BRIDGE DECK BEHAVIOUR VIS-A-VIS LONGITUDINAL GIRDERS

A DISSERTATION

submitted in partial fulfilment of the requirements for the award of the degree
of
MASTER OF ENGINEERING
in
CIVIL ENGINEERING
(With Specialization in Building Science & Technology)

By

ARUN KUMAR CHOUDHARY

DEPARTMENT OF CIVIL ENGINEERING
UNIVERSITY OF ROORKEE
ROORKEE - 247 667 (INDIA)

OCTOBER, 1989

DEDICATED
TO
MY FRIENDS
LOVE TO MY
BROTHERS & SISTERS

CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the dissertation entitled **BRIDGE DECK BEHAVIOUR VIS-A-VIS LONGITUDINAL GIRDERS** in partial fulfilment of the **MASTER OF ENEGINEERING** in Civil Engineering with specialization in **BUILDING SCIENCE AND TECHNOLOGY**, submitted in the **DEPARTMENT OF CIVIL ENGINEERING, UNIVERSITY OF ROORKEE**, is an authentic record of my own work carried out during the period Nov.1988 to oct 1989,under the supervision of Dr. Jagdish Prasad, Lecturer, Civil Engineering Department, University of Roorkee, Roorkee.

The matter embodied in this dissertation has not been submitted by me for the award of any other degree or diploma.

Arun.

(ARUN KUMAR CHOUDHARY)

Dated: Oct.28 , 1989

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

J. Prasad.

(Dr. J. PRASAD)
Lecturer
Civil Engineering Department
University of Roorkee
Roorkee

Dated: Oct.28 , 1989

A C K N O W L E D G E M E N T

I take this opportunity to express my profound gratitude to Dr. J. Prasad, Lecturer, Department of Civil Engineering, University of Roorkee, for his active guidance and lasting encouragement at every stage of this work. He has generously devoted his precious time, despite his busy schedule, even at odd hours. His consistant co-operation right from the beginning to the end, as and when sought and the immense pains he has taken in going through the dissertation and its consequent upgradation are highly acknowledged.

Sincere most thanks are also due to my friends for there moral support and co-operation throughout the Master of Engineering course and specially for this work, without which this would have been diffcult to complete.

Last but not the least, I express my thanks to Mr. P. Dayal and Mr. A. K. Binjola for their help in running of the computer program and in other associated problems.

(ARUN KUMAR CHOUDHARY)

S Y N O P S I S

Girder bridges are structurally efficient and economical for short and medium spans. A good deal of thinking goes into deciding about the sizes and arrangement of various structural elements of a bridge deck suiting a particular site. Safety and economy consideration govern the number of girders, diaphragms, cross-beams and thickness of deck slab. This necessitates the knowledge of comparative merits and demerits of different sets of combinations of deck elements. Herein,a study is made with different number of girders for a particular bridge, keeping the volume of material of the deck as constant and without varying the slab thickness so that the influence of girders alone may precipitate in the result. This has been carried out with different skew angle keeping the span constant.

Stiffness matrix method has been adopted for the deck analysis. To make the bridge deck amenable to stiffness matrix method, it is suitably discretized into proper grid work comprising longitudinal and cross structural elements.

The bridge deck is loaded with IRC class AA tracked vehicle in a manner to have the maximum effect on the girders. Two possible modes of application of loads have been studied.

Based on practice as well belief that the imposed load dispersion is solely a function of the transverse medium, it is common practice to manipulate the structural elements of the transverse medium in an effort to effectively distribute the deck loads amongst the girders. However, it is quite apparant that the overall deck behaviour will depend upon the relative stiffness of the transverse medium to that of longitudinal member. With a view to studying this aspect of deck system behaviour, eighteen girder bridge systems have been analysed by appropriately varying the longitudinal system while keeping the transverse system constant. The results are depicted in terms of bending moment, shear, deflection and support reactions with a view to deciding about the number of longitudinals and their corss-sections so as to yieldin structurally most effective deck.

N O T A T I O N S

C_p	Member Bending Moment.
C_q	Member Torsion.
E	Young's Modulus of Elasticity of Material.
G	Shear Modulus of Elasticity of Material.
G_j	Girder j.
h	Transverse Spacing of Girders.
I_j	Moment of Inertia of Cross-section of girder j.
J_j	Torsional Constant of Girder j.
J_T	Torsional Constant of Transverse System.
L	Skew/Right Span of Bridge.
P_j	Vertical Load at node j.
O_{i1}, O_{i2}	Slope of the Girder i at the ends 1 and 2.
O_p	Member slope.
O_q	Member Rotation.
λ	Skew Angle.
α, β, γ, K	Non-dimensional Structural Parameters.
$\{D\}_e$	Displacement Vector in Local Coordinate System.
$\{D\}_g$	Displacement Vector in Global Coordinate System.
$\{P\}_e$	Force Vector in Local Coordinate System.
$\{P\}_g$	Force Vector in Global Coordinate System.
[T], [R]	Transformation Matrix for Coordinate System.
$[K]_e$	Stiffness Matrix in Local Coordinate System.
$[K]_g$	Stiffness Matrix in Global Coordinate System.
B_b	Breadth of Girder Bulb.
B_d	Depth of Girder Bulb.
W_d	Depth of Girder - Web.
W_b	Breadth of Girder - Web.
N	No. of Girder.
D_b	Breadth of Diaphragm.
D_d	Depth of Diaphragm.
S_d	Depth of Slab.
S_b	Breadth of Slab.
S_{ij}	Vertical End Reaction of Girder No. i At End j.
	i=1,2,3,4 or 5.
	j=1 for Near End, 2 for Far End.

C O N T E N T S

C H A P T E R : 5

ANALYSIS AND RESULTS

C H A P T E R : 6

LIST OF FIGURES

CHAPTER - 1

INTRODUCTION

Girder bridges, right or skew, have proved to be quite popular for short and medium spans from viewpoints of structural efficiency, economy and construction. In girder bridges, the various structural components are the deck slab, cross- members (cross-beams and /or diaphragms) and the longitudinal girders. The imposed loading gets distributed amongst the various girders through the transverse medium comprising the decking slab and cross members. While the distribution among the girders, primarily depends upon the stiffness of the transverse medium, the overall deck behaviour depends upon the relative stiffness of the transverse medium to that of longitudinal girders. For overall structural efficiency, therefore, the cross-sectional dimensions and the spacing of the girders for a given bridge cross-section are important parameters. In the study presented herein, an attempt has been made to look into these aspects in an effort to achieve an efficient configuration of bridge structural components.

A brief review of literature available on the various methods of analysis commonly adopted for bridge analysis is presented in chapter 2. Suitability or otherwise of these methods to various deck structural forms is also discussed.

With easy access to digital computers and development of numerical analysis techniques, there has been a spurt in notionally subdividing the structures into skeletons and then analysing them with the help of these hardware and software. Stiffness matrix method of structural analysis has proved to be very effective in analysing bridge decks simulated as plannar grillages. Chapter 3 deals with the aspects related to the structural simulation, mathematical modelling and computer software as applied to girder bridge decks.

Chapter 4 describes the bridge systems adopted for the specific studies to be carried out. Apart from a right bridge, two skew decks with 20° and 30° skew have also been considered. Class - AA wheeled vehicle load system has been considered for deck loading. Scheme adopted for determination of equivalent nodal load is also briefly discussed.

In chapter 5 is presented the analysis of the bridge systems described in chapter 4. Analysis has been done with respect to girder moment, shear, deflection and support reactions. These parameters have been suitably processed and presented in graphical and tabular forms with a view to arriving at definite inferences and conclusions.

Chapter 6 contains the overall summary and specifie conclusions drawn on the basis of various deck analysis carried out. Scope for further study is also indicated herein.

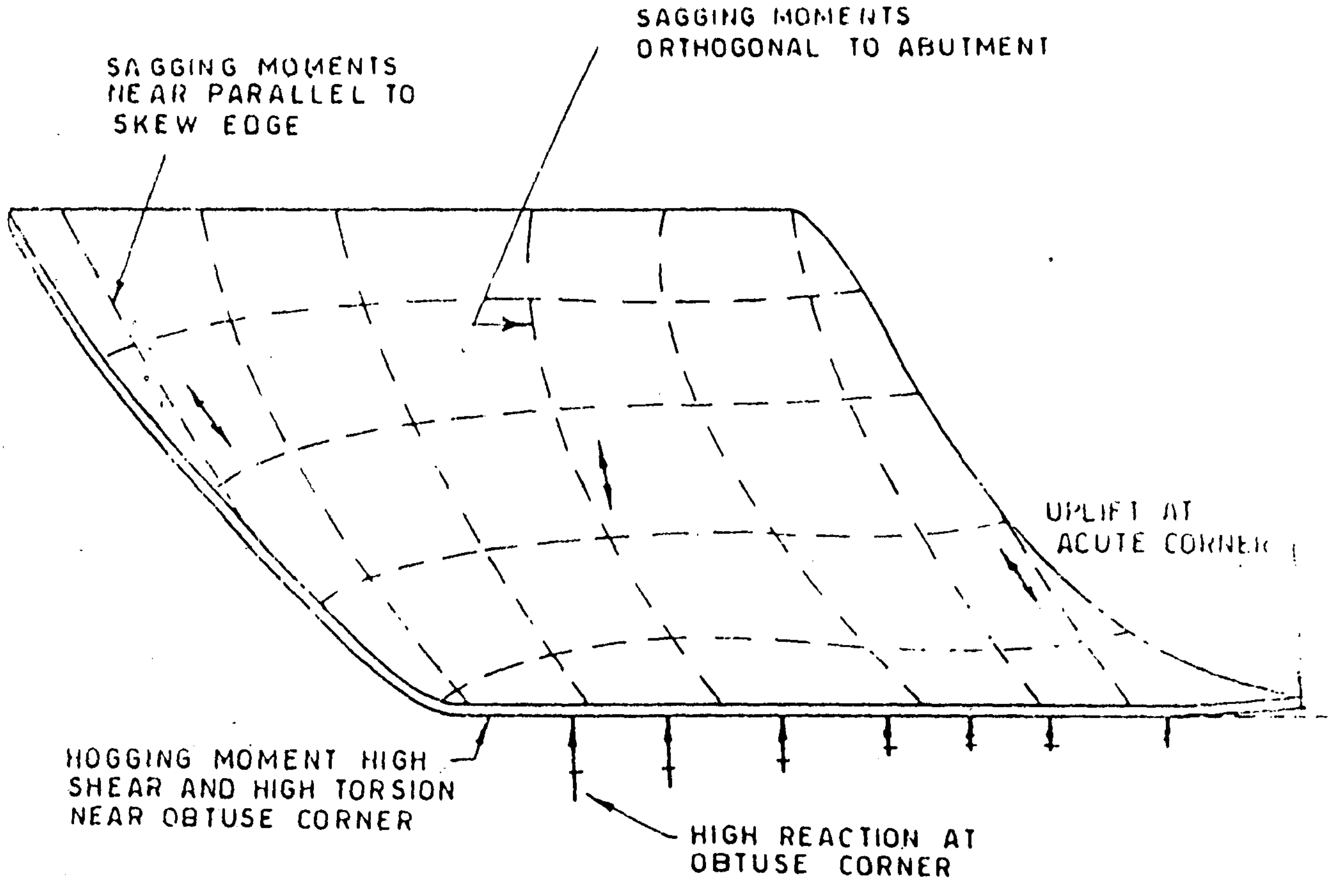
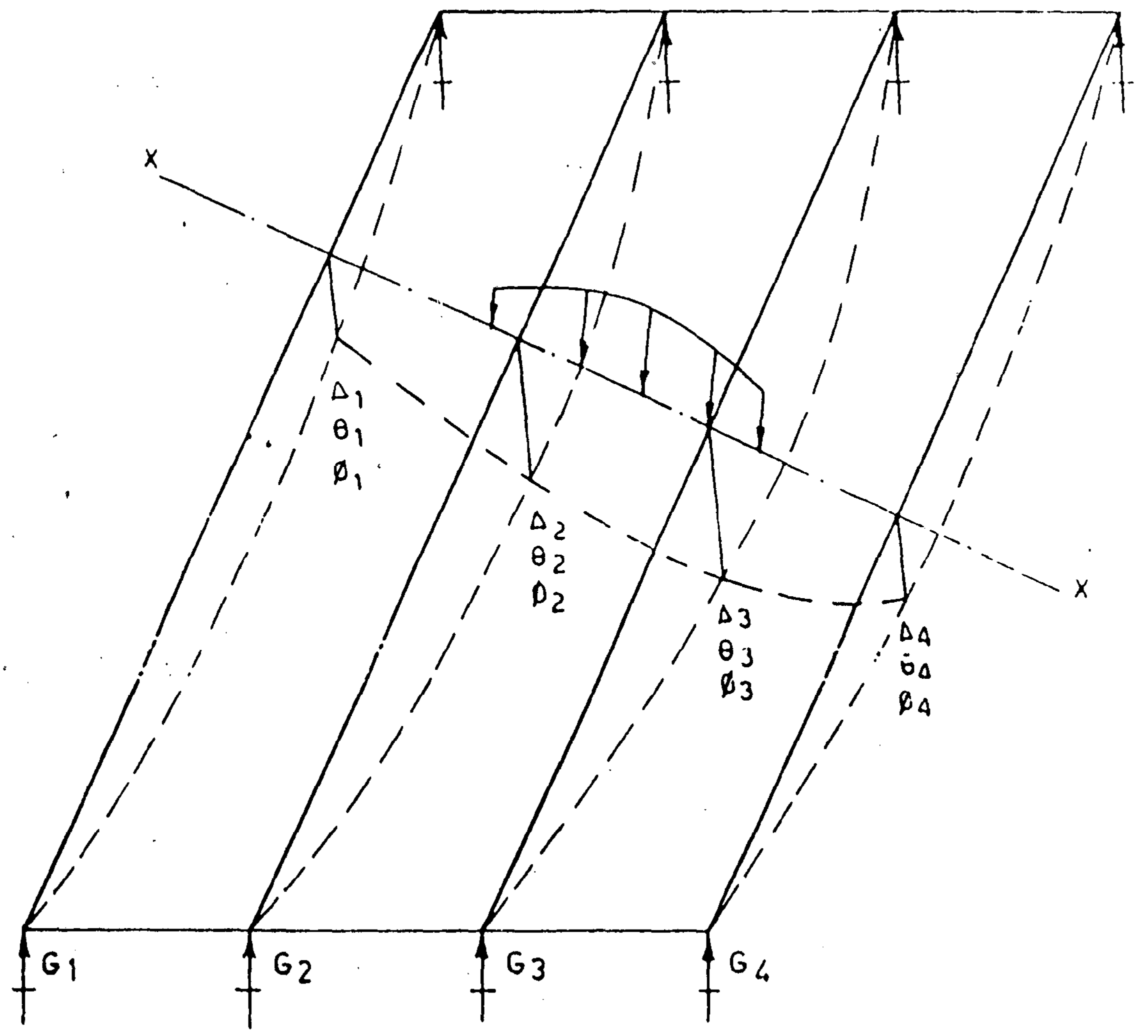


FIG. 1.1 _ SKEW SLAB - DECK BEHAVIOUR



Δ - DEFLECTIONS
θ - SLOPE
Ø - ROTATION

SIGNIFICANTLY DIFFERENT FOR ADJACENT GIRDERS

FIG. 1.2 _ GIRDER BRIDGE BEHAVIOUR

C H A P T E R - 2

LITERATURE REVIEW

2.1 Introduction

The importance of bridges is so obvious that it needs no detailed explanation. In short we can say that the whole human activities will practically come to a stand still in the absence of bridges.

Early bridge builders had little choice of matrial for their structures. Timber and masonry reigned supreme for quite a long time. They had many problems and limitations of their design techniques and materials, many a time the bridge failed and became unserviceable. The bridge disaster creates an under standable degree of public concern and leads to a great deal of new thinking by engineers and as a result other factors which have some contribution emerge at a subsequent enquiry. After a long cycle of successes and failures there was large breakthrough in all the aspects of bridge construction i.e. analysis, design, materials, workmanship and maintenance.

The improvements in methods of concrete production permitting higher working stresses and the reduction of creep coupled with the availability of high tensile steels have eventually paved the way to the widespread adoption of concrete bridges.

2.2 Basis Of Analysis And Design

To date, almost all design calculations for both concrete and steel bridges have been based on usual assumption of elastic strees- strain behaviour. Form of construction, plan geometry and support conditions are three important parameters which govern the choice of analytical techniques for the bridge deck.

The elastic methods of analysis may be divided into two- dimensional or three dimensional methods. The 2D mehtods idealises the bridge deck as a plate or as an open grillage of interconnected beams. The plates may be analysed by direct solution of the plate equation using Fourier series techniques in orthotropic plate theory or numerical solution of the equations by finite differnces. Approximate methods using design curves based on plate theory techniques have been devised. The plate may also be analysed by considering it as a number of discrete elements of triangular or quadrilateral form with finite dimensions. These finite elements are appropriately assembled to obtain the whole structure. This method of finite elements employs digital computers to quickly carry out vast amount of computations to obtain nodal displacements and element forces. Longitudinal strip elements extending through whole length of the bridge deck are employed in a specifie form of finite element method known as finite strip technique. Different techniques are discussed separately under the following headings.

2.2.1 Orthotropic Plate Analysis

An orthotropic plate is defined as one which has different specified elastic properties in two directions.
In pracice there are two forms of orthotropy (1) material orthotropy and (2)shape orthotropy. Most bridge decks are orthotropic because of shape orthotropy. More rarely there exists a combination of material and shape orthotropy.

Here the actual bridge is replaced by an equivalent orthotropic plate which is then treated according to the classical plate theory. The method thus involves the solution of the fourth order differential equation

$$D_x \frac{\partial^4 W}{\partial X^4} + 2H \frac{\partial^4 W}{\partial X^2 \partial Y^2} + D_y \frac{\partial^4}{\partial Y^4} = p(x,y)$$

where

Dx and Dy = Longitudital and transverse flexural rigidities per unit length

2 H = Total torsional rigidity

W = Deflection of bridge deck

p(x,y) = Load function

Huber appears to have been the first to use orthotropic plate theory in the analysis of reinforced concrete slabs. This was followed by Guyon who used the method to analyse a torsionless deck. Later Massonnet extended the method to include the torsional stiffness of the deck. Morrice, Little and Rowe (I) prepared the design curves based on a series of isotropic plates at a stage before the widespread availability of digital computers. Two types of charts for moment distribution and deflection of the slabs have been developed by Rowe etal (14) for no torsion and full torsion for isotoropic slab. For most bridge decks interpolation between these two is necessary. Hambly (13) has given curves in the form of influence lines for various points across the cross- section of simply supported right decks of slabs, beam - slab and girder bridges.

2.2.2 Harmonics Method

The concept of Harmonics method was first of all developed by Hendry and Jagas (10) Later it was modified by surana (11) and prasad (12) to in-corporate torsional stiffness of transverse system.

Unlike other methods, it lends itself to develop design coefficients which are used for determining girder bending moments,shear forces etc. due to any type of imposed loading on the deck through the dimensionless structural parameters and k incorporated in the formulation of this method. These parameters uniquely combine for a particular skew girder to identify it completely in so far as its structural bebaviour is concerned.

In Harmonics method, applied loading on a girder is broken into harmonic components which are easily obtainable using Fourier analysis. Each harmonic component is distributed separately amongst the girders using the design coefficients.

The bending moment for any girder is found by adding together fractions of harmonics so distributed. The method tackles the loads directly acting on the girders. Therefore, load applied between the girders are dealt with by replacing them by an equivalent system of loads acting on girders. If loads are acting on more than one girder, the equivalent moment loading relative to each are distributed separately and the total effect is found by superposition. The method also channelize the process of computing girder bending moments for any imposed loading. This results in a systemetic computation procedure making the calculation work simple and quick.

2.2.3 Finite Difference Method

Idealizing bridge as an equivalent orthotropic plate and solving the resulting equation by means of Fourier series is only convenient to standard cases of bridge decks e.g. When deck is rectangular in plan with two simple end supports. When more complex boundary conditions are encountered, the method becomes more difficult to apply. The finite difference technique has been used to advantage by westergaard (2) and lately by many others.

In this method of analysis, the deck is divided into grids of arbitrary mesh size and the deflection values at the grid points are treated as unknown quantities. The governing equation of the deck and the accompanying boundary conditions are expressed in terms of these unknown grid point deflections. The resulting simultaneous equations are then solved for the unknown deflections. Moments and shears are then determined from the known deflection pattern.

The accuracy of the results depends on the spacing of the grid points used in the analysis. The finer the mesh the more accurate is the result. The problem lies in solving the simultaneous equations formed from these grid points. This method has been applied to solve deck with different boundary conditions and skew bridge decks also. This was further extended to bridge decks curved in plan by Heins and Hails (3).

2.2.4 Finite Strip Method

The finite strip method is a hybrid procedure which combines some of the advantages of the series solution of orthtropic plates with the finite element concept. The method may be applied to Cellular decks. It was first put forward by Cheung (5) for rectangular slabs and suggested independently for the same problem by Powell and Ogder (6) Subsequently rapid develo- ment has been made in three centres - in Canada cheung, in the U.S by scordelis (7), Wiliam and Meyer and in Britain by Loo and Cusens (8) to find a displacement function applicable.

For simple support conditions it is possible to find a displacement function applicable all regions of the plate. When such a function is not conveniently obtainable, the plate may be divided into discrete longitudinal strips spanning between supports. Simple displacement interpolation functions may be used to represent displacement fields within and between individual strips.

2.2.5 Folded Plate Method

By accepted definition a folded plate is a prismatic shell formed by a series of thin plane slabs rigidly connected along their common edges. The shell is usually closed at its ends by integral diaphragms, construction is conventionally of reinforced or prestressed concrete and the structure may be simply supported or continuous over several spans.

If the box-beam deck is of multi-cell construction, orthotropic plate theory can provide an accurate estimate of load distribution as between beams. If detailed stress analysis is sought, or if the box section has less than about six cells, folded plate theory offers a logical approach. This method developed originally for the analysis of roof structures, analyses the structure in its correct form instead of replacing it by an equivalent structural system as in orthotropic plate analysis or grillage analysis.

The basic method of folded plate theory has been developed for a structure which is simply supported at its two ends. The basic method considers a structure with no interior diaphragm but the analysis may be extended to include intermediate diaphragms also.

The structure consits of a number of rectangular plates connected at longitudinal joints. Each plate is initially assumed to be fixed at the longitudinal joints. The stiffness matrix for each plate is then expressed in terms of the harmonics of a half range Fourier series. Each joint has four degree of freedom, displacement longitudinally tangential to the joint, rotation about an axis tangential to the joint, vertical displacement and horizontal displacement.

The direct stiffness method is generally used to analyse the complete structure.

The field of application of the method is restricted to right cellular bridge decks of uniform cross-section which may have intermediate diaphragms, but it must be simply supported at the extreme ends with rigid diaphragms positioned over the end supports. However, within its field of application the method is efficient in terms of computer time, is accurate and yields complete information about the elastic stresses in the structure.

2.2.6 Grillage Analysis

The approximate representation of bridge decks by a grillage of interconnected beams is a convenient way of determining the general behaviour of the bridge under load. In the past, the uses of grillage analysis were severely limited in scope since hand methods had to be used for the solution of the simultaneous equations. The general availability of the digital computer has revived the method and a number of standard programs have been written for use with all main types of computers. The direct stiffness method provides a valid and efficient technique for grillage analysis

The method of grillage analysis involves the idealization of the bridge deck through its representation as a plane grillage of discrete interconnected beams. Although, the method is necessarily approximate, it has the great advantage of almost complete generality. At the joints of the grillage any normal form of restraint to movement may be applied so that any support condition may be represented. The planform of the deck presents no real problem and skew, curved and irregular shapes may all be handled with case. (15)

Backed by extensive investigation west (4) has made recommendations on the use of grillage analysis, in which he defines methods of arranging the geometrical layout of grillage beam to simulate concrete slab and pseudo - slab bridge.

The highway engineering computer branch of British department of environment has made available a grillage program using the direct stiffness method and this will also take into account the effects of deformations due to shear. The STRUDL (IBM,U.K.LtD.London) grillage program, GRIDZJ by Gibb (9) and other recent programs also have this facility.

2.2.7 Finite Element Method

The most powerful of the techniques of analysis which arises from the direct stiffness approach is the finite element method. The finite element method employs an assemblage of discrete two and three dimensional member to represent the structure. The elements are connected at nodal points which possess an appropriate number of degrees of freedom. Many shapes of elements are available so that the method may be used to tackle complex plan - form, irregularly positioned supports, holes and other anisotropic structures may readily be incorporated in the analysis. Thus the finite element method may be seen to be very general in application and for difficult bridge deck problem, it is sometimes the only valid form of analysis. However,as presently used it has some drawbacks in terms of lengthy data preparation and computer running time. Out line of the method is as follows.

(1) Structural Idealization

A mathematical model of the structure is formulated in which it is represented as an assemblage of discrete parts, known as elements. Each element has finite dimension and properties. In order to perform the subsequent analysis, it is necessary to establish the force - displacement relationships of each element. The determination of these relationships forms the second stage of the process.

(2) Evaluation Of Elemental Properties

If a pattern of finite elements is used to represent a bridge deck as an elastic continuum, the division of the structure can be carried out in different ways. For a slab deck the elements may, for example, be taken as triangular or quadrilateral plate elements. The representation may be coarse with a small number of elements, or fine using a relatively large number of elements.

(3) Structural Analysis Of The Element Assemblage

The usual requirements to be satisfied are the following.

(a) Equilibrium of the internally and externally applied forces at each node of the element

(b) Geometric fit or compatibility of element deformations in such a way that they meet at the nodal points in the loaded configuration

(c) The internal force - displacement relationship must be established with each element as dictated by the existing geometry and material property.

In practical application of engineering problem, this method has a number of limitations as indicated below -

(1) Cumbersome to use, because it needs a time consuming and lengthy data preparation and lots of time is required for interpretation of the results.

(2) Expensive, as regards to computer time

(3) If the choice of element is incorrect, the results may be far more inaccurate than those predicted by simpler method such as grillage method. (15)

C H A P T E R - 3

STIFFNESS METHOD FOR GRID FRAMEWORK

INTRODUCTION

A girder bridge deck is composed of longitudinal members, cross beams or diaphragms and slab. This whole deck may be assumed to be a grid formed by longitudinal girders and cross members including slab which is converted into equivalent cross members. These elements are connected together at discrete nodes. These elements are represented by their stiffnesses corresponding to respective deflections and rotations. In order to structurally simulate a bridge deck, all the members entering any particular node must have the same nodal displacements. At the same time it is required that all the stiffnesses are referred to the common axes known as global coordinate axes.

3.2 Concept of Grillage Analysis

By means of some assumptions the deck is converted into a net work of longitudinal and cross beams. Besides providing rigidity, the cross members and slab facilitate the transfer of concentrated load to the longitudinal members. The longitudinal stiffness of the slab is concentrated in the nearest longitudinal girder and tranverse stiffness is concentrated in the nearest cross member. Ideally the stiffness of the beam should be such that both real and transformed bridge should give the same moments, shear force, deflections etc. under identical conditions of loading. But in actual practice both donot behave in identical ways. For the equilibrium of any element the slab requires that in the orthogonal directions torques should be indentical. But in grillage there is no physical or mathematical principle to make torques identical in orthogonal directions at joints. This shortcoming is minimised when grillage mesh is fine enough to permit the grid to deflect in a smooth surface with twist in orthogonal directions. Another shortcoming of grid analysis is that the moment in the grillage depends directly on curvature in it while in the prototype slab the moment in any direction depends on the curvature in that direction and also in orthogonal direction. However, the errors due to these are not of practical consequence. A reasonably fine gridwork tends to represent the deck well within the acceptable engineering accuracy.

3.3 Discretization of Deck

Longitudinal girders and diaphragms or cross beams and discretized slab strips constitute the net work of grid structurally representing the deck. Cross beams are connected monolithically with both girders and slabs, while diaphragms are built monolithically with girders only. Discretization should be done in such a manner that the actual structure is truly represented from structural point of view.

Girders and cross beams are monolithic with the slab, so they are represented by T- scection oriented along their lenght. Flange width of the T- section is adopted as per the recommendation of IRC Bridge code specification (Appendix-A). Length of the beams depends upon the node to node distance. The longitudinal and transverse spacing of discretized elements should be similar and the spacing of transverse member should be kept as small as possible.

The choice of orientation of the slab elements depends upon the designer. In skew system of representation, the slab element may be oriented parallel to support axes. Another choice is that the slab elements are oriented perpendicular to the girders. The discretization should be done in such a way that the constituent material is homogeneous and isotropic. If it is not so the orientation chosen should correspond to the reinforcement layout.

The discretized plan of the deck is shown in the Fig. 3.1

3.4 Mathematical Model

Grillage analysis can be performed using either straight member or curved member with or without constant cross-section. The properties of an element are given in terms of flexural modulus EI, its torsional moduls GJ, length L and its lenght mesured with respect to two coordinate axes. The deformation considered for element stiffnesses are two orthogonal rotation in the horizontal plane and vetical deflection at each node. The nodal displacements in the horizontal plane and rotation along the vertical axes are not considered to significantly contribute to the structural behaviour and hence, are ignored. A typical element and global axes is shon in Fig. (3.2)

The stiffness matrix for joint forces and displacements referred to the member axes is given by :

$$
\begin{Bmatrix} P_i \\ C_{pi} \\ C_{qi} \\ P_j \\ C_{pj} \\ C_{qj} \end{Bmatrix}
=
\begin{bmatrix}
\frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} & 0 & -\frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} & 0 \\
\frac{6\,EI}{L^2} & \frac{4\,EI}{L} & 0 & -\frac{6\,EI}{L^2} & \frac{2\,EI}{L} & 0 \\
0 & 0 & \frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} \\
-\frac{12\,EI}{L^3} & \frac{-6\,EI}{L^2} & 0 & \frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} & 0 \\
\frac{6\,EI}{L^2} & \frac{2\,EI}{L} & 0 & -\frac{6\,EI}{L^2} & \frac{4\,EI}{L} & 0 \\
0 & 0 & \frac{GJ}{L} & 0 & 0 & \frac{GJ}{L}
\end{bmatrix}
\begin{Bmatrix} W_i \\ \theta_{pi} \\ \theta_{qi} \\ W_j \\ \theta_{pj} \\ \theta_{qi} \end{Bmatrix}
$$

Alternatively

$$\{P\}_e = [K]_e \{D\}_e \qquad \ldots (3.2)$$

i and j are the nodes of the discretized structure referred to the local axes. {k} is stiffness matrix of the element. The overall stiffness matrix of the original structure is made by correctly superimposing stiffnesses of all the elements constituting the discretized structure. So the stiffness of the various elements must be expressed in terms of a common coordinate system called the global coordinate system. The forces and moments in the local axes are transfered into global co-ordinate system by transformation matrix.

$$\begin{Bmatrix} P_i \\ C_{pi} \\ C_{qi} \end{Bmatrix} = \begin{bmatrix} 1 & \emptyset & \emptyset \\ \emptyset & \text{Cos}\alpha & \text{Sin}\alpha \\ \emptyset & -\text{Sin}\alpha & \text{Cos}\alpha \end{bmatrix} \begin{Bmatrix} P_i \\ C_{xi} \\ C_{yi} \end{Bmatrix} \qquad \ldots (3.3)$$

or,

$$\{P_i\}_e = [R] \{P_i\}_g \qquad \ldots (3.4)$$

Where [R] is the rotation transformation matrix. If forces at both the ends are simultaneously considered, then

$$\begin{Bmatrix} P_i \\ P_j \end{Bmatrix} = \begin{bmatrix} R & O \\ O & R \end{bmatrix} \begin{Bmatrix} P_i \\ P_j \end{Bmatrix} \qquad \ldots (3.5)$$

or,

$$\{P\}_e = [T] \{P\}_g \qquad \ldots (3.6)$$

where,

$$[T] = \begin{bmatrix} R & O \\ O & R \end{bmatrix} \qquad \ldots (3.7)$$

Similarly deformation transformation matrix is

$$\{D\}_e = [T] \quad \{D\}_g \qquad \ldots (3.8)$$

Stiffness equations for structural systems in the local and global systems respectively are:

$$\{P\}_e = [K]_e \quad \{D\}_e \qquad \ldots (3.9)$$

and $$\{P\}_g = [K]_g \quad \{D\}_g \qquad \ldots (3.10)$$

From (3.2), (3.6) and (3.8)

$$[T] \quad \{P\}_g = [K]_e \; [T] \quad \{D\}_g \qquad \ldots (3.11)$$

or, $$\{P\}_g = [T]^{-1} \; [K]_e \; [T] \quad \{D\}_g \qquad \ldots (3.12)$$

From (3.10) and (3.12)

$$[K]_g = [T]^T \quad [K]_e \; [T] \qquad \ldots (3.13)$$

Through equation (3.13), the element stiffness matrix is expressed in global co-ordinate system

This final expression for element stiffness in global co-ordinate system is

$$
\begin{Bmatrix} P_i \\ C_{xi} \\ C_{yi} \\ P_j \\ C_{xj} \\ C_{yj} \end{Bmatrix}
=
\left[\begin{array}{ccc|ccc}
K_{11} & K_{12} & K_{13} & -K_{11} & K_{12} & K_{13} \\
K_{12} & K_{22} & K_{23} & -K_{12} & K_{25} & K_{26} \\
K_{13} & K_{23} & K_{33} & -K_{13} & K_{26} & K_{36} \\
\hline
-K_{11} & -K_{12} & -K_{13} & K_{11} & -K_{12} & -K_{13} \\
K_{12} & K_{25} & K_{26} & -K_{12} & K_{22} & K_{23} \\
K_{13} & K_{26} & K_{36} & -K_{13} & K_{23} & K_{33}
\end{array}\right]
\begin{Bmatrix} W_i \\ \theta_{xi} \\ \theta_{yi} \\ W_j \\ \theta_{xj} \\ \theta_{yj} \end{Bmatrix}
$$

Where,

$K_{11} = 12\ EI/L^3$

$K_{12} = 6EI\ \cos\alpha/L^2$

$K_{13} = 6EI\ \sin\alpha/L^2$

$K_{22} = (GJ\ \sin^2\alpha + 4EI\ \cos^2\alpha)/L$

$K_{23} = (4EI - GJ)\ \sin\alpha\ \cos\alpha\ /L$

$K_{25} = (2EI\ \cos^2\alpha - GJ\ \sin^2\alpha)/L$

$K_{26} = (GJ + 2EI)\ \sin\alpha\ .\ \cos\alpha/L$

$K_{33} = (4EI\ \sin^2\alpha + GJ\ \cos^2\alpha)/L$

$K_{36} = (2EI\ \sin^2\alpha - GJ\ \cos^2\alpha)/L$

Then stiffness matrices of all the elements constituting the discretized structure are appropriately assembled having formed the element stiffness matrix in the global co-ordinate system in such a way that all the elements entering any particular node must have tha same nodal displacements. Thus nodal stiffness matrix of the discretized structure is obtained.

Loads applied are transformed into a system of equivalent nodal forces which can be a single direct load or a combination of direct load and/or bending moment and/or torsion. The nodal stiffness matrix and equivalent forces acting at the nodes of the discretized structure being known, the equation

$$\{P\}_g = [K]_g \{D\}_g$$

Can be solved with respect to a certain support conditions. Then by back substitution the values of different functions can be computed

3.5 Computer Program

The process of discretization of bridge deck has been explained in section (3.3). After discretization, the analysis becomes very simple by the computer program. The computer program used is basically developed for the analysis of grid structure. In order to ensure minimum storage for the computer memory, the node numbering is done on the basis of right hand system. The constituting elements and the associated properties are needed as input data. The program develops nodal stiffness matrix of the discretized structure in the global co-ordinate system. The equivalent nodal loads are given, which get transferred to appropriate locations in the over-all load matrix. The displacement conditions for support nodes are given. Force matrix and global stiffness matrix being known, the displacement matrix is computed using numerical technique. With the help of nodal displacement and apporpriate element stiffnesses, element forces are finally computed.

There are five subroutines in the main program.

INPUT reads data for particular problem and performs a few miner calculations.

ASSMBL determines the member stiffness matrices for each member type, assembles the overall structural matrix and details of the member connectivity.

BCS performs the modification of the stiffness equation so that the specified displacement boundary condition is satisfied.

BSOLV solves the stiffness equation. Half of the banded stiffness matrix is stored and the solution is by cholesky decomposition

RESULTS prints the joint displacements, calculates and prints the member end moments, shear forces, torsion and the value of joint reactions.

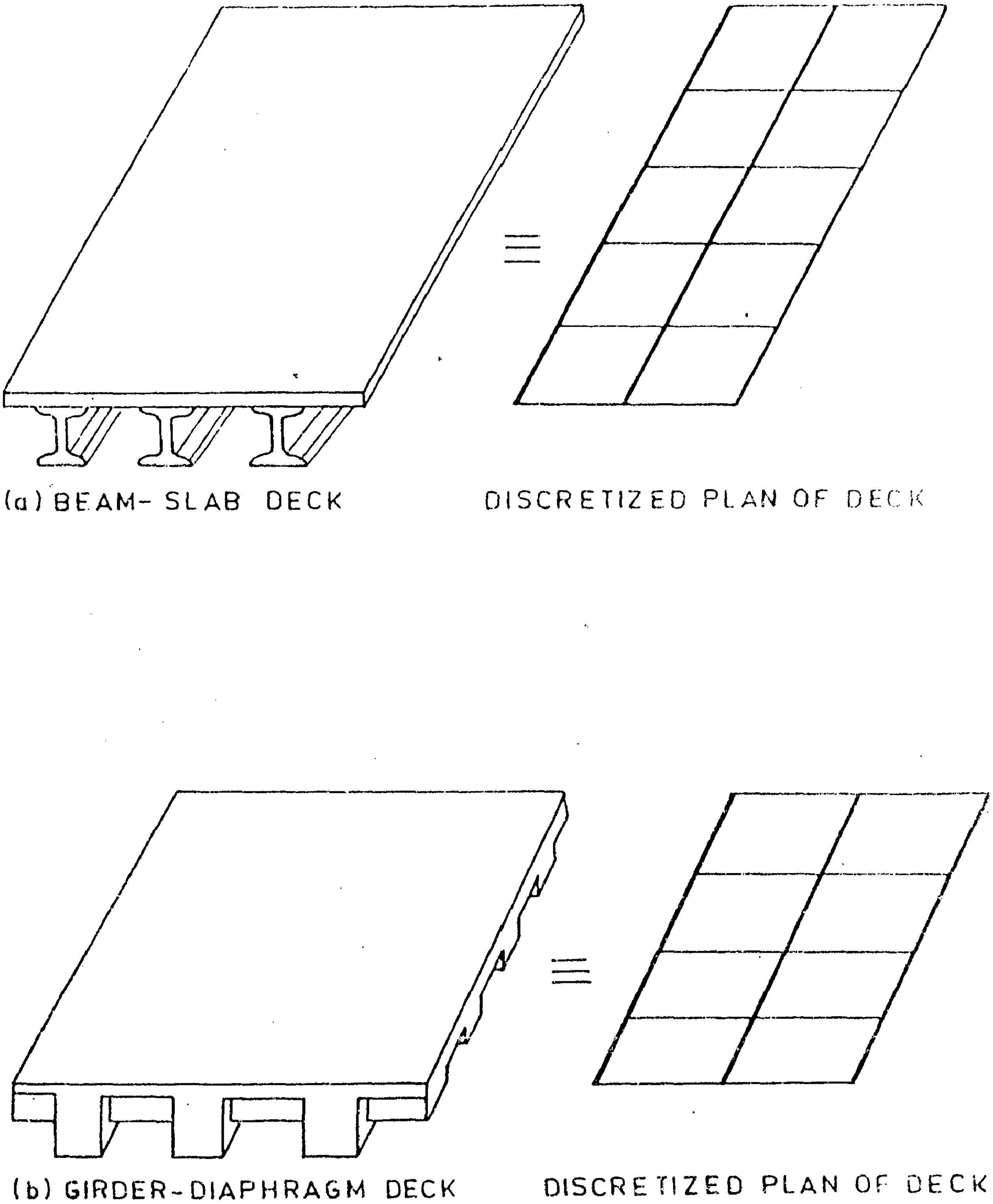
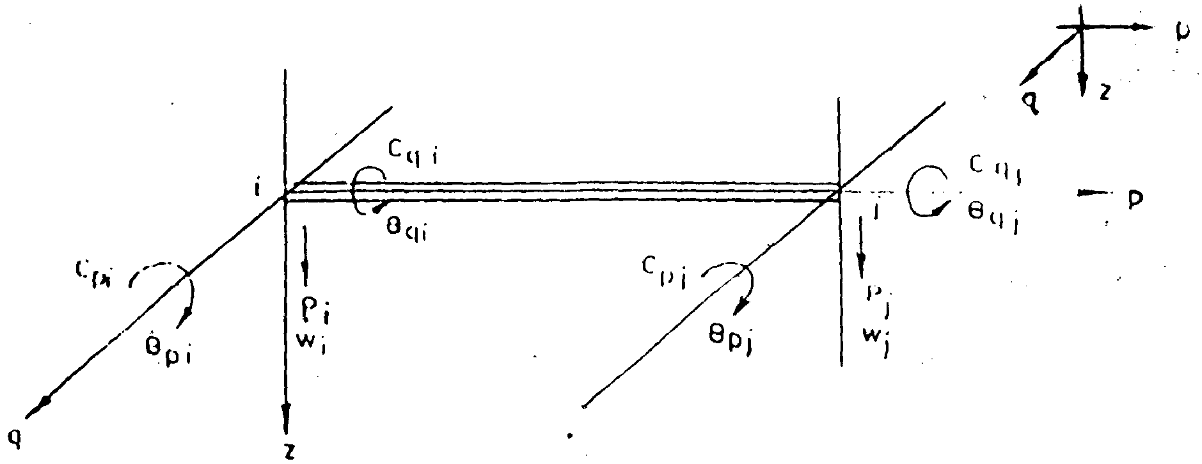
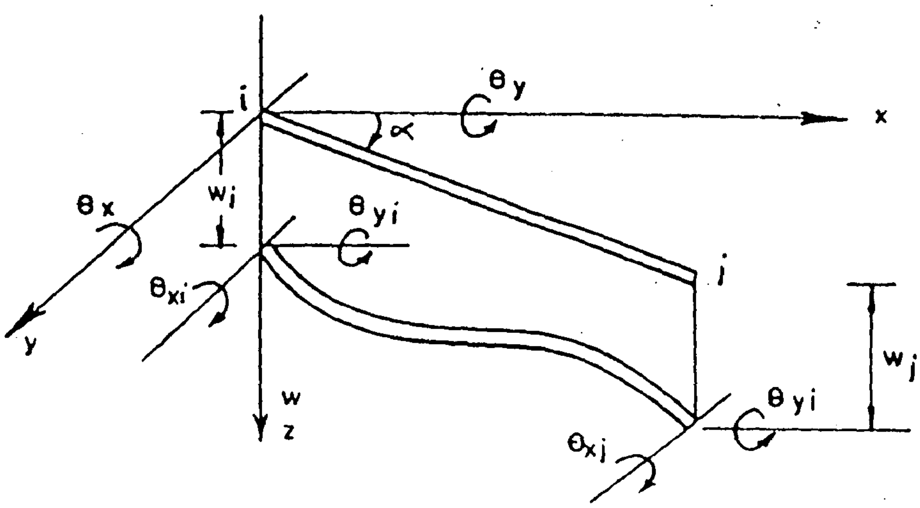


FIG. 3.1 - GIRDER BRIDGE DECK DISCRETIZATION



P, W	Nodal Vertical Load and Displacement	+
C_{p_i}, θ_{p_i}	Nodal Moment and Rotations about q - axis	+
C_{q_i}, θ_{q_i}	Nodal Moment and Rotations about p - axis	+
C_p, θ_p	Member Bending Moment and Slope	
C_q, θ_q	Member Torsion and Rotation	

(a) Nodal Forces And Displacement In Local Axes



(b) Grid Member In Global Axes

FIG.3.2 TYPICAL ELEMENT OF GRID FRAMEWORK

CHAPTER - IV

BRIDGE SYSTEM UNDER STUDY

4.1 Introduction

In girder bridges, imposed load distribution amongst the girders takes place through the transverse system interconnectioning the girders. The transverse systems consist of slabs only as in the case of slab-on girder bridges or cross beams/diaphragms along with the decking slab as in most other cases. In the latter case cross beams/diaphragms are the main agent of load distribution whereas the decking slab itself does this job in the first case. It, thus becomes clear that the arrangement and relative stiffness of cross beams/diaphragms with respect to the griders will play significant role in the study of structural efficiency in transverse load distribution. From this point of view, it is proposed to study the deck behaviour of a particular carriage width (4 lane) when the girder cross section and its number is changed without any change in the transverse system i.e, the cross-beam/diaphragm and slabs cross-sections. Girder cross- section and it number is changed in such a manner that cross- sectional area of the 4- lane bridge remains unaltered. In this process, the change in the relative stiffness of the girders vis-a-vis transverse medium is brought about through various material disposition in girder cross-section.
New girders are created while keeping the total volume of girders constant. Two ways of effecting this change is considered herein as case A and case B.

CASE A Taking matarials from the sides of the girders webs so as to create addtinal girders. Care must be taken to see that the girders do not become torsionally weak beyond the acceptable limit while reducing the web thickness.

CASE B Taking materials from the depth of the girder webs so as to create additional girders. care must-be taken to see that the deck does not become too shallow to result in excesive deflection.

4.2 Bridge System Adopted

4 - Lane girder bridge having cross beams at the supprots and three intermediate diaphragms have been chosen for the study. Birdges are simply supproted at the two ends. Span, both for the right as well of skew bridges is taken to be 25m. As per IRC recommendations, a central verge of width 1.2m and kerb of widht o.6m are provided at both the sides. Thus total width of the bridge transverse section turns out to be 17.4m including the 1.5m width of the deck slab cantilevering out from the exterior girders. Three bridges varying with respect to skew angle such as (1) right bridge $\lambda = 0°$, (2) sked bridge $\lambda = 20°$ and (3) skew bridge $\lambda = 30$ have taken. For study of the structural behaviour of the systems both for case-A and case-B, T-beams with appropriate dimensions have been provided as griders (Longitudinal girders). Initially, three girders have been provided which are subsequently transformed into four and five girders with appropriate dimensions separately for case-A and case-B. Additional girders created are placed within the same overall width of 17.4m in such a fashion that they are equally spaced in each case. Thus, the girder spacings are 7.2m, 4.8m and 3.6m for the three, four and five girder bridges both in case-A as well as case-B. Bridge dimensions in each of the cases are briefly given hereunder.

CASE - A Material is taken from the sides of the webs to create additional girders without making it torsionally weak beyond the acceptable limit. The nine bridge systems studied for case-A are briefly described hereunder.

(1) 3- Girder System Initially Adopted

Referring to Fig. 4.1, we have
Right Bridge λ=Ø°, Right Span L = 25m
No. of Girders N=3, Girder Spacing =7.2m
Girder Cross-Section= As Shown in Fig. 4.4
No. of Cross Beams = 2
Cross Beam Cross-Section = As Shown in Fig. 4.4
No. of Intermediate Diaphragm = 3
Cross-Section of Diaphragms = As shown in Fig. 4.4
Intermediate Diaphragm Spacing Spacing = 6.24m

(2) 4- Girder System Created

The 3- girder bridge system adopted in (1) is transformed into a 4-girder system without any change in the transverse system. Required amount of material is taken out from the sides of the three webs by reducing the webs thickness from Ø.55m to Ø.4m and creataing a 4-girder system. This new system has the same overall deck depth but the cross-sectional dimentions and spacing of the girders are changed. The relevant data are as follows:

All data are the same as given in (1) above except as mentioned below:

No. of Girders N=4, Girder Spacing h=4.8m
Girder Cross-Ssection = As shown in Fig. 4.4.

(3) 5-Girder System Created

The process involved in creating 4-girder systems as explained in (2) is repeated to further create a 5-girder system (Fig. 4.3). Consequently, the relevant data for the 5-girder system is as follows:

All data are the same as given in (1) except as mentioned below

No. of Girders N=5, Girder Spacing h= 3.6m

Girder Cross-Section= As shown in Fig. 4.4.

Thus, three Longitudinal girder systems for the same transverse system interconnecting the girders are formed for the adopted 25m right (λ=0°) bridge. Toatal volume of material for the girder system remains unchanged while their stiffnesses vis-a-vis transverse systems keep on changing. It, thus, becomes possible to ascertain as to which of the three girder systems is structurally most efficient.

Similarly the next three girder systems are created for (a) 25m skew bridge having skew angle = 20°. The relevant data are given in (4), (5) and (6) hereunder.

(4) <u>3- Girder Skew System Intially Adopted</u>

Referring to Fig. 4.1, we have

Skew Angle= 20°, Skew Spans L=25m,

No. of Girders N=3, Girder Spacing h=7.2m

Girder Cross-Section= As shown in Fig. 4.4,

No. of Cross Beams=2

Cross-Beams Cross-Section= As shows in Fig. 4.4.

No. of Intermediate Diaphragms = 3

Intermediate Diaphragm Cross-Sections= As shown in Fig.4.4.

Intermediate Diaphragms Spacing = 9.88m

(5) <u>4-Girder System Created</u>

The 3-girder system adopted in (4) is transformed into a 4-girder systems using the procedure explained in (2). Relevant data for the consequent 4-girder systems are as follows:

All data are the same as given in (4) except as metnioned below

No. of girders N=4, Girder spacing h=4.8m

Girder cross-section= As shown in Fig. 4.4

(6) 5- Girder System Created

The 3- girder systems adopted in (4) is further transformed into a 5-girder system using the procedure explained in (2) Relevant data for the consequent 5-girder systems are as follwos:

All data are the same as given in (4) except as mentioned below:

No. fo girders N=5, Girder spacing h=3.6m

Girder cross-section = As shown in Fig. 4.4.

Thus, for the 25m skew bridge system also, varying system of girders (N=3, 4 and 5) are created without any change in the total volume of girders. This would facilitate a study of their structural efficiency in transverse load distributions.

With a view to extending the study to large skew bridge system, skew angle is further changed to 30° while keeping the skew span the same as earlier (L= 25m). It may be noted herein that although the skew span remains unchanged, span measured along normal to the abutments are different in all the three bridge systems adopted.

(7) 3-Girder Skew System Initially Adopted

Referring to Fig. 4.1 we have

Skew Angle =30°,Skew Span L=25m

No. of Girder N=3 Girder Spacing h=7.2m

Girder Cross-Section= As shown in Fig. 4.4.

No. of Cross-Beams = 2

Cross Beams Cross-Section = As shown in Fig.4.4

No. of Intermediate Diaphragms = 3

Intermediate Diaphragm Cross-Scetion= As shown in Fig. 4.4. Intermediate Diaphragm Spacing = 8.38m

(8) **4-Girder System Adopted**

The 3-girder system adopted in (7) is transformed into a 4- girder system (Fig. 4.3) using the procedure explained in (2). Relevant data for the consequent 4-girder system are as follows:

All data are the same as given in (7) except as metnioned below:

No. of Girder N = 4, Girder Spacing h=4.8m

Girder Cross-Section = As shown in Fig. 4.4

(9) **5- Girder System Created**

The 3-girder system adopted in (7) is further transformed into a 5-girder system using the procedure explained in (2). Relevant data for the consequent 5-girder system are as follows:

All data are the same as given in (7) except the ones mentioned hereunder

No. of Girder N=3, Girder h=3.6m

Girder Cross-Section= As shown in Fig. 4.4

The above nine bridge systems forms the part of study falling in case-A. Separate nine bridge systems are created for the study falling in case B.

CASE - B In this case, the relative stiffness of girders vis-a-vis transverse system are changed by taking the material out of the depth of girder webs. While doing so, care is taken that the deck as such does not become too shallow resulting in excessive deflection. Each time enough material is taken out to create one additional girder so that we have 3,4 and 5-girder systems. With the change in girder-depths, the depth of cross beams and diaphragms also need to be cahnged in order to accommodate them within the depth of the girder-webs. This is done based on the considerations that the webs of cross beams and diaphragm have their depths equal to 80 percent of the depth of girder-webs. This process alters the stiffness of the transverse system to some extent but is inevitable. Also, the small amount of material saved from the cross beams and diaphragms are utillzed in creating new girders.

In case-B study, too nine bridge systems three for each of the skew angles =0° (right bridge), 20° and 30° are created. Salient features of each of the nine systems are givew hereunder.

(1) <u>3- Girder System Created</u>

Referring to Fig.4.1 we have, Right Span L=25m
No. of Girders = 3, Girder Spacing h = 7.2m
Girder Cross-Section= As shown in Fig. 4.5
No. of Cross Beams = 2
Cross Beams Cross- Section= As shown in Fig. 4.5.
No of Intermediate Diaphragms = 3
Intermediate Diaphragm Spacing = 6.24m

(2) <u>4- Girder System Created</u>

The 3-girder bridge system adopted in (1) is transformed into a 4-girder bridge system (Fig. 4.2) with change in transverse system. The required material is obtained from (1) depth of the web of 3 girers (2) depth of the intermediate diaphragms. By doing so the cross-section of the girder as well deaphragm changes. The spacing of the girder is also changed. In this process total volume of the matrial is kept constant. The relevant data are as follows:

All data are the same as given in (1) above except as mentioned below

No. of Girders N= 4, Girder Spacing = 4.8m
Girder Cross-Section = As shown in Fig 4.5
Intermediate Diaphragm Cross-Section= As shown in Fig. 4.5

(3) 5- Girder System Created

The process explained in (2) is repeated to create a 5-girder system(Fig. 4.3). All data are the same as given in (1) except as mentioned below:

No. of Giders = 5, Girder Spacing = 3.6m
Girder Cross-Section= As shown in Fig. 4.5
Diaphragm Cross-Section= As shown in Fig. 4.5

Similarly next three girder system are created for again 25m skew bridge having skew angle= 20°. The relevant data are given in (4), (5) and (6) hereunder

(4) 3 - Girder Skew System Initially Adopted

Referring to Fig 4.1, we have
Skew Angle = 20°, Skew Span L = 25m .
No. of Girders N = 3, Girder Spacing h = 7.2m
Girder Cross-Section= As shown in Fig. 4.5
No. of Intermediate Diaphragms = 3
Intermediate Diaphragm Cross-Section = As shown in Fig 4.5
Intermediate Diaphragm Spacing = 9.88m

(5) 4 - Girder System Created

The 3-girder system adopted in (4) is transformed into a 4-girder system (Fig. 4.2) using the procedure explained in (2). Relevant data are as follows:

All data are the same as given in (4) except as mentioned below

No. of Girder N= 4, Girder Spacing h = 4.8m
Girder Cross-Section = As shown in Fig. 4.5
Diaphragm Cross-Section= As shown in Fig. 4.5

6) 5 - Girder System Created

The 3-girder system adopted in (4) is further transformed into a 5- girder system (Fig. 4.3) using the procedure explained in (2). The relevant data for the consequent 5-girder sytem are as follows:

All data are the same as given in (4) except as mentioned below:

No. of Girders N = 5, Girder Spacing h= 3.6m
Girder Cross-Section = As shown in Fig. 4.5

Now skew angle is changed to 30° while keeping the skew span the same as earliar (L = 25m). Although, the skew span remains constant span measured along normal to the abutments is different in all the three bridge systems adopted (with respect ot skew angle).

(7) 3 - Girder Skew System Initially Adopted

Referring to Fig. 4.1, we have,
Skew Angle = 30°, Skew Span L = 25m
No. of Girders N = 3, Girder Spacing h = 7.2m

Girder Cross-Section= As shown in Fig. 4.5
No. of Cross-Beams = 2
Cross-Section of Cross-Beam = As shown in Fig. 4.5
No. of Intermediate Diaphragms = 3
Intermediate Diaphragm Cross- Section = As shown in Fig. 4.5
Intermediate Diaphragm Spacing = 8.38m

(8) <u>4 - Girder System Created</u>

The 3-girder system adopted in (7) is transformed into a 4-girder system (Fig.4.2) using the procedure explained in (2). The relevant data for the consequent 4-girder system are as follows:

All data are the same as given in (7) expect as mentioned .below

No. of Girders N = 4,Girder Spacing = 4.8m
Girder Cross-Section = As shown in Fig. 4.5
Diaphragm Cross-Section = As shown in Fig. 4.5

(9) <u>5 - Girder System Created</u>

The 3- girder system adopted in (7) is further transformed into a 5- girder system (Fig.4.3) using.The procedure explained in (2). Relevant data for the consequent 5-girder system are as follows:

All data are the same as given in (7) except the ones mentioned hereunder

No. of Girders N = 5, Girder Spacing h = 3.6m
Girder Cross-Section = As shown in Fig. 4.5

4.3 LOADING

All bridge systems have been studied for their structural behaviour under IRC loading cases. IRC class AA wheeled vehicle load system is adopted for the purpose of limited study herein. IRC class AA tracked vehicle loading could not be considered due to limitations on availability of time and volume of work. It may be considered as part of futher scope for study.

For all the bridge systems in case A as well as in case B, two extreme transverse loading arrangements have been adopted. The two positions are (1) extrême right position of the vehicle keeping it as far as possible from the support for maximum effect on internal girders. It is named as central loading (2) extreme right for one vehicle and extreme left for the other keeping as far as possible from the support for maximum effect in external girder. It is named as extreme left loading, as is appears on the left hand side of the observer here. This is explained in Fig. 4.9. These extreme positions of the vehicle have been fixed with due considertions of the minimum IRC recommended clearances. Thus, the vehicle has been positioned at 1.8m (1.2m +Ø.6m) clearance from the external edge of the kerb for the extreme left loading and similarly 1.8m from the centre of the central verge for the extreme right loading positions. This is done for the left two lanes of the 4-lane bridge systems. Thus, both the segments of the 4-lane bridge ar loaded simultaneously for the maximum moment and deflection. Longitudinally the load system is centrally located at the mid-span of the bridge.

The above described load system is used for all the girder systems both in case- A as well as case B. Thus, the loading arrangement remains unaltered with respect to the plan form of the bridges, although girder locations in 3,4 and 5-girder systems are different. This has been done with a view to sudying the structural behaviour under the same loading on the deck.

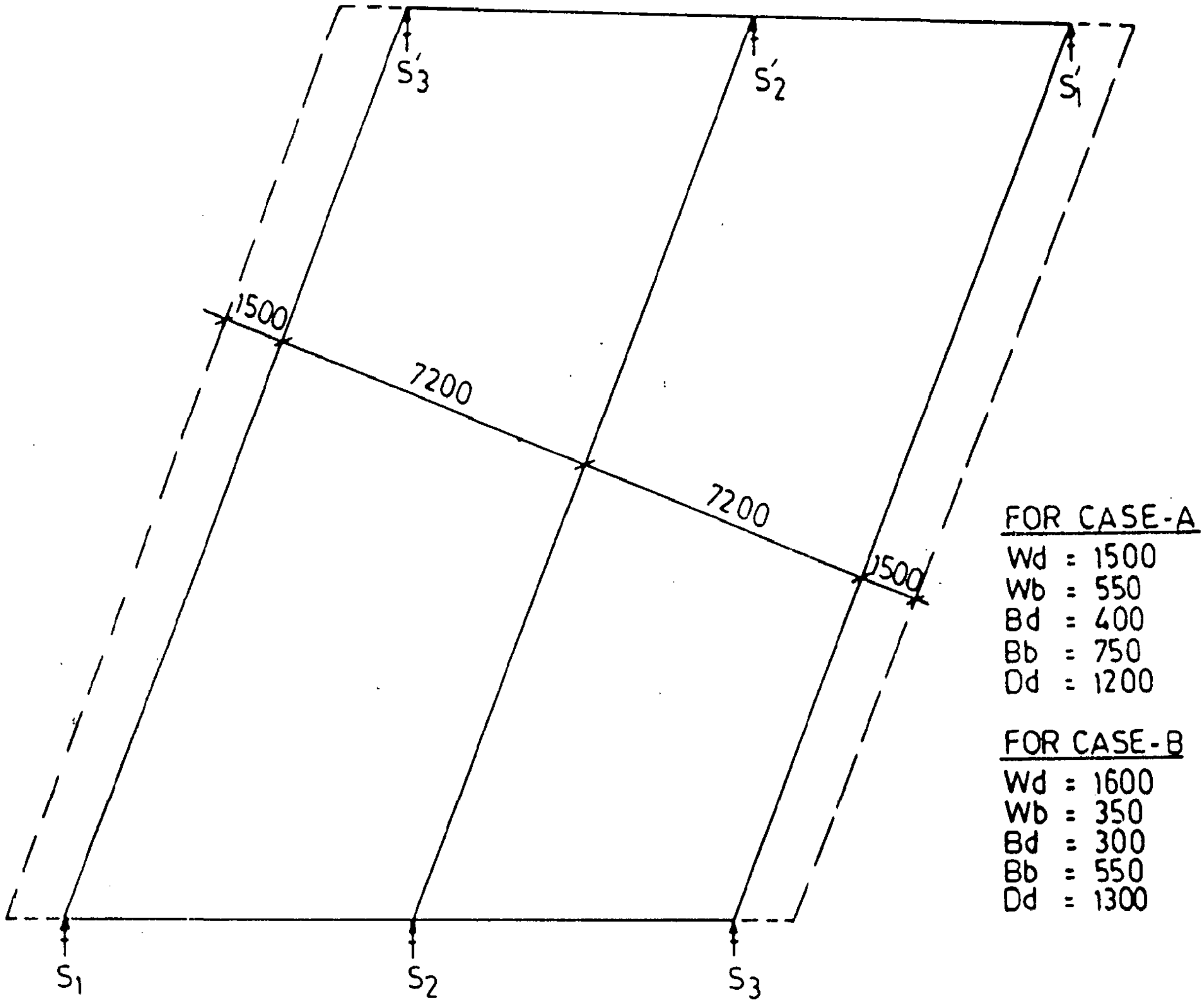
4.4 Equivalent Nodal Loads

The program requires the loads to be placed at nodes. After judiciously descretizing the bridge, the loads are suitably placed so as to cause maximum moment in the deck Keeping IRC recommendations in mind. There are very little chances that the loads coincide with one of the nodes. So, it is required that the loads should be transferred to the adjacent nodes with some assumptions. The loads are transferred to the nodes according to its distance from the nodes. Following points are kept in mind while calculating the equivalent nodal loads.

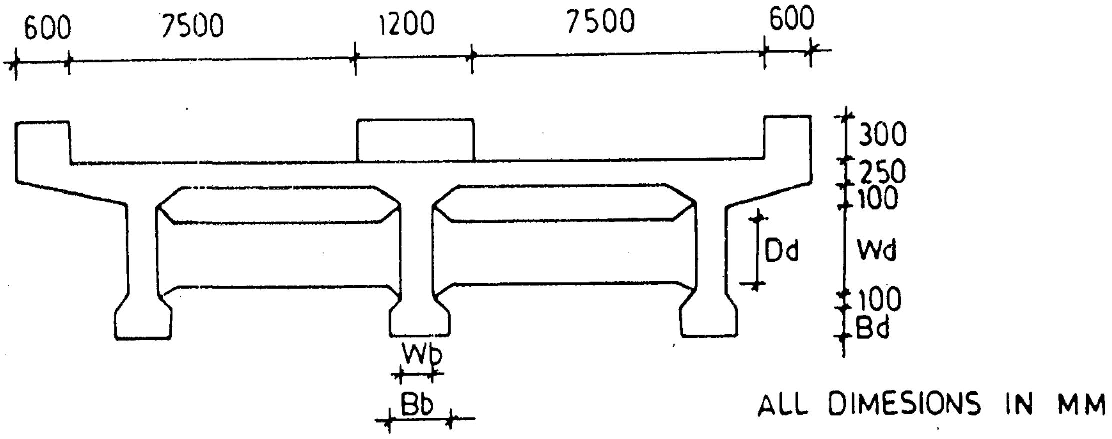
(i) A load falling in the effective flange width (App-A) of a girder is fully transferred to that girder. Again, this load is distributed between the nodes assuming the span between the two nodes as simply supproted.

(ii) If a load lies between two girders, then it is first of all distributed between the two girders in proportion of its distance from the other girder. This transferred load is again distributed between the two nodes in the same manner as (i)

The conversion of actual loads into equivalent nodal loads has been shown in Fig. 4-10

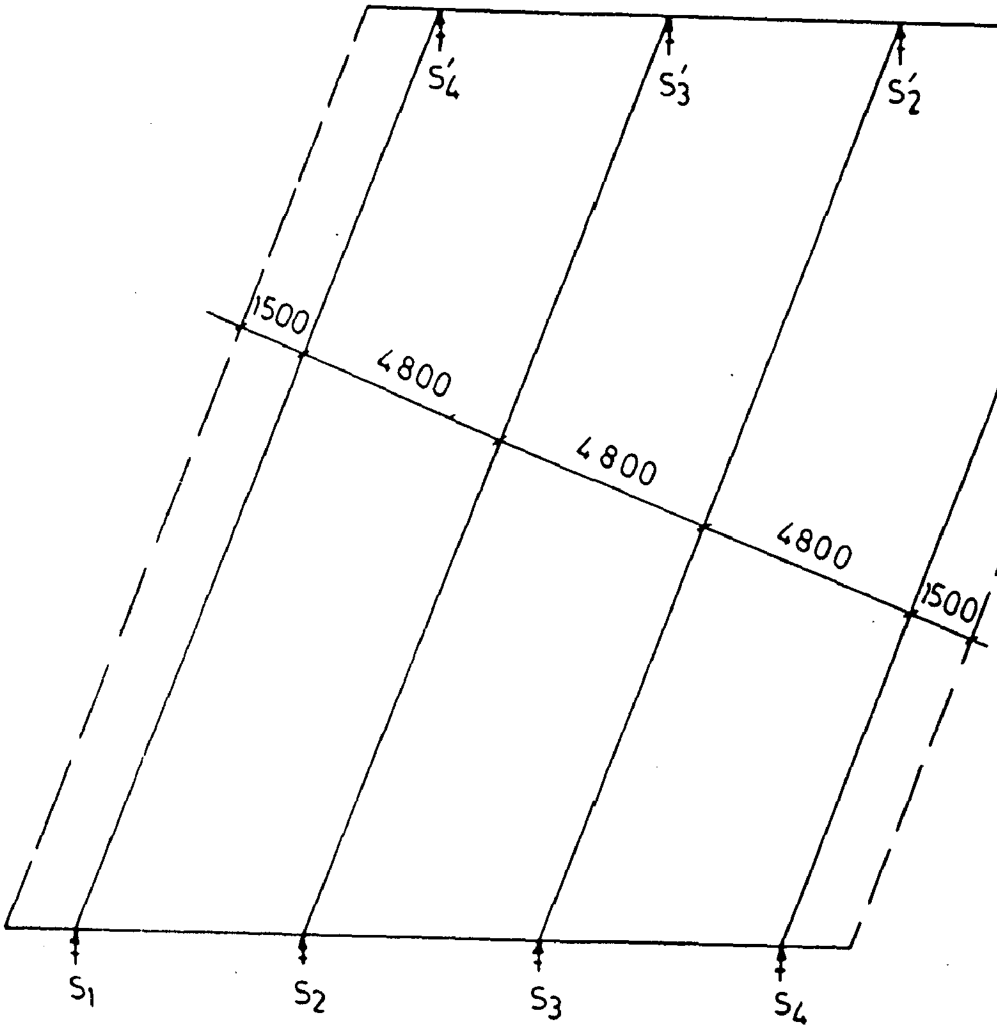


(a) - LINE PLAN OF BRIDGE



(b). MID SPAN BRIDGE CROSS-SECTION

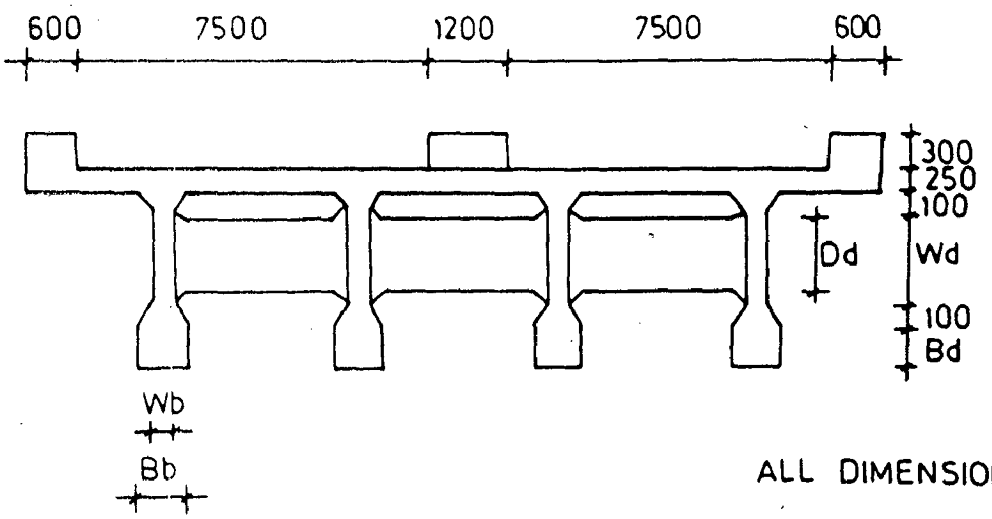
FIG. 4.1- 4-LANE, 3-GIRDER BRIDGE DECK



FOR CASE-A
Wd = 1500
Wb = 400
Bd = 400
Bb = 600
Dd = 1200

FOR CASE-B
Wd = 1150
Wb = 350
Bd = 300
Bb = 550
Db = 900

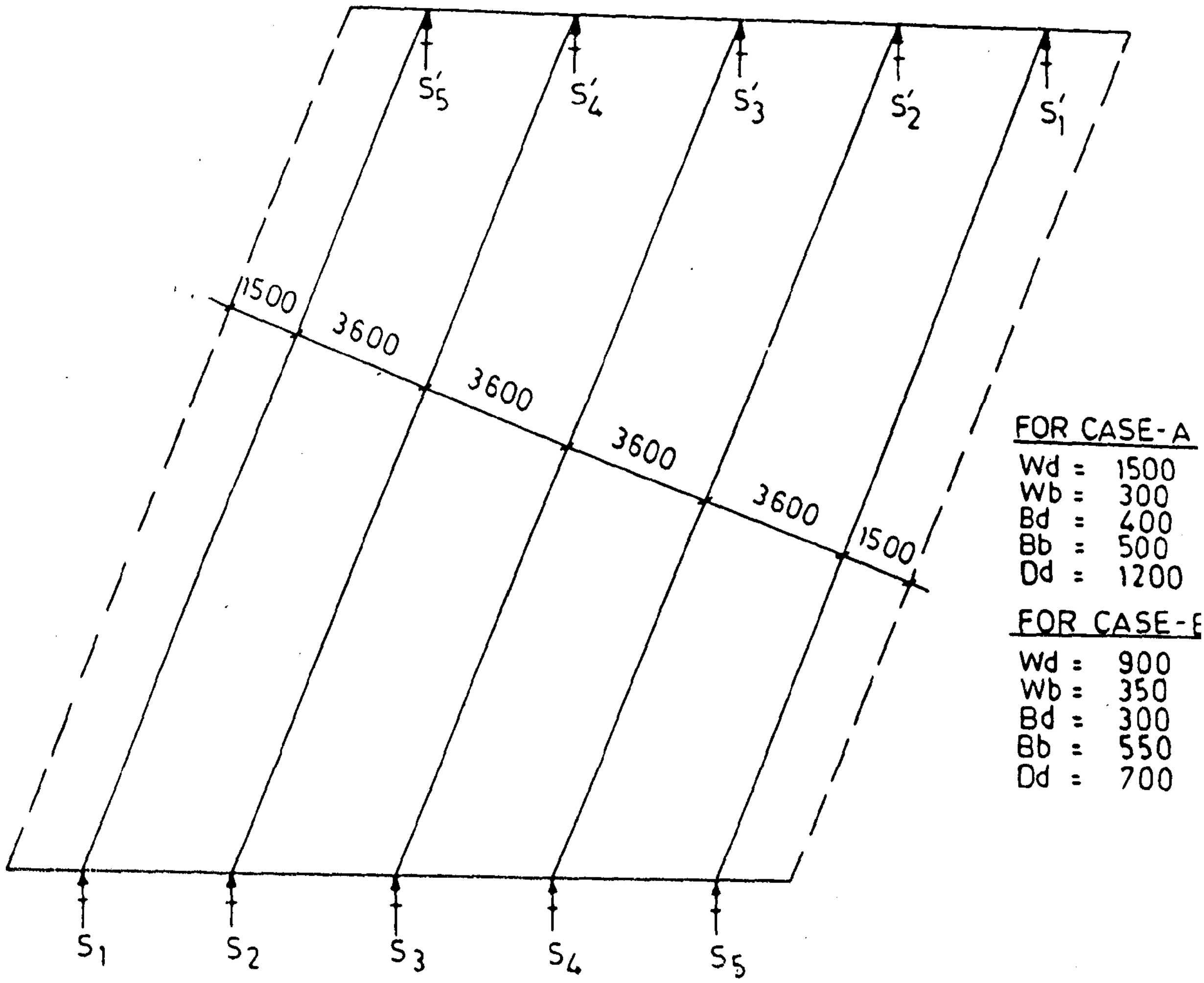
(a) - LINE PLAN OF BRIDGE



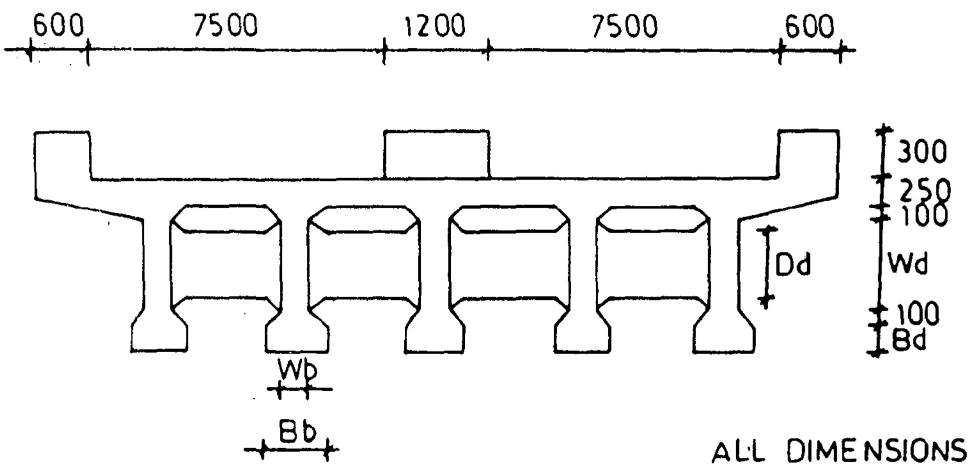
ALL DIMENSIONS IN MM

(b) - MID SPAN BRIDGE CROSS SECTION

FIG. 4.2- 4-LANE, 4-GIRDER BRIDGE DECK



(a) - LINE PLAN OF BRIDGE



ALL DIMENSIONS IN MM

(b) - MID SPAN BRIDGE CROSS SECTION

FIG. 4.3 - 4-LANE, 5-GIRDER BRIDGE DECK

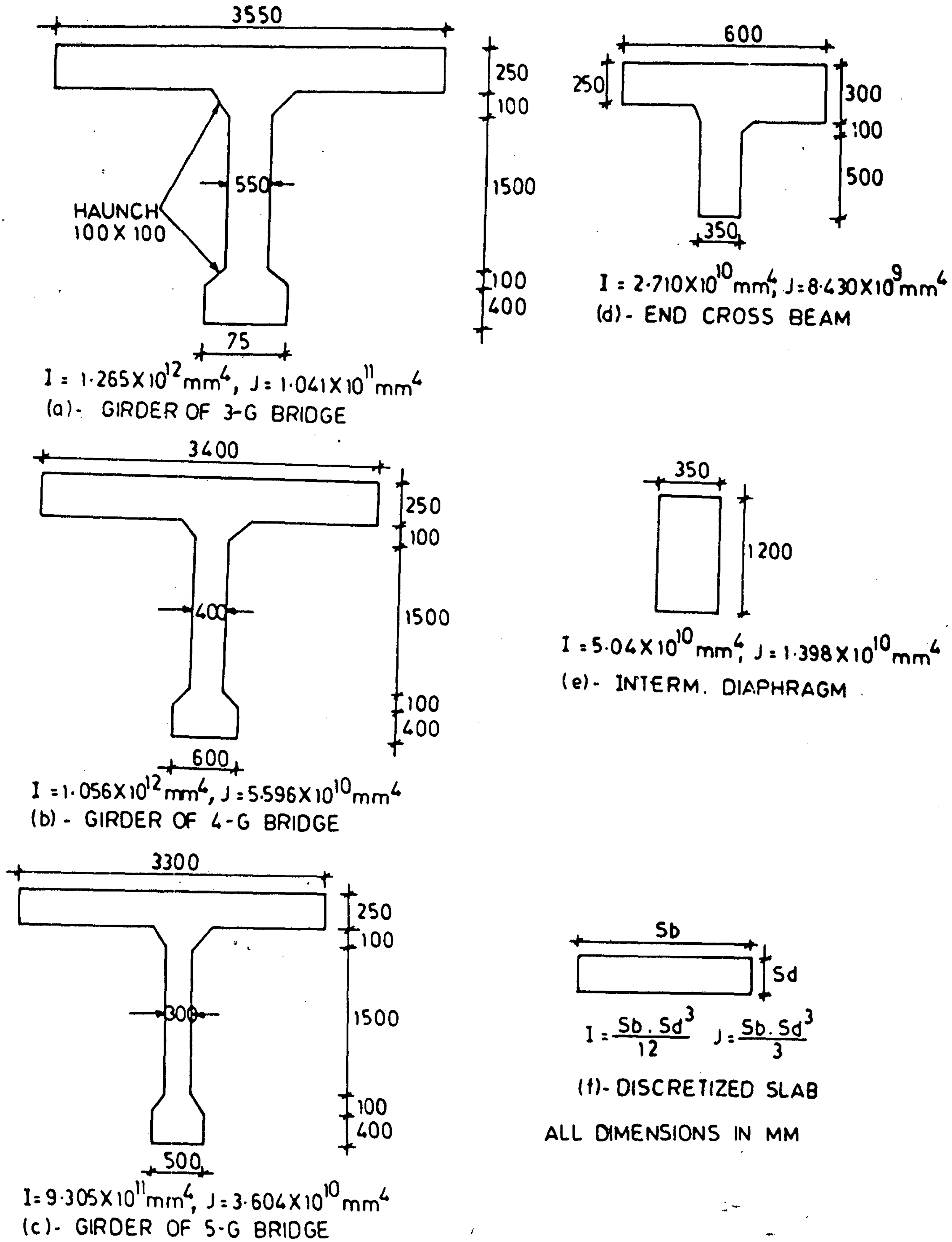


FIG.4.4-DECK STRUCTURAL ELEMENTS FOR CASE-A

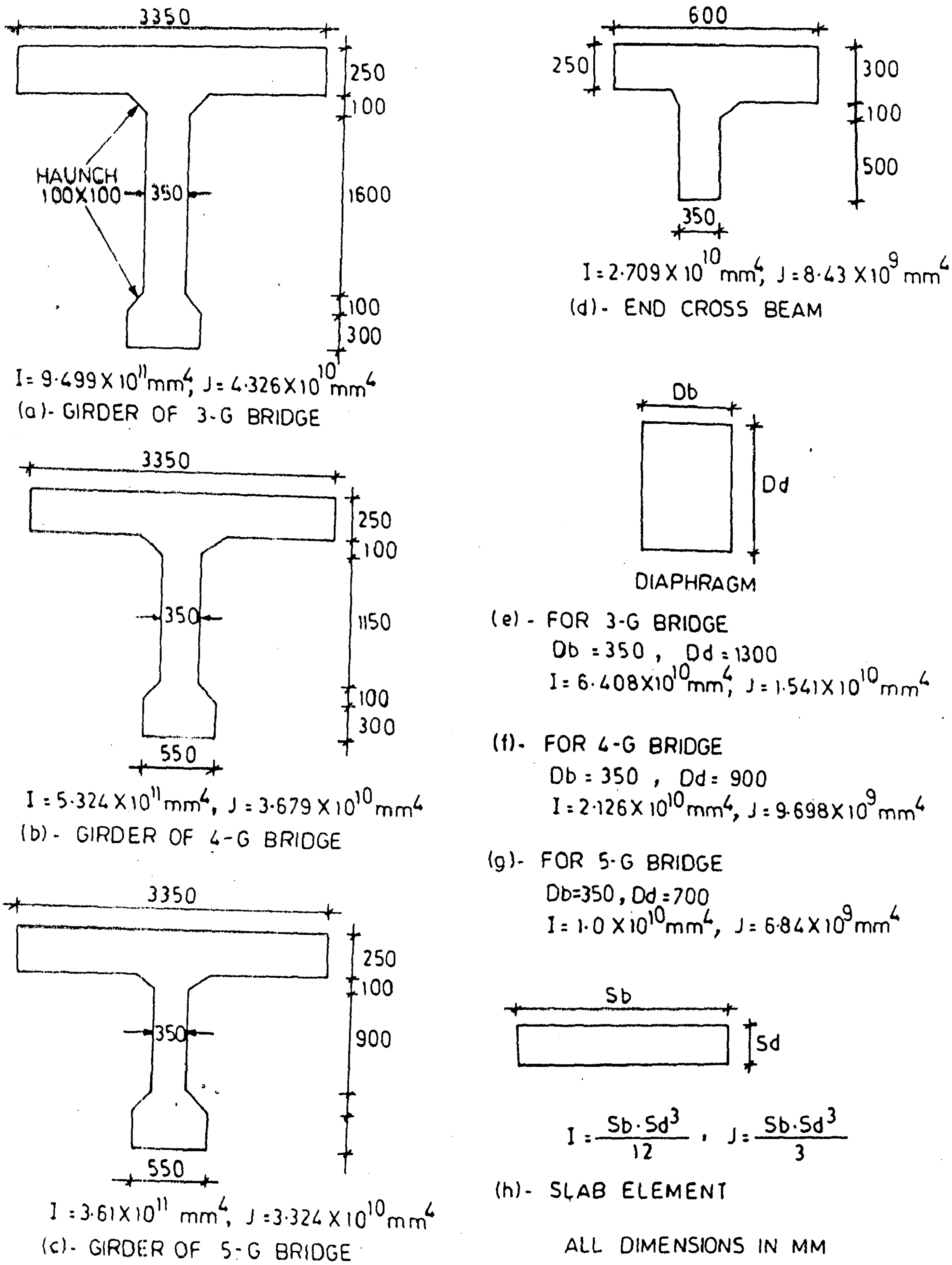
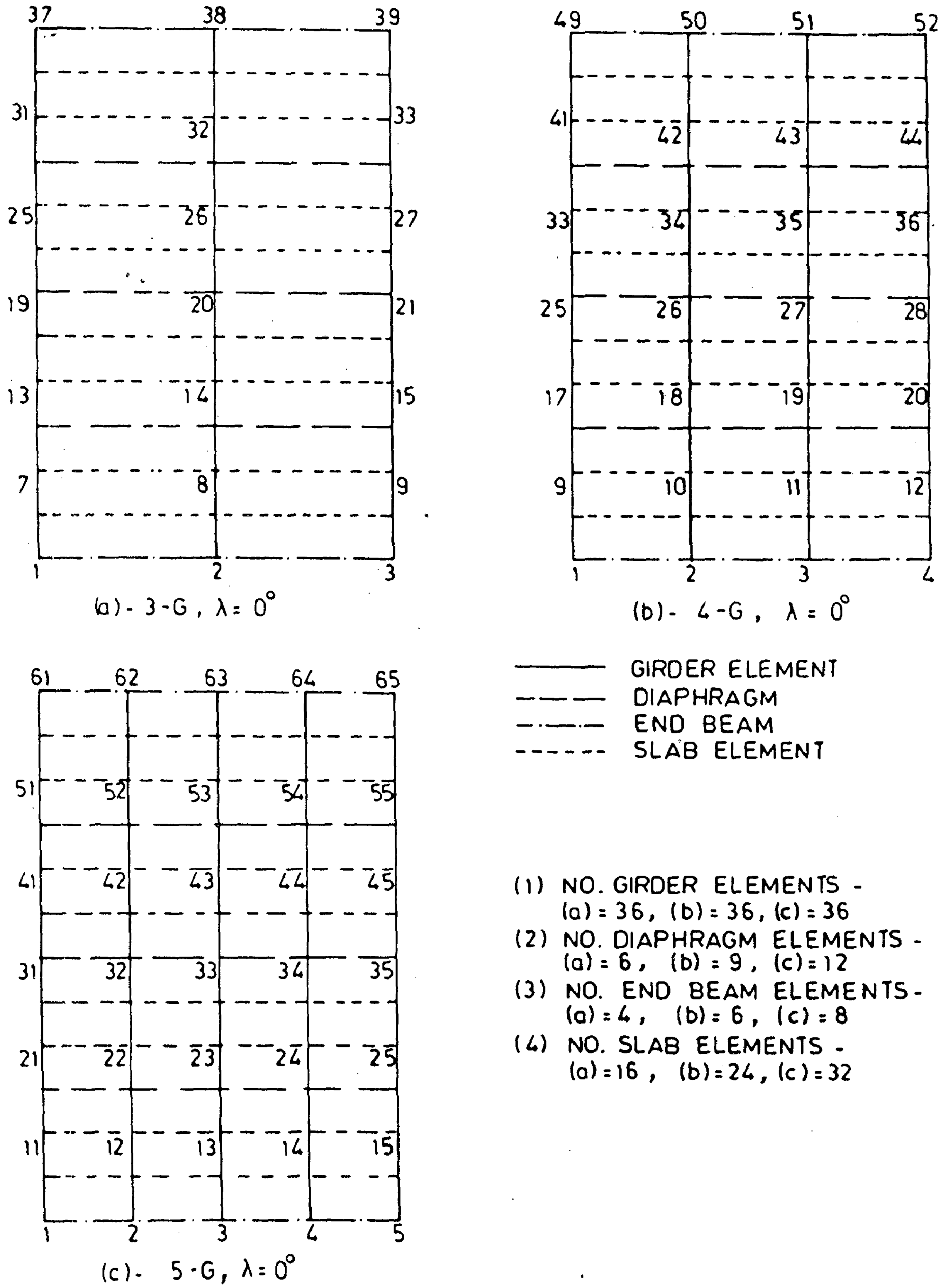


FIG. 4.5 - DECK STRUCTURAL ELEMENTS OF CASE - B



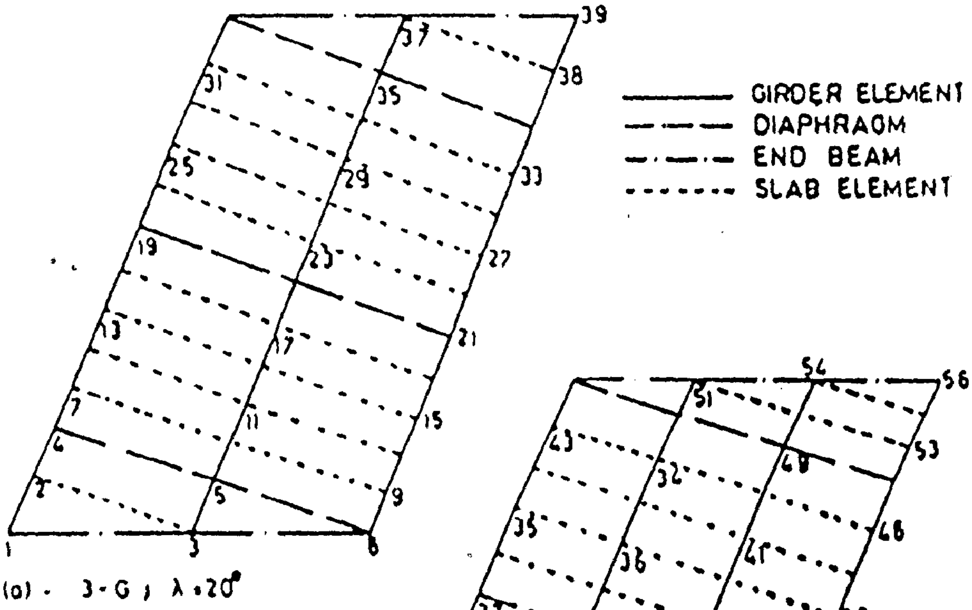
(1) NO. GIRDER ELEMENTS - (a) = 36, (b) = 36, (c) = 36

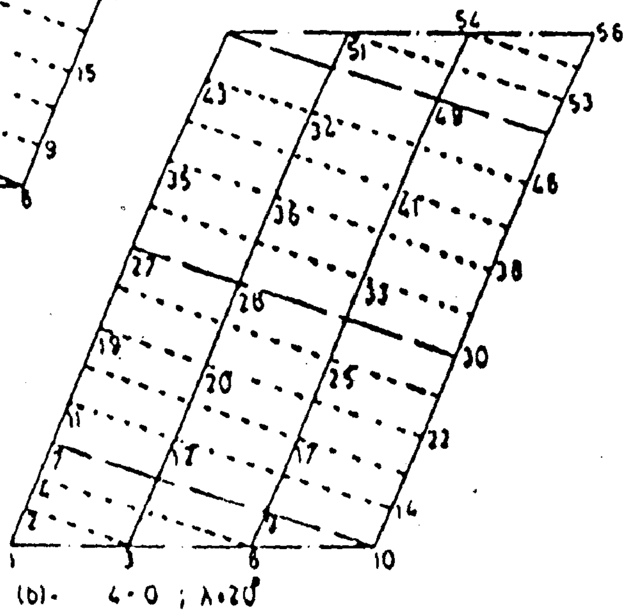
(2) NO. DIAPHRAGM ELEMENTS - (a) = 6, (b) = 9, (c) = 12

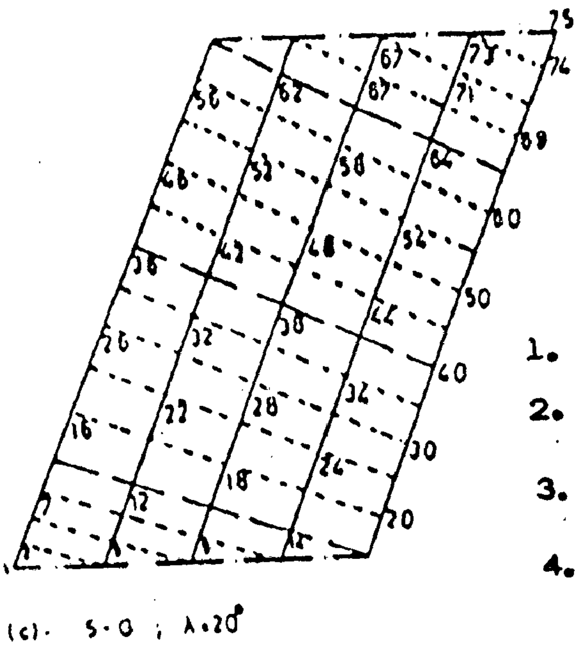
(3) NO. END BEAM ELEMENTS - (a) = 4, (b) = 6, (c) = 8

(4) NO. SLAB ELEMENTS - (a) = 16, (b) = 24, (c) = 32

FIG. 4·6 - DISCRETIZED PLAN OF RIGHT BRIDGES $\lambda = 0^\circ$







1. No. GIRDER ELEMENTS
(a) =36, (b)=39, (c)=70
2. No. DIAPHRAGM ELEMENTS
(a)=6, (b)=9, (c)=12
3. END BEAM ELEMENTS
(a)=4, (b)=6, (c)=8
4. NO. SLAB ELEMENTS
(a)=18, (b)=20, (c)=44

FIG.4.7. DISCRETIZED PLAN OF SKEW BRIDGES λ=20°

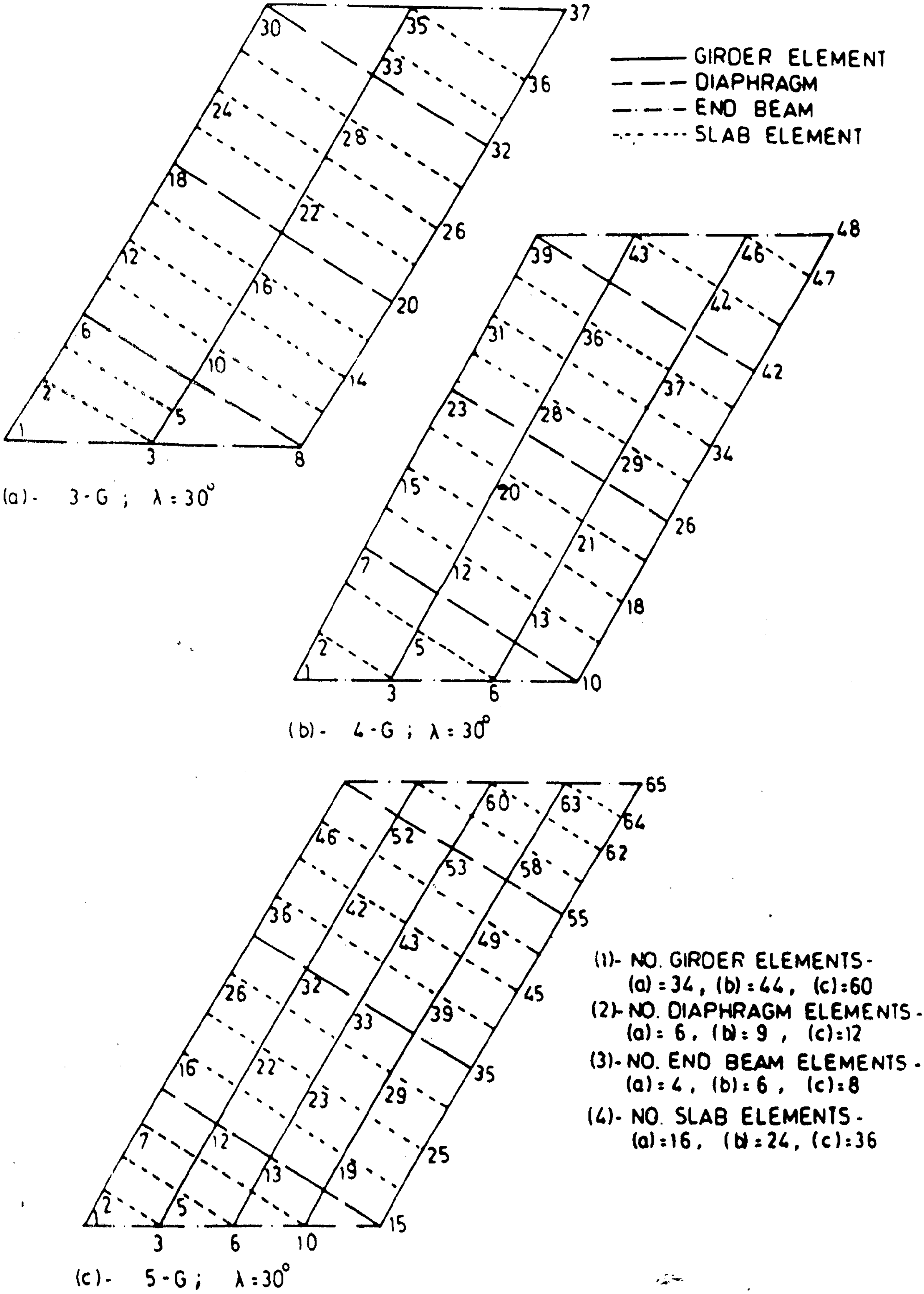
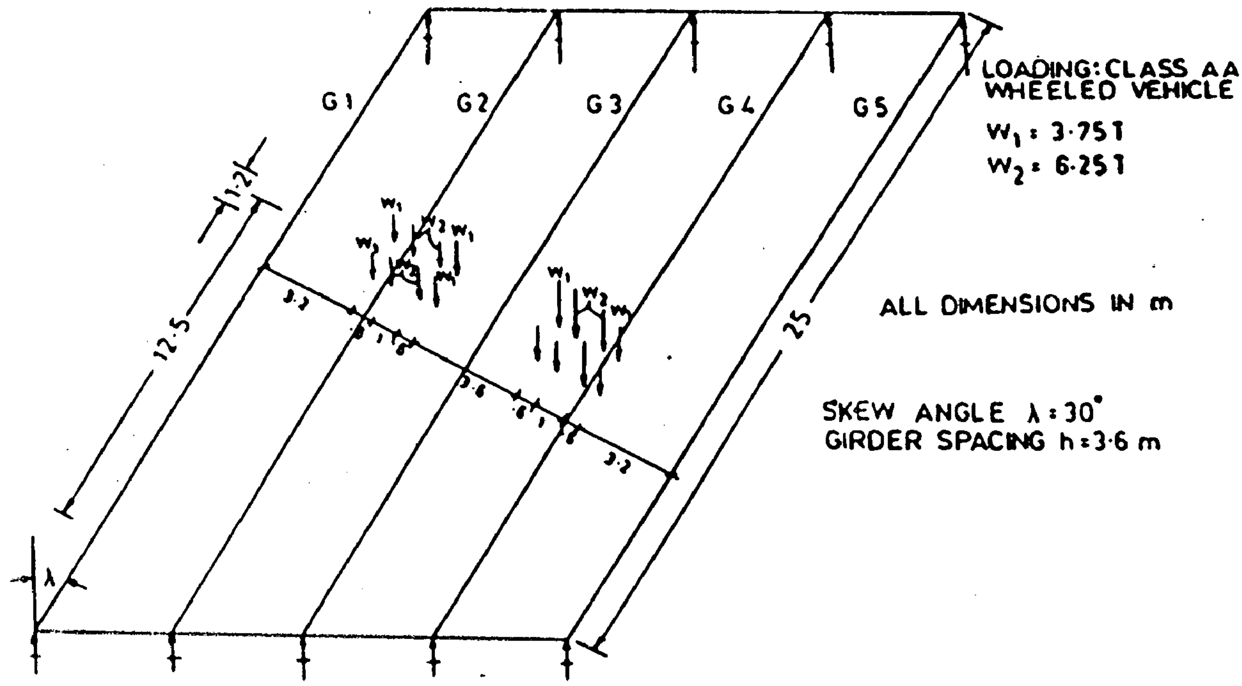
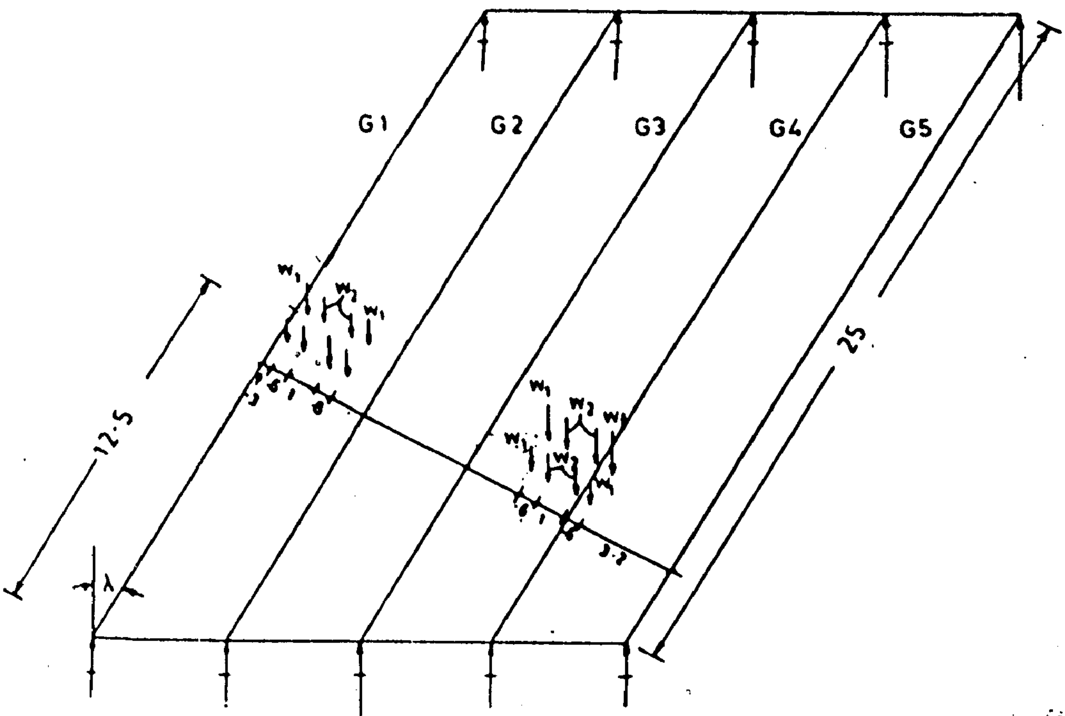


FIG. 4.8 - DISCRETIZED PLAN OF SKEW BRIDGES $\lambda = 30^{\circ}$

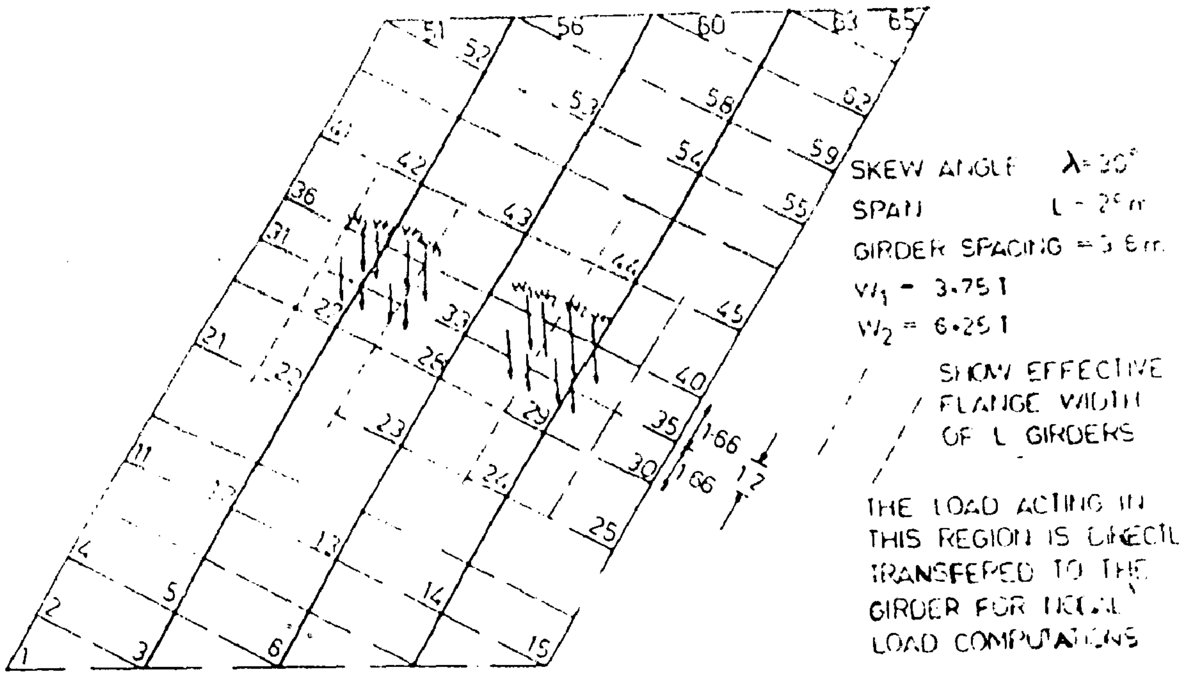


(a) CENTRAL LOADING FOR MAX. MOMENT IN INTERNAL GIRDERS



(b.)EXTREME LEFT LOADING FOR MAX. MOMENT IN EXTEREME LEFT GIRDER

FIG. 4.9 LOADING ON DECK



(a)– ACTUAL LOADING ON DECK

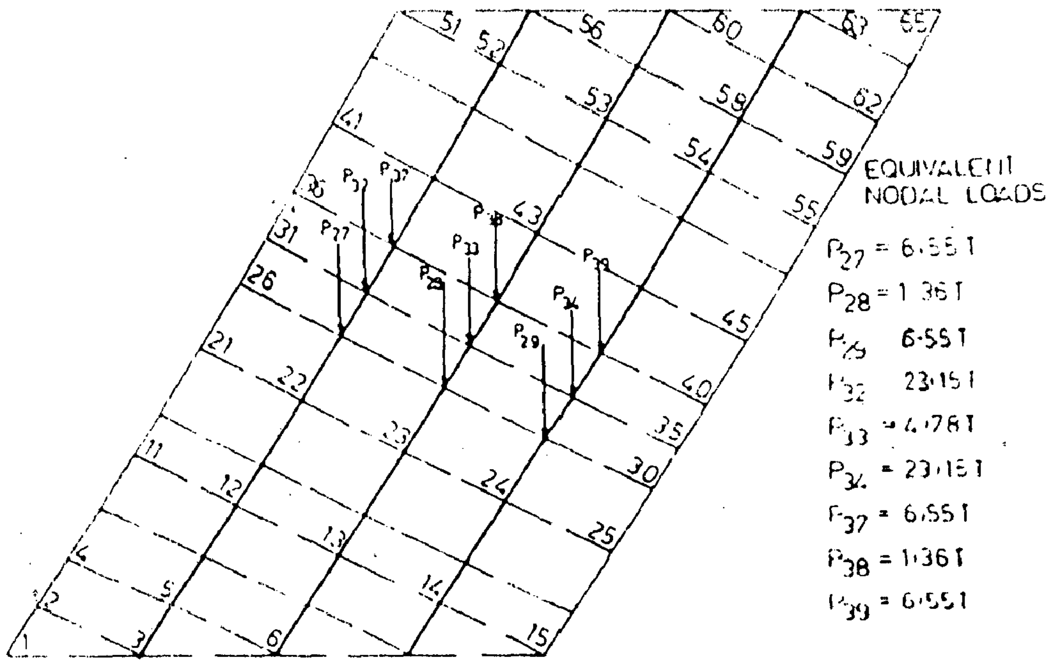


FIG 4.10(b)- EQUIVALENT NODAL LOADING FOR (a) ON DECK

C H A P T E R - 5

ANALYSIS AND RESULTS

5.1 Introduction

The eighteen bridge systems, nine each for case A and case B have been separately analysed for the two extreme load systems as discussed and presented in chapter 4. All the bridges have been loaded with the same load and load configurtation (IRC class AA wheeled vehicle), and transverse load positions on the deck slab. Thus the load system and its positions remain unchanged on the deck plan for all the bridges, although the locations of the girders change with respect to the loading in 3,4 and 5 girder bridges. Actual loads have been transferred to the nodes by computing equivalent nodal loads as discussed in chapter 4, with the increase in number of nodes as the number of girders increases, the same planer deck is discretized into increasingly fine gridwork.resulting in better nodal load computations and structural simulation. Interpretation of results, therefore, should be done keeping these factors in mind.

5.2 Analysis

Computer software used for the analysis of the bridges is based on the stiffness method of structural analysis as presented in chapter 3. The bridge is structurally simulated as planar grid. All the nodes of the grid have three degrees of freedom except the ones externally supported. Vertical deflections and element rotations about the two orthogonal member - axes are the permitted nodal displacements. For the computed nodal loads (vertical and / or moment about any of the two axes), the computer program prints out the three element forces (shear, moment and torsion) and the three nodal displacements. Each print out of the input data helps in proper identification of the bridge and loading and also in keeping record.

For the relevant study purpose here-in, bending moment, shear force and deflection print- out for the girders only are further processed to arrive at certain desired conclusions. In addition to the above, support reactions have also been studied. For a particular bridge, the girder carrying the maximum moment has been chosen for the comparative study . This has been done from the design point of view wherein the girder carrying the maximum amount governs the design. Thus, there are three graphs for bending moment, one each for the 3,4 and 5 - girder bridge systems having the same skew angle. It is pertinent to note herein that the girder carrying the maximum moment in 3,4 or 5 girder systems need not be the same. Shear force and deflection diagrams have also been plotted for the girder carrying maximum moment.

5.2.1 Girder Moment

Bending moment diagrams for the girder carrying maximum moment in the 3,4 and 5 girder systems are depicted in Fig 5.1 through Fig- 5.6 for the two load positions (central and extreme left) of all the eighteen bridge systems. For case A study, girder moment diagrams are shown in Fig 5.1 for $\lambda = 0°$ (right bridge); in Fig.5.2 for $\lambda = 20°$ and in Fig. 5.3 for $\lambda = 30°$. A close study of these six figures indicate that the transverse load distribution improves as the number of girders increases from three two five. With better load distributions, the value of maximum bending moment decreases. The decrease in bending moment values is quite pronounced in the central loading position compared to extreme left loading position. The decrease in moment values is as much as 47 percent (Fig. 5.1 a) from 3- girder system to 5-girder system . The maximum value of moment is observed in girder G-2 in case of central loading and in girder G-1 in case of extreme left loading. Again, the maximum moment values are also observed to remain almost unchanged as the skew angle increases from $\lambda = 0°$ tø $\lambda = 30°$.

The maximum value of girder moment is around 2.25 x 10^5 Kg.m for 5 - girder system, for the entire range of skew angle in excess of zero upto 30° . It may also be noted that the 4 - girder system shows only marginal improvement in moment value over the 3-girder system, where as the 5-girder system results in significantly large improvement. This is primarily due to better load distribution amongst five rather closely spaced girders compared to three girders spaced apart along the same 4- lane width with a central verge. From this observation, it becomes apparant that using five girders is more beneficial than three girders for the type of bridge systems considered.

Girder moment diagrams for case B study are presented in Fig. 5.4 through Fig. 5.6. observations made for case A above are seen to hold good in this case also with small variations in actual values. Maximum value of girder moment is observed to be around 2.1 x 10^5 Kg.m for 3-girder system and around 1.2 x 10^5 Kg.m for 5-girder, system for skew angle upto 20°. However, the 5-girder, 30° skew system shows marked reduction of 51 percent in maximum moment value (1.0 x 10^5 Kg.m, Fig. 5.6 (a)). Once again, the bridges respond more sensitively to central loading position compared to extreme left loading positions.

In a comparative study of case B against case A solely based on tbe maximum girder moment values,it may be safely inferred that the 5-girder system of case A is the most advantageous system of them all.

5.2.2 Deflection

As done in girder moment study, the girder deflection curves have been plotted for the girder showing maximum deflection. Referring to diagrams, Fig. 5.7 through Fig 5.9 depict the girder deflection curves for λ=0°, 20° and 30° respectively for case A study like wise Fig. 5.10 through Fig. 5.12 show the deflection curves for the three bridges in case B study.

A careful study of the case A deflection curves reveal that the 5- girder system undergoes much less deflection compared to the 3-girder or 4- girder system. The percentage reductions in the maximum deflection value is observed to be as much as 32 in Fig. 5.9 a. This is in consonance with our observatio about the maximum bending moment values. Thus, it may once again be inferred that a 5- girder system is structurally more efficient for a 4- lane bridge system with skew angle up to 30°.

In case B study, the situation is completely different. Girder deflection increases with the increase in the number of girders presenting a situation just the opposite of case A. It may be keenly noted, that although the moment values decrease with the increase in the number of girders, the deflection values increase. This increase in the maximum deflection value is observed to be as much as 18 percent (Fig. 5.12 a).

From the comparative study of case A and case B for girder moment and deflection, it becomes apparent that the 5 - girder system of case A is structurally the most efficient system.

5.2.3. Shearing Force

Shear force diagrams for all the eighteen bridges have been shown from Fig. 5.13 to Fig. 5.18. Fig. 5.13, Fig. 5.14 and Fig.5.15 correspond to the bridge systems with $\lambda = 0°$, $\lambda = 20°$ and $\lambda = 30°$ respectively for case A while Fig. 5.16, Fig.5.17 and Fig. 5.18. correspond to case B for bridge systems with $\lambda = 0°$, $\lambda = 20°$ and $\lambda = 30°$ respectively.

It is clear from the diagrams that shear force is maximum in 3 - girder bridge and minimum in 5 - girder bridge in all the cases of bridges and loadings. The maximum shearing force comes in central loading which occurs in 3 - girder bridge. The reduction in shear force is substantial in 5 - girder bridge from the maximum value compared to that in 4- girder bridge. There is no appreciable reduction in shear force in the extreme left loading. The central loading cases are improtant as it gives higher values.

We observe that at a few places, there is noticeable increase in shear force and even the value of shear force becomes greater than that at the support. It is because of the fact that the cross- members may act in both ways, it may load a girder or relieve its load. In the former case it transfers other girder's load to this girder and in the latter case, it transfers this girder's load to the adjacent girder. While relieving load the cross member acts as a flexible support and provides upward, reaction resulting in an increase in the shear force.

Shearing force is observed to be the minimum for the 5- girder system both in case A as well as in case B. This clearly indicates that the imposed loads get dispersed amongst the girders more uniformly in 5- girder system than in any other system. This further supports the higher structural efficiency of the 5- girder system of case of case A in comparision with the 3 and 4 - girder systems.

5.2.4. Support Reaction

Girder support reactions are presented in Table 5.1 through Table 5.2 for case A and case B. The first two tables are for case A study while the latter two are for case B. Support reaction values are given as percentage of the imposed loading and hence for each bridge the sum total is 100 percent. The symbol adopted for support reaction is sij wherein the first subscript 'i' represents the girder number and the second subscript 'j' represents the near end (j=1) or a far end (j=2).

A close study of Table 5.1 (a) and Table 5.1(b) indicates that support reactions decrease with the increase in the number of girders. It is also seen that the maximum values differ from the minimum values much more in 3-girder systems than in 5-girder systems. It may, thus, necessitate separate design for support bearings in case of 3- girder systems. The reaction values being close to each other in case of 5-girder systems, the bearing designed for the maximum support reaction could justifiably be adopted for the remaining supports. Not only it would eliminate the need of various capacity bearings, but performance of the bearings would also be much better since they would all be subjected to more or less the same structural requirements. It is pertinent here in to mention that it is usual to provide the same common bearings for all the supports in actual practice. Study of Table 5.2 leads to the similar conclusions as given just above.

Based on the overall discussions and inferences presented above, it can safely be concluded that the 5-girder system of case A is structurally most efficient of all the eighteen bridges considered. For the same transverse medium, transforming a 3-girder system into a 5-girder system is structurally more effective when the material is removed from the sides of the girder webs (case A) than from the depths (case B)

SUPPORT REACTIOINS FOR CASE-A
(AS PERCENTAGE OF TOTAL LOAD APPLIED)

TABLE 5.1(a) (CENTRAL LOADING)

BRIDGE		S11	S12	S21	S22	S31	S32	S41	S42	S51	S52
λ=0	3G	12.9	12.9	24.1	24.1	12.9	12.9	-	-	-	-
	4G	5.7	5.7	19.3	19.3	19.3	19.3	5.7	5.7	-	-
	5G	6.2	6.2	11.4	11.4	14.7	14.7	11.4	11.4	6.2	6.2
λ=20	3G	9.7	15.6	24.6	24.6	15.6	9.7	-	-	-	-
	4G	5.2	4.1	15.2	24.8	29.1	12.5	3.9	4.9	-	-
	5G	5.6	5.2	8	16.7	14.4	14.4	16.7	8	5.2	5.6
λ=30	3G	8.3	18.1	23.5	23.5	18.1	8.3	-	-	-	-
	4G	5	5.5	14.4	23.8	31.4	10.3	4.9	4.5	-	-
	5G	5.1	5.1	7.3	6.2	13.4	17.9	6.2	13.4	5.1	7.3

TABLE 5.1(b) (EXTREME LEFT LOADING)

BRIDGE		S11	S12	S21	S22	S31	S32	S41	S42	S51	S52
λ=0	3G	21.7	12.7	17.8	17.8	10.5	10.5	-	-	-	-
	4G	17	17	15.3	15.3	12.7	12.7	4.9	4.9	-	-
	5G	14	14	10.9	10.9	10.8	10.8	8.3	8.3	6.0	6.0
λ=20	3G	20.9	22.7	22.3	12.9	14.8	6.3	-	-	-	-
	4G	16.7	17.6	15.1	13.6	21.1	7.5	4.7	3.7	-	-
	5G	13.7	4.3	12.3	4	10.2	9.9	16.2	7.7	6	15.3
λ=30	3G	20.7	23.3	24.7	9.7	17.5	4	-	-	-	-
	4G	16.8	18.3	15	11	25.8	5.5	4.7	2.7	-	-
	5G	12.2	3	12.3	3.3	12	8	22.06	7.3	5.6	14.1

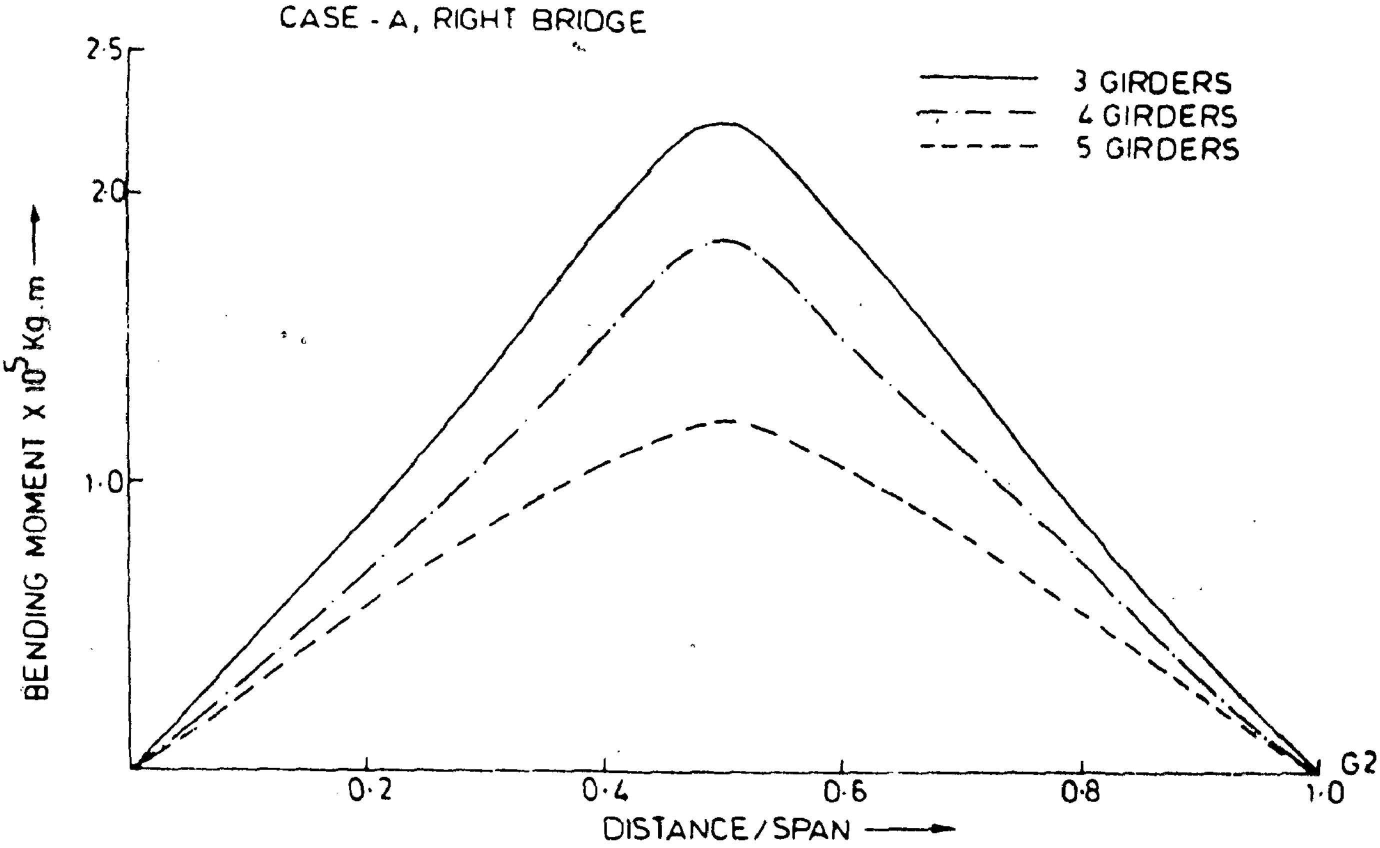
SUPPORT REACTIONS FOR CASE -B
(AS PERCENTAGE OF TOTAL LOAD APPLIED)

TABLE 5.2(a) (CENTRAL LOADING)

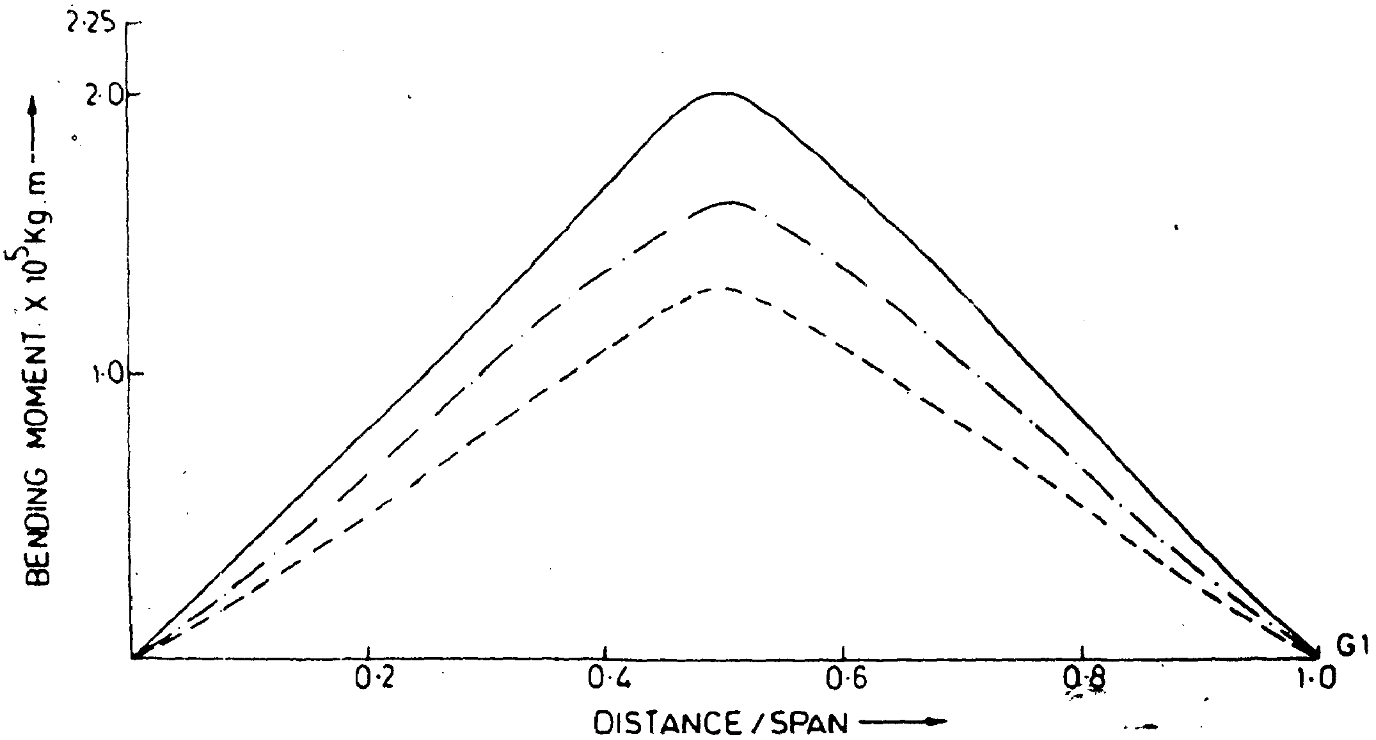
BRIDGE		S11	S12	S21	S22	S31	S32	S41	S42	S51	S52
λ=0	3G	13.9	13.9	22	22	13.9	13.9	-	-	-	-
	4G	8.1	3.1	16.9	16.9	16.9	16.9	3.1	8.1	-	-
	5G	5.1	5.1	12.6	12.6	14.6	14.6	12.6	12.6	5.1	5.1
λ=20	3G	10.5	16.8	22.6	22.6	16.8	10.5	-	-	-	-
	4G	5	4.2	15.5	24.7	28.9	12.8	4	4.7	-	-
	5G	4.6	5	9.2	16.7	14	14	16.7	9.2	5	4.6
λ=30	3G	9.3	19.6	21.1	21.1	19.6	19.3	-	-	-	-
	4G	4.9	5.4	14.3	24	31.8	10.3	4.8	4.3	-	-
	5G	4.1	5.8	8	18.4	13.5	13.5	18.4	8	5.8	4.1

TABLE 5.2(b) (EXTREME LEFT LOADING)

BRIDGE		S11	S12	S21	S22	S31	S32	S41	S42	S51	S52
λ=0	3G	21.8	21.8	17.4	17.4	10.6	10.6	-	-	-	-
	4G	17.7	17.7	14.4	14.4	12.5	12.5	5.3	5.3	-	-
	5G	14.5	14.5	9.9	9.9	10.5	10.5	9.7	9.7	5.3	5.3
λ=20	3G	21	23	22.2	12.1	14.8	6.7	-	-	-	-
	4G	16.7	17.9	14.8	13	21.5	7.8	4.6	3.5	-	-
	5G	13.8	16	11.7	7	9.8	9.7	17.4	5	5.7	3.7
λ=30	3G	20.8	23.8	24.5	8.8	17.6	4.5	-	-	-	-
	4G	16.7	18.8	14.6	10.5	24.5	5.7	4.6	2.6	-	-
	5G	11.7	12.5	12.6	3.7	11	8.1	23.8	6.7	5	14.8



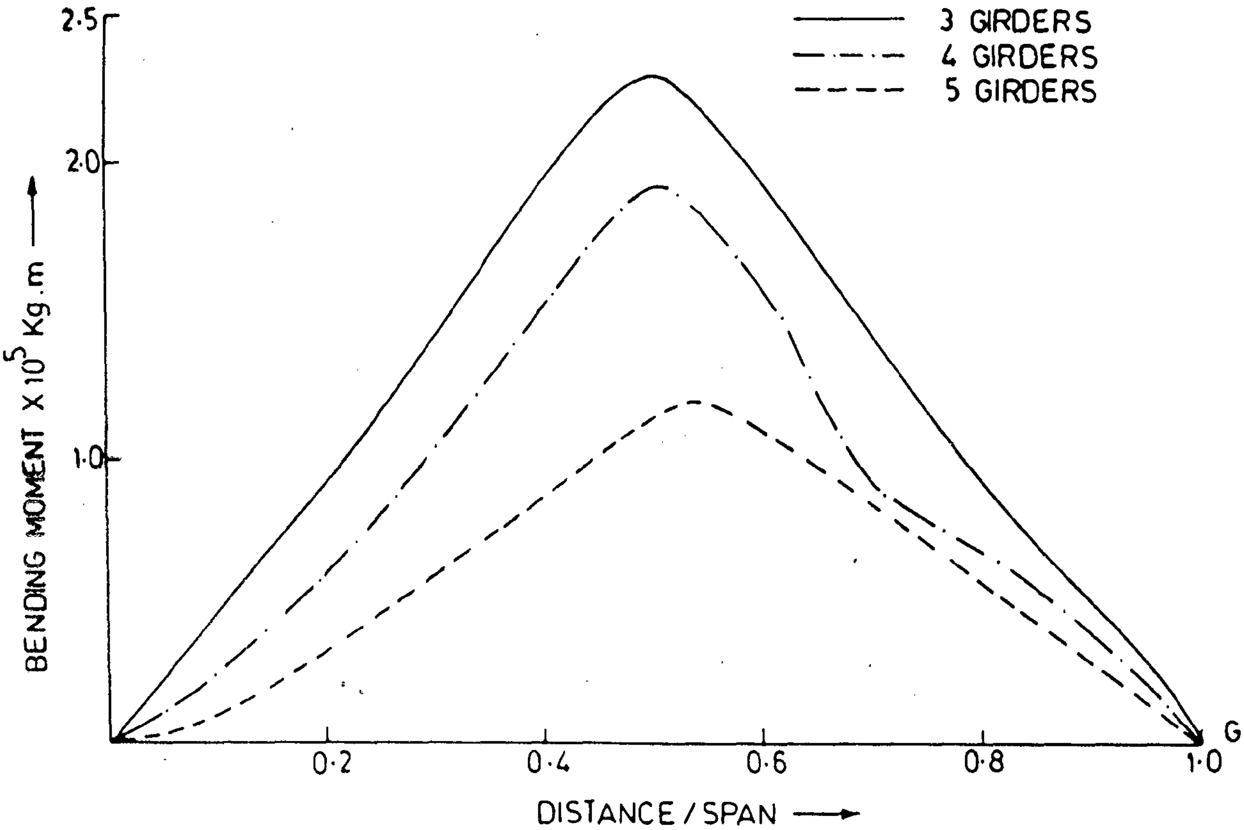
(a) - CENTRAL LOADING



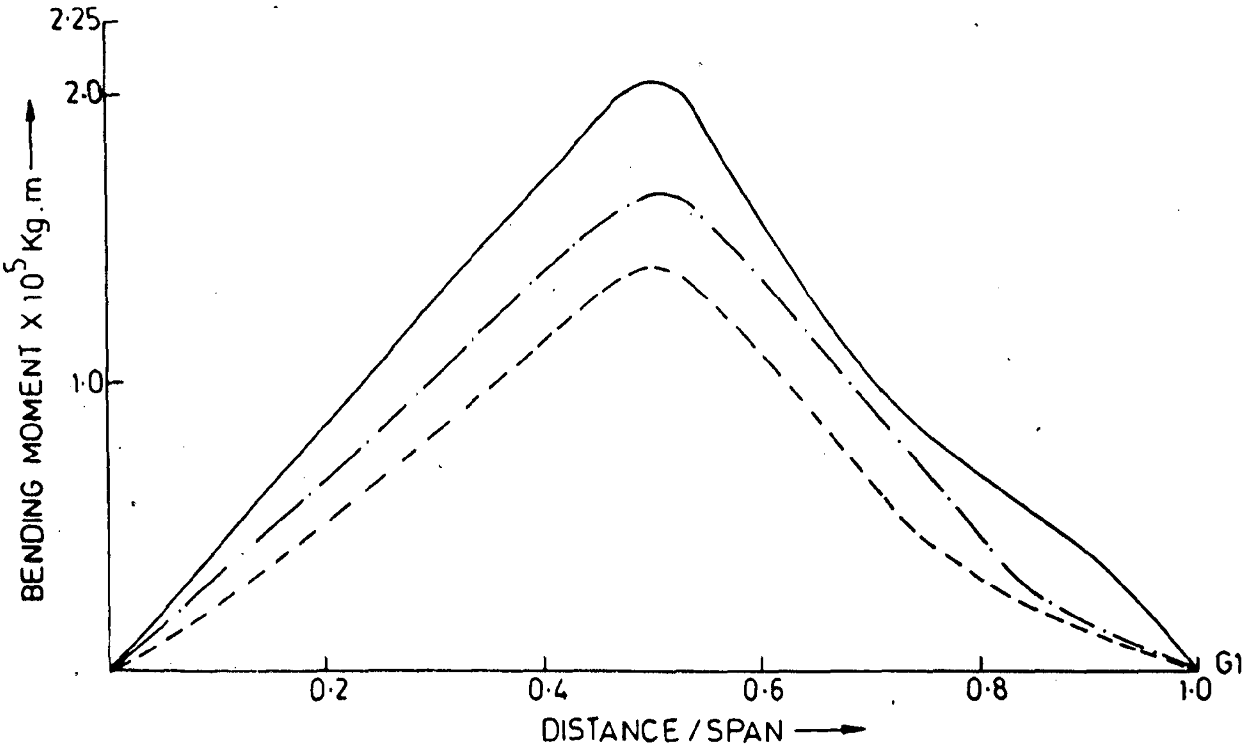
(b) - EXTREME LEFT LOADING

FIG. 5·1-GIRDER BENDING MOMENT

CASE-A, 20° SKEW BRIDGE



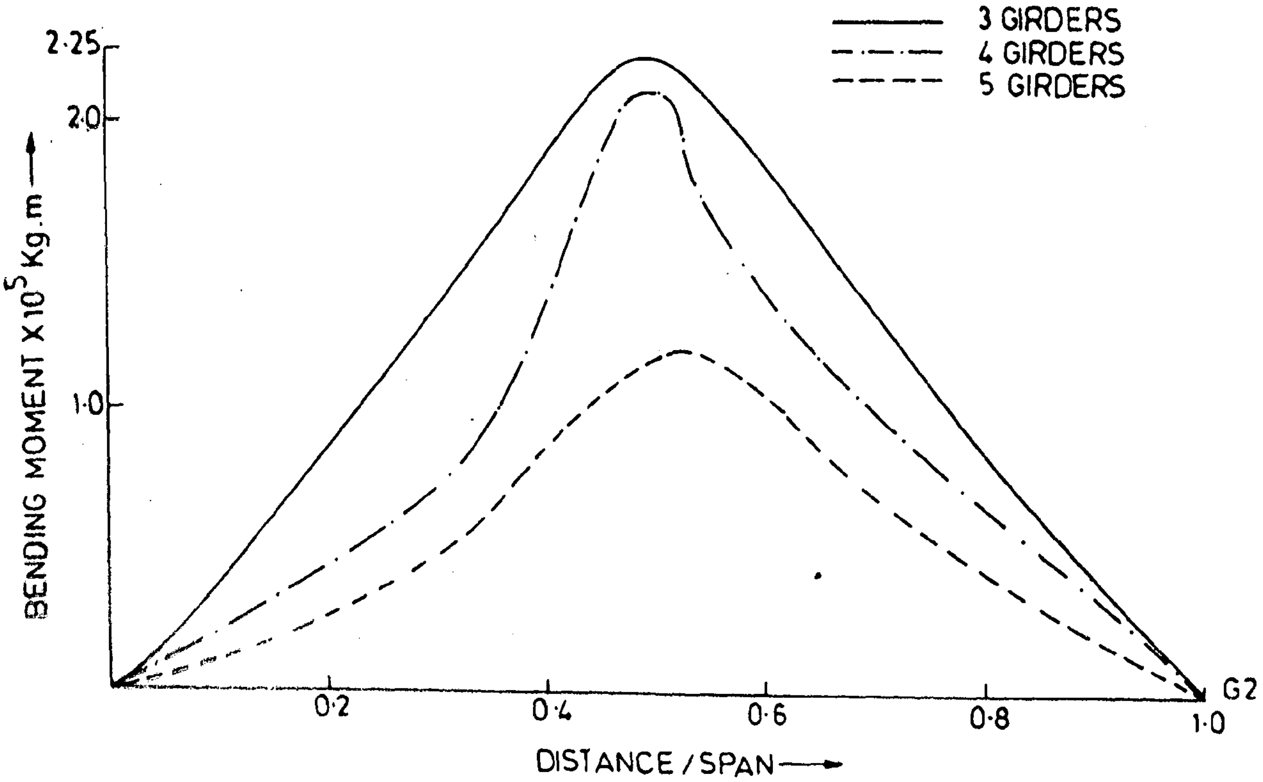
(a) - CENTRAL LOADING



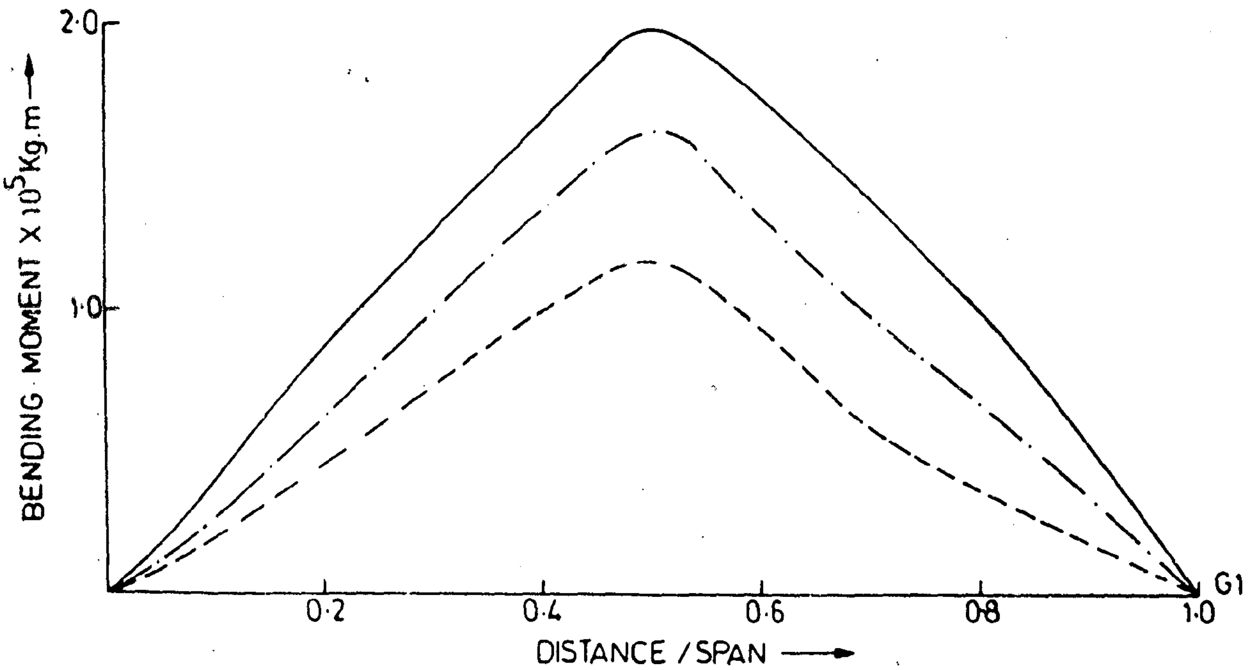
(b) - EXTREME LEFT LOADING

FIG. 5·2 - GIRDER BENDING MOMENT

CASE-A 30°, SKEW BRIDGE

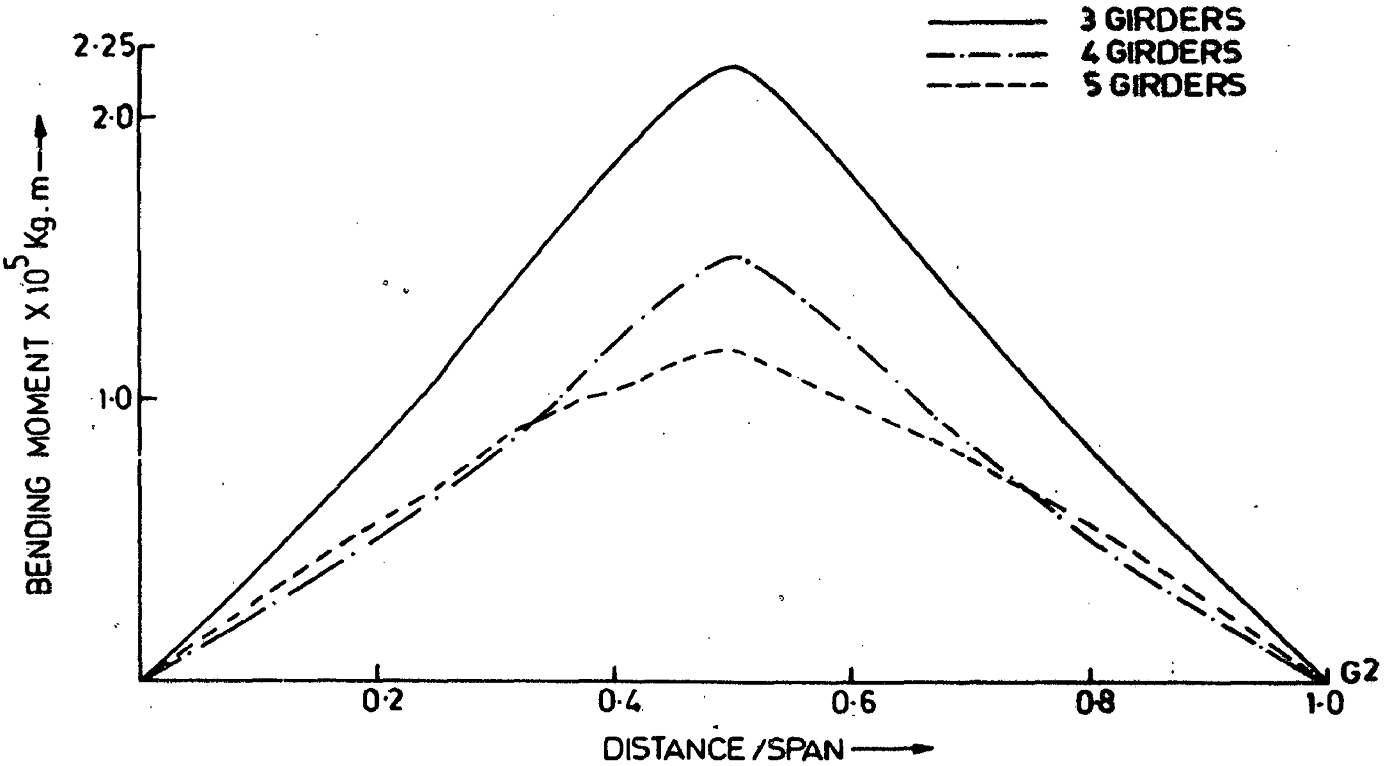


(a) - CENTRAL LOADING

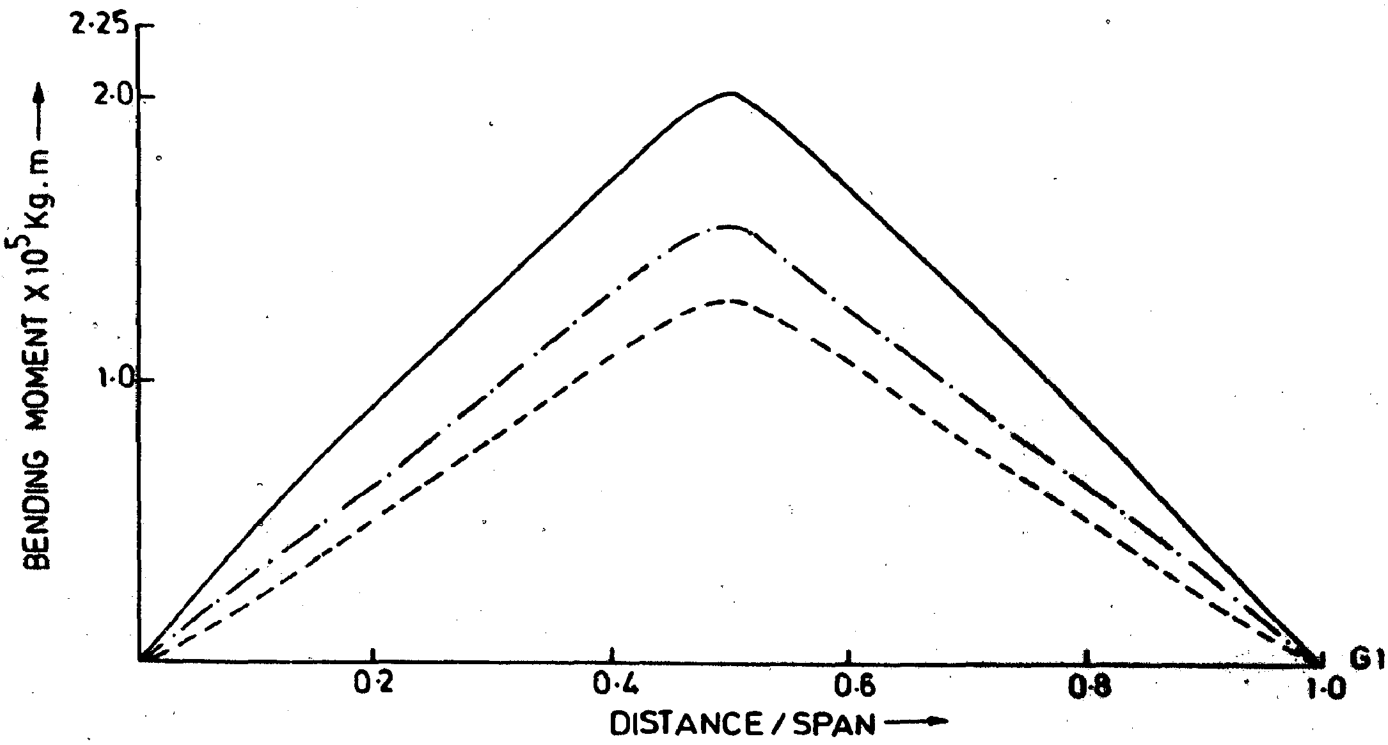


(b) - EXTREME LEFT LOADING

FIG. 5.3 - GIRDER BENDING MOMENT



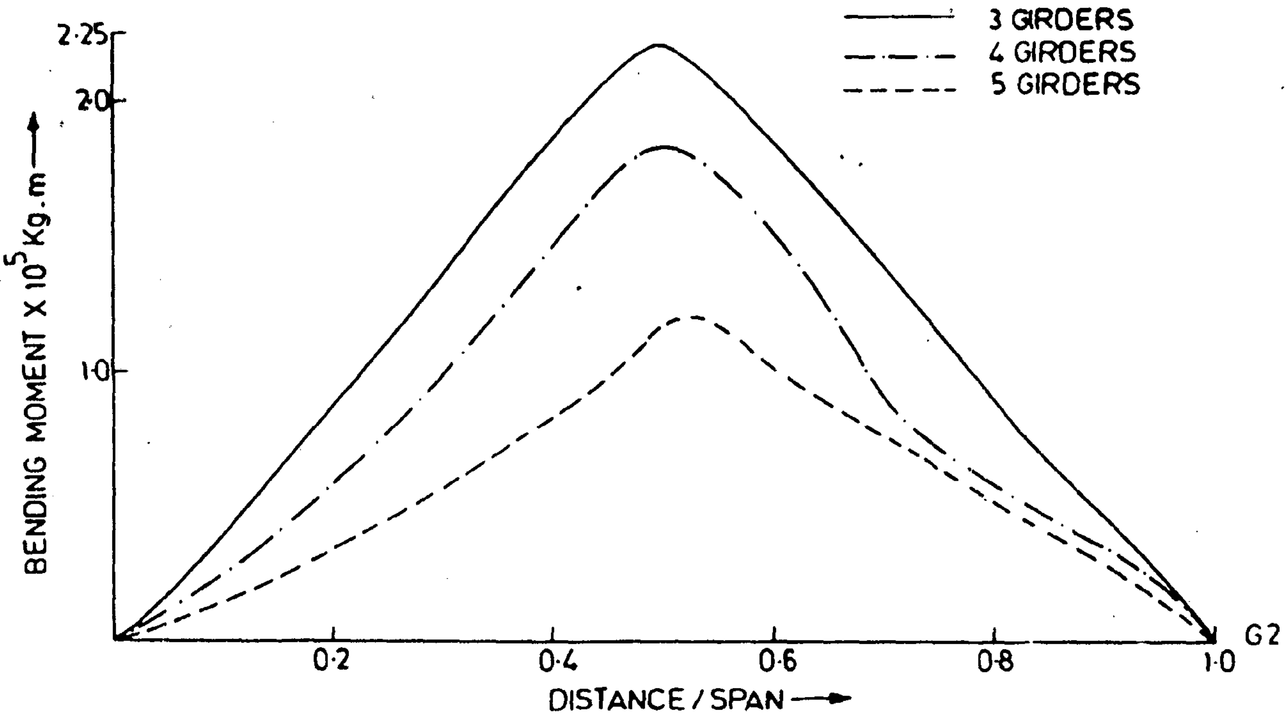
(a) - CENTRAL LOADING



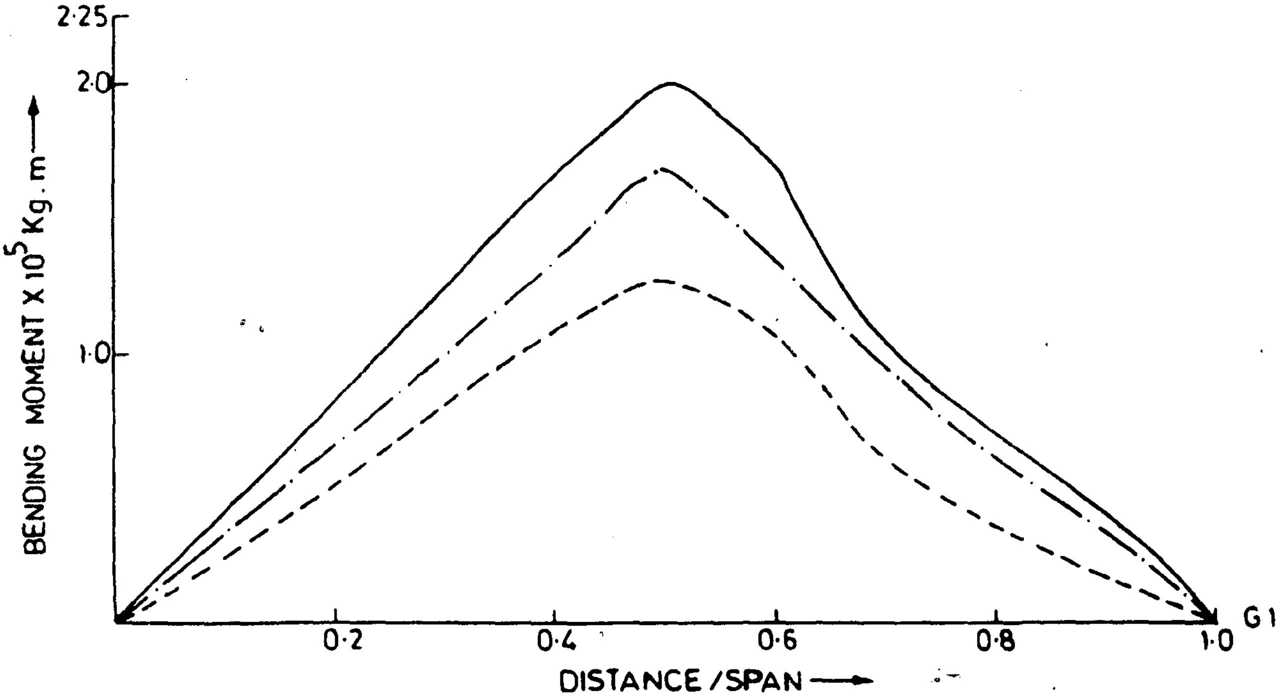
(b) - EXTREME LEFT LOADING

FIG. 5·4-GIRDER BENDING MOMENT

CASE - B, 20° SKEW BRIDGE



(a) - CENTRAL LOADING



(b) - EXTREME LEFT LOADING

FIG. 5·5 - GIRDER BENDING MOMENT

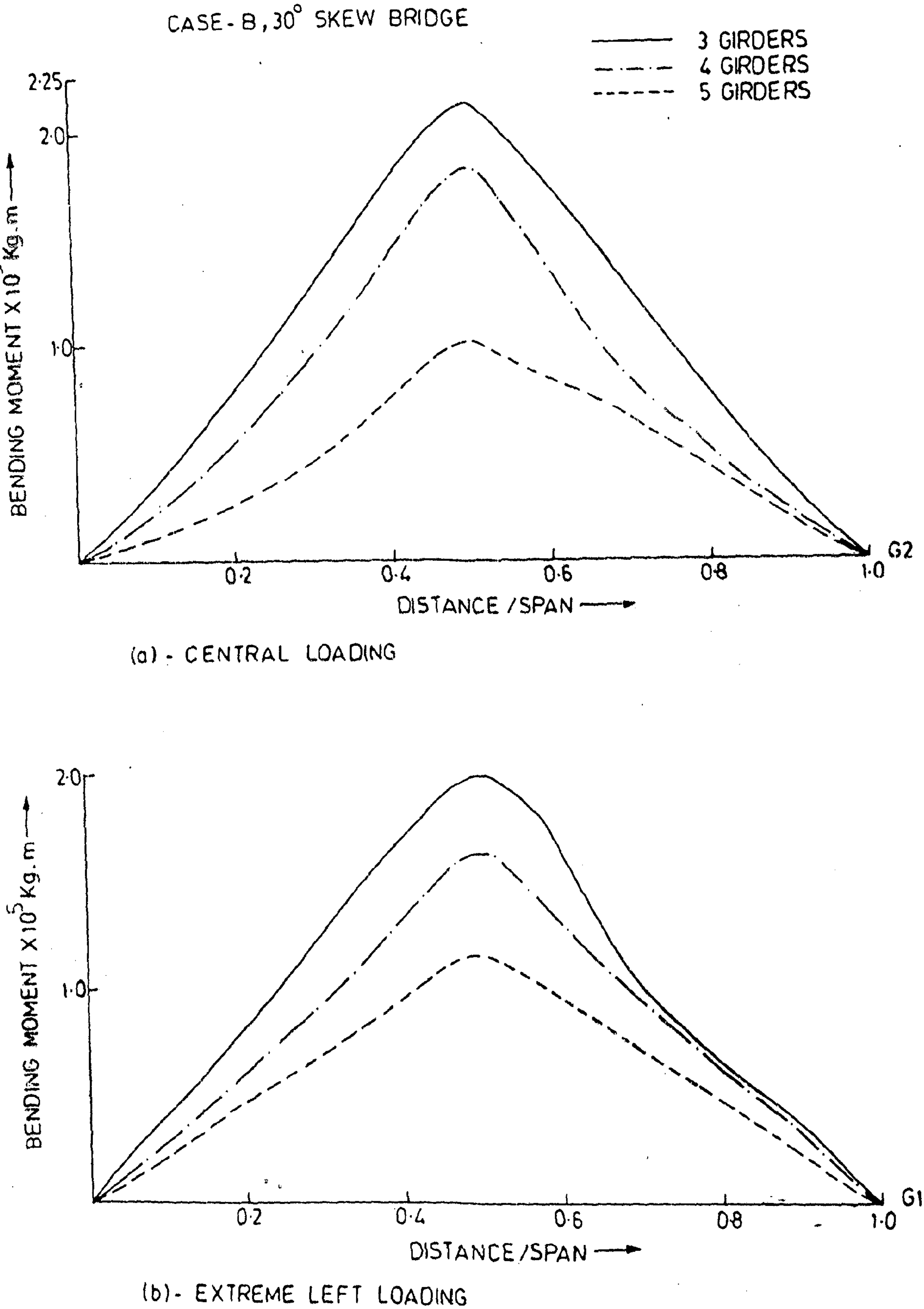
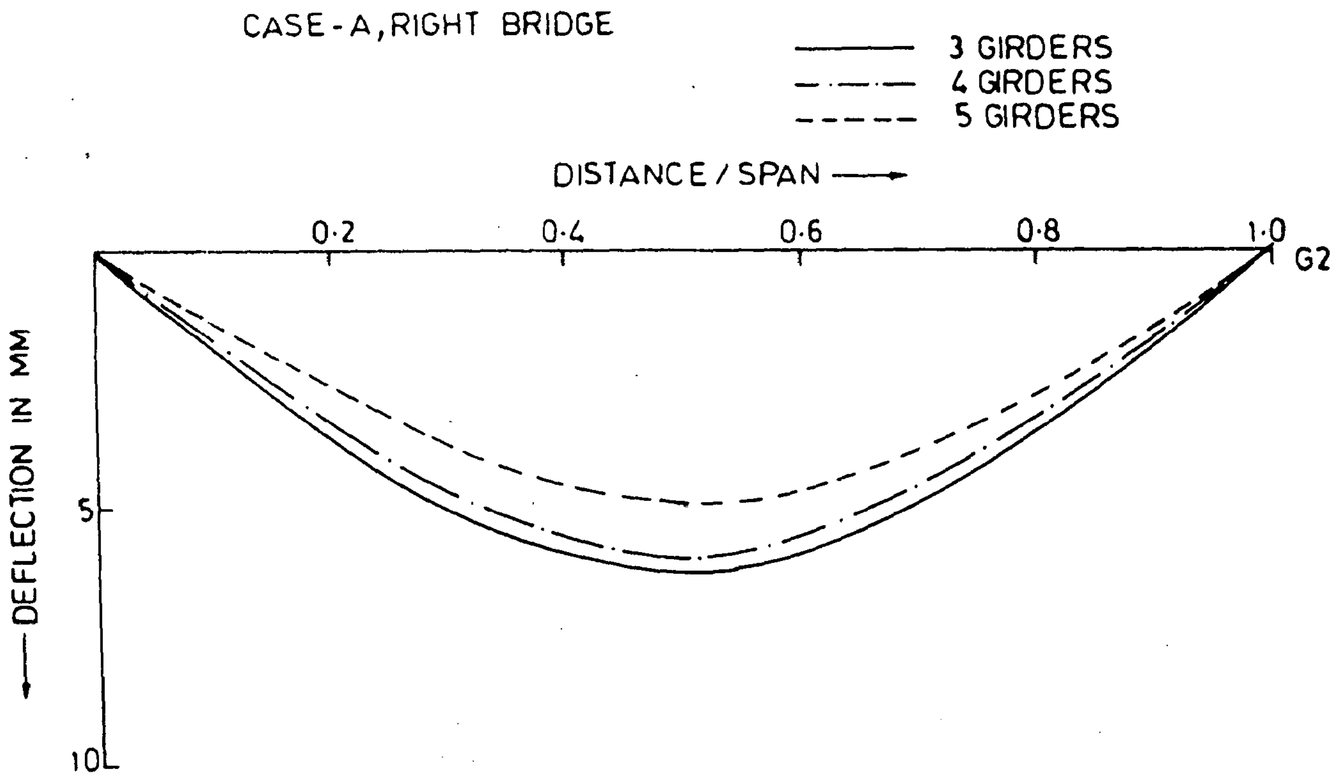
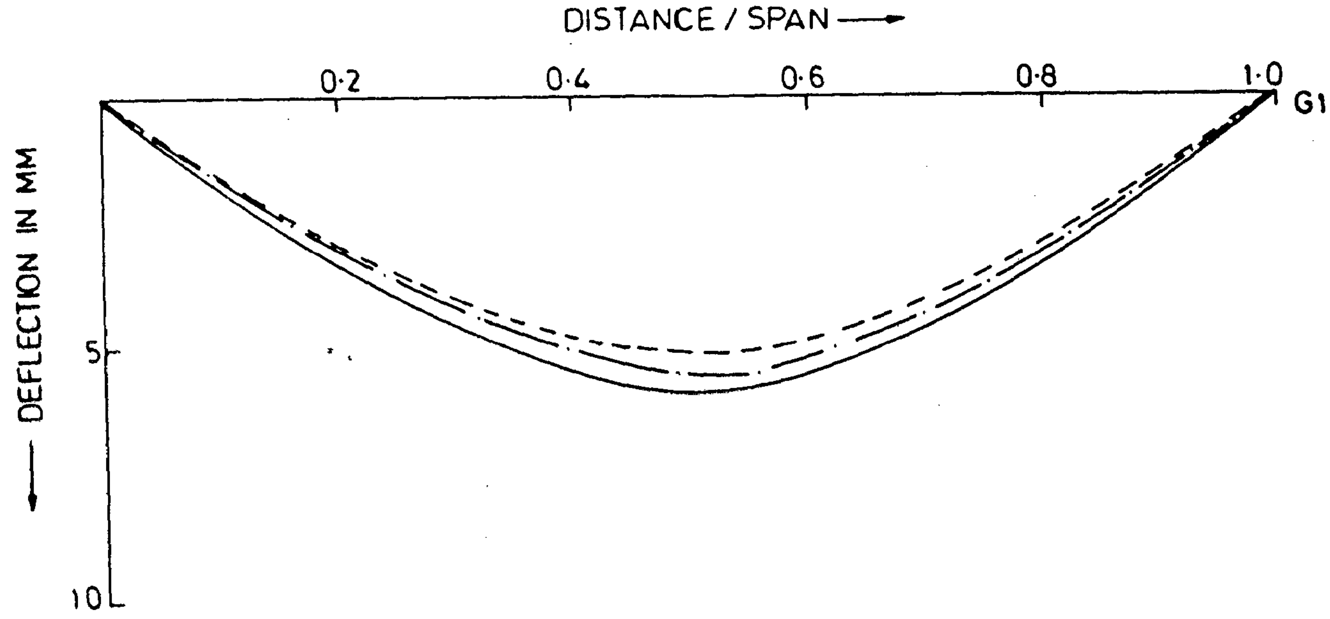


FIG. 5·6 - GIRDER BENDING MOMENT

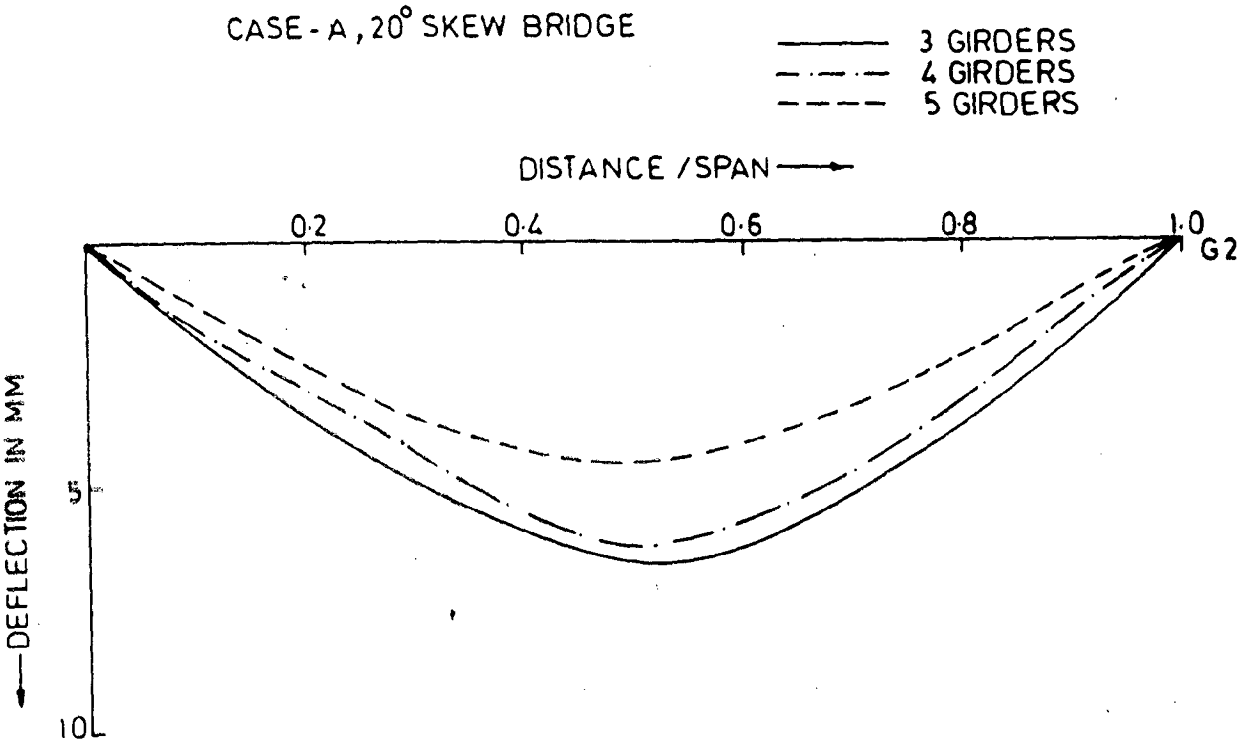


(a) - CENTRAL LOADING

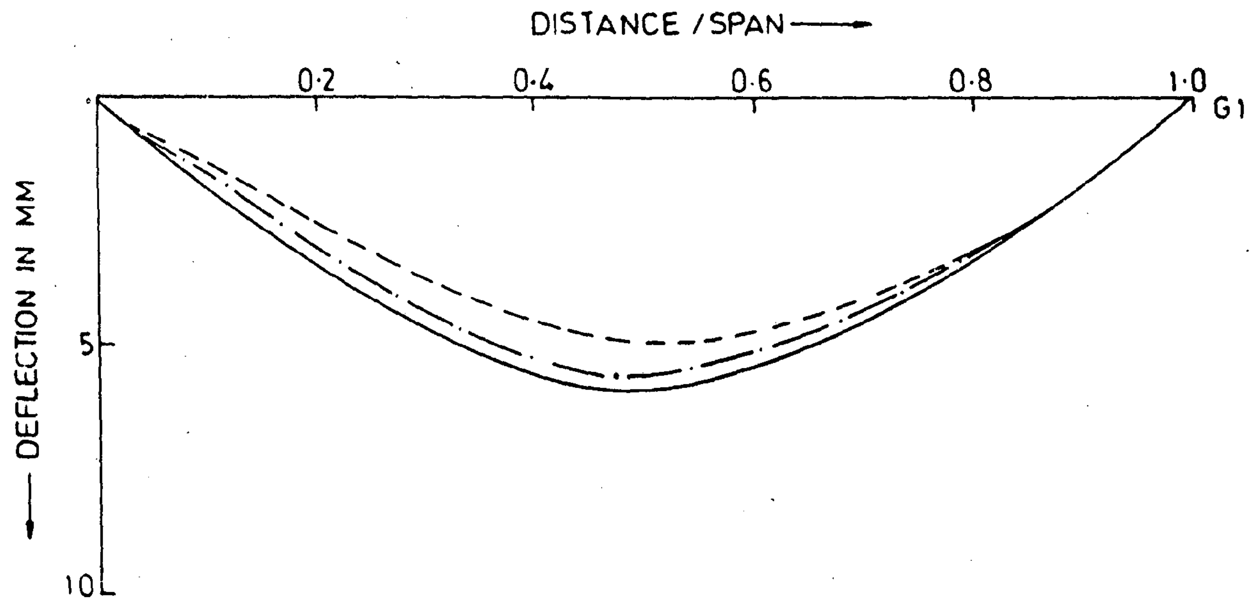


(b) - EXTREME LEFT LOADING

FIG. 5·7 - GIRDER DEFLECTION

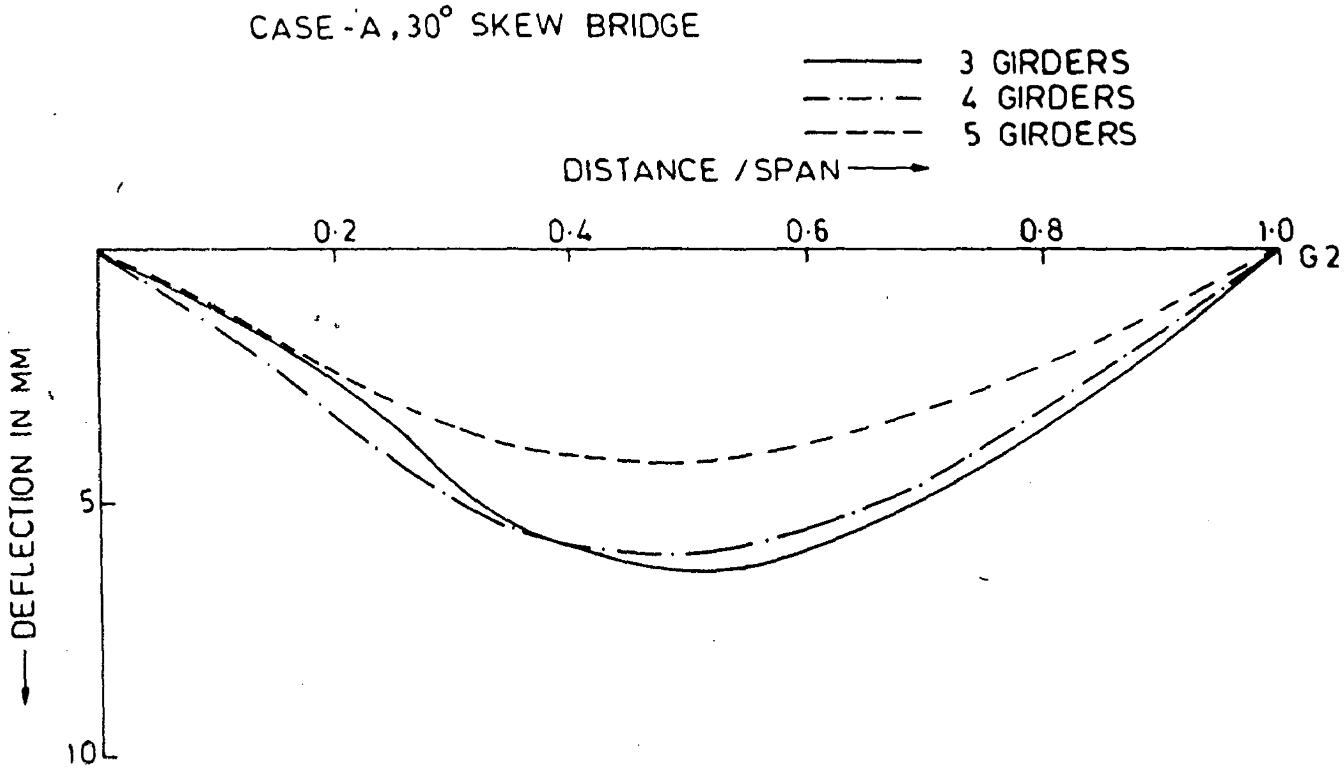


(a) - CENTRAL LOADING

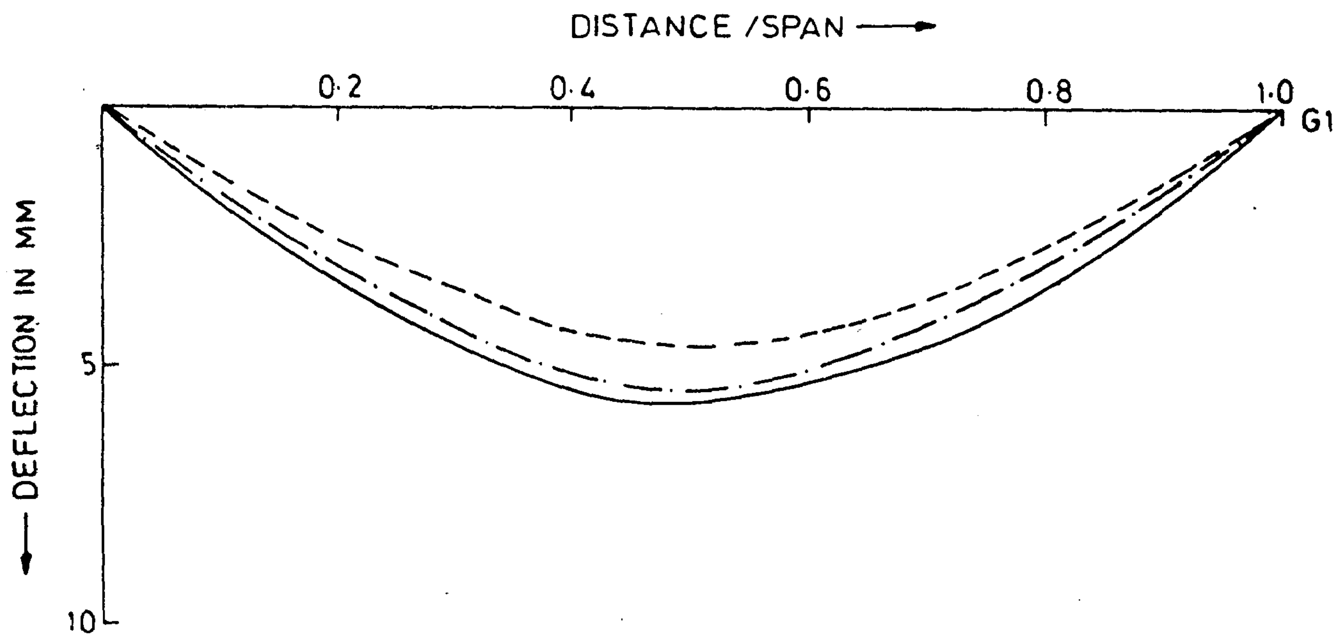


(b) - EXTREME LEFT LOADING

FIG. 5·8-GIRDER DEFLECTION



(a)- CENTRAL LOADING



(a)- EXTREME LEFT LOADING

FIG. 5·9 - GIRDER DEFLECTION

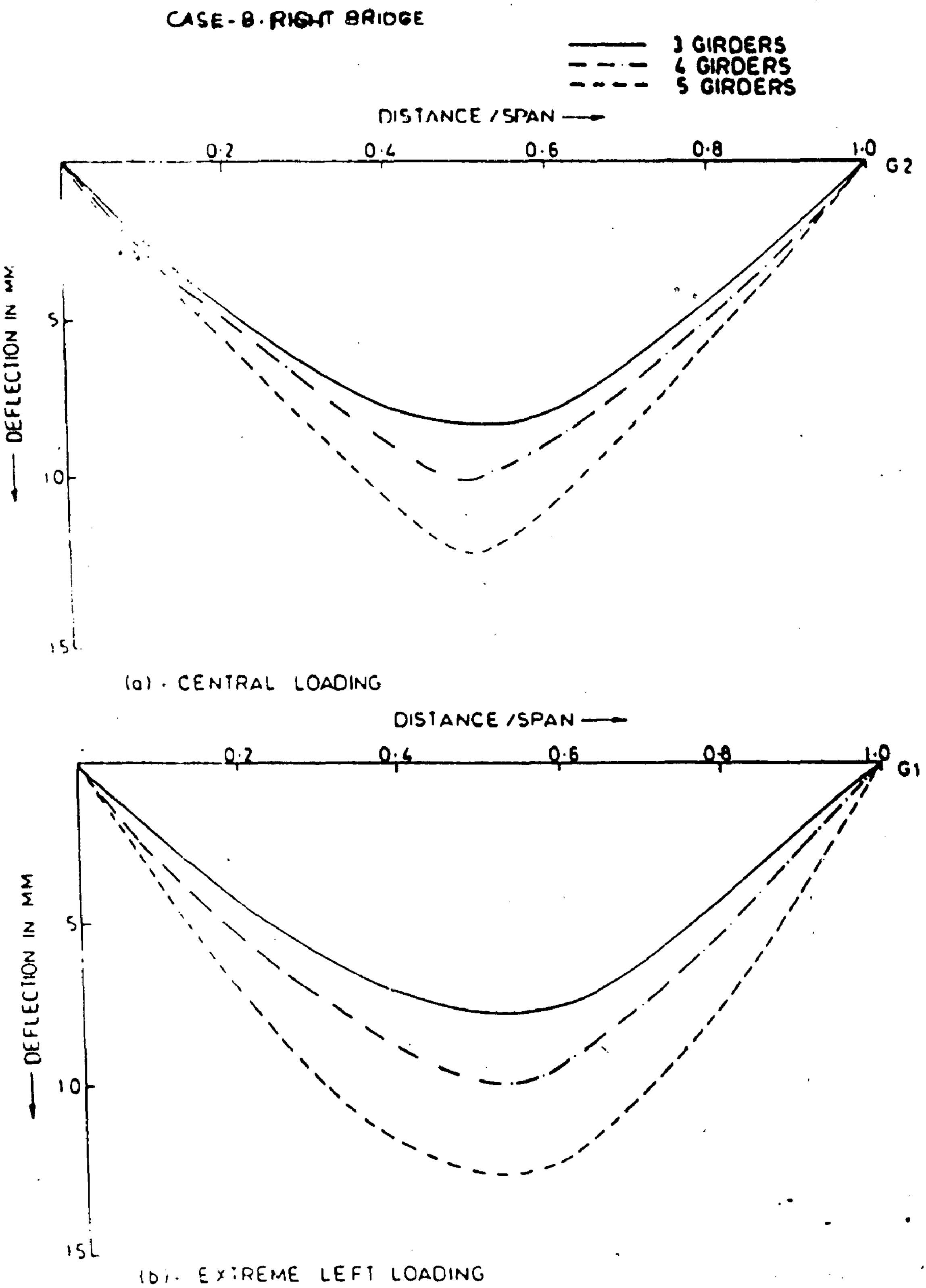


FIG. 5·10 - GIRDER DEFLECTION

CASE-B, 20° SKEW BRIDGE

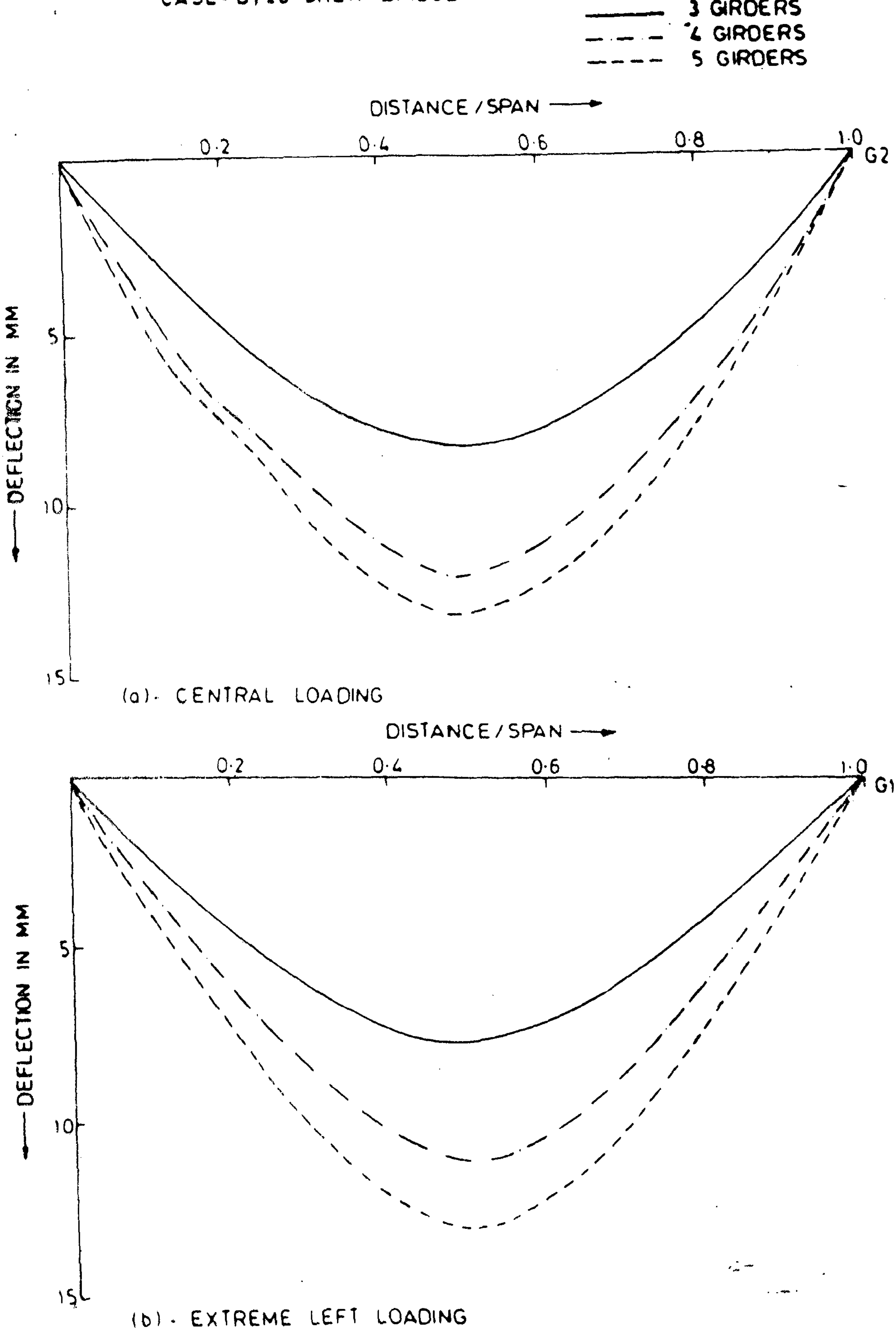


FIG. 5·11 - GIRDER DEFLECTION

CASE-B, 30° SKEW BRIDGE

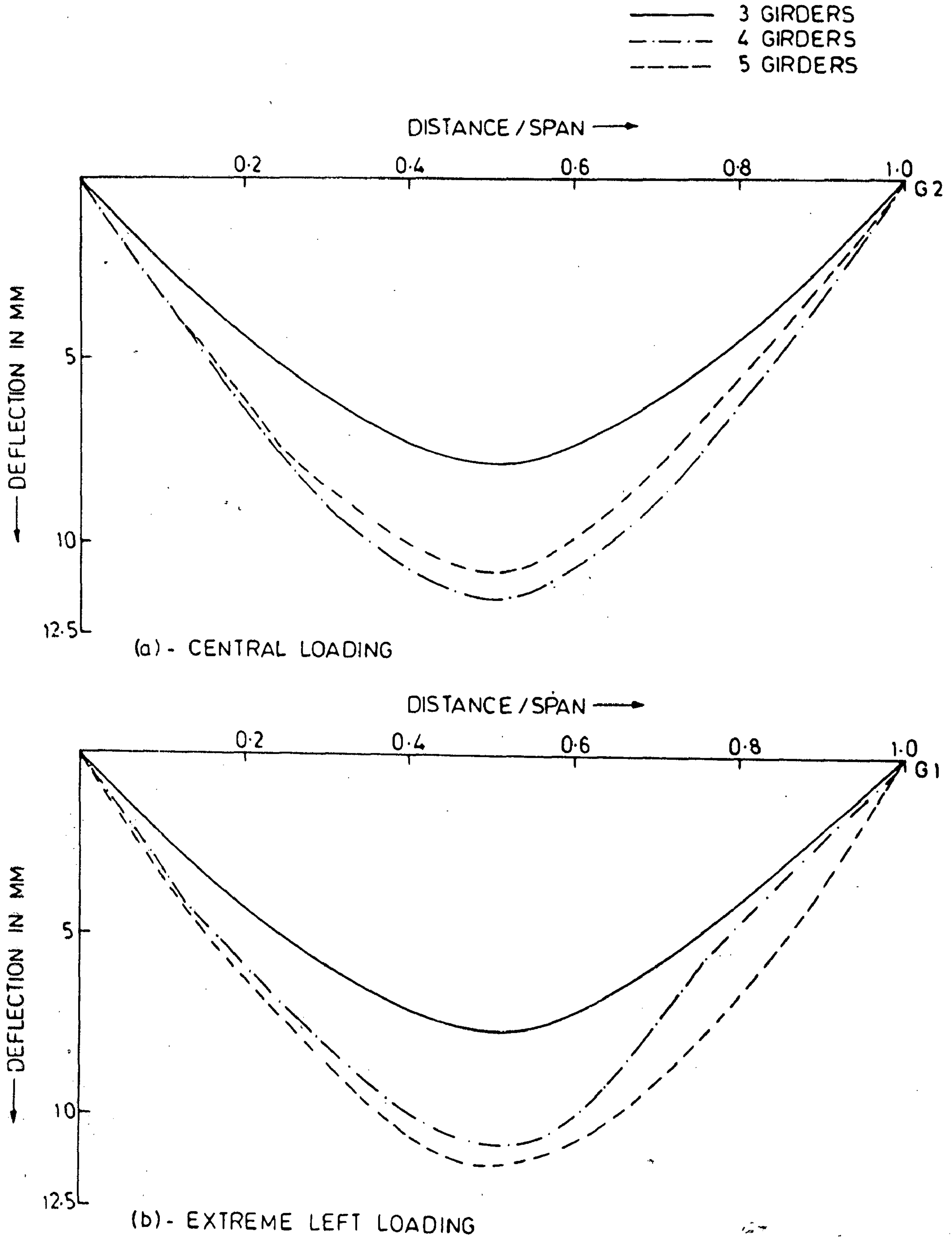
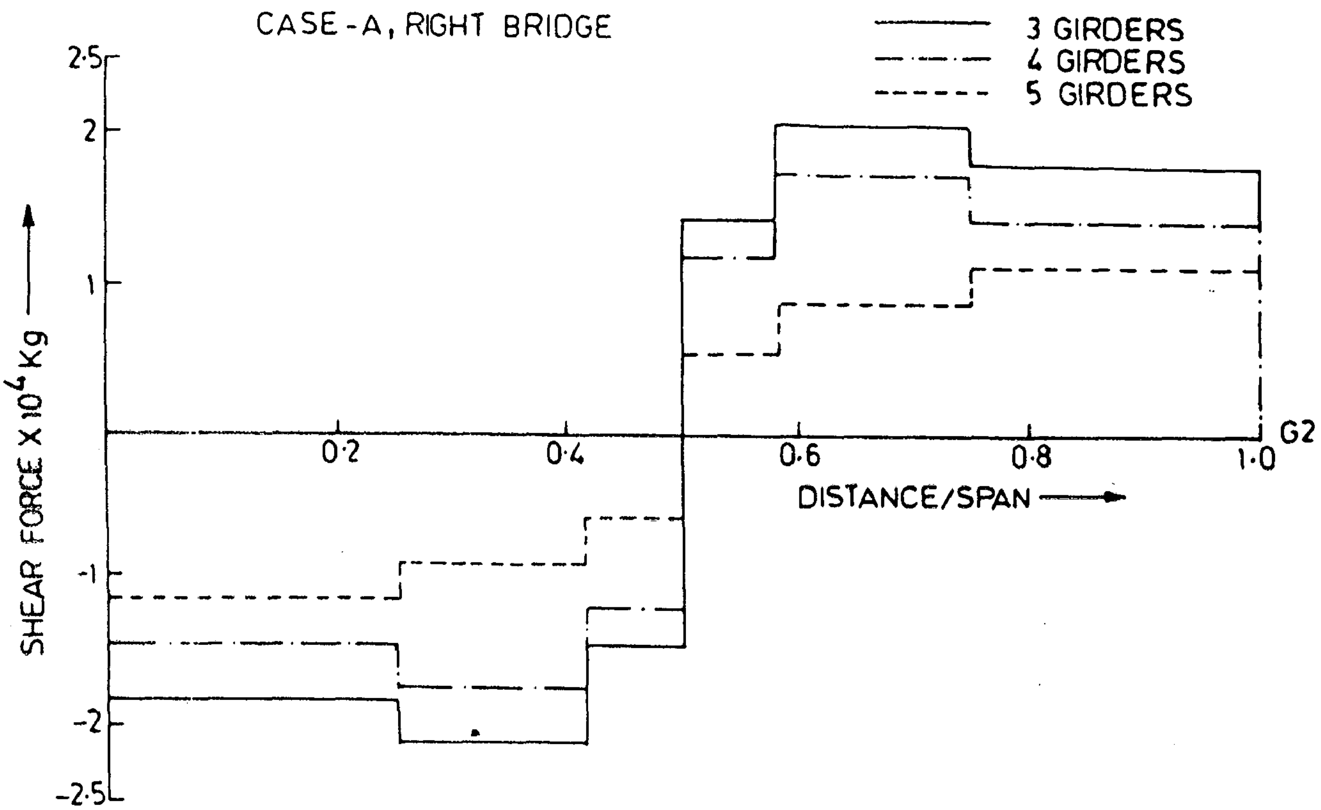
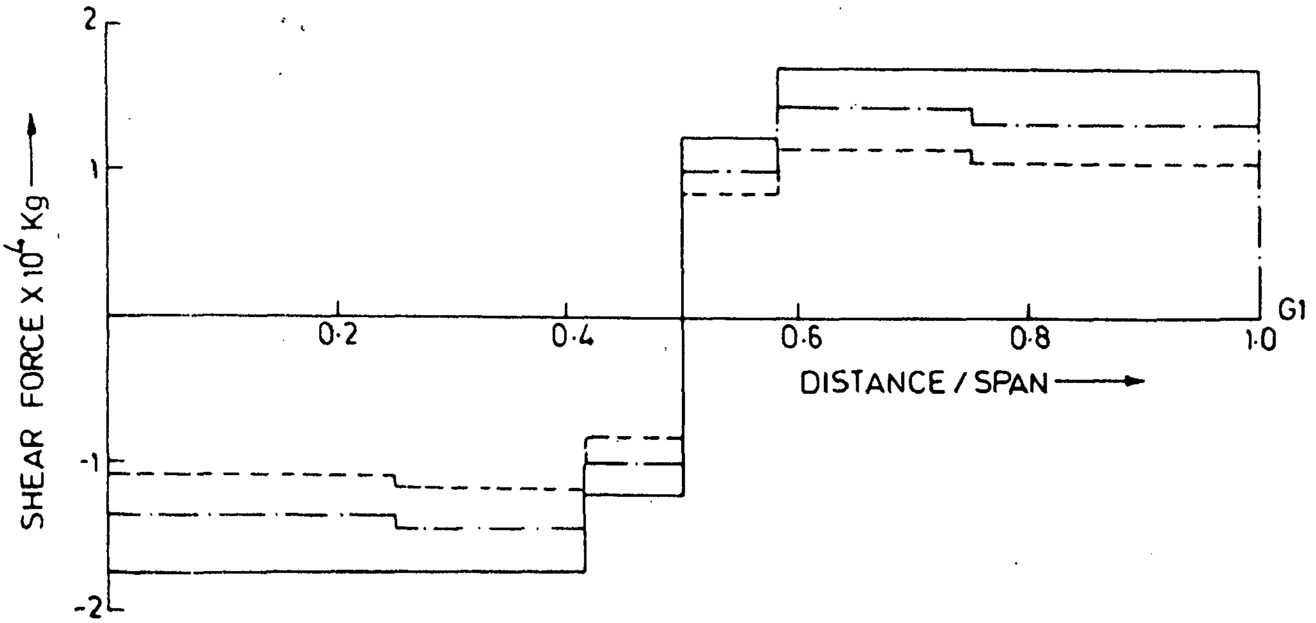


FIG. 512 GIRDER DEFLECTION

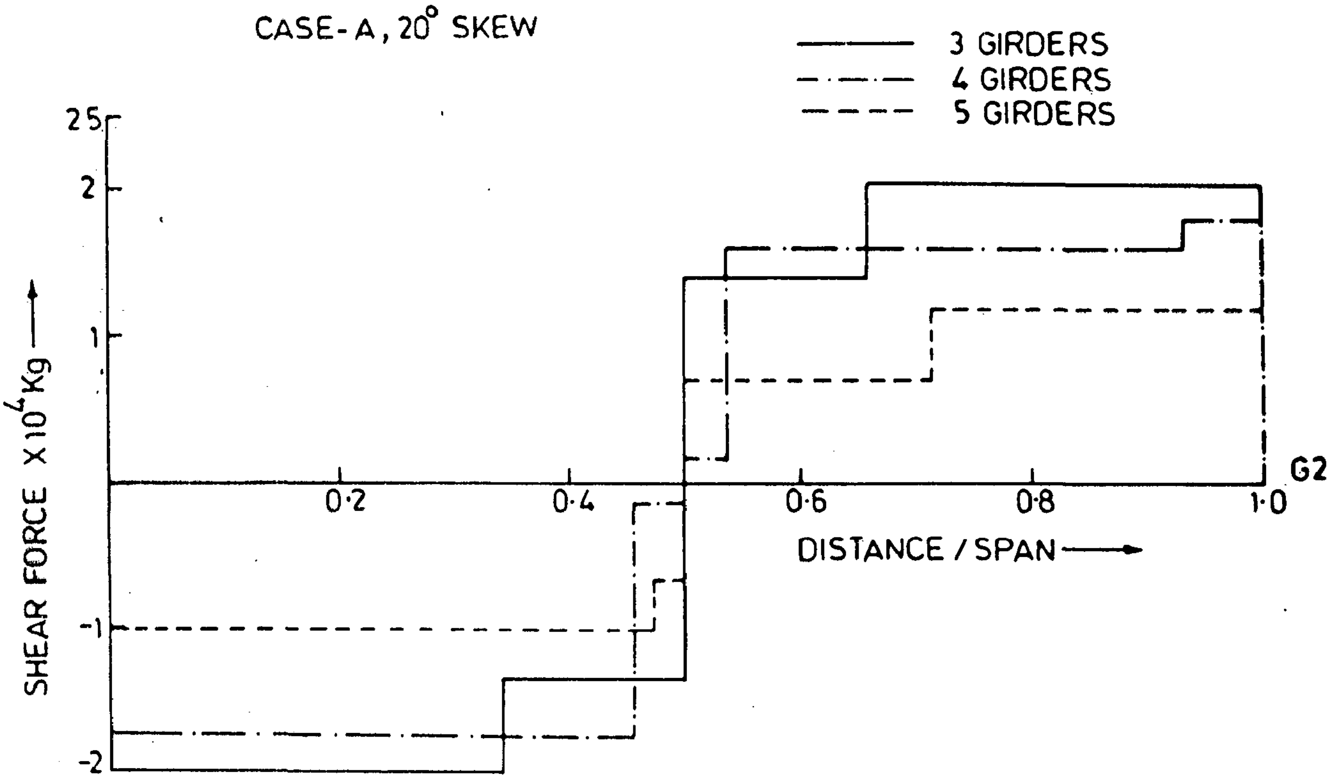


(a)- CENTRAL LOADING

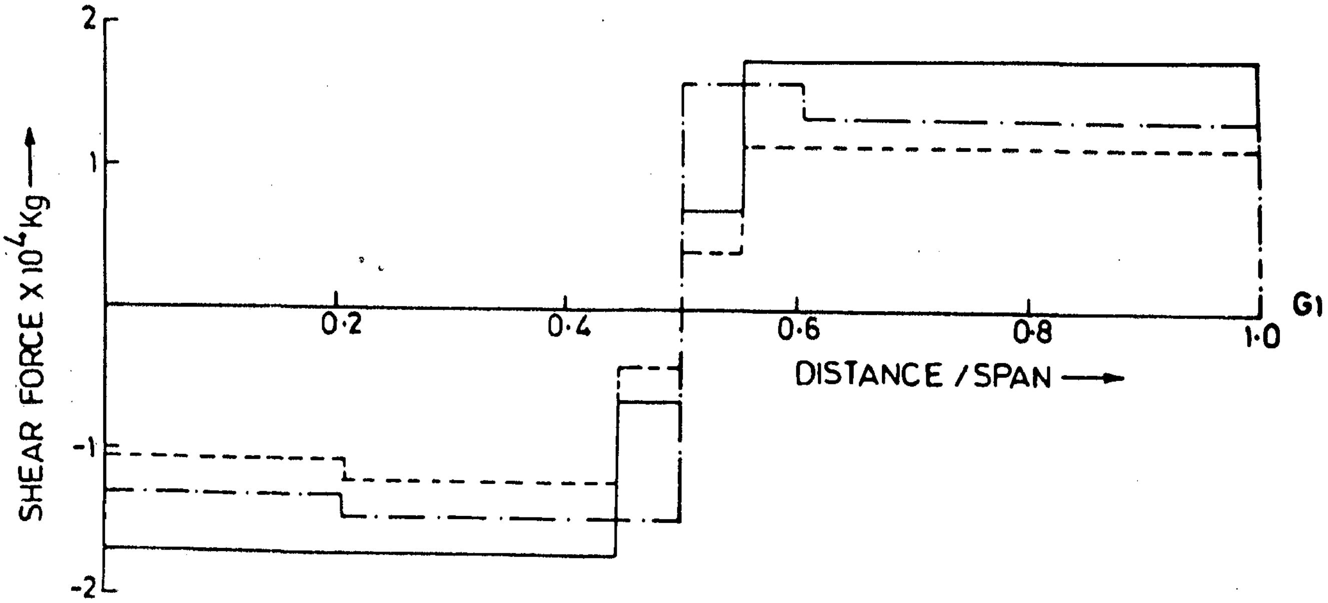


(b) - EXTREME LEFT LOADING

FIG. 5.13 - GIRDER SHEAR



(a) - CENTRAL LOADING



(b) - EXTREME LEFT LOADING

FIG. 5·14 - GIRDER SHEAR

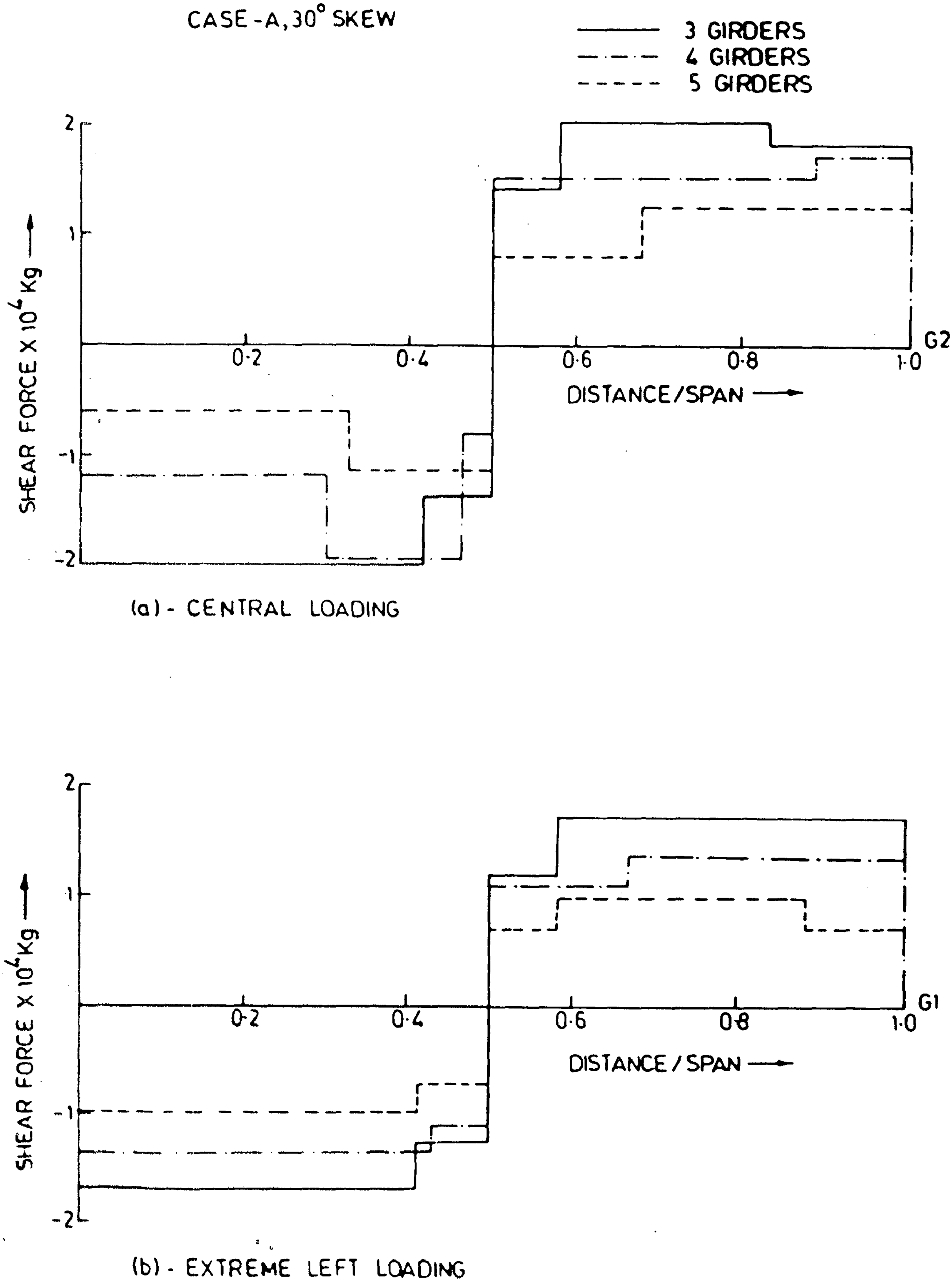
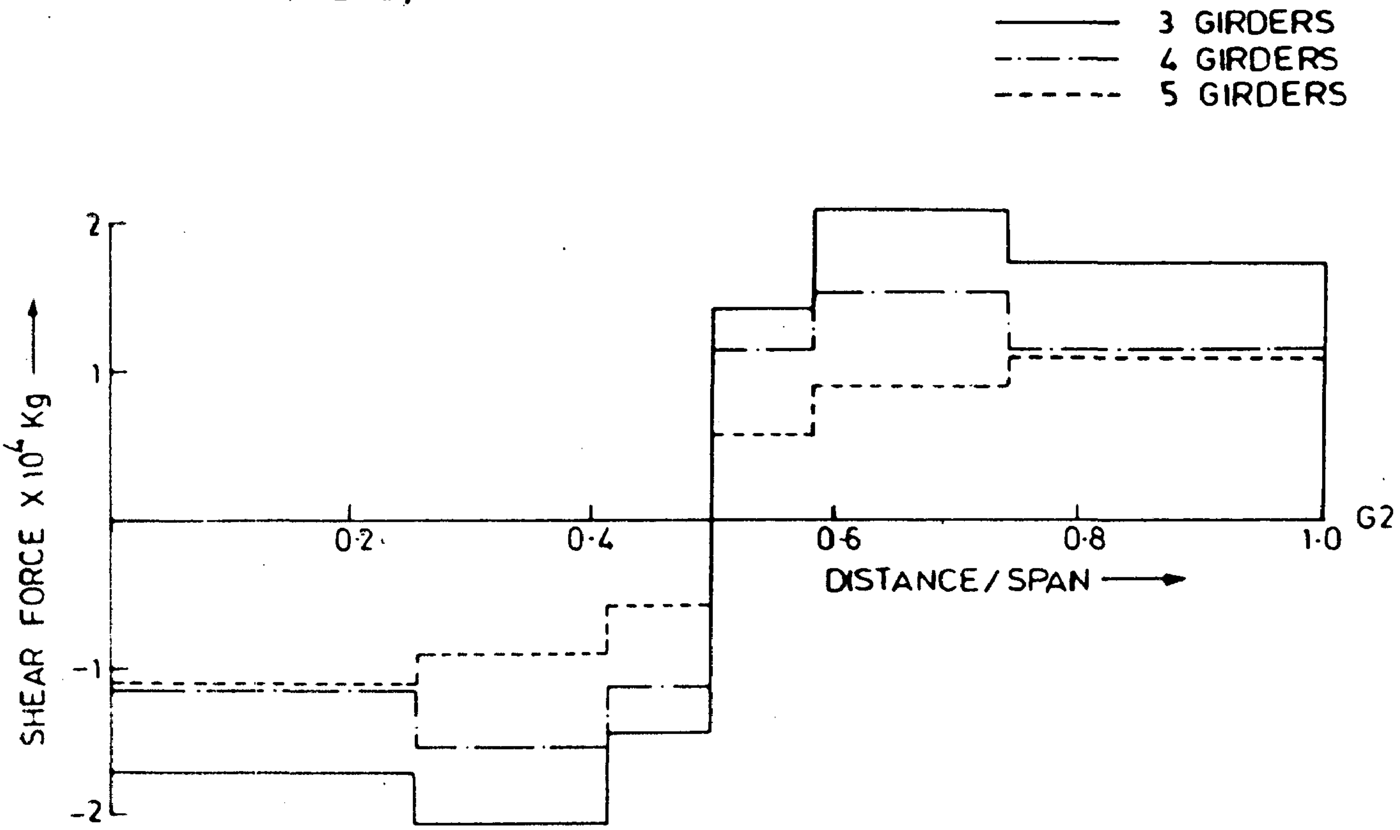
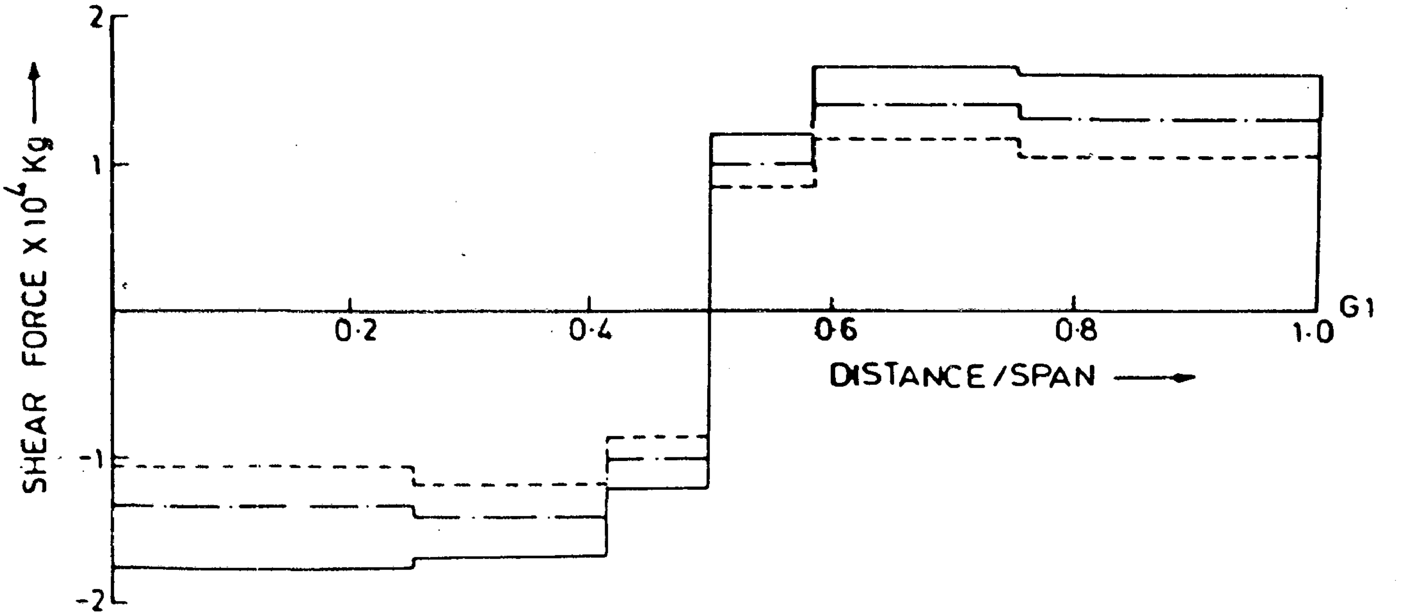


FIG. 5·15-GIRDER SHEAR

CASE - B, RIGHT BRIDGE

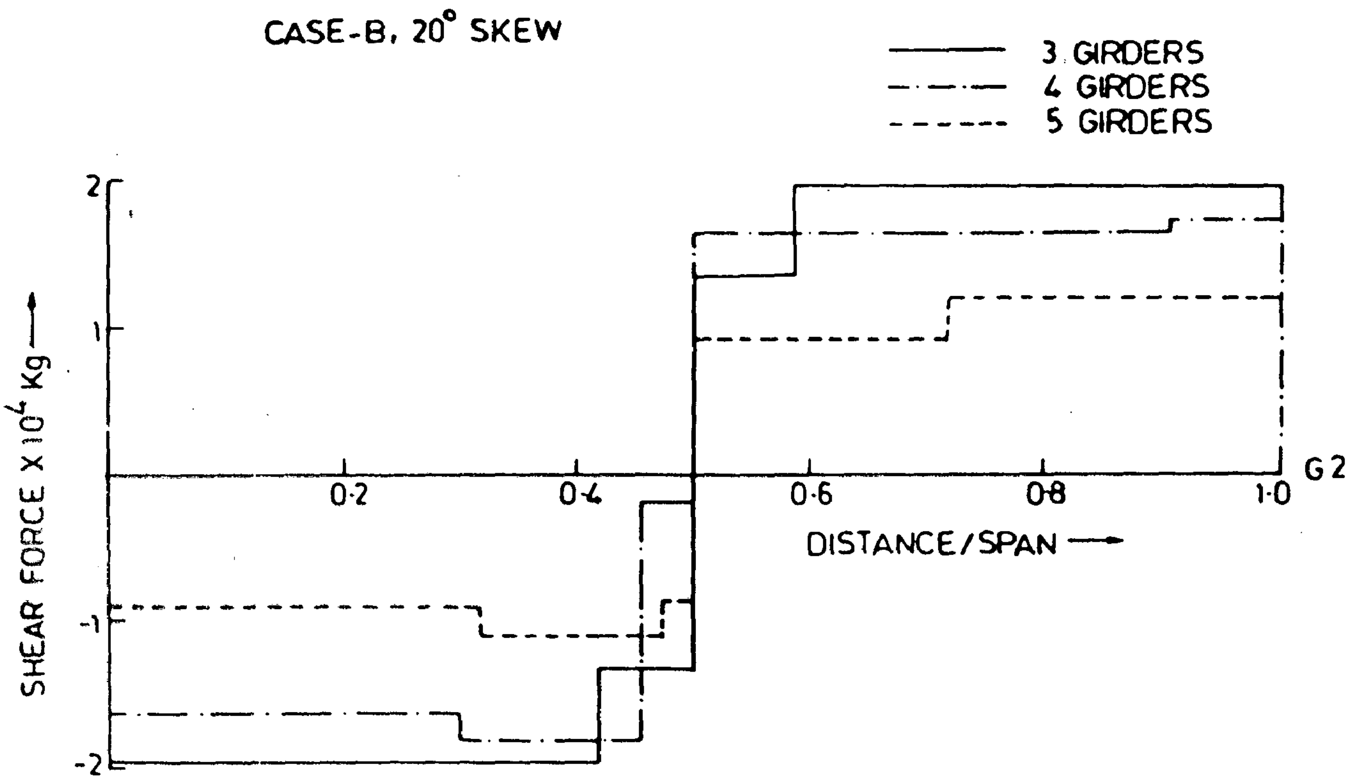


(a) - CENTRAL LOADING

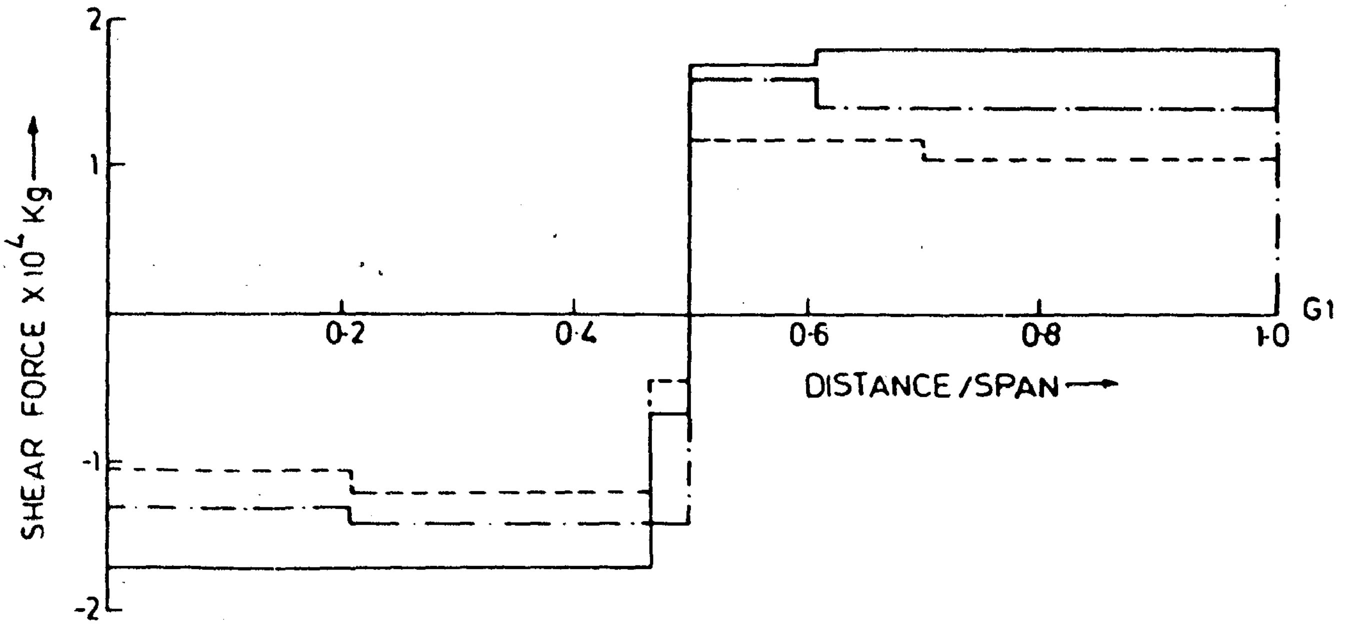


(b) - EXTREME LEFT LOADING

FIG. 5·16 - GIRDER SHEAR

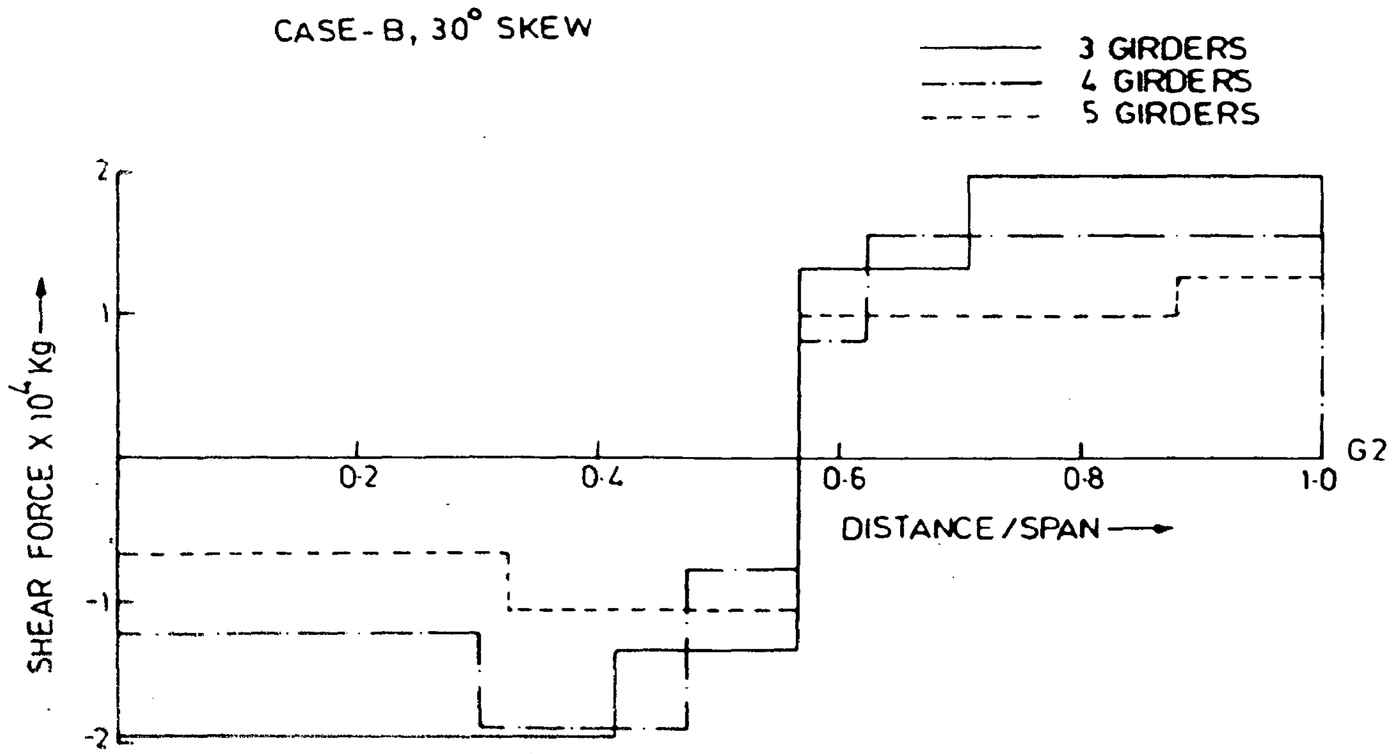


(a)- CENTRAL LOADING

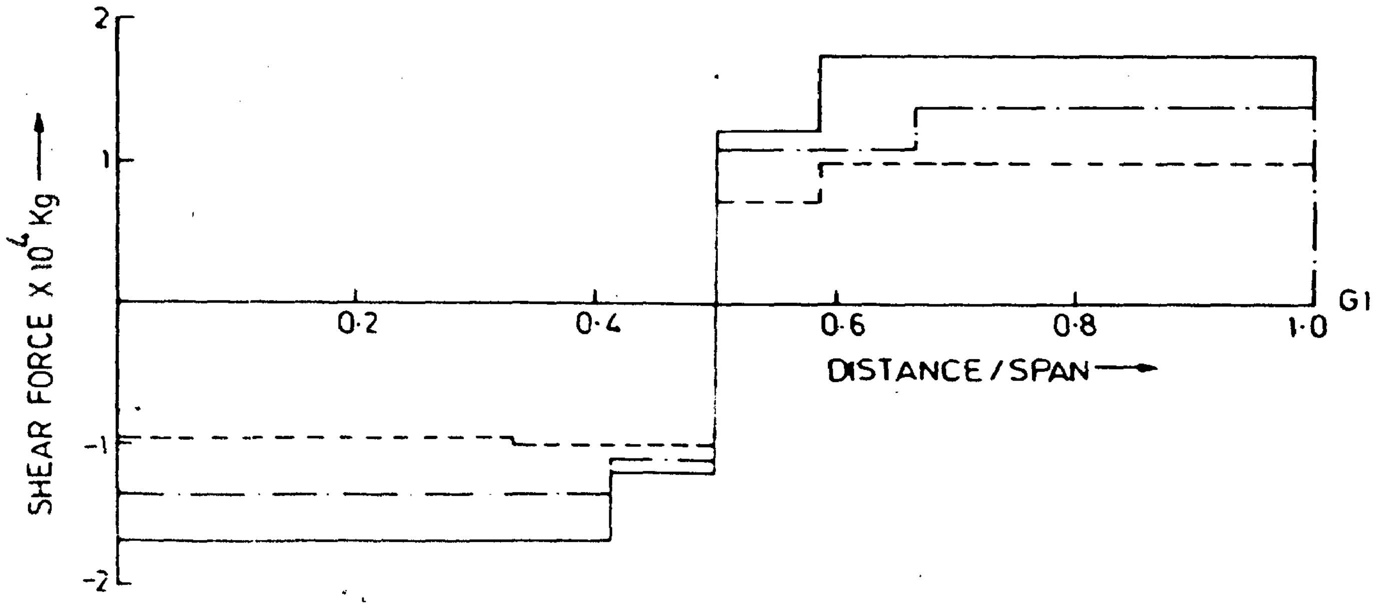


(b)- EXTREME LEFT LOADING

FIG. 5·17- GIRDER SHEAR



(a) - CENTRAL LOADING



(b) - EXTREME LEFT LOADING

FIG. 5·18 - GIRDER SHEAR

CHAPTER - 6

SUMMARY AND CONCLUSIONS

A girder bridge deck consists of two structural systems namely a transverse medium and a longitudinal medium. The transverse medium mainly consists of a deck slab and a system of cross-beams/ diaphragms. The longitudinal medium comprises a system of parallel girders usually of the same dimensions. It is common practice to manipulate the structural elements of the transverse medium in an effort to effectively distribute the deck loads amongst the girders. This is based both on practice as well as belief that the imposed load dispersion is solely a function of the transverse medium. However, it is quite apparant that the overall deck behaviour will depend on the relative stiffness of the transverse medium to that of the longitudinal medium. With a view to studying this aspect of the deck system behaviour, eighteen girder bridges have been analysed by appropriately varying the longitudinal system while keeping the transverse system constant. A 3-girder longitudinal system is appropriately transformed into a 4-girder and 5-girder systems by manipulating the girder dimensions but without changing the total area (and hence volume) of cross sections of all the girders. Two procedures are adopted to achieve the above mentioned objective. In the first procedure named as case A, material is removed from the side of the webs of the girders in order to effect the transformations from 3-girder system to subsequently 4-girder and 5-girder systems. In the second procedure, named as case B, material is removed from the depth of the girder webs to achieve the transformation. The eighteen bridge systems, thus, created for the study are presented and discussed in chapter 4. Right as well as skew bridges of four lanes with a central verge have been chosen for the study. Standard IRC class AA wheeled vehicle load system has been adopted for loading of decks.

Bridge deck has been structurally simulated as a planar grid. Despite some shortcomings, this is the most popular simulation adopted by bridge designers eversince the advent of high speed digital computers. Method of analysis is based on the stiffness method of structural analysis suitably modifed to apply to the planar grid. The method is briefly described in chapter 3.

Based on analysis, discussions and inferences presented in chapters 5, the following may generally be concluded -

(1) For effective structural behaviour of a girder bridge, variation in the longitudinal system must be studied for the same constant transverse medium.

(2) Out of the eighteen bridge systems studied, the 5-girder system in case A is structurally the most effective system of them all.

(3) Variation in the parameters (moment,shear,deflections and support reactions) values are small for skew angles up to 20°.

(4) Creating additional girders with material removed from the sides of the girder webs (case A) is more advantageous than from the depth (case B)

(5) The 5-girder systems result in more uniform support reaction compared to 3 and 4-girder systems. The structural reqirements of support bearings, thus being uniform result in much improved performance of the bearings. This further results in increased life - span for support bearings.

Scope Of Further Study

(1) Study of other parameters such as girder torsions and rotation.

(2) Study under the IRC class AA tracked vehicle and IRC 70R loadings.

(3) Study with respect to different spans

(4) Study of bridges with foot paths.

REFERENCES AND BIBLIO GRAPHY

1. Rowe, R. E., 1962 Concrete Bridge Design.

2. Westergaard, H. M., 1926 Formulas for the Design of Rectangualr Floor Slabs And Supporting Girders, Proc. AC1, 22, 1926, P. 26

3. Heins, C. P. and Hails, R. L., 1969, Behaviour of Stiffened Curve Plate Model, Journal of The Structural Division, ASCE, Stll, Nov. 9 pp 2353-2370

4. West, R. 1973, Recommendations on The Use of Grillage Analysis For Slab and Pseudo- Slab bridge decks, Report 46.017 Cement And Concrete Association And CIRI 4, London.

5. Cheung, M. S. And Cheung, Y. .K. 1971, Analysis of Curved Box Girder Bridges By Finite Strip Method, Publ. 1ABSE, 31-I, pp 1-20.

6. Powell, G. H. and odgen, D. W. 1969, Analysis of orthotropi Steel Plate Bridge Decks, Proc. ASCE 95, S T 5, May, p. 909.

7. Scordelis, A. C., 1967, Analysis of Continuous Box Girder Bridge Report SESM 67-25, University of California, Berkely.

8. Look, Y. C. And Cusen, A. R., 1971 b, Developement of The Finite Strip Method In The Analysis of Bridge Design And Coustruction, Cardiff, March-April 1971, pp. 53-72.

9. Gibb, A.1972, Grillage Analysis, Notes For Course on Bridge Deck Analysis, Civil Engineering Department University Of Dandec.

10. Hendry, A. W. and Jeager, L. G. 1958, The Analysis of Gird Frame Works And Related Structures, Chatto And Windus, London.

11. Surana, C. S., 1968, Interconnected Skew Bridge Girders, Ph.D. Thesis, University of Edinburg, V. K.

12. Prasad, J. 1983 Modified Harmonics Method For Analysis And Design of Skew Girder Bridges, Ph.D. Thesis, I.I.T. Delhi, Indial

13. Hambly., E. C. 1976, Bridge Deck Behaviour, Champman And Hall, London.

14. Morice, P. B., Little, G. and Rowe, R. E., Design Curves For The Effect of Concentrated Loads on Concrete Bridge decks, London Cement And Concrete Assiociation, July 1956.

15. Cusens, A. R. and R. P. Pama, 1975, Bridge Deck Analysis, Johb Wely And Sons.

16. Agrawal, R., 1975, Analysis and Design of Interconnected Skew Girder Bridges, ph.D. Thesis, I.I.T. New Delhi.

17. Bakht B. and L.G. Jaeger, 1985, Bridge Analysis Simplified, Mc-Graw Hill Book Comapny.

18. Bazeley, G. P.,et al, 1965 , Triangular Elements in Bending Conforming and Non-conforming Solutions, Proc. Conf. Matrix Methods in Structural Mech. Wright-Patterson Base, Ohio.

19. Beckett, D., 1973, Concrete Bridges, Surrey University Press, England.

20. Cheung, Y. K., 1969, Analysis of Box Girder Bridges by Finite Strip Method, ACI Publ, SP. 26.

21. Cope, R. J. and L. A. Clark, 1984, Analysis and Design of Concrete Slabs, Elsevier Applied Science Publication, London

22. Davies, J. D., I. J. Somervaille and O. C. zienkiewiez,1971, Analysis of Various Types of Bridges by the Finite Element

Method, Development in Bridge Design and Construction, Crosby Lockwoods and Sons, Ltd., London.

23. Clough, R. W., 1960, The Finite Element in Plane Stress Analysis, Proc. 2nd ASCE Conf. on Electronic Computations Pittsburg.

24. Ghali, A., 1969, Design of Simply Supported Skew Concrete Girder Bridges, Int. Symp. on Concrete Association, London.

25. Jackson, N., 1968, The Torsional Rigidity of Concrete Bridge Deck, Concrete, pp. 468-474.

26. Lightfoot, E. and F. Swako, 1959, Structural Frame Analysis by Electronic Computer, Grid Framework Resolved by Generalised Slope Deflection Engg.

27. Martin, H. C., 1966, Introduction of Matrix Method of Structural Analysis, Mc Graw Hill.

28. Meyer, C. and A. C. Scordelis, 1970. Computer Program for Prismatic Folded Plates With Plate and Beam Elements, Report SESM, 70-3, University of California.

29. Meyer, C., 1970, Analysis and Design of Curved Box Girder Bridges, Report, Univ. of California.

30. Morice, P. B. and G. Little, 1956, Right Bridge Decks subjected to Abnormal Loads, Cement and Conc. Assoc. London.

31. Swako, F., 1967, Computer Analysis of Grillage, Curved in Plan, Publ.IABSE, 27.

32. Scordelis, A. C., 1966, Analysis of Simply Supported Box Girder Bridges, SESM 66-17, Deptt. of Civil Engg. University of California.

33. Sriskandian, K., 1971, Case Studies in the use of BAPS Finite Program Package, Development in Bridge Design and Construction, Crosby Lockwood and Sons Ltd., London.

34. Surana, C. S., 1984, Grillage Analogy for Analysis of Superstructure of Bridges, Proc. Seminar on Modern Trends in Structural Analysis and Design B. H. U. Varanasi.

35. Turner, M. J. et al, 1956, Stiffness and Deflection Analysis of Complex Structures, J. Aero. SCI, 23.

36. Victor, D. J. 1978, Essentials of Bridge Engg. Oxford and IBH Publishing Co. New Delhi, India.

37. West, R., 1972, The use of grillage Analogy for the Analysis Slabs and Pseudo- slabn Bridge Deck, City Univ.

38. Yahai, M., 1987, Skew Effects in Girder Bridge Decks, Univ. of Roorkee, Roorkee.

39. Parwani, J., 1988, Influence of Transverse Members On Structural Behaviour of Girder Bridges, Univ. of Roorkee.

APPENDIX - A

(a) Moment of Inertia

Moment of Inertia of the beams are calculated by considering the gross cross section of the T - beam etc

(b) Torsional Constant (J)

Timoshenko has suggested a method of computation for torsional constant (J) of the T - beam. The approximate procedure for calculating J is explained for two cases below:

Case (a) Rectangular section:

Torsional constant (J) for rectangle of Fig. A -a (b) with sides b and a is given by $J = Kba^3$ where K (Timoshenko Torsion Coefficient) is function of aspect ratio b/a and is obtained by using figure A - 1 (a) and the given table.

Case (b) T - section

Torsional constant for T - section is obtained by subdividing the section into rectangle shapes and summing the value of J for these elements. Illustration for T - section (Fig. A - 1) (c) from Fig. A - 1 (a).

For element (1) $K_1 = 1/3$, $J_1 = 7.33 \times 10^5$

For element (2) $K_2 = 0.283$, $J_2 = 1.94 \times 10^6$

For element (3) $K_3 = 0.145$, $J_3 = 9.97 \times 10^5$

J for T section is, $J = J_1 + J_2 + J_3$

$J = 3.67 \times 10^6$ cm^4

(c) Flange Width of Beams

Girders of the bridge deck are considered to act as T-beams. Flange width of these T-sections are taken in acordance to IRC Bridge Code Specifications, Section I, General feature of Design and it is the minimum of the following :

(i) 1/4 of the effective span

(ii) The distance between centre to centre of the ribs of the beams.

(iii) The breadth of the rib plus twelve times the thickness of the slab.

b/a	1	1.5	2	2.5	3	4	6	10	∞
K	0.141	0.196	0.229	0.249	0.263	0.281	0.299	0.312	0.333 = 1/3

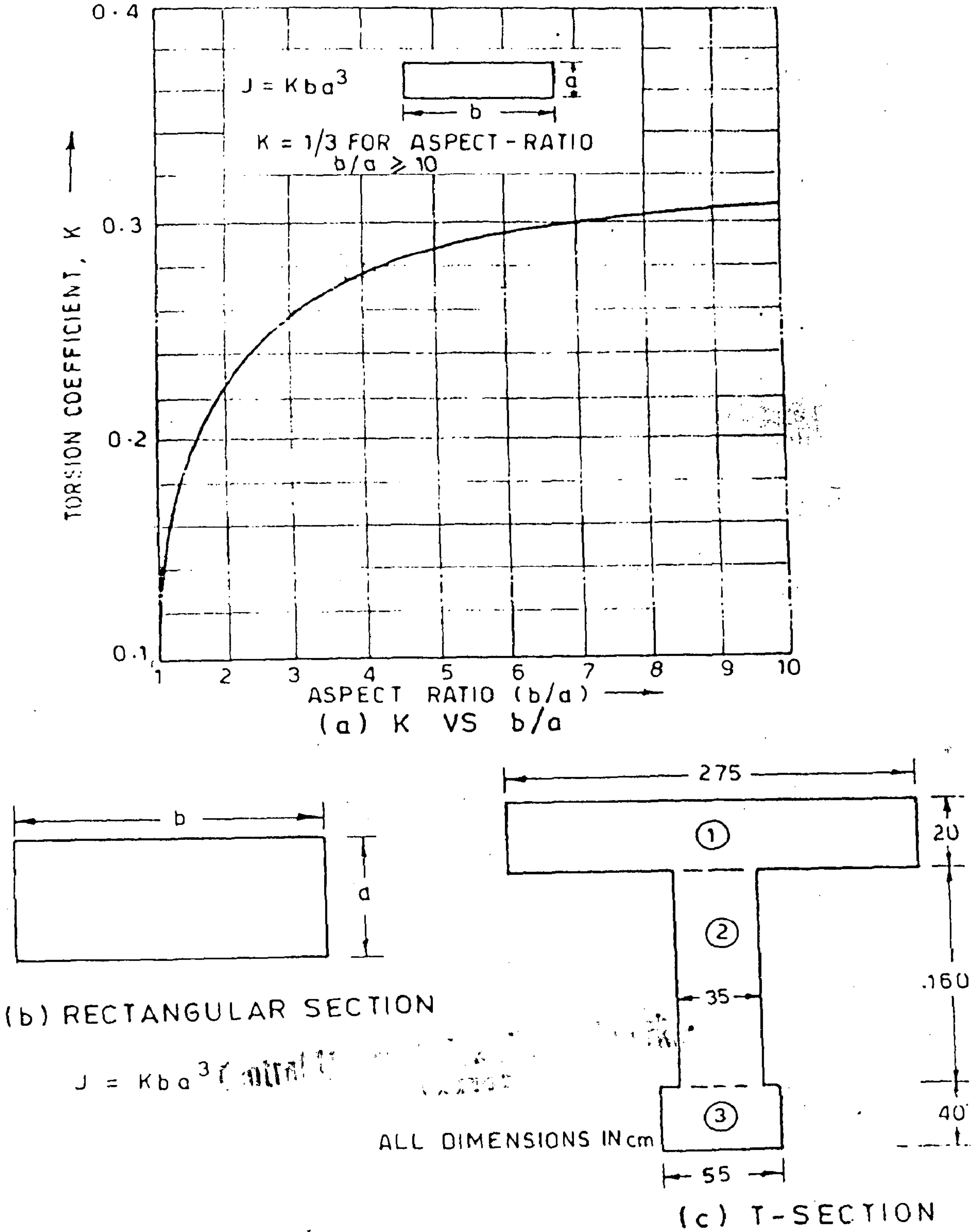


FIG. A-1 – TORSIONAL CONSTANT COMPUTATION