COMPUTER-AIDED SIMULATION STUDIES EXPLORING EFFICIENT MIXING-TYPE ESTIMATORS

A THESIS

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DOCTOR OF PHILOSOPHY

in

APPLIED MATHEMATICS

By

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TO
MY PARENTS

CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled, **"COMPUTER-AIDED SIMULATION STUDIES EXPLORING EFFICIENT MIXING-TYPE ESTIMATORS"** in fulfilment of the requirement for the award of the degree of **Doctor of Philosophy**, submitted in the Department of Mathematics of the University is an authentic record of my own work carried out during the period from ~~August~~ July 1988 to April 1992 under the supervision of **Dr. Ashok Sahai**.

The matter embodied in this thesis has not been submitted by me for the award of any other degree.

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This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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ERRATA

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ABSTRACT

In sample surveys, one area of interest has been to improve the ratio and product methods of estimation. A number of estimators has been proposed by various authors (e.g., Srivenkataramana, T. & Tracy, D.S. (1979, 1981-Statistica Neerlandica, Aust. Jr. of Stat.); Sahai, A. (1979-Statistica Neerlandica); Vos, J.W.E. (1980-statistica Neerlandica) and others) which are ratio, product and ratio-cum-product type in nature. These estimators make the use of auxiliary information to estimate the population total/mean. In the present thesis, we have proposed some new estimators as also the efficient mixings of some existing and proposed estimators.

The problem with the ratio, product and ratio-cum-product estimators proposed in the thesis as also with the usual ratio estimator is that their mean-square-errors (MSEs) could not be found analytically in a closed form. Hence, only the approximate MSEs could be the basis of comparison in terms of relative efficiencies. If we take a first order ($O(n^{-1})$; n being the sample

size) large sample approximation to the MSEs of these estimators, a comparison is algebraically intricate and the issue depending on many population parameters' values, which are unknown, it is difficult to conclude as to which one of these estimators is more efficient and when. Further, in case the sample size is that large as to justify the first order large sample approximation, regression estimator will be better motivated than the proposed families of estimators. As such, only when the sample is rather fairly large though not very large, we are motivated enough to use the proposed families of estimators and in this case we will have to go for at least a second order $(O(n^{-2}))$ large sample approximations to the MSEs of the estimators. In this case, the approximations to the MSEs turn out to be still more intractable algebraically and a comparison is just impossible. So, we have compared the various estimators through the computer-aided empirical-simulation study. In this study, we generate the random samples of desired size from a hypothetical bivariate normal population.

Sisodia & Dwivedi(1981-Jr. Ind. Soc. Agri Stat.) and Singh & Upadhyaya (1986-Proc. Nat. Acad. Sci., INDIA) proposed modified ratio and product estimators, respectively, by making the use of coefficient of variation for the auxiliary variable. Motivated by their works, we have proposed variants of ratio and product estimators with the use of sample counter part of the coefficient of variation in Chapter-2. We have also proposed a few other estimators using one design-parameter in this chapter. A part of the work in this chapter has been published in Int. Jr. of

Management and System-Volume 6, No.3 (1990); Allahabad Mathematical Society Bulletin-Second Biennial Conference (April, 1990) and The Proc. of 73^{rd} Indian Science Congress.

We have also proposed two families of ratio-cum-product estimators which make the use of two design-parameters. Thus, we will be having two degrees of freedom and it enables us to control the first and second order MSEs of these estimators. This work has been presented in Chapter-3 of the thesis. A part of this work has been published in Proc. of 47^{th} session of Int. Stat. Inst. (Aug.-Sep., 1989, PARIS).

Further, we have proposed some efficient mixings of the already existing estimators and the estimators proposed by us. These mixings are motivated by the work of Vos(1980-Statistica Neerlandica). We have improved the various estimators by mixing them with the usual mean-per-unit estimator. The weights for these mixings have been ascertained using the relative frequencies of the respective estimators to be winner in the comparison via the empirical-simulation study. Two more types of mixings of the estimators have been dealt with. In the first type of mixing, we have proposed efficient mixings of the estimators proposed/studied by us taking two of the winning estimators at a time and in the second type of mixing, we have proposed a few linear combinations of mean-per-unit estimators, ratio estimator and the two winning estimators proposed/studied by us. Again, the weights for these mixings have been ascertained as per the empirical-probabilities of winning of the mixing-estimators estimable from the relevant empirical-simulation study. We have also taken up the finer comparisons of the estimators. For this, we first have picked up various winners from the earlier study and then have compared these with each other to bring forth the various ranges of G-values and ρ-values for a particular estimator to be the best.

The concluding chapter enlists a brief review of the highlights of the work presented in the thesis. As regards to future possibilities of gainful consequences in this area, it is hoped that the various estimation strategies proposed in the thesis can be the basis of defining multivariate generalised/mixing-type estimators besides leading to possible discoveries of more gainful mixing-estimators.

CHAPTER - 1
INTRODUCTION

It is a well-established fact that a properly collected sample can be considered to be a representative of whole of the population and the characteristics of the sample can help us in making decisions about the characteristics of the population. Statistical Estimation Theory deals with the problems in which we make the inferences about the population by drawing a random sample from it.

The errors associated with a sample survey are of two-types, namely, sampling and non-sampling errors. We have confined only to the sampling errors in the present thesis. The sampling error is measured in terms of the standard error which is the positive square root of the mean square error (MSE). It is always possible to lower the MSE and consequently to increase the relative efficiency of a statistical estimator by the use of supplementary information which might be available to us in advance in terms of census, past data or in terms of experience and long association with the experimental material. In some other cases, this information could be gathered without any significant increase in the cost of sample survey. In other words, the additional cost in obtaining the supplementary information might be outweighed by the consequent gain in the precision of the estimator, to the extent of being negligible.

The supplementary information might be available with us in terms of supplementary variable(s), also called the auxiliary variable(s). Such a variable will be much cheaper, as said above, time and money-wise, to be observed on the sample units, e.g., if we want to measure the total leaf-area on certain plant, the variable leaf area will be much more expensive to observe on the sampling units, i.e., leaves of the plant than the auxiliary variable, namely, leaf-weight of the leaves in the random sample. We will henceforth call the variable under study (leaf area in the above example) as the main variable. The supplementary information may also be available with us in terms of more than one variable and that will be the case of multi-auxiliary information. However, we have not included the case of multi-auxiliary information in the present work as it would just be possible by suitable generalizations of the gainful/efficient estimators.

The present thesis consists of the author's efforts towards a more efficient utilization of the auxiliary information. The various ratio, product, ratio-cum-product and mixing-type estimators exploit the correlation between the main and auxiliary variable to estimate the population mean/total for the main variable more efficiently.

The auxiliary information can be used in two ways, either at sample selection stage or at the estimation stage. This thesis concerns with the use of auxiliary information only at the estimation stage. The ratio, product, regression/difference and the ratio-cum-product methods make the use of auxiliary information at the estimation stage. We, in the following paragraphs describe these methods in brief.

Let U_1, $U_2, \ldots,$ U_N be the N units which constitute our population (say, finite). Let us denote the main variable by Y and the auxiliary variable by X. The observations corresponding to the two variables are $Y_1, Y_2, \ldots,$ Y_n and $X_1, X_2, \ldots, X_n$ respectively for Y and X. In the sampling theory, we are generally interested in the estimation of population mean $\bar{Y}$, where

$$\bar{Y} = N^{-1}.\sum_{i=1}^{N} Y_i \qquad \ldots(1.1)$$

or

$$\text{Population Total : } T_Y = N.\bar{Y} \qquad \ldots(1.2)$$

For the estimation of $\bar{Y}$, let us draw a simple random sample of size 'n' without replacement from the above mentioned population. Thus, the sampling fraction 'f' will be equal to n/N. Let the n units thus selected for the sample be $u_1, u_2, \ldots, u_n$ and the observations corresponding to variable Y are $y_1, y_2, \ldots, y_n$ and corresponding to variable X are $x_1, x_2, \ldots, x_n$.

Let us here give some of the terms used in the thesis with their definitions.

$$\text{Sample Mean for variable Y } (\bar{y}) = n^{-1}.\sum_{i=1}^{n} y_i \qquad \ldots(1.3)$$

$$\text{Sample Total for variable Y } (T_y) = n.\bar{y} \qquad \ldots(1.4)$$

$$\text{Population Mean for variable X } (\bar{X}) = N^{-1}.\sum_{i=1}^{N} X_i \qquad \ldots(1.5)$$

Population Total for variable X (T_X) = $N.\bar{X}$ ...(1.6)

Sample Mean for variable X ($\bar{x}$) = $n^{-1}.\sum_{i=1}^{n} x_i$ ...(1.7)

Sample Total for variable X (T_x) = $n.\bar{x}$ ...(1.8)

Coefficient of variation for variable Y (C_Y) = $\sigma_Y/\bar{Y}$ ...(1.9)

Coefficient of variation for variable X (C_X) = $\sigma_X/\bar{X}$ ...(1.10)

σ_Y and σ_X being the population standard deviations for the two variables. Parallely, the sample counter-parts of C_Y and C_X can also be defined.

Population Variance for Y (S_Y^2) = $\frac{1}{N}\sum_{i=1}^{N}(Y_i-\bar{Y})^2$ ...(1.11)

Population Variance for X (S_X^2) = $\frac{1}{N}\sum_{i=1}^{N}(X_i-\bar{X})^2$ ...(1.12)

The Population Parameter $G = \rho\,\frac{C_Y}{C_X}$ ...(1.13)

where, ρ is the coefficient of correlation between the two variables Y and X.

The unbiased sample estimate of S_Y^2 (s_y^2) = $(n-1)^{-1}\sum_{i=1}^{n}(y_i-\bar{y})^2$ ...(1.14)

and the unbiased sample estimate of S_X^2 (s_x^2) = $(n-1)^{-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$ ...(1.15)

In the light of the identity $T_Y = N.\bar{Y}$, i.e., the population total being a N-multiple of population mean, we can confine our studies, without any loss of generality, to the estimation of population mean $\bar{Y}$.

When we talk of an estimator, say, t_n of an unknown parameter

Θ characterizing the sampled population, it is a function of the sample values $t_n(y_1, y_2, \ldots, y_n)$ or $t_n(y_1, x_1; y_2, x_2; \ldots; y_n, x_n)$ etc. Therefore the value of the estimator is subject to chance. The Mean Square Error (MSE) i.e. the expected error is defined as

$$MSE(t_n) = E(t_n - \Theta)^2 \qquad \ldots(1.16)$$

Thus, we will have

$$\begin{aligned} MSE(t_n) &= E(t_n - E(t_n))^2 + (E(t_n - \Theta))^2 \\ &= \text{Variance}(t_n) + (\text{Bias}(t_n))^2 \\ &= V(t_n) + B^2(t_n) \end{aligned} \qquad \ldots(1.17)$$

Here, it is worth to explain that the precision of an estimator depends upon its MSE, the lower the MSE of the estimator is, it will be more precise. The positive square root of MSE is defined as the standard error of the estimator and the reciprocal of the standard error provides us the measure of the precision of the estimator.

1.0 CONCEPT OF THE RELATIVE EFFICIENCY OF AN ESTIMATOR :

Let us take two estimators t_n and t'_n for the estimation of a parameter Θ. The relative efficiency of the estimator t_n with respect to t'_n is defined as

$$REEF(t_n, t'_n) = (MSE(t'_n) \;/\; MSE(t_n))*100\ \% \qquad \ldots(1.18)$$

For the purpose of our studies of this thesis, we have taken the estimator t'_n as the usual unbiased estimator(UUE).

1.1 USUAL UNBIASED ESTIMATOR :

The sample mean $\bar{y}$ is an estimator for the population mean $\bar{Y}$. This estimator is also known as UUE and Simple Expansion Estimator (SEE). This estimator has been termed as UUE due to the simple fact that $E(\bar{y}) = \bar{Y}$. It may also be noted that

$$V(\bar{y}) = (g/n).\ \bar{Y}^2.\ C_Y^2 \quad = (g/n).\ S_Y^2$$

Also,

$$V(\bar{x}) = (g/n).\ \bar{X}^2.\ C_X^2 = (g/n).\ S_X^2$$

and

$$\text{Covariance}(\bar{x},\ \bar{y}) = \text{cov.}(\bar{x},\bar{y})$$

$$= (g/n).\ (\bar{Y}.\bar{X}).\ \rho.C_Y.\ C_X \qquad \ldots(1.19)$$

where, $g = 1-f$.

For large populations $g \simeq 1$; so that g, called finite population correction, may be ignored for the above expressions for such populations.

Now, assuming the knowledge of $\bar{X}$ and having parallel observations on the auxiliary variable X, we have the following famous estimators for the estimation of population mean.

1.2 RATIO ESTIMATOR :

The ratio estimator $(\bar{Y}_R)$ for the estimation of $\bar{Y}$ is defined as

$$\bar{Y}_R = \hat{R}\ .\ \bar{X}$$

where, $\hat{R} = (\bar{y}/\bar{x})$ is an estimate for the ratio $R = (\bar{Y}/\bar{X})$.

This estimator should be used when the two variables Y and X are positively correlated. To be more specific and following Murthy(1964), its use might be done when $G > 0.5$.

It can be easily seen that the sampling bias and MSE of ratio estimator can not be found exactly. We can only get their large sample approximations. This can be done through Binomial series expansion of the MSE of $\bar{Y}_R$ on the assumption that the sample is at least so large as to justify $0 < |\ \bar{x}\ | < 2.|\ \bar{X}\ |$. If we assume that the observations on the variable X are all positive, we can take $0 < \bar{x} < 2.\bar{X}$. Following Murthy(1964), the bias and MSE of $\bar{Y}_R$ can be found as follows.

Let us fix,

$$e = (\bar{y} - \bar{Y}) / \bar{Y}$$

and $e_1 = (\bar{x} - \bar{X}) / \bar{X}$ ...(1.20)

Here, both e and e_1 are of the order of $O(n^{-1/2})$. Hence, we will have to retain the terms up to second(fourth) order in e and e_1 in the Binomial series expansion of MSE of $\bar{Y}_R$ in order to obtain first(second) order Bias/MSE of the estimator. Let us denote first/second order Bias and MSE of an estimator, t_n by $B_1(t_n)/B_2(t_n)$ and $M_1(t_n)/M_2(t_n)$, respectively.

Let us also denote,

$V_{ij} = E((e)^i.(e_1)^j)$, i and j being positive integers.

We can easily establish that

$$B_1(\bar{Y}_R) = \bar{Y} . (V_{20}-V_{11}),$$
$$B_2(\bar{Y}_R) = \bar{Y} . (V_{20}-V_{11}+V_{12}-V_{03}+V_{04}-V_{13}),$$
$$M_1(\bar{Y}_R) = \bar{Y}^2. (V_{20}-2V_{11}+V_{02}) \text{ and}$$
$$M_2(\bar{Y}_R) = \bar{Y}^2. (V_{20}-2V_{11}+V_{02}-2V_{21}+4V_{12}-2V_{03}+3V_{22}-6V_{13}+3V_{04})$$

...(1.21)

Thus, $\bar{Y}_R$ is a biased estimator of $\bar{Y}$. Many authors have tried to improve the ratio method of estimation in past. Some of them worth-noting are Hartley and Ross(1954), Robson(1957), Quenouille(1956), Tin(1965), Murthy and Nanjamma(1959), Lahri(1951), Midzuno(1950), Mickey(1959), Nanjamma, Murthy and Sethi(1959), Nieto de Pascual(1961), Raj(1954), Rao(1964), Rao(1965) and Sukhatme(1962). All these authors tried to improve the ratio method of estimation by controlling its bias by some technique or the other.

1.3 PRODUCT ESTIMATOR :

The product estimator for the estimation of $\bar{Y}$ is defined as

$$\bar{Y}_P = \hat{P} \,/\, \bar{X} \qquad \ldots(1.22)$$

where, $\hat{P}$ is an estimate for the product $P = (\bar{Y}.\bar{X})$. Again, following Murthy(1964), this estimator should be used when $\rho < 0$ and more precisely when $G < -0.5$. Bias and MSE of this estimator can be found exactly and these are

$$B_1(\bar{Y}_P) = \bar{Y} \,.\, V_{11},$$

$$M_1(\bar{Y}_P) = \bar{Y}^2. \, (V_{02} + 2V_{11} + V_{20}) \text{ and}$$

$$M_2(\bar{Y}_P) = \bar{Y}^2. \, (V_{02}+2V_{11}+V_{20}+2V_{21}+2V_{12}+V_{22}) \qquad \ldots(1.23)$$

Almost all the efforts made by various authors to improve ratio and product estimators were concentrated around controlling their biases, e.g., Shukla(1976), and others given above, in case of ratio estimator. However, in our present studies, we have confined our efforts in lowering down only the MSE of the estimators without considering their biases. From 1.17 it is also very clear that the bias of an estimator is included in its MSE, so, lowering down the MSE of an estimator will automatically include the lowering down of its bias.

1.4 DIFFERENCE AND LINEAR REGRESSION ESTIMATORS :

In the practical situations, it is sometimes possible that we may have a good guess of population regression coefficient of Y on X, say, B available with us. In that cases, we can use the following estimator, called the Difference estimator, for the estimation of population mean given by Hansen, Harwitz and Madow(1953).

$$\bar{Y}_D = \bar{y} + b\,(\bar{X} - \bar{x}) \qquad \ldots(1.24)$$

Where, b is the guessed value of B.

Also, if we are unable to guess the value of B closely, we can use its estimated value b^* ($= r.s_x/s_y$, r is the sample correlation coefficient, i.e., an estimate of ρ.) and in those cases we name the estimator as the Linear Regression estimator.

$$\bar{Y}_{LR} = \bar{y} + b^* \, (\bar{X} - \bar{x}) \qquad \ldots(1.25)$$

In our present study, we have not included these estimators because it is well known that these estimators perform very good only when the sample is very large and there may be certain populations where it will not be possible to get fairly a large sample as to justify the use of these estimators.

1.5 THE RATIO-CUM-PRODUCT ESTIMATORS :

In order to make the more efficient utilization of auxiliary information, various authors have proposed different ratio-cum-product type estimators. For the sake of comparisons, we have included the following ratio-cum-product estimators in our present studies proposed by Srivastava(1967), Reddy(1974) and Sahai(1979).

$$\bar{Y}_{Sr.} = \bar{Y}. \; (\bar{X} \,/\, \bar{x})^a \qquad \ldots(1.26)$$

$$\bar{Y}_{Re.} = \bar{y}.\bar{X} \,/\, (\bar{X} + a \; (\bar{x} - \bar{X}) \qquad \ldots(1.27)$$

$$\bar{Y}_{Sa.} = \bar{y}. \; ((1+a).\bar{X} + (1-a).\bar{x}) \,/\, ((1+a).\bar{x} + (1-a).\bar{X}) \qquad \ldots(1.28)$$

where, 'a' is a design-parameter. The method of minimum MSE initially given by Searls(1964) has been applied to obtain the optimal value of the design-parameter for the different estimators. Thus, minimizing first order MSE of these estimators will give us the optimal value of 'a'.

We have proposed and studied a few other similar families of ratio-cum-product estimators. The families proposed by us contain one as-well-as two design parameters. The optimal values of the design parameters in case of the families containing two design-parameters are obtained by minimising first and second order MSEs of the respective estimators.

Vos(1980) proposed some mixing estimators and compared them through some artificial populations. We note that this comparison is of limited value for inferring about the estimators' potential efficiency. On the other hand the empirical-simulation study undertaken by us is much more capable of discovering it via the comparisons. Nevertheless we do get the motivation from Vos(1980)'s work in proposing some gainful mixing estimators.

1.6 CONCEPT OF RELATIVE ERROR IN GUESSING THE VALUE OF A POPULATION PARAMETER :

The optimal values of the design-parameters for the different families of estimators include certain population parameters. The most critical parameter in almost all the expressions is G($= \rho.C_Y/C_X$). It is not always possible that we are able to guess the exact value of G in the practical situations and this very fact has lead us to incorporate some measure of overguessing/underguessing for the value of the parameter. We have given the name REG(G) to this relative error in guessing the value of a parameter and have considered five levels of REG(G). These levels are corresponding to the following five situations.

a. When the value of G has been underguessed by 20 %.

b. When the value of G has been underguessed by 10 %.

c. When the value of G has been guessed exactly.

d. When the value of G has been overguessed by 10 %.

e. When the value of G has been overguessed by 20 %.

Corresponding to these five situations, we will be having five values of REG(G), namely, -0.2, -0.1, 0.0, 0.1 and 0.2. So, whenever we use the estimated value of G, we have also considered these five levels of REG(G).

1.7 THE EMPIRICAL-SIMULATION STUDY - ITS FRAME WORK :

As stated earlier, we in our present studies have studied/proposed some ratio, product, ratio-cum-product and mixing-type estimators. These families include one or two design-parameters. The reasons for the computer-aided empirical-simulation study are two fold. Firstly, a closed-form algebraic expression for the MSE of almost all the estimators is not available. Moreover, if we take a first order ($O(n^{-1})$: n being the sample size) large sample approximation to the MSEs of these estimators, a comparison is algebraically intricate and the issue depending on many population parameters' values, which are unknown, it is difficult to conclude as to which one of these estimators is more efficient and when. Further, in case sample size is that large as to justify this first order large sample approximation, regression estimator will be better motivated than the proposed families of estimators. As such, when the sample is only fairly large and performance of regression estimator being rather unpredictable, we are motivated enough to use the proposed families of estimators. Here, we have to go for at least a second order ($O(n^{-2})$) large sample approximation to the MSE of the estimators. In this case, the approximation to the MSEs turn out to be still more intractable algebraically and a comparison is just impossible. Hence, the only alternative is to go for a computer-aided empirical-simulation study for the comparison of the estimators.

For the empirical-simulation study, we have assumed some hypothetical bivariate normal populations. We have considered two example-levels of $\bar{Y}$ (=2.0,4.0), $\bar{X}$(=1.0,2.0), σ_Y(=2.0,4.0) and σ_X(=1.0,2.0) and ten example-levels of ρ(= ±0.2, ±0.4, ±0.6, ±0.8, ±0.9) have been taken into account. In all, we will have 160 value-combinations for G out of which 80 combinations will be for positive correlation case and 80 combinations will be for negative correlation case. Here, we have implemented the concept of relative error in guessing the value of G and thus for a single value of G, we will be having five values of guessed G according to the five situations described in the earlier section. In this way, we will be having 400 different values of G for the case of positive correlation and another 400 values for the case of negative correlation. Now, for each value-combination, we have generated 100 random samples of sizes 10 and 20 each using the following transformation.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \bar{X} \\ \bar{Y} \end{bmatrix} + \begin{bmatrix} \sigma_X \cdot Z_1 \\ \sigma_Y . (\rho Z_1 + (1-\rho^2).Z_2) \end{bmatrix} \quad \ldots(1.29)$$

where, Z_1 and Z_2 are the random normal deviates between zero and unity. These normal deviates have been generated using Box-Muller (1958)'s approach. Now, over this replication of 100 samples and for different combinations, we calculate the actual values of the estimators and thus their MSEs and consequently their relative efficiencies, relative to mean-per-unit estimator. Noting the number of times a particular estimator has the maximum relative efficiency, we calculate the relative frequency of its being the winner among the other estimators in the competition.

Here, it may be noted that we had to modify the example-levels of $\bar{Y}$ and $\bar{X}$, stated above, for the empirical-simulation study in case of one of the estimators proposed by us in Chapter-2. Also, in Chapter-6 of the thesis, wherein we have carried out the finer comparisons of the estimators, we had to increase the combinations for G-value in order to ascertain the trends of different estimators. Here, we have modified the example-levels of $\bar{Y}$, $\bar{X}$, σ_X and σ_Y.

For the sake of finer comparisons, based on G-values of various estimators, we have divided the whole range of G into six intervals, namely, I1 : $G < -1$; I2: $-1 \le G < -0.5$; I3: $-0.5 \le G < 0$; I4: $0 \le G \le 0.5$; I5: $0.5 < G \le 1$ and I6: $G > 1$. Keeping the track of different intervals, we note the number of times a particular estimator has the maximum relative efficiency and thus arrive at the empirical probabilities of the different estimators. being the winner for different intervals.

In order to facilitate the gain in the relative efficiencies of the different estimators more clearly, we have carried out the graphical display of the relative efficiencies of the estimators. The graphical display has been performed by drawing bar-graphs for the various estimators in competition. For these bar-graphs, we have divided the whole range of relative efficiency (taking all the estimators in competition, into account) into ten intervals. Now, to ascertain the frequency of a particular estimator for any interval, we note the number of times the relative efficiency of the estimator falls into the interval and it is maximum when compared with the relative efficiencies of other estimators in the competition. For the sake of clarity, we have accumulated the last five intervals into a single interval as the frequencies of all the estimators were very low for these intervals in almost all the cases. The upper limits of the intervals have only been displayed on the graphs. One can easily check the lower limit of an interval as it will be nothing but the upper limit of the previous interval. The relative efficiency of $\bar{y}$ being equal to 100%, it will always lie into the interval containing relative efficiency equal to 100%. So, the frequency of this interval for $\bar{y}$ will be equal to the sum of the frequencies of any other estimator in competition and for other intervals, it will be zero. So, we have not drawn the bars corresponding to $\bar{y}$ in the graphs as there will be a single bar for only one particular interval.

1.8 AN OVERVIEW OF THE CONTENTS OF VARIOUS CHAPTERS :

The present thesis comprises of seven chapters. The present chapter, i.e., Chapter-1 is introductory in which a brief historical review and the motivation for the work has been set out with the relevant details.

In Chapter-2, various estimators have been proposed which use only one parameter in their design. The optimal value of this design-parameter is obtained by minimising the first order MSE of the concerned estimator. We have also proposed two variants, one for ratio and another for product estimator using sample counter-part of the coefficient of variation, C_X.

Chapter-3 contains the estimators which have been constructed using two design-parameters. Here, the optimal values of the two design-parameters are obtained by minimising the first and second order MSEs of the respective estimators. In quite a few cases, these estimators turn out to be better than other estimators.

Chapter-4 and Chapter-5 have been devoted to propose the efficient mixings of the existing estimators and the estimators proposed by us. In Chapter-4, we have successfully tried to improve the various estimators by mixing them with the usual mean-per-unit estimator. The weights for mixing of these estimators have been ascertained using the relative frequencies of the respective estimators to be winner in the comparison via the empirical-simulation study. Chapter-5 contains two types of mixings. In the first type, we have tried to propose efficient mixings of the estimators proposed/studied by us in Chapter-2 and Chapter-3 taking two winners at a time and in the second type, we have proposed some mixings which contain $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ and two parent-estimators which are from amongst the winners from Chapter-2 and Chapter-3. Again the weights for these mixings have been ascertained as per the empirical-probabilities of winning of the mixing estimators estimable from the relevant empirical-simulation study.

In Chapter-6, we have taken up the finer comparisons of the estimators. For this, we first have picked up various winners from the earlier chapters and then have compared these winning estimators to bring forth the various ranges of G-values for a particular estimator to be the best.

In the last chapter, we have given a brief review of the highlights of the work presented in the thesis, conclusions therefrom and some remarks indicating future possibilities of gainful investigation in this area.

CHAPTER - 2

THE ONE-PARAMETER FAMILIES OF ESTIMATORS

In this chapter, we have proposed some new families of ratio-cum-product type estimators which involve only one design-parameter. Beside these families, we have also proposed two variants, one each of ratio and product estimators.

Sisodia and Dwivedi(1981) proposed a modified ratio estimator, $\bar{Y}_{MR}$, to make it more efficient by incorporating the known value of the coefficient of variation for the auxiliary variable in a gainful manner. They established the possibility of gaining efficiency with an algebraic comparison of the modified ratio estimator with the ratio estimator through their approximate MSEs. The algebraic comparisons given by them can not be considered to be very illustrative due to the fact explained by us in section 1.7. Singh and Upadhyaya(1986) proposed an analogous estimator using the knowledge of coefficient of variation of auxiliary variable. This estimator, $\bar{Y}_{MP}$, was a dual to $\bar{Y}_{MR}$. The two estimators were,

$$\bar{Y}_{MR} = \bar{y}.(\bar{X} + C_X)/(\bar{x} + C_X) \qquad \ldots(2.1)$$

and $$\bar{Y}_{MP} = \bar{y}.(\bar{x} + C_X)/(\bar{X} + C_X) \qquad \ldots(2.2)$$

In order to come over the difficulty of algebraic comparisons of these estimators, we have studied these via the empirical-simulation studies explained in section 1.7. Here, we have ompared $\bar{Y}_{MR}$ with $\bar{y}$ & $\bar{Y}_R$ and $\bar{Y}_{MP}$ with $\bar{y}$ & $\bar{Y}_P$. Table 2.1 gives the results of the empirical-simulation studies carried out for these estimators.

For all the tables that follow in this chapter, as well as in the next chapters, RF(∘) represents the relative frequency of a particular estimator '∘' being the winner per the simulation studies. These tables detail the results of the empirical-simulation study in a summarised form. However, remarks following each table give highlights on finer details of the comparison, as revealed through this study.

TABLE 2.1

RF(∘) FOR $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ and $\bar{Y}_{MR}/\bar{Y}_{MP}$

Estimators → / Sample Sizes ↓		$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_P$	$\bar{Y}_{MR}$	$\bar{Y}_{MP}$
$\rho > 0$	n=10	0.275	0.220	-	0.505	-
	n=20	0.260	0.275	-	0.465	-
$\rho < 0$	n=10	0.235	-	0.210	-	0.555
	n=20	0.223	-	0.255	-	0.522

This is clear from this table that $\bar{Y}_{MR}$ is a better choice than $\bar{y}$ and $\bar{Y}_R$ when $\rho > 0$ and $\bar{Y}_{MP}$ is better than $\bar{y}$ and $\bar{Y}_P$ when $\rho <$ 0. We have also noted that $\bar{Y}_{MR}$ performs very good when $G \in [0.5, 1]$ and $\bar{Y}_{MP}$ is particularly good when $G \in [-1, -0.5]$. Here, $G=\rho.(C_Y/C_X)$ as defined in chapter-1. For illustrating the gain in efficiency more clearly, we have carried out the graphical display of relative efficiencies of these estimators. Graphs 2.1 to 2.4

provide a more visual display of the results of this study.

In the proposition of the estimators $\bar{Y}_{MR}$ and $\bar{Y}_{MP}$, the coefficient of variation has been assumed to be known. However, it is not always possible that C_X be known to us before hand. So, following Sisodia & Dwivedi(1981) and Singh & Upadhyaya(1986), we have proposed the following variants of ratio and product estimators.

$$\bar{Y}_{VR} = \bar{y}.(\bar{X} + c_x)/(\bar{x} + c_x) \qquad ...(2.3)$$

and $$\bar{Y}_{VP} = \bar{y}.(\bar{x} + c_x)/(\bar{X} + c_x) \qquad ...(2.4)$$

where, $c_x\left[= s_x/\bar{x}\ ;\quad s_x^2 = (n-1)^{-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \right]$ is the sample counter-part of the coefficient of variation C_X. The results of empirical-simulation study and the graphical display of relative efficiencies of $\bar{Y}_{VR}$ and $\bar{Y}_{VP}$ are given below in table 2.2 and graphs 2.5 to 2.8, respectively.

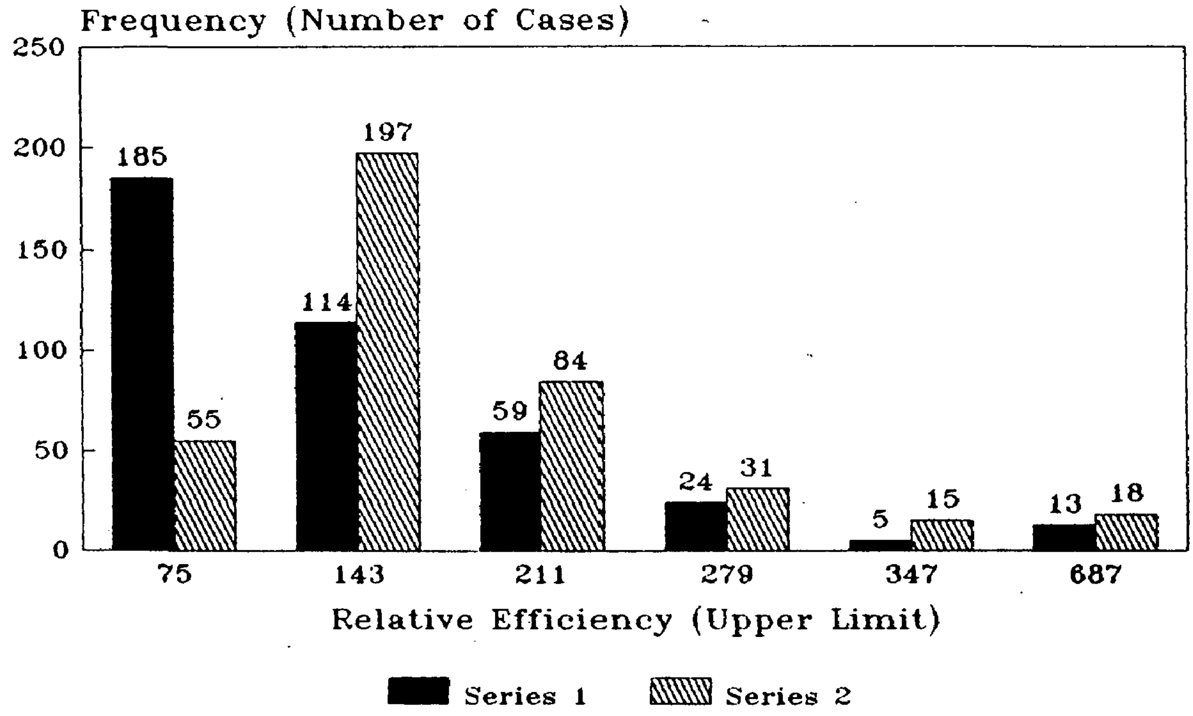
TABLE 2.2

RF(∘) FOR $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ and $\bar{Y}_{VR}/\bar{Y}_{VP}$

Estimators → / Sample Sizes ↓		$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_P$	$\bar{Y}_{VR}$	$\bar{Y}_{VP}$
$\rho > 0$	n=10	0.243	0.232	-	0.525	-
	n=20	0.220	0.262	-	0.518	-
$\rho < 0$	n=10	0.375	-	0.237	-	0.388
	n=20	0.285	-	0.280	-	0.435

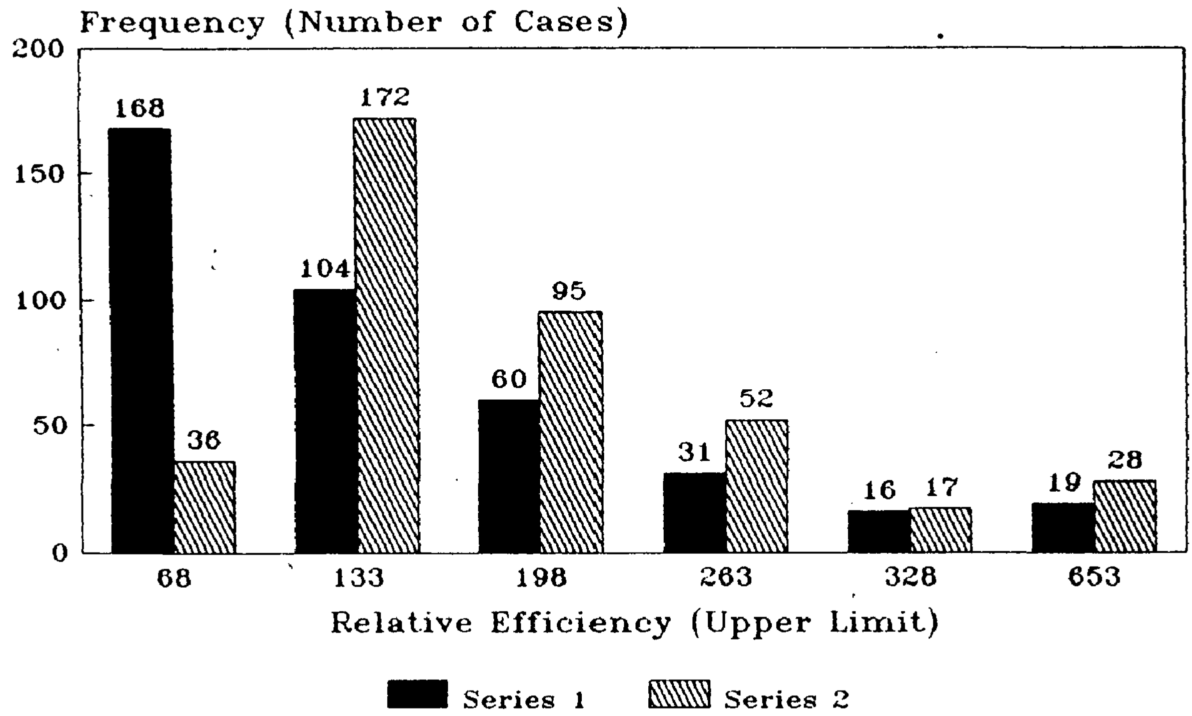
Here again, we recommend that $\bar{Y}_{VR}/\bar{Y}_{VP}$ should be preferred than $\bar{y}$ and $\bar{Y}_R/\bar{Y}_P$ for the estimation of $\bar{Y}$. Also, the finer comparisons of the relative efficiencies of these estimators show that $\bar{Y}_{VR}$ comes out to be winner much more often than $\bar{y}$ and $\bar{Y}_R$ when $G \in [0.5,\ 1]$

Series 1 for $\bar{Y}_R$ and Series 2 for $\bar{Y}_{MR}$



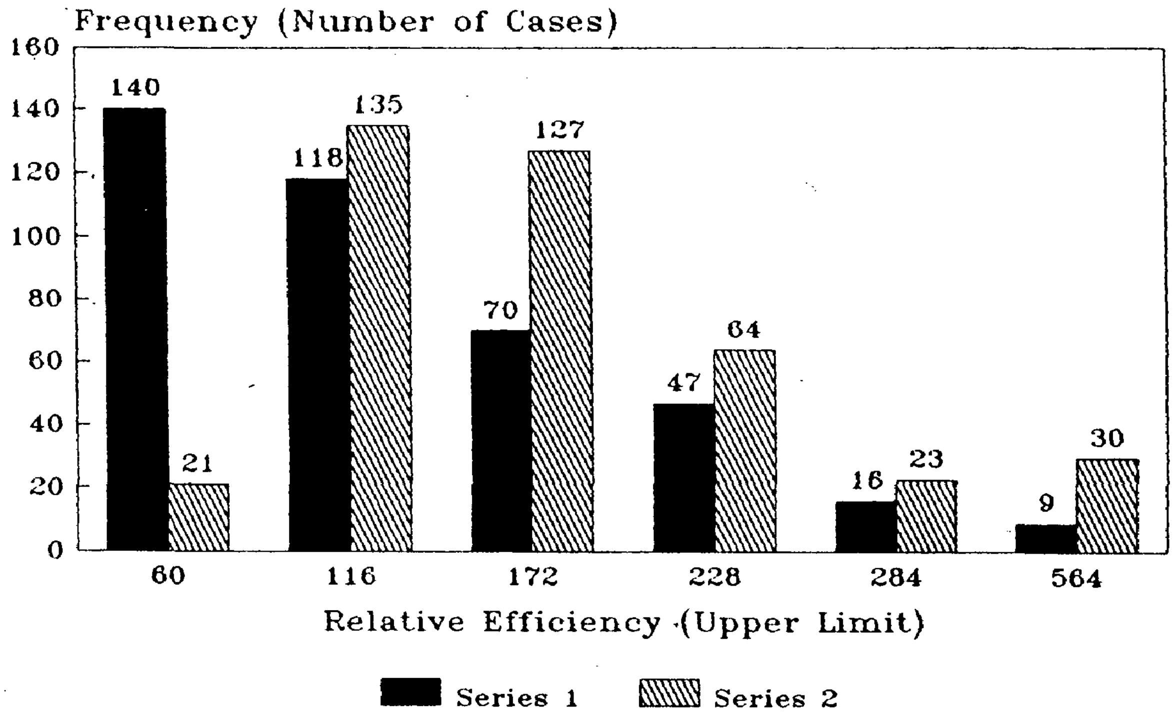
Graph – 2.1 (Sample Size = 10, $\varrho > 0$)

Series 1 for $\bar{Y}_R$ and Series 2 for $\bar{Y}_{MR}$



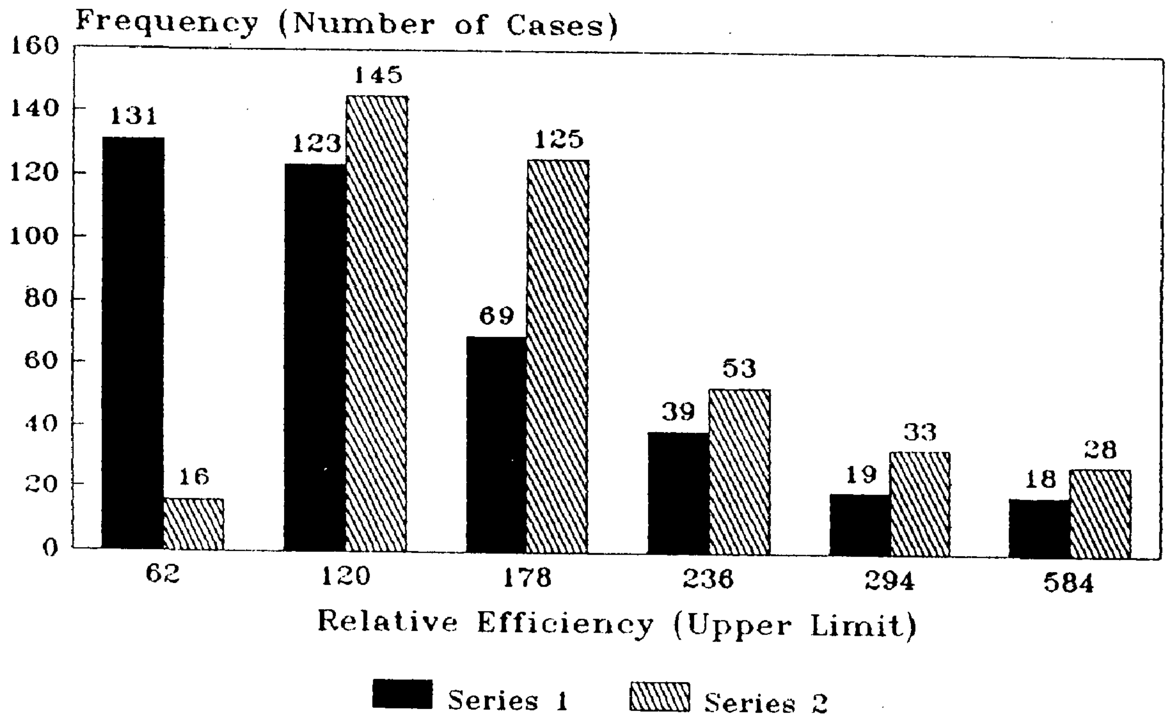
Graph – 2.2 (Sample Size = 20, $\varrho > 0$)

Series 1 for $\bar{Y}_P$ and Series 2 for $\bar{Y}_{MP}$



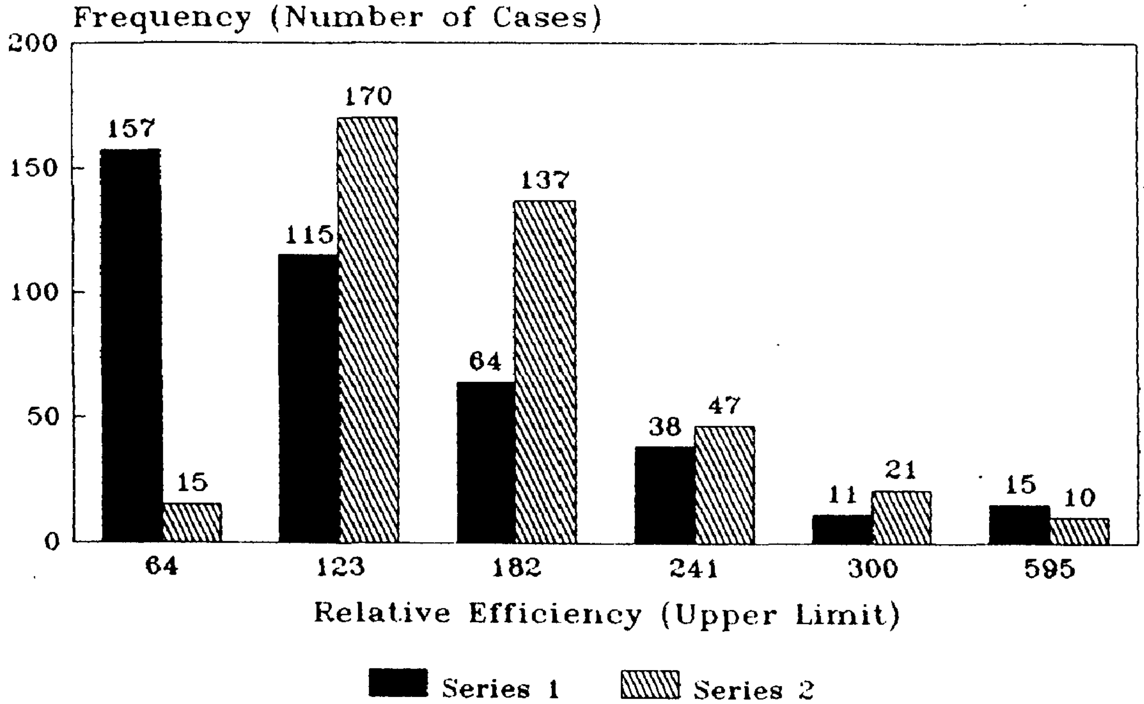
Graph – 2.3 (Sample Size = 10, ρ < 0)

Series 1 for $\bar{Y}_P$ and Series 2 for $\bar{Y}_{MP}$



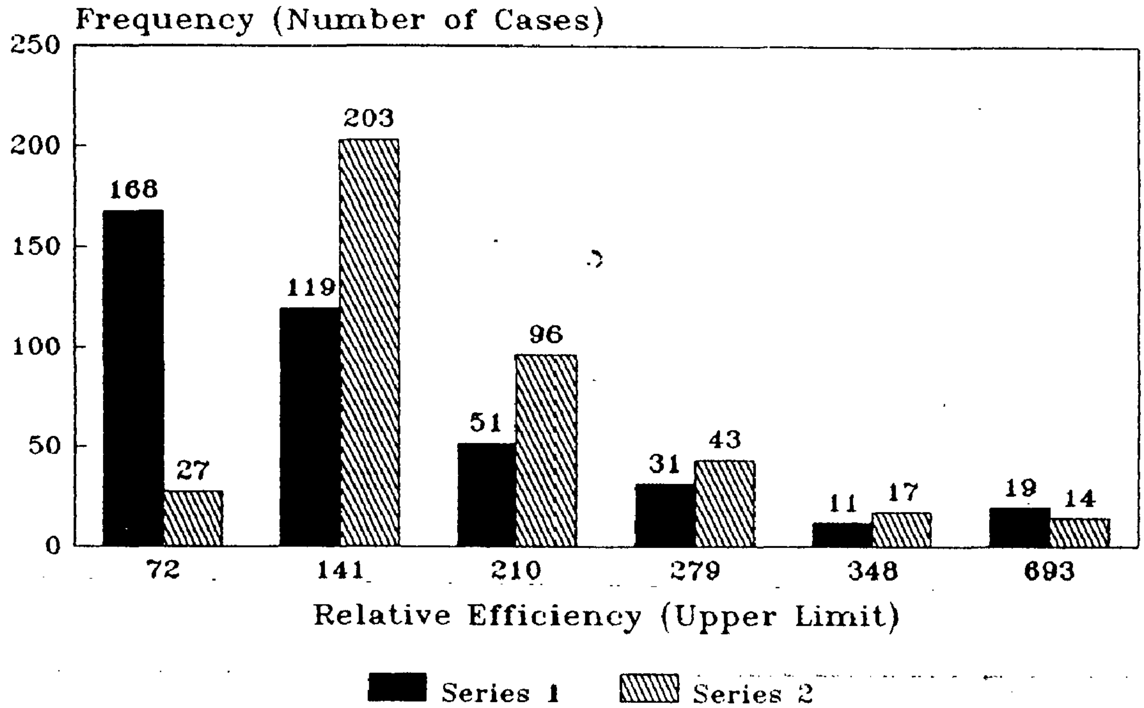
Graph – 2.4 (Sample Size = 20, ρ < 0)

Series 1 for $\overline{Y}_R$ and Series 2 for $\overline{Y}_{VR}$



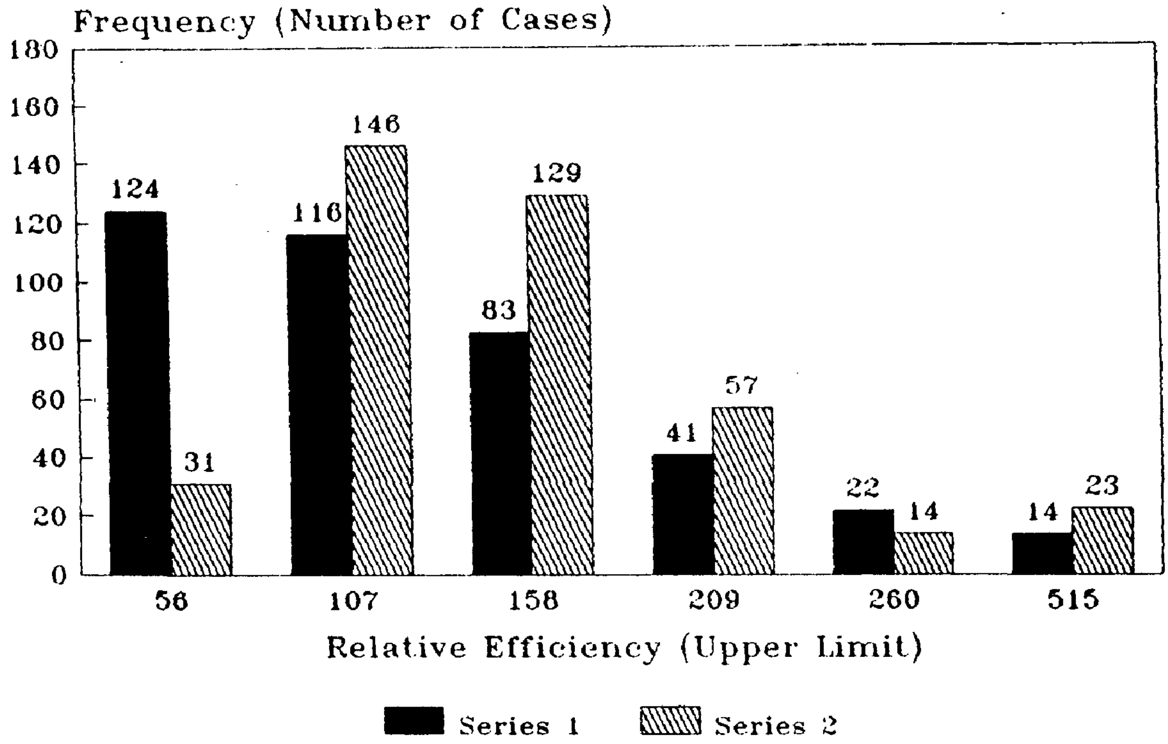
Graph - 2.5 (Sample Size = 10, ρ > 0)

Series 1 for $\overline{Y}_R$ and Series 2 for $\overline{Y}_{VR}$



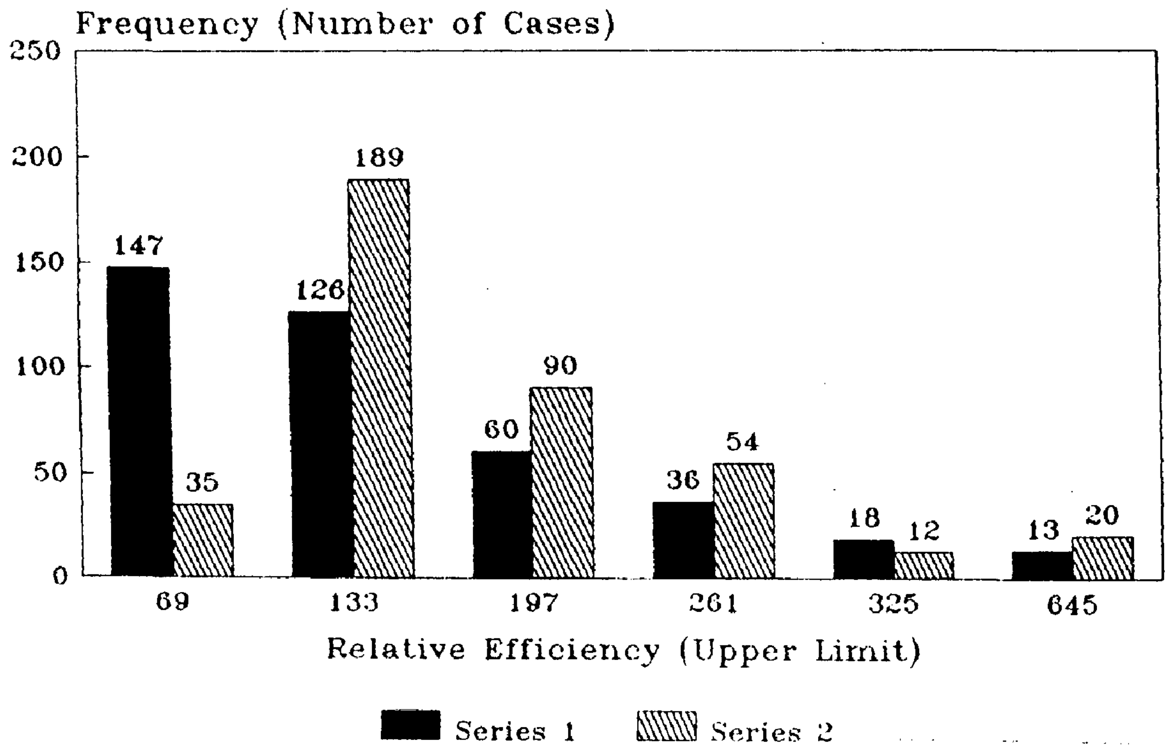
Graph - 2.6 (Sample Size = 20, ρ > 0)

Series 1 for $\overline{Y}_P$ and Series 2 for $\overline{Y}_{VP}$



Graph – 2.7 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_P$ and Series 2 for $\overline{Y}_{VP}$



Graph – 2.8 (Sample Size = 20, $\rho < 0$)

and $\bar{Y}_{VP}$ performs very often better than $\bar{y}$ and $\bar{Y}_P$ when $G \in [-1, -0.5)$.

The estimators $\bar{Y}_{MR}$, $\bar{Y}_{MP}$, $\bar{Y}_{VR}$ and $\bar{Y}_{VP}$ use either C_X or C_x in their design in a special way. This way of using C_X/C_x has lead us to propose a new one-parameter family of ratio-cum-product estimators as below.

$$\bar{Y}_{P.a} = \bar{y}.(\bar{x} + a)/(\bar{X} + a) \qquad ...(2.5)$$

where, 'a' is a non-stochastic design-parameter. The optimal value of 'a' is obtained by using the method of minimum MSE given by Searls(1964). It appears from the structure of $\bar{Y}_{P.a}$ that it is a product-type estimator but through our empirical-simulation study we have discovered that $\bar{Y}_{P.a}$ performs good even when the correlation between Y and X is positive. In fact, $\bar{Y}_{P.a}$ is essentially a ratio-cum-product type estimator.

It can easily be checked that the first order MSE of $\bar{Y}_{P.a}$ for a random sample from a bivariate normal population is given by

$$M_1(\bar{Y}_{P.a}) = [\ 1 + A.(A + 2G)\ /\ C_Z^2\ ].\bar{Y}^2.C_Y^2\ /\ n \qquad ...(2.6)$$

where, $A = \bar{X}/(\bar{X} + a)$ and $C_Z = C_Y/C_X$. Here, $M_1(\bar{Y}_{P.a})$ attains the minimum value for $A = -G$. Without any loss of generality, we can take $\bar{X} = 1$, as we can divide each observation corresponding to the auxiliary variable by $\bar{X}$, which is assumed to be known and carry out the study with the new set of auxiliary observations. The minimisation of $M_1(\bar{Y}_{P.a})$ with respect to A is equivalent to minimising $M_1(\bar{Y}_{P.a})$ with respect to 'a' as $da/dA \neq 0$. Hence, $M_1(\bar{Y}_{P.a})$ attains the minimum value for $a = -(1 + 1/G)$ with the condition that $\bar{X} = 1$.

In view of the above condition of $\bar{X}$ being unity, we have to modify the parameters of our empirical-simulation study. Here, we have considered four example-levels of $\bar{Y}(=1.0,\ 2.0,\ 3.0$ and $4.0)$ and one fixed value of $\bar{X}(=1.0)$. The other parameters have the same values as explained in section 1.7. It should be mentioned here that now on, whensoever we take the estimator $\bar{Y}_{P.a}$ into the group of competing estimators, we make the above modifications with our empirical-simulation study. This is without any loss of generality as we would have had the same number of comparisons of the estimators in this modified study too. The results of the empirical-simulation study are tabulated below in table 2.3 and are graphically displayed in the graphs 2.9 to 2.12 for $\bar{Y}_{P.a}$.

TABLE 2.3

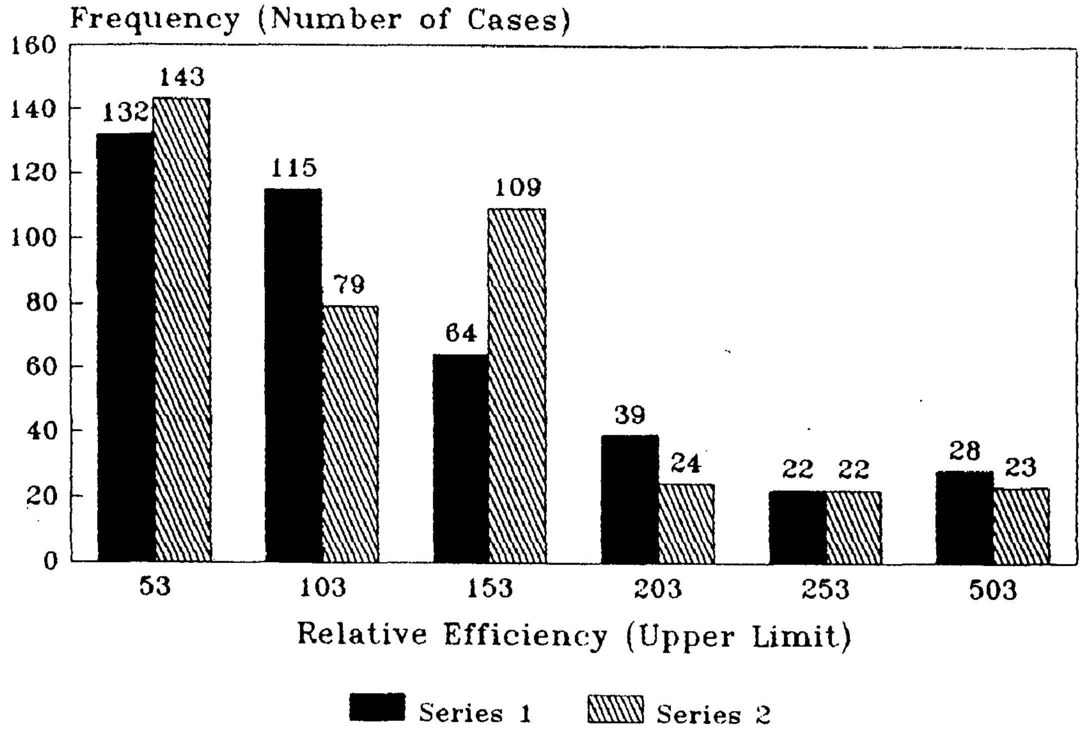
RF(∘) FOR $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ and $\bar{Y}_{P.a}$

Estimators → / Sample Sizes ↓		$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_P$	$\bar{Y}_{P.a}$
$\rho > 0$	n=10	0.248	0.332	-	0.420
	n=20	0.188	0.377	-	0.435
$\rho < 0$	n=10	0.068	-	0.115	0.817
	n=20	0.077	-	0.083	0.840

Table 2.3 reveals that $\bar{Y}_{P.a}$ performs consistently better than the other estimators in the competition. It performs better exceptionally, more often when $G \in [-0.5, 0.5]$. It has also been observed that $\bar{Y}_{P.a}$ performs better than $\bar{Y}_P$ when G lies between -1 and -0.5.

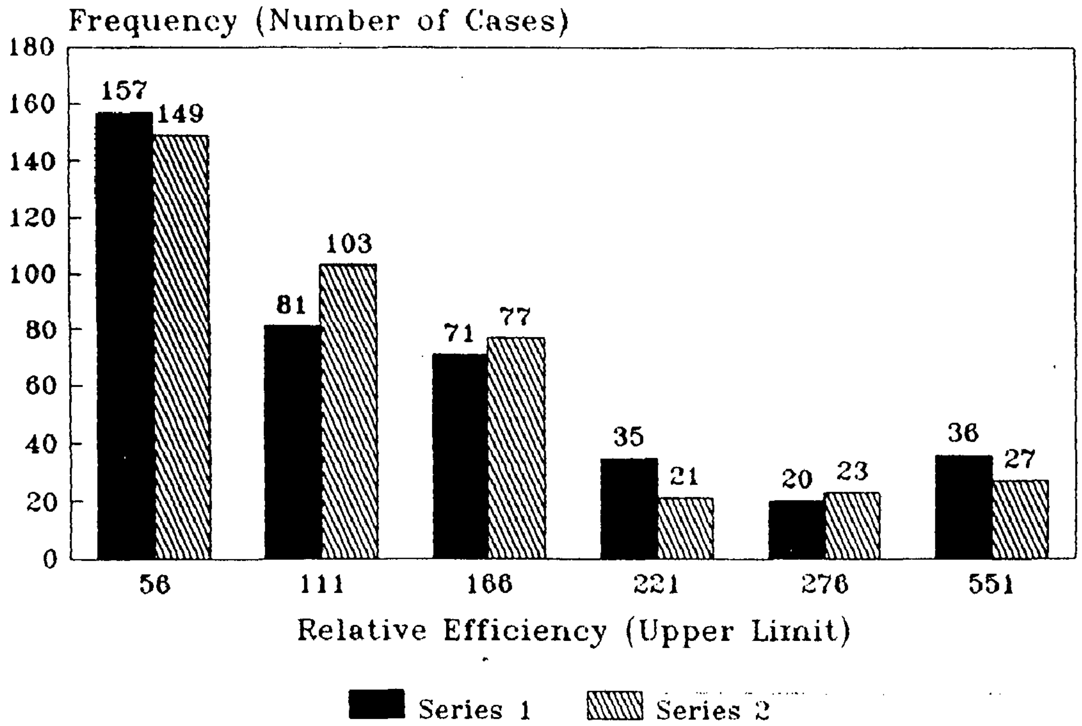
In the present study, we have also included the estimators given by Srivastava(1967), Reddy(1974) and Sahai(1979) defined earlier in the Chapter-1. It can easily be checked that the first order MSE of these estimators $\bar{Y}_{Sr.}$, $\bar{Y}_{Re.}$ and $\bar{Y}_{Sa.}$ come out to be :

Series 1 for $\bar{Y}_R$ and Series 2 for $\bar{Y}_{P.a}$



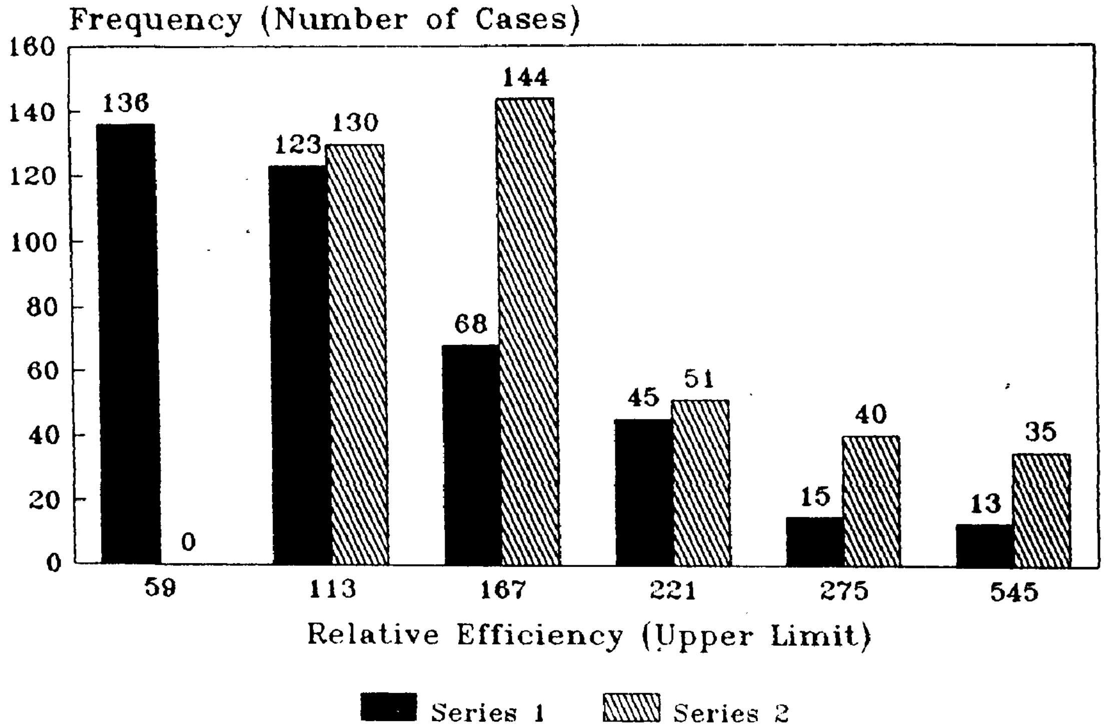
Graph – 2.9 (Sample Size = 10, $\rho > 0$)

Series 1 for $\bar{Y}_R$ and Series 2 for $\bar{Y}_{P.a}$



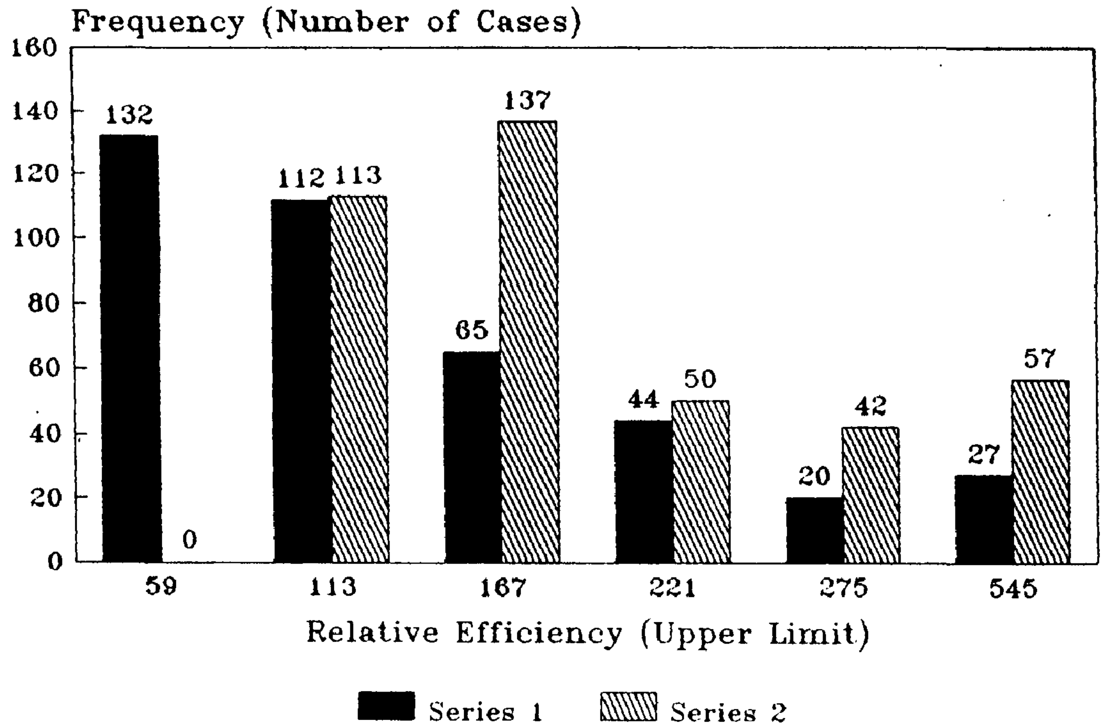
Graph – 2.10 (Sample Size = 20, $\rho > 0$)

Series 1 for $\bar{Y}_P$ and Series 2 for $\bar{Y}_{P.a}$



Graph – 2.11 (Sample Size = 10, $\rho < 0$)

Series 1 for $\bar{Y}_P$ and Series 2 for $\bar{Y}_{P.a}$



Graph – 2.12 (Sample Size = 20, $\rho < 0$)

$$M_1(\bar{Y}_{Sr.}) = M_1(\bar{Y}_{Re.})$$
$$= M_1(\bar{Y}_{Sa.})$$
$$= [1 + a.(a-2G)/C_Z^2].\ \bar{Y}^2.C_y^2/n \qquad \ldots(2.7)$$

In order to obtain the optimal value of 'a', we minimise the above mentioned first order MSE and find that it is minimum when a=G. Thus, we are able to define $\bar{Y}_{Sr.}$, $\bar{Y}_{Re.}$ and $\bar{Y}_{Sa.}$ completely. In what follows, we give the results of empirical-simulation study of these estimators and the graphical display of their relative efficiencies.

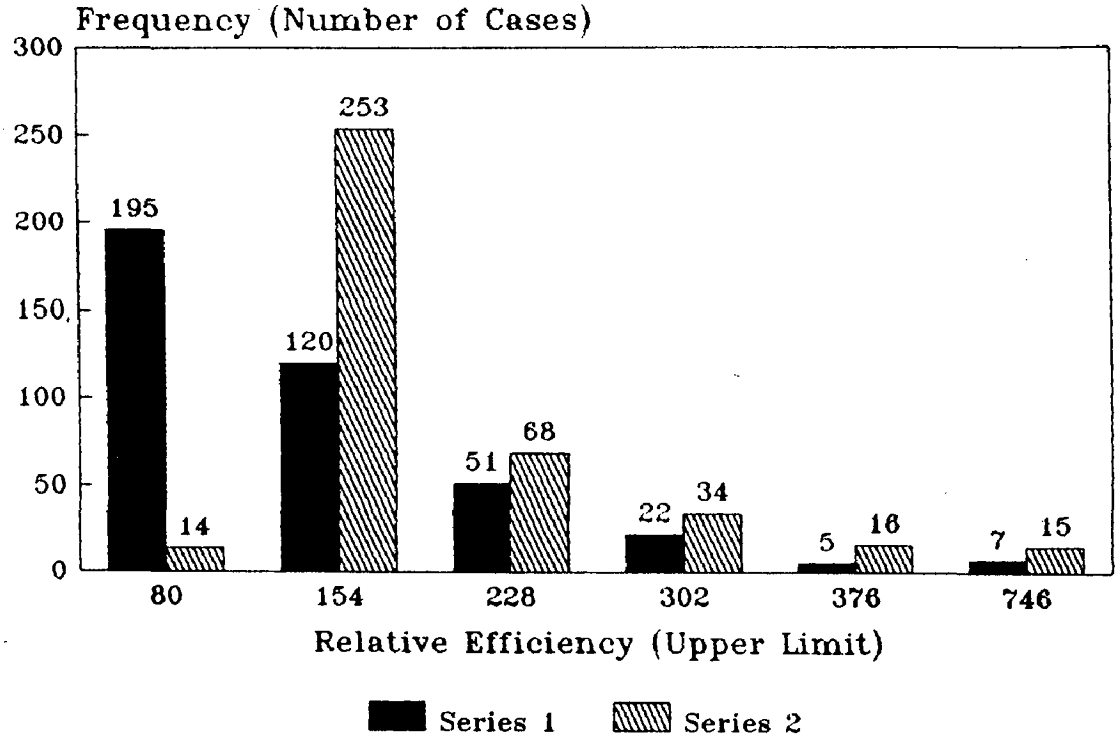
TABLE 2.4

RF(∘) FOR $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ and $\bar{Y}_{Sr.}$

Estimators → / Sample Sizes ↓		$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_P$	$\bar{Y}_{Sr.}$
$\rho > 0$	n=10	0.215	0.115	-	0.670
	n=20	0.150	0.092	-	0.758
$\rho < 0$	n=10	0.075	-	0.140	0.785
	n=20	0.055	-	0.110	0.835

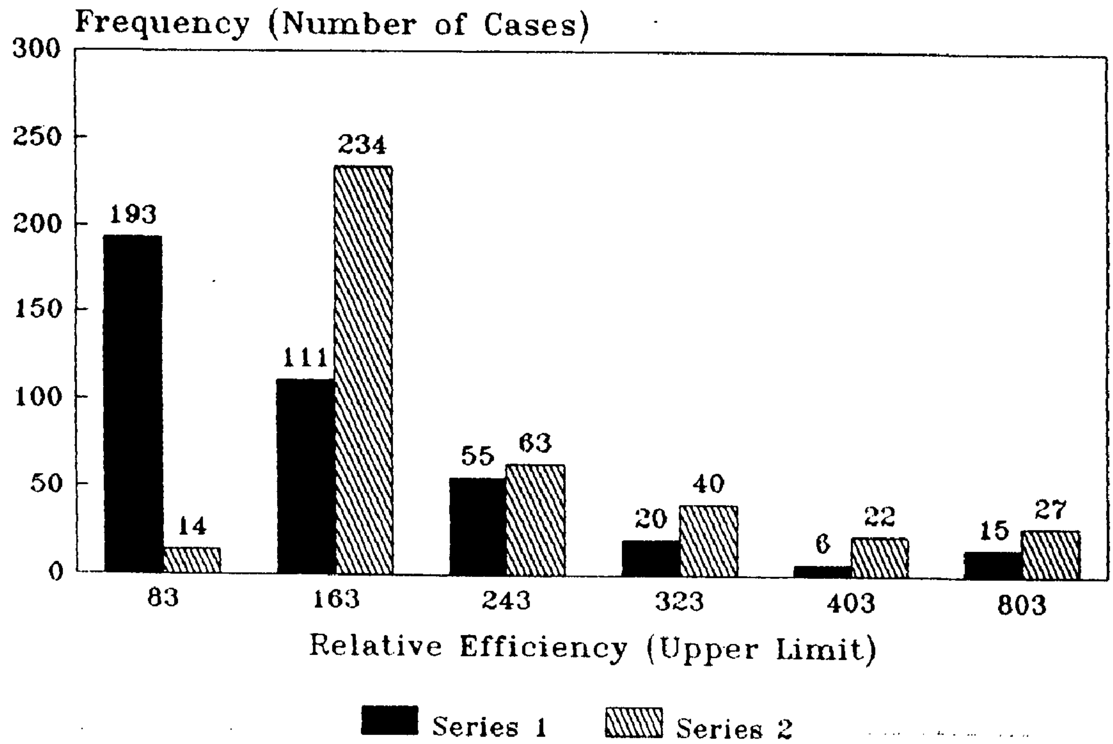
The finer comparisons of the relative efficiencies of these estimators show that $\bar{Y}_{Sr.}$ is most probably a better choice than $\bar{Y}$ and $\bar{Y}_R$ when $G \in [-1,-0.5]$. Graphical display of the relative efficiencies of these estimators is being given in the graphs 2.13 to 2.16.

Series 1 for $\bar{Y}_R$ and Series 2 for $\bar{Y}_{Sr.}$



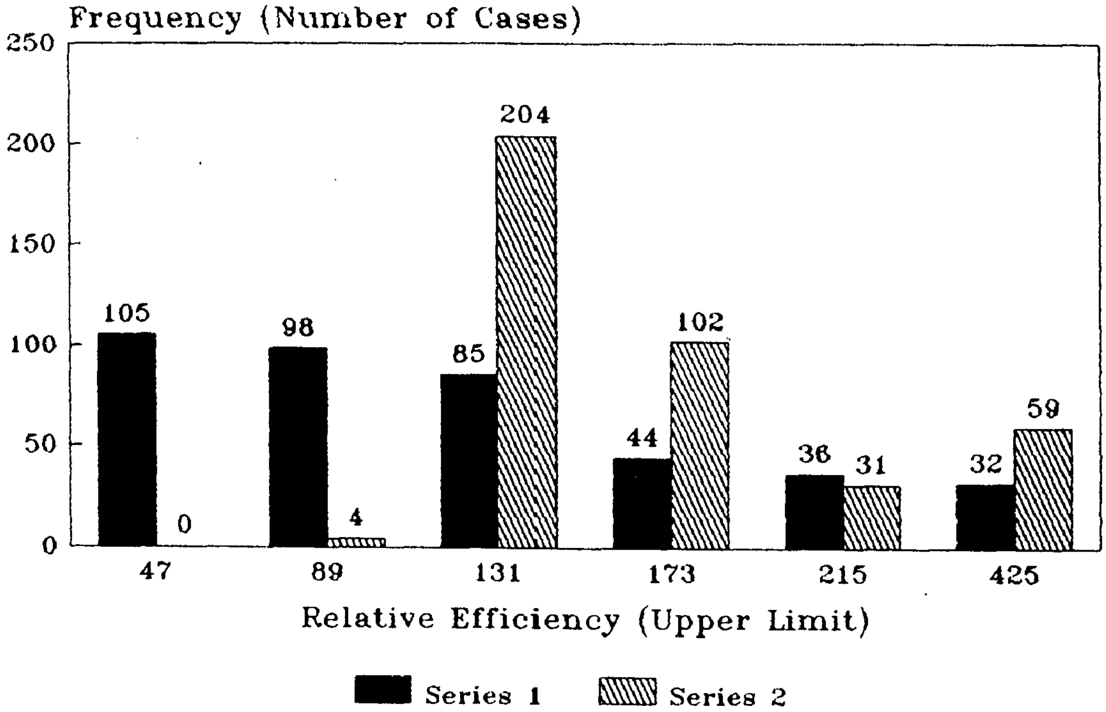
Graph - 2.13 (Sample Size = 10, $\rho > 0$)

Series 1 for $\bar{Y}_R$ and Series 2 for $\bar{Y}_{Sr.}$



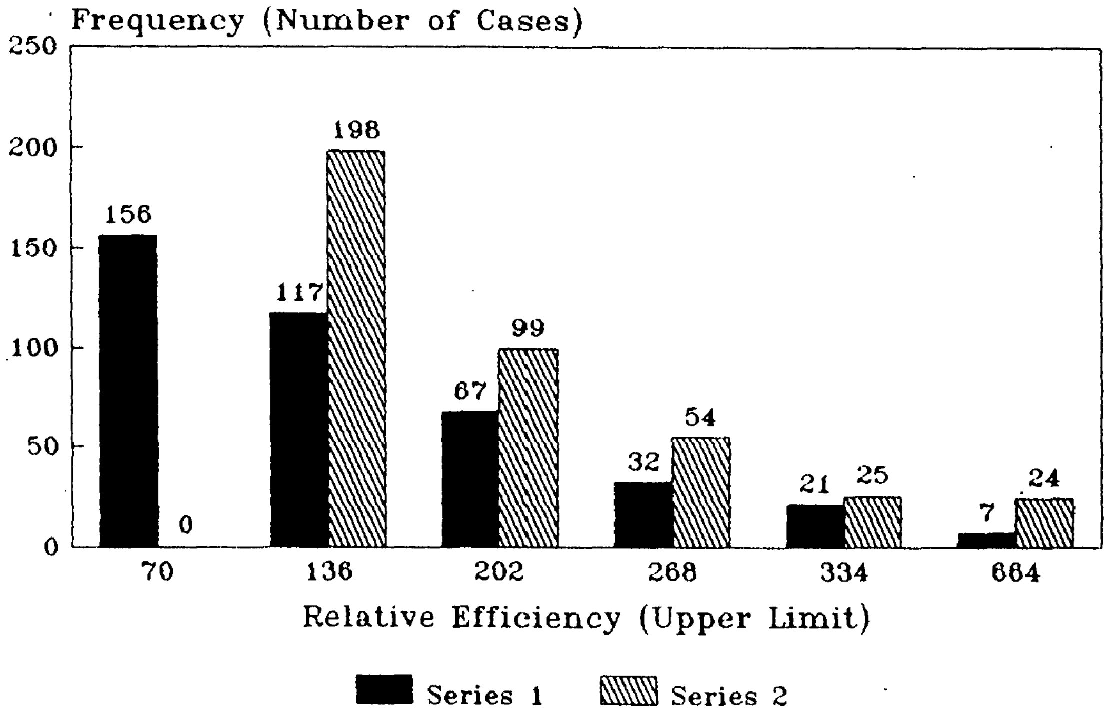
Graph - 2.14 (Sample Size = 20, $\rho > 0$)

Series 1 for $\bar{Y}_p$ and Series 2 for $\bar{Y}_{sr}$.



Graph - 2.15 (Sample Size = 10, $\rho < 0$)

Series 1 for $\bar{Y}_p$ and Series 2 for $\bar{Y}_{sr}$.



Graph - 2.16 (Sample Size = 20, $\rho < 0$)

TABLE 2.5

RF(∘) FOR $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ and $\bar{Y}_{Re.}$

Estimators → / Sample Sizes ↓		$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_P$	$\bar{Y}_{Re.}$
$\rho > 0$	n=10	0.080	0.142	-	0.778
	n=20	0.072	0.123	-	0.805
$\rho < 0$	n=10	0.090	-	0.180	0.730
	n=20	0.072	-	0.153	0.775

From the above table, we can say that $\bar{Y}_{Re.}$ is most probably a better choice than $\bar{y}$ and $\bar{Y}_R/\bar{Y}_P$. The empirical-simulation study also reveals that $\bar{Y}_{Re.}$ is better, quite often, when $G \in [-1,1]$. Graphs 2.17 to 2.20 bear a clearer view of the relative efficiencies of these estimators.

TABLE 2.6

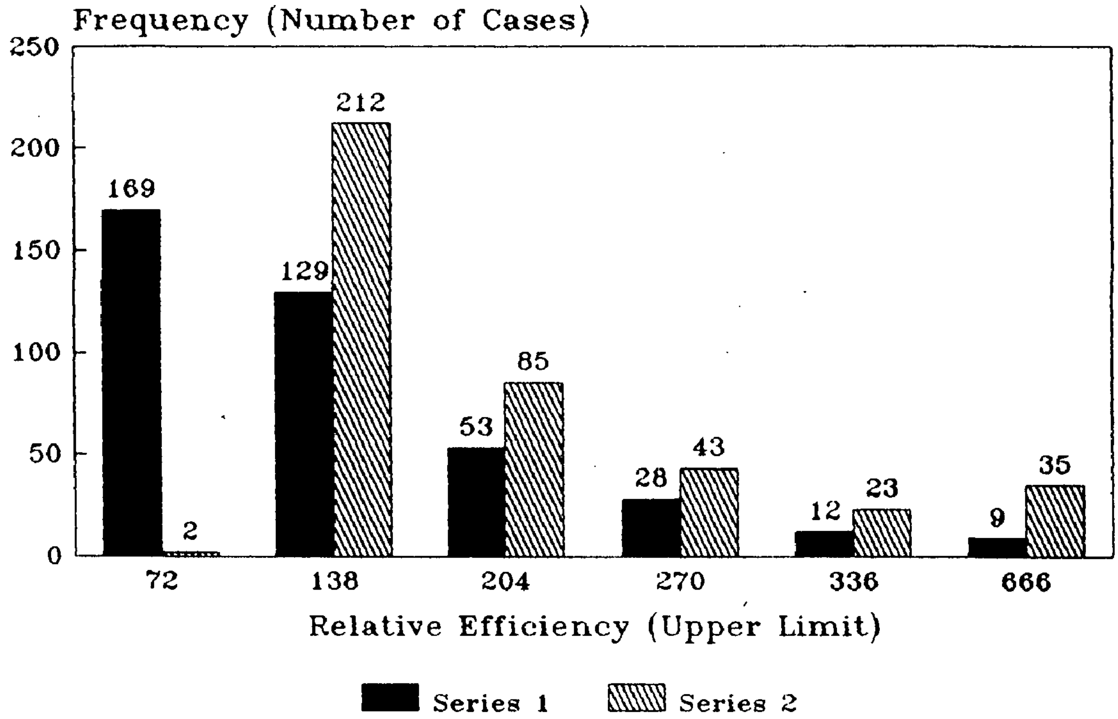
RF(∘) FOR $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ and $\bar{Y}_{Sa.}$

Estimators → / Sample Sizes ↓		$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_P$	$\bar{Y}_{Sa.}$
$\rho > 0$	n=10	0.198	0.115	-	0.687
	n=20	0.095	0.108	-	0.797
$\rho < 0$	n=10	0.145	-	0.132	0.723
	n=20	0.062	-	0.113	0.825

Here, although $\bar{Y}_{Sa.}$ performs better than $\bar{y}$ and $\bar{Y}_R/\bar{Y}_P$, it comes out to be winner much more often when G lies between -1 and 1. The gain in the relative efficiency when we use $\bar{Y}_{Sa.}$ has been displayed graphically in the graphs 2.21 to 2.24.

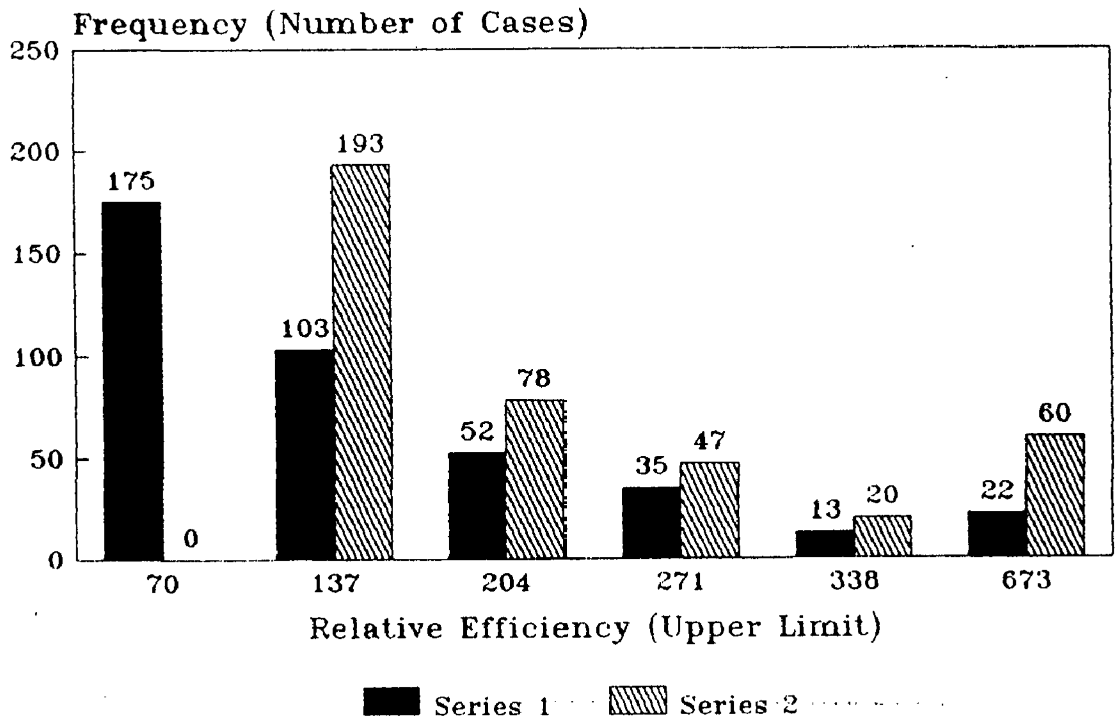
Motivated by the structure of the estimator $\bar{Y}_{Sa.}$, we propose a new family of ratio-cum-product estimators as

Series 1 for $\bar{Y}_R$ and Series 2 for $\bar{Y}_{Re}$.

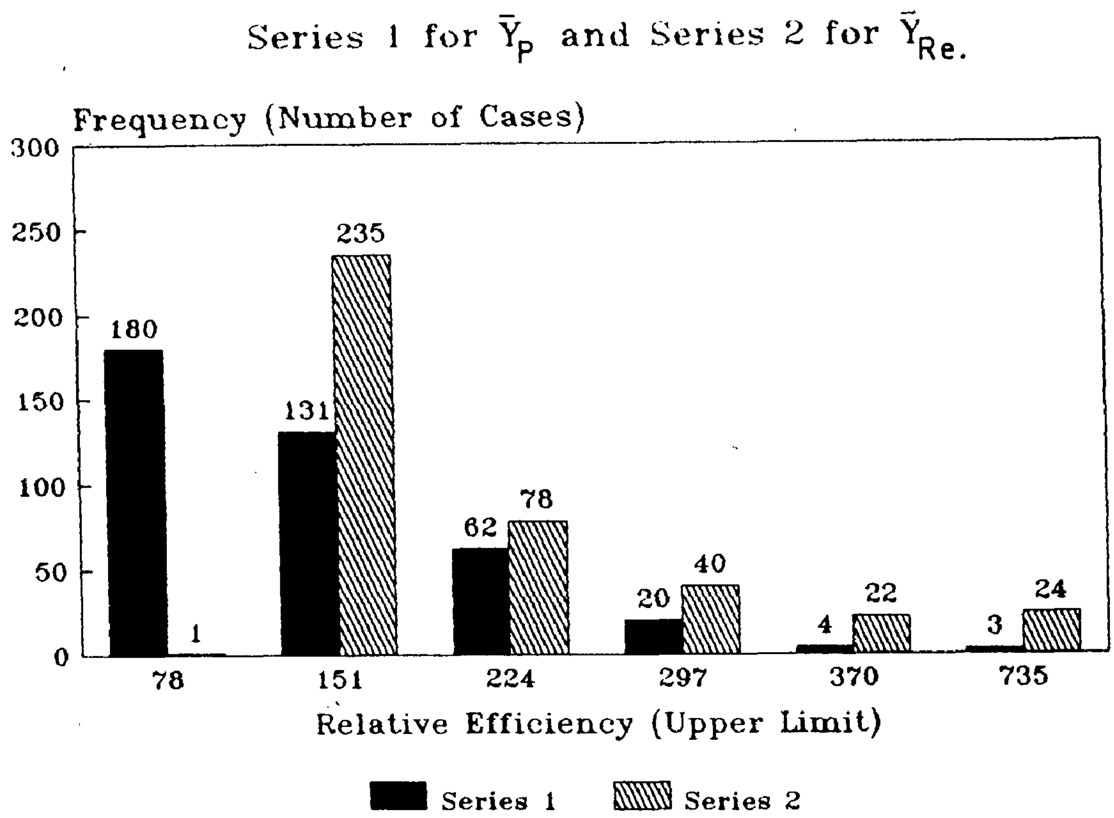


Graph - 2.17 (Sample Size = 10, $\theta > 0$)

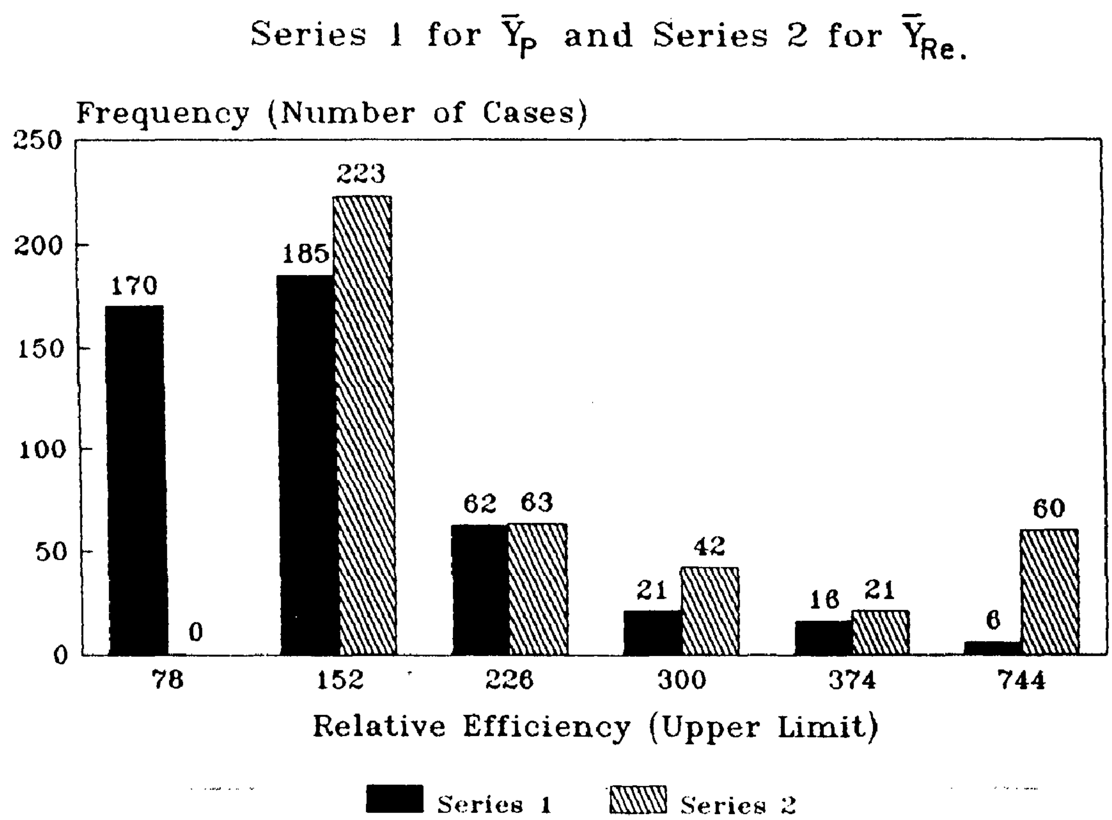
Series 1 for $\bar{Y}_R$ and Series 2 for $\bar{Y}_{Re}$.



Graph - 2.18 (Sample Size = 20, $\theta > 0$)

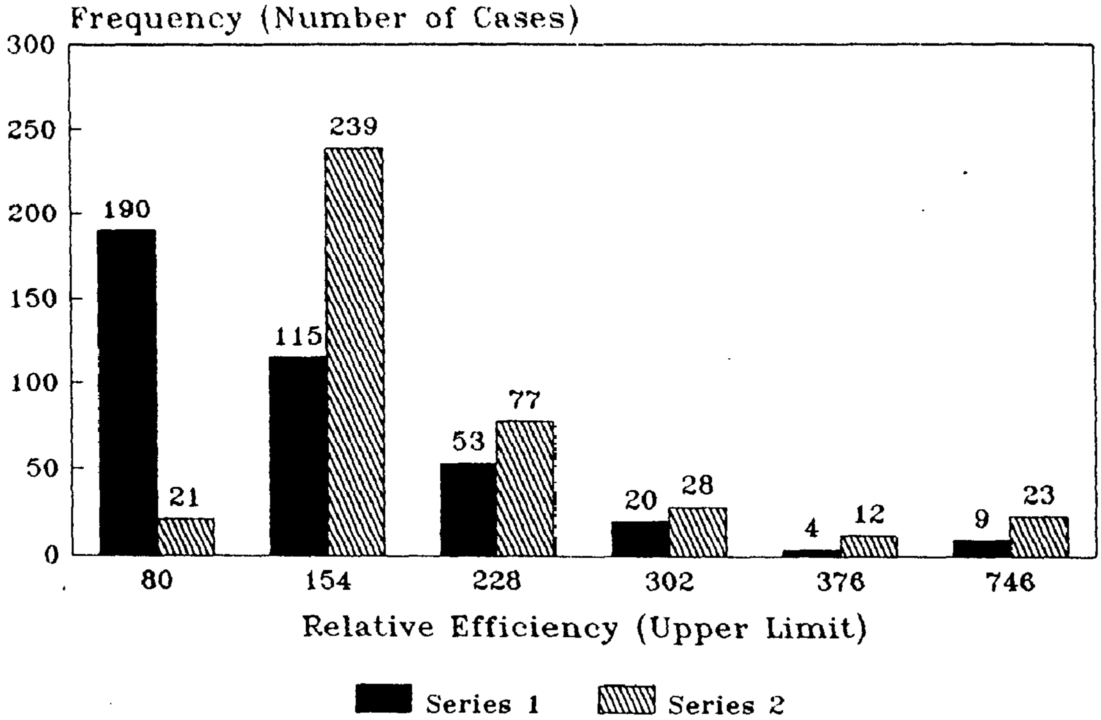


Graph - 2.19 (Sample Size = 10, $\rho < 0$)



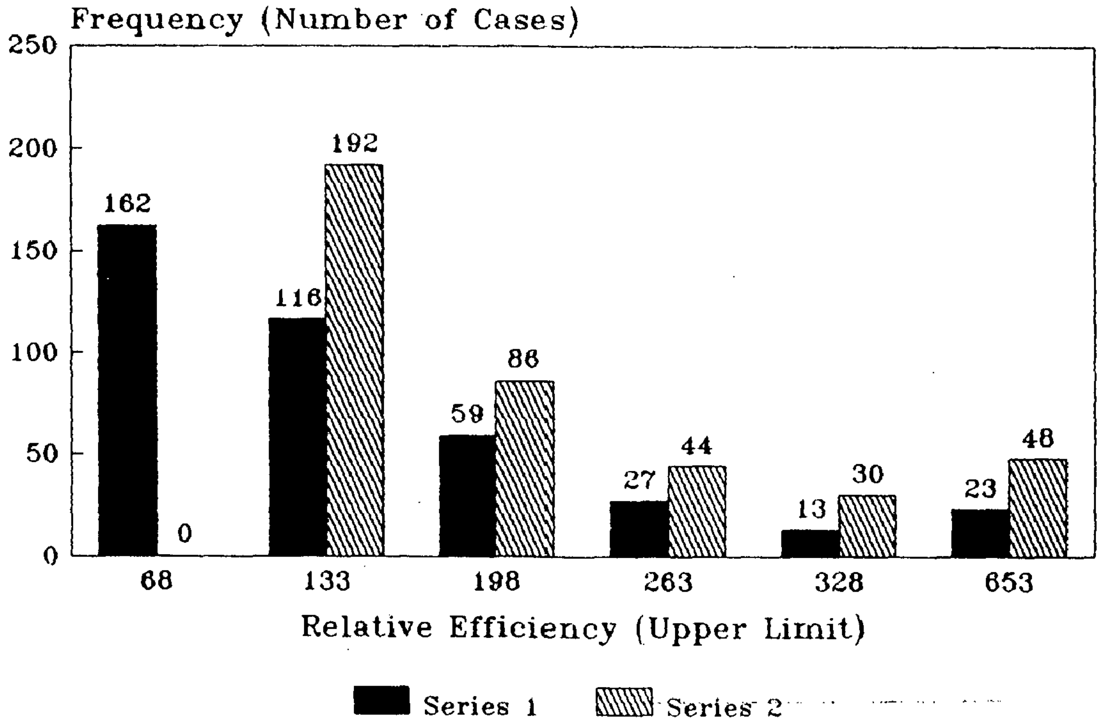
Graph - 2.20 (Sample Size = 20, $\rho < 0$)

Series 1 for $\bar{Y}_R$ and Series 2 for $\bar{Y}_{Sa}$.



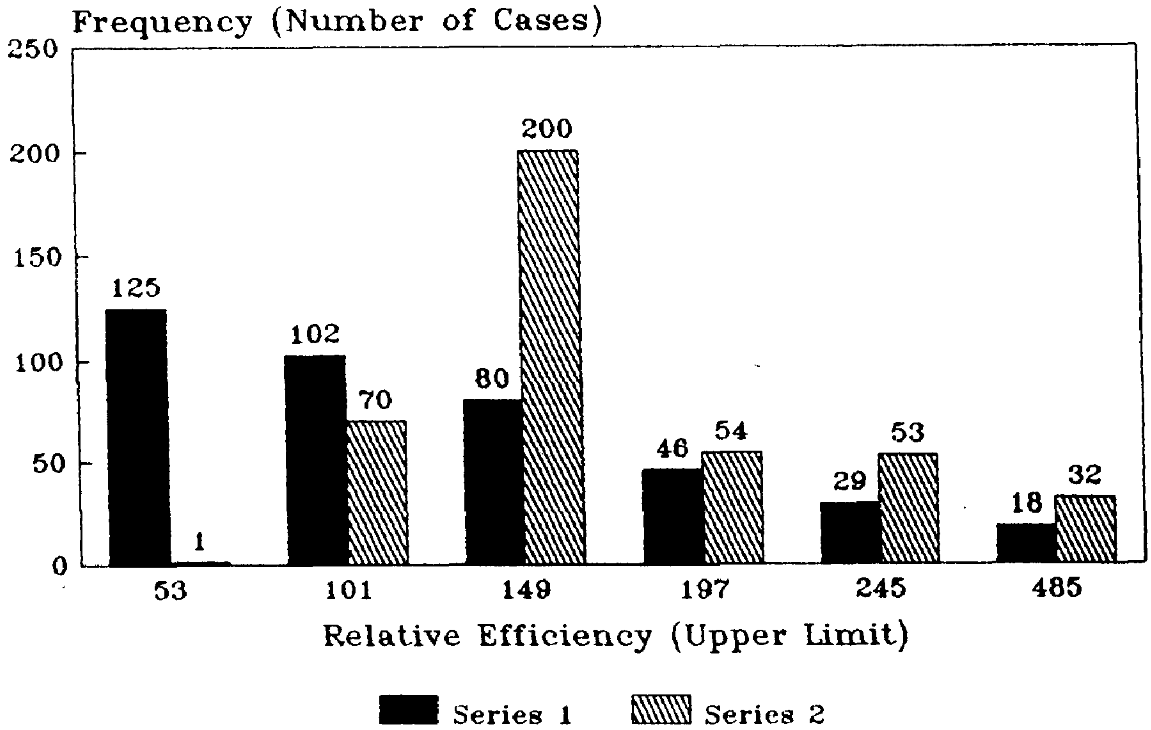
Graph – 2.21 (Sample Size = 10, ρ > 0)

Series 1 for $\bar{Y}_R$ and Series 2 for $\bar{Y}_{Sa}$.



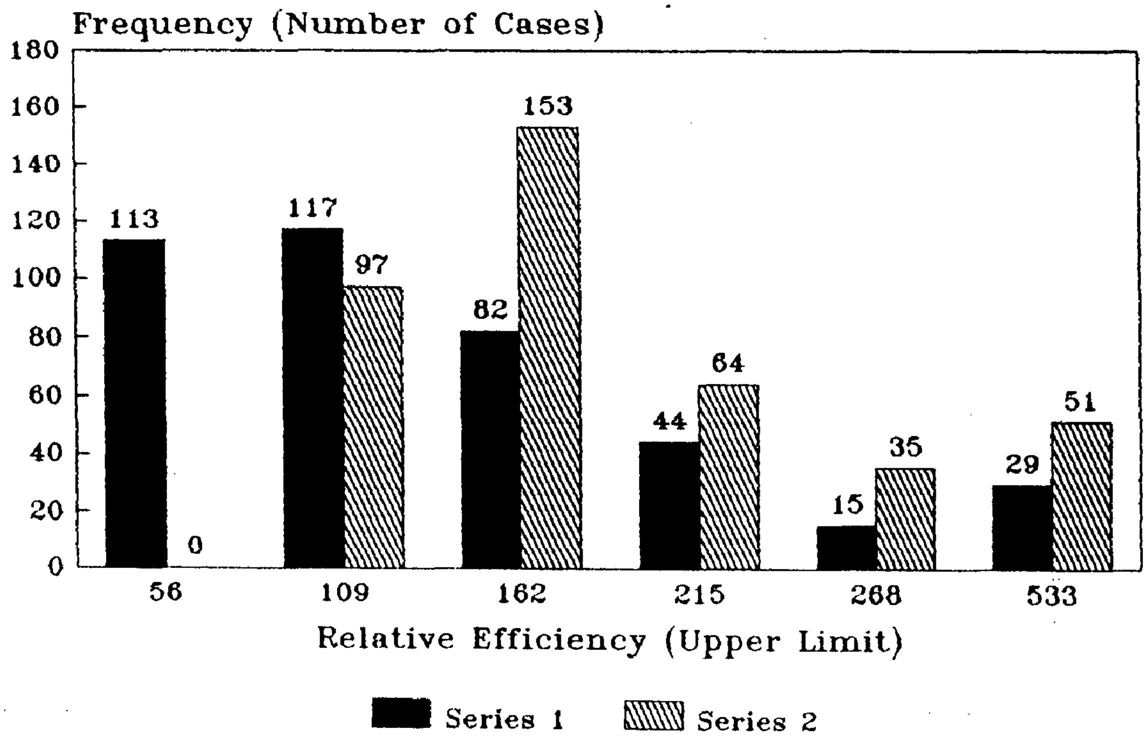
Graph – 2.22 (Sample Size = 20, ρ > 0)

Series 1 for $\bar{Y}_p$ and Series 2 for $\bar{Y}_{Sa}$.



Graph – 2.23 (Sample Size = 10, $\rho < 0$)

Series 1 for $\bar{Y}_p$ and Series 2 for $\bar{Y}_{Sa}$.



Graph – 2.24 (Sample Size = 20, $\rho < 0$)

$$\bar{Y}_{VSa.} = \bar{y}.((1+a).\bar{Y}_R + (1-a).\bar{Y}_P) \ / \ ((1+a).\bar{Y}_P + (1-a).\ \bar{Y}_R) \qquad ...(2.8)$$

This can, equivalently, be written as

$$\bar{Y}_{VSa.} = \bar{y}.((1+a).\bar{X}^2 + (1-a).\bar{x}^2) \ / \ ((1+a).\bar{x}^2 + (1-a).\bar{X}^2) \qquad ...(2.9)$$

Again, 'a' is a non-stochastic design-parameter. The first order MSE of $\bar{Y}_{VSa.}$ can be checked to be :

$$M_1(\bar{Y}_{VSa.}) = [\ 1 + 4a.(a-G)/C_Z^2\ ].\ \bar{Y}^2.C_Y^2/n \qquad ...(2.10)$$

$M_1(\bar{Y}_{VSa.})$ attains its minimum value when a = G/2. Putting this value of 'a' in 2.9 we get,

$$\bar{Y}_{VSa.} = \bar{y}.((2+G).\ \bar{X}^2 + (2-G).\ \bar{x}^2) \ / \ ((2+G).\bar{x}^2 + (2-G).\bar{X}^2) \qquad ...(2.11)$$

We now compare this estimator with $\bar{y}$ and $\bar{Y}_R/\bar{Y}_P$. The results of the comparisons are given below in table 2.7. Also, graphs 2.25 to 2.28 give a clearer view of the relative efficiencies of these estimators.

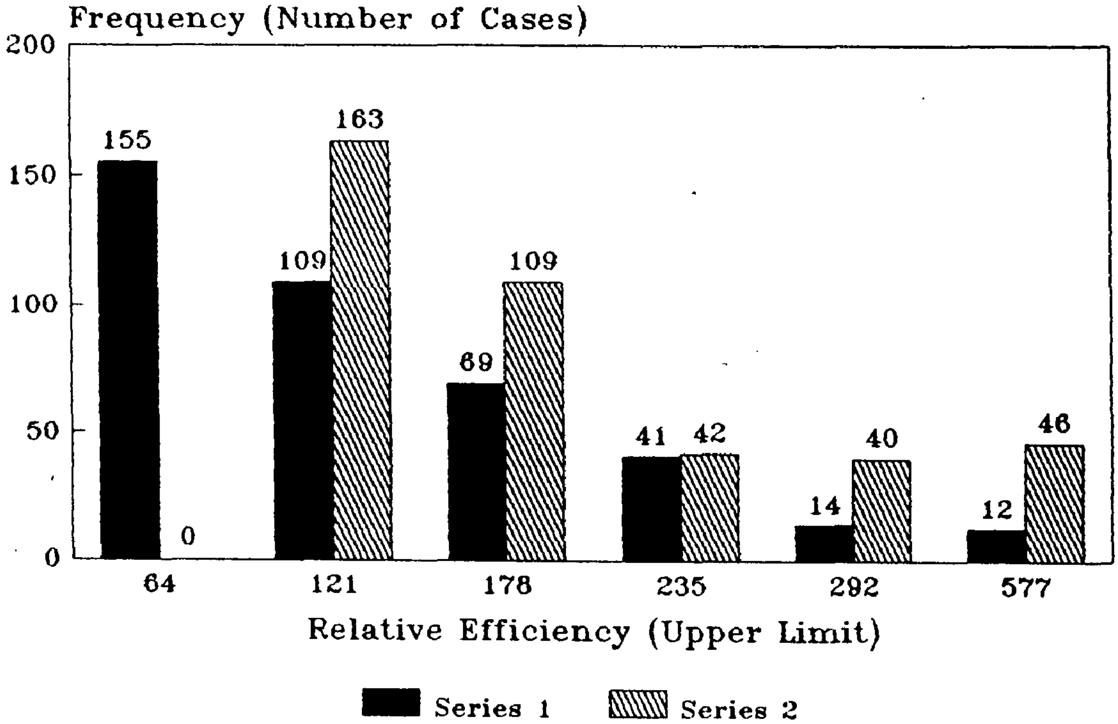
TABLE 2.7

RF(∘) FOR $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ and $\bar{Y}_{VSa.}$

Estimators → / Sample Sizes ↓		$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_P$	$\bar{Y}_{VSa.}$
$\rho > 0$	n=10	0.107	0.090	-	0.803
	n=20	0.085	0.102	-	0.813
$\rho < 0$	n=10	0.050	-	0.110	0.840
	n=20	0.040	-	0.087	0.873

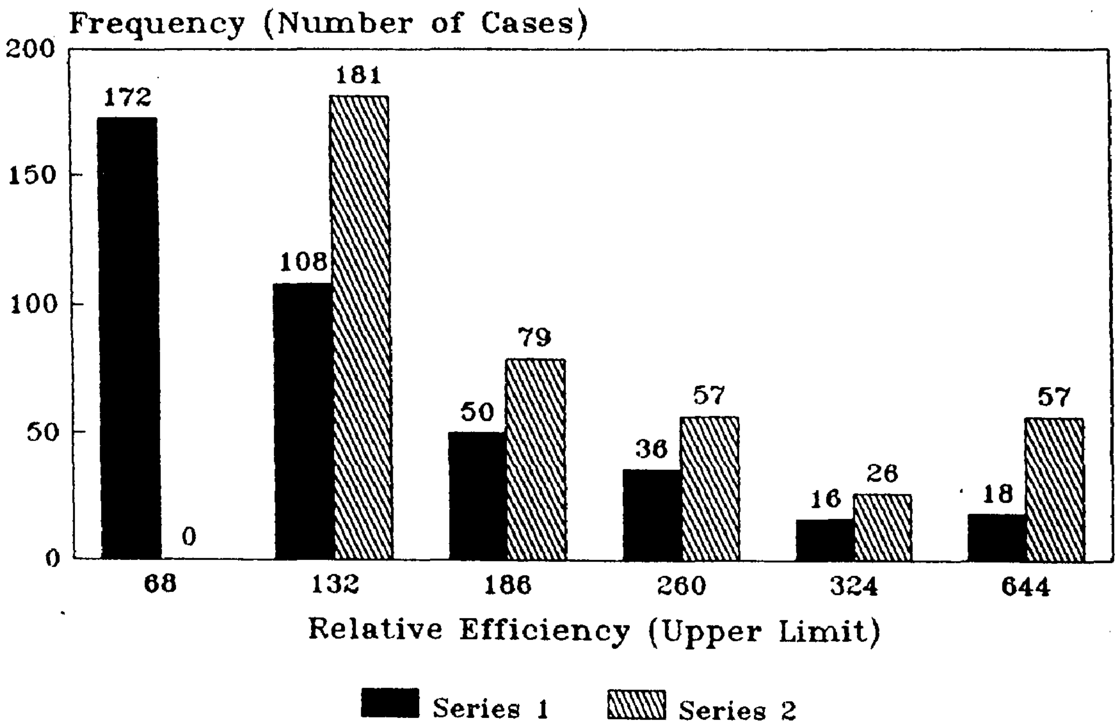
From the above table, we can strongly recommend the use of $\bar{Y}_{VSa.}$ when $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ and $\bar{Y}_{VSa.}$ are in the competition. We have also noted through the finer comparisons of the relative efficiencies of these estimators that $\bar{Y}_{VSa.}$ performs better than $\bar{y}$ and $\bar{Y}_R/\bar{Y}_P$ for the entire range of G considered by us. Thus, we can use $\bar{Y}_{VSa.}$ irrespective of the value of G.

Series 1 for $\bar{Y}_R$ and Series 2 for $\bar{Y}_{VSa}$.



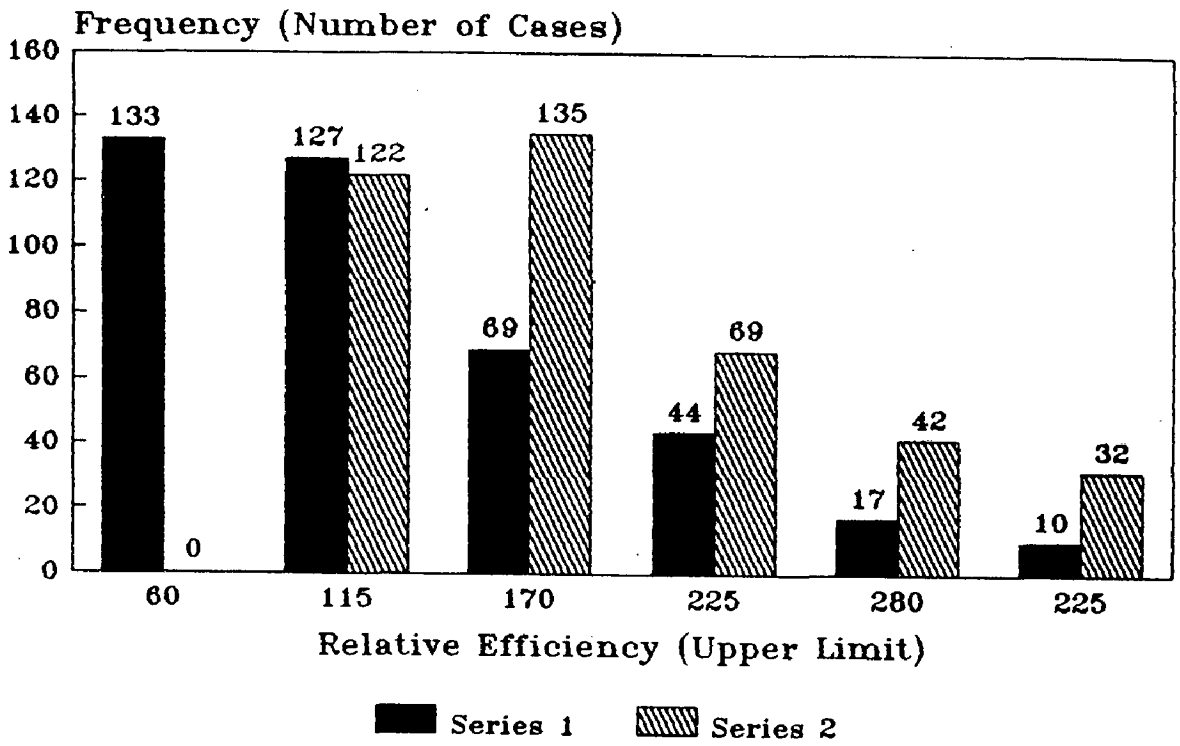
Graph – 2.25 (Sample Size = 10, $\rho > 0$)

Series 1 for $\bar{Y}_R$ and Series 2 for $\bar{Y}_{VSa}$.



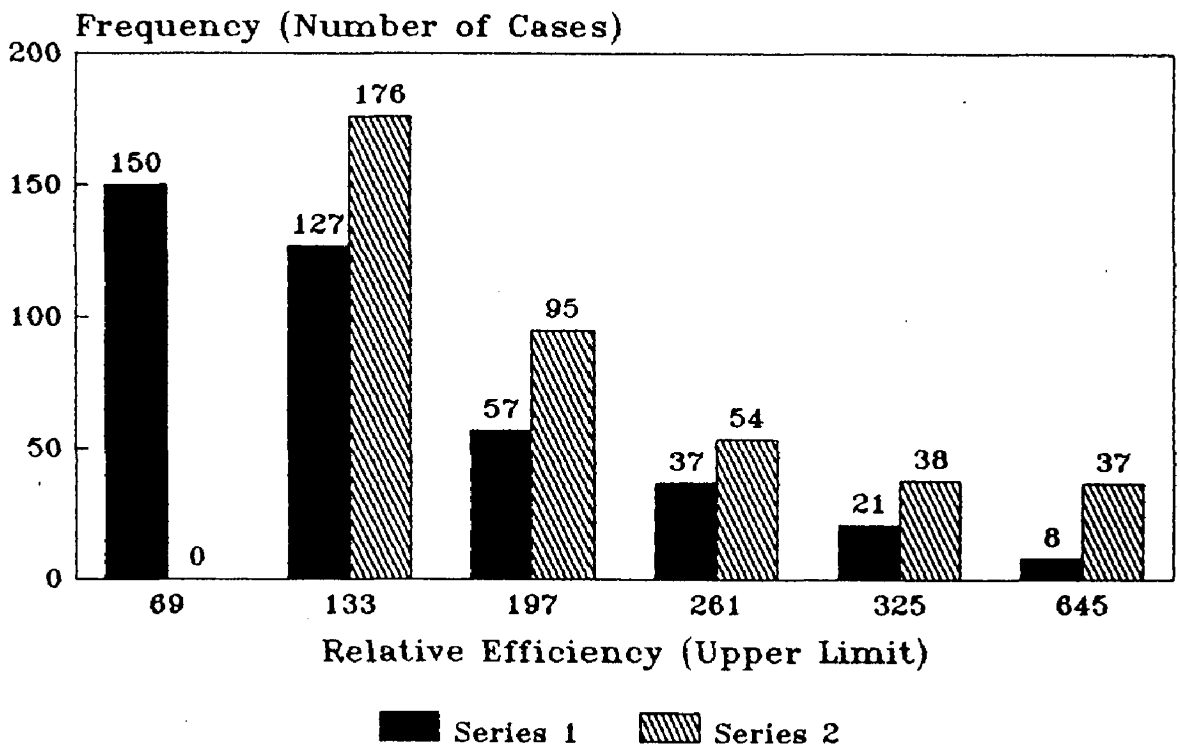
Graph – 2.26 (Sample Size = 20, $\rho > 0$)

Series 1 for $\bar{Y}_P$ and Series 2 for $\bar{Y}_{VSa}$.



Graph - 2.27 (Sample Size = 10, $\rho < 0$)

Series 1 for $\bar{Y}_P$ and Series 2 for $\bar{Y}_{VSa}$.



Graph - 2.28 (Sample Size = 20, $\rho < 0$)

CHAPTER - 3

THE TWO-PARAMETER FAMILIES OF ESTIMATORS

In Chapter-2, we proposed and studied some ratio, product and ratio-cum-product type estimators in which only one design-parameter has been used for the mixing of $\bar{y}$, $\bar{x}$ and $\bar{X}$. This parameter was assigned an optimal value, its optimality being in reference to the minimisation of the first order large sample approximation to the MSE of the estimator. In this chapter, we have tried the mixings of $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_P$ using two design-parameters rather than one. By the use of two parameters, we have two degrees of freedom for manipulation. This additional degree of freedom is used for controlling the second order large sample approximation to the MSEs' of the proposed estimators. It so happens that in this process we are not only able to use the guessed value 'g' of 'G' but also the guessed value, say, 'r' of 'ρ' which would have been implicitly used in guessing 'G' by 'g'. Thus a fuller use of the guessed values leads to gainful consequences in terms of more efficient families of estimators. We propose the following two families of ratio-cum-product estimators.

Taking 'a' and 'b' as the two non-stochastic design parameters, we propose,

$$\bar{Y}^{(1)}_{RP.ab} = (1+2a).\ \bar{y} - a.((1+b).\bar{Y}_R + (1-b).\ \bar{Y}_P) \qquad \ldots(3.1$$

For obtaining the optimal values of 'a' and 'b', we minimise the first and second order MSEs of this estimator. Let $M_1(\bar{Y}^{(1)}_{RP.ab})$ and $M_2(\bar{Y}^{(1)}_{RP.ab})$ be the first and second order MSEs of $\bar{Y}^{(1)}_{RP.ab}$ respectively, for a random sample from a bivariate normal population. We can check that $M_1(\bar{Y}^{(1)}_{RP.ab})$ and $M_2(\bar{Y}^{(1)}_{RP.ab})$ can be given by the following equations.

$$M_1(\bar{Y}^{(1)}_{RP.ab}) = [\ 1 + 4d.(G+d)/C^2_Z\ ].\ \bar{Y}^2.C^2_Y\ /n \qquad \ldots(3.2$$

where, d = a.b . Minimising $M_1(\bar{Y}^{(1)}_{RP.ab})$, the optimal value of 'd' comes out to be equal to -G/2,

$$\text{and } M_2(\bar{Y}^{(1)}_{RP.ab}) = M_1(\bar{Y}^{(1)}_{RP.ab}) + P.[2Q.(2d^2-(a+d)) + 6G.(a+d).(1-4d) + 3.(a+d).(a+5d)] \qquad \ldots(3.3$$

where, $P = C^4_X/n^2$, $Q = (1+2\rho^2).\ C^2_Z$ and $C_Z = G/\rho$. Now, using the optimal value for 'd' and minimising $M_2(\bar{Y}^{(1)}_{RP.ab})$, we see that it will be minimum for $a = [2C^2_Z + G.(3-8G)]/6$. Thus, we have the following expressions for the optimal values of 'a' and 'b' for $\bar{Y}^{(1)}_{RP.ab}$.

$$a = [2C^2_Z + G.(3-8G)]/6 \qquad \ldots(3.4)$$

$$\text{and} \qquad b = -\ G/(2a) \qquad \ldots(3.5)$$

Now, in this chapter, we have carried out the empirical-simulation study to compare $\bar{Y}^{(1)}_{RP.ab}$ (using the guessed value 'g' of 'G' and 'r' of 'ρ') with $\bar{y}$ and $\bar{Y}_R$ when $\rho > 0$ and with $\bar{y}$ and $\bar{Y}_P$ when $\rho < 0$. The results of this study are tabulated below in table 3.1. The gain in efficiency which occurs by the use of $\bar{Y}^{(1)}_{RP.ab}$ has also been displayed graphically through graphs 3.1 to 3.4.

TABLE 3.1

RF(∘) FOR $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ and $\bar{Y}^{(1)}_{RP.ab}$

Estimators → / Sample Sizes ↓		$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_P$	$\bar{Y}^{(1)}_{RP.ab}$
$\rho > 0$	n=10	0.170	0.135	-	0.695
	n=20	0.097	0.113	-	0.790
$\rho < 0$	n=10	0.305	-	0.282	0.413
	n=20	0.167	-	0.198	0.635

This table shows that $\bar{Y}^{(1)}_{RP.ab}$ performs better than $\bar{y}$ and $\bar{Y}_R$ much more often when $\rho > 0$. Although, it does not perform that very good when $\rho < 0$ and n=10, it recovers for large sample size (n=20) and dominates $\bar{y}$ and $\bar{Y}_P$, quite often. It has also been seen that $\bar{Y}^{(1)}_{RP.ab}$ performs very nicely when $G \in [0,1]$ and for this range of G, the relative efficiency of $\bar{Y}^{(1)}_{RP.ab}$ comes out to be more than that of $\bar{y}$ and $\bar{Y}_R$, quite often. This estimator also performs exceptionally better than $\bar{y}$ and $\bar{Y}_P$ when $G \in [-0.5,0]$.

One more family of ratio-cum-product type estimators by making the use of two non-stochastic design-parameters 'a' and 'b' has been proposed by us in what follows. An estimator belonging to this family is,

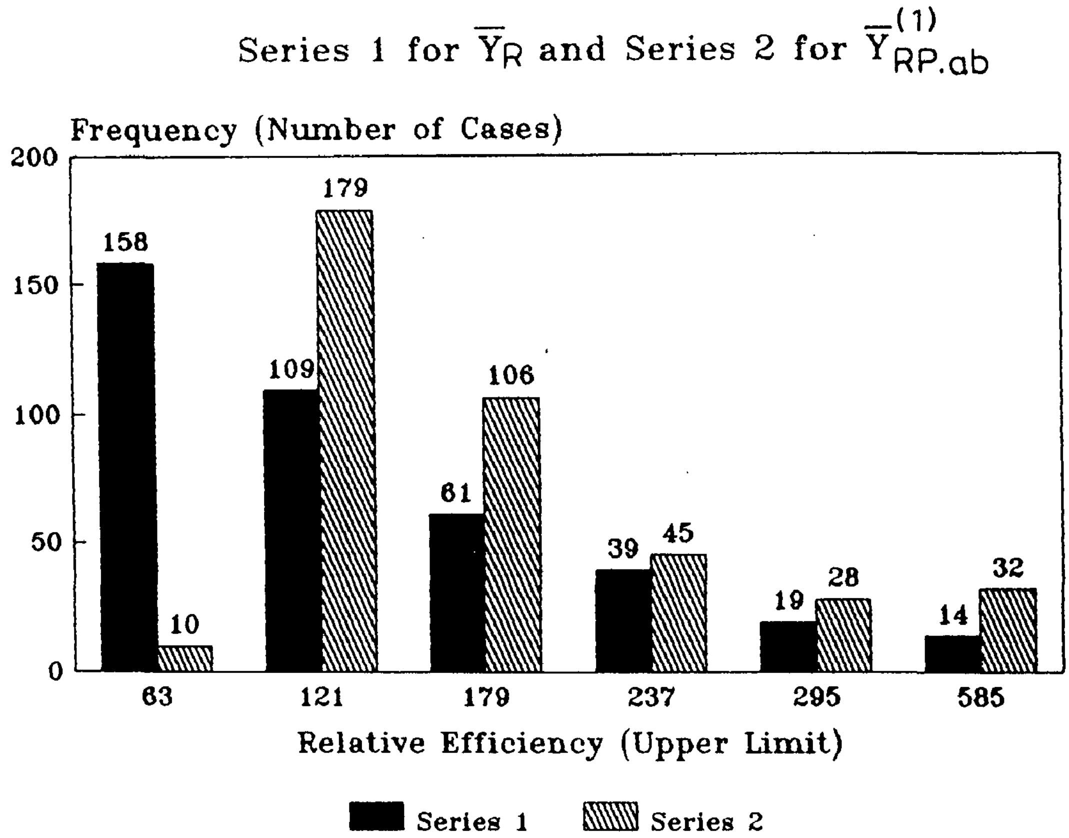
$$\bar{Y}^{(2)}_{RP.ab} = (1+a+b).\ \bar{y} - a.\bar{Y}_R - b.\bar{Y}_P \qquad \ldots(3.6)$$

The first and second order MSEs of this estimator can be checked to be :

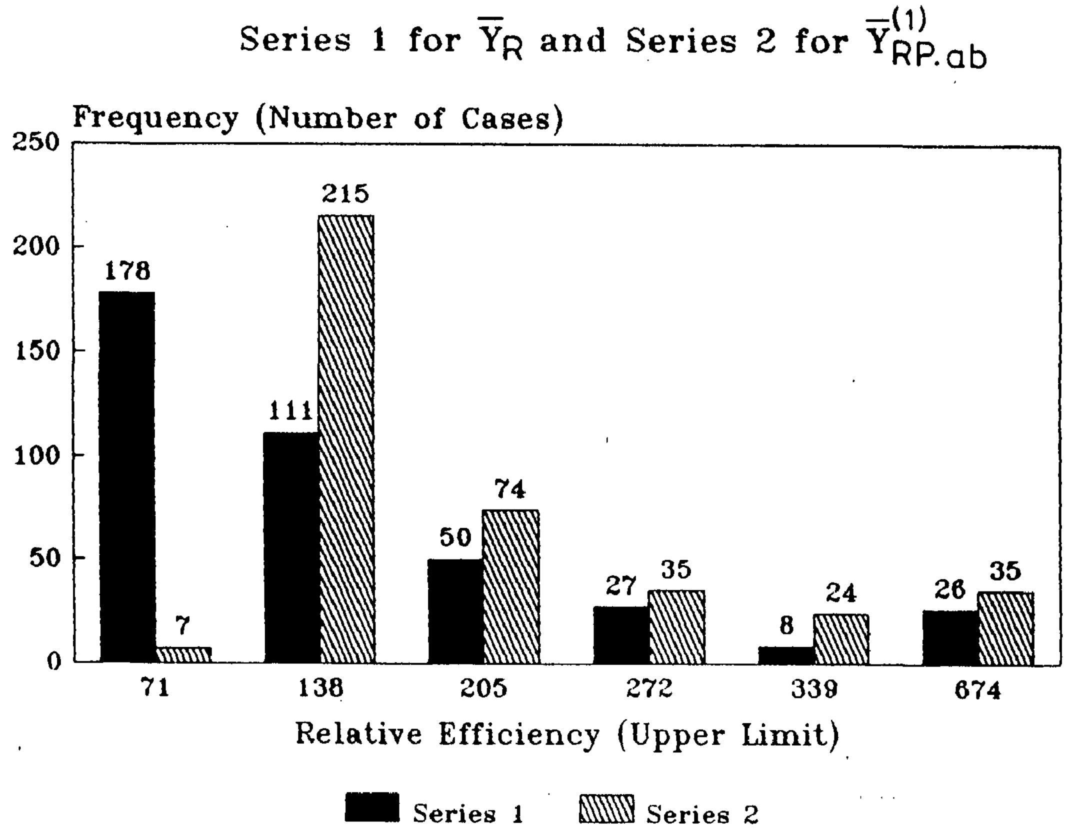
$$M_1(\bar{Y}^{(2)}_{RP.ab}) = [1 + t.(t+2G)/C_Z^2].\ \bar{Y}^2.C_Y^2/n \qquad \ldots(3.7)$$

where, $t = a-b$. $M_1(\bar{Y}^{(2)}_{RP.ab})$ takes its minimum value when $t = -G$,

and $M_2(\bar{Y}^{(2)}_{RP.ab}) = M_1(\bar{Y}^{(2)}_{RP.ab}) + P.[Q.(t^2-2a) - 6G.a.(1+2t) +$

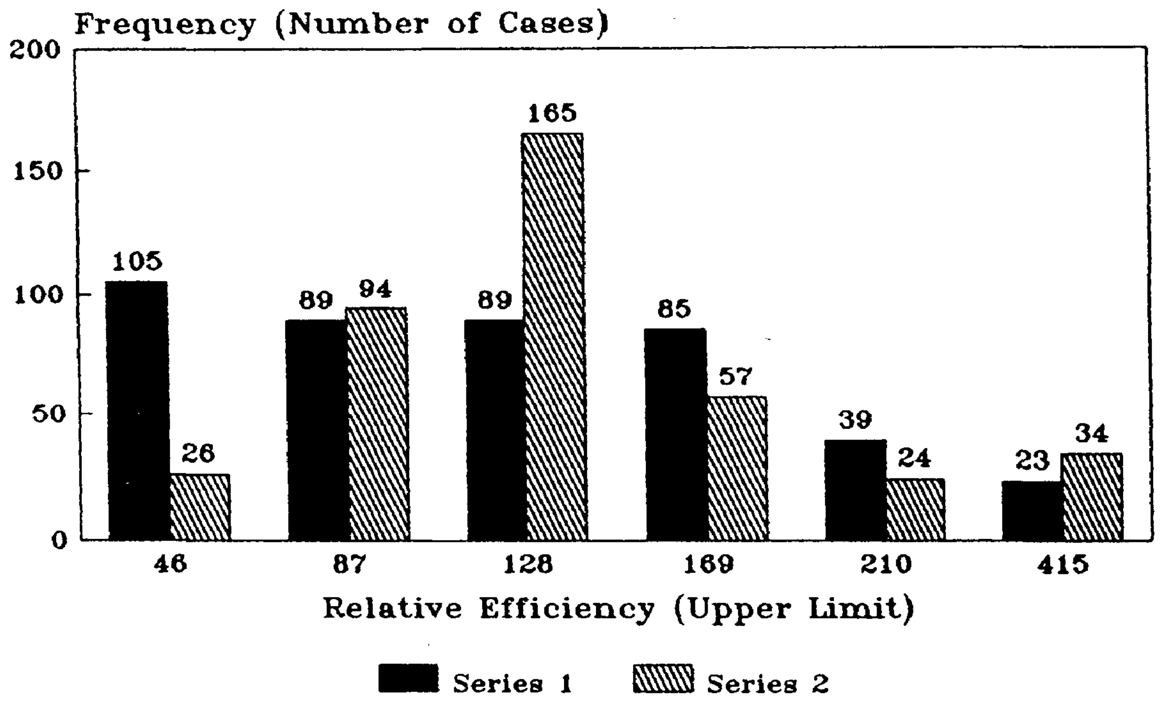


Graph – 3.1 (Sample Size = 10, $\rho > 0$)



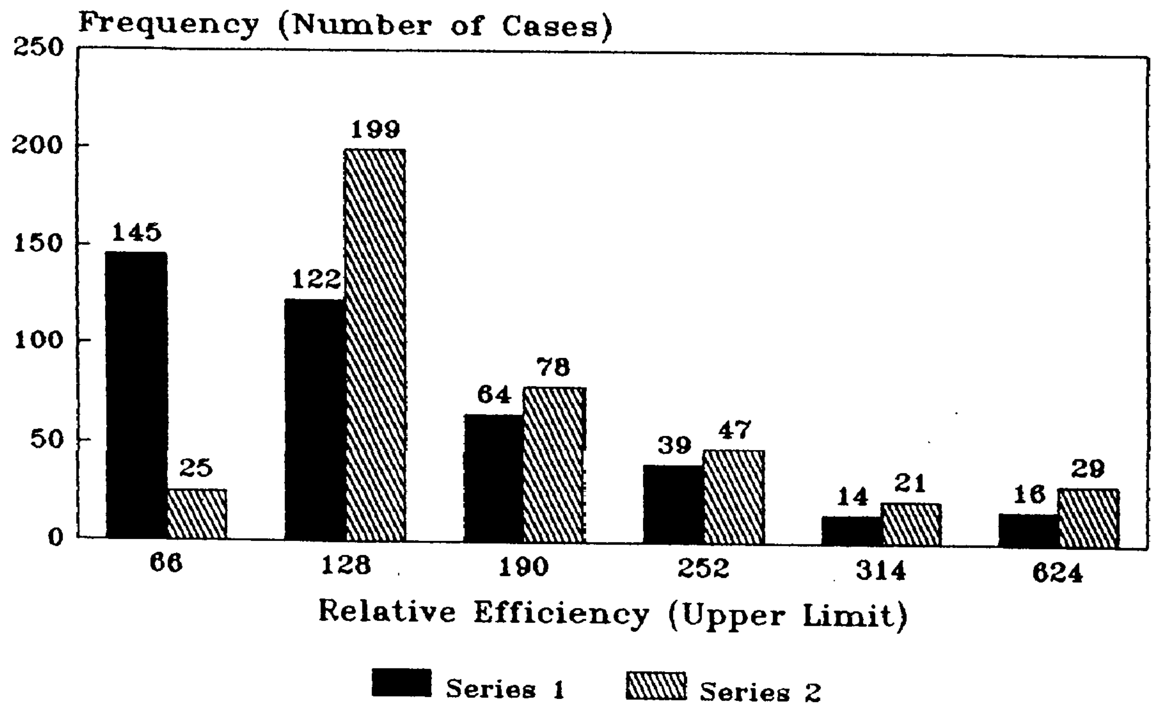
Graph – 3.2 (Sample Size = 20, $\rho > 0$)

Series 1 for $\overline{Y}_P$ and Series 2 for $\overline{Y}^{(1)}_{RP.ab}$



Graph – 3.3 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_P$ and Series 2 for $\overline{Y}^{(1)}_{RP.ab}$



Graph – 3.4 (Sample Size = 20, $\rho < 0$)

$+3a.(a-2t)]$ ...(3.8)

where, P and Q are the same as defined for equation 3.3. We minimise $M_2(\bar{Y}^{(2)}_{RP.ab})$ with respect to 'a' and see that it has its minimum value when $a = (C_Z^2 - 4G^2)/3$. So, the optimal values of 'a' and 'b' for $\bar{Y}^{(2)}_{RP.ab}$ can be given by the following equations.

$$a = (C_Z^2 - 4G^2)/3 \qquad \text{...(3.9)}$$

$$\text{and} \quad b = a + G \qquad \text{...(3.10)}$$

In this way, we have defined $\bar{Y}^{(2)}_{RP.ab}$ wherein we used guessed values 'g' of 'G' and 'r' of 'ρ' to choose optimal values of 'a' and 'b'. Now, we proceed to compare $\bar{Y}^{(2)}_{RP.ab}$ with $\bar{y}$ and $\bar{Y}_R/\bar{Y}_P$. The results of these comparisons are tabulated below in table 3.2. The relative efficiencies of these estimators have been displayed graphically through graphs 3.5 to 3.8.

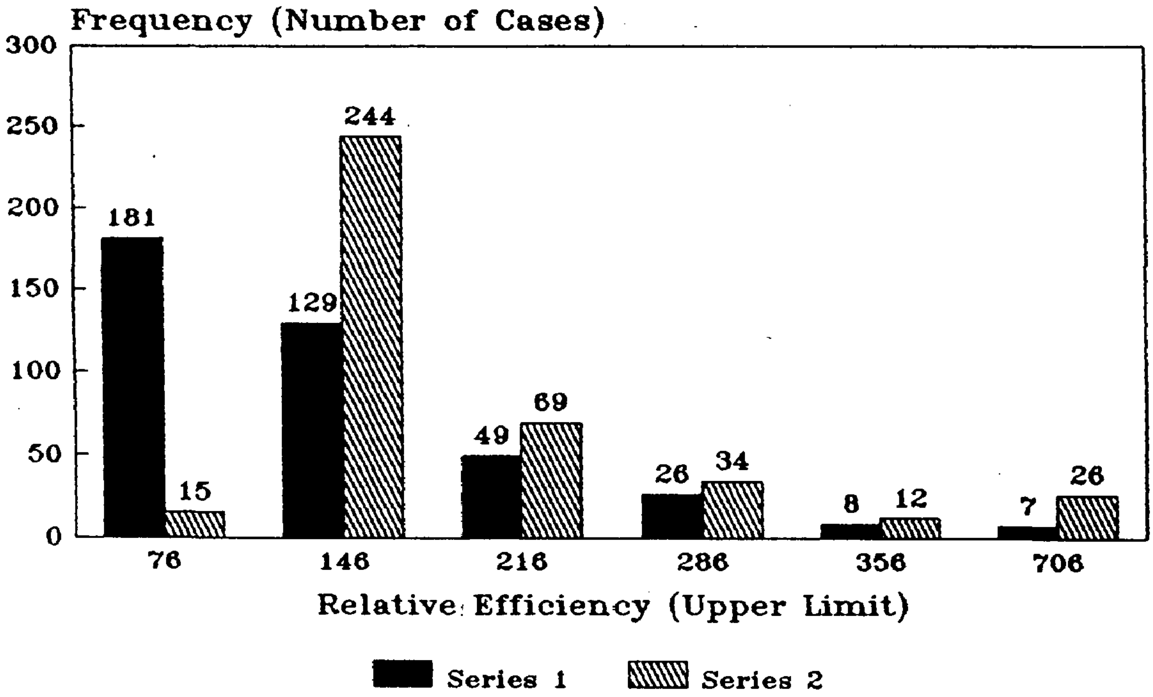
TABLE 3.2

RF(∘) FOR $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ and $\bar{Y}^{(2)}_{RP.ab}$

Estimators → / Sample Sizes ↓		$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_P$	$\bar{Y}^{(2)}_{RP.ab}$
$\rho > 0$	n=10	0.152	0.103	-	0.745
	n=20	0.092	0.113	-	0.795
$\rho < 0$	n=10	0.307	-	0.305	0.388
	n=20	0.197	-	0.188	0.615

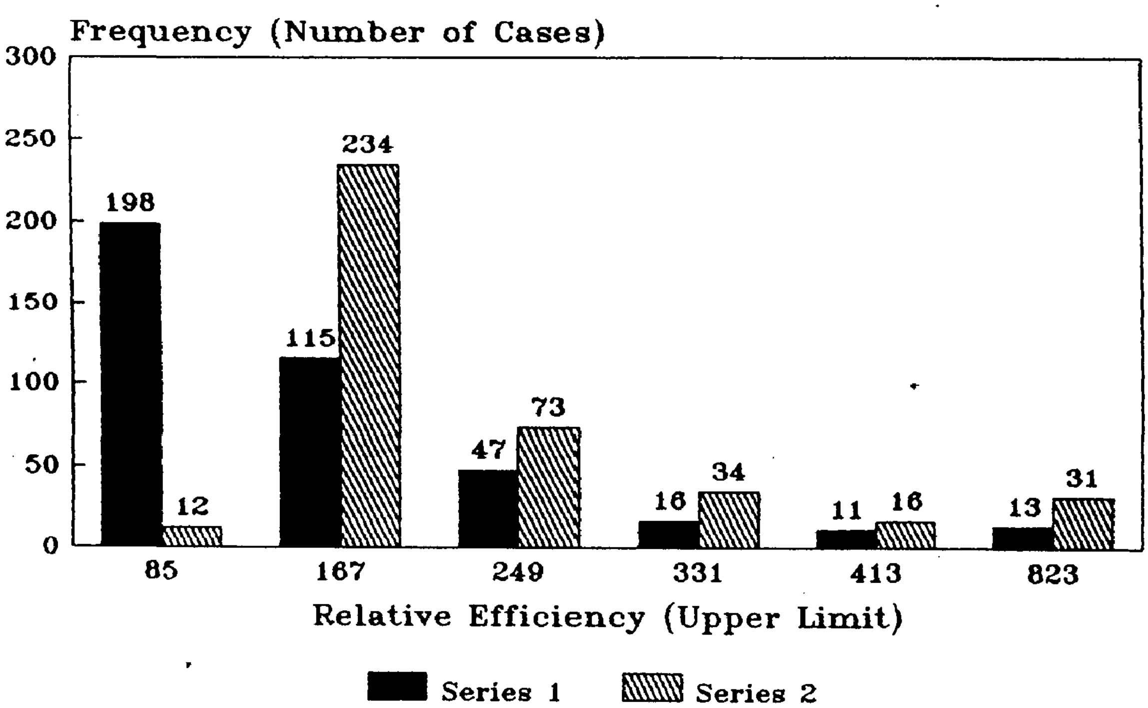
From this table, we can infer that $\bar{Y}^{(2)}_{RP.ab}$ is most probably a better choice than $\bar{y}$ and $\bar{Y}_R/\bar{Y}_P$. The finer comparisons based on G-values of these estimators reveal that when $\rho > 0$ and $G \in [0,1]$, $\bar{Y}^{(2)}_{RP.ab}$ performs exceptionally better than $\bar{y}$ and $\bar{Y}_R$. It also comes out to be winner much more often when G lies between -1 and 0.

Series 1 for $\overline{Y}_R$ and Series 2 for $\overline{Y}^{(2)}_{RP.ab}$



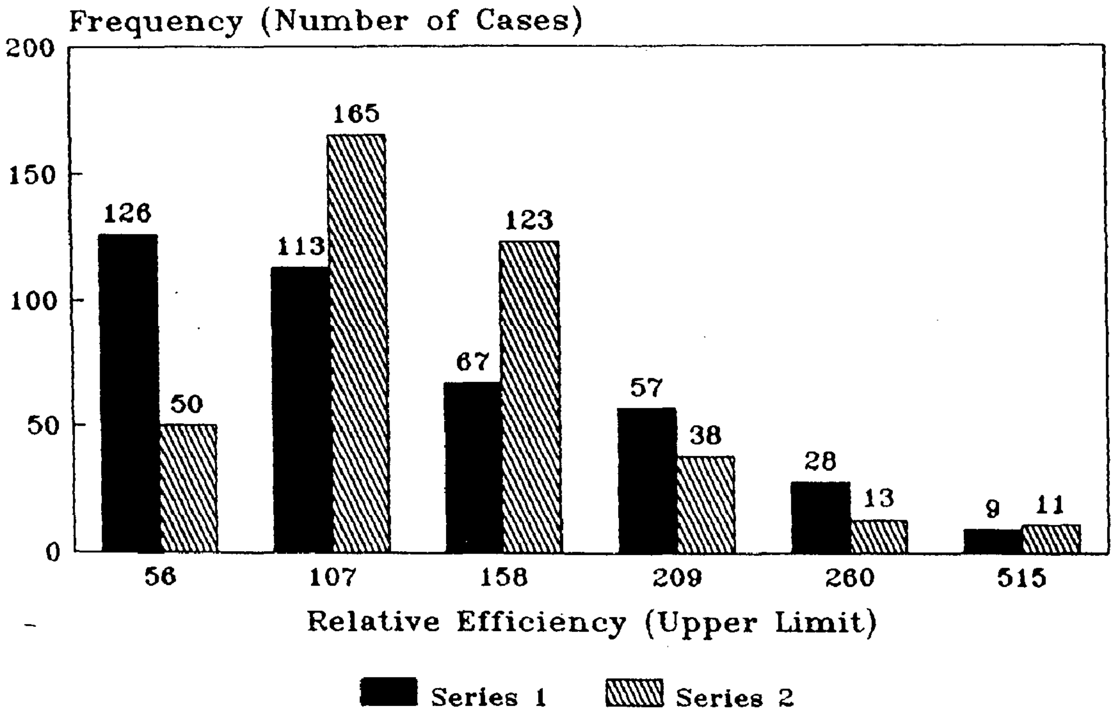
Graph - 3.5 (Sample Size = 10, $\rho > 0$)

Series 1 for $\overline{Y}_R$ and Series 2 for $\overline{Y}^{(2)}_{RP.ab}$



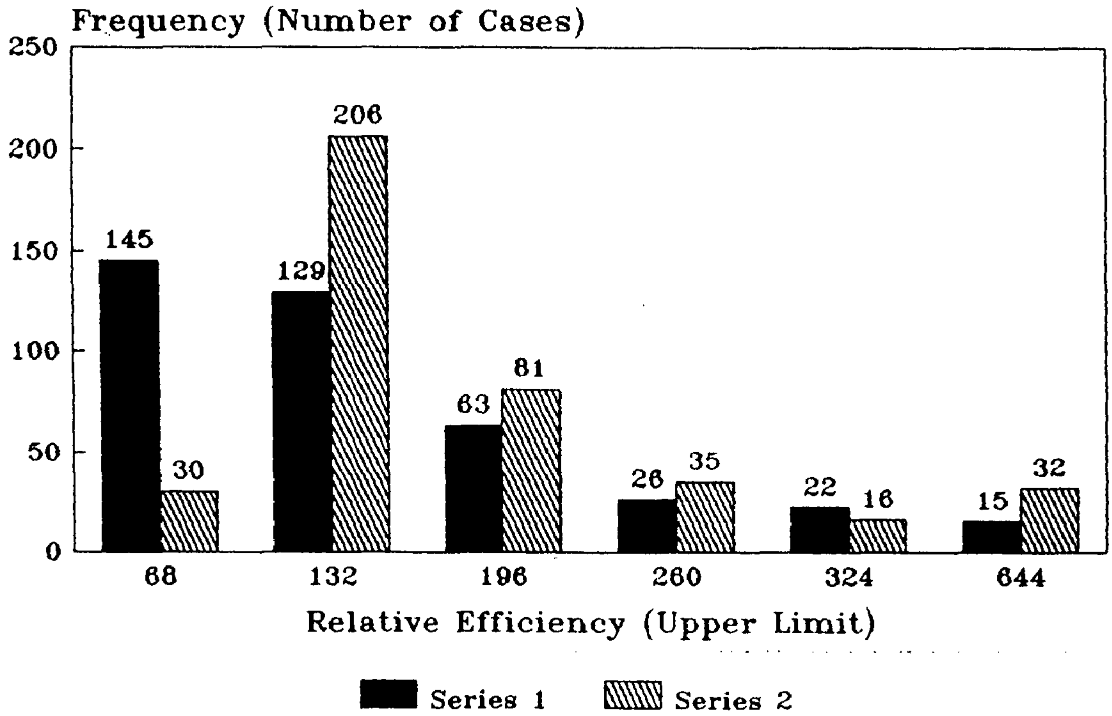
Graph - 3.6 (Sample Size = 20, $\rho > 0$)

Series 1 for $\overline{Y}_P$ and Series 2 for $\overline{Y}^{(2)}_{RP.ab}$



Graph - 3.7 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_P$ and Series 2 for $\overline{Y}^{(2)}_{RP.ab}$



Graph - 3.8 (Sample Size = 20, $\rho < 0$)

CHAPTER - 4

GAINFUL MIXINGS OF ONE OF THE ESTIMATORS WITH MEAN-PER-UNIT ESTIMATOR

Vos(1980) studied some mixing-type estimators for the efficient estimation of population mean using information via observation on an auxiliary variable. The paper considered some of the estimators obtained by mixing $\bar{y}$ with $\bar{Y}_R$ and $\bar{Y}_P$. In fact, all the estimators proposed and studied by us are nothing but the mixings of $\bar{y}$, $\bar{x}$ and $\bar{X}$ with one or two design-parameters. Motivated by the work of Vos(1980), we have proposed some gainful mixings of the various estimators proposed by us in Chapter-2 and Chapter-3 with $\bar{y}$. The analytical expressions to obtain the optimal values of mixing-parameters become very intricate and sometimes it becomes very difficult or even impossible to get the optimal values of the new design-parameters for these mixings. This fact has lead us to propose a new method of mixing of the estimators based on their performances via the empirical-simulation studies carried out in Chapter-2 and Chapter-3.

We suggest that the weight(s) for mixing estimator(s) t_n with $\bar{y}$ can be decided by the relative frequencies of the respective estimator(s) when compared with each other in the presence of

$\bar{Y}_R/\bar{Y}_P$. In the present chapter, we have tried this type of mixing of one of the different estimators and our empirical-simulation studies have revealed that the proposed mixing-estimators perform better than their parent estimators, quite often.

4.1 MIXINGS OF ONE-PARAMETER FAMILIES OF ESTIMATORS :

The mixing-estimators of $\bar{Y}_{MR}$ and $\bar{Y}_{MP}$, respectively when mixed with $\bar{y}$, are proposed to be :

$$MEST(1) = b_1.\bar{y} + (1-b_1).\ \bar{Y}_{MR} \qquad \ldots(4.1)$$

and $$MEST(2) = b_2.\bar{y} + (1-b_2).\ \bar{Y}_{MP} \qquad \ldots(4.2)$$

where, b_1 and b_2 are the respective design-parameters for mixing or the mixing-parameters. The values of b_1 and b_2 as per the simulation study carried out in chapter-2 are, respectively,

$$b_1 = 0.36;\ b_2 = 0.30 \qquad \ldots(4.3)$$

Now, we compare MEST(1)/MEST(2) with their parent-estimators (i.e., $\bar{Y}_{MR}/\bar{Y}_{MP}$). Table 4.1 contains the results of these comparisons and the graphs 4.1 to 4.4 afford us a clearer view of the relative efficiencies of these estimators.

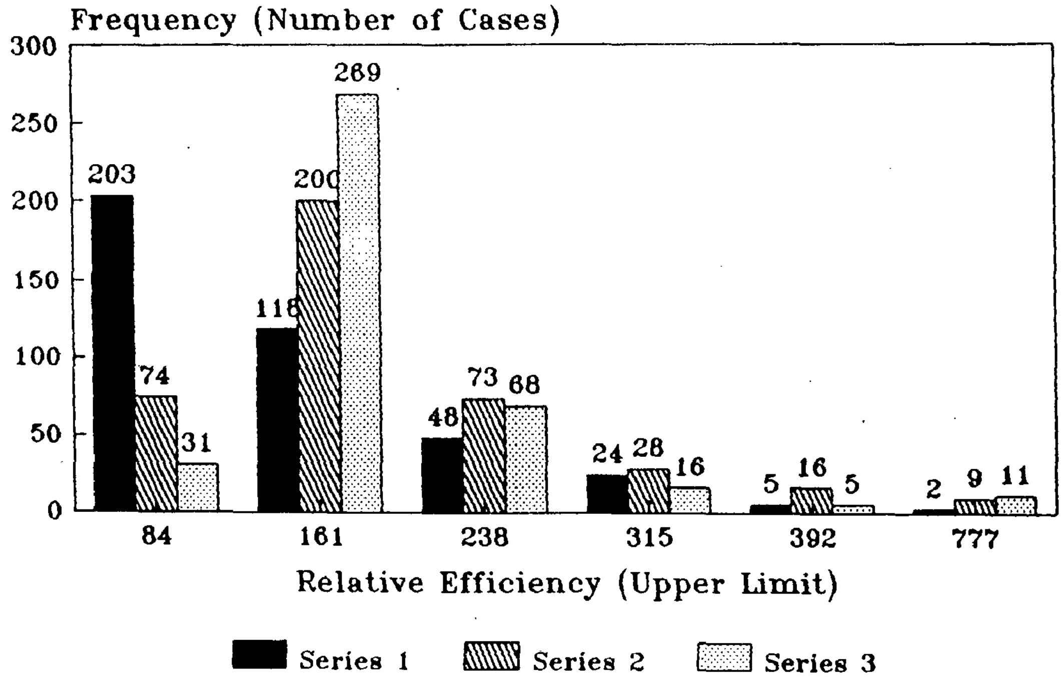
TABLE 4.1

RF(∘) FOR $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$, $\bar{Y}_{MR}/\bar{Y}_{MP}$ and MEST(1)/MEST(2)

Estimators → Sample Sizes ↓		$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_P$	$\bar{Y}_{MR}$	$\bar{Y}_{MP}$	MEST(1)	MEST(2)
$\rho > 0$	n=10	0.197	0.210	-	0.293	-	0.300	-
	n=20	0.165	0.275	-	0.282	-	0.278	-
$\rho < 0$	n=10	0.152	-	0.192	-	0.318	-	0.338
	n=20	0.142	-	0.240	-	0.298	-	0.320

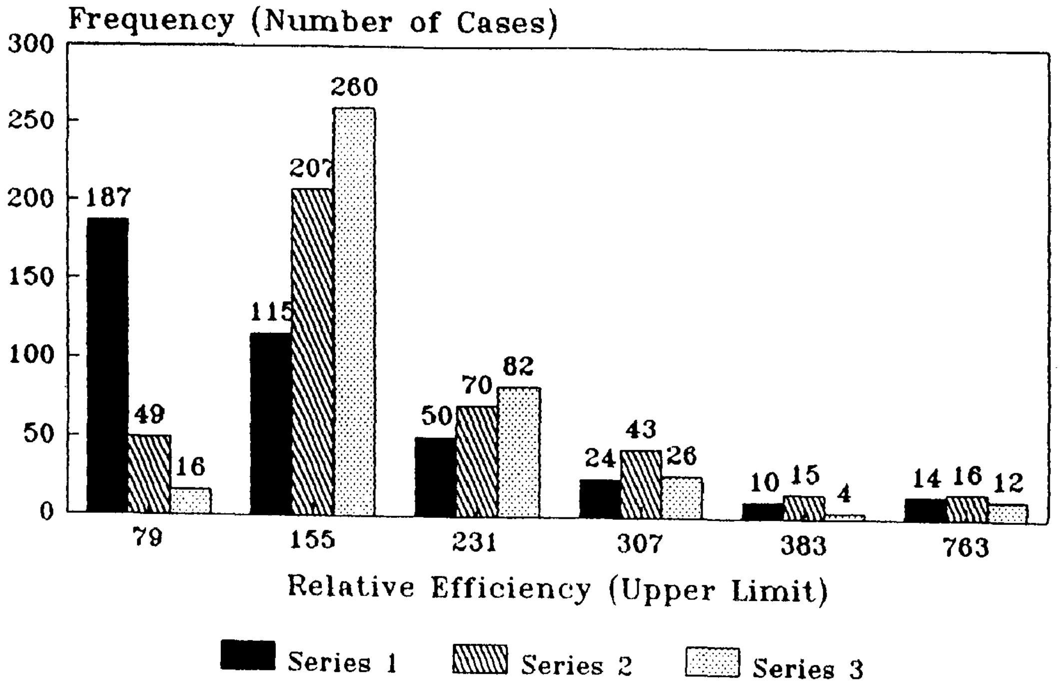
From this table, we can say that MEST(1)/MEST(2) and $\bar{Y}_{MR}/\bar{Y}_{MP}$ are quite close to each other as per the results of their comparisons.

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{MR}$ and
Series 3 for MEST(1)



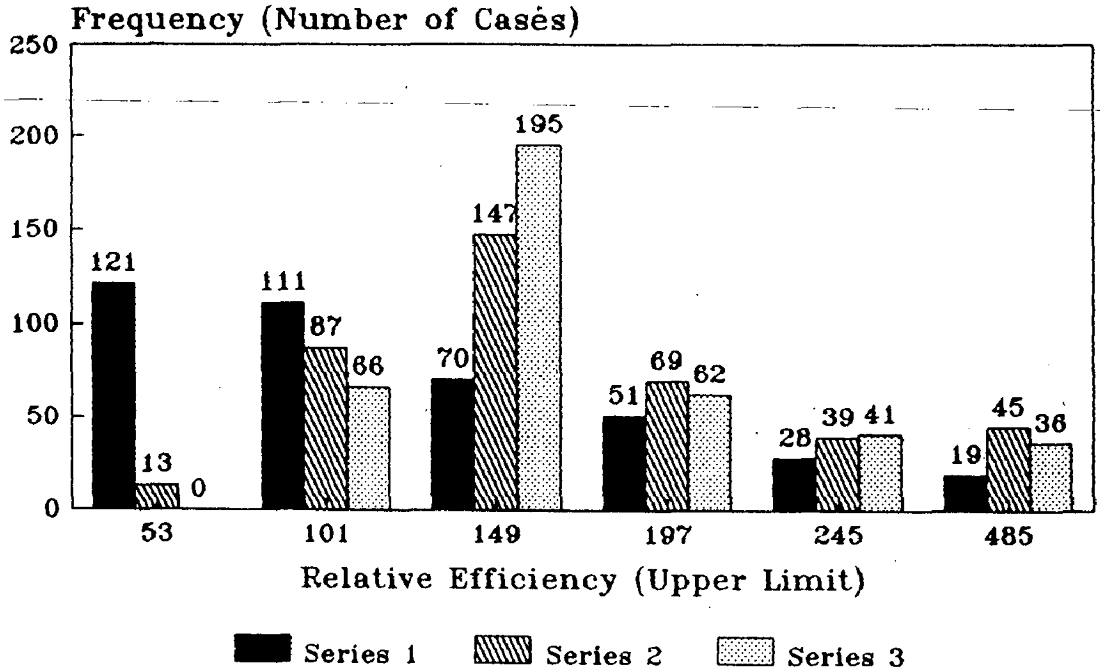
Graph – 4.1 (Sample Size = 10, $\rho > 0$)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{MR}$ and
Series 3 for MEST(1)



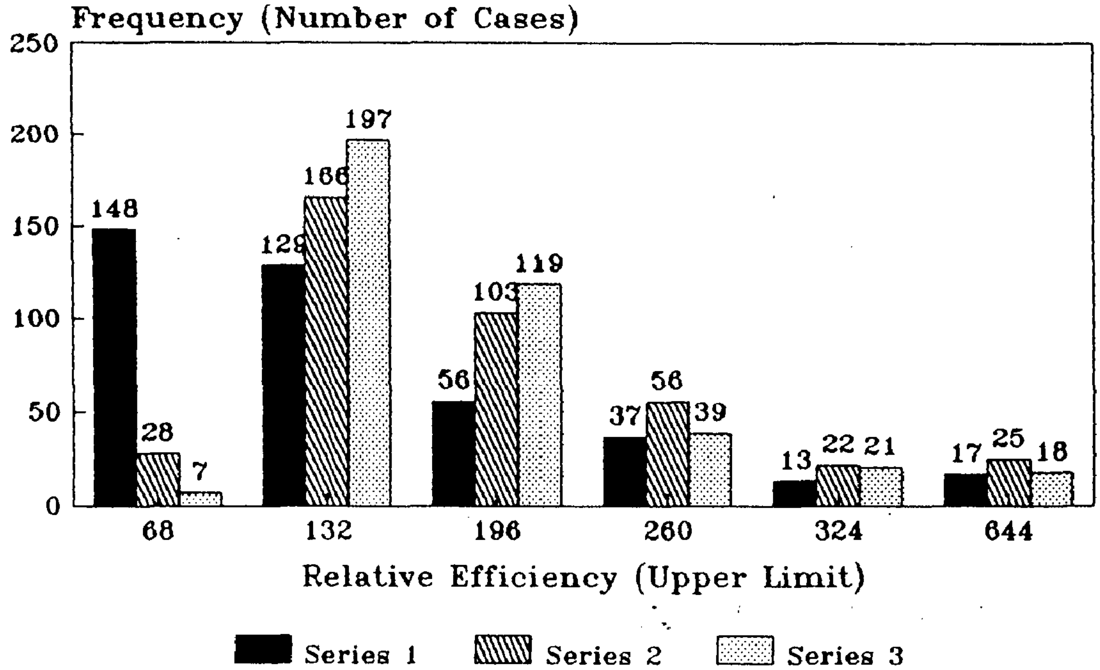
Graph – 4.2 (Sample Size = 20, $\rho > 0$)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{MP}$ and Series 3 for MEST(2)



Graph – 4.3 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{MP}$ and Series 3 for MEST(2)



Graph – 4.4 (Sample Size = 10, $\rho > 0$)

The finer comparisons based on G-values reveal, however, that MEST(1) comes out to be winner oftener (when compared with $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_{MR}$) when $G \in [0,0.5]$. Also, MEST(2) has greater relative frequency of winning than $\bar{Y}_{MP}$ when $G \in [-0.5,0]$.

The mixings of $\bar{y}$ with $\bar{Y}_{VR}$ and $\bar{Y}_{VP}$ are proposed below.

$$MEST(3) = b_3 . \bar{y} + (1-b_3) . \bar{Y}_{VR} \qquad \ldots(4.4)$$

and $$MEST(4) = b_4 . \bar{y} + (1-b_4) . \bar{Y}_{VP} \qquad \ldots(4.5)$$

where, $b_3 = 0.31$ and $b_4 = 0.45$ as per the results of earlier empirical-simulation study. In table 4.2 we tabulate the results of the comparisons of MEST(3)/MEST(4) with $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ and $\bar{Y}_{VR}/\bar{Y}_{VP}$. Graphical display of the relative efficiencies of these estimators being the winner is being given per graphs 4.5 to 4.8.

TABLE 4.2

RF(∘) FOR $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$, $\bar{Y}_{VR}/\bar{Y}_{VP}$ and MEST(3)/MEST(4)

Estimators → Sample Sizes ↓		$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_P$	$\bar{Y}_{VR}$	$\bar{Y}_{VP}$	MEST(3)	MEST(4)
$\rho > 0$	n=10	0.135	0.227	-	0.290	-	0.348	-
	n=20	0.133	0.277	-	0.290	-	0.300	-
$\rho < 0$	n=10	0.252	-	0.238	-	0.245	-	0.265
	n=20	0.140	-	0.275	-	0.312	-	0.273

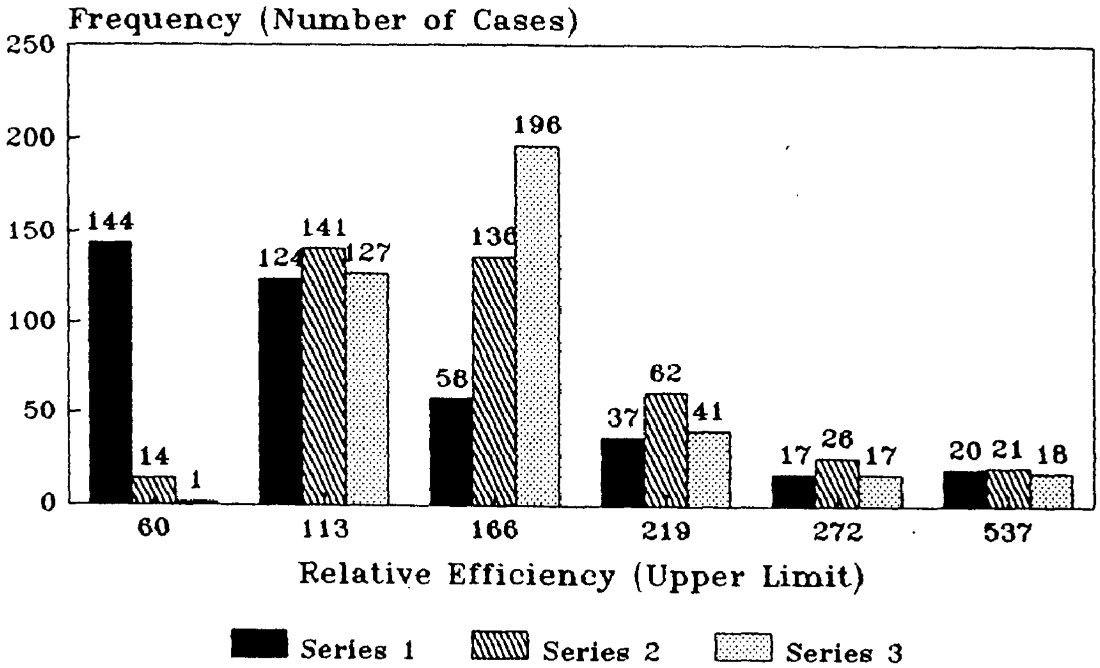
Here, MEST(3) and MEST(4) have thus provided improvement over $\bar{Y}_{VR}$ and $\bar{Y}_{VP}$ except when $\rho < 0$ and n=20. We have also observed that the relative frequency of MEST(3) being the winner in the comparisons comes out to be more, quite often when $G \in [0, 0.5]$.

We now propose this type of mixing estimator for $\bar{Y}_{P.a}$, as below.

$$MEST(5) = b_5 . \bar{y} + (1-b_5) . \bar{Y}_{P.a} \qquad \ldots(4.6)$$

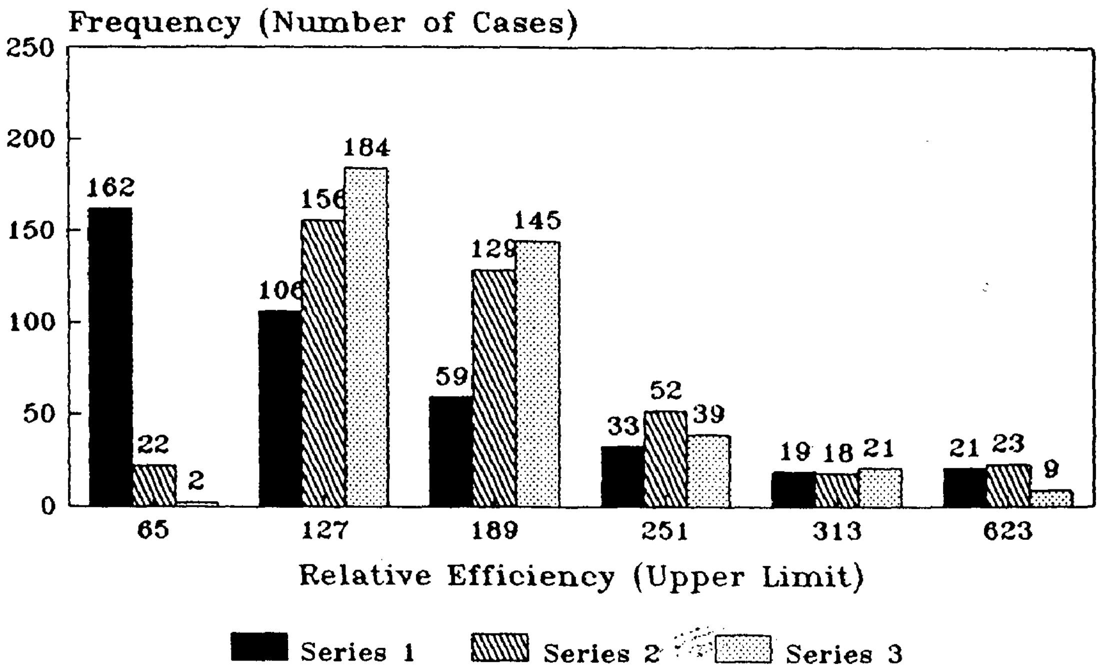
where, $b_5 = 0.09$ as per the earlier empirical-simulation study.

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{VR}$ and
Series 3 for MEST(3)



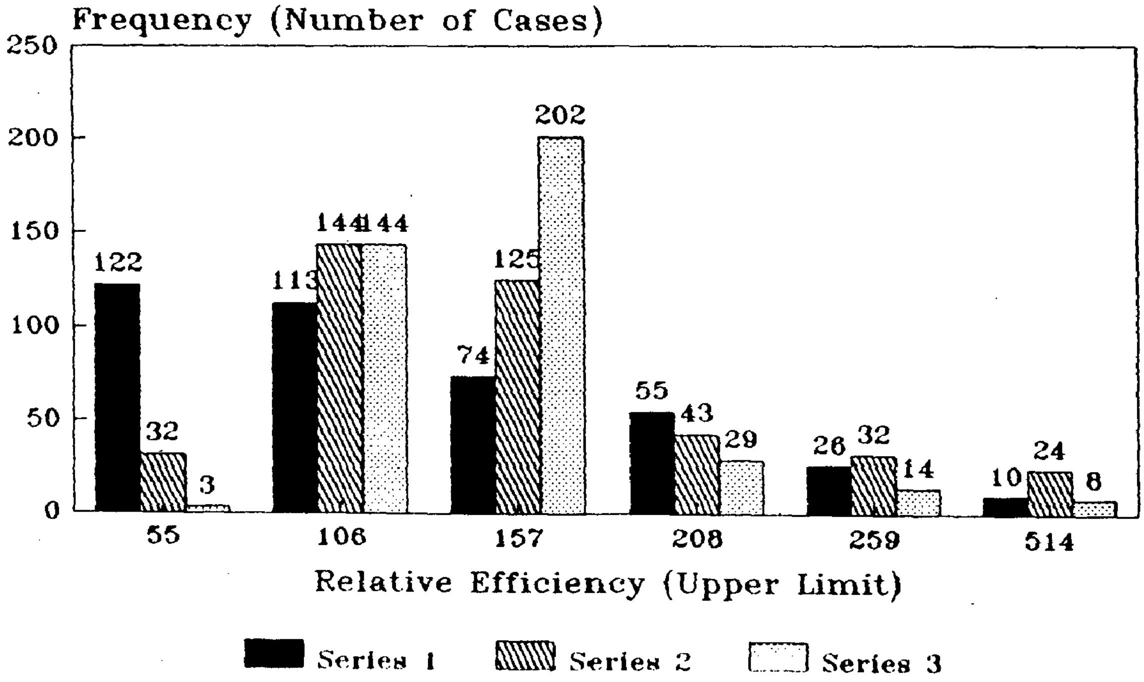
Graph - 4.5 (Sample Size = 10, $\rho > 0$)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{VR}$ and
Series 3 for MEST(3)



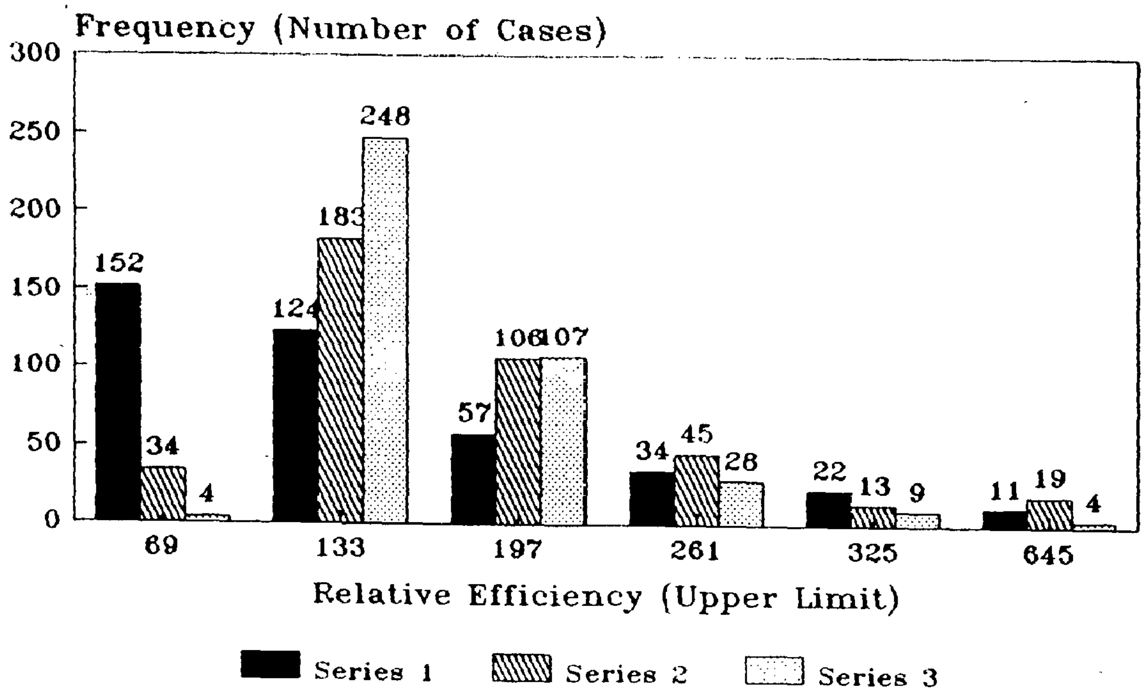
Graph - 4.6 (Sample Size = 20, $\rho > 0$)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{VP}$ and Series 3 for MEST(4)



Graph – 4.7 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{VP}$ and Series 3 for MEST(4)



Graph – 4.8 (Sample Size = 20, $\rho < 0$)

Here, we compare this estimator with $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ and $\bar{Y}_{P.a}$. Table 4.3 contains the results of the empirical-simulation study carried out for this purpose and the graphs 4.9 to 4.12 provide the graphical display of the relative efficiencies of these estimators.

TABLE 4.3

RF(∘) FOR $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$, $\bar{Y}_{P.a}$ and MEST(5)

Estimators → / Sample Sizes ↓		$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_P$	$\bar{Y}_{P.a}$	MEST(5)
$\rho > 0$	n=10	0.197	0.335	-	0.163	0.305
	n=20	0.177	0.355	-	0.218	0.250
$\rho < 0$	n=10	0.065	-	0.062	0.308	0.565
	n=20	0.052	-	0.053	0.420	0.475

It is clear from the above table that MEST(5) is an improvement over $\bar{Y}_{P.a}$. However, when the correlation between Y and X is positive, ratio estimator dominates over both $\bar{Y}_{P.a}$ and MEST(5). Here, we observe that the product-type nature of $\bar{Y}_{P.a}$ surfaces and its mixing with $\bar{y}$ comes out to be winner more often when $\rho < 0$. The finer comparisons of MEST(5) with the other estimators in competition reveal that it is most probably the best choice when G $\in [-1,0]$.

In what follows, we define the same type of mixings for $\bar{Y}_{Sr.}$, $\bar{Y}_{Re.}$ and $\bar{Y}_{Sa.}$.

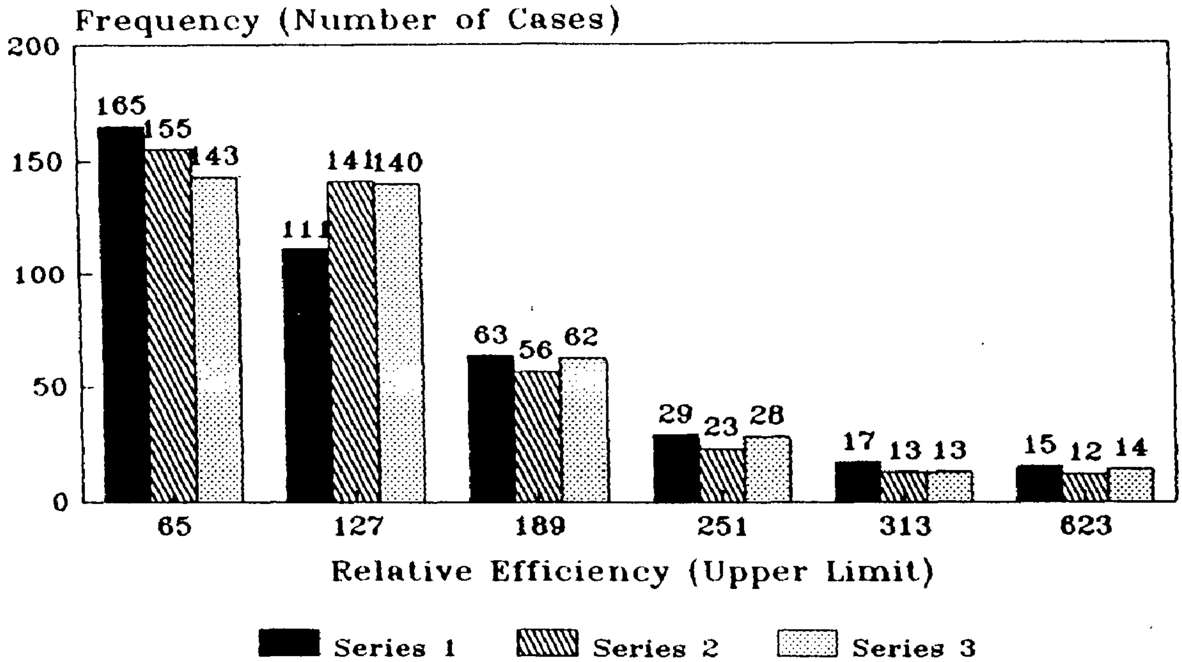
$$\text{MEST}(6) = b_6.\bar{y} + (1-b_6).\bar{Y}_{Sr.} \qquad ...(4.7)$$

$$\text{MEST}(7) = b_7.\bar{y} + (1-b_7).\bar{Y}_{Re.} \qquad ...(4.8)$$

and $$\text{MEST}(8) = b_8.\bar{y} + (1-b_8).\bar{Y}_{Sa.} \qquad ...(4.9)$$

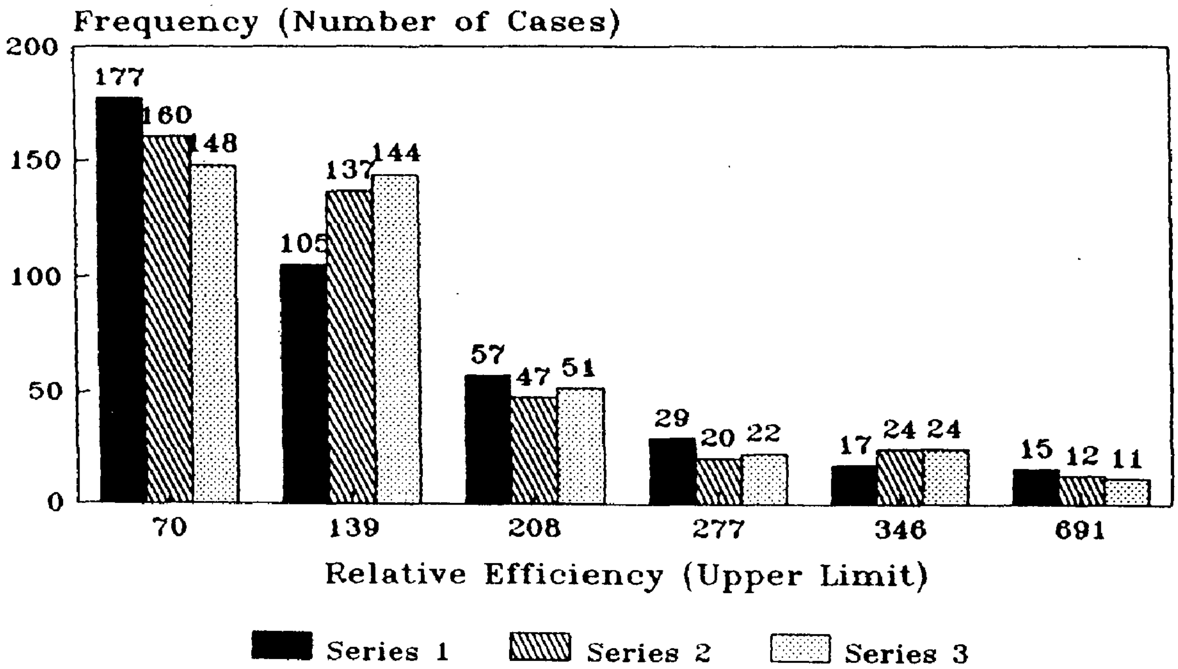
where, b_6=0.14, b_7=0.10 and b_8=0.14 as per our earlier empirical-simulation studies. The empirical-simulation study of

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{P.a}$ and Series 3 for MEST(5)



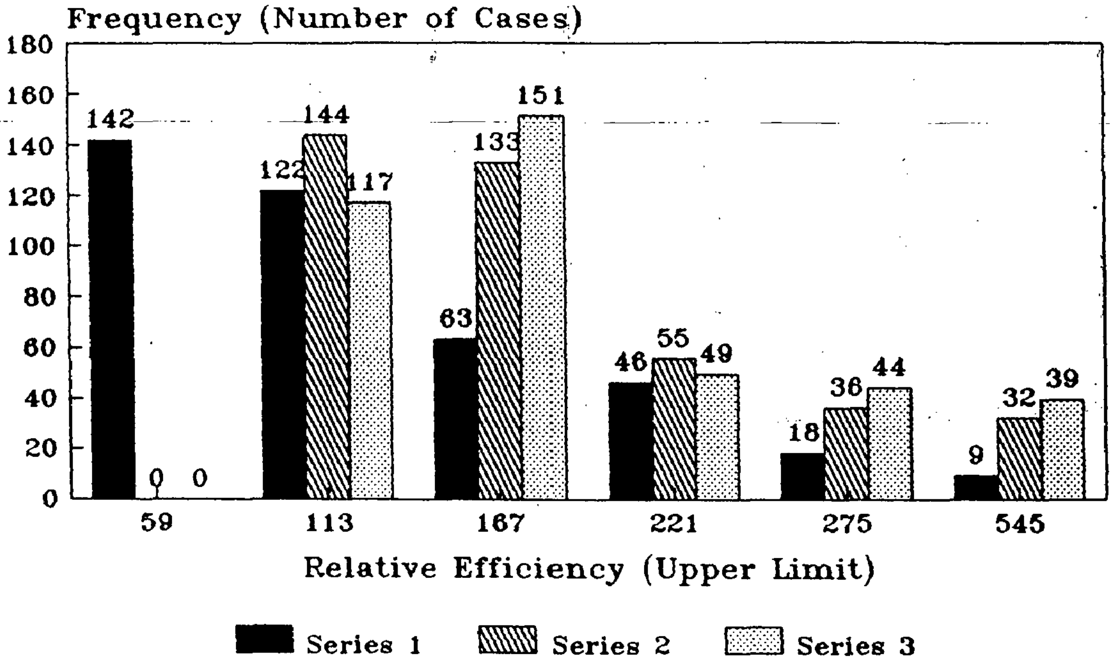
Graph - 4.9 (Sample Size = 10, $\rho > 0$)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{P.a}$ and Series 3 for MEST(5)



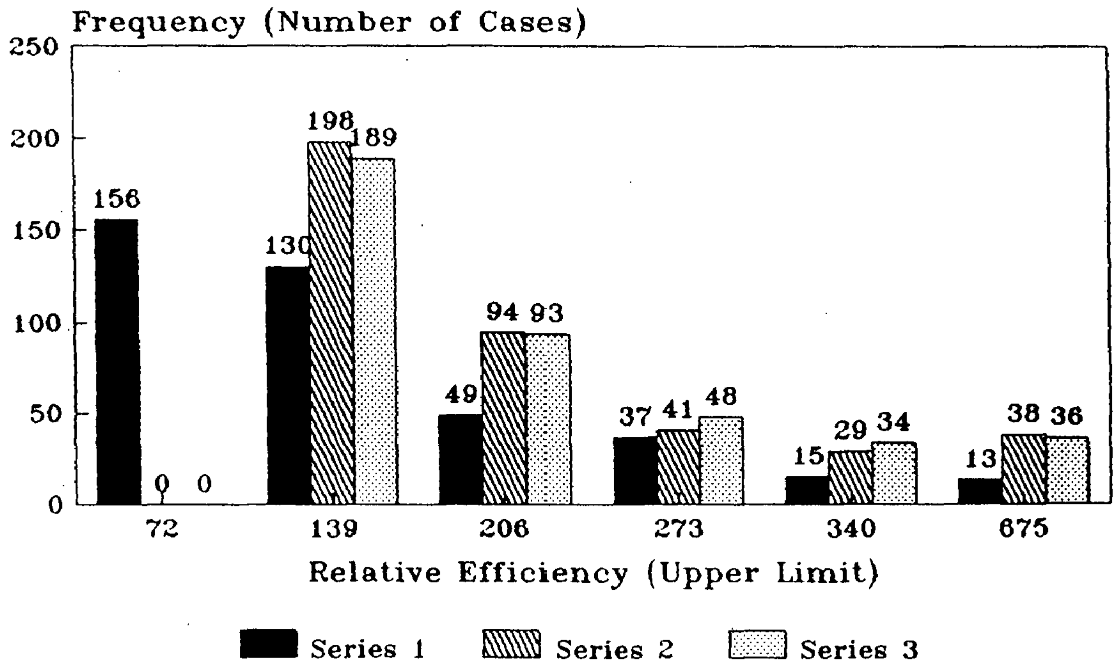
Graph - 4.10 (Sample Size = 20, $\rho > 0$)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{P.a}$ and
Series 3 for MEST(5)



Graph - 4.11 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{P.a}$ and
Series 3 for MEST(5)



Graph - 4.12 (Sample Size = 20, $\rho < 0$)

these estimators has been carried out for comparing them with their respective parent-estimators, i.e., $\bar{Y}_{Sr.}$, $\bar{Y}_{Re.}$ and $\bar{Y}_{Sa.}$ and the usual estimators $\bar{y}$ and $\bar{Y}_R/\bar{Y}_P$. The results of these comparisons have been tabulated in the tables 4.4 to 4.6 and the relative efficiencies of these estimators being the winners for the different comparisons are being displayed graphically through graphs 4.13 to 4.24.

TABLE 4.4

RF(∘) FOR $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$, $\bar{Y}_{Sr.}$ and MEST(6)

Estimators → Sample Sizes ↓		$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_P$	$\bar{Y}_{Sr.}$	MEST(6)
$\rho > 0$	n=10	0.190	0.095	-	0.225	0.490
	n=20	0.135	0.065	-	0.335	0.465
$\rho < 0$	n=10	0.035	-	0.065	0.192	0.708
	n=20	0.040	-	0.060	0.292	0.608

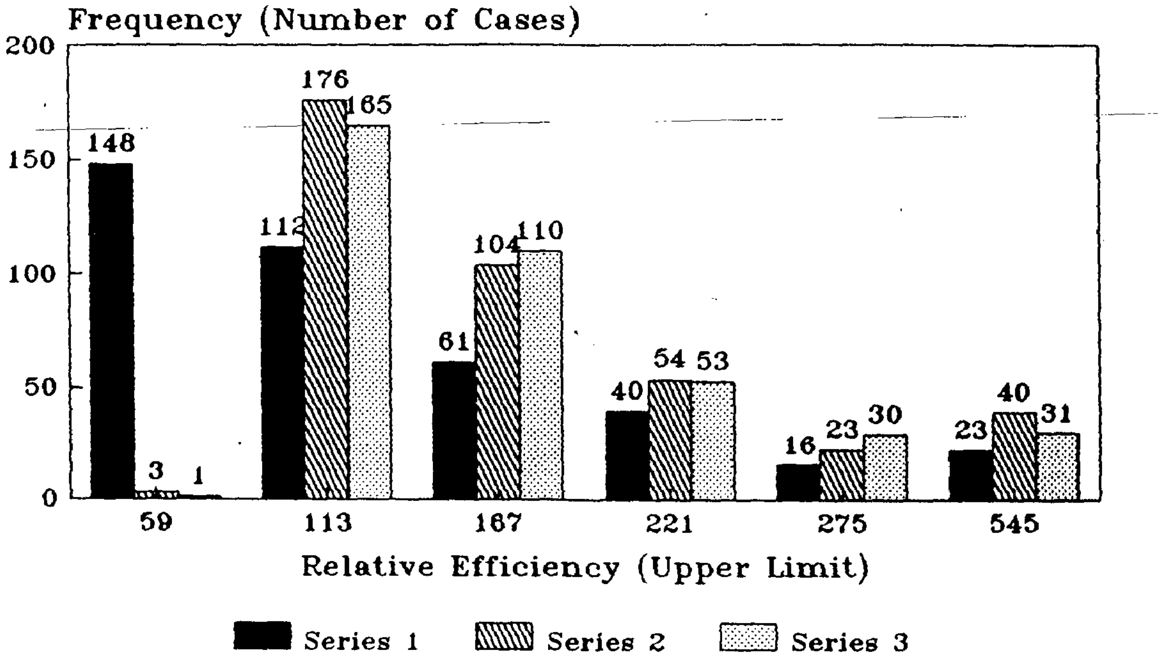
The above table indicates that we will gain considerably by the use of MEST(6) instead of using $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ and $\bar{Y}_{Sr.}$. The estimator MEST(6) performs better than the other estimators in the competition much more often for the entire range of G considered by us.

TABLE 4.5

RF(∘) FOR $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$, $\bar{Y}_{Re.}$ and MEST(7)

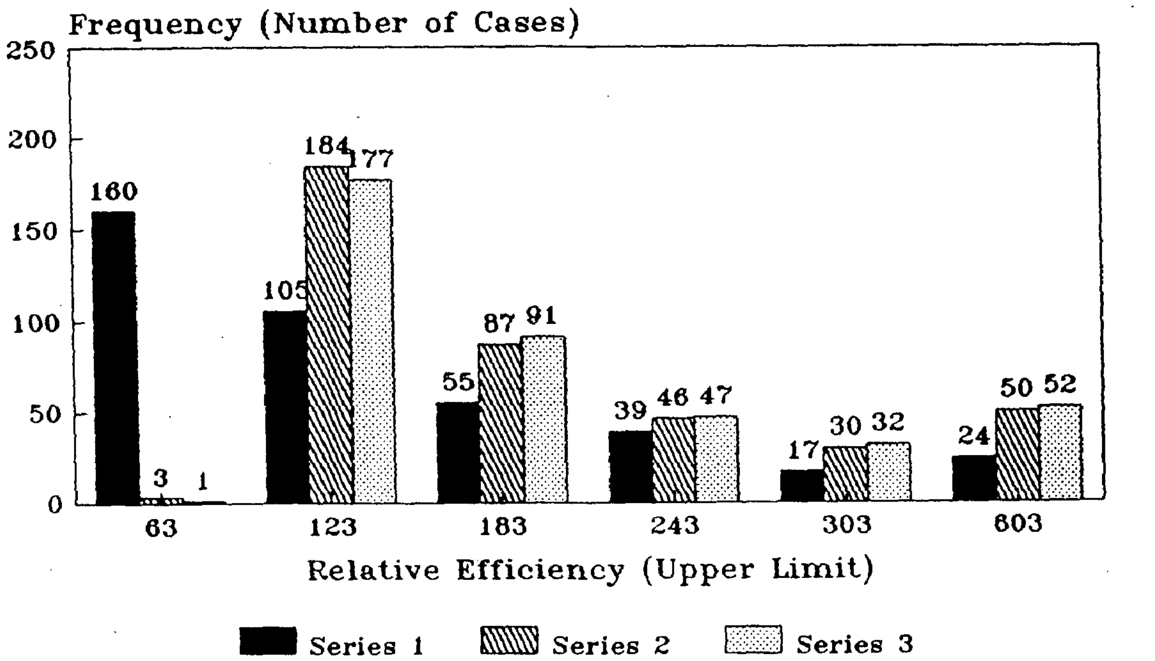
Estimators → Sample Sizes ↓		$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_P$	$\bar{Y}_{Re.}$	MEST(7)
$\rho > 0$	n=10	0.075	0.127	-	0.363	0.435
	n=20	0.057	0.108	-	0.392	0.443
$\rho < 0$	n=10	0.080	-	0.192	0.348	0.380
	n=20	0.050	-	0.125	0.417	0.408

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{Sr.}$ and
Series 3 for MEST(6)



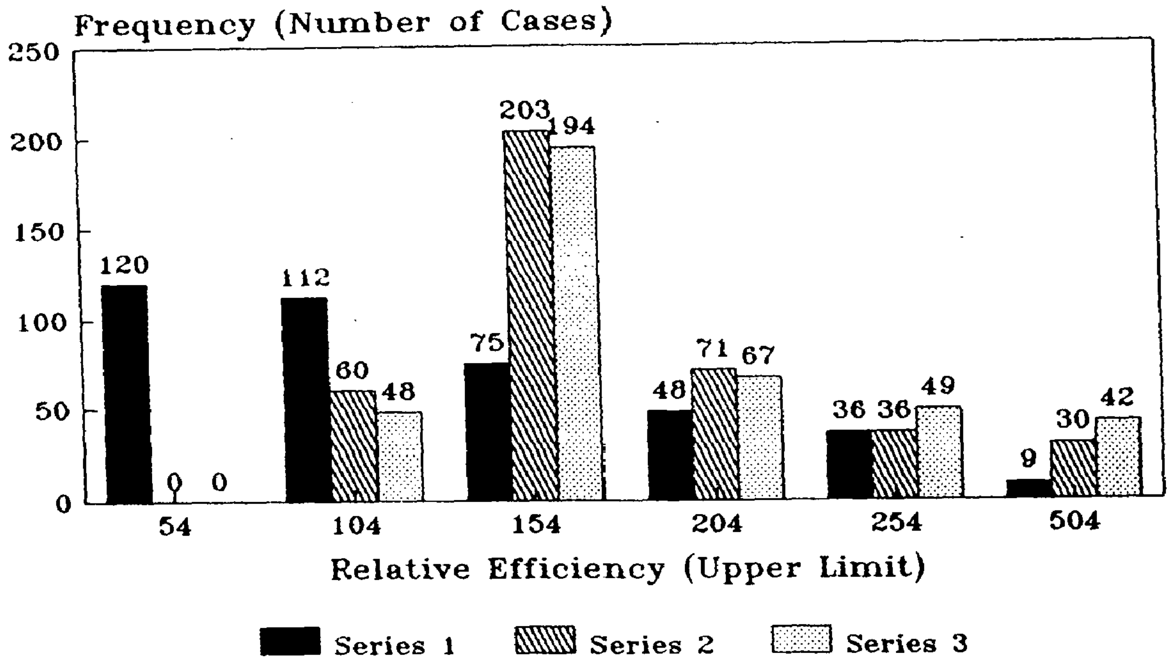
Graph – 4.13 (Sample Size = 10, $\rho > 0$)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{Sr.}$ and
Series 3 for MEST(6)



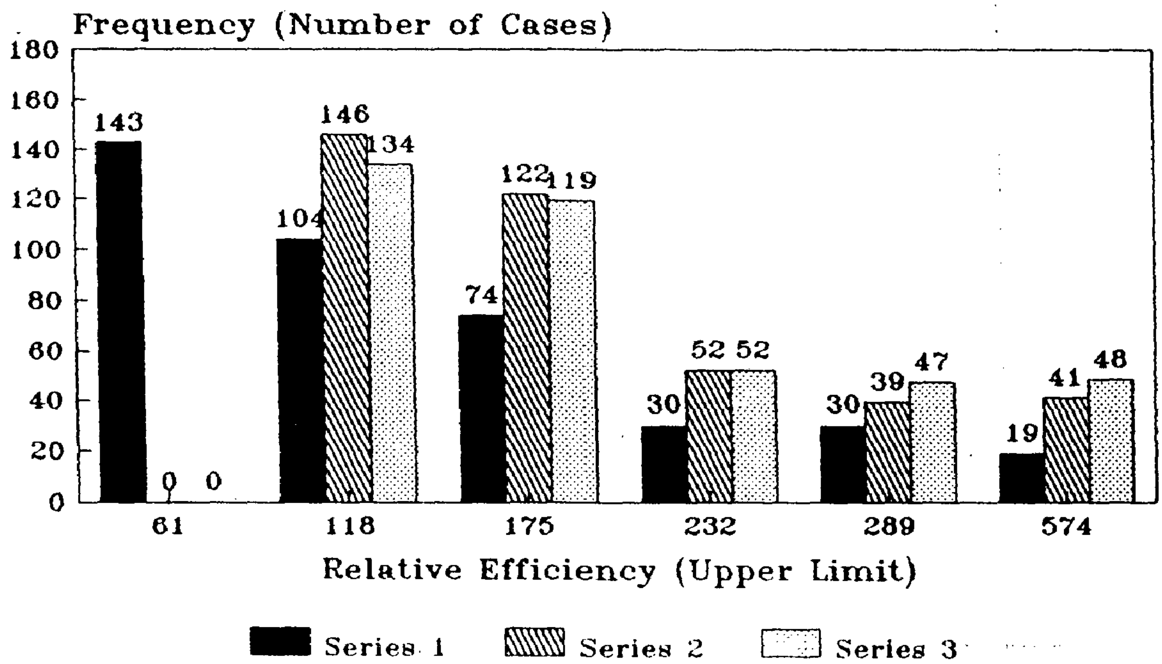
Graph – 4.14 (Sample Size = 20, $\rho > 0$)

Series 1 for $\overline{Y}_p$, Series 2 for $\overline{Y}_{Sr.}$ and
Series 3 for MEST(6)



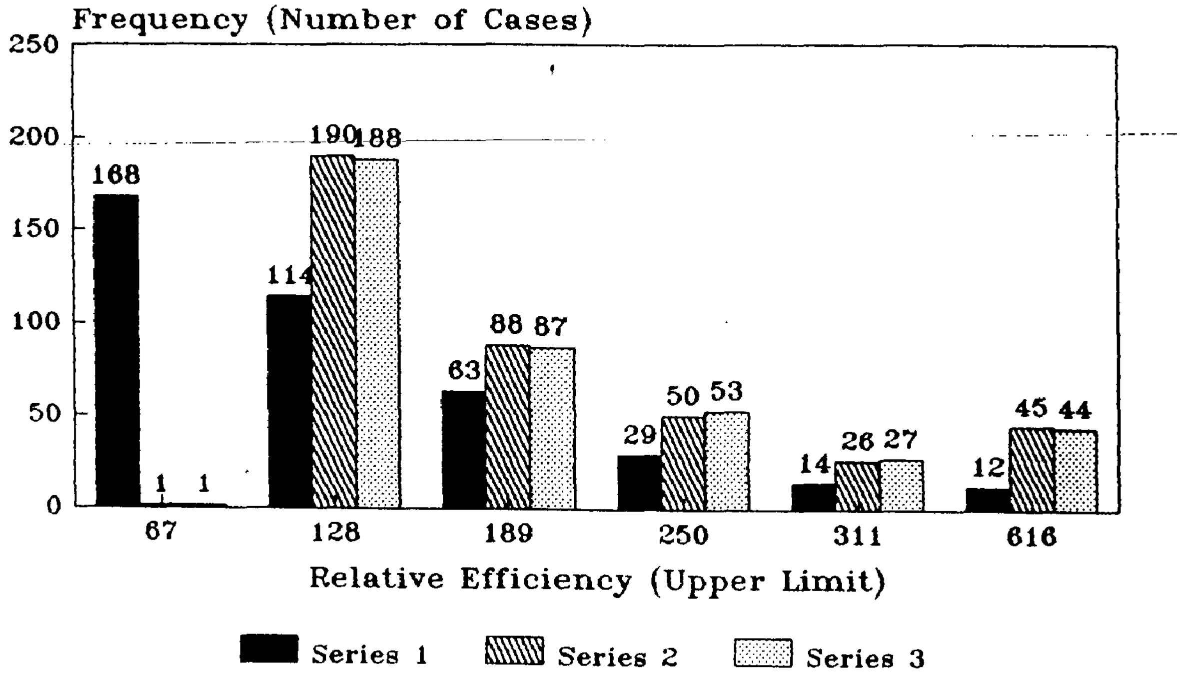
Graph – 4.15 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_p$, Series 2 for $\overline{Y}_{Sr.}$ and
Series 3 for MEST(6)



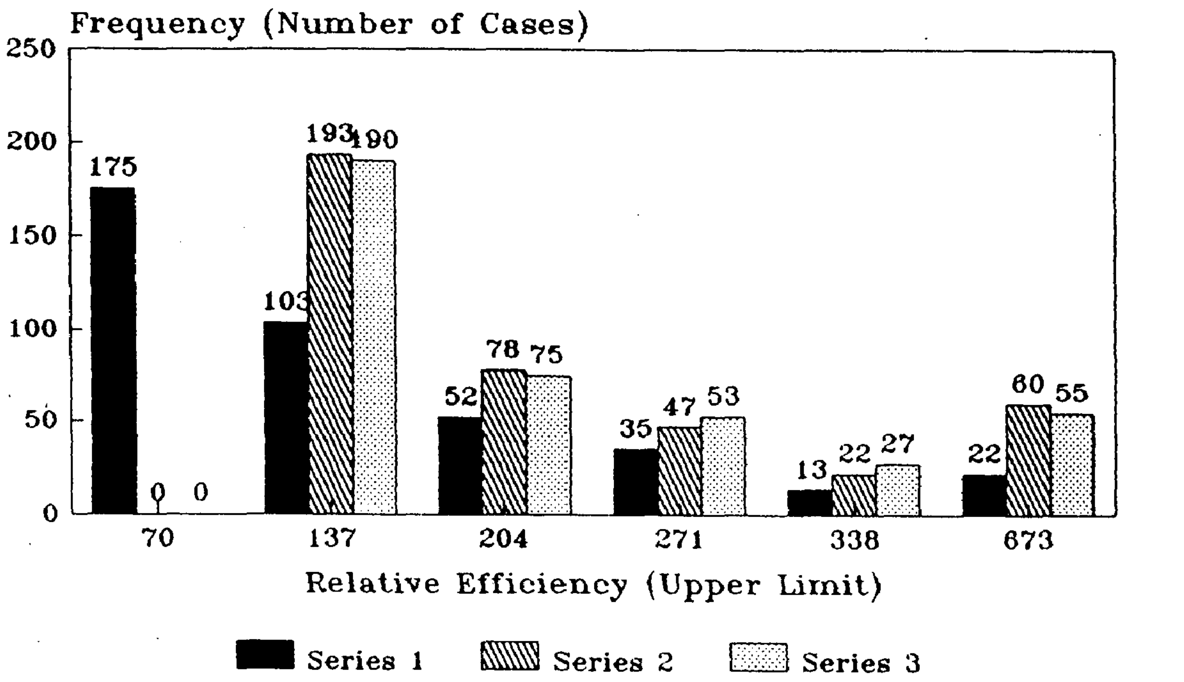
Graph – 4.16 (Sample Size = 20, $\rho < 0$)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{Re.}$ and Series 3 for MEST(7)



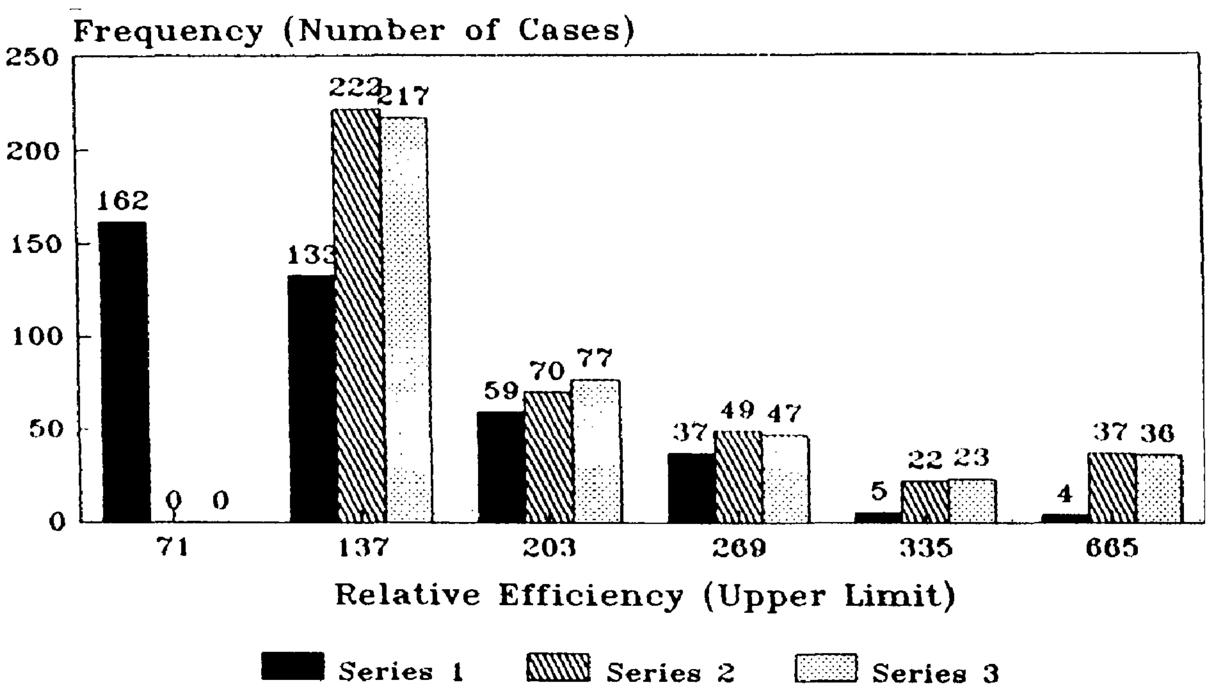
Graph – 4.17 (Sample Size = 10, $\rho > 0$)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{Re.}$ and Series 3 for MEST(7)



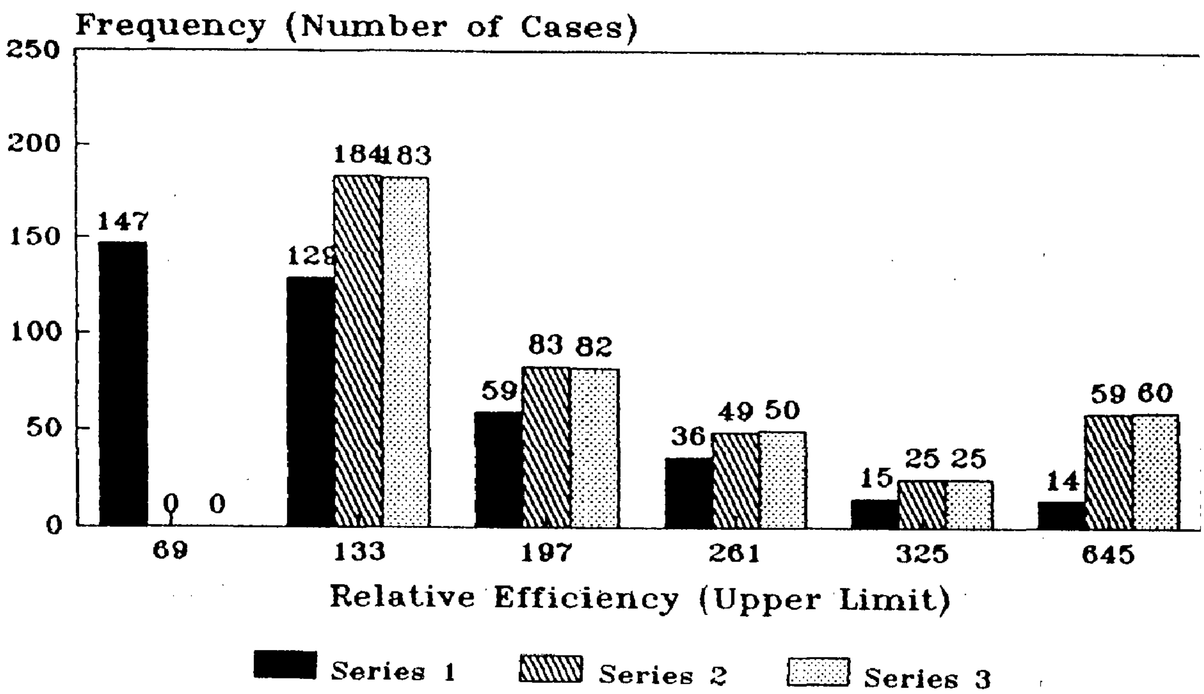
Graph – 4.18 (Sample Size = 20, $\rho > 0$)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{Re.}$ and
Series 3 for MEST(7)



Graph – 4.19 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{Re.}$ and
Series 3 for MEST(7)



Graph – 4.20 (Sample Size = 20, $\rho < 0$)

From the above table, we can infer that MEST(7) provides slight improvement over $\bar{Y}_{Re.}$ and the two estimators are quite close to each other in their performances as per the empirical-simulation study when $\rho < 0$. The finer comparisons of these estimators reveal that MEST(7) performs fairly better than $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_{Re.}$ when $G \in [0.5, 1]$. Its performance is also worth noticing as compared to $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}_{Re.}$ when G lies between -1 and -0.5.

TABLE 4.6

RF($\circ$) FOR $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$, $\bar{Y}_{Sa.}$ and MEST(8)

Estimators → / Sample Sizes ↓		$\bar{y}$	$\bar{Y}_R$	$\bar{y}_P$	$\bar{Y}_{Sa.}$	MEST(8)
$\rho > 0$	n=10	0.172	0.080	-	0.290	0.458
	n=20	0.097	0.085	-	0.358	0.460
$\rho < 0$	n=10	0.085	-	0.070	0.217	0.628
	n=20	0.032	-	0.070	0.348	0.550

The table 4.6 shows that we will be most probably a gainer in the relative efficiency if we use MEST(8) instead of $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ and $\bar{Y}_{Sa.}$ The estimator MEST(8) performs better than the other estimators much more often for the entire range of G under consideration except when G lies between 0.5 and 1, when $\bar{Y}_{Sa.}$ is quite a close competitor.

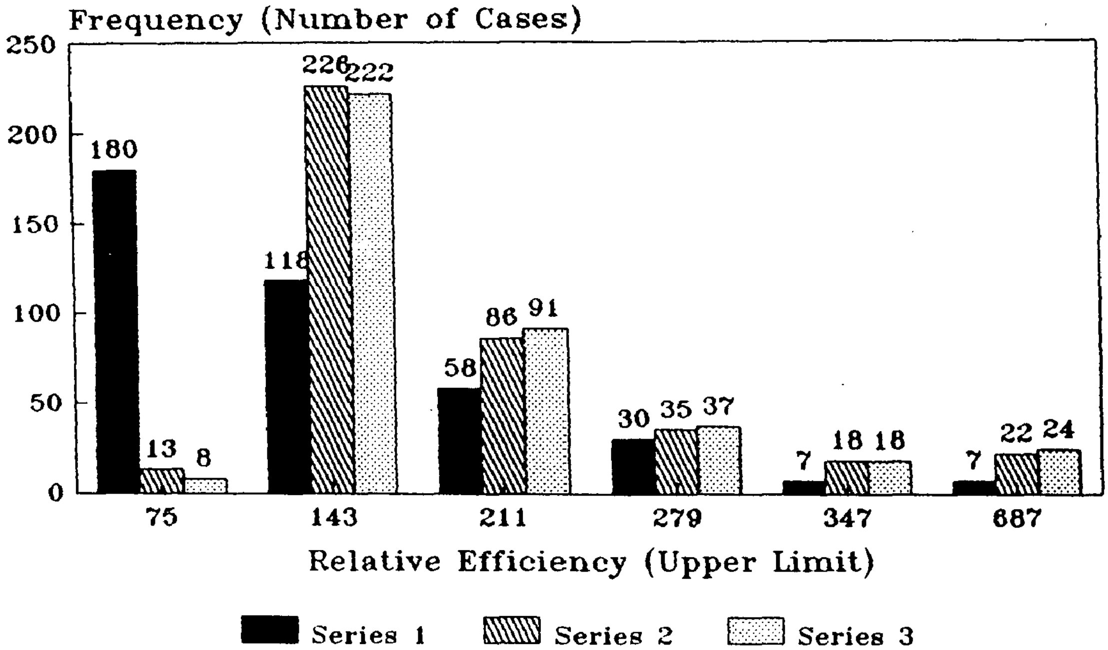
Now, we propose the same type of mixing estimator for $\bar{Y}_{VSa.}$ to be :

$$\text{MEST}(9) = b_9.\bar{y} + (1-b_9).\ \bar{Y}_{VSa.} \qquad \ldots(4.10)$$

where, b_9=0.08. We have compared this estimator with $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ and $\bar{Y}_{VSa.}$

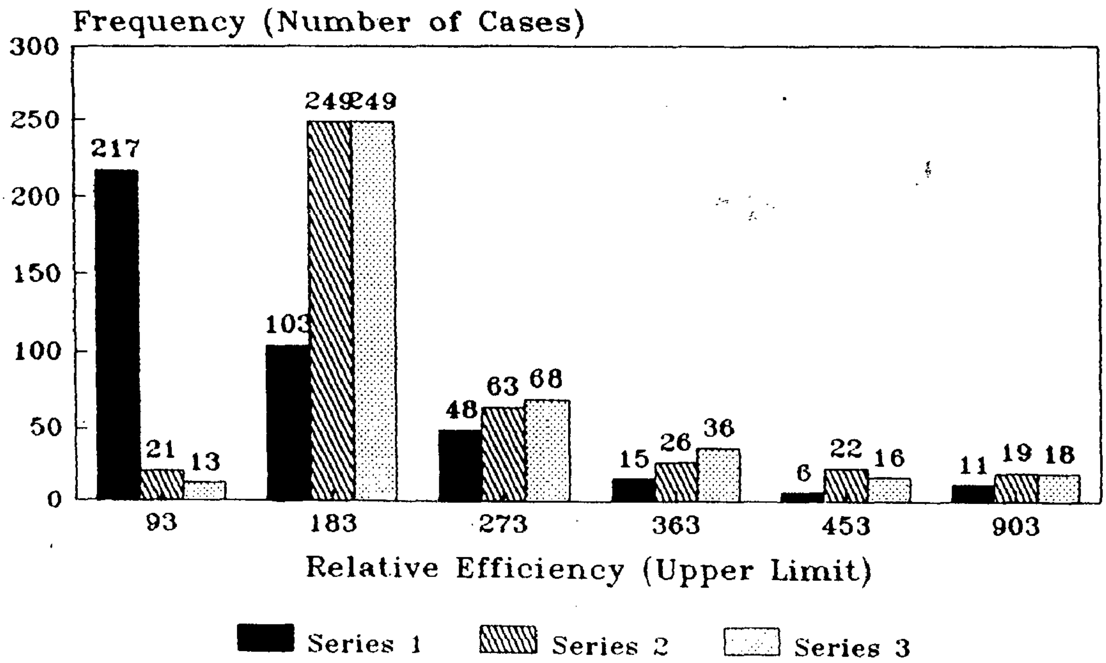
The results of these comparisons via the empirical-simulation study are tabulated below in table 4.7 and the graphical display

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{Sa}$. and
Series 3 for MEST(8)



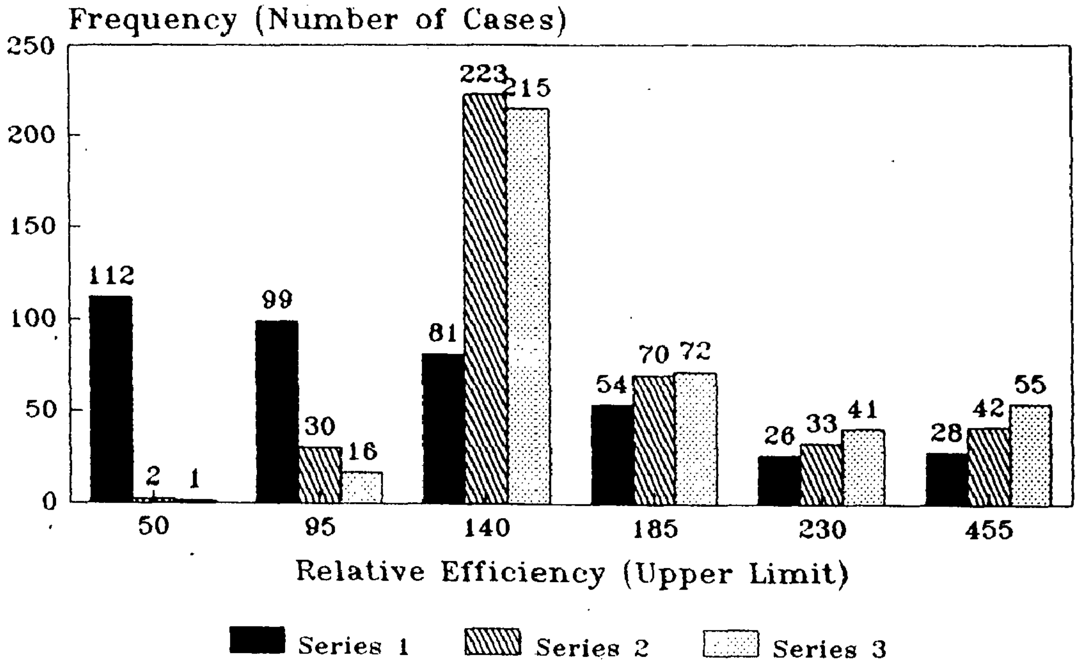
Graph - 4.21 (Sample Size = 10, $\rho > 0$)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{Sa}$. and
Series 3 for MEST(8)



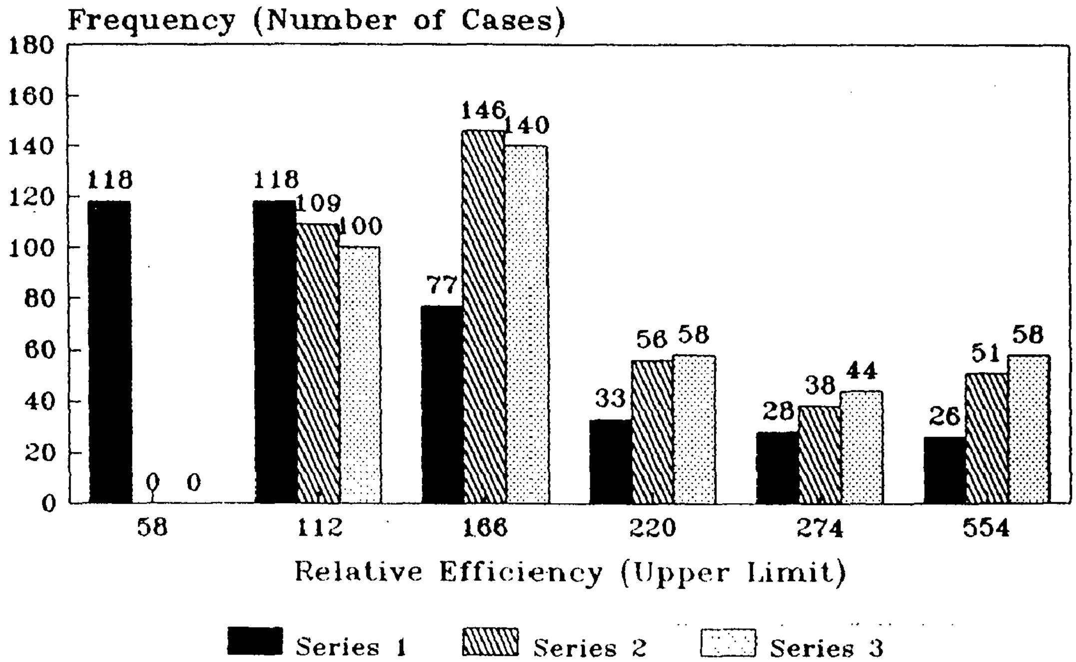
Graph - 4.22 (Sample Size = 20, $\rho > 0$)

Series 1 for $\overline{Y}_p$, Series 2 for $\overline{Y}_{Sa.}$ and
Series 3 for MEST(8)



Graph – 4.23 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_p$, Series 2 for $\overline{Y}_{Sa.}$ and
Series 3 for MEST(8)



Graph – 4.24 (Sample Size = 20, $\rho < 0$)

of the relative efficiencies of these estimators is being done through graphs 4.25 to 4.28.

TABLE 4.7

RF($\circ$) FOR $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$, $\bar{Y}_{VSa.}$ and MEST(9)

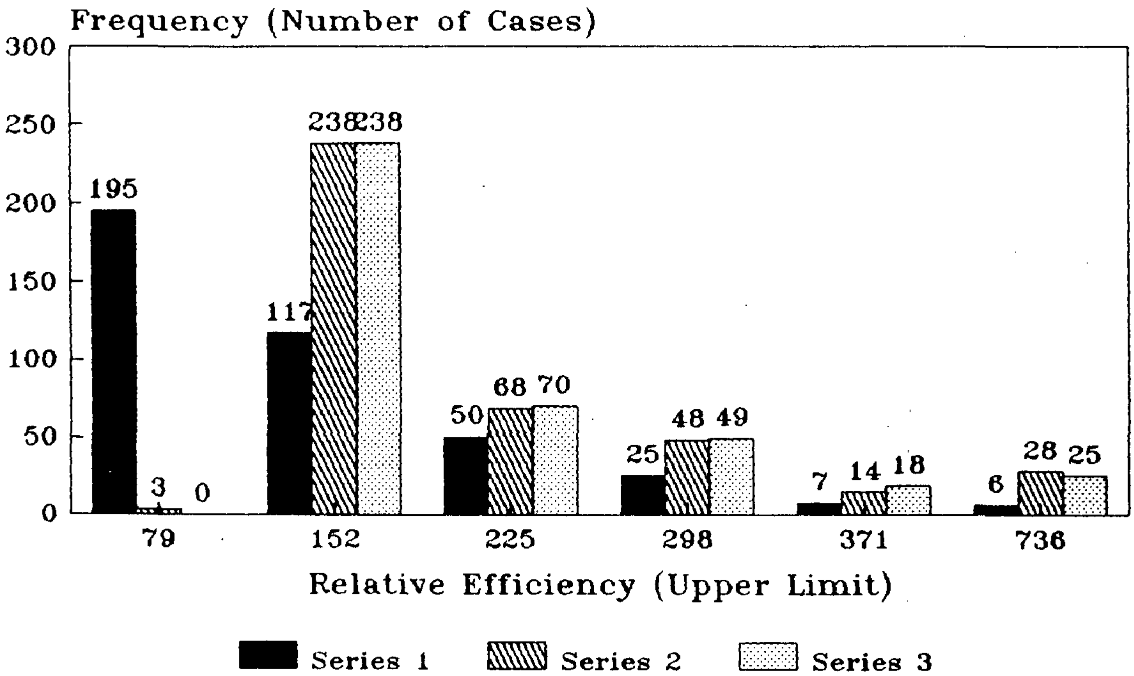
Estimators → / Sample Sizes ↓		$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_P$	$\bar{Y}_{VSa.}$	MEST(9)
$\rho > 0$	n=10	0.092	0.093	-	0.440	0.375
	n=20	0.072	0.075	-	0.468	0.385
$\rho < 0$	n=10	0.032	-	0.083	0.387	0.498
	n=20	0.040	-	0.062	0.425	0.473

From the above table, we can infer that $\bar{Y}_{VSa.}$ has been improved by mixing it with $\bar{y}$ when $\rho < 0$. This mixing does not perform very good for positive correlation case and $\bar{Y}_{VSa.}$ dominates the scene then. We have also noticed that MEST(9) takes a significant lead over all the other estimators in competition including $\bar{Y}_R$ when G > 1.0. Also, the relative frequency of MEST(9) turns out to be higher than $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}_{VSa.}$ much more often when G < -0.5. Thus, MEST(9) performs better than $\bar{Y}_R/\bar{Y}_P$ for those ranges of G where the use of $\bar{Y}_R$ or $\bar{Y}_P$ has been recommended in the literature. So, for those cases, one would recommend the use of MEST(9) instead of $\bar{Y}_R/\bar{Y}_P$.

4.2 MIXINGS OF TWO-PARAMETER FAMILIES OF ESTIMATORS :

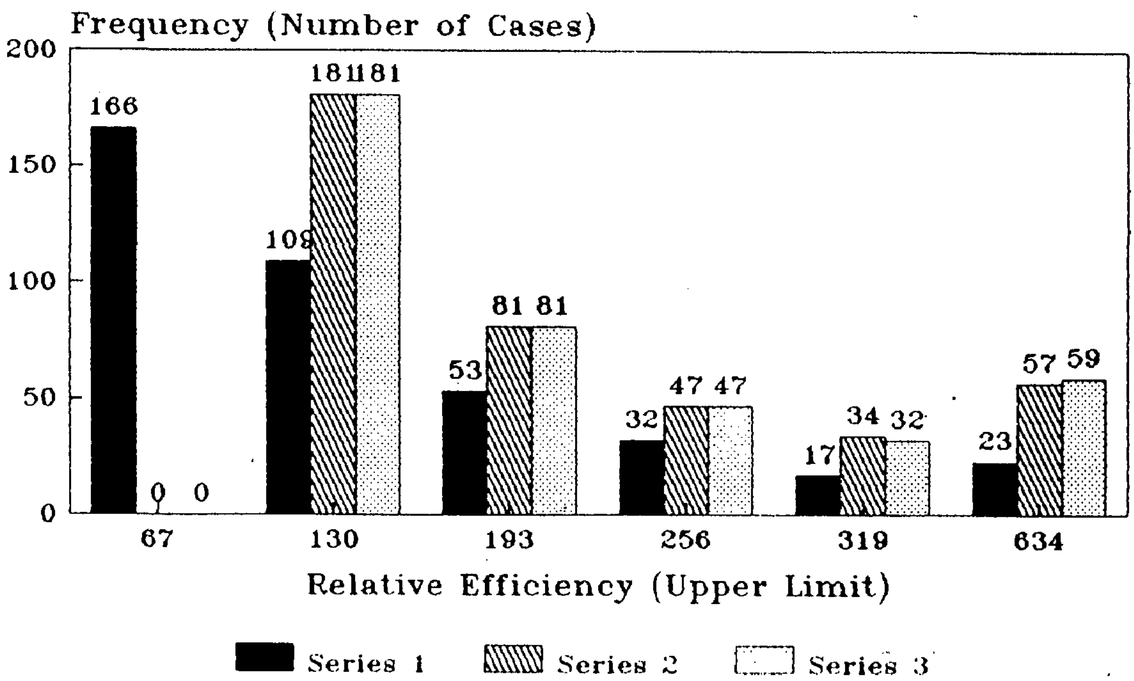
In this section, we propose the mixings of $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}^{(2)}_{RP.ab}$ with $\bar{y}$. Here also, the optimal value of mixing-parameter has been decided by the relative frequency of the respective estimator being the winner as per the empirical-simulation studies carried out by us in Chapter-3. The proposed method of mixing the

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{VSa.}$ and
Series 3 for MEST(9)



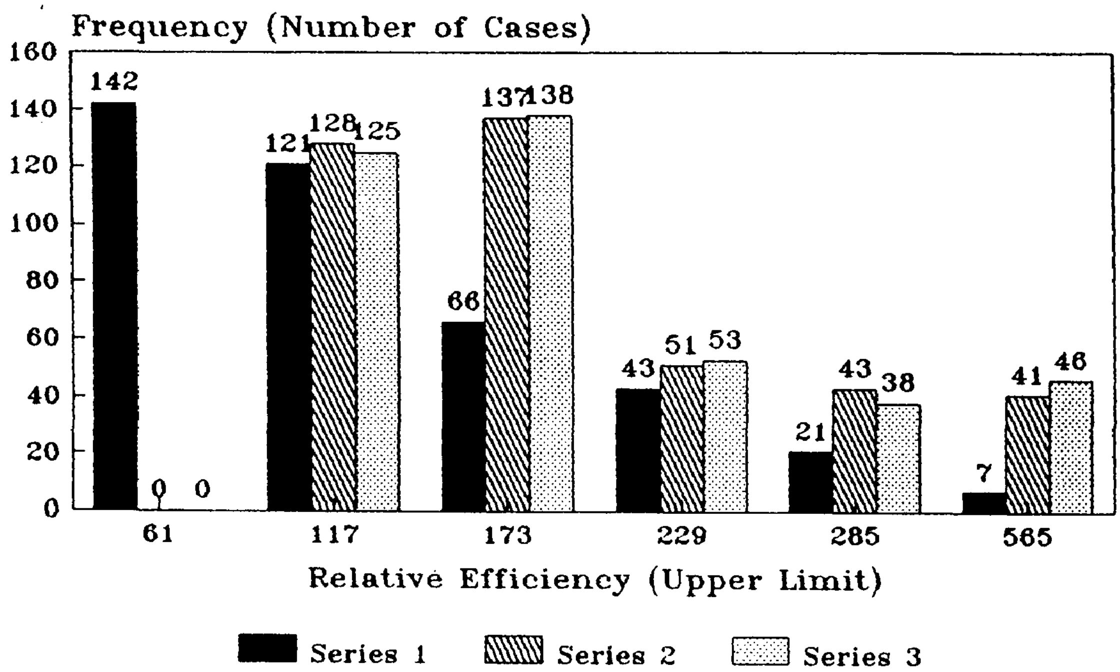
Graph – 4.25 (Sample Size = 10, $\rho > 0$)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{VSa.}$ and
Series 3 for MEST(9)



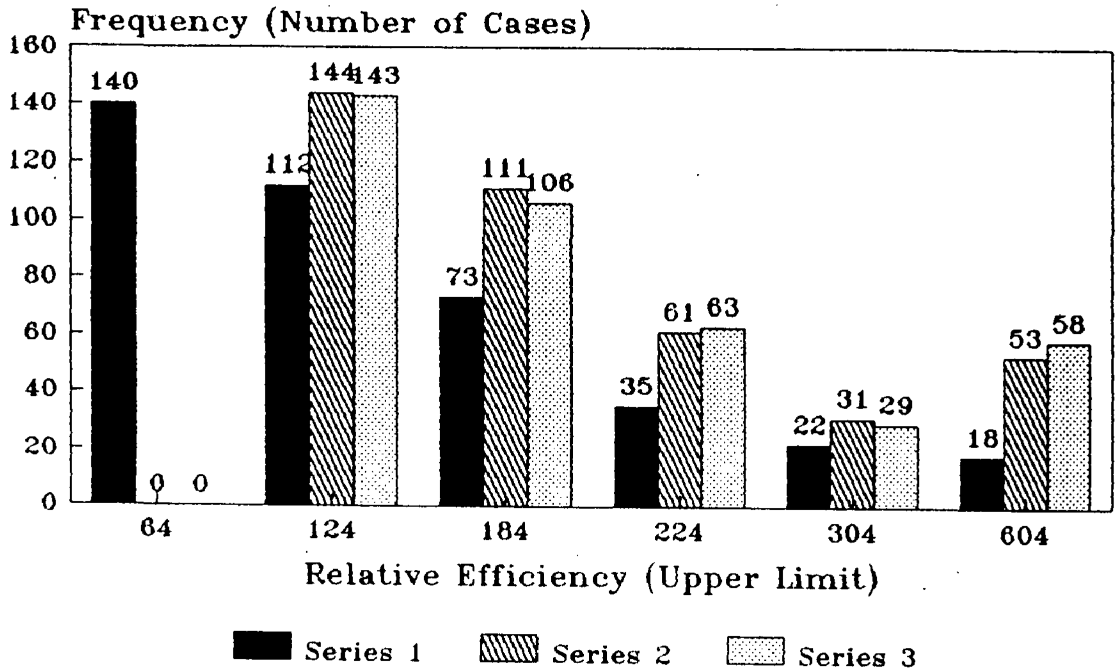
Graph – 4.26 (Sample Size = 20, $\rho > 0$)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{VSa}$ and Series 3 for MEST(9)



Graph - 4.27 (Sample Size = 10, ρ < 0)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{VSa}$ and Series 3 for MEST(9)



Graph - 4.28 (Sample Size = 20, ρ < 0) estimators is very much justifiable for the mixing-estimators of $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}^{(2)}_{RP.ab}$. If we do not follow this method of mixing $\bar{y}$ with $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}^{(2)}_{RP.ab}$, we would be having two design-parameters and one mixing-parameter. In all, we will have to find the optimal values of three parameters. This can be done by minimising the first, second and third order large sample approximate MSEs of the mixing-estimators but the expressions for the third order MSEs of these estimators become so complicated that we are unable to find the optimal values of the three design-parameters. So, following the method explained earlier, in this section we propose the mixings of $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}^{(2)}_{RP.ab}$ with $\bar{y}$ as follows.

Let 'b_{10}' be the mixing-parameter. Then the mixing of $\bar{Y}^{(1)}_{RP.ab}$ with $\bar{y}$ can be given by

$$MEST(10) = b_{10}.\bar{y} + (1-b_{10}).\bar{Y}^{(1)}_{RP.ab} \qquad ...(4.11)$$

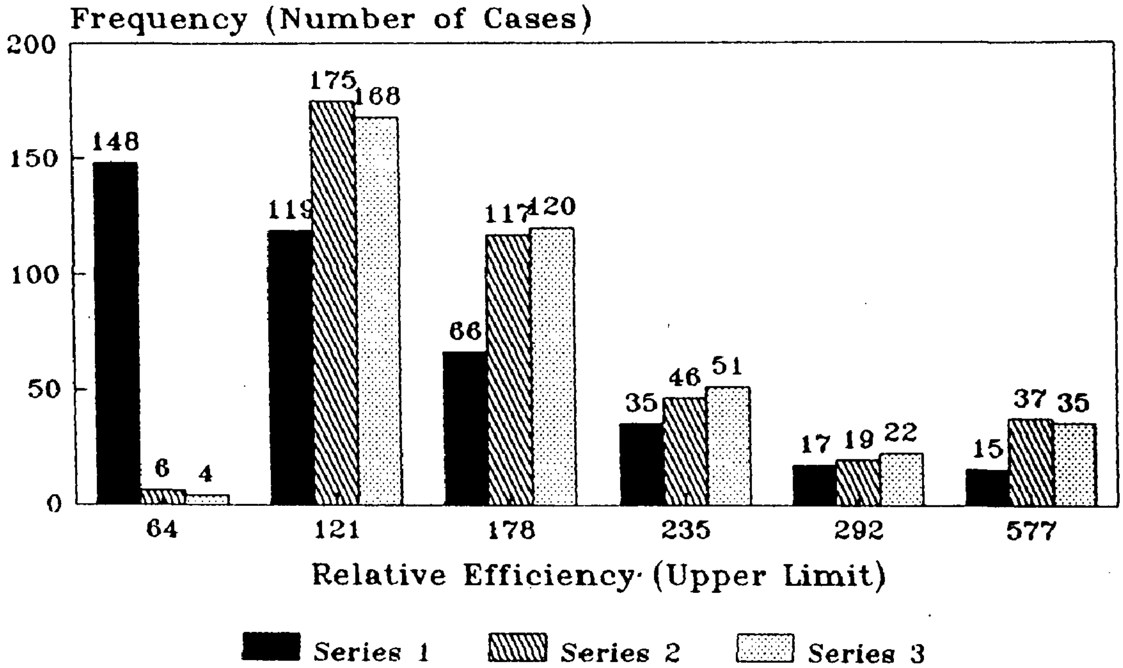
where, b_{10}=0.23 as per the empirical-simulation study carried out for $\bar{Y}^{(1)}_{RP.ab}$ in Chapter-3. Now, we compare this estimator with $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ and $\bar{Y}^{(1)}_{RP.ab}$. The results of these comparisons are tabulated below in table 4.8. The relative efficiencies of these estimators have also been displayed graphically through graphs 4.29 to 4.32.

TABLE 4.8

RF(∘) FOR $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$, $\bar{Y}^{(1)}_{RP.ab}$ and MEST(10)

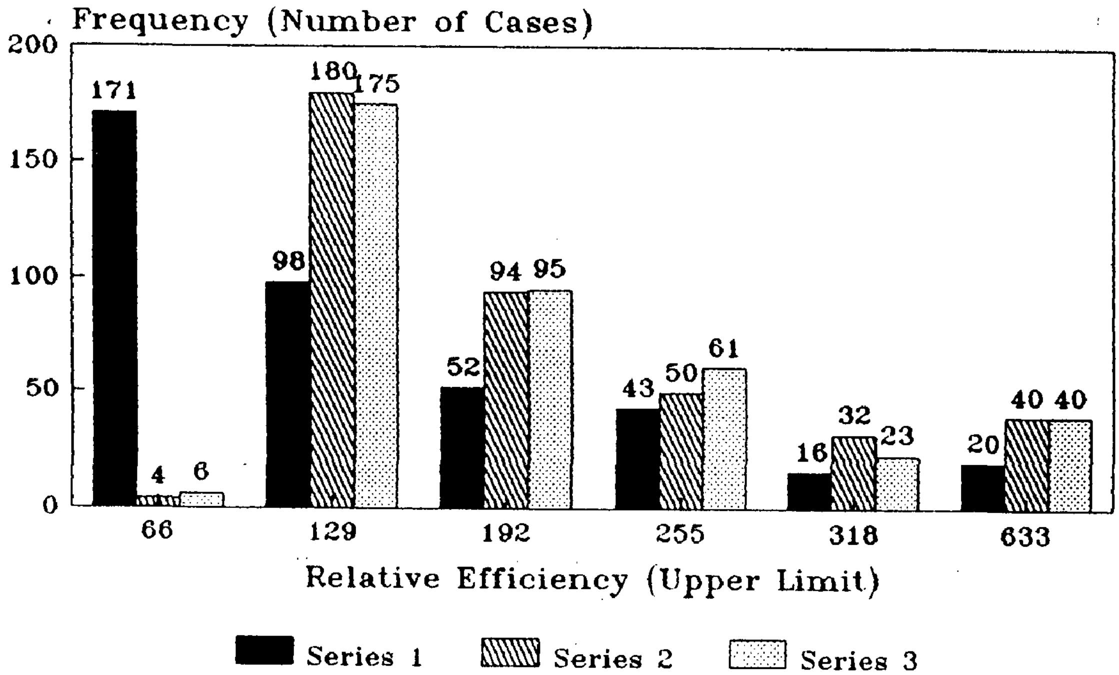
Estimators → / Sample Sizes ↓		$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_P$	$\bar{Y}^{(1)}_{RP.ab}$	MEST(10)
$\rho > 0$	n=10	0.095	0.085	-	0.370	0.450
	n=20	0.085	0.070	-	0.442	0.403
$\rho < 0$	n=10	0.122	-	0.240	0.360	0.278
	n=20	0.137	-	0.190	0.393	0.280

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{RP.ab}^{(1)}$ and
Series 3 for MEST(10)



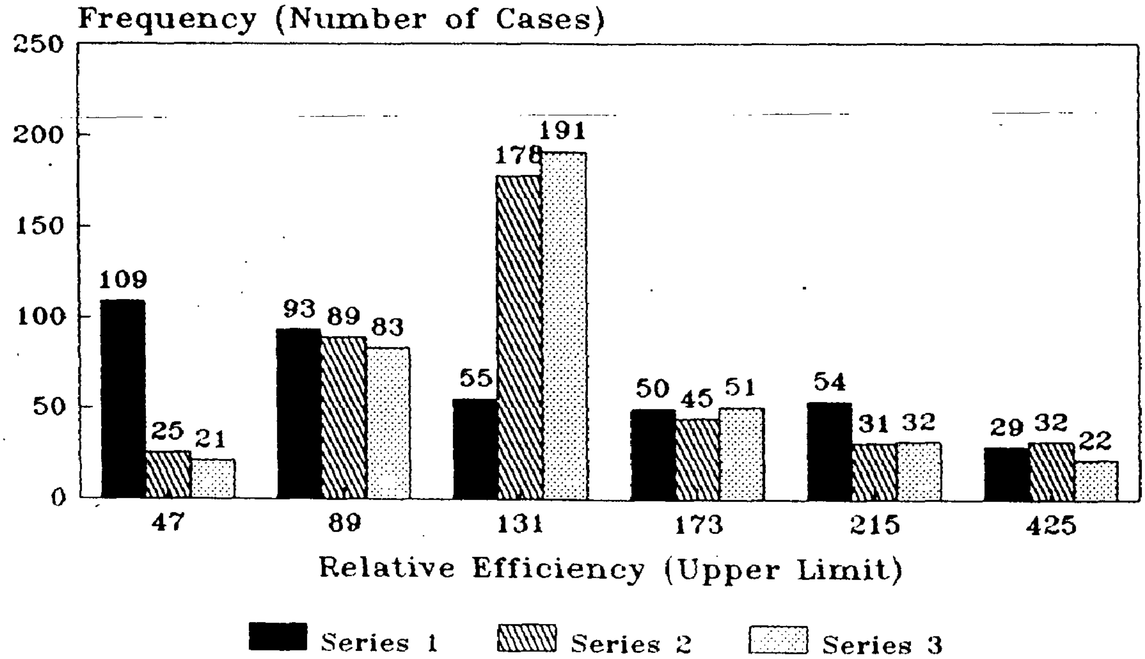
Graph - 4.29 (Sample Size = 10, ρ > 0)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{RP.ab}^{(1)}$ and
Series 3 for MEST(10)



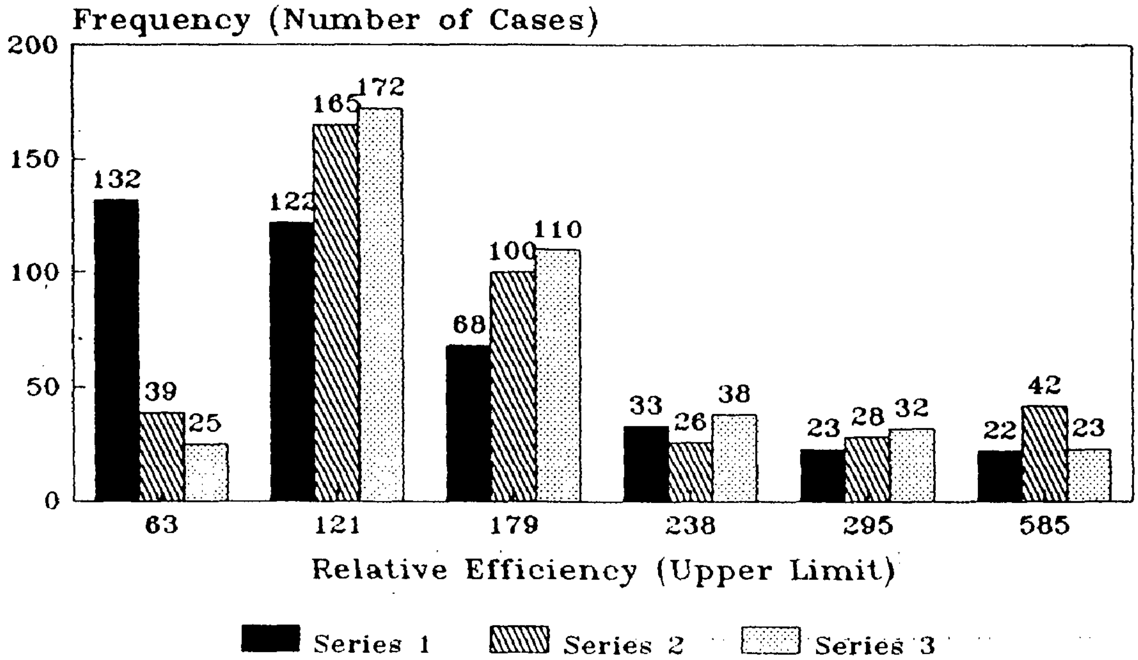
Graph - 4.30 (Sample Size = 20, ρ > 0)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}^{(1)}_{RP.ab}$ and
Series 3 for MEST(10)



Graph – 4.31 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}^{(1)}_{RP.ab}$ and
Series 3 for MEST(10)



Graph – 4.32 (Sample Size = 20, $\rho < 0$)

Table 4.8 indicates that MEST(10) performs better than the other estimators only when $\rho > 0$ and n=10. For other cases, its performance is not so good as that of $\bar{Y}^{(1)}_{RP.ab}$ but it is a strong competitor there too. It has also been observed through the finer comparisons of the relative efficiencies based on G-values that MEST(10) is most probably a better choice than $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}^{(1)}_{RP.ab}$ when $G \in [0.5, 1]$.

A similar mixing-type estimator for $\bar{Y}^{(2)}_{RP.ab}$ is proposed to be:

$$MEST(11) = b_{11}.\bar{y} + (1-b_{11}).\bar{Y}^{(2)}_{RP.ab} \qquad \ldots(4.12)$$

where, b_{11}=0.16 as per the empirical-simulation study carried out for $\bar{Y}^{(2)}_{RP.ab}$ in Chapter-3. Now, we proceed to compare this estimator with $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ and $\bar{Y}^{(2)}_{RP.ab}$. The results of these comparisons are tabulated below in table 4.9. The relative efficiencies of these estimators have also been displayed graphically through the graphs 4.33 to 4.36.

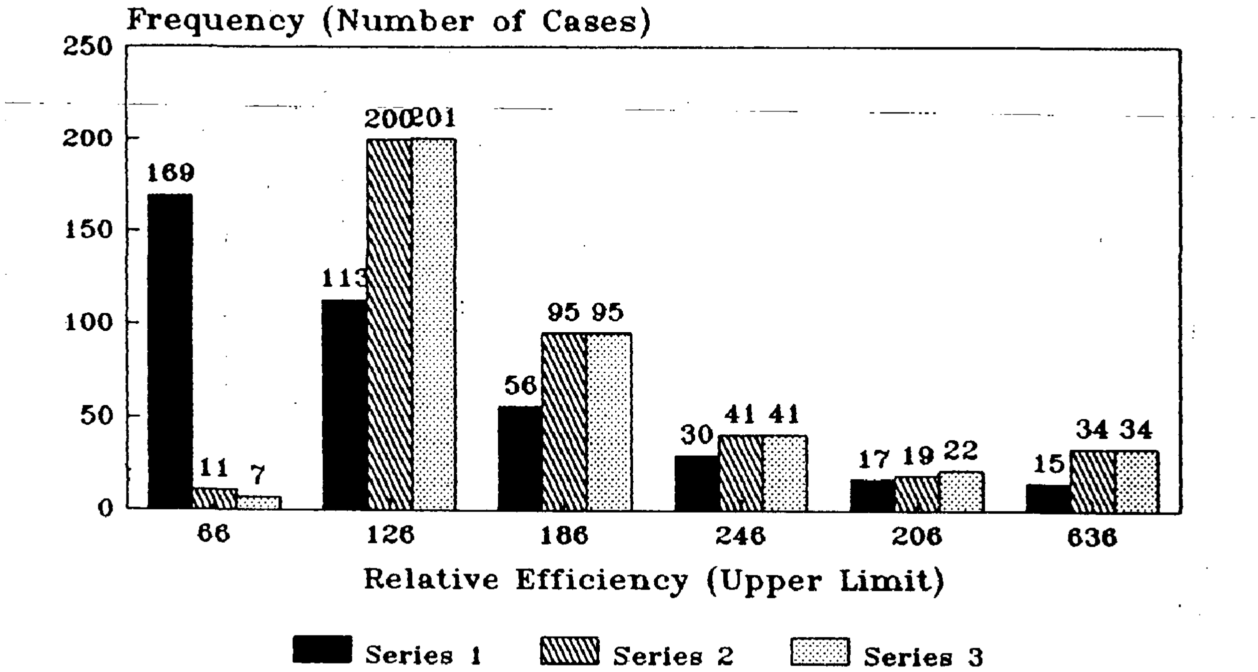
TABLE 4.9

RF(∘) FOR $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$, $\bar{Y}^{(2)}_{RP.ab}$ and MEST(11)

Estimators → SampleSizes ↓		$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_P$	$\bar{Y}^{(2)}_{RP.ab}$	MEST(11)
$\rho > 0$	n=10	0.122	0.068	-	0.332	0.478
	n=20	0.080	0.042	-	0.435	0.443
$\rho < 0$	n=10	0.255	-	0.242	0.250	0.253
	n=20	0.140	-	0.205	0.352	0.303

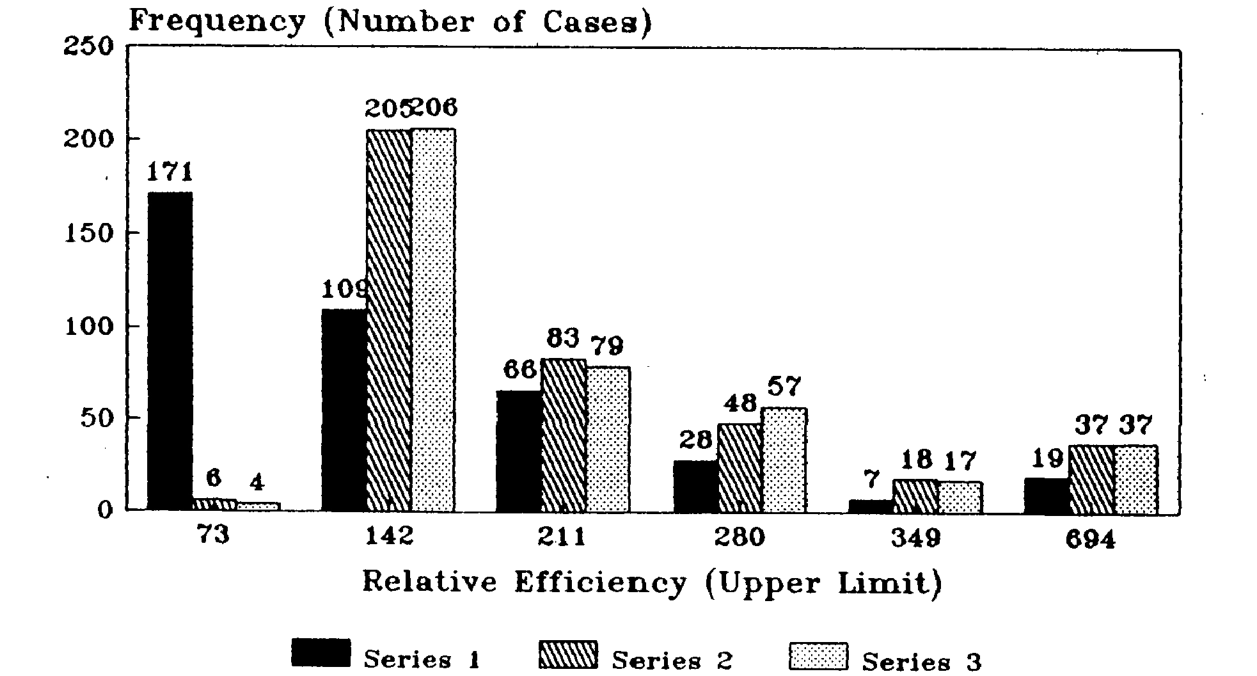
From this table, one can reach the openion that $\bar{Y}^{(2)}_{RP.ab}$ has been improved in the form of MEST(11) only when $\rho > 0$. The estimators MEST(10) and MEST(11) behave likewise here in the sense that MEST(10) was also an improved version of $\bar{Y}^{(1)}_{RP.ab}$ for the positive correlation case. It has also been observed that $\bar{Y}^{(2)}_{RP.ab}$ comes out to be winner more often as compared to other estimators when G lies between -0.5 and 1.

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}^{(2)}_{RP.ab}$ and Series 3 for MEST(11)



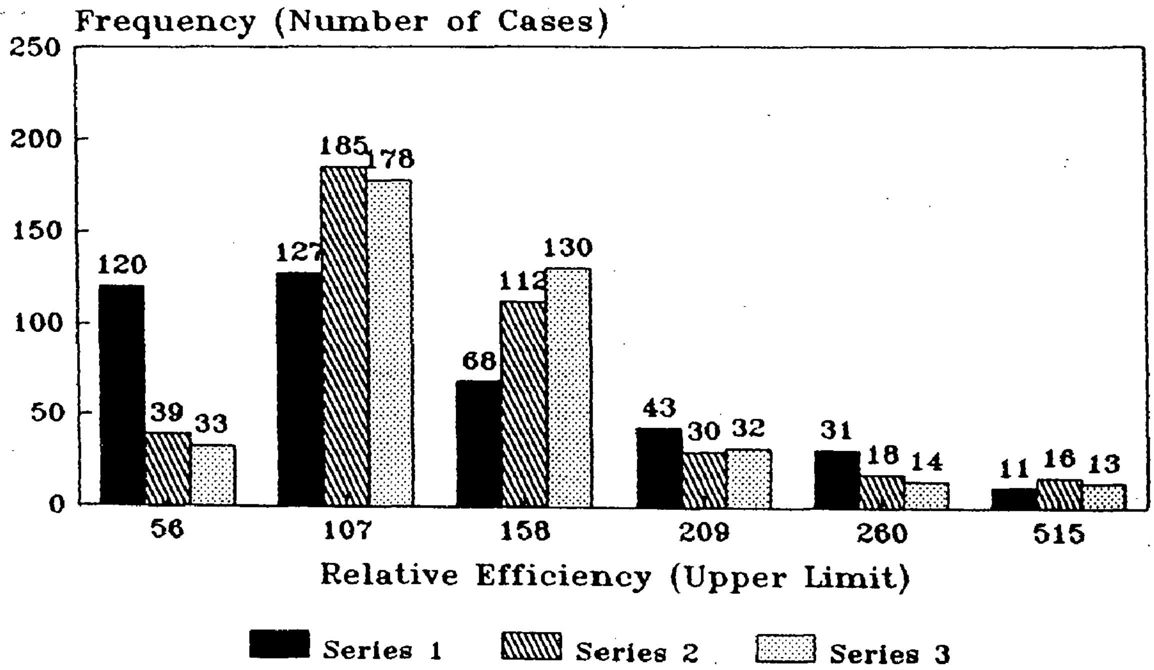
Graph - 4.33 (Sample Size = 10, $\rho > 0$)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}^{(2)}_{RP.ab}$ and Series 3 for MEST(11)



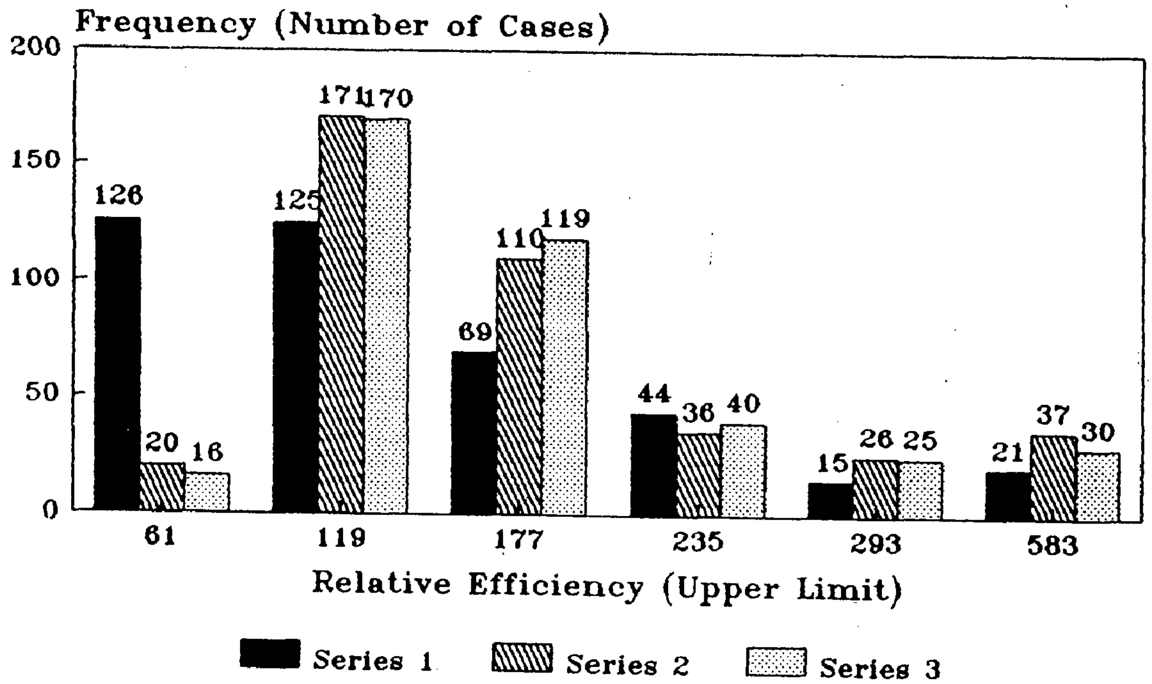
Graph - 4.34 (Sample Size = 20, $\rho > 0$)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}^{(2)}_{RP.ab}$ and Series 3 for MEST(11)



Graph - 4.35 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}^{(2)}_{RP.ab}$ and Series 3 for MEST(11)



Graph - 4.36 (Sample Size = 20, $\rho < 0$)

CHAPTER - 5

OTHER GAINFUL MIXINGS OF THE ESTIMATORS

In Chapter-4, we proposed some gainful mixings of $\bar{y}$ with various estimators which turn out to be better than their parent-estimators, quite often. There may be many other types of mixings of these estimators. We, in the present chapter propose two more types of mixings of the parent-estimators (i.e., the estimators proposed by us in Chapter-2 and Chapter-3) and observe through the empirical-simulation study that the proposed mixings perform better than their parent-estimators, quite often. In the first-type of mixing, we propose some new mixing estimators by combining the two parent estimators through a suitable mixing-parameter and in the second-type of mixing, we propose some linear combinations of $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ and the two parent-estimators.

It may be mentioned here that the performance of the estimators proposed by us in Chapter-2 and Chapter-3 is not uniform for the two cases of positive and negative correlations. So, in this Chapter, we have studied these two cases separately. For the first-type of mixing, we have not included all the parent-estimators in our present study but have considered the best five estimators when $\rho > 0$ and the best five estimators when $\rho < 0$. Subsequently, we have considered all the possible mixings of these five estimators taking two out of them at a time. This process gives rise to 10 mixing-estimators for positive correlation case and to other 10 estimators for the negative correlation case. For the second-type of mixings, we have considered the best three parent-estimators when $\rho > 0$ and the best three parent-estimators for $\rho < 0$. Thus, we will have 3 mixing-estimators for positive correlation case and other 3 for negative correlation case. To decide the best five estimators for the two cases of positive and negative correlations, we have carried out the empirical-simulation study taking all the parent-estimators simultaneously.

The best five estimators in the order of their performance turn out to be:

$$\bar{Y}_{P.a},\ \bar{Y}^{(1)}_{RP.ab},\ \bar{Y}^{(2)}_{RP.ab},\ \bar{Y}_{Re.},\ \bar{Y}_{VR} \text{ when } \rho > 0 \qquad \ldots(5.1)$$

and $\bar{Y}_{Re.},\ \bar{Y}_{VSa.},\ \bar{Y}_{MP},\ \bar{Y}^{(1)}_{RP.ab},\ \bar{Y}_{P.a}$ when $\rho < 0$ $\qquad \ldots(5.2)$

5.1 MIXINGS OF THE TWO PARENT-ESTIMATORS :

In this section, we propose the above mentioned first-type mixing of the estimators. We divide this section into two sub-sections in which we study the cases of positive and negative correlations separately.

5.1.1 MIXINGS WHEN $\rho > 0$:

The parent-estimators for this case are given in 5.1. In what follows, we propose the mixings for these parent-estimators. In order to propose a mixing-estimator for $\bar{Y}_{P.a}$ and $\bar{Y}^{(1)}_{RP.ab}$, we first carry out the empirical-simulation study to compare $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{P.a}$ and $\bar{Y}^{(1)}_{RP.ab}$. The results of this study are given in table 5.1.

TABLE 5.1

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{P.a}$ and $\bar{Y}^{(1)}_{RP.ab}$

Estimators → / Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_{P.a}$	$\bar{Y}^{(1)}_{RP.ab}$
n=10	0.062	0.168	0.465	0.305
n=20	0.027	0.135	0.463	0.375

Based on the results of the above table, we propose a mixing-estimator of $\bar{Y}_{P.a}$ and $\bar{Y}^{(1)}_{RP.ab}$ to be

$$MEST(12) = p_1 . \bar{Y}_{P.a} + (1-p_1) . \bar{Y}^{(1)}_{RP.ab} \quad \ldots(5.3)$$

where, p_1=0.58 as per the results of the table 5.1. Now, we proceed to compare MEST(12) with $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_{P.a}$ (since $\bar{Y}_{P.a}$ is the most probable winner according to table 5.1). The results of these comparisons are tabulated below in table 5.2. Graphs 5.1 and 5.2 provide the graphical display of the relative efficiencies of these estimators.

TABLE 5.2

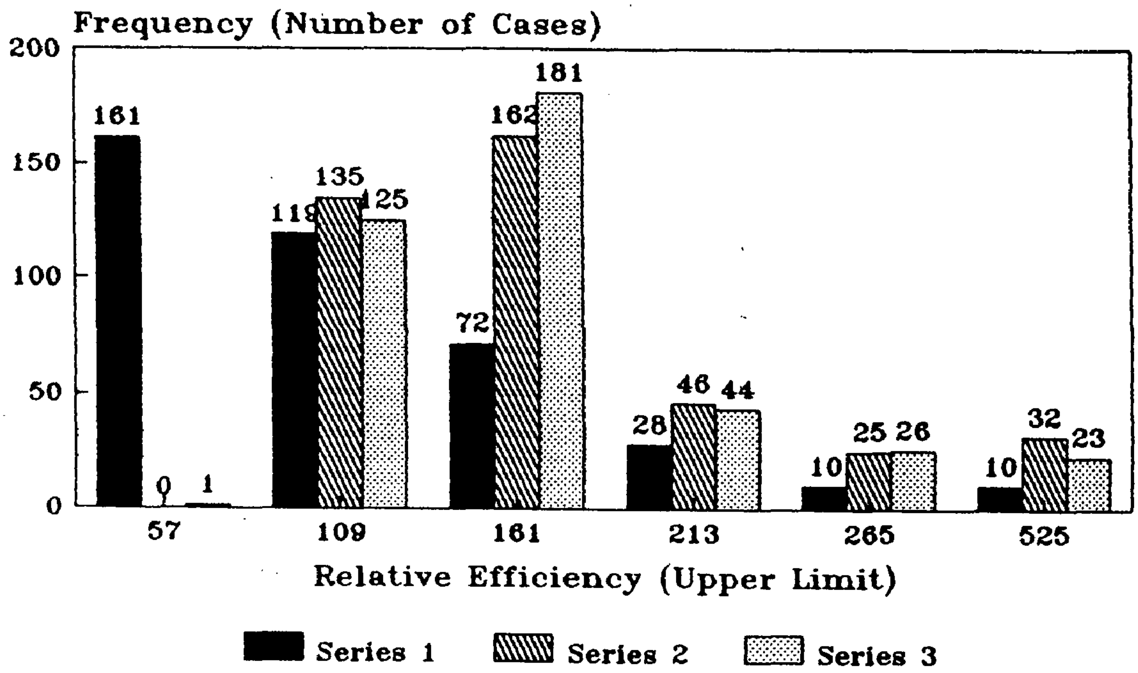
RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{P.a}$ and MEST(12)

Estimators → / Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_{P.a}$	MEST(12)
n=10	0.057	0.158	0.437	0.348
n=20	0.045	0.115	0.395	0.445

Table 5.2 reveals that we will gain in the efficiency by the use of MEST(12) for relatively larger sample sizes (i.e., n=20). It has also been noticed that this mixing-estimator performs consistently better than $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_{P.a}$ when $G \in [0.5, 1]$.

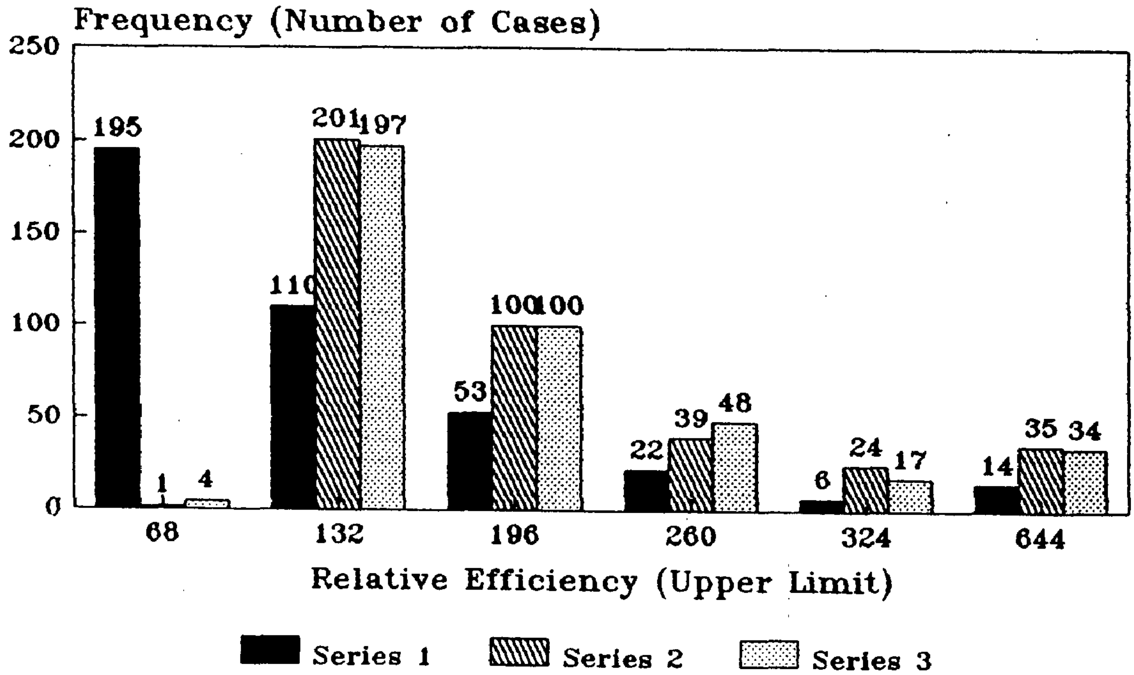
Now, we carry out the empirical-simulation study for the

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{P.a}$ and Series 3 for MEST(12)



Graph – 5.1 (Sample Size = 10, $\rho > 0$)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{P.a}$ and Series 3 for MEST(12)



Graph – 5.2 (Sample Size = 20, $\rho > 0$)

comparison of $\bar{Y}_{P.a}$ and $\bar{Y}^{(2)}_{RP.ab}$ in the presence of the usual estimators $\bar{y}$ and $\bar{Y}_R$. The results of this study are tabulated below in the table 5.3.

TABLE 5.3

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{P.a}$ and $\bar{Y}^{(2)}_{RP.ab}$

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_{P.a}$	$\bar{Y}^{(2)}_{RP.ab}$
n=10	0.035	0.132	0.525	0.308
n=20	0.037	0.113	0.445	0.405

Based on the results of the above table, we propose the mixing-estimator for $\bar{Y}_{P.a}$ and $\bar{Y}^{(2)}_{RP.ab}$ to be :

$$MEST(13) = p_2.\bar{Y}_{P.a} + (1-p_2).\bar{Y}^{(2)}_{RP.ab} \qquad \ldots(5.4)$$

where, p_2=0.58. From table 5.3, we can infer that $\bar{Y}_{P.a}$ comes out to be winner more often as compared to $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}^{(2)}_{RP.ab}$. So, in what follows, we compare MEST(13) with $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_{P.a}$. The results of these comparisons are given in table 5.4. A more comprehensive view of the relative efficiencies of these estimators is provided through graphs 5.3 and 5.4.

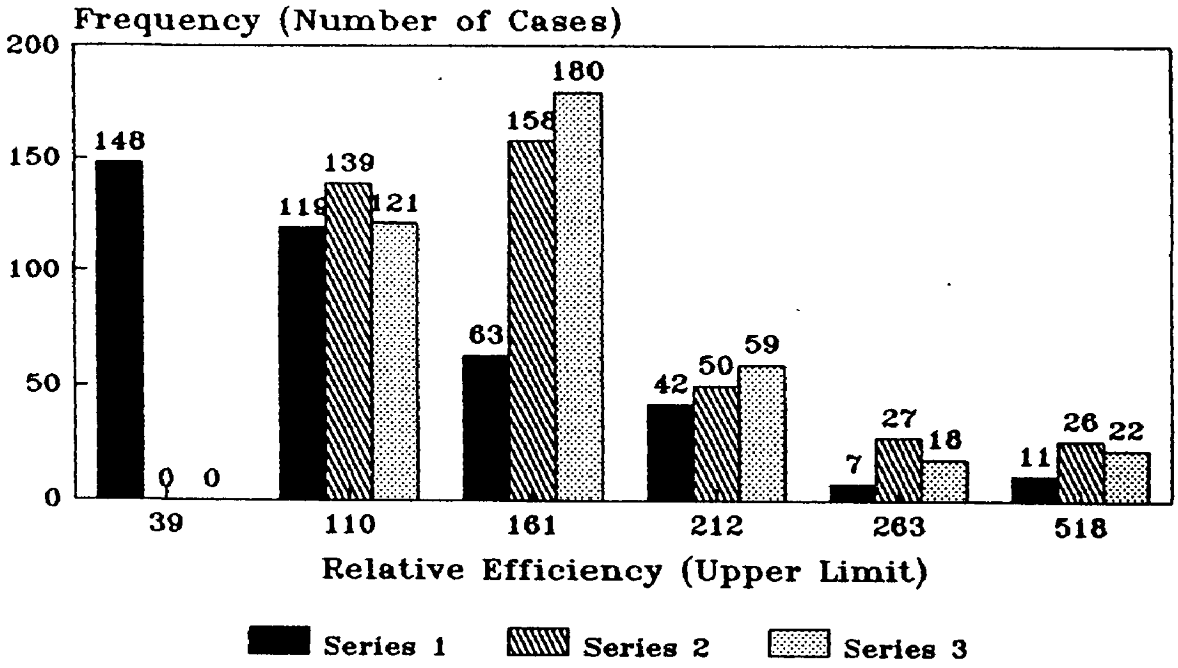
TABLE 5.4

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{P.a}$ and MEST(13)

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_{P.a}$	MEST(13)
n=10	0.037	0.180	0.428	0.355
n=20	0.032	0.128	0.380	0.460

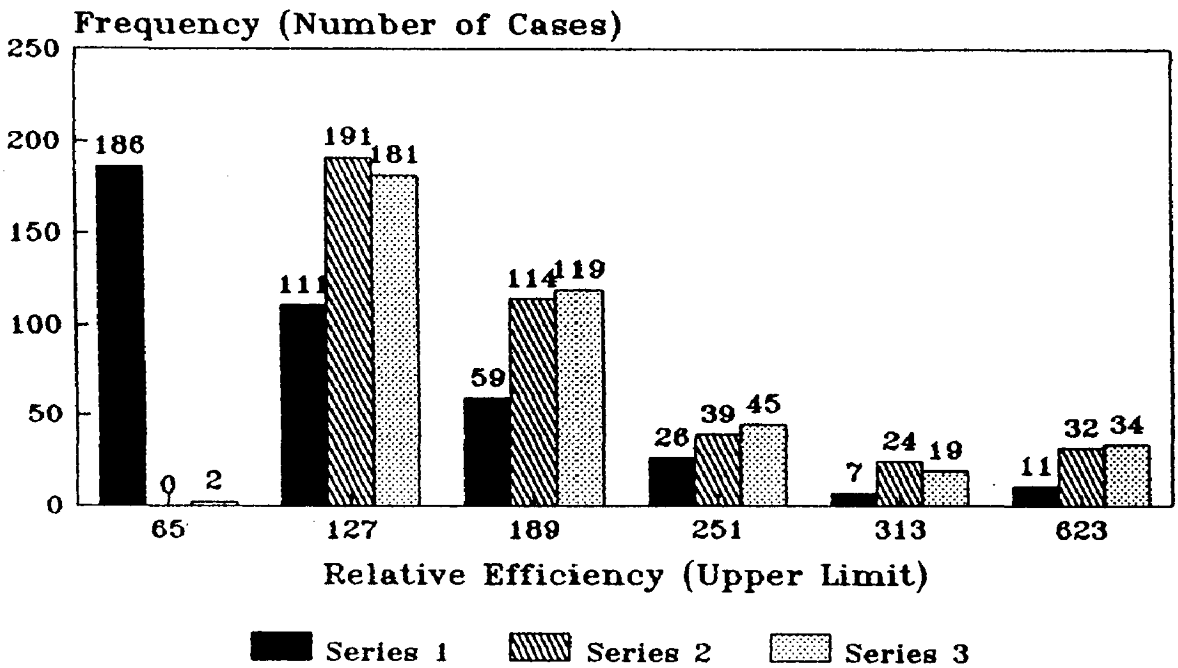
From the above table, we can infer that MEST(13) performs better than $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_{P.a}$ for rather a large sample size. The behaviour of MEST(13) is very similar to MEST(12) and it is also the winner on the comparisons much more often when $G \in [0.5, 1]$.

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{P.a}$ and
Series 3 for MEST(13)



Graph – 5.3 (Sample Size = 10, $\rho > 0$)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{P.a}$ and
Series 3 for MEST(13)



Graph – 5.4 (Sample Size = 20, $\rho > 0$)

Next, we consider the estimators $\bar{Y}_{P.a}$ and $\bar{Y}_{Re.}$. For the proposition of a mixing-estimator of these two, we first compare $\bar{Y}_{P.a}$ and $\bar{Y}_{Re.}$ in the presence of $\bar{y}$ and $\bar{Y}_R$. The results of these comparisons are tabulated below in table 5.5.

TABLE 5.5

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{P.a}$ and $\bar{Y}_{Re.}$

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_{P.a}$	$\bar{Y}_{Re.}$
n=10	0.085	0.092	0.445	0.378
n=20	0.062	0.120	0.478	0.340

The mixing-estimator for $\bar{Y}_{P.a}$ and $\bar{Y}_{Re.}$, based on the results of table 5.5 can be proposed as

$$MEST(14) = p_3.\bar{Y}_{P.a} + (1-p_3).\bar{Y}_{Re.} \qquad \ldots(5.5)$$

where, $p_3=0.56$. Thus $\bar{Y}_{P.a}$ again dominates the scene when compared with $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_{Re.}$. So, we now compare our proposed mixing with $\bar{y}$, $\bar{y}_R$ and $\bar{Y}_{P.a}$. The results of the empirical-simulation study for comparing these estimators: $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{P.a}$ and MEST(14) are given below in table 5.6. Graphs 5.5 and 5.6 provide a clearer view of the relative efficiencies of these estimators.

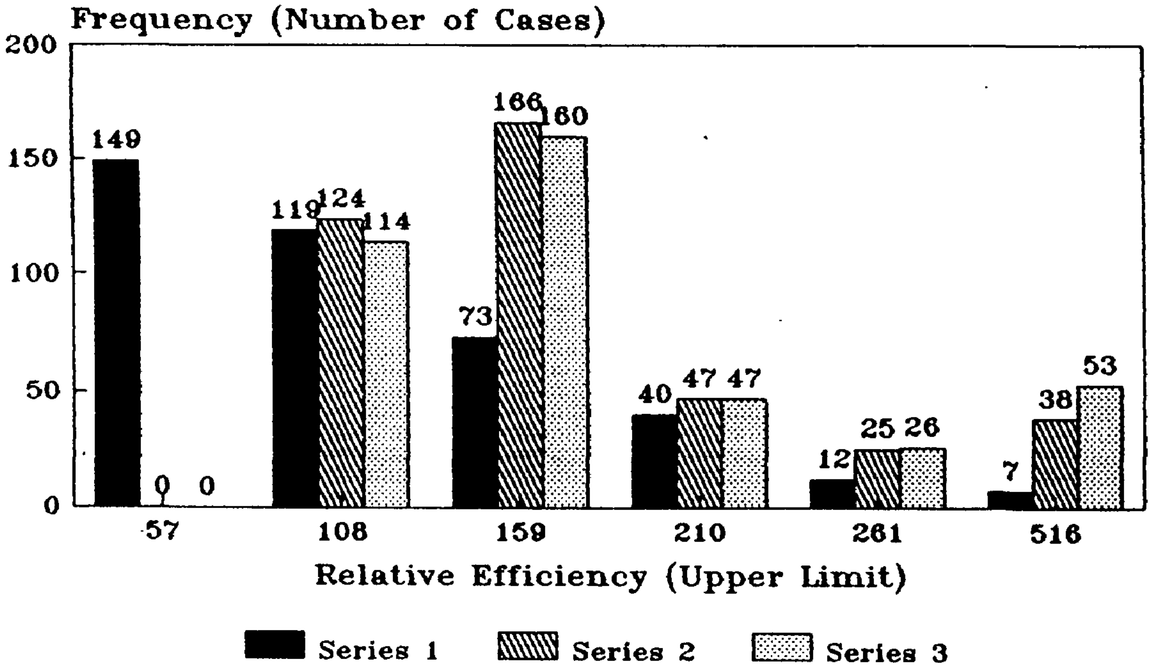
TABLE 5.6

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{P.a}$ and MEST(14)

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_{P.a}$	MEST(14)
n=10	0.070	0.165	0.372	0.393
n=20	0.040	0.035	0.442	0.483

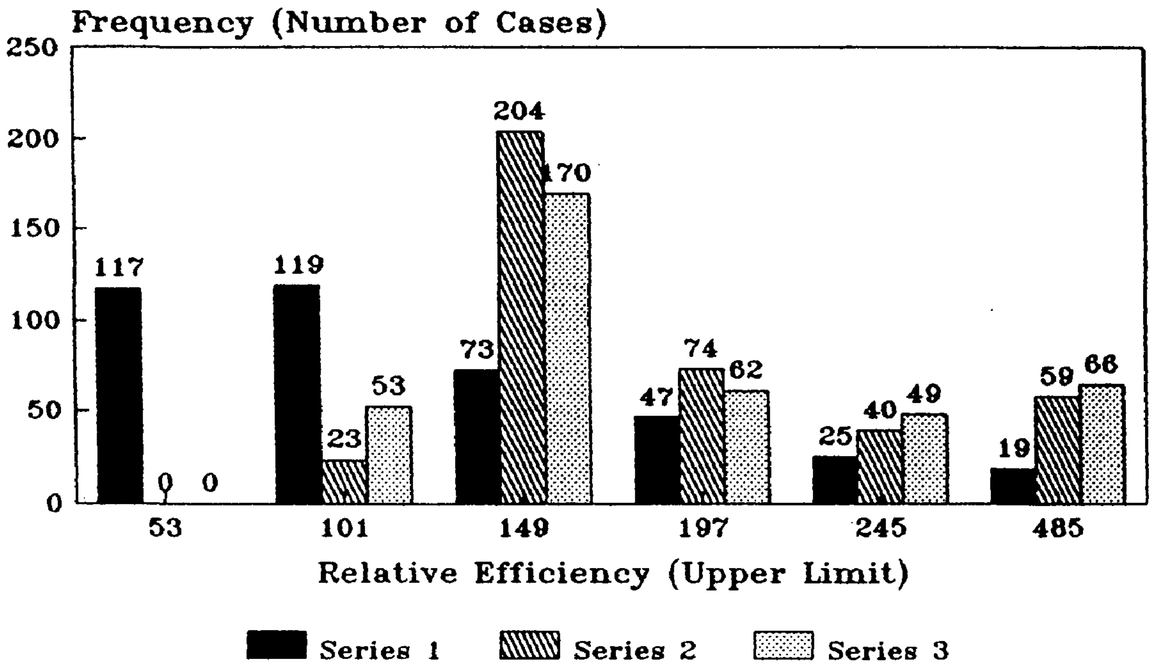
From the above table, one can conclude that MEST(14) comes out to be winner more often when compared with $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_{P.a}$. So, it is an efficient mixing of $\bar{Y}_{P.a}$ and $\bar{Y}_{Re.}$. We have also observed

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{P.a}$ and
Series 3 for MEST(14)



Graph – 5.5 (Sample Size = 10, ρ > 0)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{P.a}$ and
Series 3 for MEST(14)



Graph – 5.6 (Sample Size = 20, ρ > 0) through the finer comparisons of these estimators based on G-values that the performance of MEST(14) is quite encouraging when $G \in [0.5, 1]$.

Now, we go forward to propose a mixing of $\bar{Y}_{P.a}$ and $\bar{Y}_{VR}$. For this, we first compare $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{P.a}$ and $\bar{Y}_{VR}$ via the empirical-simulation study. The results of the study are tabulated below in table 5.7.

TABLE 5.7

RF(∘)' FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{P.a}$ and $\bar{Y}_{VR}$

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_{P.a}$	$\bar{Y}_{VR}$
n=10	0.032	0.168	0.497	0.303
n=20	0.060	0.185	0.550	0.205

Based on the figures given in the above table, we propose the following mixing estimator for $\bar{Y}_{P.a}$ and $\bar{Y}_{VR}$.

$$\text{MEST}(15) = p_4 . \bar{Y}_{P.a} + (1-p_4) . \bar{Y}_{VR} \qquad \ldots(5.6)$$

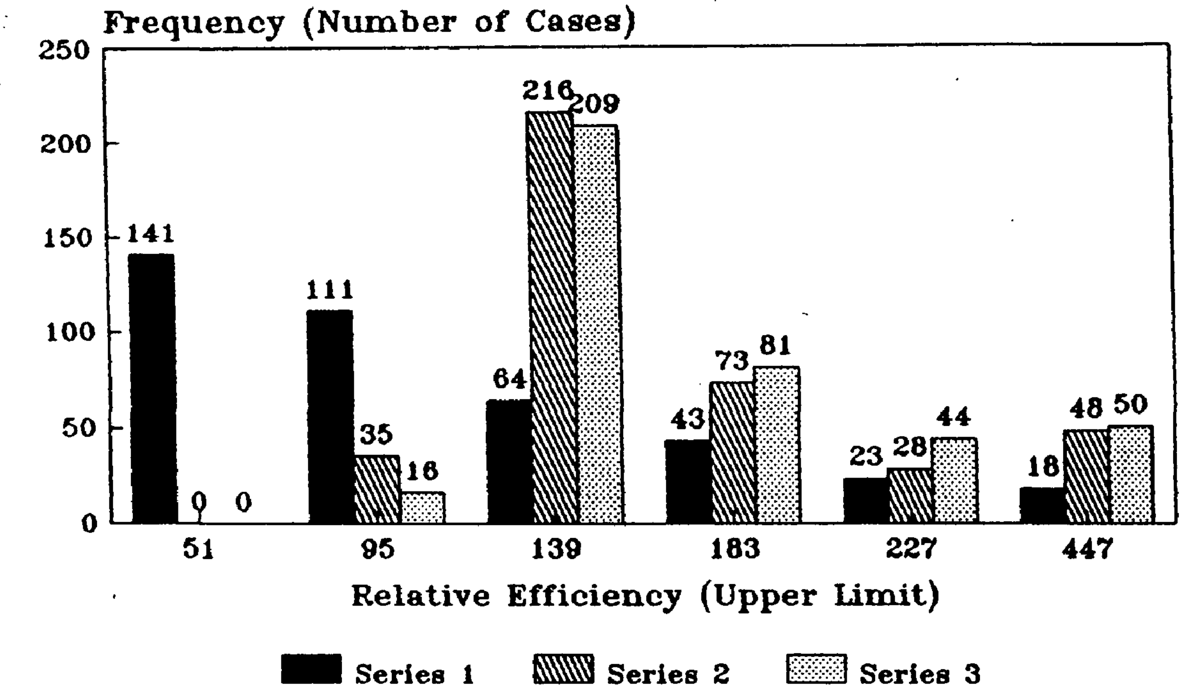
where, $p_4=0.67$. Thus, $\bar{Y}_{P.a}$ is the most probable winner when compared with $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_{VR}$. So, we compare the estimator MEST(15) with $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_{P.a}$. Table 5.8 contains the results of the empirical-simulation study carried out for these comparisons and graphs 5.7 and 5.8 afford us a clearer view of the relative efficiencies of these estimators.

TABLE 5.8

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{P.a}$ and MEST(15)

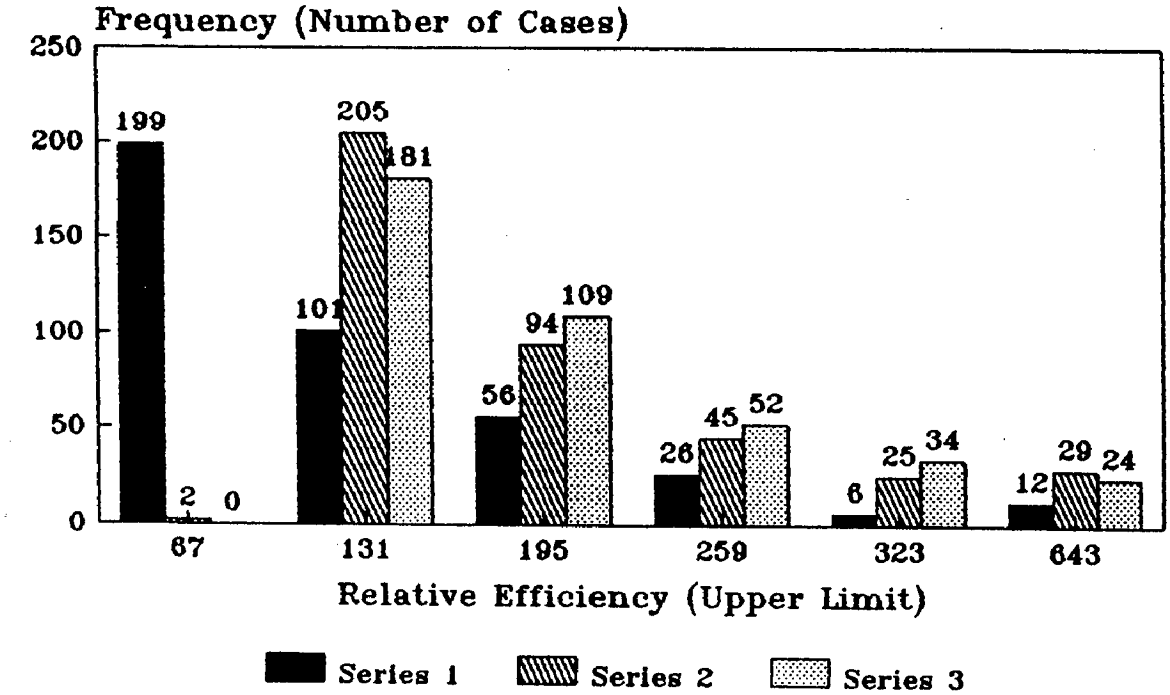
Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_{P.a}$	MEST(15)
n=10	0.052	0.165	0.300	0.483
n=20	0.057	0.130	0.355	0.458

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{P.a}$ and Series 3 for MEST(15)



Graph – 5.7 (Sample Size = 10, $\rho > 0$)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{P.a}$ and Series 3 for MEST(15)



Graph – 5.8 (Sample Size = 20, $\rho > 0$)

Table 5.8 indicates that MEST(15) is an improvement over $\bar{Y}_{P.a}$. It comes out to be winner more often as compared to $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_{P.a}$. So, one can recommend the use of MEST(15) instead of $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_{P.a}$ when $\rho > 0$. It has also been observed through the finer comparisons of $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{P.a}$ and MEST(15) that MEST(15) performs quite well in comparison with the other estimators here when $G \in [0.5, 1]$. Also, it is a close competitor of $\bar{Y}_R$ when $G > 1.0$.

For the proposition of a mixing-type estimator of $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}^{(2)}_{RP.ab}$, we now carry out the empirical-simulation study for comparing $\bar{y}$, $\bar{Y}_R$, $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}^{(2)}_{RP.ab}$. The results of this study are tabulated below in table 5.9.

TABLE 5.9

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}^{(2)}_{RP.ab}$

Estimators → / Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}^{(1)}_{RP.ab}$	$\bar{Y}^{(2)}_{RP.ab}$
n=10	0.130	0.112	0.360	0.398
n=20	0.080	0.085	0.350	0.485

Based on the above results, we propose the following mixing estimator,

$$\text{MEST}(16) = p_5 . \bar{Y}^{(1)}_{RP.ab} + (1-p_5) . \bar{Y}^{(2)}_{RP.ab} \qquad \ldots (5.7)$$

where, $p_5 = 0.45$. The value of p_5 hints that $\bar{Y}^{(2)}_{RP.ab}$ is the most probable winner when we compare $\bar{y}$, $\bar{Y}_R$, $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}^{(2)}_{RP.ab}$. So, we now take up the empirical-simulation study for comparing MEST(16) with $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}^{(2)}_{RP.ab}$. The results of this study are tabulated below in table 5.10 and the relative efficiencies of these estimators are being displayed through graphs 5.9 and 5.10.

TABLE 5.10

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}^{(2)}_{RP.ab}$ and MEST(16)

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}^{(2)}_{RP.ab}$	MEST(16)
n=10	0.120	0.100	0.402	0.378
n=20	0.067	0.085	0.470	0.378

Although, from table 5.10, one can infer that MEST(16) does not perform so good as $\bar{Y}^{(2)}_{RP.ab}$ yet it is quite a close competitor of $\bar{Y}^{(2)}_{RP.ab}$ when $G \in [0,\ 0.5]$. It even takes a lead over $\bar{Y}^{(2)}_{RP.ab}$ for this range of G for n=10.

Next, we propose the same type of mixing for $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}_{Re.}$. The results of the empirical-simulation study when we compare $\bar{y}$, $\bar{Y}_R$, $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}_{Re.}$ are tabulated below in table 5.11.

TABLE 5.11

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}_{Re.}$

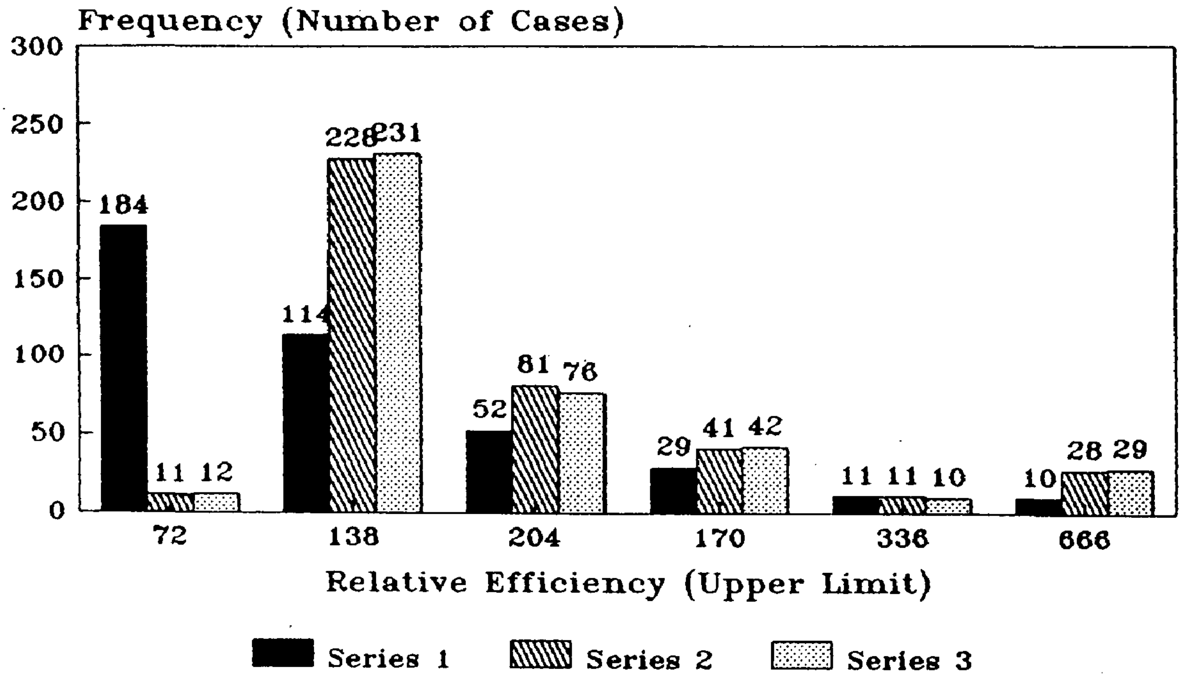
Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}^{(1)}_{RP.ab}$	$\bar{Y}_{Re.}$
n=10	0.057	0.083	0.370	0.490
n=20	0.035	0.097	0.395	0.473

Exploiting the knowledge of the performances of the two estimators $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}_{Re.}$ as indicated in table 5.11, we propose the mixing estimator for $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}_{Re.}$ to be:

$$\text{MEST(17)} = p_6.\bar{Y}^{(1)}_{RP.ab} + (1-p_6).\bar{Y}_{Re.} \qquad \ldots(5.8)$$

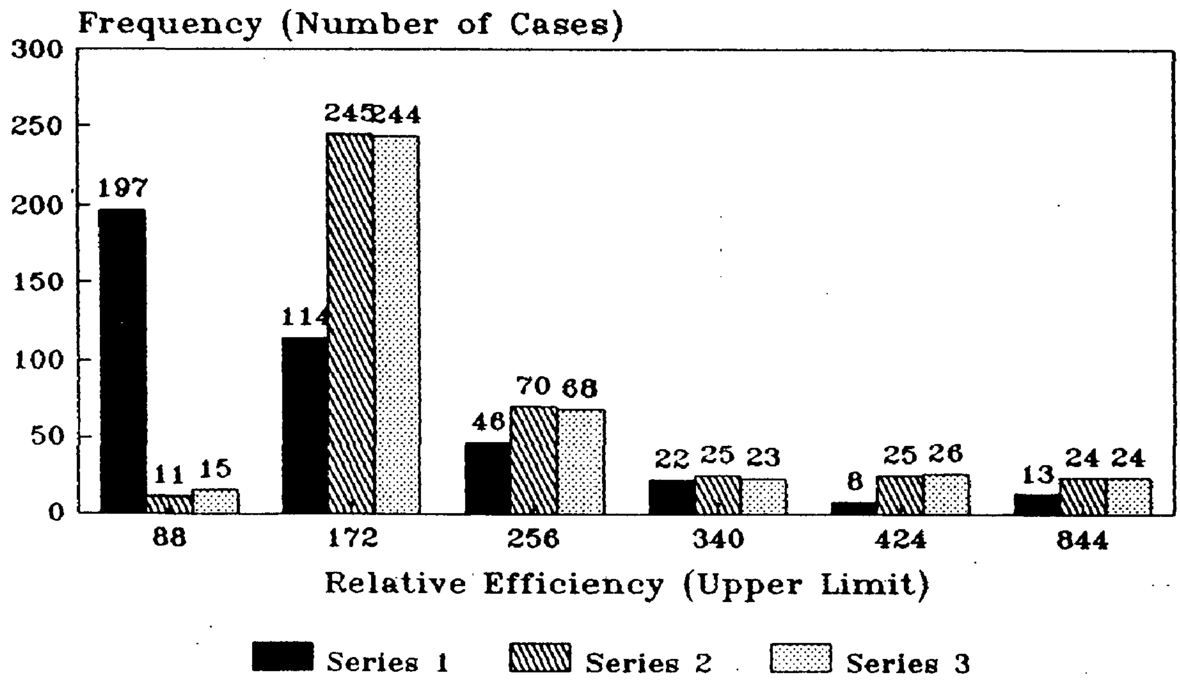
where, p_6=0.44 as per the results of table 5.11. Since p_6=0.44, we now carry out the empirical-simulation study for the comparison of $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{Re.}$ and MEST(17). Table 5.12 contains the results of this study and the graphs 5.11 and 5.12 afford us a clearer view of the relative efficiencies of these estimators.

Series 1 for $\bar{Y}_R$, Series 2 for $\bar{Y}^{(2)}_{RP.ab}$ and
Series 3 for MEST(16)



Graph – 5.9 (Sample Size = 10, $\rho > 0$)

Series 1 for $\bar{Y}_R$, Series 2 for $\bar{Y}^{(2)}_{RP.ab}$ and
Series 3 for MEST(16)



Graph – 5.10 (Sample Size = 20, $\rho > 0$)

TABLE 5.12

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{Re.}$ and MEST(17)

Estimators → / Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_{Re.}$	MEST(17)
n=10	0.067	0.098	0.375	0.460
n=20	0.042	0.098	0.367	0.493

It is clear from table 5.12 that MEST(17) performs better than $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_{Re.}$, quite often. So, one can recommend the use of MEST(17) instead of $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_{Re.}$. It has also been seen through the finer comparisons of these estimators that MEST(17) is the most probable winner when G ∈ [0.5, 1] and it takes a slight lead over $\bar{Y}_R$ when G > 1.0.

In the table 5.13 given below, we tabulate the results of the empirical-simulation study for the comparison of $\bar{y}$, $\bar{Y}_R$, $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}_{VR}$.

TABLE 5.13

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}_{VR}$

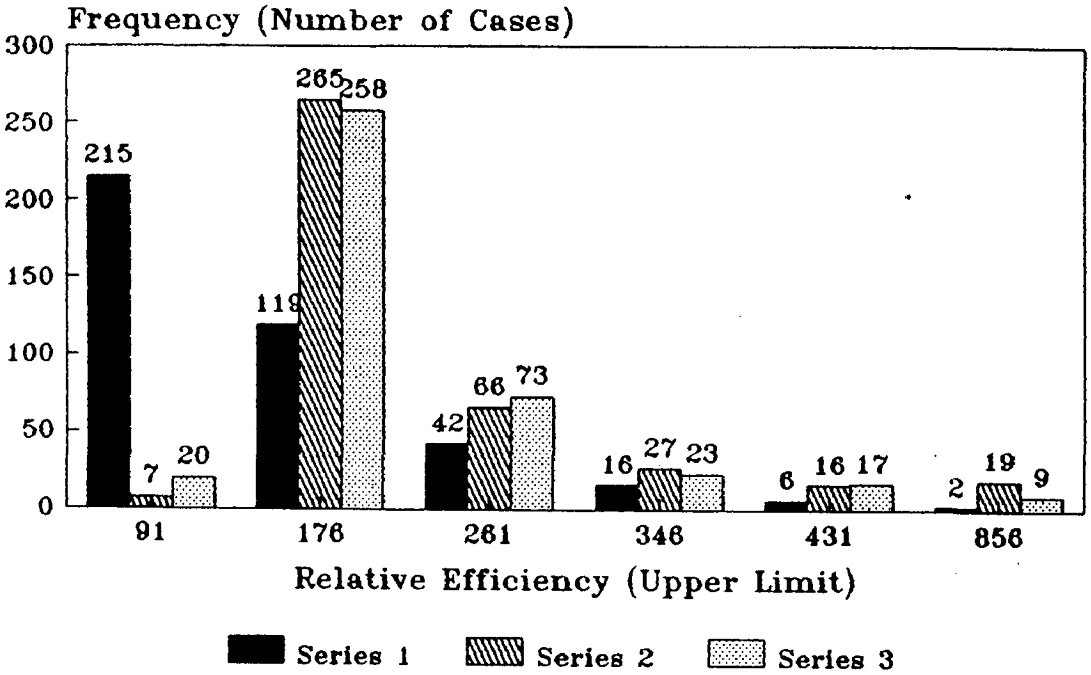
Estimators → / Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}^{(1)}_{RP.ab}$	$\bar{Y}_{VR}$
n=10	0.095	0.107	0.508	0.290
n=20	0.065	0.090	0.622	0.223

Based on the results of the above table, we propose the mixing estimator for $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}_{VR}$ as

$$\text{MEST}(18) = p_7 . \bar{Y}^{(1)}_{RP.ab} + (1-p_7) . \bar{Y}_{VR} \qquad \ldots(5.9)$$

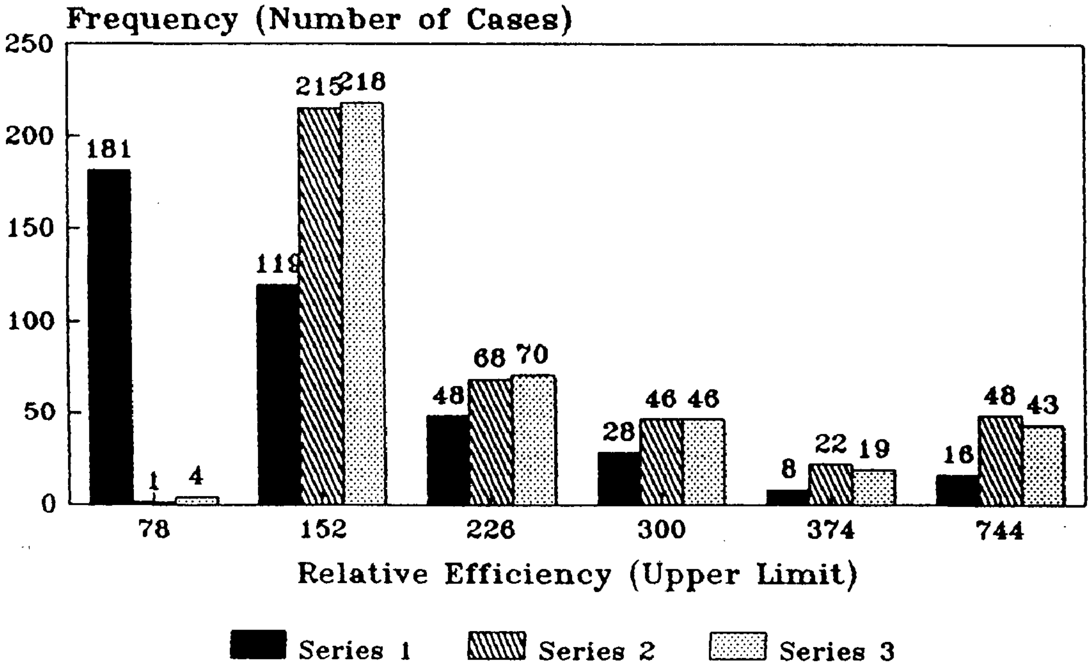
where, p_7=0.69 as per the results of table 5.13. Thus, $\bar{Y}^{(1)}_{RP.ab}$ performs better than $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_{VR}$, most often. So, we now proceed to compare MEST(18) with $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}^{(1)}_{RP.ab}$. The results of these comparisons are detailed below per table 5.14. Graphs 5.13

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{Re.}$ and
Series 3 for MEST(17)



Graph – 5.11 (Sample Size = 10, $\rho > 0$)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{Re.}$ and
Series 3 for MEST(17)



Graph – 5.12 (Sample Size = 20, $\rho > 0$)

and 5.14 afford us a clearer view of the relative efficiencies of these estimators.

TABLE 5.14

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}^{(1)}_{RP.ab}$ and MEST(18)

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}^{(1)}_{RP.ab}$	MEST(18)
n=10	0.115	0.062	0.350	0.473
n=20	0.075	0.070	0.412	0.443

Table 5.14 indicates that MEST(18) performs better than $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}^{(1)}_{RP.ab}$, quite often. We are thus able to improve $\bar{Y}^{(1)}_{RP.ab}$ in the form of mixing-type estimator MEST(18). This estimator performs exceptionally better than the other estimators when $G \in [0.5, 1]$. It is also unbeaten more often when $G > 1.0$.

In order to propose a mixing-type estimator of $\bar{Y}^{(2)}_{RP.ab}$ and $\bar{Y}_{Re.}$, we compare the two in the presence of $\bar{y}$ and $\bar{Y}_R$ via the empirical-simulation study. The results of these comparisons are tabulated below in table 5.15.

TABLE 5.15

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}^{(2)}_{RP.ab}$ and $\bar{Y}_{Re.}$

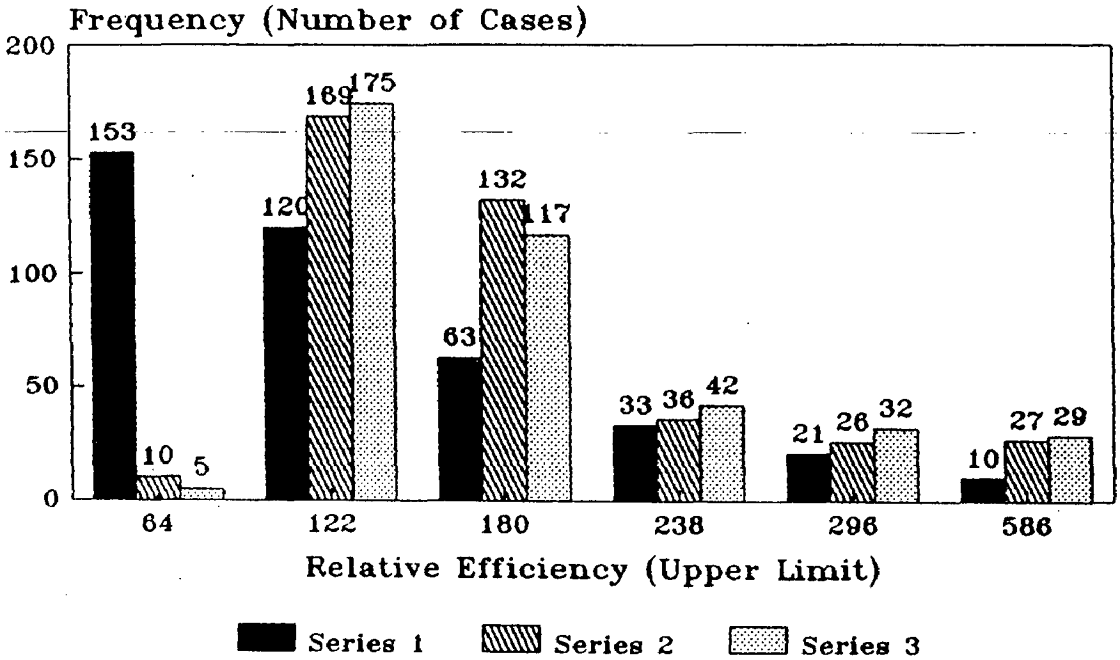
Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}^{(2)}_{RP.ab}$	$\bar{Y}_{Re.}$
n=10	0.047	0.088	0.462	0.403
n=20	0.027	0.075	0.513	0.385

Thus, the proposed mixing is:

$$\text{MEST}(19) = p_8 . \bar{Y}^{(2)}_{RP.ab} + (1-p_8) . \bar{Y}_{Re.} \qquad \ldots(5.10)$$

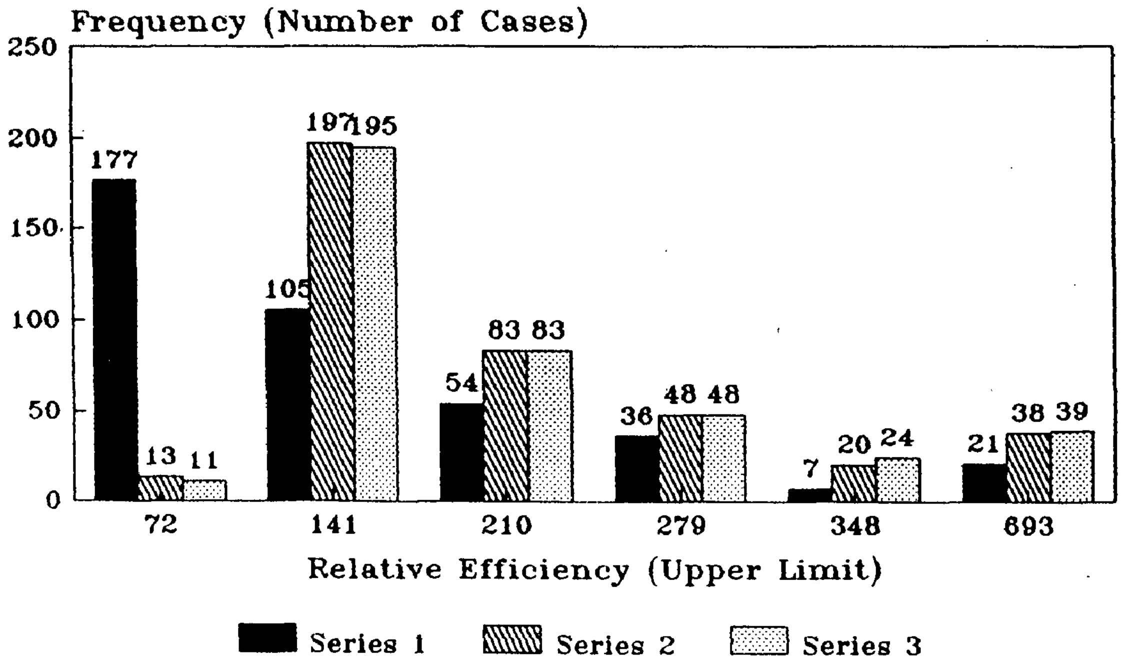
where, $p_8=0.55$ as per the results of table 5.15. Since, $\bar{Y}^{(2)}_{RP.ab}$ is the most probable winner in the above comparisons, we now compare MEST(19) with $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}^{(2)}_{RP.ab}$. The results of these comparisons

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}^{(1)}_{RP.ab}$ and Series 3 for MEST(18)



Graph – 5.13 (Sample Size = 10, $\rho > 0$)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}^{(1)}_{RP.ab}$ and Series 3 for MEST(18)



Graph – 5.14 (Sample Size = 20, $\rho > 0$)

are tabulated below in table 5.16 and the gain in the relative efficiency by using MEST(19) is also being displayed graphically per graphs 5.15 and 5.16.

TABLE 5.16

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}^{(2)}_{RP.ab}$ and MEST(19)

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}^{(2)}_{RP.ab}$	MEST(19)
n=10	0.110	0.092	0.358	0.440
n=20	0.062	0.058	0.460	0.420

One can infer from the above table that MEST(19) performs better than the other estimators for relatively smaller sample sizes (n = 10). It has also been noticed that it is unbeaten most often when $G \in [0, 0.5]$.

Now, we propose a mixing-type estimator for $\bar{Y}^{(2)}_{RP.ab}$ and $\bar{Y}_{VR}$. First, we carry out the empirical-simulation study to compare the estimators $\bar{y}$, $\bar{Y}_R$, $\bar{Y}^{(2)}_{RP.ab}$ and $\bar{Y}_{VR}$. Table 5.17 contains the results of this study.

TABLE 5.17

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}^{(2)}_{RP.ab}$ and $\bar{Y}_{VR}$

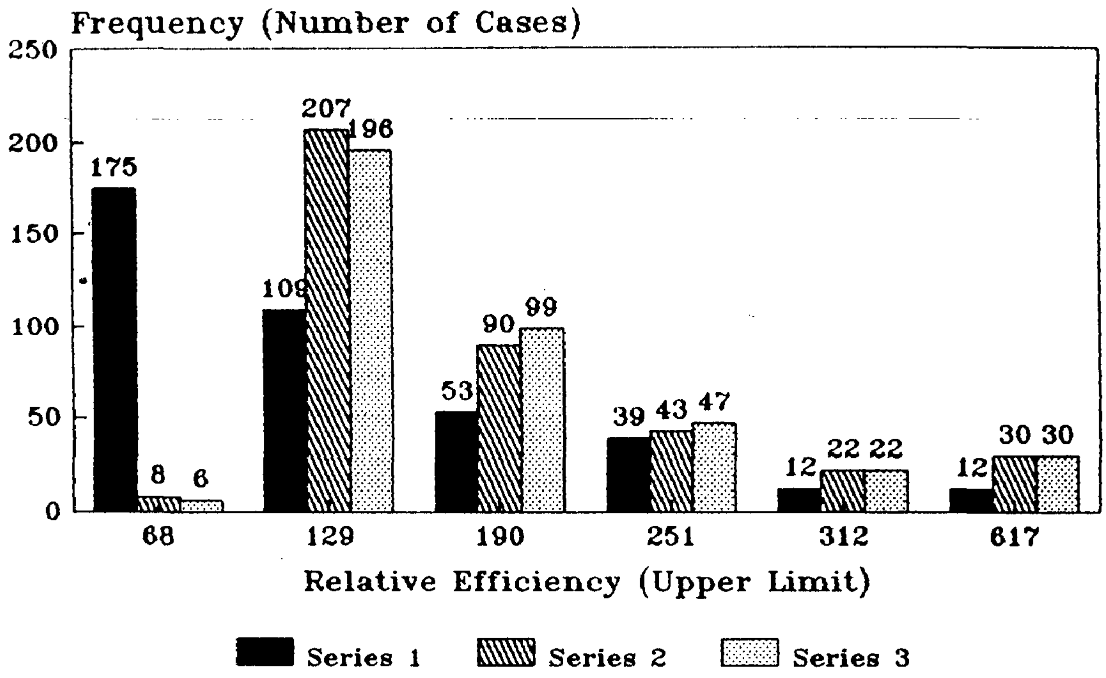
Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}^{(2)}_{RP.ab}$	$\bar{Y}_{VR}$
n=10	0.115	0.080	0.560	0.245
n=20	0.057	0.068	0.685	0.190

Thus, we propose the mixing-type estimator for $\bar{Y}^{(2)}_{RP.ab}$ and $\bar{Y}_{VR}$ to be :

$$\text{MEST}(20) = p_g \cdot \bar{Y}^{(2)}_{RP.ab} + (1-p_g) \cdot \bar{Y}_{VR} \qquad \ldots(5.11)$$

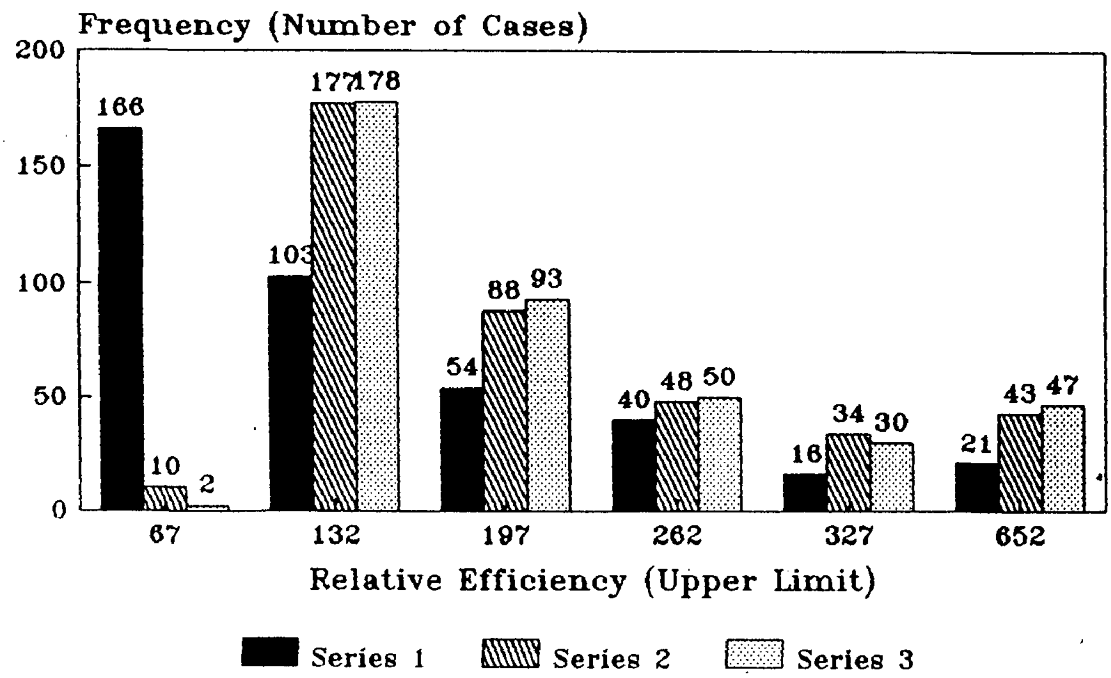
where, p_g=0.74 as per the results of table 5.17. Now, we perform the empirical-simulation study for comparing MEST(20) with $\bar{y}$, $\bar{Y}_R$

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}^{(2)}_{RP.ab}$ and Series 3 for MEST(19)



Graph – 5.15 (Sample Size = 10, $\rho > 0$)

Series 1 for $\overline{Y}$, Series 2 for $\overline{Y}^{(2)}_{RP.ab}$ and Series 3 for MEST(19)



Graph – 5.16 (Sample Size = 20, $\rho > 0$)

and $\bar{Y}^{(2)}_{RP.ab}$. The results of this study are tabulated below in table 5.18 and the relative efficiencies of the estimators being displayed through graphs 5.17 and 5.18.

TABLE 5.18

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}^{(2)}_{RP.ab}$ and MEST(20)

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}^{(2)}_{RP.ab}$	MEST(20)
n=10	0.115	0.045	0.325	0.515
n=20	0.042	0.063	0.385	0.510

Above table is the indicative of the fact that MEST(20) is an efficient mixing of $\bar{Y}^{(2)}_{RP.ab}$ and $\bar{Y}_{VR}$. It turns out to be winner, quite often when compared with $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}^{(2)}_{RP.ab}$. It has also been revealed through the finer comparisons of $\bar{y}$, $\bar{Y}_R$, $\bar{Y}^{(2)}_{RP.ab}$ and MEST(20) that MEST(20) performs better more often than the other estimators for the whole range of G considered by us.

The last mixing-type estimator for the positive correlation case has been proposed by using $\bar{Y}_{Re.}$ and $\bar{Y}_{VR}$. We first compare $\bar{Y}_{Re.}$ and $\bar{Y}_{VR}$ in the presence of the usual estimators $\bar{y}$ and $\bar{Y}_R$. Table 5.19 contains these results.

TABLE 5.19

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{Re.}$ and $\bar{Y}_{VR}$

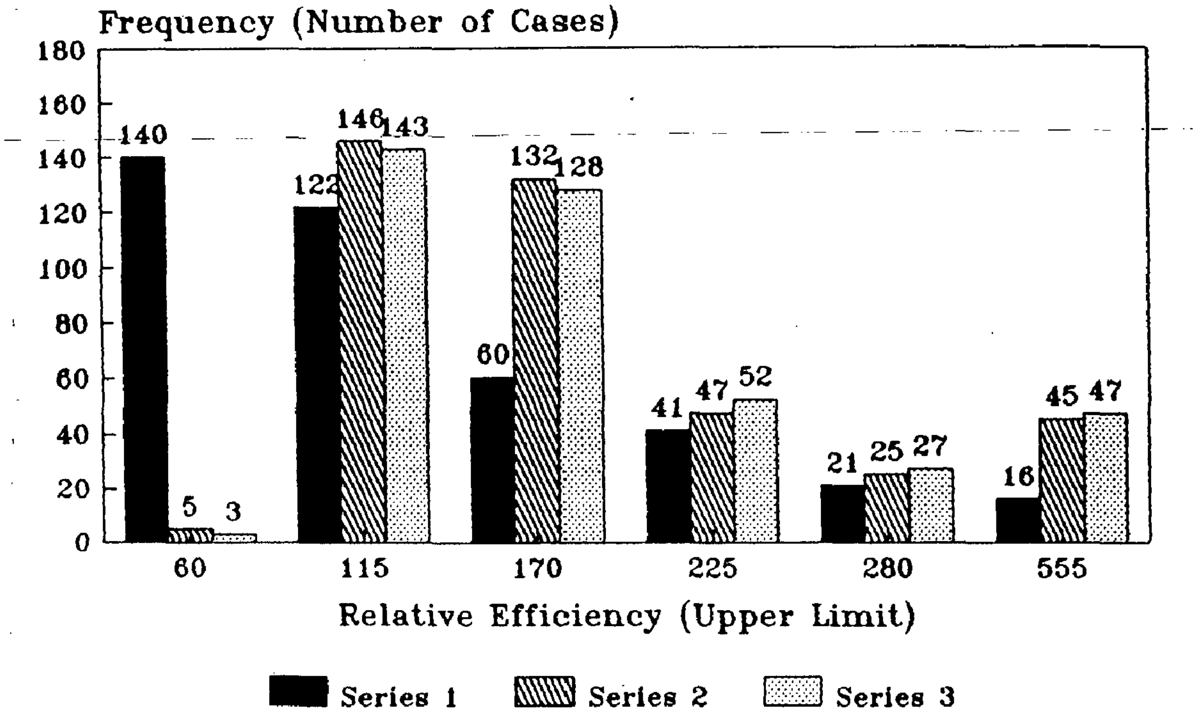
Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_{Re.}$	$\bar{Y}_{VR}$
n=10	0.037	0.088	0.632	0.243
n=20	0.060	0.095	0.660	0.185

Thus, we propose the following mixing-type estimator.

$$\text{MEST}(21) = p_{10}.\bar{Y}_{Re.} + (1-p_{10}).\bar{Y}_{VR} \qquad \ldots(5.12)$$

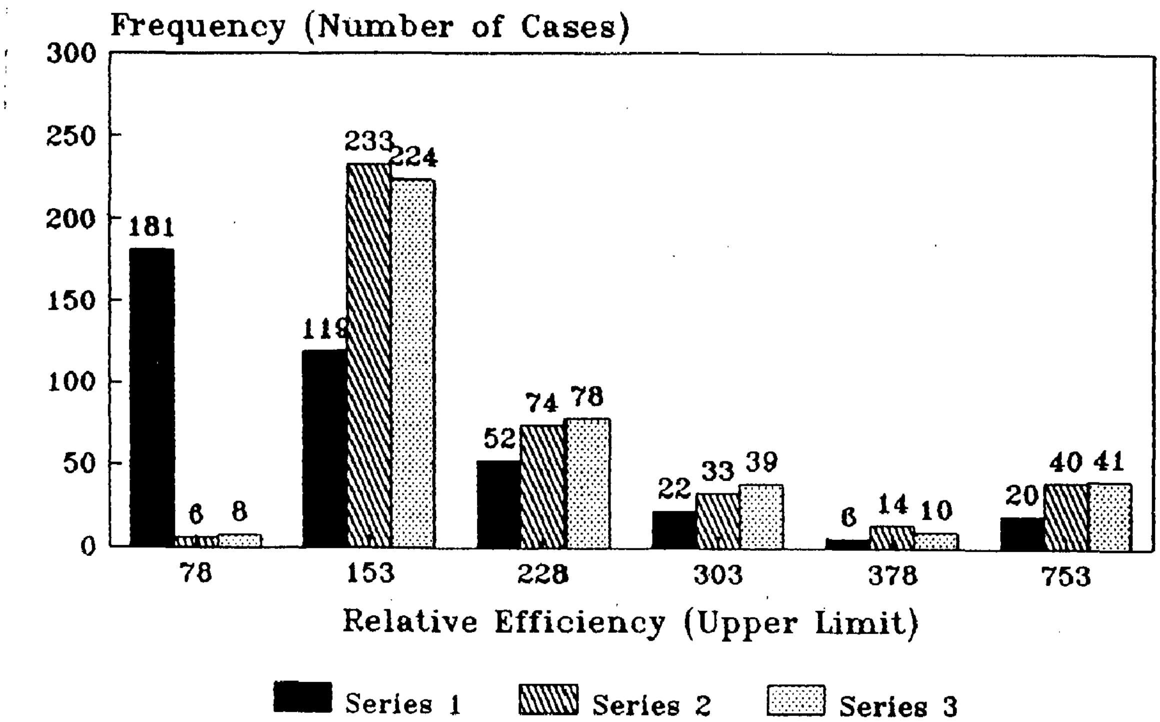
where, p_{10}=0.75 as per the results of table 5.19. Thus, $\bar{Y}_{Re.}$ is

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}^{(2)}_{RP.ab}$ and Series 3 for MEST(20)



Graph – 5.17 (Sample Size = 10, $\rho > 0$)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}^{(2)}_{RP.ab}$ and Series 3 for MEST(20)



Graph – 5.18 (Sample Size = 20, $\rho > 0$)

the most probable winner when compared with $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_{VR}$. So, we now compare MEST(21) with $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_{Re.}$. The results of the empirical-simulation study for these comparisons are tabulated below in table 5.20 and displayed per the graphs 5.19 and 5.20.

TABLE 5.20

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{Re.}$ and MEST(21)

Estimators → / Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_{Re.}$	MEST(21)
n=10	0.100	0.085	0.307	0.508
n=20	0.050	0.075	0.380	0.495

Above table indicates that MEST(21) is the winner more often when compared with $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_{Re.}$. So, it is also an efficient mixing of $\bar{Y}_{Re.}$ and $\bar{Y}_{VR}$. This mixing performs better than $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_{Re.}$ much more often for those cases when $G > 0.5$.

5.1.2 MIXINGS WHEN $\rho < 0$:

The parent-estimators for this case have been given in 5.2. The different mixing-type estimators for these parent estimators are being proposed in this sub-section.

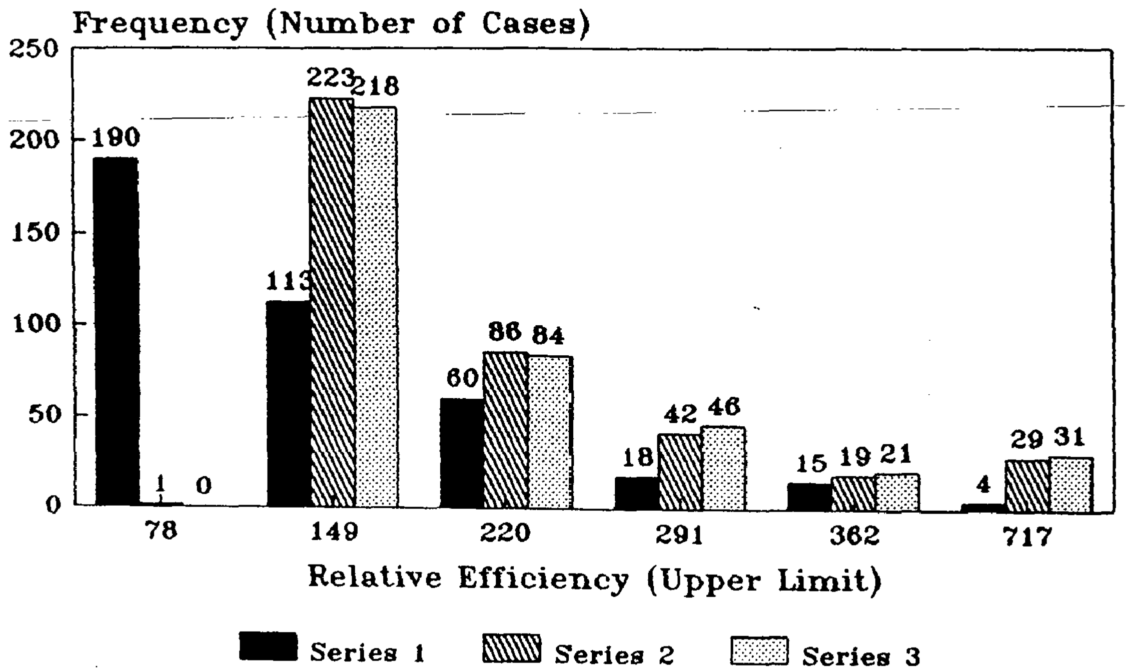
We propose the first mixing-type estimator in this sub-section as a mixing of $\bar{Y}_{Re.}$ and $\bar{Y}_{VSa.}$. For deciding the weights of $\bar{Y}_{Re.}$ and $\bar{Y}_{VSa.}$ in the proposed mixing, we first compare $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{Re.}$ and $\bar{Y}_{VSa.}$. The results of these comparisons are given below in table 5.21.

TABLE 5.21

RF(∘) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{Re.}$ and $\bar{Y}_{VSa.}$

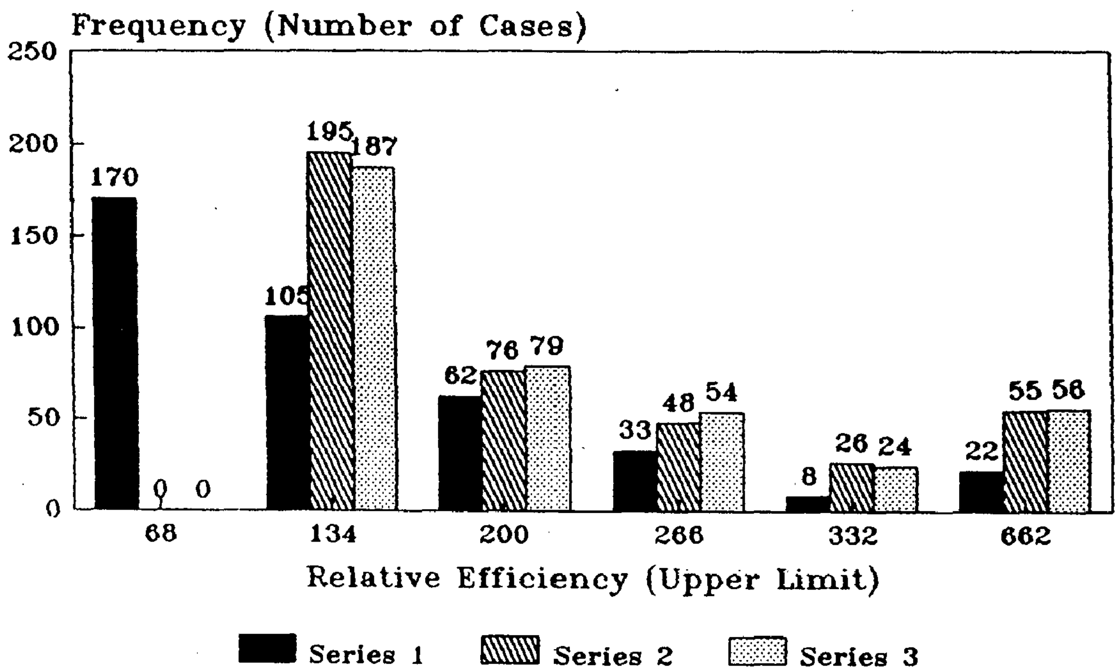
Estimators → / Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{Re.}$	$\bar{Y}_{VSa.}$
n=10	0.042	0.100	0.343	0.515
n=20	0.032	0.073	0.420	0.475

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{Re.}$ and
Series 3 for MEST(21)



Graph – 5.19 (Sample Size = 10, $\rho > 0$)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{Re.}$ and
Series 3 for MEST(21)



Graph – 5.20 (Sample Size = 20, $\rho > 0$)

Based on the above results, we propose the following mixing-type estimator.

$$MEST(22) = n_1.\bar{Y}_{Re.} + (1-n_1).\bar{Y}_{VSa.} \qquad ...(5.13)$$

where, n_1=0.44. This value of 'n_1' suggests that $\bar{Y}_{VSa.}$ is a more probable winner when compared with $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}_{Re.}$. So, we now carry out the empirical-simulation study for comparing $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{VSa.}$ and MEST(22). Table 5.22 contains the results of this study and graphs 5.21 and 5.22 afford us a clearer view of the relative efficiencies of these estimators.

TABLE 5.22

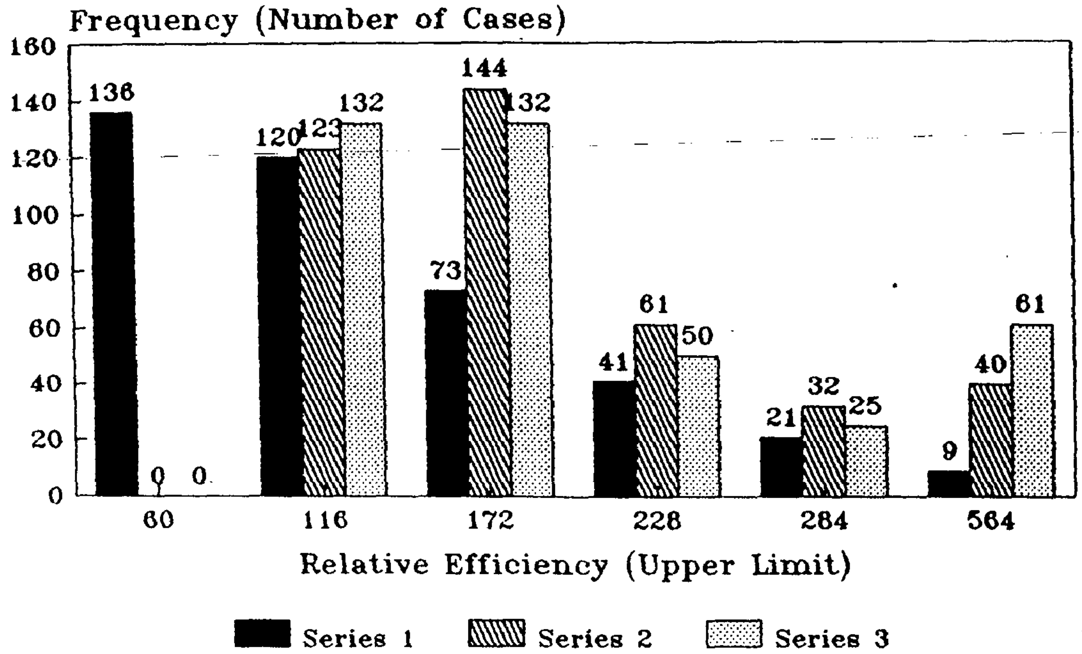
RF(∘) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{VSa.}$ and MEST(22)

Estimators → / Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{VSa.}$	MEST(22)
n=10	0.030	0.098	0.392	0.480
n=20	0.035	0.077	0.343	0.545

We note from the above table that MEST(22) turns out to be more efficient than $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}_{VSa.}$, more often. So, we can recommend the use of MEST(22) instead of $\bar{Y}_{Re.}$ and $\bar{Y}_{VSa.}$. It has also been observed through the finer comparisons of the estimators that MEST(22) performs exceptionally good when $G \in [-1, 0.5]$ and takes a slight lead over $\bar{Y}_P$ for those cases when $G < -1.0$.

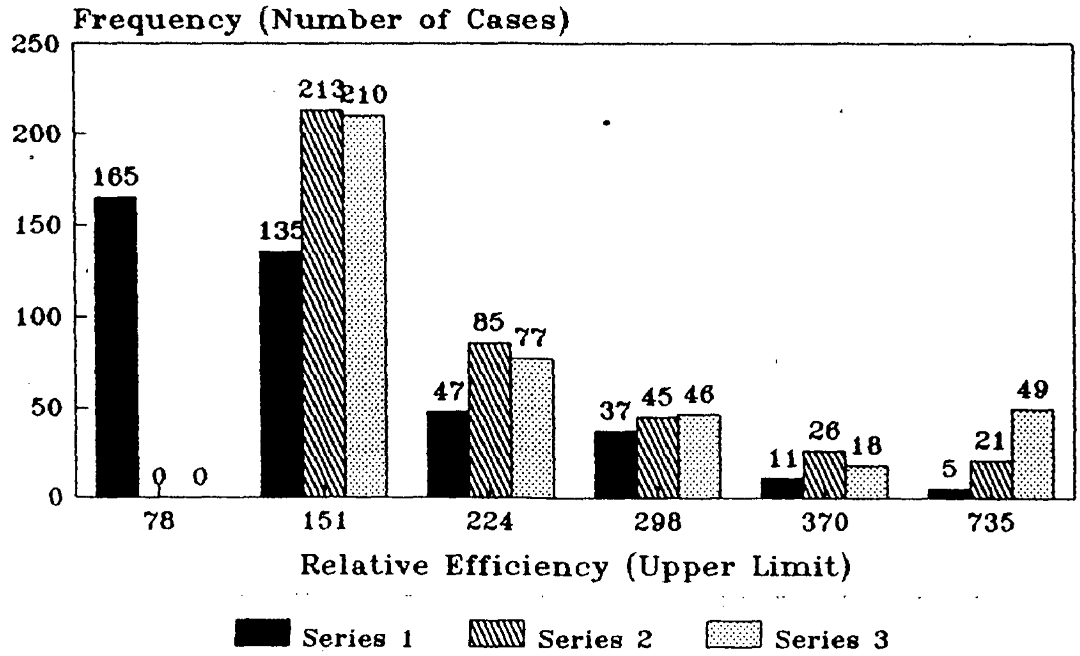
Now, we take up the simulation study for the comparison of the estimators $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{Re.}$ and $\bar{Y}_{MP}$. Table 5.23 contains the results of this study.

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{VSa}$ and
Series 3 for MEST(22)



Graph – 5.21 (Sample Size = 10, $\varrho < 0$)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{VSa}$ and
Series 3 for MEST(22)



Graph – 5.22 (Sample Size = 20, $\varrho < 0$)

TABLE 5.23

RF(∘) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{Re.}$ and $\bar{Y}_{MP}$

Estimators → / Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{Re.}$	$\bar{Y}_{MP}$
n=10	0.060	0.107	0.570	0.263
n=20	0.052	0.100	0.660	0.188

The above table indicates that $\bar{Y}_{Re.}$ performs better than $\bar{Y}_{MP}$, quite often. A mixing-type estimator for $\bar{Y}_{Re.}$ and $\bar{Y}_{MP}$ can be proposed to be:

$$\text{MEST}(23) = n_2 . \bar{Y}_{Re.} + (1-n_2) . \bar{Y}_{MP} \qquad \ldots(5.14)$$

where, n_2=0.73 as per the results of the table 5.23. We compare this estimator MEST(23) with $\bar{Y}_{Re.}$ and the usual estimators $\bar{y}$ and $\bar{Y}_P$. The results of this study are being tabulated in table 5.24. Graphs 5.23 and 5.24 contain a clearer view of the relative efficiencies of these estimators.

TABLE 5.24

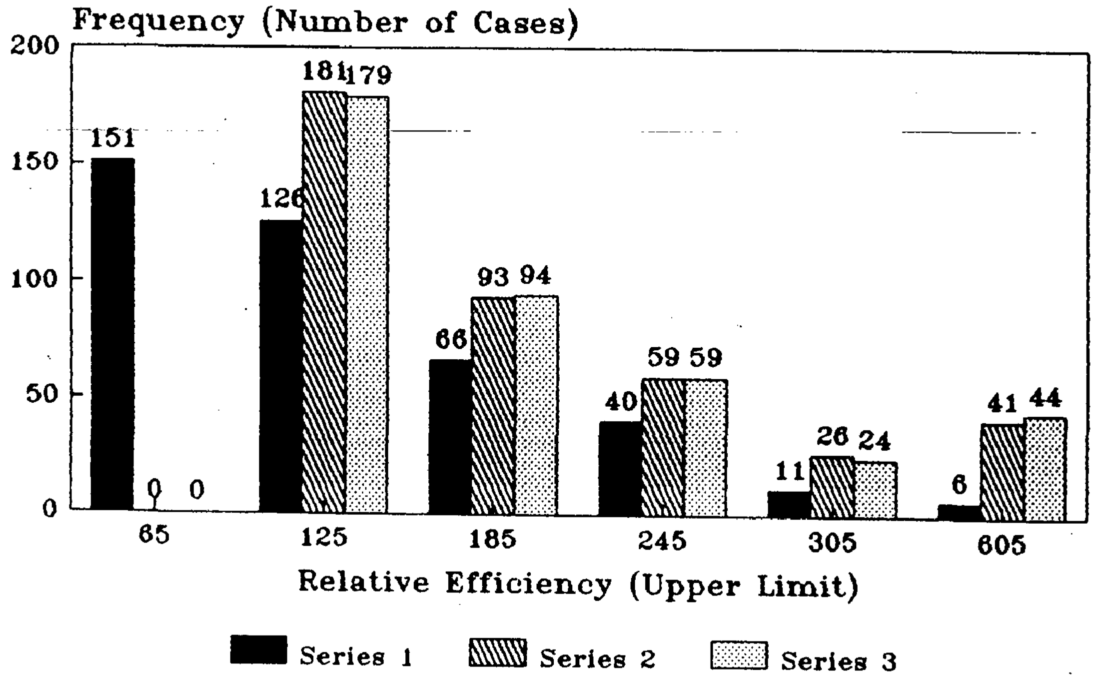
RF(∘) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{Re.}$ and MEST(23)

Estimators → / Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{Re.}$	MEST(23)
n=10	0.085	0.122	0.433	0.360
n=20	0.052	0.100	0.413	0.435

One can observe from the above table that MEST(23) will be a better choice than $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}_{Re.}$ for relatively larger sample-sizes(n=20, here) and for this case it beats the other estimators, quite often when $G \in [-1, -0.5]$ or $G < -1.0$. It does also perform better than $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}_{Re.}$ when $G \in [-1, -0.5]$ and n=10.

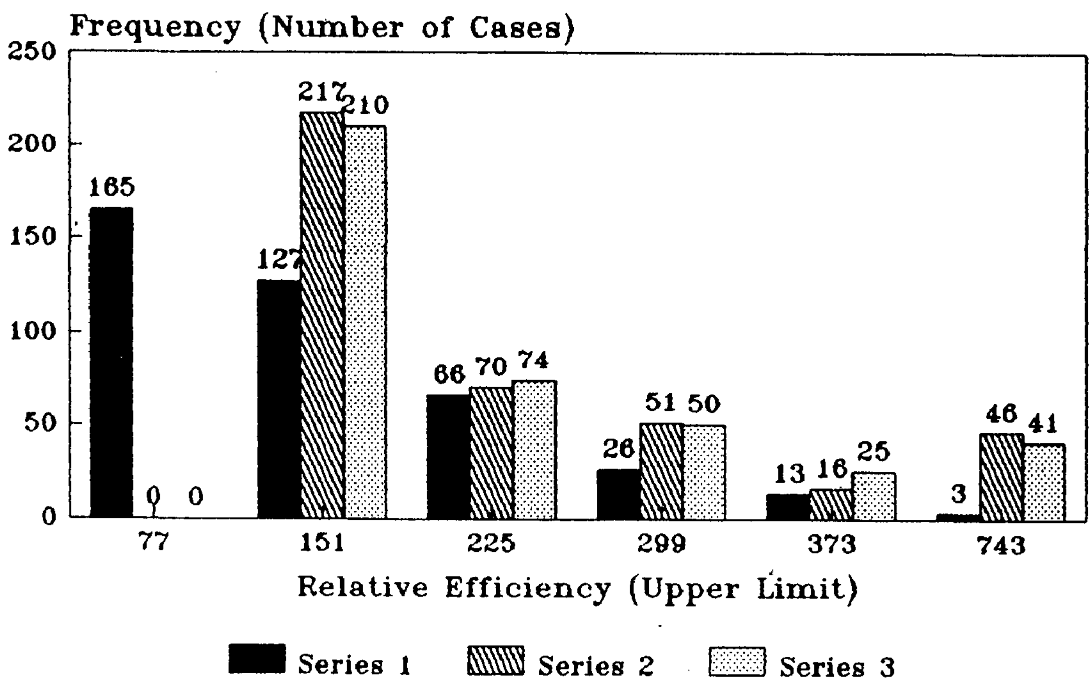
Now, we propose a mixing-type estimator of $\bar{Y}_{Re.}$ and $\bar{Y}^{(1)}_{RP.ab}$.

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{Re.}$ and
Series 3 for MEST(23)



Graph – 5.23 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{Re.}$ and
Series 3 for MEST(23)



Graph – 5.24 (Sample Size = 20, $\rho < 0$)

For this, we first carry out the simulation study to compare $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{Re.}$ and $\bar{Y}^{(1)}_{RP.ab}$. Table 5.25 contains the results of this study.

TABLE 5.25

RF(∘) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{Re.}$ and $\bar{Y}^{(1)}_{RP.ab}$

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{Re.}$	$\bar{Y}^{(1)}_{RP.ab}$
n=10	0.065	0.145	0.595	0.195
n=20	0.037	0.113	0.562	0.288

Based on the above results, we propose the following mixing-type estimator.

$$\text{MEST}(24) = n_3.\bar{Y}_{Re.} + (1-n_3).\bar{Y}^{(1)}_{RP.ab} \qquad \ldots(5.15)$$

where, n_3=0.71 as per the results of table 5.25. Since $\bar{Y}_{Re.}$ is the most probable winner when compared with $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}^{(1)}_{RP.ab}$, we now compare MEST(24) with $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}_{Re.}$. The results of these comparisons are contained in table 5.26 and the graphs 5.25 and 5.26 give us a more comprehensive view of the relative efficiencies of these estimators.

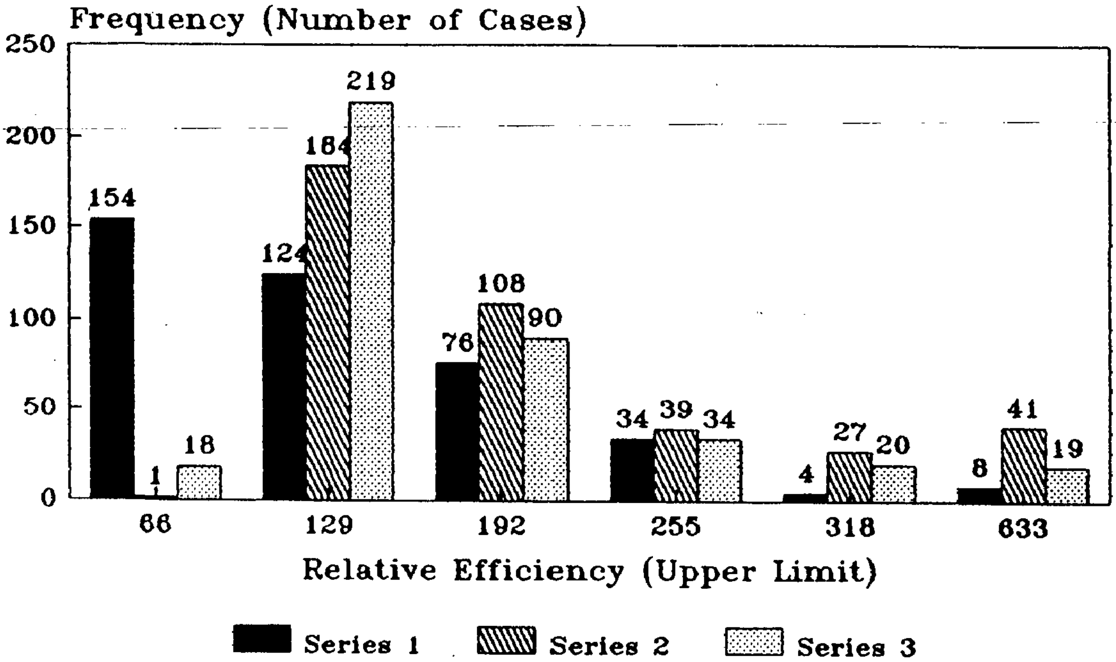
TABLE 5.26

RF(∘) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{Re.}$ and MEST(24)

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{Re.}$	MEST(24)
n=10	0.070	0.197	0.453	0.280
n=20	0.057	0.138	0.407	0.398

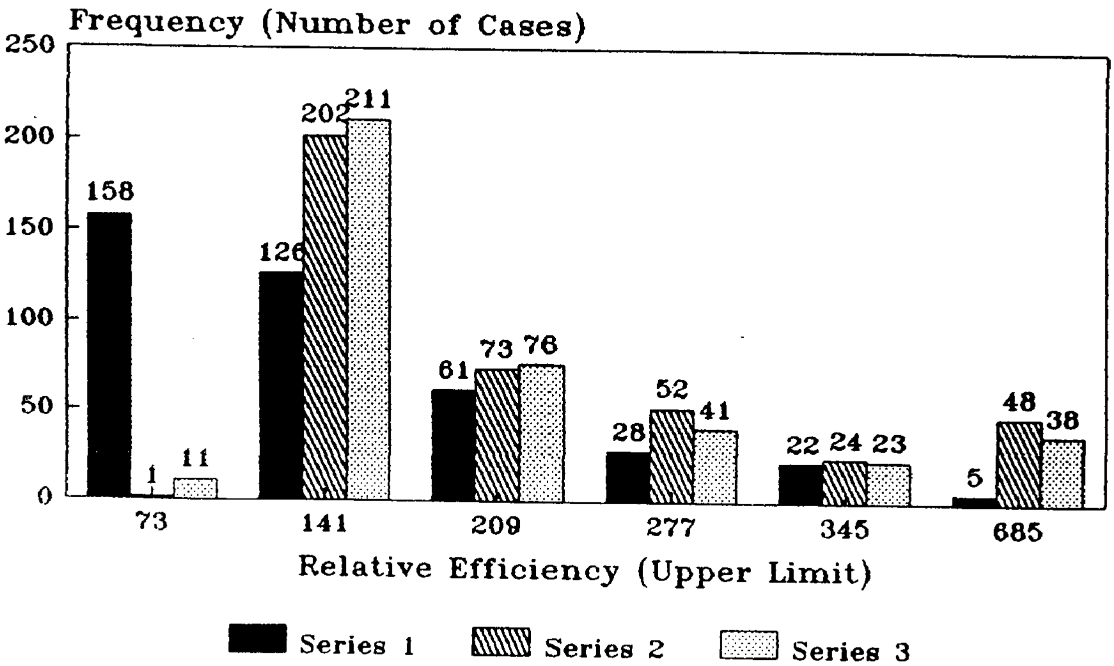
Above table is indicative of the fact that the mixing-type estimator MEST(24) is very often not more efficient than the other estimators in competition. It has also been observed that MEST(24) performs good only for those cases when $G \in [-0.5, 0]$.

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{Re.}$ and
Series 3 for MEST(24)



Graph – 5.25 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{Re.}$ and
Series 3 for MEST(24)



Graph – 5.26 (Sample Size = 20, $\rho < 0$)

Now, we proceed to propose a mixing-type estimator for $\bar{Y}_{Re.}$ and $\bar{Y}_{P.a}$. The results of the empirical-simulation study for comparing $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{Re.}$ and $\bar{Y}_{P.a}$ are being tabulated below in table 5.27.

TABLE 5.27

RF($\circ$) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{Re.}$ and $\bar{Y}_{P.a}$

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{Re.}$	$\bar{Y}_{P.a}$
n=10	0.080	0.107	0.350	0.463
n=20	0.067	0.105	0.388	0.440

Relying on the results of table 5.27, we propose the following mixing of $\bar{Y}_{Re.}$ and $\bar{Y}_{P.a}$.

$$MEST(25) = n_4.\bar{Y}_{Re.} + (1-n_4).\bar{Y}_{P.a} \qquad \ldots(5.16)$$

where, n_4=0.45. The value of n_4 suggests that $\bar{Y}_{P.a}$ is the most probable winner amongst $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{Re.}$ and $\bar{Y}_{P.a}$. So, we now proceed to compare MEST(25) with $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}_{P.a}$. The results of the empirical-simulation study for these comparisons are tabulated below in table 5.28 and the relative efficiencies of these estimators are being displayed graphically per the graphs 5.27 and 5.28.

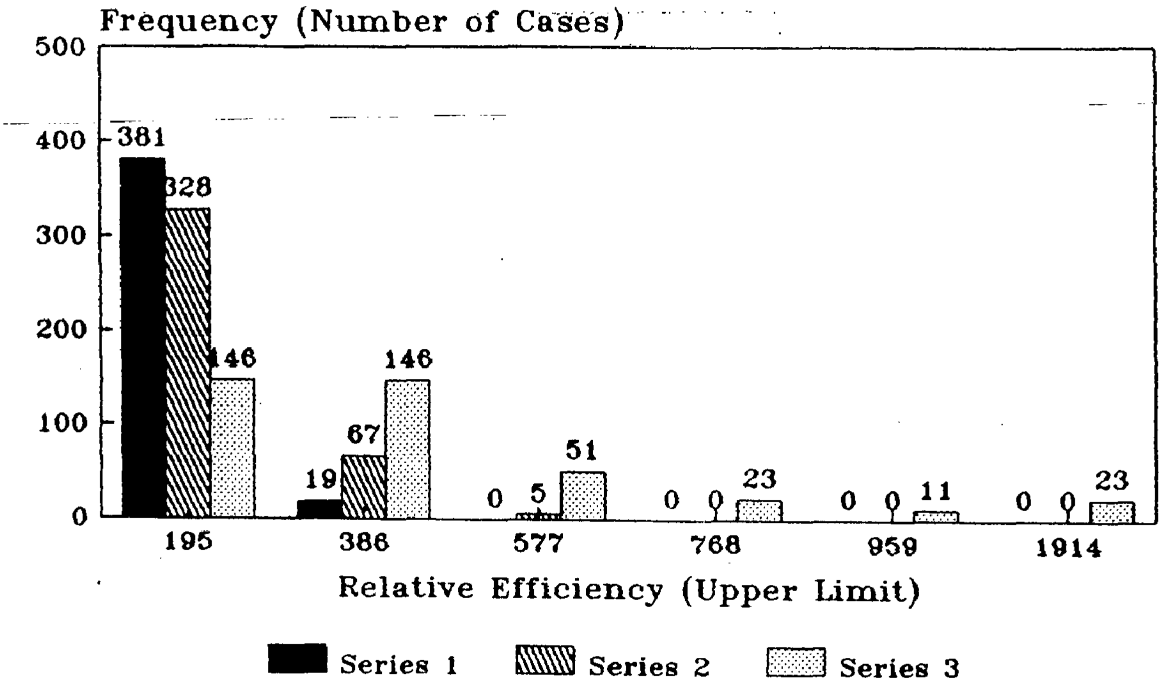
TABLE 5.28

RF($\circ$) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{P.a}$ and MEST(25)

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{P.a}$	MEST(25)
n=10	0.000	0.067	0.008	0.925
n=20	0.000	0.042	0.003	0.955

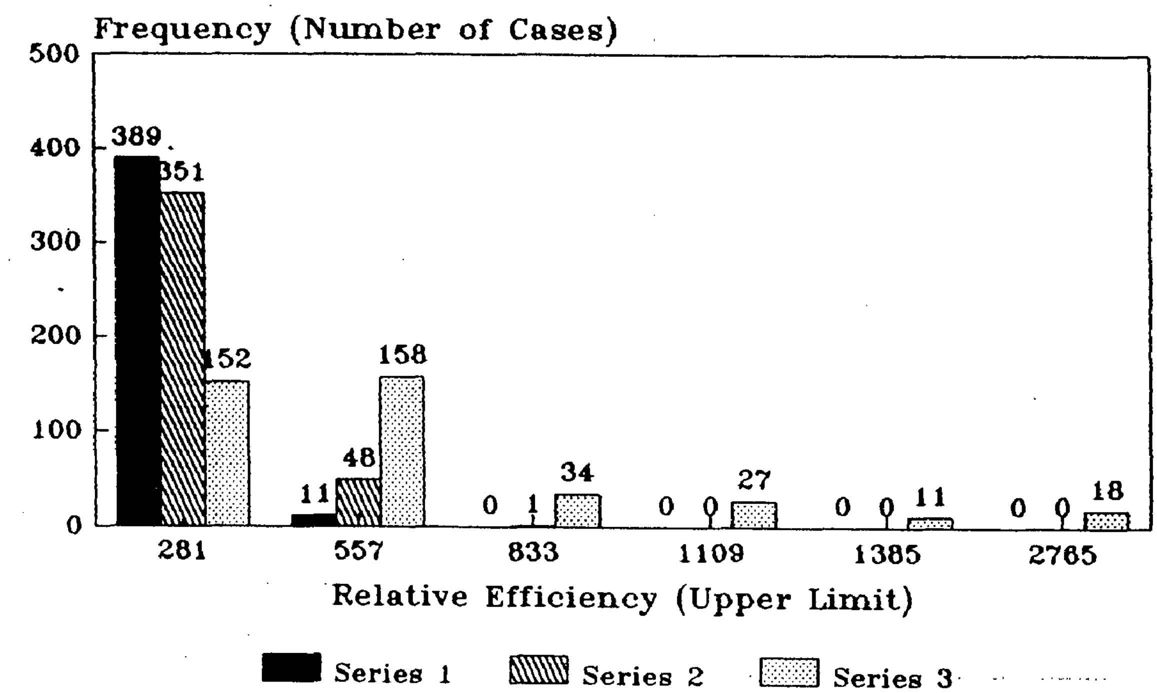
From the above table, we observe that MEST(25) is simply an excellent mixing-estimator almost always.

Series 1 for $\overline{Y}_p$, Series 2 for $\overline{Y}_{p.a}$ and
Series 3 for MEST(25)



Graph - 5.27 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_p$, Series 2 for $\overline{Y}_{p.a}$ and
Series 3 for MEST(25)



Graph - 5.28 (Sample Size = 20, $\rho < 0$)

Next, we propose a mixing-type estimator for $\bar{Y}_{VSa.}$ and $\bar{Y}_{MP}$. The results of the empirical-simulation study for the comparisons of $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{VSa.}$ and $\bar{Y}_{MP}$ are tabulated below in table 5.29.

TABLE 5.29

RF(∘) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{VSa.}$ and $\bar{Y}_{MP}$

Estimators → / Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{VSa.}$	$\bar{Y}_{MP}$
n=10	0.032	0.090	0.650	0.228
n=20	0.035	0.067	0.673	0.225

Based on the results of the above table, we propose the mixing-type estimator for $\bar{Y}_{VSa.}$ and $\bar{Y}_{MP}$ to be:

$$MEST(26) = n_5.\bar{Y}_{VSa.} + (1-n_5).\bar{Y}_{MP} \qquad \ldots(5.17)$$

where, n_5=0.75. This table also indicates that $\bar{Y}_{VSa.}$ is the winner more often when compared with $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}_{MP}$. So, we now compare MEST(26) with $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}_{VSa.}$. The results of the empirical-simulation study carried out for these comparisons are contained in table 5.30. The graphs 5.29 and 5.30 afford us a clearer view of the relative efficiencies of these estimators.

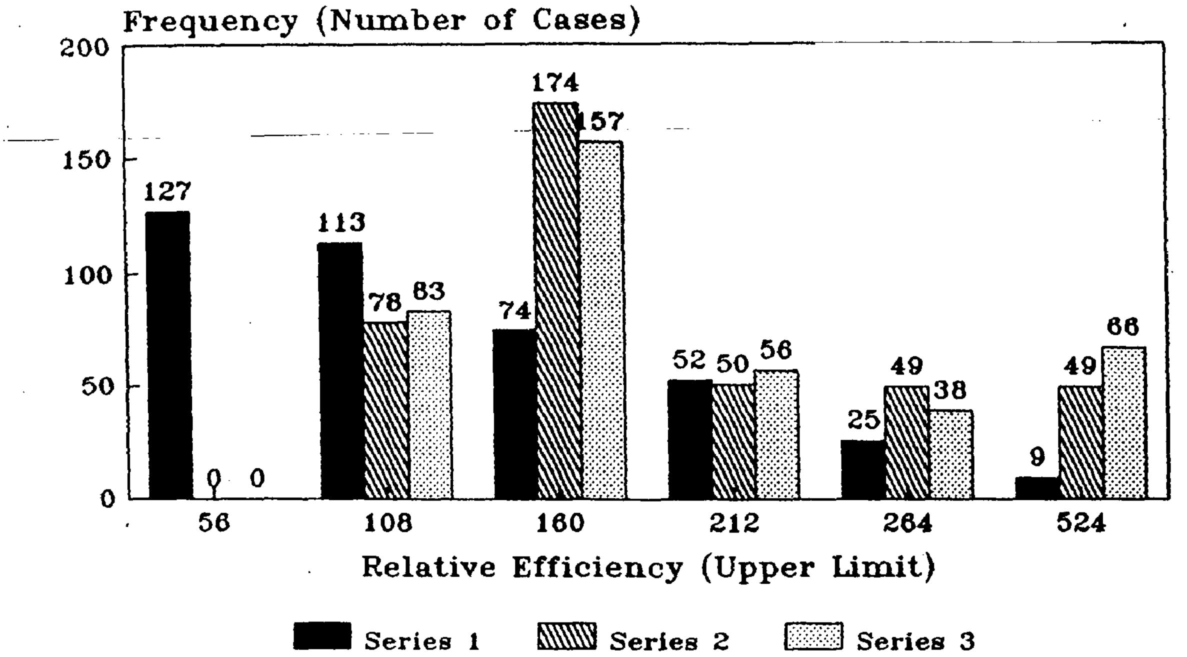
TABLE 5.30

RF(∘) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{VSa.}$ and MEST(26)

Estimators → / Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{VSa.}$	MEST(26)
n=10	0.032	0.060	0.388	0.520
n=20	0.040	0.055	0.395	0.510

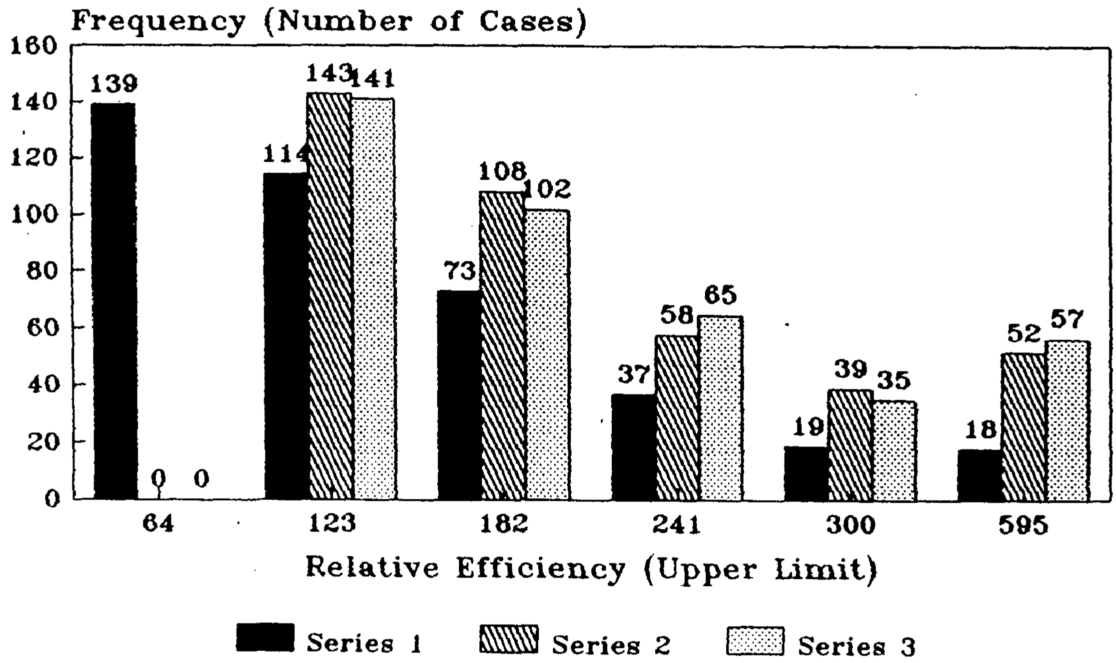
Table 5.30 indicates that MEST(26) is a more probable winner when compared with $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}_{VSa.}$. As such one could recommend the use of this mixing-type estimator instead of $\bar{Y}$, $\bar{Y}_P$, $\bar{Y}_{VSa.}$ and $\bar{Y}_{MP}$. It has also been observed through the finer comparisons of these

Series 1 for $\overline{Y}_p$, Series 2 for $\overline{Y}_{VSa.}$ and Series 3 for MEST(26)



Graph - 5.29 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_p$, Series 2 for $\overline{Y}_{VSa.}$ and Series 3 for MEST(26)



Graph - 5.30 (Sample Size = 20, $\rho < 0$)

estimators that MEST(26) comes out to be winner more often when compared with $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}_{VSa.}$ when G < -0.5.

In order to propose a mixing-type estimator of $\bar{Y}_{VSa.}$ and $\bar{Y}^{(1)}_{RP.ab}$, we now carry out the empirical-simulation study for comparing $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{VSa.}$ and $\bar{Y}^{(1)}_{RP.ab}$. Table 5.31 contains the results of this study.

TABLE 5.31

RF(∘) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{VSa.}$ and $\bar{Y}^{(1)}_{RP.ab}$

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{VSa.}$	$\bar{Y}^{(1)}_{RP.ab}$
n=10	0.027	0.085	0.690	0.198
n=20	0.035	0.075	0.560	0.330

Following the results of table 5.31, we propose the following mixing-type estimator.

$$MEST(27) = n_6 . \bar{Y}_{VSa.} + (1-n_6) . \bar{Y}^{(1)}_{RP.ab} \quad \ldots(5.18)$$

where, n_6=0.70. In order to show the betterment of MEST(27) over $\bar{Y}_{VSa.}$, which is the most probable winner of the comparisons of $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{VSa.}$ and $\bar{Y}^{(1)}_{RP.ab}$, we compare the two in the presence of usual estimators $\bar{y}$ and $\bar{Y}_P$. Table 5.32 contains the results of these comparisons. We have also displayed the relative efficiencies of these estimators through graphs 5.31 and 5.32.

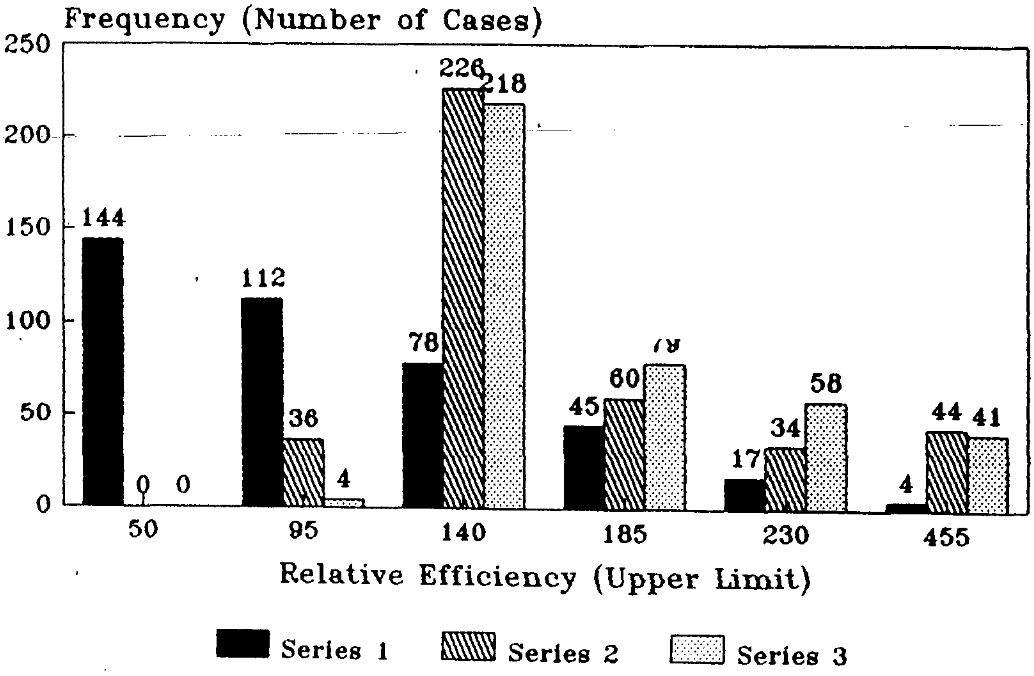
TABLE 5.32

RF(∘) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{VSa.}$ and MEST(27)

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{VSa.}$	MEST(27)
n=10	0.045	0.130	0.280	0.545
n=20	0.052	0.103	0.297	0.548

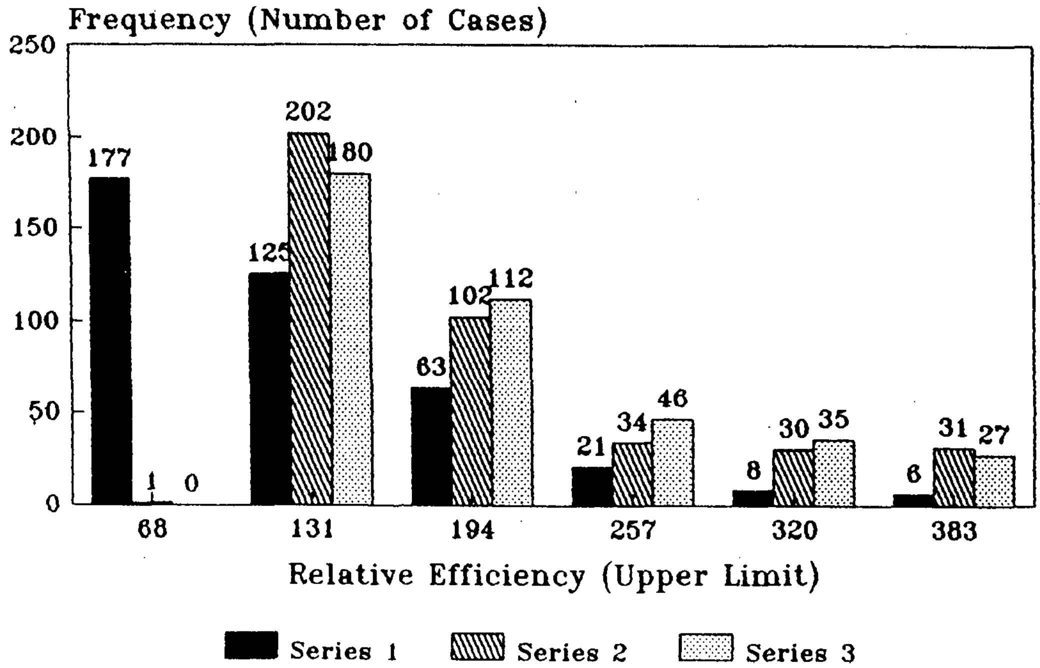
From the above table, one may conclude that MEST(27) performs

Series 1 for $\overline{Y}_p$, Series 2 for $\overline{Y}_{VSa.}$ and
Series 3 for MEST(27)



Graph – 5.31 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_p$, Series 2 for $\overline{Y}_{VSa.}$ and
Series 3 for MEST(27)



Graph – 5.32 (Sample Size = 20, $\rho < 0$)

better than $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}_{VSa.}$, quite often. So, it is an efficient mixing of $\bar{Y}_{VSa.}$ and $\bar{Y}^{(1)}_{RP.ab}$. It has also been observed through the finer comparisons of $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{VSa.}$ and MEST(27) that MEST(27) performs exceptionally good when $G \in [-1, 0]$.

Now, we carry out the empirical-simulation study for comparing the estimators $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{VSa.}$ and $\bar{Y}_{P.a}$. Table 5.33 contains the results of this study.

TABLE 5.33

RF(∘) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{VSa.}$ and $\bar{Y}_{P.a}$

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{VSa.}$	$\bar{Y}_{P.a}$
n=10	0.035	0.115	0.570	0.280
n=20	0.042	0.093	0.567	0.298

Based on the results of the above table, we propose the mixing of $\bar{Y}_{VSa.}$ and $\bar{Y}_{P.a}$ to be:

$$\text{MEST}(28) = n_7.\bar{Y}_{VSa.} + (1-n_7).\bar{Y}_{P.a} \qquad \ldots(5.19)$$

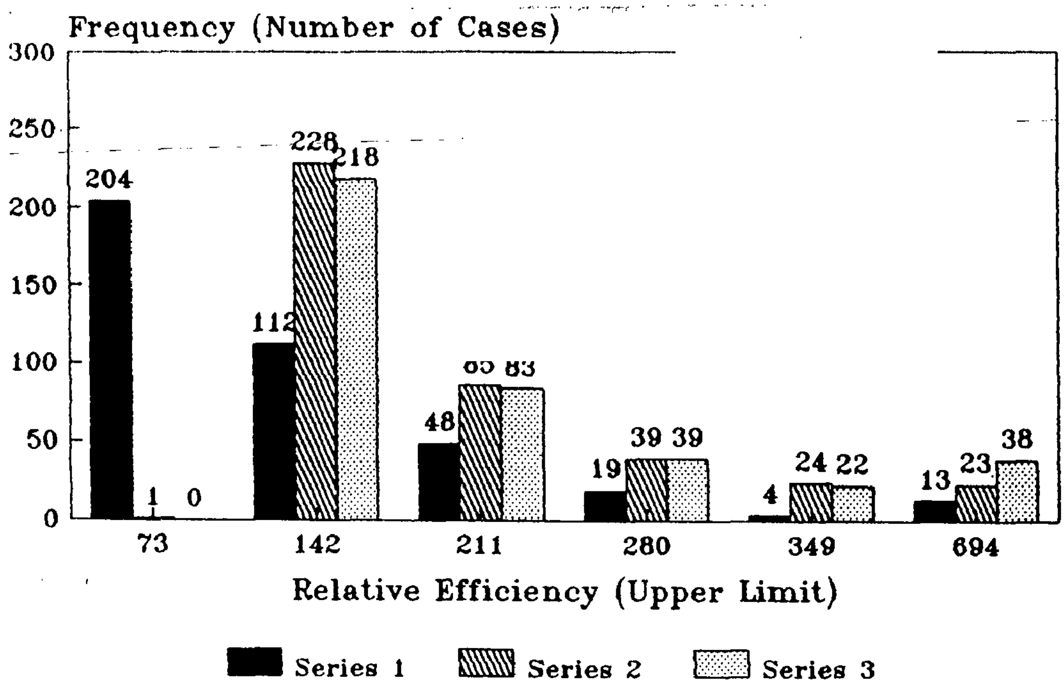
where, n_7=0.66. This mixing-type estimator MEST(28) is now compared with $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}_{Vsa.}$. The results of the empirical-simulation study carried out for these comparisons are tabulated below in table 5.34 and the relative efficiencies of the estimators in the competition has also been displayed graphically through graphs 5.33 and 5.34.

TABLE 5.34

RF(∘) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{VSa.}$ and MEST(28)

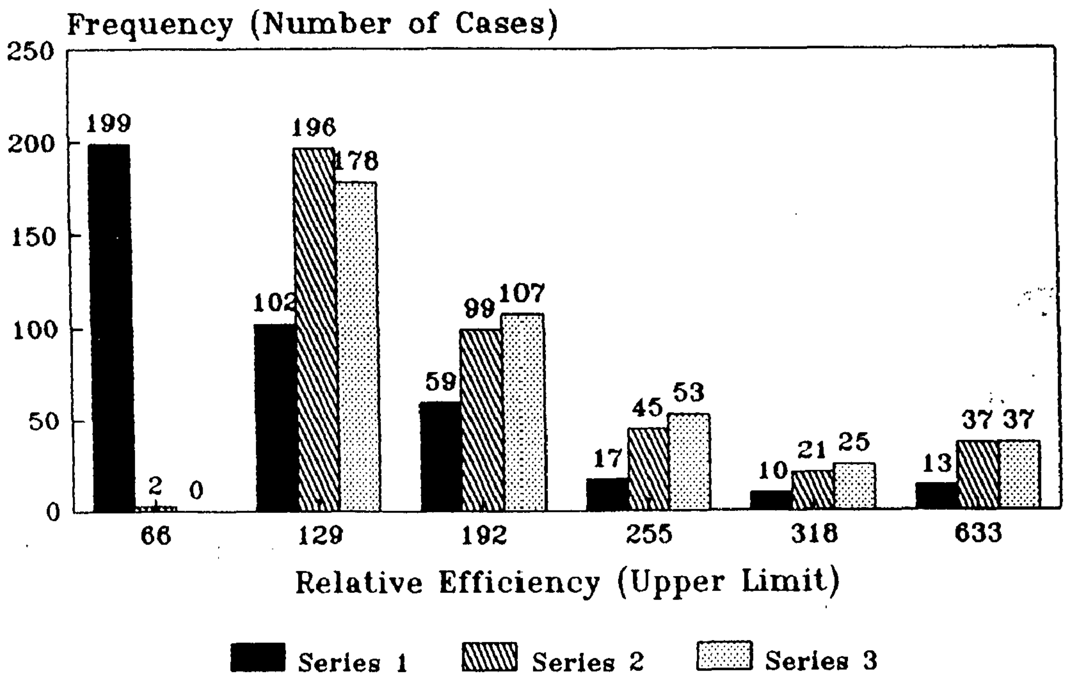
Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{VSa.}$	MEST(28)
n=10	0.057	0.158	0.395	0.390
n=20	0.057	0.115	0.378	0.450

Series 1 for $\bar{Y}_P$, Series 2 for $\bar{Y}_{VSa.}$ and Series 3 for MEST(28)



Graph - 5.33 (Sample Size = 10, $\rho < 0$)

Series 1 for $\bar{Y}_P$, Series 2 for $\bar{Y}_{VSa.}$ and Series 3 for MEST(28)



Graph - 5.34 (Sample Size = 20, $\rho < 0$)

From table 5.34, one can observe that MEST(28) is rather an efficient mixing of $\bar{Y}_{VSa.}$ and $\bar{Y}_{P.a}$. One might recommend the use of MEST(28) for relatively larger sample sizes. It has also been observed through the finer comparisons of $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{VSa.}$ and MEST(28) that MEST(28) comes out to be winner much more often for those cases when $G \in [-1, -0.5]$.

To propose a mixing-type estimator of $\bar{Y}_{MP}$ and $\bar{Y}^{(1)}_{RP.ab}$, we first compare $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{MP}$ and $\bar{Y}^{(1)}_{RP.ab}$ taking together. Table 5.35 contains the results of the empirical-simulation study carried out for these comparisons.

TABLE 5.35

RF(∘) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{MP}$ and $\bar{Y}^{(1)}_{RP.ab}$

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{MP}$	$\bar{Y}^{(1)}_{RP.ab}$
n=10	0.155	0.162	0.395	0.288
n=20	0.110	0.120	0.282	0.488

Based on the results of the above table, we propose the mixing of $\bar{Y}_{MP}$ and $\bar{Y}^{(1)}_{RP.ab}$ to be:

$$\text{MEST}(29) = n_8.\bar{Y}_{MP} + (1-n_8).\bar{Y}^{(1)}_{RP.ab} \qquad \ldots(5.20)$$

where, n_8=0.47. The value of n_8(=0.47) suggests that $\bar{Y}^{(1)}_{RP.ab}$ is a more probable winner in the comparisons of $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{MP}$ and $\bar{Y}^{(1)}_{RP.ab}$. So, we now take up the empirical-simulation study to compare MEST(29) with $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}^{(1)}_{RP.ab}$. Table 5.36 contain the results of this study. We have also presented the relative efficiencies of these estimators through graphs 5.35 and 5.36.

TABLE 5.36

RF(∘) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}^{(1)}_{RP.ab}$ and MEST(29)

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}^{(1)}_{RP.ab}$	MEST(29)
n=10	0.220	0.227	0.245	0.308
n=20	0.115	0.137	0.380	0.368

We can conclude from the above table that MEST(29) comes out to be winner when compared with $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}^{(1)}_{RP.ab}$, quite often, for relatively smaller sample-sizes. It has also been seen that MEST(29) performs exceptionally better than $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}^{(1)}_{RP.ab}$ when $G \in [-1,-0.5]$.

In the table 5.37 below, we tabulate the results of the empirical-simulation study carried out for comparing $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{MP}$ and $\bar{Y}_{P.a}$.

TABLE 5.37

RF(∘) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{MP}$ and $\bar{Y}_{P.a}$

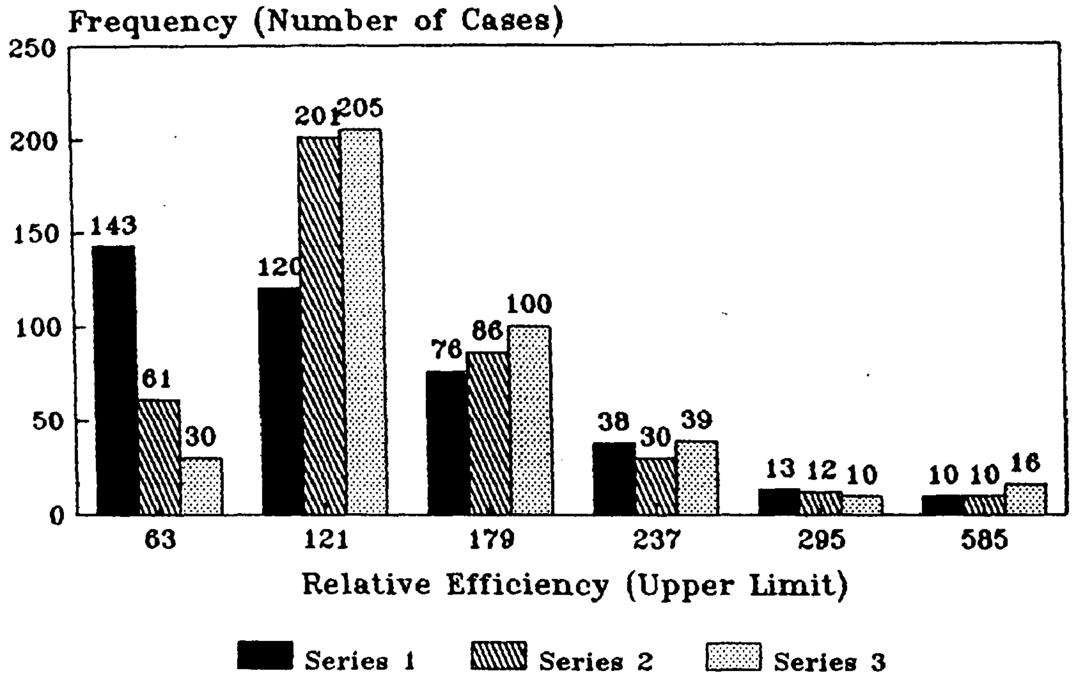
Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{MP}$	$\bar{Y}_{P.a}$
n=10	0.040	0.132	0.355	0.473
n=20	0.045	0.170	0.230	0.555

Relying on the results tabulated above, we propose the following mixing of $\bar{Y}_{MP}$ and $\bar{Y}_{P.a}$.

$$MEST(30) = n_9.\bar{Y}_{MP} + (1-n_9).\bar{Y}_{P.a} \qquad \ldots(5.21)$$

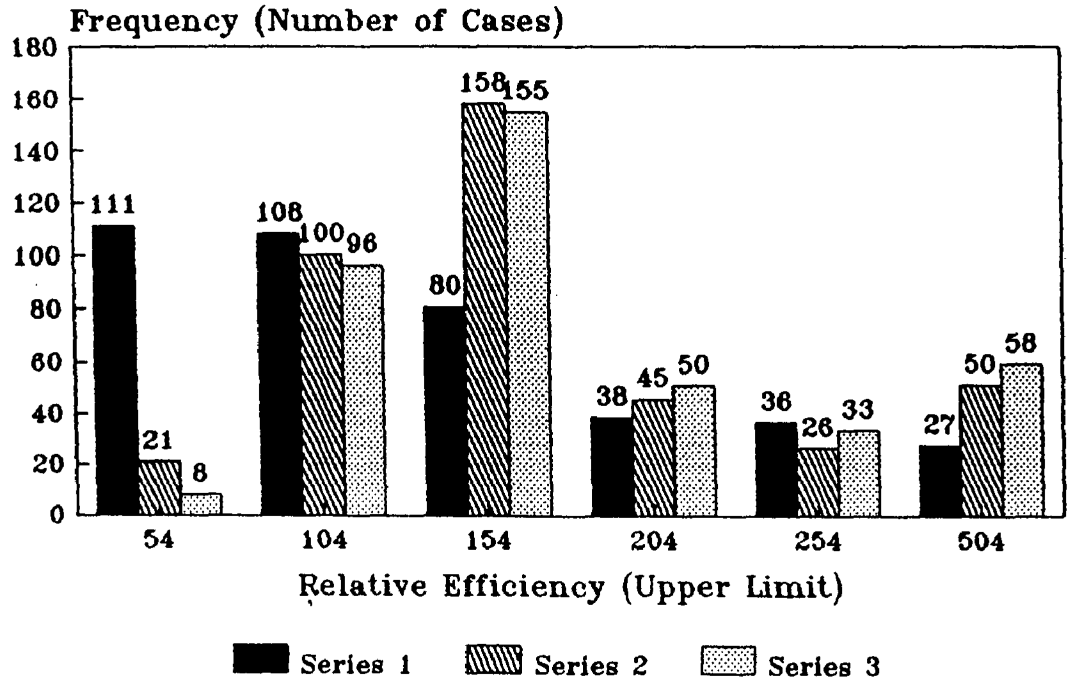
where, n_9=0.36. Table 5.37 also indicates that $\bar{Y}_{P.a}$ is the winner here, quite often. So, we now proceed to compare MEST(30) with $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}_{P.a}$. The results of these comparisons are tabulated below in table 5.38 and the graphs 5.37 and 5.38 afford us a clearer view of the relative efficiencies of these estimators.

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}^{(1)}_{RP.ab}$ and
Series 3 for MEST(29)



Graph - 5.35 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}^{(1)}_{RP.ab}$ and
Series 3 for MEST(29)



Graph - 5.36 (Sample Size = 20, $\rho < 0$)

TABLE 5.38

RF($\circ$) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{P.a}$ and MEST(30)

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{P.a}$	MEST(30)
n=10	0.060	0.127	0.335	0.478
n=20	0.052	0.098	0.327	0.423

One can observe from the above table that we will gain in the relative efficiency by the use of MEST(30) instead of $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}_{P.a}$. We have also observed through the finer comparisons of these estimators based on G-values that MEST(30) performs exceptionally better than the other estimators for those cases when $-1 \le G \le -0.5$.

The last mixing of this type has been proposed by us by mixing the estimators $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}_{P.a}$. We first tabulate below the results of the empirical-simulation study carried out for comparing $\bar{y}$, $\bar{Y}_P$, $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}_{P.a}$.

TABLE 5.39

RF($\circ$) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}_{P.a}$

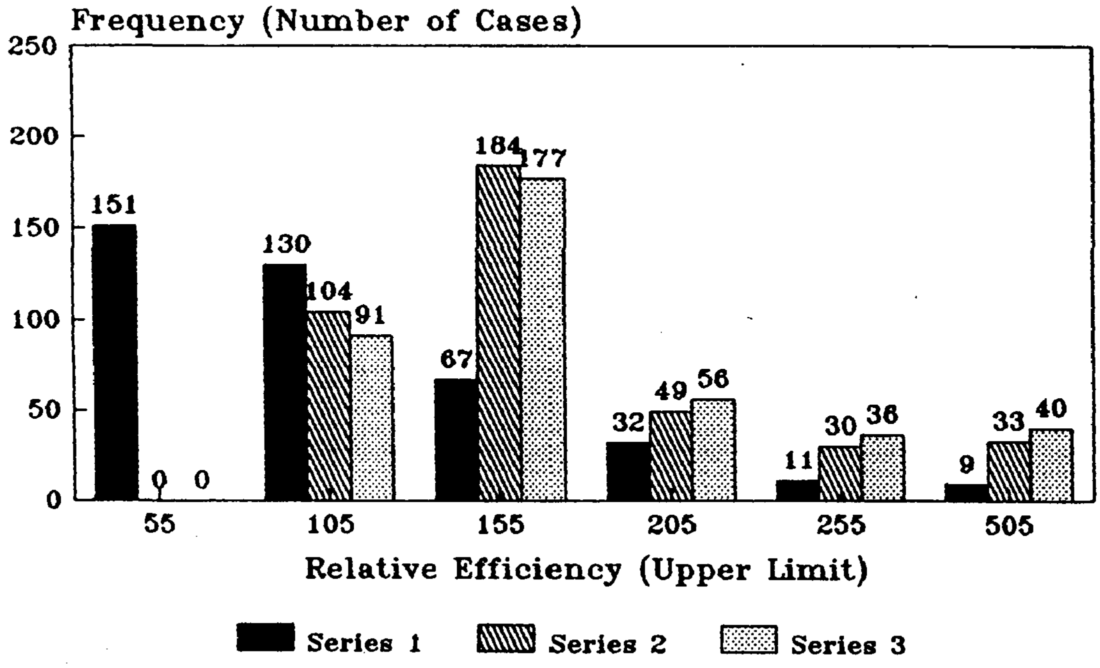
Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}^{(1)}_{RP.ab}$	$\bar{Y}_{P.a}$
n=10	0.062	0.180	0.118	0.640
n=20	0.045	0.150	0.202	0.603

Exploiting the knowledge of the performances of the estimators $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}_{P.a}$, we propose the following mixing estimator.

$$\text{MEST}(31) = n_{10}.\bar{Y}^{(1)}_{RP.ab} + (1-n_{10}).\bar{Y}_{P.a} \qquad \ldots(5.22)$$

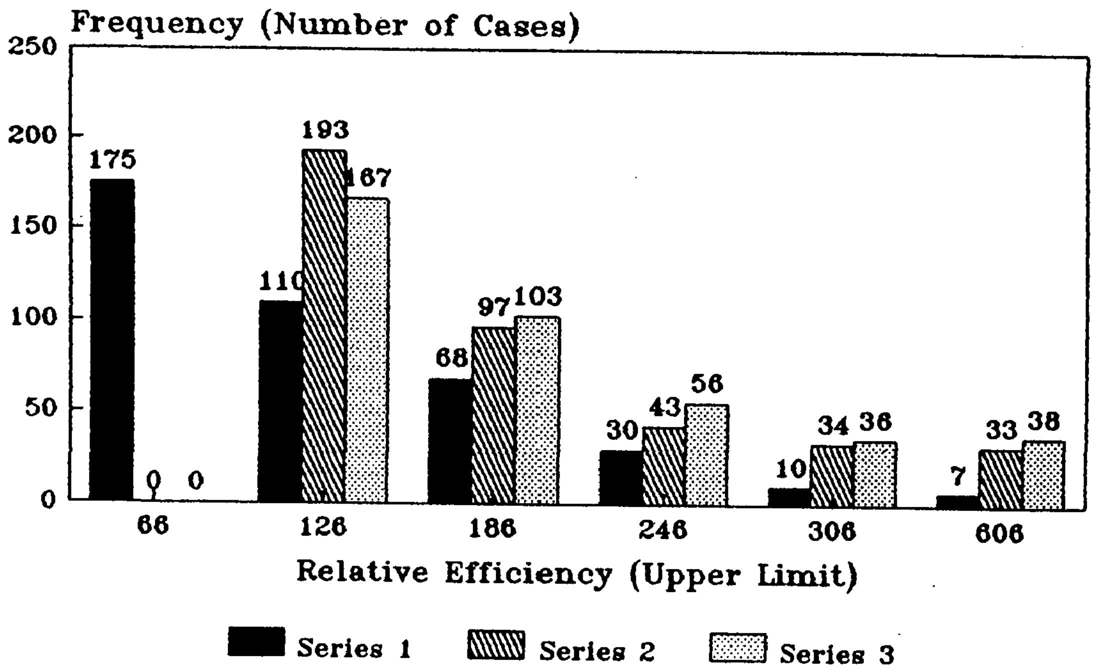
where, n_{10}=0.20 as per the results of table 5.39. Since $\bar{Y}_{P.a}$ is the most probable winner when compared to $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}^{(1)}_{RP.ab}$, we now carry out the empirical-simulation study for comparing MEST(31)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{P.a}$ and
Series 3 for MEST(30)



Graph – 5.37 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{P.a}$ and
Series 3 for MEST(30)



Graph – 5.38 (Sample Size = 20, $\rho < 0$)

with $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}_{P.a}$. Table 5.40 contains the results of this study and graphs 5.39 and 5.40 afford us a clearer view of the relative efficiencies of these estimators.

TABLE 5.40

RF(∘) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{P.a}$ and MEST(31)

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{P.a}$	MEST(31)
n=10	0.072	0.138	0.480	0.310
n=20	0.045	0.112	0.393	0.450

Table 5.40 indicates that MEST(31) is a more probable winner of the comparisons for relatively larger sample sizes when compared with $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}_{P.a}$. It has also been observed that the performance of MEST(31) is exceptionally better than the other estimators in competition here for those cases when $-1 \le G \le -0.5$.

5.2 MIXINGS OF $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ AND THE TWO PARENT-ESTIMATORS :

As mentioned earlier, in this section we propose six linear combinations of $\bar{y}$, $\bar{Y}_R/\bar{Y}_P$ and the two parent-estimators. The cases of positive and negative correlations have been dealt with separately.

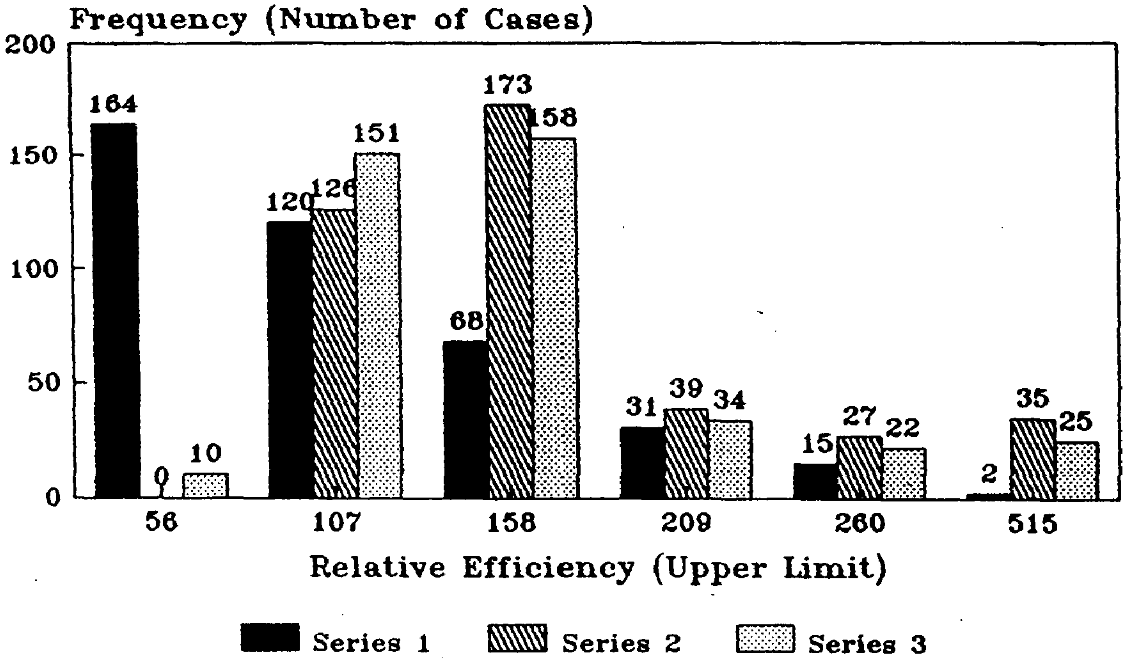
5.2.1 MIXINGS WHEN $\rho > 0$:

For this case, as per 5.1, the three estimators will be $\bar{Y}_{P.a}$, $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}^{(2)}_{RP.ab}$. In what follows, we propose their mixings (taking two of them together) with $\bar{y}$ and $\bar{Y}_R$. First, we propose the mixing of $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{P.a}$ and $\bar{Y}^{(1)}_{RP.ab}$ to be:

$$\text{MEST}(32) = e_1.\bar{y} + f_1.\bar{Y}_R + g_1.\bar{Y}_{P.a} + h_1.\bar{Y}^{(1)}_{RP.ab} \qquad \ldots(5.23)$$

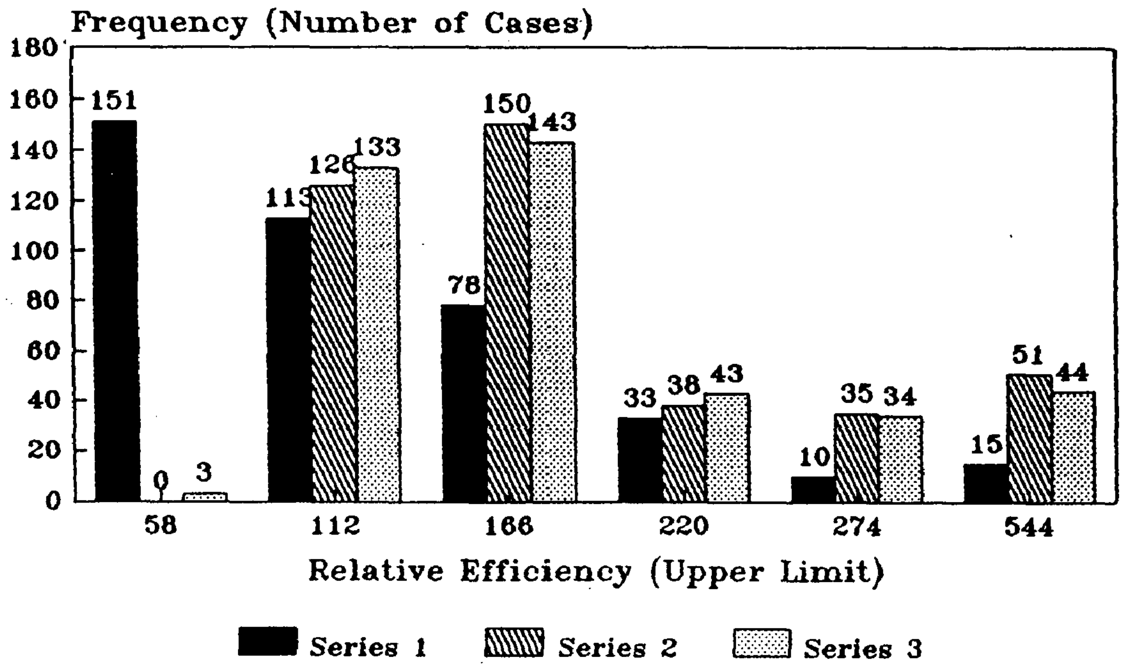
where, $e_1=0.05$, $f_1=0.15$, $g_1=0.46$ as per the results of table 5.1 and $h_1=1-(e_1+f_1+g_1)$. Now, we carry out the empirical-simulation study for comparing MEST(32) with $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}_{P.a}$. The results of

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{P.a}$ and Series 3 for MEST(31)



Graph – 5.39 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{P.a}$ and Series 3 for MEST(31)



Graph – 5.40 (Sample Size = 20, $\rho < 0$)

this study are tabulated below in table 5.41. We have also displayed the relative efficiencies of the estimators $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{P.a}$ and MEST(32) graphically through graphs 5.41 and 5.42.

TABLE 5.41

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{P.a}$ and MEST(32)

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_{P.a}$	MEST(32)
n=10	0.072	0.110	0.530	0.288
n=20	0.055	0.075	0.550	0.320

According to table 5.41, one can say that the proposed mixing does not perform better than $\bar{Y}_{P.a}$ but through the finer comparisons of these estimators, we have observed that MEST(32) comes out to be winner more often when $G \in [0.5, 1]$. It also beats the ratio estimator much more often for those cases when $G > 1.0$.

Now, we propose a mixing of $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{P.a}$ and $\bar{Y}^{(2)}_{RP.ab}$. This mixing, according to the results of table 5.3, comes out to be

$$\text{MEST}(33) = e_2.\bar{y} + f_2.\bar{Y}_R + g_2.\bar{Y}_{P.a} + h_2.\bar{Y}^{(2)}_{RP.ab} \qquad \ldots(5.24)$$

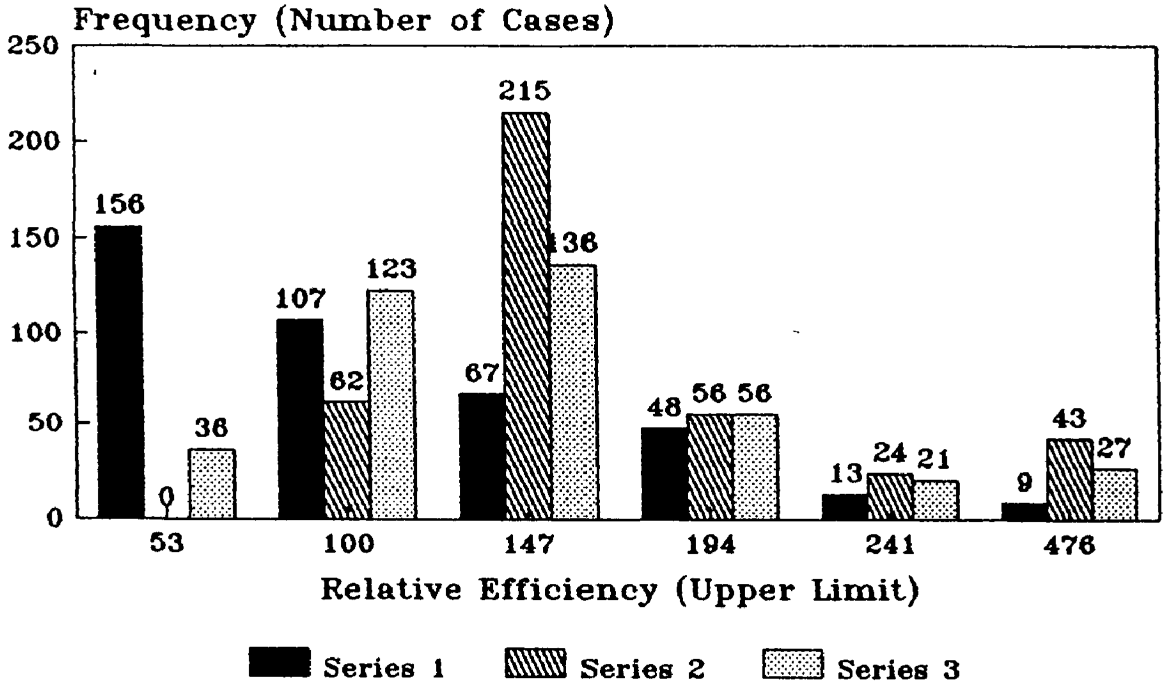
where, $e_2=0.04$, $f_2=0.12$, $g_2=0.48$ and $h_2=1-(e_2+f_2+g_2)$. These weights suggest that $\bar{Y}_{P.a}$ is the most probable winner here. So, we compare MEST(33) with $\bar{Y}_{P.a}$ in the presence of the usual estimators $\bar{y}$ and $\bar{Y}_R$. Table 5.42 contains the results of these comparisons and graphs 5.43 and 5.44 afford us a clearer view of the relative efficiencies of these estimators.

TABLE 5.42

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}_{P.a}$ and MEST(33)

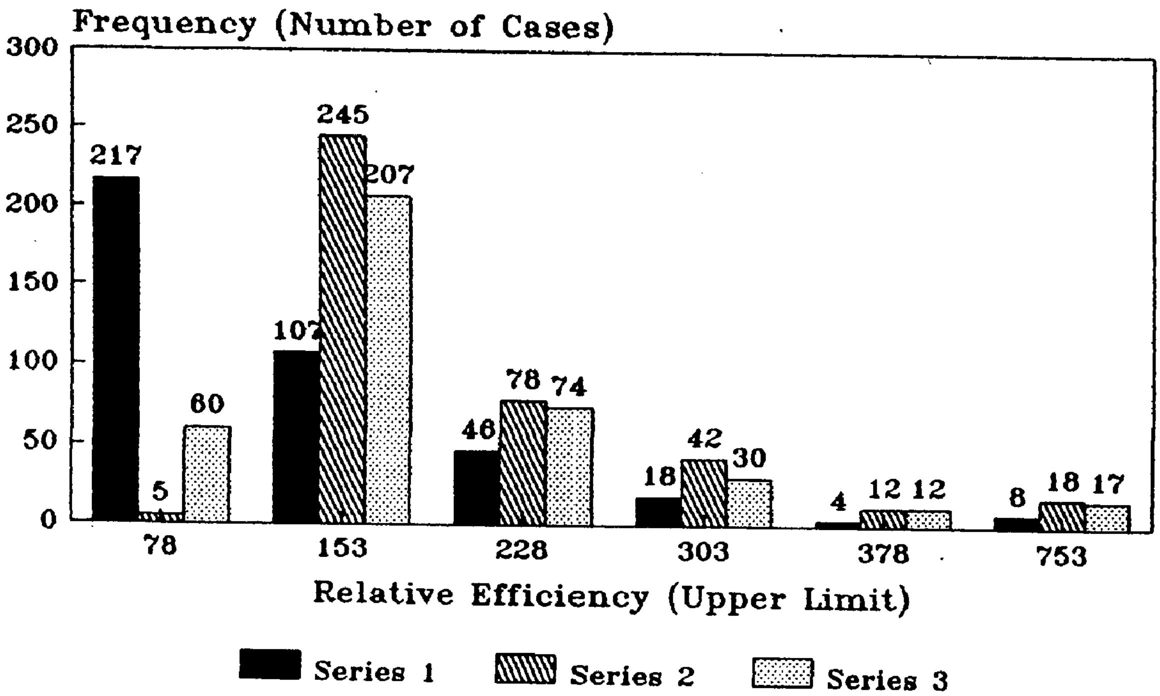
Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}_{P.a}$	MEST(33)
n=10	0.042	0.135	0.543	0.280
n=20	0.058	0.092	0.497	0.353

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{P.a}$ and
Series 3 for MEST(32)



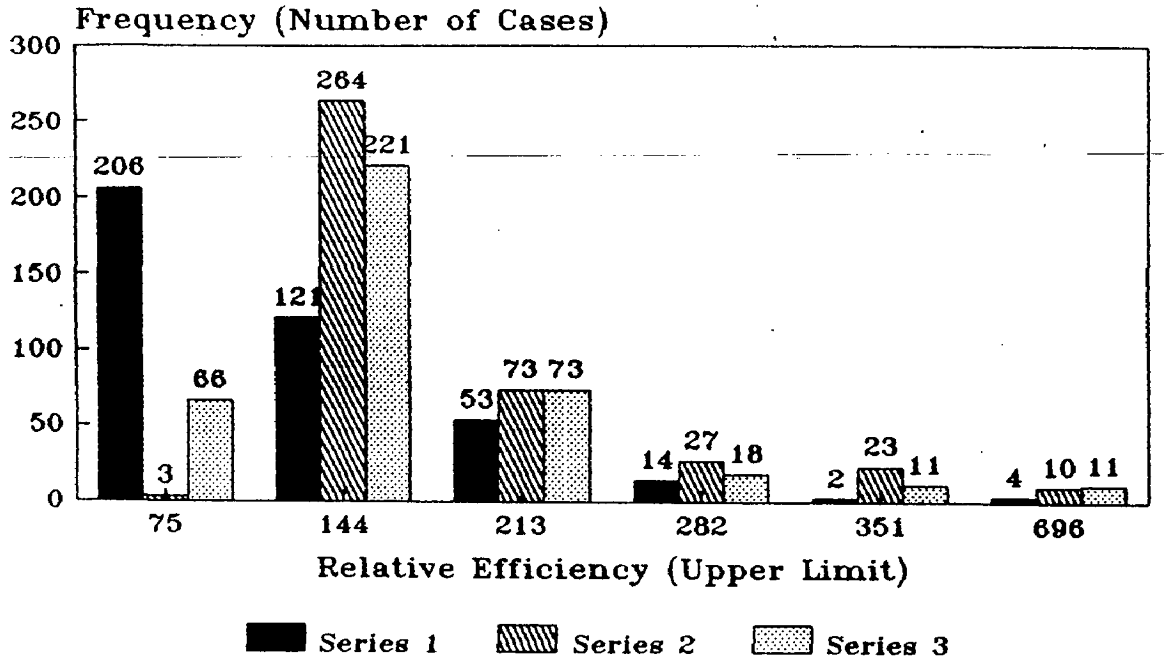
Graph – 5.41 (Sample Size = 10, $\rho > 0$)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{P.a}$ and
Series 3 for MEST(32)



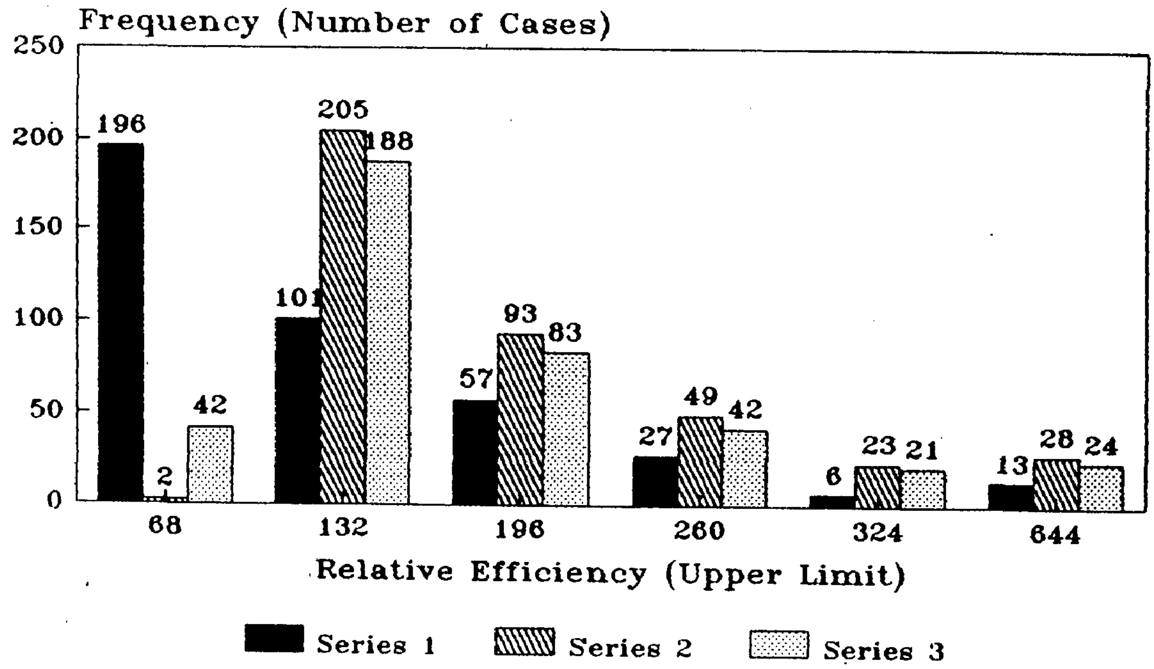
Graph – 5.42 (Sample Size = 20, $\rho > 0$)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{P.a}$ and
Series 3 for MEST(33)



Graph – 5.43 (Sample Size = 10, $\rho > 0$)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}_{P.a}$ and
Series 3 for MEST(33)



Graph – 5.44 (Sample Size = 20, $\rho > 0$)

Table 5.42 indicates that we are again unable to improve $\bar{Y}_{P.a}$. But if we consider the finer comparisons of these estimators based on G-values, we can infer that MEST(33) performs exceptionally better than the other estimators when $G \in [0.5, 1]$ and it strongly beats $\bar{Y}_R$ when $G > 1.0$ and n=20.

Based on the results of table 5.9, the mixing of $\bar{y}$, $\bar{Y}_R$, $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}^{(2)}_{RP.ab}$ can be proposed to be :

$$MEST(34) = e_3.\bar{y} + f_3.\bar{Y}_R + g_3.\bar{Y}^{(1)}_{RP.ab} + h_3.\bar{Y}^{(2)}_{RP.ab} \quad \ldots(5.25)$$

where, $e_3=0.10$, $f_3=0.10$, $g_3=0.36$ and $h_3=1-(e_3+f_3+g_3)$. We have compared MEST(34) with $\bar{Y}^{(2)}_{RP.ab}$ in the presence of usual estimators $\bar{y}$ and $\bar{Y}_R$ via the empirical-simulation study. Table 5.43 contains the results of this study and graphs 5.45 and 5.46 afford us a clearer view of the relative efficiencies of these estimators.

TABLE 5.43

RF(∘) FOR $\bar{y}$, $\bar{Y}_R$, $\bar{Y}^{(2)}_{RP.ab}$ and MEST(34)

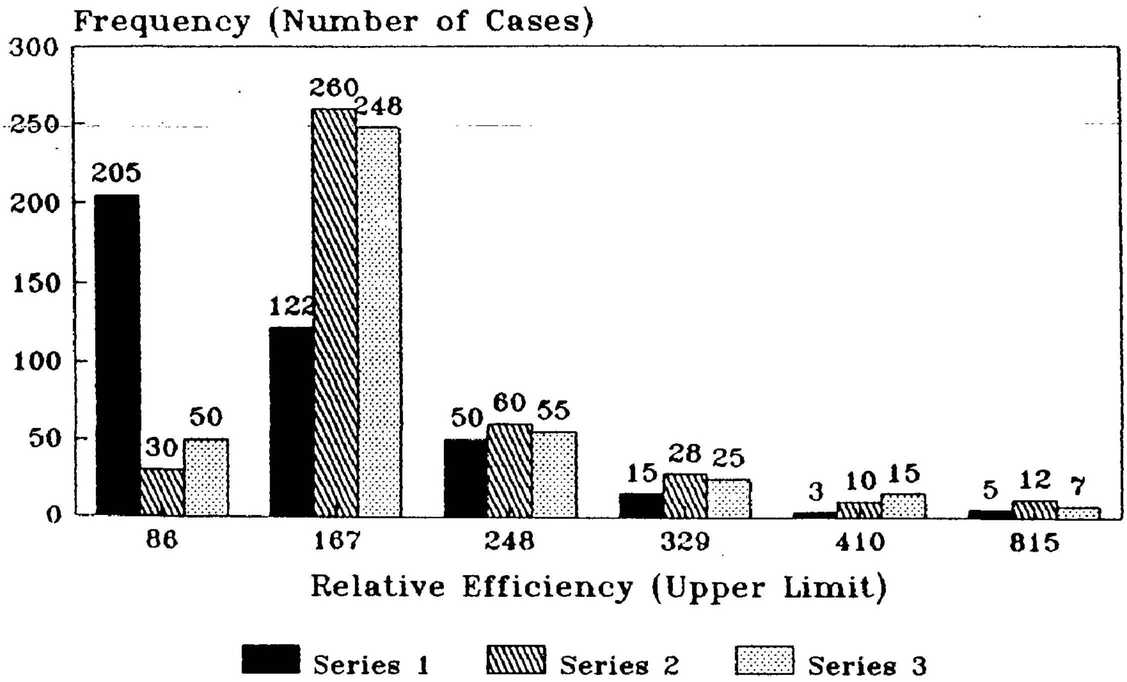
Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_R$	$\bar{Y}^{(2)}_{RP.ab}$	MEST(34)
n=10	0.102	0.083	0.447	0.368
n=20	0.052	0.053	0.517	0.378

Table 5.43 reveals that MEST(34) can not be taken to be an improvement over $\bar{Y}^{(2)}_{RP.ab}$ if we consider all the G-values simultaneously. But if we consider its finer comparisons with $\bar{y}$, $\bar{Y}_R$ and $\bar{Y}^{(2)}_{RP.ab}$, the behaviour of MEST(34) is similar to that of MEST(32) and MEST(33). It also comes out to be winner more often for those cases when $G \in [0.5, 1]$.

5.2.2 MIXINGS WHEN $\rho < 0$:

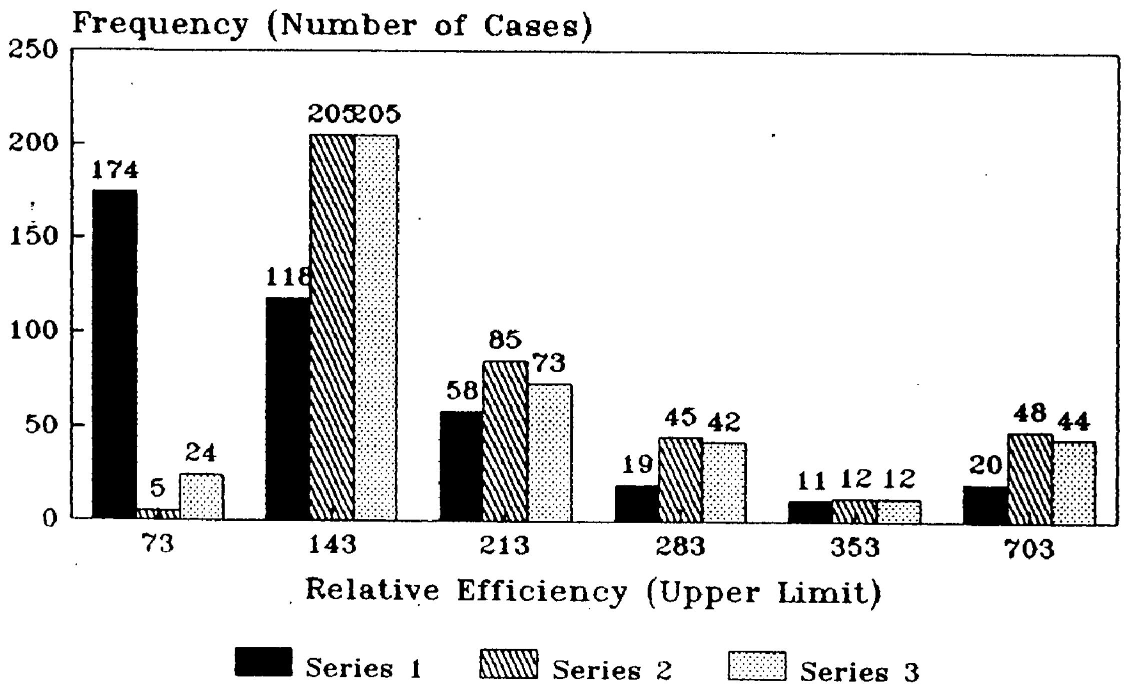
For this case, according to 5.2, the three estimators will be $\bar{Y}_{Re.}$, $\bar{Y}_{VSa.}$ and $\bar{Y}_{MP}$. We propose the following mixing estimators

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}^{(2)}_{RP.ab}$ and Series 3 for MEST(34)



Graph – 5.45 (Sample Size = 10, $\rho > 0$)

Series 1 for $\overline{Y}_R$, Series 2 for $\overline{Y}^{(2)}_{RP.ab}$ and Series 3 for MEST(34)



Graph – 5.46 (Sample Size = 20, $\rho > 0$)

using $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{Re.}$, $\bar{Y}_{VSa.}$ and $\bar{Y}_{MP}$. The mixing estimator of $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{Re.}$ and $\bar{Y}_{VSa.}$ can be proposed to be :

$$MEST(35) = e_4.\bar{y} + f_4.\bar{Y}_P + g_4.\bar{Y}_{Re.} + h_4.\bar{Y}_{VSa.} \qquad \ldots(5.26)$$

where, e_4=0.04, f_4=0.09, g_4=0.38 and $h_4=1-(e_4+f_4+g_4)$. These values of 'e_4', 'f_4' and 'g_4' are based on the results of table 5.21. Now, we compare this mixing MEST(35) with $\bar{Y}_{VSa.}$ (which is the more probable winner according to the results of table 5.21). Table 5.44 contains the results of the simulation study for comparing $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{VSa.}$ and MEST(35). We can also have a better view of the relative efficiencies of these estimators through graphs 5.47 and 5.48.

TABLE 5.44

RF(∘) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{VSa.}$ and MEST(35)

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{VSa.}$	MEST(35)
n=10	0.037	0.093	0.405	0.465
n=20	0.037	0.060	0.380	0.523

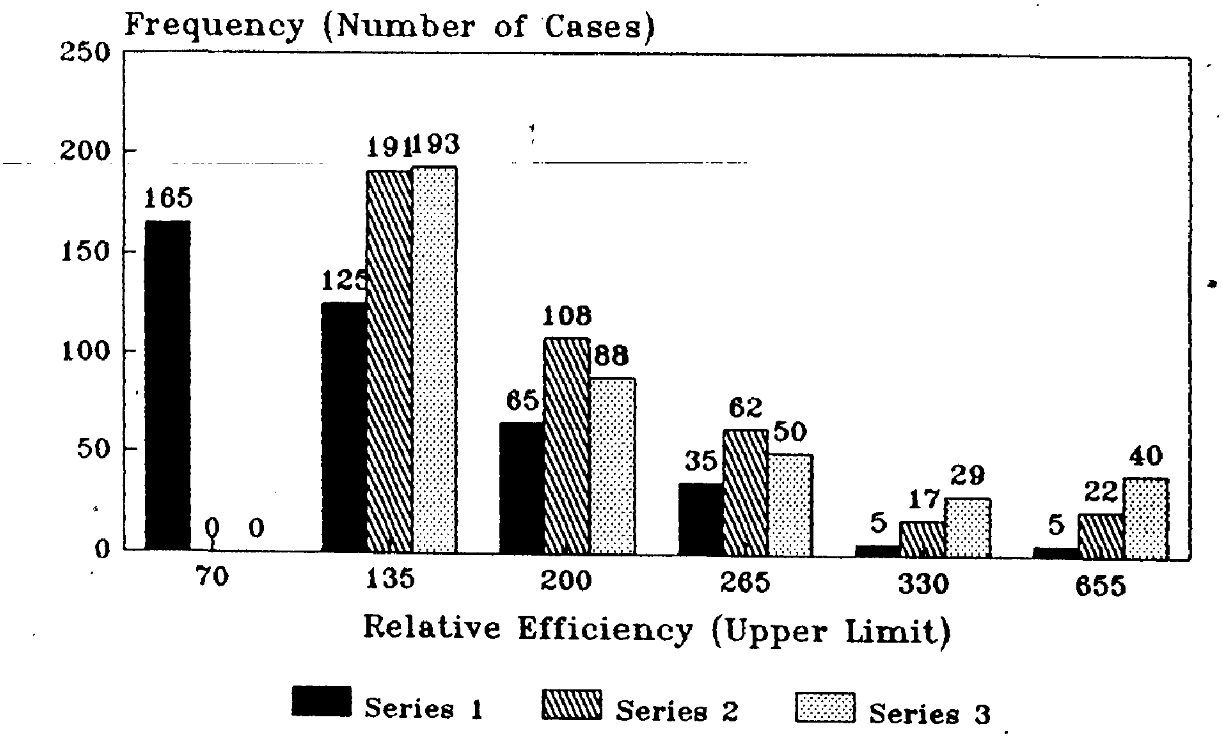
One can infer from table 5.44 that MEST(35) is more efficient than the other estimators in competition. So, we can recommend the use of MEST(35) instead of $\bar{Y}_{VSa.}$ and also instead of $\bar{Y}_{Re.}$. The finer comparisons of these estimators based on G-values reveal that MEST(35) performs exceptionally better than the other estimators for the cases when G < -0.5.

Exploiting the knowledge of the results tabulated in 5.23, we propose the following mixing estimator,

$$MEST(36) = e_5.\bar{y} + f_5.\bar{Y}_P + g_5.\bar{Y}_{Re.} + h_5.\bar{Y}_{MP} \qquad \ldots(5.27)$$

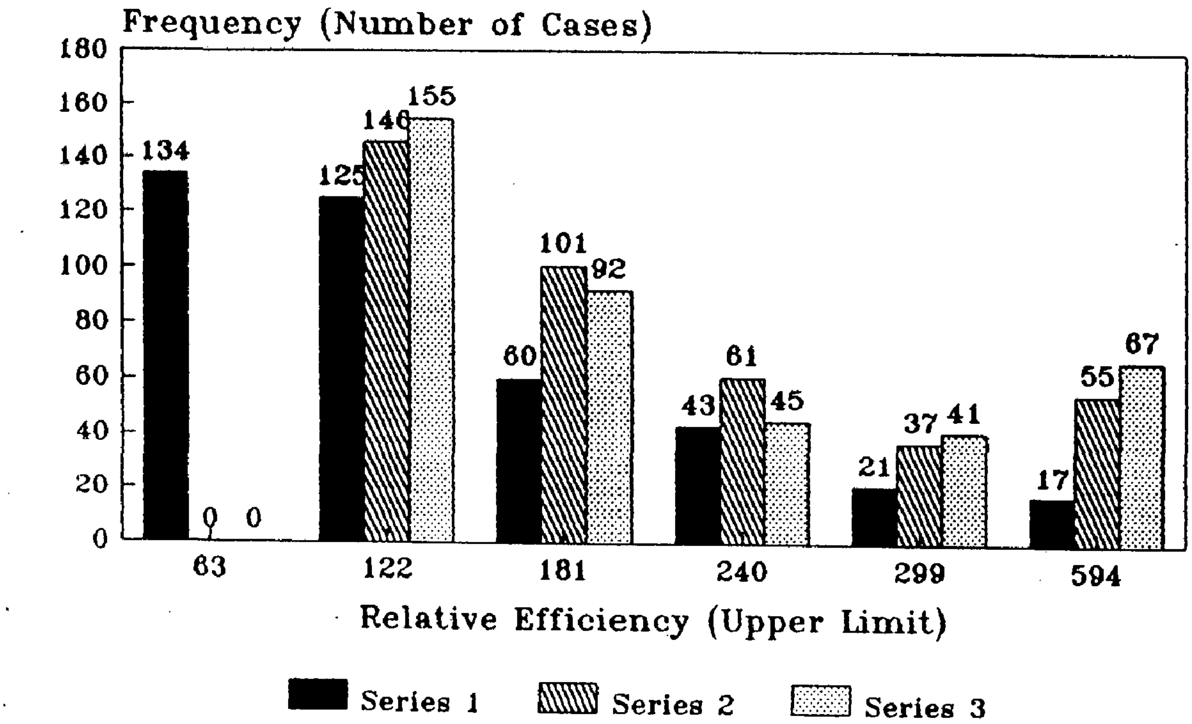
where, e_5=0.06, f_5=0.10, g_5=0.61 and $h_5=1-(e_5+f_5+g_5)$. We have compared MEST(36) with $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}_{Re.}$ via the empirical-simulation

Series 1 for $\overline{Y}_p$, Series 2 for $\overline{Y}_{VSa.}$ and
Series 3 for MEST(35)



Graph – 5.47 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_p$, Series 2 for $\overline{Y}_{VSa.}$ and
Series 3 for MEST(35)



Graph – 5.48 (Sample Size = 20, $\rho < 0$)

study. Table 5.45 below contains the results of this study. Graphs 5.49 and 5.50, which present a clearer view of relative efficiencies of these estimators, show the gain in the relative efficiency by the use of MEST(36).

TABLE 5.45

RF(∘) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{Re.}$ and MEST(36)

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{Re.}$	MEST(36)
n=10	0.075	0.115	0.415	0.395
n=20	0.047	0.093	0.472	0.388

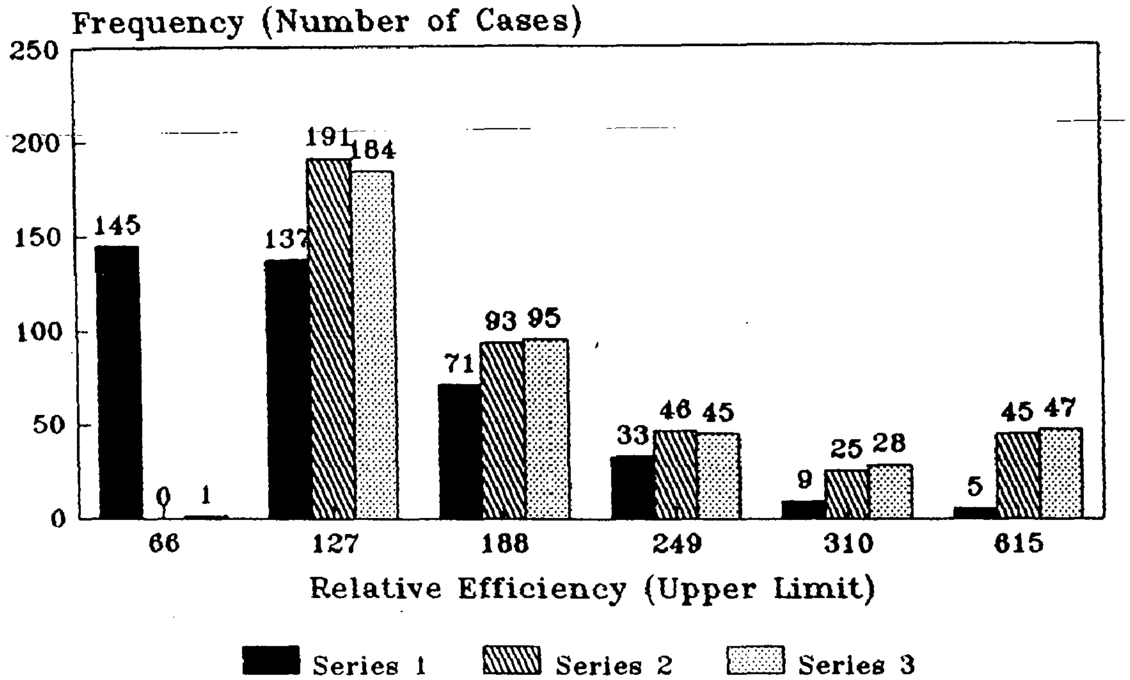
We observe from table 5.45 that MEST(36) is not more efficient than $\bar{Y}_{Re.}$ but it is still a better choice than $\bar{y}$ and $\bar{Y}_P$. Nevertheless finer comparisons reveal that MEST(36) performs better, quite often than $\bar{y}$, $\bar{Y}_P$ and $\bar{Y}_{Re.}$ when $G \in [-1, -0.5]$.

Now, based on the results of table 5.29, we propose the mixing of $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{VSa.}$ and $\bar{Y}_{MP}$ to be :

$$\text{MEST}(37) = e_6.\bar{y} + f_6.\bar{Y}_P + g_6.\bar{Y}_{VSa.} + h_6.\bar{Y}_{MP} \quad \ldots(5.28)$$

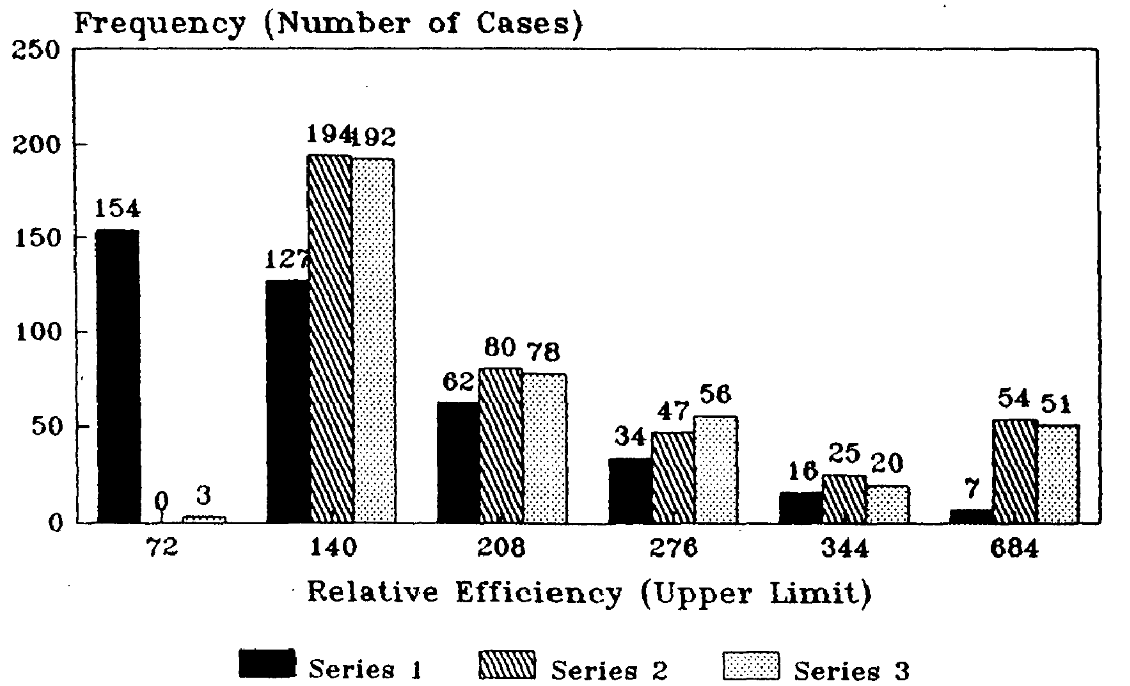
where, $e_6=0.03$, $f_6=0.08$, $g_6=0.66$ and $h_6=1-(e_6+f_6+g_6)$. It is clear from these values that $\bar{Y}_{VSa.}$ is the most probable winner here. So, we now proceed to compare MEST(37) with $\bar{Y}_{VSa.}$ in the presence of the usual estimators $\bar{y}$ and $\bar{Y}_P$. Table 5.46 contains the results of these comparisons and the graphical display of the relative efficiencies of these estimators has been afforded through graphs 5.51 and 5.52 which show a slight gain in the relative efficiency by the use of MEST(37).

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{Re.}$ and
Series 3 for MEST(36)



Graph - 5.49 (Sample Size = 10, $\rho < 0$)

Series 1 for $\overline{Y}_P$, Series 2 for $\overline{Y}_{Re.}$ and
Series 3 for MEST(36)



Graph - 5.50 (Sample Size = 20, $\rho < 0$)

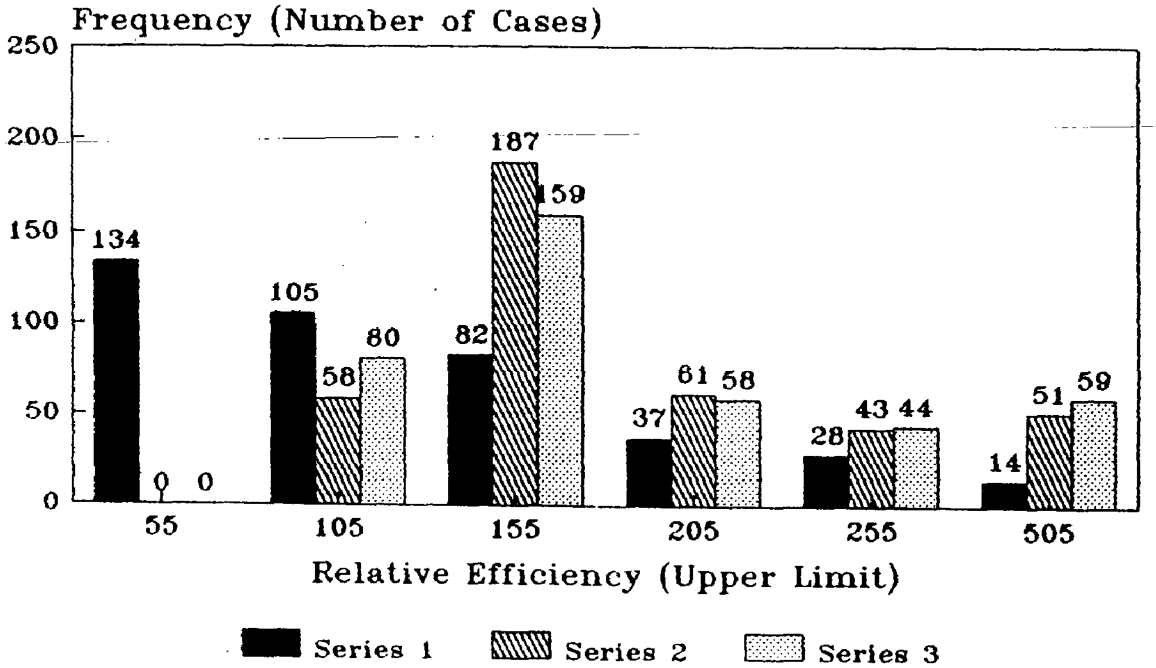
TABLE 5.46

RF(∘) FOR $\bar{y}$, $\bar{Y}_P$, $\bar{Y}_{VSa.}$ and MEST(37)

Estimators → Sample Sizes ↓	$\bar{y}$	$\bar{Y}_P$	$\bar{Y}_{VSa.}$	MEST(37)
n=10	0.035	0.060	0.450	0.455
n=20	0.047	0.050	0.468	0.435

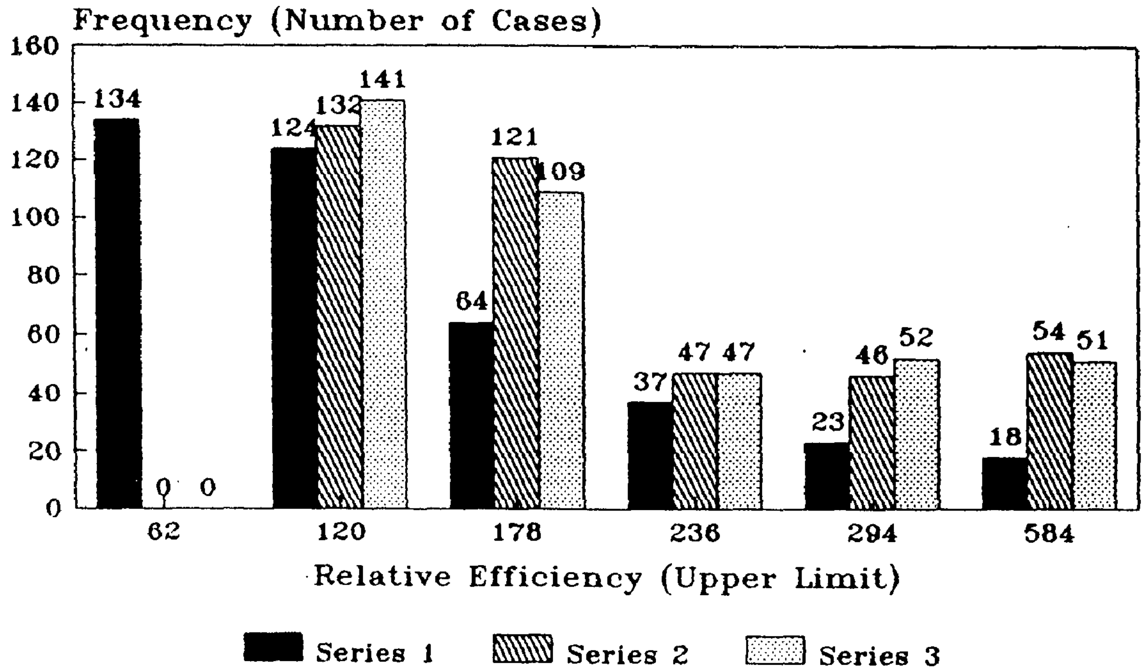
One can make the inference from table 5.46 that MEST(37) performs better, quite oftener, than $\bar{y}$ and $\bar{Y}_P$ and it is a close competitor of $\bar{Y}_{VSa.}$. When we take up the finer comparisons of these estimators, we observe that the behaviour of MEST(37) is almost similar to that of MEST(35) and MEST(36). It comes out to be winner more often for those cases when G < -0.5.

Series 1 for $\overline{Y}_p$, Series 2 for $\overline{Y}_{VSa.}$ and
Series 3 for MEST(37)



Graph – 5.51 (Sample Size = 10, $\varrho < 0$)

Series 1 for $\overline{Y}_p$, Series 2 for $\overline{Y}_{VSa.}$ and
Series 3 for MEST(37)



Graph – 5.52 (Sample Size = 20, $\varrho < 0$)

CHAPTER - 6

FINER COMPARISONS OF THE ESTIMATORS

In this chapter, we have attempted a more detailed and finer comparison of the estimators which are the winners as per the empirical-simulation study detailed earlier. To get a more comprehensive idea of the asymptotic behaviour of the efficiencies of the estimators, we have here included the case of n=50 into our empirical-simulation study. To facilitate the finer comparisons, we have considered a larger number of values of $\bar{Y}$, namely, 0.1, 0.3, 0.5, 0.7, 1.0, 2.0, 3.0, 4.0, 6.0, 8.0, 10.0 and 12.0. Thus, we have considered here twelve values of $\bar{Y}$ rather than two as earlier. Analogously, we have considered four values each of σ_X and σ_Y, namely, σ_X= 0.5, 1.0, 2.0, 3.0 and σ_Y=1.0, 2.0, 4.0, 6.0 rather than two values each in the earlier empirical-simulation study. Consequently, the total number of parametric value-combinations blows up to 57,600 rather than 2,400 as in earlier empirical-simulation studies.

Next, to make the finer details more comprehensive, we have tabulated the results for various ranges of the values of the parameter 'G' vis-a-vis particular values of ρ and n. For this purpose, we here have considered five subintervals, namely, GI1: $0 \le G \le 0.5$; GI2: $0.5 < G \le 1.0$; GI3: $1.0 < G \le 2.0$; GI4: $2.0 < G \le 3.0$ and GI5: $G > 3.0$ when $\rho > 0$ and five subintervals, namely, GI6: $G < -3.0$; GI7: $-3.0 \le G < -2.0$; GI8: $-2.0 \le G < -1.0$; GI9: $-1.0 \le G < -0.5$ and GI10: $-0.5 \le G \le 0$ when $\rho < 0$. In this chapter also, we have dealt with the two cases of positive and negative correlations separately. Moreover, we have not included all the estimators proposed till this chapter in the final comparisons but we have considered five best parent-estimators ; five mixings when the parent-estimators are mixed with $\bar{y}$; five mixings when the two parent-estimators are mixed with one another ; two mixings when the two parent-estimators are mixed with $\bar{y}$ and $\bar{Y}_R/\bar{Y}_P$ and the usual estimators $\bar{y}$ and $\bar{Y}_R/\bar{Y}_P$. Thus, we have 19 estimators for the positive correlation case and other 19 estimators for the negative correlation case in the competition.

6.1 COMPARISONS WHEN $\rho > 0$:

We first carry out the empirical-simulation studies for deciding the estimators which are to be included in the final comparisons when $\rho > 0$. We have observed through different empirical-simulation studies that for this case, we will have to consider the following 19 estimators into our study.

(a) Usual estimators : $\bar{y}$ and $\bar{Y}_R$

(b) Parent-estimators : $\bar{Y}_{P.a}$, $\bar{Y}^{(1)}_{RP.ab}$, $\bar{Y}^{(2)}_{RP.ab}$, $\bar{Y}_{Re.}$ and $\bar{Y}_{VR}$

(c) Mixings, when parent-estimators are mixed with $\bar{y}$

: MEST(5), MEST(11), MEST(10), MEST(7) and MEST(3)

(d) Mixings, when two parent-estimators are mixed with one another

: MEST(15), MEST(21), MEST(14), MEST(20) and MEST(18)

(e) Mixings, when two parent-estimators are mixed with $\bar{y}$ and $\bar{Y}_R$

: MEST(34) and MEST(33)

Now, we compare the above mentioned nineteen estimators through the empirical-simulation studies. Tables 6.1 to 6.15 contain the results of these studies. As mentioned earlier, we have tabulated these results for various ranges of G-values vis-a-vis a particular value of ρ and sample size 'n'. In the following tables, we have highlighted the winning estimator by bold-facing the entity corresponding to that estimator.

TABLE 6.1

RF($\circ$) FOR THE ESTIMATORS WHEN ρ = 0.2 AND n = 10.

Subintervals → Estimators ↓	GI1	GI2	GI3	GI4	GI5
$\bar{y}$	0.095	0.009	-	-	-
$\bar{Y}_R$	-	-	-	-	-
$\bar{Y}_{P.a}$	0.098	-	-	-	-
$\bar{Y}^{(1)}_{RP.ab}$	0.073	0.047	-	-	-
$\bar{Y}^{(2)}_{RP.ab}$	0.055	0.019	0.013	-	-
$\bar{Y}_{Re.}$	0.0670	-	-	-	-
$\bar{Y}_{VR}$	0.059	0.009	-	-	-
MEST(5)	**0.125**	-	-	-	-
MEST(11)	0.043	0.038	0.013	-	-
MEST(10)	0.064	0.142	0.037	-	-
MEST(7)	0.047	**0.557**	**0.800**	**0.970**	**0.962**
MEST(3)	0.050	-	-	-	-
MEST(15)	0.035	-	-	-	-
MEST(21)	0.047	-	-	-	0.025
MEST(14)	0.003	-	-	-	0.013
MEST(20)	0.032	0.028	0.025	-	-
MEST(18)	0.036	0.066	0.050	0.030	-
MEST(34)	0.052	0.047	-	-	-
MEST(33)	0.019	0.038	0.062	-	-

TABLE 6.2

RF(∘) FOR THE ESTIMATORS WHEN ρ = 0.2 AND n = 20.

Subintervals → / Estimators ↓	GI1	GI2	GI3	GI4	GI5
$\bar{y}$	0.086	0.009	-	-	-
$\bar{Y}_R$	0.001	0.009	-	-	-
$\bar{Y}_{P.a}$	0.088	0.009	-	-	-
$\bar{Y}^{(1)}_{RP.ab}$	0.070	0.057	-	-	-
$\bar{Y}^{(2)}_{RP.ab}$	0.065	0.028	0.025	-	-
$\bar{Y}_{Re.}$	0.103	-	-	-	-
$\bar{Y}_{VR}$	0.042	0.009	0.012	-	-
MEST(5)	0.127	0.009	-	-	-
MEST(11)	0.050	0.047	0.025	-	-
MEST(10)	0.098	0.151	0.113	-	-
MEST(7)	0.024	0.462	0.688	0.970	0.963
MEST(3)	0.059	0.009	-	-	-
MEST(15)	0.020	-	-	-	-
MEST(21)	0.045	-	-	-	0.012
MEST(14)	0.001	-	-	-	0.012
MEST(20)	0.029	0.038	0.037	-	-
MEST(18)	0.029	0.066	0.037	-	0.013
MEST(34)	0.039	0.028	-	-	-
MEST(33)	0.024	0.069	0.063	0.030	-

TABLE 6.3

RF($\circ$) FOR THE ESTIMATORS WHEN ρ = 0.2 AND n = 50.

Subintervals → / Estimators ↓	GI1	GI2	GI3	GI4	GI5
$\bar{y}$	0.100	0.019	-	-	-
$\bar{Y}_R$	-	0.009	-	-	-
$\bar{Y}_{P.a}$	0.057	-	-	-	-
$\bar{Y}^{(1)}_{RP.ab}$	0.103	0.123	0.050	-	-
$\bar{Y}^{(2)}_{RP.ab}$	0.076	0.075	0.050	-	-
$\bar{Y}_{Re.}$	0.120	0.009	-	-	-
$\bar{Y}_{VR}$	0.039	0.009	-	-	-
MEST(5)	0.100	0.009	-	-	-
MEST(11)	0.045	0.057	0.050	0.029	-
MEST(10)	0.129	0.160	0.087	0.147	0.012
MEST(7)	0.003	**0.198**	**0.438**	**0.588**	**0.925**
MEST(3)	0.044	0.038	-	-	-
MEST(15)	0.003	-	-	-	0.012
MEST(21)	0.029	-	-	-	-
MEST(14)	0.003	0.009	-	-	-
MEST(20)	0.044	0.038	0.062	0.029	-
MEST(18)	0.036	0.085	0.100	0.118	0.012
MEST(34)	0.036	0.047	0.062	0.029	-
MEST(33)	0.033	0.115	0.101	0.060	0.039

TABLE 6.4

RF(∘) FOR THE ESTIMATORS WHEN ρ = 0.4 AND n = 10.

Subintervals → / Estimators ↓	GI1	GI2	GI3	GI4	GI5
$\bar{y}$	0.004	0.008	-	-	0.007
$\bar{Y}_R$	-	0.017	-	0.020	-
$\bar{Y}_{P.a}$	0.083	0.017	-	-	-
$\bar{Y}^{(1)}_{RP.ab}$	0.072	0.042	0.019	-	-
$\bar{Y}^{(2)}_{RP.ab}$	0.079	0.051	0.009	-	-
$\bar{Y}_{Re.}$	0.109	-	-	0.039	0.014
$\bar{Y}_{VR}$	0.079	0.119	0.057	-	0.007
MEST(5)	0.105	0.008	-	-	-
MEST(11)	0.079	0.119	0.085	-	-
MEST(10)	0.125	0.144	0.170	0.059	-
MEST(7)	0.007	0.212	0.547	0.706	0.951
MEST(3)	0.035	0.025	-	-	-
MEST(15)	0.059	0.008	0.028	0.020	-
MEST(21)	0.059	-	-	-	0.007
MEST(14)	0.011	-	-	0.020	0.007
MEST(20)	0.026	0.068	0.028	0.059	0.007
MEST(18)	0.028	0.085	0.019	0.059	-
MEST(34)	0.026	0.042	0.038	0.018	-
MEST(33)	0.014	0.035	-	-	-

TABLE 6.5

RF(∘) FOR THE ESTIMATORS WHEN ρ = 0.4 AND n = 20.

Subintervals → / Estimators ↓	GI1	GI2	GI3	GI4	GI5
$\bar{y}$	0.002	0.008	-	0.020	-
$\bar{Y}_R$	0.002	0.042	-	-	0.007
$\bar{Y}_{P.a}$	0.092	-	0.019	-	-
$\bar{Y}^{(1)}_{RP.ab}$	0.081	0.068	0.028	-	-
$\bar{Y}^{(2)}_{RP.ab}$	0.074	0.034	0.019	-	-
$\bar{Y}_{Re.}$	0.138	0.025	0.009	0.020	0.007
$\bar{Y}_{VR}$	0.042	0.119	0.057	-	-
MEST(5)	0.081	0.008	-	-	0.007
MEST(11)	0.063	0.169	0.113	-	0.007
MEST(10)	0.172	0.127	0.142	0.137	-
MEST(7)	-	0.110	0.406	0.627	0.909
MEST(3)	0.037	0.059	0.009	-	-
MEST(15)	0.039	0.025	0.028	0.20	0.028
MEST(21)	0.050	-	0.028	-	0.007
MEST(14)	0.006	0.008	0.009	0.020	0.007
MEST(20)	0.029	0.093	0.047	0.098	0.014
MEST(18)	0.033	0.076	0.057	0.058	0.007
MEST(34)	0.031	0.017	0.019	-	-
MEST(33)	0.028	0.012	0.010	-	-

TABLE 6.6

RF(∘) FOR THE ESTIMATORS WHEN ρ = 0.4 AND n = 50.

Subintervals → / Estimators ↓	GI1	GI2	GI3	GI4	GI5
$\bar{y}$	0.004	-	0.009	-	-
$\bar{Y}_R$	-	0.042	0.009	0.039	0.007
$\bar{Y}_{P.a}$	0.088	0.008	-	0.019	-
$\bar{Y}^{(1)}_{RP.ab}$	0.077	**0.169**	0.066	0.039	-
$\bar{Y}^{(2)}_{RP.ab}$	0.072	0.042	0.075	-	0.007
$\bar{Y}_{Re.}$	0.148	0.017	0.028	0.019	0.007
$\bar{Y}_{VR}$	0.037	0.085	0.047	-	-
MEST(5)	0.087	0.025	0.019	-	-
MEST(11)	0.061	0.161	0.075	0.059	-
MEST(10)	0.173	0.127	0.141	0.137	0.021
MEST(7)	0.004	-	**0.179**	**0.353**	**0.727**
MEST(3)	0.033	0.025	-	-	-
MEST(15)	0.046	0.034	0.057	0.078	0.056
MEST(21)	0.042	0.025	0.009	-	-
MEST(14)	0.004	0.025	0.019	-	-
MEST(20)	0.033	0.119	0.113	0.176	0.105
MEST(18)	0.026	0.076	0.113	0.078	0.063
MEST(34)	0.046	0.010	0.018	-	0.007
MEST(33)	0.019	0.010	0.023	-	-

TABLE 6.7

RF(∘) FOR THE ESTIMATORS WHEN ρ = 0.6 AND n = 10.

Subintervals → Estimators ↓	GI1	GI2	GI3	GI4	GI5
$\bar{y}$	-	-	-	-	-
$\bar{Y}_{R}$	-	0.015	0.043	0.065	0.005
$\bar{Y}_{P.a}$	0.096	0.023	0.009	-	-
$\bar{Y}^{(1)}_{RP.ab}$	0.057	0.053	0.009	-	-
$\bar{Y}^{(2)}_{RP.ab}$	0.063	-	0.017	0.016	-
$\bar{Y}_{Re.}$	**0.211**	0.023	0.043	0.049	0.031
$\bar{Y}_{VR}$	0.033	**0.382**	0.113	0.049	0.005
MEST(5)	0.174	0.061	0.026	-	-
MEST(11)	0.052	0.046	0.017	-	-
MEST(10)	0.057	0.084	0.052	-	-
MEST(7)	-	0.031	**0.330**	**0.508**	**0.876**
MEST(3)	0.022	0.038	-	-	-
MEST(15)	0.054	0.122	0.217	0.180	0.026
MEST(21)	0.070	0.015	-	-	0.010
MEST(14)	0.054	0.015	0.026	0.049	-
MEST(20)	0.011	0.015	0.043	0.033	0.026
MEST(18)	0.013	0.023	0.035	0.033	0.021
MEST(34)	0.019	0.015	0.010	0.016	-
MEST(33)	0.014	0.039	0.010	-	-

TABLE 6.8

RF(∘) FOR THE ESTIMATORS WHEN ρ = 0.6 AND n = 20.

Subintervals → / Estimators ↓	GI1	GI2	GI3	GI4	GI5
$\bar{y}$	-	-	-	-	-
$\bar{Y}_R$	-	0.031	0.026	0.065	0.015
$\bar{Y}_{P.a}$	0.096	0.084	0.043	-	-
$\bar{Y}^{(1)}_{RP.ab}$	0.067	0.031	0.026	-	-
$\bar{Y}^{(2)}_{RP.ab}$	0.076	-	0.008	-	0.010
$\bar{Y}_{Re.}$	**0.178**	0.046	0.078	0.016	0.046
$\bar{Y}_{VR}$	0.019	**0.244**	0.069	0.033	0.005
MEST(5)	0.166	0.038	0.043	0.049	-
MEST(11)	0.061	0.046	0.017	-	0.005
MEST(10)	0.094	0.076	0.087	0.033	-
MEST(7)	-	0.008	0.183	**0.426**	**0.763**
MEST(3)	0.024	0.008	-	-	-
MEST(15)	0.059	0.214	**0.296**	0.213	0.108
MEST(21)	0.052	0.031	0.035	-	0.005
MEST(14)	0.035	0.008	0.017	0.016	0.005
MEST(20)	0.004	0.023	0.008	0.082	0.005
MEST(18)	0.015	0.053	0.026	0.016	0.015
MEST(34)	0.037	0.023	0.026	0.016	0.005
MEST(33)	0.015	0.038	0.008	0.033	0.010

TABLE 6.9

RF(∘) FOR THE ESTIMATORS WHEN ρ = 0.6 AND n = 50.

Subintervals → Estimators ↓	GI1	GI2	GI3	GI4	GI5
$\bar{y}$	-	-	-	-	-
$\bar{Y}_{R}$	-	0.030	0.035	0.033	0.010
$\bar{Y}_{P.a}$	0.104	0.084	0.052	0.033	-
$\bar{Y}^{(1)}_{RP.ab}$	0.076	0.030	0.061	0.033	-
$\bar{Y}^{(2)}_{RP.ab}$	0.074	-	0.008	-	0.005
$\bar{Y}_{Re.}$	**0.168**	0.046	0.113	0.164	0.051
$\bar{Y}_{VR}$	0.022	**0.244**	0.061	0.016	-
MEST(5)	0.157	0.038	0.043	0.016	0.005
MEST(11)	0.076	0.046	0.035	0.033	-
MEST(10)	0.089	0.076	0.069	0.016	-
MEST(7)	-	0.008	0.008	0.115	**0.774**
MEST(3)	0.009	0.008	-	-	-
MEST(15)	0.030	0.214	**0.270**	**0.279**	0.108
MEST(21)	0.081	0.030	0.026	-	0.005
MEST(14)	0.022	0.008	0.043	0.049	-
MEST(20)	0.015	0.023	0.061	0.049	0.005
MEST(18)	0.019	0.053	0.061	0.082	0.021
MEST(34)	0.039	0.023	0.043	0.049	0.005
MEST(33)	0.017	0.038	0.008	0.033	0.010

TABLE 6.10

RF(∘) FOR THE ESTIMATORS WHEN ρ = 0.8 AND n = 10.

Subintervals → Estimators ↓	GI1	GI2	GI3	GI4	GI5
$\bar{Y}_R$	-	-	-	-	-
$\bar{Y}_{P.a}$	-	0.049	0.093	0.046	0.008
$\bar{Y}^{(1)}_{RP.ab}$	0.087	-	-	-	-
$\bar{Y}^{(2)}_{RP.ab}$	0.032	0.021	0.042	0.062	-
$\bar{Y}_{Re.}$	0.037	0.021	0.042	0.046	-
$\bar{Y}_{VR}$	**0.277**	0.085	**0.144**	0.138	0.127
MEST(5)	0.006	0.170	0.034	-	-
MEST(11)	0.182	0.035	-	-	-
MEST(10)	0.045	0.071	0.042	-	-
MEST(7)	0.022	0.057	0.017	0.031	-
MEST(3)	-	-	0.085	**0.323**	**0.694**
MEST(15)	0.007	0.014	-	-	-
MEST(21)	0.017	0.071	0.085	0.015	0.013
MEST(14)	0.130	**0.206**	0.068	0.015	0.034
MEST(20)	0.115	0.028	0.025	-	-
MEST(18)	0.007	0.071	0.127	0.015	-
MEST(34)	0.002	0.035	0.076	0.092	-
MEST(33)	0.010	0.028	0.060	0.062	-
	0.023	0.035	0.060	0.154	0.123

TABLE 6.11

RF(∘) FOR THE ESTIMATORS WHEN ρ = 0.8 AND n = 20.

Subintervals → / Estimators ↓	GI1	GI2	GI3	GI4	GI5
$\bar{y}$	-	-	-	-	-
$\bar{Y}_R$	-	0.092	0.085	0.062	0.008
$\bar{Y}_{P.a}$	0.105	0.007	-	-	-
$\bar{Y}^{(1)}_{RP.ab}$	0.042	0.028	0.042	0.062	0.004
$\bar{Y}^{(2)}_{RP.ab}$	0.070	0.035	0.068	0.077	0.013
$\bar{Y}_{Re.}$	**0.212**	0.043	**0.195**	**0.246**	0.183
$\bar{Y}_{VR}$	0.005	0.043	0.034	-	-
MEST(5)	0.180	0.043	-	0.015	-
MEST(11)	0.077	0.085	0.034	-	-
MEST(10)	0.030	0.043	0.025	0.015	-
MEST(7)	-	-	-	0.108	**0.528**
MEST(3)	0.007	0.007	-	-	-
MEST(15)	0.022	**0.163**	0.102	0.092	0.004
MEST(21)	0.095	0.142	0.059	0.077	0.034
MEST(14)	0.077	0.071	0.034	0.046	0.004
MEST(20)	0.012	0.064	0.144	0.015	0.008
MEST(18)	0.005	0.035	0.051	0.062	0.025
MEST(34)	0.022	0.035	0.076	0.046	-
MEST(33)	0.038	0.064	0.051	0.077	0.187

TABLE 6.12

RF(∘) FOR THE ESTIMATORS WHEN ρ = 0.8 AND n = 50.

Subintervals → Estimators ↓	GI1	GI2	GI3	GI4	GI5
$\bar{y}$	-	-	-	-	-
$\bar{Y}_{R}$	-	0.121	0.085	0.046	0.008
$\bar{Y}_{P.a}$	0.120	0.021	0.008	-	-
$\bar{Y}^{(1)}_{RP.ab}$	0.052	0.050	0.059	0.092	-
$\bar{Y}^{(2)}_{RP.ab}$	0.102	0.057	0.051	0.046	0.008
$\bar{Y}_{Re.}$	0.147	0.099	**0.144**	**0.261**	0.155
$\bar{Y}_{VR}$	0.005	0.021	-	0.015	-
MEST(5)	**0.177**	0.035	0.008	-	-
MEST(11)	0.112	0.071	0.025	0.015	-
MEST(10)	0.037	0.057	0.034	0.031	-
MEST(7)	-	-	-	-	**0.611**
MEST(3)	0.007	-	-	-	-
MEST(15)	0.027	**0.135**	0.136	0.015	0.008
MEST(21)	0.060	0.085	0.085	0.061	0.034
MEST(14)	0.040	0.064	0.059	0.077	-
MEST(20)	0.020	0.035	0.068	0.046	0.013
MEST(18)	0.007	0.035	0.093	0.092	0.013
MEST(34)	0.032	0.064	0.068	0.046	-
MEST(33)	0.052	0.050	0.076	0.154	0.150

TABLE 6.13

RF($\circ$) FOR THE ESTIMATORS WHEN ρ = 0.9 AND n = 10.

Subintervals $\longrightarrow$ Estimators $\downarrow$	GI1	GI2	GI3	GI4	GI5
$\bar{y}$	-	-	-	-	-
$\bar{Y}_R$	-	0.071	0.156	0.089	0.008
$\bar{Y}_{P.a}$	0.029	-	-	-	-
$\bar{Y}^{(1)}_{RP.ab}$	0.029	0.035	0.049	0.060	0.012
$\bar{Y}^{(2)}_{RP.ab}$	0.037	0.085	0.098	0.030	-
$\bar{Y}_{Re.}$	**0.451**	0.113	**0.230**	**0.463**	0.341
$\bar{Y}_{VR}$	0.003	0.035	0.008	-	-
MEST(5)	0.115	0.028	-	-	-
MEST(11)	0.043	0.057	0.025	0.015	-
MEST(10)	0.008	0.028	0.008	-	0.008
MEST(7)	-	-	0.008	0.045	**0.420**
MEST(3)	0.005	-	-	-	-
MEST(15)	-	0.007	0.025	-	-
MEST(21)	0.133	**0.298**	0.074	0.075	0.031
MEST(14)	0.099	0.021	-	-	-
MEST(20)	0.019	0.092	0.115	0.045	0.004
MEST(18)	-	0.043	0.057	0.060	0.023
MEST(34)	0.013	0.064	0.082	0.015	-
MEST(33)	0.016	0.022	0.066	0.104	0.153

TABLE 6.14

RF(∘) FOR THE ESTIMATORS WHEN ρ = 0.9 AND n = 20.

Subintervals → Estimators ↓	GI1	GI2	GI3	GI4	GI5
$\bar{y}$	-	-	-	-	-
$\bar{Y}_R$	-	0.085	0.156	0.090	-
$\bar{Y}_{P.a}$	0.059	-	-	-	-
$\bar{Y}^{(1)}_{RP.ab}$	0.035	0.042	0.082	0.060	0.012
$\bar{Y}^{(2)}_{RP.ab}$	0.045	0.042	0.098	0.060	0.016
$\bar{Y}_{Re.}$	**0.408**	0.092	**0.238**	**0.358**	**0.384**
$\bar{Y}_{VR}$	-	0.028	-	-	-
MEST(5)	0.144	-	-	-	-
MEST(11)	0.053	0.128	0.057	-	0.004
MEST(10)	0.016	0.028	0.008	-	0.012
MEST(7)	-	-	-	0.015	0.310
MEST(3)	-	0.008	-	-	-
MEST(15)	0.003	0.028	0.008	-	-
MEST(21)	0.077	**0.227**	0.041	0.060	0.027
MEST(14)	0.088	0.050	0.016	0.030	0.004
MEST(20)	0.021	0.078	0.164	0.075	0.004
MEST(18)	0.003	0.057	0.033	0.045	0.008
MEST(34)	0.021	0.035	0.049	0.030	-
MEST(33)	0.027	0.071	0.049	0.179	0.219

TABLE 6.15

RF(∘) FOR THE ESTIMATORS WHEN ρ = 0.9 AND n = 50.

Subintervals → Estimators ↓	GI1	GI2	GI3	GI4	GI5
$\bar{y}$	-	-	-	-	-
$\bar{Y}_R$	-	0.099	0.107	0.060	0.004
$\bar{Y}_{P.a}$	0.056	0.007	-	-	-
$\bar{Y}^{(1)}_{RP.ab}$	0.053	0.028	0.082	0.090	0.012
$\bar{Y}^{(2)}_{RP.ab}$	0.093	0.092	0.090	0.134	0.008
$\bar{Y}_{Re.}$	**0.237**	0.135	**0.246**	**0.343**	**0.361**
$\bar{Y}_{VR}$	0.003	0.035	-	-	-
MEST(5)	0.136	0.035	-	-	-
MEST(11)	0.115	0.078	0.057	0.030	0.004
MEST(10)	0.029	0.022	0.016	0.015	0.008
MEST(7)	-	-	-	-	**0.365**
MEST(3)	0.005	-	-	-	-
MEST(15)	0.012	0.014	0.025	-	-
MEST(21)	0.085	**0.156**	0.049	0.090	0.027
MEST(14)	0.051	0.035	0.049	0.045	0.004
MEST(20)	0.024	0.071	0.123	0.060	0.004
MEST(18)	0.016	0.042	0.025	0.015	0.016
MEST(34)	0.037	0.064	0.066	-	-
MEST(33)	0.045	0.086	0.066	0.119	0.188

6.2 COMPARISONS WHEN $\rho < 0$:

Here also, we first give the nineteen estimators which are to be included in the final comparisons.

(a) Usual estimators : $\bar{y}$ and $\bar{Y}_P$

(b) Parent-estimators : $\bar{Y}_{Re.}$, $\bar{Y}_{VSa.}$, $\bar{Y}_{MP}$, $\bar{Y}^{(1)}_{RP.ab}$ and $\bar{Y}_{P.a}$

(c) Mixings, when parent-estimators are mixed with $\bar{y}$

: MEST(7), MEST(9), MEST(6), MEST(5) and MEST(10)

(d) Mixings, when two parent-estimators are mixed with one another

: MEST(26), MEST(23), MEST(28), MEST(27) and MEST(22)

(e) Mixings, when two parent-estimators are mixed with $\bar{y}$ and $\bar{Y}_P$

: MEST(35) and MEST(37)

We have compared the above mentioned nineteen estimators through the empirical-simulation studies. Tables 6.16 to 6.30 contain the results of these studies. The results have again been tabulated for various ranges of G-values vis-a-vis a particular value of ρ and sample size 'n'. Here also, we have highlighted the winning estimator by bold-facing the entity corresponding to that estimator.

TABLE 6.16

RF(∘) FOR THE ESTIMATORS WHEN $\rho = -0.2$ AND $n = 10$.

Subintervals → / Estimators ↓	GI6	GI7	GI8	GI9	GI10
$\bar{y}$	-	-	-	0.019	0.095
$\bar{Y}_P$	-	-	0.013	0.009	0.001
$\bar{Y}_{Re.}$	0.063	-	-	-	0.052
$\bar{Y}_{VSa.}$	-	-	-	0.009	0.065
$\bar{Y}_{MP}$	-	-	-	0.019	0.006
$\dot{\bar{Y}}^{(1)}_{RP.ab}$	-	-	0.038	0.048	0.073
$\bar{Y}_{P.a}$	-	-	-	-	0.068
MEST(7)	0.012	-	-	-	0.042
MEST(9)	-	-	-	-	0.059
MEST(6)	-	-	-	-	0.209
MEST(5)	0.800	1.000	0.821	0.562	0.061
MEST(10)	-	-	0.077	0.190	0.092
MEST(26)	-	-	-	-	0.039
MEST(23)	0.113	-	-	-	0.039
MEST(28)	-	-	-	-	0.012
MEST(27)	-	-	0.051	0.143	0.047
MEST(22)	-	-	-	-	0.009
MEST(35)	0.012	-	-	-	0.026
MEST(37)	-	-	-	-	0.023

TABLE 6.17

RF(∘) FOR THE ESTIMATORS WHEN ρ = -0.2 AND n = 20.

Subintervals → Estimators ↓	GI6	GI7	GI8	GI9	GI10
$\bar{y}$	-	-	-	0.048	0.097
$\bar{Y}_{P}$	-	-	-	0.009	0.003
$\bar{Y}_{Re.}$	0.012	-	-	-	0.074
$\bar{Y}_{VSa.}$	-	-	-	-	0.073
$\bar{Y}_{MP}$	-	-	-	0.048	0.008
$\bar{Y}^{(1)}_{RP.ab}$	-	-	0.038	0.086	0.098
$\bar{Y}_{P,a}$	-	-	0.013	-	0.053
MEST(7)	-	-	-	-	0.047
MEST(9)	-	-	-	0.009	0.094
MEST(6)	-	-	-	0.019	**0.152**
MEST(5)	**0.976**	**1.000**	**0.679**	**0.457**	0.026
MEST(10)	-	-	0.192	0.229	0.105
MEST(26)	-	-	-	-	0.042
MEST(23)	0.012	-	-	-	0.012
MEST(28)	-	-	-	-	0.004
MEST(27)	-	-	0.077	0.095	0.042
MEST(22)	-	-	-	-	0.011
MEST(35)	-	-	-	-	0.033
MEST(37)	-	-	-	-	0.026

TABLE 6.18

RF(∘) FOR THE ESTIMATORS WHEN ρ = -0.2 AND n = 50.

Subintervals → Estimators ↓	GI6	GI7	GI8	GI9	GI10
$\bar{y}$	-	-	0.013	0.048	0.098
$\bar{Y}_{P}$	0.012	0.027	0.038	0.028	0.006
$\bar{Y}_{Re.}$	-	-	-	-	0.073
$\bar{Y}_{VSa.}$	-	-	-	0.038	0.092
$\bar{Y}_{MP}$	-	-	-	0.019	0.005
$\bar{Y}^{(1)}_{RP.ab}$	-	0.054	0.077	0.105	**0.161**
$\bar{Y}_{P.a}$	-	0.027	0.026	-	0.008
MEST(7)	-	-	-	-	0.058
MEST(9)	-	-	-	-	0.085
MEST(6)	-	-	-	0.038	0.089
MEST(5)	**0.938**	**0.541**	**0.397**	0.229	0.001
MEST(10)	0.50	0.216	0.269	**0.343**	0.147
MEST(26)	-	-	-	-	0.020
MEST(23)	-	-	-	-	0.017
MEST(28)	-	-	-	-	0.003
MEST(27)	-	0.135	0.167	0.143	0.061
MEST(22)	-	-	-	-	0.004
MEST(35)	-	-	-	-	0.045
MEST(37)	-	-	-	-	0.027

TABLE 6.19

RF(∘) FOR THE ESTIMATORS WHEN ρ = -0.4 AND n = 10.

Subintervals → Estimators ↓	GI6	GI7	GI8	GI9	GI10
$\bar{y}$	-	-	-	0.026	0.007
$\bar{Y}_{P}$	0.014	0.077	0.095	0.017	-
$\bar{Y}_{Re.}$	0.119	-	0.010	0.017	0.081
$\bar{Y}_{VSa.}$	-	-	0.010	0.093	0.070
$\bar{Y}_{MP}$	-	0.019	0.133	0.169	0.035
$\bar{Y}^{(1)}_{RP.ab}$	-	-	0.010	0.017	0.063
$\bar{Y}_{P.a}$	-	0.019	0.019	0.008	**0.111**
MEST(7)	0.042	-	-	0.017	0.072
MEST(9)	-	-	0.010	0.085	0.087
MEST(6)	-	-	-	0.085	0.079
MEST(5)	**0.650**	**0.750**	**0.524**	**0.263**	0.004
MEST(10)	-	0.038	0.124	0.093	0.100
MEST(26)	-	0.038	-	0.026	0.090
MEST(23)	0.147	-	-	0.008	0.020
MEST(28)	-	-	-	0.017	0.041
MEST(27)	-	-	0.028	0.034	0.030
MEST(22)	-	-	-	0.008	0.033
MEST(35)	0.014	-	0.010	-	0.039
MEST(37)	0.014	0.058	0.028	0.017	0.039

TABLE 6.20

RF(∘) FOR THE ESTIMATORS WHEN $\rho = -0.4$ AND n = 20.

Subintervals → / Estimators ↓	GI6	GI7	GI8	GI9	GI10
$\bar{y}$	0.007	-	-	0.008	0.007
$\bar{Y}_{P}$	0.014	0.174	0.133	0.051	-
$\bar{Y}_{Re.}$	0.077	-	-	0.034	**0.129**
$\bar{Y}_{VSa.}$	0.007	-	0.010	0.051	0.079
$\bar{Y}_{MP}$	-	-	0.010	0.051	0.079
$\bar{Y}^{(1)}_{RP.ab}$	-	-	0.086	0.186	0.020
$\bar{Y}_{P.a}$	-	0.038	0.010	-	0.063
MEST(7)	0.049	-	-	0.008	0.072
MEST(9)	-	-	0.019	0.110	0.107
MEST(6)	-	-	-	0.059	0.082
MEST(5)	**0.741**	**0.615**	**0.448**	0.110	-
MEST(10)	0.014	0.134	0.200	**0.136**	0.103
MEST(26)	-	-	0.019	0.076	0.048
MEST(23)	0.056	-	0.010	0.017	0.022
MEST(28)	-	0.019	-	0.034	0.031
MEST(27)	-	-	0.028	0.042	0.031
MEST(22)	0.007	-	-	-	0.033
MEST(35)	0.007	-	-	0.017	0.054
MEST(37)	0.021	0.019	-	0.008	0.044

RF(∘) FOR THE ESTIMATORS WHEN ρ = -0.4 AND n = 50.

Subintervals → Estimators ↓	GI6	GI7	GI8	GI9	GI10
$\bar{y}$	0.007	-	-	-	0.013
$\bar{Y}_{P}$	0.077	0.096	0.190	0.051	-
$\bar{Y}_{Re.}$	0.021	0.019	0.028	0.034	0.137
$\bar{Y}_{VSa.}$	-	-	-	0.059	0.088
$\bar{Y}_{MP}$	-	-	0.086	0.144	0.029
$\bar{Y}^{(1)}_{RP.ab}$	-	-	0.076	0.110	0.059
$\bar{Y}_{P.a}$	0.014	0.058	0.038	-	0.024
MEST(7)	0.007	-	-	0.017	0.100
MEST(9)	-	0.019	0.028	0.042	0.085
MEST(6)	-	-	-	0.085	0.089
MEST(5)	**0.720**	**0.404**	0.162	0.008	-
MEST(10)	0.112	0.346	**0.200**	**0.220**	**0.155**
MEST(26)	-	-	0.028	0.076	0.039
MEST(23)	0.014	0.019	0.010	0.025	0.035
MEST(28)	-	-	-	0.017	0.006
MEST(27)	-	0.019	0.048	0.068	0.026
MEST(22)	0.014	-	0.010	-	0.018
MEST(35)	0.007	0.019	0.028	0.008	0.063
MEST(37)	0.007	-	0.067	0.034	0.035

TABLE 6.22

RF(∘) FOR THE ESTIMATORS WHEN ρ = -0.6 AND n = 10.

Subintervals → / Estimators ↓	GI6	GI7	GI8	GI9	GI10
$\bar{y}$	-	-	-	-	-
$\bar{Y}_{P}$	0.036	0.238	0.168	0.015	-
$\bar{Y}_{Re.}$	0.277	0.111	0.044	0.061	**0.148**
$\bar{Y}_{VSa.}$	0.005	-	0.018	0.054	0.028
$\bar{Y}_{MP}$	-	0.016	0.088	**0.254**	0.044
$\bar{Y}^{(1)}_{RP.ab}$	-	-	0.009	0.023	0.061
$\bar{Y}_{P.a}$	-	-	0.009	0.008	0.126
MEST(7)	0.031	-	-	0.062	0.109
MEST(9)	-	0.032	0.035	0.100	0.035
MEST(6)	0.077	0.048	0.009	0.023	0.033
MEST(5)	**0.318**	**0.254**	0.186	0.015	0.072
MEST(10)	0.010	0.063	0.062	0.054	0.044
MEST(26)	0.015	0.032	0.018	0.023	0.046
MEST(23)	0.087	0.032	0.026	0.062	0.046
MEST(28)	-	-	0.018	0.069	0.035
MEST(27)	-	-	0.018	0.031	0.063
MEST(22)	0.015	-	0.053	0.023	0.054
MEST(35)	0.056	0.048	0.035	0.023	0.057
MEST(37)	0.072	0.127	**0.204**	0.100	

TABLE 6.23

RF(∘) FOR THE ESTIMATORS WHEN ρ = -0.6 AND n = 50.

Subintervals → / Estimators ↓	GI6	GI7	GI8	GI9	GI10
$\bar{y}$	-	-	-	-	-
$\bar{Y}_P$	0.046	0.222	0.142	0.046	-
$\bar{Y}_{Re.}$	0.169	0.048	0.035	0.115	0.146
$\bar{Y}_{VSa.}$	0.005	0.016	0.009	0.069	0.037
$\bar{Y}_{MP}$	-	-	0.035	0.085	0.024
$\bar{Y}^{(1)}_{RP.ab}$	-	0.032	0.035	0.054	0.041
$\bar{Y}_{P.a}$	0.005	0.016	0.053	0.023	0.083
MEST(7)	0.041	0.032	0.009	0.046	**0.155**
MEST(9)	0.016	-	0.027	0.069	0.054
MEST(6)	0.077	0.048	0.027	0.054	0.039
MEST(5)	**0.364**	**0.238**	0.062	0.008	-
MEST(10)	0.026	0.032	0.044	0.008	0.054
MEST(26)	0.005	0.032	0.097	0.085	0.039
MEST(23)	0.056	0.016	0.018	0.054	0.063
MEST(28)	-	-	0.009	0.061	0.033
MEST(27)	-	0.016	0.026	0.038	0.035
MEST(22)	0.005	0.048	0.053	0.031	0.078
MEST(35)	0.087	0.048	0.071	0.023	0.067
MEST(37)	0.097	0.158	**0.248**	**0.131**	0.050

TABLE 6.24

RF(∘) FOR THE ESTIMATORS WHEN ρ = -0.6 AND n = 50.

Subintervals → / Estimators ↓	GI6	GI7	GI8	GI9	GI10
$\bar{y}$	-	-	-	-	-
$\bar{Y}_{P}$	0.067	**0.349**	**0.239**	0.069	-
$\bar{Y}_{Re.}$	0.123	0.032	0.053	**0.146**	0.135
$\bar{Y}_{VSa.}$	0.005	0.016	0.018	0.077	0.063
$\bar{Y}_{MP}$	-	-	0.018	0.085	0.033
$\bar{Y}^{(1)}_{RP.ab}$	0.005	0.032	0.035	0.023	0.063
$\bar{Y}_{P.a}$	0.005	0.079	0.071	0.023	0.044
MEST(7)	0.026	-	0.018	0.054	**0.144**
MEST(9)	0.010	0.016	0.035	0.062	0.065
MEST(6)	0.062	0.032	0.018	0.069	0.039
MEST(5)	**0.359**	0.079	0.009	-	-
MEST(10)	0.061	0.079	0.080	0.069	0.089
MEST(26)	0.031	0.063	0.026	0.108	0.039
MEST(23)	0.020	-	0.009	0.0046	0.076
MEST(28)	-	0.016	0.018	0.008	0.022
MEST(27)	-	0.032	0.035	0.046	0.044
MEST(22)	0.031	-	0.062	0.046	0.052
MEST(35)	0.061	0.048	0.018	0.038	0.065
MEST(37)	0.133	0.127	**0.239**	0.031	0.026

TABLE 6.25

RF(∘) FOR THE ESTIMATORS WHEN ρ = -0.8 AND n = 10.

Subintervals → Estimators ↓	GI6	GI7	GI8	GI9	GI10
$\bar{y}$	-	-	-	-	-
$\bar{Y}_P$	0.026	0.092	0.068	-	-
$\bar{Y}_{Re.}$	0.489	0.262	0.186	0.163	0.252
$\bar{Y}_{VSa.}$	-	-	0.008	-	0.002
$\bar{Y}_{MP}$	-	-	0.042	0.135	0.032
$\bar{Y}^{(1)}_{RP.ab}$	-	-	-	-	0.025
$\bar{Y}_{P.a}$	-	-	-	-	0.080
MEST(7)	0.034	0.046	0.017	0.064	0.224
MEST(9)	-	0.031	-	0.014	0.013
MEST(6)	0.221	0.077	0.008	0.021	0.010
MEST(5)	0.013	0.015	-	-	-
MEST(10)	0.021	-	-	0.007	0.020
MEST(26)	0.004	0.062	0.034	0.050	0.010
MEST(23)	0.021	0.031	0.068	0.135	0.152
MEST(28)	-	-	0.008	0.142	0.030
MEST(27)	-	0.015	0.051	0.057	0.013
MEST(22)	0.004	0.031	0.085	0.021	0.052
MEST(35)	0.077	0.108	0.076	0.085	0.077
MEST(37)	0.089	0.231	0.347	0.106	0.008

TABLE 6.26

RF(∘) FOR THE ESTIMATORS WHEN ρ = -0.8 AND n = 20.

Subintervals → / Estimators ↓	GI6	GI7	GI8	GI9	GI10
$\bar{y}$	-	-	-	-	-
$\bar{Y}_P$	0.017	0.077	0.017	0.028	-
$\bar{Y}_{Re.}$	**0.383**	0.108	0.144	**0.191**	**0.297**
$\bar{Y}_{VSa.}$	-	-	0.008	0.007	0.010
$\bar{Y}_{MP}$	-	-	0.008	0.043	0.022
$\bar{Y}^{(1)}_{RP.ab}$	-	-	-	0.021	0.047
$\bar{Y}_{P.a}$	-	-	-	0.014	0.062
MEST(7)	0.030	0.062	0.042	0.099	0.232
MEST(9)	0.004	0.015	0.025	0.007	0.015
MEST(6)	0.213	0.046	0.025	0.014	0.015
MEST(5)	0.017	-	-	-	-
MEST(10)	0.026	0.015	0.008	0.021	0.020
MEST(26)	0.034	0.015	0.068	0.099	0.015
MEST(23)	0.009	-	0.102	0.092	0.112
MEST(28)	-	-	-	0.035	0.013
MEST(27)	0.017	0.046	0.102	0.035	0.013
MEST(22)	0.017	0.108	0.093	0.099	0.065
MEST(35)	0.077	0.185	0.102	0.071	0.062
MEST(37)	0.157	**0.323**	**0.254**	0.121	-

TABLE 6.27

RF(∘) FOR THE ESTIMATORS WHEN ρ = -0.8 AND n = 50.

Subintervals → Estimators ↓	GI6	GI7	GI8	GI9	GI10
$\bar{y}$	-	-	-	-	-
$\bar{Y}_P$	0.051	0.123	0.034	0.050	-
$\bar{Y}_{Re.}$	0.217	0.138	0.178	**0.248**	**0.264**
$\bar{Y}_{VSa.}$	0.004	0.015	0.008	0.014	0.008
$\bar{Y}_{MP}$	-	-	-	0.043	0.027
$\bar{Y}^{(1)}_{RP.ab}$	-	-	0.051	0.043	0.035
$\bar{Y}_{P.a}$	0.004	0.015	0.025	0.043	0.082
MEST(7)	0.021	0.046	0.051	0.014	0.227
MEST(9)	0.013	0.015	0.025	0.078	0.027
MEST(6)	0.200	0.031	0.034	0.043	0.013
MEST(5)	0.038	-	-	-	-
MEST(10)	0.026	0.015	-	0.035	0.040
MEST(26)	0.038	0.077	0.068	0.057	0.010
MEST(23)	0.013	0.015	0.059	0.135	0.112
MEST(28)	-	0.015	0.008	0.007	0.010
MEST(27)	0.038	0.031	0.025	0.064	0.015
MEST(22)	0.021	0.092	0.093	0.071	0.030
MEST(35)	0.085	0.061	0.102	0.057	0.087
MEST(37)	**0.230**	**0.308**	**0.237**	0.021	0.013

TABLE 6.28

RF(∘) FOR THE ESTIMATORS WHEN ρ = -0.9 AND n = 10.

Subintervals → / Estimators ↓	GI6	GI7	GI8	GI9	GI10
$\bar{y}$	-	-	-	-	-
$\bar{Y}_{P}$	0.008	0.030	-	-	-
$\bar{Y}_{Re.}$	**0.492**	0.288	0.213	0.227	**0.320**
$\bar{Y}_{VSa.}$	-	-	-	-	-
$\bar{Y}_{MP}$	-	-	-	0.014	0.008
$\bar{Y}^{(1)}_{RP.ab}$	-	0.030	-	0.007	0.027
$\bar{Y}_{P.a}$	-	-	-	-	0.061
MEST(7)	0.051	0.015	0.057	0.121	0.275
MEST(9)	-	-	-	-	-
MEST(6)	0.316	0.091	0.008	-	0.003
MEST(5)	0.004	-	-	-	-
MEST(10)	-	-	-	0.007	-
MEST(26)	-	0.015	0.008	0.028	0.005
MEST(23)	0.012	0.076	0.180	**0.248**	0.203
MEST(28)	-	-	-	0.106	0.003
MEST(27)	0.004	0.030	0.041	0.021	0.003
MEST(22)	-	0.076	0.082	0.092	0.043
MEST(35)	0.012	0.045	0.115	0.071	0.045
MEST(37)	0.102	**0.303**	**0.295**	0.057	0.005

TABLE 6.29

RF($\circ$) FOR THE ESTIMATORS WHEN ρ = -0.9 AND n = 20.

Subintervals → / Estimators ↓	GI6	GI7	GI8	GI9	GI10
$\bar{y}$	-	-	-	-	-
$\bar{Y}_P$	0.020	0.061	-	0.014	-
$\bar{Y}_{Re.}$	0.438	0.136	0.180	0.291	0.336
$\bar{Y}_{VSa.}$	-	-	-	-	0.008
$\bar{Y}_{MP}$	-	-	0.008	0.021	0.013
$\bar{Y}^{(1)}_{RP.ab}$	0.008	-	0.008	0.014	0.067
$\bar{Y}_{P.a}$	-	-	-	-	0.267
MEST(7)	0.035	0.061	0.041	0.092	0.003
MEST(9)	-	-	-	0.007	0.008
MEST(6)	0.289	0.076	0.008	0.014	-
MEST(5)	-	-	-	-	0.019
MEST(10)	-	-	0.008	0.021	0.003
MEST(26)	0.008	0.015	0.082	0.043	0.003
MEST(23)	-	0.045	0.164	0.170	0.184
MEST(28)	-	-	-	-	0.003
MEST(27)	0.035	0.106	0.008	0.035	0.003
MEST(22)	0.008	0.091	0.107	0.092	0.048
MEST(35)	0.023	0.121	0.123	0.121	0.037
MEST(37)	0.137	0.288	0.262	0.064	-

TABLE 6.30

RF(∘) FOR THE ESTIMATORS WHEN ρ = -0.9 AND n = 50.

Subintervals → / Estimators ↓	GI6	GI7	GI8	GI9	GI10
$\bar{y}$	-	-	-	-	-
$\bar{Y}_P$	0.020	0.045	-	0.071	-
$\bar{Y}_{Re.}$	0.281	0.212	**0.254**	**0.319**	**0.304**
$\bar{Y}_{VSa.}$	-	-	-	-	0.011
$\bar{Y}_{MP}$	-	-	-	0.014	0.013
$\bar{Y}^{(1)}_{RP.ab}$	-	-	0.033	0.028	0.048
$\bar{Y}_{P.a}$	-	-	-	-	0.061
MEST(7)	0.051	0.045	0.049	0.163	0.264
MEST(9)	-	-	0.008	-	0.011
MEST(6)	0.289	0.106	0.016	0.035	0.005
MEST(5)	-	-	-	-	-
MEST(10)	-	-	-	0.014	0.035
MEST(26)	0.008	0.015	0.082	0.021	0.005
MEST(23)	0.004	0.091	0.139	0.177	0.139
MEST(28)	-	-	-	-	0.005
MEST(27)	0.074	0.076	0.049	-	0.011
MEST(22)	0.031	0.076	0.107	0.106	0.029
MEST(35)	0.043	0.076	0.107	0.050	0.053
MEST(37)	0.199	**0.258**	0.156	-	0.005

CHAPTER - 7

CONCLUSIONS

In this chapter, we highlight the performances of various estimators proposed and studied by us in the preceding chapters. We note that $\bar{Y}_{P.a}$ (table 2.3) performs very nicely with empirical probability 0.817 (n=10) and 0.840 (n=20) for $\rho < 0$ among the various estimators studied in Chapter-2. While $\bar{Y}_{Sr.}$ (table 2.4), $\bar{Y}_{Re.}$ (table 2.5) and $\bar{Y}_{Sa.}$ (table 2.6) are the best estimators available in literature, we have been able to affect a betterment over them through $\bar{Y}_{VSa.}$ (table 2.7) which happens to be the winner with a probability of at least 80% and is a still more potential winner with probability more than 87% when $\rho < 0$ and n=20. Thus, $\bar{Y}_{VSa.}$ turns out to be the best amongst all the estimators proposed in Chapter-2.

In Chapter-3, we have proposed a couple of two-parameter family of the variants of ratio-cum-product estimators. Both of the proposed families (tables 3.1 and 3.2) are found to be more potential a winner when $\rho > 0$. Nevertheless when $\rho < 0$ and sample size is not very small, i.e., n=20 (or, possibly, a bigger sample size) the two estimators are expected to perform reasonably well with their empirical probability of winning being 63.5% and 61.5%, respectively.

In Chapter-4, we have been able to evolve some gainful mixings of the estimators proposed in Chapter-2 and Chapter-3 with the mean-per-unit estimator. The work in this chapter has been motivated by that of Vos(1980). However, the type of comparison we have taken up to ascertain the potential winner is apparently rather more reasonable than that of Vos(1980). Amongst the proposed mixings of one parameter families of estimators, the mixing estimator MEST(6) (table 4.4) turns out to be the most potential gainer. It is worth noting that this estimator is much more superior to its parent estimator, namely, $\bar{Y}_{Sr.}$ particularly when sample size is rather small, say n=10 (or even smaller sample size). Another point worth noting is that the estimator $\bar{Y}_{VSa.}$ is not bettered by this mixing when $\rho > 0$ but is bettered when $\rho < 0$ particularly when n=10 (table 4.7). As far as the mixing of two-parameter family of estimator is concerned, we note that the significant betterment over parent-estimators is achieved only when $\rho > 0$ and sample size is rather small, i.e., n=10 (tables 4.8 and 4.9).

In Chapter-5, we have proposed some gainful mixings of two potential winners from amongst the estimators of Chapter-2 and Chapter-3. We have also proposed a similar mixing considering three mixing-estimators at a time mixed with $\bar{y}$. In this Chapter, for apparent reasons, we have considered the mixings separately for the two possibilities, namely, $\rho > 0$ and $\rho < 0$, respectively. We observe that the proposed mixings of two parent-estimators have been gainful except in the case of MEST(16) (table 5.10) and

MEST(24) (table 5.26). Another worth noting feature of this type of mixing of two parent-estimators as per table 5.28 is that the estimator MEST(25) excels over the better of the two parent-estimators, namely, $\bar{Y}_{P.a}$ with an empirical probability of 92.5% or even more. Amongst the other mixings of this type, we have the following observations. It turns out that while the mixing estimators MEST(19) (table 5.16) and MEST(29) (table 5.36) are more potential winners than their parent estimators for rather smaller sample size, i.e., n=10, the mixing estimators MEST(23) (table 5.24), MEST(28) (table 5.34) and MEST(31) (table 5.40) happened to be more potential winners than their parent-estimators for rather a larger sample size, i.e., n=20. Further, it has been noted that the mixing estimators MEST(20) (table 5.18), MEST(21) (table 5.20), MEST(26) (table 5.30), MEST(27) (table 5.32) and MEST(30) (table 5.38) provide a more probable betterment over their parent-estimators for rather smaller sample size, i.e., n=10 : the mixing estimator MEST(22) (table 5.22) turns out to be more potential a winner when the sample size is rather large, i.e., n=20. Next, amongst the other type of mixing, i.e., those of three parent-estimators (including $\bar{Y}_R/\bar{Y}_P$ according as ρ >/< 0) with $\bar{y}$, it is worth noting that the mixing estimators MEST(32) (table 5.41), MEST(33) (table 5.42), MEST(34) (table 5.43) and MEST(36) (table 5.45) fail to beat the parent-estimators universally, however, a finer comparison reveals the range of betterment, respectively. While the mixing estimator MEST(37) (table 5.46) provide a betterment only when n=10 (i.e., the sample size is rather small), the mixing-estimator MEST(35) (table 5.44) is capable of providing a significant betterment over the parent-estimators only when n=20.

In Chapter-6, we have first taken up the case of positive correlation. For the first example-value : ρ=0.2, the comparisons reveal that MEST(7) is rather the universally most probable winner except for the G-interval : GI1, when MEST(5) supersedes it and when the sample size is rather large : n=50, the most probable winner happens to be MEST(10). Another worth noting observation is that for the example G-interval : GI5, MEST(7) is the most probable winner for all the example-values of ρ with two exceptions : ρ=0.8; n=10, MEST(3) is the most probable winner and ρ=0.9; n=20, $\bar{Y}_{Re.}$ is the most probable winner. Next, for second example-value : ρ=0.4, MEST(7) again is the most probable winner except G $\in$ GI1 when MEST(10) is the most probable winner and G $\in$ GI2 ; n=20/n=50, when MEST(11)/$\bar{Y}^{(1)}_{RP.ab}$ is the most probable winner. For the third example-value : ρ=0.6, $\bar{Y}_{Re.}$/$\bar{Y}_{VR}$/ MEST(7) is the most probable winner when G $\in$ GI1/G $\in$ GI2/G $\in$ GI5. Also, when G $\in$ GI1 or GI2, MEST(7)/MEST(15) is the most probable winner for n=10/n=50. For the fourth example-value : ρ=0.8, when G $\in$ GI3 or GI4, $\bar{Y}_{Re.}$ is the most probable winner for n=20 and n=50. However, when the sample size is small $\bar{Y}_{VR}$/MEST(3) is the most probable winner for G $\in$ GI3/G $\in$ GI4. MEST(15)/MEST(7) is the most probable winner when G $\in$ GI2/G $\in$ GI5 for n=20 and n=50; when sample size is small : n=10, MEST(14)/MEST(3) takes over, respectively. In this case when G $\in$ GI1, $\bar{Y}_{VR}$/$\bar{Y}_{Re.}$/ MEST(5) is the most probable winner for n=10/n=20/n=50. Lastly, for the fifth example-value, $\bar{Y}_{Re.}$/MEST(21)/$\bar{Y}_{Re.}$/MEST(7) is the most probable winner for G $\in$ GI1/G $\in$ GI2/G $\in$ GI3 or GI4/G $\in$ GI5 except one case, i.e., n=20 and G $\in$ GI5 when $\bar{Y}_{Re.}$ is the most probable winner.

In the second case taken up in Chapter-6 that is that of negative correlation, the important observations are as below. In the first example-value : $\rho=-0.2$, MEST(5) is the most probable winner except when $G \in GI10$; in that case MEST(6)/ $\bar{Y}^{(1)}_{RP.ab}$ is the most probable winner when n=10 or n=20/n=50. For the second example-value : $\rho=-0.4$, MEST(5) is the most probable winner when $G \in GI6$ or GI7 or GI8 except the case of n=50 and $G \in GI8$ when MEST(10) is the most probable winner. When $G \in GI9$, MEST(5)/MEST(10) is the most probable winner for n=10/n=20 or 50. Also, when $G \in GI10$, $\bar{Y}_{P.a}$ /$\bar{Y}_{Re.}$/MEST(10) is the most probable winner for n=10/n=20/n=50. For the third example-value : $\rho=-0.6$, MEST(5) is the most probable winner when $G \in GI6$ or GI7 with the exception of the case n=50, $G \in GI7$ when $\bar{Y}_P$ is the most probable winner. When $G \in GI8$, MEST(37)/$\bar{Y}_{Re.}$ is the most probable winner for n=10 or 20/n=50. For $G \in GI10$, $\bar{Y}_{Re.}$/MEST(7) is the most probable winner for n=10/n=20 or 50. For the fourth example-value : $\rho=-0.8$, $\bar{Y}_{Re.}$ is the most probable winner for n=10 except for the case : $G \in GI8$ when MEST(37) is the most probable winner. For n=20, again $\bar{Y}_{Re.}$ is the most probable winner except for two cases : $G \in GI7$, $G \in GI8$ when MEST(37) is the most probable winner. For n=50, MEST(37) is the most probable winner except for the two cases : $G \in GI9$, $G \in GI10$ when $\bar{Y}_{Re.}$ is the most probable winner. For the last example-value : $\rho = -0.9$, $\bar{Y}_{Re.}$ is the most probable winner except the following cases. MEST(37) is the most probable winner when $G \in GI7$ or GI8; MEST(6) is the most probable winner when $G \in GI6$ and n=10; MEST(23) is the most probable winner when $G \in GI9$ and n=10.

In the last, we may conclude that even though we wanted to have a systematic comparison leading to the discovery of gainful directions in designing the generalised/mixing-type ratio-cum-product estimators for the optimal use of auxiliary information, the comparisons were still too intricate and rather partly unconclusive as apparent in the above paras. It is simply hoped that future researches in this area will unveal more powerful systematic schemes of comparisons to make them conclusive for the optimal design of the generalised/mixing-type ratio-cum-product estimation strategies. Also, we may observe that any of the estimators proposed in the thesis can be the basis of defining a corresponding multivariate generalised/mixing-type estimator (e.g., on the pattern of Olkin(1958)), to facilitate the use of multi-auxiliary information.

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