therefore, from Eq.4.36 and Eq.4.39, the following relationships were obtained.

$$\beta_r = \pi/2 \tag{4.40}$$

$$kh_1 = Mg\,[\,\sigma\,\Pi_1 + (\rho - \sigma)K\,] \tag{4.41}$$

where,

$$\Pi_1 = \Pi(\pi/2, \alpha_1^2, m)$$

$$K = F(\pi/2, m)$$

Similarly, integrating Eq.4.34 in the region AB, i.e., $1 > 0 \geq t > -\sigma > -\gamma$, [Byrd and Friedman, 1971, section 254.10 and 254.00],

$$\int_{W_A}^{W} dw = \frac{Mg}{i}\,[\,(\gamma-\sigma)\,\Pi(\beta_v, \alpha_2^2, m') + (\rho-\gamma)F(\beta_v, m')\,] \tag{4.42}$$

where,

$$\beta_v = \sin^{-1}\sqrt{[\gamma(t+\sigma)]/[\sigma(t+\gamma)]} \tag{4.43a}$$

$$m'^2 = \sigma(1+\gamma)/\{\gamma(1+\sigma)\} = 1 - m^2 \tag{4.43b}$$

$$0 < \alpha_2^2 = \sigma/\gamma < m'^2 \tag{4.43c}$$

The value of the complex potential at Point A is $W_A = iq$ and at any point on the equipotential line AB, the value of the complex potential is, $W = i\psi^*$, where ψ^* is the stream function value at the point. Integrating the left-hand side of Eq.4.42 and substituting the values of W_A and W, the following equation was obtained.

$$\int_{W_A}^{W} dw = W - W_A$$

$$= i(\psi^* - q) \qquad (4.44)$$

Substituting Eq.4.44 in Eq.4.42,

$$q - \psi^* = Mg \left[(\gamma-\sigma)\Pi(\beta_v, \alpha_2^2, m') + (\rho-\gamma)F(\beta_v, m') \right] \qquad ...(4.45)$$

Integrating the left-hand side of Eq.4.42 between points A and B, in which the upper limit is the value of the complex function at Point B, i.e., $W = W_B = 0$,

$$\int_{W_A}^{W} dw = 0 - iq = -iq \qquad (4.46)$$

At Point B, t = 0 and from Eq.4.43(a),

$$\beta_v = \pi/2 \qquad (4.47)$$

Substituting Eqs.4.46 and 4.47 in Eq.4.42 (or knowing that at Point B, $\psi^* = 0$, and hence from Eq. 4.45),

$$q = Mg\left[(\gamma-\sigma)\Pi_2' + (\rho-\gamma)K' \right] \qquad (4.48)$$

where,

$$\Pi_2' = \Pi(\pi/2, \alpha_2^2, m')$$

$$K' = F(\pi/2, m')$$

Also, integrating the left side of Eq.4.42 between points A and A', where at Point A', $W = i\psi^* = i\psi_{A'}$ and $t = -t_{A'}$ [refer Table 4.1], from Eq.4.42 (or Eq.4.45) and Eq.4.43(a) the following relationships were found.

$$\psi_{A'} = q - Mg \left[(\gamma-\sigma)\Pi(\beta_{A'}, \alpha_2^2, m') + (\rho-\gamma)F(\beta_{A'}, m')\right] \qquad ...(4.49)$$

$$\beta_{A'} = \sin^{-1}\sqrt{\gamma(\sigma - t_{A'})/\{\sigma(\gamma - t_{A'})\}} \qquad (4.50)$$

Similarly, knowing [refer Table 4.1] that at Point B', W = $i\psi^{*}$ = $i\psi_{B'}$ and t = $-t_{B'}$, integration of the left hand side of Eq.4.42 and the appropriate substitutions in Eq.4.42 (or Eq.4.45) and Eq.4.43(a), gave the following equations.

$$\psi_{B'} = q - Mg\left[(\gamma-\sigma)\Pi(\beta_{B'}, \alpha_2^2, m') + (\rho-\gamma)F(\beta_{B'}, m')\right] \qquad ...(4.51)$$

$$\beta_{B'} = \sin^{-1}\sqrt{\gamma(\sigma - t_{B'})/\{\sigma(\gamma - t_{B'})\}} \qquad (4.52)$$

. Integrating Eq.4.34 in the region EA, i.e., in the region $1 > 0 > -\sigma > t \geq -\gamma$ gave [Byrd and Friedman, 1971, section 253.11 and 253.00],

$$\int_{W}^{W_A} dw = -M\int_{\zeta}^{-\sigma} \frac{(t+\rho)dt}{\sqrt{(1-t)(0-t)(-\sigma-t)(t+\gamma)}}$$

$$= Mg\left[\sigma\Pi(\beta_L, \alpha_3^2, m) - \rho F(\beta_L, m)\right] \qquad ...(4.53)$$

where,

$$\beta_L = \sin^{-1}\sqrt{\gamma(t+\sigma)/t(\gamma-\sigma)} \qquad (4.54)$$

$$m^2 < \alpha_3^2 = (\gamma-\sigma)/\gamma < 1 \qquad (4.55)$$

The complex potential value at any point on the free surface AE is W = ky + iq, where y is the ordinate of the point. At Point A, the complex potential value is W_A = iq. Integrating the left hand side of Eq.4.53 and making the appropriate substitution,

$$\int_{W}^{W_A} dw = W_A - W$$

$$= iq - (ky + iq)$$

$$= -ky \qquad (4.56)$$

Substituting Eq.4.56 in Eq.4.53,

$$ky = Mg\left[-\sigma \Pi(\beta_L, \alpha_3^2, m) + \rho F(\beta_L, m)\right] \qquad (4.57)$$

Hence, integrating the left-hand side of Eq.4.53 between points E and A, in which $W = W_E = (kh_2 + iq)$ and $t = -\gamma$, reduced Eq.4.54 and Eq.4.57 into the following equations.

$$\beta_L = \sin^{-1}\sqrt{\gamma(-\gamma+\sigma)/\{-\gamma(\gamma-\sigma)\}}$$

$$= \pi/2 \qquad (4.58)$$

$$kh_2 = Mg\left[-\sigma \Pi(\pi/2, \alpha_3^2, m) + \rho F(\pi/2, m)\right]$$

$$= Mg\left[\rho K - \sigma \Pi_3\right] \qquad (4.59)$$

where,

$$\Pi_3 = \Pi(\pi/2, \alpha_3^2, m)$$

Integrating Eq.4.34 in the region E'E, i.e., in the region $1 > 0 > -\sigma > -\gamma > t$, we have [Byrd and Friedman, 1971, section 251.03 and 251.00],

$$\int_{W}^{W_E} dw = \frac{M}{-i}\int_{t}^{-\gamma} \frac{(t+\rho)dt}{\sqrt{(1-t)(0-t)(-\sigma-t)(-\gamma-t)}}$$

$$= \frac{Mg}{i}\left[(\gamma-\sigma)\Pi(\beta',\alpha_4^2,m') - (\rho-\sigma)F(\beta',m') \right] \quad \ldots(4.60)$$

where,

$$\beta' = \sin^{-1}\sqrt{(1+\sigma)(-\gamma-t)/\{(1+\gamma)(-\sigma-t)\}} \quad (4.61)$$

$$\alpha_4^2 = (1+\gamma)/(1+\sigma) > 1 \quad (4.62)$$

W_E and W_E' are the complex potential values at the points E and E', respectively. Hence, integrating both sides of Eq.4.34 between points E' and E, in which $W_E = (kh_2 + iq)$ and $W = W_E' = (kh_2 + iq')$ and $t = -\rho$,

$$\int_{W_E'}^{W_E} dw = W_E - W_E' = (kh_2 + iq) - (kh_2 + iq') = i(q - q') \quad \ldots(4.63)$$

and, [Byrd and Friedman, 1971, section 251.03 and 251.00],

$$i(q - q') = \frac{Mg}{i}\left[(\gamma-\sigma)\Pi(\beta_4,\alpha_4^2,m') - (\rho-\sigma)F(\beta_4,m')\right]$$

$$q' = q + Mg\left[(\gamma-\sigma)\Pi(\beta_4,\alpha_4^2,m') - (\rho-\sigma)F(\beta_4,m') \right] \quad \ldots(4.64)$$

for $t = -\rho$, from Eq.4.61,

$$\beta' = \sin^{-1}\sqrt{[(1+\sigma)(\rho-\gamma)]/[(1+\gamma)(\rho-\sigma)]}$$

$$= \beta_4 \quad (4.65)$$

4.3.4 Mapping of the ζ-plane onto the lower half of the t-plane.

As discussed above, the solution of the problem involved the mapping of the θ-plane onto the lower half of the ζ-plane [Eq.4.1(a)] and for convenience, the mapping of the w-plane onto another semi-infinite plane, i.e., the t-plane [Eq. 4.2(a)]. These mappings were done by the Schwarz-Christoffel transformation. The t-plane has been mapped onto ζ-plane to find out relationship between ζ and t. It is known [Nehari, 1952,1961] that any three values on the boundary of the half plane can be chosen to correspond to three points on the boundary of the region to be mapped. The remaining values must be determined so as to satisfy conditions of similarity. The three points selected for bilinear transformation are A, B and D. The values assigned to these points in ζ-plane are -1, 0 and ∞, respectively. The corresponding values of these points in the t-plane are $-\sigma$, 0 and ∞, respectively.

Using the cross ratio formula [section A.2], the following relationship between the ζ-plane and the t-plane was thus obtained :

$$\frac{(t - t_B)\ (t_D - t_A)}{(t - t_A)\ (t_D - t_B)} = \frac{(\zeta - \zeta_B)\ (\zeta_D - \zeta_A)}{(\zeta - \zeta_A)\ (\zeta_D - \zeta_B)} \qquad (4.66)$$

Substituting the corresponding values in Eq.4.66,

$$\frac{(t - 0)\ (\infty + \sigma)}{(t + \sigma)\ (\infty - 0)} = \frac{(\zeta - 0)\ (\infty + 1)}{(\zeta + 1)\ (\infty - 0)} \qquad (4.67)$$

Eq.4.67, after rearranging, gave

$$t = \sigma\zeta \tag{4.68}$$

If $-\lambda_1$ and $-\lambda_2$ are values at Points A' and B', respectively, in the ζ-plane [Fig.4.1(c)], then the corresponding values in the t-plane are obtained using Eq.4.68. Similarly, the corresponding values in the ζ-plane of the Points A, E and E' which are having values of $-\sigma$, $-\gamma$, and $-\rho$, respectively, in the t-plane [Fig.4.1(e)]), were obtained through Eq.4.68.

The values at the different points in the ζ-plane and the t-plane are summarized in Table 4.1(b).

4.4 DIMENSIONLESS FORM OF EQUATIONS

From Table 4.1(b), $\zeta_c = 1/\sigma$, $\zeta_E = \gamma/\sigma$ and $\zeta_{E'} = \rho/\sigma$ and substituting these in Eqs.4.15(b), 4.29(c) and 4.33(b),

$$I_1 = \int_0^{\sinh^{-1}\sqrt{1/\sigma}} [\sinh^2 u+\lambda_2]^{a_1} [\sinh(u)]^{1-2a_1} [\sinh^2(u)+\lambda_1]^{a_2} [\cosh(u)]^{1-2a_2} du \quad \ldots (4.69a)$$

$$I_5 = \int_0^{\cosh^{-1}\sqrt{\gamma/\sigma}} \left[\cosh^2 u-\lambda_2\right]^{a_1} \left[\cosh(u)\right]^{1-2a_1} \left[\cosh^2(u)-\lambda_1\right]^{a_2} \left[\sinh(u)\right]^{1-2a_2} du \quad \ldots (4.69b)$$

$$I_6 = \int_{-\rho/\sigma}^{-\gamma/\sigma} \left[\frac{\zeta + \lambda_2}{\zeta}\right]^{a_1} \left[\frac{\zeta + \lambda_1}{\zeta + 1}\right]^{a_2} d\zeta \quad \ldots (4.69c)$$

Dividing Eqs.4.15a and 4.29(b) by Eq.4.21(b), the following dimensionless relationships were obtained.

$$L_1/H = \text{cosec}(\pi a_1)\ [I_1]/[(\lambda_2) I_2] \quad (4.70)$$

$$L_2/H = \text{cosec}(\pi a_1)\ [I_5]/[(\lambda_2) I_2] \quad (4.71)$$

Similarly, Eq.4.33(a) yielded the equation,

$$L_3/H = 0.5\ \text{cosec}(\pi a_1)\ [I_6/(\lambda_2 I_2)] - [(q - q')/kh_1]\ [h_1/H] \quad \ldots (4.72)$$

Dividing Eq.4.21(b) by Eq.4.21(d),

$$\lambda_2 = \sin(\pi a_2)\ \text{cosec}(\pi a_1)[I_3]/[I_2] \quad (4.73)$$

Dividing Eqs.4.24(e) by H or its equivalent as given by Eq.4.21(b), and rearranging the resulting equation, the following relationships were found.

$$B_2/H = 0.5\ \text{cosec}(\pi a_1)\ [I_4/\{(\lambda_2) I_2\}] + [(\psi_A' - \psi_B')/kh_1]\ [h_1/H] \quad \ldots (4.74)$$

Also, dividing Eqs.4.48, 4.49, 4.51, 4.59 and 4.64 by Eq.4.41, resulted in the following relationships.

$$q/kh_1 = \frac{(\gamma-\sigma)\ \Pi_2' + (\rho-\gamma)\ K'}{\sigma\ \Pi_1 + (\rho-\sigma)\ K} \quad (4.75)$$

$$\psi_A'/kh_1 = q/kh_1 - \frac{(\gamma-\sigma)\ \Pi(\beta_A', \alpha_2^2, m') + (\rho-\gamma)\ F(\beta_A', m')}{\sigma\ \Pi_1 + (\rho - \sigma)\ K} \quad \ldots (4.76)$$

$$\psi_B'/kh_1 = q/kh_1 - \frac{(\gamma-\sigma)\ \Pi(\beta_B', \alpha_2^2, m') + (\rho-\gamma)\ F(\beta_B', m')}{\sigma\ \Pi_1 + (\rho - \sigma)\ K} \quad \ldots (4.77)$$

$$h_2/h_1 = \frac{\rho K - \sigma \Pi_3}{\sigma \Pi_1 + (\rho-\sigma)\ K} \qquad \ldots (4.78)$$

$$q'/kh_1 = q/kh_1 + \frac{(\gamma-\sigma)\ \Pi(\beta_4, \alpha_4^2, m') - (\rho-\sigma)\ F(\beta_4, m')}{\sigma \Pi_1 + (\rho-\sigma)\ K}$$

$$\ldots (4.79)$$

Rearranging Eq.4.78,

$$\rho = \frac{\sigma\ [(h_2/h_1)(\Pi_1 - K) + \Pi_3]}{(1 - h_2/h_1)\ K} \qquad \ldots (4.80)$$

Shape of the free surface : Dividing Eq.4.39 by Eq.4.39, the dimensionless form of the ordinate of a point on the free surface BC, as given below was obtained.

$$y/h_1 = \frac{\sigma \Pi(\beta_r, \alpha_1^2, m) + (\rho-\sigma)\ F(\beta_r, m)}{\sigma \Pi_1 + (\rho-\sigma)\ K}$$

$$\ldots (4.81)$$

The abscissa of the point which has the ordinate as given by Eq.4.81, was found in a non-dimensional form by dividing Eq.4.14 by Eq.4.15(a). The resulting equation is as follows.

$$x/L_1 = I_7/I_1 \qquad (4.82)$$

where,

$$I_7 = \int_0^{\sinh^{-1}\sqrt{\zeta}} [\sinh^2 u + \lambda_2]^{a_1} [\sinh(u)]^{1-2a_1} [\sinh^2(u) + \lambda_1]^{a_2} [\cosh(u)]^{1-2a_2} du$$

$$\ldots (4.83)$$

From Eq.4.68,

$$\zeta = t/\sigma \qquad 1 \geq t \geq 0. \qquad . . . (4.84)$$

Free surface AE : Dividing Eq.4.57 by Eq.4.59, the ordinate of a point on the free surface AE is expressed as given below.

$$y/h_2 = \frac{[\ \rho F(\beta_L, m) - \sigma \Pi(\beta_L, \alpha_9^2, m)\ ]}{\rho K - \sigma \Pi_3} \qquad (4.85)$$

The horizontal distance from Point A of the point on the free surface AE which has the ordinate as given by Eq.4.85, was obtained by dividing Eq.4.29(a) by Eq.4.29(b). The resulting equation is as given below.

$$\frac{X'}{L_2} = I_8/I_5 \qquad (4.86)$$

where,

$$I_8 = \int_0^{\cosh^{-1}\sqrt{-\zeta}} [\cosh^2 u - \lambda_2]^{a_1} [\cosh(u)]^{1-2a_1} [\cosh^2(u) - \lambda_1]^{a_2} [\sinh(u)]^{1-2a_2} du$$

. . . (4.87)

From Eq.4.68, $\zeta = t/\sigma$ in which , $-\sigma \geq t \geq -\gamma$.

4.5 RESULTS AND DISCUSSIONS

Transformation equations relating z-plane with the θ-plane and θ-plane with ζ-plane are derived. On combining these equations, relationship between z, w, and ζ are obtained. In the next operation, w-plane is transformed onto t-plane and t-plane in turn is transformed onto ζ-plane. Thus, equations relating w and t and t and ζ are obtained. Finally, equations relating z, w, ζ and t are established.

It is difficult to find direct relationship between w and z by eliminating ζ and t from the above equations. Therefore, the equations derived above give values of z and w in terms of the intermediate parameters such as σ, γ, λ_1 and λ_2. The procedure followed in computations is to assume the values of intermediate parameters σ, γ, λ_1 and λ_2 and determine the values of w corresponding to these parameters and then determine the value of z. This procedure may not initially give the exact desired values of parameters in the physical system such as the bed width, drainage distances, h_2/h_1, canal side slope, etc. In order to get the exact desired values of physical dimensions of canal and drainage distances, assumed values of intermediate values had to be suitably modified.

As compared to the present case, the relationships between the physical dimensions of the system and intermediate parameters for the case of negligible water depth (Chapter 3) were simple and more explicit. For the case of canal of negligible water depth, the parameter σ is related to the physical dimensions B_2 and L_1 as well as the quantity q/k [Eq.3.36]. The parameter γ is in turn related to the drainage distances L_1 and L_2 as well as to the parameter σ [Eq.3.38]. These relationships were used as an initial approximation of the parameters σ and γ in the present case, i.e. trapezoidal canal. Using the starting value of q/kh_1 = 0. the values of the parameters σ and γ were determined. These values were refined till the difference between two consecutive values of q/kh_1 as calculated by Eq.4.75 is equal to or within an acceptable value.

Then value of the parameter λ_2 which is closely related to bed width and side slope of the canal is assumed. An approximate value of λ_1 is taken as equal to $1-\lambda_1$. At this stage, the computation of ψ_A'/kh_1, ψ_B'/kh_1, a_1, a_2 and b'/H is possible through Eqs.4.76, 4.77, 4.5, 4.8 and 4.24(a) respectively. Calculated value of B_2/H through Eq.4.74 is found and compared with the desired value. The parameters λ_2 and λ_1 are adjusted till the difference between the calculated and desired values of B_2/H is negligible. Then, using Eqs.4.70 and 4.71 the values of L_1/H and L_2/H are calculated and compared with the respective desired values. If not acceptably comparable, the whole process starting from the calculation of parameters σ and γ are repeated. In this updating process, the values of σ and γ are modified in proportion to the differences in the desired and calculated values of L_1/H and L_2/H. The whole process is repeated till the desired accuracies were obtained. A flow chart [Fig.B.2] in Appendix B shows the computational sequence.

The values of seepage discharge has been calculated for various combination of the following values of bed width, water depth, drainage distances and elevations.

B_2/h_1	H/h_1	L_1/h_1 & L_2/h_1	h_2/h_1
10 20 30	0.5 0.3 0.1	10 10^2 10^3 10^4 10^5	1.0 0.9 0.8 0.7

The results of the above computations are given in Tables 4.2 to 4.5. These values are also plotted as presented in Figs.4.2 to 4.5. In order to see the effect of side slope of the canal on the seepage discharge, values of seepage discharge for side slopes 2:1 and 0.5:1 and for the various other physical dimensions are calculated. The results are presented in Tables 4.6 and 4.7. These results are plotted in Figs.4.6 and 4.7.

Dimensionless seepage discharges emerging in the left-hand side and right-hand side drainages have been calculated and given in Tables 4.8 to 4.15. The seepage discharge from the side slopes and bed of the canal have been calculated for various combinations of physical parameters and are given in Tables 4.16 to 4.25.

Coordinates of the free surface are calculated from Eqs.4.81 and 4.82 for the right-hand side and from Eqs.4.85 and 4.86 for the left-hand side. The free surface coordinates are functions of variables such as bed width, water depth, side slope, drainage distances and their elevations. It is difficult to present nomographs for determining the coordinates of free surface for various combinations of the physical dimensions of the system. However, the free surface profiles have been determined for some cases in connection with determining the effect of various physical parameters on the free surfaces. The curves of the free surfaces, for the various physical parameters are plotted in Figs.4.8, 4.9(a), 4.9(b), 4.10 and 4.11.

Table 4.2(a) Total Seepage Discharge From Trapezoidal Canal To Asymmetric Drainages

[Side slope 1:1, H/h_1 = 0.5 and h_2/h_1 = 1.0]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	q/kh_1				
	B_2/h_1 = 10				
10	1.34681	1.09532	1.05495	1.05061	1.05017
10^2	1.09532	0.73449	0.64670	0.63519	0.63399
10^3	1.05495	0.64670	0.48257	0.44220	0.43695
10^4	1.05061	0.63519	0.44220	0.35707	0.33441
10^5	1.05017	0.63399	0.43695	0.33441	0.28316
	B_2/h_1 = 20				
10	1.61989	1.28310	1.21882	1.21160	1.21088
10^2	1.28310	0.85008	0.73395	0.71857	0.71697
10^3	1.21882	0.73395	0.53235	0.48360	0.47729
10^4	1.21160	0.71857	0.48360	0.38366	0.35760
10^5	1.21088	0.71697	0.47729	0.35760	0.29958
	B_2/h_1 = 30				
10	1.80734	1.41315	1.32728	1.31726	1.31626
10^2	1.41315	0.93516	0.79722	0.77849	0.77654
10^3	1.32728	0.79722	0.56804	0.51286	0.50574
10^4	1.31726	0.77849	0.51286	0.40201	0.37348
10^5	1.31626	0.77654	0.50574	0.37348	0.31064

Table 4.2(b) Total Seepage Discharge From Trapezoidal Canal To Asymmetric Drainages

[Side slope 1:1, H/h_1 = 0.3 and h_2/h_1 = 1.0]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	q/kh_1				
B_2/h_1 = 10					
10	1.30797	1.06877	1.03053	1.02642	1.02601
10^2	1.06877	0.72266	0.63678	0.62563	0.62448
10^3	1.03053	0.63678	0.47698	0.43752	0.43238
10^4	1.02642	0.62563	0.43752	0.35402	0.33174
10^5	1.02601	0.62448	0.43238	0.33174	0.28125
B_2/h_1 = 20					
10	1.58857	1.26227	1.19999	1.19314	1.19243
10^2	1.26227	0.84022	0.72663	0.71157	0.71001
10^3	1.19999	0.72663	0.52841	0.48035	0.47413
10^4	1.19314	0.71157	0.48035	0.38161	0.35582
10^5	1.19243	0.71001	0.47413	0.35582	0.29833
B_2/h_1 = 30					
10	1.77957	1.39489	1.31111	1.30136	1.30037
10^2	1.39489	0.92679	0.79110	0.77267	0.77075
10^3	1.31111	0.79110	0.56483	0.51025	0.50320
10^4	1.30136	0.77267	0.51025	0.40039	0.37209
10^5	1.30037	0.77075	0.50320	0.37209	0.30968

Table 4.2(c) Total Seepage Discharge From Trapezoidal Canal To Asymmetric Drainages

[Side slope 1:1, $H/h_1 = 0.1$ and $h_2/h_1 = 1.0$]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	q/kh_1				
	$B_2/h_1 = 10$				
10	1.26283	1.03755	1.00167	0.99782	0.99744
10^2	1.03755	0.70789	0.62531	0.61458	0.61347
10^3	1.00167	0.62531	0.47051	0.43208	0.42708
10^4	0.99782	0.61458	0.43208	0.35047	0.32863
10^5	0.99744	0.61347	0.42708	0.32863	0.27903
	$B_2/h_1 = 20$				
10	1.55244	1.23799	1.17817	1.17148	1.17080
10^2	1.23799	0.82922	0.71838	0.70368	0.70215
10^3	1.17817	0.71838	0.52397	0.47669	0.47056
10^4	1.17148	0.70368	0.47669	0.37930	0.35382
10^5	1.17080	0.70215	0.47056	0.35382	0.29692
	$B_2/h_1 = 30$				
10	1.74753	1.37363	1.29216	1.28269	1.28173
10^2	1.37363	0.91745	0.78425	0.76616	0.76427
10^3	1.29216	0.78425	0.56125	0.50733	0.50037
10^4	1.28269	0.76616	0.50733	0.39860	0.37054
10^5	1.28173	0.76427	0.50037	0.37054	0.30860

Table 4.3(a) Total Seepage Discharge From Trapezoidal Canal To Asymmetric Drainages

[Side slope 1:1, H/h_1 = 0.5 and h_2/h_1 = 0.9]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	q/kh_1				
			B_2/h_1 = 10		
10	1.28068	1.01149	0.95860	0.94936	0.94729
10^2	1.06693	0.69914	0.59537	0.57630	0.57242
10^3	1.04276	0.63382	0.45857	0.40678	0.39621
10^4	-	-	0.43360	0.33926	0.30756
10^5	-	-	-	0.32791	0.26904
			B_2/h_1 = 20		
10	1.53973	1.18664	1.10780	1.09442	1.09157
10^2	1.24790	0.80783	0.67577	0.65186	0.64718
10^3	1.20422	0.71901	0.50583	0.44482	0.43273
10^4	-	-	0.47415	0.36451	0.32886
10^5	-	-	-	0.35064	0.28462
			B_2/h_1 = 30		
10	1.71757	1.30854	1.20695	1.18979	1.18629
10^2	1.37280	0.88869	0.73423	0.70624	0.70088
10^3	1.31084	0.78072	0.53972	0.47174	0.45851
10^4	-	-	0.50282	0.38194	0.34346
10^5	-	-	-	0.36620	0.29512

Table 4.3(b) Total Seepage Discharge From Trapezoidal Canal To Asymmetric Drainages

[Side slope 1:1, H/h_1 = 0.3 and h_2/h_1 = 0.9]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	q/kh_1				
	B_2/h_1 = 10				
10	1.24402	0.98711	0.93661	0.92771	0.92569
10^2	1.04187	0.68697	0.58625	0.56766	0.56386
10^3	1.01937	0.62413	0.45327	0.40248	0.39208
10^4	-	-	0.42901	0.33637	0.30511
10^5	-	-	-	0.32529	0.26723
	B_2/h_1 = 20				
10	1.51013	1.16748	1.09092	1.07787	1.07508
10^2	1.22835	0.79856	0.66905	0.64553	0.64091
10^3	1.18641	0.71189	0.50209	0.44184	0.42987
10^4	-	-	0.47097	0.36257	0.32723
10^5	-	-	-	0.34890	0.28343
	B_2/h_1 = 30				
10	1.69134	1.29174	1.19237	1.17555	1.17210
10^2	1.35571	0.88075	0.72859	0.70096	0.69567
10^3	1.29551	0.77476	0.53667	0.46933	0.45620
10^4	-	-	0.50026	0.38041	0.34218
10^5	-	-	-	0.36484	0.29421

Table 4.3(c) Total Seepage Discharge From Trapezoidal Canal To Asymmetric Drainages

[Side slope 1:1, $H/h_1 = 0.1$ and $h_2/h_1 = 0.9$]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	q/kh_1				
	$B_2/h_1 = 10$				
10	1.20148	0.95852	0.91064	0.90215	0.90021
10^2	1.01277	0.67297	0.57573	0.55768	0.55397
10^3	0.99208	0.61295	0.44713	0.39750	0.38728
10^4	-	-	0.42369	0.33301	0.30225
10^5	-	-	-	0.32224	0.26511
	$B_2/h_1 = 20$				
10	1.47607	1.14523	1.07118	1.05850	1.05578
10^2	1.20597	0.78813	0.66146	0.63839	0.63384
10^3	1.16591	0.70386	0.49788	0.43847	0.42664
10^4	-	-	0.46738	0.36038	0.32539
10^5	-	-	-	0.34693	0.28210
	$B_2/h_1 = 30$				
10	1.66114	1.27227	1.17530	1.15886	1.15546
10^2	1.33618	0.87190	0.72229	0.69507	0.68984
10^3	1.27789	0.76811	0.53328	0.46665	0.45364
10^4	-	-	0.49741	0.37870	0.34076
10^5	-	-	-	0.36332	0.29318

Table 4.4(a) Total Seepage Discharge From Trapezoidal Canal To Asymmetric Drainages

[Side slope 1:1, H/h_1 = 0.5 and h_2/h_1 = 0.8]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	q/kh_1				
B_2/h_1 = 10					
10	1.21450	0.92755	0.86208	0.84785	0.84410
10^2	1.04285	0.66272	0.54394	0.51730	0.51073
10^3	-	0.62106	0.43456	0.37133	0.35543
10^4	-	-	-	0.32146	0.28070
10^5	-	-	-	-	0.25491
B_2/h_1 = 20					
10	1.45954	1.09012	0.99669	0.97706	0.97209
10^2	1.21714	0.76562	0.61755	0.58507	0.57730
10^3	-	0.70423	0.47930	0.40602	0.38814
10^4	-	-	-	0.34536	0.30012
10^5	-	-	-	-	0.26965
B_2/h_1 = 30					
10	1.62782	1.20393	1.08659	1.06224	1.05625
10^2	1.33690	0.84219	0.67118	0.63391	0.62516
10^3	-	0.76436	0.51139	0.43059	0.41124
10^4	-	-	-	0.36186	0.31343
10^5	-	-	-	-	0.27959

Table 4.4(b) Total Seepage Discharge From Trapezoidal Canal To Asymmetric Drainages

[Side slope 1:1, H/h_1 = 0.3 and h_2/h_1 = 0.8]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	q/kh_1				
			B_2/h_1 = 10		
10	1.17996	0.90529	0.84242	0.82867	0.82503
10^2	1.01846	0.65122	0.53563	0.50957	0.50312
10^3	-	-	0.42954	0.36741	0.35174
10^4	-	-	-	0.31872	0.27847
10^5	-	-	-	-	0.25319
			B_2/h_1 = 20		
10	1.43165	1.07255	0.98161	0.96241	0.95752
10^2	1.19813	0.75685	0.61140	0.57941	0.57172
10^3	-	0.69726	0.47576	0.40330	0.38558
10^4	-	-	-	0.34352	0.29863
10^5	-	-	-	-	0.26854
			B_2/h_1 = 30		
10	1.60308	1.18857	1.07355	1.04962	1.04370
10^2	1.32027	0.83468	0.66603	0.62919	0.62052
10^3	-	0.75854	0.50850	0.42840	0.40918
10^4	-	-	-	0.36041	0.31226
10^5	-	-	-	-	0.27873

Table 4.4(c) Total Seepage Discharge From Trapezoidal Canal To Asymmetric Drainages

[Side slope 1:1, H/h_1 = 0.1 and h_2/h_1 = 0.8]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	q/kh_1				
			B_2/h_1 = 10		
10	1.13994	0.87930	0.81931	0.80609	0.80258
10^2	0.99013	0.63799	0.52605	0.50065	0.49434
10^3	-	-	0.42374	0.36288	0.34745
10^4	-	-	-	0.31554	0.27586
10^5	-	-	-	-	0.25118
			B_2/h_1 = 20		
10	1.39962	1.05237	0.96402	0.94529	0.94052
10^2	1.17635	0.74698	0.60448	0.57302	0.56544
10^3	-	0.68942	0.47177	0.40024	0.38269
10^4	-	-	-	0.34145	0.29695
10^5	-	-	-	-	0.26727
			B_2/h_1 = 30		
10	1.57469	1.17085	1.05835	1.03486	1.02904
10^2	1.30127	0.82631	0.66028	0.62391	0.61533
10^3	-	0.75205	0.50529	0.42595	0.40688
10^4	-	-	-	0.35880	0.31096
10^5	-	-	-	-	0.27778

Table 4.5(a) Total Seepage Discharge From Trapezoidal Canal To Asymmetric Drainages

[Side slope 1:1, H/h_1 = 0.5 and h_2/h_1 = 0.7]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	q/kh_1				
B_2/h_1 = 10					
10	1.14825	0.84348	0.76530	0.74604	0.74062
10^2	1.01871	0.62628	0.49244	0.45820	0.44893
10^3	-	-	0.41053	0.33585	0.31462
10^4	-	-	-	0.30364	0.25382
10^5	-	-	-	-	-
B_2/h_1 = 20					
10	1.37932	0.99358	0.88549	0.85958	0.85246
10^2	1.18635	0.72339	0.55926	0.51821	0.50733
10^3	-	-	0.45275	0.36721	0.34353
10^4	-	-	-	0.32621	0.27136
10^5	-	-	-	-	-
B_2/h_1 = 30					
10	1.53805	1.09932	0.96619	0.93461	0.92609
10^2	1.30097	0.79566	0.60809	0.56153	0.54936
10^3	-	-	0.48305	0.38943	0.36396
10^4	-	-	-	0.34178	0.28339
10^5	-	-	-	-	-

Table 4.5(b) Total Seepage Discharge From Trapezoidal Canal To Asymmetric Drainages

[Side slope 1:1, H/h_1 = 0.3 and h_2/h_1 = 0.7]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	q/kh_1				
			B_2/h_1 = 10		
10	1.11578	0.82332	0.74798	0.72932	0.72404
10^2	0.99503	0.61543	0.48494	0.45138	0.44227
10^3	-	-	0.40580	0.33232	0.31135
10^4	-	-	-	0.30107	0.25181
10^5	-	-	-	-	-
			B_2/h_1 = 20		
10	1.35311	0.97769	0.87217	0.84679	0.83979
10^2	1.16788	0.71511	0.55370	0.51320	0.50244
10^3	-	-	0.44941	0.36475	0.34126
10^4	-	-	-	0.32447	0.27002
10^5	-	-	-	-	-
			B_2/h_1 = 30		
10	1.51473	1.08534	0.95466	0.92359	0.91518
10^2	1.28482	0.78858	0.60342	0.55734	0.54529
10^3	-	-	0.48032	0.38745	0.36214
10^4	-	-	-	0.34042	0.28233
10^5	-	-	-	-	-

Table 4.5(c) Total Seepage Discharge From Trapezoidal Canal To Asymmetric Drainages

[Side slope 1:1, H/h_1 = 0.1 and h_2/h_1 = 0.7]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	q/kh_1				
	B_2/h_1 = 10				
10	1.07824	0.79983	0.72765	0.70966	0.70456
10^2	0.96746	0.60297	0.47629	0.44351	0.43459
10^3	-	-	0.40033	0.32822	0.30757
10^4	-	-	-	-	0.24946
10^5	-	-	-	-	-
	B_2/h_1 = 20				
10	1.32307	0.95939	0.85668	0.83188	0.82503
10^2	1.14670	0.70582	0.54744	0.50756	0.49694
10^3	-	-	0.44565	0.36198	0.33871
10^4	-	-	-	0.32251	0.26850
10^5	-	-	-	-	-
	B_2/h_1 = 30				
10	1.48817	1.06938	0.94128	0.91074	0.90246
10^2	1.26633	0.78069	0.59821	0.55268	0.54075
10^3	-	-	0.47729	0.38524	0.36011
10^4	-	-	-	0.33889	0.28116
10^5	-	-	-	-	-

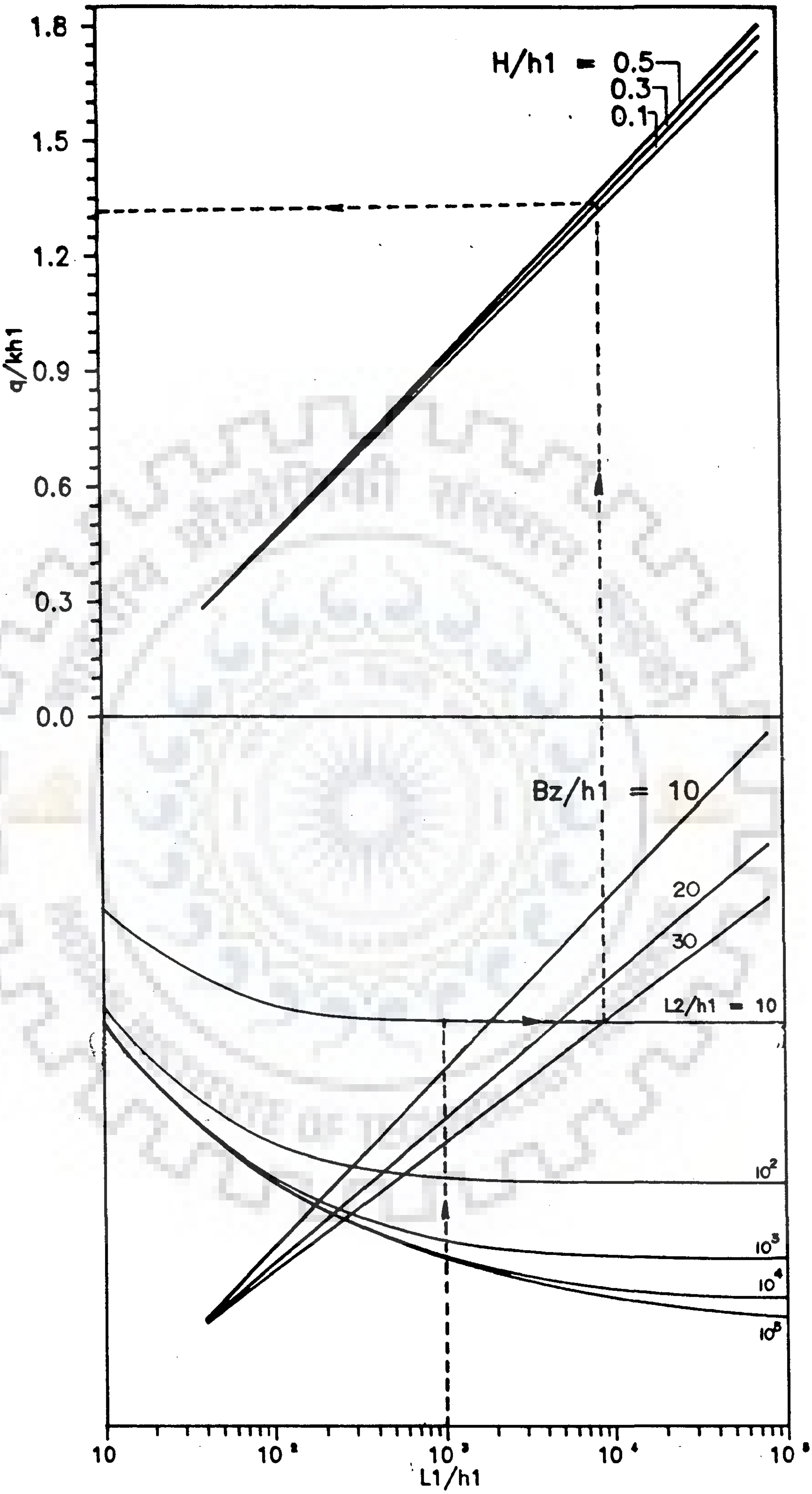


Fig.4.2 TOTAL SEEPAGE DISCHARGE TO ASYMMETRIC DRAINAGES. (slope 1:1, $h2/h1 = 1.0$)

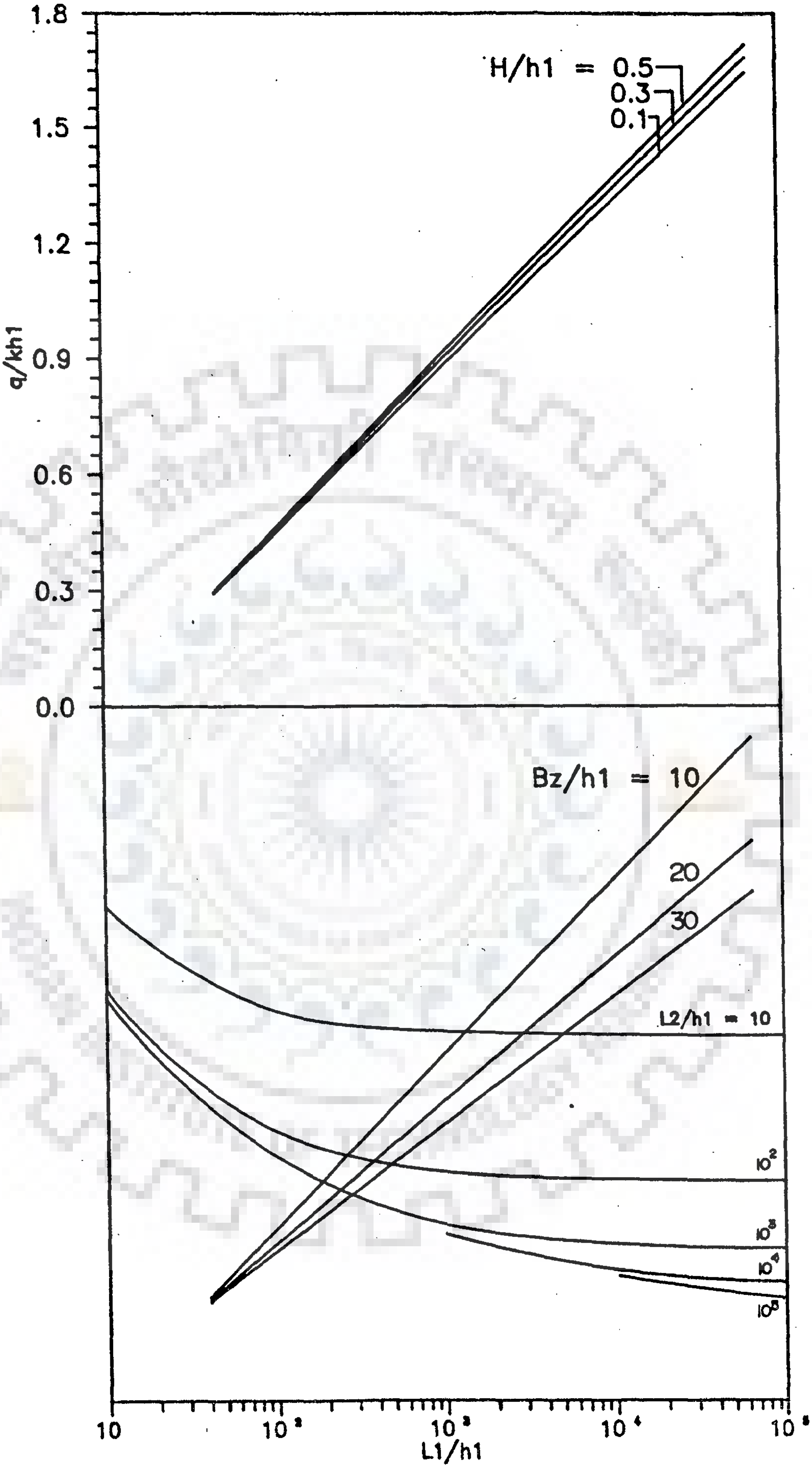


Fig.4.3 TOTAL SEEPAGE DISCHARGE TO ASYMMETRIC DRAINAGES. (slope 1:1, h2/h1 = 0.9)

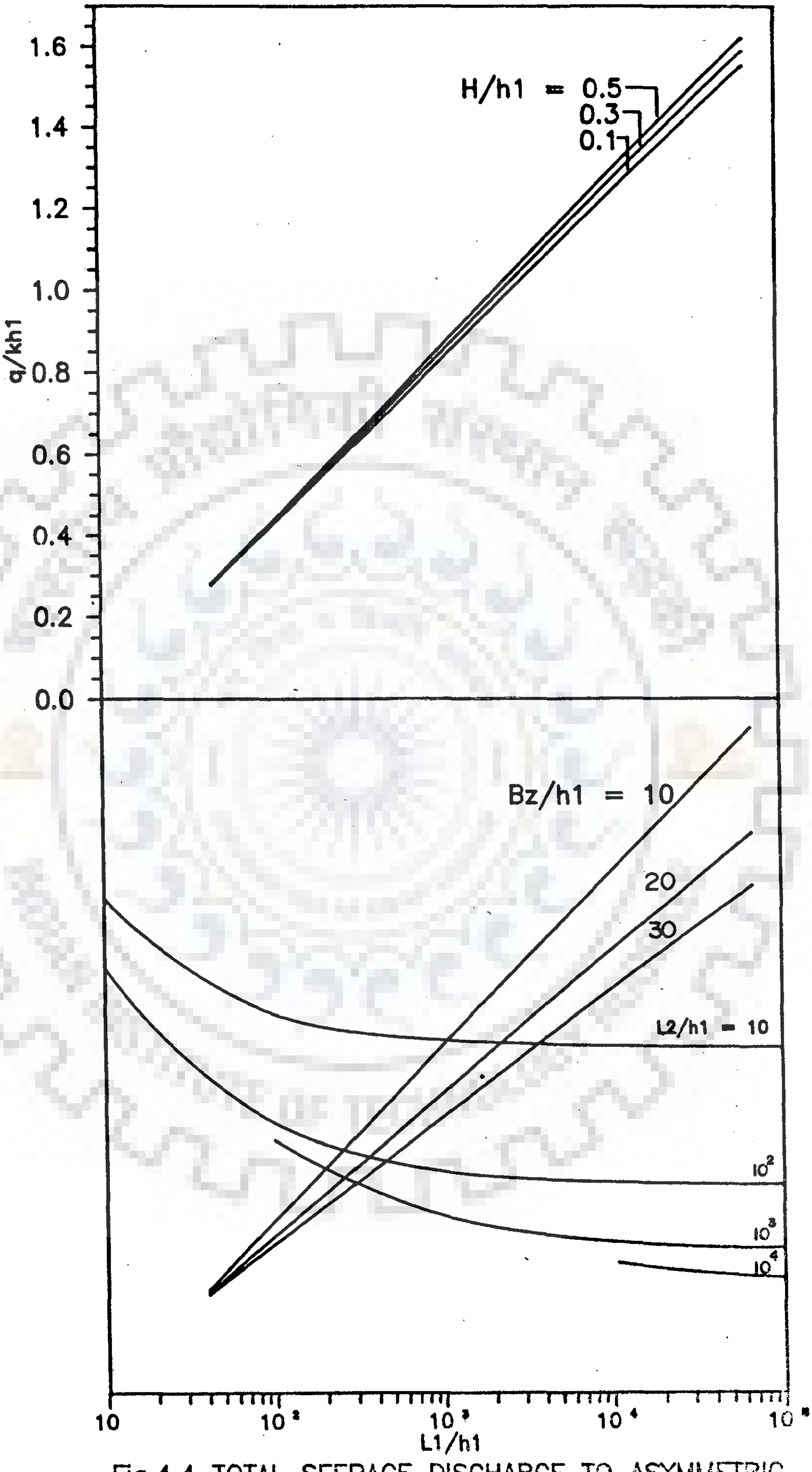


Fig.4.4 TOTAL SEEPAGE DISCHARGE TO ASYMMETRIC DRAINAGES. (slope 1:1, h2/h1 = 0.8)

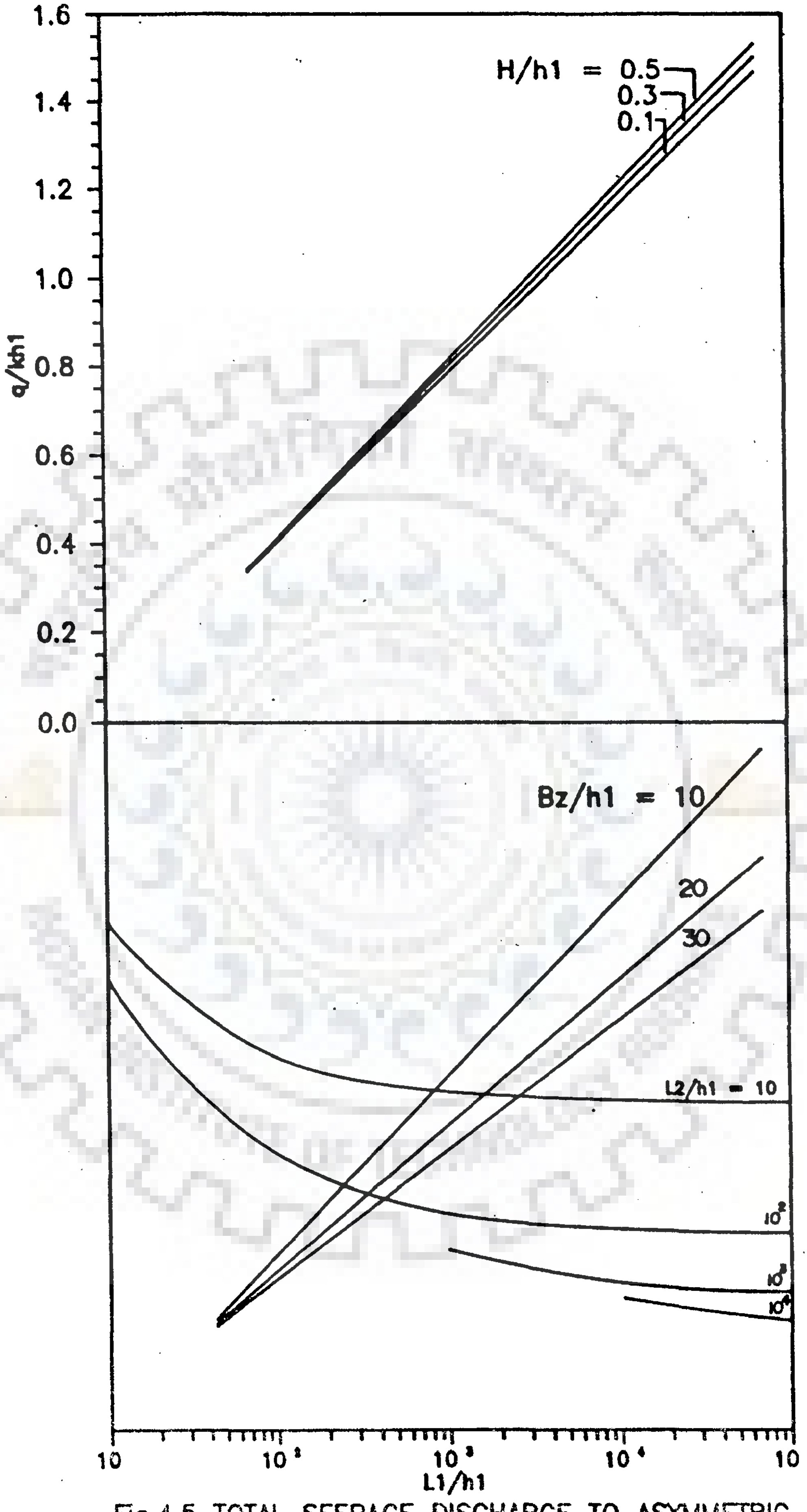


Fig.4.5 TOTAL SEEPAGE DISCHARGE TO ASYMMETRIC DRAINAGES. (slope 1:1, h2/h1 = 0.7)

Table 4.6(a) Total Seepage Discharge From Trapezoidal Canal To Asymmetric Drainages

[Side slope 2:1, H/h_1 = 0.5 and h_2/h_1 = 1.0]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	q/kh_1				
	B_2/h_1 = 10				
10	1.36527	1.10873	1.06644	1.06188	1.06141
10^2	1.10873	0.74535	0.65437	0.64254	0.64132
10^3	1.06644	0.65437	0.48709	0.44600	0.44066
10^4	1.06188	0.64254	0.44600	0.35955	0.33659
10^5	1.06141	0.64132	0.44066	0.33659	0.28471
	B_2/h_1 = 20				
10	1.62871	1.28987	1.22380	1.21636	1.21560
10^2	1.28987	0.85672	0.73903	0.72339	0.72176
10^3	1.22380	0.73903	0.53533	0.48606	0.47969
10^4	1.21636	0.72339	0.48606	0.38523	0.35896
10^5	1.21560	0.72176	0.47969	0.35896	0.30053
	B_2/h_1 = 30				
10	1.81165	1.41692	1.32941	1.31917	1.31814
10^2	1.41692	0.94043	0.80115	0.78217	0.78019
10^3	1.32941	0.80115	0.57036	0.51476	0.50759
10^4	1.31917	0.78217	0.51476	0.40319	0.37450
10^5	1.31814	0.78019	0.50759	0.37450	0.31135

Table 4.6(b) Total Seepage Discharge From Trapezoidal Canal To Asymmetric Drainages

[Side slope 2:1, H/h_1 = 0.3 and h_2/h_1 = 1.0]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	q/kh_1				
	B_2/h_1 = 10				
10	1.31946	1.07723	1.03784	1.03359	1.03317
10^2	1.07723	0.72880	0.64158	0.63024	0.62907
10^3	1.03784	0.64158	0.47983	0.43992	0.43472
10^4	1.03359	0.63024	0.43992	0.35559	0.33312
10^5	1.03317	0.62907	0.43472	0.33312	0.28225
	B_2/h_1 = 20				
10	1.59390	1.26638	1.20314	1.19604	1.19533
10^2	1.26638	0.84433	0.72975	0.71454	0.71296
10^3	1.20314	0.72975	0.53025	0.48187	0.47561
10^4	1.19604	0.71454	0.48187	0.38258	0.35667
10^5	1.19533	0.71296	0.47561	0.35667	0.29893
	B_2/h_1 = 30				
10	1.78210	1.39709	1.31232	1.30243	1.30143
10^2	1.39709	0.92999	0.79349	0.77492	0.77298
10^3	1.31232	0.79349	0.56625	0.51141	0.50433
10^4	1.30243	0.77492	0.51141	0.40112	0.37272
10^5	1.30143	0.77298	0.50433	0.37272	0.31011

Table 4.7(a) Total Seepage Discharge From Trapezoidal Canal To Asymmetric Drainages

[Side slope 0.5:1, H/h_1 = 0.5 and h_2/h_1 = 1.0]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	q/kh_1				
			B_2/h_1 = 10		
10	1.34234	1.09131	1.05170	1.04744	1.04701
10^2	1.09131	0.73174	0.64375	0.63235	0.63117
10^3	1.05170	0.64375	0.48078	0.44069	0.43548
10^4	1.04744	0.63235	0.44069	0.35607	0.33354
10^5	1.04701	0.63117	0.43548	0.33354	0.28253
			B_2/h_1 = 20		
10	1.61969	1.28196	1.21834	1.21123	1.21050
10^2	1.28196	0.84751	0.73203	0.71676	0.71517
10^3	1.21834	0.73203	0.53118	0.48263	0.47635
10^4	1.21123	0.71676	0.48263	0.38304	0.35706
10^5	1.21050	0.71517	0.47635	0.35706	0.29920
			B_2/h_1 = 30		
10	1.80914	1.41329	1.32803	1.31812	1.31711
10^2	1.41329	0.93325	0.79578	0.77714	0.77520
10^3	1.32803	0.79578	0.56713	0.51212	0.50503
10^4	1.31812	0.77714	0.51212	0.40154	0.37307
10^5	1.31711	0.77520	0.50503	0.37307	0.31035

Table 4.7(b) Total Seepage Discharge From Trapezoidal Canal To Asymmetric Drainages

[Side slope 0.5:1, H/h_1 = 0.3 and h_2/h_1 = 1.0]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	q/kh_1				
B_2/h_1 = 10					
10	1.30530	1.06616	1.02837	1.02432	1.02391
10^2	1.06616	0.72037	0.63496	0.62389	0.62274
10^3	1.02837	0.63496	0.47588	0.43658	0.43147
10^4	1.02432	0.62389	0.43658	0.35340	0.33119
10^5	1.02391	0.62274	0.43147	0.33119	0.28086
B_2/h_1 = 20					
10	1.58849	1.26143	1.19966	1.19275	1.19206
10^2	1.26143	0.83872	0.72547	0.71048	0.70892
10^3	1.19275	0.72547	0.52770	0.47976	0.47356
10^4	1.19275	0.71048	0.47976	0.38123	0.35549
10^5	1.19206	0.70892	0.47356	0.35549	0.29810
B_2/h_1 = 30					
10	1.78058	1.39485	1.31142	1.30174	1.30076
10^2	1.39485	0.92564	0.79022	0.77185	0.76994
10^3	1.31142	0.79022	0.56428	0.50980	0.50277
10^4	1.30174	0.77185	0.50980	0.40011	0.37184
10^5	1.30076	0.76994	0.50277	0.37184	0.30950

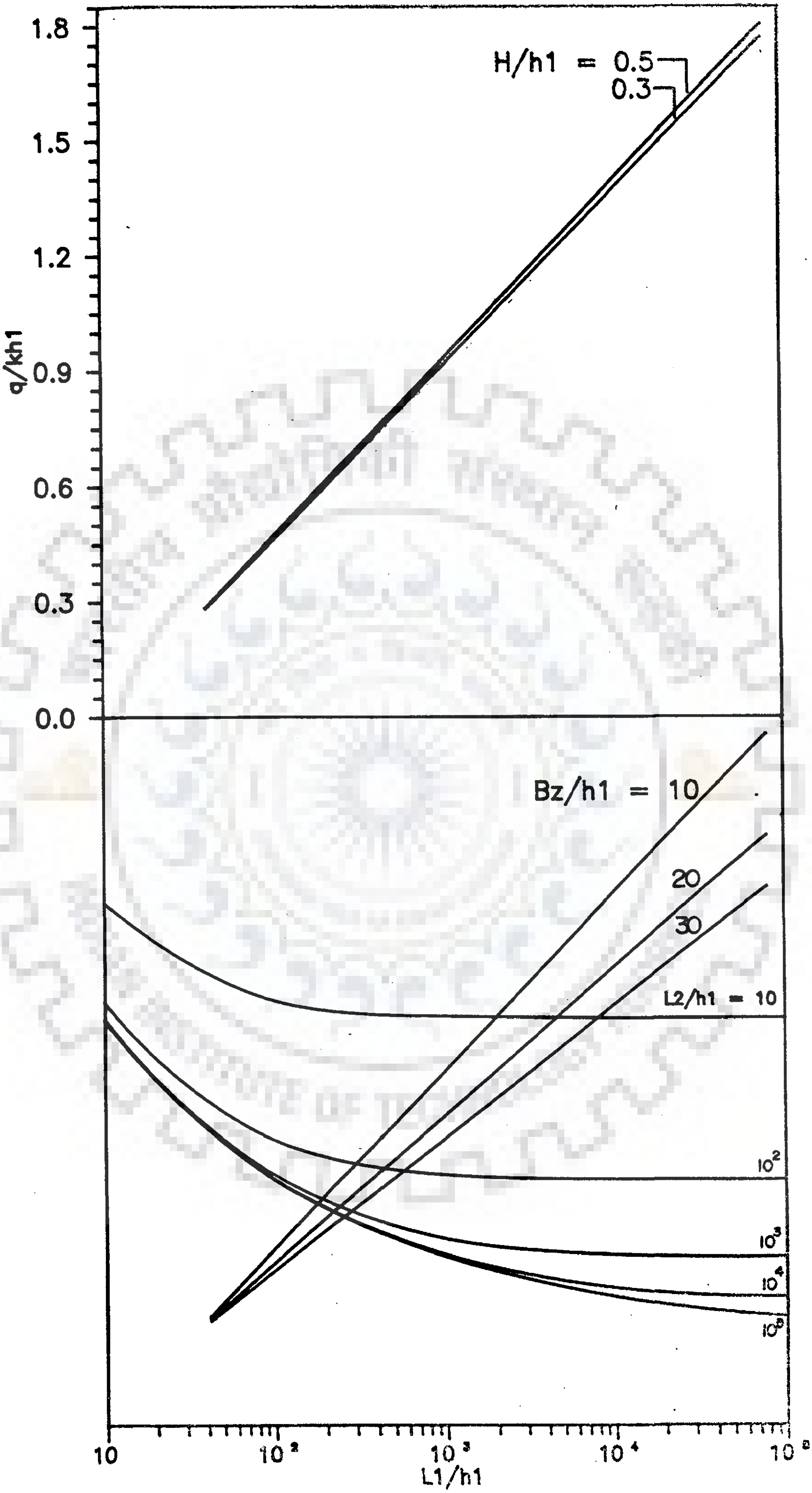


Fig.4.6 TOTAL SEEPAGE DISCHARGE TO ASYMMETRIC DRAINAGES. (slope 2:1, h2/h1 = 1.0)

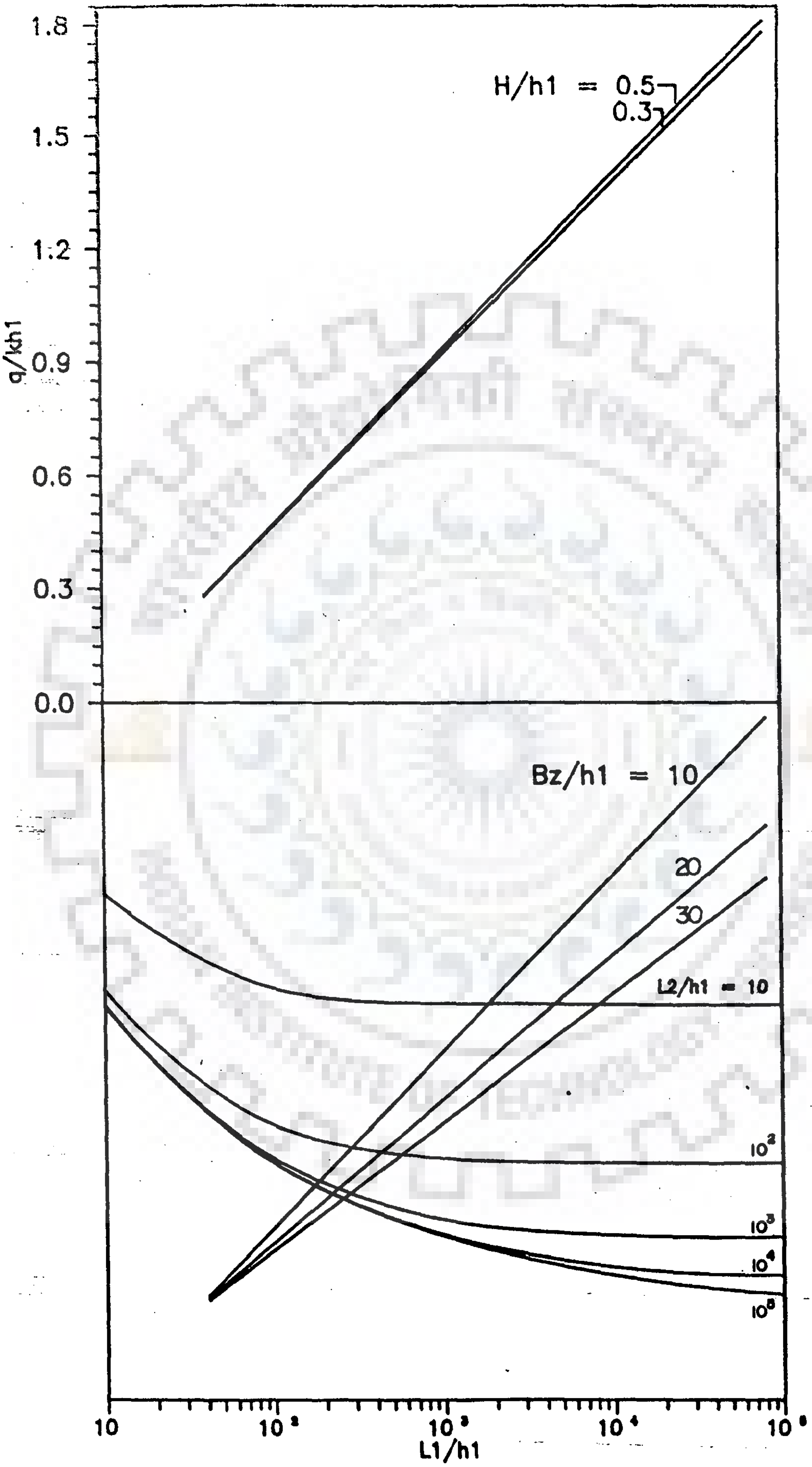


Fig.4.7 TOTAL SEEPAGE DISCHARGE TO ASYMMETRIC DRAINAGES. (slope 0.5:1, h2/h1 = 1.0)

Table 4.8 Seepage Discharge Components from Canal to the Left and Right Drainages

[Side slope 1:1 ; $H/h_1 = 0.5$; $h_2/h_1 = 1.0$]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	values* in non-dimensional form in terms of kh_1				
$B_2/h_1 = 10$					
10	0.67340 0.67340	0.84947 0.24585	0.97542 0.07953	1.02538 0.02523	1.04217 0.00800
10^2	0.24585 0.84947	0.36725 0.36725	0.51793 0.12878	0.59386 0.04133	0.62088 0.01311
10^3	0.07953 0.97542	0.12878 0.51793	0.24128 0.24128	0.35575 0.08645	0.40412 0.02783
10^4	0.02523 1.02538	0.04133 0.59386	0.08645 0.35575	0.17853 0.17853	0.26915 0.06525
10^5	0.00800 1.04217	0.01311 0.62088	0.02783 0.40412	0.06525 0.26915	0.14158 0.14158
$B_2/h_1 = 20$					
10	0.80994 0.80994	0.97120 0.31190	1.11672 0.10210	1.17917 0.03243	1.20060 0.01028
10^2	0.31190 0.97120	0.42504 0.42504	0.58513 0.14881	0.67081 0.04776	0.70182 0.01515
10^3	0.10210 1.11672	0.14881 0.58513	0.26617 0.26617	0.38886 0.09474	0.44682 0.03047
10^4	0.03243 1.17917	0.04776 0.67081	0.09474 0.38886	0.19183 0.19183	0.28781 0.06979
10^5	0.01028 1.20060	0.01515 0.70182	0.03047 0.44682	0.06979 0.28781	0.14979 0.14979
$B_2/h_1 = 30$					
10	0.90367 0.90367	1.05025 0.36290	1.20710 0.12018	1.27904 0.03822	1.30415 0.01211
10^2	0.36290 1.05025	0.46758 0.46758	0.63286 0.16436	0.72570 0.05278	0.75980 0.01674
10^3	0.12018 1.20710	0.16436 0.63286	0.28402 0.28402	0.41219 0.10067	0.47338 0.03236
10^4	0.03822 1.27904	0.05278 0.72570	0.10067 0.41219	0.20100 0.20100	0.30057 0.07291
10^5	0.01211 1.30415	0.01674 0.75980	0.03236 0.47338	0.07291 0.30057	0.15532 0.15532

* NOTE :- Top values in each row in tables 4.8 to 4.15 are seepage discharge components to the left drainage whereas the corresponding values below them are those to the right drainage.

Table 4.9 Seepage Discharge Components from Canal to the Left and Right Drainages

[Side slope 1:1 ; H/h_1 = 0.3 ; h_2/h_1 = 1.0]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	values in non-dimensional form in terms of kh_1				
B_2/h_1 = 10					
10	0.65398 0.65398	0.82961 0.23916	0.95316 0.07737	1.00187 0.02455	1.01823 0.00778
10^2	0.23916 0.82961	0.36133 0.36133	0.51007 0.12671	0.58496 0.04067	0.61157 0.01290
10^3	0.07737 0.95316	0.12671 0.51007	0.23849 0.23849	0.35199 0.08553	0.40484 0.02754
10^4	0.02455 1.00187	0.04067 0.58496	0.08553 0.35199	0.17701 0.17701	0.26700 0.06473
10^5	0.00778 1.01823	0.01290 0.61157	0.02754 0.40494	0.06473 0.26700	0.14062 0.14062
B_2/h_1 = 20					
10	0.79428 0.79428	0.95581 0.30646	1.09966 0.10033	1.16127 0.03187	1.18233 0.01010
10^2	0.30646 0.95581	0.42011 0.42011	0.57939 0.14723	0.66431 0.04726	0.69502 0.01499
10^3	0.10033 1.09966	0.14723 0.57939	0.26420 0.26420	0.38626 0.09409	0.44386 0.03027
10^4	0.03187 1.16127	0.04726 0.66431	0.09409 0.38626	0.19080 0.19080	0.28638 0.06945
10^5	0.01010 1.18233	0.01499 0.69502	0.03027 0.44386	0.06945 0.28638	0.14916 0.14916
B_2/h_1 = 30					
10	0.88978 0.88978	1.03674 0.35815	1.19250 0.11862	1.26363 0.03773	1.28842 0.01195
10^2	0.35815 1.03674	0.46339 0.46339	0.62809 0.16300	0.72032 0.05235	0.75414 0.01660
10^3	0.11862 1.19250	0.16300 0.62809	0.28241 0.28241	0.41010 0.10015	0.47100 0.03220
10^4	0.03773 1.26363	0.05235 0.72032	0.10015 0.41010	0.20019 0.20019	0.29945 0.07263
10^5	0.01195 1.28842	0.01660 0.75414	0.03220 0.47100	0.07263 0.29945	0.15484 0.15484

Table 4.10 Seepage Discharge Components from Canal to the Left and Right Drainages

[Side slope 1:1 ; H/h_1 = 0.1 ; h_2/h_1 = 1.0]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	values in non-dimensional form in terms of kh_1				
	B_2/h_1 = 10				
10	0.63141 0.63141	0.80599 0.23156	0.92674 0.07493	0.97405 0.02377	0.98990 0.00753
10^2	0.23156 0.80599	0.35394 0.35394	0.50097 0.12434	0.57466 0.03992	0.60081 0.01266
10^3	0.07493 0.92674	0.12434 0.50097	0.23525 0.23525	0.34762 0.08446	0.39988 0.02720
10^4	0.02377 0.97405	0.03992 0.57466	0.08446 0.34762	0.17523 0.17523	0.26450 0.06413
10^5	0.00753 0.98990	0.01266 0.60081	0.02720 0.39988	0.06413 0.26450	0.13951 0.13951
	B_2/h_1 = 20				
10	0.77622 0.77622	0.93759 0.30040	1.07981 0.09836	1.14021 0.03127	1.16087 0.00993
10^2	0.30040 0.93759	0.41461 0.41461	0.57291 0.14547	0.65698 0.04670	0.68734 0.01481
10^3	0.09836 1.07981	0.14547 0.57291	0.26198 0.26198	0.38332 0.09337	0.44052 0.03004
10^4	0.03127 1.14021	0.04670 0.65698	0.09337 0.38332	0.18965 0.18965	0.28476 0.06905
10^5	0.00993 1.16087	0.01481 0.68734	0.03004 0.44052	0.06905 0.28476	0.14846 0.14846
	B_2/h_1 = 30				
10	0.87376 0.87376	1.02076 0.35287	1.17527 0.11689	1.24548 0.03720	1.26993 0.01179
10^2	0.35287 1.02076	0.45872 0.45872	0.62275 0.16150	0.71428 0.05188	0.74782 0.01645
10^3	0.11689 1.17527	0.16150 0.62275	0.28062 0.28062	0.40776 0.09957	0.46835 0.03201
10^4	0.03720 1.24548	0.05188 0.71428	0.09957 0.40776	0.19930 0.19930	0.29821 0.07233
10^5	0.01179 1.26993	0.01645 0.74782	0.03201 0.46835	0.07233 0.29821	0.15430 0.15430

Table 4.11 Seepage Discharge Components from Canal to the Left and Right Drainages

[Side slope 1:1 ; H/h_1 = 0.5 ; h_2/h_1 = 0.9]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	values in non-dimensional form in terms of kh_1				
			B_2/h_1 = 10		
10	0.50386 0.77683	0.66853 0.34296	0.80077 0.15784	0.86888 0.08048	0.90367 0.04361
10^2	0.12573 0.94120	0.23380 0.46534	0.37991 0.21546	0.47010 0.10620	0.51598 0.05644
10^3	0.00462 1.03814	0.03459 0.59923	0.12704 0.33153	0.24160 0.16518	0.31148 0.08473
10^4	-	-	0.42518 0.42518	0.26217 0.26217	0.13798 0.13798
10^5	-	-	-	0.32762 0.32762	0.21956 0.21956
			B_2/h_1 = 20		
10	0.62601 0.91372	0.77521 0.41142	0.92262 0.18517	1.00186 0.09256	1.04189 0.04968
10^2	0.18186 1.06604	0.28348 0.52435	0.43773 0.23804	0.53630 0.11556	0.58624 0.06094
10^3	0.01658 1.18764	0.04900 0.67001	0.14753 0.35830	0.26948 0.17534	0.34358 0.08915
10^4	-	-	0.01286 0.46130	0.08739 0.27712	0.18491 0.14395
10^5	-	-	-	0.00142 0.34922	0.05546 0.22916
			B_2/h_1 = 30		
10	1.00751	0.46363	0.20621	0.10162	0.05416
10^2	0.22616 1.14664	0.32079 0.56790	0.47899 0.25524	0.58362 0.12261	0.63660 0.06428
10^3	0.02816 1.28269	0.06071 0.72001	0.16239 0.37733	0.28924 0.18250	0.36627 0.09224
10^4	-	-	0.01630 0.48653	0.09460 0.28733	0.19546 0.14800
10^5	-	-	-	0.00245 0.36375	0.05953 0.23559

Table 4.12 Seepage Discharge Components from Canal to the Left and Right Drainages

[Side slope 1:1 ; H/h_1 = 0.5 ; h_2/h_1 = 0.8]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	values in non-dimensional form in terms of kh_1				
B_2/h_1 = 10					
10	0.38195 0.83255	0.53351 0.39404	0.66638 0.19570	0.74342 0.10443	0.78645 0.05765
10^2	0.05700 0.98585	0.14774 0.51498	0.28539 0.25855	0.38183 0.13547	0.43663 0.07409
10^3	-	0.00002 0.62104	0.06127 0.37329	0.16901 0.20232	0.24612 0.10931
10^4	-	-	-	0.29646 0.29646	0.17070 0.17070
10^5	-	-	-	-	0.24681 0.24681
B_2/h_1 = 20					
10	0.48974 0.96980	0.62580 0.46432	0.77050 0.22620	0.85786 0.11920	0.90662 0.06547
10^2	0.10212 1.11503	0.18977 0.57585	0.33451 0.28303	0.43847 0.14661	0.49747 0.07982
10^3	-	0.00496 0.69926	0.07718 0.40212	0.19216 0.21386	0.27340 0.11475
10^4	-	-	-	0.03205 0.31331	0.12240 0.17772
10^5	-	-	-	-	0.01141 0.25824
B_2/h_1 = 30					
10	0.56415 1.06366	0.68650 0.51744	0.83733 0.24926	0.93212 0.13012	0.98505 0.07120
10^2	0.13921 1.19769	0.22177 0.62042	0.36972 0.30147	0.47900 0.15491	0.54110 0.08406
10^3	-	0.01120 0.75316	0.08898 0.42241	0.20866 0.22193	0.29271 0.11853
10^4	-	-	-	0.03714 0.32472	0.13097 0.18246
10^5	-	-	-	-	0.01380 0.26579

Table 4.13 Seepage Discharge Components from Canal to the Left and Right Drainages

[Side slope 1:1 ; H/h_1 = 0.5 ; h_2/h_1 = 0.7]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	values in non-dimensional form in terms of kh_1				
			B_2/h_1 = 10		
10	0.28071 0.86754	0.41750 0.42599	0.54652 0.21878	0.62733 0.11871	0.67466 0.06595
10^2	0.01371 1.00499	0.08303 0.54325	0.20796 0.28448	0.30530 0.15290	0.36443 0.08449
10^3	-	-	0.01854 0.39199	0.11220 0.22365	0.19093 0.12369
10^4	-	-	-	0.00012 0.30352	0.06535 0.18847
10^5	-	-	-	-	-
			B_2/h_1 = 20		
10	0.37412 1.00520	0.49605 0.49753	0.63417 0.25131	0.72447 0.13511	0.77765 0.07481
10^2	0.04548 1.14087	0.11716 0.60623	0.24889 0.31037	0.35306 0.16515	0.41639 0.09094
10^3	-	-	0.02931 0.42345	0.13095 0.23626	0.21375 0.12978
10^4	-	-	-	0.00228 0.32393	0.07502 0.19634
10^5	-	-	-	-	-
			B_2/h_1 = 30		
10	0.43891 1.09915	0.54804 0.55128	0.69048 0.27570	0.78743 0.14719	0.84481 0.08128
10^2	0.07450 1.22647	0.14373 0.65193	0.27838 0.32971	0.38727 0.17426	0.45364 0.09571
10^3	-	-	0.03774 0.44531	0.14440 0.24503	0.20996 0.13401
10^4	-	-	-	0.00453 0.33726	0.08176 0.20162
10^5	-	-	-	-	-

Table 4.14 Seepage Discharge Components from Canal to the Left and Right Drainages

[Side slope 2:1 ; H/h_1 = 0.5 ; h_2/h_1 = 1.0]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	values in non-dimensional form in terms of kh_1				
			B_2/h_1 = 10		
10	0.68263 0.68263	0.85693 0.25180	0.98485 0.08159	1.03599 0.02589	1.05320 0.00821
10^2	0.25180 0.85693	0.37267 0.37267	0.52382 0.13055	0.60064 0.04190	0.62802 0.01329
10^3	0.08159 0.98485	0.13055 0.52382	0.24354 0.24354	0.35879 0.08721	0.41259 0.02807
10^4	0.02589 1.03599	0.04190 0.60064	0.08721 0.35879	0.17977 0.17977	0.27091 0.06568
10^5	0.00821 1.05320	0.01329 0.62802	0.02807 0.41259	0.06568 0.27091	0.14235 0.14235
			B_2/h_1 = 20		
10	0.81435 0.81435	0.97360 0.31626	1.12010 0.10370	1.18341 0.03295	1.20516 0.01045
10^2	0.31626 0.97360	0.42836 0.42836	0.58892 0.15011	0.67520 0.04819	0.70648 0.01528
10^3	0.10370 1.12010	0.15011 0.58892	0.26766 0.26766	0.39082 0.09524	0.44906 0.03063
10^4	0.03295 1.18341	0.04819 0.67520	0.09524 0.39082	0.19261 0.19261	0.28890 0.07006
10^5	0.01045 1.20516	0.01528 0.70648	0.03063 0.44906	0.07006 0.28890	0.15026 0.15026
			B_2/h_1 = 30		
10	0.90582 0.90582	1.05049 0.36643	1.20788 0.12154	1.28051 0.12154	1.30586 0.03866
10^2	0.36643 1.05049	0.47021 0.47021	0.63570 0.16545	0.72903 0.05314	0.76334 0.01685
10^3	0.12154 1.20788	0.16545 0.63570	0.28518 0.28518	0.41370 0.10107	0.47510 0.03249
10^4	0.12154 1.28051	0.05314 0.72903	0.10107 0.41370	0.20159 0.20159	0.30140 0.07311
10^5	0.03866 1.30586	0.01685 0.76334	0.03249 0.47510	0.07311 0.30140	0.15567 0.15567

Table 4.15 Seepage Discharge Components from Canal to the Left and Right Drainages

[Side slope 0.5:1 ; H/h_1 = 0.5 ; h_2/h_1 = 1.0]

L_1/h_1 →	10	10^2	10^3	10^4	10^5
L_2/h_1	values in non-dimensional form in terms of kh_1				
	B_2/h_1 = 10				
10	0.67117 0.67117	0.84793 0.24338	0.97305 0.07865	1.02249 0.02495	1.03910 0.00791
10^2	0.24338 0.84793	0.36587 0.36587	0.51568 0.12806	0.59126 0.04110	0.61814 0.01304
10^3	0.07865 0.97305	0.12806 0.51568	0.24039 0.24039	0.35455 0.08615	0.40775 0.02773
10^4	0.02495 1.02249	0.04110 0.59126	0.08615 0.35455	0.17803 0.17803	0.26845 0.06508
10^5	0.00791 1.03910	0.01304 0.61814	0.02773 0.40775	0.06508 0.26845	0.14126 0.14126
	B_2/h_1 = 20				
10	0.80984 0.80984	0.97188 0.31008	1.11693 0.10141	1.17902 0.03221	1.20030 0.01021
10^2	0.31008 0.97188	0.42375 0.42375	0.58374 0.14829	0.66917 0.04759	0.70008 0.01509
10^3	0.10141 1.11693	0.14829 0.58374	0.26559 0.26559	0.38809 0.09454	0.44594 0.03041
10^4	0.03221 1.17902	0.04759 0.66917	0.09454 0.38809	0.19152 0.19152	0.28737 0.06969
10^5	0.01021 1.20030	0.01509 0.70008	0.03041 0.44594	0.06969 0.28737	0.14960 0.14960
	B_2/h_1 = 30				
10	0.90457 0.90457	1.05184 0.36146	1.20844 0.11959	1.28009 0.03803	1.30506 0.01205
10^2	0.36146 1.05184	0.46662 0.46662	0.63186 0.16392	0.72450 0.05264	0.75850 0.01669
10^3	0.11959 1.20844	0.16392 0.63186	0.28356 0.28356	0.41161 0.10052	0.47271 0.03231
10^4	0.03803 1.28009	0.05264 0.72450	0.10052 0.41161	0.20077 0.20077	0.30025 0.07283
10^5	0.01205 1.30506	0.01669 0.75850	0.03231 0.47271	0.07283 0.30025	0.15517 0.15517

Table 4.16 Seepage Discharge Components Through Canal Profile .

[h_2/h_1 = 1.0 , H/h_1 = 0.5 , side slope 1:1]

B_2/h_1 = 10

L_1/h_1 →		10	10^2	10^3	10^4	10^5
L_2/h_1		values in non-dimensional form in terms of kh_1				
10	R	0.18896	0.13070	0.12298	0.12217	0.12209
	L	0.18896	0.18265	0.18040	0.18013	0.18010
	B	0.96889	0.78197	0.75157	0.74831	0.74798
10^2	R	0.17670	0.10350	0.08930	0.08755	0.08737
	L	0.12683	0.10350	0.09376	0.09239	0.09225
	B	0.78746	0.52849	0.46364	0.45525	0.45437
10^3	R	0.17391	0.09351	0.06879	0.06302	0.06227
	L	0.11903	0.08907	0.06879	0.06335	0.06263
	B	0.75762	0.46399	0.34499	0.31583	0.31205
10^4	R	0.17360	0.09213	0.06334	0.05125	0.04805
	L	0.11822	0.08730	0.06301	0.05125	0.04808
	B	0.75440	0.45562	0.31585	0.25457	0.23828
10^5	R	0.17356	0.09198	0.06262	0.04807	0.04081
	L	0.11813	0.08711	0.06226	0.04805	0.04081
	B	0.75409	0.45477	0.31207	0.23830	0.20154

NOTE :-

R - seepage component through canal right-hand side slope,

L - seepage component through canal left-hand side slope and

B - seepage component through canal bed.

Table 4.17 Seepage Discharge Components Through Canal Profile

[h_2/h_1 = 1.0 , H/h_1 = 0.5 , side slope 1:1]

B_2/h_1 = 20

L_1/h_1 →		10	10^2	10^3	10^4	10^5
L_2/h_1		values in non-dimensional form in terms of kh_1				
10	R	0.17369	0.10615	0.09635	0.09531	0.09521
	L	0.17369	0.17343	0.17177	0.17154	0.17152
	B	1.27251	1.00352	0.95070	0.94475	0.94415
10^2	R	0.16606	0.08817	0.07329	0.07146	0.07128
	L	0.10199	0.08817	0.07999	0.07875	0.07862
	B	1.01059	0.67366	0.58067	0.56836	0.56707
10^3	R	0.16361	0.07966	0.05592	0.05067	0.05000
	L	0.09220	0.07299	0.05592	0.05007	0.05054
	B	0.95847	0.58115	0.42051	0.38286	0.37675
10^4	R	0.16331	0.07840	0.05114	0.04058	0.03785
	L	0.09116	0.07115	0.05066	0.04058	0.03789
	B	0.95262	0.56886	0.38180	0.30250	0.28186
10^5	R	0.16328	0.07826	0.05052	0.03789	0.03181
	L	0.09106	0.07096	0.04999	0.03785	0.03181
	B	0.95202	0.56760	0.37678	0.28186	0.23596

Table 4.18 Seepage Discharge Components Through Canal Profile

[h_2/h_1 = 1.0 , H/h_1 = 0.5 , side slope 1:1]

B_2/h_1 = 30

L_1/h_1 →		10	10^2	10^3	10^4	10^5
L_2/h_1		values in non-dimensional form in terms of kh_1				
10	R	0.16708	0.09387	0.08246	0.08122	0.08109
	L	0.16708	0.16987	0.16865	0.16845	0.16843
	B	1.47318	1.14941	1.07617	1.06759	1.06674
10^2	R	0.16173	0.08066	0.06501	0.06308	0.06289
	L	0.08967	0.08066	0.07348	0.07231	0.07219
	B	1.15727	0.77384	0.65873	0.64310	0.64146
10^3	R	0.15952	0.07309	0.04953	0.04451	0.04388
	L	0.07837	0.06467	0.04953	0.04515	0.04457
	B	1.08484	0.65930	0.46898	0.42320	0.41729
10^4	R	0.15922	0.07190	0.04513	0.03530	0.03281
	L	0.07715	0.06273	0.04450	0.03530	0.03285
	B	1.07638	0.64370	0.42323	0.33141	0.30782
10^5	R	0.15919	0.07177	0.04455	0.03285	0.02738
	L	0.07702	0.06253	0.04386	0.03281	0.02738
	B	1.07552	0.64207	0.41733	0.30782	0.25588

Table 4.19 Seepage Discharge Components Through Canal Profile
[h_2/h_1 = 1.0 , H/h_1 = 0.3 , side slope 1:1]

L_1/h_1 →		10	10^2	10^3	10^4	10^5
L_2/h_1		values in non-dimensional form in terms of kh_1				
B_2/h_1 = 10						
10	R	0.14182	0.10048	0.09494	0.09436	0.09430
	L	0.14181	0.13791	0.13636	0.13618	0.13616
	B	1.02434	0.83038	0.79923	0.79588	0.79555
10^2	R	0.13265	0.07978	0.06925	0.06794	0.06781
	L	0.09690	0.07978	0.07249	0.07147	0.07136
	B	0.83571	0.56310	0.49504	0.48622	0.48531
10^3	R	0.13061	0.07226	0.05366	0.04926	0.04869
	L	0.09124	0.06903	0.05366	0.04950	0.04896
	B	0.80509	0.49538	0.36966	0.33876	0.33473
10^4	R	0.13038	0.07121	0.04949	0.04022	0.03775
	L	0.09065	0.06775	0.04925	0.04022	0.03777
	B	0.80180	0.48656	0.33878	0.27358	0.25622
10^5	R	0.13036	0.07110	0.04894	0.03777	0.03215
	L	0.09059	0.06756	0.04868	0.04022	0.03775
	B	0.80147	0.48571	0.33476	0.25622	0.21695

Table 4.20 Seepage Discharge Components Through Canal Profile
[h_2/h_1 = 1.0 , H/h_1 = 0.1 , side slope 1:1]

L_1/h_1 →		10	10^2	10^3	10^4	10^5
L_2/h_1		values in non-dimensional form in terms of kh_1				
B_2/h_1 = 10						
10	R	0.07447	0.05494	0.05225	0.05197	0.05194
	L	0.07447	0.07376	0.07316	0.07308	0.07307
	B	1.11389	0.90885	0.87626	0.87277	0.87243
10^2	R	0.06985	0.04413	0.03874	0.03806	0.03799
	L	0.05212	0.04413	0.04042	0.03989	0.03983
	B	0.91340	0.61963	0.54615	0.53663	0.53565
10^3	R	0.06885	0.04021	0.03043	0.02807	0.02776
	L	0.04928	0.03854	0.03043	0.02819	0.02790
	B	0.88127	0.54649	0.40965	0.37582	0.37142
10^4	R	0.06874	0.03966	0.02818	0.02312	0.02176
	L	0.04899	0.03785	0.02805	0.02312	0.02177
	B	0.87781	0.53700	0.37585	0.30423	0.28510
10^5	R	0.06873	0.03961	0.02788	0.02176	0.01863
	L	0.04896	0.03778	0.02775	0.02175	0.01863
	B	0.87746	0.53601	0.37145	0.28512	0.24177

Table 4.21 Seepage Discharge Components Through Canal Profile

[h_2/h_1 = 0.9 , H/h_1 = 0.5 , side slope 1:1]

L_1/h_1 →		10	10^2	10^3	10^4	10^5
L_2/h_1		values in non-dimensional form in terms of kh_1				
B_2/h_1 = 10						
10	R	0.18515	0.12284	0.11269	0.11096	0.11057
	L	0.17448	0.16697	0.16390	0.16332	0.16319
	B	0.92105	0.72168	0.68201	0.67508	0.67353
10^2	R	0.17500	0.09929	0.08266	0.07974	0.07915
	L	0.12226	0.09777	0.08629	0.08401	0.08354
	B	0.76967	0.50208	0.42642	0.41255	0.40973
10^3	R	0.17335	0.09197	0.06554	0.05811	0.05660
	L	0.11758	0.08711	0.06538	0.05836	0.05691
	B	0.75183	0.45474	0.32765	0.29031	0.28270
10^4	R	-	-	0.06216	0.04875	0.04426
	L	-	-	0.06179	0.04874	0.04428
	B	-	-	0.30965	0.24177	0.21902
10^5	R	-	-	-	0.04716	0.03881
	L	-	-	-	0.04713	0.03881
	B	-	-	-	0.23362	0.19142

Table 4.22 Seepage Discharge Components Through Canal Profile
[$h_2/h_1 = 0.8$, $H/h_1 = 0.5$, side slope 1:1]

L_1/h_1 →		10	10^2	10^3	10^4	10^5
L_2/h_1		values in non-dimensional form in terms of kh_1				
		$B_2/h_1 = 10$				
10	R	0.18133	0.11495	0.10232	0.09964	0.09894
	L	0.15994	0.15116	0.14724	0.14631	0.14606
	B	0.87323	0.66144	0.61252	0.60190	0.59910
10^2	R	0.17330	0.09508	0.07598	0.07188	0.07087
	L	0.11768	0.09201	0.07877	0.07556	0.07476
	B	0.75187	0.47563	0.38919	0.36986	0.36510
10^3	R	-	0.09043	0.06227	0.05318	0.05090
	L	-	0.08515	0.06194	0.05335	0.05116
	B	-	0.44548	0.31035	0.26480	0.25337
10^4	R	-	-	-	0.04625	0.04046
	L	-	-	-	0.04622	0.04047
	B	-	-	-	0.22899	0.19977
10^5	R	-	-	-	-	0.03680
	L	-	-	-	-	0.03680
	B	-	-	-	-	0.18131

Table 4.23 Seepage Discharge Components Through Canal Profile
[h_2/h_1 = 0.7 , H/h_1 = 0.5 , side slope 1:1]

L_1/h_1 →		10	10^2	10^3	10^4	10^5
L_2/h_1		values in non-dimensional form in terms of kh_1				
B_2/h_1 = 10						
10	R	0.17753	0.10702	0.09187	0.08822	0.08719
	L	0.14535	0.13523	0.13037	0.12907	0.12871
	B	0.82537	0.60123	0.54306	0.52875	0.52472
10^2	R	0.17160	0.09084	0.06926	0.06395	0.06252
	L	0.11309	0.08622	0.07120	0.06704	0.06590
	B	0.73402	0.44922	0.35198	0.32721	0.32051
10^3	R	-	-	0.05900	0.04822	0.04516
	L	-	-	0.05850	0.04831	0.04537
	B	-	-	0.29303	0.23932	0.22409
10^4	R	-	-	-	0.04374	0.03664
	L	-	-	-	0.04369	0.03664
	B	-	-	-	0.21621	0.18054
10^5	R	-	-	-	-	-
	L	-	-	-	-	-
	B	-	-	-	-	-

Table 4.24 Seepage Discharge Components Through Canal Profile

[h_2/h_1 = 1.0 , H/h_1 = 0.5 , side slope 2:1]

L_1/h_1 →		10	10^2	10^3	10^4	10^5
L_2/h_1		values in non-dimensional form in terms of kh_1				
B_2/h_1 = 10						
10	R	0.24549	0.16898	0.15857	0.15748	0.15737
	L	0.24548	0.23651	0.23337	0.23299	0.23296
	B	0.87430	0.70324	0.67450	0.67141	0.67108
10^2	R	0.22944	0.13406	0.11531	0.11298	0.11274
	L	0.16439	0.13406	0.12128	0.11947	0.11928
	B	0.71033	0.47723	0.41778	0.41009	0.40930
10^3	R	0.22568	0.12098	0.08858	0.08105	0.08007
	L	0.15391	0.11503	0.08858	0.08149	0.08055
	B	0.68224	0.41820	0.30993	0.28346	0.28004
10^4	R	0.22525	0.11915	0.08147	0.06577	0.06161
	L	0.15282	0.11269	0.08103	0.06577	0.06165
	B	0.67920	0.41054	0.28350	0.22801	0.21333
10^5	R	0.22520	0.11896	0.08054	0.06165	0.05225
	L	0.15271	0.11245	0.08006	0.06161	0.05225
	B	0.67890	0.40974	0.28005	0.21333	0.18021

Table 4.25 Seepage Discharge Components Through Canal Profile
[h_2/h_1 = 1.0 , H/h_1 = 0.5 , side slope 0.5:1]

L_1/h_1 →		10	10^2	10^3	10^4	10^5
L_2/h_1		values in non-dimensional form in terms of kh_1				
		B_2/h_1 = 10				
10	R	0.16220	0.11156	0.10501	0.10433	0.10426
	L	0.16220	0.15605	0.15408	0.15385	0.15383
	B	1.01794	0.82370	0.79261	0.78926	0.78892
10^2	R	0.15151	0.08800	0.07593	0.07444	0.07429
	L	0.10862	0.08800	0.07967	0.07851	0.07839
	B	0.82754	0.55574	0.48815	0.47940	0.47849
10^3	R	0.14913	0.07948	0.05846	0.05355	0.05292
	L	0.10200	0.07575	0.05846	0.05383	0.05322
	B	0.79679	0.48841	0.36386	0.33331	0.32934
10^4	R	0.14886	0.07830	0.05382	0.04356	0.04083
	L	0.10131	0.07425	0.05355	0.04356	0.04086
	B	0.79351	0.47969	0.38714	0.26895	0.25185
10^5	R	0.14883	0.07818	0.05321	0.04086	0.03469
	L	0.10124	0.07409	0.05291	0.04083	0.03469
	B	0.79317	0.47879	0.32936	0.25185	0.21315

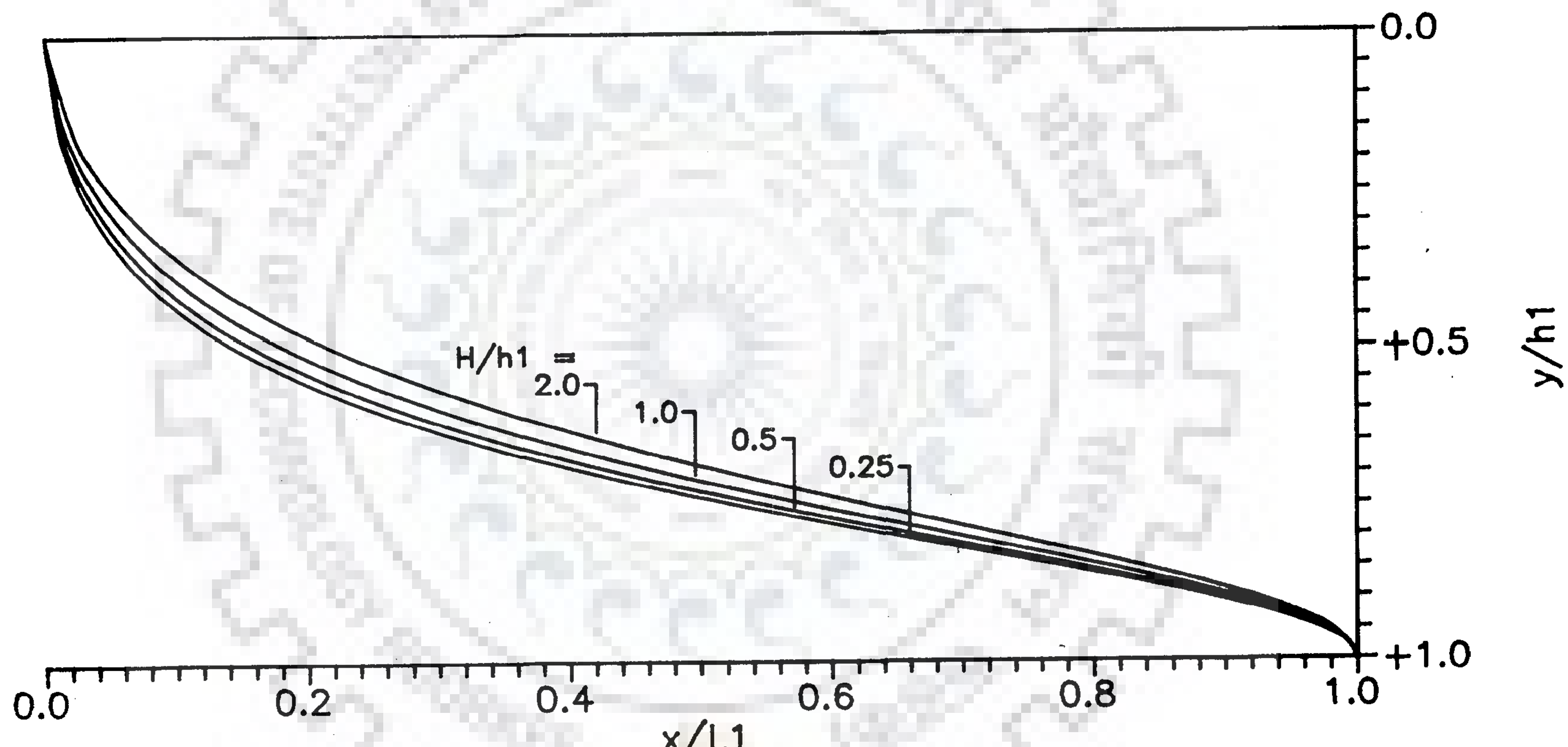


Fig.4.8 FREE SURFACE ON RIGHT-HAND SIDE — Effect of Canal Water Depth. (Slope 2:1, Bz = 5h1, L1=L2 = 100h1, h2 = h1)

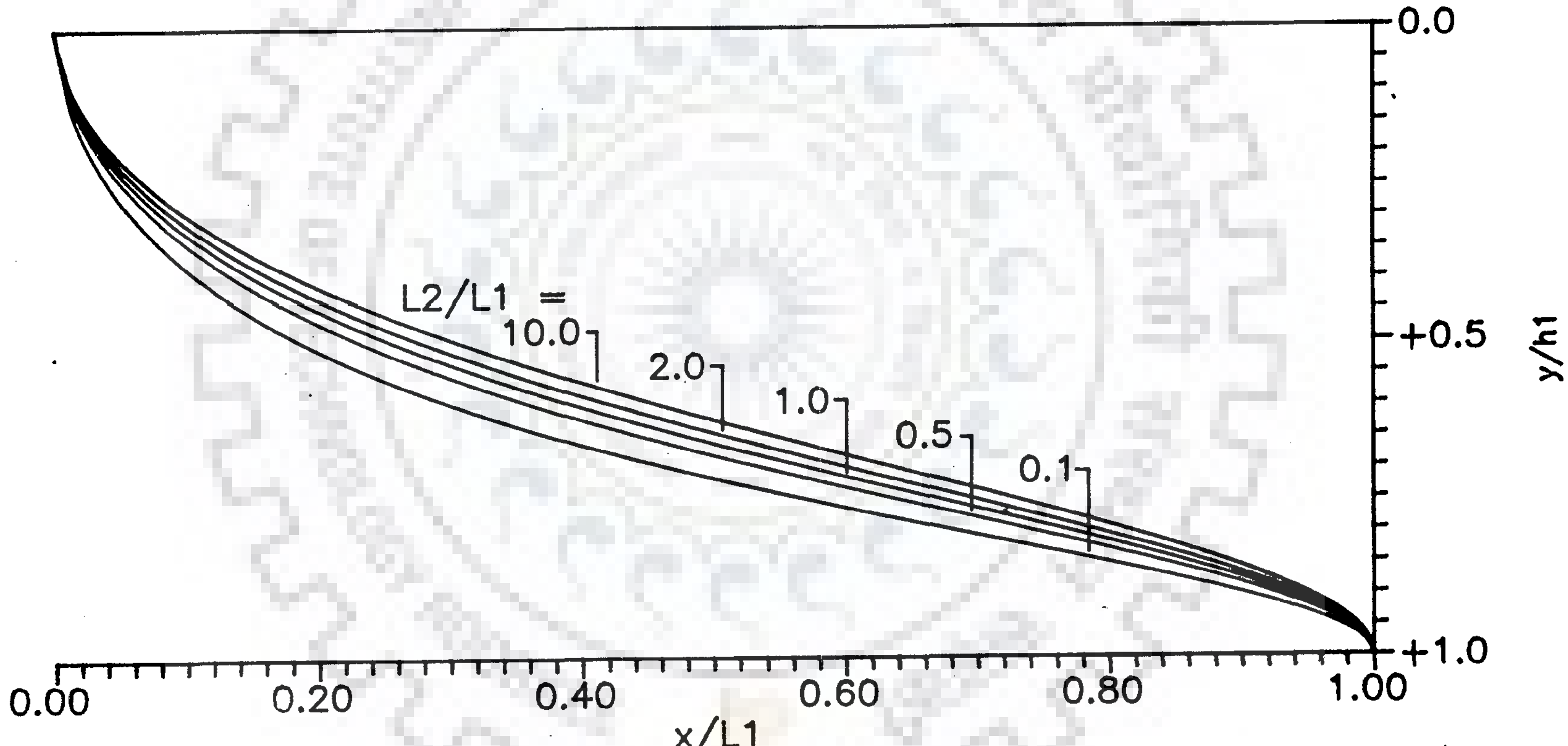


Fig.4.9(a) FREE SURFACE ON RIGHT-HAND SIDE — Effect of Left Drainage Distance. (2:1, H=0.5h1, Bz=10h1, h2=h1, L1=50h1)

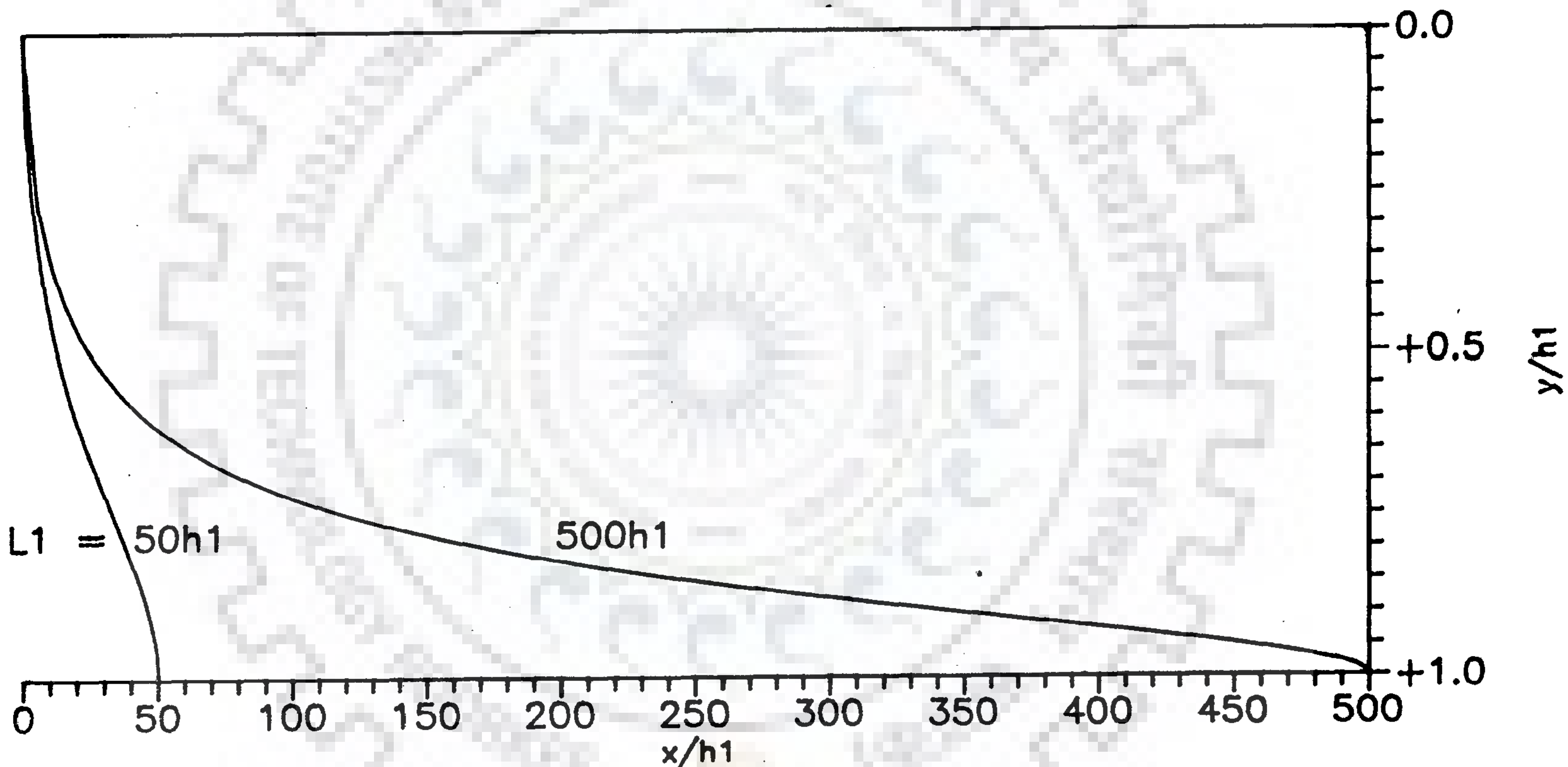


Fig.4.9(b) FREE SURFACE ON RIGHT-HAND SIDE — Effect of Right Drainage Distance. (2:1, H=0.5h1, Bz=10h1, h2=h1, L2=50h1)

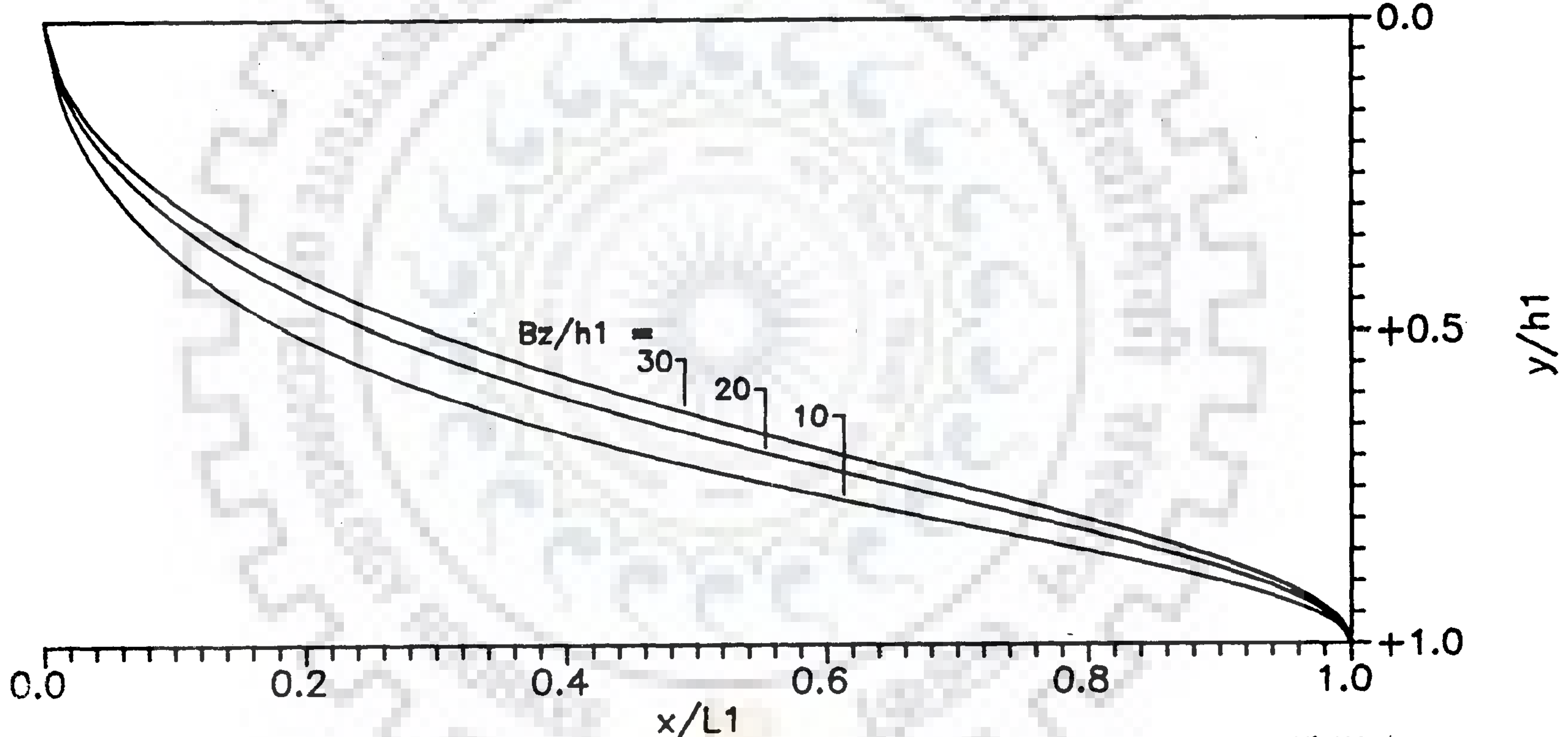


Fig.4.10 FREE SURFACE ON RIGHT-HAND SIDE – Effect of Bed Width
(Slope 2:1, H = 0.5h1, h2 = h1, L1 = 50h1, L2 = 5h1)

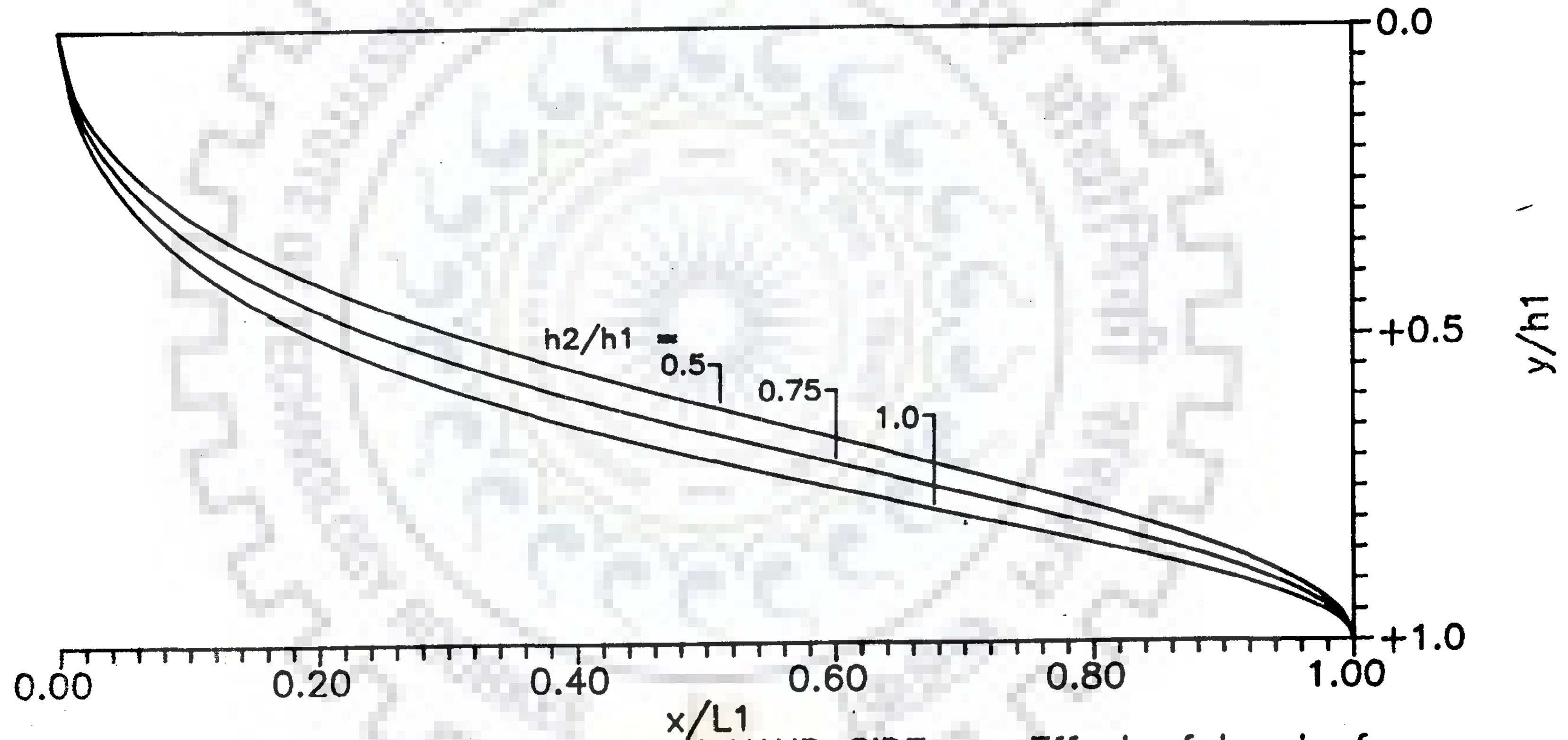


Fig.4.11 FREE SURFACE ON RIGHT-HAND SIDE — Effect of Level of Left Drainage. (2:1, H =0.5h1, Bz =10h1, L1=L2 =100h1)

As discussed earlier, the seepage from the canal flows to the drainages located at different levels and distances on either side of the canal. The seepage water emerging in the drainages will depend upon their distances and elevation in relation to canal water levels. It is seen that if level of a drainage is raised, seepage to this drainage is reduced. At certain critical level (h_c), it will become ineffective, i.e., canal seepage emerging in this drainage will be reduced to zero. Depth at which a drainage become ineffective depends on the values of parameters such as canal bed width and water depth, and relative levels and distances of the drainages. The critical drainage level ratio, h_c/h_1, for various physical parameters is presented in Figs.4.12 to 4.22.

A perusal of Tables 4.2 to 4.7 and Figs.4.2 to 4.7 indicates that the seepage discharge decreases with increase in the values of L_1/h_1 and L_2/h_1, i.e., as drainage distances increase, the seepage discharge decreases. The seepage discharge increases with the increase in bed width and water depth. Also, the seepage discharge from canal increases with flattening of side slope. However, the effect of change in the slope of the canal on the seepage discharge is very small. For example, dimensionless seepage discharges, q/kh_1, from canal with $L_1/h_1 = L_2/h_1 = 10^4$ and $B_2/h_1 = 20$, $H/h_1 = 0.5$ and $h_2/h_1 = 1.0$ for side slopes 0.5:1, 1:1 and 2:1 are 0.38304, 0.38366 and 0.38523 respectively. Similarly, the effect of water depth on the seepage discharge is small.

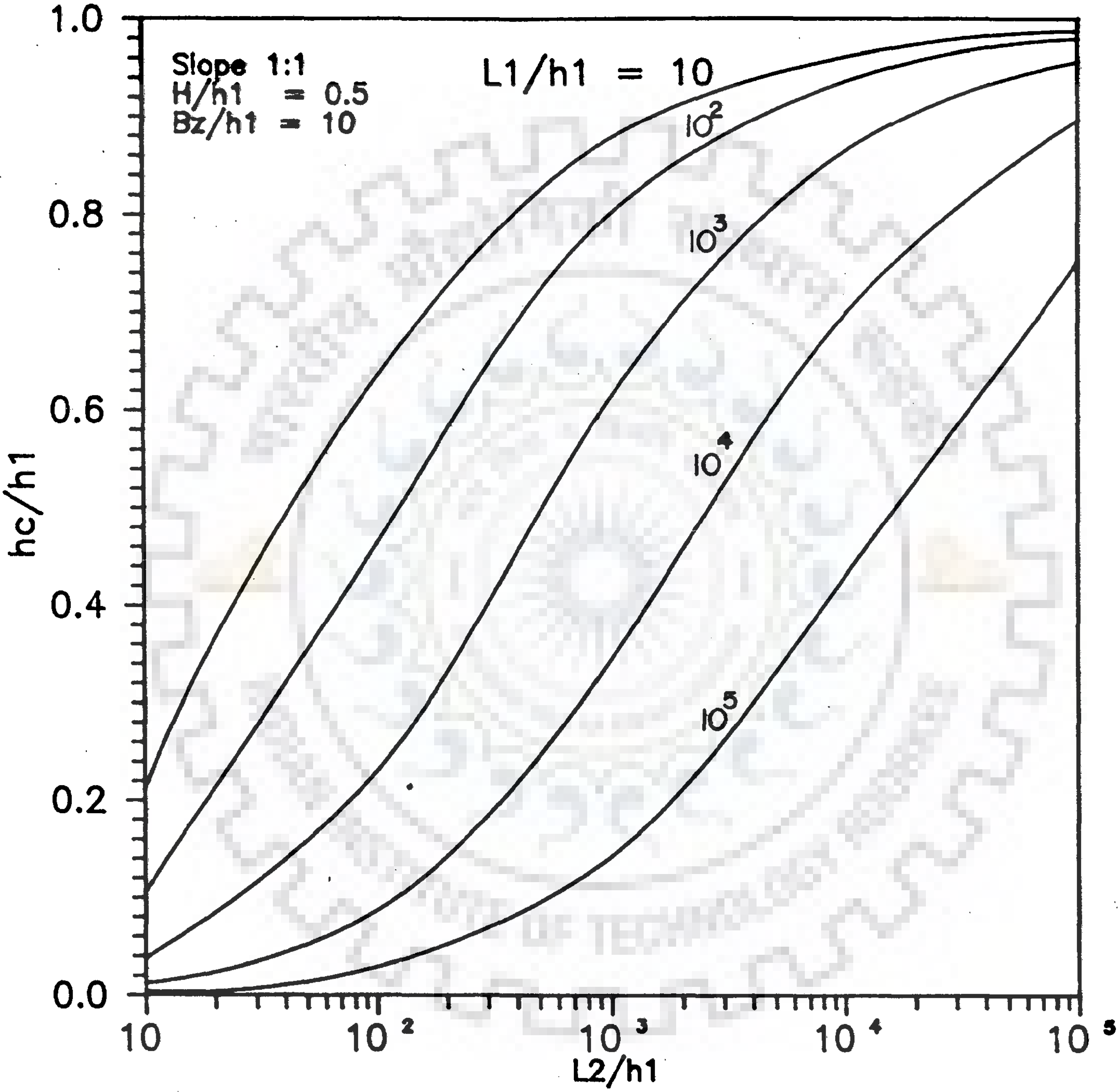


Fig.4.12 CRITICAL LEVEL OF LEFT DRAINAGE BELOW CANAL WATER LEVEL

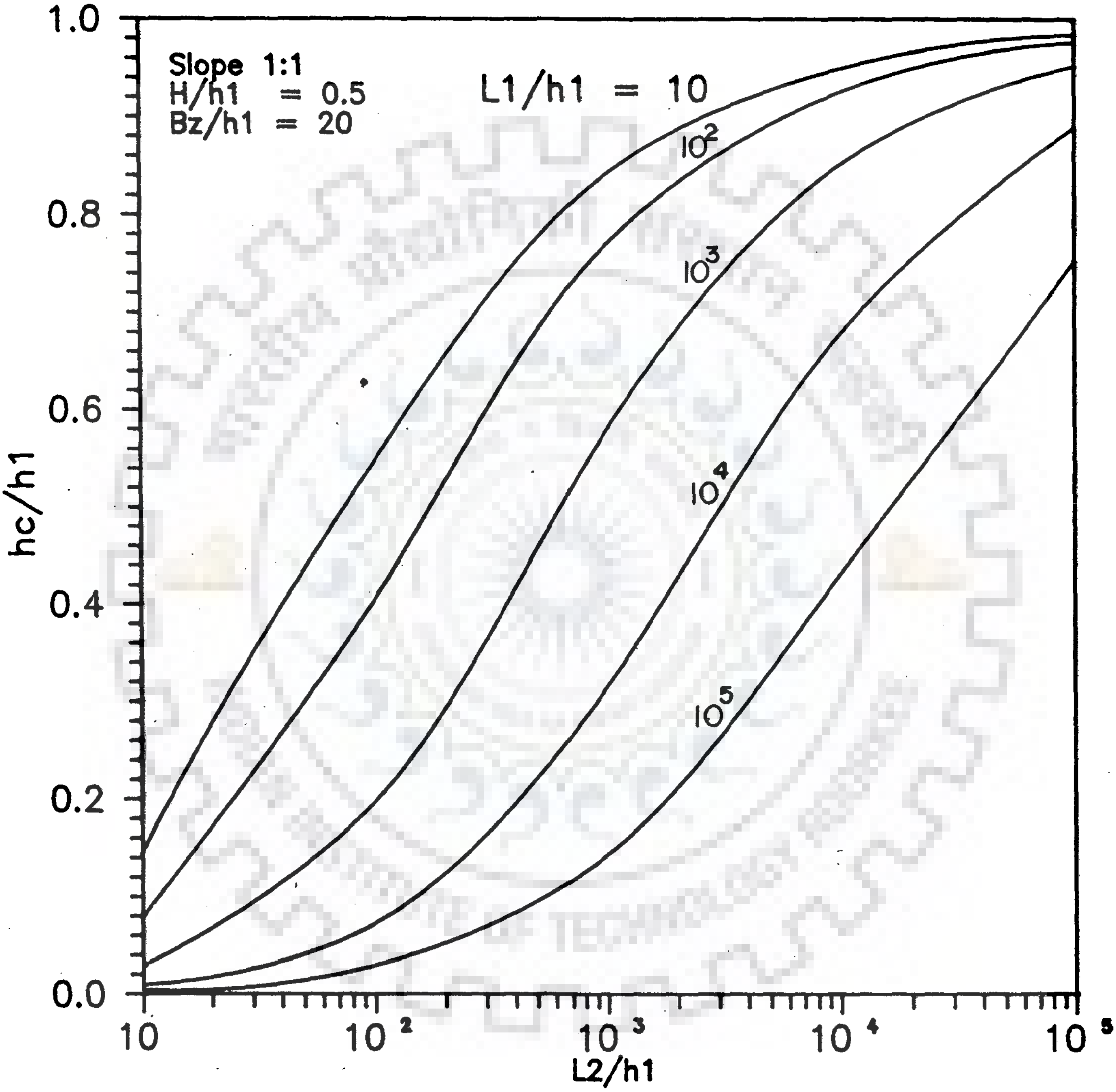


Fig.4.13 CRITICAL LEVEL OF LEFT DRAINAGE BELOW CANAL WATER LEVEL

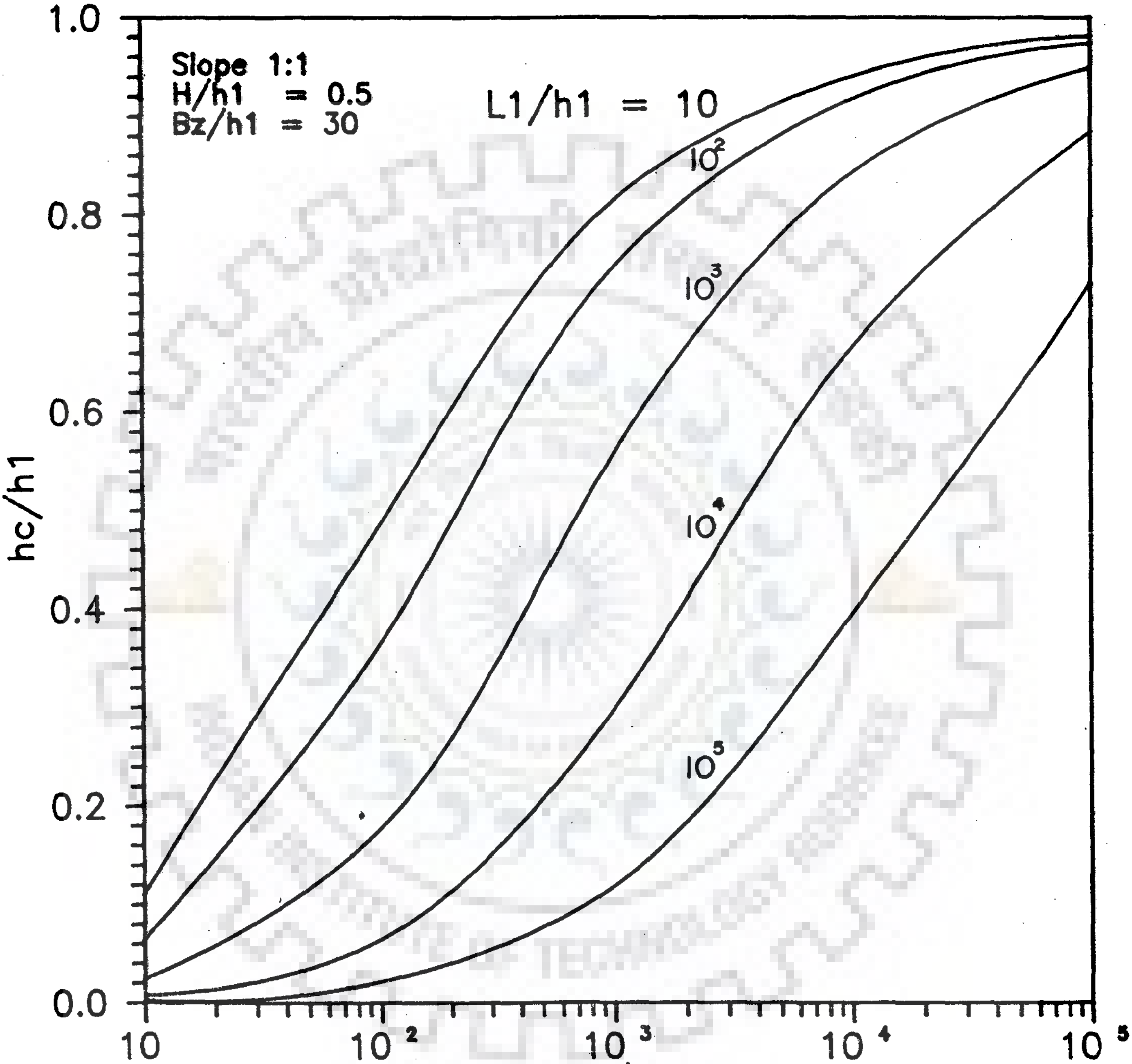


Fig.4.14 CRITICAL LEVEL OF LEFT DRAINAGE BELOW CANAL WATER LEVEL

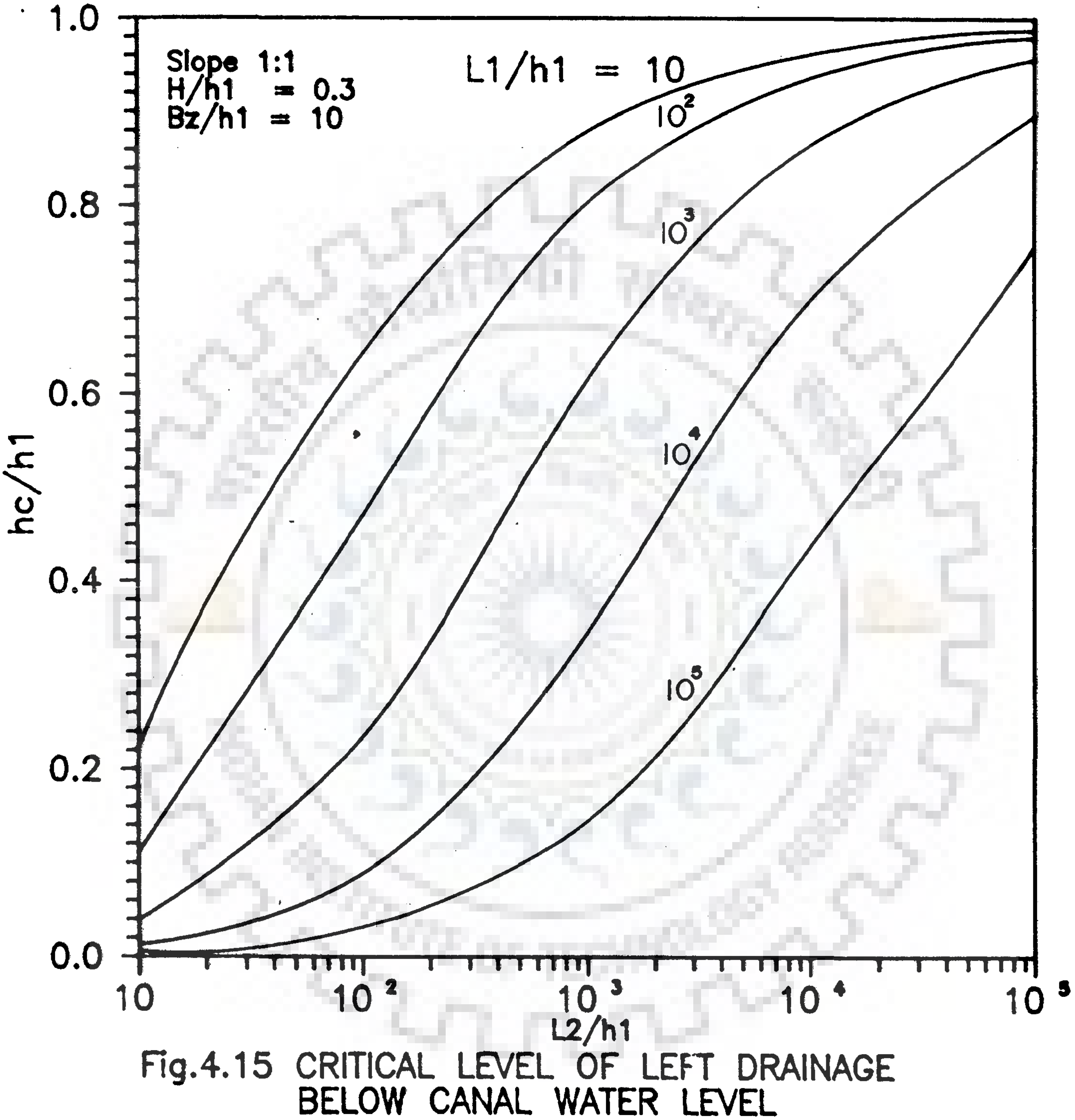


Fig.4.15 CRITICAL LEVEL OF LEFT DRAINAGE BELOW CANAL WATER LEVEL

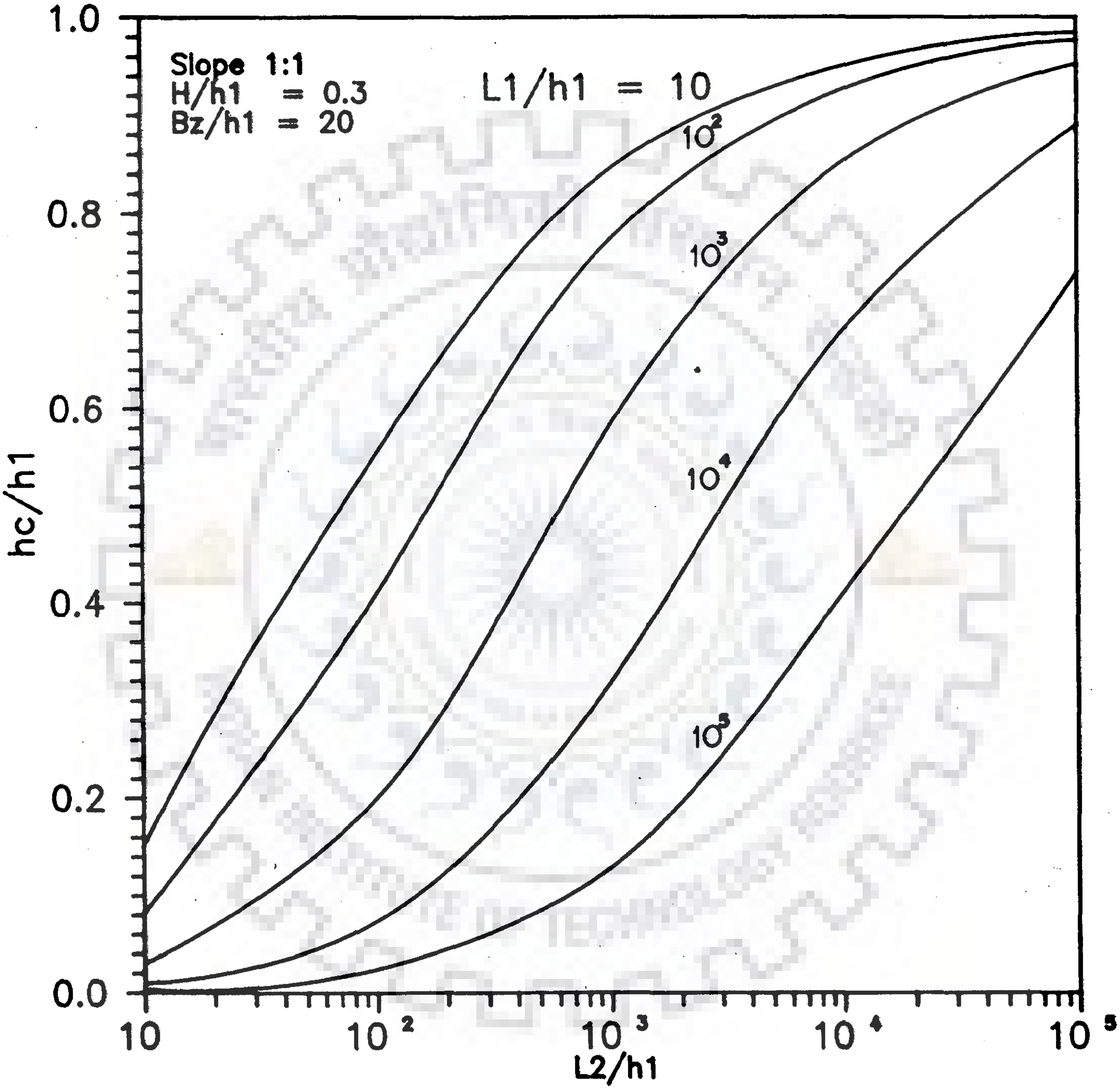


Fig.4.16 CRITICAL LEVEL OF LEFT DRAINAGE BELOW CANAL WATER LEVEL

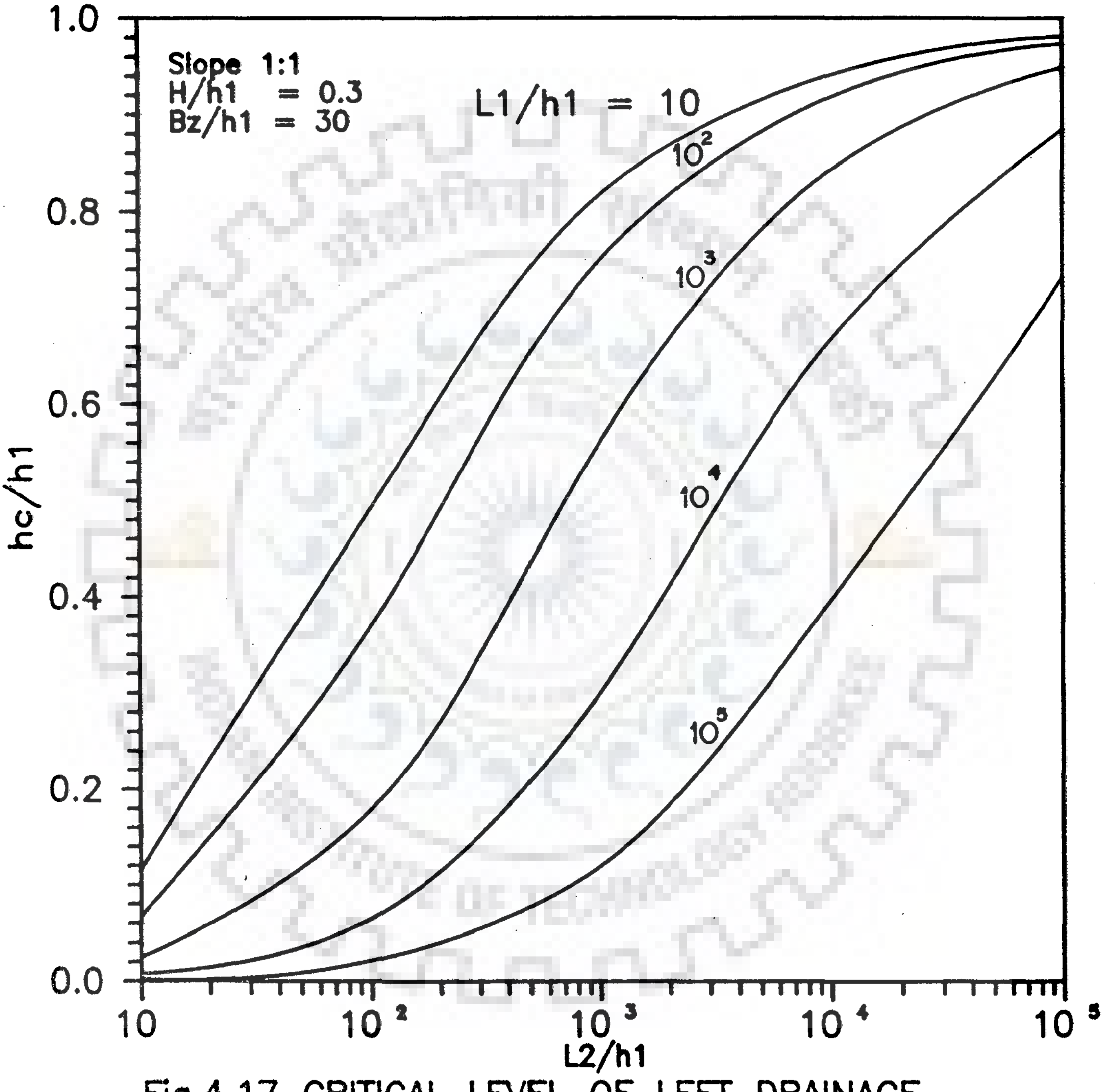


Fig.4.17 CRITICAL LEVEL OF LEFT DRAINAGE BELOW CANAL WATER LEVEL

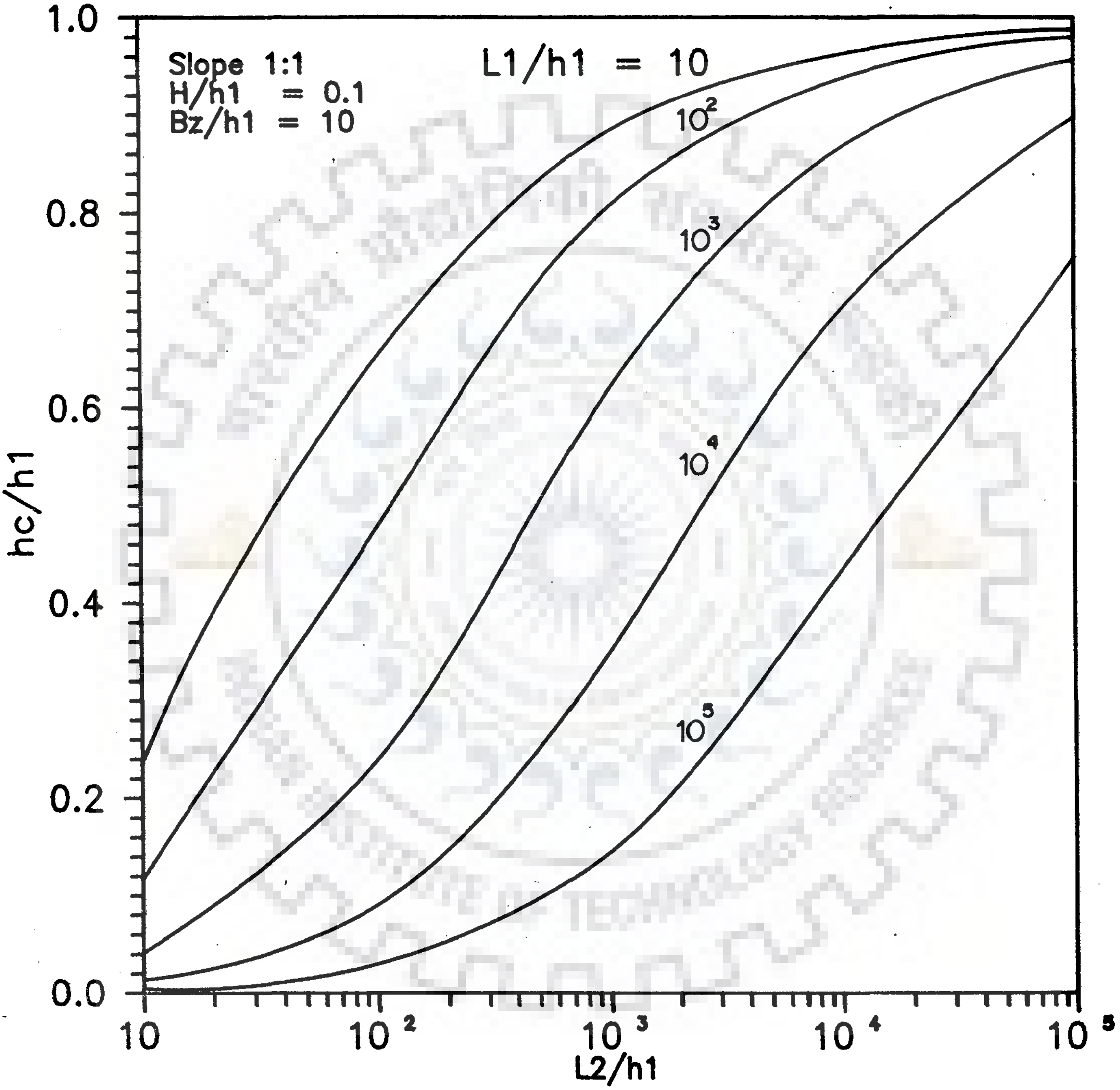


Fig.4.18 CRITICAL LEVEL OF LEFT DRAINAGE BELOW CANAL WATER LEVEL

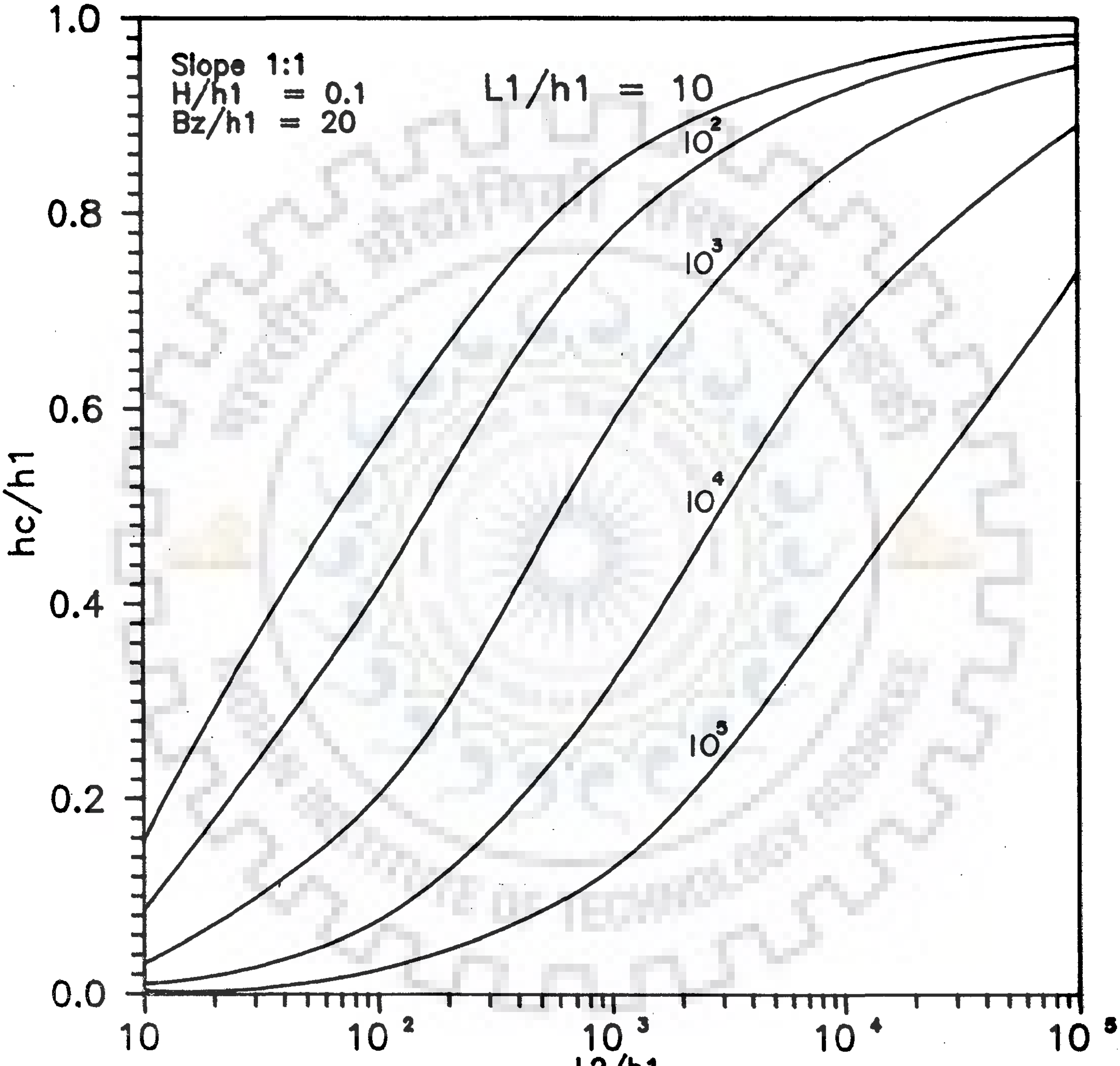


Fig.4.19 CRITICAL LEVEL OF LEFT DRAINAGE BELOW CANAL WATER LEVEL

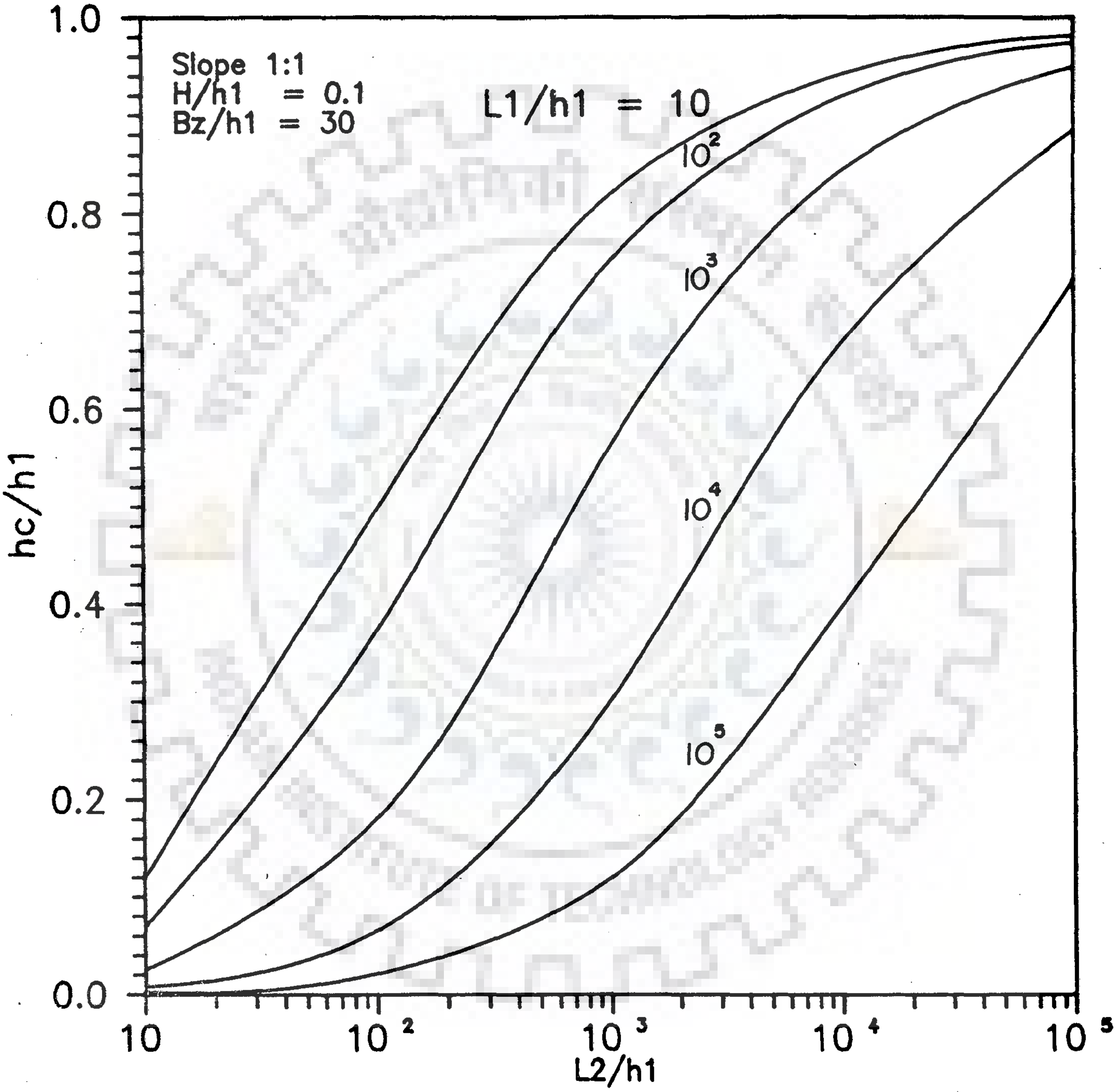


Fig.4.20 CRITICAL LEVEL OF LEFT DRAINAGE BELOW CANAL WATER LEVEL

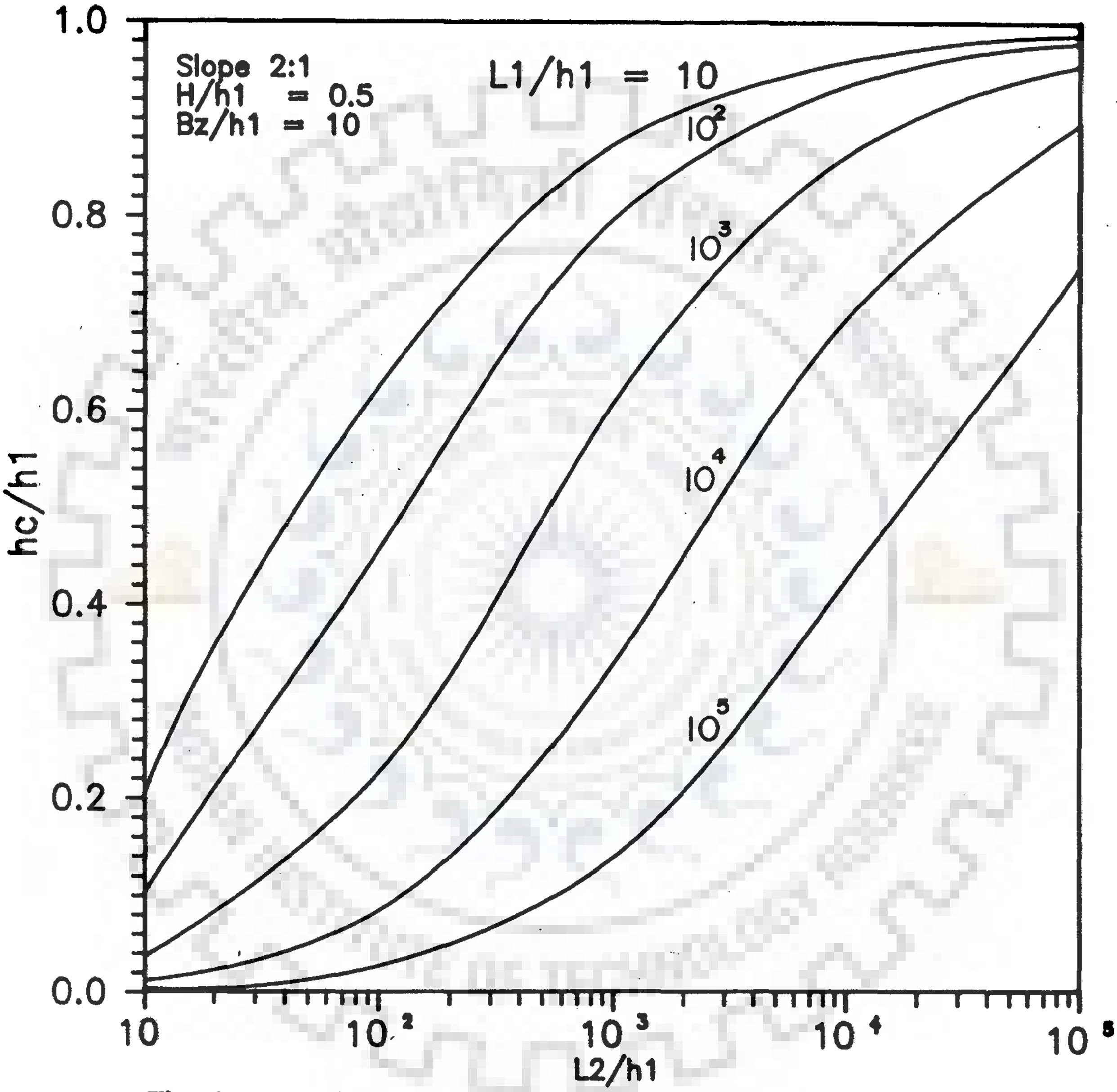


Fig.4.21 CRITICAL LEVEL OF LEFT DRAINAGE BELOW CANAL WATER LEVEL

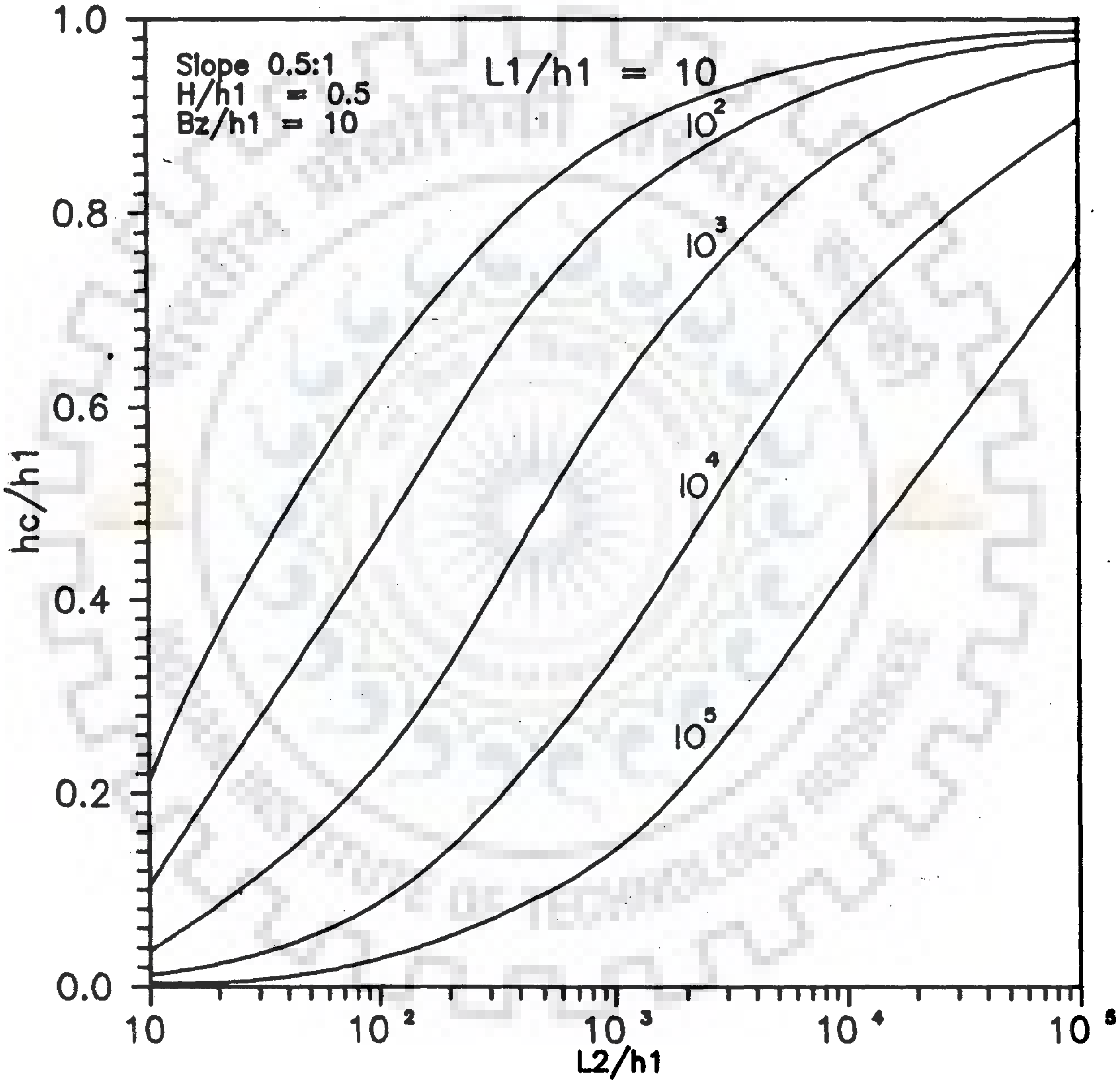


Fig.4.22 CRITICAL LEVEL OF LEFT DRAINAGE BELOW CANAL WATER LEVEL

The dimensionless seepage discharges for side slope 1:1 and for $L_1/h_1 = L_2/h_1 = 10^4$, $B_2/h_1 = 20$ and $h_2/h_1 = 1.0$ are 0.38366, 0.38161 and 0.37930 for H/h_1 = 0.5, 0.3 and 0.1 respectively . When H/h_1 is reduced to 0.0, [refer Table 3.2], the dimensionless seepage discharge for the above physical parameters worked out to be 0.37786.

It is also seen that if level of one of the drainages is raised, total seepage from the canal reduces. The seepage discharge to the raised drainage reduces and that to the drainage, level of which is fixed, increases [refer to Tables 4.8 and 4.11 to 4.13]. Seepage discharge to the drainage at lower level and nearer to the canal is more than that to the drainage at higher level and further distance [refer to Tables 4.8 to 4.15].

The case of symmetric drainages located on either side of canal is a particular case of the present study. The results obtained in this study for symmetrical drainages compare well with that presented by Garg and Chawla [1970]. The comparison is shown in Table 4.26.

The free surface profile is affected by the various parameters such as drainage distance and elevation, canal bed width, water depth and side slope. The effects of drainage distances and elevations and canal bed width on the free surface profile on either side of the canal have been discussed in the previous chapter in which the water depth inside the canal is negligible. Free surface profiles have been plotted in Fig.4.8 for H/h_1 = 2.0, 1.0, 0.5 and 0.25 for fixed values of B_2/h_1 = 5,

Table 4.26 Comparison with Previous Work

[$h_2 = h_1 = h$; $L_1 = L_2 = L$]

	L/B_z = 10 Side slope 2:1			H/h = 0.5 ; B_z/H = 10 and L/H = 50		
	H/B_z			Side slope		
	0.05	0.10	0.2	2:1	1:1	0.5:1
	q/kh			q/kh		
Present work	0.74536	0.77283	0.81792	0.91255	0.88946	0.88098
Garg & Chawla	0.745	0.77	0.81	0.906	0.885	0.875
	Side slope 2:1 and H/h = 0.5					
	L/H = 10			B_z/H = 10		
	B_z/H			L/H		
	5	10	15	10	50	100
	q/kh			q/kh		
Present work	1.24659	1.42712	1.56431	1.42712	0.91253	0.77283
Garg & Chawla	1.23	1.39	1.52	1.39	0.906	0.77

Free surface profiles have been plotted in Fig.4.8 for H/h_1 = 2.0, 1.0, 0.5 and 0.25 for fixed values of B_2/h_1 = 5, $L_1/h_1 = L_2/h_1$ = 100, h_2/h_1 = 1.0 and side slope 2:1. A perusal of the figure indicates that the free surface rises with increase in the value of H/h_1, i.e., with increase in the water depth. The effect of the change in the side slope of the canal on the free surface profile has also been studied and found that it is negligible. For h_2/h_1 = 1.0, H/h_1 = 0.5, B_2/h_1 = 20 and $L_1/h_1 = L_2/h_1$ = 1000, the free surface coordinates for side slopes 2:1, 1:1 and 0.5:1 are given in Table 4.27. Plotting of this results indicated that the profiles of the free surfaces for the above three side slopes are not distinctly separate.

The effect of the drainage distance is shown in Fig.4.9. With H/h_1 = 0.5, L_1/h_1 = 50, h_2/h_1 = 1.0, B_2/h_1 = 10 and side slope 2:1, the free surface rises with increase in the distance of the other drainage, i.e., as L_2/h_1 increases, the free surface rises as given in Fig.4.9(a). The free surface on the side of the drainage, distance of which is increased, also rises with increase in this distance as shown in Fig.4.9(b).

Free surface on the right side of the canal have been plotted in Figs.4.10 for B_2/h_1 = 10, 20 and 30 with fixed values of h_2/h_1 = 1.0, H/h_1 = 0.5, L_1/h_1 = 50, L_2/h_1 = 5 and side slope of 2:1. As shown in the figure that the free surface rises with increase in the bed width of the canal.

In order to study the effect of drainage elevation on the free surface, curves of free surface for h_2/h_1 = 1.0, 0.75 and 0.5 have been plotted in Fig.4.11 in which H/h_1 = 0.5, B_2/h_1 =

Table 4.27 Coordinates of the right-hand side free surface

(Variation with change in canal side slope)

[h_2/h_1 = 1.0 ; H/h_1 = 0.5 ; L_1 = L_2 = 10^2 h_1]

Side slope 2:1		Side slope 1:1		Side slope 0.5:1	
x/L_1	y/h_1	x/L_1	y/h_1	x/L_1	y/h_1
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.050605	0.403231	0.050723	0.405662	0.050822	0.406620
0.100597	0.506103	0.100708	0.508371	0.100802	0.509262
0.150572	0.570076	0.150677	0.572150	0.150766	0.572964
0.200542	0.616939	0.200641	0.618838	0.200725	0.619583
0.250510	0.654180	0.250603	0.655926	0.250682	0.656611
0.300478	0.685296	0.300564	0.686904	0.300638	0.687535
0.350445	0.712212	0.350525	0.713697	0.350593	0.714280
0.400411	0.736112	0.400485	0.737484	0.400548	0.738022
0.450377	0.757782	0.450445	0.759049	0.450503	0.759546
0.500343	0.777782	0.500405	0.778950	0.500458	0.779408
0.550309	0.796535	0.550365	0.797609	0.550412	0.798030
0.600275	0.814383	0.600324	0.815365	0.600367	0.815750
0.650241	0.831621	0.650284	0.832515	0.650320	0.832865
0.700206	0.848535	0.700243	0.849343	0.700275	0.849658
0.750172	0.865431	0.750203	0.866148	0.750229	0.866431
0.800138	0.882682	0.800162	0.883310	0.800183	0.883557
0.850103	0.900839	0.850122	0.901371	0.850138	0.901579
0.900068	0.920886	0.900081	0.921310	0.900092	0.921475
0.950034	0.945277	0.950040	0.945571	0.950045	0.945684
1.000000	1.000000	1.000000	1.000000	1.000000	1.000000

10, L_1/h_1 = L_2/h_1 = 100 and side slope 2:1. In the figure it is seen that the free surface rises with decrease in the value of h_2/h_1, i.e., free surface rises with rise in the elevation of drainage on the other side.

Critical depth of drainage at which it becomes ineffective have been compiled for different values of various parameters. For H/h_1 = 0.5, side slope 0.5:1, 1:1 and 2:1, L_1/h_1 = 10, 10^2, 10^3, 10^4 and 10^5, L_2/h_1 = 10, 10^2, 10^3, 10^4 and 10^5 and B_2/h_1 = 10, 20 and 30, the values of h_c/h_1 have been plotted in Figs.4.12 to 4.22. From these figures it is seen that the values of h_c/h_1 decreases with decrease in the values of L_1/h_1 and L_2/h_1. It is also found that the values of h_c/h_1 for L_1/h_1 = L_2/h_1 = 10^4 and side slope 1:1, H/h_1 = 0.5 are 0.6969, 0.6780 and 0.6652 for values of B_2/h_1 = 10, 20 and 30 respectively. It is therefore seen that the value of h_c decreases with increase in the bed width, i.e., the critical level at which drainage become ineffective is higher for larger bed width. For the value of B_2/h_1 = 20, L_1/h_1 = L_2/h_1 = 10^4, side slope 1:1, the values of h_c/h_1 are 0.6780, 0.6795, 0.6811 and 0.6821 respectively for H/h_1 = 0.5, 0.3, 0.1 and 0.0. Hence, it is seen that the critical depth of the drainage increases with decrease in the water depth of the canal although the effect is very small. For B_2/h_1 = 20, L_1/h_1 = L_2/h_1 = 10^4, H/h_1 = 0., the values of h_c/h_1 are 0.6769, 0.6780 and 0.6785 for side slope 2:1, 1:1 and 0.5:1 respectively which indicates that the critical level of the drainage is lowered with canal side slope becoming steeper.

The seepage discharge from canal takes place from the bed and sides of the channel. The percentage discharge taking place from bed and sides depends on the relative distances and levels of the drainages on either side. With increase in the bed width of the canal, the percentage seepage discharge from the bed increases. For $h_2/h_1 = 1.0$, $L_1/h_1 = L_2/h_1 = 10$ (symmetrical case), $B_2/h_1 = 10$, $H/h_1 = 0.5$ and side slope 1:1, the seepage discharge (q/kh_1) from both side slopes is 28.06 percent of the total seepage from the canal. With increase in the drainage distance on the right-hand side from $10h_1$ to 10^5 h_1, the seepage from both the side slopes is 28.775 percent of total seepage from the canal although seepage from the right side slope decreases from 14.03 to 11.63 percent due to increase in drainage distance on this side and seepage discharge component increases to 17.15 percent from the other side slope. With increase in bed width from $10h_1$ to $20h_1$ and $30h_1$, the dimensionless seepage discharge from bed of the canal increases from 71.94 to 78.555 and 81.51 percent of the total seepage, respectively. It is therefore seen that the percentage increase in seepage discharge from the canal bed is only marginal although bed width is increased to two or three times the original bed width. Keeping other parameters same, the seepage discharge from the side slopes decreases with decrease in water depth. For example, for $L_1/h_1 = L_2/h_1 = 10$, $B_2/h_1 = 10$, $h_2/h_1 = 1.0$ and side slope 1:1, the seepage discharge from the side slopes decreases from 14.06 to 11.794 percent of the total seepage discharge with decrease in water depth from $0.5h_1$ to $0.1h_1$. Change in elevation of the drainage on one side also affects the seepage from either side slope. For example with decrease in h_2 from $1.0h_1$ to $0.7h_1$, i.e. raising of left drainage, the seepage from the side slope towards the raised drainage decreases from 14.03 to 12.66 percent of total seepage from the canal and increases from 14.03 to 15.46 percent from the other side slope (Total from both side slope is 28.11 percent against original value of 28.06 percent). As expected the seepage from side slopes increases with flattening of side slope.

CHAPTER 5

SEEPAGE FROM TRAPEZOIDAL CANAL TO DRAINAGE ON ONE SIDE ONLY

Introduction

Effectiveness of drainage depends upon its distance from canal and its depth below canal water level. In case the drainage is located at very large distance it will become ineffective and the other drainage located at nearer distance will receive entire seepage from canal. Similarly, as discussed earlier, if level of the drainage is raised above certain level, it becomes ineffective. Solution of the problem of seepage from a canal of negligible water depth to a collector drainage on one side of the canal is available [Polubarinova-Kochina [1962, pp.131-132] and is presented in Appendix C. Among other factors, the seepage losses from unlined canals depend on the shape of the canal cross section and the depth of water in the canal as well. Presently, no solution is available for the seepage from a trapezoidal canal to a collector drainage on one side of the canal. Solution of this problem is given in this chapter. In the derivation, the above factors are included and exact solution of the problem of seepage from a trapezoidal canal in homogeneous medium to a collector drainage located at finite distance is presented. On the side where there is no collector drainage the free surface spreads out and approaches the level of the water in the drainage which is located at finite distance on the other side of the canal.

5.1 Boundary conditions.

In the z-plane [Fig.5.1(a)], AA'B'B is an equipotential line and corresponds to $\phi = 0$. Along the phreatic line BC which is a stream line, the value of ψ is taken to be zero. For the phreatic line AF, ψ has been assigned a value equal to q, where q is the unknown seepage loss per unit length of the canal. Along the drainage CD, which is an equipotential line, ϕ is equal to kh, in which h is the difference between water levels of the canal and the drainage. The distance of the drainage is L from the right side of the canal, i.e from the water line of the canal as shown in Fig.5.1(a).

5.2 Solution of problem.

Transformations : The physical plane and the θ-plane in this case are similar to those obtained in Chapter 4 except that Points E/E' merge with Point F. Therefore, transformation of the z-plane [Fig.5.1(a)] onto the θ-plane [Fig.5.1(b)] and θ-plane onto ζ-plane [Fig.5.1(c)] are similar to those obtained in Chapter 4. In another operation, the w-plane is mapped on the t-plane [Figs.5.1(d) and 5.1(e)]. This mapping is similar as given in Appendix C. Table 5.1 summarizes the location of the points in the different planes. The bilinear transformation that maps the ζ-plane onto the t-plane is identical as that given by Eq.4.68 and it is reproduced below.

$$t = \sigma\zeta \qquad (5.1)$$

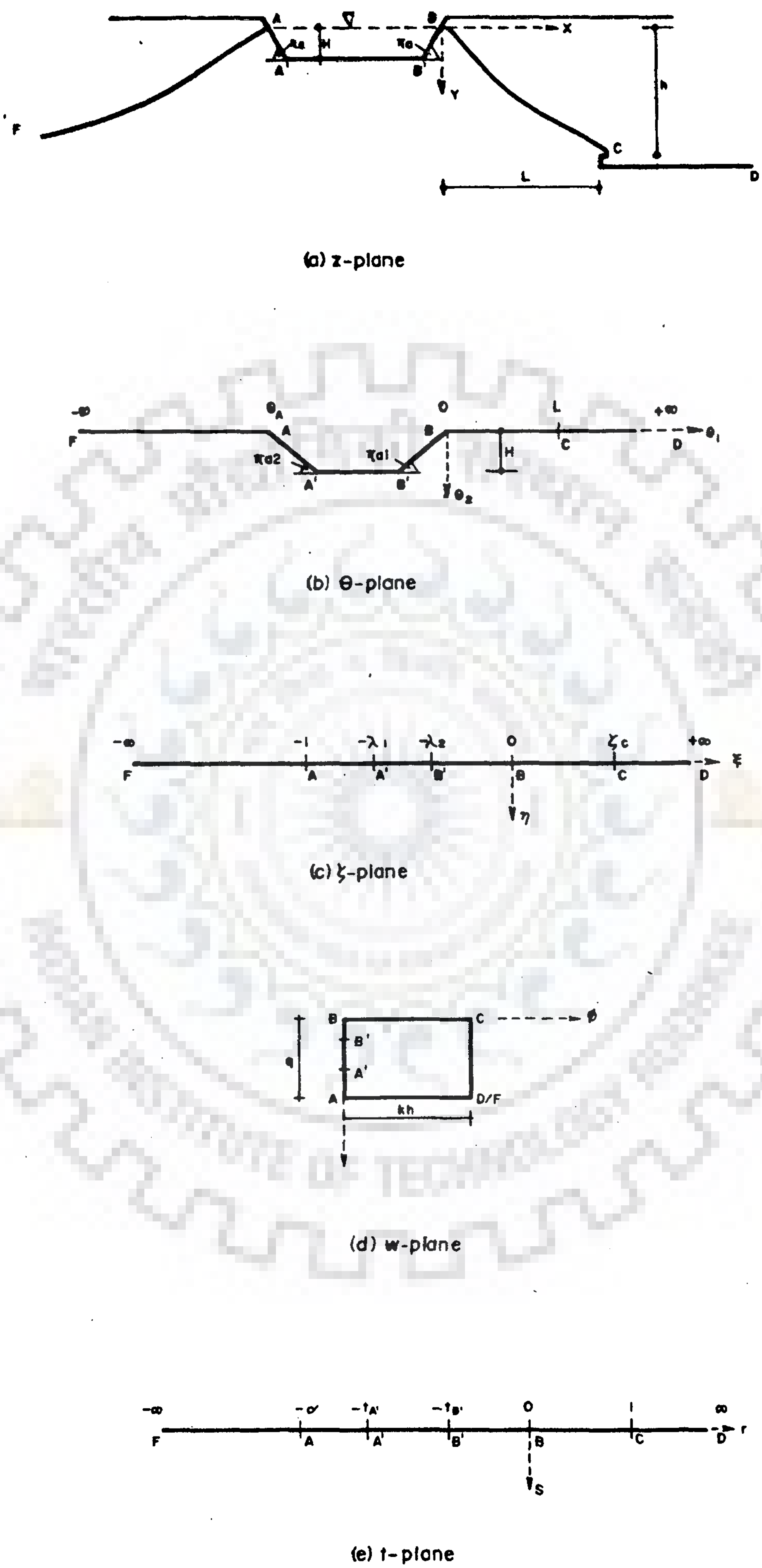


FIG. 5.1 TRANSFORMATION LAYOUT

Table 5.1(a) Points Location on the z, θ and ζ- Planes

POINT	z - plane		θ - plane		ζ - plane	
	x	y	θ_1	θ_2	ξ	η
A	$-Bz + 2H\cot(\pi a)$	0	θ_A	0	−1	0
A'	$-Bz + H\cot(\pi a)$	H	θ_A'	H	$-\lambda_1$	0
B'	$-Bz + H\cot(\pi a)$	H	θ_B'	H	$-\lambda_2$	0
B	0	0	0	0	0	0
C	L	h	L	0	ζ_c	0
D	∞	h	∞	0	∞	0
F	$-\infty$		$-\infty$	0	$-\infty$	0

Table 5.1(b) Points Location on the w and t - planes

POINT	w - plane		t - plane	
	ϕ	ψ	r	s
A	0	q	$-\sigma$	0
A'	0	ψ_A'	$-t_A'$	0
B'	0	ψ_B'	$-t_B'$	0
B	0	0	0	0
C	kh	0	1	0
D	kh	q	∞	0
F	kh	q	$-\infty$	0

Similar to Eqs.4.5 and 4.8 the following were obtained.

$$\cot(\pi a_1) = \cot(\pi a) - \psi_B'/kH \tag{5.2}$$

$$\cot(\pi a_2) = \cot(\pi a) - (q-\psi_A')/kH \tag{5.3}$$

Also, from Eq.4.70, the following relationship was found.

$$L/H = \operatorname{cosec}(\pi a_1)[I_1]/[(\lambda_2)I_2] \tag{5.4}$$

in which,

$$I_1 = \int_0^{\sinh^{-1}\sqrt{1/\sigma}} [\sinh^2 u+\lambda_2]^{a_1} [\sinh(u)]^{1-a_1} [\sinh^2(u)+\lambda_1]^{a_2} [\cosh(u)]^{1-2a_2} du \tag{5.5}$$

$$I_2 = \int_0^{\pi/2} \left[\sin(u)\right]^{1+2a_1} \left[\cos(u)\right]^{1-2a_1} \left[\frac{\lambda_1 - \lambda_2 \cos^2 u}{1 - \lambda_2 \cos^2 u}\right]^{a_2} du \tag{5.6}$$

From Eq.4.74,

$$B_z/H = 0.5 \operatorname{cosec}(\pi a_1)\ [I_4/\{(\lambda_2)I_2\}] + [(\psi_A' - \psi_B')/kh][h/H] \tag{5.7}$$

in which,

$$I_4 = M_2 \int_{-\lambda_1}^{-\lambda_2} \left[\frac{\zeta + \lambda_2}{\zeta}\right]^{a_1} \left[\frac{\zeta + \lambda_1}{\zeta + 1}\right]^{a_2} d\zeta \tag{5.8}$$

The abscissa, x, of the point on the free surface BC, is as given below [refer Eq.4.82]

$$x/L = I_7/I_1 \tag{5.9}$$

in which,

$$I_7 = \int_0^{\sinh^{-1}\sqrt{\zeta}} [\sinh^2 u + \lambda_2]^{a_1} [\sinh(u)]^{1-2a_1} [\sinh^2(u) + \lambda_1]^{a_2} [\cosh(u)]^{1-2a_2} du$$

. . . (5.10)

and $0 \le \zeta \le 1/\sigma$.

The horizontal distance, X_L, of a point on the free surface AF, from Point A is as given below [refer Eqs.4.25, 4.26, 4.27, and 4.29(a)]

$$X_L/L = I_8/I_1 \qquad (5.11)$$

in which,

$$I_8 = \int_0^{\cosh^{-1}\sqrt{-\zeta}} [\cosh^2(u) - \lambda_2]^{a_1} [\cosh(u)]^{1-2a_1} [\cosh^2 - \lambda_1]^{a_2} [\sinh(u)]^{1-2a_2} du$$

. . . (5.12)

and $-1 \ge \zeta > -\infty$.

Mapping of the W-plane onto the t-plane : There is no major change in the shapes of the rectangular flow fields presented in the w-plane for the cases of the canal with negligible water depth [Fig.C.1(c)], and the present case in which the shape of the canal and depth of the water in the canal are taken into consideration [Fig.5.1(d)]. However, for convenience the rectangular flow field, Fig.5.1(d), is mapped onto another auxiliary half-plane, t-plane, as shown in Fig.5.1(e) whereas the θ-plane is mapped on the intermediate ζ-plane. This mapping is done using the Schwarz-Christoffel conformal transformation. The Point A in the t-plane is located at $-\sigma$ and its value should be determined.

The rectangular flow field in Fig.5.1(d) is opened at Point F/D and mapped from $-\infty$ to $+\infty$ along the real axis of the t-plane. Points A, B and C are mapped at $t = -\sigma$, 0 and 1 respectively. Points A' and B' are mapped at $t = -t_{A'}$ and $-t_{B'}$ respectively. The Schwarz-Christoffel transformation equation is as given below :

$$\int dw = M_3 \int \frac{dt}{\sqrt{(t+\sigma)(t)(t-1)}} \tag{5.13}$$

where M_3 is a complex constant.

It is observed that Eq.5.13 and Eq.C.5 are similar. Therefore, the same solutions of Eq.C.5 in various segments of the t-plane hold true for Eq.5.13 as well. Hence, the resulting equations obtained on integrating Eq.5.13 along different segments of the t-plane are as given below.

The ordinate, y, of a point on the free surface BC is given by the following equation.

$$-ky = M_3 \; F(\phi_5, m) \tag{5.14}$$

in which,

$$\phi_5 = \sin^{-1}\sqrt{(1+\sigma)t/(t+\sigma)} \quad ; \quad 0 \le t < 1 \tag{5.15}$$

At Point C, y = h, t = 1. Substituting these in Eqs.5.15 and Eq.5.14, it is found that $\phi_5 = \pi/2$ and

$$kh = -M_3 \; K(m) \tag{5.16}$$

in which, $K(m)$ is the complete elliptic integral of the first kind with modulus m and

$$m^2 = 1/(1+\sigma) \quad (5.17)$$

Dividing Eq.5.14 by Eq.5.16, the following non-dimensional relationship for the ordinate of a point on the free surface BC was found.

$$y/h = F(\phi_5,m)/K(m) \quad (5.18)$$

The abscissa, x, of the point on the free surface BC can be found by making use of Eq.5.9. The relationship between the corresponding values of the point in the t-plane and ζ-plane are as given by Eq.5.1.

The ordinate, y, of the point on the free surface AF having a value of t in the t-plane can be expressed as follows [refer Eq.C.7 and Eq.5.16].

$$y/h = F(\phi_4,m)/K(m) \quad (5.19)$$

in which,

$$\phi_4 = \sin^{-1}\sqrt{(\sigma+t)/t} \quad ; \quad -\infty < t \leq -\sigma \quad (5.20)$$

The value of the abscissa of the above point can be found using Eq.5.11. The corresponding values of the point in the t-plane and ζ-plane can be found from Eq.5.1.

At Point F on the free surface AF, $t = -\infty$ and Eq.5.20 gives $\phi_4 = \pi/2$. Substituting this in Eq.5.19 and dividing the resulting equation by Eq.5.16, the equation for the ordinate of the free surface at the farthest point, Point F, i.e $y = Y_F$, as given below was obtained.

$$Y_F/h = 1 \quad (5.21)$$

As $\zeta = -\infty$ at this point, and if this is substituted in Eq.5.11, the corresponding abscissa of the point on the free surface works out to be $x = -\infty$. So, Eq.5.21 states that at infinity, the free surface on the left-hand side approaches the level of water in the collector drainage.

The stream function value at any point along AA'B'B , say ψ', is given by the following equation [refer Eq.C.21].

$$(q - \psi') = -M_B\, F(\phi_\sigma, m') \qquad (5.22)$$

in which,

$$\phi_\sigma = \sin^{-1}\sqrt{(t+\sigma)/\sigma} \qquad (5.23)$$

$$m' = \sigma/(1+\sigma) = \sqrt{(1 - m^* m)} \qquad (5.24)$$

Dividing Eq.5.22 by Eq.5.16, the following non dimensional equation was obtained.

$$q/kh - \psi'/kh = F(\phi_\sigma, m')/k(m) \qquad (5.25)$$

At Point B, $t = 0$ and $\psi' = 0$. Hence, substituting these in Eq.5.23 and Eq.5.25, the following expression for the seepage quantity, q, from the canal was obtained.

$$q/kh = F(\pi/2, m')/K(m)$$

$$= K(m')/K(m) \qquad (5.26)$$

At Point A', $\zeta = -\lambda_1$ and from Eq.5.1, $t = -\sigma\lambda_1$. Putting $\psi' = \psi_{A'}$ and substituting these in Eqs.5.23 and 5.25,

$$\psi_{A'}/kh = q/kh - F(\beta_{A'}, m')/K(m) \qquad (5.27)$$

in which,

$$\beta_{A'} = \sin^{-1}\sqrt{(1-\lambda_1)} \qquad (5.28)$$

At Point B′, $\zeta = -\lambda_2$ and from Eq.5.1, $t = -\sigma\lambda_2$ and putting $\psi' = \psi_B'$ and substituting these in Eqs.5.23 and 5.25,

$$\psi_B'/kh = q/kh - F(\beta_B', m')/K(m) \tag{5.29}$$

in which,

$$\beta_B' = \sin^{-1}\sqrt{(1-\lambda_2)} \tag{5.30}$$

5.3 Results and Discussions

As discussed in the previous chapter, it is difficult to obtain a direct relationship between z and w. Therefore, the procedure followed in computations was to assume the values of parameters in intermediate plane and then determine the values of w corresponding to these parameters and determine the various dimensions in physical plane such as Bz/h, L/h, H/h and side slopes. In case specific values of these parameters in physical plane are desired, few iterations are required by way of adjustments in the values of intermediate parameters such as σ, λ_1 and λ_2. Coordinates of the free surface on the right-hand side, i.e. BC, are determined from Eqs.5.9 and 5.18 for various values of t ($0 \le t \le 1$). Similarly, coordinates of the free surface on the left-hand side, i.e AF, are obtained from Eqs.5.11 and 5.19 for various values of t ($-\infty < t \le -\sigma$).

The values of seepage discharge had been calculated for the various combinations of the following physical parameters; Bz/h = 10, 20 and 30 ; L/h = 10, 10^2, 10^3, 10^4 and 10^5; H/h = 0.0, 0.1, 0.3 and 0.5 and side slope 1:1. The results are given in Table 5.2 and are plotted as shown in Fig.5.2.

Table 5.2 Seepage from Canal with Drainage on One Side

[Side slope 1:1]

L/h →	10	10^2	10^3	10^4	10^5
Bz/h	q/kh				
H/h = 0.0					
10	0.97679	0.60614	0.42307	0.32342	0.26160
20	1.15473	0.69702	0.46764	0.34884	0.27792
30	1.26736	0.76002	0.49784	0.36549	0.28838
H/h = 0.1					
10	0.99511	0.61328	0.42650	0.32539	0.26287
20	1.16816	0.70191	0.46986	0.35006	0.27870
30	1.27891	0.76398	0.49957	0.36642	0.28895
H/h = 0.3					
10	1.02237	0.62424	0.43179	0.32845	0.26485
20	1.18856	0.70971	0.47341	0.35203	0.27993
30	1.29641	0.77040	0.50239	0.36794	0.28988
H/h = 0.5					
10	1.04572	0.63372	0.43635	0.33106	0.26654
20	1.20628	0.71664	0.47656	0.35377	0.28104
30	1.31162	0.77616	0.50492	0.36930	0.29073

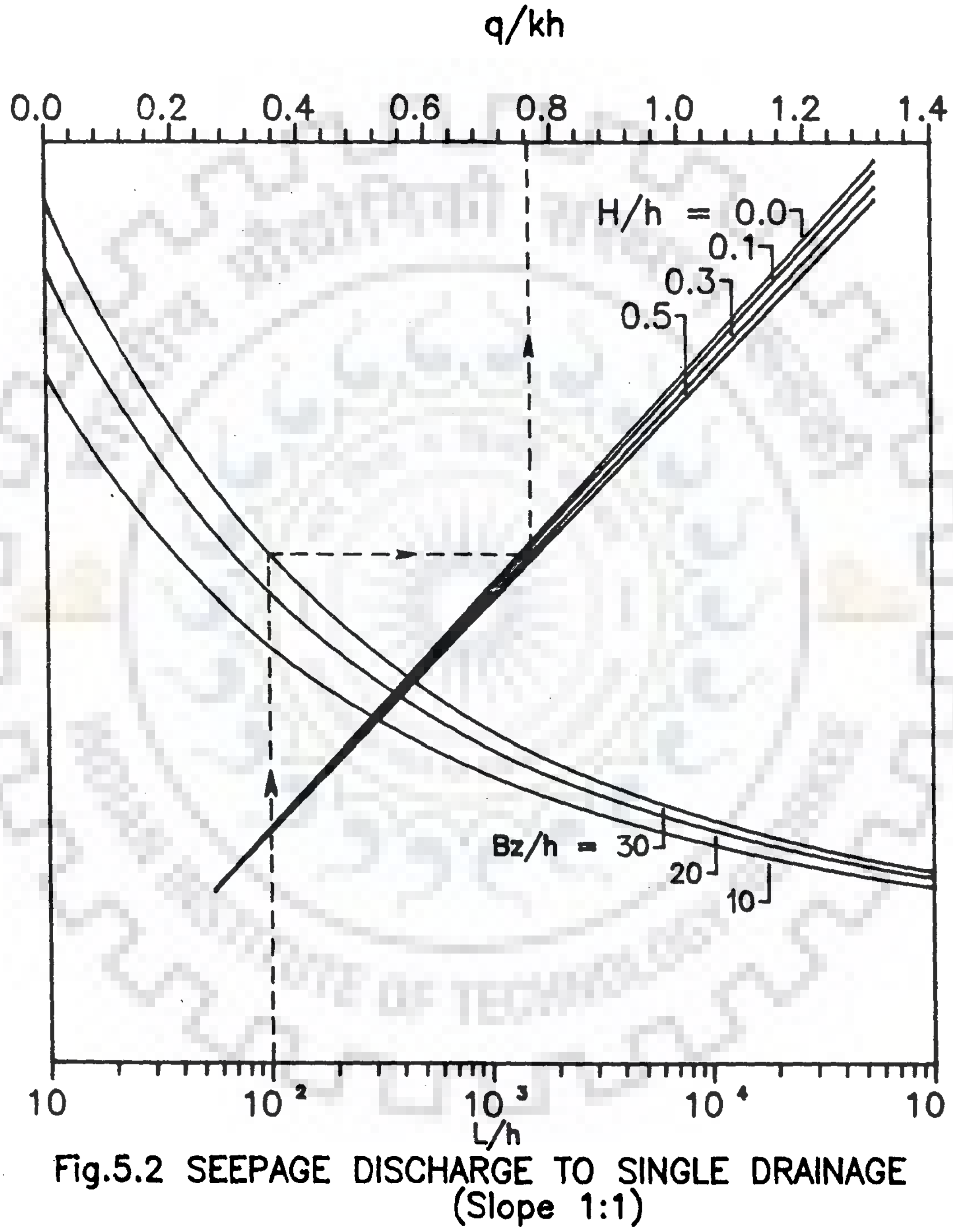


Fig.5.2 SEEPAGE DISCHARGE TO SINGLE DRAINAGE (Slope 1:1)

A comparison of seepage losses from trapezoidal canal with collector drainage on both sides (Table 3.2 and 4.2 to 4.5) and those from trapezoidal canal with collector drainage only on one side (Tables 5.2) indicates that seepage losses for the former case approach the values of seepage for the latter case if the value of $L_2/L_1 \geq 10^5$. In case the farther side collector drainage is located at a distance 10 times or more than the nearer drainage (say, $L_2/L_1 \geq 10$), the error in the seepage losses calculated by ignoring the farther drainage worked out to be less than 10 percent as compared to that obtained by considering both drainages. The magnitude of this error depends upon the bed width and distances of the drainages. A further perusal of Table 5.2 indicates that the seepage losses increase with increase in the water depth of the canal. For example, the seepage losses (q/kh) are 0.977, 0.995, 1.022 and 1.046 for H/h = 0.0, 0.1, 0.3 and 0.5 respectively for Bz/h = 10, L/h = 10 and side slope 1:1. For $L/h = 10^5$, Bz/h = 10, the seepage losses (q/kh) are 0.2616, 0.26287, 0.26485 and 0.26654 for H/h = 0.0, 0.1, 0.3 and 0.5 respectively. It is therefore seen that effect of water depth on the seepage losses reduces as L/h increases. Table 5.3 gives values of seepage losses from sides and bottom of the canal for L/h = 10, 10^2, 10^3, 10^4, and 10^5; Bz/h = 10, 20 and 30; H/h = 0.5 and side slope 1:1. It is seen from the table that the seepage from the side slopes on the drainage side (right side in this case) are higher than those from the other side. However, this differences reduce as $L/h \geq 10^3$.

Table 5.3 Seepage Discharge Components through Canal Profile
[Drainage on One Side :- slope 1:1 and H/h = 0.5]

L/h →	10	10^2	10^3	10^4	10^5
Note that Seepage component values below are in terms of kh					
Bz/h = 10					
R	0.17356	0.09197	0.06218	0.04760	0.03845
L	0.11813	0.08709	0.06218	0.04758	0.03845
B	0.75403	0.45466	0.31163	0.23588	0.18964
Bz/h = 20					
R	0.16328	0.07825	0.05045	0.03749	0.02987
L	0.09105	0.07094	0.04991	0.03745	0.02987
B	0.95195	0.56745	0.37620	0.27883	0.22130
Bz/h = 30					
R	0.15919	0.07176	0.04448	0.03249	0.02565
L	0.07701	0.06250	0.04379	0.03244	0.02564
B	1.07542	0.6419	0.41665	0.30437	0.23944

NOTE :- R = seepage component through right side slope,
L = seepage component through left side slope and
B = seepage component through canal bottom width.

The effect of drainage distance, bed width and water depth on the free surfaces on the drainage side and on the other side of the canal had also been studied.

Free surface profiles on the right-hand side and left-hand side have been plotted in Figs.5.3 and 5.4 respectively, for B/h = 10, H/h = 0.5, side slope 2:1 and L/h = 10, 50 and 100. It is seen that the free surface is higher for the larger value of L/h. Comparing the rises in the free surfaces at the right and left side of the canal, it is observed that as the drainage distance increases, the rises on the free surface, on the side on which the drainage is located, is very high. But, the free surface on the other side, i.e. in this case the left-hand side, drops steeply at the vicinity of the canal and then drops gently and does not reach the level of the drainage at the corresponding distance, i.e. L, from the canal as in the case of the right-hand side free surface. Moreover, the free surface on the drainage side is s-shaped with steep slope near the canal and the drainage with a point of contraflexture in between whereas on the left-hand side the free surface drops steeply near the canal and the slope becomes flatter as distance from the canal increases.

For the study of the effect of the bed width on the free surfaces on the left and right side of the canal, three bed width values of B_2 = $10h_1$, $20h_1$ and $30h_1$ were considered [Fig.5.5 and 5.6]. The canal side slope at 2:1, the water depth in the canal at H = 0.5h and the drainage distance at L = 50h were fixed.

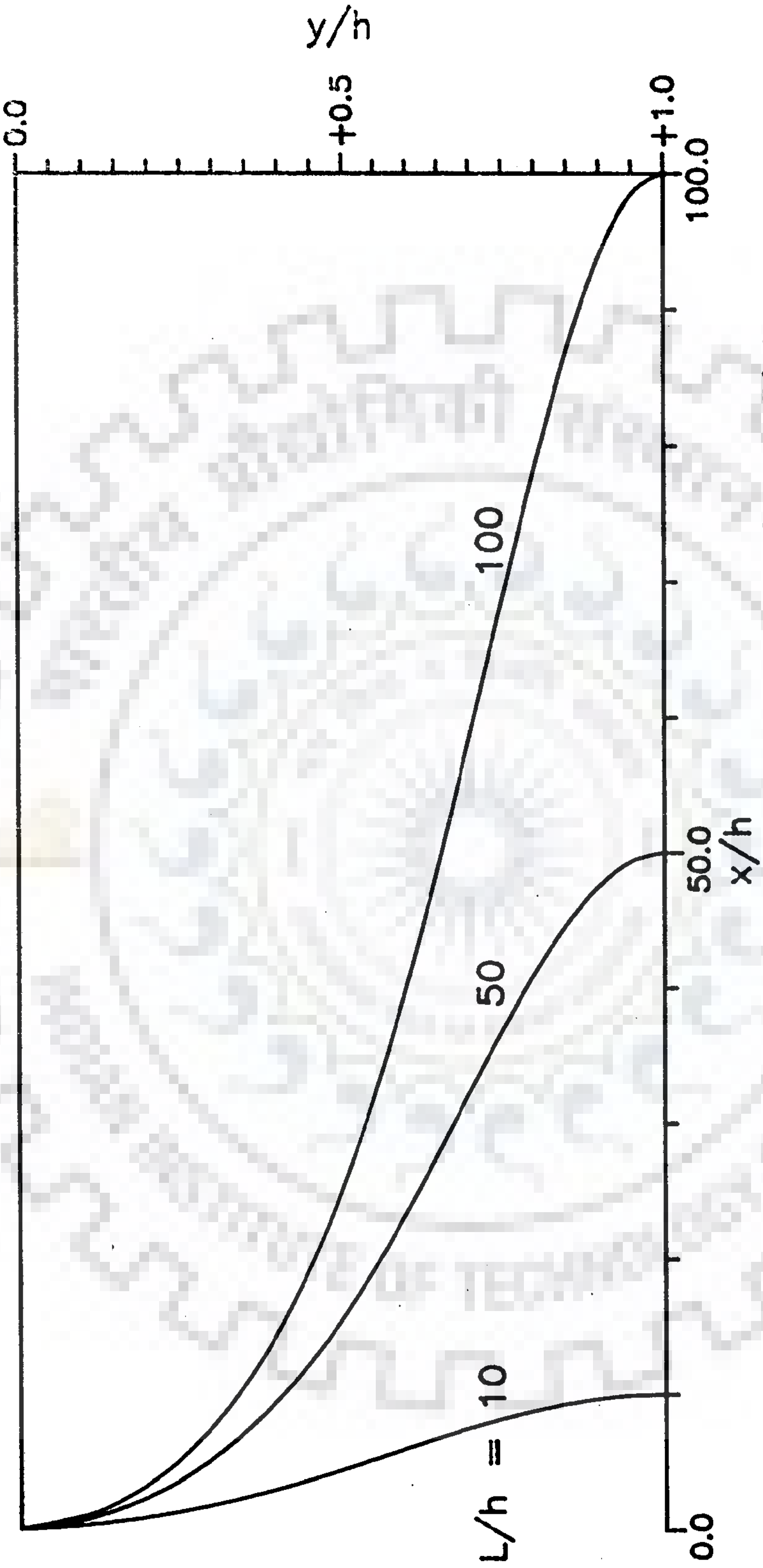


Fig.5.3 RIGHT FREE SURFACE — Effect of Drainage Distance.
(Slope 2:1, H/h = 0.5, Bz/h = 10)

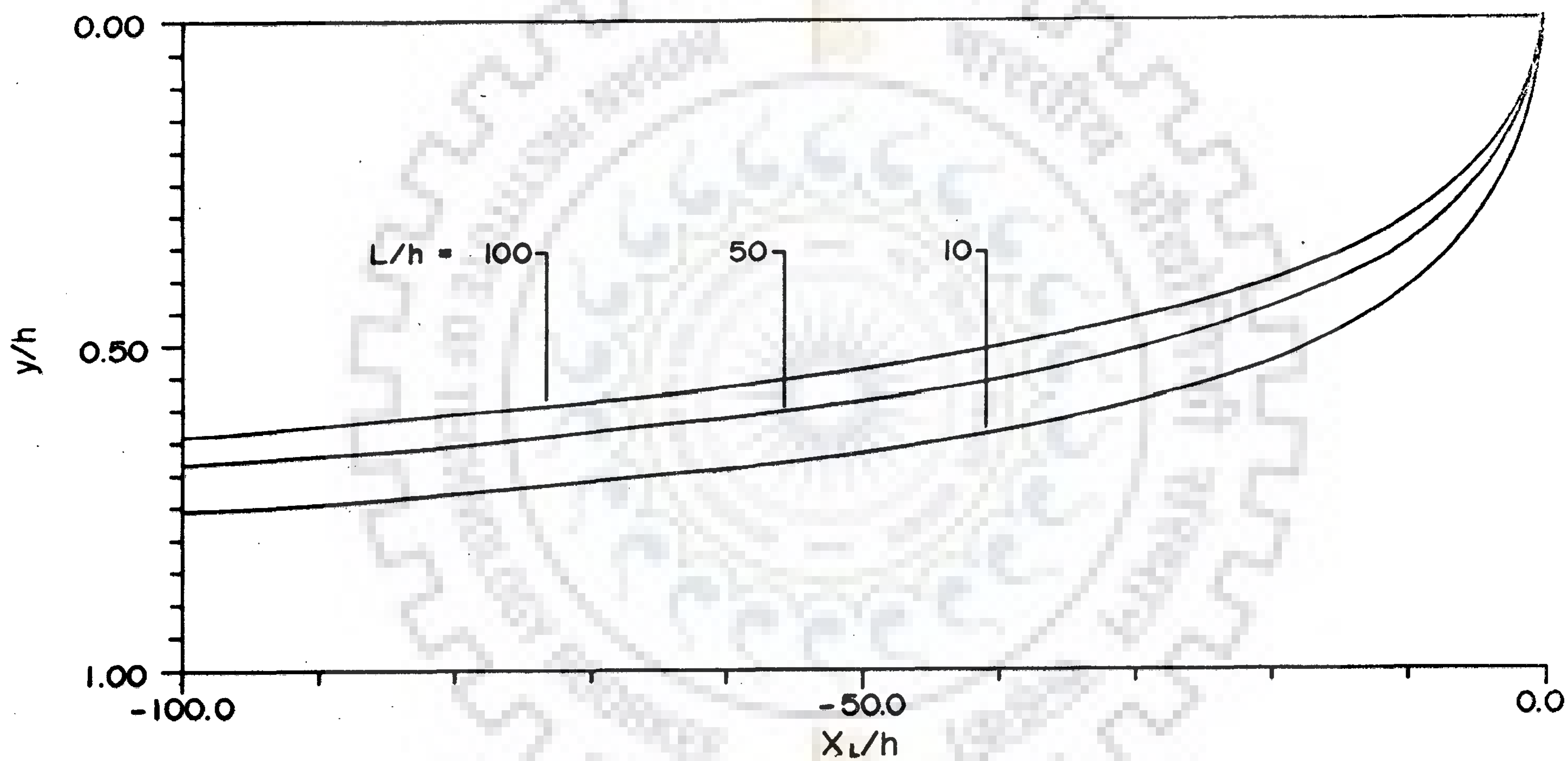


Fig.5.4 LEFT FREE SURFACE - Effect of Drainage Distance.
(Slope 2:1. H/h = 0.5. Bz/h = 10)

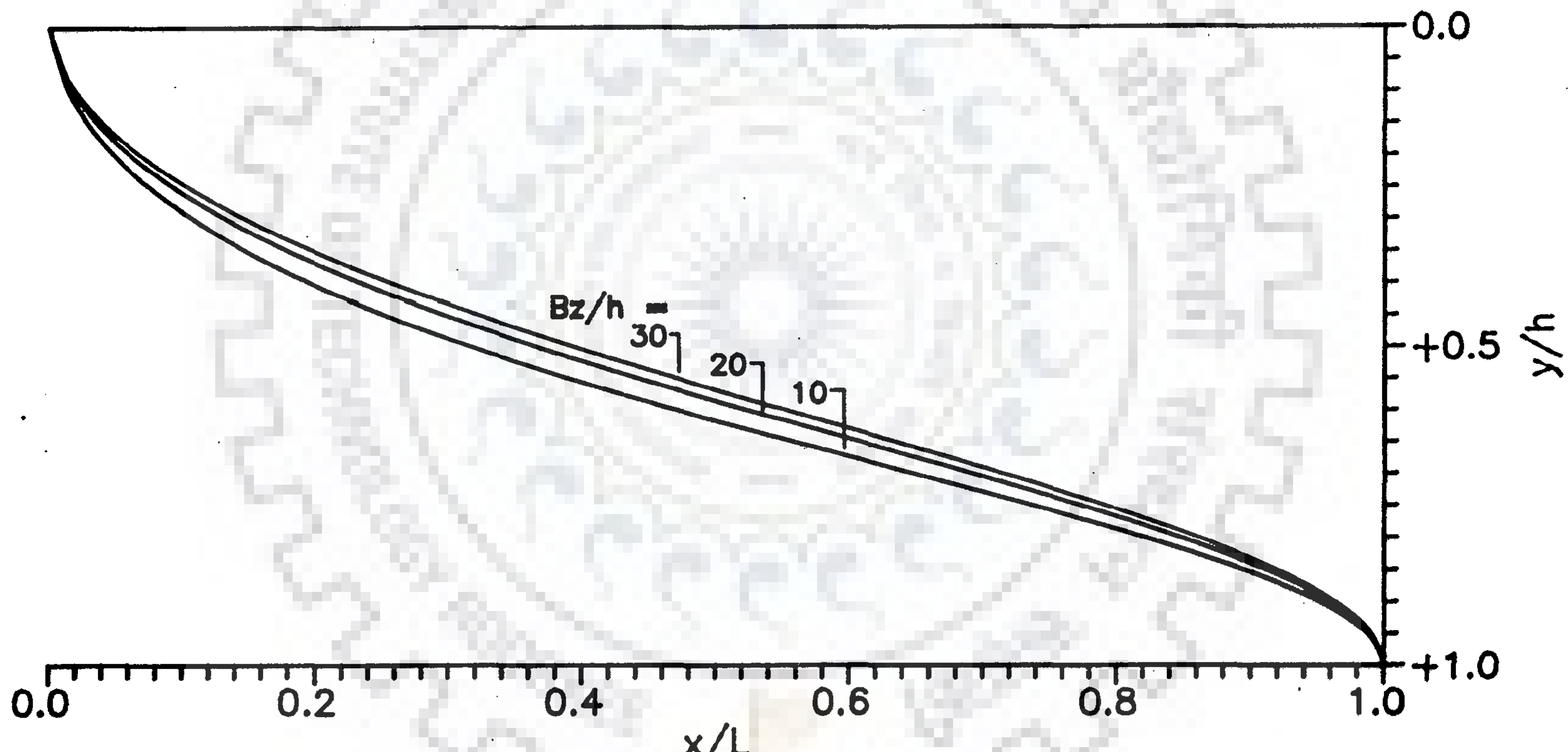


Fig.5.5 RIGHT FREE SURFACE – Effect of Bed Width
(Slope 2:1, H/h = 0.5, L/h = 50)

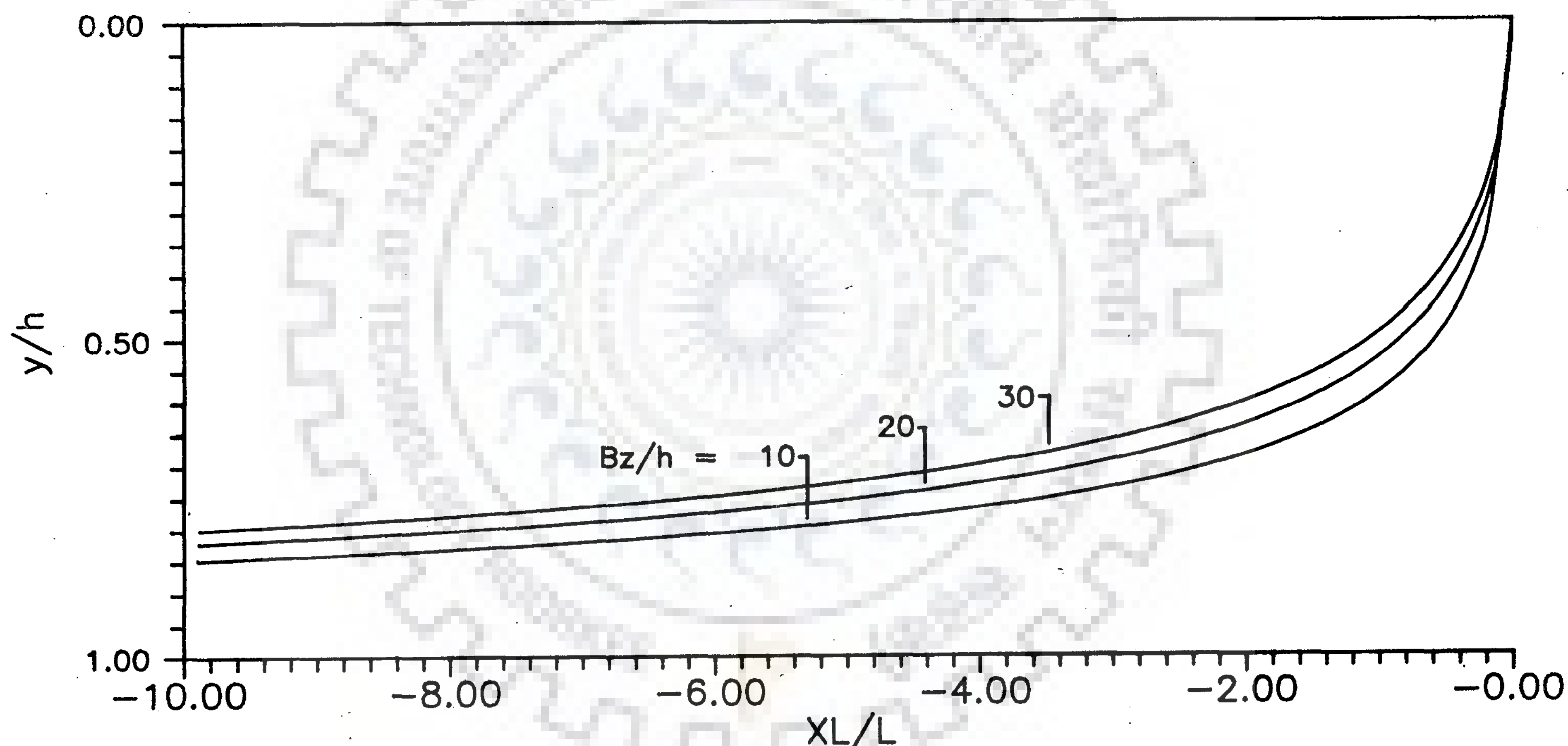


Fig.5.6 LEFT FREE SURFACE – Effect of Bed Width
(Slope 2:1. H/h = 0.5. L/h = 50)

It is seen from the above figures [Fig.5.5 and 5.6] that the free surfaces rise with increase in the bed width. The free surface on the right side reduces to the level of the left drainage at a distance of L from the canal whereas the left free surface's reduction to the level of the drainage is very gradual. From Fig.5.6, it is observed that though the left free surface is approaching the level of the drainage as distance from the canal is increased, even at a distance ten times L, it did not reach this level.

Right and Left free surface profiles have also been plotted in Fig.5.7 and 5.8 respectively, for Bz/h = 5, L/h = 100 and side slope 2:1 and H/h = 2.0, 1.0, 0.5. 0.25 and 0.0. It is seen that the free surfaces rise with increase in the water depth inside the canal.

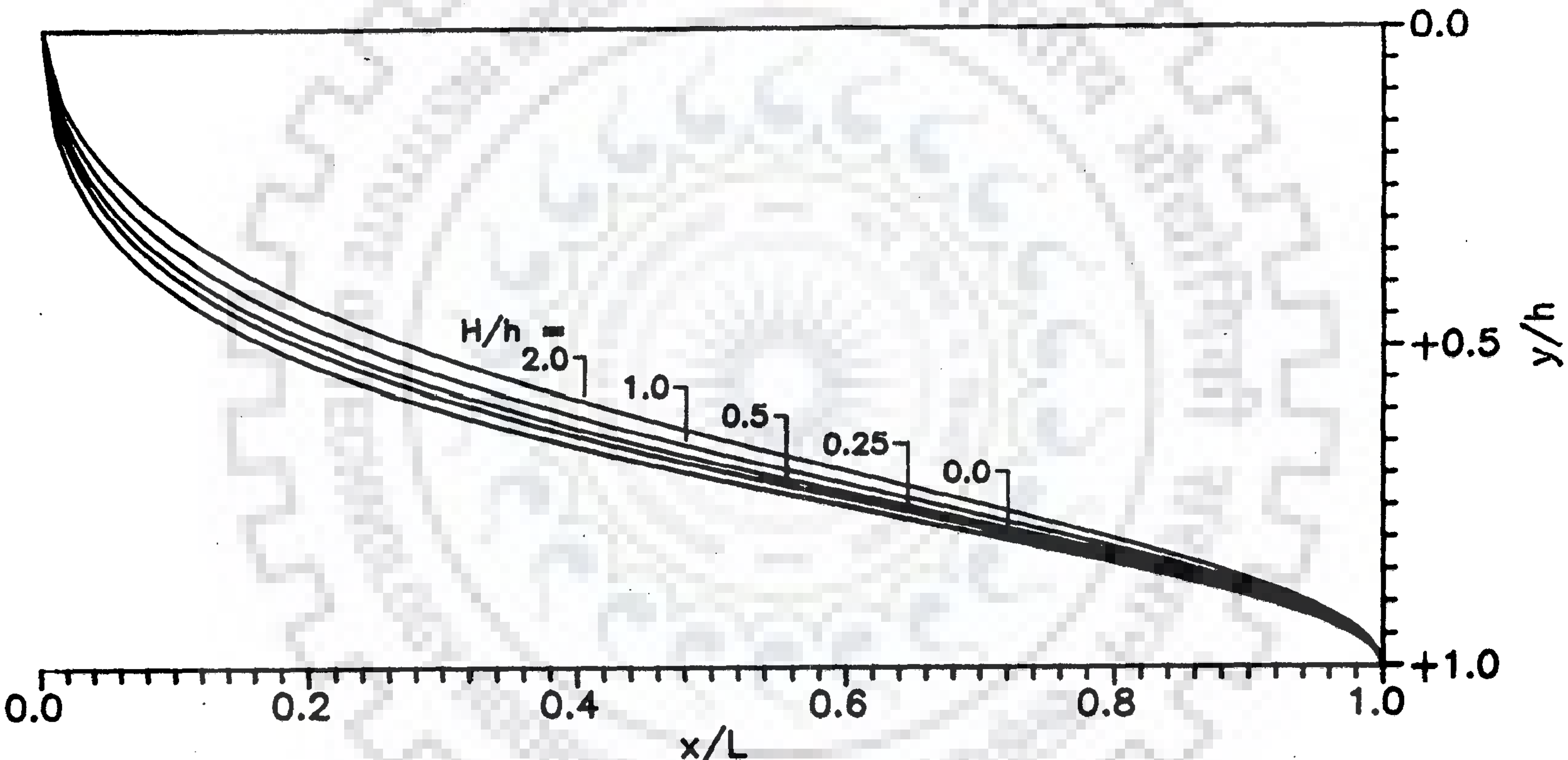


Fig.5.7 RIGHT FREE SURFACE — Effect of Canal Water Depth.
(Slope 2:1, Bz/h = 5, L/h = 100)

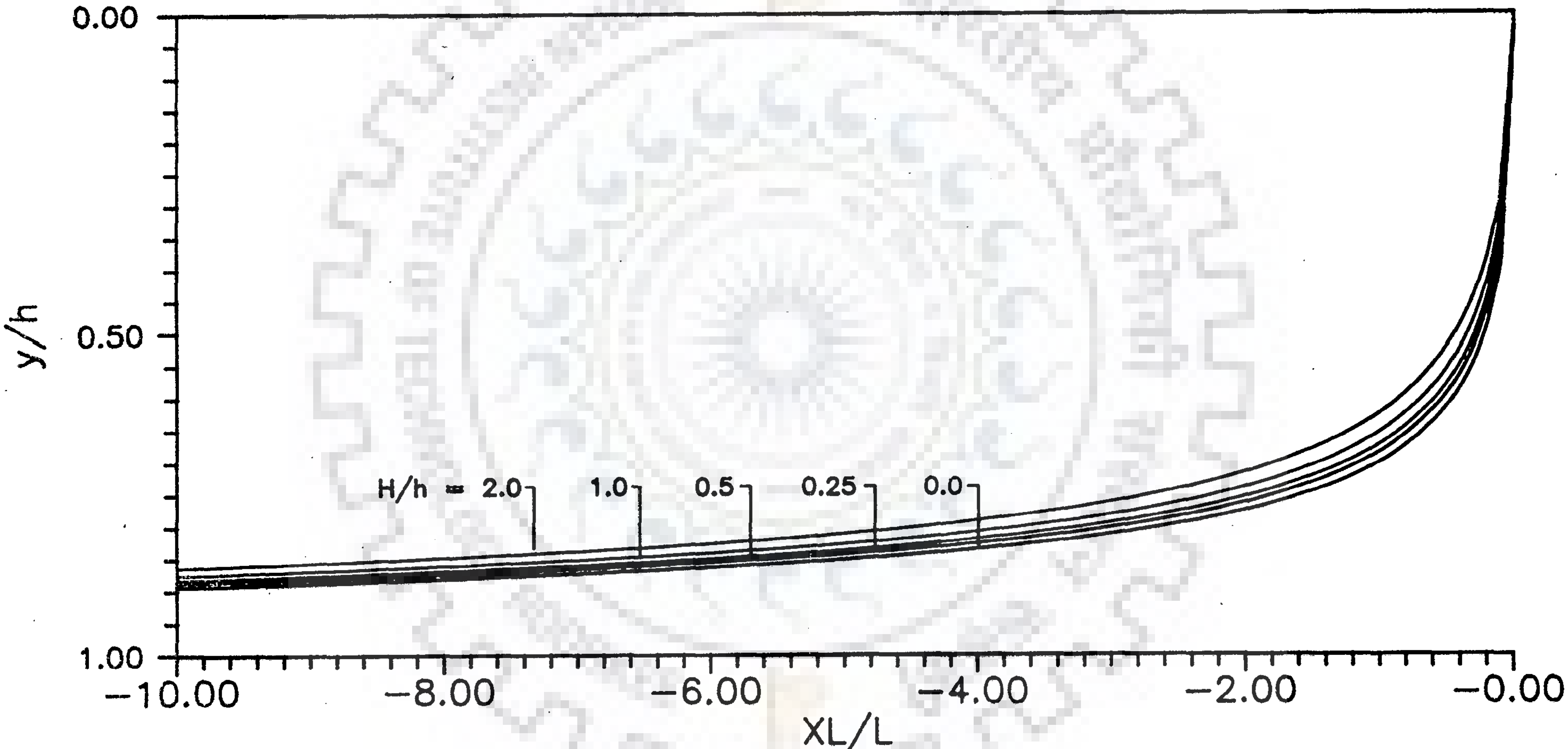


Fig.5.8 LEFT FREE SURFACE – Effect of Canal Water Depth.
(Slope 2:1, Bz/h = 5, L/h = 100)

CHAPTER 6

CONCLUSIONS

An analytical solution has been obtained with the help of Zhukovsky's function and conformal transformation for determining the phreatic surface and seepage losses from a canal with asymmetrically disposed drainages. The following conclusions are drawn from the present study :

(1) The seepage discharge for various values of physical dimensionless parameters have been prepared in easy to use curves [nomographs] for practical use. The nomograph has been presented to predict seepage loss from a canal for practical ranges of the drainage distances and levels, canal dimensions and water depths. The range considered for L_1/h_1 and L_2/h_1 is 10 to 10^5, B_2/h_1 = 10, 20 and 30 ; H/h_1 = 0.5, 0.3, 0.1, 0.0 and h_2/h_1 = 1.0, 0.9, 0.8 and 0.7. The canal side slopes considered are 2:1, 1:1 and 0.5:1. For the case of H = 0, separate nomographs for the different dimensions of the flow system are given [Chapter 3]. Also, for the case of drainage on one side [Chapter 5] a separate nomograph is made to compute seepage discharge for canal side slope 1:1 and for any value of drainage distance (L/h) between 10 to 10^5, value of bed width (B_2/h) between 10 to 30 and canal water depth (H/h) between 0 to 0.5.

(2) A perusal of the results indicates that, the seepage discharge decreases with increase in the values of L_1/h_1 and L_2/h_1, i.e as the drainage distance increases, the seepage discharge decreases. It is also seen that the seepage discharge

increases with increase in the value of B_2/h_1. However, increase in the seepage discharge due to increase in bed width is not proportional to the increase in bed width. Therefore, the practice of expressing the seepage from canals in terms of wetted perimeters irrespective of their size is not correct.

(3) Other things being the same, the seepage quantity increases as the canal side slope is made flatter but the variation is small.

(4) The seepage quantity is significantly governed by the depth of water in the canal.

(5) The seepage loss received by the drainage which is at the lower level is more than that received by the drainage whose level is higher. Lowering of any one of the drainage channels results in the increase of seepage losses from canal. A drainage higher than a certain level will not receive any seeping water from the canal. The location of the drainage channel at which it does not receive any water from the canal depends on the drainage distances, canal dimensions and the depth of water in the canal.

(6) Free surface on either side of the drainage rises with increase in the bed width and increase in drainage distances. Free surface also rises if drainage on the other side is located at a higher level. The effect of change of the side slope of canal on the free surface is negligible. However, the increase in depth of water in the canal significantly raises the free surface.

(7) Comparison of seepage losses from canal with collector drainages on one side with that from canal with drainages on both sides of the canal indicates that seepage losses for the latter case approach the values of seepage for the former case if the value of $L_2/L_1 \geq 10^5$. In case the farther side collector drainage is located at a distance ten times or more than the nearer drainage (say, $L_2/L_1 \geq 10$), the error in the seepage losses calculated by ignoring the farther drainage is less than 10% as compared to that obtained by considering both drainages. The magnitude of the error depends on the bed width and drainage distances.

(8) Free surfaces on both sides of the canal rise with increased drainage distance. Comparison of the changes in the levels of the free surface at the right and left side of the canal shows that the corresponding changes on the side where the drainage is located is very high.

The free surface on the other side drops steeply in the vicinity of the canal and then drops gently and does not reach the level of the drainage at the corresponding distance, i.e L, from the canal as in the case of the free surface on the side where the drainage is located. Moreover, the free surface on the drainage side is s-shaped with steep slope near the canal and the drainage with a point of contraflexture in between. But the free surface on the other side of the canal drops steeply near the canal and the slope becomes flatter as distance from the canal increases.

Curves of the free surfaces on both sides of the canal with collector on one side have been plotted to show the effect of the bed width and water depth of the canal. For this case also, it is seen that the free surfaces rise with increase in bed width and water depth in the canal.

Recommendations : As the personal computer (PC) is available in many offices, it may be of great interest and use if an interactive and user friendly software, which can calculate the canal seepage loss and the coordinates which define the profile of the free surfaces at either side of the canal, is developed. By giving as input the physical dimensions, namely, canal cross section, drainage locations, etc. and the soil permeability, the computer program may be developed to give the seepage loss and the free surface profiles and hence be a valuable assistance to those who are involved in the design or management of unlined irrigation canals in alluvial soil. If the latest graphic programs could be coupled with the program thus developed, one can feed the ground profile and then find the area which is likely to be water logged due to the introduction of the canal.

The solutions derived in the present thesis may be extended to deal other boundary conditions such as (a) presence of impermeable layer below the drainages, (b) effect of shape and size of the drainages. and (c) cases of infiltration/evaporation on the free surfaces. Consideration of unsteady flow condition as well as to some extent anisotropic and nonhomogeneous condition of the soil will be of some interest to determine their effect on the seepage losses and on the phreatic surfaces.

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APPENDIX A

SOLUTION TECHNIQUES

The two dimensional steady flow through homogeneous and isotropic porous medium is governed by the Laplace's equation,

$$\partial^2\phi/\partial x^2 + \partial^2\phi/\partial y^2 = 0 \qquad (A.1)$$

The function, ϕ, satisfying Eq.A.1 in a region, say R, is called harmonic. If $f(z) = \phi(x,y) + i\psi(x,y)$ is analytic in R, then ϕ and ψ are harmonic in R, i.e, ϕ and ψ satisfy Laplace's equation [Churchill, 1948 ; Nehari, 1952, 1961; Spiegel, 1981]. The functions ϕ and ψ, which depend on the spatial co-ordinates x and y, are called conjugate functions and given one, the other can be determined within an arbitrary additive constant.

If R is a simply-connected region bounded by a simple closed curve C [Fig.A.1], two types of boundary-value problems are of great importance :

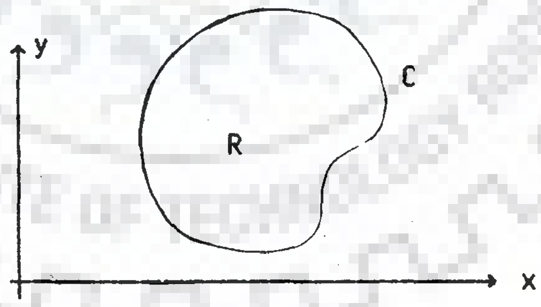


Fig.A.1 Simply-connected region R.

(1) Dirichlet's problem seeks the determination of a function ϕ which satisfies Laplace's equation (i.e., harmonic) in R and takes prescribed values on the boundary C.

(2) Neumann's problem seeks the determination of a function ϕ which satisfies Laplace's equation in R and whose normal derivative $\partial\phi/\partial n$ takes prescribed values on the boundary C.

The region R may be unbounded; for example R can be the upper half plane with the x-axis as the boundary C. Solutions to both the Dirichlet and Neumann problems exist and are unique under very mild restrictions on the boundary conditions [Spiegel, 1981].

The Dirichlet and Neumann problems can be solved for any simply-connected region R which can be mapped conformally by an analytic function on to the interior of a unit circle or half plane. Theoretically, by Riemann's mapping theorem this can always be accomplished [Nehari, 1952, 1961]. The basic ideas involved are as follows.

(i) Use the mapping function to transform the boundary-value problem for the region R into a corresponding one for the unit circle or half plane.

(ii) Solve the problem for the unit circle or half plane.

(iii) Use the solution in (ii) to solve the given problem by employing the inverse mapping function.

The usefulness of conformal mapping in two-dimensional flow problems stems from the fact that solutions of Laplace's equation remain solutions when subjected to conformal transformations [Harr, 1962].

A.1 The Schwarz-Christoffel transformation.

The Riemann mapping theorem guarantees the existence of an analytic function which maps a given simply-connected domain onto the unit circle or half plane but it does not show how to find this function [Nehari, 1952, 1961]. In groundwater problems, where it is often necessary to determine the seepage characteristics within complicated boundaries, the transformation is yet more difficult. But, though this may appear somewhat disturbing, the use of appropriate auxiliary mapping techniques enables us to transform even complicated flow regions into regular geometric shapes [Harr, 1962]. Generally these figures will be polygons having a finite number of vertices (one or more of which may be at infinity). Thus the method of mapping a polygon from one or more planes onto the upper half of another plane is of particular importance [Harr, 1962].

An explicit equation called the Schwarz-Christoffel transformation is used as a mapping function in the case in which the domain in question is a polygon. If the polygon is located in the z-plane ($z = x + iy$), then this transformation that maps it conformally onto the upper half of the t-plane ($t = r + is$) is,

$$dz = M \frac{dt}{(t-a)^{1-A/\pi}(t-b)^{1-B/\pi}(t-c)^{1-C/\pi} \dots} \quad \dots (A.2a)$$

or,

$$z = M \int \frac{dt}{(t-a)^{1-A/\pi}(t-b)^{1-B/\pi}(t-c)^{1-C/\pi} \dots} + N \quad \dots (A.2b).$$

where M and N are complex constants; A, B, C, ..., are the interior angles (in radians) of the polygon in the z-plane

[Fig.A.2(a)]; and a, b, c, ...(a < b < c < ...) are points on the real axis of the t plane corresponding to the respective vertices A, B, C, ... [Fig.A.2(b)]. It should be noted, in particular that the complex constant N corresponds to the point on the perimeter of the polygon that has its image at t = 0 [Harr, 1962].

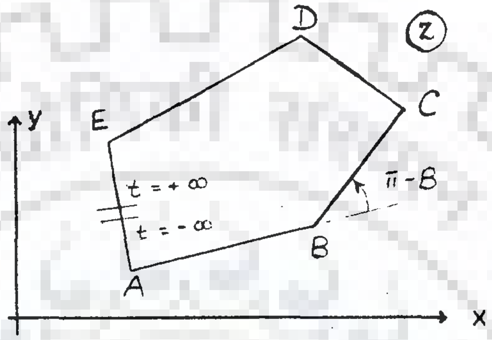


Fig. A. 2(a) Polygonal region in z-plane.

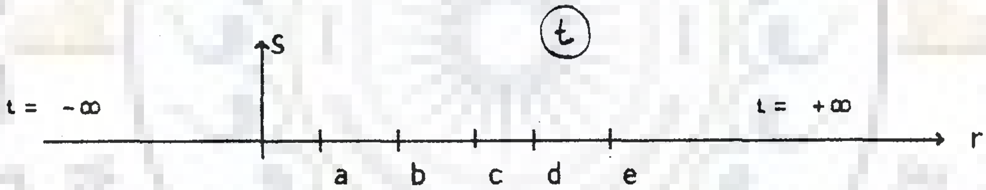


Fig. A. 2(b) t-plane

It should also be noted that if a vertex, for example A_2, tends to infinity in such a way that the adjacent sides become parallel [Fig.A.3(a)], then one must take $\alpha_2 = 0$. For further

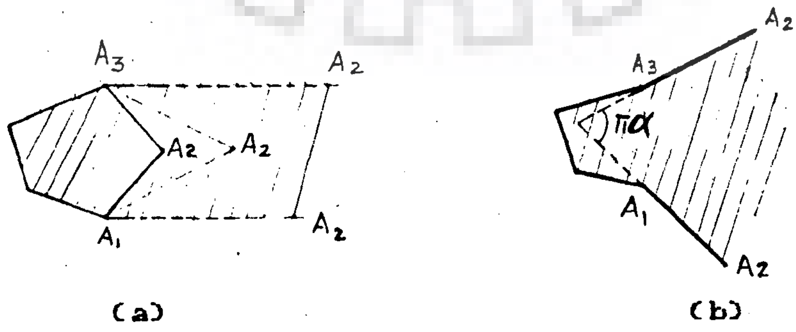


Fig. A. 3 Polygonal region in z-plane.

turning of the sides A_1A_2, when they cease to be parallel, but when the vertex A_2 remains at infinity [Fig.A.3(b)], the angle α_2 must be considered negative, and namely $\alpha_2 = -\alpha_2'$, where α_2' is the magnitude of the angle formed by the prolongation of the sides A_1A_2 and A_3A_2 [Polubarinova-Kochina, 1969, pp.70-71].

The Schwarz-Christoffel transformation can be considered as the mapping of a polygon in the z-plane onto a similar polygon in the t-plane in such a manner that the sides of the polygon in the z-plane extend through the real axis of the t-plane. This is accomplished by opening the polygon at some convenient point, say between A and E of Fig.A.2(a), and extending one side to $t = -\infty$ and the other to $t = +\infty$ and are placed along the real axis of the t-plane. The interior angle at the point of opening may be regarded as π (in the z plane) and, as noted in Eq.A.2, takes no part in the transformation. The point of opening in the z-plane is represented in the upper half of the t-plane by a semicircle with a radius of infinity. Thus the Schwarz-Christoffel transformation, in effect, maps conformally the region interior to the polygon ABC... of the z-plane into the interior of the polygon bounded by the sides ab, bc,... and a semicircle with a radius of infinity in the upper half of the t-plane, or, more simply, into the entire upper half of the t plane [Harr, 1962].

Corresponding values of a,b,c, ... and A, B, C, ... can be chosen so that the polygons in their respective planes are similar [Harr, 1962]. Any three of the values a, b, c, ... can be chosen arbitrarily to correspond to three of the vertices of the given polygon A, B, C, ... For a polygon of n sides, the (n-3) remaining values must then be determined so as to satisfy conditions of similarity. Whereas we shall often choose to map a vertex of the flow region (z-plane) into one at infinity in the t-plane, it is important to note that not only is this factor omitted from the transformation, but the number of arbitrary values is reduced by one.

For the more demonstration of the mechanism of the Schwarz-Christoffel transformation and for its derivation one is referred to Harr [1962], Nehari [1952, 1961], Cunningham [1965], Spiegel [1981], Strack [1989].

A.2 Bilinear transformation.

The general form of the bilinear transformation, sometimes called the Möbius transformation, is as follows [Harr, 1962].

$$w = \frac{az + b}{cz + d} \qquad (A.3)$$

in which, $ad - bc \neq 0$ and a, b, c, and d are complex constants and $w = \phi + i\psi$. The requirement $ad - bc \neq 0$ is necessary to ensure the conformal nature of the transformation. In addition, if $a/c = b/d$, w will be a constant irrespective of z and hence the transformation will map the entire z plane into a point in the w plane [Harr, 1962].

The bilinear transformation maps circles in z-plane onto circles in the w-plane. Straight lines are regarded as special cases of circles (namely, circles passing through the point at infinity). The point $z = -(d/c)$ is transformed by Eq.A.3 into the point $w = \infty$; accordingly, circles passing through the point $z = -(d/c)$ will transform into straight lines [Nehari, 1961, p.168].

The function given by Eq.A.3 is univalent. To demonstrate this, Eq.A.3 is rewritten as follows.

$$w = \frac{a}{c} + \frac{bc - ad}{c(cz + d)} \qquad (A.4)$$

If w_1 and w_2 are the images of the points z_1 and z_2, respectively, from Eq.A.4 we conclude that

$$w_1 - w_2 = \frac{(ad - bc)(z_1 - z_2)}{(cz_1 + d)(cz_2 + d)} \qquad (A.5)$$

This shows that $w_1 \neq w_2$ if $z_1 \neq z_2$, provided neither w_1 nor w_2 are infinite. Since by Eq.A.3, the point $w = \infty$ is the image of $z = -(d/c)$ and of no other point, the assertion follows.

If z_1, z_2, z_3 and z_4 are four distinct finite points in the z-plane (and none of these points coincide with $z = -d/c$), Eq.A.5 implies that,

$$\frac{(w_1 - w_4)(w_3 - w_2)}{(w_1 - w_2)(w_3 - w_4)} = \frac{(z_1 - z_4)(z_3 - z_2)}{(z_1 - z_2)(z_3 - z_4)} \qquad \ldots (A.6)$$

The right-hand side expression of Eq.A.6 is called the cross-ratio of the four points z_1, z_2, z_3 and z_4. The formula given by Eq.A.6 shows that the cross-ratio of four points is equal to the cross-ratio of the images of these points under a linear transformation, i.e., the cross-ratio of four points is invariant under a bilinear transformation. If one of the points w_μ, say for $\mu = 1$, i.e., w_1, approaches the point at infinity, the left-hand side of Eq.A.6 reduces to $(w_3-w_2)/(w_3-w_4)$. This expression is therefore to be regarded as the cross-ratio of the points ∞, w_2, w_3 and w_4.

Eq.A.6 makes it possible to write down the bilinear transformation which carries three given points z_1, z_2, z_3 into three preassigned points w_1, w_2, w_3, respectively. If z is any other point in the z-plane, and if w is its image under the transformation as given by Eq.A.3, it follows from Eq.A.6 that we must have,

$$\frac{(w_1 - w)(w_3 - w_2)}{(w_1 - w_2)(w_3 - w)} = \frac{(z_1 - z)(z_3 - z_2)}{(z_1 - z_2)(z_3 - z)} \quad \text{. . .(A.7)}$$

If Eq.A.7 is solved for w, the right-hand side is easily seen to be of the same form as the right-hand side of Eq.A.3. The relation given by Eq.A.7 is therefore equivalent to a bilinear transformation. It may also be noted that the three correspondences (z_1, z_2, z_3 to w_1, w_2, w_3 , respectively) determine the transformation completely.

Since a bilinear transformation maps circles onto circles, and since a circle is determined by three of its points, we can thus find bilinear transformations which carry a given circle in the z-plane into a given circle in the w-plane. We can say, moreover, that three points on the first circle be carried into three given points on the second circle. Once this is done, the transformation is completely determined. The inside of the circle C_z in the z-plane may be mapped either onto the inside of the circle C_w in the w-plane, or onto the exterior of C_w. In any given mapping, it is easy to decide which of these two cases occurs. If the point $z = -(d/c)$ lies inside C_z, the image of the inside must contain the point $w = \infty$, i.e., it is mapped onto the outside of C_w; otherwise, the interior of C_z corresponds to the interior of C_w. If C_w degenerates into a straight line, both the interior and the exterior of C_z are mapped onto half-planes bounded by this line.

To find all linear transformations which carry the real axis in the z-plane into the real axis in the w-plane, we have to consider all transformations, Eq.A.7, for which the numbers z_1, z_2, z_3, w_1, w_2, w_3 are real. Solving Eq.A.7 for w, we are led to transformations, Eq.A.3, for which the coefficients a, b, c, d are all real numbers. Conversely, if these numbers are real, Eq.A.3 will evidently carry real numbers z into real numbers w. In view of what was said above, the image of the upper half-plane $Im(z) > 0$ may be either the upper half-plane $Im(w) > 0$ or the lower half-plane $Im(w) < 0$. In order to decide between these alternatives in a given case, we test the mapping of a point in $Im(z) > 0$, say the point $z = i$. By Eq.A.3, we have,

$$w = \frac{ai + b}{ci + d}$$

$$= \frac{(ai + b)(-ci + d)}{(ci + d)(-ci + d)}$$

$$= \frac{(ac + bd) + i\,(ad - bc)}{(c^*c + d^*d)} \qquad (A.8)$$

and thus from Eq.A.8,

$$Im(w) = \frac{ad - bc}{c^*c + d^*d} \qquad (A.9)$$

Hence, $Im(z) > 0$ will be mapped onto $Im(w) > 0$ if $(ad - bc) > 0$ and onto $Im(w) < 0$ if $(ad - bc) < 0$.

For more detailed discussion on the bilinear transformation, one is referred to Nehari [1961, pp.166-173].

A.3 Zhukovsky's function.

In unconfined flow problems, in which loci of the phreatic line is not known a priori, the flow problems can be solved using a special mapping technique which makes use of an auxiliary transformation called Zhukovsky's function [Harr, 1962, Aravin and Numerov, 1965].

The velocity potential $\phi = \phi(x,y)$, in two-dimensional flow in the xy plane, i.e. in the z plane, is defined [Harr, 1962] as below :

$$\phi = -k(p/\gamma_w + y) + C$$

$$= -kh + C \qquad (A.10)$$

where C is an arbitrary constant, p is the pressure, γ_w is the unit weight of the fluid, k is the coefficient of permeability of the porous medium and h is the total head. (The derivative of the velocity potential, i.e. Eq.A.10, with respect to any direction gives the velocity of the fluid in that direction, which is the same as that stated by Darcy's law). It can also be observed that ϕ satisfies the Laplace equation, Eq.A.1.

If the arbitrary constant C is taken to be 0 and after rearranging Eq.A.10, we have, $-kp/\gamma_w = \phi + ky$. If we define :

$$\Gamma_1 = -kp/\gamma_w \qquad (A.11)$$

then Eq.A.10, reduces to the following equation.

$$\Gamma_1 = \phi + ky \qquad (A.12)$$

Γ_1 is seen to be an harmonic function of x and y as it satisfies Eq.A.1. Hence, its conjugate is the function given below [Harr, 1962].

$$\Gamma_2 = \psi - kx \qquad (A.13)$$

where ψ is the stream function. Defining $\Gamma_1 + i\Gamma_2 = \Gamma$, then Eqs.A.12 and A.13 give us the following relationship.

$$\begin{aligned}\Gamma &= \Gamma_1 + i\Gamma_2 \\ &= (\phi + ky) + i(\psi - kx) \\ &= w - ikz \qquad (A.14)\end{aligned}$$

The function defined by Eq.A.14 and any function with its real or imaginary part differing from it by a constant multiplier is called a Zhukovsky function [Harr, 1962].

Nelson-Skornyakov [1949] used the modified form of Zhukovsky's function [Harr, 1962, p.171]. He modified Eq.A.14 as given below.

$$\begin{aligned}\theta &= \theta_1 + i\theta_2 \\ &= z - iw/k \qquad (A.15)\end{aligned}$$

where,

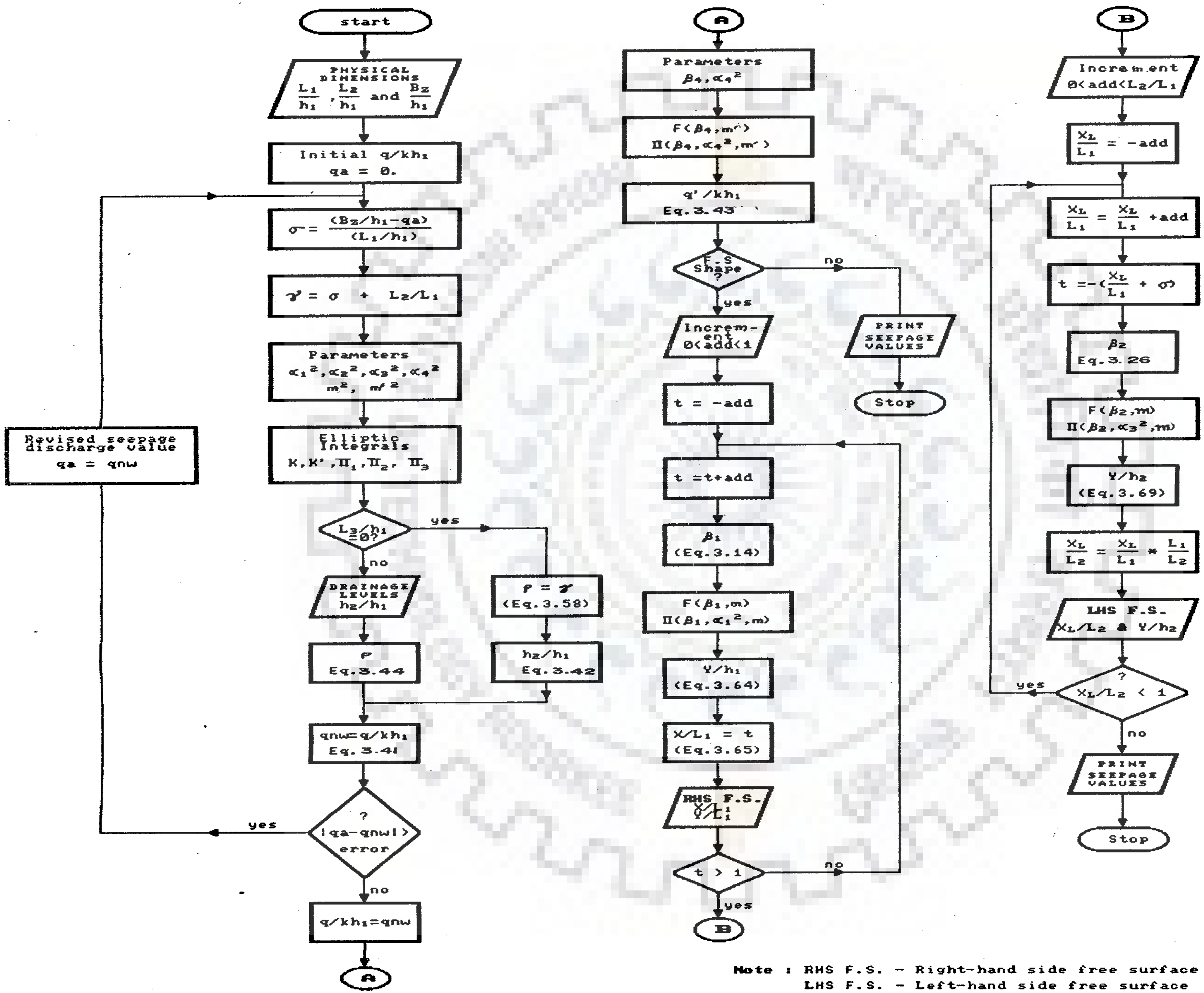
$$\theta_1 = x + \psi/k \qquad (A.16)$$

$$\theta_2 = y - \phi/k \qquad (A.17)$$

The advantage of the form of the Zhukovsky function as given by Eq.A.15 is primarily one of orientation. Whereas $p/\gamma_w = y - \phi/k = 0$ along the free surface taking the vertical axis as positive down, with the form of Eq.A.15, the image of this free surface will be along the real axis of the θ plane. The free surface is therefore represented by a straight line in the θ plane. Since the boundaries in θ plane are straight lines, the Schwarz-Christoffel conformal mapping and the bilinear transformation are applicable.

In this thesis, the modified form of the Zhukovsky function as given by Eq.A.15 is implied wherever Zhukovsky's function is mentioned.

APPENDIX B



Note : RHS F.S. – Right-hand side free surface
LHS F.S. – Left-hand side free surface

FIG. B.1. Flow Chart for Solution of Case (i)

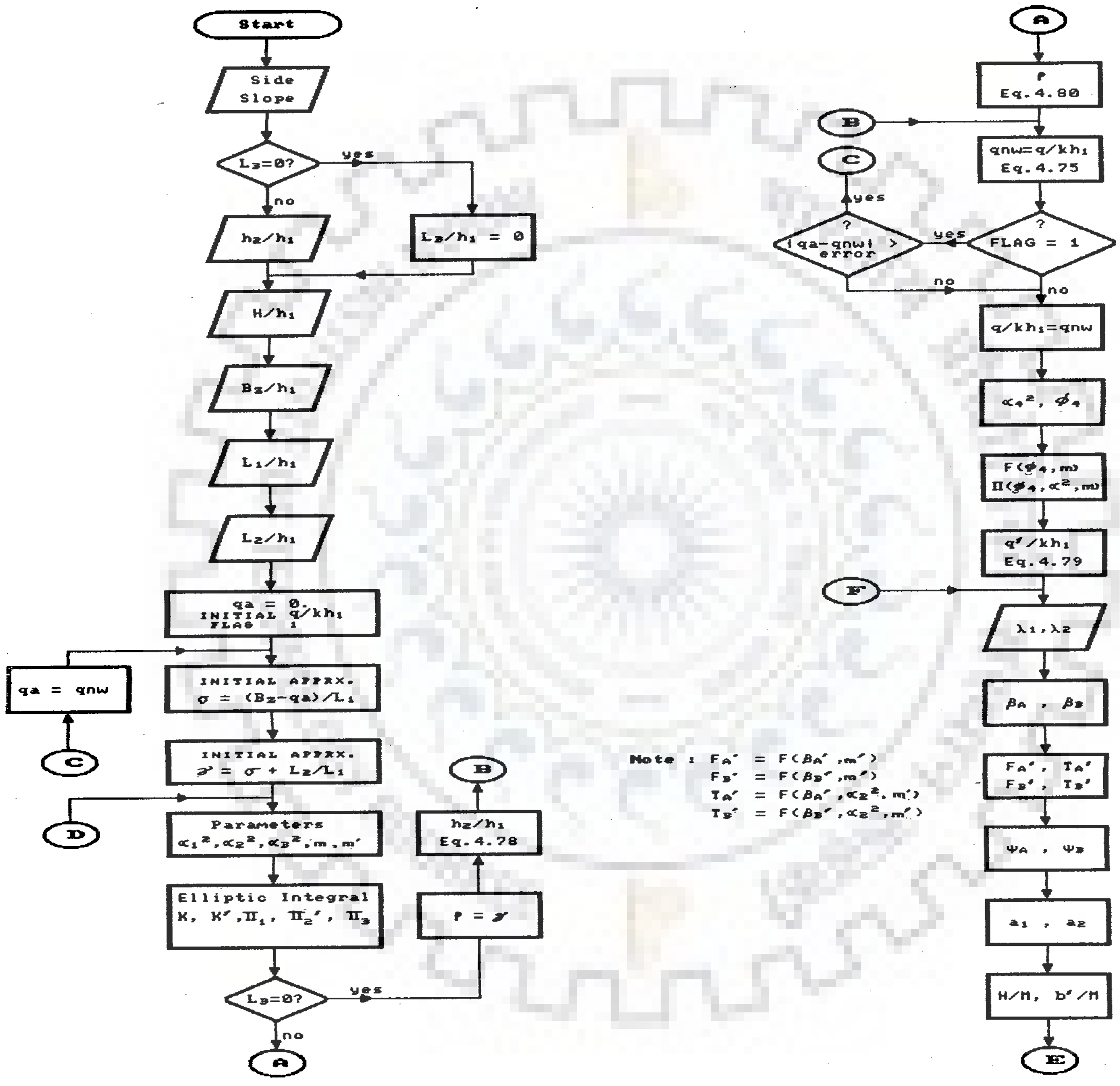


FIG.B.2.(a) Flow Chart for Solution of Case(ii)

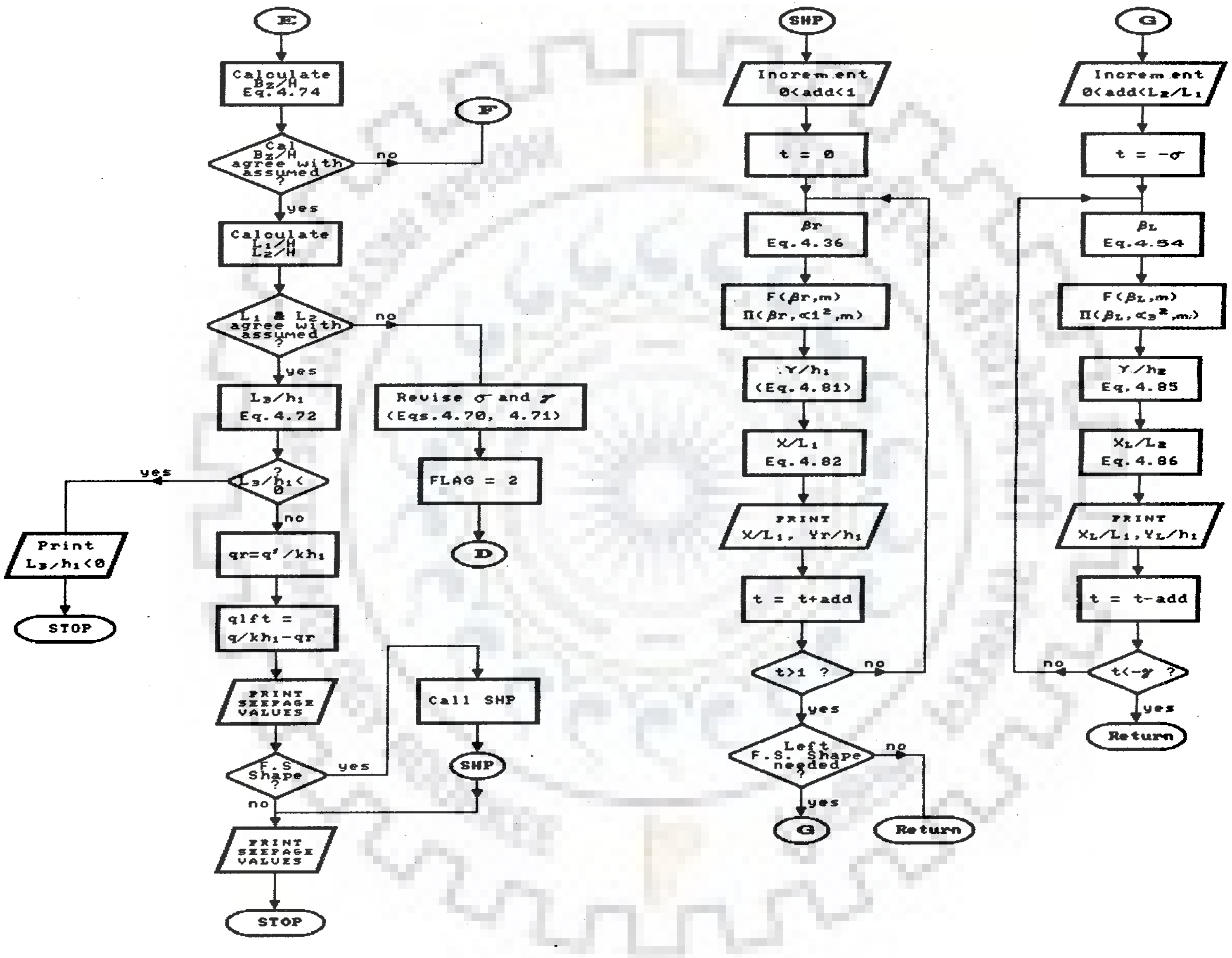


FIG.B.2(b) Flow Chart for Solution of Case (ii)

APPENDIX C

CANAL OF NEGLIGIBLE WATER DEPTH WITH COLLECTOR ON ONE SIDE

C.1. Formulation and Solution of Problem

The z-plane is as shown in Fig.C.1(a) where AB, which has a value of B length, is the bed width of the canal and is an equipotential line and corresponds to $\phi = 0$. The equipotential line CD is $\phi = kh$, where h is the difference between the canal and drain water levels. The distance of the drainage from the canal, i.e. horizontal distance BC, is L. Along the stream line BC, the value of ψ is taken to be 0. Then, $\psi = q$ on the stream line AF where q is the seepage discharge per unit length of the canal. The rectangular flow region in the w-plane is as shown in Fig.C.1(c).

As the result of Zhukovsky's transformation [Eq.A.15], the boundaries in θ-plane as shown in Fig.C.1(b) consists of straight lines. The corresponding points in the Z-plane and the θ-plane are as given in Table C.1.

Two operations were made in which the θ-plane as well as the w-plane were mapped onto the lower half of an intermediate t'-plane [Fig.C.1(d)], where $t' = r' + is'$.

Table C.1 Transformation Table
(Canal of negligible water depth)
(Drainage on one side)

POINT	θ-plane	t'-plane
A	$-Bz + q/k$	$-\sigma'$
B	0	0
C	L	1
D	∞	∞
F	$-\infty$	$-\infty$

First Operation.- In this operation the θ-plane was transformed onto the intermediate semi-infinite t'-plane using the bilinear transformation. The various points in the θ-plane were placed in the t'-plane as shown in Fig.C.1(d). Here, σ' is a transformation parameter which has to be determined.

The bilinear transformation that maps the lower half of the θ-plane into the lower half of the t'-plane was made using the cross ratio formula as given below.

$$\frac{(\theta - \theta_D)(\theta_C - \theta_B)}{(\theta - \theta_B)(\theta_C - \theta_D)} = \frac{(t' - t'_D)(t'_C - t'_B)}{(t' - t'_B)(t'_C - t'_D)} \qquad \ldots (C.1)$$

Substituting the corresponding values of the points B, C and D in the θ-plane and t'-plane in Eq.C.1,

$$\frac{(\theta - \infty)(L - 0)}{(\theta - 0)(L + \infty)} = \frac{(t' - \infty)(1 - 0)}{(t' - 0)(1 + \infty)} \qquad \ldots (C.2)$$

Eq.C.2, after simplification resulted in the following equation.

$$\theta = Lt' \qquad (C.3)$$

Referring to Table C.1, at A, $\theta = -Bz + q/k$ and $t' = -\sigma'$. Substituting these in Eq.C.3 and rearranging,

$$\sigma' = (Bz - q/k)/L \qquad \text{(C.4)}$$

The corresponding points in the θ-plane and the t'-plane are as given in Table C.1.

Second operation.- The rectangular flow field in Fig.C.1(c) was opened at Point F/D and mapped from $-\infty$ to $+\infty$ along the real axis of the t'-plane. Points A, B and C are mapped at $t' = -\sigma'$, 0 and 1 respectively. The Schwarz-Christoffel transformation equation used to map the w-plane onto the lower half of the t'-plane is as given below.

$$\int dw = M' \int \frac{dt'}{\sqrt{(t'+\sigma')(t')(t'-1)}} \qquad \text{(C.5)}$$

where M' is a complex constant.

The integration of the right-hand side of Eq.C.1 between the desired regions on the t'-plane results [Byrd and Friedman, 1971] in elliptic integrals. The integration of Eq.C.5 between the different regions is made as shown below.

Integrating Eq.C.5 between F and A, i.e. in the region $-\infty \le t' < -\sigma'$,

$$\int_{W_L}^{W_A} dw = \frac{M'}{i^3} \int_{t'}^{-\sigma'} \frac{dt}{\sqrt{(-\sigma-t')(-t')(1-t')}} \qquad \text{. . . (C.6a)}$$

$$W_A - W_L = M' F(\Phi, m)/(-i) \qquad \text{. . . (C.6b)}$$

where, $W_A = 0 + iq$ and $W_t = ky + iq$ are the complex potential values at A and at point on the free surface AF with ordinate y, respectively. Hence, Eq.C.6(b) reduces to,

$$-ky = M\ F(\Phi, m) \qquad (C.7)$$

M is a constant and,

$$\Phi = \sin^{-1}\sqrt{(\sigma' + t')/t'} \qquad (C.8)$$

$$m^2 = 1/(1+\sigma') \qquad (C.9)$$

Using Eq.A.15, on the free surface AF,

$$\theta = \theta_1 = x + q/k \qquad (C.10)$$

where x is the abscissa of the point considered.

Substituting Eq.C.3 in Eq.C.10 and rearranging, the value of t' corresponding to x is,

$$t' = x/L + q/kL \qquad (C.11)$$

At Point F on the free surface AF, $t' = -\infty$ and Eq.C.8 gives $\Phi = \pi/2$. Substituting this in Eq.C.7, the following relationship was obtained to get the ordinate, $y = Y_F$, of the Point F on the free surface AF.

$$-kY_F = M\ K(m) \qquad (C.12)$$

Integrating Eq.C.5 between B and C, i.e. in the region $0 < t' \leq 1$,

$$\int_{W_B}^{W_t} dw = \frac{M'}{i}\int_0^{t'} \frac{dt'}{\sqrt{(t'+\sigma')(t')(1-t')}} \qquad ...\ (C.13)$$

$$W_t - W_B = M\ F(\phi_2, m) \qquad ...\ (C.14)$$

where, $W_B = 0$ and $W_t = ky + 0$ are the complex potential values at B and at a point on the free surface BC, with ordinate value of y, respectively. Hence, from Eq.C.12,

$$-ky = M\ F(\phi_2, m) \qquad (C.15)$$

$$\phi_2 = \sin^{-1}\sqrt{(1+\sigma')t'/(t'+\sigma')} \qquad (C.16)$$

From Eq.A.15, on the free surface BC, $\theta = \theta_1 = x$. Substituting this in Eq.C.3, the following relationship between the abscissa, x, of the point on the free surface BC and its corresponding t' value was obtained.

$$t' = x/L \qquad (C.17)$$

At Point C, t' = 1 and from Eq.C.16, $\phi_2 = \pi/2$. Therefore, substituting these results in Eq.C.15,

$$kh = -M\ K(m) \qquad (C.18)$$

Integrating Eq.C.5 between A and B, i.e. in the region $-\sigma' < t' \leq 0$,

$$\int_{W_A}^{W_{t'}} dw = \frac{M'}{i^2} \int_{-\sigma'}^{t'} \frac{dt'}{\sqrt{(t'+\sigma')(-t')(1-t')}} \qquad \ldots (C.19)$$

$$W_{t'} - W_A = iM\ F(\phi_3, m') \qquad (C.20)$$

where $W_A = 0 + iq$ and $W_{t'} = 0 + i\psi'$ are the complex potential values at A and at any point on the equipotential line AB respectively. Here, ψ' is the stream function value at the point on the equipotential line AB. Hence, Eq.C.20 becomes :

$$(q - \psi') = -M\ F(\phi_3, m') \qquad (C.21)$$

where,

$$\phi 3 = \sin^{-1}\sqrt{(t'+\sigma')/\sigma'} \qquad (C.22)$$

$$m' = \sigma'/(1+\sigma') = \sqrt{(1 - m*m)} \qquad (C.23)$$

At Point B, t' = 0 and ψ' = 0. Hence, substituting these in Eq.C.21 and Eq.C.22, the following expression for the seepage quantity, q, from the canal was obtained.

$$q = -M\,F(\pi/2, m')$$
$$= -M\,K(m') \qquad (C.24)$$

Dividing, Eq.C.24 by Eq.C.18,

$$q/kh = K(m')/K(m) \qquad (C.25a)$$

Rearranging Eq.C.9,

$$\sigma' = (1-m^2)/m^2 \qquad (C.25b)$$

Dividing Eq.C.4 by h and substituting Eq.C.25(b) and then rearranging the resulting equation,

$$Bz/h = [(1-m^2)/m^2]\,[L/h] + q/kh \qquad (C.25c)$$

Substituting Eq.C.25(a) in Eq.C.25(c), the following expression is resulted. This is the same as that given in the solution given by Polubarinova-Kochina [1962, P.132].

$$Bz/h = [(1-m^2)/m^2]\,[L/h] + K(m')/K(m) \qquad (C.25d)$$