BETA - GAMMA - GAMMA ANGULAR CORRELATION STUDIES OF THE RADIATIONS FROM THE DECAY OF SOME RADIOACTIVE ISOTOPES

Thesis

Submitted to the University of Roorkee
for the award of the Degree
of
DOCTOR OF PHILOSOPHY
in

PHYSICS

by
INDRA VIKRAM SINGH RATHORE

DEPARTMENT OF PHYSICS
UNIVERSITY OF ROORKEE
ROORKEE INDIA
OCTOBER, 1977

To

My reverend brother, "Sri N.S. Rathore".

and

My devoted wife, "Beena"

CERTIFICATE

Certified that the thesis entitled "Beta-Gamma-Gamma Angular Correlation Studies Of The Radiations From The Decay Of Some Radio-Active Isotopes" which is being submitted by Mr.Indra Vikram Singh Rathore in fulfilment for the award of of the degree of Doctor of Philosophy in Physics of the University of Roorkee, Roorkee is a record of his own work carried out by him under my supervision and guidance. The matter embodied in this thesis has not been submitted for the award of any other degree.

Further it is certified that he has worked from Sept.23,1973 to October 1977 for preparing his thesis for the Ph.D.degree at the University.

Dated: October 11, 1977.

B. P. Singh
(B.P.Singh)
Professor,
Department of Physics,
University of Roorkee,
Roorkee -247672

ACKNOWLEDGEMENT

It is a great pleasure to express my heartiest gratitude to my research supervisor Professor, B.P.Singh for his meticulous guidance, fruitful discussions and stimulating encouragement. In fact, it is almost impossible for me to convey my feelings in words for all the help I received from him throughout my stay.

I express my obligation to Prof. S.K.Joshi, Head of the Physics Department for his interest and encouragement during the progress of this work and for providing the laboratory facilities.

The completion of this work has left me indebted to a galaxy of kith and kin whose good wishes served as beacons amidst vicissitudes which generally baffle a young research worker. Running the risk of missing many names, I acknowledge the help received from Ishwar Singh, Rajendra Prasad, Suresh Kumar, Bhushan Kumar ,Gauri Shankar, Pratap Singh, Naresh Kumar, Ajay Singh Rajesh Kumar and Bhishm Kumar. I am thankful to Mr.M.S.Negi, Senior Laboratory Technician for continuous help in the maintenance of the experimental set-up.

Thanks are due to the authorities of the Structural Engineering Research Centre, Roorkee and the Delhi University Computer Centre for providing computer facilities.

I would also like to acknowledge the financial help in the form of the fellowships awarded by Council of Scientific and Industrial Research, New Delhi and Department of Atomic Energy, Government of India.

Sri A.P.Singh and Sri V.K.Sharma deciphered my hieroglyphics intelligently and brought these to the present form. For this I say 'thank you' to both of them.

I am greatly indebted to my family for their affection, patience, encouragement and support throughout my career.

I.V.S. Rathore

(I.V.S.Rathore)

RÉSUMÉ

The matter presented in this thesis is the author's attempt to study the properties of nuclear excited states and the radiations emitted by some radioactive nuclei by beta-gamma-gamma angular correlation method. The geometrical considerations for the mounting of the three detectors are suggested in order to make angular correlation method feasible for experimental studies of the radiations from radioactive nuclei. A few radioactive isotopes have been studied. A survey of all angular correlation data have been made and analysis of $(2^{+'} \rightarrow 2^{+})$ gamma-transitions in even-even nuclei are presented to get additional insight into the structure of the excited states. The thesis has been divided into the following chapters.

CHAPTER-I INTRODUCTION

The importance of angular correlation technique has been given. The main advantages of triple beta-gamma-gamma angular correlation over the double angular correlation are summarized.

CHAPTER-II THEORY OF DIRECTIONAL CORRELATION

The outlines of the theory of angular correlation have been given. The theory has been extended to the cascade of three radiations. The general formula for the angular correlation coefficients with the geometrical considerations for the mounting of the detectors has been suggested. The triple angular correlation function has been simplified which could be readily applied

for the analysis of the angular correlation data.

CHAPTER-III EXPERIMENTAL SET-UP FOR THE STUDY OF THE CASCADE OF TWO AND THREE RADIATIONS

The electronic set-up for coincidence and angular correlation studies for the cascade of two and three radiations is given. Details of slow and fast coincidence circuits, vacuum chamber alongwith the source stand, for triple correlations are given. The assembly is illustrated in block diagram.

CHAPTER-IV BETA-GAMMA-GAMMA ANGULAR CORRELATION STUDIES OF THE RADIATIONS FROM THE DECAY OF Ru^{103}

Beta-gamma-gamma directional correlation studies for the cascades (i) β-rays of E_{max} 120 keV $\rightarrow\gamma$ -rays of 557 keV $\rightarrow$ γ-rays of 53 keV and (ii) β-rays of E_{max} 210 keV $\rightarrow\gamma$ - rays of 444 keV $\rightarrow$ γ - rays of 53 keV have been made. The spin values for the 93-, 537- and 650- keV energy levels in Rh^{103} are deduced and the most suitable values are 9/2, 5/2 and 7/2 respectively. It is also concluded that 444 keV gamma-transition is pure quadrupole and 557 keV gamma-transition is the mixture of dipole and quadrupole with mixing ratio 'δ' (between 0.1 to 0.25)

CHAPTER-V BETA-GAMMA-GAMMA ANGULAR CORRELATION STUDIES OF THE RADIATIONS FROM THE DECAY OF Eu^{152} AND Eu^{154} AND STUDIES OF THE NEGATIVE PARITY STATES IN Gd^{152}

Beta-gamma-gamma angular correlation studies are done for the radiations from the decay of Eu^{154} and Eu^{152} for the following cascades (i) β-rays of E_{max} 590 keV→ γ - rays of 1278 keV→γ -rays of 123 keV and (ii) β- rays of E_{max} 360 keV→γ -rays of 779 keV →γ - rays of 344 keV. The angular correlation coefficients thus obtained are analysed and spins of the excited states and multipolarities of gamma-transitions are determined. A method for the estimation of the mixture of dipole + quadrupole + octupole for triple angular correlation coefficients is also given.

CHAPTER-VI NEGATIVE PARITY STATES IN Nd^{144} AND MULTIPOLARITY OF GAMMA-TRANSITION FROM 1^- TO 2^+ STATES BY BETA-GAMMA-GAMMA-ANGULAR CORRELATION METHOD

Some of the low lying states in many isotopes (Nd^{144}, Sm^{148}, Gd^{152} and Gd^{156}) show a similar typical behaviour. The first 2^+ is regarded as a single quadrupole phonon state and 3^- as a single octupole phonon state. The levels with the spins and parities of 1^-, 5^-, 3^-, 4^- etc. are considered due to the simultaneous excitation of quadrupole and octupole phonons. If this consideration is correct, then the transition from J^- to 2^+ states must contain an appreciable E3 content.

The beta-gamma-gamma angular correlation coefficients for the cascade of β-rays of E_{max} 800 keV → γ- rays of 1489 keV → γ -rays of 696 keV are used to estimate E3 content in E1 transition in Nd^{144} and estimate is also made for other nuclei [Sm^{148}, Sm^{152} and Gd^{156}] by gamma-gamma angular correlation data.

CHAPTER-VII STUDY OF THE DOUBLET AT 1128 AND 1134-keV LEVELS IN Pd^{106} FROM THE DECAY OF 30sec Rh^{106} BY BETA-GAMMA-GAMMA ANGULAR CORRELATION METHOD

The beta-gamma-gamma angular correlation for the cascade of β-rays of E_{max} 2400 keV → γ-rays of (622 + 616) keV → γ-rays of 512 keV has been carried out to study the doublet at 1128- and 1134- keV levels in Pd^{106} from the decay of 30 sec Rh^{106}. The branching of two beta-groups (β_1 and β_2) to these levels is determined and the percentage of former is $\leq$ 5 percent that of the latter. The spin values are confirmed as 0^+ and 2^+ for the 1134 and 1128-keV levels.

CHAPTER-VIII A METHOD FOR THE DETERMINATION OF A_2^β, THE ANGULAR CORRELATION COEFFICIENT BY BETA-GAMMA-GAMMA ANGULAR CORRELATION STUDIES

A method of beta-gamma-gamma angular correlation has been tried to obtain A_2^β , the angular correlation coefficient 'b_k' particle parameter terms in triple angular correlation are included. A typical case of the decay of Tb^{160} to Dy^{160} is given for the cascade of β-rays of E_{max} 566 keV → γ-rays of 298 keV → γ-rays of 966 keV. The results obtained by this triple angular correlation method are compared with those obtained by beta-gamma angular correlation method.

CHAPTER-IX SYSTEMATICS IN E2/M1 MULTIPOLE MIXING RATIOS OF ($2'^{+} \rightarrow 2^{+}$) GAMMA-TRANSITIONS IN EVEN-EVEN NUCLEI

$A_2^{Expt.}$ and $A_4^{Expt.}$ angular correlation coefficients for the cascade $2'^{+} \rightarrow 2^{+} \rightarrow 0^{+}$ for almost all the even-even nuclei have been collected and reanalysed to calculate the amplitude mixing ratios (δ) for $2'^{+} \rightarrow 2^{+}$ gamma-transitions. There are two methods for analysing the data viz. Arns and Wiedenbeck method [Phys. Rev.111, 1631 (1958)] using A_2 values or A_4 values and that of Coleman [Nucl. Phys.5, 495 (1958)] using A_2 and A_4 values. The phases and values which are common in both the analysis, are taken for the analysis. The plot of $\ln |\delta/E_\gamma|$ versus mass number (A) is given alongwith the values predicted by Greiner's model (74) and that of Krane (73). Many systematics obtained in the values and phases of δ are presented.

LIST OF THE PUBLICATIONS

1. On the decay of 30sec Rh^{106}.
(Indian J.Pure and Appl.Phys.May Vol.12.No.5,p.391-92 (1974)).

2. Beta-gamma-gamma angular correlation studies in Rh^{103} from the decay of Ru^{103}(Pramana Vol.8,No.1,91 (1977).

3. A method for the determination of A_2^{β}, 74 Angular correlation coefficient By Beta-Gamma-Gamma Angular correlation studies (accepted for publication in Indian.J.of Physics).

4. Multipolarity of gamma-transition in Rh^{103} from the decay of Ru^{103} by the method of beta-gamma-gamma angular correlation (International Conference on Gamma-ray transition probabilities Nov.11-15, 1974).

5. Negative Parity States in Nd^{144} and multipolarity of gamma-transition from 1^- to 2^+ states. (Communicated).

6. Beta-gamma-gamma angular correlation studies of the radiations from decay of Eu^{152} and Eu^{154} and negative parity states in Gd^{152} (Communicated).

7. Systematics in E2/M1 multipole mixing ratios of $(2'^{+} \rightarrow 2^{+})$ gamma-transitions in even-even nuclei (Communicated).

CONTENTS

CHAPTER-1

INTRODUCTION:

The directional correlation studies between successively emitted gamma-quanta from radioactive nuclei are particularly most informative in assigning the spin values of the excited energy levels and the multipole orders for the gamma-radiations. The experiments for the study of directional correlation of successively emitted nuclear radiations have been performed by using the coincidence technique.

The first theoretical study of the angular correlation of a cascade of gamma-rays as given by Hamilton (1) has proved to be a valuable tool in nuclear spectroscopy. Utilizing the angular correlation technique, the properties of the emitted radiations were first investigated by Dunworth (2) and then Brady and Deutsch (3). The large number of papers and review articles that have appeared, have either been important in the development of the field or contain essential information. The early angular correlation experiments and an elementary formulation of the theory are reviewed by Deutsch (4) and by Frauenfelder (5). Biedenharn and Rose, in one joint (6) and two separate contributions (7,8) have treated the complete theory of angular correlations. Devons and Goldfarb, in their Encyclopedia article (9) give a comprehensive discussion of the theory and experiments pertaining to cascade correlations and to angular distributions in nuclear reactions. A brief introduction to the theory of gamma-gamma angular correlations and a description of corresponding experiments is contained in a survey by Deutsch and Kofoed-Hansen (10). The experimental aspects of angular correlation are summarized by Frauenfelder (11).

The incentive for this rigourous theoretical development has come from the remarkably rapid improvement in the technique of studying angular correlations. A major step in this direction was the introduction of the scintillation counters and big size Ge(Li) detectors into the angular correlation work. The scintillation counters have three major advantages; high counting efficiency, speed, and energy sensitivity. The development of fast electronics at the same time permitted full use of these three properties. The improved experimental technique combined with efficient scintillation counter makes angular correlation studies a powerful tool with wide scope for the investigation of the properties of the excited states of the nucleus and radiations emitted. The theoretical interpretation of the experimental data is greatly facilitated by the compilation of many useful tables of algebraic functions.

The theory of beta-gamma angular correlation was developed by Falkoff and Uhlenbeck (12). Their theory had some limitations which were later removed by many investigators (13). The expression generally used for β (allowed)$\rightarrow\gamma$-directional correlation was given by Morita (14). For allowed β-transitions, neglecting higher order effects, beta-gamma directional correlation is isotropic.

Biedenharn, Arfken and Rose (15) have extended the theory of angular correlation between the radiations involved in the three successive nuclear transitions. It is applicable to the case of three successively emitted radiations as well as to the case in which one of the emission process is replaced by an absorption process.

They have treated angular correlation between radiations involved in the first and third transitions with the intermediate state unobserved. Ferguson (16) has outlined some possible triple angular correlation studies in which all the three radiations are detected. The interest for doing such complicated angular correlation is its advantage in the identification of the nuclear states. The usefulness of triple coincidence and angular correlation studies may be seen by comparing it with double correlation study. They are as following:

(i) Additional confirmation of the decay scheme is obtained by beta-gamma-gamma studies.

(ii) Cascade of three radiations can be separated for angular correlation studies in a complicated decay scheme consisting of many cascades of two radiations.

(iii) Compton backgrounds of other unwanted gamma cascades in a gamma-gamma angular correlation study are angle dependent. Therefore there is a need of the correction for the unwanted gamma-cascade which is difficult to estimate in a complicated decay scheme. This is completely eliminated (or reduced to the minimum).

(iv) Many possible permutations of spin assignments to the excited energy levels can be minimized by triple angular correlation studies as compared to gamma-gamma angular correlation studies.

(v) The value of 'δ', the multipole mixing ratio of gamma-transition can be determined by the directional correlation study for the cascade of three radiations as is done in a study for the cascade of two radiations but the uncertainty in the sign of 'δ' (when 'δ' is determined for the middle gamma-transition of the cascade of three radiations) can be removed in triple directional correlation study.

Angle function for triple correlation is given by Ferguson (16) as:

$$P_{k_1k_2k_3}(\Omega_1,\Omega_2,\Omega_3)=(4\pi)^{3/2}\ \frac{i^{k_1+k_2-k_3}}{k_1k_2k_3^2}\ \sum_{N_1N_2N_3}\ (-1)^{N_3}$$

$$\langle k_1N_1,k_2N_2|k_3N_3\rangle\ Y_{k_1}^{N_1}(\Omega_1)\ Y_{k_2}^{N_2}(\Omega_2)\ Y_{k_3}^{N_3}(\Omega_3)$$

where $\Omega_1, \Omega_2, \Omega_3$ are the directions of first, second and third radiations. It is assumed that the excited states have definite parities so that k_1, k_2 and k_3 are even. If k_1, k_2 and k_3 are restricted to ≤ 4, then fifteen angle functions with $k_1k_2k_3$ = 000, 022, 044, 202, 220, 222, 224, 242, 244, 404, 422, 424, 440, 442, 444 are obtained giving, in general fourteen constants. These constants can be minimized by certain restrictions on Ω_1, Ω_2 and Ω_3. The simple restrictions which are experimentally feasible, are $\Omega_1=0$ i.e. $\theta_1 = 0$ and $\phi_1 = 0$, θ_2 or θ_3 is 90^0 and $\phi(=\phi_2 - \phi_3)$ is 90^0 or 180^0. With these restrictions, the function becomes.

$$P_{k_1k_2k_3}(\Omega_1, \Omega_2, \Omega_3) = \sum_k \alpha^i_{k_1k_2k_3k} P_k(\cos\theta)$$

The $\alpha^i_{k_1k_2k_3k}$ are the coefficients giving the restrictions and θ is the angle of the movable detector.

In triple angular correlation study, following two types of the mountings of the detectors can be considered. These mountings are discussed in the present work.

(i) All the three detectors are mounted in the plane of the table. Two of them are fixed at right angle to each other and third detector is movable in the opposite quadrant of the detectors. All the three detectors are equidistant from the source at the centre.

(ii) Two of the detectors are in the plane of the table and third is perpendicular to the plane of the table. One of the detectors in the plane of the table is fixed while the other is movable. The source is kept at the centre equidistant from all the three detectors.

In the present work, the radiations from the decays of a few radioactive nuclei i.e. Ru^{103}, Eu^{152} and Eu^{154}, Pr^{144}, Rh^{106} and Tb^{160} have been investigated by the method of beta-gamma-gamma angular correlation studies. The choice of the nuclei was determined by their availability from Bhabha Atomic Research Centre, Trombay, Bombay, the only place to produce radioactive isotopes in India and was thus random. But still then, usefulness of this method of beta-gamma-gamma angular correlation has been tried and results have been obtained. Some of the important considerations for the method of beta-gamma-gamma correlations are:

(i) Will it be possible to rule out many permutations of spin assignment for the excited levels ? (Decay of Ru^{103})

(ii) Can we separate certain gamma-cascades in complicated decay scheme and get information which otherwise was difficult to obtain ? (Decay of Eu^{152} and Eu^{154})

(iii) Will it be possible to ascertain more informations regarding the multipolarity of gamma-ray transition if gamma cascades are separated ? (Decay of Pr^{144})

(iv) Will it be possible to separate two beta groups of close energies producing doublet or doublet formed, may not be due to the beta decay but formed by cascade of gamma-transitions from higher excited states ? (Decay of Rh^{106})

(v) Are we in a position to determine the higher order effects in allowed beta-transitions ? (Decay of Tb^{160})

'Multipolarity' of gamma-ray transition is important in all angular correlation studies. Therefore a survey of all angular correlation data have been made but results and analysis of $(2^{\prime +} \rightarrow 2^{+})$ gamma-transitions in even-even nuclei are presented to understand certain systematics.

CHAPTER-2

THEORY OF DIRECTIONAL CORRELATION

2.1 INTRODUCTION

Directional correlation is a valuable tool for the determination of the spins of the nuclear excited states and multipolarities of gamma-transitions. In general the radiations observed from the randomly oriented nuclei are isotropic and an isotropic radiation pattern can be observed only from an ensemble of nuclei which are not randomly oriented. Anisotropic distribution of radiations can also be obtained by picking out only those nuclei whose spins happen to lie in a preferred direction. We can realize this if the nuclei decay through successive emission of two radiations R_1 and R_2. We observe the radiation say R_1 in a fixed direction $\vec{K}_1$ and study the distribution of R_2 with respect to the direction $\vec{K}_1$ of the first radiation. Then the succeeding radiation R_2 shows a definite angular distribution with respect to $\vec{K}_1$.

The relative probability $W(\theta)d\Omega$ that the second radiation R_2 is emitted into the solid angle $d\Omega$ at an angle θ with respect to $\vec{K}_1$, is of interest. The theoretical expression for the correlation function $W(\theta)$ for two successively emitted radiations has been worked out by many investigators (13). We write down the main points of the theory for the cascade of two radiations and then extend it to the cascade of three radiations and for beta-gamma-gamma angular correlation

2.2 THEORY OF DIRECTIONAL CORRELATION FOR THE CASCADE OF TWO RADIATIONS:-

We derive the directional correlation function $W(\vec{K}_1,\vec{K}_2)$. Where $W(\vec{K}_1, \vec{K}_2)d\,\Omega_1 d\Omega_2$ is defined as the probability that the radiations R_1 and R_2 are emitted successively in the directions $\vec{K}_1$ and $\vec{K}_2$ respectively into the solid angles $d\,\Omega_1$ and $d\,\Omega_2$. The angular correlation function is derived by making use of the concept of density matrices.

(i) DENSITY MATRIX FORMULATION:-

Let us assume that the eigenstates of some operators form a complete orthonormal set $|m\rangle$. Assume that we have a system in a pure state $|n\rangle$ which is the function of spin quantum number a, can be expressed as

$$|n\rangle = \sum_m a_{nm}|m\rangle \qquad (2.1)$$

The expectation value of an operator F is given by

$$\langle n|F|n\rangle = \sum_{m',m} a^*_{nm'}\, a_{nm} \langle m'|F|m\rangle$$

In angular correlation studies, we are dealing with a group of nuclei in a various states and these nuclei behave independently of each other. This system must be given by an incoherent set of pure states $|n\rangle$ with weight g_n. The expectation value of operator F for this system is then obviously

$$\langle F\rangle = \sum_n g_n \langle n|F|n\rangle$$

$$= \sum_{n,m',m} g_n a^*_{nm'}\, a_{nm} \langle m'|F|m\rangle \qquad (2.2)$$

These coefficients $(g_n a^*_{nm}, a_{nm})$ describe the state completely. We define the matrix element of a density operator ρ by

$$\langle m|\rho|m'\rangle = \sum_n g_n \, a^*_{nm'} \, a_{nm}$$

Then

$$\langle F\rangle = \sum_{m,m'} \langle m|\rho|m'\rangle \langle m'|F|m\rangle$$

$$\langle F\rangle = T_r(\rho F) = T_r(F\rho) \qquad (2.3)$$

$\langle m|\rho|m'\rangle$ is the density matrix. If the states $|n\rangle$ are not normalized, then $\langle F\rangle = \dfrac{T_r(\rho F)}{T_r(\rho)}$. The density operator of a state can be defined in a Dirac notation as

$$\rho = \sum_n |n\rangle \, g_n \, \langle n| \qquad (2.4)$$

The probability of finding the pure system $|n\rangle$ in the state $|m\rangle$ which is described by the incoherent superposition of states $|n\rangle$ is

$$P(m) = \langle m|\rho|m\rangle$$

Assume a transition from level A to level B. Let a, a'..... set of quantum number describing the eigenstates of level A. The operator H inducing the transition from the levels A to B is assumed to be linear but it does not have to be Hermitian.

Adjoint H^* of H;

$$\langle f|H|i\rangle^* = \langle i|H^*|f\rangle$$

$$|f\rangle = H|i\rangle \;, \; \langle f| = \langle i|H^* \qquad (2.5)$$

$$|f\rangle = H|a\rangle$$

In general, the initial state of the system is described by a density operator

$$\rho_A = \sum_n |n\rangle\, g_n \,\langle n| \tag{2.6}$$

After the transition which is induced by H, the density operator in level B is

$$\rho_B = \sum_n H|n\rangle\, g_n \,\langle n|H^* \tag{2.7}$$

Writing the matrix element in terms of $|b\rangle$

$$\langle b|\rho_B|b'\rangle = \sum_n \langle b|H|n\rangle\, g_n \langle n|H^*|b'\rangle$$

By applying $|n\rangle = \sum_a |a\rangle\langle a|n\rangle$, we have

$$\langle b|\rho_B|b'\rangle = \sum_{\substack{a,a'\\n}} \langle b|H|a\rangle\langle a|n\rangle g_n \langle n|a'\rangle\langle a'|H^*|b'\rangle$$

$$= \sum_{\substack{a,a'\\n}} \langle b|H|a\rangle\langle a|\rho_A|a'\rangle\langle b'|H|a'\rangle^* \tag{2.8}$$

(ii) DIRECTIONAL CORRELATION FUNCTION AND DESCRIPTION OF THE PROCESS:-

The angular correlation function $W(\vec{K}_1,\vec{K}_2)$ is derived by the density matrix formulation. Let us assume that the nucleus emitts radiations R_1 and R_2 in a cascade which is $I_i \xrightarrow{R_1} I \xrightarrow{R_2} I_f$.

The density matrix ρ_i describes the initial state I_i while $\rho(\vec{K}_1)$ describes the intermediate state which is the final state of the first transition but the initial state of the second transition. $\rho(\vec{K}_1, \vec{K}_2)$ describes the final state (nuclear ensemble + emitted radiation).

In order to derive the correlation function $W(\vec{K}_1, \vec{K}_2)$ we first consider the transition $I_i \longrightarrow I$. Applying equation (2.8) to this transition by putting $a = m_i$, $b = m$, we obtain the density matrix for the intermediate state

$$\langle m|\rho(\vec{K}_1)|m'\rangle = S_1 \sum_{m_i, m_i'} \langle m|H_1|m_i\rangle \langle m_i|\rho_i|m_i'\rangle$$
$$\times \langle m'|H_1|m_i'\rangle^* \qquad (2.9)$$

The symbol S_1 denotes summation over all unobserved properties of the radiations e.g., spin, polarization etc. The matrix element $\langle m|H_1|m_i\rangle$ stands for $\langle I m\,\vec{K}_1\sigma_1|H_1|I_i m_i\rangle$; σ_1 is the component of intrinsic spin of quantum of radiation field along $\vec{K}_1$ and H_1 is the interaction operator for the emission of R_1 into direction $\vec{K}_1$ with polarization σ_1.

Applying (2.8) to the second transition and obtain the density matrix for the final state by setting $a = m$, $b = m_f$

$$\langle m_f|\rho(\vec{K}_1,\vec{K}_2)|m_f'\rangle = S_1 S_2 \sum_{\substack{m,m' \\ m_i,m_i'}} \langle m_f|H_2|m\rangle \langle m|H_1|m_i\rangle\langle m_i|\rho_i|m_i'\rangle$$
$$\times \langle m'|H_1|m_i'\rangle^* \langle m_f'|H_2|m'\rangle^* \qquad (2.10)$$

Since the probability of finding the nucleus in the final state m_f, while the two radiations R_1 and R_2 are observed in the direction $\vec{K}_1$ and $\vec{K}_2$, is given by the diagonal elements of $\rho(\vec{K}_1, \vec{K}_2)$

$$P_f(m_f) = \langle m_f|\rho(\vec{K}_1,\vec{K}_2)|m_f\rangle \qquad (2.11)$$

In a directional correlation experiment, we observe the final m_f states with equal efficiency and no distinction is made between them.

Therefore

$$W(\vec{K}_1, \vec{K}_2) = \sum_{m_f} P_f(m_f)$$

$$= \sum_{m_f} \langle m_f | \rho(\vec{K}_1,\vec{K}_2) | m_f \rangle \qquad (2.12)$$

Using equation (2.10), the directional correlation function becomes

$$W(\vec{K}_1, \vec{K}_2) = S_1 S_2 \sum_{\substack{m_f, m \\ m', m_i}} \langle m_f | H_2 | m \rangle \langle m | H_1 | m_i \rangle$$

$$X \langle m' | H_1 | m_i \rangle^* \langle m_f | H_2 | m' \rangle^* \qquad (2.13)$$

Rewritting the matrices in the following way

$$\langle m | \rho(\vec{K}_1) | m' \rangle = S_1 \sum_{m_i} \langle m | H_1 | m_i \rangle^* \qquad (2.14)$$

and

$$\langle m' | \rho(\vec{K}_2) | m \rangle = S_2 \sum_{m_f} \langle m_f | H_2 | m \rangle \langle m_f | H_2 | m_i' \rangle^* \qquad (2.15)$$

Therefore equation (2.13) is written as

$$W(\vec{K}_1, \vec{K}_2) = \sum_{m, m'} \langle m | \rho(\vec{K}_1) | m' \rangle \langle m' | \rho(\vec{K}_2) | m \rangle$$

$$= T_r \left[\rho(\vec{K}_1) , \rho(\vec{K}_2) \right] \qquad (2.16)$$

The (2.16) equation is valid only if the intermediate state after the emission of the first transition is not changed before the emission of the second radiation and the initial state is randomly oriented.

We wish to express the density matrix $\langle m | H | m_i \rangle$ $= \langle I m \vec{K} \sigma | H | I_i m_i \rangle$. This matrix element is represented in terms of the nuclear spin and the direction $\vec{K}$ and the polarization σ of the emitted radiation. The transformation from the plane wave representation $\langle\vec{K}\sigma|$ to the angular momentum or spherical representation $\langle LM\pi|$ is given by a unitary transformation

$$\langle\vec{K}\sigma| = \sum_{LM\pi} \langle\vec{K}\,\sigma|LM\pi\rangle\ \langle LM\pi| \qquad (2.17)$$

Therefore we get

$$\langle I\ m\ \vec{K}\sigma|H|I_i m_i\rangle = \sum_{LM\pi} \langle\vec{K}\sigma|LM\pi\rangle\ \langle I\ m\ LM\pi|H|I_i m_i\rangle \qquad (2.18)$$

(If parity is conserved, the sum over π reduces to one term. For β transitions, parity is not conserved, therefore the sum over π can not be omitted.). In this equation, the first factor $\langle\vec{K}\sigma|LM\pi\rangle$ describes the eigenfunction of the radiation R corresponding to the eigenvalues L, M and π of the operators of angular momentum, Z component of the angular momentum and parity respectively. The second factor, the matrix element $\langle I\ m\ LM\pi|H|I_i m_i\rangle$ corresponds to the vector addition $I_i = I+L$, $m_i = m + M$. Thus we can express the angular part of the state $\langle I\ m\ LM\pi| \equiv \langle Im\ |\langle LM\pi|$ in terms of states $\langle I'_i\ m'_i|$

$$\langle I\ m\ LM\pi|H|I_i m_i\rangle = \sum_{I'_i m'_i} \langle I\ mLM|I'_i m'_i\rangle\ \langle I'_i m'_i IL\pi|H|I_i m_i\rangle$$

Applying Wigner-Eckart theorem to $\langle I'_i m'_i IL\pi|H|I_i m_i\rangle$,
We get

$$\langle I'_i m'_i IL\pi|H|I_i m_i\rangle = (-1)^{I'_i - m'_i} \begin{pmatrix} I'_i & 0 & I_i \\ -m'_i & 0 & m_i \end{pmatrix}$$

$$\times \langle I'_i||H(L,\pi)||\ I_i\rangle \qquad (2.19)$$

Therefore $\langle I\ m\ LM\pi|H|I_i m_i\rangle = \langle I\ m\ LM|I_i m_i\rangle$

$$\times\ \langle I||L\pi||I_i\rangle \qquad (2.20)$$

on the right hand side of equation (2.20), the first term

$\langle I\ m\ LM|I_i m_i\rangle$ is a Clebsch-Gordan coefficient (geometrical factor) and the second term $\langle I||L\pi||I_i\rangle$ the reduced matrix element of a scaler operator H, is a nuclear factor.

We apply the rotational matrix to equation (2.18) i.e. $\langle \vec{K}\sigma|LM\pi\rangle = \sum_{\mu} \langle 0\ \sigma|L\mu\pi\rangle D^{L*}_{M\mu}(\vec{Z}\rightarrow\vec{K})$ and replacing the Clebsch-Gordan coefficients into Wigner-3J symbol,

We get

$$\langle m|H|m_i\rangle = \sum_{L\pi\mu} (-1)^{-I+L-m_i} \begin{pmatrix} I & L & I_i \\ m & M & -m_i \end{pmatrix}$$

$$X \langle 0\sigma|L\mu\pi\rangle\langle I||L\pi||I_i\rangle D^{L*}_{M\mu}\ (\vec{Z}\rightarrow\vec{K}) \qquad (2.21)$$

According to equation (2.21), the density matrix for the first transition is written as

$$\langle m|\rho(\vec{K}_1)|m'\rangle = S_1 \sum_{m_i} \sum_{\substack{L_1\mu_1\\ \pi_1}} \sum_{\substack{L_1'\mu_1'\\ \pi_1'}} (-1)^{I+L_1-m_i} \begin{pmatrix} I & L_1 & I_i \\ m & M_1 & -m_i \end{pmatrix}$$

$$X \langle 0\sigma_1|L_1\mu_1\pi_1\rangle\langle I||L_1\pi_1||I_i\rangle D^{L*}(Z\rightarrow\vec{K}_1)\ (-1)^{I+L_1'-m_i}$$

$$X \begin{pmatrix} I & L_1' & I_i \\ m' & M_1' & -m_i \end{pmatrix} \langle 0\sigma_1'|L_1'\mu_1\pi_1'\rangle\langle I||L_1'\pi_1'||I_i\rangle^* D^{L_1'}_{M_1'\mu_1}(Z\rightarrow\vec{K}_1) \qquad (2.22)$$

Now applying Clebsch-Gordan series after changing $D^{(L)}_{M_1'\mu_1'}$ into $D^{(L)*}_{-M_1-\mu_1'}$, we get after some manipulation by setting $N_1 = M_1' - M_1 = (m_i-m') - (m_i-m) = m - m'$ and $\tau_1 = \mu_1' - \mu_1$

$$\langle m|\rho(\vec{K}_1)|m'\rangle = S_1 \sum_{\substack{L_1\mu_1\pi_1\\ L_1'\mu_1'\pi_1'}} \sum_{k_1 n_1 \tau_1} (-1)^{2I-I_i+m-\mu_1'} (2k_1+1)$$

$$\times\begin{pmatrix} I & I & K_1 \\ m' & -m & N_1 \end{pmatrix}\begin{Bmatrix} I & I & K_1 \\ L_1 & L_1' & I_i \end{Bmatrix}\begin{pmatrix} L_1 & L_1' & k_1 \\ \mu_1 & -\mu_1' & \tau_1 \end{pmatrix}\langle 0\sigma_1|L_1\mu_1\pi_1\rangle$$

$$\times\langle 0\sigma_1'|L_1'\mu_1'\ \pi_1'\rangle^*\langle I||L_1\pi_1||I_i\rangle\langle I||L_1'\pi_1'||I_i\rangle^*\ D^{k_1}_{N_1\tau_1}(\vec{Z}\rightarrow\vec{K}_1)$$

(2.23)

In this equation, some factors depend only on the properties of the particular radiation emitted. To single out these factors clearly, we contract these factors into Racah radiation parameter $C_{k\tau}$ which is defined by

$$C_{k\tau}(L'L) = S\sum_{\mu,\mu'}(-)^{L_1'-\mu_1'}(2k+1)^{1/2}\begin{pmatrix} L & L' & k \\ \mu & -\mu' & \tau \end{pmatrix}\langle 0\sigma|LM\pi\rangle$$

$$\times\langle 0\sigma'|L'M'\pi'\rangle^* \qquad (2.24)$$

Now the final expression for the density matrix of the first transition becomes

$$\langle m|\rho(\vec{K}_1)|m'\rangle = \sum_{L_1L_1'}\ \sum_{k_1N_1\tau_1}(-1)^{2I-I_i+m+L_1'}(2k_1+1)^{1/2}$$

$$\times C_{k_1\tau}(L_1'L_1)\begin{pmatrix} I & I & k_1 \\ m' & -m & N_1 \end{pmatrix}\begin{Bmatrix} I & I & k_1 \\ L_1 & L_1' & I_i \end{Bmatrix}\langle I||L_1\pi_1||I_i\rangle$$

$$\times\langle I||L_1'\pi_1'||I_i\rangle^*\ D^{k_1}_{N_1\tau_1}(\vec{Z}\rightarrow\vec{K}_1) \qquad (2.25)$$

Similarly we obtain

$$\langle m'|\rho(\vec{K}_2)|m\rangle = \sum_{L_2L_2'}\ \sum_{k_2N_2\tau_2}(-1)^{k_2-I_f-m-L_2'}(2k_2+1)^{1/2}C^*_{k_2\tau_2}(L_2L_2')$$

$$\times\begin{pmatrix} I & I & k_2 \\ m' & -m & N_2 \end{pmatrix}\begin{Bmatrix} I & I & k_2 \\ L_2 & L_2' & I_f \end{Bmatrix}\langle I_f||L_2\pi_2||I\rangle\langle I_f||L_2'\pi_2'||I\rangle^* D^{k_2}_{N_2\tau_2}(\vec{Z}\rightarrow\vec{K}_2)$$

(2.26)

We now evaluate the directional correlation funotion $W(\vec{K}_1, \vec{K}_2)$ in the following manner

$$W(\vec{K}_1,\vec{K}_2) = (-1)^{2I-I_i-I_f} \sum_{k} \sum_{\substack{L_1L_1' \\ L_2L_2'}} \sum_{\tau_1\tau_2} (-1)^{k-L_1'-L_2'}$$

$$\times \begin{Bmatrix} I & I & k \\ L_1 & L_1' & I_i \end{Bmatrix} \begin{Bmatrix} I & I & k \\ L_2 & L_2' & I_f \end{Bmatrix} \langle I_f || L_2\pi_2 || I\rangle \langle I_f || L_2'\pi_2' || I\rangle^*$$

$$\times \langle I || L_1\pi_1 || I_i\rangle \langle I || L_1'\pi_1' || I_i\rangle^* \, C_{k\tau_1}(L_1'L_1) \, C^*_{k\tau_2}(L_2L_2') D^k_{\tau_2\tau_1}(\vec{K}_2 \rightarrow \vec{K}_1)$$

(2.27)

The rotation $\vec{K}_2 \rightarrow \vec{K}_1$ carries the coordinate system of the second radiation R_2 into that of the first radiation R_1. The properties of 6-J symbols lead to the condition for $\vec{K}$.

$$0 \leq k \leq \text{Min}\ (2I,\ L_1+L_1',\ L_2+L_2')$$

andfor pure multipoles

$$0 \leq k \leq \text{Min}\ (2I,\ 2L_1,\ 2L_2)$$

For pure multipole radiations ($L_1 = L_1'$, $L_2 = L_2'$), the reduced matrix elements in equation (2.27) can be omitted and the sums over L_1, L_1', L_2 and L_2' all degenerate into one term. For mixed radiations, L and L' vary independently over the range of allowed angular momenta.

In angular correlation experiments, we observe only the directions and not the polarization of two radiations R_1 and R_2. In this case, the angular correlation function $W(\theta)$ independent of the Euler angles γ_1 and γ_2 which denote the rotation about t directions of propagation $\vec{K}_1$ and $\vec{K}_2$. The independent of γ means $\tau_1 = \tau_2 = 0$ and the radiation parameter $C_{k\tau}$ (LL') is

$$C_{k0}(LL') = (-1)^{L-1} (2L+1)^{1/2} (2L'+1)^{1/2} (2k+1)^{1/2} \begin{pmatrix} L & L' & k \\ 1 & -1 & 0 \end{pmatrix}$$

Ferguson (16) has defined Z coefficient which is written as

$$Z_1(L_1 I L_1' I, I_i k) = (-1)^{k-L_1+L_1'-1} \hat{L}_1 \hat{L}_1' \hat{I} \hat{I} \begin{pmatrix} L_1 & L_1' & k \\ 1 & -1 & 0 \end{pmatrix} \begin{Bmatrix} I & I & k \\ L_1 & L_1' & I_i \end{Bmatrix} \qquad (2.28)$$

Therefore the expression for $W(\theta)$ can be written as

$$W(\theta) = \sum_{kL_1L_1'L_2L_2'} (-1)^{I_i-I_f-L_1-L_1'} Z_1(L_1IL_1'I;I_ik) Z_1(L_2IL_2'I;I_fk)$$

$$\times \langle I||L_1\pi_1||I_i\rangle \langle I||L_1'\pi_1'||I_i\rangle^* \langle I_f||L_2\pi_2||I\rangle$$

$$\times \langle I_f||L_2'\pi_2'||I\rangle^* \, Q_k(1)\, Q_k(2)\, P_k(\mathrm{Cos}\,\theta) \qquad (2.29)$$

Where $Q_k(1)$ and $Q_k(2)$ are the attenuation coefficients for the first and second radiations. Introducing the multipole mixing ratios

$$\delta_1 = \frac{\langle I||L_1'\pi_1'||I_i\rangle}{\langle I||L_1\pi_1||I_i\rangle} \quad \text{and} \quad \delta_2 = \frac{\langle I_f||L_2'\pi_2'||I\rangle}{\langle I_f||L_2\pi_2||I\rangle}$$

The correlation function can be written as

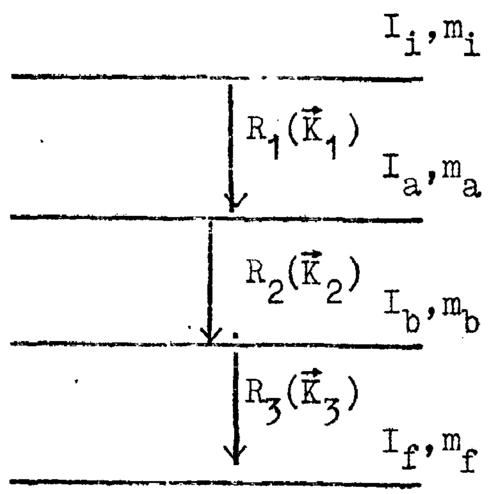
$$W(\theta) = \sum_{kL_1L_1'L_2L_2'} (-1)^{I_i-I_f-L_1-L_1'} Z_1(L_1IL_1'I;I_ik)\, Z(L_2IL_2'I;I_fk)$$

$$\times \delta_1^{r_1} \delta_2^{r_2} \, Q_k(1)\, Q_k(2)\, P_k(\mathrm{Cos}\,\theta) \qquad (2.30)$$

Where r_1 is the exponent having values 0,1 and 2 depending on whether the term is of the type (L_1L_1), (L_1L_1') or $(L_1' L_1')$. θ is the angle between the two radiations R_1 and R_2 when the direction of R_1 is fixed.

2.3 EXTENSION TO THE CASCADE OF THREE RADIATIONS:-

Let us consider that nucleus decays through successive emission of three radiations R_1, R_2 and R_3 into the directions $\vec{K}_1$, $\vec{K}_2$ and $\vec{K}_3$. Let I_i, I_a, I_b and I_f be the spin quantum numbers and m_i, m_a, m_b and m_f be the magnetic quantum numbers of the states.



Let us compute $R_2 - R_3$ correlation using as ρ_i the density matrix $\rho(k_1)$. From equation (2.10), setting $m_i = m_a$, $m = m_b$, We obtain the density matrix $\rho_f(\vec{K}_1, \vec{K}_2, \vec{K}_3)$ of the final state

$$\langle m_f | \rho_f (\vec{K}_1, \vec{K}_2, \vec{K}_3) | m'_f \rangle = S_2 S_3 \sum_{\substack{m_a, m'_a \\ m_b, m'_b}} \langle m_f | H_3 | m_b \rangle \langle m_b | H_2 | m_a \rangle$$

$$\times \langle m_a | \rho(\vec{K}_1) | m'_a \rangle \langle m'_f | H_3 | m'_b \rangle^* \langle m'_b | H_2 | m_a \rangle \qquad (2.31)$$

Now we sum over m_f and obtain the triple directional correlation $(m'_f = m_f)$.

$$W(\vec{K}_1,\vec{K}_2,\vec{K}_3) = S_2 \sum_{\substack{m_a, m'_a \\ m_b\, m'_b}} \langle m_a | \rho(\vec{K}_1) | m'_a \rangle \langle m_b | H_2 | m_a \rangle$$

$$\times \langle m'_b | H_2 | m'_a \rangle^* \langle m_b | \rho(\vec{K}_3) | m'_b \rangle$$

$$= \sum_{\substack{m_a m'_a \\ m_b m'_b}} \langle m_a | \rho(\vec{K}_1) | m'_a \rangle \langle m_a m'_a | G(\vec{K}_2) | m_b m'_b \rangle \langle m_b | \rho(\vec{K}_3) | m'_b \rangle \qquad (2.32)$$

The matrix elements $\langle m_a | \rho(\vec{K}_1) | m'_a \rangle$ are given by (2.25) and the $\langle m_b | \rho(\vec{K}_3) | m'_b \rangle$ are obtained from equation (2.26) by substituting 3 for 2. and the coupling matrix $G(\vec{K}_2)$ is given by

$$\langle m_a m'_a | G(\vec{K}_2) | m_b m'_b \rangle = S_2 \langle m_b | H_2 | m_a \rangle \langle m'_b | H_2 | m'_a \rangle^* \qquad (2.33)$$

This coupling matrix represents the unitary transformation expressing the evaluation of the state b from a, i.e.

$$\rho_a \xrightarrow{G(\vec{K}_2)} \rho_b$$

Substituting equation (2.21) into (2.33) equation and replacing Clebsch-Gordan coefficients into 3-J symbols, we obtain the explicit expression for the coupling matrix $G(\vec{K}_2)$

$$\langle m_a m'_a | G(\vec{K}_2) | m_b m'_b \rangle = \sum_{L_2 L'_2\, M_2 M'_2} (-1)^{k_2 - 2I_b - L_2 - m_a - m'_a + M_2}$$

$$\times (2k_2+1)^{1/2}\, C^*_{k_2 0}(L_2 L'_2) \begin{pmatrix} I_b & L_2 & I_a \\ m_b & M_2 & -m_a \end{pmatrix} \begin{pmatrix} I_b & L'_2 & I_a \\ m'_b & M'_2 & -m_a \end{pmatrix} \begin{pmatrix} L_2 & L'_2 & k_2 \\ -M_2 & M'_2 & N_2 \end{pmatrix}$$

$$\times \langle I_b || L_2 \pi_2 || I_a \rangle \langle I_b || L'_2 \pi'_2 || I_a \rangle^* D^{k_2}_{N_2 0} (\vec{Z} \to \vec{K}_2) \qquad (2.34)$$

The equation (2.34) is substituted into (2.32) equation and again using Z_1 coefficients as given by Ferguson (16) and Racah radiation parameters $C_{k0}(LL')$, introducing $G\gamma$ and $P_{k_1k_2k_3}(\vec{K}_1\vec{K}_2\vec{K}_3)$ which are defined by Ferguson (16),

$$G\gamma\begin{Bmatrix} I_b & L_2 & I_a \\ I_b & L_2' & I_a \\ k_3 & k_2 & k_1 \end{Bmatrix} = (-1)^{I_a+I_b-L_2'+1+-\frac{1}{2}(k_2+k_3-k_1)}$$

$$X\ (\hat{I}_a)^2\ L_2L_2'\ \hat{k}_1\ \hat{k}_2\ \hat{k}_3\ \langle L_2 1 L_2' -1|k_2 0\rangle \begin{Bmatrix} I_b & L_2 & I_a \\ I_b & L_2' & I_a \\ k_3 & k_2 & k_1 \end{Bmatrix} \qquad (2.35)$$

and

$$P_{k_1k_2k_3}(\vec{K}_1\vec{K}_2\vec{K}_3) = \Sigma \langle k_1N_1k_2N_2|k_3N_3\rangle\ D^{K_1}_{N_1 0}(\vec{Z}\to\vec{K}_1)$$

$$X\ D^{k_2}_{N_2 0}(\vec{Z}\to\vec{K}_2)\ D^{k_3}_{N_3 0}(\vec{Z}\to\vec{K}_3) \qquad (2.36)$$

We get

$$W(\vec{K}_1\vec{K}_2\vec{K}_3) = (-1)^{I_i+I_f}\Sigma(-1)^{L_1-L_1'+k_1} Z_1(L_1I_aL_1'I_a;I_ik_1)$$

$$X\ G\gamma\begin{Bmatrix} I_b & L_2 & I_a \\ I_b & L_2' & I_a \\ k_3 & k_2 & k_1 \end{Bmatrix} Z_1(L_3I_bL_3'\ I_b;I_fk_3)\ \delta_1^{r_1}\delta_2^{r_2}\delta_3^{r_3}\ P_{k_1k_2k_3}(\vec{K}_1\vec{K}_2\vec{K}_3) \qquad (2.37)$$

With the summation over $L_1L_1'\ L_2L_2'\ L_3L_3'\ k_1\ k_2\ k_3$. Where multipole mixing ratios are given by

$$\delta_1 = \frac{\langle I_a||L_1'\ \pi_1'\ ||\ I_i\rangle}{\langle I_a||L_1\ \pi_1\ ||\ I_i\rangle}$$

$$\delta_2 = \frac{\langle I_b||L_2'\ \pi_2'\ ||\ I_a\rangle}{\langle I_b||L_2\ \pi_2\ ||\ I_a\rangle}$$

$$\delta_3 = \frac{\langle I_f || L_3' \pi_3' || I_b \rangle}{\langle I_f || L_3 \pi_3 || I_b \rangle}$$

The D matrices in equation (2.36) are written in terms of the spherical harmonics by the relation

$$D^{k_1}_{N_1 0} (\vec{Z} \to \vec{K}_1) = \left(\frac{4\pi}{2k_1+1}\right)^{1/2} Y^{N_1}_{k_1}(\vec{Z} \to \vec{K}_1) \qquad (2.38)$$

The similar expression for the other D matrices can also be written. Therefore $P_{k_1k_2k_3}(\vec{K}_1\vec{K}_2\vec{K}_3)$ can be written as

$$P_{k_1k_2k_3}(\vec{K}_1\vec{K}_2\vec{K}_3) = \frac{(4\pi)^{3/2}}{\hat{k}_1\hat{k}_2(\hat{k}_3)^2} \Sigma \langle k_1N_1, k_2N_2|k_3N_3\rangle$$

$$X \; Y^{N_1}_{k_1}(\vec{Z} \to \vec{K}_1) \; Y^{N_2}_{k_2}(\vec{Z} \to \vec{K}_2) \; Y^{N_3}_{k_3} (\vec{Z} \to \vec{K}_3) \qquad (2.39)$$

For $k_1 + k_2 + k_3$ = odd integer ($k_1k_2k_3$ = 111, 122, 133, 144, 212, 221, 223, 232, 234, 243, 322, 324, 333, 342, 344, 414, 423, 432, 434, 441 and 443), $P_{k_1k_2k_3}(\vec{K}_1\vec{K}_2\vec{K}_3)$ are not real. To make it real, we take $i^{k_1}Y^{N_1}_{k_1}(\vec{Z} \to \vec{K}_1)$ as the basic function. Using the properties of spherical harmonics and also replacing the angular position of the three detectors by Ω_1, Ω_2 and Ω_3, we get as given by Ferguson (16),

$$P_{k_1k_2k_3}(\vec{K}_1\vec{K}_2\vec{K}_3) = (4\pi)^{3/2} \frac{i^{k_1+k_2-k_3}}{\hat{k}_1 \hat{k}_2 (\hat{k}_3)^2} \sum_{k_1k_2k_3} (-1)^{N_3}$$

$$X \langle k_1N_1, k_2N_2|k_3N_3\rangle \; Y^{N_1}_{k_1}(\vec{Z} \to \vec{K}_1) \; Y^{N_2}_{k_2} (\vec{Z} \to \vec{K}_2) \; Y^{N_3}_{k_3}(\vec{Z} \to \vec{K}_3) \qquad (2.40)$$

and the sum will have 19 terms with $(k_1k_2k_3)$ =(000),(022),(044),(202),(220),(222),(224),(242), (404),(422),(424),(440),(442),(444),(624),(642),(644) and (844).

In these derivations, we have assumed a point source and point detectors but in practice, it can not be so. The detectors are of definite size and they substend finite solid angles at the source. In order to compare the experimental results with the theoretical calculations, it is necessary to apply solid angle correction. Angular correlation function is multiplied by the attenuation factors for each detector. Let Ω_1 , Ω_2 and Ω_3 be the solid angles substended by the three detectors whose attenuation factors are $Q_k(1)$, $Q_k(2)$ and $Q_k(3)$. The tables of these attenuation factors are given by Ferguson (16) and a computer programme is also made on the lines given by Rose (17). Including solid angle correction, the angular correlation function would be

$$W(\Omega_1\ \Omega_2\ \Omega_3)=\left(\frac{1}{4\pi}\right)^{3/2}(-)^{I_i+I_f}\Sigma(-)^{L_1-L_1'+k_1}\ Z_1(L_1I_aL_1'I_a;I_ik_1)$$

$$X\ G\begin{Bmatrix} I_b & L_2 & I_a \\ I_b & L_2' & I_a \\ k_3 & k_2 & k_1 \end{Bmatrix} Z_1(L_3I_bL_3'\ I_b;I_fk_3)\ \delta_1^{r_1}\delta_2^{r_2}\delta_3^{r_3}\ Q_k(1)\ Q_k(2)$$

$$X\ Q_k(3)\ P_{k_1k_2k_3}(\Omega_1\ \Omega_2\ \Omega_3) \qquad (2.41)$$

With the summation over $L_1L_1'L_2L_2'L_3L_3'\ k_1k_2k_3$

2.4 THEORY OF BETA_ GAMMA AND BETA_GAMMA_GAMMA DIRECTIONAL CORRELATION:-

(i) BETA_GAMMA DIRECTIONAL CORRELATION:-

In beta-gamma angular correlation, an electron and a neutrino are emitted simultaneously and we observe experimentally the direction of the electron while the neutrino escapes unobserved. The theoretical calculation necessitates an averaging over all neutrino directions and over the spins of neutrino and electron. Inallowed beta-transition between the states of equal angular momenta, the Fermi and Gamow-Teller components contribute and two matrix elements <1> and <σ>. If higher order effects are neglected, the beta-gamma directional correlation is isotropic. The beta-gamma directional correlation function, considering higher order effects is given by Morita (14). The beta-gamma directional correlation function is

$$W(\theta) = (-1)^{I_i - I_b} \sum_{J, L_2, L_2' k} b_k(JJ) Z_1(J\ I_a\ J\ I_a;\ I_i k_1)$$

$$\times\ Z_1(L_2 I_b L_2'\ I_b; I_f k_2)\ \delta_2^{\,r_2} P_k(\cos\theta) \qquad (2.42)$$

Where $b_k(JJ)$ are considered by Morita (14) which include higher order effects.

(ii) <u>BETA-GAMMA-GAMMA DIRECTIONAL CORRELATION</u>:-

According to equation (2.41), the directional correlation function for β (allowed) → γ → γ cascade can be written as

$$W(\Omega_1\ \Omega_2\ \Omega_3) = \left(\frac{1}{4\pi}\right)^{3/2} (-1)^{I_i + I_f} \sum (-1)^{k_1}\ b_k(JJ)\ Z_1(J I_a J\ I_a; I_i k_i)$$

$$\times\ G_\gamma \begin{Bmatrix} I_b & L_2 & I_a \\ I_b & L_2' & I_a \\ k_3 & k_2 & k_1 \end{Bmatrix} Z_1(L_3 I_b L_3' I_b; I_f k_3)\ \delta_2^{\,r_2}\ \delta_3^{\,r_3} Q_k(1)\ Q_k(2) Q_k(3)$$

$$\times\ P_{k_1 k_2 k_3}(\Omega_1\ \Omega_2\ \Omega_3) \qquad (2.43)$$

With the summation over $JL_2L_2'L_3L_3'k_1k_2k_3$. If we neglect higher order effects in the allowed β-transitions, $b_2(JJ)$ and $b_4(JJ)$ are zero and $b_0(JJ)$ will not be zero.

Thus

$$W(\Omega_1\,\Omega_2\,\Omega_3) = \left(\frac{1}{4\pi}\right)^{3/2}(-1)^{I_i+I_f}\,\Sigma\,(-1)^{k_1}\,b_0(JJ)$$

$$Z_1(JI_aJI_a;I_i0)\,\mathcal{G}\begin{Bmatrix} I_b & L_2 & I_a \\ I_b & L'_2 & I_a \\ k_3 & k_2 & k_1 \end{Bmatrix} Z_1(L_3I_bL_3'\,I_b;I_fk_3)$$

$$X\,\delta_2^{r_2}\,\delta_3^{r_3}\,Q_{k=0}(1)\,Q_k(2)\,Q_k(3)\,P_{k_1k_2k_3}(\Omega_1\,\Omega_2\,\Omega_3) \qquad (2.44)$$

With the summation over $JL_2L_2'L_3L_3'\,k_1k_2k_3$

2.5 THE GENERAL FORMULA FOR ANGULAR CORRELATION COEFFICIENTS WITH THE GEOMETRICAL CONSIDERATIONS FOR THE MOUNTING OF THE DETECTORS IN THE STUDIES OF TRIPLE CASCADES:-

The function for angle in triple correlation function is given in equation (2.40) as

$$P_{k_1k_2k_3}(\Omega_1\,\Omega_2\,\Omega_3) = (4\pi)^{3/2}\,\frac{i^{k_1+k_2-k_3}}{\hat{k}_1\,\hat{k}_2\,\hat{k}_3^2}\,\sum_{N_1N_2N_3}(-1)^{N_3}$$

$$X\,\langle k_1N_1,k_2N_2|k_3N_3\rangle\,Y_{k_1}^{N_1}(\Omega_1)\,Y_{k_2}^{N_2}(\Omega_2)\,Y_{k_3}^{N_3}(\Omega_3)$$

The function $P_{k_1k_2k_3}(\Omega_1\Omega_2\Omega_3)$ can be simplified if one of the angles say Ω_1 is zero i.e. $\theta_1 = 0$, $\phi_1 = 0$, then

$$Y_{k_1}^{N_1}(\Omega_1) = Y_{k_1}^{N_1}(0,0) = \frac{\hat{k}_1\delta N_1 0}{(4\pi)^{1/2}}$$

Therefore

$$P_{k_1k_2k_3}(0\,\Omega_2\,\Omega_3) = \frac{4\pi\, i^{k_1+k_2-k_3}}{\hat{k}_2\;\hat{k}_3^{\,2}} \sum_N (-1)^N \langle k_1 0, k_2 N | k_3 N\rangle$$

$$\times\, Y^N_{k_2}(\Omega_2)\, Y^{-N}_{k_3}(\Omega_3) \qquad (2.45)$$

The spherical harmonics can be written in terms of the associated Legendre Polynomials

$$Y^N_k(\theta,\phi) = (-)^{1/2(N+|N|)} \frac{\hat{k}}{(4\pi)^{1/2}} \left[\frac{(k-|N|)!}{(k+|N|)!}\right]^{1/2} P^{|N|}_k(\cos\theta)\, e^{(iN\phi)} \qquad (2.46)$$

Using equation (2.46), we can write equation (2.45) as

$$P_{k_1k_2k_3}(0\,\Omega_2\,\Omega_3) = \frac{i^{k_1+k_2-k_3}}{\hat{k}_2\;\hat{k}_3^{\,2}} \sum_{N\geq 0} (2-\delta N0)\langle k_1 0, k_2 N | k_3 N\rangle$$

$$\times\, X^N_{k_2k_3}(\theta_2,\theta_3,\phi) \qquad (2.47)$$

Where

$$X^N_{k_2k_3}(\theta_2,\theta_3,\phi) = k_2\, k_3 \left[\frac{(k_2-N)!(k_3-N)!}{(k_2+N)!(k_3+N)!}\right]^{1/2} P^N_{k_2}(\cos\theta_2)$$

$$\times\, P^N_{k_3}(\cos\theta_3)\, \cos N\phi \qquad (2.48)$$

and $\phi_2 - \phi_3 = \phi$

The associated Legendre function with N even can be expressed in the form

$$P^N_k(\cos\theta) = \sum_k f^N_{k'k}\, P_k(\cos\theta) \qquad (2.49)$$

With $k \leq k'$ and even. The coefficients $f^N_{k'k}$ for $k' = 0,2$ and 4 are given in table I.

If we impose certain restrictions on Ω_2 and Ω_3, then it is feasible to do the angular correlation of triple cascade. The restrictions are summarized in table II.

TABLE I

Coefficients $f^N_{k'k}$ for the expansion of the associated Legendre function

k'N	k = 0	k = 2	k = 4
00	1	0	0
20	0	1	0
22	2	-2	0
40	0	0	1
42	2	10	-12
44	56	-80	24

TABLE II

θ^0 is variable

Geometry	θ_2	θ_3	$\phi = \phi_2 - \phi_3$
A_1	θ^0	90^0	180^0
A_2	90^0	θ^0	180^0
C_1	θ^0	90^0	90^0
C_2	90^0	θ^0	90^0

From table II, we see that $\phi = \phi_2 - \phi_3$ is either 90^0 or 180^0 and angle θ_2(or θ_3) is fixed and θ_3(or θ_2) is variable.

Using these restrictions on Ω_2 and Ω_3 after inserting equations (2.48),(2.49) in equation (2.47), we get for $P_{k_1k_2k_3}(0\ ,\ \Omega_2,\ \Omega_3)$ as

$$P_{k_1k_2k_3}(0,\ \Omega_2,\ \Omega_3) = \sum_k \alpha^i_{k_1k_2k_3k}\ \ P_k(\mathrm{Cos}\,\theta) \qquad (2.50)$$

Where i identifies the cases of the mounting of the detectors as given in table II. The $\alpha^i_{k_1k_2k_3k}$ coefficients of $P_k(\mathrm{Cos}\ \theta)$ are given by Ferguson (16).

The directional correlation function for beta (allowed)-gamma-gamma cascade can be written as

$$W(\theta) = \sum_{k_1k_2k_3k} a_{k_1k_2k_3}\ \alpha^i_{k_1k_2k_3k}\ \ P_k(\mathrm{Cos}\,\theta) \qquad (2.51)$$

Where

$$a_{k_1k_2k_3} = \left(\frac{1}{4\pi}\right)^{3/2} (-1)^{I_i+I_f} \sum (-1)^{k_1}\ b_0(JJ)\ Z_1(JI_a\ JI_a;I_i0)$$

$$X\ G_\gamma \begin{Bmatrix} I_b & L_2 & I_a \\ I_b & L_2' & I_a \\ k_3 & k_2 & k_1 \end{Bmatrix} Z(L_3I_bL_3'I_b;I_fk_3)\ \ \delta_2^{r_2}\ \ \delta_3^{r_3}\ \ Q_{k=0}(1)$$

$$X\ Q_k(2)\ Q_k(3) \qquad (2.52)$$

Let

$$a_k = a_{k_1k_2k_3}\ \alpha^i_{k_1k_2k_3k} \qquad (2.53)$$

then

$$W(\theta) = \sum_k a_kP_k(\mathrm{Cos}\theta) \qquad (2.54)$$

In case, all the radiations are pure, then $L_2 = L_2'$ and $L_3=L_3'$ and therefore $r_2 = 0$ or 2 and $r_3 = 0$ or 2. a_0, a_2 and a_4 can be calculated from equation (2.53) using equation (2.52).

One can mount the detectors easily in two geometrical considerations i.e.,(i) In the first geometrical consideration, all the three detectors are in the plane of the table, referred as A_1 and A_2 geometries. In these geometries, the two detectors are fixed and are perpendicular to each other and third detector is movable.

(ii) In the second geometrical consideration, the two detectors are in the plane of the table and third is perpendicular to the plane of the table, referred as C_1 and C_2 geometries, The movable detector moves in the plane of the table. The detector which is fixed in the plane of the table, always detects the first radiation of the cascade. These geometrical considerations are shown in fig.2.1(a) and 2.1(b).

If we neglect the higher order effects in the allowed β- transitions and consider only the allowed β- transitions, in this way we find the values of $\alpha^{i}_{k_1k_2k_3k}$ coefficients which are given in table III.

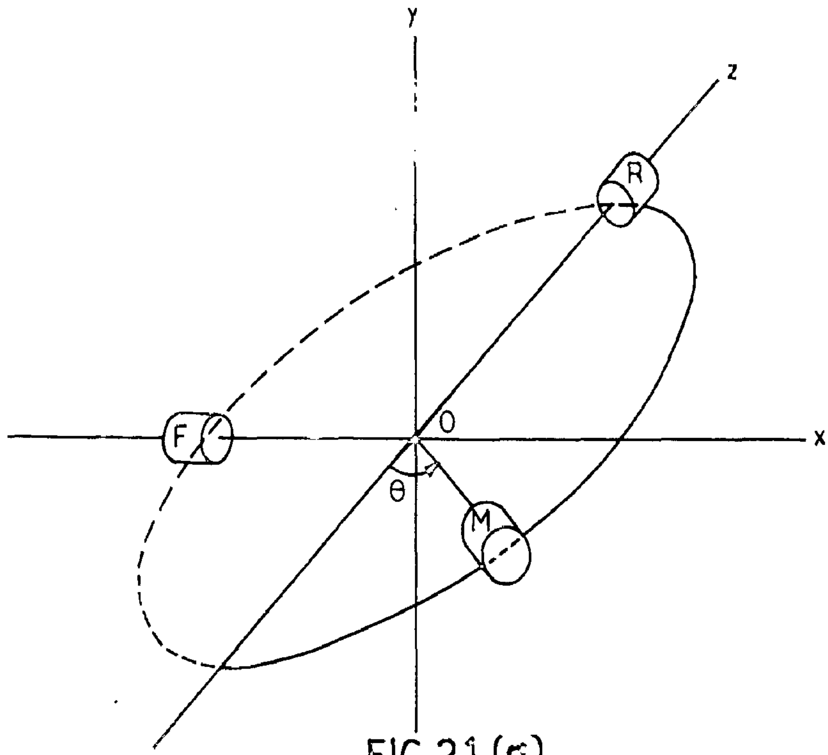


FIG 2.1 (a)

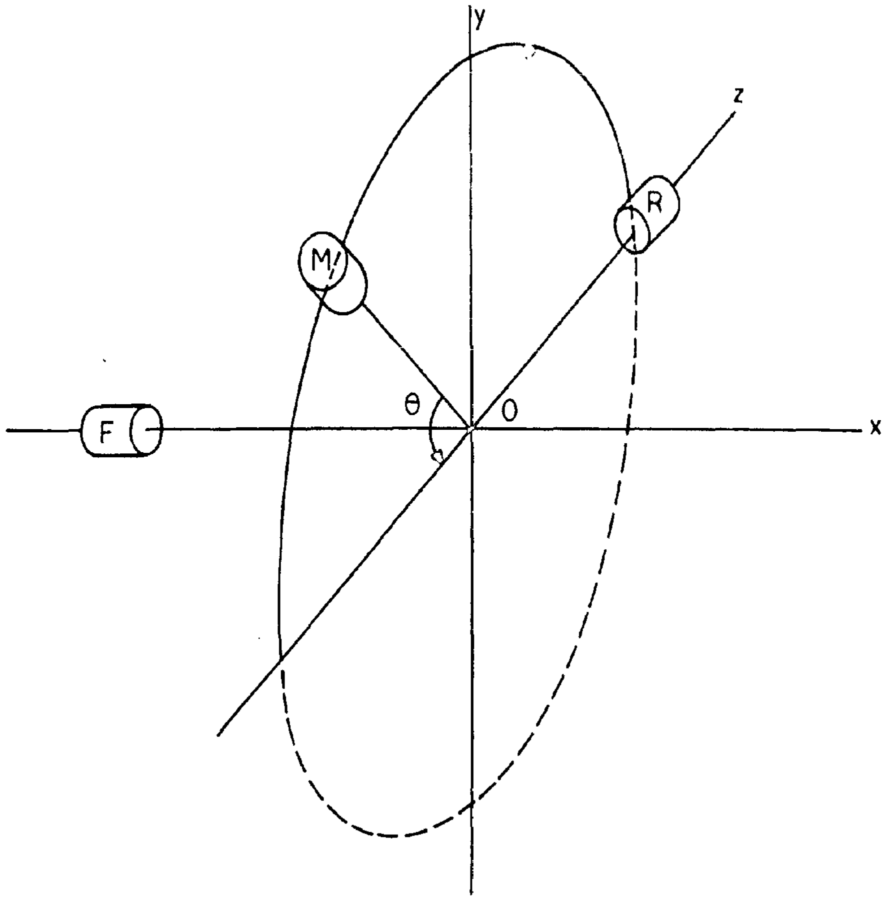


FIG. 2.1 (b)

GEOMETRIES OF THE MOUNTING OF THE DETECTOR FOR TRIPLE COINCIDENCE AND ANGULAR CORRELATION STUDIES. R & F ARE FIXED DETECTORS AND M IS A MOVABLE DETECTOR.

TABLE III

Tables for $\alpha^{i}_{k_1k_2k_3k}$ where i stands for various geometries

	Geometry A_1			Geometry A_2		
$k_1k_2k_3$	k = 0	2	4	0	2	4
000	1.0000	0	0	1.00000	0	0
022	0.22361	-0.44721	0	0.22361	-0.44721	0
044	0.06944	-0.27778	0.33333	0.06944	-0.27778	0.33333

	Geometry C_1			Geometry C_2		
$k_1k_2k_3$	k = 0	2	4	0	2	4
000	1.00000	0	0	1.00000	0	0
022	-0.22361	0	0	-0.22361	0	0
044	0.12500	0	0	0.12500	0	0

$\alpha^{i}_{k_1k_2k_3k}$ is zero for k = 2 and k = 4 in C_1 and C_2 geometries, if we consider only allowed beta transitions. Therefore the beta-gamma-gamma angular correlation will be isotropic. This will not be the case if we consider the higher order effects in the allowed beta-transitions and there will be contribution from other $(k_1k_2k_3)$, as given in equation (2.40) $\alpha^{i}_{k_1k_2k_3k}$ is not zero for k = 2 and k = 4 in A_1 and A_2 geometries for the allowed beta-transitions again from table III. Therefore there will be anisotropy in this geometry of mounting of the detectors. This has been taken in the present study.

2.6 THEORETICAL PLOT OF ANGULAR CORRELATION COEFFICIENTS VERSUS MULTIPOLE MIXING RATIO:-

Let us consider the case when one of the transitions (say second) is mixture of dipole and quadrupole in the triple beta-gamma-gamma cascade taking the first to be allowed β-transition and third transition to be pure. Therefore $J = J'$ and $L_3 = L_3'$ respectively and

$$G\gamma \begin{Bmatrix} I_b & L_2 & I_a \\ I_b & L_2' & I_a \\ k_3 & k_2 & k_1 \end{Bmatrix}$$ alongwith the δ_2 is written as

$$G\gamma \begin{Bmatrix} I_b & L_2 & I_a \\ I_b & L_2 & I_a \\ k_3 & k_2 & k_1 \end{Bmatrix} + 2\delta_2\, G\gamma \begin{Bmatrix} I_b & L_2 & I_a \\ I_b & L_2+1 & I_a \\ k_3 & k_2 & k_1 \end{Bmatrix} + \delta_2^2\, G\gamma \begin{Bmatrix} I_b & L_2+1 & I_a \\ I_b & L_2+1 & I_a \\ k_3 & k_2 & k_1 \end{Bmatrix} \qquad (2.55)$$

Considering the first term of the expression (2.55) in $a_{k_1k_2k_3}$ the equation (2.54) can be written in the form

$$W(\theta) = a_0 + a_2P_2(\text{Cos}\,\theta) + a_4\, P_4(\text{Cos}\,\theta)$$

Similarly putting second and third terms of the expression (2.55) in equation (2.54), the values of angular correlation coefficients a'_0, a'_2 and a'_4 and a''_0, a''_2 and a''_4 are obtained. Therefore $W(\theta)$ is written as

$$W(\theta) = \left[a_0 + a_2\, P_2(\text{Cos}\theta) + a_4\, P_4(\text{Cos}\theta)\right] + 2\delta_2\left[a'_0 + a'_2\, P_2(\text{Cos}\theta) + a'_4\, P_4(\text{Cos}\theta)\right] + \delta_2^2\left[a''_0 + a''_2\, P_2(\text{Cos}\theta) + a''_4\, P_4(\text{Cos}\theta)\right] \qquad (2.56)$$

Writing this in the form

$$W(\theta) = 1 + A_2 P_2(\cos\theta) + A_4 P_4(\cos\theta) \tag{2.57}$$

The values of A_2 and A_4 (defining $Q = \dfrac{\delta_2^2}{1 + \delta_2^2}$) are

$$A_2 = \frac{a_2 + 2\delta_2\, a_2' + \delta_2^2\, a_2''}{a_0 + 2\delta_2\, a_0' + \delta_2^2\, a_0''}$$

or

$$A_2 = \frac{(1-Q)\, a_2 \pm 2\sqrt{Q(1-Q)}\, a_2' + Q\, a_2''}{(1-Q)\, a_0 \pm 2\sqrt{Q\,(1-Q)}\, a_0' + Q\, a_0''}$$

and

$$A_4 = \frac{a_4 + 2\delta_2\, a_4' + \delta_2^2\, a_4''}{a_0 + 2\delta_2\, a_0' + \delta_2^2\, a_0''}$$

or

$$A_4 = \frac{(1-Q)a_4 \pm 2\sqrt{Q\,(1-Q)}\, a_4' + Qa_4''}{(1-Q)a_0 \pm 2\sqrt{Q\,(1-Q)}\, a_0' + Qa_0''}$$

For each value of Q, we shall get two values of A_2 depending on (+) or (-) sign of square root. The typical curves are obtained fromthe plot of A_2 versus Q. In a similar manner, the dipole-quadrupole mixture can also be considered in the third transition taking the other transition to be pure.

CHAPTER - 3

EXPERIMENTAL SET-UP FOR THE STUDY OF THE CASCADE OF TWO AND THREE RADIATIONS:-

3.1 INTRODUCTION:-

The scintillators, optically coupled with the photomultipliers are useful tools for the detection and spectroscopy of alpha, beta, and gamma-rays. When an energetic charged particle interacts with the molecules of the scintillator, it dissipates a part of its energy in the ionization and excitation of the molecules. A fraction of this energy is emitted into the form of light (scintillations). The photons (scintillations) produced are emitted in all directions, some of them are made to fall on the photocathode of the photomultiplier, thereby ejecting electrons from its surface. These electrons are multiplied at each dynode of the photomultiplier and after multiplication at each stage, an avalanche of electrons arrives at the anode where it produces a voltage pulse across a resistance. This voltage pulse is amplified and studied.

The process of the scintillation emission is divided into five stages.

(i) The absorption of the incident radiations by the scintillator.

(ii) The scintillation process in which the energy dissipated in the scintillator is converted into the luminescence emission of photons.

(iii) The transit of the emitted photons to the cathode of the photomultiplier tube.

(iv) The absorption of the photons at the cathode, the emission of photoelectrons and their collection at the first dynode.

(v) The electron multiplication process.

Wide varities of scintillators are in use today. Some of the commonly employed scintillators are :

(i) Organic crystals like anthracene, trans-stilbene and plastic for the spectrometry of beta-particles and

(ii) Inorganic crystals like sodium iodide activated with 0.1 percent mole fraction of the thallium for the spectrometry of gamma-rays.

3.2 BRIEF DESCRIPTION OF UNITS:

(i) SCINTILLATOR-PHOTOMULTIPLIER ASSEMBLY:-

For gamma-ray spectrometry, the cylendrical NaI(Tl) detectors whose dimensions are 3.8 cm in length and 3.8 cm in diameter, have been employed. These crystals were obtained from Harshaw Chemical Company. The lower surface of the crystal is optically coupled to the face of the photomultiplier tube RCA 6810 A with a thin film of transparent fluid (silicon grease having viscosity 60,000centistokes). The cylindrical Mu-magnetic shields around the photomultipliers are used. The detector and photomultiplier tube assemblies are made light tight by using black tape.

The organic scintillator (plastic scintillator) is used for the detection of beta-particles and it was obtained from Bhabha Atomic Research Centre,Bombay. The plastic scintillator is cut and polished using acetone and is optically coupled to RCA6810A photomultiplier using silicon grease. The plastic scintillator whose dimensions are 3cm in diameter and 0.3cm in thickness, is kept alongwith the source in vacuum chamber which is shown in fig.3.1. The source was dried on the cellotape and it is mounted on the perspex stand. The source on the cellotape alongwith the stand is kept in the vacuum chamber. The vacuum is obtained by the rotary vacuum pump obtained from Electrical Construction and Equipment Co.Ltd. Calcutta (India)

(ii) SINGLE CHANNEL SCINTILLATION SPECTROMETER:

The scintillator-photomultiplier tube alongwith the cathode follower and voltage divider network to feed power to the different dynodes of the tube are put in one assembly. The output pulses from the cathode follower are fed to the amplifier and then to the analyser and finally to the scaler and recorder. Such assemblies are referred as a single channel scintillation spectrometer. Some details of the various units used are as follows:

(a) EHT POWER SUPPLY:

High voltage units (type No.H.V.201C) obtained from Electronic Corporation of India Ltd., Hyderabad, providing electronically regulated high voltage of 1.5kV to 2.5kV at a maximum load current of ten milliamperes, are used. The ripple and noise contents are less than 15 millivolts peak to peak. The output variation is less than 0.002%. for one percent change in line voltage between 218 and 250 volts.

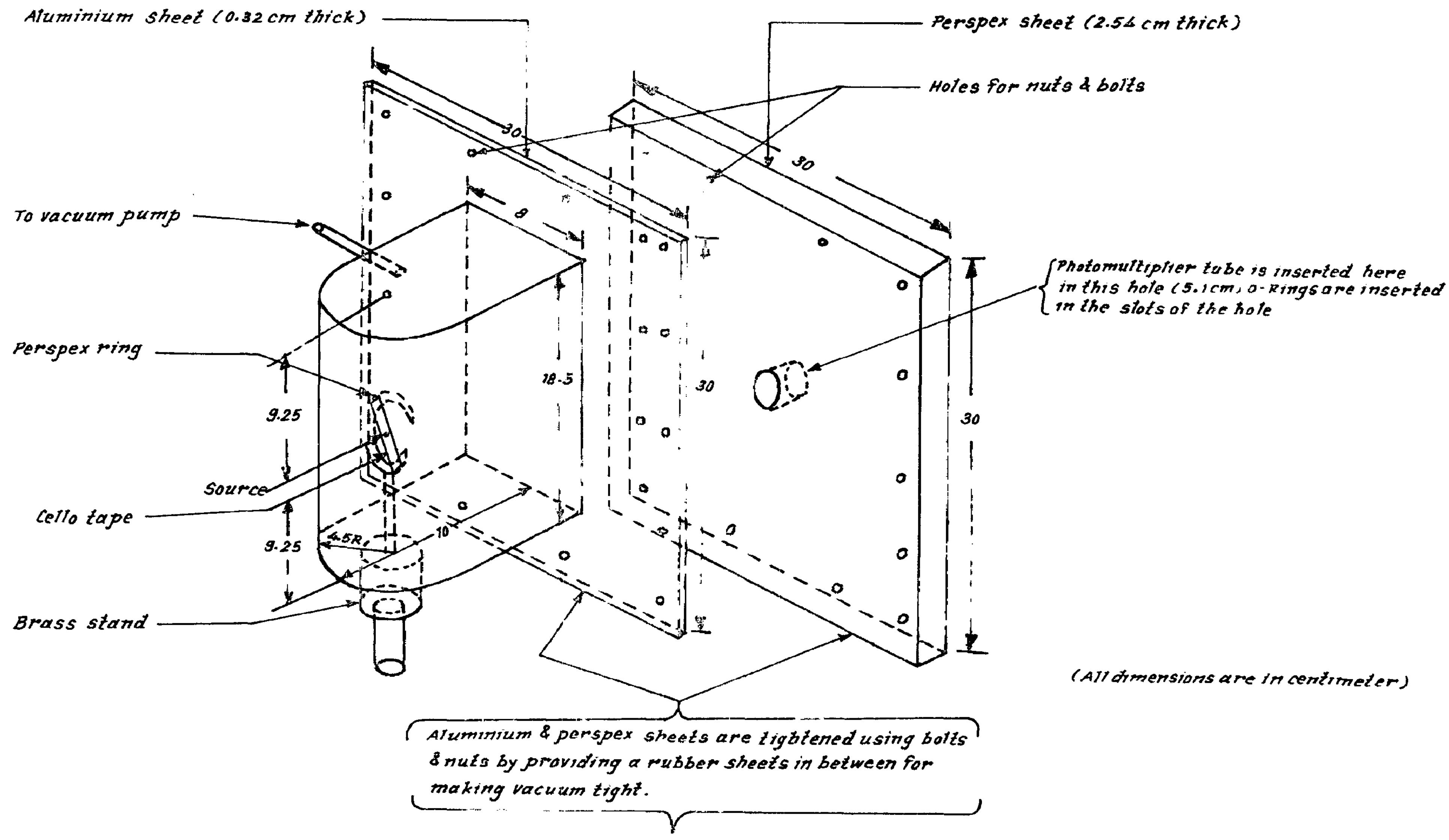


FIG 3.1 _ DETAILS OF THE VACUUM CHAMBER.

(b) LINEAR AMPLIFIER-ANALYSER:

Model 250 amplifier-analyser was obtained from Baird Atomic Inc.Mass. This model 250 is a combination of amplifier and analyser. The gain of the amplifier ranges from 72 to 6400 with coarse and fine gain control. The pulses required for the analyser portion is from 5 to 100 volts. The analyser can be used as integral and also as differential with the channel setting from 0.5 volts to 20 volts.

(c) SCALER AND ELECTRONIC TIMER:

Scaler and electronic timer are obtained from Electronic Corporation of India Ltd., Hyderabad. Scaler model DS325 may be operated either manually with an electronic timer. The electronic timer type ET450A is a preset timer with time setting from 1 second to 9,999 seconds.

(iii) SLOW-FAST COINCIDENCE SET-UP:

The arrangement of a slow-fast coincidence set-up is shown in fig.3.2. The main features of the fast coincidence unit as shown in fig.3.3 are:

(i) pulse shaping by shorted cable (RG176/u) and

(ii) coincidence circuit using 6BN6 tube;

and those of slow-coincidence set-up as shown in fig.3.4 are:

(i) pulse shaping using 6J6 tube (univibrator) and

(ii) Rossi type coincidence circuit using IN34 diodes.

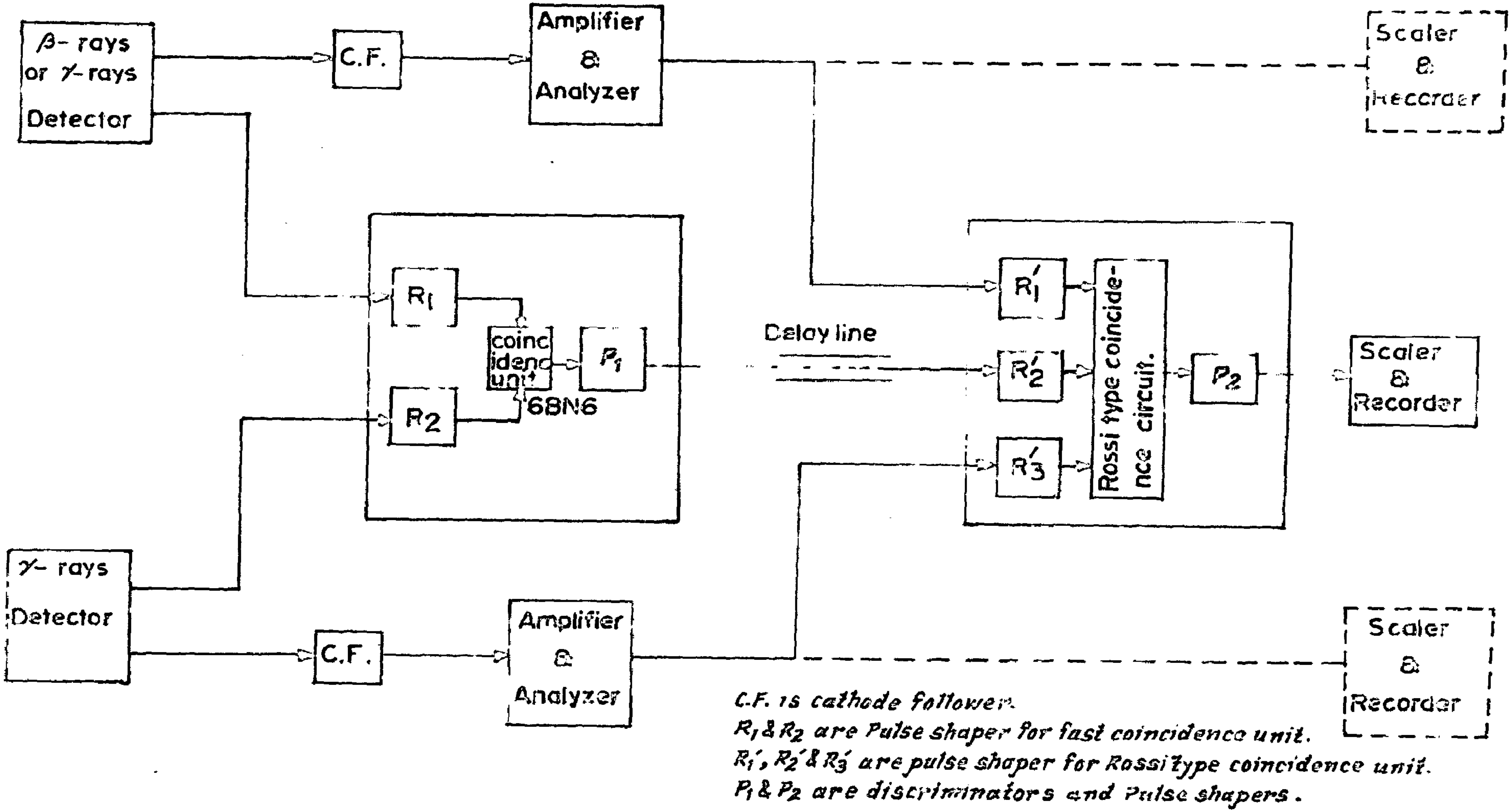


FIG.3.2 BLOCK DIAGRAM OF SLOW-FAST COINCIDENCE SET-UP.

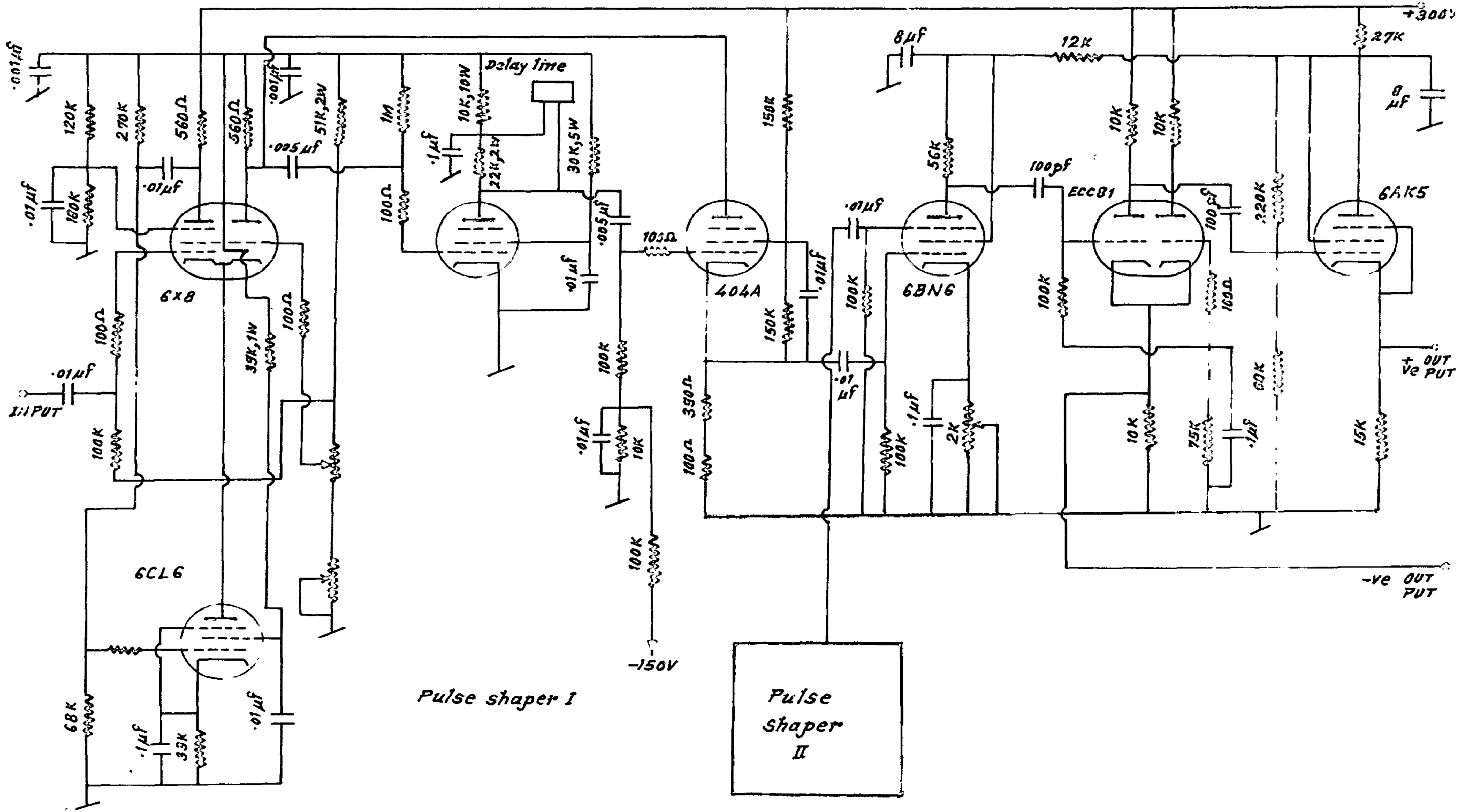


FIG.3.3_ CIRCUIT DIAGRAM OF FAST COINCIDENCE UNIT.

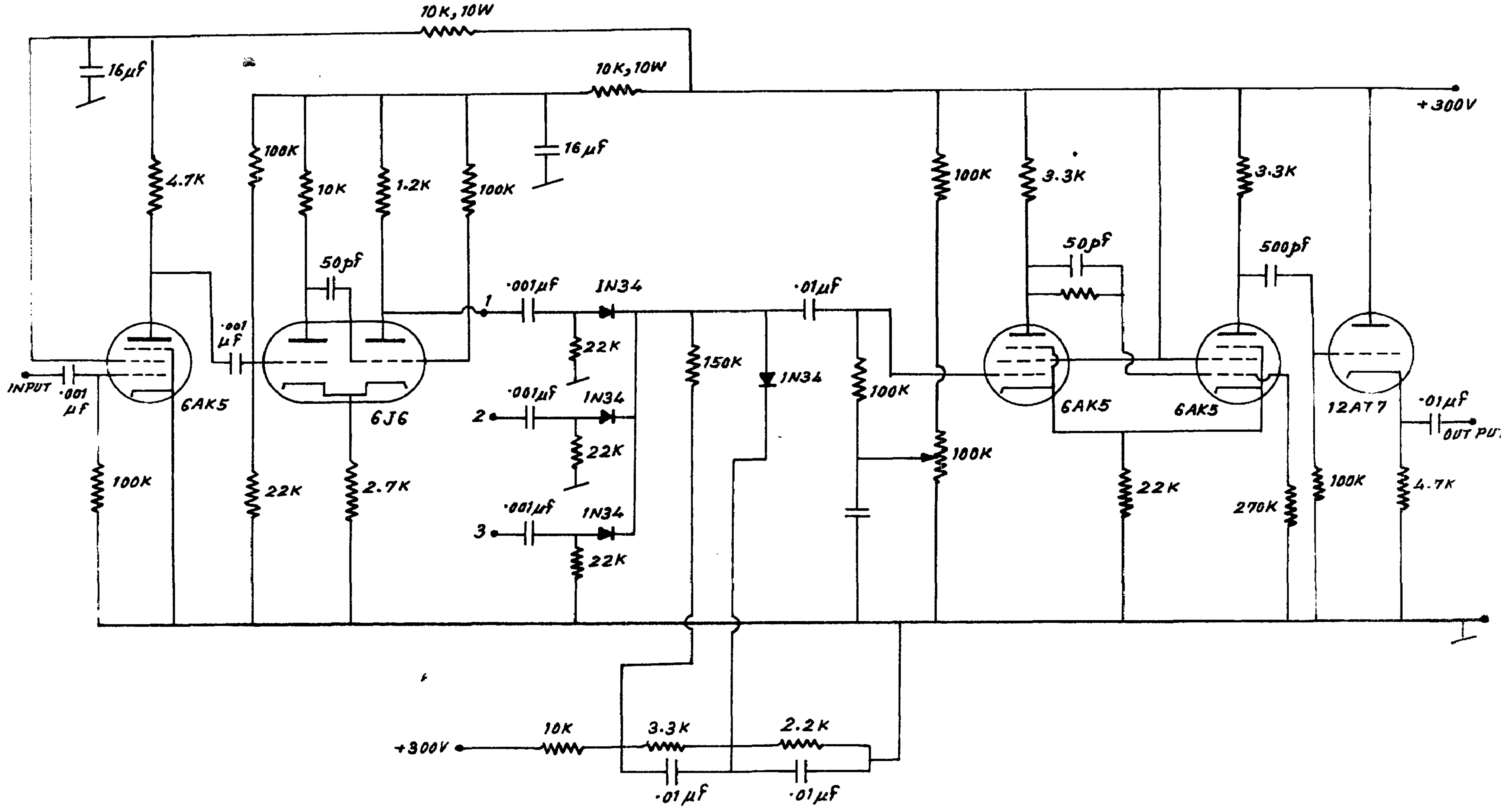


FIG. 3.4 - CIRCUIT DIAGRAM OF TRIPLE COINCIDENCE UNIT.

(a) FAST COINCIDENCE SET-UP:

The circuit diagram of fast coincidence unit is shown in figure 3.3. The input requirement of fast coincidence unit is negative pulses. These negative pulses are amplified by using (6 X 8) tubes so as to trigger the pulse shaping tubes (404A) and in the pulse forming section, the tube (404A) is biased at zero and its plate load consists of a shorted section of delay line. The delay line (RG176/u) used is 2.5 inches in length i.e. both ways time delay is 5×10^{-8} sec. The pulse shapers I and II are of the same type. The output pulses from the shapers I and II are applied to the first and third grid respectively of 6BN6 coincidence tube. The coincidence output is taken using ECC81 and EF95 tubes as shown in fig.3.3.

(b) SLOW-COINCIDENCE UNIT:

The figure 3.4 shows the circuit-diagram of the slow coincidence unit. The negative pulses are given to the inputs of this slow coincidence unit. These negative pulses are inverted and amplified by 6AK5 tubes so as to trigger the pulse shapers (univibrators) using 6J6 tubes. Rossi type coincidence circuit using IN34 diodes has been used, followed by the discriminator which is not triggered by single or double input pulses but only triggered by triple input pulses in coincidence. The triple coincidence circuit has been used as shown by block diagram in fig.3.2. The delay cable (RG176/u) has been introduced after fast coincidence circuit-before it is fed to the pulse shaper of triple coincidence unit in order to compensate the delay from other two inputs.

3.3 BETA-GAMMA-GAMMA COINCIDENCE STUDIES:

The slow-fast coincidence set-up as described above is used for making the coincidence of the cascade of beta-and gamma- rays. In this arrangement, the output pulse of each detector, one from plastic scintillator and other from NaI(Tl) detector is fed to a fast coincidence circuit and is also fed to the single channel analyser using cathode followers. The output pulses from two single channel analysers and the output of the fast coincidence unit are fed to slow triple coincidence unit. The output of the slow-fast coincidence set-up is shaped using shorted cable as given in circuit-diagram in fig.3.2. The pulses from the third NaI(Tl) detector (single channel spectrometer) are also shaped similarly using shorted cable and are fed to the coincidence unit (using 6BN6 tube). The output of this coincidence circuit is fed to the scaler and recorder. This output therefore is triple coincidence. The delay line has been inserted as shown in fig.3.5(a) after the output of slow-fast coincidence set-up. This is provided in order to compensate the artificial delay, also for recording chance coincidence by delaying the real coincidences. The figures 3.5(a) and 3.5(b) show the block diagram of the general set-up.

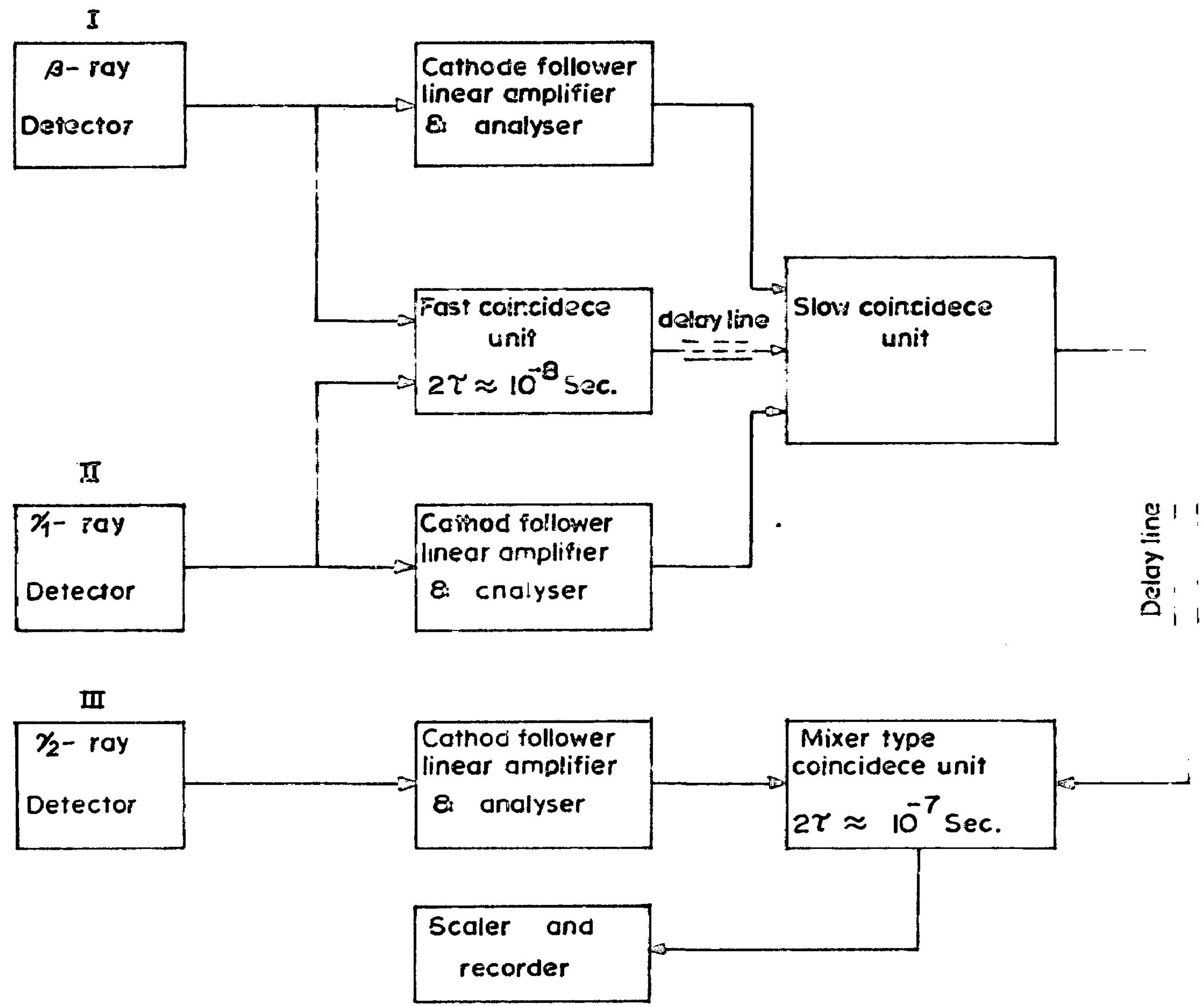


FIG.3.5(a) BLOCK DIAGRAM OF EXPERIMENTAL ARRANGEMENT FOR BETA-GAMMA-GAMMA COINCIDENCE & ANGULAR CORRELATION STUDIES.

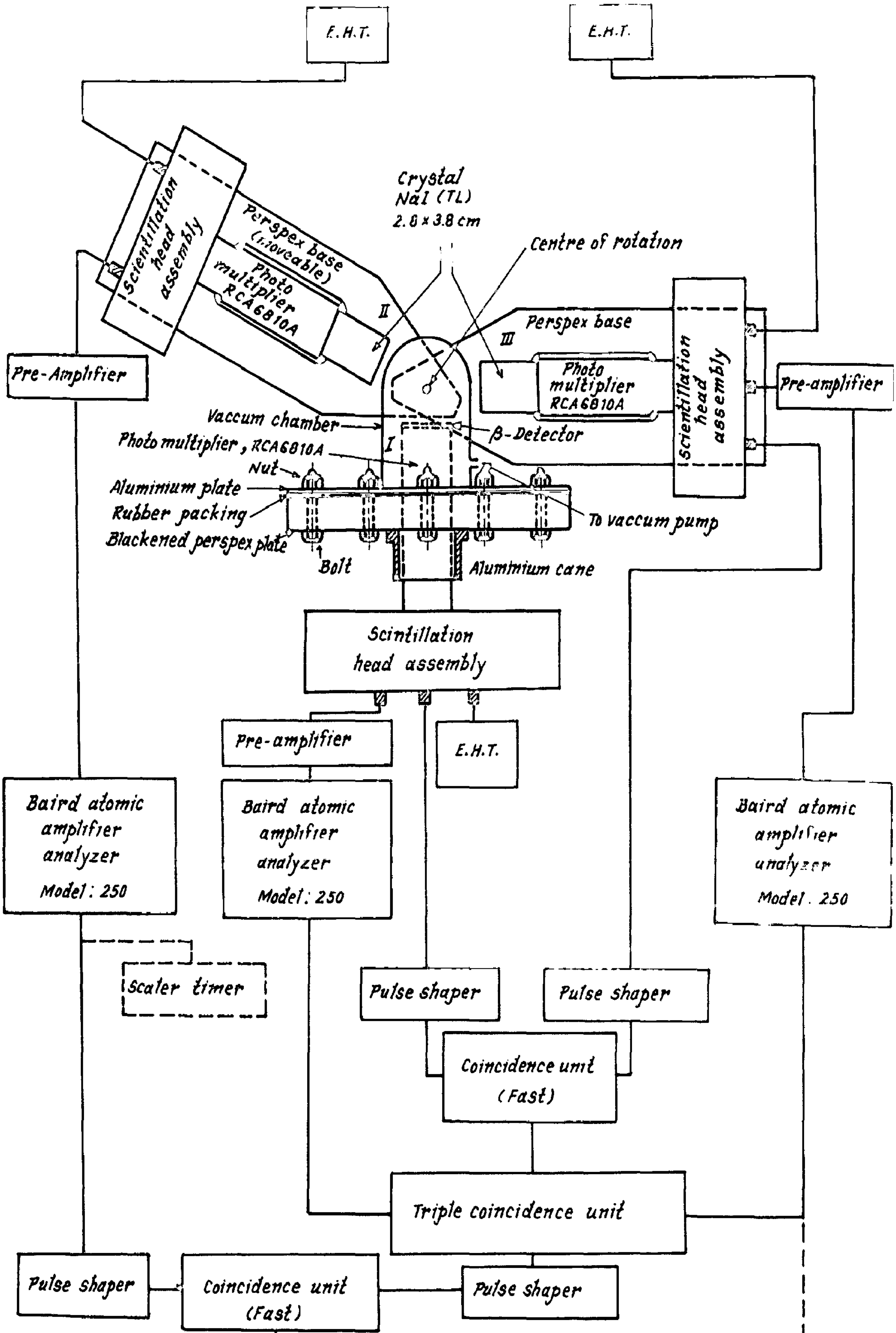


FIG.3.5 (b)_ BLOCK DIAGRAM OF β-γ-γ COINCIDENCE AND ANGULAR CORRELATION SET-UP.

3.4 CHECKING OF THE MOUNTING OF THE DETECTORS FOR ANGULAR CORRELATION STUDIES:

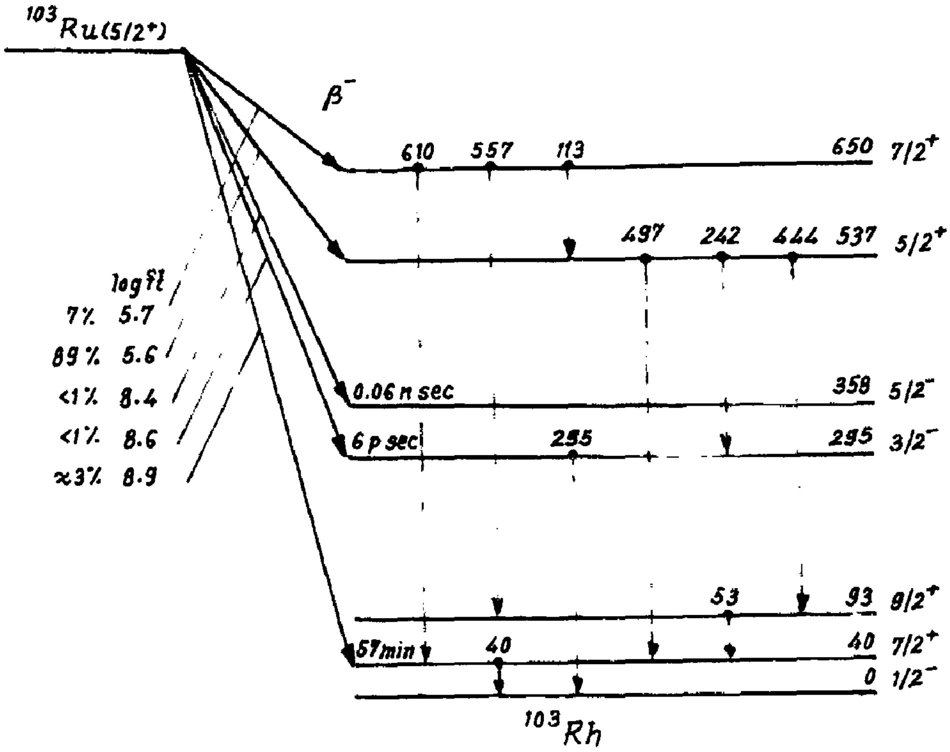
The experimental arrangement described earlier, was calibrated by measuring the angular correlation function for the 1.17- and 1.33 MeV gamma- ray cascade emitted in the decay of Co^{60}. The detector mounted perpendicular to the plane of the table has been checked by the following method. The coincidences were recorded for the 1.17meV→ 1.33meV gamma-ray cascade using the fixed detector perpendicular to the plane of the table and movable detector in the plane of the table. The coincidence counting rate was found to be the same (within the statistical error) when the movable detector was kept at the various angles in the plane of the table. Similarly the coincidence counting rate was found to be the same in the reverse.

CHAPTER - 4

BETA-GAMMA-GAMMA DIRECTIONAL CORRELATION STUDIES OF THE RADIATIONS FROM THE DECAY OF Ru^{103}

4.1 INTRODUCTION:

The excited energy levels in ^{103}Rh from the decay of ^{103}Ru have been well established as given in fig. 4.1 by many previous investigators (18, 19). The spins and parities of excited states in ^{103}Rh are uniquely assigned except for the 93- , 537- , and 650- keV levels. The spins and parities of the excited states in ^{103}Rh have been assigned on the basis of gamma- gamma directional correlation and internal conversion coefficient studies by many workers (20 - 27). Bargholtz etal (27) have studied the properties of the positive parity states in ^{103}Rh on the basis of unified vibrational model and have measured directional correlation involving the transition deexciting the 537- and 650-keV levels. Avignone III and Frey (26) claimed to have conclusively assigned the parities and spins of the 93-, 537-, and 650-keV levels by gamma-gamma angular correlation and internal conversion coefficient studies. One of the serious difficulties pointed out by Avignone III and Frey (26) was due to the interference of the gamma-cascades from the decay of ^{106}Ru resulting from use of fission product sources. ^{106}Ru decays to ^{106}Pd via ^{106}Rh. The coincidence counting rate due to gamma-radiations from ^{106}Rh can not be eliminated if it is present as an impurity in the coincidence



DECAY SCHEME OF ^{103}Rh PROPOSED ON THE GROUNDS OF THIS AND EARLIER WORK. ALL ENERGIES ARE IN keV

FIG. 4.1

of gamma- rays of the interest from the decay of ^{103}Ru. But one can eliminate it (or reduce it to the extent of elimination) by beta-gamma-gamma coincidences, selecting beta-radiations in a fixed energy intervals. Therefore an attempt has been made to reinvestigate it by the method of beta-gamma-gamma angular correlation, so as to either eliminate or reduce the gamma-cascades of ^{106}Rh.

4.2 EXPERIMENTAL:

The radioactive isotope material $RuCl_3$ in dilute HCl solution was obtained from Bhabha Atomic Research Centre, Bombay. The specific activity was 10mc/gm Ru. A few drops of the source were dried on a cellotape. The source was spread in 2mm dia. and mounting was done using perspex stand. The source alongwith the plastic scintillator was placed in a vacuum. The source was 3.5cm from the beta detector and 6cm from each of the gamma detectors.

4.3 (i) COINCIDENCE AND ANGULAR CORRELATION STUDIES FOR THE CASCADE OF β-RAYS OF E_{max} 120 keV → γ -RAYS OF 557 keV → γ -RAYS OF 53 keV:

The beta-ray spectrometer is used as an integral spectrometer for selecting β-rays between 50-keV and 1MeV and gamma-ray spectrometer (fixed one) is used as a differential spectrometer which scans the spectrum in the region of gamma- ray photopeak of 557 keV in one volt channel width (1V = 14.2 keV). These two spectrometers are used for coincidences of beta-gamma rays using slow-fast coincidence set-up. The output of this forms the gate of coincidences of β-rays of E_{max} 120-keV and gamma-rays of 557 keV for one of the inputs of mixer type coincidence unit. The second input is from the third gamma-ray spectrometer which selects the gamma-rays in photopeak of 53-keV in 7V channel width (1Volt = 3.6 keV). Therefore the output is the triple coincidence as shown in fig. 4.2 which establishes a triple cascade of β-group of E_{max} 120 keV → γ-rays of 557 keV → γ -rays of 53 keV. The angular correlation studies are done by selecting 557-keV gamma-ray above 497 keV energy (using the spectrometer in the integral position above 35 volts pulse height as shown in fig.4.2) and 53-keV gamma-ray at the photopeak in 7V channel width as mentioned above.

(ii) COINCIDENCE AND ANGULAR CORRELATION STUDIES FOR THE CASCADE OF β-RAYS OF E_{max} 210 keV → γ -RAYS OF 444-keV → γ -RAYS OF 53-keV:

The coincidence studies for this cascade have been made in the same as done for the previous cascade. The coincidence spectrum alongwith the single spectrum is shown in fig. 4.2. The angular correlation studies are done by selecting the 444-keV gamma-ray at the photopeak(using the spectrometer as differential with the setting at 28Volt pulse height in 7V channel width) and 53-keV at the photopeak. The compton contribution of triple β-557 keV →53 keV cascade at the photopeak of 444 keV has been taken by keeping the position

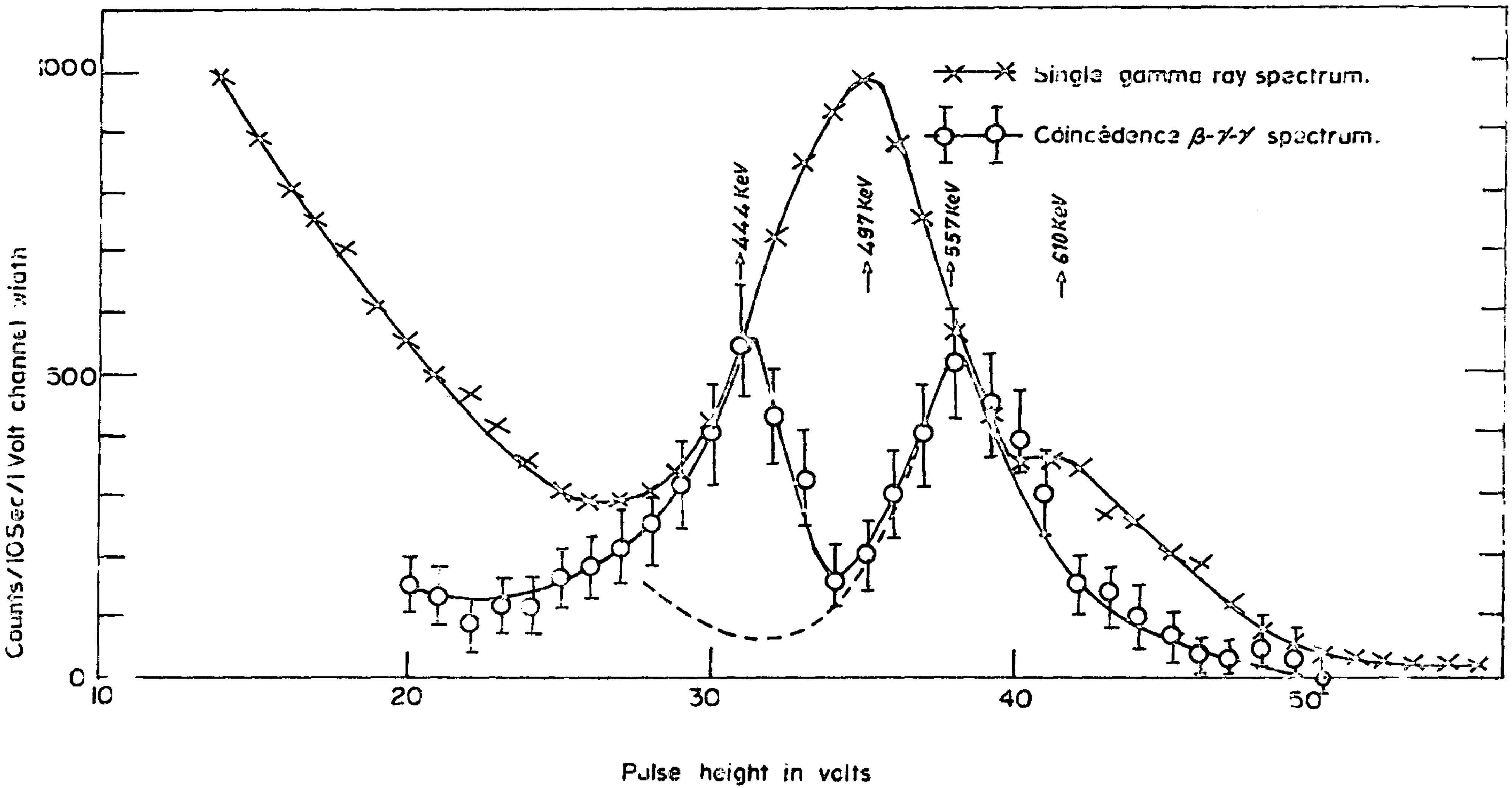


FIG.4.2 BETA-GAMMA-GAMMA COINCIDENCE SPECTRUM ALONGWITH THE SINGLE SPECTRUM.

of movable detector at 112.5^{0} with respect to the first detector. The compton contribution at other angles was calculated using the above experimental correlation function for this cascade. The comption contribution is substracted from the counting rates of second cascade of β-444 keV→53 keV and the angular correlation function was obtained.

4.4 ANGULAR CORRELATION RESULTS:

The angular correlation functions $W(\theta)$ obtained by the method of least square fit (without applying solid angle correction which has been considered in the theoretical calculations) are as follows:

For the cascade, β-rays of E_{max} 120 keV →γ -rays of 557-keV → γ -rays of 53 keV

$$W(\theta) = 1+(-0.153 \pm 0.031)\ P_2(Cos\theta) + (0.004 \pm 0.035) P_4(Cos\theta)$$

and for the cascade, β-rays of E_{max} 210 keV →γ -rays of 444-keV → γ - rays of 53 keV

$$W(\theta) = 1+(0.163 \pm 0.042)\ P_2(Cos\theta) + (-0.035 \pm 0.058) P_4(Cos\theta)$$

4.5 ANALYSIS OF ANGULAR CORRELATION DATA AND DISCUSSION OF THE RESULTS:

The spin and parity of the ground state of ^{103}Ru have been reported by earlier workers [Kuhn and Woodgate (28), Goldhaber and Hill (29) and Mason etal (30)] to be $7/2^{+}$.

53 keV gamma-ray transition from 93 keV level to 40 keV

level has been considered predominantly to be M1 transition with the little mixture of E2 ($\delta \leq 0.02$) by internal conversion coefficient measurements. In the present analysis, 53 keV transition is taken to be a mixture of M1 + E2 with δ=0.02.

Singh (21) has considered 3/2, 5/2 or 7/2 as possible spin values for 537-and 650-keV levels and 5/2,7/2 or 9/2 for 93 keV level. These possibilities of spin assignments for 93-, 537-, and 650-keV levels are taken on the basis of 'log ft values', half lives of the excited states and partly on the basis of internal conversion coefficient measurements of various transitions. George etal (24) have considered 9/2,11/2 or 13/2 as well. These spin assignments to the 537-,and 650-keV levels which are seen to be compatible by gamma-gamma angular correlation results but are ruled out on the basis of observed beta-transition probabilities from the ground state of ^{103}Ru to the top most levels in ^{103}Rh. This is quite plausible reason to rule out these possibilities. The various spin possibilities as mentioned by Singh (21) are to be considered in the present analysis of beta-gamma-gamma angular correlation studies.

Beta-gamma-gamma angular correlation coefficients A_2 and A_4 were calculated for all the possible spin sequences as given in table I, considering the transitions either to be pure dipole or pure quadrupole. The plots of A_2 versus Q($Q = \frac{\delta^2}{1+\delta^2}$) have been done for all the cascades. Three of such plots are given in figures 4.3, 4.4 and 4.5 when the experimental values cut the curve. In other cases, experimental values are large

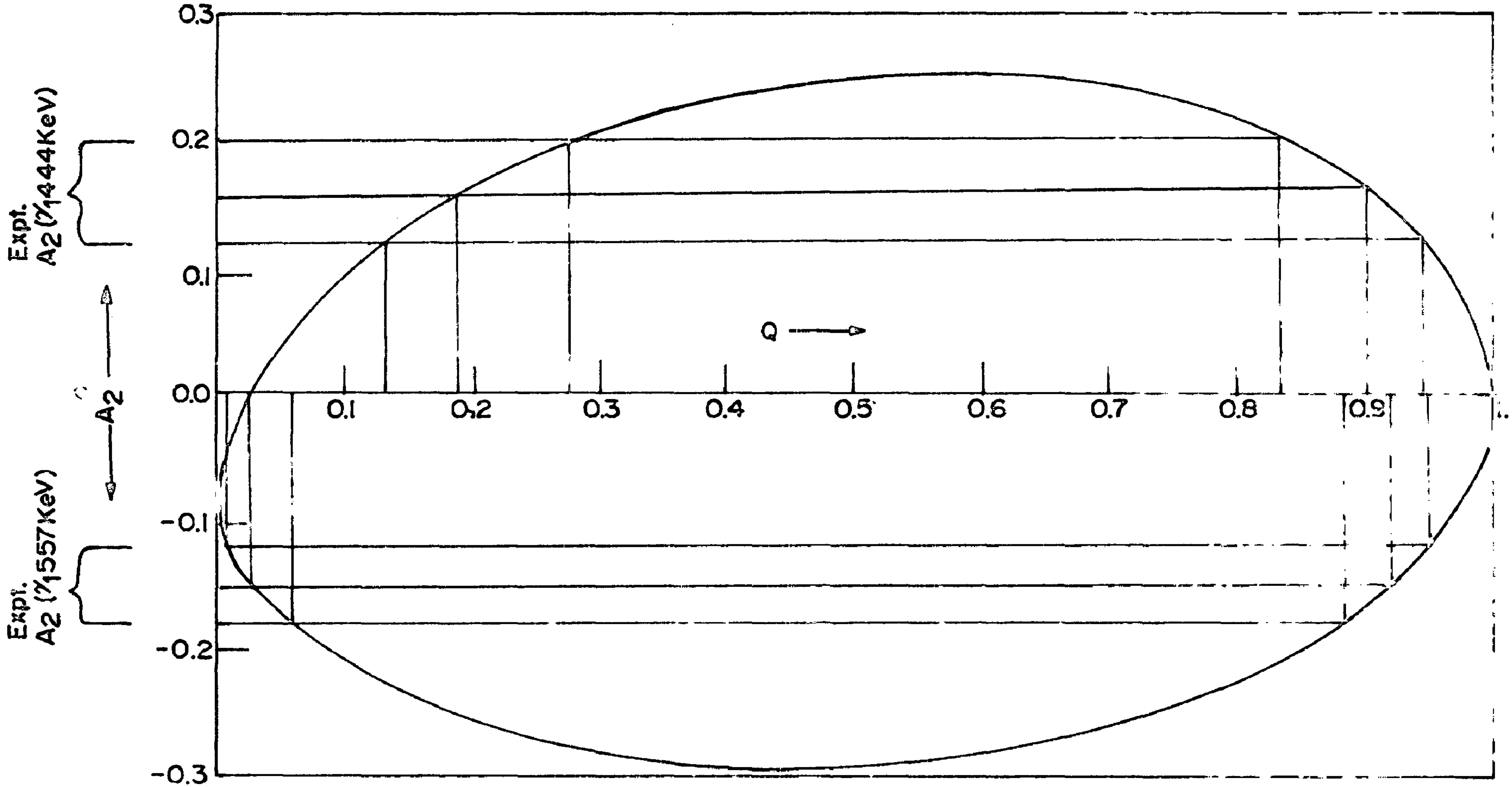


FIG. 4.3 THEORETICAL PLOT OF A_2 Vs Q FOR THE CASCADE

$$5/2 \xrightarrow[\text{allowed}]{\beta} 7/2 \xrightarrow{(557 \text{ or } 444 \text{ KeV})} 9/2 \xrightarrow[\delta = 0.02]{53 \text{ KeV}} 7/2$$

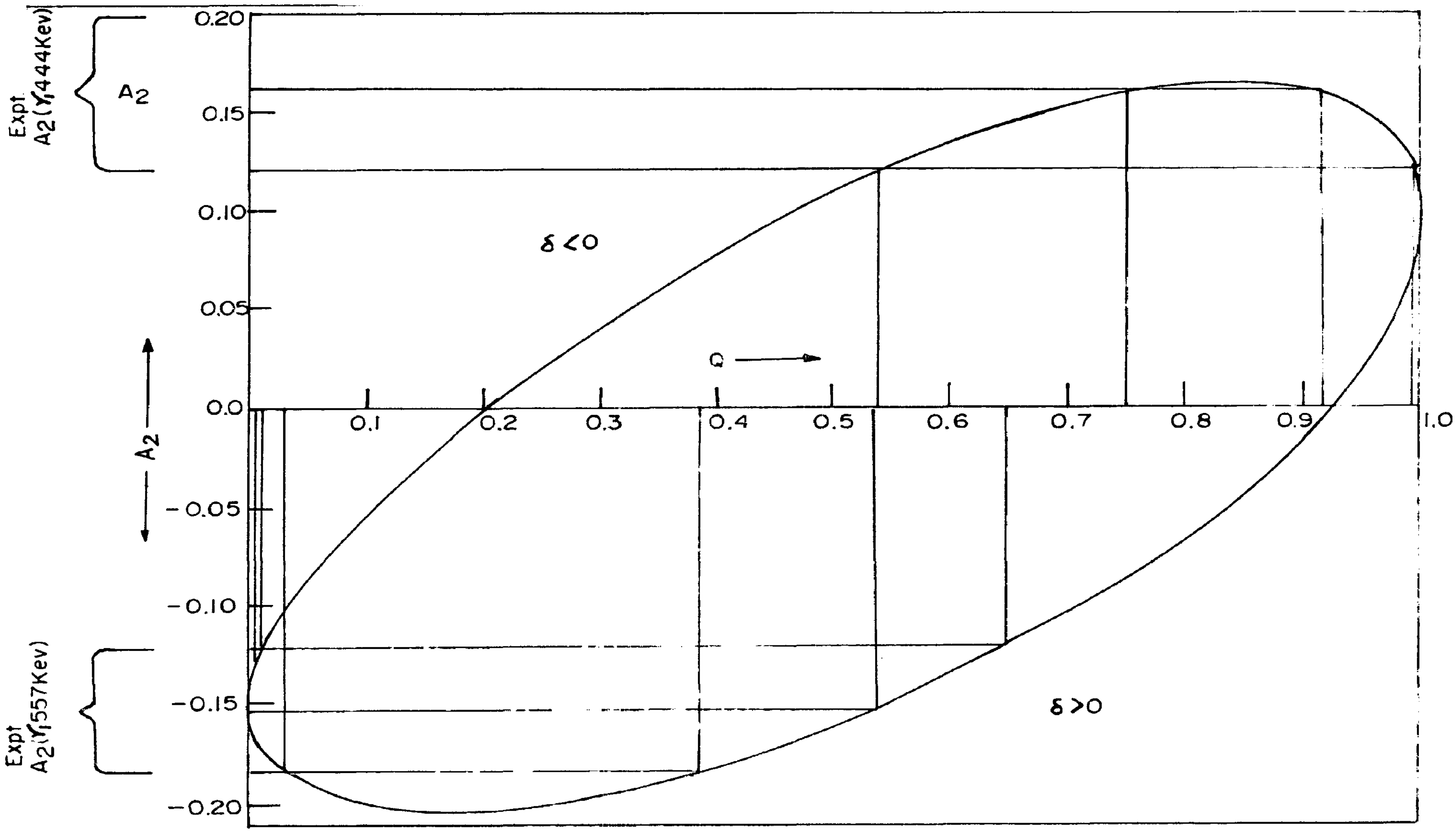


FIG.4.4 THEORETICAL PLOT OF A_2 Vs Q FOR THE CASCADE

$5/2 \xrightarrow[\text{allowed}]{\beta} 7/2 \xrightarrow{\gamma_1 (557 \text{ or } 444 \text{ Kev})} 7/2 \xrightarrow{\gamma_2 (53 \text{ Kev})} 7/2$

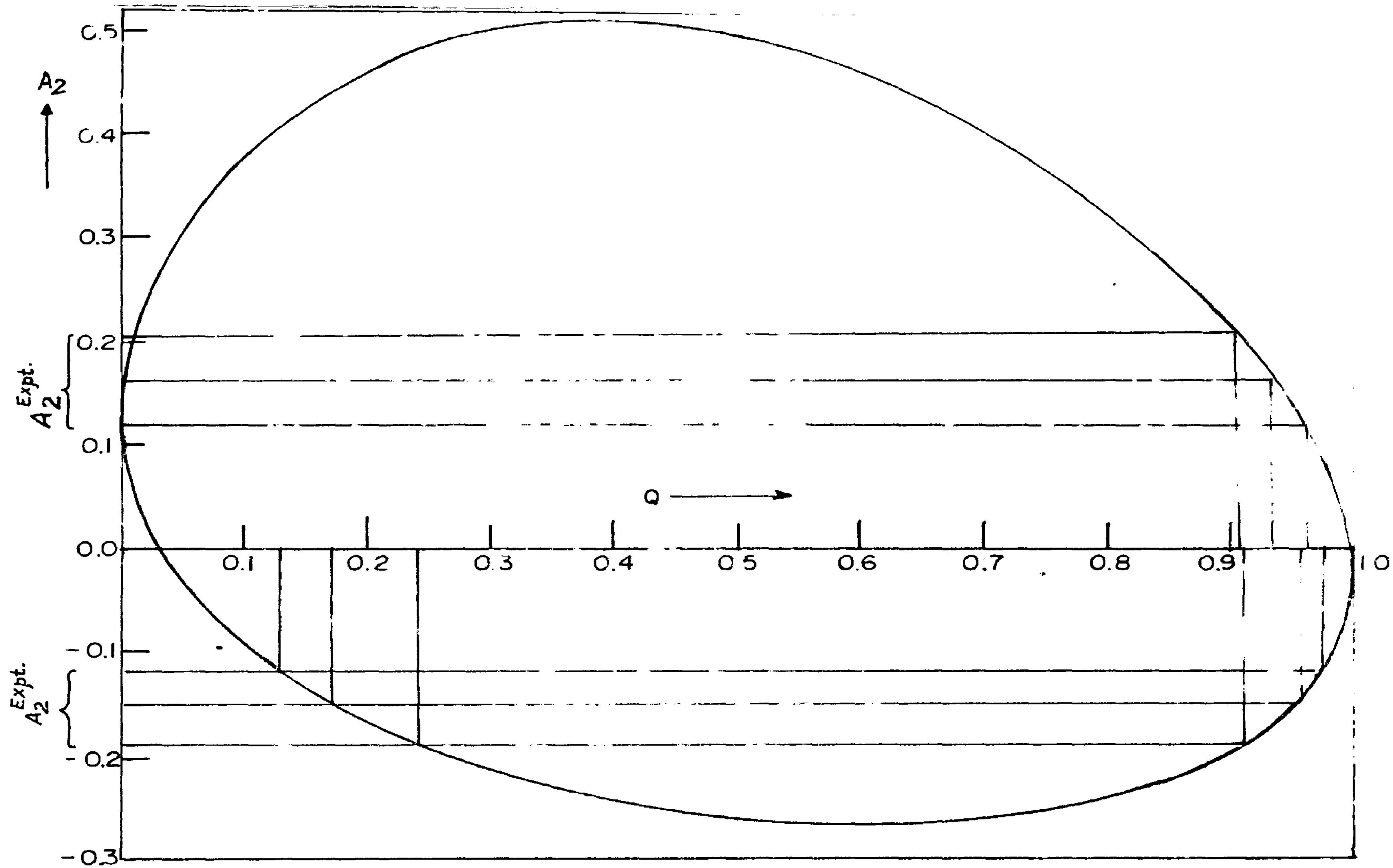


FIG. 4.5 THEORETICAL PLOT OF A_2 Vs Q FOR THE CASCADE

$5/2 \xrightarrow[\text{allowed}]{\beta} 5/2 \xrightarrow{\gamma_1 (557 \text{ or } 444 \text{ Kev})} 7/2 \xrightarrow[\delta = 0.02]{\gamma_2 (53 \text{ Kev})} 7/2$

(within the experimental range) and are not given here. There are two cases when the gamma-ray transition is considered to be pure E2 for 5/2 → 9/2 or 3/2 → 7/2 transitions. These two cases are serial number (1) and (6) in table I.

As summarized in Nuclear Data sheets (19) and based on reference there in, the 537-keV level decays to 295-keV level (J^{π} = 3/2) via 242 keV gamma-ray. Since the 537-keV level must have positive parity from log ft = 5.7 in Ru^{103} decay, the 242-keV gamma ray must be E1 or M2 or E3. The conversion electron studies of Pettersson etal (25) and also Avignone III and Frey (26) show that 242-keV gamma-ray is E1 in character. Furthermore Pettersson etal (25) also show that the 497-keV gamma-ray between the 537- and 40-keV levels is mostly M1. Therefore it is reasonable to conclude that 537-keV levels $J^{\pi} = 5/2^{+}$. This spin value can be assigned by the present beta-gamma-gamma angular correlation studies but in that case 9/2 or 7/2 are to be considered for 93-keV level (as given in table I and II). From table II, it is further noticed that 9/2 is taken for 93-keV level, then 444 keV transition is pure quadrupole and if 7/2 is taken, then 444-keV transition is either almost pure dipole or the mixture of quadrupole and dipole. Avignone III and Frey (26) reported by the internal conversion coefficient studies that 444-keV transition is pure quadrupole. In that case, one can reasonably assign 9/2 for 93-keV level with 5/2 for 537-keV level by the present studies.

As given in table II, 5/2 is not possible for 650keV level taking 9/2 for 93keV level. But 7/2 can be assigned for 650-keV level with the certain mixture of dipole and quadrupole (i.e.δ to be 0.1 to 0.25 or 0.77 to 4.36). If 557-keV transition is taken predominantly dipole in character, the present value of δ (between 0.1 to 0.25) and the value of 'δ' deduced by Avignone III and Frey (26) i.e.-0.32 ± 0.03 are approximately to be of the same order in magnitude. The sign of 'δ' also agrees if we follow the same sign convention [Avignone III and Frey (26) have taken the sign convention of Krane etal (31)] . The complete absence of β-transition to 93-keV level indicates that 9/2 is preferred to 93 keV level. Therefore spin values of 9/2, 5/2 and 7/2 are assigned to the 93-, 537-, and 650-keV levels in ^{103}Rh respectively by the present analysis.

Bargholtz etal (27) had done the calculations to find the properties of excited states in Rh^{103}, discribing it by unified vibrational model. Employing the formalism of Heyde and Brussard (32) and considering the quasiparticle aspects of Castel etal (33), Bargholtz etal (27) had by these calculations, suggested 5/2 for 537-keV and 7/2 for 650-keV level. The present analysis of beta-gamma-gamma angular correlations agrees with their theoretical predictions.

TABLE I

Spin sequence for the β-γ-γ cascade of a $\xrightarrow[\text{allowed}]{\beta\text{-transition}}$ b $\xrightarrow{\gamma_1}$ c $\xrightarrow{\gamma_2}$ d

Sl. No.	Spin value for b	c	Multipolarity of gamma-transition from b to c	Triple angular correlation coefficient	
1	5/2	9/2	quadrupole	$A_2=0.1327$	$A_4=0.001$
2	7/2	9/2	dipole	$A_2=-0.0874$	$A_4=0.000$
	7/2	9/2	quadrupole	$A_2=0.059$	$A_4=0.0001$
3	3/2	5/2	dipole	$A_2=-0.0346$	$A_4=0.000$
	3/2	5/2	quadrupole	$A_2=0.0181$	$A_4=0.000$
4	5/2	5/2	dipole	$A_2=0.0410$	$A_4=0.000$
	5/2	5/2	quadrupole	$A_2=-0.0177$	$A_4=0.000$
5	7/2	5/2	dipole	$A_2=-0.0302$	$A_4=0.000$
	7/2	5/2	quadrupole	$A_2=0.0074$	$A_4=0.000$
6	3/2	7/2	quadrupole	$A_2=-0.1606$	$A_4=0.0001$
7	5/2	7/2	dipole	$A_2=0.1303$	$A_4=0.000$
	5/2	7/2	quadrupole	$A_2=-0.0287$	$A_4=0.000$
8	7/2	7/2	dipole	$A_2=-0.1508$	$A_4=0.000$
	7/2	7/2	quadrupole	$A_2=0.0927$	$A_4=-0.001$
(i)	β-rays of E_{max} 120keV-γ-rays of 557-keV-γ-rays of 53-keV			$A_2^{Expt.}=-0.153\pm0.031$ $A_4^{Expt.}=0.004\pm0.035$	
(ii)	β-rays of E_{max} 210keV-γ rays of 444-keV-γ-rays of 53keV			$A_2^{Expt.}=0.163\pm0.042$ $A_4^{Expt.}=-0.035\pm0.038$	

γ_1 is 557 or 444-keV gamma-transition and γ_2 is 53keV gamma-transition which is taken to be a mixture of M1+E2 with mixing ratio $\delta=0.02$. Spin values a and d are fixed and are taken to be 5/2 and 7/2 respectively. The different spin assignments to b and c are possible for the two levels i.e.(i) 650keV and (ii)537-keV. The values of angular correlation coefficients A_2 and A_4 are calculated by taking gamma-ray transition to be pure dipole or quadrupole.

TABLE-2

Multipole mixing ratio in 444 and 557-keV transitions

Spin sequence $b \xrightarrow{\gamma_1} c \xrightarrow{\gamma_2} d$	δ^2 in 444 keV transition $\beta \rightarrow 444keV \rightarrow 53keV$ angular correlation results
5/2→9/2→7/2	Possible (table I)
7/2→9/2→7/2	$0.149 \leq \delta_1^2 \leq 0.379$ $5.060 \leq \delta_2^2 \leq 10.0$
3/2→7/2→7/2	Not possible
5/2→7/2→7/2	$0 \leq \delta_1^2 \leq 0.010$ $9.52 \leq \delta_2^2 \leq 24.0$
7/2→7/2→7/2	$1.150 \leq \delta_1^2$ $\delta_2^2 \leq \infty$
3/2 5/2→5/2→7/2 7/2	Not possible

Spin sequence $b \xrightarrow{\gamma_1} c \xrightarrow{\gamma_2} d$	δ^2 in 557-keV transition β-557-keV→53-keV angular correlation results
5/2→9/2→7/2	Not possible
7/2→9/2→7/2	$0.010 \leq \delta_1^2 \leq 0.063$ $7.695 \leq \delta_2^2 \leq 19.0$
3/2→7/2→7/2	Possible (Table I)
5/2→7/2→7/2	$0.15 \leq \delta_1^2 \leq 0.25$ $10.11 \leq \delta_2^2 \leq 39.0$
7/2→7/2→7/2	$0.0049 \leq \delta_1^2 \leq 0.031$ $0.626 \leq \delta_2^2 \leq 1.857$
3/2 5/2→5/2→7/2 7/2	Not possible

CHAPTER-5

BETA-GAMMA-GAMMA ANGULAR CORRELATION STUDIES OF THE RADIATIONS FROM THE DECAY OF Eu^{152} AND Eu^{154} AND STUDIES OF NEGATIVE PARITY STATES IN Gd^{152}

5.1 INTRODUCTION:

Eu^{152} decays to Gd^{152} by β^- emission and to Sm^{152} by EC and β^+ emission. Also this isotope can not be easily obtained free from Eu^{154} which decays to Gd^{154} by β^- emission and to Sm^{154} by EC and β^+ emission. Therefore gamma-ray spectra from the decay of Eu^{152} and Eu^{154} are quite complicated and large correction factors are needed for the study of gamma-gamma angular correlation work as done earlier [Helppi etal (34), Barrette etal (35), Kalfas etal (36), Sen Gupta and Mukherji (37), Whitlock etal (38), Rud and Neilsen (39), Manning etal (40), Debrunner etal (41), Stiening etal (42)]. These correction factors can be eliminated or reduced to the minimum by the method of beta-gamma-gamma angular correlation.

There are two main beta-gamma-gamma cascades in the decays of Eu^{154} and Eu^{152}.

(i) β - rays of E_{max} 590 keV $\rightarrow$ γ-rays of 1278 keV $\rightarrow$ γ-rays 123 keV.

(ii) β - rays of E_{max} 360 keV $\rightarrow$ γ rays of 779 keV $\rightarrow$ γ-rays of 344 keV.

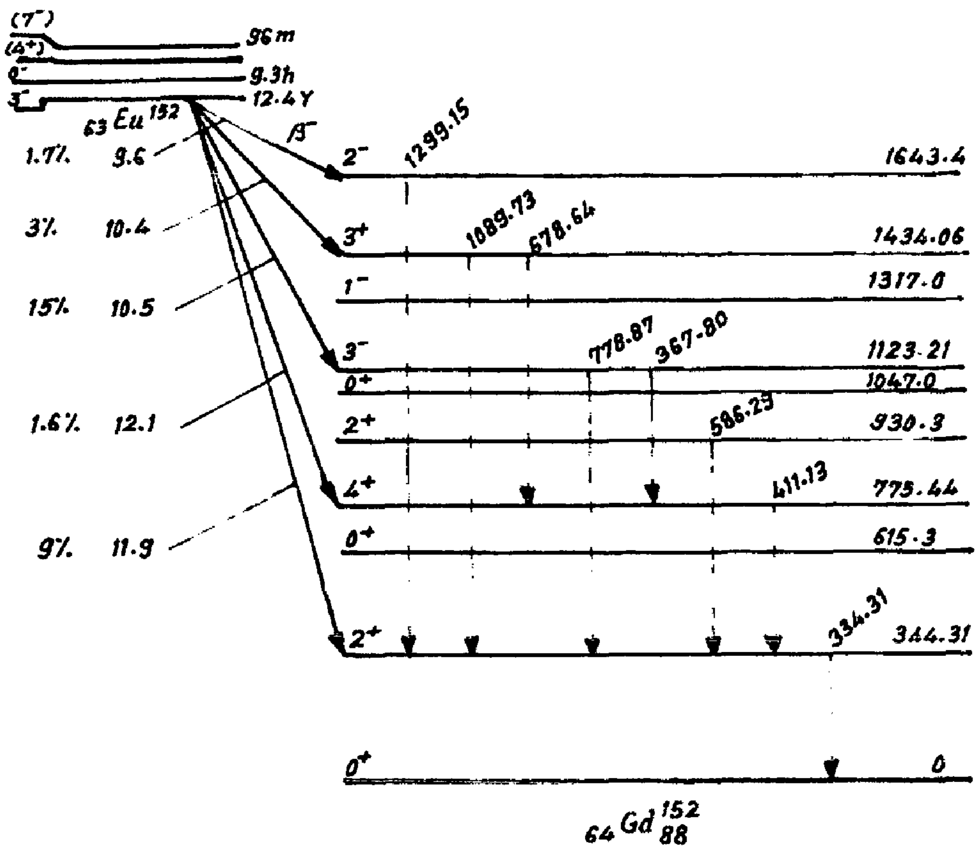
The beta-transitions in both the cascades are allowed having ΔI = 0, logft = 10.5 in Gd^{152} and ΔI = 1, logft = 10.0 in Gd^{154}.

Gamma-ray transitions in Gd^{154} are quite well established (18, 43). Therefore this case can be taken for the checking of the experimental set-up and also for the confirmation of the spins of the levels and multipolarities of gamma-transitions. The gamma-ray transition involving 779 keV gamma-ray is of importance, firstly since the multipolarity of 779 keV gamma-transition is not well established and secondly due to the theoretical importance of the transition from octupole quasi vibrational band to quasi rotational band. Helppi etal (34) and Barrette etal (35) have measured gamma-gamma directional correlations to determine the multipole mixing ratios of gamma-transitions and spins of the excited states in Gd^{152} from the decay of Eu^{154}. Kalfas etal (36) have attempted to interpret the level structure of Gd^{152} from the decay of Tb^{152} by determining the multipole character of some gamma-transitions. A search is needed to establish the possible M2 and E3 admixtures in predominantly E1 transition from 3^- to 2^+ levels in Gd^{152} from the decay of Eu^{152}.

The level schemes of Gd^{152} from the decay of Eu^{152} and Gd^{154} from the decay of Eu^{154} are shown in figures 5.1 and 5.2.

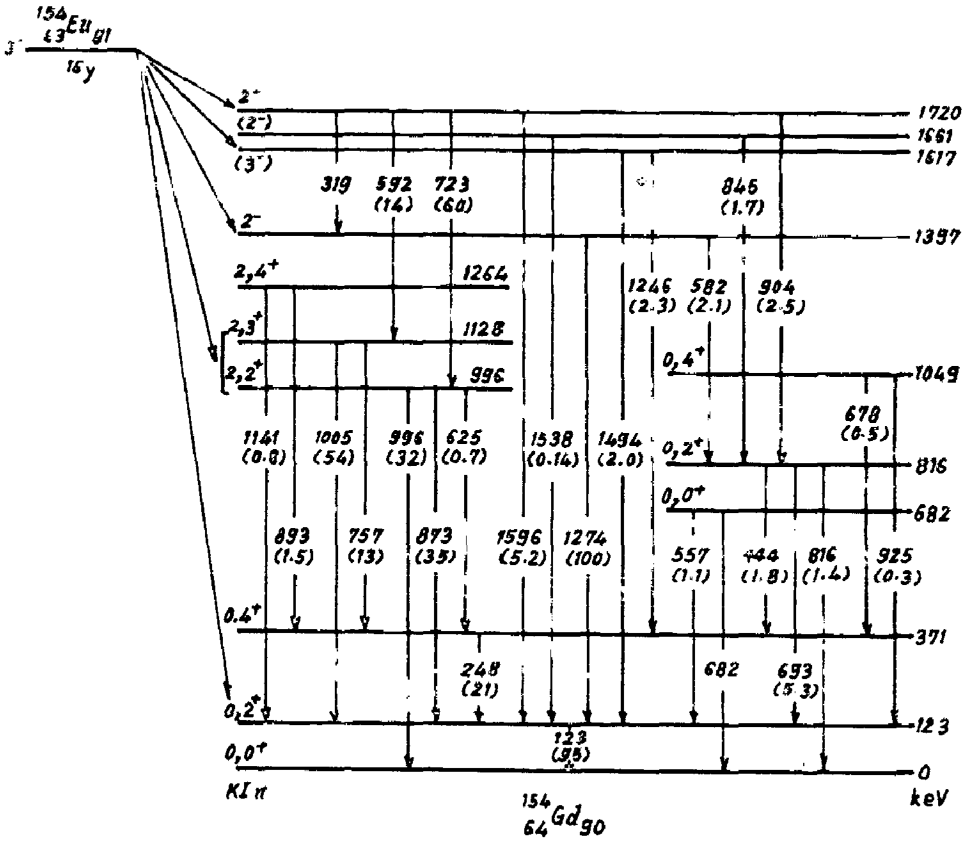
5.2 EXPERIMENTAL:

The source in the form of $EuCl_3$ in dilute HCl solution was obtained from Bhabha Atomic Research Centre, Bombay and its specific activity was 1000mc/gm Eu. A few drops of the source were dried on a cellotape. The source was spread in 2mm diameter and mounting was done using perspex stand. The source alongwith



LEVEL SCHEME OF $^{152}_{64}Gd$ FROM THE DECAY OF $_{63}Eu^{152}$. LEVEL TRANSITION ENERGIES ARE IN keV.

FIG. 5.1



A PARTIAL DECAY SCHEME OF ^{154}Eu. ALL ENERGIES ARE IN KeV.

FIG. 5.2

the plastic scintillator was placed in vacuum. The source was 3.5cm from the beta-detector and 6cm from each of the gamma detectors.

5.3 (i) COINCIDENCE AND ANGULAR CORRELATION STUDIES FOR THE CASCADE OF β-RAYS OF E_{max} 590 keV →γ-RAYS OF 1278 keV→ γ-RAYS OF 123 keV FROM THE DECAY OF Eu^{154}:

The beta-rays are selected between 380- and 608-keV energies using differential spectrometer and gamma-rays above 1014 keV (using integral spectrometer) for beta-gamma coincidences using slow-fast coincidence set-up and output of this forms one of the inputs of mixer type coincidence unit. The second input of the coincidence circuit is from the second gamma-ray spectrometer which scanes the gamma-ray spectrum and a clear photopeak at 123 keV is obtained as shown in Fig.5.3. Since there is no other peak in the coincidence spectrum, therefore it clearly confirms the coincidences due to the cascade of β-rays of E_{max} 590-keV →γ-rays of 1278 keV→γ-rays of 123 keV.

The angular correlation study is done by selecting 123 keV gamma-ray at the photopeak in 5V channel width (1V = 7.7 keV) and other beta rays of E_{max} 590 keV and gamma-ray of 1278 keV of the cascade as mentioned above.

(ii) COINCIDENCE AND ANGULAR CORRELATION STUDIES FOR THE CASCADE OF β-RAYS OF E_{max} 360 keV→γ-RAYS OF 779 keV→ γ-RAYS OF 344 keV FROM THE DECAY OF Eu^{152}:

Beta-gamma coincidence study is done selecting beta-rays between 237- and 457-keV energies (using differential

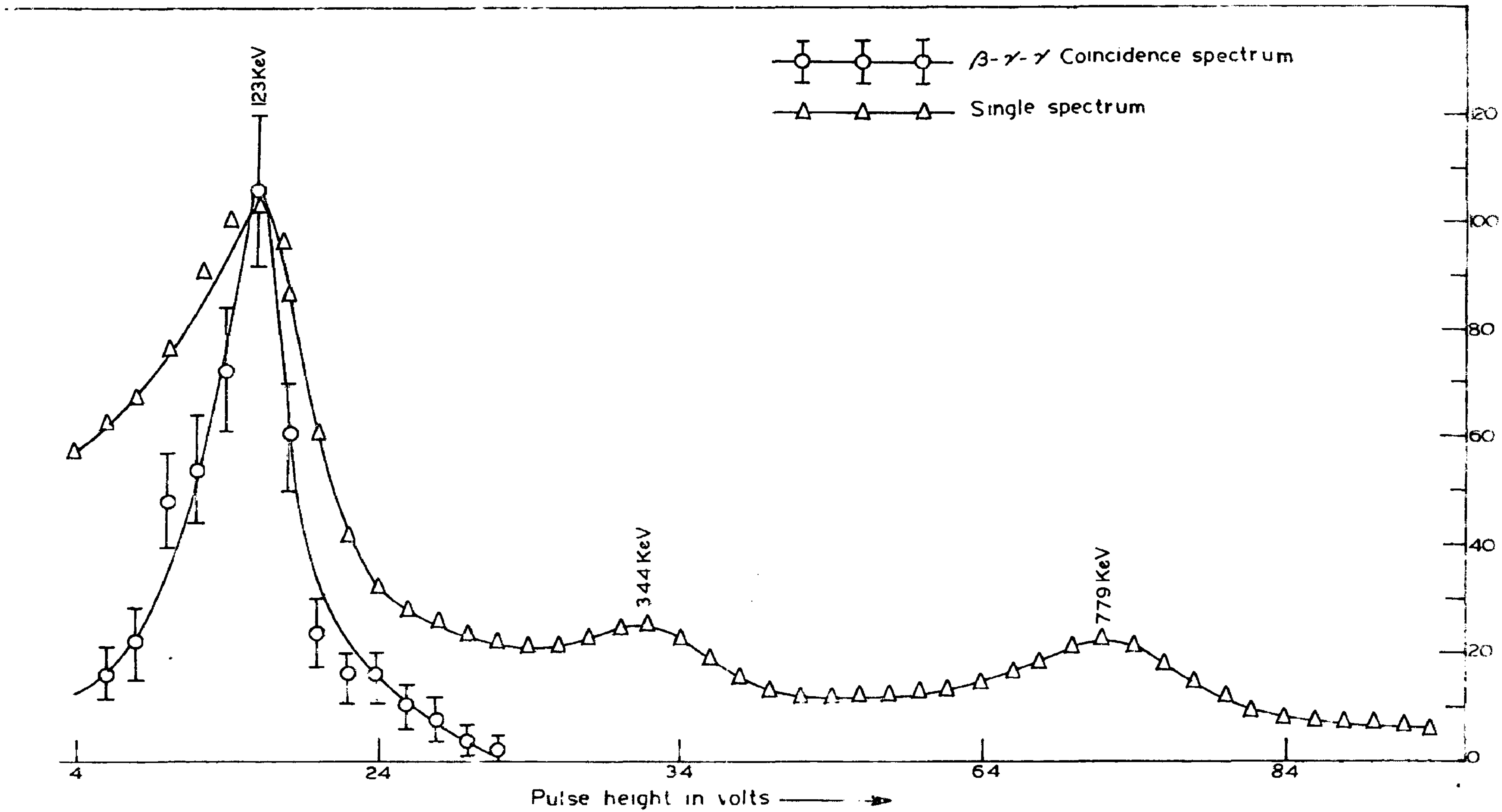


FIG.5.3 _ BETA-GAMMA-GAMMA COINCIDENCE SPECTRUM ALONG WITH THE SINGLE SPECTRUM.

spectrometer) and gamma-rays at 779 keV photopeak in 6V channel width (1V = 22.9keV) using slow-fast coincidence circuit. The output of this forms one of the inputs of the mixer type coincidence unit and the second input of the coincidence circuit is from the movable gamma-ray spectrometer which scans the spectrum in one volt channel width in the region of 344 keV gamma-rays. The triple coincidence spectrum clearly confirms the peak at 344 keV as shown in figure 5.4.

The angular correlation study is done selecting 344 keV gamma-ray at the photopeak in 7V channel width (1V = 22.9 keV) and the beta and gamma-rays of the cascade mentioned above.

5.4 ANGULAR CORRELATION RESULTS:

The angular correlation functions $W(\theta)$ obtained by the method of least square fit (without applying solid angle correction which has been included in the theoretical calculations) are as follows:

For the cascade of β - rays of E_{max} 590 keV $\rightarrow\gamma$-rays of 1278 keV $\rightarrow\gamma$- rays of 123 keV from the decay of Eu^{154}

$$W(\theta)=1-(0.193\pm0.040)P_2(\mathrm{Cos}\theta)+(0.043\pm0.046)P_4(\mathrm{Cos}\theta)$$

and for the cascade of β-rays of E_{max} 360 keV $\rightarrow\gamma$-rays of 779keV$\rightarrow$ γ-rays of 344 keV from the decay of Eu^{152}.

$$W(\theta)=1+(0.145\pm0.019)P_2(\mathrm{Cos}\theta)+(0.024\pm0.023)P_4(\mathrm{Cos}\theta)$$

5.5 DISCUSSION:

The theoretical values of A_2 and A_4 for the cascade

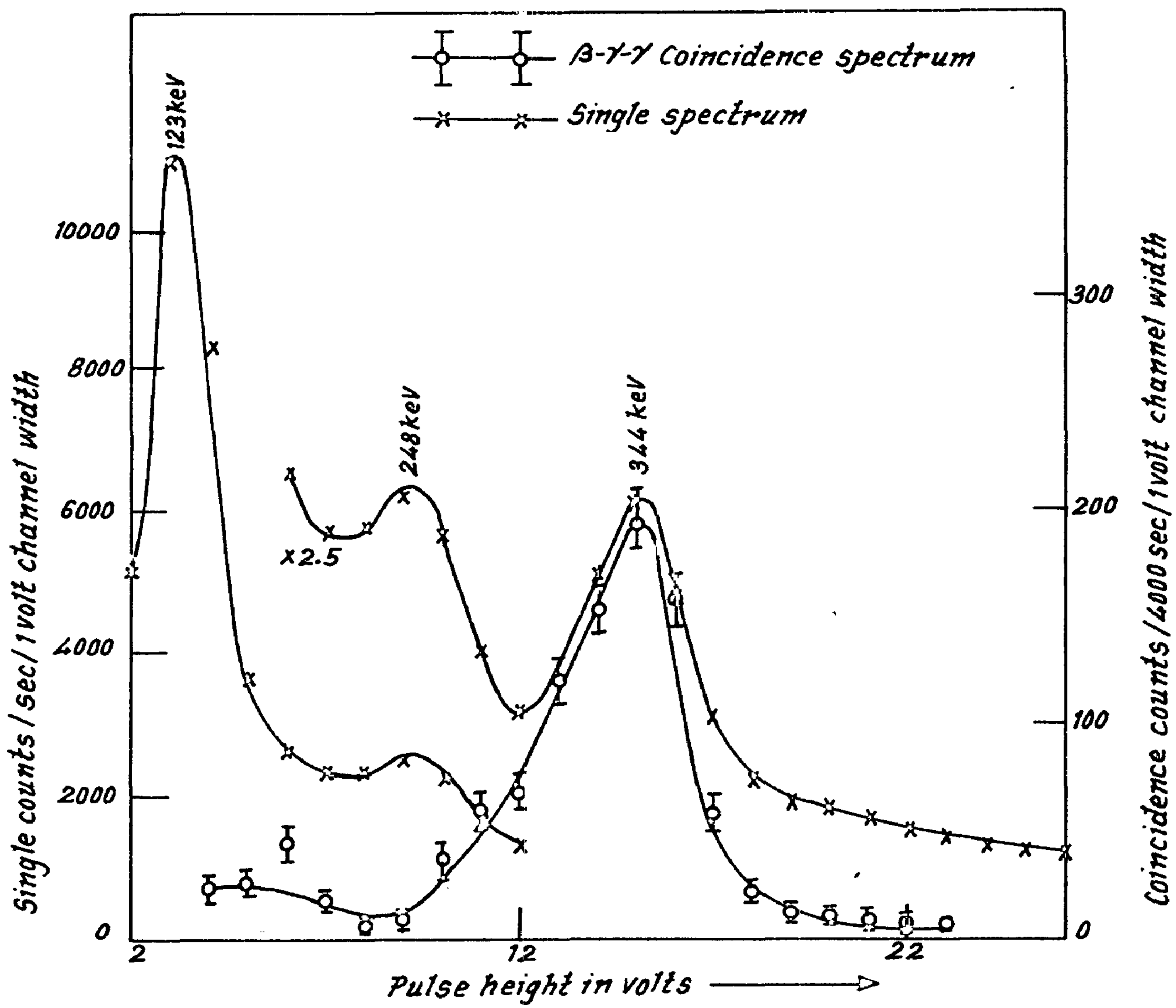


FIG.5.4 - β-γ-γ COINCIDENCE SPECTRUM ALONG WITH THE SINGLE SPECTRUM.

including solid angle correction (from the decay of Eu^{154})

$$3^- \xrightarrow[E_{max}\ 590\ keV]{\beta - transition} 2^- \xrightarrow[1278\ keV]{\gamma_1} 2^+ \xrightarrow[123\ keV]{\gamma_2} 0^+$$ are as follows if we take β-transition to be allowed and γ_2(123keV) radiation to be pure quadrupole and 1278 keV gamma transition to be :

(i) pure dipole

$$A_2 = -0.207, \quad A_4 = 0.0$$

(ii) pure quadrupole

$$A_2 = -0.137, \quad A_4 = 0.248$$

Comparing these theoretical values with the experimental values, we find that 1278 keV gamma-transition is almost pure dipole. But in order to determine the mixture of dipole and quadrupole, it has been done by the plot of A_2 versus $Q\left[\left(=\frac{\delta^2}{1+\delta^2}\right) \text{ where } \delta = \frac{\langle f||L+1||i\rangle}{\langle f||L||\ i\ \rangle}\right]$ which gives the values:

$Q_1 \leq 0.009$, $\qquad |\delta_1| \leq 0.09$

or

$Q_2 \leq 0.999$, $\qquad |\delta_2| \leq 38.198$

The values of 'δ' obtained by earlier workers for 1278keV gamma-transition are as follows:

Author	Multipole mixing ratio 'δ'	Multipolarity
Whitlock etal (38)	+ 0.024	E1+(0.06%) M2
Rud and Nielsen (39)	-0.013 ± 0.014	E1($\leq$0.08%M2)
Manning etal (40)	$\lvert\delta\rvert\leq 0.1$	E1
Debrunner etal (41)	$\lvert\delta\rvert\leq 0.1$	E1
Stiening etal (42)	-0.031	E1+(0.1%)M2
Present work	-0.026(Mean value) or $\lvert\delta\rvert\leq 0.09$	E1+(~0.07%)M2

This clearly indicates the consistancy of the present results by the method of beta-gamma-gamma angular correlation.

The theoretical values of A_2 and A_4 for the cascade including solid angle correction (from the decay of Eu^{152}) $3^- \xrightarrow[E_{max}\ 360\ keV]{\beta\text{-transition}} 3^- \xrightarrow[779\ keV]{\gamma_1} 2^+ \xrightarrow[344\ keV]{\gamma_2} 0^+$ are as follows if we take β- transition to be allowed and the γ_2(344-keV) transition to be pure quadrupole and γ_1(779 keV) transition to be

(i) pure dipole

$$A_2 = 0.072, \quad A_4 = 0.0$$

(ii) pure quadrupole

$$A_2 = 0.285, \quad A_4 = -0.071$$

Comparing these theoretical values with the experimental values, we find that one can not take 779 keV gamma-transition to be pure dipole or pure quadrupole but can be taken to be a mixture of dipole and quadrupole. This can be done by the plot of A_2

versus Q $(=\frac{\delta^2}{1+\delta^2})$, where δ is the multipole mixing ratio) as shown in fig.5.5. The values obtained are as follows:

$$Q_1 = 0.007^{+0.004}_{-0.003}, \quad \delta_1 = +(0.084^{+0.021}_{-0.021})$$

or

$$Q_2 = 0.978^{+0.006}_{-0.007}, \quad \delta_2 = -(6.700^{+1.128}_{-0.864})$$

One can also consider 779 keV gamma-transition to be mixture of dipole + quadrupole + octupole. The present triple angular correlation data can be analysed in the following manner:

Let us consider $\delta_{21}(=\frac{\langle f||M2||i\rangle}{\langle f||E1||i\rangle})$ and $\delta_{31}\ (=\frac{\langle f||E3||i\rangle}{\langle f||E1||i\rangle})$.

and

$$A_2^{Expt} = A_2' + A_2'' \qquad (5.1)$$

where

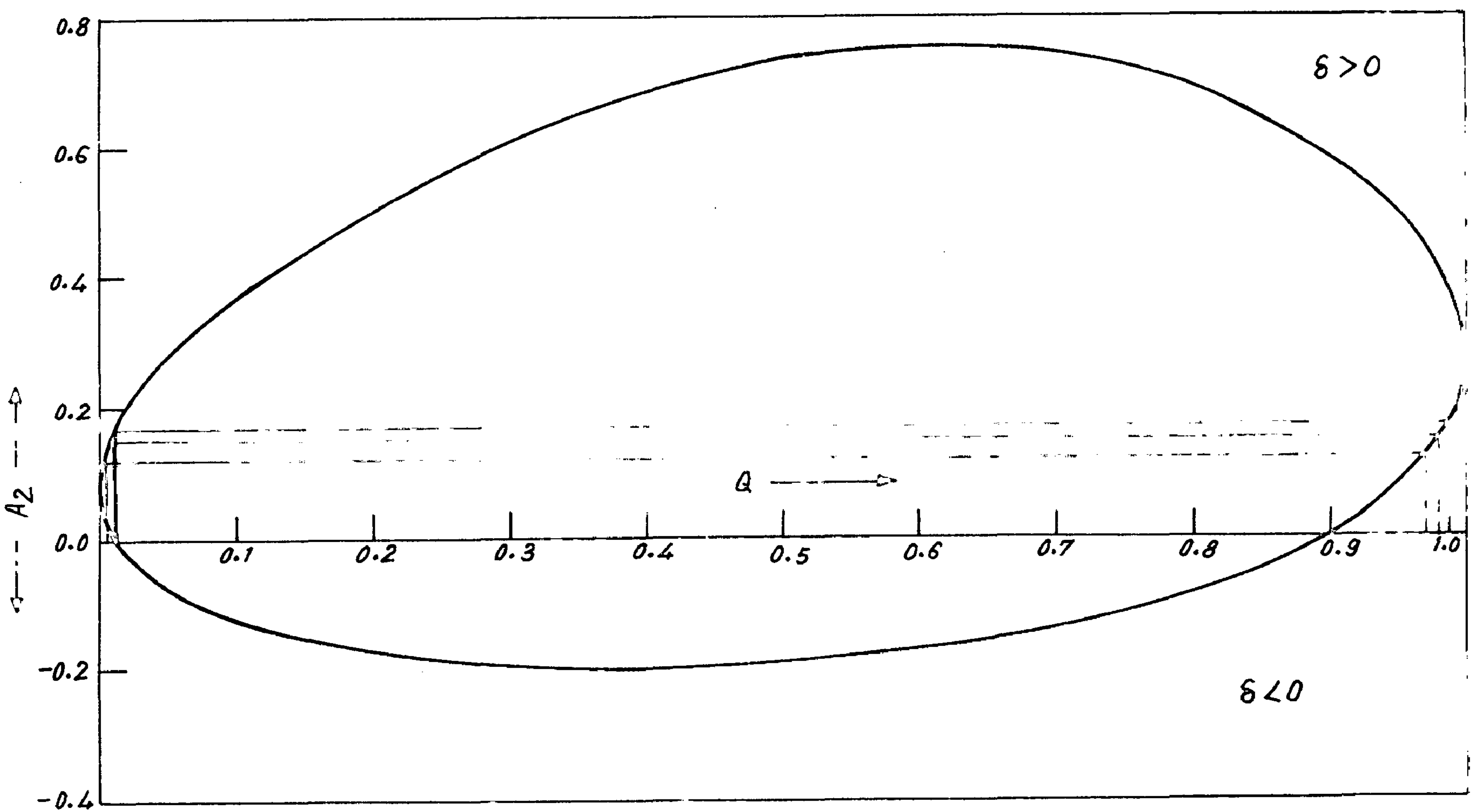
$$A_2' = \frac{a_2 + 2\delta_{21}a_2' + \delta_{21}^2 a_2''}{a_0 + 2\delta_{21}\ a_0' + \delta_{21}^2\ a_0''}, (M2 : E1) \qquad (5.2)$$

$$\left[a_2 = -0.0944, a_2' = -0.5169,\ a_2'' = -0.3412 \right.$$
$$\left. a_0 = -1.3028,\ a_0' = 0.2585,\ a_0'' = -1.1973 \right]$$

and

$$A_2'' = \frac{a_2 + 2\delta_{31}\ a_2' + \delta_{31}^2\ a_2''}{a_0 + 2\delta_{31}\ a_0' + \delta_{31}^2\ a_0''}, (E3 : E1) \qquad (5.3)$$

$$\left[a_2 = -0.0944,\ a_2' = -0.6721,\ a_2'' = -0.3427 \right.$$
$$\left. a_0 = -1.3027,\ a_0' = 0.3193,\ a_0'' = -\ 1994 \right]$$



G.5.5– THEORETICAL PLOT OF A_2 VERSUS Q, THE QUADRUPOLE CONTENT IN 779 keV γ-TRANSITION FOR THE CASCADE OF

$$3^- \xrightarrow[\text{allowed}]{\beta\text{-rays of } E_{max.}\, 360\, keV} 3^- \xrightarrow[M2+E1]{\gamma\text{-rays of } 779\, keV} 2^+ \xrightarrow[E2]{\gamma\text{-rays of } 344\, keV} 0^+$$

Taking certain value of δ_{21}, A_2' is obtained from equation (5.2) and now taking this value of A_2' (and since A_2^{Exp} is determined), A_2'' is obtained from equation (5.1). Substituting this value of A_2'' in equation (5.3), δ_{31} is obtained. The plot of δ_{21} versus δ_{31} is given in figure 5.6. In the similar manner using the K shell internal conversion coefficient given by Gromov etal (44), a plot of δ_{21} versus δ_{31} for the value of $\alpha_K\left[=0.00204(36)\right]$ is drawn. The interception of these two plots (figure 5.6) gives the values of δ_{21} and δ_{31} i.e.

$$\delta_{21}(M2 : E1) = +(0.135^{+0.045}_{-0.070}),\quad \delta_{31}(E3 : E1) = -(0.125^{+0.050}_{-0.070})$$

or

$$\delta_{21}(M2 : E1) = -(0.145^{+0.055}_{-0.075}),\quad \delta_{31}(E3 : E1) = +(0.090^{+0.030}_{-0.045})$$

Odd-spin negative parity states to spin 17 have been identified in Gd^{152} from in-beam spectra of $(\alpha,4n)$ reaction by Zolnowski etal (45) and these states are compared, considering quadrupole-octupole coupling model. The lowest 3^- state in Gd^{152} is considered to have strongly collective character. Therefore we expect the dominant multipolarity in such a transition is expected to be E1. The angular correlation coefficient for the triple cascade, as in present studies having pure E1 for the transition from $3^- \rightarrow 2^+$, as calculated is $A_2 = 0.072$ but experimental value of A_2 is 0.145 ± 0.019. This clearly indicates that 779 keV gamma transition can not be

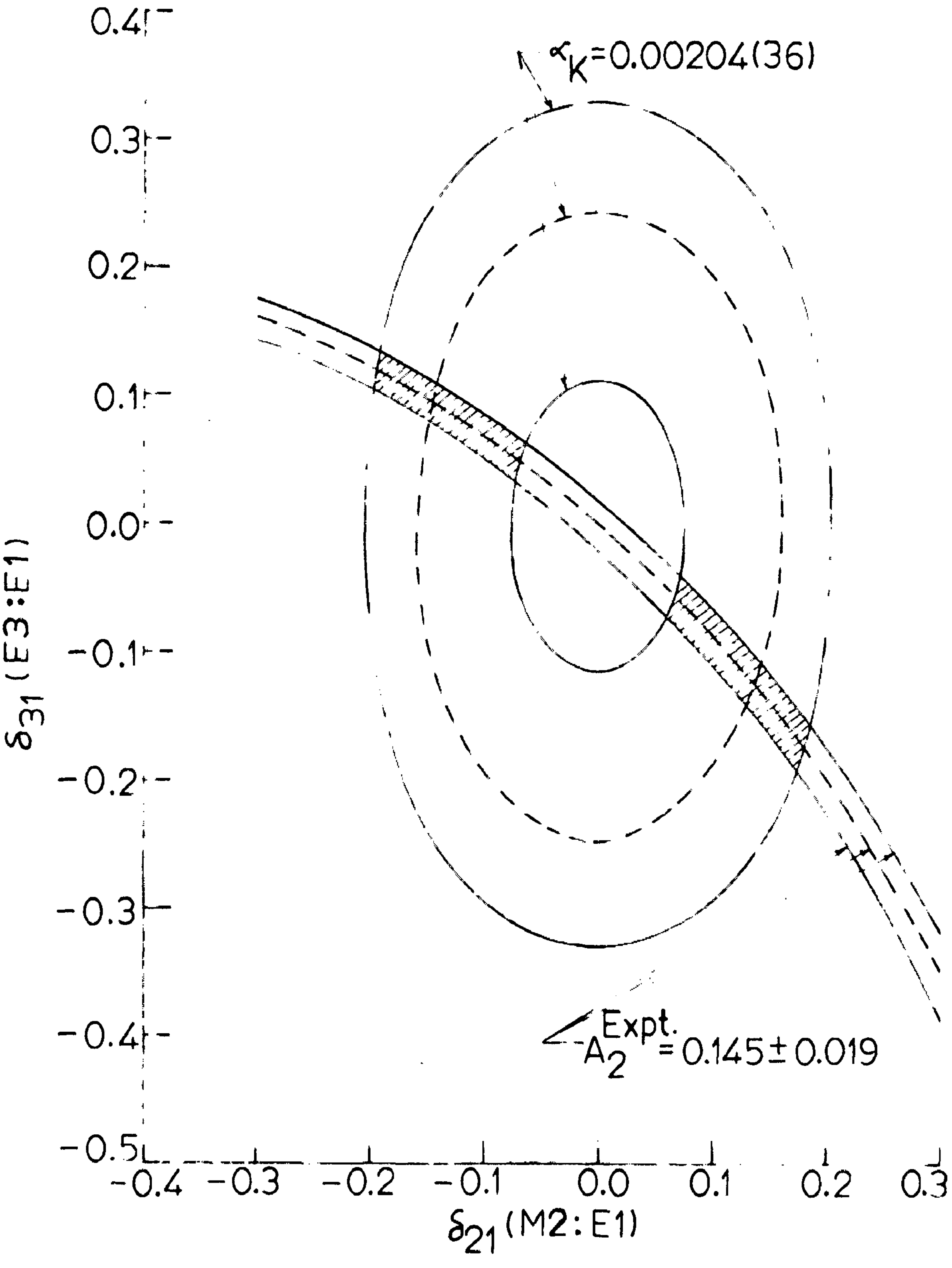


FIG. 5.6 – QUADRATIC PLOT OF δ_{21} (M2:E1) VERSUS δ_{31} (E3:E1).

pure E1 but mixture of M2 or M2 + E3 (the analysis is presented above).

The calculations for the 'δ' multipole mixing ratio for the transition between $3^- \rightarrow 2^+$ states using particular model are not available to compare the above results but these results suggest that the multipole mixing ratios for $5^- \rightarrow 4^+$, $7^- \rightarrow 6^+$, $9^- \rightarrow 8^+$, $11^- \rightarrow 10^+$ and $13^- \rightarrow 12^+$ in Gd^{152} by either angular correlation or angular distribution method are required in order to further understand this coupling and transitions.

CHAPTER - 6

NEGATIVE PARITY STATES IN Nd^{144} AND MULTIPOLARITY OF GAMMA- TRANSITION FROM 1^- to 2^+ STATES BY BETA-GAMMA-GAMMA ANGULAR CORRELATION METHOD

6.1 INTRODUCTION:

Some of the low lying states in many isotopes (Nd^{144}, Sm^{148}, Sm^{152}, Gd^{152} and Gd^{156}) show a similar typical behaviour. The first 2^+ is regarded as a single quadrupole phonon state and 3^- as a single octupole phonon state. The levels with the spins and parities of $1^-, 5^-, 3^-, 4^-$ are around an energy which is very close to the sum of energies of 2^+ and 3^- states. Therefore these negative parity states are considered due to simultaneous excitation of quadrupole and octupole phonon (46-48). Bhatt (46) has calculated the spectrum considering the coupling of quadrupole to octupole phonon by giving the wavefunction of this state as

$$|J^-\rangle = |\left[2^+,3^-\right] J^-\rangle \ , \ J = 1,2,3,4,5$$

Where the bracket $\left[2^+,3^-\right]$ indicates vector coupling of the angular momenta 2 and 3 to give the resultant J. The energy splitting of these negative parity states is given by

$$\triangle E_J = - CW(J232;32) \qquad (6.1)$$

Where W(J232;32) is the Racah coefficient and C is a constant which determines the strength of interaction. He compared it with the experimental one and found it to be valid in Sm^{148}. One of the important predictions made by him is that the transition from any member $|J^{-}\rangle$ of quadrupole-octupole multiplet to the quadrupole $|2^{+}\rangle$ state would occur through the collapse of the octupole phonon. Therefore the corresponding transition should have an appreciable E3 content. The measurement of this appreciable E3 content will be of great advantage to provide a clue for this type of coupling and formation of the negative parity states.

One of the interesting case is in Nd^{144} from the decay of Pr^{144} which is formed from the decay of Ce^{144}. The decay scheme and level sequence, as shown in figure 6.1, are well established by many investigators (47, 49 , 50, 51). The 1^{-} level at 2185.68 keV lies very near to the sum of energies of 2^{+} state at 696.49 keV and 3^{-} state at 1510.65 keV and therefore the study of gamma-transition from 1^{-} to 2^{+} states may provide the necessary information.

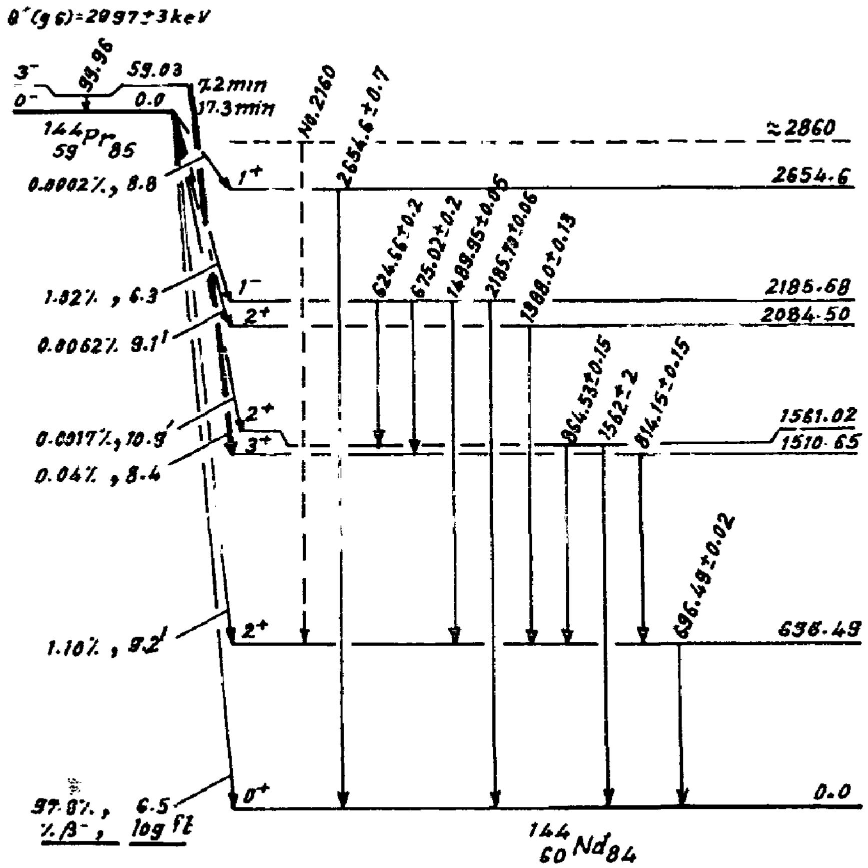
There are indirect ways of estimating the higher multipole content in gamma-transition. The more prevalent methods are based on

(i) internal conversion coefficient data

(ii) half life measurements i.e. considering the transition probabilities .

(iii) angular correlation data.

The decay being complicated, the methods (i) and (ii) are difficult to be tried. The gamma-gamma angular correlation method is also not clean due to interference of unwanted gamma-gamma cascades. But the unwanted gamma-gamma cascades can be



LEVEL SCHEME FOR ^{144}Nd FROM ^{144}Pr DECAY.

ALL ENERGIES ARE IN keV.

FIG. 6.1

avoided by the method of beta-gamma-gamma angular correlation. Therefore this study is under taken.

6.2 EXPERIMENTAL:

The source in the form of $CeCl_3$ in dilute HCl solution was obtained from Bhabha Atomic Research Centre, Bombay. Ce^{144} decays through β- emission with a half life of 284 days to Pr^{144} which in turn decays to Nd^{144}. The source dried on the cellotape, is mounted on the perspex stand at the centre of rotation of the three detectors. The source (in vacuum alongwith the plastic scintillator) was 3.5cm from beta-detector and 6cm from each of the gamma detectors.

6.3 COINCIDENCE AND ANGULAR CORRELATION STUDIES FOR THE CASCADE OF β-RAYS OF E_{max} 800 keV→γ -RAYS OF 1489keV→ γ- RAYS OF 696keV:

Beta-gamma ray (selecting γ -rays at the photopeak of 1489keV in a fixed detector and using beta-ray spectrometer as integral above 108keV energy) coincidences are obtained using slow-fast coincidence set-up. Then beta-gamma coincidences form the gate for one of the inputs of other coincidences (mixer-type) and the second input is from the movable gamma-ray spectrometer detecting gamma-rays in the photopeak of 696keV. The output of this coincidence unit will give the beta-gamma-gamma coincidence spectrum which is shown in fig.6.2. The angular correlation study is done selecting 696-keV gamma-ray in 4V channel width (1V=30.9keV). The movable detector is kept at

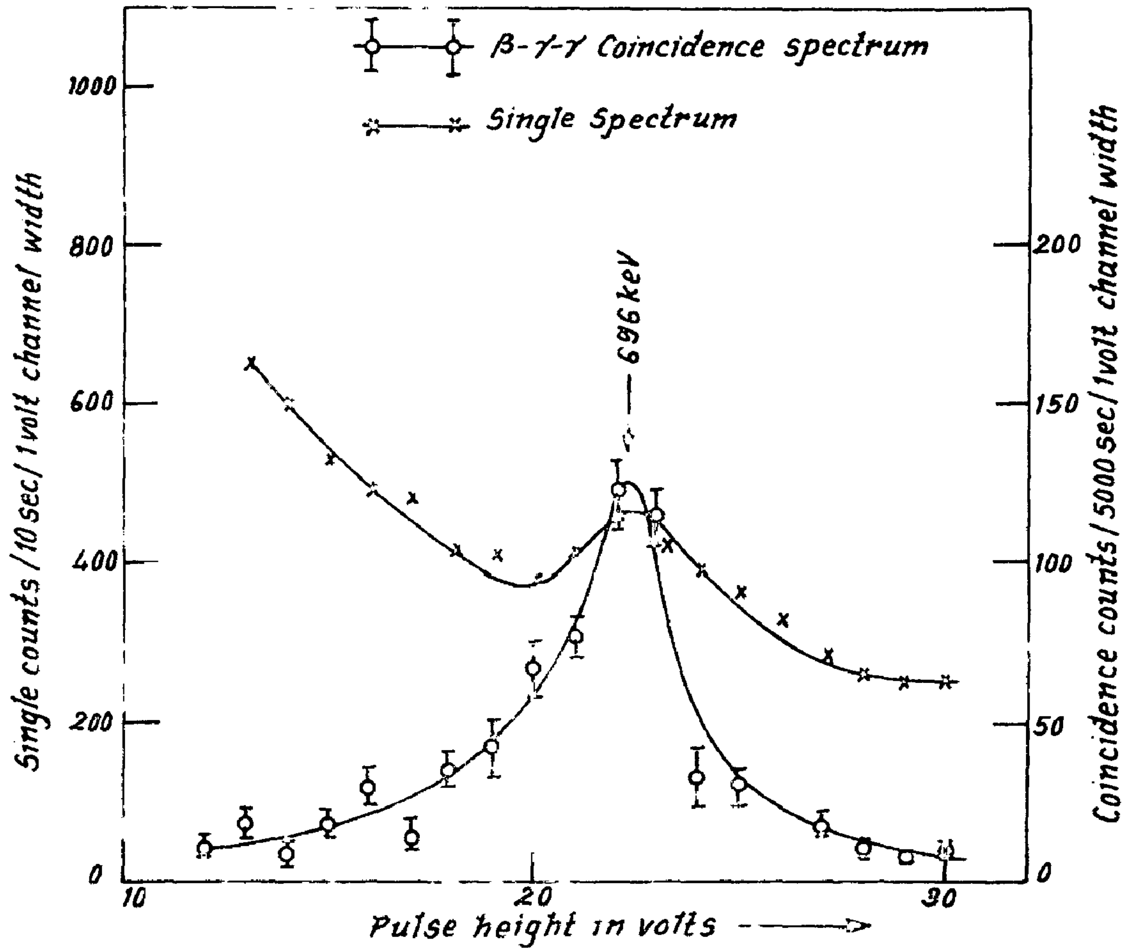


FIG. 6.2 – β-γ-γ COINCIDENCE SPECTRUM ALONG WITH THE SINGLE SPECTRUM.

several angles between 90^{0} to 180^{0} at the intervals of 22.5^{0}.

6.4 ANGULAR CORRELATION RESULTS:

The angular correlation function $W(\theta)$ obtained by the method of least square fit (without applying solid angle correction which has been considered in the theoretical calculations) for the cascade of β-rays of E_{max} 800keV→γ -rays of 1489-keV →γ -rays of 696-keV is as follows:

$$W(\theta) = 1+(0.219 \pm 0.019)\ P_2(\cos\theta) + (0.069 \pm 0.023)\ P_4(\cos\theta)$$

6.5 DISCUSSION:

Taking 696-keV gamma-ray transition to be pure quadrupole, one can consider 1489-keV gamma-ray transition to be mixture of dipole and quadrupole. Using the method of Arns and Wiedenbeck (52) and extending it to beta-gamma-gamma angular correlation, hence the usual plots are obtained for A_2 versus $Q(Q= \frac{\delta^2}{1+\delta^2}$ where $\delta = \frac{\langle f||L+1||i\rangle}{\langle f||L||i\rangle})$ as shown in figures 6.3 and 6.4. In this way we obtain, the values

$\delta_{21} = +(0.034\pm0.015)$, $Q_{21} = 0.001\pm0.001$, if E3 admixture is not present and considering the mixture of E1+M2;

and

$\delta_{31} = -(0.026\pm0.011)$, $Q_{31}=0.0007^{+0.0007}_{-0.0005}$, if M2 admixture is not present and considering the mixture of E1 + E3.

But if one wishes to include the contribution of E3 together with M2, the method of Arns and Wiedenbeck (52) can not be applied directly. This can be modified and the contribution of E3 content is determined by the following way.

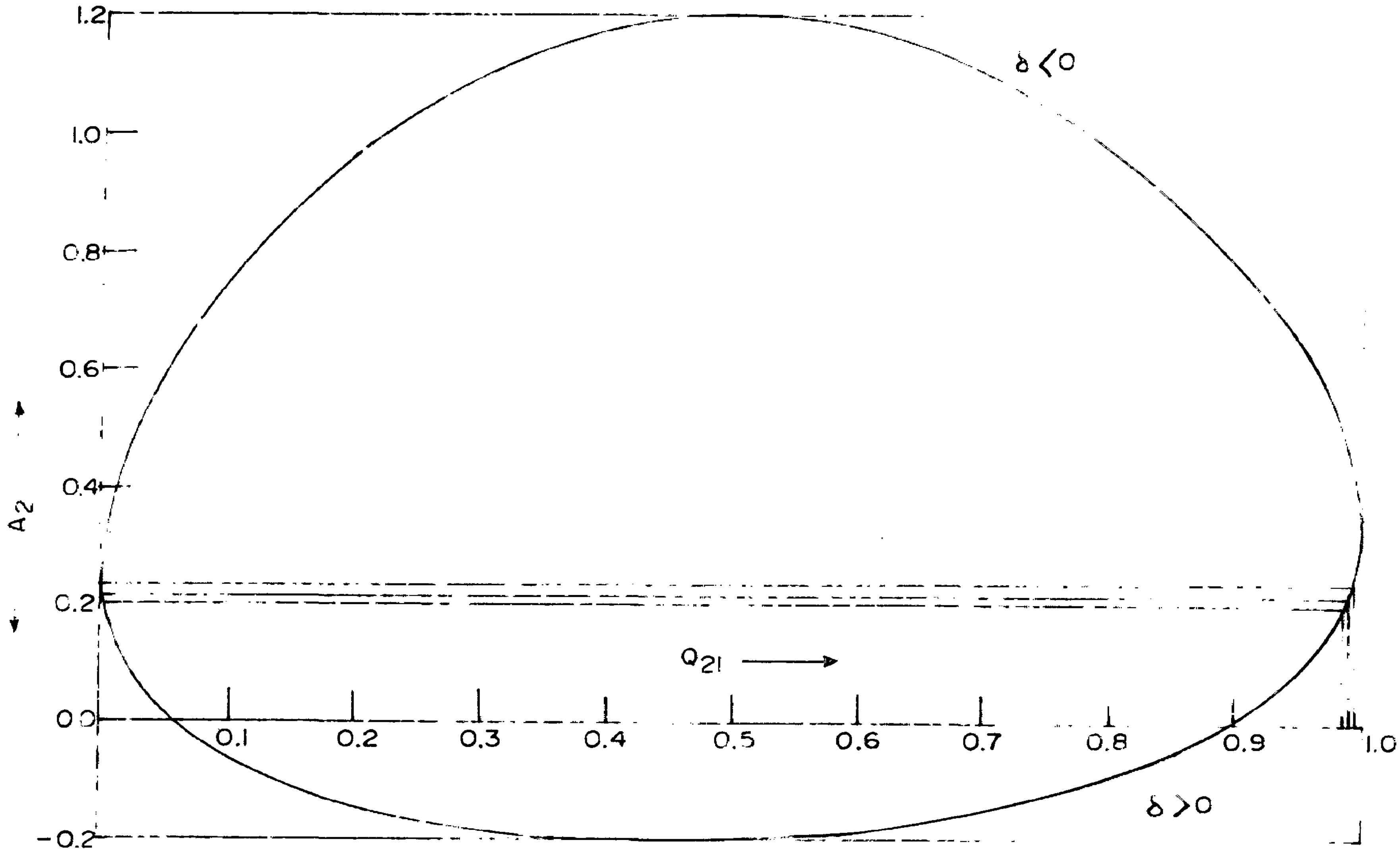


FIG.6.3 THEORETICAL PLOT OF A_2 Vs Q_{21}, THE QUADRUPOLE CONTENT IN DIPOLE FOR THE CASCADE OF

$$0^- \xrightarrow[\text{Allowed}]{\beta\text{-rays of Emax } 800\text{ Kev}} 1^- \xrightarrow[M2 + E1]{\gamma\text{-rays of } 1489\text{ Kev}} 2^+ \xrightarrow[E2]{\gamma\text{-rays of } 696\text{ Kev}} 0^+$$

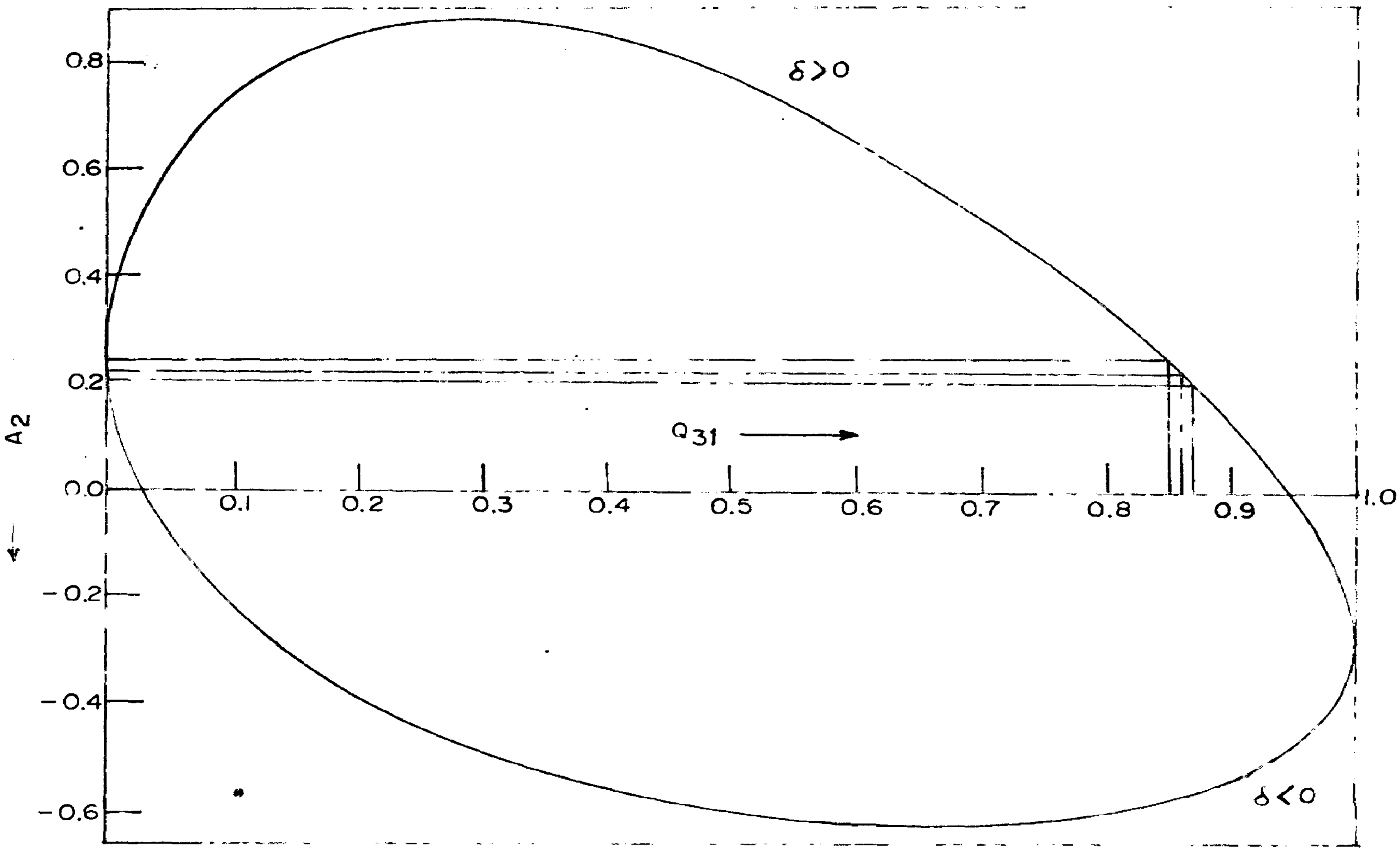


FIG.6.4 THE THEORETICAL PLOT OF A_2 Vs Q_{31}, THE OCTUPOLE CONTENT IN DIPOLE FOR THE CASCADE OF 0^- $\xrightarrow[\text{allowed}]{\beta\text{-rays of Emax x 800 Kev}}$ 1^- $\xrightarrow[\text{E1+E3}]{\gamma\text{-rays of 1489 Kev}}$ 2^+ $\xrightarrow[\text{E2}]{\gamma\text{-rays of 696 Kev}}$ 0^+

Let

$$A_2^{Expt} = A_2' + A_2'' \qquad (6.2(a))$$

$$A_4^{Expt} = A_4' + A_4'' \qquad (6.2(b))$$

Where A_2' , A_2'' , A'_4 and A_4'' are the angular correlation coefficients and are written as for the transition $(1^- \to 2^+)$ from the cascade $0^- \xrightarrow[\text{allowed}]{\beta} 1^- \xrightarrow[1489\text{keV}]{\gamma_1} 2^+ \xrightarrow[696\text{keV}]{\gamma_2} 0^+$.

Where β-transition is taken as allowed and γ_2 as pure E2 transition. Defining $Q_{21}\left[= \frac{\delta^2}{1+\delta^2} \text{ where } \delta = \frac{\langle f||M2||i\rangle}{\langle f||E1||i\rangle}\right]$ and $Q_{31}\left[=\frac{\delta^2}{1+\delta^2}, \text{ where } \delta = \frac{\langle f||E3||i\rangle}{\langle f||E1||i\rangle}\right]$, we have

$$A_2' = \frac{(1-Q_{21})\, a_2 \pm 2\sqrt{Q_{21}(1-Q_{21})}a_2'+Q_{21}a_2''}{(1-Q_{21})a_0 \pm 2\sqrt{Q_{21}(1-Q_{21})}a_0' + Q_{21}a_0''}, \text{ (M2:E1)} \qquad (6.3(a))$$

with

$a_2 = -0.4022$, $a_2' = 0.8944$, $a_2'' = -0.5708$ and

$a_0 = -1.5310$, $a_0' = -0.4497$, $a_0'' = -1.6612$

$$A_4' = \frac{(1-Q_{21})\, a_4 \pm 2\sqrt{Q_{21}(1-Q_{21})}a_4'+Q_{21}a_4''}{(1-Q_{21})\, a_0 \pm 2\sqrt{Q_{21}(1-Q_{21})}\, a_0' +Q_{21}a_0''}, \text{ (M2:E1)} \qquad (6.3(b))$$

Where

$a_4 = 0.0$, $a_4' = 0.0$, $a_4'' = 1.0295$ and

$a_0 = 1.5310$, $a_0' = -0.4497$, $a_0'' = -1.6612$

and

$$A_2'' = \frac{(1-Q_{31})a_2 \pm 2\sqrt{Q_{31}(1-Q_{31})}\, a_2' \pm Q_{31}a_2''}{(1-Q_{31})a_0 \pm 2\sqrt{Q_{31}(1-Q_{31})}\, a_0' \pm Q_{31}a_0''}, \text{(E3:E1)} \qquad (6.4(a))$$

Where

$a_2 = -0.4022$, $a_2' = -1.1950$, $a_2'' = 0.5823$ and

$a_0 = -1.5310$, $a_0' = -0.3562$, $a_0' = -2.0500$;

$$A_4'' = \frac{(1-Q_{31})a_4 \pm 2\sqrt{Q_{31}(1-Q_{31})}a_4' \pm Q_{31}\ a_4''}{(1-Q_{31})a_0 \pm 2\sqrt{Q_{31}(1-Q_{31})}a_0' \pm Q_{31}\ a_0''} \quad (E3 : E1) \qquad (6.4(b))$$

with

$a_4 = 0.0$, $a_4' = 1.1582$, $a_4'' = 0.1287$ and

$a_0 = -1.5310$, $a_0' = 0.3562$, $a_0'' = -2.0500$.

Choosing a value of Q_{21} lying between 0 and 1, substituting it in [6.3(a)] alongwith the other coefficients, we find A_2' which, in turn, is used to determine A_2'' from [6.2(a)]. This value of A_2'' is employed to find out Q_{31} with the help of equation [6.4(a)]. A similar procedure is followed to obtain Q_{31} from A_4^{Expt} through equations [6.3(b)], [6.2(b)] and [6.4(b)]. The values of Q_{31} and hence δ_{31} obtained through these two procedures are plotted against Q_{21} or δ_{21} values, selecting the experimental values within the error and the graphs are depicted in Figs.6.5(a) and 6.5(b)- The interception of these quadratic plots based on two procedures gives the possible solutions for Q_{21}(or δ_{21}) and Q_{31} (or δ_{31}). The hatched portion in these plots gives the values:

$Q_{21}(M2:E1) = 0.031^{+0.019}_{-0.011}$, $Q_{31}(E3:E1) = 0.004^{+0.001}_{-0.002}$

and

$\delta_{21}(M2:E1) = +(0.18^{+0.06}_{-0.04})$, $\delta_{31}(E3:E1) = -(0.06^{+0.015}_{-0.010})$

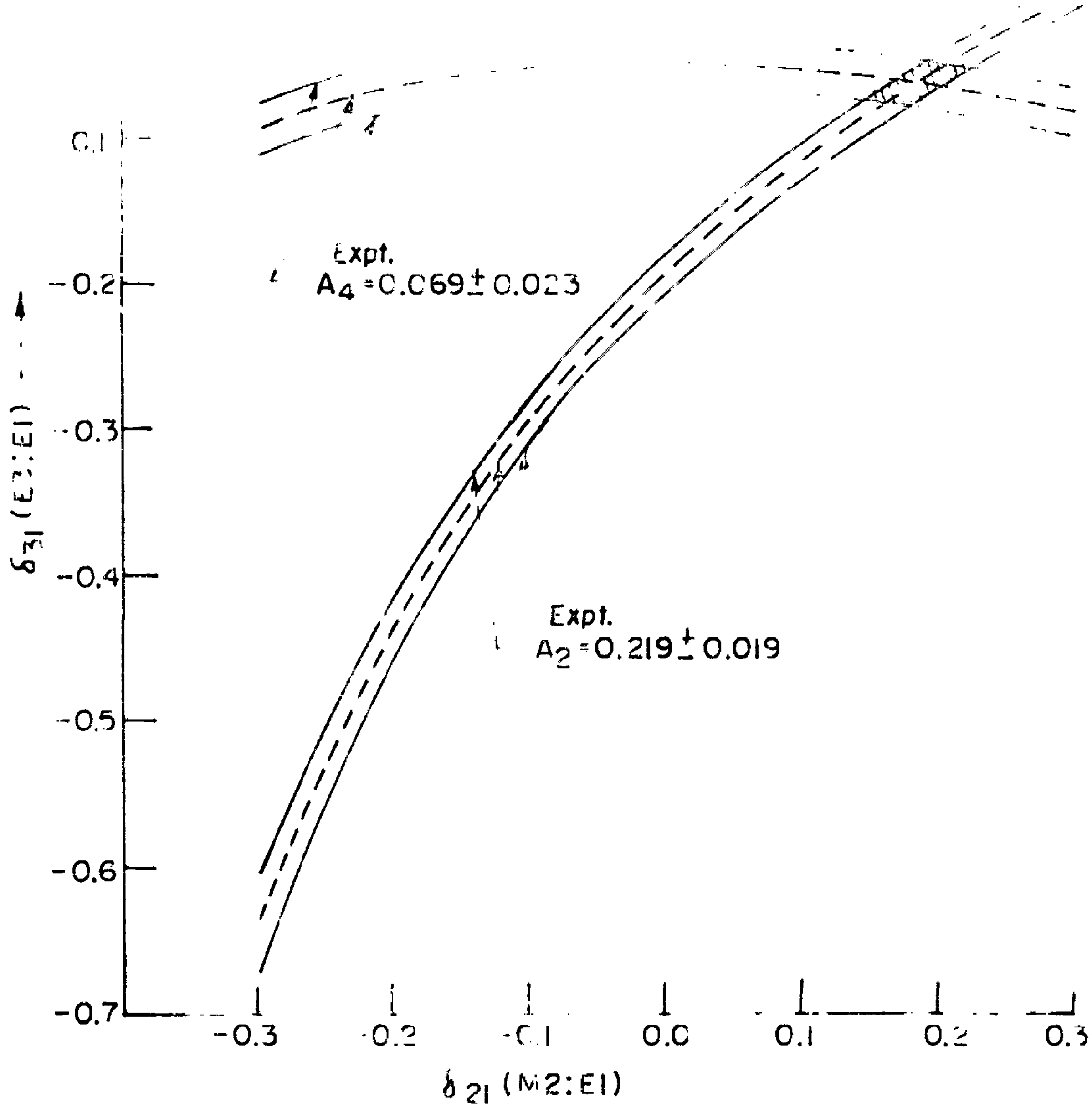


FIG.6.5(a) QUADRATIC PLOT OF δ_{21}(M2:E1) Vs δ_{31}(E3:E1) USING EXPERIMENTAL VALUES OF A_2 & A_4

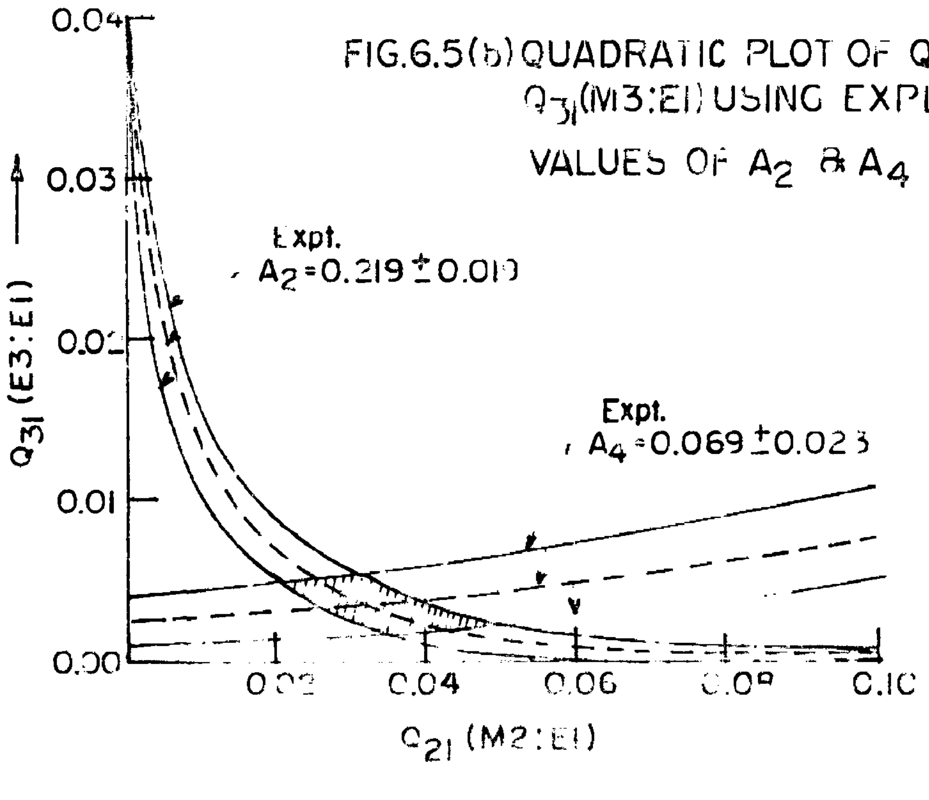


FIG.6.5(b) QUADRATIC PLOT OF Q_{21}(M2:E1) Vs Q_{31}(M3:E1) USING EXPERIMENTAL VALUES OF A_2 & A_4

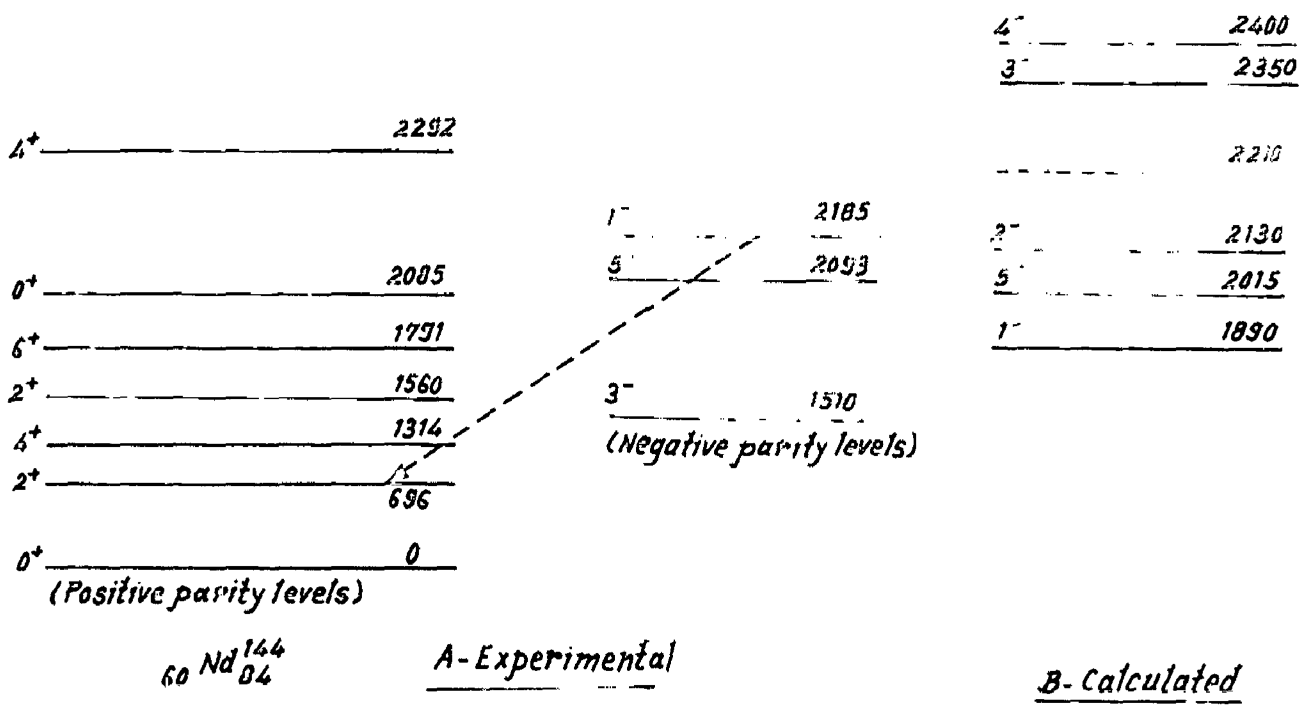
Therefore this clearly indicates $\sim 0.4\%$ contribution of E3 transition. If this contribution is appreciable, then model suggested by Bhatt (46) can be taken for further calculations.

6.6 ANALYSIS OF GAMMA-GAMMA ANGULAR CORRELATION DATA FOR $1^- \rightarrow 2^+ \rightarrow 0^+$ CASCADE:

Apart from Nd^{144}, we have four more isotopes i.e. Sm^{148}, Sm^{152}, Gd^{152} and Gd^{156}, where we find the similar negative parity states, having the cascade $1^- \rightarrow 2^+ \rightarrow 0^+$. All these nuclei lie in the rare earth region. The decay schemes of these isotopes are given in figures 6.6(a,b,c,d) which also include calculated energy levels of negative parity states by the approximation of Bhatt (46).

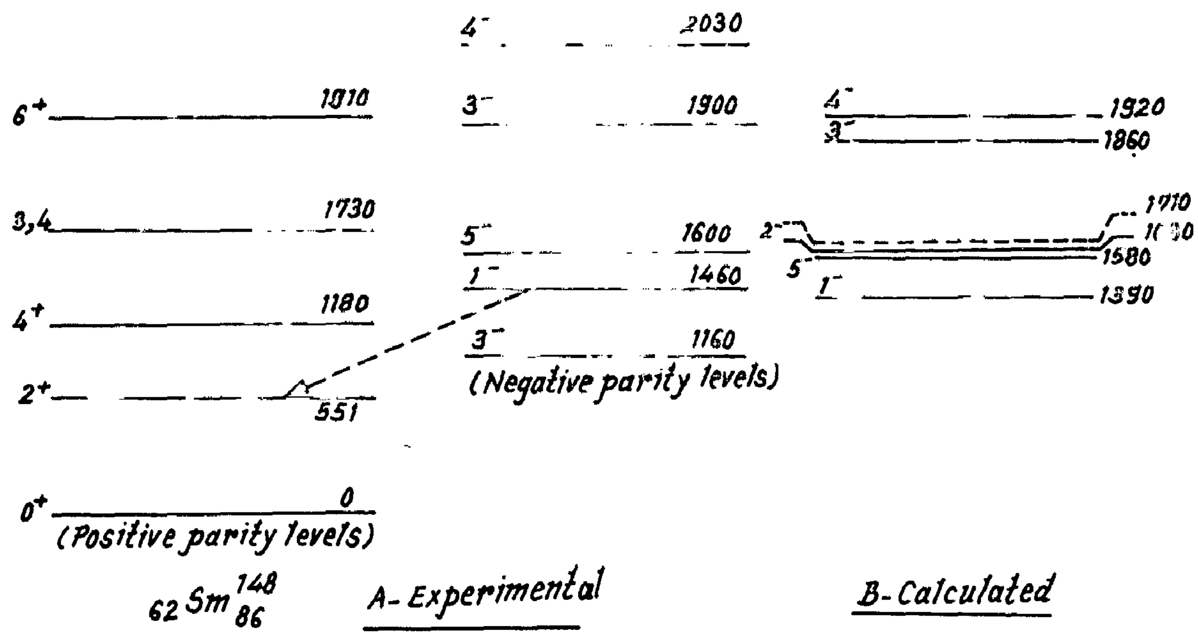
Gamma-gamma angular correlation coefficients have already been reported by various workers and are given in the following table:

Nucleus	Cascade $1^- \rightarrow 2^+ \rightarrow 0^+$ (Energy in keV)	A_2	A_4	Reference
Sm^{148}	912-551	-0.242 ± 0.024	0.033 ± 0.046	Horpster etal(53)
Sm^{152}	842-122	-0.245 ± 0.021	0.003 ± 0.014	Debrunner etal(54)
Gd^{156}	1153-89	-0.254 ± 0.010	0.004 ± 0.009	Hamilton etal (55)
Gd^{152}	Not done, since this state is formed from the decay of excited states of Eu^{152} having half life 9.3h.			



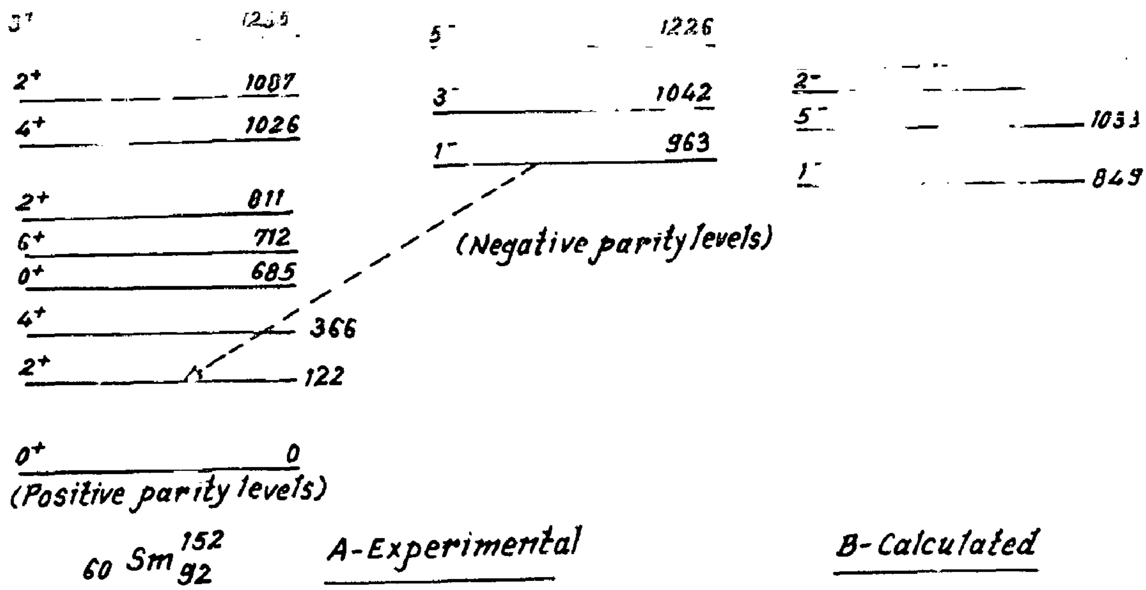
THE LEVEL STRUCTURE OF Nd^{144} BY THE THEORETICAL PREDICTIONS OF BHATT (46).

FIG. 6.6 (a)



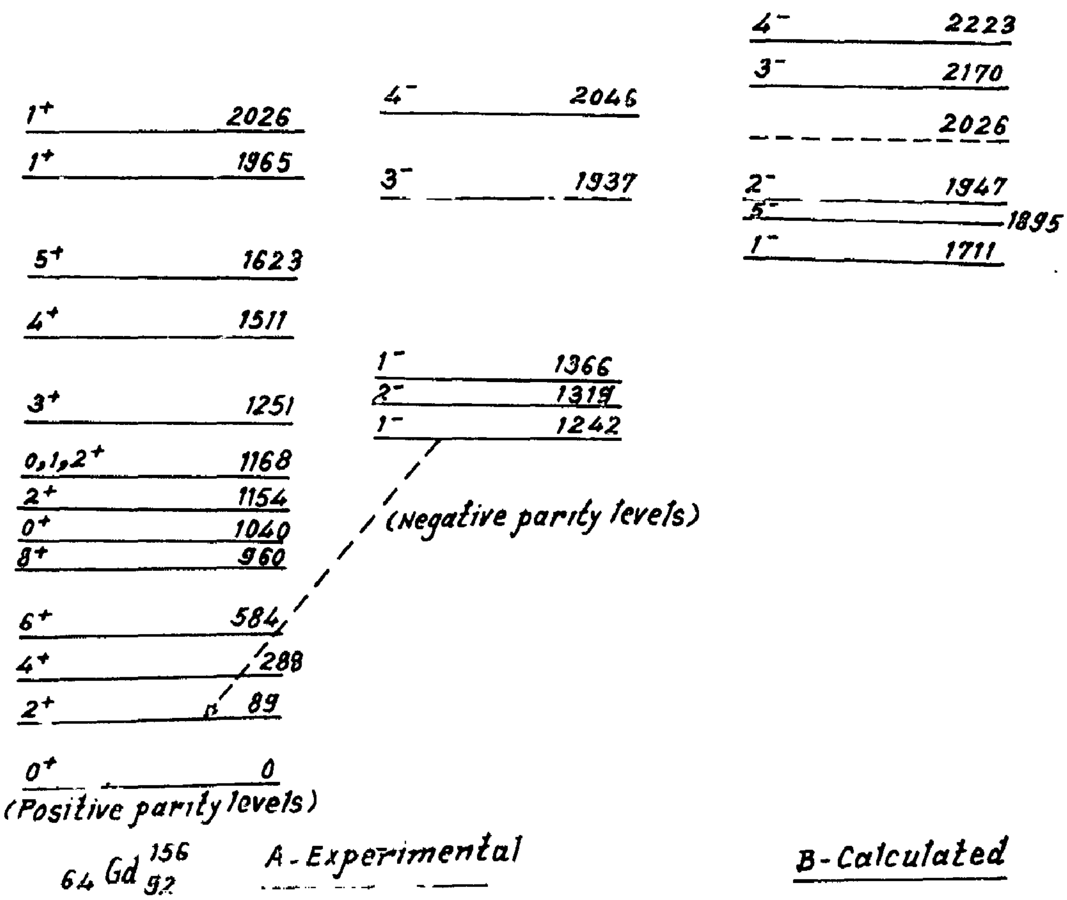
THE LEVEL STRUCTURE OF Sm^{148} BY THE THEORETICAL PREDICTIONS OF BHATT (46).

FIG. 6.6 (b)



THE LEVEL STRUCTURE OF Sm^{152} BY THE THEORETICAL PREDICTIONS OF BHATT (46).

FIG. 6.6 (c)



THE LEVEL STRUCTURE OF Gd^{156} BY THE THEORETICAL PREDICTIONS OF BHATT (46).

FIG. 6.6 (d)

The data of A_2 and A_4 for these cascades have been analysed similarly as given above for A_2 and A_4 obtained by beta-gamma-gamma angular correlation, for E1+ M2+ E3 content. The results are given in figures 6.7(a), 6.7(b) and 6.7(c). The following table gives the M2 and E3 contribution.

Nucleus	Cascade $1^- \to 2^+ \to 0^+$ (Energy in keV)	Q_{21}(M2:E1)	Q_{31}(E3:E1)	Reference
Sm^{148}	912-551	$0.044^{+0.012}_{-0.013}$	$0.001^{+0.002}_{-0.002}$	Horpster etal (53)
Sm^{152}	842-122	$0.044^{+0.006}_{-0.006}$	$0.0005^{+0.0005}_{-0.0004}$	Debrunner etal (54)
Gd^{156}	1153-89	$0.042^{+0.004}_{-0.005}$	$0.0004^{+0.0005}_{-0.0003}$	Hamilton etal (55)
Nd^{144}	1489-696	$0.031^{+0.019}_{-0.011}$	$0.004^{+0.001}_{-0.002}$	Present work.

This analysis of gamma-gamma angular correlation data reveals that there may be E3 content as well, but experimental error are quite large which do not warrant to jump for this conclusion by this analysis. The present study of Nd^{144}, positively shows the $0.4^{+0.1}_{-0.2}$ % E3 content in E1 alongwith M2. These studies need to be supplemented by other methods;but again these estimates indicate that other methods are very difficult to give conclusive results. More refined theoretical calculations are needed to estimate the E3 content in E1 alongwith M2.

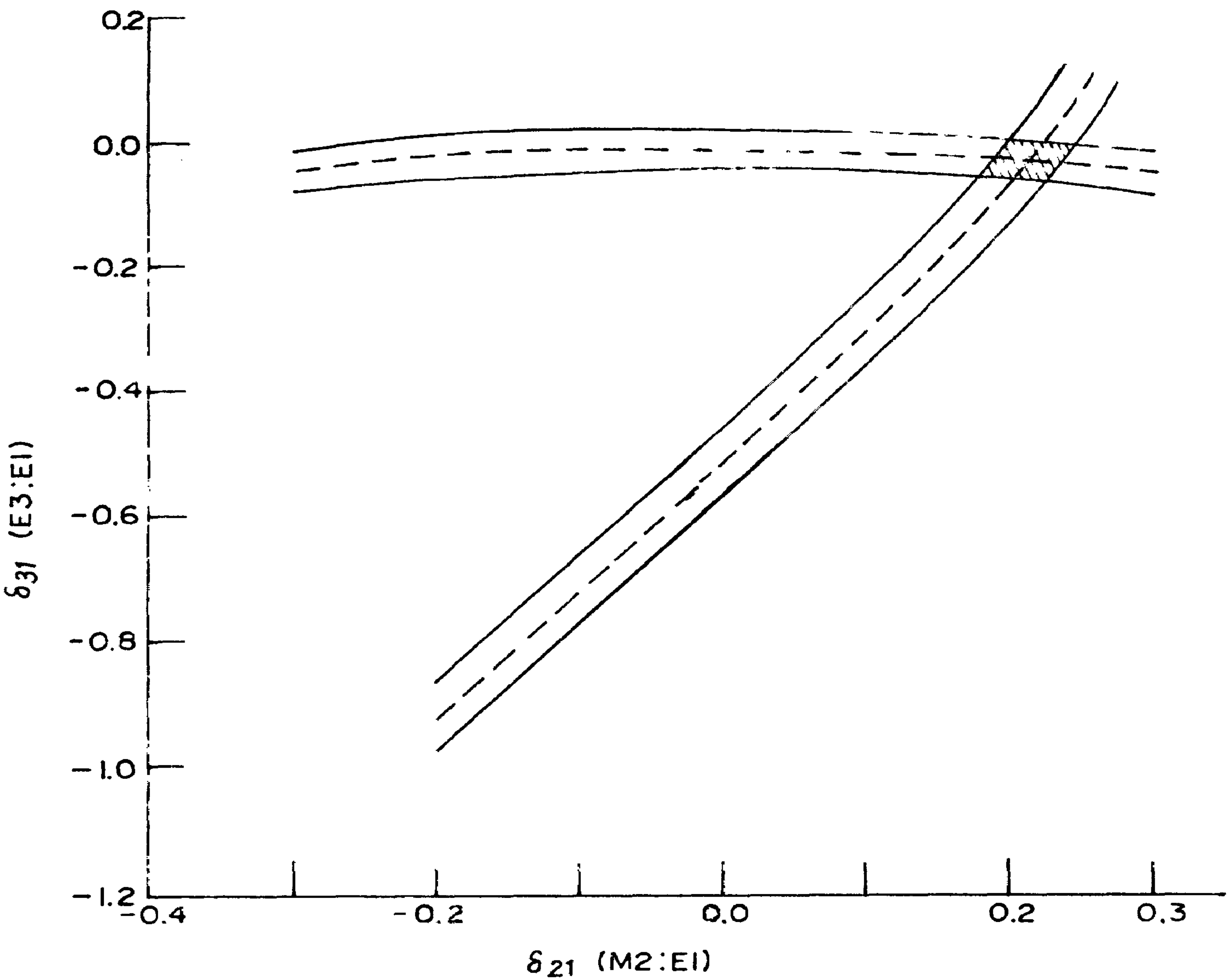


FIG. 6.7(a) QUADRATIC PLOT OF δ_{21} (M2:E1) Vs δ_{31} (E3:E1) USING THE EXPERIMENTAL VALUES A_4 GIVEN BY HARPSTER et. al (53)

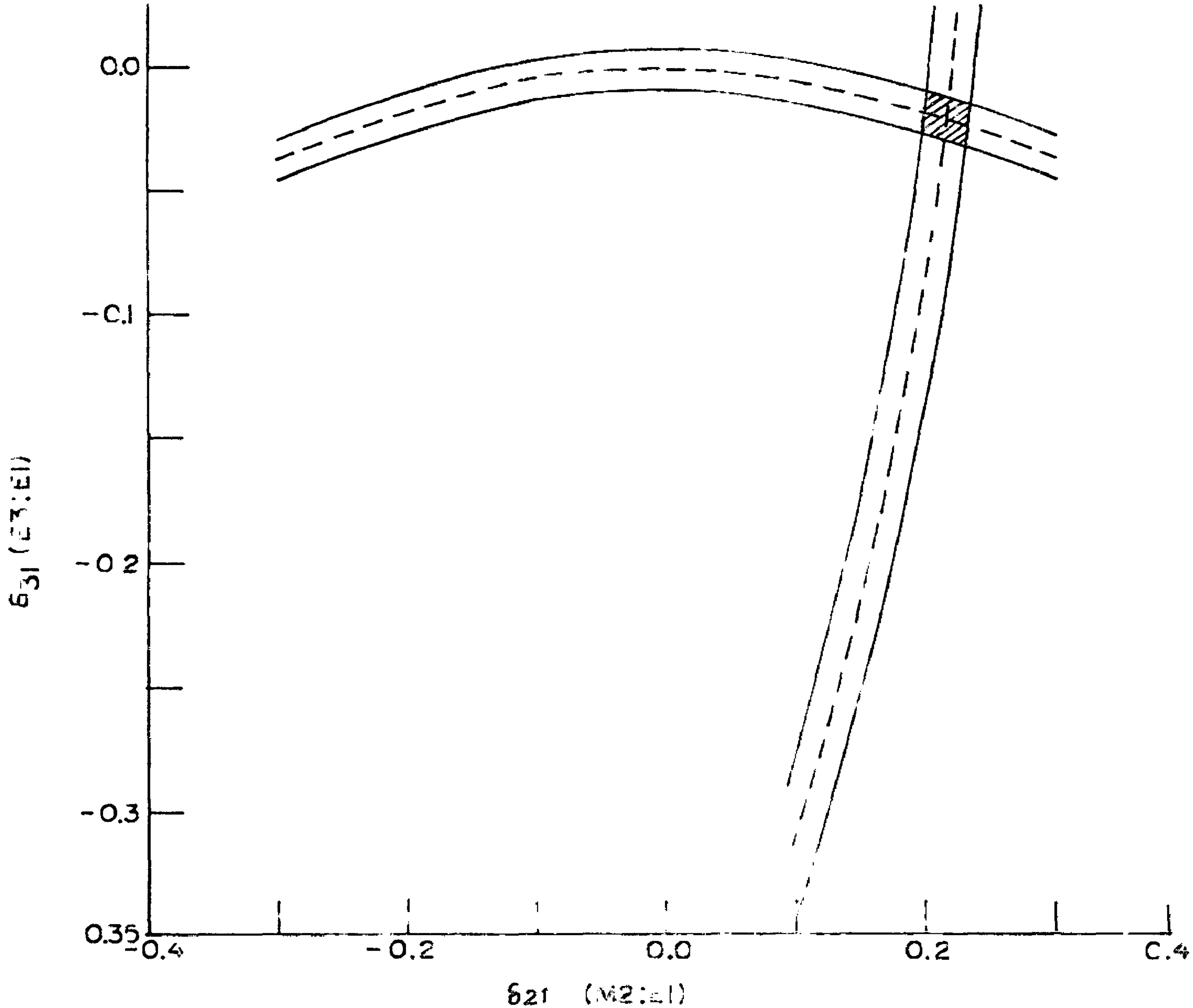


FIG.6.7(b) QUADRATIC PLOT OF δ_{21} (M2:E1) VS δ_{31} (E3:E1) USING THE EXPERIMENTAL VALUES OF A_2 AND A_4 GIVEN BY DERRUNNER et.al. (54)

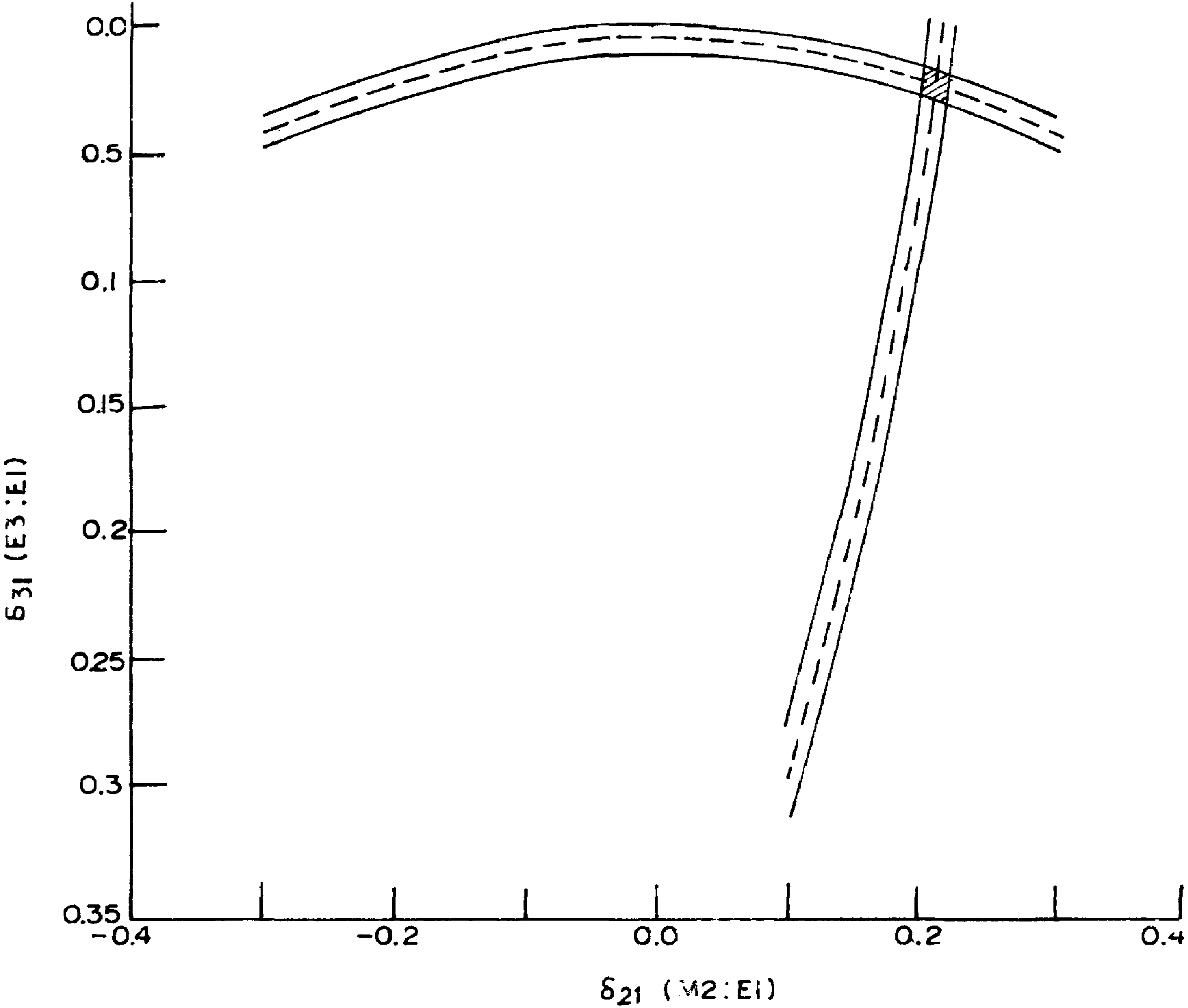


FIG.6.7(c) QUADRATIC PLOT OF δ_{21} (M2:E1) Vs δ_{31} (E3:E1) USING THE EXPERIMENTAL VALUES OF A_2 AND A_4 GIVEN BY HAMILTON et. al. (55)

CHAPTER - 7

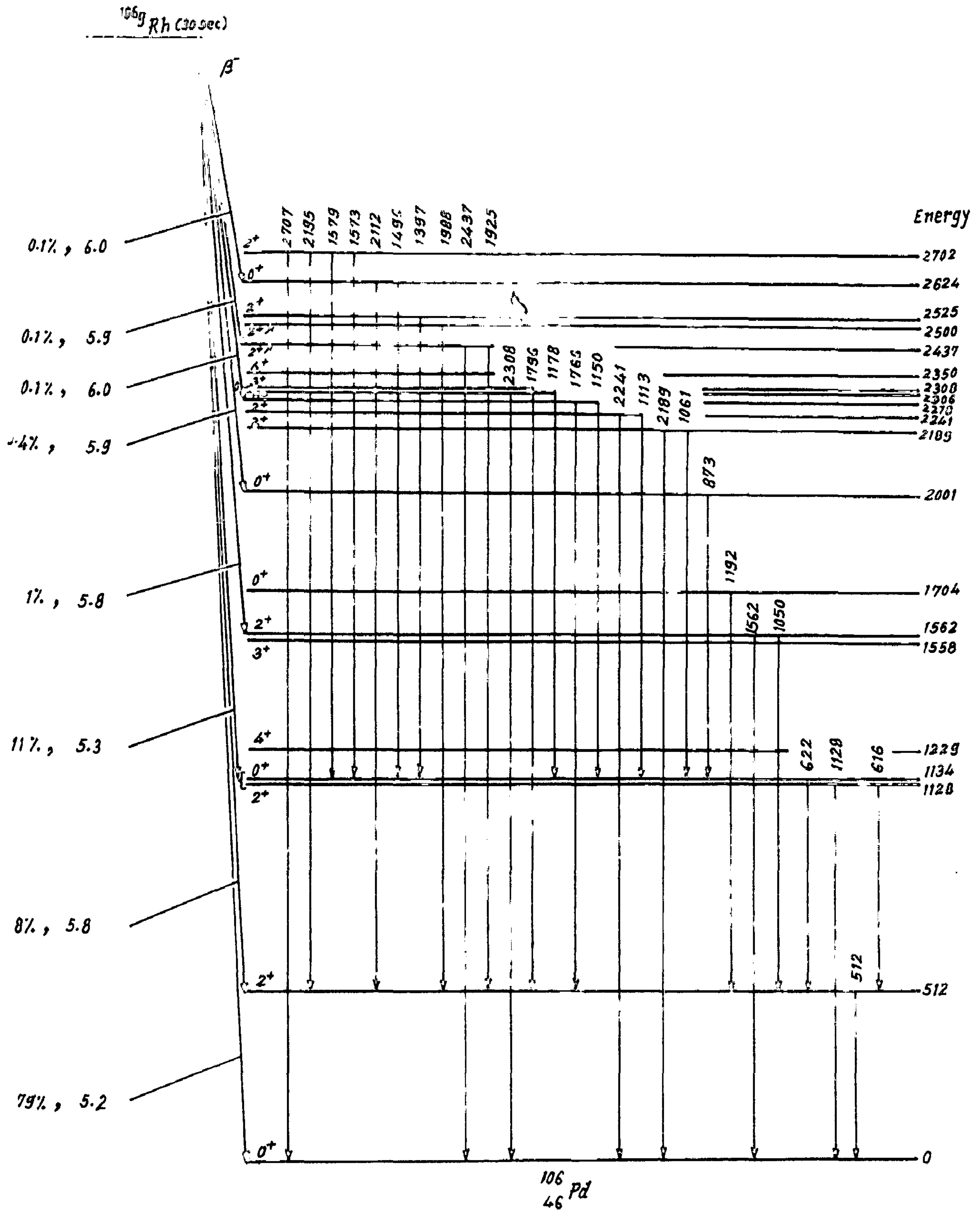
STUDY OF THE DOUBLET AT 1128-AND 1134-keV ENERGY LEVELS IN Pd^{106} FROM THE DECAY OF 30sec Rh^{106} BY BETA-GAMMA-GAMMA ANGULAR CORRELATION METHOD

7.1 INTRODUCTION:

The radiations from the decay of 30 sec Rh^{106} have been investigated as given in figure 7.1 by many investigators (18 ,56). Alburger (57) determined the ground state spin of Rh^{106} by the investigation of the β-ray spectra. The gamma-gamma directional correlation studies (58 ,59 ,60 ,61,62) have led to the many spin and parity assignments to the energy levels of Pd^{106}. Hattula and Liukkonen (58) studied them in order to understand the level structure above the vibrational 0^+, 2^+, 4^+ triplet at 1.1 - 1.2 MeV. The main purpose of the investigations of Avignone III and Pinkerton (59) was to measure the directional correlations of gamma-ray cascades for fixing the spins of the excited energy levels. The technique of beta-gamma-gamma directional correlation was used to study the doublet at 1128- and 1133-keV levels and to ascertain if they are due to β-transitions and if so what the branching ratio is.

7.2 EXPERIMENTAL:

The source in the form of $RuCl_3$ in dilute HCl solution was obtained from Bhabha Atomic Research Centre, Bombay. A few drops of the source were dried on the cellotape and it is mounted on a perspex stand. The source alongwith the plastic scintillator was placed in vacuum. The source was 3.5cm from



THE DECAY SCHEME OF $^{106}_{46}Pd$. TRANSITION ENERGIES ARE IN keV.

FIG. 7.1

the plastic scintillator and 6 cm from each of the NaI(Tl) detectors.

7.3 COINCIDENCE AND ANGULAR CORRELATION STUDIES FOR THE CASCADE OF β-RAYS OF E_{max} 2.4 MeV → γ - RAYS OF 622- AND 616-keV → γ - RAYS OF 512 keV.

A beta-ray spectrometer is used as integral spectrometer selecting β-rays above 2.0 MeV energy and a γ- ray spectrometer (fixed one) is used as differential spectrometer in the photopeak at 616-keV in 4V channel width (1V = 15keV). These two spectrometers were used for coincidences of beta-and gamma-rays using slow-fast coincidence set up and output of this forms the gate for one of the inputs of mixer type coincidence unit of resolving time of the order of 5×10^{-8} sec. The second input of this unit is from the second gamma-ray spectrometer (movable) which scans the spectrum in 1V channel width. The output of this mixer type coincidence unit is due to triple β - γ - γ cascade and it is fed to a scaler and recorder. The single and triple coincidence spectrum are shown in fig.7.2. The chance counting rate is taken by putting the delay in the mixer type coincidence unit. This establishes the cascade of β-rays of E_{max} 2.4 MeV → γ-rays of 622 and 616-keV→ γ-rays of 512-keV.Since there is only one peak at 512-keV and coincidence spectrum is not extended, it appears certain that there is no contribution of either β-groups or any other γ- ray to this coincidence spectrum.

For angular correlation studies, the third gamma-ray

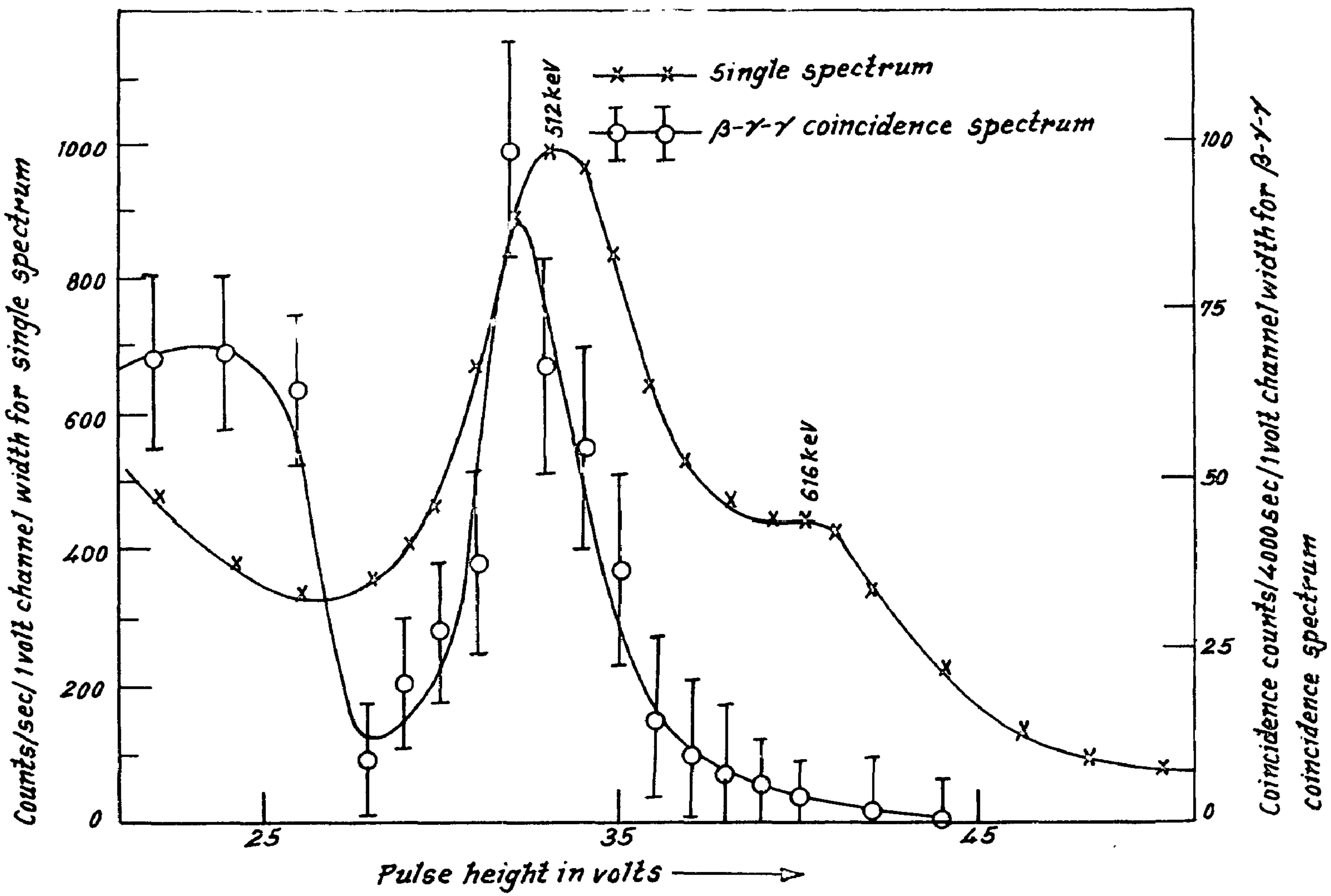


FIG.7.2.- β-γ-γ COINCIDENCE SPECTRUM ALONG WITH THE SINGLE SPECTRUM.

spectrometer which is movable, was used as a differential spectrometer selecting γ -rays of 512-keV in 4V channel width. The selection of other β-and γ-rays was done as mentioned above. The movable detector is kept at various angles in the interval of 22.5° between 90° to 180°.

7.4 <u>ANGULAR CORRELATION RESULTS</u>:

The angular correlation function $W(\theta)$ was obtained by the method of least square fit (without applying solid angle correction which is considered in the theoretical consideration). The angular correlation results obtained are as follows:

$$W(\theta) = 1 - (0.734 \pm 0.046)\ P_2(\mathrm{Cos}\theta) + (0.664 \pm 0.048) P_4(\mathrm{Cos}\theta)$$

7.5 <u>DISCUSSION</u>:

If we take β-transitions to be allowed, 622- and 512-keV gamma-transitions to be pure E2 and consider the spin value of 0^{+} for 1134-keV level, the theoretical values of the angular correlation coefficients A_2 and A_4 are as follows:

$$A_2 = -0.7854 \text{ and } A_4 = +0.6486$$

The present experimental values are slightly on the lower side which suggests that a slight mixture of another β-group can also be considered. Let us consider that the β-transition not only goes to 1134-keV level but also to 1128-keV level of ^{106}Pd (referring β_1 to the former and β_2 to the latter). Considering the spin assignment 2^{+} to 1128-keV level and 616-keV γ - ray from this level to be either pure E2 or M1 or mixture of M1 and E2 (taking β- transition to be allowed and 512-keV to be pure E2), the theoretical values for angular correlation coefficients are as follows:

$A_2 = -0.207$ and $A_4 = 0.0$ (for 616-keV to be pure M1)

$A_2 = -0.135$ and $A_4 = +0.246$ (for 616-keV to be pure E2)

The following estimates are made considering two (β_1 and β_2) groups to satisfy the experimental values:

(i) Taking 616-keV γ -transition to be pure E2;

considering A_2 value, we get (4.8 ± 4.3) % of β_2 in β_1

considering A_4 value, we get ≤ 4.9% of β_2 in β_1

(ii) Taking 616-keV γ -transition to be pure M1:

considering A_2 value, we get (9 ± 8)% of β_2 in β_1

considering A_4 value, we get ≤ 5% of β_2 in β_1

(iii) Taking 616-keV γ -transition to be mixture of M1 and E2

The percentage of mixture of M1 in E2 for 616-keV γ -transition as compiled by Lederer etal (18) is ≤ 0.5% . This small percentage of M1 in E2 for 616-keV transition shall not change the estimates of two β-groups (considering theoretical values of A_2 as mentioned above), if we take 616-keV transition to be pure E2.

Therefore we find that there are two beta-groups, one corresponding to 1128-keV and other to 1134-keV and the percentage of former β-group is $\leq 5\%$ to that of the latter. Further as mentioned above, the spin values of the levels involved are confirmed by the present studies of $\beta - \gamma - \gamma$ angular correlation, viz 0^+ and 2^+ for the 1134 and 1128-keV excited energy levels respectively.

CHAPTER-8

A METHOD FOR THE DETERMINATION OF A_2^β, THE ANGULAR CORRELATION COEFFICIENT BY BETA-GAMMA-GAMMA ANGULAR CORRELATION STUDIES

8.1 INTRODUCTION:

The decay of Tb^{160} to the levels in Dy^{160} has been extensively studied by many investigators (18,63). The gamma-gamma angular correlation studies (64-68) have been performed in order to verify the current spin assignments to the excited energy levels and multipolarities of the gamma-transitions. The log ft values are found to be very large and in such cases, A_2^β the angular correlation factor is not zero and has been measured. One of the difficulties in A_2^β measurements is due to other competitive beta-gamma cascades (coincidence of beta-spectrum and compton distribution of gamma-ray spectrum). Further the angular correlation of these beta-gamma coincidences are difficult to account if the beta-transition is the first forbidden. But on the otherhand, the beta-gamma-gamma angular correlation studies can be tried and by this technique, the unwanted beta-gamma cascades can be either eliminated or reduced for the determination of A_2^β in allowed cases.

The decay of Tb^{160} to Dy^{160} has been studied by this method. This decay is not simple but involves large number of beta (allowed and forbidden) and gamma-transitions. Therefore complicated corrections are needed if one wants to obtain A_2^β by beta - gamma - gamma angular correlation studies.

Cipolla etal (69) determined A_2^{β} = 0.041 ± 0.015 in the decay of Tb^{160} for the β-group of E_{max} 566keV.

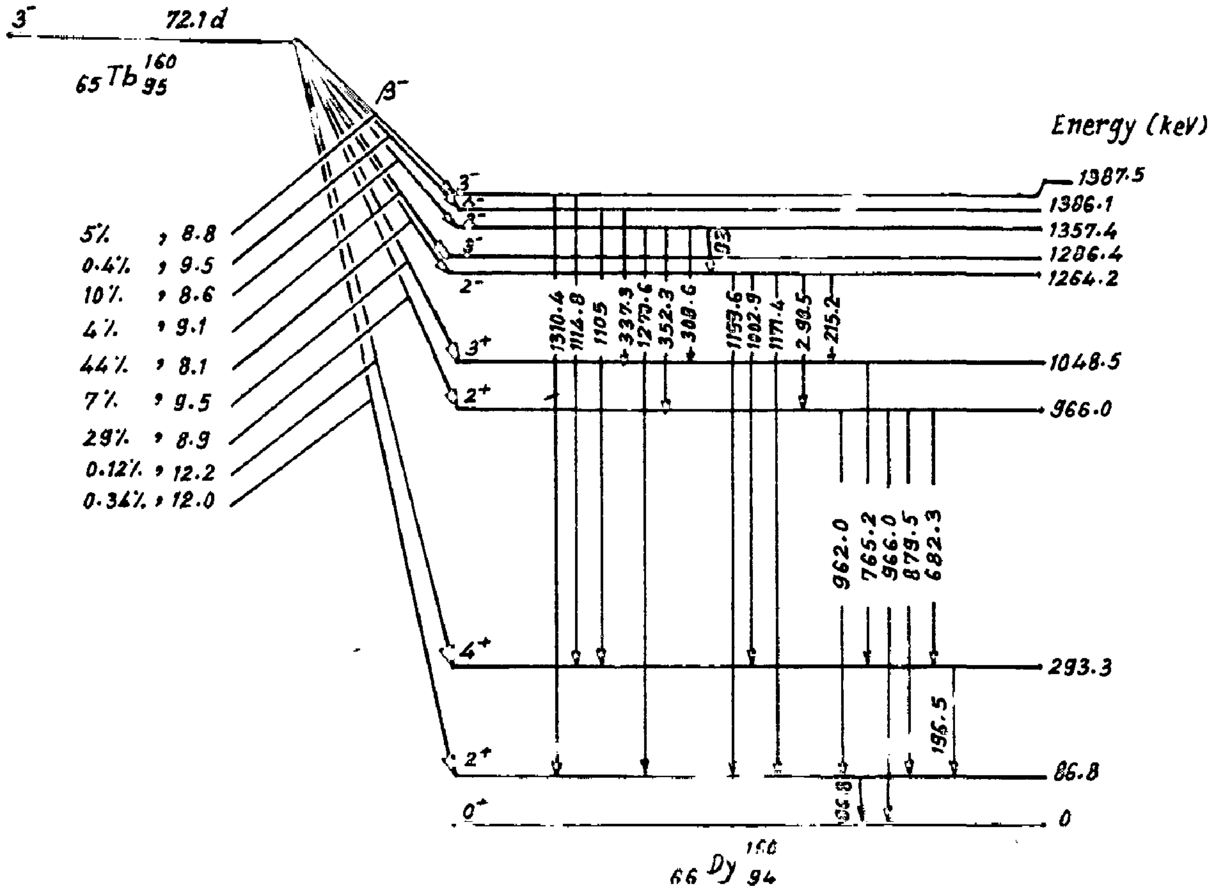
The principle features of the Tb^{160} decay scheme are well known from the previous works and the decay scheme of Tb^{160} is presented in fig.8.1.

8.2 EXPERIMENTAL:

The source in the form of $TbCl_3$ in dilute HCl solution was obtained from Bhabha Atomic Research Centre, Bombay. The specific activity of the source was 150mc/gm Tb. A few drops of the source were dried on the cellotape and it was spread in 2mm diameter. The cellotape alongwith the source was mounted on a perspex stand and it was kept in vacuum alongwith the plastic scintillator. The source was 3.5cm from the beta-detector and 6cm from each of the NaI(Tl) detectors.

8.3 COINCIDENCE AND ANGULAR CORRELATION STUDIES FOR THE CASCADE OF β-RAYS OF E_{max} 566keV → γ-RAYS OF 298keV→ γ-RAYS OF 966-keV:

A beta-ray spectrometer is used as an integral spectrometer selecting beta-rays above 360 keV energy and gamma-ray spectrometer (fixed one) is used as a differential spectrometer scanning the spectrum in 1V channel width (1V = 10.6keV) in the region of gamma-rays of 298-keV and a photopeak of 298-keV was obtained. The output of the slow-fast coincidence circuit (selecting gamma-rays in photopeak of 298keV) forms the gate for one of the inputs of the mixer type coincidence unit and the second input of this unit from the movable gamma-ray



DECAY SCHEME OF Tb^{160}. ALL ENERGIES ARE IN keV.

FIG. 8.1

spectrometer which scans the gamma-rays above 924keV energy (1V = 22keV). The triple coincidence spectrum was obtained clearly indicating a photopeak of 966keV. The beta-gamma-gamma coincidence spectrum alongwith the single spectrum is shown in fig.8.2.

The angular correlation studies for the cascade of β-rays of E_{max} 566keV → γ-rays of 298keV → γ-rays of 966keV are done by selecting 298- and 966 keV energies at the photo-peak. The movable detector is kept at the various angles in intervals of 22.5° between 90° to 180°.

8.4 ANGULAR CORRELATION RESULTS:

The angular correlation function W(θ) was obtained by the method of least square fit (without applying solid angle correction which has been included in the theoretical calculations) and the results thus obtained are as follows:

$$W(\theta)=1+(-0.199 \pm 0.016)\ P_2(\cos\theta)+(0.019 \pm 0.018)P_4(\cos\theta)$$

8.5 THEORETICAL ANALYSIS:

The beta-gamma-gamma angular correlation function W(θ) as given in equation (2.51) of chapter 2, has been modified to calculate the β-correlation factor A_2^β.

Let us consider the case when the second transition is pure dipole and third transition is pure quadrupole, respectively i.e. $L_2 = L_2' = 1$ and $L_3 = L_3' = 2$ and therefore $\delta_2 = \delta_3 = 1$. Considering the allowed β-transitions (J=J' =1), the equation (2.51) can be expanded by summing over k_1.

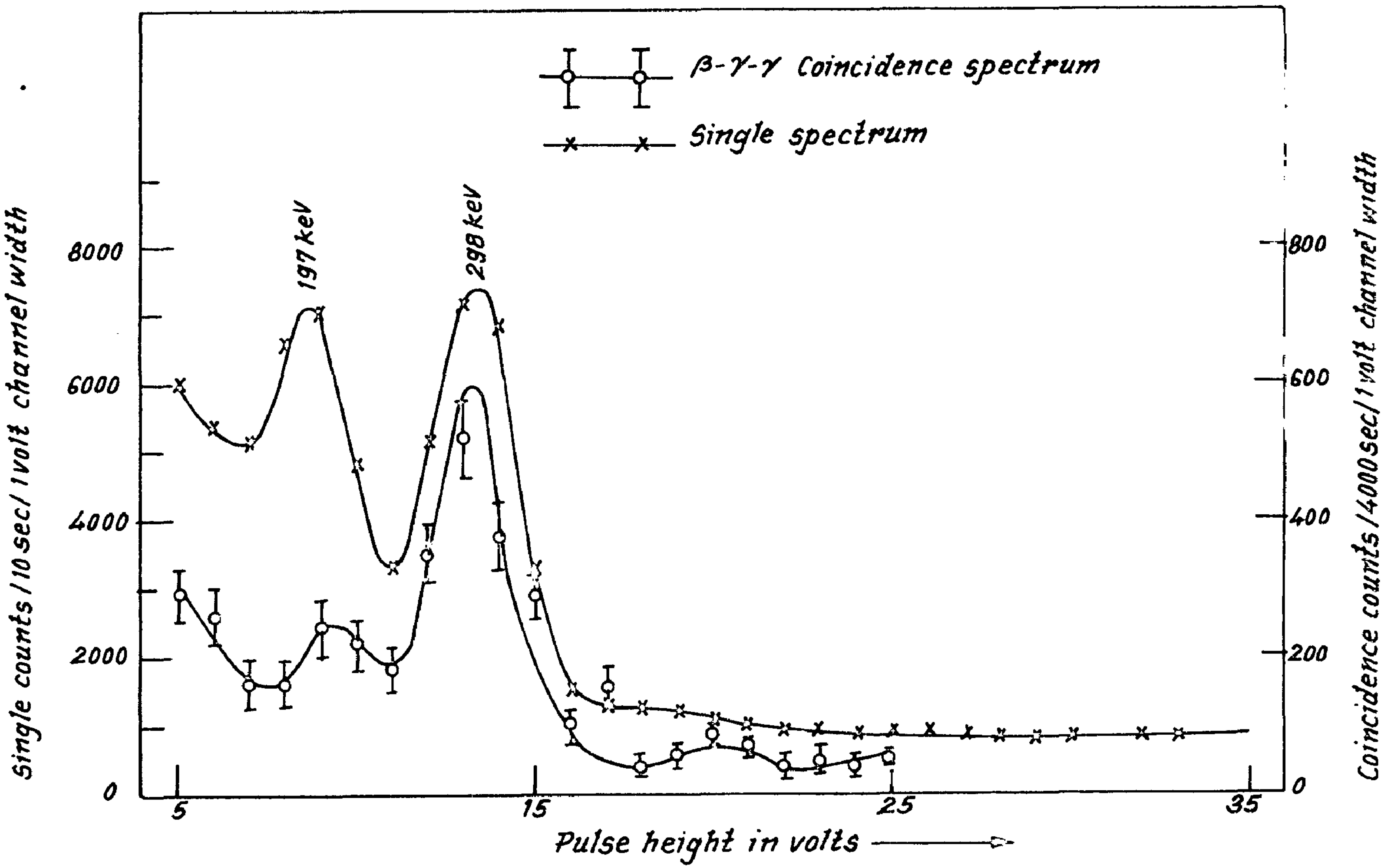


FIG. 8.2 – β-γ-γ COINCIDENCE SPECTRUM ALONG WITH THE SINGLE SPECTRUM.

$$W(\theta) = \left(\frac{1}{4\pi}\right)^{3/2} (-)^{I_i+I_f} \sum_{k_2k_3k} \Big[b0(JJ)\ Z\ (JI_aJI_a;I_i0)\ x$$

$$G\gamma \begin{Bmatrix} I_b & L_2 & I_a \\ I_b & L_2 & I_a \\ k_3 & k_2 & 0 \end{Bmatrix} Z_1(L_3I_bL_3I_b;I_fk_3)\ Q_0\ Q_{k_2}\ Q_{k_3}\ \alpha^{i}_{0k_2k_3k} P_k(\mathrm{Cos}\theta)$$

$$+b2(JJ)Z(JI_aJI_a;I_i2)\ G\gamma \begin{Bmatrix} I_b & L_2 & I_a \\ I_a & L_2 & I_a \\ k_3 & k_2 & 2 \end{Bmatrix} Z_1(L_3I_bL_3I_b;I_fk_3)\ x$$

$$Q_2\ Q_{k_2}\ Q_{k_3}\ \alpha^{i}_{2k_2k_3k}\ P_k(\mathrm{Cos}\theta) + b4(JJ)Z_1(JI_aJI_a;I_i4)x$$

$$G\gamma \begin{Bmatrix} I_b & L_2 & I_a \\ I_b & L_2 & I_a \\ k_3 & k_2 & 4 \end{Bmatrix} Z_1(L_3I_bL_3I_b;I_fk_3)\ Q_4Q_{k_2}Q_{k_3}\ \alpha^{i}_{4k_2k_3k} P_k(\mathrm{Cos}\theta) \Big] \tag{8.1}$$

The first term of the equation (8.1) can be written in the following way by summing over k i.e. taking k = 0,2, and 4 values

$$\sum_{k_2k_3} \Big[\ b0\ (JJ)\ Z_1\ (JI_aJI_a;I_i0)\ G\gamma \begin{Bmatrix} I_b & L_2 & I_a \\ I_b & L_2 & I_a \\ k_3 & k_2 & 0 \end{Bmatrix}\ x$$

$$Z(L_3I_bL_3I_b;I_fk_3)Q_0Q_{k_2}Q_{k_3}\ \ (\ \alpha^{i}_{0k_2k_30}\ P_0(\mathrm{Cos}\theta) + \alpha^{i}_{0k_2k_32} P_2(\mathrm{Cos}\theta)$$

$$+ \alpha^{i}_{0k_2k_34} P_4(\mathrm{Cos}\theta)) \Big] \tag{8.2}$$

Writting α_0, α_2 and α_4 for the coefficients of b0 $P_0(\cos\theta)$, b0 $P_2(\cos\theta)$ and b0 $P_4(\cos\theta)$ and therefore the first term of equation (8.2) is written as

$$b0 \left[\alpha_0 P_0(\cos\theta) + \alpha_2 P_2(\cos\theta) + \alpha_4 P_4(\cos\theta) \right] \qquad (8.3)$$

Similarly taking second term having b2(JJ) in the expression (8.1) and summing over k, one can write the coefficients to be β_0, β_2, and β_4 and third term having b4(JJ), the coefficients can be taken as γ_0, γ_2 and γ_4 respectively.

Therefore angular correlation function $W(\theta)$ is written as:

$$W(\theta)=(\alpha_0 b0 + \beta_0 b2 + \gamma_0\, b4)\, P_0(\cos\theta) + (\alpha_2 b0 + \beta_2 b2 + \gamma_2 b4)\, P_2(\cos\theta) + (\alpha_4 b0 + \beta_4 b2 + \gamma_4\, b4)\, P_4(\cos\theta) \qquad (8.4)$$

In order to compare it with the experiment, the coefficient A_2 and A_4 are written in the following manner

$$A_2 = \frac{\alpha_2 b0 + \beta_2 b2 + \gamma_2 b4}{\alpha_0 b0 + \beta_0\, b2 + \gamma_0\, b4} \qquad (8.5(a))$$

and

$$A_4 = \frac{\alpha_4 b0 + \beta_4 b2 + \gamma_4 b4}{\alpha_0 b0 + \beta_0 b2 + \gamma_0 b4}$$

Further A_2 can be written as

$$A_2 = \frac{\alpha_2}{\alpha_0} \left[\left(1 + \frac{\beta_2}{\alpha_2}\,\frac{b2}{b0} + \frac{\gamma_2}{\alpha_2}\,\frac{b4}{b0}\right) \left(1 + \frac{\beta_0}{\alpha_0}\,\frac{b2}{b0} + \frac{\gamma_0}{\alpha_0}\,\frac{b4}{b0}\right)^{-1} \right]$$

(8.6)

Taking the approximation in the expression, we have

$$A_2 = \frac{\alpha_2}{\alpha_0}\left[1 + \left(\frac{\beta_2}{\alpha_2} - \frac{\beta_0}{\alpha_0}\right)\frac{b2}{b0} + \left(\frac{\gamma_2}{\alpha_2} - \frac{\gamma_0}{\alpha_0}\right)\frac{b4}{b0}\right] \quad (8.7)$$

$\alpha_0\alpha_2$; $\beta_0\beta_2$; and $\gamma_0\gamma_2$ are the co efficients which are known for the particular spin sequence and multipolarities of gamma-transitions.

Considering the cascade $3^- \xrightarrow[\beta]{J=1} 2^- \xrightarrow[\gamma_1]{L_2=1} 2^+ \xrightarrow[\gamma_2]{L_3=2} 0^+$, the theoretical value of A_2 is obtained to be

$$A_2 = -\ 0.208\ \left(1 + 0.132\ \frac{b2}{b0} + 0.0\ \frac{b4}{b0}\right)$$

8.6 RESULTS AND DISCUSSION:

Taking the experimental value A_2 to be -0.199 ± 0.016, we find the mean value of $\frac{b2}{b0} = -\ 0.313 \pm 0.60$. In order to compare it with the value obtained by Cipolla etal (69) we take $F_k(LL;I_iI_a)$ to be 0.120 and taking proper sign to change $Z_1(JI_a\ JI_a;\ I_ik_1)$, as defined by Ferguson (16) into $F_k(LL;I_iI_a)$, as defined by Rose (6), we get the mean value to be $A_2^{\beta} = 0.037$ This value is comparable with that given by Cipolla etal (69) as 0.041 ± 0.015.

CHAPTER-9

SYSTEMATICS IN E2/M1 MULTIPOLE MIXING RATIOS OF $(2'^{+} \rightarrow 2^{+})$ GAMMA-TRANSITIONS IN EVEN-EVEN NUCLEI

9.1 INTRODUCTION

Many attempts [Potnis and Rao (70), Grechukhin (71), Tamura and Yoshida (72), Krane (73), Greiner (74), Davydov and Filippov (75), Hamilton (76), and Kumar (77)] in the past have been made to compile E2/M1 multipole mixing ratios (≡'δ') and relative phase. Kumar (77) has summarized in review article entitled "What can we learn from the transition probabilities and mixing ratios of atomic nuclei". In all these attempts, the magnitude of 'δ' has been the main consideration. Krane (73) has compiled the number of cases with positive and negative phases alongwith their magnitudes. Krane (73) has further reported that there is no correlation or any systematic in phases of 'δ'. An attempt has been made in order to investigate the phases of 'δ'. Another objective of the present study is to find out the systematics in the magnitudes and phases which may reveal the nuclear structure.

9.2 PHASE CONVENTION AND DEFINITIONS:

In extracting the mixing ratios from the quoted angular correlation coefficients, the sign convention of Biedenharn and Rose (6) has been followed, in which the first transition is the absorption to form an intermediate state followed by the second transition, consisting of the emission.

The angular correlation coefficients for the cascade $I_i \xrightarrow{\gamma_1} I \xrightarrow{\gamma_2} I_f$ in which the initial transition is a mixture of M1 and E2, are written as:

$$A_k = \frac{Z(L_1IL_1I;I_ik) + 2\delta\ Z(L_1IL_1'I;I_ik) + \delta^2\ Z(L_1'IL_1'I;I_ik)}{1+\delta^2}$$

$$X \quad Z(L_2IL_2I;I_fk) \qquad (9.1)$$

Where 'δ' the amplitude mixing ratio, is defined as

$$\delta = \frac{\langle I||L_1'\pi_1'||\ I_i\rangle}{\langle I||\ L_1\pi_1\ ||\ I_i\rangle} \qquad (9.2)$$

The sign convention of Biedenharn and Rose (6) is compared with Krane (31) and Rose-Brink (78) conventions for a cascade as

$$\delta(\gamma_1)_{BR} = -\ \delta(\gamma_1)_{KS}\ ,\ \delta(\gamma_1)_{RB} = -\ \delta(\gamma_1)_{KS}$$

$$\delta(\gamma_2)_{BR} = \delta(\gamma_2)_{KS},\ \delta(\gamma_2)_{RB} = -\ \delta(\gamma_2)_{KS} \qquad (9.3)$$

The mixing ratio 'δ' may be compared with theoretical values through the expression, as defined by Krane (73)

$$\frac{\delta}{E_\gamma} = 0.835\ \frac{\langle I||m(E2)||\ I_i\rangle}{\langle I||m(M1)||\ I_i\rangle} \qquad (9.4)$$

Where $E\gamma$ is the gamma-ray energy in MeV. For the purpose of theoretical comparison, it is useful to define the mixing ratio $\Delta\left(=\frac{\delta}{0.835\ E\gamma}\right)$ as

$$\Delta = \frac{\langle I||m(E2)||\ I_i\rangle}{\langle I||m(M1)||\ I_i\rangle} \qquad (9.5)$$

Where Δ is given in units of eb/μ_N

9.3 COMPILATION AND ANALYSIS OF THE ANGULAR CORRELATION COEFFICIENTS

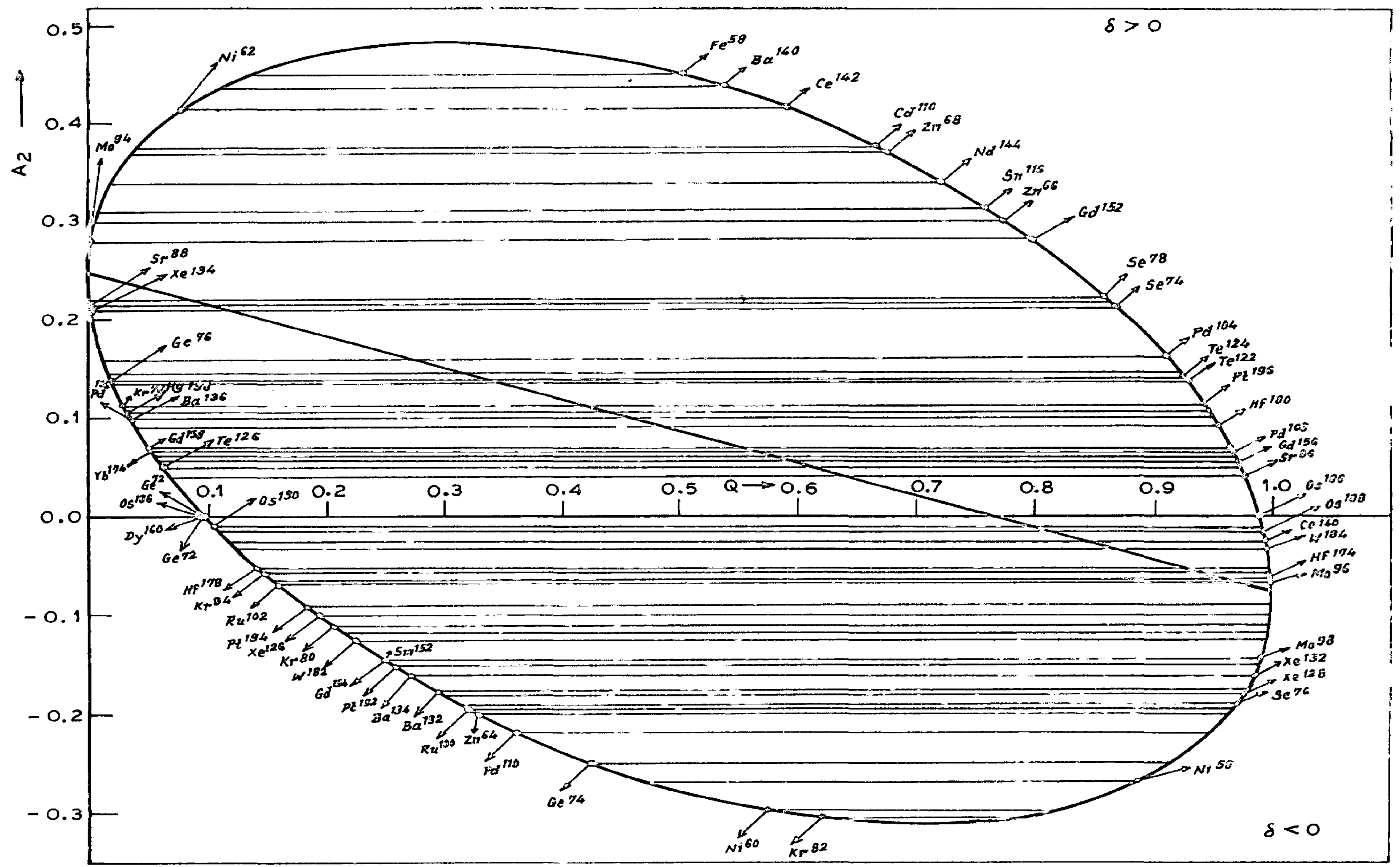
The angular correlation coefficients (A_2 and A_4) for the cascade $2^{+'} \rightarrow 2^+ \rightarrow 0^+$ from almost all the radioactive disintegrations as reported by various authors are collected and are given in table I. The details of the cascade i.e. energies of γ_1 and γ_2 alongwith the references are also given. The analysis of the angular correlation data has been done by two methods which are as follows.

(i) THE ANALYSIS OF THE MULTIPOLE MIXING RATIOS BY THE METHOD OF ARNS AND WIEDENBECK

Arns and Wiedenbeck (52) have suggested the method to determine the magnitude and phase of the multipole mixing ratio if one transition of gamma-gamma cascade is mixed. The theoretical plot of A_2 versus Q $(= \frac{\delta^2}{1+\delta^2})$ is done and an ellipse is obtained. The multipole mixing ratio 'δ' is determined if the experimental value with the error cuts the curve. Such type of the plot is given in figure 9.1. The values of mixing ratios as obtained by this analysis are given in table II.

(ii) ANALYSIS OF THE MULTIPOLE MIXING RATIO BY THE GRAPHICAL METHOD OF COLEMAN

Coleman (79) has introduced a method for evaluating the multipole mixing ratio 'δ' from gamma-gamma directional correlation experiment if only one transition is mixed by using both A_2



Theoretical plot of A_2 versus Q [$\equiv \delta^2/(1+\delta^2)$, where '$\delta$' is the mixing ratio in ($2'^+ \rightarrow 2^+$) γ-transition and other ($2^+ \rightarrow 0^+$) transition is pure E2] for the spin sequence $2'^+ \rightarrow 2^+ \rightarrow 0^+$. This is done by the method of Arns and Wiedendeck (52).

FIG.9.1

TABLE-I

Angular correlation coefficients for $2'^{+} \rightarrow 2^{+} \rightarrow 0^{+}$ cascade

Nucleus	E_1 (keV)	E_2 (keV)	$\hbar w$ (keV)	Expt. A_2	Expt. A_4	Reference
$_{26}Fe^{58}_{32}$	1675	810	865	0.45±0.04	0.08±0.09	R.A.Fox etal,P.R.C5, 853 (1972).
$_{28}Ni^{58}_{30}$	2775	1454	1321	-0.27±0.03	0.16±0.04	D.F.H.Start etal.,N.P. A162,49 (1971).
$_{28}Ni^{60}_{32}$	2156	1330	826	-0.30±0.03	0.15±0.05	S.M.Shaforth etal.,P.R. 149,827 (1966).
$_{28}Ni^{62}_{34}$	2301	1173	1128	0.415±0.053	0.082±0.131	D.M.Van Patter etal., N.P.A178,355 (1972).
$_{30}Zn^{64}_{34}$	1800	992	810	-0.200±0.022	-0.020±0.032	A.K.Sen Gupta etal., P.L.3,355 (1963).
$_{30}Zn^{66}_{36}$	1872	1039	833	0.30±0.05	0.21±0.07	T.Hayashi etal., J.P.S. (J)27,1375 (1969).
$_{30}Zn^{68}_{38}$	1883	1077	806	0.369±0.025	0.234±0.034	J.Lange etal.,P.R.C7, 177 (1973).
$_{32}Ge^{72}_{40}$	1464	834	630	-0.002±0.009	0.311±0.012	H.Chen etal.,N.P.A219, 365 (1974).
$_{32}Ge^{74}_{42}$	1204	596	609	-0.251±0.015	0.27±0.04	M.C.Cambiaggio etal., Z.P.A275,183 (1975).
$_{32}Ge^{76}_{44}$	1108	563	545	0.14±0.04	-	K.C.Chung etal., P.R. C2,139 (1970).
$_{34}Se^{74}_{40}$	1270	635	635	0.213±0.022	0.272±0.044	M.C.Cambiaggio etal., Z.P.A275, 183 (1975).
$_{34}Se^{76}_{42}$	1216	559	657	-0.185±0.012	0.305±0.021	T.Nagahara etal.,J.P.S (J)34, 579 (1973).
$_{34}Se^{78}_{44}$	1306	613	693	0.22±0.03	0.03±0.04	R.M.Lieder etal., P.R. C2, 531 (1970).
$_{36}Kr^{78}_{42}$	1119	455	664	0.11±0.04	0.03±0.07	N.E.Andernson etal., J.P.A(GB)7,1156(1974).
$_{36}Kr^{80}_{44}$	1256	616	639	-0.12±0.04	0.21±0.07	T.Hayashi etal.,J.P.S (J)27,1375 (1969).
$_{36}Kr^{82}_{46}$	1475	777	698	-0.307±0.015	0.239±0.010	J.Kotch etal,N.P.103, 300 (1967).
$_{36}Kr^{84}_{48}$	1897	881	1016	-0.056±0.050	0.426±0.089	J.P.Roalsvig etal., P.R.138,B1378 (1965).

Nucleus	E_1 (keV)	E_2 (keV)	$\hbar w$ (keV)	$A_2^{Expt.}$	$A_4^{Expt.}$	Reference
${}_{38}Sr^{86}_{48}$	1854	1077	777	0.040±0.023	0.017±0.037	R.G.Arns etal,N.P. A148,625 (1970).
${}_{38}Sr^{88}_{50}$	3223	1836	1387	0.217±0.014	-0.004±0.016	Y.Kawase etal.,N.P. A154, 127 (1970).
${}_{42}Mo^{94}_{52}$	1864	871	993	0.28±0.04	0.22±0.011	N.K.Aras etal,N.P.A112, 609 (1968).
${}_{42}Mo^{96}_{54}$	1498	778	720	-0,072±0.020	0.056±0.036	D.Heck etal, N.P. A159, 49 (1970).
${}_{42}Mo^{98}_{56}$	1432	787	645	-0.147±0.020	0.060±0.035	D.Heck etal,N.P. A165,327 (1971).
${}_{44}Ru^{100}_{56}$	1360	540	820	-0.196±0.024	0.324±0.030	H.Kawakanic etal, J.P.S.(J)24,614 (1968).
${}_{44}Ru^{102}_{58}$	1103	475	628	-0.069±0.017	0.347±0.033	B.Singh etal,N.P.A155, 90 (1970).
${}_{46}Pd^{104}_{58}$	1323	556	767	0.16±0.06	0.02±0.04	N.C.Singhal etal.,P.R. C5, 948 (1972).
${}_{46}Pd^{106}_{60}$	1125	513	612	0.100±0.040	0.310±0.007	J.Kotch etal, N.P.103, 300 (1967).
${}_{46}Pd^{108}_{62}$	1441	434	1007	0.069±0.024	0.055±0.033	K.Okano etal, N.P.A164, 545 (1971).
${}_{46}Pd^{110}_{64}$	810	374	436	-0.220±0.037	-	R.L.Robinson etal, N.P.A166, 141 (1971).
${}_{48}Cd^{110}_{62}$	1476	658	818	0.375±0.050	0.180±0.050	K.S.Krane etal,P.R.C2, 724 (1970).
${}_{50}Sn^{116}_{66}$	2109	1293	819	0.31±0.02	0.27±0.05	G.Garncia Barmudiz etal, P.R.C9,1060 (1974).
${}_{52}Te^{122}_{70}$	1250	564	686	0.142±0.006	0.298±0.005	J.Kotch etal, N.P.103, 300 (1967).
${}_{52}Te^{124}_{72}$	1325	603	723	0.136±0.009	0.270±0.015	K.R.Baker etal, N.P. A189, 493 (1972).
${}_{52}Te^{126}_{74}$	1421	667	754	0.052±0.028	0.291±0.035	Z.W.Grabowski etal, P.R.C3,1649 (1971).
${}_{54}Xe^{126}_{72}$	880	389	491	-0.110±0.012	0.341±0.030	Z.W.Grabowski etal, P.R.C3,1649 (1971).
${}_{54}Xe^{128}_{74}$	955	540	415	-0.18±0.01	0.36±0.02	T.Hayashi etal, N.I.M 53,123 (1967).
${}_{54}Xe^{132}_{78}$	1297	667	630	-0.162±0.047	0.363±0.092	H.W.Taylor etal,C.J.P. 49,2724 (1971).
${}_{54}Xe^{134}_{80}$	1613	847	766	0.220±0.043	0.062±0.065	J.M.Gualda etal,N.P. A234, 357 (1974).

TABLE-II

E2/M1 multipole mixing ratios from the quoted angular correlation coefficients.

Nucleus	Mean value of δ^a	Mean value of δ^b				Mean value of δ taken	Value of δ^c from the literature	Δ (eb/μ_N)
		Q_1	δ_1	Q_2	δ_2			
$_{26}Fe^{58}_{32}$	+0.26	0.13	+0.39	0.5	+1.0	+1.0	$+(1.0^{+0.60}_{-0.20})$	+1.385
$_{28}Ni^{58}_{30}$	-1.0	0.465	-0.93	0.885	-2.77	-1.0	-1.1 ± 0.2	-0.91
$_{28}Ni^{60}_{32}$	-1.0	0.575	-1.16	0.82	-2.13	-1.0	$-1.3\leq\delta\leq-0.9$	-1.45
$_{28}Ni^{62}_{34}$	+0.26	0.07	+0.27	0.59	+1.20	+0.27	Not deduced	+0.29
$_{30}Zn^{64}_{34}$	-0.66	0.325	-0.69	0.965	-5.25	-0.69	Not deduced	-1.02
$_{30}Zn^{66}_{36}$	+0.37	0.05	+0.07	0.775	+1.85	+1.85	2.0	+2.66
$_{30}Zn^{68}_{38}$	+0.37	0.035	+0.19	0.675	+1.44	+1.44	$+1.45\pm0.14$	+2.14
$_{32}Ge^{72}_{40}$	+0.47	0.1	-0.33	0.99	+9.95	+9.95	10.3 ± 1.3	+18.92
$_{32}Ge^{74}_{42}$	-2.33	0.425	-0.86	0.82	-2.13	-2.13	-3.4 ± 0.4	-4.19
$_{32}Ge^{76}_{44}$	-0.11	0.02	-0.14	0.925	+3.51	-0.14	-0.1 ± 0.1	-0.31
$_{34}Se^{74}_{40}$	+0.41	0.003	-0.05	0.865	+2.53	+2.53	$+2.6\pm0.2$	+4.77
$_{34}Se^{76}_{42}$	-5.67	0.315	-0.68	0.97	-5.69	-5.69	Not deduced	-10.38
$_{34}Se^{78}_{44}$	-0.05	0.003	-0.05	0.860	+2.48	-0.05	Not deduced	-0.08
$_{36}Kr^{78}_{42}$	-0.17	0.03	-0.17	0.945	+4.14	-0.17	Not deduced	-0.31
$_{36}Kr^{80}_{44}$	-12.33	0.21	-0.51	0.995	-14.11	-14.11	Not deduced	-26.47
$_{36}Kr^{82}_{46}$	-1.86	0.62	-1.28	0.77	-1.83	-1.83	-1.65 ± 0.15	-3.14
$_{36}Kr^{84}_{48}$	+0.49	0.145	-0.41	0.995	+14.11	+14.11	Not deduced	+16.64
$_{38}Sr^{86}_{48}$	-0.25	0.07	-0.27	0.98	+7.0	-0.27	$0.05\leq Q\leq0.09$	-0.42
$_{38}Sr^{88}_{50}$	-0.05	0.003	-0.05	0.86	+2.48	-0.05	-0.04 ± 0.02	-0.04
$_{42}Mo^{94}_{52}$	+0.39	0.003	+0.05	0.8	+2.0	+2.0	2	+2.41
$_{42}Mo^{96}_{54}$	-0.43	0.16	-0.44	0.997	+18.23	-0.44	$-0.44^{+0.04}_{-0.03}$	-0.73
$_{42}Mo^{98}_{56}$	-0.54	0.25	-0.58	0.97	-5.69	-0.58	-0.58 ± 0.05	-1.08

* Mean value of δ

(a) From the plot of A_2 versus A_4

(b) From the plot of A_2 versus Q

(c) Sign convention of Biedenharn and Rose (6)

Nucleus	Mean value of δ^a	Mean value of δ^b				Mean value of δ taken	Value of δ^c from the literature	Δ (eb/μ_N)
		Q_1	δ_1	Q_2	δ_2			
$_{44}Ru^{100}_{56}$	-5.67	0.32	-0.69	0.97	-5.69	-5.69	$-6^{+2.5}_{-1.5}$	-8.31
$_{44}Ru^{102}_{58}$	∞	0.16	-0.44	0.997	+18.23	+18.23	60 ± 20	+34.79
$_{46}Pd^{104}_{58}$	-0.11	0.01	-0.10	0.91	+3.18	-0.10	Not deduced	-0.16
$_{46}Pd^{106}_{60}$	+0.44	0.035	-0.19	0.945	+4.14	+4.14	$+4.35\pm0.80$	+8.10
$_{46}Pd^{108}_{62}$	-0.25	0.05	-0.23	0.97	+5.69	-0.23	-0.24 ± 0.04	-0.27
$_{46}Pd^{110}_{64}$		0.36	-0.75	0.95	-4.36	-0.75	-0.77 ± 0.11	-2.04
$_{48}Cd^{110}_{62}$	+0.35	0.04	+0.20	0.665	+1.41	+1.41	$+1.20\pm0.15$	+2.06
$_{50}Sn^{116}_{66}$	+0.39	0.01	+0.10	0.755	+1.75	+1.75	$+1.8\pm0.2$	+2.56
$_{52}Te^{122}_{70}$	+0.44	0.02	-0.14	0.925	+3.51	+3.51	$+3.52\pm0.10$	+6.12
$_{52}Te^{124}_{72}$	+0.43	0.02	-0.14	0.93	+3.64	+3.64	$+3.3\pm0.2$	+6.02
$_{52}Te^{126}_{74}$	+0.46	0.065	-0.26	0.975	+6.24	+6.24	$+5.5^{+0.4}_{-0.3}$	+9.90
$_{54}Xe^{126}_{72}$	-19.0	0.205	-0.50	0.997	-18.23	-18.23	-27^{+30}_{-9}	-44.46
$_{54}Xe^{128}_{74}$	-5.67	0.30	-0.65	0.975	-6.24	-6.24	Not deduced	-18.03
$_{54}Xe^{132}_{78}$	-9.0	0.275	-0.62	0.99	-9.95	-9.95	-18^{+50}_{-9}	-18.91
$_{54}Xe^{134}_{80}$	-0.05	0.003	-0.05	0.86	+2.48	-0.05	-2.4 ± 02	-0.08
$_{56}Ba^{132}_{76}$	-5.67	0.30	-0.65	0.975	-6.24	-6.24	-9^{+7}_{-8}	-13.19
$_{56}Ba^{134}_{78}$	-7.00	0.275	-0.61	0.99	-9.95	-9.95	$\delta\leq-30$	-21.17
$_{56}Ba^{136}_{80}$	-0.17	0.035	-0.19	0.95	+4.36	-0.19	$-0.21\leq\delta\leq-0.19$	-0.18
$_{56}Ba^{140}_{84}$	+0.31	0.105	+0.34	0.54	+1.08	+0.34	$+1.1^{+0.14}_{-0.10}$	+0.45
$_{58}Ce^{140}_{82}$	-0.43	0.125	-0.38	0.995	+14.11	-0.38	$-0.3^{+0.01}_{-0.02}$	-0.61
$_{58}Ce^{142}_{84}$	+0.26	0.075	+0.28	0.59	+1.20	+0.28	$+0.61\pm0.018$	+0.37
$_{60}Nd^{144}_{84}$	+0.37	0.015	+0.12	0.72	+1.60	+1.60	$+1.6\pm0.5$	+2.22
$_{62}Sm^{152}_{90}$	-9.0	0.275	-0.62	0.99	-9.95	-9.95	-8^{+9}_{-3}	-17.33

Nucleus	Mean value of δ^a	Mean value of δ^b				Mean value of δ taken	Value of δ^c from the literature	Δ (eb/μ_N)
		Q_1	δ_1	Q_2	δ_2			
${}_{64}Gd^{152}_{88}$	+0.39	0.003	+0.05	0.8	+2.0	+2.0	2.0 ± 0.5	+4.09
${}_{64}Gd^{154}_{90}$	-9.0	0.25	-0.58	0.99	-9.95	-9.95	-11 ± 3	-17.18
${}_{64}Gd^{156}_{92}$	+0.46	0.06	-0.26	0.975	+6.24	+6.24	$5.9^{+1.4}_{-2.8}$	+7.19
${}_{64}Gd^{158}_{94}$	-0.17	0.05	-0.23	0.97	+5.69	-0.23	Not deduced	-0.29
${}_{66}Dy^{160}_{94}$	+0.47	0.1	-0.33	0.99	+9.95	+9.95	$+(9.7^{+2.5}_{-1.7})$	+13.56
${}_{70}Yb^{174}_{104}$	-0.25	0.06	-0.25	0.965	+5.25	-0.25	-0.26 ± 0.04	-0.24
${}_{72}Hf^{174}_{102}$	-0.48	0.155	-0.43	0.997	+18.23	-0.43	$0\geq\delta^{-1}\leq-0.24$	-0.64
${}_{72}Hf^{178}_{106}$	-0.43	0.145	-0.41	0.997	+18.23	-0.41	$0.410^{+0.036}_{-0.035}$	-0.42
${}_{72}Hf^{180}_{108}$	-0.17	0.035	-0.19	0.955	+4.61	-0.19	-0.20	-1.06
${}_{74}W^{182}_{108}$	-9.0	0.275	-0.54	0.995	-14.11	-14.11	-12^{+2}_{-1}	-15.07
${}_{74}W^{184}_{110}$	+0.49	0.12	-0.37	0.995	+14.11	+14.11	+15	+21.31
${}_{76}Os^{186}_{110}$	+0.47	0.1	-0.33	0.99	+9.95	+9.95	$+10^{+15}_{-4}$	+18.92
${}_{76}Os^{188}_{112}$	+0.47	0.105	-0.34	0.995	+14.11	+14.11	$+12.3\pm2.8$	+35.36
${}_{76}Os^{190}_{114}$	+0.47	0.105	-0.34	0.995	+14.11	+14.11	$+12^{+3}_{-5}$	+45.52
${}_{78}Pt^{192}_{114}$	-9.0	0.26	-0.59	0.99	-9.95	-9.95	$+8.8\pm0.3$	-40.28
${}_{78}Pt^{194}_{116}$	-39	0.18	-0.47	0.995	-14.11	-14.11	-45^{+55}_{-19}	-57.59
${}_{78}Pt^{196}_{118}$	+0.44	0.02	-0.16	0.98	+7.00	+7.00	$+4.3\pm0.2$	+25.18
${}_{80}Hg^{198}_{118}$	-0.17	0.035	-0.19	0.95	+4.36	-0.19	-0.19	-0.36
${}_{80}Hg^{200}_{120}$	-0.17	0.035	-0.19	0.95	+4.36	-0.19	-0.19 ± 0.01	-0.39

and A_4 values. One plots the curve between A_2 versus A_4 for a $2'^{+} \rightarrow 2^{+} \rightarrow 0^{+}$ cascade as shown in fig.9.2. This curve is labelled with the values of 'δ' (or more conveniently with the values of the parameter $\delta' = |\frac{\delta}{1-\delta}|$) The measured experimental values with the errors for both A_2 and A_4 are depicted in the plot which gives the best fit value of 'δ' with phases, as shown in fig.9.2. The values thus obtained are given in table II.

9.4 MULTIPOLE MIXING RATIOS ON THE BASIS OF GREINER'S MODEL

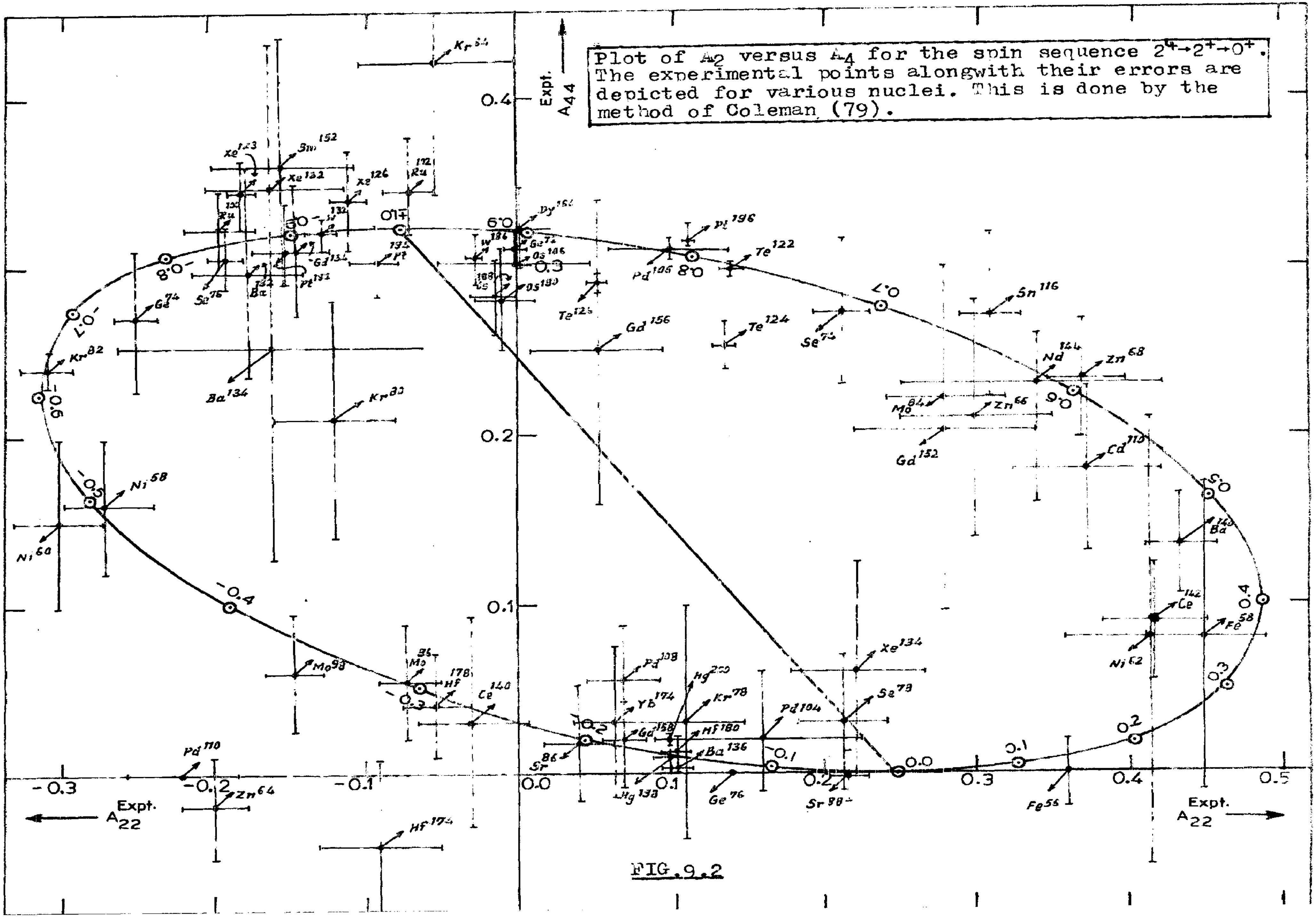
Walter Greiner (74) has reported a model to calculate the g-factors and E2/M1 multipole mixing ratios in vibrational nuclei. The basic idea of the proposed model is the assumption that the proton distribution in nuclei is less deformed than the neutron distribution because the pairing force acting between protons is larger than acting between the neutrons. Let G_p and G_n be the pairing forces for protons and neutrons respectively. Nilsson and Prior (80) have found the values for $G_p = \frac{25}{A}$ MeV and $G_n = \frac{18}{A}$ MeV while Marschalek and Rasmussen (81) use $G_p = \frac{30}{A}$ MeV and $G_n = \frac{20}{A}$ MeV.

The average deformation β_0 of the mass distribution is defined as

$$\beta_0 = \frac{N\beta_0(n) + Z\beta_0(p)}{A} \qquad (9.6)$$

Where $\beta_0(n)$ and $\beta_0(p)$ are the neutron and proton deformations.

Greiner (74) has given the expression for the calculation of Δ ($\simeq \delta/E_\gamma$) and is written as



Plot of A_2 versus A_4 for the spin sequence $2^+ \rightarrow 2^+ \rightarrow 0^+$. The experimental points alongwith their errors are depicted for various nuclei. This is done by the method of Coleman (79).

FIG.9.2

$$\Delta = \frac{(1.1 \times 10^{-3}) A^{5/3} \beta_0}{f(1-2f)} \quad (9.7)$$

Where f gives the difference in proton and neutron deformations

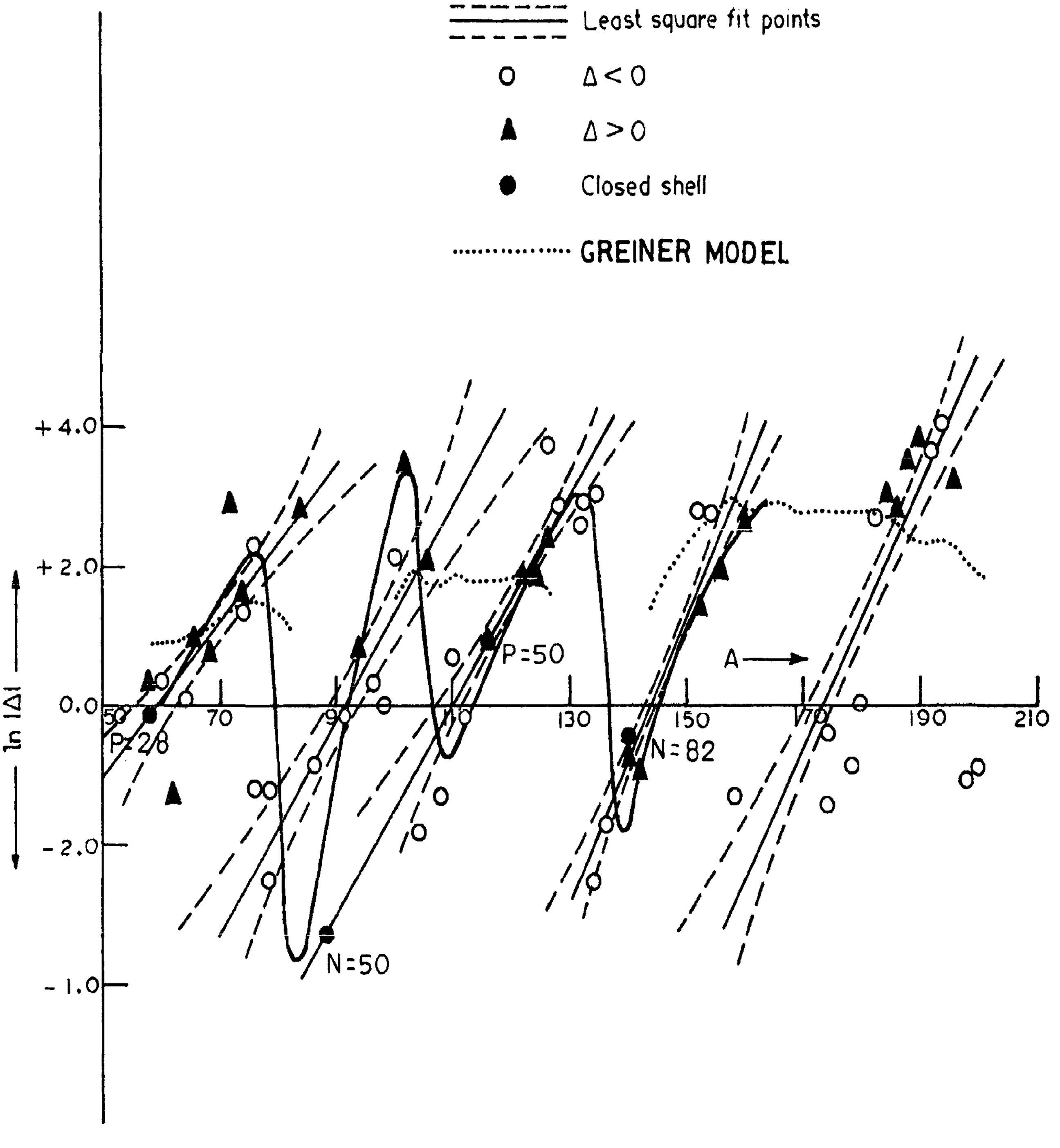
$$f = \frac{N}{A} \left[\frac{\beta_0(n)}{\beta_0(p)} - 1 \right] \quad (9.8)$$

The parameter f is calculated from equation (9.8). The root mean square deformation β_0 for the first excited state is given by Greiner (82). Putting the values of these parameters in equation (9.7), the value of multipole mixing ratio $\Delta (\equiv \delta/E_\gamma)$ is obtained. The mixing ratios for a number of nuclei have been calculated and it is found that the sign of Δ is always positive.

9.5 THE MAGNITUDE OF 'δ'

The plot of Δ $(\equiv \delta/E_\gamma)$ versus mass number A is given in figure 9.3. The calculated values of Δ on the basis of Greiner's model (74), are also given in the figure 9.3 and are denoted by dotted line. The plot as done by Krane (73) is also given which accordingly to him indicates the trend of the measured values and shows a pronounced minima in the vicinity of the closed shells.

In the present analysis, it is indicated that we can divide the nuclei in five groups by drawing straight lines as given in figure 9.3. These straight lines are obtained by the method of least square fit for values for nuclei in the groups. The error is calculated by the method given in Daniel and Wood (83) and indicated by the broken lines. Almost all the values of Δ lie within the groups. In the first four groups, the values of Δ approaches to zero when



Plot of ln|Δ| [≡ δ/Eγ, where Eγ is the gamma-ray energy] versus mass number (A). The solid curve indicates the trend of measured values as given by Krane (73) whereas calculations through Greiner's model (74) are represented by dotts. The straight lines obtained by the method of least square fit in order to divide in five groups are depicted by the solid lines and their errors by the broken lines.

FIG. 9.3

the neutron numbers or proton numbers are at or around shell closure, except in the fifth group , indicating as if there is one more shell type structure. The slopes of these groups with the errors are as follows:

$$\frac{\ln|\Delta|}{A} = \underset{\text{group I}}{0.1154 \pm 0.0262},\ \underset{\text{group II}}{0.1554 \pm 0.0403},\ \underset{\text{group III}}{0.1505 \pm 0.0212},\ \underset{\text{group IV}}{0.2070 \pm 0.0244}$$

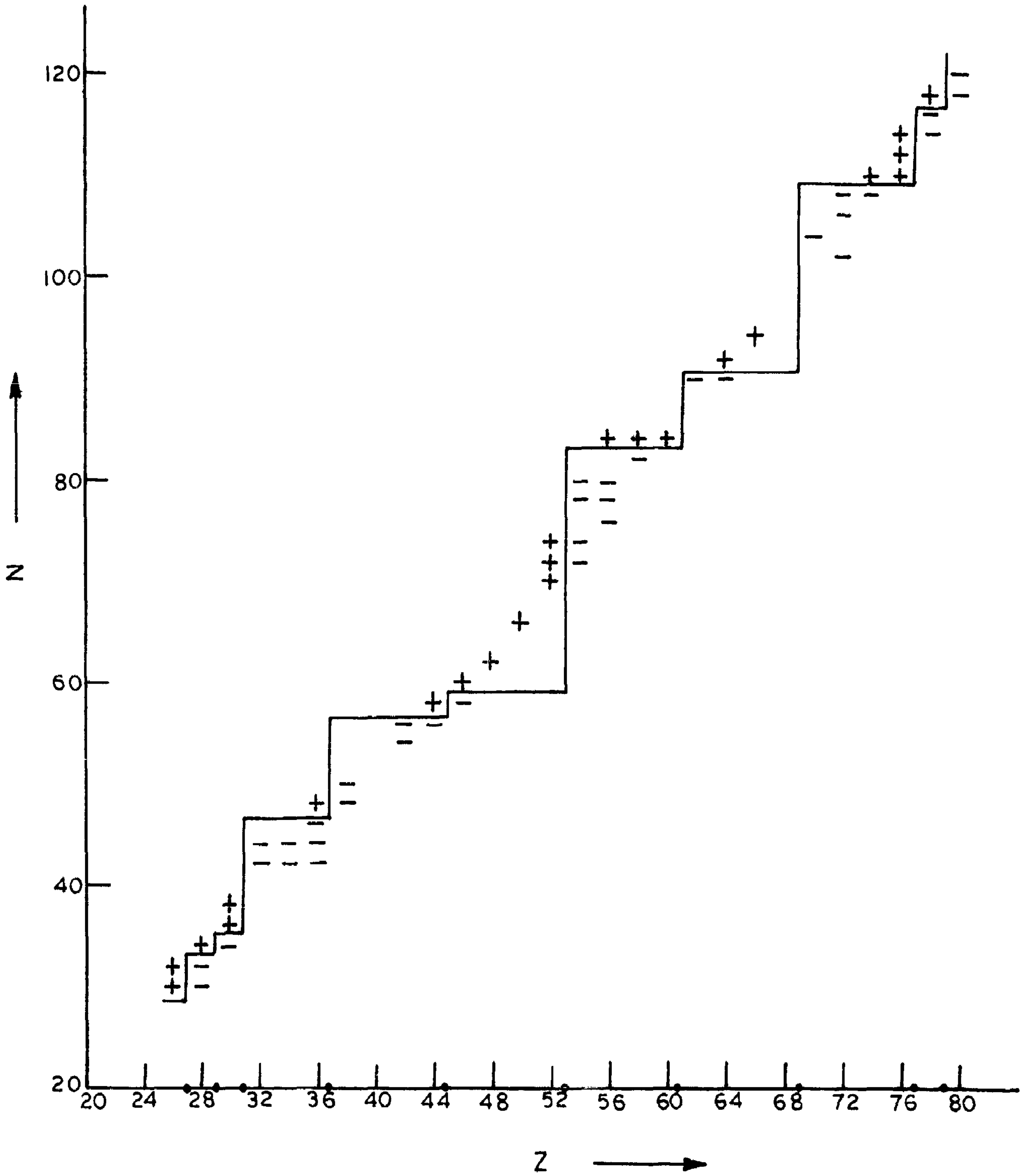
$$\frac{\ln|\Delta|}{A} = \underset{\text{group V}}{0.1877 \pm 0.0330}$$

9.6 THE PHASES OF 'δ'

The phases of 'δ' are as given in the table II and are also plotted (figure 9.4). The X-and Y-axes are taken Z(atomic number) and N(neutron number) respectively. The phase (+) or (-) is given for almost all nuclei expect four cases (Ge^{72}, Se^{74}, Mo^{94} and Gd^{152}). The plot clearly indicates a certain system. The line as given in figure 9.4, separates positive and negative phases. For every eight protons, we find a change as indicated in the figure. These proton numbers are at 37, 45, 53, 61, 69 and 77. There is a change at 29 but again at 39 as well. Below 29 and above 77, there is no such type of structure.

9.7 ENERGY OF $2'^{+}$ EXCITED STATES

In all these nuclei, the first excited state is 2^{+} and second excited is not $2'^{+}$ but this lies above 4^{+} or 6^{+} or 8^{+} or above 0^{+}. 0^{+} may lie between 2^{+} and $2'^{+}$, or 4^{+} and $2'^{+}$ or between 2^{+} and 4^{+}. All these cases can be divided in five main



The phases of 'δ' are given for various nuclei by plotting 'Z' (atomic number) along X-axis and 'N' (Neutron number) along Y-axis.

FIG.9.4

groups as given in figure 9.5. In seven cases which are as follows: Sr^{88}, Ru^{100}, Ce^{140}, Sm^{152}, Gd^{152}, Gd^{154} and Hf^{174}, the decay schemes of the isotopes do not lie in the five groups but are different. The following table gives the number of cases in various categories alongwith (+) and (–) phases and large and small values of 'δ'.

Group		A	B	C	D	E
Number of cases,		33	11	6	2	4
Number of cases, having phases	+	12	7	2	2	3
	–	21	4	5	–	1
Number of cases, large values with phases	+	10	7	2	–	2
	–	11	–	1	–	1
small values with phases	+	2	–	–	–	1
	–	10	4	4	–	–

The almost 50 percent cases are in A group where second excited state is 2^{+}. This classification may be considered on the basis of β-and γ- vibrational bands and may also help in microscopic calculations for the nuclear structure studies.

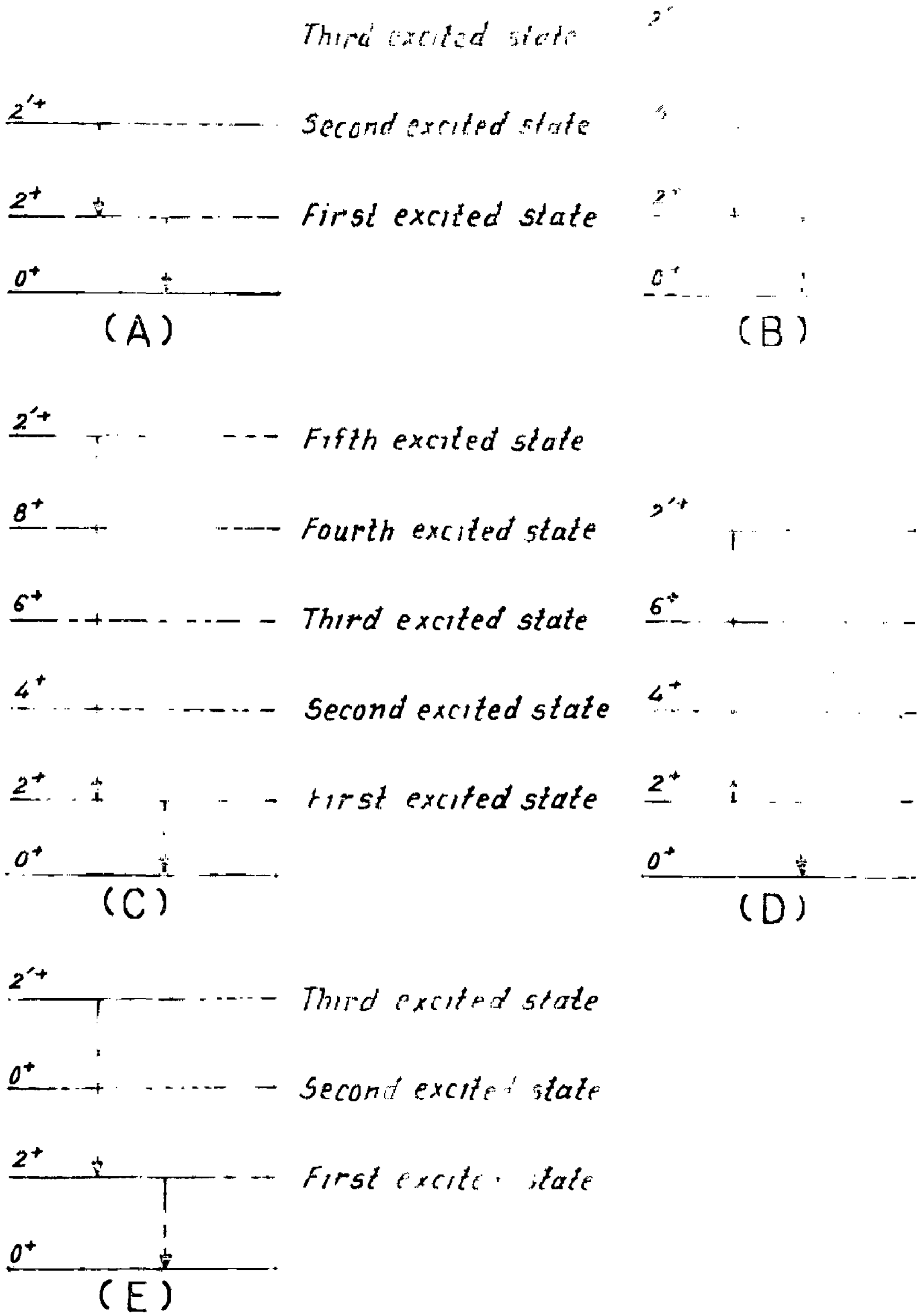


FIG.9.5_ LOWEST EXCITED STATES OF EVEN-EVEN NUCLEI

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