EFFECT OF INTERMEDIATE DRAINS ON STABILITY OF STRUCTURES FOUNDED ON SOIL OF FINITE DEPTH

A THESIS

submitted in fulfilment of the
requirements for the award of the degree
of
DOCTOR OF PHILOSOPHY
in
WATER RESOURCES DEVELOPMENT

By

AKHILESH KUMAR

WATER RESOURCES DEVELOPMENT TRAINING CENTRE
UNIVERSITY OF ROORKEE
ROORKEE-247 667 (INDIA)

May, 1985

CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled 'Effect of Intermediate Drains on Stability of Structure Founded on Soil of Finite Depth' in fulfilment of the requirement for the award of Degree of Doctor of Philosophy, submitted in the department of Water Resources Development Training Centre of the University, is an authentic record of my own work carried out during the period from December 1979 to April 1985 under the supervision of Dr.Bharat Singh and Dr.A.S.Chawla.

The matter embodied in this thesis has not been submitted by me for the award of any other degree.

(AKHILESH KUMAR)
Candidate's Signature

This is to certify that the above statement made by the candidate is correct to the best of our knowledge.

(A.S.CHAWLA)
Professor
Water Resources Development
Training Centre, University
of Roorkee, Roorkee(U.P.)
INDIA.

(BHARAT SINGH)
Vice-Chancellor
University of Roorkee
Roorkee(U.P.),INDIA.

ROORKEE

DATED: May 14 ,1985.

ACKNOWLEDGEMENT

The author expresses his deep sense of gratitude and profound regards to Dr.Bharat Singh, Vice-Chancellor, University of Roorkee and Dr.A.S.Chawla, Professor, Water Resources Development Training Centre, University of Roorkee, Roorkee for their unstinted inspiration and invaluable guidance.

The author is grateful to the Professor & Head, Water Resources Developing Training Centre, University of Roorkee for providing facilities for the study.

The author also has the pleasure to acknowledge the patience and understanding of his wife Indira and encouragement by other family members who have always been a source of inspiration to him.

Secretarial assistance and other facilities used at Water Resources Development Training Centre are also acknowledged thankfully.

SYNOPSIS

The stability of hydraulic structures founded on permeable soil has to be ensured against forces caused by percolating water. The seepage water exerts uplift pressure below the structure and tends to remove the soil particles at the exit and which may ultimately lead to piping. Intermediate filters or drains can be provided below the floor of hydraulic structures founded on permeable soil to reduce uplift pressures resulting in appreciable savings. A solution was obtained by Chawla (6) for a drain of any dimension located anywhere between two cutoffs of a flat floor founded on permeable soil of infinite depth. Until now no solution was available to determine the effect of plane drainage located anywhere between two cutoffs of a flat floor founded on permeable soil of finite depth.

In many cases an impermeable layer exists at shallow depth and downstream sheetpile cutoff may extend upto impermeable layer due to inadequate subsurface investigations or as a safe gaurd against scour. In such a situation uplift pressures equal to upstream water head would develop below the central portion of the floor unless the upstream sheet pile is also driven upto the impermeable layer or an alternative arrangement is made to release the pressure. It is impossible to provide a vertical cutoff completely free ffom leakage, unless it is made of concrete and is monalithic with the floor. In steel sheet-piles leakage may occur due to improper interlocking or corrosion of sheet-pile and therefore even

providing a complete cutoff at the upstream end of the floor to intercept seepage cannot be relied upon. In such a situation, an intermediate filter could be provided to release the uplift pressures.

In the present work, the problems of seepage below a flat floor, with a partial cutoff at the upstream end and a partial or complete cutoff at the downstream end, founded on permeable soil stratum of finite depth with an intermediate filter of any dimension located anywhere in between have been solved with the help of conformal mapping. In addition the solution has also been obtained using finite element method for the seepage below a flat floor with two partial cutoffs and an intermediate filter founded on subsoil underlain by sloping impermeable layer.

The uplift pressures at key points and exit gradient at the end of the floor have been calculated for various combinations of the parameters involved. The results have been plotted in the form of curves which can be conveniently used for design.

The results indicate that the uplift pressures on the floor and exit gradient at the end of floor reduce considerably with the provision of an intermediate filter of even very small length. Further reduction in pressure and exit gradient with increase in length of the filter is less as compared to the initial reduction. The uplift pressures decrease on the downstream and increase on the upstream side of the filter as the location of filter is shifted downstream.

The uplift pressures on the floor and exit gradient at the end of floor decrease with decrease in the depth of pervious stratum and increase with increase in the inclination of the impervious layer. The uplift pressures decrease with increase in the depth of upstream cutoff. The effect of depth of the downstream cutoff on uplift pressure is very small, however, the exit gradient reduces significantly with increase in depth of the downstream cutoff.

An illustrative example of Narora barrage, using the design curves developed in the present work has also been included in Appendix A-1.The redesign with intermediate drainage shows 44% reduction in sectional area of concrete.

NOTATIONS

A	=	Area of flow region
A_c	=	Area of an element
a_F, a_G, a_j	=	Transformation parameters
b	=	Length of floor with end cutoffs
b_1	=	Distance of filter from upstream cutoff
d_1	=	Depth of upstream cutoff
d_2	=	Depth of downstream cutoff
$F(\varphi, m)$, $F(\varphi_1, m_1)$	=	Elliptic integral of first kind
$\{F\}^e$	=	Flux vector for an element
$\{F\}$	=	Global flux vector
F_s	=	Factor of safety against heave
f	=	Length of filter
G_E	=	Exit gradient at end of floor
g, g'	=	Constant
H_1	=	Head in filter measured above downstream water level
H_2	=	Head in upstream bed measured above downstream water level
h	=	Head at any point measured above downstream water level
h_{av}	=	Average head
h_w	=	Piezometric head at any point
$\{h\}^e$	=	Head vector for an element
$\{h\}$	=	Global head vector
I	=	Gradient at any point
i, j, k	=	Nodal point of an element

K	=	$F(\pi/2, m)$ = complete elliptic integral of first kind
K_1	=	$F(\pi/2, m_1)$ = complete elliplic integral of first kind
K'	=	$F(\pi/2, m')$ = Associated complete elliptic integral of first kind
K_{ij}	=	Element of stiffness matrix at i^{th} row and j^{th} column
$[K]^e$	=	Stiffness matrix of an element
$[K]$	=	Global stiffness matrix
k	=	Coefficient of permeability
k_1	=	Coefficient of permeability of upper layer of soil
k_2	=	Coefficient of permeability of lower layer of soil
k_{xx}	=	Coefficien of permeability in x direction
k_{yy}	=	Coefficient of permeability in y direction
M	=	Constant
m, m_1	=	Modulus of elliplic integrals
m'	=	$\sqrt{1-m^2}$ = comodulus of elliptic integrals
N	=	Constant
N_i	=	Interpolation function (Shape function) for the ith node
n_x, n_y	=	x and y component of unit vector normal to boundary
q	=	Discharge per unit width across a boundary surface
q_1	=	Seepage discharge per unit width normal to direction of flow through filter
q_2	=	Total seepage discharge per unit width normal to direction of flow
R_i	=	Residual at i^{th} node
s	=	Surface area of flow region
T	=	Depth of pervious stratum

t	=	$r+is$ complex variable representing semi infinite plane
W	=	Addition weight per unit area over filter
w	=	$\phi+i\psi$ = complex variable representing flow field
w_i	=	Weightage function of ith node
x_j, y_j	=	Coordinate of point j (point of maximum uplift pressure downstream of filter)
z	=	$x+iy$ = complex variable representing physical plane
α	=	Substitution parameter, (Chapter-2)
$\alpha_1, \alpha_2, \alpha_3$	=	Parameters of ellipic integral of the third kind
$\delta, \lambda, \mu, \rho, \sigma$	=	Transformation parameters
γ_w	=	Unit weight of water
γ_{sub}	=	Submerged weight of soil

$\Pi(\varphi_1, \alpha_1^2, m), \Pi(\varphi_2, \alpha_2^2, m'), \Pi(\varphi_3, \alpha_3^2, m)$ = Elliptic integrals third kind

$\Pi_1 = \Pi(\pi/2, \alpha_1^2, m)$ = Complete elliptic integral of third kind

$\Pi_2 = \Pi(\pi/2, \alpha_2^2, m)$ = Complete elliptic integral of third kind

$\Pi_3 = \Pi(\pi/2, \alpha_3^2, m)$ = Complete elliplic integral of third kind

θ	=	A substitution parameter (Chapter 2)
θ	=	Inclination of impervious stratum with horizontal (Chapter 4)
ϕ	=	Potential function
$\varphi_1, \varphi_2, \varphi_3, \varphi', \varphi_1'$	=	Argument of elliplic integrals
φ_D'	=	Argument of elliplic integrals at point D'
φ_F	=	Argument of elliplic integrals at point F
φ_G	=	Argument of elliplic integrals at poing G
ψ	=	Stream function.

LIST OF TABLES

LIST OF FIGURES

CONTENTS

downstream end. The upstream cut-off is provided to safegaurd against possible scour and to reduce uplift pressure below the floor and the downstream cutoff is provided to safegaurd against piping and scour at the downstream end. Intermediate filters or drains can be provided below hydraulic structures to reduce the uplift pressure resulting in appreciable saving.

1.4 A solution of considerable importance was obtained by Maleshchenko (11) and Numerov (13) in which they included the effect of one or two drainage holes in the impervious floor. The effect of plane drainage connected to downstream bed in case of seepage below a flat apron or a single overfall founded on infinite depth of permeable soil was obtained by Zamarin(19). Sangal (16) determined the extent of reduction in pressure affected by a flat and deep filter of particular dimensions below the foundation of a barrage with the help of electrical anology model. Chawla (6) determined the effect of drainage of any dimension located anywhere between two cut-offs of a flat floor founded on infinite depth of permeable soil. Until now, no solution was available to determine the effect of plane drainage located anywhere between the two cut-offs of a flat floor founded on permeable soil of finite depth.

1.5 In many cases where an impermeable layer exists at shallow depth, it is not necessary to take the downstream cut-off upto impermeable layer from piping consideration. However, in some cases the sheet pile cut-off might have been driven upto

CHAPTER - 1

INTRODUCTION

1.1 The design of hydraulic structures founded on permeable soils presents problems of complex nature. Besides testing such structures for forces due to surface flow, their stability has to be ensured against forces caused by percolating water. The seepage water exerts uplift pressure below the structure and tends to wash away soil under it leading to piping. In most of the cases, the damage to such structures may be attributed to the destructive effect of the seepage. The importance of proper design and planning of hydraulic structures founded on permeable soil is, therefore, obvious.

1.2 General solution to the problem of seepage below structures, which is a confined flow problem, by conformal mapping was indicated by Pavlovsky (14) and Khosla(10). Based on conformal transformation, general solutions for various boundaries have been given by Musket (12), Harr (9), Aravin and Numerov(1), Polubrarinova - Kochina (15) and Chawla (6). These solutions are applicable for flat floor with an asymmetrical cut-off, inclined floors, inclined cut-offs, depressed floor with symmetric or asymmetric cut-offs, with infinite and finite pervious reaches. Some of these solutions have been obtained for anisotropic soils also.

1.3 It is seen that generally the design engineer has to provide two cut-offs, one at the upstream end and other at the downstream end. The upstream cut-off is provided to safegaurd against possible scour and to reduce uplift pressure below the floor and the downstream cutoff is provided to safegaurd against piping and scour at the downstream end. Intermediate filters or drains can be provided below hydraulic structures to reduce the uplift pressure resulting in appreciable saving.

1.4 A solution of considerable importance was obtained by Maleshchenko (11) and Numerov (13) in which they included the effect of one or two drainage holes in the impervious floor. The effect of plane drainage connected to downstream bed in case of seepage below a flat apron or a single overfall founded on infinite depth of permeable soil was obtained by Zamarin(19). Sangal (16) determined the extent of reduction in pressure affected by a flat and deep filter of particular dimensions below the foundation of a barrage with the help of electrical anology model. Chawla (6) determined the effect of drainage of any dimension located anywhere between two cut-offs of a flat floor founded on infinite depth of permeable soil. Until now, no solution was available to determine the effect of plane drainage located anywhere between the two cut-offs of a flat floor founded on permeable soil of finite depth.

1.5 In many cases where an impermeable layer exists at shallow depth, it is not necessary to take the downstream cut-off upto impermeable layer from piping consideration. However, in some cases the sheet pile cut-off might have been driven upto impermeable layer due to inadequate sub-surface investigations or to safe guard against scour. In such a situation uplift pressures equivalent to upstream water level would develop below the floor unless an arrangement is made to release the pressure. Alternately a fully penetrating cut-off may be provided at the upstream end of the floor also to check the seepage below the floor thereby reducing uplift pressures. However, providing a fully penetrating cut-off at the upstream end for intercepting the seepage below floor is not a positive and foolproof measure and therefore, cannot be relied upon. In such a situation, intermediate filters could, however, be provided to release uplift pressure.

1.6 In the present study, an attempt has been made to study the effect of intermediate filter with different boundary conditions. The cases with the following boundary conditions have been studied:

(i) Flow under a weir with unequal partial cut-offs at both ends of the floor and an intermediate filter founded on permeable soil of finite depth (Chapter 2).

(ii) Flow under a weir with a partial cut-off at upstream end, a complete cut-off at downstream end of the floor and an intermediate filter founded on permeable soil of finite depth (Chapter 3).

(iii) Flow under a weir with unequal partial cut-offs at both ends of the floor and an intermediate filter founded on permeable soil underlain by a sloping impervious stratum (Chapter 4).

1.7 The solution of the problems of first two cases has been obtained with the help of conformal mapping. The transformation equations have been integrated numerically using Simpson's formula. A computer programme has been developed to evaluate the intermediate transformation parameters and uplift pressure all along the floor and also exit gradient at the end of floor.

The solution of the problem in the third case is obtained by solving the Laplace equation using finite element method. A computer programme has been developed to compute the uplift pressure all along the floor and exit gradient at the end of the floor. The factor of safety against heave below the filter is also determined to study the safety against piping below the filter.

CHAPTER - 2

THEORETICAL SOLUTION FOR INTERMEDIATE FILTER WITH HORIZONTAL IMPERVIOUS LAYER

2.1 INTRODUCTION

The stability of hydraulic structure founded on permeable soil has to be ensured against uplift pressure and piping. Intermediate filters or drains can be provided below hydraulic structures founded on permeable soil to reduce uplift pressures resulting in appreciable savings. Chawla (6) determined the effect of drainage of any dimension located anywhere between two end cutoffs of a flat floor founded on infinite depth of permeable soil. Until now, no solution was available to determine the effect of plane drainage located anywhere between the two end cutoffs of a flat floor founded on permeable soil of finite depth. In this chapter, a solution has been obtained using conformal mapping for seepage below a flat floor with two unequal cutoffs and a plane drainage located anywhere between the two cut offs and founded on permeable soil of finite depth.

2.2 LAYOUT, BOUNDARY CONDITIONS AND METHOD OF SOLUTION

Consider an impervious floor AB of length b founded on a homogeneous permeable soil of depth T. The floor has a cutoff, C'D' of depth d_1 at the upstream end and a cutoff ED of depth d_2 at the downstream end of the floor. The floor is drained by an intermediate filter FG of length f at distance b_1 from the upstream cutoff. The structure is founded on permeable soil of depth T under-lain by an impervious stratum MM_1. The profile is represented in the z- plane as shown in Fig. 2.1(a).

The steady seepage through the pervious foundation of the structure that causes uplift is governedby Laplace equation

$$\nabla^2 \phi = 0 \qquad \ldots(2.1)$$

in which ϕ = velocity function = $-kh$, h = the head, and k = the hydraulic conductivity.

Without loss of generality, the head h along the down-stream bed can be taken as zero and head measured above this $h = H_1$ as the head in the filter and $h = H_2$ as the head in the upstream bed. Then along the upstream bed MA, $\phi = -kH_2$ and along the downstream bed BM_1, $\phi = 0$. The foundation profile AD'C'F, forms the inner boundary of upstream flow and therefore can be taken as the stream line $\psi = 0$, in which ψ is stream function. The lower profile of filter FG, is an equipotential line, $\phi = -kH_1$. The foundation profile GEDB forms the inner boundary of downstream flow and may be taken as stream line, $\psi = q_1$ in which q_1 = discharge per unit width normal to direction of flow, drained through the filter. Starting from somewhere at the upstream end the streamline $\psi = q_1$, would meet the floor GEDB, at some point J where it would divide into two streamlines, one along JG emerging at G and other along JEDB emerging at B. The potential along the floor GJEDB would be maximum at J. The impervious boundary MM_1 forms another streamline $\psi = q_2$ in which q_2 = total discharge seeping below the foundation. The complex potential is represented by

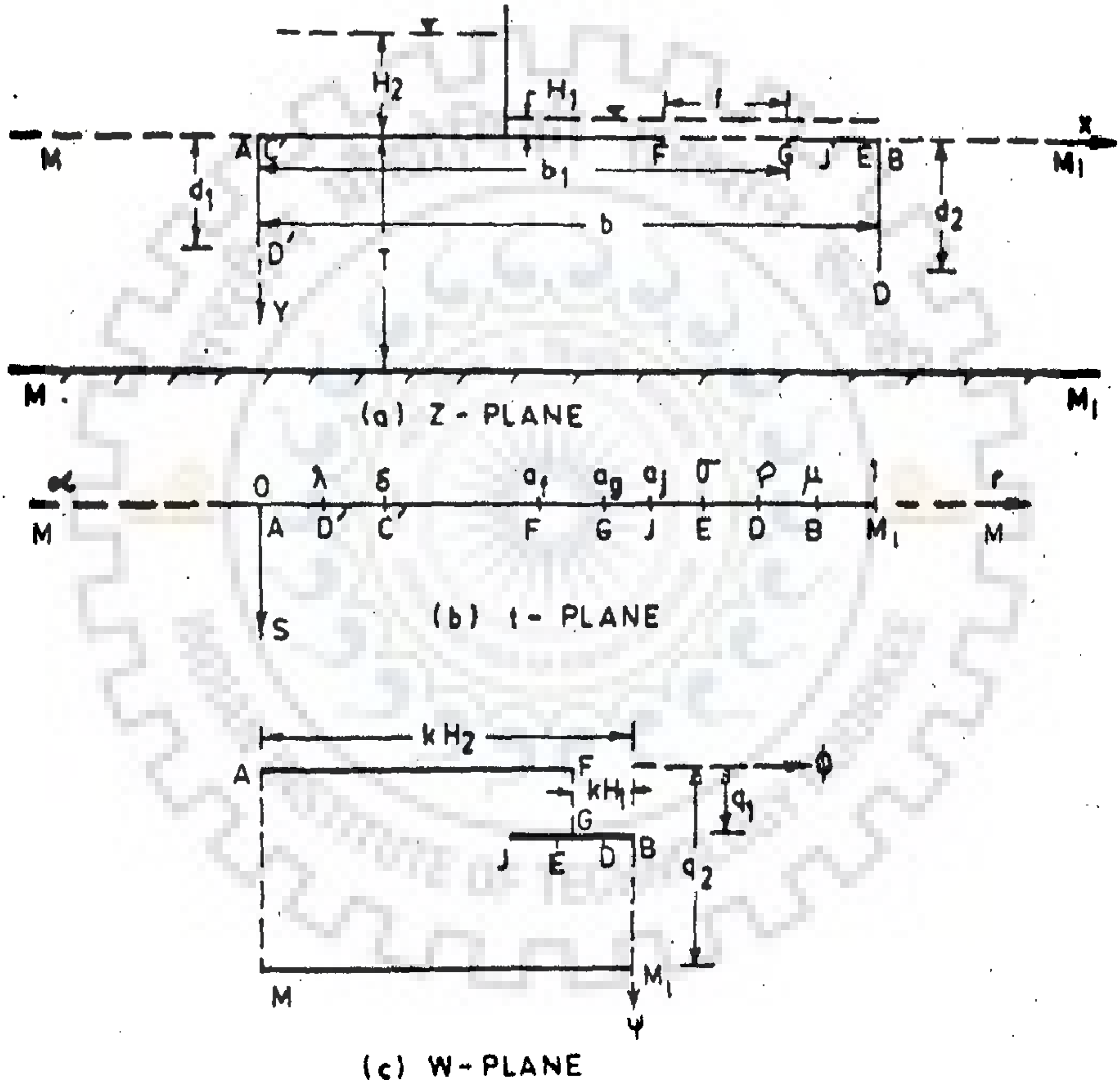


FIG. 2.1 - TRANSFORMATION LAYOUT

$w = \phi + i\psi$. The layout of various boundaries in the w-plane is shown in Fig. 2.1(c). The region of complex seepage potential is the area in w-plane between the two vertical lines AM ($\phi = -kH_2$) and BM_1($\phi = 0$) and bounded on the upper side by the lines AF($\psi = 0$), FG($\phi = -kH_1$) and GJEDB ($\psi = q_1$) and on the lower side by MM_1($\psi = q_2$).

To obtain the solution, both the profiles of structure in the z-plane and the complex seepage potential in the w-plane have been transformed into lower half of same semi-infinite t-plane using the Schwartz-Christoffel transformation. The following relations are thus obtained:

$$z = f(t) \qquad \ldots(2.2)$$

$$w = F(t) \qquad \ldots(2.3)$$

combining Eqs. 2.2 and 2.3

$$z = f(t) = fF^{-1}(w) \qquad \ldots(2.4)$$

and

$$w = F(t) = Ff^{-1}(z) \qquad \ldots(2.5)$$

in which $z = x + iy$ represents the physical plane, $t = r + is$ represents the intermediate semi-infinite plane, and $w = \phi + i\psi$ represents the flow field.

2.3 THEORETICAL SOLUTION

2.3.1 First Operation z = f(t)

In this operation the profile of the hydraulic structure in the z-plane is transformed onto the real axis of the t-plane. On the t-plane, the points A and M_1, are placed at 0 and 1 and the points D',C', F,G,E,D, and B lie at λ, δ, a_F, a_G, σ, ρ

and μ respectively. The values of these parameters are to be determined. The Schwarz-Christoffel transformation that gives the afore-mentioned mapping is:

$$\frac{dz}{dt} = M \frac{(t-\lambda)\ (t-\rho)}{(1-t)\ \sqrt{t(t-\delta)(t-\sigma)(t-\mu)}} \qquad \ldots(2.6)$$

This equation has been integrated in various portions of the seepage boundaries.

Along Upstream Cutoff AD'C' ($0 < t \leq \delta$):

For integration of Eq. 2.6 along the upstream cutoff AD'C', a substitution $t = \delta \mathrm{Sin}^2\ \theta$ is made and the Eq. 2.6 yields

$$y = 2M \int_0^{\theta} \frac{f_1(\theta)}{f_2(\theta)}\ d\theta \qquad \ldots(2.7)$$

in which

$$f_1(\theta) = (\delta \mathrm{Sin}^2\theta - \lambda)\ (\delta \mathrm{Sin}^2\theta - \rho) \qquad \ldots(2.8)$$

$$f_2(\theta) = (1-\delta \mathrm{Sin}^2\theta)\ \sqrt{(\sigma-\delta \mathrm{Sin}^2\theta)(\mu-\delta \mathrm{Sin}^2\theta)} \qquad \ldots(2.9)$$

At point D', $y = d_1$ and $t = \lambda$ i.e. $\theta = \mathrm{Sin}^{-1}\sqrt{\lambda/\delta}$, the Eqn. 2.7 reduces to

$$\frac{d_1}{2M} = \int_0^{\mathrm{Sin}^{-1}\sqrt{\lambda/\delta}} \frac{f_1(\theta)}{f_2(\theta)}\ d\theta \qquad \ldots(2.10)$$

At point C', $y = 0$ and $t = \delta$ i.e. $\theta = \pi/2$, the Eqn. 2.7 reduces to

$$\int_0^{\pi/2} \frac{f_1(\theta)}{f_2(\theta)}\, d\theta = 0 \qquad \ldots(2.11)$$

Eq. 2.11 can be written as

$$(\lambda+\rho)\int_0^{\pi/2} \frac{\delta \operatorname{Sin}^2\theta}{f_2(\theta)}\, d\theta - \lambda\rho\int_0^{\pi/2} \frac{d\theta}{f_2(\theta)} = \int_0^{\pi/2} \frac{\delta^2 \operatorname{Sin}^4\theta}{f_2(\theta)}\, d\theta \ldots(2.12)$$

Along the downstream cutoff EDB ($\sigma < t \leqslant \mu$):

For integrating Eq. 2.6 along downstream cutoff EDB the substitution $t = \mu \operatorname{Sin}^2\theta + \sigma \operatorname{Cos}^2\theta$ is made and Eq. 2.6 reduces to

$$y = 2M \int_0^{\theta} -\frac{f_3(\theta)}{f_4(\theta)}\, d\theta \qquad \ldots(2.13)$$

in which

$$f_3(\theta) = (\mu \operatorname{Sin}^2\theta + \sigma \operatorname{Cos}^2\theta - \lambda)(\mu \operatorname{Sin}^2\theta + \sigma \operatorname{Cos}^2\theta - \rho) \ldots(2.14)$$

$$f_4(\theta) = (1-\mu \operatorname{Sin}^2\theta - \sigma \operatorname{Cos}^2\theta)\sqrt{(\mu \operatorname{Sin}^2\theta + \sigma \operatorname{Cos}^2\theta)(\mu \operatorname{Sin}^2\theta + \sigma \operatorname{Cos}^2\theta - \delta)}$$

$$\ldots(2.15)$$

At point D, $y = d_2$ and $t=\rho$ i.e. $\theta = \operatorname{Sin}^{-1}\sqrt{\frac{\rho-\sigma}{\mu-\sigma}}$, the Eq. 2.13 reduces to,

$$\frac{d_2}{2M} = \int_0^{\operatorname{Sin}^{-1}\sqrt{\frac{\rho-\sigma}{\mu-\sigma}}} -\frac{f_3(\theta)}{f_4(\theta)}\, d\theta \qquad \ldots(2.16)$$

At point B, $y = 0$ and $t = \mu$ i.e. $\theta = \pi/2$, the Eqn. 2.13 reduces to

$$\int_0^{\pi/2} \frac{f_3(\theta)}{f_4(\theta)}\, d\theta = 0 \qquad \ldots(2.17)$$

Eq. 2.17 can be written as

$$(\lambda+\rho)\int_0^{\pi/2} \frac{(\mu \mathrm{Sin}^2\theta+\sigma\ \mathrm{Cos}^2\theta)}{f_4(\theta)}\, d\theta - \lambda\rho\int_0^{\pi/2} \frac{d\theta}{f_4(\theta)}$$

$$= \int_0^{\pi/2} \frac{(\mu \mathrm{Sin}^2\theta+\sigma\ \mathrm{Cos}^2\theta)^2}{f_4(\theta)}\, d\theta \qquad \ldots(2.18)$$

Along the floor C'E ($\delta < t \leqslant \sigma$):

For integrating along the floor the substitution $t = \sigma\ \mathrm{Sin}^2\theta+\delta \mathrm{Cos}^2\theta$ is made and Eq. 2.6 reduces to:

$$\frac{x}{2M} = \int_0^{\theta} -\frac{f_5(\theta)}{f_6(\theta)}\, d\theta \qquad \ldots(2.19)$$

in which

$$f_5(\theta) = (\sigma \mathrm{Sin}^2\theta+\delta \mathrm{Cos}^2\theta-\lambda)(\sigma \mathrm{Sin}^2\theta+\delta\ \mathrm{Cos}^2\theta-\rho) \qquad \ldots(2.20)$$

$$f_6(\theta) = (1-\sigma \mathrm{Sin}^2\theta-\delta \mathrm{Cos}^2\theta)\sqrt{(\sigma \mathrm{Sin}^2\theta+\delta \mathrm{Cos}^2\theta)(\mu-\sigma \mathrm{Sin}^2\theta-\delta \mathrm{Cos}^2\theta)} \qquad \ldots(2.21)$$

At point E, x = b and t = σ i.e. $\theta=\pi/2$, the Eq. 2.19 reduces to

$$\frac{b}{2M} = \int_0^{\pi/2} -\frac{f_5(\theta)}{f_6(\theta)}\, d\theta \qquad \ldots(2.22)$$

At point M_1 $(t = 1)$:

At point, M_1, as t passes around a semi-circle of small radius $r = (t-1)\, e^{-i\theta}$, the corresponding change in z is iT. The Eq. 2.6 can be written as

$$iT = \lim_{r\to 0} \int_{\pi}^{0} \frac{M(1+re^{i\theta}-\lambda)(1+r\, e^{i\theta}-\rho)ire^{i\theta}}{-re^{i\theta}\sqrt{(1+re^{i\theta})(1+re^{i\theta}-\delta)(1+re^{i\theta}-\sigma)(1+re^{i\theta}-\mu)}}\, d\theta$$

On simplification, this equation reduces to

$$\frac{T}{2M} = \frac{0.5\ \pi(1-\lambda)(1-\rho)}{\sqrt{(1-\delta)(1-\sigma)(1-\mu)}} \qquad \text{...(2.23)}$$

On dividing the Eqs. 2.10, 2.22, 2.23 and 2.19 by Eq. 2.16, these are reduced to

$$\frac{d_1}{d_2} = \frac{1}{F(\theta)} \int_{0}^{\sin^{-1}\sqrt{\lambda/\delta}} - \frac{f_1(\theta)}{f_2(\theta)}\, d\theta \qquad \text{...(2.24)}$$

$$\frac{b}{d_2} = \frac{1}{F(\theta)} \int_{0}^{\pi/2} \frac{f_5(\theta)}{f_6(\theta)}\, d\theta \qquad \text{...(2.25)}$$

$$\frac{T}{d_2} = \frac{-1}{F(\theta)} \frac{0.5(1-\lambda)(1-\rho)\pi}{\sqrt{(1-\delta)(1-\sigma)(1-\mu)}} \qquad \text{...(2.26)}$$

$$\frac{x}{d_2} = \frac{1}{F(\theta)} \int_{0}^{\theta} \frac{f_5(\theta)}{f_6(\theta)}\, d\theta \qquad \text{...(2.27)}$$

in which

$$F(\theta) = \int_{0}^{\sin^{-1}\sqrt{\frac{\rho-\sigma}{\mu-\sigma}}} \frac{f_3(\theta)}{f_4(\theta)}\, d\theta \qquad \text{...(2.28)}$$

At point F, $x=b_1 - f$ and $t = a_F$ therefore, Eq. 2.27 gives

$$\frac{b_1 - f}{d_2} = \frac{1}{F(\theta)} \int_0^{\sin^{-1}\sqrt{\frac{a_F-\delta}{\sigma-\delta}}} \frac{f_5(\theta)}{f_6(\theta)} \, d\theta \qquad \ldots(2.29)$$

At point G, $x = b_1$, and $t = a_G$ therefore Eq. 2.27 gives

$$\frac{b_1}{d_2} = \frac{1}{F(\theta)} \int_0^{\sin^{-1}\sqrt{\frac{a_G-\delta}{\sigma-\delta}}} \frac{f_5(\theta)}{f_6(\theta)} \, d\theta \qquad \ldots(2.30)$$

Eqs. 2.12, 2.18, 2.24, 2.25, 2.26, 2.29 and 2.30 make it possible to determine the value of $\lambda, \rho, \delta, \sigma, \mu$, a_F and a_G

Since these equations are not explicit in transformation parameters, direct solution of these equations for the values of these parameters is not convenient. These equations can, however, be conveniently solved for physical dimensions b/d_2, d_1/d_2, x/d_2 and T/d_2 for assumed values of δ, σ, μ and t.

2.3.2 Second Operation $w = F(t)$

In this operation the flow field in the w-plane, shown in Fig. 1(c) is transformed on the semi-infinite t-plane, shown in Fig. 1(b). The transformation of the polygon in w-plane on the t-plane is given by

$$\frac{dw}{dt} = N \frac{(t-a_j)}{\sqrt{t(t-a_F)(t-a_G)(\mu-t)(1-t)}} \qquad \ldots(2.31)$$

Integrating Eq. 2.31 along GJB ($a_G \leq t < \mu$):

Making substitution $t = \mu \sin^2\alpha + a_G \cos^2\alpha$ to remove the singularities at points G and B and integrating the Eq. 2.31 along GJB, it yields

$$\phi = -2N \int_{\alpha_t}^{\pi/2} \frac{f_1(\alpha)}{f_2(\alpha)} \, d\alpha \qquad \ldots(2.32)$$

in which $\alpha_t = \mathrm{Sin}^{-1}\sqrt{\frac{t-a_G}{\mu-a_G}}$

$$f_1(\alpha) = (a_j-\mu \mathrm{Sin}^2\alpha - a_G \mathrm{Cos}^2\alpha)$$

$$f_2(\alpha) = \sqrt{(\mu \mathrm{Sin}^2\alpha + a_G \mathrm{Cos}^2\alpha)\, R}$$

$$R = (\mu \mathrm{Sin}^2\alpha + a_G \mathrm{Cos}^2\alpha - a_F)(1-\mu \mathrm{Sin}^2\alpha - a_G \mathrm{Cos}^2\alpha)$$

At point G, $\phi = -kH_1$ and $t = a_G$, therefore

$$kH_1 = 2N \int_{0}^{\pi/2} \frac{f_1(\alpha)}{f_2(\alpha)} \, d\alpha \qquad \ldots(2.34)$$

Integrating Eq. 2.31 along AF ($0 < t \leqslant a_F$):

Integrating the Eq. 2.31 along AF after substituting $t = a_F \mathrm{Sin}^2\alpha$, we obtain

$$\phi + kH_2 = 2N \int_{0}^{\mathrm{Sin}^{-1}\sqrt{t/a_F}} \frac{f_3(\alpha)}{f_4(\alpha)} \, d\alpha \qquad \ldots(2.35)$$

in which

$$f_3(\alpha) = (a_F \mathrm{Sin}^2\alpha - a_j) \qquad \ldots(2.36)$$

$$f_4(\alpha) = \sqrt{(a_F \mathrm{Sin}^2\alpha - a_G)\,(\mu - a_F \mathrm{Sin}^2\alpha)(1-a_F \mathrm{Sin}^2\alpha)} \qquad \ldots(2.37)$$

At point F, $t = a_F$ and $\phi = -kH_1$, therefore

$$k(H_2-H_1) = 2N \int_{0}^{\pi/2} \frac{f_3(\alpha)}{f_4(\alpha)} \, d\alpha \qquad \ldots(2.38)$$

From Eqs. 2.34 and 2.38

$$r = \frac{H_1}{H_2-H_1} = \frac{\int_0^{\pi/2} \frac{f_1(\alpha)}{f_2(\alpha)}\, d\alpha}{\int_0^{\pi/2} \frac{f_3(\alpha)}{f_4(\alpha)}\, d\alpha} \qquad \ldots(2.39)$$

Solving for a_j, we obtain

$$a_j = \frac{r\int_0^{\pi/2} \frac{a_F \mathrm{Sin}^2\alpha \, d\alpha}{f_4(\alpha)} + \int_0^{\pi/2} \frac{(\mu \mathrm{Sin}^2\alpha + a_G \mathrm{Cos}^2\alpha)\, d\alpha}{f_2(\alpha)}}{r\int_0^{\pi/2} \frac{d\alpha}{f_4(\alpha)} + \int_0^{\pi/2} \frac{d\alpha}{f_2(\alpha)}} \qquad \ldots(2.40)$$

From Eq. 2.40 value of a_j can be determined

2.4 UPLIFT PRESSURES

Uplift pressures below the upstream floor AD'C'F can be determined from Eq. 2.35 after substituting the value of N from Eq. 2.38 as below

$$\frac{\phi + kH_2}{k(H_2-H_1)} = \frac{\int_0^{\mathrm{Sin}^{-1}\sqrt{t/a_F}} \frac{f_3(\alpha)}{f_4(\alpha)}\, d\alpha}{\int_0^{\pi/2} \frac{f_3(\alpha)}{f_4(\alpha)}\, d\alpha} \qquad \ldots(2.41)$$

Uplift pressure below downstream floor, GJEDB can be determined from Eq. 2.32 after substituting the value of N from Eq. 2.38 as below

$$\frac{\phi}{k(H_2-H_1)} = \frac{\int_{\alpha_t}^{\pi/2} \frac{f_1(\alpha)}{f_2(\alpha)} d\alpha}{\int_{0}^{\pi/2} \frac{f_3(\alpha)}{f_4(\alpha)} d\alpha} \qquad \ldots(2.42)$$

in which

$$\alpha_t = \sin^{-1}\sqrt{\frac{t-a_G}{\mu-a_G}}$$

Knowing the values of a_F, a_G, a_j, μ, the uplift pressure can be determined at any point below the floor by substituting the corresponding value of t in Eq. 2.41 or 2.42 depending whether point is upstream or downstream of filter FG.

2.5 EXIT GRADIENT

In addition to the uplift pressure, it is also important to know the hydraulic gradient at the downstream end of the floor i.e. at point B.

Gradient I at any point is given by

$$I = \frac{dh}{ds} \qquad \ldots(2.43)$$

in which h = the head at any point along the floor or the cut-off and s = distance measured along the streamline passing that point. Eq. 2.43 can be written as

$$I = \frac{1}{k}\frac{d\phi}{ds} = \frac{1}{k} \cdot \frac{d\phi}{dt} \cdot \frac{dt}{dz} \cdot \frac{dz}{ds} \qquad \ldots(2.44)$$

Along the outer face of the downstream cutoff,

$$\frac{dz}{ds} = i \qquad \ldots(2.45)$$

Also along the floor and cutoff Ψ = constant, therefore

$$\frac{d\phi}{dt} = \frac{dw}{dt} \qquad \ldots(2.46)$$

Substituting the values of constants M and N, the values of dt/dz and dw/dt can be determined from Eqs. 2.6 and 2.31 respectively, therefore

$$\frac{dt}{dz} = \frac{\pi(1-t)(1-\lambda)(1-\rho)}{T(t-\lambda)\ (t-\rho)} \sqrt{\frac{(t(t-\delta)(t-\sigma)(t-\mu)}{(1-\delta)(1-\sigma)(1-\mu)}} \qquad \ldots(2.47)$$

$$\frac{dw}{dt} = \frac{0.5\ k(H_2-H_1)}{\int_0^{\pi/2} \frac{(a_F \text{Sin}^2\alpha - a_j)\ d\alpha}{\sqrt{(a_F\text{Sin}^2\alpha - a_G)(\mu - a_F\text{Sin}^2\alpha)(1-a_F\text{Sin}^2\alpha)}}} \text{ x } \frac{(t-a_j)}{\sqrt{t(t-a_F)(t-a_G)(\mu-t)(1-t)}} \qquad \ldots(2.48)$$

Combining the Eqs. 2.44 to 2.48 and simplifying the value of GE the exit gradient at point B ($t=\mu$) is given by

$$GE = \frac{0.5\pi(H_2-H_1)(1-\lambda)(1-\rho)(\mu-a_j)}{T(\mu-\lambda)\ (\mu-\rho)\ IG} \sqrt{\frac{(\mu-\delta)\ (\mu-\sigma)\ (1-\mu)}{(1-\delta)(1-\sigma)(1-\mu)(\mu-a_F)(\mu-a_G)}} \qquad \ldots(2.49)$$

in which

$$IG = \int_0^{\pi/2} \frac{(a_F\text{Sin}^2\alpha - a_j)\ d\alpha}{(a_F\text{Sin}^2\alpha - a_G)(\mu - a_F\text{Sin}^2\ \alpha)(1-a_F\text{Sin}^2\alpha)} \qquad \ldots(2.50)$$

Exit gradient GE can be expressed in terms of downstream cutoff depth d_2 also by combining Eqs. 2.49 and 2.26.

2.6 RESULTS

The equations derived herein have been used for computations of uplift pressures and exit gradient. These calculations involve the evaluation of many integrals which were computed on digital computer by numerical methods. For assumed values of δ, σ and μ the physical dimensions d_1/d_2, b/d_2 and T/d_2 are worked out from Eqs. 2.24, 2.25 and 2.26 respectively after finding the values of λ and ρ from Eqs. 2.12 and 2.18. The values of b_1/d_2 and f/d_2 are then worked out from Eqs. 2.29, and 2.30 for assumed values of a_F and a_G. The uplift pressure at key points corresponding to these physical dimensions of the structure are determined from Eqs. 2.41 and 2.42 after determining the value of a_j from the Eq. 2.40. Pressure at any point along the floor can be obtained from Eq. 2.41 or 2.42 for assumed value of t and the corresponding value of x/d_2 is obtained from the Eq. 2.27 in which x is the distance of the point from the upstream cutoff. The exit gradient at point B is then worked out from the Eq. 2.49.

To facilitate the use of various equations for finding out the uplift pressure and exit gradient, the values of uplift pressures at key points and exit gradient have been computed for different combinations of floor length, depth of upstream and downstream cutoffs and depth of soil strata and have been plotted in the form of curves. The results obtained herein can be used for practical design. The curves cover the values of b/d_2 = 2.5, 5 and 10, d_2/d_1 = 1, 2, 3 and 4, f/d_2 = 0.0 to 1.0, anywhere between two cutoffs and T/d_2 = 1.5, 2 and 4. The uplift pressures at D', C', E , D and J have been plotted in Figs. 2.2, 2.3, 2.4, 2.5 and 2.6 respectively. The location of point J is obtained in Fig. 2.7. The exit gradient at point B is plotted in Fig. 2.8.

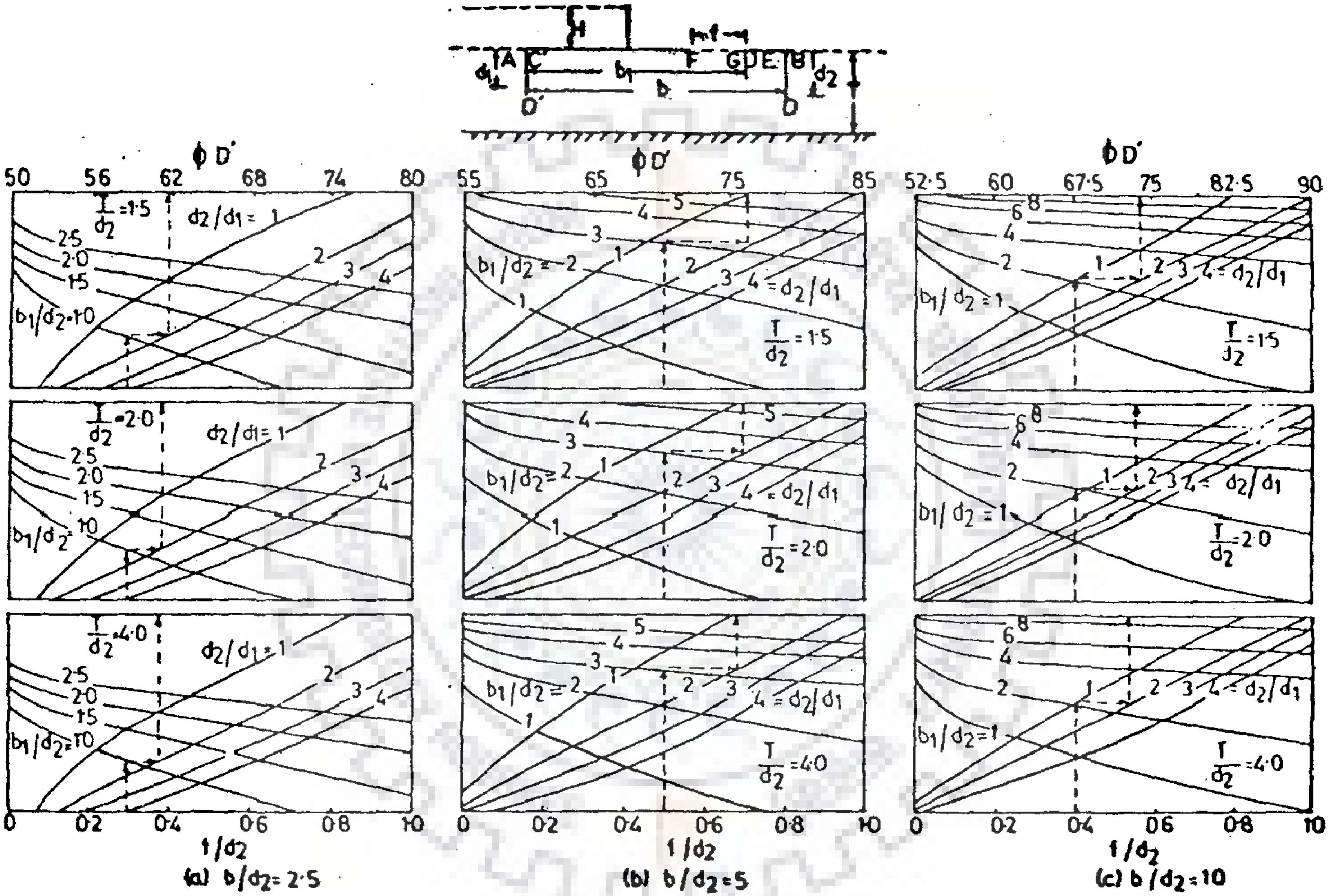
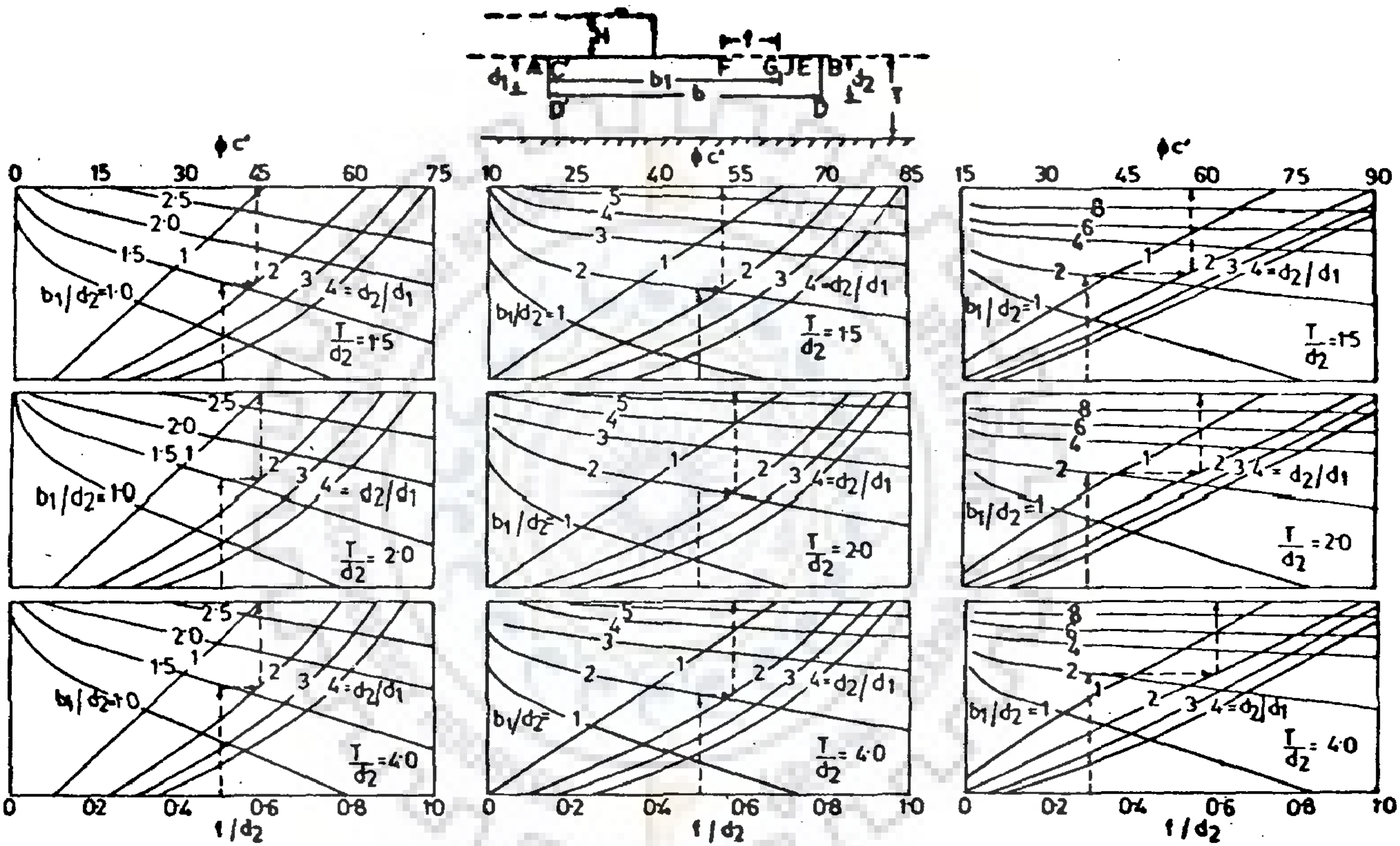
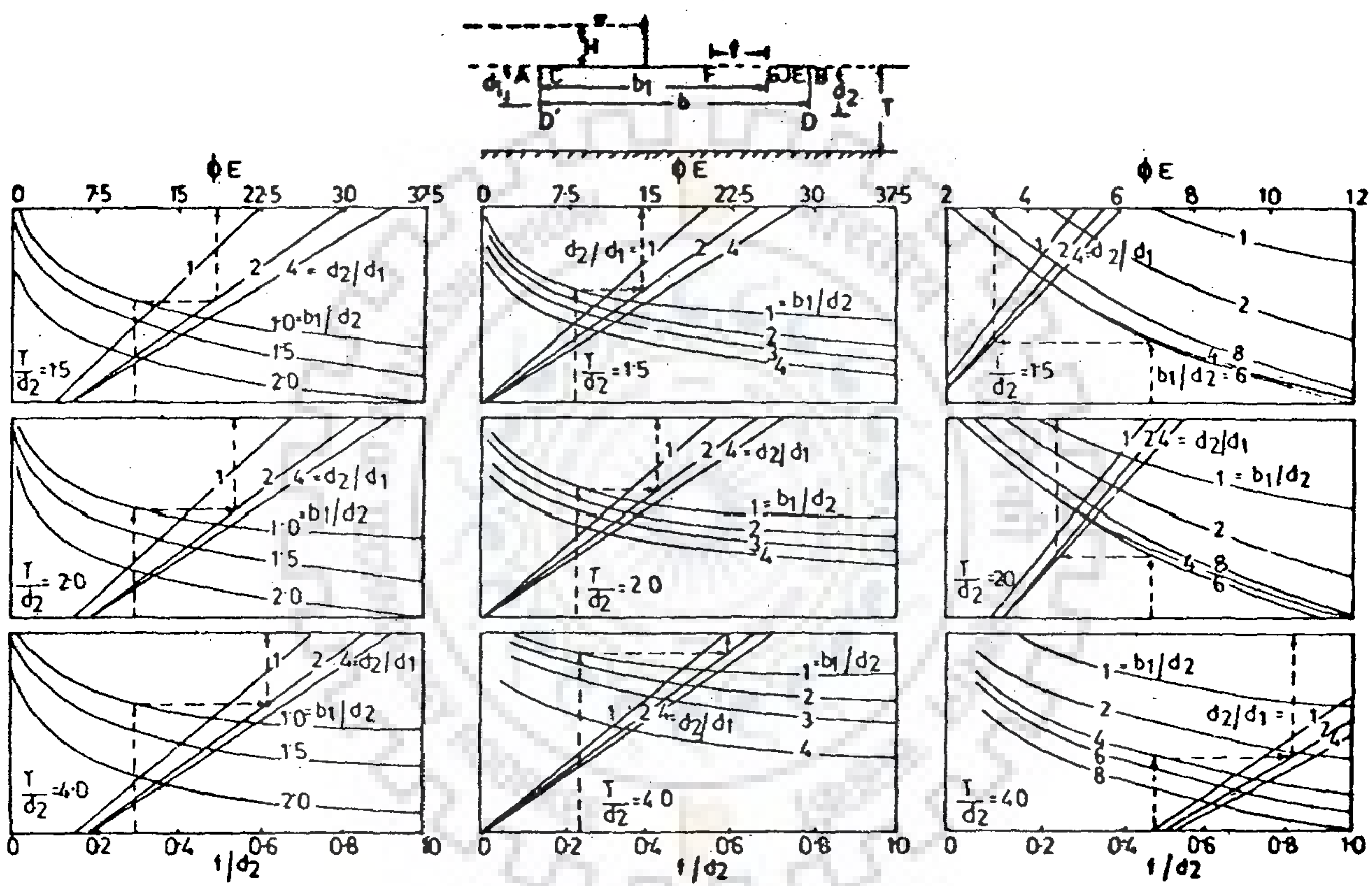


FIG 2.2-UPLIFT PRESSURE AT D' IN PERCENT OF H



(a) $b/d_2 = 2.5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG 2.3-UP LIFT PRESSURE AT C' IN PERCENT OF H



(a) $b/d_2 = 2.5$ (b) $b/d_2 = 5.0$ (c) $b/d_2 = 10.0$

FIG 2.4-UPLIFT PRESSURE AT E IN PERCENT OF H

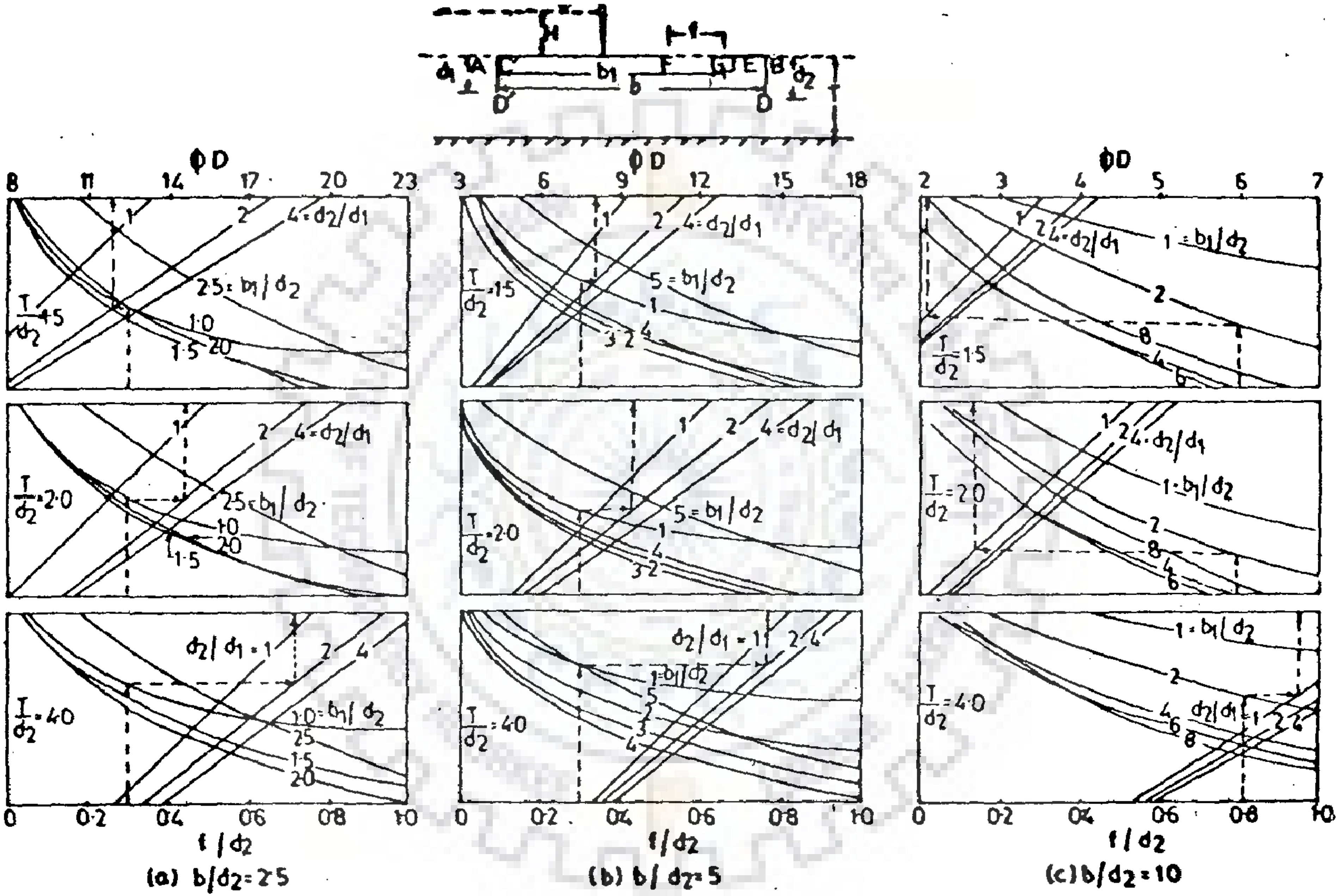


FIG.2.5-UPLIFT PRESSURE AT D IN PERCENT OF H

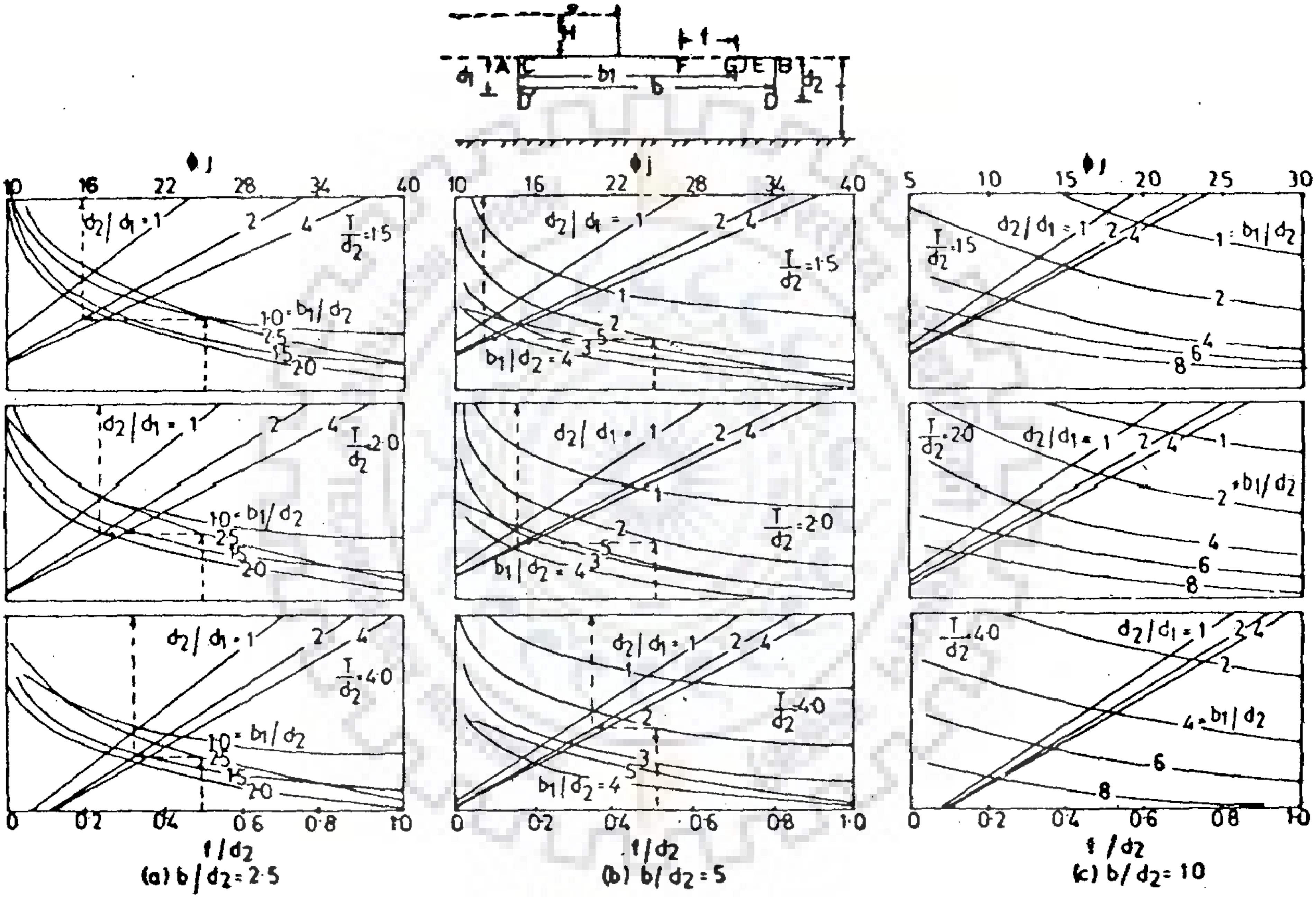


FIG 2.6 - UPLIFT PRESSURE AT J IN PERCENT OF H

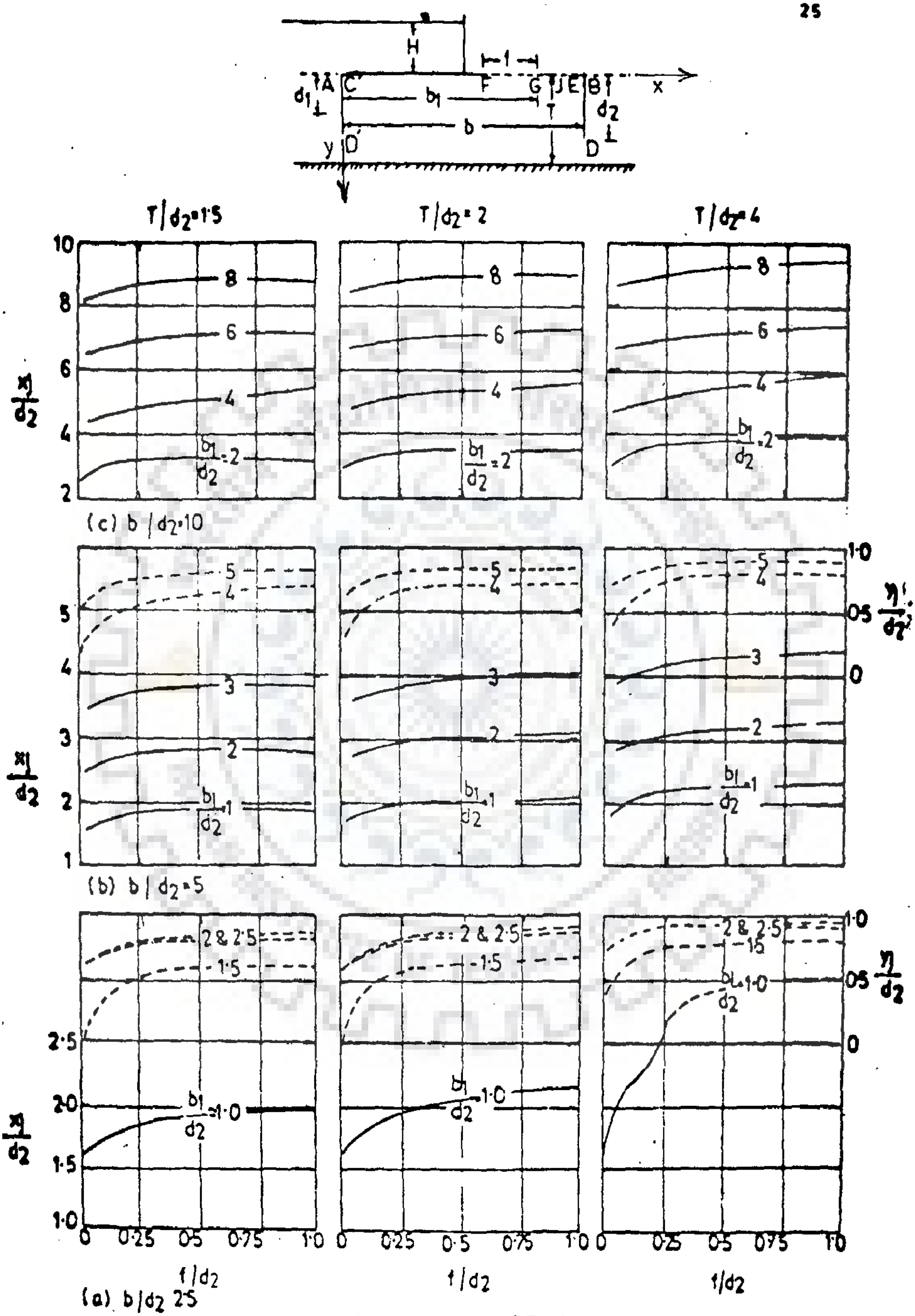


FIG.2.7-LOCATION OF J.

X_1 = ABSCISSA OF POINT J, Y_1 = ORDINATE OF POINT J

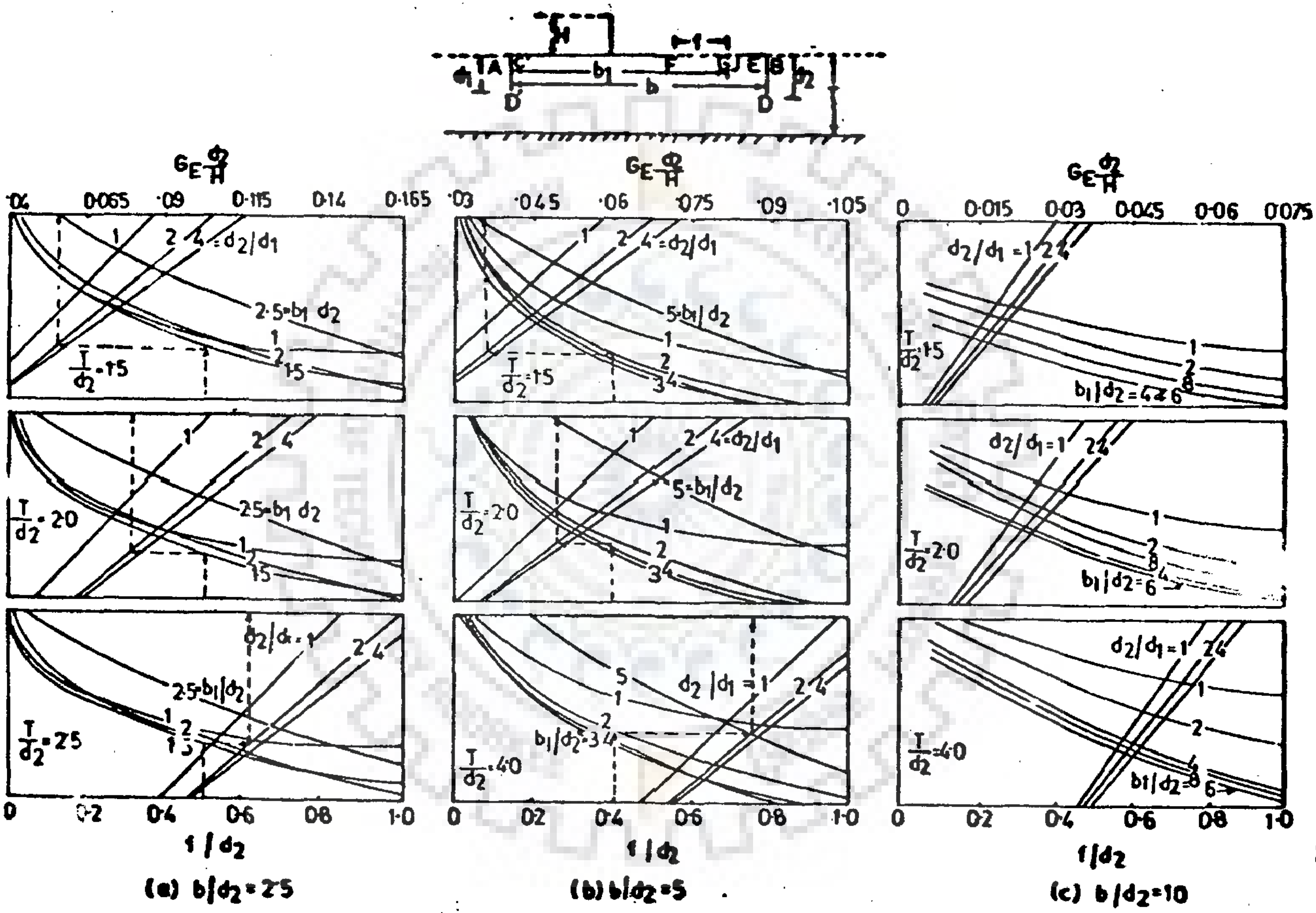


FIG.28 - EXIT GRADIENT AT B

CHAPTER - 3

SOLUTION FOR INTERMEDIATE FILTERS WITH FULLY PENETRATING DOWNSTREAM CUTOFF

3.1 INTRODUCTION

Sheetpile cutoff is provided at the downstream end of the impervious floor of structures founded on permeable soil to ensure stability against piping and scour due to surface flow. Deeper cutoff, although, improves safety against piping but results in increased uplift pressures and higher cost of the structure. Cutoff depth is,therefore, kept minimum to ensure adequate safety factor against scour and piping. In case an impermeable layer exists at shallow depth and the cutoff penetrates the impermeable layer, uplift pressures below the floor will approach pressures equilvalent to upstream waterlevel. From piping consideration, it is not necessary to take the cutoff upto impermeable layer. However, in many cases the sheetpile cutoff might have been driven upto impermeable layer due to inadequate subsurface investigations or to safeguard against scour. In such a situation full uplifts would develop unless an arrangement is made to release the pressures or a fully penetrating cutoff is provided at the upstream end of the floor to intercept the seepage below the floor.

Providing a fully penetrating cutoff at the upstream end for intercepting the seepage below floor is not a foolproof measure. It cannot be relied upon due to possible leakage which would lead to increased pressures. Intermediate filter can be provided to release the uplift pressures. An analysis has been presented in this Chapter to determine the uplift pressure below a floor founded on permeable soil of finite depth with fully penetrating cutoff on the downstream end of the floor and an intermediate filter for release of pressures.

3.2 LAYOUT AND BOUNDARY CONDITIONS

Consider an impervious floor AE of length b founded on a homogenous permeable soil of finite depth T underlain by an impervious stratum. The floor has a cutoff C'D' of depth d at the upstream end and cutoff ED at the downstream end penetrating into the impervious stratum at a depth T. The floor has an intermediate filter FG of length f, at a distance b_1 from the upstream cutoff (i.e. $AG=b_1$).Pervious bed on the upstream and downstream of the floor extends upto infinity. The profile is represented in z-plane and is shown in Fig. 3.1(a).

We shall take head h = 0 on the downstream water level and measured above it $h = H_1$ as the head in the filter and $h=H_2$ as the head in upstream bed. Then along AM, $\phi = -kH_2$ and along the filter FG, $\phi = -kH_1$, where ϕ is the velocity function.

The upstream foundation profile A D' C' F forms the inner boundary of the flow and therefore can be taken as stream line $\psi = 0$, where ψ is the stream function. The lower profile of filter FG is an equipotential line $\phi = -kH_1$. The downstream foundation profile M'DEG forms the another boundary of flow and may be taken as stream line $\psi = q_1$, where q_1 is discharge drained

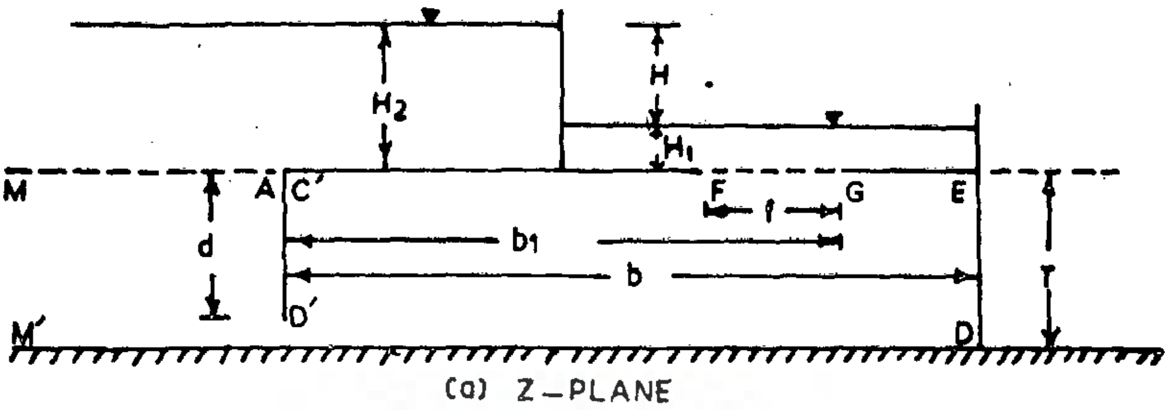


FIG. 3·1 TRANSFORMATION LAYOUT

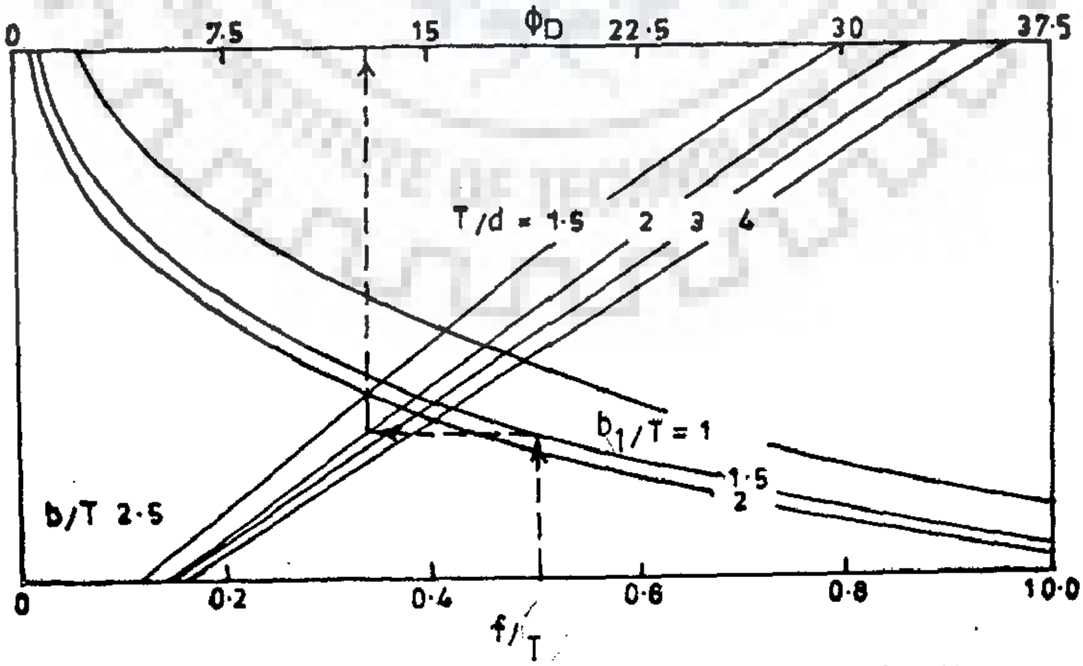


FIG. 3·2 UPLIFT PRESSURE AT D IN PERCENT OF H through the filter per unit width normal to direction of flow.

$w = \phi + i\psi$ represents the complex potential. Layout in the w - plane is shown in Fig. 3.1(c). The region of complex potential is the area in the w - plane between the two vertical lines AM ($\phi = -kH_2$) and FG($\phi = -kH_1$) and bounded by the lines AF ($\psi = 0$) GM' ($\psi = q_1$).

3.3 THEORETICAL SOLUTION

3.3.1 First Operation z = f(t)

In this operation the profile of the structure in the z - plane is transformed onto the real axis of the t - plane (Fig. 3.1(b)). The point A and C' are placed at O and 1, the points D', F, G, E and D lie at λ, a_F, a_G, δ and σ. The values of these five transformation parameters have to be determined.

The Schwartz - Christoffel transformation that gives the above mapping is,

$$\frac{dz}{dt} = M \frac{(t-\lambda)}{\sqrt{t(t-1)(t-\delta)(t-\sigma)}} \qquad \ldots(3.1)$$

This equation can be integrated along various regions to determine the values of the constants.

<u>Along upstream cutoff A D' C' ($0 < t \leq 1$):</u>

$$y = M\left[\int_0^t \sqrt{\frac{t}{(1-t)(\delta-t)(\sigma-t)}}\, dt - \int_0^t \frac{\lambda\, dt}{\sqrt{t(1-t)(\delta-t)(\sigma-t)}}\right]$$

Making use of standard formulae (Ref.(3), Eqs. 252.00, 252.14 and 337.01).

$$\frac{y}{Mg} = (\sigma-\lambda)\ F(\varphi_1, m) - \sigma \Pi\ (\varphi_1, \alpha_1^2, m) \qquad ...(3.02)$$

in which $F(\varphi_1, m)$ and $\Pi(\varphi_1, \alpha_1^2, m)$ are elliptic integral of first and third kind respectively, and

$$g = \frac{2}{\sqrt{(\sigma-1)}} \qquad ...(3.04)$$

$$\alpha_1^2 = \frac{1}{1-\sigma} \qquad ...(3.5)$$

$$\varphi_1 = \mathrm{Sin}^{-1} \sqrt{\frac{(\sigma-1)t}{(\sigma-t)}} \qquad ...(3.6)$$

$$m^2 = \frac{(\sigma-\delta)}{(\sigma-1)\delta} \qquad ...(3.6)$$

At point C', y = 0 and t = 1 therefore, Eq. 3.2 gives

$$\lambda = \sigma(1 - \frac{\Pi_1}{K}) \qquad ...(3.7)$$

in which

$\Pi_1 = \Pi\ (\frac{\pi}{2}, \alpha_1^2, m)$ = complete elliplic integral of third kind.

$K = F\ (\frac{\pi}{2}, m)$ = complete elliplic integral of first kind.

At point D', y = d and t = λ, therefore Eq. 3.2 gives

$$\frac{d}{Mg} = (\sigma-\lambda)\ F\ (\varphi_{D'}, m) - \sigma \Pi(\varphi_{D'}, \alpha_1^2, m) \qquad ...(3.8)$$

in which

$$\varphi_{D'} = \mathrm{Sin}^{-1} \sqrt{\frac{(\sigma-1)\lambda}{(\sigma-\lambda)}} \qquad ...(3.9)$$

Along the floor C'FGE, $(1 < t \leq \delta)$:

Integrating the Eq. 3.1 along the floor

$$x = \frac{M}{i^2}\left[\int_1^t \sqrt{\frac{t}{(t-1)(\delta-t)(\sigma-t)}}\,dt - \int_1^t \frac{\lambda\,dt}{\sqrt{t(1-t)(\delta-t)(\sigma-t)}}\right]$$

Making use of standard formula (Ref. (3) Eq. 254.00, 254.02)

$$\frac{x}{Mg} = \lambda F(\varphi_2, m') - \Pi(\varphi_2, \alpha_2^2, m') \qquad \ldots(3.10)$$

in which

$$\varphi_2 = \operatorname{Sin}^{-1}\sqrt{\frac{(t-1)\delta}{(\delta-1)t}} \qquad \ldots(3.11)$$

$$\alpha_2^2 = \frac{\delta-1}{\delta} \qquad \ldots(3.12)$$

$$m'^2 = 1-m^2 = \frac{(\delta-1)\sigma}{(\sigma-1)\delta} \qquad \ldots(3.13)$$

At point E, $x = b$ and $t = \delta$, therefore Eq. 3.10 gives

$$\frac{b}{Mg} = \lambda K' - \Pi_2' \qquad \ldots(3.14)$$

in which

$$K' = F(\tfrac{\pi}{2}, m') \qquad \ldots(3.15)$$

$$\Pi_2' = \Pi(\tfrac{\pi}{2}, \alpha_2^2, m') \qquad \ldots(3.16)$$

At point, F, $x = b_1 - f$ and $t = a_F$, therefore Eq. 3.10 yields

$$\frac{b_1-f}{Mg} = \lambda F(\varphi_F\, m) - \Pi(\varphi_F, \alpha_2^2, m') \qquad \ldots(3.17)$$

in which

$$\varphi F = \operatorname{Sin}^{-1}\sqrt{\frac{\delta(a_F-1)}{(\delta-1)a_F}} \qquad \ldots(3.18)$$

At point G, $x = b_1$ and $t = a_G$, therefore Eq. 3.10 reduces to

$$\frac{b_1}{Mg} = \lambda\, F(\varphi_G, m') - \Pi\ (\varphi_G, \alpha_2^2, m') \qquad \ldots(3.19)$$

in which

$$\varphi_G = \mathrm{Sin}^{-1} \sqrt{\frac{\delta(a_G-1)}{(\delta-1)a_G}} \qquad \ldots(3.20)$$

Along downstream cutoff ED ($\delta < t \leqslant \sigma$):

Integrating Eq. 3.1 along downstream cutoff ED

$$iy = \frac{M}{i}\left[\int_\delta^t \sqrt{\frac{t}{(t-1)(t-\delta)(\sigma-t)}}\, dt - \int_\delta^t \frac{\lambda\, dt}{\sqrt{t(t-1)(t-\delta)(\sigma-t)}}\right]$$

Making use of standard formulae Ref. (3), Eqs. 256.00, 256.13 and 339.01)

$$\frac{y}{Mg} = \left(\lambda - \frac{\delta m^2}{\alpha_3^2}\right) F\ (\varphi_3, m) - \frac{\delta(\alpha_3^2 - m^2)}{\alpha_3^2}\ \Pi\ (\varphi_3, \alpha_3^2, m) \qquad \ldots(3.21)$$

in which

$$\alpha_3^2 = \frac{\sigma-\delta}{\sigma-1} \qquad \ldots(3.22)$$

$$\varphi_3 = \mathrm{Sin}^{-1} \sqrt{\frac{(\sigma-1)(t-\delta)}{(\sigma-\delta)(t-1)}} \qquad \ldots(3.23)$$

Putting the value of α_3^2 and m in Eq. 3.21, it reduces

$$\frac{y}{Mg} = (\lambda-1)\ F(\varphi_3, m) - (\delta-1)\,\Pi\ (\varphi_3, \alpha_3^2, m) \qquad \ldots(3.24)$$

At point D, $y = T$, and $t = \sigma$, $\varphi_3 = \pi/2$, Eq. 3.24 reduces to

$$T/Mg = (\lambda-1)K - (\delta-1)\Pi_3 \qquad \ldots(3.25)$$

in which

$$\Pi_3 = \Pi\ \left(\frac{\pi}{2},\ \alpha_3^2,\ m\right) \qquad \ldots(3.26)$$

From Eqs. 3.8,3.10,3.14,3.17,3.19 and 3.25

$$\frac{d}{T} = \frac{(\sigma-\lambda)F(\varphi_D', m) - \sigma\,(\varphi_D', \alpha_1^2, m)}{(\lambda-1)\;K-(\delta-1)\Pi_3} \qquad ...(3.27)$$

$$\frac{x}{T} = \frac{\lambda F(\varphi_2, m') - \Pi(\varphi_2, \alpha_2^2, m')}{(\lambda-1)\;K-(\delta-i)\Pi_3} \qquad ...(3.28)$$

$$\frac{b}{T} = \frac{\lambda K' - \Pi_2'}{(\lambda-1)K-(\delta-1)\Pi_3} \qquad ...(3.29)$$

$$\frac{b_1 - f}{T} = \frac{\lambda F(\varphi_F, m') - \Pi\;(\varphi_F, \alpha_2^2, m')}{(\lambda-1)\;K-(\delta-1)\Pi_3} \qquad ...(3.30)$$

and

$$\frac{b_1}{T} = \frac{\lambda F(\varphi_G, m') - \Pi\;(\varphi_G, \alpha_2^2, m')}{(\lambda-1)K\;-\;(\delta-1)\Pi_3} \qquad ...(3.31)$$

Eqs. 3.7, 3.27, 3.29,3.30 and 3.31 enable us to determine the values of λ ,δ,σ, a_F and a_G.

3.3.2 Second Operation $w = F(t)$

In this operation the flow field in w-plane (Fig. 3.1(c)) is transformed on to the same semi-infinite t-plane. The transformation of polygon AFGM in w-plane onto the t-plane is given by

$$\frac{dw}{dt} = N\frac{1}{\sqrt{t(t-a_F)(t-a_G)}} \qquad ...(3.32)$$

This equation can be integrated along various regions to determine the value of the uplift pressures.

<u>Along the portion AD' C'F ($0 \leq t < a_F$) :</u>

Integrating Eq. 3.32 along portion AD'C'F

$$\int_{\phi}^{-kH_1} dw = \frac{N}{i^2} \int_{t}^{a_F} \frac{dt}{\sqrt{t(a_F-t)(a_G-t)}}$$

Making use of standard Formula (Ref. (3), Eq. 233.00)

$$\phi + kH_1 = Ng' \; F(\varphi', m_1) \qquad \ldots(3.33)$$

in which

$$g' = \frac{2}{\sqrt{a_G}} \qquad \ldots(3.4\;)$$

$$\varphi' = \mathrm{Sin}^{-1} \sqrt{\frac{a_G(a_F-t)}{a_F(a_G-t)}} \qquad \ldots(3.35)$$

$$m_1^2 = \frac{a_F}{a_G} \qquad \ldots(3.36)$$

At A, $\phi = -kH_2$ and $t = 0$, therefore, Eq. 3.33 gives

$$Ng' = -\frac{k(H_2-H_1)}{K_1} \qquad \ldots(3.37)$$

where

$$K_1 = F(\tfrac{\pi}{2}, m_1) \qquad \ldots(3.38)$$

<u>Along GM' ($t > a_G > a_F > 0$):</u>

Integrating the Eq. 3.32 along GM'

$$\int_{-kH_1}^{\phi} dw = N \int_{a_G}^{t} \frac{dt}{\sqrt{t(t-a_F)(t-a_G)}}$$

Making use of Standard Formula (Ref. (3), Eq. 237.00)

$$\phi + kH_1 = Ng' \; F(\varphi_1', m_1) \qquad \ldots(3.39)$$

in which

$$\varphi_1' = \sin^{-1} \sqrt{\frac{t-a_G}{t-a_F}} \qquad \ldots(3.40)$$

3.4 UPLIFT PRESSURE

3.4.1 Uplift Pressure Below Upstream Floor AD'C'F

The uplift pressure below the upstream floor can be determined from Eq. 3.33 after substituting the value of Ng'

$$\frac{\phi}{k(H_2 - H_1)} = -\left(\frac{F(\varphi', m_1)}{K_1} + \frac{H_1}{H_2 - H_1}\right) \qquad \ldots(3.41)$$

In case the release in the filter is provided at the downstream water level i.e. $H_1 = 0$, then the Eq. 3.41 reduces to

$$\frac{\phi}{kH_2} = -\frac{F(\varphi', m_1)}{K_1} \qquad \ldots(3.42)$$

4.3.2 Uplift Pressure Below Downstream Floor GED

The uplift below downstream floor GED can be determined from Eq. 3.39 after substituting the value of Ng'

$$\frac{\phi + kH_1}{k(H_2 - H_1)} = -\frac{F(\varphi_1', m_1)}{K_1}$$

or

$$\frac{\phi}{k(H_2 - H_1)} = -\left(\frac{F(\varphi_1', m_1)}{K_1} + \frac{H_1}{H_2 - H_1}\right) \qquad \ldots(3.43)$$

In case release in the filter is provided at the downstream level, $H_1 = 0$, Eq. 3.43 reduces to

$$\frac{\phi}{kH_2} = -\frac{F(\varphi_1', m_1)}{K_1} \qquad \ldots(3.44)$$

Knowing the values of a_F, a_G, the uplift pressure can be determined at any point below the floor by substituting the corresponding value of t in Eq. 3.41 or 3.43 depending upon whether the point is upstream or downstream of filter FG.

3.5 RESULTS

The equations derived above have been used for computation of uplift pressures. Since the equations 3.27 to 3.31 are not explicit in the transformation parameters direct solution of these equations for the values of these parameters is not convenient. These equations, however, can be conveniently used for computing the physical dimensions b/T, d/T and x/T for assumed value of parameters δ, σ and t. The uplift pressure at the corresponding point below the floor can be determined by using the Eq. 3.41 or Eq. 3.43.

A computer programme was written for computation of uplift pressure for various combinations of the values of the variables involved. The calculations involve the use of elliptic functions of first and third kind which were computed by a subroute EFUN.

The uplift pressure at key points was calculated for assumed values of δ and σ and various values of a_F and a_G so as to cover all the possible dimensions of a structure and plotted in the form of curves. The computations have been made for values of b/T = 2.5, 5 and 10, T/d = 1.5, 2, 3 and 4, f/T from 0.0 to 1.0 and various values of b_1/T.

The uplift pressures at D, E, C' and D' obtained for the above values have been plotted in Figs. 3.2, 3.3, 3.4 and 3.5 respectively assuming water level in filter corresponding to the downstream water level (i.e. $H_1 = 0$). The pressure at point D is approximately same as that at point E for $b/T = 5$ and 10. Therefore, uplift pressure at D in Fig.3.2 are plotted only for $b/T = 2.5$ and can be taken same as for point E for $b/T = 5$ and 10.

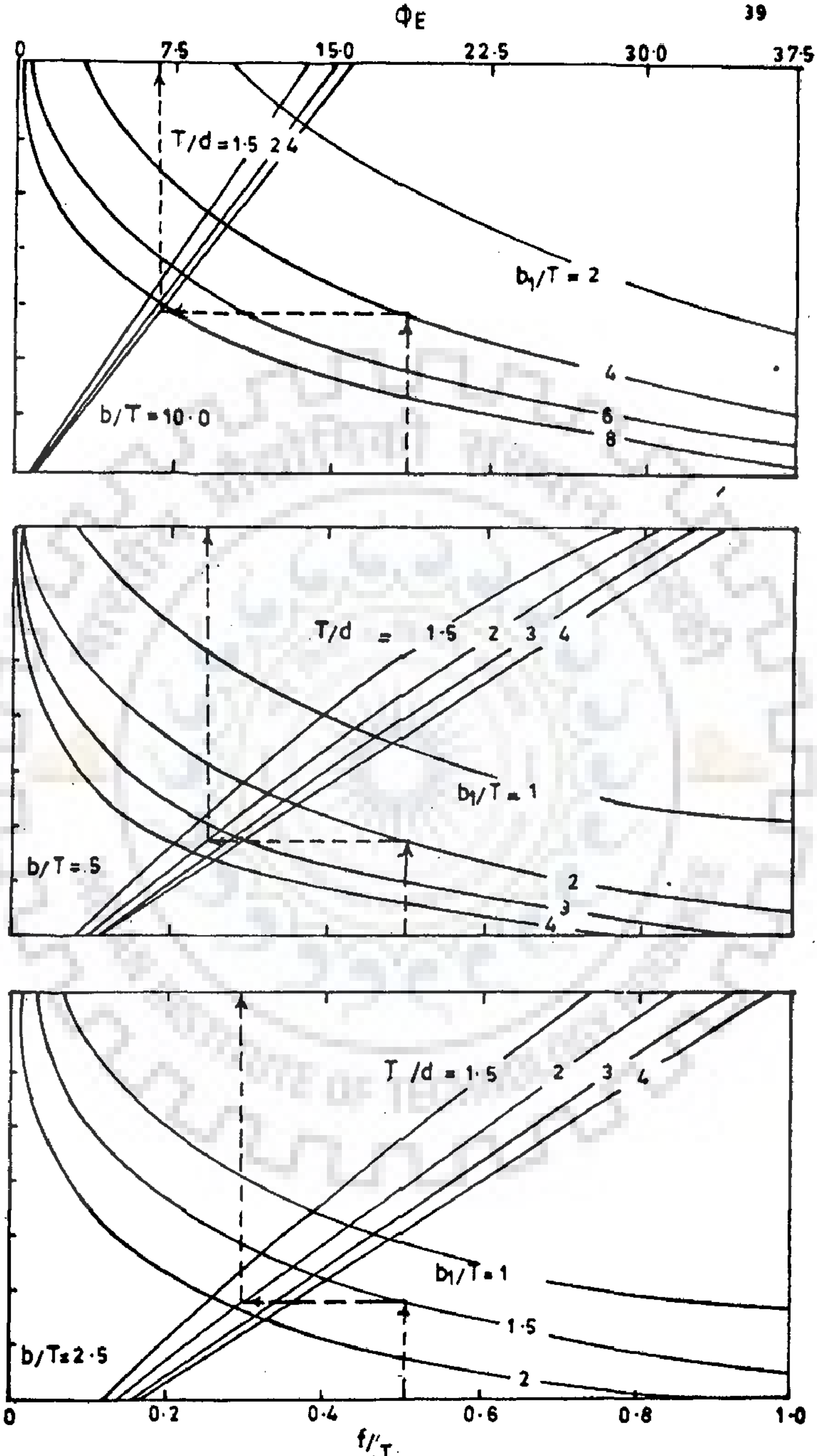


FIG. 3·3 – UPLIFT PRESSURE AT E IN PERCENT OF H

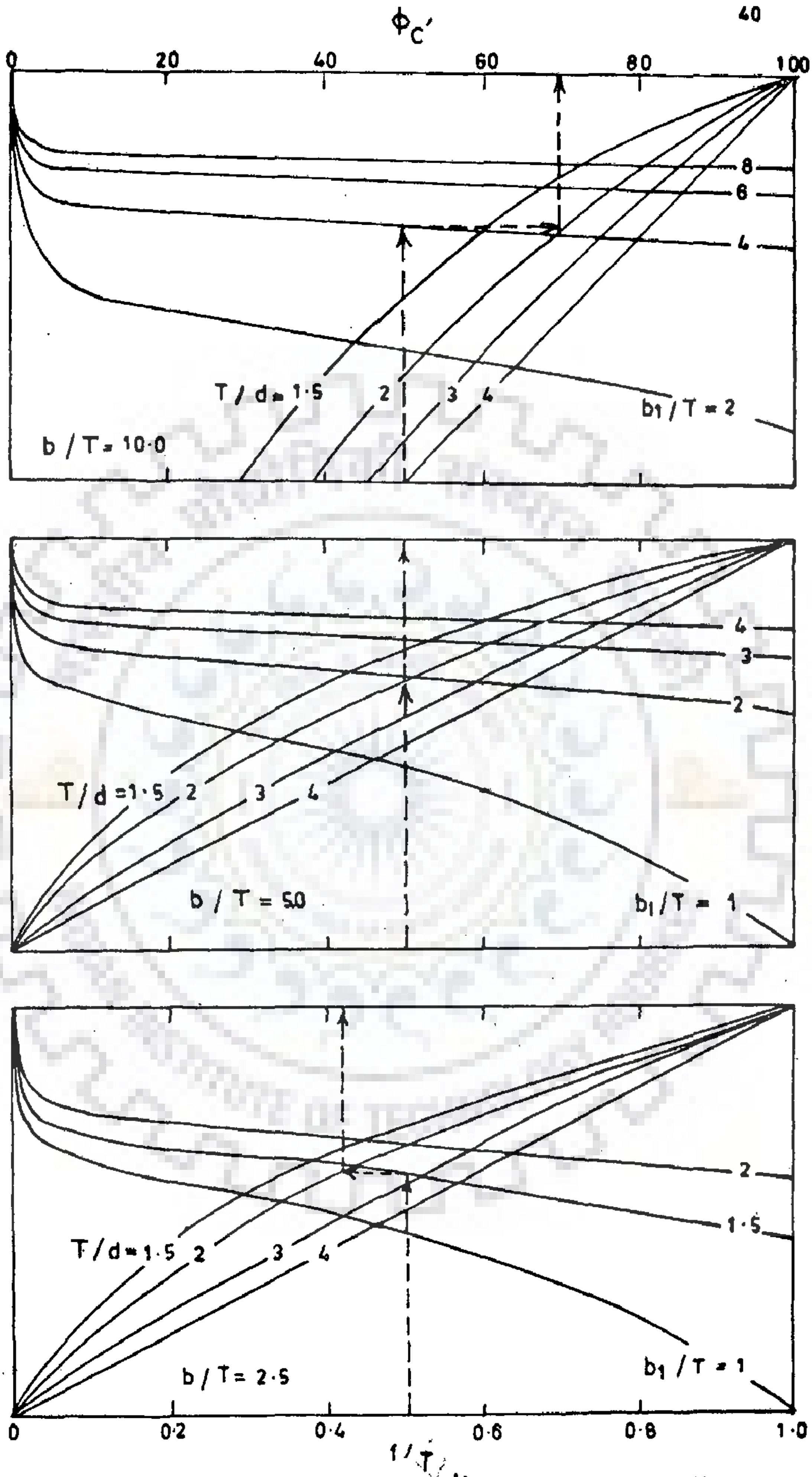


FIG. 3·4 UPLIFT PRESSURE AT 'C'' IN PERCENT OF H

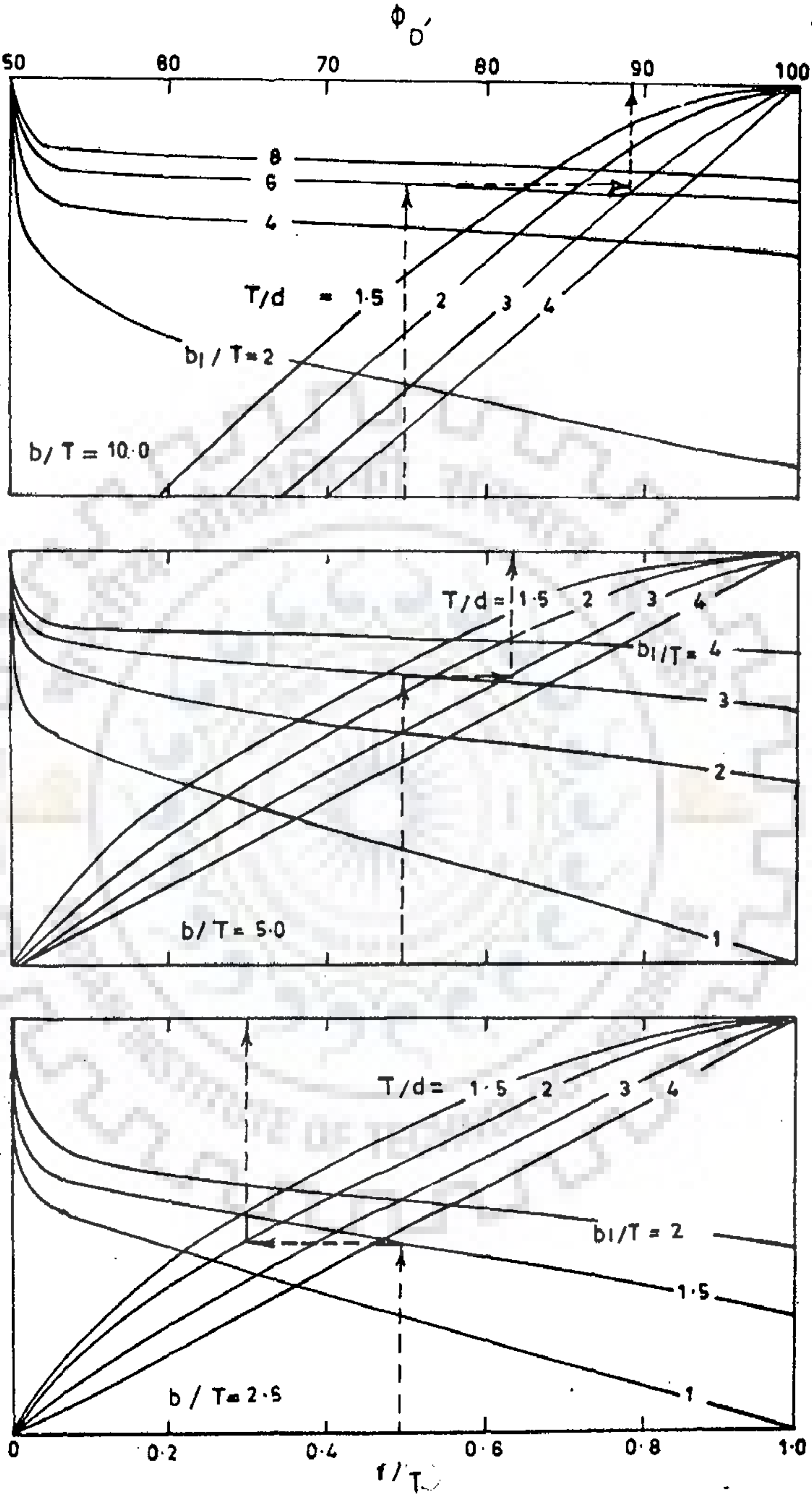


FIG. 3·5 UPLIFT PRESSURE AT 'D′' IN PERCENT OE H

CHAPTER - 4

NUMERICAL SOLUTION FOR INTERMEDIATE FILTER WITH SLOPING IMPERVIOUS STRATUM

4.1 INTRODUCTION

In actual field problems, the impervious strata may not be horizontal. They may be sloping either upstream or downstream. The solution of the problem by method of conformal mapping involves a large number of transform parameters and it is very difficult to solve the equations which are not explicit in transformation parameters. The equations are to be solved indirectly for assumed values of transformation parameters and involve many trials to obtain the desired values of physical dimensions. In recent years, numerical methods have been used extensively for solving seepage problems. Galerkin's method is a weighted residual numerical method for approximate solution of a differential equation with some defined boundary conditions. This method with finite element technique is used here to solve the Laplace equation which governs the flow in this case.

4.2 LAYOUT

Consider an impervious floor AB of length b founded on homogenious permeable soil underlain by an impervious stratum sloping at an angle θ with the horizontal. The floor has a cutoff C'D' of depth d_1 at upstream end and a cutoff ED of depth d_2 at the downstream end of the floor. The floor is drained by an intermediate filter FG of length f at a distance

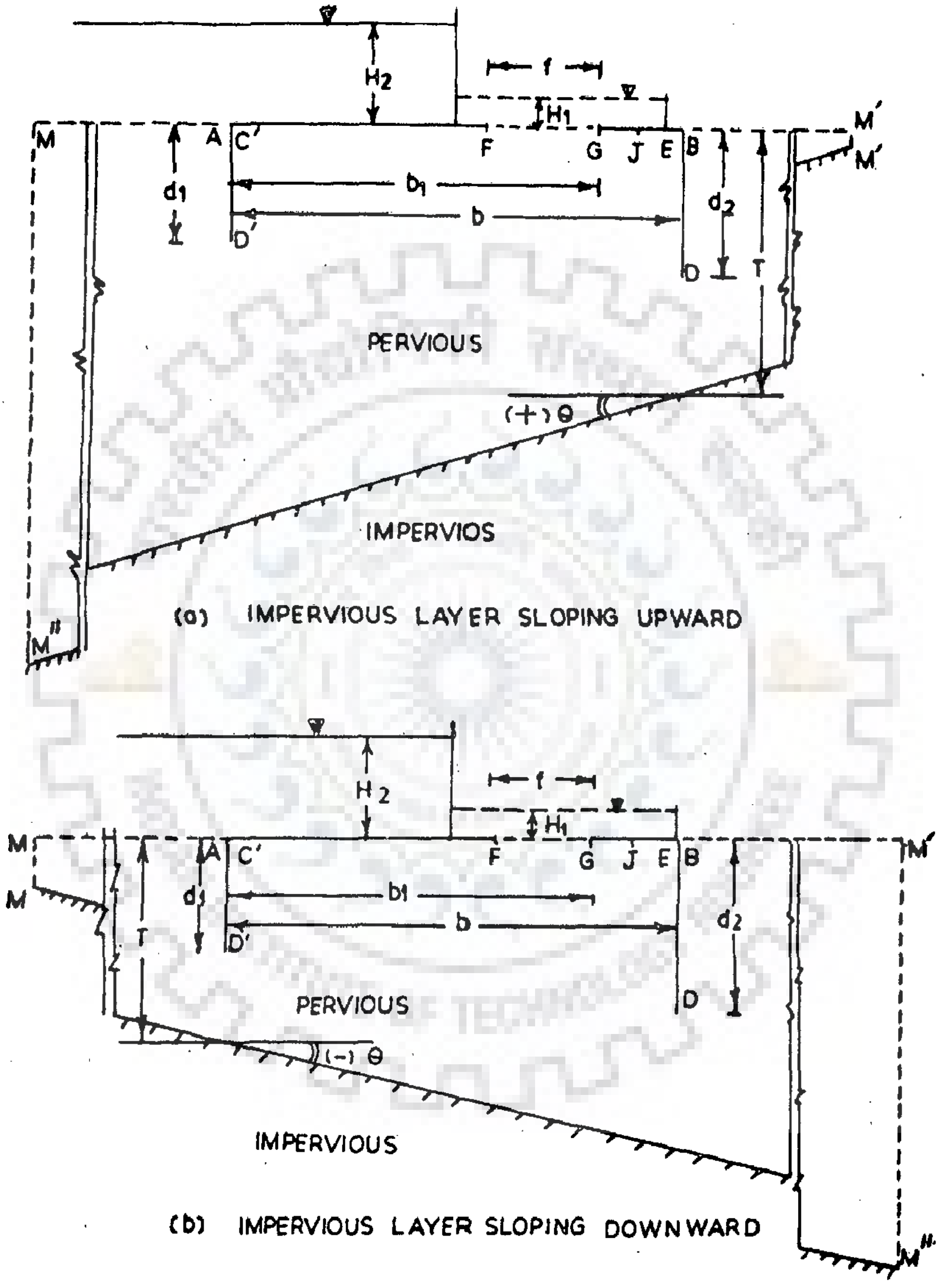


FIG. 4.1 LAYOUT DETAILS

b_1 from the upstream cutoff. The thickness of pervious strata is T below upstream or downstream end of the floor depending upon whether the impervious layer is sloping downward or upward respectively. The profile is represented in the z-plane and is shown in Fig. 4.1.

4.3 FORMULATION OF THE PROBLEM

The velocity potential ϕ of the flow through the pervious medium satisfied the two dimensional Laplace's equation

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad \ldots(4.1)$$

where $\phi = -kh$, h is the head and k is coefficient of percolation. In weighted residual method, the function ϕ is approximated by a series as below

$$\phi = \hat{\phi} = \sum_{i=1}^{n} N_i \phi_i \qquad \ldots(4.2)$$

where N_i are undetermined parameters and ϕ_i are unknown parameters. $\hat{\phi}$ is called a trial function. Substituting the trial function into the Eq. 4.1, it produces an error or residual R such that

$$R = \nabla^2 (\hat{\phi}) \qquad \ldots(4.3)$$

The best solution will be one which reduces the residual R to a least value at all the points of region under consideration. An obvious way to achieve this is to make use of the fact that if R is identically zero every where, then

$$R_o = \int_A wR \, dA = 0$$

or $$R_o = \int_A \nabla^2 (\hat{\phi}) \, w \, dA = 0 \qquad \ldots(4.4)$$

where w is a weightage function of the coordinates and A is Area of the region under consideration. If the number of unknown parameters (ϕ_i) is n, then n weightage functions (w_i) are chosen and the corresponding number of equations are written as follows

$$R_i = \int_A \nabla^2 (\phi) \; w_i \, dA = 0 \qquad \ldots(4.5)$$

For anisotropic soil Eq. 4.5 can be written as

$$R_i = \int_A k_{xx} \frac{\partial^2 h}{\partial x^2} \; w_i \, dA + \int_A k_{yy} \frac{\partial^2 h}{\partial y^2} w_i \, dA = 0$$

Integrating this by parts

$$R_i = \int_s [(k_{xx} \frac{\partial h}{\partial x} n_x) + (k_{yy} \frac{\partial h}{\partial y} n_y)] w_i ds$$

$$- \int_A (k_{xx} \frac{\partial w_i}{\partial x} \; \frac{\partial h}{\partial x} + k_{yy} \frac{\partial w_i}{\partial y} \frac{\partial h}{\partial y}) \, dA = 0$$

or

$$R_i = \int_s q w_i \, ds - \int_A (k_{xx} \frac{\partial w_i}{\partial x} \; \frac{\partial h}{\partial x} + k_{yy} \frac{\partial w_i}{\partial y} \; \frac{\partial h}{\partial y}) dA = 0$$

or

$$\int_A (k_{xx} \frac{\partial h}{\partial x} \frac{\partial wi}{\partial x} + k_{yy} \frac{\partial h}{\partial y} \; \frac{\partial w_i}{\partial y}) dA = \int_s q w_i \, ds \qquad \ldots(4.6)$$

where n_x and n_y are the x and y component of the unit vector normal to the boundary and q is the boundary flux.

Galerkin's chosen weight function w_i is same as interpolation function N_i (i.e. $w_i = N_i$) and therefore,

$$\int_A [(k_{xx} \frac{\partial h}{\partial x} \frac{\partial N_i}{\partial x}) + (k_{yy} \frac{\partial h}{\partial y} \frac{\partial N_i}{\partial y})] \, dA = \int_s qN_i \, ds \qquad ...(4.7)$$

Substituting $\phi = - kh$ in Eq. 4.2 and differentiating it with respect to x and y, we have

$$\frac{\partial h}{\partial x} = \sum_{j=1}^{n} h_j \frac{\partial N_j}{\partial x} \qquad ...(4.8)$$

and

$$\frac{\partial h}{\partial y} = \sum_{j=1}^{n} h_j \frac{\partial N_j}{\partial y} \qquad ...(4.9)$$

Substituting this in Eq. (4.7).

$$\int_A [k_{xx}(\sum_{j=1}^{n} h_j \frac{\partial N_j}{\partial x} \frac{\partial N_i}{\partial x}) + k_{yy}(\sum_{j=1}^{n} h_j \frac{\partial N_j}{\partial y} \frac{\partial N_i}{\partial y})] dA = \int_s qN_i ds$$

or

$$\int_A \sum_{j=1}^{n} [k_{xx} \frac{\partial N_j}{\partial x} \frac{\partial N_i}{\partial x} + K_{yy} \frac{\partial N_j}{\partial y} \frac{\partial N_i}{\partial y}] h_j \, dA = \int_s q N_i \, ds$$

or

$$\sum_{j=1}^{n} h_j \int_A (k_{xx} \frac{\partial N_j}{\partial x} \frac{\partial N_i}{\partial x} + k_{yy} \frac{\partial N_j}{\partial y} \frac{\partial N_i}{\partial y}) dA = \int_s q N_i \, ds$$

or

$$\sum_{j=1}^{n} K_{ij} h_j = F_i \qquad ...(4.10)$$

$$i = 1,2,3 \text{ --------} n$$

where

$$K_{ij} = \int_A (k_{xx} \frac{\partial N_j}{\partial x} \frac{\partial N_i}{\partial x} + k_{yy} \frac{\partial N_j}{\partial y} \frac{\partial N_i}{\partial y}) \, dA \qquad ...(4.11)$$

and $$F_i = \int_S q\, N_i\, ds \qquad \ldots(4.12)$$

Thus we can write n equations , which can be solved to obtain the values of h at all the n points with known boundary conditions.

4.4 SOLUTION BY FINITE ELEMENT METHOD

The main difficulty in direct application of Galerkin method is the choice of the global functions. These functions have not only to satisfy the essential boundary conditions of the problem but also to be adequate to describe the geometry, material and other characteristics of the problem. All these conditions are generally very difficult to fulfil. Therefore, the method in classical sense has limited use. With the advent of high speed digital computer, the idea of the approximating functions being localised in a small region (element) was developed. In this way simpler interpolation functions can be used. These localised functions can be better implemented if each element is considered separately thus originating the finite element technique. In this technique we consider each individual element unrelated to the others and then joined together, satisfying the necessary continuity and the global boundary conditions. Therefore, the unknown function h is defined element by element.

In general triangular, rectangular and quadrilateral types of elements are used. The choice of element shape and the interpolation functions is an important part of finite element analysis and the accuracy of the results achieved depends to a large extent on these factors. In general higher order polynomials yield better results but are taxing to computer as well as difficult to handle. The choice is, thus, made by overall consideration of these factors. Simplest elements in two dimensional problems are triangular elements with linear interpolation function and are taken for the analysis, in this case.

Equation 4.10 can be discretized using finite element to give a matrix notation as

$$[K]^e \{h\}^e = \{F\}^e \qquad \ldots(4.13)$$

where

$$[K]^e = [K_{ij}] = \begin{bmatrix} K_{ii} & K_{ij} & K_{ik} \\ K_{ji} & K_{jj} & K_{jk} \\ K_{ki} & K_{kj} & K_{kk} \end{bmatrix} \qquad \ldots(4.14)$$

$$\{h\}^e = \begin{Bmatrix} h_i \\ h_j \\ h_k \end{Bmatrix} \qquad \ldots(4.15)$$

$$\{F\}^e = \begin{Bmatrix} F_i \\ F_j \\ F_k \end{Bmatrix} \qquad \ldots(4.16)$$

and

i, j, k are the nodal numbers of the element.

For a triangular element, the interpolation function can be taken as,

$$h = \alpha_1 + \alpha_2 x + \alpha_3 y \qquad ...(4.17)$$

which varies linearly in the triangle and produces three unknowns per element. These unknowns can be related to the nodal values at nodes i, j and k and we have

$$\begin{Bmatrix} h_i \\ h_j \\ h_k \end{Bmatrix} = \begin{bmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{Bmatrix} \qquad ...(4.18)$$

The inverse of the Eqs. (4.18) gives relationship between α_i and h_i as below

$$\begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{Bmatrix} = \frac{1}{2A_e} \begin{bmatrix} 2A_i^o & 2A_j^o & 2A_k^o \\ b_i & b_j & b_k \\ a_i & a_j & a_k \end{bmatrix} \begin{Bmatrix} h_i \\ h_j \\ h_k \end{Bmatrix} \qquad ...(4.19)$$

where

$$a_i = x_k - x_j, \; a_j = x_i - x_k, \; a_k = x_j - x_i$$

$$b_i = y_j - y_k, \; b_j = y_k - y_i, \; b_k = y_i - y_j \qquad ...(4.20)$$

$$2A_i^o = x_j y_k - x_k y_j, \; 2A_j^o = x_k y_i - x_i y_k, \; 2A_k^o = x_i y_j - x_j y_i$$

$$2A_e = b_i a_j - b_j a_i, \; (A_e = \text{Area of triangular element})$$

Substituting the values of α_1, α_2 and α_3 in Eq. (4.17) and collecting the terms of h_i, h_j and h_k together we have,

$$h = \frac{1}{2A_e}[(2A_i^o + b_i x + a_i y)h_i + (2A_j^o + b_j x + a_j y)h_j + (2A_k^o + b_k x + a_k y)h_k]$$

or

$$h = N_i h_i + N_j h_j + N_k h_k \qquad ...(4.21)$$

where

$$N_i = \frac{1}{2A_e}(2A_i^o + b_i x + a_i y)$$

$$N_j = \frac{1}{2A_e}(2A_j^o + b_j x + a_j y) \qquad \ldots(4.22)$$

$$N_k = \frac{1}{2A_e}(2A_k^o + b_k x + a_k y)$$

In matrix notations Eq. 4.21 can be written as

$$\{h\} = \{N\}^T \{h\}^e \qquad \ldots(4.23)$$

where $\{N\}^T = \{N_i \quad N_j \quad N_k\}$ ...(4.24)

Now
$$\left\{\frac{\partial N}{\partial x}\right\} = \frac{1}{2A_e}\{b\} = \frac{1}{2A_e}\begin{Bmatrix} b_i \\ b_j \\ b_k \end{Bmatrix} \qquad \ldots(4.25)$$

and
$$\left\{\frac{\partial N}{\partial y}\right\} = \frac{1}{2A_e}\{a\} = \frac{1}{2A_e}\begin{Bmatrix} a_i \\ a_j \\ a_k \end{Bmatrix} \qquad \ldots(4.26)$$

Substituting the values of $\frac{\partial N}{\partial x}$ and $\frac{\partial N}{\partial y}$ from Eqs. 4.25 and 4.26 in Eq. 4.11

$$K_{ij} = \int_A \frac{1}{4A_e^2}(k_{xx}\, b_j\, b_i + k_{yy}\, a_j\, a_i)\,dA$$

$$= \frac{1}{4A_e}(k_{xx}\, b_j\, b_i + k_{yy}\, a_j\, a_i) \qquad \ldots(4.27)$$

The values of K_{ij} can be substituted in Eq. 4.14 to obtain the value of the matrix $[K]^e$ for a finite element.

Matrix equations for the complete system can be obtained by systematic superposition as

$$[K] \{h\} = \{F\} \qquad \ldots(4.28)$$

with the degree of freedom of system M, equal to the total number of nodal points. At the boundary nodal points, either the pressure is defined and known ($h = h_o$) if the point is on a constant potential line or the flow q and therefore F is zero if point is on a flow line. The Eq. (4.28) can be rearranged corresponding to active and boundary points as below

$$\begin{bmatrix} \underset{m \times m}{[K_{11}]} & \underset{m \times p}{[K_{12}]} \\ \underset{p \times m}{[K_{21}]} & \underset{p \times p}{[K_{22}]} \end{bmatrix} \begin{Bmatrix} \underset{m \times 1}{\{h_1\}} \\ \underset{p \times 1}{\{h_2\}} \end{Bmatrix} = \begin{Bmatrix} \underset{m \times 1}{\{F_1\}} \\ \underset{p \times 1}{\{F_2\}} \end{Bmatrix} \qquad \ldots(4.29)$$

where $\{h_1\}$ is the vector of unconstrained or free nodes and $\{h_2\}$ is the vector of specifid values of potential at boundary. p is the number of boundary nodal point on which pressure is known and m is the number of free nodes on which pressure is to be computed.

The problem is now reduced to solution of the first set of equations (4.29).

$$[K_1] \{h_1\} = \{F_1\} - [K_{12}] \{h_2\} \qquad \ldots(4.30)$$

The value of F_2 at the constrained boundaries can be computed from the second set of equations (4.29)

$$\{F_2\} = [K_{21}] \{h_1\} + [K_{22}] \{h_2\} \qquad \ldots(4.31)$$

A more practical way of forming the modified equilibrium equations is to write the equations in the following partitioned form

$$\begin{bmatrix} \underset{m\times m}{[K_{11}]} & \underset{m\times p}{[0]} \\ \underset{p\times m}{[0]} & \underset{p\times p}{[I]} \end{bmatrix} \begin{Bmatrix} \underset{m\times 1}{\{h_1\}} \\ \underset{p\times 1}{\{h_2\}} \end{Bmatrix} = \begin{Bmatrix} \underset{m\times 1}{\{F_1\} - [K_{12}]\{h_2\}} \\ \underset{p\times 1}{\{h_2\}} \end{Bmatrix} \quad \ldots(4.32)$$

In order to preserve the banded nature of the equations, the process is performed without reordering the equations implied by the partitioning. For a specified nodal potential h_j occuring at the jth degree of freedom, the above process is summarized as:

$$\begin{aligned} F_i &= F_i - K_{ij} \; h_j \qquad \text{for } i = 1,2,3\ldots M \\ K_{jm} &= K_{mj} = 0 \quad \text{for } m = 1,2,3\ldots\ldots\ldots\ldots M \quad \ldots(4.33) \\ K_{jj} &= 1 \\ F_j &= h_j \end{aligned}$$

h_j is known value of potential at jth boundary node. These operations ensure that the equilibrium equations remain symmetrical.

4.5 BOUNDARY CONDITIONS

Without loss of generality we shall take h = 0 on the downstreamm water level, and measured above it $h = H_1$ as the head in the filter and $h = H_2$ as the head in the upstream bed. Then on the upstream bed AM, $\phi = -kH_2$ and on downstream bed BM', $\phi = 0$. The foundation profile A D' C' F forms the inner boundary of the stream line $\psi = 0$ where ψ is the stream function. There will not be any flow across this and therefore the value of F along this can be taken as zero. The profile of the filter FG is an equipotential $\phi = -kH_1$. The foundation profile GEDB forms the inner boundary of the downstream flow and may be taken as the sstreamline $\psi = q_1$ where q_1 is the discharge per unit width normal to the direction of flow drained through the intermediate filter. Therefore, the value of F along this boundary will also be zero. The impervious boundary M'M'' forms another streamline $\Psi = q_2$ in which q_2 is total discharge seeping below the foundation. Therefore the value of F along this is also zero. For finding the solution to the problem the boundary MM'' is assumed as a streamline $\psi = q_2$ so that the value of F is zero. This assumption is not likely to involve substantial error as the pervious length of bed is assumed to extend, three times the floor length from the floor end.

4.6 MESH SIZE

The seepage problem with the boundary conditions specified as above has been solved by using triangular element. The accuracy of results achieved depends to a large measure on the size of element. In general smaller size yields better results but requires more computer time. Therefore, a detailed study has been carried out to find the optimal size of the element for the following case

$$\frac{b}{d_2} = 10, \frac{d_2}{d_1} = 2, \frac{T}{d_2} = 4, \frac{b_1}{d_2} = 8, \frac{f}{d_2} = 1.0 \text{ and } \Theta = 0^\circ$$

The uplift pressures for the different mesh sizes were computed and compared with the results obtained by the closed-form solution in Chapter 2 (Table 4.1). It is found that as the mesh size is reduced the uplift pressure approach to those obtained by closed form solution, but the CPU time increases. The differences in the uplift pressures at the key points obtained by FEM and those obtained from closed form solution are very small with the mesh size equal to b/40 and further improvement in the uplift pressure with the reduction in mesh size to b/50 is not appreciable. Therefore the mesh size adopted for computing the uplift pressure for various combinations of the floor lengths and cutoff depths has been taken as b/50 which gives uplift pressures very close to those obtained by the closed form solution.

Table-4.1: Comparison of pressures with different mesh sizes

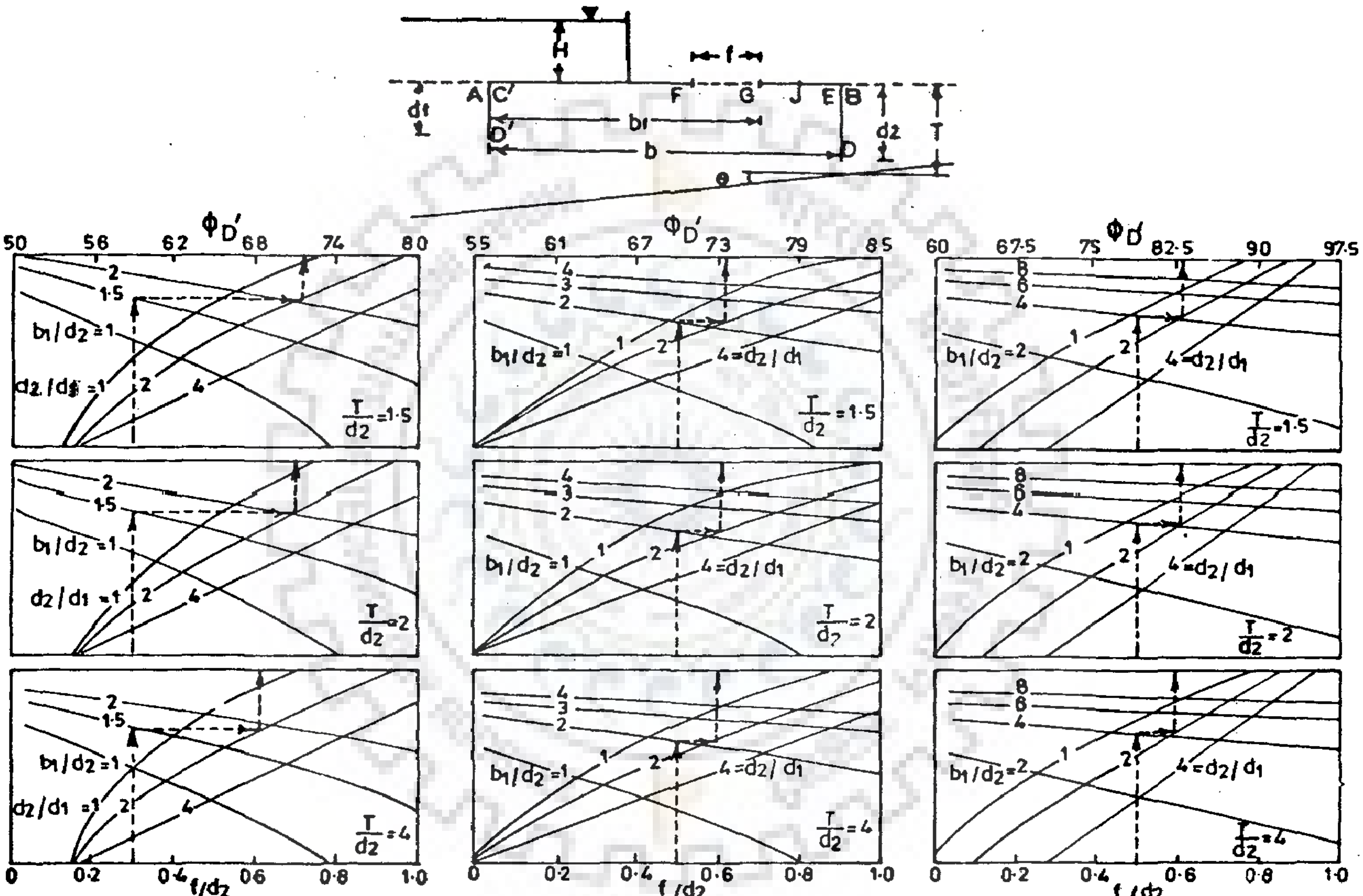
Mesh size	Uplift pressures in per-cent of differential head H					Location of point J	Exit gradient	CPU Time in sec.
	$h_{D'}$	$h_{C'}$	h_E	h_D	h_j	x_j/d_2	$G_E \frac{d_2}{H}$	
b/30	86.25	81.25	6.16	4.89	6.28	9.31	0.0382	8.53
b/40	86.09	80.72	6.33	4.97	6.44	9.31	0.0381	11.70
b/50	86.02	80.45	6.42	5.02	6.55	9.17	0.0380	13.02
Closed form solution	85.98	80.36	6.48	5.07	6.59	9.17	0.0379	-

4.7 RESULTS

A computer programme has been written to generate the mesh with given boundary conditions and to calculate the uplift pressure at all the nodal points. The values of exit gradient and uplift pressures at key points have been computed for different combinations of floor length, depths of upstream and downstream cutoffs, depth of soil strata and slope of impervious layer and have been plotted in the form of curves. The results obtained here can be used for practical design. The curves cover the values of b/d_2 = 2.5, 5 and 10, d_2/d_1 =1,2,3 and 4, f/d_2 = 0.0 to 1.0, anywhere between two cutoffs, T/d_2= 1.5, 2 and 4 and $\theta = -15^\circ$, -30°, -45°, 15°, 30° and 45°.

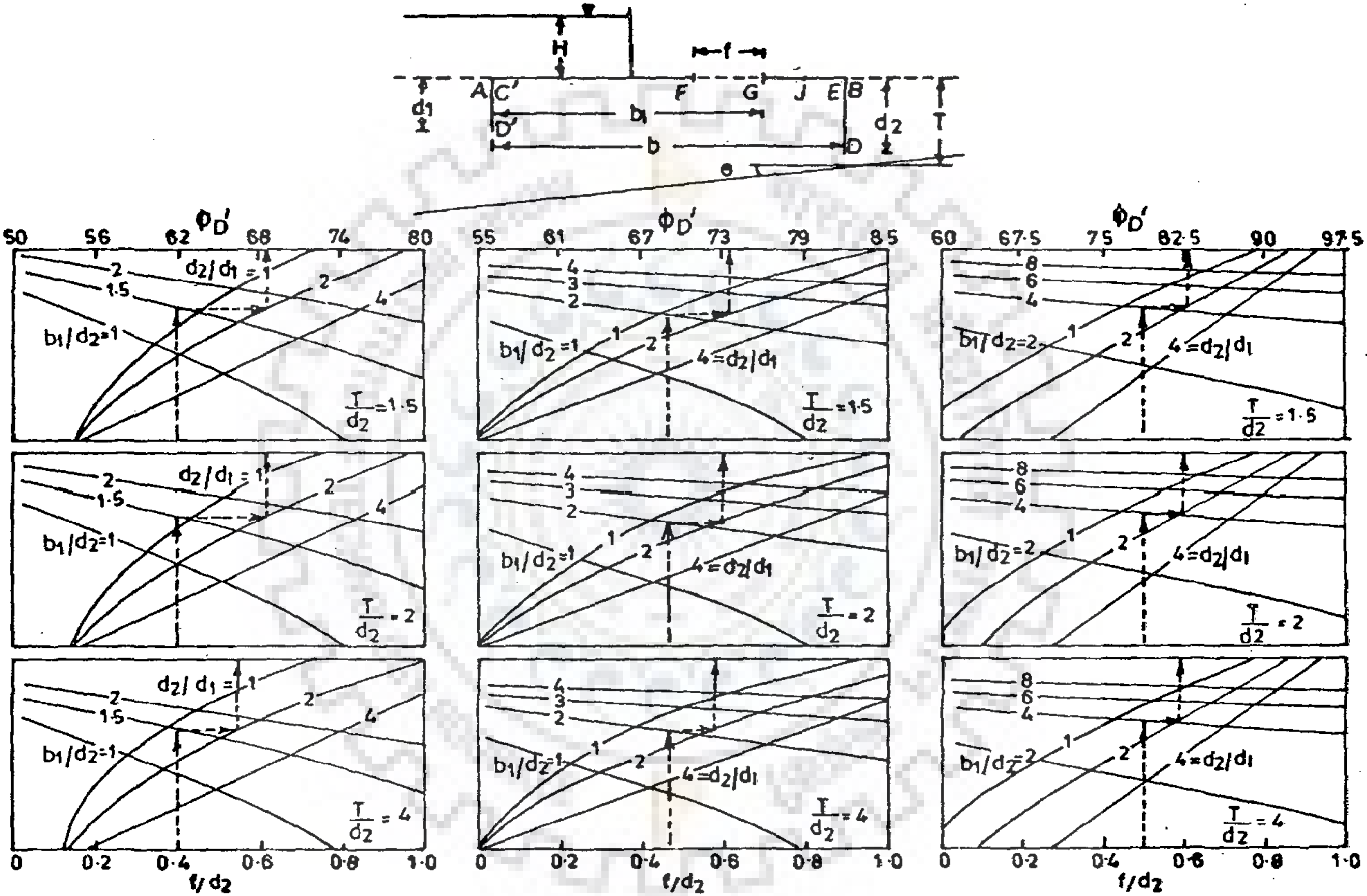
At the filter the pressure would correspond to the release level, which is normally the downstream water level, but as all the streamlines are not intercepted by the filter and some of them proceed to the downstream discharge surface, pressure again builds up on the floor between the filter and the downstream discharge surface. Starting from somewhere at the upstream, the stream-line $\psi = q_1$, would meet the floor GEDB at some point J where it would divide into two parts, one along JG, emerging at G and other along JEDB emerging at B. The potential along the floor GJDB would be maximum at J.

The uplift pressure at D', C', E, D and J have been plotted in figures 4.2, 4.3, 4.4, 4.5 and 4.6 respectively. The location of point J is obtained in figure 4.7. The exit gradient at point B is plotted in figure 4.8.



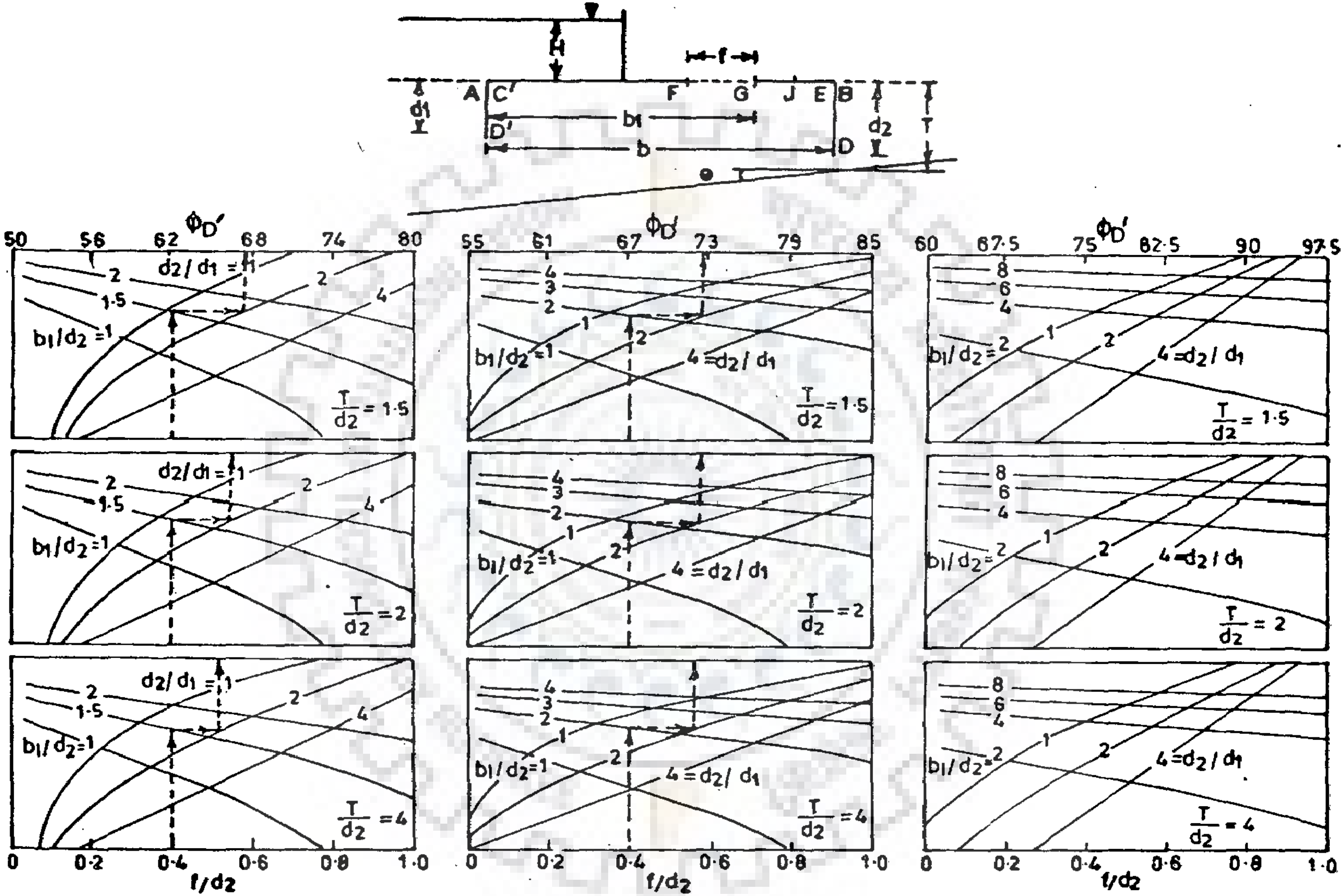
(a) $b/d_2 = 2.5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG. 4.2 (i) UPLIFT PRESSURE AT D' ($\theta = 45°$)



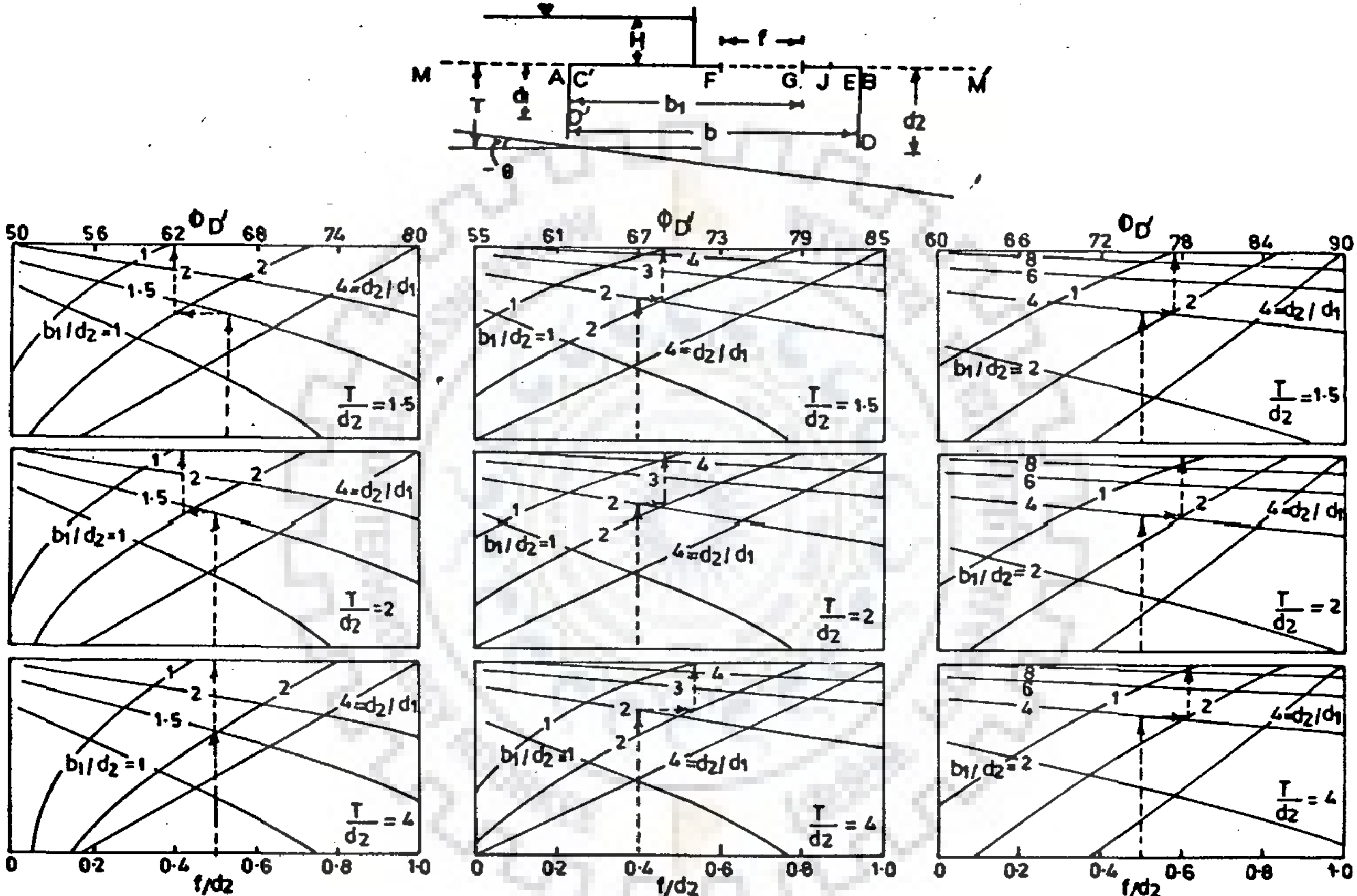
(a) b/d2=2.5 (b) b/d2 = 5 (a) b/d2 = 10

FIG.4.2 (ii) UPLIFT PRESSURE AT D′ (θ 30°)



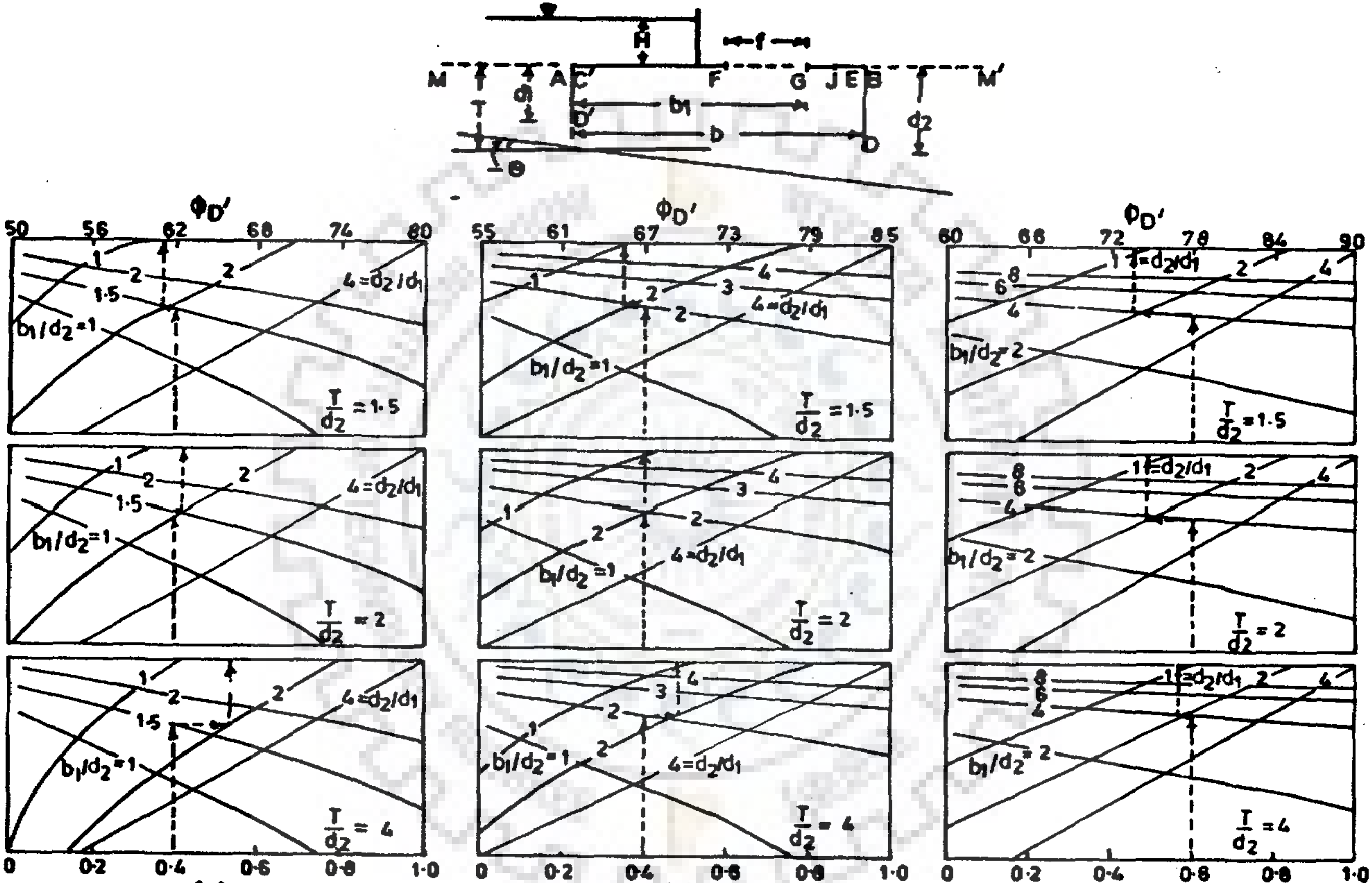
(a) $b/d_2 = 2.5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG. 4.2 (iii) UPLIFT PRESSURE AT D′ ($\theta = 15°$)



(a) $b/d_2 = 2.5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG. 4 2 (iv) UPLIFT PRESSURE AT D′ ($\theta = -15°$)



(a) $b/d_2 = 2.5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG. 4.2 (V) UPLIFT PRESSURE AT D' ($\theta = -30°$)

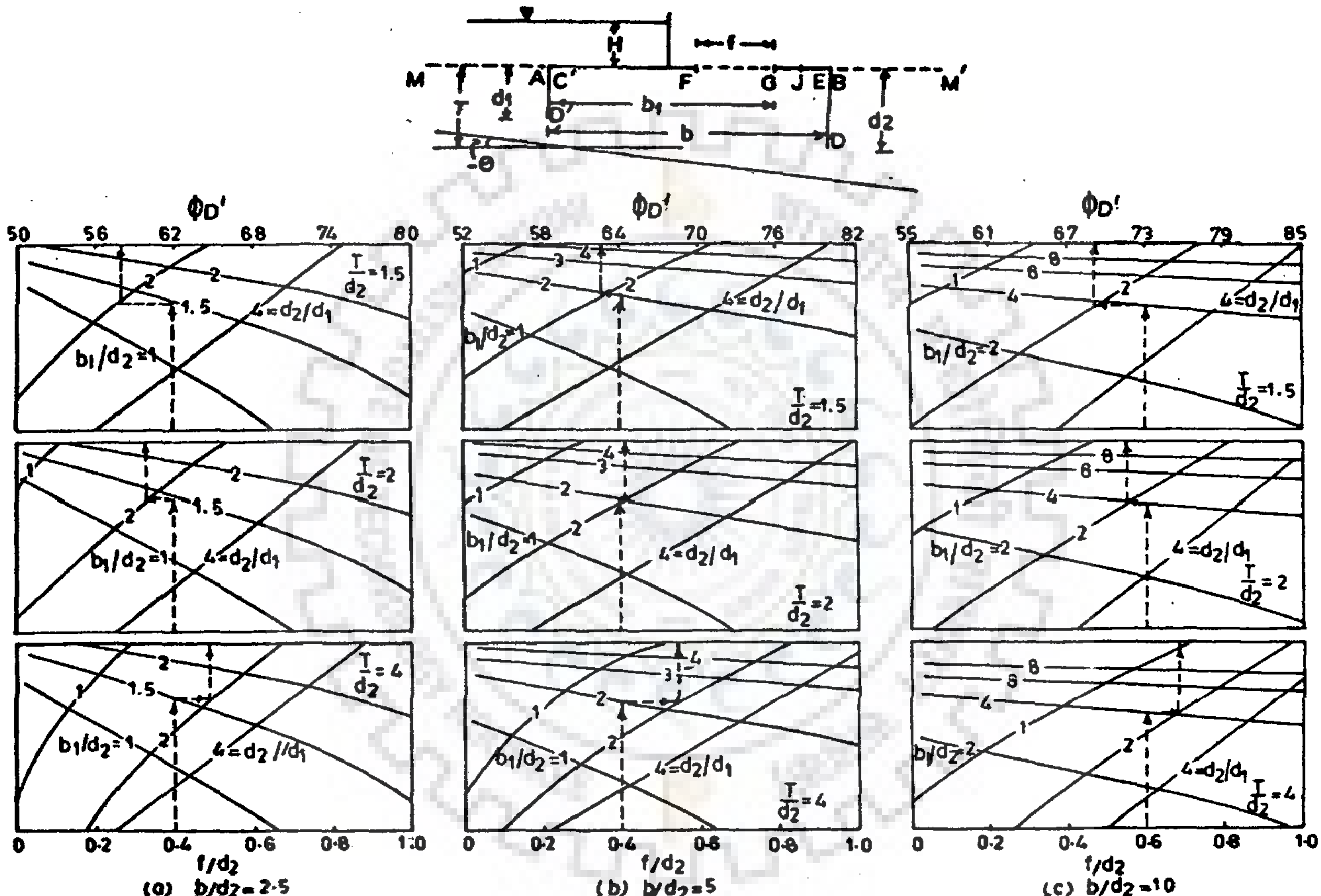


FIG. 4·2 (vi) UPLIFT PRESSURE AT D' (θ = −45°)

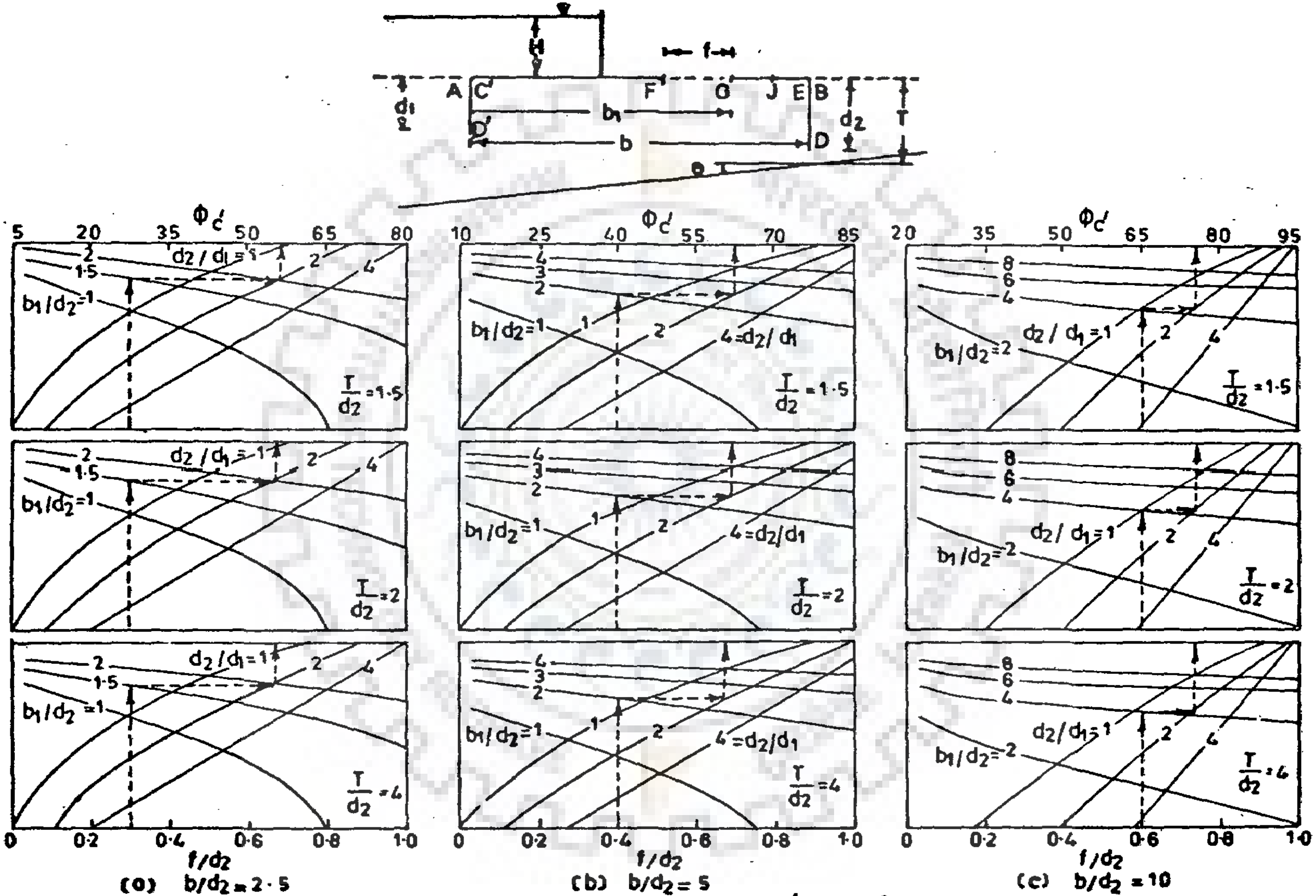
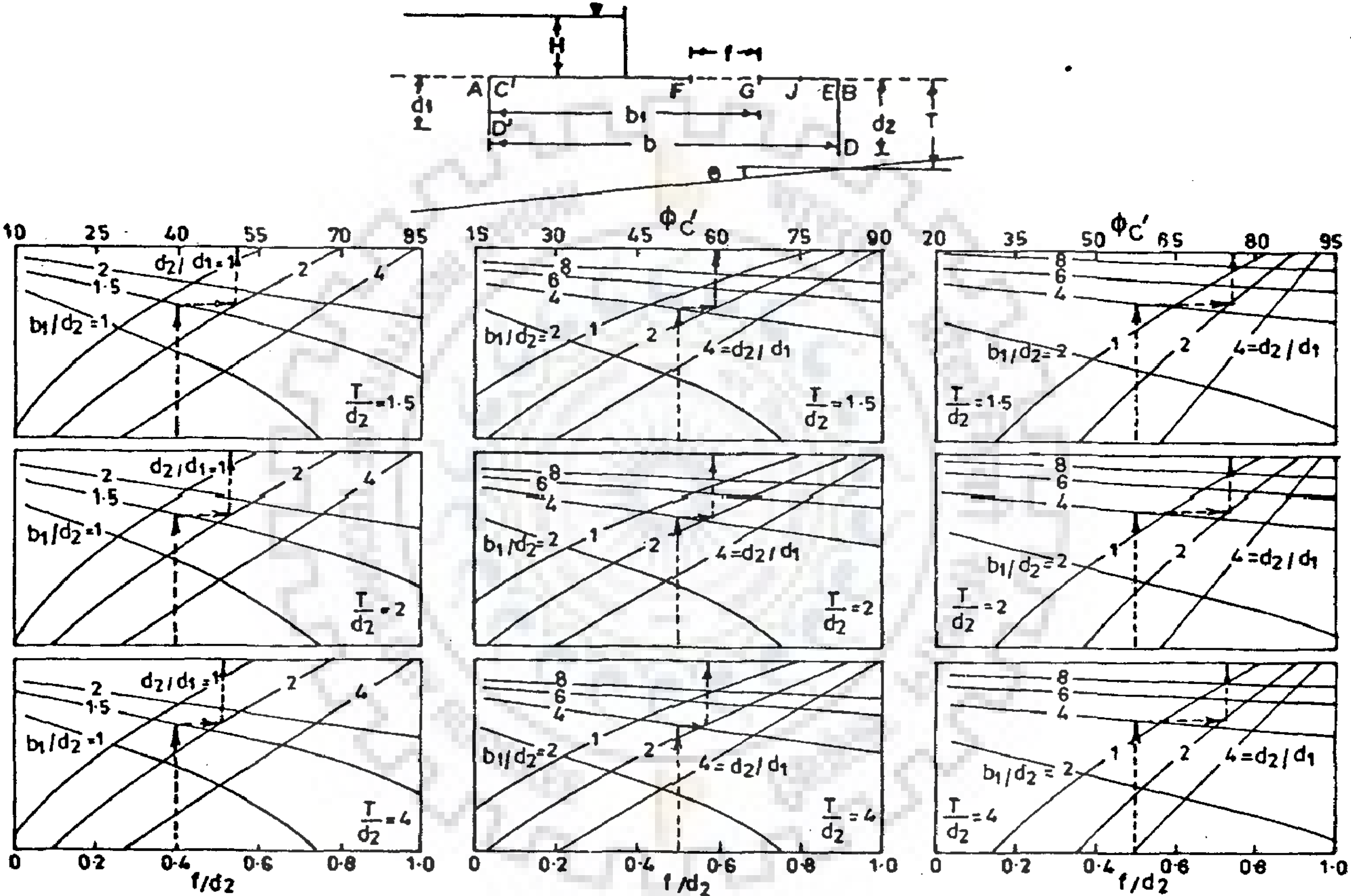


FIG.4.3 (i) UPLIFT PRESSURE AT C' ($\theta=45°$)



(a) $b/d_2 = 2.5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG.4.3 (ii) UPLIFT PRESSURE AT C' ($\theta = 30°$)

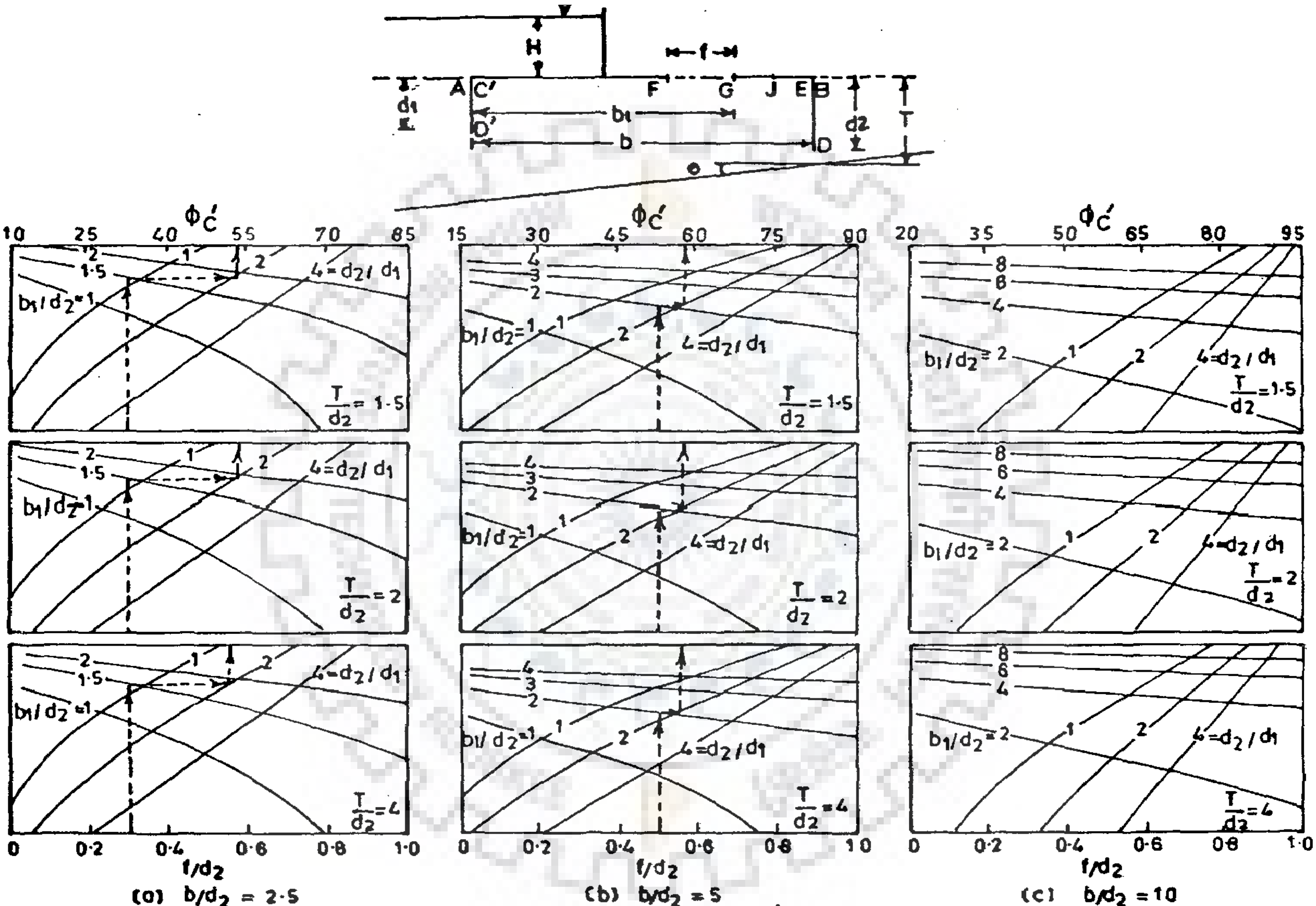


FIG. 43 (iii) UPLIFT PRESSURE AT C′ (θ = 15°)

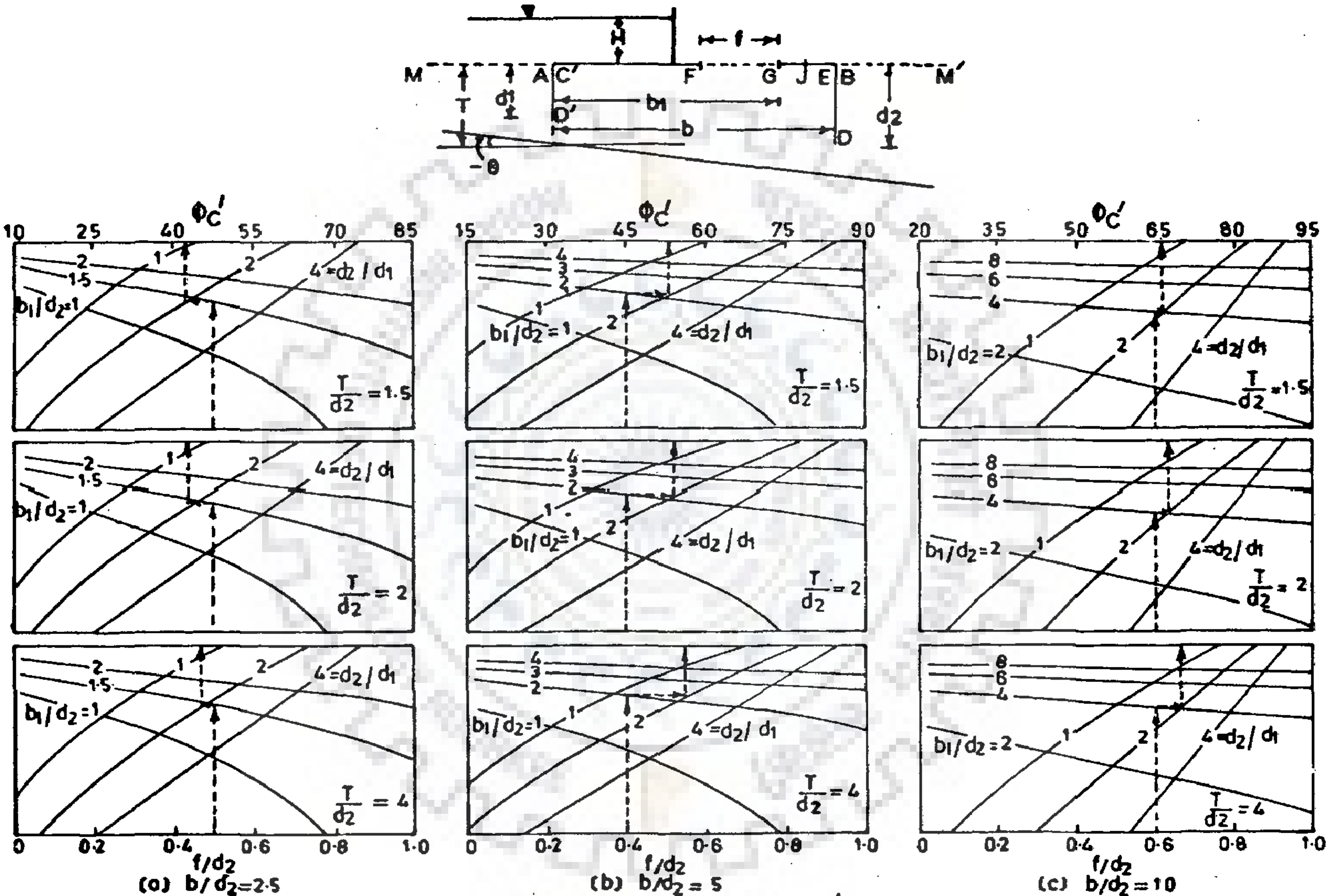
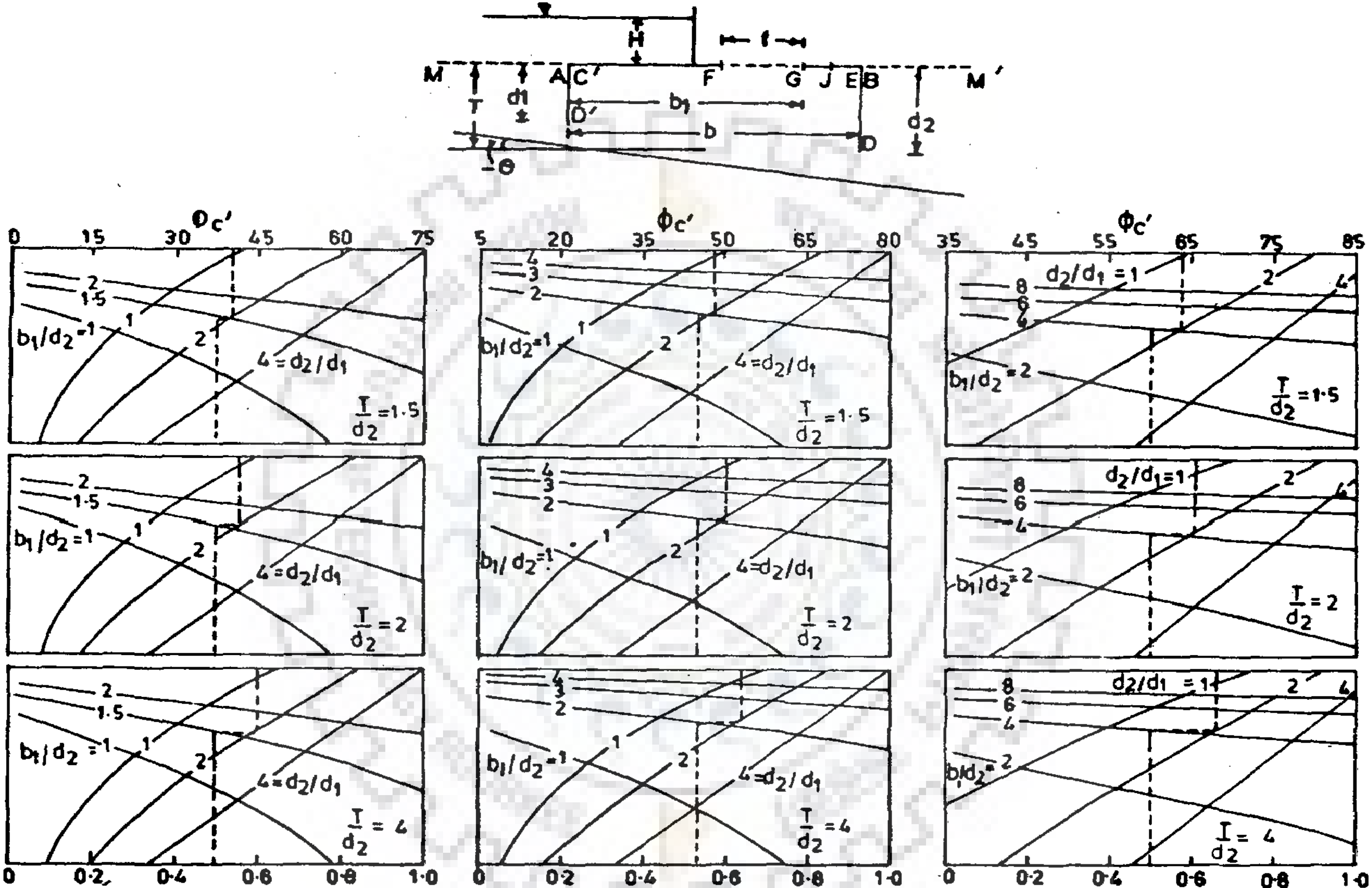


FIG. 4.3 (iv) UPLIFT PRESSURE AT C′ (θ = −15°)



(a) $b/d_2 = 2{\cdot}5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG. 4·3 (V) UPLIFT PRESSURE AT C' ($\theta = -30^\circ$)

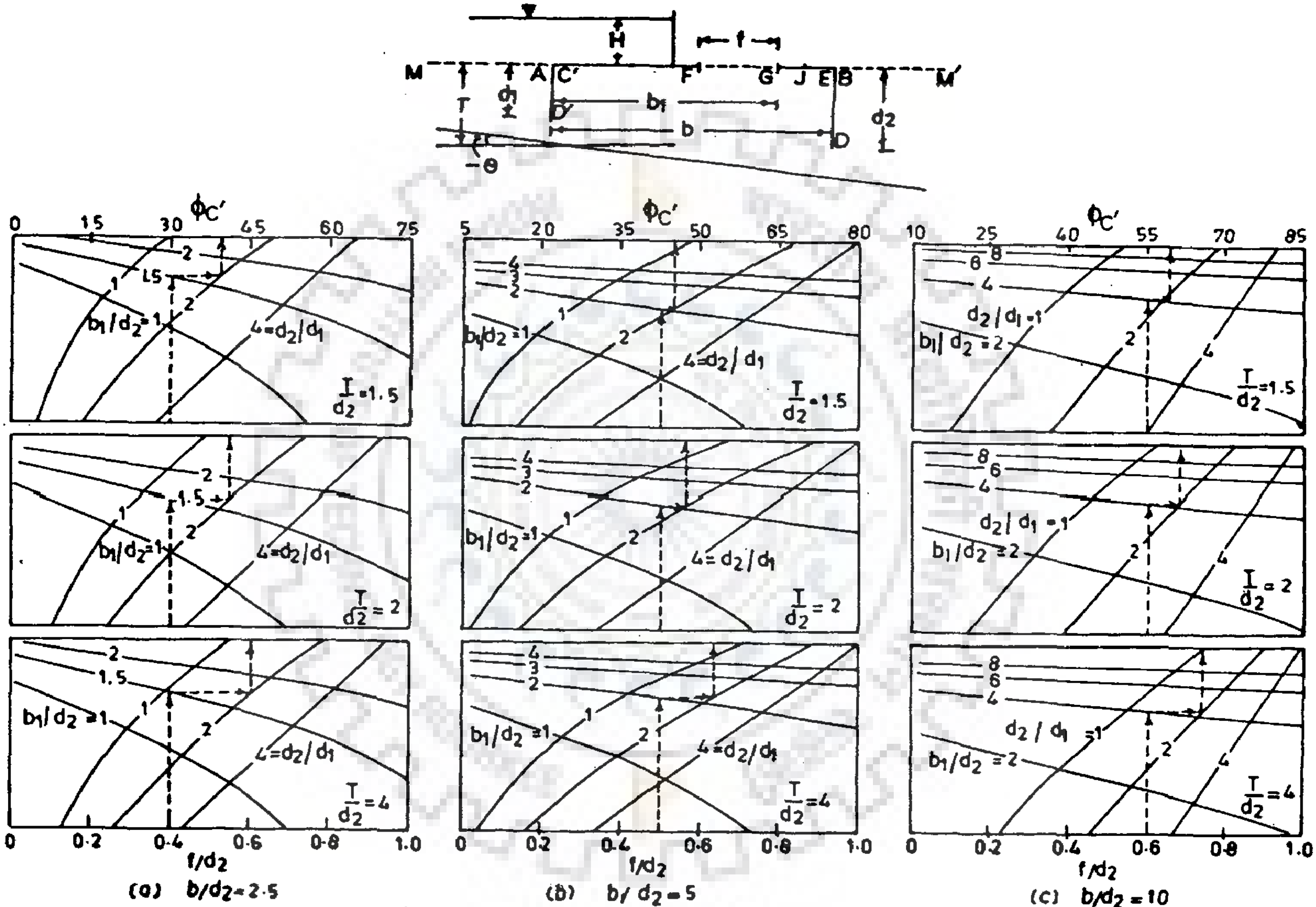


FIG. 4.3 (Vi) UPLIFT PRESSURE AT C' ($\theta=-45°$)

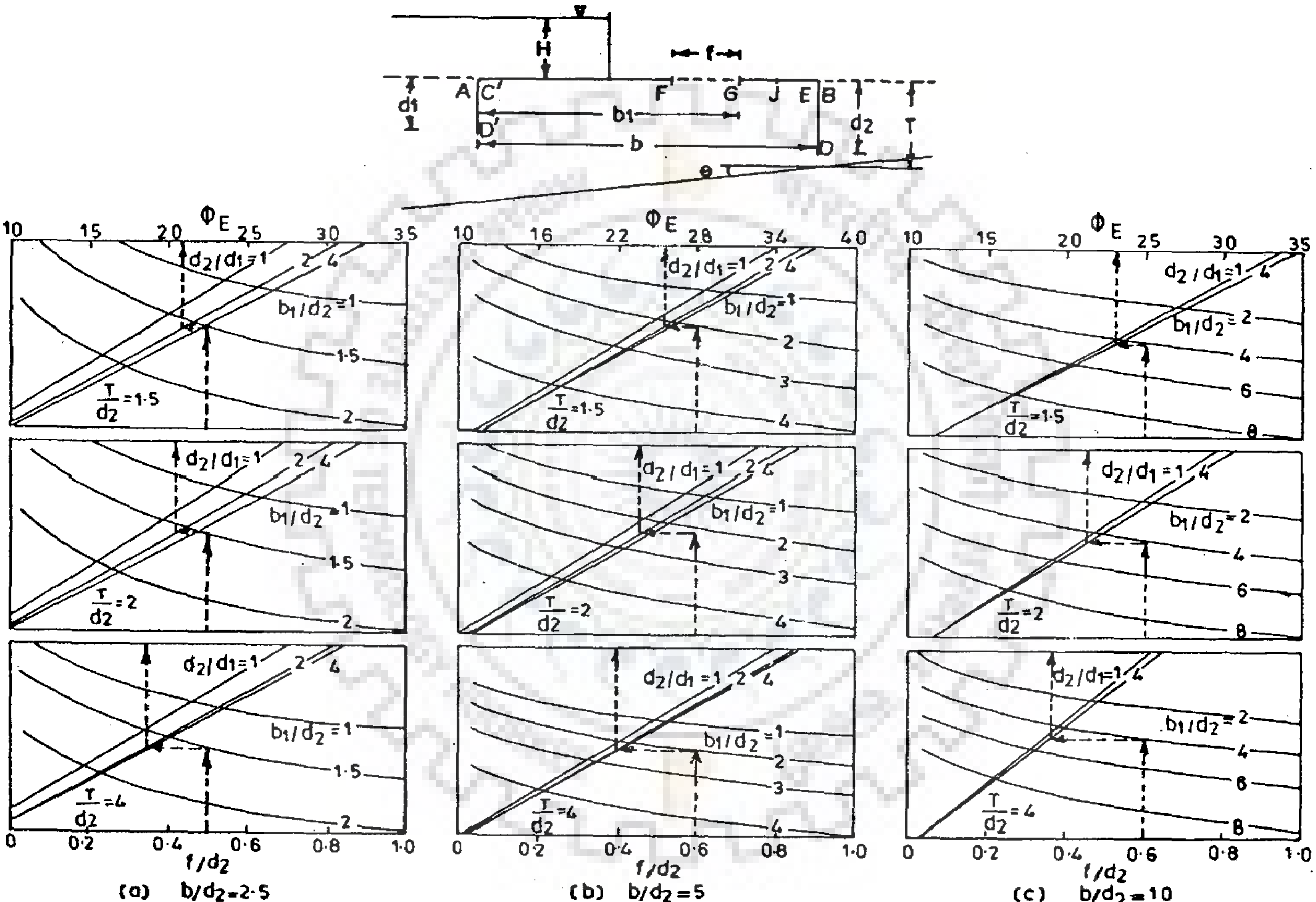


FIG. 4.4 (i) UPLIFT PRESSURE AT E ($\theta = 45°$)

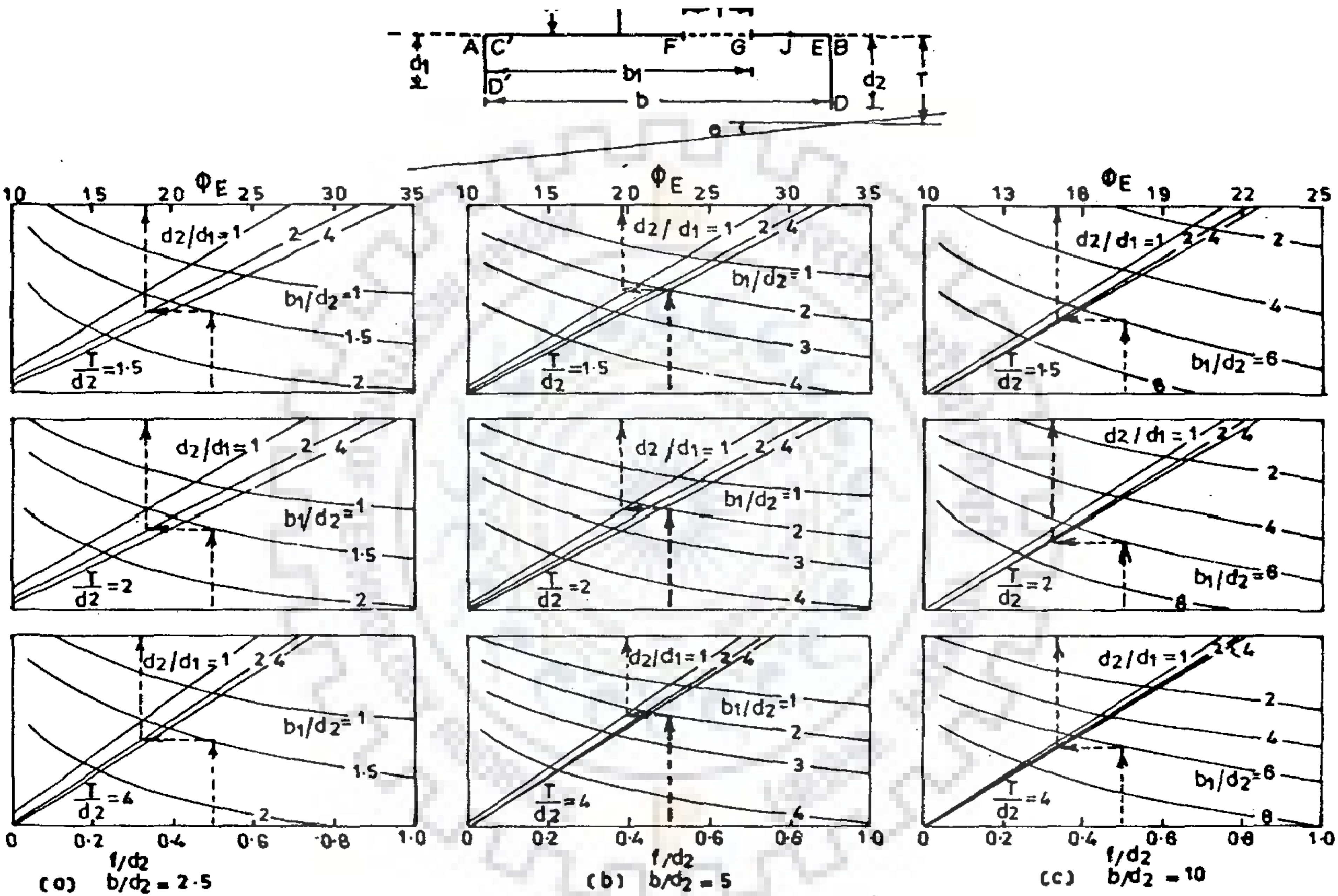


FIG. 4.4 (ii) UPLIFT PROSSURE AT E ($\theta = 30°$)

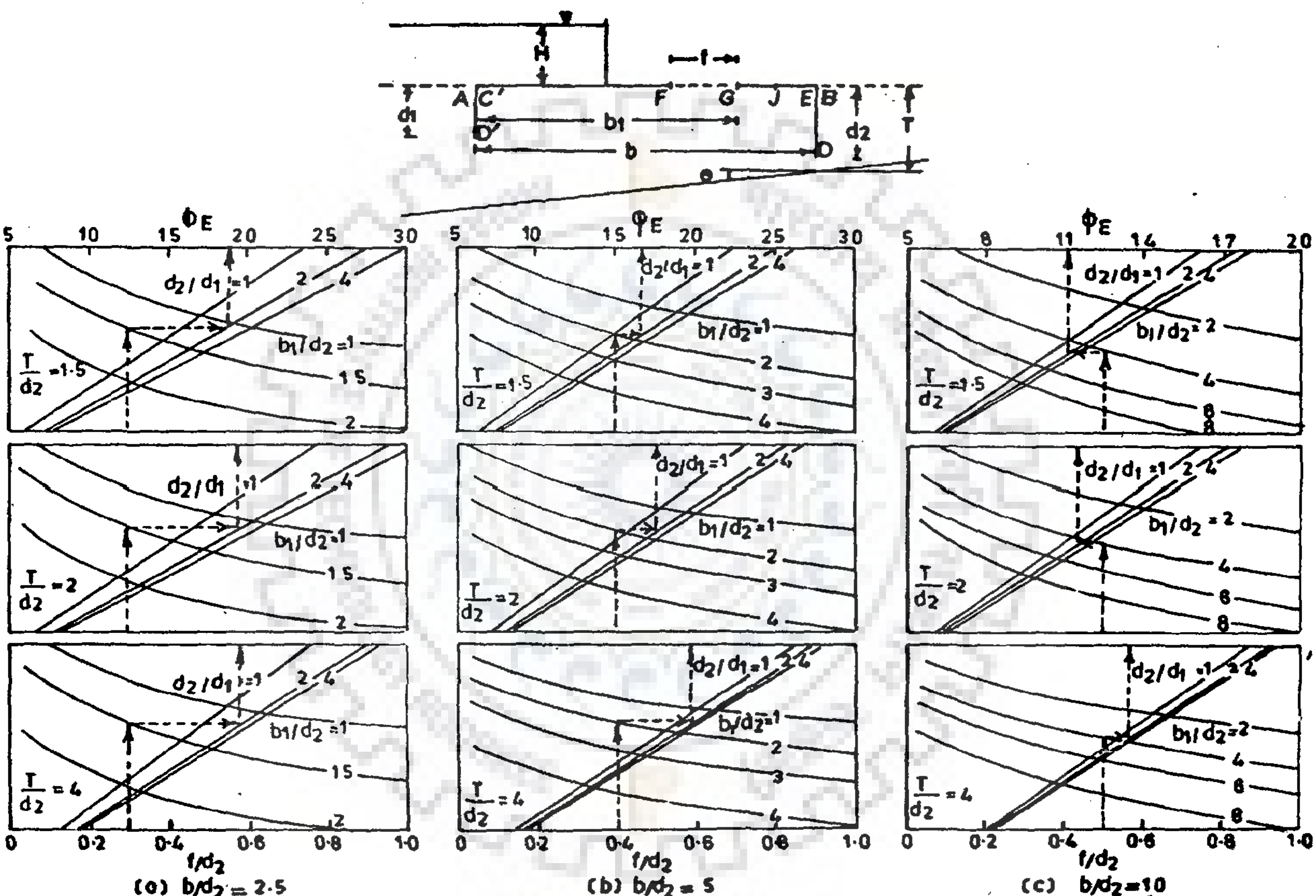


FIG. 4.4(iii) UPLIFT PRESSURE AT E ($\theta = 15^\circ$)

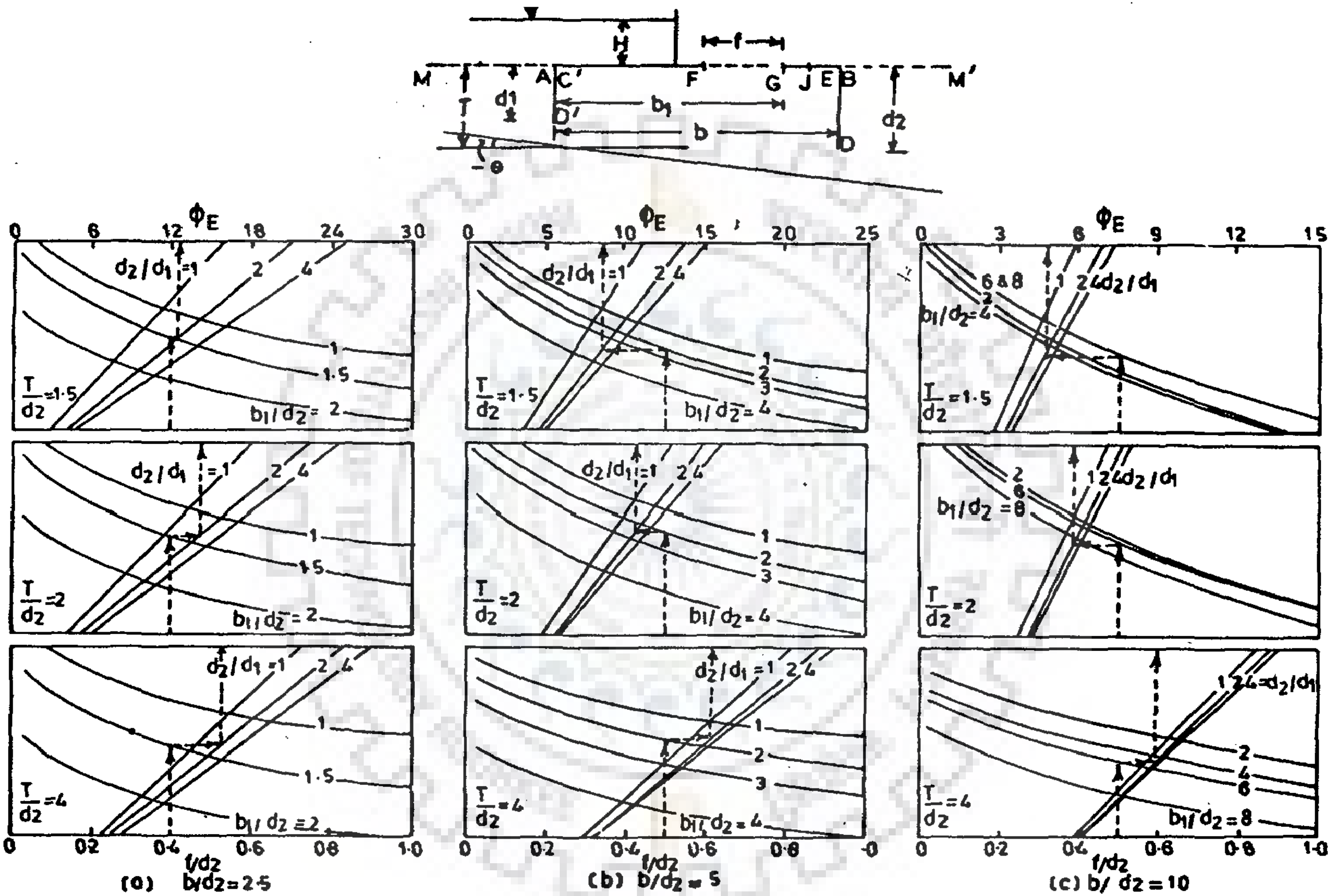
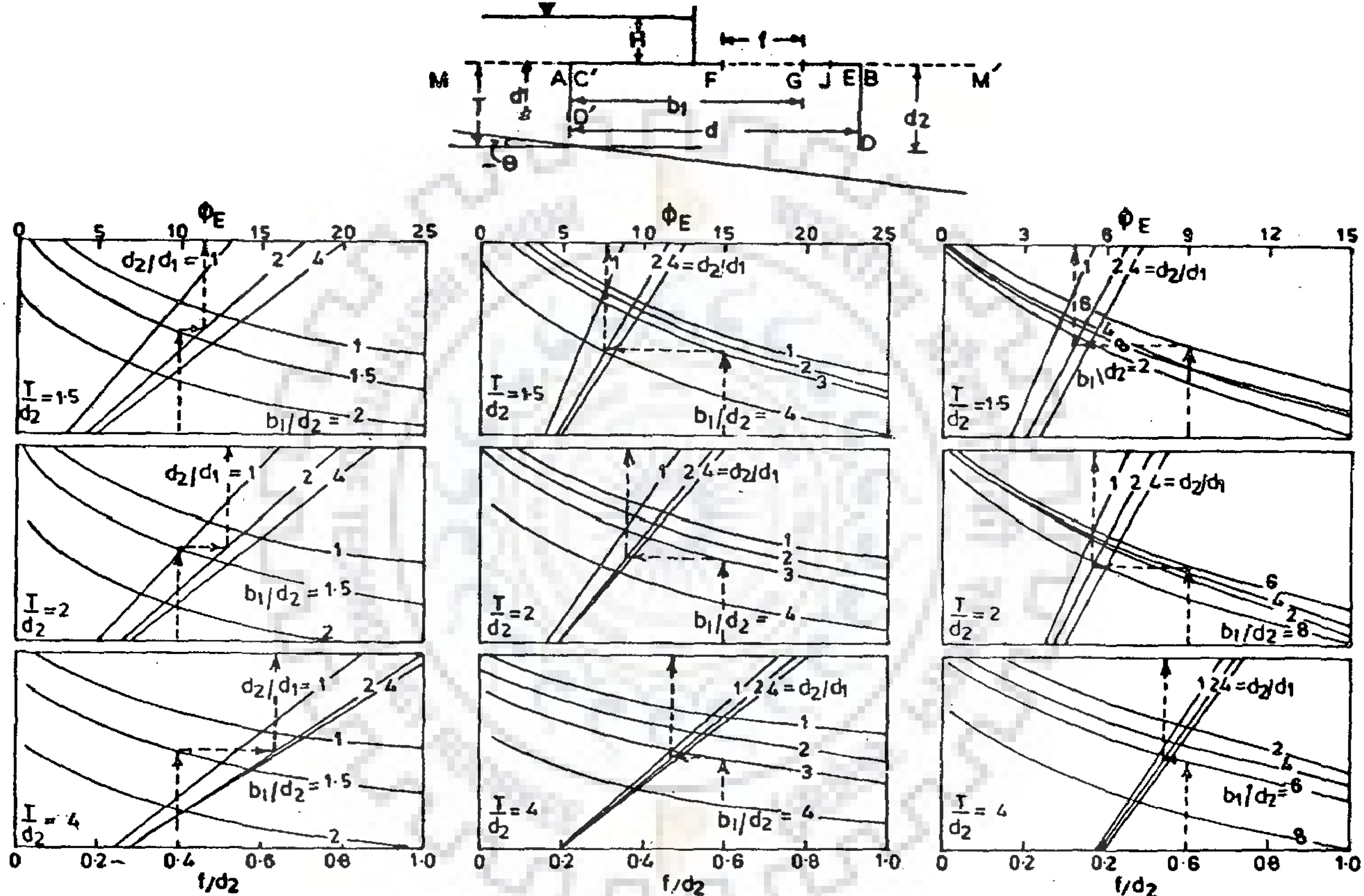
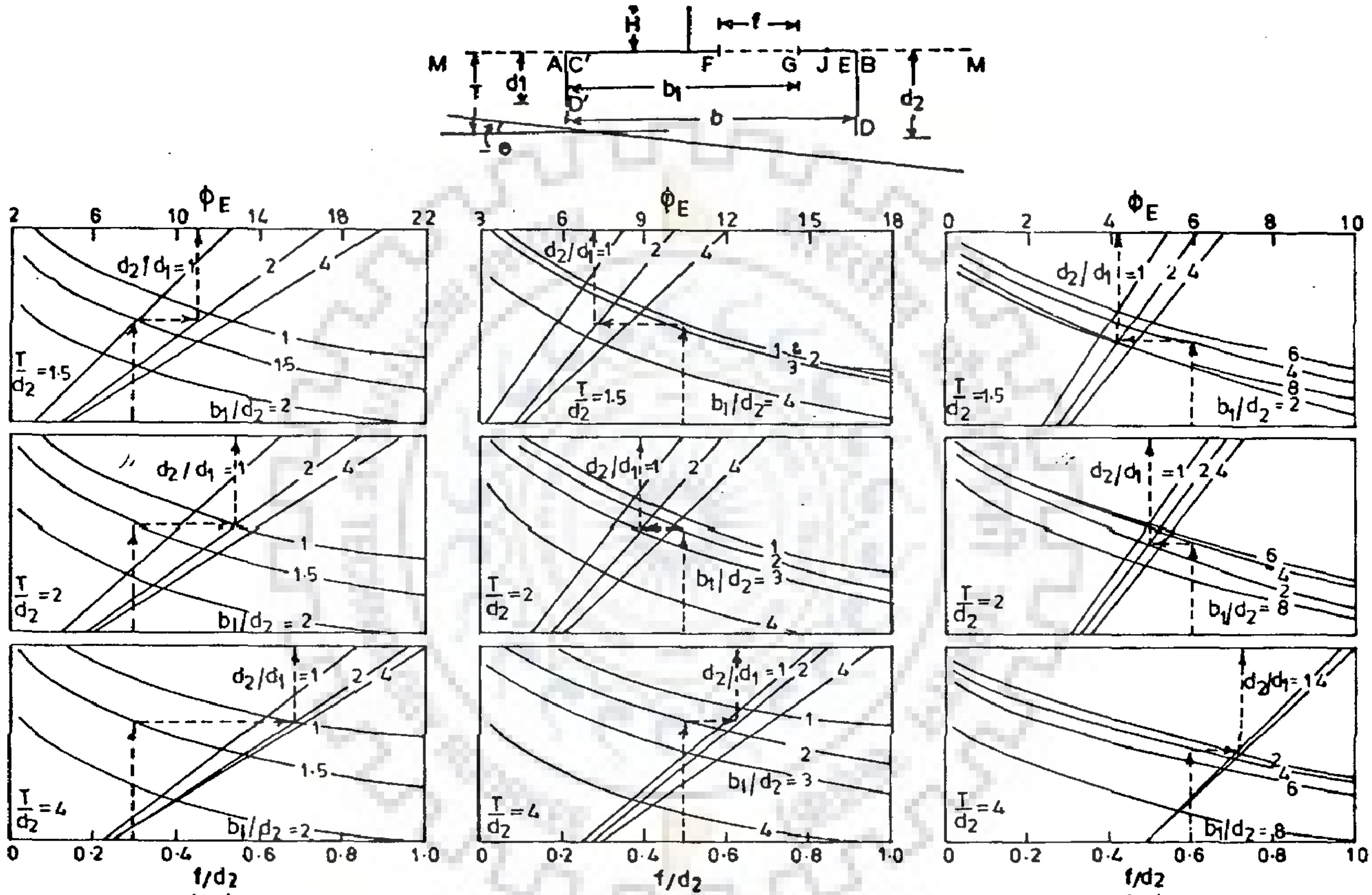


FIG. 4·4 (IV) UPLIFT PRESSURE AT E ($\theta = -15°$)



(a) $b/d_2 = 2{\cdot}5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG. 4·4 (V) UPLIFT PRESSURE AT E ($\theta = -30°$)



(a) $b/d_2 = 2.5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG. 4.4 (vi) UPLIFT PRESSURE AT E ($\theta = -45°$)

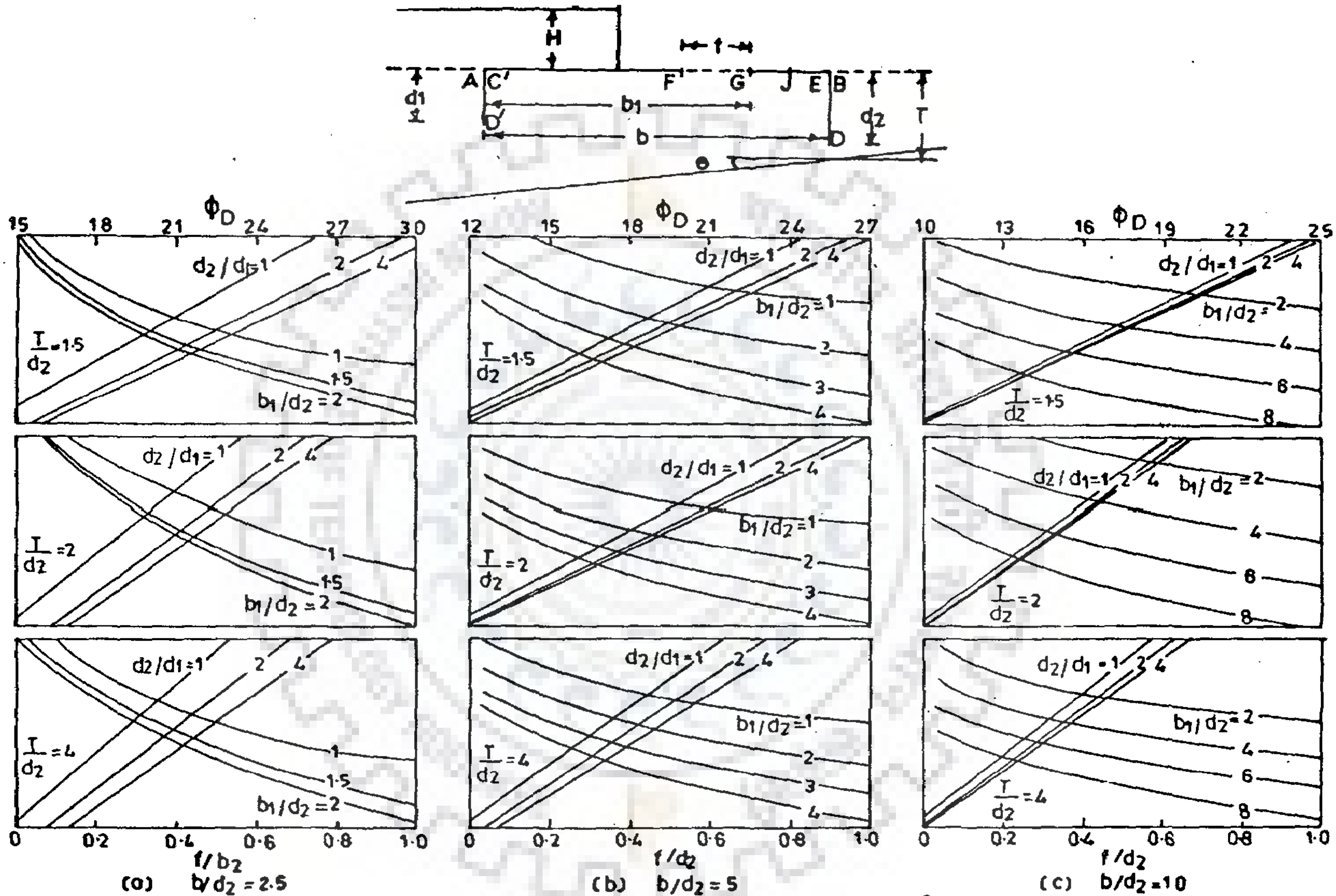


FIG. 4.5 (i) UPLIFT PRESSURE AT D (θ = 45°)

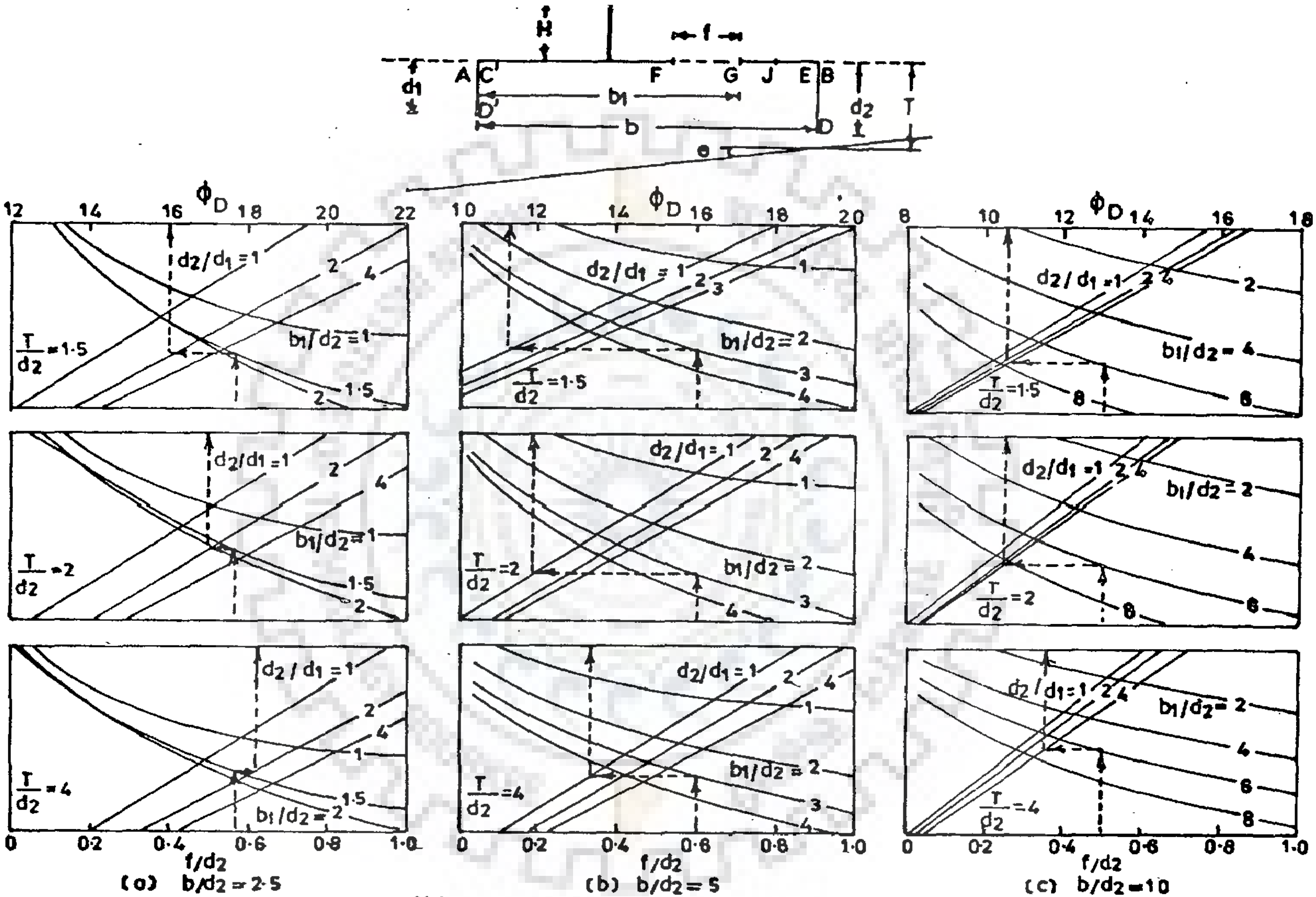
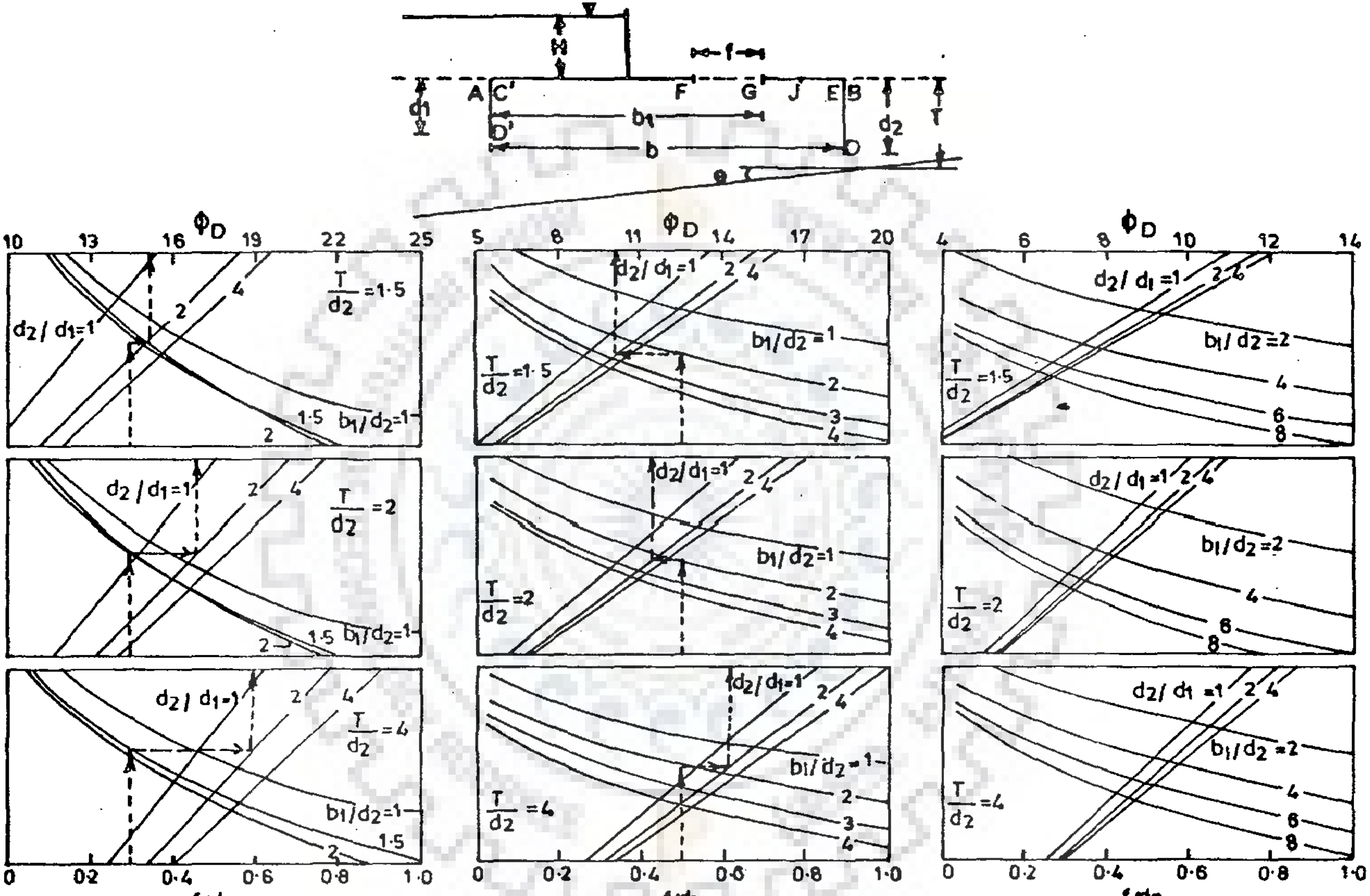
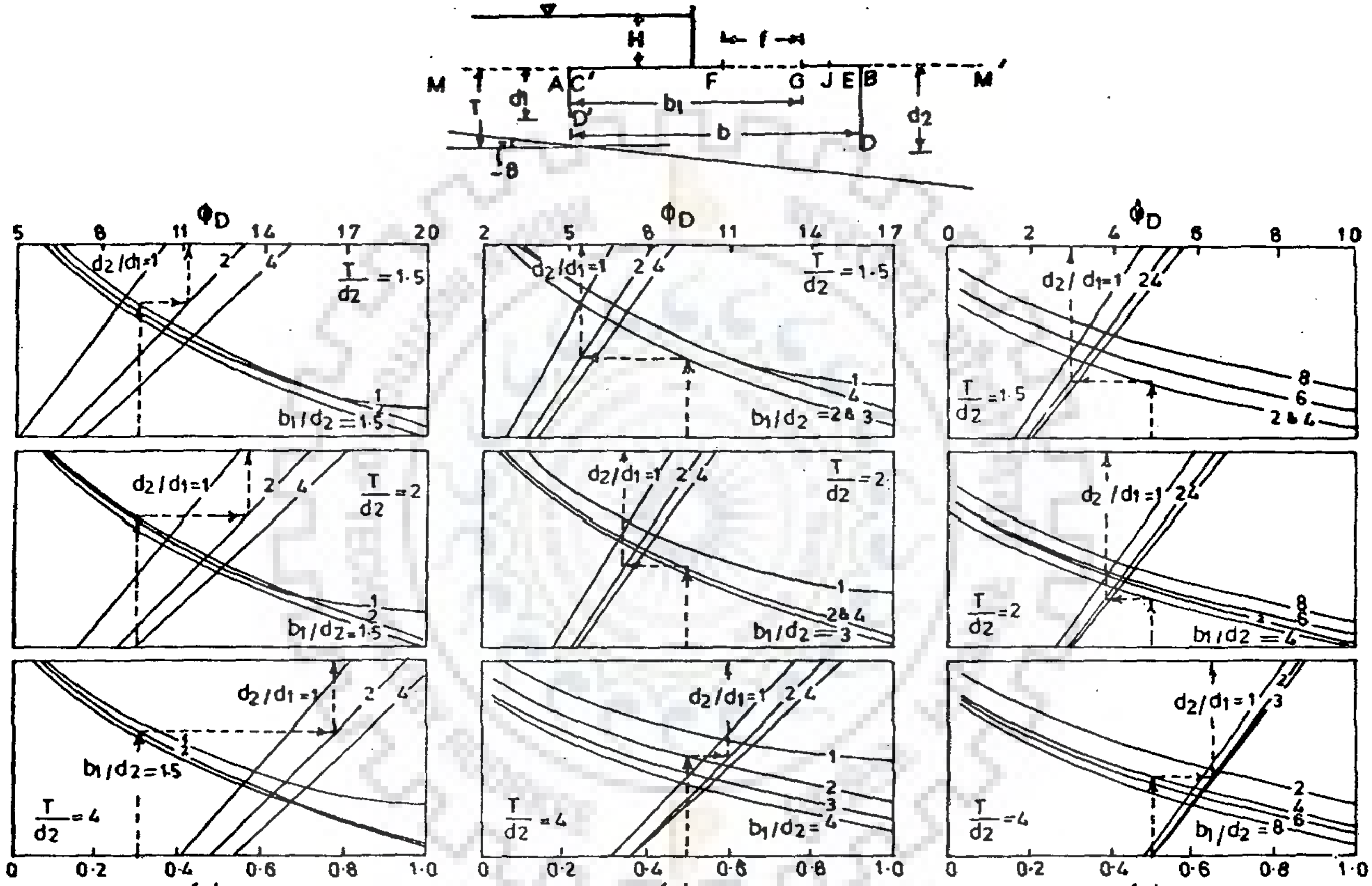


FIG. 4.5 (ii) UPLIFT PRESSURE AT D ($\theta = 30^\circ$)



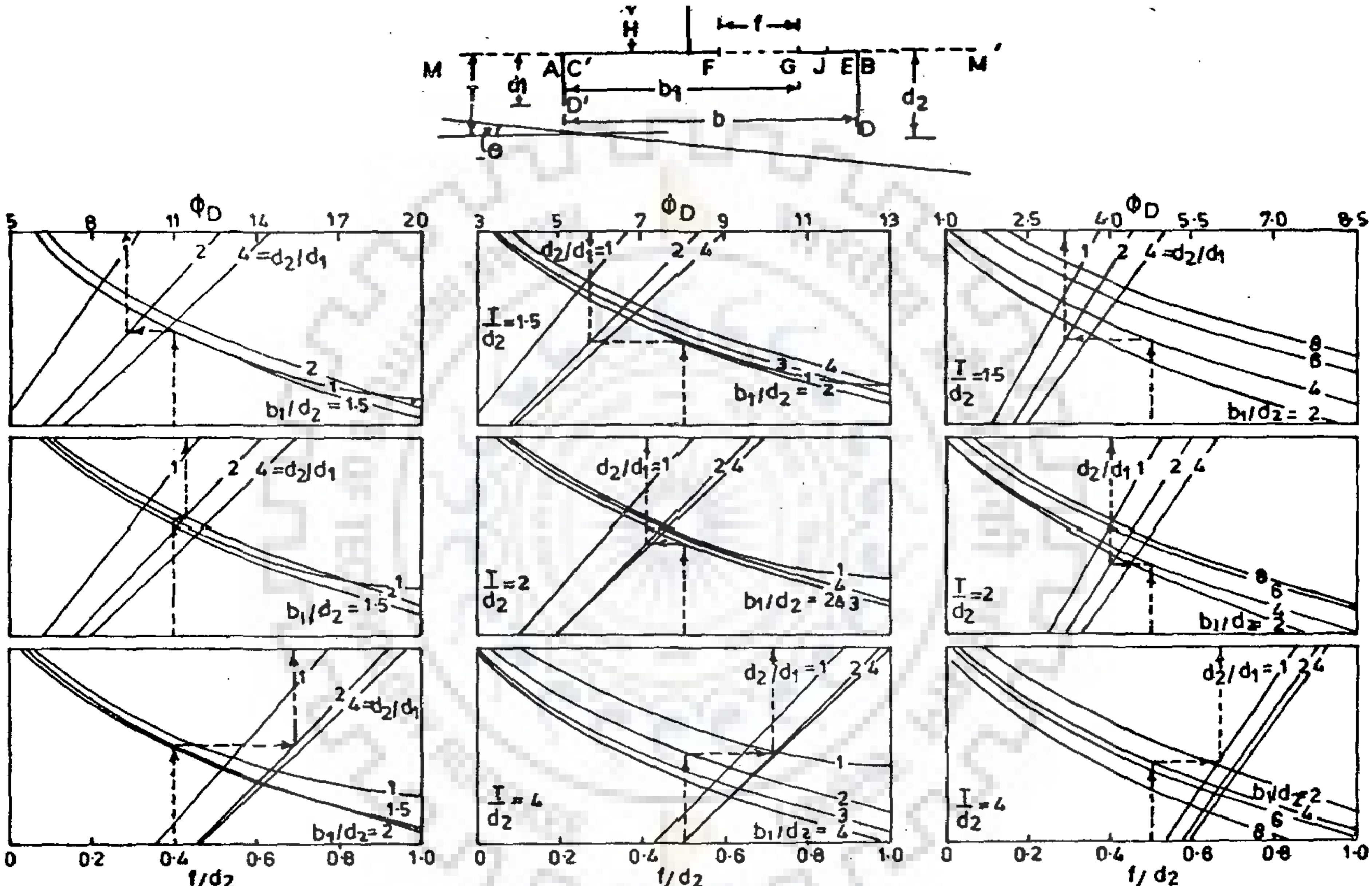
(a) $b/d_2 = 2.5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG.4.5 (iii) UPLIFT PRESSURE AT D ($\theta = 15°$)



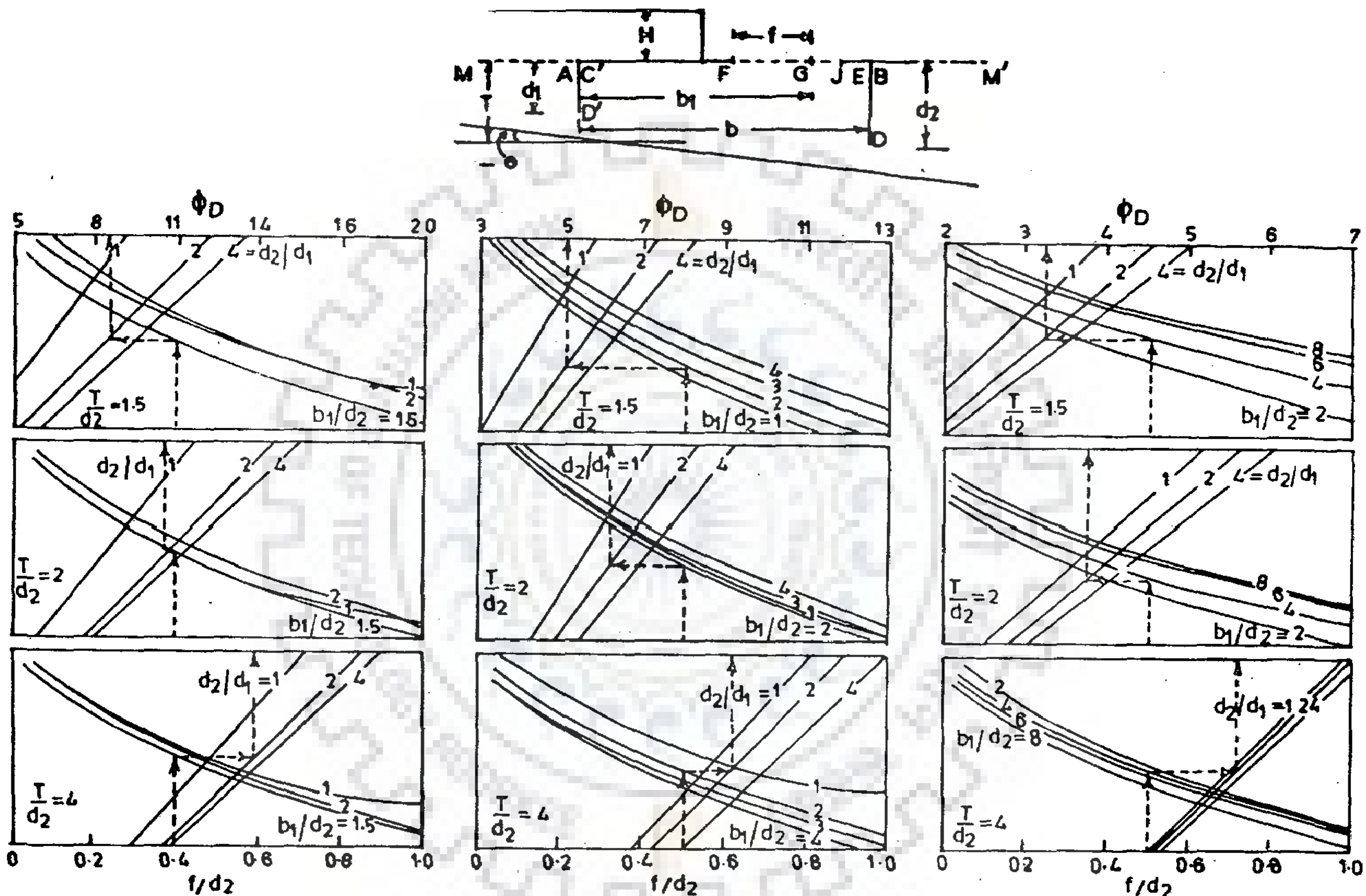
(a) $b/d_2 = 2{\cdot}5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG. 4·5 (IV) UPLIFT PRESSURE AT D ($\theta = -15°$)



(a) $b/d_2 = 2.5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG. 4.5 (V) UPLIFT PRESSURE AT D ($\theta = -30°$)



(a) $b/d_2 = 2.5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG 4.5(vi) UPLIFT PRESSURE AT D ($\theta = -45°$)

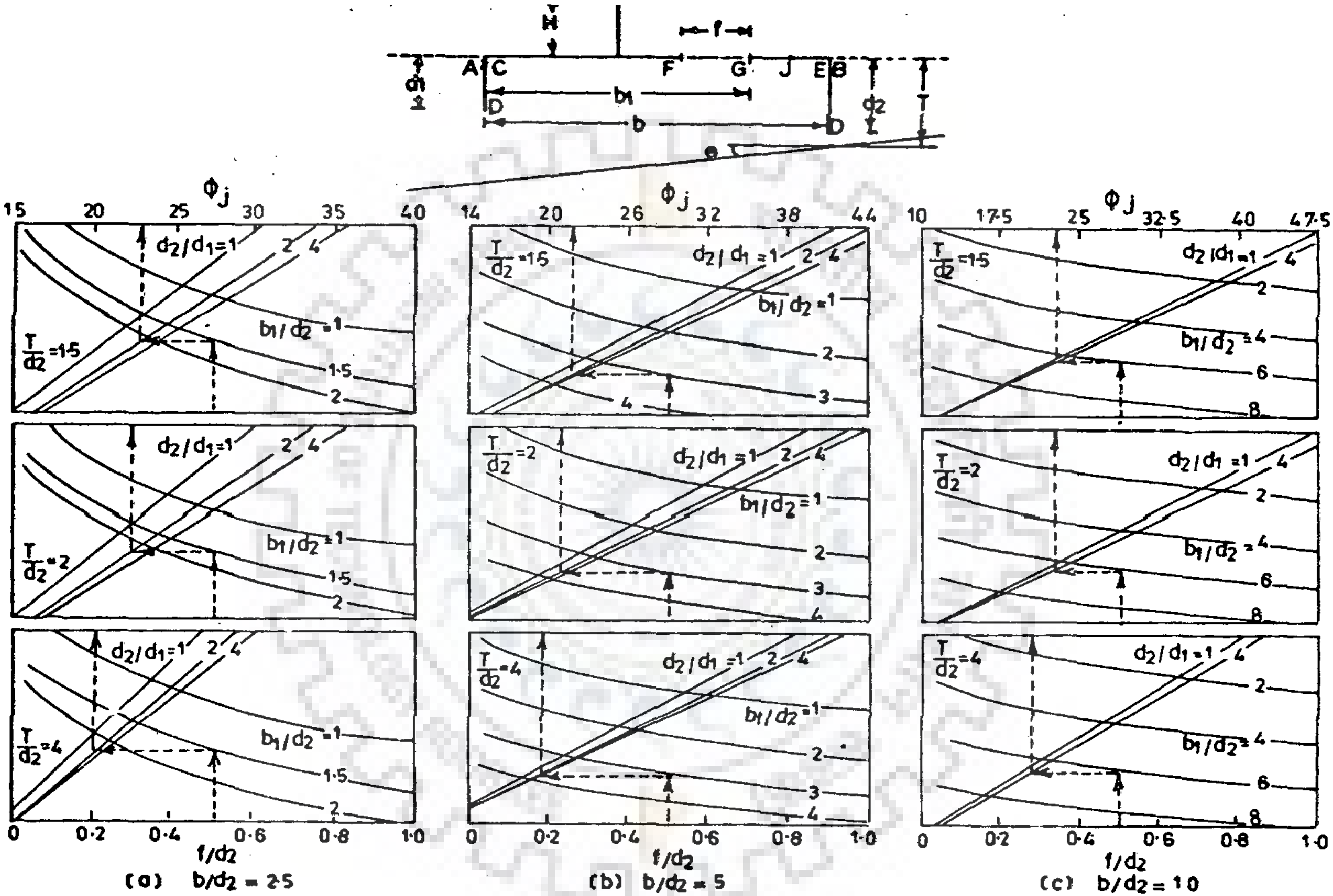


FIG 4.8(i) UPLIFT PRESSURE AT J (θ=45°)

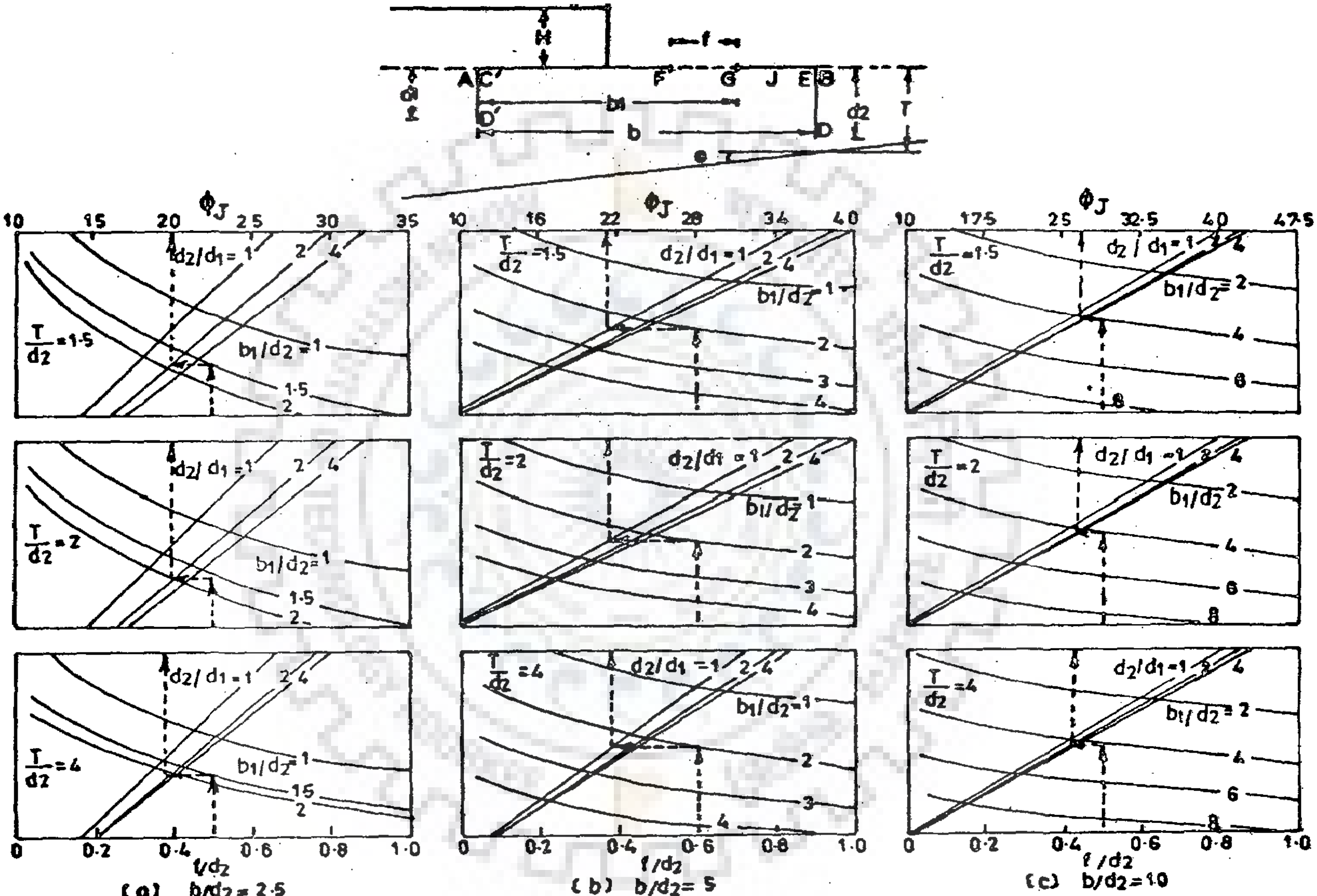
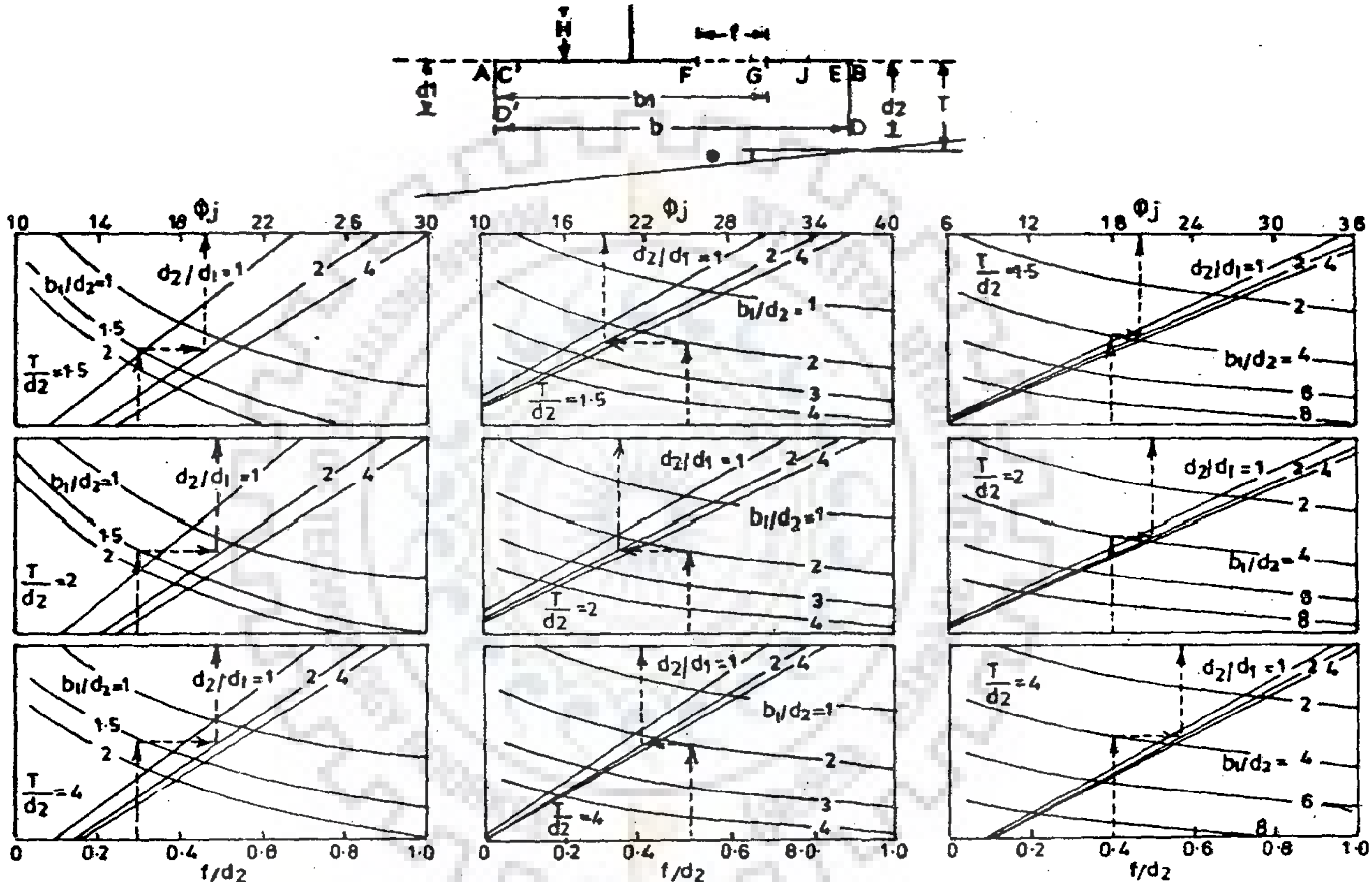


FIG. 4-6 (ii) UPLIFT PRESSURE AT J (θ = 30°)



(a) $b/d_2 = 2.5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG. 4.8 (iii) UPLIFT PRESSURE AT J ($\theta = 15^\circ$)

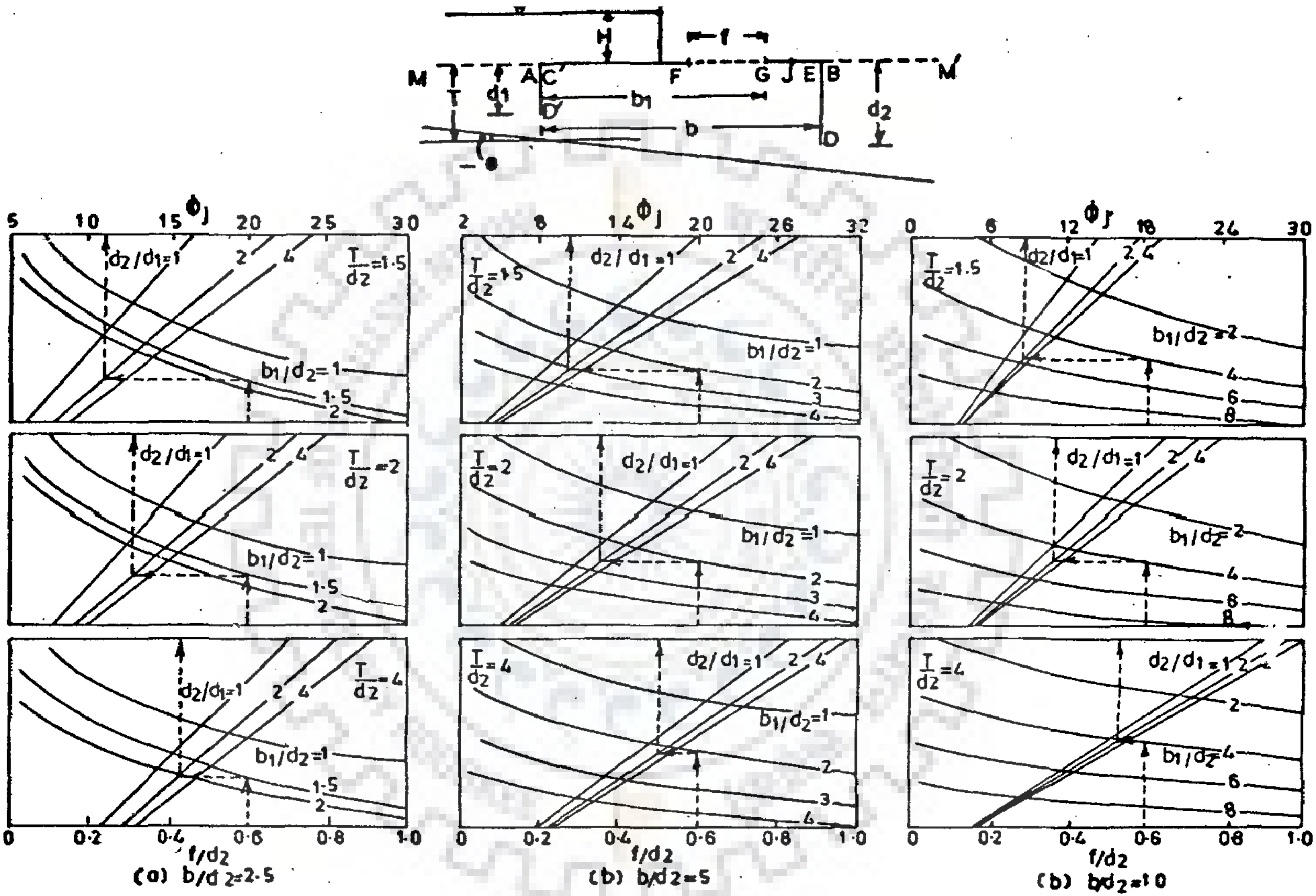
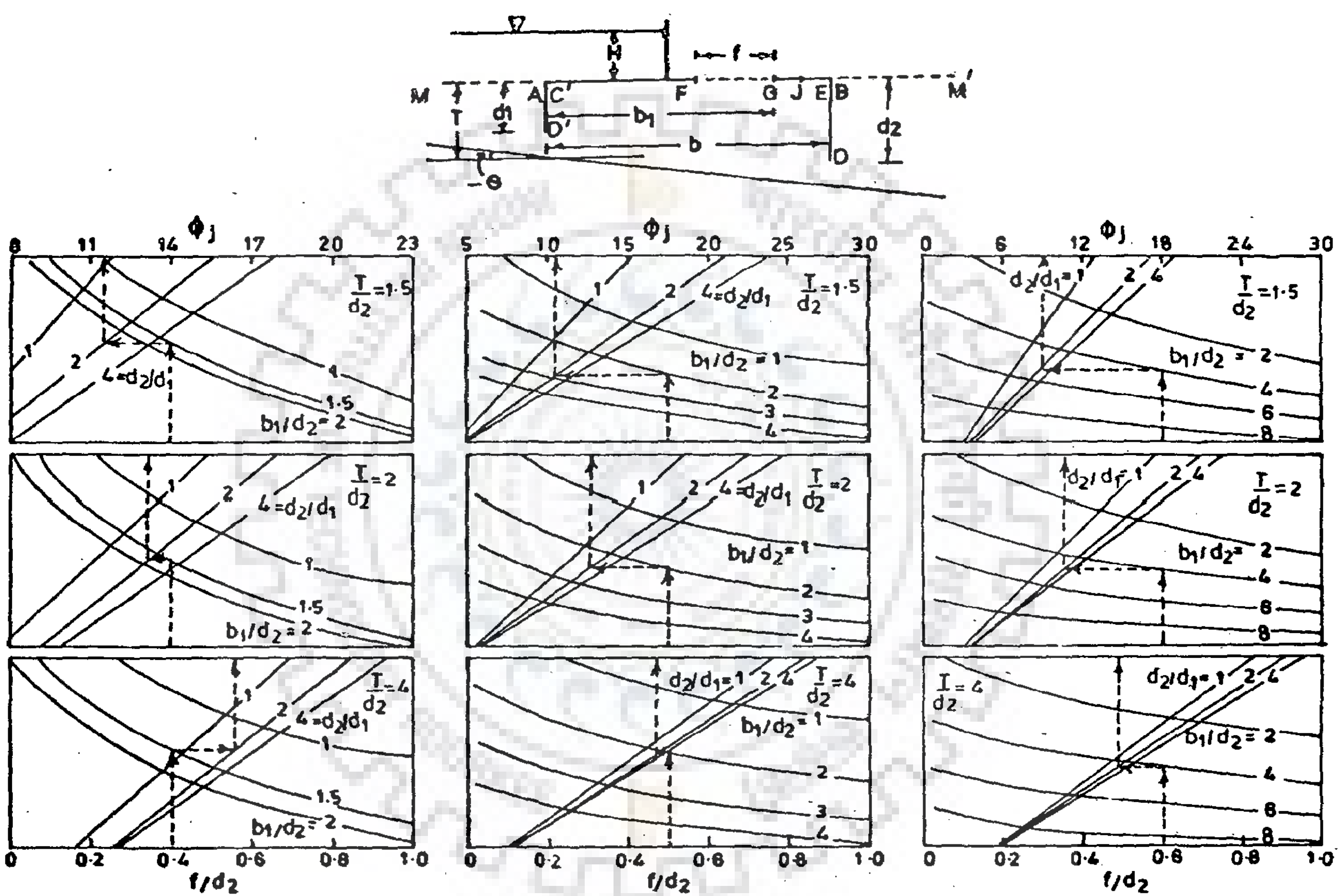
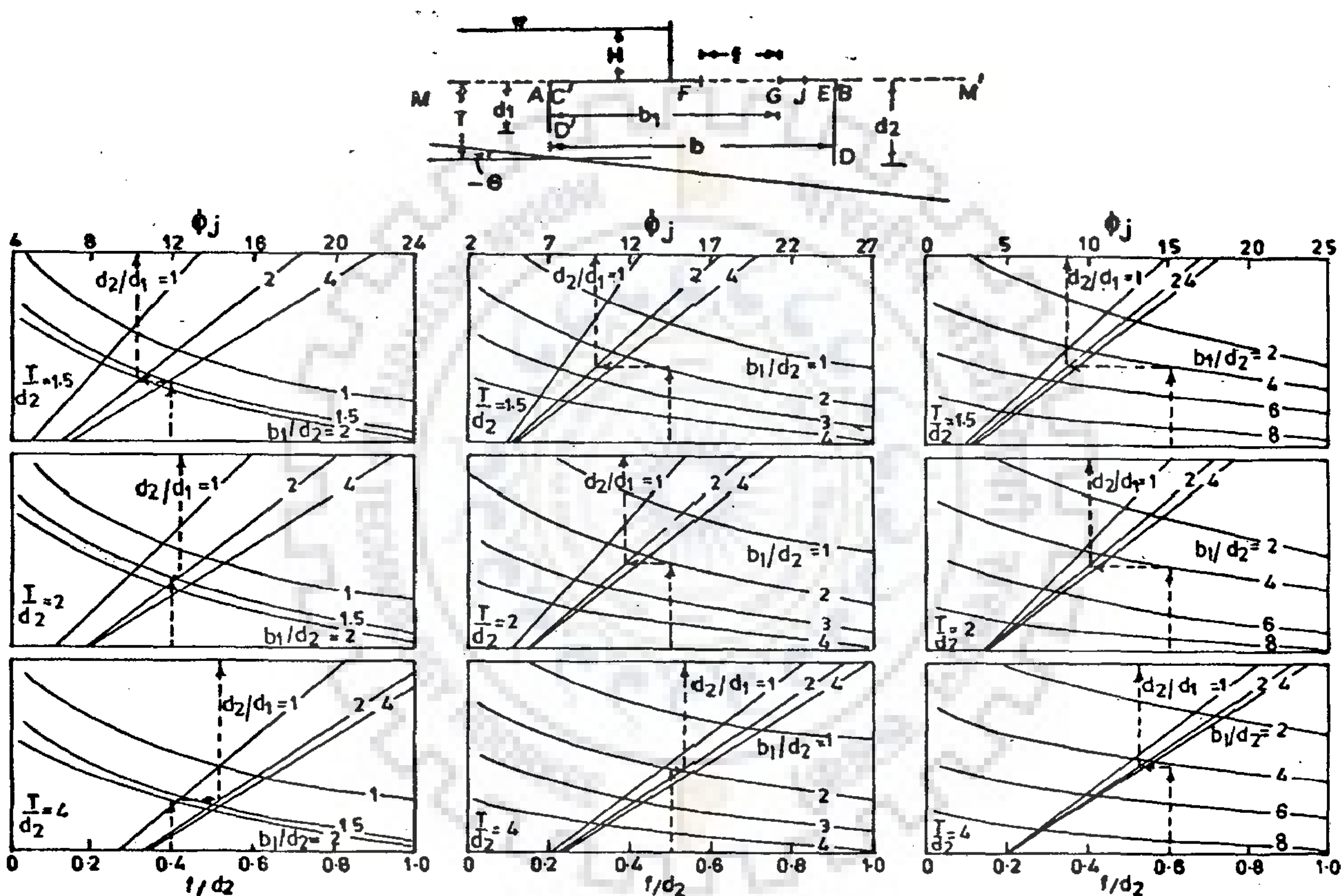


FIG.4·6 (iv) UPLIFT PRESSURE AT J (θ = −15°)



(a) $b/d_2 = 2.5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG. 4·6 (V) UPLIFT PRESSURE AT J $(\theta = -30^\circ)$



(a) $b/d_2 = 2{\cdot}5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG. 4·6 (vi) UPLIFT PRESSURE AT J ($\theta = -45°$)

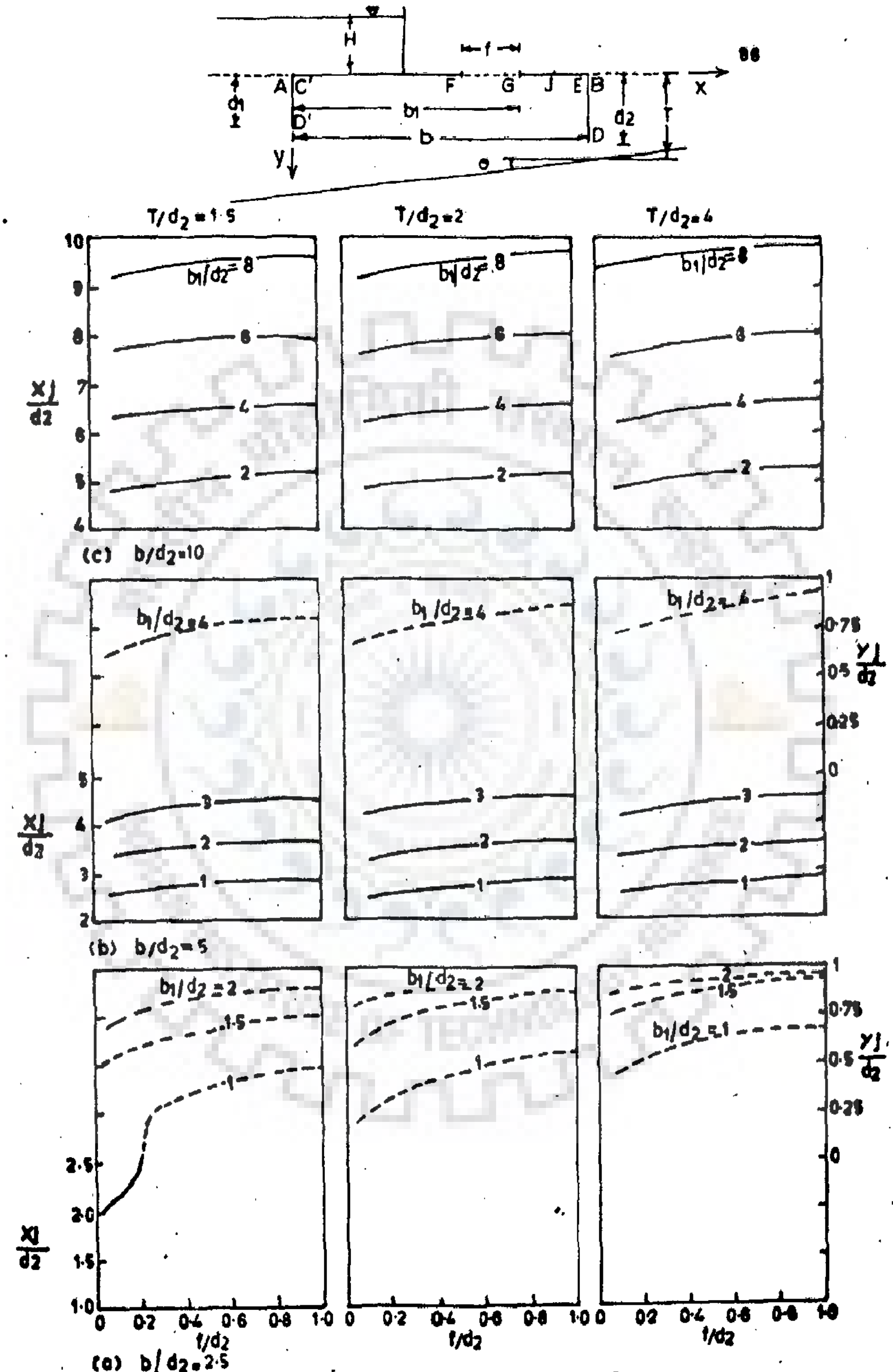


FIG.4.7 (I) LOCATION OF J (θ = 45°)

Xj = ABSCISSA OF POINT J , Yj = ORDINATE OF POINT J

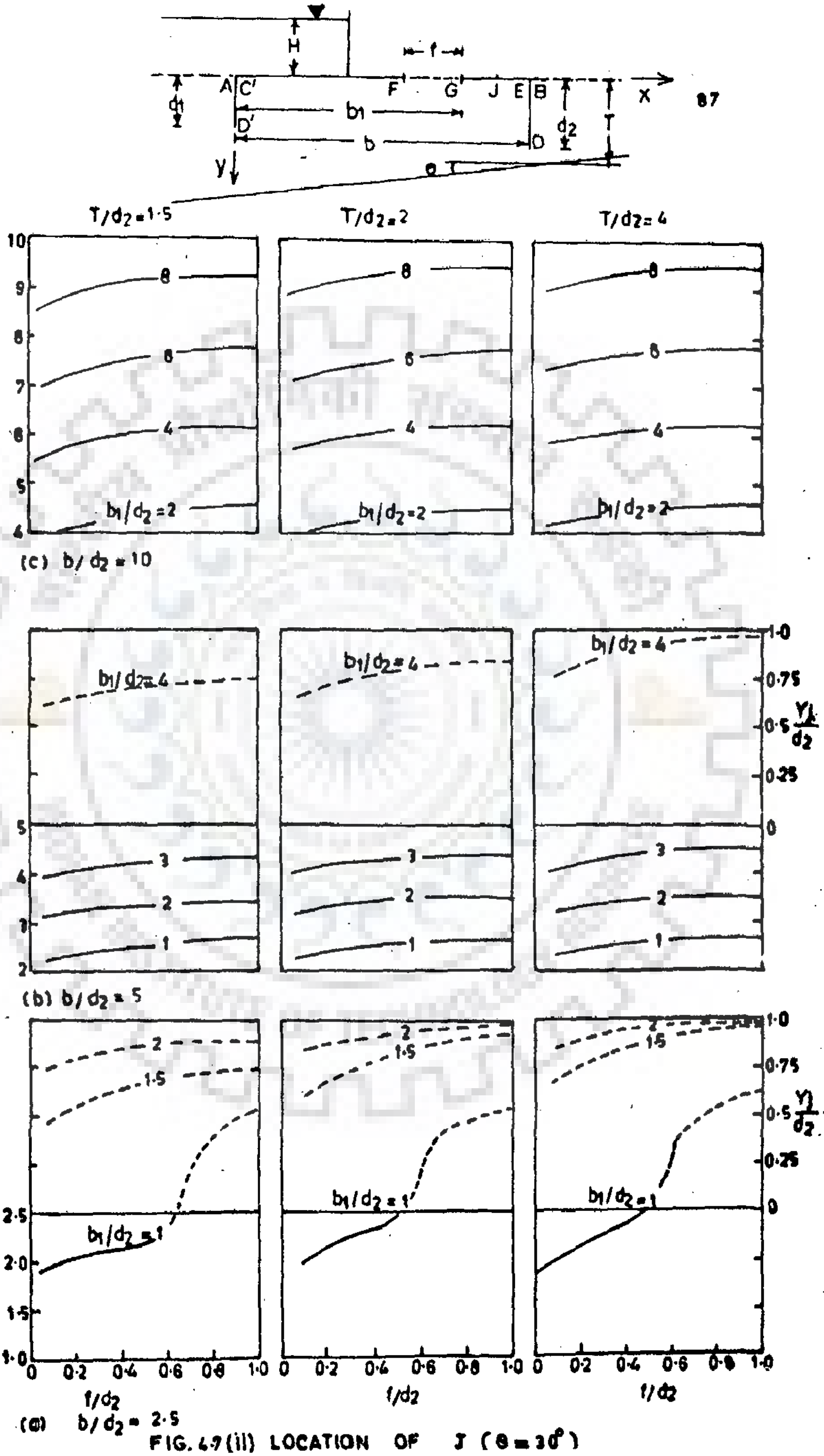


FIG. 4.7 (ii) LOCATION OF J (θ = 30°)

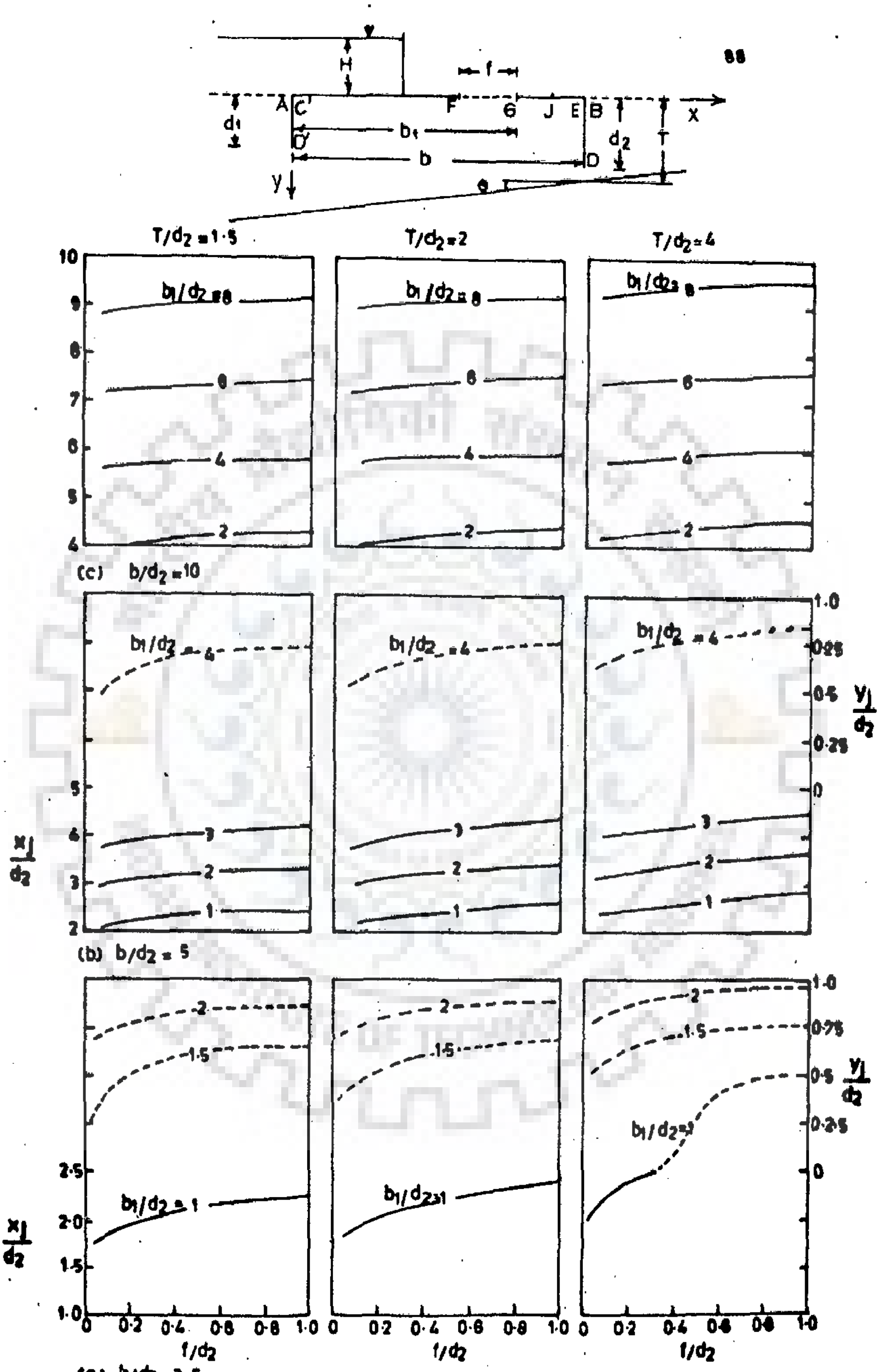


FIG. 4.7(iii) LOCATION OF J (θ = 15°)

x_J = ABSCISSA OF POINT J, y_J = ORDINATE OF POINT J

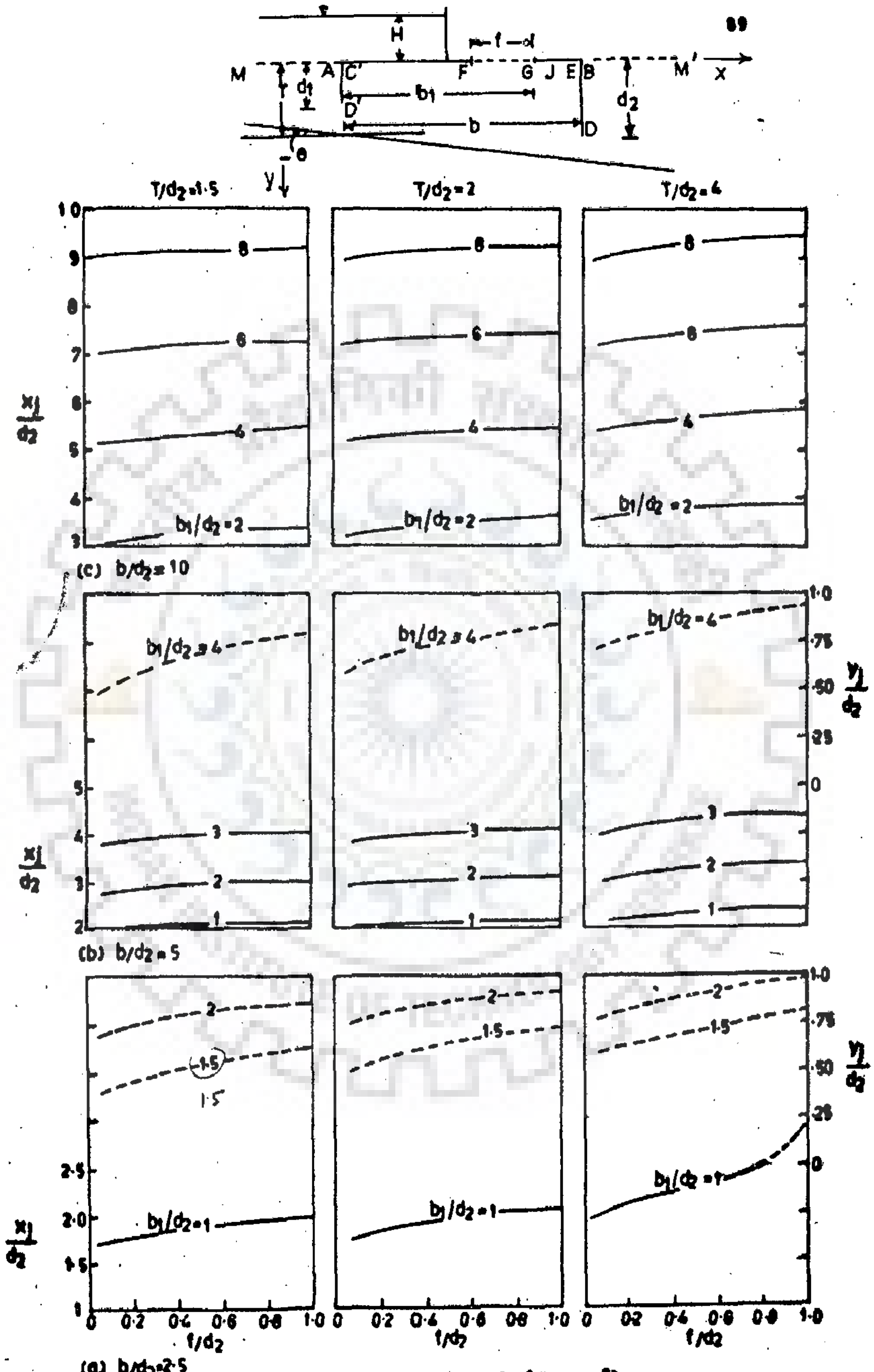


FIG.4.7 (IV) LOCATION OF J (θ = −15°)

X_j = ABSCISSA OF POINT J, Y_j = ORDINATE OF POINT J

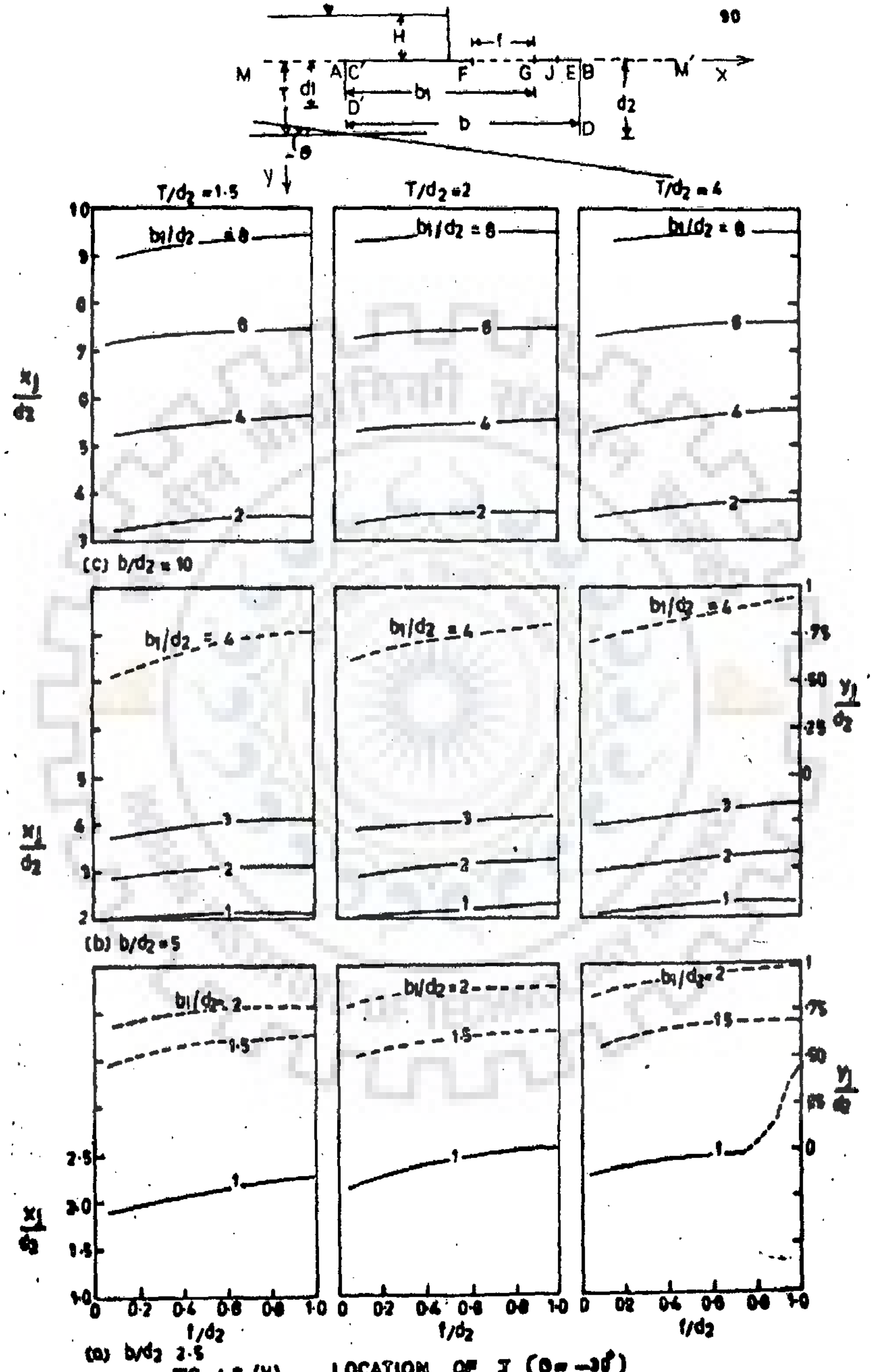
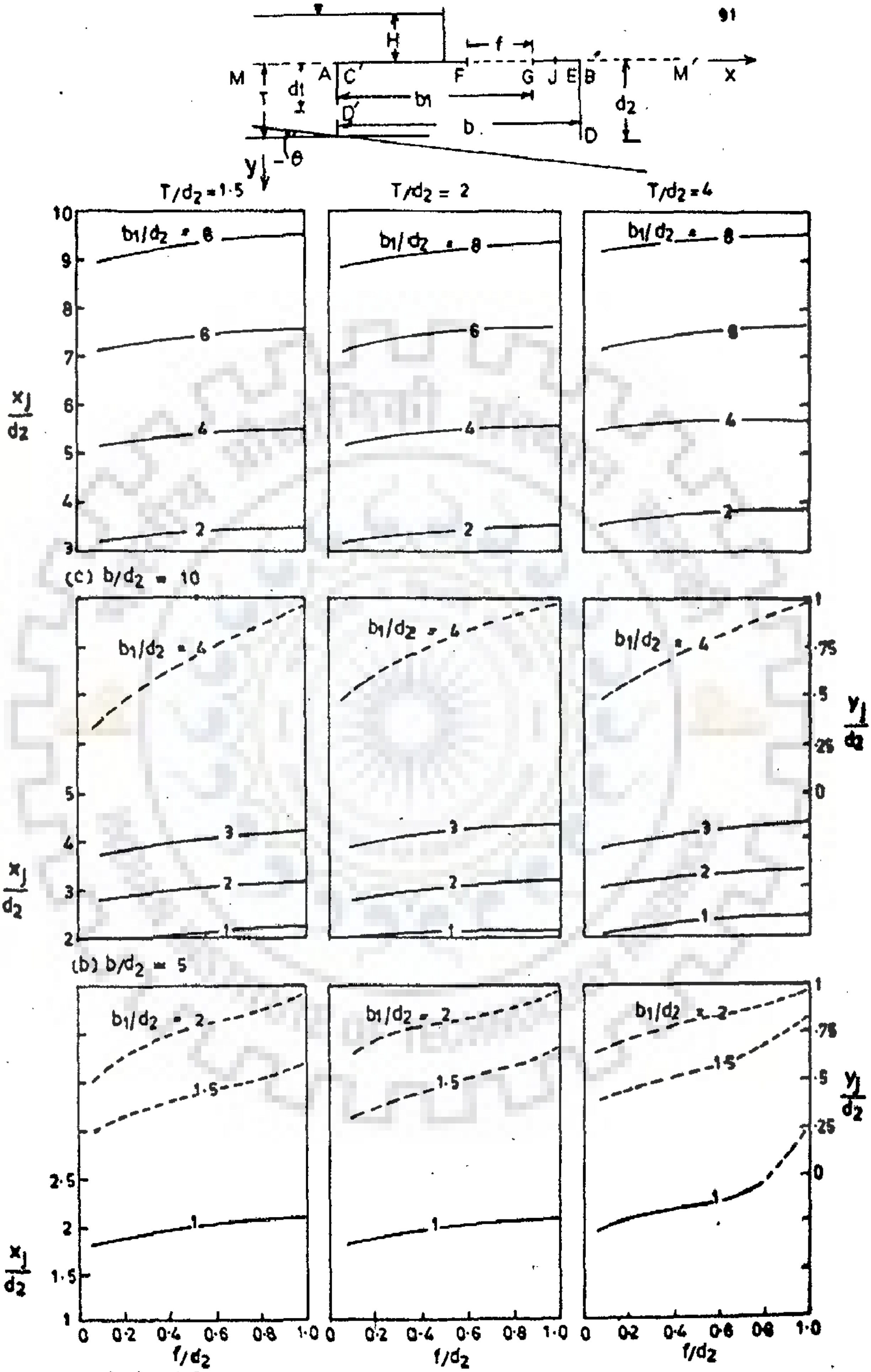


FIG. 4·7 (V) LOCATION OF J ($\theta = -30°$)

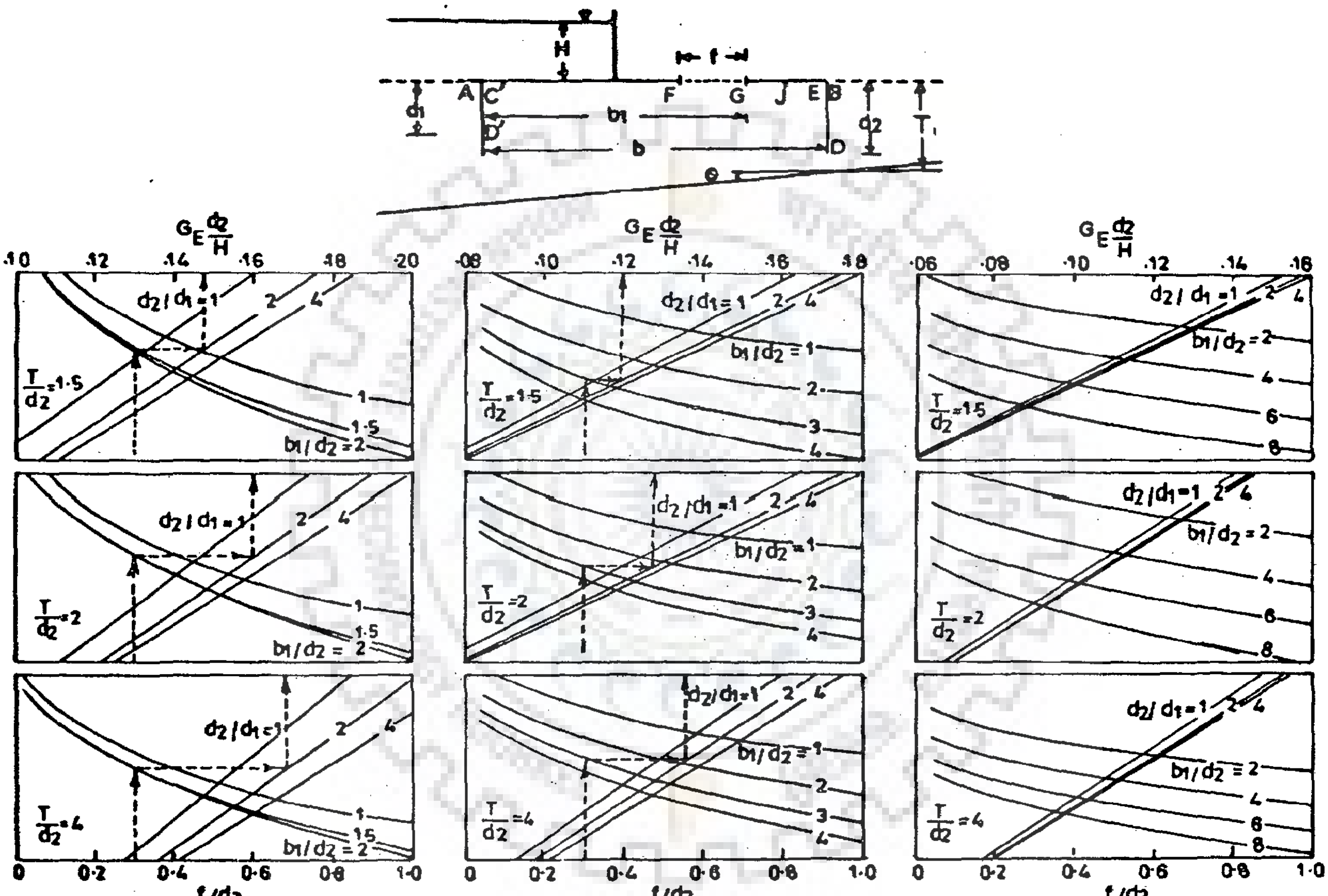
X_J : ABSCISSA OF POINT J, Y_J : ORDINATE OF POINT J.



(a) b/d2 2·5

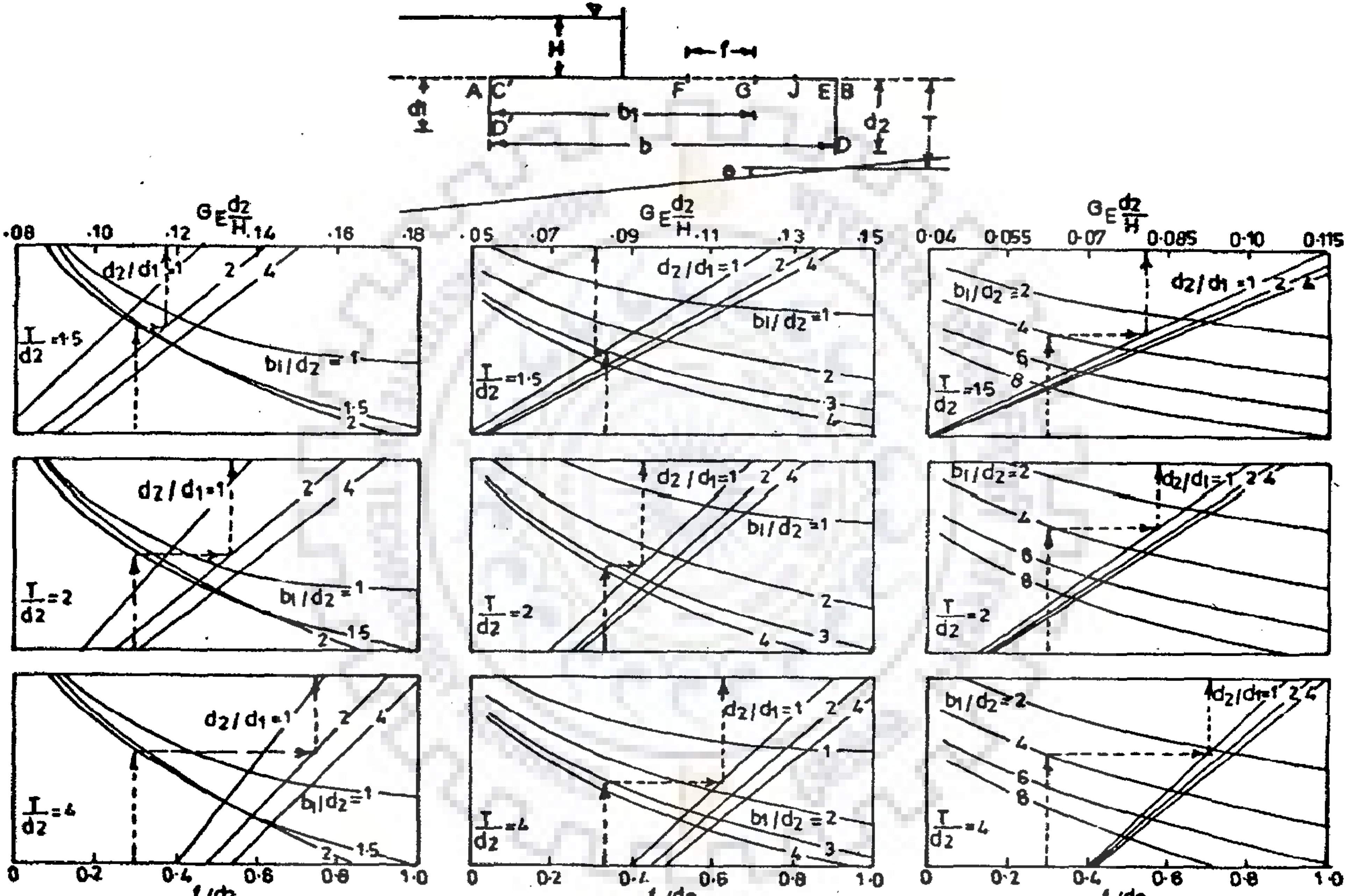
FIG 4·7 (VI) LOCATION OF J ($\theta = -45°$)

X_J = ABSCISSA OF POINT J, Y_J = ORDINATE OF POINT J



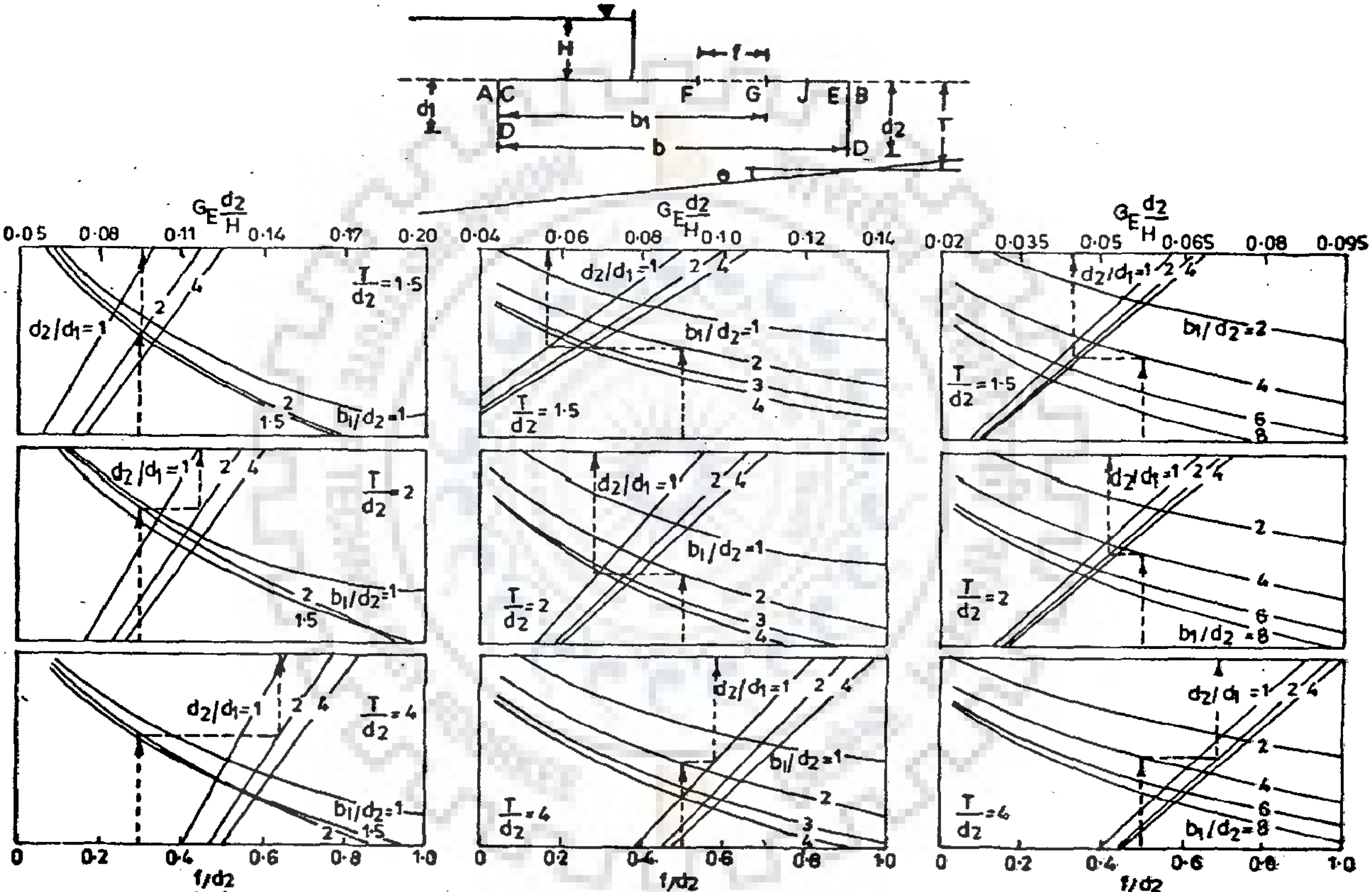
(a) $b/d_2 = 2{\cdot}5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG. 4·8 (i) EXIT GRADIENT AT B ($\theta = 45°$)



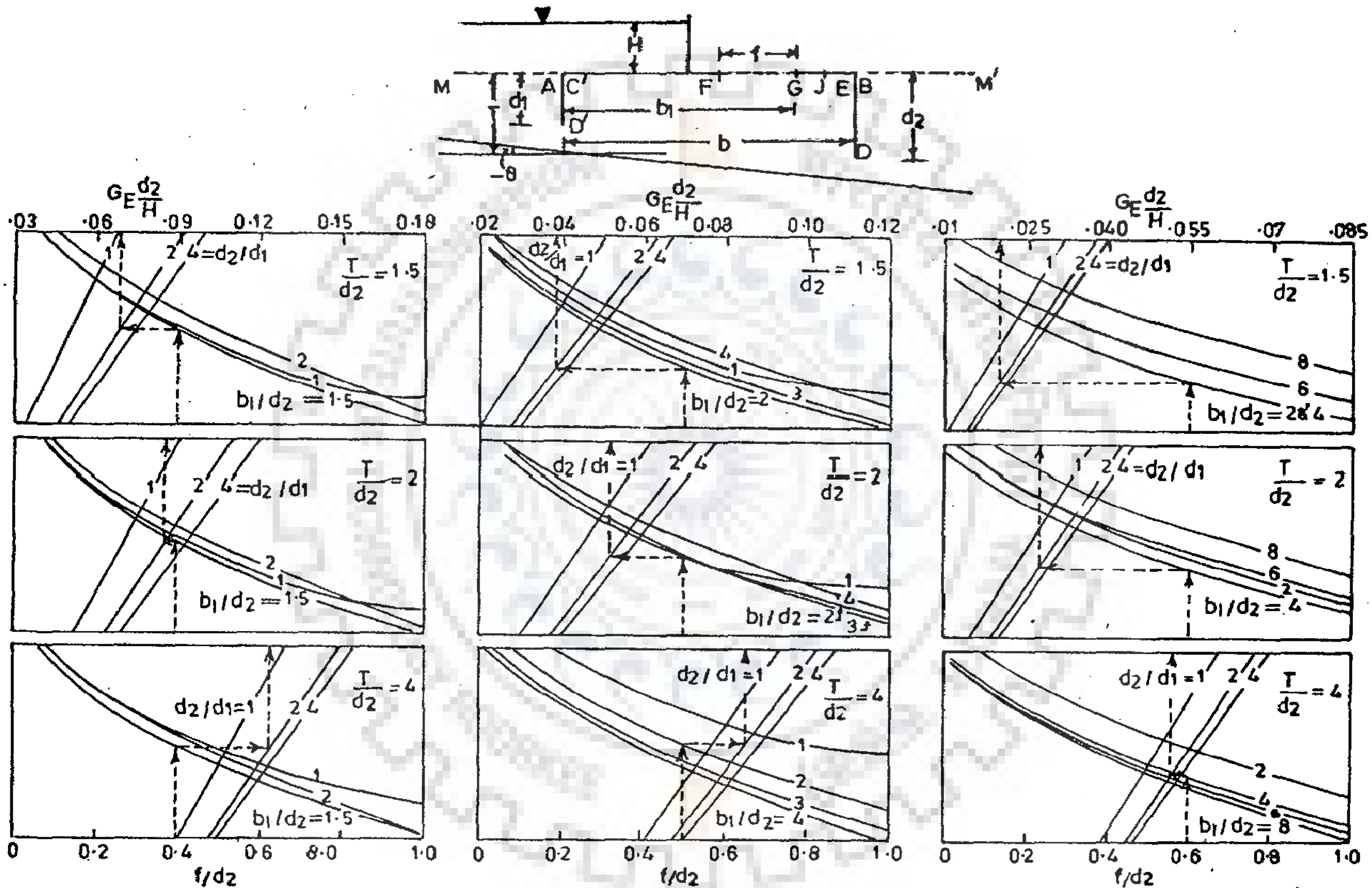
(a) $b/d_2 = 2.5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG.4.8 (ii) EXIT GRADIENT AT 'B' ($\theta = 30°$)



(a) $b/d_2 = 2\cdot5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG. 4·8 (iii) EXIT GRADIENT AT B ($\theta = 15°$)



(a) $b/d_2 = 2.5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG. 4·8 (iV) EXIT GRADIENT AT B ($\theta = -15°$)

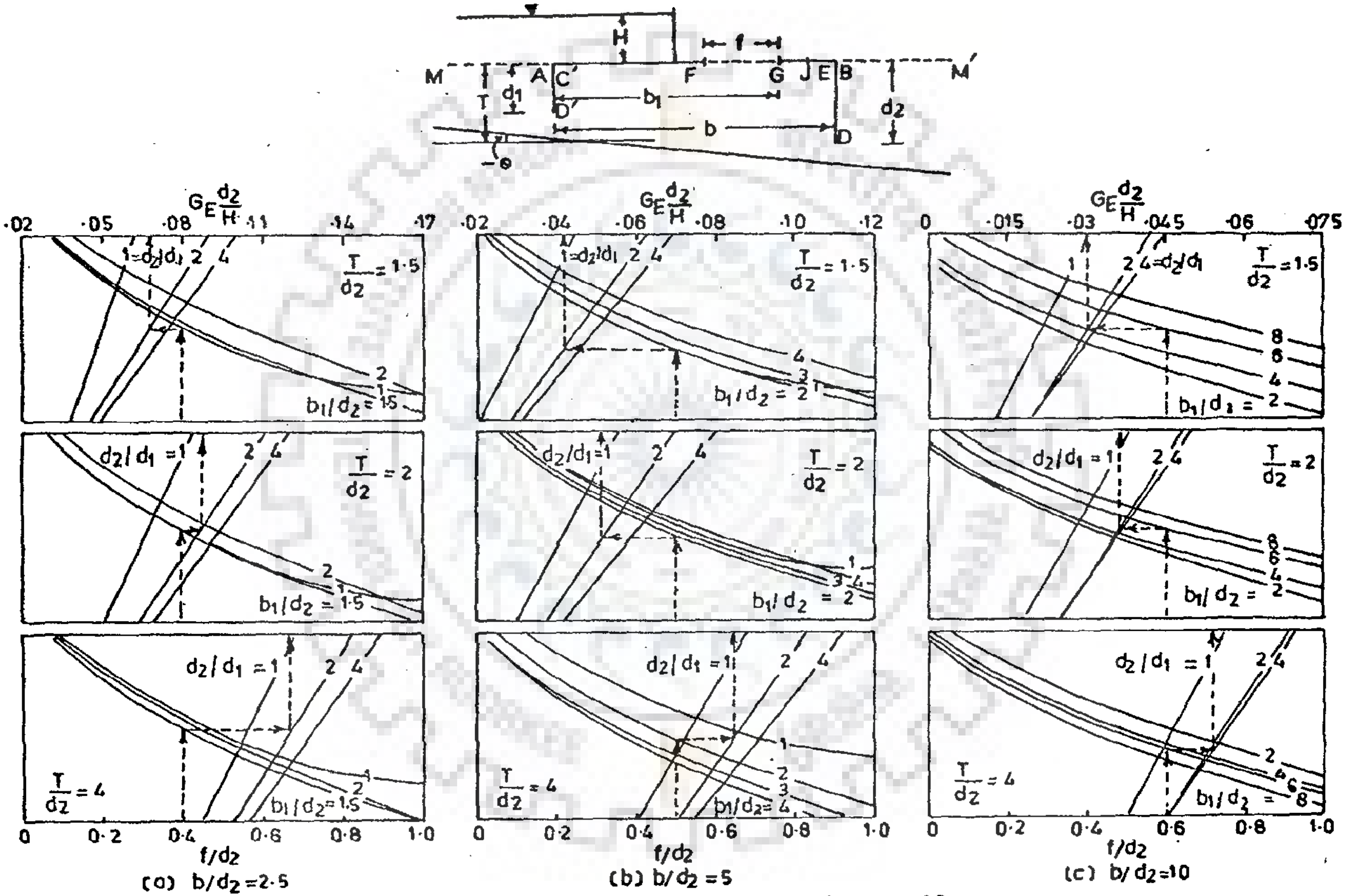
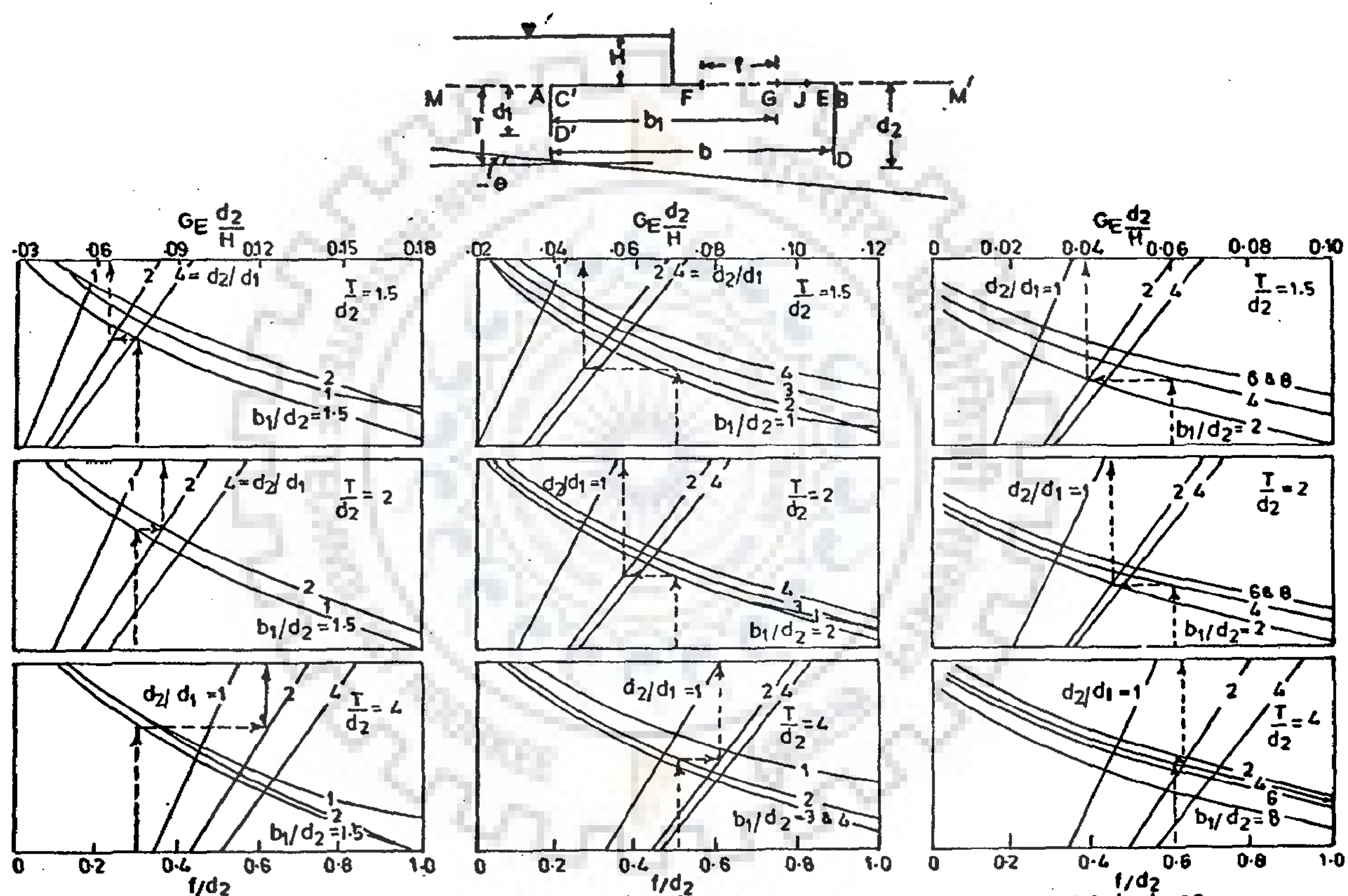


FIG. 4·8 (V) EXIT GRADIENT AT B ($\theta = -30°$)



(a) $b/d_2 = 2.5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

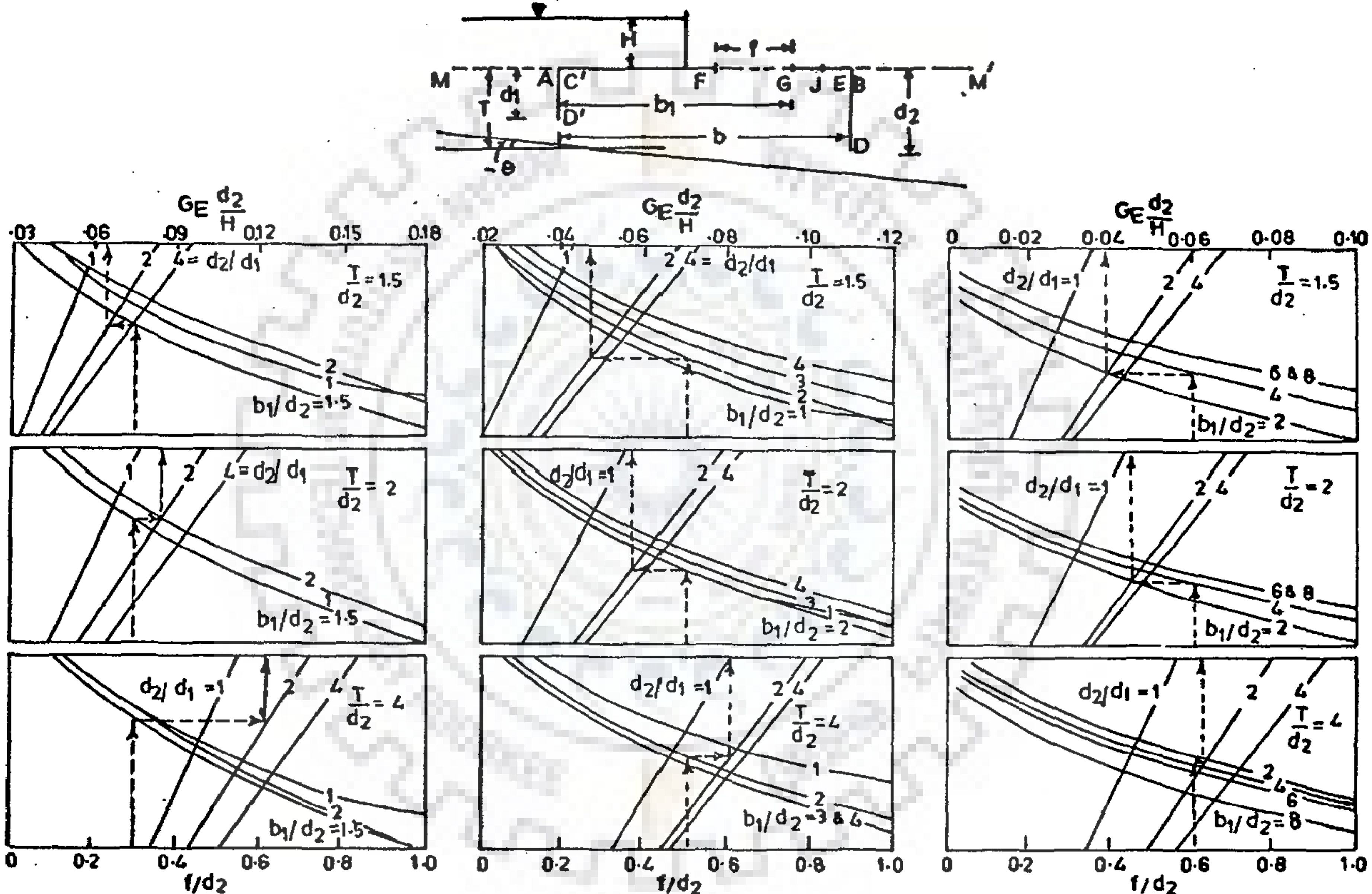
FIG. 4·8 (vi) EXIT GRADIENT AT B ($\theta = -45°$)

filter and may cause failure of structure due to piping, starting from filter towards upstream of floor.

5.2 FACTOR OF SAFETY AGAINST SUBSURFACE EROSION

Practically all piping failures on record have been caused by subsurface erosion involving the progressive removal of material through springs. The erosion of subsurface soil is supposed to result by the high velocities of flow of water through it, when such velocities exceed a certain limit. But this concept of piping is incomplete. In 1925 Professor Charles Terzaghi explained the causes of subsurface erosion by pressure - gradient. Water has a certain residual force in the direction of flow and is proportional to the pressure gradient at that point. At the release point, this force is upwards and will tend to lift up the soil particle if it is more than the submerged weight of the latter. Once the surface particles are disturbed the resistence against upward pressure of water will be further reduced, tending to progressive disruption of subsoil. At the instant of just stable condition the residual force will have its critical value which will be just resisted by downward weight of soil. The slightest increase in the value of residual force will lead instability and soil particle will start to be lifted up. Therefore, the exit gradient provides a design criterian for the factor of safety with respect to piping caused by subsurface erosion initiated at exit point.

At both the ends F and G of the intermediate filter the



(a) $b/d_2 = 2.5$ (b) $b/d_2 = 5$ (c) $b/d_2 = 10$

FIG. 4·8 (vi) EXIT GRADIENT AT B ($\theta = -45°$)

CHAPTER - 5

SAFETY AGAINST PIPING BELOW FILTER

5.1 INTRODUCTION

A number of structures founded on previous soil have failed apparently by sudden formation of a pipe shaped discharge channel or tunnel located between the soil and the foundation. As the stored water rushed out of the reservoir into the outlet passage, the width and depth of passage increased rapidly until the structure collapsed. An event of this type is known as failure by piping.

Failures by piping can be caused by two different processes. They may be due to subsurface erosion that starts at springs near the downstream toe and proceeds upstream along the base of the structure or some bedding plane. Failure occurs as the upstream end of eroded hole approaches the bottom of the reservoir. Piping failures have also been initiated by the sudden rise of a large body of soil adjoining the downstream toe of the structure. A failure of this kind occures only if the seepage pressure of the water that percolates upward through the soil beneath the toe becomes greater than the effective weight of the soil. Failures of the first category are referred to as failures by sub-surface erosion and those of the second as failures by heave (17,18).

In case of intermediate filter, percolating water excercises an upward pressure on the soil till it emerges at the filter and may cause failure of structure due to piping, starting from filter towards upstream of floor.

5.2 FACTOR OF SAFETY AGAINST SUBSURFACE EROSION

Practically all piping failures on record have been caused by subsurface erosion involving the progressive removal of material through springs. The erosion of subsurface soil is supposed to result by the high velocities of flow of water through it, when such velocities exceed a certain limit. But this concept of piping is incomplete. In 1925 Professor Charles Terzaghi explained the causes of subsurface erosion by pressure - gradient. Water has a certain residual force in the direction of flow and is proportional to the pressure gradient at that point. At the release point, this force is upwards and will tend to lift up the soil particle if it is more than the submerged weight of the latter. Once the surface particles are disturbed the resistance against upward pressure of water will be further reduced, tending to progressive disruption of subsoil. At the instant of just stable condition the residual force will have its critical value which will be just resisted by downward weight of soil. The slightest increase in the value of residual force will lead instability and soil particle will start to be lifted up. Therefore, the exit gradient provides a design criterian for the factor of safety with respect to piping caused by subsurface erosion initiated at exit point.

At both the ends F and G of the intermediate filter the exit gradient will be very high (Theoretically it will be infinite as the filter depth is taken zero for the analysis) but it rapidly falls down at the adjacent points. Thus the factor of safety with respect to piping by subsurface erosion cannot be evaluated by any practicable means. However, if the removal of subsurface material is reliably prevented, the conditions for the validity of the theory are satisfied. The foremost requirement is to avoid local concentration of flow lines. The removal of subsurface material and concentration of flow lines can be avoided by well designed inverted filter of sufficient depth with no sharp edges.

5.3 FACTOR OF SAFETY AGAINST HEAVE

The piping by heave is initiated by an expansion and followed by an expulsion of the soil out of a zone. No such phenomenon could occur unless the water pressure overcomes the weight of the soil located within the zone of expulsion. With sufficient accuracy we can assume that the body of soil which is lifted up by the water has the shape of a prism with a width equal to Δx and a horizontal base at some depth y below the surface. The rise of prism is resisted by the weight of prism and by the friction along the vertical sides of prism. At the instant of failure the effective horizontal pressure on the sides of the prism and the corresponding frictional resistance are practically zero. Therefore, the prism rises as soon as the total water pressure on its base becomes equal to the sum of the weight of the prism, soil and water combined.

The neutral stress at any arbitrary point within the seepage zone at depth y from the surface is equal to the height h_w to which the water rises above that point in a piezometric tube multiplied by the unit weight γ_w of the water. The height h_w can be divided into two parts y and h and then the neutral stress is as below

$$u_w = h_w \gamma_w = (y + h)\gamma_w \qquad ...(5.1)$$

The first part $y \gamma_w$ represents the hydrostatic uplift. Its mechanical effect consists in reducing the effective unit weight of the soil from saturated unit weight to submerged unit weight. The second part $h \gamma_w$ is the excess hydrostatic pressure in the water with reference to free water level on the downstream side and h is hydrostatic head at the point under consideration. Therefore, the condition for the rise of the prism of soil is that the total excess hydrostatic pressure on the base of the prism should be greater than the submerged weight of the prism. If the average hydrostatic head on the base of prism is h_{av}, then the total excess hydrostatic pressure on this base is

$$u_e = \gamma_w h_{av} \Delta x \qquad ...(5.2)$$

and the submerged weight of the prism is

$$w' = y \Delta x \gamma_{sub} \qquad ...(5.3)$$

The factor of safety F_s against heave at any depth y can be determined by the ratio between the submerged weight W' and the excess hydrostatic pressure, i.e.

$$F_s = \frac{W'}{U_e} = \frac{y\gamma_{sub}}{h_{av}\gamma_w} \qquad ...(6.4)$$

The distribution of hydrostatic pressure h and hence the value of h_{av} can be obtained from the analysis presented in the preceeding chapters. The stability must be investigated at various horizontal sections at different depths to determine the minimum factor of fsafety. Generally a factor of safety of 4 to 5 is considered adequate. If the factor of safety is too small, it may be increased by placing an addition weight W per unit area on the prism and then factor of safety will be given by

$$F_s = \frac{y\ \gamma_{sub} + W}{h_{av}\ \gamma_w} \qquad ...(5.5)$$

5.4 RESULTS

In order to compute the factor of safety against heave below the filter, the prism, may be assumed of base width equal to length of filter. The intensity of the excess hydrostatic pressure can be determined at every point below the base of prism at any depth y by constructing a flow-net.

In the Chapter 4 the complete seepage region is divided into a grid and the excess hydrostatic pressure as percent of total head H is computed at every nodal point for different combination of b/d_2, b_1/d_2, T/d_2, d_1/d_2, f/d_2 and θ by using finite element technique. The average of excess hydrostatic pressure at different depths on the base of prism is then calculated by dividing the total uplift on the base by the base width. Total uplift pressure is calculated by using Simpson's rule. The value of submerged unit weight of soil and unit weight of water has been taken as 1.0 gm/cc. The factor of safety against heave is calculated by equation 5.4 and have been plotted at different depth for different combination of b/d_2, b_1/d_2, T/d_2, f/d_2, d_2/d_1 and θ in Fig. 5.1.

The results indicate that the factor of safety against heave is minimum at the bottom of filter (ignoring weight of the filter) and increases as the depth of section under consideration increases. Desired factor of safety below the filter can be obtained by providing adequate thickness of the filter.

The results indicate that the factor of safety increases with increase in filter length, by shifting the filter towards the downstream cut-off and by increasing the depth of upstream cutoff. The effect of downstream cutoff on the factor of safety against heave is not significant. Factor of safety against heave decreases with increase in value of inclination (θ) of the impervious layer.

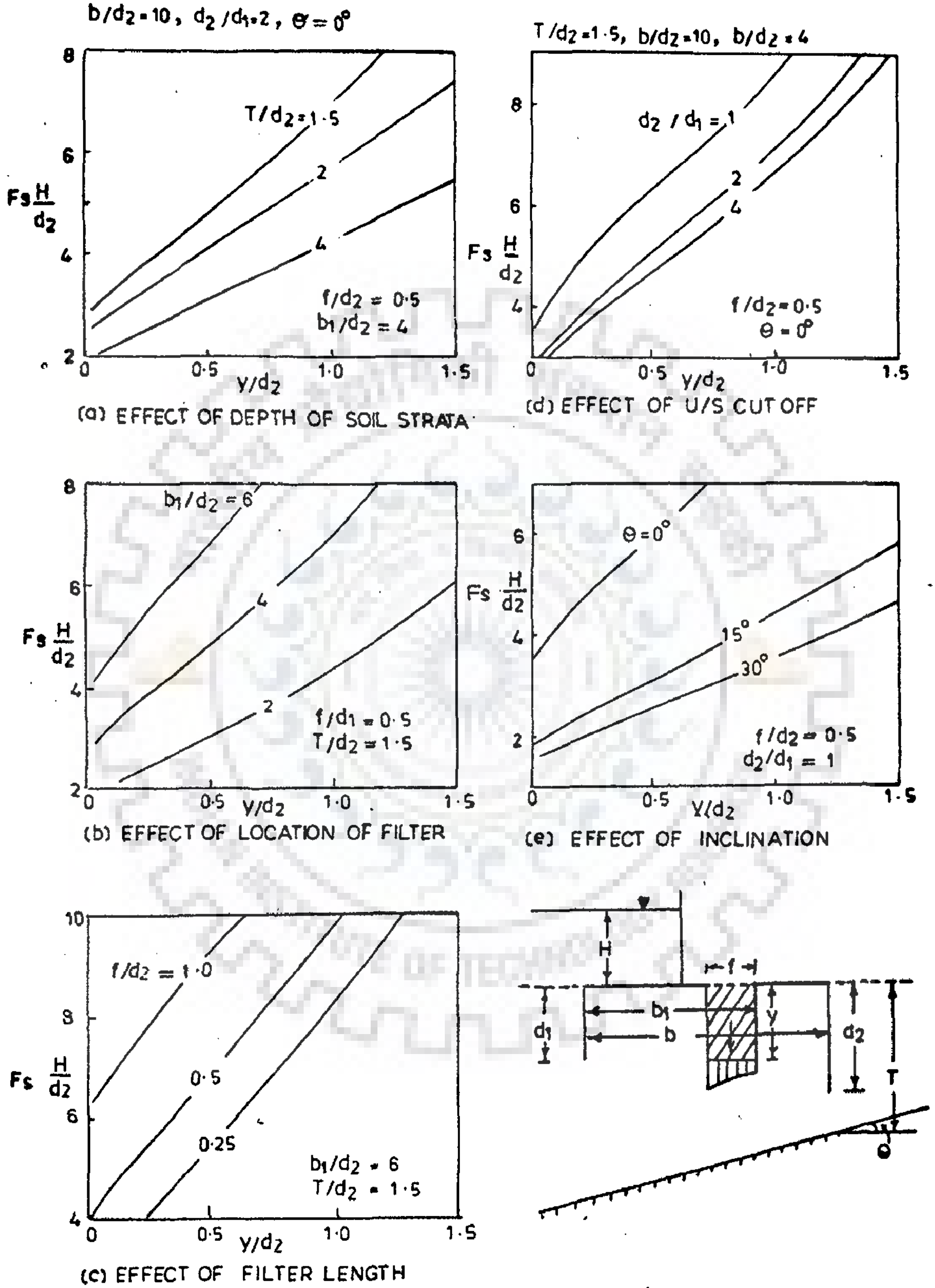


FIG. 5·1 FACTOR OF SAFETY AGAINST HEAVE

CHAPTER - 6

DISCUSSION

6.1 GENERAL

The pressure at the filter would correspond to the release level which is normally the downstream water level, but as all the stream lines are not intercepted by the filter and some of them proceed to the downstream discharge surface, pressures are again built up on the floor between filter and downstream discharge surface. These pressures are lower than those that would have developed without the filter. The extent of reduction depends upon the length and position of the filter. Uplift pressures also depend upon the depth of cutoffs, depth of permeable soil stratum and inclination of impervious layer.

These aspects are discussed in succeeding paragraphs.

6.2 EFFECT OF FILTER LENGTH

A perusal of figures 2.2 to 2.6, 3.2 to 3.5 and 4.2 to 4.6 indicates that uplift pressures at points D', C', E, D and J decrease with increase in the length of filter. However, to clearly bring out the effect of increase in filter length on the pressures along the entire length of floor, the uplift pressures for $b/d_2 = 10$, $d_2/d_1 = 1$, $b_1/d_2 = 6$, $T/d_2 = 1.5$, $\theta = 0^{o}$ and different lengths of filter viz. $f/d_2 = 0.1$, 0.5 and 1.0 have been given in table 6.1 and plotted in figure 6.1. The uplift pressures without filter (5) have also been

TABLE - 6.1

EFFECT OF LENGTH OF FILTER

Co-ordinate of the point with origin at upstream cutoff			Uplift pressure as percent of total head H for $T/d_2 = 1.5$, $b/d_2 = 10$, $b_1/d_2 = 6$, $d_2/d_1 = 1$			
			With Filter			Without Filter
Key Point	X/d_2	Y/d_2	$f/d_2 = 0.1$	0.5	1.0	-
	0.0	0.25	96.98	96.76	96.53	98.11
	0.0	0.50	93.61	93.14	92.66	96.01
	0.0	0.75	89.12	88.33	87.50	93.20
D'	0.0	1.00	80.57	79.17	77.70	87.88
	0.0	0.75	73.14	71.19	69.04	83.16
	0.0	0.50	70.03	67.86	65.49	81.23
	0.0	0.25	68.57	66.21	63.81	80.32
C'	0.0	0.0	68.13	65.81	63.31	80.04
	2.50	0.0	46.86	42.87	39.20	66.94
	4.00	0.0	30.80	25.49	20.08	56.79
	5.50	0.0	12.75	0.0	0.0	46.65
	6.00	0.0	10.00	0.0	0.0	43.21
	7.0	0.0	7.66	4.54	2.71	36.43
	8.0	0.0	6.61	3.97	2.38	29.70
	9.0	0.0	5.25	3.16	1.89	23.40
E	10.0	0.0	4.48	2.69	1.62	19.95
	10.0	0.25	4.42	2.66	1.60	19.68
	10.0	0.50	4.22	2.54	1.52	18.77
	10.0	0.75	3.78	2.28	1.37	16.84
D	10.0	1.0	2.73	1.64	0.98	12.12
	10.0	0.75	1.53	0.92	0.55	6.80
	10.0	0.50	0.90	0.54	0.32	3.99
	10.0	0.25	0.42	0.25	0.15	1.89
ϕ_j			7.67	4.54	2.71	
X_j/d_2			6.80	7.00	7.00	
Exit gradient ($GE \frac{d_2}{H}$)			0.017	0.010	0.006	0.074

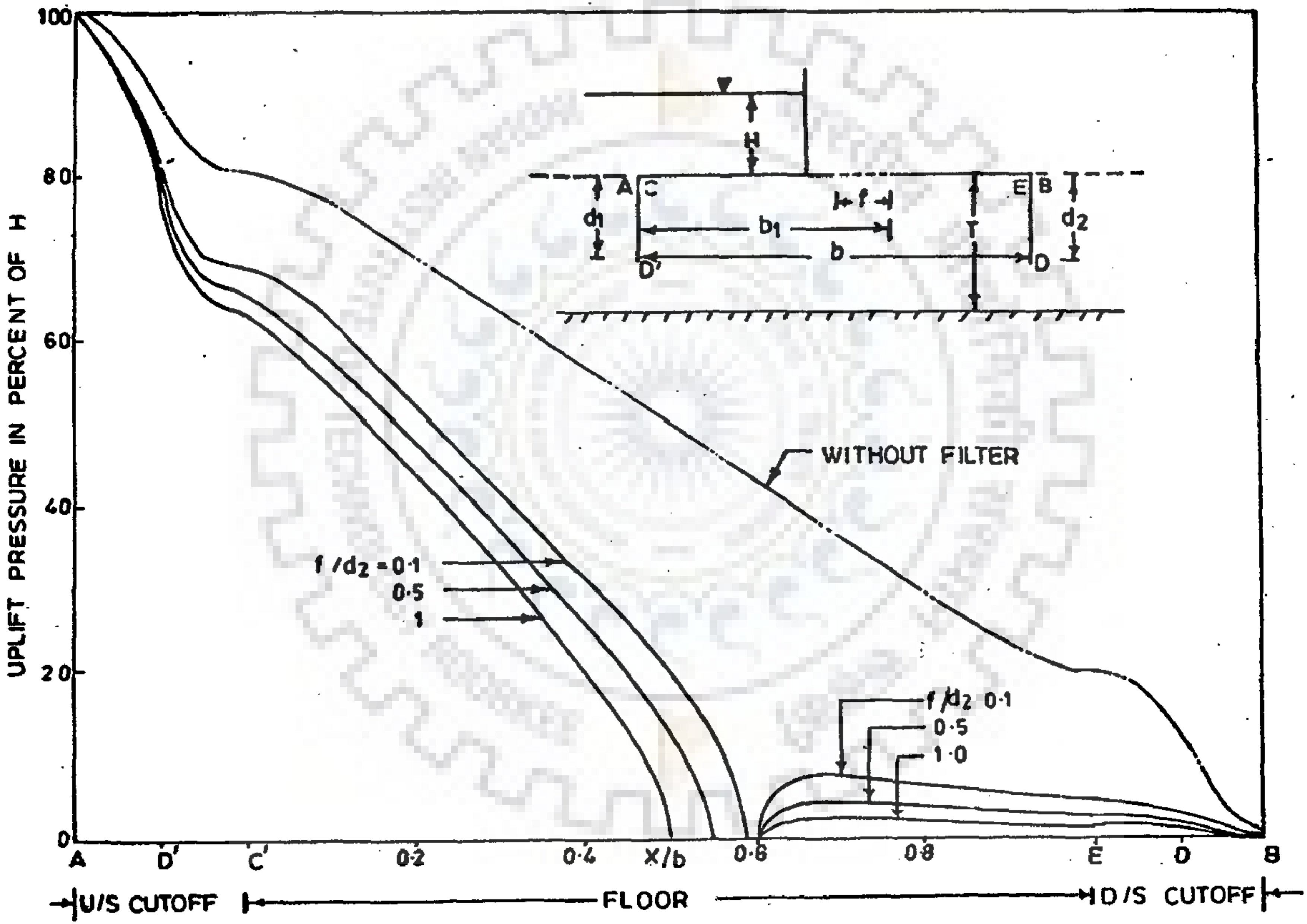


FIG. 6·1 EFFECT OF LENGTH OF FILTER ($b/d_2 = 10$, $b_1/d_2 = 6$, $d_2/d_1 = 1$, $T/d_2 = 1{\cdot}5$)

plotted in figure 6.1 for comparison. A perusal of this figure indicates that uplift pressures decrease with the increase in the length of filter, along the entire length of floor and cutoffs. The decrease in pressures along the outer faces of the cutoffs is very small. The maximum reduction in pressure takes place near the filter. With the provision of filter of as small a length as 0.1 d_2, the decrease in pressure along the floor downstream of filter ranges between 29.01 and 15.5 percent of the total differential head. With subsequent increase in filter lengths to 0.5 d_2 and 1.0 d_2, the further decrease in pressure is less and reduction in pressure rangesbetween 32% to 17% and 34% to 18% of total differential head respectively. The uplift pressuresalso decrease along the floor upstream of the filter. A perusal of figures2.8 and 4.8 indicates that the exit gradient at downstream end of floor (point B) decreases with increase in filter length. Figure 5.1(c) indicates that factor of safety against heave below filter increases with increase in filter length.

6.3 EFFECT OF FILTER LOCATION

From figures 2.2 to 2.6, 3.2 to 3.5 and 4.2 to 4.6, it is possible to evaluate the effect of filter location on the uplift pressures at points D', C', E, D and J. The uplift pressures increase at D' and C' and decrease at E, D and J as filter is moved downstream from a upstream position. However, the decrease in pressure at D as the filter is moved

TABLE – 6.2

EFFECT OF LOCATION OF FILTER

Co-ordinate of the point with origin at upstream cutoff			Uplift pressure as percentage of total head H for $T/d_2 = 1.5$, $b/d_2 = 10$, $d_2/d_1 = 1$			
			with filter ($\frac{F}{d_2} = 1.0$)			without filter
Key Point	X/d_2	Y/d_2	$\frac{b}{d_2} = 6.0$	5.0	4.0	-
	0.0	0.25	96.53	96.04	95.38	98.11
	0.0	0.50	92.66	91.61	90.22	96.01
	0.0	0.75	87.50	85.72	83.35	93.20
D'	0.0	1.0	77.70	74.53	70.31	87.88
	0.0	0.75	69.04	64.63	58.76	83.16
	0.0	0.50	65.49	60.57	54.02	81.23
	0.0	0.25	63.81	58.66	51.78	80.32
C'	0.0	0.0	63.31	58.08	51.10	80.04
	2.5	0.0	39.20	30.38	17.01	66.94
	4.0	0.0	20.08	0.00	0.0	56.79
	5.0	0.0	0.0	0.00	3.87	46.21
	6.0	0.0	0.0	3.22	3.60	43.21
	7.0	0.0	2.71	2.92	3.06	36.43
	8.0	0.0	2.38	2.41	2.50	29.70
	9.0	0.0	1.89	1.90	1.97	23.40
E	10.0	0.0	1.62	1.62	1.68	19.95
	10.0	0.25	1.60	1.60	1.66	19.68
	10.0	0.50	1.52	1.52	1.58	18.77
	10.0	0.75	1.37	1.37	1.42	16.84
D	10.0	1.0	0.98	0.98	1.02	12.12
	10.0	0.75	0.55	0.55	0.57	6.80
	10.0	0.5	0.32	0.32	0.34	3.99
	10.0	0.25	0.15	0.15	0.16	1.89
ϕ_j			2.71	3.22	3.88	-
X_j/d_2			7.00	6.00	5.20	-
The Value of ($GE \frac{d_2}{H}$)			0.0060	0.0060	0.0063	0.074

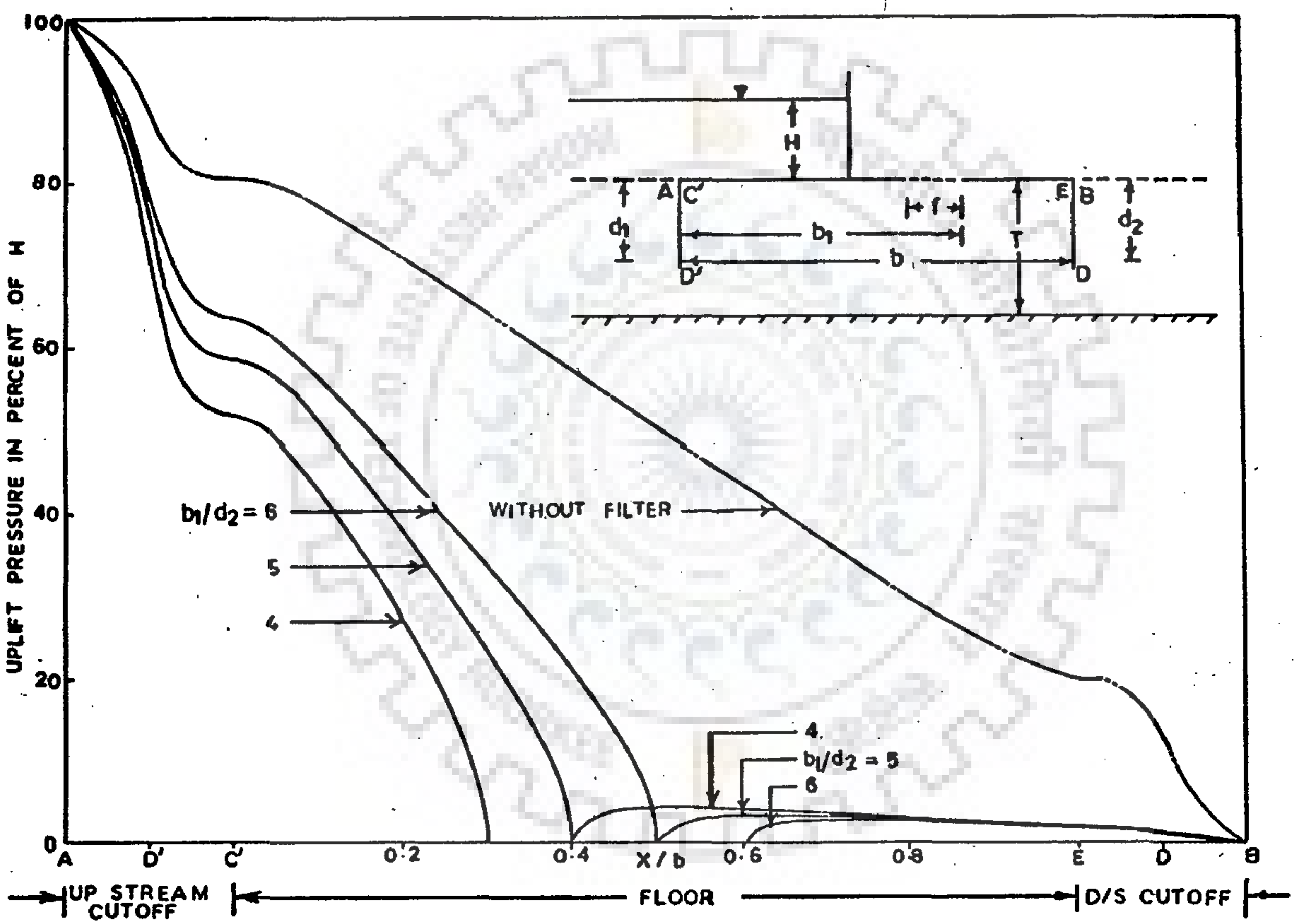


FIG. 6·2 EFFECT OF LOCATION OF FILTER ($T/d_2 = 1{\cdot}5$, $b/d_2 = 10{\cdot}0$, $d_2/d_1 = 1{\cdot}0$, $f/d_2 = 1{\cdot}0$.)

downstream continues upto a certain point beyond which the pressure at D starts increasing if filter is further moved downstream. To clearly bring out the effect of location of filter on the uplift pressure all along the floor length, the uplift pressures for $b/d_2 = 10$, $d_2/d_1 = 1$, $T/d_2 = 1.5$, $\theta = 0^o$ and $b_1/d_2 = 4.0$, 5.0 and 6.0 have been given in Table 6.2 and plotted in Figure 6.2. The uplift pressures without filter have also been given (5). A perusal of the figure indicates that uplift pressures decrease on the downstream side and increase on the upstream side of filter as the filter is moved downstream from a upstream position. However, the change in pressure on downstream of filter and along outer faces of both cutoffs is less than that on the floor upstream of the filter. Therefore, it is advantageous to provide the filter as close to the gate line as possible so that total uplift on the floor downstream of the gate line is minimum. A perusal of figures 2.8 and 4.8 indicates that exit gradient at downstream end of floor decrease with the shifting of filter towards downstream cut off upto a certain point and it starts increasing if filter is further moved towards E. Figure 5.1(b) indicates that factor of safety against heave below filter increases as the filter is shifted towards the downstream cutoff.

6.4 EFFECT OF UPSTREAM CUTOFF

A perusal of figures 2.2 to 2.6, 3.2 to 3.5 and 4.2 to 4.6 indicates that the pressures at key points D', C', E, D

TABLE - 6.3

EFFECT OF UPSTREAM CUTOFF

Co-ordinate with origin at junction of upstream cutoff			Uplift pressure as percentage of total head H for $T/d_2 = 1.5, b/d_2 = 10.0, b_1/b_2 = 6, f/d_2 = 10$		
Key Point	X/d_2	Y/d_2	$d_2/d_1 = 1$	$d_2/d_1 = 2$	$d_2/d_1 = 3$
	0.0	0.25	96.53	94.13	91.84
	0.0	0.50	92.66	85.01	-
	0.0	0.75	87.50	-	-
	0.0	1.00	77.70	-	-
D'	-	-	77.70	85.01	88.10
	0.0	0.75	69.04	-	-
	0.0	0.50	65.49	85.01	-
	0.0	0.25	63.81	79.06	85.54
C'	0.0	0.0	63.31	77.74	83.10
	2.5	0.0	39.20	45.78	47.47
	4.0	0.0	20.08	23.43	24.29
F	5.0	0.0	0.0	0.0	0.0
G	6.0	0.0	0.0	0.0	0.0
	7.0	0.0	2.71	3.16	3.27
	8.0	0.0	2.38	2.78	2.88
	9.0	0.0	1.89	2.21	2.29
E	10.0	0.0	1.62	1.89	1.96
	10.0	0.25	1.60	1.86	1.93
	10.0	0.50	1.52	1.78	1.84
	10.0	0.75	1.37	1.59	1.65
D	10.0	1.0	0.98	1.15	1.19
	10.0	0.75	0.55	0.64	0.67
	10.0	0.5	0.32	0.38	0.39
	10.0	0.25	0.15	0.18	0.19
ϕ_j			2.71	3.16	3.27
X_j/d_2			7.00	7.00	7.00
Value of $(GE \frac{d_2}{H})$			0.0060	0.0070	0.0073

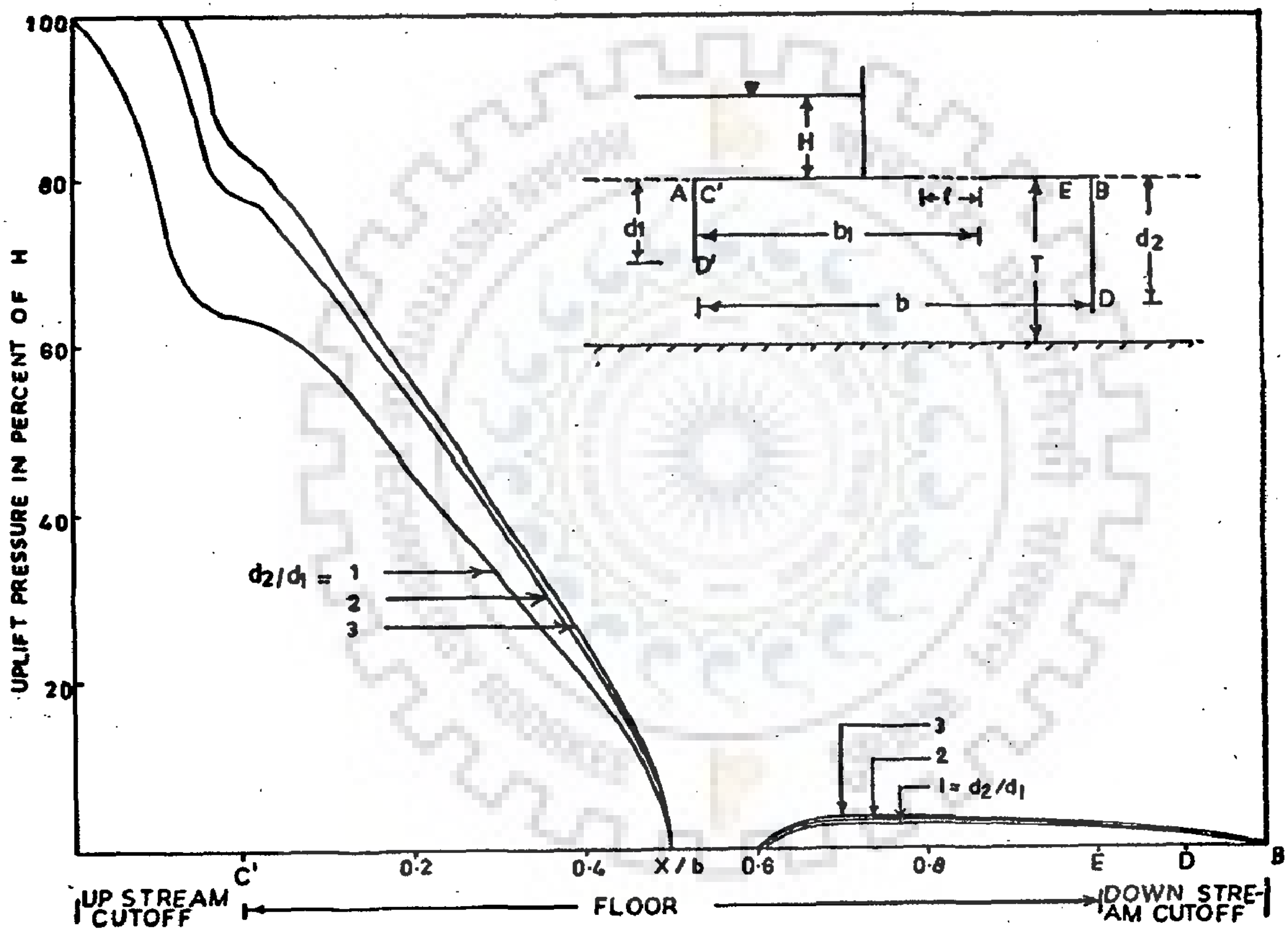


FIG. 6·3 EFFECT OF UPSTREAM CUTOFF DEPTH ($b/d_2=10$, $T/d_2=1{\cdot}5$, $b_1/d_2=6$, $\ell/d_2=1$)

and J decrease with increase in depth of upstream cutoff. To clearly bring out the effect of upstream cutoff on the pressure all along the floor, the uplift pressures for $\frac{b}{d_2} = 10$, $\frac{T}{d_2} = 1.5$, $\frac{b_1}{d_2} = 6$, $f/d_2 = 1$, $\theta = 0$ and $d_2/d_1 = 3.0$, 2.0 and 1 are given in table 6.3 and are plotted in figure 6.3. The figure indicates that the pressures decrease all along the floor and cutoffs with increase in depth of upstream cutoff. The pressure at junction of floor with upstream cutoff (point C') decreases from 83.10% to 77.74% and 63.31% of the total differential head if the depth of upstream cutoff is increased from 0.33 d_2 to 0.5 d_2 and 1.0 d_2 respectively. However, the decrease in pressure on downstream of filter is insignificant. Figures 2.8 and 4.8 indicate that exit gradient also decreases with increase in depth of upstream cutoff. The factor of safety against heave below filter increases with increase in depth of upstream cutoff.

6.5 EFFECT OF DOWNSTREAM CUTOFF

To see the effect of downstream cutoff on the uplift pressures below the floor, the uplift pressures for $b/d_1 = 20$, $T/d_1 = 4$, $b_1/d_1 = 12$, $f/d_1 = 2$, $\theta = 0^o$ and $d_2/d_1 = 1$, 2 and 3 are given in table 6.4 and are plotted in figure 6.4. The results indicate that uplift pressures increase all along the floor with increase in depth of downstream cutoff, though the increase in pressure is very small. Figures 2.8

TABLE - 6.4

EFFECT OF DOWNSTREAM CUTOFF

Co-ordinate of point with origin at upstream cutoff			Uplift Pressure as percentage of total head H for $T/d_1 = 4$, $b/d_1 = 20.0$, $b_1/d_1 = 12$, $f/d_1 = 2.0$		
Key Point	X/d_1	Y/d_1	$d_2/d_1 = 1$	$d_2/d_1 = 2$	$d_2/d_1 = 3$
	0.0	0.33	95.88	95.88	95.88
	0.0	0.66	91.15	91.16	91.17
D'	0.0	1.0	84.21	84.22	84.21
	0.0	0.66	79.59	79.61	79.63
	0.0	0.33	77.67	77.69	77.11
C'	0.0	0.0	77.09	77.11	77.14
	4.0	0.0	54.30	53.44	53.49
	8.0	0.0	24.86	24.91	24.98
	10.0	0.0	0.0	0.0	0.0
	12.0	0.0	0.0	0.0	0.0
	14.0	0.0	4.60	4.88	5.23
	16.0	0.0	4.00	4.46	5.03
	18.0	0.0	2.91	3.62	4.41
E'	20.0	0.0	2.04	3.12	4.09
	20.0	0.33	1.82	3.10	4.08
	20.0	0.66	1.99	3.05	4.05
	20.0	1.0	1.41	2.95	3.99
	20.0	2.0	-	1.99	3.64
	20.0	3.0	-	-	2.43
	20.0	2.0	-	1.99	1.08
	20.0	1.0	1.41	0.70	0.48
	20.0	0.66	0.79	0.45	0.31
	20.0	0.33	0.37	0.22	0.15
	ϕ_j		4.60	4.89	5.29
	X_j/d_1		14.00	14.40	14.40
Exit gradient $(GE\frac{d_1}{H})$			0.0107	0.0065	0.0046

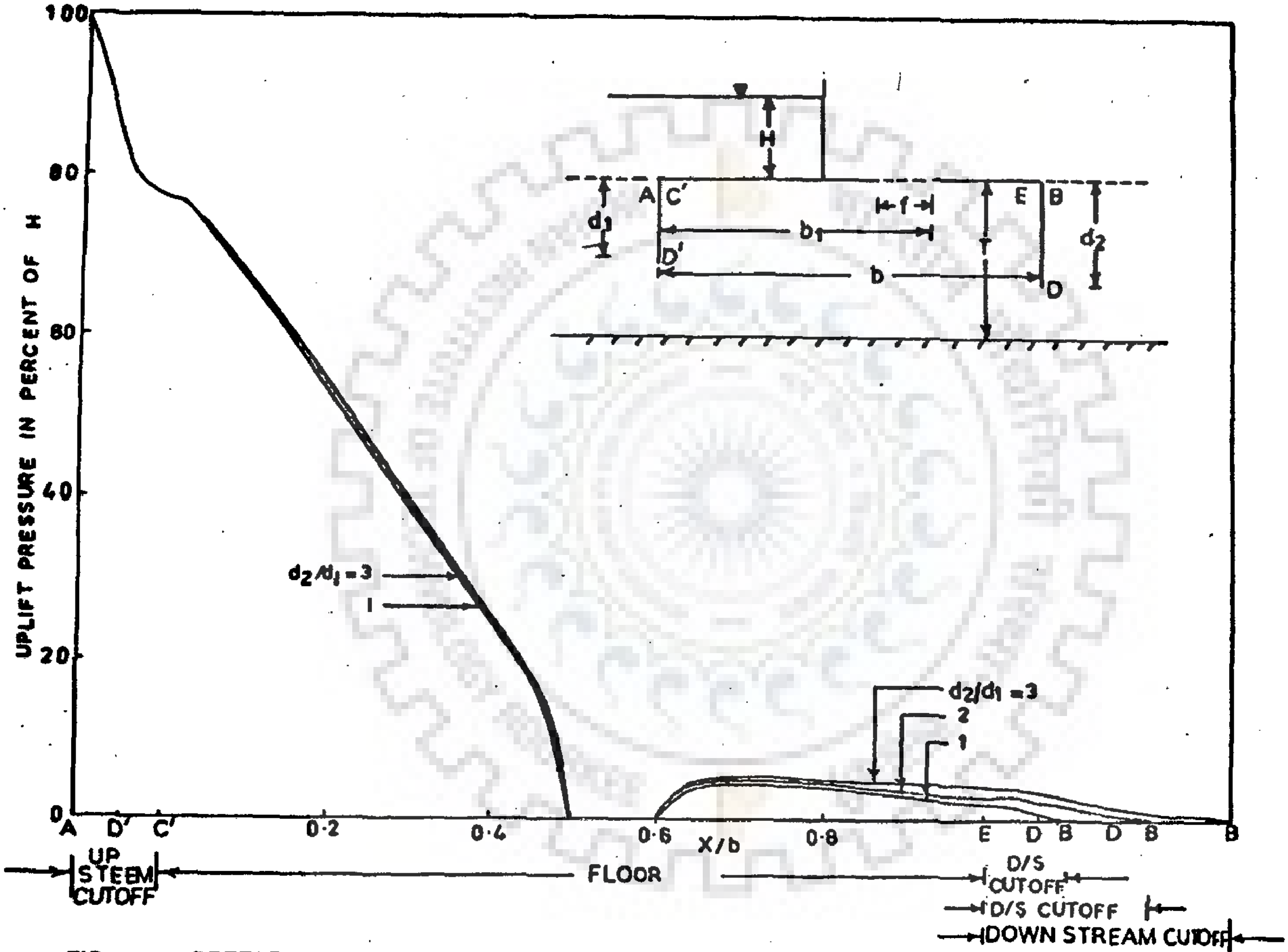


FIG. 6·4 EFFECT OF DOWN STREAM CUTOFF DEPTH ($b/d_1 = 20, T/d_1 = 4, b_1/d_1 = 12, f/d_1 = 2$)

and 4.8 indicate that the exit gradient at the end of the floor (point B) decreases with increase in depth of downstream cutoff. The effect of downstream cutoff on the factor of safety against heave below filter is insignificant.

6.6 EFFECT OF DEPTH OF PERMEABLE STRATUM

A perusal of figures 2.2 to 2.6, 3.2 to 3.5 and 4.2 to 4.6 indicates that the uplift pressures at points E, D and J decrease while those at D' and C' increase with decrease in depth of permeable stratum. To clarify the effect of depth of permeable stratum, the uplift pressure for $b/d_2 = 10.0$, $d_2/d_1 = 2.0$, $b_1/d_2 = 6.0$, $f/d_2 = 1.0$, $\theta = 0^0$ and different depths of permeable soil stratum viz. $T/d_2 = 1.1$, 1.5, 2 and 4 have been given in table 6.5 and plotted in figure 6.5. The uplift pressure for infinite depth of soil stratum have also been shown (6). A perusal of the figure indicates that uplift pressures decrease along the floor except along the upstream cutoff and some portion of floor near it with decrease in the depth of pervious stratum. The maximum reduction in pressure takes place downstream of filter. The reduction in pressure upstream of filter is less as compared to downstream of the filter. The maximum pressure downstream of the filter reduces by 7.2%, 11.8%, 13.5% and 15.2% of total differential head, while increase in pressure at C' is 1.33%, 2.11%, 2.74% and 3.35% of total differential head, if the depth of pervious stratum is reduced from infinite to 4,2,1.5 and 1.1 times of depth of downstream cutoff respectively.

TABLE - 6.5

EFFECT OF DEPTH OF PERVIOUS STRATA

Co-ordinate with origin at upstream cutoff			Uplift pressure as percentage of total head H for $b/d_2 = 10, b_1/d_2 = 6, d_2/d_1 = 2.0, f/d_2 = 1.00$				
Key Point	X/d_2	Y/d_2	$T/d_2=1.1$	$T/d_2=1.5$	$T/d_2=2.0$	$T/d_2=4.0$	$T/d_2=\infty$
	0.0	0.18	96.18	95.75	95.80	95.10	
	0.0	0.37	91.36	90.41	92.00	88.70	
D'	0.0	0.50	85.94	85.01	84.22	83.41	82.50
	0.0	0.37	81.63	81.00	80.20	79.10	
	0.0	0.18	79.06	78.50	77.80	77.00	
C'	0.0	0.0	78.35	77.74	77.11	76.33	75.00
	2.0	0.0	52.54	53.06	53.44	54.25	
	4.0	0.0	21.87	23.43	24.91	41.79	
	5.0	0.0	0.0	0.0	0.0	0.0	
	6.0	0.0	0.0	0.0	0.0	0.0	
	7.0	0.0	1.73	3.16	4.88	9.38	
	8.0	0.0	1.51	2.78	4.46	9.50	
	9.0	0.0	1.22	2.21	3.62	8.29	
E	10.0	0.0	1.05	1.89	3.12	7.38	15.0
	10.0	0.25	1.04	1.86	3.08	7.30	
	10.0	0.50	0.99	1.78	2.95	6.98	
	10.0	0.75	0.91	1.59	2.64	6.63	
D	10.0	1.00	0.60	1.15	1.99	5.18	12.0
	10.0	0.75	0.30	0.64	1.18	3.40	
	10.0	0.50	0.17	0.38	0.70	2.50	
	10.0	0.25	0.09	0.18	0.33	0.96	
ϕ_j			1.74	3.16	4.89	9.73	19.0
X_j/d_2			6.80	7.00	7.20	7.40	7.70
$G_E \frac{d_2}{H}$			0.0030	.0070	0.0131	0.0381	-

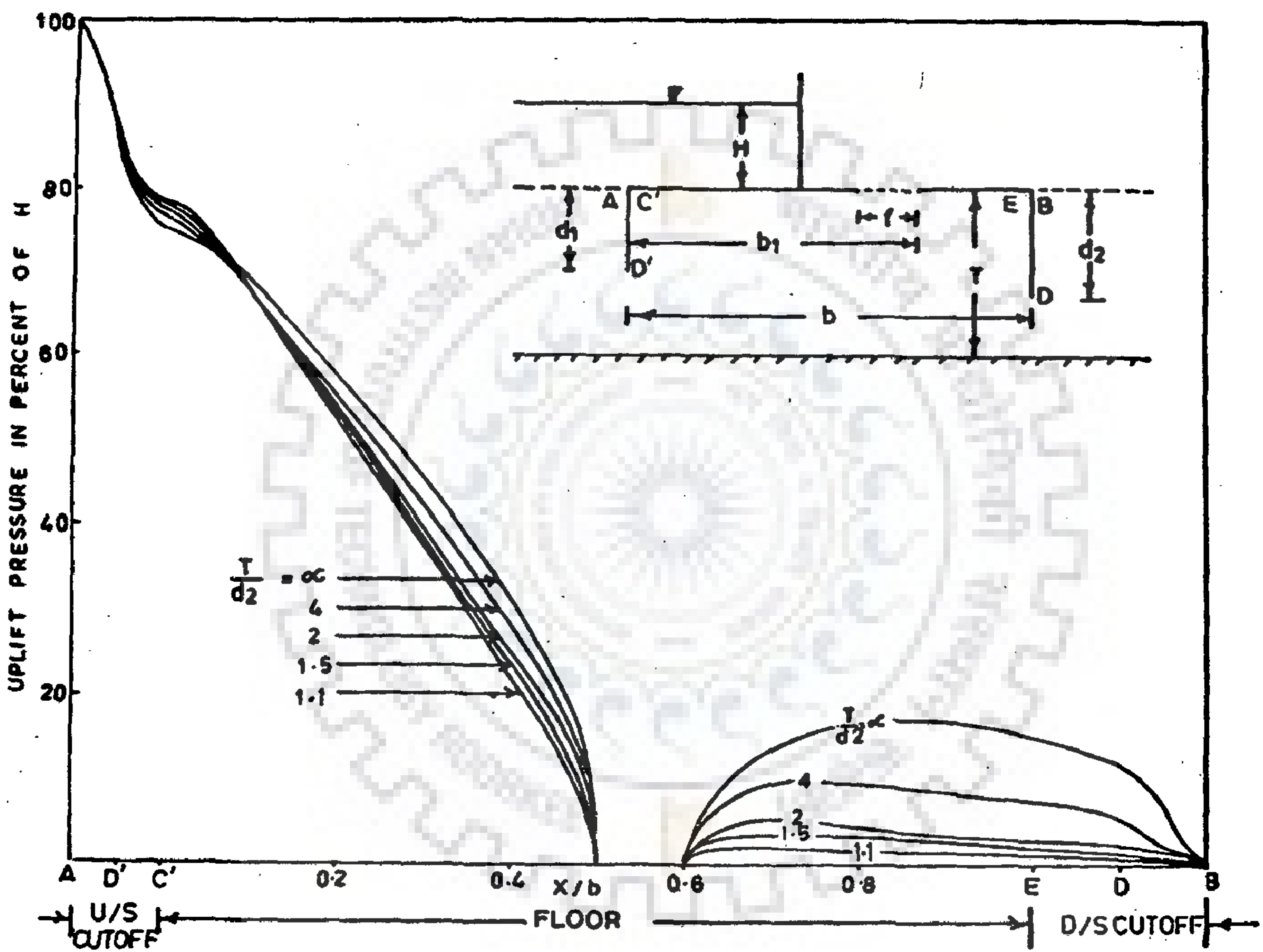


FIG. 8·5 EFFECT OF DEPTH OF PERVIOUS STRATA ($b/d_2 = 10$, $d_2/d_1 = 2{\cdot}0$, $b_1/d_2 = 6{\cdot}0$, $f/d_2 = 1{\cdot}0$)

The figures 2.8 and 4.8 indicate that the exit gradient at end of floor also decreases as the depth of pervious soil stratum decreases. The factor of safety against heave below filter increases with decrease in depth of pervious soil stratum.

6.7 EFFECT OF INCLINATION OF IMPERVIOUS LAYER

The perusal of figures 4.2 to 4.6 indicates that the uplift pressure increases at all the key points D', C', E, D and J with increase in inclination of impervious layer. To clarify the effect, the uplift pressures for $b/d_2 = 10$, $d_2/d_1 = 2$, $b_1/d_2 = 6$, $f/d_2 = 1$, $T/d_2 = 1.5$ and different slopes of impervious strata, viz. $\theta = 0^o$, 10^o, 20^o and 30^o have been shown in table 6.6 and are plotted in figure 6.6. The results indicate that uplift pressures increase all along the floor with increase in inclination of impervious layer. The increase in pressure upstream of filter is less as compared to increase in pressure on downstream of filter. The maximum increase in pressure downstream of the filter is 4.3%, 8.4% and 12% of total differential head while maximum increase on upstream of filter is 5.2%, 7.6% and 9% of total differential head if the inclination is increased from 0^o to 10^o, 20^o and 30^o respectively. Figure 4.8 indicates that exit gradient at end of floor also increases with increase in inclination of impervious layer. The factor of safety against heave below filter decreases with increase in inclination.

TABLE - 6.6.

EFFECT OF INCLINATION OF IMPERVIOUS STRATA

Co-ordinate with origin at upstream cutoff			Uplift pressure as percentage of total head H for $T/d_2 = 1.5$, $b/d_2 = 10$, $b_1/d_2 = 6$, f/d_2 and $d_2/d_1 = 2$			
Key Point	x/d_2	Y/d_2	$\theta = 0^\circ$	$\theta = 10^\circ$	$\theta = 20^\circ$	$\theta = 30^\circ$
	0.0	0.20	-	94.80	94.80	-
D'	0.0	0.50	85.01	84.92	85.17	84.83
	0.0	0.20	-	79.32	79.60	-
C'	0.0	0.0	77.74	78.49	78.80	78.94
	2.0	0.0	53.06	56.80	58.25	59.03
	4.0	0.0	23.43	28.66	31.07	32.43
	5.0	0.0	0.0	0.0	0.0	0.0
	6.0	0.0	0.0	0.0	0.0	0.0
	7.0	0.0	3.16	7.41	11.07	13.97
	8.0	0.0	2.78	7.08	11.22	14.79
	9.0	0.0	2.21	5.92	9.77	13.34
E	10.0	0.0	1.89	5.17	8.69	12.06
	10.00	0.25	1.86	5.11	8.59	11.94
	10.0	0.5	1.78	4.90	8.28	11.54
	10.0	0.75	-	4.44	7.58	10.65
D	10.0	1.0	1.15	3.34	5.89	8.55
	10.0	0.75	-	1.80	3.19	4.70
	10.0	0.5	0.38	1.07	1.91	2.81
	10.0	0.25	0.18	0.51	0.90	1.33
$\bar{\phi}_j$			3.16	7.50	11.48	14.89
x_j/d_2			7.00	7.20	7.40	7.80
$GE \frac{d_2}{H}$			0.0070	0.0201	0.0357	0.0525

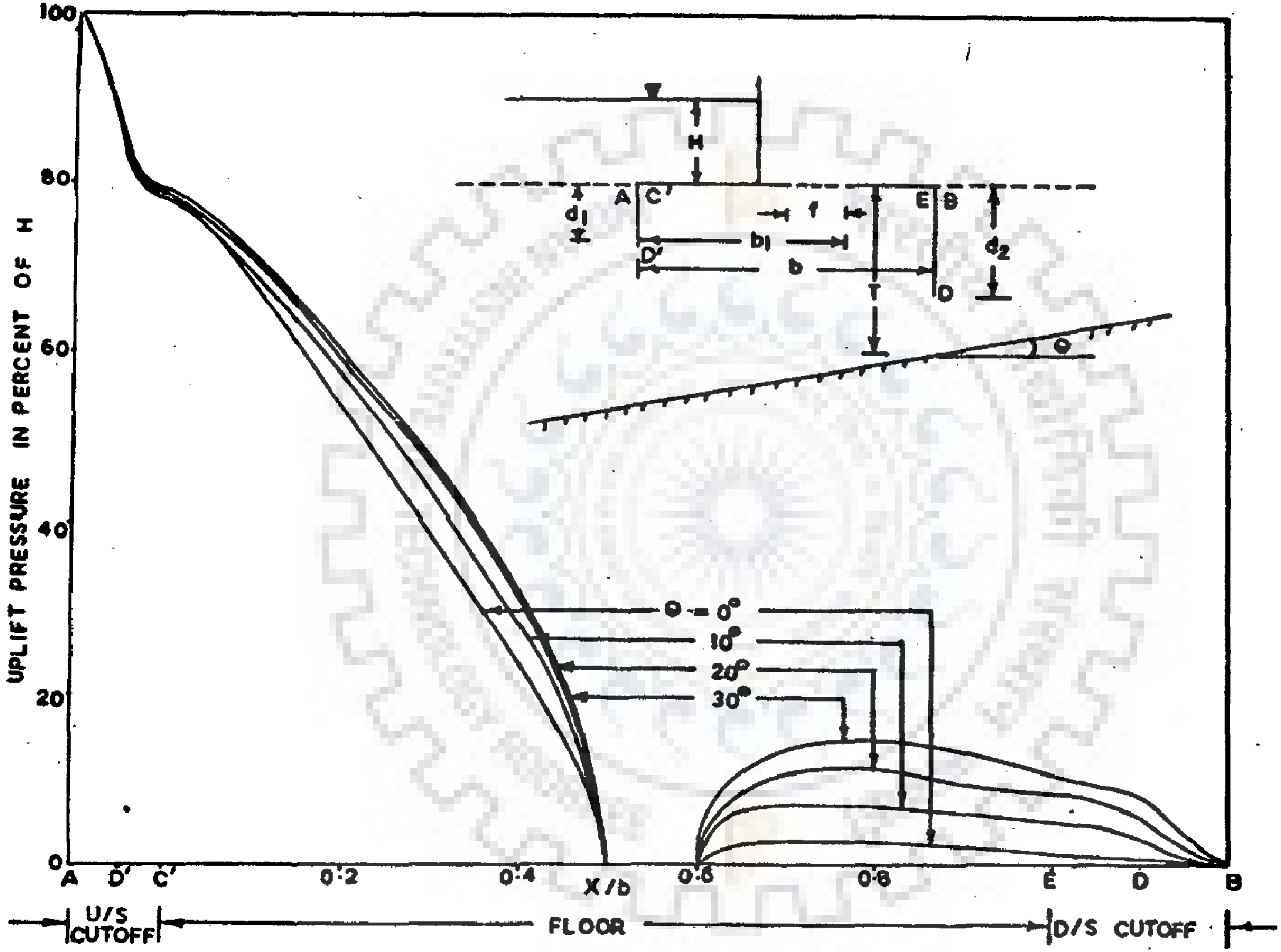


FIG. 6·8 EFFECT OF INCLINATION OF IMPERVIOUS LAYER ($b/d_2=10$, $d_2/d_1=2$, $b_1/d_2=6$, $T/d_2=1·5$, $f/d_2=1$)

6.8 EFFECT OF STRATIFICATION

To see the effect of stratification on the uplift pressure, the uplift pressures below the floor with intermediate filter and two cutoff with $b/d_2 = 10$, $d_2/d_1 = 2$, $b_1/d_2 = 5$, $f/d_2 = 1$, founded on stratified foundation of two pervious layers of equal depth ($T = 1.5d_2$), with different permeability, (k_1 and k_2) have been shown in table 6.7 and are plotted in figure 6.7 for $k_2/k_1 = 0.1$, 1.0, 10.0 and 100.0 The results indicate that the pressures close to filter increase while along upstream cutoff and in some part of the floor close to this cutoff decrease as the permeability of lower layer is increased. The increase in pressure downstream of filter is very large as compared to increase in pressure upstream of the filter. The maximum pressure downstream of filter is increased by 13% and 27% of total differential head if the permeability of lower layer is increased to 10 & 100 times of the permeability of the upper layer and decreased by 4% if the permeability of lower layer is reduced to 1/10 times of permeability of the upper layer. This indicates that filter is not very effective if the structure is founded on stratified layer with lower layer being more pervious as compared to the uppte layer.

TABLE - 6.7

EFFECT OF STRATIFICATION

Co-ordinate with origin at upstream cutoff			Uplift pressure as percentage of total head H for T/d_2 = 3.0, b/d_2 = 10, d_2/d_1 = 2.0, b/d_2 = 6.0			
Key Point	X/d_2	Y/d_2	K_2/k_1 =0.10	k_2/k_1 = 1.0	k_2/k_1 =10.0	k_2/k_1 = 100.0
	0.00	0.25	93.78	93.01	91.06	88.81
D'	0.00	0.50	84.94	83.35	79.38	74.67
	0.00	0.25	79.43	77.90	74.14	69.29
C'	0.00	0.0	78.00	76.51	72.82	67.97
	1.00	0.0	67.56	66.70	64.60	60.59
	2.00	0.0	53.35	53.95	55.50	54.07
	3.00	0.0	38.91	41.01	46.37	47.89
	4.00	0.0	23.91	26.80	34.21	37.78
	5.00	0.0	0.0	0.0	0.0	0.0
	6.0	0.0	0.0	0.0	0.0	0.0
	7.0	0.0	3.68	7.59	19.84	30.54
	8.0	0.0	3.31	7.40	21.25	34.94
	9.0	0.0	2.60	6.26	19.69	34.63
E	10.00	0.0	2.23	5.51	18.30	33.44
	10.00	0.25	2.25	5.44	18.17	33.31
	10.00	0.50	2.14	5.24	17.75	32.87
	10.00	0.75	1.92	4.79	16.80	31.76
D	10.00	1.00	1.42	3.72	14.34	28.66
	10.00	0.75	0.82	2.27	9.44	19.54
	10.00	0.50	0.49	1.38	5.96	12.58
	10.00	0.25	0.23	0.66	2.92	6.21
ϕ_j			3.68	7.74	21.33	35.14
X_j/d_2			7.00	7.40	7.80	8.40
$GE \frac{d_2}{H}$			0.0091	.0262	0.1160	0.2473

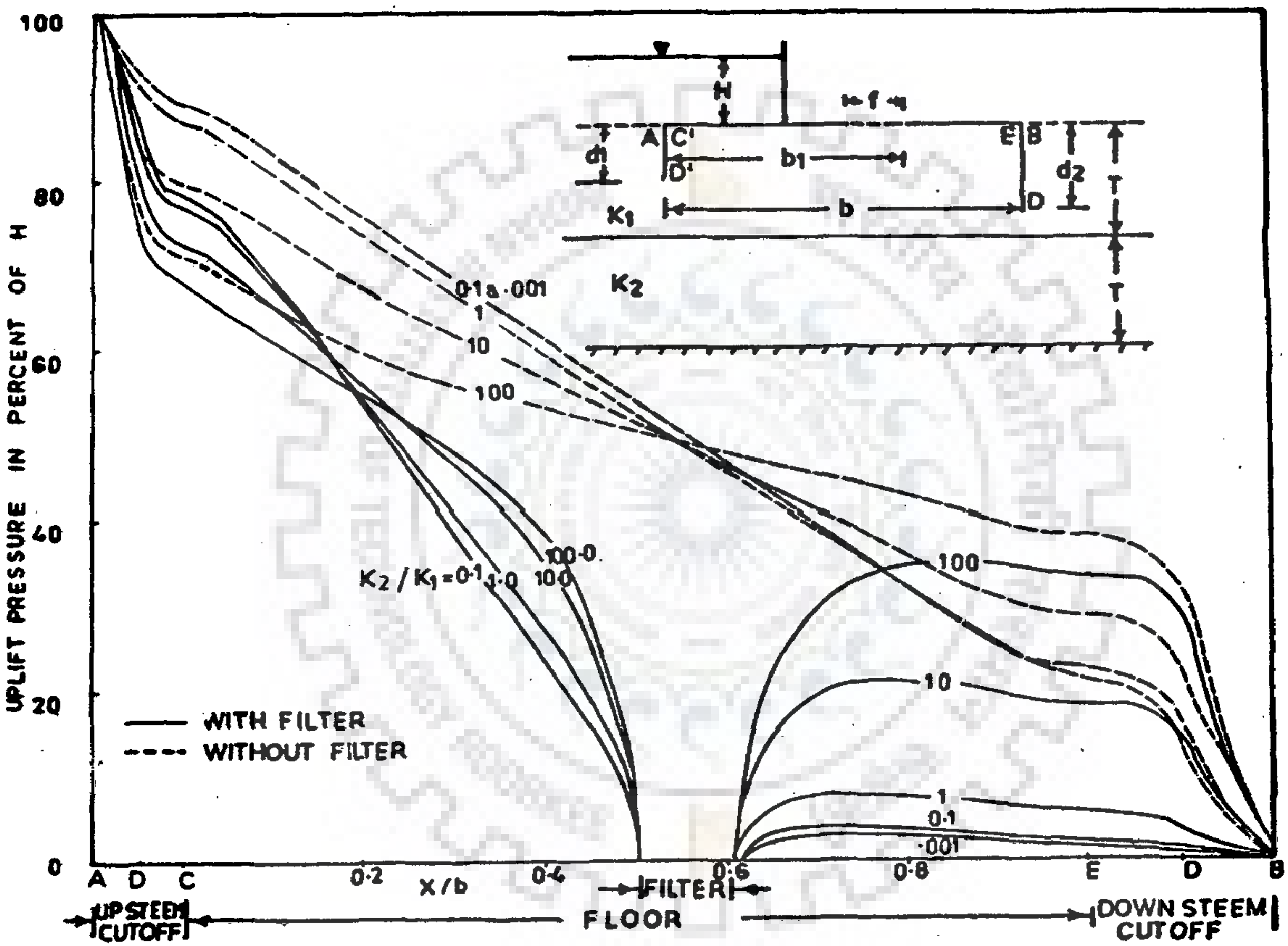


FIG. 6·7 EFFECT OF STRATIFICATION ($b/d_2=10$, $d_2/d_1=2$, $T/d_2=1{\cdot}5$, $b_1/d_2=6$, $f/d_2=1$)

CHAPTER - 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 CONCLUSIONS

Exact solutions have been obtained for the problem of two dimensional seepage flow below a hydraulic structure founded on permeable soil of finite depth with the help of conformal mapping for the following boundary conditions.

a) A flat floor with two partial cutoffs at ends and a horizontal filter of finite length located anywhere between the two cutoffs.

b) A flat floor with fully penetrating cutoff at the downstream end of the floor, partial cutoff at the upstream end of the floor and a horizontal filter of finite length located anywhere between the two cutoffs.

A numerical solution has also been obtained for seepage flow below a structure founded on permeable soil under lain by a sloping impervious stratum by finite element method for the boundary conditions specified at (a) above.

The equations derived have been used for the computation of pressures at the key points and exit gradient at the end of floor. The results have been plotted in the form of design curves for various combinations of parameters involved. The effect of various parameters on the uplift pressures and exit gradient has been studied.

As discussed in Chapter 6 the uplift pressures reduce considerably along the entire profile of the structure with the provision of filter of even very small length. The uplift pressures reduce with increase in the length of filter. However, further reduction in pressures with subsequent increase in length of filter is less as compared to initial reduction. The exit gradient at end of floor also decreases considerably with provision of filter. The uplift pressures decrease on the downstream side and increase on the upstream of the filter as the filter is moved from upstream to the downstream side. The filter should be located close to the gate line to minimise the uplift pressure downstream of the gate line. Exit gradient at the end of the structure decreases with shifting of filter towards the downstream cut-off upto a certain point and then starts increasing with further shifting towards the downstream end.

Uplift pressure decreases with increase in depth of upstream cutoff while it is reverse for the downstream cutoff. The increase in pressure with increase in depth of downstream cutoff is very small. The exit gradient at end of floor decreases with increase in depth of either cutoff. The effect of downstream cutoff on the exit gradient is more pronounced.

The uplift pressure decreases along the floor except along the upstream cutoff and some portion of the floor near it, with decrease in the depth of pervious strata. The maximum reduction takes place on downstream of filter.

The exit gradient at end of floor also decreases with decrease in the thickness of pervious stratum. The uplift pressures all along the floor and exit gradient at end of floor increase with increase in the inclination of the lower impervious layer.

The filter is less effective if the structure is founded on stratified soil with lower layer more pervious than the upper layer.

The design procedure based on present work has been illustrated by an actual example of Narora barrage in Uttar-pradesh (appendix A-1) and the section proposed is compared with the actual section provided at site. The volume of concrete is considerably reduced by providing a filter of 2.5 m length. Because of reduction in concrete thickness the excavation is also reduced resulting in considerable savings.

7.2 RECOMMENDATIONS

Intermediate filter should be provided below the hydraulic structures to reduce the uplift pressures and exit gradient at downstream end of the structure and thereby afford reduction in cost of structure. A lot of reliance is placed on proper functioning of filters in case of earth and rockfill dam, though it is not possible to take any remedial measure if the filter gets choked. In the case of hydraulic structures like barrages, it is possible to adopt some remedial measures if the filter gets choked. It is therefore recommended that the filter should be relied upon for release of uplift pressures below hydraulic structures founded on permeable soil. If the filter is properly designed and constructed, there should be no possibility of failure. In order to monitor the behaviour of the filter, piezometric pipes should be provided and a record of uplift pressure below the structure and seepage discharge through the filter should be kept.

In actual practice the filter would not be of negligible depth as assumed in the analysis. However, for design purposes it would be safe to assume the filter to be of negligible depth.

7.3 SCOPE FOR FURTHER WORK

The work presented in the thesis may be extended as follows:

a) Solution of seepage below a flat floor with two end cutoffs and a deep drain founded on finite or infinite depth of pervious soil strata.

b) Solution of seepage below a depressed floor with a sloping glacis, two end cutoffs and a filter founded on pervious soil of finite and infinite depth.

c) Solution of seepage for anisotropic soil strata with boundary conditions, as at (a) and (b) above.

APPENDIX - 1

ILLUSTRATVE EXAMPLE

A-1.1 General

The foundation of Narora barrage consists of fine sand to medium sand underlain by a stiff clay layer at a depth of about 7.0 m below river bed. The design procedure based on the present work has been illustrated by redesigning the floor of Narora Barrage with a horizontal filter. The uplift pressures below the floor and the exit gradient with and without filter have been workd out and compared. The existing floor thickness provided to withstand against uplift pressure without filter has been compared with those with filter to have an idea of reduction in excavation and volume of concrete with the provision of filter.

A.1.2 Details of Existing Barrage

Narora barrage is located across the river Ganga at Narora in Bulandshaher District of Uttar Pradesh. It is designed to pass a flood discharge of 14150 cumecs. It is 924.2m long and consists of 7 undersluice bays each 15.2 m wide separated by 3.1 m thick piers and 54 barrage bays 12.2m wide separated by 2.4m thick piers. The canal head regulator is inclined at an angle of 107^{o} with the barrage axis and consists of 7 bays of 7.6 m each separated by 1.52 m thick piers. The canal carries a discharge of 165 cumecs.

The barrage portion is designed for a flood discharge of 11320 cumecs with an afflux of 0.91 m. The cistern level is worked out such that hydraulic jump is forming on the glacis for all discharges. The length of cistern is kept approximately equal to 5.5 times the height of hydraulic jump. Upstream sheet pile has been provided for the scour corresponding to 1.05 times of scour depth R below high flood level. Downstream sheet pile has been provided for the scour corresponding to 1.20 times the scour depth R below the high flood. Lower value of factor of safety against scour has been taken due to the clay layer encountered at a shallow depth (RL 167.38 m). Concrete blocks over the filter and launching apron have been provided on upstream and downstream of the floor to safeguard against scour and heave. The floor length has been provided to obtain an exit gradient equal to 1/6 at downstream end of the floor. The uplift pressures on the floor and the exit gradient have been worked out from the Khosla's curves. Gravity floor has been provided and the maximum thickness of concrete at toe of glacis worked out to 3.0 m. The hydraulic profile and the uplift pressures have been checked on models by U.P. Irrigation Research Institute Roorkee. It was found that the observed uplift pressures on upstream side of floor are slightly more and on downstream end are less than calculated pressures by Khosla's curves. The section of existing barrage is shown in figure A-1.1. The uplift pressure line is also shown in this figure. The salient features of the barrage bays are as below,

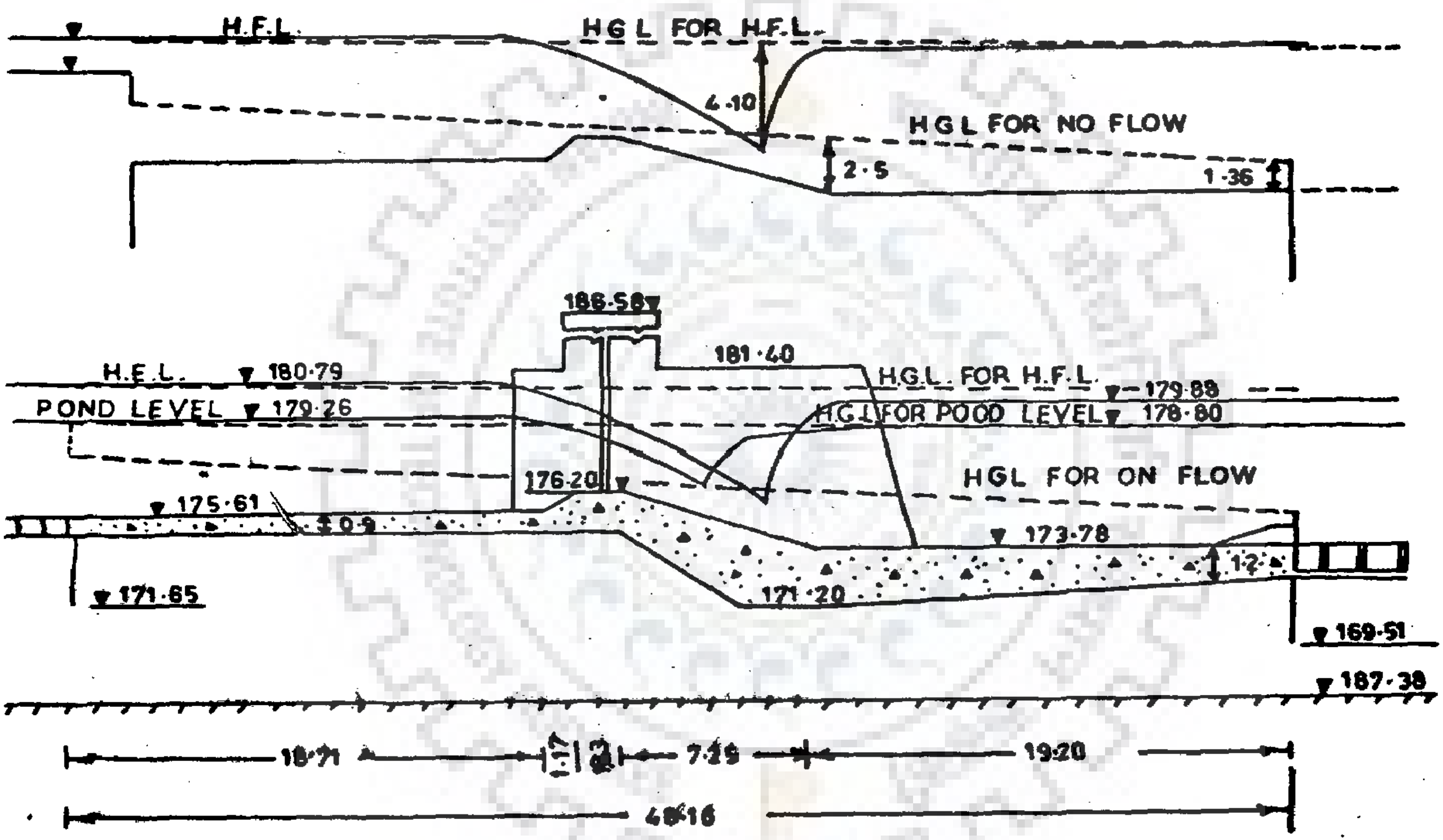


FIG. A-1.1 SECTION OF NARORA BARRAGE

Upstream high flood level	180.79 m
Downstream high flood level	179.88 m
Pond level	179.26 m
Crest level	176.20 m
Upstream floor level	175.61 m
Downstream floor level	173.78 m
Bottom of upstream sheet pile	171.65 m
Bottom of downstream sheet pile	169.51 m
Total floor length	48.16 m
Length of cistern	19.20 m
Slope of glacis	3H:IV

A.1.3 Design With Filter

The hydraulic profile downstream of crest has been kept same as for existing barrage. An intermediate filter of 2.5 m length is provided at a distance of 23.00 m from the downstream end. The length of upstream floor has been reduced to 4.71 m so that total floor length is 34.16 m. Total length of the floor has been reduced by 14.0 m. The thickness of floor has been kept as 0.9 m which is considered to be the minimum required from practical considerations. details of the floor are shown in figure A-1.2.

The uplift pressures at key points D', C', E, D, and J are calculated from figures 2.2 to 2.6 and are as below

For upstream cutoff

$T = 175.61 - 167.38 = 8.23$ m

$b = 34.16$ m

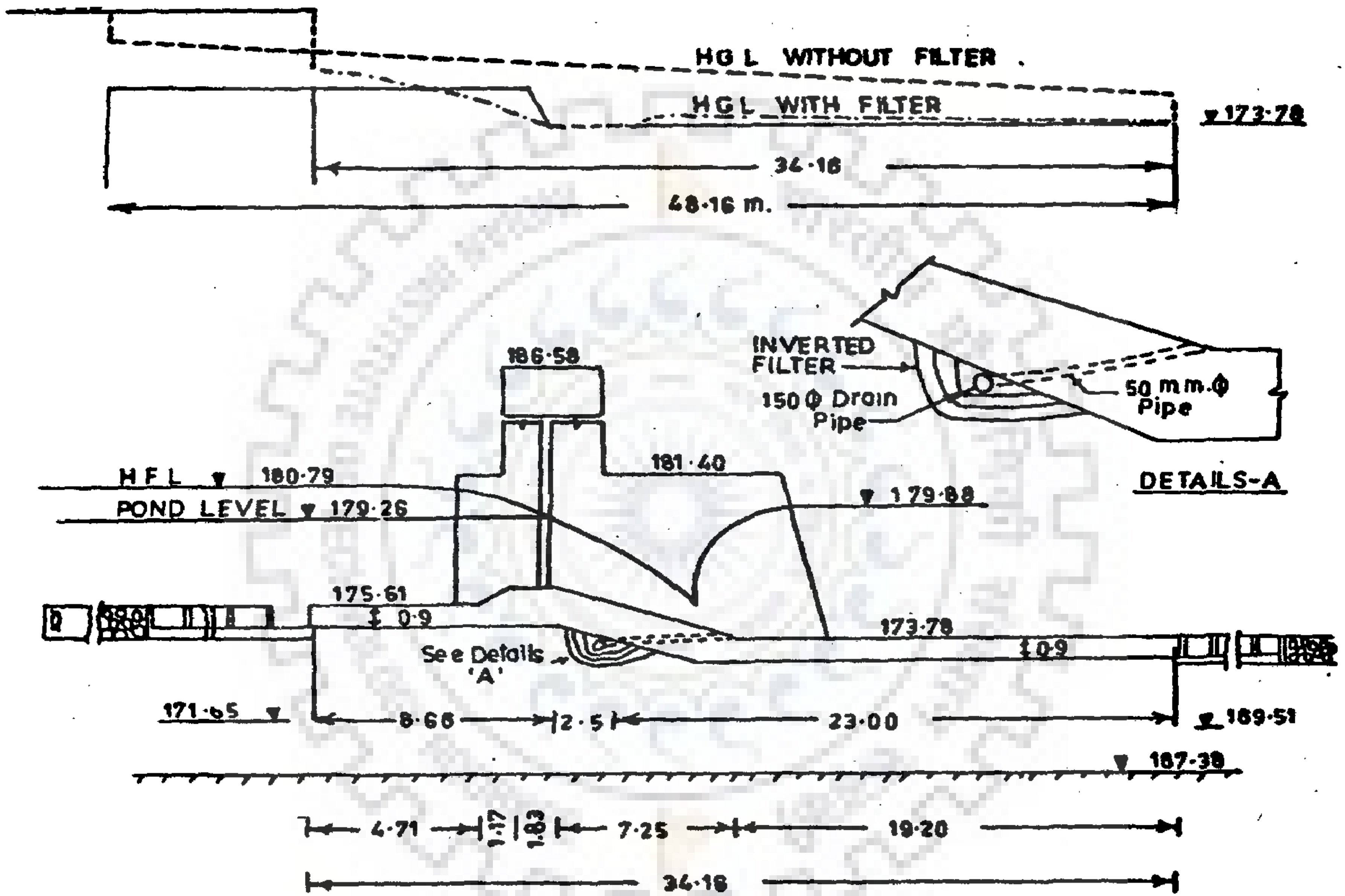


FIG. A- 1·2 PROPOSED SECTION OF NARORA BARRAGE

$b_1 = 11.16$ m

$f = 2.5$ m

$d_1 = 175.61 - 171.65 = 3.96$ m

$d_2 = 175.61 - 169.51 = 6.1$ m

$\frac{T}{d_2} = \frac{8.23}{6.1} = 1.35$

$\frac{b}{d_2} = \frac{31.16}{6.1} = 5.6$

$\frac{b_1}{d_2} = \frac{11.16}{6.1} = 1.83$

$\frac{f}{d_2} = \frac{2.5}{6.1} = 0.41$

From figures 2.2 and 2.3 for $b/d_2 = 5$, $\frac{b_1}{d_2} = 1.83$,

$\frac{f}{d_2} = 0.41$, $d_2/d_1 = 1.54$ and $T/d_2 = 1.5$

$\phi_{C'} = 42$ %

$\phi_{D'} = 67$ %

Correction for thickness $= \frac{(67-42) \times 0.9}{3.96} = 5.7$ %

Corrected value of pressure at the junction of upstream sheet pile with floor, $\phi_{C'} = 42 + 5.7 = 47.7$ %

For downstream cutoff

$T = 173.78 - 167.38 = 6.4$ m

$b = 34.16$ m

$b_1 = 11.16$ m

$f = 2.5$ m

$d_1 = 173.78 - 171.65 = 2.13$ m

$d_2 = 173.78 - 169.51 = 4.27$ m

$$\frac{T}{d_2} = \frac{6.4}{4.27} = 1.5$$

$$\frac{b}{d_2} = \frac{34.16}{4.27} = 8$$

$$\frac{b_1}{d_2} = \frac{11.16}{4.27} = 2.61$$

$$\frac{f}{d_2} = \frac{2.5}{4.27} = 0.58$$

$$\frac{d_2}{d_1} = \frac{4.27}{2.13} = 2$$

From figures 2.4, 2.5 and 2.6, 2.7 and 2.8

For $\frac{b_1}{d_2} = 2.61$, $\frac{T}{d_2} = 1.5$, $\frac{f}{d_2} = 0.58$, $\frac{d_2}{d_1} = 2$

and $b/d_2 = 5$ and 10.

$\phi_E = 3.6$ % for $b/d_2 = 10$ and
7.5 % for $b/d_2 = 5$

$\phi_D = 2.2$ % for $b/d_2 = 10$ and
4.8 % for $b/d_2 = 5$

Interpolating linearly

$$\phi_E = 3.6 + \frac{7.5 - 3.6}{5} \times 2 = 5.16 \text{ for } b/d_2 = 8$$

$$\phi_D = 2.2 + \frac{4.8 - 2.2}{5} \times 2 = 3.76 \text{ \% for } b/d_2 = 8$$

$$\text{Correction for thickness} = \frac{5.16 - 3.76}{4.27} \times 0.9 = 0.3$$

Corrected value of pressure at junction of downstream cutoff with floor $\phi_E = 5.16 - 0.3 = 4.86$ %

$\phi_j = 9$ % for $b/d_2 = 10$ and
10 % for $b/d_2 = 5$

$$X_j/d_2 = 3.4 \text{ for } b/d_2 = 5 \text{ and}$$
$$3.8 \text{ for } b/d_2 = 10$$

Interpolating linearly

$$\phi_j = 9 + \frac{10-9}{5} \times 2 = 9.4\ \% \text{ for } b/d_2 = 8$$

$$X_j/d_2 = 3.4 + \frac{0.4}{5} \times 3 = 3.54 \text{ for } b/d_2 = 8$$

$$X_j = 3.54 \times 4.27 = 15.11 \text{ m from u/s cutoff.}$$

$$GE\frac{d_2}{H} = 0.015 \text{ for } b/d_2 = 10 \text{ and}$$
$$0.02 \text{ for } b/d_2 = 5$$

Interpolating linearly

$$GE\frac{d_2}{H} = 0.015 + \frac{0.02 - 0.015}{5} \times 2 = 0.017 \text{ for } b/d_2 = 8$$

U/S water level = 179.26 m

D/S water level = 173.78 m

Total Head = 179.26 - 173.78 = 5.48 m

TABLE-A-1.1: UPLIFT PRESSURES

Point	Percentage head	Head above D/S floor(m)	Elevation of energy line(m)
D'	67.0	3.67	177.45
C'	47.7	2.61	176.39
E	4.86	0.27	174.05
D	3.76	0.20	173.98
J	9.49	0.51	174.21

The uplift pressure lines with filter and without filter are shown in figure A-1.2. It is clear from figure that uplift pressures are reduced considerably.

The floor thickness required on downstream of filter

$$= \frac{0.51}{1.3} = 0.39 \text{ m}$$

A minimum of 90 cm thick floor is provided

The exit gradient at downstreame end

$$G_E = 0.017 \times \frac{H}{d_2}$$

$$= 0.017 \times \frac{5.48}{4.27} = 0.0218$$

$$= \frac{1}{45} \text{ (Safe)}$$

The section is shown in figure A-1.2. The details of filter are also shown in fig. A-1.2. A drain pipe of 150mm dia shall be provided with open joints and the outlet shall be provided on the chute blocks to reduce the chances of choking

The uplift pressures are also worked out for the above profile for filter length of 0.5, 1.5 and 3.75 m and are given in Table A-1.2, ahead. The uplift pressure for the proposed floor without filter are also shown in the table.

It is seen from the table that even with a filter length of 0.5 m length, the uplift pressures are considerably less and the proposed concrete thickness is sufficient to withstand the pressures. However, it is suggested that a filter length of 2.5 m may be provided so that even if part of the filter gets choked , the remaining portion will effectively reduce the uplift pressures.

TABLE-A-1.2 : UPLIFT PRESSURE AS PERCENT OF TOTAL HEAD FOR DIFFERENT FILTER LENGTHS

	Filter Length				Without filter
	0.5 m	1.5 m	2.5 m	3.75 m	
$\phi_{C'}$	53.02	49.21	45.88	41.58	74.40
$\phi_{D'}$	70.78	68.50	66.56	64.14	79.00
ϕ_E	6.75	5.05	4.06	3.17	28.83
ϕ_D	4.33	3.29	2.64	2.06	22.00
ϕ_j	14.60	10.91	8.72	6.78	52.47
X_j/d_2	3.86	3.59	3.59	3.59	3.59
$G_E \frac{d_2}{H}$	0.0259	0.0196	0.0157	0.0123	0.149

A.1.4 Savings In Concrete

By providing the filter below the glacis, the floor thickness is reduced from 2.6 m to 0.9 m at toe of glacis and 1.2 m to 0.9 m at the end of floor. The length of upstream floor is also reduced by 14 m. The total concrete volume is reduced to 41.2 m^3/m length of barrage from 73.8 m^3/ length of barrage i.e. a reduction of nearly 44 % in concrete volume. Because of reduction in floor thickness, the excavation quantities and dewatering efforts are also reduced considerably. Therefore, net saving will be substantial.

FLOOR FOUNDED ON PERVIOS FOUNDATION OF FINITE DEPTH

```
******************************************************
  DEL=    A TRANSFORMATION PARAMETER
  SIGMA=              -DO-
  EMU=                -DO-
  ROE=                -DO-
  ALAM=               -DO-
  AG=                 -DO
  T=                  -DO-
  N= NUMBER OF STRIPPS FOR INTEGRATION. IT IS AN EVEN NUMBER
  M= 1 FOR LAST DATA SET OTHERWISE = 0
  NN=1            -DO-
  MM=1            -DO-
  COMMON/A1/AJ
  COMMON/A2/AF,AG,EMU
  COMMON/A3/THETA,X,Y,Z,W,WW,VING
  COMMON/A4/SUMX,SUMY,SUMZ
  COMMON/A5/ALAM,ROE
  OPEN(UNIT=1,DEVICE='DSK',FILE='PHD.DAT')
  ---------------------------------------------
91  PRINT 92
  -------------------------
  READ (1,90)N
  -------------------------
101 READ(1,102)DEL,SIGMA,EMU,M
  -------------------------
  PI=3.1415926
  X=0.0 ;Y=DEL ;Z=SIGMA ;W=EMU ;THETA=0.0
  CALL ALRO(N)
  S2=SUMX ;S1=SUMY ;S3=SUMZ
  X=SIGMA ;Y=EMU ;Z=0.0 ;W=DEL ;THETA=0.0
  CALL ALRO(N)
  S5=SUMX ;S4=SUMY ;S6=SUMZ
  C1=(S3*S5-S6*S2)/(S1*S5-S4*S2)
  C2=(S6*S1-S3*S4)/(S2*S4-S5*S1)
  FUN=C1*C1-4.*C2
```

```
      IF(FUN)1000,1001,1001
      -------------------------
)00   PRINT1002,DEL,SIGMA,EMU,FUN
      -------------------------
      GO TO1100
)01   FUN=SQRT(  FUN)
      ROE=0.5*(C1+FUN)
      ALAM=.5*(C1-FUN)
      IF(ROE.LE.SIGMA)GO TO 907
      IF(ROE.GE.EMU)GO TO 907
      IF(ALAM.LE.0.0)GO TO 907
      IF(ALAM.GE.DEL)GO TO 907
      X=SIGMA ;Y=EMU ;Z=0.0 ;W=DEL
      THETA=ASIN(SQRT((ROE-SIGMA)/(EMU-SIGMA)))
      CALL VINGT(N)
      D2M=VING
      X=0.0 ;Y=DEL ;Z=SIGMA ;W=EMU
      THETA=ASIN(SQRT(ALAM/DEL))
      CALL VINGT(N)
      D1M=-VING ;D1D2=D1M/D2M ;D2D1=D2M/D1M
      X=DEL ;Y=SIGMA ;Z=0.0 ;W=EMU ;THETA=PI/2.
      CALL VINGT(N)
      BM=VING ;BD2=BM/D2M ;BD1=BM/D1M
      TM=-0.5*PI*(1.-ALAM)*(1.-ROE)/SQRT((1.-DEL)*(1.-SIGMA)*(1.-EMU))
      TD2=TM/D2M ;TD1=TM/D1M
      -----------------------------------------------------
)5    PRINT 96,DEL,SIGMA,EMU,ALAM,ROE,TD1,TD2,BD1,BD2,D2D1
      -----------------------------------------------------
      PRINT 917
      -----------------------------------------------------
)05   READ(1,97)AG,NN
      -------------------
      INDEX=1
      T=AG
)00   THETA=ASIN(SQRT((T-DEL)/(SIGMA-DEL)))
      X=DEL ;Y=SIGMA ;Z=0.0 ;W=EMU
```

```
      CALL VINGT(N)
      IF(INDEX.EQ.2) GO TO 902
      B1D2=VING/D2M
      INDEX=2
C     ----------------------
 901  READ(1,97) T,MM
C     ----------------------
      GO TO 900
 902  B1FD2=VING/D2M
      FD2=B1D2-B1FD2
      AF=T
C   :*******************************:
C      PRESSURE CALCULATION
C   :*******************************:
      WW=1.0
      CALL CALAJ(N)
      IF(AJ.LT.AG) STOP
      IF(AJ.GT.EMU)STOP
      THETA=0.0
      X=0.0 ;Y=AF ;Z=AG ;W=EMU
      CALL VINT(N)
      VINTO=VING
      THETA=ASIN(SQRT(ALAM/AF))
      CALL VINT(N)
      PD1=VING/VINTO
      THETA=ASIN(SQRT(DEL/AF))
      CALL VINT(N)
      PC1=VING/VINTO
      X=AG ;Y=EMU ;Z=0.0 ;W=AF
      THETA=ASIN(SQRT((AJ-AG)/(EMU-AG)))
      CALL VINT(N)
      PJ=-VING/VINTO
      THETA=ASIN(SQRT((SIGMA-AG)/(EMU-AG)))
      CALL VINT(N)
      PE=-VING/VINTO
      THETA=ASIN(SQRT((RDE-AG)/(EMU-AG)))
```

```
      CALL VINT(N)
      PD=-VING/VINTO
C    :*******************************:
C       CALCULATION OF EXIT GRADIENT
C    :*******************************:
     T=1.01*AF
     J=-1
 708 TERM1=(T-DEL)*(T-SIGMA)*(1.-T)
     TERM2=(T-AF)*(T-AG)*(1.-DEL)*(1.-SIGMA)*(1.-EMU)
     TERM=SQRT(ABS(TERM1/TERM2))
     GT=0.5*PI*(1.-ROE)*(1.-ALAM)*(T-AJ)*TERM/(VINTO*(T-ALAM)*(T-ROE))
     GD=GT/TD2
     IF(J)709,710,711
 709 GEF=GD
     J=0
     T=0.99*AG
     GO TO 708
 710 GEG=GD
     J=1
     T=EMU
     GO TO 708
 711 GEB=GD
C    :***************************:
C       CALCULATION OF DISCHARGE
C    :***************************:
      X=AF;Y=AG;Z=0.0;W=EMU;THETA=0.0
      CALL VINT(N)
      Q1=VING/VINTO
      X=EMU ;Y=1.0 ;Z=0.0 ;W=AF ;WW=AG ;THETA=0.0
      CALL VINT(N)
      Q2Q1=VING/VINTO
C    :*********************************:
C      LOCATION OF POINT 'J' IN Z- PLANE
C    :*******************************:
      IF(AJ-SIGMA)713,712,714
 712  XJD2=D2 ;XJD2=0.0
```

```
      GO TO 715
 713  THETA=ASIN(SQRT((AJ-DEL)/(SIGMA-DEL)))
      X=DEL ;Y=SIGMA ;Z=0.0 ;W=EMU
      CALL VINGT(N)
      XJD2=VING/D2M ;YJD2=0.0
      GO TO 715
 714  X=SIGMA ;Y=EMU ;Z=0.0 ;W=DEL
      THETA=ASIN(SQRT((AJ-SIGMA)/(EMU-SIGMA)))
      CALL VINGT(N)
      YJD2=VING/D2M ;XJD2=BD2
C     ----------------------------------------------------------------------
 715  PRINT 716,AG,AF,AJ,B1D2,FD2,XJD2,YJD2,PD1,PC1,PJ,PE,PD,GEF.GEG,
    1 GEB,Q1,Q2Q1
C     ----------------------------------------------------------------------
717   IF(MM)901,901,904
904   IF(NN)905,905,1100
907   PRINT 908,ALAM,ROE
1100  IF(M)91,91,108
908   FORMAT(60X,2F7.5)
 90   FORMAT(I5)
 92   FORMAT(//8X,5HDELTA,5X,5HSIGMA,5X,3HMUE,7X,5HLAMDA,5X,3HROE,8X,
    1 4HT/D1,6X,4HT/D2,6X,4HB/D1,6X,4HB/D2,6X,5HD2/D1)
102   FORMAT(3F10.5,I1)
1002  FORMAT(15HFUN IS NEGATIVE,4F10.6)
 917  FORMAT(/4X,2HAG,7X,2HAF,7X,2HAJ,7X,5HB1/D2,3X,4HF/D2,2X,5HXJ/D2,2X,
    1 5HYJ/D2,3X,3HPD1,4X,3HPC1,4X,2HPJ,5X,2HPE,5X,2HPD,4X,3HGEF,4X,
    2 3HGEG,4X,3HGEB,3X,2HQ1,4X,5HQ2-Q1/)
 96   FORMAT(/5X,5F10.7,5F10.4/)
 97   FORMAT(F10.7,I5)
716   FORMAT(2X,3F9.6,14F7.3)
 108  CLOSE(UNIT=1)
      PRINT 98,N
 98   FORMAT(//10X,'**********  N=', I4,'  *************')
      STOP
      END
```

```
      SUBROUTINE VINT(N)
      COMMON/A1/AJ
      COMMON/A3/THETA,X,Y,Z,W,WW,VING
      AN=N
      PI=3.1415926
      H=(PI*.5-THETA)/AN
      SUM=0.0
      M=N+1
      DO 20 I=1,M
      WF=4-2*(I-I/2*2)
      IF(I-1)21,21,22
22    IF(I-M)23,21,21
21    WF=1.
23    AI=I-1
      BETA=THETA+AI*H
      SI=SIN(BETA)
      T=(Y-X)*SI*SI+X
      FUN=(AJ-T)/SQRT(ABS((T-Z)*(T-W)*(WW-T)))
      SUM=SUM+FUN*WF
20    CONTINUE
      VING=SUM*H/3.
      RETURN
      END
```

```
      SUBROUTINE CALAJ(N)
      COMMON/A1/AJ
      COMMON/A2/AF,AG,EMU
      AN=N
      A=0. ;AB=EMU-AG ;PI=3.1415926
      SUMA=0.0;SUMB=0.0
      H=PI*0.5/AN
      M=N+1
      DO 20I=1,M
      WF=4-2*(I-I/2*2)
      IF(I-1)21,21,22
22    IF(I-M)23,21,21
21    WF=1
23    AI=I-1
      THETA=A+AI*H ;SI=SIN(THETA) ;T=AB*SI*SI+AG
      FUN=SQRT(T*(T-AF)*(1.-T))
      FUN1=1./FUN
      FUN2=T/FUN
      SUMA=SUMA+WF*FUN1
      SUMB=SUMB+WF*FUN2
20    CONTINUE
      AJ=SUMB/SUMA
      RETURN
      END
```

```
      SUBROUTINE VINGT(N)
      COMMON/A5/ALAM,ROE
      COMMON/A3/THETA,X,Y,Z,W,WW,VING
      AN=N
      PI=3.1415926
      H=THETA/AN
      SUM=0.0
      M=N+1
      DO 20 I=1,M
      WF=4-2*(I-1/2*2)
      IF(I-1)21,21,22
2     IF(I-M)23,21,21
1     WF=1.
3     AI=I-1
      BETA=AI*H
      SI=SIN(BETA)
      T=(Y-X)*SI*SI+X
      FUN=(T-ALAM)*(T-ROE)/((1.-T)*SQRT(ABS((Z-T)*(W-T))))
      SUM=SUM+FUN*WF
0     CONTINUE
      VING=SUM*H/3.
      RETURN
      END
```

```
      SUBROUTINE ALRO(N)
      COMMON /A4/SUMX,SUMY,SUMZ
      COMMON /A3/THETA,X,Y,Z,W,WW,VING
      AN=N
      PI=3.1415926
      H=(0.5*PI-THETA)/AN
      SUMX=0.0 ;SUMY=0.0 ;SUMZ=0.0
      M=N+1
      DO 20 I=1,M
      WF=4-2*(I-I/2*2)
      IF(I-1)21,21,22
22    IF(I-M)23,21,21
21    WF=1.
23    AI=I-1
      BETA=THETA+AI*H
      SI=SIN(BETA)
      T=(Y-X)*SI*SI+X
      FUN=(1.-T)*SQRT((Z-T)*(W-T))
      FUN=1./FUN
      SUMX=SUMX+FUN*WF
      SUMY=SUMY+FUN*WF*T
      SUMZ=SUMZ+FUN*WF*T*T
20    CONTINUE
      SUMX=SUMX*H/3.
      SUMY=SUMY*H/3.
      SUMZ=SUMZ*H/3.
      RETURN
      END
```

```
C     UPLIFT PRESSURE U  FLOOR  ITH COMPLETE CUT OFF A D INTER EDIATE FIL
C*************************************************************************
C     DEL=     A TRA SFOR ATIO  PARAMETER
C     SIG A=          -DO-
C     AG=             -DO-
C     AF=             -DO-
C      = 1  FOR THE L-ST  AIR SET OTHERWISE = 0
C       =1            -DO-                  = 0
      OPE  (U I =1,DEVICE='DSK',FILE='DATA.DAT')
C     ----------------------------------------------
      PRI T 1
C     ----------------------------------------------
      G= .   1
      PI=3.1 159265
C     ----------------------------------------------
1.    REA  (1,1  )DEL,SIG A,
C     ----------------------------------------------
      XK=(1.-SIG A)*DEL/((1.-DEL)*SIG A)
      ALFA1=DEL/(DEL-1.)
      ALFA2=(SIG A-DEL)/SIGMA
      ALFA3=(1.-SIG A)/(1.-DEL)
      JK=0
      PHI=PI/2.
      XKI=1.-XK
      CALL EFU (JK,PHI,XK,ALFA1,G,U,EU,CK,CE,CKD,CED,CPYE1)
      JK=1
      ALA DA=1.-CPYE1/CK
      CALL EFU (JK,PHI,XK,ALFA3,G,U,EU,CK,CE,CKD,CED,CPYE3)
      DE  U=(ALA DA-DEL)*CK-(SIG A-DEL)*CPYE3
      X=(1.-DEL)*ALAMDA/((1.-ALAMDA)*DEL)
      X=SQRT(X/(1.-X))
      PHI I=ATA (X)
      JK=-1
      CALL EFU (JK,PHID1,XK,ALFA1,G,U,EU,CK,CE,CKD,CED,PYED1)
      A   =(1.-ALA DA)*U-PYED1
      D1 2=A   /DE
```

```
      PHI=PI/2.
      CALL EFUN(JK,PHI,XK1,ALFA2,G,U,EU,CKD,CED,CK,CE,CPYE2)
      ANUM =ALAMDA*CKD-DEL*CPYE2
      BD2=ANUM/DENO
C     ------------------------------------------------
      PRINT 2[illegible],DEL,SIGMA,ALAMDA,B1D2,BD2
C     ------------------------------------------------
      GO TO 97
2     READ(1,2[illegible])AG
C     ------------------------------------------------
      T=AG;ICHECK=1
3     X=SIGMA*(1-DEL)/((SIGMA-DEL)*T)
      X=SQRT(X/(1.-X))
      PHIX=ATAN(X)
      CALL EFUN(JK,PHIX,XK1,ALFA2,G,U,EU,CKD,CED,CK,CE,PYE2X)
      ANUM=ALAMDA*U-DEL*PYE2X
      IF(ICHECK.GT.1)GOTO40
      B1D2=ANUM/DENO
C     ------------------------------------------------
35    READ(1,2[illegible])AF,[illegible]
C     ------------------------------------------------
      T=AF
      ICHECK=2
      GO TO 3
4[illegible] FD2=B1D2-ANUM/DENO
C     ------------------------------------------------
      PRINT 3[illegible],B1D2,FD2
C     ------------------------------------------------
      IF([illegible])35,35,97
      JK=2
      XK=AF/AG
      T=ALAMDA;IC=1
5     X=AG*(AF-T)/(AF*(AG-T))
      X=SQRT(X/(1.-X))
      PHI=ATAN(X)
      ALFA=[illegible]
```

```
5      CALL EFUN(JK,PHI,XK,ALFA,G,U,EU,CK1,CE1,CKD1,CED1,PYE)
       IF(IC.GT.1)GO TO 7
       HD1=100.*U/CK1
       T=DEL;JK=-1;IC=2
       GO TO 50
7      IF(IC.GT.2)GO TO 90
       HC1=100.*U/CK1
       T=SIGMA;IC=3
8      X=(T-AG)/(T-AF)
       X=SQRT(X/(1.-X))
       PHI=ATAN(X)
       GO TO 60
9      IF(IC.GT.3)GO TO 95
       HE=100.*U/CK1
       T=1.
       IC=4
       GO TO 80
95     HD=100.*U/CK1
C      --------------------------------------------------------------------
       PRINT 4000, DEL,SIGMA,ALAMDA,D1D2,BD2,B1D2,FD2,HD1,HC1,HE,HD
C      --------------------------------------------------------------------
97     IF(M)10,10,98
98     STOP
1000   FORMAT(2F10.4,I5)
2000   FORMAT(F10.4,I5)
1000   FORMAT(/8X'DELTA    SIGMA   ALAMDA    D1D2      B/D2     B1/D2'
      1,'  F/D2    HD1      HC1      HE      HD'/)
2000   FORMAT(5X,3F8.5,2F8.3)
3000   FORMAT(45X,2F8.2)
4000   FORMAT(11F10.4)
       END
```

```
C         SUBROUTINE TO CALCULATE ELLIPTIC INTEGRALS
C         **********************************************************
          SUBROUTINE EFCA(JK,PHI,XK,ALFA,G,U,EU,CK,CE,CKD,CED,PYE)
      SK=SQRT(XK)
      PI=3.1415926
      IF(XK-[illegible])505,505,500
500   IF(XK-0.9999)509,501,501
501   ALFAS=SQRT(ABS(ALFA))
      CK=ALOG(4./SQRT(1.-XK))
      CE=1.
      CKD=PI/2.
      IF(PHI-PI/2.)503,502,503
502   U=0.
      GO TO 504
503   U=ALOG((1.+SIN(PHI))/COS(PHI))
504   EU=SIN(PHI)
      IF(ALFA)610,104,611
610   AS=ABS(ALFAS)
      PYE=(U+AS*ATAN (AS*SIN(PHI)))/(1.-ALFA)
      GO TO 2
611   IF(ALFA-1.)613,612,613
612   CKD=1.-XK
      GO TO 116
613   PYE=SQRT((1.+ALFAS*SIN(PHI))/(1.-ALFAS*SIN(PHI)))
      PYE=(U-ALFAS*ALOG(PYE))/(1.-ALFA)
      GO TO 2
505   CK=PI/2. ;CE=PI/2. ;CED=1. ;U=PHI ;EU=PHI
      CKD=ALOG(4./SK)
      IF(ALFA-1.)499,612,508
499   IF(PHI-PI/2.)507,506,507
506   PYE=PI/(2.*SQRT(1.-ALFA))
      GO TO 2
507   PYE=ATAN((SIN(PHI)/COS(PHI))*SQRT(1.-ALFA))/SQRT(1.-ALFA)
      GO TO 2
508   SQ=SQRT(ALFA-1.)
      FF=COS(PHI)-SQ*SIN(PHI)
```

```
9       IF(M-1)10,10,11
10      CAD=F
        CED=E
        GO TO 35
11      M=1
        EK=CKD
        F=PI/2.0
35      IF(ALFA)101,104,109
101     IF(-ALFA-SK)102,105,103
102     J=-2
        TH=ATAN(SQRT(ALFA/(-XK)))
        GOTO118
103     J=-1
        TH=ATAN(SQRT(1.0/(-ALFA)))
        GOTO118
104     J=4
        PYE=0
        GOTO 2
105     J=6
        IF(PHI-PI/2.0)106,107,106
106     AT=SQRT(1.0-XK*SIN(PHI)*SIN(PHI))
        PYE=C/2.+ATAN((1.-ALFA)*SIN(PHI)/(COS(PHI)*AT))/(2.*(1.-ALFA))
        GOTO 2
107     PYE=CK/2.+PI/(4.*(1.-ALFA))
        GOTO 2
109     IF(ALFA-XK)110,111,112
110     J=0
        TH=ATAN(SQRT(ALFA/(XK-ALFA)))
        EK=XK
        GOTO118
111     J=5
        AT=SQRT(1.-XK*SIN(PHI)*SIN(PHI))
        PYE=(EU-XK*SIN(PHI)*COS(PHI)/AT)/EKD
        GOTO 2
112     IF(ALFA-SK)113,105,114
113     J=1
```

```
C       SUBROUTINE TO CALCULATE ELLIPTIC INTEGRALS
C       ****************************************************
        SUBROUTINE EFUN(UK,PHI,XK,ALFA,G,U,EU,CK,CE,CKD,CED,PYE)
        SK = SQRT(XK )
        PI=3.1415926
        IF(XK-.     1)505,505,50.
 5      IF(  - .99995)5 9,501,501
 5 1    ALFAS=SQRT(ABS(ALFA))
        CK= ALOG(4./SQRT(1.-XK))
        CE=1.
        CKD=PI/2.
        IF(PHI-PI/2.)503,502,503
 5.     U=C.
        GO TO 5.4
 5 3    U=ALOG((1.+SIN(PHI))/COS(PHI))
 5 4    EU=SIN(PHI)
        IF(ALFA)61.,104,611
 61     AS=ABS(ALFAS)
        PYE=(U+AS*ATAN (AS*SIN(PHI)))/(1.-ALFA)
        GO TO 2.
 .11    IF(ALFA-1.)613,612,613
 61     CED=1.-XK
        GO TO 116
 .13    PIE=SQRT((1.+ALFAS*SIN(PHI))/(1.-ALFAS*SIN(PHI)))
        PYE=(U-ALFAS*ALOG(PIE))/(1.-ALFA)
        GO TO 2.
 5 5    CK=PI/2. ;CE=PI/2. ;CED=1. ;U=PHI ;EU=PHI
        CKD=ALOG(4./SK)
        IF( LFA-1.)499,612,5..
 49.    IF(PHI-PI/2.)5.7,506,507
 5 6    PYE=PI/(2.*SQRT(1.-ALFA))
        GO TO 2
 5 7    PYE=ATAN ((SIN(PHI)/COS(PHI))*SQRT(1.-ALFA))/SQRT(1.-ALFA)
        GO TO 2
 5 8    SQ=SQRT(ALFA-1.)
        FE=COS(PHI)-SQ*SIN(PHI)
```

```
      IF(FF)511,511,510
51    FFA=COS(PHI)+SQ*SIN(PHI)
      PYF=ALOG(FFA/FF)/(2.*SQ)
      GO TO 20
511   PRINT 512,PHI,ALFA,XK
512   FORMAT(3X,'VALUE OF FYE IMAGINARY',5X,3F12.7)
      STOP
5 9    =-1
      P=PHI ;EK=XK ;EKD=1.-EK
      IF(JK)3,3,40
3     A=1.  ;B=1. ;S=P ;F=P ;E=P ;N=1
4     XN=N
      YN=2*N-1
      ZN=2*N
      S=S*YN/ZN-COS(P)*(SIN(P))**YN/ZN
      A=A*EK*YN/ZN
      B=B*EK*(XN-1.5)/XN
      F=F+A*S
      E=E+B*S
      CHEK=ABS(A*S)
      IF(CHEK-G)6,6,5
5     N=N+1
      GOTO4
6     IF(M)7,8,9
7     U=F ;EU=E
      IF(JK)40,35,40
35    IF(PHI-PI/2.0)1,2,1
1     P=PI/2.0
      M=M+1
      GOTO3
2      =M+1
8     CK=F
      CE=E
      EK=EKD
       =  +1
      GOTO 3
```

```
9       IF(*-1)10,10,11
10      CKD=F
        CED=E
        GO TO 35
4.      *=1
        EK=FKD
        P=PI/2.0
35      IF(ALFA)101,104,109
101     IF(-ALFA-SK)102,105,103
102     J=-2
        TH=ATAN(SQRT(ALFA/(-XK)))
        GOTO118
103     J=-1
        TH=ATAN(SQRT(1.0/(-ALFA)))
        GOTO118
104     J=4
        PYE=0
        GOTO20
105     J=6
        IF(PHI-PI/2.0)106,107,106
106     AT=SQRT(1.0-XK*SIN(PHI)*SIN(PHI))
        PYE=U/2.+ATA ((1.-ALFA)*SIN(PHI)/(COS(PHI)*AT))/(2.*(1.-ALFA))
        GOTO20
107     PYE=CK/2.+PI/(4.*(1.-ALFA))
        GOTO20
109     IF(ALFA-XK)110,111,112
110     J=0
        TH=ATAN(SQRT(ALFA/(XK-ALFA)))
        EK=XK
        GOTO118
111     J=5
        AT=SQRT(1.-XK*SIN(PHI)*SIN(PHI))
        PYE=(AJ-XK*SIN(PHI)*COS(PHI)/AT)/FKD
        GOTO2
112     IF(ALFA-SK)113,105,114
113     J=1
```

```
      TH=ATAN(SQRT((ALFA-XK)/(XK*(1.-ALFA))))
      GOTO118
114   IF(ALFA-1.)115,116,117
115   J=2
      TH=ATAN(SQRT((1.-ALFA)/(ALFA-XK)))
      GOTO118
116   J=7
      IF(PHI-PI/2.)515,513,515
 513  PYE=9.9999999E25
      GO TO 2
515   AT=SQRT(1.-XK*SIN(PHI)*SIN(PHI))
      PYE=U-(EU-SIN(PHI)*AT/COS(PHI))/EKD
      GOTO2
117   J=3
      TH=ATAN(1./SQRT(ALFA-1.))
      EK=XK
118   P=TH
      N=N+1
      GOTO3
11    EK=XK
      UT=E
      EUT=E
      V=PI*U/(2.*CK)
      W=PI*UT/(2.*CK)
      PE=PI*CKD/(2.*CK)
      Q=EXP(-2.*PE)
      SUMA=0.0 ;SUMB=0.0 ;SUMC=0.0
      N=1
12    XN=N
      NS=N*N
      XV=2.*XN*V
      IF(J)121,124,125
121   IF(J+1)122,123,124
122   X =2.*X *
      GOTO 128
123   X =2.*X *(PE-.)
```

```
      GOTO128
12.   X =2.*XN*
      XP=2.*XN*PE
      SIN =(EXP(2.*XP)-1.)/(2.*EXP(XP))
      SUM=SU +SI (X )*SI (X )/(X *SINH)
      CHEK=ABS(SIN(XV)*SIN(XN)/(XN*SINH))
      IF(CHEK-G)16,16,15
125   IF(U-2)126,127,134
126   X =2. *XN*
      GOTO128
127   X =2. *X *(PE- )
128   SINH=(EXP(2.*X )-1.)/(2.*EXP(X ))
      COSH=(EXP(2.*X )+1.)/(2.*EXP(X ))
      IF(U)129,132,132
129   NH=N/2
      IF( -2* H)130,130,131
130   SUM =SU  -G**NS*SI (XV)*SINH
      SUMD=SUMD+G**NS*COS(XV)*COSH
      GOTO133
131   SUM =SUM +G**NS*SI (XV)*SINH
      SUMD=SU D-G** S*COS(XV)*COSH
      GOTO133
132   SUM =SU  +G** S*SIN(X )*SINH
      SUMD=SU D+G**NS*COS(XV)*COSH
133   CHEK=ABS(G**NS*SI (XV)*SINH)
      IF(CHEK-G)14,14,15
14    CHEK=ABS(G**NS*COS(XV)*COSH)
      IF(CHEK-G)16,16,15
15    N=N+1
      GOTO 12
134   X =2.*X *
      XP=2.*X *PE
      SIN =(EXP(2.*XP)-1.)/(2.*EXP(XP))
      SUM=SU +G**X *SIN(X )*SIN(XV)/(XN*SINH)
      CHEK=ABS(G**X *SIN(X )*SI (XV)/(X *SINH))
      IF(CHEK-G)16,16,15
```

```
16     IF(J)136,135,136
135    Q =SUM
       ELL=EUT-CL*U1/CK
       D=SQRT(ALFA*(1.-ALFA)*(XK-ALFA))
       PYE=U+ALFA*(U*ELL-Q )/D
       GOTO 2
136    IF(J-3)138,137,138
137    SPV=.+V
       . V=.-V
       Q = .5*ALOG(SIN(SPV)/SIN(SMV))+SUM
       EL =EUT-CL*U1/CK
       D=SQRT(ALFA*(ALFA-1.)*(ALFA-XK))
       PYE=-ALFA*(U*EL -Q.)/D
       GOT 2
138    D=2.*C *SQRT(ALFA*(1.-ALFA)*(ALF -XK))
       EL =(CK*U1+CK*EUT-CA*J1)*2./PI
       S =2.*SU  /(1.+2.*S  D)
       IF(J+1)139,140,141
139    Q =2.*C *ATAN(S  )/PI
       PYE=U*XK/(XK-ALFA)-PI*ALFA*(U*EL +Q )/D
       GOTO 2
140    Q = +2.*CK*ATAN(S  )/PI
       PYE=U/(1.-ALFA)+PI*ALFA*(U*EL -Q )/D
       GOTO 2
141    IF(J-1)142,142,143
142    Q =2.*CK*ATAN(SU )/PI
       PYE=ALFA*PI*(U*EL -Q )/D
       GOTO 2
143    Q = -2.*CK*ATAN(S  )/PI
       PYE=U+PI*ALFA*(Q -U*EL )/D
2      RETURN
       END
```

```
C        PROGRAMME FOR UPLIFT PRESSURE BELOW HYDRAULIC STRUCTURE BY FEM
C       *************************************************************
C       NDATA= NO. OF DATA SET
C       INTER= 1 IF INTERMEDIATE RESULTS NEEDED OTHERWISE = 0
C       T= DEPTH OF SOIL STRATUM
C       B= LENGTH OF FLOOR
C       B1= DISTANCE OF FILTER FROM UPSTREAM CUTOFF
C       FL= LENGTH OF FILTER
C       D1= DEPTH OF UPSTREAM CUTOFF
C       D2= DEPTH OF DOWNSTREAM CUTOFF
C       THETA= INCLINATION OF IMPERVIOUS LAYER
C       N1=NO. OF STRIPPS DOWNSTREAM OF FLOOR
C       N2=NO. OF STRIPPS BETWEEN FILTER AND D/S CUTOFF
C       N3=NO. OF STRIPPS IN FILTER LENGTH
C       N4=NO. OF STRIPPS BETWEEN FILTER AND U/S CUTOFF
C       N5=NO. OF STRIPPS U/S OF FLOOR
C       M= NO. OF HORIZONTAL STRIPPS
C       AKK1= PERMEABILITY OF UPPER LAYER
C       AKK2= PERMEABILITY OF LOWER LAYER
C       DEPTH1= DEPTH OF UPPER LAYER
        COMMON/A/ T,B,B1,FL,D1,D2,THETA,N1,N2,N3,N4,N5,M,MB
        COMMON/B/ X(2200),Y(2200),NI(4400,3),INDEX(2200)
        COMMON/C/ NNP,NEL,NBC
        COMMON/D/ PHI(2200)
        COMMON/E/ BL(2200,40),F(2200)
        COMMON/F/K1,K2,K3,K4,K5,K6,K7
        COMMON/G/PHID1,PHIC,PHIE,PHID,PHIJ,XJ,YJ,GE
        DIMENSION TITLE(20),S(3,3)
        OPEN (UNIT=1,DEVICE='DSK',FILE='FEM.DAT')
C       ------------------------------------------
        READ (1,100) (TITLE(I),I=1,20)
C       ------------------------------------------
        PRINT 100,(TITLE(I),I=1,20)
        IF (INTER.EQ.1) PRINT 9000
C       ------------------------------------------
        READ(1,400) NDATA,INTER
```

```
C       -----------------------------------------------
          DO 20 NSET=1,NDATA
          IF(INTER.NE.0)PRINT 7000,NSET
C       -----------------------------------------------
          READ(1,200)T,B,B1,FL,D1,D2,THETA
C       ----------------------------------------------
          IF(INTER.NE.0)PRINT 5000,T,B,B1,FL,D1,D2,THETA
          TD2=T/D2;BD2=B/D2;DD2=D1/D2;B1D2=B1/D2;FD2=FL/D2
C       ------------------------------------------------
          READ(1,300)N1,N2,N3,N4,N5,M
C       ------------------------------------------------
          IF(INTER.NE.0)PRINT 300,N1,N2,N3,N4,N5,M
C       -----------------------------------------------
C         READ(1,500)  AKK1,AKK2,DEPTH1
C       ------------------------------------------------
C         IF(INTER.NE.0)PRINT 8000,AKK1,AKK2,DEPTH1
C        ***************************************
C               GENERATION OF MESH
C       ******************************************
          CALL MESH
C       ****************************************
C         INITIAL VALUE OF BL AND F MATRIX
C        ***************************************
          DO 5 I=1,NP
          F(I)=0.0
          DO 5 J=1,NB
          BL(I,J)=0.0
   5      CONTINUE
C       ****************************************
C             GENERATION OF 'S' MATRIX
C       *****************************************
         DO 7 I=1,NEL
         NPI=NI(I,1)
         NPJ=NI(I,2)
         NPK=NI(I,3)
         AJ=X(NPK)-X(NPJ)
```

```
      AJ=X(NPI)-X(NPK)
      AK=X(NPJ)-X(NPI)
      BI=Y(NPJ)-Y(NPK)
      BJ=Y(NPK)-Y(NPI)
      BK=Y(NPI)-Y(NPJ)
      AREA=0.5*(BI*AJ-BJ*AI)
      TT=0.25/AREA
C     AKK=AKK1
C     IF(Y(NPI).GT.DEPTH1.OR.Y(NPJ).GT.DEPTH1)  AKK=AKK2
C     IF(Y(NPK).GT.DEPTH1)  AKK=AKK2
C     TT=AKK*TT
      S(1,1)=(BI*BI+AI*AI)*TT
      S(2,2)=(BJ*BJ+AJ*AJ)*TT
      S(3,3)=(BK*BK+AK*AK)*TT
      S(2,1)=(BJ*BI+AJ*AI)*TT
      S(3,1)=(BK*BI+AK*AI)*TT
      S(3,2) =(BK*BJ+AK*AJ)*TT
      S(1,2)=S(2,1)
      S(1,3)=S(3,1)
      S(2,3)=S(3,2)
C    ****************************************************
C          ASSEMBLE OF GLOBAL BANDED MATRIX'BL'
C    ****************************************************
       DO 6IM=1,3
       IR=NI(I,IM)
       DO 6IN=1,3
       IC=NI(I,IN)
       ID=IC-(IR-1)
       IF(ID.LT.1) GO TO 6
       BL(IR,ID)=BL(IR,ID)+S(IM,IN)
    6  CONTINUE
    7  CONTINUE
C    ****************************************************************
C        BOUNDARY CONDITIONS AND GENERATION OF RHS 'F' COL. VECTOR
C    *****************************************************************
       DO 11 J=1,NP
```

```
      IF(INDEX(J).EQ.0) GO TO 11
      DO 10 I=1,(MB-1)
      IR=J-I ; II=I+1
      IF(IR.LT.1) GO TO 9
      F(IR)=F(IR)-BL(IR,II)*PHI(J)
      BL(IR,II)=0.0
 9    IR=J+I
      IF(IR.GT.NNP) GO TO 10
      F(IR)=F(IR)-BL(J,II)*PHI(J)
      BL(J,II)=0.0
 10   CONTINUE
      BL(J,1)=1.0
      F(J)=PHI(J)
 11   CONTINUE
C    ***************************************
C          SOLUTION OF EQUATIONS
C    ****************************************
      CALL BSOLVE(NNP,MB)
      IF(INTER.NE.0)PRINT 2000,NNP,NEL,NBC
      CALL RESULT (D2,M)
      IF(INTER.NE.0)PRINT 9000
      PRINT6000,TD2,BD2,DD2,B1D2,FD2,PHID1,PHIC,PHIE,PHID,PHIJ,XJ,YJ,GE
      IF(INTER.EQ.0)GO TO 20
      PRINT 5000
      DO 13 I=1,NNP
      IF(X(I).LT.0.0.OR.X(I).GT.X(K4))GO TO 13
      IF (X(I).EQ.0.0.AND.Y(I).LE.D1)GO TO 12
      IF (X(I).EQ.X(K4).AND.Y(I).LE.D2) GO TO 12
      IF(Y(I).NE.0.0)GO TO 13
 12   PRINT 4000,I,X(I),Y(I),F(I)
 13   CONTINUE
      DO 14 K=1,(N3+1)
      L=(K-1)*(M+4)+K5-1
      DO 15 J=1,(M+4)
      I=L+J
 15   PRINT 4000,I,X(I),Y(I),F(I)
```

```
   14   CONTINUE
   20   CONTINUE
  100   FORMAT(20A4)
  200   FORMAT(7F10.3)
  300   FORMAT(6I5)
  400   FORMAT (2I5)
  500   FORMAT(3F10.3)
 1000   FORMAT(//26X,20A4//)
 2000   FORMAT(//10X,'NO. OF NODAL POINT=',I4//10X,'NO. OF ELEMENT=',
      1 I4//10X,'NO. PF BOUNDARY POINTS ON CONSTANT HEAD=',I3//)
 3000   FORMAT(//10X,'NODE NO. (I)',7X, 'X(I)',9X, 'Y(I)',7X, 'PHI(I)'//)
 4000   FORMAT(/15X,I5,7X,F6.2,7X,F6.2,7X,F6.2/)
 5000   FORMAT(//3X,'T=',F5.2,'  B=',F5.2,'  B1=',F5.2,'  F=',F5.2
      1 ,'  D1=',F5.2,'  D2=',F5.2,'  THETA=',F5.2,' DEG'//)
 6000   FORMAT(/3X,12F7.2,F10.4)
 7000   FORMAT(/'***  SET NO.',I3,'  **'/'-------------------')
 8000   FORMAT('    K1=',F7.3,'   K2=',F7.3,'  DEPTH OF TOP LAYER=',F7.3)
 9000   FORMAT(/6X,'T/D2   B/D2  D1/D2   B1/D2   F/D2   HD1    HC1     HE
      1     HJ   XJ/D2  YJ/D2   GE*D2/H'/)
        CLOSE (UNIT=1)
        STOP
        END
```

```
C          SOLUTION OF EQUATIONS (BANDED) BY GAUSS ELIMINATION METHOD
C         ***********************************************************
          SUBROUTINE BSOLVE(N,M)
          COMMON/E/ BL(2200,40),F(2200)
          DIMENSION C(50)
          DO 10 K=1,N
          F(K)=F(K)/BL(K,1)
          IF(K.EQ.N) GO TO 100
          DO 20 J=2,M
          C(J)=BL(K,J)
          BL(K,J)=BL(K,J)/BL(K,1)
   20     CONTINUE
          DO 30 L=2,M
          I=K+L-1
          IF(N.LT.I) GO TO 30
          J=0
          DO 40 LL=L,M
          J=J+1
          BL(I,J)=BL(I,J)-C(L)*BL(K,LL)
   40     CONTINUE
          F(I)=F(I)-C(L)*F(K)
   30     CONTINUE
   10     CONTINUE
          K=N
  100     K=K-1
          IF(K.EQ.0) GO TO 200
          DO 50 J=2,M
          L=K+J-1
          IF(N.LT.L) GO TO 50
          F(K)=F(K)-BL(K,J)*F(L)
   50     CONTINUE
          GO TO 100
  200     RETURN
          END
```

```
C       GENERATION OF MESH FOR FINIT ELEMENT ANALYSIS
C       ***************************************************
        SUBROUTINE MESH
        COMMON/B/ X(2200),Y(2200),NI(4400,3),INDEX(2200)
        COMMON/C/ NNP,NEL,NBC
        COMMON/A/ T,B,E1,FL,D1,D2,THETA,N1,N2,N3,N4,N5,M,NB
        COMMON/D/ PHI(2200)
        COMMON/F/K1,K2,K3,K4,K5,K6,K7
        THETA=3.141592*THETA/180.
        N=N1;AM=M;AN=N
        Z3=3.*B ;Z2=3.*B
        IF(THETA.EQ.0.0) GO TO 5
        Z1=T*COS(THETA)/SIN(THETA)-Z2
        T1=Z1*SIN(THETA)/COS(THETA)
        L=1 ;NC=1 ; NCC=0
        IF(Z1.GT.0.0) GO TO 10
        Z1=0.1*T*COS(THETA)/SIN(THETA)
        Z2=9.*Z1 ;T1=0.1*T
        N1=AN*Z2/Z3
        IF(N1.LT.12) N1=12
        N=N1 ;  AN=N
        GO TO 10
  5     Z1=0.0
        T1=T;L=0 ;NC=0 ;NCC=0
 10     XXB=Z1+Z2+B ;X(1)=XXB ;Y(1)=0.0
        XX=Z2/AN
        Z=Z1 ;N=N1-1
 12     DO 15 I=1,N
        AI=I ; INDEX1=0
        TT=T1+(AI-1.)*XX*SIN(THETA)/COS(THETA)
        YY=TT/AM
        IF(NC.GT.3) YY=T/AM
        ZZ=Z+(AI-1.)*XX
        DO 15 J=1,(M+4)
        AJ=J
        K=(I-1)*(M+4)+J+L
```

```
      X(K)=XXB-ZZ
      IF(INDEX1.EQ.1) GO TO 13
      Y(K)=(AJ-4.)*YY
      IF(J.LE.5) Y(K)=(AJ-1.)*YY/4.
      IF(Y(K).LT.D1.OR.NC.LE.3) GO TO 15
      Y(K)=D1 ;INDEX1=1  ;YY=(TT-D1)/(AM+4-AJ)
      GO TO 15
13    Y(K)=Y(K-1)+YY
15    CONTINUE
      IF(NC.GT.3) GO TO 35
16    IF(NCC.GE.1) GO TO 18
      L=N*(M+4)+L
      NCC=1 ;N=2
17    XX=XX/2.
      Z=ZZ+XX
      T1=TT+XX*SIN(THETA)/COS(THETA)
      GO TO 12
18    IF(NCC.EQ.2) GO TO 19
      L=N*(M+4)+L
      NCC=2 ;N=3
      GO TO 17
19    T1=T ;D=D2 ;Z=Z1+Z2 ; INDEX2=0
20    YY=T1/AM
      IF(INDEX2.EQ.1) YY=T/AM
      L=N*(M+4)+L
      INDEX1=0
      DO 30 J=1,(M+4)
      AJ=J ;K=J+L
      X(K)=XXB-Z
      IF(INDEX1.EQ.1) GO TO 25
      JK=J ;KK=K+M+4
      X(KK)=X(K)
      Y(K)=(AJ-4.)*YY
      IF(J.LE.5) Y(K)=(AJ-1.)*YY/4.
      Y(KK)=Y(K)
      IF(Y(K).LT.D) GO TO 30
```

```
      Y(K)=0
      DEND=N+4-J
      YY=(T1-D)/DEND
      INDEX1=1
      GO TO 30
25    Y(K)=Y(K-1)+YY
30    CONTINUE
      IF(INDEX2.EQ.1)GO TO 60
      K1=L+1  ;K2=  K1+1 ;K3=L+JK ;K4=  K1+M+4
      JK1=JK-1
      M1=NC+N1+6
      NC=M1
      L=KK-1
35    IF(NC.NE.M1) GO TO 40
      K5=K4
      IF(B.EQ.B1)GO TO 45
      N=N2 ;AN=N
      XX=(B-B1)/AN
      Z=Z1+Z2+XX
      T1=Z*SIN(THETA)/COS(THETA)
      IF(THETA.EQ.0.0) T1=T
      M2=NC+N ;NC=M2
      GO TO 12
40    IF(NC.NE.M2) GO TO 50
      L=L+N2*(M+4)
      K5=L-(M+3)
45    IF(FL.EQ.0.0) GO TO 52
      N=N3 ;AN=N ;XX=FL/AN
      Z=Z1+Z2+B-B1+XX
      T1=Z*SIN(THETA)/COS(THETA)
      IF(THETA.EQ.0.0) T1=T
      M3=NC+N ;NC=M3
      GO TO 12
50    IF(NC.NE.M3) GO TO 55
      L=L+N3*(M+4)
52    N=N4-1
```

```
      AN=N4 ;XX=(H1-FL)/AN
      Z=Z1+Z2+B-H1+FL+XX
      T1=Z*SIN(THETA)/COS(THETA)
      IF(THETA.EQ.0.0) T1=T
      M4=NC+N ;NC=M4
      GO TO 12
55    IF(NC.NE.M4) GO TO 65
      Z=Z1+Z2+B
      T1=Z*SIN(THETA)/COS(THETA)
      IF(THETA.EQ.0.0) T1=T
      D=D1 ;NC=NC+2
      INDEX2=1
      GO TO 20
60    K6=L+1 ;K7=L+JK ;L=KK-1
      JK2=JK-1
      N=N5 ;AN=N ;XX=Z3/AN ;XX=XX/4. ;N=4
      Z=Z1+Z2+B+XX
      T1=Z*SIN(THETA)/COS(THETA)
      IF(THETA.EQ.0.0) T1=T
      M5=NC+N ;NC=M5
      GO TO 12
65    IF(NC.NE.M5) GO TO 75
      L=2*(N+4)+L ;M6=NC+N ;NC=M6 ;N=2
70    XX=2.*XX
      Z=ZZ+XX
      T1=TT+XX*SIN(THETA)/COS(THETA)
      GO TO 12
75    IF(NC.NE.M6) GO TO 80
      L=2*(N+4)+L
      M7=NC+N ;NC=M7 ;N=N5-2
      GO TO 70
80    MP=K ; M8=N+6+JK1
      L=0 ; ME=0
      N=N1+N2+N3+N4+N5+8
      M1=N1+5
      M2=N1+N2+N3+N4+5
```

```
      IF(THETA.EQ.0.) GO TO 125
      DO 120 J=1,(M+3)
      NE=J
      NI(NE,1)=1
      NI(NE,2)=J+1
      NI(NE,3)=J+2
120   CONTINUE
      L=1
125   DO 170 I=1,N
      K=(I-1)*(M+4)+L
      IF(I.EQ.M1)K=I*(M+4)+L
      IF(I.GT.M1) K=(I-1)*(M+4)+L+JK1
      IF(I.EQ.M2) K=I*(M+4)+L+JK1
      IF(I.GT.M2) K=(I-1)*(M+4)+L+JK1+JK2
      DO 165 J=1,(M+3)
130   NE=NE+1
      NI(NE,1)=K+J
      NI(NE,2)=NI(NE,1)+M+4
      IF(I.NE.M1) GO TO 135
      IF(J.GT.JK1) NI(NE,1)=NI(NE,1)-M-4
      NI(NE,2)=K+J+JK1
135   IF(I.NE.M2) GO TO 140
      IF(J.GT.JK2)NI(NE,1)=NI(NE,1)-M-4
      NI(NE,2)=K+J+JK2
140   NI(NE,3)=NI(NE,2)+1
      NE=NE+1
145   NEE=NE-1
      NI(NE,1)=NI(NEE,1) ; NI(NE,2)=NI(NEE,3)
      NI(NE,3)=NI(NE,1)+1
      IF(I.NE.M1) GO TO 150
      IF(J.EQ.JK1) NI(NE,3)=NI(NE,1)-M-3
150   IF(I.NE.M2) GO TO 165
      IF(J.EQ.JK2) NI(NE,3)=NI(NE,1)-M-3
165   CONTINUE
170   CONTINUE
      NEL=NE
```

```
C      SUBROUTINE FOR CALCULATING FINAL RESULTS AT KEY POINTS
C      ************************************************************
       SUBROUTINE RESULT (D,M)
       COMMON/B/ X(2200),Y(2200),NI(4400,3),INDEX(2200)
       COMMON/E/ BL(2200,40),F(2200)
       COMMON/F/K1,K2,K3,K4,K5,K6,K7
       COMMON/G/PHID1,PHIC,PHIE,PHID,PHIJ,XJ,YJ,GE
       PHID=F(K3);PHIF=F(K4);PHIC=F(K6);PHID1=F(K7)
       GE=0.61*D*(F(K2)-F(K1))/Y(K2)
       PHIJ=F(K5)
       I=K5 ; YJ=0.0
20     I=I-(M+4)
       IF(X(I).GE.X(K4)) GO TO 30
       IF(F(I).GE.PHIJ) GO TO 50
       PHIJ=F(I)
       XJ=X(I)
       GO TO 20
30     I=K4-1 ;YJ=0.0
40     I=I+1
       IF (F(I).LT.PHIJ) GO TO 50
       PHIJ=F(I) ; XJ=X(I)  ;YJ=Y(I)
       GO TO 40
50     XJ=XJ/D ; YJ=YJ/D
       RETURN
       END
```

```
      IF(THETA.EQ.0.) GO TO 125
      DO 120 J=1,(M+3)
      NE=J
      NI(NE,1)=1
      NI(NE,2)=J+1
      NI(NE,3)=J+2
120   CONTINUE
      L=1
125   DO 170 I=1,N
      K=(I-1)*(M+4)+L
      IF(I.EQ.M1)K=I*(M+4)+L
      IF(I.GT.M1) K=(I-1)*(M+4)+L+JK1
      IF(I.EQ.M2) K=I*(M+4)+L+JK1
      IF(I.GT.M2) K=(I-1)*(M+4)+L+JK1+JK2
      DO 165 J=1,(M+3)
130   NE=NE+1
      NI(NE,1)=K+J
      NI(NE,2)=NI(NE,1)+M+4
      IF(I.NE.M1) GO TO 135
      IF(J.GT.JK1) NI(NE,1)=NI(NE,1)-M-4
      NI(NE,2)=K+J+JK1
135   IF(I.NE.M2) GO TO 140
      IF(J.GT.JK2)NI(NE,1)=NI(NE,1)-M-4
      NI(NE,2)=K+J+JK2
140   NI(NE,3)=NI(NE,2)+1
      NE=NE+1
145   NEE=NE-1
      NI(NE,1)=NI(NEE,1) ; NI(NE,2)=NI(NEE,3)
      NI(NE,3)=NI(NE,1)+1
      IF(I.NE.M1) GO TO 150
      IF(J.EQ.JK1) NI(NE,3)=NI(NE,1)-M-3
150   IF(I.NE.M2) GO TO 165
      IF(J.EQ.JK2) NI(NE,3)=NI(NE,1)-M-3
165   CONTINUE
170   CONTINUE
      NEL=NE
```

```
      NBC=N1+N3+N5+11+L
      IF(N3.EQ.0) NBC=N1+N5+10+L
      DO 175 I=1,NNP
      PHI(I)=0.0 ;INDEX(I)=0
 175  CONTINUE
      L=L+1
      PHI(1)=0.0 ;INDEX(1)=0
      DO 220 I=L,NBC
      IF(I.GT.(N1+L+4)) GO TO 185
      K=(I-L)*(M+4)+L
 180  PHI(K)=0.0
      INDEX(K)=1
      GO TO 220
 185  IF(N3.EQ.0) GO TO 195
      IF(I.GT.(N1+N3+5+L)) GO TO 205
      IF(N2.EQ.0) GO TO 190
      IF(I.EQ.(N1+L+5)) K=(N1+N2+4)*(M+4)+JK1+L
      IF(I.GT.(N1+5+L))K=K+M+4
      GO TO 180
 190  IF(I.EQ.(N1+L+5))K=(N1+5)*(M+4)+L
      IF(I.EQ.(N1+L+6)) K=K+JK1
      IF(I.GT.(N1+L+6)) K=K+M+4
      GO TO 180
 195  IF(I.EQ.(N1+L+5)) K=(N1+N2+N4+5)*(M+4)+JK1+L
      IF(I.EQ.(N1+L+6)) K=K+JK2
      IF(I.GT.(N1+L+6)) K=K+M+4
      GO TO 210
 205  IF(I.EQ.(N1+N3+L+6)) K=(N1+N2+N3+N4+5)*(M+4)+JK1+L
      IF(I.EQ.(N1+N3+L+7)) K=K+JK2
      IF(I.GT.(N1+N3+L+7)) K=K+M+4
 210  PHI(K)=100.0
      INDEX(K)=1
 220  CONTINUE
      RETURN
      END
```

```
C         SUBROUTINE FOR CALCULATING FINAL RESULTS AT KEY POINTS
C        *****************************************************************
          SUBROUTINE RESULT (D,M)
          COMMON/B/ X(2200),Y(2200),NI(4400,3),INDEX(2200)
          COMMON/E/ BL(2200,40),F(2200)
          COMMON/F/K1,K2,K3,K4,K5,K6,K7
          COMMON/G/PHID1,PHIC,PHIE,PHID,PHIJ,XJ,YJ,GE
          PHID=F(K3);PHIE=F(K4);PHIC=F(K6);PHID1=F(K7)
          GE=0.01*D*(F(K2)-F(K1))/Y(K2)
          PHIJ=F(K5)
          I=K5 ; YJ=0.0
   20     I=I-(M+4)
          IF(X(I).GE.X(K4)) GO TO 30
          IF(F(I).LE.PHIJ) GO TO 50
          PHIJ=F(I)
          XJ=X(I)
          GO TO 20
   30     I=K4-1 ;YJ=0.0
   40     I=I+1
          IF (F(I).LT.PHIJ) GO TO 50
          PHIJ=F(I) ; XJ=X(I)  ;YJ=Y(I)
          GO TO 40
   50     XJ=XJ/D ; YJ=YJ/D
          RETURN
          END
```

APPENDIX A-3

REFERENCES

1. Aravin, V.I. and Numerov, S.N.,'Theory of fluidflow in undeformable porous media', translated from Russion by Isreal Program for Scientific Translation, 1965.

2. Brebbia, C.A. and Connor, J.J. 'Finite element techniques for fluid flow' Newnes-Butterworths, London.

3. Byrd, P.F. and Fried-man, M.E. 'Hand book of Elliplic Integrals for Engineers and Scientists' springer - verlag, Berlin, West Germany, 1971.

4. Chawla, A.S. and Akhilesh Kumar, 'Stability of structure with two cutoffs', Journal of the Institution of Engineers (India), Civil Engineering Division,March,1983.

5. Chawla, A.S. and Akhilesh Kumar, 'Stability of structure with two cutoffs founded on pervious strata of finite depth', accepted for publication in the Journal of Central Board of Irrigation and Power (India) New Delhi, April, 1985.

6. Chawla, A.S. 'Stability of structure with intermediate filters', Journal of Hydraulic Division, ASCE, vol.101 No.HY2', Proc. Paper 11139, February, 1985.

7. Desai, C.S. and ABEL, J.F. 'Introduction to the finite element method', Van Nostrand Rein-hold Company,New York.

8. Garg, S.P. and Chawla, A.S. 'Stability of structures on permeable foundation' Journal of the Hydraulic Division, ASCE, Vol. 95, No.HY4, Proc.Paper 6648,July,1969.

9. Harr, M.E. 'Ground Water and Seepage', Mc Graw-Hill book company, New York, 1962.

10. Khosla, A.N., Bose, N.K. and Taylor, E.M. 'Design of weirs on permeable foundations' publication No.12 (reprint) Central Board of Irrigation and Power, India, New Delhi, January, 1957.

11. Meleshchenko, N.T. 'Computations of ground water flow under structures fitted with drain-holes', Investiya Nauchno-Issledovatelskogo Institute, Gidrokekhniki, Vol. 19, 1936.

12. Muskat, M. 'The flow of homogeneous fluids through porous media' Mc Graw-Hill Book co. New York,N.Y.,1937.

13. Numeror, S.N. and Arvin, V.I. 'Seepage computations in Hydraulic Structures', Cosstrolizedet, U.S.S.R.,1948.

14. Pavlovsky, N.N. 'The theory of movement of ground Water under Hydraulic Structure and its main applications', Petrogrod, USSR, 1922.

15. Polubarinova-Kochina, P.Y. 'Theory of groundwater movement' translated by J.M. Roger Dewiest, Princeton University Press, Princeton, N.Y. 1962.

16. Sangal, S.P. 'Efficiency and location of intermediate filters in wier and barrage floors' thesis submitted in partial fulfilment of requirement for degree of M.E. in Civil Engineering, University of Roorkee, 1964.

17. Terzaghi, K. and Peck, R.B. 'Soil Mechanics in Engineering Practice' John Willy & Son's, INC, New York.

18. Terzaghi, K. 'Theoritical soil mechanics', John Willy & Son's INC, New York.

19. Zamarin, E.A. 'The flow of ground water under Hydraulics structures' Tashkent, 1931.

20. Zienkiewicz, O.C. 'The finite element method in Engineering Science' Mc Graw-Hill-Book Co.,London.