DISTORTIONS
IN
AERIAL PHOTOGRAPHS

A Dissertation submitted to the
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Photogrammetric Engineering.

BY

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1960

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(Y.R.NAGARAJA)

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DISTORTIONS IN AERIAL PHOTOGRAPHS

SYNOPSIS

The increasing need for accuracy in Photogrammetric methods has brought the question of distortion to the forefront as a problem of the first order.

In the present paper an attempt has been made to deal with all the sources contributing towards the introduction of distortion and their behaviour and effect on the final product "the topographical map". The methods and means for elimination of distortion are reviewed and the effect of not completely eliminated (residual) distortion on mapping,&aerial triangulation is investigated.

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: INTRODUCTION :

Nowhere in the field of optics, is required a higher degree of accuracy of the lens systems than in Photogrammetry. From its definition alone - "The science or art of obtaining reliable measurements by means of Photography" - Photogrammetry precludes any photographic system which is not controlled to attain the highest degree of accuracy.

Photogrammetry is becoming more and more a precise method of surveying, so that it is a very important factor in large scale mapping. Yet, to replace the older methods of surveying, the science should be applied with the utmost care bearing in mind all the possible sources of errors.

IMPORTANCE OF THE STUDY OF DISTORTION AT THE PRESENT STAGE

The new method of preparing maps by Photogrammetric methods meets the need of the present day. Development programmes are being executed so quickly nowadays that a comparable speed in the preparation of maps has assumed much importance. The conventional Ground Survey methods are costly, laborious and take much time. Photogrammetric methods on the contrary are quick and economical.

The question that may be asked is whether the aerial survey method is sufficiently accurate particularly in making large scale plans. It is found that maps prepared by photogrammetric methods could be as accurate as those prepared by the ground survey methods. The new methods is better when contours are to be positioned in their true places. The Photogrammetric method has several advantages over ground survey methods. It gives a far truer picture of the actual shape of the greund in question; contours can be observed at every point along their path. On the other hand in the ground surveys, contours must be interpolated in a grid of points (spot levels), thus they cannot follow the exact shape of the ground. Once the area has been photographed maps can be prepared quickly in the office. The contours can normally be guaranteed to be accurate to within one half of the contour interval.

"Photogrammetry can always provide details to within plottable limits of the publication scale. It has been found from experience, that details plotted from aerial photographs are in general at least as good as can be represented by field measurements and in many cases much better upto a scale of 1/500 or 40-ft. to an inch."(1)*

Where contours are required there is no doubt that photogrammetric method is superior to ground survey method and in addition it has the advantage of speed and economy

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* For this and further citations see References at the end.

in man-power.

The accuracy of maps made by photogrammetric methods, however, depends on the following factors: (2)

1. The accuracy of the aerial negatives used
2. The processing of the negatives,
3. The instruments and methods employed in the preparation of maps,
4. The ground control, and
5. The error standards.

Distortion, with which this paper deals, is a source of error and is the important factor determining the accuracy of aerial photographs. Although improvements have been effected in the several stages of the preparation of maps like the design of the aerial cameras, the processing of aerial photographs, the stereoscopic plotters etc., there is still a need to further improve the photograph which forms the raw material in all photogrammetric operations. Distortion is the chief limiting source of error on which depends the accuracy of the aerial photograph.

It is therefore the objective of modern effort to study the causes of distortion and to eliminate or reduce distortion in aerial photographs, so that the accuracy and efficiency of photogrammetric methods could be increased.

Full justice to a review of this subject could not be done as several published contributions were not available in this country.

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CHAPTER - I

GENERAL DEFINITION OF DISTORTION

In an aerial photograph distortion is the displacement of the image of an object from its true position. The image due to several reasons is not at the position where it should be in a true perspective (Fig.1). This distortion is measured in terms of a micron-(symbol μ -) which is one thousandth of one millimetre. (0.001)mm.

This displacement may occur in the photograph, while it is being taken during flight, or it may occur in the next stage of processing the aerial negative or during projection in the plotting instrument. It might also be caused by a combined effect of the above stages.

At the moment of exposure, in the air, the distortion called the Photogoniometric error -- may be caused due to the distortion in the Lens itself, due to the unflatness of the negative plane and due to the effect of temperature and also refraction and curvature of the earth. We may call this the actual distortion.

During the processing stage, distortion errors can be caused by improper handling and drying of the aerial negative. At the plotting stage, errors are introduced from the plotting instrument itself, and the sag of a glass negative in a

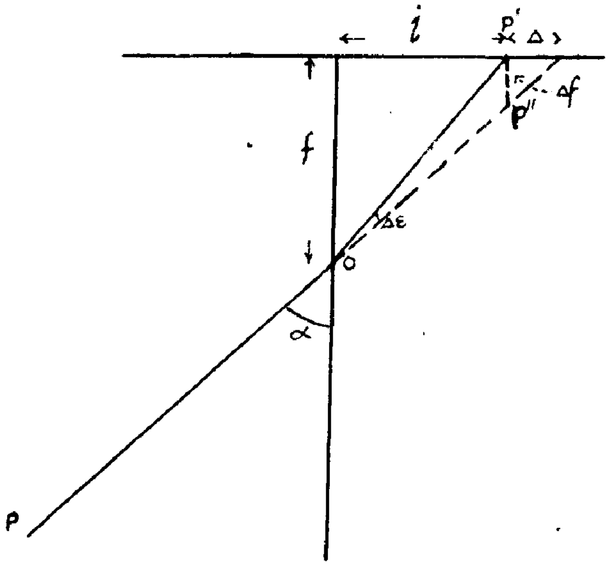


fig. 1 . Displacement of the image
Δe = Angle of distortion.

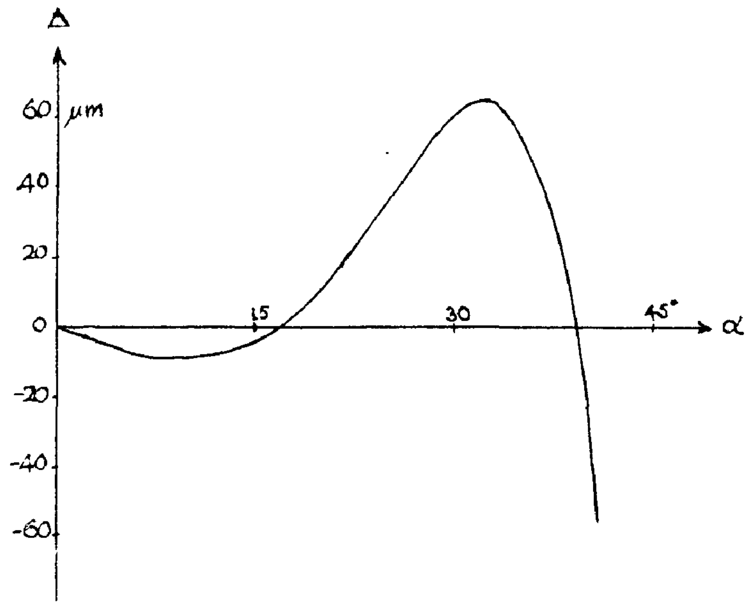


fig. 2 Typical curve of distortion.

plotter, the inaccuracies of the reconstruction of the object bundle of rays, are some of the causes. This may be referred to as the vertical distortion.

The effect of one or all of the above causes is to falsify the actual position of the image of an object, on the aerial photograph and on the subsequent map which is the goal of all topographical photogrammetric operations.

Such distortion can happen in any direction on a photograph. It is a vector quantity having a radial and a tangential component. The distortion is considered positive if the image is moved away from the principal point, and is considered negative if the movement is towards the principal point.

In photogrammetry, aerial photographs are used as a means to set up a model of the terrain photographed, from which topographical maps one produced. To creat a model, the object bundle of rays from two successive and overlapping photographs should be reconstructed as they were at the moment of exposure. To realise this, at the time of plotting, the processes of Inner, Relative and Absolute Orientations have to be performed for each pair of photographs.

In these operations the role of distortion is of importance as it affects the reconstruction of the object bundle. of rays.

We shall now deal with the effect of distortion on the above processes.

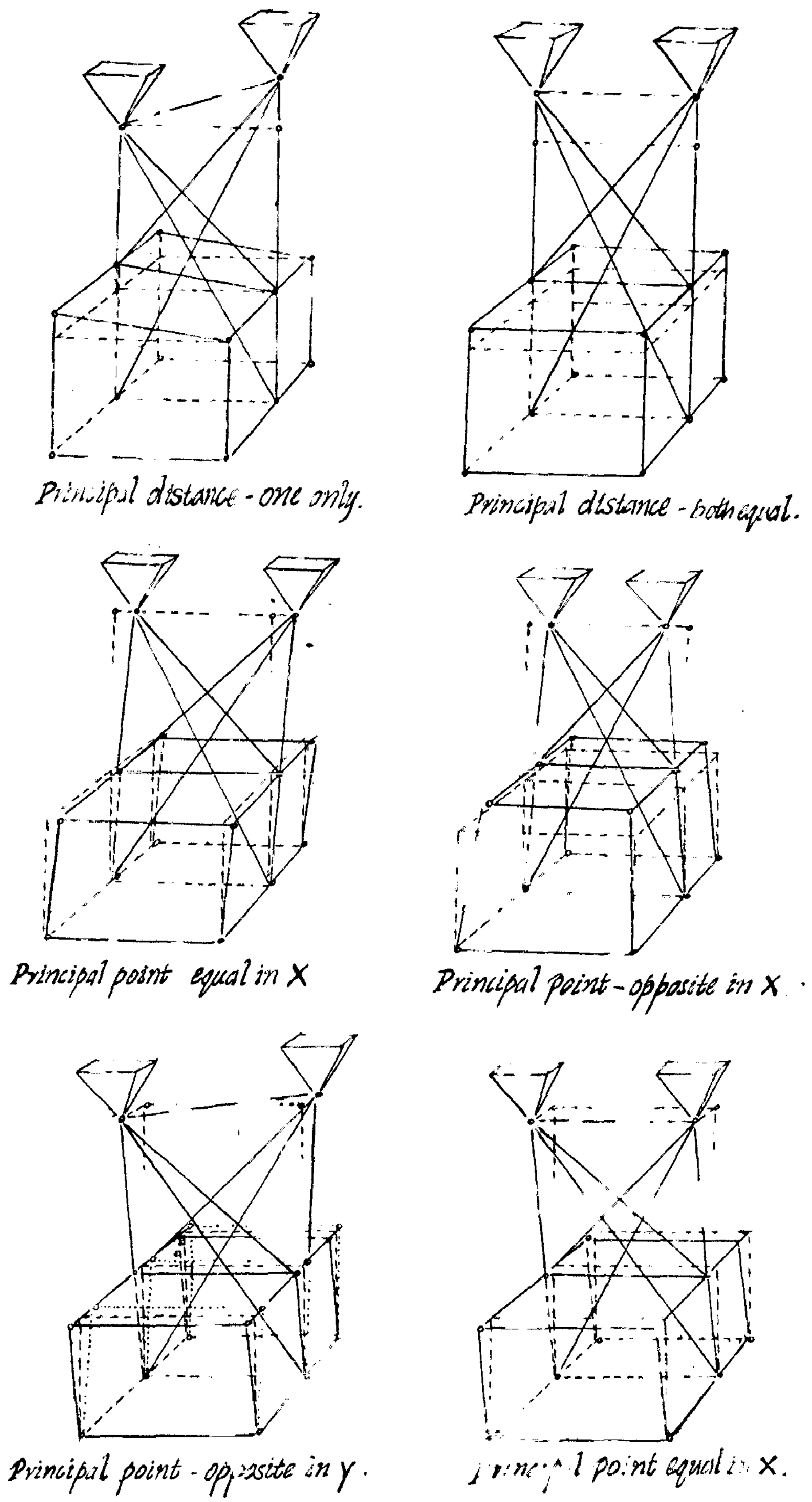
B. PHOTOGRAMMETRIC EFFECTS OF DISTORTION

B.1 Effect of Distortion on Inner Orientation:

Distortion arising from improper calibration of the aerial camera or of the projection printer will result in small errors in the principal distance and the location of the principal point. If the negative is not aligned properly in a diapositive printer, errors in the location of the principal point will result. These errors cause model defermations (see fig: 3 : The errors have been greately exaggerated for illustrative purposes).

The effect of the principal distance error is to alter the vertical scale of the model with respect to the horizontal scale.

When unequal x-components occur due to a displacement of the principal point, errors in vertical scale result. Due to unequal y-components on account of displacement of the principal point in the y-direction, y-parallax is introduced throughout the model introducing a slight cylindrical deformation. Equal y-components will only



Principal distance - one only.

Principal distance - both equal.

Principal point equal in X

Principal point - opposite in X.

Principal point - opposite in Y.

Principal point equal in X.

fig. 3 . Effects of interior orientation errors.

tilt the model effecting neither scale change nor the introduction of y-parrallax. Small errors in the interior orientation get usually compensated during the process of absolute orientation.

B.2 <u>Effect of distortion on Relative Orientation</u>:

During the process of Relative orientation, the y-parrallax at the same six points of orientation, cannot be eliminated completely. One of the reasons for this is the distortion.

Corresponding light rays say r_1 & r_2 (see fig.45) from two photographs I and II, which, when the photographs were taken originated from an object point, will no longer intersect at a point during the process of relative orientation. They cross in space when the pencil of rays is reconstructed in the plotting instrument. This discrepancy will result in the form of a parallax Δp in the image plane. This parallax can be divided into the horizontal component Δp_h and the vertical component Δp_v. The vertical component Δp_v affects the process of relative orientation in that it requires additional y tilts or ϕ to remove the y-parrallax. This additional ϕ warps the model imparting a cylindrical deformation (see fig.4). The effect of this is a systematic error in height, in addition to the height errors caused by the horizontal component Δp_h of the parallax Δp.

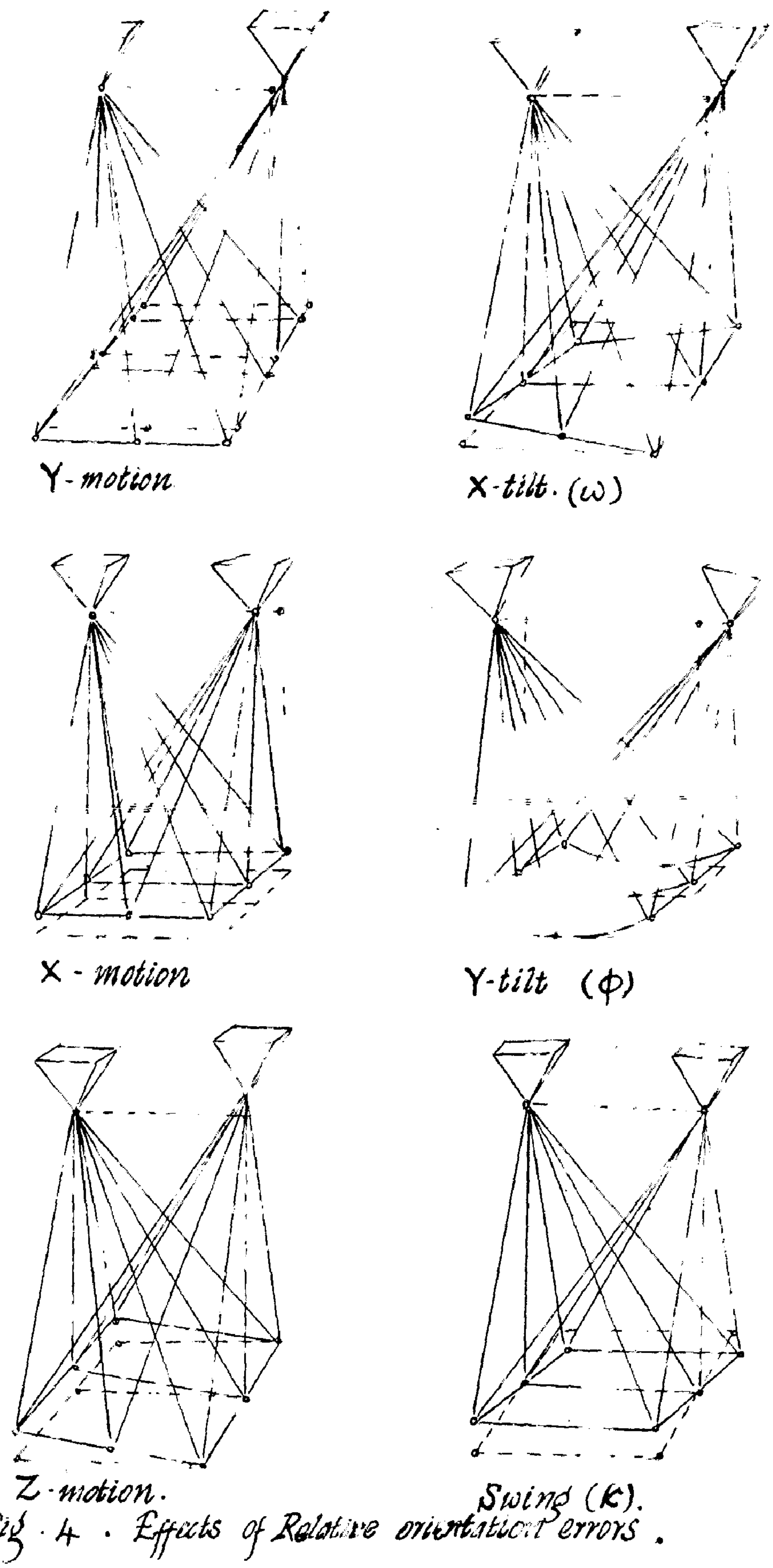


fig. 4. Effects of Relative orientation errors.

. However the effect of distortion is not appreciable in a single model. But in case of aerial triangulation, its effect cannot be neglected and elimination of distortion becomes absolutely necessary.

B.3 Effects of distortion on the Stereoscopic Model (Model deformation)

Errors caused by distortion in the elements of Relative orientation have been illustrated in fig.4. In the figure their effects have been greately exaggerated for illustrative purpose. Their effects are more pronounced and serious in the case of elevations than in planimetry.

Out of the six errors, the errors due to tilt about the x and y axes are important in that they are more significant.

These deformations may not be serious in a single model, but are of great importance in aerial triangulation by bridging, in that they influence the determination of each new image position. (For further treatment (42) can be referred).

It has been proved by Dr. M.Zeller that it is primarily the distortion and instrumental errors which cause larger errors in the relative orientation compared to the Accidental errors.

B.4 Effect of Distortion on Aerial Triangulation:

The process of aerial triangulation is one of continuity and any error in the first model will be carried forward. Among factors contributing to these errors distortion is also one. Though the first model is levelled carefully, it is a common experience that the succeeding models tend to warp, such that the datum of the whole strip would be arched upward in the centre. This vertical bow is chiefly the result of small distortions - coming mainly from the lens - and in case of multiplex, is the most persistant deformation.

While the vertical component of the distortion as seen before, affects the process of orientation, The horizontal componant affects the scale of the model. A small error in the scale in the first model will accumulate as a large error in the long extension of several models.

Deformations in the X Y plane i.e., horizontal deformation is known to be caused by tangential lens distortion which is just a manufacturing error resulting from small decentrations of the lens elements. This distortion may be either in the camera lens or in case of multiplex in the printer lens also.

The errors are the horizontal and the differential bow of the horizontal datum. (Figs 5 and 6)

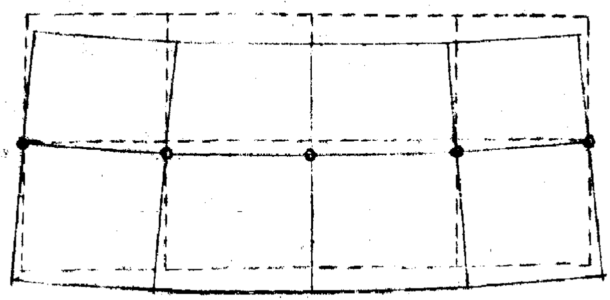


fig. 5 . Horizontal bow

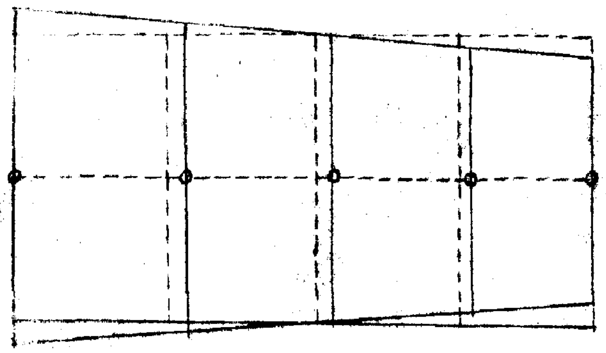


fig. 6 . Differential bow

The horizontal bow is indicated by the end models appearing to be slightly swung in opposite directions with respect to their control. The horizontal bow is caused by swing errors, is roughly like a bow and circular similar to the vertical bow which is caused by Y-tilts. This will cause a discrepancy in the Y-direction at a control point in the centre of the strip.

The differential bow is evidenced by a change in the scale of the model progressively. The scale of the model at one end will be too large and at the other end it will be too small compared to the control. This error causes a discrepancy in the X direction at a control point in the centre of the strip.

Thus distortion together with instrumental errors can cause considerable errors in the co-ordinates of the mirror control points between their readings in one pair and a subsequent pair. But however, these large errors can be brought down to a very small amount if the same position of the control point is always chosen for the process of relative orientation.

B.5 Effect of Distortion on Graphical Methods of Plotting:

In most graphical methods of plotting, the effect of lens distortion is usually neglected. This is well as far as planimetry is concerned but when elevations are measured this effect is more marked.

In graphical methods, the distortion correction can be effected as follows: The distortion pattern can be considered as lines of equal amount of distortion in the shape of concentric ends. These can be placed down on transparent material and laid on the photograph which is used for plotting. The corrections can be read for an image in terms of X and Y co-ordinates. This method will give better results as compared with the case where no correction is made at all.

Radial lens distortion causes no errors in Radial Plotting or graphic triangulation as the radial assumption is that an angle subtended by two images at the principal point on the photograph is equal to the horizontal angle subtended by the corresponding objects on the ground. This is true when the photographs are exactly vertical, because then the image displacement radiates from the nadir point. But if the photograph has a tilt, then the image displacement is radial not from the nadir point, but from the isocentre. The error, arising from a tilt less than 3^{o} is considered negligible because then the principal point, the nadir point and the isocentre can be regarded as coincident; the error due to distortion will come in this limit and so it is also neglected

The error due to distortion is usually small compared to the error that caused by the tilt. For the radial plot, these errors may not be serious, but when the same photographs are used for compilation of topographic map, these errors, do cause discrepancy in the final map positions.

In the foregoing chapter, we have seen how distortion affects the photogrammetric operations. The processes of Interior and Relative Orientations are two processes which are performed successively and an error in the process of Interior orientation will effect the process of Relative Orientation. Further the stereoscopic model is affected and finally the accuracy of the result. It is evident from this how important it is to eliminate distortion.

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CHAPTER - II

SOURCES OF DISTORTION

In this chapter, we shall deal with the sources which contribute towards the introduction of distortion in the aerial photograph.

The following are the sources of distortion:-

1. Lens distortion
2. Effect of Temperature
3. Refraction and Curvature of Earth
4. Unflatness of Negative Plane.

1. LENS DISTORTION

The ray PO from an object P passing through the projectbn centre O should continue straight. In an aerial camera it passes through the camera lens which has a certain distortion. Due to this the ray is deviated at O and it strikes the image plane at P' instead of at P'' (Fig.1)

The radial displacement in the image plan is Δ. This displacement is known as Lens Distortion. The angle of distortion is Δ_2.

1.(a) Methods of defining distortion and their Comparison:

1.(a-1) The Lens Manufacturer's Method:

The usual definition of distortion Δ is

$$\Delta = x - f\tan\alpha$$

This type of definition is called the "Lens Manufacturer's formula"

1.(a-2) The French Method:

At the "Institute Geographique Nationale" (France) the distortion is considered as a variation Δf in the focal focal length f.

This type $f = x\cot\alpha$ or $x = f\tan\alpha$ is ued in France and Sweden.

1.(a-3) The Ministry of Supply Formula:

The"Ministry of Supply (Britain) Formula" gives the distortion as $d = \frac{\Delta x}{x}$

where $\Delta x = x - f\tan\alpha$

We shall now deal with the three methods in detail:

1.(a-1) METHOD (1)

In this formula f is called the calibrated Focal length and is defined as that value of the focal length (principal distance) which is so chosen as to eliminate the distortion at a selected annular zone in the photograph.

Fig.2 shows a distortion curve based on the calibrated focal length.

To obtain the data required to plot this curve it is required to assume a focal length and consider the distortion as the difference between the displacement actually measured on the photograph taken through the lens and the displacement resulting in a distortion-free photograph of this focal length. Thus this arbitrarily assured value of the focal length determines the displacement or distortion (3)

The question arising out of this consideration is what is the value of the focal length that should be selected?

In Fig.2, the displacement is negative until about 15^0 and then it changes sign becoming positive upto 45^0 and then again it is negative. (If in case, another value is chosen for the calibrated focal length* the nature of the curve would entirely be different). This suggests that images are displaced toward the axis in the region from 0^0 to 15^0, and displacements is outward between 15^0 and 45^0 etc. Also since the curve touches the X axis at 0^0, 15^0 and 45^0, it might be assumed that at these three values of α , the scale of the photograph is the same. Such is not the case (5)

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* Called calibrated Principal distance in England.

Also the calibrated focal length does not give any further information than that given by the function $r/\tan\alpha$ The selection of the Calibrated Focal Length is rather arbitrary and also it leads one to assume that the lens distortion can be changed by changing the Calibrated Focal Length.

It is evident that unless something physical in the lens is changed, the behaviour of the lens is constant and cannot be changed.

Also, it is assumed that by changing Calibrated Focal Length, a photograph can be made suitable for use in a given plotting process.*

From the point of view of distortion, an ideal lens, is one in which the displacement X in the ~~exter~~ focal plane is equal to $f\tan\alpha$ where α is the external angle. The departure from this ideal condition is a measure of distortion. The formula $\Delta = x - f\tan\alpha$ is thus simple to understand by a layman and as such has a commercial advantage. Otherwise the formula has nothing to recommend it.

1.(a-2) <u>METHOD</u> (2) <u>Formula of the Institute Geographique Nationale</u>:

In this method distances X are measured from the Principal

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* For further remarks, reference may be made to the original Papers of E.H.Thompson [3].

point to points having known angles α. The measurement is done on any arbitrarily chosen plane, but usually on the plane of best average definition

Then $f = \frac{x}{\tan\alpha}$ (fig. 7)

for different measurements of X at different angles α, we get a number of values for f. In a distortion free lens f should have constant values for all the number of measurements. The departure from constancy will then be a measure of the distortion.

In Fig.8, a distortion curve based on this method is shown (5).

The method uses only measured quantities X and α and no assumption is made about the focal length. The Principal point and the Principal distance can be defined using the above quantities X and α while avoiding the definition of distortion in terms of the perpendicular from the vertex which cannot be recovered in a real camera. The Principal point will be the origin for which the function X/tan α will be a constant, the value of this constant being the principal distance. The variation in the values of function f (x/tan α) is then a measure of the departure of the image from a true perspective representation and can be called the distortion(4)

The curve of Fig.8, shows that the scales at 15° and 43°

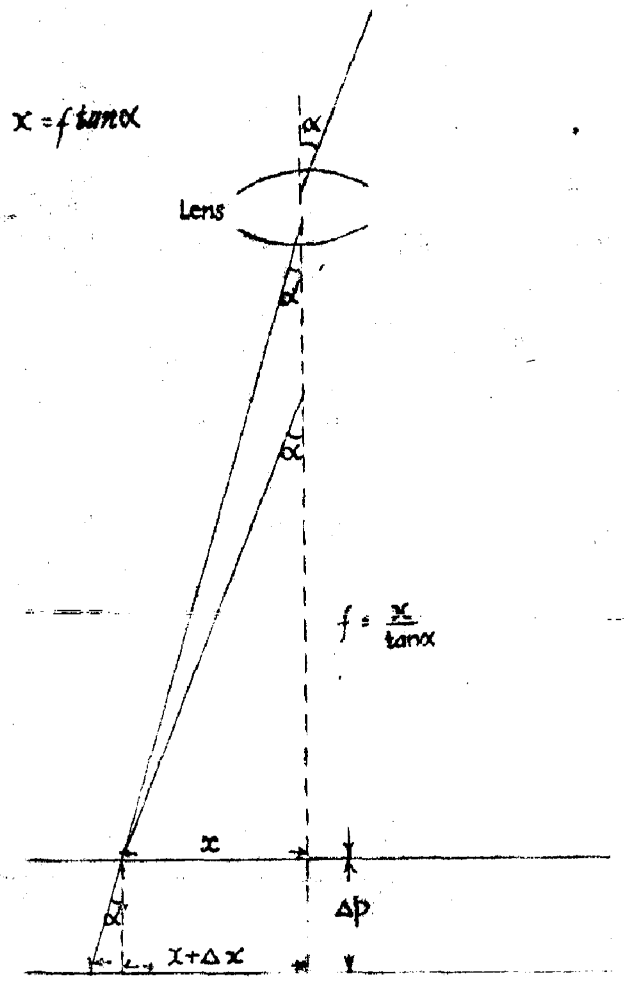

fig. 7

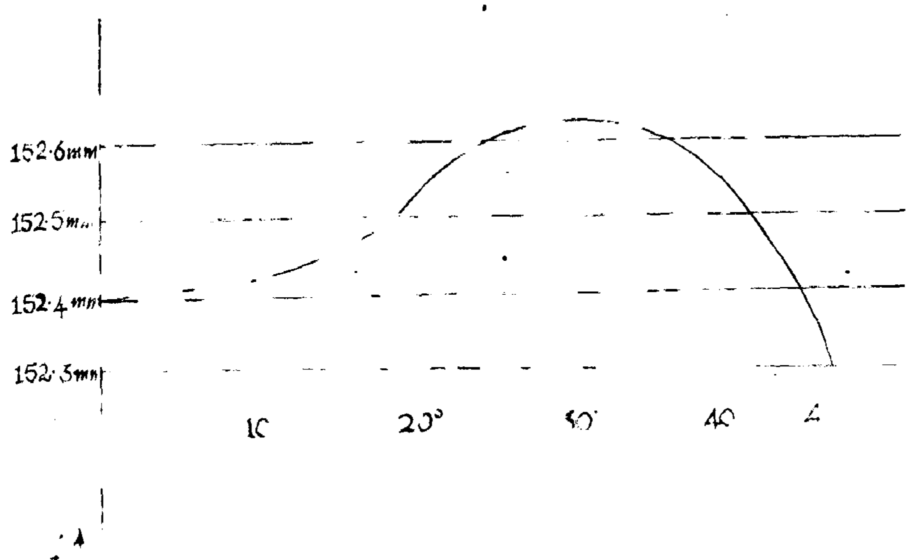

fig. 8 . Variation of EFL. with angle α .

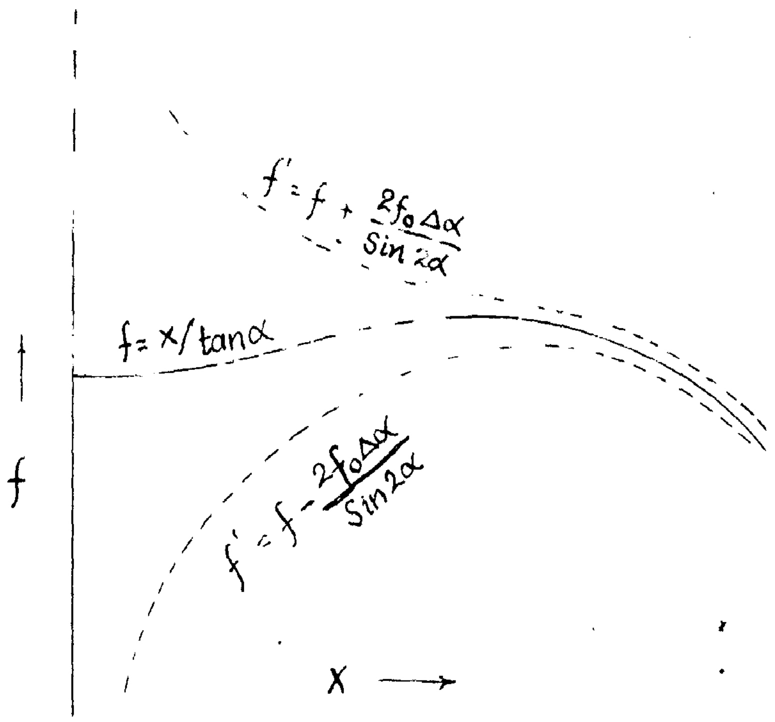

fig. 9

are equal. The focal length or scale increases upto about 32^0 and then decreases upto 45^0. There is no real displacement of images toward the axis in the region of 0^0 to 15^0.

Suppose in Fig.7, the arbitrary plane is shifted along the axis by Δp

Then the corresponding change in x will be

$$\Delta x = \Delta p \tan\alpha'$$

But since α' is very nearly equal to α

$$\Delta x = \Delta p \tan\alpha$$

If Δf is the change resulting in f, then

$$f + \Delta f = \frac{x + \Delta x}{\tan\alpha}$$

$$= \frac{x + \Delta p \tan\alpha}{\tan\alpha}$$

$$= \frac{x}{\tan\alpha} + \Delta p$$

$$= f + \Delta p$$

Hence the effect of changing plane of measurement by a small amount Δp is to simply add an equal amount to all the f.

Thus the form of the distortion curve, f against x, remains unchanged.

Choosing of suitable lenses from the point of view of distortion, is easy when based on this formula.

If a small departure $\pm\Delta\alpha$ is allowable in the direction of a ray from the direction α' given by another lens or by "typical lens".

Then $x = f_0 \tan\alpha$

where f_0 is some mean value of the principal distances

$$\Delta x = f_0 \cdot \sec^2\alpha \cdot \Delta\alpha$$

$$f' = f + \Delta f = \frac{x \pm \Delta x}{\tan\alpha}$$

$$= \frac{x}{\tan\alpha} \pm f_0 \cdot \Delta\alpha \cdot \frac{\sec^2\alpha}{\tan\alpha}$$

$$\text{ie, } f' = \frac{x}{\tan\alpha} \pm \frac{2f_0 \cdot \Delta\alpha}{\sin 2\alpha}$$

The two curves of tolerance of f' against x are shown in fig.9(4)

The full line gives the curve of the typical lens and the dashed lines give the upper and lower tolerances.

If the curve of a lens can be fitted between the two tolerance curves, always allowing a displacement in the f direction if necessary, then from a geometrical view point the lens is acceptable. Thus we know that, it is possible to bring the curve of the lens within a tolerable distance of the typical curve by a suitable movement of the plate on which measurements are done, from the plane of best definition. Also we can judge from an optical point of view, whether this shift could be allowed i.e., whether this shift is within the depth of focus of the lens.

1.(a-3)(2) $_0$ Ministry of Supply Formula:

The distortion is given as $d = \frac{\Delta x}{x}$

where $\Delta x = x - f\tan\alpha$

Here the quanity f is an arbitrarily chosen value as in Method (1) and is taken somewhere near the calibrated principal distance.

If as before Δp is a small change, then

$$d + \Delta d = \frac{x + \Delta p\tan\alpha - f\tan\alpha}{x} = \frac{\Delta x}{x} + \frac{\Delta p\tan\alpha}{x} \quad (\text{approx})$$

since the $\frac{\Delta p\tan\alpha}{x}$ is small, we can put $\frac{\tan\alpha}{x} = \frac{1}{f}$

thus $d + \Delta d = \frac{\Delta x}{x} + \frac{\Delta p}{f} = d + \frac{\Delta p}{f}$

Hence the curve, d against x, remains unchanged.

The disadvantage of the method, compared to the Method(3) is that values of d are of no use in themselves for deciding whether a certain lens can be used to give results within the required tolerance.

The use of a formula which involves an arbitrary function f leads to confusion when comparing one lens with another. A formula which is satisfactory for the above, should contain only such variables which can be measured physically in a particular lens-plate combination. Also the change of that physical quantity should be as direct as possible. Of the three formulae discussed above, the French method ($x = f \tan\alpha$) above fulfils the two conditions.

Thus this method opens the way to a universal calibration specification. At present the results got are not on a comparable basis, since for the same lens differing results are obtained from different methods.

B. LENS DISTORTIONS COMPARED TO OTHER ABERRATIONS:

Before dealing with the factors contributing towards lens distortion, we should see whether the other aberrations like Spherical aberration etc., are serious compared to lens distortion. But this verification is confined only as far as our requirements are concerned.

All photographic lenses have inherent defects in them like Spherical aberration, Chromatic aberration, astigmatism and distortion etc.

But however, the above aberrations in the lens with the exception of distortion impair the definition of the photograph only. On the other hand, if the lens has distortion, the images formed will not be a mathematical, central perspective. It is because the lens exhibiting distortion,

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For further information and discussion on excellent paper by Prof.E.H.Thompson (4) can be referred.

will focus the images at different focal lengths and thus has consequently different image scales. The images of all straight lines which do not pass through the optical axis are curved lines.

Thus it is seen that distortion affects the measured values whereas the other aberrations only impair the definition of the photography.

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The following are the items contributing towards lens distortion.

C. LIMITATIONS IN THE MANUFACTURING PROCESS OF THE LENS:

Any type of lens will have a distortion curve according to its design. This can be called the theoretical curve of distortion. But when a lens is tested after it is manufactured it has been found that the distortion curve will be different from the theoretical one. The difference in the theoretical and practical distortions is caused by the inevitable limitations in the manufacturing process of the lens like incorrect centering of the various optical parts, deviations from the spherical shape of one or more of the component parts etc.

"Excessively steep surface curvatures are not conducive to good optical performance, at acceptable lens speeds. Thee should be as weak refractions as possible and aberrations being compensated by a suitable correcting means through a favourable distribution. This should be followed by a readjustment of all parameters to produce optimum system performance. If steep curvatures are provided in the design often one finds that even the most careful balancing of aberrations will not produce acceptable performance (6).

Since the Metrogon lens has steep surface curvatures a very careful balancing of residual aberrations is of utmost importance even for minimum performance. The strong refractions of the chief rays indicate that the residual distortion is likely to be appreciable.

The strong refractions of the chief rays in the Metrogon and allied designs like the Topogon etc., lead to a zonal term in the distortion curve. But in the Aviogon, though the distortion curve may have a zonal term, yet it is of such small magnitude that it produces negligible residual distortion.

The recent designs of wide angle lenses which aim at complete elimination of distortion, have resulted in bulkier lenses having more number of optical parts which result in difficulties of realising and keeping under service conditions of use the centring of the numerous elements. Incorrect centring of the various parts will result in "Tangential Distortion" and also the distortion curves obtained over a number of radial banks differ i.e., unsymmetrical distortion with respect to the radial centre will result. Further, with increasing number of elements in the lens, reflection losses also increase. The more the number of glass-air surfaces the more is the loss. Also the brilliance of the images will be reduced due to the internal reflection.

Fig.10 shows distortion obtained over 4 radial banks in a Williamson Ross lens and Fig.12 shows that of a Aviogon lens over 8 radial banks. Both cases show some unsymmetrical distortion (7).

For lenses of symmetrical distortions, the compensation is based on the representative curve for that lens. But in case of lenses showing unsymmetrical distortion, the compensating device is to be based on an average or represantative curve since this will result in minimum of residual errors. Figs.11 and 13 give the standard diviation (7) ie. the graphs provide the mean square erros for each angle of incidance when the aveage curve is used for the compensating divice.

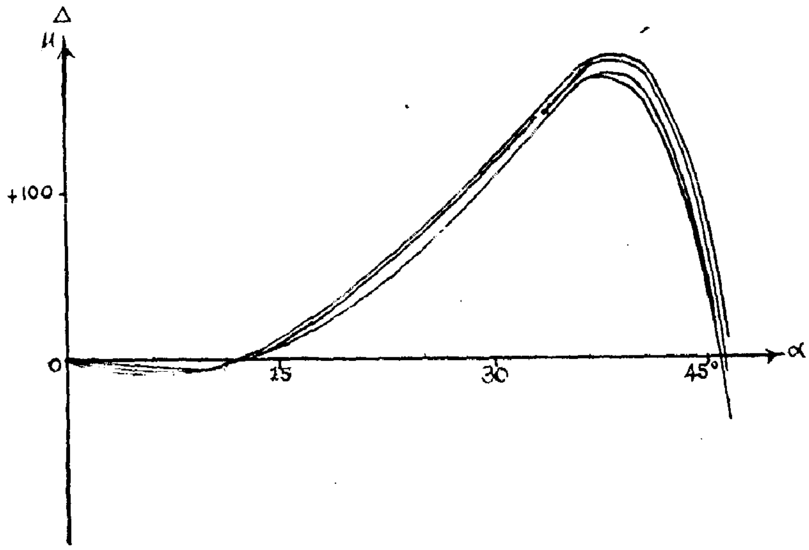

fig. 10 Distortion of 4 radial banks in Ross lens No. 66405
f = 6 inches.

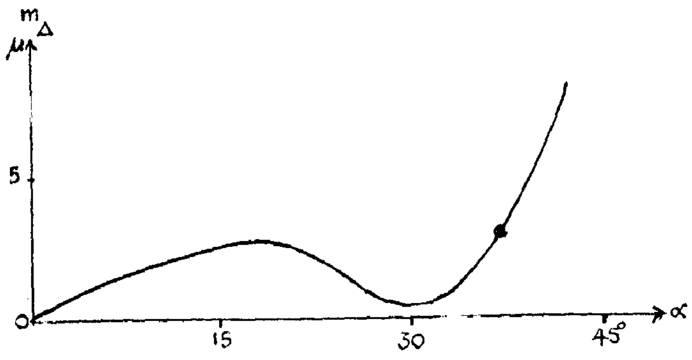

fig. 11. Standard deviation for fig 10.

Any lens should be carefully checked for any unsymmetrical distortion before it is used for a precise mapping work since errors due to unsymmetrical distortion cannot be eliminated precisely and easily. For elimination of unsymmetrical distortion a correction plate can be ground manually with respect to the average curve of distortion. But this is a very difficult process and it is best to have a lens without unsymmetrical distortion.

The distortion curve of each lens in a batch of lenses of the same type differ. In fig.14, distortion curves of 3 Ross 6" f/5.6 lenses of the particular series are shown. The standard deviation of each curve with respect to the average curve is plotted for each angle of incidence in fig.15 (7).

This difference in distortion properties between lenses of two types or batches results because it is very difficult to control the quality of the optical glass from batch to batch. It is to be found out from individual cases whether a single compensation device could be used with photos taken with lenses of different production series.

Fig.15 makes it clear that the residual error is $\pm 5\mu$ for all photos using a single compensating device and

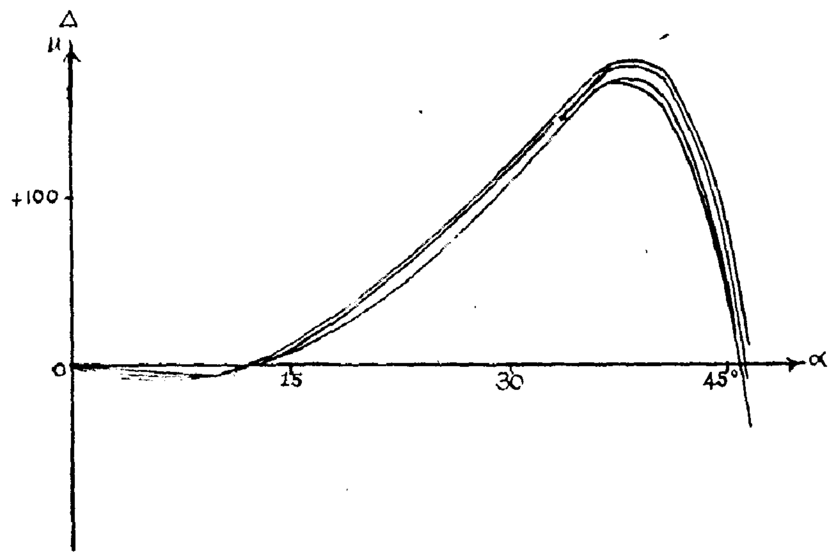

fig. 10 Distortion of 4 radial banks in Ross lens No. 66405
f = 6 inches

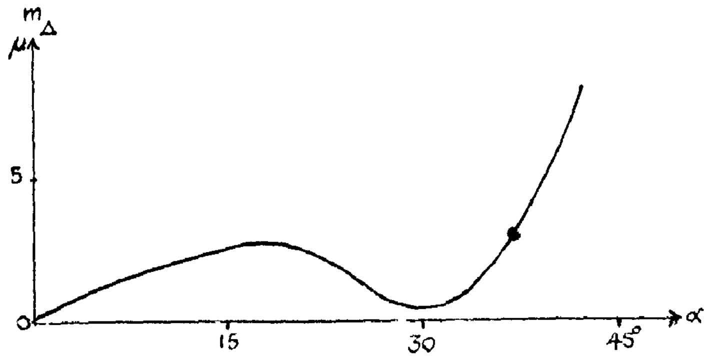

fig. 11. Standard deviation for fig 10.

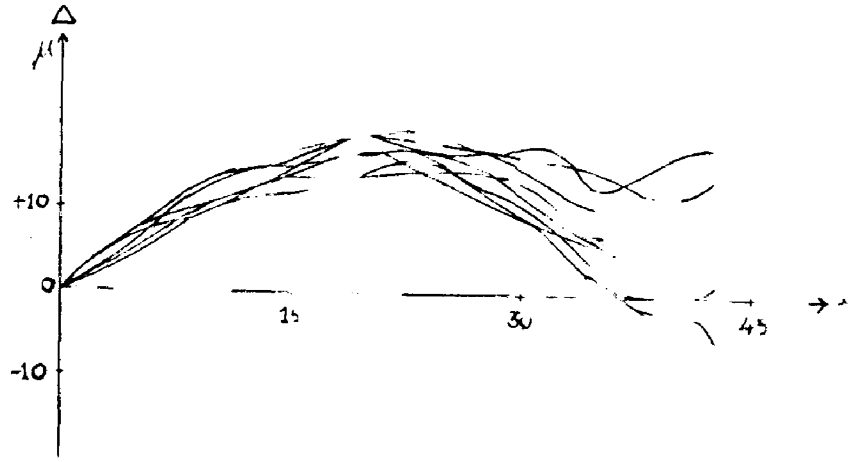

fig 12 Distortion of 8 radial bunks in an Aviogon lens f = 115 mm.

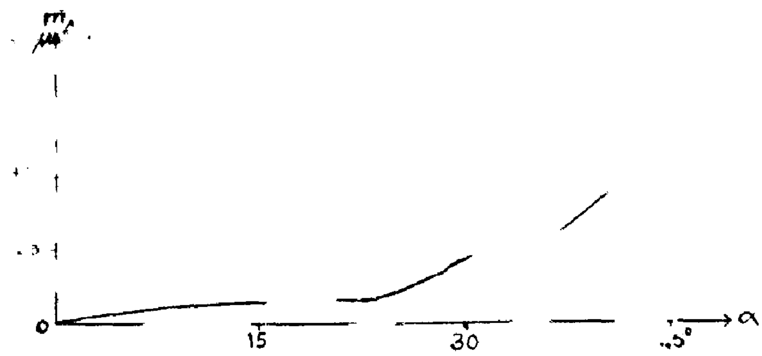

fig 13 standard deviation for fig 12

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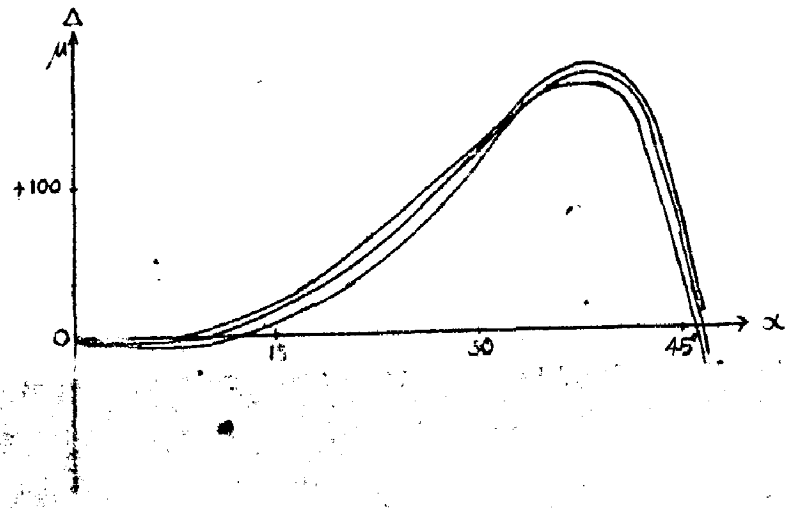
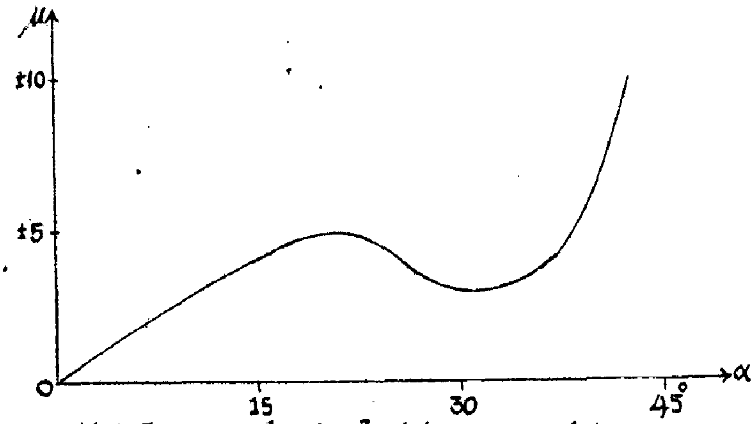


fig 14 Distortion of 9 Ross 6 inch $f = 5{\cdot}6$ lenses from one production series



fig·15. Standard deviation for fig·14.

increases only in the last end between 40^0 to $42^030'$ angle of incidence to $\pm 10\mu$. The points used for relative orientation and aerial triangulation lie in this region and thus suffer a residual error due to distortion of $\pm 10\mu$.

Sewell (8) has tested 276 Metrogon lenses. It was found that with modern stereoplotting instruments, uncompensated distortion in excess of 10 microns will cause noticeable errors in final map positions and elevations.

All of the 276 lenses' distortion curves were plotted with special attention being given to their shapes between 40 and 45 degrees since the most important points selected for orientation purposes lie in this region. During this investigation the average curve of the 276 distortion curves was determined by averaging the distortions for each of the collimator positions. Around this plotted curve an envelope of ± 10 microns was placed (see fig.16) except at $42\frac{1}{2}^0$ where it was made zero and at 45^0 where a spread of ± 20 microns was allowed.

It was desired that all the 276 lenses fit within this envelope. On comparing the 276 curves it was found that only 48% of the total curves fell within the envelope. 15.6% of curves had one point outside the envelope, 13.0%

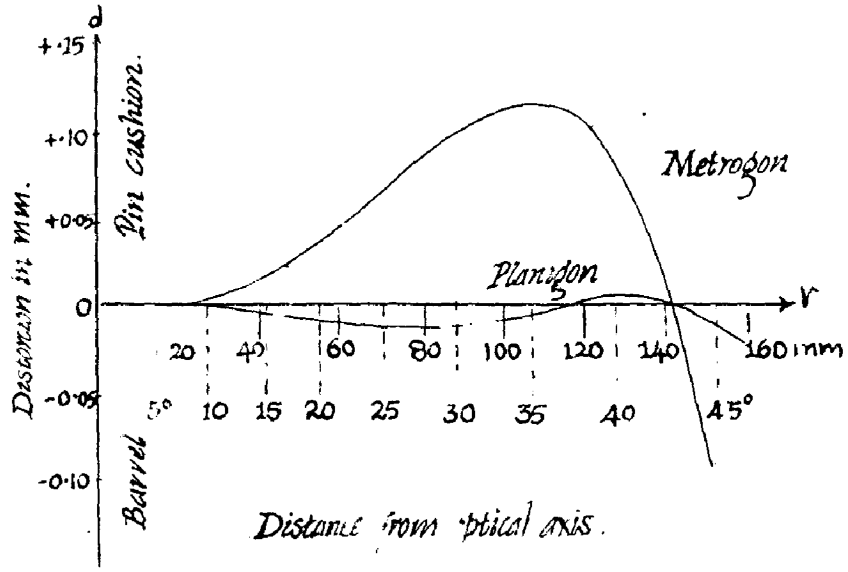


fig 16a. Nominal distortion curves for Metrogon and Planigon lenses

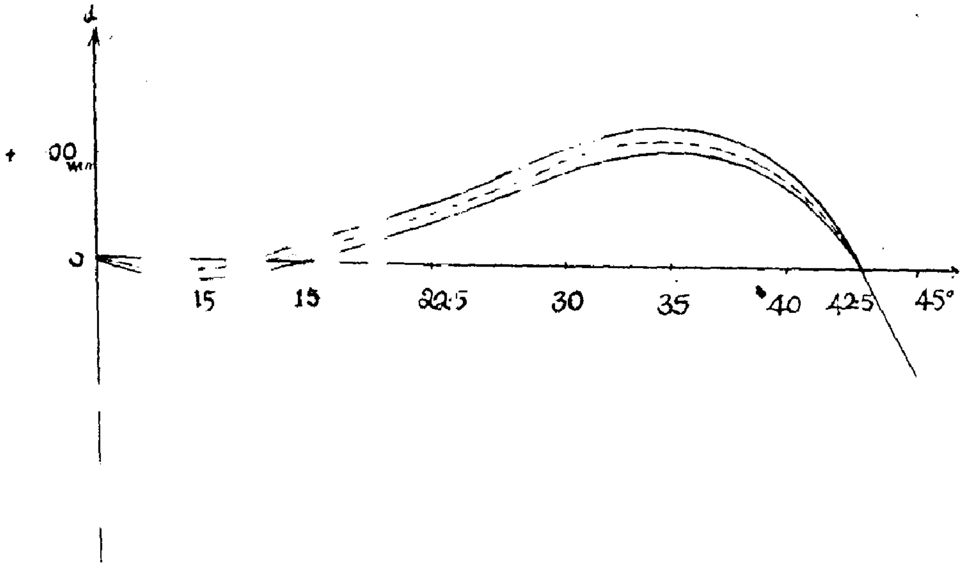


fig. 16b. +0·010 mm distortion Envelope.

had two points outside, 6.9% had three points, 12.7% had 4 points and 3.6% had 5 points outside the ±10μ envelope. A large number of points which fell outside the envelope were from 22 cameras which had failed to meet the specifications of the camera. Sewell points out that the cause for the excessive distortions was that the shape of the envelope around the characteristic curve was arbitrarily chosen. The standard deviation of each curve with respect to the average of the 276 curves was found to be between ±8μ to ±12μ with a maximum spread of 50μ . It was found that within the ±10μ belt 3 different compensating device, are necessary for elimination of distortion of all the 276 lenses within a tolerance of ±10 microns.

The specification for the new planigon lens states"that radial distortion shall not exceed ±20 microns". Though it was originally thought that no compensating device is needed for this lens, since the new plotting machines are capable of measuring closer to 15 to 20μ distortion, a compensating device has been suggested for the planigon lens. In fact the distortion specification itself makes it so. Fig.17 shows the distortion curves of some Planigon lenses. Fig.18 gives the standard deviation of each curve with respect to the average of the 13 curves. The squared mean

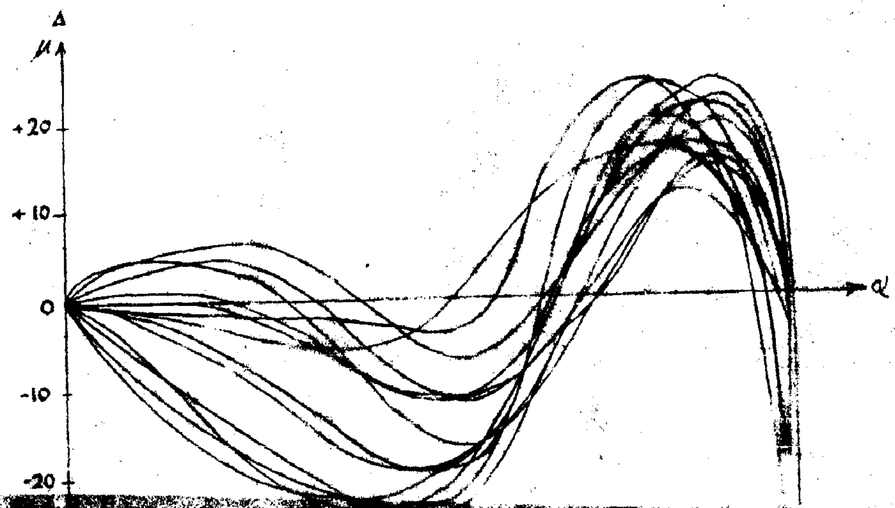


fig [illegible]

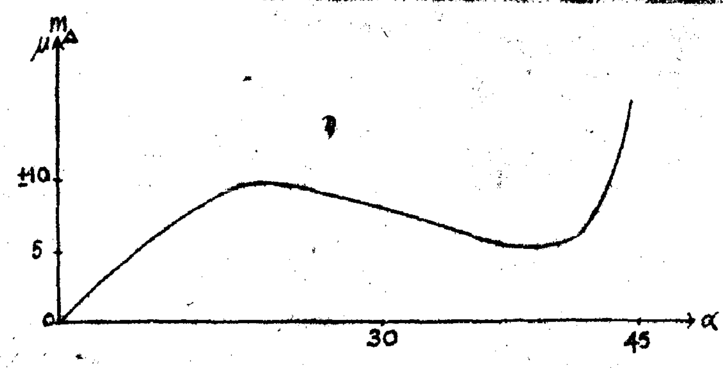


fig 18. Standard deviation for fig 17.

of the standard deviation is $\pm 7.5\mu$ (7).

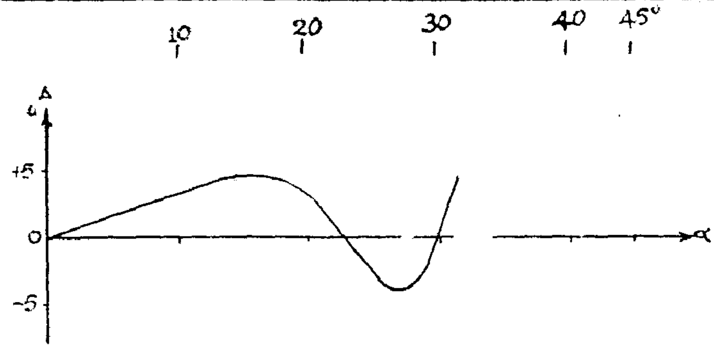
Some data regarding zeiss lenses Topar and Pleogon have been recorded from (7).

The following data shows the squared mean Δ_m of the distortion and the squared mean m_Δ of the standard deviation of some lenses with respect to the average curves.

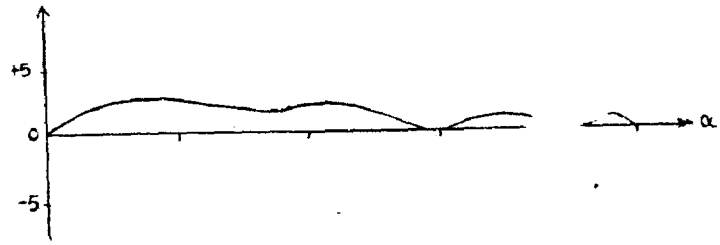
			Δ_m	m_Δ
Topar	f	210 mm	$\pm 4\mu$	$\pm 2\mu$
Pleogon	f	115 mm	$\pm 3\mu$	$\pm 3\mu$
Pleogon	f	6 ins.	$\pm 3\mu$	$\pm 2\mu$

Fig.19 shows the distortion curves of the 3 lenses quoted above.

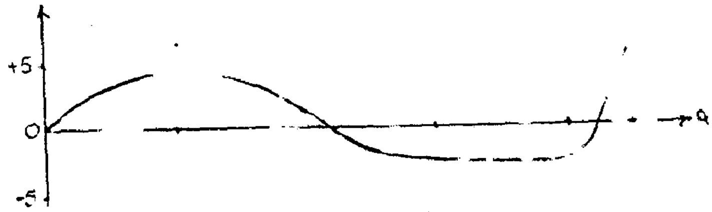
The presence of a prism in a lens camera combination arising either from a filter of nonparallel surfaces or from small errors in the centering of the different elements of the lens is evidenced by a displacement of the principal point of autocollimation from the true principal point and the appearance of Asymmetric distortion values. By this prism effect, the values of focal length and distortion are affected. The asymmetric distortion affects the relative location of images in the image plane even as does the



Distortion of Topar, f = 210 mm



Distortion of Pleogon, f = 115 mm



Distortion of Pleogon, f = 150 mm.

fig 19

distortion inherent in the lens arising from lens abberrations.

Washer (9) states that in a distortion-free lens, an effective prism angle of 30 minutes may produce an error of ±0.003 mms. in the values of the distortion referred to the calibrated focal length. For a lens having no prism effect, the central image (image of central target) and the principal point coincide. However, if the lens suffers from prims effect, the central image is displaced from the principal point by an amount depending on the displacement of the axial ray and the calibrated focal length. The measured values of the distortion will be asymmetric.

Washer (9) has dealt with the subject. He adds that cameras showing excessive amount of prims effect should not be used for precise photogrammetric methods.

In addition to the radial asymmetric distortion produced by the prism effect, tangential distortion is also produced. But it has been found that however, the tangential distortion is small compared to the radial asymmetric distortion.

Tangential distortion is caused by lens elements not perfectly centered. It produces a displacement of a point in a direction at right angles to the radius drawn from the centre of the picture. Modern lens mounting techniques are so accurate that exact centering of the lens elements within the specified distortion of 10 microns has been possible. Because of these low distortions and the random nature, it is not usually compensated for during compilation work. However this tangential distortion is no physical property of the lens, but is just a manufacturing error. There is, so far, no means to correct this error and a high quality lens which shows tangential distortion should not be accepted for use in precise mapping or aerial triangulation.

1.(d) DISTORTION FROM CAMERA CALIBRATION

The purpose of calibrating a photogrammetric camera is to provide essential information like the Distortion characteristics, the position of principal point and resolution etc. to the user of the camera.

The calibration of a camera is mainly done in two methods:

1. Visual method
2. Photographic method (a) Field method
   (b) Collimator method.

The first method is mostly used in Europe while the photographic method is used in the U.S.A. and Canada (7).

Recently it has been found out that results of the two methods vary significantly from each other. The Eighth International Congress of Photogrammetry at Stockholm, Sweden, has agreed that the calibration should preferably be done by the Photographic method under conditions approaching those which the camera will encounter in service.

The advantage of the Photographic method is that it brings the camera itself into function while in the visual method only the lens is in function.

The advantage of the visual method is that the angles of incidence can be determined more precisely than in the Photographic method.

Washer(10) has given some values for the standard [illegible]- deviation of the length 'l' and standard deviation of the angle 'α' for the collimator method. He gives $m_l = \pm 3\mu$ and $m_\alpha = \pm 4''$ for his camera calibrator.

Hothmer(7) has given curves of standard deviation based on the above values. Fig. 20 shows the standard deviation The curve n=1 gives values in determining the distortion from a single radial bank while n=4 holds for the case that 4 radial banks are used.

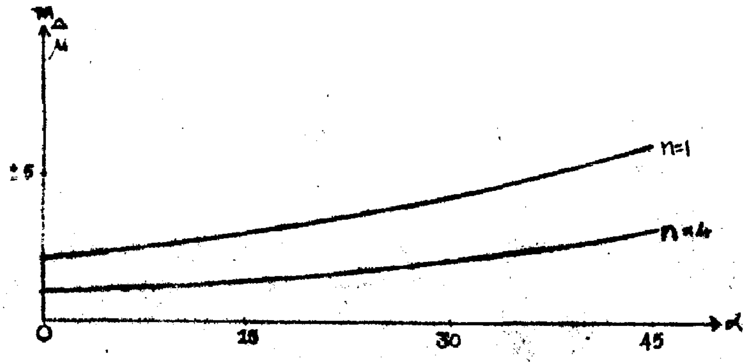


fig 20 Standard deviation in distortion for measurements made with the Camera Calibrator $f = 150$ mm

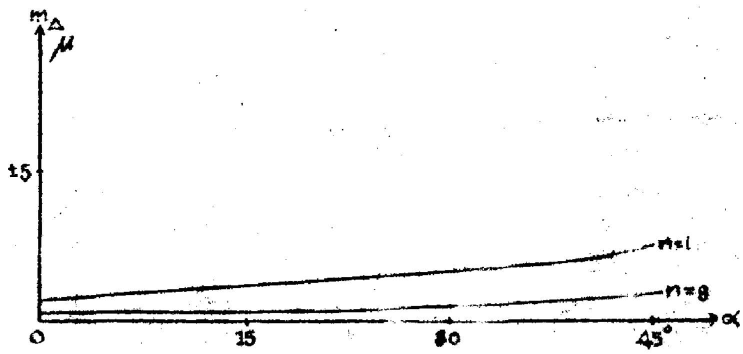


fig 21 Standard deviation in distortion for measurements made with the Goniometer, $f = 150$ mm.

As regards the visual method of calibration, its accuracy has been investigated by Roelofs. He gets $m_{l} = \pm 0.71\mu$ and $m_{\alpha} = \pm 1.7''$ Again the standard deviation, as in fig.21, has been given (7) for n=1 and n=8 the distortion being determined over 8 radial banks.

Until recently the hesitation as regards applying the photographic method was the precision of the angle α the standard deviation m_{α} being as high as $m_{\alpha} = \pm 9''$. Now with the camera calibrator of Washer the accuracy has been increased and the standard deviation $m_{\alpha} = \pm 4''$ provided that n=4. That means to get such an accuracy a measurement should be carried out 4 times on one radical bank or measurements be done once on each of the 4 radial banks assuming the distortion to be symmetrical with respect to the radial centre. This is mever true due to the inherent limitations in the manufacturing process.

Any test should be carried out under the same conditions as the object tested would be used in practice. This principle applies to camera calibration also. In practice the aerial camera is used with a minus blue (yellow) filter. Hence the aerial camera should be calibrated Photographically with a minus blue filter and also under the same conditions of light as far as possible. The collimator method of calibration with the recent accuracy in determining the angles of incidence ($m_{\alpha} = \pm 4''$) comes upto this demand. In many countries the visual method is still used as it allows a more accurate determination of the angle α.

Carman and Brown (11) of the National Research Council of Canada have investigated the difference between the Visual and Photographic methods of Calibration for a Wild Aviogon lens.

Photographic calibration data of Wild RC5a cameras tested at the National Research Council of Canada was found to be differing constantly from the Visual Calibration data furnished by the manufacturer. Both the calibrated focal length and distortion showed small but persistant discrepancies. The photographic value of the calibrated focal length usually exceeded the manufacturers (Visual) values by 0.01 to 0.02 mm. In case of the radial distortion the maximum difference was of the order of 10 to 17 microns, averaging 5 or 6 microns higher than the manufacturer's data. The light used in the visual method has a mean wavelength of 550 mμ whereas the effective light in the photographic method including the minus the blue filter has a mean wavelength of 605 mμ. Fig.22 shows the difference between the two methods.

This difference between the two methods is attributed to the chromatic difference of distortion of the lens in different conditions of light. Recently due to these differences the Aviogon has been replaced by the Infra-gon for Infra-red photography. This investigation points out the necessity of calibrating cameras in conditions which are close to the conditions in which they will be used.

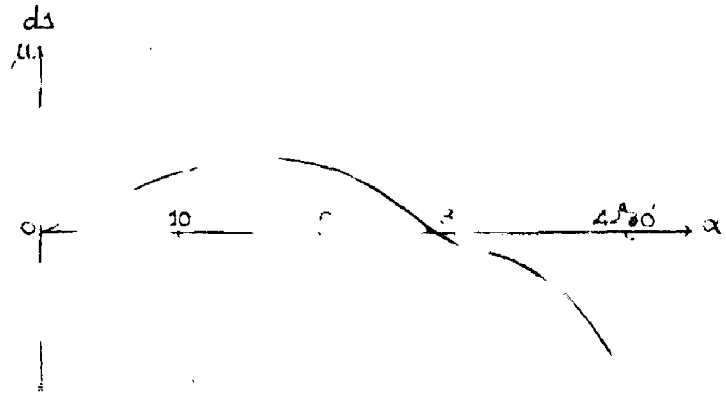


fig. 22. Difference between photographic and visual calibration for an Aviogon, $f = 152.88$ mm

Calibration of cameras are not performed at the same aperture stops of the lens as will be used in practice and further the camera will not be used for photographing at the same aperture stop. Hothmer (7) states that practical results obtained by Roelofs and David seem to confirm that the diaphragm setting has no effect on distortion. An S.O.M. aquilor lens f 125 mmwas calibrated and distortions for f:/6.2 as well as for f:/14 were determined. The maximum difference between the two was not more than 3 microns.

While calibration is done photographically, the emulsion used should have the same colour sensitivity as the emulsion used in practice.

E. METHODS FOR ELIMINATION OF THE LENS DISTORTION:

We shall now deal in this chapter the different methods in vogue for elimination of lens distortion. However, the distortions produced by the other sources dealt in subsequent chapters can also be eliminated by the above methods.

The following are the methods:

E.1. Porro-Koppe method

E.2. Compensation plates

E.3. Use of Aspheric surfaces which have been given the desired distortion.

E.4. A correction to the Plotting Principal Distance.

E.1. PORRO-KOPPE PRINCIPAL:

This principle developed by I Porro, an Italian Geodesist and C.Koppe a German Professor is the oldest method for elimination of the distortion. The bundle of rays in the object space is re-established so that it is congruent to the bundle of rays which produced the image through the aerial camera lens by means of a lens having the same distortion characteristics as the aerial camera lens.

The ray PO (Fig.23) suffered a deviation when passing through the survey camera lens resulting in an image at P'. This ray is re-established in the plotting instrument so that P' is imaged at P.

This method is not without its inherent shortcomings. It is almost impossible to manufacture a lens similar to the aerial camera lens having similar distortion properties. (reference is made to Section II c)

E.2. COMPENSATION PLATES:

This is an optical method for elimination of distortion. The method consists in using a plate ground on one face to the required shape so that it refracts the ray (fig.24) passing through it by the required amount. The other face on which the photograph lies is exactly plane. The curvature is so calculated that a slight additional refraction is given to the bundle of rays.

This method has been used by R.David since 1938 for the Wild Instruments like A5, A7 etc. The plate has the advantage that any desired distortion can be removed by giving the suitable shape at that point.

E.3. USE OF ASPHERIC PLATES WHICH HAVE BEEN GIVEN THE DESIRED DISTORTION

E.3-1. When projection diapositives are produced from aerial negatives any desired distortion can be removed. Fig.25 explains the principle, In the fig.

f_1 stands for the principal distance (C.F.L.)

f_2 stands for the plotting principal distance

f_0 stands for the focal length of the projection lens.

The ray is established when it passes through the aspheric surface and is imaged in its proper place on the projection diapositive. However we have to consider any distortion say $\Delta\varepsilon_{pr}$ that may be present in the projection lens itself. Thus the aspheric plate has to compensate the

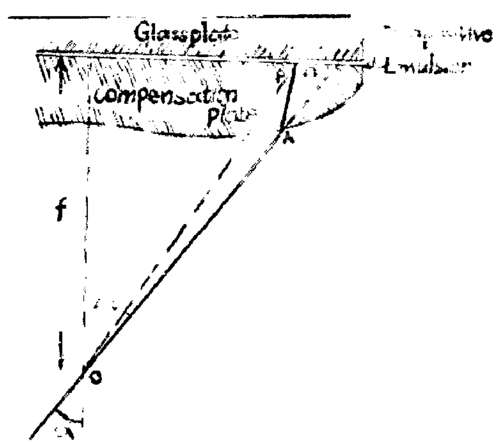


fig. 24. Principle of compensation plates

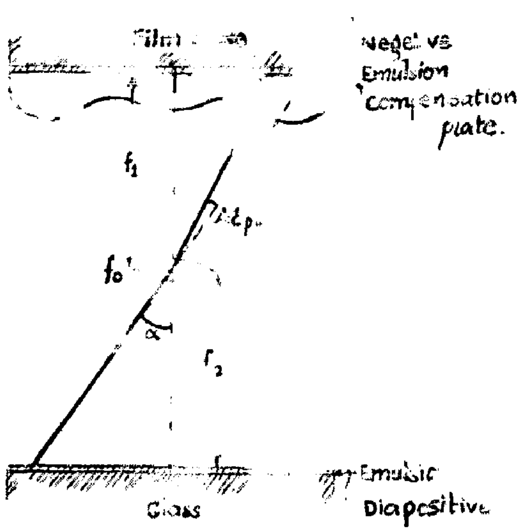


fig. 25. Elimination of the distortion when taking a projected diapositive

combined effect of the distortions due to the aerial camera lens and the projection lens.

This method of removal can be further extended in its application. Any other distortions say due to the unflatness of suction plate, the earth curvature (radial symmetrical deviation) etc., can be removed along with the distortion due to the lenses.

As Hothmer (7) has put it, this probably will be the method for elimination of distortion in the future. Further investigation is necessary regarding the effect of astigmatism etc. of the aspheric element used.

Hothmer (7) has given the standard deviation of the residual distortion m_Δ obtainable with the aspheric plates in a projection printer to be about $m_\Delta = \pm 2\mu \cdots\cdots \pm 3\mu$.

This principle for elimination of distortion has been applied in the recent Wild U3 and Zeiss Reductor.

E.3-2. Tham (12) has suggested distortion free negatives. The pressure plate in the magazine of the camera is to be given such a curve that it directly eliminates the distortion due to the aerial camera lens. Fig.26 illustrates the principle. An experiment conducted by Tham has proved its usefulness. However further investigation is needed as to the standard deviation of the elimination of distortion.

E.3-3. Distortion can also be removed by using two cameras working on the Porro-Koppe principle (7). Fig.27 shows the arrangement. The camera O_1 has the same focal length f_1 as that of the Surrey camera and the lens of O_1 has the same distortion properties as the lens of the aerial camera. Similarly the camera O2 has the same focal length (calibrated focal length), f_2 as that of the projection instrument (eg. Multiplex) and the lens of O_2 having the same distortion properties as the lens of the projection instrument. The combined effect will reconstruct the bundle of rays as they exist if there is no distortion in the aerial camera lens. The application of two cameras working on Porro-Koppe principle has been used by Nistri in his Printer-rectifier.

E.4. A CORRECTION TO THE PLOTTING PRINCIPAL DISTANCE:

The distortion can be eliminated by a correction Δf to the plotting principal distance. There are two possibilities of doing this (7).

E.4-1. Correction being applied to the image points

E.4-2. Correction being applied to the projection Centre.

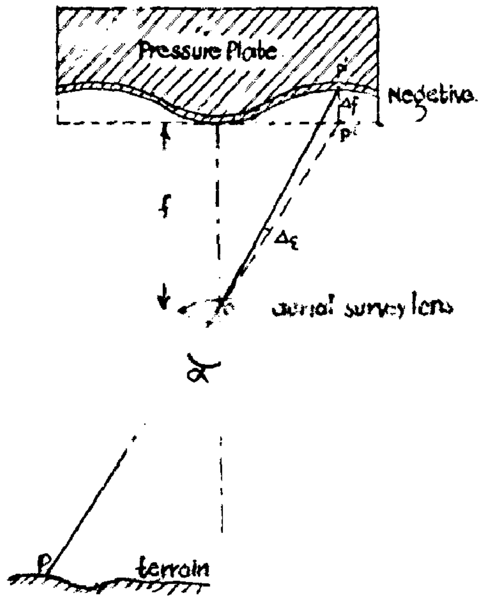


fig. 26 . System for obtaining negatives free of distortion.

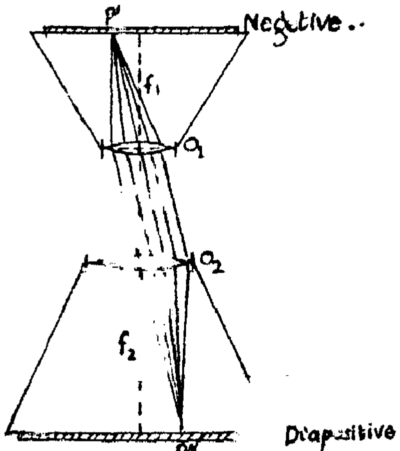


fig. 27. Telescopic printer

E.4-1. If the image point P is replaced to P_2 by a correction Δf_1 to the principal distance f then we have the image point located at the correct distance from the optical axis ($MP'' = M_1P_2$). As a result the light ray will strike the image plane (here displaced) at exactly the same radial distance as if the lens were free from distortion (fig.28).

If Δ is the radial distortion and α the image angle then the Δf correction is given by the equation.

$$\Delta f_1 = \frac{\Delta}{\tan\alpha}$$

E.4-2. The form of the bundle of rays which represent the relationship between the object and the image is determined by the position of the projection ~~with~~ centre O with respect to the ground points. The task of Interior orientation is reconstruction of this bundle of rays with reference to a fixed projection centre. Thus special attention is necessary to the above fact when eliminating the distortion by a correction Δf_c to the projection centre (7).

In fig.28 to correct for the distortion the projection centre O has to be displaced to O' so that the restituted ray passes through P maintaining the relation $PN = Z \tan\alpha$

Suppose now the plotting distance is Z_1 instead of Z, then the correct position of P will be P_1 on the new plane

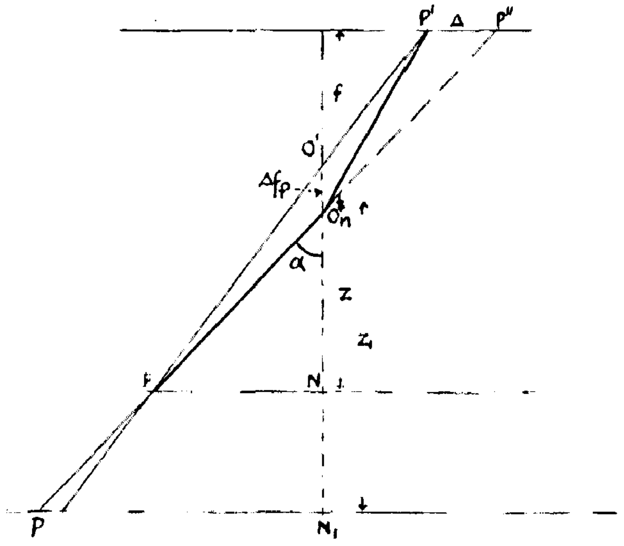


fig. 28

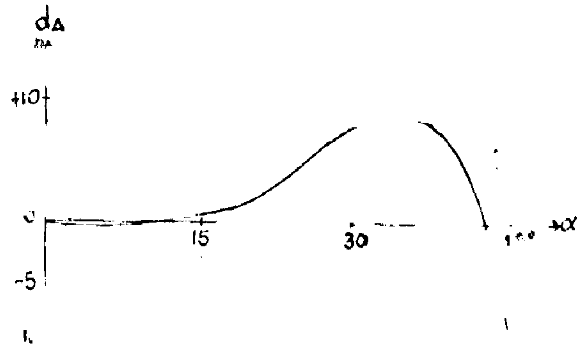


fig 29 Example of an error dΔ which can occur if the distortion is eliminated by a displacement Δf_c of the projection centre. Maximum Values.

such that $P_1N_1 = z_1 \tan \alpha$ But the projection ray P'O' will not strike at P_1. Hence the correction Δf_c to the projection centre holds good for only a certain projection distance.

The equation for Δf_c has been given in (7).

$$\Delta f_c = \frac{z \cdot \Delta}{\tan (f+z)}$$

Photogrammetric instruments now in use have a range of projection distance from $1{\cdot}5 f < z < 3 f$

It is evident that if plotting is done at any other projection distance other than that prescribed errors will be introduced.

If the compensation device has been made for Δf_c for an average projection distance $z_0 = 2{\cdot}25 f$, then the error introduced by using this device at the maximum and minimum projection distances has been given by (7) for the Metrogon distortion.

Fig.29 shows the result. The maximum error in radial distortion is 18μ. The corresponding error for other lenses will be smaller as $d\Delta$ is a linear function of Δ.

2. EFFECT OF TEMPERATURE

Many a good camera on the ground has yielded poor photographs in the air. One of the reasons for this is the effect of temperature on the lens elements.Cameras are usually calibrated in the laboratories at a temperature of about +20°C. The resulting distortion curve is a representative at the laboratory temperature. The ground temperature on a photographic day may be between +30°C to +40°C. While the temperature of the air at the flying height may be as low as -20°C to -40°C. Thus a temperature gradient of as much as 80°C is acting on the aerial camera. This temperature change certainly introduces internal strains in the lens elements. This gradient will take a long time to die away depending on the length of the whole lens. Also the camera takes some time to adopt itself to the environment. During this period we can certainly expect small changes in the calibration data.

The temperature change causes a definite and readily measurable shift in the focal plane. According to (13) this is due to a combination of a physical change in the radii of curvature and the thickness of the lens elements as a result of the thermal expansion. A change in the lens separation distances, change of refractive index etc., are all due to this temperature gradient.

(13) has given a table (Table I from E.B. Woodford and R.N.Nierenberg*) for the measured shift in the flange focus of some lenses for a temperature change of 1°C. Though the focus shift can be compensated, the only solution to battle the ill effects of temperature is to fully thermostat the aerial camera. (For table I refer page 44)

Very little investigation has been done regarding the effect of temperature on distortion. A Topar and a Pleogon were put in a refrigerator for 3 hours at a temperature of -20°C, measurements were then made on them (7). It was found that the variation of distortion was of the order of ±5 microns for both types of lenses.

The thermostating of the camera has been successfuly done by Dr.J.G.Baker in the Harward 40-inch f/5.0 Telephoto lens.

Quite apart from the effects of temperature, pressure difference also has an effect on the lens elements and thereby on the distortion (13). At higher altitudes, the density of the air falls off. The refractive index of the air inside will change leading to a shift in the focal plane. Some values regarding this shift has been given in Table I (vide Page 44).

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*Woodford, E.B., and Nierenberg, R.N., J.Opt. Soc of America Vol.XXXV, 1945.

TABLE - I

Lens	Flange focus shift for 1°C	Shift of focus for 1 mm change in atmospheric pressure
48-in f:6.3(telephoto)	-0.0195 mm	
40-in f:8.0(telephoto)	-0.0334 mm	0.00134 mm
Same lens in Invar Barrel	-0.0177 mm	
36-in f:8.0(telephoto)	-0.0294 mm	
*24-in f:6.0 (Tessar type)	-0.0214 mm	0.00049 mm
*24-in f:6.0 (Tessar type)	-0.0090 mm	0.00043 mm
12-in f:5.0 (Tessar type)	-0.0105 mm	0.00013 mm

* Different designs by different manufacturers.

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Since holding the lens in a chamber maintaining the ground pressure is impractical some means should be found out to keep the pressure constant. In Dr.Baker's lens, the rear element moves automatically to maintain the correct focusing distance.

From the Table-I, it is evident that temperature and pressure have opposite effects thus compensating the effect to a small degree. The data for the 40-in f:8 lens shows at a flight height of 40,000-ft. the pressure drops from 750-mm to 150-mm leading to a shortening of the flange focus of the lens by 0.8-mm. On such conditions the temperature drop will be usually from +25°C to -55°C leading to a lengthening of the flange focus by about 1.5 mm. These values compensate each other to some extent. If the cone is of Aluminium, the temperature difference of 80°C will shrink it by about 0.9 mm making the situation worse, resulting in an error of +1.6 mm in the focus.

In certain survey airplanes, a camera window of glass is used. This usually will be of a thickness of the order of $\frac{1}{4}$-in. A temperature gradient of 60°C to 80°C might cause the window to be bowed to such an extent as to form a lens of the glass sheet causing a shift in the optical image (14). Quite apart from this, internal stresses and strains will be developed in the camera window aggravating the situation. If the glass used is not optically flat, the unevenass will introduce random errors.

The standard windows used in American Military Mapping Aircrafts called a Group M Window is of $\frac{3}{4}$-in thick glass. (15) gives some data about this. The surfaces of the glass are parallel and have a wedge angle of less than 4 seconds and a wedge angle differential of less than 4 seconds of arc. Thus with a glass of refractive index 1.5 a ray of light at 35^0 off the axis will be displaced by an amount as great as 18μ as the focal plane quite apart from the effects of temperature and pressure.

It seems thus the camera window causes more of damage to the optical quality of the photograph for the commercial usage. The distortion from this will lead to a deformation of the plastic model. Though the measuring of the temperatures outside and inside the glass window is not difficult, one is not certain exactly what is being measured.

The difficulty to investigate the problem is that in the laboratories simulating conditions equivalent to that in the air are not easy of realising.

Thus it is felt that for commercial usage it is better to do away with the camera window at the present conditions, though a military mission is bound to use the camera window.

The temperature effect will also be caused due to the aerodynamic heating resulting from high speeds of aircrafts. This causes deterioration of materials as well as distortion characteristics. Information about this is unfortunately lacking.

The foregoing review quite justifies further research work on the effect of temperature on distortion. As was previously said change of distortion characteristics due to this effect is very little known . Modern photogrammetric lenses like the Aviotar, Aviogon, Planigon,Topar, Pleogon etc., should be thoroughly tested in laboratories to find out their behaviour at a range of temperature which acts on them when they are put to use. Photographic surveying is being conducted for scientific research work at the Arctic and Antarctic regions and the extremely low temperatures necessitate thermostating of the cameras. A solution to fight the ill effects of temperature is to use a material, for the various elements of the camera having the same coefficient of thermal expansion as glass.

As seen earlier, apart from temperature, pressure differences also have their effect on distortion. But fortunately they tend to balance slightly the effect of temperature. This subject is still a matter for further investigation. Regarding camera windows, considerable work has been carried out at the University College, London. But the author regrets the nonavailability of the information to him while writing this dissertation.

3. EFFECT OF REFRACTION AND CURVATURE OF EARTH

Between the lens in the aerial camera and the terrain that is being photographed are interposed the Refraction and Curvature of the Earth. These affect the restitution of the object bundle of rays.

The effect of Refraction is to cause displacements of the image points in the direction away from the image centre i.e. to cause a positive distortion.

Ekelund (16) has converted the magnitude of Refraction expressed in angular measure into radial displacements in the plane of the image for various altitudes and a focal length of 150 mms (Fig.30).

The refraction in photogrammetry can be treated similar to the lens distortion. Tham (17) has converted the refraction effect into horizontal and vertical parallaxes by a graphical treatment (See figs.31 and 32). These horizontal parallaxes can be connected into height corrections.

Distortion curves for the Aviogon lens, were plotted using the photographic method (figs.33 and 34) by the United States Geological Survey. The photography was done over a test field area in Ohio, U.S.A. Separate measurements were made on a stereocomparator and a Wild Autograph A7.

These curves were compared with the curves supplied by the Wild Co. for the respective lenses. The curves showed an increasing positive distortion with increasing radii in comparison with the Company's distortion curves, Suggesting the effect of Refraction.

Lejonhufvud (18) has dealt with photogrammetric refraction thoroughly. The maximum height errors due to refraction and curvature are shown in fig.35.

Though the parallaxes caused by refraction are small, it should not be neglected in precise works.

The magnitude of the image movement due to atmospheric refraction becomes more appreciable as the flying height is increased the wide angle photography due to the obliquity of the marginal rays and also due to the increased differential of temperature and pressure.

On some photographic occasions the lower atmosphere may be covered with Haze (very fine particles such as ice crystals, smoke, droplets of water suspended in air). The image forming light rays suffer refraction, reflection and diffraction through this part of the atmosphere due to the Haze. The degree through which the light is scattered depends on the wavelength of the light and the size of the particles.

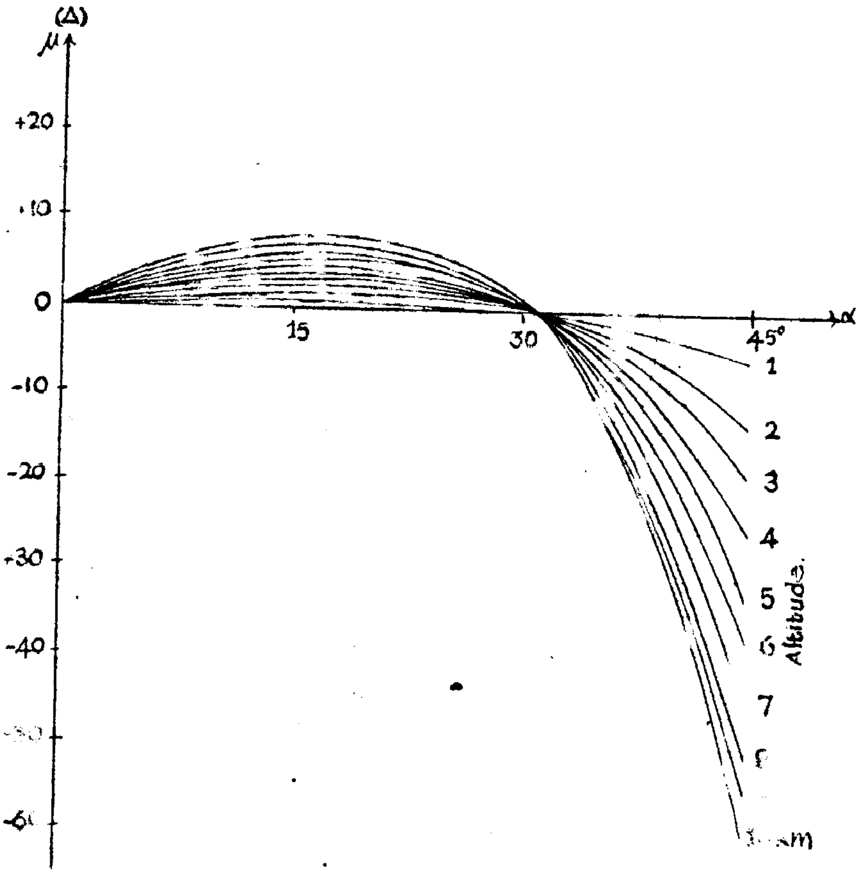


fig. 30. Radial displacement (Δ) caused by earth curvature and refraction, f = 150 mm.

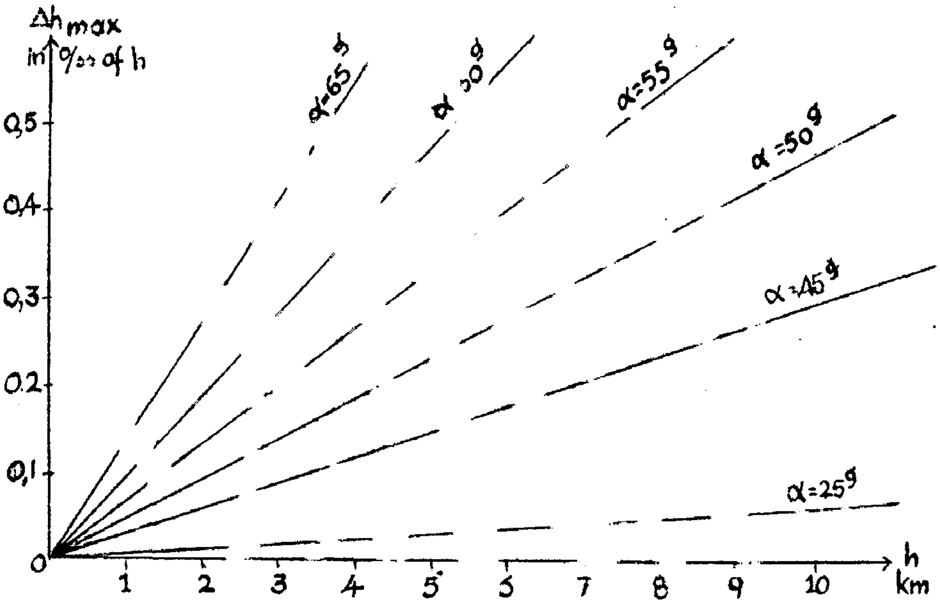


fig. 35 . Maximum height error Δh_{max} in percentage of the flying for different angles α of incident. Δh caused by earth curvature and refraction.

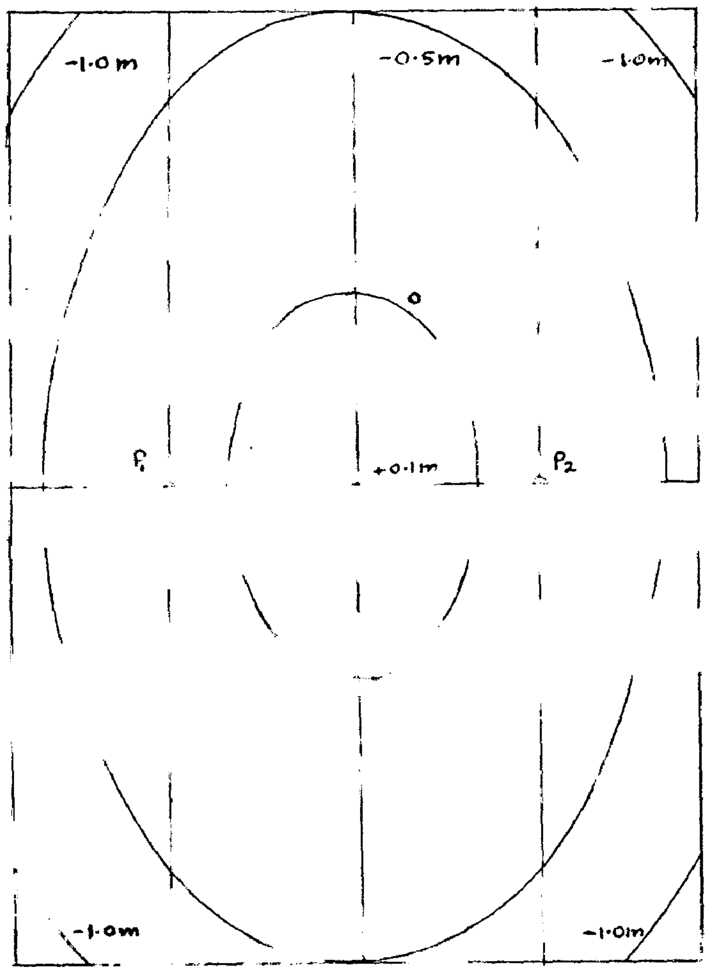


fig. 31. The height [illegible] caused by refraction and curvature. Height of flight 400 m., f 200 mm. overlap = 65%.

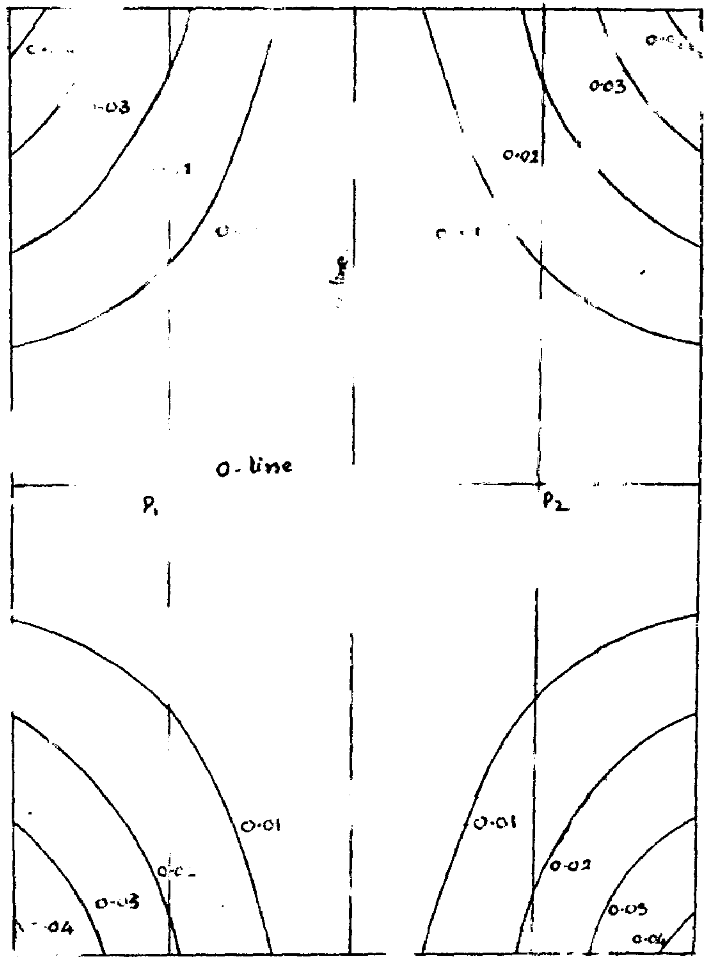


fig. 32. The corresponding Vertical parallax of fig.

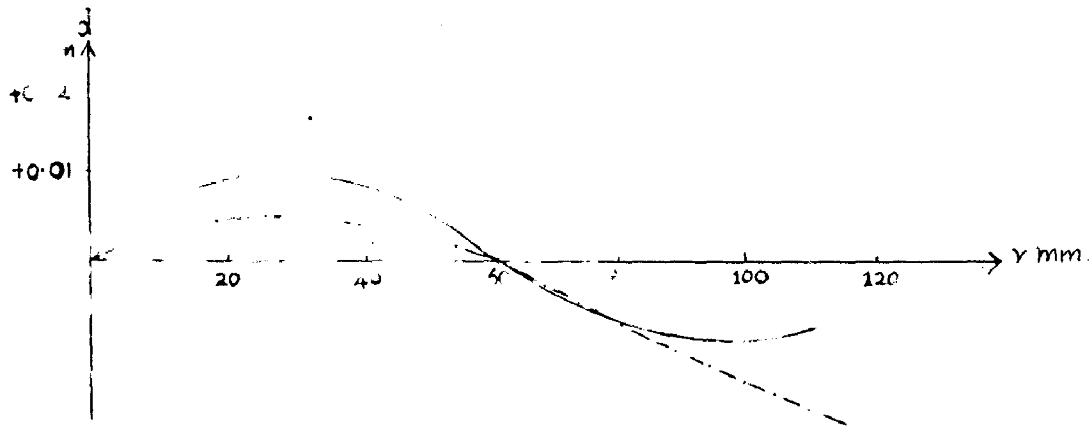


fig 33. From measurements in a Coordinatograph, f = 153·02 mm.

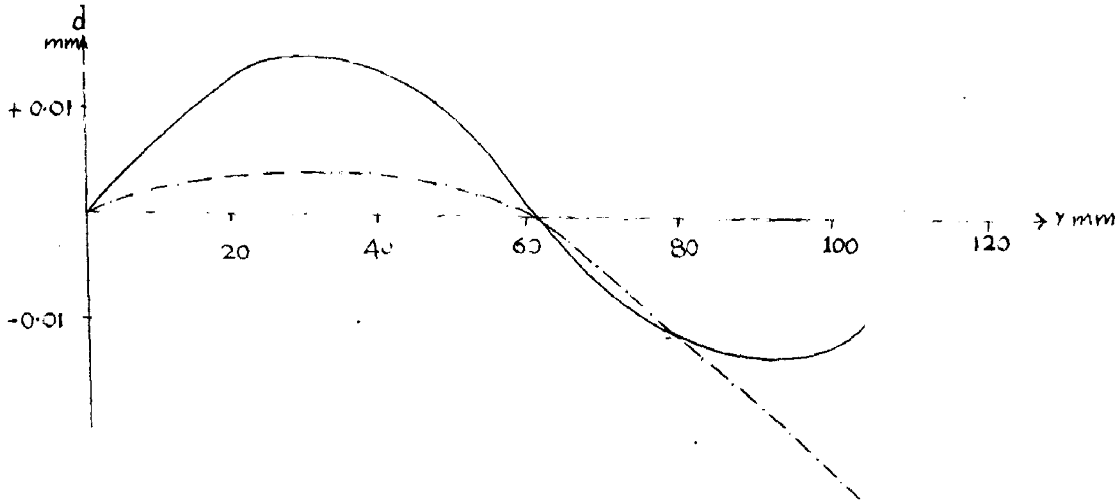


fig. 34. From measurements in Autograph, A-7.

—— Radial distortion in picture.

-·-·- Radial distortion according to Wild Co.

Little is known about the complex behaviour of the atmospheric Haze with regard to its optical properties. To look through the atmospheric Haze, yellow or orange filters are used before the lens to absorb the short wave rays as these are the one scattered more.

If a tangent plane to the earth is placed through the Nadir point the distance 'd' from any point on the surface of the earth to this place can be computed from the following expression (16)

From fig.36

$$OP^2 = OQ^2 + PQ^2$$

$$R^2 = (R-d)^2 + L^2$$

$$R^2 = R^2 + d^2 - 2Rd + L^2$$

d is small and hence d^2 is negligible.

Therefore $d = \frac{L^2}{2R}$

Where R = Radius of the earth

L = Distance of the point from the nadir

The distance d corresponds to a radial displacement x in the tangent plane

$$x = \frac{L}{H} \cdot d = \frac{L^3}{2RH}$$

the distortion corresponding to this displacement in the image plane is given by

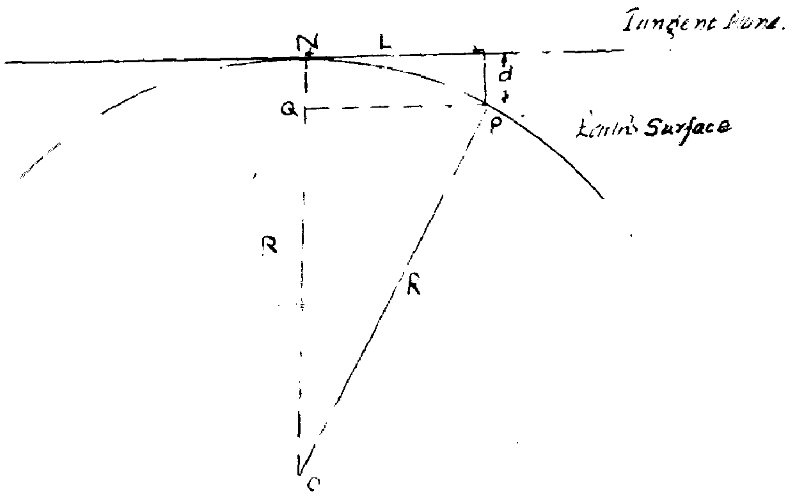


fig.36 .

$$dr = \frac{f}{H} \cdot x \quad \doteq \quad \frac{fL^3}{2RH^2} \quad \text{if} \quad l = \frac{f}{H} \cdot L$$

$$\text{then} \quad dr = \frac{H}{2Rf^2} \cdot l^3$$

Ekelund (16) has given curves for radial displacement for a focal length of 150 mms. and different values of H. Fig.30 shows the combined effect of both the Refraction and curvature of earth. The curvature of earth causes negative distortion i.e. displacement of the image point towards the image centre.

As was shown earlier, distortion plays a very significant role in establishing the object bundle of rays. The Refraction and curvature of Earth both affect this restitution in that they cause systematic radial displacement.

This radial displacement causes height errors as shown in fig.35. The process of elimination of this radial displacement could be suitably combined along with the process of elimination of lens distortion. But then, the compensation device holds good for only a definite flying height.

4. EFFECT OF UNFLATNESS OF THE IMAGE PLANE

This aspect is a chief source of distortion. The process of photography introduces errors at the plotting stage when the object bundle of rays are reconstructed.

There are three items which involve the above aspect.

4. a. Flatness of the Negative Plane at the moment of of Exposure.

This involves two sources:

4. a-1. Insufficient suction of the film at the moment of Exposure

4. a-2. Unflatness of the Suction (pressure) plate

4. b. Dimensional stability of the photographic material

4. c. Flatness of the photo carriers in the Stereoscopic plotting instrument (plotting image plane).

4. a. FLATNESS OF THE NEGATIVE PLANE AT THE MOMENT OF EXPOSURE:

4. a-1: Insufficient Suction of the Film at the moment of exposure:

In view of the high accuracy of the plotting instruments it is very important that the film, in an Aerial Survey Camera using film as a photographic material, should be a perfect plane at the instance of exposure. In such cameras, the film is pressed against a suction plate (locating back) so that it lies flat at the moment of exposure. This is done either by glass pressure plates or air pressure against the film or by maintaining a vacuum.

In America the "Oblique Ray Magnification Principle" is used to find out if the film is plane at the moment of exposure. The principle is that when a highly oblique ray focuses a grid on a flat plane, deviation of that surface from a true plane will distort the image. This has been dealt in detail by (19). If the grid lines are straight and parallel to the edges of the frame, it is evident that the focal plane was in proper contact with the locating back thus presenting a true plane.

A test devised by Worton as quoted by Attwell (14) makes use of a good principle to find out whether the film is plane at the moment of exposure. The principle is to coat the register glass of the camera with a thin film of engineers' blue. While in operation, when the film is pressed against the register glass by vacuum, the film will carry an impression of the ink, of points where contact was established. It has given good results (as shown by Attwell in his paper through photographs) for different types of cameras. The method does not give a quantitative measurement of the lack of flatness but however it reveals the danger spots.

The errors due to insufficient suction are extremely difficult to measure and also they are irregular and does not yield to any systematic treatment.

4. a-2: Unflatness of the Suction Plate:

This is one of the imperfections of the camera manufacture. The Plate against which the film is held flat at the moment of exposure itself should be a true plane to ensure the flatness of the film.

Regarding this, Ekelund (16) checked some plates and found that all suction plates had relatively large errors. Deviations upto 70 microns from flatness were observed. The plates were concave throughout and the concavity being relatively symmetrical.

If the flatness error is taken as an error df in the camera constant f then the distortion at a distance l from the image centre can be written as

$$dr = \frac{l}{f} .df$$

Fig.37, gives the distortion curve corresponding to the flatness errors of an actual suction plate (16)

When tests were conducted by Washer (20) on suction plates, the average variation from flatness for different thickness of plates were observed to be as follows:

For 1/16	inch	plates	± 0.016 mm
For 1/8	inch	plates	± 0.008 mm
For 1/4	inch	plates	± 0.004 mm

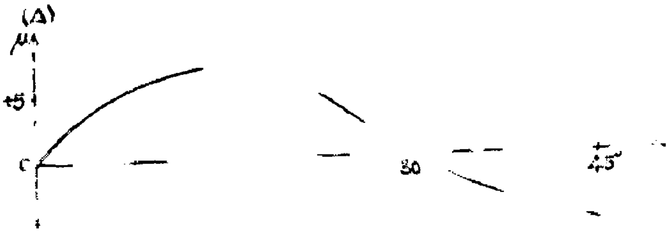


fig. 37. Radial displacement caused by unflatness of a suction plate. f = 150 mm.

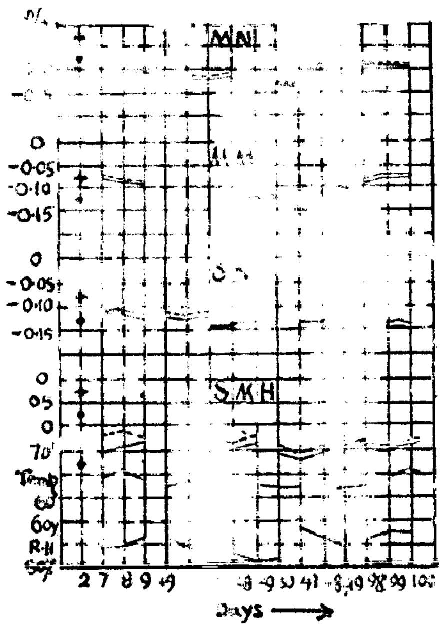
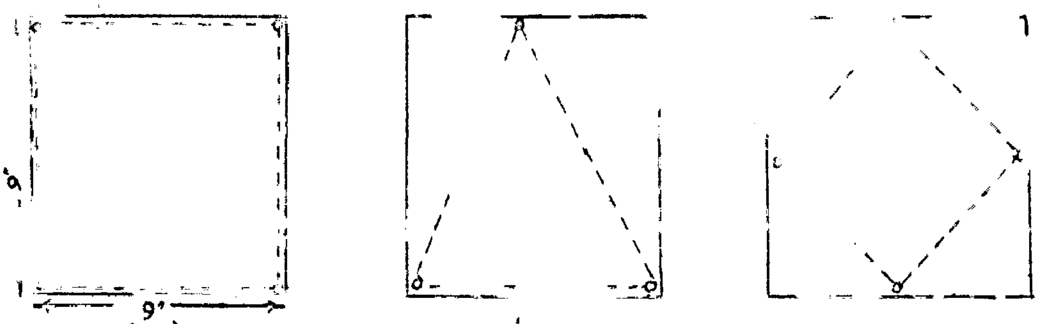


fig. 38. Variation of dx and dy with time and processing.

dx = Dimensional change in the X axis [X = length of roll film

dy = Dimensional change in the y-axis [Y = width of roll film



(a) Supports at the four corners. (b) Supports at 3 points. (c) Supports at the mid-points of Sides.

fig. 39.

The maximum variation in distortion allowed in America due to the unflatness of the suction plate is about 12μ. With the $\frac{1}{4}$-inch plate we may expect a maximum value of 5 to 6μ, suggesting the suitability of $\frac{1}{4}$-inch plates as suction plates. But however, one cannot be sure as to the behaviour of the suction plate at the field conditions where temperature gradients of 60°C to 80°C are not uncommon. The author regrets that no data are available regarding this subject.

Sometimes a plane parallel glass plate is used in an aerial camera either as a pressure plate or as a filter (eg. minus blue filter) in front of the lens. When used as a pressure plate, it is pressed against the film so that the film remains flat at the moment of exposure. To ensure the best results, this plate should be optically flat.

The insertion of a glass plate between the lens and the focal plane, displaces the focal plane away from the lens by about one third the thickness of the plate used. This displacement of the focal plane results in radial displacement of images towards or away from the centre of the field depending on whether the plate is used between the lens and image (in a pressure plate) or between the lens and object (i.e. a filter) respectively. This displacement varies with the refractive index of the glass used for the plate (21). Also this glass plate might introduce chromatic aberration in the system.

When a glass plate is used, the distortion arising may be compensated by using a compensation plate.

4. b. <u>DIMENSIONAL STABILITY OF THE PHOTOGRAPHIC MATERIAL</u>

The Photographic material essentially contains two parts namely the Base material and the Sensitive Emulsion.

The movement of the emulsion relative to the plate or film namely "creep" is very small/ It is of the order of a few microns and is comparitively negligible. It is the base which determines the distortion characteristics of the negative.

The essentials of a base for survey proposes are:

i) Distortion between exposure and time of printing should be as low as possible,

ii) Lateral and longitudinal shrinkage should be uniform and equal,

iii) Humidity effect as low as possible.

Film distortion as also the distortion of the Printing Paper is usually erratic in direction. But film distortion is smaller, more systematic and more uniform than paper distortion; but the effects of the two are very similar (21). The effects of distortion on a Photographic Print are related to the water content of the materials, the length of time of storage after Photography, the relative humidity of the atmosphere while storage, mechanical treatment of the film while processing and the directions of rolling the film and paper while manufacture. The change in the dimensions is not necessarily the same in all directions. Usually one dimension changes more than the other; one diagonal frequently changes differently than the other. In some cases one dimension has been noticed to elongate while the other has shrunk. Thus the term "differential distortion" is often used. With the return of the environment to the original condition does not necessarily mean, a return to the original dimensions of the film or paper.

The behaviour of film as a Photographic base has been treated exhaustively by Calhoun (23).

Though the results available so far are not conclusive, a feir idea about the probable errors that can be expected has been possible. The effect of various types of processing of films, of different types of films etc. can be deduced.

The dimensional stability of Photographic films is strongly dependent upon the procedures of exposure and processing of the negative. It is very important that these conditions must be carefully controlled if distortion is to be a minimum.

The film records the data of the terrain under varying conditions of temperature and humidity. After exposure, it is processed, dried and stored and used at some later date by the Photogrammetrist. When the container of the film is opened the dimensions of the film are in a continuous stage of change from then on. This environment affects the dimensional stability of the film.

It has been found that immediately after processing the distortions are at the maximum. They will diminish from then on, and will not reach stability for a period of 1 to 2 months. Within the first 2 or 3 days there is a considerable drop and from 20 to 30 days the distortion is reaching a stabilised figure. Hence it is better if the film is used after its has reached a stabilised state.

(23) gives the irregular shrinkage resulting from storing the film for a few weeks (min - 1 week) to be about $\pm 5\mu$. as an average, with maximum values of $\pm 20\mu$.

Some investigations conducted by (14) have been given here. Equal lengths of films were processed after exposure and dried by different methods at the same time. They were stored together and measured at the same intervals of time. Fig. 38 shows the results obtained. The figure shows the linear changes along the two directions, the seperation between the dx and dy lines giving a measure of the differential distortion. The change is temperature and humidity conditions during storage are given at the bottom.

The results showed that there was a decrease in the differential distortion with storage and also a shrinkage of film on processing.

The film showed suseeptibility to greatest contraction under the combination of lowest temperature and lowest humidity and the least contraction occured under the combination of highest temperature and highest humidity. The changes in temperature and humidity were small being within $\pm 5°F$ and ± 5 percent.

The most important point in the handling of the film, is its drying; whatever method is used for the drying it is of utmost importance that every drop of surface water be removed from both the sides of the film since unequal drying rate of the moisture produce local stresses in the film.

On the basis of the results obtained Attwell (14) has given the following suggestions to improve the accuracy through a better control of the various conditions.

i) Diapositives for plotting machines should not be made until several weeks after processing, during which the film should be stored loosely wound in conditions of fairly constant temperature and humidity.

ii) There is no objection to mechanical processing provided excessive heat is not used.

iii) A basically safe method under all circumstances is spool tank development with the film dried by hanging on edge.

The conditions actually under which the film is used, are very different from those at the Laboratories. The conditions equivalent to that of the field is very difficult to be realised in the laboratory. But it is highly important that one should know what actually is happening before one can think of a remedy. It is known that low humidity is conducive to the formation of static on the film. The important question is to know the geometrical shape of the film at the moment of exposure and the geometrical shape it has at the stage of plotting.

To assess the effect of temperature and humidity, it is felt that a series of pictures be taken on different films on the same day and on different days, and a detailed investigation be conducted. The differential distortion is causing more anxiety than linear distortion.

Glass Plates used as Photographic base are more stable than films and hence are used for large scale maps. Glass negatives are more suited for large scale plotting of the order of 1 : 500 to 1 : 5000. But for smaller scale plotting on economic grounds, films are better suited. But, glass negatives are heavy and unwieldy to carry in large numbers and require very careful handling.

Glass Plates for Photographic base are manufactured by the Gavaert Company. Their "Ultra plan" Plates have a maximum deviation from flatness of 22μ for both the 15 x 15 Cm. and 19 x 19 cm. formats. This 22μ deviation will produce radial distortion smaller than ± 5 microns. The process of developing and drying has a tremendous influence on the distortion as found out by (22) An error of 50μ was caused in some plates by developing and drying thus bringing an additional radial displacement of 20μ at the points of relative orientation (7).

The average deviation of flatness, from a mean reference plane of a large number of diapositive plates checked by the "Bundesant für Eich-und Vermessungswesen" in Vienna was

$\pm 50\mu$ (7)

The growing use of Photography for the preparation of large scale maps increases the requirement for flatness in the plates.

To find out the deviation from flatness of Photographic Plates, two methods are used,

1. Interference method,
2. Pneumatic method.

Meeus and Thiriar (48) have dealt with the first method and is considered to be more rapid and economic compared to the Pneumatic method. The Pneumatic method has been dealt by J.C. Evans and I.G. Morgan (38)

Film distortion is considered to be controlled if glass plates are used in photography and if the map is compiled from the original negatives or from prints on glass. Film distortion is corrected in some cases by having a rectangular glass grid in the focal plane of the camera (Reseau Method of Ordnance Survey, England). Thus any unknown distortion is confined within the area of a single grid square.

The positype (Acetate impregnated) paper used for prints has almost the same distortion characteristics as the film. In recent times prints are made on metal mounted paper

(Correctostat Paper). The distortion due to this may be said to be only due to temperature effects, and is of negligible amount under normal conditions.

4. c. FLATNESS OF THE PLOTTING (IMAGE) PLANE:

It is highly important for accurate photogrammetric operations that the photo carriers in a plotter on which the photograph is supported for stereoscopic viewing should be perfectly flat. Otherwise the restitution of the object bundle of rays will not be true, consequently introducing errors.

Photographic Plates (Diapositives and negatives on glass) have a tendency to sag under their own weight. This introduces errors in the Photogrammetric operations of plotting maps and supplying control through aerial triangulation. Oswal (24) has investigated the errors which arise due to different modes of support and their effect on photogrammetric operations. But this applies only to instruments of optical projection principle, since in case of instruments based on mechanical projection principle, rays are orthagonally collected and as such vertical displacements of points are ineffective in causing errors of projection.

In his investigations the following modes of support were tested and the subsequent errors on plotting and aerial triangulation were found out. (Figs 39)

Finally it has been deduced that the type (c) of support was the best for Photogrammetric operations. The deflection of the plate in (c) was the least. It was only 1/6th of that for (a).

TABLE II

Thickness of Plate 2mm. Summary of Results Size of plate 9 1/2-in.sq.

Mode of support	Maximum deflections	Max. Projection error: seconds		Max. residual error: seconds		Optimum adjustment of principal distance: μ	Maximum error in dy: μ	Minimum thickness of plate mm
		f=6"	f=12"	f=6" Fp=7"	f=12" Fp=7"			
(a)	75.0	31.4	9.6	17.9	1.4	26.0	-25.3 +19.2	5
(b)	58.3	36.5	12.0	20.8	1.7	27.8	nil	5.5
(c)*	12.5	4.8	1.4	2.7	0.2	14.7	-2.3 +1.8	2

REMARKS: (c)* Photogrammetrically the best.

Fp = focal length of projection printer.

If a diapositive is produced by a projection printer instead of by a contact printer the above errors can be eliminated completely, if the plate is supported in the printer in the same way as in the Photogrammetric Plotter and the focal lengths of the printer and the plotter be the same.

Lastly, the plotting plane in which the drawing paper is put for plotting the details should be a perfect plane. Otherwise the deviations from flatness will introduce errors in heights measured. It is not easy to detect this and the temperature changes will warp the surface irregularly. All the plotting instruments in recent times are always housed in airconditioned rooms thus avoiding this source of error.

The drawing sheet should also be kept perfectly flat. Usually some heavy weights are placed at the corners to see that the sheet is held taut.

The sources of error in different instruments using the Mechanical, Optical and Optical-Mechanical principles, using film and plates have been noted by Hothmer (7). They have been tabulated here for reference (refer Table III in pages No.66 and 67).

Errors arising
Unflatness of the negative plate
Unflatness of the negative plate
Unflatness of the diapositive plate
Unflatness of the negative plane; Irregular shrinkage of film;alternation in dimensional stability of film
Unflatness of the negative plane; Irregular shrinkage of film.
Unflatness of the negative plane; Irregular shrinkage of film; unflatness of the diapositive plate
None of the various errors are effective. This method is applied at the Institute Geographique Nationale, Paris.
Unflatness of both the negative and diapositive plates.
Unflatness of diapositive plate.

Errors arising
Unflatness of negative plane; Irregular shrinkage of film; Alternation in dimensional stability of film.
Unflatness of negative plane and diapositive plate; Irregular shrinkage of film.
Unflatness of negative plane and diapositive plate; Irregular shrinkage of film.

CHAPTER - III

MEANS APPLIED FOR ELIMINATION OF DISTORTION IN DIFFERENT PLOTTING INSTRUMENTS:

1. Stereotopograph of Poivilliers, Thompson Watts Plotter, Nistri Fotobeta:

All the three instruments do not have any means to eliminate distortion and hence should apply either the Porro Koppe principle or distortion free diapositives.

The most accurate elimination of distortion is reported (7) to be obtained at the Institute Geographique Nationale at Paris with the Stereotopograph.

The Stereotopograph uses the ideal Porro Koppe principle for elimination of distortion. According to Cruset (25) the standard deviation for elimination of distortion is $m_{\Delta_\varepsilon} = \pm 4''$ corresponding to a $m_\Delta = \pm 2.5\mu$ for $f = 125$ mm. The original glass plate negatives are used in the instrument and hence the question of errors due to the unflatness of the negative plane does not arise. But however, the errors due to the effect of different wave lengths, the effect of temperature gradient and effect of refraction and curvature of earth have not been considered.

In applying the Porro Koppe principle only photographs which have been taken with a lens having similar distortion characteristics as that of the plotting instrument can be plotted.

In case of the Thompson Watts Plotter, the user is advised to use distortion-free diapositives. This will give better results as it is very difficult to apply the Porro Koppe principle in its purest form due to the liminations in obtaining the proper lens.

The Fotobeta of Nistri utilises either the Porro Koppe principle or distortion free diapositives for elimination of distortion. For the production of these diapositives, we have the Orthoscopic Photoprinter of Nistri (exposed at the Stockholm Congress of 1956).

Hothmer (7) conducted some experiments on the Nistri Fotobeta. The results proved that both the projection lenses are distortion free within the accuracy of measurement.

2. WILD INSTRUMENTS:

All the stereoplotting instruments of Wild apply the compensation plates for elimination of distortion. The standard deviation for elimination of distortion is $m_{\Delta} = \pm 2\mu$ (7).

3. STEREOPLANIGRAPH C8:

This instrument was originally designed for the application of Porro Koppe principle for elimination of distortion. But from the last ten years, the C8 has also been supplied with compensation Plates. Thus the plotting lenses only correspond to the aerial camera lens approximately in respect of the focal length. But a pair of compensation plates goes with a certain lens. The compensation plate has to eliminate the resultant distortion between the distortion of the aerial camera lens and that due to the projection lens.

On each of the plate the lens with which it goes is indicated. The left and right plates should not be interchanged.

The standard deviation for eliniation of distortion is $m_{\Delta} = \pm 2\mu \ldots \pm 3\mu$ (7).

4. <u>KELSH PLOTTER</u>:

This simple instrument has a complicated system for elimination of distortion.

The elements for elimination of distortion are a projection lens which moves up and down along the axis (Δ fc) and a glass plate which behaves like a compensation plate.

The diapositives in this instrument are used with emulsion up. Otherwise with the projection and contact diapositives which are made emulsion against emulsion (Fig. 25) a mirror reversed plot will result. The projection ray thus passes through the glass plate which acts as a compensation plate (fig.40). The projection lens may have a distortion $\Delta \varepsilon_{p\gamma}$ The projection lens can be displaced along the direction of the principal distance to remove the distortion. In the Kelsh Plotter the whole distortion is removed by an aspheric ball cam which is ground so as to make the sum of all the distortions equal to zero.

In (1.e-4.2) we have seen that removal of distortion by a correction fc to the projection centre holds good for only a certain projection distance z. In the Kelsh Plotter the average projection distance is z_0 = 75 cm and the maximum and minimum distances are z_{max} = 90 cm and z_{min} = 60 cm.

The error in radial distortion which crops up if the ball cam is ground for the average distance and used at the maximum orminimum projection distances is given in fig.41 (7)

The radial displacement due to the glass plate is given by (fig.42).

$a = d\,(\tan\alpha - \tan\beta)$ where d = thickness of plate.

The error Δp of distortion due to the variation of d is given in (7) by the equation.

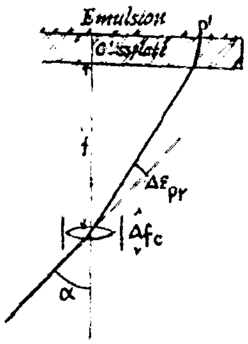


fig. 40. Distortion in Kelsh Plotter

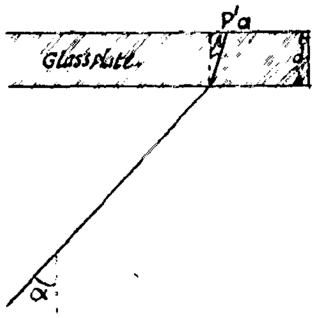


fig 42.

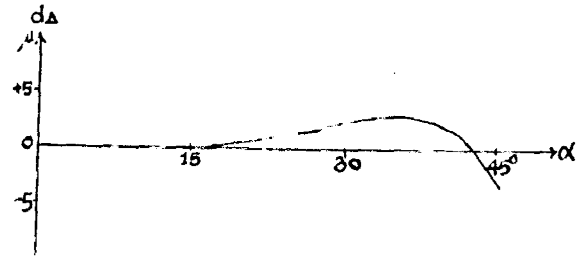


fig 41. Error in radial distortion if Metrogon photography of mountainous terrain is plotted in Kelsh

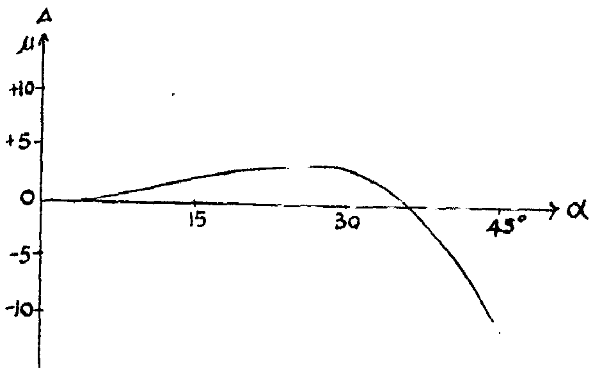


fig 43. Error in radial distortion in Kelsh Plotter caused by a variation of 0·1 mm. of the thickness of the diapositive glass plate.

$\Delta p = d\ (\tan\alpha - \tan\beta - 0.008\,\alpha^{o})$

Where α^{o} is in sexagesimal degrees. The error Δp due to a variation of 0.1 mm in d is given in fig.43. The amount of error at orientation points is about 10μ. This with the error arising due to the unflatness of diapositive plates i.e., about 50μ produces large errors. Thus it is of utmost importance that the ball cam has to be ground for each thickness of the glass plate.

(7) suggests the use of diapositives with emulsion down. This then does not cause the mirror reversed plot. The diapositives are made by projection through the film, the film emulsion being away from the diapositive thus projection through the film base. This involves projection through plane parallel plastic plate i.e. the film base and a thickness of 0.12 mm. Consequently the error Δp will be smaller than that in fig.43. But no information is available as to the error that may be caused due to a variation in the refractive index (plastic has a smaller refractive index than glass). Also the question whether there would be a loss in image resolution in this operation has to be investigated.

At the present stage no consideration has been given to the effect of variation in distortion for every lens in a single batch, the effect of refraction and curvature of earth, and the errors due to unflatness of the diapositive plane.

Errors will arise if a ball cam ground for the average curve of distortion say Metrogon type and is used with other lenses of the same type.

As Hothmer (7) has suggested the use of distortion free projection diapositive seems to be a good remedy to combat the many errors arising due to the use of aspheric ball cam and a projection lens moving.

5. MULTIPLEX:

The Multiplex uses reduced diapositive obtained from a projection printer. The lens of the pointer has distortion characteristics opposite to that of the aerial camera lens and hence the diapositives got are distortion free. The sum of the distortions of the aerial camera lens, the projection lens and the Multiplex projection lens is zero.

The above condition thus restricts the use of a projection printer with a particular aerial camera.

Here also no consideration has been given to the effect of different distortion curves of a type of lens, to the effects of temperature,&refraction curvature of earth.

6. SANTONI INSTRUMENTS:

The instruments of Santoni (ie. the Stereosimplex II Stereosimplex III and Stereocartograph IV) use a correction to the plotting principal distance, to eliminate the distortion. This correction is realised in the form of very small ball cams ($\frac{1}{4}$-in. dia) acting on the mechanically realised photopoint and are inserted at the picture points end of the projection bar. These cams can be ground to match any desired distortion curve. These cams are ground to the exact distortion of the lens of the particular aerial camera used. This feature is of a considerable advantage in that a mean curve is not used to correct for distortion.

A very simple cutting machine which is supplied by the manufacturer can be used by the customer to grind the cams to any desired distortion. The cams are relatively cheap a pair of them for the stereosimplex III costing about $40.

Ball cams of the Stereosimplex II and III have the same sign but are of different magnitude. But the cams of Stereocartograph IV have to be ground with oppositive sign as the projection bar approaches the effective photo plane from above.

A test conducted by P.Meli on a Stereocartograph IV (7) resulted in a standard deviation for elimination of distortion of $\pm 5\mu$.

The effects of different items like refraction, curvature of earth, unflatness of negative plane etc., can be included while grinding the ball cam thereby extending the use of the cams. Le Divelec (36) has dealth with the application of ball cams for removal of distortion arising from the effects of refraction and curvature of earth.

To avoid errors which may creep in unnoticed some suggestions by (7) may be quoted. After grinding, the ball cams have to be hardened to minimise the mechanical wear of the ball cams. But this process calls for extreme care as a little invisible fracture in the cams might cause considerable errors. The small sphere which is always in contact with the ball cam should also be treated with similar care and this should be removed after a year's service for best results.

While interchanging the ball cams, case should be taken not to drop them on the floor as this may result in small fracture. Everytime the ball cams should be checked for such occurance. A compensation plate will be carried to check if any fracture or breakage occurs but this is not the case with the metallic ball cams. Care should be taken to insert the ball cam properly in its suggost in the projection bar. Otherwise errors in the shape of fig.44 can occur(26)

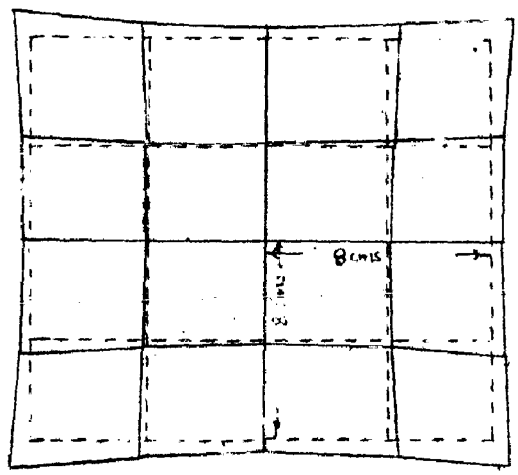


fig. 44:

CHAPTER - IV

EFFECT OF NOT COMPLETELY ELIMINATED DISTORTION

As seen earlier in Chapter II, the standard deviation for elimination of distortion varies with different methods of elimination. Also the amount of distortion depends on the type of lens since some type of lenses like Zeiss Topar etc., are superior to other lenses like Metrogon etc. But it is a well known fact that each lens will leave some amount of residual distortion after the compensation has been performed.

Thus the corresponding rays of light say r_1 and r_2 of a pencil of rays in a pair of photographs I and II, which, when the photographs were taken originated from an object point, will no longer intersect at a point. They cross in space when the pencil of rays is reconstructed in a plotting machine. This discrepancy will result in the form of a parallax Δp in the image plane having two components, a horizontal component Δp_h and a vertical component. Δp_v (Fig. 45).

The effect of these components on planimetry and height measurements in a photogrammetric operation depends on the types of lenses.

On the basis of the range of distortion lenses can be grouped in four classes (7)

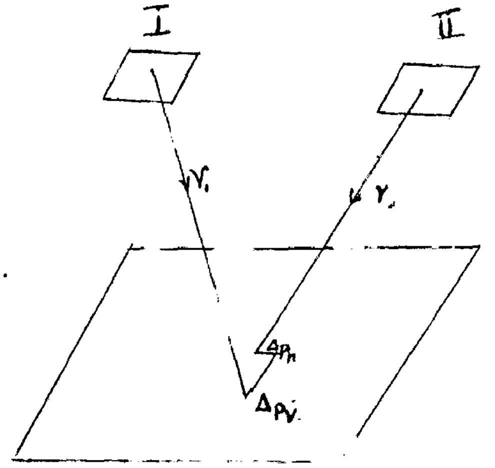


fig. 45. Horizontal and Vertical components of parallax Δp.

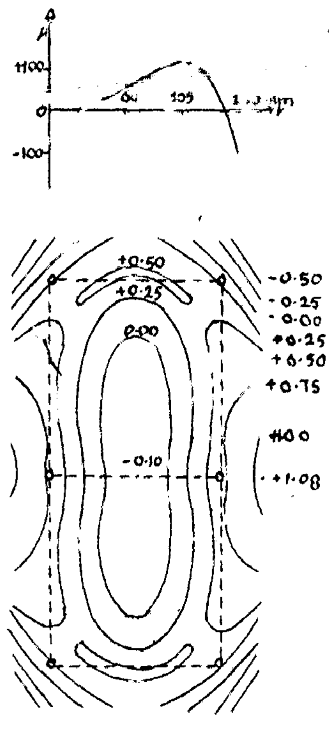


fig 46. Height deformation caused by distortion of Metrogon, in ‰ of the flying height Contourline interval 0.25 ‰ h

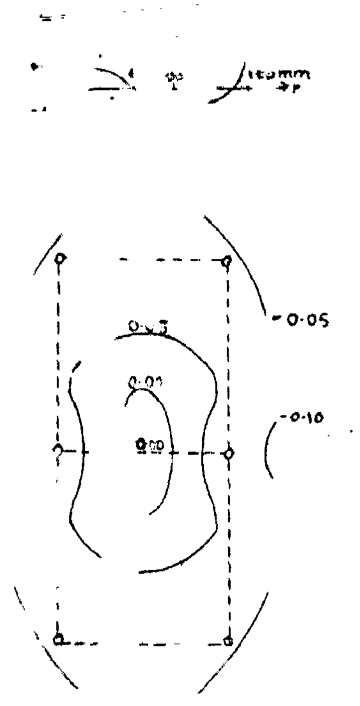


fig 47. Height deformation caused by distortion of Pleogon f=150 mm, in ‰ of the flying height.

1. Topogon type lenses (Metrogon, S.O.M.-Aquilor, Williamson Ross, Nistri Riegel), distortion ranging between $\pm 100\mu$

2. Planigon lens, distortion ranging between $\pm 20\mu$

3. Wild lenses Aviotar and Aviogon, distortion ranging between $\pm 10\mu$

4. Zeiss lenses Topon and Pleogon, distortion ranging between $\pm 5\mu$

These horizontal parallax affects the height measurement at the time of plotting while the vertical parallax causes discrepancies in relative orientation.

The horizontal parallax is usually dealth with by a correction for the height at the point in question and Figs. 46&47 show the height deformation caused by the residual distortion for two of the above classes of lenses. The distortion curves with respect to which the height deformation has been calculated is given. These graphs depend on the length of the base line. The six orientation points are also shown in the graphs.

The neat model area is bounded by the six orientation points whereas the whole model consists of all in the 60 percent overlapping.

TABLE IV

LENS	MAXIMUM ERROR			
	NEAT MODEL		TOTAL MODEL	
	o/ooh	h = 2000m	o/00h	h = 2000m
Metrogon	0.80	1.60 m	1.60	3.20 m
Planigon	0.30	0.60 m	0.44	0.88 m
Aviogon	0.20	0.40 m	0.24	0.48 m
Pleagon	0.05	0.10 m	0.10	0.20 m

It is seen that the maximum errors in height in the three types of lenses exceed the error in height measurement (i.e. 0.03 °/oo) as well as instrumental errors (0.10 °/oo). Thus distortion should be eliminated. But in the case of the Pleogon the distortion of $\pm 5\mu$ produces errors within the order of instrumental errors and thus there is no need for further elimination of distortion (7).

The asymmetric distortion causes some height deformations. This has been shown in fig.48. The maximum value of the height deformation occurs near a orientation point (top left) and is 0.24 °/ooh. The figures in bracket are the height deformations after the model has been levelled with respect to the six orientation points (7).

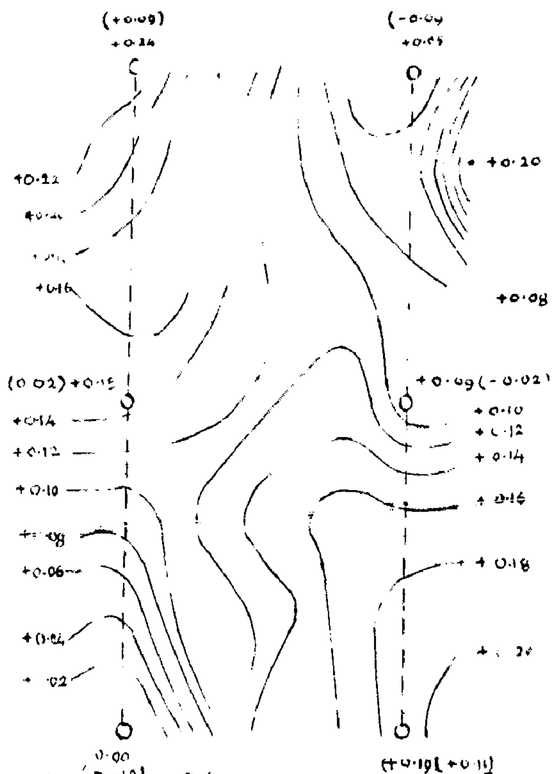


fig 48. Height deformation caused by non-symmetrical distortion in an Aviogon lens in ‰ of flying height contour line interval 0.02 ‰ h.

Distortion due to refraction and curvature of earth are readily transferable to radical displacement in the horizontal. From these radical displacements, the height deformations due to earth curvature can be calculated.

The height deformation due to the earth curvature will be within the order of the accuracy of the instrument for a wide angle objective below a flying height of 5000 m. But after this they start exceeding the instrument accuracy. Height deformations for a wide angle lens of field angle 90^0 and for h = 3000 m and h = 5000 m are shown in figs.49&50 (7)

TABLE V

h	1	2	3	4	5	6	7	8	9	10 Km
$\Delta\phi$:	0.5	1.0	1.4	1.9	2.4	2.8	3.3	4.0	4.7	5.2^c

ϕ Cracks for field angle 90° as caused by Earth Curvature and Refraction.

Although generally the errors due to the effect of refraction and curvature of earth are not eliminated for photogrammetric operations of high accuracy it is a must. In the process of aerial triangulation, it is mainly required. Table V shows the ϕ cracks $\Delta\phi = \phi_i - \phi_{i-1}$ for different flying heights as caused by Refraction and curvature of earth.

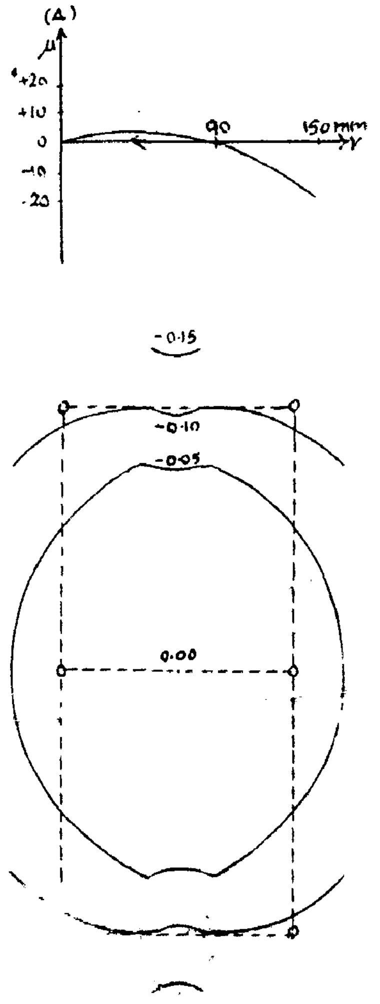


fig.49. Height deformation caused by refraction and curvature of earth in ‰ of ‰ of flying height. The Graph stands for h = 3000 m. f = 150mm.

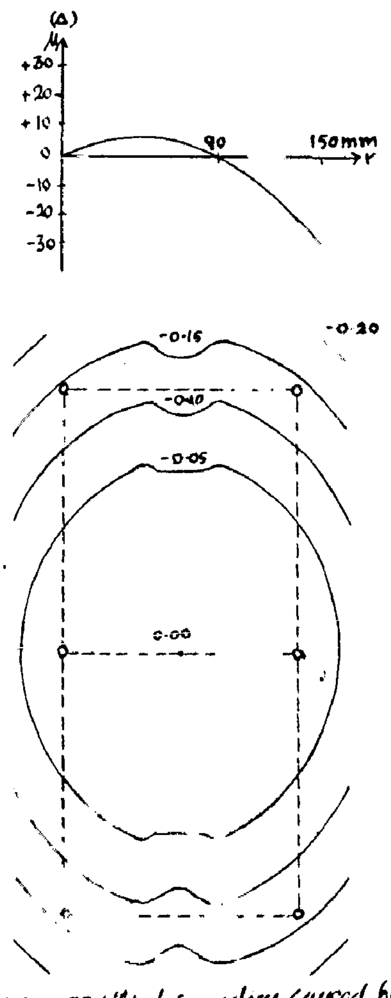


fig.50. Height deformation caused by refraction and curvature of the earth. in ‰ of flying height. The graph stands for h = 5000 m, f = 150mm.

For minimum amount of errors to be present, the best remedy is that the compensation device should be based on the distortion curve of the particular lens.

AERIAL TRIANGULATION:

The process of aerial triangulation being one of continuity, the errors which appear in the first model will be carried forward till the end. The horizontal component of the residual distortion will introduce errors in height measurement and the vertical component will bring discrepancies in the process of orientation. These produce Y- parallax at the orientation points. When there are height errors at the principal points, then difference will constitute a scale jump (7). This scale jump exists between all the models and will be carried forward.

Tham (12) states that sometimes if the lens distortion curve is favourable as to its stereoscopic effect, it would be advisable to execute the aerial triangulation without any compensation. This may be true only when the photographs got from such a lens are used for aerial triangulation alone. But since such a condition will rarely arise, the validity of distortion compensation is evident.

****

* * * * * * * *
* CONCLUSION: *
* * * * * * * *

The most important problem in Photogrammetry is the setting up of the stereoscopic model in the plotting machine in exact accordance with the terrain photographed. This requires the accurate reconstruction of the object bundle of rays. The primary limiting factors in such exact reconstruction of the object bundle of rays are distortion and the precision of the plotting machine. The first order instruments like, the Wild Autograph A7, the Zeiss Stereoplanigraph C8 etc., have a standard deviation in planimetry - m_p, from $m_p = \pm 5\mu$ to $\pm 10\mu$; and the standard deviation in height measurement - m_h from $m_h = \pm 0.03\,^{0}/_{00}\,h$ to $\pm 0.10\,^{0}/_{00}\,h$, where h is the flying height. If the residual distortion is smaller than this accuracy of the plotting machine, then comparatively it can be neglected, so that the accuracy of the plotting machine becomes the chief limiting factor for such a reconstruction. To achieve this objective, it is necessary to control the various sources contributing towards the introduction of distortion.

The better the photogrammetric materials, like the lenses cameras and sensitive materials and higher the accuracy of the plotting machines the greater could be the flying height and the consequent economy. If the flying height is increased, say doubled, then four times the area will be included in each photograph thus bringing down the cost of photography and of subsequent operations. However, the scale of the photograph will be reduced to one half and the photographic details correspondingly reduced. This increase in flying height cannot be done indefinitely as there is a definite Resolving Power for both the lens and the negative emulsion. Also there is a ceiling which limits the satisfactory performance of the photographic aircraft and the crew. Though the flying height is increasing rapidly, equipment for such flying is becoming increasingly complex. There is a practical limit to benefits from increased flight height. Without increasing the flying altitude, a greater coverage of area is possible by the use of the Wide Angle lens. Since with the use of Wide Angle lens, the photographic base is increased, a larger base to height ratio is obtainable and hence a more accurate process of Relative Orientation with the consequent smaller deformation of the model. But, by the use of a Wide Angle lens, problems like increased lens distortion, the masking of detail in mountainous areas and an unevenness of exposure due to the falling off in light transmission to the outer portions of the negative (21) are encountered.

There is a general belief that lens distortion from a normal angle lens is negligible and need not be compensated. This is not true, since when dealing with large scale plans and with demands of high accuracy, we have seen that distortion is considerable and in practice it has to be eliminated.

Tham and Samsioe (12) have investigated the magnitude of distortion using a f/4.2, normal angle Wild Aviotar lens of focal length 170 mms. and with a flying altitude of 3400 metres. Fig.51 shows the height corrections caused by the horizontal component of the parallax due to the lens distortion. Fig.52 shows the corresponding vertical parallax caused by lens distortion.

The correction nomogram of fig.51 shows that at the central portion of the stereoarea, the correction varies from -0.15m. at the principal points to +0.75 m. at the centre. Away from the centre the curves show larger and larger negative values. For Stereophotogrammetric plotting machines of the highest precision the mean error is about 0.20^{0}/oo to 0.25^{0}/oo of the flying height which in this case about 0.75m. Compared to this value the systematic error caused by the lens distortion of the Aviotar and amounting to a total of 1.40m. is definitely too large to be neglected.

The vertical parallax nomogram of fig.52 has two symmetric axes which at the same time are zero lines. The

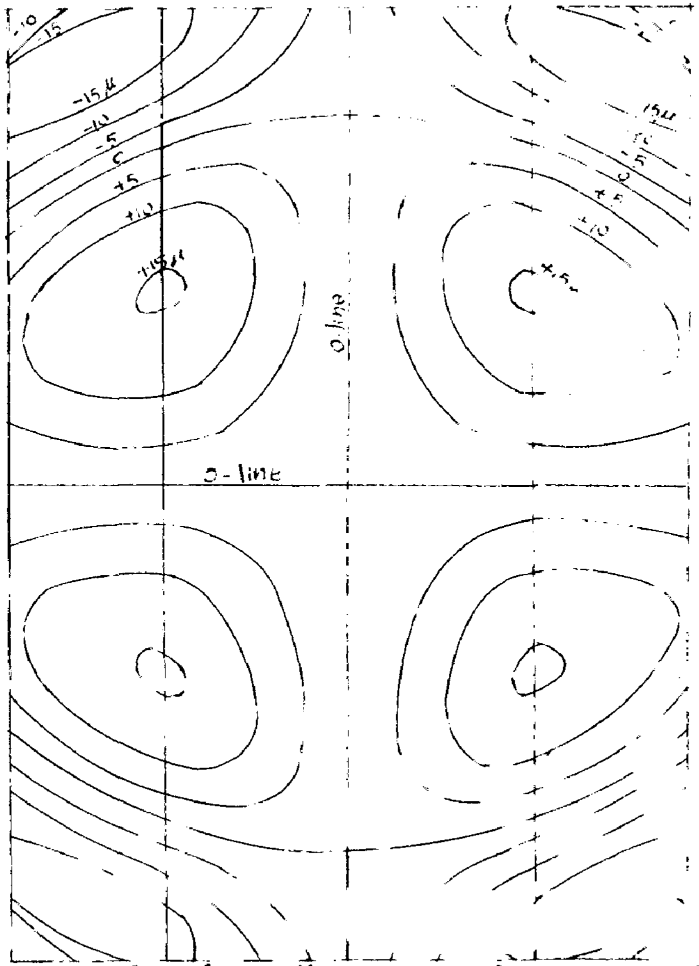


fig. 51. Vertical parallax indicated in µ caused by the lens distortion of the normal angle Aviotar lens f/4.2/170 mm. overlap 65 %.

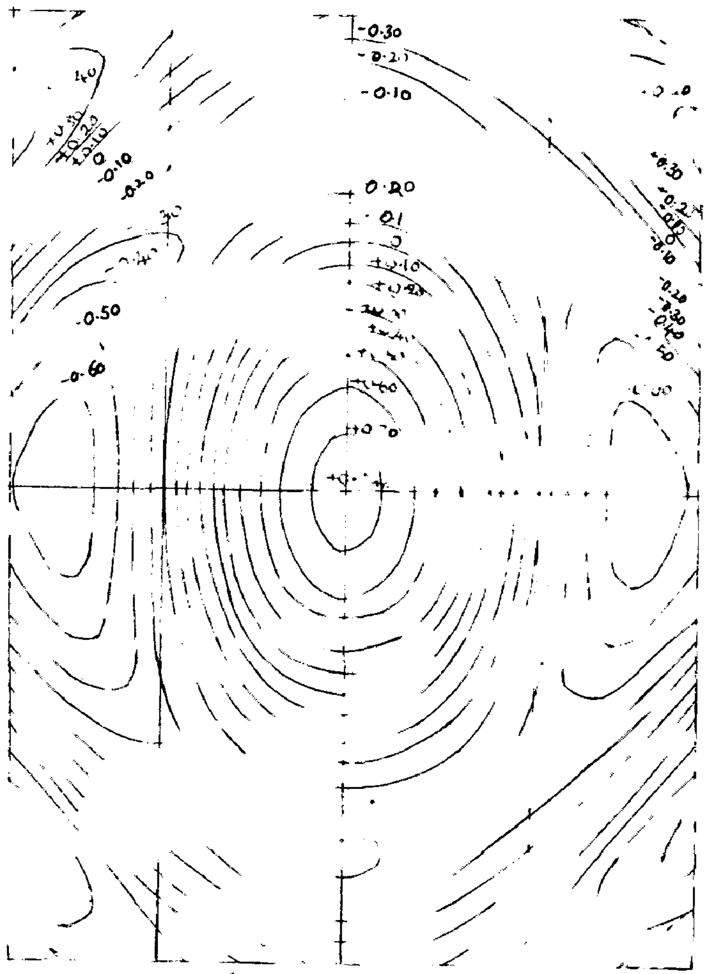


fig. 52. Height corrections indicated in metres caused by the lens distortion of the normal angle. Aviation lens f/4.2/170 mm overlap: 65 %.

vertical parallaxes are fairly small. But the position of the zero lines are rather unfavourable to the selection of points for Relative Orientation.

In Table VI (7) all factors contributing towards distortion have been listed. This is to provide a comparative basis. Here *m* stands for the standard deviation. According to the theory of errors the maximum value is equal to two or three times m. Both are plus or minus, thus the maximum spread is twice as much as the maximum value.

(For Table No.VI, refer page No.85)

Concluding we may summarise as below:

The tolerances allowed for photogrammetric operations in recent times are very small and they can only be met by high quality lenses.

We have treated asymmetrical distortion and found that there is no precise and economic method to eliminate it. Since they produce height deformations in a model, lenses showing unsymmetrical distortion should never be used in precision work.

Imperfect centering of the various lens elements causes tangential distortion and since so far there are no means

TABLE - VI

Subject.	Radial displacement		
	m	Maximum value	Maximum spread.
Calibration with Collimator	2	3	6
Calibration with Goniometer	1	2.5	5
Asymmetrical distortion	3	8	16
Various lenses of same type	5	10	20
Different wavelengths	4	--	15
Different diaphragms	-	3	6
Different temperature	-	5	10
Unflatness of suction plate	4.5	--	12
Unflatness of best glass plates	-	--	10
Unflatness of glass plate due to developing	-	--	25
Unflatness of diapositive glass plates	-	--	25
Irregular film shrinkage	5	20	40
Earth curvature h=1000 m	-	--	7
Earth Curvature h=8000 m	-	--	58
Accuracy of plotting instruments	5-10	20	40
Compensation device in plotting instrument	2-5	--	25

to correct this, lenses which show tangential distortion should not be used in precise operations. Both the unsymmetrical distortion and tangential distortion are caused by prism effect in lens. Thus lenses which show even a small amount of prism effect should be discarded for photogrammetric operations.

Tests should be in accurate compliance with internationally agreed lines. Only then the results will be on a comparable basis since results of the same subject obtained by different methods differ.

Calibration should be performed photographically with the minus blue filter. Emulsion used for calibration should have the same colour sensitivity as the emulsion used in practice. It should be developed to a gamma (γ) of 1.2±0.1. It should be as fine as possible. The data of the emulsion should be stated in the calibration report (27). If calibration is done visually care should be taken to see that the light of the same mean wavelength is used as that of natural light. The camera should be calibrated at different working operatives and at full aperture. For better results, the results of all the four diagonals should be given instead of only the mean distortion curve.

Tests conducted by Olsson (28) show no significant differences in the geometrical properties of three kinds of films namely Panchromatic, Infrared and colour film. As for the colour film, it seems to be suitable for photogrammetric purposes. The average value of irregular film shrinkage is found to be $\pm 5\mu$ (See table VI). The maximum distortions can be avoided if meticulous care is taken in handling the film while processing. Accurate correction of film and paper distortion is normally almost impossible because a complete knowledge of the distortion pattern is lacking. Care must be taken to see that in an aerial camera using air pressure to maintain the film flat during exposure the excess air pressure acting on the opposite side of supporting plate during the exposure, should not be so high as to cause the resultent of the air pressure to become equal to the force applied by the spring.

In photogrammetric operations restitution of the object bundle of rays is performed by giving a compensating refraction. This method suffers from the inconvenience that the camera objective and the plotting machine should have to fit each other from an optical point of view (12).

As reported earlier, Tham and Samsioe (12) have attempted to get distortion free negatives by using pressure plates prepared with an initial curve so as to compensate for the lens distortion. Using a Zeiss RMK/2030 camera with a

Topogon lens an experiment was conducted using an ordinary pressure plate as well as a curved one in the magazine. The test area was a part of the Archipelago of Stockholm.

Since the distortion has a more marked influence on the height co-ordinates than on the X and Y co-ordinates, stereomeasurements of heights of the checkpoints was made and compared. Although all the checkpoints had a zero level the discrepancy of 1 metre was found with the ordinary pressure plates, With the curved plates, however, at most of the points zero level was obtained (refer fig.53).

Regarding the methods for elimination of distortion in the case of Porro Koppe method, it is almost impossible to get a lens identical in distortion characteristics with the aerial camera lens. Thus the use of aspheric plates to get distortion free projection diapositives seems to be the most practical method. Any radial distortion can be suitably combined and removed. Cams which have been given a desired distortion shape are however inferior to the distortion free projection diapositives. Though they are relatively cheap and easy of grinding, they are not durable. The balls have to be suitably hardened and require very careful handling and constant care. Even a very small fracture will impair the accuracy of the Cams.

Considering the variety of items contributing towards the interior orientation and their complicated nature, it is obvious to think of other methods of determining distortion. There are two methods to do this in a model; we can do it either with the residual x-parallaxes or residual y-parallaxes. If a model is without any errors, all height will read to the correct values. If not so, then the residual height cross shuld be suitably used to derive radial symmetrical corrections i.e. distortions. This has been dealt by Ekelund (16). After relative orientation has been carried out, a model should be free from y-parallaxes. The residual y-parallaxes can then be used for compilation of a distortion curve. This has been dealt by Hallert (29).

The two methods have many advantages. In these methods the difference between visual and photographic methods is not inherent. The effects of temperature, curvature of earth and refraction are no longer sources of errors. However, the unflatness errors have not been taken into consideration. Also it is a disadvantage to make measurements in the plotting machine itself. The x-components of the residual projection errors of the instruments will cause some model deformation. This defect will be improved in the x-parallax method (16) but will be made even worse in the y-parallax method (7).

Distortion characteristics as far as possible, should be determined under flight conditions. Also distortion to the particular lens in question should be eliminated and not merely the nominal distortion of such type of lens (7).

A distortion of ±5 microns produces height deformations in the order of measurement accuracy (refer fig.30) consequently, we have the ±5μ belt as the limit of our possibilities for elimination of the distortion, and it is still lee way to achieve better results than this.

Lastly, we have to mention the effect of draughtsmanship on the accuracy of the map. The accuracy of human draughtsmanship is 0.01 inch. If this accuracy could be increased, better accuracy in the map is possible. In Eurppe and U.S.A. the scribing technique is being tried (30). This technique consists of dilineating the lines and symbols representing map detail with sharp tools. The sharp tool cuts out the opaque paint spread upon a transparent sheet. This leaves a pattern of transparent map detail, the base remaining opaque except where it has been scribed. This new process cuts down the cost and time of production of maps and at the same time has enhanced the quality of printing with a better control of accuracy. The lines drawn are smooth and of uniform width. However, the transparent sheet should be stable in dimensions under various conditions of ~~trans~~ temperature and humidity. Normally glass is the best material, but it is fragile and difficult to handle and store. Researchs are being carried out to find a suitable substitute for glass. In Europe and U.S.A. different plastics are used as scribing base. In India this method has not yet been introduced. Mapping organisations in India have not so far been able to take advantage of this new method though it has revolutionaized modern mapping technique in Europe and U.S.A. However, there are many difficulties to introduce this at present. To get supplies of proper equipment is a problem. A suitable plastic material resistant to the varying tropical weather conditions of India has to be found out. To get the proper tools for scribing is also a problem. But this technique has to be introduced in India in the long run, for better maps.

SUBJECTS FOR FURTHER STUDY AND INVESTIGATION:

The following problems need immediate attention:

1. Astigmatism, Chromatic Aberration etc., of the Aspheric plates used for producing distortion free projection diapositives,
2. The effects of temperature gradient on distortion. For this, conditions similar to the field conditions should be obtained in the laboratory.

Topogon lens an experiment was conducted using an ordinary pressure plate as well as a curved one in the magazine. The test area was a part of the Archipelago of Stockholm.

Since the distortion has a more marked influence on the height co-ordinates than on the X and Y co-ordinates, stereomeasurements of heights of the checkpoints was made and compared. Although all the checkpoints had a zero level the discrepancy of 1 metre was found with the ordinary pressure plates, With the curved plates, however, at most of the points zero level was obtained (refer fig.53).

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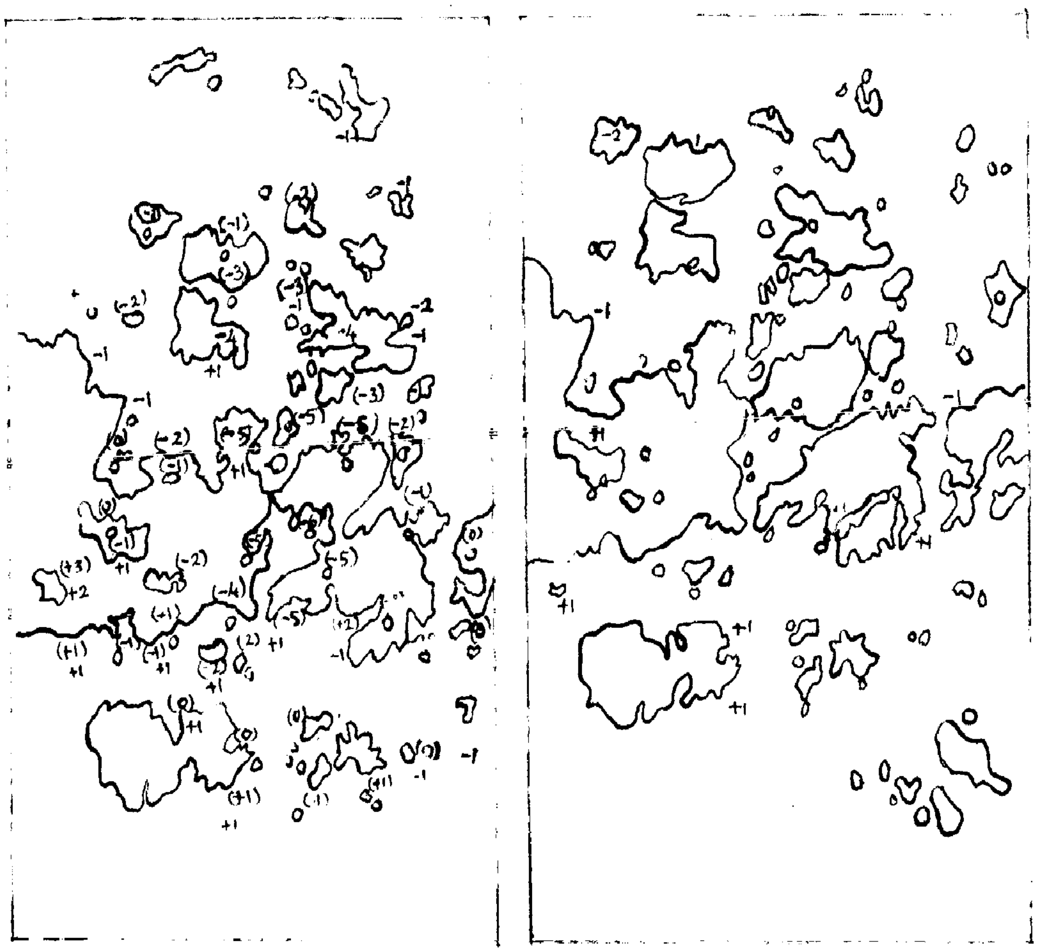

fig. 53 . Result of measurements in the Archipelago of Stockholm, Sweeden
Left: "Ordinary" pictures; Right: "Curved" Pictures

Considering the variety of items contributing towards the interior orientation and their complicated nature, it is obvious to think of other methods of determining distortion. There are two methods to do this in a model; we can do it either with the residual x-parallaxes or residual y-parallaxes. If a model is without any errors, all height will read to the correct values. If not so, then the residual height cross shuld be suitably used to derive radial symmetrical corrections i.e. distortions. This has been dealt by Ekelund (16). After relative orientation has been carried out, a model should be free from y-parallaxes. The residual y-parallaxes can then be used for compilation of a distortion curve. This has been dealt by Hallert (29).

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Distortion characteristics as far as possible, should be determined under flight conditions. Also distortion to the particular lens in question should be eliminated and not merely the nominal distortion of such type of lens (7).

A distortion of ± 5 microns produces height deformations in the order of measurement accuracy (refer fig.30) consequently, we have the $\pm 5\mu$ belt as the limit of our possibilities for elimination of the distortion, and it is still lee way to achieve better results than this.

Lastly, we have to mention the effect of draughtsmanship on the accuracy of the map. The accuracy of human draughtsmanship is 0.01 inch. If this accuracy could be increased, better accuracy in the map is possible. In Eurppe and U.S.A. the scribing technique is being tried (30). This technique consists of dilineating the lines and symbols representing map detail with sharp tools. The sharp tool cuts out the opaque paint spread upon a transparent sheet. This leaves a pattern of transparent map detail, the base remaining opaque except where it has been scribed. This new process cuts down the cost and time of production of maps and at the same time has enhanced the quality of printing with a better control of accuracy. The lines drawn are smooth and of uniform width. However, the transparent sheet should be stable in dimensions under various conditions of ~~trans~~ temperature and humidity. Normally glass is the best material, but it is fragile and difficult to handle and store. Researchs are being carried out to find a suitable substitute for glass. In Europe and U.S.A. different plastics are used as scribing base. In India this method has not yet been introduced. Mapping organisations in India have not so far been able to take advantage of this new method though it has revolutionized modern mapping technique in Europe and U.S.A. However, there are many difficulties to introduce this at present. To get supplies of proper equipment is a problem. A suitable plastic material resistant to the varying tropical weather conditions of India has to be found out. To get the proper tools for scribing is also a problem. But this technique has to be introduced in India in the long run, for better maps.

SUBJECTS FOR FURTHER STUDY AND INVESTIGATION:

The following problems need immediate attention:

1. Astigmatism, Chromatic Aberration etc., of the Aspheric plates used for producing distortion free projection diapositives,
2. The effects of temperature gradient on distortion. For this, conditions similar to the field conditions should be obtained in the laboratory.

3. Research to find out the possibility of fully thermostating the aerial camera.
4. Research work regarding the effect of pressure differences on distortion.
5. Investigation into the complex behaviour of the atmospheric haze and its optical properties.
6. Investigation on vacuum conditions in aerial cameras using vacuum, to hold the film flat during exposure - due to the temperature and pressure differences.
7. Investigations on the relationship between the geometrical shapes of the film at the time of exposure and at the time of plotting in the laboratory.
8. Research on the nature of the distortion curve as it approaches the origin. This is of paramount importance as the influence of distortion on the stereomeasurements is completely dependent upon the type of distortion curve. Not until the year 1940 could the true distortion curve be obtained. In less than 20 years, it has revolutionized the science of Photogrammetry.

It is expected that many important discoveries and advances in the Science of Photogrammetry will come to light at the International Congress of Photogrammetry to be held in London, this year.

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