

# ANALYSIS OF MULTISTOREY FRAMED BUILDINGS FOR THE LATERAL LOADS

**A DISSERTATION**

*submitted in partial fulfilment of the  
requirements for the award of the degree*

*of*

**MASTER OF ENGINEERING**

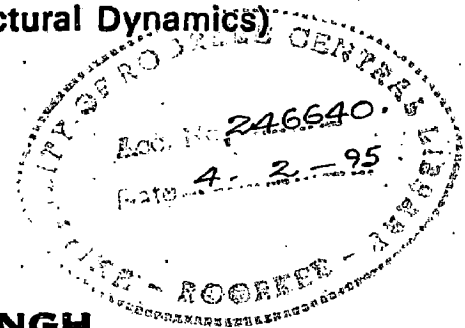
*in*

**EARTHQUAKE ENGINEERING**

**(With Specialization in Structural Dynamics)**

By

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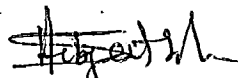
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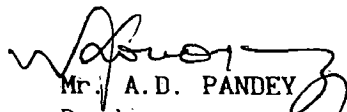
I hereby certify that the work which is being presented in this dissertation entitled "ANALYSIS OF MULTISTOREY FRAMED BUILDINGS FOR THE LATERAL LOADS" in partial fulfillment of the requirements for the award of degree of Master of Engineering with specialization in STRUCTURAL DYNAMICS submitted in the Department of Earthquake Engineering, University of Roorkee, Roorkee, India, is an authentic record of my own work carried out for a period of from October 1993 to March 1994 under the supervision of Mr. A. D. Pandey, Reader and Dr. (Mrs.) P. R. Bose, Reader, Department of Earthquake Engineering, University of Roorkee, Roorkee, India.

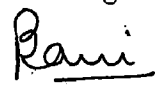
The matter embodied in this thesis has not been submitted by me for the award of any other degree or diploma.

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This is to certify that the above statement made by the candidate is correct to the best of our knowledge.

  
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## ACKNOWLEDGEMENT

I express my deep gratitude and sincere thanks to my guides Mr. A.D. PANDEY, Reader and Dr. (Mrs.) P. R. Bose, Reader, Department of Earthquake Engineering University of Roorkee, Roorkee, for their expert guidance and valuable suggestions. I do not have words to express my obligation for their benevolent gesture of support and motivation provided throughout the course of this work.

I am also thankful to Dr. D. K. Paul, Professor, Department of Earthquake Engineering for providing the freely extending computer facilities till the end of this work.

Appreciation and special thanks are also extended specially to my senior Mr. M. Laghate, Mr. Khalid and my colleagues, Mr. Ajai Varshney, Mr. Akash Gupta, Mr. J.S. Rajpurohit and Mr. R.K. Goel who helped me in various forms at every pace of this thesis work.

  
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## ABSTRACT

A multistoreyed building frame consists of a rectangular gridwork of columns and beams in space. The lateral loads on the structure are produced by wind action and earthquake. Under the action of wind forces and earthquake forces acting in the transverse and longitudinal direction of the building, the whole building frame acts as one integral unit. When the conditions of the frame are fairly uniform from one plane frame idealised in the longitudinal direction to the other i.e. idealised in the transverse direction, it is easy and reasonable to analyse the forces and moments in the plane frames taken separately in the directions of the building rather than in the form of an integral space frame.

Though the phenomena of wind and earthquakes are dynamic in nature, the force produced by these actions can be reduced to static loads in an approximate analysis. Exact methods which may be employed by using vast number of unknowns are in most cases considered as time consuming and impractical for preliminary dimensioning in design offices. A number of approximate methods have been developed for the analysis of multistorey framed structures. These methods take into account various characteristics of the structures and yield approximate but reliable results for most of the cases met in practice.

The present work addresses the analysis of idealised plane frames which have been analysed by some of the approximate methods as well as by the stiffness matrix method. All the dimensions and the lateral loadings used in different methods are same. The results obtained by the various approximate methods have been compared with the results of the stiffness matrix method.

The design forces worked out for a symmetric frame by approximate methods are in error in general. The maximum error is about thirty percent except at few sections. For the unsymmetric frames, the errors in the worked out designed forces are not found in a symmetric pattern. Kani's method and Kloucek's method gives the acceptable values but they are found to be time consuming for manual implementation. Modified approximate method such as Factor method, Bowman's method, Blume et al's method yields result nominally superior.

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## INTRODUCTION

Multistorey building frames represent one of the most complex types of structural systems encountered in ordinary civil engineering practice. The ever growing metropolitan cities all over the world have been experiencing shortage of space for further expansion. Further, the necessity for locating their important offices and trade centers in the down-town area led them to raise the building or go up towards the sky. Thus sky scrapers or high rise towers are found in almost all big metropolitan cities. Such tall structures are subjected to two dominant horizontal forces: (1) Wind forces and (2) Earthquake forces, in addition to other loads.

### WIND FORCES

Wind is air in motion relative to the surface of the earth. The liability of a building to high wind pressure depends not only upon the geographical location and proximity of other obstruction to air flow, but also upon the characteristics of the structure itself. The effect of wind on the structure as a whole is determined by the combined action of external and internal pressure acting on it. The effect of wind is calculated on the basis of basic pressures on the entire height of the building, with due regard to mean retarding surface and variation in wind pressure with height.

### EARTHQUAKE FORCES

Earthquake causes random motion of ground which can be resolved in three mutually perpendicular directions. This motion causes the structure to vibrate. The predominant direction of vibration is horizontal.

The vibration intensity of ground expected at any location depends upon the magnitude of earthquake, the depth of focus, distance from epicenter and strata on which structure stands. In the case of a structure designed for horizontal seismic force only, the forces are considered to act in any one direction at a time and in case where both horizontal and vertical forces are taken into account, horizontal forces in any one direction at a time are considered along with vertical forces.

The analysis of indeterminate structures depends on a knowledge of member proportions which are unknown at the time the design analysis cycle is begun. Therefore it becomes necessary to perform some approximate analysis to arrive at an estimate of member sizes. With appropriate assumptions, statically indeterminate structures can be reduced to statically determinate systems, which can then be analysed for approximate member forces and reactions using statics.

In analysing the building frame for the lateral loads, the loads due to wind and earthquake are approximated as static loads. In analysing a statically indeterminate structure by approximate procedures, one assumption is made for each degree of indeterminacy. These assumptions are based on logical interpretations of the structural response to the given loading. The exact analysis of the plane frame can be done by stiffness or flexibility methods using matrices.

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### APPROXIMATION OF LATERAL LOADS

#### 2.1 Wind loads:

Wind is essentially a random phenomenon. In the past, the wind pressure, corresponding to the highest wind speed that had been recorded at the meteorological stations nearest to the place concerned, was applied statically. This was an erroneous practice since wind loading varies with time. Moreover, the wind speed depends on several factors such as density of obstructions in the terrain, size of gust, return period, and probable life of structure etc. thus no deterministic method can do justice with the wind loading.

The recent revisions in IS: 875-1987 (Part III) are based on two considerations:-

1. The statistical and probabilistic approach to the evaluation of wind loads and
2. Due recognition to the dynamic component of wind loading and its interaction with the dynamic characteristic of the structure.

The design wind speed  $V_d$  at a given site is expressed as a product of five parameters.

$$V_d = V_b \cdot k_1 \cdot k_2 \cdot k_3 \cdot k_4$$

where,

$V_b$  = basic wind speed in (m/sec) given in map shown in IS: 875-1987 Part III.

$k_1$  = Probability or risk factor

$k_2$  = terrain and height factor

$k_3$  = local topography factor

$k_4$  = gust factor

$V_b$ , the basic wind speed represents the extreme value which is likely to be exceeded on an average only once during a specified return period. It is arrived at by statistical analysis. The wind speeds based on peak gust velocity averaged over 3 seconds have been worked out for a 50 year return period.

The probability or risk factor ' $k_1$ ', is based on statistical concepts which takes account of the degree of security required and probability of exceedence of basic wind speed. It increases with the increase in mean probable life of a structure. The factor ' $k_1$ ' for different class of structure for the purpose of design in given in table (2.1.1).

TABLE 2.1.1

RISK COEFFICIENTS FOR DIFFERENT CLASSES OF STRUCTURES IN DIFFERENT WIND SPEED ZONES  
( Clause 5.3.1 )

CLASS OF STRUCTURE	MEAN PROBABLE DESIGN LIFE OF STRUCTURE IN YEARS	$k_1$ FACTOR FOR BASIC WIND SPEED (m/s) OF					
		33	39	44	47	50	55
All general buildings and structures	50	1.0	1.0	1.0	1.0	1.0	1.0
Temporary sheds, structures such as those used during construction operations ( for example, formwork and falsework ), structures during construction stages and boundary walls	5	0.82	0.76	0.73	0.71	0.70	0.67
Buildings and structures presenting a low degree of hazard to life and property in the event of failure, such as isolated towers in wooded areas, farm buildings other than residential buildings	25	0.94	0.92	0.91	0.90	0.90	0.89
Important buildings and structures such as hospitals communication buildings / towers, power plant structures	100	1.05	1.06	1.07	1.07	1.08	1.08

The terrain factor ' $k_2$ ' takes due regard to the effect of roughness, or height and density of structures on the basic wind speed. Terrains are classified in four categories.

Categories

Terrain type

1. Exposed open terrain with few or no obstruction in which the average height of any object surrounding the structure is less than 1.5 m.
2. Open terrain with well scattered obstruction having heights generally between 1.5 to 10.0m.
3. Terrain with numerous closely spaced obstruction having the size of building structures upto 10m in heights with or without a few isolated tall structure.
4. Terrain with numerous large, high, closely spaced obstructions.

The 'k<sub>2</sub>' factor for different categories of terrain is given in table (2.1.2).

TABLE 2.1.2

TABLE 2.1.2, FACTORS TO OBTAIN DESIGN WIND SPEED VARIATION WITH HEIGHT IN DIFFERENT TERRAINS FOR DIFFERENT CLASSES OF BUILDINGS/STRUCTURES (Clause 5.3.2.2)

HEIGHT m	TERRAIN CATEGORY 1 CLASS			TERRAIN CATEGORY 2 CLASS			TERRAIN CATEGORY 3 CLASS			TERRAIN CATEGORY 4 CLASS		
	A (2)	B (3)	C (4)	A (5)	B (6)	C (7)	A (8)	B (9)	C (10)	A (11)	B (12)	C (13)
10	1.05	1.03	0.99	1.00	0.98	0.93	0.91	0.88	0.82	0.80	0.76	0.67
15	1.09	1.07	1.03	1.05	1.02	0.97	0.97	0.94	0.87	0.80	0.76	0.67
20	1.12	1.10	1.06	1.07	1.05	1.00	1.01	0.98	0.91	0.80	0.76	0.67
30	1.15	1.13	1.09	1.12	1.10	1.04	1.06	1.03	0.96	0.97	0.93	0.83
50	1.20	1.18	1.14	1.17	1.15	1.10	1.12	1.09	1.02	1.10	1.05	0.95
100	1.26	1.24	1.20	1.24	1.22	1.17	1.20	1.17	1.10	1.20	1.15	1.05
150	1.30	1.28	1.24	1.28	1.25	1.21	1.24	1.21	1.15	1.24	1.20	1.10
200	1.32	1.30	1.26	1.30	1.28	1.24	1.27	1.24	1.18	1.27	1.22	1.13
250	1.34	1.32	1.28	1.32	1.31	1.26	1.29	1.26	1.20	1.28	1.24	1.16
300	1.35	1.34	1.30	1.34	1.32	1.28	1.31	1.28	1.22	1.30	1.26	1.17
350	1.37	1.35	1.31	1.36	1.34	1.29	1.32	1.30	1.24	1.31	1.27	1.19
400	1.38	1.36	1.32	1.37	1.35	1.30	1.34	1.31	1.25	1.32	1.28	1.20
450	1.39	1.37	1.33	1.38	1.36	1.31	1.35	1.32	1.26	1.33	1.29	1.21
500	1.40	1.38	1.34	1.39	1.37	1.32	1.36	1.33	1.28	1.34	1.30	1.22

The effect of topography is to accelerate wind near the summits of hills or crests of cliffs, escarpments or ridge and decelerate the wind in valleys or near the foot of cliffs, steep escarpments or ridges

The topography factor  $k_3$  is given by

$$k_3 = 1 + C S$$

where C has the following value

Slope	C
$3^\circ < \theta \leq 17^\circ$	$1.2(Z/L)$
$> 17^\circ$	0.36

where,

Z → effective height of the feature

$\theta$  → upwind slope in wind direction

L → actual length of the upwind slope in the wind direction

S → is a factor determined from fig. (1) and fig. (2) appropriate to the height, H above mean ground level and the distance, X from the summit or crest relative to the effective length,  $L_e$ . The effective length  $L_e$  depends on the slope as follows

Slope	$L_e$
$3^\circ < \theta \leq 17^\circ$	L
$> 17^\circ$	$Z/0.3$



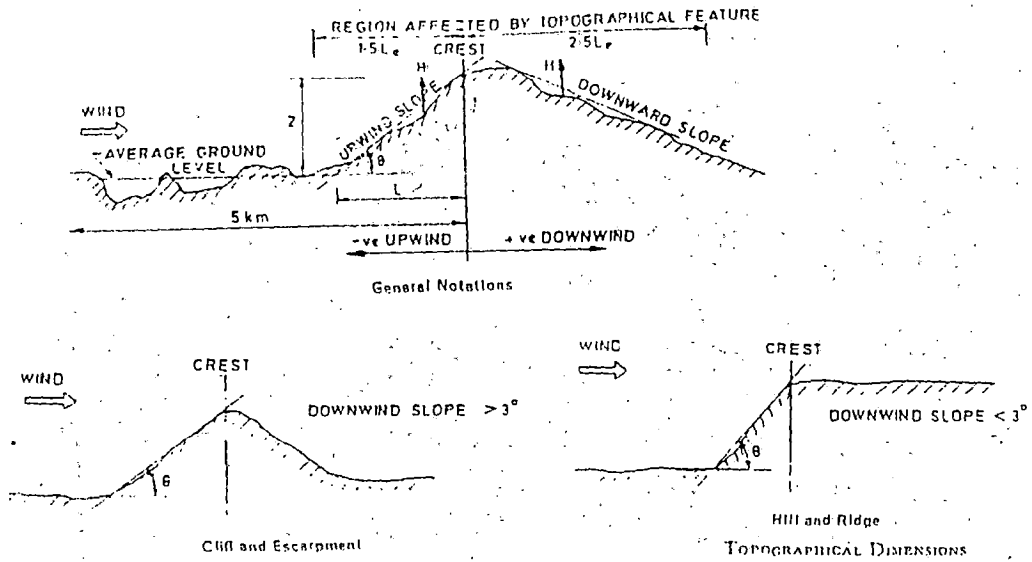


Fig. 1

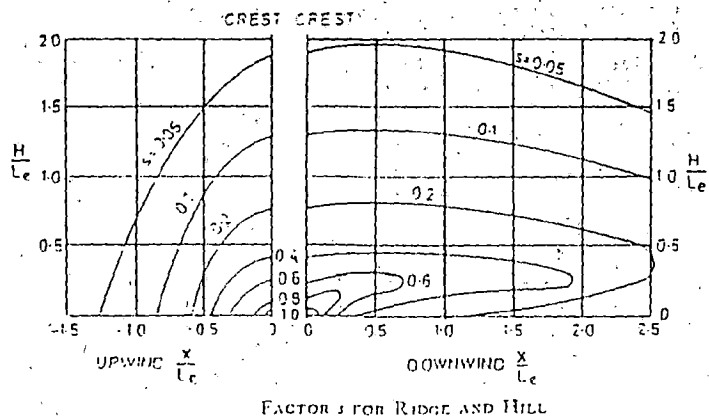
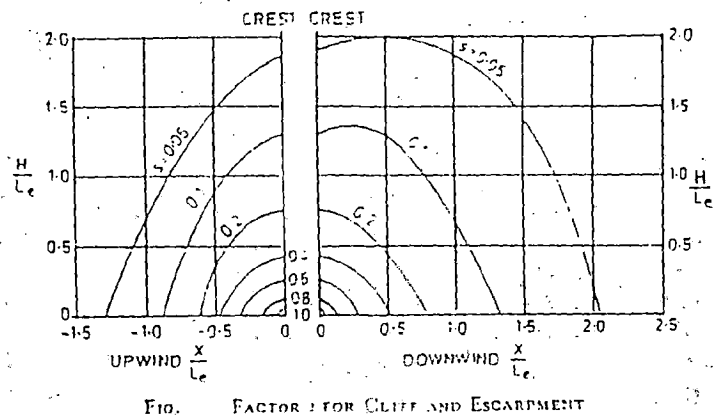


Fig. 2

Gusts of small duration can produce large wind loads if they could develop simultaneously over the whole structure. The Gust factor  $k_4$  is given by ( $k_4 = \text{peak load} / \text{mean load}$ ) and can be calculated from the specification given in IS: 875-1987 part III

A reduction in wind forces is permitted if a building is large as compared to a gust of 3 seconds duration which cannot completely envelope it. The factor  $k_4$  takes care of size of building.

The design wind pressure  $p_d$  in  $N/m^2$  is given by relation

$$p_d = 0.6 V_d^2$$

The coefficient "0.6" depends upon mass density of air

$V_d$  = design wind velocity (m/sec).

The wind force  $F$  on a complete building can be calculated as follows

$$F = C_f A_e P_d$$

$C_f$  = force coeff. for the building shown in fig (3).

$A_e$  = effective exposed area.

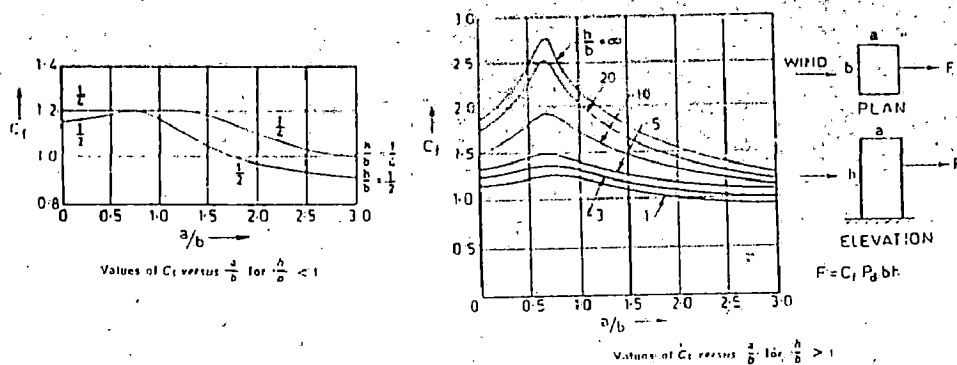


Fig. 3 Force coefficient for rectangular Clad Buildings

## 2.2 Earthquake loads

Earthquake or seismic load on a building depends upon its geographical location, lateral stiffness and mass, and is reversible. Its effect should also be considered along both axes of a building taken one at a time. However, wind loads and earthquake loads are assumed not to act simultaneously. Therefore, a building should be designed for only one of the two loads, whichever is critical. The analysis of a building for earthquake loads is done in accordance with IS: 1893-1984. A building may be analyzed either by seismic coefficient method or by response spectrum method. The first step in the seismic analysis is to determine the horizontal shear at the base of a frame which is also known as base shear. It is given by:

$$V_B = K.C. \alpha_h . W$$

where,  $\alpha_h$  = design seismic coefficient

K = Performance factor depending on the structural framing system and brittleness or ductility of construction given in table (2.2.1)

C = Coefficient defining the flexibility of structure with the increase in number of storeys depending upon fundamental time period (T). Fig. (4)

W = total dead load plus appropriate reduced live load on the whole building frame as given in Table (2.2.2).

TABLE 2.2.1

VALUES OF PERFORMANCE FACTOR, K

(Clause 4.2.1.1)

Sl. No.	STRUCTURAL FRAMING SYSTEM	VALUES OF PERFORMANCE FACTOR, K	REMARKS
(1)	(2)	(3)	(4)
i)	a) Moment resistant frame with appropriate ductility details as given in IS: 4326-1976* in reinforced concrete or steel	1.0	These factors will apply only if the steel bracing members and the infill panels are taken into consideration in stiffness as well lateral strength calculations provided that the frame acting alone will be able to resist at least 25 percent of the design seismic forces
	b) Frame as above with R. C. shear walls or steel bracing members designed for ductility	1.0	
ii)	a) Frame as in (i) (2) with either steel bracing members or plain or nominally reinforced concrete infill panels	1.3	
	b) Frame as in (i) (2) in combination with masonry infills	1.6	
iii)	Reinforced concrete framed buildings [ Not covered by (i) or (ii) above ]	1.6	

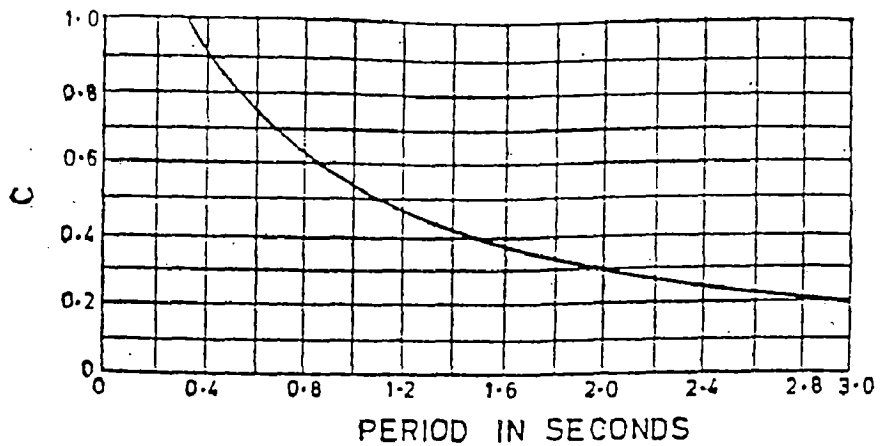


FIG. 4. *C Versus PERIOD*

Table (2.2.2) Reduced live load during earthquakes

Load class	Percentage of design live load
200, 250, 300	25
400, 500, 750, 1000	50

The design seismic coefficient  $\alpha_h$  is calculated as follows:

(a) Seismic coefficient approach

$$\alpha_h = \beta \cdot I \cdot \alpha_0$$

(b) Spectral acceleration approach

$$\alpha_h = \beta \cdot I \cdot F \cdot S_a / g$$

where;

$\alpha_0$  = basic horizontal seismic coefficient specified for various seismic zones as given in Tables (2.2.3)

F = seismic zone factor specified for various zones as given in Table (2.2.3)

Table 2.2.3 Seismic zones, coefficients and factors

Zone No.	Basic seismic coefficient $\alpha_0$	Zone factor F
V	0.08	0.40
IV	0.05	0.25
III	0.04	0.20
II	0.02	0.10
I	0.01	0.05

$\beta$  = a factor depending on the soil foundation systems as given in Table (2.2.4)

I = importance factor depending on the life and function of the structure. Given in table (2.2.5)

$S_{a/g}$  = can be calculated from acceleration spectra from fig. (5)

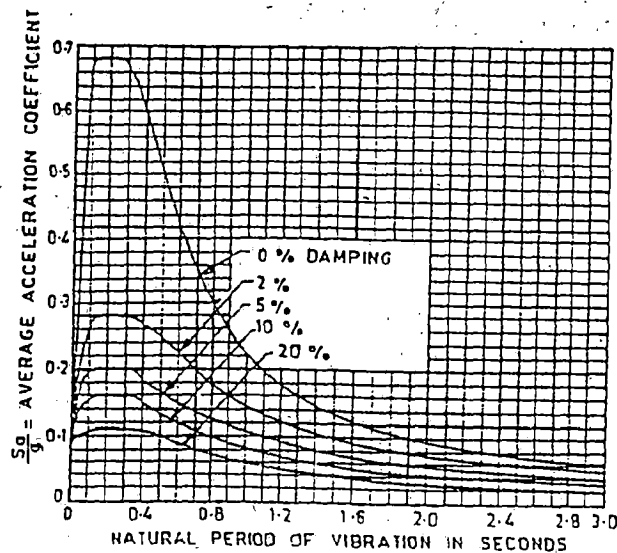


FIG. 5 AVERAGE ACCELERATION SPECTRA

TABLE 2-2-4

TABLE VALUES OF  $\beta$  FOR DIFFERENT SOIL-FOUNDATION SYSTEMS  
( Clause 3.4.3 )

SL No.	TYPE OF SOIL MAINLY CONSTITUTING THE FOUNDATION	VALUES OF $\beta$ FOR					
		Piles Passing Through Any Soil, but Resting on Soil Type I	Piles Not Covered Under Col 3	Raft Foundations	Combined or Isolated RCC Footings with Tie Beams	Isolated RCC Footings Without Tie Beams or Unreinforced Strip Foundations	Well Foundations
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
i)	Type I Rock or hard soils	1.0	—	1.0	1.0	1.0	1.0
ii)	Type II Medium soils	1.0	1.0	1.0	1.0	1.2	1.2
iii)	Type III Soft soils	1.0	1.2	1.0	1.2	1.5	1.5

NOTE — The value of  $\beta$  for dams shall be taken as 1.0.

TABLE 2-2-5

TABLE VALUES OF IMPORTANCE FACTOR,  $I$   
( Clauses 3.4.2.3 and 3.4.4 )

SL No.	STRUCTURE	VALUE OF IMPORTANCE FACTOR, $I$ ( see Note )
(1)	(2)	(3)
i)	Dams ( all types )	3.0
ii)	Containers of inflammable or poisonous gases or liquids	2.0
iii)	Important service and community structures, such as hospitals; water towers and tanks; schools; important bridges; important power houses; monumental structures; emergency buildings like telephone exchange and fire bridge; large assembly structures like cinemas, assembly halls and subway stations	1.5
iv)	All others	1.0

NOTE — The values of importance factor,  $I$  given in this table are for guidance. A designer may choose suitable values depending on the importance based on economy, strategy and other considerations.

T = estimated natural or fundamental time period of the building in seconds. It may be estimated from Eq. (1) and for unbraced buildings or from Eq. (2) for braced buildings:

$$T = 0.1N \quad (1)$$

And for all others

$$T = 0.09H/\sqrt{d} \quad (2)$$

H = Total height of the main structure of the building in meters and

d = maximum base dimension of building in meters in a direction parallel to the applied seismic force

After calculating the base shear  $V_B$ , the distribution of earthquake force on different floors is determined as follows:

$$F_i = \frac{W_i h_i^2}{\left[ \sum_{i=1}^n W_i h_i^2 \right]} V_B$$

where;

$F_i$  = horizontal force acting at any floor i

$W_i$  = weight of i the storey assumed to be lumped at ith floor

$h_i$  = height of the floor above base of frame

n = number of storeys

METHODS OF ANALYSIS

The building plane frame can be analysed by-

- A. Approximate methods
- B. Exact method(Stiffness matrix method)

APPROXIMATE METHODS FOR LATERAL LOAD ANALYSIS:

The commonly used methods for the approximate analysis of building frames subjected to lateral loadings are-

3.1 PORTAL METHOD

In the portal method the assumptions to analyse the building frame for the lateral loads are based on the consideration of the behaviour and the deflected shape of the building frame under the action of lateral loads which is shown below :

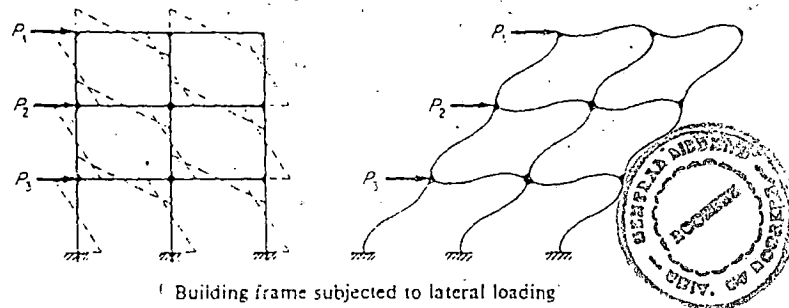


Fig. 6

It is seen from the deflected shape that there is a point of inflection near the centre of each girder and column. That is why in the first two assumptions it is assumed the point of contraflexure at the middle of the members and the another which is assumed reasonably that interior columns carry shear twice of the exterior columns considering each storey to be made up of a series of Portals.

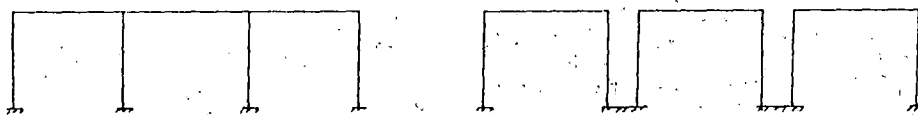


Fig. 7

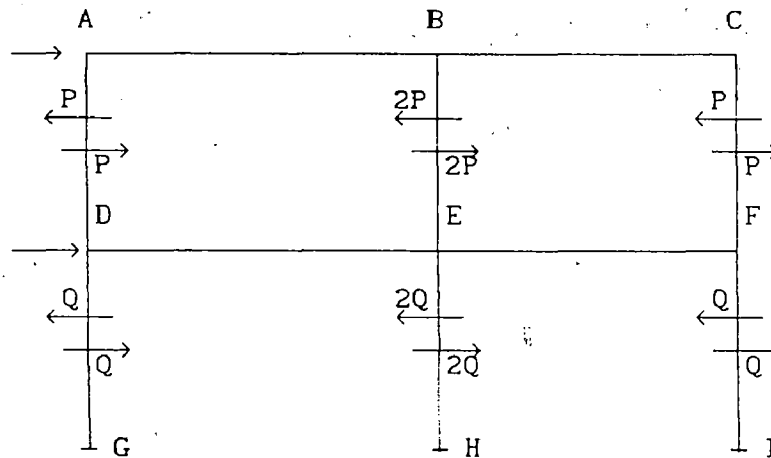
Series of portals equivalent to building frame



The assumptions made in this method are as follows :

1. An inflection point is located at mid height of each column.
2. An inflection point is located at mid span of each beam.
3. The horizontal shear is divided among the columns in the ratio of one part to exterior columns and two parts to interior columns.

The third assumption is a result of considering each level of the frame to be composed of individual portals. Thus by this assumption an interior column is in effect resisting the shear of two columns of individual portals.



### Procedure of Analysis

The application of the Portal method is a numerical problem can be made in the following steps

1. Determine the shear force in each of the columns in each storey.
2. Knowing the point of contraflexure in the columns, compute the column end moments which are counter clockwise.
3. Applying the joint equilibrium condition, i.e. sum of counter clockwise column end moments must be equal to the numerical

sum of clockwise beam end moments, compute the beam end moments.

4. Solve for the axial forces in the column and beams from the free body diagram of the frame.

### 3.2 CANTILEVER METHOD

For the approximate analysis by this method, first two assumptions are made as for the Portal method keeping the consideration of the deflected shape of the building frame. The third assumption is arrived at by considering the the column axial stress intensities which can be obtained by a method analogous, to that used for determining the distribution of normal stress intensities on a transverse section of a cantilever beam as shown below :

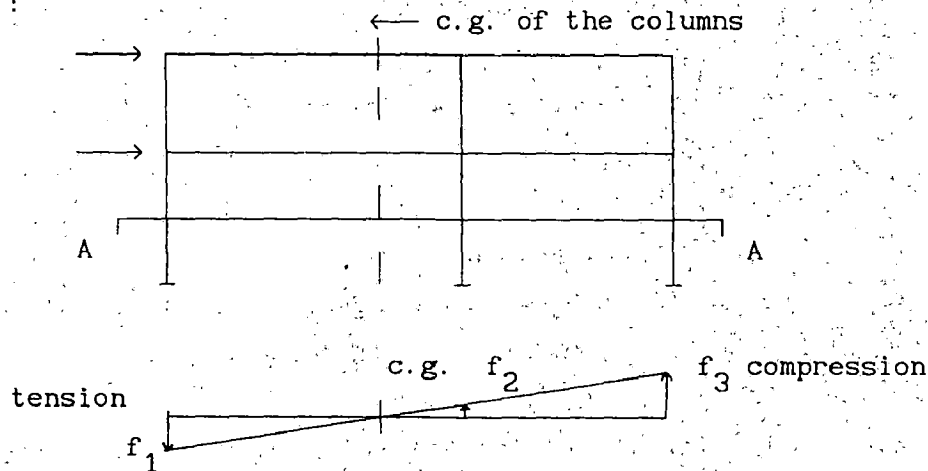


FIG. 8

(AXIAL STRESS IN COLUMNS AT A-A)

The axial stresses in the columns on the section A-A are assumed to vary linearly as on a cross section of the cantilever beam. The axial force in a column is equal to the product of the assumed axial stress and its cross sectional area.

The assumption made in the analysis are as follows :

1. An inflection point is located at the mid height of the column in each storey.

2. An inflection point is located at the mid point of each beam and,
3. The axial force in each column is proportional to its distance from the centroid of the areas of the column group at that level.

### Steps of Procedure

The application of the cantilever method in a numerical problem can be made in the following steps :

1. Determine the location of the centroid of the columns area.
2. Solve for the axial forces in the columns of each storey by applying moments equation to the free body from the top of frame to the inflection point of the columns in that storey.
3. Draw the shear diagrams for the beams in each storey and from them compute the beam end moments in that storey.
4. Solve for the column end moments using the conditions that the sum of the column end moments must be equal to the sum of the beam end moments at each joints.
5. Solve for the horizontal resisting shears at the lower ends of all columns in each storey and check the total value against the total lateral load above that level.

### 3.3 KANIS' METHOD

This method was developed by by Gasper Kani of Germany in 1947. the method is an excellent extension of the slope deflection method. It has the simplicity of moment distribution. The steps of this method are:

1. All F.E.M, restraint moments, storey shear forces and storey moments are calculated. Storey moments are indicated at mid height of the storey.
2. The rotation and displacement factors are calculated.

$$\text{Rotation factor} = -1/2(\text{distribution factor})$$

$$\text{displacement factor} = -\frac{3}{2} \left[ \frac{\text{stiffness of the column}}{\text{sum of all the column stiffnesses in that storey}} \right]$$

3. The calculation of displacement and rotation contribution begins with basic operation for the calculation of linear displacement contributions. After this basic operation is carried out for all storeys, the application of basic operation follows, for rotation contributions at all joints. There two basic operation carried out one after another until results reach desired accuracy.
4. The determination of end moments from final contribution is performed.

$$M_{AB} = M_{AB}^F + 2 M'_{AB} + M'_{BA} + M''_{AB}$$

where,  $M_{AB}^F$  = fixed end moment

$M'_{AB}$  = rotation moment

$M''_{AB}$  = translation moment

Some special points:-

1. The sum of the rotation factor at a joint is  $(-\frac{1}{2})$ .

2. If an end of a member is fixed, the rotation at that end being zero, the rotation moment is also zero.
3. If an end of a member is hinged or pinned, it is convenient to consider it as fixed and take the relative stiffness as  $(3/4) \frac{I}{L}$ .

The analysis of a multistorey building frame with horizontal loading differs from that of a frame with vertical loading only by the fact that in performing the basic operation for determination of the translation moments, the sum of the rotation moments of all member ends of the storey must also contain storey moment.

### 3.4 BOWMAN'S METHOD

This method is based on the study of numerous bents by slope-deflection method suggested by Bowman. The following assumptions are made.

1. Points of contraflexure in exterior girders are located at 0.55 of their length from their outer ends and in other girders at their mid points except (a) in the center bay, where the total number of bays is odd and (b) in the two bays nearest the center where the total number of bays is even. In these excepted cases the points of contraflexure in girders will be located as required by conditions of symmetry and equilibrium.
2.
  - a. In bents of one or more storeys, the points of contraflexure in bottom storey columns are at 0.60 height from the base.
  - b. In of two or more storeys the point of contraflexure in top storey columns are at 0.65 height from the top.
  - c. In bents of three or more storeys, the point of contraflexure in the columns of the storey next to the top are 0.60 height

from the upper end.

- d. In bents of four or more storeys the points of contraflexure in the columns of the second storey from the top are located at 0.55 height from the upper end, and
- e. In bents of five or more storeys, the points of contraflexure in the columns of storeys not provided for above are at 0.50 height.

3.

- a. For Bottom Storey :

An amount of shear

$$= \frac{(\text{No. of bays} - 1/2)}{(\text{No. of columns})} * \text{total shear in the storey}$$

is divided equally among the columns of the bottom storey and the remaining shear in the bottom storey is divided among the bays inversely as their width and the shear in bays is divided equally between the two columns adjacent to the bay.

- b. For Other Storeys :

There is divided equally among the columns of other storeys an amount of shear equal to  $(\text{no. of bays} - 2) / \text{no. of columns}$  times the total shear in the storey.

The remaining shear in the storey is divided among the bays inversely as their widths and the shear in the bay is divided equally between the two columns adjacent to the bay.

#### Steps of Procedure

From the above assumptions, the steps in the analysis are

1. Locate the points of contraflexure in the members according to the assumptions first and second.
2. Distribute the storey shears into the columns of that storey

according to third assumption.

3. Calculate the column end moments knowing the column shear and the points of contraflexure in the columns.
4. Applying the condition that at each joint sum of the beam end moments must be equal to the sum of column end moments, calculate the beam end moments.
5. The axial forces in the members can be calculated from the free body diagram.

### 3.5 THE FACTOR METHOD

This method may be considered as an approximate slope deflection method using correct relative stiffness values of the members of the frame. The following steps are involved in this method.

- 1). For each joint compute the girder factor  $g$  by the following

$$g = \frac{\Sigma k_c}{\Sigma k}$$

where;

$\Sigma k_c$  = sum of  $k$  values for the columns meeting at that joint.

$\Sigma k$  = sum of the  $k$  values for all members of that joint.

Write each value of ' $g$ ' thus obtained at the near end of each girder meeting at the joint where it is computed.

2. For each joint, compute the column factor ' $c$ ' by the following  $c=1-g$ , where  $g$  is the girder factor for that joint

as computed in step 1. Write each value of 'c' thus obtained at the shear end of each column meeting at the joint where it is computed. For the fixed column basis of the first storey, take  $c=1$ .

3. From step 1 and 2, there is a number at each end of each member of the bent. To each of these numbers, add half of the number at the other end of the member.
4. Multiply each sum obtained from step 3 by the  $k$  value for the member in which the sum occurs. For columns, call this product the column moment factor 'C'; for girders call this product the girder moment factor 'G'.
5. The column moment factors 'C' from step 4 are actually approximate relative values for column end moments for the storey in which they occur. The sum of the column end moments in a given storey may be shown by statics to equal the total horizontal shear on that storey multiplied by the storey height. Hence the column moment factor 'C' may be converted into column end moments by direct proportion for each storey.
6. The girder moment factor 'G' from step 4 are actually approximate relative values for girder end moments at each joint. The sum of the girder end moments at each joint equals by statics, the sum of the column end-moments at that joint which can be obtained from step 5. Hence the girder moment factors G may be converted to girder end moments by direct proportion for each joint.

### 3.6 KLOUCEKS' METHOD (DISTRIBUTION OF DEFORMATION METHOD)

The distribution of deformation (developed by C.V. Kloucek of Czechoslovakia) is a method based on the slope-deflection equations.



Whereas the solutions by both Cross (Moment distribution method) and Kani yield the end moments of the structural members directly, Klaucek, determines their end-deformation (the end rotations and the lateral end displacements) through an ingenious solution of the joint equilibrium equations identical with those already used in the slope deflection method. The end-moments of the members are then calculated from the slope deflection equation.

Kloucek's method involves using only two basic formulae. One of them gives the rotation of the joint subjected to an applied moment and the other gives the proportion of this rotation carried over to the adjacent joint. This method is specially useful in multistoreyed frames.

### Step's Involved

The following steps are involved in the analysis of the structure

1. Calculate F.E.Ms. Obtain the restraint moment  $M_i$  at the joint  $i$  as the algebraic sum  $\sum M_{ij}^F$  where  $j$  denotes joints adjacent to  $i$
2. Calculate the member stiffness and joint stiffness values

$$K_{ij} = \frac{I_{ij}}{L_{ij}}, \text{ stiffness of the member } ij$$

$$K_i = \sum K_{ij}; \text{ stiffness of the joint 'i'}$$

3. Calculate the factor 'a' for all member

$$a_{ij} = a_{ji} = \frac{K_{ij}^2}{K_i K_j}$$

4. Calculate factors  $a'$

The  $a'$  factors are replaced by 'a' factor approximation assuming all joints except the two ends  $i$  &  $j$  of the member are fixed against rotation.

5. Calculate the primary deformation ( $d_i^0$ ) at the joint  $i$  produced by the restraint moment  $M_i$

$$d_i^0 = - \frac{M_i}{K_i (1 - \sum_j a'_{ij})}$$

6. Calculate the secondary deformations, i.e. the parts of  $d_i^0$  carried over to the other joints:

$$d_{ij} = -d_i^0 \frac{K_{ij}}{K_j (1 - \sum_k a'_{jk})}$$

$$d_{jk} = -d_{ij} \frac{K_{jk}}{K_k (1 - \sum_l a'_{kl})} \quad \text{etc.}$$

where  $\sum_k$  denotes the sum of all joints adjacent to 'j', excluding 'i',  $\sum$  denotes the sum over all joints to  $K$  excluding  $j$ .

7. Calculate the total deformation at all the joints as the algebraic sum of their primary and secondary deformations.
8. Calculate the end moments of the structural members from the slope deflection equations

$$M_{ij} = M_{ij}^F + K_{ij} (2d_i + d_j)$$

### 3.7 BLUME et.als'. Method

Analysis of framed buildings subjected to lateral loads, such as those generated by earthquake motion and high wind, requires knowledge of lateral stiffness for calculation of lateral displacement in static analysis, and calculation of lateral displacement and dynamic properties in dynamic analysis.

The concept of lateral stiffness concludes that a single value can be used to represent the stiffness of a storey in an elastic, rectangular frame with fixed base that is subjected to regular distributions of lateral load. It is found that this method is applicable only for uniform frames with girders that are flexurally stiffer than columns. The expressions for approximating the lateral stiffness is limited to rectangular frames that are fixed at the base and for which only flexural deformations are important.

In the analysis of framed buildings subjected to wind or earthquake loads, it is generally assumed that lateral loads are distributed in a regular manner.

Blume et.al. (1961) present another procedure whereby the method of moment distribution is used to determine how much the lateral stiffness of each column in a shear building is softened in proportion to girder flexibility.

The apparent stiffness  $K_s$  of the storey is obtained by adding the contributions of all columns ( $K_s = \sum k_c$ ). Blume et.al. assumes that the typical column is in a regular frame and that the column end rotations are equal. Fixed end moment for a column are calculated based on rigid girders and an arbitrary selected storey drift. Only a single cycle of moment distribution is needed because member stiffness are modified to reflect equal end rotations for which case carry over movements are equal to zero.

The resulting column moments are used to calculate the resisting shear force in the column and from this shear force, apparent stiffness  $K_c$  of the column is approximated as

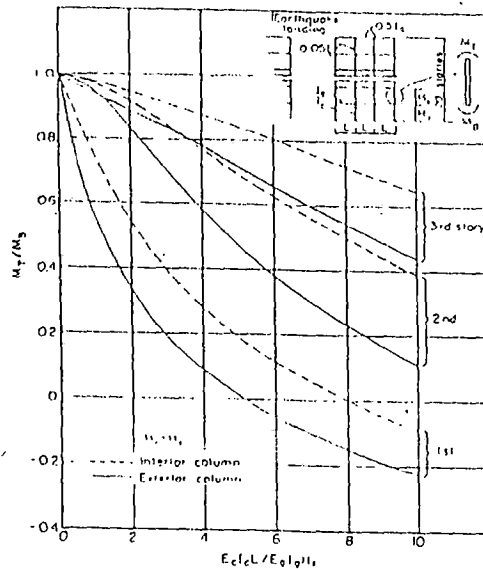
$$K_c = \left( \frac{12E_c I_c}{H^3} \right) \left[ 1 - \left( \frac{K_c}{\Sigma K_a} \right) - \left( \frac{K_c}{\Sigma K_b} \right) \right]$$

where ;

$K_c$  -> flexural stiffness of the column.

The sum of the stiffness of all connecting members in the joints above and below the column are given by  $\Sigma K_a$  and  $\Sigma K_b$  respectively.

Blume et.al. recognise that this approximation breaks down at boundary stories i.e. at the top and base of a frame. The disturbances introduced by abrupt termination of the frame and the fixed base are not consistent with the assumption of equal end rotations. To compensate for this shortcoming, Blume et.al. recommend the use of charts that summarise multiplicative factors for modifying column end moments at boundary stories.



Effect of relative stiffness of girder to column on location of point of inflection in exterior and interior columns fixed at the base and having equal stiffness.

FIG. 9

### Steps of Procedure

The application of the Blume et al's method in a numerical problem can be made in the following steps :

1. Calculate apparent column stiffness from the formula approximated in this method.
2. Compute the apparent storet stiffness
$$K_s = \sum k_c$$
3. Distribute the storey shear into the columns in the ratio of their apparent column stiffnesses.
4. For each column find the value of  $M_T/M_B$  i.e. the ratio of top and bottom end moments of the columns from the curve given by Blume.
5. Knowing the sum of end moments of each column and the ratio of top and bottom end moment, calculate the end moments separately.
6. Applying the condition at each joint that column end moments must be equal to the beam end moments calculate the values of beam end moments.
7. The shears and the axial forces can be calculated from the moments calculated above and drawing free body diagram.

### 3.8 STIFFNESS MATRIX METHOD

Stiffness is important property which characterise the response of a structure by means of the force displacement relationship. In general stiffness is defined as the force required for a unit displacement. A structural member can have

four types of displacements :

- (i) Axial displacement
- (ii) Transverse displacement
- (iii) Bending or Flexural displacement
- (iv) Torsional displacement

If 1, 2, ..... n be the system of coordinates chosen to express the system of forces  $P_1, P_2, \dots, P_n$ . Proceeding displacements  $\Delta_1, \Delta_2, \dots, \Delta_n$ . If a unit displacement is given at coordinate "j" without any displacement at other coordinates. The force required at coordinates 1, 2, ..... n may be represented by  $k_{1j}, k_{2j}, \dots, k_{nj}$ . These are the forces which must act at coordinates 1, 2, ..... n to hold the structure in this specific deformed position in which  $\Delta_j = 1$  and  $\Delta_i (i \neq j) = 0$ . In other words  $k_{1j}, k_{2j}, \dots, k_{nj}$  are the forces required at coordinates 1, 2, ..... n respectively in order to produce a unit displacement at coordinate "j" and zero displacement at all other coordinates. Thus  $k_{ij}$  is the force at coordinate "i" due to a unit displacement at coordinate "j" only. The total force  $P_i$  at coordinate i due to displacement  $\Delta_1, \Delta_2, \dots, \Delta_n$  may be computed by using a principle of superposition.

$$P_i = k_{i1} \Delta_1 + k_{i2} \Delta_2 + \dots + k_{in} \Delta_n$$

Similar equations can be written for the forces at other coordinates resulting in the following set of equations.

$$P_1 = k_{11} \Delta_1 + k_{12} \Delta_2 + \dots + k_{1j} \Delta_j + \dots + k_{1n} \Delta_n$$

$$P_2 = k_{21} \Delta_1 + k_{22} \Delta_2 + \dots + k_{2j} \Delta_j + \dots + k_{2n} \Delta_n$$

$$P_i = k_{i1} \Delta_1 + k_{i2} \Delta_2 + \dots + k_{ij} \Delta_j + \dots + k_{in} \Delta_n$$

$$P_n = k_{n1} \Delta_1 + k_{n2} \Delta_2 + \dots + k_{nj} \Delta_j + \dots + k_{nn} \Delta_n$$

The above equations may be expressed in the matrix form

$$\begin{bmatrix} P_1 \\ P_2 \\ \cdot \\ \cdot \\ P_i \\ \cdot \\ \cdot \\ P_n \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & \dots & k_{1j} & \dots & k_{1n} \\ k_{21} & k_{22} & \dots & k_{2j} & \dots & k_{2n} \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ k_{i1} & k_{i2} & \dots & k_{ij} & \dots & k_{in} \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ k_{n1} & k_{n2} & \dots & k_{nj} & \dots & k_{nn} \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \cdot \\ \cdot \\ \Delta_i \\ \cdot \\ \cdot \\ \Delta_n \end{bmatrix}$$

This equation may be written in the compact form

$$[p] = [k] [\Delta]$$

where  $[k]$  = a square matrix of order  $n$  known as stiffness matrix

#### Steps of procedure

The following steps are written below for analysis by stiffness matrix method

1. Determine the degree of the kinematic determinacy,  $n$
2. Identify the independent displacement component.
3. Assign coordinate 1,2,...,n to the independent displacement component.
4. Present all the independent displacement component to obtain the restrained structure.
5. Determine  $[P']$  the force at the coordinates in the restrained structure due to loads other than those acting at the coordinates.
6. Determine  $[P_{\Delta}]$ , the force required at the coordinates in the unrestrained structure to cause independent displacement component ( $\Delta$ ).
7. Compute the net forces  $[P] = [P] + [P_{\Delta}]$
8. Use the conditions of equilibrium of forces to compute the displacements  

$$[\Delta] = [k]^{-1} \{ [P] - [P'] \}$$
9. Knowing the displacements, compute the internal member forces by using slope deflection equation.

## CASE STUDY

For a building frame the lateral loads which acts due to earthquake and wind action can be calculated by some approximation as mentioned in the previous Chapter "Approximation of the lateral loads". The cases which have been considered here are the building frames subjected to earthquake loadings from which the external loading taken as point loads on each floor level.

Some symmetric and unsymmetric frames have been analysed by commonly used methods and for one of the frames, a comprehensive study has been made by using various approximate methods and by using stiffness matrix method i.e.

a) A comprehensive study of the frame A.

The frame which has been analysed is two-bay double-storied as shown below.

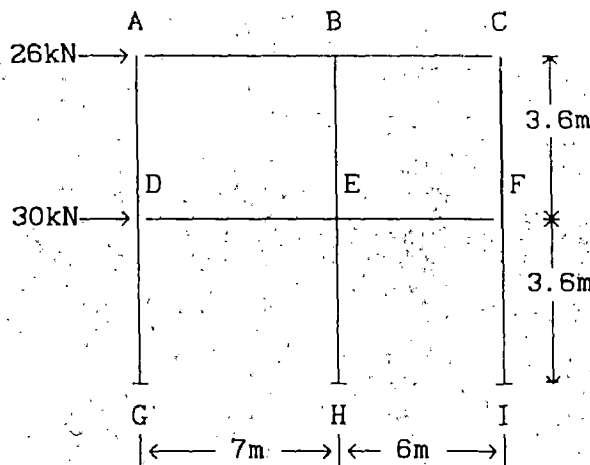


FIG-10 FRAME -A

The widths of bays are 7m and 6m respectively. Storey height of each storey is 3.6m. The lateral loads are 26 KN and 30 KN acting on the upper roof level and first floor level respectively. The column and beam sizes are 35 cm x 35 cm and 35 cm x 65 cm respectively which gives the moment of inertia  $1.25 \times 10^{-3} \text{ m}^4$  and  $8.01 \times 10^{-3} \text{ m}^4$  respectively. The values of relative stiffnesses

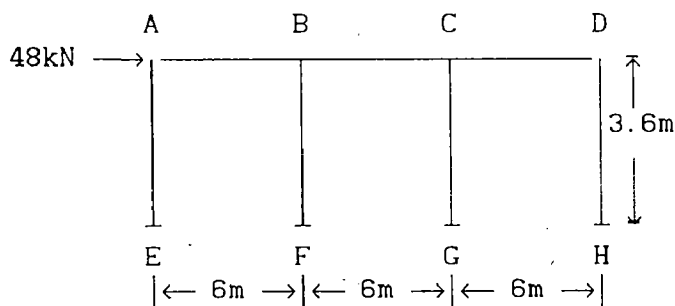


are shown in the middle of the frame members as in the form of their ratio i.e. for columns it is shown by "k" and for beams it is shown by 3.3 k and 3.8 k respectively.

The detailed analysis is done by eight methods i.e. (i) Portal method, (ii) Cantilever method, (iii) Kani's method, (iv) Bowman's method, (v) Factor Method, (vi) Kloucek's method, (vii) Blume et. al's method and (viii) Stiffness matrix method. The steps of analysis are shown in Appendix B.

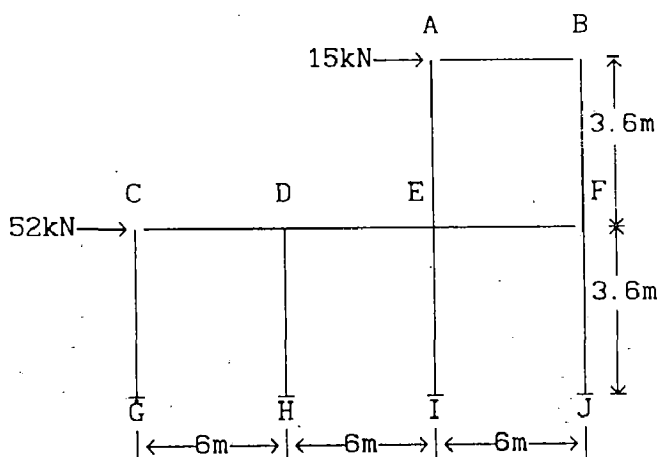
**(b) Study of Some Frames by Commonly Used Methods**

Some other frames including unsymmetric frames have been studied and analysed by commonly used approximate methods and by the stiffness matrix method. These all frames are having a maximum of three bays in each storey and a maximum number of three stories. The frames are :



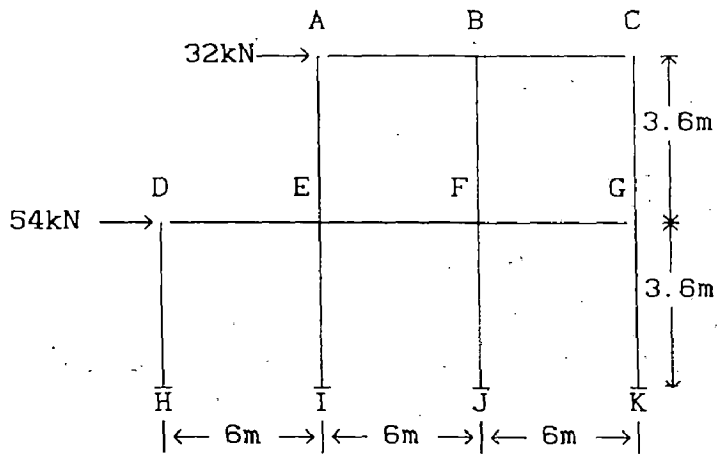
FRAME -1

FIG -11



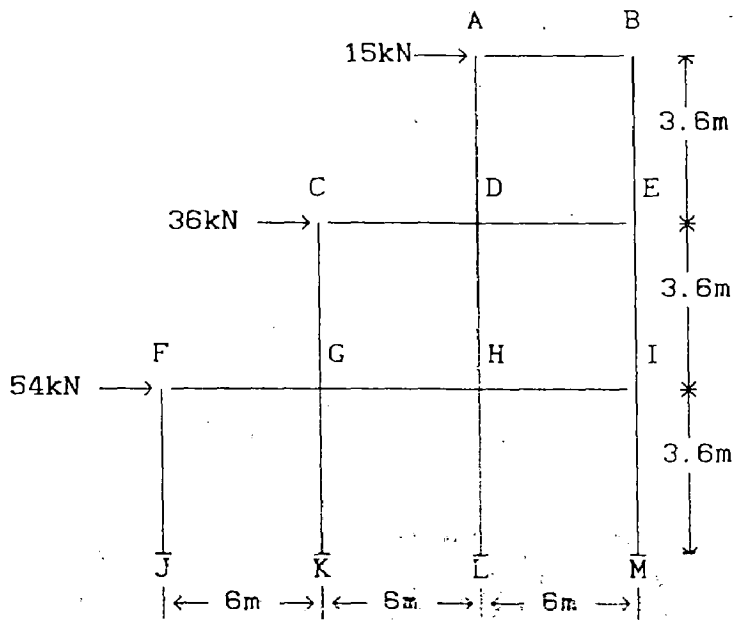
FRAME -2

FIG -12



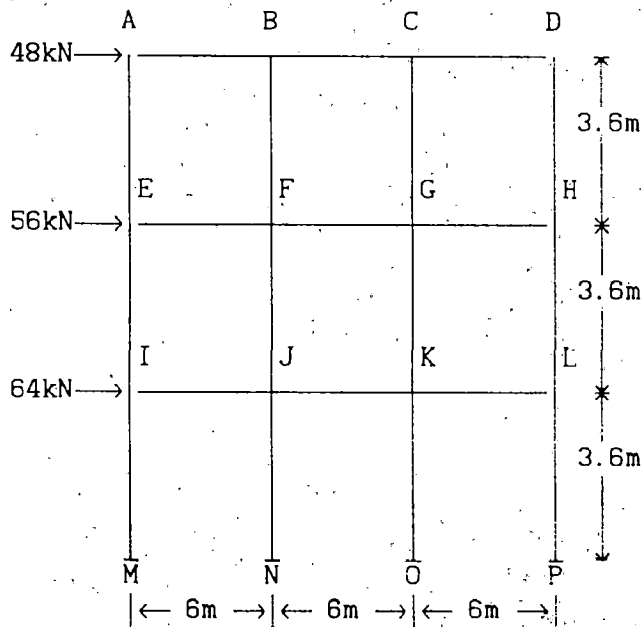
FRAME -3

FIG -13



FRAME -4

FIG -14



FRAME -5

FIG-15

The methods used are (i) Portal method, (ii) Cantilever method, (iii) Kani's method and (iv) stiffness matrix method.

The frames are of different heights and shapes but the bay width and storey heights taken are the same and are 6 m and 3.6 m each

respectively. The size of columns and beams are 35 cm x 35 cm and 35 cm x 65 cm respectively and moment of inertia are  $1.25 \times 10^{-3} \text{ m}^4$  and  $8.01 \times 10^{-3} \text{ m}^4$  respectively. The relative stiffnesses shown at the middle of the each member are in the form of ratio i.e. for columns it is "k" and for beams it is "3.8 k" respectively.

## RESULTS AND DISCUSSION

## 5.1 Comparison of Results of Frame A with Comprehensive Study

The comprehensive analysis of the Frame A has been made by various approximate methods and by the stiffness matrix method. Comparative study of the results of frame A can be made by Table 5.1.1.

TABLE 5.1.1 Results of Comparison by Various Methods

Storey		Portal Method	Cantilever Method	Kani's Method	Bowman's Method	The Factor Method	Klouceks' Method	Blume et als Method	Exact Method
	Moment in Beams								
Top	M <sub>AB</sub>	11.7	12.2	14.38	14.04	14.96	12.90	13.93	14.37
	M <sub>BA</sub>	11.7	12.2	10.01	11.48	8.36	9.77	9.10	10.09
	M <sub>BC</sub>	11.7	10.5	10.86	13.39	9.23	11.21	9.10	10.75
	M <sub>CB</sub>	11.7	10.5	15.23	16.38	15.51	14.78	14.40	15.24
Bottom	M <sub>DB</sub>	36.9	40.67	39.77	30.30	43.79	36.14	45.75	39.93
	M <sub>BD</sub>	36.9	40.67	27.12	24.80	24.29	26.73	26.50	27.19
	M <sub>EF</sub>	36.9	33.12	29.31	27.01	26.63	30.78	26.50	29.15
	M <sub>FB</sub>	36.9	33.12	41.96	33.11	45.49	41.61	47.10	41.85
	Moment in Columns								
Top	M <sub>AD</sub>	11.7	12.92	14.38	14.04	14.96	14.72	13.93	14.37
	M <sub>DA</sub>	11.7	12.92	10.98	7.56	13.99	11.74	14.15	10.94
	M <sub>BE</sub>	23.4	23.42	20.87	30.42	17.59	19.47	18.11	20.84
	M <sub>EB</sub>	23.4	23.42	19.98	16.38	16.90	18.39	18.39	19.97
	M <sub>CF</sub>	11.7	10.5	15.23	16.38	15.51	14.74	14.40	15.24
	M <sub>FC</sub>	11.7	10.5	12.16	8.82	19.68	11.75	14.62	12.24
Bottom	M <sub>DG</sub>	25.2	27.75	28.78	27.74	29.75	29.52	31.60	28.99
	M <sub>GD</sub>	25.2	27.75	38.08	34.11	34.79	34.12	32.91	34.35
	M <sub>EH</sub>	50.4	50.37	36.45	33.60	34.00	35.22	34.56	36.37
	M <sub>HE</sub>	50.4	50.37	37.91	50.39	36.92	36.97	36.00	37.86
	M <sub>FI</sub>	25.2	22.62	29.79	24.29	30.81	29.52	32.48	29.61
	M <sub>IF</sub>	25.2	22.62	34.56	36.44	35.32	34.12	33.83	34.41
	Shear in Beams (KN)								
Top	V <sub>AB</sub>	3.34	3.69	3.48	3.64	3.32	3.24	3.29	3.49
	V <sub>BC</sub>	3.90	3.50	4.35	4.96	4.13	4.33	3.91	4.33
Bottom	V <sub>DB</sub>	10.54	11.62	9.55	7.86	9.72	7.69	10.35	9.59
	V <sub>BF</sub>	12.30	11.04	11.88	10.03	12.06	12.46	12.29	11.83

Shear in Columns									
Top	V <sub>AD</sub>	6.50	7.17	7.05	6.00	8.04	7.83	7.83	7.03
	V <sub>BE</sub>	13.00	13.00	11.34	13.00	9.90	10.36	10.08	11.34
	V <sub>CF</sub>	6.50	5.83	7.62	7.00	8.06	7.84	8.09	7.63
Bottom	V <sub>DG</sub>	14.00	15.42	17.46	15.79	17.93	17.36	17.95	17.59
	V <sub>EH</sub>	28.00	27.99	27.65	23.33	19.66	20.72	19.64	20.62
	V <sub>FI</sub>	14.00	12.57	17.88	16.87	18.36	17.36	18.45	17.78
Axial Force in Beams									
Top	AB	19.5	18.93	18.95	20.00	17.96	18.17	18.17	18.97
	BC	6.5	5.83	7.61	7.00	8.06	7.81	8.09	7.63
Bottom	DE	22.5	21.75	19.59	20.21	20.11	20.47	19.88	19.44
	EF	7.5	6.76	10.28	9.88	10.34	9.52	10.36	10.15
Axial Force in Columns									
Top	AD	3.34	3.64	3.48	3.64	3.32	3.24	3.29	3.49
	BE	0.56	(-0.19)	0.87	1.32	0.78	1.12	0.59	0.84
	CF	3.9	3.5	4.35	4.96	4.13	4.33	3.91	4.33
Bottom	DG	13.88	15.31	13.03	11.50	13.04	13.93	13.64	13.08
	EH	2.32	(-0.77)	3.2	3.49	3.12	2.89	2.54	13.08
	FI	16.2	14.54	16.23	14.99	16.19	16.79	16.2	16.16

## 5.2 Comparison of Results of the frames studied by commonly used methods

As comprehensive study has been made for frame A, five other frames have also been analysed by the commonly used approximate methods, that are Portal method, Cantilever method and Kani's method and have been also analysed by stiffness matrix method. To compare the results the values after analysis are shown in tabular form as Table 5.2.1.

Table for Frame 1  
(5.2.1)

Storey	Portal Method	Cantilever Method	Kani's Method	Stiffness Matrix Method
Moment (KN-m)				
BEAM				
M <sub>AB</sub>	14.40	12.96	19.41	19.89
M <sub>BA</sub>	14.40	12.96	13.91	13.93
M <sub>BC</sub>	14.40	17.28	8.41	8.52
M <sub>CB</sub>	14.40	17.21	8.41	8.54

	V <sub>BF</sub>	7.50	7.50	6.75	6.74
	BEAM				
B	V <sub>CD</sub>	6.70	8.73	7.69	7.74
O	V <sub>DE</sub>	6.70	11.64	4.99	5.13
	V <sub>EF</sub>	11.20	4.23	10.80	10.66
T	V <sub>CG</sub>	11.17	14.55	16.15	16.39
T	V <sub>DH</sub>	22.34	33.95	18.00	18.12
O	V <sub>EI</sub>	22.34	18.95	17.32	17.20
M	V <sub>FJ</sub>	11.17	-0.45	15.52	15.28
	AXIAL FORCE (KN)				
	BEAM				
T	F <sub>AB</sub>	7.50	7.50	6.75	6.74
O	COLUMN				
P	F <sub>AE</sub>	-4.50	-4.50	-4.77	-4.75
	F <sub>BF</sub>	4.50	4.50	4.77	4.75
B	BEAM				
O	F <sub>CD</sub>	40.83	37.45	35.85	35.60
T	F <sub>DE</sub>	18.49	3.50	17.85	17.48
T	F <sub>EF</sub>	3.65	7.95	8.79	8.54
O	COLUMN				
M	F <sub>CG</sub>	-6.70	-8.73	-7.69	-7.74
	F <sub>DH</sub>	0.00	-2.91	2.70	2.61
	F <sub>EI</sub>	-9.00	2.91	-10.58	-10.28
	F <sub>FJ</sub>	15.70	8.73	15.57	15.41

Table for Frame 3

Storey		Portal Method	Cantilever Method	Kani's Method	Stiffness Matrix Method
Moment (KN-m)					
BEAM					
T	M <sub>AB</sub>	14.40	14.40	19.40	19.49
O	M <sub>BA</sub>	14.40	14.40	12.26	12.50
P	M <sub>BC</sub>	14.40	14.40	11.17	10.86
S	M <sub>CB</sub>	14.40	14.40	17.21	17.11
T COLUMN					
O	M <sub>AE</sub>	14.40	14.40	19.41	19.49
R	M <sub>EA</sub>	14.40	14.40	19.34	19.43
E	M <sub>BF</sub>	28.80	28.80	23.43	23.37
Y	M <sub>FB</sub>	28.80	28.80	21.75	21.73
	M <sub>CG</sub>	14.40	14.40	17.21	17.12
	M <sub>GC</sub>	14.40	14.40	14.07	14.07
BEAM					
B	M <sub>DE</sub>	25.80	40.50	36.48	36.94
O	M <sub>ED</sub>	25.80	40.50	31.91	31.97
T	M <sub>EF</sub>	40.17	39.60	26.32	26.49
T	M <sub>FE</sub>	40.17	39.60	25.31	25.57
O	M <sub>FG</sub>	40.20	26.10	35.86	35.35
M	M <sub>GF</sub>	40.20	26.10	47.42	47.03
S COLUMN					
T	M <sub>DH</sub>	25.79	40.50	36.48	36.94
O	M <sub>HD</sub>	25.79	40.50	40.08	40.61
R	M <sub>EI</sub>	51.59	65.70	38.89	39.04
E	M <sub>IE</sub>	51.59	65.70	41.29	41.41
Y	M <sub>FJ</sub>	25.79	36.90	39.43	39.20
	M <sub>JF</sub>	25.79	26.90	41.56	41.32
	M <sub>GK</sub>	51.59	11.70	33.34	32.95
	M <sub>KG</sub>	51.59	11.70	38.52	38.12

		SHEAR(KN)			
T O P	BEAM				
	V <sub>AB</sub>	4.80	4.80	5.27	5.33
	V <sub>BC</sub>	4.80	4.80	4.73	4.66
COLUMN					
	V <sub>AE</sub>	8.00	8.00	10.78	10.81
	V <sub>BF</sub>	16.00	16.00	12.53	12.52
	V <sub>CG</sub>	8.00	8.00	8.69	8.66
B O T T O M	BEAM				
	V <sub>DE</sub>	8.60	13.50	11.40	11.49
	V <sub>EF</sub>	13.39	13.20	8.60	8.68
	V <sub>FG</sub>	13.40	8.70	13.91	13.73
COLUMN					
	V <sub>DH</sub>	14.33	22.50	21.21	21.54
	V <sub>EI</sub>	28.66	36.50	22.22	22.34
	V <sub>FJ</sub>	28.66	20.50	22.53	22.36
	V <sub>GK</sub>	14.33	6.50	19.96	19.74
		AXIAL FORCE(KN)			
BEAM					
	F <sub>AB</sub>	24.00	24.00	21.22	21.19
	F <sub>BC</sub>	8.00	8.00	8.69	8.66
COLUMN					
	F <sub>AE</sub>	-4.80	-4.80	-5.27	-5.33
	F <sub>BF</sub>	0.00	0.00	0.54	0.66
	F <sub>CG</sub>	4.80	4.80	4.73	4.66
BEAM					
	F <sub>DE</sub>	39.67	31.50	32.79	32.46
	F <sub>EF</sub>	19.05	3.00	21.35	20.92
	F <sub>FG</sub>	6.35	-1.50	11.35	11.08
COLUMN					
	F <sub>DH</sub>	-8.60	-13.50	-11.40	-11.49
	F <sub>EI</sub>	-9.49	-4.50	-2.47	-2.52
	F <sub>FJ</sub>	0.00	4.50	-4.77	-4.38
	F <sub>GK</sub>	18.20	13.50	18.64	18.39



Table for Frame 4

Storey	Portal Method	Cantilever Method	Kani's Method	Stiffness Matrix Method
Moment (KN-m)				
BEAM				
M <sub>AB</sub>	13.50	13.50	15.13	15.11
M <sub>BA</sub>	13.50	13.50	13.40	13.33
COLUMN				
M <sub>AD</sub>	13.50	13.50	15.13	15.12
M <sub>DA</sub>	13.50	13.50	15.13	15.27
M <sub>BE</sub>	13.50	13.50	13.40	13.33
M <sub>EB</sub>	13.50	13.50	10.32	10.29
BEAM				
M <sub>CD</sub>	22.95	36.45	31.49	31.18
M <sub>DC</sub>	22.95	36.45	24.16	24.99
M <sub>DE</sub>	36.45	22.95	26.85	26.04
M <sub>ED</sub>	36.45	22.95	36.85	36.53
COLUMN				
M <sub>CG</sub>	22.95	36.95	31.49	31.78
M <sub>GC</sub>	22.95	36.95	31.51	31.77
M <sub>DH</sub>	45.90	45.90	35.88	35.77
M <sub>HD</sub>	45.90	45.90	34.51	34.48
M <sub>EI</sub>	22.95	9.45	26.52	26.24
M <sub>IE</sub>	22.95	9.45	27.69	23.55
BEAM				
M <sub>FG</sub>	31.50	63.99	45.27	45.69
M <sub>GF</sub>	31.50	63.99	41.94	41.99
M <sub>GH</sub>	22.95	48.87	36.58	36.98
M <sub>HG</sub>	22.95	48.87	34.54	35.15
M <sub>HI</sub>	65.85	27.54	48.05	47.19
M <sub>IH</sub>	65.85	27.54	63.59	63.03
COLUMN				
M <sub>FJ</sub>	31.50	63.99	45.27	45.69

M <sub>JF</sub>	31.50	63.99	49.53	49.99
M <sub>GK</sub>	63.00	76.41	47.01	47.19
M <sub>KG</sub>	63.00	76.41	50.40	50.54
M <sub>HL</sub>	63.00	30.51	48.09	47.87
M <sub>LH</sub>	63.00	30.51	50.94	50.74
M <sub>IM</sub>	31.50	18.09	39.91	39.50
M <sub>MI</sub>	31.50	18.09	46.85	46.47

**SHEAR (KN)**

**BEAM**

V <sub>AB</sub>	4.50	4.50	4.76	4.74
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**COLUMN**

V <sub>AD</sub>	7.50	7.50	8.41	8.44
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V <sub>BE</sub>	7.50	7.50	6.60	6.56
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**BEAM**

V <sub>CD</sub>	7.65	12.15	9.26	9.46
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V <sub>DE</sub>	12.15	7.65	10.61	10.42
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**COLUMN**

V <sub>CG</sub>	12.75	20.25	17.49	17.65
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V <sub>DH</sub>	25.50	25.50	19.50	19.51
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V <sub>EI</sub>	12.75	5.25	13.96	13.83
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**BEAM**

V <sub>FG</sub>	10.50	21.33	14.56	14.61
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V <sub>GH</sub>	7.65	16.29	11.84	12.02
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V <sub>HI</sub>	21.15	9.18	18.62	18.37
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**COLUMN**

V <sub>FJ</sub>	17.50	35.55	26.32	26.58
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V <sub>GK</sub>	35.00	42.45	27.02	27.15
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V <sub>HL</sub>	35.00	16.95	27.48	27.39
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V <sub>IM</sub>	17.50	10.05	24.04	23.87
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**AXIAL FORCE (KN)**

**BEAM**

F <sub>AB</sub>	7.50	7.50	6.59	6.56
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**COLUMN**

F <sub>AD</sub>	-4.50	-4.50	-4.76	-4.76
-----------------	-------	-------	-------	-------

F <sub>BE</sub>	4.50	4.50	4.76	4.74
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BEAM				
F <sub>CD</sub>	23.25	15.75	18.51	18.34
F <sub>DE</sub>	5.25	-2.25	7.42	7.27
COLUMN				
F <sub>CG</sub>	-7.65	-12.15	-9.26	-9.46
F <sub>DH</sub>	-9.00	0.00	-5.85	-5.70
F <sub>EI</sub>	16.65	12.15	15.11	15.17
BEAM				
F <sub>FG</sub>	36.50	18.45	27.68	27.41
F <sub>GH</sub>	14.25	-3.75	18.15	17.92
F <sub>HI</sub>	4.75	4.80	10.17	10.04
F <sub>FJ</sub>	-10.50	-21.33	-14.56	-14.61
F <sub>GK</sub>	-4.80	-7.11	-6.54	-6.87
F <sub>HL</sub>	-22.50	7.11	-12.63	-12.05
F <sub>IM</sub>	37.80	21.33	33.73	33.54

Table for Frame 5

Storey	Portal Method	Cantilever Method	Kani's Method	Stiffness Matrix Method
<b>Moment (KN-m)</b>				
<b>BEAM</b>				
$M_{AB}$	14.40	12.96	20.20	19.60
$M_{BA}$	14.40	12.96	15.40	13.34
$M_{BC}$	14.40	17.28	10.60	13.10
$M_{CB}$	14.40	17.28	10.60	13.10
$M_{CD}$	14.40	12.96	15.40	13.40
$M_{DC}$	14.40	12.96	20.20	19.71
<b>COLUMN</b>				
$M_{AE}$	14.40	12.96	20.20	19.60
$M_{EA}$	14.40	12.96	16.19	15.73
$M_{BF}$	28.80	30.24	26.00	26.45
$M_{FB}$	28.80	30.24	24.01	24.46
$M_{CG}$	28.80	30.24	26.00	26.51
$M_{GC}$	28.80	30.24	24.01	24.52
$M_{DH}$	14.40	12.96	20.19	19.71
$M_{HD}$	14.40	12.96	16.19	15.82
<b>BEAM</b>				
$M_{EF}$	45.60	41.04	58.17	57.24
$M_{FE}$	45.60	41.04	45.72	43.76
$M_{FG}$	45.60	54.72	33.26	35.92
$M_{GF}$	45.60	54.72	33.26	35.91
$M_{GH}$	45.60	41.04	45.71	43.89
$M_{HG}$	45.60	41.04	58.17	57.53
<b>COLUMN</b>				
$M_{EI}$	31.20	28.08	41.98	41.51
$M_{IE}$	31.20	28.08	37.06	36.64
$M_{FJ}$	62.40	65.52	54.96	52.22
$M_{JF}$	62.40	65.52	53.20	53.51

M <sub>GK</sub>	62.40	65.52	54.96	55.28
M <sub>KG</sub>	62.40	65.52	53.20	53.57
M <sub>HL</sub>	31.20	28.08	41.98	41.70
M <sub>LH</sub>	31.20	28.08	37.06	36.99
BEAM				
M <sub>IJ</sub>	81.60	73.44	102.28	102.36
M <sub>JI</sub>	81.60	73.44	77.85	76.72
M <sub>JK</sub>	81.60	97.92	53.42	55.15
M <sub>KJ</sub>	81.60	97.92	53.42	55.15
M <sub>KL</sub>	81.60	73.44	77.85	76.29
M <sub>LK</sub>	81.60	73.44	102.28	101.49
COLUMN				
M <sub>IM</sub>	50.40	45.36	65.22	65.72
M <sub>MI</sub>	50.40	45.36	76.33	77.05
M <sub>JN</sub>	100.80	105.84	78.07	78.36
M <sub>NJ</sub>	100.80	105.84	82.76	83.02
M <sub>KO</sub>	100.80	105.84	78.07	77.88
M <sub>OK</sub>	100.80	105.84	82.76	82.54
M <sub>LP</sub>	50.40	45.36	65.22	64.51
M <sub>PL</sub>	50.40	45.36	76.33	75.73
<b>SHEAR(KN)</b>				
BEAM				
V <sub>AB</sub>	4.80	4.32	5.94	5.49
V <sub>BC</sub>	4.80	5.76	3.53	4.36
V <sub>CD</sub>	4.80	4.32	5.92	5.51
COLUMN				
V <sub>AE</sub>	8.00	7.20	10.15	9.81
V <sub>BF</sub>	16.00	16.80	13.89	14.14
V <sub>CG</sub>	16.00	16.80	13.89	14.18
V <sub>DH</sub>	8.00	7.20	10.15	9.87
BEAM				
V <sub>EF</sub>	15.20	13.68	17.31	16.83
V <sub>FG</sub>	15.20	18.24	11.09	11.97
V <sub>GH</sub>	15.20	13.68	17.31	16.90

COLUMN				
V <sub>EI</sub>	17.33	15.60	21.98	21.71
V <sub>FJ</sub>	34.67	36.40	30.03	30.20
V <sub>GH</sub>	34.67	36.40	30.03	30.23
V <sub>HL</sub>	17.33	15.60	21.98	21.86
BEAM				
V <sub>IJ</sub>	27.20	24.48	29.99	29.85
V <sub>JK</sub>	27.20	32.64	17.81	18.38
V <sub>KL</sub>	27.20	24.48	30.06	29.63
COLUMN				
V <sub>IM</sub>	28.00	25.20	39.53	39.66
V <sub>JN</sub>	56.00	58.80	44.61	44.83
V <sub>KO</sub>	56.00	58.80	44.61	44.56
V <sub>LP</sub>	28.00	25.20	39.53	38.95
AXIAL FORCE				
BEAM				
F <sub>AB</sub>	40.00	40.80	37.85	38.18
F <sub>BC</sub>	24.00	24.00	23.96	24.04
F <sub>CD</sub>	8.00	-0.72	10.07	9.87
COLUMN				
F <sub>AE</sub>	-4.80	-4.32	-5.94	-5.49
F <sub>BF</sub>	0.00	-1.44	2.42	1.12
F <sub>CG</sub>	0.00	1.44	-2.4	-1.15
F <sub>DH</sub>	4.80	4.32	5.92	5.52
BEAM				
F <sub>EF</sub>	46.67	47.60	44.17	44.11
F <sub>FG</sub>	28.00	28.00	28.03	28.04
F <sub>GH</sub>	9.33	8.40	11.89	11.99
COLUMN				
F <sub>EI</sub>	-20.00	-18.00	-23.25	-22.32
F <sub>FJ</sub>	0.00	6.00	-8.64	5.98
F <sub>GK</sub>	0.00	6.00	8.64	-6.08
F <sub>HL</sub>	20.00	18.00	23.29	22.42

BEAM				
F <sub>IJ</sub>	53.33	54.40	46.45	46.64
F <sub>JK</sub>	32.00	32.00	31.87	31.42
F <sub>KL</sub>	10.67	9.60	17.29	17.09
F <sub>IM</sub>	-47.20	-42.28	-53.28	-52.16
F <sub>JH</sub>	0.00	-14.16	20.82	17.44
F <sub>KO</sub>	0.00	14.16	20.86	-17.32
F <sub>LP</sub>	47.20	42.48	53.35	52.05

### 5.3 Discussion of Results

For the frame A difference in each value of the result has been compared with the exact value. This variation has been calculated in the percentage difference giving the (+)ve sign for over estimated and (-)ve sign for under estimated values as tabulated in table 5.3.1.

#### PERCENTAGE VARIATION OF THE RESULTS

TABLE 5.3.1  
(BEAM MOMENT)

Methods	Storey	Near end		Far end	
		ext.	int.	int.	ext.
Portal	T O P	-18.58	+15.96	+8.84	-23.23
Canti.		-15.10	+20.91	-2.33	-31.10
Kani.		+0.07	-0.71	+1.02	-0.07
Bowm.		-2.30	+13.78	+24.56	-15.35
Factor		+4.11	-17.15	-14.14	+1.77
Klouc.		-10.23	-3.17	+4.28	-3.02
Blum.		-3.06	-9.81	-15.35	-5.51

## (BEAM MOMENTS)

Methods	Storey	Near end		Far end	
		ext.	int.	int.	ext.
Portal	B O T T O M	-7.59	+35.71	+26.59	-11.83
Canti.		+1.85	+49.58	+13.62	-20.86
Kani.		-0.40	-0.26	+0.55	+0.26
Bowm.		-24.12	-8.79	-7.34	-20.88
Factor		+9.54	-10.67	-8.64	+8.70
Klouc.		-9.49	-1.69	+5.59	-0.57
Blum.		+14.58	-2.54	-9.09	+12.54

## (COLUMN MOMENTS)

Methods	Storey	ext.		middle		ext.	
		top	bottom	top	bottom	top	bottom
Portal	T O P	-18.58	+6.95	+12.28	+17.18	-23.23	-4.41
Canti.		-10.09	+18.10	+12.38	+17.28	-31.10	-14.22
Kani.		+0.07	+0.37	+0.14	+0.05	-0.07	-0.65
Bowm.		-2.30	-30.90	+45.97	-17.98	+7.48	-27.94
Factor		+4.11	+27.88	-15.60	-15.37	+1.77	+19.93
Klouc.		+2.94	+7.31	+6.57	-7.91	-3.28	-4.00
Blum.		-3.06	+29.34	-13.10	-7.91	-5.55	+19.44

## (COLUMN MOMENTS)

Methods	Storey	ext.		middle		ext.	
		top	bottom	top	bottom	top	bottom
Portal	B O T T O M	-13.07	-26.64	+38.58	+33.12	-15.07	-26.77
Canti.		-4.28	-19.21	+38.49	+33.04	-23.60	-34.26
Kani.		-0.72	-0.79	+0.22	+0.13	+0.40	+0.49
Bowm.		-4.31	-0.70	-7.62	+33.10	-18.13	+5.90
Factor		+2.62	+1.28	-6.46	-2.48	+3.84	+2.64
Klouc.		+1.83	-0.67	-3.16	-2.35	-0.51	-0.84
Blum.		+9.00	-4.19	-4.98	-4.91	+9.47	-1.69



## (SHEAR IN BEAMS)

Methods	Storey	Near	Far
Portal	T O P	-4.30	-9.93
Canti.		+5.73	-19.17
Kani.		-0.29	+0.46
Bowm.		+4.30	+4.55
Factor		-4.87	-4.62
Klouc.		-7.16	0.00
Blum.		-5.73	-9.70

## (SHEAR IN BEAMS)

Methods	Storey	Near	Far
Portal	B O T T O M	+9.91	+3.97
Canti.		+21.17	-6.68
Kani.		-0.42	+0.42
Bowm.		-18.04	-15.22
Factor		+1.36	+1.94
Klouc.		-19.81	+5.33
Blum.		+7.92	+3.89

## (SHEAR IN COLUMNS)

Methods	Storey	ext.	middle	ext.
Portal	T O P	-7.54	+14.64	-14.81
Canti.		+1.99	+14.64	-23.59
Kani.		+0.28	0.00	-0.13
Bowm.		-14.65	+14.64	-8.26
Factor		+14.37	-12.70	+5.37
Klouc.		+11.38	-8.64	+2.75
Blum.		+11.38	-11.11	+6.07

## (SHEAR IN COLUMNS)

Methods	Storey	ext.	middle	ext.
Portal	B O T T O M	-20.41	+35.79	-21.26
Canti.		-13.59	+35.74	-29.30
Kani.		-0.74	+0.15	+0.56
Bowm.		-10.23	+13.14	-5.12
Factor		+1.93	-4.66	+3.26
Klouc.		-1.31	+0.48	-2.36
Blum.		+2.05	-4.75	+3.77

## (AXIAL FORCE IN BEAMS)

Methods	Storey	Near	Far
Portal	T O P	+2.79	-14.81
Canti.		-0.21	-23.59
Kani.		-0.11	-0.26
Bowm.		+5.43	-8.26
Factor		-5.32	+5.64
Klouc.		-4.22	+2.36
Blum.		-4.22	+6.03

## (AXIAL FORCE IN BEAMS)

Methods	Storey	Near	Far
Portal	B O T T O M	+15.74	-26.11
Canti.		+11.88	-33.41
Kani.		+0.77	+1.28
Bowm.		+3.96	-2.66
Factor		+3.45	+1.87
Klouc.		+5.30	-6.21
Blum.		+2.26	+2.07



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## (AXIAL FORCE IN COLUMNS)

Methods	Storey	Near	middle	Far.
Portal	T O P	-4.30	-33.33	-9.93
Canti.		+4.30	-122.62	-19.17
Kani.		-0.29	+3.57	+0.46
Bowm.		+4.30	+57.14	+14.55
Factor		-4.87	-7.14	-4.62
Klouc.		-7.16	+33.33	0.00
Blum.		-5.73	-29.76	-9.70

## (AXIAL FORCE IN COLUMNS)

Methods	Storey	Near	Middle	Far.
Portal	B O T T O M	+6.12	-24.68	+0.25
Canti.		+17.05	-125.00	-10.02
Kani.		-0.38	+3.90	+0.43
Bowm.		-12.08	+16.56	-7.24
Factor		-0.31	+1.30	+0.19
Klouc.		+6.50	-6.17	+3.90
Blum.		+4.28	-17.53	+0.25

Similarly for other frames these calculated values are in the percentage difference with the exact values giving the underestimation and overestimation as (+)ve and (-)ve sign respectively shown in Table 5.3.2.

FRAME 4

PERCENTAGE DIFFERENCE OF TOP BEAM MOMENTS  
BEAM

	ext	int
1.	10.66	-1.28
2.	10.66	-1.28
3.	-0.13	-0.53

FRAME 4

PERCENTAGE DIFFERENCE OF MIDDLE BEAM MOMENTS  
Near End Far End

	ext1	int1	int1	int2
1.	28.24	8.16	-40.36	0.22
2.	-16.90	-45.86	11.87	37.17
3.	-0.99	3.32	-3.11	-0.88

FRAME 4

PERCENTAGE DIFFERENCE OF BOTTOM BEAM MOMENTS

	Near End		Middle		Far End	
	ext1	int1	int1	int2	int2	ext2
1.	31.06	24.98	37.94	34.71	-39.54	-4.47
2.	-40.05	-52.39	-32.15	-39.03	41.64	56.31
3.	0.92	0.12	-35.58	1.74	-1.82	-0.89

FRAME 5

PERCENTAGE DIFFERENCE OF TOP BEAM MOMENTS

	Near End		Middle		Far End	
	ext1	int1	int1	int2	int2	ext2
1.	26.53	-7.95	-9.92	-9.92	-7.46	26.94
2.	33.88	2.85	-31.91	-31.91	3.28	34.25
3.	-3.06	-15.44	19.08	19.08	-14.93	-2.49

FRAME 5

PERCENTAGE DIFFERENCE OF MIDDLE BEAM MOMENTS

	Near End		Middle		Far End	
	ext1	int1	int1	int2	int2	ext2
1.	20.34	-4.20	-26.95	-26.98	-3.90	20.74
2.	28.30	6.22	-52.34	-52.38	6.49	28.66
3.	-1.62	-4.48	7.41	7.38	-4.15	-1.11

FRAME 5

PERCENTAGE DIFFERENCE OF BOTTOM BEAM MOMENTS

	Near End		Middle		Far End	
	ext1	int1	int1	int2	int2	ext2
1.	20.28	-6.36	-47.96	-47.96	-6.96	19.60
2.	28.25	4.28	-77.55	-77.55	3.74	27.64
3.	0.08	-1.47	3.14	3.14	-2.04	-0.78

FRAME 1

PERCENTAGE DIFFERENCE OF COLUMN MOMENTS

	Near End		due near		due far		Far End	
	top	bottom	top	bottom	top	bottom	top	bottom
1.	27.60	35.05	-28.29	-24.30	-30.43	-26.32	24.37	32.08
2.	34.84	41.54	-34.70	-30.51	-36.96	-32.63	31.93	38.87
3.	2.36	2.53	0.58	0.47	-1.09	-1.14	-2.00	-1.93

FRAME 2

PERCENTAGE DIFFERENCE OF TOP COLUMN MOMENTS

	Near End		Far End	
	top	bottom	top	bottom
1.	9.82	8.54	0.22	-25.82
2.	9.82	8.54	0.22	-25.82
3.	-0.27	0.41	-0.37	0.28

FRAME 2

PERCENTAGE DIFFERENCE OF BOTTOM COLUMN MOMENTS

	Near End		due near		due far		Far End	
	top	bottom	top	bottom	top	bottom	top	bottom
1.	27.87	35.42	-24.91	-21.74	-33.63	-26.25	21.20	31.81
2.	6.06	15.90	-89.84	-85.01	-13.36	-7.10	103.17	102.75
3.	1.51	1.64	0.43	0.30	-0.66	-0.63	-1.57	-1.25

FRAME 3

PERCENTAGE DIFFERENCE OF TOP COLUMN MOMENTS

	Near End		Middle		Far End	
	top	bottom	top	bottom	top	bottom
1.	26.12	25.89	-23.23	-32.54	15.89	-2.35
2.	26.12	25.89	-23.23	-32.54	15.89	-2.35
3.	0.41	0.46	-0.26	-0.09	-0.53	0.00

FRAME 3

PERCENTAGE DIFFERENCE OF BOTTOM COLUMN MOMENTS

	Near End		due near		due far		Far End	
	top	bottom	top	bottom	top	bottom	top	bottom
1.	30.18	36.49	-32.15	-24.58	34.21	37.58	-56.57	-35.34
2.	-9.64	0.27	-68.29	-58.66	5.87	34.90	64.49	69.31
3.	1.25	1.31	0.38	0.29	-0.59	-0.58	-1.18	-1.05

FRAME 4

PERCENTAGE DIFFERENCE OF TOP COLUMN MOMENTS

	Near End		Far End	
	top	bottom	top	bottom
1.	10.71	11.59	-1.28	-31.20
2.	10.71	11.59	-1.28	-19.57
3.	-0.07	0.92	-0.53	-0.29

FRAME 4

PERCENTAGE DIFFERENCE OF MIDDLE COLUMN MOMENTS

	Near End		Middle		Far End	
	top	bottom	top	bottom	top	bottom
1.	27.78	27.76	-28.32	-31.59	12.54	2.55
2.	-16.27	-16.30	-28.32	-33.12	63.99	59.87
3.	0.91	0.82	-0.31	-0.09	-1.07	-17.58

FRAME 4

PERCENTAGE DIFFERENCE OF COLUMN MOMENTS

	Near End		due near		due far		Far End	
	top	bottom	top	bottom	top	bottom	top	bottom
1.	31.06	36.99	-33.50	-24.65	-31.61	-24.16	20.25	32.21
2.	40.05	-28.01	-61.92	-51.19	36.26	39.87	54.20	61.07
3.	0.92	0.92	0.38	0.28	-0.46	-0.39	-1.04	-0.82

FRAME 5

PERCENTAGE DIFFERENCE OF TOP COLUMN MOMENTS

	Near End		due near		due far		Far End	
	top	bottom	top	bottom	top	bottom	top	bottom
1.	26.53	8.46	-8.88	-17.74	-8.64	-17.46	26.94	8.98
2.	33.88	17.61	-14.33	-23.63	-14.07	-23.33	34.25	18.08
3.	-3.06	-2.92	1.70	1.84	1.92	2.08	-2.49	-21.30

FRAME 5

PERCENTAGE DIFFERENCE OF MIDDLE COLUMN MOMENTS

	Near End		due near		due far		Far End	
	top	bottom	top	bottom	top	bottom	top	bottom
1.	24.84	14.85	-19.49	-16.61	-12.88	-16.48	25.18	15.65
2.	32.35	23.36	-25.47	-22.44	-18.52	-22.31	32.66	24.09
3.	-1.13	-1.15	-5.25	0.58	0.58	0.69	-0.67	-0.19

FRAME 5

PERCENTAGE DIFFERENCE OF BOTTOM COLUMN MOMENTS

	Near End		due near		due far		Far End	
	top	bottom	top	bottom	top	bottom	top	bottom
1.	23.31	34.59	-28.64	-21.42	-29.43	-22.12	21.87	33.45
2.	30.98	41.13	-35.07	-27.49	-35.90	-28.23	29.69	40.10
3.	0.76	0.93	0.36	0.31	-0.24	-0.27	-1.10	-0.81

PERCENTAGE DIFFERENCE OF THE VALUES OBTAINED BY VARIOUS

METHODS WITH RESPECT TO EXACT VALUES

Shear in Beams

FRAME 1

Method	Storey	Near	Middle	Far
PORTAL		14.89	-69.0	11.60
CANTILEVER		23.40	-102.82	1.41
KANI		1.59	11.36	-2.40

FRAME 2

PORTAL	T		5.26	
CANTILEVER	O		5.26	
KANI	P		-0.42	
PORTAL	B	13.436	30.60	-5.065
CANTILEVER	O	-12.79	-126.90	13.41
KANI	T	0.645	22.73	-1.31

FRAME 3

PORTAL	T	9.9		-3.0
CANTILEVER	O	9.9		-3.0
KANI	P	1.125		-1.5
PORTAL	B	25.15	-54.26	2.40
CANTILEVER	O	-17.5	-52.07	36.63
KANI	T	0.7833	0.922	-1.311

FRAME 4

PORTAL	T		5.06	
CANTILEVER	O		5.06	
KANI	P		-0.42	
PORTAL	M	19.13		-16.60
CANTILEVER	I	-28.44		-26.58
KANI	D	2.11		-1.82
PORTAL	B	28.13	-36.36	-15.13
CANTILEVER	O	-46.00	-35.52	50.03
KANI	T	0.34	1.50	-1.36

FRAME 5

PORTAL	T	12.56	-10.10	-13.04
CANTILEVER	O	21.31	-32.11	21.74
KANI	P	-8.20	19.04	-7.24
PORTAL	M	9.68	-27.00	10.00
CANTILEVER	I	18.71	-52.38	19.65
KANI	D	-2.85	7.35	-2.42
PORTAL	B	8.877	-48.00	8.20
CANTILEVER	O	17.98	-77.60	17.38
KANI	T	0.47	3.10	-1.45

Axial Forces in Beams

FRAME 1

Method	Storey	Near	Middle	Far
PORTAL	T	-10.13	- 1.48	28.44
CANTILEVER	O	-12.33	- 1.48	35.60
KANI	P	-0.716	- 1.14	-1.61

FRAME 2

PORTAL	T		-11.27	
CANTILEVER	O		-11.27	
KANI	P		- 0.148	
PORTAL	B	-14.70	- 5.8	57.26
CANTILEVER	O	-5.20	-80.00	6.908
KANI	T	-0.702	- 2.10	-2.90



FRAME 3

PORTAL	T	-13.26		7.62
CANTILEVER	O	-13.26		7.62
KANI	P	- 0.1415		-3.46

PORTAL	B	-22.20	8.93	42.69
CANTILEVER	O	2.90	85.66	113.53
KANI	T	-1.01	- 2.00	-2.40

FRAME 4

PORTAL	T		-14.33	
CANTILEVER	O		-14.37	
KANI	P		- 0.46	

PORTAL	M	-26.77		27.90
CANTILEVER	I	4.12		130.95
KANI	D	- 0.93		-2.06

PORTAL	B	-33.16	20.48	52.69
CANTILEVER	O	32.69	120.93	52.19
KANI	T	-0.99	- 1.28	-1.29

FRAME 5

PORTAL	T	- 4.77	0.17	18.95
CANTILEVER	O	- 6.86	0.17	92.71
KANI	P	0.86	0.33	-2.03

PORTAL	M	-5.80	0.14	22.00
CANTILEVER	I	-7.91	0.14	29.94
KANI	D	-0.13	0.0356	1.25

PORTAL	B	-14.35	-1.845	37.56
CANTILEVER	O	-16.63	-1.845	43.82
KANI	T	2.55	-1.43	-1.17

Shear in Column

FRAME 1

	EXT1.	INT1	INT2	EXT2
PORTAL	31.50	-26.28	-28.31	28.44
CANTILEVER	38.35	-32.60	-28.31	35.60
KANI	2.22	0.473	-1.122	- 2.147

FRAME 2

PORTAL	T	9.2			-11.27
CANTILEVER	O	9.2			-11.27
KANI	P	0.121			- 0.148

PORTAL	B	31.04	-23.30	-29.88	26.90
CANTILEVER	O	11.23	-87.36	-10.17	102.94
KANI	T	1.46	0.622	-0.7	- 1.57

FRAME 3

PORTAL	T	26.00	-27.80		7.62
CANTILEVER	O	26.00	-27.80		7.62
KANI	P	3.05	-0.08		- 0.346

PORTAL	B	33.47	-28.30		-28.17
CANTILEVER	O	-4.457	-63.38		8.32
KANI	T	1.532	0.537		- 0.76

FRAME 4

PORTAL	T	11.14			-14.33
CANTILEVER	O	11.14			-14.33
KANI	P	0.36			- 0.61

PORTAL	M	27.76	-30.70		7.81
CANTILEVER	I	-14.73	-30.70		62.04
KANI	D	0.91	0.05		- 0.94

PORTAL	B	34.16	-28.91	-27.78	26.69
CANTILEVER	O	-33.75	-56.35	38.12	57.90
KANI	T	1.00	0.48	-0.32	- 0.71

FRAME 5

PORTAL	T	18.45	-13.15	-12.835	18.94
CANTILEVER	O	26.60	-18.81	-18.476	27.05
KANI	P	-3.46	1.77	-2.045	- 2.836

PORTAL	M	20.17	-14.80	-14.68	20.72
CANTILEVER	I	28.14	-20.53	-20.41	28.63
KANI	D	-0.414	0.563	0.6616	- 0.549

PORTAL	B	29.40	-24.91	-11.44	28.11
CANTILEVER	O	36.46	-31.16	-31.95	35.30
KANI	T	0.32	0.49	-0.1122	- 1.49

Axial Force in Column

		EXT1.	INT1	INT2	EXT2
FRAME 1					
PORTAL	T	14.90	100.00	100.00	11.60
CANTILEVER	O	23.40	151.60	156.25	20.44
KANI	P	1.60	1.43	7.812	-2.40
FRAME 2					
PORTAL	T	5.26			5.26
CANTILEVER	O	5.26			5.26
KANI	P	-0.42			-0.42
PORTAL	B	13.43	100.00	12.45	-1.88
CANTILEVER	O	-12.80	211.50	128.30	43.35
KANI	T	0.646	-3.45	-3.00	-1.04
FRAME 3					
PORTAL	T	9.90	100.00		-3.00
CANTILEVER	O	9.90	100.00		-3.00
KANI	P	1.12	225.14		-1.50
PORTAL	B	25.15	-280.50	100.00	1.033
CANTILEVER	O	-17.50	-78.57	202.74	26.60
KANI	T	0.7833	2.00	-8.90	-1.36
FRAME 4					
PORTAL	T	5.46			5.06
CANTILEVER	O	5.46			5.06
KANI	P	0.08			-0.42
PORTAL	M	19.13	-57.90		-9.76
CANTILEVER	I	-28.43	100.00		19.91
KANI	D	2.11	-2.63		0.40
PORTAL	B	28.13	30.13	-86.72	-93.38
CANTILEVER	O	-45.00	-3.49	159.00	36.02
KANI	T	0.34	4.80	-4.81	-1.23
FRAME 5					
PORTAL	T	12.57	100.00	100.00	13.04
CANTILEVER	O	21.31	228.57	225.22	21.74
KANI	P	8.19	-116.07	108.70	7.25
PORTAL	M	10.39	100.00	100.00	10.80
CANTILEVER	I	19.35	-5.633	198.60	19.71
KANI	D	-4.167	252.10	242.10	-3.88

PORTAL	B	9.31	100.00	100.00	9.31
CANTILEVER	O	18.55	181.20	181.70	18.386
KANI	T	-2.07	-19.38	220.43	-2.497

We can very well point out some comparative points after studying the variation of values of frame A obtained by different methods w.r.t. exact values.

In frame A by the portal method, the top storey beam moments and the bottom storey beam moments are underestimated at the exterior, and are overestimated at interior joints. Moments at the top and the bottom of the outer column are underestimated in each storey except at the bottom end of top storey near end column. The moments are overestimated in middle column. The moments are overestimated by a larger amount in the bottom storey middle column as compared to the top storey middle column.

#### CANTILEVER METHOD

The top and the bottom storey beam moments are found underestimated at the exterior and the far ends except in the bottom storey exterior column but amount of error is very less.

The interior near and far end moments are overestimated in both storeys except in the upper storey far end beams but this error is very less.

The exterior top and column end moments are underestimated in both storeys. The internal columns top and bottom end moments are overestimated.

#### KANI'S METHOD

The interior near end beam moments in both the storeys are underestimated and interior far end beam moments in both the storeys are overestimated. The exterior near and far end beam moments are overestimated and underestimated respectively in top storey and vice-versa in bottom storey.

The interior columns top and bottom end moments are overestimated in both the storeys. The exterior near end and far end column moments are overestimated and underestimated

respectively in top storey and vice-versa in bottom storey.

#### **BOWMAN'S METHOD**

The exterior near and far end beam moments are underestimated in both the storeys. The interior near and far end beam moments are overestimated in top storey and underestimated in bottom storey.

The external near end column moments are underestimated in both the storeys. The middle and exterior far end column moments are overestimated at top end and underestimated at bottom end in the top storey and vice-versa in bottom storey.

#### **FACTOR METHOD**

The exterior near end and far end and interior near and far end beam moments are overestimated and underestimated respectively in both the storeys.

The exterior and interior column end moments are overestimated and underestimated respectively in both the storeys.

#### **KLOUCEK'S METHOD**

All the beam moments are underestimated except the interior far end beam moments in both the storeys.

The exterior near end column moments are overestimated except the bottom end of the bottom storey. Middle and exterior far end column moments are underestimated.

#### **BLUME'et al's METHOD**

The exterior near and far end beam moments are underestimated and overestimated in top and bottom storey respectively. The interior near and far end beam moments are underestimated in both the storeys.

The exterior near and far end column moments are underestimated and overestimated in top storey and vice-versa in bottom storey. The middle column end moments are underestimated in both the storeys.

The results from the calculation of shear forces are discussed as below:

#### **PORTAL METHOD**

Shears are underestimated in top storey beams and overestimated in bottom storey beams.

Shears are overestimated in middle columns and underestimated in exterior columns in both the storeys.

#### **CANTILEVER METHOD**

Shears in near and far end are overestimated and underestimated respectively in both the storeys.

Interior column shear is overestimated and exterior column shears are underestimated except in the top storey exterior near end column but the overestimated value is very less.

#### **KANI'S METHOD**

Near end beam shears are underestimated and far end beam shears are overestimated in both the storeys.

The exterior near and far end column shears are overestimated and underestimated respectively in top storey and vice-versa in bottom storey.

#### **BOWMAN'S METHOD**

In the top storey, the shears are overestimated and in bottom storey the shears are underestimated.

Shears in the exterior column are underestimated while these are overestimated in the interior column in both the storeys.

#### **FACTOR METHOD**

Shears in upper storey beams are underestimated and the shears in bottom storey beams are overestimated.

The exterior column shears are overestimated and interior shear is underestimated in both the storeys.

#### **KLOUCEK'S METHOD**

The near end beam shears are underestimated and far end beam moments are overestimated.

In the upper storey, the exterior column shears are overestimated and interior column shears are underestimated in the bottom storey.

#### BLUME'S METHODS

The top storey beam shears are underestimated and bottom storey beam shears are overestimated.

The external column shears are overestimated and the internal column shears are underestimated in both the storeys.

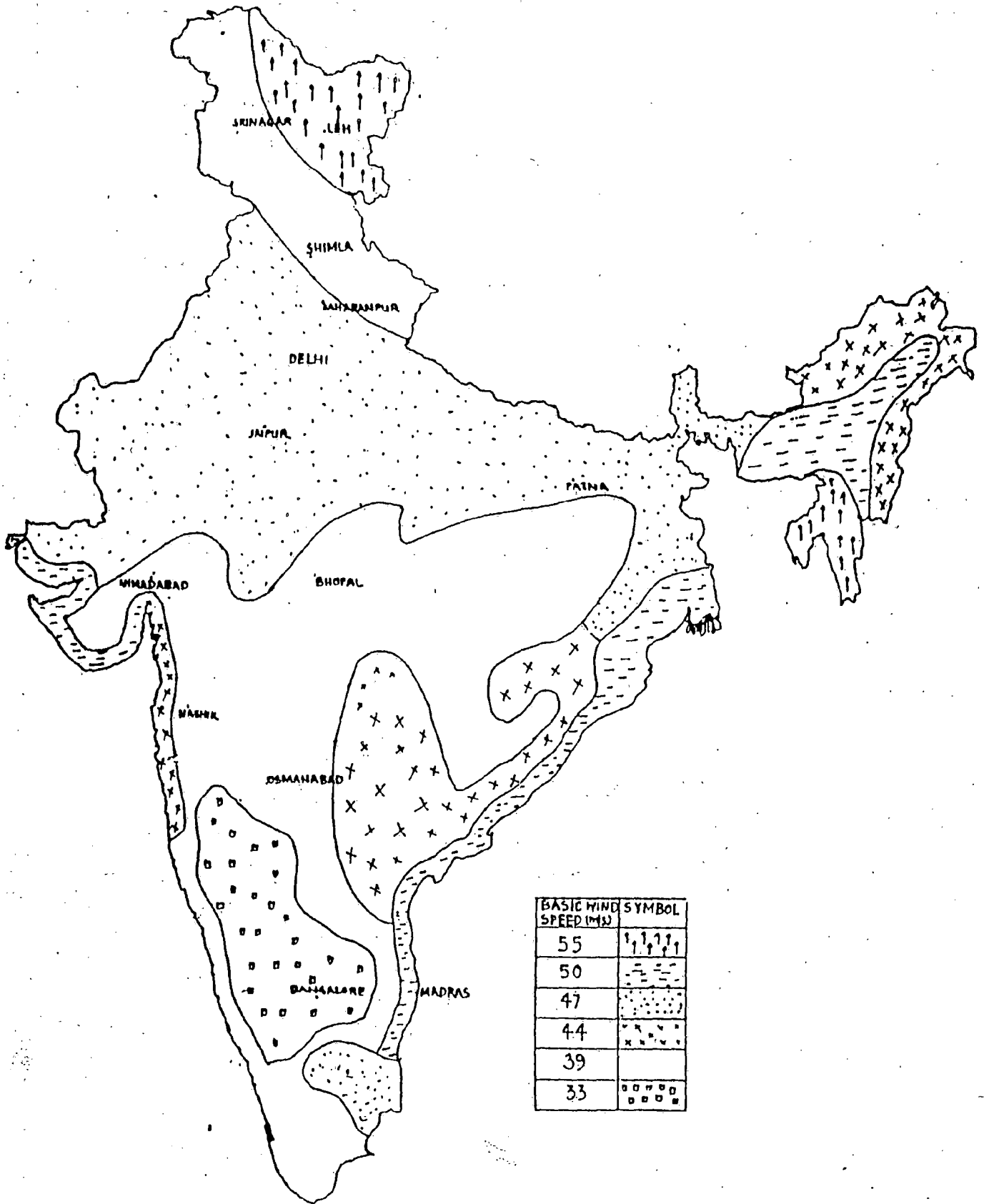
The maximum error in the axial forces in beam is about 20 to 35% while the error in the axial forces in columns are very unrealistic by cantilever method only, specifically in the middle column otherwise the maximum error in the axial force is also about 20 to 30% by approximate method.

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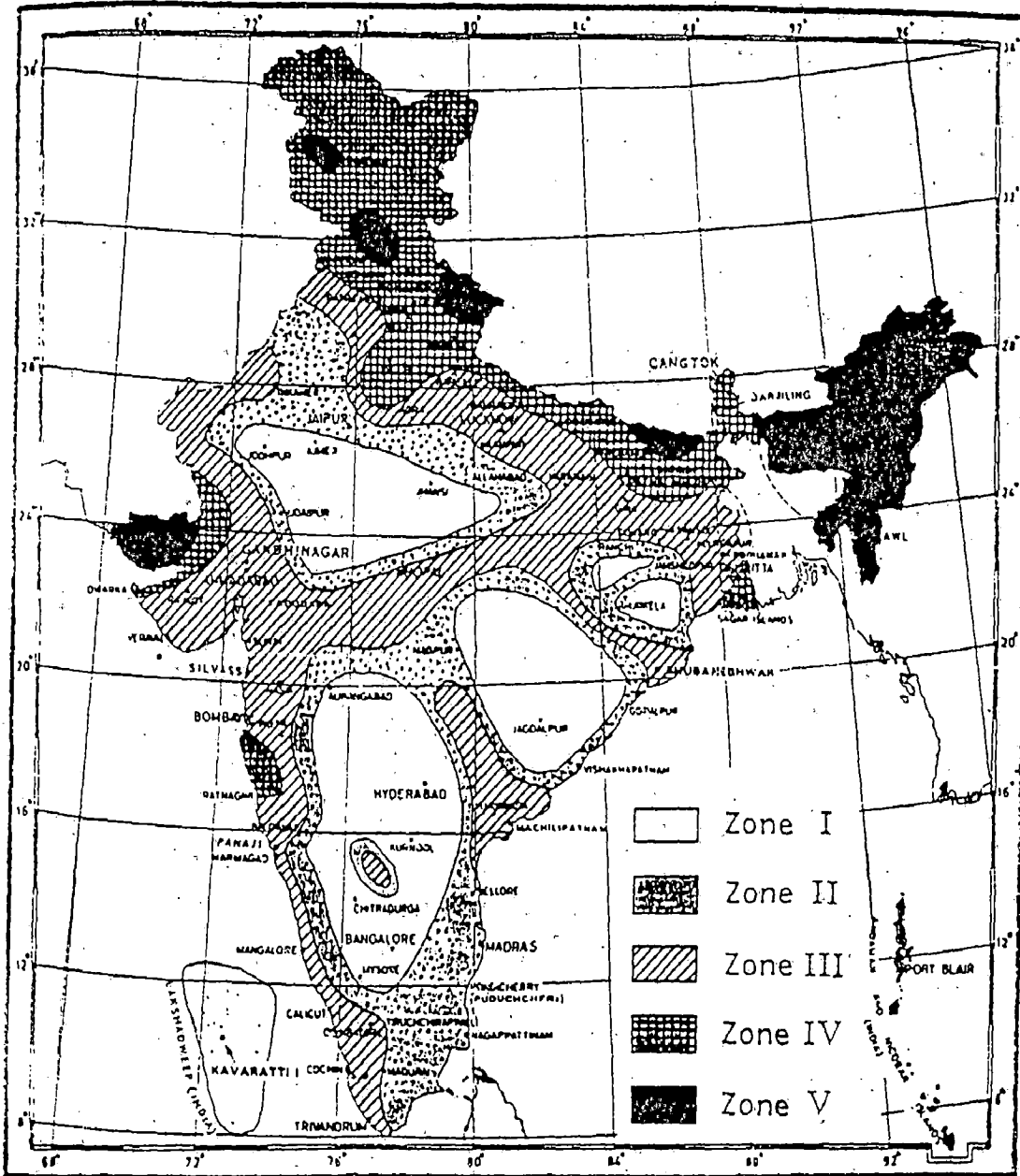
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APPENDIX-A



BASIC WIND SPEED (in m/s)

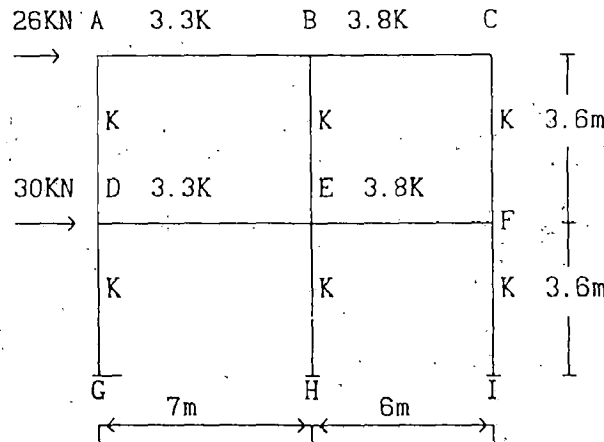


SEISMIC ZONING MAP OF INDIA

APPENDIX - B

Comprehensive Analysis of Symmetric Two Bay Double Storey Building Frame

The frame shown in figure is a double storey, double panel building frame. The lateral loads are 30 KN and 26 KN at the first floor level respectively and the upper roof level. Each storey height is 3.6 m, and the panel widths are 7 m and 6 m. The relative stiffness are shown in the figure. The size of all the columns are same as 35 cm x 35 cm and the size of the beams are 35 cm x 65 cm.



( Building frame )

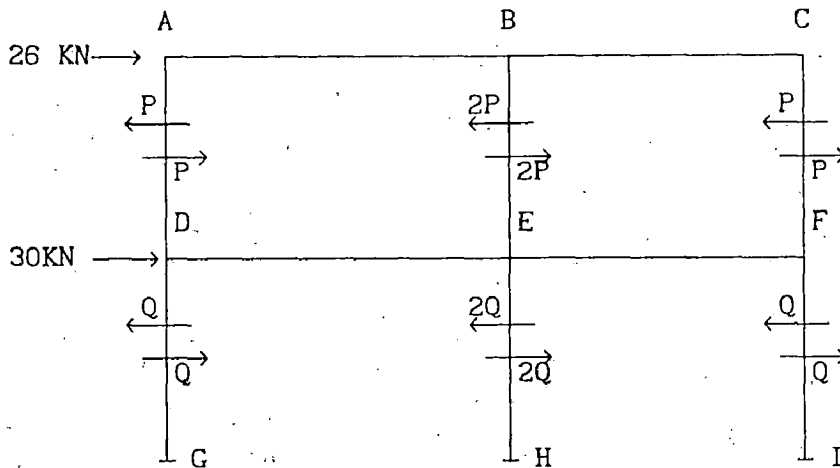
The above frame structure is analysed by different methods. Firstly the analysis is done for the seven approximate methods and then is done by stiffness matrix method i.e. exact method.

# 1. ANALYSIS OF FRAME BY PORTAL METHOD

## STEPS INVOLVED IN ANALYSIS :

The analysis steps are :

### (1a) Shear in Columns :



Top storey vertical members are AD, BE, & CF. Let the horizontal shear for the outer columns AD and CF be 'P' each, while that for the inner column BE be '2P'

The value 'P' is given by

$$26 = P + 2P + P$$

$$P = 6.5 \text{ KN} \quad \text{-----} \rightarrow (1)$$

Similarly considering the bottom storey, the exterior columns are DG and FI have shear 'Q', and the inner column EH has the shear 2Q, then

$$26 + 30 = Q + 2Q + Q$$

$$Q = 14.0 \text{ KN} \quad \text{-----} \rightarrow (2)$$

### (1b) Shear in Beams

$$F_{AB} = \frac{M_{AB}}{(7/2)} = 11.7/3.5 = 3.34 \text{ KN}$$

$$F_{BA} = 3.34 \text{ KN}$$

$$F_{BC} = 11.7/3 = 3.90 \text{ KN}$$

$$F_{CB} = 3.90 \text{ KN}$$

$$F_{DE} = 36.9/3.5 = 10.54 \text{ KN}$$

$$F_{ED} = 10.54 \text{ KN}$$

$$F_{EF} = 36.9/3 = 12.30 \text{ KN}$$

$$F_{FE} = 12.30 \text{ KN}$$

(2a) Moments at the ends of columns

top storey

$$\begin{aligned} M_{DA} = M_{AD} = M_{FC} = M_{CF} &= P * h/2 = 6.5 * 3.6/2 \\ &= 11.7 \text{ KN - m} \end{aligned}$$

$$\begin{aligned} M_{EB} = M_{BE} &= 2P * h/2 = 6.5 * 3.6 \\ &= 23.4 \text{ KN - m} \end{aligned}$$

bottom storey

$$\begin{aligned} M_{GD} = M_{DG} = M_{IF} = M_{FI} &= Q * h/2 = 14.0 * 3.6/2 \\ &= 25.2 \text{ KN - m} \end{aligned}$$

$$\begin{aligned} M_{HE} = M_{EH} &= 2Q * h/2 = 14.0 * 3.6 \\ &= 50.4 \text{ KN - m} \end{aligned}$$

(2b) Moments at the ends of beams

Top storey

$$\begin{aligned}M_{AB} = M_{BA} = M_{BC} = M_{CB} &= P \cdot h/2 = 6.5 \cdot 1.8 \\ &= 11.7 \text{ KN} - \text{m}\end{aligned}$$

First floor beam

$$\begin{aligned}M_{DE} = M_{ED} = M_{EF} = M_{FE} &= M_{DA} + M_{DG} \\ &= 11.7 + 25.2 \\ &= 36.9 \text{ KN} - \text{m}\end{aligned}$$

(4) Axial Force in Columns and Beams

$$\text{Axial force in column AD} = \text{Shear in AB} = 3.34 \text{ KN}$$

$$\begin{aligned}\text{Axial force in column DG} &= \text{Axial force in AD} \\ &+ \text{Shear in DE} \\ &= 3.34 + 10.54 = 13.88 \text{ KN}\end{aligned}$$

$$\begin{aligned}\text{Axial force in column BE} &= \text{Shear in BC} \\ &- \text{Shear in BA} \\ &= 3.90 - 3.34 = 0.56 \text{ KN}\end{aligned}$$

$$\begin{aligned}\text{Axial force in column EH} &= \text{Axial force in BE} \\ &+ \text{Shear ED} - \text{Shear in EF} \\ &= 0.56 + 10.54 - 12.30 \\ &= 1.2 \text{ KN}\end{aligned}$$

$$\text{Axial force in column CF} = \text{Shear in CB} = 3.9 \text{ KN}$$

$$\begin{aligned}\text{Axial force in column FI} &= \text{Axial force in column CF} \\ &+ \text{Shear in FE} \\ &= 3.90 + 12.30 = 16.20 \text{ KN}\end{aligned}$$

$$\text{Axial force in AB} = 26 - 6.5 = 19.5$$

$$\text{Axial force in BC} = 6.5$$

$$\text{Axial force in DE} = 22.5$$

$$\text{Axial force in EF} = 7.5$$

## 2. ANALYSIS OF BUILDING FRAME BY CANTILEVER METHOD

### STEPS INVOLVED IN ANALYSIS :

From the assumptions the points of contraflexure are shown :

#### (1) Location of the centroidal axis of the column

Let the centroidal axis be at a distance  $\bar{x}$  from the column line ADG. Taking moment of areas of the columns about ADG,

we get :

$$\bar{x} = \frac{7 + 13}{3} = 6.67$$

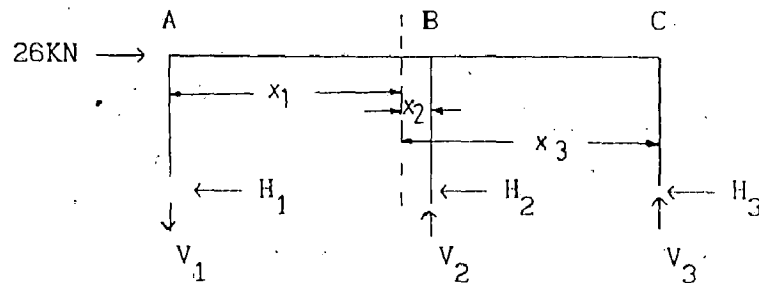
Therefore

$$x_1 = 6.67$$

$$x_2 = 7 - 6.67 = 0.33$$

$$x_3 = 6 + 0.33 = 6.33$$

#### (2)(a) Axial forces in columns of top storey

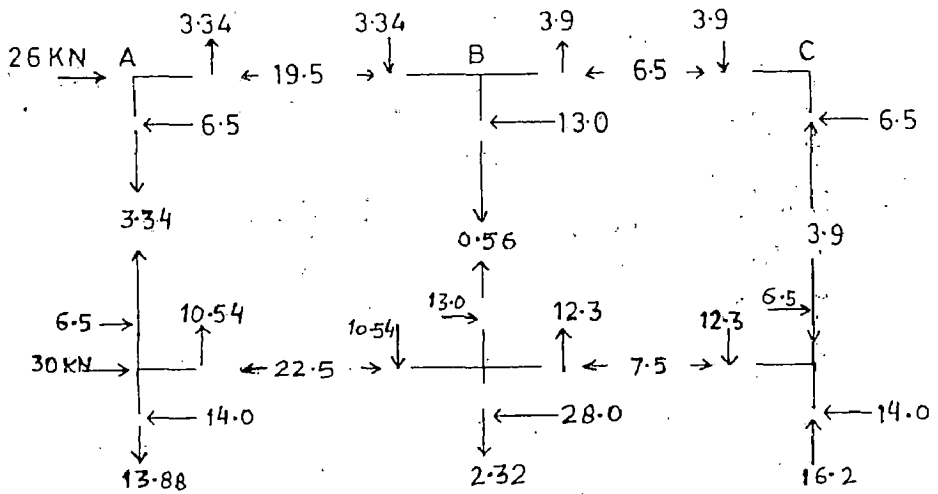


From the assumption (3) we can write the following relationship for the axial forces in the columns. Since the areas are equal, the axial forces in columns will be in proportion to their distances from the centroidal axis

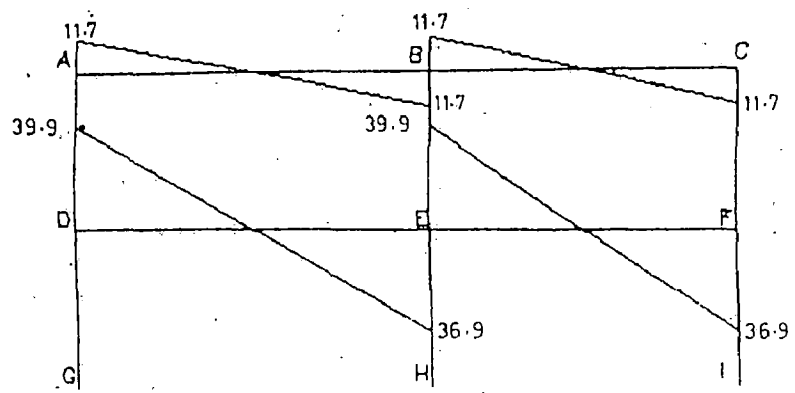
$$\frac{V_1}{x_1} = \frac{V_2}{x_2} = \frac{V_3}{x_3}$$

$$V_2 = V_1 \frac{x_1}{x_2} = V_1 \cdot \frac{0.33}{6.67} = 0.05 V_1$$

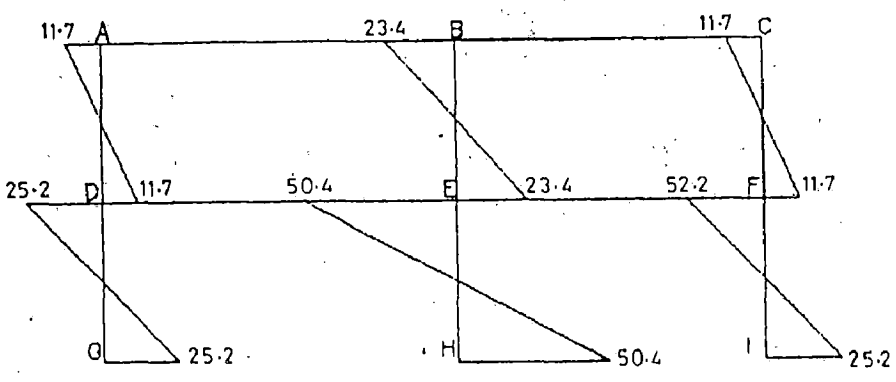




The bending moment diagrams are shown in figure.



B.M.D. FOR BEAMS



B.M.D. FOR COLUMNS

$$V_3 = V_1 \frac{x_3}{x_1} = V_1 \frac{6.33}{6.67} = 0.95 V_1$$

Taking the moment of all the forces about the point of contraflexure of the right hand column

$$26.0 * 1.8 + V_2 * 6.0 - V_1 * 13.0 = 0$$

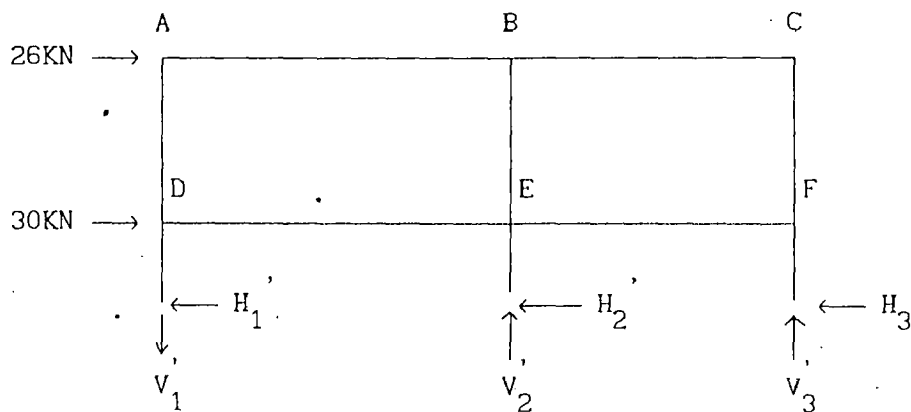
$$\text{or } 46.8 + 0.30 * V_1 - 13.0 * V_1 = 0$$

$$\text{or } V_1 = 3.69 \text{ KN (tensile)}$$

$$\text{Hence } V_2 = 0.185 \text{ KN or } 0.19 \text{ KN (Compressive)}$$

$$V_3 = 3.50 \text{ KN (Compressive)}$$

(2b) Axial force in bottom storey



$$\text{From } \frac{V_1'}{x_1} = \frac{V_2'}{x_2} = \frac{V_3'}{x_3}$$

$$V_2' = 0.05 V_1'$$

$$V_3' = 0.95 V_1'$$

Taking moment of all forces about the point of contraflexure of the right hand column

$$26.0 * 5.4 + 30.0 * 1.8 + V_2' * 6 - V_1' * 13 = 0$$

We get  $V_1' = 15.31 \text{ KN (Tensile)}$

$$V_2' = 0.77 \text{ KN (Compressive)}$$

$$V_3' = 14.54 \text{ KN (Compressive)}$$

### 3(a) Shear at the end of the columns

Top story: In order to determine  $H_1$ , the moment about the point of contraflexure between AB,

$$H_1 * 1.8 = V_1' * 3.5$$

$$H_1 = 7.17 \text{ KN}$$

similarly taking moment about point of contraflexure between BC

$$H_1 * 1.8 + H_2 * 1.8 + V_2' * 3 = V_1' * 10$$

$$H_2 = \frac{3.69 * 10 - 7.17 * 1.8 - 0.19 * 3}{1.8}$$

$$H_2 = 13.0 \text{ KN}$$

for  $H_3$ , taking moment about the point of contraflexure between BC,

$$H_3 * 1.8 = V_3' * 3$$

$$H_3 = 5.83 \text{ KN}$$

Bottom story: Taking moment about point of contraflexure between AB,

$$H_1 * 1.8 + 7.17 * 1.8 + 3.69 * 3.5 = 15.31 * 3.5$$

$$H_1 = 15.42$$

and for  $H_2$ ,

$$H_2 * 1.8 + 13.0 * 1.8 + 0.77 * 3 = 11.62 * 6.5 + 0.19 * 3$$

$$H_2 = 27.99 \text{ KN}$$

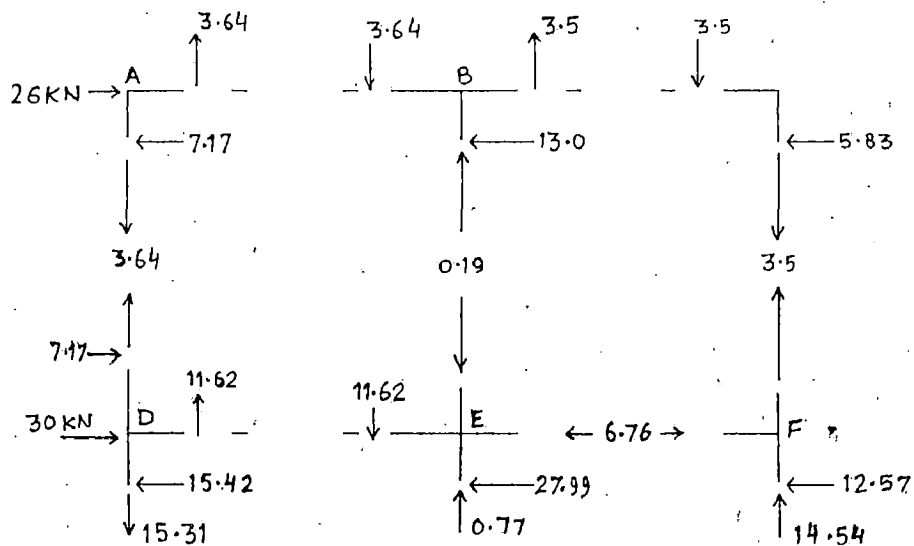
and for  $H_3$ ,

$$H_3 * 1.8 + 3.5 * 3 + 5.83 * 1.8 = 14.54 * 3$$

$$H_3 = 12.57 \text{ KN}$$

### (3)(b). Shears at the ends of beams

The shears at the ends of beams can be determined from the axial force in the columns at various joints as shown in figure :



### (4) Moments at the ends of beams

Since there is a point of contraflexure at the middle of each beam, the moment at the end of beam is formed by multiplying shear

at the end by half the length of the beam

(a) Top story

$$M_{AB} = M_{BA} = 3.69 * 3.5 = 12.92 \text{ KN-m}$$

$$M_{BC} = M_{CB} = 3.50 * 3.0 = 10.50 \text{ KN-m}$$

(b) Bottom story

$$M_{DE} = M_{ED} = 11.62 * 3.5 = 40.67 \text{ KN-m}$$

$$M_{EF} = M_{FE} = 11.04 * 3.0 = 33.12 \text{ KN-m}$$

(5) **Moments at the ends of columns**

(a) Top story

$$M_{AD} = M_{DA} = 12.92 \text{ KN-m}$$

$$M_{BE} = M_{BA} + M_{BC} = 12.92 + 10.5 = 23.42 \text{ KN-m}$$

$$M_{EB} = 23.42 \text{ KN-m}$$

$$M_{CF} = M_{CB} = 10.5 \text{ KN-m} = M_{FC}$$

(b) Bottom story

$$M_{DG} + M_{DA} = M_{DE}$$

$$M_{DG} = M_{DE} - M_{DA} = 40.67 - 12.92 = 27.75 \text{ KN-m}$$

$$M_{GD} = 27.75 \text{ KN-m}$$

$$M_{EH} + M_{EB} = M_{ED} + M_{EF}$$

$$M_{EH} = 40.67 + 33.12 - 23.42 = 50.37 \text{ KN-m}$$

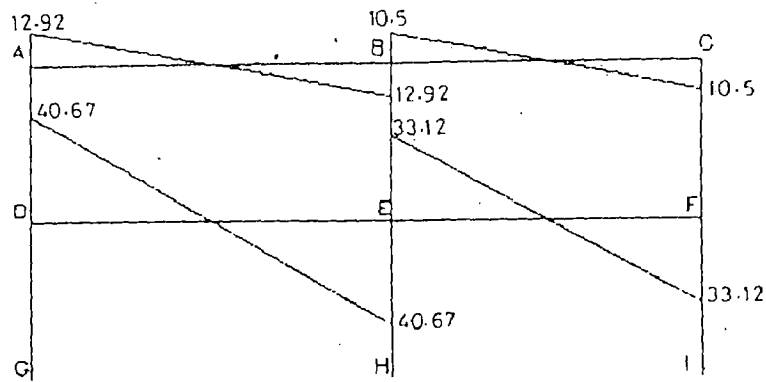
$$M_{HE} = 50.37 \text{ KN-m}$$

$$M_{FI} + M_{FG} = M_{FE}$$

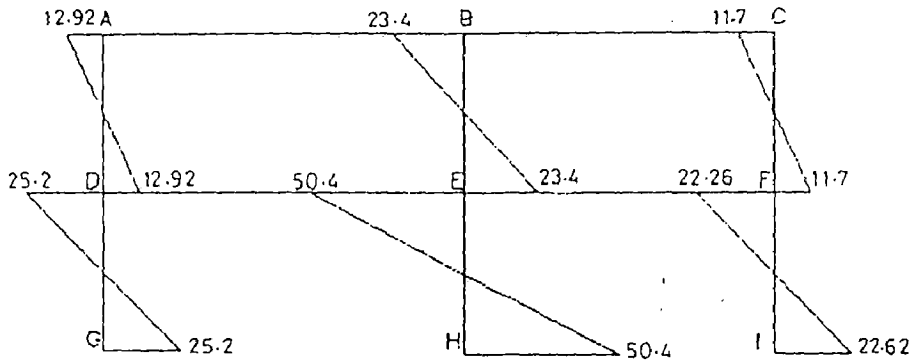
$$M_{FI} = 33.12 - 10.5 = 22.62 \text{ KN-m}$$

$$M_{IF} = 22.62 \text{ KN-m}$$

The bending moment diagram is shown below :



B.M.D. FOR BEAMS



B.M.D. FOR COLUMNS

### 3. ANALYSIS OF BUILDING FRAME BY KANI'S METHOD

#### STEPS INVOLVED IN ANALYSIS :

Displacement factor in each storey

$$\begin{aligned} D_{ij} &= \frac{K}{\Sigma K} \times (-3/2) \\ &= 1/3 \times -3/2 = -1/2 \end{aligned}$$

First Storey Moment

$$M = - \frac{26 \times 3.6}{3} = -31.2 \text{ KN-M}$$

Bottom Storey Moment

$$M = - \frac{56 \times 3.6}{3} = -67.2 \text{ KN-M}$$

These storey moments are shown entered in small boxes at the mid height of each storey to the left of the first column line.

The rotation moments are initially taken to be zero and using equation the translation moments are calculated.

$$M''_{ij} = D_{ij} \left\{ M_r^F + \sum_j (M'_{ij} + M'_{ji}) \right\}$$

where

$M''_{ij}$  → Translation moment of a column in a storey

$D_{ij}$  → Displacement factor of member ij

$M_r^F$  →  $\frac{Q_r \text{ hr}}{3}$  storey moment

and  $\sum (M'_{ij} + M'_{ji})$  represents the sum of the rotation moments for all the columns in rth storey.

Point	Member	Relative stiffness (K)	$\Sigma K$	Distribution factor	Rotational factor
A	AB	3.3	4.3	0.767	-0.384
	AD	1.0		0.233	-0.116
B	BA	3.3	8.1	0.407	-0.204
	BC	3.8		0.469	-0.234
	BE	1		0.124	-0.062
C	CB	3.8	4.8	0.792	-0.396
	CF	1		0.208	-0.104
D	DA	1	5.3	0.189	-0.095
	DE	3.3		0.622	-0.311
	DG	1		0.189	-0.096
E	EB	1	9.1	0.109	-0.055
	ED	3.3		0.364	-0.182
	EF	3.8		0.418	-0.209
	EH	1		0.109	-0.054
F	FC	1	5.8	0.172	-0.086
	FE	3.8		0.656	-0.328
	FI	1		0.172	-0.086

Top Storey :

$$M_{AD}'' = M_{BE}'' = M_{CF}'' = -1/2 \times (-31.2) = +15.6 \text{ KN-M}$$



Bottom Storey :

$$M_{DG} = M_{EH} = M_{FI} = -1/2 \times (-67.2) = +33.6 \text{ KN-M}$$

We shall now proceed to determine the rotation moments ( $M'_{ij}$ ):

Calculation for rotation moments:

Joint D

$$M_{DA} = M_{DG} = -0.095 \times (33.6 + 15.6) = -4.674$$

$$M_{DE} = -0.311 \times (33.6 + 15.6) = -15.301$$

Joint E

$$M_{EB} = M_{EH} = -0.055 \times (33.6 + 15.6 - 15.301) = -1.864$$

$$M_{ED} = -0.182 (33.6 + 15.6 - 15.301) = -6.1696 = -6.170$$

$$M_{EF} = -0.209 (33.6 + 15.6 - 15.301) = -7.085$$

Joint F:

$$M_{FC} = M_{FI} = -0.086 \times (33.6 + 15.6 - 7.085) = -3.622$$

$$M_{FE} = -0.328 (33.6 + 15.6 - 7.085) = -13.81$$

Joint A :

$$M_{AD} = -0.116 \times (15.6 - 4.674) = -1.267$$

$$M_{AB} = -0.384(15.6 - 4.674) = -4.196$$

Joint B :

$$M_{BA} = -0.204 \times (15.6 - 4.196 - 1.864) = -1.946$$

$$M_{BE} = -0.062(15.6 - 4.196 - 1.864) = -0.591$$

$$M_{BC} = -0.234(15.6 - 4.196 - 1.864) = -2.232$$

Joint C :

$$M_{CB} = -0.396 \times (15.6 - 2.232 - 3.622) = -3.859$$

$$M_{CF} = -0.104(9.746) = -1.014$$

This completes one cycle. The rotation and translation moments have been shown entered in figure as the first row in each entry.

Cycle 2:

The second cycle again starts with the determination of translation moments using the above equation, the improved values of translation moments are :

Top Storey :

$$M_{AD}'' = M_{BE}'' = M_{CF}'' = -1/2 (-31.2 - 1.267 - 4.674 - 0.591 - 1.864 - 1.014 - 3.622) = +22.116 \text{ KN-M}$$

Bottom Storey :

$$M_{DG}'' = M_{EH}'' = M_{FI}'' = -1/2 (-67.2 - 4.674 - 1.864 - 3.622) = +38.68 \text{ KN-M}$$

Rotation Moments :

Joint D

$$M_{DA} = M_{DG} = -0.095 (22.12 + 38.68 - 6.170 - 1.267) = -5.069$$

$$M_{DE} = -0.311 (+53.363) = -16.5958 = -16.596$$

Joint E

$$M_{EB} = M_{EH} = -0.055 (38.68 + 22.12 - 16.596 - 0.591 - 13.81) = -1.639$$

$$M_{ED} = -0.182 (29.803) = -5.424$$

$$M_{EF} = -0.209 (29.803) = -6.229$$

Joint F:

$$M_{FC} = M_{FI} = -0.086 (38.68 + 22.112 - 6.229 - 1.014 - 0) = -4.606$$

$$M_{FE} = -0.328 (53.557) = -17.567$$

Joint A:

$$M_{AD} = -0.116 (22.12 - 1.946 - 5.069) = -1.752$$

$$M_{AB} = -0.384(15.105) = -5.800$$

Joint B:

$$M_{BA} = -0.204 (22.12 - 5.800 - 3.859 - 1.639) = -2.208$$

$$M_{BE} = -0.062(10.822) = -0.671$$

$$M_{BC} = -0.234(10.822) = -2.532$$

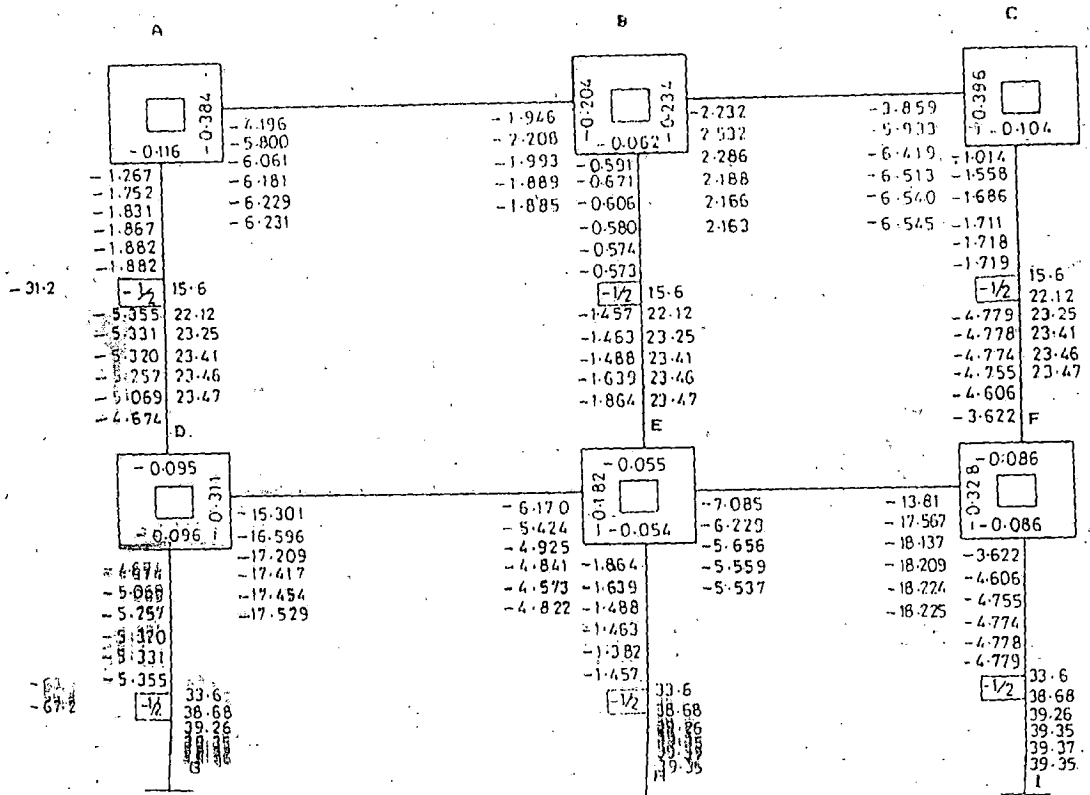
Joint C:

$$M_{CB} = -0.396 (22.12 - 2.532 - 4.606) = -5.933$$

$$M_{CF} = -0.104(14.982) = -1.558$$

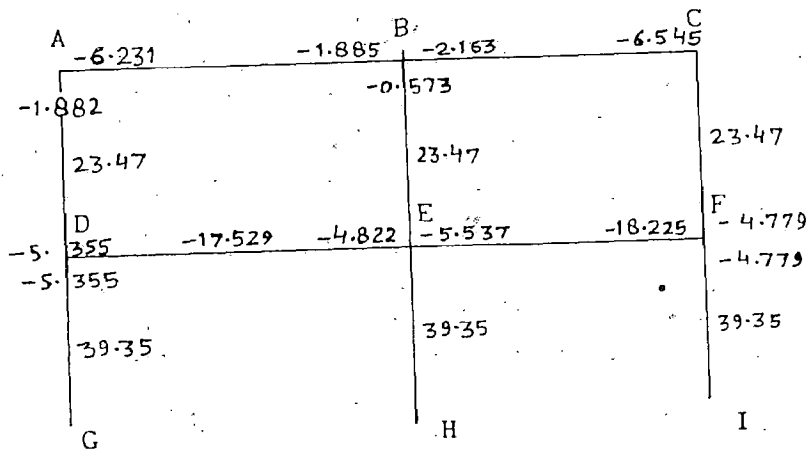
This completes the second cycle. In the similar manner computations are carried out in the subsequent cycles of iteration.

The values of the rotation and translation moments upto five cycles have been shown entered in figure.



CALCULATION CYCLES BY KANJ'S METHOD

The values obtained in the fifth cycles are taken as acceptable and all the values in the previous cycles are ignored.



Rotation and Translation Moments in 5th Cycle

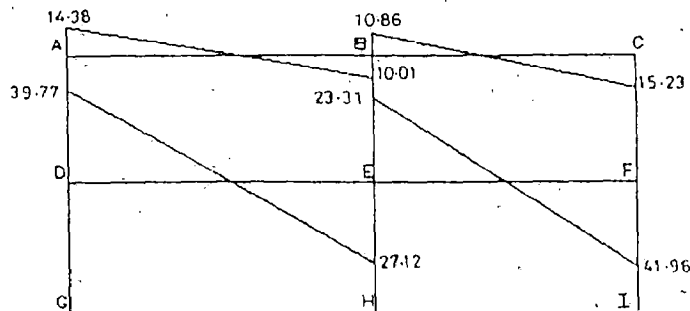
The final moments are worked out as usual using equation.

$$M_{ij} = M_{ij}^F + 2M_{ij}^T + M_{ji}^T + M_{ij}^R$$

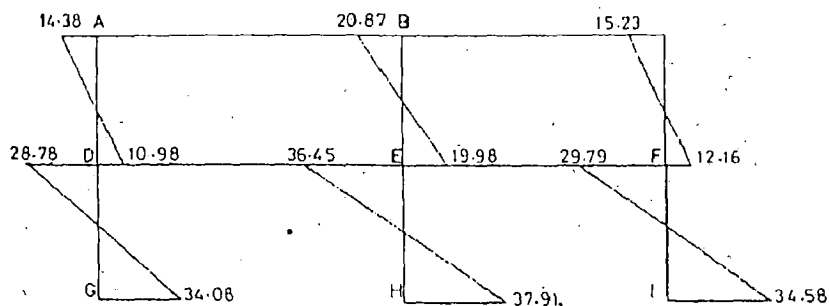
where  $M_{ij}^T$  → rotation moment at 'i'

$M_{ij}^R$  → Translation moment for any column in the storey

The bending moments in the members are calculated and shown in figure

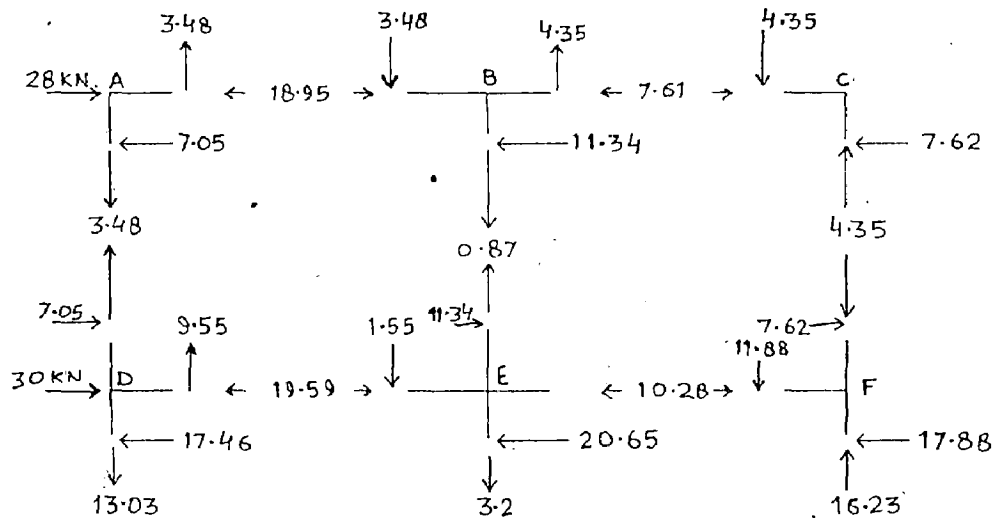


B.M.D. FOR BEAMS



B.M.D. FOR COLUMNS

Applying the static equilibrium equation at the joints, the shears and the axial forces are calculated and shown in Figure.

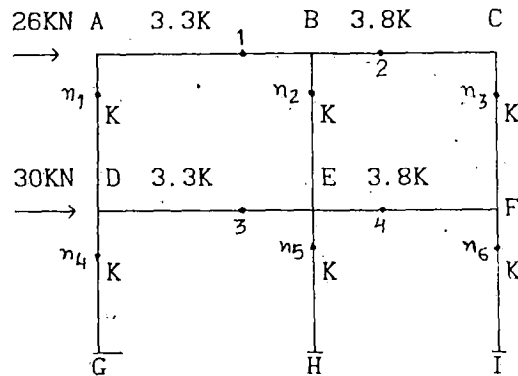


#### 4. ANALYSIS OF BUILDING FRAME BY BOWMAN'S METHOD

##### STEPS INVOLVED IN ANALYSIS :

Point of Contraflexure

1, 2, 3, 4 are the point of contraflexure in Beam AB, BC, DE, EF respectively and  $n_1, n_2, n_3, n_4, n_5, n_6$  are the point of contraflexure in column.



$$x_1 = x_4 = 0.55 \times 7 = 3.85 \text{ m}$$

$$x_2 = x_3 = 0.55 \times 6 = 3.30 \text{ m}$$

$$y_1 = y_2 = y_3 = 0.65 \times 3.6 = 2.34 \text{ m}$$

$$y_4 = y_5 = y_6 = 0.60 \times 3.6 = 2.16 \text{ m}$$

##### (1a) SHEAR IN BOTTOM STOREY :

As from the Bowman method, the shear in the bottom storey column are equally divided by an amount of shear.

$$= \frac{\text{No. of bays} - 1/2}{\text{No. of column}} \times \text{total shear in that storey}$$

$$= \frac{2 - 1/2}{3} \times 56 = 28 \text{ kN}$$

Shear in column DG, EH and FI are  $28/3$  i.e., 9.33 kN

The remaining shear i.e.,  $56 - 28 = 28$  kN will be divided around the bays inversely as their width.

Shear in the bay GDEH, =  $28 \times 6/13 = 12.92$  KN

Shear in the bay HEFI =  $28 \times 2/13 = 15.08$  KN

Additional shear in column DG =  $12.92/2 = 6.46$  KN

Additional shear in column EH =  $12.92/2 = 6.46$  KN

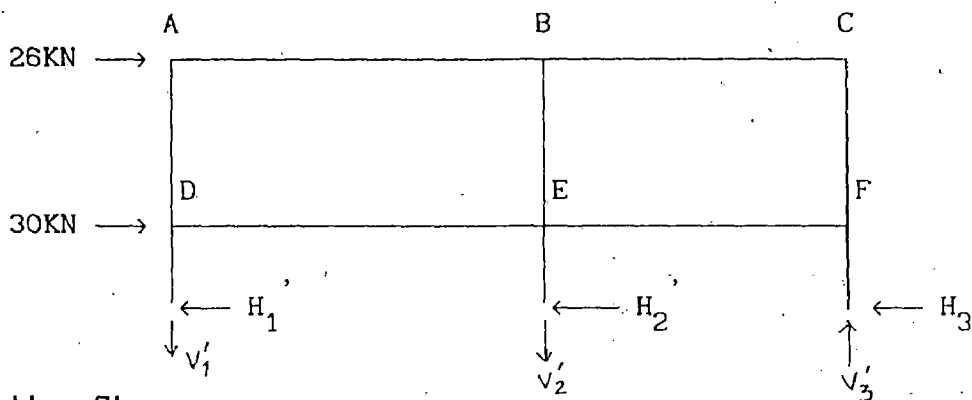
Additional shear in column FI =  $15.08/2 = 7.54$  KN

Additional shear in column EH =  $15.08/2 = 7.54$  KN

Final shear in column DG =  $9.33 + 6.46 = 15.79$  KN

Final shear in column EH =  $9.33 + 6.46 + 7.54 = 23.33$  KN

Final shear in column FI =  $9.33 + 7.54 = 16.87$  KN



Bottom Storey :

Taking moment about the point of contraflexure in beam DE

$$6 \times 1.26 + 15.79 \times 1.44 = (v_1' - 3.64) \times 3.85$$

$$v_1' = 11.50 \text{ KN}$$

Taking moment about point of contraflexure in beam EF

$$7 \times 1.26 + 16.87 \times 1.44 = (v_3' - 4.96) \times 3.3$$

$$v_3' = 11.99$$

From equilibrium of Joint E

$$v_2' = 3.49$$

(1b) SHEAR IN TOP STOREY :

Shear which is to be divided among the columns equally

$$= \frac{\text{No. of bays} - 2}{\text{No. of column}} \times \text{Total Shear}$$
$$= \frac{2 - 2}{3} \times 26 = 0$$

Hence the total 26 KN shear will be then divided in each bays inversely of their width.

$$\text{Shear in bay ABED} = 26 \times 6/13 = 12 \text{ KN}$$

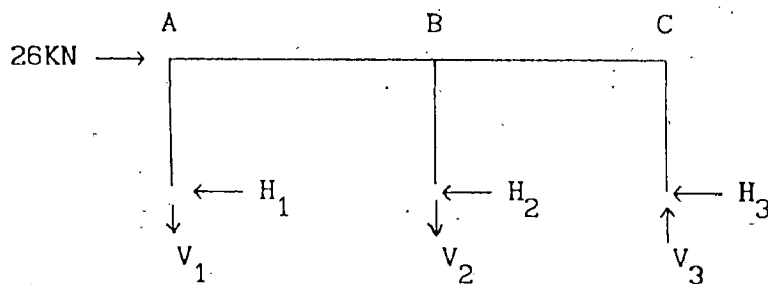
$$\text{Shear in bay BCFE} = 26 \times 7/13 = 14 \text{ KN}$$

$$\text{Hence shear in column AD} = 12/2 = 6 \text{ KN}$$

$$\text{Shear in column BE} = 12/2 \times 14/2 = 6 + 7 = 13 \text{ KN}$$

$$\text{Shear in column CF} = 14/2 = 7 \text{ KN}$$

Hence the freebody diagram :



Taking moment about 1

$$v_1 \times 3.85 = 6 \times 2.34$$

$$v_1 = 3.64 \text{ KN}$$

Taking moment about 2

$$v_3 \times 3.3 = 7 \times 2.34$$

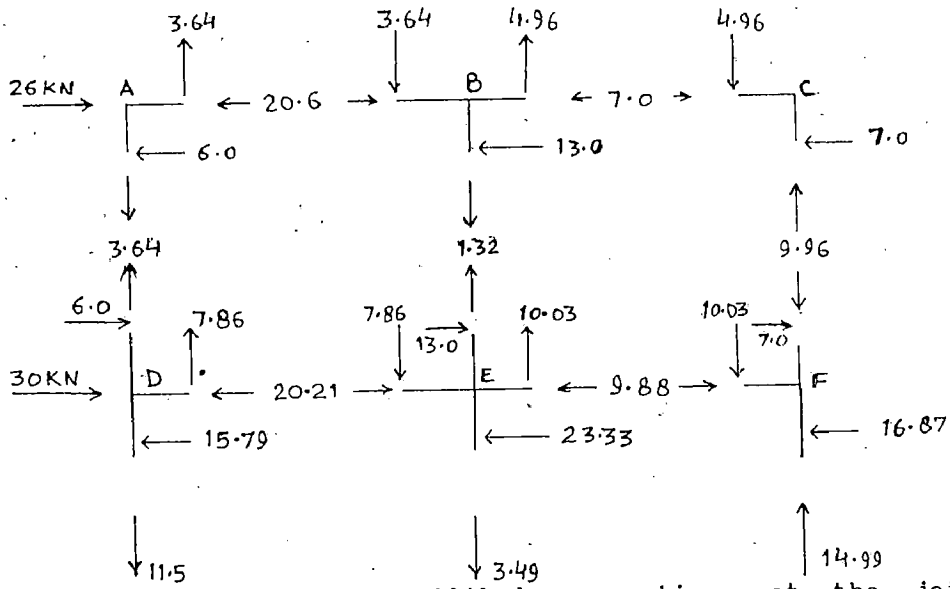
$$v_3 = 4.96 \text{ KN}$$

and from balancing the vertical forces

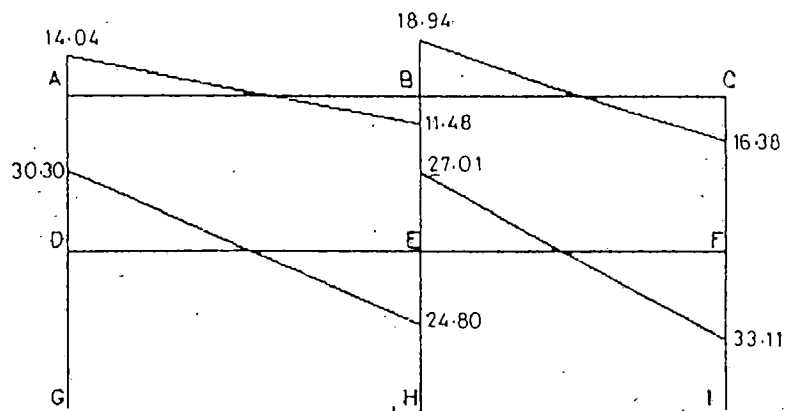
$$v_2 = 4.96 - 3.64 = 1.32 \text{ KN}$$



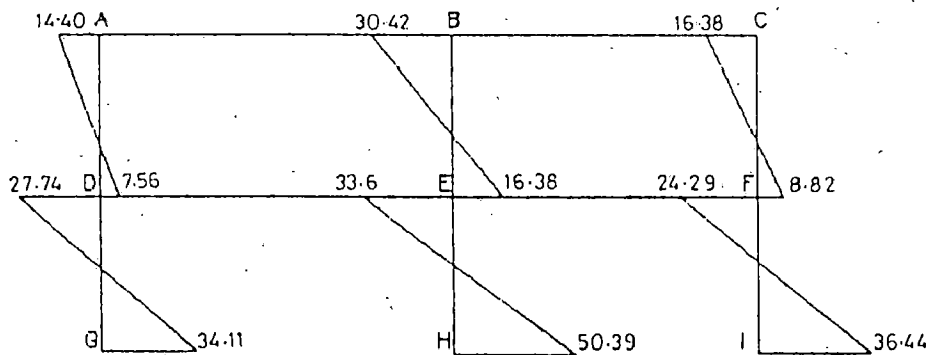
The shears and the axial forces are shown in figure.



From the static equilibrium equations at the joint the bending moments of the members are calculated and shown in figure.



B.M.D. FOR BEAMS



B.M.D. FOR COLUMNS

5. ANALYSIS OF BUILDING FRAME BY FACTOR METHOD

STEPS INVOLVED IN ANALYSIS :

Column stiffness = K each

Girder stiffness =  $K_{AB} = K_{DE} = 3.3 K$  and  $K_{BC} = K_{EF} = 3.8 K$

$$K_2 = K_4 = 0.51K, \quad K_1 = K_7 = 0.44K$$

As formulae are mentioned above we can write the results in tabular form .

Joint (1)	Memb <sup>r</sup> (2)	Col/Gird <sup>r</sup> factor c or g (3)	Half value of factor at opp end of memb. (4)	Add (3)&(4) (5)	K=I/L (6)	Col/gird <sup>r</sup> Mt. factor C or G=(5)*(6) (7)
A	AB	0.23	0.06	0.29	3.3K	0.957K
	AD	0.77	0.31	1.08	K	1.08K
B	BA	0.12	0.115	0.235	3.3K	0.775K
	BC	0.12	0.105	0.225	3.8K	0.855K
	BE	0.88	0.39	1.27	K	1.27 K
C	CB	0.21	0.06	0.27	3.8K	1.026K
	CF	0.79	0.33	1.12	K	1.12 K
D	DA	0.62	0.385	1.01	K	1.01 K
	DG	0.62	0.50	1.12	K	1.12 K
	DE	0.38	0.11	0.49	3.3K	1.62 K

contd....

Joint (1)	Memb <sup>r</sup> (2)	Col/Gird <sup>r</sup> factor c or g (3)	Half value of factor at opp end of memb. (4)	Add (3)&(4) (5)	K=I/L (6)	Col/gird <sup>r</sup> Mt. factor C or G=(5)*(6) (7)
E	EB	0.78	0.44	1.22	K	1.22 K
	ED	0.22	0.19	0.41	3.3K	1.35 K
	EF	0.22	0.17	0.39	3.8K	1.48 K
	EH	0.78	0.50	1.28	K	1.28 K
F	FC	0.66	0.395	1.06	K	1.06 K
	FE	0.34	0.11	0.45	3.8K	1.71 K
	FI	0.66	0.50	1.16	K	1.16 K
G	GD	1.00	0.31	1.31	K	1.31 K
H	HE	1.00	0.39	1.39	K	1.39 K
I	IF	1.00	0.33	1.33	K	1.33 K

CALCULATION OF COLUMN MOMENTS :

Total column moments for each storey

$$A = \frac{H \cdot h}{\sum C}$$

Where, H → Total horizontal force above the storey considered.

h → Height of the storey.

C → Sum of column end moment factors of that storey.

A<sub>0</sub> → Total column moments of ground storey.

$A_1$  → Total column moments of first storey (top storey).

$$A_0 = \frac{(30+26) * 3.6}{C_{DG} + C_{GD} + C_{EH} + C_{HE} + C_{FI} + C_{IF}}$$

$$= \frac{56 * 3.6}{(1.12+1.31+1.28+1.39+1.16+1.33)K}$$

$$= \frac{26.56}{K}$$

$$A_1 = \frac{26 * 3.6}{C_{AD} + C_{DA} + C_{BE} + C_{EB} + C_{CF} + C_{FC}}$$

$$= \frac{26 * 3.6}{(1.08+1.01+1.27+1.22+1.12+1.06)K}$$

$$= \frac{13.85}{K}$$

**COLUMN MOMENTS :**

For Top Storey :  $A_1 = \frac{13.85}{K}$

$$M_{AD} = A_1 * C_{AD} = \frac{13.85}{K} * 1.08K = 14.96 \text{ KN m.}$$

$$M_{DA} = A_1 * C_{DA} = \frac{13.85}{K} * 1.01K = 13.99 \text{ KN m.}$$

$$M_{BE} = A_1 * C_{BE} = \frac{13.852.57}{K} * 1.27 K = 17.59 \text{ KN m.}$$

$$M_{EB} = A_1 * C_{EB} = \frac{13.85}{K} * 1.22K = 16.90 \text{ KN m.}$$

$$M_{CF} = A_1 * C_{CF} = \frac{13.85}{K} * 1.12K = 15.51 \text{ KN m.}$$

$$M_{FC} = A_1 * C_{FC} = \frac{13.85}{K} * 1.06K = 14.68 \text{ KN m.}$$

For Bottom Storey :

$$M_{DG} = A_0 * C_{DG} = \frac{26.56}{K} * 1.12K = 29.75 \text{ KN m.}$$

$$M_{GD} = A_0 * C_{GD} = \frac{26.56}{K} * 1.31K = 34.74 \text{ KN m.}$$

$$M_{EH} = A_0 * C_{EH} = \frac{26.56}{K} * 1.28K = 34.00 \text{ KN m.}$$

$$M_{HE} = A_0 * C_{HE} = \frac{26.56}{K} * 1.39K = 36.92 \text{ KN m.}$$

$$M_{FI} = A_0 * C_{FI} = \frac{26.56}{K} * 1.16K = 30.81 \text{ KN m.}$$

$$M_{IF} = A_0 * C_{IF} = \frac{26.56}{K} * 1.33 K = 35.32 \text{ KN m.}$$

CALCULATION OF BEAM MOMENTS :

---

$$\text{Constant B} = \frac{\text{Sum of column moments at joints}}{\text{Sum of girder moment factor at that joint}}$$

$$\text{Joint A : } B_A = \frac{M_{AD}}{G_{AB}} = \frac{14.96}{0.96} = 15.58$$

$$\text{Joint B : } B_B = \frac{M_{BE}}{G_{BA} + G_{BC}} = \frac{17.59}{0.78 + 0.86} = 10.73$$

$$\text{Joint C : } B_C = \frac{M_{CF}}{G_{CB}} = \frac{15.51}{1.03} = 15.06$$

$$\text{Joint D : } B_D = \frac{M_{DA} + M_{DG}}{G_{DE}} = \frac{13.99 + 29.75}{1.62} = 27.00$$

$$\text{Joint E : } B_E = \frac{M_{EB} + M_{EH}}{G_{ED} + G_{EF}} = \frac{16.9 + 34.00}{1.35 + 1.48} = 17.99$$

$$\text{Joint F : } B_F = \frac{M_{FC} + M_{FI}}{G_{FE}} = \frac{14.68 + 30.81}{1.71} = 26.60$$

BEAM MOMENTS :

Beam Moments = B \* Girder Moment Factor

$$M_{AB} = B_A * G_{AB} = 15.58 * 0.96 = 14.96 \text{ KN m.}$$

$$M_{BA} = B_B * G_{BA} = 10.73 * 0.78 = 8.37 \text{ KN m.}$$

$$M_{BC} = B_B * G_{BC} = 10.73 * 0.86 = 9.23 \text{ KN m.}$$

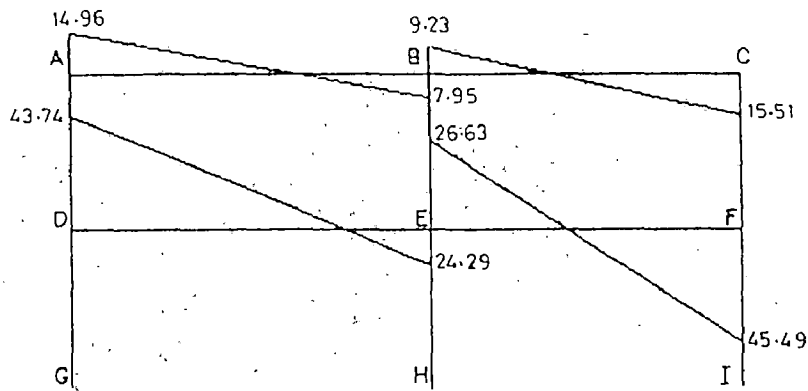
$$M_{CB} = B_C * G_{CB} = 15.06 * 1.03 = 15.51 \text{ KN m.}$$

$$M_{DE} = B_D * G_{DE} = 27.00 * 1.62 = 43.74 \text{ KN m.}$$

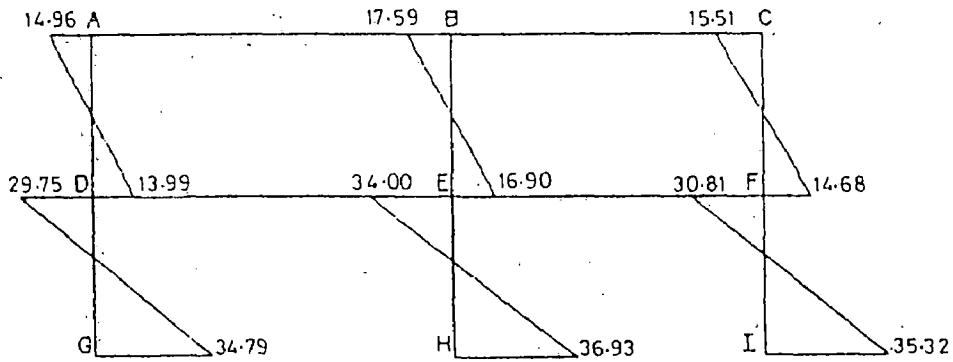
$$M_{ED} = B_E * G_{ED} = 17.99 * 1.35 = 24.29 \text{ KN m.}$$

$$M_{EF} = B_E * G_{EF} = 17.99 * 1.48 = 26.63 \text{ KN m.}$$

$$M_{FE} = B_F * G_{FE} = 26.60 * 1.71 = 45.49 \text{ KN m.}$$



B.M.D. FOR BEAMS



B.M.D. FOR COLUMNS

POINT OF CONTRAFLEXURE IN BEAMS :

$$X_1 = \frac{14.96 * 7}{(14.96 + 8.36)} = 4.5$$

Similarly

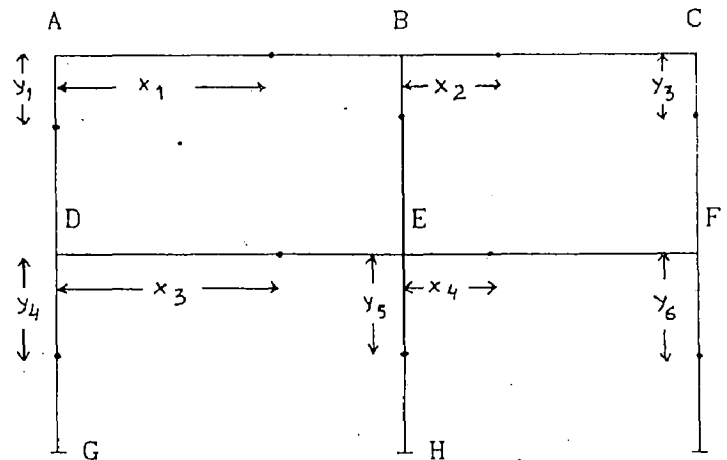
$$X_2 = 2.24, \quad X_3 = 4.5, \quad X_4 = 2.22$$

POINT OF CONTRAFLEXURE IN COLUMNS :

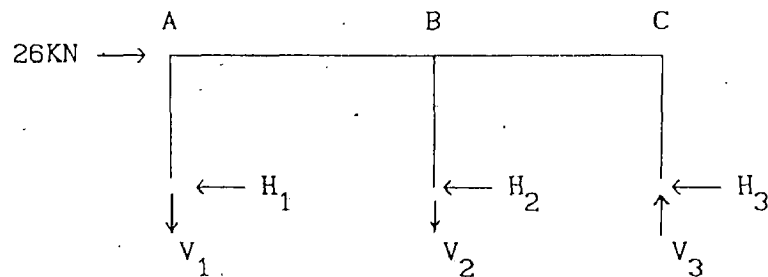
$$Y_1 = \frac{14.96 * 3.6}{14.96 + 13.99} = 1.86$$

Similarly

$$Y_2 = 1.78, \quad Y_3 = 1.8, \quad Y_4 = 1.66, \quad Y_5 = 1.73, \quad Y_6 = 1.68$$



CALCULATION OF SHEAR :





$$M_{AD} = H_1 * 1.86$$

$$\text{or } H_1 = 14.96/1.86 = 8.04 \text{ KN.}$$

$$\text{Similarly } H_2 = 9.34 \text{ KN.}, \quad H_3 = 8.62 \text{ KN.}$$

$$\text{And from } M_{AD} = V_1 * 4.5$$

$$\text{or } V_1 = 14.96/4.5 = 3.32 \text{ KN.}$$

$$\text{Similarly } V_3 = 4.13 \text{ KN.}$$

Taking moment about the point of contraflexure of the windward direction column.

$$V_3 * 13 = 26 * 1.86 + V_2 * 7$$

$$\text{or } V_2 = 0.76 \text{ KN.}$$

$$M_{DG} = H_1 * 1.66$$

$$\text{or } H_1 = 24.75/1.66 = 17.93 \text{ KN.}$$

$$\text{Similarly } H_2 = 19.66 \text{ KN.} \quad H_3 = 18.36 \text{ KN.}$$

Again from the figure

$$M_{DG} + M_{DA} = (V_1 - 3.32) * 4.5$$

$$\text{or } V_1 = \frac{29.75 + 13.99 + 4.5 * 3.32}{4.5} = 13.04 \text{ KN.}$$

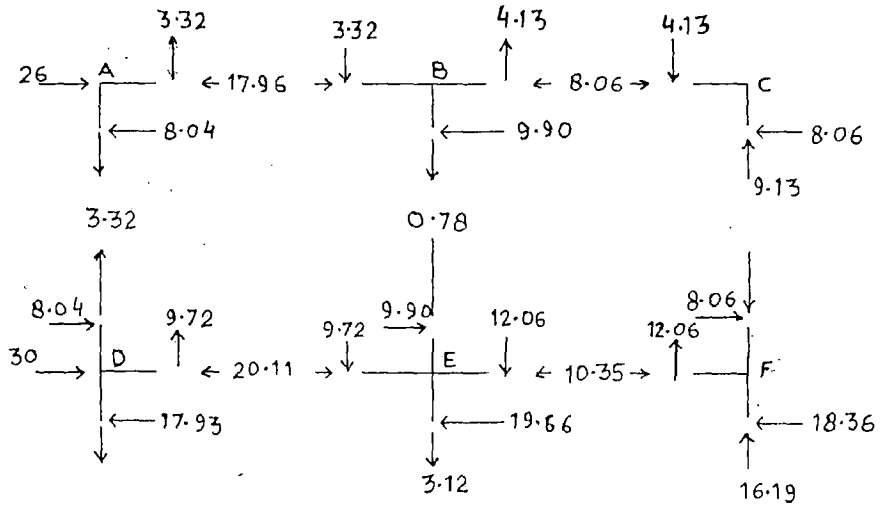
$$V_3 = \frac{M_{FC} + M_{FI} + 3.78 * 3.32}{3.78} = 16.38 \text{ KN.}$$

Taking moment about the point of contraflexure in column FI we

have

$$V_2 = 3.12 \text{ KN.}$$

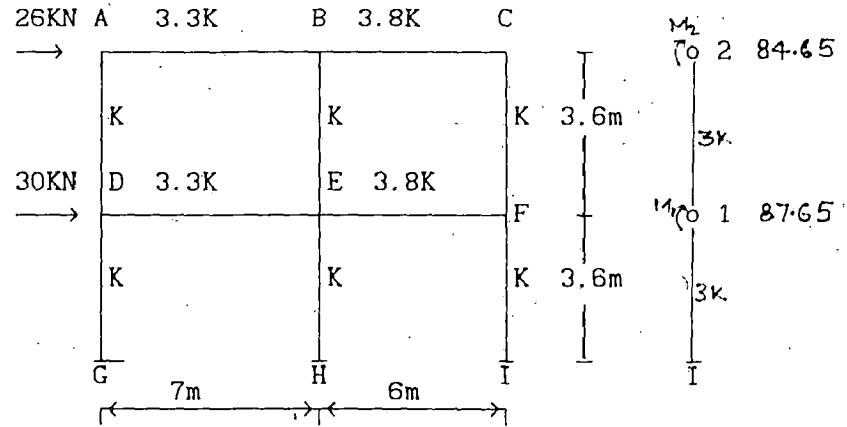
The shears and the axial forces in the members are calculated and shown in figure.



## 6. ANALYSIS OF BUILDING FRAME BY KLOUCEK'S METHOD

### STEPS INVOLVED IN ANALYSIS :

The frame is



All fixed end moments are zero

$$K_{ij} = I_{ij}/L_{ij}$$

All values of the stiffness are shown in figure

$$K_A = \frac{1}{2} K_{AB} \text{ i.e. } K_A = K_{AB} + K_{AD} = 3.3 K + K = 4.3 K$$

$$K_B = 3.3 K + 3.8 K + K = 8.1 K$$

$$K_D = K + 3.3 K + K = 5.3 K$$

$$K_E = 3.3 K + K + 3.8 K + K = 9.1 K$$

$$K_F = K + k + 3.8 K = 5.8 K$$

$$a_{AB} = a_{AB} = \frac{K_{AB}^2}{K_A K_B} = \frac{3.3^2 k^2}{4.3 k \times 8.1} = 0.313$$

$$a_{BC} = \frac{K_{BC}^2}{K_B K_C} = \frac{3.8^2}{4.8 \times 8.1} = 0.371$$

$$a_{AD} = \frac{K_{AD}^2}{K_A K_D} = \frac{1}{4.3 \times 5.3} = 0.044$$

$$a_{DE} = \frac{K_{DE}^2}{K_D K_E} = \frac{3.3^2}{5.3 \times 9.1} = 0.226$$

$$a_{DG} = \frac{K_{DG}^2}{K_D K_G} = \frac{1}{5.3 \times 1} = 0.189$$

$$a_{EB} = \frac{K_{EB}^2}{K_E K_B} = \frac{1}{9.1 \times 8.1} = 0.014$$

$$a_{EH} = \frac{K_{EH}^2}{K_E K_H} = \frac{1}{9.1 \times 1} = 0.110$$

$$a_{EF} = \frac{K_{EF}^2}{K_E K_F} = \frac{3.8^2}{4.1 \times 5.8} = 0.274$$

$$a_{FC} = \frac{K_{FC}^2}{K_F K_C} = \frac{1}{5.8 \times 4.8} = 0.036$$

$$a_{FI} = \frac{K_{FI}^2}{K_F K_I} = \frac{1}{5.8 \times 1} = 0.172$$

Joint stiffness factors of the frame system =  $2 \frac{1}{I} K_i$  are

Joint	Joint stiffness factor
A	8.6
B	16.2
C	9.6
D	10.6
E	18.2
F	11.6
G	2.0
H	2.0
I	2.0

The storey load members 'S' are

$$S_1 = -\frac{56 \times 3.6}{3} = -67.2 \text{ KN-m}$$

$$S_2 = -\frac{26 \times 3.6}{3} = -31.2 \text{ KN-m}$$

The coefficient of the displacement equation

$$X_1 = 2/3(1+1+1) = 2$$

$$X_2 = 2/3(1+1+1) = 2$$

From the above values 'S' we calculate the moment loadings for the substitute cantilever

$$m_1 = 3(S_1 + S_2) = 3(67.2 + 31.2) = 295.2 \text{ KN-m}$$

$$m_2 = 3(S_2 + 0) = 3(31.2) = 93.6 \text{ KN-m}$$

Member stiffness factors of the substitute cantilever are obtained according to figure

$$K_{I-1} = (1+1+1) = 3$$

$$K_{I-2} = (1+1+1) = 3$$

Due to irregular beam stiffness factors as in figure, the floor coefficient  $A_n$  would be taken

$$A_n = 1.15$$

i.e.,  $A_1 = A_2 = 11.5$

Which gives Knot stiffness factor of the substitute cantilever.

$$K_1 = 3 + 3 + 11.5 (3.3 + 3.8) = 87.65$$

$$K_2 = 3 + 11.5 (3.3 + 3.8) = 84.65$$

After rough determination of the constant for the cantilever,

$$a_{1-2} = \frac{3^2}{87.65 \times 84.65} = 0.001213$$

We calculate the primary deformation of the knots.

$$d_1^{\circ} = \frac{M_x}{K_1 (1 - \sum_{j=1}^n a_{1j})}$$

$$d_1^{\circ} = 295.2 / 87.65 (1 - 0.001213) = 3.37$$

$$d_2^{\circ} = 93.6 / 84.65 \times 0.998 = 1.11$$

By distributing these deformations throughout the cantilever, we obtain the knot deformation.

1 —————>2

$$3.37 \times \frac{3}{84.65 \times 1} = 0.119$$

$$0.042 = \frac{3}{87.65} \times \frac{1.110}{1.229}$$

Total deformation

$$d_1 = 3.3 + 0.042 = 3.412$$

$$d_2 = 1.11 + 0.119 = 1.229$$

and from the displacement equation, the sideways  $\mu^u = \mu$  (for  $d_1=0$ )

$$\mu_1 = -\frac{3}{2} \left[ \frac{67.2}{3} + 0 + 3.412 \right] = -38.718$$

$$\mu_2 = -\frac{3}{2} \left[ \frac{31.2}{3} + 1.229 + 3.412 \right] = -22.562$$

we can now calculate the six joint moments

$$M_D = - (1 \times \mu_1 + 1 \times \mu_2) = 38.718 + 22.562 = 61.28$$

$$M_E = - (1 \times \mu_1 + 1 \times \mu_2) = 61.28$$

$$M_F = - (1 \times \mu_1 + 1 \times \mu_2) = 61.28$$

$$M_A = -(1 \times \mu_2 + 0) = 22.56$$

$$M_B = -(1 \times \mu_2) = 22.56$$

$$M_C = -(1 \times \mu_2) = 22.56$$

For corrected value of defromation at Joint A

$$M_A = 22.56$$

$$-1 \times 4.6 = -4.6$$

$$-3.8 \times 1.229 = -4.056$$

$$13.9043 : 2 \sum K_A = 13.904/2 \times 4.3 = 1.62 = d_A'$$

For corrected value of defromation at Joint B

$$M_B = 22.56$$

$$-3.3 \times 1.62 = -5.346$$

$$-3.8 \times 1.229 = -4.67$$

$$-1 \times 1.75 = -1.75$$

$$10.79 : 2 \sum K_B = 10.79/2 \times 8.1 = 0.67 = d_B'$$

For corrected value of defromation at Joint C

$$\begin{aligned}M_C &= 22.56 \\-3.8 \times 0.67 &= -2.546 \\-1 \times 4.6 &= -4.6\end{aligned}$$

---

$$15.414 : 2 \sum K_C = 15.414 / 2 \times 4.8 = 1.61 = d'_C$$

As initial values for the iteration solution of the equations, we take

$$\begin{aligned}d_D &= d_E = d_F = d_1 = 3.412 \\d_A &= d_B = d_C = d_2 = 1.229 \\\mu_1 &= -38.718 \\\mu_2 &= -22.562\end{aligned}$$

We obtain the corrected deformation of joint D

$$\begin{aligned}M_D &= 61.28 \\-3.3 \times d_1 &= -3.3 \times 3.412 \\-1 \times d_2 &= -1 \times 1.229\end{aligned}$$

---

$$48.79 : 2 \sum K_D = \frac{48.79}{2 \times 5.3} = 4.60 = d'_D$$

Corrected deformation of E

$$\begin{aligned}M_E &= 61.28 \\-3.3 \times 4.60 &= -15.18 \\-3.8 \times 3.412 &= -12.96 \\-1 \times 1.229 &= -1.229\end{aligned}$$

---

$$31.905 : 2 \sum K_E = \frac{31.905}{2 \times 9.1} = 1.75 = d'_E$$



For Corrected deformation at joint F

$$M_F = 61.28$$

$$-3.8 \times 1.75 = -6.65$$

$$-1 \times 1.229 = -1.229$$

---

$$53.401 : 2K_F = \frac{53.401}{2 \times 5.8} = 4.60 = d_F$$

Calculation of end-moments from the corrected deformations:

Beam Moments :

$$M_{AB} = 3.3(2 \times 1.62 + 0.67) = 12.90 \text{ KN-m}$$

$$M_{BA} = 3.3(2 \times 0.67 + 1.62) = 9.77 \text{ KN-m}$$

$$M_{BC} = 3.8(2 \times 0.67 + 1.61) = 11.21 \text{ KN-m}$$

$$M_{CB} = 3.8(2 \times 1.61 + 0.67) = 14.78 \text{ KN-m}$$

$$M_{DE} = 3.3(2 \times 4.6 + 1.75) = 36.14 \text{ KN-m}$$

$$M_{ED} = 3.3(2 \times 1.75 + 4.60) = 26.73 \text{ KN-m}$$

$$M_{EF} = 3.8(2 \times 1.75 + 4.6) = 30.78 \text{ KN-m}$$

$$M_{FE} = 3.8(2 \times 4.6 + 1.75) = 41.61 \text{ KN-m}$$

Column end moments :

$$\begin{aligned} M_{AD} &= K_{AD}(2d_A + d_D + \mu_2) \\ &= 1(2 \times 1.62 + 4.6 - 22.562) = -14.72 \text{ KN-m} \end{aligned}$$

$$M_{DA} = 1(2 \times 4.6 + 1.62 - 22.562) = -11.74 \text{ KN-m}$$

$$M_{BE} = 1(2 \times 0.67 + 1.75 - 22.562) = -19.47 \text{ KN-m}$$

$$M_{EB} = 1(2 \times 1.75 + 0.67 - 22.562) = -18.39 \text{ KN-m}$$

$$M_{CF} = 1(2 \times 1.61 + 4.6 - 22.562) = -14.74 \text{ KN-m}$$

$$M_{FC} = 1(2 \times 4.6 + 1.61 - 22.562) = -11.75 \text{ KN-m}$$

$$M_{DG} = 1(2 \times 4.6 - 38.718) = -29.52 \text{ KN-m}$$

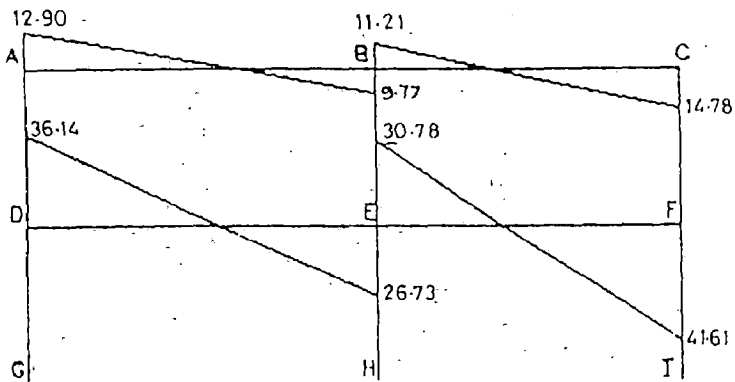
$$M_{GD} = 1(4.6 - 38.718) = -34.12 \text{ KN-m}$$

$$M_{EH} = 1(2 \times 1.75 - 38.718) = -35.22 \text{ KN-m}$$

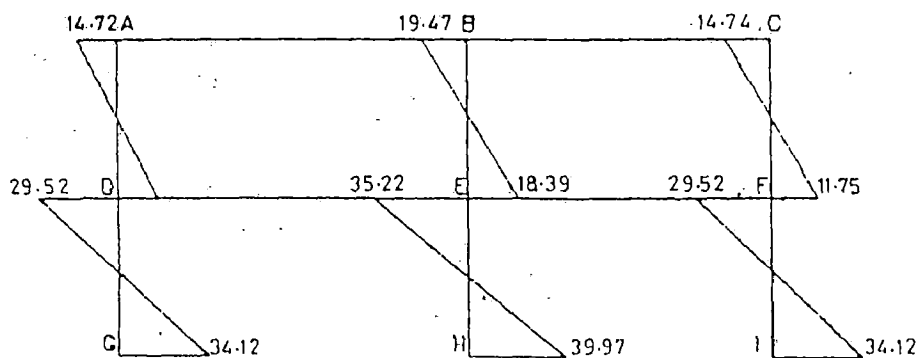
$$M_{HE} = 1(1.75 - 38.718) = -36.97 \text{ KN-m}$$

$$M_{FI} = 1(2 \times 4.6 - 38.718) = -29.52 \text{ KN-m}$$

$$M_{IF} = 1(4.6 - 38.718) = -34.12 \text{ KN-m}$$

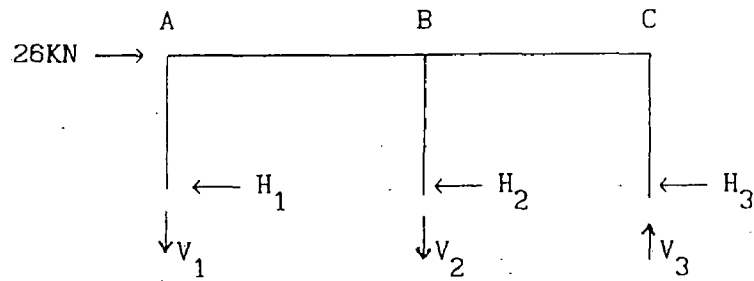


B.M.D. FOR BEAMS



B.M.D. FOR COLUMNS

$$V_2 = 1.13$$



Similarly,

$$H_1 = 29.52/1.7 = 17.36$$

$$H_2 = 35.22/1.76 = 20.72$$

$$H_3 = 29.52/1.7 = 17.36$$

$$(V_1 - 3.24) \times 4.02 = 7.83 \times (3.6 - 1.88) + 17.36 \times 1.7$$

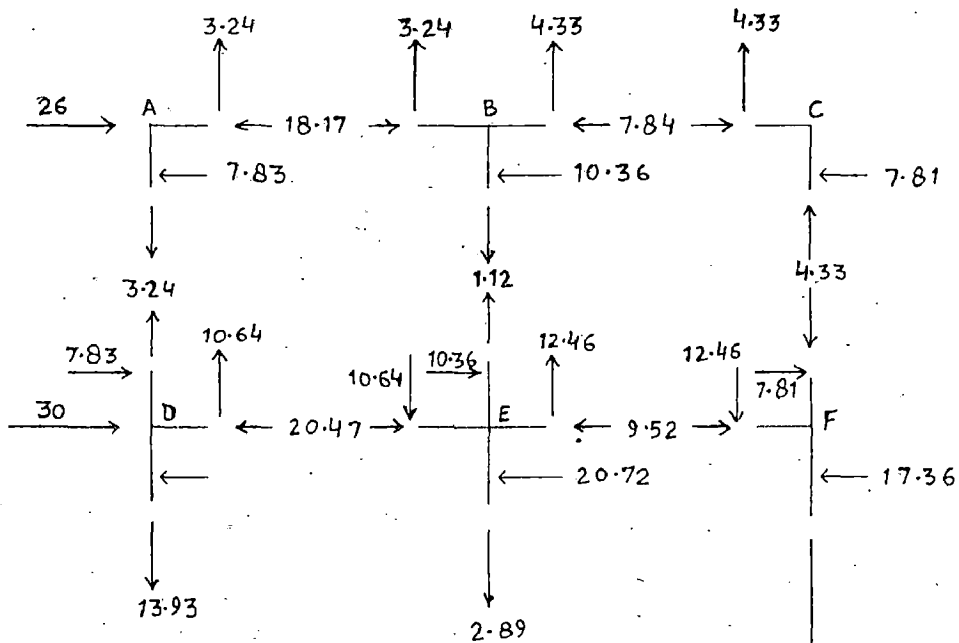
$$V_1 = 13.93$$

$$(V_3 - 4.33) \times 3.45 = 7.84 \times (3.6 - 1.88) + 17.36 \times 1.7$$

$$V_3 = 16.79$$

$$\text{and } V_2 = 2.89$$

The calculated shears and axial force in different members are shown in figure



## 7. ANALYSIS OF BUILDING FRAME BY BLUME et als. METHOD

### STEPS INVOLVED IN ANALYSIS

The calculations of moments in the top and bottom of the columns are shown in tabular form:

Storey	Column	Column	Apparent	Storey	$\frac{K_c}{\sum K_c}$	Shear in	$(M_T + M_B)$	$M_T/M_B$	$M_B = \frac{V_2 H_s}{1 + M_T/M_B}$	$(M_T = V_2 H_s - M_B)$	
	Stiffness	Stiff.	Storey	Shear		Column	$= V_2 H_s$				
Top	AD	K	0.54 K	26KN	0.30	7.8	28.08	0.985	14.15	13.93	
	BE	K	0.71 K		1.82 K	0.39	10.14	-36.50	0.985	18.39	18.11
	CF	K	0.57 K			0.31	8.06	29.02	0.985	14.62	14.40
Bottom	DG	K	0.75 K	56KN	0.32	17.92	64.51	0.960	32.91	31.60	
	EH	K	0.82 K		2.34 K	0.35	19.6	70.56	0.960	36.00	34.56
	FI	K	0.77 K			0.33	18.48	66.31	0.960	33.83	32.48

For the Top Storey- point of contraflexure :

$$y_1 = \frac{13.93 \times 3.6}{13.93 + 14.15} = 1.78$$

Similarly

$$y_2 = 1.78, y_3 = 1.78, y_5 = 1.76, y_6 = 1.76, y_7 = 1.76$$

Horizontal Shear

$$H_1 = \frac{13.93}{1.78} = 7.83$$

$$H_3 = \frac{14.40}{1.78} = 8.09$$

$$H_2 = 10.08 \text{ KN}$$

From the joint equilibrium

$$M_{AB} = 13.93$$

$$M_{BA} + M_{BC} = 18.11$$

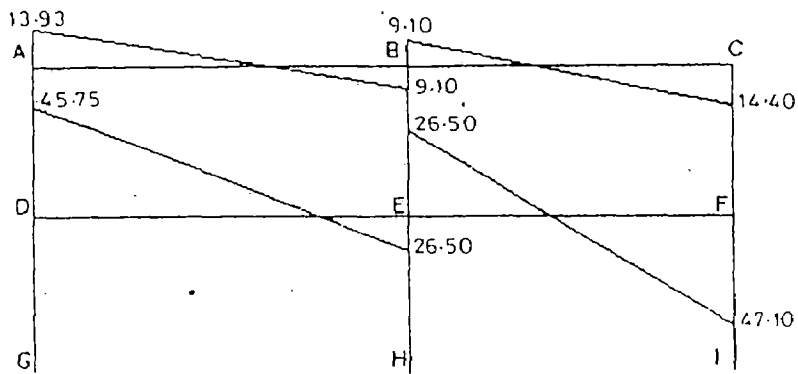
$$M_{CB} = 14.4$$

$$M_{DE} = M_{DA} + M_{DG} = 14.15 + 31.6 = 45.75$$

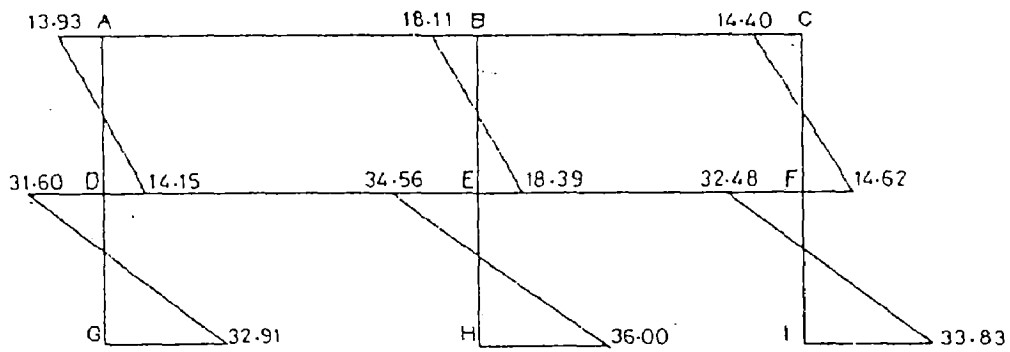
$$M_{ED} + M_{EF} = 34.56 + 18.39 = 52.95$$

$$M_{FE} = 14.62 + 32.48 = 47.1$$

The calculated bending moments are shown in BMD.



B.M.D. FOR BEAMS



B.M.D. FOR COLUMNS

Point of contraflexure in AB,

$$X_1 = \frac{13.93 \times 7}{13.93 + 9.1} = 4.23$$

Point of contraflexure in BC,

$$X_2 = \frac{9.1 \times 6}{9.1 + 14.4} = 2.32$$

Point of contraflexure in DE,

$$X_3 = \frac{45.75 \times 7}{45.75 + 26.5} = 4.43$$

Point of contraflexure in EF,

$$X_4 = \frac{26.5 \times 6}{26.5 + 47.1} = 2.16$$

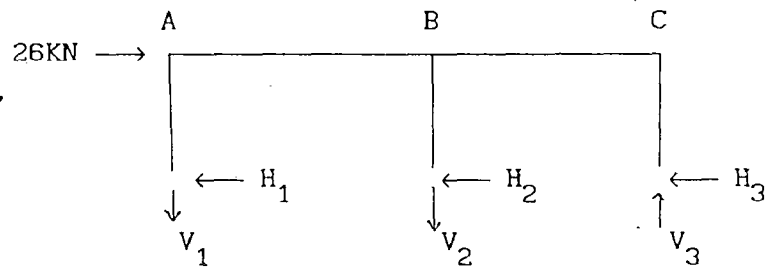
Values of  $V_1$ ,  $V_2$  and  $V_3$  are given by

$$V_1 = \frac{13.93}{4.23} = 3.29$$

$$V_3 = \frac{14.4}{(6-2.32)} = 3.91$$

$$3.29 \times 13 + V_2 \times 6 = 26 \times 1.78$$

$$V_2 = 0.59$$



$$H_1 = 31.6/1.76 = 17.95$$

$$H_3 = 32.48/1.76 = 18.45$$

$$H_2 = 34.56/1.76 = 19.64$$

$$(V_1 - 3.29) \times 4.43 = 7.83 \times 1.82 + 17.95 \times 1.76$$

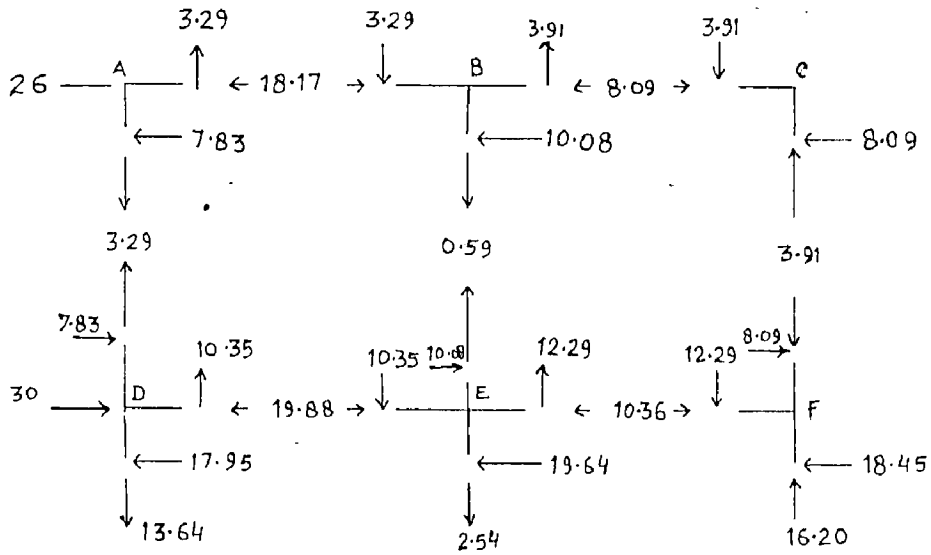
$$V_1' = 13.64$$

$$(V_3' - 3.91) \times 3.84 = 8.09 \times 1.82 + 18.45 \times 1.76$$

$$V_3' = 16.20$$

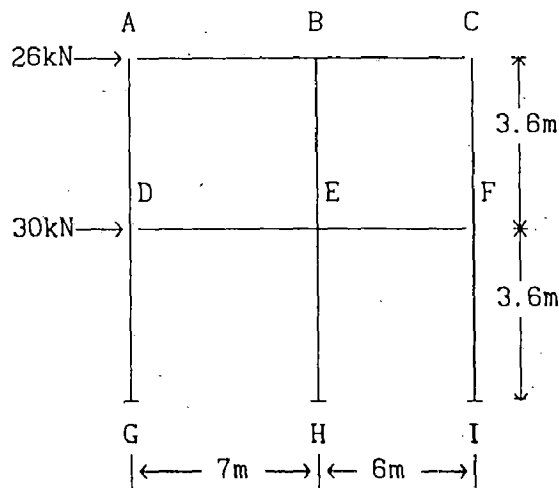
$$V_2' = 2.54$$

The calculated shears and axial forces are shown below :



## 8. STIFFNESS MATRIX METHOD

In the problem the horizontal displacements of joints "A" and "D" and the rotations of joints A, B, C, D, E, F are independent displacement components. It may be noted that the horizontal displacements of joints B and C are the same as that of joint A and the horizontal displacements of joints E and F are the same as that of Joint D. Hence the degree of freedom of frame is eight.



FRAME -A

The stiffness matrix can be developed by giving a unit displacement successively at coordinate 1 to 8 without any displacement at other coordinates. To generate the first column of the stiffness matrix; give a unit displacement at coordinate 1 without any displacement at other coordinates.

The elements of the stiffness matrix are calculated and given below :



$$K=EI \begin{bmatrix} 2.78 & -0.46 & -0.46 & 0.46 & 2.78 & -0.46 & 0.46 & -0.46 \\ -0.46 & 3.00 & 0.94 & 0.00 & 0.46 & 0.56 & 0.00 & 0.00 \\ -0.46 & 0.94 & 5.75 & 1.27 & 0.46 & 0.00 & 0.56 & 0.00 \\ -0.46 & 0.00 & 1.27 & 3.64 & 0.46 & 0.00 & 0.00 & 0.56 \\ -2.78 & 0.46 & 0.46 & 0.46 & 5.56 & 0.00 & 0.00 & 0.00 \\ -0.46 & 0.56 & 0.00 & 0.00 & 0.00 & 5.89 & 0.94 & 0.00 \\ -0.46 & 0.00 & 0.56 & 0.00 & 0.00 & 0.94 & 6.64 & 1.27 \\ -0.46 & 0.00 & 0.00 & 0.56 & 0.00 & 0.00 & 1.27 & 4.76 \end{bmatrix}$$

substituting this value in the stiffness matrix equation as shown in chapter 3.

In the above problem the nodal forces

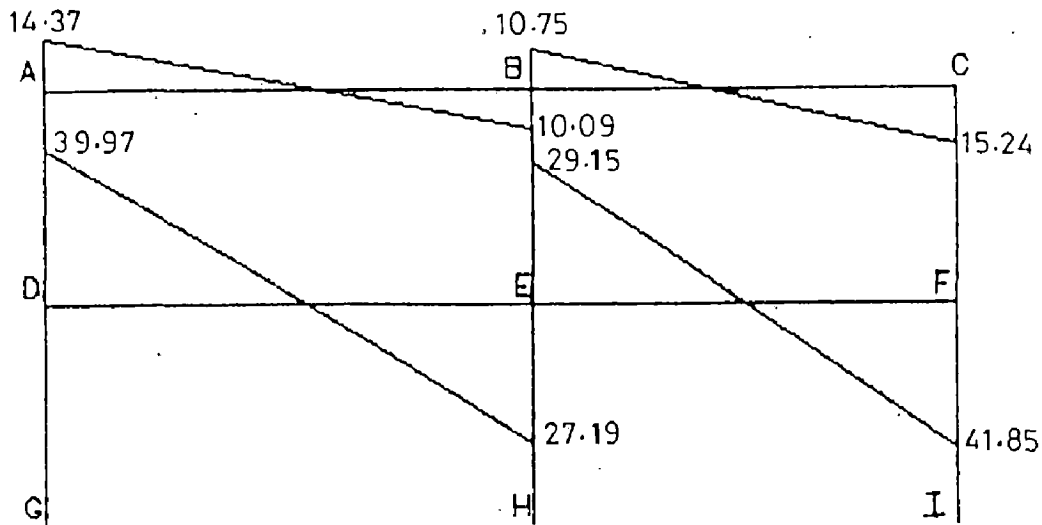
$$P_1=26, \quad P_2=P_3=P_4=P_6=P_7=P_8=0, \quad P_5=30 \text{ KN}$$

and the values of non nodal forces

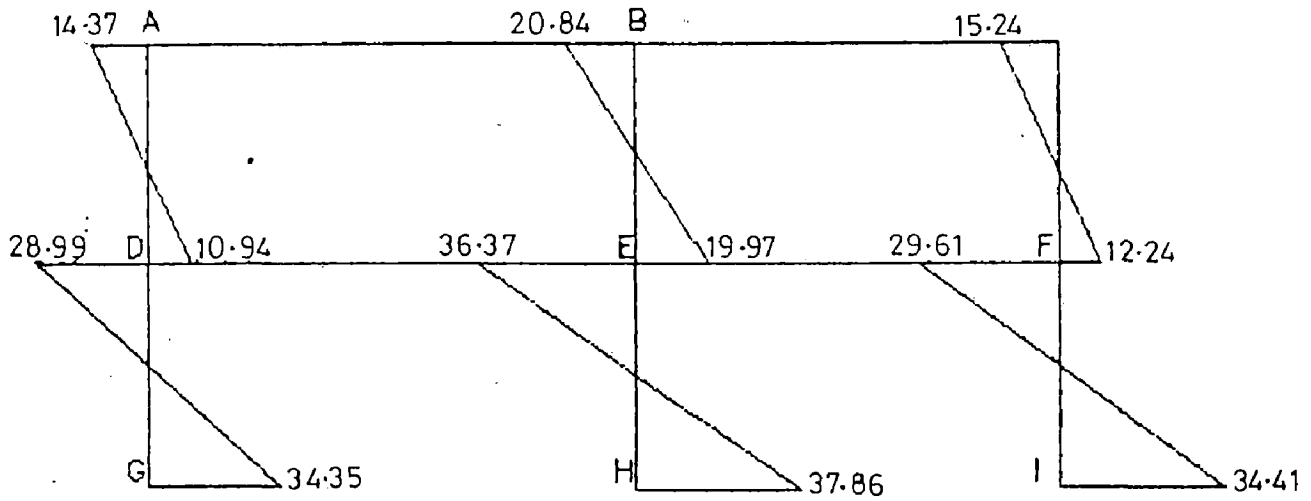
$$P'_1=P'_2=P'_3=P'_4=P'_5=P'_6=P'_7=P'_8=0$$

putting in the equation  $[\Delta] = [K]^{-1}\{[P]-[P']\}$

knowing the displacement component the end moments are computed by slope deflection equations.



B.M.D. FOR BEAMS



B.M.D. FOR COLUMNS