ACCIDENTAL TORSION ANALYSIS IN FRAMED BUILDINGS

A DISSERTATION

submitted in partial fulfilment of the requirements for the award of the degree

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EARTHQUAKE ENGINEERING

(With Specialisation in STRUCTURAL DYNAMICS)

By

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CANDIDATE'S DECLARATION

I hereby certify that the work which is presented in this disseration entitled "**ACCIDENTAL TORSION ANALYSIS IN FRAMED BUILDINGS**", in partial fulfilment of the requirement for the award of the degree of **MASTER OF ENGINEERING** in **EARTHQUAKE ENGINEERING** with specialization in **STRUCTURAL DYNAMICS**, submitted in the Department of Earthquake Engineering, University of Roorkee, Roorkee, is an authentic record of my own work carried out for a period of about five months from December, 1992 to May 1993 under the supervision of Dr. Pankaj, Lecturer, Department of Earthquake Engineering, University of Roorkee, Roorkee, India.

The matter embodied in this dissertation has not been submitted by me for the award of any other degree or diploma.

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DATED: May, 26 1993 (N.K. SINGH) 26-5-93

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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ABSTRACT

Building frames have to be designed to resist torsional moment that may arise during an earthquake. A structure may face torsional forces due to planned eccentricity or due to accidental causes. This study predominantly focusss on later aspect.

Many codes of practice incorporate accidental torsion by including an additional eccentricity in the design eccentricity calculations. The analysis is then done using a purely state method. In this study a mass perturbation methodology is discussed and employed for anlysing buildings for accidental torsion. Mass perturbation alters the mass distribution thereby creating an additional eccentricity. The building is then analysed using a dynamic method.

Two example buildings are analysed one symmetric and the other asymmetric. Comparisions are made using mass perturbation methodology and static torsional analysis. It is concluded that a static torsional analysis may not be good enough and the mass perturbation method should be used.

CONTENTS

CHAPTER - 1

INTRODUCTION

Structures subjected to seismic excitation exhibit torsional response in addition to lateral vibrations. The torsional response arises due to a variety of reasons. The best recognised source of torsional response is asymmetric distribution of mass and the lateral load resisting elements in the plan of structure, which induces an eccentricity between the centres of mass and stiffness. In symmetrical structures torsional response may arise due to (a) torsional component of ground motion, (b) non-uniform ground motion along the foundation of structure or (c) due to accidental causes introduced during construction or during occupancy (uneven distribution of the load).

Most codes of practice recommend a separate static analysis to take into account torsional response that may arise in structures. This static analysis invariably involves application of a torsional moment at each floor level. The calculation of this moment, in turn, requires, the calculation of the so called design eccentricity. The design eccentricity is calculated using an amplified static eccentricity plus accidental torsional eccentricity which depends on plan dimensions.

For asymmetric buildings it has been shown that the maximum value of story torque due to dynamic excitation can be significantly larger than the static torque, calculated by the product of static eccentricity and story shear of the corresponding concentric structure. Several procedures to take into account this amplification have been suggested (1,2,3).

However, comparatively less work has been done to take into account the torsional response that may arise due to accidental eccentricity (4,5,6,19). Owing to the uncertain nature of sources that cause accidental torsion it is difficult to account for them using a direct deterministic approach. Many building codes recommend application of an additional static torque to account for torsional response arising out of accidental eccentricity (7). The second approach to take into account accidental torsional response emanates from the introduction of a shift in the centre of mass (4,5,6,19).

In case of symmetrical structures no contribution from torsional modes can be obtained due to the translational components of ground motion (4). Thus in order to account for accidental torsional response something more than a mere dynamic analysis is essential.

In this study a mass perturbation procedure is considered and the possibility of avoiding a dual static and dynamic analysis to account for additional torsional response studied. A comparative study is conducted using the conventional quasi-static approach and the perturbation approach which calls for dynamic analysis only.

CHAPTER - 2

REVIEW OF LITERATURE

2.1 Introduction

As discussed in the previous chapter torsional response can arise due to a variety of reasons. This study predominantly deals with torsion arising out of accidental causes such as those due to uncertainties in location of loads. These cause an accidental eccentricity which is envisaged to be of the order of 5 to 10 % of plan dimension by many seismic codes. The more researched cause of torsion i.e. torsion arising due to asymmetry is also discussed.

2.2 Codal provisions for evaluation of torsional response.

Most buildings codes require provisions to take into account torsional response arising out of planned or accidental causes. They require designing for a torsional moment which is proportional to the so called design eccentricity e_d which in turn is given by the following formula.

$$e_d = \alpha_1 e + \beta_1 b \qquad 2.1(a)$$

$$e_d = \alpha_2 e - \beta_2 b \qquad 2.1(b)$$

In the above formula 'e' is the static eccentricity i.e. the distance between the centre of mass and centre of stiffness and 'b' is the plan dimension perpendicular to the applied earthquake direction. The values of $\alpha_1, \alpha_2, \beta_1$ and β_2 as recommended by various codes (7) are given in table 2.1.

The factors α_1 and α_2 are basically amplification factors that amplify static eccentricity to obtain the expected values of dynamic eccentricity. It has been shown that dynamic eccentricity can be much more than predicted by these factors especially for small values of static eccentricity (1).

The second part of equation 2.1 are meant to take into account accidental eccentricity. No firm comment on the choice of factors β_1 and β_2 can be made due to the uncertain nature of accidental torsional response. However, it is important to note that the Indian code (IS: 1893-1984) does not provide for accidental torsion. It has been shown that for a symmetric building translational earthquake motion can not excite torsional modes of vibration (4,5,6,19). Therefore, in the absence of planned eccentricity design for torsional response is not required as per IS: 1893-1984.

Most codes of practice also do not clearly state as to whether the torsional moments at various floor levels in a multistorey buildings need to be applied simultaneously or separately at each floor level. The codes are also silent about the direction in which these moments should be applied in case they are applied simultaneously. For example, for a two storey building it is possible to apply torsional moments at both the floors simultaneously but it is not clear as to whether the direction of the two moments should be identical or opposite.

2.3 Previous Studies

Effects of torsion on the linear-elastic and non-linear responses on account of seismic excitation have drawn the attention of many investigators over the past few years [1]. Most of the studies are based on an idealized single storey structure.

The responses of buildings are seen to be influenced by basic controlling parameters of the system, such as static eccentricity ratio (e/r), uncoupled translational frequencies (w_y) and uncoupled frequency ratio (UFR).

The studies conducted by Kan and Chopra (11) have shown that the linear-elastic torsional response of multi-storey structures belonging to a special class of buildings, can be determined by the study of a single storey torsionally coupled system along with an N-storey torsionally uncoupled counterpart of the actual building. In this context a minimum of two independent degrees of freedom (one translational and one rotational) are sufficient to model the special class of torsionally coupled systems.

After analysing such a system with an idealized response spectra technique, Kan and Chopra (10) showed that a relationship between base shear and torque in a coupled system to the base shear of an uncoupled system, can be worked out by means of simple interactive equations like,

$$\left[\frac{V_x}{V_{yo}}\right]^2 + \left[\frac{V_y}{V_{yo}}\right]^2 + \left[\frac{T_m}{V_{yo}r}\right]^2 = 1$$

where V_x and V_y are base shears in the x and y directions, respectively ; V_{yo} is maximum base shear of the corresponding uncoupled system in the y direction , T_m is base torque around the centre of mass ; and r is radius of gyration about a vertical axis passing through centre of mass. Obviously, for a system symmetric about the X- axis this interaction equation reduces to.

$$\left[\frac{V_y}{V_{yo}}\right]^2 + \left[\frac{T_m}{V_{yo}r}\right]^2 = 1$$

From these interaction equations they deduced that the value of total base shear in a torsionally coupled system is less than that in the corresponding uncoupled system.

A further study was performed by Chandler and Hutchinson [14] using the time history approach. They showed that in response to individual earthquakes, over some ranges of the parameters (e/r) and UFR, some small increase in dynamic shear may result from lateral-torsional coupling ; and hence the design storey shear forces do not always provide a conservative estimate of response irrespective of the presence of eccentricity.

The parameters e/r and UFR are of much significance as revealed by many investigators. It has been shown using both a time history analysis approach and a response spectra approach, that for small static eccentricity ratios and values of UFR close to unity, the dynamic amplification of static eccentricity is considerably large (8,9,10,12,13,14). Tso and Dempsey (8), in their assessment of the recommendations of five major building codes for seismic design of torsionally coupled buildings, showed that except Germany, all the codes significantly underestimate the torsional moment and edge displacement of a typical linear-elastic single-storey system, when the eccentricity ratio is small and UFR close to unity. Pertaining to this a bilinear relationship between the dynamic and static eccentricities is perhaps more suitable to represent the dynamic eccentricity in the building codes. An example of this type of relationship suggested by Tso and Dempsey (8) is as under.

Flat spectrum

$$\frac{e_d}{r} = \frac{6e}{r} \qquad \frac{e}{r} < 0.1$$

$$\frac{e_d}{r} = 0.6 + 0.6\left[\frac{e}{r} - 0.1\right] \qquad \frac{e}{r} > 0.1$$

Hyperbolic spectrum

$$\frac{e_d}{r} = \frac{6e}{r} \qquad \frac{e}{r} < 0.1$$

$$\frac{e_d}{r} = 0.6 + 0.4\left[\frac{e}{r} - 0.1\right] \qquad \frac{e}{r} > 0.1$$

where e_d is dynamic eccentricity ; e is static eccentricity and r is radius of gyration.

In the study by Chandler and Hutchinson [14] same results were obtained through the use of a linear-elastic time history analysis. They showed that the lateral seismic force, taking into account the accidental eccentricities, is underestimated by ATC-3 (15) and the Canadian (17), Mexican, (7) and New Zealand codes (18) for small to moderate eccentricities and UFR in the range of 0.8 to 1.7. All the above codes provide overconservative estimate of lateral force response for U.F.R. smaller than 0.8, and a reasonable estimate for U.F.R. greater than 1.7. This also confirms the results of Kan and Chopra (12) that, for values of U.F.R greater than two, lateral displacement is essentially uneffected by torsion, and the torsional deformation is proportional to e/r, indicating no dynamic amplification.

Most of the above research has been directed towards structures that do have an eccentricity. Comparatively less work has been done to understand the provisions for accidental eccentricities (4,5,6,19). A symmetrical structure, in which the so called centre of mass and centre of rigidity coincides, will have pure torsional modes and no translational earthquake will be able to excite these modes (4,5). In other words mode participation factors for these modes will be zero. Thus a dynamic analysis using a three dimensional idealization would not help. However, even in these "symmetrical" structures eccentricity may arise due to accidental causes which are predominantly due to uncertain location of loads. If a dynamic analysis does not help a dual approach - a dynamic analysis followed by a static analysis to take into account accidental torsion, becomes essential, unless an eccentricity is introduced. The US code (7) suggests mass perturbation to take into account accidental eccentricities but does not specifically state the methodology. Mass perturbation has also been suggested by other investigators (4,5) and will be discussed in the following chapter.

CHAPTER - 3

ANALYSIS OF ACCIDENTAL TORSION - MASS PERTURBATION APPROACH

3.1 Introduction

As discussed in the last chapter many codes of practice have provisions to take into account response due to accidental torsion. Generally, a static method has been suggested as a mere dynamic analysis could not take into account accidental torsion. The US code has suggested that an additional eccentricity be introduced by perturbing masses i.e. changing the mass distribution in the structure. However, no clear methodology in this regard has been suggested.

In this chapter a mass perturbation methodology will be discussed to account for accidental torsion response. As is the usual practice it will be assumed that the dynamic analysis is conducted by applying translational base excitation to the structure.

3.2 Mass perturbation methodology

In a symmetrical structure a translational excitation will be unable to excite torsional modes (4,5). Records of spectra of torsional ground motions are as yet not available. An analyst has, therefore, to confine himself to using translational earthquake excitations for analysis. In fact, this eases the analytical computations considerably.

Mass perturbation is a process which is used to change the mass distribution of the structural model, thereby introducing an eccentricity for symmetrical structures and an additional eccentricity for asymmetric structures. The methodology being discussed in this section was employed by Bicanic et al (5) and Singh (4).

Consider the mass distribution at a floor level as shown in figure 3.1. The radius of the circles are proportional to mass that it represents and the centre of mass is indicated as CM. An axis of mass perturbation w at any angle is selected. The mass perturbation axis is perpendicular to the direction in which the earthquake excitation is to be applied. Pankaj and Singh (19) suggest that a critical earthquake direction be first evaluated and for best effect the perturbation axis chosen perpendicular to it. The centre of mass is now moved to its new position $\overline{CM}$ on axis $\bar{P}$ while preserving total mass. Both P and $\bar{P}$ are axes perpendicular to ω. This shift is equal to a chosen percentage of gyratarn radius as shown in Figure 3.1. The two conditions-- (a) Preservation of total mass and (b) Shift of mass centre lead to two scaling factors on two sides (L and R) of axis P (figure 3.1). From the first condition one obtains.

$$\Sigma m_i = M = (1 - \beta_1) \Sigma m_i^L + (1 + \beta_2) \Sigma m_i^R \quad - 3.1$$

while from the second one gets

$$M (a_o + e) = (1 - \beta_1)\Sigma m_i^L a_i^L + (1 + \beta_2)\Sigma m_i^R a_i^R \quad - \quad 3.2$$

Solving the above two equations for β_1 and β_2 one gets

$$\beta_1 = \frac{M e \Sigma m_i^R}{\Sigma m_i^L \Sigma m_i^R a_i^R - \Sigma m_i^R \Sigma m_i^L a_i^L}$$

$$\beta_2 = \frac{M e \Sigma m_i^L}{\Sigma m_i^L \Sigma m_i^R a_i^R - \Sigma m_i^R \Sigma m_i^L a_i^L} \quad --- \quad 3.3$$

The two simultaneous conditions in practice may lead to movement of $\overline{CM}$ along $\bar{P}$ away from the perturbation axis thus resulting in an eccentricity which is more than e.

The above procedure eliminates the need to evaluate the centre of rigidity and takes into account accidental eccentricities. Moreover, the analysis does not require "static" torsional analysis as the complete analysis can take place in a dynamic domain.

3.3 Discussion of the methodology

The methodology discussed can be used to introduce eccentricities in symmetrical structures and to increase (or decrease) the eccentricities in an asymmetric structure. The procedure eliminates the need to evaluate the centre of rigidities. The choice of the eccentricities desired to be introduced lies with the analyst. The eccentricity to be introduced may be a fraction of the radius of gyration as shown in figure 3.1 or be a fraction of the plan dimension perpendicular to the direction of the applied earthquakes as suggested by several codes (7).

Asymmetric structures are also termed as coupled structures because the response due to torsion and translation are coupled. Symmetrical structures on the other hand exhibit pure torsional and translational modes which are uncoupled. At times it may be necessary for the sake of comparison to obtain uncoupled response in an asymmetric structure due to a particular earthquake direction. This can be easily achieved using the mass perturbation procedure discussed Consider for example the plan of a single storey structure as shown in figure 3.2 where the rectangles indicate columns.

The masses may be lumped at column tops. Let the CM and CR be as shown. If an earthquake is applied along direction 1 as shown torsional modes would not be excited as in this direction there is no eccentricity between centre of mass and centre of rigidity. Thus pure translational response can be expected. However, for any other direction say direction 2 as shown the response would be coupled. Mass perturbation can be performed to obtain an uncoupled response for earthquake in direction 2. In order to do this all that is required to be done is that a CM be moved to same point on line P from line P. All that is required to be known is the perpendicular distance between P and $\bar{P}$ which is the eccentricity 'e' to be introduced in this case.

Let, the equation of line P be

$$ax + by + c = 0 \tag{3.4}$$

Now in order to perform mass perturbation one needs to evaluate β_1 and β_2 using equation 3.3. For this purpose the perpendicular distances between the various mass points and P need to be known. This can easily be found using the basic equations of co-ordinate geometry wherein, the perpendicular distance a_i of a point (x_i, y_i) from the line defind by equation 3.4 is given by.

$$a_i = \frac{ax_i + by_i + c}{\sqrt{a^2 + b^2}}$$

Thus once the direction of excitation and value of the eccentricity required to be introduced is determined the mass perturbation method discussed can easily be applied.

CHAPTER - 4

PROBLEMS AND RESULTS

4.1 Introduction

In order to study the concept of mass perturbation and its efficacy in dealing with accidental torsion two buildings were considered. The first building is symmetrical while the second is asymmetric. In each case masses are assumed to be lumped at beam and column joints. A diagonal mass matrix with negligible mass moment of inertia was employed. All problems were solved using a three dimensional space frame idealization. The seismic loading used was in the form of an acceleration spectra with 5 % of critical damping as given in IS:1893,1984 (20).

4.2 Problem 1 - Symmetrical building.

A two storey building square in plan as shown in fig.4.1 was analysed.

The mathematical model of the building is shown in figure 4.2. This problem is similar to the one analysed by Singh (4) and Pankaj and Singh (19). However, the earlier investigations did not study some of the aspects of accidental torsion which are highlighted in this work.

This building is symmetrical as the centre of rigidity and centre of mass coincide and fall on a single vertical line. Therefore, torsional modes could not be excited by translational excitation. Thus in order to introduce a provision for accidental torsion mass perturbation as discussed in the last chapter was employed. The direction of excitation was assumed to be along the X-axis (fig. 4.2)

The eccentricity assumed for perturbation was 0.4 m which is 10% of the plan dimension perpendicular to the direction of earthquake. Something about which the mass perturbation methodology does not provide any clues is the manner in which perturbation is to be performed for multistory buildings such as this. It is quite possible to shift the centre of mass in the same direction on both floors or shift them in the opposite direction. Both these possibilities are illustrated by figure 4.3.

For the earthquake direction and eccentricity employed the use of equations 3.3 give $\beta_1 = 0.2$ and $\beta_2 = 0.2$. Thus in order to introduce an eccentricity of 0.4 m, masses on one side were increased by 20 % while the other decreased by 20 %. The only difference between perturbation 1 and perturbation 2 was (figure 4.3) that the increase was made on the same side for the two floors in case of perturbation 1 (fig.4.3a) and on the opposite sides on two floors for perturbation 2 (fig. 4.3b).

4.2.1. Natural Frequencies

The natural frequencies obtained for the unperturbed building and after employing perturbation 1 and perturbation 2 are given in table 4.1. It can be seen that there is negligible difference in natural frequencies before and after perturbation. Thus, the perturbation employed does not appear to influence, the natural frequencies of the structure significantly. This supports the procedure discussed. For a response spectra analysis the spectral values corresponding to different modes will, therefore, not change significantly due to perturbation.

4.2.2 Mode participation factors.

A comparison of mode participation factors for x direction excitations is given in Table 4.2. It can be seen that out of the first six modes, only modes 1 and 5 have non-zero mode participation factors in the unperturbed structure. The zero participation factors indicate that these modes do not contribute to the response under an X-direction earthquake. If a particular mode is purely a torsional mode, then in a symmetrical structure, one would obtain a zero participation factor for excitation in any direction. However, if a mode is purely translational then for earthquake excitation in directions orthogonal to its direction of translation zero participation factors would be obtained.

After perturbation the modes representing the torsional and translational response become coupled. As a result, non-zero participation factors are obtained for most modes. The reason why the second mode under perturbation 1 and the third mode under perturbation 2 have zero participation factors is clearly that these modes represent pure translational response in Y-direction. Since, there is no eccentricity in the Y-direction the mode representing translation in this direction remains uncoupled.

The response due to accidental torsion is obtained due to coupling of modes as discussed above.

4.2.3 Member forces

In order to study the response due to accidental torsion, dynamic and static analysis are performed. First, the unperturbed structure was analysed for earthquake forces under an X-direction earthquake. Response of first six modes of vibration and their SRSS (Square Root of Sum of Squares) values were computed. Then the earthquake was applied to the perturbed structures shown in figure 4.3. Once again the response in the first six modes and SRSS response was computed. Clearly the difference between the response after perturbation and the response prior to perturbation represents accidental torsional response. This difference was compared with that obtained using static torsional analysis. In order to conduct a static torsional analysis a torsional moment was applied at the two floors. The torsional moment was computed using

$$M_1 = V_{x\,1} \times e$$

$$M_2 = V_{x\,2} \times e$$

where M_1 and M_2 represent torsional moment at floors 1 and 2; e is eccentricity (0.4 m) and Vx_1 and V_{x2} represent shears in the first and second storys in the X-direction. Vx_1 and Vx_2 were obtained from the three dimensional unperturbed analysis. The static analysis was done for six cases. The cases are described below.

Case 1 Vx_1 and Vx_2 computed from mode 1 response of unperturbed structure and M_1 and M_2 applied in the same direction at the two floors.

Case 2 Vx_1 and Vx_2 computed from mode 1 response of unperturbed structure and M_1 and M_2 applied in the opposite direction at the two floors.

Case 3 Vx_1 and Vx_2 computed from mode 5 response of unperturbed structure and M_1 and M_2 applied in the same direction.

Case 4 Vx_1 and Vx_2 computed from mode 5 response of unperturbed structure and M_1 and M_2 applied in the opposite direction.

Case 5 Vx_1 and Vx_2 computed from SRSS response of unperturbed structure and M_1 and M_2 applied in the same direction.

Case 6 Vx_1 and Vx_2 computed from SRSS response of unperturbed structure and M_1 and M_2 applied in the opposite direction.

The results obtained by the three dynamic analyses (one prior to perturbation and two after perturbation) and static analysis for the six cases are given in tables 4.3 to 4.10.

It can be seen that the additional member forces due to accidental torsional response are because of perturbation 1 for first story columns and first floor beams (Tables 4.3 to 4.6). On the other hand the second story columns and second floor beams get influenced by perturbation 2 (Table 4.7 to 4.10), with regard to the additional member forces due to accidental torsion.

This feature is also reflected by the static analysis. The maximum torsional response in the first storey and first floor members is obtained when static torsional moments are applied in the sane direction at the two floors (Tables 4.3 to 4.6). Application of moments in the same direction corresponds to Perturbation 1. Similarly the torsional response in the second storey and second floor members is maximum when static torsion moments are applied in the opposite direction on the two floors (Tables 4.7 to 4.10). This corresponds to Perturbation 2.

In general it can be seen that the order of maximum difference between the perturbed and unperturbed dynamic analysis is of the same order as that obtained from static analysis. Moreover, in dynamic analysis the torsional response is governed by just two modes i.e. modes 1 and 3 for perturbation 1 and modes 1 and 2 for perturbation 2. It also appears from the study done that perhaps it would be adequate to use SRSS shears for static analysis (cases 5 and 6 of static analysis). However, if one has to conduct a static torsional analysis then it would be more appropriate to follow the following procedure.

(a) From an unperturbed dynamic analysis obtain storey shears for different modes.

(b) For each mode calculate the torsional moment to be applied at each floor.

(c) Separately apply torsional moment obtained from different modes and calculate member forces which now corresponds to different modes.

(d) Compute the SRSS of the member forces computed in step c for different modes. This should truly represent the member forces due to member torsion.

In the example studied it can be seen that the use of SRSS storey shears (for calculation of torsional moments) yielded almost identical results as compared to those obtained using the above procedure.

In general it can be seen that mass perturbation provides a method for accidental torsion which has several advantages over static torsional analysis. First, it does not require the use of a dual static and dynamic analyses. Secondly, it is physically meaningful as it accounts for the actual manner in which accidental eccentricity could be introduced. The perturbation procedure suggested does not change appreciably the natural frequency characteristics of the structure. The third advantage implies that one is still analysing the original structure as far as the natural frequency content is concerned. One aspect which needs further study is the direction of the perturbation for different floors in a multistorey building. In fact this aspect also needs to be examined for static analysis in which direction of torsional moment application is involved.

4.3 Problem 2 - Asymmetric Building.

A single story building of rectangular plan as shown in figure 4.4 was selected. The mathematical model of this building is shown in figure 4.5.

Due to the unequal column sizes the building is asymmetric. The first floor of the building is assumed to be very stiff. This was acheived in the mathematical model by providing bracing members at first floor level as shown in figure 4.5. Masses were assumed again to be lumped at beam and column joints and for the sake of simplicity equal masses were assumed at all nodes. Thus the centre of mass (CM) was located at the centre of the plan as shown in figure 4.4. The static centre of rigidity calculated was found to be located at CR as shown in figure 4.4. Thus, along both transverse and longitudinal directions the structure has eccentricities. As a result in both these direction the torsional response would be coupled with the translational response.

The dynamic analysis of a structure was conducted for four cases. In the first case the structure was analysed for an X-direction earthquake, in the form it is, without any perturbation. This analysis would be subsequently referred to as analysis pertaining to the **"original"** structure. The second dynamic analysis was conducted for the same earthquake after removing the eccentricity in the X-direction. The eccentricity was removed by shifting the centre of mass in the Y-direction so that it lies on a line parallel to the X-axis and passing through CR. This was acheived through mass perturbation, where, the use of equations 3.3 yield β_1 = 0.157 and β_2 = 0.157. Thus, masses on the right of CM were increased by 15.7 % and on the left decreased by the same amount. This perturbation uncouples the X-direction modes from the torsional modes and would be referred to as **"uncoupled"** analysis.

The third dynamic analysis was conducted after introducing in the original structure an additional eccentricity of 10 % of the plan dimension i.e. 0.8 m, to take into account the possibility of accidental torsion. In this case the masses on the left of CM of the original structure was increased while those on the right decreased. Thus, the total eccentricity in X-direction become 1.219 m. To acheive this, once again, equation 3.3 was employed and yielded $\beta_1 = \beta_2 = 0.3$. This case would be referred to as **"perturbation 1"**.

The fourth dynamic analysis was similar to perturbation 1 except that the new CM was moved towards the right by a distance of 0.8 m, from the centre of plan. This will be referred as perturbation 2.

The shift in the centre of mass for all these cases is illustrated in figure 4.6.

4.3.1 Frequencies

The first six natural frequencies of the structure for various cases illustrated in figure 4.6 are given in Table 4.11. It can be seen that basically the first three modes are of real interest. Further, it can be observed that various perturbation do not alter the natural frequencies significantly.

4.3.2 Mode shapes and mode participation factors

The first four modes for various dynamic cases were plotted in plan and are shown in figure 4.7 to fig. 4.22. For all cases it can be seen that the fourth mode corresponds to a very high frequency (Table 4.11) and is associated with floor distortion of the very stiff floor figs. 4.10, 4.14, 4.18 and 4.22. Thus, from this point onwards attention will be confined to the first three modes which are of practical interest.

The first mode of the original structure (fig.4.7) and under perturbation 1 and 2 (fig.4.15 and 4.19) are coupled modes that represent predominantly translational displacements in the X-direction and torsion in that order. The first mode in the uncoupled structure, as expected, is an X-directional translational mode. Thus, under earthquake excitation in X-direction, the first mode will contribute towards translational and torsional response in the original and perturbed structures while it will contribute only towards translational response in uncoupled structure. This is also apparent from Table 4.12 which gives mode participation factors for X-direction excitation. The maximum participation factors can be seen to be for the uncoupled structure in the first mode.

Similarly, the second mode in all cases except uncoupled case can be seen to be predominantly torsional and X-directional in that order (Fig. 4.8, 4.16 and 4.20). The second mode for the uncoupled case (fig. 4.12) is a coupled torsional and Y directional mode in that order. Once again from table 4.12 it can be seen that for the uncoupled case mode 2 has a negligible participation factor.

The third mode in all cases in predominantly Y-directional and torsional in that order (Fig. 4.9, 4.13, 4.17 and 4.21). As a result, this mode has the least participation factors for an X directional earthquake for all cases as seen from Table 4.12.

For a symmetrical single story structure the first three modes are expected to represent translation in the X and Y direction and pure torsion separately. However, in this case the first three modes represent X and Y direction translation and torsion in a coupled manner. Only in case of an uncoupled structure coupling of any response with translational response in X direction has been prevented through mass perturbation. Thus, it can be said that, for an X-directional earthquake the uncoupled structure would respond to only to mode 1 out of the first three modes.

4.3.3 Member forces.

The structure was analysed for the four dynamic cases as discussed earlier and various forces in the nine columns evaluated. The dynamic analysis of the original structure under an X direction earthquake would give member forces due to translational response and due to the torsion induced because of asymmetry. It would obviously not include response due to accidental torsion. The dynamic analysis under perturbation 1 and 2 is expected to incorporate torsional response due to accidental torsion as well. The dynamic analysis for the uncoupled structure would give only translational response for X direction earthquake excitation. Thus, the difference between the response obtained under perturbation 1 and 2 and that obtained using the original structure would give the torsional response due to accidental torsion. Similarly the difference between the response obtained under perturbation 1 and 2 that obtained using the uncoupled structure would give the combined torsional response due to asymmetry and accidental causes. It is this later case which was compared with a static torsional analysis.

For the static analysis the static moment to be applied was computed using base shear of the uncoupled analysis. As the Indian code does not incorporate provision for accidental torsion the Canadian code was used as it is most stringent of all codes studied (Table 2.1). As per the Canadian code design eccentricity e_d due to asymmetry and accidental causes is given by the following formulas:

$$e_d = 1.5\ e_s + 0.1\ b$$

$$e_d = 0.5\ e_s - 0.1\ b$$

where e_s is the static eccentricity and b is the plan dimension perpendicular to earthquake excitation. Thus, the Canadian code envisages accidental torsion of the order of 10 % of the plan dimension which is also the quantity that has been taken under perturbation 1 and 2 in this study. Static torsional analysis using the above formulas was carried out. The results obtained for various cases of dynamic analysis and for the two cases of static torsional analysis (using the two formulas above) are given in Table 4.13 to 4.15. While the dynamic analysis are expected to be more realistic the static analysis should give comparable or at least more conservative values.

Table 4.13 to 4.15 also compare the SRSS member forces obtained due to static torsional analysis and the maximum difference between the results obtained under perturbation 1 or 2 and the uncoupled structure. The comparision is valid because in both cases the torsional response due to planned and accidental eccentricities has been incorporated.

Table 4.13 to 4.15 show that the static torsional analysis does not give conservative values of torsional response in all cases. It can be seen that while the torsional responses predicted by the static analysis are highly conservative for some columns they are no so for others. The responses obtained due to static torsional analysis are higher for columns 14, 15, 17, 18, 20 and 21 than the maximum difference between perturbed and unperturbed dynamic analysis. The member forces for the static cases are smaller than the above difference for columns 13, 16 and 19. Thus it appears that a static torsional analysis is not adequate.

CHAPTER - 5

CONCLUSIONS

From the study conducted following conclusions can be drawn.

1. In addition to a planned eccentricity accidental eccentricity may exist in the structure giving rise to an additional torsional response. Many codes of practice of various countries have included provisions to account for such accidental torsion and generally propose a static method of analysis. The mass perturbation method can provide a good way of taking into account accidental torsion through a purely dynamic method.

2. The mass perturbation method discussed does not alter the natural frequencies of the structure significantly. Therefore, corresponding spectral value would also not change much. A symmetrical structure which has uncoupled translational and torsional modes and in which torsional modes would not be excited due to a translational earthquake starts to exhibit coupled torsional and translational response after perturbation. This torsional response is included in the dynamic analysis.

3. In asymmetric buildings that have coupled torsional and translational response, mass perturbation can provide a method uncoupling responses.

4. From the study of the two story symmetrical building it is seen that after introducing an eccentricity of 10 % through mass perturbation the torsional response with regard to member forces is close to that obtained through a static analysis with 10 % design eccentricity. However, the static analysis does not yield conservative values in all cases.

5. In a single story asymmetric building comparisons were made to study the combined torsional response due to planned and accidental eccentricity using dynamic and static analysis.

   The member forces obtained due to the combined torsional response were considerably different when a static method was employed and when dynamic analysis was used. Moreover, the static method did not yield conservative results in all cases.

6. As an earthquake is a dynamic phenomenon the dynamic method of analysis is suitable for accidental torsions analysis and mass perturbation as discussed appears to provide a methodology that introduces accidental eccentricity.

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TABLE 2.1- VALUES OF α_1, α_2, β_1, β_2, RECOMMENDED BY VARIOUS SEISMIC CODES

COUNTRY	α_1	β_1	α_2	β_2
Argentina	1.5	0.1	1.5	0.1
Canada	1.5	0.1	0.5	0.1
Ethopia	1.5	0.05	1.0	0.05
Indonesia	1.5	0.05	1.0	0.05
India	1.5	0	1.5	0

Table 4.1 : Natural frequencies (rad/sec) before and after perturbation for first 6 modes. (Problem 1)

Mode	Original	perturbation 1	perturbation 2
1	36.0	35.3	35.3
2	45.1	46.1	45.5
3	46.1	46.5	46.1
4	93.7	94.7	92.4
5	107.8	103	109.6
6	123.9	132	128.5

Table 4.2 : Comparison of mode participation factors before and after perturbation for an X-direction earthquake (Problem 1)

Mode	Original	Perturbation 1	Perturbation 2
1	93.6	91.3	92.2
2	-	-	17.2
3	-	20.6	-
4	-	1.8	12.1
5	28.9	26.9	24.1
6	0.0	10.7	9.2

Table 4.3 : Maximum shear Force in X-direction in 1st storey columns (Problem 1) (   )

MODE	Dynamic Analysis (Force)			Maximum difference
	Unperturbed	Perturbation 1	Perturbation 2	
1	1751	2010	1837	259
2	-	-	222	222
3	-	265	-	265
4	-	4	47	47
5	133	166	107	33
6	-	46	40	46
SRSS	1756	2019	1842	263

STATIC ANALYSIS	
Case	FORCE
1	285
2	107
3	27
4	3
5	286
6	107
SRSS 1 and 3	286
SRSS 2 and 4	107

Table 4.4 : Maximum moment about Y-axis at the base of 1st storey columns. Problem 1) (   )

MODE	Dynamic Analysis (Moments)			Maximum difference
	Unperturbed	Perturbation 1	Perturbation 2	
1	3457	3957	3640	500
2	-	-	429	429
3	-	518	-	518
4	-	7	86	86
5	230	285	180	55
6	-	79	69	79
SRSS	3465	3971	3649	506

CASE	STATIC ANALYSIS
	MOMENTS
1	540
2	179
3	51
4	4
5	543
6	178
SRSS 1 and 3	542
SRSS 2 and 4	179

Table 4.5 : Maximum vertical (z direction) shear forces in beams 1 and 3 (Ref. Figure 4.2) ( )

MODE	Dynamic Analysis (FORCES)			Maximum difference
	Unperturbed	Perturbation 1	Perturbation 2	
1	2244	2528	2439	284
2	-	-	238	238
3	-	316	-	316
4	-	5	38	38
5	21	26	41	20
6	-	7	4	7
SRSS	2244	2530	2442	286

CASE	STATIC ANALYSIS
	FORCES
1	244
2	45
3	27
4	11
5	246
6	46
SRSS 1 and 3	245
SRSS 2 and 4	46

Table 4.6 : Maximum moment in beams 1 and 3 in Y-direction. (   )

MODE	DYNAMIC ANALYSIS (MOMENTS)			Maximum difference
	Unperturbed	PERTURBATION 1	PERTURBATION 2	
1	4487	5055	4487	568
2	-	-	473	473
3	-	631	-	631
4	-	8	40	40
5	41	53	82	41
6	-	15	8	15
SRSS	4488	5059	4884	571

CASE	STATIC ANALYSIS
	MOMENTS
1	489
2	89
3	53
4	22
5	491
6	91
SRSS 1 and 3	492
SRSS 2 and 4	92

TABLE 4.7 : **Maximum shear forces in X-direction in 2nd storey columns.** (N)

MODE	DYNAMIC ANALYSIS (FORCES)			MAXIMUM DIFFERENCE
	UNPERTURBED	PERTURBATION 1	PERTURBATION 2	
1	1146	1315	1345	199
2	-	-	113	113
3	-	174	-	174
4	-	2	30	30
5	149	138	154	39
6	-	53	41	53
SRSS	1156	1330	1350	194

CASE	STATIC ANALYSIS
	FORCES
1	74
2	162
3	12
4	19
5	74
6	164
SRSS 1 and 3	75
SRSS 2 and 4	163

TABLE 4.8 Maximum Moment about Y-axis at Ist floor Level in 2nd Story Columns.

MODE	DYNAMIC ANALYSIS (MOMENTS)			MAXIMUM DIFFERENCE
	UNPERTURBED	PERTURBATION 1	PERTURBATION 2	
1	1815	2085	2157	342
2	-	-	174	174
3	-	277	-	277
4	-	4	54	54
5	279	350	284	71
6	-	98	77	98
SRSS	1837	2117	2167	330

CASE	STATIC ANALYSIS
	MOMENTS
1	99
2	297
3	18
4	34
5	100
6	299
SRSS 1 and 3	101
SRSS 2 and 4	299

Table : 4.9 : **Maximum vertical (z direction) shear forces in beams 5 and 7 (Refer Figure 4.2).**

MODE	Dynamic Analysis (FORCES)			Maximum difference
	Unperturbed	Perturbation 1	Perturbation 2	
1	1098	1233	1252	154
2	-	-	99	99
3	-	153	-	153
4	-	2	25	25
5	122	148	123	26
6	-	40	31	40
SRSS	1104	1242	1255	151

CASE	Static Analysis
	FORCES
1	70
2	119
3	10
4	14
5	70
6	119
SRSS 1 and 3	71
SRSS 2 and 4	120

Table 4.10 : Maximum Moment about Y-axis in beams 5 and 7.

MODE	Dynamic Analysis (MOMENTS)			Maximum difference
	Unperturbed	Perturbation 1	Perturbation 2	
1	2195	2465	2503	308
2	-	-	198	198
3	-	306	-	306
4	-	4	50	50
5	244	296	245	52
6	-	14	61	61
SRSS	2209	2485	2511	302

CASE	Static Analysis
	MOMENTS
1	139
2	237
3	21
4	28
5	141
6	239
SRSS 1 and 3	141
SRSS 2 and 4	239

Table 4.11 : Natural frequencies (rad/sec) of the original uncoupled and perturbed structure (Problem 2).

MODE	Original	Un Coupled	Perturbation 1	Perturbation 2
1	42.7	43.1	40.5	42.7
2	52.9	52.8	56.3	54.0
3	59.8	59.8	60.4	59.9
4	414.3	414.9	421.8	421.6
5	490.1	476.3	438.7	447.9
6	498.8	487.6	448.7	468.6

Table 4.12 : Shear forces in X-direction in columns under various conditions (Problem 2).

Members	Original	Uncoupled	Pert. 1	Pert. 2	Max pert. Unc	Static case 1	Static case 2
13	1776	1527	1958	1231	431	371	154
14	1548	1564	1430	1494	-134	37	15
15	3168	3809	2274	4240	431	725	300
16	2119	1807	2354	1443	547	462	191
17	1773	1793	1636	1715	-78	40	17
18	1544	1887	1085	2127	240	394	163
19	4407	3793	4853	3063	1060	909	376
20	1533	1552	1415	1484	-68	34	14
21	3170	3825	2278	4270	445	734	304

Table 4.13 : Comparison of mode participation factors under an X-direction earthquake (Problem 2).

Mode	Original	Uncoupled	Perturbation 1	Perturbation 2
1	162	164	153	162
2	28	2	53	28
3	7	2	27	4

Table 4.14 : Moment about Y-axis at column base under various conditions (Problem 2).

Members	Original	Uncoupled	Pert. 1	Pert. 2	Max pert.Unc	Static case 1	Static case 2
13	3766	3231	4161	2598	930	796	329
14	3252	3288	3303	3144	15	76	31
15	7343	8881	5231	9930	1049	1745	722
16	4221	3602	4686	2878	1084	916	379
17	3552	3594	3277	3437	-157	79	33
18	3049	3722	2145	4193	471	773	320
19	10270	8812	11343	7088	2531	2157	892
20	3233	3272	2983	3130	-142	72	30
21	7347	8902	5239	9971	1069	1758	727

Table 4.15 : Moments about Y axis at column tops.

Members	Original	Uncoupled	Pert. 1	Pert. 2	Max pert.Unc	Static case 1	Static case 2
13	3337	2878	3670	2328	792	689	285
14	2938	2967	2717	2833	-134	73	30
15	5328	6356	3865	7029	673	1154	477
16	4254	3626	4729	2892	1103	930	385
17	3540	3580	3267	3423	-157	81	33
18	3129	3827	2196	4317	490	803	332
19	7359	6360	8070	5163	1710	1479	612
20	2900	2934	2676	2807	-127	65	27
21	5333	6396	3874	7109	713	1179	488

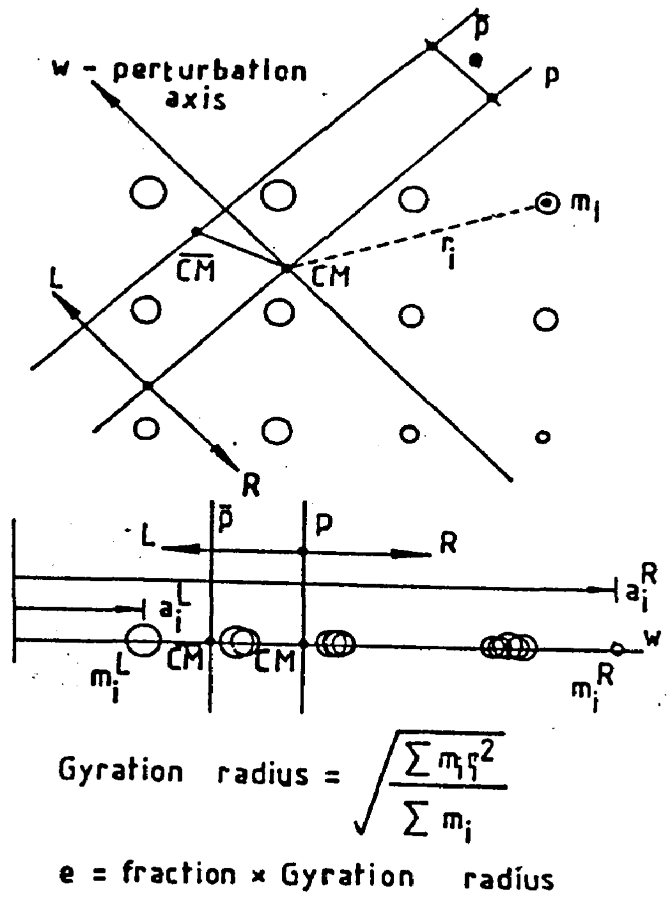


FIG. 3.1 - MASS PERTURBTION PROCEDURE (19)

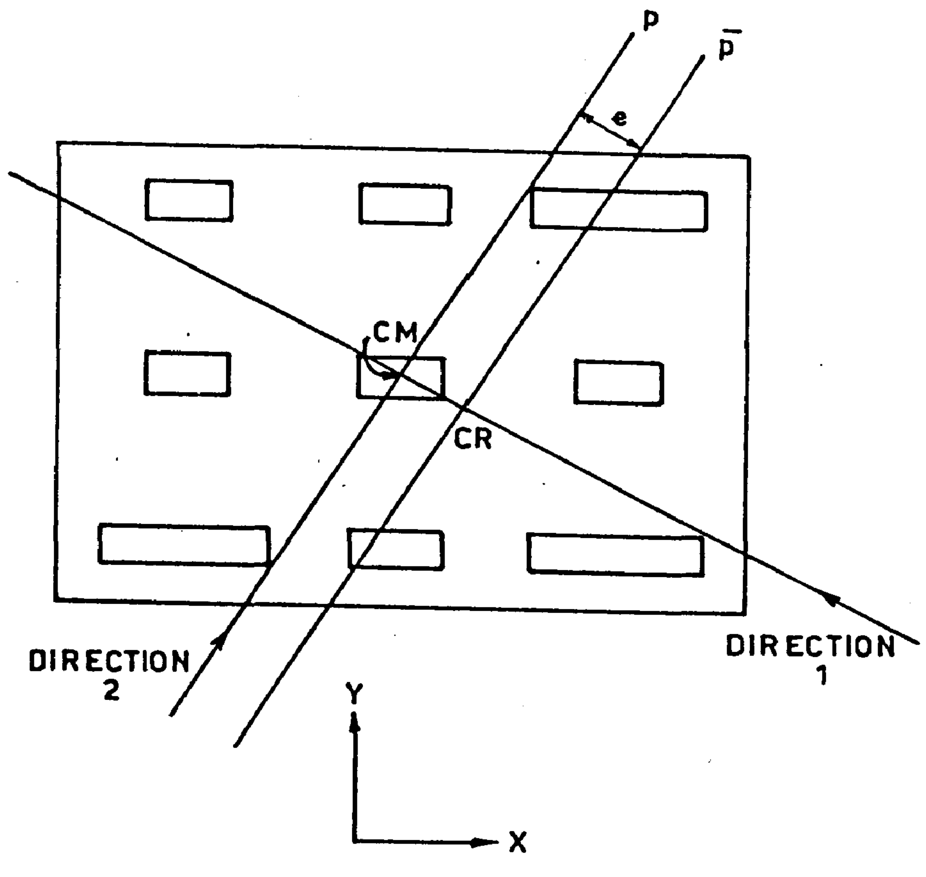


FIG. 3.2 - PLAN OF A SINGLE STOREY ASYMMETRIC STRUCTURE
RECTANGLES INDICATE COLUMNS

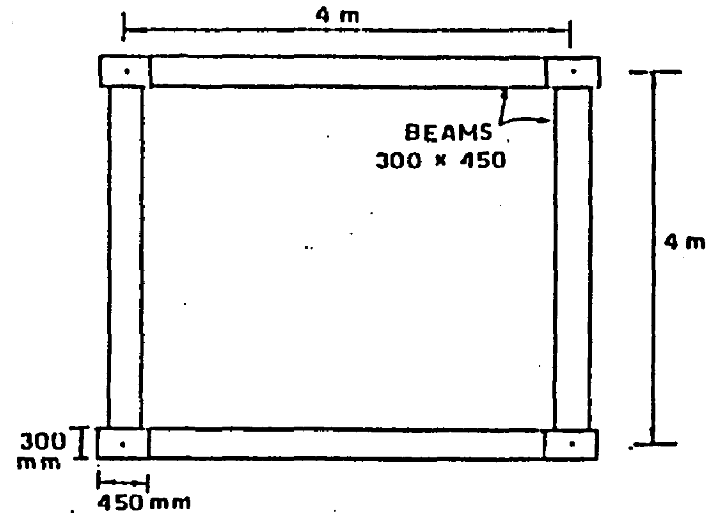


FIG. 4.1_ PLAN OF THE BUILDING

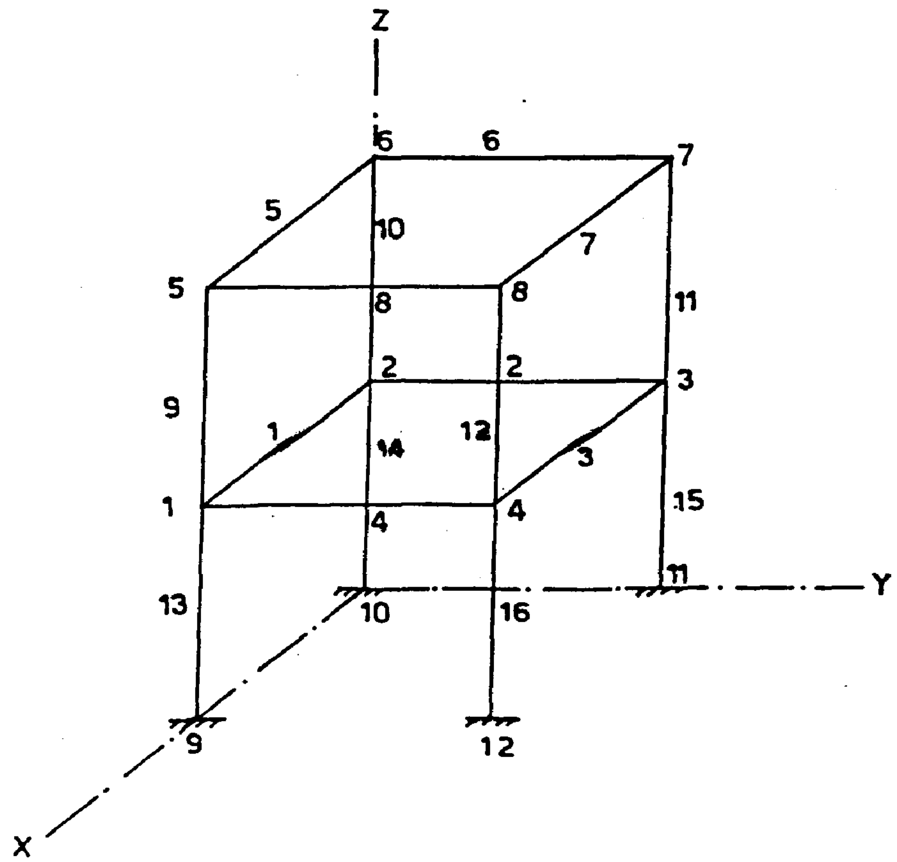
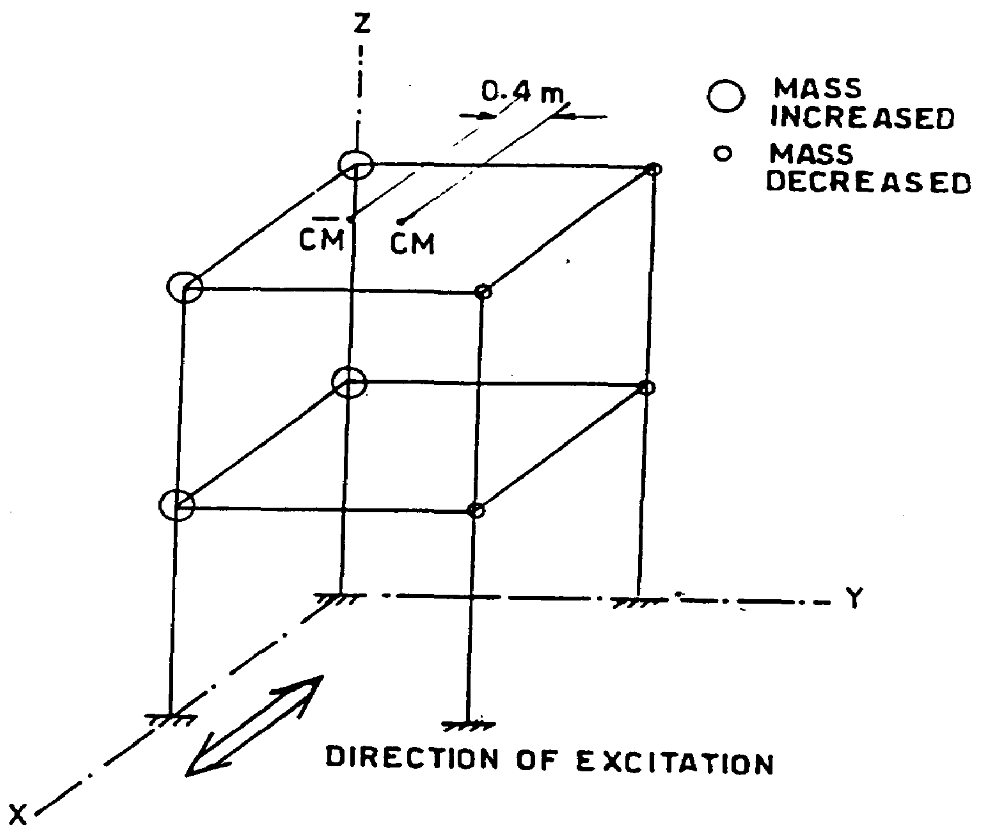
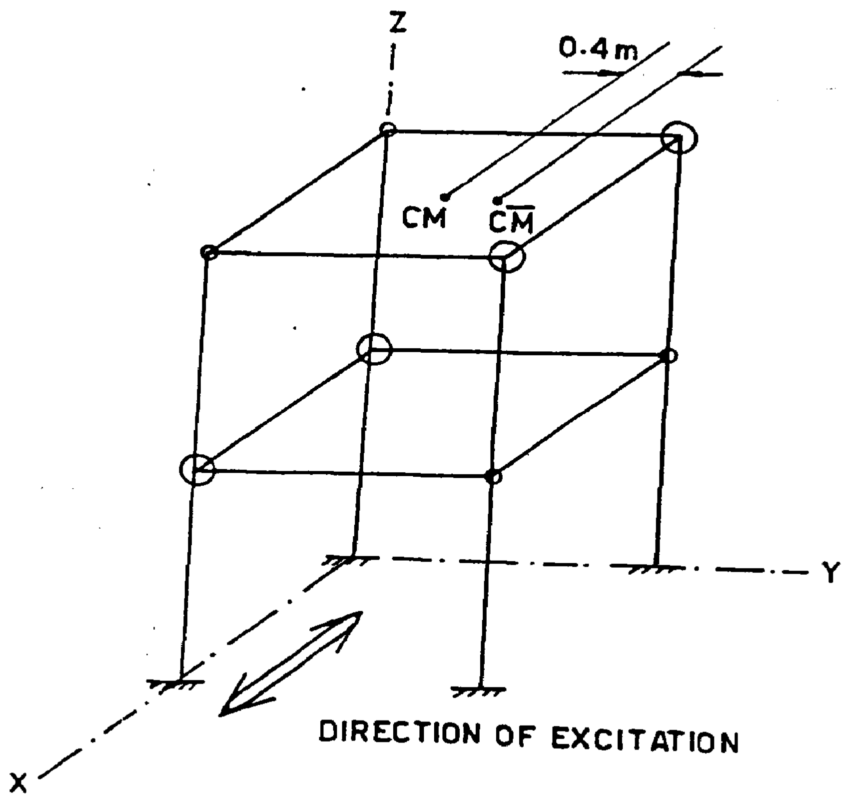


FIG. 4.2 _MATHEMATICAL MODEL OF THE BUILDING



(a) PERTURBATION 1 _ MASS PERTURBATION IN SAME DIRECTION ON BOTH FLOORS



(b) PERTURBATION 2 _ MASS PERTURBATION IN OPPOSITE DIRECTIONS ON THE BOTH FLOORS

FIGURE _ 4.3

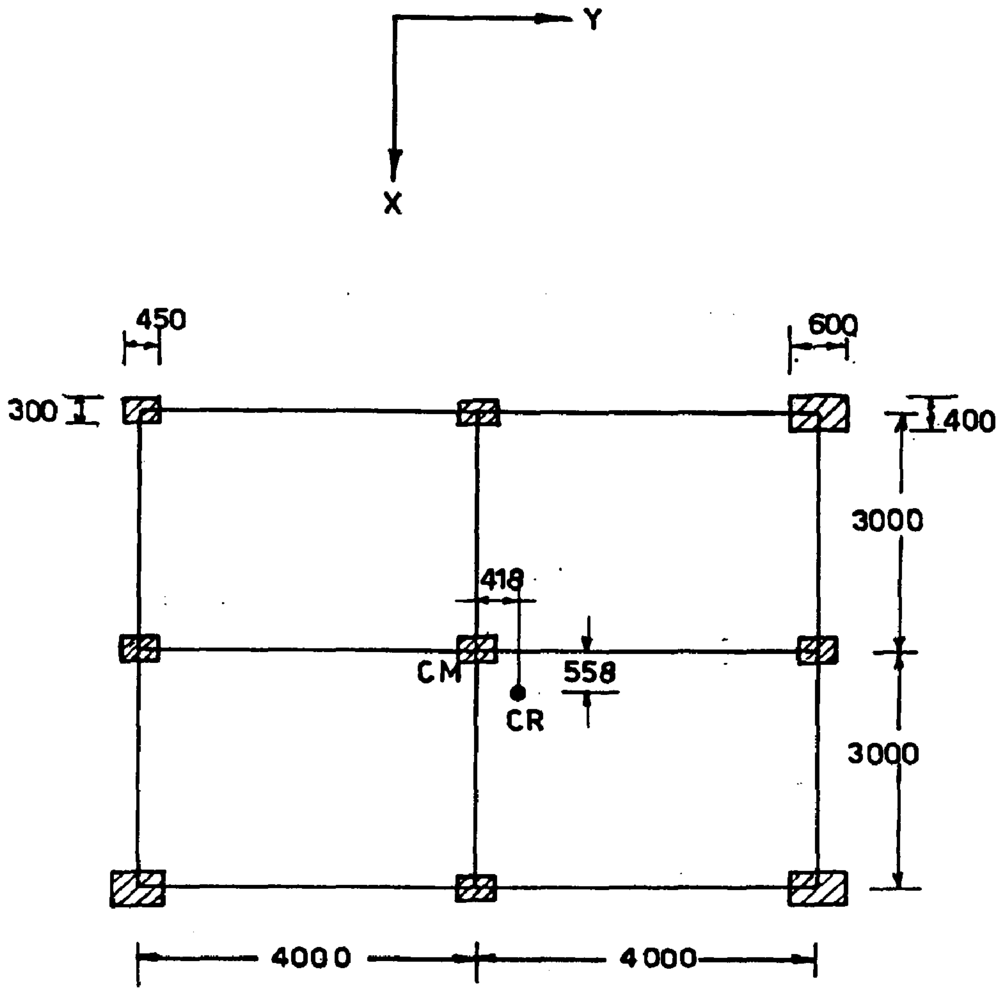


FIG. 4. 4 _ PROBLEM 2 - PLAN OF THE ASYMMETRIC BUILDING

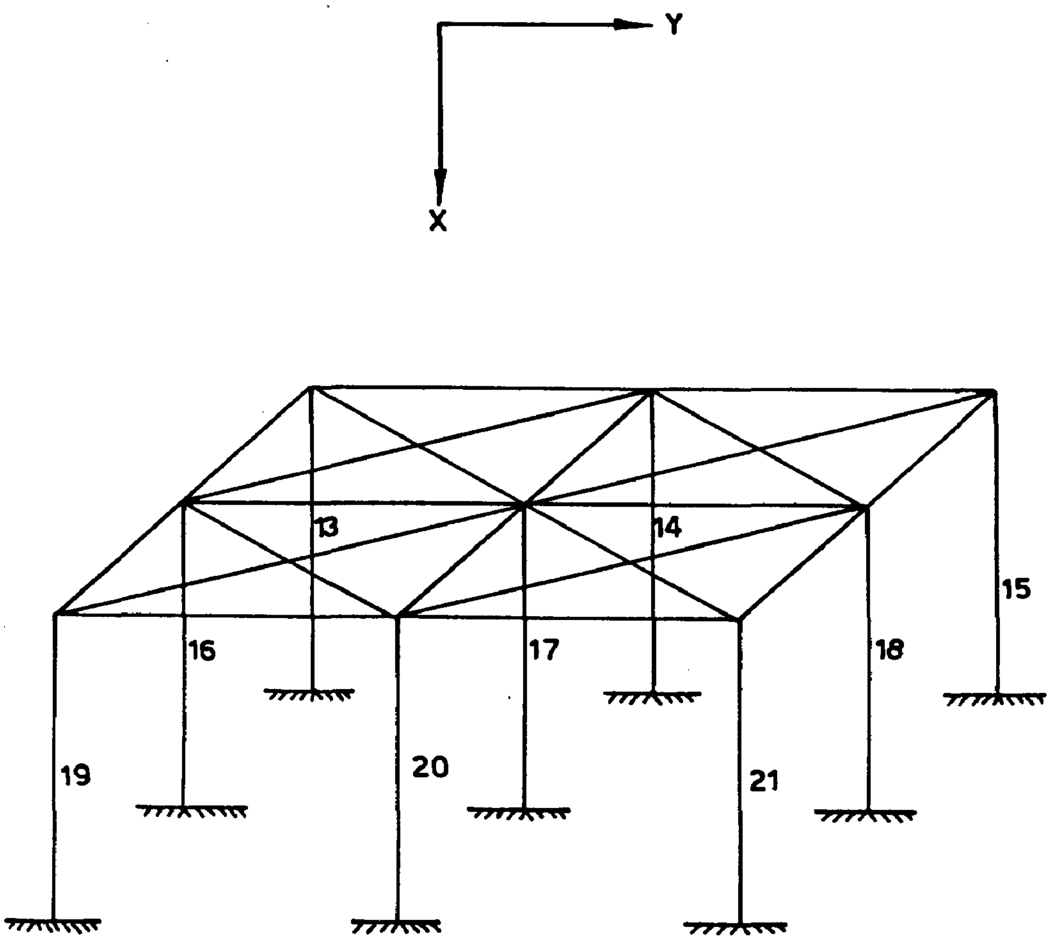
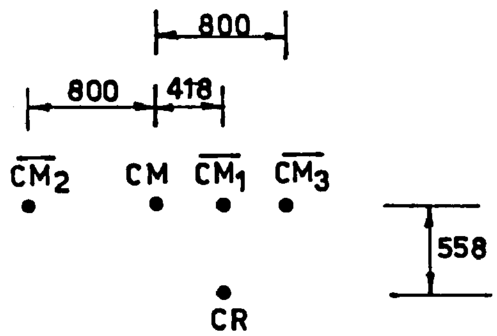


FIG. 4.5 _ PROBLEM 2 _ THE MATHEMATICAL MODEL OF BUILDING

(ONLY COLUMNS ARE NUMBERED)



CR – CENTRE OF RIGIDITY

CM – CENTRE OF MASS IN THE ORIGINAL STRUCTURE

$\overline{CM}_1$ – CENTRE OF MASS FOR UNCOUPLED CASE

$\overline{CM}_2$ – CENTRE OF MASS FOR PERTURBATION 1

$\overline{CM}_3$ – CENTRE OF MASS FOR PERTURBATION 2

FIG. 4.6 – POSITION OF CENTRE OF MASS FOR VARIOUS CASES

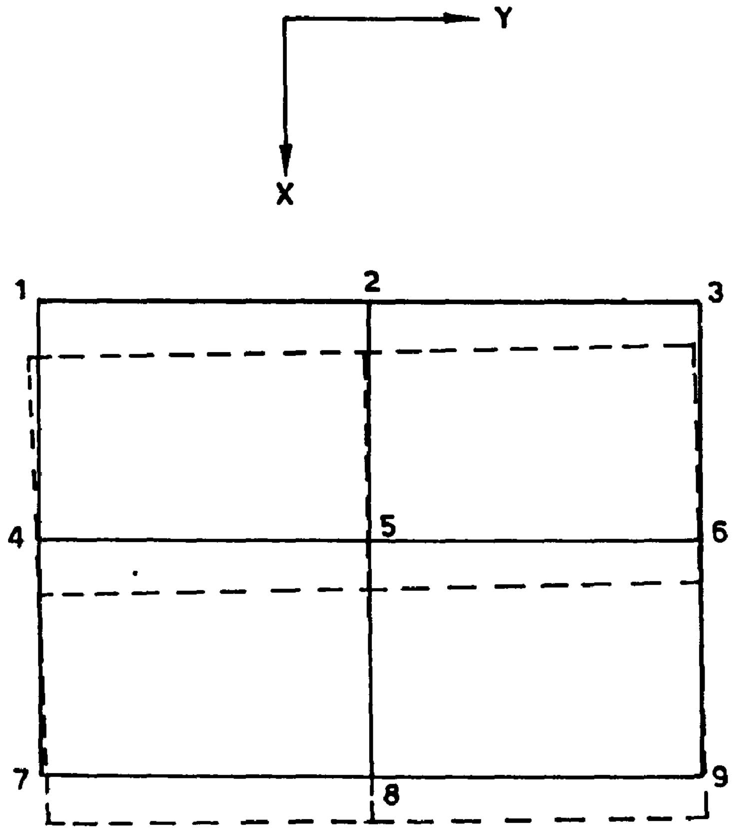


FIG. 4.7 _ MODE 1—ORIGINAL STRUCTURE

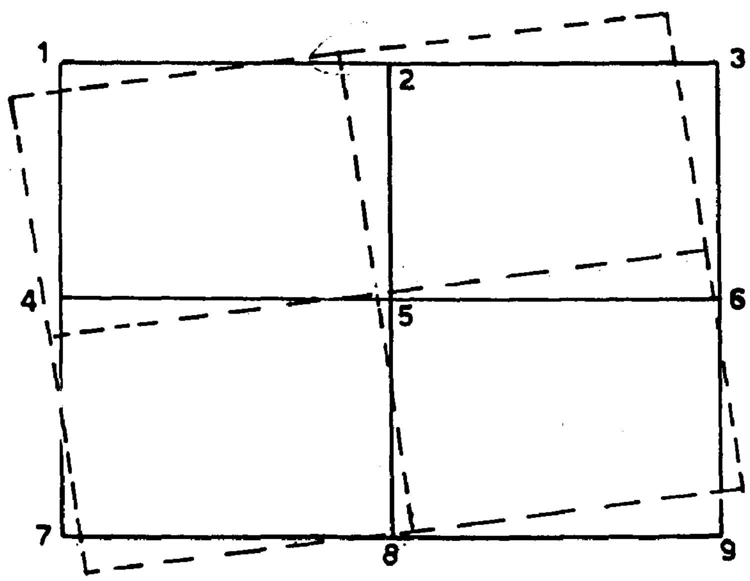


FIG. 4.8 _ MODE 2—ORIGINAL STRUCTURE

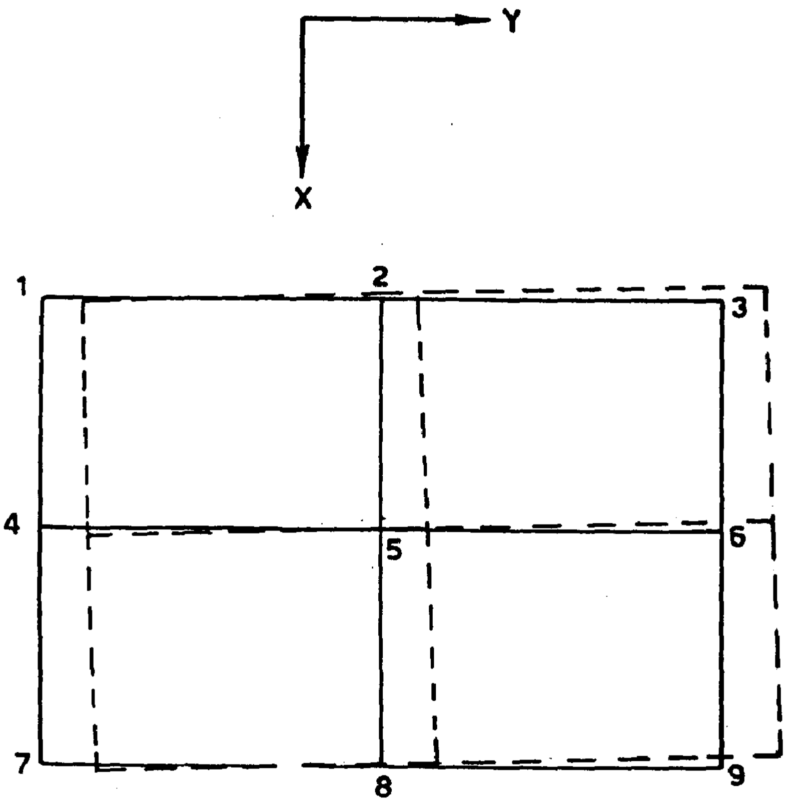


FIG. 4.9 _ MODE 3-ORIGINAL STRUCTURE

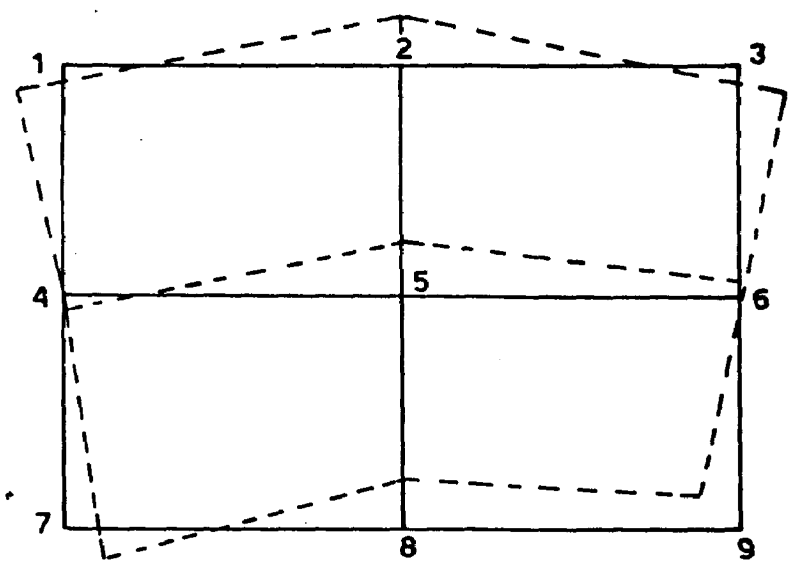


FIG. 4.10 _ MODE 4-ORIGINAL STRUCTURE

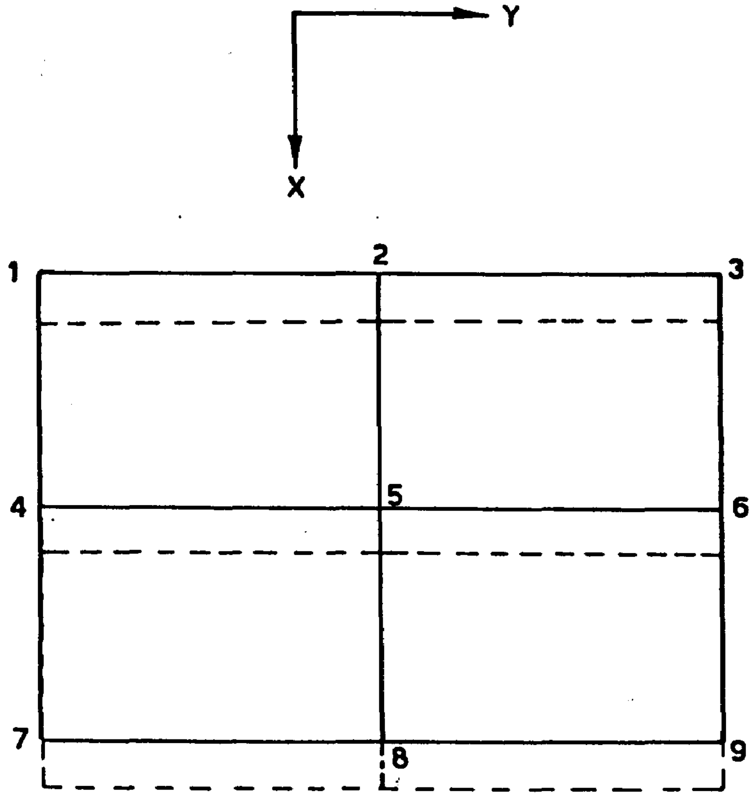


FIG. 4.11 _ MODE 1 — UNCOUPLED CASE

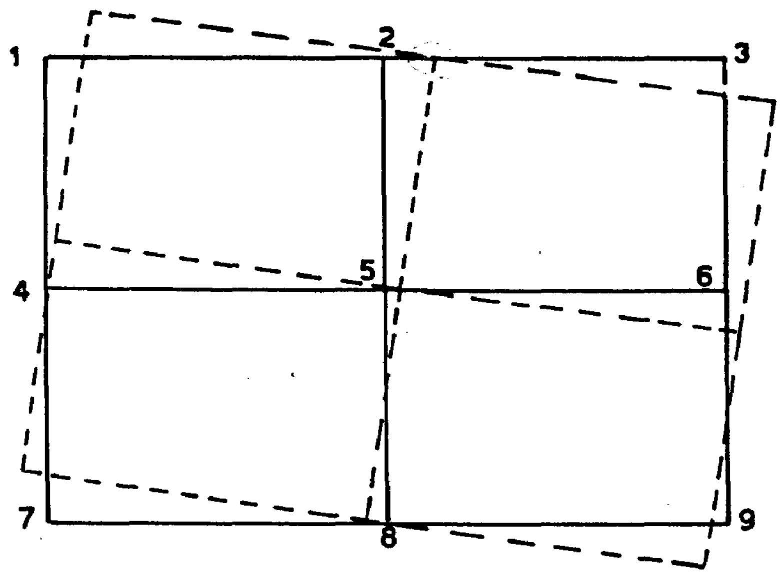


FIG. 4.12 _ MODE 2 — UNCOUPLED CASE

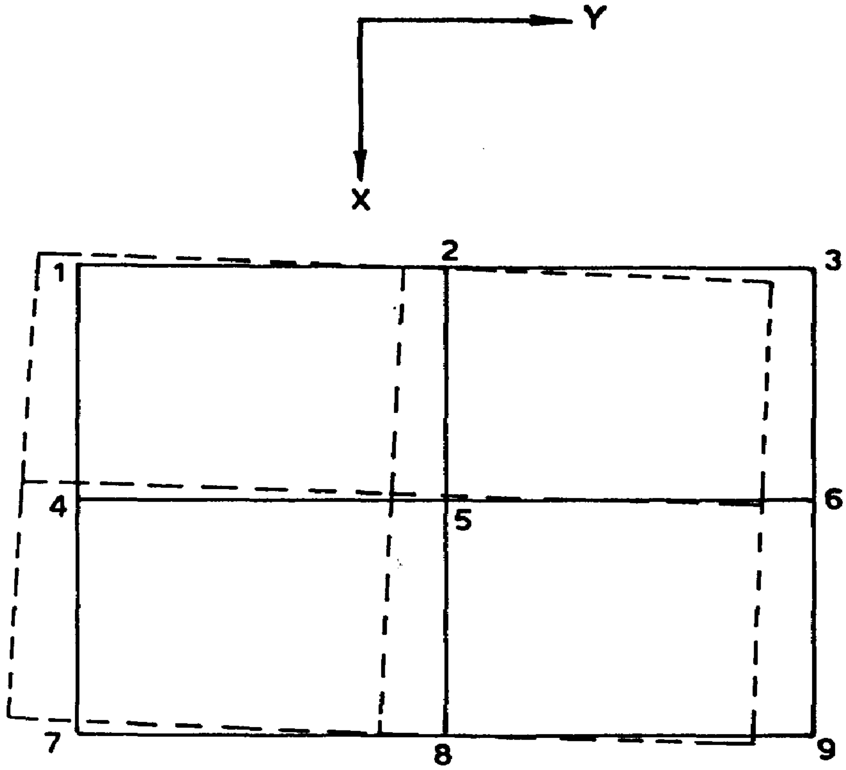


FIG. 4.13_ MODE 3 — UNCOUPLED CASE

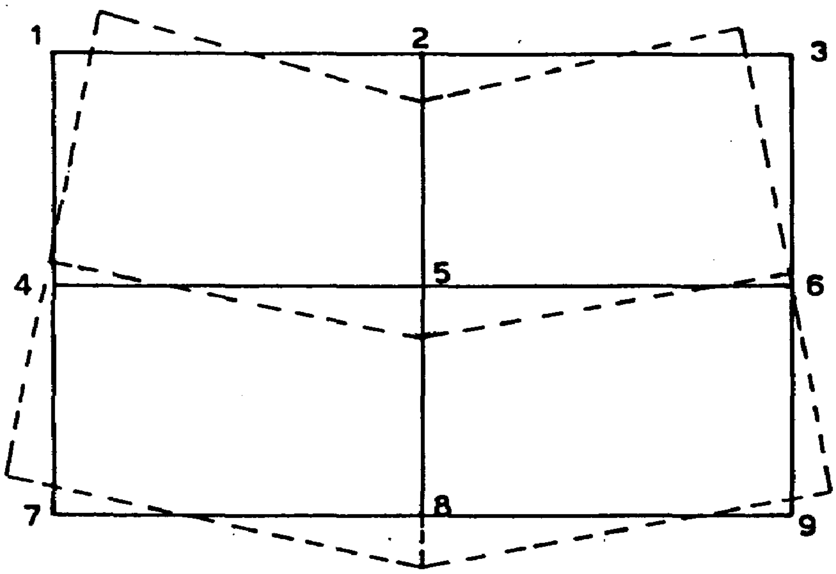


FIG. 4.14_ MODE 4 — UNCOUPLED CASE

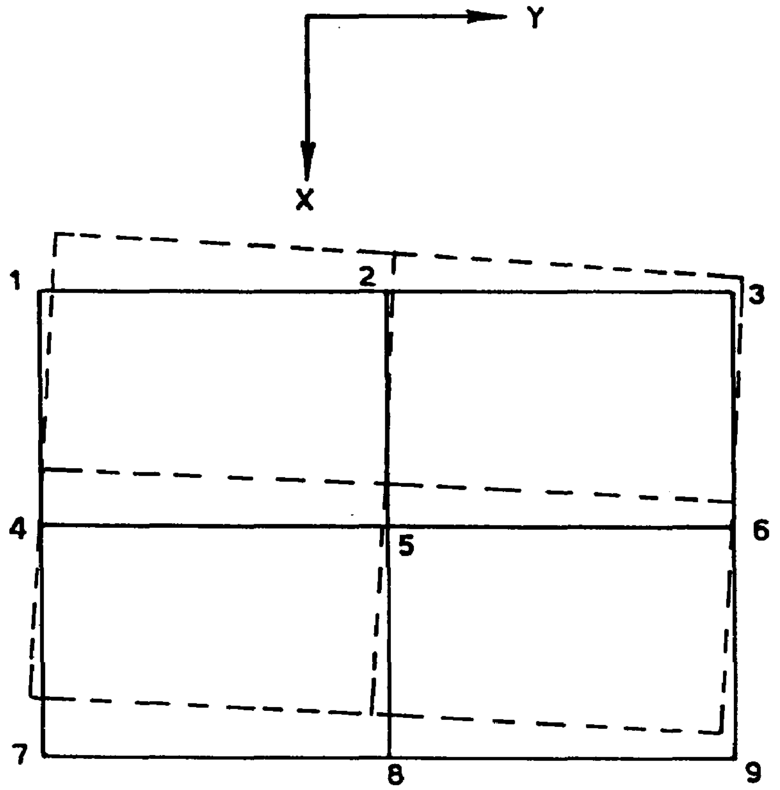


FIG. 4.15 _ MODE 1 – PERTURBATION 1

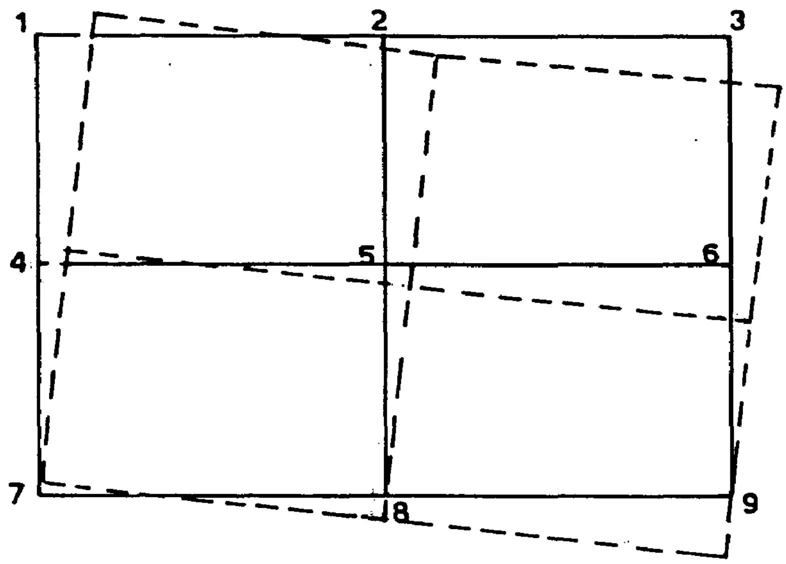


FIG. 4.16 _ MODE 2 – PERTURBATION 1

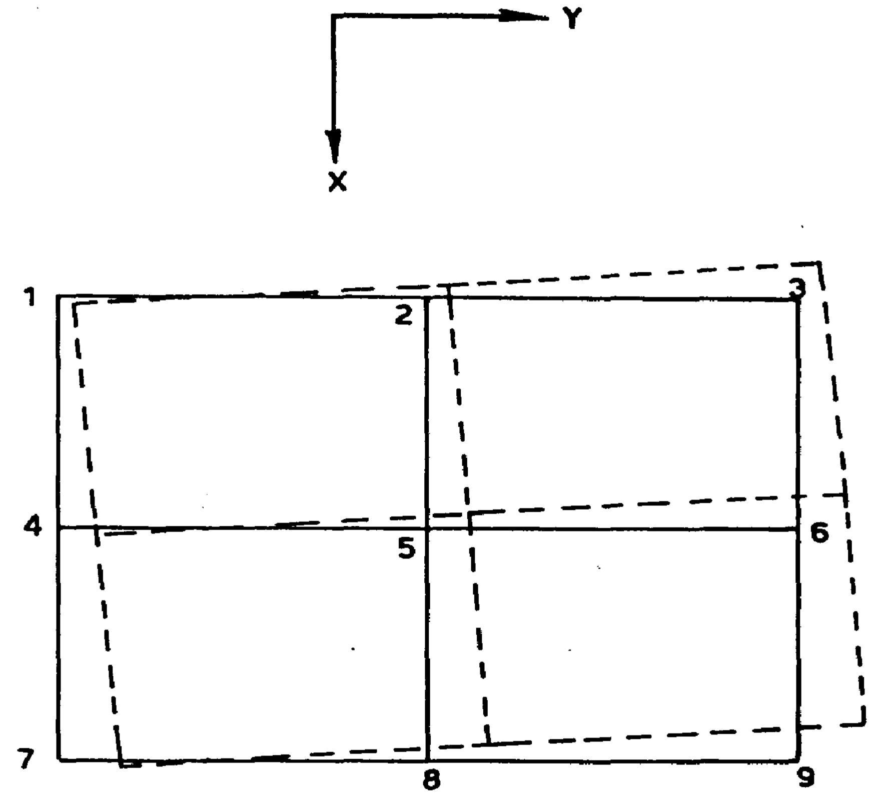


FIG. 4.17 _ MODE 3 - PERTURBATION 1

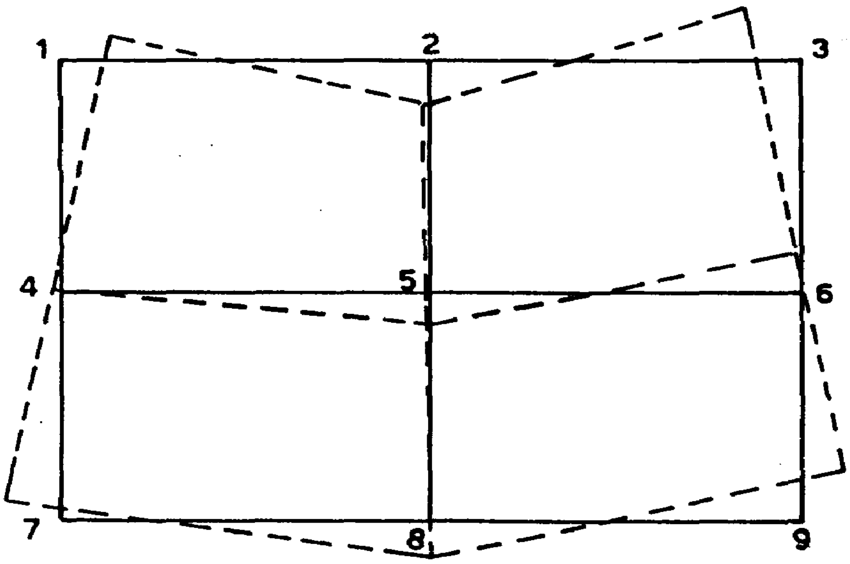


FIG. 4.18 _ MODE 4 - PERTURBATION 1

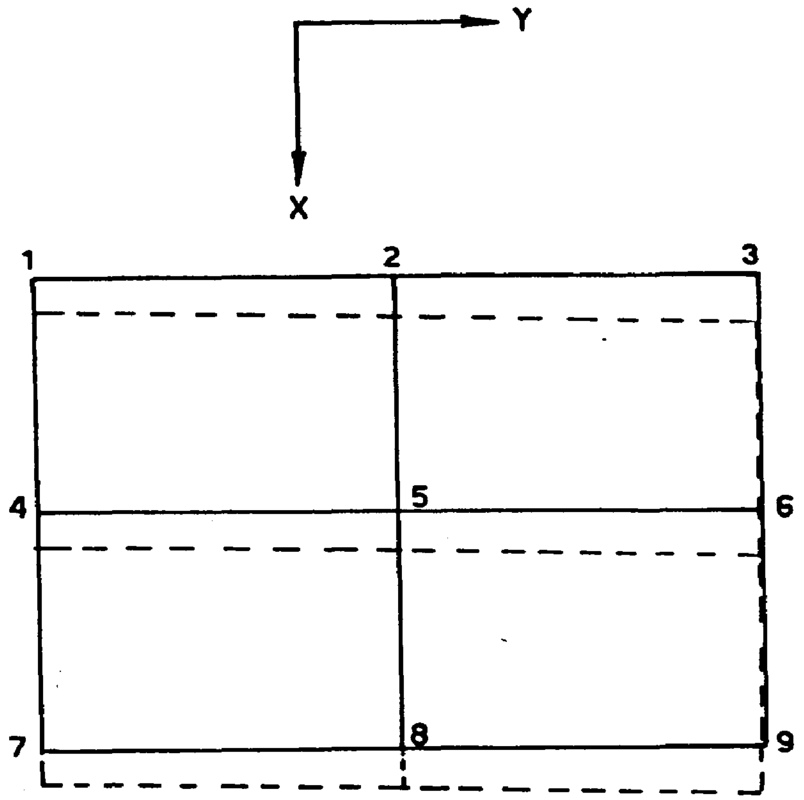


FIG. 4.19 _ MODE 1 - PERTURBATION 2

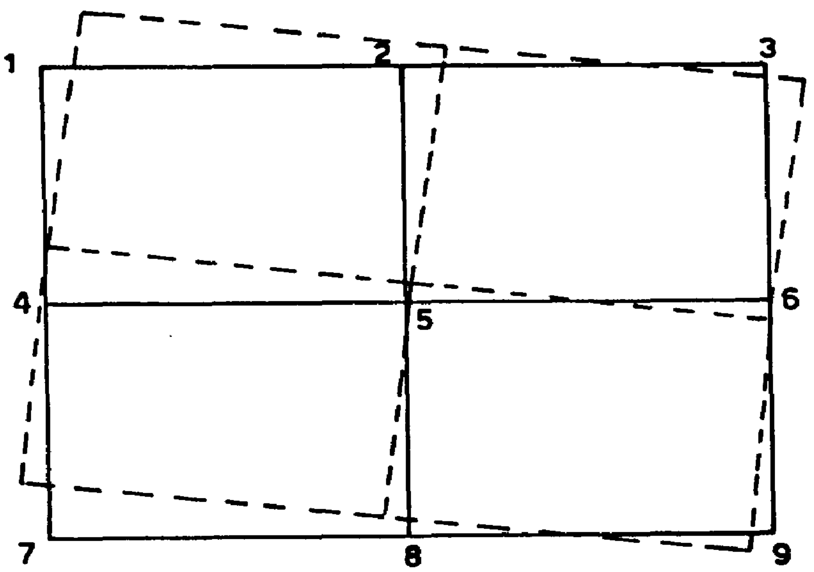


FIG. 4.20 _ MODE 2 - PERTURBATION 2

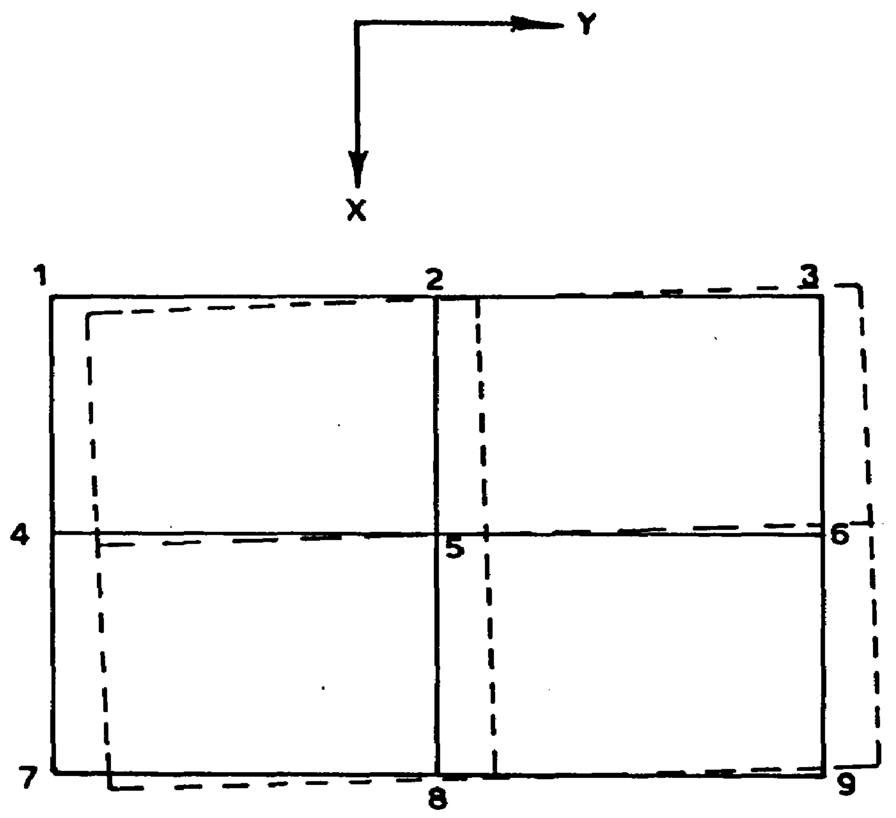


FIG. 4.21 _ MODE 3 - PERTURBATION 2

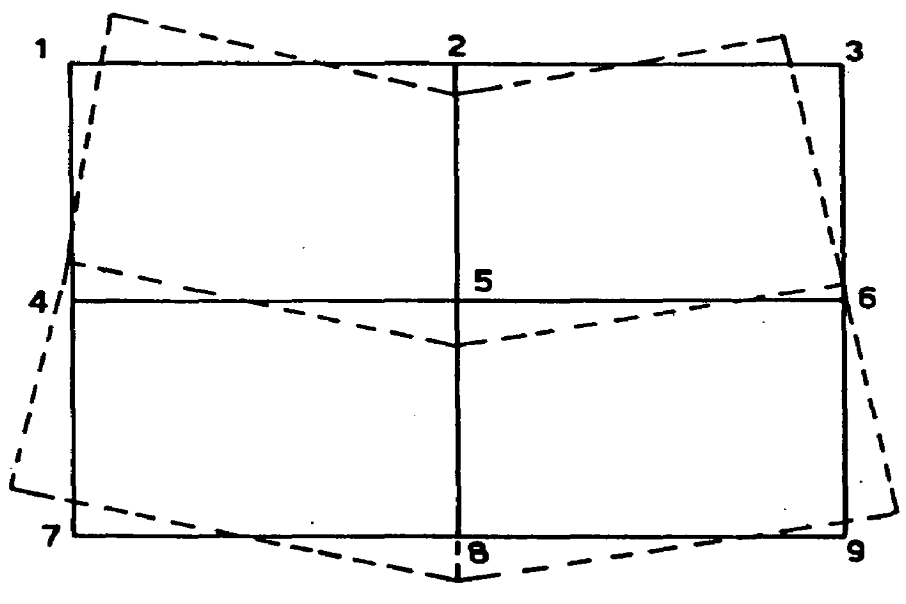


FIG. 4.22 _ MODE 4 - PERTURBATION 2