ANALYSIS AND OPTIMISATION OF FLOW LINE PRODUCTION SYSTEMS

A THESIS

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled 'ANALYSIS AND OPTIMISATION OF FLOW LINE PRODUCTION SYSTEMS' in fulfilment of the requirement for the award of the Degree of Doctor of Philosophy, submitted in the Department of Mechanical and Industrial Engineering of the University is an authentic record of my own work carried out during a period from ~~July 1980~~ Jan. 1981 to July 1983 under the supervision of Dr. P.C.Pandey.

The matter embodied in this thesis has not been submitted by me for the award of any other degree.

(DILBAGH SINGH HIRA)

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ABSTRACT

A flow line production system is characterised by a number of stages arranged in series, where the work material passes through all the stages in a fixed sequence. The variability in the production rates of the stages causes delays and hence loss in production. A number of factors, external and internal to the system, may be responsible for the variability in production rates of the stages. Two factors, which have generally been considered in the literature are :

(i) Random variations in operation times of the stages.
(ii) Breakdowns of the stages.

It has been found that in most of the existing literature, only one of these two causes of production line inefficiency has been considered. In each case the exact analytical models of line efficiency are available for two or three stage lines only. The application of numerical methods is also limited to small problems only. The empirical models, developed from the simulation data, have been found to be specific in nature and limited in application. A little work has been done to analyse the lines having variable operation times, as well as subject to breakdowns. Also, not much work has been done to study the performance of unbalanced lines.

In this thesis, simulation, the most versatile tool for analysing the large complex systems, has been employed. Flow line systems having ;

The applicability of the empirical models has been demonstrated with the help of illustrative examples and a case study. In each case the sensitivity analysis has been performed to examine the effect of system parameters on the optimum size of the inprocess buffers. The sensitivity of the net gain from the system, to deviations from the optimum values of in-process buffer size and the number of repairmen has also been examined.

The subject matter of the thesis has been presented under the following heads :

1. Introduction
2. Review of past work
3. System description and design of simulation experiments
4. Balanced flow lines with random operation times
5. Unbalanced flow lines with random operation times
6. Balanced flow lines with unreliable stages
7. Unbalanced flow lines with unreliable stages
8. Flow lines with random operation times and unreliable stages
9. Effect of repair policy and crew size on line performance
10. Conclusions and scope for future work, and Bibliography

(i) Random operation times (manual flow lines),

(ii) Unreliable stages (automatic transfer lines), and

(iii) Random operation times as well as unreliable stages,

have been simulated in FORTRAN IV to study the influence of system parameters on the line efficiency and the work-in-process inventory.

Validity of the simulation models has been tested by comparing the simulation results with the available data reported by others. Simulation experiments have been designed to ensure a reasonable degree of confidence in the results. The data thus obtained has been analysed to develop simple and reliable empirical models for the efficiency of balanced manual flow lines, as well as, for the balanced automatic transfer lines. While the empirical models in case of manual flow lines are valid for normal and Erlang distributions of operation times of the stages, the empirical model for the efficiency of automatic transfer lines is applicable when the failure and repair times are exponential. In each case the cost analysis of the system has been conducted and expressions for the optimum inprocess buffer size derived.

The empirical models of line efficiency for the two cases, viz., manual flow lines and automatic transfer lines, have been incorporated in an approximate formula for determining the efficiency of the system, when both the cases of production rate variability viz., random operation times and breakdowns of stages are present simultaneously. The validity of the approximate model has been proved by comparing its results with the simulation results.

In case of unbalanced lines, simulation experiments have been planned to determine the effect of various types of imbalances on the line efficiency and the work-in-process inventory. A number of line configurations involving the following types of imbalances have been examined.

(i) Unequal mean operation times (in case of lines having variable operation times).

(ii) Unequal reliabilities (in case of lines having unreliable stages).

(iii) Unequal inprocess buffers.

(iv) Unequal coefficients of variation of the operation, failure and repair times.

From the simulation results some conclusions have been derived, which would prove useful in the design of flow line systems.

The influence of number of repairmen on the line performance has also been studied. A systematic search has been employed to optimise the size of the inprocess buffers and the number of repairmen, for a given set of cost factors. In addition, the effect of some repair priority rules on the systems efficiency has been compared.

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D. S. Hira

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NOMENCLATURE

A	Availability of a work station
ABL	Average buffer level in an inprocess storage
ABU	Average buffer utilisation
ACT	Average failure cycle time of a stage
AR	Additional Revenue
BL	Buffer level (instantaneous) in a buffer storage
BU	Buffer utilisation (instantaneous)
C	Constant
C_1	Cost of buffer space occupied by one work unit/unit time
C_2	Inventory carrying cost per work unit/unit time
C_3	Cost of material handling facilities/storage point/unit time
C_4	Buffer handling cost/work unit/unit time
C_f	Fixed inprocess buffer cost
C_v, C'_v	Variable inprocess buffer cost
C_r	Repair cost/repairman
CV	Coefficient of variation
D	Delay in production
E(.)	Expected value of .
E	Buffer Effectiveness
FCFS	First come first served
f	Frequency of failures of a stage
G, GAIN	Gain from the system
g	Number of workstations grouped into one stage
HC	Buffer handling cost
IC	Inprocess inventory cost

J,K,K'	Constants
k	Number of work stations in a line
KS	Inprocess storage size as a multiple of mean down time production.
L	Constant
LFS	Longest failure cycle
M	Number of inprocess buffers in a line
MCT	Mean cycle time
MFT	Mean failure time
MRT	Mean repair time
N	Number of stages in a line
n	Sample size
NRM	Number of repairmen
P	Profit per unit produced
p	Probability
p.d.f.	Probability density function
R	Reliability or Efficiency of a stage
REV	Revenue from the system
RPi	Repair policy i
S	Inprocess buffer size when $S_j = S \; \forall \; j$
S_j	Size of buffer storage j
s	Variance
SC	Buffer storage space cost
SE	Standard error
t	Accepted tolerance limit
TC	Total cost
WIP	Work-in-process inventory
WUP	Warm up period

$\bar{x}_d$	Mean difference
$Y_{1-\alpha/2}$	Two tailed standard normal statistic for the probability $1-\alpha$.
Z	Total inprocess buffer capacity $= \sum_{j=2}^{N} S_j$
α	Probability of making type I error or level of significance
β	Sensitivity ratio for buffer size(S/S^*,KS/KS^*)
ξ	Sensitivity ratio for number of repairmen(NRM/NRM^*)
ϕ	Sensitivity ratio for GAIN(GAIN/GAIN*)
μ	Processing time of a stage
τ, τ_S^N	Production rate of a N-stage line having inprocess buffers of size S each
ρ	Processing rate of a stage
η, η_s^N	Efficiency of an N-stage line having inprocess buffers of size S each
η_d	Difference, $(\eta_{SIM} - \eta_{APP})$
$\bar{\eta}_d$	Average value of η_d
σ	Standard deviation
δ	Percentage error $\frac{(\eta_{SIM}-\eta_{APP})}{\eta_{SIM}} \times 100$
λ	Breakdown rate of a stage
$\Delta sys, \Delta\eta$	System deficiency factor $(\eta_{\infty} - \eta_0)$
$\forall$	For all values of
π	Product operator

Suffixes

i	Stage number
j	Inprocess buffer storage number, stage with smallest processing time (PT_j)

SIM	Simulated value
APP	Approximate value
EMP	Empirical value
BD	Value, when operation times are fixed and line is subject to breakdowns.
VT	Value, when operation times are variable and there are no breakdowns.
*	Optimal value

In addition, the following abbreviations have also been used in flow charts :

BDT(I)	Blockade time of stage i
CLOK, KLOK, CLOKK, CLOKMX	Measures of elapsed time
CONT, KONT, KOUNT	Count of units produced
EXP	Exponentially distributed Times
IRT(I), DT(I)	Repair time of stage i
IS(I)	Level of buffer in storage i
ISMN(I)	Minimum level in storage i
ISMX(I)	Maximum level in storage i
MS(I)	Capacity of storage i
PT(I)	Processing time of stage i
PTMN	Minimum processing time
RUN, LOR	Length of run
SB(I)	Service beginning time at stage i
SE(I)	Service ending time at stage i
WT(I)	Waiting time at stage i

CHAPTER - 1

INTRODUCTION

Inventories in production systems have been known to play an important role. A multistage production inventory system would exist whenever a product requires the use of more than one processing facilities. Material processed at one stage and waiting for operation at the next stage, constitutes the inprocess inventory. Depending upon the operational characteristics, the multistage production inventory systems can be of various forms, which can broadly be grouped into batch production and continuous production systems. The object of work inprocess inventory in case of batch production systems is to reduce the cost of production, whereas, in case of continuous production, the inprocess inventory buffers help to reduce the idle time of the stages and increase the utilisation of the system.

The continuous production systems can be classified into the following types :

(i) Job shop systems

(ii) Flow line systems or manual flow lines

(iii) Transfer lines or Automatic production lines.

In a job shop, a workpiece can follow any desired sequence of machines, while in a flow or transfer line, all workpieces must follow the same sequence of operations. The flow lines, commonly referred to as manual flow lines are characterised by random processing times at different stages, whereas, the transfer line comprises of automatic work stations (fixed operation times). The idle time in the transfer line system arises due to random breakdowns and subsequent repairs of the stages, whereas, both the breakdowns and variability in operation times force the stages to idle in case of flow lines.

Three varieties of manual flow lines can be identified as :

(i) Single model lines, on which a single item is produced.

(ii) Multi model lines, on which two or more similar items are produced in separate batches.

(iii) Mixed model lines, on which two or more similar items are produced simultaneously.

Flow lines can also be classified as non-mechanical and moving belt lines. In case of moving belt line the work-pieces may be fixed on the line or removable but the operations are paced, since the belt moves at a constant speed. Operations on non-mechanical lines are normally free of any mechanical pacing effects, hence the use of buffer stocks of items between the workstations is an important feature of such lines. In contrast, physical buffer stocks of items are rarely practicable on the moving belt type lines. The research reported in this thesis has considered the unpaced non-mechanical flow lines only.

There are three general configurations of the automatic transfer lines (also called fixed cycle production lines)

(i) Rotary type or circular indexing type - the flow of work units is usually around a rotating dial or table.

(ii) In-line type - stages are arranged in a straight flow line configuration.

(iii) Link-line - the workstations are linked by conveyors.

The production line is a common material handling and processing device of the modern industry and is used for mass production. Such systems involve huge capital investment and provide little flexibility for the machines and operators to do productive work during the idle periods caused by the imbalances in the system. A large capital investment held idle causes high opportunity cost, and even a small improvement in the utilisation of such system, would have a drastic effect on their overall economy. The design of flow line systems has thus become an important activity in Industrial Engineering.

Three major problems in the design and operation of production lines, as emphasised by Buchan and Koenigsberg [17] are :

(1) sequencing of jobs into stages and determination of the number of stages,

(2) location of bunkers or storage spaces for inprocess inventory,

(3) determination of the size or capacity of the pulsating stores.

A major part of the research embodied in this thesis deals with problems 2 and 3 above. One way of improving the efficiency of the flow line systems is by the provision of in-process buffers, which in turn has associated with it high costs; as the cost of storage space, cost of holding inprocess inventory and material handling costs. Thus, the selection of appropriate number of storage points, the total inprocess buffer and its distribution among the storage points is essential for maximising the gain from the system. The problem of buffer capacity distribution becomes more critical specially when the line is unbalanced.

The requirement of inprocess buffer arises due to the variations in the production rates of the different stages in the line. The variability may be due to a number of factors external and internal to the system. Two causes of variability which depend upon the characteristics of the stages have been considered in literature :

(i) Random variation in operation times of stages.
(ii) Breakdown of the stages.

In majority of the existing research, the two causes of variability have been treated independently by assuming only one of the two to be present at a time. Studies dealing with the breakdown of workstations are confined to the automatic transfer lines only, whereas, the studies related to production lines take into account the variability in operation times only and assume that the stages are 100 percent reliable. However, in the manually operated flow lines, both the sources of variability are present.

To optimise the profits from a flow line system, it is essential to know its efficiency and the average inprocess inventory, in addition to the various cost functions. Analytical models have been employed for determining the efficiency of small systems having two or three stages only and with a small capacity of the inprocess buffers. For the larger real life systems, the only possible approach of analysis seems to be the simulation, which in turn consumes considerably large computation time, and in some cases, the computation cost could make its use uneconomical. Hence the use of heuristic and empirical models.

In the research work embodied in this thesis, simulation models of the following types of flow lines have been developed.

(i) Flow lines with variable processing times.

(ii) Flow lines with unreliable stages (Automatic transfer lines).

(iii) Flow lines with variable processing times and unreliable stages.

The data generated by simulation has been used to develop empirical models for the efficiency of balanced manual flow lines as well as automatic transfer lines. In case of a flow line having variable operation times and stages subject to failures, the simulation model has been employed to test the applicability of a simplified method for determining the efficiency of a system. The empirical models have been used to optimise the size of the inprocess buffers.

Extensive simulation experiments have been undertaken to gain better insight into the behaviour of unbalanced lines. A number of unbalancing policies involving unequal operation times of the stages (in case of manual flow lines), unequal reliabilities of the stages (in case of automatic transfer lines), unequal inprocess buffers, and unequal coefficients of variation of the operation times and reliabilities, have been examined with regard to their efficiency and the average inprocess inventory. Based on the simulation results some guide lines have been developed, which would prove very useful in the design of flow line systems.

In the case of automatic transfer lines, the size of the repair crew, is an important parameter which largely affects the system costs. A search procedure has been incorporated in the simulation model to determine the optimum combination of the crew size and the size of inprocess buffers. In addition, the effect of a number of repair priority rules on the performance of the system has been studied.

The simulation programmes have been written in FORTRAN IV and simulation experiments conducted on DEC-20 computer system.

CHAPTER - 2

REVIEW OF PAST WORK

2.1 INTRODUCTION

Research publications dealing with various aspects of the production inventory systems are quite exhaustive. The survey of literature concerning this has been published by a number of researchers. A good account of the work relating to the inventory theory and applications can be found in Scarf [115], Veinott [128], Iglehart [72], Lampkin [83], and Aggarwal [1]. Research work concerning the problem of economic lot scheduling has been extensively reviewed by Elmaghraby [39]. Clark [32] on the other hand has presented a survey of multi-echlon inventory theory. This covers publications upto the year 1971. Hollier and Vrat [70] have updated the work of Clark, till the year 1975. A review of the research work on multistage production inventory systems can be found in the work of Kumar [82].

Since the literature dealing with production inventory systems is vast, the survey of literature, presented in this thesis is restricted to flow line production systems only. Except some earlier reviews [80,92,114] exhaustive survey of literature dealing with multistage production inventory systems, of the flow line type, is not available.

2.2 CLASSIFICATION OF RESEARCH EFFORTS

Researchers have modelled the flow line systems using different assumptions and a number of analytical techniques have been employed to analyse them. The research efforts in this direction can be grouped under the following categories, depending upon the characteristics of the stages in the line.

(1) Random operation times and no stage failures.

(2) Fixed operation times with random failures.

(3) Random operation times with random failures.

It is quite difficult to classify a particular system, exactly under one of the above categories, however, the author has made an effort to group the available literature, based on the techniques used for analysing the systems. The various techniques used have been identified as analytical, numerical, empirical, heuristic and simulation. However, for the sake of convenience, the available literature can be broadly divided into the following two categories :

(i) Analytical models

(ii) Simulation models

2.3 LINES WITH RANDOM OPERATION TIMES AND NO STAGE FAILURES

In this section the research effort dealing with the flow line systems, in which the stages are assumed to be

100 percent reliable, and having variable processing times defined by some probability density function have been reviewed. The models whether analytical or simulation are normally based on a number of assumptions. Some of the assumptions are related to the,

(i) nature of variabilities in the processing times of the stages, often described by p.d.f. of the processing time

(ii) type of material flow-discrete or continuous

(iii) nature of the input process characterised by infinite queues before the first station with random arrivals of inexhaustible supply of parts to the Ist station

(iv) type of queue formation viz., open ended or cyclic

(v) type of line-balanced or unbalanced.

In addition, other assumptions which are common to all types of models are :

(i) There is always some storage space available in which the last stage can discharge.

(ii) Only one type of product flows into the system.

(iii) No spoilage of workpieces and hence no rejections at any of the stages

(iv) Stages are always set up for the product.

2.3.1 Analytical Models

In analytical models, the system is often described by the various states it can attain. For exponentially distributed service times of the stages, the vector defining the states is a markov process, whose state probabilities, in statistical equilibrium, satisfy a set of linear equations. Solution of these equations yields the steady state probabilities and the production rate is determined by considering all states in which the last stage (N) is busy, and the production rate(τ) that can be attained is given by the equation (2.1).

$$\tau = \frac{p\ [\text{stage N is busy}]}{E\ (\rho_N)} \qquad \ldots\ (2.1)$$

Pioneering studies on the performance analysis of production lines was initiated by Hunt [71]. He considered the effect of interstage storage restrictions by modelling them as infinite queues, finite queues, and no queues between the stages, and developed expressions for the maximum utilisation of the system and the corresponding number of customers in the system.

2.3.1.1 Infinite queues before the stages--random arrivals

Production lines with infinite queues between the stages have been studied by a number of researchers as reported in [21]. A survey of the work done in general area of queues in series has also been presented by Saaty [114]. Most of his work relates to queueing systems with infinite queues, Poisson input and exponential service times, Burke [19] showed that if the input process to the first stage of the system was Poisson, the output process and hence the input process to the next stage would also be Poisson. This result enabled the application to such systems of the well established formulae for single stage, single server queues. Disney, et al. [37], assuming a general service time distribution, have found that the departure process would be a renewal process if and only if the queue is in steady state and waiting line capacity is zero.

Hilderband [62,63] analysed the systems having infinite queues with random arrivals, by separating the effects of various stations with respect to starving, blocking and vacant times, but he could not provide new results pertaining to the system performance or queue lengths. Similar studies have also been reported by Neuts [102,103] and Goode and Saltzman [57].

2.3.1.2 Infinite queue before the first stage and finite interstage queues

A two stage system having infinite queue before the first stage and no queue between the stages has been studied by Avi-Itzhak and Yadin [8]. They assumed the operation times of both the stages either fixed or exponentially distributed and showed that the sequence of the two stages did not influence the output rate of the system. Avi-Itzhak [7] extended the analysis to N-stage system with finite queues and fixed service times of all the stages. A similar investigation was independently conducted by Friedman [46]. Makino [89] slightly extended Hunt's results for two and three stage lines for the case of zero in-process buffers. Analytical methods, employed in the study of finite queues have also been discussed by Patterson [108].

2.3.1.3 Inexhaustible supply to first stage and finite queues between others

According to Hunt [71], for a two-stage system with exponential service times, and interstage storage of capacity S, the production rate can be obtained from equation (2.2)

$$\tau = \frac{\rho_1 \rho_2 (\rho_1^{S+2} - \rho_2^{S+2})}{\rho_1^{S+3} - \rho_2^{S+3}} \qquad \ldots (2.2)$$

For the situation, $\rho_1 = \rho_2 = 1$, using L'Hopital's rule this reduces to

$$\tau = \frac{S+2}{S+3} \qquad \ldots (2.3)$$

Hunt's model for the line with an inexhaustible storage before the first stage has been extended by a number of researchers [61, 65-67]. Hillier and Boling [65] applied the numerical methods to solve a set of simultaneous equations for obtaining steady state probabilities. They admitted that the numerical approach was computationally feasible only for small problems. Motivated by the result of Burke [19] which states that for infinite interstage queues, each stage in the line has Poisson input process, Hillier and Boling [65] developed an approximate method for determining the performance of the system having finite interstage queues. The results obtained were quite accurate for smaller values of N and for larger values of S, but for larger N and smaller S, the approximate method resulted in over estimation.

A three stage system was analysed by Hatcher [61], who obtained closed form expressions for the state probabilities. However, because of lengthy computations involved,his model like that of Hunt [71] was difficult to solve for,beyond three stages, and validity of this model was disproved by Knott [77]. In another study, Knott [78] modelled a production line with identical stations. He used delay in production as a measure of system performance, and suggested an approximate formula, equation (2.4),for computation of system delay.

$$D = (CV^2.X)/(S+J) \qquad \ldots (2.4)$$

where J was a slowly varying function of S, and of the service time distribution and its CV, and

$$X = 2(N-1)\ N \qquad \ldots (2.5)$$

Knott showed that delay D was proportional to X for binomial, uniform, cup and cap shaped service time distributions, while in case of Erlang distribution D/(1+D) was proportional to X. The results of the approximate formula were compared with that obtained by others [4,65], and by Simulation. Simulation results did not support the approximation of Hillier and Boling [65] for larger systems with zero queue capacities, and for the same conditions, considerable difference was found between the simulation and approximate results of Knott [78].

The problem of flow lines with random processing times has also been investigated by Kraemer and Love [81] and Muth [96,97]. Muth's [96] model, unlike others, is based on, not the states of the system, but the service times, blocking periods, and idle times of individual items, as they progressed through the system. He replaced the system of N stages having total buffer of Z work units, by a system of (N+Z) stages with zero buffers. The Z stations having zero processing times, were located in place of buffers in the former line. He established the upper and lower bounds (R_U and R_L) of the mean production rate of the lines as follows :

$$R_U = \frac{1}{\max\left[E(\rho_i)\right]} \qquad \text{... (2.6)}$$

$$R_L = \frac{1}{E\left[\max(\rho_i)\right]} \qquad \text{... (2.7)}$$

A method for determining the intermediate buffers was also developed, but this suffered from computational limitation as in case of other models. Muth further demonstrated that the use of exponentially distributed service times gave highly pessimistic results, whereas, Erlang distribution gave more realistic results.

Disney [36] analysed a large flow line by decomposing it into independent single channel queues and obtained approximate results. Related studies are also reported in Cinlar and Disney [31] and Pritsker [110].

2.3.1.4 Systems modelled for mean cycle time

Some researchers [24,112,113] have developed expressions for the mean cycle time for the system, which is reciprocal of production rate of the line. Buzacott [24] considered a two stage line with deterministic time (T) for the first stage and exponentially distributed operation time (ρ) for the second stage and obtained the following relationship :

$$MCT = T + [\rho.\exp(-T/\rho)] \left[\frac{\exp(-T/\mu)}{1-\frac{T}{\mu}\exp(-\frac{T}{\mu})} \right] \quad \ldots (2.8)$$

Since this model is based on the assumption of a single unit buffer in the line, its applicability is limited. Rao [111] using normal and Erlang distributions for processing times for the two stage system without any interstage buffer, showed that for such a system,

$$MCT = \int_0^\infty (MCT)_1\, g_1(t_1)dt_1 = \int_0^\infty (MCT)_2\, g_2(t_2)dt_2 \quad \ldots (2.9)$$

where $g_1(t_1)$ and $g_2(t_2)$ are the probability density functions of the service times t_1 and t_2; $(MCT)_1$ and $(MCT)_2$ can be obtained by fixing the values of t_1 and t_2 respectively. In another study [112], he assumed exponential service time at the first stage keeping normal or Erlang at the second and combined his approach of [111] with the state probabilities of the system. It was shown that Erlang and normal distributions give very nearly the same results for the same coefficient of variation, while uniform distribution gave significantly different results.

For series production systems with identical Erlang service times, Rao [113] proposed an alternative approach. This involves the solution of the integral equations developed by Muth [96]. The computational burden limits the application of this method to smaller lines only. The number of simultaneous equations to be solved in this case increases with the Erlang shape factor, k.

2.3.1.5 Systems with feed back loop

Koenigsberg [79] and Buchan and Koenigsberg [17] have investigated the buffer storage problem as a system of 'cyclic queue' as illustrated in Fig. 2.1. This forms a closed loop system within which a finite number of units that have completed the service at the last stage return to the first stage. This model is particularly applicable to production lines in which the workpiece is loaded to a jig or fixture or in chemical processes in which a solvent is reprocessed and used again.

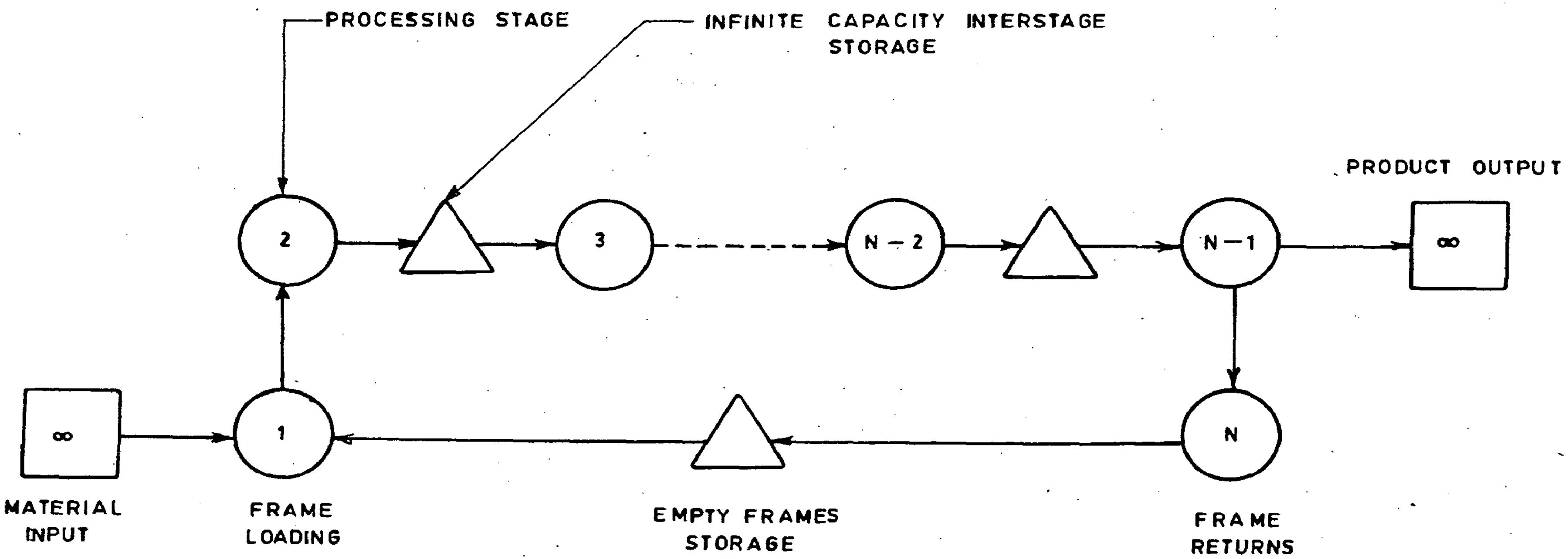

FIG. 2.1 THE FLOW LINE WITH FEED BACK LOOP

For M number of frames, and with no storage restriction between the stages, the utilisation, U, of the line can be given by,

$$U = \frac{M}{M+N-1} \text{ with } U_i = U \ \forall \ i \qquad \ldots (2.10)$$

Equation 2.10 has been used by Basu [11] in his cost analysis model of the manual assembly line. Basu's model gives significantly higher values of the interstage storage, as compared to those obtained by using the results of Anderson and Moodie [4], Hillier and Boling [65] and Knott [78]. As the number of frames is increased, the accuracy of the Basu's formula increases, because the system tends to behave like an open ended queue, with reduced restrictions on the input to the first stage.

2.3.2 Simulation Models

Simulation has been used by a number of researchers to validate their analytical and approximate models, to develop empirical models, and also as a tool to analyse the behaviour of the large and complex systems. Some of the situations where simulation has been used extensively are discussed in the following subsections.

2.3.2.1 Inexhaustible supply to first stage and finite inprocess buffers

One of the earliest simulation study on flow lines is due to Barten [10], who simulated an open ended, balanced production line with inexhaustible supply of material to the first stage, and normally distributed operation times of the stages. He developed equation (2.11) to depict the behaviour of the system and to act as a basis for the simulation programme.

$$D_{N,i} = \left[\sum_{j=1}^{N-1} R_{N-j,i} \prod_{L=1}^{j} P(E^{i}_{N-L}) \right] \left[1-P(E^{i}_{N-(L+1)}) \right] - R_{N,i-1}\, P(E^{i}_{N-i}) \quad @ \qquad \ldots (2.11)$$

where, $D_{N,i}$ is the delay time of N stage line to make i^{th} item

$R_{N,i}$ is the production rate of Nth stage for ith item, and

$P(E^{i}_{N})$ is the probability that stage Nth is busy at time i.

Barten's results show that with increase in storage capacity, the variations in the mean output time decreases. Anderson and Moodie [4] continued this line of investigation by simulating a 5-stage line with equal buffer capacities and by regression analysis of the results, developed the following empirical equations.

$$\text{(Average Delay) } D_{N,S} = \frac{\alpha_1}{S + \alpha_2} \qquad \ldots(2.12)$$

where α_1 and α_2 are functions of N

$$\text{(Average inprocess inventory) } I_{N,S} = -0.13-0.32S+0.98N+0.45NS \qquad \ldots(2.13)$$

@ Symbol π has been used as 'product operator' in this thesis.

Incorporating these two equations alongwith the cost parameters, he developed a model to optimize the size of in-process inventory.

It can be shown that the delay (eqn.2.12) model is similar to the model of Hunt [71] except for the constants α_1 and α_2 which are specific in nature and limit the application of the model to the situation when CV = 0.3. In another study [5] they presented a method of approximating the optimum inprocess inventory by taking averaged value of CV for the lines with unequal coefficient of variation of operation times at various stages. But the results obtained were significantly different from the simulation results. Warnecke, et al. [131] also used the idea of averaged coefficient of variation in their empirical model for the computation of the production rate and average work content in the line. Their model is based on simulation results obtained by using normally distributed operation times with CV $\leq$ 0.4. For the considered situation they found the optimum inprocess buffers size to be between 3 and 4 units.

Knott [78] employed simulation to verify his results obtained from the approximate model, and also checked the results of Hillier and Boling's [65] approximate formula.

2.3.2.2 Lines with infinite inprocess buffers

A 20-stage production line with infinite storage space between the stages was simulated by Payne, et al. [109].

They showed that for the situation when the line was balanced, stations towards the end incurred more idle time as compared to those towards the beginning of the line. However, these results included the transient effects, as the interstage stores were assumed to be empty at the start of the simulation run.

2.3.2.3 Random arrivals to first stage

Mcgee and Webster [91] simulated a two stage system where, in addition to the operation times, the interarrival times were also independently and normally distributed, but were not necessarily equal. Allowing infinite buffer capacity between the stages, they tried to obtain a suitable distribution for the output process from the first stage. If the input and output processes of a stage could be determined, the whole line could be analysed by studying the stages independently. Though the study did not lead to any concrete suggestions, the results did indicate that, the output process was not identical to the input process. They developed a regression equation for the expected time, an item spends in the system,as a function of mean interarrival time and mean service times. However, this equation has limited applicability.

2.3.2.4 Some case studies

A three stage conveyor serviced system was simulated by Crisp, et al. [33]. They concluded that the general assumption of stationary Bernoulli distribution of units on conveyors is not established. A cotton spinning system, characterised by a series of stages with different number of parallel machines at each stage has been simulated by Hollier and Satir [69], wherein, they investigated the influence of various technological and operational factors on the performance of the system.

2.3.3 Unbalanced Lines

A balanced line has been defined as the one having identical operation time distributions at various stages and provided with equal capacity interstage buffers. A line would be defined as unbalanced, if it has any one or more of the following unbalancing properties :

(i) Unequal mean operation times.

(ii) Unequal variances of the operation times.

(iii) Unequal capacities of the interstage buffers.

For comparing the performance of unbalanced lines with the balanced lines or within the unbalanced lines, the sum of variable parameter over the length of the line is kept a constant, i.e.,

$$\sum_{i=1}^{N} \mu_i = N.\mu$$

$$\sum_{i=1}^{N} CV_i = N.CV$$

$$\sum_{j=2}^{N} S_j = (N-1).S.$$

Unless mentioned otherwise, the assumptions made in section 2.3, hold here also.

In the year 1966, Hillier and Boling [64] demonstrated that the performance of a production line could be improved by deliberately unbalancing it. As a consequence of this, a number of research papers dealing with the unbalanced lines appeared in literature. Exponential or Erlang distributions of operation times have generally been assumed in the analytical models, while normally distributed timeshave commonly been employed in simulation studies.

2.3.3.1 Unequal mean operation times

Hillier and Boling [64] analysed symmetrically unbalanced lines, having unequal operation times at various stages, and demonstrated that a small line with middle stages slightly faster than the outer ones, is more productive as compared to a fully balanced line. They termed it as the 'Bowl Phenomenon'. In subsequent publications [66-68], they reported the effect of the number of stages, capacities of interstage buffers, and the variances of the operation times on the performance of unbalanced lines. They showed that the advantage gained by unbalancing the line increases for larger lines and diminishes with increase in $S(S_j = S \; \forall \; j)$ and decrease in CV $(CV_i = CV \; \forall \; i)$. Magazine and Silver [88] used heuristic procedure to obtain the production rate of a system, and obtained results similar to Hillier and Boling [64]. They suggest that largest gain due to unbalancing occurs for intermediate values of N(= 5 to 20) and small values of S(=0) and k(=1). For other situations, they did not consider it computationally economical to search the optimal unbalanced allocations. The bowl phenomenon has also been confirmed by some other researchers [29,98,126,136]. Muth and Mehta [99] demonstrated that the conjecture of reversibility i.e.,

$$R(W) = R(W') \qquad \ldots(2.14)$$

where, $W = \mu_1, \mu_2, \ldots \mu_N$

and $W' = \mu_N, \mu_{N-1} \ldots \mu_1$,

holds for a general class of production lines.

Payne, et al. [109] based on their simulation study confirmed the results of Davis [34], who was of the opinion that in order to achieve maximum output, very fast, fast and slow stages should respectively be placed in the beginning, middle and at the end of the line. Their study, however, includes the transient effects, since they started the line with empty buffers and did not impose any restrictions on their maximum capacity.

2.3.3.2 Unequal variances of operation times

The consequences of positioning a single workstation having high variance as compared to others, at various positions in a line was investigated by Kala and Hitchings [75].

They proposed that station with largest variability should be put at the end of the line. On the other hand Payne, et al. [109] determined the maximum deterioration in line performance, when the variance of operation times increased progressively towards the end of the line. These two studies imposed no restrictions on interstage storage capacity. Flow lines with finite interstage buffers, and unequal variabilities of operation times defined by Weibull distribution was simulated by Carnall and Wild [29] and by Wyche and Wild [136]. They observed that line with less variable stages in the middle performs better, and that placement of steadier and variable stages alternatively gave results closer to the balanced line.

Paced and unpaced flow lines having operators with different service time distributions have been investigated by Buffa [18] Sury [123-125], Davis [34] and Buxey and Sadjadi [20]. In each case the provision of inprocess buffer and increase in tolerance time, resulted in vast improvements in the system output and operator utilisation. Buxey and Sadjadi [20] discussed alternative methods of incorporating buffer storage space to complement tolerance time in paced assembly lines, thereby reducing the frequency of 'misses' and increasing the productivity.

Wild and Slack [134] investigated the advantage of substituting one line with two operators at each stage, for two lines with one operator at each stage, and found the arrangement beneficial in case of lines having large number of stages, low inprocess buffer capacity and large operator time variability.

2.3.3.3 Unequal interstage buffers

The influence of providing unequal interstage buffers, was examined by Hatcher [61]. He determined analytically that equal size buffers lead to optimum performance in a three stage line having identical operation times and recommended that, for maximum gain, if an additional unit is to be added to the inprocess buffer, it should be added to the second bunker. This result was however, contradicted by Knott [77]. While Payne, et al.[109] questioned the propriety of providing equal buffers in the line, Carnall and Wild [29] and Wyche and Wild [136] suggested that buffers should either be uniformly distributed or concentrated slightly in the middle of the line. They further showed that the values of buffer allocation strategy were independent of the length of the line or the degree of station time variability. Smith and Brumbaugh [120] determined that the positive effect of unequal buffers, in the three stage line, was more significant at lower levels of inprocess inventory.

2.3.3.4 More than one parameter unbalanced

Effect of more than one factor leading to imbalance in the line has also been considered by some researchers [29,112, 120]. Rao [112], based on the results of his analytical study of a two stage line, suggested that the efficiency of a line could be improved by assigning slightly higher work loads to the less variable stages. Smith and Brumbaugh [120] and Brumbaugh and Smith [14] based on a simulation study of 3-stage line, proposed that largest buffers be placed in the vicinity of the more variable stages. On the other hand, Carnall and Wild [29] observed that the effect of buffer distribution was independent of the station time variability.

2.4 LINES WITH FIXED OPERATION TIMES AND SUBJECT TO BREAKDOWNS

The research work relating to the flow lines where the processing times of the stages are deterministic, but the stages are subject to random breakdowns, has been reviewed in this section. Some of the assumptions which have generally been made by the various researchers in this area, are :

(1) The first stage is never starved and the last is never blocked.

(2) Only one type of product flows into the system.

(3) Up times and down times of the stages are independent of each other.

In most of the analytical studies, the failure (up) and repair (down) times have been assumed to be exponentially or geometrically distributed, whereas exponential distribution has mostly been used in simulation studies.

The various models differ in their assumptions about;

(1) Causes of failures - operation dependent or time dependent failures.

(2) Limitations on the number of stages being down at a time - simultaneous repairs allowed or not allowed.

(3) Repair priorities.

(4) Work transfer mechanism - sequential relay or serial relay.

2.4.1 Analytical Models

2.4.1.1. Automatic transfer lines with zero or infinite buffer

The efficiency of automatic transfer lines with no in-process buffer and with interstage buffers of infinite capacity, has been studied by many researchers [9,23,59,96]. For operation dependent failures (i.e. a stage can not fail when in forced down state), the efficiency of the line without in-process buffer, can be obtained from the equation 2.15 (Muth [96]).

$$\eta_o = \left[1 + \sum_{i=1}^{N} \frac{1-R_i}{R_i} \right]^{-1} \qquad \ldots(2.15)$$

When failure of the stages are time dependent i.e., the failure mechanism remains active during the forced down time as well [27, 127], the system efficiency is given by,

$$\eta_o = \prod_{i=1}^{N} R_i \qquad \ldots(2.16)$$

Automatic transfer line without inprocess buffers has also been studied by Groover [59]. He modelled it for the frequency of stops per cycle (F) caused by the stage failures. If p_i is the probability that a part would jam at stage i due to stage breakdowns, and each failure caused the workpiece on the stage to be rejected, then

$$F = 1 - \prod_{i=1}^{N} (1-p_i) \qquad \text{... (2.17)}$$

for a case where breakdowns do not cause the part to be removed from the line,

$$F = \sum_{i=1}^{N} p_i \qquad \text{... (2.18)}$$

and the efficiency of the line could be expressed as

$$\eta = (MFT)/(MFT+F\ (MRT)) \qquad \text{... (2.19)}$$

Difficulty in the use of the equations (2.17 - 2.19) is mainly in obtaining accurate values of p_i for various stages.

In case of lines having infinite capacity buffers, the line efficiency is determined by the most inefficient stage [22] and is given by equation (2.20).

$$\eta_{\infty} = \min_{1 \leq i \leq N} [R_i] \qquad \text{... (2.20)}$$

Okamura and Yamashina [105] tried to use equation (2.20) in their model of a two stage line, and observed that this equation does not always hold true. The disagreement as brought out by Shanthikumar [117], resulted from Okamura and Yamashina's assumption that the workpiece on a machine when it fails, is scrapped. In Buzacott's model [22], whereas, the failures do not cause the workpieces to be scrapped. Shanthikumar derived a generally applicable formula for the production rate of lines with infinite buffers and with possible rejection of workpieces.

2.4.1.2 Lines with finite inprocess buffers

Production lines having automatic stages and finite inprocess buffers, have been analysed by several researchers [3,22,26,48-51,53-55,73,86,116]. Finch, as reported by Buchan and Koenigsberg [17] modelled the effect of inprocess inventory on the output of a production line having exponential failure and repair times. He considered the failures of the stages to be time dependent and number of repairmen to be sufficiently large, so that a stage does not wait for repairs. His model of two stage line showed that gain in output increased with increase in inprocess buffer upto a certain extent and then became constant.

Another study on production lines with inprocess storage, quoted by Buchan and Koenigsberg [17] is due to Vladzievskii, who assumed that only one station could be down at a time. He employed embedded Markov chain approach to obtain the probability distribution of the inventory levels, from which a loss transfer coefficient and the efficiency of the line could be estimated. The applicability of this approach is limited, since it does not account for the storage space required in the forward direction in order to allow output from previous stage to be stored, when the succeeding stage is occupied or down. Sevastyonov [See 27] extended Vladzievskii's model to lines with non-identical stages. But because of the assumption of loss propagation in forward direction only, the model is of limited value.

Assuming identical repair time distributions, and operation dependent failures, Buzacott [22] developed an analytical model for the effectiveness of buffer in a two stage line. He found that for the same performance of the line, the buffer capacity required for a geometric repair time distribution was about double of that required for a constant repair time distribution. The analytical approach, however, could not be extended to larger systems, and instead an approximate method was developed. Buzacott [22] assumed that the probability of two events, failures or repairs, in one cycle was negligible. He extended the analysis [25] to the exact operation dependent failures in which more than one event could occur in a cycle, and presented the following model for the efficiency of a two stage line..

$$\eta = \frac{2 - \gamma + KS(1+x)}{2(1+2x) - \gamma(1+x)^2 + KS(1+x)^2} \qquad \ldots(2.21)$$

where, $x = (1 - R_i)/R_i$ ...(2.22)

In another study Buzacott [24] developed a simple model to show that for a two stage line, similar buffer capacities were required when one of the repair or breakdown times were random, and that the buffer capacity requirements were almost doubled, when both were random. He suggested that the line in general be divided into 4 or 5 stages only, and the buffer capacities should not be in excess of 4 or 5 times the mean repair time production of the stage.

From analytical studies of a two stage line, Okamura and Yamashina [105, 106] concluded that the difference in breakdown rates of the stages, reduced the effect of in-process buffer, whereas the differences in repair times did not.

Two stage automatic transfer lines having geometrically distributed failure and repair times and finite inter-stage buffer, have also been modelled as Markov chains by Gershwin [48], Gershwin and Schick [54], Gershwin and Berman [51], Berman [12]. Schick and Gershwin [116] obtained the steady state probability distribution of the states of the Markov chain, which could in turn be used to determine the average efficiency and the average inprocess inventory of the system. The proposed method is computationally more efficient than the conventional method of solving the linear transition equations for the states of the Markov chain. Gershwin and Schick [53, 55] extended the application of their method to three stage lines with two inprocess buffers. However, for larger lines and for larger capacity buffers, their technique too was found to be computationally infeasible because of high computation time, memory and precision requirements [53].

The methodology proposed in [53,55] has been employed by Ammar [3] to the study of assembly merger networks (AMN). Treating transfer line as a special case of AMN, he confirmed the result of reversibility of transfer lines [68, 98,99], according to which the efficiency of the line remains unchanged when the order of stages in the line is reversed.

Ignall and Silver [73] making use of the known results of Barlow and Proschan [9] for lines without buffer (equation 2.15), and of Buzacott [23] for lines having infinite inprocess buffer (equation 2.20) developed an approximate model to predict the output rate of a two stage line, where each stage comprised of a number of automatic machines. His model is characterized by the following equation.

$$\tau_s = \tau_o + (\tau_\infty - \tau_o) m(s) \qquad \ldots (2.23)$$

where $m(s)$ is the weighting factor increasing from $m(0) = 0$ to $m(\infty) = 1$, as S increases, and is given by equation (2.24)

$$m(S) = \max\left(\frac{\lambda_1}{\lambda_2}, 1\right) \frac{\left(1 - \left(\frac{\lambda_1}{\lambda_2}\right)^{gsr}\right)}{1 - \left(\frac{\lambda_1}{\lambda_2}\right)^{gsr+1}} \qquad \ldots (2.24)$$

where, g is a function of the repair time distribution.

A two stage line having deterministic but unequal production rates has been modelled by Wijngaard [133]. He employed negative exponential distribution for the failure and repair times of the stages, and assumed that stage 2 starved and stage 1 blocked, had the same failure rates as running. For the case $\mu_1 = \mu_2 = 1$, he obtained

$$\tau = \frac{\gamma_1 \gamma_2}{(\lambda_1+\gamma_1)(\lambda_2+\gamma_2)} \left[1 + \frac{\lambda_2}{\gamma_2} \left\{ 1 - \frac{\frac{\lambda_1}{\gamma_1} - \frac{\lambda_2}{\gamma_2}}{(\frac{\lambda_1}{\gamma_1}) \exp(r_f - r_g)S - \frac{\lambda_2}{\gamma_2}} \right\} \right] \quad \ldots (2.25)$$

where, $r_f - r_g = (\frac{\lambda_1}{\gamma_1} - \frac{\lambda_2}{\gamma_2})\ \sigma$

$$\text{with} \quad \sigma = \gamma_1 \gamma_2 \frac{(\lambda_1+\lambda_2+\gamma_1+\gamma_2)}{(\lambda_1+\lambda_2)(\gamma_1+\gamma_2)} \quad \ldots (2.26)$$

This is similar to the result obtained by Finch, given in [17], except that the value of σ differs, which according to Wijngaard was due to some error in Finch's equation.

2.4.1.3 Models with deterministic failure and repair times

Transfer lines with deterministic failure, repair, and operation times have been modelled by Canuto, et al. [28] and Villa, et al. [129], for unequal production rates of the stages. Their models have been based on the analysis of a module comprising of two stages with an intermediate buffer. The cumulative effect of all modules in the line, has been analysed by analytical as well as graphical methods, to determine the size of the inprocess buffers for the desired production rate.

2.4.1.4 Models in which simultaneous repair of stagesis not permitted

Fox and Zerbe [43] analysed a line in which some stages could have buffers, while some others could be provided with back up units. Their methodology consisted of determining the equivalent availabilities of different stages, which were then substituted in Barlow and Proschan's [9] model of system availability for lines with zero in process buffers. They also assumed that each buffer (surge) protected only one stage on its downstream side, and that a repaired stage could speed to restore the surge. In their two stage system, they assumed the second stage to be much faster as compared to the first, such that buffer was emptied after each repair.

Murphy [93] extended Fox and Zerbe's [43] model and considered the case where the protected stage (stage with buffer), could fail before its upstream buffer is emptied. He treated the buffer level as a random variable and developed recursive procedure to estimate its expected value (T). The same was used in conjunction with the results of Fox and Zerbe to obtain approximately the effect of buffer on system availability. Murphy [94] further modified the methodology by improving the assumption that 'repair' instead of 'operation or repair' of the output device is suspended in the event, the input device fails. He presented an approximate method to estimate the output of a line having more than one inprocess buffer. In another study [95], he showed that the stochastic behaviour of buffer protection (T), in a single buffer series system could be modelled with arbitrary closeness of a Markov chain, and presented a method of obtaining the limiting distribution of T, as well as its transient probabailities. It may be pointed out that the common assumption that more than one stage cannot be repaired at a time [43,93-95] is quite restrictive and would not hold true in majority of the larger lines, since the number of stages down at a time, will increase with the length of the line, and also with increase in the in-process buffer capacity. Similarly the assumption [94,95] that operation of the output device is suspended in case the input device fails, is not valid in practice, and can be made only for the purpose of computational tractibility.

A two stage system similar to the one discussed in [93-95] has been analysed by Bryant and Murphy [15,16] with slightly less restrictive assumptions. However, they also did not allow simultaneous repairs. They gave priority to the repair of the slower stage, i.e. in the event of the failure of the slower stage, repair of the faster stage is suspended.

2.4.1.5 Location of inprocess buffer in automatic transfer lines

The problem of location of the inprocess buffer storages in automatic transfer lines has been studied by a number of researchers [22,121,122]. In a line with even number of identically reliable machines, the optimal placement of a single buffer is exactly in the middle [22]. A formal proof of this has been given by Soyster and Toof [122]. For a line with stages not necessarily identically reliable, the single buffer (j^*) should be so placed as to minimise the absolute value of the difference between the reliabilities of the two parts of the line [85].

$$\prod_{i=1}^{J^*} R_i - \prod_{i=j^*+1}^{N} R_i = \min \left| \prod_{i=1}^{j} R_i - \prod_{i=j+1}^{N} R_i \right| \qquad \ldots(2.27)$$

For the location of two or more buffers in the line, Lev and Toof [85] evaluated the steady state probabilities from the one step transition probability matrix. However, this approach is feasible for small problems only. Buzacott [22] proposed that inprocess buffers should be located so that the stages (groups of work stations between the two successive buffers), have approximately the same breakdown rate.

2.4.1.6 Sequential relay model

A sequential relay automatic transfer line in which all events including station failures, repairs and transfer of parts occured at the same point (epoch) in time, has been considered by Sheskin [118]. He presented an approximate algorithm for predicting the output rate of a line, and obtained numerical solutions for three and four stage lines. Sequential relay model has also been employed by Soyster and Toof [122] in the analysis of a two stage transfer line. They have derived the equation (2.28) for the computation of the probability ($f^*(S)$) of obtaining a unit per cycle from the system, having inprocess buffer of capacity S.

$$f^*(S) = \begin{cases} q_1 q_2 (1-\phi^S)/(q_2 - q_1 \phi^S) & \text{if } q_1 \neq q_2 \\ qS/(S+1-q) & \text{if } q_1 = q_2 = q \end{cases} \quad \ldots (2.28)$$

where,

$$\phi = \frac{q_1(1-q_2)}{q_2(1-q_1)}$$

q_1 and q_2 are the probabilities of the respective stages being in order.

Soyster, et al. [121] extended the work of Soyster and Toof [122] to N stage production lines, and computed the upper and lower bounds on the steady state reliability $f^*(S)$ (eqn. 2.29).

$$\prod_{i=1}^{N} q_i \leq f^*(S) \leq \min_{1 \leq i \leq N} q_i \quad \ldots (2.29)$$

In order to obtain a compromise solution between the upper and lower bounds, the use of the following surrogate function for the system availability has been proposed [121].

$$g(S) = \prod_{i=1}^{N-1} f_i(S_i)^{[\min q_i, q_{i+1}]^{-1}} \qquad ...(2.30)$$

where, $f_i(S_i)$ is the steady state reliability of the two stage subsystem comprising stages i and i-1 with interstage buffer of capacity S_i. Evaluation of the objective function and its upper and lower bounds was however, possible by simulation only.

2.4.2 Simulation Models

Simulation has been used, in the context of automatic transfer lines, mostly for the purpose of validation of analytical and approximate models [60,73,94,105,121,137] and sparingly for the purpose of laying down heuristic rules for the determination of the system performance [45,84,90].

Using the actual data from a production line, Hanifin et al. [60] carried out simulation studies pertaining to the effect of inprocess buffer on the output rate of a two stage line,having unequal reliabilities of the stages. Comparison of his results with the analytical data of Buzacott [25] revealed that the analytical results were quite different from the simulation results. The difference was small when the inprocess buffer was zero and increased with the size of the buffer.

Murphy [94] and Ignall and Silver [73] employed simulation technique to validate their approximate predictive models of two stage balanced and unbalanced lines. Soyster, et al. [121] integrated their analytical model of the optimisation process into a simulation model for the purpose of evaluation and comparison. A two stage unbalanced line was simulated by Okamura and Yamashina [106] for obtaining some useful qualitative results. They showed that the buffer was most effective in a balanced line, and that the effectiveness of the buffer decreased with increase in imbalance.

A three stage automated production line with exponentially distributed failures and repair times of the stages was simulated by Freeman [45]. He determined, that like a two stage line, the buffer capacity required to achieve a predetermined gain in the efficiency of a three stage balanced line, is a linear function of λ/γ. He also used simulation to study the behaviour of unbalanced lines. Masso and Smith [90] employed simulation technique to find the total inprocess buffer required in a three stage line, having constant production rates and exponentially distributed repair and breakdown times of the stages. They developed linear regression equation (2.31) for the required buffer capacity, as a function of the system deficiency factor (Δ_{sys})

$$Z = K \cdot \Delta_{sys} \qquad \ldots (2.31)$$

The value of the constant K, is specific to the problem considered. They further developed an approximate method based on the availabilities of the stages, to distribute the total buffer capacity between the two interstage storages. The analysis was limited to three stage lines, with time dependent failure, and hence is of limited practical value.

2.4.2.1 Some case studies

Singh, et al. [119] simulated the working of a flow shop with 40 machines having exponentially distributed inter-breakdown and repair times. They studied the effect of buffer on the performance of line and proposed a buffer stock policy, along with the installation of some backup units, to improve the system output.

An automatic bottling plant of a brewery was simulated by Kay [76]. The bottling line comprised of 5 machines connected by belt conveyors. It was shown that a moderate improvement in buffer resulted in drastic improvement in system efficiency. The efficiency however, falls as the buffers are enlarged still further.

2.4.3 Some Results of Unbalanced Transfer Lines

A transfer line having identical stages with regard to their breakdown and repair times and provided with equal in-process buffers is considered to be balanced. Thus, if any one or more of the following unbalancing properties are present, the line is said to be unbalanced;

(i) Non-identical probability density functions of failure times of stages.

(ii) Non-identical probability density functions of repair times of stages.

(iii) Unequal inprocess buffer capacities.

The analytical models reviewed in section 2.4.1 has generally been developed by considering unequal up and down times of the stages, and the formulae for balanced lines have been derived as special cases. In the case of two stage line, since there is only one buffer, the line can be unbalanced by having non-identical reliabilities of the stages. It has generally been shown that the balanced lines are the most efficient, and that the effect of inprocess buffer decreases as the imbalance between the stage is increased. There are only a few available studies, analytical as well as simulation, which relate to more than two stage lines. In this section some important results pertaining to the unbalanced transfer lines have been presented.

Freeman [45] has recommended that a bad stage in the line should be surrounded by good stages, and that larger buffers should be placed close to the bad stages. He further observed that placing of a bad stage at the end of the line is more critical. Sheskin [118], on the other hand, analytically demonstrated, that a bad stage put either in the beginning or the end of the line has same effect in case of symmetric lines with reliabilities arranged in increasing or decreasing order. This result has also been proved by Ammar [3] and Gershwin and Ammar [50].

Okamura and Yamashina [105] concluded that the effect of interchanging the two stages, which have different production rates is almost negligible. However, they suggested that, when the difference in the production rates of the two stages is large the faster stage should be located first. In another study [106] they determined that the difference in breakdown rates reduces the effect of installing a buffer, while that of repair rates does not.

The studies on two stage lines support the intuition that the line with identically reliable stages is the most efficient. For larger lines, Buzacott [24] is of the opinion that there is a slight advantage in placing stages with some what higher breakdown rates in the beginning or the end of the line.

Sheskin [118] determined that for lines having identically reliable stages, the production rate could be maximised by providing buffers of approximately equal capacity, and in case of non-identical stages larger share of buffer be allocated to less reliable stages. The later result of Sheskin has also been confirmed by Soyster, et al. [121], Ohmi [104] and Okamura and Yamashina [105]. Ohmi [104] proposed that locally averaged allocation of buffers be made in order to achieve maximum yield from a line with stages of different reliabilities.

2.5 LINES WITH RANDOM OPERATION TIMES AND SUBJECT TO BREAKDOWNS

Extensive search of the published literature has revealed that only two research papers have been published which deal with the combined effect of variable processing times and breakdowns of the stages in a line. Buzacott [26] using Markov chain analysis developed an exact analytical model for computing the production rate of a two stage system. Gershwin and Berman [52] also used the Markov chain to model a two stage system. Their model differed from that of Buzacott [26] in respect of the assumption regarding the failure mechanism. Buzacott assumed geometrically distributed failure times, whereas Gershwin and Berman employed exponential distribution for the same. The processing and repair times were exponential in both the cases. Since the analytical model was found to be very cumbersome to use, Buzacott [26] proposed an approximate method for determining the average cycle time of the system. The method comprised of combining the analytical model valid for fixed processing time and random failures [22] with that of Hillier and Boling's [65] for exponential processing times and no breakdowns. The mean cycle time, then could be approximated by,

$$MCT = \frac{1}{\rho} + \left[(MCT)_{VT} - \frac{1}{\rho} \right] + \left[(MCT)_{BD} - \frac{1}{\rho} \right] \quad \ldots (2.32)$$

where, $MCT_{VT} = \frac{1}{\mu} \cdot \frac{S + 3}{S + 2}$ ... (2.33) and $$(MCT)_{BD} = \frac{1}{\mu} \left[1+ \frac{\alpha}{\gamma} + \frac{\alpha(2-\alpha-\gamma)}{\gamma(2+(S-1)(\alpha+\gamma)}\right] \qquad \ldots (2.34)$$

Where, α is the probability of a breakdown during the processing time μ.

The results of the approximate model compare closely with the results of analytical model in case of two stage system. However, the validity of the approximate method for larger systems has not been verified.

2.6 MODELS WITH REPAIR PRIORITIES

In the models of automatic transfer lines subject to breakdowns, reviewed in section 2.4.1.1 to 2.4.1.3, the number of repairmen employed has been assumed to be sufficiently large, so that no stage has to wait for repairs. In section 2.4.1.4, some two stage models have been discussed in which only one stage could be repaired at a time. The repair priority rules in [9,43,93-95] has been employed only for the purpose of computational tractability, and not as a parameter affecting system performance. Very few studies [38, 40] have been devoted to examine the effect of repair priority rules on the production efficiency of the system. Dudick [38] analysed a two stage transfer line having inprocess buffer and considered three repair policies, viz., repair priority to Ist stage, repair priority to 2nd stage and priority dependent upon buffer occupancy. Elsayed and Turley [40] also analysed a two stage system, in which each stage could fail in two modes. They compared three repair policies in which breakdowns were serviced according to (i) First come first served (FCFS) rule, (ii) priority to type I failures, and (iii) priority to type II failures. They found that the FCFS rule yielded better results.

2.7 PROBLEM FORMULATION

From the review of literature, the following observations can be made :

(1) Two types of flow lines, one having random operation times and completely reliable stages (manual flow lines) and second having fixed operation times, but stages subject to breakdowns (automatic transfer lines), have generally been treated separately, though in practice, most of the manual flow lines have variable operation times and are subject to random failures.

(2) For the types of flow lines above, analytical models could be developed for 2 or 3 stage systems only. Because of computational burden, use of numerical methods is also restricted to smaller problems only.

(3) The simulation technique has generally been employed to analyse the specific situations, or to validate the analytical and approximate models. In a few cases, simulation data has been used to develop empirical models, which are generally specific to the situations considered.

(4) The work relating to unbalanced lines is comparatively limited, and in many cases, the available results are contradictory.

(5) The effect of number of repairmen employed, and of the repair priority rules have not been given much attention.

(6) In most of the investigations on balanced and unbalanced lines, the system has been modelled for predicting the production rate or the production efficiency of the lines. The work-in-process inventory, and the size of the inprocess buffers, has not received the due attention.

In this thesis, an attempt has been made to investigate large, balanced and unbalanced, flow line systems, with regard to their production efficiency and the WIP inventory. System simulation has been employed as the tool of analysis, and simulation models have been developed for the following cases:-

(i) Flow lines having variable operation times and no stage failures.

(ii) Flow lines having fixed operation times, but stages subject to random failures.

(iii) Flow lines having variable operation times and stage subject to random failures.

Based on the simulation data, empirical models have been developed, for the system efficiency and the optimum size of the inprocess buffer storages, valid for balanced lines of the types (i) and (ii) above.

An attempt has been made to conduct a thorough investigation of the unbalanced lines, and to recognise the line layouts and inprocess buffer distribution policies, which may result in maximisation of the system efficiency and help to keep the WIP inventory low.

An approximate formula for predicting the efficiency of a balanced flow line having random operation times and unreliable stages, has been examined and found valid over a wide range of system parameters. A search procedure has been presented to optimise the size of interstage buffers.

The influence of the number of repairmen on the line performance has been studied. A search procedure has been presented to optimise the size of inprocess storages and the number of repairmen. A number of repair priority rules have been compared to determine the best one.

CHAPTER-3

SYSTEM DESCRIPTION AND DESIGN OF SIMULATION EXPERIMENTS

3.1 INTRODUCTION

In this chapter, a general description of the flow line system has been presented and the various terms used have been defined. The assumptions that are common to most of the chapters have been given. This chapter also presents some of the important features of the analytical technique employed.

3.2 STRUCTURE OF THE SYSTEM

The system considered for analysis comprises of a number of workstations arranged in series, so that each work unit passes through all the stations in a fixed sequence. In between some or all the workstations inprocess inventory or buffers are provided (Fig.3.1). The stations grouped in between the inprocess buffers, but having no buffer within the group are regarded as one stage. The line is of open ended type with an inexhaustible supply of material before the first stage and there is infinite space to accomodate the output from the last stage.

At each stage certain operations are performed on the work material. The specific nature of operations is of little consequence in the present analysis. However, the operation times of the stages may be fixed or random. Each stage is also liable to breakdowns. Since several factors are responsible for the failure of a particular stage, the downtimes of the failed stages, like

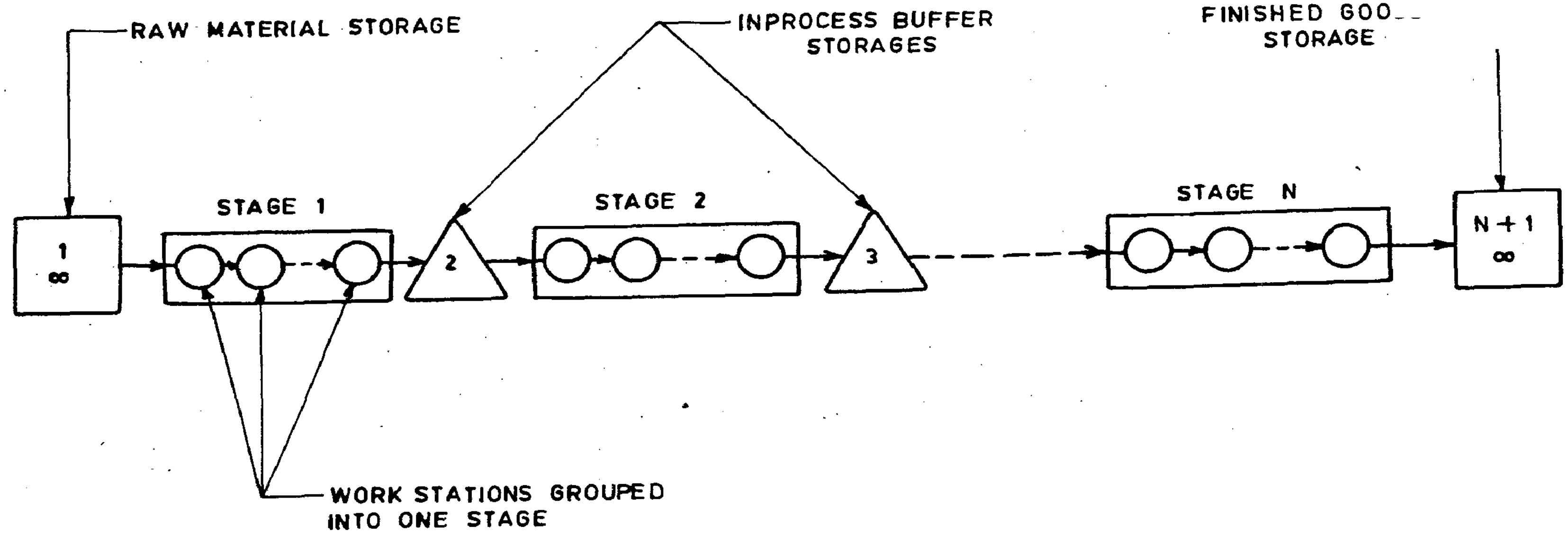

FIG. 3.1 SCHEMATIC REPRESENTATION OF THE FLOW LINE SYSTEM

the uptimes of the operating stages are random variables. The variability in the operation times and the failures of the stages, cause interference between the working of the stages, giving rise to production delays due to forced down times. If a stage is forced to stop due to non-availability of work material in its upstream storage, the stage is said to be starved. Similarly when a stage is forced to stop due to its downstream buffers being full, it is referred to as blocked. These effects propagate up and down the line, if the interference between the stages is large.

Interstage buffers partially decouple the adjacent stages and help to mitigate the effect of failures and variability in the operation times of one stage on the operation of others. However, when the buffers are empty or full, this decoupling can not take place. Thus, as the capacities of the inprocess buffers increase, the probability of their being empty or full decreases, and the efficiency of the line increases. However, increase in size of buffers, give rise to the inprocess inventory, since more partially completed material is present between the stages.

3.3 STAGE CHARACTERISTICS

3.3.1 Processing Times

The processing or operation time of a stage is defined as the time taken by the stage to process one work unit. It is reciprocal of the production rate of the stage. In case of automated lines, the processing times of various stages may be constant, but in case of manually operated lines, the processing time will vary from operator to operator and from time to time. In most of the analytical studies, exponential distribution has been used to define the processing times [7,20,61,65-67,114]. Some researchers [4,78,96] consider it to be highly pessimistic. Normal distribution has generally been employed to define the processing times in simulation models [4,10,18,20,109,131]. In the present work both the Erlang and normal distributions have been considered.

3.3.2 Reliability of a Stage

If a stage is completely decoupled from the adjacent stages, the fraction of time, it is expected to be operational, defines the availability of the stage. This has also been used as a measure of the efficiency of an isolated stage, and is given by equation (3.1).

$$R_i = \frac{MFT}{MFT + MRT} \qquad .. \qquad (3.1)$$

The time unit adopted is the average time required by a stage to process one work unit. The up and down times, as defined below have been measured in these units.

3.3.3 Mean Failure Time (MFT)

The average number of operations (work units) completed between two successive breakdowns.

3.3.4 Mean Repair Time (MRT)

The average number of operations (work units) which could be completed during the time, the stage remained under repair.

Note that the terms MFT and MRT do not include the forced down time of the stages or the time for which a failed stage may have to wait before its repair is undertaken.

3.3.5 Mean Cycle Time (ACT)

If the number of repairmen are sufficiently large, so that the repair could start as soon as a stage breaks down, then the average time between two successive failures of an isolated stage is defined as the cycle time.

$$ACT = MFT + MRT \qquad .. \qquad (3.2)$$

3.3.6 Failure and Repair Times Distribution

The failure and repair times of a stage are random variables. Various types of distributions have been used by researchers to define these terms. The selection of the distribution mainly depends upon the analytical technique employed. Geometric and exponential distribution have commonly been used in analytical models, whereas a variety of distributions have been used in simulation studies.

Each stage of the system comprises of a group of several types of machines and components and since each of them have their own failure characteristics, the resulting overall failure distribution for such a complex system could closely be approximated by exponential distribution. This phenomenon has extensively been analysed and discussed by Von Alven [130].

3.4 LINE EFFICIENCY

The steady state efficiency of the flow line has been defined as,

$$\eta = \frac{\text{Actual output over time T}}{\text{Ideal output over time T}} \; ; \; T \rightarrow \infty \qquad .. \quad (3.3)$$

where ideal output is the output of a hypothetical line, having 100 percent reliable stages and inprocess buffers of infinite size.

Since the output from the line is the number of units discharged from the Nth stage, hence,

$$\eta_s = \frac{\tau_s(N)}{\tau_{\infty}(N)} = \frac{\tau_s(N)}{\mu_N} \qquad .. \quad (3.4)$$

3.5 WORK-IN-PROCESS INVENTORY

The level of semifinished goods in different interstage buffers will fluctuate between zero and maximum (full). If S_j is the capacity of the buffer storage j, and IS_j is the time averaged value of the buffer held in it then the average buffer utilisation can be given by,

$$BU_j = IS_j / S_j \qquad j = 2,3,\ldots N \qquad .. \quad (3.5)$$

The average buffer utilisation for the entire line can be written as,

$$ABU = \frac{1}{(N-1)} \sum_{j=2}^{N} BU_j \qquad .. \quad (3.6)$$

and the average work-in-process inventory is given by,

$$WIP = \sum_{j=2}^{N} IS_j = ABU. \sum_{j=2}^{N} S_j \qquad .. \quad (3.7)$$

3.6 ASSUMPTIONS

Some of the assumptions which are common to various cases considered in Chapters 4-9 are as follows:

1. Only one type of product flows in the system, that is the mean values of processing, failure and repair times of the stages do not change with time, and their distributions are stationary.

2. Workpieces are not reworked or rejected at any of the stages in the line. Partially finished workpieces do not enter the line.

3. First stage is never starved and the last stage is never blocked.

4. Time to move the workunits between the stages and in and out of the buffers is negligible as compared to the operation times.

3.7 FEATURES OF SIMULATION MODEL

After the formulation of the problem, the important steps in the present study are,

i) Development of computer simulation methodology, flow charting and programming.

ii) Validation of the simulation programme.

iii) Design of the simulation experiments.

iv) Analysis of the results.

Since the work embodied in this thesis comprises of the development of a number of simulation models, with varied parameters and assumptions, it would be more appropriate to discuss the details of model building and validation of different models separately. Some aspects of the simulation models and design of simulation experiment, which are common to all, have been briefly described here.

3.7.1 Simulation Language

In addition to the general purpose langauges such as FORTRAN, ALGOL, COBOL etc., a number of special purpose simulation langauges such as GPSS, SIMSCRIPT, GASP, SIMPAC, DYNAMO, SIMULA etc., have been developed. The selection of a langauge depends upon a number of factors, first and foremost being the availability. Since the compilers for the special purpose langauges were not readily available, FORTRAN IV has been used for writing the simulation programmes extensively. Advantage of the FORTRAN is that it is well known and is considered to be more efficient than the special simulation langauges with regard to the computation time, as well as storage [101]. However, it is harder and time consuming to prepare programmes in FORTRAN for complex systems.

In this thesis all the programmes have been written in Fortran IV.

3.7.2 Length of Simulation Run

One of the most important features of the design of stochastic simulation experiment is the choice of correct length of simulation run, which should give results within some specified tolerance limits and at a reasonable degree of confidence. Length of run (LOR) is analogus to the sample size in physical experiments. A number of statistical methods are available to estimate accurately the sample size for experiments in science, engineering and other fields. The most commonly used relationship which gives the sample size 'n' for an experiment is as follows :

$$n = \frac{(y_{1-\alpha/2})^2 \, \sigma^2}{t^2} \qquad .. \quad (3.8)$$

Derivation of the equation (3.8) is based on the following two assumptions:

i) The distribution of the variable being observed is stationary.

ii) The samples are statistically independent.

These two assumptions are generally satisfied in the physical experiments and also in case of static stochastic experiments. But in case of dynamic stochastic experiments, both of these assumptions have to be cared for, since such a system exhibits very prominent and distinct transient behaviour, and its output data may be auto-correlated.

3.7.3 Elimination of Initial Bias in Simulation Experiments

The length of transients will depend upon the starting conditions of the system. Fig.3.2 shows the transient behaviour of the system for three starting conditions viz. with all inter-stage buffers empty, full and half full.

Proper selection of the starting conditions, which would be as close to the steady state as possible, will eliminate the initial bias to a certain extent or alternatively, the observations recorded in the beginning over a period of time, which may include the transients, be ignored. The simulation studies reported here has used an approach which incorporates both the above methods.

3.7.4 Statistical Independence of Observations

When the recorded value of a variable at a certain instant of time is likely to be influenced by its value recorded earlier, the data is said to be serially correlated or autocorrelated. Mathematical techniques are available to analyse the serially correlated data, but to start with it is essential to know the chances of the data being autocorrelated.

In the present work in all the cases the main response has been chosen to be the system efficiency or the output from the Nth stage of the system. The processing times have been assumed to be random and follow a prescribed distribution. The input rate of units to the Nth stage is not dependent upon one single factor but a host of random variables, which combine to make the input as well as output process random. Hence, it would be safe to assume that the observations of the response are statistically independent. Secondly, each observation corresponds to averaged value of a group of observations, which helps to reduce the chances of autocorrelation between them. Further, to make sure that the estimated length of run takes into account, if at all, any autocorrelation is present, the method of ' blocking', which is considered to be most suitable to handle the correlated data [35,58] has been used to determine the simulation run lengths.

3.7.4.1 Estimation of LOR

A pilot experiment was run on each of the simulation models considered in the thesis, in which after allowing sufficient warm up time, the observations were recorded. After a specified run length (5 batches) the standard error (S.E.) of the mean was computed. If the error was found to be within the prescribed limits, the simulation was terminated, otherwise it was continued for the next batch of 1000 time units. Standard error of the observed values was computed as follows:-

$$\text{Mean value of efficiency} = \eta = \frac{1}{b} \sum_{i=1}^{b} \eta_i \quad .. \quad (3.9)$$

where b is the number of batches

η_i is the average efficiency of each batch.

$$\text{Variance of the blocked mean, } \bar{\sigma}^2 = \sigma_\eta^2 / b \quad .. \quad (3.10)$$

$$S.E. = (1.96\ \bar{\sigma}/n)^{1/2}$$

$$\text{Then LOR} = n = \frac{1.96\ \bar{\sigma}}{(S.E.)^2} \qquad .. \quad (3.11)$$

The length of run required for a particular experiment depends upon the parameters of the system. Since the parameters considered in each experiment have been varied over a large range (N= 2 to 20, S = 0 to 20, CV= 0 to 1.0), the LOR required also varies widely. The influence of various factor levels on LOR (for t = ± 1 percent and α = 0.05) can be judged from Fig.3.3 and Table 3.1. Since it is not computationally economical to determine LOR for each factor level combination, the values given in Table 3.1 have been used as a guide to fix the LOR's for various line arrangements.

3.7.5 Variance Reduction

A number of variance reduction techniques (VRT) are given in standard text books on statistics but the circumstances under which each of these techniques viz., antithetic sampling, correlated sampling, importance sampling, and stratified sampling etc. can be employed satisfactorily and their comparative gains are still not fully known [101]. Application of any of the VRT is not getting something for nothing, additional information about the system has to be gathered which consumes considerable computer time. In certain situations the use of VRT could worsen rather than improve the precision of the results.

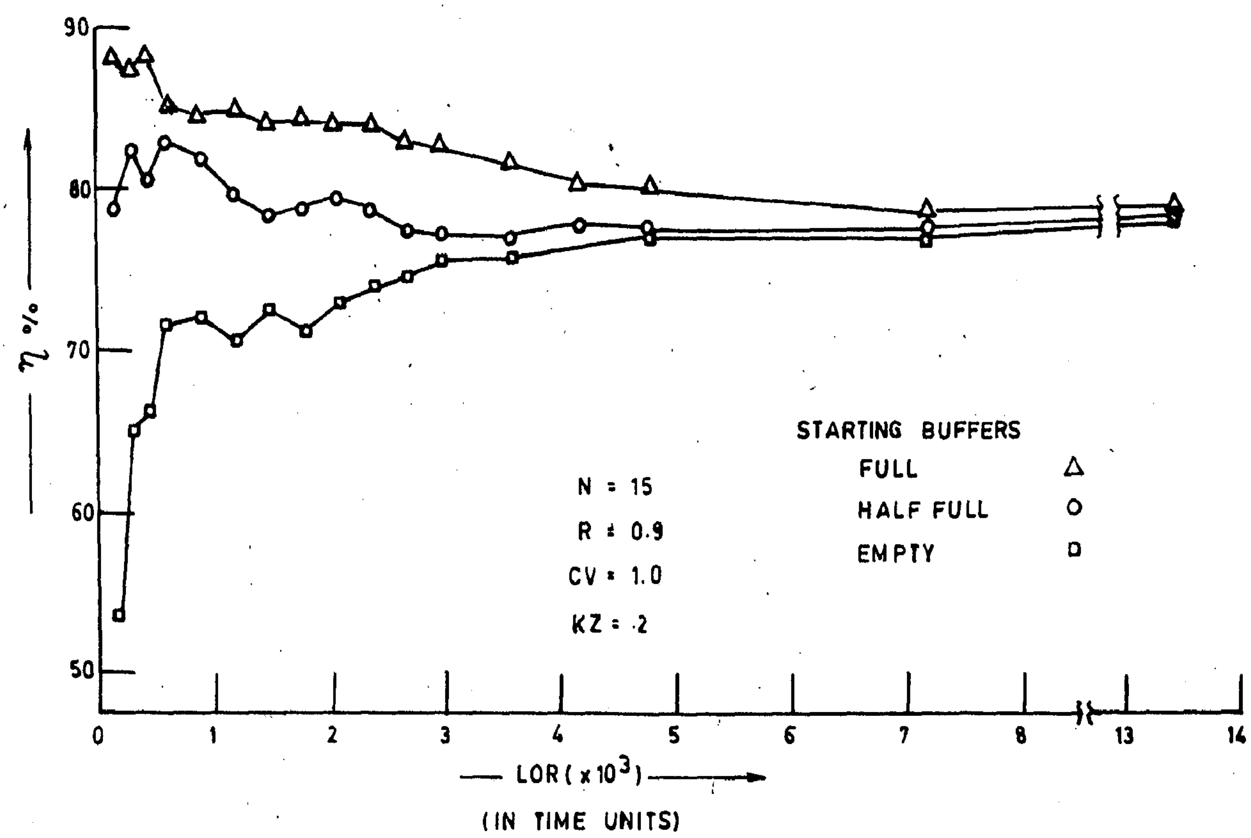

FIG. 3.2 EFFECT OF STARTING CONDITIONS ON SYSTEM EFFICIENCY

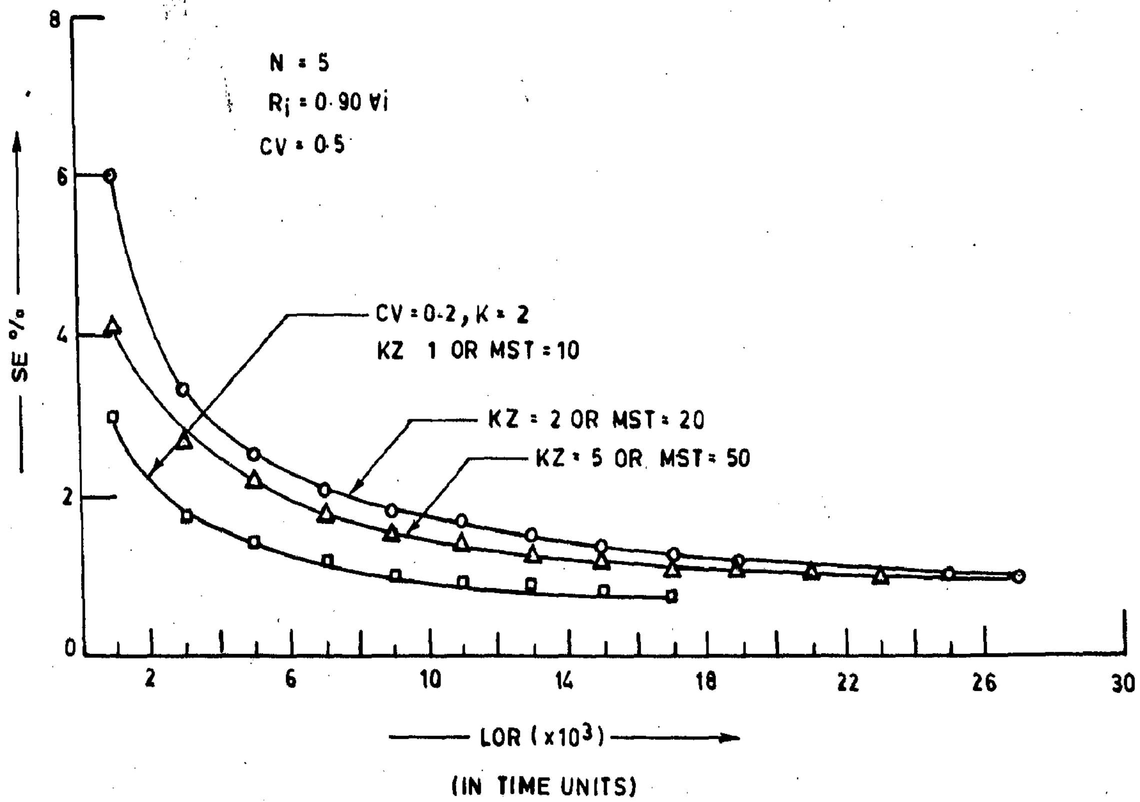

FIG. 3.3 STANDARD ERROR (SE) OF THE MEAN EFFICIENCY AS A FUNCTION OF LOR

TABLE 3.1 Estimated length of simulation run(in thousands of time units)

CV	Random processing times No breakdowns Accepted error=0.5 percent of mean η					Fixed processing times Random breakdowns Accepted error = 1.0 percent of mean η									
							$R_i = 0.80\ \bar{V}_i$					$R_i = 0.90\ \bar{V}_i$			
	S \ N	5	10	15	20	KS \ N	5	10	15	20	KS \ N	5	10	15	20
0.2	2	1.5	2.0	1.5	1.5	1	35	44	44	47	1	12	17	15	19
	4	7.5	7.5	8.0	3.0	2	34	38	35	36	2	10	11	9	11
	10	8.0	7.5	7.0	3.0	5	30	33	29	33	5	9	10	10	10
	20	1.5	7.5	3.0	3.0	10	32	34	31	33	10	10	9	8	9
0.5	2	15	16.5	20	14.5	1	91	91	92	91	1	33	37	35	45
	4	26	26.5	21	26.5	2	83	89	88	90	2	27	35	32	40
	10	27	33	35	33	5	84	75	85	84	5	26	24	25	22
	20	32	39	33	35	10	80	75	75	77	10	25	27	25	21
1.0	2	33	30	33	33	1	175	163	165	167	1	85	91	92	95
	4	45	57	51	45	2	173	189	185	196	2	79	82	84	84
	10	84	84	105	100	5	170	171	175	175	5	69	68	69	72
	20	115	105	116	120	10	165	173	167	166	10	55	65	55	53

The common random numbers (CRN) is the most widely used VRT, where two or more alternatives have to be compared with each other. It is logical that comparisons be made under identical conditions, so that the variance of the difference between two population estimates be the minimum. Though Wright and Ramsay [135] point out that in complex systems, the use of CRN may not show the desired effect, it has been shown by Wemmerlov [132] that CRN is more effective and computationally much more efficient than any other VRT. Because of its simplicity and intutive appeal, the CRN has been employed in this study where the comparison of certain alternatives, as of unbalanced lines, line repair policies and probability density functions, etc., has to be made.

3.7.6. Analysis Techniques

Since the total work involves several types of different experiments separate analytical technique has been employed in each case. Regression analysis has been used to develop empirical models for the efficiency of balanced lines with variable times and/or with breakdown of stages. Analysis of variance techniques such as T-test, and F-test has been used in comparing the alternative strategies. Nonparametric tests such as signs test and Wilcoxon T-test has also been used. Search procedures have been developed to locate the optimum points on the response surface. Details of analysis are discussed at the appropriate places.

3.7.7 Time Flow Mechanism

A fundamental part of simulation technique is the time flow mechanism. For this purpose, after each simulation, we must advance time, determine the state of the system at new points in time, keep track of the elapsed time and terminate the experiment at the end of specified simulation periods. Time flow mechanism, timing routines, event scheduling procedures or simulation executives all terms that are widely used, have been catagorised as belonging to one of the two general methods, i.e. fixed time increment or next event increment [30]. The definitions,applications and their comparative advantages and disadvantages are discussed in a number of text books [35,58, 101]. Nance [100] pointed out certain ambiguities and inconsistencies in the use of fixed time and next event classification, in the case of a patrolling repairman problem. He advanced the concept of a continum of time flow methods between the fixed time increment and next event increment methods. However, there is no yard stick to decide which method would be computationally more efficient for a given simulation model. The only way to find the better method is to experiment with the alternative methods, which is seldom justified by the amount of labour involved. The programmer's judgement and programming convenience are thus the most important criteria for the selection of time flow mechanism. In the present study in some of the models fixed time increment and in others next event increment methods,as given below,have been used.

Simulation models	Time flow mechanism
System with variable operation times	Next event increment
System with breakdowns of stages	Fixed event increment
System with variable operation time and breakdowns of stages	Next event increment.

CHAPTER-4

BALANCED FLOW LINES WITH RANDOM OPERATION TIMES

4.1 INTRODUCTION

The production efficiency of a flow line system having 100 percent reliable stages depends upon the number of stages, capacities of the inprocess buffers, the production rates of the stages and the variability in the processing times, of the stages. The processing times may remain constant for machine controlled operations, and in such a case the slowest stage determines the output rate of the line. If the stages involve manual operations, considerable variation in processing times of the stages, is unavoidable. The variability in operation times disrupts the flow of material, causes idleness of the stages and reduces the output from the system. For such situations provision of adequate size buffers can help in increasing the system output. Theoretically, an ideal solution would be to provide interstage storages of infinite capacities such that a stage is never starved or blocked. This would ensure that the stages become independent and work uninterrupted. However, an infinite storage is not practicable because of very high cost. On the other hand, if no storages are provided, the stages will be completely coupled and variability in one operation time would influence the entire line, leading to frequent idling of the stages. As a compromise finite interstage storages are commonly provided in such systems.

One of the problems in flow line design, which eludes a satisfactory solution, is the determination of the production rate and hence the efficiency of the system. Because of the large number of interacting parameters that influence its performance, it has not been possible to develop analytical models for the efficiency of large systems.

In this chapter, a simulation model of the flow line is presented, which enables us to study the influence of various parameters on the system performance. Based on the simulation results, empirical models have been developed, which very closely predict the efficiency of a balanced flow line.

4.2 SYSTEM MODELLING

The flow line system considered here, consists of N stages arranged sequentially with interstage buffers between them. The system works as follows:

The work unit enters the line at the first stage and after passing through all the stages leaves at the end of the line as a finished product. After the operation at one particular stage have been completed the work unit moves to the following inprocess store if a vacancy exists, otherwise, the stage gets blocked. As soon as the stage becomes free, i.e. relieved of the previous work unit, a fresh work unit is drawn from the predecessor store, if available, otherwise, the stage starves.

4.2.1 Assumptions

The system has been considered to be balanced with all stages having identical operation time distributions, and all inprocess buffers of the equal capacity. In addition, the following assumptions have been made:

1) Only one type of product flows in the system.
2) The probability of a stage breakdown during the production run is zero.
3) There are no work rejections and rework at the stages.
4) First stage is never starved and last is never blocked.
5) The transit time of the work units between the stages and in and out of the buffers is negligible, compared to the operation times.
6) The operation times of the stages are random and independently distributed.

For the sake of convenience, the mean operation time of the stages has been taken as one time unit ($\mu_i = 1.0\ \forall i$), and hence the production rate and production efficiency of the line are synonymous.

4.3 SIMULATION MODEL

A generalised computer simulation model has been developed for an N stage line with non-identical operation times of the stages and with N+1 buffers. The first and the last storages are of infinite capacity whereas, the size of inter-stage storages may be finite or infinite. The level of raw material in the first storage at the start of the run is assigned such a high value that it does not get depleted during the length of simulation run. The finished goods storage is taken to be empty at the start of the run. The operation times of the stages are generated by employing the random variate generators. Sub-programmes to generate the normal and Erlang variates have been incorporated in the simulation. The simulation programme, however, can be used for any type of operation time distribution.

On account of programming convenience and high computational efficiency, two different simulation programmes have been developed for the following cases:

Case 1 - Line without inprocess buffers

Case 2 - Line with finite inprocess buffers

4.3.1 Case 1 - Line Without Buffers

Simulation flow chart for this model is given in Fig.4.1. In this case all the stages in the line are idle initially so that the first work unit will pass through the line without any delay at any of the stages. For each work unit, the processing times at different stages are generated and then its progress through the line is followed from first to the last stage, where it emerges as a finished product. In each pass through the line, the service beginning and service ending times of the stages are updated. A clock is employed to register the elapsed time, while the number of units added to the $(N+1)^{th}$ storage give

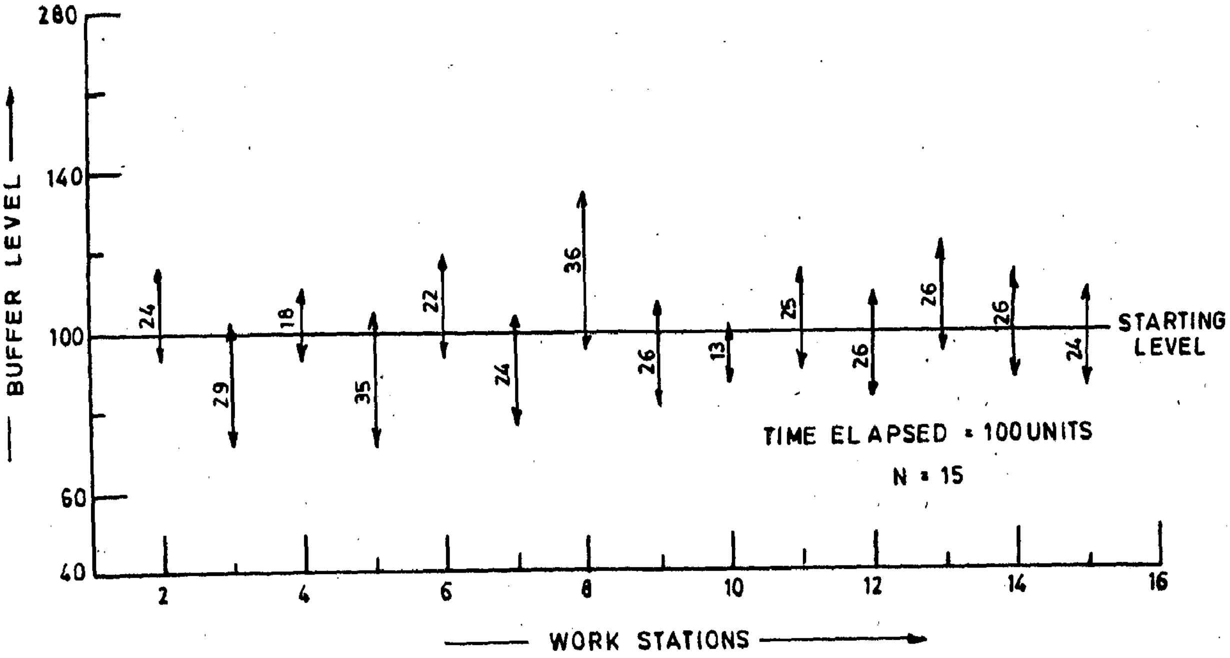

FIG. 4. 4 BUFFER LEVEL FLUCTUATIONS IN FLOW LINE WITH INFINITE STORAGE

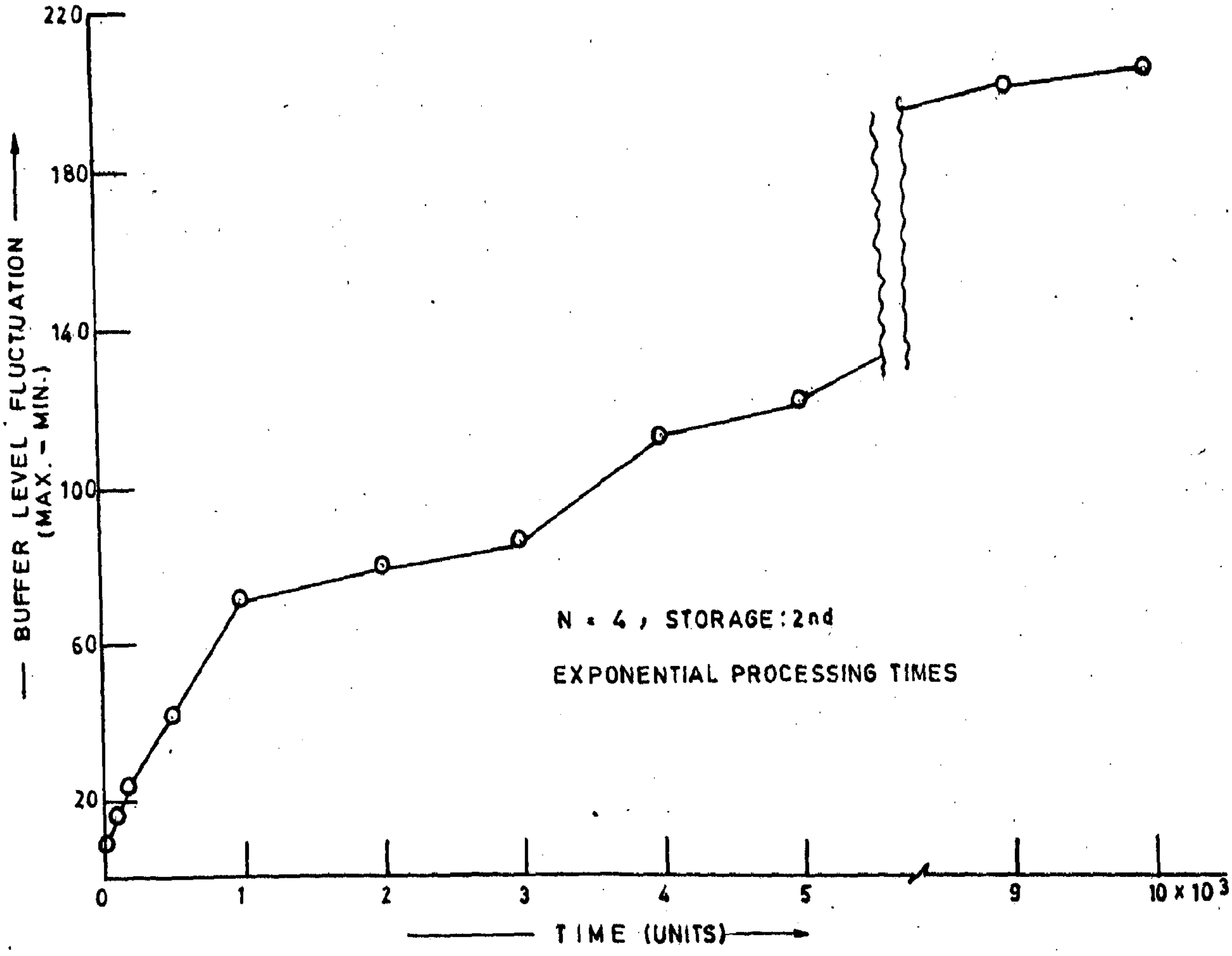

FIG. 4.5 VARIATION IN (Max – Min) LEVEL RANGE OF BUFFER WITH TIME

substantial increase in system efficiency. With exponentially distributed operation times of the stages, even very high storage capacity would not be able to ensure 100 percent production efficiency.

4.5 FLOW LINE EFFICIENCY WITH FINITE INPROCESS BUFFERS

The object of this experiment was to generate data for developing the empirical predictive equations for the efficiency of the balanced flow lines. To ensure the applicability of the empirical model to a wide range of practical flow lines, the range of parameters selected for the purpose are quite wide. It is known from the previous researchers [4,65,78,131] that the line efficiency is more sensitive to the changes in N and S when their values are small, and hence, more weightage has been given to their lower levels. The coefficient of variation of the processing times has, however, been found to have a significant effect on the line efficiency at all levels over the range considered. To get a good number of data points simulation was run for all the combinations of N,S and CV for the following levels:

N	2,3, ...10,12,15,20
S	0,1,.....6,8,...12,15,20
CV	0.1, 0.2, ...1.0

4.5.1 Influence of System Parameters on η

Variation in system efficiency as a function of the number of stages, for several values of interstage buffer capacities, has been shown in Fig.4.7, for exponentially

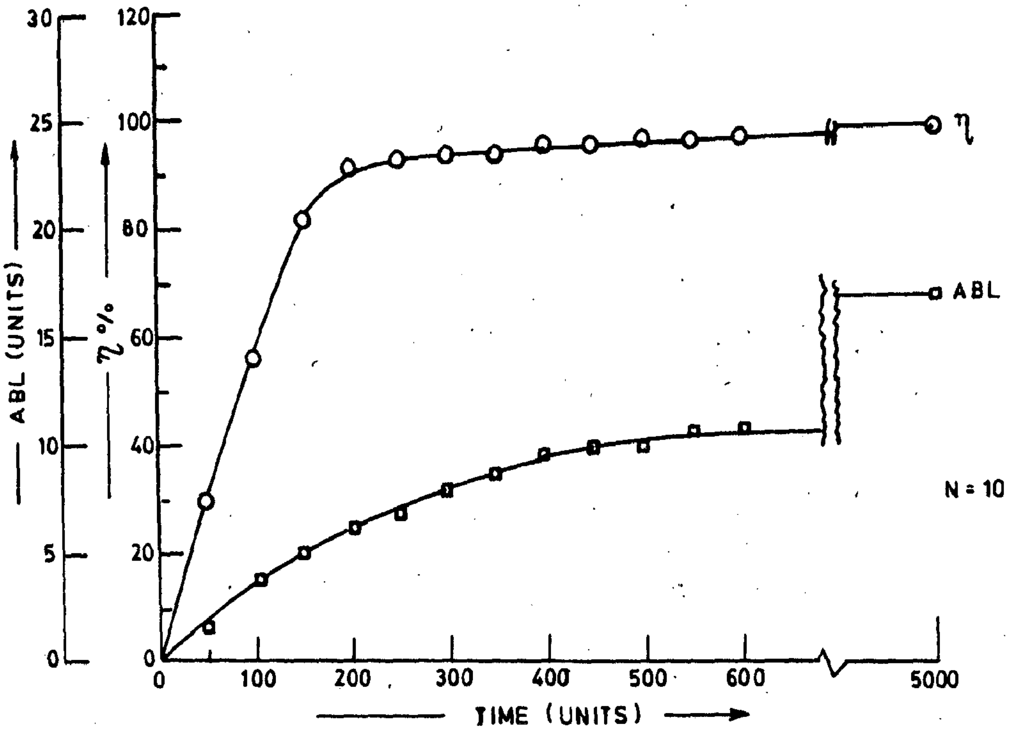

FIG. 4.6 GROWTH OF η AND ABL WITH TIME

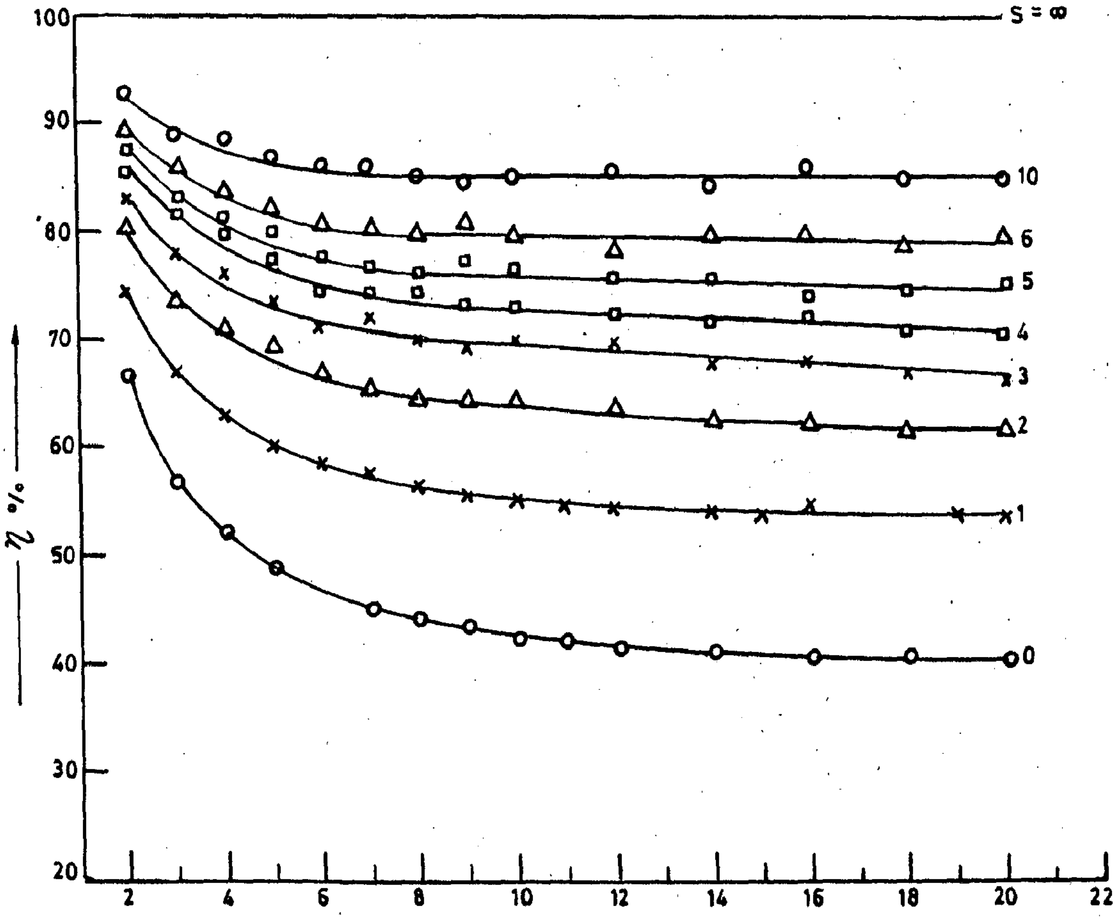

FIG. 4.7 EFFECT OF NUMBER OF STAGES ON SYSTEM EFFICIENCY (EXPONENTIALLY DISTRIBUTED PROCESSING TIMES)

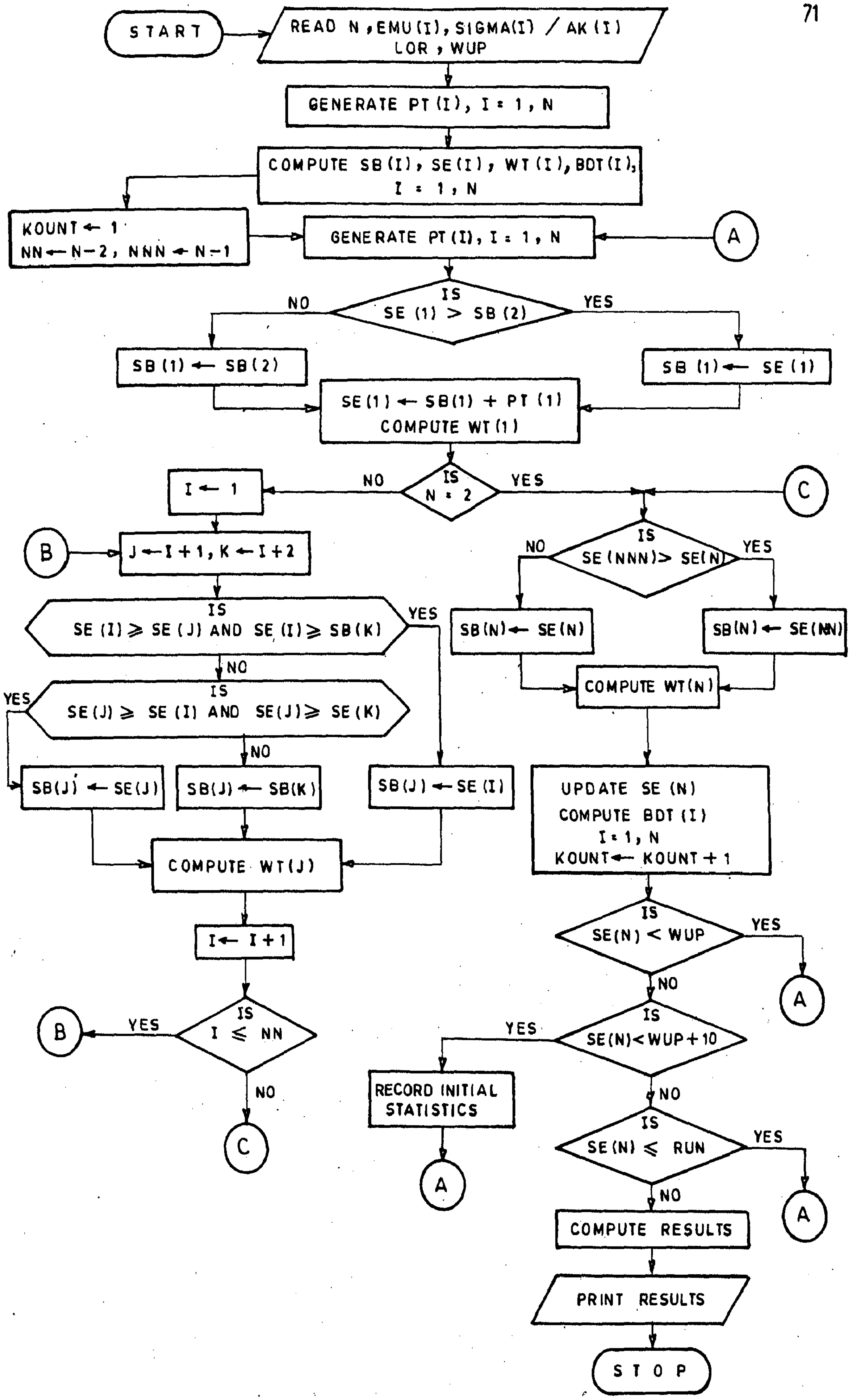

FIG. 4.1 SIMULATION FLOW CHART OF FLOW LINE WITHOUT INPROCESS BUFFERS

the number of units produced. Every time a work unit is completed, the clock is updated. Since the stages are idle in the beginning, the production of first few units is ignored to remove the initial bias. A warming up period of $10 \sum_{i=1}^{N} \mu_i$ time units has been found to be satisfactory. After the warming up of the line, the efficiency of the line was computed and statistics of interest were recorded at intervals of 25 time units. After a fixed length of run of 5000 time units, the mean efficiency, standard deviation, and standard error of the mean efficiency were computed. If the error at this stage was within the specified limits of ± 0.5 percent, the simulation run was terminated, otherwise it was incremented further by 1000 time units. For a 20-stage line, with coefficient of variation of operation times equal to 0.5, the experiment terminated after about 25000 time units of run.

4.3.2 Case 2 - Line With Finite Inprocess Buffers

The flow chart of the simulation model for this case is given in Fig.4.2. For developing this programme, event to event variable time model was used. In this case the clock, which follows the progress of the experiment, is updated at the occurence of each of the subsequent earliest events. After initialising the variables, the processing times of all the stages are generated, and the stage (j) with the smallest processing time is recognised. Since all the stages start working at the same time, completion of operation on stage j marks the

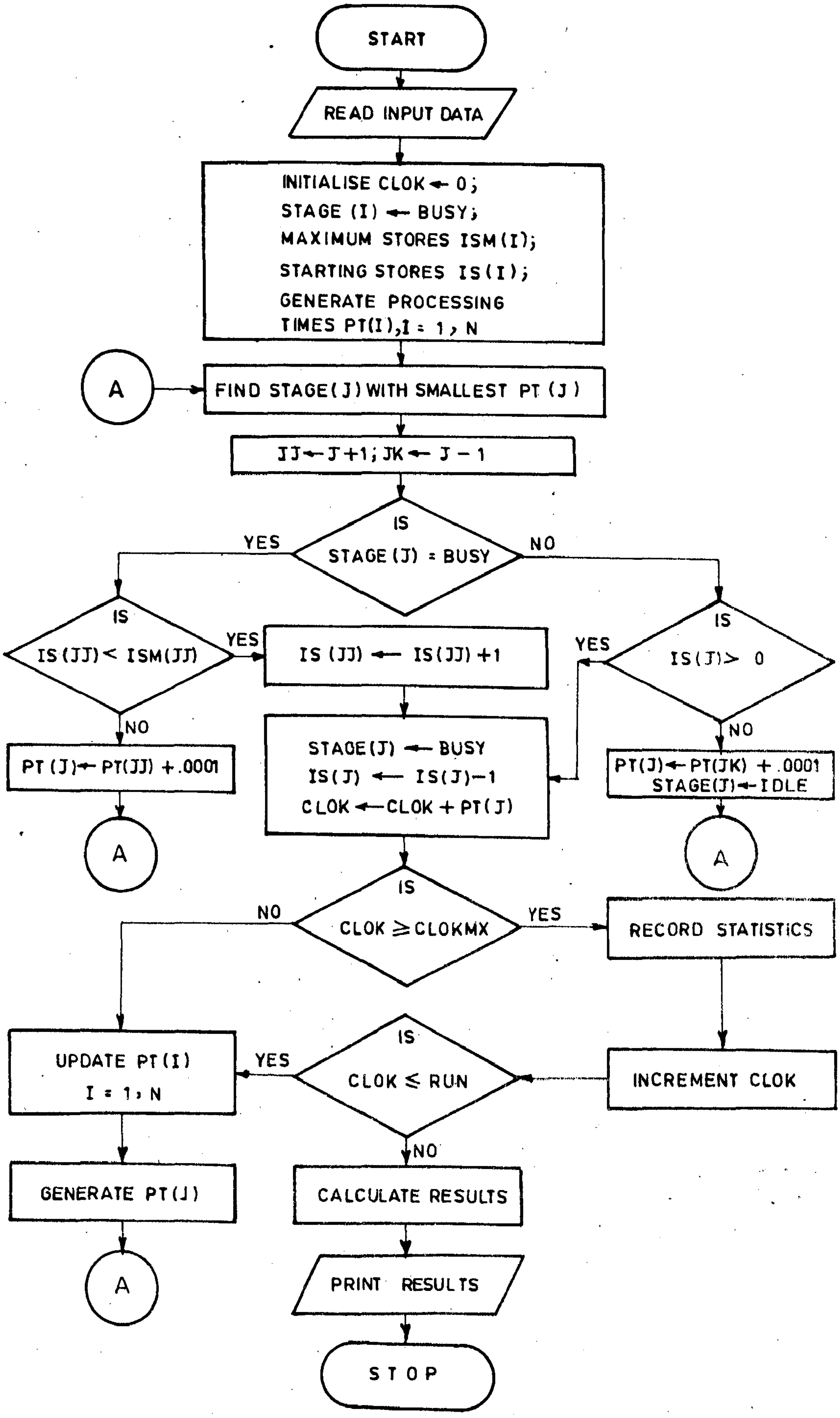

FIG. 4.2 SIMULATION FLOW CHART OF THE FLOW LINE HAVING FINITE INPROCESS BUFFERS

earliest event. The clock is advanced by the time PT_j and states of all the stages and the two storages on the sides of stage j are updated. Then the processing time for the next unit on stage j is generated and the line is scanned to find the next earliest event. After every 25 time units the statistics of interest were computed and recorded, and after a predetermined length of run (see section 3.7.2), the efficiency of the line and the WIP were computed.

4.3.2.1 Starting conditions

The starting state of the simulated system depends upon the object of the experiment. When the experiment is designed to study the growth of buffer in the line as a function of LOR, the system is started with empty buffers. On the other hand, when the steady state performance of the flow line is to be obtained, it is essential to ensure that the system reaches a steady state before the record of data is obtained. For that purpose the following steps have been taken:-

i) All interstage buffers were set to half full in the beginning.

ii) All the stages were started at the same time.

iii) To eliminate the initial bias, results of initial 1500 time units of simulation run were ignored.

4.4 GROWTH OF 'WIP' IN INFINITE CAPACITY BUFFERS

A simulation experiment was planned to study the behaviour of the interstage buffers of infinite capacity, in a balanced line having exponential processing times, such that it could operate at 100 percent efficiency. The simulation model of the line having finite inprocess buffers was modified to study the fluctuations in the buffer levels. The flow chart for the same is given in Fig.4.3. The line was started with arbitrarily selected high levels of interstage buffers ($IS_j = 100 \; \forall \; j$), and all the stages started operating at the same time. The maximum and minimum levels of the buffers were continuously monitored. After a fixed interval of 10 time units the storage states were recorded for determining the average level of the individual buffers. Fig.4.4 shows the maximum and minimum buffer levels attained at different stages during a run of 100 time units. As the experimental run progressed, the buffer levels required for 100 percent production were found to increase continuously and did not attain a steady state even after a run of 10,000 time units. Fig.4.5 shows the increase in the range of buffer fluctuation with time at the 2nd stage of a 4-stage line. To determine the growth of WIP and η with time, a 10 stage production line having exponentially distributed operation times, was started with empty buffers and was run for 5000 time units. Fig.4.6 shows that within 200 time units of run, the efficiency could attain a value of 90 percent for an average buffer level of 7 units only. With further increase in time, the increase in efficiency was very small, whereas, the ABL continued to grow rapidly. These results confirm the view that the line efficiency is sensitive to variations in buffer only when its value is low, and that with increase in buffer the WIP increases without

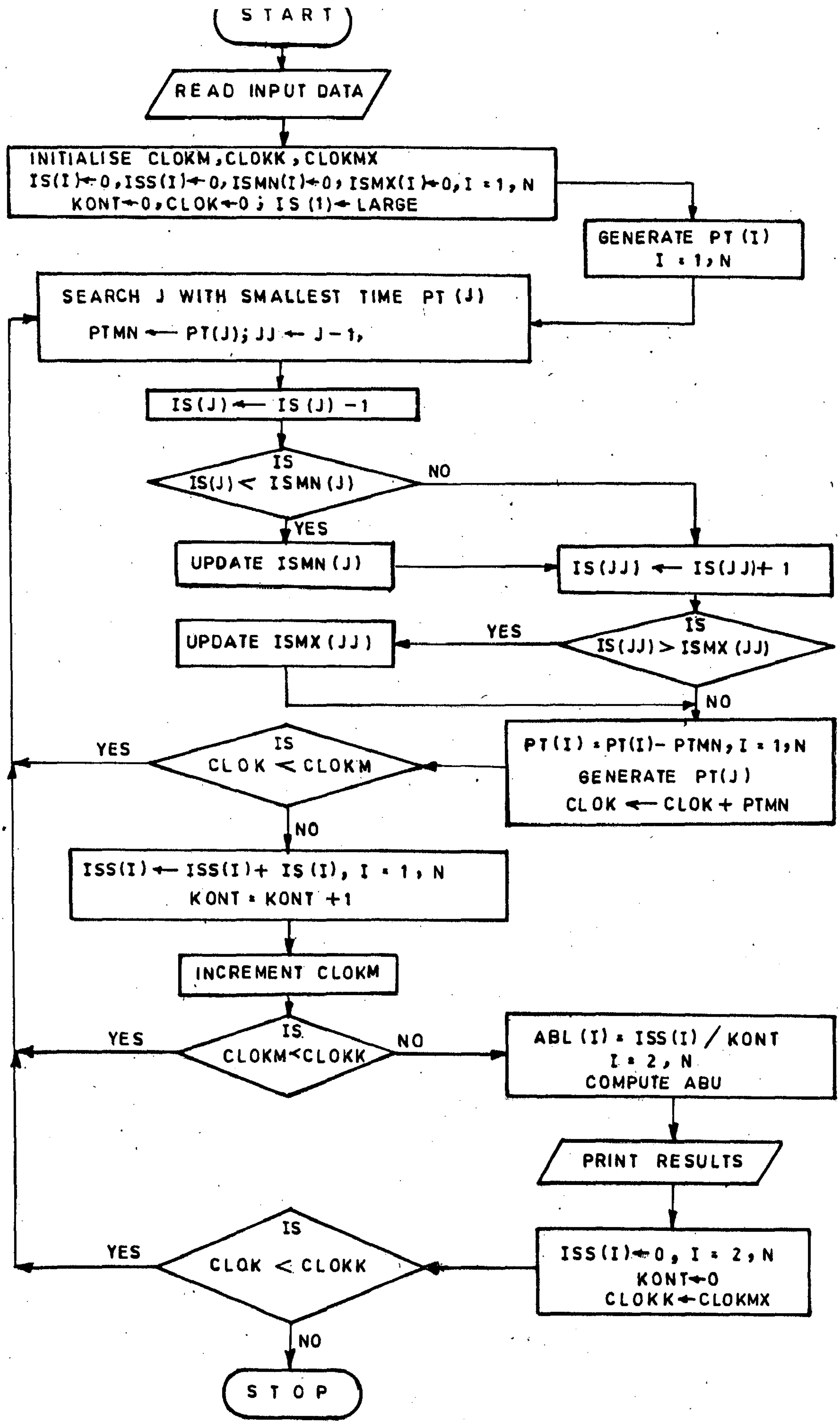

FIG. 4.3 SIMULATION FLOW CHART OF FLOW LINE WITH INFINITE STORAGE

distributed processing times. Fig.4.8 illustrates the influence of N on η at different values of S for normally distributed operation times with different values of coefficient of variation of operation times. It can be observed that in all the cases (Figs.4.7 and 4.8), as the number of stages increases, the efficiency decreases, the rate of decrease being significantly large for smaller values of N, and gradually diminishes as N increases. The line efficiency is also influenced by S and CV. Fig.4.9 shows a plot of η as a function of S for several values of N with exponentially distributed operation times. Similar results for the line with normally distributed operation times are illustrated in Fig.4.10. It can be noticed that provision of a small buffer is significantly beneficial, but the benefit of the buffer gradually diminishes as S increases. This effect is more prominent when the variability in processing times of the stages is low (Fig.4.10). It can be noticed from Fig.4.10, that for CV = 0.1, when inprocess buffers of only one unit capacity are provided, the efficiency of a 20 stage line increases from 90 to 98 percent, while the increase in S from 1 to 5 units results into an η improvements of $<$ 1.5 percent.

The effect of variability in processing times of the stages for lines with different number of stages (N) and inprocess storage capacities(S) is illustrated in Fig.4.11.

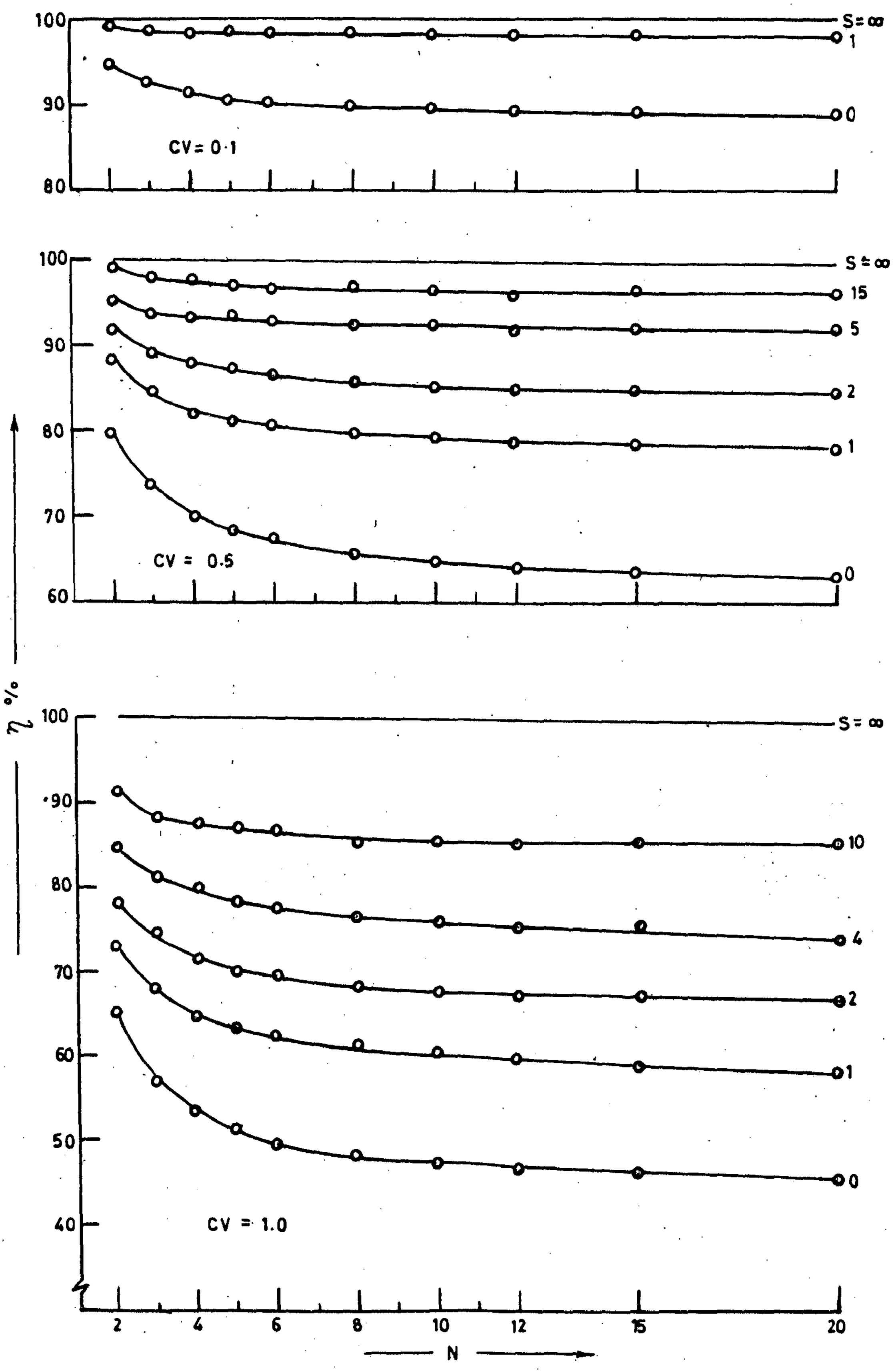

FIG. 4.8 EFFECT OF NUMBER OF STAGES ON SYSTEM EFFICIENCY (NORMALLY DISTRIBUTED PROCESSING TIMES)

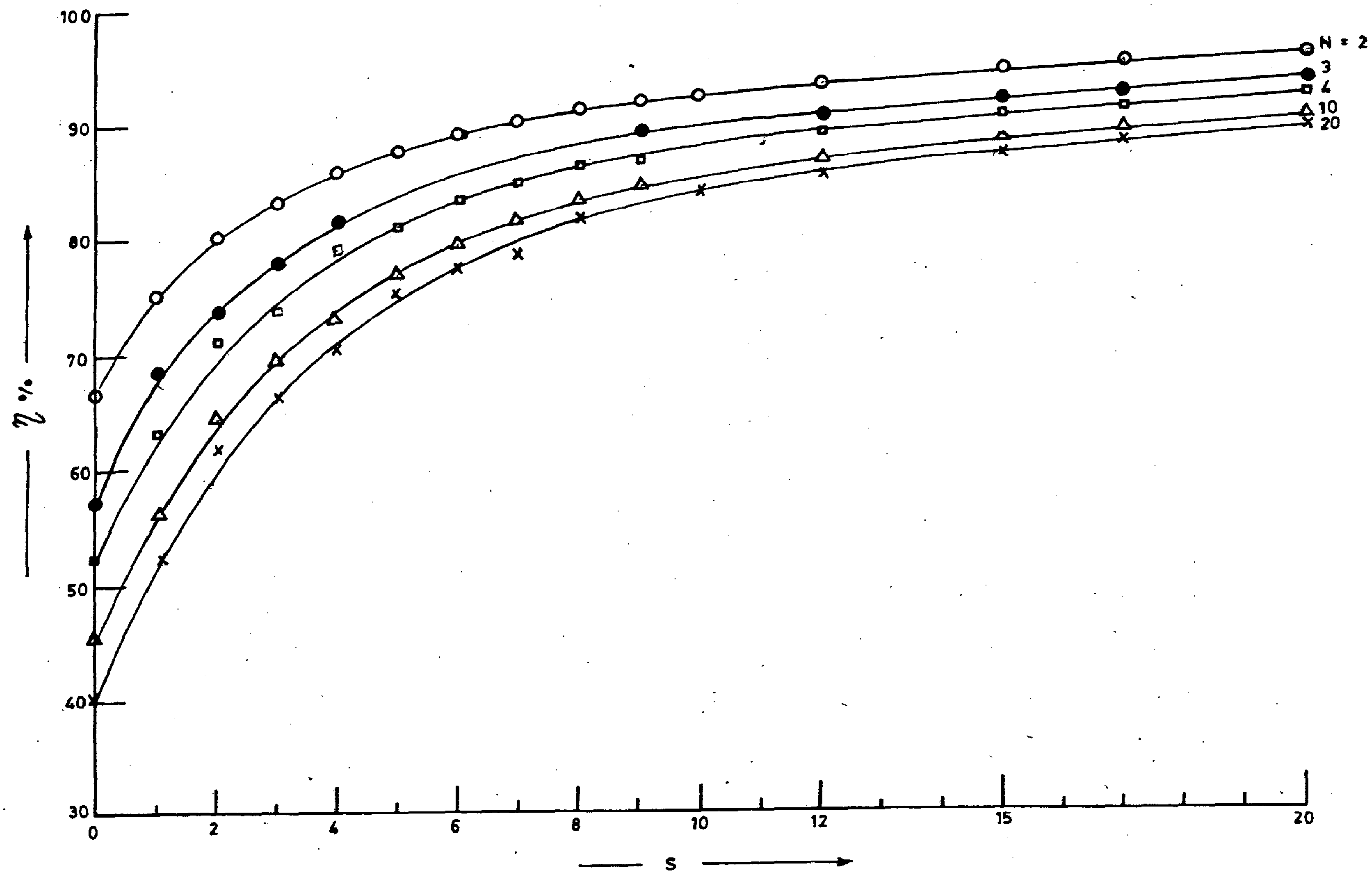

FIG. 4.9 INFLUENCE OF INPROCESS BUFFER ON SYSTEM EFFICIENCY
(EXPONENTIAL PROCESSING TIMES)

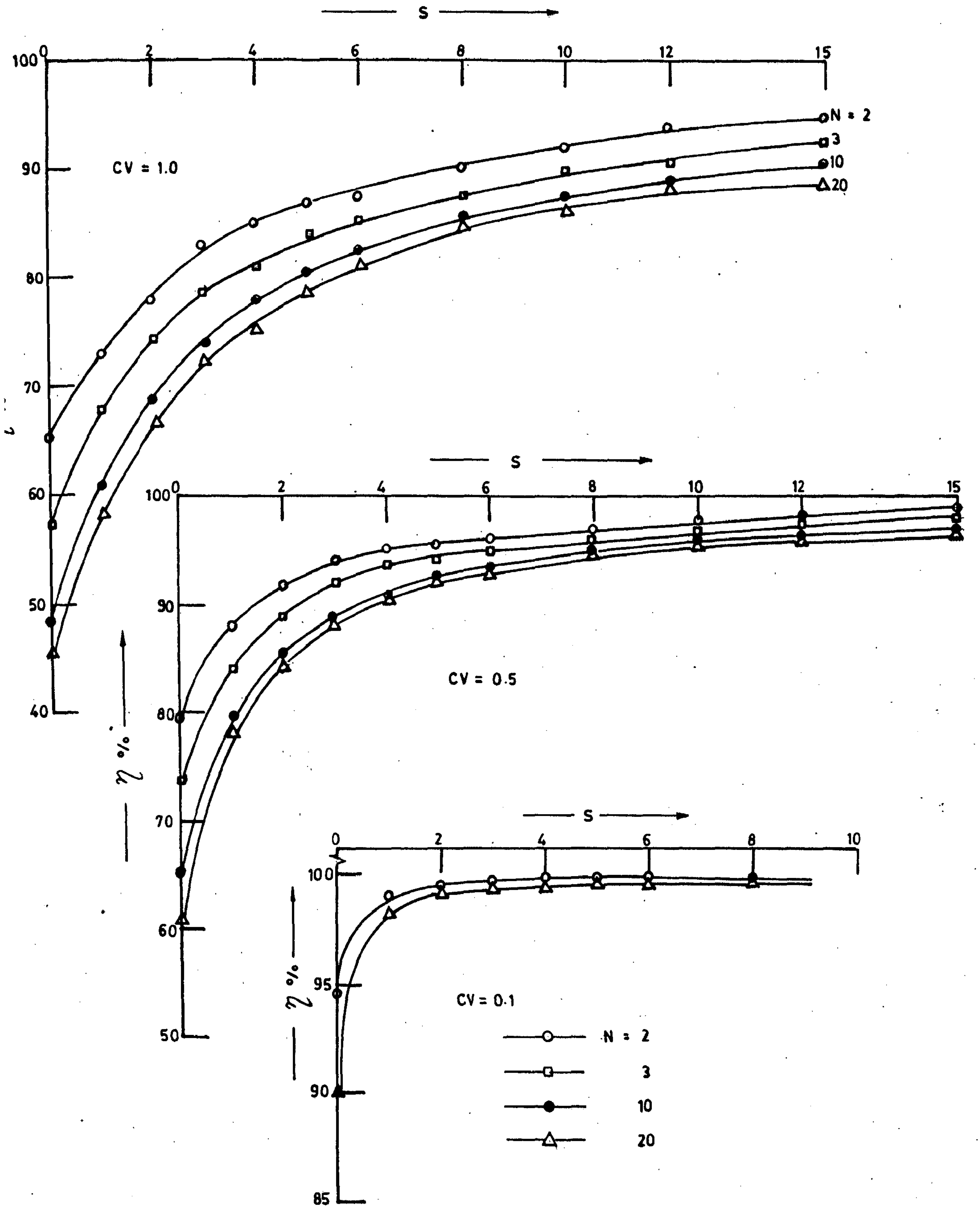

FIG.4.10 INFLUENCE OF INPROCESS BUFFER ON SYSTEM EFFICIENCY (NORMALLY DISTRIBUTED PROCESSING TIMES)

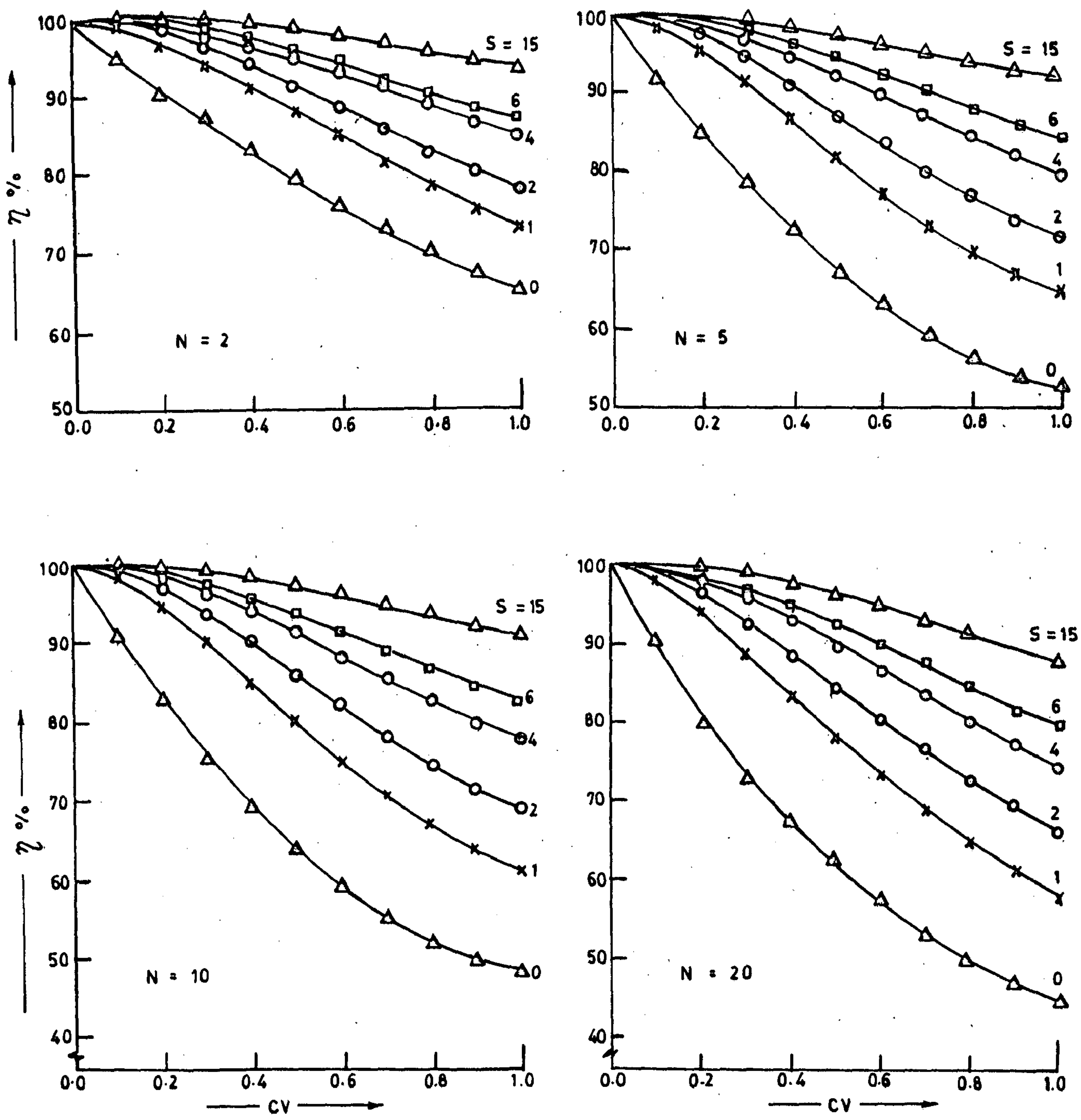

FIG. 4.11 EFFECT OF OPERATION TIME VARIABILITY ON SYSTEM EFFICIENCY

The increase in coefficient of variation of processing times, causes deterioration in system efficiency, the effect being more prominent when the number of stagesis large and inprocess buffer is small. Thus, for a fixed length of line, the effect of variability in operation times is consumed by the inprocess buffer. Hence, for a particular line configuration, the amount of inprocess buffer required would be strongly influenced by the variability in operation times.

4.6 COMPARISON OF SIMULATION RESULTS WITH OTHER STUDIES

4.6.1 Exponential Times

Before analysing the simulation results it would be in order to compare the results obtained, with those available in some existing studies. Exact analytical results have been obtained by Hillier and Boling [65], for small lines with small S and exponentially distributed operation times. The same has been given in Table 4.1, alongwith the simulation results. By employing Wilcoxon T-test it has been found that there is no significant difference between the two sets of data (at $\alpha = 0.05$). This confirms the validity of the simulation model.

A comparison of the results with those obtained numerically by Hillier and Boling [65], and empirically by Panwalkar and Smith [107] is given in Fig.4.12. For smaller values of S, the efficiency obtained by Hillier and Boling is higher as compared to the simulation results, while it is lower in case of Panwalkar and Smith [107]. The difference increases as the

TABLE 4.1 - Comparison of Simulation Efficiency with the Results of Hillier and Boling [65] (Simulation values in brackets)

Exponential Processing Times

S	Number of Stages (N)			
	3	4	5	6
0	.5641 (0.5667)	.5148 (.5173)	.4858 (.4896)	.4667 (.4635)
1	.6705 (.6731)	.6312 (.6299)	.6076 (.6044)	.5918 (.5928)
2	.7340 (.7349)	.7007 (.6985)	.6805 (.6857)	
3	.7767 (.7749)	.7477 (.7492)		
4	.8075 (.8071)	.7818 (.7858)		
5	.8308 (.8294)	.8077 (.8098)		
6	.8490 (.8477)			
7	.8637 (.8601)			
8	.8757 (.8767)			
9	.8858 (.8907)			
10	.8944 (.8920)			

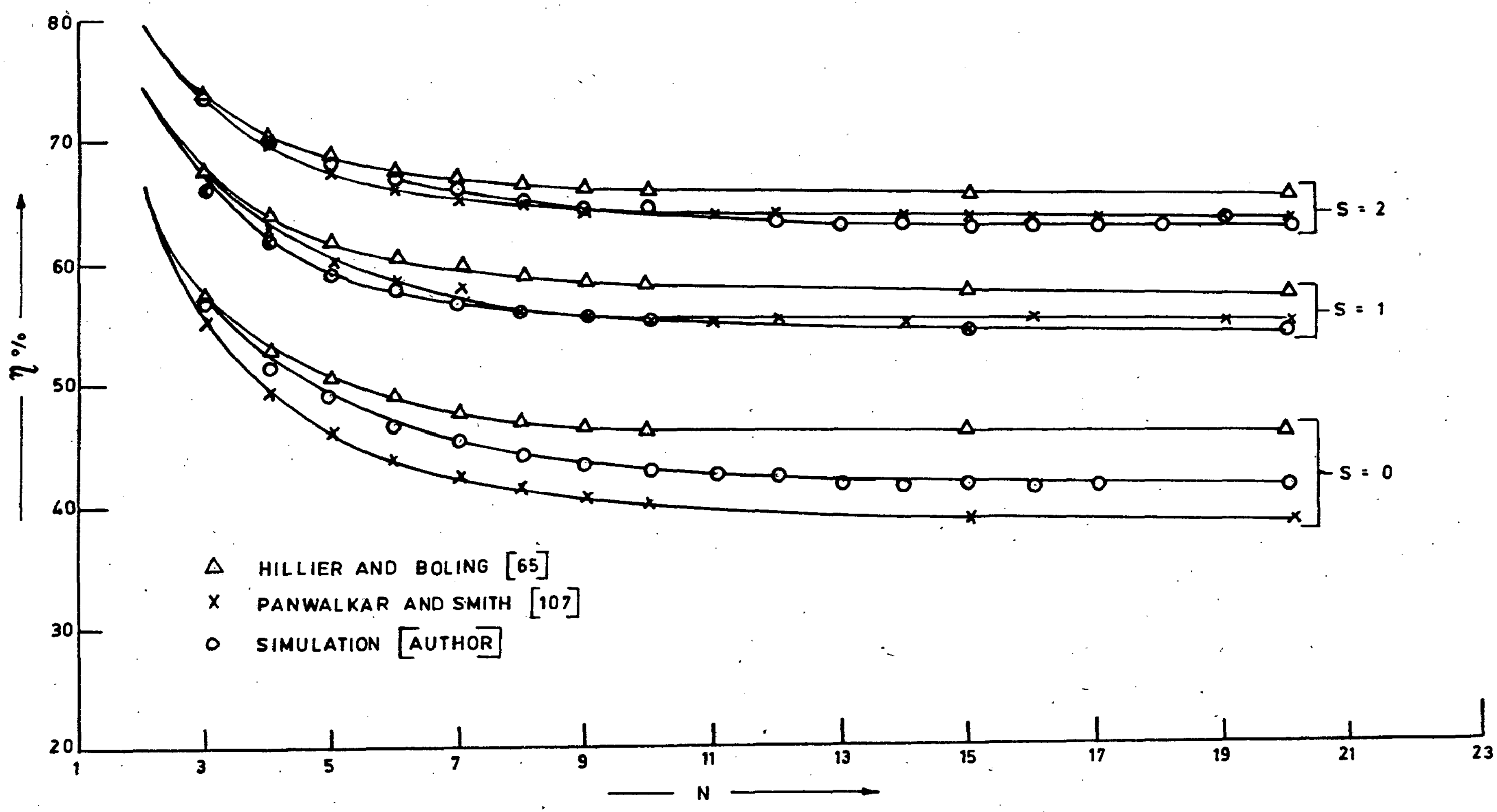

FIG. 4.12 COMPARISON OF SIMULATION RESULTS WITH OTHER STUDIES

length of the line increases. For larger values of S, the results of the two studies agree well with those given by simulation.

4.6.2 Normally Distributed Times

In case of production lines having normally distributed processing times, the simulation results have been compared with the approximate results of Anderson and Moodie [4] and of Knott [78]. As illustrated in Table 4.2, the approximate results are quite different from each other and from simulation results at lower levels of S. The difference increases with increase in N. At higher levels of S the difference between the three studies diminishes.

4.7 DEVELOPMENT OF PREDICTION EQUATION

It has been demonstrated analytically by Hunt [71] that for a two stage line, the line efficiency can be given by equation:

$$\eta_S^2 = \frac{S + 2}{S + 3} \qquad \text{.. (4.1)}$$

A thorough investigation of the results revealed that the efficiency of a line with any number of stages could be plotted as a linear function of a parameter $\frac{S + K}{S+K+1}$. The value of the constant K varied with the coefficient of variation of the operation times. However, it was found to remain constant for a particular value of CV. Some of the representative plots for values of CV = 0.1, 1.0 and 0.5, are shown in Figs.4.13 and 4.14 respectively.

TABLE 4.2 - Efficiency of line with operation time distribution normal C_v=0.3

N \ S	0			1			2			3			4		
	SIM	A & M	KNOTT	SIM	A & M	KNOTT	SIM	A & M	KNOTT	SIM	A & M	KNOTT	SIM	A & M	KNOTT
2	8590	8718	8569	9406	9740	9443	9661	9662	9652	9742	9756	9746	9797	9804	9804
4	7896	8244	7974	9103	9208	9191	9518	9488	9497	9666	9625	9634	9658	9699	9709
6	7633	8097	7800	9036	9124	9116	9371	9425	9443	9557	9579	9587	9635	9602	9681
8	7520	8032	7716	9006	9074	9066	9342	9398	9416	9477	9551	9569	9609	9643	9662
10	7433	7994	7661	8931	9041	9041	9331	9372	9398	9545	9533	9560	9611	9624	9652
12	7403	7968	7634	8905	9025	9025	9327	9363	9390	9519	9524	9551	9600	9615	9652

SIM - Simulation results (Author)
A&M - Empirical results of Anderson and Moodie [4],
KNOTT- Approximation of Knott [78]

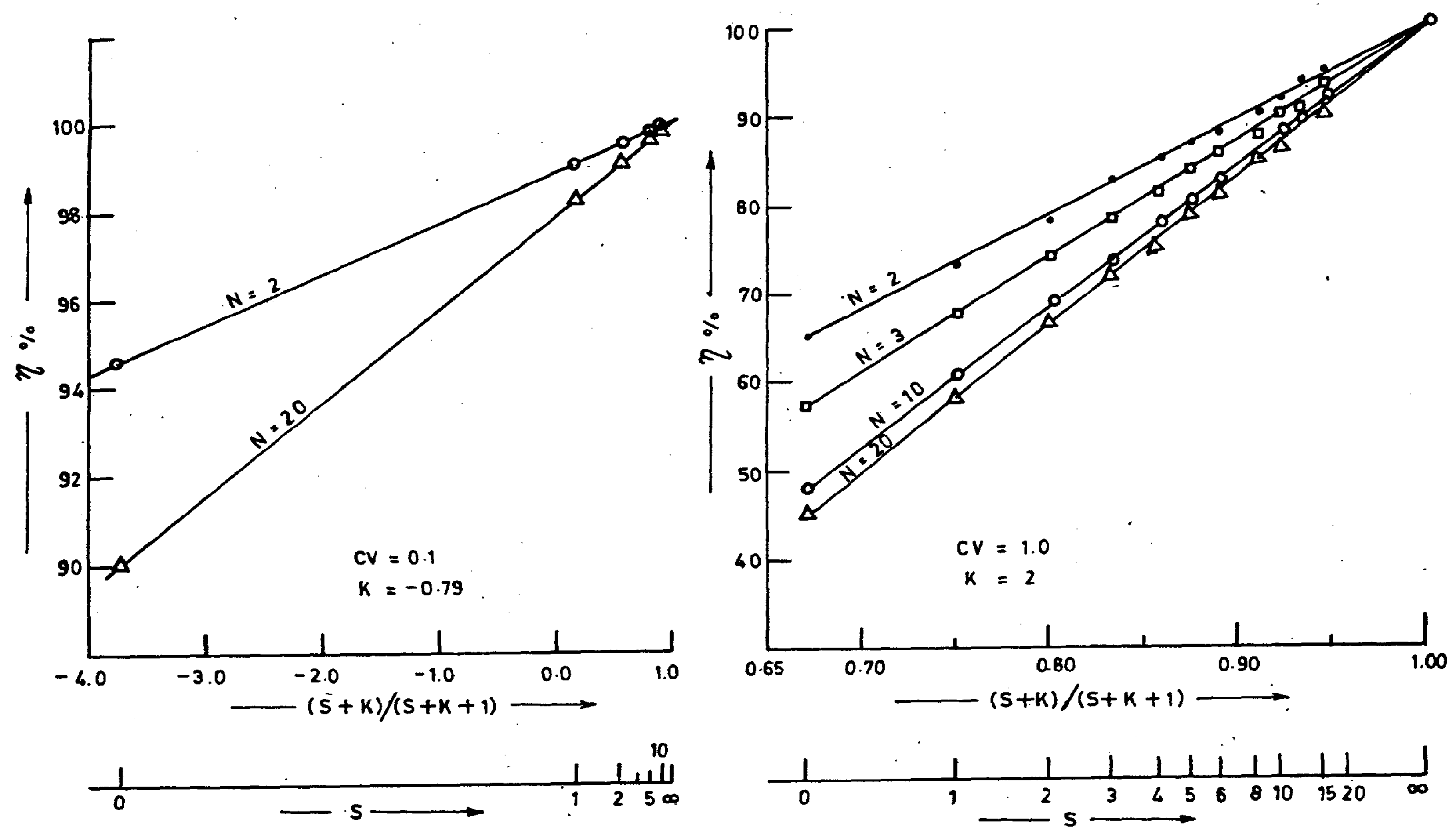

FIG. 4.13 EFFICIENCY AS A LINEAR FUNCTION OF $(S+K)/(S+K+1)$

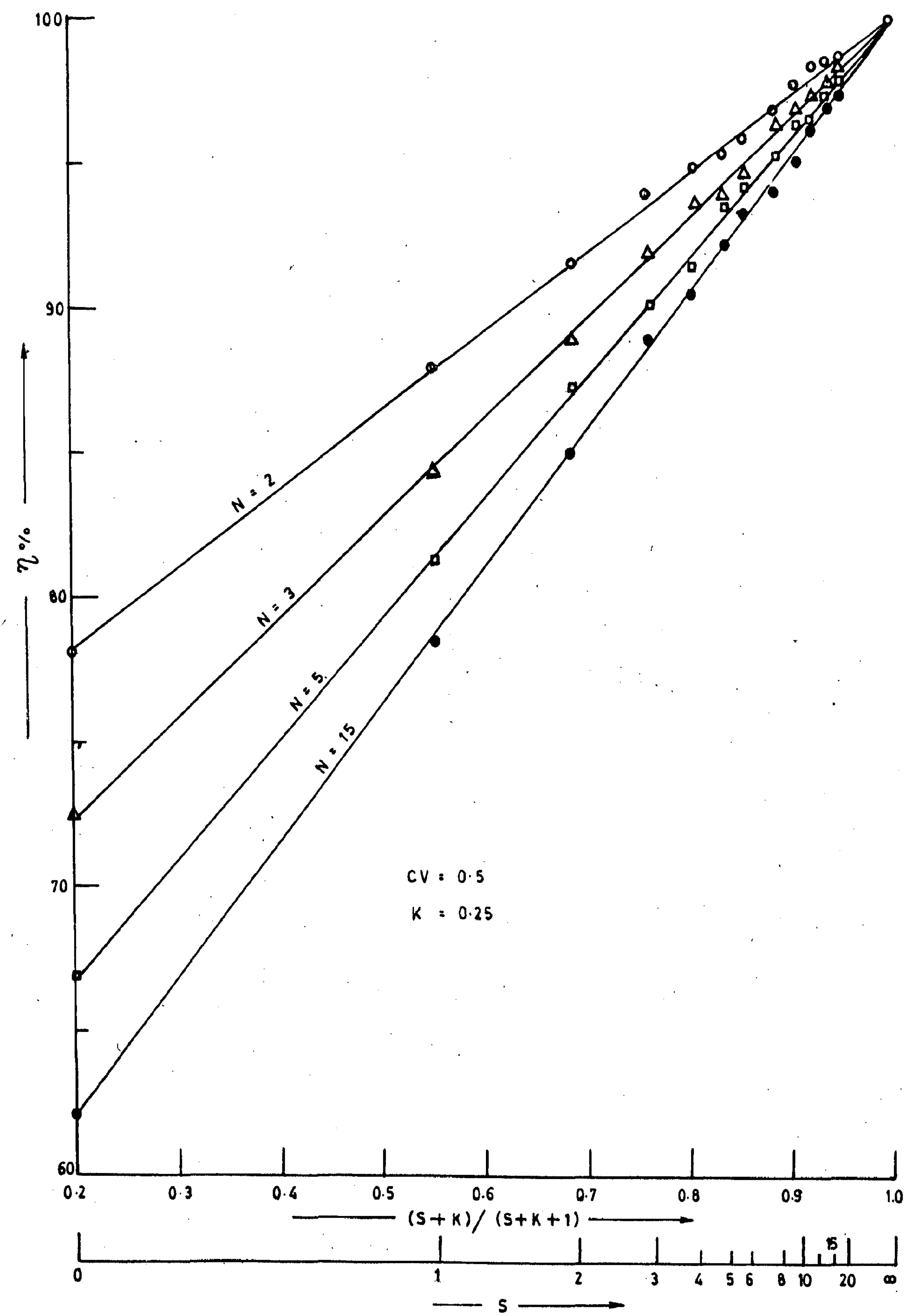

FIG.4.14 EFFICIENCY AS A LINEAR FUNCTION OF (S+K)/(S+K+1)

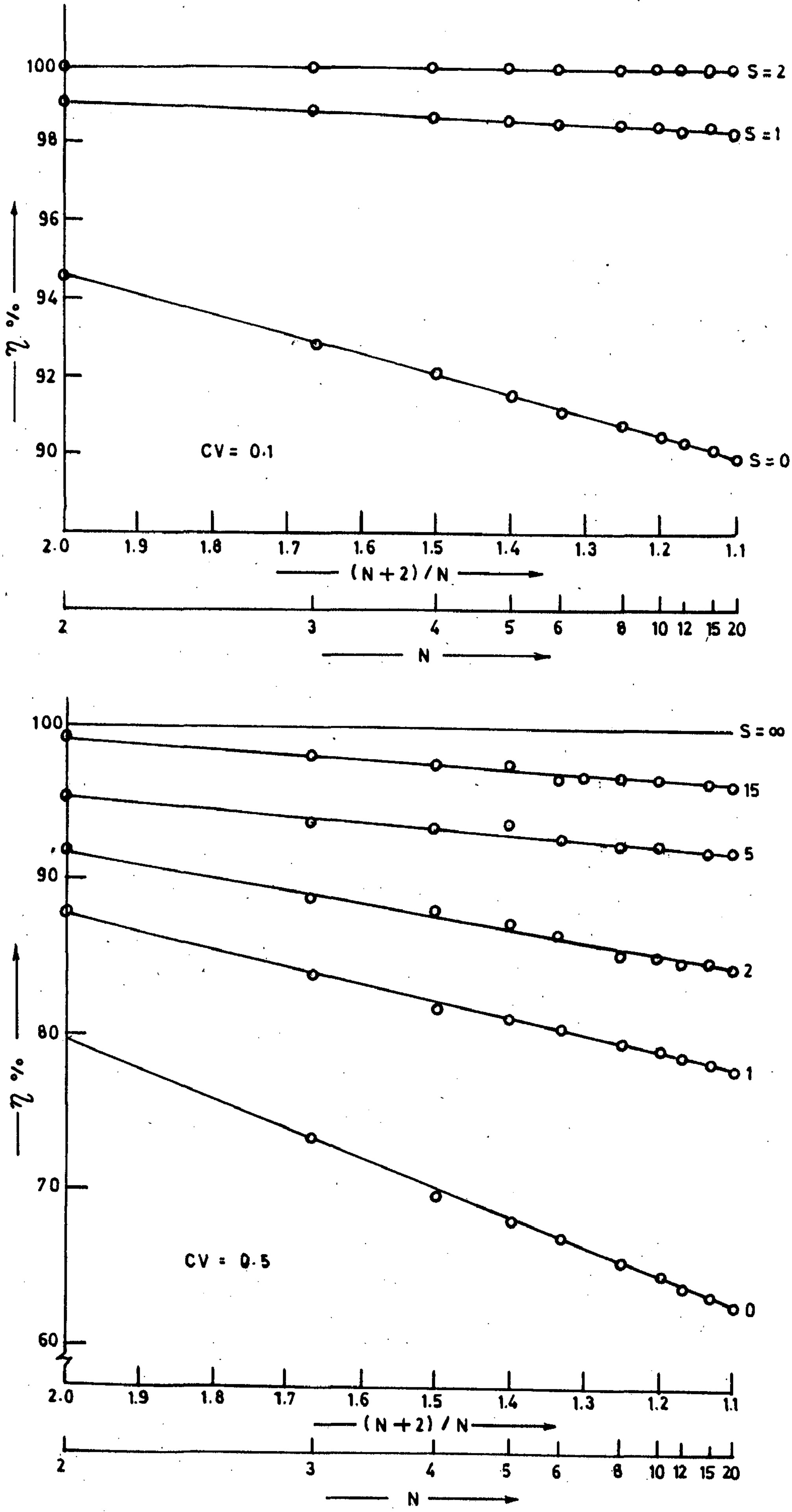

FIG. 4.15 RELATIONSHIP BETWEEN THE LINE EFFICIENCY AND (N+2)/ N

To determine the efficiency of the line for the given set of the operating parameters, values of K, η_o^2 and η_o^{20} are required. As can be seen from Figs.4.13 and 4.14 that the value of K varies with coefficient of variation of the processing times, and that η_o^2 and η_o^{20} are also functions of CV. By performing polynomial regression analysis of the simulation data for 2 and 20 stage lines having zero inprocess buffers, following expressions have been obtained.

$$\eta_o^2 = .99561 - .505193\ CV + .149002\ CV^2 \quad .. \quad (4.6)$$

$$\eta_o^{20} = .990139 - .94981CV + .422493\ CV^2 \quad .. \quad (4.7)$$

The coefficients of correlation for the equations (4.6) and (4.7) respectively were 0.9994 and 0.9995.

To determine the value of K for each value of CV, a search procedure was employed. The flow diagram of the computer programme for this purpose is given in Appendix A 4.1. The search started with an arbitrarily selected values of K which was incremented/ decremented in steps. For each value of K, the efficiencies at the selected parameter levels were computed and the sum of the squares of the differences between computed and simulated values was determined. Search continued until the value of K corresponding to the minimum sum of squares was obtained. The coefficient of correlation between the predicted and simulated values was also determined. At each value of CV, value of K was determined at different levels of N separately. The difference in values of K was found to be quite small, and hence the mean value was

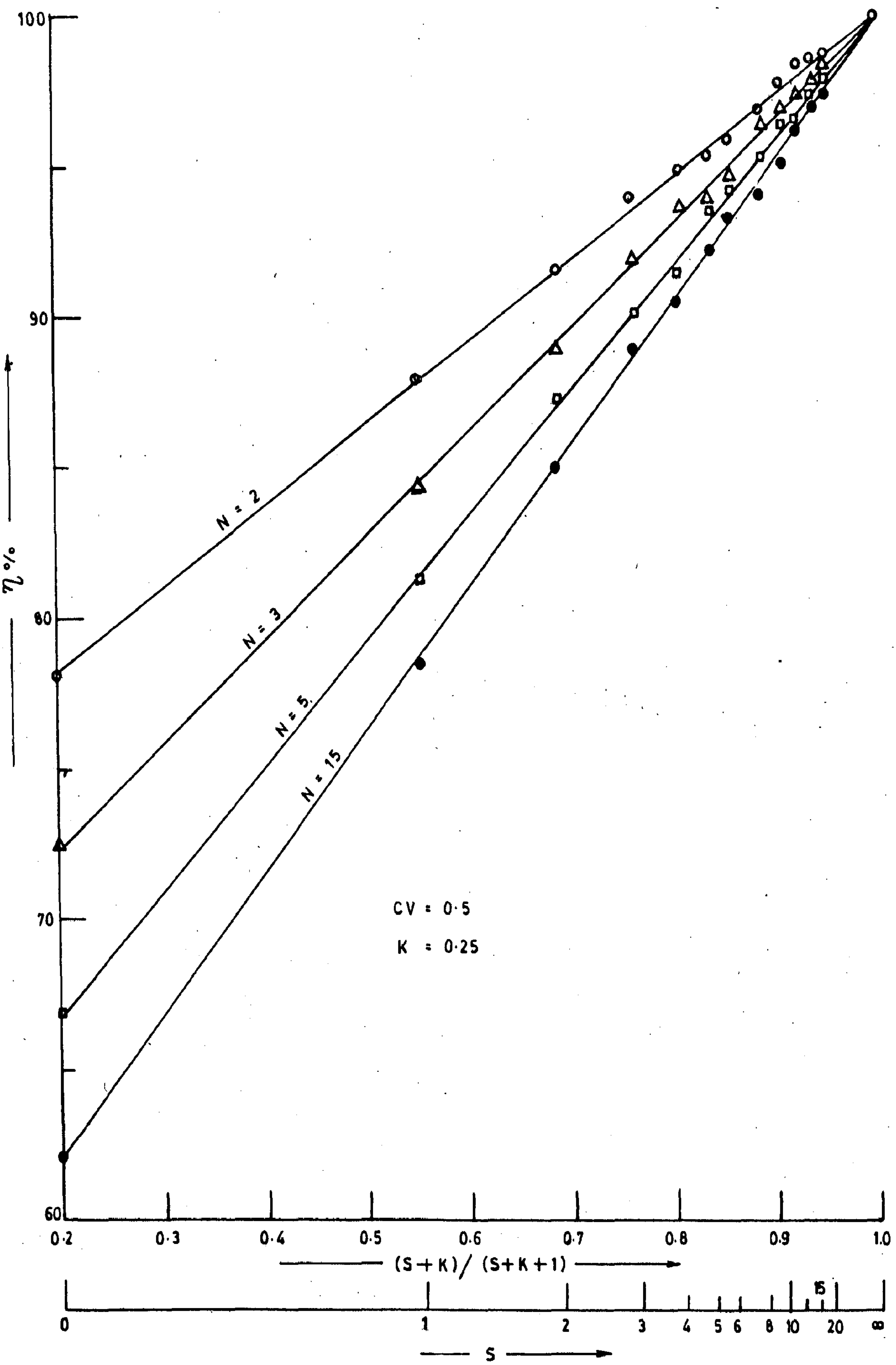

FIG.4.14 EFFICIENCY AS A LINEAR FUNCTION OF (S+K)/(S+K+1)

Similarly it is possible to obtain linear relationship between η and (N+ 2)/N (Figs.4.15 and 4.16).

For a particular value of S, the efficiency of a N stage line can be expressed in the form ,

$$\eta_S^N = \eta_s^2 - \frac{(\eta_s^2 - \eta_s^{20})}{2-1.1} (2-N')$$

$$= \eta_s^2 - \frac{(2-N')}{0.9} (\eta_s^2 - \eta_s^{20}) \qquad .. \qquad (4.2)$$

The efficiency of an N stage line as a function of S can be written as,

$$\eta_S^N = 1 - \frac{1-\eta_o^N}{1 - \frac{K}{K+1}} \left[1- \frac{S+K}{S+K+1}\right]$$

$$= 1 - \frac{K+1}{S+K+1} \left[1-\eta_o^N\right]$$

$$= \frac{S+(K+1)\eta_o^N}{S+K+1}$$

$$\therefore \ \eta_s^2 = \frac{S+(K+1)\eta_o^2}{S + K+1} \qquad .. \qquad (4.3)$$

$$\text{and } \eta_s^{20} = \frac{S+(K+1)\eta_o^{20}}{S+K+1} \qquad .. \qquad (4.4)$$

Substituting the values of η_s^2 and η_s^{20} in equation (4.2), we obtain,

$$\eta_S^N = \frac{S+(K+1)\eta_o^2}{S+K+1} - \frac{(2-N')}{0.9} \frac{K+1}{S+K+1} (\eta_o^2 - \eta_o^{20})$$

$$= \frac{1}{S+K+1} \left[S+(K+1)\eta_o^2 - \frac{(N-2)}{0.9\,N}(K+1) (\eta_o^2 - \eta_o^{20})\right] \qquad .. \qquad (4.5)$$

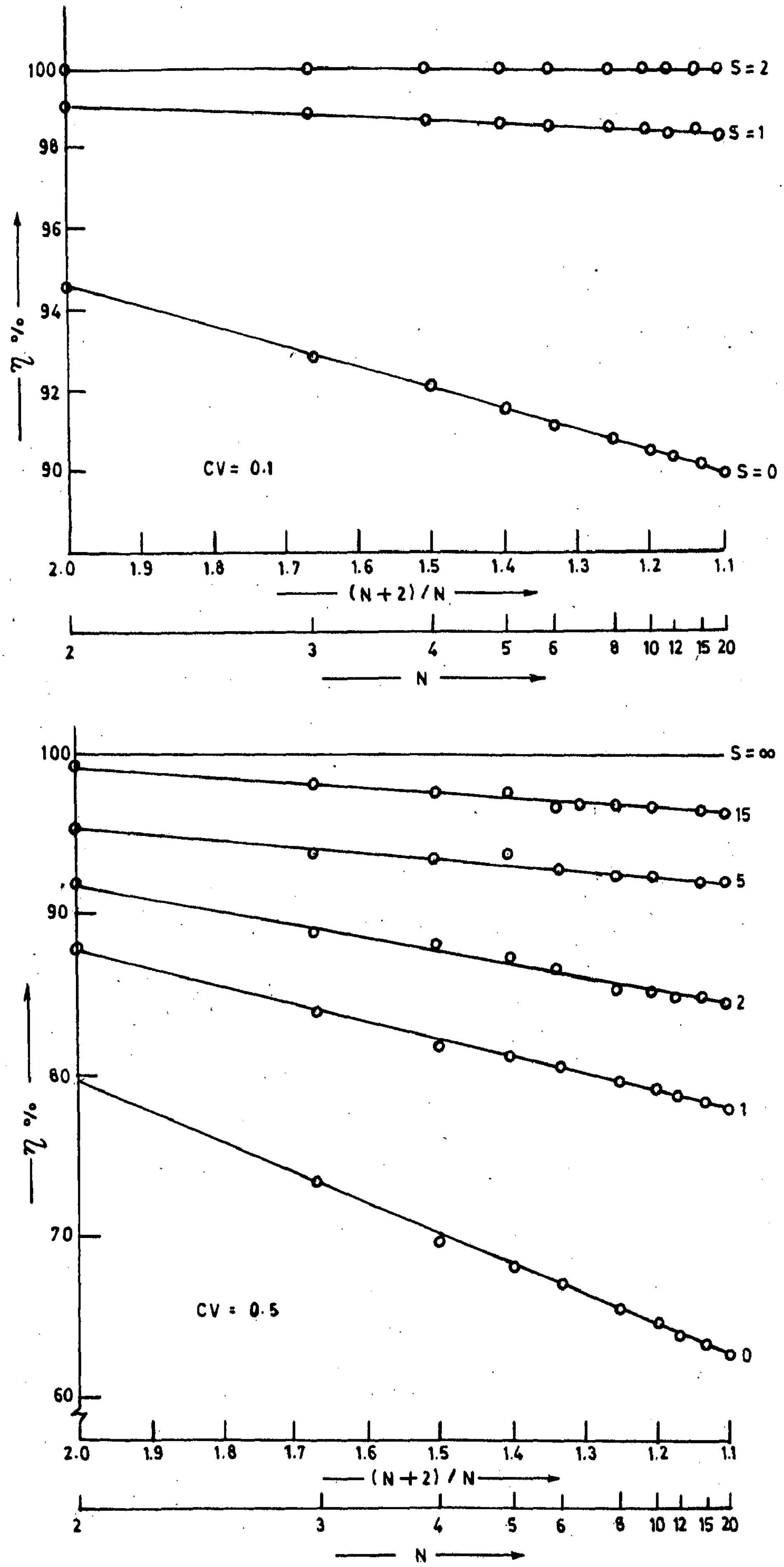

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TABLE 4.3 - Values of K for Different Values of N for CV=0.4

N	2	3	4	5	6	8	10	12	15	20
K	0.07	-0.16	-0.07	-0.03	0.0	-0.11	-0.09	0.05	0.13	-0.10

Mean values of K =-0.03

The value of constant K varied with CV as shown in Fig.4.17 Regression analysis of the data, yielded the following relationship between K and CV

$$K = -0.987 + 1.988\ CV + 1.01\ CV^2 \quad .. \quad (4.8)$$

A coefficient of correlation of 0.99853 was obtained for this expression. The constants in equation (4.8) could be rounded off to the nearest whole numbers as follow;

$$K = -1.0 + 2\ CV + CV^2 \quad .. \quad (4.9)$$

with little effect on the degree of correlation (CR = 0.998123).

Thus equations (4.5) to (4.7) and 4.9 can be employed to compute the efficiency of a balanced line.

4.7.1. Adequacy of the Model

To test the adequacy of the proposed model, predicted and simulated results for 100 combinations of the factor levels

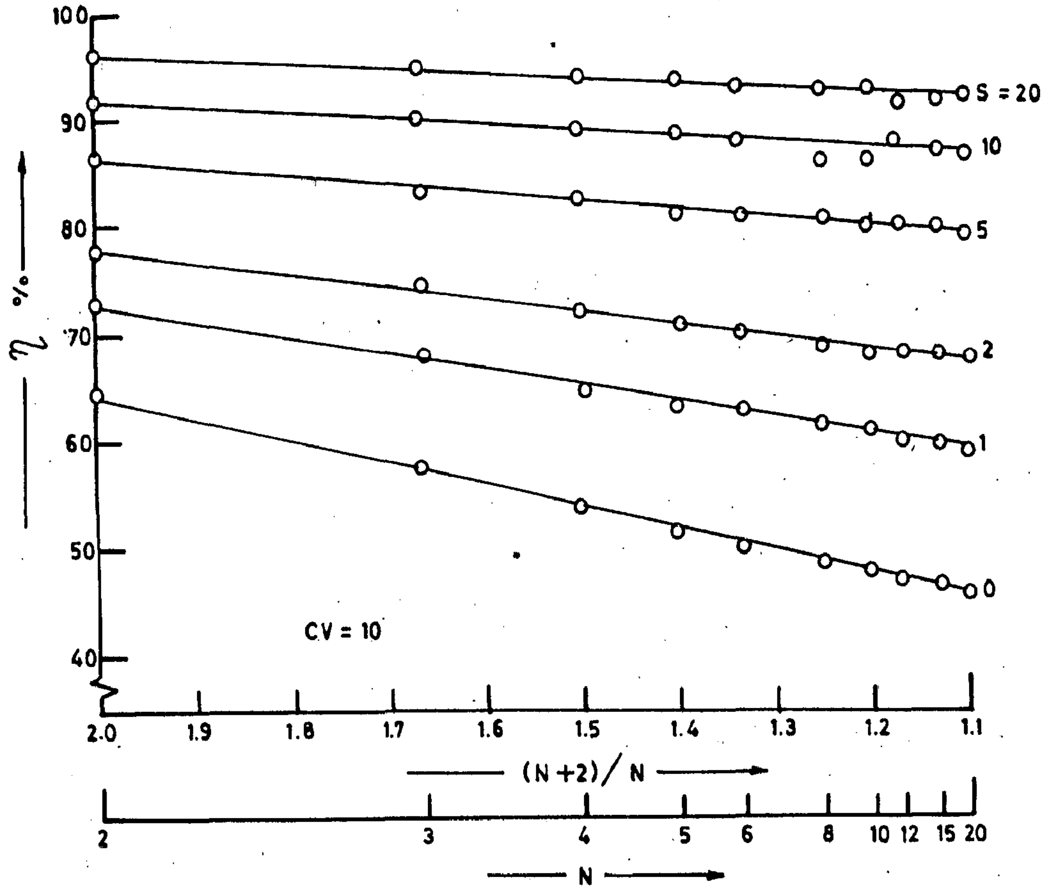


FIG.4.16 RELATIONSHIP BETWEEN LINE EFFICIENCY AND (N+2)/N

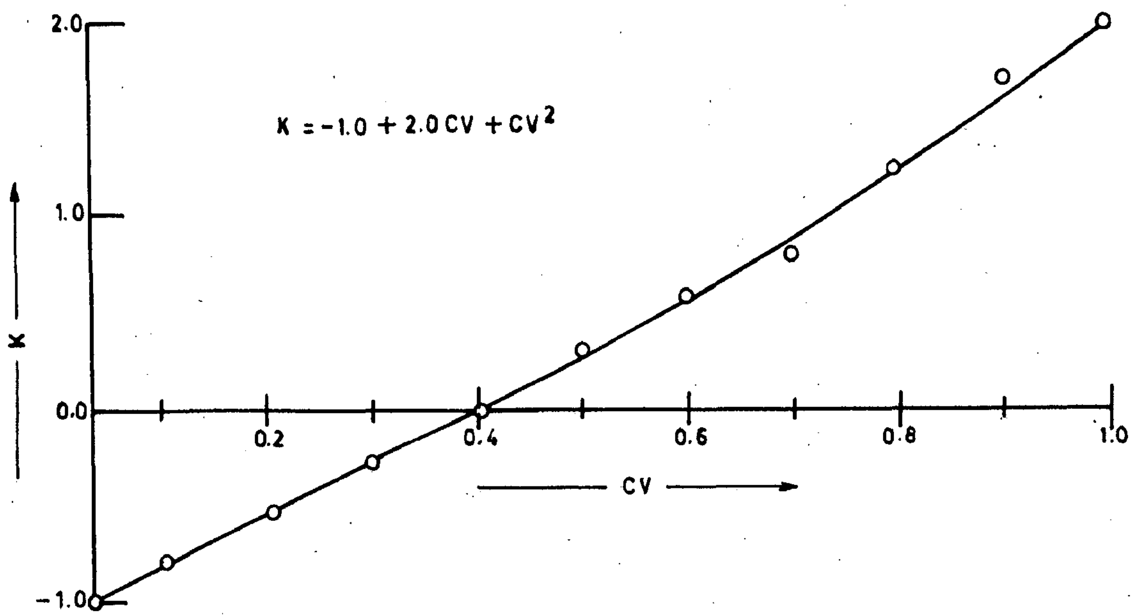


FIG.4.17 K AS A FUNCTION OF CV

were compared for 100 factor level combinations randomly selected over the range $2 \leq N \leq 20, 0 \leq S \leq 20$ and $0.1 \leq CV \leq 0.5$. Both the Students t-test and F-test were employed to check the difference between the means and the variances of the efficiencies, obtained in the two cases. It was found that the two sets of values were significantly the same at 99 percent confidence level. However, when the sets of values corresponding to situation CV > 0.5 were compared separately, it was found that normal distribution gave better efficiency as compared to Erlang. The difference increased with increase in CV and was significantly large at CV = 1.0, i.e. for exponential distribution. Thus for larger values of CV (CV > 0.5), the empirical model (Equations 4.5 to 4.9) would give approximate results for Erlang distribution.

4.7.3 Exponentially Distributed Times

Analysis of the simulation results revealed that the same transformations as used in the case of normally distributed operation times could be employed for exponential time distribution as well. (Figs.4.18 and 4.19). The value of K(=2) obtained in this case is the same as for the normally distributed operation times with CV = 1.0. Since the efficiency of a two stage line with exponentially distributed operation times is given by equation (4.1), equation (4.2) can be written for the exponential times as:

$$\eta_S^N = \frac{S+2}{S+3} - \left(\frac{N-2}{0.9\,N}\right)\left[\frac{S+2}{S+3} - \eta_S^{20}\right] \quad .. \quad (4.10)$$

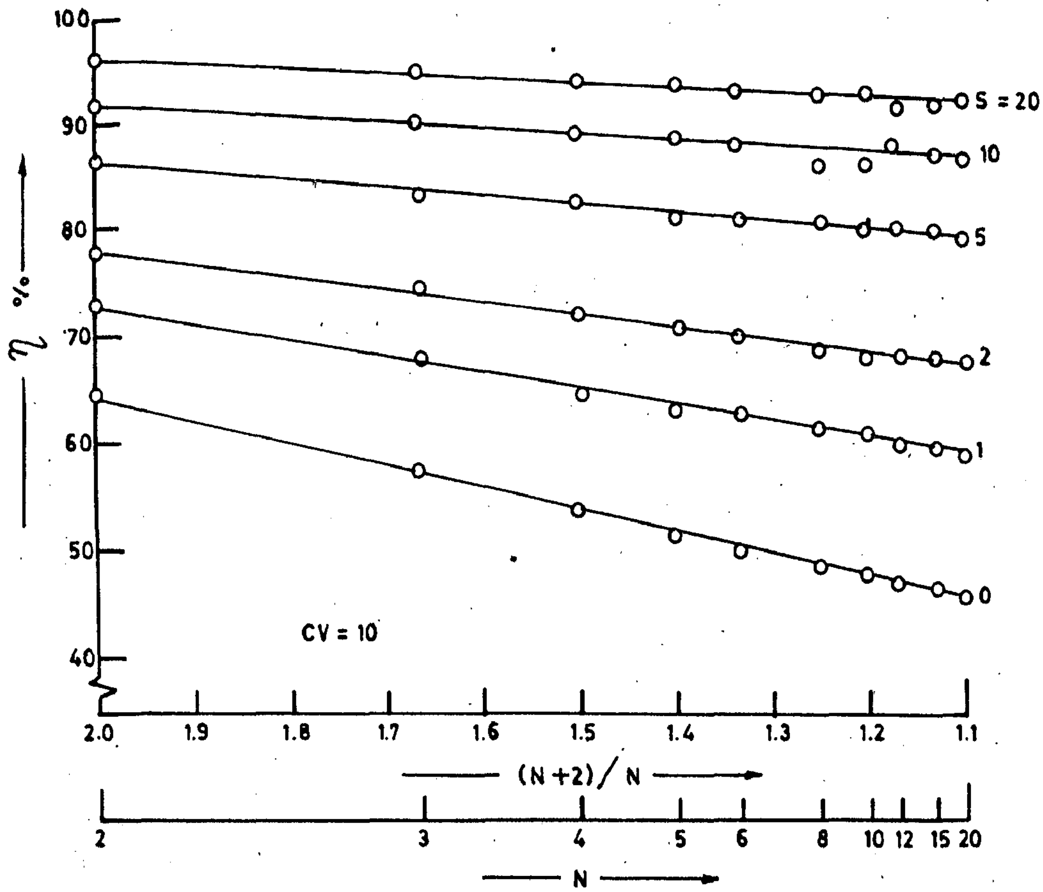


FIG.4.16 RELATIONSHIP BETWEEN LINE EFFICIENCY AND (N+2)/N

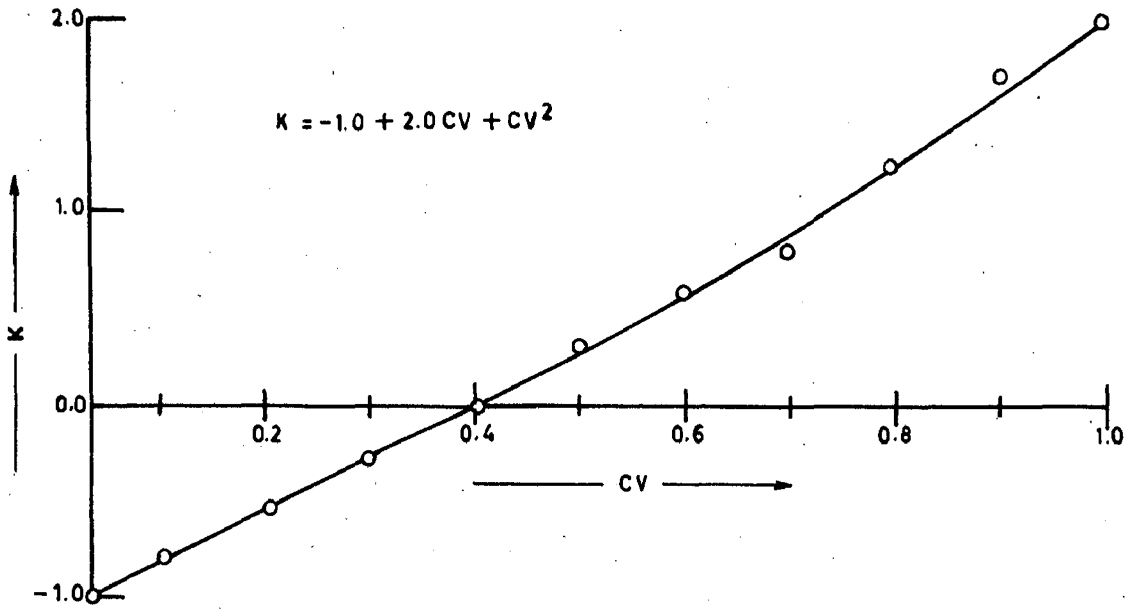


FIG.4.17 K AS A FUNCTION OF CV

between the range of N = 2 and 20, S = 0 and 20 and CV = 0.1 and 1.0 (Appendix A 4.2) were compared employing the Students' t-test. The difference in the results was found to be insignificant at 99 percent confidence level, and the simulated and predicted results yielded the coefficient of correlation equal to 0.9662.

In the present case, although the model has been developed on the basis of the data generated for lines of upto 20 stages, but it is expected to yield significantly accurate results even for lines with $N > 20$ and over the range $0 \leq S \leq \infty$.

4.7.2 Erlangian Processing Times of Stages

The empirical model developed in section 4.7 is based on the simulation data obtained by assuming normally distributed operation times. However, it can also be applied with considerable accuracy to situations when the operation times follow Erlang distribution. Rao [112] using a two stage line demonstrated that at lower values of CV, the Erlang and normal distributions give practically identical results. This is because the Erlang and normal curves have similar shape when the coefficient of variation is low. It has also been indicated that, the presence of intermediate storages helps to conceal the effect of the specific forms of distribution curve. Thus, for a line with finite storage and smaller variability in processing times it is the value of CV which is important, and not the type of distribution. To check the validity of this statement, efficiencies, obtained by assuming normal and Erlang time distributions, were compared for 100 factor level combinations randomly selected over the range $2 \leq N \leq 20, 0 \leq S \leq 20$ and $0.1 \leq CV \leq 0.5$. Both the Students t-test and F-test were employed to check the difference between the means and the variances of the efficiencies, obtained in the two cases. It was found that the two sets of values were significantly the same at 99 percent confidence level. However, when the sets of values corresponding to situation $CV > 0.5$ were compared separately, it was found that normal distribution gave better efficiency as compared to Erlang. The difference increased with increase in CV and was significantly large at CV = 1.0, i.e. for exponential distribution. Thus for larger values of CV ($CV > 0.5$), the empirical model (Equations 4.5 to 4.9) would give approximate results for Erlang distribution.

4.7.3 Exponentially Distributed Times

Analysis of the simulation results revealed that the same transformations as used in the case of normally distributed operation times could be employed for exponential time distribution as well. (Figs.4.18 and 4.19). The value of K(=2) obtained in this case is the same as for the normally distributed operation times with CV = 1.0. Since the efficiency of a two stage line with exponentially distributed operation times is given by equation (4.1), equation (4.2) can be written for the exponential times as:

$$\eta_S^N = \frac{S+2}{S+3} - \left(\frac{N-2}{0.9\ N}\right) \left[\frac{S+2}{S+3} - \eta_S^{20}\right] \quad .. \quad (4.10)$$

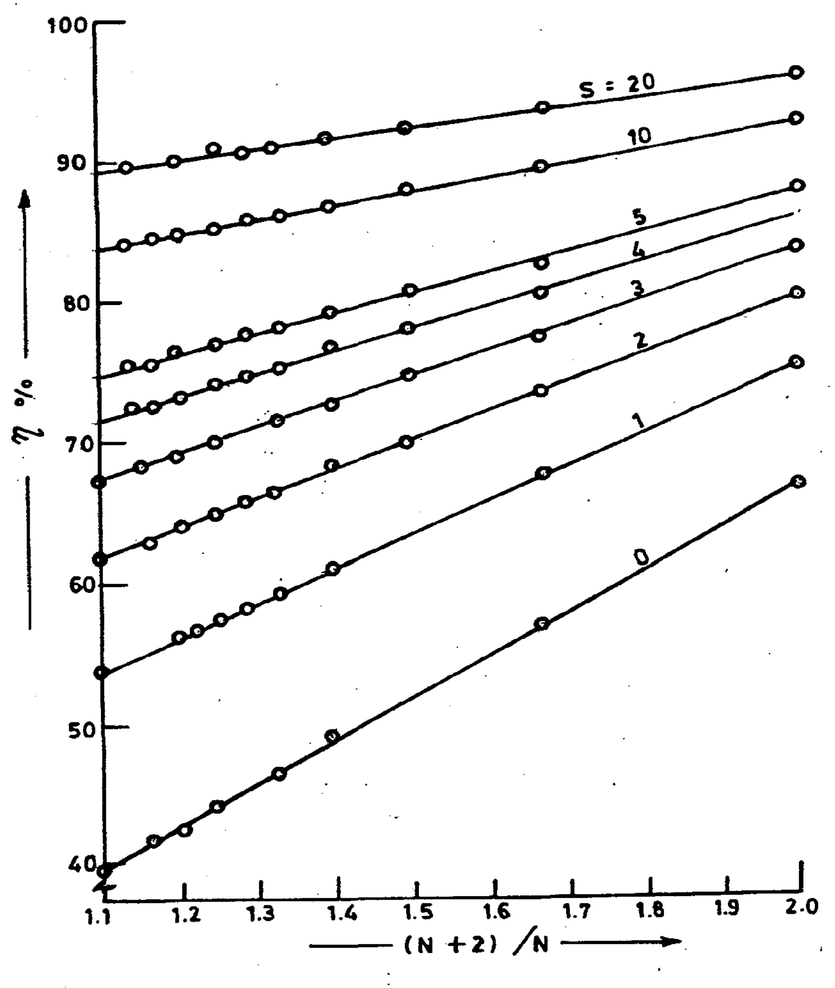


FIG. 4.18 EFFICIENCY AS A LINEAR FUNCTION OF $(N+2)/N$
(EXPONENTIAL PROCESSING TIMES)

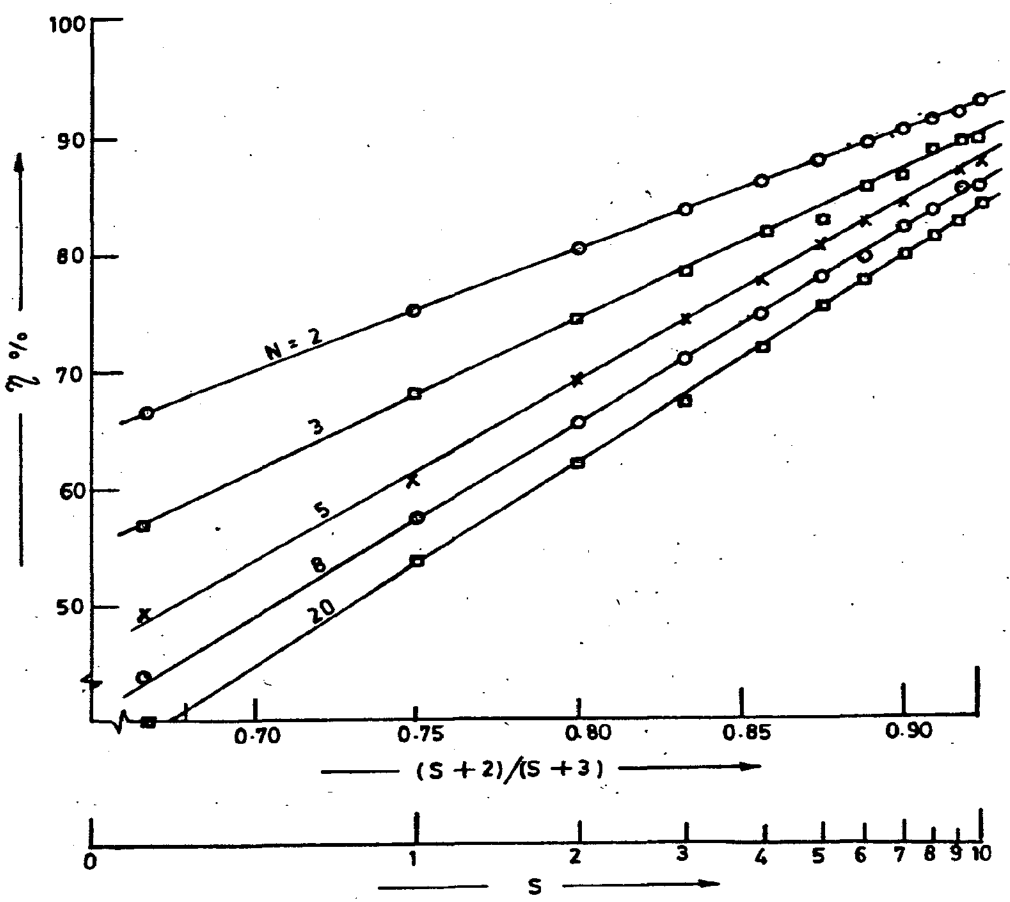


FIG. 4.19 RELATIONSHIP BETWEEN η AND $(S+2)/(S+3)$

By regression analysis of the data for a 20 stage line and $0 \leq S \leq 20$, the following expression was obtained

$$\eta_S^{20} = 0.3938 + .576 \left(\frac{S}{S + 3} \right) \qquad .. \qquad (4.11)$$

with a coefficient of correlation of 0.9978.

Thus, with the help of equations (4.10) and (4.11) the efficiency of a balanced line, having exponentially distributed operation times could be obtained. For values of N ranging from 2 to 20 and $0 \leq S \leq 20$ a coefficient of correlation of 0.9798 was obtained between the predicted and simulated results.

4.8 INPROCESS BUFFER STORAGE COSTS

The provision of inprocess buffer in a flow line helps to increase its production efficiency. It however, involves expenditure of money in terms of the value of space occupied, the cost of handling the material and the cost of inprocess inventory. For a given operating configuration, it would therefore be desirable to obtain the buffer capacity that would maximize the overall gain from the system. The following cost parameters pertaining to the inprocess inventory have been considered.

i) Storage space cost (SC)

The space required for the inprocess storage would depend upon the number of storage points in the line, as well as on the capacity of each. The space requirement of the work unit may vary as it progresses through the line, but in most of the cases it can be assumed to remain constant. If one work unit occupies one unit space, and C_1 is the cost of unit space per unit time, then for a balanced line, the cost of the storage space can be expressed as:

$$SC = C_1 . S(N-1) / \text{ unit time} \qquad .. \quad (4.12)$$

ii) Inventory holding cost (IC)

Inprocess inventory carrying cost will depend upon the amount of inprocess inventory and its cost. In a balanced line, the average inprocess inventory has been found to remain at about 50 percent of the provided buffer capacity. Since each operation adds to the value of the work unit, the intrinsic value of the material increases as it progresses through the line. Taking the average value, let the inventory carrying cost be C_2 per work unit per unit time. Then IC can be written as:

$$IC = C_2 \frac{S(N-1)}{2} / \text{ unit time} \qquad .. \quad (4.13)$$

iii) Storage handling cost (HC)

This will comprise of the cost on the investment in the material handling equipment and the operating cost. For each storage the first has been assumed as fixed while the later may vary as a linear function of the storage size.

$$HC = C_3(N-1) + C_4 \, S(N-1) / \text{ unit time} \qquad .. \quad (4.14)$$

where, C_3 is the fixed cost of each storage per unit time, and C_4 is the variable cost per work unit per unit time.

iv) Additional Revenue (AR)

Increase in capacity of the inprocess storage contributes towards improving the system efficiency and the output. If P is the profit from each unit produced, then the increase in revenue due to the provision of buffer can be expressed as,

$$AR = (\eta_s^N - \eta_o^N)\ P/\text{unit time} \qquad .. \qquad (4.15)$$

v) Net gain

Net gain from the system per unit time would be equal to,

$$GAIN = AR - (SC + IC + HC) \qquad ... \qquad (4.16)$$

Fig.4.20 shows the variation in various cost elements as a function of the inprocess buffer capacity for a 10 stage line with normally distributed operation times (μ=1.0 and CV = 0.5). In this case the various cost factors were assumed as given below.

C_1 = 0.125/ unit capacity/ 100 time units

C_2 = 0.25/ unit capacity/ 100 time units

C_3 = 0.70/ buffer/ 100 time units

P = 1.25/ unit produced

The optimum capacity of each buffer for the situation considered comes out to be as 4 units, beyond which the net gain from the system decreases and for S > 15, it becomes negative.

4.9 OPTIMUM BUFFER CAPACITY MODEL

In a balanced flow line of the type modelled in this chapter, the system efficiency and hence the revenue from the system would depend upon N,CV and S and for a particular N stage line having fixed CV, the revenue is a function of S only. The various cost parameters are also functions of S alone. Therefore, it is possible to derive an expression (Eqn. 4.17) for the optimum capacity of the inprocess buffers (S^*), in terms of the line parameters and the buffer costs.

$$S^* = \sqrt{\frac{(K+1-L)\ P}{(N-1)\ C}} - (K+1) \qquad .. \qquad (4.17)$$

Where, $C = C_1 + C_2 .\ ABU + C_4$

and $$L = (K+1)\ \eta_o^2 - \left(\frac{2-N'}{0.9\ N}\right)(K+1)\ (\eta_o^2 - \eta_o^{20}) \qquad ... \qquad (4.18)$$

Detailed derivation of the equation (4.17) is given in Appendix A 4.3.

The utility of the model can best be illustrated by an example.

4.10 ILLUSTRATIVE EXAMPLE

The cost data given in Section 4.8 pertains to a 10 stage balanced line having normally distributed operation times with $CV_i = 0.5\ \forall i$. Since the costs are given for 100 times units, P would also be scaled to 100 time units.

Since the average buffer utilisation in case of balanced lines is about 50 percent, we have,

$$C = C_1 + 0.5\ C_2 + C_4 = 0.25$$

$$P = 1.25 \times 100 \qquad = 125$$

From equations (4.6),(4.7) and (4.9), we get,

$\eta_o^2 = 0.78026$

$\eta_o^{20} = 0.62086$

$K = 0.25$

Substituting the values in equation (4.18), we get

$L = 0.79822$

and the optimum inprocess buffers size is

$$S^* = \sqrt{\frac{0.25 + 1 - .79822}{9\ (0.25)}} - 1.25 = 3.76$$

$$\simeq 4 \text{ units}$$

Same value of S^* is obtained from Fig.4.20 as well.

4.11 SENSITIVITY ANALYSIS

For a line with fixed number of stages, the optimum capacity of the interstage storage (S^*) would depend upon the variability in processing times and the profit and cost factors. Fig. 4.21 illustrates that the value of S^* increases with increase in coefficient of variation and that the increase is almost linear. It is evident from equation (4.17) that S^* will be proportional to $\sqrt{P}$ and inversely proportional to $\sqrt{C}$. The effect of P and C for the assumed situation is illustrated in Fig.4.22. When the profit per unit produced is high, larger size buffer should be provided to get the maximum gain from the system, while the reverse is true when the costs associated with the buffer are large.

Due to practical considerations, it might not always be convenient to use the optimal buffer size or it might not be possible to accurately estimate the values of different parameters

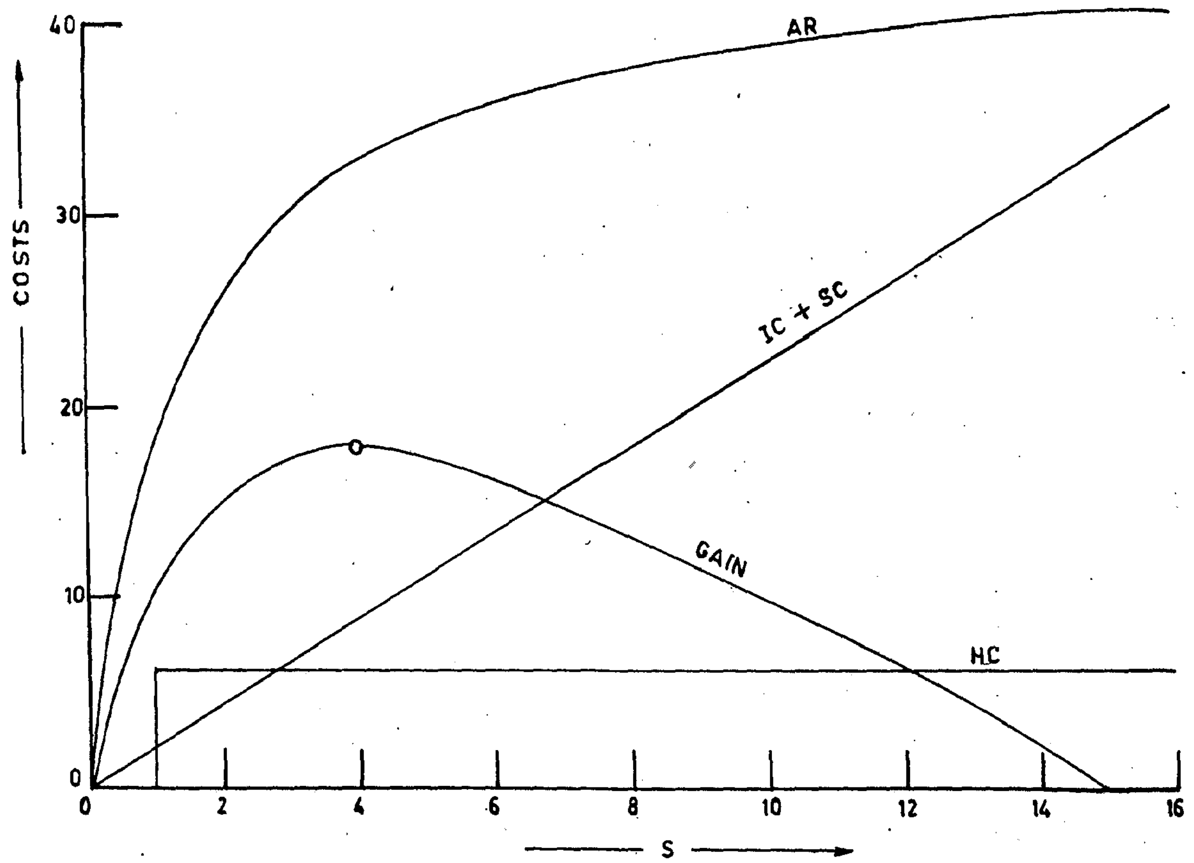


FIG. 4.20 EFFECT OF BUFFER CAPACITY ON SYSTEM COSTS

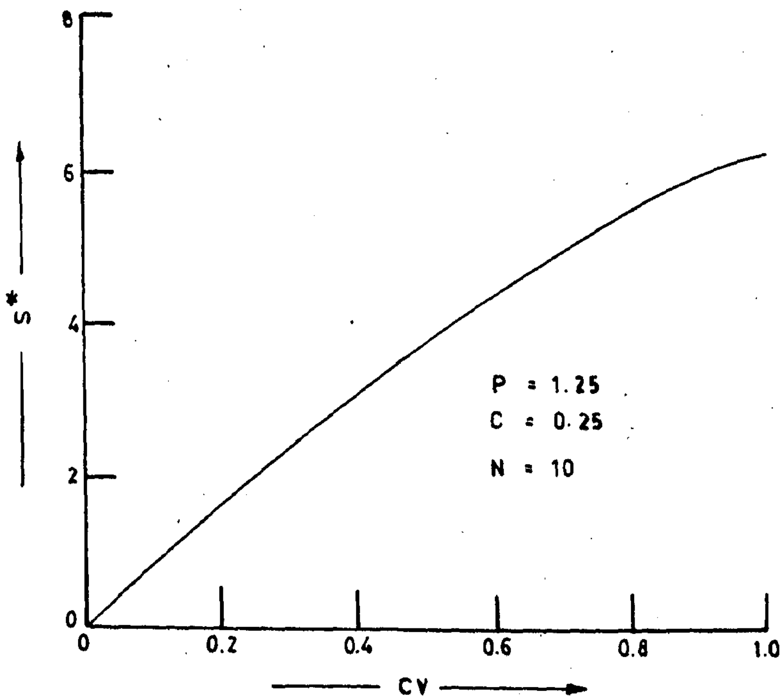


FIG. 4.21 EFFECT OF OPERATION TIMES VARIABILITY ON S*

used in the model. Under these circumstances, the total gain from the system would not be optimum. Fig.4.23 shows the variation in gain as a function of S for different combinations of P and C. It can be observed that when the cost parameters result in higher value of S^*, the gain curve obtained is flatter around S^* as compared to the case when the value of S^* is low. This effect is more clearly demonstrated in Fig. 4.24. In case when S^* is low ($S^* = 2$), the deviation from S^* causes greater percentage loss in gain, as compared to the cases where S^* is large ($S^* = 4$ or 6). It can also be observed that under estimation of S^* is more harmful as compared to its overestimation. Hence, it is very important to provide inprocess buffers as close to the optimum as possible, specially in cases, where the line parameters are such as to require smaller size buffers.

4.12 COMPARISON WITH ANDERSON AND MOODIE'S MODEL [5]

Anderson and Moodie [5] developed an empirical model for the efficiency and the optimum buffer storage capacity for the situation, when CV = 0.3. For other values of CV, they recommended the use of equation (4.19)

$$S^*_{CV} = S^*_{0.3} \cdot \frac{CV}{0.3} \qquad .. \quad (4.19)$$

The results obtained by the use of this equation have been compared with those obtained from equation (4.17), in Fig.4.25. The cost data employed, has been adopted from Anderson and Moodie [5]. It can be observed that at lower levels of CV, the difference is negligibly small, which goes on increasing with

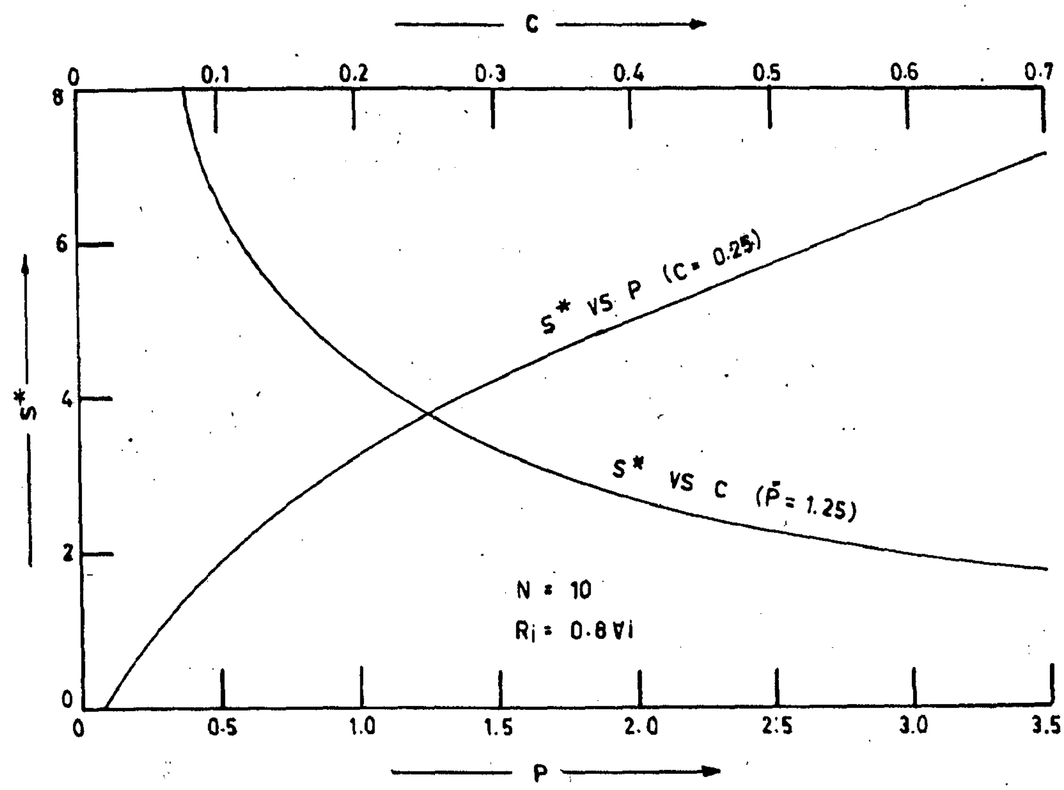


FIG. 4.22 EFFECT OF VARIATIONS IN COSTS (P AND C) ON S^*

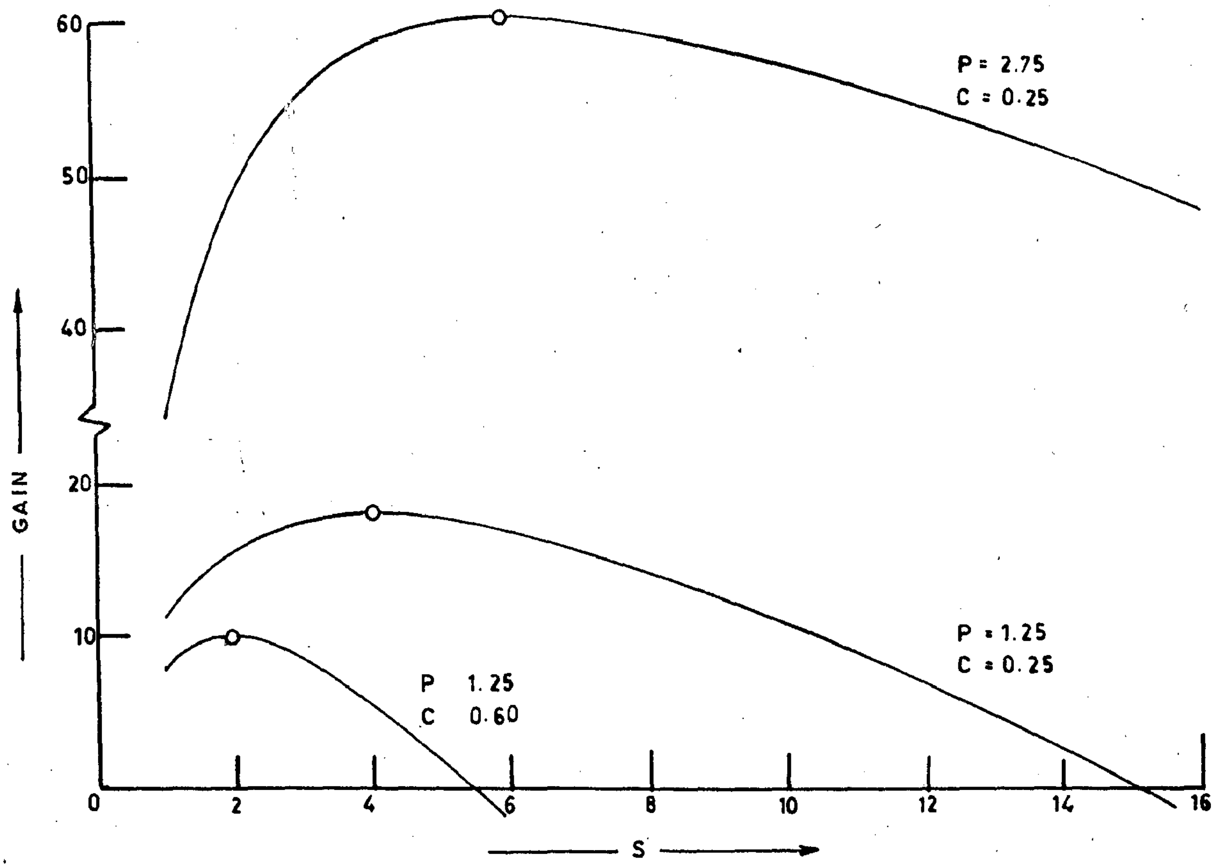


FIG. 4.23 GAIN AS A FUNCTION OF S FOR VARIOUS COMBINATIONS OF P AND C

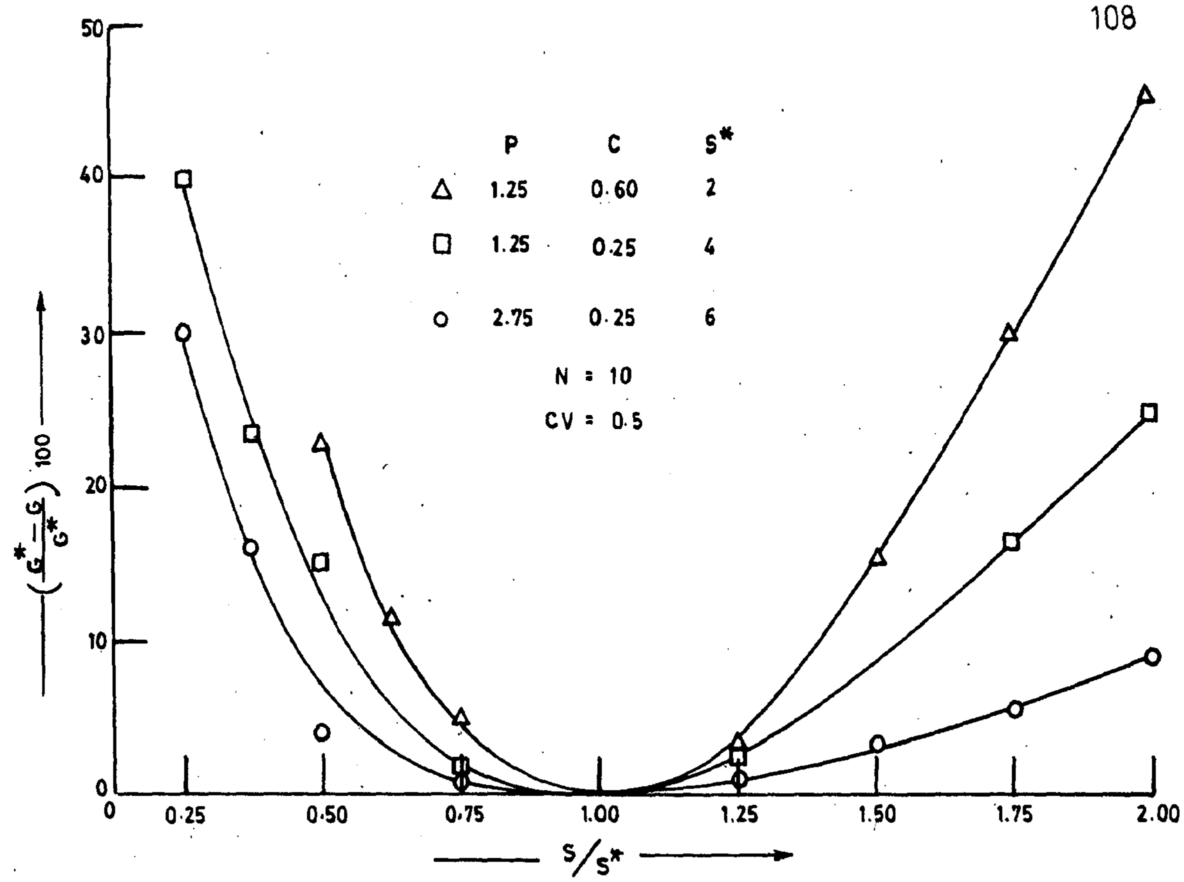


FIG. 4.24 EFFECT OF DEVIATION FROM S*

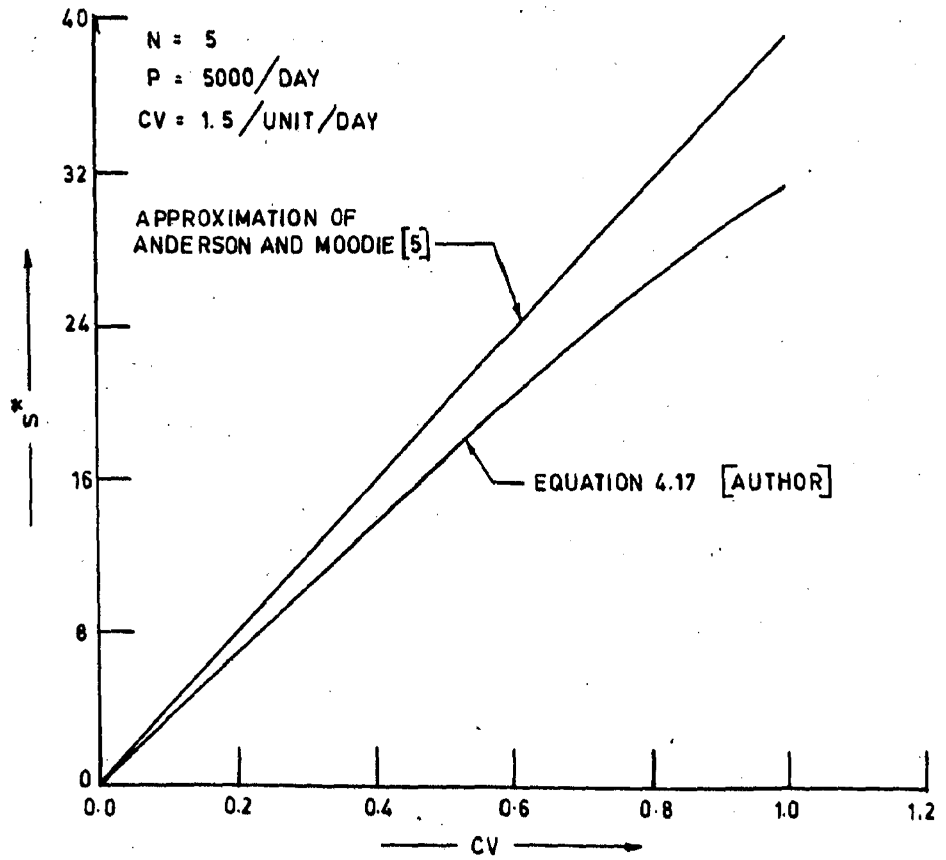


FIG. 4.25 OPTIMUM BUFFER CAPACITY AS A FUNCTION OF CV (ANDERSON AND MOODIE's [5] COST DATA)

increase in CV. The constants in [5] are specific for CV=0.3, for which the value of S^* is in close agreement with the one given by the present model, but Anderson and Moodie's approximation of linearity between S^* and CV leads to over estimation of buffer size at higher values of CV.

4.13 CONCLUSIONS

Based on the results given in this chapter the following conclusions can be drawn :

1) Estimation of buffer capacities for 100 percent efficiency of flow line is not feasible, since their values go on increasing as the length of production run is increased. The efficiency soon gets saturated for inprocess buffers of 4 or 5 units, while the ABL or WIP continues to grow with time.

2) The system efficiency is very sensitive to changes in S and N when their values are small. At higher levels, as S and N are increased, their effect on η gradually vanishes. However, with increase in CV, η gradually decreases at all levels, the effect being more for small S and large N.

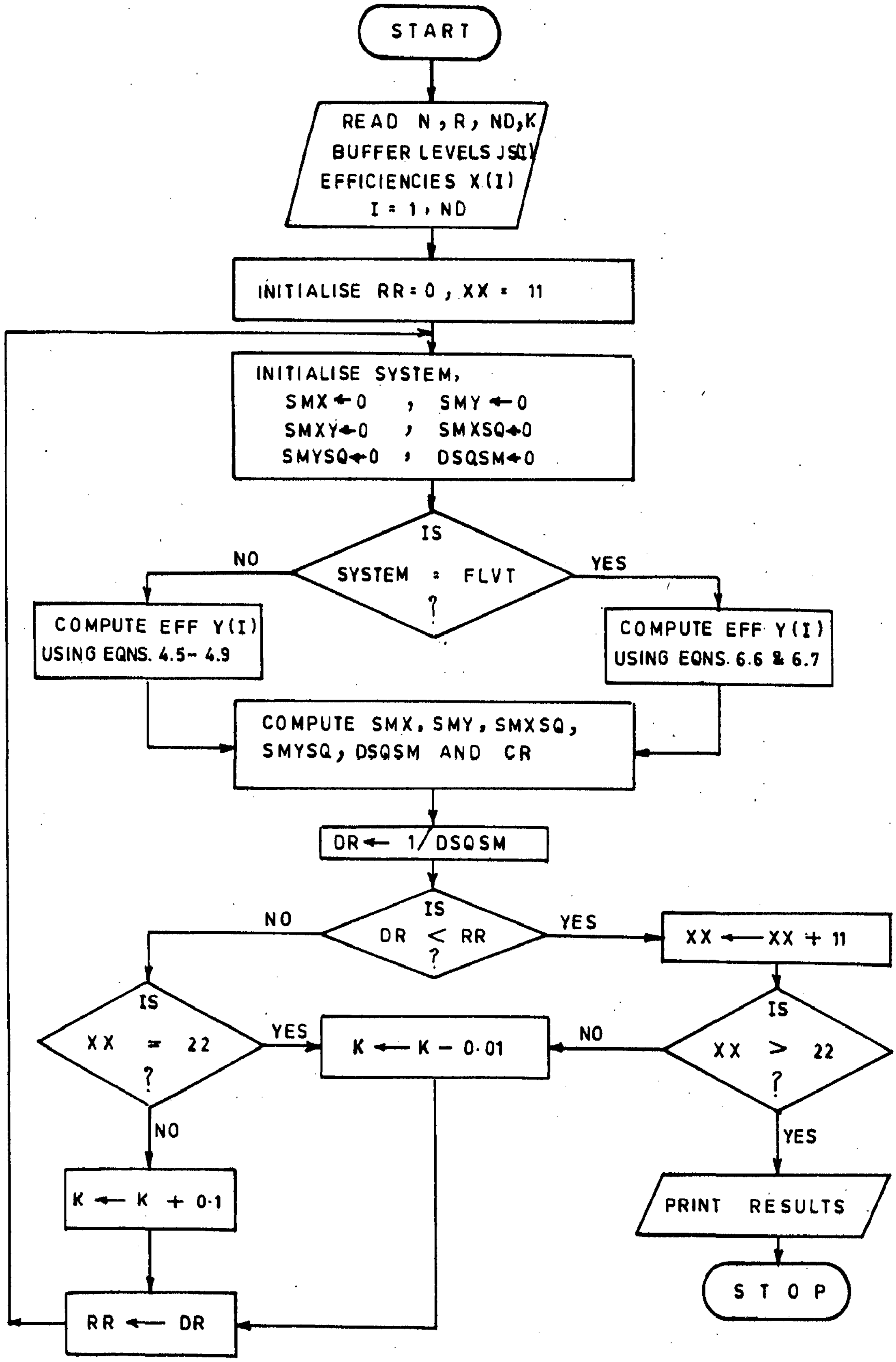
3) Based on the simulation results, empirical model of line η has been developed, which can very efficiently be employed to determine η for exponential, normal and Erlangian operation times of stages.

4) The optimum size of inprocess buffers, can easily and accurately be computed by employing the empirical model (Eqn.4.17) developed here.

5) The value of S^* is more sensitive to variations in costs at their lower levels, while it increases almost uniformly with increase in CV.

6) The underestimation of S^* causes greater decrease in gain from the system as compared to its overestimation.

APPENDIX A4.1



FLOW CHART FOR DETERMING THE VALUE OF CONSTANT K IN PARAMETER (S+K)/(S+K+1)

APPENDIX A 4.2

SIMULATED AND EMPIRICAL VALUES OF FLOW LINE EFFICIENCY
WHEN THE OPERATION TIMES ARE DISTRIBUTED NORMALLY

N	S	η_{SIM}	η_{EMP}	N	S	η_{SIM}	η_{EMP}
		CV = 0.1				CV = 0.3	
6	10	.9971	.9981	6	3	.9600	.9570
4	6	.9962	.9972	5	8	.9836	.9824
20	15	.9992	.9985	10	0	.7584	.7559
5	20	.9998	.9991	3	5	.9993	.9918
12	2	.9887	.9903	12	6	.9704	.9740
6	12	.9973	.9984	15	1	.8915	.8960
10	5	.9978	.9959	2	20	.9966	.9952
15	3	.9946	.9932	20	12	.9877	.9858
8	0	.8993	.9072	8	2	.9381	.9383
2	8	.9983	.9986	5	10	.9806	.9857
		CV = 0.2				CV = 0.4	
8	3	.9766	.9779	5	3	9283	9327
4	1	.9547	.9547	9	15	9807	9816
10	8	.9914	.9907	2	20	9994	9916
6	4	.9821	.9837	6	8	9722	9691
2	0	.8986	.9005	12	1	8421	8465
15	20	.9967	.9960	3	5	9635	9619
20	5	.9830	.9849	15	0	6805	6830
3	10	.9953	.9944	10	12	9785	9771
8	6	.9376	.9882	5	6	9568	9616
10	15	.9943	.9949	20	4	9280	9371
		CV = 0.5				CV = 0.8	
8	6	.9379	.9388	5	8	.8972	.9040
10	8	.9492	.9508	3	1	.7306	.7355
5	0	.6329	.6740	12	12	.9203	.9233
20	4	.8998	.9092	2	15	.9676	.9592
4	12	.9656	.9707	6	3	.8001	.8067
3	20	.9889	.9835	15	0	.5107	.5076
12	3	.8893	.8914	20	6	.8463	.8638
2	10	.9817	.9754	4	5	.8757	.8707
6	1	.8106	.8117	10	20	.9520	.9516
15	15	.9700	.9711	8	10	.9012	.9140

CV = 0.6

10	15	.9667	.9615	3
5	3	.8606	.8730	8
2	20	.9825	.9815	5
6	1	.7650	.7663	15
20	12	.8567	.8505	2
15	5	.9003	.8994	20
8	2	.8213	.8247	12
20	8	.9242	.9299	4
12	0	.6005	.5852	10
4	10	.9521	.9525	6

CV = 0.9

2	7698	7694
5	8230	8309
10	8903	9043
0	4900	4843
20	9619	9608
3	7438	7564
12	8917	9088
8	8830	8913
6	8326	8474
15	9298	9294

CV = 0.7

12	3	.8242	.8239	2
20	5	.8625	.8712	15
3	15	.9662	.9604	3
2	6	.9293	.9315	12
4	20	.9657	.9665	6
6	12	.9430	.9426	5
10	4	.8573	.8559	10
5	8	.9202	.9219	8
15	2	.7778	.7755	20
8	0	.5656	.5627	4

CV = 1.0

5	8674	8645
3	7224	7342
0	5763	5740
20	9163	9314
1	6230	6311
8	8559	8692
6	8288	8271
10	8704	8825
2	6683	6772
12	9091	9080

APPENDIX A 4.3

OPTIMUM BUFFER CAPACITY MODEL

For the cost parameters discussed in section 4.8, the net gain from the system can be expressed as :

$$G = AR - (SC+IC+HC)$$

$$=(\eta_s^N - \eta_0^N) P - [S(N+1)(C_1+C_2.ABU+C_4)+C_3(N-1)] \quad .. (A4.1)$$

For a given line, N, C_1,C_2, ABU, C_3, and C_4 are constants, and let,

$$C = C_1 + C_2 \text{ ABU} + C_4 \quad .. (A4.2)$$

$$\overline{N} = N-1 \quad .. (A4.3)$$

When η is taken as percent, all costs are scaled to 100 time units.

$$G = (\eta_s^N - \eta_0^N) P-S\overline{N}C-C_3 \overline{N}$$

or $$G = \eta_s^N P - S\overline{N}C -(\eta_0^N P + C_3 \overline{N}) \quad .. (A4.4)$$

$$\text{Here, } \eta_0^N P + C_3 \overline{N} = \text{constant} = K_1 \text{ say} \quad .. (A4.5)$$

and from equation (4.5)

$$\eta_s^N = \frac{1}{S+K+1} [S+(K+1) \eta_2^0 - \frac{(2-N')}{0.9} (K+1) \eta_0^2 - \eta_0^{20})]$$

Since S is the only variable, we can put

$$(K+1) \eta_0^2 - \frac{2-N'}{0.9} (K+1) (\eta_0^2 - \eta_0^{20}) = L = \text{constant} \quad (A4.6)$$

$$\therefore \quad \eta_s^N = \frac{1}{S+K'} (S+L) \quad .. (A4.7)$$

Where, K' = K+ 1

Hence, from equations (A4.4),(A4.5) and (A4.7), we get

$$G = \frac{(S+L)}{S+K'} P - S \bar{N} C - K_1 \qquad (A4.8)$$

Differentiating equation(A 4.8)w.r.t S and equating to zero

$$P \left[\frac{(S+K') - (S+L)}{(S+K')^2} \right] - \bar{N} C = 0$$

or $$(S+K')^2 = \frac{(K'-L)P}{N'C}$$

or $$S^* = S = \sqrt{\frac{(K'-L)P}{\bar{N}C}} - K' \qquad (A4.9)$$

The model for the optimum buffer capacity could also be obtained by considering P as the delay cost per unit time, Then the total cost could be written as,

$$TC = (1 - \eta_S^N) P + S(N-1)(C_1+C_2+C_3) - (N-1)C_4$$

$$= \left(1 + \frac{S+L}{S+K'}\right) P + S\bar{N}C - \bar{N} C_4$$

or, $$TC = \left(\frac{K'-L}{S+K}\right) P - SNC - \bar{N} C_4 \qquad (A4.10)$$

By differentiating equation (A 4.10) w.r.t S and equating to zero, the same model as in Eqn. (A4.9) is obtained.

CHAPTER - 5

UNBALANCED FLOW LINES WITH RANDOM OPERATION TIMES

5.1 INTRODUCTION

It has already been defined in Chapter 4 that a flow line is considered to be balanced, only when all the stages have identical operation times and the interstage storages are of equal capacity. In practice, it may not be possible to have completely balanced lines, and hence, it is necessary to understand the behaviour of the unbalanced lines.

Survey of literature (Chapter 2, Sec. 2.3.3) reveals that relatively less attention has been paid to the study of unbalanced production lines. Some of the researchers [34,64, 109,136], have proposed that slight unbalancing of the line improves its production rate, but their opinions about the choice of unbalancing policy are contradictory to each other in some of the cases. In most of the studies, analytical as well as simulation, the efficiency or the production rate of the line has generally been adopted as the criterion of system performance, while the amount of inprocess inventory, which has a great bearing on the system economy, has not been given due attention.

The simulation results reported in this chapter pertain to a flow line system of Fig. 3.1, wherein, several policies of the line unbalancing have been examined in detail, specially as regards to their production efficiency and inprocess inventory.

5.2 UNBALANCING POLICIES

The following line unbalancing policies have been examined by assigning :

(i) Unequal mean operation times to stages,

(ii) unequal variability in operation times at different stages,

(iii) unequal interstage buffers.

For the purpose of conducting this study very large number of combinations of the system parameters are possible. However, they have been varied, only in the following systematic orders, so that certain general conclusions could be drawn :

(a) Variable increasing along the line.

(b) Variable decreasing along the line.

(c) Variable increasing upto middle of the line and then decreasing.

(d) Variable decreasing upto middle of the line and then increasing.

In all the experiments, the values of the operating parameters have been selected so as to satisfy the following relationships :

$$\sum_{i=1}^{N} \mu_i = \mu N = \text{constant}$$

$$\sum_{j=2}^{N} S_i = S(N-1) = \text{constant}$$

$$\sum_{i=1}^{N} CV_i = CV(N-1) = \text{constant}$$

where, μ, S and CV are the average values, used in case of balanced line.

5.3 OPERATION TIME DISTRIBUTION

Simulation runs have been made for exponential, Erlang and normally distributed operation times of the stages. The Erlang and normally distributed times have been used to demonstrate the influence of operation time variability on the performance of unbalanced lines, whereas exponential time distribution was used to check the validity of Hillier and Boling's Bowl phenomenon [64].

5.4 LENGTH OF SIMULATION RUN

System efficiency has been chosen as the response of major concern and the same has been used as the basis for determining LOR, as given in Sec. 3.7.2. Four replications of each run were obtained so as to reduce the error to within $\pm$ 0.25 percent at 95 percent confidence level.

The details of the system model, simulation model and assumptions are the same as given in Chapter 4.

5.5 RESULTS AND DISCUSSION

5.5.1 Interstage Storage Utilisation

To study the variations in the average buffer level (ABL) from stage to stage along the line, a balanced production line having exponential processing times, and finite interstage storage capacities was examined for several values of S and N. The percentage buffer capacity utilisation at each stage, for a few typical line configuration is shown in Fig.5.1. In all these cases the average capacity utilisation of the whole line (ABU) was found to be about 50 percent. The capacity utilisation was higher for the stages in the beginning of the line and decreased towards the end. This is caused by the fact that each stage is dependent upon its predecessor for its supply and hence with larger number of predecessors, the probability of a stage being starved increases. Similarly, the probability of a stage being blocked increases towards the front end, causing the buffers to remain at higher levels. Based on similar observations, Payne,et al. [109] suggested, that in order to reduce total station idle time, operations with greater time variability should be allocated to positions towards the end of the line.

5.5.2 Unbalanced Production Lines

5.5.2.1 Unequal operation times

For this study the total work load of the line was divided among the stages in a manner such that they had unequal mean processing times. The coefficient of variation and the interstage buffer capacities were kept same for all the stages.

5.5.2.1a. Increasing/decreasing operation times

The simulation results for 3 and 5 stage lines with uniformly increasing, constant, and **uniformly** decreasing processing times (processing rate is inverse of processing time) are given in Table 5.1 for exponentially distributed times. In each case the production rate of the middle stage has been kept as one unit per unit time, and $\sum_{i=1}^{N} \mu_i = N$. Some results for lines with normally distributed operation times are illustrated in Fig. 5.2. It can be noticed from Table 5.1 and Fig. 5.2, that amongst the cases examined here, the balanced line appears to be the most efficient, and that as the amount of imbalance is increased, the efficiency of the line deteriorates. The efficiency curves are symmetrical about the equal times ordinate, which proves the following reversibility property.

$$\eta(\mu_1, \mu_2 \ldots \mu_N) = \eta(\mu_N, \mu_{N-1}, \ldots \mu_1) \qquad \ldots (5.1)$$

It can be observed from Fig. 5.2, that a line with smaller number of work stations is more sensitive to imbalance as compared to larger lines and that the sensitivity increases with increase in S and decrease in CV. Thus, we see that the results do not support the findings of Payne, et al. [109], who reported that a line with fast, medium and slow workstation in the beginning, middle and end respectively would lead to smaller percentage of idle time of the stages. On the contrary, this arrangement of stages not only lowers the efficiency of the system, but results in higher work inprocess inventory, as

Table 5.1 Efficiency and ABU of unbalanced lines (Unequal exponential processing times and equal buffers, $S_j = S \ \forall \ j$)

Processing rates	N = 3		N = 5		N = 9	
	η %	ABU %	η %	ABU %	η %	ABU %
0.6 - 1.4	55.00	43.63	49.12	30.77	44.77	25.63
0.8 - 1.2	62.53	46.08	58.22	40.39	52.07	35.50
1.0 - 1.0	68.56	49.61	62.48	50.00	55.85	51.50
1.2 - 0.8	64.10	50.98	57.40	62.50	53.39	69.50
1.4 - 0.6	53.32	51.67	47.25	68.27	44.97	78.63

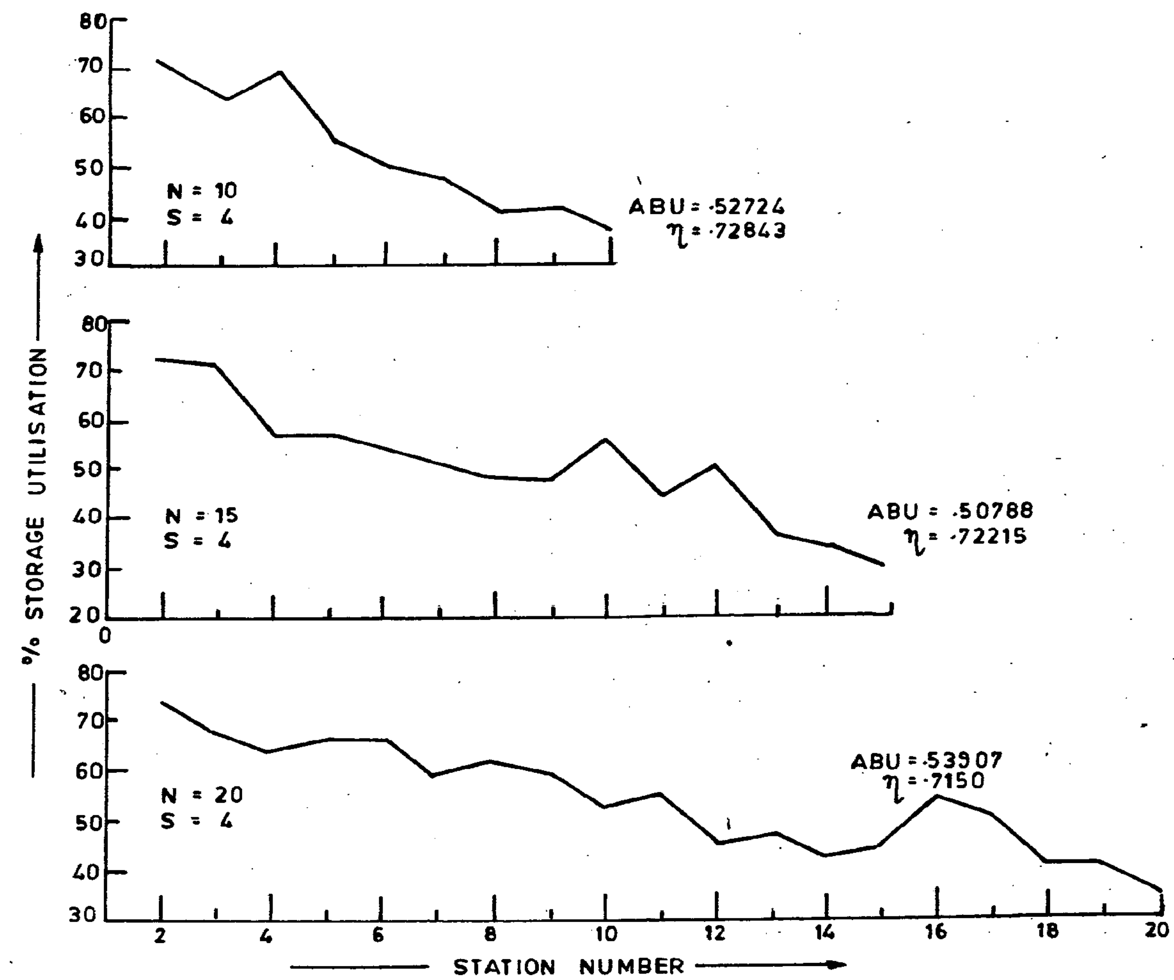


FIG.5.1 BUFFER CAPACITY UTILISATION ALONG THE LINE

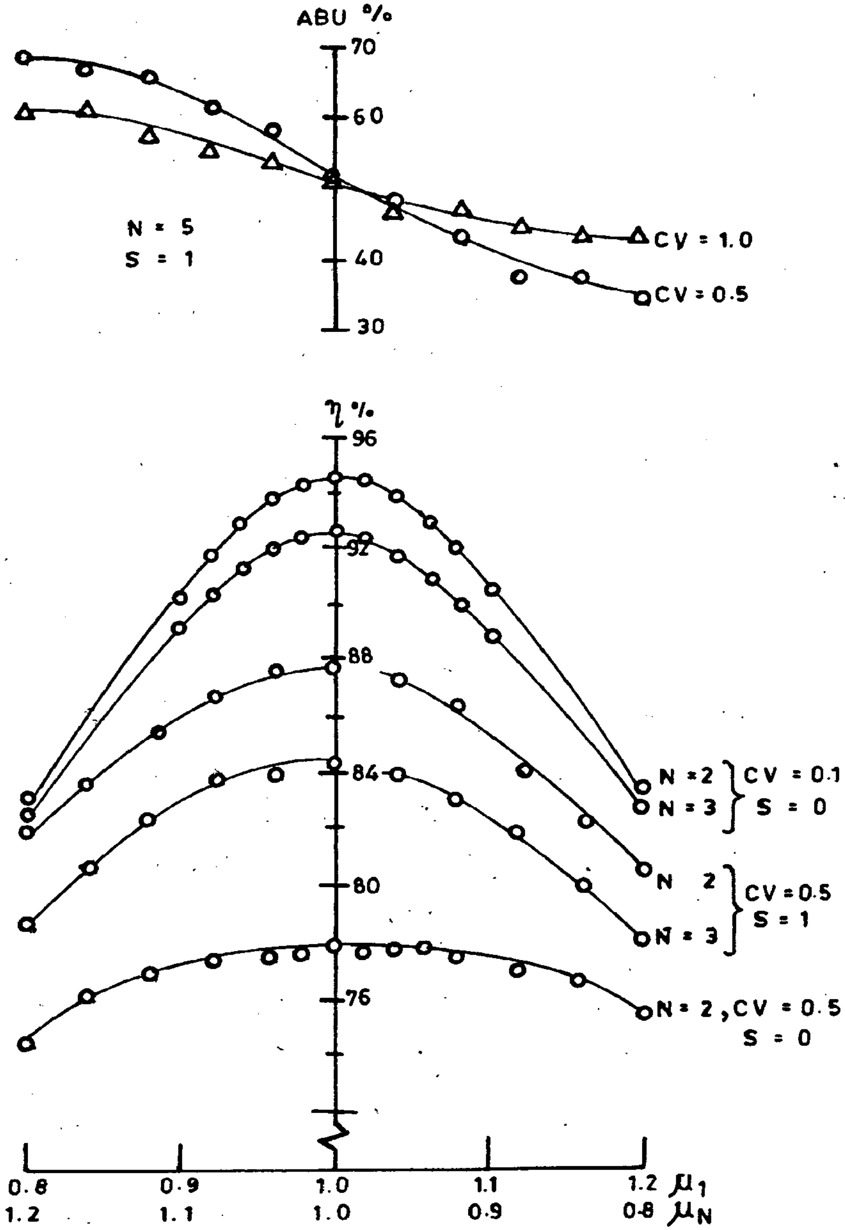


FIG.5.2 EFFECT OF UNEQUAL (GRADUALLY INCREASING/DECREASING) OPERATION TIMES ON THE η AND ABU

is evident from Table 5.1 and Fig. 5.2. A fast stage in the beginning of the line pumps into the system work material at a faster rate, whereas, the slow stage towards the end turns out the work at a slower rate, thus leading to an increase in the level of inprocess inventory. The reverse would happen, when the fast stage is located towards the end of the line and slower stage is placed in the beginning. Fig. 5.2 also shows that the effect of imbalance on ABU is more when the value of CV is small. In case of balanced lines the average buffer in the line has been found to remain at about 50 percent of the provided capacity.

5.5.2.1b. Bowl/inverse bowl formation

To investigate into the Hillier and Bolings' [64] bowl phenomenon, extensive experimentation was performed by assigning unequal processing times to the stages. The line configuration with middle stage having smaller work load as compared to the outer and arranged symmetrically about the centre has been termed as 'Bowl formation'. Figs.5.3 and 5.4 show some of the results obtained when the processing rates of the stages were respectively exponential and Erlang, while the results for normally distributed processing times are given in Figs. 5.5 and 5.6. In Figs. 5.3 - 5.6, in almost all the cases, the maximum efficiency point can be seen to lie slightly towards the left of the equal rates ordinate. Though, the improvement in efficiency due to unbalancing is very small, the results do show a trend which confirms the Hillier and Boling's [64] .

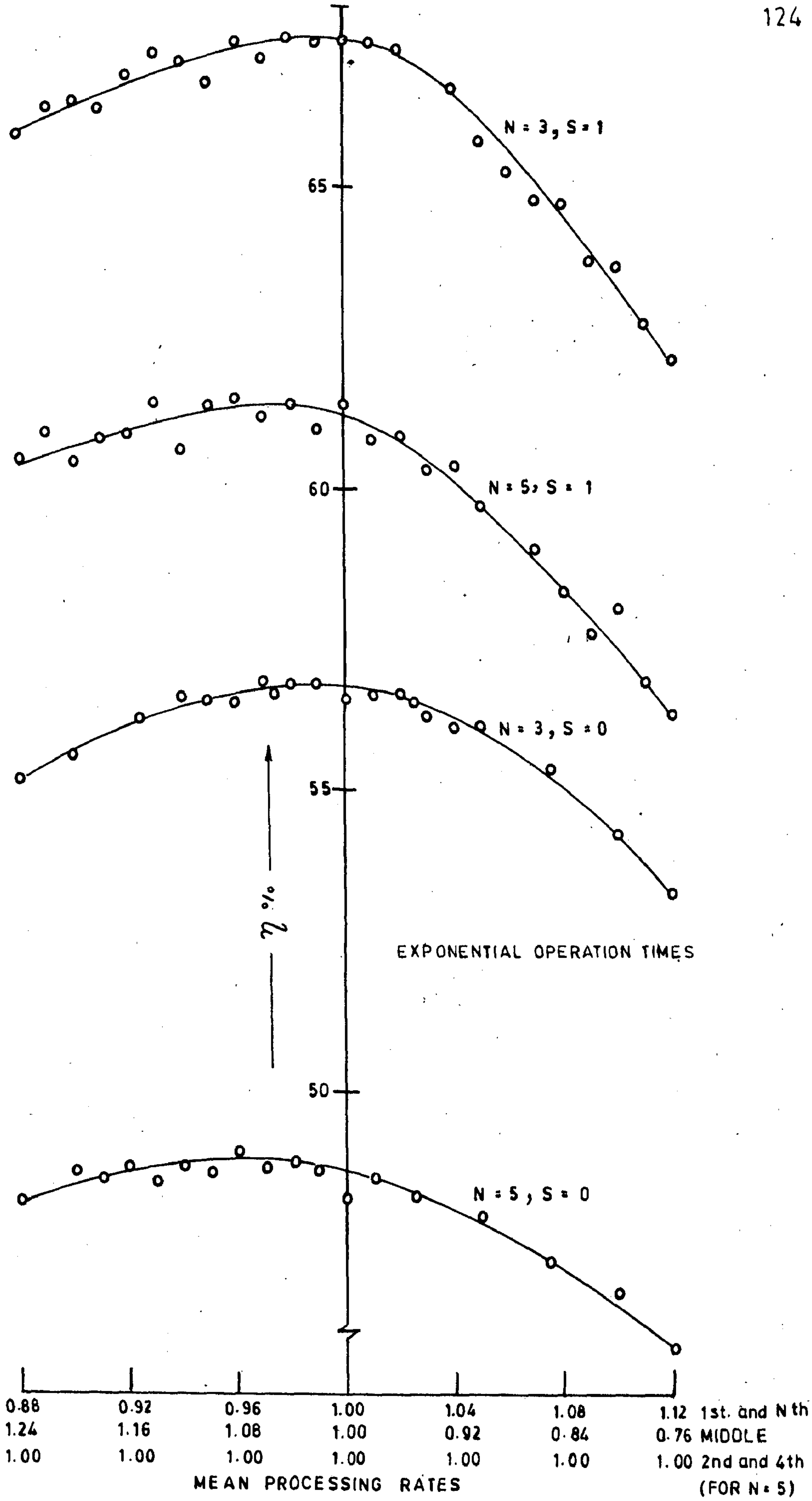


FIG. 5.3 EFFECT OF UNEQUAL (BOWL/INVERSE BOWL FORMATIONS) PROCESSING RATES ON THE η

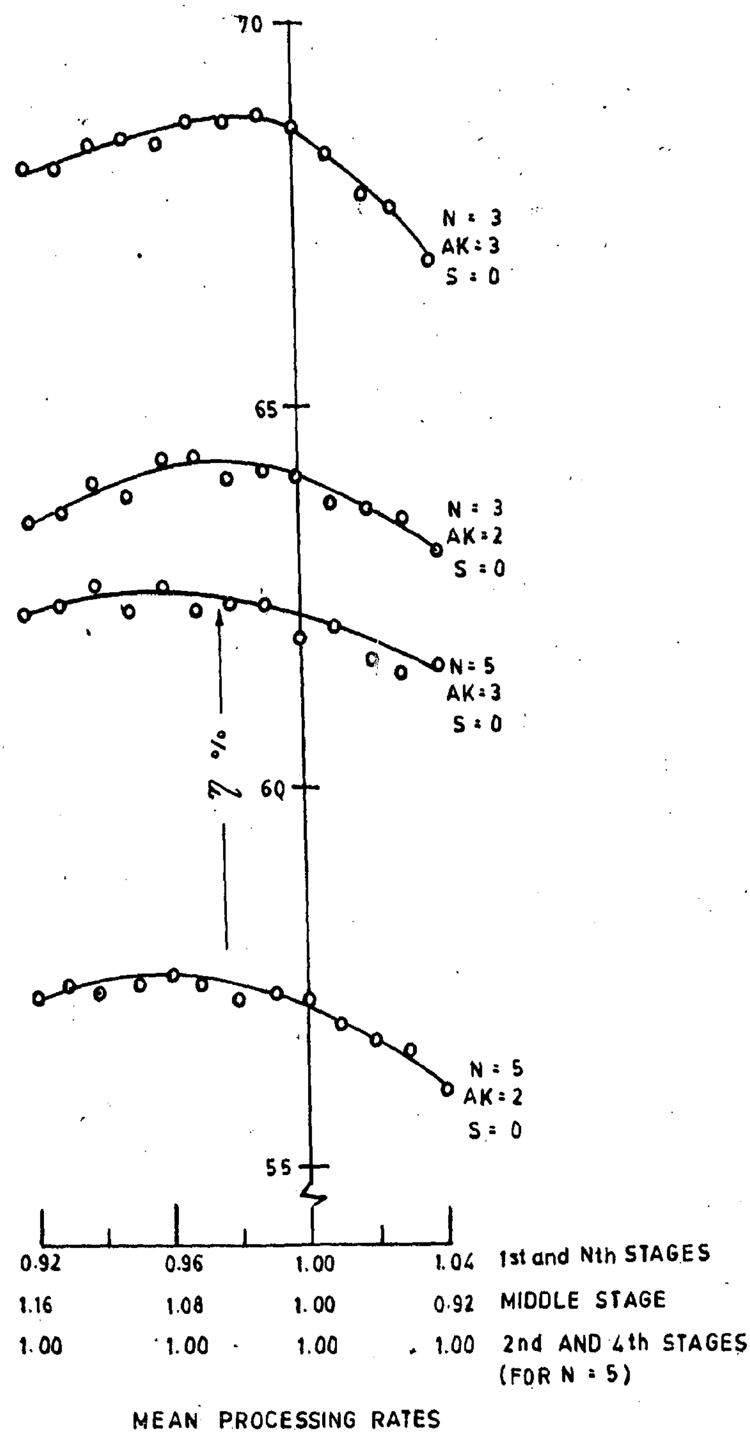


FIG. 5.4 EFFECT OF UNEQUAL (BOWL/INVERSE BOWL FORMS) PROCESSING RATES ON THE η

(ERLANG OPERATION TIMES)

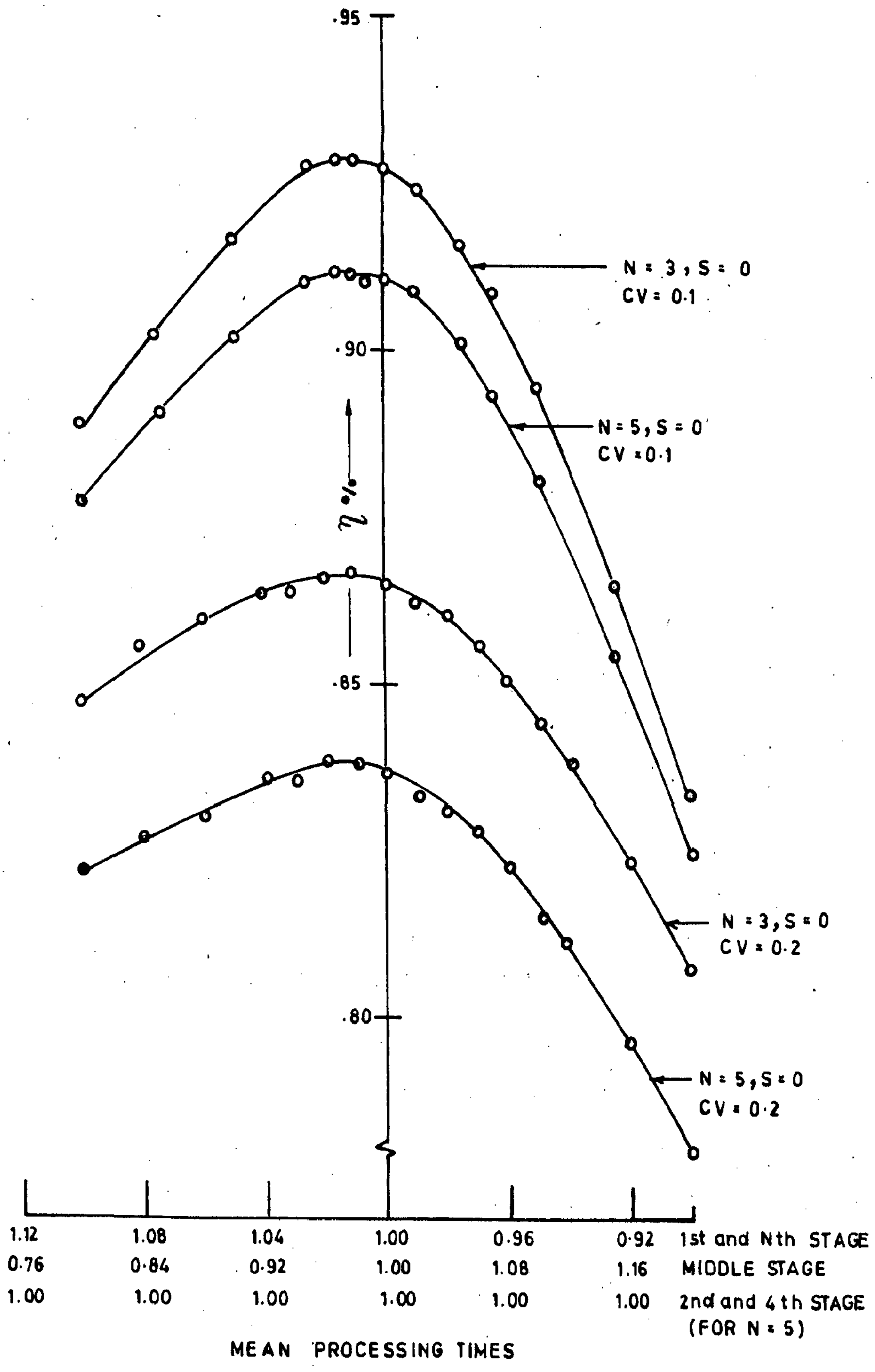


FIG.5.5 EFFECT OF UNEQUAL (BOWL INVERSE BOWL FORMS) OPERATION TIMES ON THE η

(NORMALLY DISTRIBUTED OPERATION TIMES)

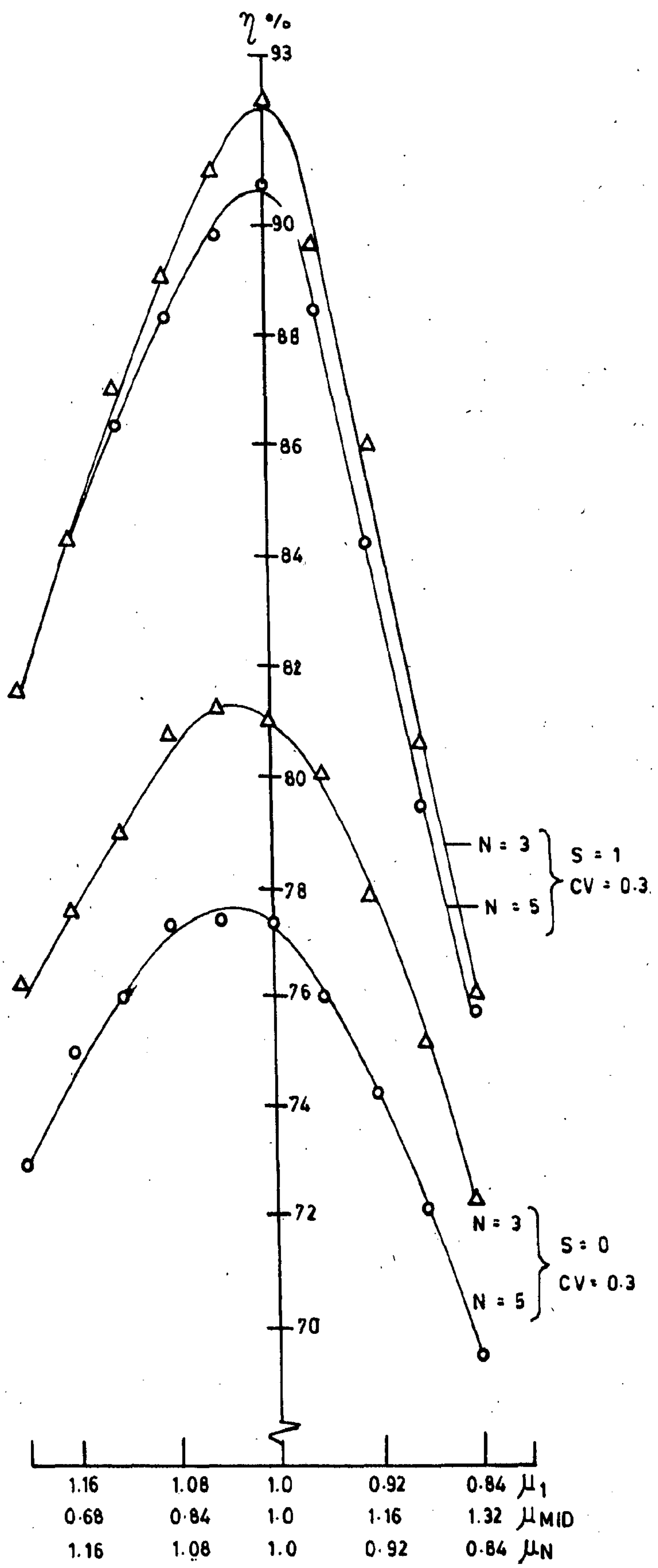


FIG. 5.6 FFFECT OF UNEQUAL (BOWL/INVERSE BOWL FORMS) OPERATION TIMES ON THE η

(NORMALLY DISTRIBUTED OPERATION TIMES)

finding that an unbalanced line with middle stages slightly faster than the outer ones would yield better results.

Fig. 5.5 reveals that the system with smaller variability in processing times is more sensitive to unbalancing as compared to the one having larger variability. The figure shows further that when the variability in the processing times is decreased, the point of maximum efficiency moves closer to the equal times ordinate. A similar effect could also be achieved by increasing the size of inprocess buffer, (Fig.5.6). When the value of S is increased from 0 to 1, the point of maximum efficiency shifts to the equal times ordinate. Thus we find that a slight increase in efficiency can be obtained by unbalancing the line in bowl form, only when the variability in operation times is high and the inprocess buffers are low. The ABU remains at about 50 percent in all the cases where the line is arranged symmetrically about the centre.

5.5.2.2 Unequal coefficients of variation

In this study, the mean operation times of the stages and the interstage buffer capacities were kept constant throughout the line, while the coefficient of variation of the operation times was varied from stage to stage. Effects of the following types of variations in CV along the line have been examined.

5.5.2.2a. CV increasing/decreasing along the line

Fig. 5.7 shows the results for different allocations of CV_i ($i = 1, N$) with $\sum_{i=1}^{N} CV_i = 0.5\,N$. It can be observed that in this case too, the balanced line with $CV_i = 0.5\ \forall\, i$ leads to maximum efficiency. Comparison of results for $N = 2$ and $N = 3$ at $S = 0$ show that smaller lines are more sensitive to imbalance as compared to the larger ones. Fig. 5.7 also reveals that as in the case of Fig. 5.6, the sensitivity increases with increase in S. The present results lead us to conclude that :

$$\eta(CV_1, CV_2 \ldots CV_N) = \eta(CV_N, CV_{N-1}, \ldots CV_1) \quad \ldots (5.2)$$

The ABU is about 50 percent for the balanced cases, and increases as the CV_i ($i = 1,N$) are arranged in the increasing order, larger being the increase in ABU for larger imbalances (Fig. 5.7). The reverse is true when CV_i are arranged in the decreasing order. Thus a stage with smaller variability in operation times, behaves in the similar fashion as a stage with smaller processing times.

5.5.2.2b. Bowl/inverse bowl formation

Results for this arrangement are shown in Fig. 5.8. It can be seen that when the stages are arranged with middle stages having smaller coefficient of variation as compared to the outer, the efficiency of the system slightly improves. Fig. 5.8 also shows that slightly larger imbalance can be afforded in larger lines, as compared to that in smaller lines.

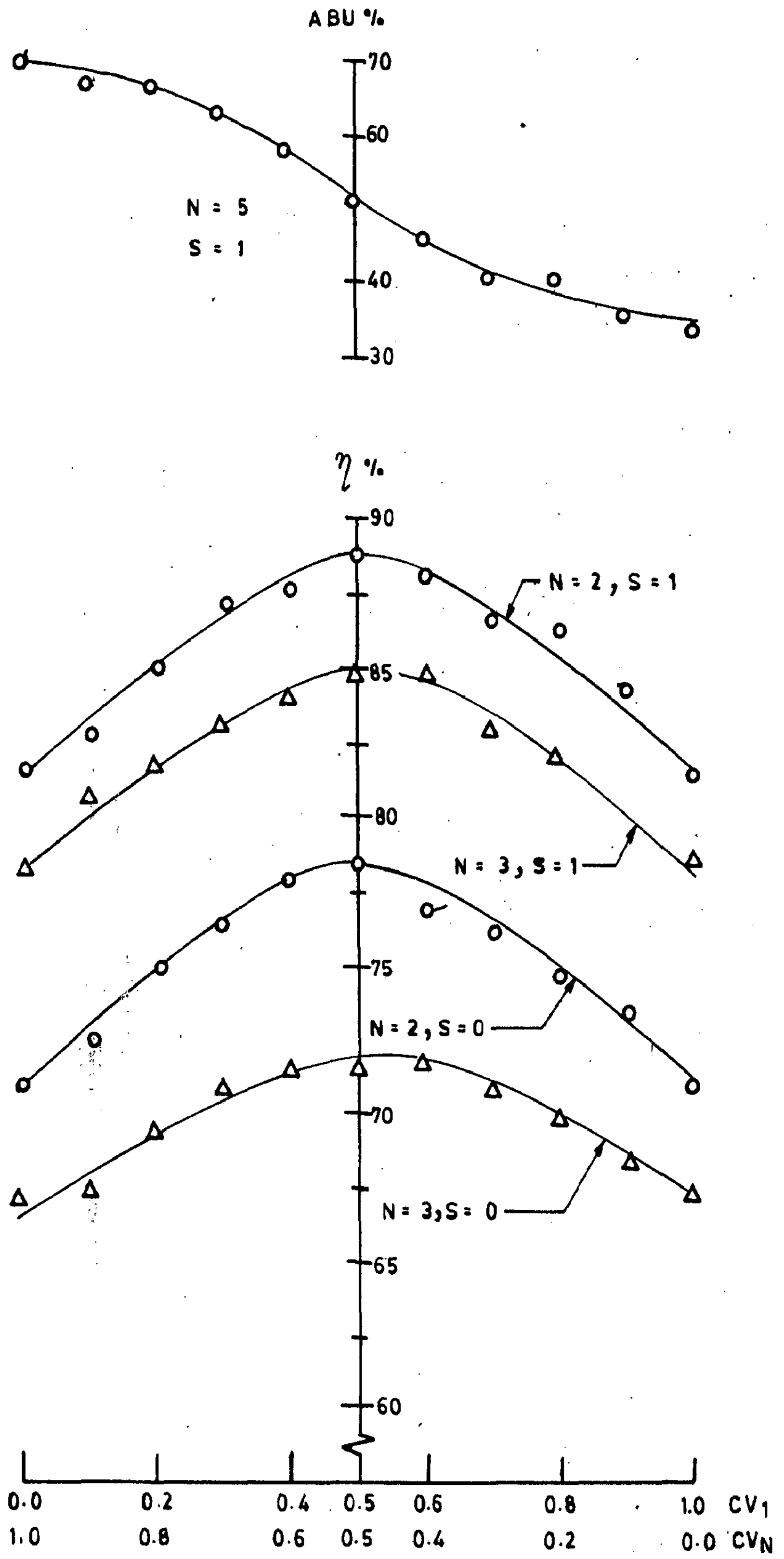


FIG.5.7 INFLUENCE OF UNEQUAL (INCREASING/DECREASING) COEFFICIENTS OF VARIATION ON THE η AND ABU

Since the line was symmetrical about the centre, the average buffer capacity utilisation in this case too was 50 per-cent for all the N and S combinations.

5.5.2.3 Unequal mean operation times and unequal coefficients of variation

In these experiments the line was arranged with CV_i decreasing in equal steps along the line from 0.525 to 0.075, and the mean processing times varied from gradually increasing (0.8 to 1.2) to gradually decreasing (1.2 to 0.8), with $\sum_{i=1}^{N} \mu_i = N$, as shown in Fig. 5.9. It can be noticed that for the situation under consideration, the point of maximum efficiency lies towards the left of the equal mean times ordinate, suggesting that for maximum efficiency, stages with larger variability in operation times should have smaller operation times. Also with increase in N, the point of maximum efficiency shifts further away from the equal times ordinate, and the improvement in efficiency over the balanced line, increases. This would imply, that larger imbalance in operation times is required to achieve maximum output from a long line with unequal variabilities in operation times. These results suggest that less work load should be assigned to the more variable stages.

Fig. 5.10 illustrates the influence of line unbalancing in bowl or inverse bowl formations. It can be noticed that when the variability of the outer stages is small as compared to the middle stages, the maximum efficiency occurs for an arrangement

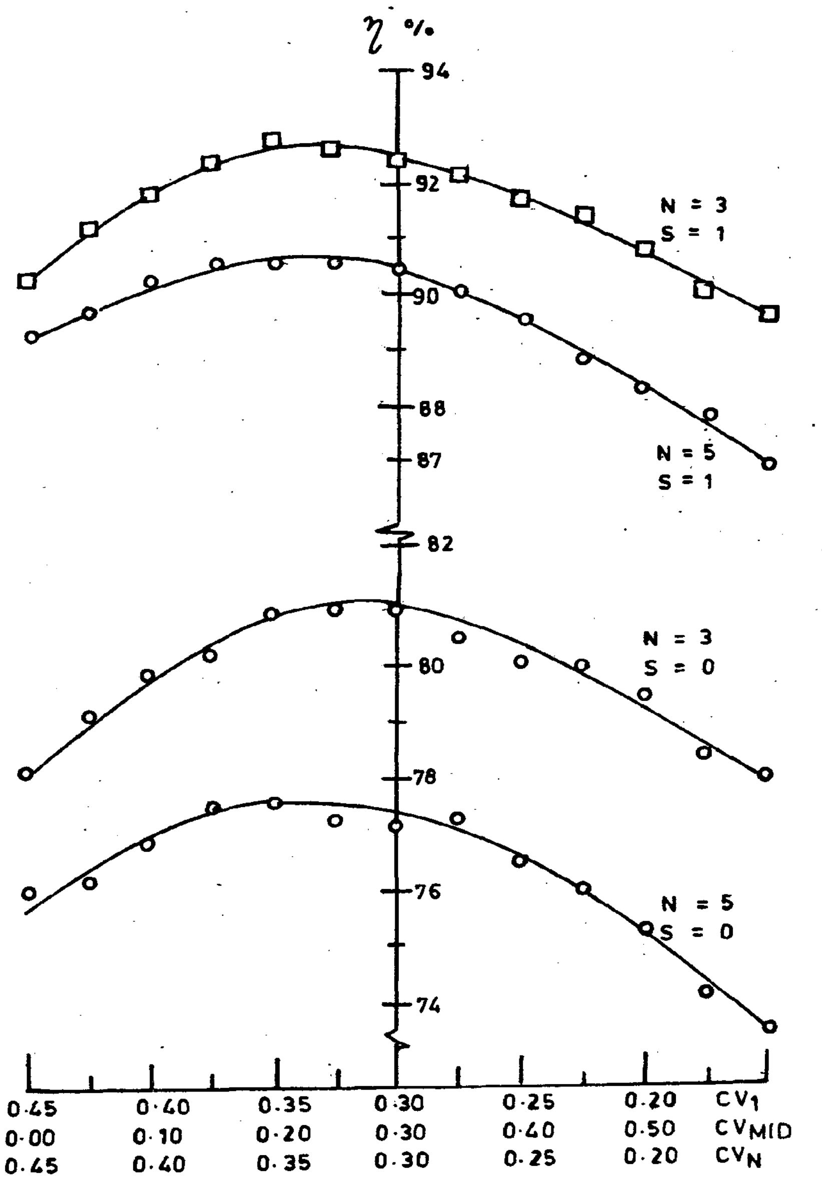


FIG. 5.8 INFLUENCE OF UNEQUAL (BOWL/INVERSE BOWL FORMS) CV's ON THE η

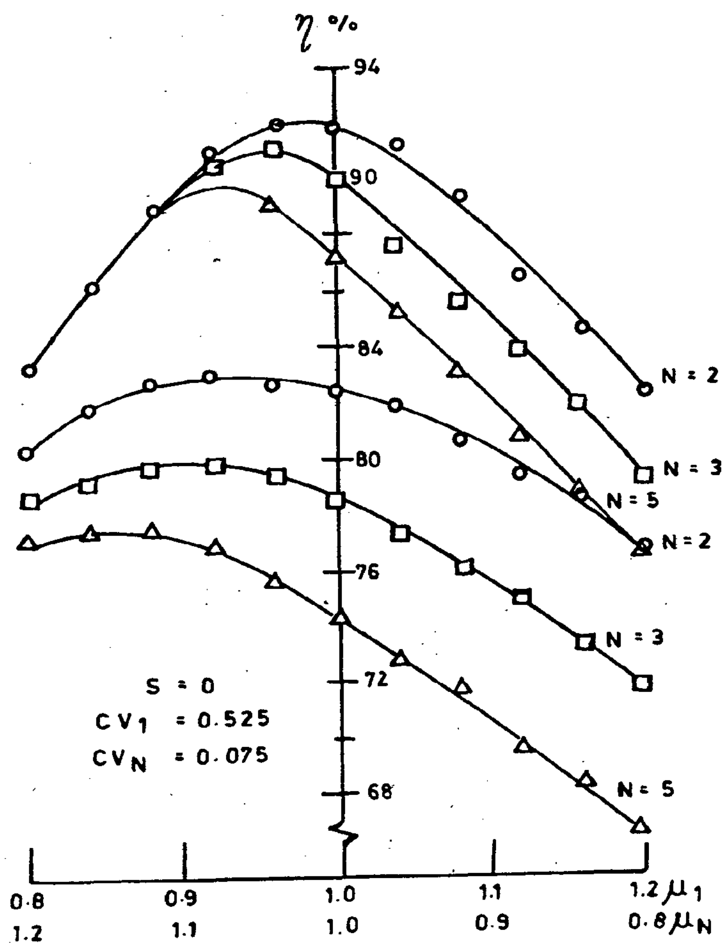


FIG. 5.9 EFFICIENCY OF THE LINE WITH UNEQUAL OPERATION TIMES AND UNEQUAL VARIABILITIES (CV's)

where middle stages have less work load in comparison to the outer. The reverse is true when the operation times of the middle stages are less variable than the outer stages. Figs. 5.9 and 5.10 also reveal that an increase in S helps to shift the point of maximum efficiency towards the equal times ordinate. At higher values of S, the line with equal mean times gives the optimum efficiency inspite of unequal coefficients of variation at different stages.

5.5.2.4. Unequal interstage buffers

5.5.2.4a. Buffers arranged in increasing/decreasing order

In order to study the effect of unequal buffers on the efficiency and the work inprocess inventory of the system, the processing times were taken as identical for all the stages. The line was divided in three approximately equal segments, to which different inprocess buffer capacities were allocated. Results for some line layouts having exponential processing times are given in Table 5.2, and for normally distributed processing times (CV = 0.2) are illustrated in Fig. 5.11. These results show that provision of unequal buffers leads to reduction in efficiency, and that effect of unbalancing is more in case of larger lines. For any set of inprocess buffers, η remains unchanged, when the order of buffers is reversed i.e.

$$\eta(S_1, S_2, \ldots S_N) = \eta(S_N, S_{N+1} \ldots S_1) \qquad \ldots (5.3)$$

Table 5.2 Effect of Unbalancing the Buffers

Buffer capacities in line segments			N = 4		N = 10		N = 15	
1	2	3	η %	ABU %	η %	ABU %	η %	ABU %
2	4	6	75.89	37.06	69.38	30.86	67.19	29.72
3	4	5	77.56	44.56	71.76	40.77	71.59	43.04
4	4	4	78.13	52.61	72.86	49.92	72.16	51.54
5	4	3	77.67	56.33	71.93	62.36	71.28	62.43
6	4	2	77.12	63.15	69.03	72.46	67.66	73.66
3	6	3	78.04	50.44	72.08	52.17	71.50	54.42
4	5	3	77.97	52.70	72.91	58.91	71.54	62.96
3	5	4	77.90	48.99	72.77	49.65	71.88	44.66

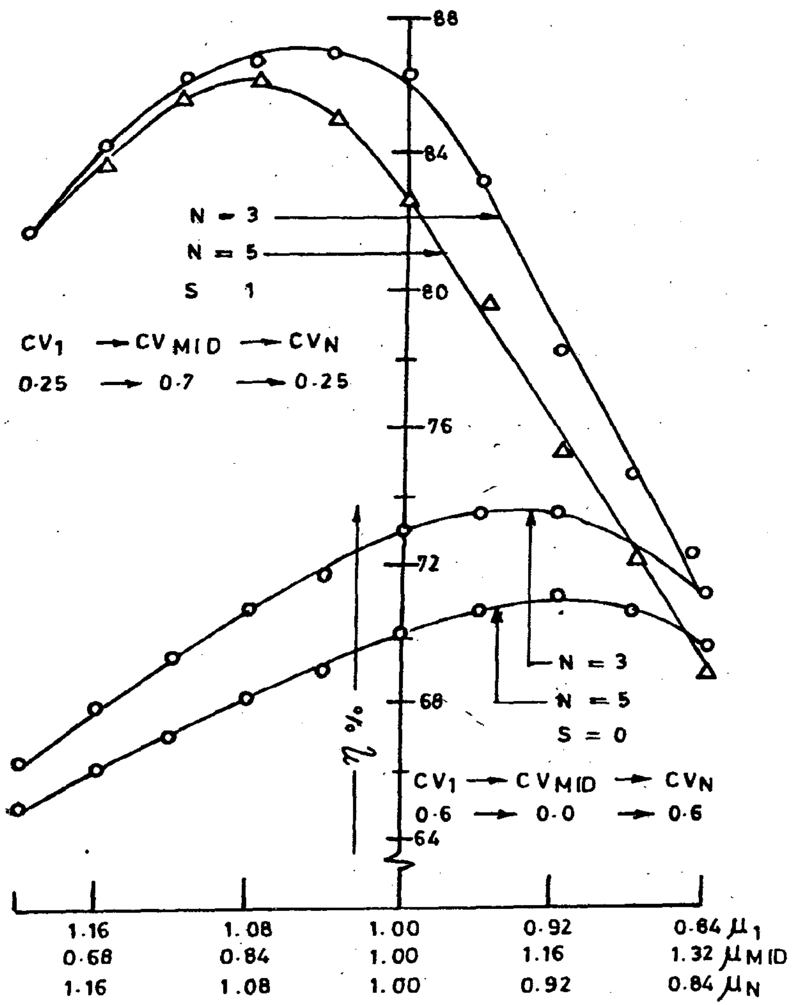


FIG. 5.10 EFFICIENCY OF THE LINES WITH OPERATION TIMES AND VARIABILITIES IN BOWL/INVERSE BOWL FORMATIONS

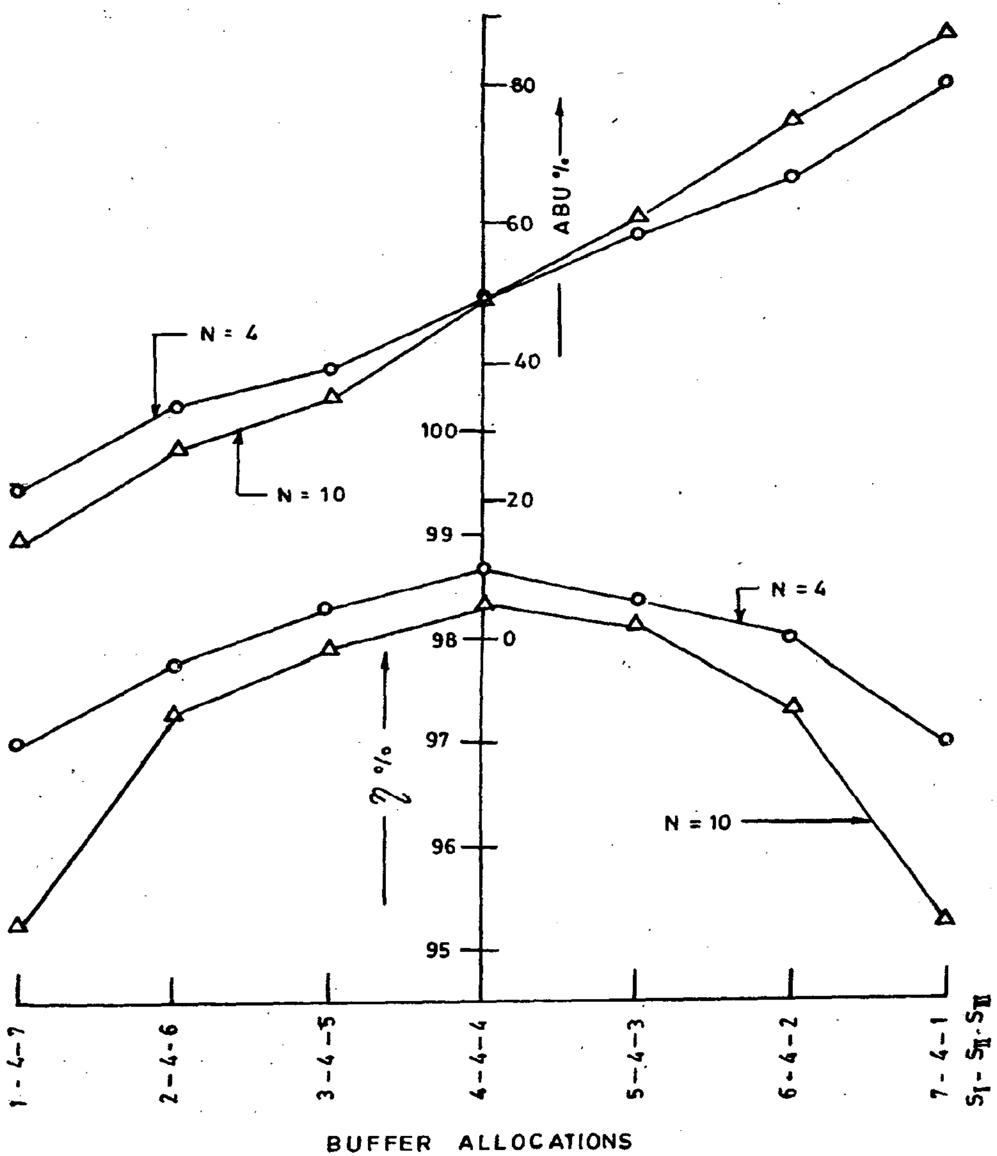


FIG. 5.11 EFFECT OF UNEQUAL BUFFERS ON THE η AND ABU

The work inprocess inventory depends upon the arrangement of the buffers. For the case of high, medium and low allocations in the first, second and third segments respectively, the ABU increases, while the reverse is true for low, medium and high allocations.

5.5.2.4b Buffers in bowl/inverse bowl formations

Fig. 5.12 illustrates the results when the inprocess buffer is distributed in the bowl (smaller buffers in the middle) and inverse bowl formations. As in the case of unequal operation times and variabilities, in this case too, a slightly better efficiency can be obtained by allocating larger buffer capacities to the middle stages of the line. At larger imbalances the efficiency falls. The ABU remains at 50 percent in almost all the symmetrical cases.

Simulation results illustrated in Fig. 5.13 disapprove Hatcher's [61] contention that for maximum gain in production of a 3-stage line, capacity of the second buffer should be increased. As can be noted from this figure, almost equal gain in efficiency is obtained, whether additional units of capacity are added to the first or second bunkers.

5.5.2.5 Unequal buffers and unequal variabilities

It has already been established that buffers help to smoothen out the effect of variabilities in operation times. Fig. 5.14 demonstrates the effect of providing unequal buffers in the line segments, where the variabilities of the operation

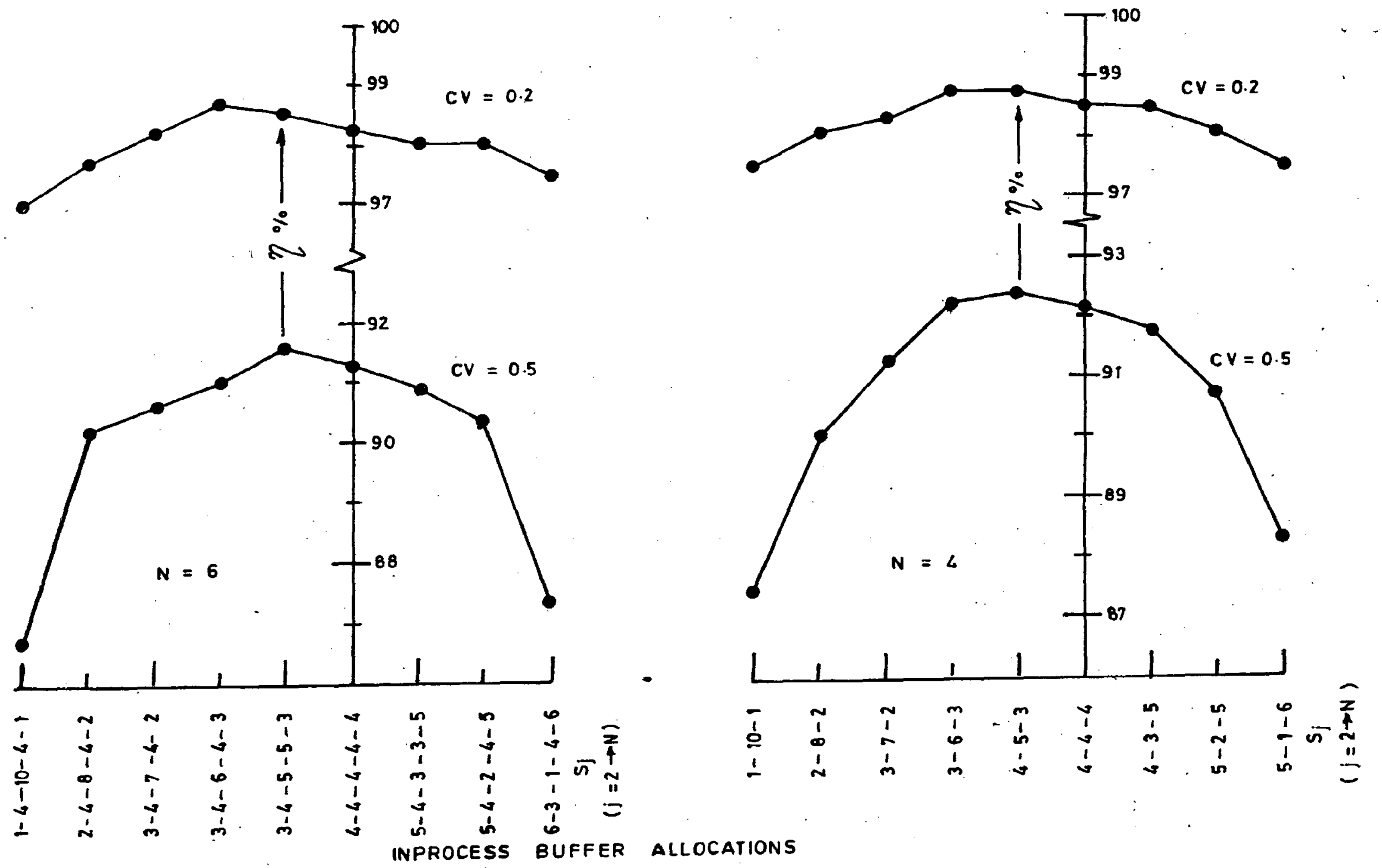


FIG. 5.12 EFFECT OF UNEQUAL BUFFERS (BOWL/INVERSE BOWL FORMATION) ON THE η

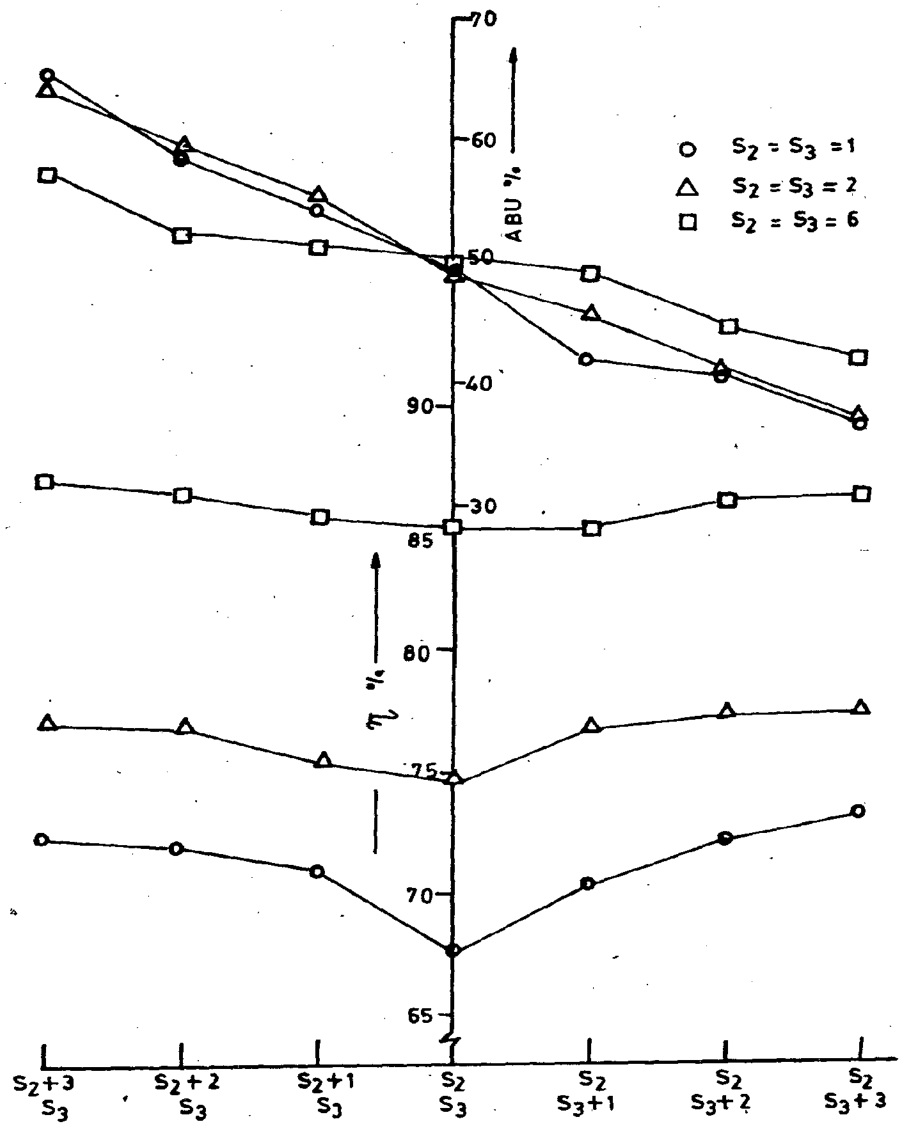


FIG. 5.13 INFLUENCE OF UNEQUAL BUFFERS ON THE η AND ABU OF A 3-STAGE LINE

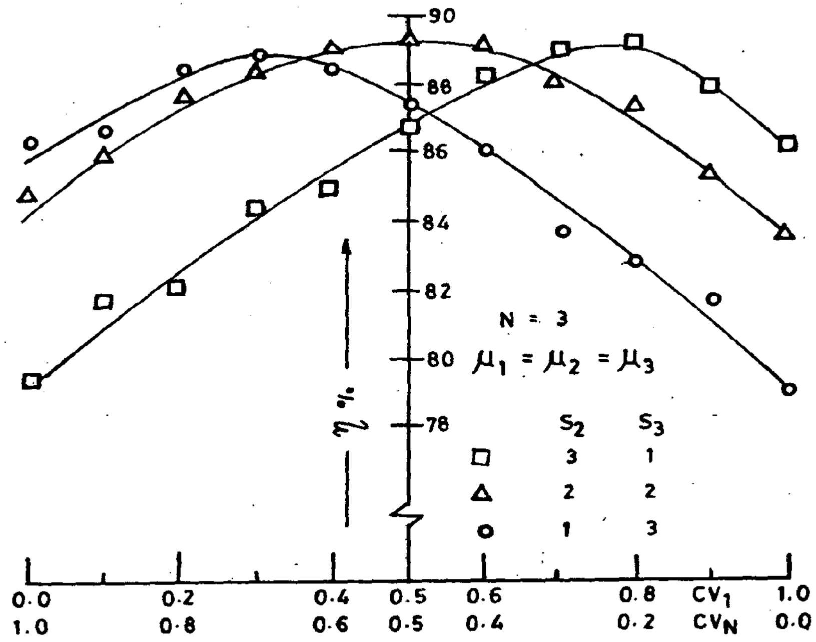


FIG. 5.14 INFLUENCE OF UNEQUAL BUFFERS ON THE η OF THE LINE HAVING UNEQUAL CV OF OPERATION TIMES

times are also different in different segments. It is clear from the figure that to achieve the maximum from the system, larger buffers be allocated in those sections of the line where the variability in operation times is large.

5.6 CONCLUSIONS

From the results reported in this chapter, the following conclusions can be drawn :

(1) The average inventory held in various buffers in a balanced line is higher in the beginning and decreases towards the end of the line, with the WIP for the line being around 50 percent of the provided buffer capacity.

(2) Flow line layouts with larger buffers in the beginning and smaller towards the end or with faster stages in the beginning and slower towards the end, or having stages with larger variability towards the end result in higher WIP, while their reverse arrangements reduce the work in process.

(3) When the line has μ_i, CV_i or S_j symmetrically arranged about the centre, the WIP remain at about 50 percent of the provided buffer capacity.

(4) When the layout of a line is reversed so that the new layout is a mirror image of the original, the line efficiency remains unchanged,

$$\text{i.e. } \eta(\mu_1 CV_1 S_1,\ \mu_2 CV_2 S_2 \ldots\ \mu_N, CV_N S_N)$$

$$= \eta(\mu_N CV_N S_N,\ \mu_{N-1}, CV_{N-1}, S_{N-1} \ldots \mu_1 CV_1 S_1) \quad \ldots (5.4)$$

(5) System with smaller variability and/or with larger inprocess buffer i.e. having greater efficiency are more sensitive to unbalancing.

(6) Results confirm the Hillier and Boling's 'Bowl Phenomenon' that a line with middle stages slightly faster than the outer is more efficient than the balanced line. The application of this rule has also been proved for variability in operation times and inprocess buffer capacities. The results, however, show that the phenomenon occurs only when either CV is large or when S is small. In majority of the practical situations where $S > 2$ and or $CV < 0.4$, the balanced line is the most efficient.

(7) Lesser work load or larger inprocess buffers should be allocated to stages having larger variability in operation times.

CHAPTER - 6

BALANCED FLOW LINES WITH UNRELIABLE STAGES

6.1 INTRODUCTION

Flow line system modelled in this chapter comprises of automatic stages arranged in series. Each stage in the line may consist of a number of automatic machines which are subject to random breakdowns and requiring random repair times. In such a case, the inefficiency of the system is caused not by the variabilities in the processing times but by the failures of the stages. When one stage fails, it may force the other stages to stop work and lead to loss of production. The forced down time of the line would depend upon the failure characteristics of the stages, number of stages and the capacities of the inprocess buffers.

A simulation model of the flow line, having unreliable stages with random interbreakdown and repair times and having fixed processing times at different stages, has been presented in this chapter. Based on the simulation data, an empirical model has been developed to predict the efficiency of the balanced line, which is valid for a wide range of operating parameters. The utility of the empirical model has been demonstrated by its application to the study of a flow line system for the mass production of crankshafts.

6.2 SYSTEM MODELLING

The flow line modelled in this chapter comprises of sequentially arranged automatic stages. Each stage may comprise of several machines, but the operating characteristics of all the stages have been assumed to be identical, and in-process buffers are assumed to be of equal capacity i.e.,

$$R_i = R \ \forall \ i$$

$$(MFT)_i = MFT \ \forall \ i$$

$$(MRT)_i = MRT \ \forall \ i$$

$$KS_j = KS \ \forall \ j$$

6.2.1 Operating Policy

Each work unit enters the line at the first stage and passes through all the stages in a fixed sequence. In case of line without buffers, work unit is transferred from stage to stage, while in case of line with finite inprocess buffers, the work is transferred from one stage to the next through the intermediate store. After all the operations at one particular stage have been completed, the work unit moves to the following inprocess store, if a vacancy exists, otherwise the stage gets blocked. As soon as a stage becomes free, a fresh work unit is drawn from the predecessor store, if available, otherwise the stage starves. A stage maintains normal (constant) speed at all times when it is operating. When all the stages are working, there is no change in the buffer levels. When a stage breaksdown, other stages in the line continue to work till they are forced down (blocked or starved). Thus, the level of the buffers would change only during the breakdown of the stages.

6.2.2 Classification of Failures

The system failures can be categorised into total line failures or single stage failures [27]. The total line failure is caused by events outside the boundary of the system. These may be power supply failures or the non-availability of raw materials etc. The single stage failures are caused due to factors present within the system. These would range from machine stoppage for tool adjustment to major breakdowns. The failures can also be classified as follows :

(i) Operation dependent failures : Some failure mechanism, such as tool wear, remain active only during the time a machine is operating. Such failures cannot develop during the forced down periods.

(ii) Time dependent failures : Such failures occur due to some phenomenon, which remains active all the time, including the operating and forced down periods, e.g., electronic control equipments may fail even when the machine is forced down.

Hanifin as reported by Buzacott and Hanifin [27], based on a case study of transfer lines has reported that 85 percent of the failures were single station failures, and out of these in 93 percent of the cases, the cause of failure was operation dependent. Hence in this study the failures have been assumed to be operation dependent only.

6.3 THE SIMULATION MODEL

For the sake of convenience and computational efficiency, the simulation model of a flow line subject to failures was developed in two blocks as described below and the flow charts for which are given in Figs. 6.1 and 6.2. The following assumptions have been made in order to develop the model :

(1) System processes only one type of product.

(2) The failures of the stages are operation dependent.

(3) The first stage is never starved and the last is never blocked.

(4) There are no work rejections or rework at any of the stages.

(5) Enough repairmen are available, and each breakdown is attended to immediately without waiting.

(6) Up and down times of the stages are mutually independent and are exponentially distributed.

(7) Repairs do not impair the performance of the stages, i.e. a repaired stage performance is as good as of a new machine.

6.3.1 Block 1 : Line Without In-Process Buffers

The line without inprocess buffers was simulated by employing the event to event variable time increment model

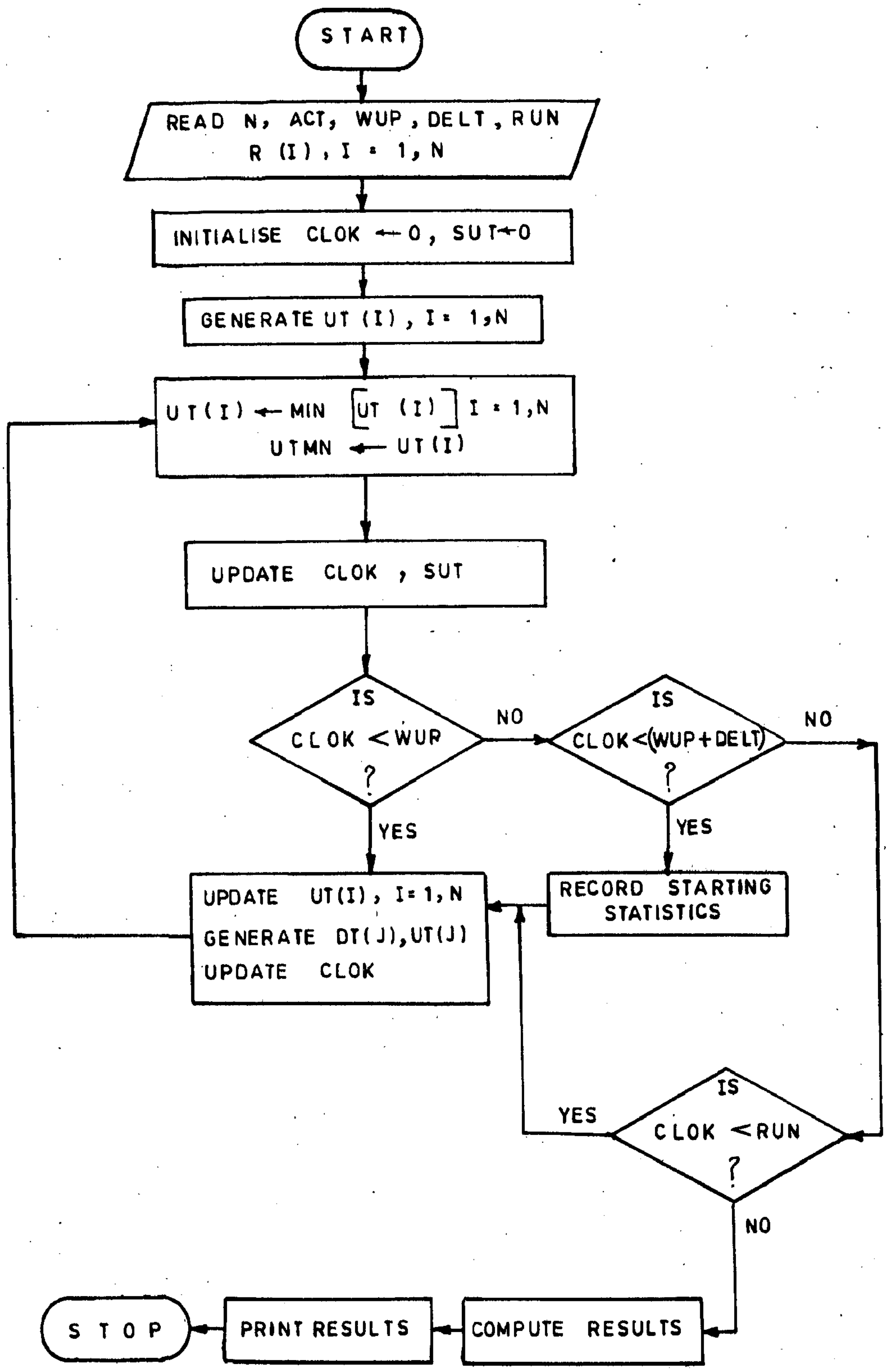


FIG. 6.1 SIMULATION FLOW CHART OF FLOW LINE HAVING UNRELIABLE STAGES AND NO INPROCESS BUFFER

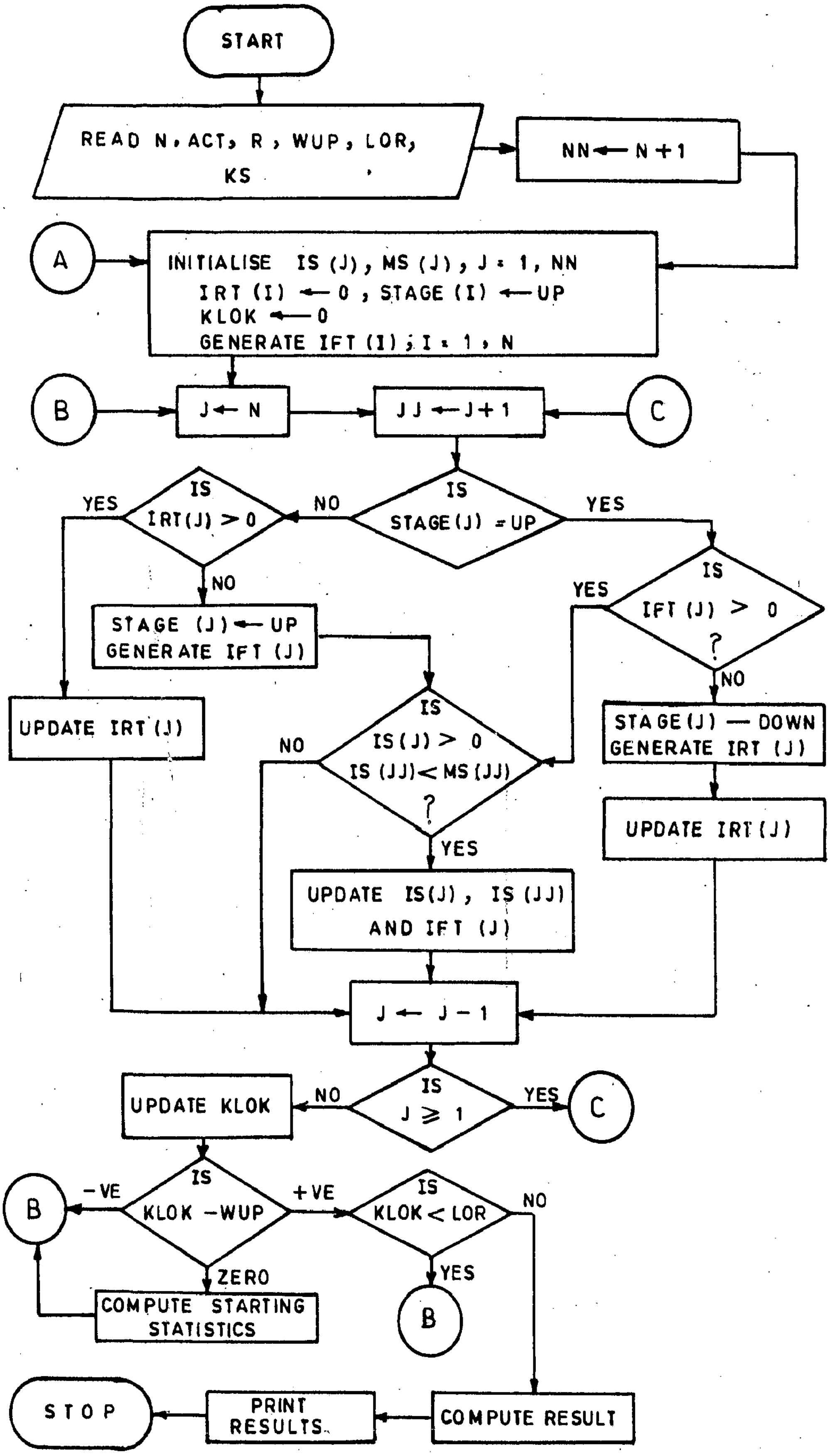


FIG. 6.2 SIMULATION FLOW CHART OF FLOW LINE HAVING UNRELIABLE STAGES WITH FINITE INPROCESS BUFFERS

(Fig.6.1). The breakdown of a stage would mark the occurrence of an event, while the state of the system is defined by CLOK, SUT and a vector UT(I).

where, CLOK : Total time elapsed since the start of the run

SUT : Sum of the elapsed up times of the line

UT(I) : Time to failure of stage I, (I = 1,N).

After the occurrence of an event the repair time of the down stage is generated and the state of the system is updated. Next the time to failure (uptime) of the repaired stage is generated and the system is scanned to search for the next earliest event. The number of units processed are stored in the buffer at the end of the line, while CLOK keeps a record of the elapsed time. After allowing for the warming up time, to enable the system to reach steady state, the statistics of interest were recorded after fixed time intervals of 100 time units. After the estimated length of run (LOR), the standard error of the mean efficiency was computed. If acceptable, the simulation was terminated, otherwise LOR was incremented by 1000 time units and sequential testing was employed.

6.3.2 Block 2 : Line With Finite In-Process Buffers

In this case the system has been simulated by using the fixed increment time flow mechanism (Fig. 6.2). One time unit, which has also been taken as the unit processing time at the stages, has been selected as the fixed time increment. The cycle time (MFT + MRT) has been taken as 100 time units and to increase the computational efficiency the time to failure (TTF) and the time to repair (TTR) are rounded off to integer values. After every one time unit, the line is scanned to check the states of the stages and the inprocess buffers. If a stage has broken down, its repair time is generated and if the repair of a down stage has been completed, its next time to failure is generated. The input and output buffers at the stages are checked to determine if a stage is in operating or forced down state. After updating the states of all the stages and the inprocess buffers, the CLOK is advanced by one time unit.

After allowing the system to warm up, initial statistics of interest were recorded and simulation test performed for the predetermined length of run (Chapter 3). The length of the simulation run was selected so as to achieve an accuracy of $\pm$ 1 percent at $\alpha = 0.05$, in computing the mean efficiency. After computing the results, the buffer level was updated for the next simulation.

For each simulation run, the system was started with all the interstage buffers half full and all the stages started simultaneously.

6.4 EFFICIENCY OF LINE WITHOUT BUFFERS

In case of automated production lines, with no inprocess buffers, analytical models are available for predicting their efficiency. These models do not account for the nature of up and down time distributions of the stages, they however, consider the type of failure.

When a line is subject to time dependent failure, that is the failure mechanism remains active during the forced down time also, the line efficiency is given by the well known formula,

$$\eta_0 = \prod_{i=1}^{N} R_i \qquad \ldots (6.1)$$

However, when the failures are operation dependent, that is a forced down stage cannot fail, the efficiency of the line is given by equation (6.2), (Barlow and Proschan [9])

$$\eta_0 = \frac{1}{1 + \sum_{i=1}^{N} \frac{(1-R_i)}{R_i}} \qquad \ldots (6.2)$$

Equations (6.1) and (6.2) yield significantly different results except for the case when $N = 1$ and/or $R_i = 1.0 \ \forall \ i$. The difference can be important especially when R_i is low and N is large. For example, when $R_i = 0.8 \ \forall \ i$ and $N = 3$ the system efficiencies calculated for the time dependent and operation dependent cases are 0.5120 and 0.5714 respectively.

The values of η obtained from equation (6.1) and (6.2) are compared with the simulated values in Table 6.1. Student's t-test revealed, that there is no significant difference between the results given by simulation and the equation (6.2)

Table 6.1 : Efficiency of Line with Zero Inprocess Buffers

R_i (i = 1,6) = 0.8, 0.55, 0.85, 0.8, 0.75, 0.8

N	Predicted η Equation(6.1)	Predicted η Equation (6.2)	Simulated η
2	.6800	.7010	.7027
3	.5780	.6238	.6209
4	.4624	.5397	.5390
5	.3468	.4574	.4552
6	.2774	.4105	.4121

6.5 EFFICIENCY OF LINE WITH INFINITE CAPACITY BUFFERS

When breakdown of one stage of the system has no effect on the working of other stages in the line (i.e. the forced down time of stages is zero), then all the stages work independently and in such a case the slowest stage determines the efficiency of the line. For such a situation, the efficiency is given by equation (6.3)

$$\eta_{\infty} = \min (R_i, \quad i = 1,N) \qquad \ldots (6.3)$$

6.6 EFFECT OF BUFFER DISTRIBUTION

Consider an automated production line with k identical workstations and having total inprocess buffer capacity of Z units. The k-station line can be divided into N stages by grouping $k' = k/N$ stations together. The total buffer would then be divided into $M = N-1$ inprocess storages. If A is the efficiency of each station, then the efficiency of each stage (R) is given by :

$$R = \left[1 + k' \frac{(1-A)}{A}\right]^{-1}$$

If the number of stations grouped into a single stage is large, the stage efficiency would be low. Effect of the number of inprocess buffer storages (M) on the efficiency of a line with k = 12 and z = 330 is illustrated in Fig. 6.3. In this figure it can be seen that as the parameter M is increased, the value of η_∞ increases, thus increasing the scope for improvement in the system performance due to the inprocess buffers. The value of η_z (Fig. 6.3) also goes on increasing as M is increased.

In order to study the effectiveness of the buffer, let us define the buffer effectiveness (E) by equation (6.4)

$$E = \frac{\eta - \eta_0}{\eta_\infty - \eta_0} \qquad \ldots (6.4)$$

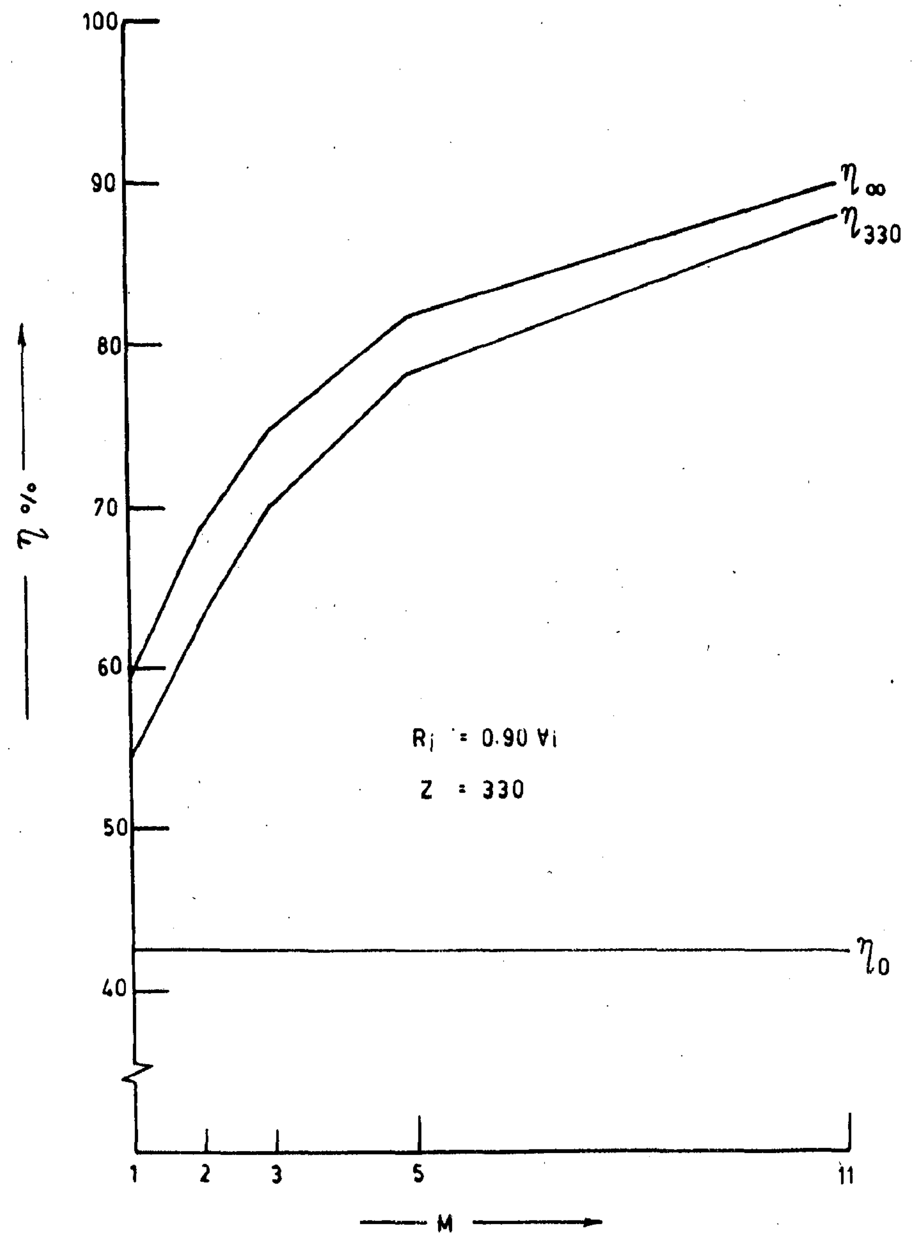


FIG. 6.3 EFFECT OF BUFFER DISTRIBUTION ON THE LINE EFFICIENCY

For the total buffer of 330 units, when the line is divided into two parts with a single inprocess buffer, E comes out to be as 0.71, while E = 0.96 when the line is divided into 12 stages allocating 33 units capacity to each inprocess buffer. Thus for achieving high buffer effectiveness, effort should be made to divide the line into as many stages as possible. However, the reliability of all the stages should almost be identical.

6.7 SIMULATION RESULTS

The object of simulation experiment was to generate data for developing an empirical model of the system efficiency. Since the efficiency is more sensitive to changes in N and S, specially when their values are small, more weightage has been given to their lower levels. Simulations were run for all the factor combinations at the following levels :

N : 2, 3, 4, ...6, 8....12, 15, 20

KS : 0, 1, 2, ...6, 8, ...12, 15

R : 0.8, 0.85, 0.90, 0.95

6.7.1 Influence of Line Parameters on Efficiency

The effect of number of stages on the efficiency for various values of KS is illustrated in Fig. 6.4 for R_i=0.8, 0.85, 0.9 and 0.95. As the length of the line is increased, the η decreases, the effect being more at lower levels of N, and for smaller values of inprocess buffer. When KS is large,

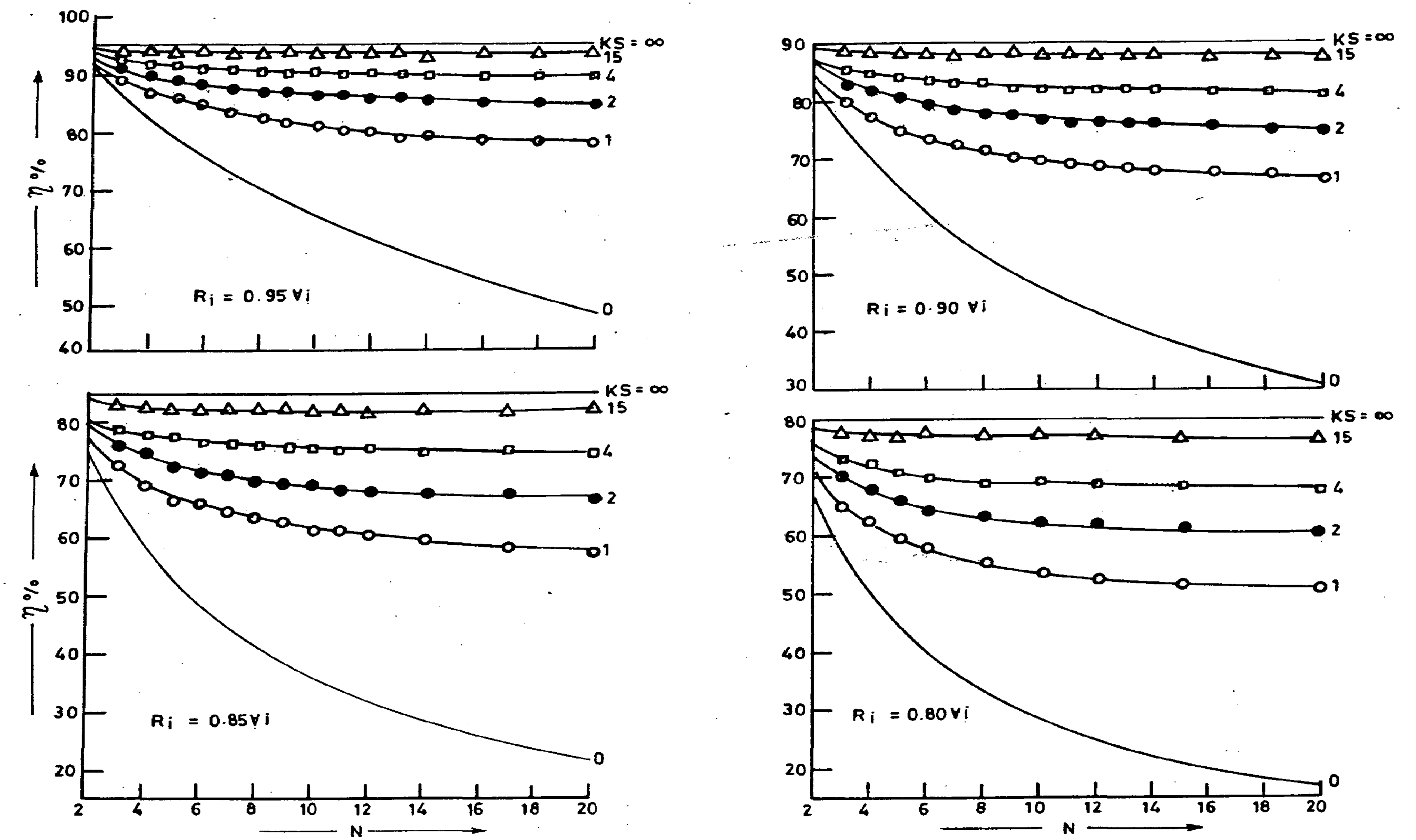


FIG. 6.4 EFFECT OF NUMBER OF STAGES ON LINE EFFICIENCY FOR VARIOUS VALUES OF KS AND R

the increase in the length of line beyond a certain level, has insignificant effect on the system efficiency. Fig. 6.4 shows that the sensitivity of η to changes in N and KS decreases as the reliability of the stages is increased.

From Fig.6.5 it can be noted that the production efficiency increases rapidly with increasing inprocess storage size, however, beyond S = 3KS, the improvement in the rate of increase in efficiency slows down considerably. It can also be observed that the gain due to inprocess storage is remarkable in case of longer lines.

Fig. 6.6 illustrates the effect of MFT on the line efficiency for the selected values of MRT. As the MFT is increased, the reliability of the stages increases, causing increase in the line efficiency. In this case again the system is more sensitive to changes in MFT at lower levels. The effect of MRT on the η for a 5 stage line with $S_i = 10 \; \forall \; i$ is illustrated in Fig. 6.7 for several values of MFT. For a fixed value of MFT, increase in MRT reduces the reliability of the stages and hence, a loss in efficiency.

6.8 DEVELOPMENT OF EMPIRICAL MODEL

The graphical analysis of the simulation data has revealed that the η vs KS curves (Fig. 6.5) for various values of N and R could be plotted as straight lines by transforming the scale of KS to a newly defined parameter (KS + K)/(KS+K+1).

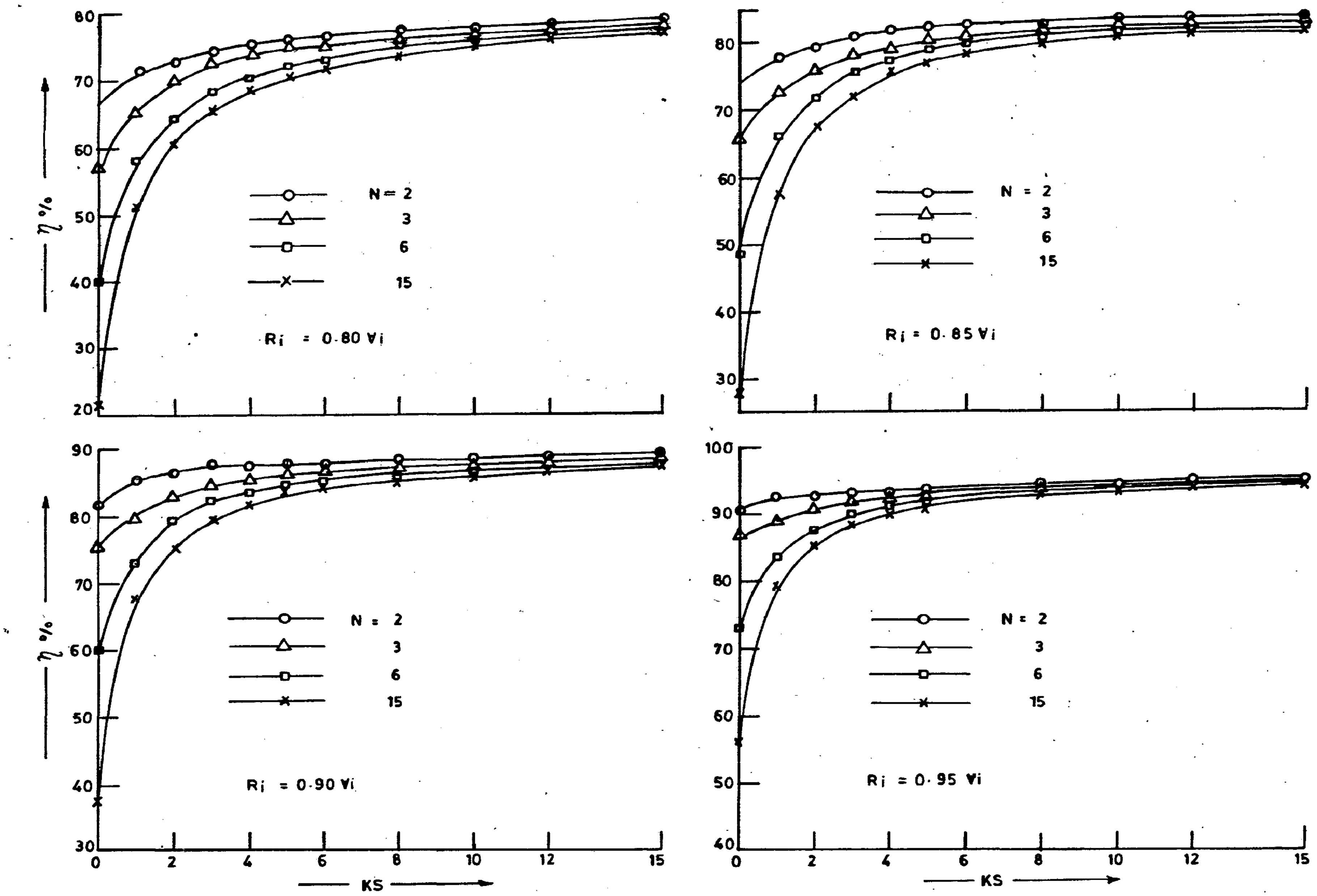


FIG.6.5 EFFECT OF INPROCESS BUFFER ON LINE EFFICIENCY FOR VARIOUS VALUES OF N AND R

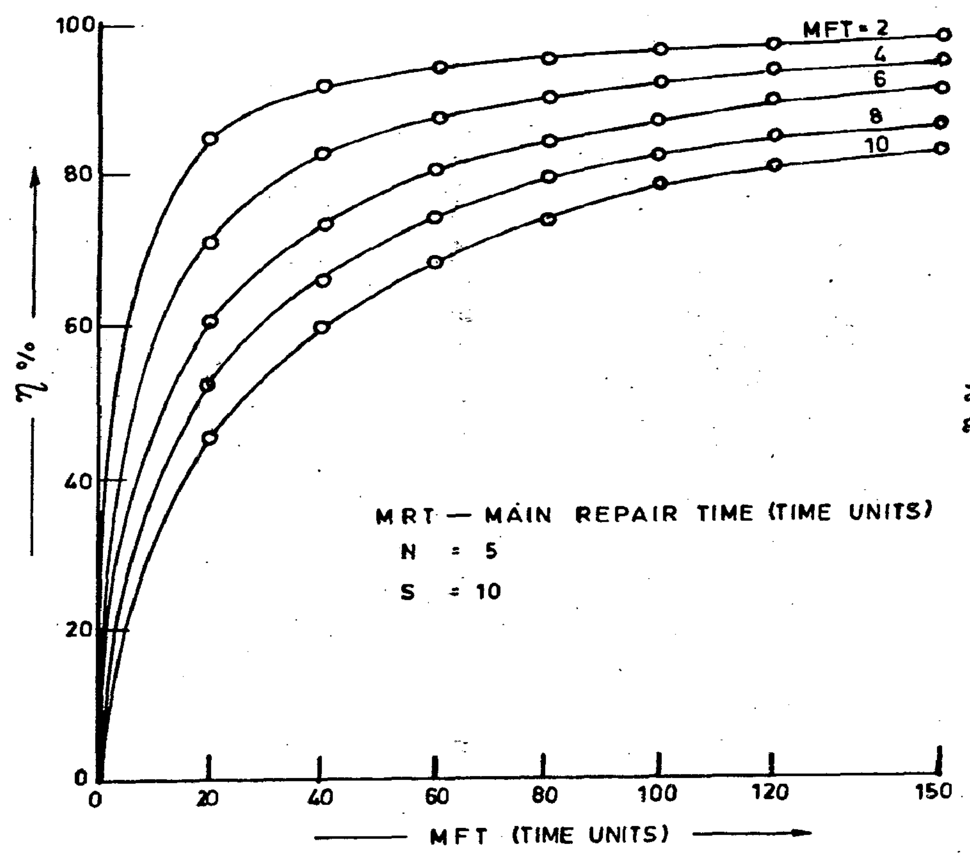


FIG. 6.6 EFFECT OF MFT ON LINE η

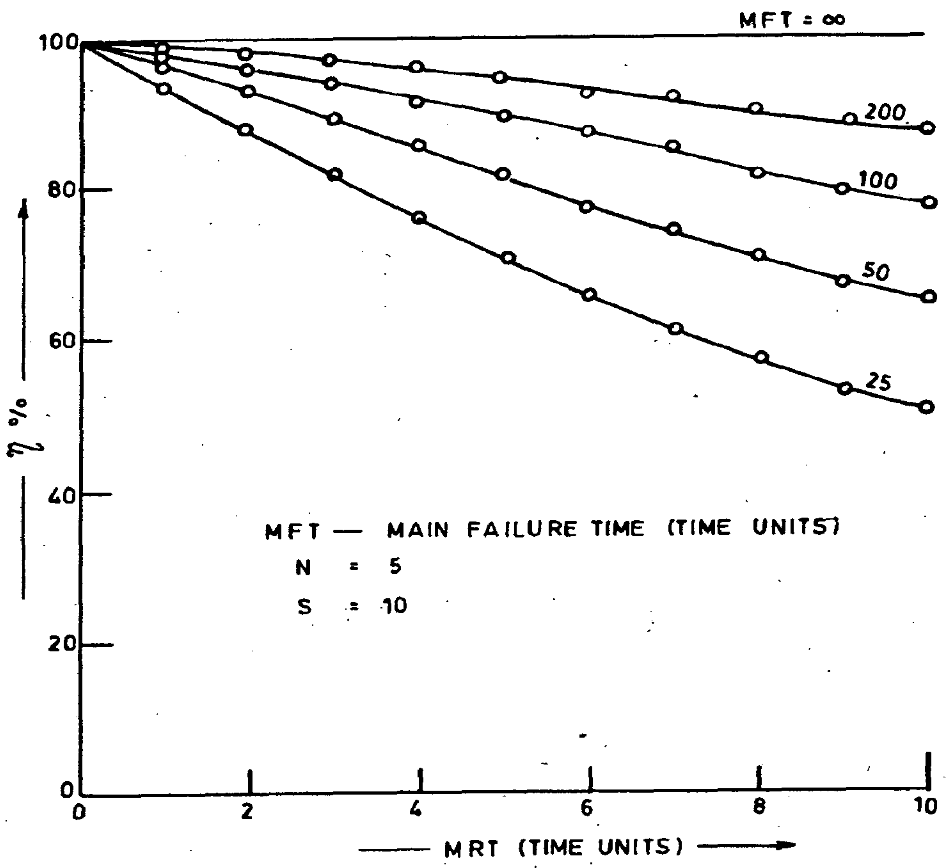


FIG. 6.7 EFFECT OF MRT ON LINE η

Where the value of the constant K would vary with N and R. For each combination of N and R, the value of K, which gave the least sum of the squares of the differences between the simulated and predicted values, was obtained by employing a systematic search procedure. (Appendix A4.1, Chapter 4). In each case the coefficient of correlation between the simulation and predicted values was greater than 0.995.

Figs. 6.8(a) and (b) show the plot of efficiency as a linear function of (KS + K)/(KS+K+1) for some combinations of N and R which had the values of K close to 0.0 and -0.11 respectively.

The nature of variation in K with N, for several values of R is illustrated in Fig. 6.9. The relationship between K and N for each value of R could also be transformed to linear form by plotting (N+J)/N instead of N against K. The value of J was found to be very close to 2, for the range of R considered in this study. The relationship between K and (N+2)/N is illustrated in Fig. 6.10.

From Fig. 6.8, the following expression for the efficiency of the system could be derived:

$$\eta^N_{KS} = \eta^N_{\infty} - \frac{(\eta^N_{\infty} - \eta^N_o)}{(1 + \frac{K}{K+1})} \left(1 - \frac{KS + K}{KS+K+1}\right)$$

$$\eta^N_{KS} = \eta^N_{\infty} - (\eta^N_{\infty} - \eta^N_o)\left(\frac{K + 1}{KS + K + 1}\right) \qquad \ldots (6.5)$$

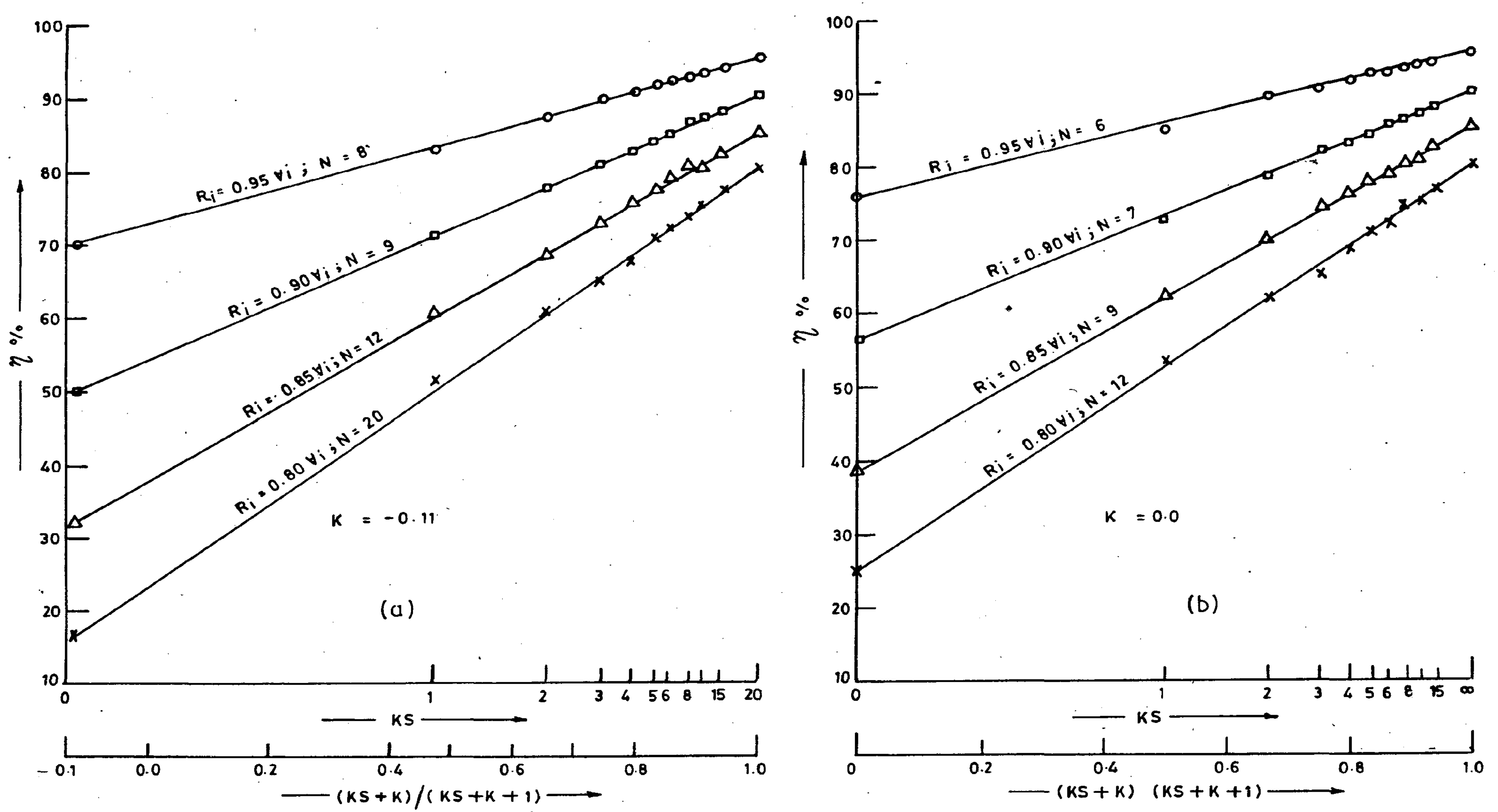


FIG. 6.8 LINE EFFICIENCY AS A LINEAR FUNCTION OF (KS + K) / (KS + K + 1)

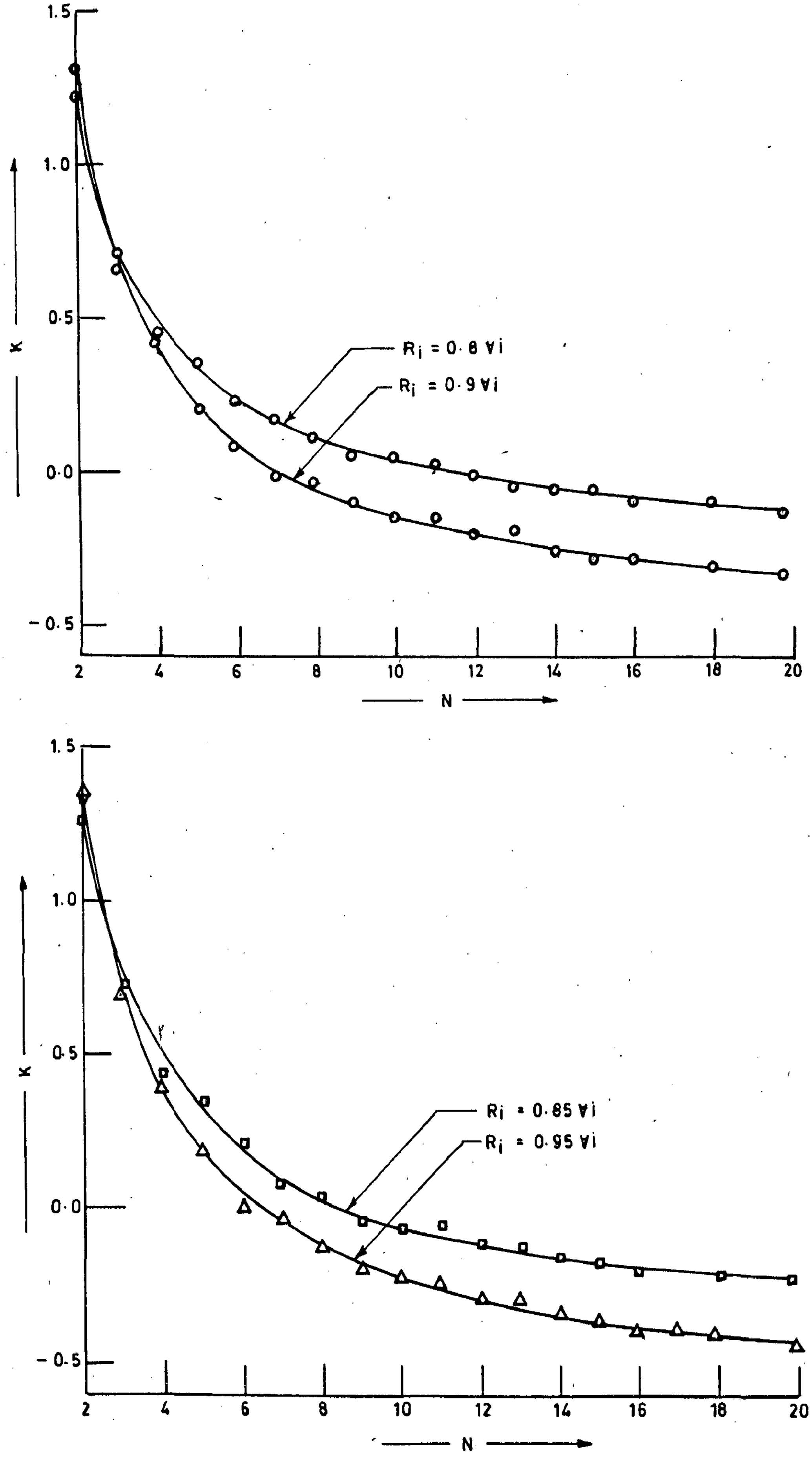


FIG. 6.9 VARIATION IN THE VALUE OF K WITH N

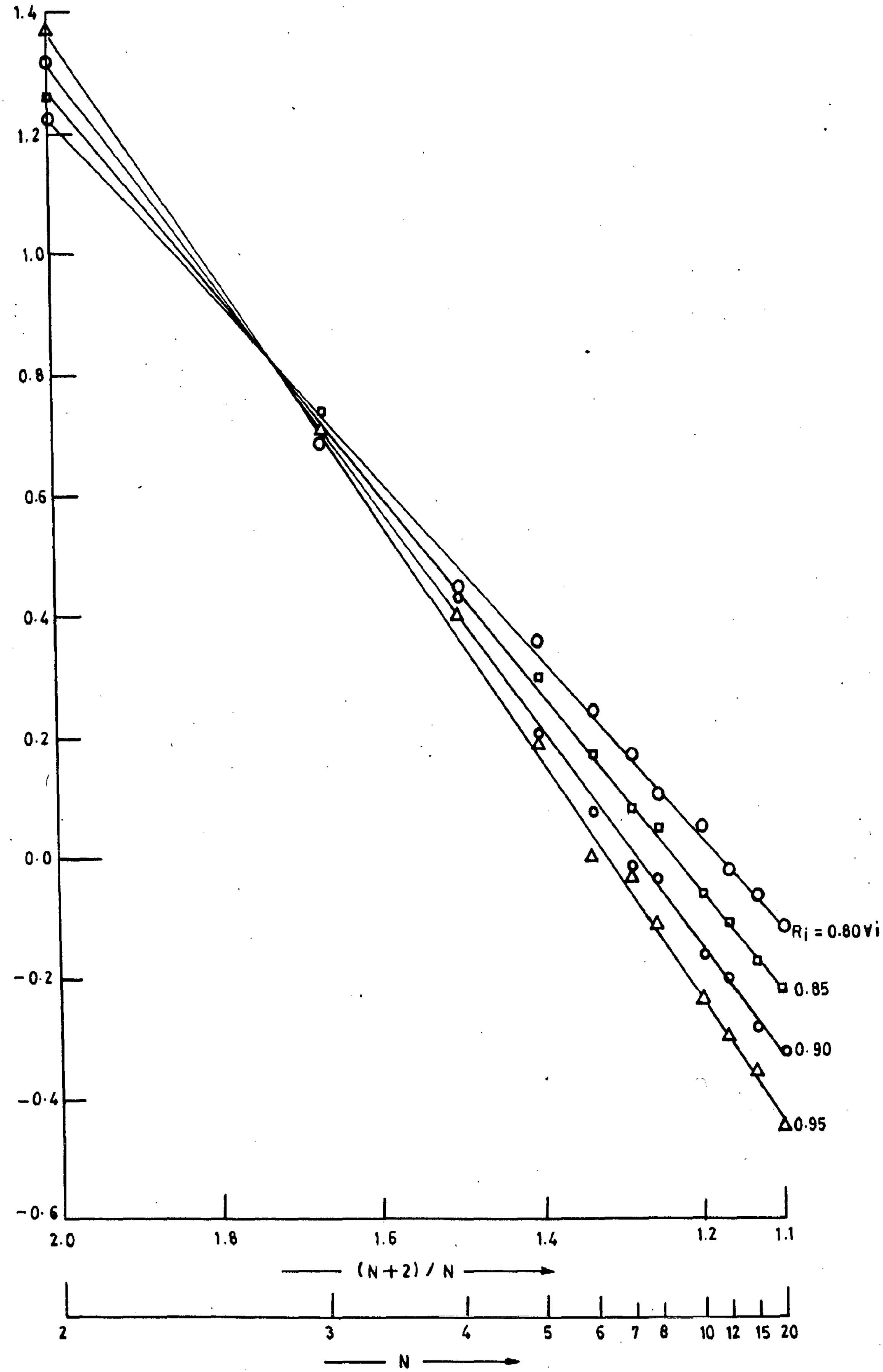


FIG. 6.10 RELATIONSHIP BETWEEN K AND (N+2)/N

Since $R_i = R \ \forall \ i$

$$\eta_{\infty}^{N} = R$$

and
$$\eta_{o}^{N} = \left[1 + \sum_{i=1}^{N} \frac{(1-R)}{R}\right]^{-1}$$

$$= \frac{R}{R + N(1-R)}$$

Substituting the values of η_{∞}^{N} and η_{o}^{N} in equation (6.5), the following expression for η_{KS}^{N} is obtained.

$$\eta_{KS}^{N} = \frac{R}{KS + K + 1}\left[KS + \frac{K + 1}{R + N\,(1-R)}\right] \quad \ldots (6.6)$$

From Fig. 6.10 for each value of R, the value of K could be expressed as,

$$K = A + B\,(N+2)/N \quad \ldots (6.7)$$

The values of the constants A and B for various values of R obtained by the method of least squares, along with the coefficients of correlation obtained, are given in Table 6.2.

Table 6.2 : Values of Constants in Equation (6.7)

R	A	B	CR
0.80	-1.75	1.490	.9879
0.85	-2.04	1.6556	.9907
0.90	-2.35	1.8335	.9885
0.95	-2.63	2.000	.9928

The constants A and B can further be expressed as linear functions of R as follows :

$$A = 3.05 - 6.0\,R \qquad CR = 0.9998 \qquad \ldots (6.8)$$

$$B = -1.23 + 3.4\,R \qquad CR = 0.9999 \qquad \ldots (6.9)$$

6.8.1 Adequacy of the Model

The adequacy of the empirical model of line efficiency, developed in this chapter, has been checked by making a comparison between the empirical and simulation results. Data for randomly selected 24 factor level combinations of N and KS has been compared by employing the two tailed t-test for each value of R = 0.80, 0.85, 0.90, 0.95, as given in Appendix A.6.1. In each set of the related data, difference between the predicted and simulated values was found to be insignificant at $\alpha = 0.05$.

6.9 AVERAGE WORK INPROCESS

The total work inprocess inventory in a line would depend upon the number of inprocess buffers, their capacities and their time averaged levels. The average buffer level or the average utilisation of the buffer depends upon the line operating policy. In a flow line the material may move from stage to stage directly or through the intermediate storage, as illustrated in Fig. 6.11(a) and (b) respectively. In the first case (Fig. 6.11a), when stages complete one operation the workpiece moves from stage i to stage j directly. The storage J is operated only when one of the stages is forced down. In the second case (Fig. 6.11b) in one transfer cycle workpiece moves from i to J or from J to j only and not from i to j. Thus, there would be instances, when storage changes state from 0 to 1, that stage j would starve for one time unit. In case of small capacity buffers, this will lead to average buffer levels greater than 50 percent and may slightly affect the efficiency of the line. But when the inprocess buffers are of adequate size (KS = 2 or 3), the difference between the two work transfer mechanisms would become negligible. Thus for all practical purposes, in case of balanced lines, the WIP can be taken as 50 percent of the provided capacity.

6.10 OPTIMUM SIZE OF INPROCESS BUFFERS

The provision of inprocess buffers in the line involves expenditure. The major costs associated with this would be the cost of storage space, cost of material handling and the inprocess inventory carrying cost etc. These costs can be divided into two groups :

(i) Fixed cost (C_f) : which will depend upon the number of inprocess buffers, but not on their capacities. This includes the cost of storage space, cost of handling equipment and a part of the operation costs.

(ii) Variable cost (C_v') : This will depend upon the number and capacity of the inprocess buffers, and the inprocess inventory and operational costs.

$$C_v' = C_v.(N-1).MRT \qquad \ldots (6.10)$$

Then total cost per unit time

$$TC = C_f + C'_v . KS$$

If P is the profit on each unit produced, then addition in revenue due to inprocess buffer can be written as,

$$AR = (\eta - \eta_0)\,P$$

and the net gain from the system per unit time is,

$$G = (\eta - \eta_0)P - C_f - C'_v\, KS \qquad \ldots (6.11)$$

Substituting the value of η from equation (6.6), and solving, we get,

$$G = \Delta\eta\, P\, \frac{KS}{KS + K + 1} - C_f - C'_v\, KS \qquad \ldots (6.12)$$

where, $\Delta\eta = \eta_\infty - \eta_0$

Differentiating equation (6.12) w.r.t. KS and equating to zero, the following expression for the optimum size of inprocess buffers, can be obtained.

$$KS^* = \sqrt{\frac{\Delta\eta\,(K+1)P}{C'_v}} - (K+1) \qquad \ldots (6.13)$$

Then $\quad S^* = KS^* \, . \, MRT \qquad \ldots (6.14)$

On account of the term (K+1), equation (6.13) will underestimate the value of KS^* slightly, when the value of $\Delta\eta \rightarrow 0$. However, this will happen only for small lines with highly reliable stages, where the buffer required is very small.

6.10.1 Illustrative Example

To illustrate the utility of the model of optimum in-process buffer size (equation 6.13), a 10 stage line with the following parameters is considered:

MFT = 90 time units

MRT = 10 time units

μ = 1.0

C_v = 5.00/unit buffer capacity/100 time units

P = 300/unit produced

Since there would be 9 inprocess buffers in the line hence, $C'_v = 5 \times 9 \times 10 = 450$ per 10 unit capacity/100 time units. For 10 stage line with $R_i = 0.9 \; \forall \; i$,

$$\eta_{\infty} = 0.9$$

$$\eta_0 = [1 + 10(.1/.9)]^{-1} = .4737$$

$$\Delta\eta = .4263 = 42.63 \text{ percent}$$

and $$K = -2.35 + 1.8335\ (12/10) = -0.1498$$

$$KS^* = (42.63(1-.1498)\ 300/450)^{1/2} - .8502$$

$$= 4.065$$

or, $$S^* = 41 \text{ units}$$

6.10.2 Sensitivity Analysis

In practice, it may not be practicable to determine all the parameters of the line very accurately and hence a sensitivity analysis of the parameters affecting the system performance, would be of practical utility.

The value of the optimum size of the inprocess buffer, S^*, depends upon $\Delta\eta$, K, P and C_v. An overestimation of P will result in higher values of S^*, while overestimation of C_v will result in lower values of S^*. Since KS^* is a function of $\sqrt{P/C_v}$, a small change in P or C_v will not have much effect on S^*. The reliabilities of the stages (R), effect the value of $\Delta\eta$ as well as of K, and the value of R is affected by the error in estimation of mean failure and repair times. For a fixed value of ACT, the effect of variation in R on the values of KS^* and S^* is illustrated in Fig. 6.12. Figure shows that S^* is highly sensitive to changes in R.

6.11 CASE STUDY

The case considered here is that of a crankshaft manufacturing line at Tata Engineering and Locomotive Company, Jamshedpur. The data has been taken from a published case study reported by Singh, R.D., et al. [119]. The line comprises of 40 machines, the average breakdown and repair times of which are given in Table 6.3. The processing time on each machine is deterministic, while the interbreakdown times and the repair times are exponentially distributed. When the queue length at each machine is allowed to be as large as the number of units produced per shift of 8 hours and the line works for 26 days a month @ 20 hrs a day, the annual output of the line is 24000 crankshafts.

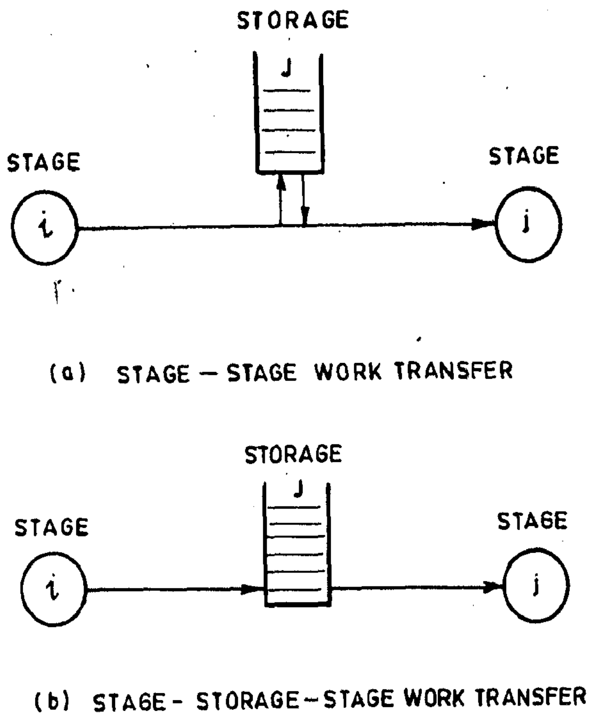


FIG. 6.11 WORK TRANSFER MECHANISMS

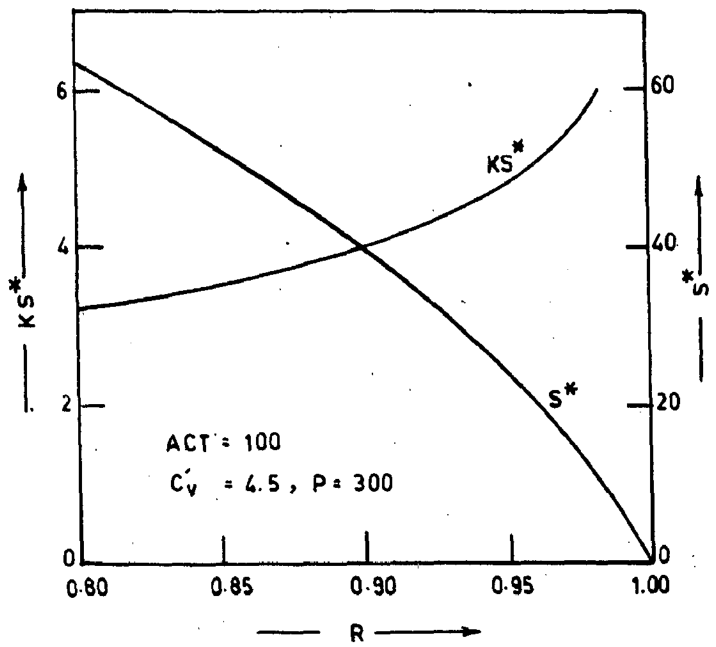


FIG. 6.12 EFFECT OF RELIABILITY OF STAGES ON THE OPTIMUM SIZE OF INPROCESS BUFFERS

TABLE 6.3 : Breakdown data of machines [119]
(MFT and MRT are in hours)

Machine No.	MRT	MFT	A
1	-	-	1.0
2	7.075	271.5	.9746
3	5.1	271.5	.9816
4	1.59	543.0	.9971
5	2.8	4344.0	.9994
6	3.69	394.9	.9907
7	5.76	217.2	.9742
8	3.06	482.0	.9937
9	2.53	1448.0	.9983
10	0.50	4344.0	.9999
11	5.5	724.0	.9925
12	15.02	120.7	.8893
13	2.4	2172.0	.9989
14	9.72	434.4	.9781
15	-	-	1.0
16	5.79	289.6	.9804
17	5.08	1086.0	.9953
18	3.04	620.6	.9951
19	14.17	1448.0	.9903
20	4.87	63.0	.9282
21	-	-	1.0
22	-	-	1.0
23	8.29	289.6	.9722
24	4.20	2172.0	.9981
25	-	-	1.0
26	26.08	1085.8	.9765
27	2.3	620.6	.9963
28	-	-	1.0
29	-	-	1.0
30	2.57	724.0	.9965
31	7.12	188.8	.9637
32	-	-	1.0
33	1.0	4344.0	.9998
34	5.83	482.7	.9881
35	2.7	4344.0	.9994
36	-	-	1.0
37	3.83	1086.0	.9965
38	2.0	4244.0	.9995
39	3.68	434.4	.9916
40	5.74	620.6	.9908

6.11.1 Line With No Buffer

If the line considered is allowed to run with zero in-process buffers, that is with machines completely coupled to each other, then the line efficiency would be the lowest. Assuming the failures to be operation dependent and the processing times to be equal to the processing time of the least reliable machine, the line efficiency can be obtained as :

$$\eta_0 = \left[1 + \sum_{i=1}^{40} \frac{1 - A_i}{A_i}\right]^{-1} = 0.66743 \text{ or } 66.74 \text{ percent}$$

6.11.2 When Infinite Buffer is Provided

The machines No.12, has the lowest availability, and that would be the upper limit of the line efficiency. In this case $\eta_\infty = 0.8893$. Thus the steady state efficiency of the line under the given conditions could be increased from 66.74 percent to 88.93 percent.

Examination of the breakdown data of the machines in Table 6.3 shows a large imbalance in the availability of the machines. Since, it is known that the inprocess buffer is most effective only when the line is balanced, therefore, instead of providing buffers between all the machines, they should be located such that the reliability of all the stages becomes more or less equal. It can be seen that 29 out of 40 machines have reliability greater than 99 percent, while there are only two stages with reliability less than 96 percent. Thus, the line performance could drastically be improved by making the following improvements in the design of the system.

(i) Improve the reliability of machine numbers 12 and 20, either by putting additional machines in parallel or by replacement with better machines.

(ii) Locate inprocess buffer storages in such a way that the line becomes approximately balanced.

Since cost data of the line is not available, the effect of improving the reliability of machines 12 and 20, on the overall economy of the system cannot be evaluated, but the cost involved is expected to be much smaller as compared to the gain from the increased production. Now the lowest reliability in the line is 0.96.

$\therefore \quad \eta_{\infty} = 96$ percent

and $\quad \eta_{0} = 72.25$percent from equation (6.2)

Since the line has been assumed to be balanced with regard to the processing times, there is no advantage of some stages being highly reliable as compared to the others. Hence, we divide the line into stages so that reliability of each stage is as close to 0.96 as possible. This reduces the 40 machines line to 11 stage line. The reliabilities of the stages and the grouped machines are given in Table 6.4.

6.11.3 Average Failure and Repair Times of Stages

In each stage the average up and down times of different machines differ from each other significantly. In order to determine the mean failure and repair times of a stage, the frequency of failures of each machine in the longest failure cycle of the stage was determined, and then the frequency weighted average value of the repair times of the stage was calculated. For example, in stage 2, there are four machines (Nos. 3,4,5 and 6) and machine No.5 has the longest failure cycle (LFC) = (MFT + MRT) = 4346.8 hrs. Mean failure and repair times are determined as under :

Machines in stage 2	3	4	5	6
Average down time of machines (MRT)	5.1	1.59	2.8	3.69
Average up time of machines (MFT)	271.5	543.0	4344.0	394.9
Frequency of failures (f) in LFC = 4346.8	15.715	7.98	1.0	10.905

$\Sigma f = 35.6$

$\Sigma f.MRT = 135.8742$

$$\text{Mean repair time of stage 2} = \frac{\Sigma f.MRT}{\Sigma f} = 3.8167$$

$$\text{Mean failure time of stage 2} = \frac{LFC}{\Sigma f} - MRT = 118.284$$

The availability of the stage determined from the failure and repair times could be checked with the availability determined by equation (6.2). The values of the MFT and MRT for different stages are given in columns 4 and 5 in Table 6.4.

6.11.4 Allocation of Inprocess Buffer

The 11-stage line is now almost balanced with respect to the availability of the stages, but the MFT and MRT are different for different stages. Since the effect of an inter-stage buffer storage is maximum on the adjoining stages, its capacity can be fixed as a multiple of the average of the MRT's of the stages on its up and down stream sides, i.e.,

$$S_j = \frac{KS}{2} (MRT_{j-1} + MRT_j)$$

The values of inprocess buffers to be provided when KS = 1 are given in Table 6.4.

6.11.5 Efficiency of Line with Finite Inprocess Buffers

For determining the efficiency of the line corresponding to various values of KS, the empirical model developed in Section 6.8 can be used.

$$\eta^N_{KS} = \eta^N_{\infty} - \frac{1+K}{KS+K+1} (\eta^N_{\infty} - \eta^N_0)$$

For N = 11, and R = 0.96, from equations (6.2), (6.3) and (6.7) we get,

$$\eta_{\infty} = 0.96$$
$$\eta_0 = 0.7235$$
$$K = -0.18314$$

Then efficiency for finite inprocess buffers of size KS will be given by

$$\eta_{KS} = 0.96 - \frac{0.1931883}{KS+0.81686}$$

The values of η_{KS} determined from the above equation have been compared with the simulation results in Table 6.5. The difference has been found to be less than 1.0 percent in most of the cases. The mean difference is 0.515 percent only. The slightly higher values obtained by simulation may be attributed to the error involved in rounding up the mean failure and repair times to integer values. The error involved, however, is so small that it will not effect the optimum size of the inprocess buffers. The optimum value of inprocess buffer size can easily be obtained by employing the equation (6.13), once the costs involved in the provision of inprocess buffers are known.

Similarly by improving upon some more stages the values of η_0 and η_∞ could further be improved. That will require larger number of storage points. But the utility of such improvements can be assessed only by cost analysis.

6.12 CONCLUSIONS

From the results reported in this chapter the following conclusions can be drawn :

(1) The distribution of inprocess buffer in a flow line having unreliable stages and fixed processing times, have a significant influence on the line performance. A large number of small capacity buffers are comparatively more effective than a few large sized buffers.

Table 6.4 : Breakdown Data of 11-stage line

Stage No.	Machines grouped	R	MRT	MFT	Buffer size for KS = 1
1	1 - 2	.9746	7.025	271.5	α
2	3 - 6	.9692	3.817	118.28	5
3	7 - 11	.9600	4.93	108.32	4
4	12	.9600	5.03	120.7	5
5	13 - 15	.9770	8.48	360.3	7
6	16 - 19	.9600	5.89	145.86	7
7	20	.9600	4.87	116.88	5
8	21 - 24	.9704	7.80	254.23	6
9	25 - 30	.9697	7.91	249.75	8
10	31	.9637	7.12	188.8	8
11	32 - 40	.9665	4.55	128.08	6

Table 6.5 : Predicted and Simulated Efficiency of n-stage line

KS	Predicted η	Simulated η	percentage Error
0	72.35	73.13	1.07
1	85.37	86.07	0.81
2	89.14	90.15	1.12
3	90.94	91.47	0.58
4	91.99	92.23	0.26
5	92.68	92.98	0.32
7	93.53	93.97	0.47
10	94.21	94.35	0.15
15	94.78	95.00	0.23
25	95.25	95.87	0.65
∞	96.00	96.00	0

(2) The efficiency of the flow line system having unreliable stages, can very closely and efficiently be predicted by employing the empirical model presented in this chapter.

(3) If the costs associated with the inprocess buffers are available then the size of the inprocess buffer can be optimised by employing the relation proposed in this chapter (equation 6.13).

The models presented are applicable over a wide range of the line operating parameters, and hence can be applied to a variety of practical system.

COMPARISON OF SIMULATED AND PREDICTED RESULTS FOR AUTOMATED LINES WITH STAGES SUBJECT TO FAILURES

$R_i = 0.8\ \forall\ i$			$R_i = 0.85\ \forall\ i$			$R_i = 0.90\ \forall\ i$			$R_i = 0.95\ \forall\ i$		
N,Kz	η_{SIM}	η_{EMP}	N,Kz	η_{SIM}	η_{EMP}	N,Kz	η_{SIM}	η_{EMP}	N,Kz	η_{SIM}	η_{EMP}
2,1	71.81	70.79	2,4	81.99	80.98	2,8	88.21	88.16	2,2	92.44	92.55
2,4	75.14	75.22	2,1	77.83	77.30	2,3	87.59	86.43	2,12	94.39	94.25
3,2	69.99	69.39	3,6	80.58	80.63	3,5	86.37	86.18	3,5	92.95	92.81
3,4	73.74	73.09	3,10	82.34	82.12	3,10	88.41	87.81	3,8	93.79	93.48
4,15	78.12	77.30	4,0	58.62	58.62	4,2	82.19	81.45	4,1	87.02	87.84
4,10	75.66	76.12	4,12	81.61	82.17	4,12	87.88	87.82	4,15	93.96	93.96
5,12	76.18	76.44	5,2	72.61	72.57	5,6	85.34	85.66	5,3	90.20	90.56
5,0	44.44	44.44	5,8	80.40	80.61	5,8	85.83	86.60	5,8	92.68	92.98
6,3	68.55	68.32	6,1	66.09	65.38	6,3	81.95	81.94	6,5	92.20	91.74
6,12	76.35	76.26	6,8	80.11	80.36	6,10	86.81	87.05	6,8	92.91	92.82
7,8	75.25	74.45	7,10	81.14	81.05	7,1	72.61	73.06	7,12	93.30	93.41
7,3	66.77	67.79	7,2	71.00	70.81	7,3	81.70	81.52	7,4	90.80	90.82
8,1	56.07	56.68	8,5	77.64	77,57	8,4	82.71	82.94	8,3	89.52	89.46
8,10	75.04	75.33	8,4	76.25	76.09	8,15	87.81	87.81	9,6	91.74	91.76
9,6	71.51	72.54	9,3	73.98	73.55	9,10	86.80	86.73	10,5	91.46	91.07
9,15	78.13	76.72	9,12	81.72	81.49	9,6	84.80	84.83	11,3	88,95	88.78
10,0	28.57	28.57	10,5	76.56	77.23	10,1	69.44	70.41	12,10	92.74	92.79
10,8	73.00	74.09	10,6	78.17	78.35	11,15	88.01	87.68	13,15	93.63	93.46
12,5	71.09	70.92	12,15	81.80	82.03	12,5	82.98	83.57	14,4	89.70	89.73
12,10	75.23	75.05	12,3	72.15	72.88	14,6	84.22	84.38	15,15	93.79	93.41
15,6	71.68	72.03	15,8	79.96	79.55	15,12	87.65	87.00	16,1	78.80	79.42
15,12	76.53	75.72	15,12	80.93	81.25	17,2	74.95	75.65	17,6	91.46	91.13
20,2	60.45	60.51	20,10	80.16	80.44	18,5	83.31	83.15	18,2	85.74	85.03
20,5	70.03	70.44	20,4	74.42	74.72	20,4	81.24	81,57	20,10	92.95	92.57

CHAPTER-7

UNBALANCED FLOW LINES WITH UNRELIABLE STAGES

7.1 INTRODUCTION

A flow line having constant operation times and unreliable stages has been defined to be unbalanced, when the breakdown characteristics of the stages are non-identical and/or the inprocess buffers are of unequal size. In practice, it may not be possible to have perfectly balanced lines. Some stages may have comparatively higher reliability (good stages) than others (bad stages). The location of a bad/good stage in a line would affect its efficiency as well as the average work inprocess inventory (WIP). Similarly, the distribution of total inprocess buffer capacity in between the stages would influence the system efficiency and the WIP. Provision of inprocess buffers involves huge expenditure in terms of money and hence, determination of the optimum buffer distribution plan is one of the important aspects of the flow line design.

Review of literature (Chapter 2) reveals, that relatively limited attention has been paid towards the study of the performance behaviour of unbalanced automatic transfer lines. Most of the reported studies are limited to the analysis of two stage systems only and some of the findings are contradictory. Further, the performance behaviour of a few of the important line configurations such as with stages arranged in the bowl(worst stage in the middle) or inverse bowl (best stage in the middle) formations, remain unanalysed. Studies reported so far[24,45, 104-106,118], relate to the effect of the line layout and buffer allocations on the production rate of the system, whereas, the effect of buffer allocation policies and the type of line layout, on the WIP inventory has not been investigated satisfactorily.

The simulation results presented in this chapter relate to the effect of line layout and buffer allocation policies on the unbalanced line efficiency and WIP inventory.

7.2 SYSTEM MODEL

The flow line system analysed in this chapter is similar to the one modeled in Chapter 6. Operating policy for the system and assumptions given in Sections 6.2.1 and 6.3 respectively are also valid in the present case, except, that different reliability values and inprocess buffers, have been assigned to various stages of the line. The production rates of all the stages are identical and constant, and have been taken to be as unity. While the MFT and MRT would be different for different stages, the ACT has been kept constant and equal to 100 time units.

7.3 DISTRIBUTION OF FAILURE AND REPAIR TIMES

It was pointed out in Chapter 3 that failure and repair time distributions of a stage, has mostly been assumed to be exponential. However, to study the influence of the coefficient of variation of the failure and repair times, normal distribution has also been employed in addition to the exponential distribution. In addition, in some of the cases Erlang distribution has been used, but it has been found to consume larger CPU time as compared to normal distribution case, especially at higher values of Erlang shape factor.

7.4 LENGTH OF SIMULATION RUN

In this study, the system efficiency has been taken as the major response, and based on the same, the length of simulation run has been fixed as per method discussed in Chapter 3. The LOR depends upon the combination of system parameters, and could vary widely with N,R or CV. For the range of N and R considered in this study, the following values of LOR have been found to give the mean efficiency well within ± 1 percent at $\alpha = 0.05$.

Distribution of failure and repair times	LOR (Time units)
Exponential	80,000
Normal, CV = 0.5	35,000
CV = 0.2	17,000

Four replications of each run were obtained so as to bring the accepted error within ± 0.05 percent. The details of the simulation model are the same as in Chapter 6.

7.5 RESULTS AND DISCUSSION

The following results have been presented in this chapter.

i) Effect of the location of the good/bad stage on line efficiency.

ii) Efficiency of line with unequal reliabilities of stages.

iii) Efficiency of line with unequal buffers.

iv) Efficiency of line with unequal reliabilities and unequal buffers.

v) Work inprocess inventory.

7.5.1 Effect of the Location of the Good/Bad Stage on Line Efficiency

The influence of the location of a particularly good or bad stage in a flow line has been studied by employing in-process buffers of equal capacity ($S_j = S \; \forall j$). Simulation results have been obtained for three and five stage lines. With values of S ranging from 1 to 50 and for the exponential and normal distributions (CV = 0.05) of the failure and repair times of the stages. The values of efficiency of the line with bad stage (R = 0.85) placed in the beginning, middle and at the end of the line are given in Table 7.1. Each set of data was analysed by employing the Wilcoxon T-test and in each case it was found that at $\alpha = 0.05$, the layout with bad stage at either of the two ends was better than the one with the bad stage in the middle. The layouts with bad stages in the beginning and end were found to be equally efficient. From these results it can be concluded, that installation of a bad stage in the centre of the line should be avoided and conversely good stages should be located in the middle of the line.

TABLE 7.1 Efficiency of Production Lines with Non-identical Reliabilities of Stages

(ACT = 100, $S_i = S \ \forall i$, $R_i = 0.95 \ \forall$ i except $i=j, R_j = 0.85$)

	Exponential failure and repair times					
	N = 3			N = 5		
S \ j	1	2	3	1	3	5
1	78.17	77.34	77.86	71.07	71.40	72.40
2	78.43	77.95	79.10	75.00	74.58	74.68
3	79.52	79.85	79.83	76.14	75.55	75.08
5	81.29	79.33	80.81	78.94	78.48	78.85
7	81.78	81.16	82.01	79.86	79.37	79.88
10	82.73	82.43	83.08	81.18	81.04	82.47
15	82.83	82.94	83.81	83.65	82.45	83.41
25	84.21	83.97	84.03	84.54	83.87	85.09
50	84.65	84.62	84.64	85.08	85.03	85.54

	Normal failure and repair Times(C_v= 0.5)					
	N = 3			N = 5		
j	1	2	3	1	3	5
1	79.38	79.19	79.03	74.77	73.97	74.07
2	80.38	80.20	80.67	76.77	76.70	76.77
3	81.71	81.23	81.14	78.96	78.88	78.68
5	83.51	83.17	83.49	82.60	81.85	82.51
7	84.55	84.08	84.19	84.34	82.95	84.59
10	84.80	84.84	85.02	85.03	84.79	84.95
15	85.28	85.05	85.21	85.04	84.97	85.01
25	85.49	85.27	85.44	85.46	85.39	85.47
50	85.62	85.45	85.62	85.50	85.55	85.76

7.5.2 Efficiency of Line with Unequal Reliabilities of Stages

In case of flow lines, where the reliabilities of all the stages differ, arrangement of stages so as to achieve the maximum production efficiency should be determined. In order to investigate into this problem, the performance of the following four configurations of a 6-stage line, having equal inprocess buffers ($S_j = S \ \forall j$) were compared.

i) Stages arranged in a manner such that their reliabilities increase from beginning towards end.

ii) Reliabilities of the stages decreasing from beginning towards end.

iii) Reliabilities of the stages decreasing upto centre and then increasing (bowl formation).

iv) Reliabilities of the stages increase upto centre and then decrease (inverse bowl formation).

Simulation results for the above mentioned configurations are given in Table 7.2. The line configurations with reliabilities arranged in the increasing or decreasing orders are equally efficient. Thus, when the order of stages in a line is reversed, the efficiency of the system remains unchanged, or we can write :

$$\eta\ (R_1, R_2 \ldots R_{N-1}, R_N) = \eta\ (R_N, R_{N-1} \ldots R_2, R_1) \qquad \ldots (7.1)$$

This result supports the findings of Sheskin [118]. However, the results of Table 7.2 indicate that the line with reliabilities arranged in inverse bowl configuration yields better efficiency, as compared to others, whereas, the efficiency in case of bowl configuration is the least.

7.5.3 Efficiency of Line with Unequal Buffers

It is well known that the provision of inprocess buffers helps to improve the efficiency of the flow line systems. The distribution of the total inprocess buffer between the various intermediate storages however, is an important problem in the flow line design. Sheskin [118] suggested that, in a line with equally reliable stages, equal inprocess buffers would lead to maximum production rate. The simulation results reported in the previous sections of this chapter, however, indicate that the middle of the line is more critical and required better attention. To investigate into the influence of unequal buffers on the performance of a line having identically reliable stages, simulation was run for a 6-stage line, for several arrangements of buffer allocations pertaining to the bowl and inverse bowl configurations. In each arrangement it was assumed that

$$\sum_{j=2}^{N} S_j = 25 \text{ units}$$

Fig.7.1 illustrates the results for a line with exponentially distributed failure and repair times of the stages whereas, Figs.7.2 and 7.3 relate to simulation results corresponding to the normally distributed failure and repair times. In Figs.7.1 to 7.3, the point of maximum efficiency can be seen to lie towards the left of equal buffers ordinate at R = 0.85 and 0.90, while for R = 0.95, the maximum efficiency corresponds

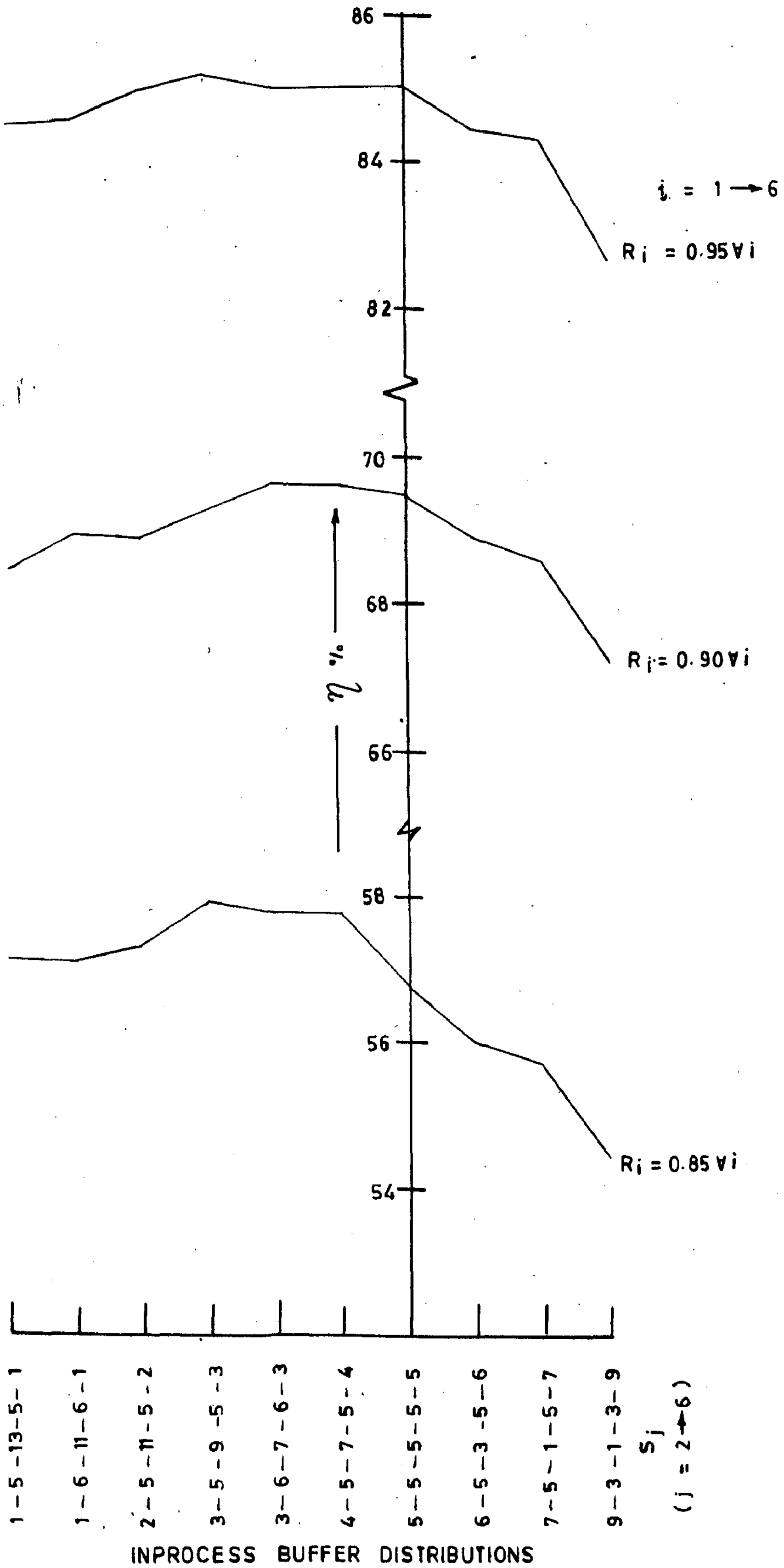


FIG.7.1 INFLUENCE OF BUFFER ALLOCATION ON THE η OF LINE
(EXPONENTIAL UP AND DOWN TIMES)

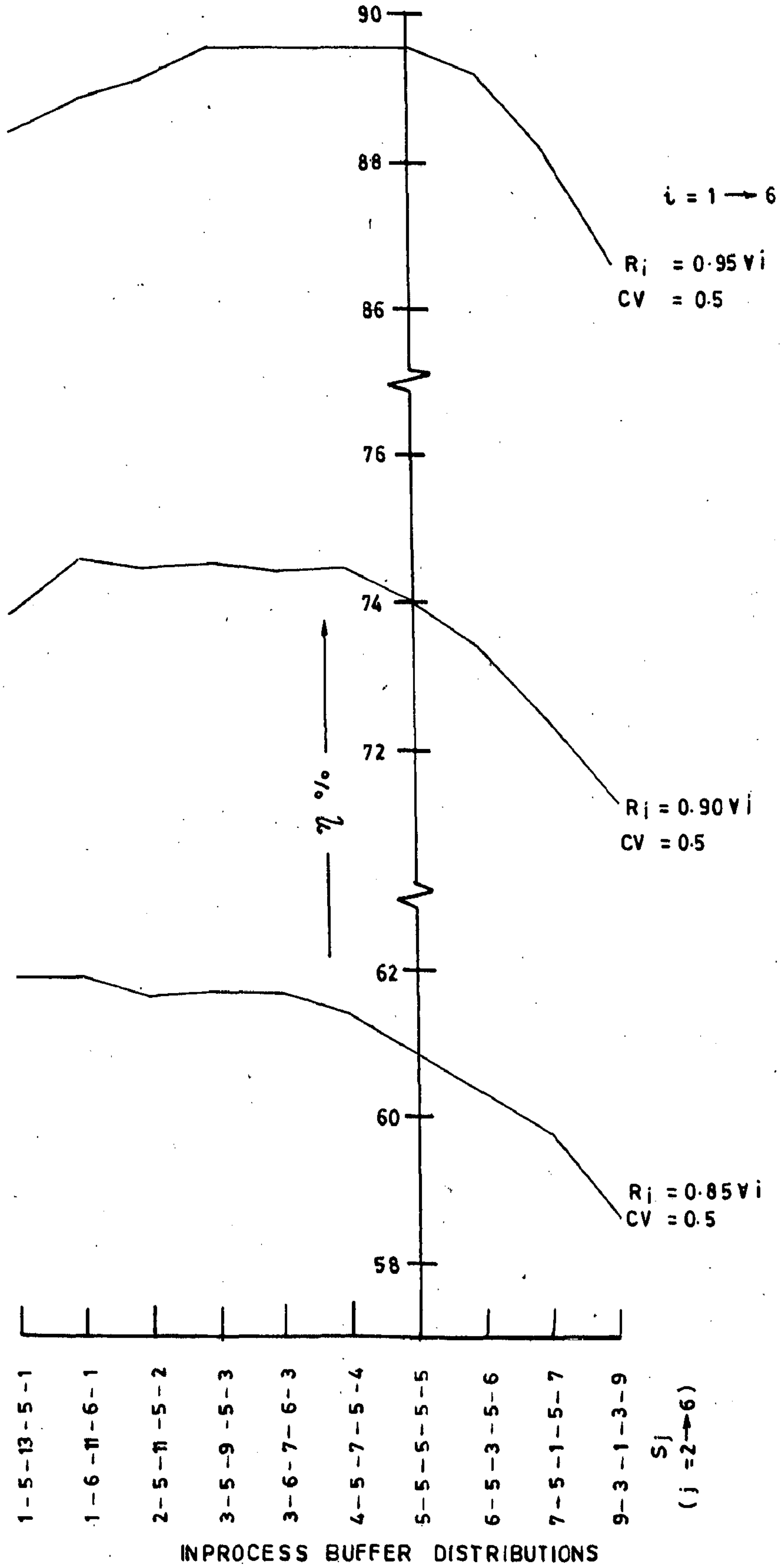


FIG.7.2 INFLUENCE OF BUFFER ALLOCATIONS ON THE η OF LINE
(NORMALLY DISTRIBUTED UP AND DOWN TIMES)

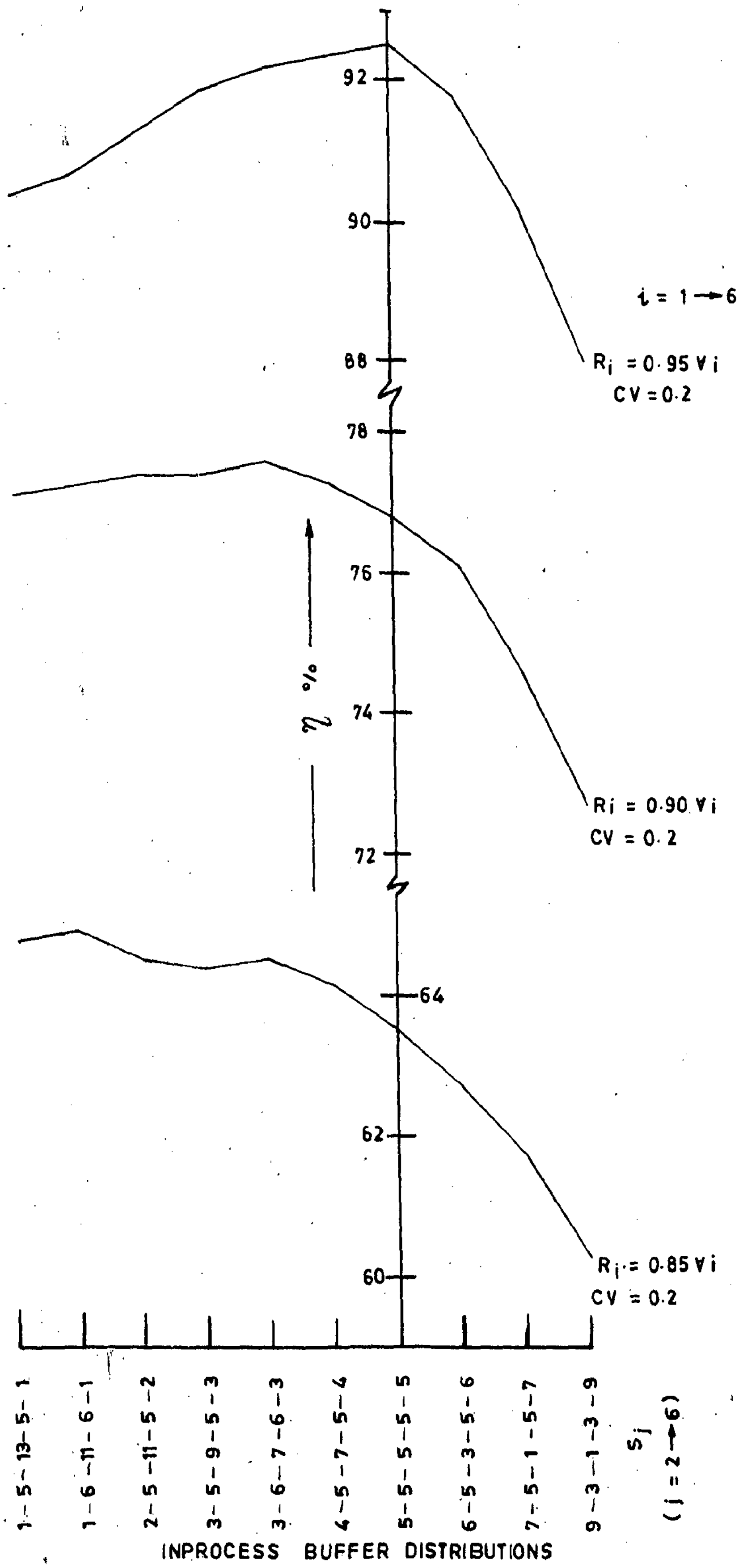


FIG. 7.3 INFLUENCE OF BUFFER ALLOCATIONS ON THE η OF LINE
(NORMALLY DISTRIBUTED UP AND DOWN TIMES)

to the equal buffers arrangement. Thus, in case of lines having smaller reliability of the stages, maximum efficiency is obtained by allocating larger share of the inprocess buffer to the middle of the line (inverse bowl formation of buffers). This contradicts the results of Sheskin [118]. Examination of the results in each of the Figs.7.1 to 7.3 shows that smaller the reliability of the stages, greater is the imbalance required in buffers, to achieve the maximum output from the system. Figs.7.1 to 7.3 also reveal that the improvement obtained in efficiency by providing unequal buffers is comparatively more prominent when coefficient of variation of the failure and repair times is low.

7.5.4 Efficiency of Line with Unequal Reliabilities and Unequal Buffers

When reliabilities of the stages in a production line are unequal, interstage buffers of unequal capacity have to be provided to achieve maximum output from the line. It has generally been the practice [24,45,104,118], that larger buffers be placed close to the stations with lower reliabilities. To check the applicability of this conjecture, extensive simulation runs were made for various line configurations, using a number of buffer allocation policies in each case. The results for lines with N=3,5 and 7 with one stage particularly good/bad are illustrated in Figs.7.4-7.6. Fig.7.4 indicates that when R_i=(.95,.85,.85) a slight improvement in efficiency, over that attainable for a line with equal buffers is possible by allocating larger sized buffers to the second bunker, and that a larger buffer in the first bunker improves the efficiency when R_i = (.85,.85,.95).

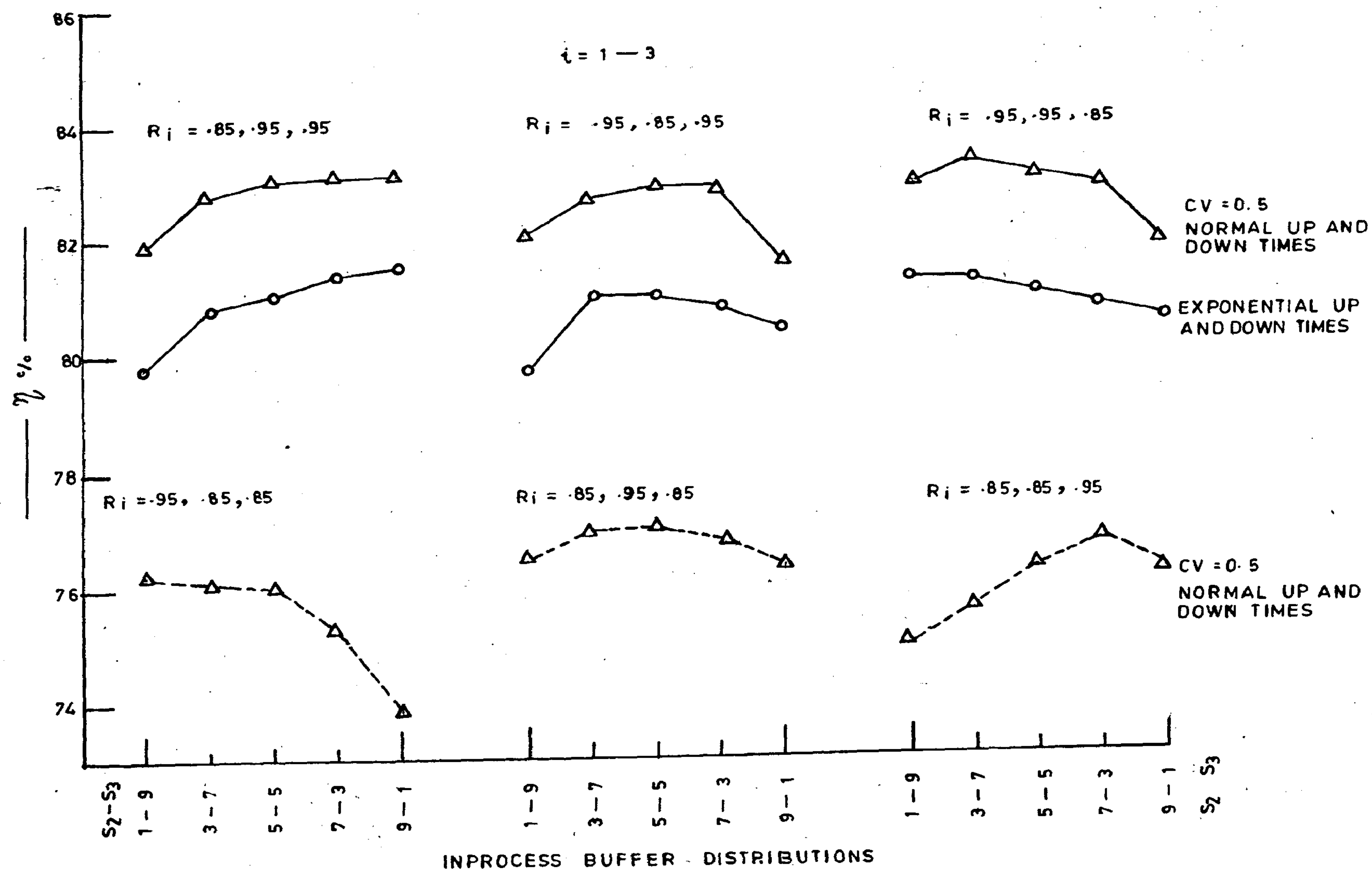


FIG. 7.4 EFFECT OF BUFFER DISTRIBUTION ON THE η OF A 3-STAGE LINE WITH ONE BAD/GOOD STAGE

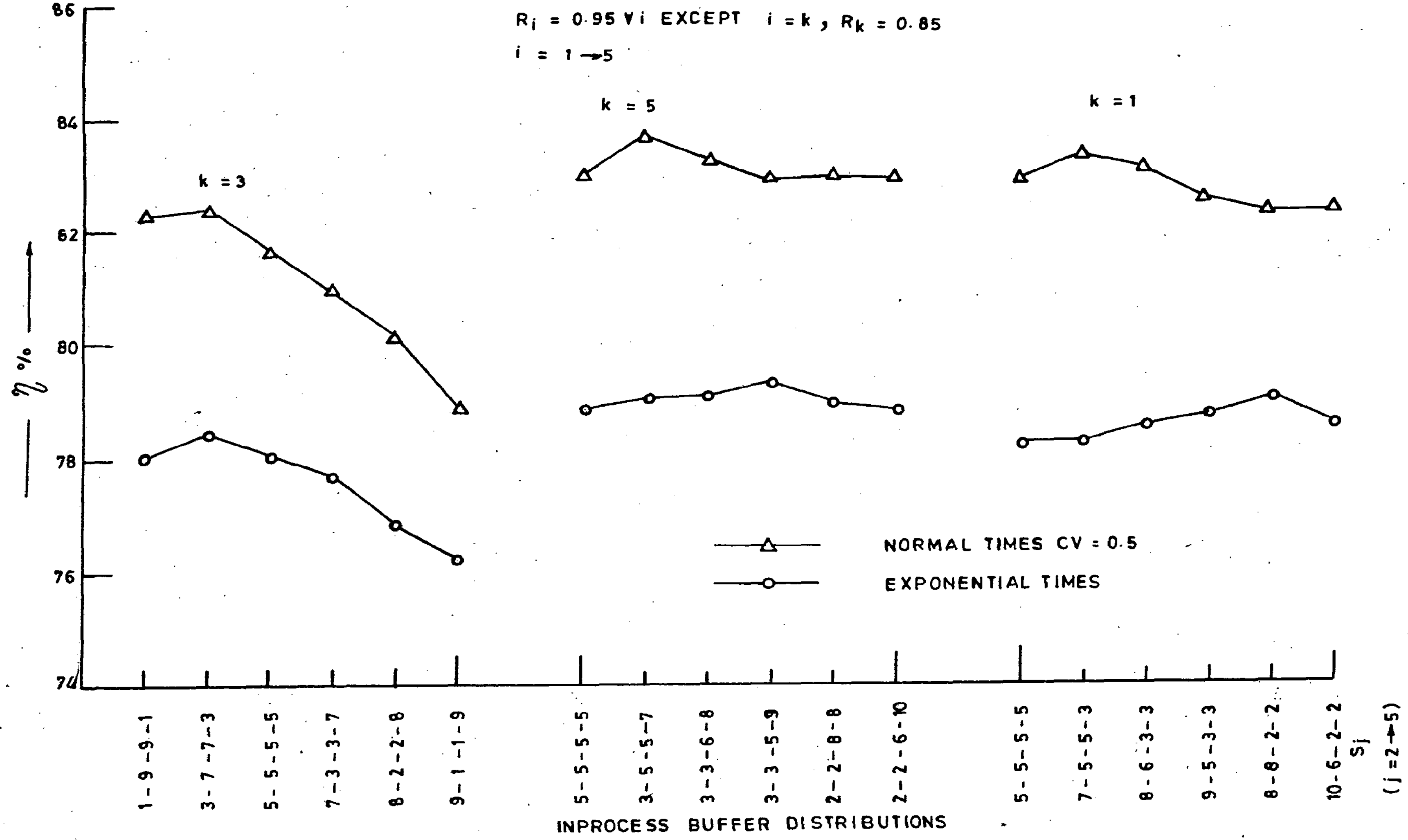


FIG. 7.5 EFFECT OF BUFFER DISTRIBUTION ON THE η OF A 5-STAGE LINE WITH ONE BAD/GOOD STAGE

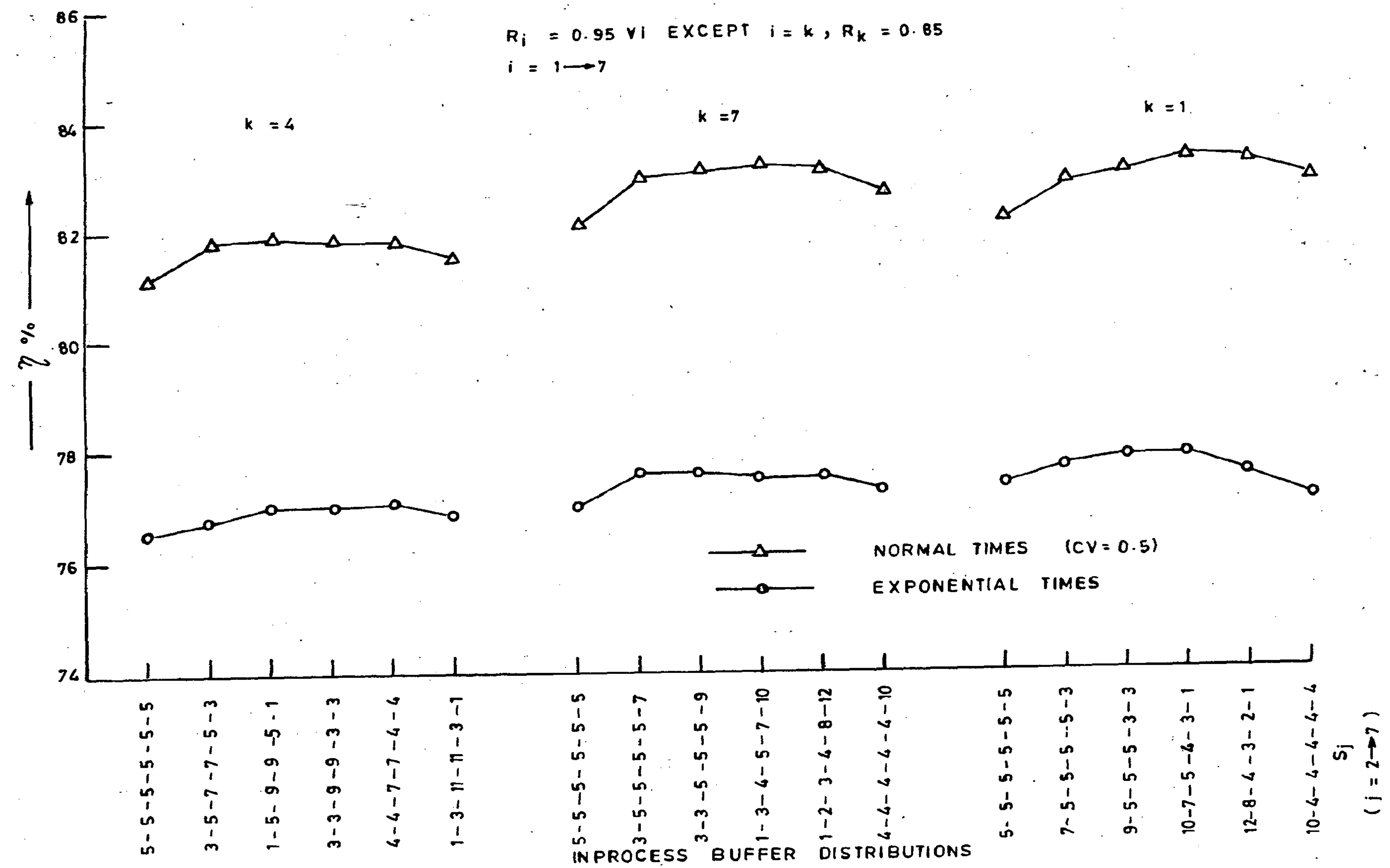


FIG. 7.6 EFFECT OF BUFFER DISTRIBUTION ON THE η OF A 7-STAGE LINE WITH ONE BAD/GOOD STAGE

Similarly, performance of the system could be improved by allocating larger share of the buffer to the first and second bunkers in case of R_i =(.85,.95,.95) and (.95,.95,.85) respectively. In case of symmetrical arrangements, R = (.85,.95,.85) and (.95, .85,.95), equal size of the two buffers led to maximum efficiency. Though the improvement in efficiency has been found to be quite small, results do support the conjecture that larger share of buffers be allocated to the storages close to the bad stages. This result has further been confirmed in Figs.7.5 and 7.6. In case of 5-stage line (Fig.7.5), when the middle stage is less reliable (R_3= .85), buffer allocation of 3-7-7-3 results in maximum efficiency, whereas, buffer allocations of 7-5-5-3 and 3-5-5-7 gave the maximum efficiency when the bad stage was in the beginning and at the end of the line respectively. A similar trend is also demonstrated by the data given in Fig.7.6. It should however be noted that in all the cases very large imbalances bring down the production efficiency.

Fig.7.7 shows the efficiency of the line when the reliabilities are arranged in the,

a) decreasing order

b) increasing order

c) bowl configuration

d) inverse bowl configuration

It should be noted that in case (a) the buffer allocation S_j= 1,3,5,9,7 gives better efficiency than the allocation S_j= 1,3,5,7,9, and in case (b) the allocation policy S_j=7,9,5, 3,1 is better than S_j=9,7,5,3,1. In case (c) inverse bowl

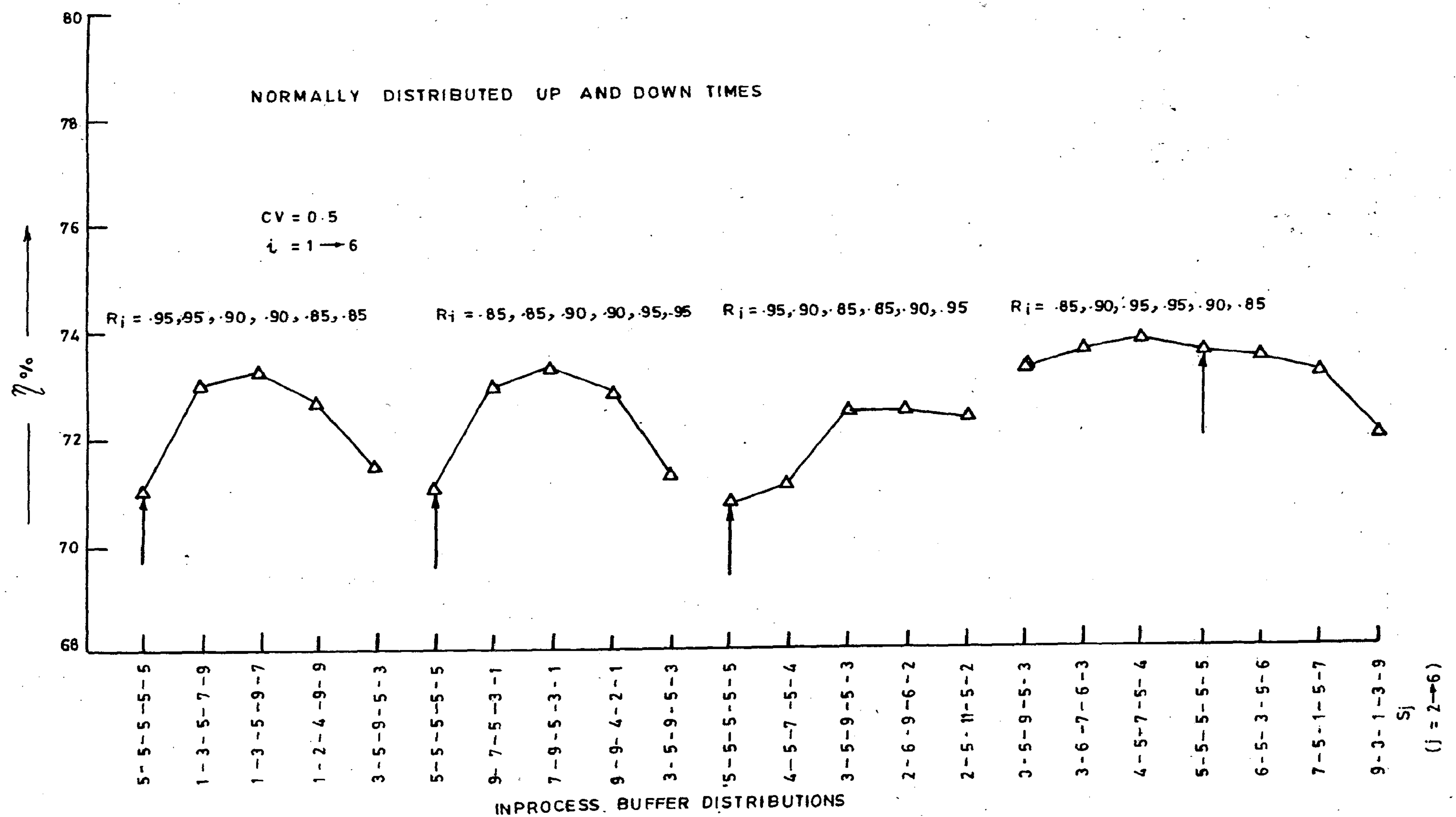


FIG. 7.7 INFLUENCE OF BUFFER ALLOCATIONS ON THE η OF SOME CONFIGURATIONS OF A 6-STAGE LINE

formation of buffers has proved to be beneficial, however, the reverse of this is not true. When the reliabilities are in the inverse bowl form, case (d), a very small improvement in efficiency is possible, and that too corresponding to a small imbalance in the inverse bowl formation of buffers. Thus, the rule that larger buffers be allocated to less reliable stages does not hold true in all the cases, and has to be applied with caution.

In almost all the cases it has been observed that the central part of the line is more critical. Stages in the middle of the line should have higher reliability or the larger share of inprocess buffer be allocated to them. Bad stages or smaller buffers near the ends of the line do not have significant effect on the line performance. When bad stages are in the middle of the line (Fig.7.7c) large improvements in efficiency are possible by making proper allocation of inprocess buffers. However, the maximum efficiency obtained in this case is not comparable to the efficiency of the line when more reliable stages are in the middle (Fig.7.7d).

7.5.5 Work In-Process Inventory

The average inprocess inventory in a flow line depends upon a number of factors, including the breakdown characteristics of the stages, layout of the line, capacities of the inter-stage buffers and the buffer operating policy. The WIP has been found to differ considerably for different line layouts and buffer allocations.

Fig.7.8 shows the variation in WIP for a three stage line, where the stages with unequal reliabilities are arranged in different orders. It can be observed that when the first stage is less reliable than others, R_i = (.85, .95, .95), the inventory held in line remains at lower level, while in case, when R_i= .95, .95, .85, inprocess inventory remains high. The distribution of the inprocess buffer between the intermediate storages also has a great bearing on WIP. With total inprocess buffer of 10 units ($S_2 + S_3 = 10$), when $S_2 < S_3$, WIP is low, and larger the difference (S_3-S_2), the smaller is the WIP. The reverse is true when $S_2 > S_3$. In Fig.7.8 it can be seen, that the variation in WIP due to change in buffer allocation is more prominent, when the reliabilities of the stages are symmetrical about the centre (R_i = .95, .85, .95) and for the case R_i = .95 $\forall i$. When the total inprocess buffer is increased from 10 to 100 units, the variation in WIP with a change in buffer distribution (Fig.7.9) is quite small for the cases, R_i = .85, .95, .95 and R_i= .95,.95,.85, but the change is significant in case of the symmetric lines, R_i = .95, .85, .95 and R_i = .95 $\forall i$.

Some WIP results for a 6-stage line having unequal reliabilities of the stages and equal inprocess buffers have been given in Table 7.2. From the results of Figs.7.7 and 7.8, and Table 7.2, it can be concluded that a bad stage at the end or a good stage in the beginning of the line results in higher inprocess inventory, and reverse is true when a good stage is placed at the end or a bad stage is put in the beginning of the line.

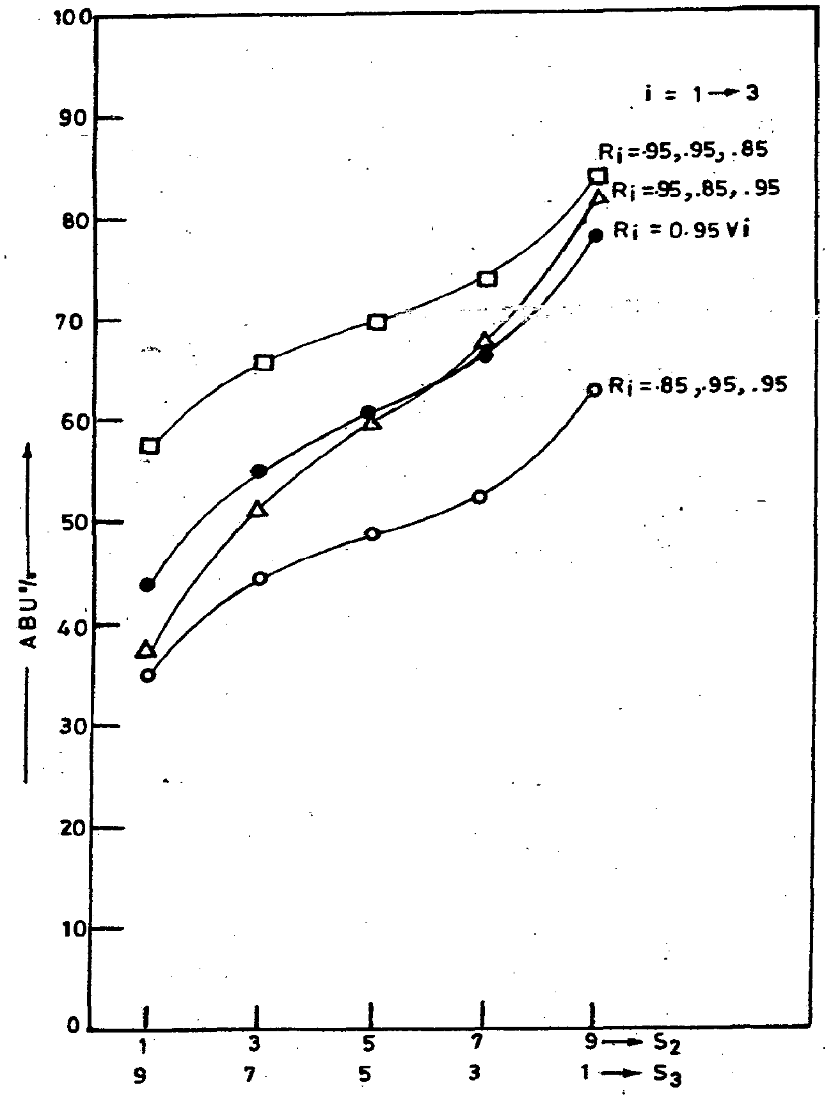


FIG.7.8 INFLUENCE OF BUFFER ALLOCATIONS ON THE WIP IN A 3-STAGE LINE WITH ONE BAD/GOOD STAGE

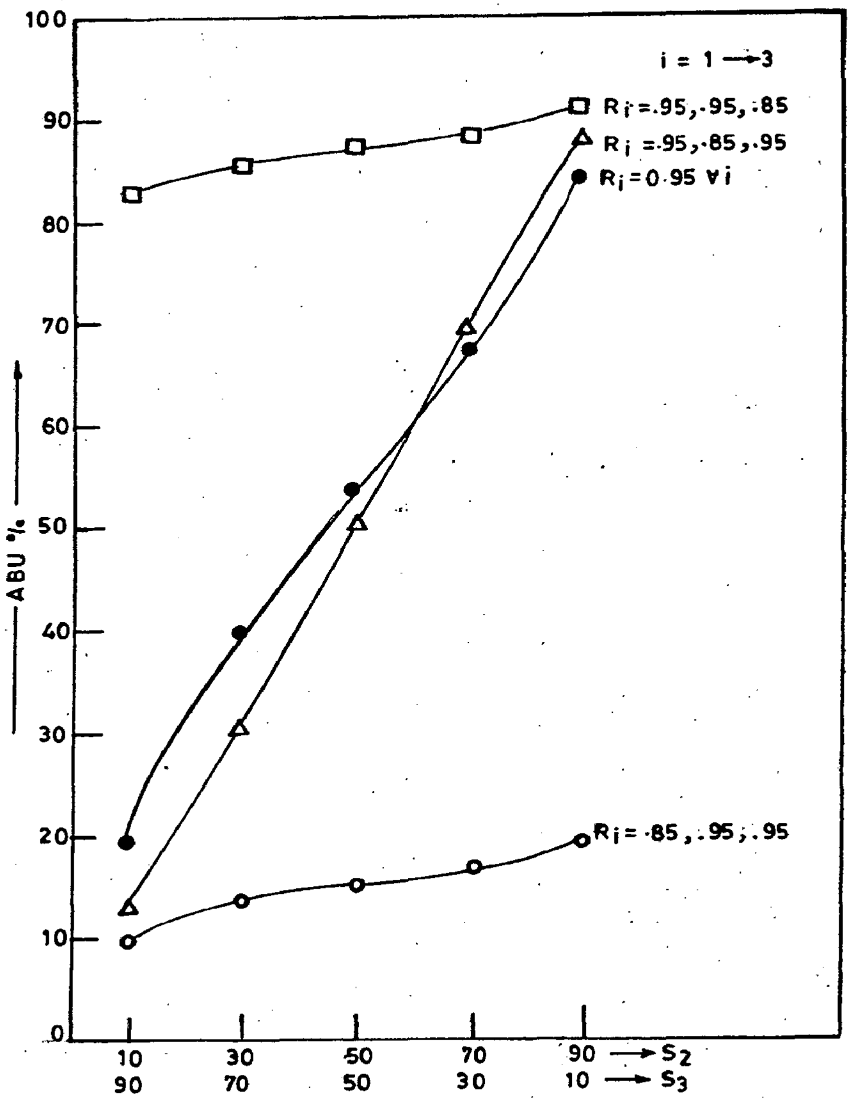


FIG.7.9 INFLUENCE OF BUFFER ALLOCATIONS ON THE WIP IN A 3-STAGE LINE WITH ONE BAD/GOOD STAGE

TABLE 7.2 Comparison of Some Line Layouts

(N = 6, CV = 0.5 $S_j = 5 \ \forall j$)

R_i(i=1,N)	η percent	WIP percent
.85,.85,.90,.90, .95,.95	71.03	31.24
.95,.95,.90,90, .85,.85	71.00	62.38
.95,.90,.85,.85, .90,.95	70.71	50.08
.85,.90,.95,.95, .90,.85	73.48	49.96

Fig.7.10 shows some of the results for a 5-stage line, for three layouts, with bad stage in the end, beginning and middle respectively. In addition to the effect of buffer distribution, the effect of variabilities in failure and repair times has also been illustrated in this figure. For the case of lines with less reliable stages in the end (R_5= .85), the WIP for various buffer allocations remains high for exponentially distributed failure and repair times, as compared to the case when failure and repair times are normally distributed with CV = 0.5 and CV = 0.2. The reverse of this happens when the first

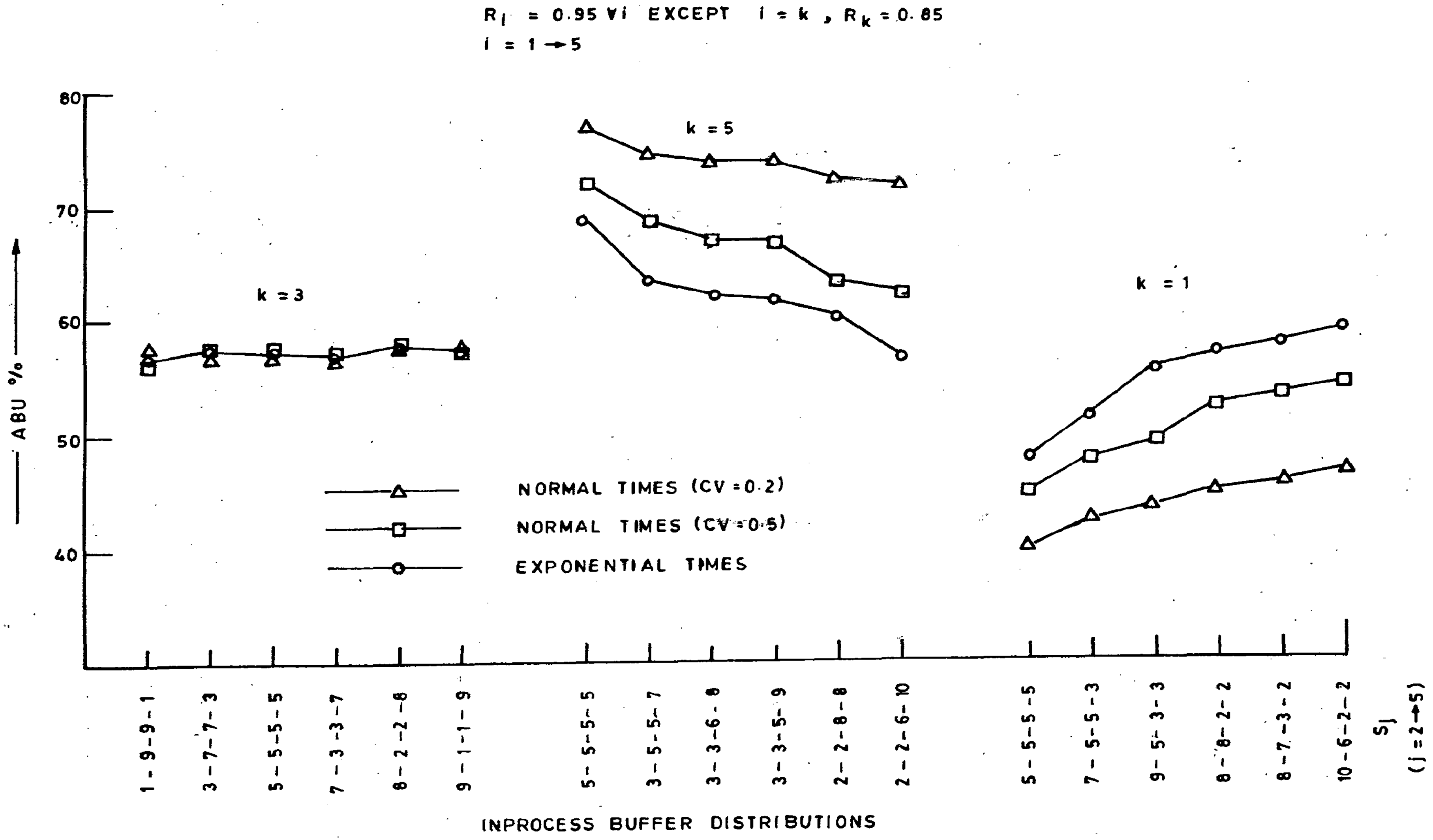


FIG.7.10 INFLUENCE OF BUFFER ALLOCATIONS ON WIP IN SOME CONFIGURATIONS OF A 5-STAGE LINE

stage is less reliable ($R_1=.85$). This shows that when the variability of the failure and repair times increases, the performance of the bad stage further deteriorates, causing frequent **blocking** in the material flow within the line. Fig.7.10 shows that irrespective of the arrangements of the stages, larger share of buffer in the beginning of the line leads to increase in the WIP, while the larger share of buffer towards the end helps to reduce the WIP.

In case of symmetric lines, with buffers symmetrically distributed about the centre, the WIP is affected by the operating policy of the line as discussed in Chapter-6. Since stage-storage-stage material transfer mechanism has been employed in this study, the WIP remains a little higher than 50 percent, when the inprocess buffers are small. A comparison of symmetric cases in Figs.7.8 and 7.9 will show that while WIP is nearly 60 percent in case of $S_2 = S_3 = 5$, it reduces down to 50 percent when $S_2 = S_3 = 50$. In Fig.7.10, when $\sum_{j=1}^{n} S_j = 20$, the WIP remains at about 57 percent in all the symmetric arrangements.

7.6 CONCLUSIONS

The following conclusions can be drawn from the results reported in this chapter.

1) A bad stage located in the middle of the line results in lower efficiency as compared to its location at the ends of the line.

2) A good stage located either in the beginning or in the end of the line has similar effect on the line efficiency. However, a good stage in the beginning results in higher inprocess inventory as compared to its location in the middle or end of the line.

3) Buffers arranged in decreasing order lead to higher WIP inventory as compared to their arrangement in increasing order. However, both the allocations are equally efficient.

4) When the reliabilities of all the stages in a line are identical, provision of slightly higher buffers in the middle of the line leads to some what better performance.

5) When the reliabilities of the stages are unequal, following allocations are recommended.

Reliabilities	Buffer allocation
Increasing order	Inverse bowl, skewed to right
Decreasing order	Inverse bowl, skewed to left
Bowl formation	Inverse bowl
Inverse bowl formation	Inverse bowl or equal

CHAPTER - 8

FLOW LINES WITH RANDOM OPERATION TIMES AND UNRELIABLE STAGES

8.1 INTRODUCTION

It has been discussed in Chapters 4 and 6 that the inefficiency of a flow line system is mainly on account of delay in production, which in turn is caused due to variability in production rates of the stages. Out of numerous external and internal factors, which affect the production rate of a stage, two factors which contribute to the inefficiency, in a major way, have been considered independently in Chapters 4 to 7.

In Chapters 4 and 5, the variability in operation times of the stages has only been considered, while the stages were assumed to be 100 percent reliable. In Chapters 6 and 7, the operation times of the stages were assumed to be fixed and only the breakdowns of the stages were accounted for. In practical situations, however, both the causes of variability would be present simultaneously. This aspect of flow line systems has not been investigated in detail.

It has been indicated in Chapter 2, that only two research papers [26,52] have been reported in literature, which consider the operation times to be random as well as the stages to be unreliable. For such a situation, development of the analytical model is restricted to two stage systems only. An approximate method for the determination of the average cycle time has been proposed by Buzacott [26]. In this chapter, an indepth study of Buzacott's approximate model has been made, using simulation methodology, so as to check its validity and applicability to the system considered.

8.2 SYSTEM MODELLING AND ASSUMPTIONS

The system chosen for this study incorporates the design features of the systems modelled in Chapters 4 and 6. The processing times of the stages have been assumed to be random and each stage is liable to fail. The failure and repair times of the stages has also been taken to be random. The line has been assumed to be balanced with

$$\mu_i = \mu \;\forall\; i$$

$$MFT_i = MFT \;\forall\; i$$

$$MRT_i = MRT \;\forall\; i$$

$$S_j = S \;\forall\; j$$

$$i = 1 \rightarrow N \quad \text{and} \quad J = 2 \rightarrow N$$

The failure and repair times have been assumed to be exponentially distributed, while exponential as well as normal distributions have been employed to define the processing times.

8.3 SYSTEM SIMULATION

The simulation model employed in this chapter, is a combination of the simulation models employed in Chapters 4 and 6. In this case, the system has been simulated in two blocks, one for line without buffers and the second for line with finite buffers.

8.3.1 Line Without Buffers

For this simulation study variable increment time flow mechanism has been employed (Fig. 8.1). The system starts with all the stages in working condition and in idle state. To begin with failure times of all the stages, FT(I), are generated. For each work unit the processing times at all the stages (PT(I)) are generated, and the progress of the work unit through the line is followed. Starting with the first stage, the state of each stage is checked and the service beginning times SB(I), service ending times (SE(I)), and failure times are updated. As the work unit leaves the Nth stage, a counter, employed to register the number of units produced, is advanced by unity. As soon as the failure time of a stage reduces to zero, i.e., a stage breaksdown, its repair time is generated. This corresponds to forced down time of the line. After updating the state of the failed stage, its next failure time is generated.

To eliminate the initial bias, sufficiently long warm up period is allowed, after which the system is run for the estimated length. The mean efficiency and the standard error of the mean efficiency are then determined. If the error is within the accepted limits, the simulation experiment is

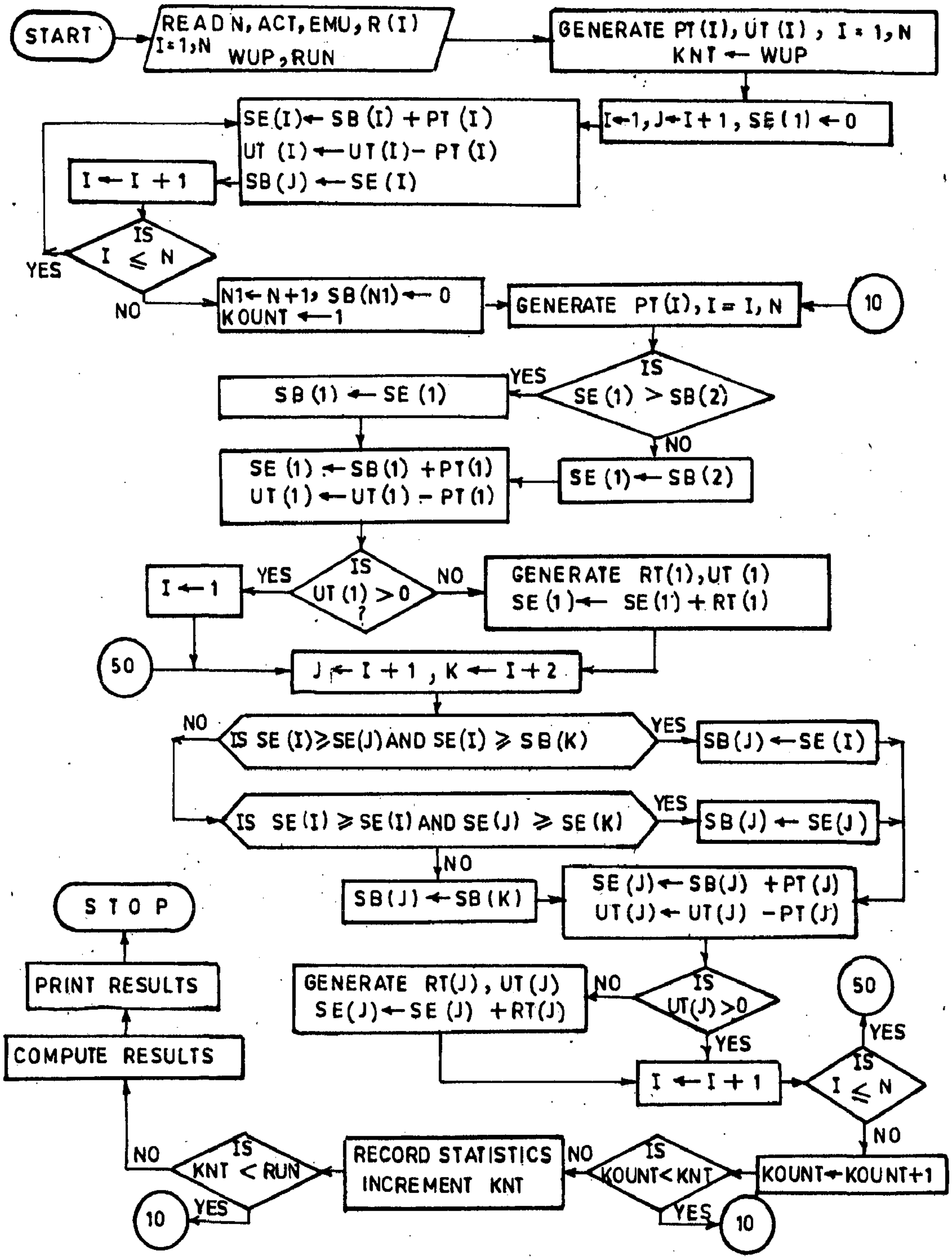


FIG. 8.1 FLOW CHART FOR SIMULATION OF FLOW LINE WITHOUT INPROCESS BUFFERS

terminated, otherwise the LOR is incremented by 1000 time units.

8.3.2. Line with Inprocess Buffers

In order to study the performance of a flow line, having variable operation times with stages liable to breakdowns, and provided with intermediate buffers, simulation model shown in Fig. 8.2 was employed. In this case, to start with, all the stages were assumed to be in working condition with inprocess buffers half full. The times to failure and the processing times of the stages were generated. The completion of operation on a work unit by a stage marks an event. At the occurrence of earliest event, the state of the affected stage (j) is checked. Its time to failure, (FT_j) input and output buffers, (IS_j and IS_{j+1}) and the processing times of all the stages are updated. The processing time of the stage j is generated next and the line scanned to identify the next earliest event. As soon as a stage breaksdown, its repair time is generated and is added to the processing time of the work unit being processed at that instant. The output from the system is measured by the number of finished work units added to the store at the end of the line. A clock is employed to register the elapsed time. After allowing the system to warm up for 5000 time units, simulation experiments were run for the estimated LOR as discussed in Chapter 3.

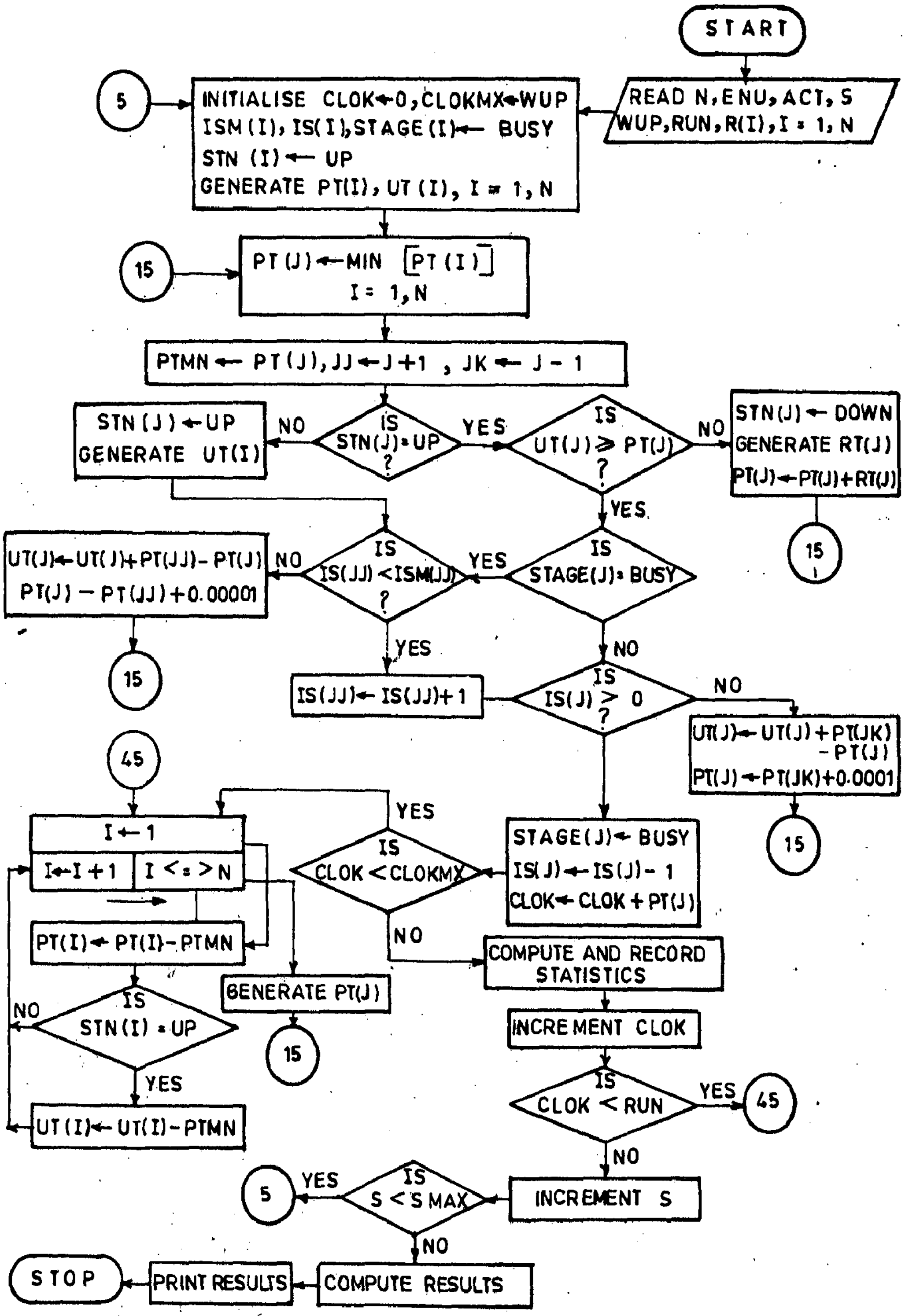


FIG. 8.2 FLOW CHART FOR SIMULATION OF FLOW LINE HAVING FINITE INPROCESS BUFFERS

8.4 EFFICIENCY OF THE LINE

The exact analytical model for the flow lines, where both the causes of production rate variability, i.e., variability in operation times as well as the breakdown of stages, are present, is difficult to formulate. Such models are limited to two stage systems only. Simulation on the other hand consumes large computation time and may not be economical under certain practical situations. This led Buzacott [26] to develop an approximate method for computing the efficiency of such systems. His method requires the determination of the effect of two causes of inefficiency on the system performance separately, and then the two effects are combined to give equation (8.1).

$$\bar{F} = \frac{1}{\rho} + (E - \frac{1}{\rho}) + (D - \frac{1}{\rho}) \qquad \ldots (8.1)$$

where $\bar{F}$ is the approximate mean cycle time for variable operation times and unreliable stages

E is the mean cycle time for variable operation times but no breakdowns

D is the mean cycle time for fixed operation times and unreliable stages

ρ is the production rate of the stages

Buzacott assumed the operation times and repair times to be exponential, whereas, failure times were geometrically distributed. He compared the approximate results with the exact analytical results available for two stage lines for $0 \leq S \leq \infty$, and found that the deviation between the results obtained from the use of analytical model and the equation (8.1) was within 1.0 percent.

In the present study, the failure and repair times of the stages have been assumed to be exponentially distributed, while the processing times of the stages has been taken to be exponential as well as normally distributed.

8.4.1 Buzacott's Approximate Model in Terms of Efficiency

Since the mean cycle time of the line is the reciprocal of its production rate, equation (8.1) can be written as

$$\frac{1}{\tau_{APP}} = \frac{1}{\tau_{VT}} + \frac{1}{\tau_{BD}} - \frac{1}{\rho} \qquad \ldots (8.2)$$

The efficiency of the line has been defined (Chapter 3) as the ratio of actual production rate to the ideal production rate, i.e.,

$$\eta = \frac{\tau}{\rho} = \tau \text{ (for } \rho = 1)$$

$$\therefore \quad \frac{1}{\eta_{APP}} = \frac{1}{\eta_{VT}} + \frac{1}{\eta_{BD}} - 1$$

or

$$\eta_{APP} = \left[\frac{1}{\eta_{VT}} + \frac{1}{\eta_{BD}} - 1\right]^{-1} \qquad \ldots (8.3)$$

8.5 SIMULATION RESULTS

Simulation results have been obtained for a large number of parameter combinations, over the range; $0.8 \leq R \leq 0.95$,

$0 \leq S \leq 150$ and $N = 3,5,10,15,20$. Results for a few line configurations are given in Fig.8.3. This figure illustrates the effect of inprocess buffers size on the system efficiency. In this case the operation times of the stages, as well as their failure and repair times have been considered to be exponentially distributed. Alongwith the simulation efficiency of the system (η_{SIM}), the efficiencies for the cases where only one of the two sources of production rate variability is present, has also been plotted. Efficiency of the line having exponential processing times and no breakdowns (η_{VT}) have been obtained from equation (4.10) (Chapter 4), whereas the values of efficiency for unreliable stages having fixed processing time (η_{BD}) have been computed from equation (6.6)(Chapter 6). It can be observed from Fig. 8.3, that when the number of stages in the lines is small (Figs. 8.3a and 8.3b) the effect of inprocess buffer on η_{BD} is quite small as compared to its effect on η_{VT}. That is the variability in processing times is the major cause of inefficiency in case of smaller lines. On the other hand, when N is large (Figs. 8.3c and 8.3d) both η_{VT} and η_{BD} increase as the sizes of the inprocess buffers are enlarged. In all the cases, as η_{VT} approaches 100 percent, the simulation efficiency of the line approaches the η_{BD}.

8.6 COMPARISON OF APPROXIMATE AND SIMULATION RESULTS

A comparison between the approximate values of the efficiency (η_{APP}) obtained from equations (8.3), and of the

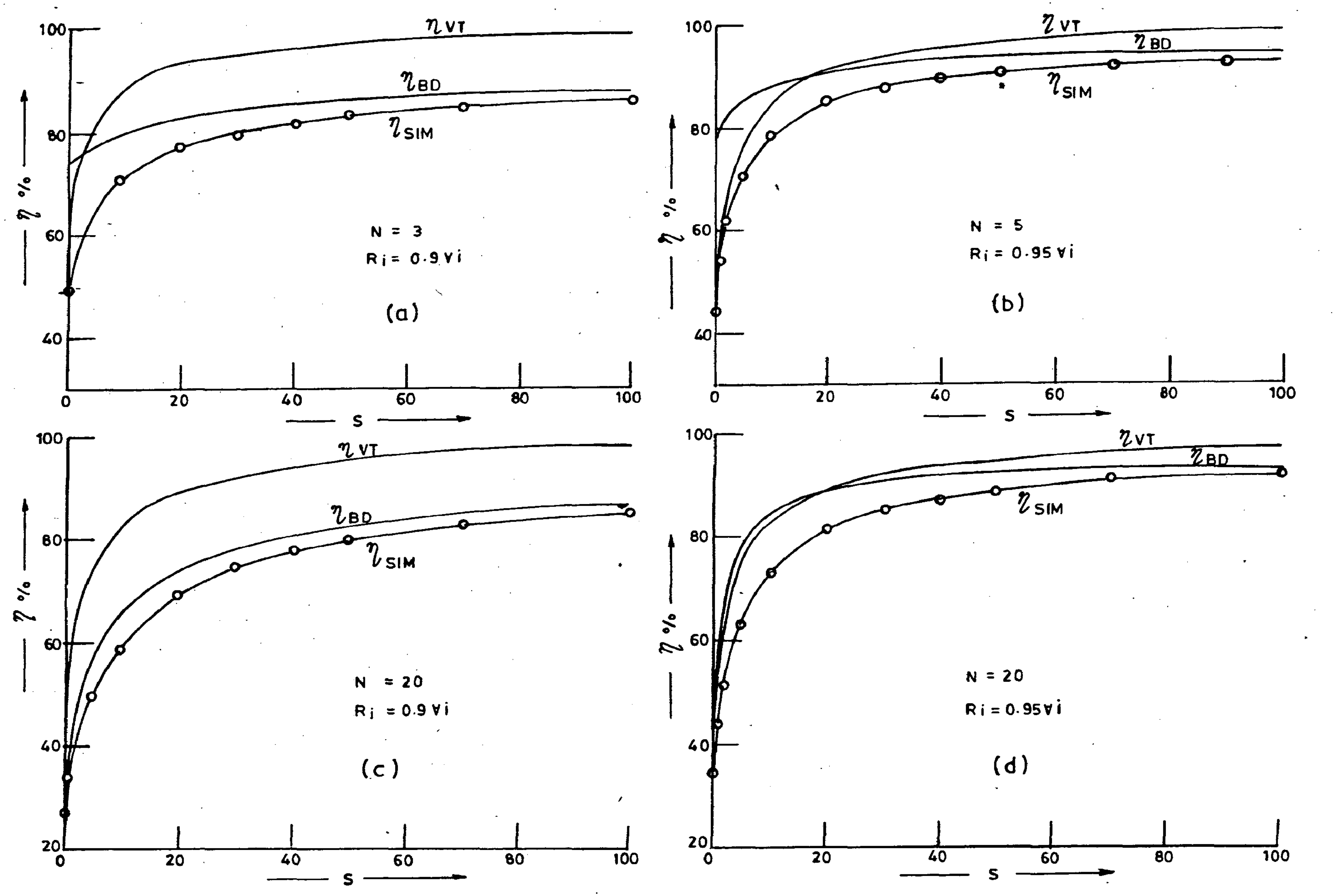


FIG. 8.3 FLOW LINE EFFICIENCY AS A FUNCTION OF INPROCESS BUFFER

simulation results (η_{SIM}) from the model in this Chapter, has been made in Fig. 8.4. It can be noticed that when a little or no inprocess buffer is provided, η_{APP} is less than η_{SIM}, and the difference between the two increases as the length of the line increased. However, when the size of the inprocess buffers is increased, the difference between η_{SIM} and η_{APP} diminishes. Simulated and approximate efficiencies are almost equal at all buffer levels for N = 2. Similarly, when infinite interstage buffers are provided, the difference η_{SIM} - η_{APP} becomes zero for all the levels of N.

The variation of ($\eta_{SIM} - \eta_{APP}$) with the size of the inprocess buffer, at smaller buffer levels, has further been illustrated in Fig. 8.5. In this case the percentage of approximation, defined by equation 8.4, has been plotted against the inprocess buffers size (S)

$$\delta = \frac{(\eta_{SIM} - \eta_{APP})}{\eta_{SIM}} \times 100 \qquad \ldots (8.4)$$

The error 'δ' in Fig. 8.5 can be seen to be maximum when inprocess buffer is zero, and is larger for longer lines. As the inprocess buffer size is increased, δ rapidly decreases and becomes very small as KS $\rightarrow$ 1.0. A comparison of various sets of curves plotted in Fig. 8.5, shows that the reliability of the stages also influences the error of approximation. When reliability of the stages is low, the error (δ) is comparatively larger, but as illustrated in Fig. 8.6, its overall effect

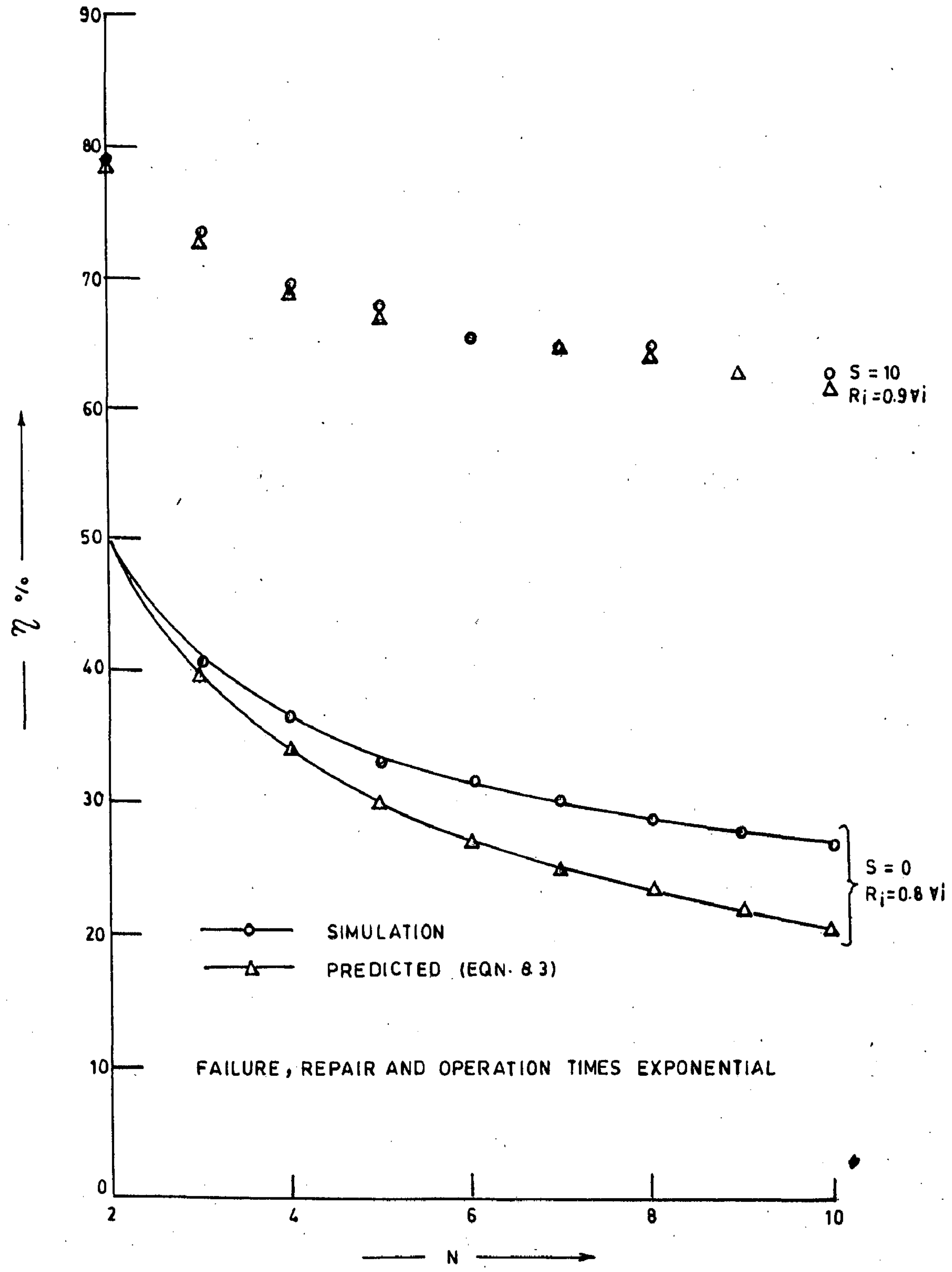
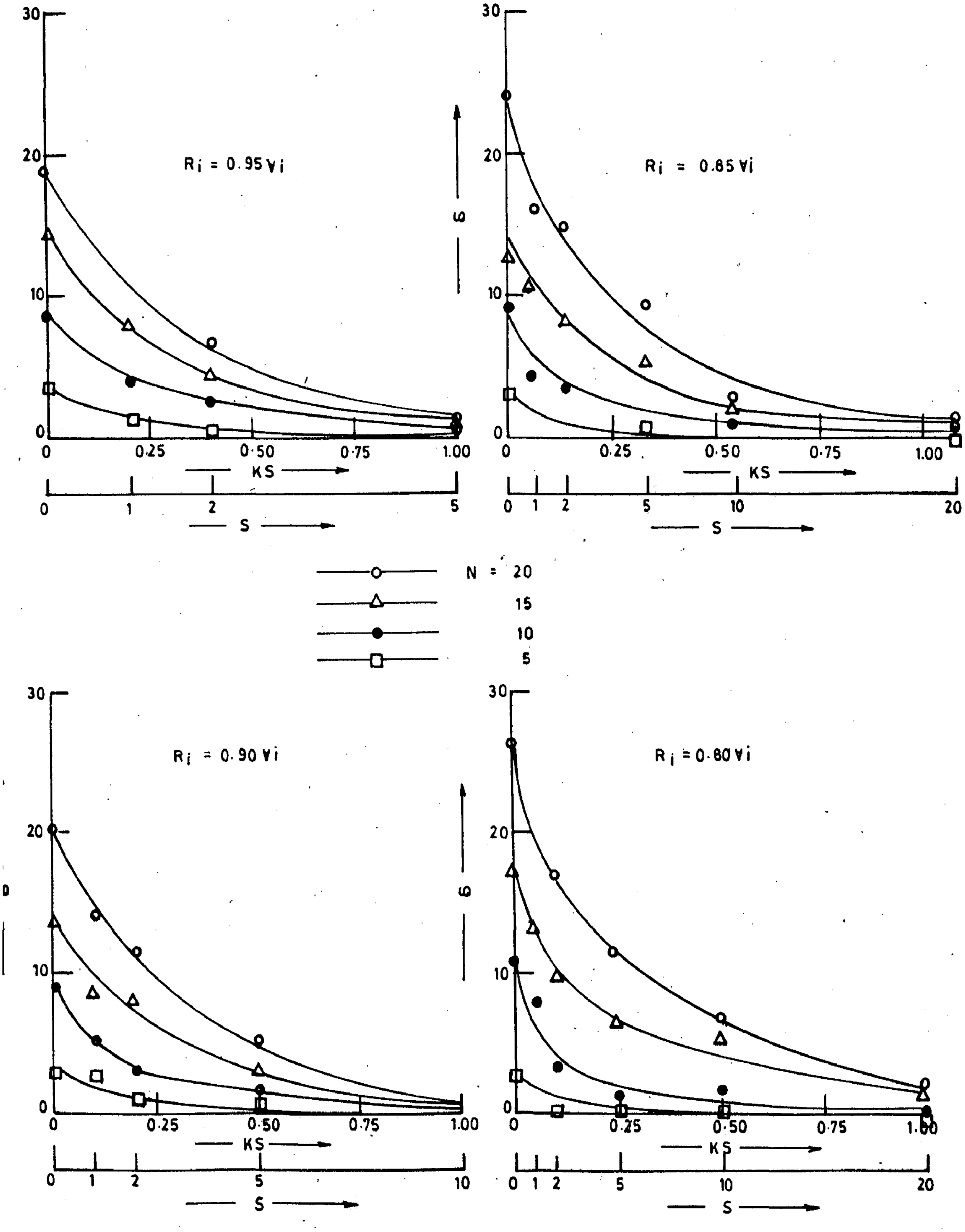


FIG. 8.4 COMPARISON OF SIMULATION AND PREDICTED RESULTS



.8.5 EFFECT OF INPROCESS BUFFER ON THE ERROR OF APPROXIMATION FOR VARIOUS COMBINATIONS OF N AND R is quite small. Fig. 8.6 shows the result for a 20 stage line. For smaller lines, the effect of reliability of stages on the error of approximation, would still be smaller and hence negligible.

The effect of variability in processing times of a 20 stage line having stages with 80 percent reliability is illustrated in Fig. 8.7. As the coefficient of variation of the processing times increases, the error of approximation increases, though slightly. Since in practical situations, the variability in processing times would rarely be as large as CV = 1.0, the approximate formula will give results still closer to the actual. Similarly a reduction in the variability in the failure and repair times would also help to reduce the error of approximation.

From the results discussed above, it can be concluded that the error of approximation is effected mainly by the number of stages in the line, and that too only when the in-process buffers are very small. However, it has been determined in Chapter 6, that the optimum size of the inprocess buffer is generally more than two times the down time production ($K S^{*} > 2$). Simulation and Approximate Results for KS > 1, for several combinations of N and R are given in Table 8.1.

A comparison of the results obtained by Simulation and approximate methods, employing Wilcoxon T-test has revealed that the approximate formula gives efficiency values lower than

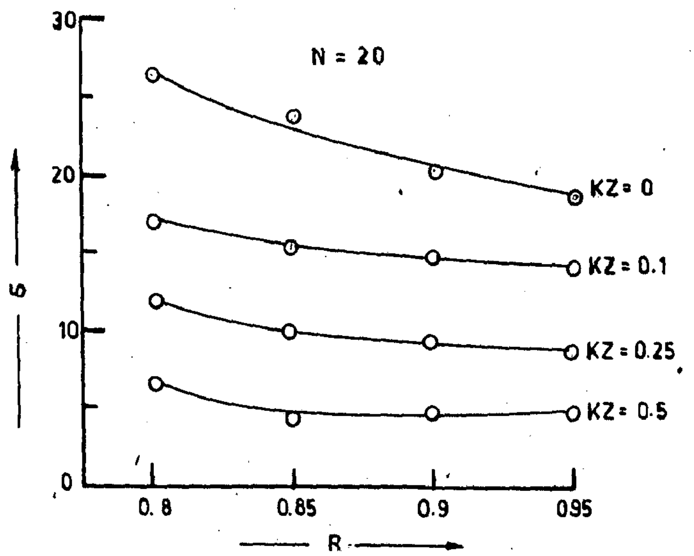


FIG.8.6 EFFECT OF RELIABILITY OF STAGES ON б

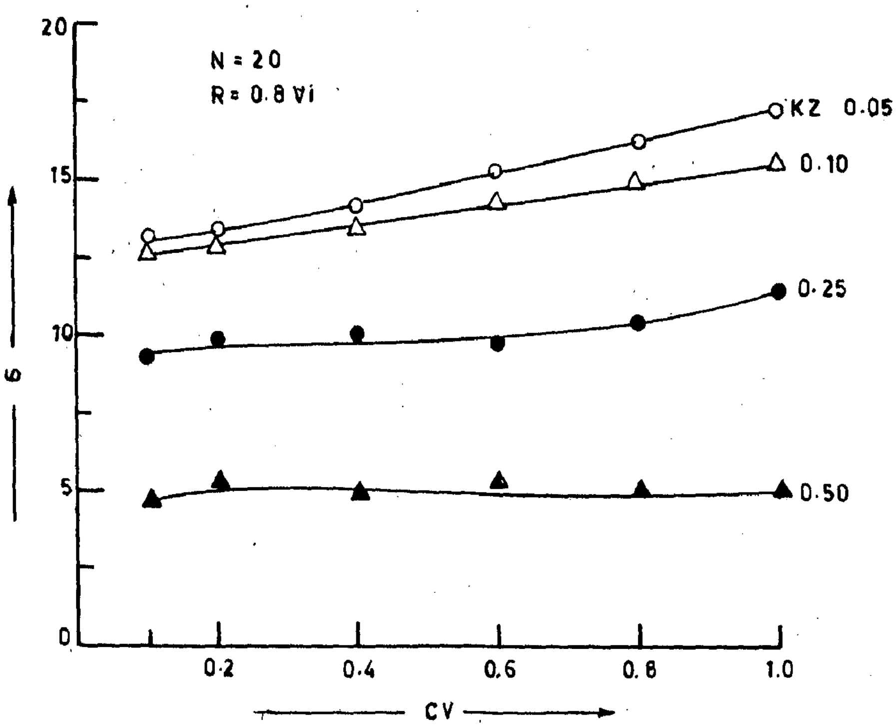


FIG.8.7 EFFECT OF OPERATION TIMES VARIABILITY ON б
(NORMALLY DISTRIBUTED OPERATION TIMES)

Table 8.1 : Comparison of Simulated and Predicted Efficiency (Equation 8.3)

N	R = .80			R = .85			R = 0.90			R = 0.95		
	S	η_{SIM}	η_{APP}	S	η_{SIM}	η_{APP}	S	η_{SIM}	η_{APP}	S	η_{SIM}	η_{APP}
5	20	56.78	56.54	20	65.69	65.21	10	68.26	68.06	5	70.94	70.63
	40	64.00	63.71	40	72.54	72.06	20	75.56	74.89	10	78.43	78.32
	70	69.23	69.12	70	76.53	76.84	40	80.83	80.87	20	85.53	84.28
	100	72.14	71.71	100	79.35	78.91	70	84.74	84.81	50	90.15	89.85
10	20	50.67	50.82	20	60.38	60.45	10	62.83	62.63	5	66.21	65.77
	40	60.47	60.26	40	69.80	69.32	20	71.67	71.49	10	75.84	74.99
	70	67.90	67.13	70	75.77	75.36	40	78.22	79.03	20	82.92	82.14
	100	70.58	70.31	100	77.35	77.87	70	84.81	83.91	50	87.71	88.81
15	20	49.47	48.59	20	59.44	58.57	10	61.41	60.31	5	64.60	63.60
	40	58.27	58.99	40	69.07	68.30	20	71.13	70.12	10	74.71	73.56
	70	66.73	66.40	70	75.53	74.82	40	79.05	78.32	20	82.26	81.28
	100	70.26	69.81	100	78.14	77.50	70	83.93	83.56	50	89.35	88.39
20	20	48.36	47.39	20	58.59	57.36	10	59.41	59.00	5	63.23	62.28
	40	58.38	58.11	40	68.11	67.48	20	69.70	69.37	10	73.64	72.72
	70	66.52	66.04	70	73.57	74.53	40	78.43	77.91	20	81.65	80.74
	100	69.34	69.56	100	78.04	77.31	70	83.44	83.36	50	88.94	87.86
		$\bar{\bar{x}}_d$ = .2881			$\bar{\bar{x}}_d$ = .3769			$\bar{\bar{x}}_d$ = 0.3613			$\bar{\bar{x}}_d$ = 0.6794	

simulation under identical conditions. However, the difference between the two was generally within 1 percent, and the average difference was only 0.4264 percent. Thus in case of practical flow lines, the approximate formula can be employed to predict the system efficiency with reasonable degree of accuracy.

8.7 OPTIMISATION OF INPROCESS BUFFER SIZE

The object of computing the system efficiency is mainly to analyse the economics of the system and to optimize the size of the interstage buffers. It is well established that increase in inprocess buffers size leads to increased production from the system. This, however, results into increased costs associated with the buffer, and hence the need to optimize the buffer size. The various costs and gains, associated with the provision of buffers, have already been discussed in Chapters 4 and 6. In this chapter, the effect of the approximation in computing the system efficiency by equation (8.3) on the gain has been studied.

The gain from the system can be expressed as :

$$\text{GAIN} = \Delta\eta . P - C_v . S - C_f \qquad \ldots (8.5)$$

The first term on the R.H.S.i.e., the revenue from the system is a function of η_s and η_o. Since the difference between η_{SIM} and η_{APP} is appreciable at zero buffer level and negligible at higher buffer levels, $\Delta\eta$ would be considerably larger when the approximate formula is employed, as compared to the case when simulated efficiencies are used. Thus $(AR)_{APP}$ as well as $(GAIN)_{APP}$ would be larger than the actual (simulation) values. This is illustrated in Fig. 8.8 for 10 and 20 stage lines having the following assumed cost values :

C_v = 0.1/unit buffer/100 time units.

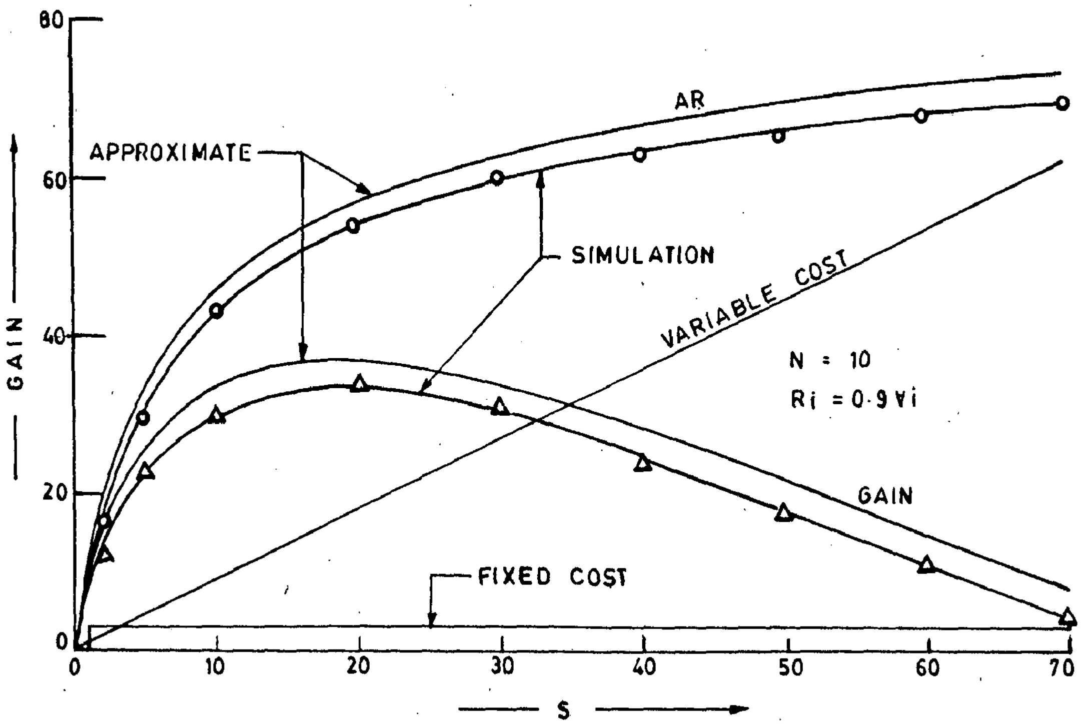
C_f = 0.3/storage point/100 time units.

P = 1.35/unit produced.

It can be observed from Fig. 8.8, that error of approximation has little or no effect on the optimum value of buffer size. The gain curves for 5 and 20 stage lines, obtained by employing simulation and approximate model of efficiency, have been compared in Fig. 8.9 for several values of stage reliabilities. This figure shows that there is little or no effect of approximating the values of η_s and η_0 on the optimum size of the interstage buffers.

8.7.1 Comparison Between KS^* and KS^*_{BD}

A comparison of KS^*_{APP} (hereafter taken as KS^*) values with the KS^*_{BD} values (where only the breakdowns of the stages are considered) has been made in Table 8.2. Values of KS^*_{BD} has been obtained by employing equation (6.13). For all the factor level combinations considered, KS^* has been found to be greater than KS^*_{BD}. However, at higher levels of N the difference between the two is quite small.



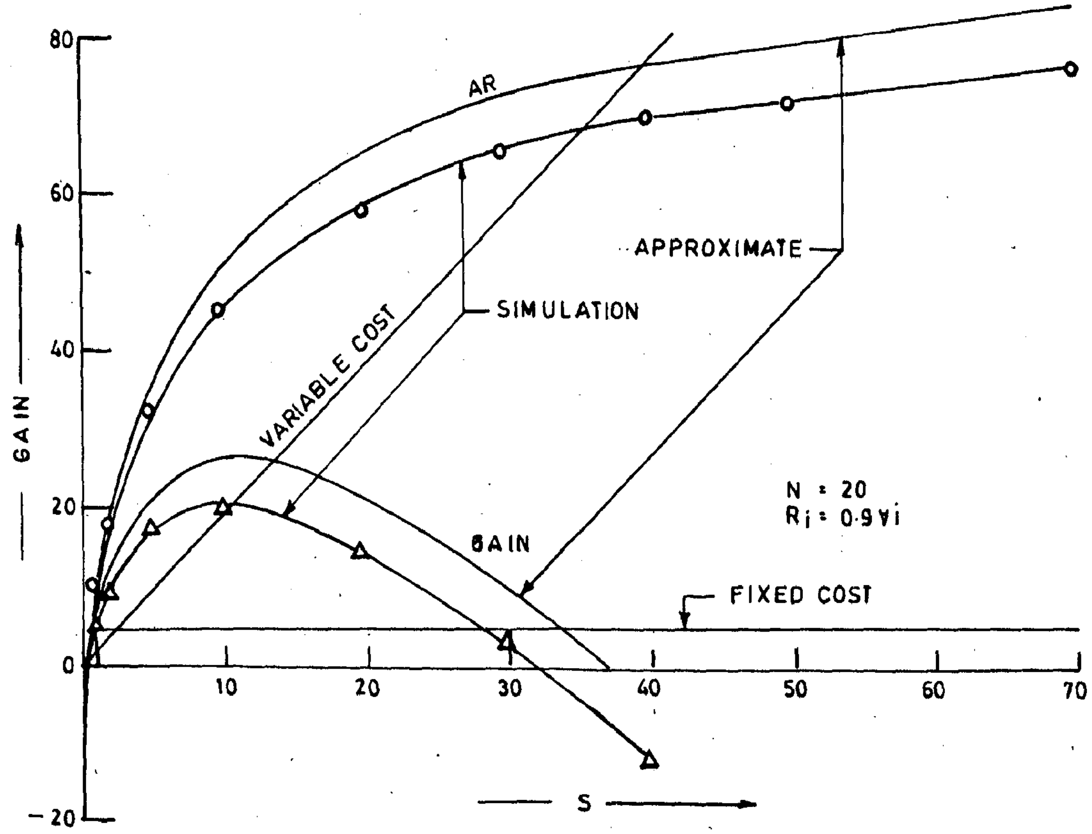


FIG. 8.8 COMPARISON OF COST CURVES OBTAINED FROM SIMULATION AND APPROXIMATE EFFICIENCY MODELS

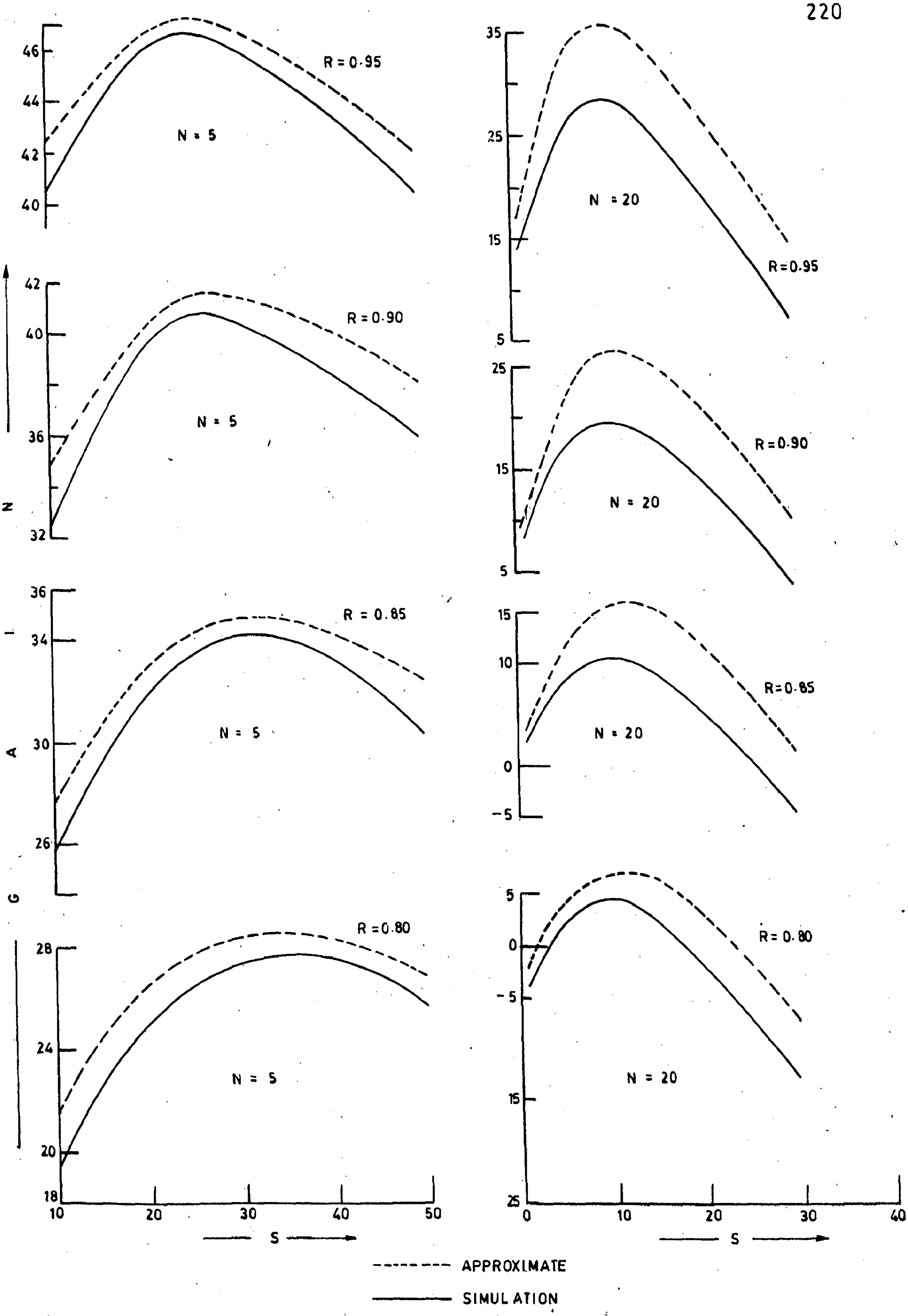


FIG.8.9 COMPARISON OF GAIN OBTAINED FROM SIMULATION AND APPROXIMATE MODELS

Table 8.2 : Optimum size of inprocess buffers

S^{*}_{BD} - Computed from empirical equation (6.13)

S^{*}_{APP} - Computed from approximate model (8.3)

($P=1.35$, $C_v = 0.1$, $C_f = 0.3$)

R	N = 5		N = 10		N = 15		N = 20	
	S^{*}_{BD}	S^{*}_{APP}	S^{*}_{BD}	S^{*}_{APP}	S^{*}_{BD}	S^{*}_{APP}	S^{*}_{BD}	S^{*}_{APP}
0.80	30	35	19	23	14	15	11	12
0.85	26	32	18	20	14	15	11	11
0.90	20	27	16	18	12	15	10	11
0.95	12	24	9	15	8	10	7	10

8.7.2 Search Procedure to Optimise KS

As discussed above, for a particular line configuration the value of KS^{*}_{BD} is less than KS^{*}, and hence, it can be used as a base to start the search for KS^{*}. A simple and efficient search procedure, flow charted in Fig. 8.10, can be employed for this purpose. The search starts by computing $KS = KS^{*}_{BD}$ from equation (6.13). By employing the empirical models (equations (4.10), (6.6) and (8.3)), η_{VT}, η_{BD} and η_{APP} are determined corresponding to buffer size KS and for zero buffers. Then corresponding to the given cost factors, the GAIN from the system is computed. The value of KS is incremented in steps and the gain computed so long as the gain increases. Then KS is decremented in smaller steps. The process is continued till the step in increment or decrement of KS reduces to one unit (S = 1). Since KS^{*}_{BD} is quite close to KS^{*}, in most of the cases, it helps to make the search very efficient.

8.8 CONCLUSIONS

The efficiency of a flow line system with random operation times, and unreliable stages, can be predicted with reasonable degree of accuracy by employing Buzacott's approximate formula (eqn.8.3). This equation has been found to be applicable over a wide range of system parameters, which would encompass the majority of practical flow line systems. When $KS \geq 1$, the error of approximation is less than 1 percent of the simulated efficiency and has little effect on the optimum size of the inprocess buffer.

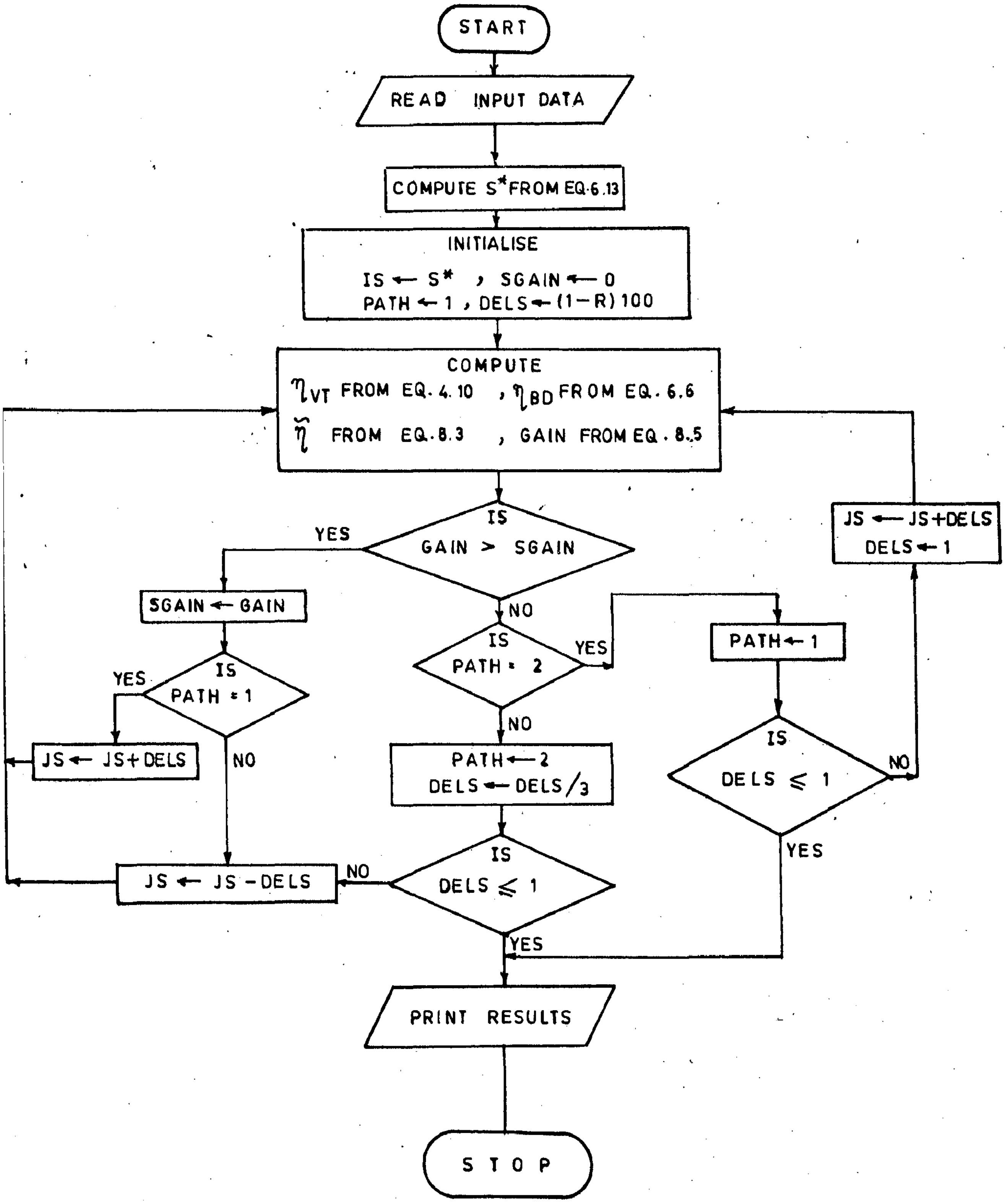


FIG. 8.10 INPROCESS BUFFER OPTIMISATION-SEARCH METHOD

CHAPTER -9

EFFECT OF REPAIR POLICY AND CREW SIZE ON LINE PERFORMANCE

9.1 INTRODUCTION

In the analysis of flow line production systems, having fixed processing times of the stages and when the stages are subject to random breakdowns, one of the following two assumptions is generally made;

i) Only one stage can be down at a time.

ii) Enough repairmen are available to repair the failed stages simultaneously.

Majority of the existing literature pertaining to the automatic production lines, relates to two or three stage systems only. In such systems, the above assumptions can easily be justified. But in case of flow lines comprising of a large number of stages with inprocess storages between them, the probability of more than one stage being down is large. The frequency of breakdowns of the line increases with the number of stages. The second assumption would imply that a very large work force and unlimited repair facility is available. This would lead to large idle times of the repairmen and under utilisation of the repair facilities. On the other hand, work force smaller than the required would mean idling of the failed stages and loss of production. Thus the choice of repair crew size would have a large bearing on the system economy. On the other hand, the size of inprocess buffer would have significant effect on the system performance. This was studied independently in Chapter 6. The size of the inprocess buffer is also expected to influence the number of repairmen to be employed. Hence, to achieve maximum gain from the system, it would be essential to optimise the inprocess buffer and the repair crew size.

With limited number of repairmen, during actual operation, a number of stages are likely to queue up for repairs. In such a situation, the repair policy employed would also influence the system performance. As discussed in Chapter 2, Section 2.6, only a little effort has been made towards the study of the effect of repair priority rules on the performance of flow line production systems. The available studies apply to two stage systems only.

In this chapter, a simulation model for evaluating the performance of large automatic production lines, having finite inprocess buffers, and limited number of repairmen, has been presented. The effect of the size of inprocess buffers and of the number of repairmen on the system efficiency has been studied. In addition, a search procedure has been developed to optimise the crew size for maximum gain attainable from the system. Simulation model has also been used to examine the performance of five different repair policies. The analysis has been supported by means of an illustrative example.

9.2 SYSTEM MODELLING

The system considered in this chapter is similar to the one modelled in Chapter 6, except that the number of repairmen employed in this case is limited, and hence, a failed stage may assume any one of the following states :

i) Repair in progress.

ii) Waiting for repairs : No repairman is available to undertake the repairs.

If at a particular instant of time, more than one stage is waiting for repairs, repairs are undertaken by following a predetermined policy. A repair once started is never suspended inbetween to take up some other repair.

All other assumptions and definitions used in this chapter are the same as in Chapter 6.

9.3 REPAIR POLICIES

The effect of the following five repair policies, which depend upon the order of failures, repair time durations, and the buffer occupancy has been examined with regard to the system efficiency.

RP1. First Come First Served (FCFS)

If more than one stage is waiting for repairs, the stage with the largest waiting time is given priority over the others.

RP2. Minor Repairs First

The repair time lesser than a fixed value (T_{mnr}) has been treated as minor. Out of the minor repairs, the failed stage requiring the smallest repair time is given priority. If no stage has minor repair time, FCFS rule is applicable.

RP3. Major Repairs First

The repair time greater than a fixed value (T_{mjr}) has been treated as the major repair. Out of the major repairs, the stage requiring maximum repair time is attended first. In case of no major repairs being present, FCFS rule is applicable.

RP4. Stage with Largest Empty Space in the Following Storage First

The states of buffers following the broken down stages are checked, and the stage having largest empty space in the following storage gets repair priority over the others.

RP 5. Stage with Largest Buffer in the Preceding Storage First

The buffers preceding the broken down stages are checked, and the one with highest buffer level gets priority over the other failed stages.

9.4 SIMULATION MODEL

The simulation model presented in Chapter 6, has been modified to include the effect of limited number of repairmen for the present studies. The flow chart of the modified simulation model is shown in Fig.9.1. The model is based on the fixed

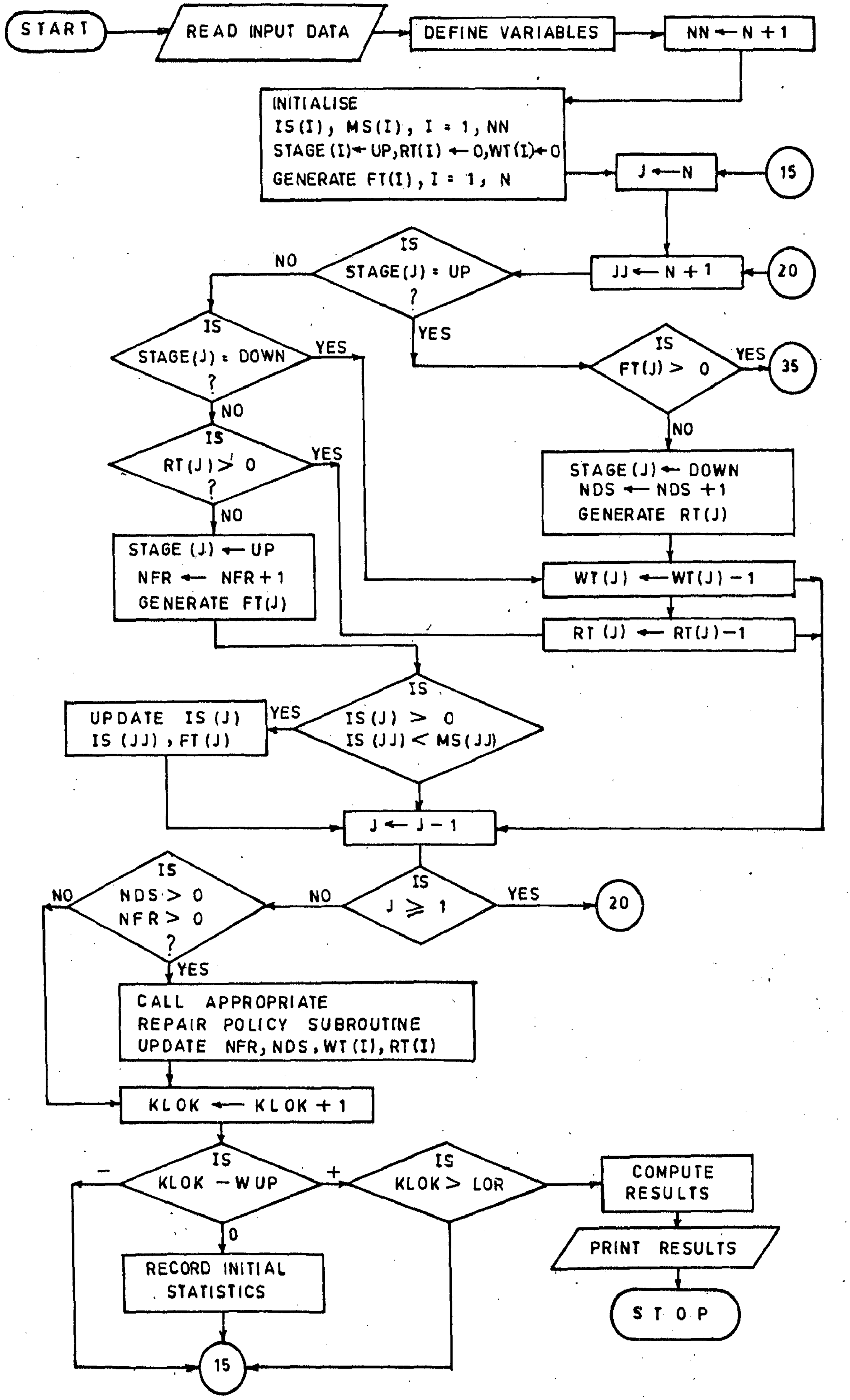


FIG.9.1 SIMULATION FLOW CHART OF FLOW LINE HAVING VARIABLE OPERATION TIMES AND UNRELIABLE STAGES

After every one time unit, the system is scanned to determine if any stage has broken down or the repair of a failed stage has been completed. The states of the inprocess storages, waiting times of the down stages, and times to completion of repairs being done, are updated. The appropriate repair policy is employed and the number of free repairmen and number of down stages waiting for repairs are updated, before the clock is advanced to the next step. The flow charts for the various repair policy subroutines are given in Figs.9.2-9.4. After allowing for the warming up period (WUP), the starting statistics of the system are recorded. The length of simulation run and the starting conditions were the same as in case of simulation model of Chapter 6. However, in case of repair policies, as the difference between their effectiveness was expected to be small, four replications of each run were obtained. Thus the efficiency obtained was within ± 0.25 percent of the mean at 95 percent confidence level.

9.5 RESULTS AND DISCUSSION

9.5.1 Efficiency With No Inprocess Buffers

In flow line systems where the stages are completely coupled to each other, i.e. with zero inprocess buffer, failure of one stage forces the entire line to stop. Since the failures have been assumed to be operation dependent only, no stage can fail during its forced down state. There is little probability

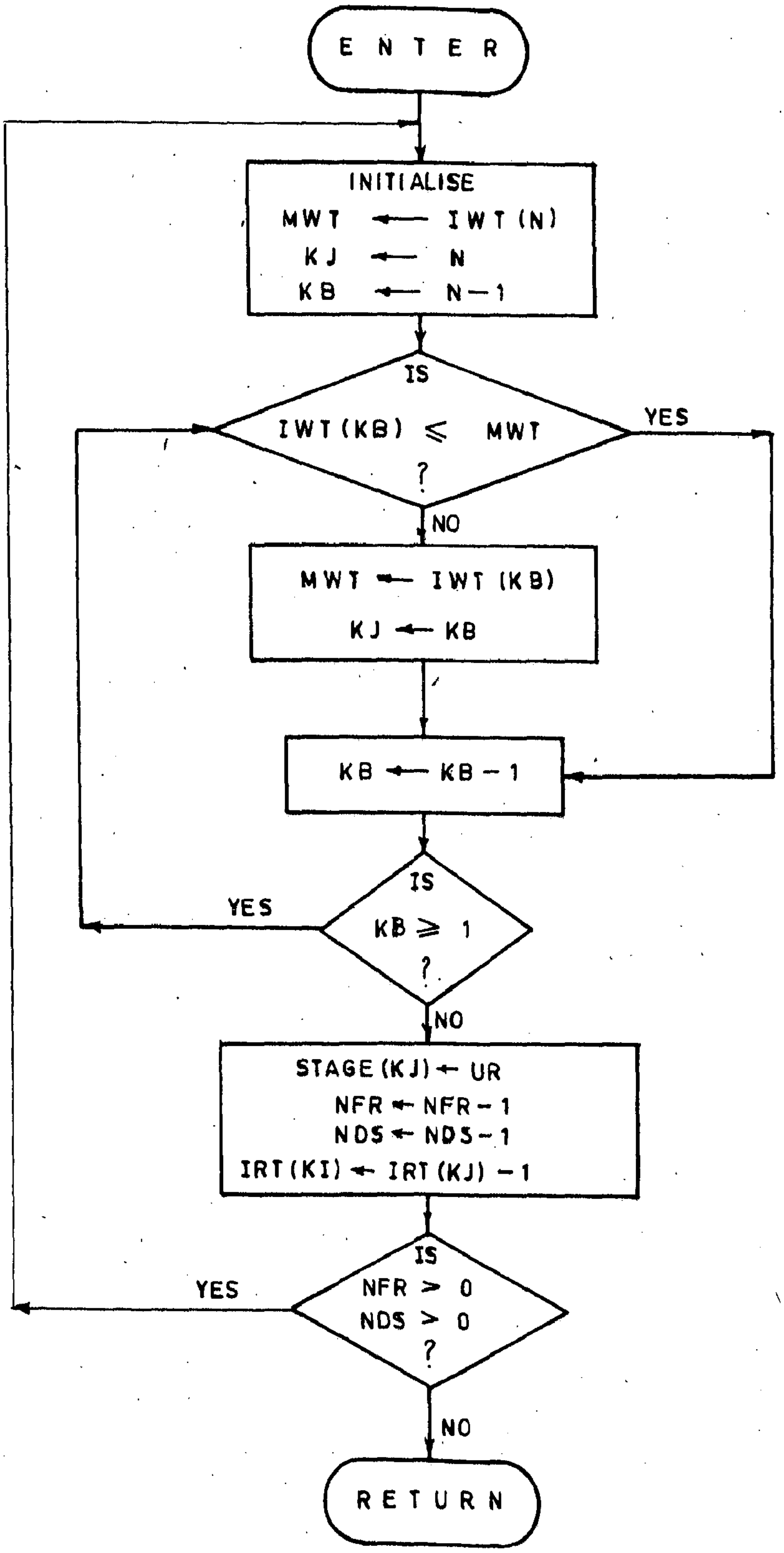


FIG. 9.2 FLOW CHART OF SUBROUTINE RP1

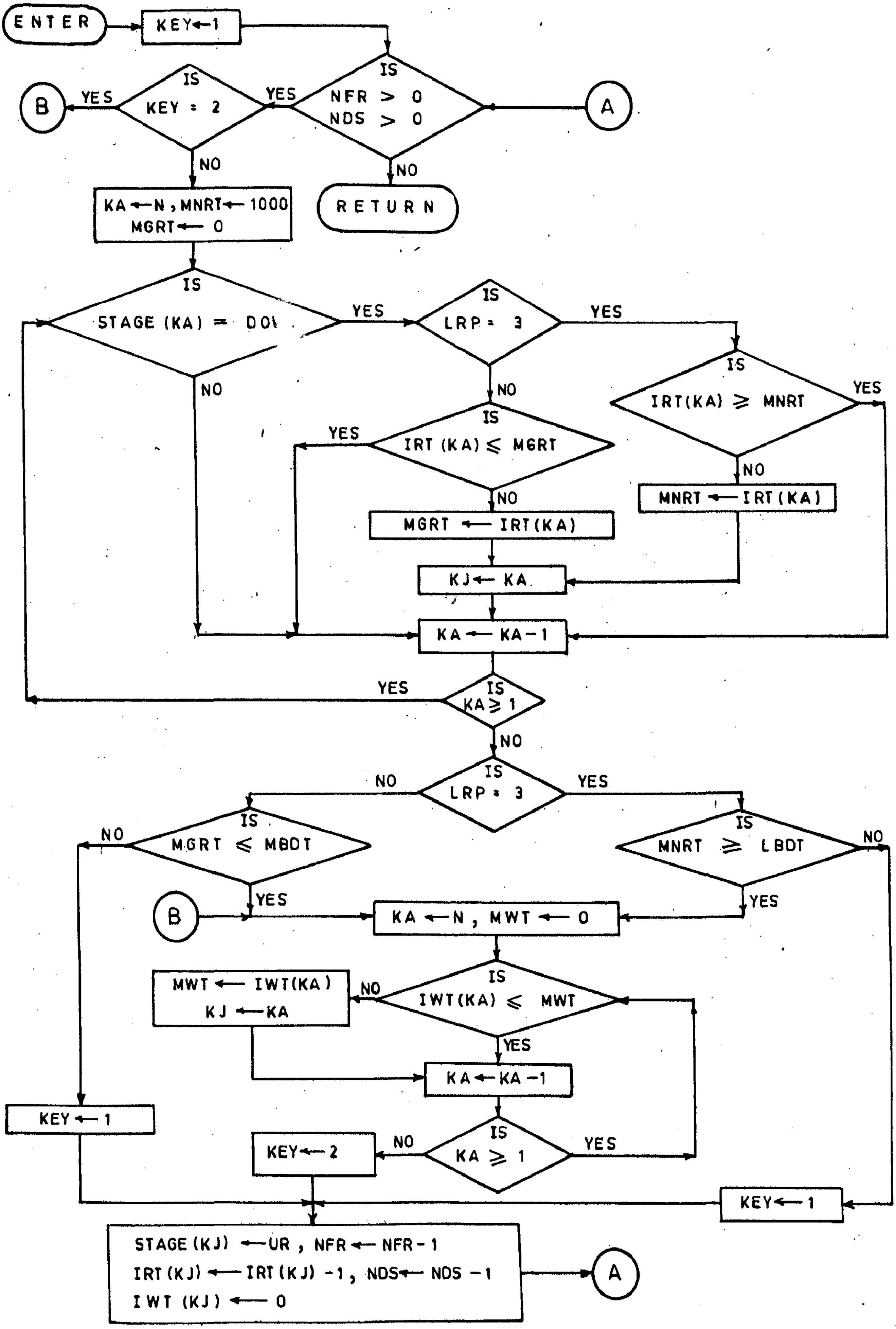


FIG.9.3 FLOW CHART OF SUBROUTINE RP 2-RP3

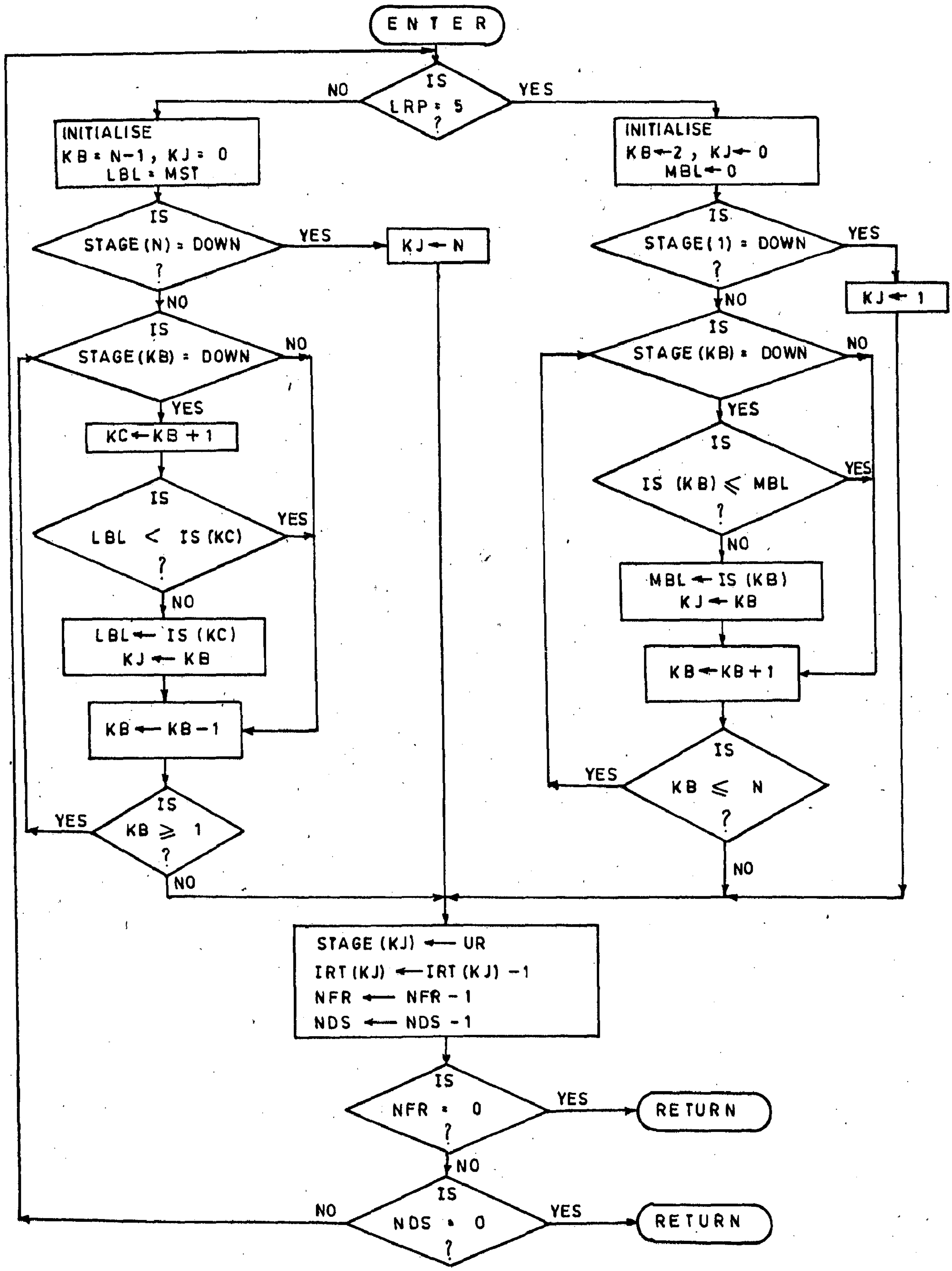


FIG. 9.4 FLOW CHART OF SUBROUTINE RP4-RP5

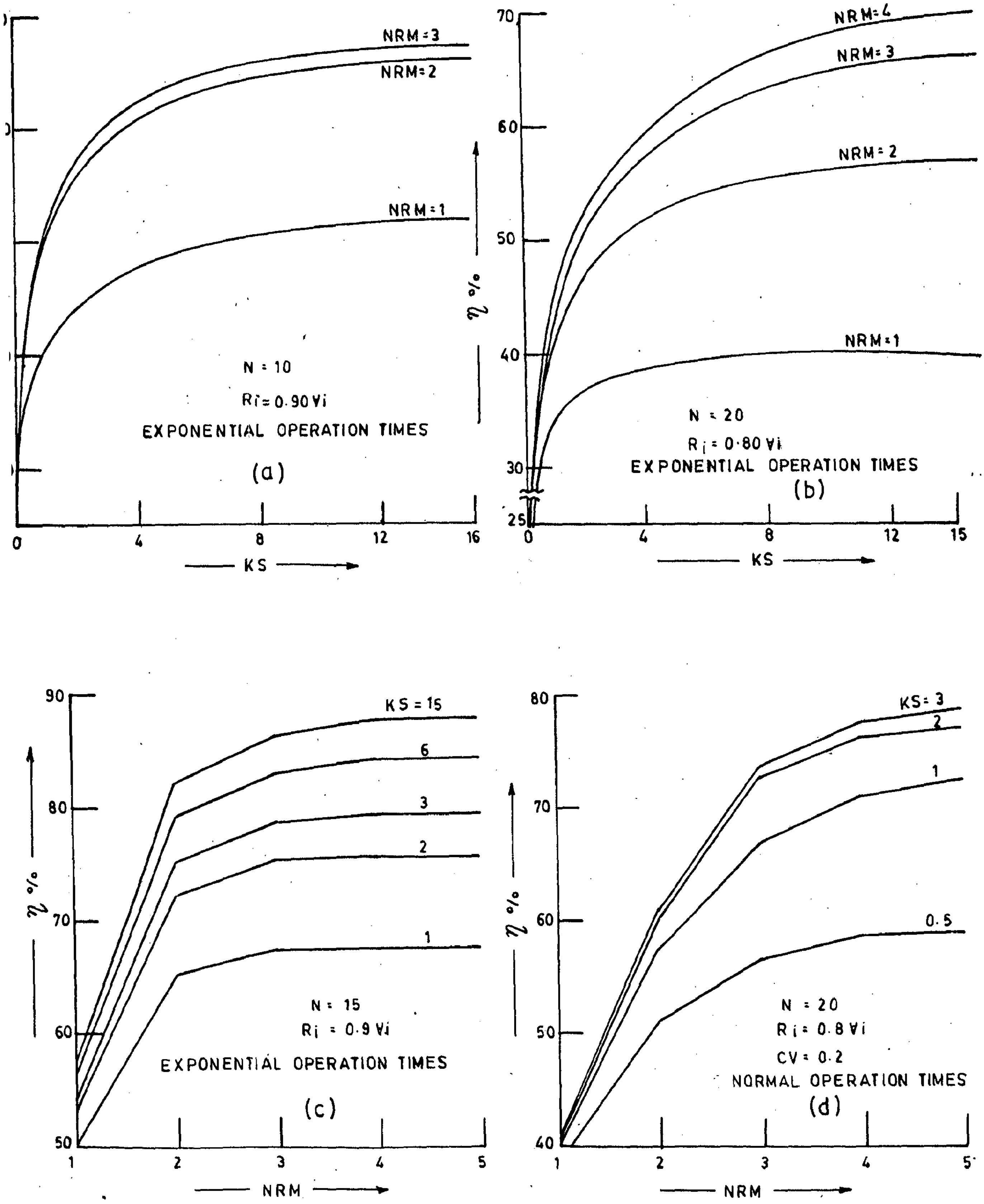
of two or more stages breaking down at the same instant, and hence in lines with zero inprocess buffer, more than one stage will not be down at any instant of time. Therefore, in such systems only one repairman would be required. The efficiency of the system can be determined from equation (6.2).

9.5.2 Effect of Inprocess Buffer and 'NRM' on Efficiency

Provision of inprocess buffers between the stages would enable them to operate somewhat independently and hence would reduce their forced down time and lead to an improvement in the production efficiency. The other factor which contributes to the line inefficiency, is the waiting time of the failed stages, which can be reduced by increasing the number of repairmen.

Some results pertaining to the effect of inprocess buffer on line efficiency are given in Fig. 9.5(a and b). It can be observed that while operating with small inprocess buffers, even a slight increase in the buffer size has a drastic effect on the efficiency of the line. However, with larger buffers, a small increase in their size has little or no effect on the η. Figs. 9.5 (a and b) also illustrate that increasing the buffer size is more beneficial when NRM is high. In both the 10 and 20 stage lines, the increase in η with an increase in KS for NRM $\geq$ 2 is much larger as compared to the situation when NRM = 1.

Figs. 9.5 (c and d) illustrate the effect of NRM on η for several values of the inprocess buffer size. It can be observed that a major gain in η is obtained when NRM is increased



.9.5 EFFECT OF INPROCESS BUFFER AND NRM ON SYSTEM η.

within the lower level range and when the inprocess buffers are of large size. In Fig.9.5(c), where the operation times are exponential, and the reliability of the stages high, improvement in η with NRM at KS $\leq$ 2 is negligible for NRM > 3. For KS = 15, significantly large improvement in η can be obtained by increasing NRM upto 4. When the length of the line is large, and reliability of the stages is low, comparatively larger number of repairmen would be required. The effect of NRM on the η of a 20 stage line having R_i = 0.8 $\forall i$ for several values of KS is illustrated in Fig.9.5 (d).

The influence of the variability of the failure and repair times of the stages on the effectiveness of the number of repairmen is illustrated in Fig.9.6, where results for normally distributed failure and repair times have been plotted. It can be seen that improvement in η, for each additional repairman is more at lower levels of NRM. For all the buffer levels, increase in η by increasing the NRM is more when the variability of up and down times is low (CV = 0.2). In the line considered, the difference is maximum when NRM = 1 or 2, and diminishes as NRM is increased further.

The results given in Figs. 9.5 and 9.6 are based on the FCFS repair policy.

9.6 OPTIMISATION OF 'S' AND 'NRM'

As discussed above, the efficiency of the system can be increased by increasing the size of the inprocess buffers as well as the number of repairmen. However, they have high costs associated with them, and the maximum GAIN from the system would correspond only to an optimum combination of S and NRM. The various costs involved in the provision of inprocess buffer are : the storage space cost, inprocess inventory carrying cost and buffer handling costs, and these have already been discussed in Chapters 4 and 6. In the present case, in addition to the above, the cost factor which must be accounted for is the repair cost (RC). This would include the wages of the repairmen and the cost due to repair facilities.

The repair cost can be assumed to be a linear function of NRM and can be expressed as :

$$RC = C_R . NRM \qquad .. \quad (9.1)$$

If profit on each unit produced is P, then revenue from the system (REV), would be given by equation (9.2),

$$REV = \eta . P/\text{time unit} \qquad .. \quad (9.2)$$

and the net gain from the system would be

$$GAIN = REV - (SC+IC+HC+RC)$$

or

$$GAIN = \eta P - C_f - C_v . S - C_R . NRM \qquad .. \quad (9.3)$$

Since the system efficiency is a function of the size of the inprocess buffers, and/or the number of repairmen, the GAIN would also be influenced by them. The effect of S and NRM on the GAIN can best be illustrated by the following example.

9.6.1 Illustrative Example

The following assumed cost data, pertains to a 20 stage automatic production line, having all stages identically reliable ($R_i = 0.8 \ \forall \ i$).

C_f = 38/100 time units

C_v = 0.57/ unit buffer/100 time units.

C_R = 5.0/repairman/ 100 time units

P = 2.0 / unit produced

The failure and repair times are distributed normally with CV = 0.5. The repairs are attended according to the FCFS rule.

Since the ACT = MFT+MRT = 100, the MRT = 20 and thus S = 20 KS. Then gain from the system would be

$$\text{GAIN} = 2.0\ \eta - 38.0 - 11.4\ \text{KS} - 5.0\ \text{NRM}$$

The values of η corresponding to several values of KS and NRM have been obtained from the simulation model. The variation in GAIN with respect to KS and NRM, for the above cost data, is given in Figs. 9.7 and 9.8 respectively. Fig. 9.7 shows that at lower values of NRM (= 1 and 2) KS = 1 leads to larger value of the gain and for NRM $\geq$ 3, KS = 2 gives the largest value of gain. Similarly, from Fig.9.8 it can be observed that the best value of gain is obtained corresponding to NRM = 4 and KS = 2.

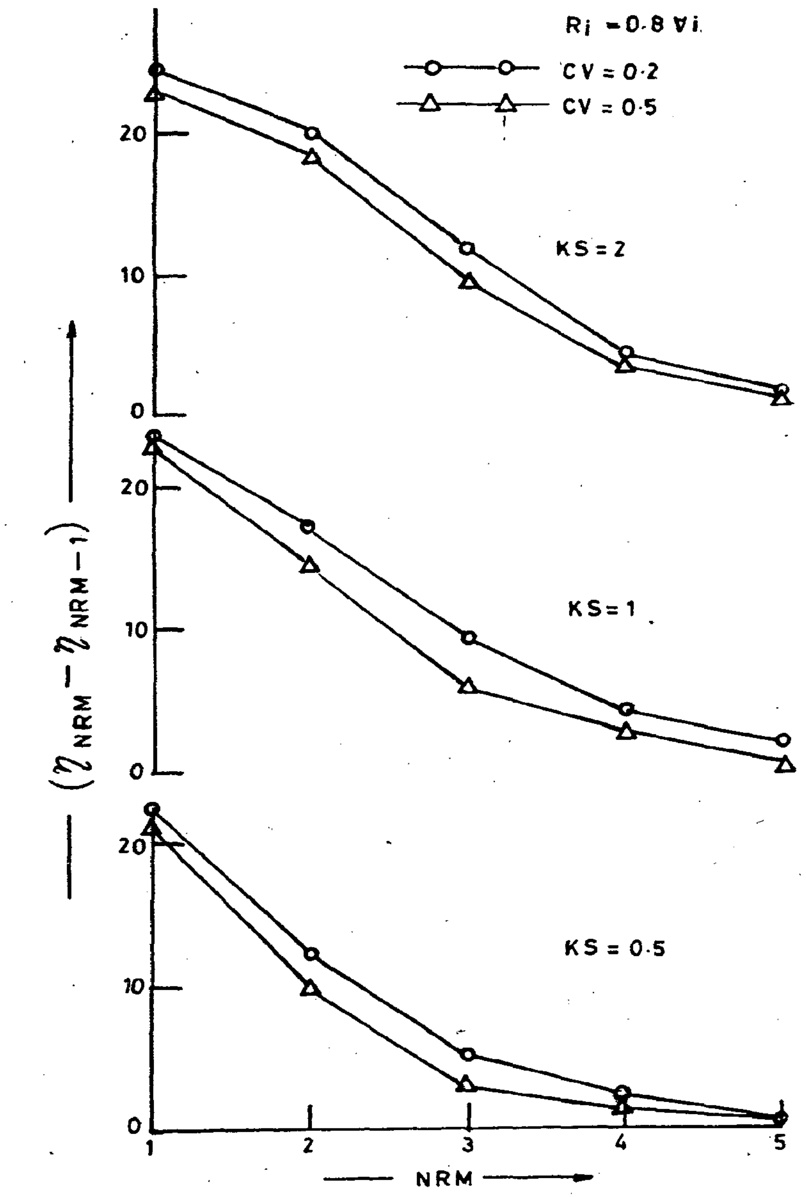


FIG.9.6 INFLUENCE OF CV OF FAILURE AND REPAIR TIMES ON THE EFFECTIVENESS OF NRM

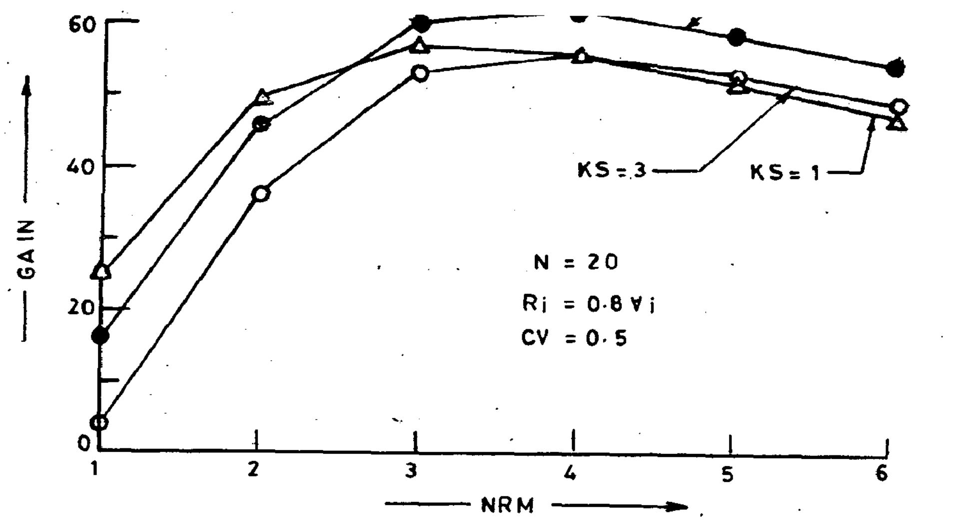


FIG.9.7 INFLUENCE OF NRM ON THE GAIN FROM THE SYSTEM

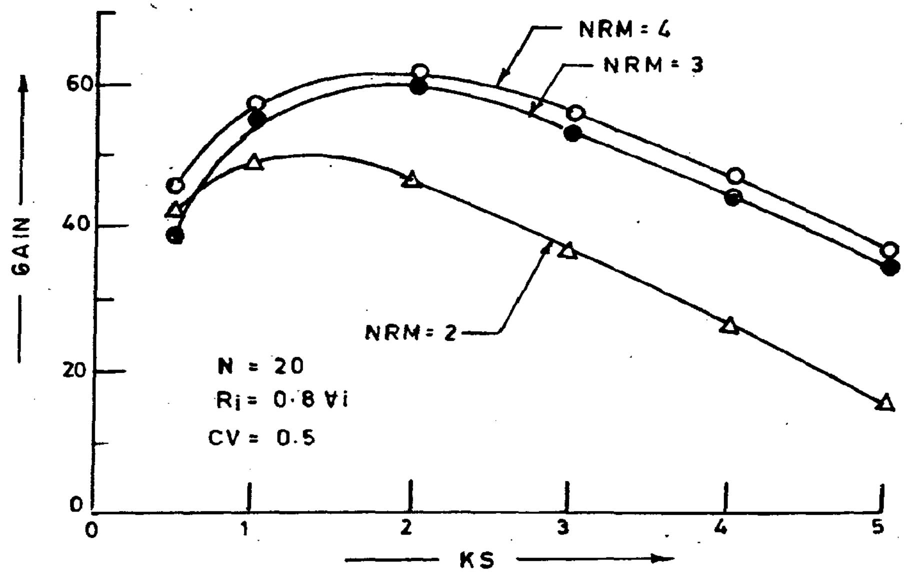


FIG.9.8 INFLUENCE OF INPROCESS BUFFER ON THE GAIN FROM THE SYSTEM

9.6.2 Search Procedure

A three dimensional plot of the GAIN, results in the generation of a hump type response surface (Fig.9.9), which has a single maxima. To reach the highest point in Fig.9.9, a search procedure has been developed. The flow chart for the same is given in Fig.9.10. The search for KS^* and NRM^* starts by initialising NRM = 1 and KS = 1(or some other values smaller than KS^* and NRM^*). The simulation program is run to determine η and the corresponding gain. The value of NRM is incremental by 1. Efficiency and gain corresponding to new NRM are computed. The increase in NRM is continued so long as the gain increases. Then KS is incremented by one unit and the η and gain are computed. If the gain increases, the value of NRM is incremented or decremented to find the one corresponding to the highest gain on the new KS line. The iterative procedure continues so long as the gain increases, say upto NRM' and KS'. Then KS' is incremented/decremented in smaller steps (DELKS) to check if the gain further increases. Once the gain is saturated KS^* and NRM^* are obtained, and the search is terminated. Since the η for each combination of KS and NRM is determined by simulation KS should be incremented in large steps in the beginning. The computer time consumed will depend upon the starting conditions and the selection of incremental values of KS. It took a little less than 3 min CPU time for the example considered above, where the maximum gain corresponded to $NRM^* = 4$ and $KS^* = 1.75$ or $S^* = 35$.

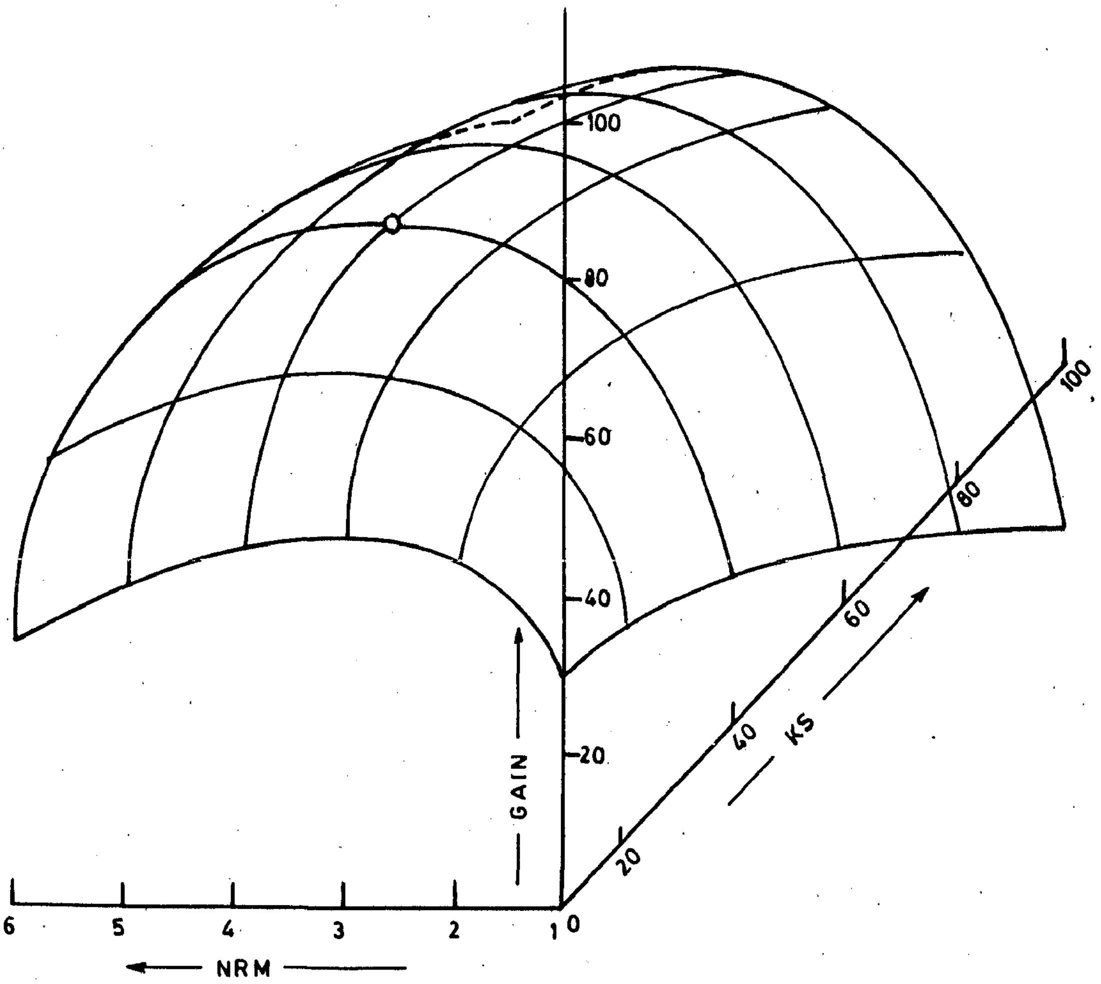


FIG. 9.9 RELATIONSHIP BETWEEN KS, NRM AND GAIN

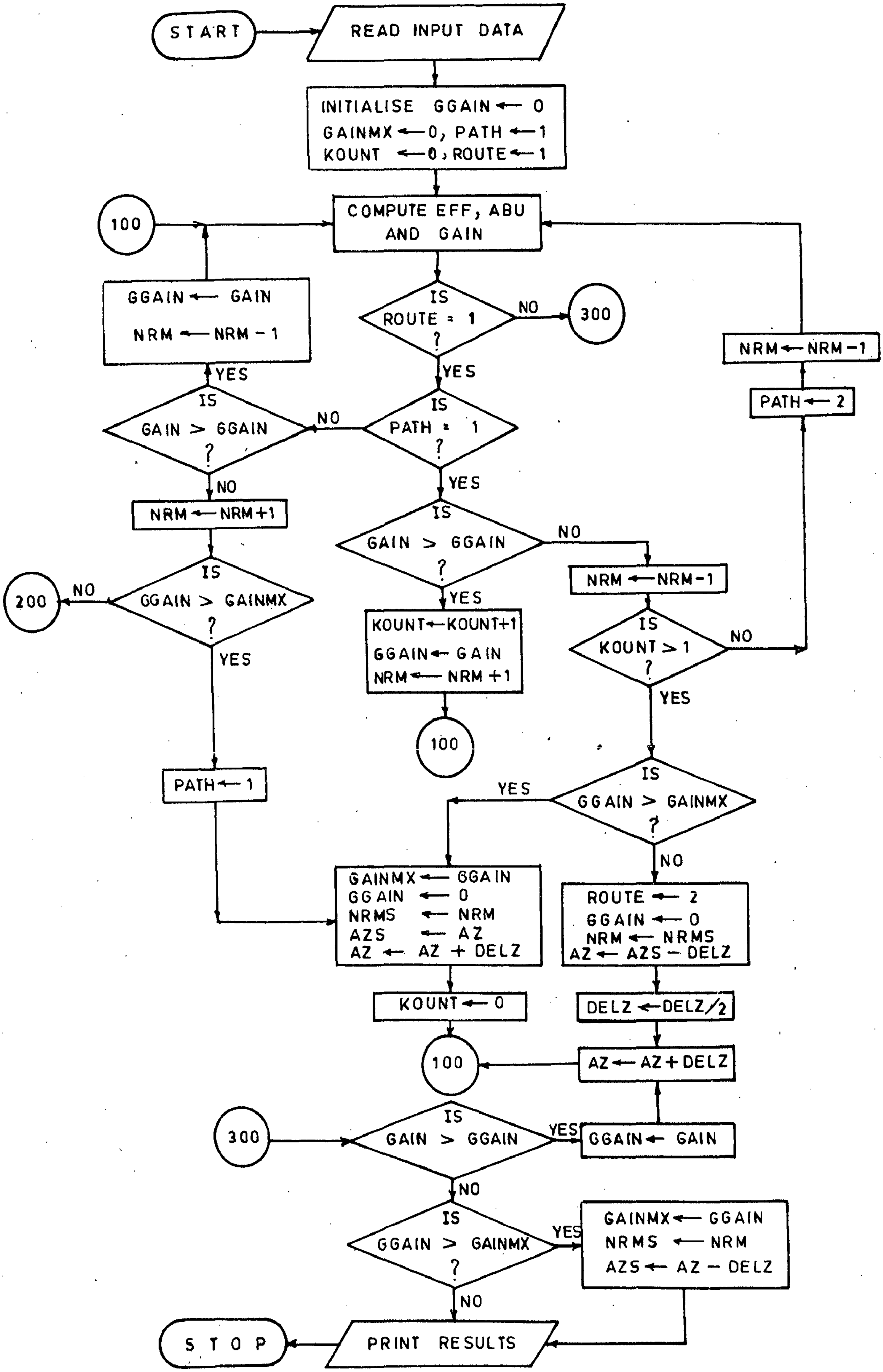


FIG. 9.10 FLOW CHART FOR THE OPTIMISATION SEARCH PROCEDURE

9.7. SENSITIVITY ANALYSIS

In practice it may not always be possible to determine the various cost factors very accurately, and they may vary from time to time, and hence it is important to know the effect of variations in cost factors on the values of KS^* and NRM^*. A sensitivity analysis of the system has revealed that NRM^* remain practically unaffected even under considerably large variation in costs. In the 20-stage line analysed above, NRM^* was found to lie between 3 and 4 over a wide range of C_R, C_V and P, except at very low values of C_R (Fig.9.11). The value of KS^* was found to remain unaffected by variations in C_R (Fig.9.11a), while it gradually decreased with increase in C_V (Fig.9.11b), and increased with increase in P (Fig.9.11c).

The effect of deviations from the optimum values of NRM and KS on the gain from the system is illustrated in Fig.9.12, where the ratio $\phi = \frac{GAIN}{GAIN^*}$, has been plotted as a function of the ratios $\xi = NRM/NRM^*$ and $\beta = KS/KS^*$. The β and ξ curves tend to be flat in the vicinity of their optimum values, and hence gain is not very sensitive to small deviations from KS^* and NRM^*. Also ϕ is less sensitive to variations in KS as compared to variations in NRM, especially when the variability in the failure and repair times is low (CV = 0.2). From Fig.9.12, it can be observed that in the vicinity of $\xi = \beta = 1$, the ϕ is quite insensitive to a little over or under estimation of KS and NRM. On the other hand, when deviations from the optimum are large, under estimation of KS and NRM drastically cuts the gain, while their over-estimation is comparatively less harmful.

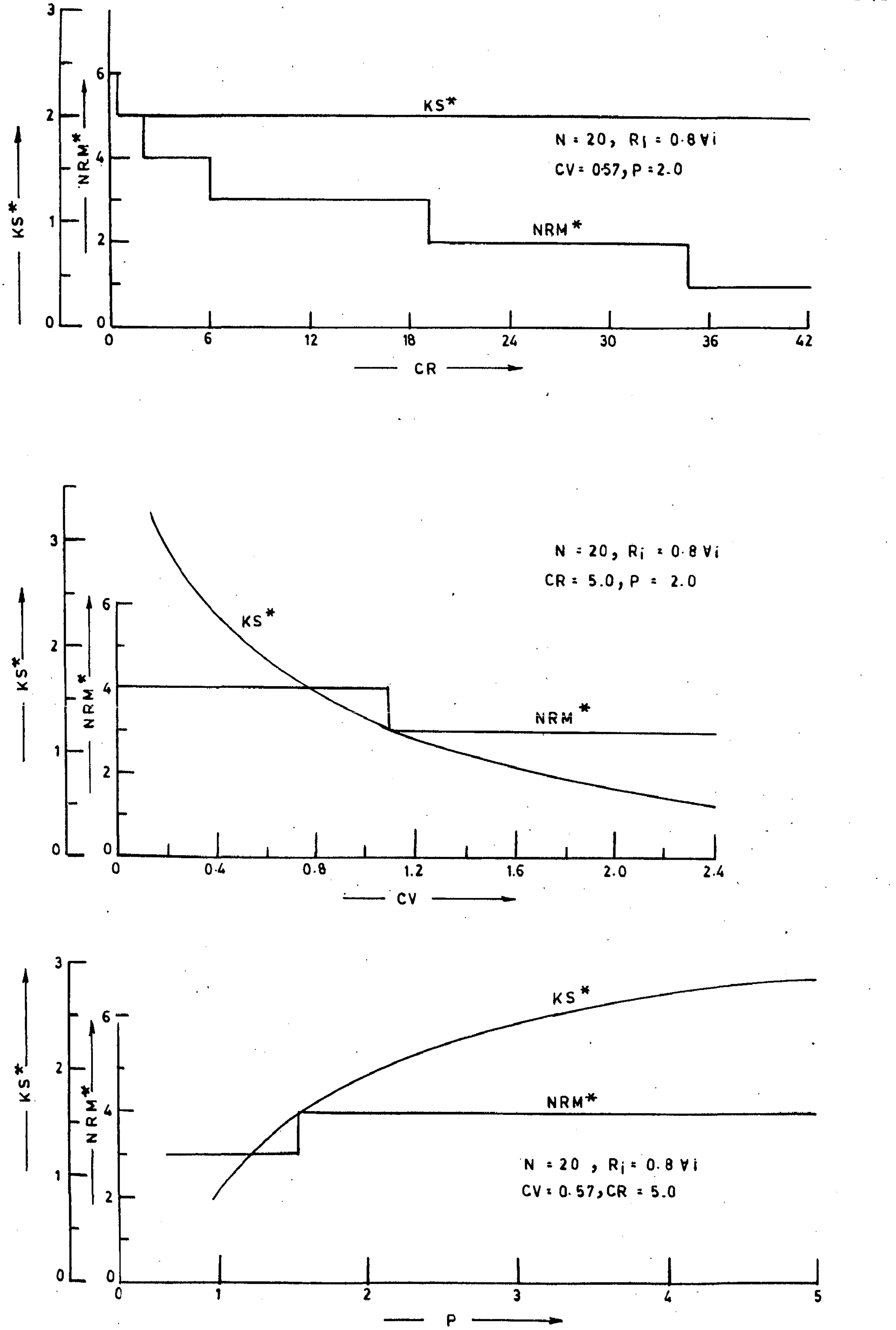


FIG. 9.11 EFFECT OF SYSTEM COSTS ON NRM* AND KS*

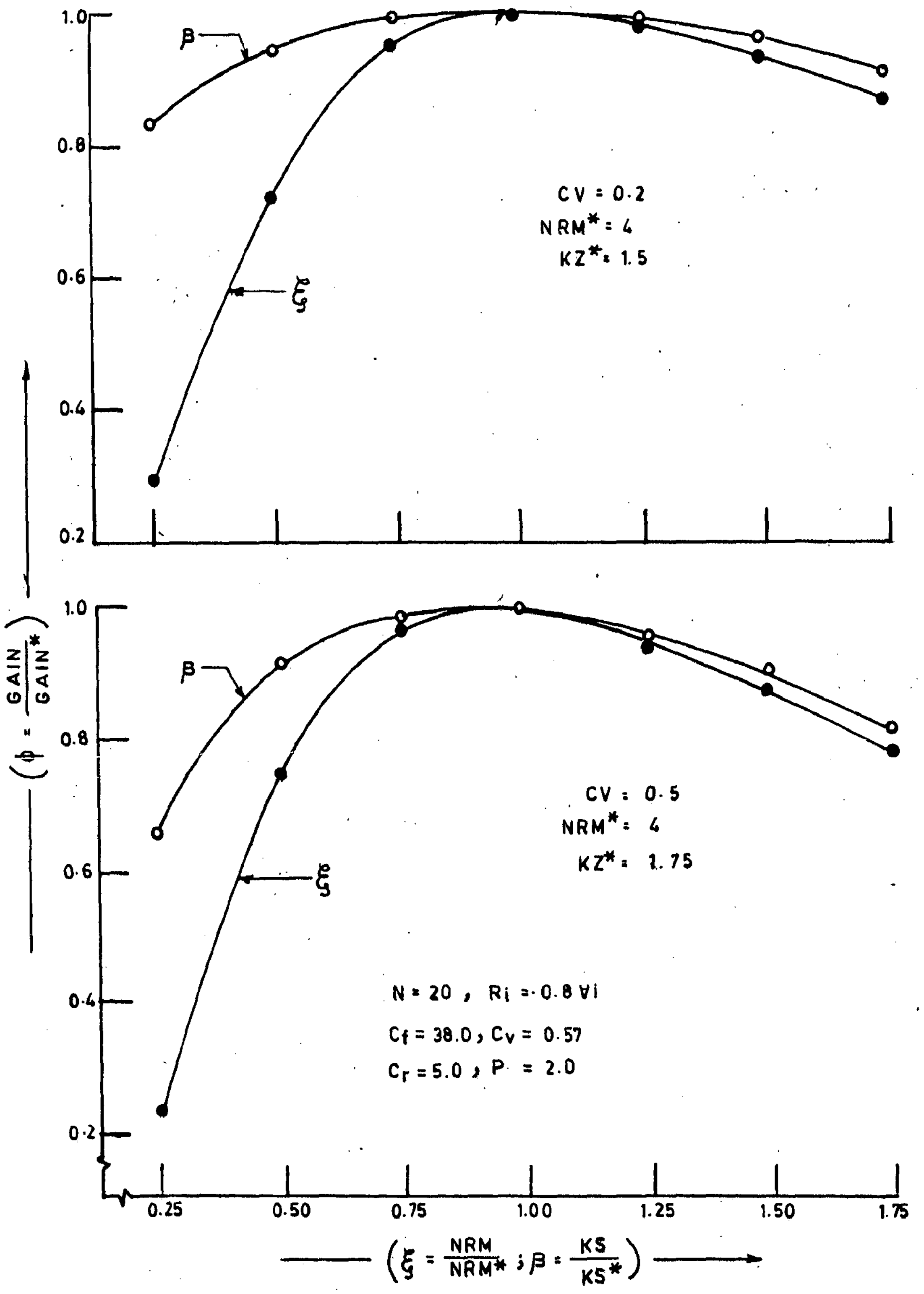


FIG. 9.12 SENSITIVITY OF GAIN TO DEVIATIONS FROM NRM* AND KS*

9.8 COMPARISON OF REPAIR POLICIES.

In order to compare the effectiveness of various repair policies considered in this study, simulation was performed for a 20 stage line having identically reliable stages with $R_i = 0.8$ $\forall i$. The reliability of the stages has been taken low (0.8), since in case of highly reliable lines, the difference between the performance of various repair policies does not seem to be obvious. For the same reason a longer line with N = 20 has been selected. The failure and repair times of the stages have been considered to be exponentially as well as normally distributed. The results have been obtained over a large range of inprocess buffer and number of repairmen. For all the repair policies, simulation runs were performed with the similar starting conditions and by employing the same random number series. The simulation results obtained are given in Appendices A 9.1- A 9.3. The different pairs of repair policies have been compared by employing the signs test as well as the Wilcoxon T-test, at $\alpha = 0.05$, for each value of NRM, for different values of variability of failure and repair times.

The simulation results obtained by employing exponentially distributed failure and repair times did indicate differences between the effectiveness of some of the repair policies, but these were found to be insignificant and no conclusions could be derived. Since large variabilities in failure and repair times of the stages were expected to camouflage the differences in the performance due to various repair policies, experiments were repeated by employing normally distributed times with CV = 0.5 and 0.2 (Appendices A 9.2 and A 9.3). It has been observed that for the 20 stage line with NRM > 5, the five repair policies are equally effective. A comparison between RP2 and RP3 shows no significant difference at different NRM values. On the other hand, the repair policy RP4 was found to be better than RP5 within the range $1 \leq NRM \leq 5$.

The repair policy RP1 (FCFS), has been found to be better than RP2 and RP3 (Appendices A 9.2 and A 9.3). The difference depended upon the values of T_{mnr} and T_{mjr} i.e. the limits on the minor and major repair times. When the limit T_{mnr} is increased in case of RP2 and T_{mjr} is reduced in case of RP3, the performance of line with repair policies RP2 and RP3 further deteriorates. On the other hand, as a limit $T_{mnr} \longrightarrow 0$, RP2 $\longrightarrow$ RP1 (FCFS) and as $T_{mjr} \longrightarrow \infty$, RP3 $\longrightarrow$ RP1. The repair policy RP1 has also proved to be better than RP5 within the range $1 \leq NRM \leq 5$.

Comparison of RP1 and RP4, revealed very interesting results (Appendices A 9.1 - A 9.3 and Fig.9.13). At NRM = 1, the difference between the two was insignificant. RP1 was found to be better than RP4 for NRM = 2, as shown in Fig.9.13. However, for NRM = 3 and 4, the repair policy RP4 proved to be significantly better than RP1, while at higher values of NRM, the difference between RP1 and RP4 vanishes. Thus it can be concluded that RP4 is the best among the five repair policies

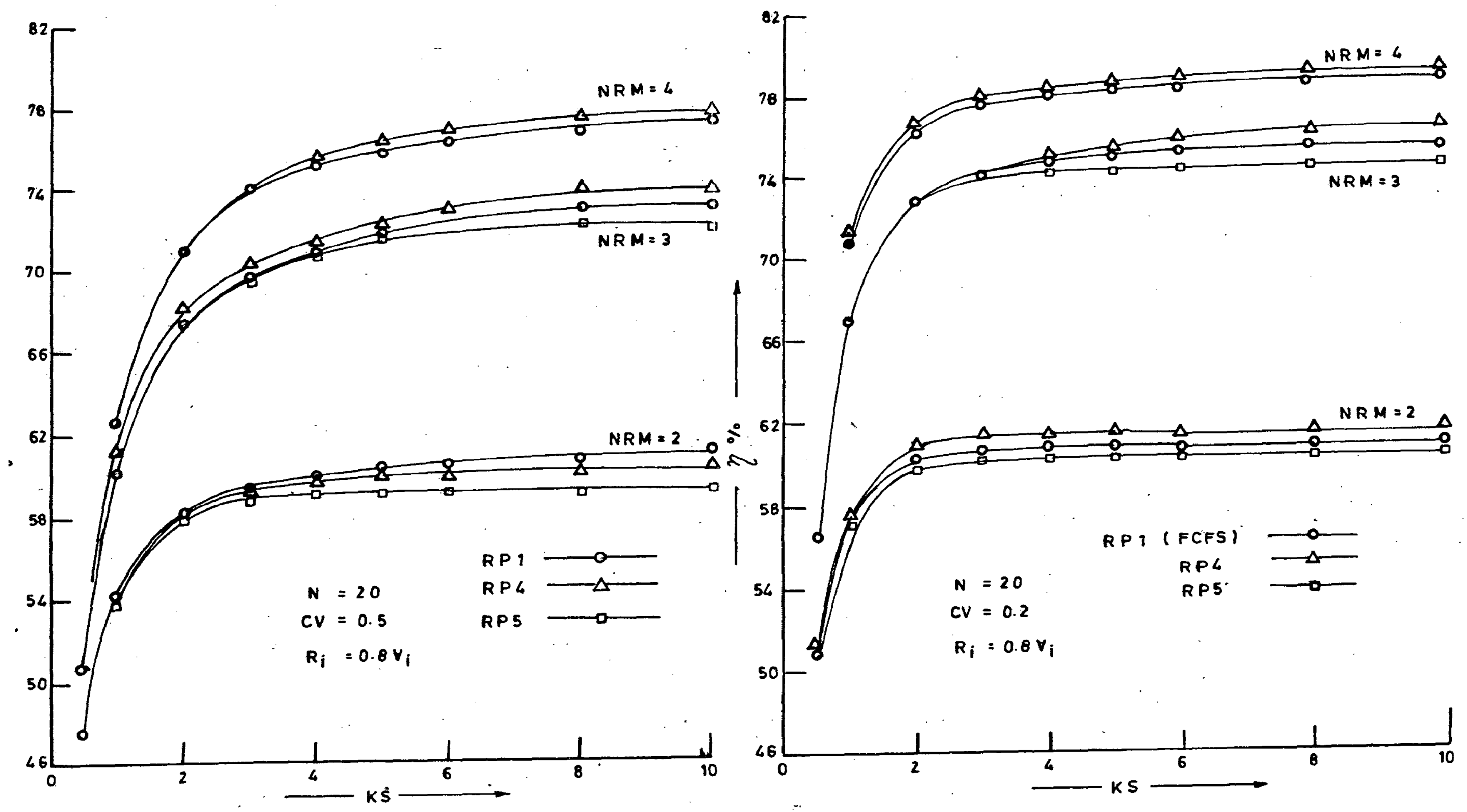


FIG. 9.13 COMPARISON OF SOME REPAIR POLICIES

considered, and the FCFS(RP1) is better than all others except RP4.

The repair policy RP4 gives priority to the repair of a down stage having the largest empty space in the following in-process buffer. In a balanced line, the buffer levels remain low towards the end of the line and high in the beginning. Thus according to this policy failures towards the end of the line would receive better attention as compared to those in the beginning of the line.

9.9 CONCLUSIONS

The efficiency of a flow line having unreliable stages is highly sensitive to changes in the size of inprocess storages and the number of repairmen at their lower levels. The simulation model and search procedure presented in this chapter can efficiently be employed to optimise the inprocess buffer size and the number of repairmen to be employed.

The optimum number of repairmen is quite insensitive to small variations in system costs. The optimum size of inprocess buffers is totally insensitive to changes in repair costs, while it varies considerably with variations in other cost factors.

The maximum gain from the system is not affected much by small deviation in NRM and KS from their optimum values, but is highly sensitive to large under-estimations.

The repair priority rule based on the buffer occupancy is more effective as compared to those based on the extent of repair times. The policy, in which repair priority is assigned to the down stage having largest empty space in the following buffer, has been found to be the best among all considered.

APPENDIX A 9.1

Simulation Efficiency of 20-stage Line with $R_i = 0.8\ \forall i$
(Exponential Failure and Repair Times of Stages)

	NRM = 1					NRM = 2				
S	RP1	RP2	RP3	RP4	RP5	RP1	RP2	RP3	RP4	RP5
10	34.53	35.02	34.01	34.93	33.60	40.18	39.93	40.61	40.07	40.47
20	37.21	37.47	36.78	37.20	37.90	47.22	47.39	46.49	47.25	46.82
40	39.04	38.56	39.27	38.81	38.31	51.84	51.45	51.95	52.36	52.68
60	39.50	38.76	38.24	40.06	38.81	54.50	54.28	54.00	54.40	54.20
80	39.01	39.92	39.71	40.23	39.44	54.58	55.29	54.96	55.24	54.71
100	39.39	39.23	40.02	40.31	38.37	56.36	56.20	56.51	57.67	56.34
120	39.51	39.68	38.87	39.12	38.90	56.08	55.35	56.89	56.57	55.77
160	39.70	40.96	39.23	39.78	39.01	58.21	56.57	57.03	57.35	56.68
200	39.78	40.03	39.41	40.06	38.42	58.14	56.89	57.12	57.31	56.82
	NRM = 3					NRM = 4				
10	41.58	41.71	41.51	41.80	41.37	42.12	42.11	41.72	42.91	42.13
20	50.12	50.14	49.90	49.91	50.16	51.37	51.26	51.07	51.37	50.34
40	57.84	58.02	58.33	57.89	57.81	59.42	59.57	59.78	60.28	59.70
60	61.43	60.68	61.67	61.59	61.24	63.73	63.75	63.34	63.84	64.06
80	63.55	63.85	63.55	64.01	64.06	66.98	66.43	67.61	67.33	67.29
100	64.55	64.40	64.14	65.34	65.24	68.23	68.96	68.41	68.90	68.84
120	65.05	65.63	65.74	66.59	66.20	69.35	69.19	69.29	69.52	69.39
160	67.06	67.70	67.95	68.42	67.15	71.12	71.18	71.02	72.51	71.03
200	69.01	69.39	68.92	69.36	68.94	72.54	72.70	72.87	73.62	73.34
	NRM = 5					NRM = 6				
10	42.27	42.15	40.99	42.41	42.09	41.79	41.93	41.14	42.44	41.72
20	50.72	51.26	50.82	51.04	51.00	51.44	51.09	50.54	50.75	51.25
40	60.56	59.57	59.49	60.74	59.18	60.44	60.10	60.75	60.00	59.51
60	65.71	64.15	65.03	65.78	64.10	64.72	65.16	64.44	65.58	64.98
80	68.18	66.43	67.51	67.95	67.50	68.02	67.70	68.11	67.90	68.39
100	69.56	68.96	69.19	69.87	69.79	70.18	69.65	69.72	70.08	69.21
120	71.67	69.59	70.51	71.08	70.62	71.44	71.44	71.58	72.10	71.84
160	73.87	71.18	73.02	73.52	72.99	73.66	73.17	72.91	73.23	73.31
200	74.26	72.70	74.22	74.66	74.47	74.45	73.83	73.44	74.92	74.03

APPENDIX A 9.2

Simulation Efficiency of 20-Stage Line with R_i=0.8 Vi

(Normally distributed Up and Down Times CV=0.5)

S	RP1	RP2	RP3	RP4	RP5	RP1	RP2	RP3	RP4	RP5
	NRM=1					NRM = 2				
10	37.83	37.54	37.36	37.61	36.98	47.78	47.42	47.16	47.43	47.40
20	39.74	39.85	39.61	39.90	39.45	54.19	53.71	53.71	54.31	53.67
40	40.85	40.54	40.61	40.94	39.70	58.13	57.68	58.09	58.29	57.73
60	40.92	40.52	40.46	40.40	39.43	58.90	58.23	58.83	59.13	58.82
80	40.43	40.20	40.47	40.86	40.13	59.98	58.47	59.68	59.90	59.04
100	40.67	39.92	40.55	40.61	40.44	60.21	59.37	59.88	60.14	59.14
120	41.00	40.49	39.77	41.27	40.37	60.61	59.52	60.53	59.49	59.11
160	41.68	40.73	41.21	40.87	40.73	60.36	59.92	61.35	59.95	58.92
200	40.71	40.89	40.69	40.81	40.50	61.34	60.78	61.26	60.64	59.12
	NRM=3					NRM = 4				
10	50.68	51.19	51.04	51.51	50.71	51.98	51.69	51.69	51.45	51.44
20	60.19	60.64	60.50	61.04	60.75	62.91	63.10	63.06	62.65	62.98
40	67.46	66.99	66.93	67.96	67.48	70.94	70.72	70.71	70.94	70.76
60	69.56	69.44	70.03	70.09	69.97	73.86	73.12	73.53	74.08	73.75
80	70.72	70.49	70.69	71.20	71.10	75.10	74.80	74.63	74.94	75.05
100	71.28	70.66	72.23	71.96	71.44	75.63	75.33	75.79	76.39	75.47
120	72.57	71.58	72.13	72.54	71.52	76.10	75.93	76.16	76.67	76.05
160	73.01	72.20	72.50	73.82	72.02	76.70	76.39	76.71	77.20	77.32
200	72.81	72.74	73.80	73.50	71.64	77.10	77.07	77.08	77.52	77.37
	NRM=5					NRM=6				
10	52.53	52.32	52.00	52.50	51.95	52.26	52.29	52.16	52.33	51.68
20	63.71	63.54	63.19	63.64	63.39	63.95	63.65	63.97	63.89	63.45
40	71.94	71.95	71.84	71.99	72.01	72.33	72.44	72.26	72.21	72.01
60	74.65	74.37	74.42	74.65	74.90	74.82	74.80	74.55	74.85	75.07
80	76.24	75.90	75.83	76.00	75.87	76.74	76.04	76.29	76.71	76.29
100	76.58	76.45	76.70	76.91	76.67	77.81	77.29	77.12	77.42	76.82
120	77.54	77.43	77.29	77.44	77.58	78.09	77.95	77.82	78.21	77.64
160	78.16	78.24	78.46	77.96	77.91	78.56	78.36	78.34	78.41	78.17
200	78.36	78.34	78.53	78.75	78.78	78.92	79.26	78.79	78.57	78.97

APPENDIX A.9.3

Simulation Efficiency of 20-Stage Line with R_i = 0.8 $\bar{V}$i

(Normally distributed Up and Down Times, CV=0.2)

	NRM=1					NRM = 2				
S	RP1	RP2	RP3	RP4	RP5	RP1	RP2	RP3	RP4	RP5
10	39.06	38.70	38.63	39.01	38.63	51.12	51.03	50.98	51.07	50.93
20	40.40	40.25	40.38	40.48	39.64	57.56	56.99	57.02	57.60	56.98
40	40.85	40.72	40.82	40.63	40.23	60.86	59.90	59.77	60.38	59.71
60	40.87	40.81	40.63	40.91	40.34	61.35	60.52	60.96	60.67	60.25
80	40.79	40.53	40.99	40.70	40.49	61.23	61.16	60.96	60.46	60.58
100	40.65	40.86	40.76	40.85	40.26	61.51	60.93	61.10	60.65	60.56
120	40.95	40.71	40.49	40.90	40.25	60.88	61.01	60.97	60.75	60.41
160	41.04	40.61	41.17	40.81	40.49	61.51	61.33	61.02	60.93	60.26
200	40.83	40.70	41.05	40.97	40.42	61.59	60.82	61.57	60.83	60.22

	NRM = 3					NRM = 4				
10	56.65	56.48	56.31	56.63	56.44	58.51	58.26	58.19	58.25	57.93
20	66.96	67.16	66.49	67.04	67.02	71.00	71.15	70.67	70.86	71.13
40	72.75	72.56	72.12	72.60	72.64	76.38	76.40	76.32	76.69	76.55
60	73.83	73.92	73.80	74.14	73.82	77.57	77.63	77.54	77.85	77.52
80	74.63	74.30	74.22	74.79	74.05	77.94	77.96	77.96	78.27	77.92
100	74.80	74.66	74.86	75.29	73.84	78.31	78.13	78.20	78.52	78.19
120	75.28	74.93	75.14	75.81	74.22	78.52	78.51	78.34	78.79	78.17
160	75.39	75.19	75.73	76.13	74.13	78.83	78.62	78.63	78.99	78.70
200	75.31	75.60	75.86	76.65	74.13	78.81	79.07	78.61	79.24	78.71

	NRM = 5				
10	58.70	58.84	58.83	58.88	58.56
20	72.50	72.46	72.33	72.39	72.27
40	77.84	77.84	77.64	77.92	77.84
60	78.76	78.75	78.68	78.79	78.81
80	79.08	79.05	79.08	79.17	79.10
100	79.38	79.25	79.48	79.41	79.29
120	79.52	79.43	79.59	79.54	79.27
160	79.70	79.75	79.55	79.71	79.80
200	79.77	79.72	79.88	79.74	79.78

CHAPTER - 10

CONCLUSIONS AND SCOPE FOR FUTURE WORK

10.1 INTRODUCTION

The work presented in this thesis has been summarised in this chapter. Also some suggestions regarding the possible directions for further research in the field of flow line systems have been given.

10.2 THESIS SUMMARY

In this thesis,balanced and unbalanced flow line production systems, having random operation times and/or unreliable stages have been analysed with regard to their production efficiency and the work inprocess inventory.

From the review of literature in Chapter 2, it has been observed that analytical models are unsuitable for large systems. There also appears to be the absence of suitable empirical models to predict the performance of the flow line systems. In case of unbalanced lines the available literature is comparatively small, and several contradictions in results have been observed.

The structure of the flow line system, various terms used, and some assumptions common to all the chapters, have been discussed in Chapter 3. In this chapter, the salient features of simulation experimental design have also been given.

Simulation model of the flow line having variable operation times and 100 percent reliable stages has been presented in Chapter 4. Based on simulation data, empirical models have been developed for predicting the efficiency of the balanced systems having exponential (equations 4.10 and 4.11) and normally distributed operation times (equations 4.5 to 4.9). The empirical models (equations 4.5 to 4.9) could also be employed in case of operation times defined by Erlang distributions, especially when the Erlang shape factor is large. The utilisation of inprocess buffer, along with the growth of efficiency and WIP inventory with time has been studied. A model (equation 4.17), for determining the optimum size of the inprocess buffers has also been presented in Chapter 4.

In Chapter 5, unbalanced flow lines of the type modelled in Chapter 4 have been studied. Results confirm the Hillier and Boling's [66] bowl phenomenon, that the line having slightly faster stages in the middle is more productive than a balanced line Results show that lesser work load be assigned to stages having larger variability in operation times. Faster stage in the beginning of the line and/or smaller buffers towards the end help to keep the WIP low, however, they result in less efficient arrangement. In case of symmetric lines, the WIP remains at about 50 percent of the provided buffer capacity.

Flow lines having fixed operation times, but unreliable stages have been modelled in Chapter 6. The effect of number of inprocess storage locations on the performance of the system has been studied, which shows that large number of small sized buffers are more effective than a few large sized ones. Empirical model (equation 6.6 to 6.9) has been developed to predict the efficiency of the system having identically reliable stages with exponential failure and repair times. An empirical model for determining the optimum size of the inprocess buffers has also been presented (equation 6.13). The applicability of the models (equation 6.6 and 6.13) have been demonstrated by an illustrative example and a case study.

Simulation results for flow lines having non-identical unreliable stages with fixed and equal operation times, have been presented in Chapter 7. The central part of the line has been found to be more critical, and the performance of the line could be improved by providing more reliable stages or larger inprocess buffers or stages having smaller variability in failure and repair times, in the middle part of the line. Larger inprocess buffer should be placed in the vicinity of less reliable stages, except, when the less reliable stages are at the ends of the line. The average WIP has been found to be a little more than 50 percent of the provided buffer capacity in case of symmetric lines, while a bad stage near the end and/or a good stage in the beginning of the line or larger inprocess buffers in the beginning could lead to increased WIP.

The flow lines with random operation times of the stages as well as stages subject to failures, have been simulated in Chapter 8. The approximate formula (equation 8.3), proposed by Buzacott [26] has been found to be valid for evaluating the efficiency of the system considered. Except for very small values of the inprocess buffers, the formula predicts the η of the system with an error of less than 1 percent. This error has little effect on the optimum size of the inprocess buffers, which could be determined by employing the search procedure presented in this chapter.

In Chapter 9, the influence of number of repairmen and some repair priority rules on the line efficiency, has been analysed. A search procedure for optimising the NRM and inprocess buffer has been presented. Comparison of five different repair policies has revealed that the repair priority rules based on buffer occupancy are more effective as compared to those based on the extent of repair time. The policy in which repair priority is assigned to the down stage having largest empty space in its downstream buffer has been found to be the best among all.

The applicability of the empirical models has been demonstrated by illustrative examples. Sensitivity analysis has been conducted in each case to study the effect of system parameters on its performance and the optimum size of inprocess buffers.

10.3 SCOPE FOR FUTURE WORK

The research work reported in this thesis may possibly be extended as given below :

(1) The simulation model presented in Chapter 4 can be extended to consider situations of cyclic queues, from where the effect of number of frames (jigs and fixtures), on the system efficiency may be investigated. The simulation model of Chapter 6 may be extended to incorporate the effect of rejections and rework at various stages. There is also scope of extending the simulation model of Chapter 8 so as to apply to unbalanced lines, and in Chapter 9, some more repair priority rules, based on the production rate or reliabilities of the unbalanced lines can be investigated.

(2) The approach employed in developing the empirical models in Chapter 4, should be extended to the cases where operation times are defined by p.d.f's other than exponential, Erlang and normal, and also to the modelling of cyclic queues. The empirical models similar to the one presented in Chapter 6, may be developed for the lines, having operation, failure and repair times distributions other than exponential, and incorporating the

rejections and rework at various stages. Some case studies may be taken up to further validate the models presented in this thesis.

(3) The predicting of the efficiency of the unbalanced lines has always been a problem. It may be possible to extend the empirical models of Chapters 4 and 6 to incorporate the effect of imbalances. However, in this case the imbalance would have to be quantified as a system parameter.

(4) More work needs to be undertaken in case of lines having variable operation times, as well as, unreliable stages, especially where the stages have non-identical operating and failure characteristics.

(5) It should be possible to construct an empirical model for the prediction of line efficiency, when the number of repairmen is taken as a system parameter. The same may further be extended to determine the optimum size of repair crew and inprocess buffers.

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