INVESTIGATION OF RIGID PAVEMENT BEHAVIOUR UNDER STATIC LOADS BY MODEL TESTS

A THESIS SUBMITTED IN PARTAIL FULFILMENT
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C E R T I F I C A T E

Certified that the Thesis entitled, " INVESTIGATION OF RIGID PAVEMENT BEHAVIOUR UNDER STATIC LOADS BY MODEL TESTS " which is being submitted by Shri CHHOTE LAL SARAF in partial fulfilment for the award of the Degree of Master of Engineering in HIGHWAY ENGINEERING of University of Roorkee is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this Thesis has not been submitted for the award of any other Degree or Diploma.

This is further to certify that he has worked for a period of about seven months from 20th April, 1962 to 15th November, 1962, for preparing Thesis for Master of Engineering Degree at the University.

N Vaswani

Roorkee:
Dated 29th November , 1962.

(N. K. VASWANI)
Reader in Civil Engineering.

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C O N T E N T S

SYNOPSIS

An accurate prediction of Rigid Pavement behaviour under static loads is important from their design point of view. Model experiments can be employed to arrive at such objectives.

The present investigations were carried out on a rigid pavement model designed and constructed in the laboratory for the purpose of studying the stress distribution pattern on the pavement due to static loads placed at three critical positions. The model consisted of soil subgrade (with rubber solution binder) and Bakelite pavement. The same were adopted after a few trials with other materials.

The test results revealed that the Westergaard's theoretical analysis is applicable in case of pavements of this type. Out of the three critical loading positions, edge loading is important and produces the maximum critical stresses on the pavement, when considering no temperature gradient and slab is in contact with subgrade.

The modified empirical equation has been developed for critical distance in case of corner loading which is

$$x_1 = 2.2 \; (a_1 \, l)^{\frac{1}{2}}$$

Charts have been developed on the basis of model tests to show the stress in case of three critical load positions for different l/a ratios. These charts cover most of cases of pavements generally met in the field. Hence they are very useful from pavement design point of view.

1-

INTRODUCTION

2

THEORY OF MODELS AND ITS APPLICATION IN RIGID PAVEMENT DESIGN

1. INTRODUCTION

Verification of cumbersome equations with number of variables becomes very difficult and costly in prototype slabs. Therefore, it becomes essential to check or to verify the equations by model tests. Very little work has been done in this direction.

It was therefore proposed to investigate by model tests the application of the Westergaard's equations.

Experiments carried out in the past were based on completely elastic support. As the soil is semi-elastic in nature it is proposed to investigate a mix design to simulate the field conditions by various trial and error methods.

Also, the model pavements used in previous tests were not satisfactory. A suitable pavement material was proposed for investigation.

No investigator has determined stresses for all the three critical positions. This was found to be essential and all the three locations are proposed to be tried.

The field tests, besides being costly and cumbersome, have got many drawbacks. The modulus of subgrade reaction of the subgrade changes with the change in the moisture content. Once the pavement is laid, one have to depend on the previous tests for the value of 'k'. The temperature stresses, how-so-ever small they may be, can not be avoided. Models on the other hand are free from these defects. Once we employ them for determining the critical stress due to static loads they are sure to indicate the static load stresses only.

It is very difficult to obtain representative value of surface strains in case of prototype slabs because of its non-homogeneous characteristics. Models on the otherhand are free from this drawback also and if a suitable material is chosen for pavement, it will give the most representative value of surface strains.

SCOPE OF PRESENT STUDIES.

The present studies of models deals with the experimental determination of strains and hence stresses in the model slabs due to the three critical loadings when the pavement is completely supported by subgrade i.e. for no warping conditions. The object of such studies is to investigate the pattern of stress distribution over the pavement due to wheel loads of varying intensity.

Previous reports on such tests indicate that the critical stresses does not vary much due to the change in the shape of the contact area and therefore circular contact areas were chosen for the present investigations.

A wide range of contact areas was adopted for the present investigation to cover most of the prototype slabs and contact area usually met with. Thus the results of present investigations become important from practical point of view.

An attempt has been made to correlate the experimental stresses with the theoretical stress calculated by Westergaard's analysis.

3.

REVIEW AND NECESSITY OF THE PRESENT INVESTIGATIONS.

2. THEORY OF MODELS AND ITS APPLICATION IN RIGID PAVEMENT DESIGN

2.1. GENERAL

Engineering problems may be broadly divided into following three groups :-

(a) Problems which can be easily solved by the direct application of well known laws based on equilibrium or other condition of state.

(b) Problems which require a mathematically cumbersome solution because of their complex nature and involving a number of variables.

(c) Problems for which the application of well known general laws is yet unknown and analytical solution is not available.

Problems of first group create no difficulty so far their solution is concerned. But problems of latter two groups are difficult to solve and in order to get at a quick solution, model studies are usually recommended. Based on these model test results analysed with the help of certain rules, the solution of a problem becomes easier.

2.2. DEFINITION AND TYPES OF MODELS :

A model is usually defined as a device which is so related to a physical system that the observations made on the model may be used to predict accurately the performance of the physical system in the desired respect. The physical system for which the predictions are to be made is called a "Prototype".

The behaviour of a prototype can be predicted with the help of more than one type of model but often only one type of model is best suited for predicting the required characteristics of a prototype. Usually a choice is made out of the following four types of models :

(a) True models.

(b) Adequate models.

(c) Distorted models.

and (d) Dissimilar models.

(a) True Models :

True models are models in which all significant characteristics of the prototype are faithfully reproduced to scale. In addition to being geometrically similar to the prototype, the model must satisfy other restrictions introduced by the design conditions. In general it may be used to obtain data suitable for predicting characteristics related to those for which it was designed.

(b) Adequate Models :

They are just sufficient to predict only one characteristic of prototype for which they are designed and may not yield accurate predictions of other characteristics.

(c) Distorted Models :

Because of certain restrictions introduced in the construction of models, it may not be possible to follow a uniform scale throughout and a distortion becomes necessary, resulting in distorted models. A model of River is usually distorted because the depth scale could not be kept the same as the length and breadth scale and so on.

(d) Dissimilar Models :

They bear no apparent resemblance to the prototype but through suitable anologies, they can be used to predict accurately the behaviour of prototype. Seepage net of Dam Section is usually determined by electrical analogy method and is an example of dissimilar model.

2.3. DESIGN AND ANALYSIS OF MODELS :

Design and Analysis of models is based on the theory of similitude. Theory of similitude includes a consideration of the conditions under which the behaviour of two separate entities or systems will be similar, and the techniques of accurately predicting results on the one from observations on the other.

Theory of similitude is developed by dimensional analysis. Dimensional analysis on the other hand is developed from a consideration of dimensions in which each of the pertinent quantities involved in a phenomenon is expressed.

Dimensional analysis is based on the following two axioms that are inherent in our methods of measurement and evaluation of quantities.

AXIOM 1. Absolute numerical equality of quantities may exist only when the quantities are similar qualitatively.

That is, a general relationship may be established between two quantities only when the two quantities have the same dimensions. For example, a quantity that is measured in terms of force can be equal only to a quantity that is evaluated in terms of force, and can not be equal to a quantity having dimensions of length, time, mass, velocity or anything else except force.

AXIOM II. The ratio of the magnitudes of two like quantities is indepent of the units used in their measurement. Provided that the same units are used for evaluating each.

For example, the ratio of the length of a table to its width is the same, regardless of whether the dimensions are measured in inches, feet or meters.

Dimensional analysis, developed from these two axioms, differs from other types of analysis in that it is based solely on the relationships that must exist among the pertinent variables because of their dimensions instead of being based on Newton's Laws of Motion or other so-called natural laws. In itself, dimensional analysis gives qualitative rather than quantitative relationships, but when combined with experimental procedures it may be made to supply quantitative results and accurate prediction equations.

2.4. DESIGN OF PAVEMENT MODELS WITH THE HELP OF THEORY OF SIMILITUDE :

Rigid Pavement models may be developed on the basis of the above theory in a manner described below :-

The object of pavement models in the present case is to study the pattern of stress distribution over a pavement due to the static load distributed in the form of circular contact areas of different sizes placed at three different critical positions.

In order that the results of model tests may be employed to predict the behaviour of prototype slabs, Westergaard's theoretical approach may be taken as the basis for the design of Pavement models. For the purposes of simulation of the model with the prototype, the following basic assumptions of Westergaard's theory may be considered and the model may be developed accordingly.

(a) The Sub-grade consists of a homogeneous material uniform in character and provide uniform support to the slab resting over it.

(b) The slab consists of a homogeneous, isotropic, elastic material of uniform thickness.

(c) The critical stresses remain within the elastic limits of both the slab and the subgrade.

(d) The slab is infinite in horizontal extent.

(e) The depth of the subgrade is infinite.

Westergaard (4) introduced a quantity denoted by 'l' in his analysis. He stated :-

" A certain quantity which is a measure of the stiffness of the slab relative to that of the subgrade occurs repeatedly in the analysis. It is of the nature of a linear dimension, like, for example, the radius of gyration. It will be called the radius of relative stiffness. Denoted by l, it is expressed by the formula

$$l = \sqrt[4]{\frac{E h^3}{12 (1 - \mu^2) k}}$$

where E is the modulus of elasticity of the concrete, and μ is Poisson's Ratio of lateral expansion to longitudinal Shortening."

This quantity l thus can be taken as the basis for expressing the other terms in the dimensional analysis. Assuming a basic relationship

$$\frac{l}{a} = \frac{l_m}{a_m} \qquad \ldots\ldots \qquad (1)$$

where l = radius of relative stiffness of prototype.

a = radius of circular loaded area corresponding to prototype.

(and the quantities expressed by subscript m are the corresponding quantities for model.)

the length scale $n = \dfrac{l}{l_m}$ ...... (2)

Now on the basis of relationship (1) other relationships may be developed as indicated below :

CORNER LOADING

Westergaard (4) gave the following equation for the determination of maximum stress at the corner :

$$\sigma_c = \frac{3P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{l}\right)^{0.6}\right] \quad \ldots\ldots \quad (3)$$

where P = load on the foot print.

h = slab thickness.

a = Radius of footprint.

l = Radius of relative stiffness of slab.

In the above equation he assumed that the loading is symmetrical about the diagonal bisector of the corner angle and that the critical corner stress occurs in the top of the slab along the diagonal bisector and acts in a direction parallel to the diagonal bisector.

The distance x_1 from the corner, at which the maximum stress occurs is given by

$$x_1 = 2.3785\ (a\ l)^{1/2} \quad \ldots\ldots \quad \ldots\ldots \quad (4)$$

Let σ_{cm} be the critical corner stress observed in the model, then from the equation (3) we get :

$$\sigma_{cm} = \frac{3P_m}{h_m^2}\left[1 - \left(\frac{a_m\sqrt{2}}{l_m}\right)^{0.6}\right] \quad \ldots\ldots \quad (5)$$

Using the relationship (1) and dividing (3) by (5)

$$\frac{\sigma_c}{\sigma_{cm}} = \frac{P\,h_m^2}{P_m\,h^2} \quad \ldots\ldots \quad \ldots\ldots \quad (6)$$

As σ_{cm}, h_m and P_m for the model is known, the stress σ_c in the prototype of thickness h due to wheel load P can be easily determined.

EDGE LOADING :

Westergaard (5) equation for maximum edge stress is

$$\sigma_e = .529\ (1 + .54\,\mu)\ \frac{P}{h^2}\ \log_{10}\left(\frac{E\,h^3}{k\,b^4}\right) - 0.71 \quad .. \quad (7)$$

where μ = Poisson's ration of concrete or pavement material,

b = Radius of equivalent distribution of pressure. It is expressed by equations $b = (1.6\,a^2 + h^2)^{1/2} - 0.675\,h$ when $a < 1.724\,h$

$b = a$ when $a > 1.724\,h$

and the other terms have their usual meaning.

Let σ_{em} be the critical Edge stress observed in the model slab then ,

$$\sigma_e = \sigma_{em}\,\frac{P\,h_m^2}{P_m\,h^2}\ \frac{(1 + .54\,\mu)}{(1 + .54\,\mu_m)}\ \frac{\log_{10}\left(\frac{E\,h^3}{k\,b^4}\right) - 0.71}{\log_{10}\left(\frac{E_m\,h_m^3}{k_m\,b_m^4}\right) - 0.71} \quad .. \quad (8)$$

and may be used to obtain the values of critical edge-stresses by model Tests.

INTERIOR LOADING.

Westergaard's equation (5) for critical interior load stress is given by the relationship :

$$\sigma_i = 0.275\ (1 + \mu)\ \frac{P}{h^2}\ \log_{10}\left(\frac{E\,h^3}{k\,b^4}\right)$$

Using the same method as in other cases, the critical interior stresses σ_i may be calculated from the relationship

$$\sigma_i = \sigma_{im} \frac{P\, h_m^2}{P_m\, h^2} \quad \frac{(1+\mu)}{(1+\mu_m)} \quad \frac{\log_{10} \dfrac{E\, h^3}{k\, b^4}}{\log_{10} \dfrac{E_m\, h_m^3}{k_m\, b_m^4}} \quad \ldots \quad (9)$$

based on the observed stresses σ_{im} in the model.

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3. REVIEW AND NECESSITY OF PRESENT INVESTIGATION

3.1. GENERAL :

Cement concrete Road came into existence with the inception of automobiles in the early years of twentieth century. Since then a number of investigations have devoted themselves to improve their designs and methods of construction. The short of space and the objective of the problem here may not permit an account of all of them but it will not be out of place to mention here a few of them whose works and/or theories have been in wide use even to-day.

3.2. DR. H.M. WESTERGAARD :

Westergaard, begining in 1925, published a series of articles on the theoretical analysis of concrete pavements slabs that has largely been accepted as fundamental . He propogated a theory commonly known as "Thick Liquid Subgrade Theory". In his assumptions, he considered the subgrade consisting of a material similar to thick liquid. Further he assumed the pavement to be similar to a thin plate of homogeneous, isotropic, elastic solid material in equilibrium. These assumptions lead him to assume again that the reaction of the subgrade is vertical and is proportional to the deflection of the slab. Thus the analysis was reduced to a problem of the Mathematical theory of Elasticity. He introduced two new factors 'k' and 'l' defined (4) as "modulus of subgrade reaction" and "radius of relative stiffness"; in his analysis. Formulae were given by him for calculating the stresses and deflections produced by three critical loading positions.

E.F. Kelley (9), based on the experimental results of Arlington tests, revised the corner load formula as

$$\sigma_c = \frac{3P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{l}\right)^{1.2}\right] \qquad \ldots\ldots \qquad (1)$$

The above theory of Westergaard has certain drawbacks which in short are as follows :-

(a) The theory is based on the assumptions that may not be justified in all circumstances.

(b) Secondary stresses caused by temperature gradients or other causes, which may be much more severe than that are due to wheel loads (especially in thick slabs) have been ignored in the derivations of the formulae.

(c) The stresses produced in the subgrade has not been calculated and it has been assumed that the subgrade has its effect on the stresses in the concrete only. In some cases the stresses produced in subgrade might well exceed the strength of the soil.

3.3. A.H.A. HOGG AND D.L. HALL :

Hogg and Hall came forward in the year 1938 with a new conception of pavement design. Their theories (10,12 and 13) assumes the subgrade similar to an elastic solid and pavement like a thin plate of homogeneous, isotropic, elastic material.

3.4. L.A. PALMER AND E.S. BARBER:

Palmer and Barber's two layer theory (14) gave an approximate formula for the vertical elastic displacement at the surface of subgrade and pavement. He gave the final formula for the displacement at the surface of the pavement, Δ as

$$\Delta = \frac{1.5\ p\ a}{E_2}\ F_w' \qquad \ldots\ldots \qquad \ldots\ldots \qquad (2)$$

where F_w' is a displacement factor given by

$$F_w' = \left[\frac{a}{\left\{ \left(a^2 + h^2 \left(\frac{E_1}{E_2} \right)^{2/3} \right) \right\}^{1/2}} \left(1 - \frac{E_2}{E_1} \right) + \frac{E_2}{E_1} \right] \ldots\ldots \qquad (3)$$

3.5. D.M. BURMISTER.

Burmister (15) analysed the stresses and strains in a two-layered system, consisting of an elastic slab, infinite in the horizontal plane only, placed on a semi-infinite solid of lower modulus of elasticity. This system is being subjected to a uniformly distributed load acting over a circular area and applied to the upper surface of the slab. The relation used for finding out the vertical elastic displacement (Δ) at the surface is given by him as follows :-

$$\Delta = \frac{1.5\ p\ a}{E_2}\ F_w \qquad \ldots\ldots \qquad \ldots\ldots \qquad (4)$$

where F_w is a displacement factor which can be determined from the charts supplied by him.

Burmister's theory is also not freem from the defects and one can not depend upon his analysis because of the following reasons :-

(a) No account has been taken for the non-elastic behaviour of materials.

(b) By limiting the displacement he has assumed the failure to be dependent only on displacement but in certain cases the failure may be due to excessive stresses produced because of the displacement.

(c) Effect of Traffic intensity and consolidation due to the traffic has been ignored.

3.6. GERALD PICKETT :

Pickett (16) in the year 1951 published a number of influence charts based on Westergaards liquid subgrade theory and Hogg and Hall's elastic solid subgrade theory. With the help of these charts, deflections and stresses can be determined at a point over the slab due to any type of load over the pavement He also gave a semiempirical formula for determining the stresses in a pavement due to circular load at its corner. This formula is as follows :-

$$\sigma_c = \frac{4.2\,P}{h^2}\left[1 - \frac{\sqrt{a/l}}{0.925 + 0.22\,a/l}\right] \quad \ldots\ldots \quad (5)$$

For protected corners he suggested a reduction of 20% stresses as calculated by the above formula.

MODEL TESTS

3.7. GENERAL.

Although full scale model tests were carried out by many investigators like F.F. Kelley, Teller and Sutherland etc. in the past, the need and importance of small scale model tests of pavements were not fully realised in those days. Thus only a few investigators have diverted their attention to such tests. Following paragraphs indicate the type of work done in this connection.

3.8. F.M. MELLINGER AND P.E. CARLTON:

Mellinger and Carlton (1) conducted the model tests to determine the possibility of application of models to design studies of concrete airfield pavements. They used Westergaard's analysis in their investigations. They tested the models for very high pressures in the corresponding prototypes (50 - Kips on single wheel and 100 Kips on dual wheels). Effect of multiple wheel loading was studied and curves were drawn for converting multiple wheel loading into an equivalent single wheel load.

The model was prepared out of natural rubber subgrade (size 24" x 24" x 12" deep) and hydrostone gypsum cement mortar pavement (size 15" x 15" x 1/8" thick). In order to measure stresses in the slab, A-7 type S-R-4 bonded resistance wire strain gauges were used.

Maximum stresses for both interior and edge loadings were determined for a wide range of foot-print radii and were compared with the theoretical results obtained by Westergaard's equations. As reported in the paper, the results obtained were quite comparable within the limits of experimental error.

With the help of the multiple wheel load tests, stresses in the pavement were determined and then converted into an equivalent single wheel load having a constant contact area. Thus, this indicates the possibility of basing the design and evaluation of airfield pavements on a single equivalent wheel load having a constant area .

3.9. A.C. WHIFFIN :

A.C. Whiffin (17) in his tests maintained the mechanical properties of concrete used in model slabs identical to those of prototype slabs by providing big size slabs. The value of modulii of the subgrade reaction were also same for model as well as prototype.

The scale of the model was kept constant for length, breadth and thickness of the slab and thus the stresses were in inverse proportion to the square of slab thickness.

The shape of contact area was similar to the prototype and dimensions were proportional to the relative thicknesses of the model and Full size slabs (i.e. to the scale factor).

Tests were carried out with three different scales models viz. full size, half size and one third size model slabs, by choosing prototype slabs of different thicknesses.

The wheel load 'P' was changed for each slab but the intensity of wheel. load pressure was kept constant by changing the contact area according to the scale ratio.

He states "It is desired that the stresses induced in the model slab shall not exceed those in the full size slab by more than 10% and the distribution of stress shall not depart from true scaled values by more than 10% , the scale factors used must not be less than those summarized in Table below " :

Quantity	Notes	a/h	Minimum scale factor to be used at value of k of 100lbs/in³	700lbs/in³
Stresses due to interior loading	Minimum scale factor varies with a/h and k	1	0.52	0.61
		0.67	0.44	0.53
		0.50	0.40	0.49
Stresses due to corner loading	Minimum scale factor varies with a/h and k	1	0.49	0.68
		0.67	0.15	0.53
		0.50	No lower limit	0.33
Radius of relative stiffness	Independent of a/h and k		0.68	
Distance along bi-sector of corner angle to point of maximum stress under corner loading	Independent of a/h and k		0.53	

3.10. P.F. CARLTON AND RUTH M. BEHRMANN :

Carlton and Behrmann (2, 3) in their investigations used models similar to those used in previous studies as reported by Hallinger and Carlton (1) The Hydro Stone Gypsum Cement Slab of size 15" x 15" x 0.125" was used over the natural rubber subgrade of 24" x 24" x 12" thick. The physical quantities like l, k, E and μ for the model were determined and then Edge and Corner stresses were determined by using SR-4 electrical strain gauges.

Two types of foot prints were used by them in their investigations (2), viz. circular and elliptical. Different sizes of contact areas were used to cover a wide range of Area/l^2 ratio which includes most prototype conditions.

They observed that in case of edge the variation in maximum observed stresses was of the order of 10 to 12 % from the theoretical stresses. But in the case of corner the stresses were 65% to 75% as great as those determined from Westergaard's equations.

A good aggrement was obtained between the model and the theory with respect to the location of the maximum stress along the corner diagonal for corner loadings.

As was reported in the paper, "The shape of the loaded area, whether circular or elliptical has little effect on the pavement stress for a given value of Area / l^2. Actually, the shape of the foot print result in less than 5% variation in stress . Since prototype footprint shapes are actually somewhere between being circular or elliptical, the average data obtained from circular and elliptical loadings can be applied to prototype with negligible error".

They concluded that the maximum corner stress is only 45% to 53% of the maximum edge stress.

They extended the model studies on prestressed rigid pavements for airfields (3).

The same type of subgrade was used as reported in previous studies. The slabs were made out of Hydrostone gypsum cement of size 16.6" x 16.6" x 0.2" thick. The slabs were prestressed longitudinally and transversely by tensioned music wires positioned at the neutral axis of each slab. Based on previous tests on the tensioning of wire, in the slab the wires of 0.02" Diameter were tensioned by means of 75 lbs. load which gave an initial tension of 239,000 psi about 65% of the ultimate tensile strength of the wire. The wires were spaced @ 0.6" centre to centre and this developed a maximum tensile stress of 600 psi in model slabs.

Various sizes of foot prints (circular as well as elliptical) were used to load the test slabs. For measuring strains in the test slabs, electrical strain gauges (Type A-7, SR-4) were used. Deflections were measured with the help of dial gauges reading direct to 0.0001 inches.

In order to determine the net amount of prestress developed in the slab, SR-4 gauges were mounted prior to the release of wires.

Crack pattern development was studied by loading the test slabs with increasing amounts of loads. It was observed that the development of cracks was almost radial in nature and originated from the bottom side of the slab under the centre of the loaded area. Prior to complete failure, cracking was confined to the bottom side of the prestressed slabs.

Both edge and interior loadings revealed that prestressing in the slab permitted them to sustain greater loads when subjected to interior loadings as compared to loadings at a free edge.

For loadings beyond the elastic range, greater deflections were observed in the plain slabs than in prestressed slabs subjected to the same loading.

NECESSITY OF PRESENT STUDIES.

1. In the previous model tests, the subgrade used was of rubber which was perfectly elastic in nature. However, the subgrade in the field is different and has visco-elastic properties. Thus the previous investigations were not much of help to predict the behaviour of prototypes in the field. Hence in the present investigations, an attempt was made to simulate the field conditions by using a subgrade similar to one usually available in the field.

2. The models were either too big and when small, cement mortar was used with the result that the slab thickness could be 0.2" of the cement mortar. Such slabs are bound to develop hair cracks and hence affecting the test results. Bakelite sheet used in present investigation eleminated the question of hair cracks in model pavement.

3. Simultaneous measurement of strains for the three critical positions of load was not done so far and hence the same was proposed in the present investigations.

4. Radial strains along perpendicular to the edge (in case of edge-loading) were not measured. The same has been done in present case.

4.

DESIGN AND CONSTRUCTION OF MODEL

4. DESIGN AND CONSTRUCTION OF MODEL.

4.1. GENERAL :

Present investigations were carried out on a small scale RIGID PAVEMENT MODEL, designed and constructed in the Highway Engineering Laboratory of the University of Roorkee. A general view of the model is shown in Fig. 18. The design of the model was completed in two stages viz., i) Design of model proper, and ii) Instrumentation of the model.

4.2. DESIGN OF MODEL :

The model proper consists of the following components :

Reaction Beam.

Subgrade Material.

Subgrade mould.

Pavement.

4.3. REACTION BEAM DESIGN

In order that the load may be conveniently applied to the foot-print area without disturbing the system, a reaction beam (See Fig. 1 and 3) was designed and prepared in the laboratory. The main considerations in its design were :

(a) It should be light in weight (as far as possible).

(b) It should be strong enough to withstand the amount of load expected to be transferred through it during testing.

(c) It must be capable of applying the vertical loads to the required

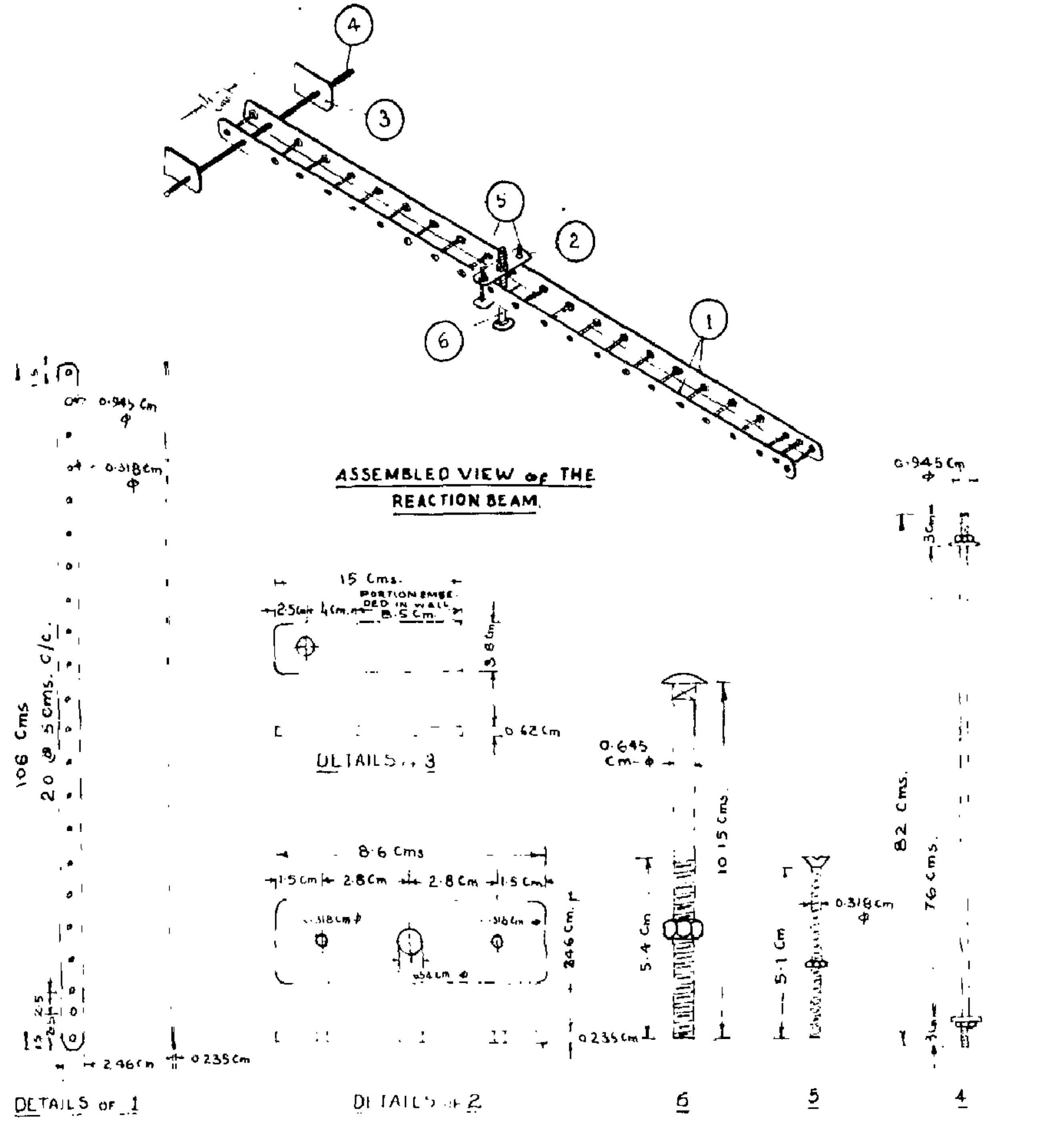


FIG 1.

DETAILS OF MATERIAL USED IN REACTION BEAM

PART NUMBER	DESCRIPTION	SIZE IN CMS. LENGTH	BREADTH	THICKNESS	QUANTITY USED
1	[illegible]	106	2.46	[illegible]	[illegible] No.
2	ALUMINIUM FLAT	[illegible] 6.	2.46	[illegible]	2 No.
3	MILD STEEL FLAT	15.0[illegible]	[illegible]	0.620	4 Nos.
4	MILD STEEL ROUND	82	0.945 cm DIA		1 No.
5	M.S. SCREW WITH 3 NUTS	[illegible]	[illegible] cm. DIA		24 Nos.
6.	M.S. BOLT WITH 3 NUTS AND 2 WASHERS	10.15	0.645 cm DIA		1 No.

RIGID PAVEMENT MODEL
DETAILS OF REACTION BEAM

DESIGNED AND DRAWN
C.L. SARAF
P.G. (HIGHWAYS)
1961 - 62

APPROVED
[signature]

DRWG. No. RB-1.

DRAWINGS NOT TO SCALE.

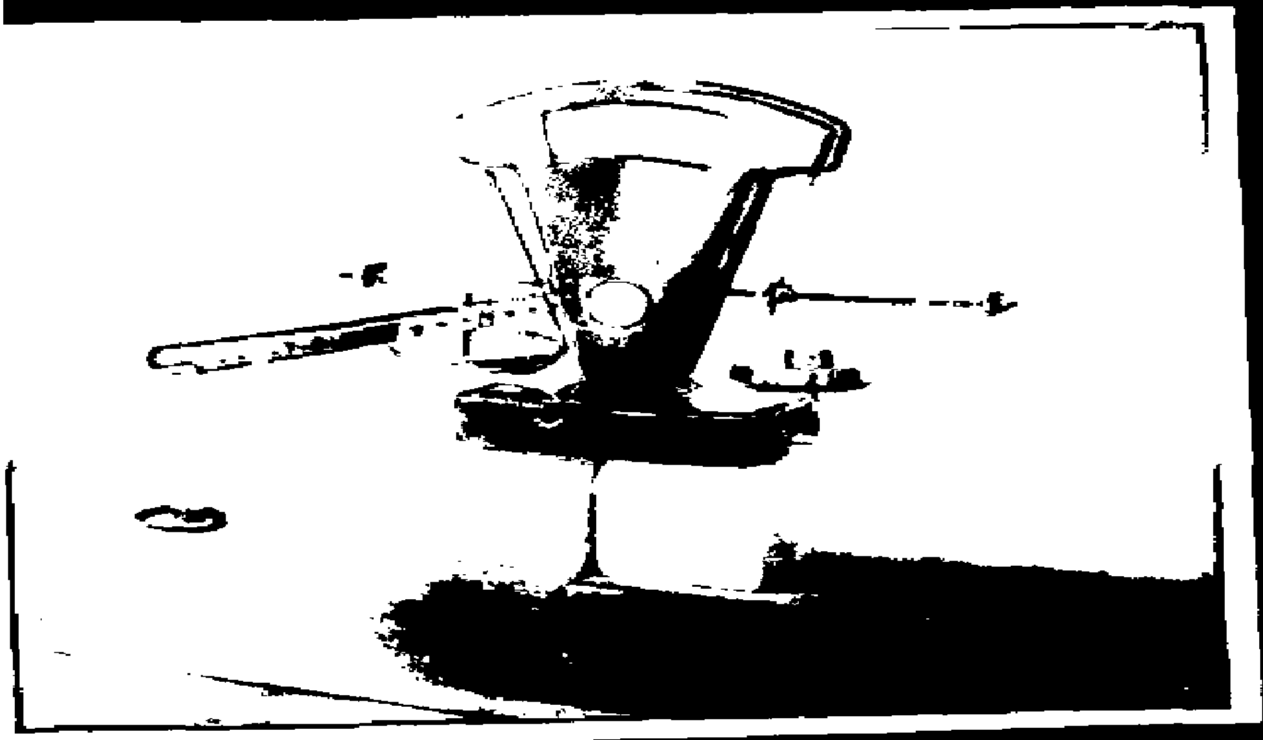


FIG. 2 - CALIBRATION OF REACTION BEAM.

FIG. 3 - A VIEW OF THE ASSEMBLED REACTION BEAM AND SUBGRADE MOULD.

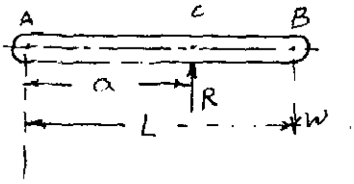
size foot prints (smallest as well as biggest).

(d) Its construction should be simple and easily manageable within the limits of the facilities available.

Keeping all the above factors in mind, a reaction beam was designed, the details of which are shown in Figure 1. Length of the beam was fixed from the considerations of the size of subgrade and the position of the supports were fixed on considerations of the rigidity of hinge points and maximum coverage of the test area by the beam. It was made out of Aluminium metal strips. The total weight of the beam was about 400 gms.

4.4. CALIBRATION OF REACTION BEAM

Neglecting the self weight of the beam and the friction at the hinge point, a theoretical relationship may be established between the applied load



W at the end B of the Beam AB and the resulting reaction R at C due to the load W, as follows :

Let AC = a

AB = L (length of Beam)

Taking moment about the point A and applying the general conditions of equilibrium,

A C. R = AB. L

or R.a = W. L or R = L/a . W ... (1)

In order to check the validity of above relation, an actual test was carried out with the help of an arrangement shown in Fig. 2 . The load W (any) was put on the load pan at the end B and the reaction R at the point C was directly determined with the help of the balance as shown in figure. The point

C was chosen at different places like L/3, L/2, 2L/3 from A and R was determined theoretically as well as experimentally. The agreement between the two values was excellent and it was found that for loads more than 200 gms at the end B (i.e. $W \geqslant$ 200 gms) the effect due to friction at the hinge A was almost negligible (within $\pm$ 0.2% maximum).

Thus, relation (1) was used in the further investigations for the purpose of calculating the Reaction R.

4.5. INVESTIGATIONS FOR SUBGRADE MATERIAL.

The choice of a proper subgrade material depends upon many factors, the most important of which are :

- (a) Uniformity in character.
- (b) Material to resemble the semi-elastic and other properties widely obtained in soils.
- (c) The Physical properties and size of the pavement.
- (d) Easily workable.
- (e) Less liability of its getting affected by weather conditions like temperature and humidity.

In case of this model studies the value of l/a was taken as constant to compare with the prototype conditions. Hence , the value of l was defined on the basis of scale ratio. This value of l depend on the values of E, μ and k. The material chosen for the model slab was proposed to be bakelite or perspex because of the reasons explained later. The value of E and μ for bakelite or perspex used is a fixed quantity and hence the k value of the sub-grade had to fall within a certain range. To achieve this type of subgrade and also to satisfy the factors mentioned above different materials were investigated. They are given below .

i) Cork (Graded) with Bitumen and/or Rubber Solution as binder.

ii) Hemp with Bitumen and/or Rubber solution as binder and sand or lime as filler.

iii) Sand (graded as well as ungraded) with Rubber Solution as binder.

iv) Sand (graded) with Roorkee Soil (sieved) with Rubber solution as binder.

v) Roorkee soil with Rubber solution as binder.

For the purpose of preparing samples, in general a 4" (10.16 cm) diameter mould was used and the weight of material was kept constant in each type of mix. used. The compaction of the samples in most of the cases was done with the help of standard hammer weighing 5½ lbs. (about 2.5 kgm) and having a total fall of 12" (30.48 cm).

In order to get an idea of the xa k value of a particular type of mix, load, deflection tests were carried out on each sample. Fig. 4 shows the general arrangement made for testing each sample in the laboratory. All the tests were carried out by loading the samples with the help of 2" diameter contact area placed centerly over the specimen and loaded from the top by means of reaction beam upto 5 kgm load in most of the cases. The settlement of the contact area or bearing plate was measured with the help of two dial gauges (reading upto 0.01 mm.) placed diametrically opposite. Modulus of subgrade reaction in kgm per cm^2 per cm of deflection was then determined in each case for each sample. The values are summarised in Table 4.13.

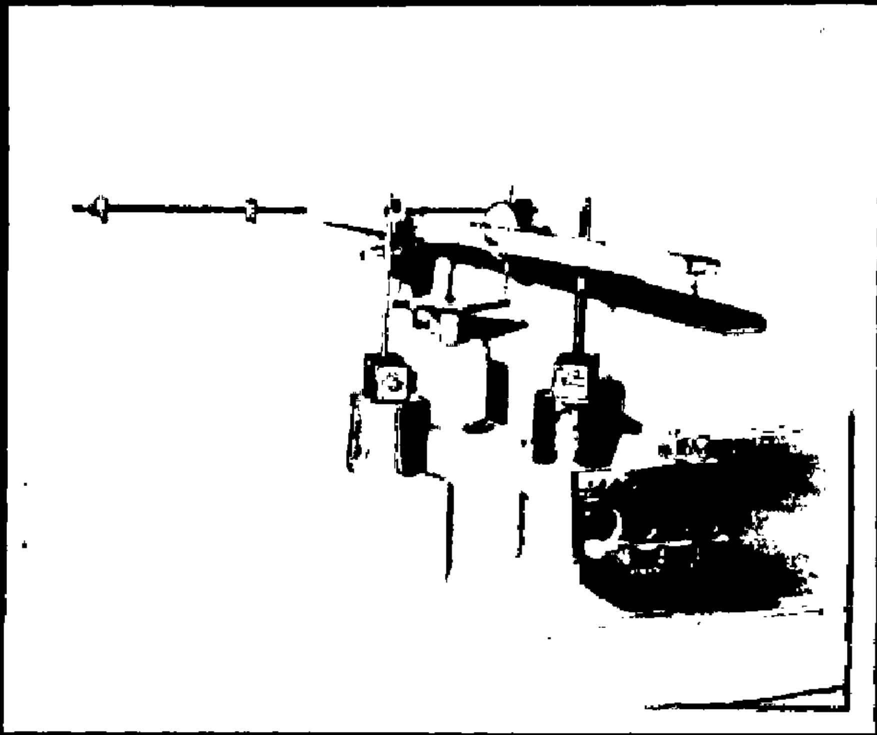


FIG. 4 - DETERMINATION OF 'k' VALUE FOR SAMPLES OF SUBGRADE MATERIAL.

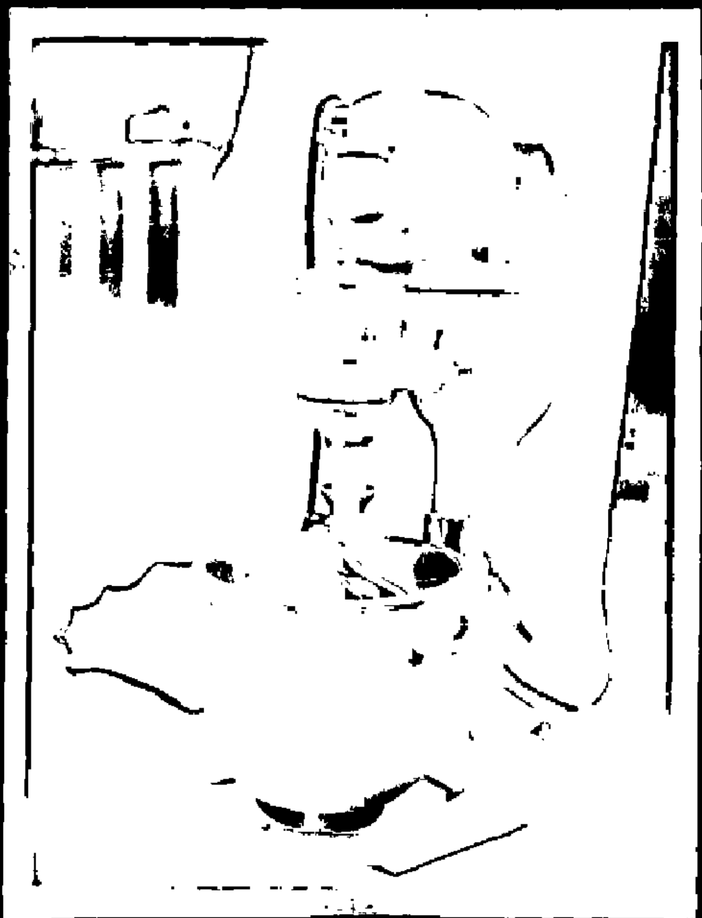


FIG. 5 - MECHANICAL MIXER

(for Preparation of Subgrade Material)

To arrive at an approximate value of k, the values determined for each sample for 2" diameter plate were converted into values corresponding to bearing plate of 30" diameter. The relationship may be derived on the basis of Dr. H.M. Westergaard's conception of modulus of subgrade reaction k and D.M.Burmister analysis of deflection in layered systems as follows :

The value of k, according to Dr. H.M. Westergaard may be expressed as :

$$k = \frac{\text{Load applied to the sample.}}{\text{Area of contact plate} \times \text{deflection of plate.}}$$

$$= \frac{P}{A \times \Delta} = \frac{p}{\Delta} \quad \ldots\ldots \quad \ldots\ldots \quad (2)$$

Also according to D.M. Burmister,

$$\Delta = \frac{1.18\ p\ a}{E_2}\ F_w \quad \ldots\ldots \quad \ldots\ldots \quad (3)$$

where Δ = vertical displacement (cms)

p = contact pressure (kgm/cm^2)

P = Total load (kgms.)

a = Radius of contact plate (cms.)

E_2 = Modulus of Elasticity of bottom layer (subgrade) (Kgm./cm^2)

F_w = Displacement factor.

Thus, from (3) we get,

$$\frac{p}{\Delta} = \frac{E_2}{1.18\ a\ F_w} = K \quad \text{(From 2.)}$$

$$\text{or} \quad k = \frac{E_2}{1.18\ F_w} \cdot \frac{1}{a}$$

Being a single layer system in the present case, $F_w = 1$ and therefore,

$$k = \left(\frac{E_2}{1.18}\right) \times \frac{1}{a} \quad \ldots\ldots \quad (4)$$

For a particulat type of soil, the value of E_2 is constant and thus the quantity within brackets on the right of equation (4) is constant for a particular type of soil and may be substituted by a constant C. The relationship (4) then assumes the form :

$$k = C. \frac{1}{a}$$

$$\text{or} \quad k \propto \frac{1}{a} \quad \ldots\ldots \quad \ldots\ldots \quad (5)$$

Based on the above logical formula the k value of 30" diameter plate will be 1/15 times the corresponding value for 2" diameter plate. The values for 30" diameter plate are given in Col. 9 of Table 4.13.

In all, 32 different types of samples were prepared and tested in the laboratory as briefly described above. Although the limited space here may not permit a detailed account of each sample tested, a brief description of them may be given as follows :

CORK SAMPLES :

Three cork samples were prepared by adopting the grading shown in Table 4.1. below :-

TABLE 4.1. <u>GRADING OF CORK (C)</u>

TABLE 4.1. <u>GRADING OF CORK (C)</u>

Sieve Size.			
Passing from	Retained on	Percentage of total weight.	Amount in (gms) taken for each sample.
¾"	½"	55	55
½"	3/8"	15	15
3/8"	¼"	10	10
¼"	-	20	20
	Total	100	100

The constituents of each of the samples were as follows :

TABLE 4.2. <u>CORK SAMPLES (CONSTITUENTS)</u>

Sample No.	Wt. of Cork graded (gms)	Wt. of Rubber solution * (gms)	Wt. of Bitumen (30/40) (gms)	Compaction by (5½ lbs. - 12" fall) hammer in no. of blows.
1.	100.00	150.00	-	150 in one layer
2.	100.00	100.00	50.00	150 in one layer
3.	50.00	50.00	12.5	150 in one layer

Fig. 6 shows the appearance of a compacted sample as well as of the uncompacted material. The test results are summarised in Table 4.13.

HEMP SAMPLES :

Hemp samples were prepared by mixing Hemp and binder. To achieve through mixing, the hemp was cut into small pieces (about 2 to 4 cm. long).

* See Appendix B. for specifications of this material.

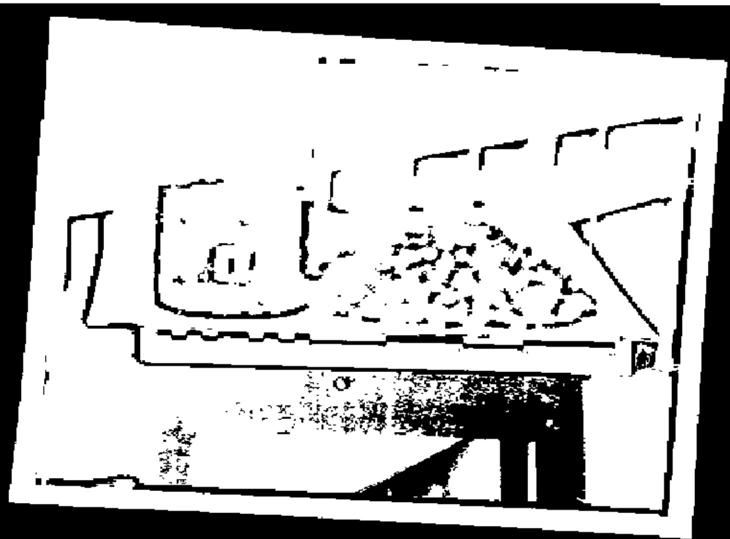


FIG.6- CORK SAMPLE.

FIG.7- HEMP SAMPLES.

FIG. 8 - SOIL SAMPLES.

FIG. 9. BORE-HOLE EQUIPMENT ALONG WITH SAMPLES.

The constituents of each sample are shown in Table 4.3. below :

TABLE 4.3. HEMP SAMPLES (CONSTITUENTS)

Sample No.	Wt. of Hemp (gms)	Wt. of Rubber solution (gms)	Wt. of Bitumen (30/40) (gms.)	Wt. of Fillers. Sand passing U.S sieve No 16. (gms)	Wt. of Fillers. Lime passing U.S. sieve No. 200.	compaction by 5½ Lbs. 12" fall hammer. (no. of blows).
4.	100.00	50.0	50.0	-	-	Total 200 in Two layers.
5.	100.00	150.0	-	-	-	Total 200 in Two layers.
6.	100.00	75.0	25.0	-	-	Total 200 in Two layers.
7.	100.00	180.0	12.5	-	-	Total 200 in Two layers.
8.	100.00	150.0	-	-	-	Total 200 in Two layers.
10.	100.00	150.0	-	50.0	-	Total 250 in Two layers.
11.	100.00	150.0	-	-	50.0	Total 250 in Two layers.

Fig. 7 shows the appearance of compacted specimen along with the uncompacted material. The test results are summarised in Table 4.13.

4.6. SOIL SAMPLE

A soil sample (sample No. 9) was prepared by mixing 100 gms of Roorkee soil (as available) with 1250 gms of Rubber solution and 100 c.c. of Petrol (Thinner). The mix was compacted under 20 blows of the 5½lbs. hammer in two layers. This gave a sample of very high k value as can be seen from the test results shown in Table 4.13.

4.7. SAND SAMPLES (Ungraded)

Ungraded sand samples were prepared as follows :

TABLE 4.4. UNGRADED SAND SAMPLES (CONSTITUENTS)

Sample No.	Wt. of Sand (gms) P30R 50	P 50 R100	Wt. of Rubber Soil (gms)	Compaction.
12	1000	-	100	Light tamping by means of an iron rod in three layers.
13	1000	-	50	— Do —
14	750	250	100	— Do —
17	1000	Cement P200 50 gm.	175	by 5½lb. 12" fall hammer given 25 blows in Total.
19	1000	Cement P200 50 gm.	130	Total blows of 5½lb 12" fall hammer 35 in two layers.

4.8. COARSE GRADED SAND SAMPLES :

The following grading was adopted for preparing the samples of this group.

TABLE 4.5. COARSE GRADED SAND (S_1)

Sieve Size U. S. -- Passing from	Retained on	% of Total weight	Amount of each portion in 1000 gms sample.
No. 8	No. 16	30	300
No. 16	No. 30	30	300
No. 30	No. 50	20	200
No. 50	No.100	10	100
No.100	No.200	10	100
	Total	100	1000

The constituents of each of the samples were as follows :-

Table No. 4.6 COARSE GRADED SAND SAMPLES (Constituents)

Sample No.	Wt. of sand S_1 (gms.)	Wt. of Cement p200 (gms.)	Wt. of Rubber Solution.	Compaction by standard 5½ lbs. 12" fall hammer.
15	1000	50	130	10 blows to first layer and 35 blows to second layer.
16	1000	50	130	Light tamping in two layers.

Test results are shown in Table 4.13.

4.9. SAND SOIL SAMPLES.

The constituents of this group of samples were as indicated below :

Table No. 4.7. SAND + SOIL SAMPLES (Constituents)

Sample No.	Sand 'S_1' (gms.)	Roorkee soil (gms.)	Cement p 200 (gms.)	Rubber solution (gms.)	Compaction.
18	500	500 (P 4)	25	130	by 5½ lb. 12" fall hammer in two layers
20	500	500 (P 4)	25	130	each given 15 blows of hammer.

The test results are shown in Table 4.13.

4.1 0 DENSE GRADED SAND SAMPLE

The following grading was used to prepare samples of this group.

TABLE 4.8. DENSE GRADED SAND (S_2)

Sieve Size		% of Total weight.	Amount of each portion in 1000 gms. sampele.
Passing from	Retained on		
3/4"	No. 4	15	150
No. 4	No. 8	5	50
No. 8	No. 16	10	100
No. 16	No. 30	15	150
No. 30	No. 50	25	250
No. 50	No.100	20	200
No.100	No.200	10	100
	Total	100	1000

Sample No. 22 was prepared by using 1000 gms. of sand S_2 (Table 4.8) and 40 gms. of cement (P 200) binded by 130 gms. of Rubber Solution. The compaction of the sampel was as for sample No. 18 and the test results are shown in Table 4.13.

4.11. ROORKEE SOIL SAMPLES

Roorkee Soil Sample were prepared as follows :

TABLE 4.9. ROORKEE SOIL SAMPLES (CONSTITUENTS)

Material		Weight of material in each samples (gms.			
		21	23	24	25
1		2	3	4	5
Roorkee soil (sieved)	P 16 R 30	-	40	55	100
	P 30 R 50	-	400	550	900
	P 50 R 100	-	560	395	-
	P 4 R 100	1,000	-	-	-

Contd....37

Table 4.9. (Contd....)

1	2	3	4	5
Rubber Solution.	125	125	125	125.
Compaction by 5½ lbs. 12" fall standard hammer.	Compaction in two layers each given 15 blows of the hammer.			

For test results please refer table 4.13.

Sample No. 20 gave the k value of 0.687 kgm/cm^3 which was within the range required for testing. Moreover it produced homogeneous, isotropic mass of soil and rubber adhesive having desirable recovery.

To obtain the further specifications for quality control in the preparation of subgrade more samples (No. 26 to 32) were prepared with slight modifications as given below :-

Sample No. 26.

The constituents of this sample were as follows :

1. Coarse sand (S_1) (Table 4.5) 500 gms.
2. Roorkee Soil (P @ R 200) (U.S. Sieve) 500 gms.
3. Cement (p 200) 25 gms.
4. Rubber Solution. 130 gms.

The compaction was done as usual by 30 blows of hammer. Before testing, its bulk density was determined and found out to be 1.83 gms/c.c. Table 4.13 shows the test results of the sample.

Sample No. 27 .

Sample No. 27.

This sample was, again prepared out of A 50% sand S_1 (Table 4.5) and 50% soil (R 200). To check the epercentage of each size of their gradient present in the sample, the sample (1000 gms.) was sieved. The results were as follows :-

TABLE 4.10. MECHANICAL ANALYSIS OF SAND + SOIL SAMPLE

Sieve size (U.S.) Passing from	Retained on	wt. of each portion in gms.	% of total weight
No. 4	No. 8	12.0	1.2
No. 8	No. 16	169.0	16.9
No.16	No. 30	137.0	13.7
No.30	No. 50	269.0	26.9
No.50	No.100	290.0	29.0
No.100	No.200	88.0	8.8
No.200	-	35.0	3.5
	Total	1000.00	100.00

The sample was mixed with 130 gms of Rubber solution and was compacted by the standard hammer giving 30 blows in two layers.

The Bulk density of the mix was 1.53 gm/c.c. The other test results are shown in Table 4.13.

SAMPLE No. 28 and 29.

The mechanical analysis of the sand + Soil mix (50% each) used for preparing these samples was as shown below :

TABLE. 4.11. MECHANICAL ANALYSIS OF SAND & SOIL MIXTURE.

Sieve size (U.S.) Passing from	Retained on	Wt. of each portion in (gms.)	Percent of total weight.
No. 4	No. 8	11.0	1.1
No. 8	No. 16	169.0	16.9
No. 16	No. 30	155.0	15.5
No. 30	No. 50	282.0	28.2
No. 50	No.100	261.0	26.1
No.100	No.200	91.0	9.1
No.200	-	31.0	3.1
	Total	1000.0	100.0

For sample No. 28, 130 gms. of rubber solution was used while for No. 29 200 gms of solution was used with 1000 gms. of soil sand mixture in each case. The compaction was done by 5½ lbs. 12" fall hammer in two layers (15 blows per layer)

The density analysis showed :

Bulk density of sample 28 1.57 gms/c.c.

Bulk density of sample 29 1.51 gms/c.c.

The other test results are shown in Table 4.13.

SAMPLES 30, 31 and 32.

As can be seen from Table 4.13, the test results of samples 26 to 29 were not uniform. Further tests were therefore necessary in order to obtain better control over the uniformity of the subgrade material. Keeping the other factors constant compaction was varied in each sample and its effect over bulk density and 'k' value was studied. The test results were as follows :-

TABLE 4.12. SAND SOIL SAMPLES (COMPARISON)

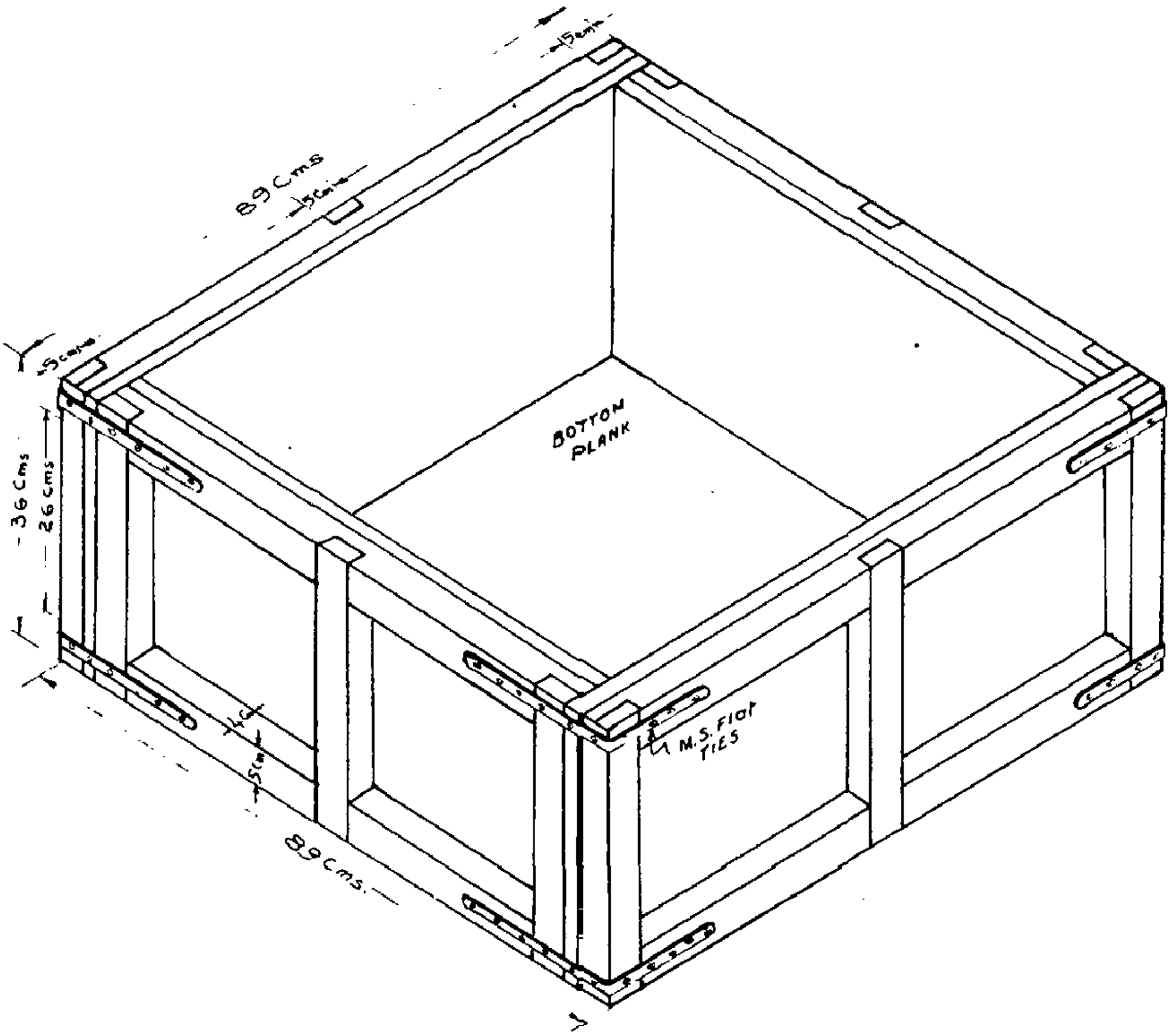
Sample No.	Compaction by standard 5½lb hammer 12" fall.	Bulk density gms/c.c.	k value (Theoretical) corresponding to 30" plate.
30	In one layer with 30 blows	1.38	0.713
31	In two layers each 15 blows	1.57	1.100
32	In two layers each 5 blows	1.35	0.749

N.B. For preparing each samples 1000 gms. of mixture was mixed with 150 gms. of Rubber solution.

The properties of finally selected material were found to be similar to that of sample No. 32. For grading of the material refer Appendix A at the end.of Fig. 6 shows the appearance of compacted samples.

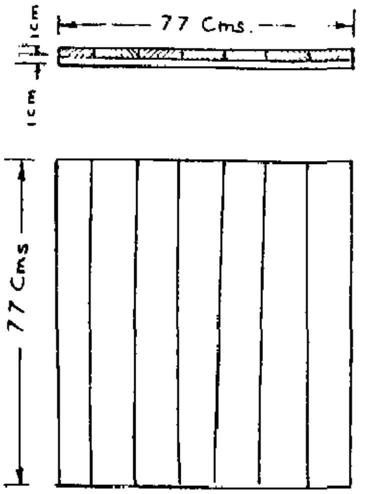
4.12. DESIGN OF SUBGRADE MOULD.

Subgrade of the model pavement was prepared in a mould. This mould facilitated the compaction of subgrade material during its preparation. Obviously, the mould was designed to be able to take the compacting loads without the walls giving way. The details of the subgrade mould are shown in Drawing No. S G 2 (Fig. 10). Fig. 3 shows the subgrade mould along with the reaction beam in the initial stage before filling the mould with the subgrade material.



SUBGRADE MOULD WITH BOTTOM PLANK IN POSITION

(SCALE 1 CM = 10 CMS.)



BOTTOM PLANK DETAILS.

(SCALE 1 CM = 20 CMS)

MATERIAL USED – CHIR WOOD

FIG. 1Q

RIGID PAVEMENT MODEL.	
DETAILS OF SUBGRADE MOULD	
DESIGNED and DRAWN – C.L. SARAF P.G. (HIGHWAYS) 1961-62	APPROVED: N. Vasavan
DRWG.NO. SG-2	SCALE AS SHOWN

TABLE 4.13:- SUMMARIZED DATA OF SUBGRADE INVESTIGATION OF TEST SAMPLES 10.16 Cm.ϕ LOADED WITH 5.08 Cm.ϕ BEARING PLATE

SAMPLE No.	MATERIALS USED IN Gms. AGGREGATE	FILLER	BINDER	COMPACTION	LOAD Kgm	TEST RESULTS AVG. SETTLEMENT OF BEARING PL. 2" ϕ mm.	'K' Kgm/cm³	K VALUE THEORETICAL CORRESPONDING TO 30" ϕ PB Kgm/cm³	COMMENTS
1	2	3	4	5	6	7	8	9	10
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6.	[illegible]	[illegible]	[illegible]	-do-	-	-	-	-	[illegible] of proper binding between the fibres distorted the shape of solid sample. Therefore could not be tested and hence rejected.
7.	[illegible]	[illegible]	[illegible]	-do-	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible] for [illegible] Triplicate
[illegible]	[illegible]	[illegible]	[illegible] petr.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible] subgrade material [illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible] sample [illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible] be tested.
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	-do-	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible] not proper and [illegible]
17. [illegible]	[illegible]	-do-	[illegible] 175	[illegible] in [illegible] layers	-	-	-		-do-
18. [illegible] Rk. soil [illegible] 500gms.	Cement [illegible] 25gm	[illegible]	[illegible]	[illegible] blows in 2 [illegible] layers.	5	[illegible]	13.7	[illegible] 45	[illegible] 'k' value but the material [illegible] [illegible] after [illegible] as done in Sample No. 2 [illegible]
19. Sand [illegible] 1000 [illegible]	Cement [illegible]	-do	30 [illegible] in [illegible] layers	[illegible]	[illegible]	[illegible]		From the sample it appeared that the grains of sand will make the subgrade friable and hence rejected.
20. [illegible] Rk.S. [illegible]	Cement [illegible]	-do-	30 blows in 2 layers.	5	[illegible]	[illegible]	[illegible]	[illegible] compacted [illegible] better [illegible] after the load is removed selected for subgrade preparations after few more trials.
21. Rk Soil (Ref.T.[illegible]) [illegible]	-	[illegible] 125	[illegible]	5	[illegible]	3.[illegible]		Very [illegible] value and therefore rejected.
22. [illegible] (Ref. [illegible]) [illegible]	Cement [illegible]	[illegible] 130	-do-	5	[illegible]	7.[illegible]	[illegible]	[illegible] particles present in the [illegible] the sample non-homogeneous and hence rejected.
23. Rk. Soil (Ref. T4 [illegible]) 1000 gm.	-	[illegible] 125	[illegible]	5	[illegible]	[illegible]	[illegible] 43	Very 'k' value hence rejected.
24.	-do-	-	do-	-do-	5	[illegible]	41.[illegible]	[illegible]	-do-
25.	-do-	-	-do-	[illegible]	5	[illegible]	24.7	1.[illegible]	High 'k' value, rejected
26. S_1 500gm (Ref.[illegible]) Rk. [illegible] 500gms.	[illegible] Cement [illegible] 25gm.	[illegible] 130	do	5	[illegible]	[illegible]	[illegible]	Very [illegible] 'k' value rejected.
27. S_1+Rk.S. (Ref.T4.[illegible]) 1000 gm.		-do-	-do-	5	[illegible]	17.64	[illegible]	High 'k' value, rejected
[illegible] S_1 + Rk.S (Ref.T.[illegible])	-	-do-	[illegible]	[illegible]	[illegible]	27.85	[illegible]	High 'k' value, rejected.
[illegible]	-do-		[illegible]	do-	5	[illegible]	[illegible]	[illegible]	Desirable [illegible] selected.
30. S_1 + Rk.S. (Refer Appendix A)	-	[illegible] 150	[illegible] blow in [illegible]	5	0.23	1[illegible].70	[illegible] 13	Compaction not uniform throughout the thickness of sample, it is more at the top than at the bottom, and hence rejected.
[illegible]	-do-	-	-do-	[illegible] blows in [illegible] layers.	5	[illegible]	16.[illegible]		[illegible] tively [illegible] more work is required to compact the specimen [illegible] [illegible] 'k' value [illegible]
32.	-do-	-		[illegible] blows in [illegible] layers.	5	[illegible]	11.23	[illegible]	[illegible] throughout and desirable 'k' value [illegible] finally for preparing [illegible]

N.B. 1. STANDARD HAMMER REFERS TO 5½ lb 12" fall hammer. 2. R.S. = Rubber Solution. 3. Rk. Soil = Roorkee Soil

A detachable bottom made out of laminated wooden planks was provided so that mould may be taken out leaving the subgrade in position if required .

4.13. PREPARATION OF SUBGRADE.

As mentioned earlier, the material for preparing subgrade consisted of sand, Roorkee Soil and Rubber Solution.

As it was decided to have the subgrade similar to Sample No. 32, the quantities of materials required for the construction of 77 cm x 77 cm x 30 cm. thick subgrade were calculated and are given in table No. 4.14. The quantities shown in Table No. 4.14 include an allowance of about 10% for the wastage of material during mixing and handling.

TABLE. 4.14. QUANTITIES OF INGREDIENTS USED FOR PREPARING SAND SOIL MIX.

Material	Properties of material	% to be mixed.	Total quantity required (Kgm.)
Sand	P 8 R 16	15	45.0
	P 16 R 30	15	45.0
	P 30 R 50	10	30.0
	P 50 R100	5	15.0
	P100 R200	5	15.0
Roorkee soil	P 4 R200	50	150.0
	Total.	100	300.0

The ingredients after sieving separately were weighed and then mixed together throughly . Three samples were then taken out of the heap and sieved.

The average of this sieve analysis is shown in Appendix A.

In order to obtain uniform compaction, the subgrade was prepared in layers as was done during sample testing. An approximate thickness of 2.0 cms. for each layer was adopted . The materials were mixed, in a mechanical mixer (see Fig. 5) holding a charge of 9 Kgm. sand soil and 1.35 kgm Rubber solution. Table 4.15 shows the actual quantities of materials used in each layer of sub-grade prepared.

TABLE 4.15. QUANTITIES OF MATERIALS USED IN EACH LAYER OF SUBGRADE.

Layer No.	Material used in Kgm. Sand-soil.	Rubber solution.	App. thickness of layer at the end of compaction (Inches)	(cms.)
1.	36	5.40	1.9	4.8
2.	35	5.25	3.7	9.4
3.	36	5.40	5.5	14.0
4.	36	5.40	6.8	17.3
5.	36	5.45	8.5	21.6
6.	36	5.40	10.6	27.0

An attempt was made to achieve a compaction similar to that of sample No. 32. and the number of blows of the standard hammer to be given to the subgrade were calculated on the basis of the exposed area of the sample for purposes of compaction. Table 4.16 shows the number of blows given to each layer and the results of bore hole tests. The test samples were obtained by means of a cork borer (See Fig. 9) and density was determined for each of them by mercury displacement method. In order to avoid weak spots formed by the removal of

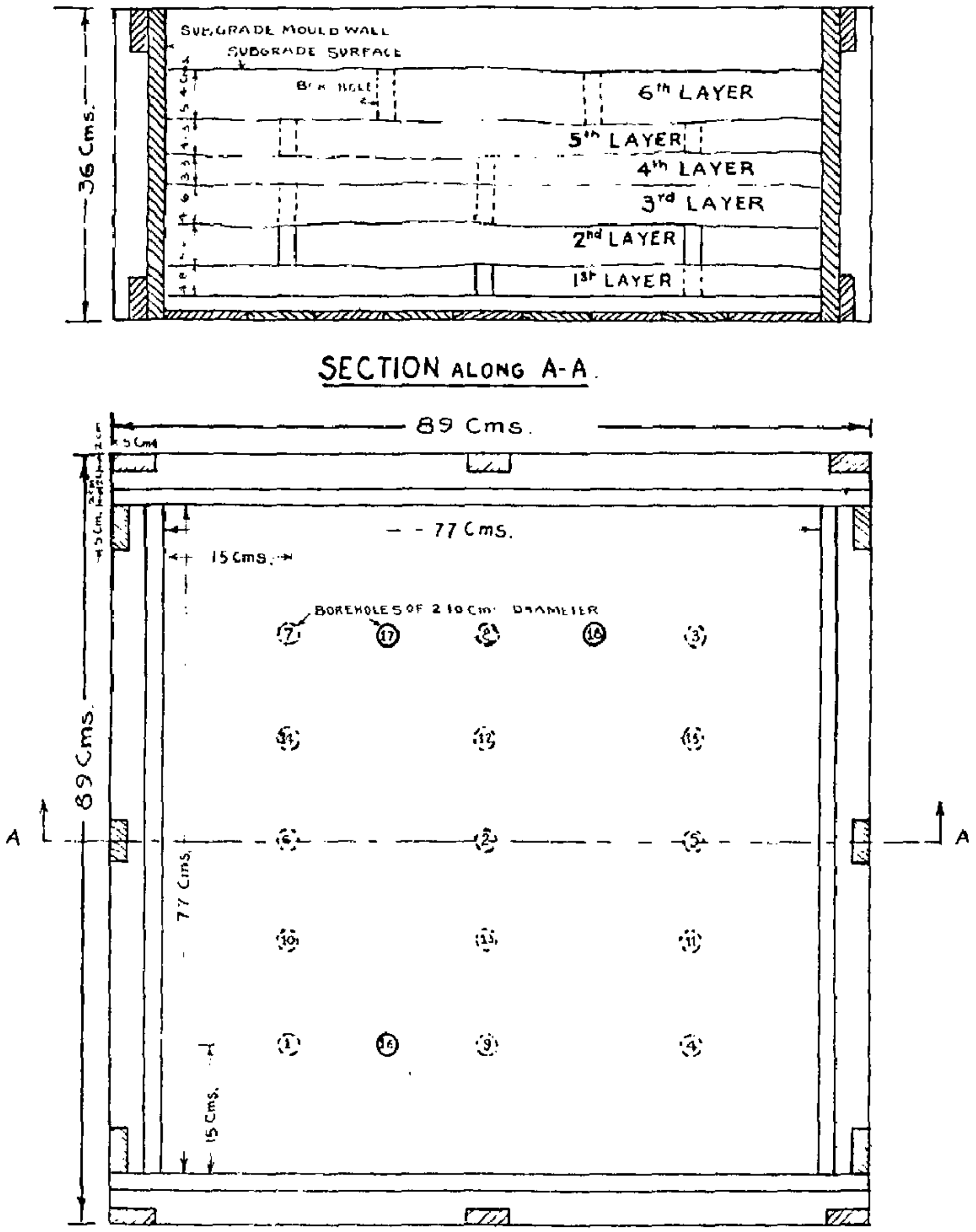


FIG. 11 - DETAILS OF BOREHOLES SUNK IN SUBGRADE

(3 Boreholes in each Layer)

CMS. 5 0 5 10 CMS.

SCALE.

material from various points, the test holes were staggard as shown in Fig. 11. Each hole was filled back by the same type of material before laying the next layer.

TABLE 4.16 Details of compaction and bore hole tests.

Layer No.	Number of blows of standard hammer for the layer.	Bore hole sample No.	Average Density of bore hole sample in gms/cc (Av. of 3 determinations)	Average Density of the layer in gms/c.c.
1.	1,424	1	1.42	1.41
		2	1.45	
		3	1.36	
2.	1,520	4	1.39	1.43
		5	1.45	
		6	1.45	
3.	1,165	7	1.44	1.47
		8	1.44	
		9	1.53	
4.	850	10	1.38	1.41
		11	1.42	
		12	1.42	
5.	550	13	1.33	1.34
		14	1.35	
		15	1.33	
6.	500	16	1.33	1.46
		17	1.39	
		18	1.65	

The top most layer was finished smooth to provide a uniform contact for pavement.

4.14 DESIGN OF PAVEMENT SLAB.

The considerations while undertaking the design of pavement slab were as follows :-

i) The slab must consist of homogeneous, isotropic, elastic material.

ii) It must be less susceptible to temperature and humidity changes.

iii) The modulus of elasticity should be such that the dimensions of the slab are neither too large nor too small. (Scale ratio is dependent on 'l' value and l is dependent on E, refer Chapter 2).

Keeping the above considerations in mind two materials viz. perspex and Bakelite were chosen for preliminary investigations and the values of modulus of elasticity were determined by beam test loaded at the centre. The average value of modulii of elasticity 'E' obtained at the room temperature (about $40^{\circ}C$) were as follows :-

1.	Bakelite	E	$= 129 \times 10^3$ kgm/cm^2
2.	Perspex	E	$= 26 \times 10^3$ kgm/cm^2

Based on the above approximate values of E the dimensions of an infinite slab were calculated into cases and it was found that with the available thickness of perspex sheet, the dimensions of the slab become too small and the instrumentation will become difficult. Hence Bakelite was chosen for the model pavement slab.

A slab of the size 38cm x 38 cm was taken for preliminary investigations (the thickness of such slab was 0.7886 cm). It was loaded centrally by a 5 kgm load over contact area of 2 cm. diameter and it was found that the spread of deflection in the pavement was just within the dimensions of the slab and hence this size was adopted.

5.

INSTRUMENTATION AND TESTING PROCEDURE

5. INSTRUMENTATION AND TESTING PROCEDURE

5.1. GENERAL :

Proper instrumentation of the model is necessary for measurement of strains and deflections of the pavement slab due to static loads. For this purpose, the complete instrumentation of the model may be divided into the following components.

1. REQUIREMENTS FOR MEASUREMENT OF DEFLECTION :
   - (a) Proper loading device capable of transmitting the required load over the pavement.
   - (b) Contact areas or foot prints of required size which are loaded through loading device and which would transmit the same to the pavement.
   - (c) Datum frame for fixing the dial gauges.
   - (d) Dial gauges of required sensitivity to record the deflections of the pavement at various points.

2. REQUIREMENTS FOR MEASUREMENT OF STRAINS:
   - (a) Proper loading device to load the pavement.
   - (b) Foot prints of proper sizes.
   - (c) Electrical strain - gauges to measure the strains of the pavement.
   - (d) Strain Indicator to record the strains directly, along with the connection units.

5.2. Reaction Beam as described earlier was used as the loading device for transmitting the required load through the foot-print to the pavement. (See Fig.3.)

5.3. Circular contact areas were used in the present investigations. The sizes of the foot prints were fixed on the basis such that the test results obtained from them may cover the entire range of prototype conditions.The sizes of the foot prints used were 0.5 cm, 1.0 cm. , 2.0 cm., 3.0 cm., 4.0 cm., 5.0Cm., 6.0 Cm and 7.0 cm. , in diameter.

5.4. Keeping in view the size of dial gauges available, it was not possible to keep them closer than 8 cm. centre to centre. Thus except for interior loading where the deflection contours are circular, the deflection measurements for other loadings were not possible. In the case of interior loading advantage of symmetry was taken and the dial gauges were staggard to be able to measure the deflections at 2 cm. apart. With this object in view the datum frame was chosen out of a heavy M.S. Flat (See Fig. 14) . Supported on the subgrade mould and secured in position by means of heavy weight as shown.

5.5. The dial gauges capable of reading direct upto 0.01 mm were chosen for the purpose and in all 6 Dial Gauges were used to measure the deflection due to interior loading (see fig. 14).

5.6. The same reaction beam was used as a load transfer device while measuring strains (see Fig. 1 and 3).

5.7. Circular contact areas of sizes of 0.5 cm, 1.0 cm, 2.0 cm, 3.0 cm., 4.0 cm., 5.0 cm, 6.0 cm and 7.0 cm diameter were used,through which the load was transferred to the pavement.

5.8. Electrical strain gauges were used to measure the strains in the pavement slab. The smallest size available in the market was chosen so that the

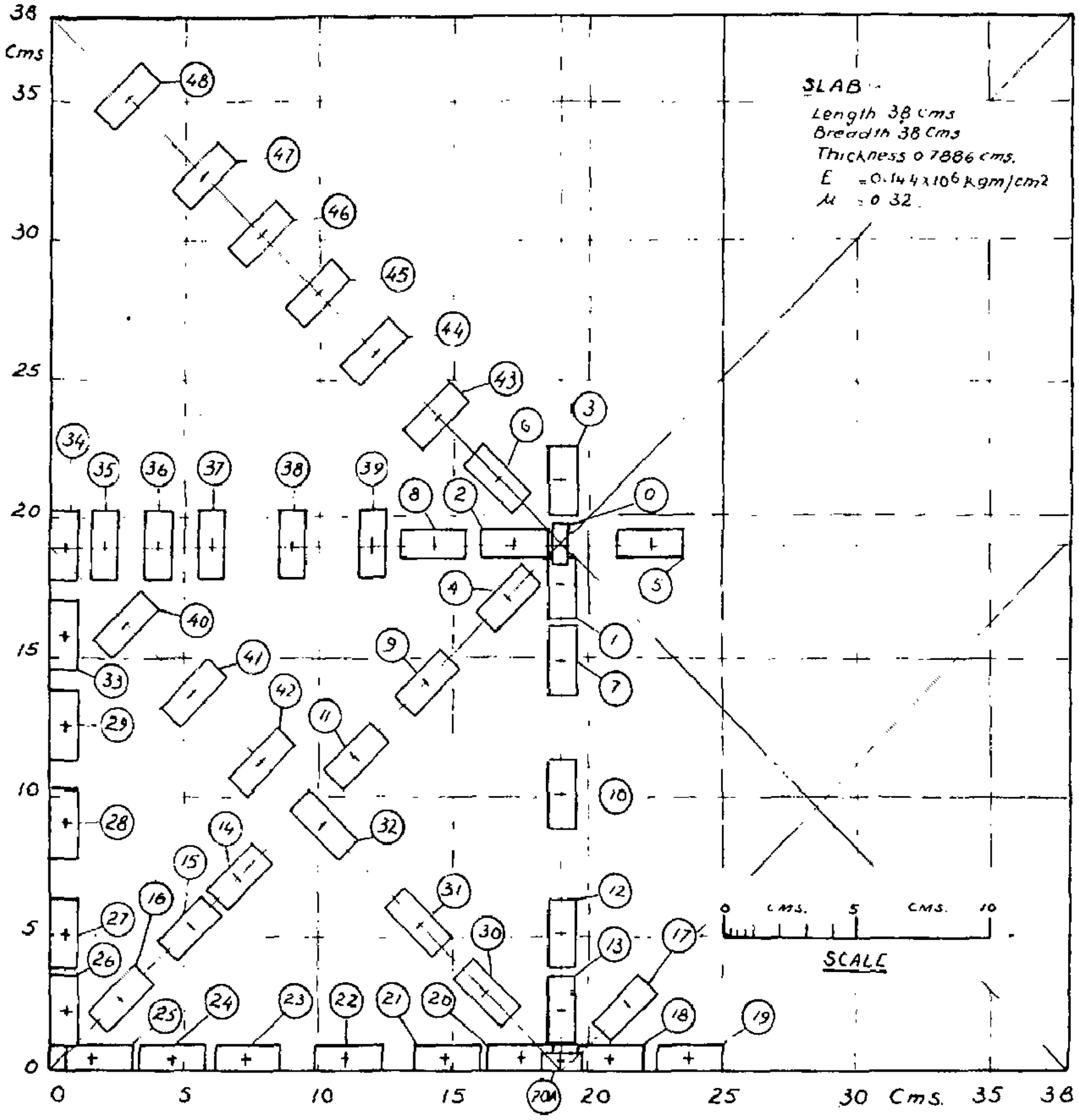


FIG. 12 - ARRANGEMENT AND LOCATIONS OF STRAIN-GAUGES ON THE MODEL SLAB.

STRAIN GAUGE No	DISTANCE PARALLEL TO X-AXIS (CMS)	DISTANCE PARALLEL TO Y AXIS (CMS)	STRAIN GAUGE No	DISTANCE PARALLEL TO X AXIS (CMS)	DISTANCE PARALLEL TO Y AXIS (CMS)	STRAIN GAUGE No	DISTANCE PARALLEL TO X AXIS (CMS)	DISTANCE PARALLEL TO Y AXIS (CMS)	STRAIN GAUGE No.	DISTANCE PARALLEL TO X AXIS (CMS)	DISTANCE PARALLEL TO Y AXIS (CMS)
0	19.00	19.00	13	19.00	2.25	25.	1.50	0.50	38	9.00	19.00
1	19.00	17.90	14	7.20	7.20	26	0.50	2.25	39	12.00	19.00
2	17.25	19.00	15	6.50	6.50	27	0.50	5.00	40	2.83	16.17
3.	19.00	21.25	16	3.00	3.00	28	0.50	9.00	41.	5.30	13.70
4	17.25	17.25	17	21.48	2.48	29	0.50	12.50	42	7.78	11.22
5	22.25	19.00	18	20.75	0.50	30	16.17	2.83	48.	2.83	35.17
6	16.62	21.38	19.	23.40	0.50	31	13.70	5.30	47	5.66	32.34
7	19.00	14.75	20.	20.25	0.50	32	10.15	8.85	46	7.78	30.22
8.	14.25	19.00	20A	19.00	0.38	33	0.50	15.75	45	9.90	28.10
9	14.30	14.30	21	14.75	0.50	34	0.50	19.00	44	12.00	26.00
10	19.00	10.00	22	11.00	0.50	35.	2.00	19.00	43	14.30	23.70
11	11.40	11.40	23	7.50	0.50	36	4.00	19.00			
12	19.00	5.00	24.	4.50	0.50	37.	6.00	19.00			

maximum number of gauges may be fixed in the available area.

The positions of the gauges fixed are shown in Fig. 12. while fixing the gauge advantage of symmetry was taken sometimes and the gauges were staggard in order tomeasure the strains more closely. Fig. 16 shows the strain-gauges in position along with the wiring system used. The specifications of the gauge used were as follows :

(a) Make Rohit & Co, Roorkee.

(b) Type Flat, bonded wire.

(c) Size of grid About 12.0 mm (½") long.

(d) size of Gauges 24 mm x 10 mm.

(e) Gauge factor 2.9 ± 2%.

(f) Resistance (varied for different gauges)

i)	For gauges No. 1 to 21 (except No. 3& 5)	117.5 ± 0.5%	Ohms
ii)	For gauges No. 3 and 5	119.0 ± 0.5%	"
iii)	For gauges No. [illegible] and 20 A	116.5 ± 0.5%	"
iv)	For gauges No. 22 to 33	116.0 ± 0.5%	"
v)	For gauges No. 34 to 42	115.5 ± 0.5%	"
vi)	For gauges No. 43 to 48	117.0 ± 0.5%	"

5.0. To record the strains, a highly precise strain indicator Bridge was used. The specifications of which were :

(a) Make Huggenberger. Zürich.

(b) Type Tepic indicator Type I T 1[illegible] (Portable transistor High precision strain indicator for static and dynamic measurements.

(c) Calibrations : Strains in 5 micro-units.

In order to make the test procedure easier an additional unit "Tepic Compensator" was used along with the indicator (See Fig. 7) . This unit facilitated the connection of 12 - strain gauges which could then be read simultaneously by means of a change over knob as shown in the above mentioned figure.

5.9. TESTING PROCEDURE.

Dr. H.M. Westergaard in his theoretical analysis of stresses in concrete pavements (4) has mentioned three cases which are of particular interest for calculating the critical stresses. He stated :-

> " In case I a wheel load acts close to a rectangular corner of a large panel of the slab. This load tends to produce a corner break. The critical stress is a tension at the top of the slab. The resultant pressure is assumed to be on the bisector of the right angle of the corner at the small distance a from each of the two intersecting edges; the distance from the corner, accordingly is $a_1 = a\sqrt{2}$. In case II the wheel load is at a considerable distance from the edges. The pressure is assumed to be distributed uniformly over the area of a small circle with radius a. The critical tension occurs at the bottom of the slab under the centre of the circle. In case III the wheel load is at the edge, but at a considerable distance from any corner. The pressure is assumed to be distributed uniformly over the area of a small semicircle with the centre at the edge and with radius a . The critical stress is a tension at the bottom under the centre of the circle ".

In the present investigations, the same three critical load positions have been investigated.

The present series of Tests consisted of the followings :

(a) Subgrade Investigations :

i) Investigations for the proper type of material (already discussed in the previous chapter).

ii) Tests for the grading and density etc. of subgrade material (already done in previous chapter.)

iii) Determination of modulus of subgrade reaction.

(b) Pavement investigations :

i) Determination of the modulus of elasticity and poisson's ratio of the pavement material.

ii) Strain distribution pattern on the pavement due to wheel loads placed at various positions.

DETERMINATION OF THE MODULUS OF SUBGRADE REACTION 'K'.

L.W. Teller and Earl C. Sutherland (17) have suggested three methods for determination of the modulus of subgrade reaction which in short are as follows :-

i) Load displacement tests : in which loads are applied at the centre of rigid circular plates of relatively small size, the pressure intensity on the soil being uniform over the entire area of the plate. In these tests the applied load, the mean vertical plate displacement and usually the time intervals are measured.

ii) Load displacement or load deflection tests in which the load is applied at the centre of slightly flexible rectangular or circular plates of relatively large dimensions. In this case some bending of the plate (or slab) occurs and the pressure intensity under the plate is not uniform throughout the area of its contact with the soil. The load, the vertical displacement of various points throughout the area of the plate and possibly time intervals are measured.

iii) Load-deflection tests on full size pavement slabs in which the load deflection data are obtained by measurement and used in Westergaard's deflection formulae to provide a value for the modulus of subgrade reaction 'k'.

Out of the above three methods mentioned above, second one was found to be most suitable for the present case and the modulus of subgrade reaction for the subgrade was investigated by the method (ii) out lined above. The pavement slab was used as a flexible plate (square shape) and the deflections of the plate at various points were determined. Figure 14 shows the general arrangement of the apparatus for this purpose. Figure 13 shows the deflection contours and calculations. The value of 'k' has been determined as follows :-

Modulus of subgrade displacement

$$= \frac{\text{Total vertical load applied to the subgrade.}}{\text{Volume of the subgrade displaced due to the applied load}}$$

$$= \frac{W}{V}$$

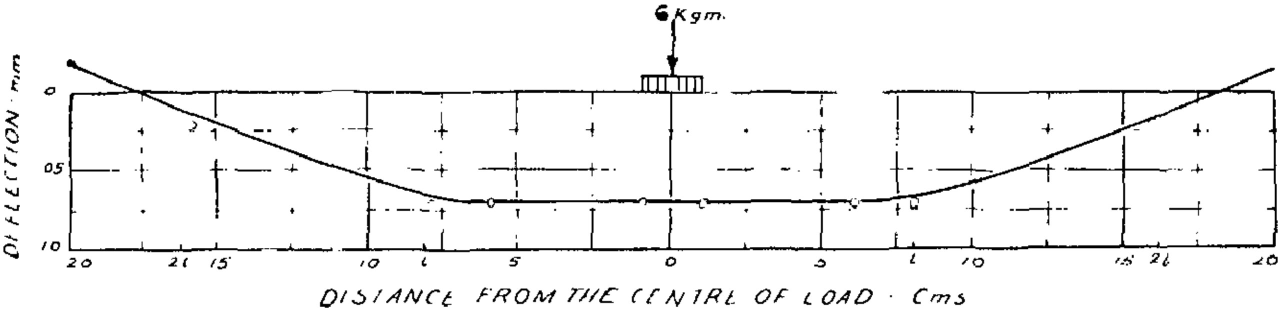
where W = Total load in Kgms.

V = Volume of subgrade displaced in cc.

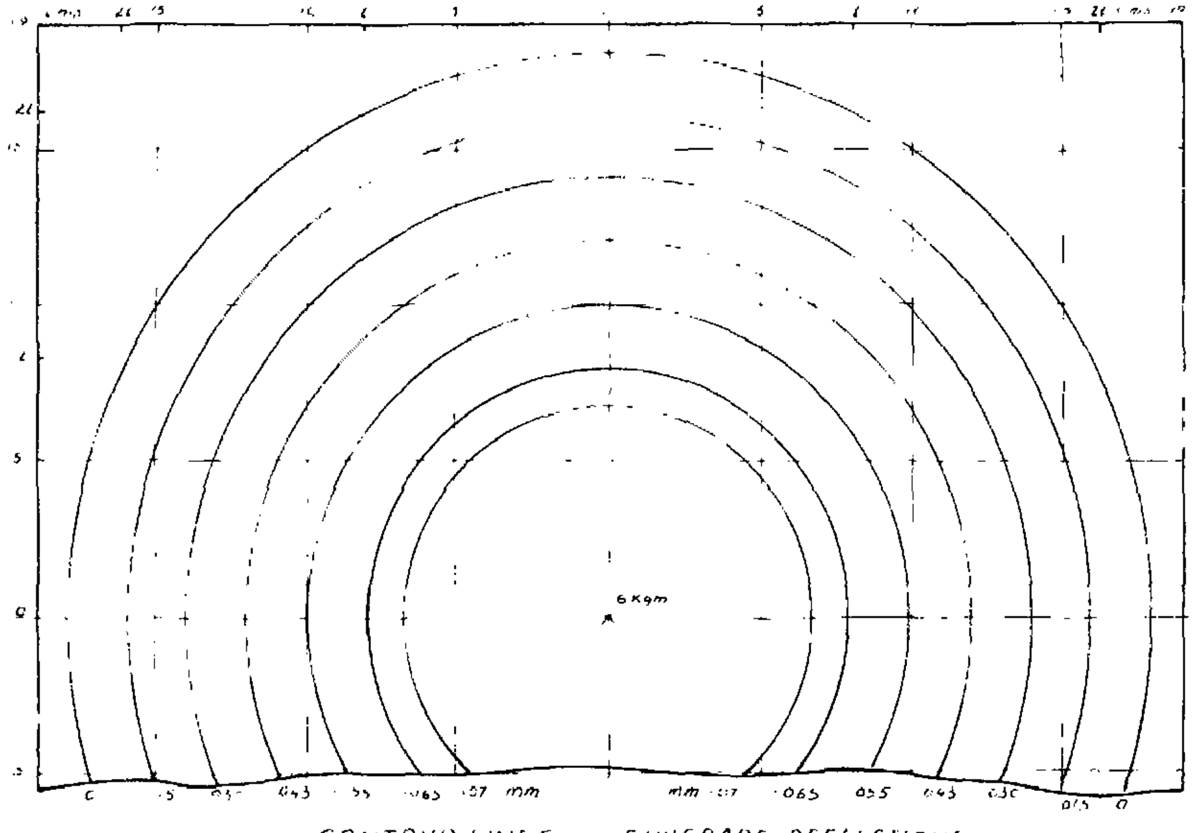
In all eight sets of observations were made to determine the 'k' value and the calculations similar to those shown in fig. 13 were made for each observation An average of these was then taken as the 'k' value of subgrade which was 1.5 Kgm/cm^3 . The other values were within $\pm$ 2% of this average value.

DETERMINATION OF E AND μ OF PAVEMENT MATERIAL i.e. BAKELITE.

The E and μ values of the Bakelite were determined in the laboratory by a beam test on the material. For this purpose a test beam 38 cm long and 2.527 cm wide was prepared out the same material as used in pavement. This had a thickness of 0.7866 cm. This beam was simply supported (see fig. 15) between two points 32 cm. apart. It was load at two points (each 8 cm. from the point



DEFLECTIONS ALONG A SECTION IN SUBGRADE



CONTOUR LINES FOR SUBGRADE DEFLECTIONS

FIG. 13 - DETERMINATION OF 'k' VALUE.

VOLUMETRIC AND OTHER COMPUTATIONS.

DISTANCE OF THE CONTOUR FROM THE CENTRE [illegible] (Cms)	DEFLECTION (Cms)	VOLUME OF THE SUBGRADE DISPLACED BETWEEN THE CONTOURS (C.C.)	MODULUS OF SUBGRADE REACTION 'K'
[illegible]	[illegible]		
7	[illegible]	[illegible]	$k = \frac{6.00}{3.953}$
8	·065	[illegible]	
10	[illegible]	[illegible]	1.515 kgm/cm^3
12	[illegible]	[illegible]	
14	[illegible]	[illegible]	
16	[illegible]	[illegible]	
18	[illegible]	[illegible]	
	TOTAL	3·953	

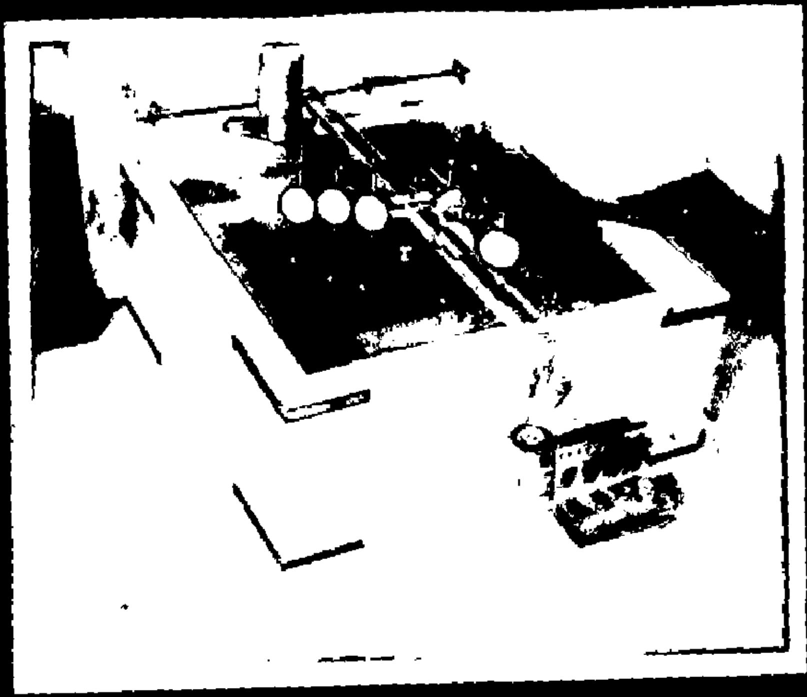


FIG. 14.- AN ARRANGEMENT MADE FOR PAVEMENT DEFLECTION MEASUREMENTS.

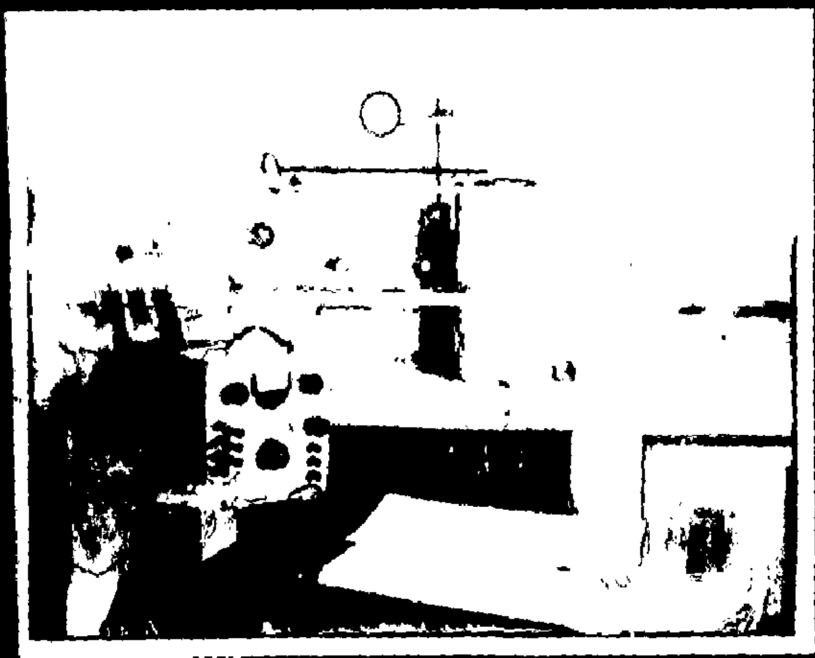


FIG. 15.- BEAM TEST FOR 'E' & 'μ' VALUES OF BAKELITE.

of support) and its deflections at the centre were measured by a direct reading dial gauge of sensitivity 0.01 mm. With the help of these load deflection data obtained, the E value was determined by the formula :

$$E = \frac{11\ W\ L^3}{384\ I\ Y}$$

where E = Modulus of elasticity of the material of beam in Kgm/Cm^2

W = Load in Kgm at each point.

L = Total length of Beam in Cms. (32 Cms. in present case).

Y = Deflections at the centre of beam due to load W (Cms.)

For getting μ value, two strain gauges (electrical) were pasted (one on each side of beam) at the centre of beam. The 6" long strain gauge was pasted in the longitudinal direction and ½" straingauge was pasted in the transverse direction to measure the strains in the corresponding direction. The Poisson's ratio μ was then determined as follows :-

$$\mu = \frac{\text{Unit strain in lateral direction.}}{\text{Unit strain in longitudinal direction.}}$$

The average of 9 observations for each gave the following results :

$E = 0.144 \times 10^6$ Kgm/Cm^2

$\mu = 0.32$

N.B. The experiments were carried out at the room temperature of 30° C.

MEASUREMENT OF STRAINS ON THE PAVEMENTS.

As described earlier, to get the strains on the pavement surface 50

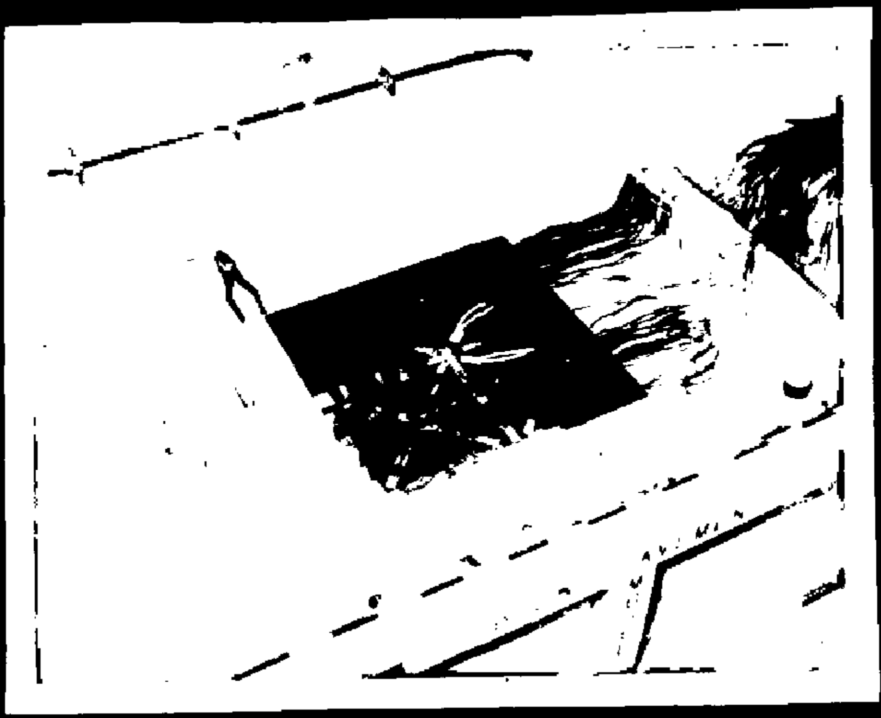


FIG.16 - POSITIONS OF STRAIN GAUGES WITH WIRING SYSTEM USED.

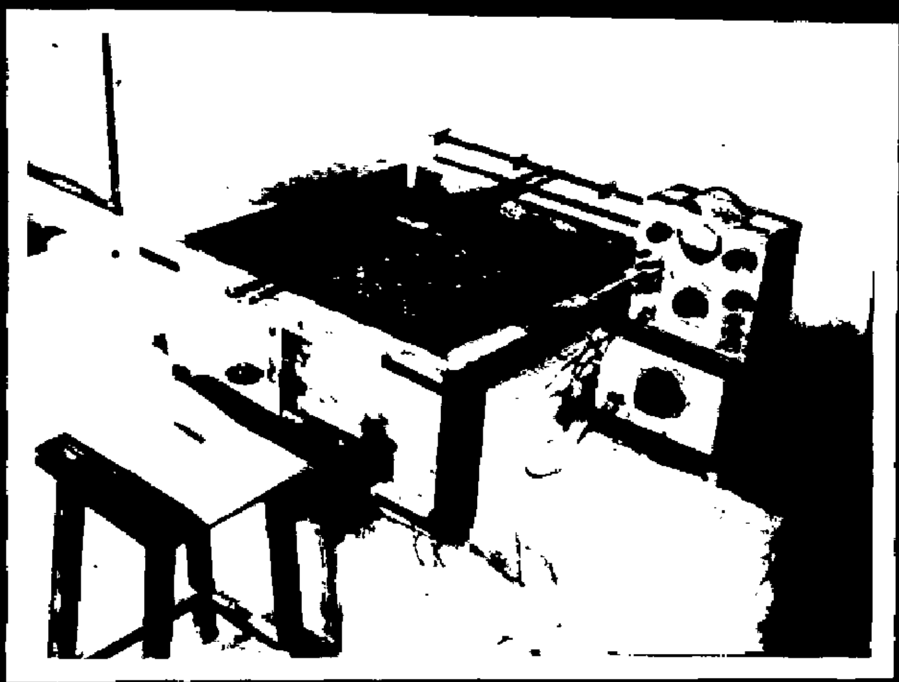


FIG. 17 - AN ARRANGEMENT MADE FOR INTERIOR LOADING TEST.
(TEPIC INDICATOR & COMPENSATOR) ON THE RIGHT.

FIG. 18 - AN ARRANGEMENT MADE FOR EDGE LOADING TEST.

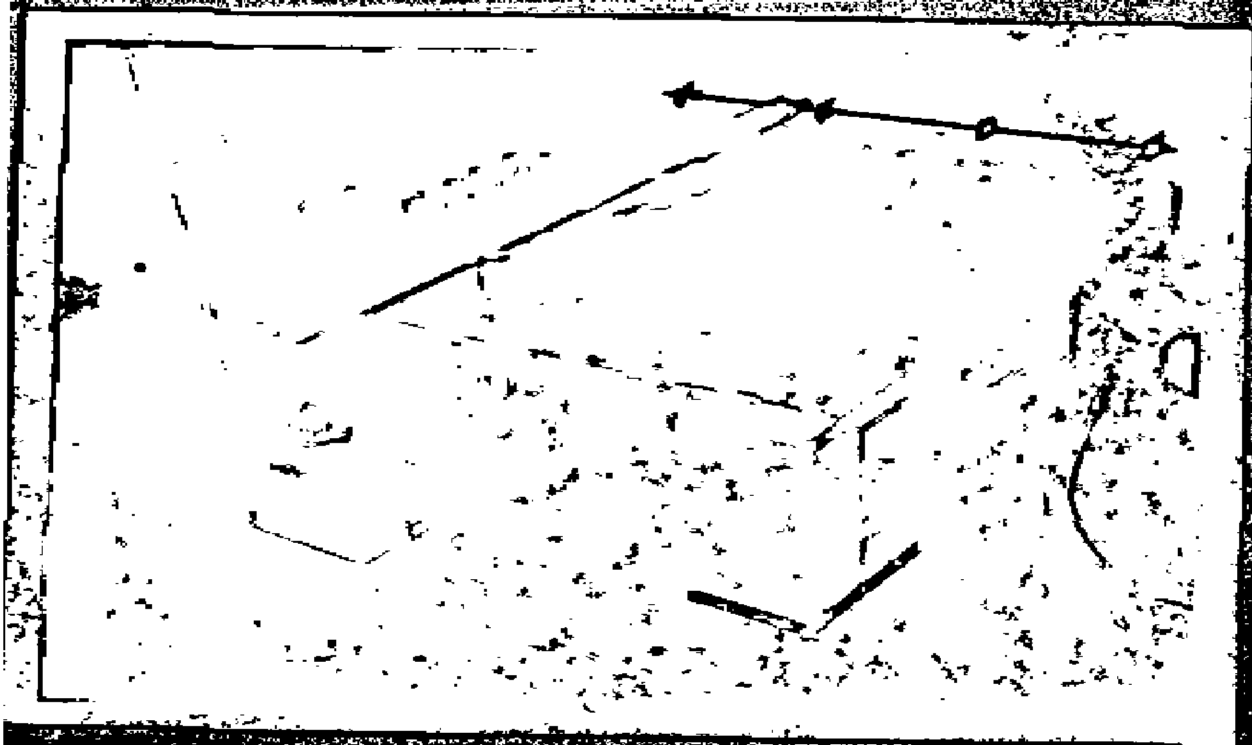


FIG. 19 - AN ARRANGEMENT MADE FOR

strain gauges were employed (See Fig. 12) with the help of these strain gauges, strains were recorded due to a load of 5 Kgm. acting over the foot prints. Data were obtained for strains for the following cases :

1) Load of 5 Kgm. acting on foot prints of 8 different sizes (0.5 cm, 1.0 cm, 2 cm. to 8 cm. diameter) placed at the interior of slab seperately.

2) Similar data when the load was acting at the edge of the slab.

3) Similar data when the load was acting at the corner of the slab.

Appendix C, D and E show the Typical data sheets for the three different cases of loading. Other data were recorded similarly.

Figures 17, 18 and 19 show the general arrangements made during the observations for strain measurements at three different critical positions.

6

ANALYSIS OF TESTS AND DISCUSSION OF TEST RESULTS

6. ANALYSIS OF TESTS AND DISCUSSION OF TEST RESULTS.

6.1. GENERAL CONDITIONS DURING TESTING.

An attempt was made to obtain a full subgrade support during testing so that the results obtained may be compared with the theoretical values, based on Westergaard's analysis. For this purpose, the top of the subgrade was finished smooth and a very thin layer of sand (fine) was used to fill up any irregularities. In order to prevent the lifting of the pavement during loading, lead cubes of 2Cm. size were distributed uniformly over the entire pavement. Thus, besides acting as a preventing device for the lifting of pavement, it also helped in simulating the field conditions.

The model slab used during the tests was thin and therefore no warping due to difference in top and bottom temperatures can be expected. Moreover the model was tested under roof. However, a minute change in 'E' value of the pavement material may result due to variations in temperature of bakelite sheet used on pavement. Thus, the tests were adjusted in such periods during which the temperature variation was minimum and close to that at which the 'E' value of pavement material was determined for the purpose of calculations. The maximum temperature during the tests was about 33°C and minimum was about 25°C. The average room temperature at which the 'E' value of pavement material was determined was about 30°C . The Poisson's ratio of the bakelite used was also tested at this temperature.

The modulus of subgrade reaction i.e. 'k' value changes in the field with the change of moisture content. In the present investigations, the subgrade was prepared out of such materials which eleminated this factor. Hence a constant 'k' value of 0.144×10^{6} Kgm/Cm^{3} (determined by loading the pavement) was assumed to be maintained in all the tests.

6.2. LIMITATIONS IN OBSERVATIONS.

The deflections of the pavement were measured only in the case of interior loading. For this purpose direct reading dial gauges having least count of 0.01 mm. were employed. By reading to the nearest half a division the observation were recorded within a variation of $\pm$ 0.005 mm. The size of the dial gauges did not permit to record the deflections, nearest than 8 cm. centre to centre but by staggering them, observations could be made @ 4 cm. centre to centre in case of interior loading. An arrangement similar to one shown in Fig. 5 was made separately to record the deflection at the centre of load by averaging the values obtained from two diametrically opposite dial gauges. Because of the difficulties mentioned above , the deflections could not be recorded for other loadings.

Strains on the top of pavement surface were recorded by means of electrical strain-gauges of gauge (or grid) length 12 mm. and it was assumed that the strain recorded by the gauge was acting at the centre of the strain-gauge grid. For the measurement of strains. Huggenberger, Tapic Indicator was used which was calibrated to read strains directly upto 5×10^{-6} centimetre per centimetre. Because of the size of the strain-gauges (with paper carrier) the minimum distance at which two adjacent strain gauges could be laid was 3 cms. The maximum distance varied according to requirement. The distances are shown in Fig. 12.

The 'E' and ' μ ' of the pavement material was determined in the laboratory by a two point load test on a beam. The stresses for the purpose of ' μ ' determination were recorded by means of electrical strain-gauges under the conditions similar to those mentioned above. For 'E' value calculations direct reading dial gauge of the type mentioned above was used. As these instruments (i.e. the dial gauge and strain recording bridge) have their own limitations the accuracy of tests is also limited and ultimately, the stresses calculated by use of these values of 'E' and ' μ ' will also have some limitations. These limitations can be considered as negligible and not affecting the stress values calculated as is evident from Westergaard's equations (4).

6.3. METHOD OF STRESS DETERMINATION OUTLINED.

The experimental stresses were determined in these investigations from the strains measured during the tests. For this purpose, strains in two perpendicular directions were measured . If e_r and e_t be such strains then from well known Hook's Law we can write :-

$$E\, e_r = S_r - \mu S_t \quad \ldots\ldots \quad \ldots\ldots \quad (1)$$

$$E\, e_t = S_t - \mu S_r \quad \ldots\ldots \quad \ldots\ldots \quad (2)$$

where E = Modulus of elasticity of the pavement material.

μ = Poisson's ratio for the pavement material.

S_r and S_t = Stresses in the directions corresponding to e_r and e_t respectively.

Combining the above two equations we get,

$$S_r = \frac{E}{1-\mu^2}\left(e_r + \mu e_t\right) \quad \ldots\ldots \quad (3)$$

$$S_t = \frac{E}{1-\mu^2}\left(e_t + \mu e_r\right) \quad \ldots\ldots \quad (4)$$

Equations (3) and (4) are general equations and can be applied for analysing any two perpendicular directional strains measured at the same point. At certain positions, they can be simplified as follows :-

MAXIMUM INTERIOR-LOAD-STRESSES :

The maximum interior load-stresses occur below the centre of load and the stresses are equal in all directions, or mathematically

$$S_r = S_t \quad \ldots\ldots \quad \ldots\ldots \quad ($$

thus, from equation (3) and (4) we get

$$e_r = e_t$$

which gives :

$$S_r = \frac{E\, e_r}{1 - \mu} \quad \ldots\ldots \quad \ldots\ldots \quad (5)$$

Substituting the conventional sign ' ' for ' S_r ' , we get the maximum interior stress as :

$$\sigma_i = \frac{E\, e_r}{1 - \mu} \quad \ldots\ldots \quad \ldots\ldots \quad (5\ a)$$

EDGE AND CORNER-EDGE STRESSES

For loading at a free edge the stress in a direction perpendicular to the edge is zero i.e. $S_t = 0$ Thus, from equation (4) we get,

$e_t = -\mu\, e_r$ and substituting this value of e_t in equation (3) we get,

$$S_r = E\, e_r \quad \ldots\ldots \quad \ldots\ldots \quad (6)$$

This relationship may be used for calculating the edge and corner - edge stresses.

6.4. ANALYSIS OF INTERIOR LOAD STRESS DATA :

Figs 20 to 27 represent the tangential and radial stresses due to interior loading with different contact areas under a load of 5 Kgms. These stresses were obtained with the help of strains measured on the surface of the pavement (Refer equations 1 to 6 para 6.3). A positive stress in the diagram represents tensions while a negative stress represents a compression in the slab. Each figure represents the stress distribution pattern for each foot print used during testing.

These diagrams show that

i) For all the contact areas investigated, the maximum stress occurs at the centre of the load.

ii) The stress contours are circular with load point as the centre. Both the radial and tangential stresses decrease with increase in distance from the centre. The negative (compressive) radial stresses on the top of the slab reach a zero value at a certain distance from the centre and then become tensile which also reduce to zero at another distance from the centre. The tangential stresses also reach zero value at a certain distance from the centre and later do not become tensile. The point beyond which no stress takes place is same for radial and tangential stresses.

iii) In all the cases, the compressive radial stresses are less than the tangential stresses outside the loaded area.

iv) The radial stresses become zero at a distance of about 7.25 cm. to 7.5 cm. or at about 0.9 l from the centre of load with varying contact area and varying load intensity.

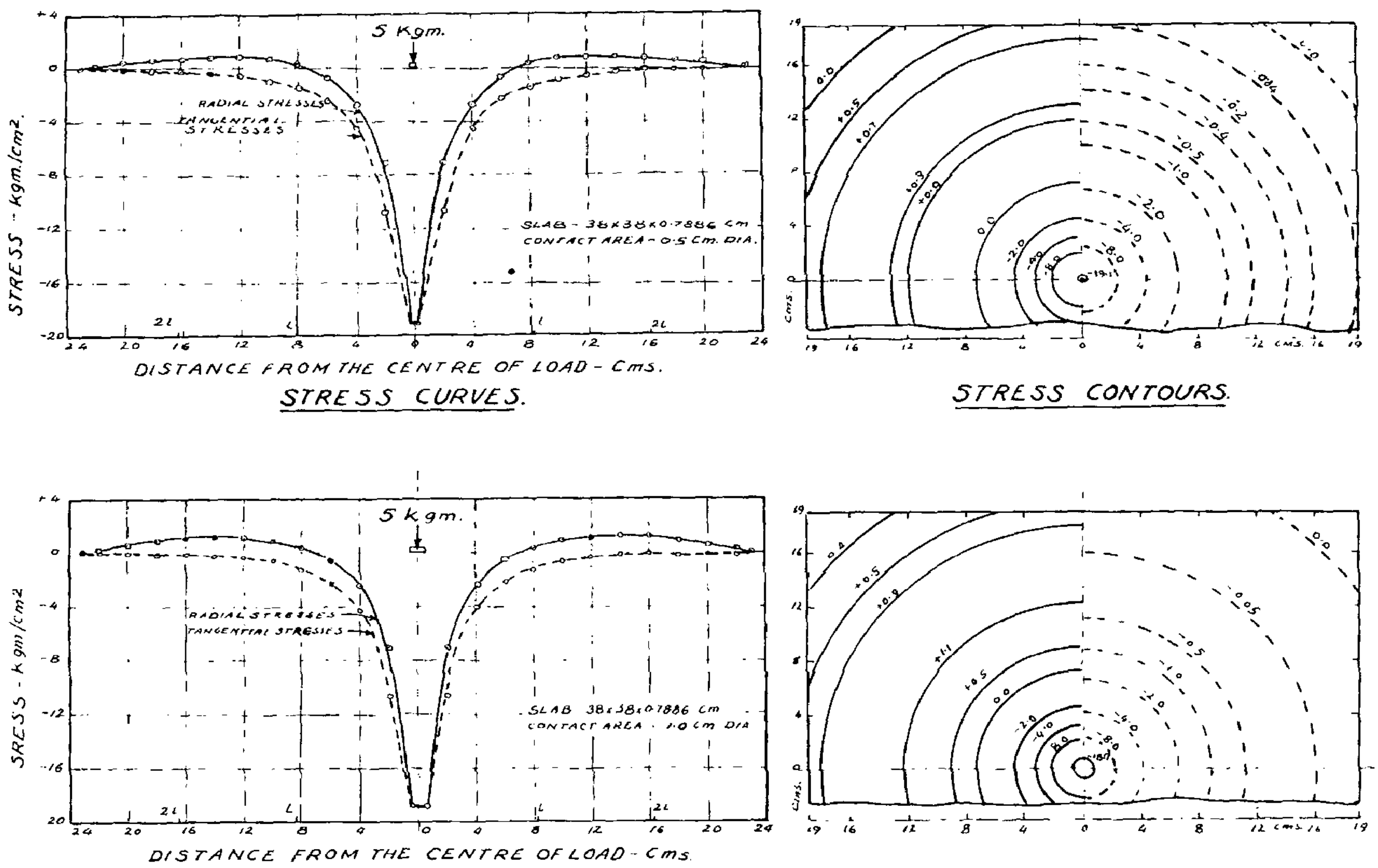


FIG. 20 & 21 - VARIATION IN TANGENTIAL AND RADIAL STRESSES DUE TO INTERIOR LOADING.

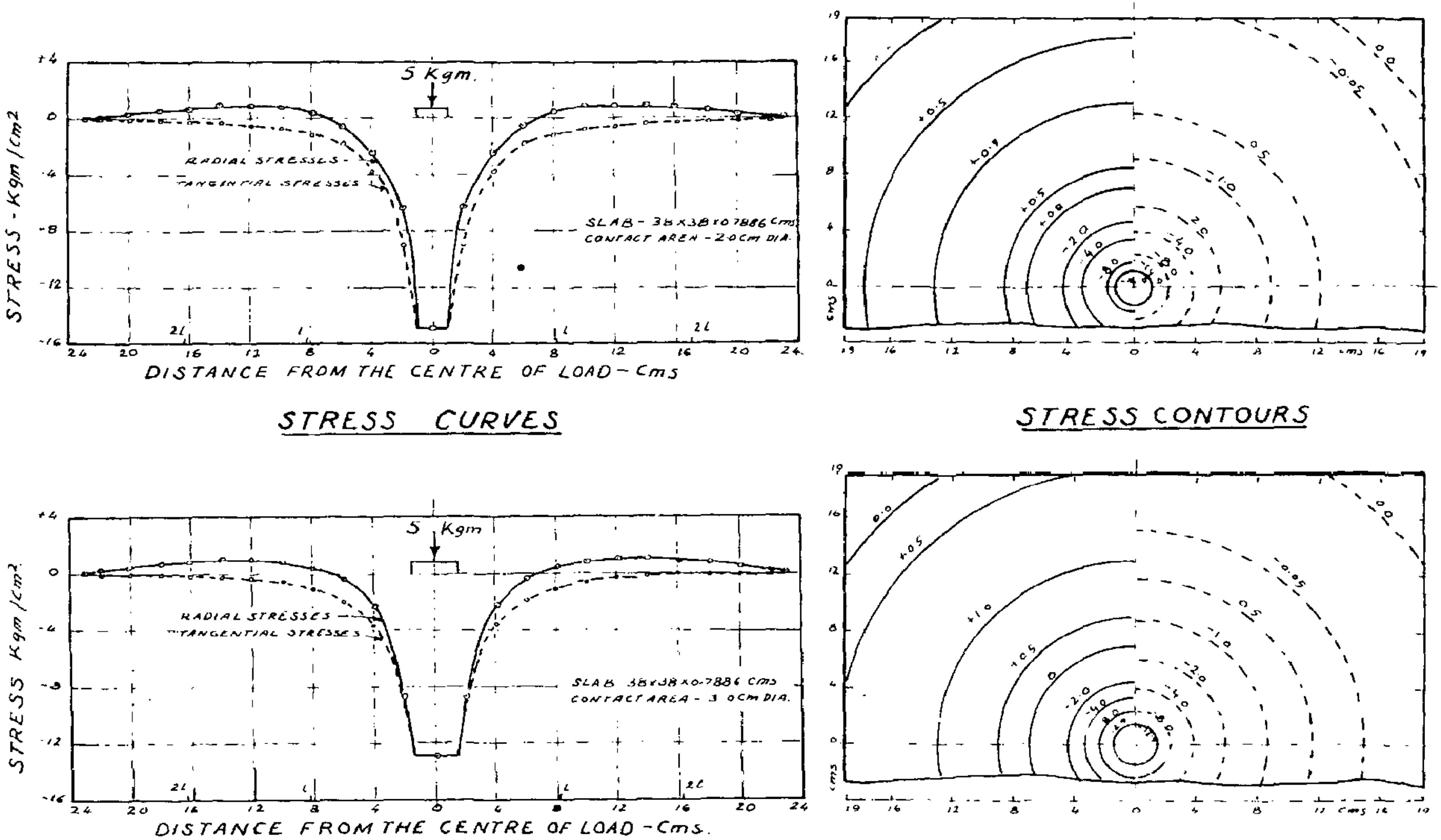


FIG. 22 & 23 - VARIATION IN TANGENTIAL AND RADIAL STRESSES DUE TO INTERIOR LOADING.

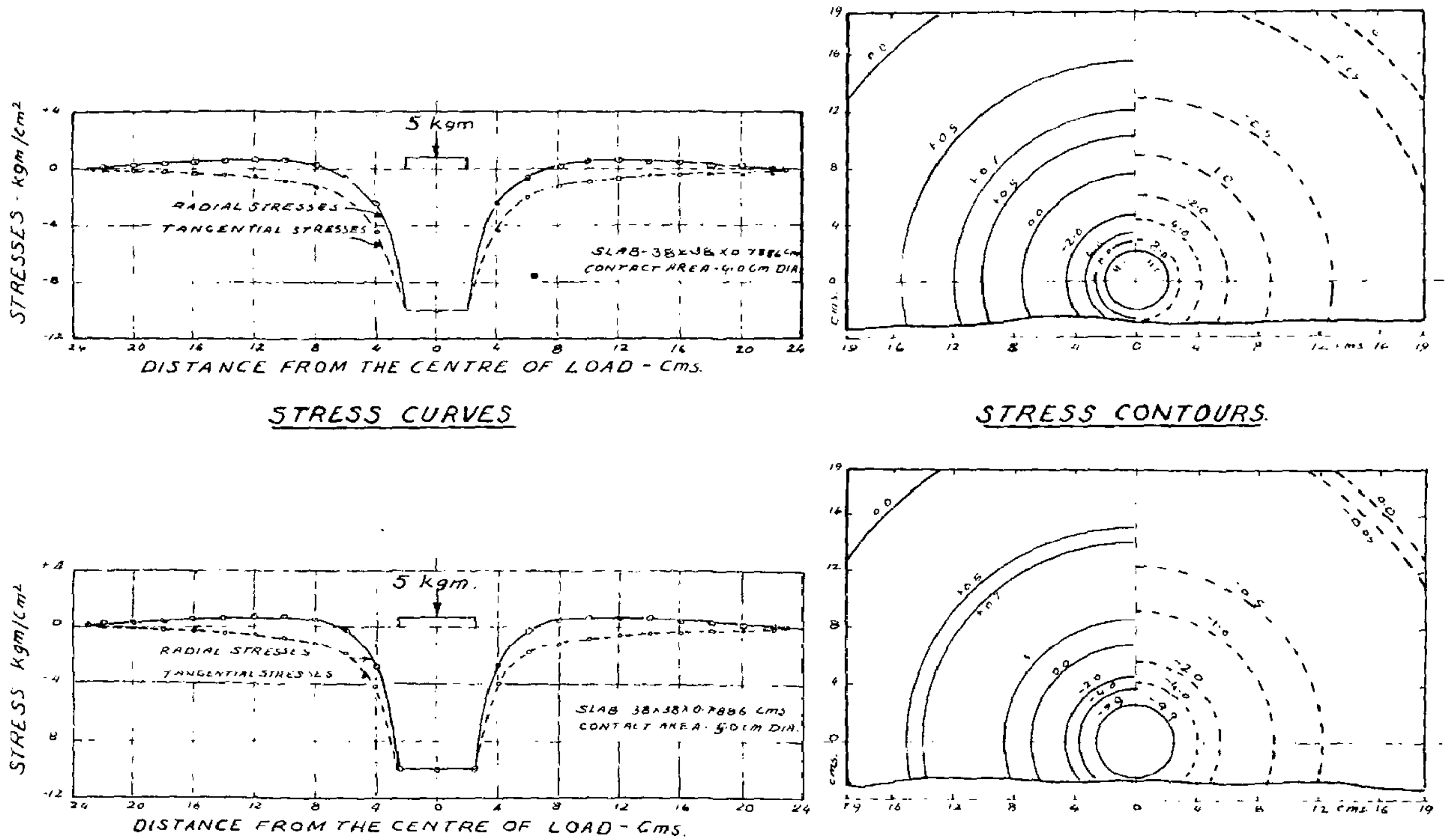


FIG. 24 & 25 - VARIATION IN TANGENTIAL AND RADIAL STRESSES DUE TO INTERIOR LOADING.

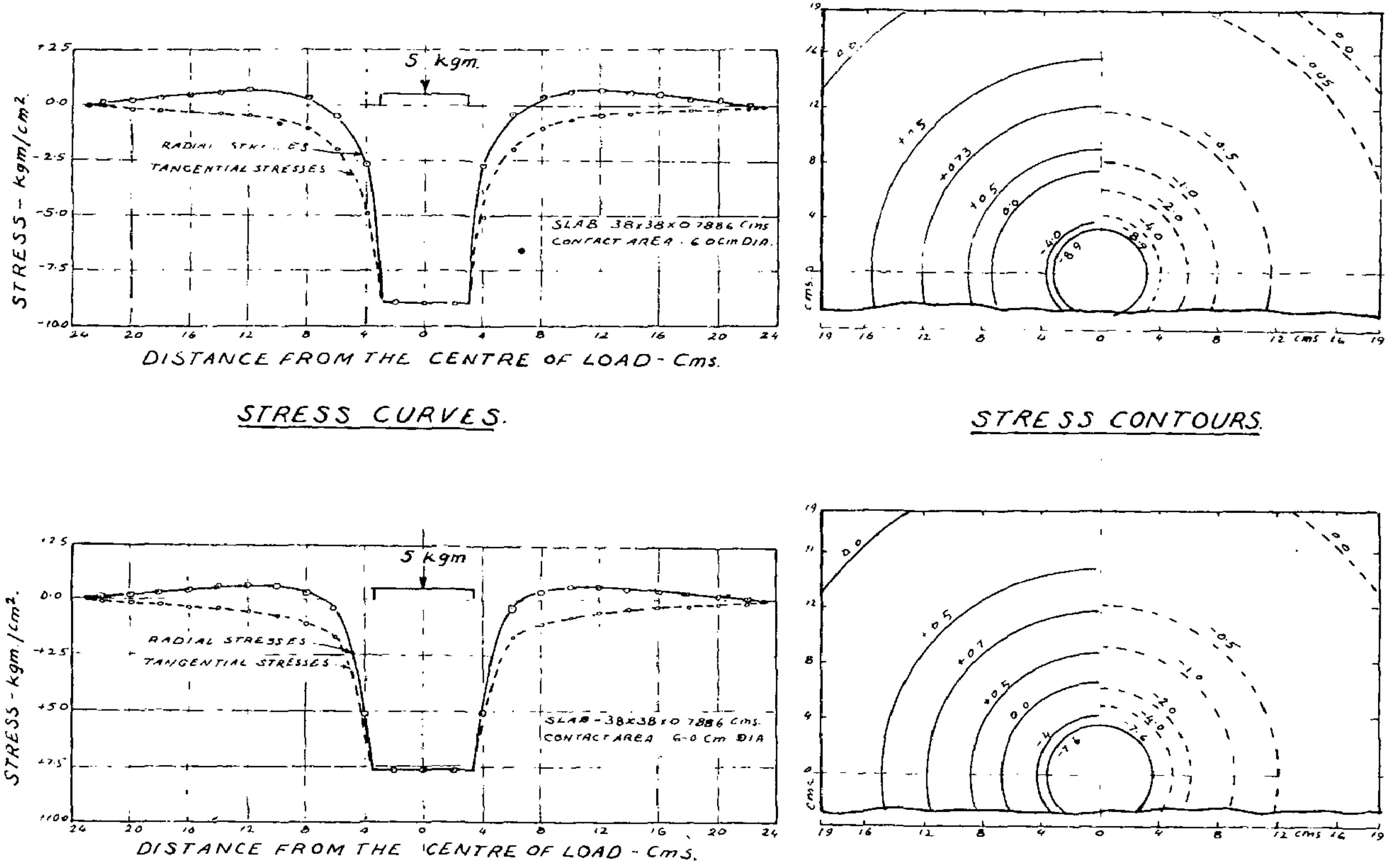


FIG. 26 & 27 - VARIATION IN TANGENTIAL AND RADIAL STRESSES DUE TO INTERIOR LOADING.

v) The stresses are concentrated in the vicinity of loaded area.

vi) The tangential stresses which are always compressive become zero at about 2.8 l in all cases of varying contact areas and varying load intensity. The radial stresses also tend to become zero again at this point (i.e. 2.8l).

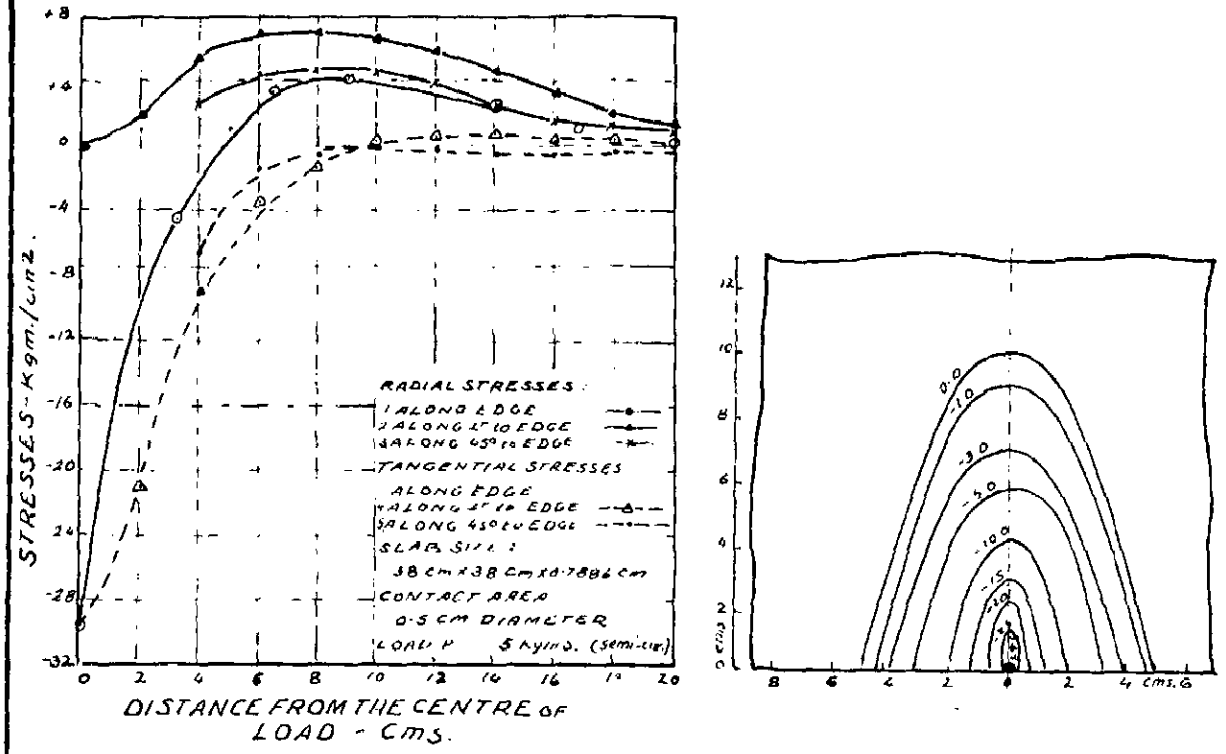
The theoretical (based on Westergaard's (4) equation) and observed critical stresses have been compared in figure 44. It can be seen from the figure that except for a slight variation of -1% to + 8% both the theoretical and experimental curves agree. The observed stresses are slightly higher than the theoretical stresses given by Westergaard's equations based on liquid subgrade theory. It can therefore be concluded that the Westergaard's analysis is applicable to interior loading.

The effect of the size of the contact area on the critical stresses can be studied from fig. 44. It is clear that the increase in the size of contact area or in other words decrease in l/a ratio leads to the reduction in the critical stress i.e. point loads are more dangerous from point of view of stresses in the pavement.

The spread of stress zone is independent of the size of contact area and load intensity.

4.5. ANALYSIS OF EDGE LOAD STRESS DATA.

Figs. 28 to 35 present the tangential and radial load stresses due to edge loading with different contact areas under a load of 5 Kgms. These stresses were obtained with the help of strains measured on the surface of pavement.



STRESS CURVES

STRESS CONTOURS

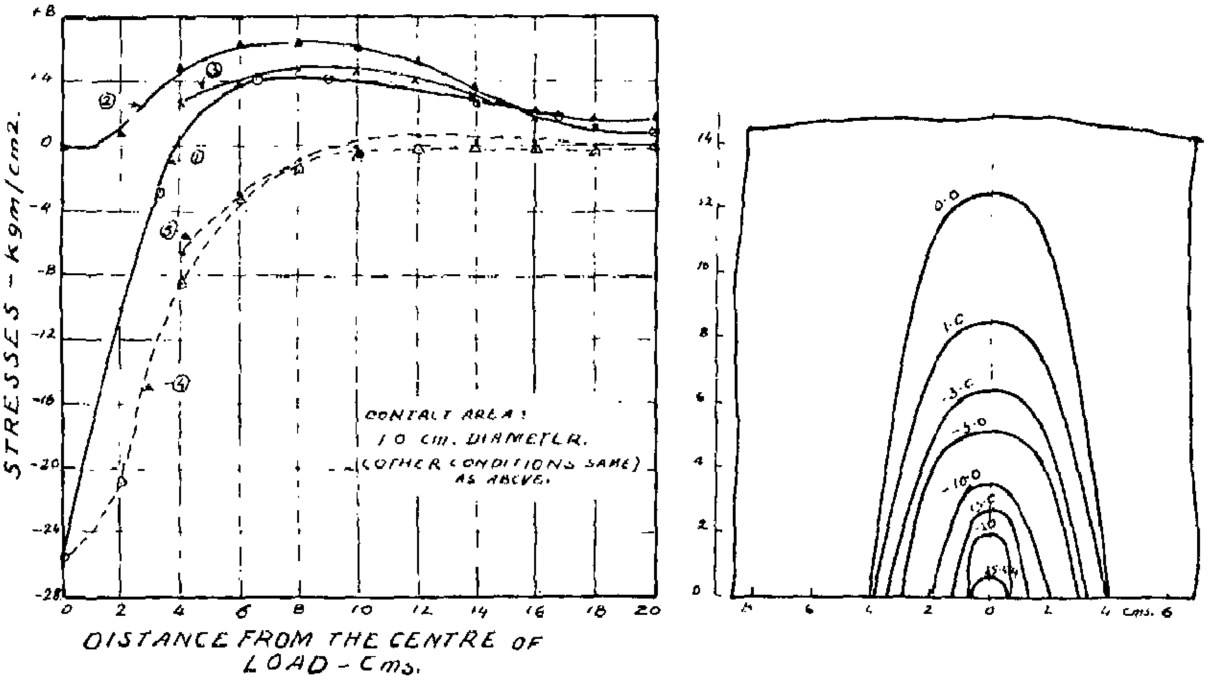
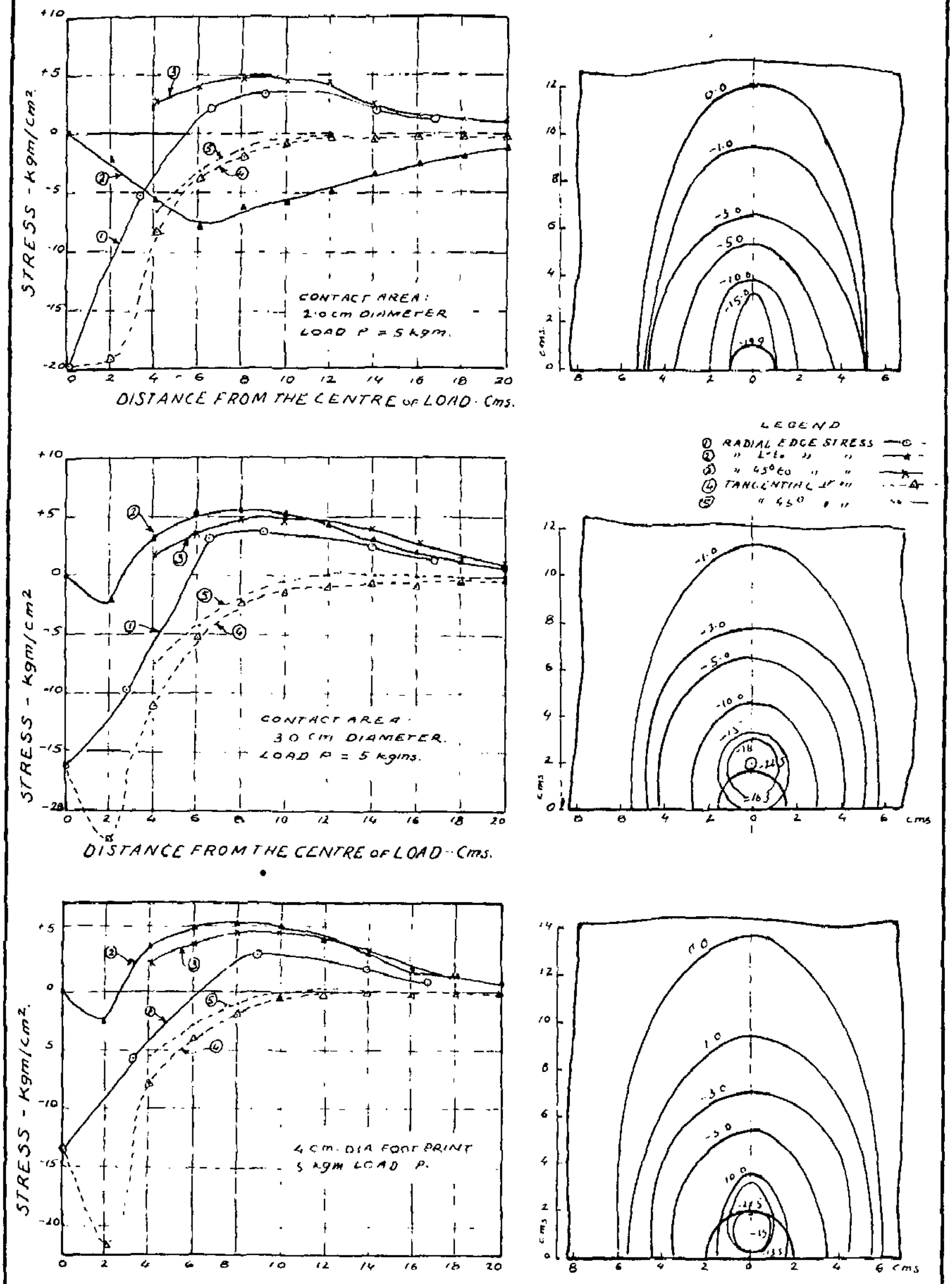


FIG. 28 & 29 - VARIATION IN STRESSES DUE TO EDGE LOADING.



STRESS CURVES

STRESS CONTOURS.

FIG. 30, 31 & 32 - VARIATION IN STRESSES DUE TO EDGE LOADING.

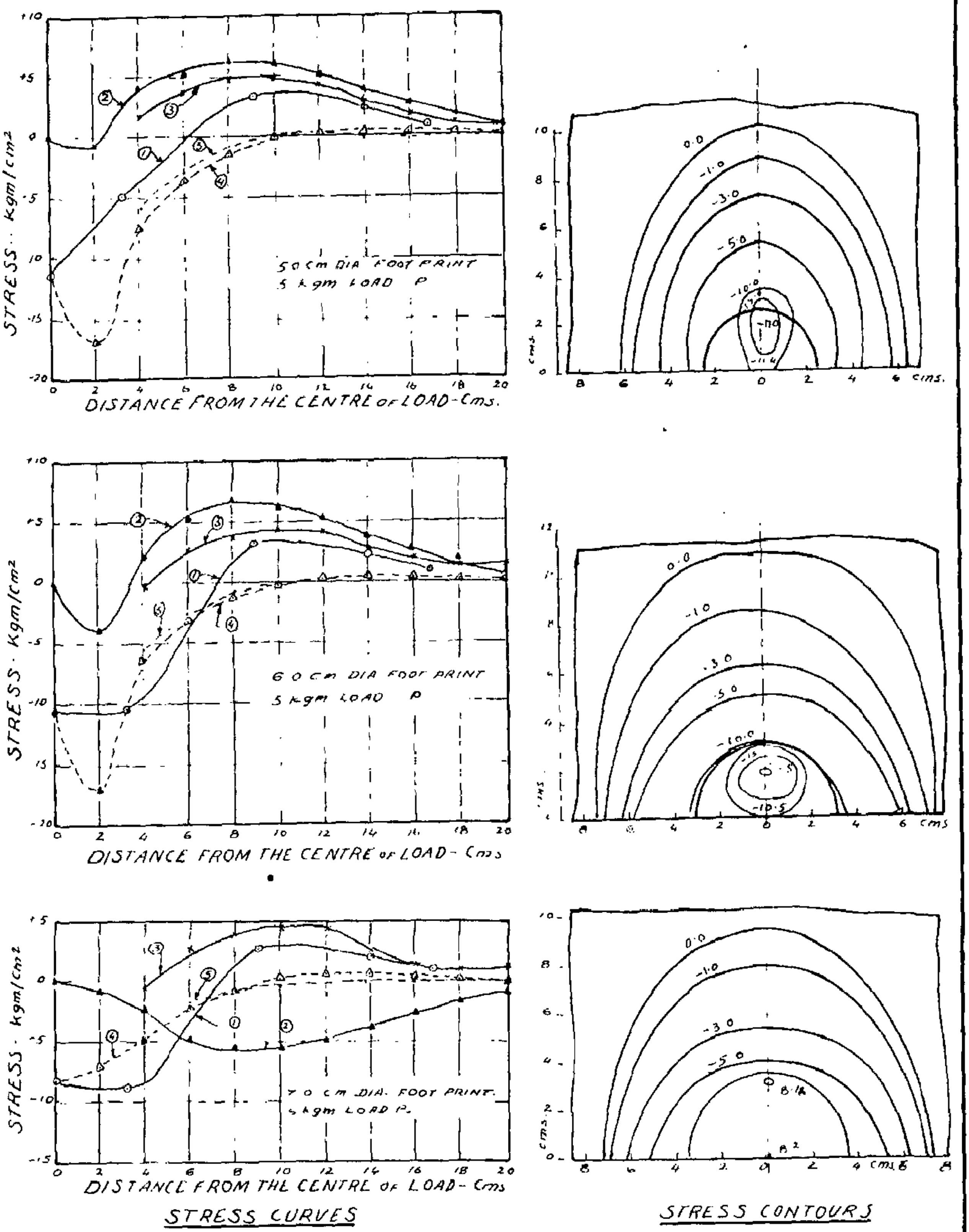


FIG. 33, 34 & 35 · VARIATION IN STRESSES DUE TO EDGE LOADING.

LEGEND
1. RADIAL EDGE STRESS
2. " 1" to " "
3. " 45° to " "
4. TANGENTIAL 1" to " "
5. " 45° to " "

(refer equations 1 to 6 para 6.3). The same convention as described in para 6.4 has been adopted here to represent the stresses on the diagrams.

The figures represent stress distribution pattern along three lines i.e. along edge, along a perpendicular to the edge, and along a line 45° to the edge. These diagrams show that :-

i) The critical stress occurs right at the edge under the centre of the load for foot prints of 0.5 , 1.0 and 2.0 cm. (or equivalent to 0.0615 l, 0.123 l and 0.246 l cms. on prototype slabs), but for other higher contact areas, the critical stress starts shifting along the perpendicular line (possibily) at the centre of gravity of the contact area). However, the critical stresses always remain within the area of foot print.

ii) The critical stresses are always compressive.

iii) The magnitude of compressive tangential stresses along perpendicular is greater than the magnitude of stresses in other direction in most of the cases. However, the concentration of stresses is near the load point.

iiii) The zero stress occurs along the edge at a distance of about 5 to 7.25 cm. (or 0.6 l to 0.9 l) i.e. the spread of stresses along the edge is within a distance of l on either side of the load.

iv) The stresses are spread widely along the perpendicular line. The critical tangential stresses along the perpendicular line occur near the edge and are zero at a distance of about 10 cm. or 1.25 l. The radial stresses are zero at the edge and become maximum at a distance of about 8 cm. or l.

(vi) So far the magnitude of stresses is concerned, the compressive stresses are maximum and occur along and parallel to the edge. The maximum tensile stresses occur along the perpendicular to the edge and are in radial direction from the centre of load.

vii) As the slab dimensions are infinite for this case, this pattern of stresses will hold good for all infinite slabs. Stresses infinite slab cases require seperate investigation.

The theoretical (based on Westergaard's equation) and observed critical stresses have been compared in Fig. 44. It can be seen from the figure that both the curves are almost identical, the maximum variation is 8% from the theoretical values calculated by Westergaard's equation. The observed stresses are less than the theoretical stresses.

The effect of thesize of contact area on the critical stresses may be studied from Fig. 44 which shows that the increase in the size of the contact area or decrease in l/a ratio leads to the reduction in the critical stress and hence smaller contact areas are more dangerous.

6.6. ANALYSIS OF CORNER LOAD STRESS DATA.

Fig. 36 to 43 present the tangential and radial stress due to corner loading with different contact areas under a load of 5 Kgms. These stresses were obtained with the help of strains measured on the surface of pavement (Refer equations 1 to 6 para 6.3.). The same convention as described in para 6.4 has been adopted here to represent the stresses on the diagrams.

The figures represent stress distribution pattern along the corner-edge and corner-bisector. These diagrams show that :

i) The critical stresses in case of loads placed symmetrically on the corner of a pavement occur along the corner bisector.

ii) In case of higher contact areas the region of maximum stress is wider

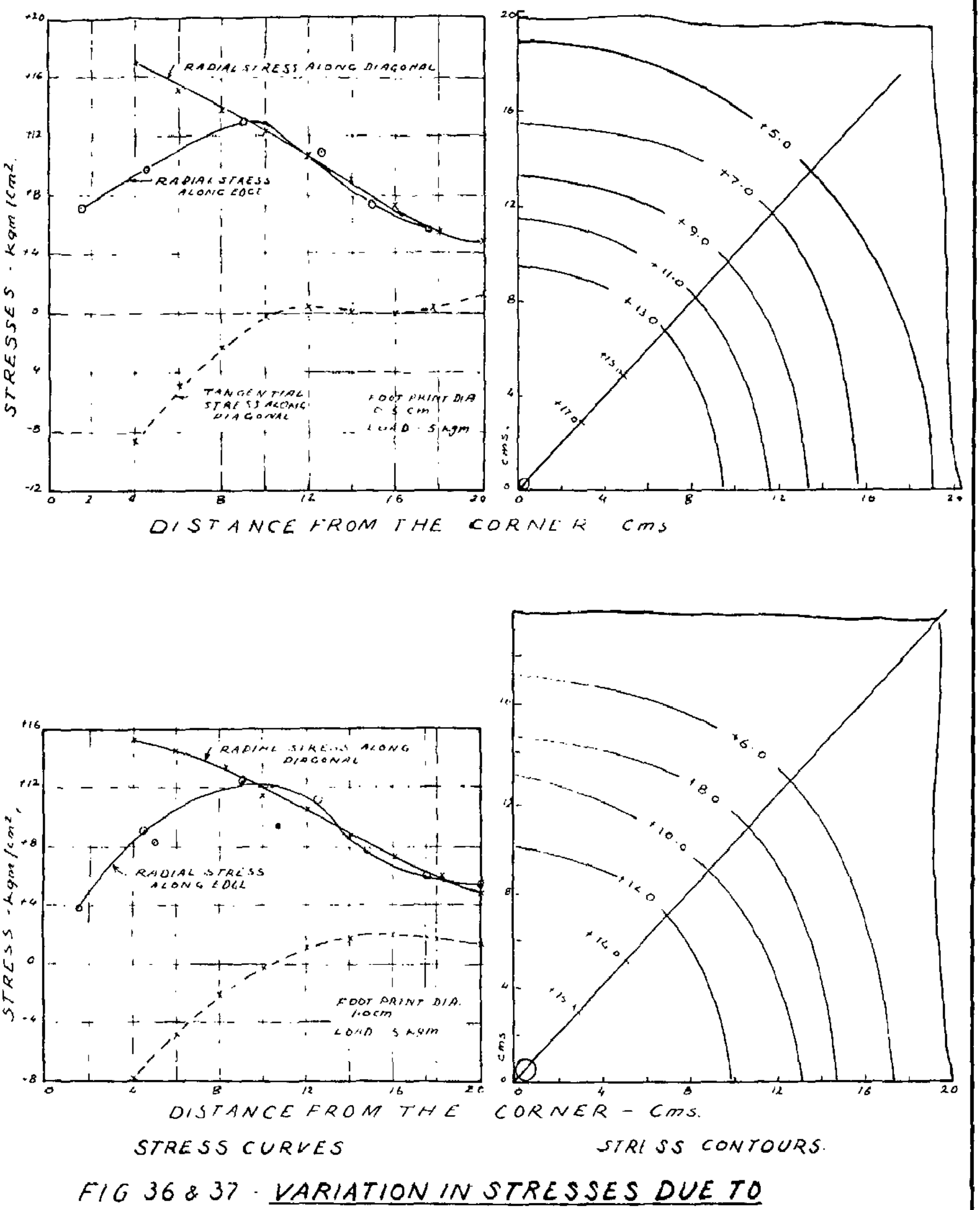


FIG 36 & 37 · VARIATION IN STRESSES DUE TO CORNER LOADING.

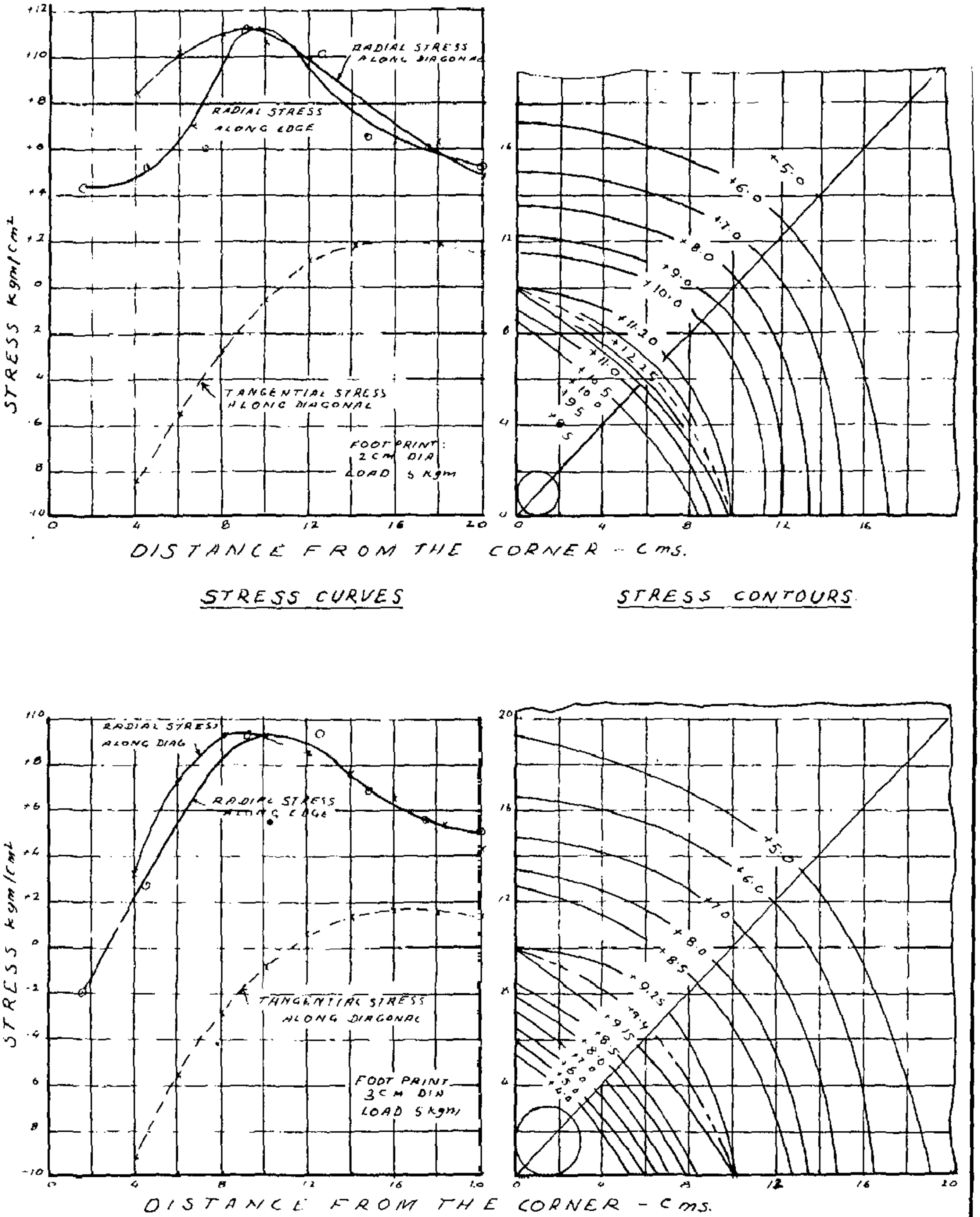
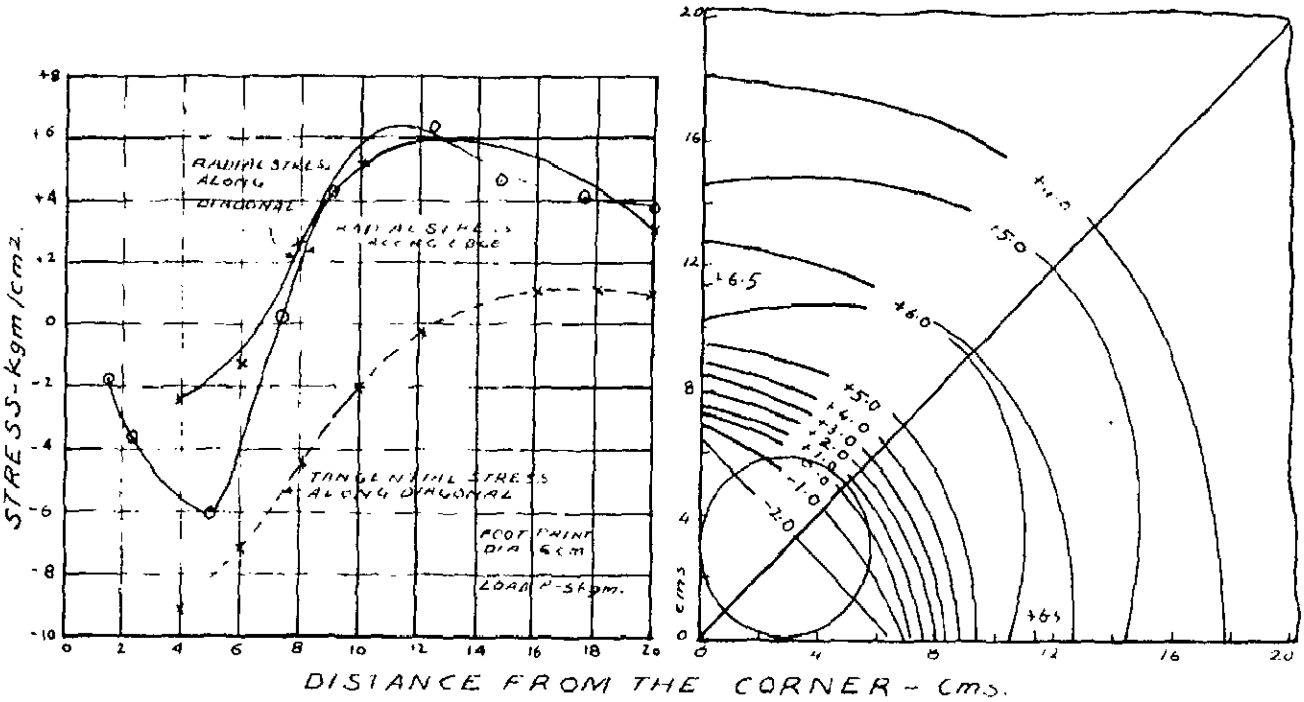


FIG 38 & 39 - VARIATION IN STRESSES DUE TO CORNER LOADING



STRESS CURVES

STRESS CONTOURS

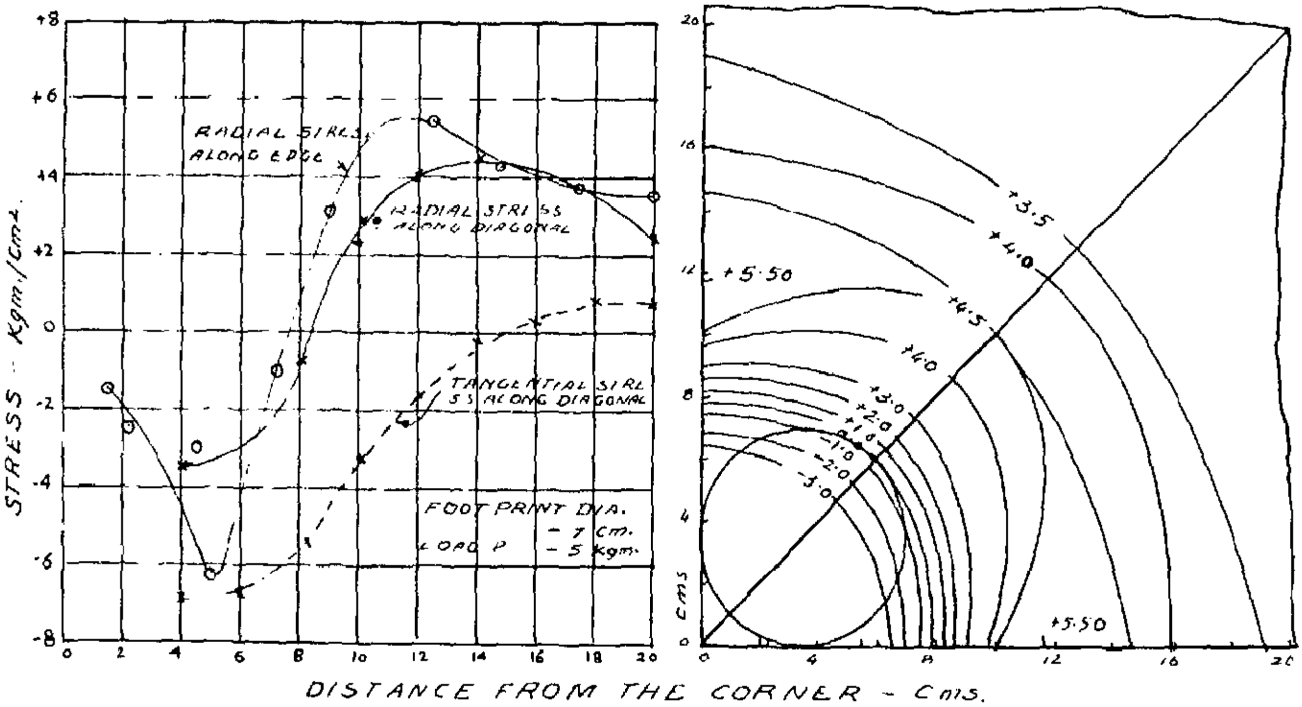
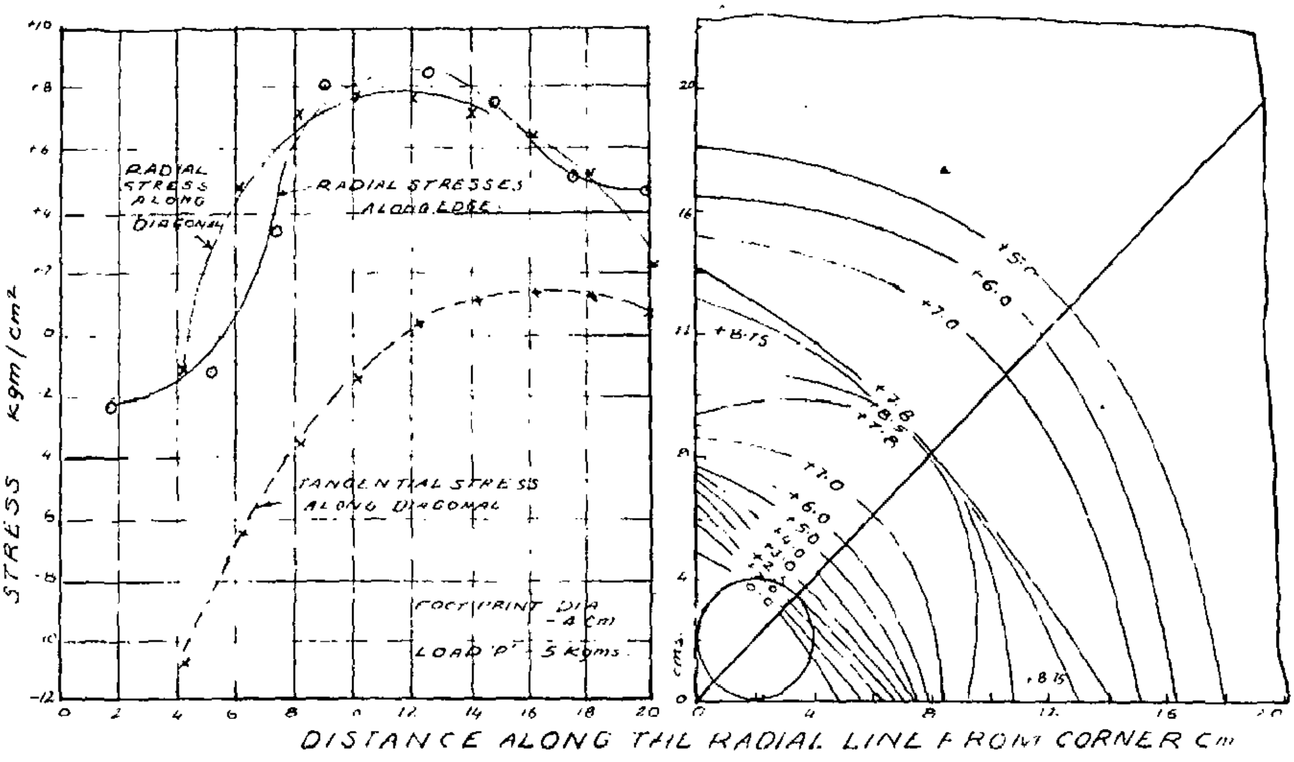


FIG. 42 & 43 VARIATION IN STRESSES DUE TO CORNER LOADING.



STRESS CURVES.

STRESS CONTOURS.

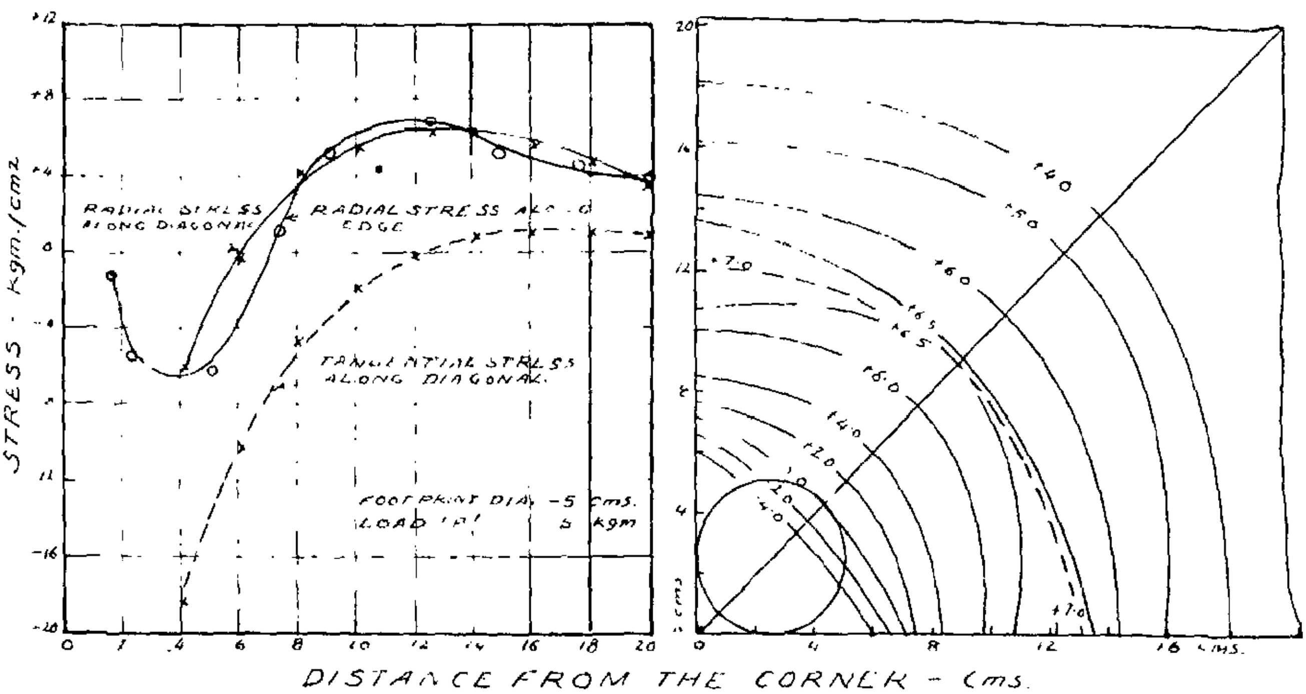


FIG. 40 & 41 - VARIATION IN STRESSES DUE TO CORNER LOADING.

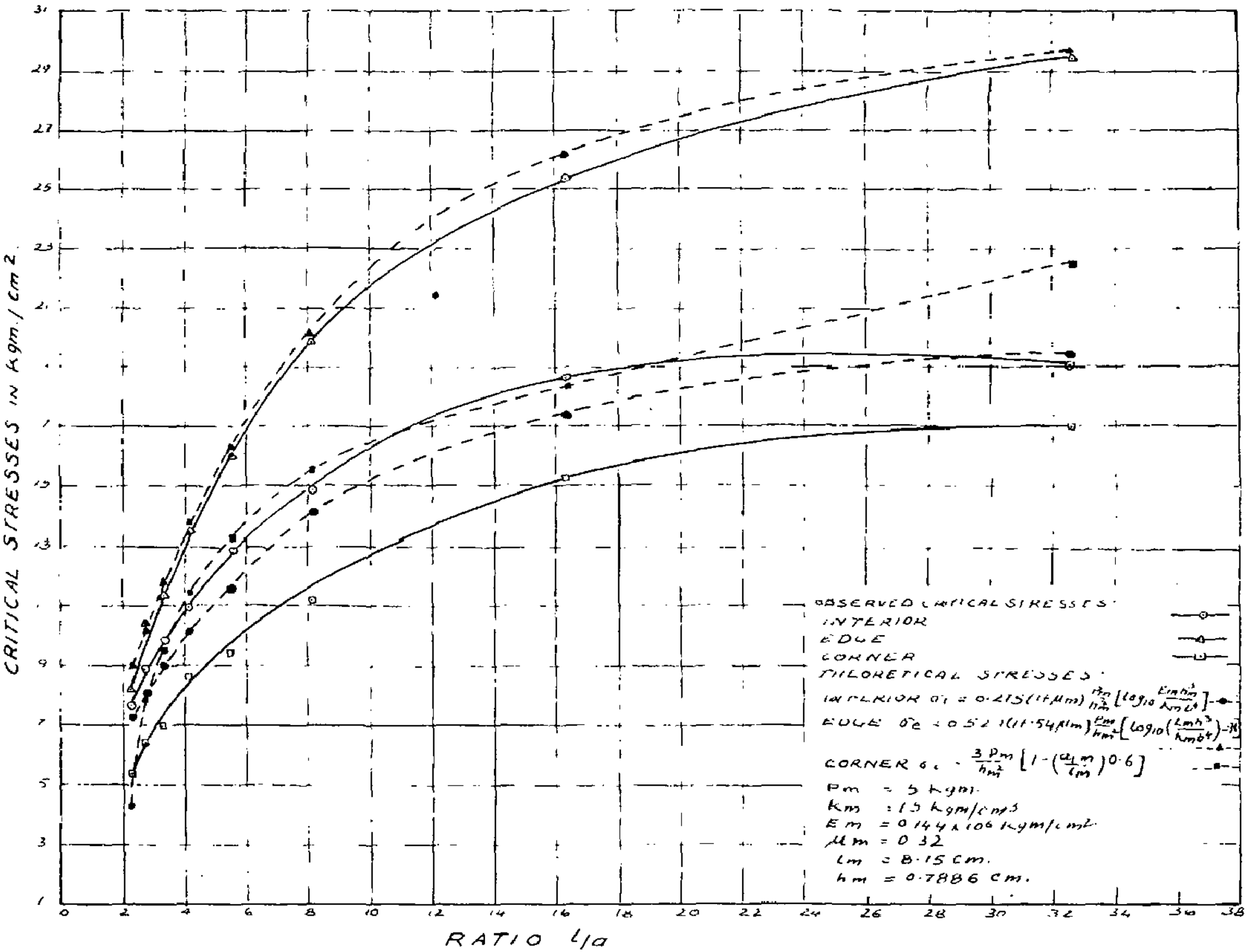


FIG. 44 - OBSERVED AND THEORETICAL CRITICAL STRESSES.

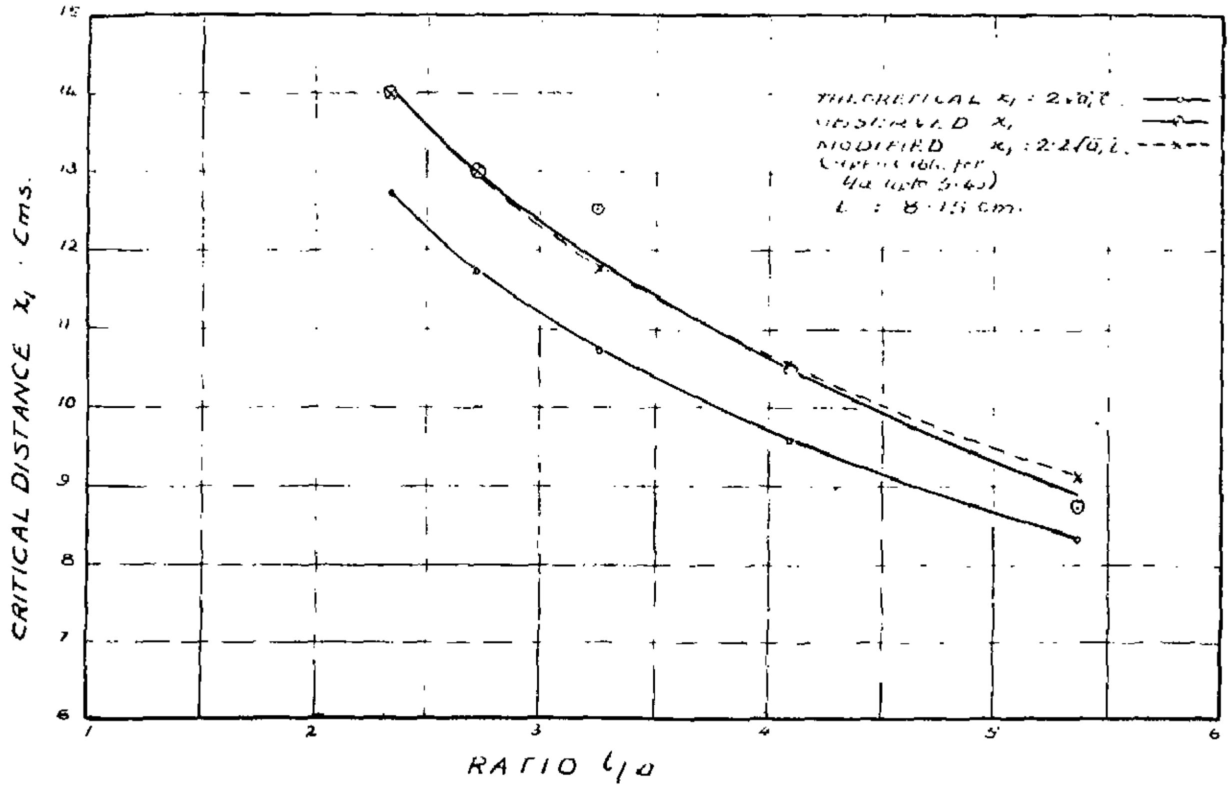


FIG. 45 - OBSERVED AND THEORETICAL VALUES OF X_1

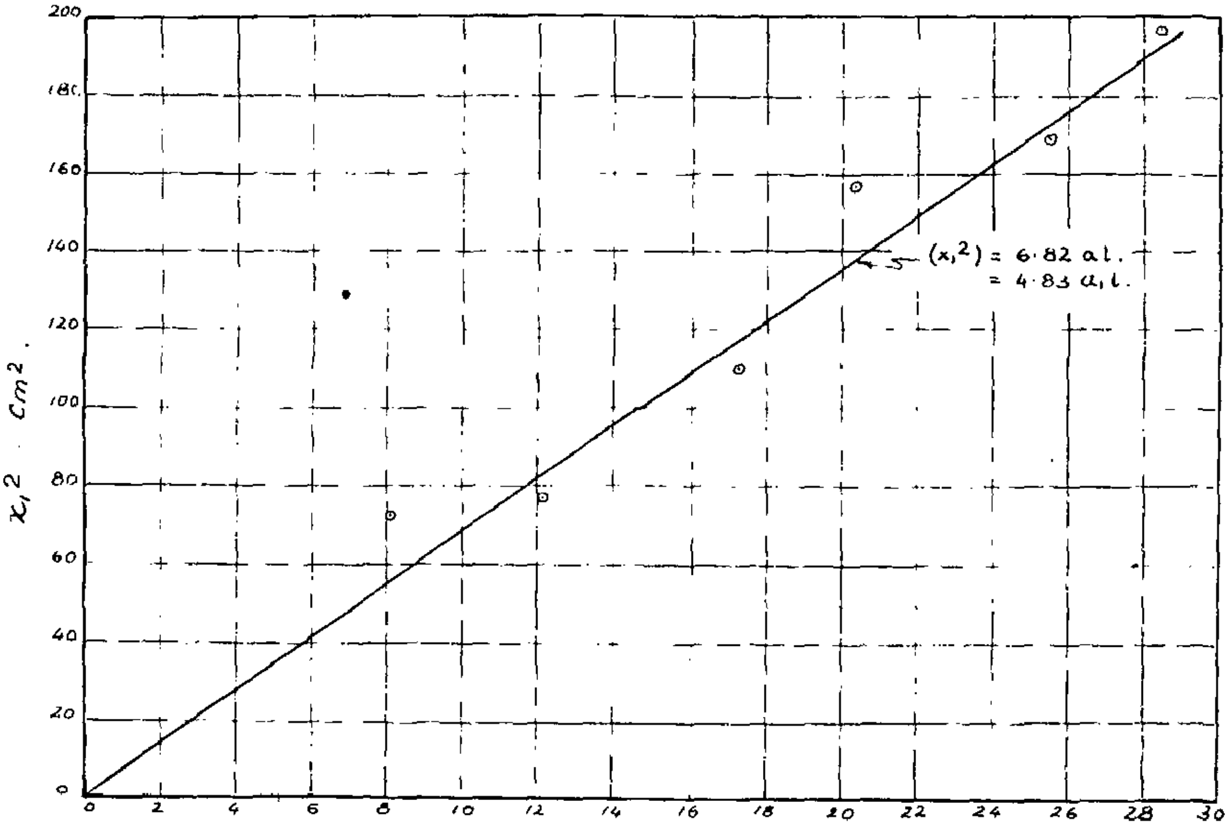


FIG. 46 - DEVELOPMENT OF EMPIRICAL EQUATION FOR CRITICAL DISTACE X_1.

at the edges than at the corner-bisector. It may be possible that in such cases the critical stresses may lie along the edge. Nothing definite can be said at this stage and hence it requires further investigation by employing smaller strain gauges at closed intervals.

iii) The stresses contours reveal that the stresses are concentrated between the point at which the critical stress occurs and the corner. Then the shape of contours changes rapidly. The contours are flat curves near the contact area which gain curvature till the critical distance is reached and then they again start flattening out.

iv) The maximum stresses are tensile and are radial. The tangential stresses are compressive upto a distance of about 1.25 to 1.50 l where they are zero and then they also become tensile. However, the critical stresses (radial) are always tensile and greater than tangential stresses.

The theoretical (based on Westergaard's equations) and observed critical stresses have been compared in figure 44. It can be seen that there is an appreciable variation in the two stress values and the values determined are less by 20% to 25% from the theoretical values, (except for 7.0 cm diameter contact area).

As will be seen in the following paragraph, the critical distance has increased by about 20%. Thus, it appears that the total stress has been spread over a wider area than calculated by Dr. H.M. Westergaard and hence the probable reason for the reduction in critical stresses.

Fig. 45 shows the comparison between the observed and theoretical values of the distance x_1 i.e. the distance at which the critical corner load stress occurs. ~~From the figure it is clear that the observed values of~~ x_1

are always slightly more than the theoretical values of x_1 (except for the contact area of 1.0 cm. diameter) Based on these test results, the Westergaard's equation of critical distance x_1 i.e. $x_1 = 2\ (a_1.l)^{½}$ has been modified to follow the observed values (see Fig. 45). The procedure for arriving at the modified equation $x_1 = 2.2\ (a_1\ .l)^{½}$ is diagrammatically represented in Fig. 46.

It is to be observed that out of the three types of loading the stresses decrease in the following order :

Edge stress,

interior stress and

corner stress ,

though the variation depends upon the value of l/a. This holds good for theoretical as well as observed stresses.

6.7. SUBGRADE MODULUS.

The 'k' value of the subgrade was determined in the laboratory by interior loading of the slab (Refer Chapter 5) From the data obtained in this studies, it is clear that the effective subgrade modulus is almost the same for interior and edge loading. In case of corner loading, the stresses (observed) are less than the theoretical stress calculated by Westergaard's analysis based on liquid subgrade assumption. Three probable reasons may be assigned for this deviation :

i) The effective subgrade modulus may be higher than substituted in the Westergaard's equation for purpose of getting the theoretical values of critical stresses ;

ii) There may be a plastic flow in the pavement due to load which reduces the pavement stresses.

iii) Additional support of soil obtained at the free edges of corner may lead to decrease the stresses in the pavement due to corner loading.

It is important to note here that, the same phenomenon was observed by other investigations also (2). An extensive study will therefore be necessary to find out the actual reason for such a phenomenon.

6.8. APPLICATION OF MODEL TEST RESULTS.

It was pointed out in the previous chapters that the model studies are very useful for predicting the behaviour of prototype slabs. This is illustrated by means of few practical examples below :-

EXAMPLE NO. 1. Data. :

Wheel load P = 4000 Kgm (app. 8800 lbs.)

Radius of foot print a = 15 Cm. (app. 6")

Thickness of slab h = 15 Cm. (app. 6")

(Poisson's ratio of concrete) $\mu = 0.15$

Modulus of subgrade reaction k = 3 Kgm/Cm^3 (app. 100 lbs/in^3)

Modulus of Elasticity of concrete E = 0.20×10^6 Kgm/Cm^2

(app. 29×10^6 lbs/in^2)

(Slab infinite in dimensions).

Let it be required to predict the critical stresses when the wheel is placed at the interior of slab, then

$$\text{(i)} \qquad l^4 = \frac{E\, h^3}{12\,(1 - \mu^2)\, k}$$

and we get,

$$l = 66 \text{ cm.}$$

ii) $b = (1.6 a^2 + h^2)^{1/2} - 0.675 h$ when

$a < 1.724 h$, and $b = a$ when $a > 1.724 h$.

therefore in present case $b = a = 15$ Cm.

iii) Ratio l / a for prototype $= 66/15 = 4.4$

$$\therefore \quad a_m = \frac{l_m}{4.4} = \frac{8.15}{4.4}$$

$$= 1.85 \text{ Cm.}$$

$$\therefore \quad b_m = 1.85 \text{ cm as } a > 1.724 h$$

$$\text{or } a > 1.36 \text{ cm.}$$

iv) From Fig. 44, for a value of $l/a = 4.4$ we get, the stresses in the model slab i.e. σ_{im} as 11.4 kgm/cm^2., thus from equation (10) of Chapter 2

$$\sigma_i = \sigma_{im} \; \frac{P h_m^2}{P_m h^2} \; \frac{(1 + \mu) \cdot \log_{10} \dfrac{E h^3}{k b^4}}{(1 + \mu_m) \cdot \log_{10} \dfrac{E_m h_m^3}{k_m b_m^4}}$$

$$= 22.3 \text{ kgm/cm}^2$$

EXAMPLE 2.

With the same data, to obtain the critical Edge - load stresses.

i) From the Fig. 44 for $l/a = 4.4$, we get

the model edge stresses

$$\sigma_{em} = 13.8 \text{ kgm/cm}^2$$

Therefore from equation (9) Chapter 2.

$$\sigma e = \sigma e_m \frac{(1 + .54\mu)}{(1 + .54\mu_m)} \frac{P}{P_m} \frac{h_m^2}{h^2} \frac{\left(\log_{10} \frac{E h^3}{k b^4} - 0.71\right)}{\left(\log_{10} \frac{E_m h_m^3}{k_m b_m^4} - 0.71\right)}$$

$$= 28.3 \text{ kgm/cm}^2$$

EXAMPLE 3.

With the same data, to obtain the critical corner load stresses from fig. 44. for $l/a = 4.4$, we get the model edge stresses $\sigma c_m = 8.8$ kgm/cm^2

Therefore from equation (6) of chapter 2) we get

$$\sigma c = \sigma c_m \frac{P h^2}{P_m h_m^2} = 19.3 \text{ kgm/cm}^2$$

similarly, many other problems of prototype slabs may be solved with the help of fig. 44.

6.9 CONCLUSIONS DERIVED FROM TEST RESULTS.

The following conclusions apply to the model tests carried out on semi-elastic subgrade with bakelite pavement.

In case of interior loading :

i) The stress contours are circular.

ii) The critical observed stresses are slightly higher than calculated by Westergaard's equation in most of the cases. The variation is not more than 8% in any case.

iii) The critical stresses increase with the decrease in the size of the foot-prints.

iv) Radial stresses become zero at a distance of about 0.9 l from the centre of loaded area.

v) Tangential stresses are greater than the Radial stresses and become zero at a distance of about 2.8 l from the centre of load. Radial stresses are once again zero at this distance from the tension side.

vi) A slab having an approximate dimensions of 6 l x 6 l is likely to behave similar to an infinite slab.

vii) The subgrade behaves in a manner similar to one assumed by Westergaard (4).

viii) The spread of stress-zone is independent of the size of contact area and load intensity.

In case of Edge loading :-

i) The stress contours are conic-sectional in shape.

ii) The critical stresses are within the contact area and near about the centre of gravity of the foot-print.

iii) Stress are concentrated along the edge near the load point.

iv) The zero stresses along the edge occur within a distance of about 0.6 l to 0.9 l and from the centre of load.

v) The zero stresses along the perpendicular to the edge occur at a distance of about 1 .25 l from the edge, therefore a slab of size 2 l x 1.5 l is likely to behave similar to an infinite slab.

vi) The increase in size of contact area (with the same load) reduces the amount of critical stresses on the pavement.

vii) The behaviour of subgrade is similar to that assumed by Westergaard (4) within a maximum variation of 0%. The observed stresses are lesser than the theoretical stresses.

In case of corner loading :

i) The stress contours are almost circular in shape.

ii) The region of maximum stresses is wider along the edges than along the corner-bisector.

iii) The observed values of distances x_1 at which the critical stresses occur are slightly higher than obtained from the Westergaards equation (4). A modified empirical equation has been developed which is

$$x = 2.2. \; (a_1 \; 1)^{1/2}$$

This equation is applicable for values of l/a upto 5.43.

iv) The observed critical stresses are less than the theoretical stresses (4). The variation is 20% to 25%.

v) The stresses are concentrated within the critical distances.

vi) Out of the three types of loading, the stresses decrease in the following order :

edge stress,

interior stress, and

corner stress .

through the variation depends upon the value of l/a for theoretical as well as observed values of stresses.

SUGGESTED PROBLEMS FOR FURTHER STUDY.

1. To verify Burmister's equation based on pavement Deflection by Model Tests.

2. Repetition of this investigation with different subgrade and subgrade support and pavement s.

3. Determination of maximum principle stresses.

4. Effect of change in the size of subgrade.

5. Investigation of Rigid Pavement behaviour under dynamic loads.

6. Effect of higher contact pressures on the rigid pavements.

7. Load Transfer capacity of joints in the pavements.

ANNEXURES

APPENDIX A

SPECIFICATIONS OF SUBGRADE MATERIAL

1. MECHANICAL ANALYSIS OF SAND-SOIL USED FOR SUBGRADE.

(Average of three determinations)

U. S. Sieve Size		% of Total Wt.
Passing	Retained	
No. 4	No. 8	00.6
No. 8	No. 16	13.1
No. 16	No. 30	14.2
No. 30	No. 50	24.5
No. 50	No. 100	33.4
No. 100	No. 200	09.6
No. 200	-	04.6
	Total	100.0

2. Specific gravity of sand-soil particles = 2.73 gms/c.c.

3. Modulus of subgrade reaction of the compacted material

'k' = 1.5 kgm/cm^3

APPENDIX B.

SPECIFICATIONS OF RUBBER SOLUTION USED AS BINDER FOR SUBGRADE MATERIAL.

1. Make - Dunlop Rubber Company (India) Ltd.
2. Specific gravity. .... 0.72 gm/c.c.
3. Percentage volatile matter .. .. 96.00 %

APPENDIX C

STATIC LOAD STRAINS FOR MODEL SLAB 30 Cm x 30 Cm x 0.7886 Cm THICK

Date 18.10.1962

Time 10.30 A.M.

Atmospheric Temp. $28^{\circ}C$

Observation No. 1

Position of load - Interior.

Foot print Diameter 2.0 cm.

Amount of load 5 Kgm.

(Radial Strains)

Strain gauge No.	Strain gauge reading Initial ‰	Final ‰	Unit strain ‰	Remarks
0.	25.270	25.200	- 0.070	
1.	14.105	14.070	- 0.035	
2.	15.770	15.735	- 0.035	
3.	7.828	7.802	- 0.026	
4.	15.833	15.812	- 0.021	Lead cubes dis-
5.	18.690	18.670	- 0.020	tributed uniform-
6.	16.310	16.298	- 0.012	ly over the
7.	20.090	20.080	- 0.010	pavement.
8.	18.710	18.700	- 0.010	
9.	18.090	18.090	- 0.00	
10.	18.735	18.744	+ 0.009	
11.	19.255	19.260	+ 0.005	
12.	18.192	18.198	+ 0.006	
13.	13.635	13.640	+ 0.005	
14.	04.990	04.994	+ 0.004	
15.	03.545	03.548	+ 0.003	
16.	04.820	04.820	0.000	

APPENDIX 'C' (Contd...)

Date 24.10.1962 Position of load - Interior.

Time 1.30 P.M. Foot print Diameter 2.0 cm.

Atmospheric Temp. 27° C Amount of load 5 Kgm.

Observation No. 1 (Tangential Strains)

Strain gauge No.	Strain gauge reading: Initial %	Strain gauge reading: Final %	Unit strain %	Remarks.
0	25.270	25.200	- 0.078	
43	15.003	14.986	- 0.017	Lead cubes uni-
44	14.590	14.580	- 0.010	formly distributed
45	14.850	14.845	- 0.005	over the slab.
46	16.055	16.050	- 0.005	
47	14.962	14.960	-0. 002	
48	09.425	09426	+ 0.001	

N.B. Three observations were recorded similarly for each set and the average was determined in each case.

APPENDIX D

STATIC LOAD STRAINS FOR MODEL SLAB 38 Cm x 38 Cm x 0.7886 Cm THICK

Date 6.10.1962 — Position of load - Edge.

Time 10.00 A.M. — Foot print Diameter 2.00 Cm.

Atmospheric Temp. 28° C. — Amount of load (Radial Strain) 5 Kgm.

Observation No. 1.

Strain gauge No.	Strain gauge reading Initial %	Strain gauge reading Final %	Unit strain %	Remarks.
20 A	18.805	18.668	- 0.137	
20	19.724	19.633	- 0.091	
18	19.010	18.930	- 0.080	
21	20.392	20.370	- 0.022	Load cubes uniformly distributed over the slab.
19	19.345	19.327	- 0.018	
22	14.974	14.980	+ 0.006	
23	15.850	15.856	+ 0.006	
24	12.441	12.450	+ 0.009	
25	11.747	11.750	+ 0.003	
17	16.416	16.454	+ 0.038	
30	12.050	12.083	+ 0.033	
31	10.893	10.928	+ 0.035	
32	12.498	12.520	+ 0.022	
13	02.702	02.760	+ 0.058	
18	03.487	03.544	+ 0.057	
10	04.055	04.097	+ 0.042	
7	05.445	05.465	+ 0.020	
1	03.243	03.259	+ 0.016	
3	07.945	07.952	+ 0.007	

Appendix 'd' (Contd....)

Date 17.10.1962
Time 10.30 A.M.
Atmospheric Temp. 26° C

Observation No. 1.

Position of load Edge.

Foot Print Diameter 2.00 cm.

Amount of load 5 Kgs.
(Radial & Tangential strain).

Strain gauge No.	Strain gauge reading: Initial ‰	Final ‰	Unit strain ‰.	Remarks
34	14.593	14.458	− 0.137	
33	1.504	1.546	− 0.038	
29	0.705	0.720	+ 0.015	
28	5.511	5.534	+ 0.023	
27	3.325	3.337	+ 0.012	Lead cubes uni-
26	0.346	0.354	+ 0.008	formly distributed
35	14.500	14.362	− 0.138	over the slab.
36	13.080	13.010	− 0.070	
37	13.776	13.735	− 0.041	
38	13.320	13.300	− 0.020	
39	15.262	15.250	− 0.012	
0	10.335	10.335	0	
40	10.840	10.790	− 0.050	
41	9.420	9.400	− 0.020	
42	9.330	9.320	− 0.010	

APPENDIX E

STATIC LOAD STRAINS FOR MODEL SLAB 38 Cm x 38 Cm x 0.7866 Cm THICK

Date	4 . 10. 1962	Position of load.	Corner
Time	1.00 P.M.	Foot Print Diameter	2.00 cm.
Atmospheric Temp.	31.5°C	Amount of load (Radial Strain)	5 Kgm.
Observation No.	1.		

Strain Gauge No.	Strain gauge reading: Initial ‰	Strain gauge reading: Final ‰	Unit strain ‰	Remarks.
25	15.478	15.447	-.031	
26	15.100	15.050	-.050	Lead cubes
24	16.150	16.185	+.035	uniformly
27	18.025	18.055	+.030	distributed
23	15.840	15.800	+.040	over the slab.
28	20.180	20.258	+.078	
22	00.115	00.166	+.051	
29	00.566	00.636	+.070	
21	05.615	05.665	+.050	
20	04.958	05.000	+.042	
18	04.245	04.282	+.037	
19	04.615	04.641	+.026	
16	15.700	15.858	+.078	
15	14.495	14.580	+.085	
14	15.926	16.002	+.076	
11	15.508	15.558	+.050	
9	14.300	14.330	+.030	
4	14.808	14.828	+.020	

Appendix 'E' (Contd....)

Date 30.10.1962

Time 11.30 A.M.

Atmospheric Temp. 26°C

Observation No. 1.

Position of load corner.

Foot Print Diameter 2.00 cm

Amount of load 5 Kg.

(Tangential strain)

Strain gauge No.	Strain gauge reading Initial ‰	Strain gauge reading Final ‰	Unit strain ‰	Remarks.
40	9.385	9.307	-0.078	
47	14.934	14.890	-0.044	Lead cubes uniformly distributed over the slab.
46	15.014	15.995	-0.019	
45	14.8[illegible]	14.809	-0.006	
44	14.546	14.545	-0.001	
43	14.950	14.950	-	

N O T A T I O N S

σ_i = Critical Interior stress (Kgm/Cm^2)

σ_e = Critical Edge Stress (Kgm/Cm^2)

σ_c = Critical corner stress (Kgm/Cm^2)

P = Wheel load (single) (Kgms.)

h = Thickness of slab (cms.)

l = Radius of Relative Stiffness (cms.)

Expressed by

$$l = \left(\frac{Eh^3}{12(1-\mu^2)^{1/2}} \right)^{1/4}$$

E = Modulus of Elasticity of Pavement material (Kgm./Cm^2)

μ = Poisson's Ratio for Pavement material.

k = Modulus of subgrade reaction (Kgm/Cm^3)

b = Radius of equivalent distribution of Pressure, (Cms.) expressed by

$$b = (1.6\,a^2 + h^2)^{1/2} - 0.675\,h$$

where $a < 1.724\,h$

$b = a \qquad a > 1.724\,h$

a = Radius of Foot Print (Cm.)

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