

ANALYSIS AND FEATURE EXTRACTION OF VIBRATIONAL SIGNAL FROM ELECTRICAL MACHINES

A DISSERTATION

submitted in partial fulfilment of the
requirements for the award of the degree

of

MASTER OF ENGINEERING

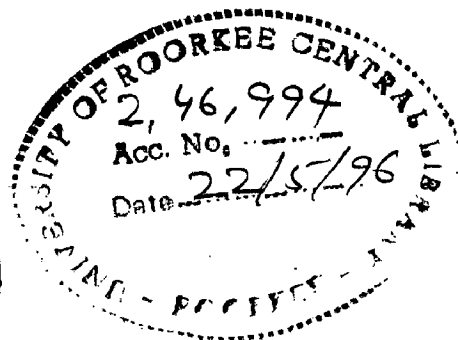
in

ELECTRICAL ENGINEERING

(With Specialization in Measurement and Instrumentation)

by

YOGENDRA SINGH



**DEPARTMENT OF ELECTRICAL ENGINEERING
UNIVERSITY OF ROORKEE
ROORKEE-247 667 (INDIA)**

MARCH, 1995

CANDIDATE'S DECLARATION

M/T/95/SCS/4
n/v

I hereby certify that the work which is being presented in this thesis entitled "ANALYSIS AND FEATURE EXTRACTION OF VIBRATIONAL SIGNAL FROM ELECTRICAL MACHINES" in partial fulfillment of the requirement for the award of the degree of "MASTER OF ENGINEERING" with specialization in "MEASUREMENT AND INSTRUMENTATION" submitted in the Department of Electrical Engineering, University of Roorkee, Roorkee is an authentic record of my own work carried out for a period from 1st september 1994 to 2 th March 1995, under the supervision of Prof. S.C. Saxena, Department of Electrical Engineering, University of Roorkee, Roorkee.

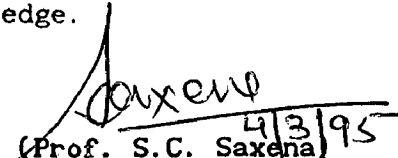
The matter embodied in this dissertation has not been submitted by me for the award of any other degree or diploma.

Roorkee

Dated: 4 th March 1995.

Yogendra Singh
(YOGENDRA SINGH)

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.


(Prof. S.C. Saxena) 4/3/95
Electrical Engg. Deptt.
University of Roorkee
Roorkee (U.P) 247667

ACKNOWLEDGMENTS

I wish to express my deep sense of gratitude to my guide Prof. S.C. Saxena, Department of Electrical Engineering, University of Roorkee, Roorkee for his expert guidance, constructive criticism and constant encouragement during the preparation of this dissertation.

I am thankful to Dr. V.K. Verma, Professor & Head, Department of Electrical Engineering, University of Roorkee, Roorkee for extending all facilities and help in the department.

I would like to extend my sincere thanks to all my friends, for their help and cooperation.

Roorkee

Dated: 4 th March 1995.

Yogendra Singh
(YOGENDRA SINGH)

ACKNOWLEDGMENTS

I wish to express my deep sense of gratitude to my guide Prof. S.C. Saxena, Department of Electrical Engineering, University of Roorkee, Roorkee for his expert guidance, constructive criticism and constant encouragement during the preparation of this dissertation.

I am thankful to Dr. V.K. Verma, Professor & Head, Department of Electrical Engineering, University of Roorkee, Roorkee for extending all facilities and help in the department.

I would like to extend my sincere thanks to all my friends, for their help and cooperation.

Roorkee

Dated: 4 th March 1995.

Yogendra Singh
(YOGENDRA SINGH)

SYNOPSIS

The nature of the vibration signals from the rotating electrical machines depends upon their mechanical assembly, rotational speed, base alignment, states of the bearings, and electrical conditions. The vibrations affect the mechanical life span and electrical performance characteristics, and may bring about a situation resulting in the failure of overall machine system. It is essential to carry out the detailed analysis of the recorded vibration signal for taking the necessary precautions for avoiding the mechanical and electrical problems due to vibrations in electrical machines. The present work deals with the development of software for the analysis of the vibration signals recorded from an electrical machine. By using the developed software, the detailed analysis and feature extraction from the vibration signal in respect of amplitude, duration, velocity, acceleration, and periodicity have been carried out in this work. As the vibration transducer of high frequency response was not available to record the signal from electrical machines, the developed software has been tested using vibration signal from the vibration table. The transducer used was a piezoresistive based seismic accelerometer having a frequency range upto 100 Hz. The vibration signal was picked up using this accelerometer and processed and digitized by an ADC card fitted within the computer system. The digitized signal was stored and analyzed by using the developed software. The test results are consistent and reliable and establishes the authenticity of the method to be used for real time vibration signal analysis of rotating electrical machines.

CONTENTS

CANDIDATE'S DECELERATION	i
ACKNOWLEDGEMENTS	ii
SYNOPSIS	iii
CHAPTER I : INTRODUCTION	1
1.1 General	1
1.2 Causes of Vibration	2
1.3 The Transducer used for Vibration signal recording	2
1.4 Organisation of the Thesis	3
CHAPTER II : VIBRATION MEASUREMENT	5
2.1 General	5
2.2 Vibration Sensors	5
2.2.1 Displacement Transducers	7
2.2.2 Velocity Transducers	9
2.2.3 Accelerometers	11
2.3 Mounting of Sensors	14
2.4 Charge Generator Model of the Accelerometer with Cable	16
2.5 Signal Conditioners	17
2.5.1 Voltage Amplifier	19
2.5.2 Charge Amplifier	22
CHAPTER III VIBRATION OF ROTATING ELECTRICAL MACHINES	24
3.1 General	24
3.2 Stator Core Response	24
3.3 Rotor Dynamics	25
3.3.1 Transverse Response	25
3.3.2 Torsional Response	28
3.4 Bearing Response	28
3.4.1 Rolling Element Bearings	30
3.4.2 Sleeve Bearings	32
CHAPTER IV VIBRATION MONITORING OF ROTATING ELECTRICAL MACHINES	34
4.1 Overall Level Monitoring	34
4.2 Frequency Spectrum Monitoring	39

	4.3 Special Vibration Monitoring Techniques	44
CHAPTER V	FAST FOURIER TRANSFORM (FFT)	47
	5.1 General	47
	5.2 The Discrete Fourier Transform	47
	5.3 The Fast Fourier Transform	52
	5.3.1 Intuitive Development	53
	5.3.2 Signal Flow Graph	59
	5.3.3 Dual Modes	60
	5.3.4 W^P Determination	64
	5.3.5 Unscrambling the FFT	66
CHAPTER VI	ALGORITHM, FLOWCHART, AND SOFTWARE	68
	6.1 General	68
	6.2 Analysis of Vibration Signal	68
	6.2.1 Fourier Transform Computation	71
	6.2.2 Integration of Vibration Signal	74
	6.2.3 Determination of Velocity from Acceleration	78
	6.2.4 Determination of Displacement from Velocity	78
	6.2.5 Cepstrum Analysis	78
	6.2.6 Overall Level of Vibration Velocity	80
	6.3 Programming	82
	6.4 Organisation of the Program	82
CHAPTER VII	VIBRATION ANALYSIS, TEST RESULTS AND DISCUSSION	85
	7.1 Monitoring of Rotating Electrical Machines using Vibration Analysis	85
	7.2 Test Results	89
	7.3 Discussion	104
CHAPTER VIII	CONCLUSIONS AND FURTHER SCOPE	106
	8.1 Conclusions	106

main methods are discussed: (i) overall level monitoring, (ii) frequency spectrum monitoring, and (iii) cepstrum monitoring.

In overall level monitoring, the measurement taken is the rms value of the vibration level over preselected bandwidth. The strength of the overall level technique is its simplicity. It requires only the simplest of the instrumentation. The use of this method is a common feature in any installations. The accuracy of the method depends on the operator. The sensitivity of the method is also low, particularly when a defect is at an early stage.

Frequency spectrum analysis is a 'post' time domain signal analysis. This method of analysis is generally employed once one's suspicions are aroused by anomalies in time domain signals taken using overall level monitoring.

Cepstrum analysis is a 'post' spectral analysis tool. The cepstrum analysis is used in examining the behaviour of gear boxes. By using this method of analysis one can easily find out side bands present in the vibration spectra of the gear boxes.

Chapter 5 lays down the foundations for building FFT computer program. It provides a thorough description of Fourier transform, DFT, and FFT algorithm.

Chapter 6 provides a detail discussion of various algorithms and flowcharts developed for the analysis of vibration signals from rotating electrical machines.

Chapter 7 provides several results taken by using software developed in chapter 6.

Chapter 8 gives the conclusion of the present work carried out in the dissertation. It also suggests the scope for the future work.

CHAPTER 1

INTRODUCTION

1.1 GENERAL

From the moment the rotating electrical machines are put into use, they are subjected to wear and tear. There is continuous deterioration in the state of the machines and may finally result in the break down. Thus sooner or later, there arise the necessity to provide their replacement. If the machine is simple, cheap and easily detachable from the other machines, the most straight forward way is to keep a reserve machine in the store for its immediate replacement. It would be good to know the real conditions of the sources of defect being developed during the normal operations of the machine. Knowing this, it would be possible to carry out the repair, overhaul or replacement whenever it is required in a planned manner.

Several decades of international professional experiences and practices have dictated that the mechanical vibrations of electrical machines are the right parameters to be measured to indicate the best technical condition of the machine. Thus it is in the best interest that we must know the trends of changes in the technical condition and variational characteristics of the machine.

The vibration signal of a machine to be tested can be obtained by using vibration transducers and data acquisition system. The fault inside machine changes amplitude and frequency of the vibration signal recorded from time to time from the machine. The analysis of the vibration signals from the machine is much sensitive method for indicating the defects. Another advantage of the spectrum analysis of the vibration signal is the general ability to identify the causes of vibrations (the sources of the defects) on the basis of frequency domain analysis during normal operation without dismantling the machine system. Thus

the engineers can know before starting their maintenance work as what has to be looked into when the machine is switched off. Of course, this procedure may also shorten the maintenance time.

1.2 CAUSES OF VIBRATION

The causes of vibration in a machines on the basis of their controllability are classified as follows:

- (i) Predictable and Controllable
 - (a) existing force harmonics
 - (b) slot harmonics
 - (c) Saturation harmonics
 - (d) vent sections in the rotor and stator
 - (e) fan blade design
- (ii) Predictable, specifiable, verifiable, but variable
 - (a) supply with harmonic content
 - (b) eccentric assembly of rotor and stator
 - (c) dynamic eccentricity of the rotor
- (iii) Random verifiable only by in-process quality control
 - (a) asymmetry in the stator or rotor winding
 - (b) bearing defect
 - (c) bent shaft end
 - (d) journal ovality
 - (e) loose assembly

1.3 THE TRANSDUCER USED FOR VIBRATION SIGNAL RECORDING

According to the requirements of the measuring technique and, in particular the modern signal processing technique, the mechanical signal is converted into an electrical signal by using vibration transducer.

There are two types of vibration-measuring transducers. The first type are non-contact transducer. There are two kinds of non-contact transducer in general use : electrodynamic and capacitive transducers. The second group of vibration-measuring

transducer includes the so-called contact transducers. In this case, the transducer is directly and rigidly fixed to the vibrating body. The commonest type of contact transducer is the piezoelectric transducer.

1.4 ORGANIZATION OF THE THESIS

After introduction in first chapter, chapter 2 provides a general discussion of the type of transducers and conditioners needed to effectively measure the vibrations. Vibration sensing revolves around the measurement of three related-quantities :

- (i) displacement,
- (ii) velocity, and
- (iii) acceleration.

Which quantity one should measure depends on the size of the machine being monitored, and the frequency range in which one is interested. The signal conditioner system comprises the cable connecting the transducer to the amplifier. There are two general types of pre-amplifier used for this purpose, one voltage amplifier and the other one is charge amplifier.

Chapter 3 gives an introduction to the vibration in rotating electrical machines. The principle areas of vibration in electrical machines are : (i) the stator core response to the attractive force developed between rotor and stator, (ii) the response of the stator end windings to the electromagnetic forces on the conductors, (iii) the dynamic behaviour of the rotor, and (iv) the response of the shaft bearings to vibration transmitted from the rotor.

Chapter 4 provides a fairly general discussion on the vibration monitoring of rotating electrical machines.

The defects of rotating electrical machines can be identified by more than one methods of vibration signal analysis. Here three

main methods are discussed: (i) overall level monitoring, (ii) frequency spectrum monitoring, and (iii) cepstrum monitoring.

In overall level monitoring, the measurement taken is the rms value of the vibration level over preselected bandwidth. The strength of the overall level technique is its simplicity. It requires only the simplest of the instrumentation. The use of this method is a common feature in any installations. The accuracy of the method depends on the operator. The sensitivity of the method is also low, particularly when a defect is at an early stage.

Frequency spectrum analysis is a 'post' time domain signal analysis. This method of analysis is generally employed once one's suspicions are aroused by anomalies in time domain signals taken using overall level monitoring.

Cepstrum analysis is a 'post' spectral analysis tool. The cepstrum analysis is used in examining the behaviour of gear boxes. By using this method of analysis one can easily find out side bands present in the vibration spectra of the gear boxes.

Chapter 5 lays down the foundations for building FFT computer program. It provides a thorough description of Fourier transform, DFT, and FFT algorithm.

Chapter 6 provides a detail discussion of various algorithms and flowcharts developed for the analysis of vibration signals from rotating electrical machines.

Chapter 7 provides several results taken by using software developed in chapter 6.

Chapter 8 gives the conclusion of the present work carried out in the dissertation. It also suggests the scope for the future work.

CHAPTER : 2

VIBRATION MEASUREMENT

2.1 GENERAL

Vibration is related to the oscillatory motion in a physical system. It is generally interpreted as symmetrical or nonsymmetrical fluctuations in the rate at which acceleration is applied to an object. Vibrations may be periodic or non-periodic (random) in nature. The measurement of vibrations includes determination of its displacement, velocity, acceleration, and frequencies. The measuring device is usually selected as per the expected ranges of frequencies. Its ease and ability to obtain accurate results are also important considerations. With low-frequency vibration, there are advantages in the measurement of displacement. At the higher frequencies, the excursion or displacement range usually becomes quite small, and therefore there is advantage in the measurement of acceleration. It may be difficult to measure displacement accurately in high frequency ranges.

2.2 VIBRATION SENSORS

Vibration sensing is one of the most important monitoring tool available to the operator of electromechanical plant. The procedures of vibration measurements have reached to a high degree of sophistication and generally revolve around the measurement of following three related quantities:

- (i) displacement,
- (ii) velocity, and
- (iii) acceleration.

Out of the above three Which quantity one should measure, depends on the size of the physical system being monitored and the frequency range in which one is interested. If we take the example

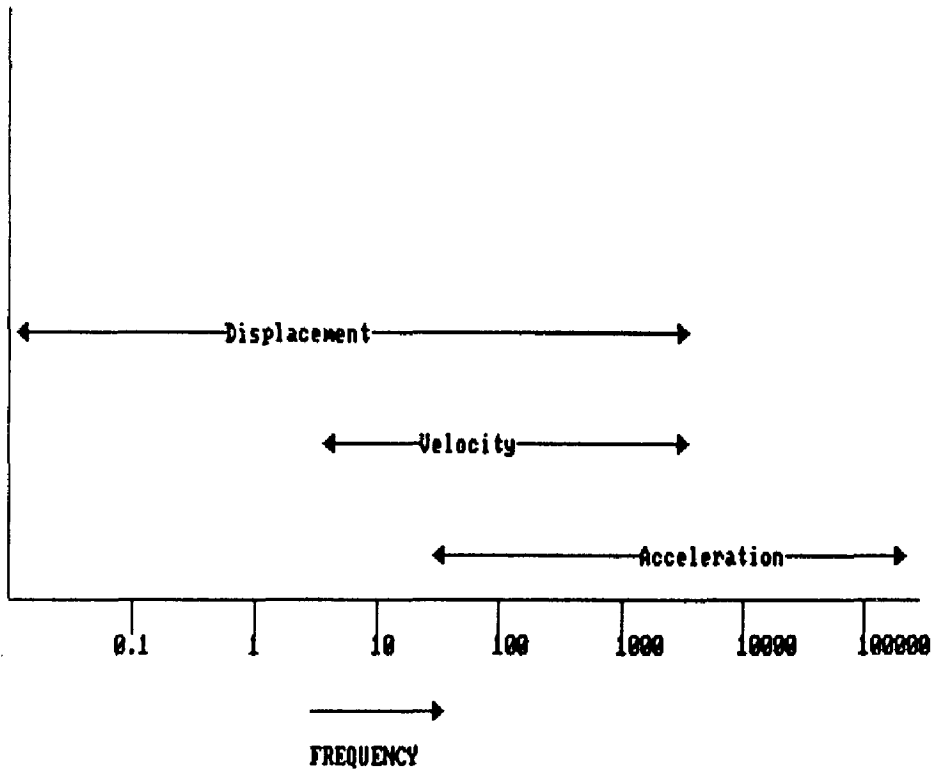


Figure 2.1 : Normal ranges of applicability for vibration ranges

of machines of similar type and size, it can be stated that there may be more or less constant vibrational velocity. It is observed that as the speed increases, there is the fall in the level of displacement and increase in the level of accelerations. This suggests that as the frequencies of interest rise it is better to progress from a displacement device to a velocity transducer, and ultimately to an accelerometer for the measurements. As a guide the approximate frequency ranges of application are as shown in figure (2.1)[1].

Care must be exercised, however, when monitoring systems which have small moving masses for in such circumstances transmitted forces may be small, and displacement will usually provide the best indication of condition.

The vibration transducers are characterized according to the quantity they measure.

2.2.1 DISPLACEMENT TRANSDUCERS

These are basically non-contacting probes or proximeters. These devices operate by using an high frequency source to generate an electromagnetic field at the probe tip. The system energy is thus dependent upon the local geometry of the area surrounding the probe tip. If it changes, for example, when the target surface moves with respect to the probe, the system energy also changes. This change is readily measured and is related to the displacement of the target surface from the probe tip. It should be noted that such systems measure the relative motion between the probe and the target; hence the vibration of the housing in which the probe is mounted is not readily measured by this technique [1].

Sensitivities of the order of 10 mv/micron displacement are easily achievable with displacement probes, and they find wide application in situations where heavy housing ensure small external movements. The measurements of eccentricity and

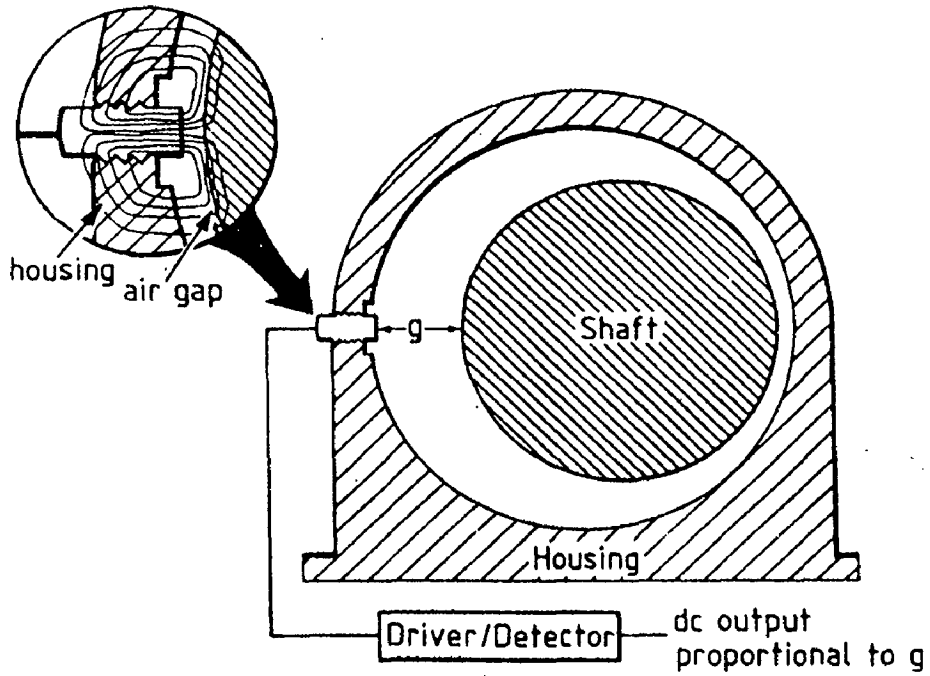


FIGURE 2.2 : OPERATION OF THE PROXIMITY PROBE

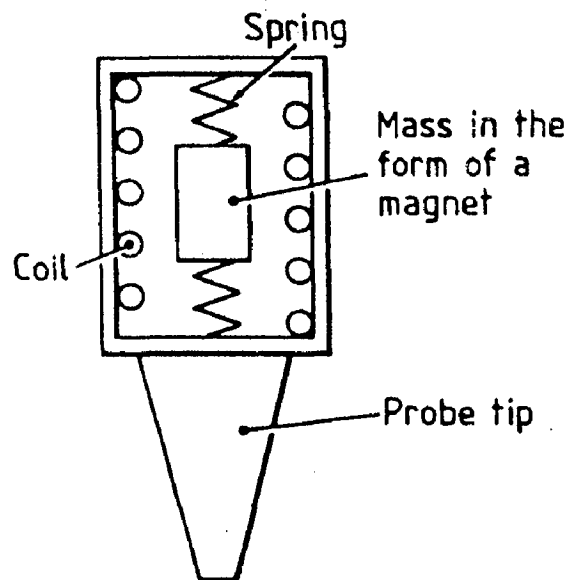


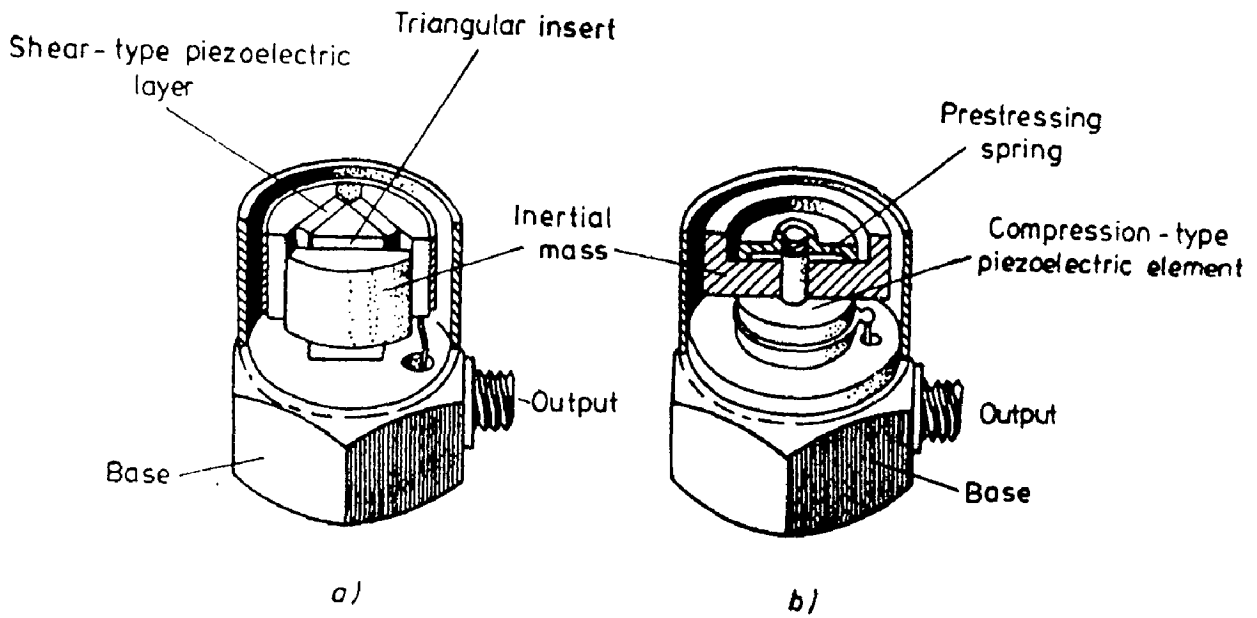
FIGURE 2.3 : PRINCIPLE OF THE VELOCITY PROBE

differential movements due to expansion are therefore most easily achieved using proximity transducers. They can also be effectively applied to measure rotational speed by sensing the passage of key ways on shafts.

As mentioned earlier, displacement is most effectively measured at the lower frequencies even though the frequency range of eddy current systems can extend above 10kHz. They are relatively robust transducers and the driving and detection circuits are straightforward. Essentially the high frequency signal applied to the probe is modulated by the passage of the target, and the demodulated signal used as the measurement quantity. Consideration of figure(2.2), which illustrates the basic displacement measurement principle, shows that the output of the system will depend not only on the displacement between the probe and the target, but also on the material of which target is made. This is because the eddy current reaction of the target, and hence the system energy, is dependent upon both the conductivity and permeability of the material. Proximity probes generally requires calibration for each target material. Care must also be taken when mounting the probe to ensure that conducting and magnetic surfaces around the probe tip do not cause unnecessary disturbance of the applied high frequency field and that the target surface is smooth with no surface or magnetic disturbances [2].

2.2.2 VELOCITY TRANSDUCERS

Velocity transducers are used in the frequency range from 10 Hz to 1 KHz. These are designed using a spring mass system with a natural frequency less than 10 Hz, and letting the mass take the form of a permanent magnet, as illustrated is Fig.(2.3) [2]. The magnet is then surrounded by a coil which is securely attached to the housing. Whenever the housing is placed in contact with a vibrating surface, the housing and coil move with respect to the magnet and cause an emf to be induced in the coil, as per the expression



**FIGURE 2.4 : TWO TYPES OF GENERAL-PURPOSE ACCELEROMETERS:
 (a) SHEAR TYPE (b) COMPRESSION TYPE**

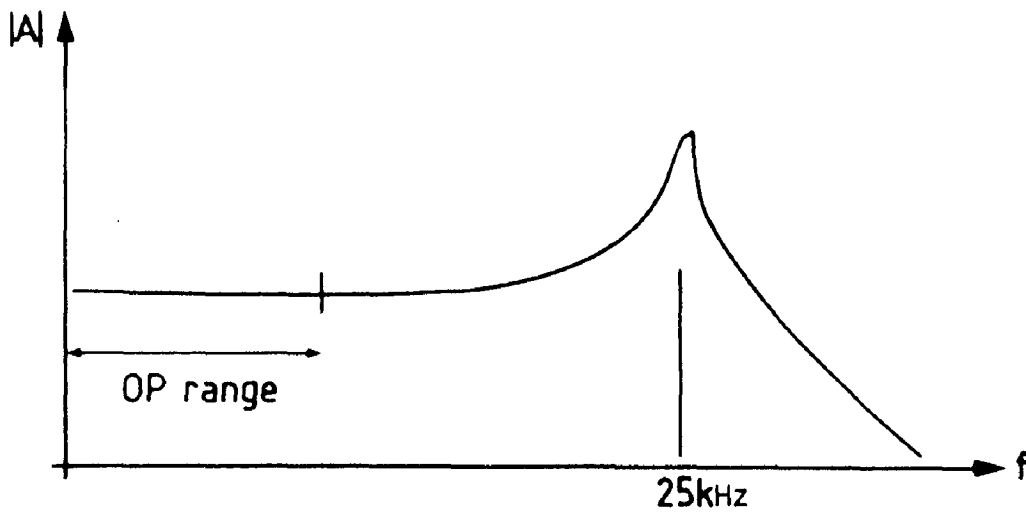


FIGURE 2.5 : ACCELEROMETER RESPONSE

$$e = l\bar{B} * \bar{V} \quad (2.1)$$

where e is the induced emf , l is the effective length of the conductor in the coil, \bar{B} is the radially directed flux density (which is constant), and \bar{V} is the velocity in the axial direction.

The transducers are relatively delicate but have the advantage of producing large output signal thereby requiring little or no signal conditioning .

2.2.3 Accelerometers

As per practice, the velocity and displacement are commonly measured using accelerometers and other required parameters are derived by integration.

Accelerometers produce an electrical output which is directly proportional to the acceleration to which they are subjected. In present time the piezoelectric device has become almost universally accepted accelerometric transducer. These are used for all the specialised vibration measurements. These are physically robust than the velocity transducer and have higher frequency range. This has become more important as techniques involving frequencies well above 1 KHz have been adopted.

The construction of a typical piezoelectric accelerometer is shown in Figure (2.4) [2]. When the transducer is subjected to vibration, the seismic mass, which is held against the piezoelectric element, exerts a force upon it. This force is proportional to the acceleration. Under such conditions the piezoelectric elements, which is usually a polarised ceramic material, generates a proportional electric charge across its faces. The output can then be conditioned using a charge amplifier. The velocity or displacement signals are recovered by integration. The device has the advantage of being a self generating source. Now integrated circuit piezoelectric devices

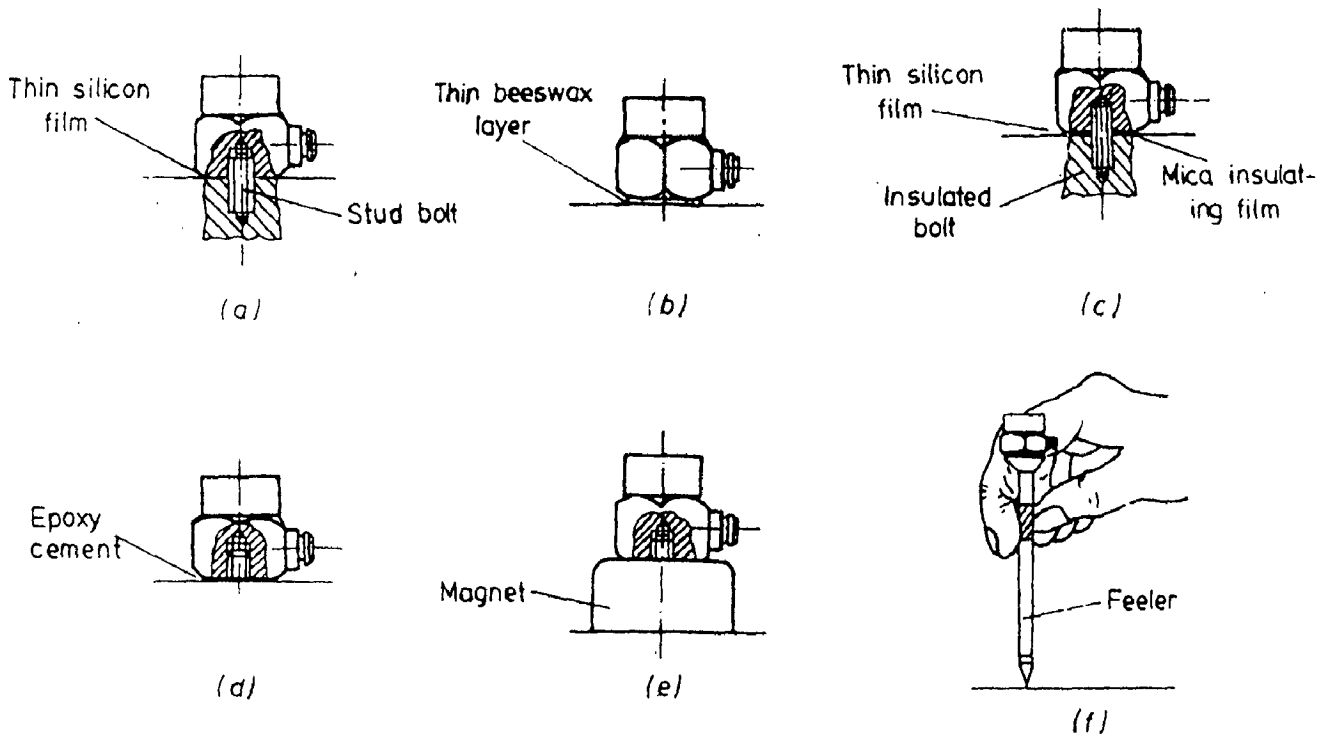


FIGURE 2.6 : METHODS OF FIXATION FOR ACCELEROMETERS.

The ratio of the mechanical natural frequency f_N of the system comprising the fixing element and the accelerometer to the accelerometer to the natural frequency f_{N1} of the transducer in the various cases:

fixation	a	b	c	d	e	f
f_N / f_{N1}	1.0	0.98	0.96	0.95	0.28	0.07

TABLE 2.1 : APPLICATION OF VIBRATION TECHNIQUES

Application	Transducer Type
Motor /pump drives	Velocity or acceleration
Motor /fan drives	Displacement or velocity
Motor connected to gear boxes (rolling element bearings)	Acceleration
Motor with oil film bearings)	Displacement
Generators/steam turbines	Displacement
Overall vibration levels on all of the above	Velocity

are available which have the output signal conditioning resident in the accelerometer encapsulation. When using piezoelectric accelerometers the natural frequency of the device is designed to be above the usual operating range. A typical frequency response is given in Figure (2.5) [3]. This limits the useful operating range to around 30% of the natural frequency. Being low output at low frequencies, the normal operation range of piezoelectric accelerometers is generally from 1 KHz to 8 KHz. Some small devices are available for ranges extending 200 KHz [3].

There is an extremely wide range of piezoelectric accelerometers available today, from very small devices that will measure shocks of high acceleration, in excess of 10^6 ms^{-2} , to large devices with sensitivities greater than 1000 pc/ms^{-2} [4]. Highly sensitive devices, on the other hand, are physically large so as to accommodate the increased seismic mass required to generate the high output. In all such cases, care must be taken when mounting accelerometers since they can be easily destroyed through over-tightening.

Table 2.1 provides a short summary of the area of application of each of the transducer types discussed above [4].

2.3 Mounting of sensors

The piezoelectric accelerometer are brought into contact with the vibrating body. There are various methods of fixation, which are illustrated in Figure (2.6) [5]. Out of these, the method employing a stud bolt is the best. Beeswax or even epoxy resin gluing is also very good. The quality of fixation depends on how much the fixation modifies the natural frequency of the transducer. The methods shown in Figure (2.6)a,b,c and d do not alter the operating frequency range of the transducer in practice. A fast method for attaching the sensor to bodies made of magnetizable material, like most electrical machines, is the magnetic mounting shown in Figure (2.6)e. Of course, this latter solution does not provide a mechanical coupling as good as a

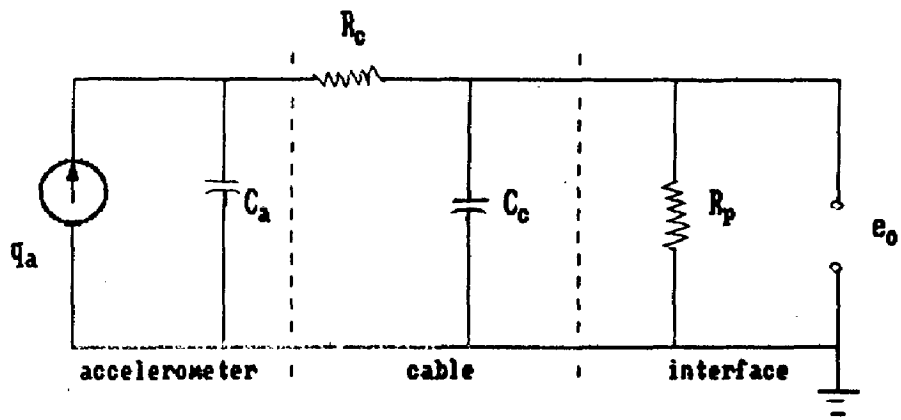


Figure 2.7 : Charge generator model of the piezoelectric accelerometer

bolted mounting, so the operating frequency range of the sensor is reduced to less than one third. For standard transducers, this range still gives a relatively high cut-off frequency (about 2 to 3 KHz) that meets the requirements of standard vibration measurements [5].

Feeler mounting is acceptable only up to 600 Hz. When the measuring system is set up, due attention is paid to avoid the formation of a ground loop through the sensor housing and the ground wire of the signal cable, since the ground current flowing through the instrument might invalidate the results of the measurements. An insulated bolt and an insulating disk under the sensor provides an acceptable solution as shown in Figure(2.6)c.

2.4 Charge generator model of the accelerometer with cable

A good approximation of the charge generator model for frequencies well below the natural frequencies is shown in Figure (2.7) [6]. The electrical characteristics of the connecting coaxial cable between the piezoelectric accelerometer and the signal conditioner is included in the mathematical representation of the complete accelerometer model, since these parameters directly affect the overall transfer function. The output voltage e_o for the system shown in figure(2.7) can then be expressed as

$$e_o = \frac{Ksx}{R_c C_a C_c s^2 + (C_c + C_a + C_a \frac{R_c}{R_p})s + \frac{1}{R_p}} \quad (2.2)$$

where, C_a = internal capacitance of accelerometer, R_c = cable resistance, C_c = cable capacitance, R_p = interface input resistance, K = proportionality constant, and X = deflection in the crystal.

The deflection X is related to the equivalent charge q_a as $q_a = D_q X$ where D_q is the characteristic constant of the piezoelectric material.

The basic limitation of this type of accelerometer can be inferred from Eqn. (2.2) wherein

$$e_o(j\omega) \Big|_{\omega=0} = 0 \quad (2.3)$$

To compensate for this lack of dc response, several schemes of circuitry are possible in the signal conditioner that interfaces with the accelerometer. One approach is to integrate the signal so that integrated output

$$e_c(s) = e_o(s)/s \quad (2.4)$$

A second method (voltage amplifier) uses an amplifier with an extremely high input impedance to minimize the loading. Assuming that $R_c = 0$ Eq(2.2) becomes

$$e_o(s) = \frac{Ks x}{(C_a + C_c)s + \frac{1}{R_p}} \quad (2.5)$$

If the input impedance of the signal conditioning stage R_p is very large, the circuit provides a very low cut-off frequency.

In terms of the input acceleration \ddot{y} , the overall transfer function (including the dynamic response of the spring mass system) of the complete system may be shown to be

$$\frac{e_o}{y} = \frac{Ks}{\{R_c C_a C_c s^2 + (C_c + C_a + C_a \frac{R_c}{R_p})s + \frac{1}{R_p}\} (s^2 + 2\zeta\omega_n s + \omega_n^2)} \quad (2.6)$$

2.5 Signal conditioners

The signal conditioning system employing a piezoelectric transducer comprises the cable connecting the transducer to the amplifier, signal amplifier and signal modifier.

The input cable connecting the transducer is usually low

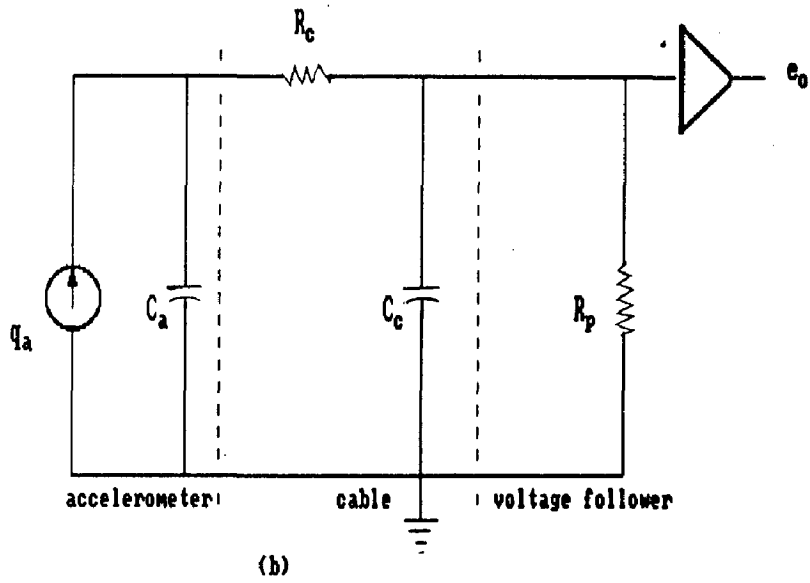
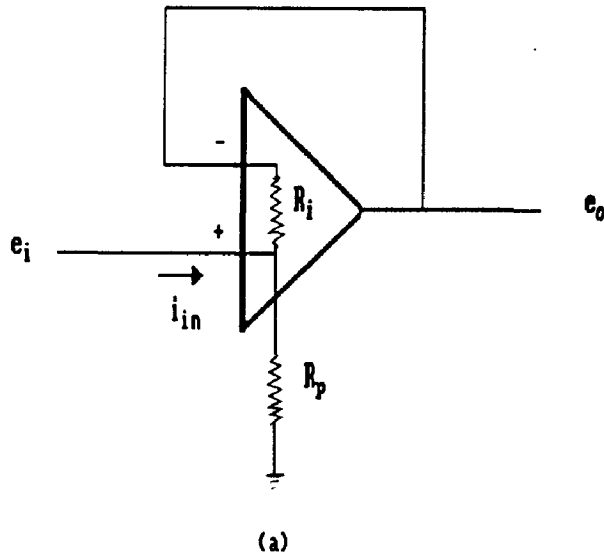


Figure 2.8 : Equivalent representation of : (a) voltage follower
 (b) accelerometer and voltage follower combination

capacitance, low noise, low microphonic coaxial cable with high insulation resistance between the leads.

The output signal of the piezoelectric crystal, that is the potential difference across the crystal armature, is directly proportional to the acceleration of the transducer housing and, consequently, to that of the vibrating body. This signal, however, is very small, and so it requires amplification. There are two general types of pre-amplifier, namely voltage amplifier and charge amplifier. The output voltage of voltage amplifiers is proportional to the input voltage. These have simple construction and are relatively cheap. At low frequencies, these are quite sensitive to the resistance and capacitance of the cable connecting the sensor with the pre-amplifier. In a charge amplifier, the output voltage is proportional to the charge on the sensor. This pre-amplifier is more expensive but it is insensitive to the cable impedance, so we can use longer cables to connect the sensor with the amplifier and the pre-amplifier may be located far away from the vibrating body. At the same time, the pre-amplifier also serves as an impedance transformer, matching the sensor with the instrument following the amplifier.

2.5.1 Voltage amplifier

The voltage follower amplifier suitable for piezoelectric devices is shown in Figure (2.8)a, which is designed for non-inverting operation with unity gain [6]. Figure (2.8)b shows the equivalent circuit of the accelerometer along with the connecting cable and voltage follower. This configuration takes advantage of the high input impedance of the amplifier, which results in very low leakage path across the transducer and cable capacitance. The low frequency response of the transducer, cable, and amplifier combination is thus improved. With operational amplifiers having FET input stages, an input impedance of the order of 10^{11} ohms can be obtained.

If the other characteristics of the amplifier are assumed to

be ideal, then the input current of the amplifier i_{in} (figure 2.8a) can be determined as

$$\frac{e_i - e_o}{R_i} + \frac{e_i}{R_p} = i_{in} \quad (2.7)$$

where R_i is the internal input resistance of the amplifier and R_p is the resistance to ground. Since the open loop dc gain of the amplifier is very large (order 10^6), it can be shown that $e_o \cong e_i$. Then

$$\frac{e_i}{R_p} = i_{in} \quad (2.8)$$

Under such assumptions, the voltage follower can be approximated by the equivalent circuit shown in Figure (2.8)b. Thus the complete transfer function given by Eq. (2.6) can be rewritten (neglecting cable resistance) as

$$e_o(s) = \frac{Ks}{(C_c + C_a)s + \frac{1}{R_p}} \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \ddot{y} \quad (2.9)$$

And equation (2.9) can be further written as

$$e_o(s) = \frac{K}{(C_c + C_a)} \frac{(C_c + C_a) R_p s}{R_p (C_c + C_a) s + 1} \frac{1}{(s^2 + 2\zeta\omega_n s + \omega_n^2)} \ddot{y} \quad (2.10)$$

$$= \left[\frac{K}{C} \right] \left[\frac{\tau_1 s}{\tau_1 s + 1} \right] \frac{1}{s^2 + 2\xi\omega_n^2 s + \omega_n^2} \ddot{y} \quad (2.11)$$

Where $C = C_c + C_a$, and $\tau_1 = R_p(C_c + C_a)$

It can be seen from Eq. (2.11) that the static sensitivity is lower for higher values of cable and piezoelectric crystal capacitances. Apart from this, the steady-state response is zero. Thus, static response cannot be obtained in this mode of operation. In practice, the value of R_p and C are suitably chosen to obtain reasonably good sensitivity as well as required low frequency response. The high frequency limitation in this

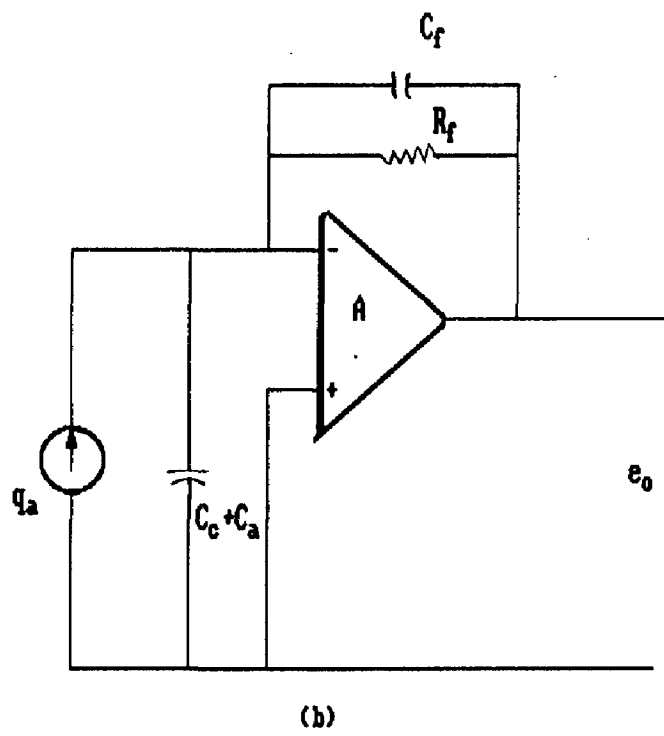
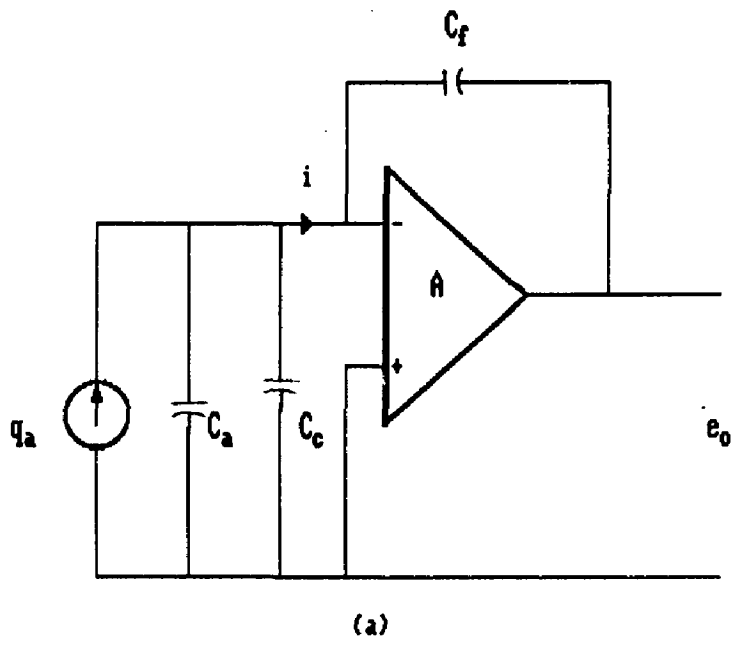


Figure 2.9 : Charge amplifier (a) basic configuration;
 (b) modified configuration

configuration is imposed by the response of the spring-mass system.

2.5.2 Charge amplifier

In the voltage amplifier configuration, it is seen that the frequency response depends on the cable capacitance. It means that for each cable length only a particular calibration is valid. To avoid such difficulty, a charge amplifier configuration, where the charge generated by the device is converted to a proportional output voltages, is preferred.

The circuit arrangement of a basic charge amplifier is as shown in Figure (2.9a) [6]. Assuming ideal conditions under which there would be no current flow into the input terminal of the operational amplifier,

$$\frac{dq}{dt} = i = k\dot{x} \quad (2.12)$$

where x is the deflection of the crystal due to input acceleration and K is a proportionality constant. The voltage across the feedback capacitor C_f (under ideal conditions) can be expressed as

$$e_o = \frac{-1}{C_f} \int i dt \quad (2.13)$$

$$= \frac{-kx}{C_f} \quad (2.14)$$

As mentioned earlier, the above equation represents an ideal condition. In practice, such a circuit would go to saturation because of the bias currents through the operational amplifier, charging the capacitor C_f . To prevent saturation, a resistor R_f providing an alternate path is connected across C_f as shown in figure (2.9b). The transfer function for the circuit is written as

$$\frac{e_o}{x}(s) = \frac{Ks\tau_2}{s\tau_2 + 1} \quad (2.15)$$

where,

$$\tau_2 = R_f C_f$$

The overall transfer function is obtained as,

$$\frac{e_o}{\ddot{y}} = K \frac{s\tau_2}{s\tau_2 + 1} \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (2.16)$$

Eq. (2.16) is identical in form with that obtained for a voltage amplifier and piezoelectric accelerometer combination given by Eq. (2.11). This also exhibits similar low frequency response limitations. However, the distinct advantage here is that both static sensitivity and low-frequency response are independent of the cable capacitance and crystal capacitance. Low-frequency response depends only on the circuit components C_f and R_f . In this case, the static sensitivity is not lost even with longer cables.

In general, it is advisable to include signal filtering after the signal amplifier stage (either voltage follower or charge amplifier) in the instrumentation system. It consists of a low-pass filter adjusted to the cut-off frequency of the most frequency-restrictive component in the system, which is usually the accelerometer. The signal conditioner is generally designed with a low-pass filter network to eliminate the accelerometer resonance errors and influences of extraneous noise signals.

Signal transmission from the filter output to the recorder or other display units is usually through a cable whose effect can be ignored as the signals are of high amplitudes are fed from low impedance sources.

CHAPTER-3

VIBRATION OF ROTATING ELECTRICAL MACHINES

3.1 GENERAL

The electrical machine, its support and base structure, and the load connected to its shaft form a complete mechanical system. The vibration frequency of the overall system depends on many factors. The machine is free to vibrate at its own natural frequency, or can vibrate at any other frequency. This results in a complex vibration which may be unacceptable and its progressive increase may damage the machine and may even result in total failure. The vibration in electrical machines may be in the following areas[7]:

- (i) the stator core may respond to the attractive force developed between rotor and stator,
- (ii) there may be response of the stator end windings to the electromagnetic force on the conductors,
- (iii) these may be due to the dynamic behaviour of the rotor,
- (iv) the response of the shaft bearings to vibration transmitted from the rotor may induce vibration.

These four areas are inter-related. The bearing mis-alignment or wear may quite easily result in eccentric running which in turn may stimulate the vibrational modes of the stator[7].

3.2 STATOR CORE RESPONSE

The stator and its support structure comprise a thick-walled cylinder which is slotted at the bore and rests inside a thin-walled structure. The unit may or may not be cylindrical [8].

The forces acting on the stator core are a result of the

interaction between the air gap flux wave and the currents flowing in the windings embedded in the stator slots. The forces acting on the end winding are due to the interaction between the end leakage flux and the winding currents. The precise nature of the applied force is a function of the form of the current distribution and the geometry of the air gap and end region. Disturbances to either, due to rotor eccentricity or damaged areas of the rotor for example, alter the harmonic components of the force wave and initiate a different response from the stator core, particularly if the applied forces stimulate any of the natural modes of the system [8].

3.3 Rotor dynamics

The motion of the rotor in response to transverse unbalanced forces, and to torsional forces, applied either due to system disturbances on the electrical side or to rotor defects within the machine may contribute to vibrations.

3.3.1. Transverse response

To examine the response of the rotor to unbalanced forces, a distinction is drawn between rigid and flexible rotors. Rigid rotors are considered as a single mass acting at the bearings. It has been shown that the rotor and its bearings can be modelled by the differential equation[9],

$$(M + m_s)X + Cx + Kx = mr\omega^2 \quad (3.1)$$

where,

M = mass of the rotating disc at the bearings

m = equivalent unbalance mass on the shaft,

m_s = support system mass,

r = effective radius of the equivalent unbalanced mass,

C = damping constant of the support system, and

K = stiffness of the support system.

The peak displacement for sinusoidal motion is given by the solution to (3.1), and is [9],

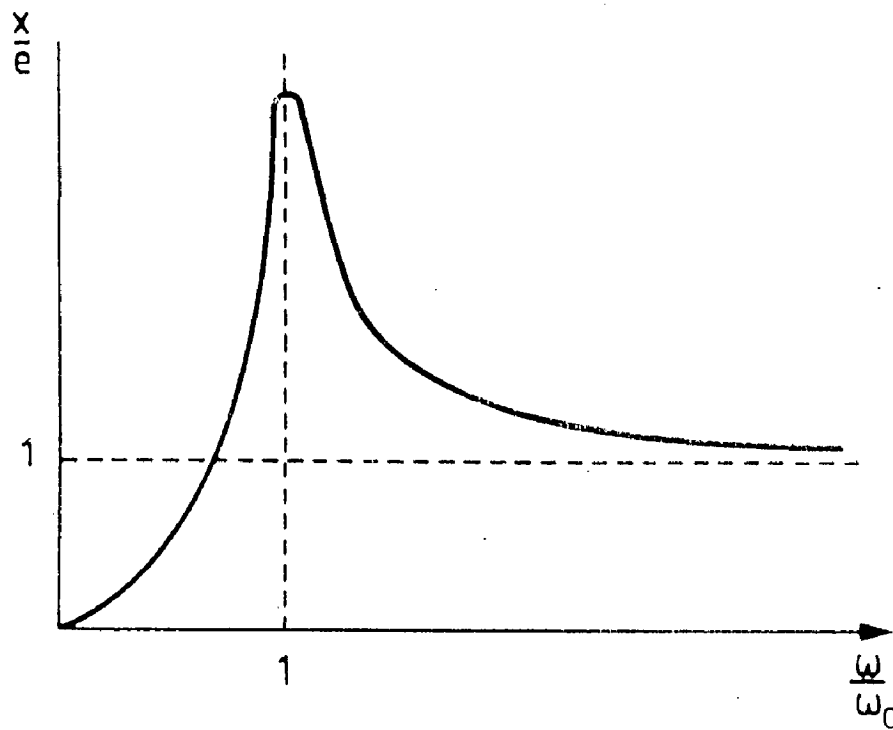


FIGURE 3.1 : DISPLACEMENT PER SPECIFIC UNBALANCE VERSUS NORMALISED FREQUENCY

$$X = \frac{mr \left(\frac{\omega}{\omega_0} \right)^2}{(M+ms) \sqrt{\left[1 - \left(\frac{\omega}{\omega_0} \right)^2 \right]^2 + 4D^2 \left(\frac{\omega}{\omega_0} \right)^2}} \quad (3.2)$$

Here, ω_0 the natural frequency of the rotor support system. The natural frequency is given as

$$\sqrt{\left(\frac{K}{M + ms} \right)}, \quad \text{and} \quad D + \frac{C}{2 \sqrt{K(M + ms)}}$$

By dividing the displacement the specific unbalance e , we have

$$e = \frac{mr}{M} \quad (3.3)$$

Figure (3.1) shows the displacement per specific unbalance versus normalized frequency.

In case of long slender rotors operating at higher speeds (as in two pole machines), specifically in case of the larger turbogenerators which have restricted rotor raddi, the foregoing analysis is insufficient. In these case, the distribution of unbalance is considered. It has been shown that it is possible to calculate the natural frequencies for general problems of a rotor with a flexural rigidity EI , and mass per unit length m , which are both function of position (x) [9]. The displacement u , for any x , is given by the solution of ,

$$\frac{d^2}{dx^2} \left\{ EI(x) \frac{d^2 u}{dx^2} \right\} - \omega^2 m(x) u = \sum_{n=1}^{\infty} f_n \quad (3.4)$$

where f_n is the n th unbalanced force. And

$$\omega_n^2 = \int m(x) g_n^2(x) dx, \quad (3.5)$$

with $g_n(x)$ the n th solution of equation (3.4).

The solution for coupled system comprising several rotors including the electrical machine and the machine it is driving or driven by is extremely complex. It is usual to assume that the stiffness of the couplings between rotors are low, and therefore, decouple each rotor, allowing them to be considered individually as described above [9].

3.3.2 Torsional response

The torsional oscillatory behaviour of a turbine generator, is complicated. The generator is effectively linking a complex steam raising plant and prime mover to a large interconnected electrical network in which huge quantities of energy are being transported. The possibility for forced torsional oscillation of the rotor of the generator is clearly high because of its great length and relatively small radius [10]. The nature of such oscillations depends upon the form of disruptions that occur in either the mechanical or the electrical system. Disturbances in the electrical system are very important. It has been established that under some circumstances, they can give rise to conditions which may consume shaft life due to fatigue.

3.4 Bearing response

The bearings are mainly responsible for the transfer of the rotor force to the stator. It is important to be able to determine the vibrational response of the bearings to these external forces so that there may not be any confusion with vibrational frequencies generated by the defects in the bearings. External forces may also result in a relative vibration of the rotor with respect to the housing and an absolute vibration of the complete bearing housing[11].

This action needs consideration for both rolling element bearings and oil-lubricated sleeve bearings.

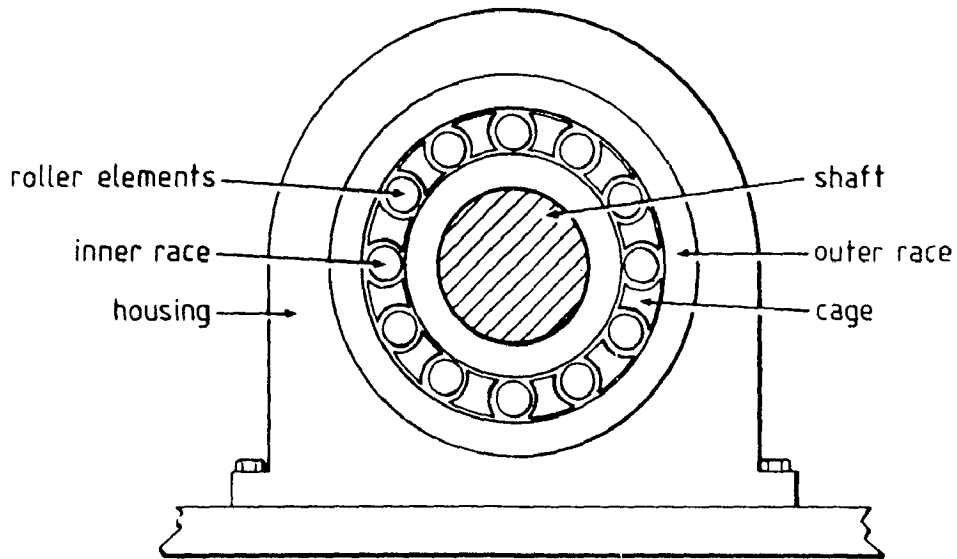
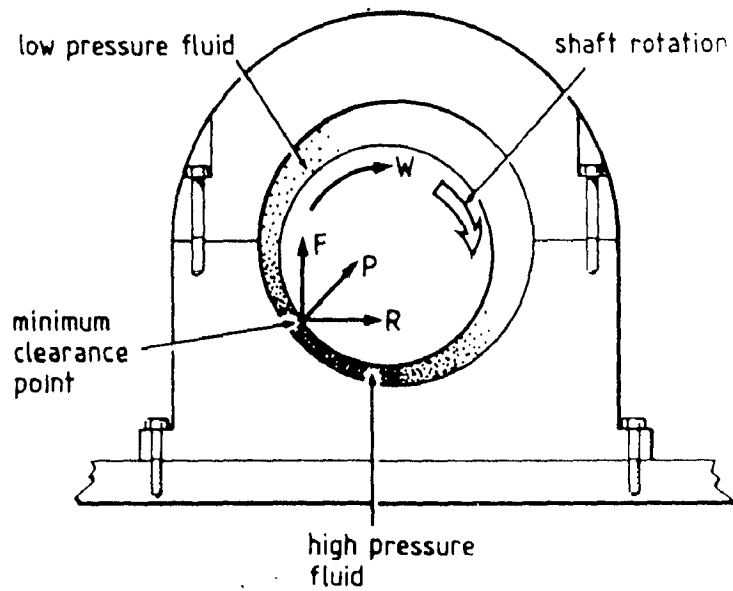


FIGURE 3.2 : ROLLING ELEMENT BEARING ASSEMBLY



- R = Reaction
- F = Destabilizing components
- P = Pressure
- W = Whirl force

FIGURE 3.3 : FORCES ACTING UPON A SHAFT IN A SLEEVE BEARING

3.4.1 Rolling Element Bearings

The rolling element bearings produce very precisely identifiable vibrational frequencies. Due to there oil film, the relative motion between the housing and the shaft is small. It is possible to detect the vibrations associated with the bearings using an accelerometer mounted directly on the bearing housing.

The characteristic frequencies of such bearings depend on the geometrical size of the various elements and are reported by many works. Table 3.1 summarizes these frequencies and their origins [11]. A schematic view of a typical rolling element bearing is shown in figure (3.2).

Besides the frequencies given in Table 3.1, there are also higher frequencies generated by electric deformation of the rolling elements, and the excitation of the natural modes of the rings that comprise the inner and outer races. These effects are secondary to the principal components.

The magnitudes of the components given in Table 3.1 are often lost in the general noise background when the degree of damage is small, but because of their precise nature they present an effective route for monitoring progressive bearing degradation. A simple instrument could be devised using an accelerometer mounted on the bearing housing to detect the amplitude of vibration at these characteristic frequencies. Once the characteristic frequencies are known, it is possible to enhance the performance of the instrument by the use of highly selective filters and weighting functions so as to be able to identify bearing faults at an earlier stage.

When monitoring the vibration due to rolling element bearing, it is always prudent to try and achieve good base-line data. This is because once the bearing becomes significantly worn, the spectrum of vibration it emits becomes more random again, although at a much higher level than the base value for a good bearing.

Defect	Frequency	Comment
Outer race	$\frac{n}{2} \cdot \frac{N}{60} \cdot (1 - \frac{d}{D} \cos\phi)$	Ball passing frequency on the outer race.
Inner race	$\frac{n}{2} \cdot \frac{N}{60} \cdot (1 + \frac{d}{D} \cos\phi)$	Ball passing frequency on the inner race.
Ball defective	$\frac{D}{2d} \cdot \frac{N}{60} (1 - (\frac{d}{D})^2 \cos^2\phi)$	Ball spin frequency.
Train defect	$\frac{1}{2} \cdot \frac{N}{60} (1 - \frac{d}{D} \cos\phi)$	Caused by irregularity in the train.
<p>n = number of balls N = rotational speed in rev/min d = ball diameter D = ball pitch diameter ϕ = ball contact angle with the race</p>		

TABLE 3.1 CHARACTERISTICS FREQUENCY OF ROLLING ELEMENT BEARINGS

Clearly if no base line is available and no history has been built up, it is possible for specific defects to be masked by the increase in general background level [11].

Machinery exhibit a small degree of unbalance. This tends to modulate the characteristic frequencies of the bearings and produces side bands at the rotational frequency.

Vibration monitoring is obviously highly suitable for monitoring the performance of rolling element bearings and has gained wide acceptance throughout the industry.

3.4.2 Sleeve Bearings

The shaft is supported by a fluid film pumped in under high pressure between the bearing liner and the shaft by the motion of the shaft in sheer bearings.

Now because of the compliance of the oil film and the limited flexibility of the bearing housing itself, vibrations measured at the housing are of low amplitude [12]. Also, because the liners of the bearing will inevitably be a soft material such as white metal, small defects are difficult to identify by measuring the absolute vibration of the housing. These factors point to the use of displacement transducers as the effective tool. But these are useful at the lower frequencies only. Higher frequencies (above 3 times the rotational frequency say) are best measured absolutely with an accelerometer mounted on the bearing housing.

It is worth keeping in mind, however, that as the bearing is more heavily loaded, due to an increase in rotor load, the thickness of the oil film will decrease with a commensurate loss in flexibility. This increases the vibration detectable at the bearing and will allow more information to be derived from the measurement.

A much more important cause for concern in the sleeve bearing

is the onset of instability in the oil film. This can result in oil whirl and subsequently oil whip, in response to unusual loading of the bearing. Figure (3.3) shows the forces acting upon the shaft in a sleeve bearing, and illustrates that the shaft is supported by a wedge of oil just at the point of minimum clearance.

The oil film circulates at half the shaft speed. But because of the pressure difference on either side of the minimum clearance point, the shaft precesses at just below half speed. This motion is called oil whirl and is a direct result of the pressure difference which is due to viscous loss in the lubricant.

Instabilities occur when the whirl frequency corresponds to the natural frequency of the shaft. Under such conditions the oil film may no longer be able to support the weight of the shaft [12].

Details of the mechanisms involved in oil whirl, and its development into the more serious instability called oil whip, which occurs when the shaft speed is twice its natural frequency [12].

Thus care must be taken such that that either the machine does not operate at a speed higher than twice the first critical speed of the rotor shaft, or if it must, then oil whirl must be suppressed.

CHAPTER - 4

VIBRATION MONITORING OF ROTATING ELECTRICAL MACHINES

4.1 OVERALL LEVEL MONITORING

This simple form of monitoring is the most commonly used technique for vibrations measurements. But as an aid to the diagnosis of fault in electrical machines, it may have fairly limited utility. In this method, the measurement taken is simply the rms value of the vibration level over a preselected band width. The usual band width is 10 Hz to 1 kHz, or 10Hz to 10kHz, and in practice the measurement of parameter is vibration velocity taken at the bearing cap of the machine under surveillance [4]. The technique is in use because over the years a considerable statistical base regarding machinery failures has been built up. This has resulted in the publication of recommended running vibration standards [13]. These standards give diagnostic information as an indication of overall health. Many operators use a strategy, based on such information, to aid maintenance scheduling. The guidance given by vibration standard VDI 2056 is given in Table (4.1) [13]. These criteria are based solely on machine rating and support systems and utilize a 10Hz to 1 kHz bend width. Essentially it is recommended that when vibration levels change by 8 dB or more, care must be exercised, and when the change exceeds 20 dB action should follow. These limits can be relaxed in case subsequent frequency analysis shows that the cause of the increase in level is due to a rise in the higher frequency components. In such cases change of 16 dB and 40 dB respectively are more appropriate figures. Another useful set of criteria is given in the canadian government specification CDA/MS/NVSH107 [4]. This specification relates primarily to measurements taken on bearings, and it is here that overall level measurement is most commonly employed. This specification has a broader bandwidth than VDI 2056, namely 10 Hz to 10 KHz, but still relies on overall velocity vibration measurement. Table (4.2)

TABLE 4.1 : VIBRATIONAL STANDARD VDI 2056

Vibration Severity Criteria (10Hz-1kHz)

		VIBRATIONAL VELOCITY (m/s)	
Not Permissible	Not Permissible	Not Permissible	45
			28
Just Tolerable 8 dB(x2.5)	Just Tolerable	Just Tolerable	18
			11.2
Allowable	Allowable	Allowable	7.1
			4.5
Good Small machines up to 15kw	Good	Good	2.8
			1.8
Group K	Group M	Group G	1.12
			0.71
			0.45
			0.28
			0.18

**TABLE 4.2 : VIBRATION LIMITS FOR MAINTENANCE AS GIVEN
IN CANADIAN GOVERNMENT STANDARD CAD/MS
/NUSH107**

Type of Plant	New Machines		Worn Machines (full load operation) *	
	hr life 100-1000	hr life 1000-10000	Service Level	Overhaul immediately -1
	All measurements in mm sec			
Boiler auxiliaries	1.0	3.2	5.6	10.0
Large steam turbines	1.8	18.0	18.0	32.0
Motor-generator sets	1.0	3.2	5.6	10.0
pump drives	1.4	5.6	10.0	18.0
Fan drives at far end	1.0	3.2	5.6	10.0
Motors (general)	0.25	1.8	3.2	5.6

* In this column the levels must not be exceeded in any octave band.

TABLE 4.3 : DIAGNOSIS USING OVERALL LEVEL MEASUREMENTS

Value of F	Trend	Defect
F < 1	Decreasing	Oil whirl.
F = 1	Steady	Unbalance-indicative of eccentricity or perhaps faulty rotor cage.
F > 1	Increasing	Misalignment-static eccentricity.

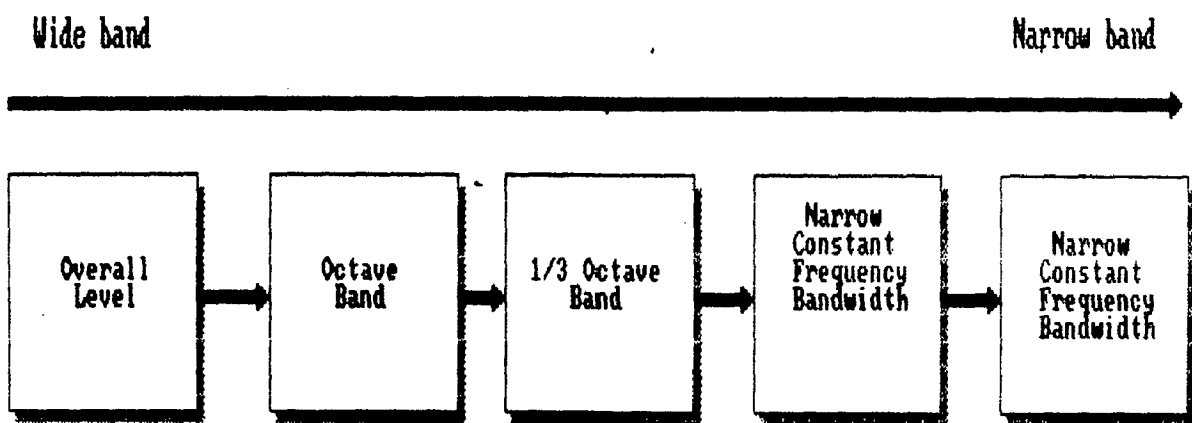


Figure 4.1 : Levels of spectral analysis

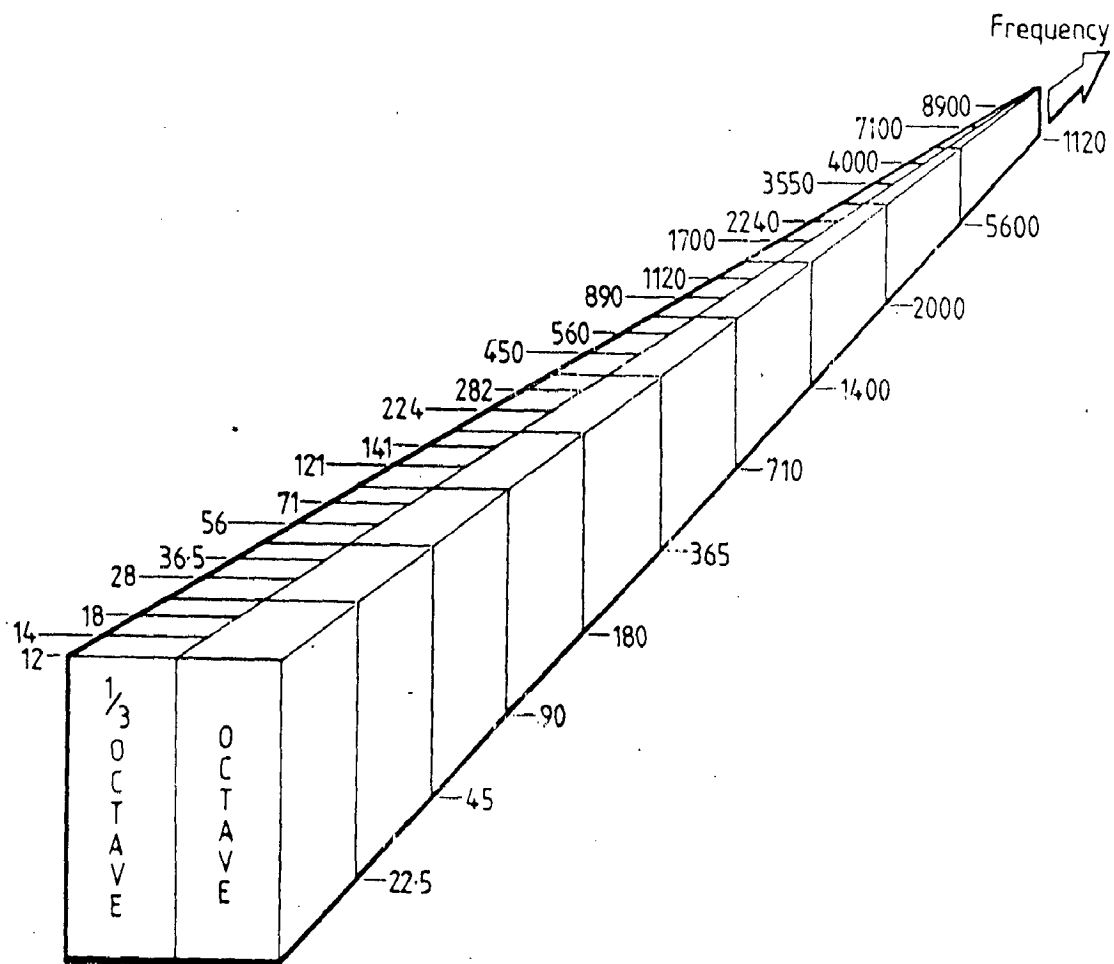


TABLE 4.4 : OCTAVE AND THIRD OCTAVE BANDS

gives a section of the specification related to electrical machines. The overall level technique is simple. It requires only the simple instrumentation. It also provides an ideal method for use with portable instruments, but it makes heavy demands upon technical personnel. The sensitivity of the technique is also low, particularly when a defect is at an early stage, and there is little help on offer to aid diagnosis without further sophisticated techniques being employed [4].

However, it has been as indicated that it may be possible to effect a limited diagnosis by taking two overall level measurement: V_a , the peak velocity, and x the peak to peak displacement [4.2]. These quantities are then used to define the parameter

$$F = \frac{0.52 N x}{V_a} \quad (4.1)$$

Where, N is the speed, in rev/min, of the machine. Accordingly, the interpretations suggested by Table (4.3) may be appropriate.

4.2 Frequency spectrum Monitoring

There are various levels of spectral analysis, these may be regarded as a continuum extending from the overall level reading to the narrow band with constant frequency bandwidth presentation, as shown in Figure (4.1) [14].

In the octave band and 1/3 octave band techniques, the spectrum is split into discrete bands, as defined by Table 4.4. The bands are by definition, such that when the frequency is scaled logarithmically, the bands are of equal width. The constant percentage band is one which is always the same percentages of the central frequency, whilst the constant frequency band width is an absolutely fixed band width form of analysis which can give very high resolution, provided the instrumentation is of a sufficiently high specification.

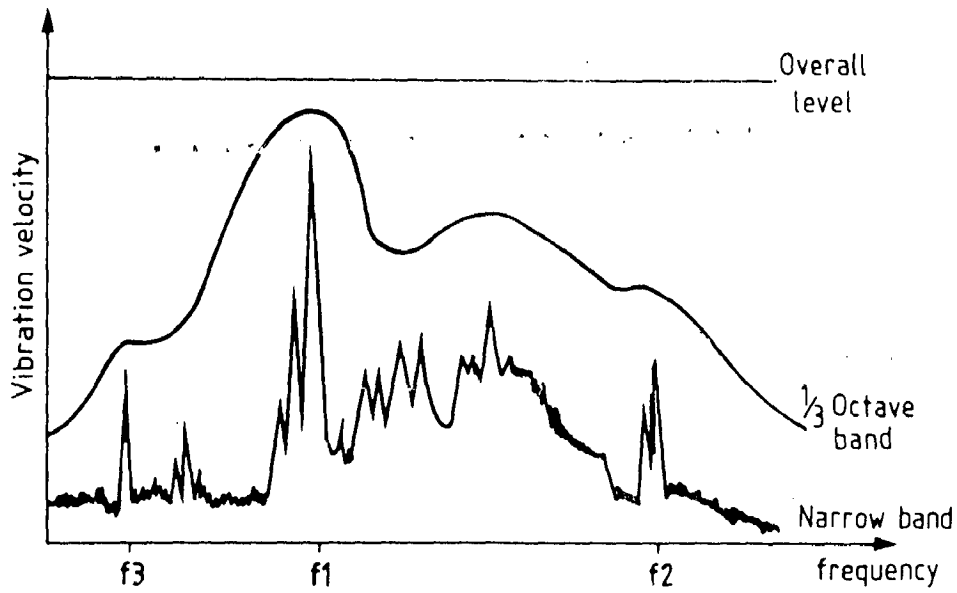


FIGURE 4.2 : EFFECT OF CHANGE IN BANDWIDTH ON SPECTRAL RESPONSE

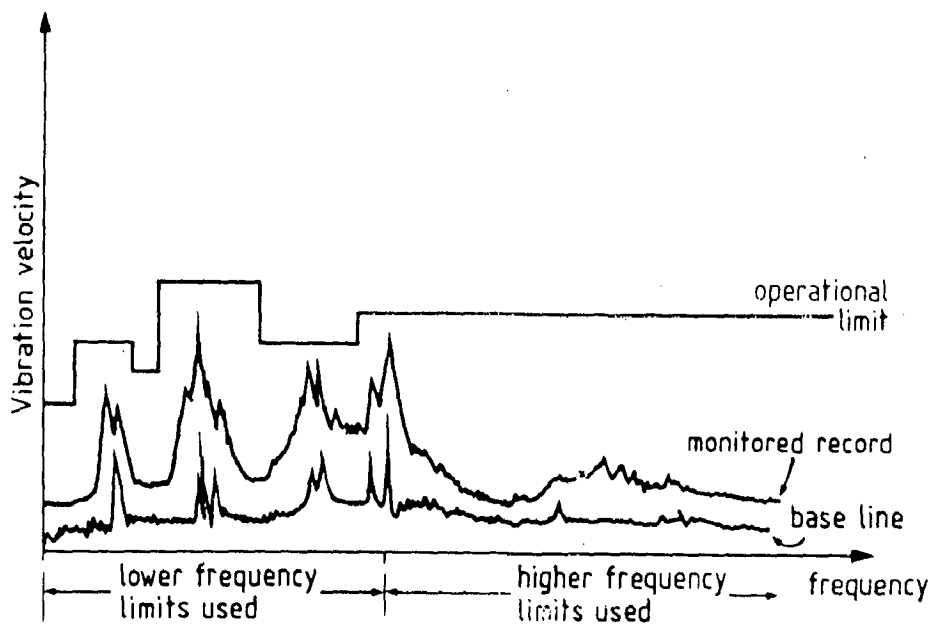


FIGURE 4.3 : OPERATIONAL ENVELOPE AROUND A SPECTRAL RESPONSE

The effect that the change of bandwidth has on the processed signal output highlights precisely why the narrow band techniques are superior to the overall level technique as diagnostic tools [14]. For example, a certain transducer may provide an output that may be interpreted in the ways shown in Figure 4.2. It is apparent that the components around frequency f_1 dominate the overall level reading, and the shape of the 1/3 octave result. Important changes, say of f_2 and f_3 , or even the presence of other components, could go largely unnoticed except by the use of narrow band methods. This is crucial because the flexibility of the system may be such that important components are masked by those closer to resonance in the mechanical structure of the machine.

The narrow band spectrum also allows the operator to trend the condition of the machine most effectively. This requires that an initial base-line spectrum is taken and subsequent spectrum are compared with it. The use of digitally derived spectra means that the results of such comparisons can be computed quickly since the spectra reduce to a simple sequence of numbers at discrete frequencies, as closely spaced as required within the limitations of the instrumentation. In this way criteria such as VDI 2056 can be applied for each frequency.

Because of the large amounts of data generated using narrow band methods, it is frequently convenient to predetermine the operational limits, on the basis of one of the vibration standards, and to construct an operational envelop around the base line spectrum. This can take account of the wider limits allowable at higher frequencies. It can also be used to automatically flag warnings when maintenance limits are reached. The basic of this technique is shown in Figure (4.3) [14].

The techniques discussed up to now are general types. In order to identify not just unsatisfactory overall performance, but to pinpoint specific problems, it is necessary to examine discrete frequencies, or groups of frequencies. Induction motors in particular require a high degree of frequency resolution applied

TABLE 4.5 : VIBRATION FREQUENCIES RELATED TO SPECIFIC ELECTRICAL MACHINE FAULTS

Fault type	Important Frequencies	Comments
Unbalanced rotor	f_r	Very common also causes unbalanced magnetic pull which gives $2f_r$ vibration.
Misalignment of rotor shaft	$f_r, 2f_r, 3f_r, 4f_r$	Also manifests as static eccentricity; therefore see components generated from this source, below.
Generates looseness of shaft in bearing housing	$f_r, f_r, 2f_r$	Generates a clipped time waveform; therefore produces a high number of harmonics.
Oil whirl and whip in sleeve bearings	$(0.43 \text{ to } 0.48)f_r$	Pressure fed bearings only.
Rolling element bearing damage	See equations in Table 3.1 for exact frequencies in the range 2-60 kHz due to element resonance.	Common source of vibration. Bearing faults can also be diagnosed using the Shock Pulse Method.
General electrical problems	nf_r, nf_s	A problem can usually be identified as having electrical origins by simply removing the supply.

TABLE 4.5 (contd.)

Fault type	Important Frequencies	Comments
Broken rotor bars in induction machines	$f_r \pm 2sf_s$	<p>If the fault disappears then the problem is associated with electrical aspect of the machine.</p> <p>May be difficult to detect due to the low level. Speed, leakage field or current changes may be preferred as a monitoring parameter.</p>
Stator winding faults	$f_s, 2f_s, 4f_s$	Difficult to differentiate between fault types using vibration monitoring alone.
D.C. machines commutator faults	kf_r	Unbalanced rotor components can also be generated.
<p>f_r = rotational frequency f_s = supply frequency n = an integer k = number of commutation sequence s = slip</p>		

to their vibration signals since the speed of rotation is close to the electrical supply frequency. This tends to generate side bands spaced at s and $2s$ around the harmonics of the supply frequency, where s is the slip frequency of the machine.

Obviously, vibration can occur in electrical machinery as a result of either electrical or mechanical action. In order to summarise the dominant frequencies for a given defect, Table 4.5 has been compiled, principally by distilling the information [13,3].

4.3 Special Vibration Monitoring Technique

In addition to the more conventional vibration monitoring techniques described in section 4.2 and 4.3, there are a number of specialised techniques which have established themselves as powerful diagnostic tool.

Cepstrum Analysis

Mathematically the cepstrum, $C(\tau)$ of a function is described as the inverse Fourier transform of the logarithm of the power spectrum of the function; i.e. if we define the power spectrum $P(f)$ of a function $g(t)$ as

$$P_{gg}(f) = F\{g(t)\}^2 \quad (4.2)$$

then the corresponding cepstrum is

$$C(\tau) = F^{-1}\{\log P_{gg}(f)\} \quad (4.3)$$

Where F and F^{-1} represent the forward and backward Fourier transforms.

The dimension of the parameter τ is time. The magnitude of the cepstrum with respect to time intervals is displayed in the same way as the spectrum is illustrated with respect to frequency.

The use of the cepstrum has found favour in examining the behaviour of gear boxes because such items of equipment tend to produce many families of side bands in their vibration spectra, due to the variety of meshing frequencies and shaft speeds that may be present [15]. The cepstrum essentially highlights periodicity in complicated signals, and hence identifies clearly various families of side bands. The identification of various side bands in a rich signals may be practically impossible using spectral analysis, but these are picked up by cepstrum as shown in figure(4.4). This figure provides an excellent review of the use of cepstrum analysis applied to gearboxes [15].

We note in passing that although the horizontal scales of the cepstrum are in seconds it is usual practice to refer to the horizontal quantity as quefrequency, and the peaks in the cepstrum as the rahmonics. This is done to firmly identify the methodology with that of spectral analysis.

Clearly if this technique can identify side bands with ease, then it may hold significant possibilities for the identification of faults in induction machines, particularly when they are fed from harmonic rich inverters. It is a 'post' spectral analysis tool, in much the same way as spectral analysis is generally employed once one's suspicions are aroused by anomalies in time domain signals taken using overall level monitoring.

CHAPTER - 5

FAST FOURIER TRANSFORM (FFT)

5.1 General

The FFT is a computationally efficient method for computing the discrete Fourier transform (DFT) that can compute the discrete Fourier transform much more rapidly than other available algorithms.

5.2 The Discrete Fourier Transform

The sine and cosine components of a periodic waveform are determined by the discrete Fourier transform (DFT). These components find more use than the shape of the waveform itself. The waveform $f(t)$ is sampled at N time intervals $t_0 = 0, t_1 = T, t_k = kT, \dots, t_{N-1} = (N-1)T$. The full sampling interval is $S = NT$ (Figure 5.1).

By using the notation $f_k = f(t_k)$, the DFT of f_k is defined as

$$F_n = \sum_{k=0}^{N-1} f_k e^{-i\pi rk/N} \quad (5.1)$$

It may be noted that

$$e^{i\phi} = \cos \phi + i \sin \phi, \quad e^{-i\phi} = \cos \phi - i \sin \phi, \quad e^0 = e^{i2\pi} = 1, \quad \text{and} \quad e^{-i\pi} = -1.$$

The significance of the DFT coefficients is that F_0 is the Fourier coefficient at frequency 0 (dc component), F_1 is the Fourier coefficient at frequency 1 (1 cycle per S), and F_n is the Fourier coefficient at frequency n (n cycles per S).

To see that this is so, let us calculate a few Fourier coefficients [16] :

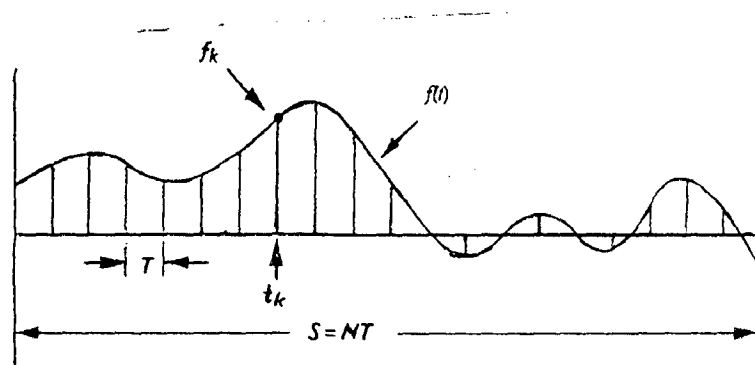


FIGURE 5.1 : WAVEFORM SAMPLED AT UNIFORM TIME INTERVALS T .

$$F_0 = \sum_{k=0}^{N-1} f_k, \quad (\text{the sum of all amplitudes})$$

Consider the case where $f_k = C$ (a constant). Then $F_0 = NC$ and all other Fourier coefficients are zero. Let us next consider the example of a sine wave with M complete cycles per sampling interval S :

$$f_k = \sin (2\pi kM/N)$$

$$F_n = \sum_{k=0}^{N-1} \sin (2\pi kM/N) [\cos (2\pi kn/N) - i \sin (2\pi kn/N)]$$

Due to the orthogonal nature of the sine and cosine series for this f_k ,

$$F_M = \sum_{k=0}^{N-1} i \sin^2 (2\pi kM/N) = -\frac{iN}{2}$$

$$F_{N-M} = \sum_{k=0}^{N-1} i \sin^2 (2\pi kM/N) = -\frac{iN}{2}$$

and all the other Fourier coefficients are zero. It can be seen that n th Fourier coefficient describes the amplitude of any sine wave component having n complete cycles per sampling interval. The coefficient a_j and b_j are related as

$$f_k = \sum_{j=0}^{N-1} a_j \cos (2\pi jk/N) + b_j \sin (2\pi jk/N)$$

NOW let us perform the discrete Fourier transform of f_k :

$$F_n = \sum_{k=0}^{N-1} \left[\sum_{j=0}^{N-1} a_j \cos (2\pi jk/N) + b_j \sin (2\pi jk/N) \right] K$$

$$\times [\cos (2\pi nk/N) - i \sin (2\pi nk/N)]$$

since

$$\begin{aligned}
 F_{(N+n)} &= \sum_{k=0}^{N-1} f_k e^{-i2\pi k} e^{-2\pi kn/N} \\
 &= \sum_{k=0}^{N-1} f_k e^{-i2\pi kn/N} = F_n
 \end{aligned}$$

Between $N/2$ and N Samples per S , we have the following results :

$$\begin{aligned}
 F_{(N-n)} &= \sum_{k=0}^{N-1} f_k e^{-i2\pi k} e^{+2\pi kn/N} \\
 &= \sum_{k=0}^{N-1} f_k e^{+i2\pi kn/N}
 \end{aligned}$$

If f_k is real, then $F_{(N-n)} = F_n^*$ and $F_{N/2}$ is real (* denotes the complex conjugate). $F_{(N/2)}$ is the Fourier coefficient at frequency $N/2$ (1 cycle per $2T$). This is the highest frequency that the DFT can determine. All Fourier coefficients for higher frequencies are either equal to or the complex conjugates of coefficients for lower frequencies. Thus, there are only $N/2$ independent Fourier coefficients.

If the sampling frequency is inadequate, higher-frequency components of the true waveform $f(t)$ will appear as lower frequency components in the DFT. This is called frequency aliasing. There is no way to correct the data after the sampling has been performed. The usual solution to this problem is to use a low-pass analog filter (anti-aliasing filter) that eliminates all frequencies above $f_s/2$ before sampling [16].

Sampling Theorem : The sampling frequency f_s must be at least twice the highest frequency in the signal to recover completely the continuous signal from its sampled counterpart. Each Fourier coefficient F_n is in general complex, the real part describing the cosine-line amplitude and the imaginary part describing the sine-like amplitude. The modulus, or magnitude F_n is defined as

$$G_n = \text{Re}(F_n)^2 + \text{Im}(F_n)^2$$

and the phase angle ϕ_n is given by $\tan \phi_n = \text{Im}(F_n) / \text{Re}(F_n)$.

$$f_k = \sum_{n=0}^{N-1} \frac{F_n}{N} e^{+i2\pi nk/N}$$

The same function is obtained by first performing the forward and then the inverse transformations given before.

Let us see as what happens if we evaluate f_k outside the sampling interval S .

$$f_{(N+k)} = \sum_{n=0}^{N-1} \frac{F_n}{N} e^{+i2\pi n(N+k)/N} = f_k$$

We see that, given a set of N Fourier coefficients, the constructed function repeats endlessly with a periodicity $S = NT$. This is analogous to the previous result that, given a set of N samples, the Fourier coefficients repeat endlessly with a periodicity $N = S/T$.

5.3 The Fast Fourier Transform

Consider the discrete Fourier transform.

$$X(n) = \sum_{k=0}^{N-1} x_0(k) e^{-i2\pi nk/N} \quad n = 0, 1, \dots, N-1 \quad (5.2)$$

It may be noted that equation (5.2) describes the computation of N equations. For example, if $N = 4$ and if we let

$$W = e^{-i2\pi/N} \quad (5.3)$$

then equation (5.2) can be written as

$$\begin{aligned}
X(0) &= x_0(0) W^0 + x_0(1) W^0 + x_0(2) W^0 + x_0(3) W^0 \\
X(1) &= x_0(0) W^0 + x_0(1) W^1 + x_0(2) W^2 + x_0(3) W^3 \\
X(2) &= x_0(0) W^0 + x_0(1) W^2 + x_0(2) W^4 + x_0(3) W^6 \\
X(3) &= x_0(0) W^0 + x_0(1) W^3 + x_0(2) W^6 + x_0(3) W^9
\end{aligned} \tag{5.4}$$

Equations (5.4) can be more easily represented in matrix form as

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} W^0 & W^0 & W^0 & W^0 \\ W^0 & W^1 & W^2 & W^3 \\ W^0 & W^2 & W^4 & W^6 \\ W^0 & W^3 & W^6 & W^9 \end{bmatrix} \begin{bmatrix} x_0(0) \\ x_0(1) \\ x_0(2) \\ x_0(3) \end{bmatrix} \tag{5.5}$$

or more compactly as

$$X(n) = W^{nk} x_0(k) \tag{5.6}$$

In equation (5.5) W and $x_0(k)$ are complex. It requires N^2 complex multiplications and $(N)(N-1)$ complex additions to perform the required matrix computation. The FFT reduces the number of multiplications and additions required in the computation of equation (5.5).

5.3.1 Intuitive Development

To illustrate the FFT algorithm, it is convenient to choose the number of sample points of $x_0(k)$ according to the relation $N=2^\gamma$, where γ is an integer. Recall that Eq. (5.5) results from the choice of $N=4=2\gamma = 2^2$; therefore, we can apply the FFT to the computation of equation (5.5).

The first step in developing the FFT algorithm for this example is to rewrite equation (5.5) as

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W^1 & W^2 & W^3 \\ 1 & W^2 & W^0 & W^2 \\ 1 & W^3 & W^2 & W^1 \end{bmatrix} \begin{bmatrix} x_0(0) \\ x_0(1) \\ x_0(2) \\ x_0(3) \end{bmatrix} \quad (5.7)$$

Matrix equation (5.7) is derived from (5.5) by using the relationship $W^{nk} = W^{nk \bmod(N)}$. It may be recalled that $[nk \bmod(N)]$ is the remainder upon division of nk

$$W^6 = W^2 \quad (5.8)$$

since

$$\begin{aligned} W^{nk} &= W^6 = \exp \left[\left(\frac{-j2\pi}{4} \right) (6) \right] = \exp [-j3\pi] \\ &= W^{nk \bmod N} = W^2 = \exp \left[\left(\frac{-j2\pi}{4} \right) (2) \right] = \exp [-j\pi] \end{aligned} \quad (5.9)$$

The second step in the development is to factor the square matrix in equation (5.7) as follows :

$$\begin{bmatrix} X(0) \\ X(2) \\ X(1) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & W^0 & 0 & 0 \\ 1 & W^2 & 0 & 0 \\ 0 & 0 & 1 & W^1 \\ 0 & 0 & 1 & W^3 \end{bmatrix} \begin{bmatrix} 1 & 0 & W^0 & 0 \\ 0 & 1 & 0 & W^0 \\ 1 & 0 & W^2 & 0 \\ 0 & 1 & 0 & W^2 \end{bmatrix} \begin{bmatrix} x_0(0) \\ x_0(1) \\ x_0(2) \\ x_0(3) \end{bmatrix} \quad (5.10)$$

Multiplication of the two square matrices of equation (5.10) yields the square matrix of equation (5.7) with the exception that rows 1 and 2 have been interchanged (The rows are numbered 0,1,2, and 3). Note that this interchange has been taken into account in (5.10) by rewriting the column vector $X(n)$. Let the row interchanged vector be denoted by

$$\overline{X(n)} = \begin{bmatrix} X(0) \\ X(2) \\ X(1) \\ X(3) \end{bmatrix} \quad (5.11)$$

This factorization is the key to the efficiency of the FFT algorithm. Having accepted the fact that equation (5.10) is correct, one can examine the number of multiplications required to compute the equation. First let

$$\begin{bmatrix} X(0) \\ X_1(1) \\ X_1(2) \\ X_1(3) \end{bmatrix} = \begin{bmatrix} 1 & 0 & W^0 & 0 \\ 0 & 1 & 0 & W^0 \\ 1 & 0 & W^2 & 0 \\ 0 & 1 & 0 & W^2 \end{bmatrix} \begin{bmatrix} x_0(0) \\ x_0(1) \\ x_0(2) \\ x_0(3) \end{bmatrix} \quad (5.12)$$

That is, column vector $x_1(k)$ is equal to the product of the two matrices on the right in equation (5.10).

Element $X_1(0)$ is computed by one complex multiplication and one complex addition (W^0 is not reduced to unity in order to develop a generalized result).

$$x_1(0) = x_0(0) + W^0 x_0(2) \quad (5.13)$$

Element $x_1(1)$ is also determined by one complex multiplication and addition. Only one complex addition is required to compute $x_1(2)$. This follows from the fact that $W^0 = -W^2$; hence

$$\begin{aligned} x_1(2) &= x_0(0) + W^2 x_0(2) \\ &\quad x_0(0) - W^0 x_0(2) \end{aligned} \quad (5.14)$$

where the complex multiplication $W^0 x_0(2)$ has already been computed in the determination of $x_1(0)$ (Eq. 5.13). By the same reasoning, $x_1(3)$ is computed by only one complex addition and no multiplications. The intermediate vector $x_1(k)$ is then determined by four complex additions and two complex multiplications [17].

Term $x_2(0)$ is determined by one complex multiplication and addition. Element $x_2(1)$ is computed by one addition because $W^0 = -W^2$. by similar reasoning, $x_2(2)$ is determined by one complex multiplication and addition, and $x_2(3)$ by only one addition.

By continuing the computation of equation (5.10), we have

$$\begin{bmatrix} X(0) \\ X(2) \\ X(1) \\ X(3) \end{bmatrix} = \begin{bmatrix} x_2(0) \\ x_2(1) \\ x_2(2) \\ x_2(3) \end{bmatrix} = \begin{bmatrix} 1 & W^0 & 0 & 0 & 0 \\ 1 & W^2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & W^1 \\ 0 & 0 & 0 & 1 & W^3 \end{bmatrix} \begin{bmatrix} x_0(0) \\ x_1(1) \\ x_1(2) \\ x_1(3) \end{bmatrix} \quad (5.15)$$

$$x_2(0) = x_1(0) + W^0 x_1(1) \quad (5.16)$$

Computation of $\overline{X(n)}$ by means of equation (5.10) requires a total of four complex multiplications and eight complex additions. Computation of $X(n)$ by (5.5) requires sixteen complex multiplications and twelve complex additions. Note that the matrix factorization process introduces zeros into the factored matrices and, as a result, reduces the required number of multiplications. For this example, the matrix factorization process reduced the number of multiplications by a factor of two. Since computation time is largely governed by the required number of multiplications, we see the reason for the efficiency of the FFT algorithm.

For $N = 2^\gamma$ the FFT algorithm is then simply a procedure for factoring an $N \times N$ matrix into γ matrices (each $N \times N$) such that each of the factored matrices has the special property of minimizing the number of complex multiplications and additions. If we extend the results of the previous example, we note the FFT requires $N\gamma/2 = 4$ complex multiplications and $N\gamma = 8$ complex additions, whereas the direct method (Eq. (5.5) requires N^2 complex multiplications and $N(N-1)$ complex additions. If we assume that computing time

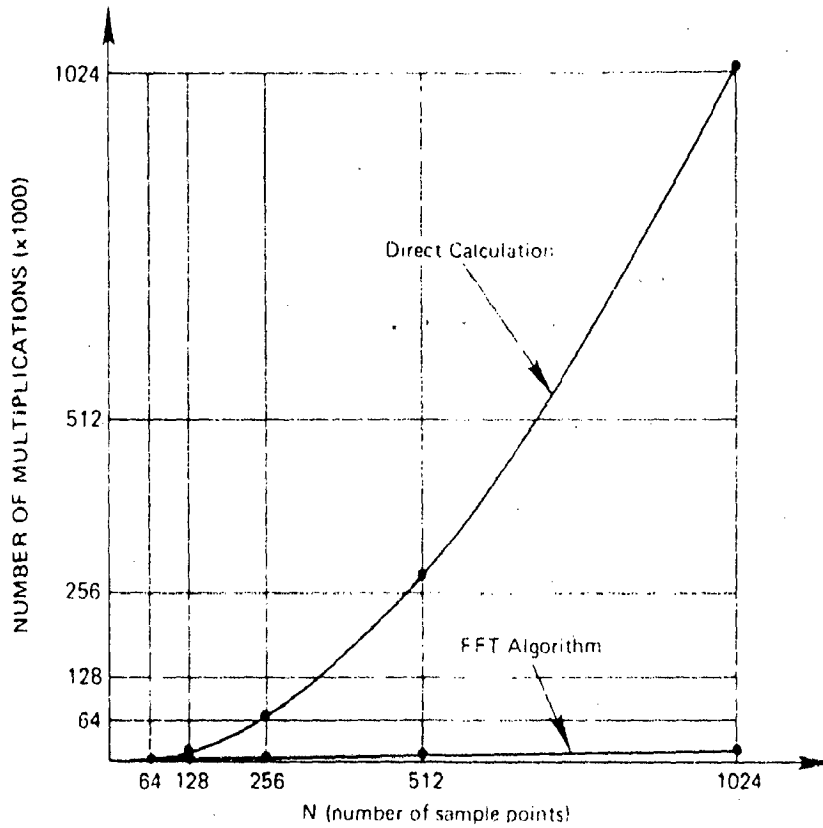


FIGURE 5.2 : COMPARISON OF MULTIPLICATIONS REQUIRED BY DIRECT CALCULATION AND FFT ALGORITHM.

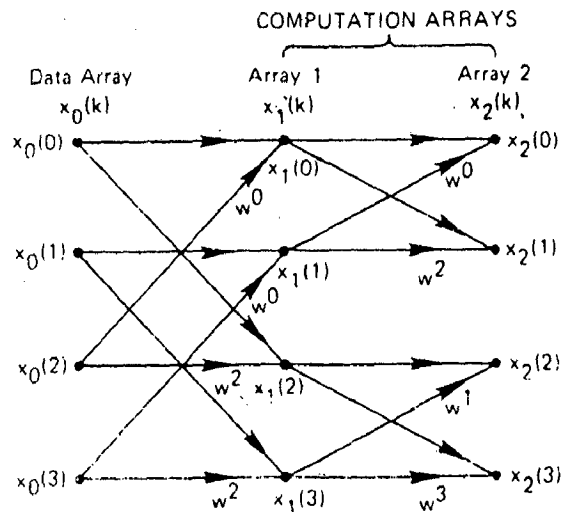


FIGURE 5.3 : FFT SIGNAL FLOW GRAPH, N = 4.



is proportional to the number of multiplications, then the approximate ratio of direct to FFT computing time is given by

$$\frac{N}{N\gamma/2} = \frac{2N}{\gamma} \quad (5.17)$$

which for $N = 1024 = 2^{10}$ is a computational reduction of more than 200 to 1. Figure (5.2) shows the relationship between the number of multiplications required using the FFT compared to direct method.

The matrix factoring procedure does introduce one discrepancy. Recall that the computation of equation (5.10) yields $\overline{X(n)}$ instead of $X(n)$; that is

$$\overline{X(n)} = \begin{bmatrix} X(0) \\ X(2) \\ x(1) \\ X(3) \end{bmatrix} \quad \text{instead of } X(n) = \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} \quad (5.18)$$

This rearrangement is inherent in the matrix factoring process and is a minor problem because it is straight forward to generalize a technique for unscrambling $\overline{X(n)}$ to obtain $X(n)$.

Rewrite $X(n)$ by replacing argument n with its binary equivalent

$$\begin{bmatrix} X(0) \\ X(2) \\ x(1) \\ X(3) \end{bmatrix} \quad \text{becomes} \quad \begin{bmatrix} X(00) \\ X(10) \\ X(01) \\ X(11) \end{bmatrix} \quad (5.19)$$

Observe that if the binary arguments of equation (5.19) are flipped or bit reversed (i.e. 01 becomes 10, 10 becomes 01, etc.) then

$$\overline{x(n)} = \begin{bmatrix} X(00) \\ X(10) \\ x(01) \\ X(11) \end{bmatrix} \text{ flips to } x(n) = \begin{bmatrix} X(00) \\ X(01) \\ X(10) \\ X(11) \end{bmatrix} \quad (5.20)$$

It is straightforward to develop a generalized result for unscrambling the FFT. For N greater than 4, it is cumbersome to describe the matrix factorization process analogous to equation (5.10). For this reason we interpret equation (5.10) in a graphical manner. Using this graphical formulation we describe sufficient generalities to develop a flow graph for a computer program.

5.3.2 Signal flow graph

Figure (5.3) shows the conversion of equation (5.10) into the signal flow graph. The data vector or array $x_0(k)$ are represented by a vertical column of nodes on the left of the graph. The second vertical array of nodes is the vector $x_1(k)$ computed in equation (5.12), and the next vertical array corresponds to the vector $x_2(k) = X(n)$, equation (5.15). In general, there will be γ computational arrays where $N = 2^\gamma$.

The signal flow graph is interpreted as follows. Each node is entered by two solid lines representing transmission paths from previous nodes. A path transmits or brings a quantity from a node in one array, multiplies the quantity by W^P , and inputs the result into the node in the next array. Factor W^P appears near the arrowhead of the transmission path; absence of this factor implies that $W^P = 1$. Results entering a node from the two transmission paths are combined additively.

To illustrate the interpretation of the signal flow graph, consider node $x_1(2)$ in Fig. (5.3). According to the rules for interpreting the signal flow graph,

$$x_1(2) = x_0(0) + w^2 x_0(2) \quad (5.21)$$

which is simply equation (5.14) . Each node of the signal flow graph is expressed similarly.

The signal flow graph is then a concise method for representing the computations required in the factored matrix equation (5.10) . Each computational column of the graph corresponds to a factored matrix; γ vertical arrays of N points each ($N=2^\gamma$) are required. This graphical presentation describe clearly the matrix factoring process for large N .

Figure(5.4) shows the signal flow graph for $N=16$. With a flow graph of this size, it is possible to develop general properties concerning the matrix factorization process and thus provide a framework for developing a FFT computer program flow chart.

5.3.3 Dual nodes

Figure(5.4) reveals that in every array we can always find two nodes whose input transmission paths stem from the same pair of nodes in the previous array. For example, nodes $x_1(0)$ and $x_1(8)$ are computed in terms of nodes $x_0(0)$ and $x_0(8)$. It may be noted that nodes $x_0(8)$ do not enter into the computation of any other node. We define two such nodes as a dual node pair.

Since the computation of a dual node pair is independent of other nodes, it is possible to perform in-place computation. To illustrate, note from Fig. 5.4 that we can simultaneously compute $x_1(0)$ and $x_1(8)$ in terms of $x_0(0)$ and $x_0(8)$ and return the results to the storage locations previously occupied by $x_0(0)$ and $x_0(8)$. Thus the storage requirements are limited to the data array $x_0(k)$ only. As each array is computed, the results are returned to this array.

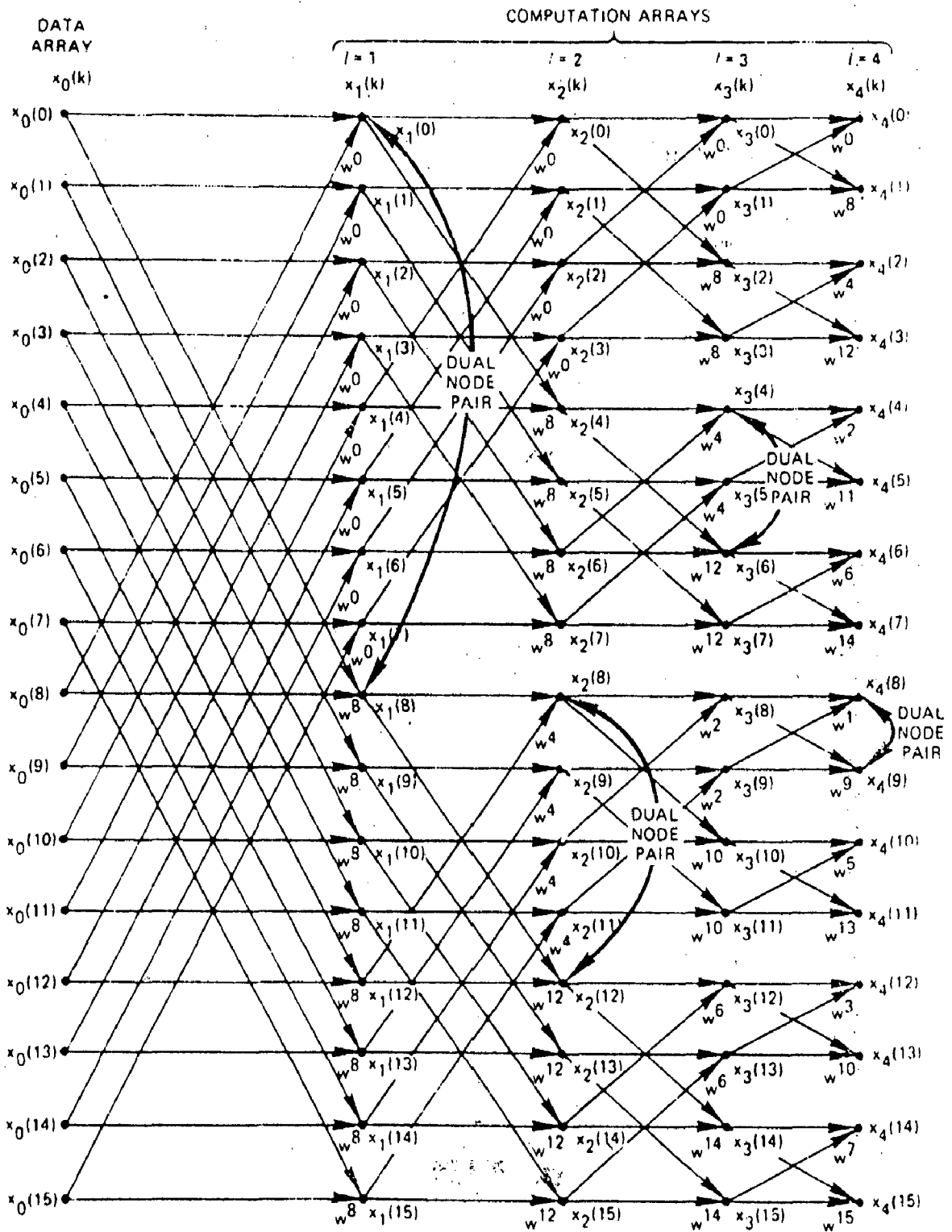


FIGURE 5.4 : EXAMPLE OF DUAL NODES.

Dual Node Spacing

Let us now investigate the spacing (measured vertically in terms of the index k) between a dual node pair. The following discussion will refer to Fig. 5.4. First, in array $l = 1$, a dual node pair, say $x_1(0); x_1(8)$, is separated by $k = 8 = N/2^1 = N/2^1$. In array $l = 2$, a dual node pair, say $x_2(8); x_2(12)$, is separated by $k = 4 = N/2^2 = N/2^2$. Similarly, a dual node pair, $x_3(4); x_3(6)$, in array $l = 3$ is separated by $k = 2 = N/2^3 = N/2^3$, and in array $l = 4$, a dual node pair, $x_4(8); x_4(9)$, is separated by $k = 1 = N/2^4 = N/2^4$.

Generalizing these results, we observe that the spacing between dual nodes in array l is given by $N/2^l$. Thus, if we consider a particular node $x_l(k)$, then its dual node is $x_l(k + N/2^l)$. This property helps to identify a dual node pair.

Dual Node Computation

The computation of a dual node pair requires only one complex multiplication. To clarify this point, consider node $x_2(8)$ and its dual $x_2(12)$, shown in Fig. 5.4. The transmission path stemming from node $x_1(12)$ are multiplied by W^4 and W^{12} prior to input at nodes $x_2(8)$ and $x_2(12)$, respectively. It is important to note that $W^4 = -W^{12}$ and that only one multiplication is required since the same data $x_1(12)$ is to be multiplied by these terms. In general, if the weighting factor at one node is W^p , then the weighting factor at the dual node is $W^{p+N/2}$. Because $W^p = -W^{p+N/2}$, only one multiplication is required in the computation of a dual node pair. The computation of any dual node pair is given by the equation pair [17]

$$x_l(k) = x_{l-1}(k) + W^p x_{l-1}(k + N/2^l) \quad (5.22)$$

$$x_l(k + N/2^l) = x_{l-1}(k + N/2^l) - W^p x_{l-1}(k + N/2^l)$$

In computing an array, we normally begin with node $k = 0$ and

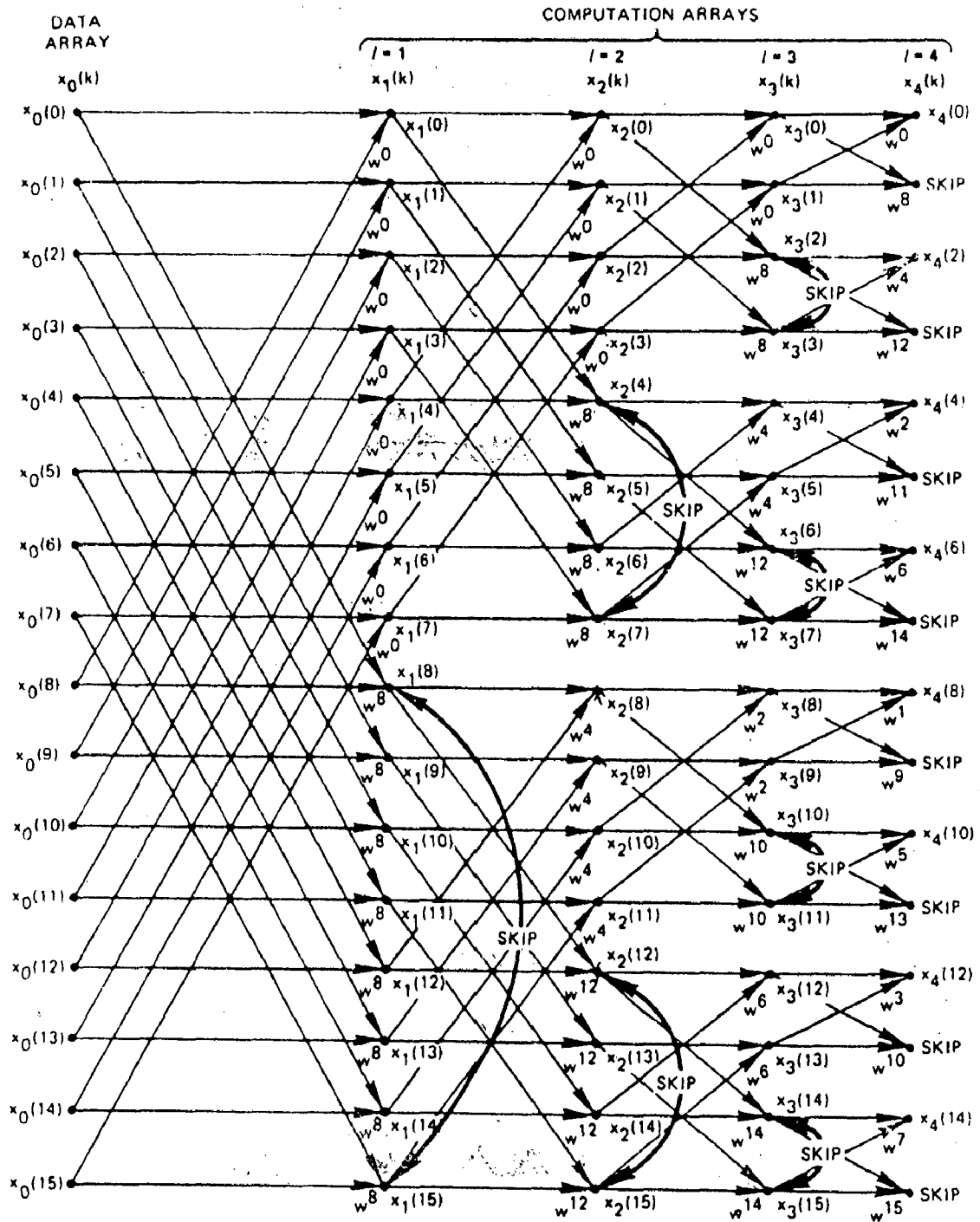


FIGURE 5.5 : EXAMPLE OF NODES TO BE SKIPPED WHEN COMPUTING SIGNAL FLOW GRAPH.

sequentially work down the array, computing the equation pair (5.22). As stated previously, the dual of any node in the l^{th} array is always down $N/2^l$ in the array. Since the spacing is $N/2^l$, then it follows that we must skip after every $N/2^l$ node. To illustrate this point, consider array $l = 2$ in Fig. 5.5. If we begin with node $k = 0$, then according to our previous discussions, the dual node is always located down by 4 in the array until we reach node 4. At this point a set of nodes previously encountered; that is, these nodes are the dual for nodes $k = 0, 1, 2$, and 3. It is necessary to skip-over nodes $k = 4, 5, 6$ and 7. Nodes 8, 9, 10, and 11 follow the original convention of the dual node being located 4 down in the array. In general, if we work from the top down in array l , then we will compute equation (5.22) for the first $N/2^l$ nodes, skip the next $N/2^l$, etc. We know to stop skipping when we reach a node index greater than $N-1$.

5.3.4 W^p Determination

Based on the preceding discussions, properties of each array have been defined with the exception of the value of p in equation (5.22). The value of p is determined by (a) writing the index k in binary form with γ bits, (b) scaling or sliding this binary number $\gamma - 1$ bits to the right and filling the newly opened bit position on the left with zeros, and (c) reversing the order of the bits. This bit-reversed number is the term p .

To illustrate this procedure, refer to Fig. 5.5 and consider node $x_3(8)$. Since $\gamma = 4, k = 8, l = 3$, then k in binary is 1000. We scale this number $\gamma - 1 = 4 - 3 = 1$ places to the right and fill in zeros; the result each 0100. We then reverse the order of the bits to yield 0010 or integer 2. The value of p is then 2.

Let us now consider a procedure for implementing this bit-reversing operation. We know that a binary number, say $a_4 a_3 a_2 a_1$, can be written in base 10 as $a_4 \times 2^3 + a_3 \times 2^2 + a_2 \times 2^1 + a_1 \times 2^0$. The bit reversed number which we are trying to describe

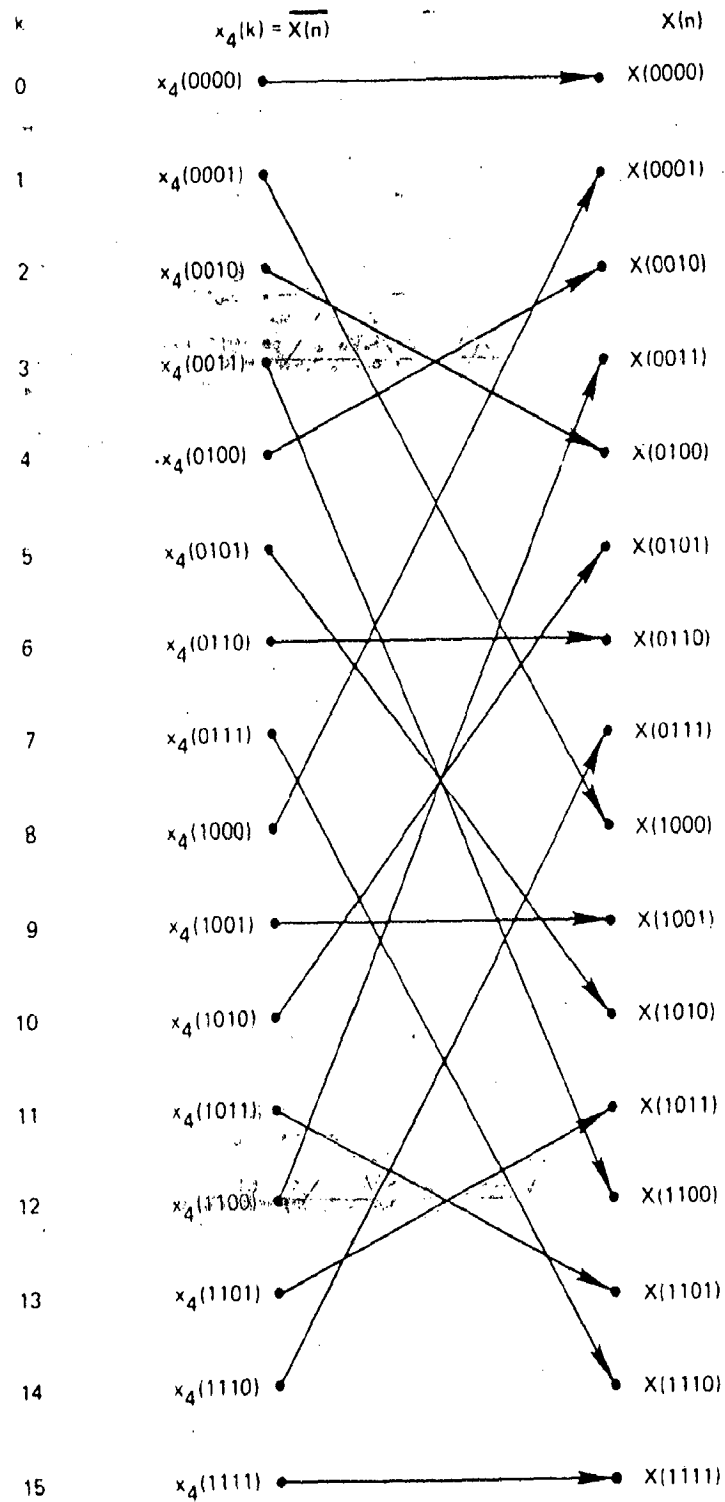


FIGURE 5.6 : EXAMPLE OF BIT-REVERSING OPERATION FOR $N = 16$.

is then given by $a_1 x 2^3 + a_2 x 2^2 + a_3 x 2^1 + a_4 x 2^0$. If we describe a technique for determining the binary bits a_4, a_3, a_2 , and a_1 , then we have defined a bit-reversing operation.

Now assume that M is a binary number equal to $a_4 a_3 a_2 a_1$. Divide M by 2, truncate, and multiply the truncated results by 2. Then compute $[a_4 a_3 a_2 a_1 \cdot 2 - (a_4 a_3 a_2)]$. If the bit a_1 is 0, then this difference will be zero because division by 2, truncation, and subsequent multiplication by 2 does not alter M . However, if the bit a_1 is 1, truncation changes the value of M and the above difference expression will be non-zero. We observe that by this technique we can determine if the bit a_1 is 0 or 1.

We can identify the bit a_2 in a similar manner. The appropriate difference expression is $[a_4 a_3 a_2 \cdot 2 - (a_4 a_3)]$. If this difference is zero, then a_2 is zero. Bits a_3 and a_4 are determined similarly. This procedure will form the basis for developing a bit-reversing computer routine.

5.3.5 Unscrambling the FFT

The final step in computing the FFT is to unscramble the results analogous to equation (5.20). Recall that the procedure for unscrambling the vector $X(n)$ is to write in binary and reverse or flip the binary number. We show in Fig. 5.6 the results of this bit-reversing operation; terms $x_4(k)$ and $x_4(i)$ have simply been interchanged where i is the integer obtained by bit-reversing the integer k .

Note that a situation similar to the dual node concept exists when we unscramble the output array. If we proceed down the array, interchanging $x(k)$ with the appropriate $x(i)$, we will eventually encounter a node which has previously been interchanged. For example, in Fig. 5.6, node $k = 0$ remains in its location, nodes $k = 1, 2$ and 3 are interchanged with nodes $8, 4$, and 12 , respectively. The next node to be considered is node 4, but this

node was previously interchanged with node 2. To eliminate the possibility of considering a node that has previously been interchanged, we simply check to see if i (the integer obtained by bit-reversing k) is less than k . If so, this implies that the node has been interchanged by a previous operation. With this check we can insure a straightforward unscrambling procedure [17].

CHAPTER - 6

ALGORITHM, FLOWCHART AND SOFTWARE

6.1 General

In this dissertation a software package has been developed for the analysis of vibration signals. The computer program is written in 'C' language. The program is written for the extraction of the following features of vibration signals.

- (i) Acceleration,
- (ii) Velocity,
- (iii) Displacement,
- (iv) Fundamental frequency component and harmonics, and their frequencies,
- (v) Periodicity, and
- (vi) Overall level of vibration velocity.

This chapter gives the details of algorithms, flowcharts and software. The complete computer program is listed in appendix-2.

6.2 Analysis of Vibration Signal

In vibration measurements, the quantity to be measure is vibration acceleration (from the surface of the electrical machines) from which vibration velocity and vibration displacement are obtained by integration, then later being the quantities to be processed and evaluated. The frequency spectrum of the acceleration (or, either velocity or displacement if considered better) is determined by means of the software. The cepstrum of the vibration is determined to fined out periodicity and side bands of the vibration. The resultant r.m.s vibration velocity in frequency range 10 Hz to 1000 Hz is determined by using frequency spectrum of vibration velocity. The algorithms and flow charts developed are discussed as follows. The complete flow chart is shown in Fig (6.6).

FIGURE 6.1 : FFT FLOW CHART

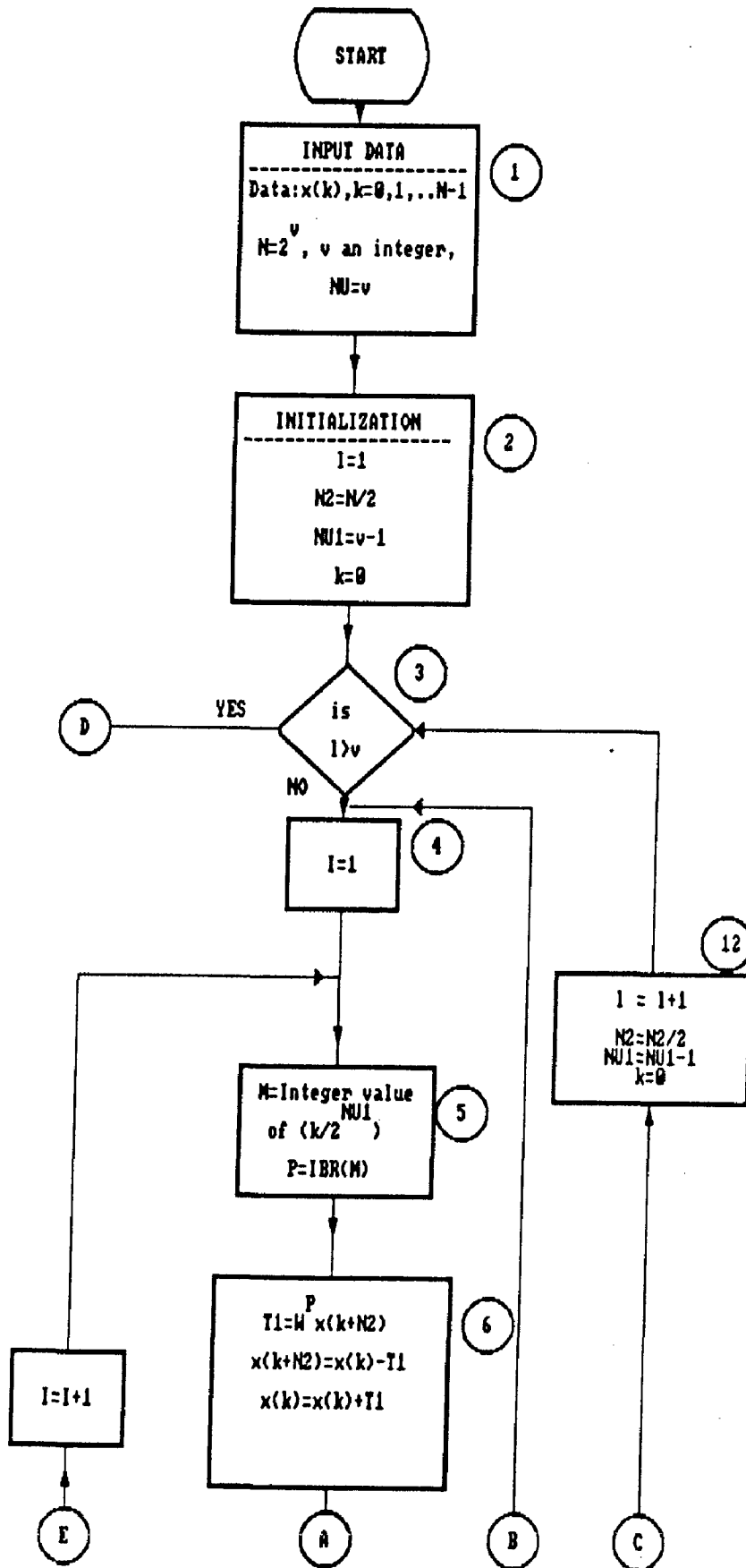
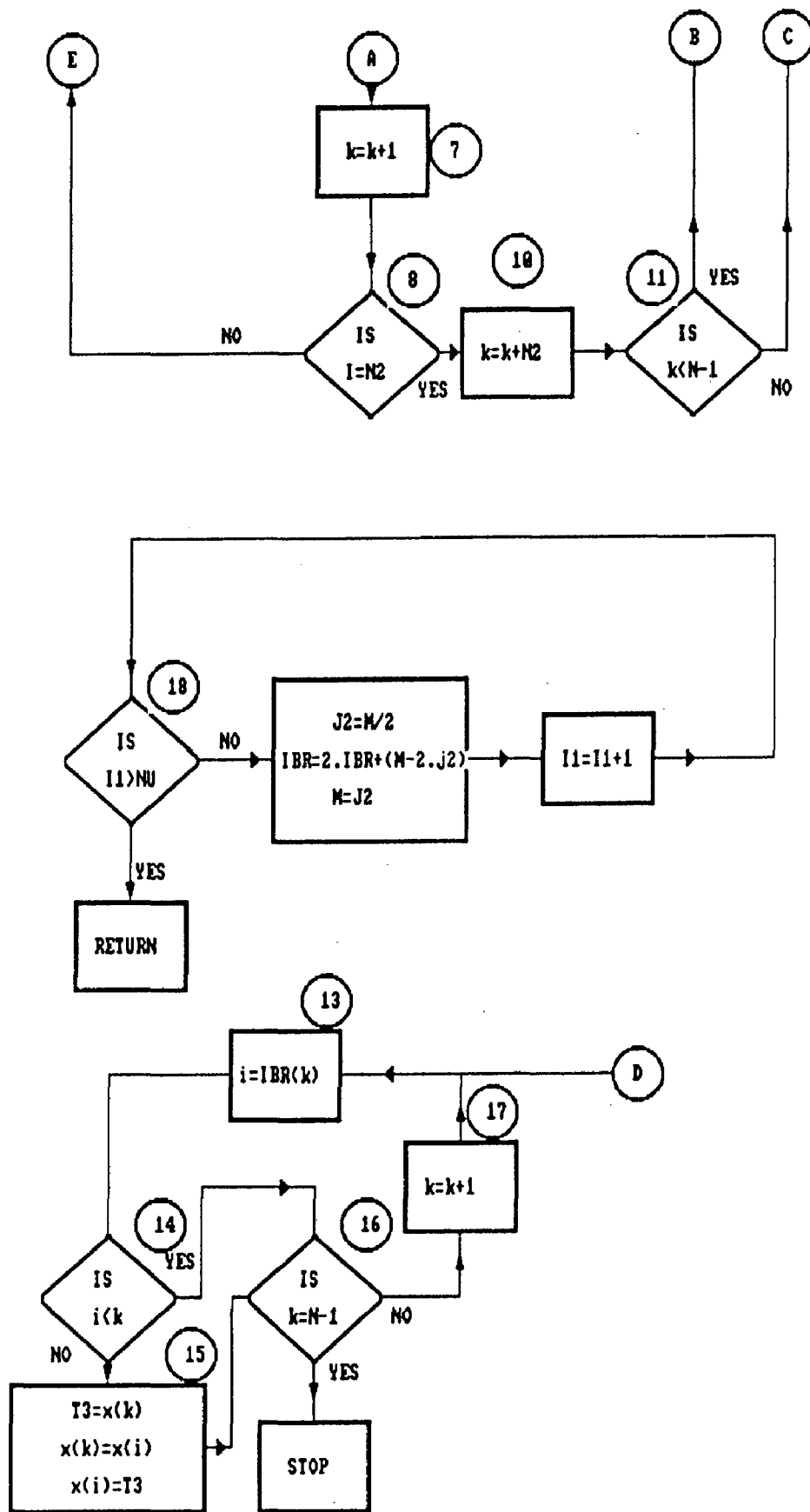


FIGURE 6.1 : (CONTD.)



6.2.1 Fourier Transform Computation

The details of algorithm and mathematical background of Fourier transform have already been discussed in chapter 5. Fourier transform computation has been performed with the FFT algorithm.

FFT computation flow chart

We first compute array $l = 1$ by starting at node $k = 0$ and working down the array. At each node k , we compute the equation pair (5.22) where p is determined by the procedure described in chapter 5. We continue down the array computing the equation pair (5.22) until we reach a region of nodes which must be skipped-over. We skip over the appropriate nodes and continue until we have computed the entire array. We then proceed to compute the remaining arrays using the same procedures. Finally we unscramble the final array to obtain the desired results. Figure 6.1 shows a flow chart for computer programming the FFT algorithm.

Box 1 describes the necessary input data. Data vector $x_0(k)$ is assumed to be complex and is indexed as $k = 0, 1, \dots, N-1$. If $x_0(k)$ is real, then the imaginary part should be set to zero. The number of sample points N must satisfy the relationship $N = 2^\gamma$, where γ is an integer.

Initialization of the various program parameters is accomplished in Box 2. Parameter l is the array number being considered. We start with array $l=1$. The spacing between dual nodes is given by the parameter $N2$; for array $l = 1$, $N2 = N/2$ and is initialized as such. Parameter $N1$ is the right shift required when determining the value of p in Eq. (5.22); $N1$ is initialized to $\gamma-1$. The index k of the array is initialized to $k = 0$; thus we will work from the top and progress down the array.

Box 3 checks to see if the array l to be computed is greater

than γ . If yes, then the program branches to Box 13 to unscramble the computed results by bit inversion. If all arrays have not been computed, then we proceed to Box 4.

Box 4 sets a counter $I = 1$. This counter monitors the number of dual node pairs which have been considered. Recall from Sec. 5.3.4 that it is necessary to skip certain nodes in order to ensure that previously considered nodes are not encountered a second time. Counter I is the control for determining when the program must skip.

Boxes 5 and 6 perform the computation of Eq. (5.22). Since k and l have been initialized to 0 and 1, respectively, the initial node considered is the first node of the first array. To determine the factor p for this node, recall that we must first scale the binary number k to the right $\gamma-1$ bits. To accomplish this, we compute the integer value of $k/2^{\gamma-1} = k/2^{NU1}$ and set the result to M as shown in box 5. According to the procedure for determining p , we must bit reverse M where M is represented by $\gamma = NU$ bits. The function $IBR(M)$ denoted in Box 5 is a special function routine for bit inversion; this routine will be described later.

Box 6 is the computation of Eq. (5.22). We compute the product $W^p_x(k + N2)$ and assign the result to a temporary storage location. Next we add and subtract this term according to Eq. (5.22). The result is the dual node output.

We then proceed down the array to the next node. As shown in Box 7, k is incremented by 1.

To avoid recomputing a dual node that has been considered previously, we check in Box 8 to determine if the counter I is equal to $N2$. For array 1, the number of nodes that can be considered consecutively without skipping is equal to $N/2 = N2$. Box 8 determines this condition. If I is not equal to $N2$, then we proceed down the array and increment the counter I as shown in Box

9. Recall that we have already incremented k in Box 7. Boxes 5 and 6 are then repeated for the new value of k .

If $I = N/2$ in Box 8, then we know that we have reached a node previously considered. We then skip $N/2$ nodes by setting $k = k + N/2$. Because k has already been incremented by 1 in box 7, it is sufficient to skip the previously considered nodes by incrementing k by $N/2$.

Before we perform the required computations indicated by Boxes 5 and 6 for the new node $k = k + N/2$, we must first check to see that we have not exceeded the array size. As shown in box 11, if k is less than $N-1$ (recall k is indexed from 0 to $N-1$), then we reset the counter I to 1 in Box 4 and repeat Boxes 5 and 6.

If $k > N-1$ in Box 11, we know that we must proceed to the next array. Hence, as shown in Box 12, I is indexed by 1. The new spacing $N/2$ is simply $N/2^l$ (recall the spacing is $N/2^l$). $NU1$ is decremented by 1 ($NU1$ is equal to $\gamma-1$), and k is reset to zero. We then check in Box 3 to see if all arrays have been computed. If so, then we proceed to unscramble the final results. This operation is performed by Boxes 13 through 17.

Box 13 bit-reverses the integer k to obtain the integer i . Again we use the bit-reversing function $IBR(k)$ which is to be explained later. Recall that to unscramble the FFT we simply interchange $x(k)$ and $x(i)$. This manipulation is performed by the operations indicated in Box 15. However, before box 15 is entered it is necessary to determine, as shown in Box 14, if i is less than or equal to k . This step is necessary to prohibit the altering of previously unscrambled nodes.

Box 16 determines when all nodes have been unscrambled and Box 17 is simply an index for k .

In Box 18, we describe the logic of the bit-reversing function $IBR(k)$. We have implemented the bit-reversing procedure discussed in Sec. 5.3.5.

When one proceeds to implement the flow graph of Fig. 6.1 into a computer program, it is necessary to consider the variables $x(k)$ and $N/2$ as complex numbers and they must be handled accordingly.

6.2.2 Integration of Vibration Signal

Integration of vibration signal can be performed by using any numerical method of integration. In this case the vibration acceleration (which is in time domain) is integrated to obtain vibration velocity and the vibration velocity is integrated to obtain vibration displacement. In this work, a different approach of integration has been used. In this approach the acceleration is integrated in frequency domain instead of time domain. The method is discussed as follows:

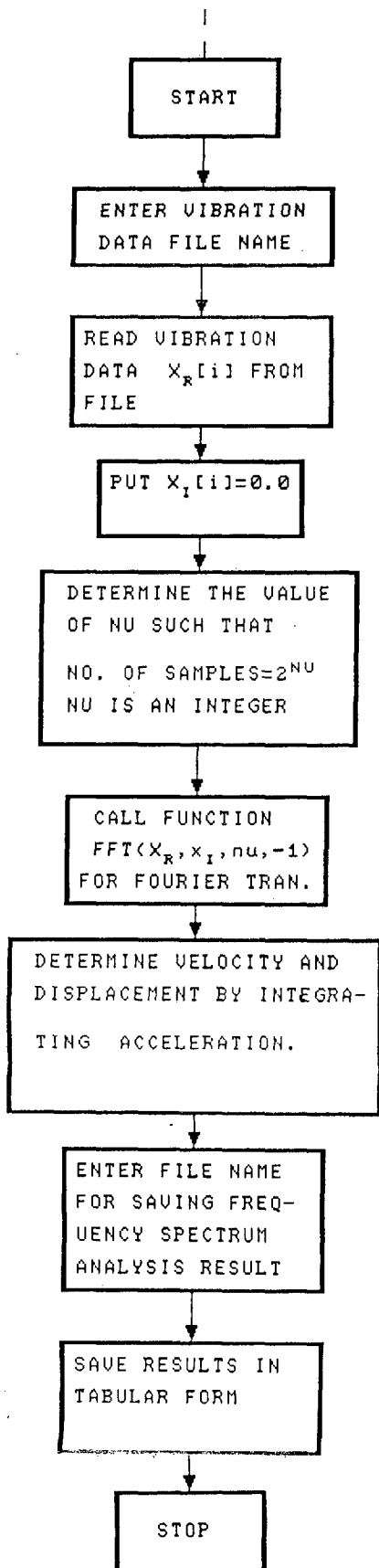
In Fourier analysis, any steady state complex vibration signal, however complex it may be, is represented as a combination of number of pure sinusoidal motions with harmonically related frequencies, as expressed by the following equation :

$$F(t) = x_1 \sin(\omega t + \phi_1) + x_2 \sin(2\omega t + \phi_2) + x_3 \sin(3\omega t + \phi_3) + \dots + x_n \sin(n\omega t + \phi_n) \quad (6.1)$$

As more and more terms are added to the above series, the description of the non-harmonic periodic vibration becomes more precise. By integrating equation (6.1), we have

$$\int F(t) dt = \frac{x_1}{\omega} \sin(\omega t + \phi_1 - \pi/2) + \frac{x_2}{2\omega} \sin(2\omega t + \phi_2 - \pi/2) + \dots + \frac{x_n}{n\omega} \sin(n\omega t + \phi_n - \pi/2) \quad (6.2)$$

From equation (6.2), it is clear that after integrating acceleration the acceleration vector is rotated backward 90° in complex plane, and its amplitude is modified by dividing it by its corresponding radial frequency.



Fig(6.2) FLOW CHART FOR FREQUENCY SPECTRUM ANALYSIS

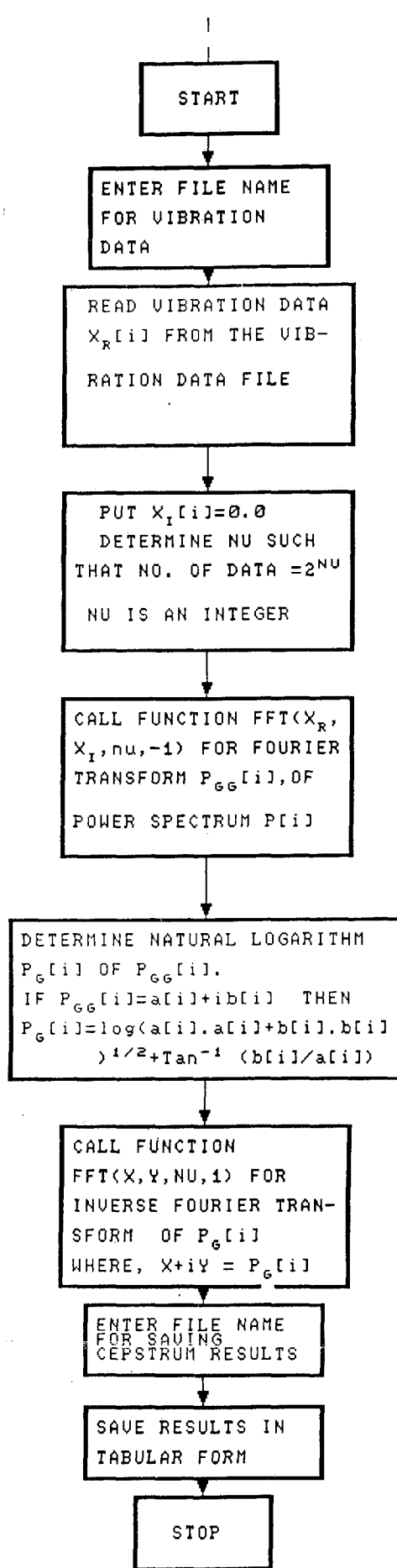


Fig. (6.4) FLOW CHART FOR CEPSTRUM COMPUTATION

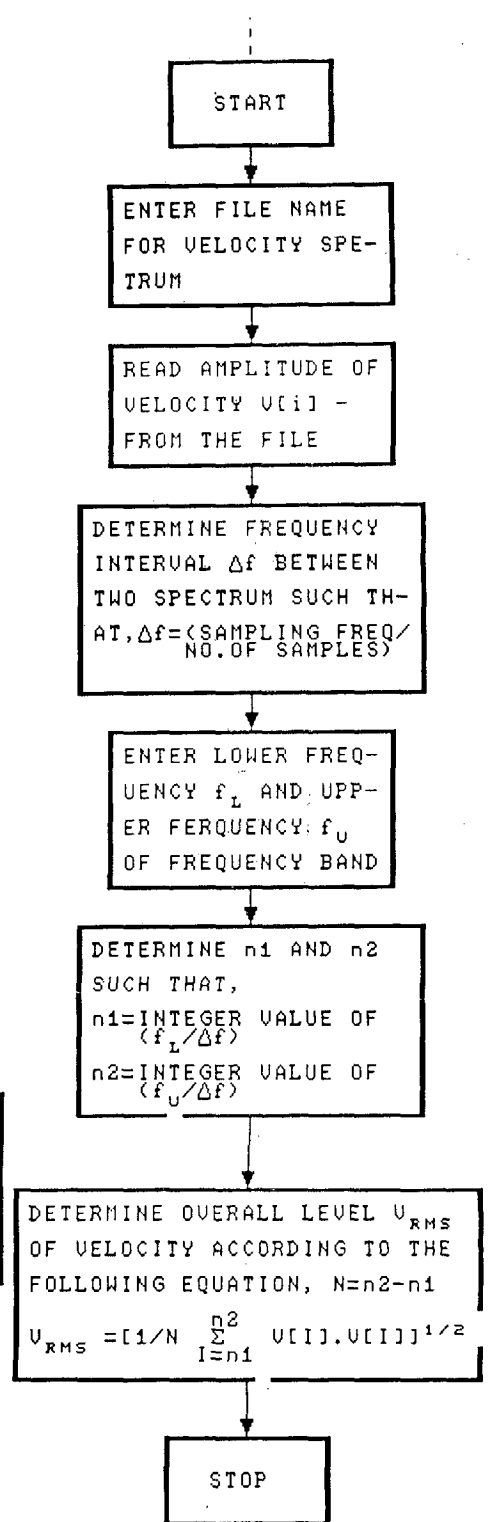


Fig. (6.5) FLOW CHART FOR OVERALL VELOCITY COMPUTATION

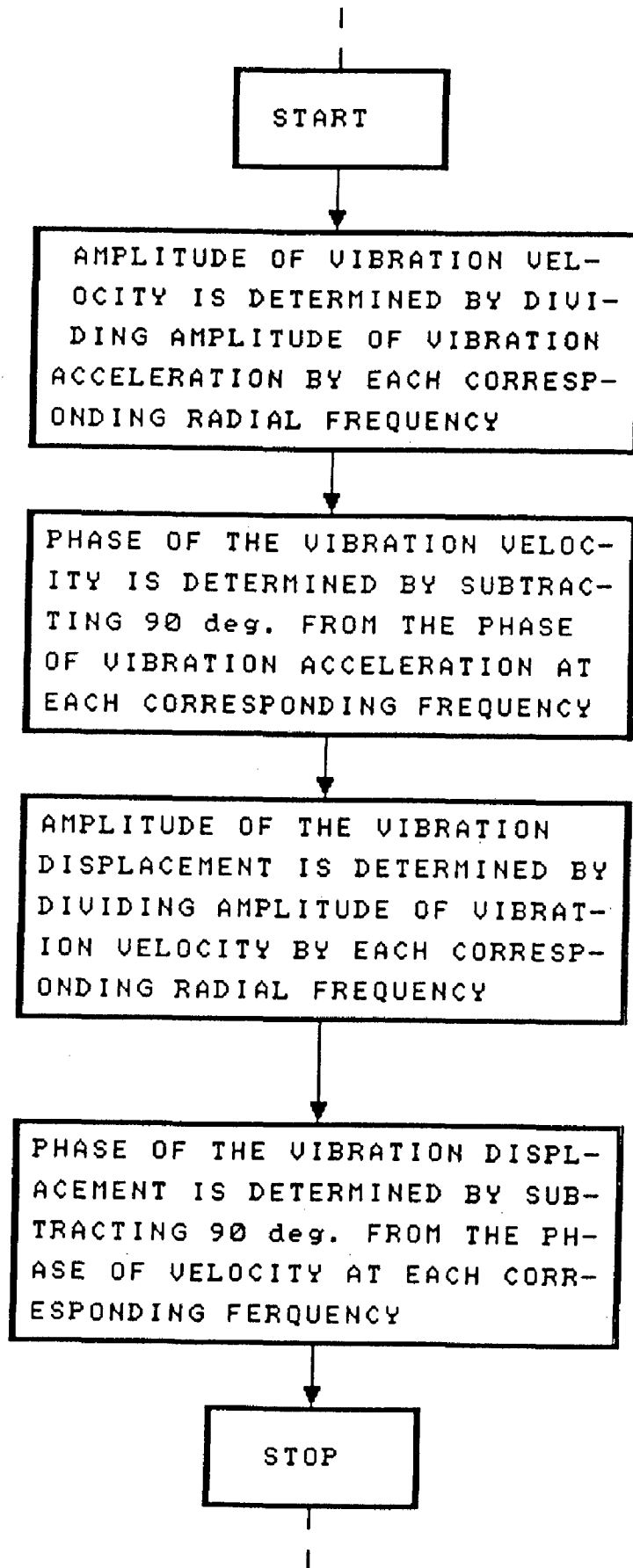
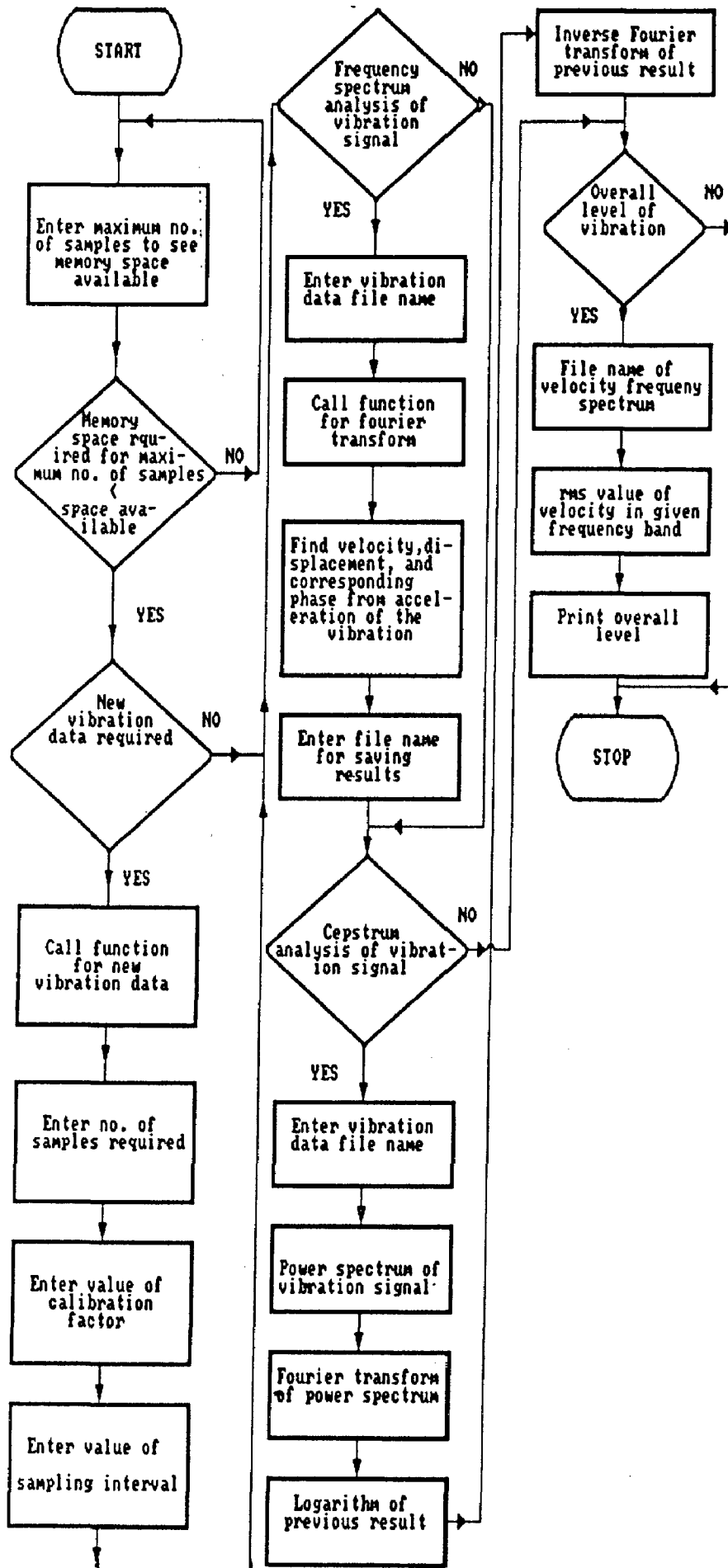


Fig.(6.3) FLOW CHART FOR INTEGRATION

FIGURE 6.6 : MAIN PROGRAM FLOW CHART



6.2.3 Determination of Velocity from Acceleration

The method discussed in section 6.2.2 has been used here for the determination of vibration velocity from vibration acceleration in frequency domain. The amplitude of the velocity at any frequency is determined by dividing the amplitude of acceleration by its corresponding radial frequency. The phase of the velocity at corresponding frequency is determined by subtracting 90° from the phase of acceleration.

6.2.4 Determination of Displacement from Velocity

The amplitude of displacement at any frequency is determined by dividing the amplitude of velocity by its corresponding radial frequency. The phase of the displacement at any frequency is determined by subtracting 90° from the phase of the velocity at its corresponding frequency.

The flow chart for determination of velocity from acceleration, and displacement from velocity is shown in Fig. (6.3).

6.2.5 Cepstrum Analysis

The cepstrum analysis, which is discussed in detail in chapter 4 (section 4.4.), is performed in following steps :

- (i) determine power spectrum $P(t)$ of vibration signal $g(t)$,
- (ii) determine Fourier transform $P_{gg}(f)$ of $P(t)$,
- (iii) determine natural logarithm of $P_{gg}(f)$, and
- (iv) determine inverse Fourier transform of $\log P_{gg}(f)$.

Power spectrum of vibration data is determined by multiplying each vibration data by itself. If $x[i]$ is a vibration data at sample i , then power spectrum $P[i]$ is determined as

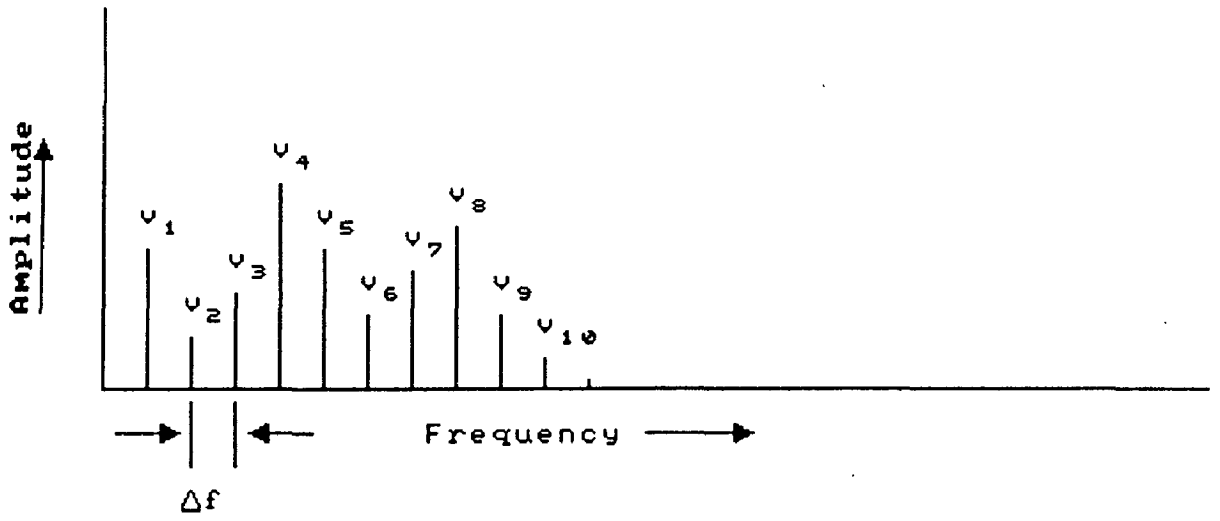


FIGURE 6.7 : FREQUENCY SPECTRUM

$$P[i] = x [i] \cdot x [i]$$

The Fourier transform of power spectrum has been performed by using FFT computer program. If Fourier transform of power spectrum $P_{gg}(f)$ is denoted by

$$P_{gg}(f) = x + iy,$$

then the natural logarithm of $P_{gg}(f)$ is given by relation

$$\begin{aligned} \log \{P_{gg}(f)\} &= \log |P_{gg}(f)| + i \arg P_{gg}(f) \\ &= \log \sqrt{(x^2+y^2)} + i \tan^{-1}(y/x) \end{aligned}$$

The flow chart for the determination of cepstrum is shown in Fig. (6.4).

6.2.6 Overall Level of Vibration Velocity

Overall level of vibration velocity shows r.m.s value of vibration velocity in preselected frequency band. By using hardware the overall level of vibration velocity is calculated, first by generating the effective value of the individual harmonic vibration components as given by following equation

$$V_{ie} = \left[\frac{1}{T} \int_0^T V_i(t) dt \right]^{\frac{1}{2}} \quad (6.3)$$

where V_{ie} - the effective value of the i th harmonic vibration components,

$V_i(t)$ - the instantaneous value of the i th harmonic vibration component,

t - time, and

T - the duration of effective value generation.

The resultant r.m.s value of harmonic components falling

within the range 10 Hz to 1000 Hz is found by the following equation

$$V_{\text{rms}} = \left[\sum_{i=1}^n V_{ie}^2 \right]^{\frac{1}{2}} \quad (6.4)$$

where V_{rms} - the resultant r.m.s vibration velocity,
 i - the i th component of harmonic vibration within the frequency range 10 Hz to 1000 Hz, and
 n - the number of such components.

The r.m.s given by equation (6.4) is produced by means of band-pass filter with cut-off frequencies of $f_l = 10$ Hz and $f_u = 1000$ Hz.

The overall vibration velocity level, in software, is determined by first taking frequency spectrum of vibration velocity. The method which is used in this dissertation is explained with the help of Fig. (6.7).

The resultant r.m.s value of velocity is determined by following equation.

$$V_{\text{rms}} = \left[\frac{1}{N} \sum_{i=n1}^{n2} V_i^2 \right]^{\frac{1}{2}} \quad (6.5)$$

where V_{rms} - the resultant r.m.s vibration velocity,
 V_i - the effective value of the i th harmonic vibration component,

N - the total no. of harmonics determined by $(n1-n2)$

i - the i th component of harmonic vibration within the frequency range 10 Hz to 1000 Hz.

$n1$ & $n2$ are determined by the following relations:

$n1 = \text{integer value of } (10 \text{ Hz}/\Delta f)$

$n2 = \text{integer value of } (1000\text{Hz}/\Delta f)$

where $\Delta f = (f_2 - f_1) = (f_3 - f_2) = \dots = (f_n - f_{n-1})$

The flow chart for the determination of overall value is given in Fig. (6.5).

6.3 Programming

In the software size of all the arrays used are defined dynamically. In dynamic location of memory, the array size is changed according to the requirement. Separate functions are written for a particular job. This improves the clarity of logic implemented and readability of the program. This type of approach helps in debugging the program during the development process.

Almost all variables are global type and initialized at the time of their declaration. The global variables are specifically used because it is not required to pass parameters to the function. By this way unnecessary stack operations are avoided and program speed is slightly increased.

6.4 Organization of the Program

The normal sequence of execution of program is as follows :

- (i) Message displays for maximum number of samples. Before starting analysis it is important to know that how much memory is available. By entering the maximum number of samples required, approximate memory space available can be determined.
- (ii) Message displays for new vibration data. If new vibration data is required, press key 'y' otherwise press key 'n'.
- (iii) If new vibration data required, then
 - (a) Message displays for number of samples. Here required number of vibration samples are entered.
 - (b) Message for channel number. The channel number for ADC is entered.
 - (c) Message displays for sampling period. Here the value of sampling period is entered.
 - (d) Message displays for calibration factor. Here the value of calibration factor is entered so that the amplitude

of vibration directly shows actual vibration acceleration in mm s^{-2} .

(e) Message displays for file name for saving vibration data.

If new vibration data is require message (iv) displays

(iv) Message displays for frequency spectrum analysis. If frequency spectrum analysis is required, key 'y' is pressed otherwise key 'n' is pressed.

If spectrum analysis is required, then

(a) Message displays for vibration data file name for analysis.

(b) The file name of vibration data is entered through keyboard.

(c) The vibration data is converted into frequency spectrum, by using Fourier transform function(see flow chart 6.1).

(d) Vibration velocity is obtained by integrating vibration acceleration(see flow chart 6.3).

(e) Vibration displacement is obtained by integrating vibration velocity.

(f) The results are stored in tabular form.

(g) Message displays for file name for saving frequency spectrum analysis result.

(v) Message displays for cepstrum analysis. If cepstrum analysis is required, key 'y' is pressed otherwise key 'n' is pressed.

If cepstrum analysis is required, then.

(a) Message displays for vibration data file name.

(b) Cepstrum analysis is performed according to the flow chart shown in Fig. (6.3).

(c) Message displays for file name for saving cepstrum analysis result. The result is stored in tabular form.

(vi) Message displays for overall value. If overall value of vibration velocity is required, key 'y' is pressed otherwise key 'n' is pressed. If key 'y' is pressed, then.

(a) The message displays for velocity spectrum file name.

(b) The velocity spectrum file name is entered through key board.

(c) The overall velocity level is performed by using flow chart shown in Fig. (6.4).

(d) The overall velocity level is displayed in screen.

(vii) Program is terminated.

The complete computer program flow chart is shown in Fig. (6.6.).

CHAPTER - 7

VIBRATION ANALYSIS, TEST RESULTS, DISCUSSION

7.1 Monitoring of Rotating Electrical Machines Using Vibration Analysis:

The principal method of monitoring the state of rotating electrical machines is the spectral analysis of vibration velocity signal recorded at its surface and bearing housing. Vibration data is collected at suitable points. The vibration data is coded to identify the measurement point, the load condition of machine, the collection date, and the instrumentation settings. The vibration data recorded is analysed by using spectral analysis. The vibration velocity spectrum of current data is compared with a base line spectrum. The base line spectrum is, the spectrum which is, recorded when machine was new and healthy. Figure 7.1 shows a diagram of a electrical machine and the points where vibration recording is done. The mounting of the transducer has already been discussed in section 2.3.

Two case studies are discussed below in which incipient faults on motors were identified using vibration analysis.

Case 1: Out of Balance Vibration [19]

This study refers to a sea water injection pump set, comprising the pump driven by a 6.6 kv, 3.6 mw, 3560 rpm cage induction machine.

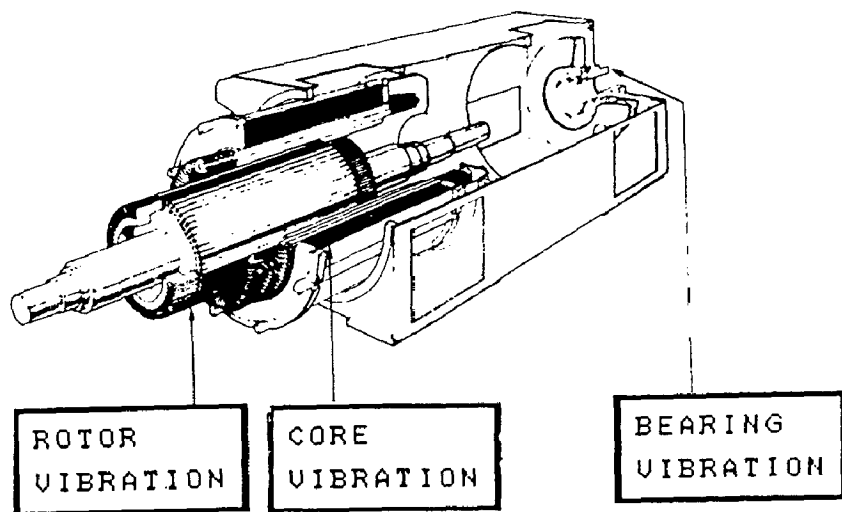


FIGURE 7.1 : POINTS OF VIBRATION MEASUREMENT FOR A TYPICAL LARGE ELECTRICAL MACHINE.

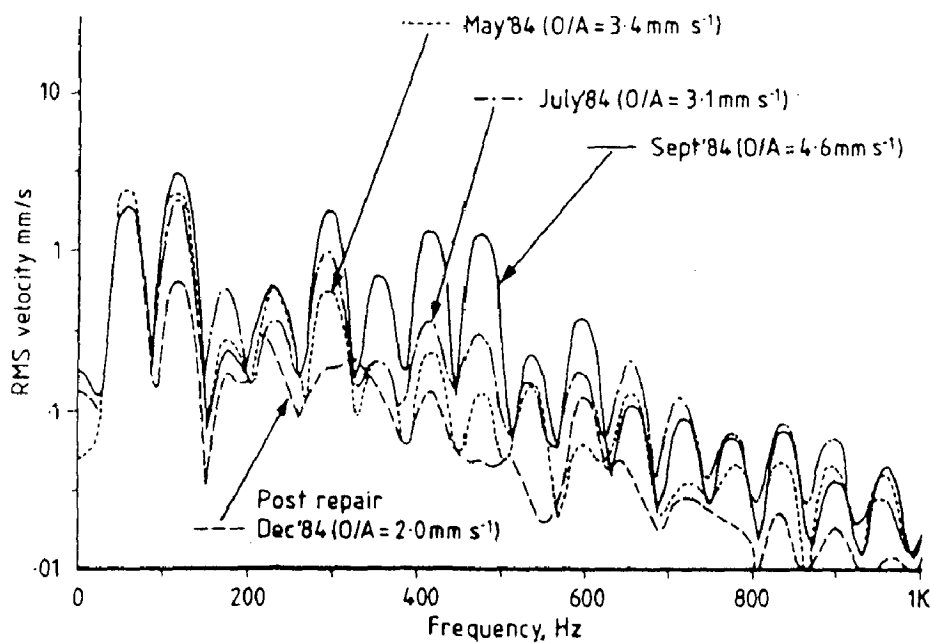


FIGURE 7.3 : PROGRESSIVE DEGRADATION DUE TO ROTOR-BAR LOOSENESS.

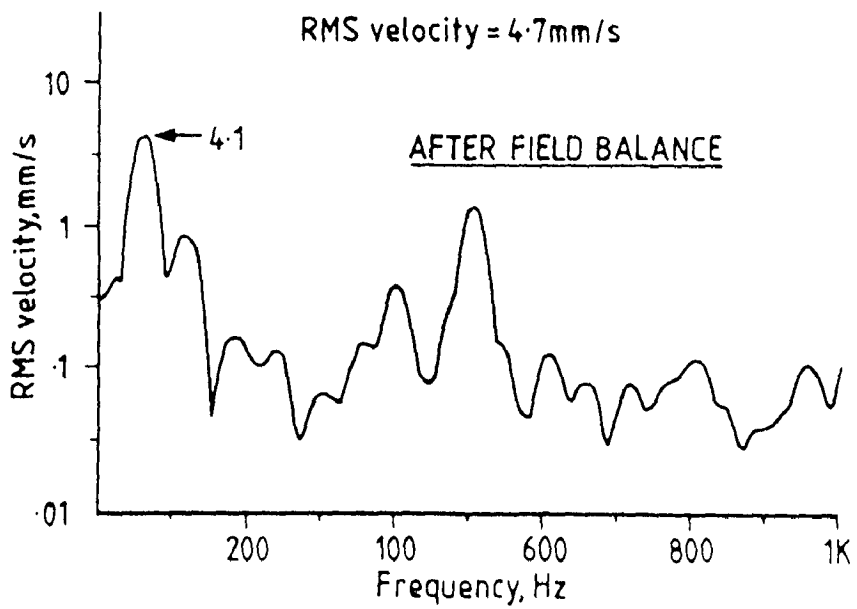
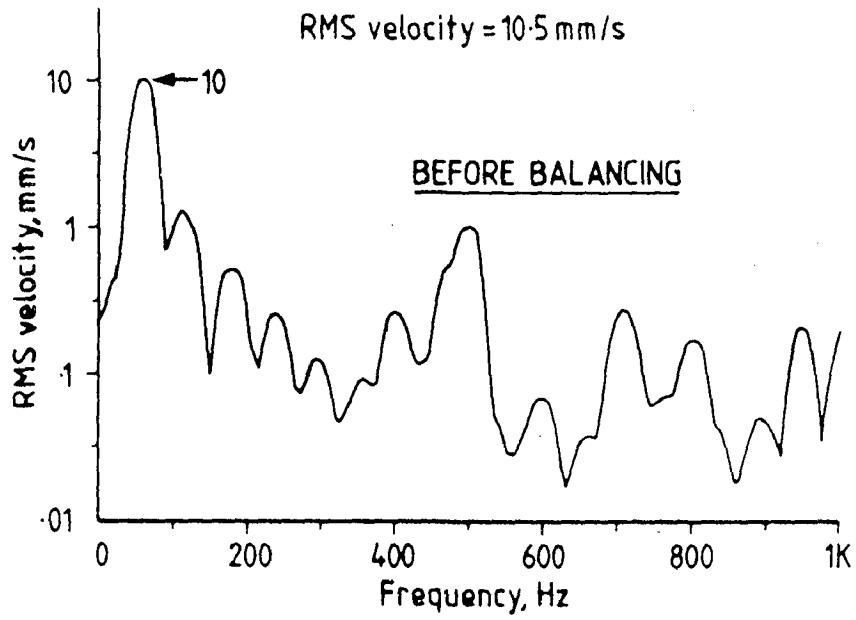


FIGURE 7.2 : THE EFFECT OF BALANCING ON BEARING VIBRATION [19].

After the analysis of the vibration signal of machine it was found that the overall radial vibration level, measured at the motor bearings, was too high and that the principal component was at the fundamental rotational frequency. This was diagnosed as the case of rotor unbalance and the machine was subsequently rebalanced.

Figure 7.2 shows the records produced by the monitoring system before and after balancing. The overall vibration level before rotor balancing was 10.5 mms^{-1} and after rotor balancing 4.7 mms^{-1} . The level of fundamental component was reduced to 4.1 mms^{-1} , from 10.0 mms^{-1} after balancing rotor.

Case 2: Identification of cracked Rotor Bars [19]

This study indicated the presence of loose bars in the rotor of a gas booster compressor drive. The machine is a 2-pole, 6.6 kv cage induction machine with a nominal speed of 3570 rpm.

The vibration spectra from this machine indicated a degree of electromagnetically induced components, principally the twice line frequency peak at 120 Hz, and the higher frequencies at 5, 6, 7, and 8 times the running speed. Although the overall level of vibration was still relatively low, the characteristics of the spectra held the key to the diagnosis of broken bars. A shut down and subsequent inspection of the rotor revealed several faulty bars, and it was considered likely that, if the machine had been allowed to continue operating in this condition, then a catastrophic failure would have resulted, due to rotor/stator

contact.

The spectral changes at a 2 monthly intervals shown in figure 7.3. For comparison, the post repair spectrum is also shown.

7.2 Test Results

Efforts were made to record the real vibration data from a rotating electrical machine surface. But since, the vibration transducer (in this case piezoresistive transducer) available in the lab, was of frequency response up to 100 Hz, it could not become possible to record the vibrations of the rotating electrical machines where were in the range from 500 Hz to 1000 Hz. So the vibration signals for the performance evolution of software were recorded from a vibration table with the available transducer (in frequency range from 20 Hz to 80 Hz).

For testing the developed software two sets of vibration signals were recorded from vibration table for different settings of frequency and amplitude. Two standard signals were also recorded from signal generator for testing the authenticity of the developed software. One sinusoidal signal of 200 Hz frequency, and another square signal of 2000 Hz frequency. These four signals were tested in respect of frequency spectrum, cepstrum, and overall level of vibrational velocity. The results are discussed in the following sections:

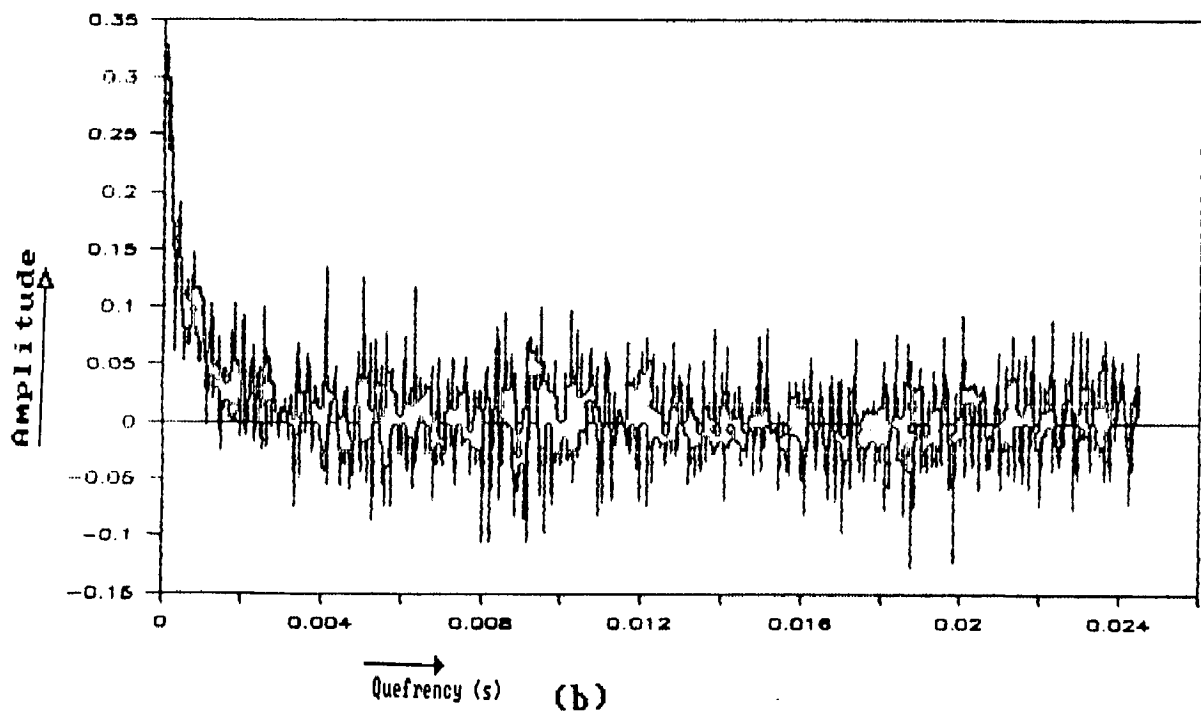
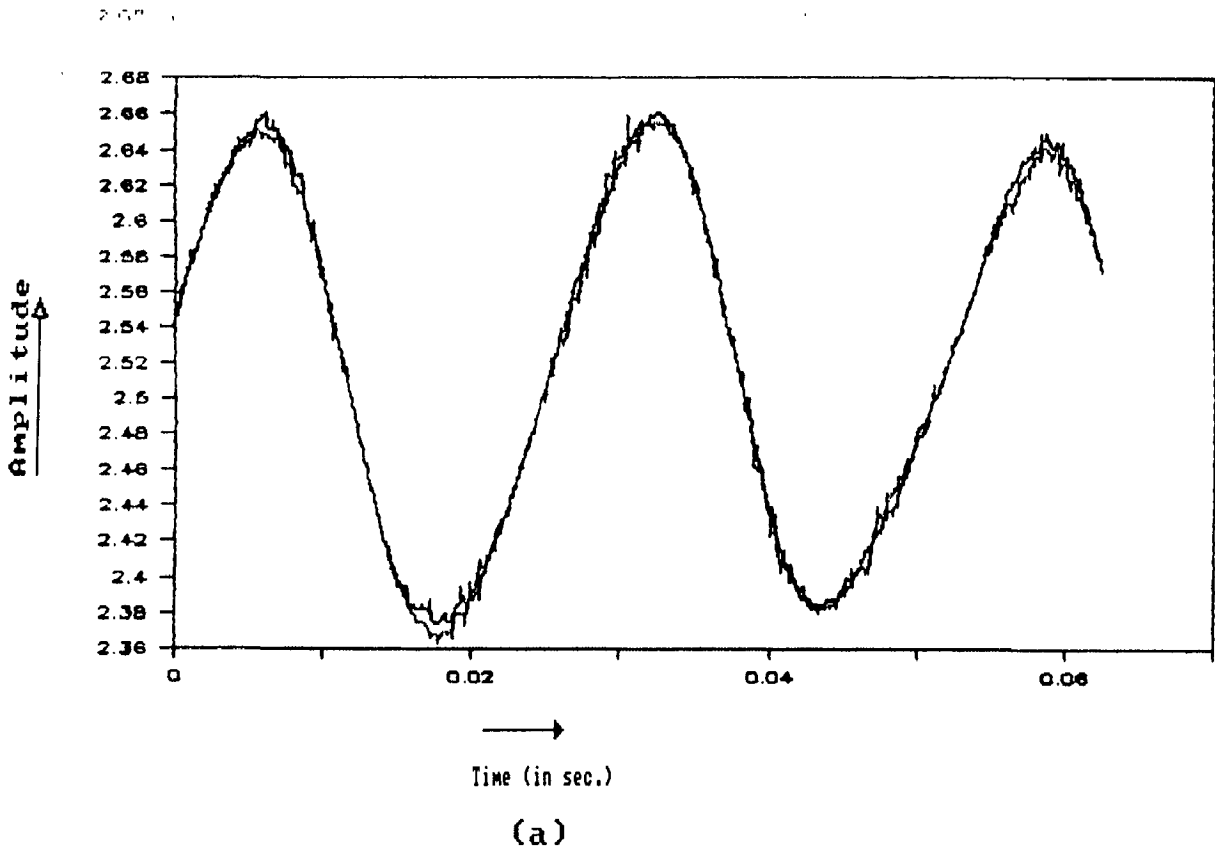
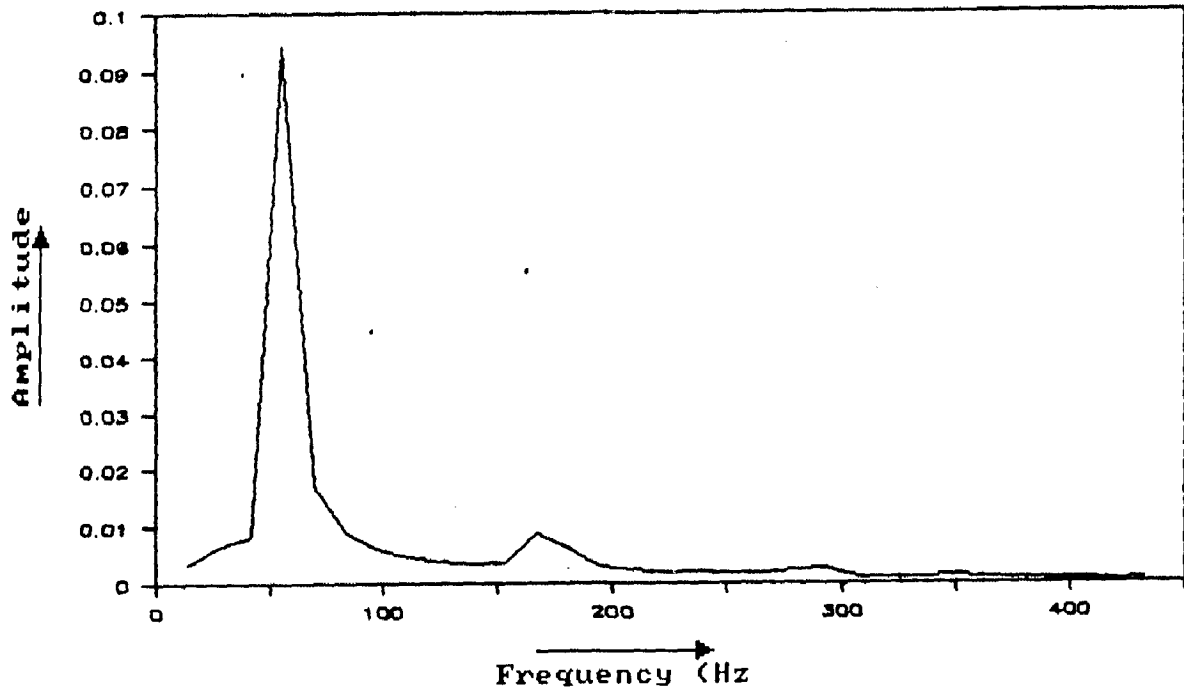
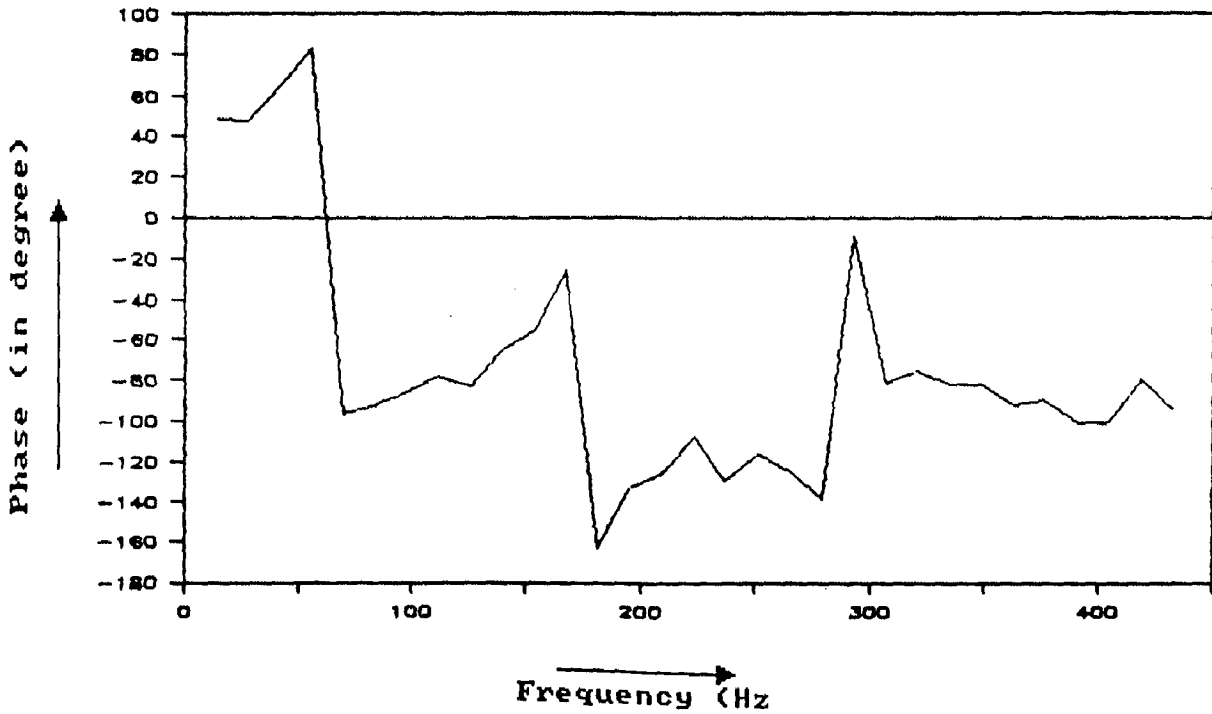


FIGURE 7.4 : VIBRATION SIGNAL RECORDED FROM VIBRATION TABLE
 (a) SIGNAL IN TIME DONAIN
 (b) CEPSTRUM



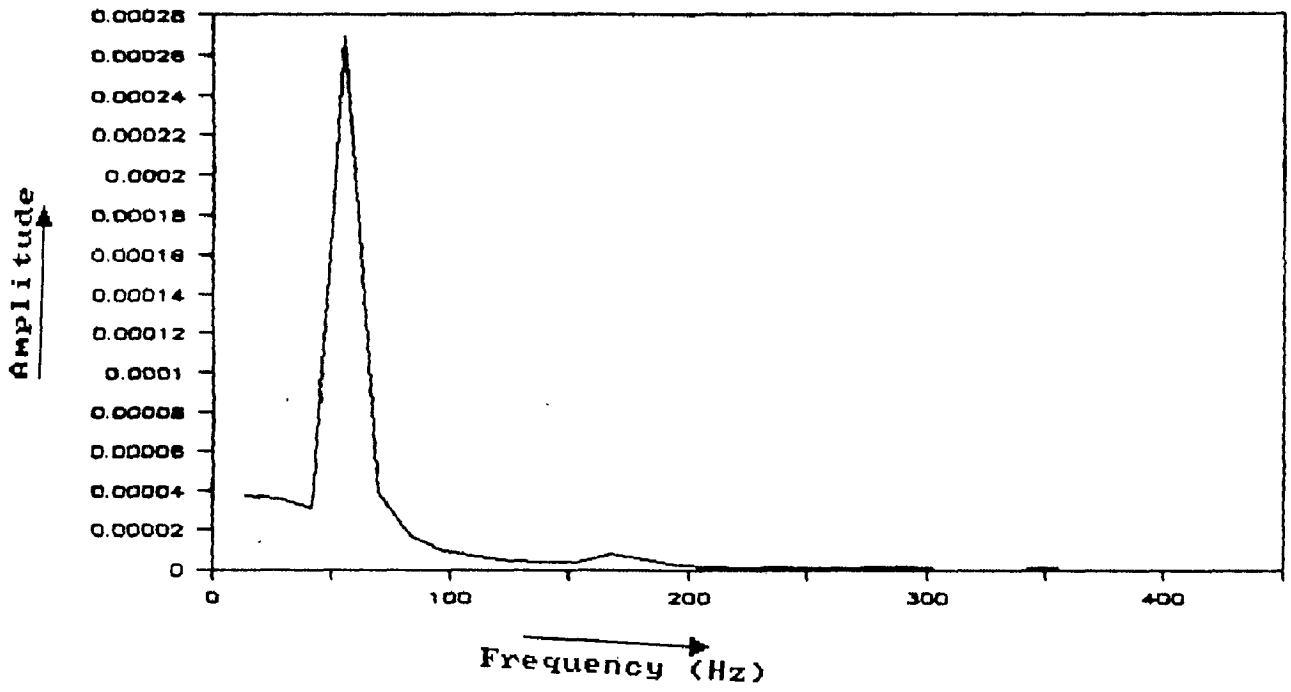
(c)



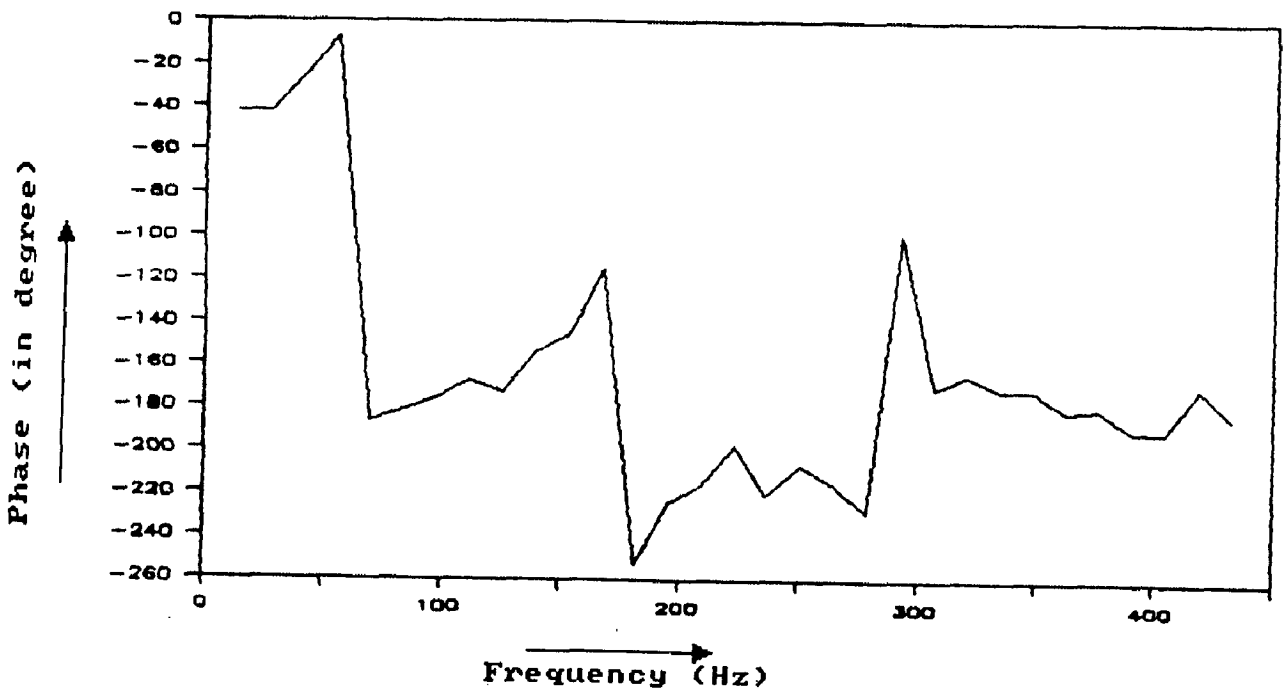
(d)

FIGURE 7.4 : CONTD.

VIBRATION ACCELERATION: (c) AMPLITUDE (d) PHASE.



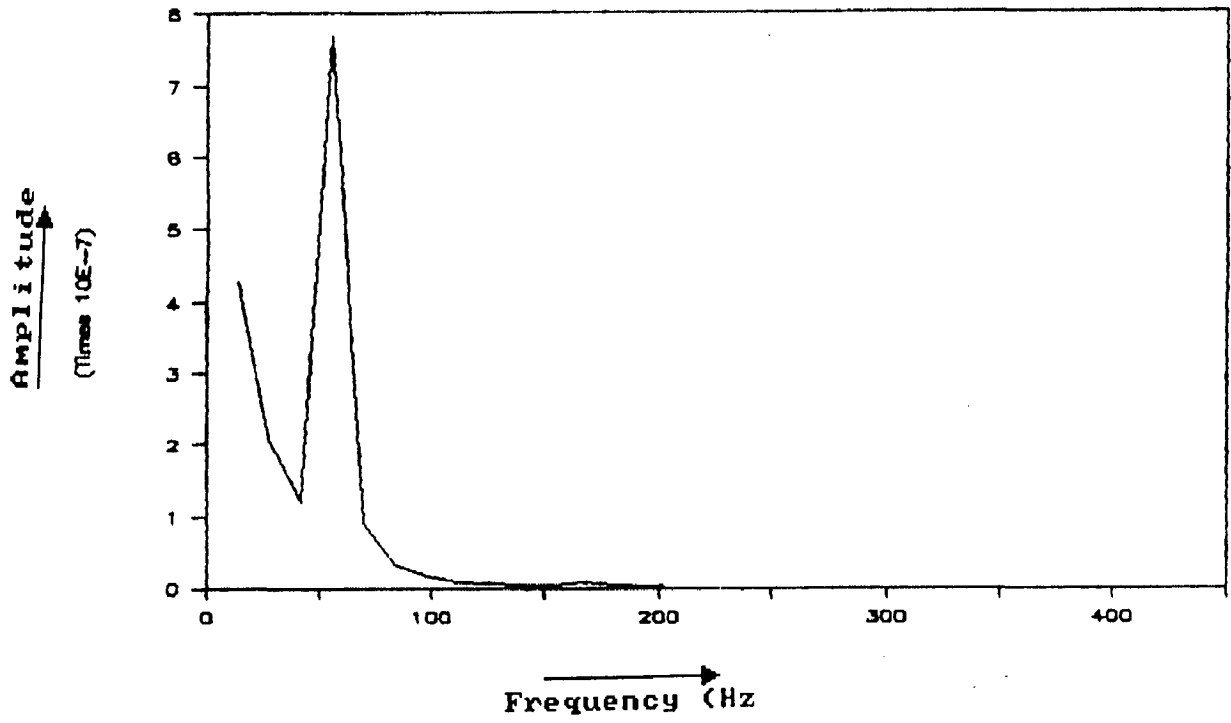
(e)



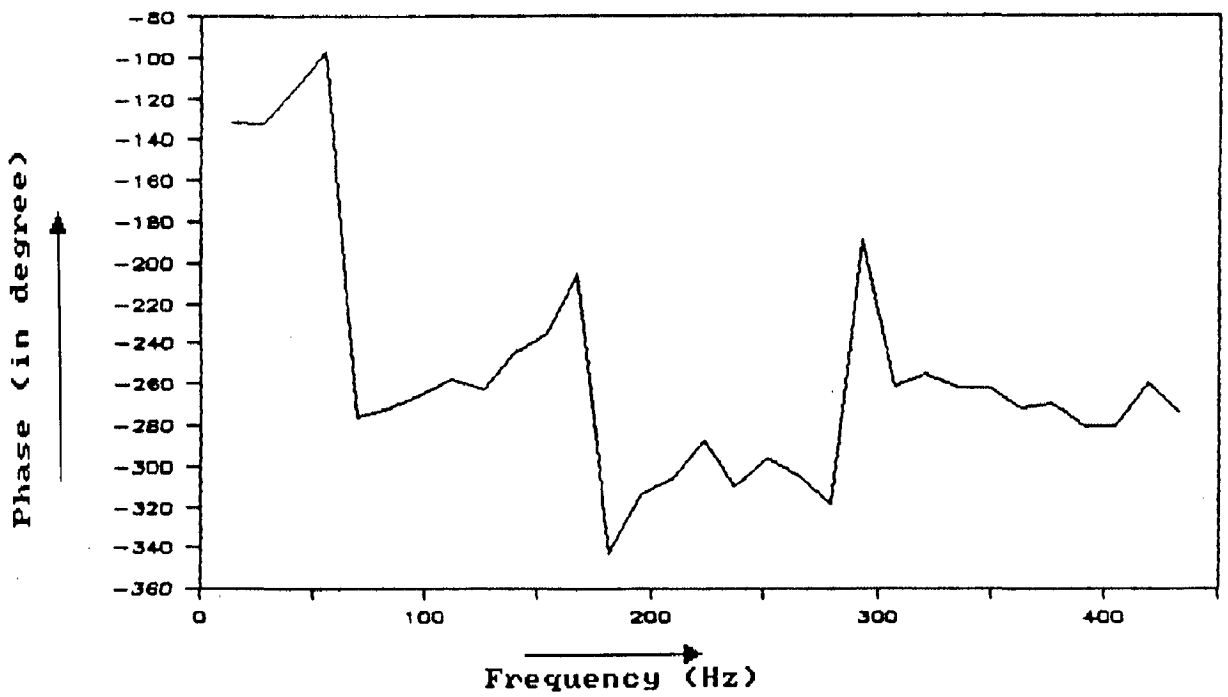
(f)

FIGURE 7.4 : CONTD.

VIBRATION VELOCITY: (e) AMPLITUDE (f) PHASE.



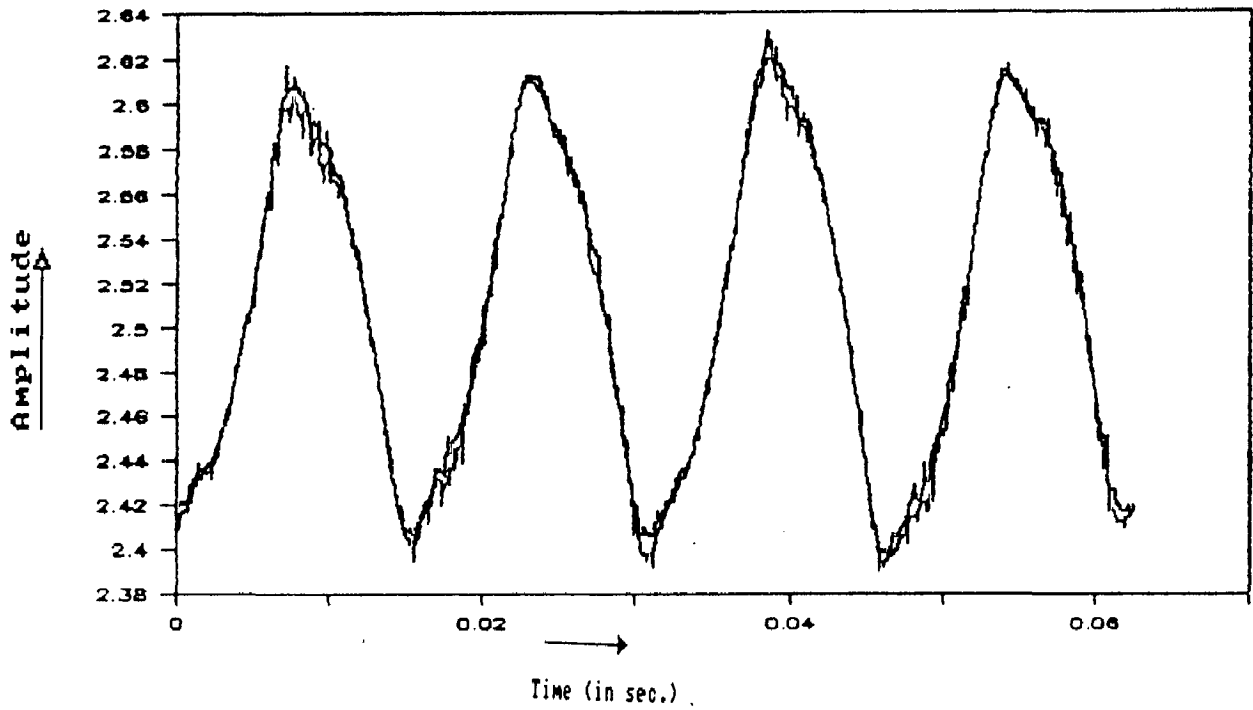
(g)



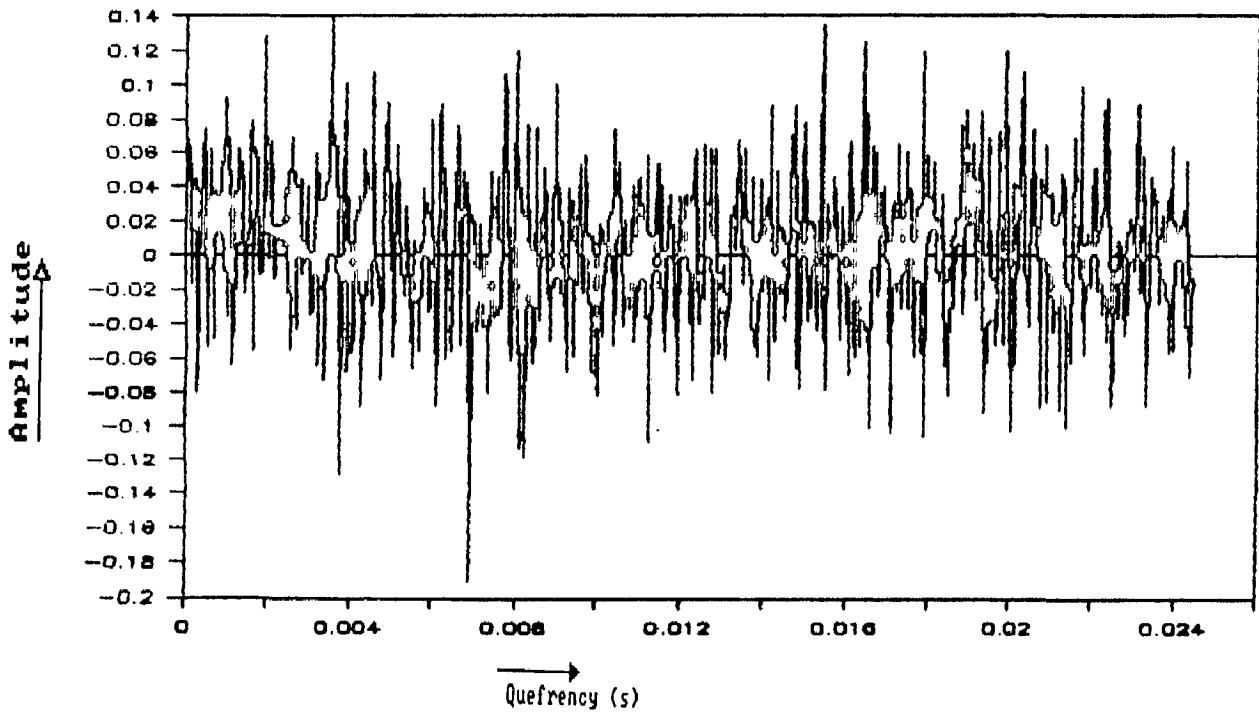
(h)

FIGURE 7.4 : CONTD.

VIBRATION DISPLACEMENT: (g) AMPLITUDE (h) PHASE.

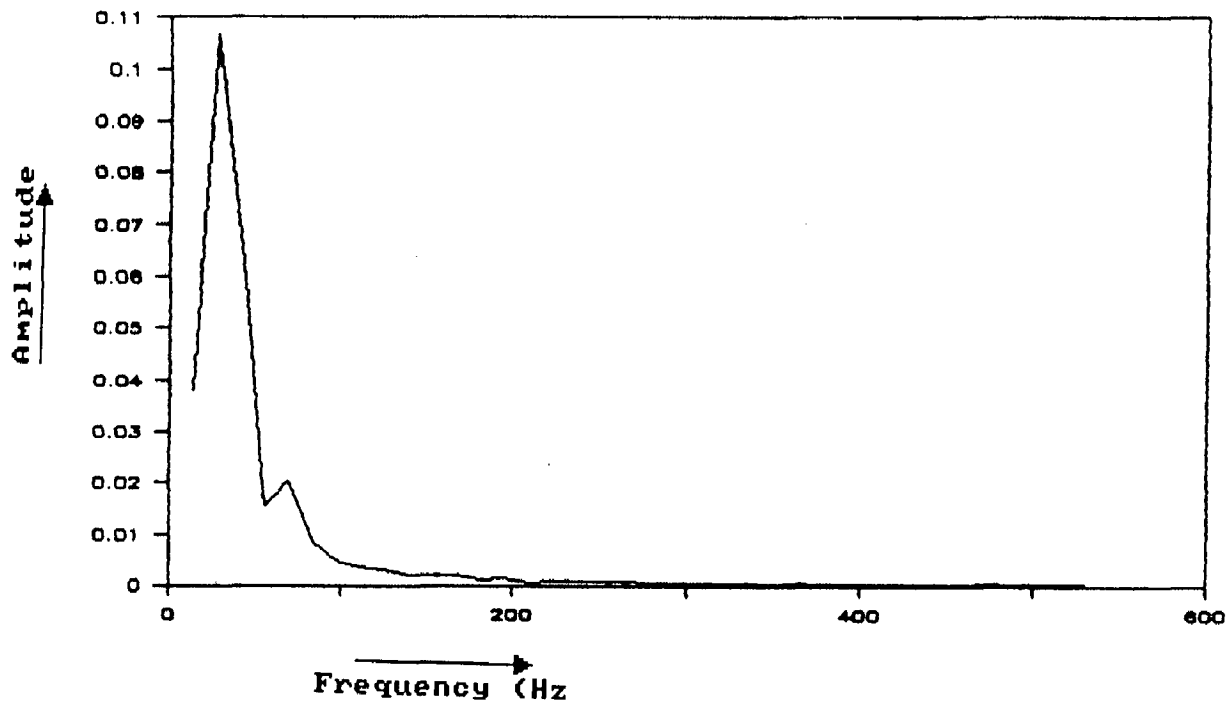


(a)

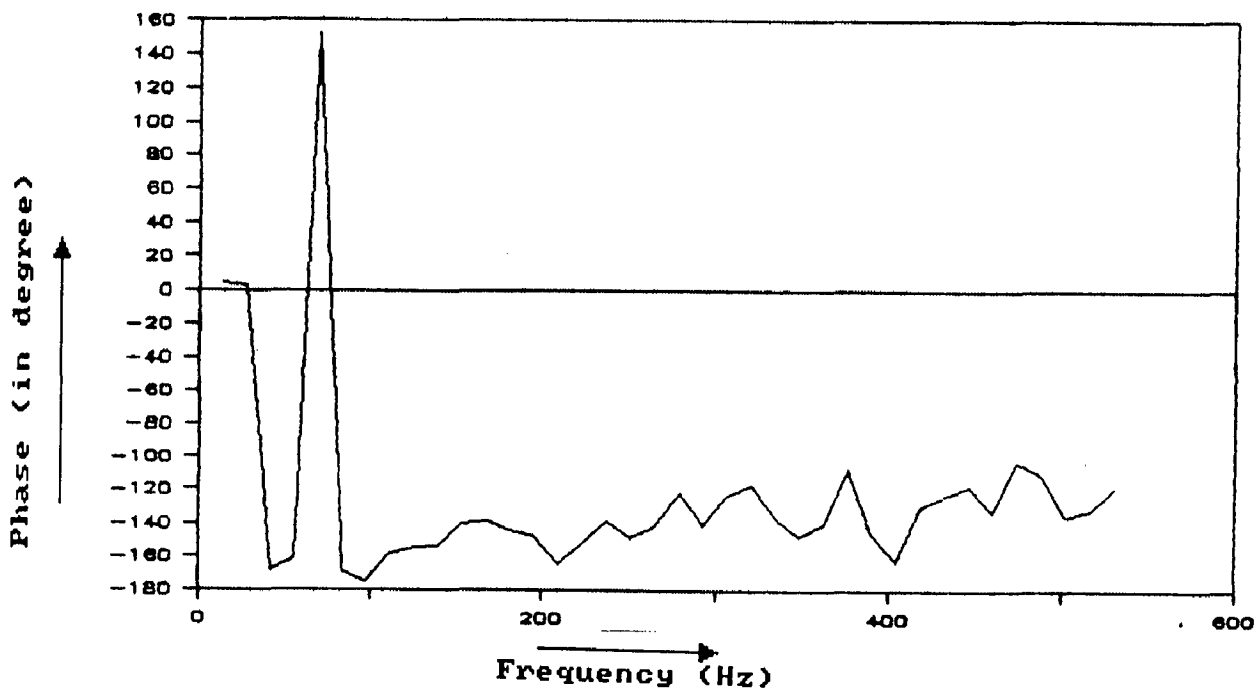


(b)

FIGURE 7.5 : VIBRATION SIGNAL RECORDED FROM VIBRATION TABLE
 (a) SIGNAL IN TIME DOMAIN
 (b) CEPSTRUM.



(c)



(d)

FIGURE 7.5 : CONTD.

VIBRATIONAL ACCELERATION (c) AMPLITUDE (d) PHASE.

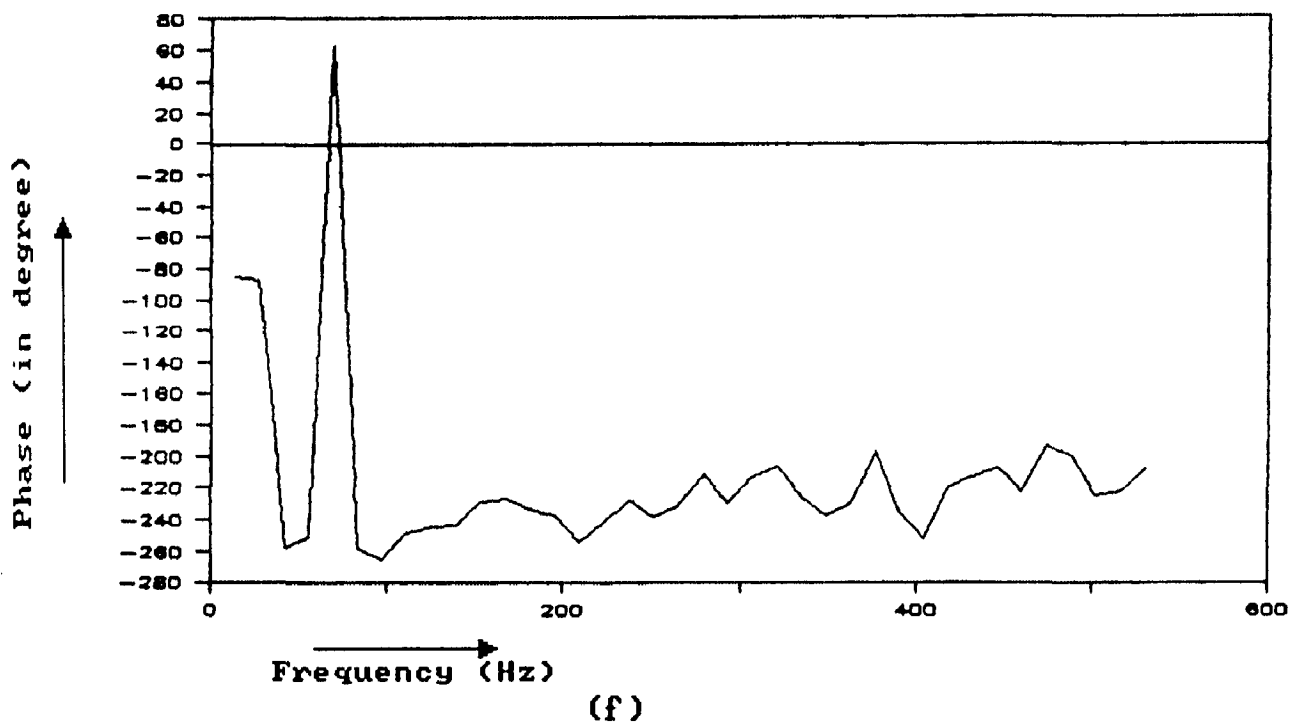
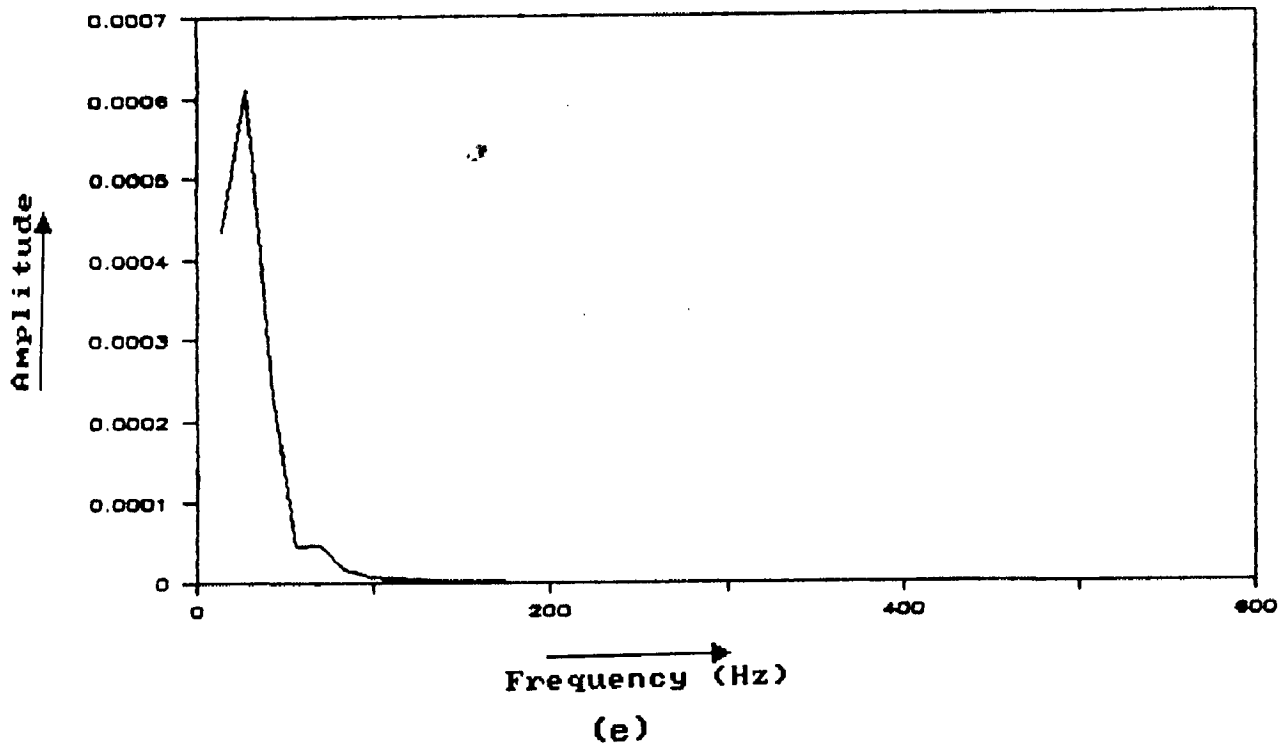
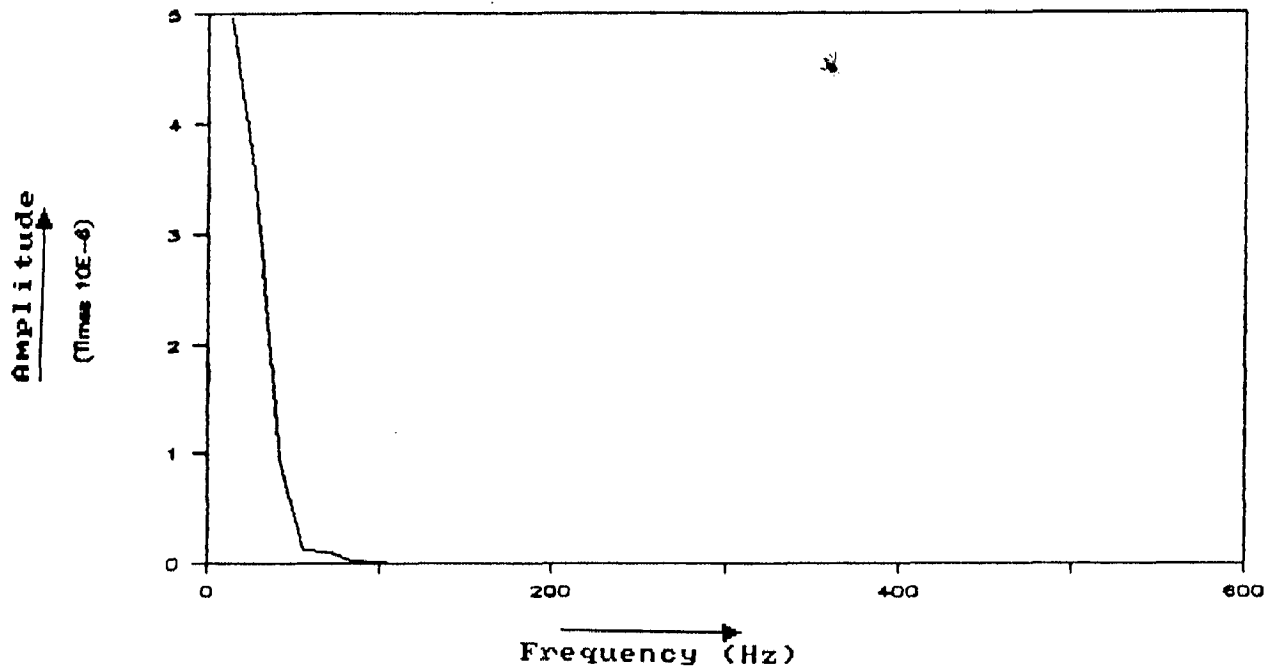
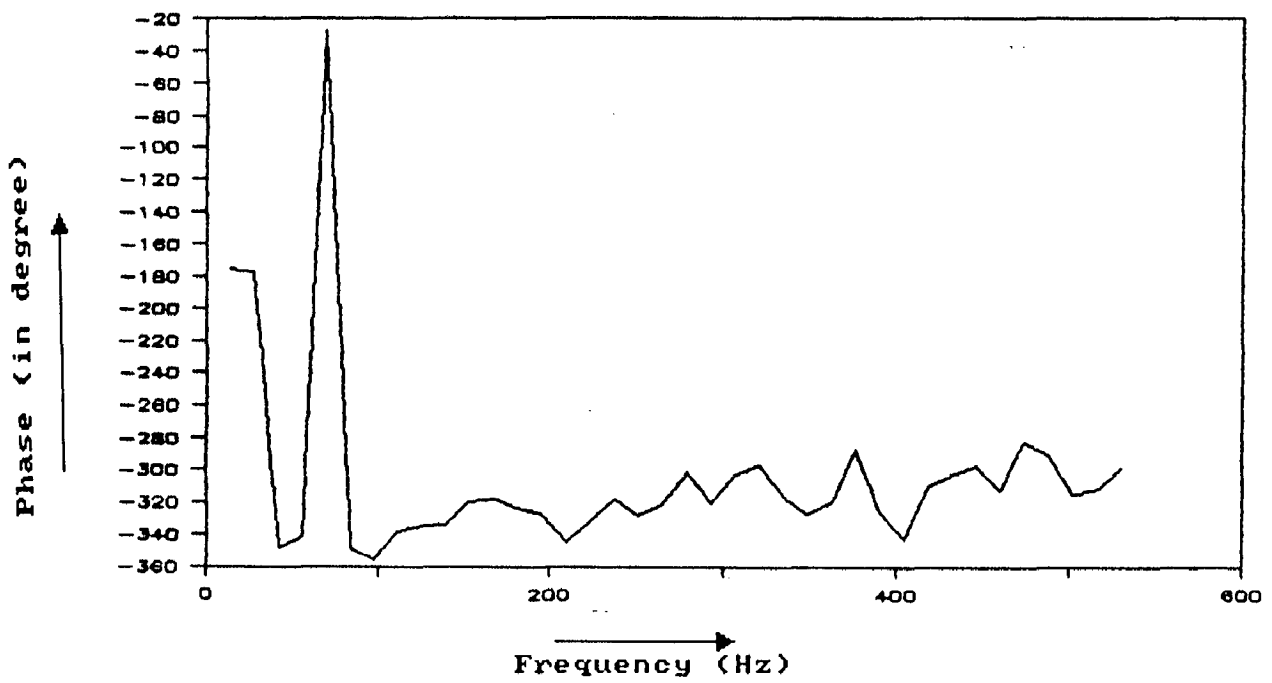


FIGURE 7.5 : CONTD.
 VIBRATIONAL VELOCITY (e) AMPLITUDE (f) PHASE.



(g)



(h)

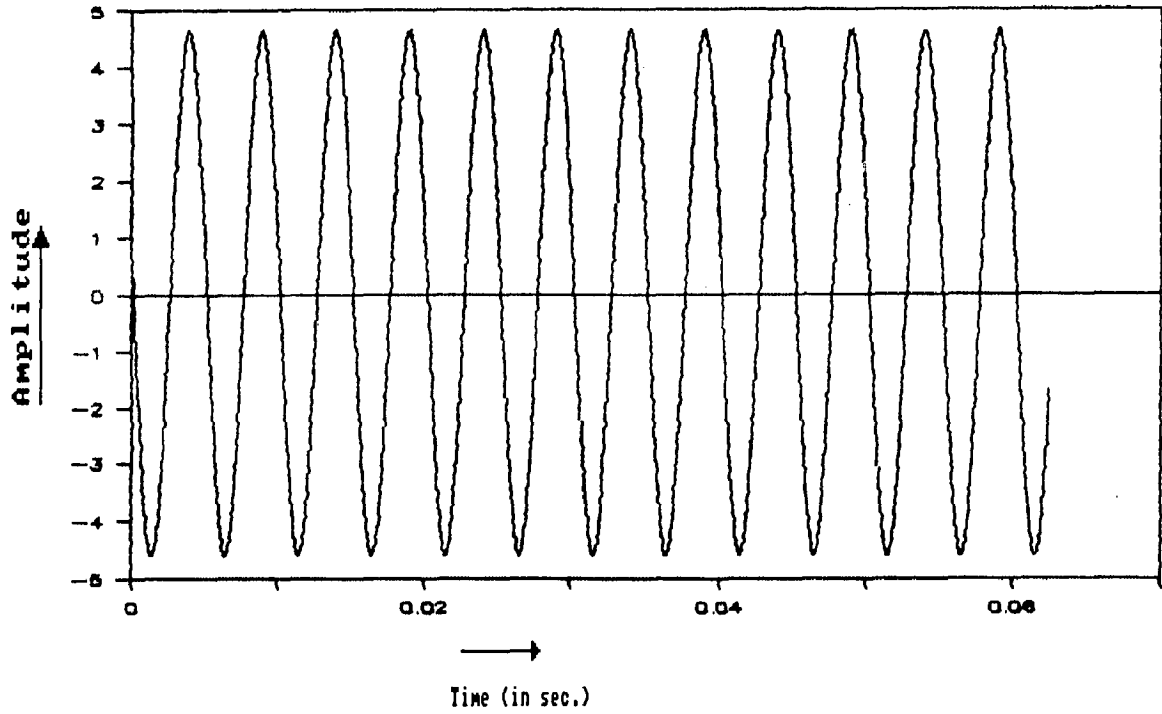
FIGURE 7.5 : CONTD.

VIBRATIONAL DISPLACEMENT (g) AMPLITUDE (h) PHASE.

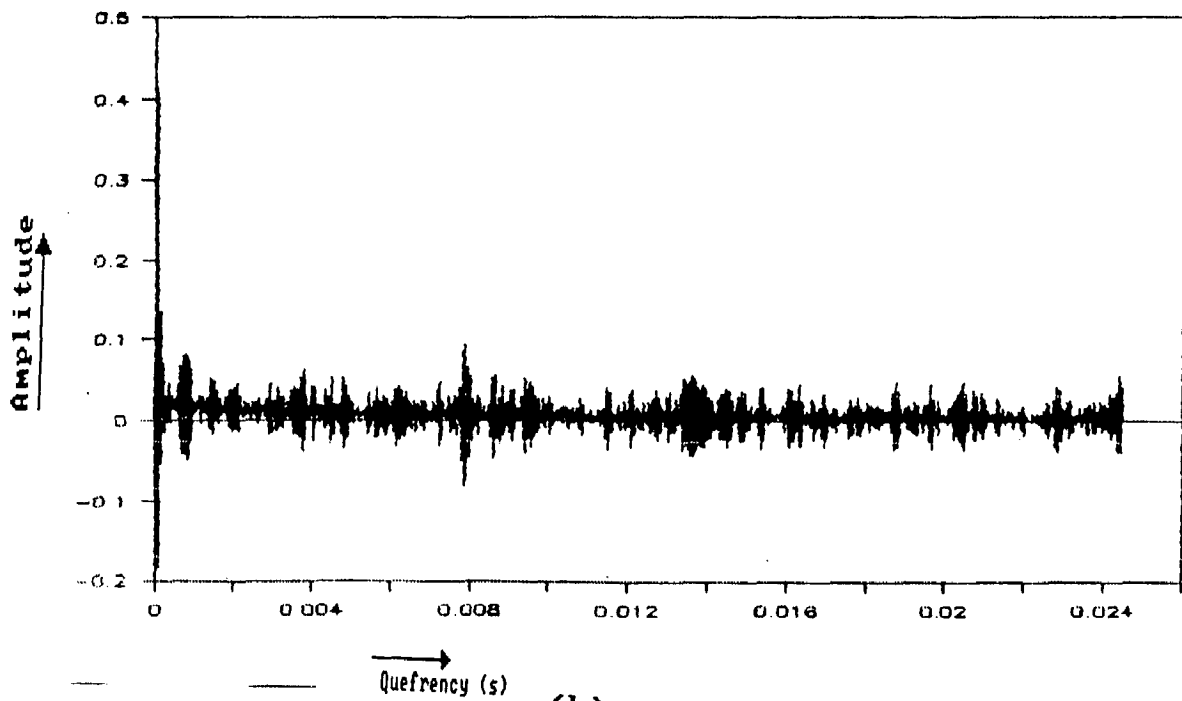
Case 1: Vibration signals recorded from the vibration table:

Vibration signal recorded for one setting of frequency and amplitude is shown in figure (7.4)a. The cepstrum of this vibration signal is shown in figure (7.4)b. From the cepstrum of this vibration it is clear that the vibration signal shown in the figure (7.4)a is periodic and having no side bands. Figures (7.4)c and d show the amplitude and the phase of vibration acceleration in frequency spectrum respectively. From these two figures it is clear that the fundamental component has frequency approximately 60 Hz, and there are no harmonic components. Figures (7.4)e and f show the amplitude and the phase of vibration velocity respectively. The amplitude of the vibration velocity is, given by the modification of amplitude of vibration acceleration by dividing it by its radial frequency. The phase of the vibration velocity is 90° backward to the phase of the vibration acceleration. Figures (7.4)g and h show the amplitude and the phase of vibration displacement respectively. From these two figures it is clear that the amplitude of displacement is a modification of the amplitude of velocity by dividing it by its radial frequency. The phase of displacement is 90° backward to the phase of the velocity.

In figure (7.5)a vibration signal for another setting of the frequency and amplitude, recorded from vibration table, is shown. The cepstrum of this vibration signal is shown in figure (7.5)b. The cepstrum of this vibration signal shows that vibration is periodic in nature with no side bands. The figures (7.5)c and d



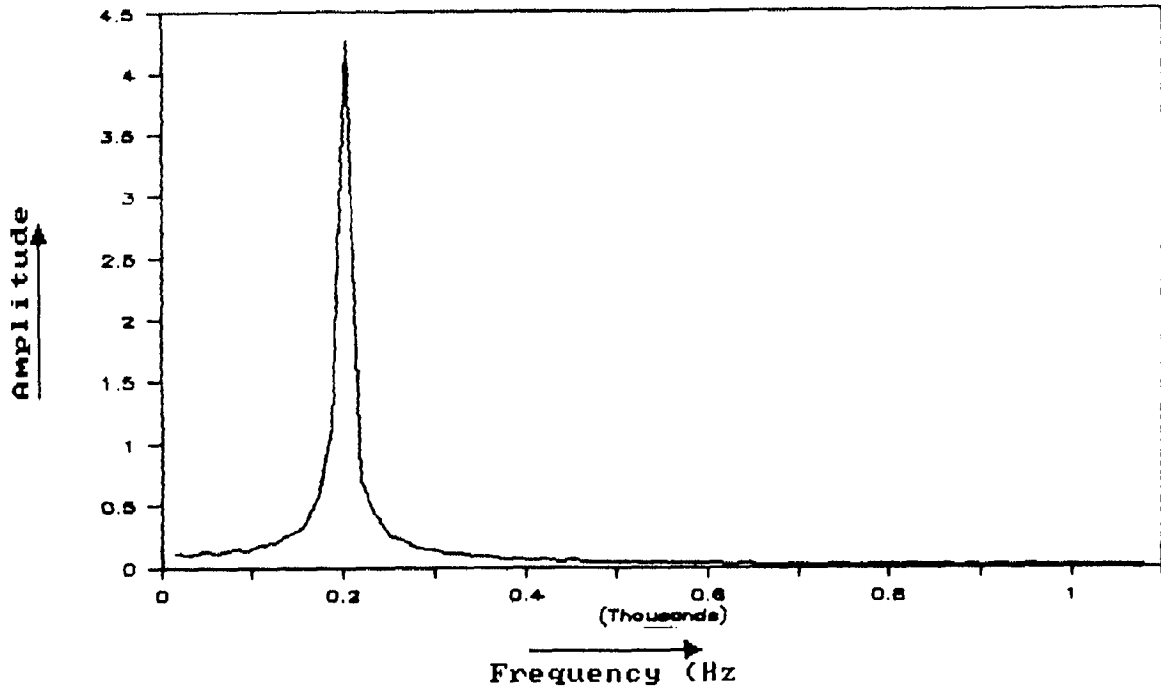
(a)



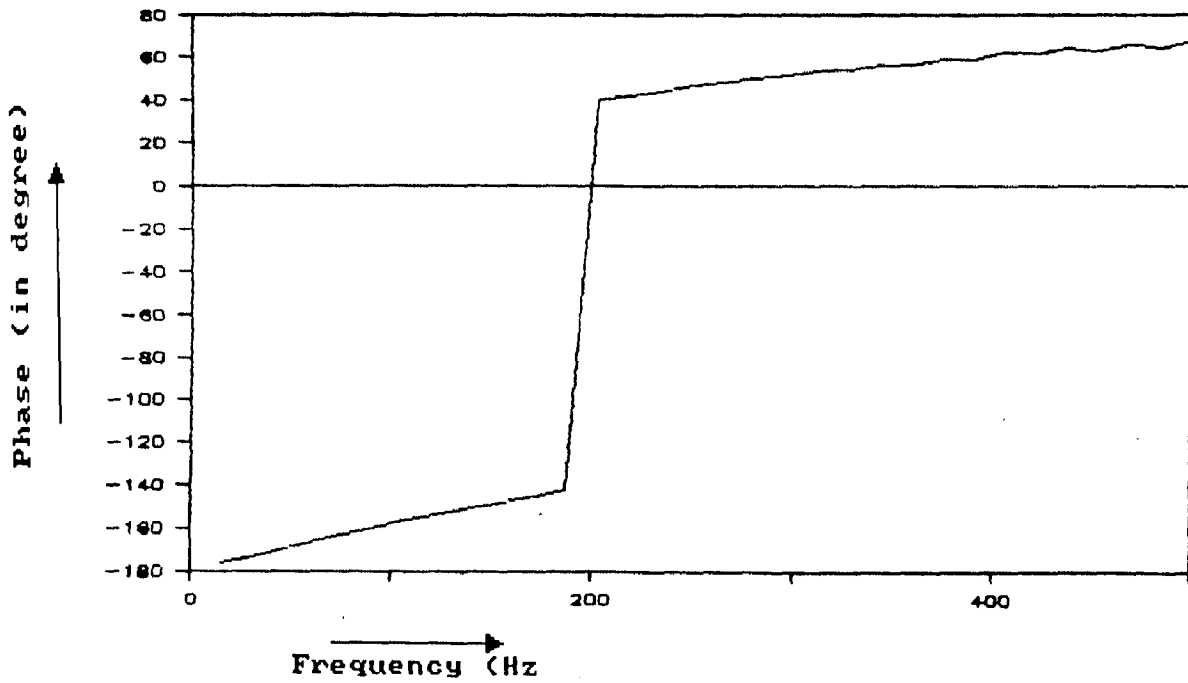
(b)

FIGURE 7.6 : SIGNAL RECORDED FROM SIGNAL GENERATOR
 (a) SIGNAL IN TIME DOMAIN
 (b) CEPSTRUM.

sine wave excitation



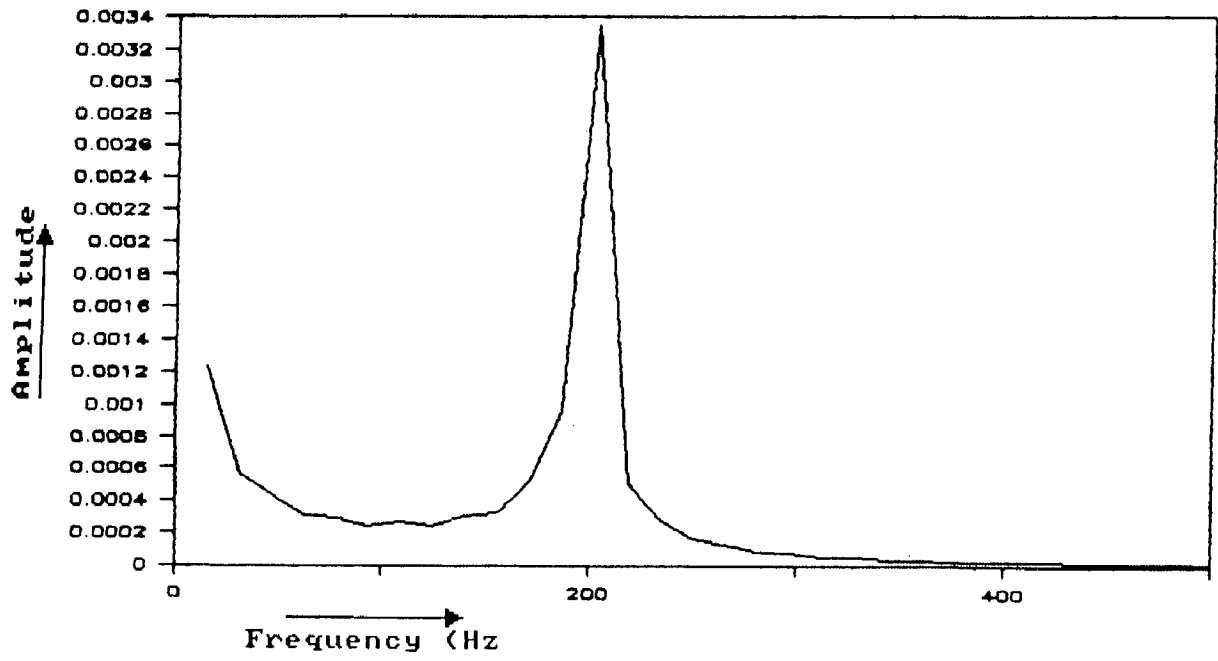
(c)



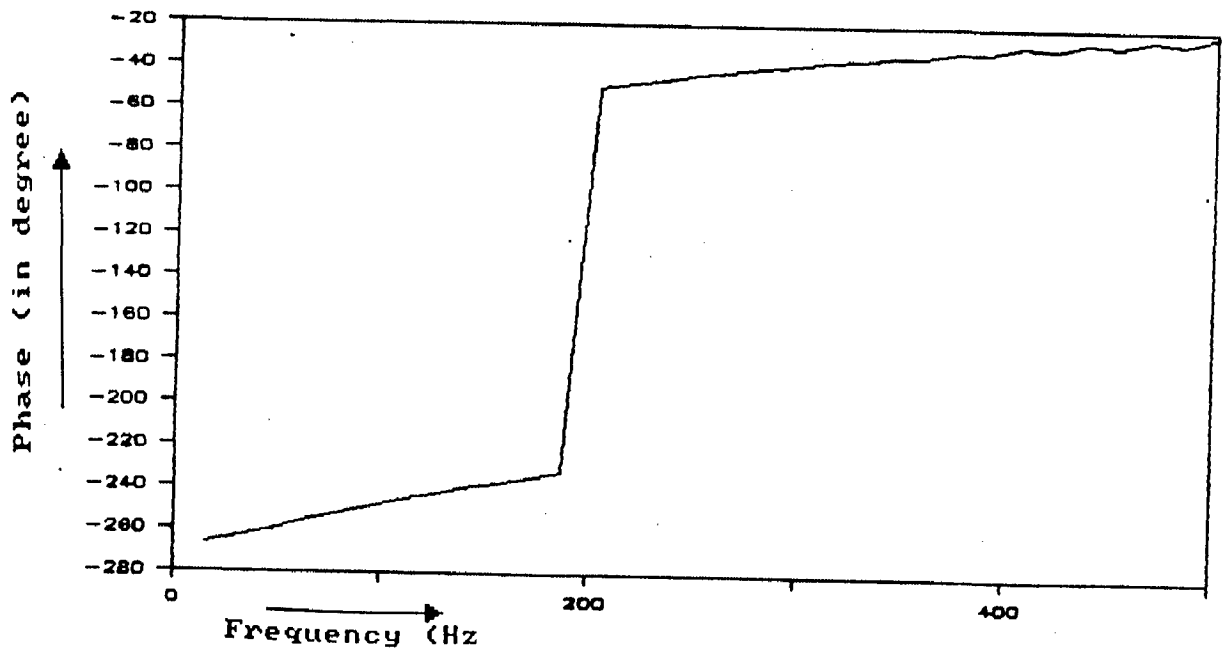
(d)

FIGURE 7.6 : CONTD.

VIBRATIONAL ACCELERATION (c) AMPLITUDE (d) PHASE.



(e)



(f)

FIGURE 7.6: CONTD.
VIBRATIONAL VELOCITY (e) AMPLITUDE (f) PHASE.

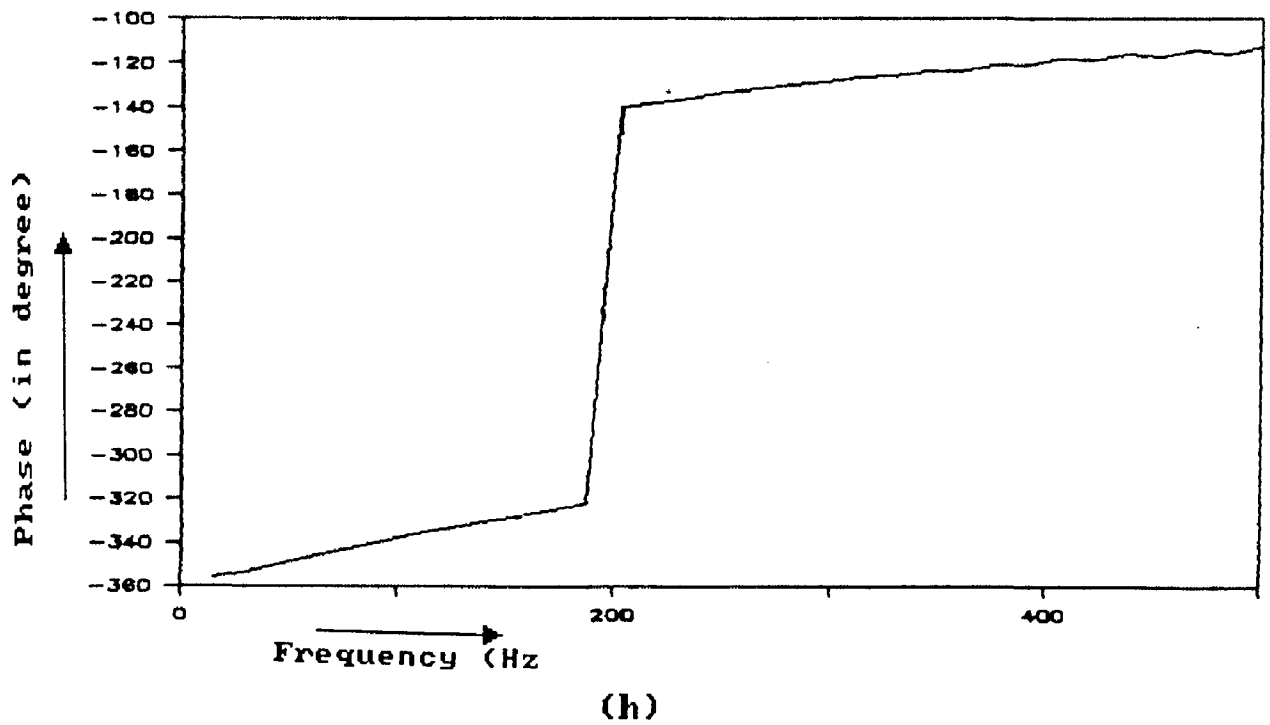
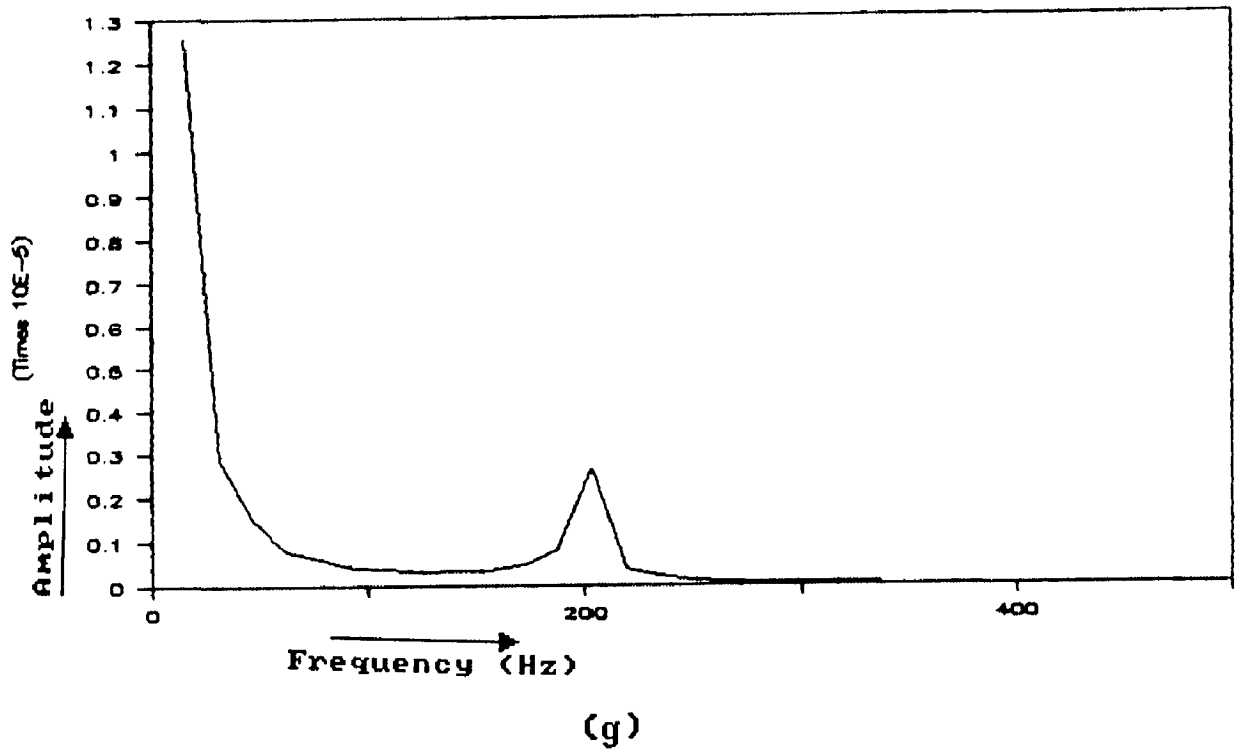


FIGURE 7.6 : CONTD.
 VIBRATIONAL DISPLACEMENT (g) AMPLITUDE (h) PHASE.

show the amplitude and the phase of the vibration acceleration in frequency spectrum respectively. From these figures it is clear that vibration has fundamental component at 35 Hz, and there is no harmonic components. Figures (7.5)e and f show the amplitude and phase of vibration velocity in frequency spectrum respectively. The amplitude of the velocity is modification of amplitude of acceleration by dividing it with its radial frequency. The phase of velocity is 90° backward to the phase of acceleration. Figures (7.5)g and h show the amplitude and the phase of vibration displacement in frequency spectrum. The amplitude of vibration displacement is modified by dividing the amplitude of velocity by its radial frequency. The phase of displacement is 90° backward to the phase of velocity.

Case 2: Signal recorded from signal generator (recorded signal is assumed as acceleration of the signal):

Vibration signal recorded for one setting of frequency and amplitude is shown in figure (7.6)a. The cepstrum of this vibration signal is shown in figure (7.6)b. From the cepstrum of this vibration it is clear that the vibration signal shown in the figure (7.6)a is periodic and having no side bands. Figures (7.6)c and d show the amplitude and the phase of vibration acceleration in frequency spectrum respectively. From these two figures it is clear that the fundamental component has frequency approximately 200 Hz, and there are no harmonic components. Figure (7.6)e and f show the amplitude and phase of vibration velocity respectively. The amplitude of the vibration velocity

acceleration by dividing it by its radial frequency. The phase of the vibration velocity is 90° backward to the phase of vibration acceleration. Figures (7.6)g and h show the amplitude and the phase of vibration displacement respectively. From these two figures it is clear that the amplitude of displacement is a modification of the amplitude of velocity by dividing it by its radial frequency. The phase of displacement is 90° backward to the phase of velocity.

7.3 Discussion

Spectrum analysis of the vibration signal recorded from the surface of the electrical rotating machine is carried out to provide information on the frequency composition of the vibration. In vibration monitoring, the cause of the vibration (the source of the defect) is identified by using spectrum analysis, on the basis of characteristic frequency during normal operation. The frequency analysis breaks down the vibration into its harmonic components. If the energy contents of the individual frequency components are nearly the same, then the analysis of vibration velocity is performed. If the energy content of the individual frequency components decreases with increasing frequency, then the analysis of vibration acceleration is performed.

The unbalanced magnetic pull generates frequency components at 1,2, and 4 times the fundamental frequency. Dynamic unbalance and coupling misalignment also produce this effect. Even orders

of the fundamental frequency occur in the frame vibration spectrum due to inter-turn winding faults on the stator.

The vibration of the end winding is due to the electromagnetic forces on the coils, due to current flowing in them. The resultant displacement are at twice the supply frequency, $2f$. For monitoring of the amplitude of vibration of end winding, accelerometers are installed on the end windings. The vibration recorded from these accelerometers are used to monitor the amplitudes of $2f$ vibration at widely spaced intervals of time in order to check that the end winding has not slackened. The displacement of end windings on large turbogenerators up to $150 \mu\text{m}$ is considered safe.

The vibration associated with the bearings fault are detected by an accelerometer mounted directly on the bearing housing. The characteristic frequency of bearings fault depend on the geometrical size of the various elements of the bearing.

Unbalanced rotor generates vibration of frequency f_r (f_r is rotational frequency). It also causes unbalanced magnetic pull which gives $2f_r$ vibration. Misalignment of rotor shaft generates frequencies f_r , $2f_r$, $3f_r$ and $4f_r$. Broken rotor bars in induction machines generates frequency components of frequencies $f_r + 2sf_s$ (s is slip and f_s is supply frequency).

CHAPTER - 8

CONCLUSIONS AND FURTHER SCOPE

8.1 Conclusions

The work has been carried out for developing a software package for the measurement, monitoring, analysis, and feature extraction of vibrational signal. The work was intended to be tested on a rotating electrical machine. But due to non-availability of the suitable transducer, the software has been tested on the standard signals from a function generator and real time vibration signals from a vibration table using a piezoresistive accelerometer. The results of analysis are consistent and reliable and establishes the authenticity of the method to be used for real time vibration signal analysis from rotating electrical machines. It has been explained in the work that the spectral analysis of the vibration signal recorded from the surface of rotating electrical machine are one of the best method of detecting defect inside machine without dismantling it.

Thus the vibration monitoring can be used to find out following defaults inside machine:

- (i) The unbalance of the rotor,
- (ii) The unbalance magnetic pull,
- (iii) Crack in rotor bars,
- (iv) The looseness of end winding,

- (v) Fault inside bearings, and
- (vi) Dynamic unbalance and coupling misalignment.

8.2 Further scope

In this dissertation the software, developed for analysis of vibration signal has been tested on the vibration signals recorded from vibration table and signals recorded from signal generator. In future the work can be done on the analysis of real vibration signals recorded by using high frequency response accelerometers from rotating electrical machines surface. The vibration signals from machine without defects and machine with defects can be compared to detect various frequency components present due to defect inside the machine. The monitoring of electrical machine can be done on the basis of frequency spectrum analysis of the sound of the machine. A work can be done on developing a software which can compare frequency spectrum of the machine with a baseline spectrum and if the pre-programmed comparison criterion is exceeded then a warning signal is generated automatically.

REFERENCES

1. Allocca J.A., Stuart A "Transducers : Theory and Application" Reston Publishing Co., Reston, 1984.
2. Oliver F.J. "Practical Instrumentation Transducers" Pitman, London, 1972.
3. Bruel and Kjaer "Measuring Vibration" Bruel and Kjaer, Naerum, 1984.
4. Buel and Kjaer "Machine Health Monitoring" Bruel and Kjaer, Naerum, 1984.
5. Timar P.L. "Noise and Vibration of Electrical Machines" Elsevier, 1989.
6. Rangan C.S. "Instrumentation Devices and Systems" TMH, New Delhi, 1983.
7. Alger P.L. "Induction Machine" John Wiley and Sons, New York, 1981.
8. Jurden H. "The Low Noise Electric Motor" Essen, Germany, 1982.
9. Wort J.F.G. "The Fundamentals of Industrial Balancing Machines and their Applications" Bruel and Kajer Review No. 1, 1981, pp 3-33.
10. Walker D.N., Adams S.L., Plaeck P.J. "Torsional Vibration and Fatigue of Turbine Generator Shafts" IEEE Trans. (PAS), Vol. PAS-102, No. 6, 1983, pp 1552-1565.
11. Calacott R.A. "Vibration Monitoring and Diagnostics" George Godwin Ltd., London, 1979.
12. Ehrich. E.F. "Identification and Avoidance of Instabilities and Self Excited Vibrations in Rotating Machinery, ASME Paper No. 72-DE-21.
13. Neale M. and Associates "A Guide to the Condition Monitoring of Machinery" HMSO Publication, 1979.
14. Mayes I.W. Steer A.G. and Thomas G.B. "The Application of Vibration monitoring for Fault Diagnosis in Large Turbo Generators" 6th Thermal Generation Specialists Meeting, Madrid, 5-6 May 1981.

15. Randall R.B. "Cepstrum Analysis and Gearbox Fault Diagnosis" Bruel and Kjaer Publications, Application Note 233, 1980.
16. Stephen E.D. "Interfacing" Prentice Hall, Englewood Cliffs, 1990.
17. Brigham E.O. "The Fast Fourier Transform and Its applications" Prentice Hall, Englewood Chiffs, 1988.
18. Tavner P.J., Gaydun B.G., and Ward D.M., "Monitoring Generators and Large Motors, Proc. IEE, Vol. 133, Pt. B, No. 3, May 1986, pp 169-180.
19. Hodge J.M., Miller T., Roberts A. and Steel J.G. "Generator Monitoring Systems in the United Kingdom" CIGRE, Paris, France, 1-9 September, 1982, Paper 11-08.

APPENDIX-1
VIBRATION DATA

TABLE-1
FREQUENCY SPECTRUM ANALYSIS DATA FOR FIG. 7.4

FREQUENCY (Hz)	ACCELERATION		VELOCITY		DISPLACEMENT	
	AMPLITUDE	PHASE (degree)	AMPLITUDE	PHASE (degree)	AMPLITUDE	PHASE (degree)
0.0	2.510e+00	0.0	2.510e+00	0.0	2.510e+00	0.0
14.0	3.296e-03	48.0	3.760e-05	-42.0	4.289e-07	-132.0
27.9	6.396e-03	47.7	3.648e-05	-42.3	2.081e-07	-132.3
41.9	8.282e-03	65.0	3.149e-05	-25.0	1.198e-07	-115.0
55.8	9.432e-02	82.7	2.690e-04	-7.3	7.672e-07	-97.3
69.8	1.708e-02	-96.4	3.897e-05	-186.4	8.891e-08	-276.4
83.7	9.032e-03	-91.8	1.717e-05	-181.8	3.265e-08	-271.8
97.7	6.070e-03	-85.4	9.893e-06	-175.4	1.612e-08	-265.4
111.6	4.740e-03	-77.6	6.759e-06	-167.6	9.638e-09	-257.6
125.6	3.808e-03	-82.8	4.827e-06	-172.8	6.119e-09	-262.8
139.5	3.560e-03	-64.4	4.062e-06	-154.4	4.634e-09	-244.4
153.5	3.571e-03	-55.5	3.704e-06	-145.5	3.841e-09	-235.5
167.4	8.751e-03	-25.6	8.320e-06	-115.6	7.910e-09	-205.6
181.4	6.028e-03	-162.8	5.290e-06	-252.8	4.642e-09	-342.8
195.3	2.864e-03	-134.0	2.334e-06	-224.0	1.902e-09	-314.0
209.3	2.372e-03	-125.9	1.804e-06	-215.9	1.372e-09	-305.9
223.2	1.848e-03	-107.7	1.318e-06	-197.7	9.397e-10	-287.7
237.2	2.154e-03	-130.3	1.445e-06	-220.3	9.700e-10	-310.3
251.1	1.810e-03	-116.4	1.147e-06	-206.4	7.272e-10	-296.4
265.1	1.941e-03	-125.4	1.165e-06	-215.4	6.997e-10	-305.4
279.0	2.398e-03	-138.4	1.368e-06	-228.4	7.803e-10	-318.4
293.0	2.569e-03	-9.2	1.396e-06	-99.2	7.583e-10	-189.2
306.9	1.018e-03	-81.0	5.277e-07	-171.0	2.737e-10	-261.0
320.9	1.035e-03	-75.2	5.131e-07	-165.2	2.545e-10	-255.2
334.8	1.119e-03	-81.8	5.321e-07	-171.8	2.529e-10	-261.8

TABLE-1 (contd.)

348.8	1.401e-03	-81.9	6.395e-07	-171.9	2.918e-10	-261.9
362.7	1.148e-03	-91.8	5.036e-07	-181.8	2.210e-10	-271.8
376.7	8.170e-04	-90.1	3.452e-07	-180.1	1.459e-10	-270.1
390.6	8.347e-04	-100.4	3.401e-07	-190.4	1.386e-10	-280.4
404.6	7.992e-04	-100.8	3.144e-07	-190.8	1.237e-10	-280.8
418.5	7.434e-04	-80.1	2.827e-07	-170.1	1.075e-10	-260.1
432.5	8.185e-04	-94.5	3.012e-07	-184.5	1.108e-10	-274.5
446.4	1.005e-03	-90.2	3.583e-07	-180.2	1.277e-10	-270.2
460.4	8.089e-04	-87.0	2.796e-07	-177.0	9.667e-11	-267.0
474.3	9.684e-04	-80.7	3.249e-07	-170.7	1.090e-10	-260.7
488.3	6.792e-04	-90.0	2.214e-07	-180.0	7.216e-11	-270.0
502.2	8.424e-04	-80.4	2.669e-07	-170.4	8.459e-11	-260.4
516.2	6.848e-04	-77.8	2.111e-07	-167.8	6.510e-11	-257.8
530.1	6.557e-04	-96.5	1.969e-07	-186.5	5.910e-11	-276.5
544.1	7.987e-04	-101.5	2.336e-07	-191.5	6.834e-11	-281.5
558.0	5.872e-04	-94.9	1.675e-07	-184.9	4.776e-11	-274.9
572.0	7.114e-04	-88.3	1.979e-07	-178.3	5.508e-11	-268.3
585.9	5.660e-04	-70.7	1.537e-07	-160.7	4.176e-11	-250.7
599.9	7.710e-04	-88.5	2.045e-07	-178.5	5.427e-11	-268.5
613.8	7.402e-04	-103.8	1.919e-07	-193.8	4.976e-11	-283.8
627.8	5.744e-04	-86.6	1.456e-07	-176.6	3.692e-11	-266.6
641.7	7.096e-04	-100.5	1.760e-07	-190.5	4.364e-11	-280.5
655.7	4.785e-04	-69.6	1.161e-07	-159.6	2.819e-11	-249.6
669.6	4.704e-04	-88.2	1.118e-07	-178.2	2.657e-11	-268.2
683.6	4.839e-04	-74.3	1.127e-07	-164.3	2.623e-11	-254.3
697.5	7.955e-04	-51.1	1.815e-07	-141.1	4.142e-11	-231.1
711.5	2.712e-04	-89.5	6.068e-08	-179.5	1.357e-11	-269.5
725.4	5.099e-04	-64.9	1.119e-07	-154.9	2.454e-11	-244.9
739.4	3.285e-04	-104.9	7.072e-08	-194.9	1.522e-11	-284.9
753.3	5.499e-04	-81.1	1.162e-07	-171.1	2.454e-11	-261.1
767.3	6.135e-04	-94.4	1.273e-07	-184.4	2.640e-11	-274.4
781.3	7.637e-04	-108.8	1.556e-07	-198.8	3.170e-11	-288.8

TABLE-2

FREQUENCY SPECTRUM ANALYSIS DATA FOR FIG. 7.5

Frequency (Hz)	ACCELERATION		VELOCITY		DISPLACEMENT	
	AMPLITUDE	PHASE (degree)	AMPLITUDE	PHASE (degree)	AMPLITUDE	PHASE (degree)
0.0	2.508e+00	0.0	2.508e+00	0.0	2.508e+00	0.0
14.0	1.714e-03	175.4	1.956e-05	85.4	2.231e-07	-4.6
27.9	1.298e-03	169.4	7.404e-06	79.4	4.224e-08	-10.6
41.9	3.792e-03	-160.5	1.442e-05	-250.5	5.484e-08	-340.5
55.8	2.135e-03	114.8	6.089e-06	24.8	1.737e-08	-65.2
69.8	3.670e-03	126.9	8.373e-06	36.9	1.910e-08	-53.1
83.7	1.124e-02	120.4	2.136e-05	30.4	4.062e-08	-59.6
97.7	9.437e-03	-60.4	1.538e-05	-150.4	2.507e-08	-240.4
111.6	3.430e-03	-63.3	4.892e-06	-153.3	6.976e-09	-243.3
125.6	2.420e-03	-70.6	3.067e-06	-160.6	3.888e-09	-250.6
139.5	1.196e-03	-80.0	1.365e-06	-170.0	1.557e-09	-260.0
153.5	9.873e-04	-91.2	1.024e-06	-181.2	1.062e-09	-271.2
167.4	9.771e-04	-80.7	9.289e-07	-170.7	8.831e-10	-260.7
181.4	2.310e-03	103.4	2.027e-06	13.4	1.779e-09	-76.6
195.3	2.616e-04	175.3	2.132e-07	85.3	1.737e-10	-4.7
209.3	9.916e-05	-152.5	7.542e-08	-242.5	5.736e-11	-332.5
223.2	2.168e-04	-136.9	1.546e-07	-226.9	1.102e-10	-316.9
237.2	3.660e-04	147.6	2.456e-07	57.6	1.648e-10	-32.4
251.1	1.250e-03	135.3	7.925e-07	45.3	5.023e-10	-44.7
265.1	5.854e-03	124.9	3.515e-06	34.9	2.110e-09	-55.1
279.0	3.991e-03	-60.5	2.276e-06	-150.5	1.298e-09	-240.5
293.0	1.643e-03	-57.9	8.926e-07	-147.9	4.849e-10	-237.9
306.9	1.318e-03	-66.1	6.833e-07	-156.1	3.543e-10	-246.1
320.9	1.037e-03	-64.3	5.143e-07	-154.3	2.551e-10	-244.3

TABLE-2 (contd.)

334.8	7.825e-04	-69.3	3.719e-07	-159.3	1.768e-10	-249.3
348.8	5.439e-04	-71.7	2.482e-07	-161.7	1.132e-10	-251.7
362.7	1.175e-03	-66.1	5.155e-07	-156.1	2.262e-10	-246.1
376.7	5.830e-04	-76.0	2.463e-07	-166.0	1.041e-10	-256.0
390.6	6.827e-04	-68.2	2.782e-07	-158.2	1.133e-10	-248.2
404.6	4.076e-04	-78.3	1.604e-07	-168.3	6.308e-11	-258.3
418.5	6.500e-04	-88.8	2.472e-07	-178.8	9.399e-11	-268.8
432.5	6.442e-04	-68.4	2.371e-07	-158.4	8.725e-11	-248.4
446.4	7.759e-04	-95.8	2.766e-07	-185.8	9.861e-11	-275.8
460.4	2.929e-04	-55.0	1.013e-07	-145.0	3.501e-11	-235.0
474.3	1.795e-04	-67.9	6.023e-08	-157.9	2.021e-11	-247.9
488.3	1.963e-04	-56.7	6.398e-08	-146.7	2.086e-11	-236.7
502.2	3.261e-04	-46.8	1.033e-07	-136.8	3.274e-11	-226.8
516.2	3.777e-04	-69.6	1.165e-07	-159.6	3.591e-11	-249.6
530.1	3.401e-04	-75.5	1.021e-07	-165.5	3.065e-11	-255.5
544.1	4.129e-04	-40.0	1.208e-07	-130.0	3.533e-11	-220.0
558.0	4.298e-04	-63.0	1.226e-07	-153.0	3.496e-11	-243.0
572.0	3.957e-04	-57.9	1.101e-07	-147.9	3.063e-11	-237.9
585.9	2.090e-04	-96.3	5.678e-08	-186.3	1.542e-11	-276.3
599.9	1.634e-04	-54.1	4.334e-08	-144.1	1.150e-11	-234.1
613.8	3.522e-04	-43.4	9.131e-08	-133.4	2.368e-11	-223.4
627.8	7.036e-04	-67.4	1.784e-07	-157.4	4.522e-11	-247.4
641.7	2.020e-04	-65.3	5.010e-08	-155.3	1.243e-11	-245.3
655.7	2.473e-04	-98.6	6.003e-08	-188.6	1.457e-11	-278.6
669.6	2.218e-04	-35.7	5.273e-08	-125.7	1.253e-11	-215.7
683.6	2.633e-04	-87.9	6.129e-08	-177.9	1.427e-11	-267.9
697.5	3.316e-04	-61.2	7.567e-08	-151.2	1.726e-11	-241.2
711.5	2.027e-04	-116.5	4.535e-08	-206.5	1.014e-11	-296.5
725.4	2.533e-04	-122.2	5.557e-08	-212.2	1.219e-11	-302.2

TABLE-2 (contd.)

739.4	8.414e-05	-26.8	1.811e-08	-116.8	3.898e-12	-206.8
753.3	3.879e-04	-82.0	8.195e-08	-172.0	1.731e-11	-262.0
767.3	9.376e-05	-3.9	1.945e-08	-93.9	4.034e-12	-183.9
781.3	1.003e-04	-173.9	2.043e-08	-263.9	4.162e-12	-353.9
795.2	4.210e-04	-45.9	8.425e-08	-135.9	1.686e-11	-225.9
809.2	3.859e-04	-72.3	7.590e-08	-162.3	1.493e-11	-252.3
823.1	2.432e-04	-84.3	4.703e-08	-174.3	9.094e-12	-264.3
837.1	1.801e-04	16.6	3.424e-08	-73.4	6.510e-12	-163.4
851.0	2.997e-04	-46.7	5.606e-08	-136.7	1.048e-11	-226.7
865.0	2.551e-04	-144.6	4.693e-08	-234.6	8.636e-12	-324.6
878.9	1.494e-04	88.2	2.706e-08	-1.8	4.900e-12	-91.8
892.9	2.557e-04	-78.9	4.557e-08	-168.9	8.124e-12	-258.9
906.8	3.510e-04	-92.5	6.161e-08	-182.5	1.081e-11	-272.5
920.8	2.097e-04	-123.8	3.624e-08	-213.8	6.265e-12	-303.8
934.7	2.490e-04	-91.5	4.239e-08	-181.5	7.218e-12	-271.5
948.7	3.673e-05	16.6	6.162e-09	-73.4	1.034e-12	-163.4
962.6	3.792e-04	-113.6	6.270e-08	-203.6	1.037e-11	-293.6
976.6	4.461e-04	92.2	7.271e-08	2.2	1.185e-11	-87.8
990.5	4.569e-04	-35.6	7.342e-08	-125.6	1.180e-11	-215.6
004.5	2.031e-04	-40.3	3.217e-08	-130.3	5.098e-12	-220.3
1018.4	2.786e-04	-70.5	4.354e-08	-160.5	6.805e-12	-250.5
1032.4	4.589e-04	-40.9	7.074e-08	-130.9	1.091e-11	-220.9
1046.3	2.499e-04	-51.6	3.800e-08	-141.6	5.781e-12	-231.6
1060.3	4.605e-04	-71.6	6.912e-08	-161.6	1.038e-11	-251.6
1074.2	4.341e-04	127.6	6.431e-08	37.6	9.528e-12	-52.4
1088.2	1.858e-04	-50.4	2.718e-08	-140.4	3.975e-12	-230.4
1102.1	1.526e-04	179.1	2.203e-08	89.1	3.181e-12	-0.9
1116.1	7.616e-05	171.1	1.086e-08	81.1	1.549e-12	-8.9
1130.0	2.069e-04	-104.2	2.913e-08	-194.2	4.103e-12	-284.2

TABLE-3
FREQUENCY SPECTRUM ANALYSIS DATA FOR FIG. 7.6

Frequency (Hz)	ACCELERATION		VELOCITY		DISPLACEMENT	
	AMPLITUDE	PHASE (in degree)	AMPLITUDE	PHASE (in degree)	AMPLITUDE	PHASE (in degree)
0.0	2.526e+00	0.0	2.526e+00	0.0	2.526e+00	0.0
14.0	3.807e-02	4.6	4.343e-04	-85.4	4.955e-06	-175.4
27.9	1.069e-01	3.0	2.098e-04	-87.0	3.479e-06	-177.0
41.9	6.550e-02	-167.3	2.491e-04	-257.3	9.471e-07	-347.3
55.8	1.568e-02	-161.3	4.472e-05	-251.3	1.275e-07	-341.3
69.8	2.052e-02	151.9	4.682e-05	61.9	1.068e-07	-28.1
83.7	8.493e-03	-168.2	1.615e-05	-258.2	3.070e-08	-348.2
97.7	4.966e-03	-175.3	8.093e-06	-265.3	1.319e-08	-355.3
111.6	3.826e-03	-157.8	5.457e-06	-247.8	7.781e-09	-337.8
125.6	3.085e-03	-154.4	3.910e-06	-244.4	4.956e-09	-334.4
139.5	2.187e-03	-153.7	2.495e-06	-243.7	2.846e-09	-333.7
153.5	1.972e-03	-139.1	2.045e-06	-229.1	2.121e-09	-319.1
167.4	2.134e-03	-137.4	2.028e-06	-227.4	1.928e-09	-317.4
181.4	1.383e-03	-143.7	1.214e-06	-233.7	1.065e-09	-323.7
195.3	1.591e-03	-147.1	1.296e-06	-237.1	1.056e-09	-327.1
209.3	7.321e-04	-164.2	5.568e-07	-254.2	4.235e-10	-344.2
223.2	1.035e-03	-151.6	7.378e-07	-241.6	5.261e-10	-331.6
237.2	9.732e-04	-137.6	6.531e-07	-227.6	4.383e-10	-317.6
251.1	8.885e-04	-148.4	5.631e-07	-238.4	3.569e-10	-328.4
265.1	8.991e-04	-141.4	5.398e-07	-231.4	3.241e-10	-321.4
279.0	5.844e-04	-121.6	3.334e-07	-211.6	1.902e-10	-301.6
293.0	6.909e-04	-140.0	3.753e-07	-230.0	2.039e-10	-320.0
306.9	6.890e-04	-123.3	3.573e-07	-213.3	1.853e-10	-303.3
320.9	5.398e-04	-116.4	2.678e-07	-206.4	1.328e-10	-296.4
334.8	5.724e-04	-135.7	2.721e-07	-225.7	1.293e-10	-315.7

TABLE-3 (contd.)

348.8	5.040e-04	-147.6	2.300e-07	-237.6	1.050e-10	-327.6
362.7	6.089e-04	-139.9	2.672e-07	-229.9	1.172e-10	-319.9
376.7	3.264e-04	-107.2	1.379e-07	-197.2	5.827e-11	-287.2
390.6	3.500e-04	-145.6	1.426e-07	-235.6	5.810e-11	-325.6
404.6	4.600e-04	-162.1	1.810e-07	-252.1	7.119e-11	-342.1
418.5	3.834e-04	-129.7	1.458e-07	-219.7	5.544e-11	-309.7
432.5	4.361e-04	-122.9	1.605e-07	-212.9	5.907e-11	-302.9
446.4	3.187e-04	-117.7	1.136e-07	-207.7	4.050e-11	-297.7
460.4	3.284e-04	-132.6	1.135e-07	-222.6	3.925e-11	-312.6
474.3	5.938e-04	-103.3	1.992e-07	-193.3	6.685e-11	-283.3
488.3	2.881e-04	-110.5	9.390e-08	-200.5	3.061e-11	-290.5
502.2	4.886e-04	-134.9	1.548e-07	-224.9	4.907e-11	-314.9
516.2	4.698e-04	-132.0	1.449e-07	-222.0	4.466e-11	-312.0
530.1	2.570e-04	-118.6	7.716e-08	-208.6	2.316e-11	-298.6
544.1	3.644e-04	-126.1	1.066e-07	-216.1	3.118e-11	-306.1
558.0	2.979e-04	-127.3	8.497e-08	-217.3	2.423e-11	-307.3
572.0	2.798e-04	-123.1	7.787e-08	-213.1	2.167e-11	-303.1
585.9	1.973e-04	-139.3	5.360e-08	-229.3	1.456e-11	-319.3
599.9	1.823e-04	-163.4	4.838e-08	-253.4	1.283e-11	-343.4
613.8	3.217e-04	-112.3	8.340e-08	-202.3	2.162e-11	-292.3
627.8	1.460e-04	-68.3	3.701e-08	-158.3	9.383e-12	-248.3
641.7	1.763e-04	-93.1	4.372e-08	-183.1	1.084e-11	-273.1
655.7	1.871e-04	-77.0	4.542e-08	-167.0	1.102e-11	-257.0
669.6	3.729e-04	-114.4	8.862e-08	-204.4	2.106e-11	-294.4
683.6	1.223e-04	-100.1	2.846e-08	-190.1	6.627e-12	-280.1
697.5	2.260e-04	-57.7	5.157e-08	-147.7	1.177e-11	-237.7
711.5	2.936e-04	-138.7	6.567e-08	-228.7	1.469e-11	-318.7
725.4	3.463e-04	178.7	7.597e-08	88.7	1.667e-11	-1.3
739.4	3.803e-04	-92.2	8.186e-08	-182.2	1.762e-11	-272.2
753.3	2.456e-04	-29.4	5.189e-08	-119.4	1.096e-11	-209.4
767.3	2.083e-04	-67.9	4.320e-08	-157.9	8.960e-12	-247.9
781.3	1.547e-04	-155.0	3.152e-08	-245.0	6.422e-12	-335.0

SOFTWARE FOR VIBRATION SIGNAL ANALYSIS

```

#include "stdio.h"
#include "dos.h"
#include "math.h"
#include "alloc.h"

int fft( double *xr, double *xi, int nu, int ie );
int ibitr( int j, int nu );
int write_ffile(double *xr,double *xi, int n, char *ffl);

double *xr, *xi,N,F,NF,*f,*xr_a,*xr_v,*xr_d,*a_a,*a_v,*a_d;
double *xa,*xb,*v;

double total;
char filef[10];
char fileg[10];
char fileh[10];
char filei[10];
char filej[10];
char filek[10];
char filel[10];
char filem[10];
FILE *fi,*ff,*f2,*f3,*f4,*f5;
#define PORT 0x300
void adc(int ch , int ns );
unsigned char *data1,*data2;
int data;
double tin =35.0e-06,fmax,cal_f;
FILE *fl;
char fln[30];

```

xr, xi, N, F, NF, f, xr_a, xr_v, xr_d, a_a, a_v, a_d, and v are defined as double. Where

xr is the real part of the vibration data,

xi is the imaginary part of the vibration data,

$N = 2^{nu}$ nu is an integer,

f is the frequency in Hz,

xr_a is the vibrational acceleration,

xr_v is the vibrational velocity,

xr_d is the vibrational displacement,

v is the frequency in radian,

a_a is the phase of the vibrational acceleration,

a_v is the phase of the vibrational velocity, and

a_d is the phase of the vibrational displacement.

MAIN PROGRAMME

```

main()
{

```

```

int fuen,nu,i;

int ch,no_samples ;

clrscr();

printf(" ENTER THE VALUE OF MAXIMUM NO OF SAMPLES  : ");
scanf("%d",&no_samples);

printf(" ENTER THE VALUE OF SAMPLING INTERVAL : ");
scanf("%lf",&tin);

printf(" ENTER THE VALUE OF CALIBRATION FACTOR :");
scanf("%lf",&cal_f);

fuen =( sizeof( unsigned char )*(no_samples + 1);
if ( ( data1 = ( unsigned char *)fmalloc((long)fuen))==0)
    {
printf("Memory not enough. Only ! ");
printf("%lu",farcoreleft());
printf(" Requirement is %lu",fuen);
exit(1);
    }
if ( ( data2 = ( unsigned char * )fmalloc((long)fuen))== 0 )
    {
printf("Memory not enough. Only ! ");
printf("%lu",farcoreleft());
printf(" Requirement is %lu",fuen);
exit(1);
    }
fuen = (sizeof(double))*(no_samples + 1);
if ( (  xr = ( double * )fmalloc( (long )fuen ) )== 0 )
    {
printf("Memory not enough. Only ! ");
printf("%lu",farcoreleft());
printf(" Requirement is %lu",fuen);
exit(1);
    }
if ( (  xi = ( double * )fmalloc( (long )fuen ) )== 0 )

```

dynamic memory location for xr

dynamic memory location for xi

```

{
printf("Memory not enough. Only ! ");
printf("%lu",farcoreleft());
printf(" Requirement is %lu",fmem);
exit(1);
}
if ( ( f = ( double * )fmalloc( (long )fmem ) )== 0 )
{
printf("Memory not enough. Only ! ");
printf("%lu",farcoreleft());
printf(" Requirement is %lu",fmem);
exit(1);
}
if ( ( xr_a = ( double * )fmalloc( (long )fmem ) )== 0 )
dynamic memory location for xr_a
{
printf("Memory not enough. Only ! ");
printf("%lu",farcoreleft());
printf(" Requirement is %lu",fmem);
exit(1);
}
if ( ( a_a = ( double * )fmalloc( (long )fmem ) )== 0 )
dynamic memory location for a_a
{
printf("Memory not enough. Only ! ");
printf("%lu",farcoreleft());
printf(" Requirement is %lu",fmem);
exit(1);
}
if ( ( xr_v = ( double * )fmalloc( (long )fmem ) )== 0 )
dynamic memory location for xr_v
{
printf("Memory not enough. Only ! ");
printf("%lu",farcoreleft());
printf(" Requirement is %lu",fmem);
exit(1);
}

```

```

}
if ( ( a_v = ( double * )fmalloc( (long )fmem ) )== 0 )           dynamic memory location for a_v
{
printf("Memory not enough. Only ! ");
printf("%lu",farcoreleft());
printf(" Requirement is %lu",fmem);
exit(1);
}
if ( ( xr_d = ( double * )fmalloc( (long )fmem ) )== 0 )           dynamic memory location for xr_d
{
printf("Memory not enough. Only ! ");
printf("%lu",farcoreleft());
printf(" Requirement is %lu",fmem);
exit(1);
}
if ( ( a_d = ( double * )fmalloc( (long )fmem ) )== 0 )           dynamic memory location for a_d
{
printf("Memory not enough. Only ! ");
printf("%lu",farcoreleft());
printf(" Requirement is %lu",fmem);
exit(1);
}
if ( ( xa = ( double * )fmalloc( (long )fmem ) )== 0 )
{
printf("Memory not enough. Only ! ");
printf("%lu",farcoreleft());
printf(" Requirement is %lu",fmem);
exit(1);
}
if ( ( xb = ( double * )fmalloc( (long )fmem ) )== 0 )
{
printf("Memory not enough. Only ! ");
printf("%lu",farcoreleft());
}

```

```

printf(" Requirement is %lu",fmen);
exit(1);
}
if ( ( v = ( double * )fmalloc( (long )fmen ) )== 0 )
{
printf("Memory not enough. Only ! ");
printf("%lu",farcoreleft());
printf(" Requirement is %lu",fmen);
exit(1);
}
printf(" DO YOU WANT TO TAKE NEW VIBRATION DATA  y/n ");
if ( getch() != 'y' )
goto ahead;
printf("ENTER FILE NAME FOR SAVING NEW VIBRATION DATA :");
scanf("%s",fln);
fl = fopen(fl,"w+");
if ( fl==0)
{
printf(" Can not open ");
exit(1);
}
printf(" ENTER THE CH. NO.  AND  NO. OF SAMPLES  : ");
scanf("%d%d",&ch,&no_samples);
adc(ch,no_samples);
for( i=0 ; i<no_samples ; i++ )
{
data =(int ) ( (data2[i]<<4)|(data1[i]>>4) ) - 2048;
xr[i]= 5.0*cal_f*((double)(data)/(double)2048);
fprintf(fl,"%lg  %lf",i*tin,xr[i]);
/*printf("%d  %lf",data,xr[i]);*/
}
fclose(fl);
ahead :

```

```

printf("DO YOU WANT FREQUENCY SPECTRUM OF VIBRATION SIGNAL y/n ");

if ( getch() != 'y' )
    exit(1);

printf("ENTER FILE NAME OF VIBRATION DATA :");
scanf("%s",fileg);
no_samples = read_ffile(xr);
for( i=0 ; i< no_samples ; i++)
    xi[i] = 0.0;
if( no_samples < 2 )
    nu = 0;
else
    for( nu = 0 ; ; nu++ )
        {
        no_samples = no_samples/2;
        if( no_samples == 0 )
            break;
        }
N=no_samples = 1<(nu;
fft(xr,xi,nu,-1 );
printf("ENTER FILE NAME FOR SAVING FREQUENCY SPECTRUM DATA :");
scanf("%s",filef);
if( (ff= fopen(filef, "w+")) == NULL )
    {
    printf("CAN'T OPEN FILE-- in function write_ffile ");
    getch();
    exit(1);
    }
i=0;
xr_a[i] = sqrt(xr[i]*xr[i]+xi[i]*xi[i])/((double)(no_samples);
f[i]=((double)((double)(i)/((double)(no_samples)*tin));
a_a[i]=0.0;
xr_v[i]=xr_a[i];
a_v[i]=0.0;

```



```

xr_d[i]=xr_v[i];
a_d[i]=0.0;
fprintf(ff,"%7.1f %11.4e %5.1f %11.4e %5.1f %11.4e %5.1f",
f[i],xr_a[i], a_a[i],xr_v[i],a_v[i],xr_d[i],a_d[i]);
for(i=1;i<(no_samples/2.0);i++)
{
xr_a[i]=2.0*sqrt(xr[i]*xr[i]+xi[i]*xi[i])/((double)(no_samples));
f[i]=(double)((double)(i)/((double)(no_samples)*tin));
v[i]=(double)(2.0*3.1416*f[i]);
if(xr[i]==0)
a_a[i]=90.0;
else
a_a[i] = (atan2(xi[i],xr[i])*180.0/3.1416);
xr_v[i]=xr_a[i]/v[i];
a_v[i]=(a_a[i]-90.0);
xr_d[i]=xr_v[i]/v[i];
a_d[i]=(a_v[i]-90.0);
fprintf(ff,"%7.1f %11.4e %5.1f %11.4e %5.1f %11.4e %5.1f",
f[i],xr_a[i], a_a[i],xr_v[i],a_v[i],xr_d[i],a_d[i]);
}
fclose(ff);
printf("%1u",farcoreleft());
printf(" DO YOU WANT CEPSTRUM ANALYSIS OF VIBRATION SIGNAL Y/N");
if(getch()!='y')
goto overall ;
printf(" ENTER VIBRATION DATA FILE NAME :");
scanf("%s",fileh);
no_samples = read_file(xr);
for(i=0; i<(no_samples);i++)
{
xi[i]=0.0;
xr[i]=xr[i]*xr[i];
}

```

dc value of signal does not have any important roll in vibration signal analysis.

The results of the above operation are stored in file in following sequence:-

(i) frequency, (ii) amplitude of the vibrational acceleration, (iii) phase of the vibrational acceleration, (iv) amplitude of the vibrational velocity, (v) phase of the vibrational velocity, (vi) amplitude of the vibrational displacement, and (vii) phase of the vibrational displacement.

Cepstrum analysis is required, if it is not possible to determine periodicity and side bands of the vibration signal in frequency spectrum analysis. Cepstrum analysis is used to analyse a vibration signal which has large no. of harmonics. It is a 'post' spectral analysis. For detail see chapter 4 section .

```

if(no_samples/2)
nu=0;
else
for(nu=0; ;nu++)
{
no_samples=no_samples/2;
if(no_samples==0)
break;
}
N=no_samples-1((nu;
fft(xr,xi,nu,-1);
for(i=0;i<no_samples;i++)
{
xa[i]=log(sqrt(xr[i]*xr[i]+xi[i]*xi[i]));
xb[i]=atan2(xi[i],xr[i]);
}
fft(xa,xb,nu,1);
printf(" ENTER FILE NAME FOR SAVING CEPSTRUM RESULT :");
scanf("%s",filek);
if((f3=fopen(filek,"w+"))==NULL)
{
printf("can't open file in function write_ffile ");
getch();
exit(1);
}
for(i=0;i<no_samples ;i++)
{
fprintf(f3,"%11.4e %11.4e %11.4e ", i*tau,xa[i],xb[i]);
}
fclose(f3);
printf("%w",farcoreleft());
over_all:
printf(" DO YOU WANT OVER ALL LEVEL OF VIBRATION VELOCITY Y/N");

```

Here we find out no. of samples such that

No. of samples = 2^{nu} where nu is an integer.

Function `fft(xr,xi,nu,-1)` is used for forward fourier transform.

function `fft(xa,xb,nu,1)` is used for inverse fourier transform.

The results of cepstrum is stored in file in following sequence:-

(i) time, and (ii) amplitude.

```

if(getch()!='y')
exit(1);
printf(" ENTER FREQUENCY SPECTRUM DATA FILE NAME :");
scanf("%s",file);
no_samples = read_lfile(xr);
printf("%d",no_samples);
printf(" ENTER FILE NAME FOR SAVING RESULT :");
scanf("%s",file);
if((f5=fopen(file,"w"))==NULL)
{
printf("can't open file in function write_lfile ");
getch();
exit(1);
}
total=0.0;
for(i=0;i<no_samples ;i++)
{
total=total+xr[i]*xr[i];
}
total=sqrt(total/ (double)no_samples);
fprintf(f5,"OVER ALL VIBRATION VELOCITY LEVEL =%lf",total);
fclose(f5);
printf("%u",farcoreleft());
}
}

```

FUNCTION FOR NEW VIBRATION DATA

```

void adc( ch , ns )
int ch,ns ;
{
register int i;
int lobyte,hibyte;

```

Here we find out the value of vibrational velocity and vibrational displacement from vibrational acceleration. The amplitude of vibrational velocity is determined by dividing amplitude of vibrational acceleration by the corresponding radial frequency and the amplitude of vibrational displacement is determined by dividing vibrational velocity by corresponding radial frequency. The phase of vibrational velocity is determined by subtracting 90° from the phase of the vibrational acceleration and the phase of vibrational displacement is determined by subtracting 90° from the phase of vibrational velocity. Here amplitude and phase at zero frequency are neglected because the overall vibration level is the rms value of the vibrational velocity in predetermined frequency band. The lower and upper frequency limit of the band is entered through the key board.

overall vibration level is directly displayed on screen.

```

unsigned char *ptr1,*ptr2;

ptr1=data1;

ptr2=data2;

outport(PORT+2,ch);

for( i=0 ; i ( ns ; i++ )
{
outport(PORT+0,0);

while(inportb(PORT+8) >127 );

*(ptr2++) = inportb(PORT+1);

*(ptr1++) = inportb(PORT+0);

}
}

```

FUNCTION FOR FOURIER TRANSFORM

```

int fft( double *xr, double *xi, int nu, int ie )
{
int n,n1,n2,nul,p,klm2,i,k,l;
double arg,c,s,tr,ti;
double *stab, *ctab;

#define TWOPI 6.283185307
#define PITWO 1.570796327

n = 1<<nu;
n2 = n/2;
nul = nu-1;
if ( ( stab = ( double * )fmalloc( (long )8*(n+1)))== 0 )
{
printf("Memory not enough. Only ! ");
printf("%lu",farcoreleft());
printf(" Requirement is %u",n+1);
exit(1);
}
if ( ( ctab = ( double * )fmalloc( (long )8*(n+1) ) )== 0 )
{

```

int fft(double *xr, double *xi, int nu, int ie) is a function, which is developed for forward and reverse fourier transform.

xr, and, xi are the real and imaginary parts, respectively of a coefficient array having 2^{nu} elements. If $ie < 0$, the forward is performed, and if $ie > 0$, the reverse transform is performed.

Data vector is assumed to be complex and is indexed as $k = 0, 1, \dots, (n-1)$. If data vector is real, then the imaginary part should be set to zero. the number of sample point must satisfy the relation $n = 2^{nu}$, nu is an integer value. Parameter $l = nu-1$ is array number being considered. We start with array $l=0$. The spacing between dual nodes is given by the parameter $n2$; for array $l=0$, $n2 = n/2$ and is initialized as such. Parameter nul is the right shift required when determining the value of p , nul is initialized to $nu-1$. The index k of the array is initialized to $k=0$: thus we will work from the top and program down the array.

```

printf("Memory not enough. Only ! ");
printf("%lu", farcoreleft());
printf(" Requirement is %u", n+1);
exit(1);
|
for( i=0; i<n; ++i)
{
arg=TWOPi*i/n;
stab[i] = sin(arg);
ctab[i] = sin( arg + PI/2 );
|
k=0;
for( l=0; l<nu; l++ )
{
while( k<n )
{
for( i=0; i<n2 ; ++i )
{
n1 = 1<<(nu-l);
p = ibitr(k/n1, nu);
s = stab[p];
c = ctab[p];
if( i<0 ) s = -s;
k1n2 = k + n2;
tr = xr[k1n2]*c + xi[k1n2]*s;
ti = xi[k1n2]*c - xr[k1n2]*s;
xr[k1n2] = xr[k] - tr;
xi[k1n2] = xi[k] - ti;
xr[k] = xr[k] + tr;
xi[k] = xi[k] + ti;
k = k + 1;
|
k = k + n2;

```

Here counter i is set to $i=0$. This counter monitors the number of dual nodes pairs. It is necessary to skip certain nodes in order to insure that previously considered nodes are not encounter a second time. Counter i is the control for determining when the program must skip. To avoid recomputing a dual node that has been considered previously, we determine if the counter i is less than $n2$. For array the number of nodes that can be considered consecutively without skipping is equal to $n/2=n2$. Here this condition is determined by condition $i < n2$. If i is less than $n2$, then we proceed down the array and increment the counter i , by 1. If $i=n2$, then we know that we reached a node previously considered we then skip n nodes by setting $k=k+n2$ because k has already been incremented by 1, it is sufficient to skip the previously considered nodes incrementing k by $n2$. Since k and l have been initialized to 0 and 0, respectively, the initial node considered is the first node of the first array. To determine the factor p for this node, we must first scale the binary number k to the right $nu-1$ bits. To accomplish this we compute the integer value of $k/2^{nu-1}$.

```

    }
    k = 0;
    nu1 = nu1 - 1;
    nu2 = nu2/2;
    }
    for( k = 0 ; k < n ; ++k)
    {
        i = ibitr(k,nu);
        if( i > k )
        {
            tr = xr[k];
            ti = xi[k];
            xr[k] = xr[i];
            xi[k] = xi[i];
            xr[i] = tr;
            xi[i] = ti;
        }
    }
    if ( ie > 0 )
    {
        for( i=0 ; i<n ; ++i)
        {
            xr[i] = xr[i]/n;
            xi[i] = xi[i]/n;
        }
    }
    return 1;
}

int ibitr( int j, int nu )
{
    int ra,ln,i,bitr;
    ra = 1;
    ln = 1<<(nu-1);

```

The function ibitr () reverse bit of $k/2^{nu1}$ for determination of p.

```

bitr = 0 ;
for( i=0 ; i<nu ; ++i )
{
if ( ( j&rn ) != 0 ) bitr = bitr|ln;
rn = rn<<1;
ln = ln>>1;
}
return (bitr);
}

```

FUNCTIONS FOR READING DATA FILE

```

int read_ffile(double *v )
{
int i;
double tan;
if( (fi= fopen(fileg, "r")) == NULL )
{
printf("CAN'T OPEN FILE-- in function read_file ");
getch();
return 0;
}
i = 0 ;
while( !feof(fi) )
{
fscanf(fi, "%lf%lf", &tan, &v[i]);
i++;
}
fclose(fi);
return i ;
}
int read_file(double*v)

```

Function read_file (xr) reads vibration data from file. The pointer returns the value of no. of samples.

```

int i;
double tan;
if((f2=fopen(fileh,"r"))==NULL)
{
printf(" can't open file in function read_file ");
getch();
return 0;
}
i=0;
while(!feof(f2))
{
fscanf(f2,"%lf%lf",&tan,&v[i]);
i++;
}
fclose(f2);
return i;
}
int read_lfile(double *v )
{
int i;
double f,xr_a,a_a;
if( (f4= fopen(fileh, "r")) == NULL )
{
printf("File not found ");
getch();
return 0;
}
i = 0 ;
while( !feof(f4) )
{
fscanf(f4,"%lf%lf%lf%lf",&f,&xr_a,&a_a,&v[i]);
i++;
}

```