AMBIENT VIBRATION TESTING OF WATER TOWERS

A DISSERTATION

Submitted in partial fulfilment of the requirements for the award of the degree

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EARTHQUAKE ENGINEERING

(With Specialization in Structural Dynamics)

By

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in this dissertation entitled, "**AMBIENT VIBRATION TESTING OF WATER TOWERS**" in partial fulfilment of the requirements for the award of the degree of MASTER OF ENGINEERING in EARTHQUAKE ENGINEERING with specialization in STRUCTURAL DYNAMICS, submitted in the department of Earthquake Engineering, University of Roorkee, Roorkee, is an authentic record of my own work carried out for a period of about six months from October, 1994 to March, 1995 under the guidance of Dr. S.K. Thakkar, Professor and Head, Mr. R.N. Dubey, Lecturer, Department of Earthquake Engineering, University of Roorkee, Roorkee, India.

The matter embodied in this dissertation has not been submitted by me for award of any other degree or diploma.

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Dated : March 30 , 1995 (SUNIL KUMAR SINHA)

This is to certify that the above statement made by the candidate is correct to the best of our knowledge.

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ABSTRACT

The ambient vibration testing of structures is a direct and practical method of determining the dynamic characteristics of structures such as natural frequencies, mode shapes and damping. In this testing, structure is excited by natural microtremors, wind, traffic load and machine or man made blast. This technique can be used to verify the mathematical models and to investigate changes in resonant frequency before and after earthquake.

This thesis presents the results of ambient vibration test of two water towers (i) near the G.P.Hostel and (ii) near the Sarojini Bhawan. The vibrations produced by wind were picked upby a Ranger Seismometers (SS-1) and recorded in a Solid State Recorder (SSR-1). The recorded data were then transferred to Note-Book computer and analyzed by using Seismic Workstation Software (SWS). The experimentally measured fundamental time periods were compared with the values obtained from mathematical model for the two water towers. A reasonably close agreement in fundamental time period between theoretically and experimentally measured values has been observed for the tank near G.P. Hostel, while for the tank near Sarojini Bhawan difference has been observed between experimental and theoretical time periods. The possible reason for the difference in the time period could be attributed to the fact that high frequency was picked up by seismometers due to weak wind velocity.

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CHAPTER - 1

INTRODUCTION

1.1 BACKGROUND

The Ambient Vibration Testing of structure is a direct and practical method of determining the dynamic characteristics of a structure such as natural frequencies, mode shapes and damping. The damping values obtained from these tests are at a very low strain level and are therefore of limited use. In the ambient vibration test the structure is excited by natural microtremors, wind, traffic load and machine or man made blast. Shakers or other potentially destructive methods of inducing forced vibration are not required. Knowledge of the above properties is to understand and interpret structural response during wind excitation or microtremors and compare observation with theoretical results. These dynamic characteristics are used to improve the Earthquake Resistant Design measures. This technique can be used to verify mathematical models and to investigate changes in resonant frequency before and after earthquake. Sensitive transducers are positioned at numerous locations on the structure and connected to the portable recorder through multichannel signal conditioner.

The ambient vibration testing technique has a wide application that includes suspension bridge, rotating machinery supports, dams, floors, nuclear power plant, offshore platforms, high rise buildings and water towers etc.

Measurement of time period of structure before and after the earthquake shocks can often indicate the damage or distress occurred in the building due to shocks. In the case of large structures where a failure would have a serious local impact, i.e., a nuclear reactor, any differences in "as designed" and "as constructed" properties can be evaluated and problem areas can be rectified.

1.2 PURPOSE OF AMBIENT VIBRATION SURVEY

The codes of practice often provide some empirical formula for determining time period of tall buildings. The seismic coefficient are worked out using response spectra from these time periods. It is possible to arrive at the more reasonable expression of time periods from the results of ambient vibration testing. There are possibilities of obtaining following information from ambient vibration testing :

(i) Data base on dynamic characteristics of different type of structures.

(ii) Experimental verification of theoretical models.

(iii) Study of soil structure interaction effect on dynamic characteristics.

A typical test setup of such a test of cantilever structure is shown in Figure 1. The analysis of data is carried out by vibration survey software to determine dynamic characteristics.

1.3 MERITS OF AMBIENT VIBRATION SURVEY

There are some features which makes this method particularly attractive over the other methods.

(i) The instrumentation is light weight and portable.

(ii) This permits data to be recorded at a large number of locations with very little interference with the normal flow of activity.

(iii) There is no requirement for attachment of equipment to the structure with the resulting patchwork which have to be done.

(iv) The level of vibration recorded are those that the structure is constantly being subjected to, and owners have no fear of damage during testing.

(v) The instrumentation will cover a wide frequency range.

(vi) The range of vibration amplitude which can be handled is also large.

(vii) The recording data contain all the resonances at a location.

(viii) This method is also economical over other methods.

1.4 OBJECTIVES OF AMBIENT VIBRATION TESTING

The following are the objectives of carrying out ambient vibration testing.

(i) To carry out the ambient vibration testing of the water towers, (1) near the G.P. Hostel and (2) near the Sarojini Bhawan in the campus of University of Roorkee.

(ii) To analyse the data obtained experimently.

(iii) To carry out analytical studies using mathematical models for determining dynamic characteristics of the water towers.

(iv) To compare the experimental and analytical results.

1.5 SCOPE OF THE INVESTIGATION

The ambient vibration testing was done for two water towers (i) near G.P. Hostel and (ii) near Sarojini Bhawan in the campus of University of Roorkee. Both water towers were tested for wind excitation and recorded by Kinemetrics Solid State Recorder through Ranger Seismometers (SS-1). The time period of the water towers were experimently determined and compared with the theoretical result which was obtained by modelling of the water towers and analysing by 3D space frame program.

1.6 COMPOSITION OF THESIS

Chapter 1 : describes the introduction to ambient vibration testing, purpose, objective and scope of the testing.

Chapter 2 : In this chapter the work of different authors on ambient vibration test of different structures is discussed.

Chapter 3 : describes the brief description and features of ambient vibration measuring equipments.

Chapter 4 : describes the software which is used for the analysis of recorded data.

Chapter 5 : describes the steps followed for the ambient vibration testing of two water towers.

Chapter 6 : describes the steps followed for analytical investigation of water towers, i.e., mathematical modelling, methods used for analysis and results and discussions.

Chapter 7 : describes the summary and conclusions drawn after the testing of water towers.

CHAPTER - 2

REVIEW OF LITERATURE

2.1 AMBIENT VIBRATION TESTS OF THE MAXICALI GENERAL HOSPITAL [12]

The Maxicali General Hospital is a public hospital which is situated in the vicinity of Maxicali. The hospital complex consists of an eight-storey reinforced concrete tower surrounded by a one-storey reinforced concrete structure (Figure 2). This structure had suffered some damages during 1987 Superstition Hills earthquake. The tests were conducted on April 20, 1989 prior to any change in structure and on August 17, 1989 after the north facade of the building and the concrete outside the reinforcement in some of the first level columns had been removed.

In this testing four horizontal and two vertical Kinemetrics 5-sec seismometers and a Kinemetrics SSR-1 portable digital event recorder were used and placed at different levels of the hospital.

The frequencies, modal ratios and mode shapes of some of the longitudinal, transverse and torsional modes were determined. It was found that the removal of the north facade of the building resulted in a reduction of longitudinal stiffness of the structure of the order of 12%. Measurement of the translational and rocking of the base indicate that the soil-structure interaction effect play a moderate role in the transverse response (about 10%) and a negligible (1%) role in its longitudinal response.

2.2 AMBIENT VIBRATION TESTING OF THREE STOREYED BUILDING [1]

The ambient vibration testing of three storeyed building (height 11.8 m) of Department of Earthquake Engineering was carried out by recording wind induced vibrations. The building is framed structure, the beams are constructed in reinforced concrete while columns are made in reinforced brick. Figure 3 shows first floor plan and front elevation of the building.

Kinemetrics 1-sec (natural period) seismometers SS-1 and a Kinemetrics SSR-1 portable digital event recorder were used in the study. The limited number of instruments available required an experimental scheme in which simultaneous measurements were made at few locations at a time. The positioning of the seismometers during tests is shown in Figure 3.

The experimentally measured time period in transverse direction of building was observed to be 0.308 sec. and the time period obtained from mathematical model in first three modes are 0.34, 0.12 and 0.080 sec. respectively. The results show a reasonably close agreement in experimental and theoretical value of frequencies.

2.3 AMBIENT VIBRATION TESTING OF COMMUNICATION TOWER OF ELECTRONICS AND COMPUTER ENGINEERING DEPARTMENT [1]

The ambient vibration testing of thirteen storeyed communication tower (height 42.6 m) of Department of Electronics and Computer Engineering was carried out by recording wind induced vibration. The tower consists a coupled shear wall in longitudinal direction and lift and a staircase.

Six Kinemetrics 1-sec seismometers SS-1 and a Kinemetrics SSR-1 portable digital event recorder were used in the study.

After obtaining the results from experimental and analytical study, it was found that the values of time period obtained from analytical study are of the order of 10% to 15% higher than those of experimentally obtained. This may be due to crude modelling of the structure.

2.4 COMPARISON BETWEEN AMBIENT AND FORCED VIBRATION EXPERIMENT OF THE NINE STOREY REINFORCED CONCRETE BUILDING [19]

The ambient and forced vibration testing of Millikan library, a nine storey reinforced concrete building with basement was carried out. It is located in the campus of the California Institute of Technology in Pasadena, California. The NS section and a typical floor plan are shown in Figure 4.

Vibrator induced motions may be several orders of magnitude greater than the ambient vibrations. Hence, there is possibility that the above methods might lead to different results if the structure behaves differently at various levels of excitation. The main objective of this paper was to explore this important point by comparing results from ambient and forced vibration tests performed on the same structure.

In this test four Earth Science Ranger Seismometers were used to measure wind and microtremor vibration in the building. For forced vibration testing excitation was provided by two shaker.

It was found that the results obtained from ambient and forced vibration test agree very closely but experimental results for only one building do not, of course, justify generalizations. The field effort involved in the ambient vibration studies is significantly smaller than for the forced vibration experiments and the total number of measurements in ambient tests is also significantly smaller.

2.5 FULL SCALE AMBIENT VIBRATION MEASUREMENTS OF THE GOLDEN GATE SUSPENSION BRIDGE - INSTRUMENTATION AND DATA ACQUISITION [13]

The Golden Gate Bridge lies across the entrance to the San Francisco Bay and joins the northern and southern peninsulas. The bridge consists of a mid span of 4200 ft and two side spans of 1125 ft. each. The span are suspended from the main cable, which is supported by the two towers (height 690 ft each).

Additional modifications were made to the lateral bracing system to increase the rigidity of the roadway after a strong wind storm caused some structural damage to the bridge. Typical elevation and span cross-section are shown in Figure 5.

Extensive measurements were made of the wind and traffic induced vibration of the mid span, one side span and one tower of the Golden Gate Bridge. The purpose of these measurements was to experimentally determine modal parameters of the structure. Due to bridge dimensions and design, field conditions, and the extent of the data required, many unique and challenging problems were present in the design of the instrumentation system and in the logistics of the data acquisition.

During field portion of the ambient vibration measurements of the Golden Gate Bridge, many problems were encountered in the design of the instrumentation and in data acquisition. These included sensor choice, sensor mounting, cable design and protection from environment.

2.6 FULL-SCALE DYNAMIC TESTING AND ANALYSIS OF A RESERVOIR INTAKE TOWER [3]

Dynamic tests were conducted on a 48 m high intake tower at Wimbleball dam situated in the south west of England. Its intake tower is constructed of reinforced concrete and it houses the pipe work which extracts water for domestic supply. The tower is about 48 m high with a control house of significant mass (375 tonnes) at its top. Sections through buttress of dam and tower is shown in Figure 6. The results were compared against predictions from a corresponding numerical model. The aim of this work was to validate the assumption that the compressibility of the reservoir water is not a significant factor in the seismic analysis of intake tower.

Three sets of tests were conducted on different occasions with different water levels in the reservoir. In the first two tests, modal characteristics of the tower were determined from the measured responses under ambient, hammer and human excitation. These results were used in planning the final set of tests, where rotating eccentric mass exciters were used to vibrate the tower.

The finite element method was used to develop a numerical model for Wimbleball tower. The tower was discretized with traditional solid elements and the reservoir with incompressible fluid elements. This model was analysed to predict the modal characteristics and harmonic responses of the tower and reservoir under the various conditions imposed during the dynamic tests.

Theoretical predictions of the tower's accelerations and hydrodynamic pressure in the reservoir were compared against the test results. Excellent agreement was found for the natural frequencies and mode shapes while predictions of the harmonic responses were only fair.

The observed responses of the tower and reservoir support the assumption that reservoir compressibility is not a significant factor in the seismic analysis of towers of this size.

2.7 WATER TOWER IN SEISMIC ZONES [10]

In this paper special problems concerning water towers in seismic zones were discussed. Results of experimental study on prototype reinforced concrete water towers reported and corresponding analytical solutions are discussed.

Thirteen modern reinforced concrete water towers, six of the tanks were empty, five of them were partially full and one of them was full, wind excited vibration were recorded. For one of the towers, measurements were made with various levels of water in the tank. For four towers, free vibration caused by pull test were also recorded.

It was concluded that provision should be made in the design for energy absorption during severe earthquake. Measurements of period of concrete water towers indicate that effective flexural rigidity changes considerably with the vertical load. It is significant that the most critical condition for design of water towers against earthquake forces is usually the one with tank full.

DESCRIPTION OF VIBRATION MEASURING EQUIPMENT

3.1 ACCELERATION LEVEL IN STRUCTURES

The vibration amplitudes to be picked up in structures is usually quite low. The natural time periods of masonary buildings and concrete gravity dams is of the order of 0.25 or low, the acceleration levels in these structures under ambient vibration is of the order of 10^{-5} g to 10^{-3} g. In flexible structures like multistoreyed buildings and tall structures, the natural time periods are of the order of 1.05 or more, the acceleration levels under ambient vibration is of the order of 10^{-4} g to 10^{-2} g. The ranger seismometers with high sensitivity are suitable for measuring frequencies of stiff structures in the range of 1 Hz to 100 Hz. Force balance accelerometers are suitable for making frequency measurement of flexible structures in the range of dC to 50 Hz.

3.2 BRIEF DESCRIPTION OF SOLID STATE RECORDER (SSR-1)

The SSR-1 is a highly flexible digital seismographic event recorder which records into solid-state RAM. It utilizes a 16 bit A to D converter to provide 96 dB of dynamic range.

It can support upto six sensors which can be passive or active sample rates of 1000 sps for a single channel, 500 sps for 3 channels and 200 sps for all six channels can be selected.

The low power design of the SSR-1 offers the user a selection between two 12 volts 6.5 amp hour sealed lead dioxide batteries or 27 D-size alkaline/manganese cells, or any external 12 volt supply. Accurate timing is provided using high stability TCXO clock. The whole system is housed in a rugged aluminum case and sealed against dust and moisture. Separately housed in a polyethylene box, the batteries and fuses are accessible without opening the instrument compartment. All I/O, power and timing connectors are recessed into a front panel for protection. Because of its design, the SSR-1 can e normally operated without opening the housing.

3.2.1 Specification of SSR-1

RAM	4 MB
Dynamic range	96 dB with 16 bit accuracy
Sampling rate	200 sps

3.3 OPERATING INSTRUCTIONS FOR RANGER SEISMOMETER (SS-1)

3.3.1 Brief Description of SS-1

The SS-1 Ranger Seismometer is a versatile, high-sensitivity, portable seismometer specially designed for a variety of seismic field applications under adverse environmental conditions. The ranger combines high sensitivity, field selectable mode (horizontal or vertical) and rugged water-tight construction, in a package measuring only 5.5 inches in diameter by 12 inches long and weighing only 10.9 pounds. A separate calibration coil in the base provides a simple means of field calibrating the ranger using only a known voltage battery and a fixed precision resister.

The ranger is a spring-mass instrument with electromagnetic transduction. Its permanent magnet assembly is the seismic mass while the coil is attached to the frame. The ranger can be used either horizontally or vertically and is well suited to field or laboratory use. The relationship between major parts is shown schematically in Fig. 7. The mass is supported by two circular flexures which constrain it to a single degree of freedom. A helical spring is used to suspend the mass. When the seismometer is used vertically, the suspension spring is fully extended; when used horizontally, the spring is unstressed. The force of the suspension spring is controlled by positioning a hanger rod attached to the spring. The basic natural period of the mass, flexures, and suspension spring is extended by the addition of small rod-magnets installed around the mass. There period extending magnets interact with the magnetic field of the mass, effectively producing a negative restoring force. In order to achieve the desired period, the field strength and position of the period-extending magnets are carefully adjusted at the factory.

3.3.2 Specification of Ranger Seismometer (SS-1)

Positioning	Longitudinal, Transverse and vertical direction
Natural Period	1 Sec.
Coil Resistance	5500 ohms
Critical Damping Resistance	6500 ohms
Generator constant	340 volts/m/sec
Total Mass Travel	2 mm
Weight	1.45 kg

3.3.3 Adjustment of SS-1

The SS-1 Ranger Seismometer should not be installed within six inches of any steel or magnetic object.

To unclamp the mass, turn the transport lock (Fig. 8) fully counter clockwise. Apply full finger torque to seat the transport lock against its gasket inside the case. This makes the seismometer weather proof in its operating mode.

The mass is brought to the centre of its span of travel by means of the spring hanger rod at the top of the instrument. After unclamping the mass by turning the transport lock fully counter clockwise, make this adjustment as follows :

1. Unscrew and remove the access cover/handle.
2. While holding the spring hanger knob with one hand, loosen the collect nut with the other hand.
3. Move the spring hanger rod with the mass is fairly near centre. Centering is determined by the coincidence of two lines which

are visible through the viewing part. With the mass reasonably centered, tighten the collect nut.

4. Fine centering of the mass is now achieved by means of the mass centering nut. Turn this nut until the two lines, as seen through the viewing port, coincide.
5. Replace the access cover/handle, being sure that its gasket is properly seated.

VIBRATION SURVEY SOFTWARE

4.1 INTRODUCTION

The Kinemetrics Seismic Workstation Software is a collection of programs written for the IBM PC and designed to provide the user with commonly required data processing programs and to provide a software core to which custom programs for specific applications may be added. The programs are centered around a common data format (CDF) for multi-channel time series data so that programming redundancy is minimized. The block diagram of the program relationships are shown in Fig. 9.

4.2 INTRODUCTION TO QUICKTALK (QT) (SSR-1 COMMUNICATIONS PROGRAM)

QuickTalk, QT, is an integrated environment for communicating with kinemetrics SSR-1 Solid State Recorder using an IBM or 100% compatible computer. It provides for direct communications as well as remote access over telephone lines and modems. A spreadsheet like parameter Worksheet is built into the program to ease the setup of experiments which require periodic changes in the configuration of the recorder. Quicklook, is also available from within QuickTalk to provide graphical display of received event files.

What QuickTalk really does that standard PC communications programs don't is to mainly "insulate" the user from the simple (and often times cryptic!) SSR-1 two-character commands and 187 numeric parameters. Many of these commands are accessed from the "user friendly" pop up Worksheet complete with context-sensitive help in many areas.

Since QuickTalk directly access the PC's serial port, interrupt controller, and display screen memory, 100% hardware compatibility with the IBM PC is absolutely required. Also, the Bios (Basic Input Output System) ROM must be compatible. QuickTalk supports colour display adapters, monochrome adapters and monitors as well as the Liquid Crystal and Plasma displays used on many Laptop computers. Communications at speeds upto 115.2 kilobaud are supported on faster computers.

4.3 INTRODUCTION TO QUICKLOOK

QuickLook (QL16) gives the field user a quick visual presentation of seismic waveform data recorded on Kinemetrics SSR-1 recorders. Maximum amplitude, event duration and predominant frequencies can all be readily determined.

It provides an immediate visual presentation of the data with no additional processing. It is also especially helpful in quick determining which records require further analysis. Finally, QL16 can be used during periodic inspections to display functional test records to verify proper operation on the spot.

The program QL16 is provided with Kinemetrics SSR-1 digital seismic recording instruments. The program allows seismic records to be displayed graphically on the screen of an IBM PC (or 100% compatible) computer. The program is compatible with both 12 bit (SSA-1) data and 16 bit (SSR-1) data. The event can be viewed as soon as they are transferred to the PC.

QL16 automatically scales the event to fit on the screen in both x and y axes. Once initially displayed, channels which are temporarily not wanted can be removed from the display and areas of interest can be quickly examined in greater detail by working with the cursor and zooming it.

4.4 TIME SERIES FILE PROCESSING

The first step in data processing on the Seismic Workstation is to acquire raw instrument data and convert it into the Workstation common data format (CDF). This stage of processing is handled by the Instrument Interface Software as shown on the block diagram in Fig. 9.

Once the raw instrument data has been converted to a CDF file (with default file extension "D16"), the next few processing steps are to decode the serial time code (if necessary), and to edit the raw data down to a subset for processing by the Vibration Survey programs.

4.5 SPECTRAL ANALYSIS

The program for spectral analysis include FFT and QFFT. Both programs perform non-averaged fast Fourier transforms on up to four channels of data from CDF files. The program FFT differs from QFFT only in the respect that FFT allows 4 times larger transforms to be computed compared to QFFT. However, FFT utilizes disk files for storage of intermediate results, thus slowing down processing.

The QFFT program is described in the following sections. However the command descriptions are equally applicable to the FFT program commands.

4.6 INTRODUCTION TO QFFT

The interactive spectral analysis program QFFT provides the user with the ability to view pre-recorded data graphically in the frequency domain. QFFT also provides the user with the capability for graphically zooming displays for detailed viewing, and producing labeled hard-copy output on the optional printer or pen plotter. Detailed frequency auto-plots are scaled both horizontally and vertically. Time series plots have units of volts and seconds. Frequency amplitude plots have units of Fourier amplitude and Hertz.

4.6.1 Starting the Analysis Session

QFFT is started with a command of the form :

```
QFFT filename
```

where "file name" is the name of the CDF time series data file to be processed. The program then responds with a series of questions to determine the processing parameters which are to be used in processing the file which is shown in Fig. 10.

As indicated in Fig. 10 the first question requests to enter the number of channels to be analysed. The acceptable range of values are shown in square brackets and the user response must be typed in integer format (no decimal point is allowed). In the example, the acceptable range is 1 through 4 channels, and the user has elected single-channel processing mode. Note that the larger the number of channels to be processes d, the smaller the allowable transform size. This is due to data array space limitations of QFFT.

The second question printed by QFFT requests the transform order (size) to be entered. This is the power of two of the number of input data points which will be used in the FFT computation. Thus, a selection of 11 means that 2048 data points will be processed, Yielding an input spectrum with 1024 points from 0 Hz (DC) to the Nyquist frequency (1/2 x sampling rate). Note that the upper limit of the acceptable range which is displayed is adjusted for the number of channels which has been selected in the preceeding question.

The third question requests to select the transform window desired. Refer to any digital signal processing text for an explanation of the theory and appropriate use of a window function.

At this point, QFFT searches for the specified data file, reads the header information, and displays the comment, sample rate used, and the assigned channel names.

Next, QFFT requests the number of the data file channel to be processed for each of the QFFT analyzer channels selected. In the example above, since only 1 analyzer channel was selected, only one data channel needed to be entered (channel 2 was entered).

The final question is used by QFFT to determine the point within the data file at which processing is to begin. This is useful for comparing transforms of data at different points within a given file, as well as for skipping past a bad section of the file. The maximum time offset which QFFT will accept is 30.0 seconds, and the user input may contain a decimal point.

At this point the initial dialog is complete, and QFFT proceeds to read the input data file and to execute the operations specified. The progress of the computations are indicated as shown in Figure 10. Note that a transform of order 10 (1024 input points) will take approximately 20 seconds. After FFT calculations are complete, the Seismic Workstation screen is cleared, and the initial plot is displayed as shown in Fig. 16.

The initial QFFT plot consists of two "frames" in which functions are displayed. The upper frame (frame #1) is a linear-linear auto-scaled plot of the input time series for analyzer channel 1, while the lower frame (frame #2) contains an auto-scaled semi-log plot of the amplitude spectrum of the data. In the case of frame #1, the horizontal axis (time) is scaled in seconds, while the amplitude is normalized to full-scale. For frame #2, the horizontal axis is frequency in Hertz. Note that the top of the plot is also labeled (from left to right) with the data file name, the comment found in the header of the file and the analyzer channel number.

4.6.2 Interrupting & Terminating QFFT

The analysis session is terminated by two methods, depending on the current input mode of QFFT. When QFFT is requesting input during the initial dialog, the program may be aborted by typing a <control-c> as the first character of the input. If QFFT is requesting input via the graphics cross-hairs, typing the character <Q> will "quit" the analysis session.

CHAPTER - 5

EXPERIMENTAL ANALYSIS OF WATER TOWERS

5.1 AMBIENT VIBRATION TESTING OF WATER TOWER NEAR G.P. HOSTEL

5.1.1 Description of the Water Tower

Ambient vibration testing of water tower (Intz tank) of capacity 525 KL was carried out on 7th March, 1995. The height of the staging is 20 m. Tank is supported on 8 columns of diameter 500 mm. Braces are connected to the columns at 4 places i.e. columns are divided in 5 bays. Dimension of the bracing is 200mm x 600mm. The foundation of the tank is annular raft foundation . Half-section half elevation and half foundation plan-half plan at brace level are shown in figure 11 and figure 12 respectively. The detail dimensions of different components of the water tower are shown in figure 11. At the time of testing the height of water in tank was 8 ft. approximately.

5.1.2 Instrumentation of Tank

Two Kinemetrics 1-sec (natural period) seismometers SS-1 and Kinemetrics SSR-1 portable digital event recorder were used in the testing. The seismometers were placed at first bracing near the column of the tank which is at a height of 2.72 m from ground level. The position of the seismometers (S_1 & S_2) are shown in figure 11. Both seismometers were placed parallel to the bracing.

5.1.3 Excitation in Tank

In this testing wind excited vibrations were recorded.

5.1.4 Description of the Tests

The excitation caused by wind was recorded by Kinemetrics SSR-1 digital recorder in digital form. The digital data was then transferred to the hard disk of Note-Book computer by using program, **QuickTalk**, which is already discussed in section 4.2. Eleven events were recorded and amongst them event no. 3 was analysed. Using option **QL16.exe** the plot shown in Fig. 13 was obtained, which is complete graphical representation of digital data recorded earlier in two channels of event number 3 (File name Tan00003,SSR).

For getting natural frequency, option **fd.exe** was used. Two points were selected on frequency plot of channel no. 2 as shown in Fig. 14. After entering the number of cycles between the above two selected points, the value of natural frequency was displayed on frequency plot. The value of natural frequency was obtained as 0.8423 cyc/sec. The plot parameters for frequency is shown in Fig. 15.

The plot of **QFFT** from samples of the recorded data of event no. 3 are shown in Fig. 16. Steps have already been described in section 4.6.

The result obtained after testing of this tank is tabulated in table -1.

5.2 AMBIENT VIBRATION TESTING OF ELEVATED WATER TANK NEAR SAROJINI BHAWAN

5.2.1 Description of Tank

Ambient Vibration Testing of elevated water tank of capacity 525 KL was carried out on 7th march 1995. The height of staging is 20 m. Tank is supported on 9 columns of dia 500 mm. Braces are connected to the columns at 4 places i.e. columns are divided in 5 bays. Dimension of braces are 200 mm x 600 mm. The foundation of tank is annular raft foundation.. Sectional view and plan of the water tower are shown in Fig. 17 and Fig. 18 respectively. Detail dimension of the components of the water tower is shown in Fig 17. At the time of testing, tanks were empty.

5.2.2 Instrumentation of Tank

Three Kinemetrics 1-sec (natural period) seismometers SS-1 and Kinemetrics SSR-1 portable digital event recorder were used in the testing. The seismometers were placed at first bracing near the column at the height of 2.9 m above the ground level. Two seismometers (S_1 & S_2) were placed parallel to the bracing and one Seismometer (S_3) was placed in the transverse direction of bracing. The position of seismometers are shown in Fig. 17.

5.2.3 EXCITATION OF TANK

In this testing wind excited vibration were recorded.

5.2.4 Description of the Test

Description of the test have already been discussed in section 5.1.4 . The QL16.exe plot of all three channels for three seismometers for the event no. 4 (File name SJT00004.SSR) are shown in Fig. 19. The frequency plot for channel no. 3 of event no. 4 are shown in Fig. 20. The value of natural frequency which has been obtained is 3.79 cys/sec. The plot parameters of frequency is shown in Fig. 21. The plot of QFFT for this event is shown in Fig. 22. The results of the test is tabulated in table 1.

5.3 RESULTS OF THE TESTING

The experimentally measured values of fundamental time periods of two water towers are tabulated below -

Table 1 Experimental Time Period of Water Towers

Water Tower	Depth of water	Time period (sec)
near G.P.Hostel	8 ft.(approx.)	1.19
near Sarojini Bhawan	empty	0.2638

CHAPTER - 6

ANALYTICAL INVESTIGATION OF WATER TOWERS

6.1 GENERAL

The two water towers (i) near G.P. Hostel (ii) near Sarojini Bhawan were considered as 3D framed structure and analysed by 3D space frame program. Both the water towers, tested experimentally are modelled for analytical purposes. The mass of the water tower is discretized and lumped at different nodal points. The body of tank including bottom ring beam are assumed to be rigid and the mass is lumped at the top nodal points. It is also assumed that the top of the column is attached by rigid link. The staging at the ground level is assumed to be rigidily fixed at base. Staircase of the water tower was not considered in modelling.

6.2 MATHEMATICAL MODEL OF WATER TOWER NEAR G.P. HOSTEL

The mathematical model of the water tower is shown in Fig. 23 which is circular in shape. There are 8 columns of height 19.1 m. The effective length of columns are shown in Fig. 23. Effective length of braces are 3.44 m. The staging has five bays. Water tower is discretized into 48 nodes and 80 members, in which member 1 to 8 are assumed to be as rigid. The weight of water (depth of water in tank 8 ft. approximately) including dead weight of tank and top ring beam are lumped at top of the column from node number 1 to 8.

The tank is also analysed for 6.5 ft. depth of water and for empty condition also and the results are tabulated in table-2. The detail dimension of the water tower are shown in Figure 11.

6.3 MATHEMATICAL MODEL OF WATER TOWER NEAR SAROJINI BHAWAN

The mathematical model of the tank is shown in Fig. 24 which is circular in shape and resting on 9 columns of height 19.3 m. The effective length of braces are 3.42 m. Effective length of columns are shown in Fig. 24. The staging has five bays. The water tower is discretized into 54 nodes and 90 members in which member number 1 to 9 are assumed to be rigid link.

Since the tank was empty hence only dead weight of tank and bottom ring beam are assumed to be lumped at the column at a height of 19.3 (top of the column). Detail dimensions of the tank are shown in Figure 17.

6.4 MATRIX METHOD OF ANALYSIS

6.4.1 General

The analysis of structures, static or dynamic requires the solution of algebraic equations, or the calculation of the eigen values of the system. In either case the problem can be handled in a systematic manner in compact matrix notation. The structure is idealized into a skeleton system which retains the properties of the original structure. The stiffness matrix of the structure as a whole is assembled from the stiffness matrix of the individual members. The resulting equation can then be solved for time periods and modal amplitudes in dynamic analysis.

6.4.2 Assumptions

The assumptions involved are essentially such as to facilitate mathematical modelling of a real system in such a manner that the behaviour of the prototype structure can be simulated. The assumptions involved in a linear elastic analysis are :

(a) The structural material is homogeneous and isotropic.

(b) All special members are replaced by line member oriented along the centroidal axis of the original member.

(c) The line members, however, retain all properties of the original member i.e. length, inclination, area and moment of inertia.

(d) The member intersections are infinitesimal in size.

(e) Members having a common junction are assumed to be concentric. (errors so introduced in either member lengths or inclination do not cause significant errors in the analysis).

(f) The structural material has a well defined linear relationship between applied load and resultant displacement.

6.4.3 Member Stiffness Matrix

The stiffness matrix method of analysis is one in which compatibility of displacements is assumed and the equilibrium equations at the nodes are formulated in terms of the nodal displacement components. The method proceeds from part to whole i.,e. member stiffness matrices are generated and contribute to the assembly of the overall stiffness matrix of the structure.

The stiffness matrix of a rigid frame member arbitrarily oriented in a 3D space having six degree of freedom at each end namely translation along x, y and z axis, and rotation about x,y and z axis. It can be derived by imposing a unit displacement along each degree of freedom and computing the induced forces corresponding to all other degrees of freedom. The resulting stiffness matrix of the member in local coordinate system is shown is Appendix-A. Member stiffness matrix thus generated are then contributed to the assembly of the overall stiffness matrix of the structure.

6.4.4 Transformation Matrix

In order to establish the equilibrium equations it is essential that force components at nodes of the members meeting at a node, be in the same directions. This transformation of force components from local coordinate system to the global coordinate system is achieved by means of a transformation matrix (Appendix-B). The stiffness matrix of a member is originally derived in local coordinate system and then modified so as to represent the stiffness in global coordinate system by using equation :

$$[K_G] = [R][K_L][R^T]$$

K_G = Stiffness in global coordinate system

K_L = Stiffness in local coordinate system

R = Transformation Matrix

6.4.5 Free Vibration Characteristics

The global stiffness matrix [K] is obtained as explained in earlier section. The generalized mass matrix is assumed to be diagonal and the diagonal elements at each node correspond to the three translational and three rotational degree of freedom. The inertial effects due to rotational degrees of freedom have been neglected.

The equation of motion for free vibration can be expressed in the form

$$[K]\{\phi\} = p^2[M]\{\phi\}$$

where p is undamped natural frequency and ϕ is the mode shape factor. Above equation can be expressed in the following form i.e.

$$A\ X = \lambda\ X$$

which represents the eigen value problem whose solution leads to evaluation of natural frequencies and corresponding mode shapes.

The forms adopted are

$$[M^{-1}]\ [K]\ \{\phi\} = p^2\{\phi\}$$

or

$$[K^{-1}]\ [M]\ \{\phi\} = (1/p^2)\ \{\phi\}$$

Equation is generally preferred for the sequential determination of eigen pairs.

6.5 COMPARISION OF RESULTS AND DISCUSSION

The experimental and theoretical fundamental time period of the water towers are taulated below

Table-2 : Experimental and Theoretical Fundamental Time Period of the tanks

Name of The Tanks	Theoretical Fundamental Time Period (Sec.)			Experimental Fudamental Time Period (Sec.)
	Depth of Water in Tank		Empty Tank	
	8 ft.	6.5 ft.		
1) Water Tank near G.P. Hostel	1.37	1.27	0.984	1.19 (for approx. 8 ft depth of water)
2) Watwr Tank near Sarojani Bhawan (Empty Tank)	--	--	1.06	0.264

The theoretical time period of the water tower near G.P.Hostel is slightly more than experimental time period. This difference may be due to approximate estimation of water depth in the tank because exact depth of water in the tank was not known.

There is. large difference in experimental and theoretical fundamental time period of the water tank near Sarojini Bhawan.

Owing to the weak velocity of the wind, the vibration caused in the tank was of low magnitude. So, the seismometer picked up the frequency of high range.

The other possible reason for the difference in the time period could be due to the assumptions made in mathematical modelling of the water towers.

SUMMARY AND CONCLUSION

7.1 SUMMARY

Ambient vibration testing of two water towers (i) near G.P. Hostel (ii) near Sarojini Bhawan, were carried out. The capacity of each tank is 525 KL and height of the staging is 20 m. The experimentally obtained time periods were compared with those obtained from corresponding mathematical models.

The time period of the towers were determined from the measured responses under wind excitation. The vibration were picked up by Ranger Seismometer (SS-1) and recorded by Solid State Recorder (SSR-1). The recorded data was then analysed by using Seismic Workstation Software (SWS).

The 3D beam element was used to develop a mathematical model for the water towers. The mass of tower was discretized and lumped at different nodes. The tank of the water tower was assumed to be rigid and its weight was lumped at the top of the columns. The columns at the base were assumed to be fixed. The free vibration analysis of mathematical model was carried out to determine fundamental time period. A comparison between experimentally measured time periods and theoretically computed period was made.

7.2 CONCLUSIONS

Following conclusion can be drawn from the study :

(1) A reasonably close agreement between experimental and theoretical value of fundamental periods has been observed for the water towers near G.P. Hostel.

(2) For the other tower near Sarojini Bhawan there is a difference between experimental and theoretical value of period. The possible reason for the difference is the high frequency vibrations recorded under weak intensity of wind at the time of testing.

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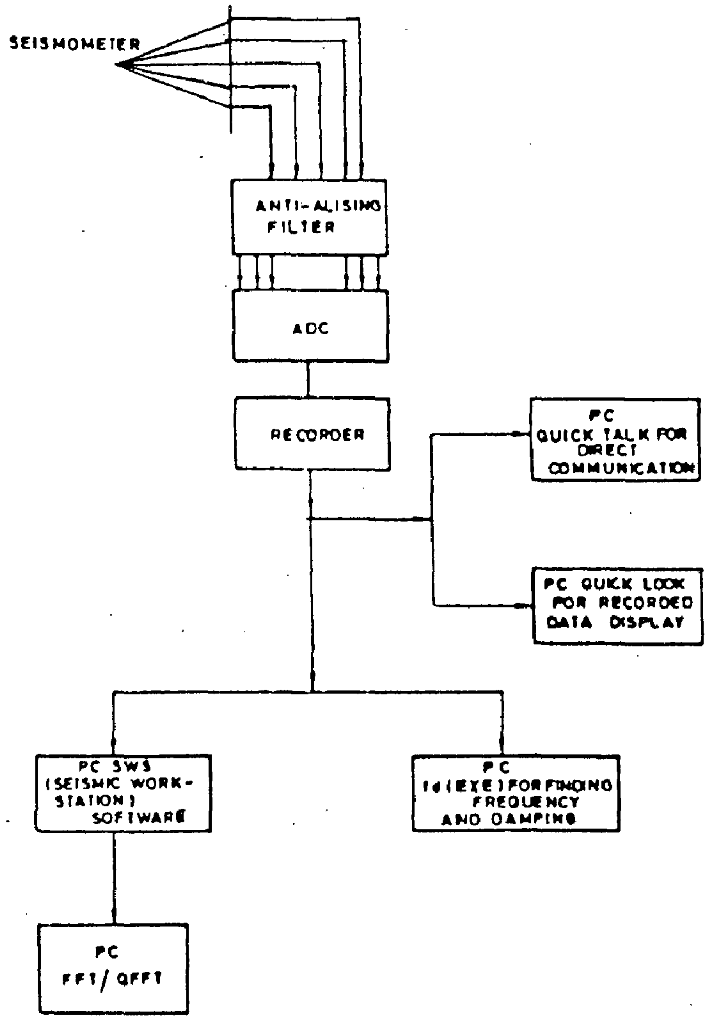


FIG.1 AMBIENT VIBRATION TEST SETUP

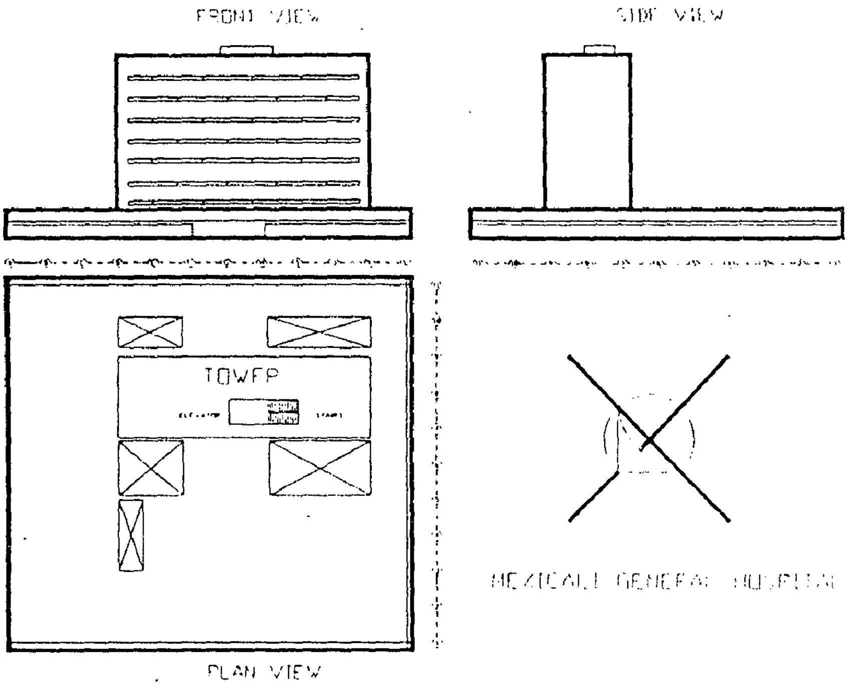


FIG.2 PLAN, FRONT AND SIDE VIEWS OF THE MAXICALI GENERAL HOSPITAL

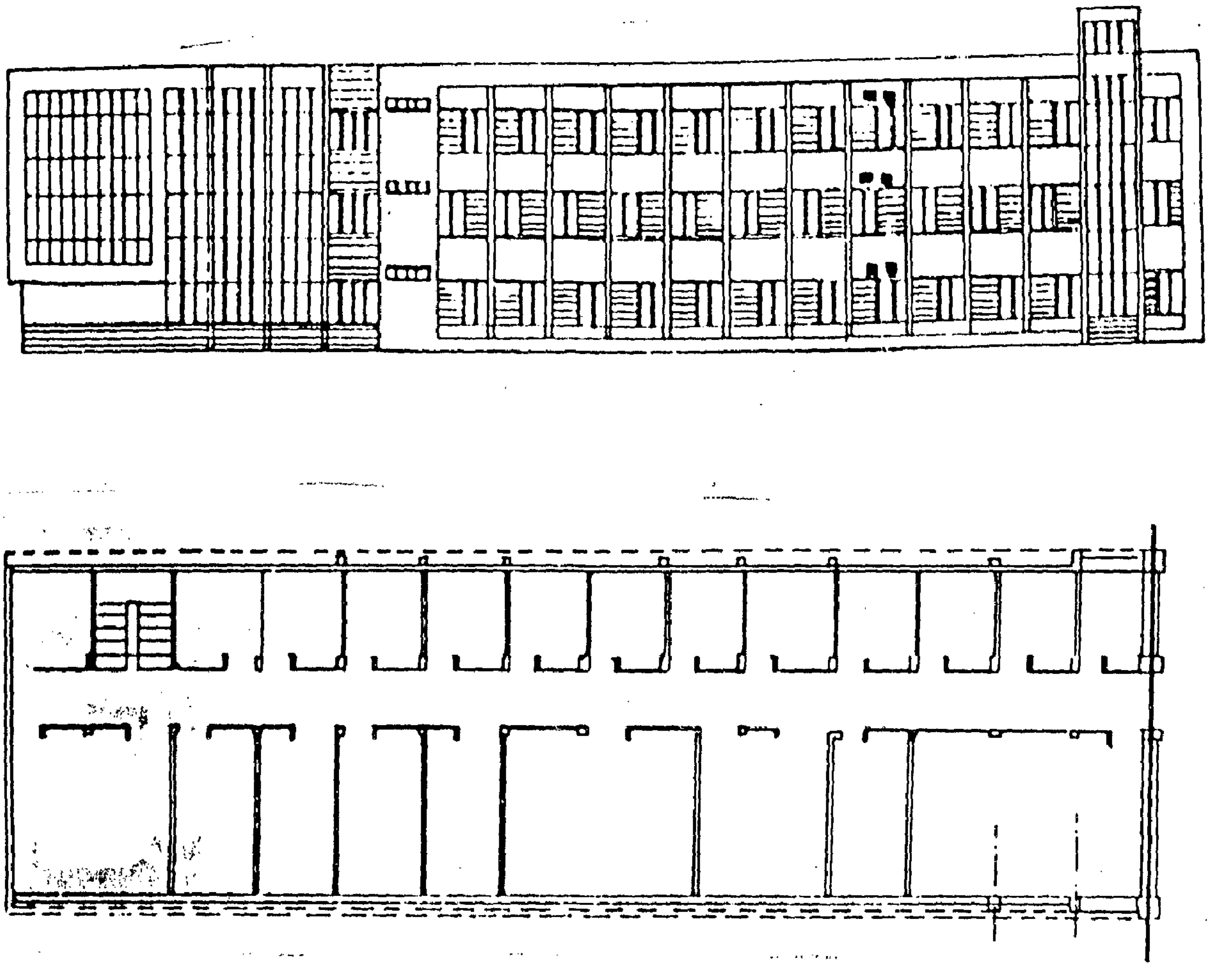


FIG.3 FIRST FLOOR PLAN AND FRONT ELEVATION
OF THE THREE STOREYED BUILDING

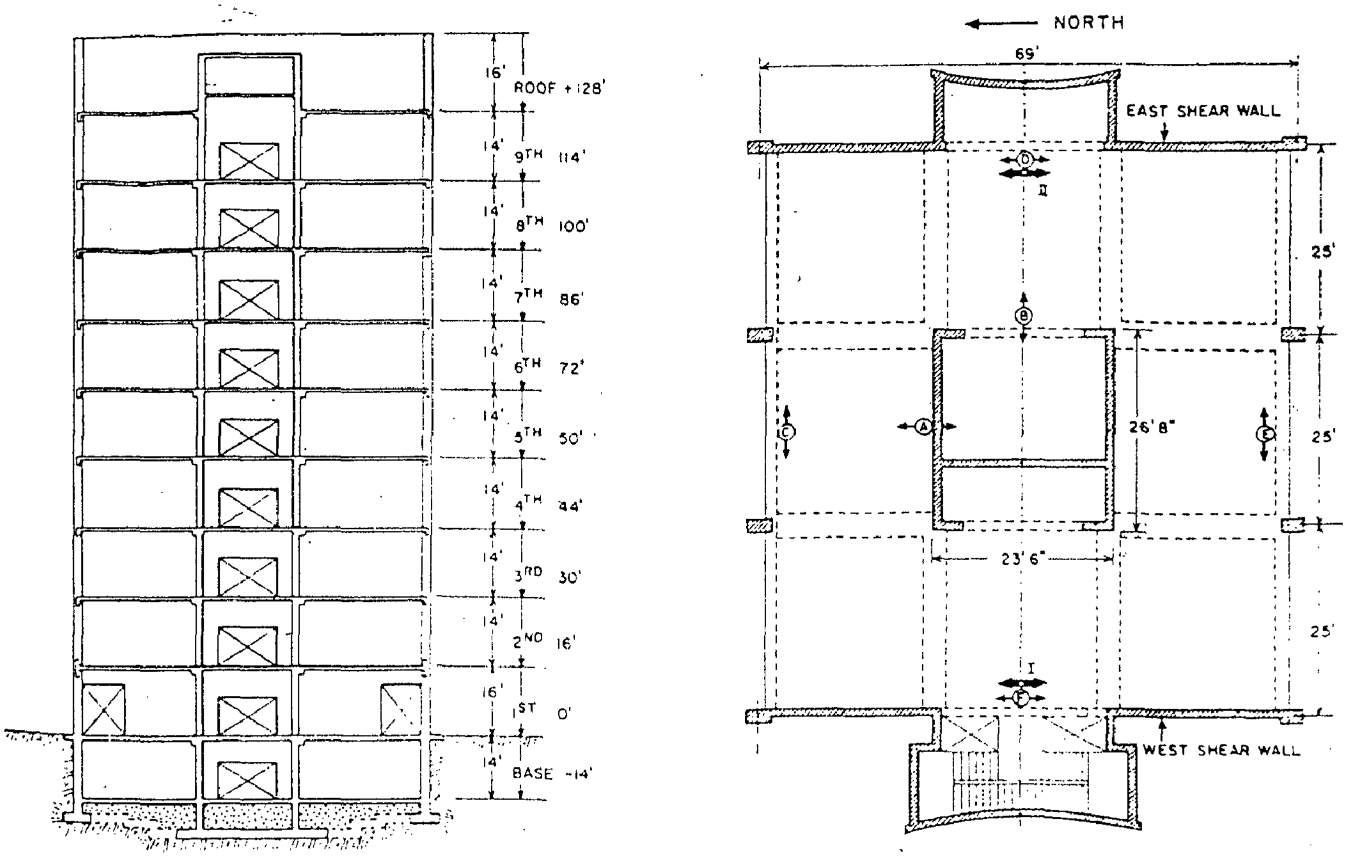
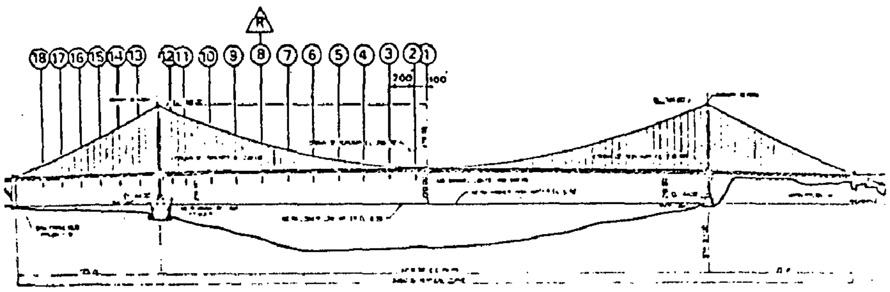


FIG.4 A NORTH SOUTH SECTION AND A TYPICAL FLOOR PLAN OF THE BUILDING OF MILLIKAN LIBRARY



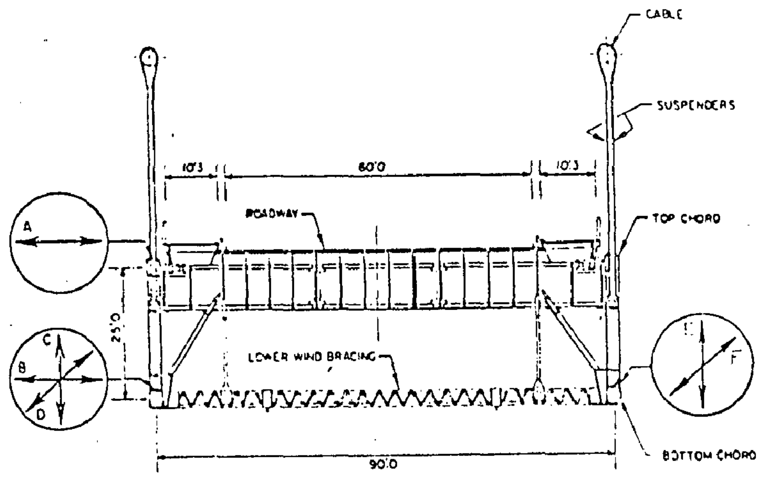


FIG.5 TYPICAL ELEVATION AND SPAN CROSS-SECTION OF GOLDEN GATE SUSPENSION BRIDGE SHOWING SENSOR PLACEMENT AND MEASUREMENT LOCATION

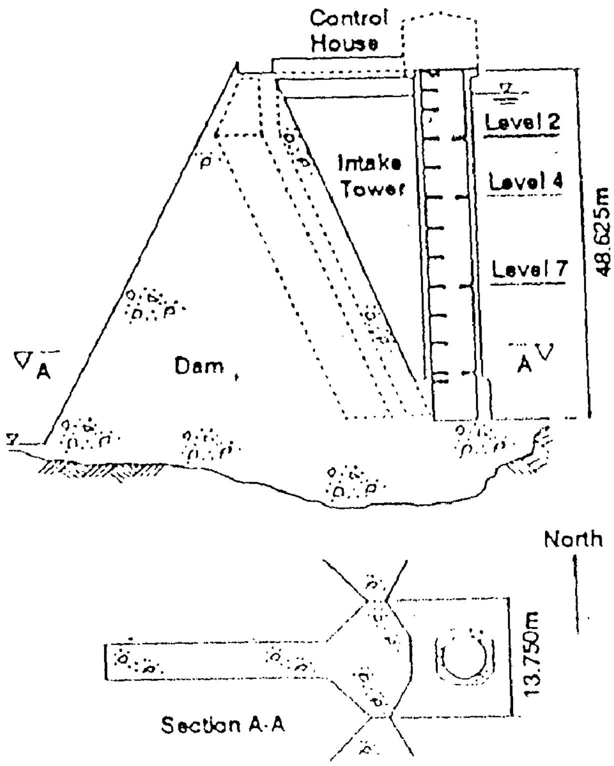


FIG.6 SECTION THROUGH BUTTRESS OF DAM AND TOWER

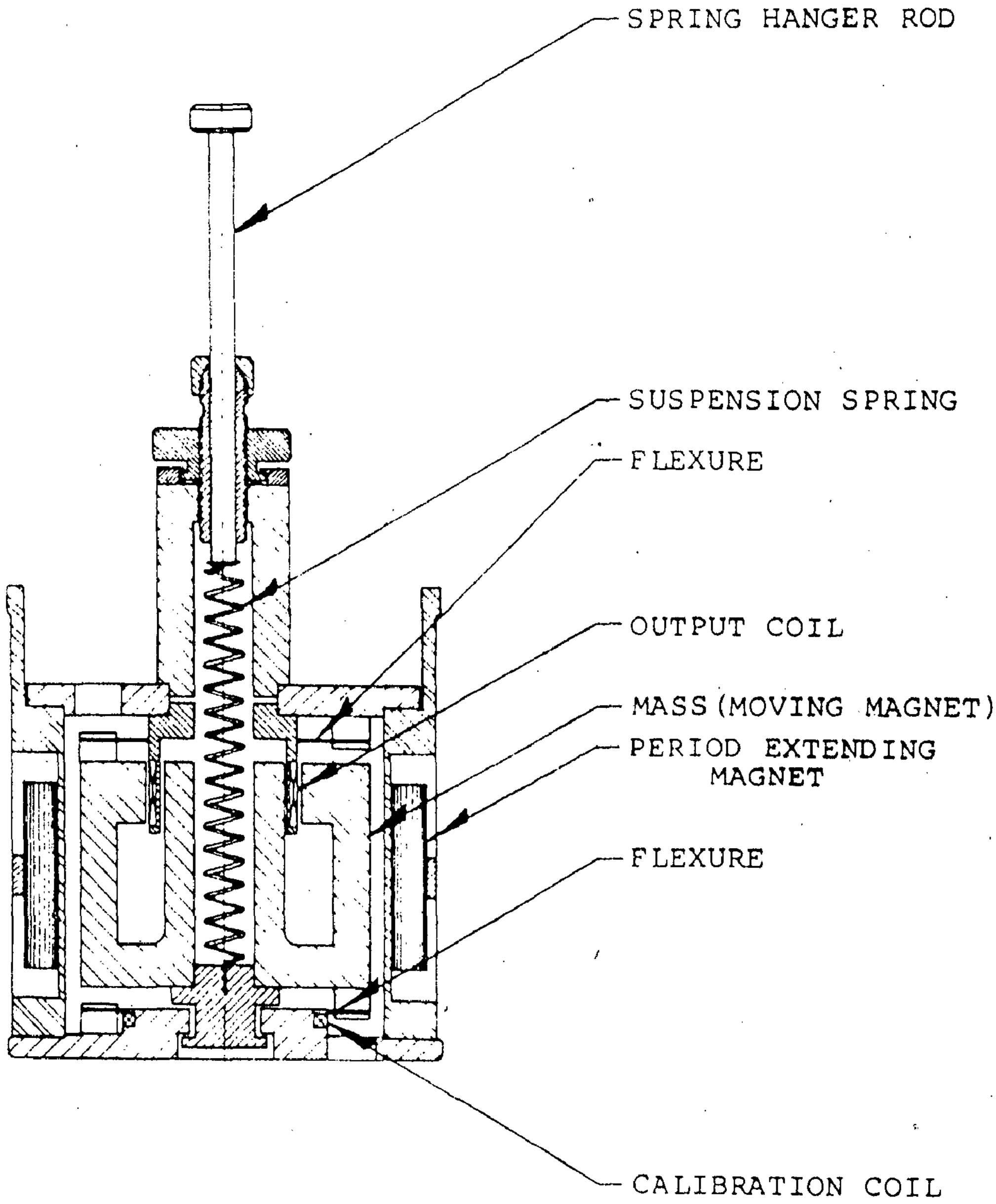


FIG.7 GENERAL CONSTRUCTION OF RANGER SEISMOMETER

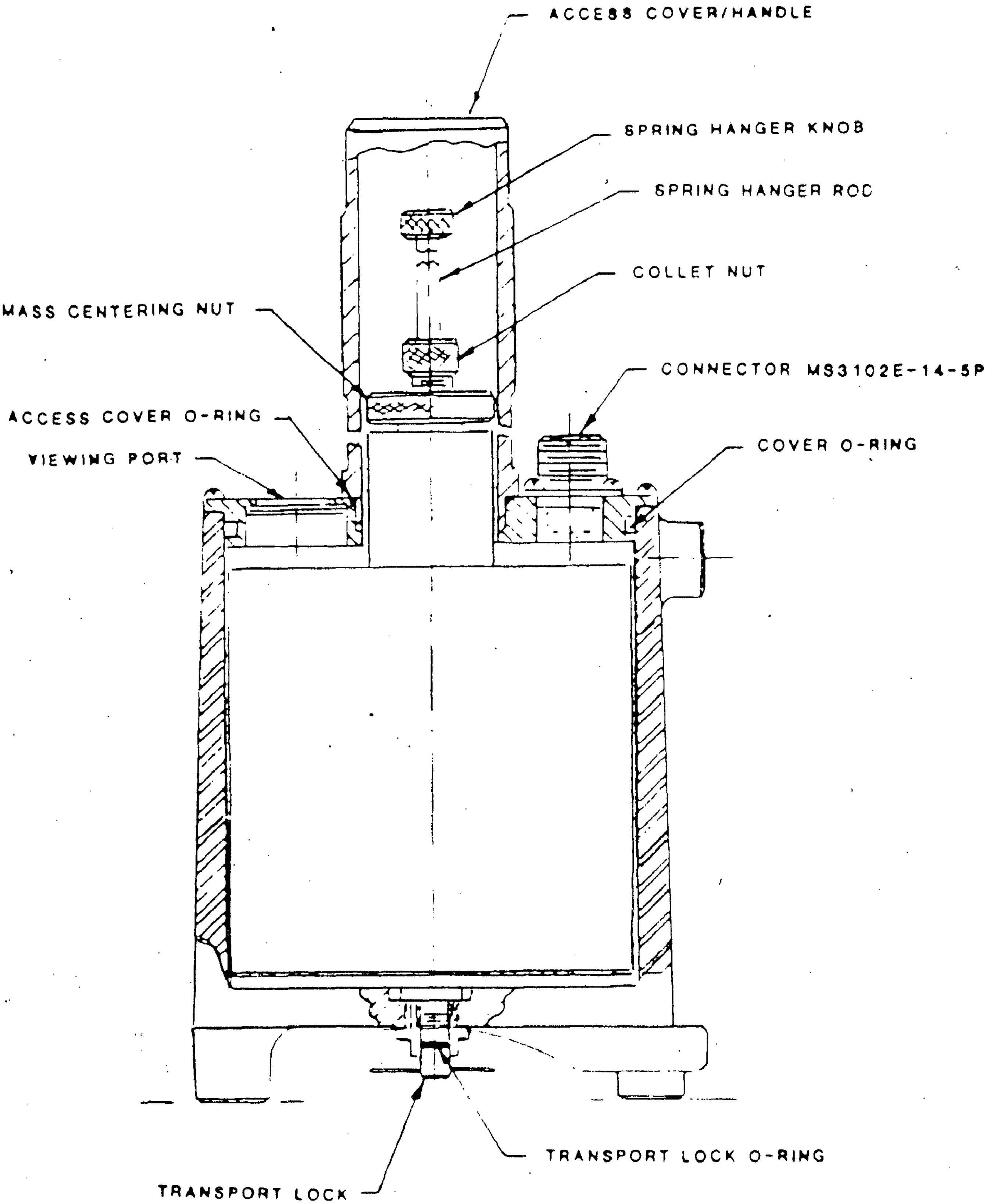


FIG.8 SEISMOMETER AND CASE ASSEMBLY

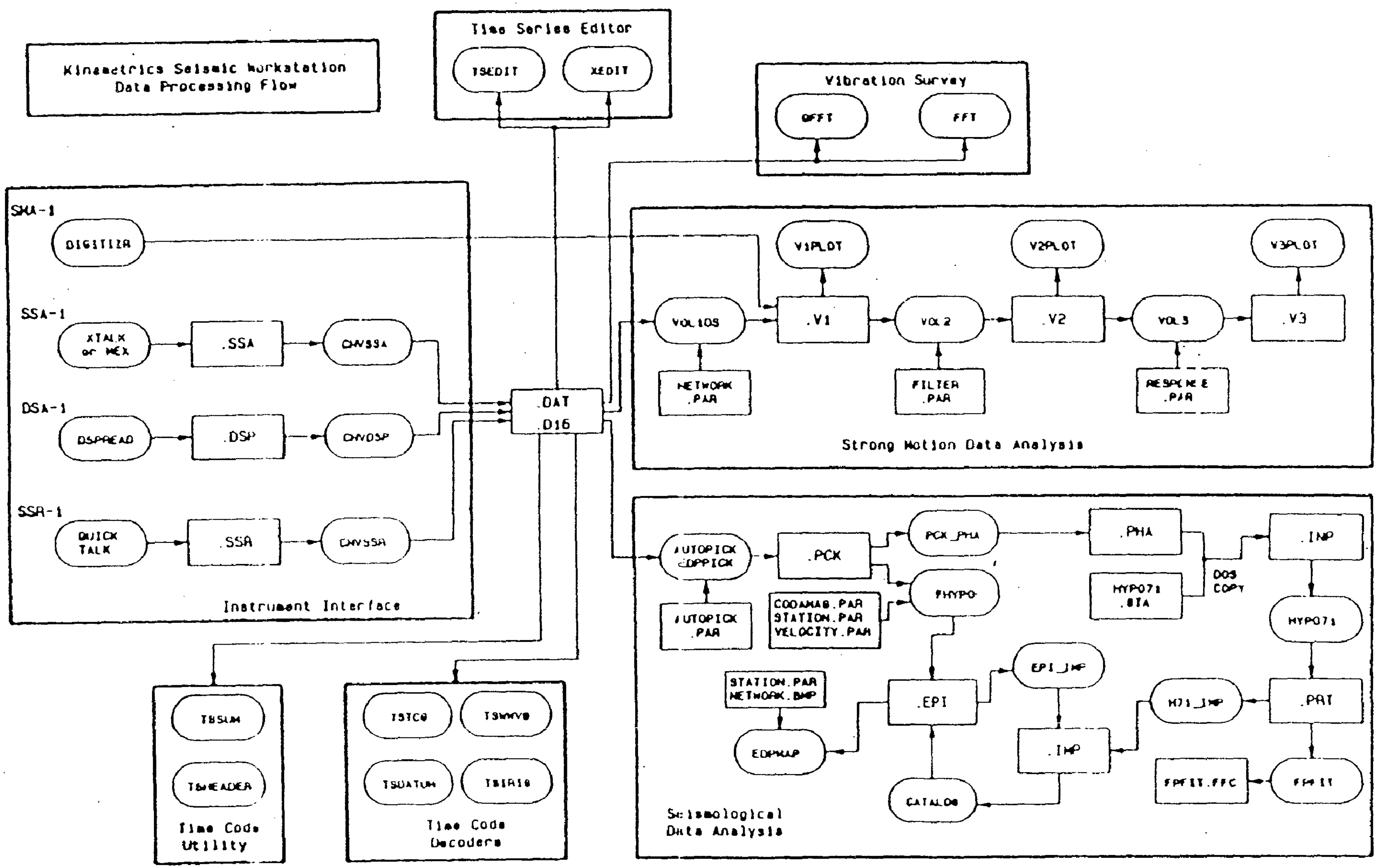


FIG.9 BLOCK DIAGRAM OF SEISMIC WORKSTATION SOFTWARE

```
D:\WORKSTN\EXE>qfft.exe
QFFT- Fast Fourier Transform Program (12/16 Bits)
SWS-1   Seismic Workstation Software Rev. F
        Seismic Workstation Libraries Rev. E (12/16 bits)
        Copyright (c) 1986,1987  Kinemetrics Systems
Enter input source file name: tan00003.d16
Number of analyzer channels [1..4]? 1
Transform order [5..11]? 9
Window [1=none,2=Hanning,3=Hamming]? 1
Editing file ---> tan00003.d16
Data width ---> 16 bits
                                            sample rate:  200.0
Channels:
 1 TIME    3          5          7
 2         4          6
Select channel (#1) [1.. 7] -2
Time offset (seconds)? 0.0
Reading data file...
Calculating ch 1 fft...
```

FIG.10 QFFT INITIAL DIALOG EXAMPLE

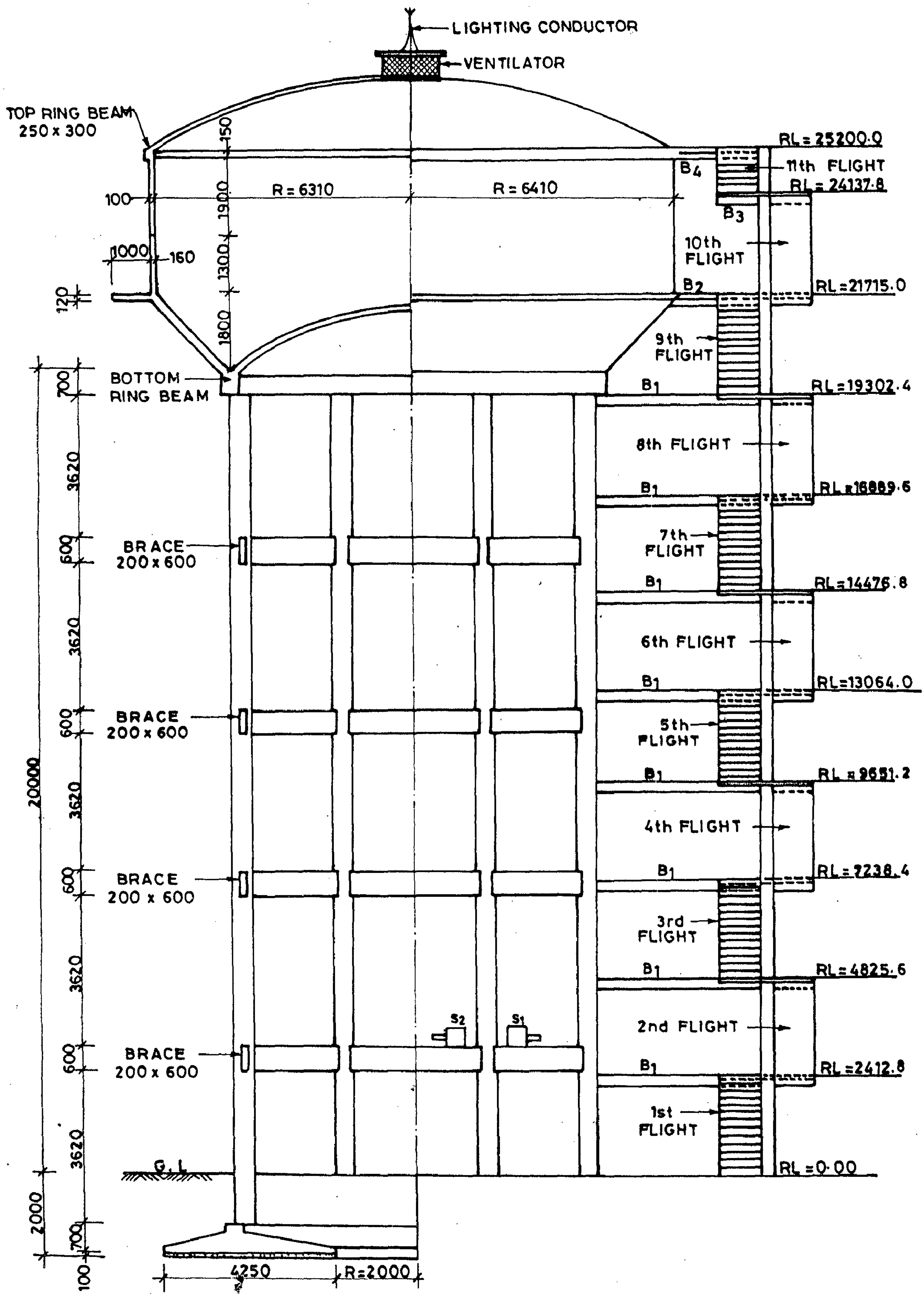


FIG. 11 HALF SECTION – HALF ELEVATION OF WATER TOWER NEAR G.P. HOSTEL

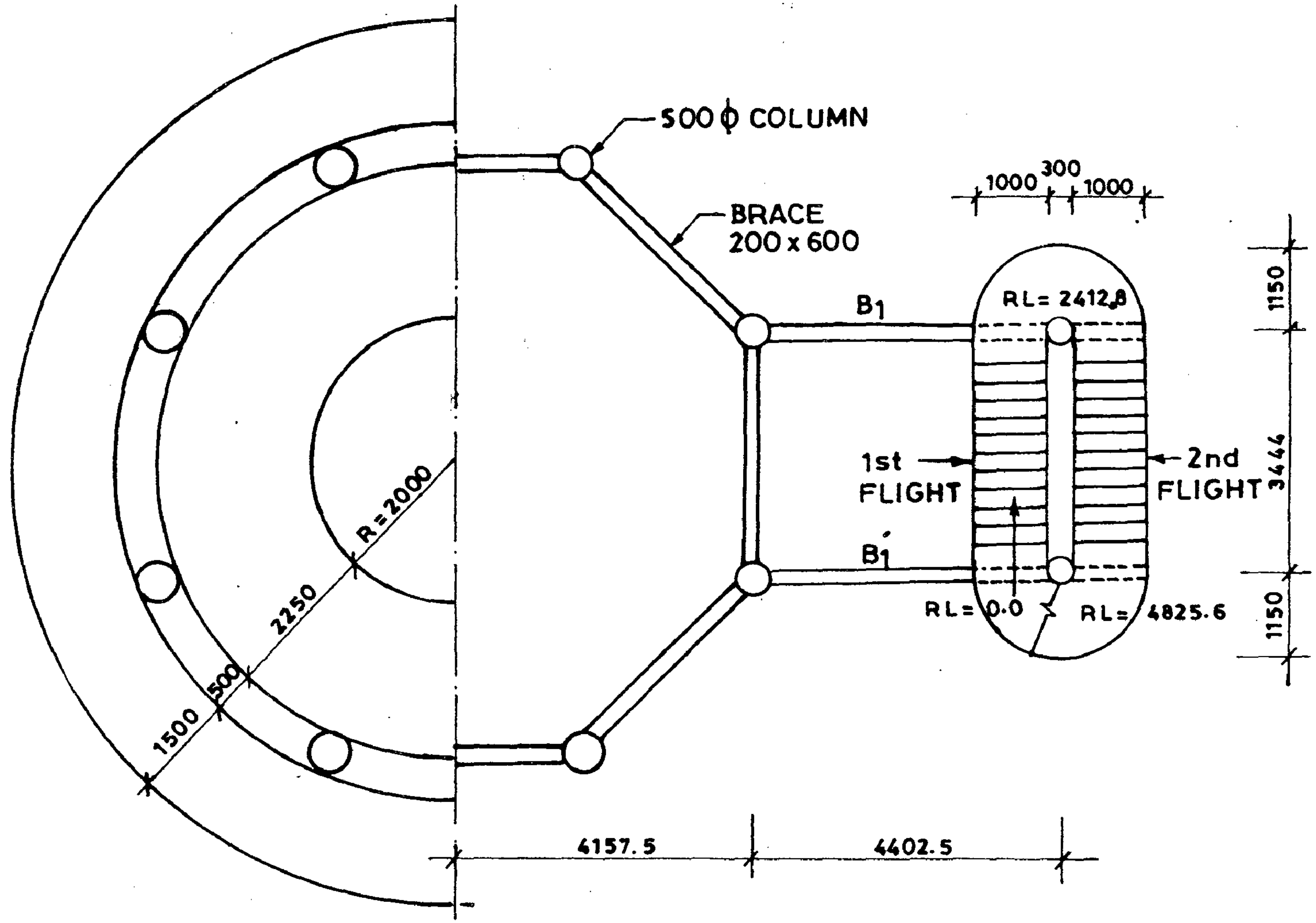


FIG. 12 HALF FOUNDATION PLAN– HALF PLAN OF WATER TOWER NEAR G.P. HOSTEL

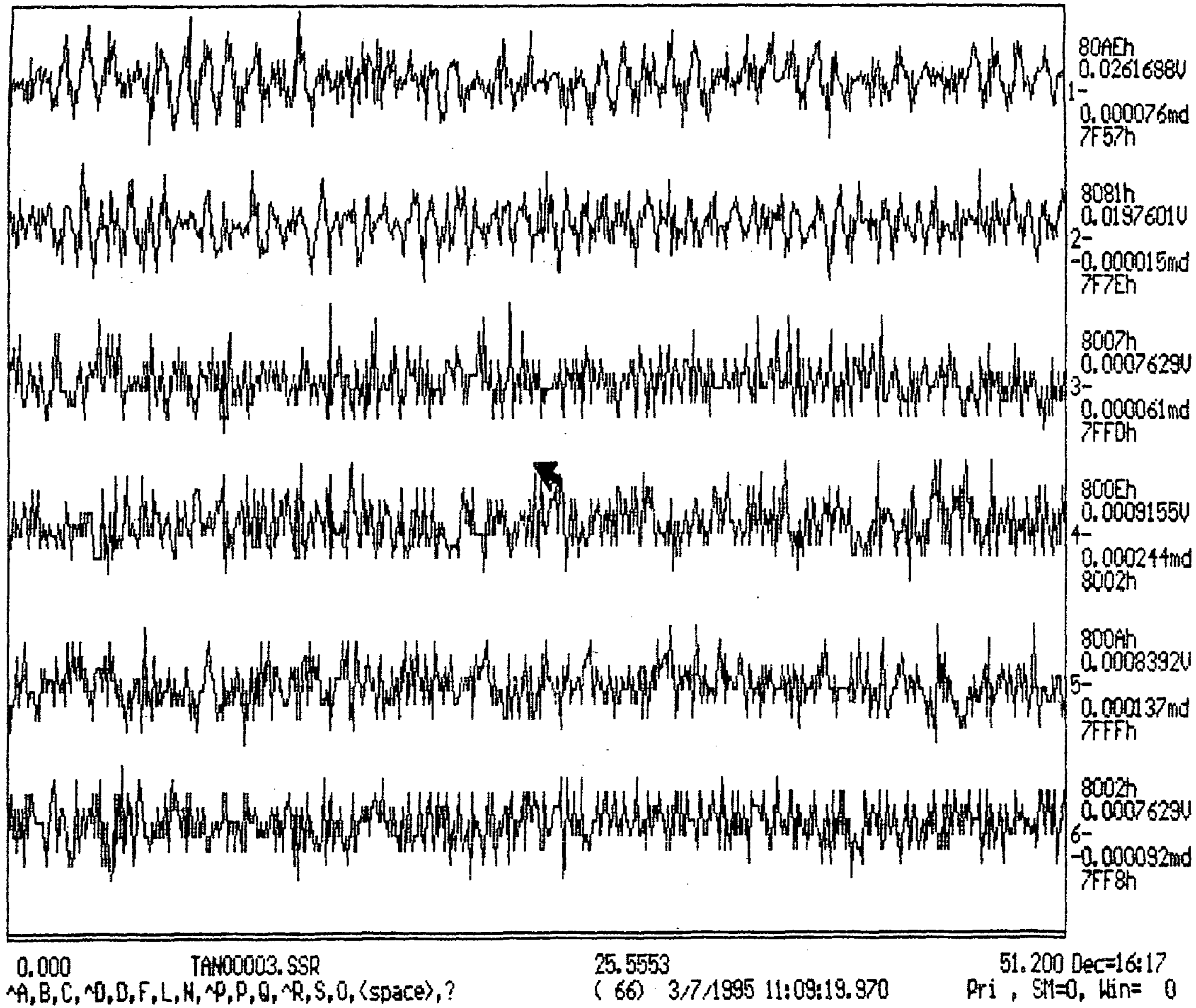


FIG.13 AMBIENT VIBRATION RECORD OF EVENT NO. 3 FOR WATER TOWER NEAR G.P.HOSTEL (THROUGH QL)

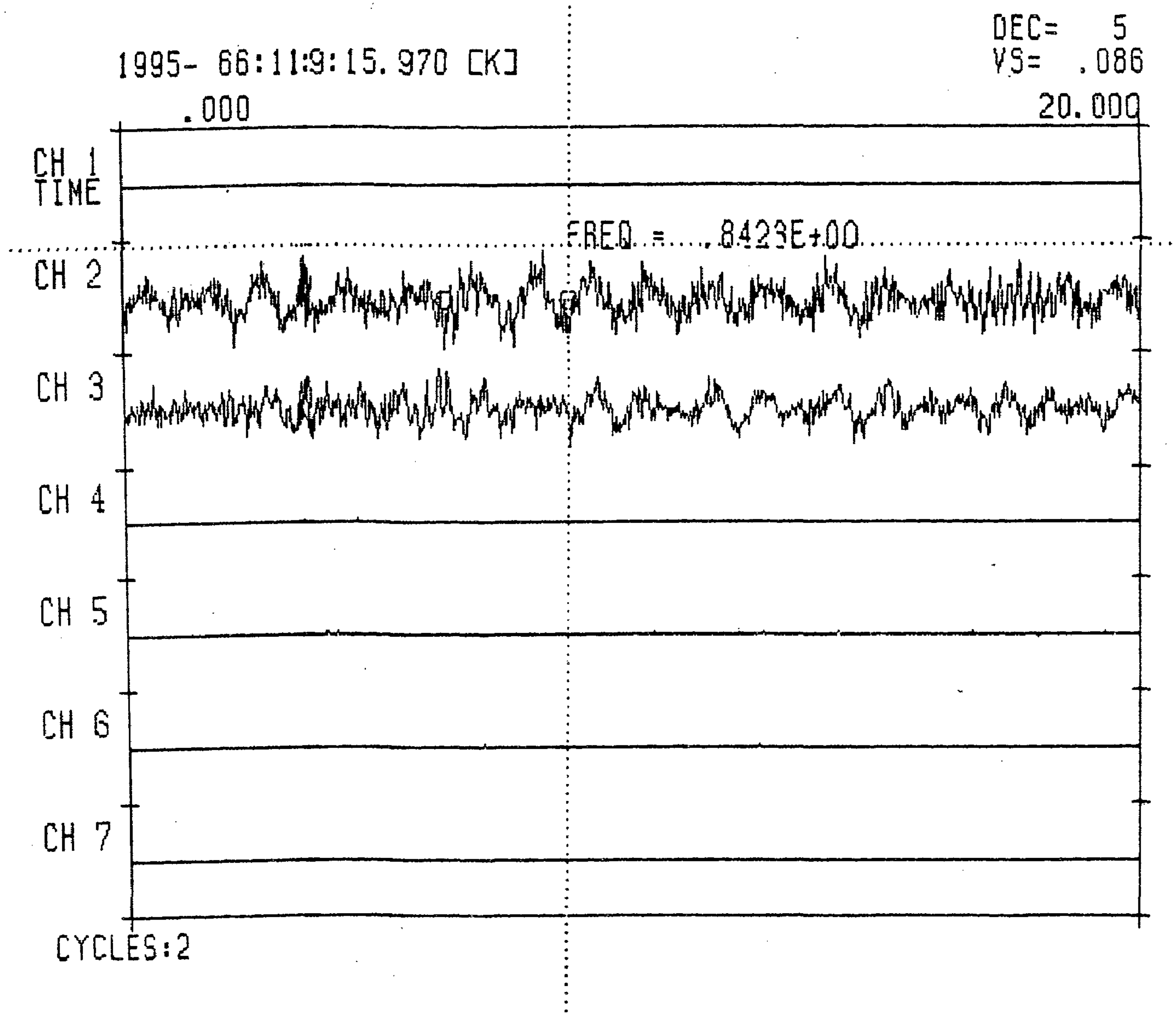


FIG.14 RECORD SHOWING FREQUENCY OF WATER TOWER NEAR G.P.HOSTEL

```
c Ch    x1          y1          x2          y2          cycle     freq        name
   1    .0000E+00   .0000E+00   .0000E+00   .0000E+00     0       .0000E+00  TIME
   2    .6803E+01   .3102E+01   .9177E+01   .3102E+01     2       .8423E+00
   3    .0000E+00   .0000E+00   .0000E+00   .0000E+00     0       .0000E+00
   4    .0000E+00   .0000E+00   .0000E+00   .0000E+00     0       .0000E+00
   5    .0000E+00   .0000E+00   .0000E+00   .0000E+00     0       .0000E+00
   6    .0000E+00   .0000E+00   .0000E+00   .0000E+00     0       .0000E+00
   7    .0000E+00   .0000E+00   .0000E+00   .0000E+00     0       .0000E+00
c
c Damping Markers
c Ch    x1         y1         x2         y2         x3         y3          Damping
   1   .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00
   2   .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00
   3   .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00
   4   .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00
   5   .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00
   6   .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00
   7   .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00  .0000E+00

Total no. of channels =  7
Sample rate      =  200.000
Display width      =   20.000
Display height     =  353.820
Press <return> when ready
```

FIG.15 PLOT PARAMETERS FOR FREQUENCY OF WATER TOWER NEAR G.P.HOSTEL

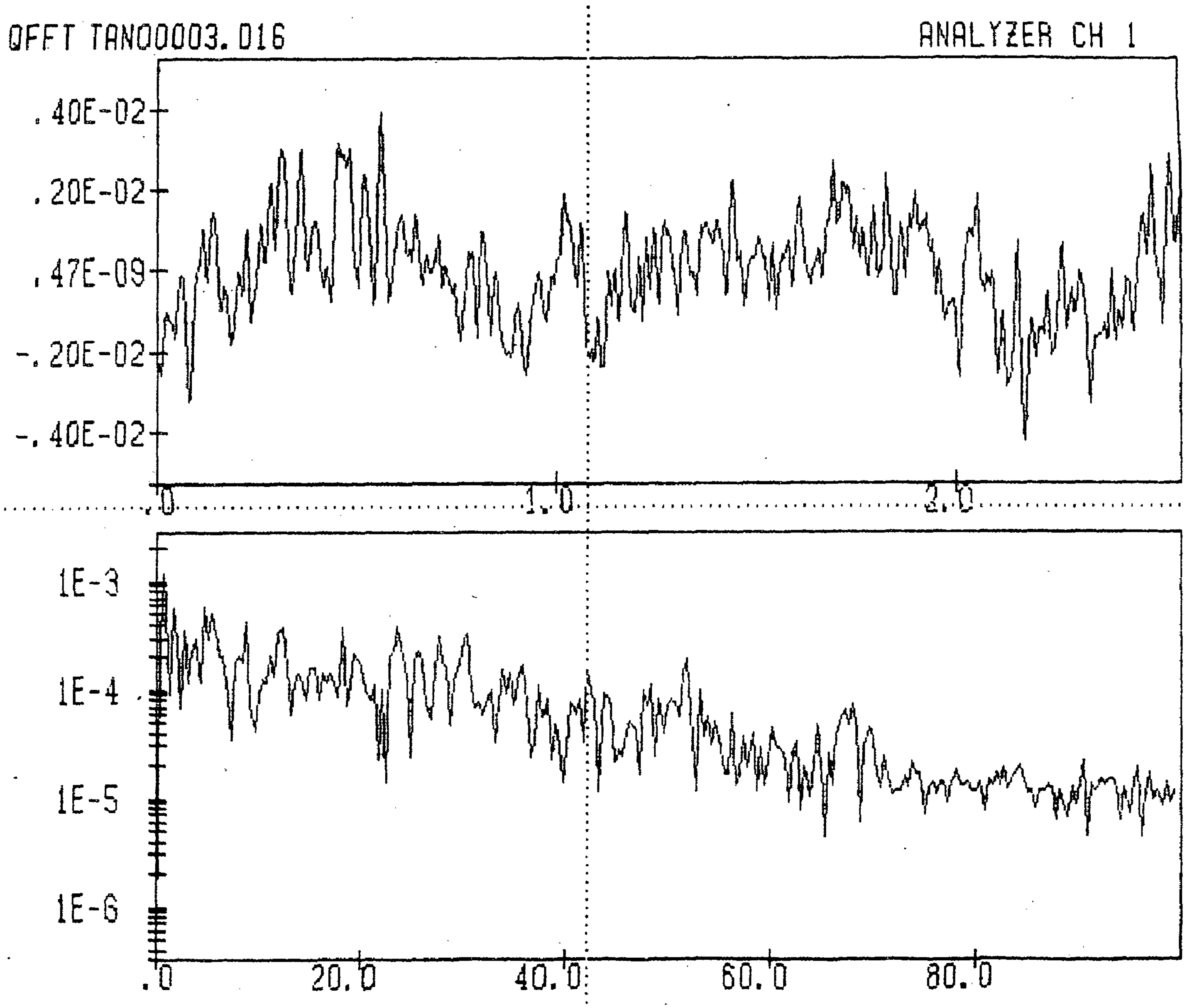


FIG.16 QFFT OF EVENT NO. 3 FOR WATER TOWER NEAR G.P.HOSTEL

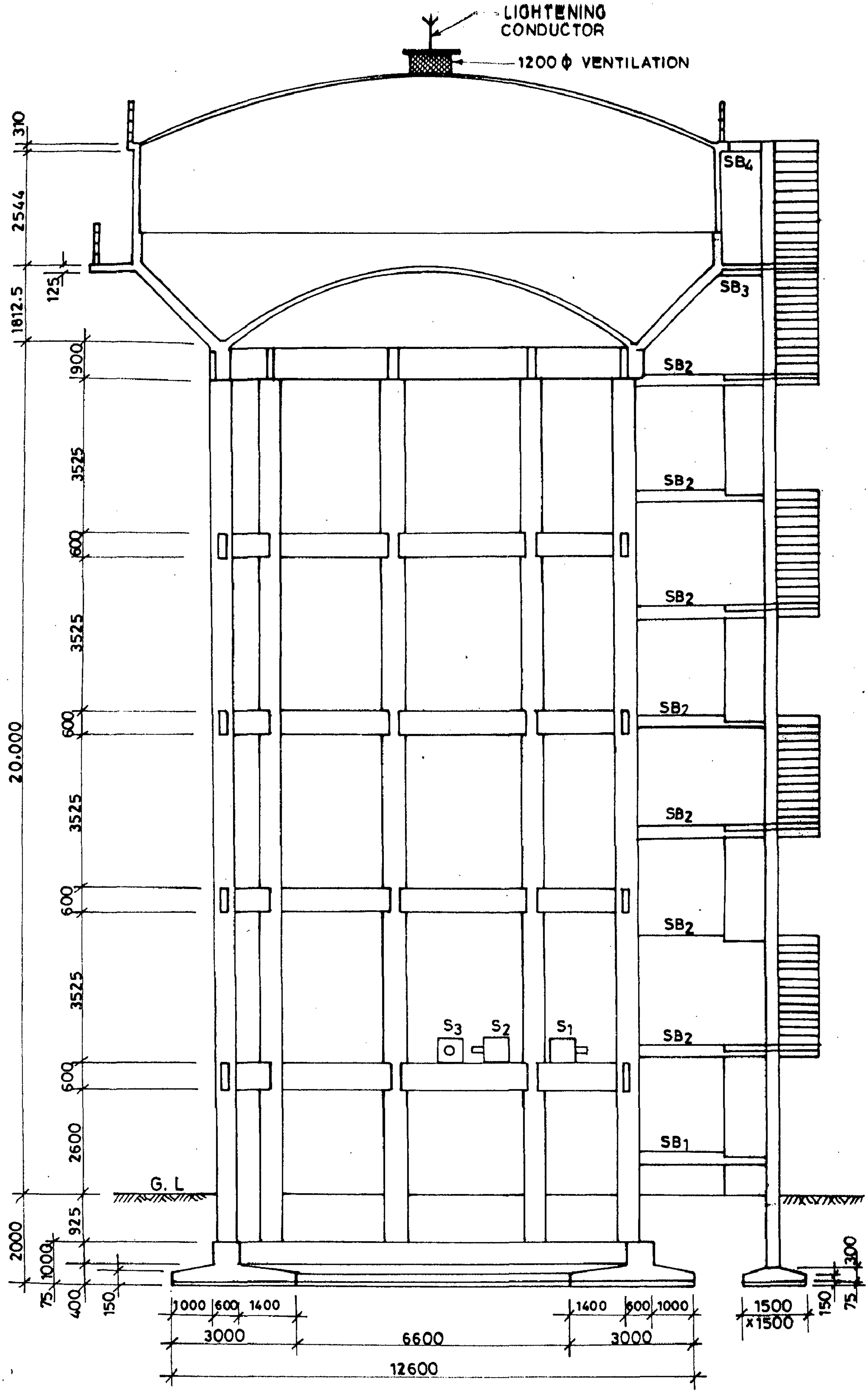


FIG. 17 SECTION OF WATER TOWER NEAR SAROJINI BHAWAN

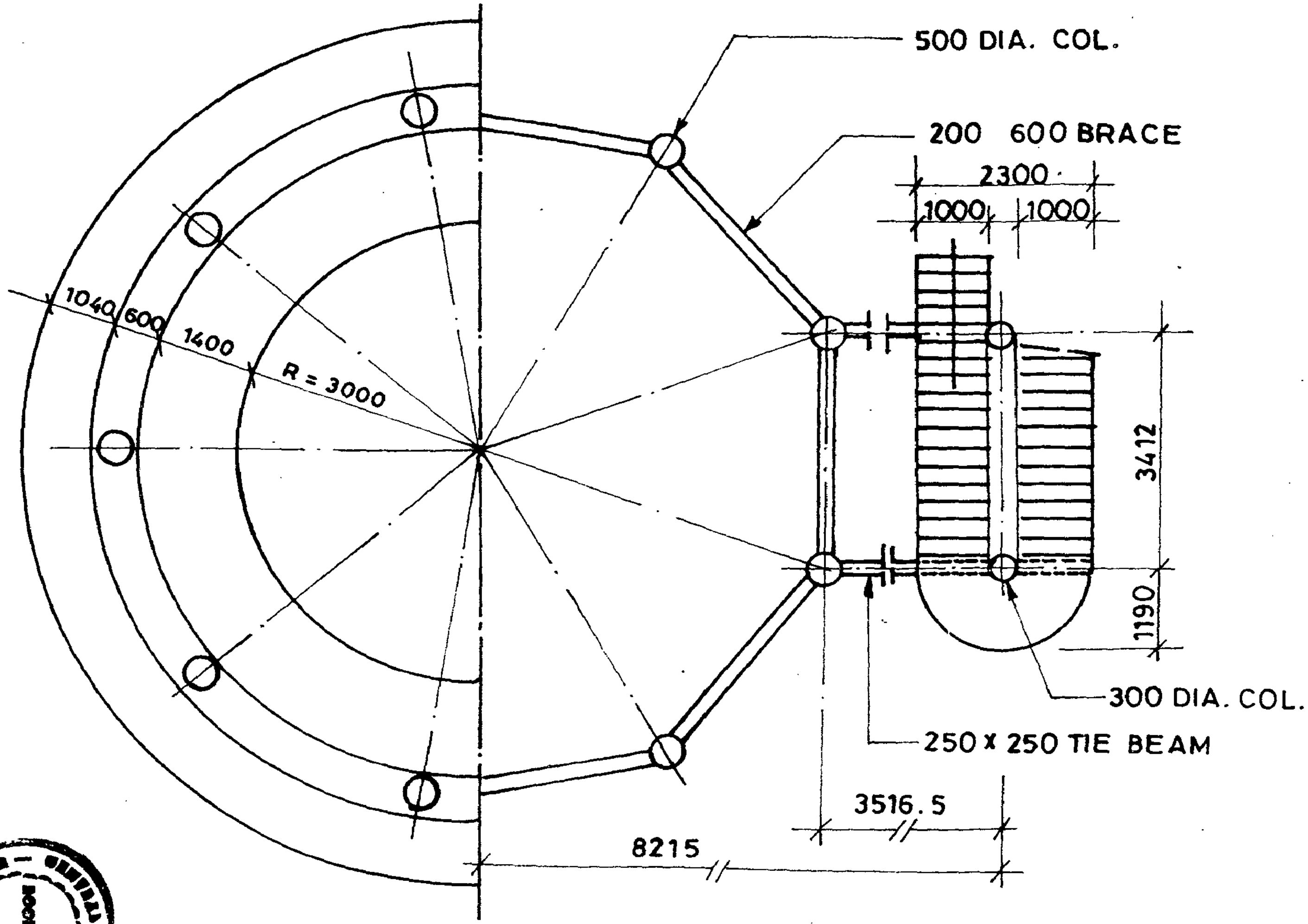


FIG.18 PLAN OF THE WATER TOWER NEAR SAROJINI BHAWAN

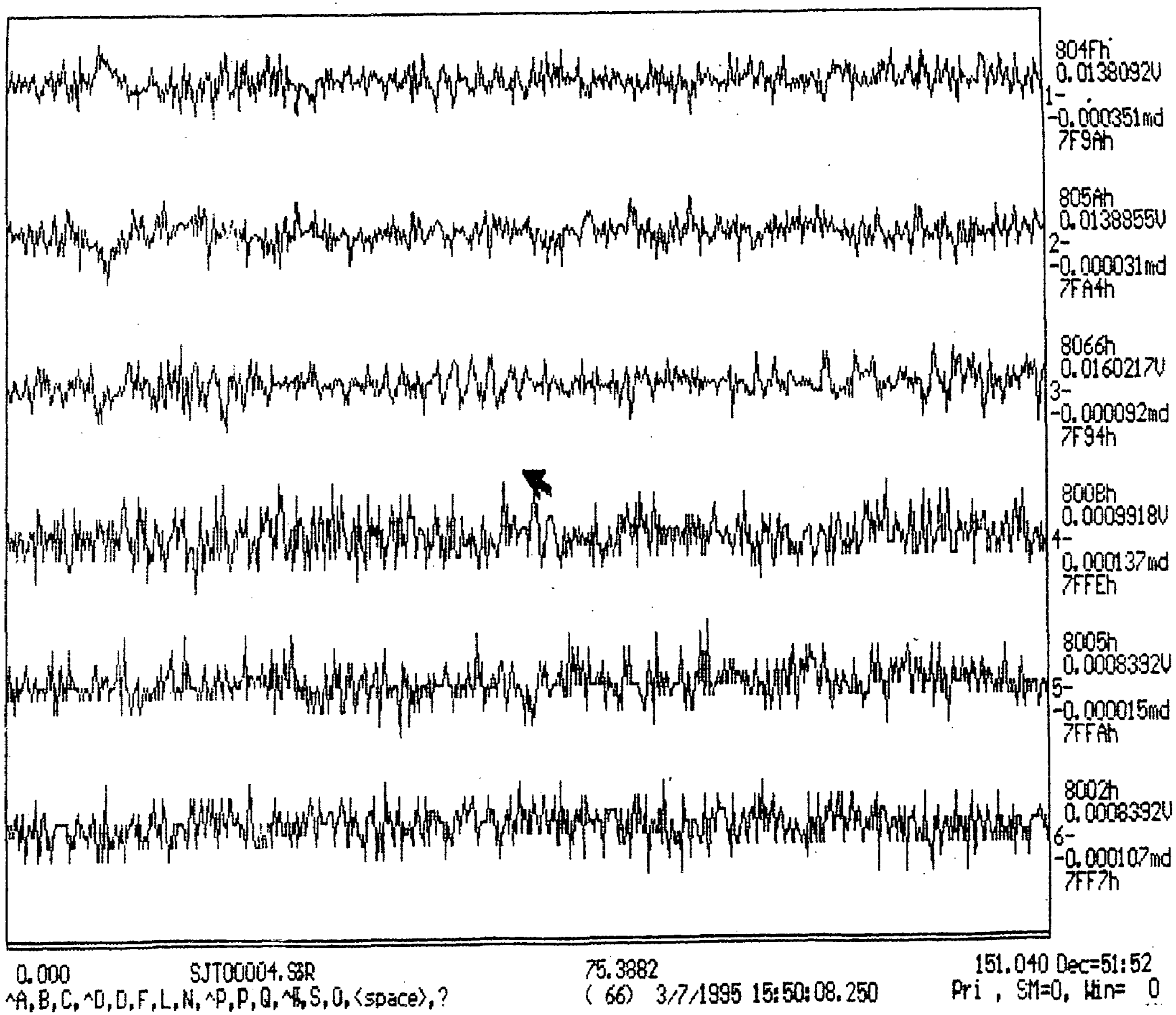


FIG.19 AMBIENT VIBRATION RECORDS OF EVENT NO. 4 FOR WATER TOWER NEAR SAROJINI BHAWAN (THROUGH QL)

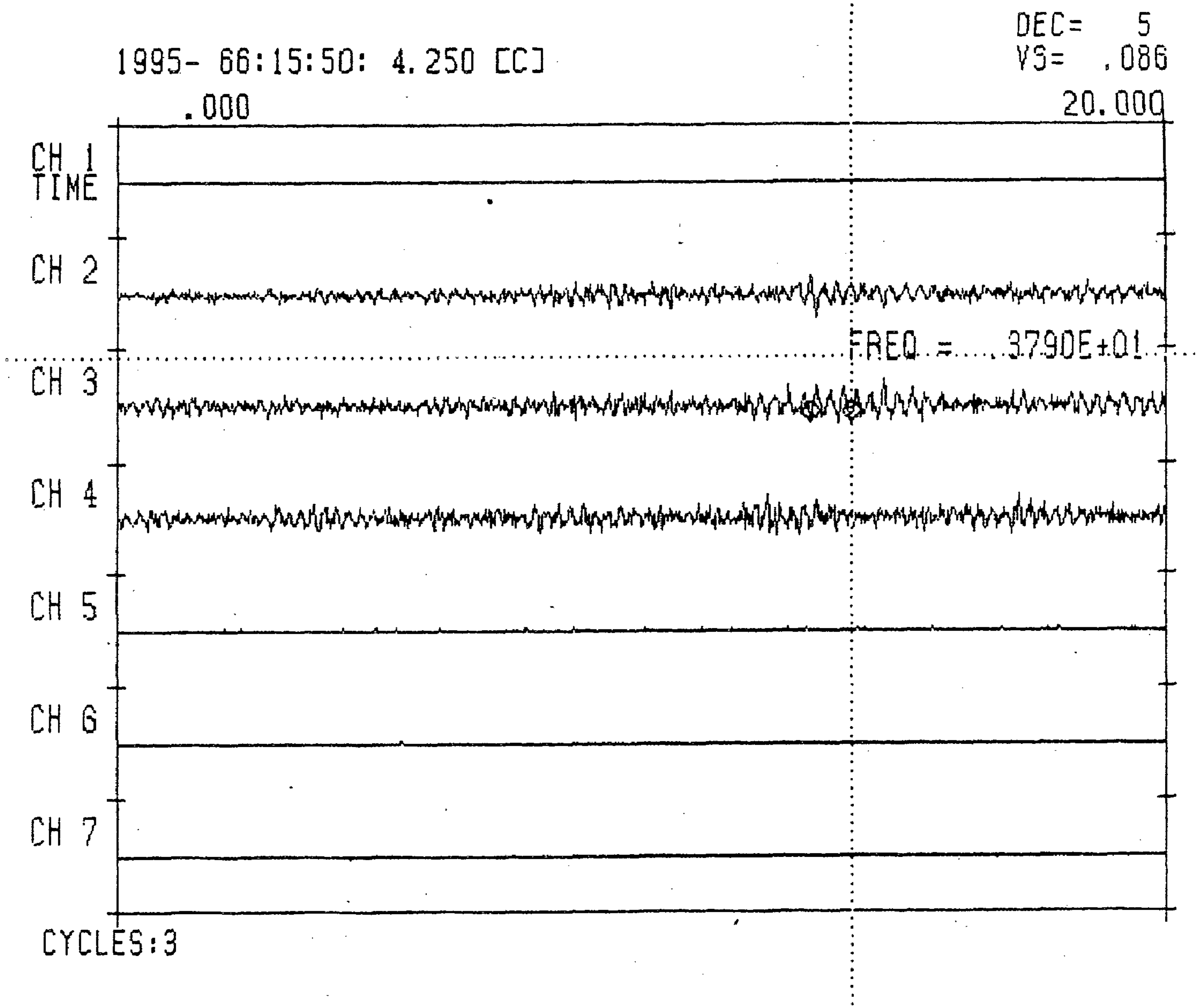


FIG.20 RECORDS SHOWING FREQUENCY OF WATER TOWER NEAR SAROJINI BHAWAN

c Ch	x1	y1	x2	y2	cycle	freq	name
1	.0000E+00	.0000E+00	.0000E+00	.0000E+00	0	.0000E+00	TIME
2	.0000E+00	.0000E+00	.0000E+00	.0000E+00	0	.0000E+00	
3	.1328E+02	-.1718E+02	.1407E+02	-.1718E+02	3	.3790E+01	
4	.0000E+00	.0000E+00	.0000E+00	.0000E+00	0	.0000E+00	
5	.0000E+00	.0000E+00	.0000E+00	.0000E+00	0	.0000E+00	
6	.0000E+00	.0000E+00	.0000E+00	.0000E+00	0	.0000E+00	
7	.0000E+00	.0000E+00	.0000E+00	.0000E+00	0	.0000E+00	

c
c Damping Markers

c Ch	x1	y1	x2	y2	x3	y3	Damping
1	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00
2	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00
3	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00
4	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00
5	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00
6	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00
7	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00

Total no. of channels = 7
Sample rate = 200.000
Display width = 20.000
Display height = 353.820
Press <return> when ready

FIG.21 PLOT PARAMETERS FOR FREQUENCY OF WATER TOWER NEAR SAROJINI BHAWAN

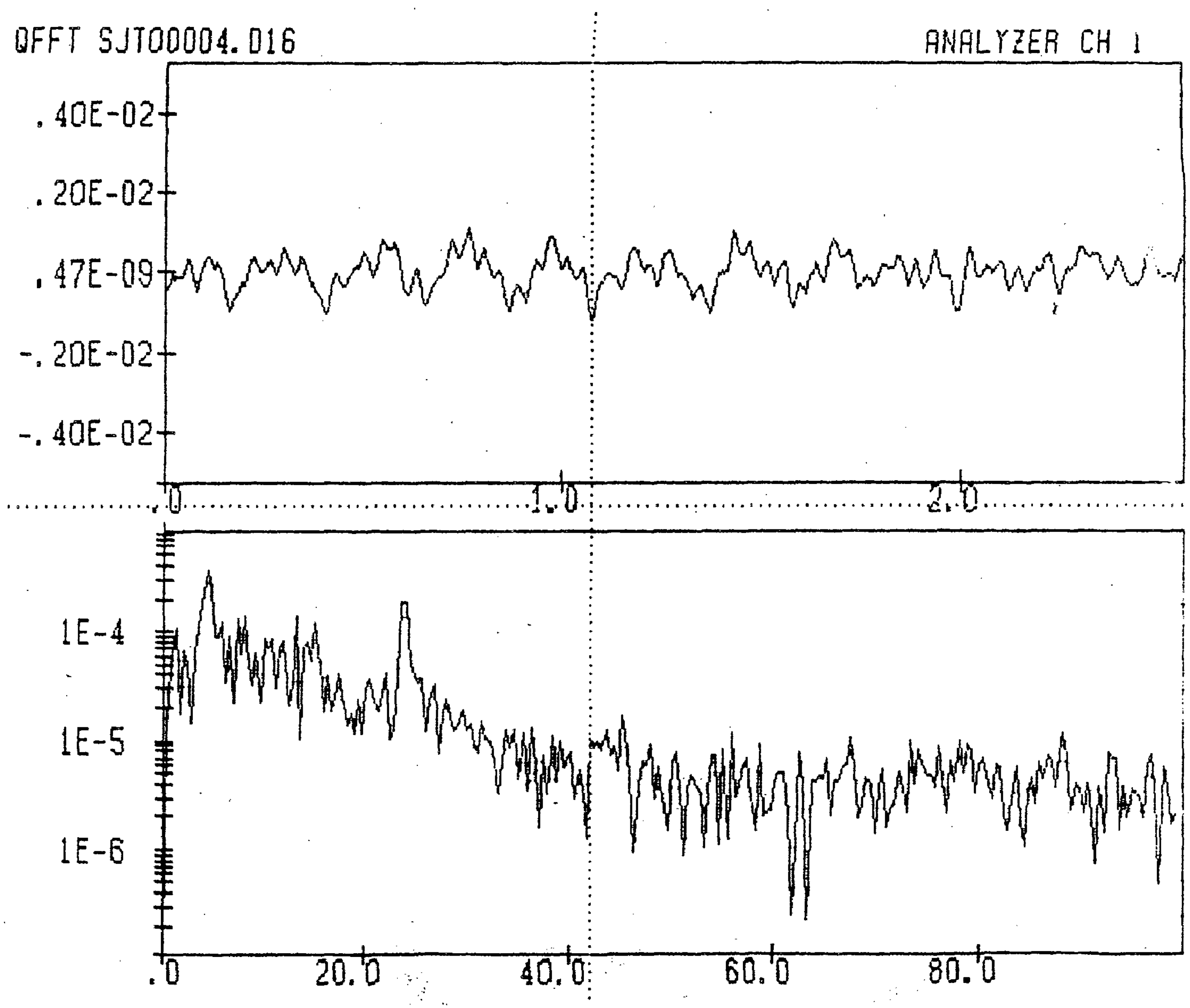


FIG.22 QFFT OF EVENT NO. 4 FOR WATER TOWER NEAR SAROJINI BHAWAN

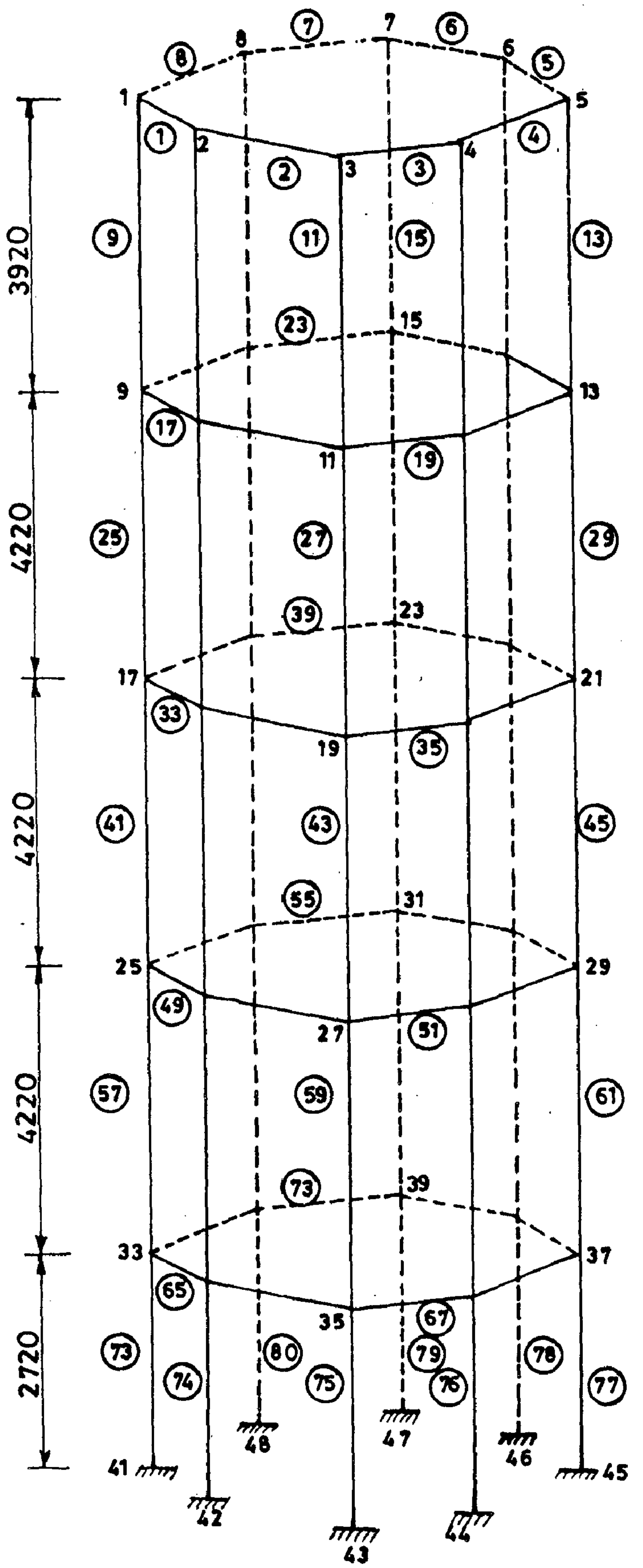


FIG.23 MATHEMATICAL MODEL OF WATER TOWER NEAR G.P.HOSTEL

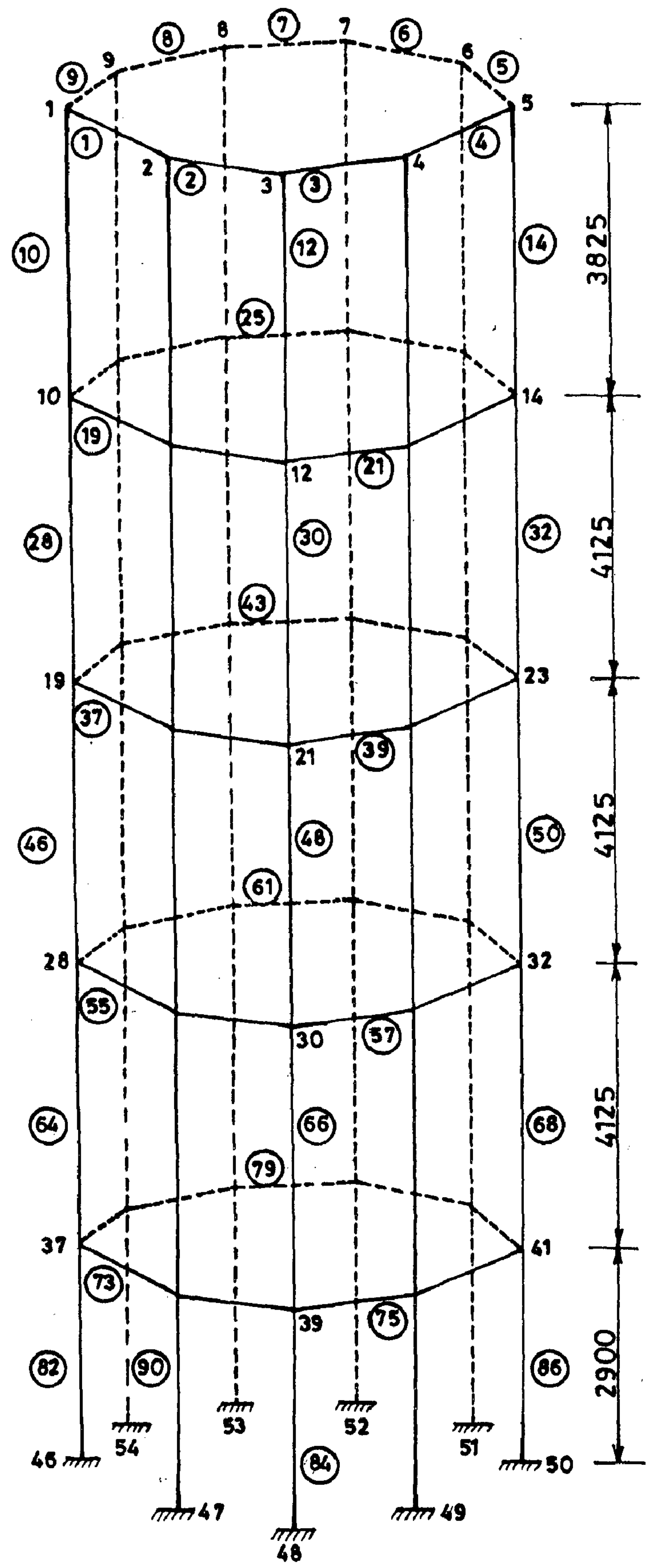


FIG.24 MATHEMATICAL MODEL OF WATER TOWER NEAR SAROJINI BHAWAN

$$
\begin{bmatrix}
\frac{EA}{l} \\
0 & \frac{12EI_z}{l^3(1+\Phi_y)} \\
0 & 0 & \frac{12EI_y}{l^3(1+\Phi_z)} \\
0 & 0 & 0 & \frac{GJ}{l} \\
0 & 0 & \frac{-6EI_y}{l^2(1+\Phi_z)} & 0 & \frac{(4+\Phi_z)EI_y}{l(1+\Phi_z)} \\
0 & \frac{6EI_z}{l^2(1+\Phi_y)} & 0 & 0 & 0 & \frac{(4+\Phi_y)EI_z}{l(1+\Phi_y)} \\
\frac{-EA}{l} & 0 & 0 & 0 & 0 & 0 & \frac{AE}{l} \\
0 & \frac{-12EI_z}{l^3(1+\Phi_y)} & 0 & 0 & 0 & \frac{-6EI_z}{l^2(1+\Phi_y)} & 0 & \frac{12EI_z}{l^3(1+\Phi_y)} \\
0 & 0 & \frac{-12EI_y}{l^3(1+\Phi_z)} & 0 & \frac{6EI_y}{l^2(1+\Phi_z)} & 0 & 0 & 0 & \frac{12EI_y}{l^3(1+\Phi_z)} \\
0 & 0 & 0 & \frac{-GJ}{l} & 0 & 0 & 0 & 0 & 0 & \frac{GJ}{l} \\
0 & 0 & \frac{-6EI_y}{l^2(1+\Phi_z)} & 0 & \frac{(2-\Phi_z)EI_y}{l(1+\Phi_z)} & 0 & 0 & 0 & \frac{6EI_y}{l^2(1+\Phi_z)} & 0 & \frac{(4+\Phi_z)EI_y}{l(1+\Phi_z)} \\
0 & \frac{6EI_z}{l^2(1+\Phi_y)} & 0 & 0 & 0 & \frac{(2-\Phi_y)EI_z}{l(1+\Phi_y)} & 0 & \frac{-6EI_z}{l^2(1+\Phi_y)} & 0 & 0 & 0 & \frac{(4+\Phi_y)EI_z}{l(1+\Phi_y)}
\end{bmatrix}
$$

where, $\phi_Y = \frac{12EI_Z}{GA_Y l^2}$

$\phi_Z = \frac{12EI_Y}{GA_Z l^2}$

Symmetric

APPENDIX A-SPACE FRAME MEMBER STIFFNESS MATRIX

APPENDIX – B

The rotation transformation matrix R_T for a space frame member can be shown to take the following form:

$$R_T = \begin{bmatrix} R & 0 & 0 & 0 \\ 0 & R & 0 & 0 \\ 0 & 0 & R & 0 \\ 0 & 0 & 0 & R \end{bmatrix}$$

Where R =

$$\begin{bmatrix} C_x & C_y & C_z \\ \dfrac{-C_xC_y \cos\alpha - C_z \sin\alpha}{\sqrt{C_x^2 + C_z^2}} & \sqrt{C_x^2 + C_z^2}\,\cos\alpha & \dfrac{-C_yC_z \cos\alpha + C_x \sin\alpha}{\sqrt{C_x^2 + C_z^2}} \\ \dfrac{C_xC_y \sin\alpha - C_z \cos\alpha}{\sqrt{C_x^2 + C_z^2}} & \sqrt{C_x^2 + C_z^2}\,\sin\alpha & \dfrac{C_yC_z \sin\alpha + C_x \cos\alpha}{\sqrt{C_x^2 + C_z^2}} \end{bmatrix}$$

This rotation matrix is expressed in terms of the direction cosines of the member (which are readily computed from the coordinates of the joints) and the angle α, which must be given as part of the description of the structure itself.