STUDY OF

INTERCONNECTED BRIDGE GIRDERS

CHANDRA SINGH SURANA

CHECKED
1495

UNIVERSITY of ROORKEE CENTRAL LIBRARY
Library Acc. No: 62,460
Date 22.11.62
ROORKEE

M. E. THESIS

DEPARTMENT OF CIVIL ENGINEERING
UNIVERSITY OF ROORKEE
ROORKEE

1962

D416
K2..
DE

C E R T I F I C A T E

Certified that the dissertation entitled " Study of Interconnected Bridge Girders" which is being submitted by Sri Chandra Singh Surana in partial fulfilment for the award of the Degree of Master of Engineering in Structural Engineering including Concrete Technology of University of Roorkee is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other Degree or Diploma.

It is further to certify that he has worked for a period of one year from July 1961 to July 1962 for preparing dissertation for Master of Engineering Degree at the University.

A. Arya

(A. S. Arya)
Reader in Civil Engineering
University of Roorkee,
Roorkee.

Roorkee.
July 27 1962.

A C K N O W L E D G E M E N T

The work of this thesis has been carried out in the Department of Civil Engineering at the University of Roorkee for the award of M.E.Degree under the able and esteemed guidance of Dr.A.S.Arya to whom the author is highly grateful. He also expresses his deep gratitude to Prof. Jai Krishna and Prof. O.P. Jain for their valuable consultations.

Sincere thanks are extended to Workshop and laboratory staff of Civil Engineering Department for their cooperation during th testing programme.

The author also thanks all others who have in one way or the other helped him or encouraged him in his work.

SYNOPSIS

This thesis presents a study of load distribution of concentrated loads to the various longitudinal girders of interconnected girder bridge. The method of analysis of Hendry-Jaeger, which has been adopted here as the basis of theoretical computations, has been discussed in detail. Based on this method, influence lines for fraction of unit load, carried by various longitudinal girders of the bridge, have been drawn for the range of bridge parameters usually occuring in practice. Using these influence lines, approximate formulae have been derived for calculating the distribution of moments in various girders for two lane I.R.C. class A and class B loadings. These formulae could be adopted for ready computation of design moments in girders of such bridges.

To verify the applicability of Hendry-Jaeger method and also to investigate if the lateral load distribution at working loads will hold at ultimate loads also, a one-fifth scale reinforced concrete model of a 60 ft. span two-lane bridge was tested to destruction. On the basis of the test it may be concluded that the experimental results are in close conformity with the theoretical ones. Also, the distribution of moment to various girders was found to be almost constant at all loads.

CONTENTS

TABLES

Page

F I G U R E S

Page

Page

I. INTRODUCTION

1. Introduction

The wide use of interconnected bridges together with an increasing awareness of their inherent complexity and the introduction of heavier loads has emphasised the need for a better understanding of the way in which they function. Of particular interest is the determination of fraction of free bending moment carried by the various longitudinals due to a train of wheel loads.

During recent years, several methods have been advanced but the basic assumptions, on which these various methods are based, are not identical and consequently the results differ and one does not know which method givesmore appropriate results.

Designs based without considering distribution of load due to inter-connection or as provided in the I.R.C. Bridge Code[1] are likely to give values of free bending moment carried by different longitudinals considerably different from the actual ones which has necessiated the study of the subject.

2. Object and Scope

The objectives of this thesis are:

(i) To obtain influence lines for moments in longitudinal beams when a unit load is placed at various positions on a transverse section of the bridge for the usual range of bridge parameters.

(ii) To determine design moments in the longitudinal beams due to two lanes of I.R.C. class A loading.

---

* Numbers in the parentheses refer tothe list of referencesat the end.

(iii) To suggest simplified design formulae for obtaining these design moments.

(iv) To verify experimentally, the theoretical distribution of bending moments in an inter-connected bridge.

The study has been restricted to simple span right bridges consisting of three and four longitudinals, spaced equidistantly and having both flexural and torsional stiffnesses.

3. Notation

The letter symbols are defined in the text where they first appear and principal letter symbols are listed here for convenient reference.

A	Amplitude of the first harmonic component of free deflection of the inner beam carrying full load.
a	Distance from the support to the point of action of concentrated load.
a_1, a_2, a_3	Amplitudes of harmonics of beam deflection curves.
C	Shearing modulus of elasticity of concrete.
E, E_T	Young's modulus of elasticity of concrete.
h	Transverse spacing of longitudinal beams.
I, I_1	Flexural moment of inertia of the cross-section of the longitudinal beams.
I_T	Total flexural moment of inertia of transverse medium about longitudinal axis.
J	Torsional stiffness factor.
L	Span length of the bridge or longitudinal beam.
M	Free bending moment on the bridge or longitudinal beam, at a section 'x' from the support.
m	Total number of longitudinal beams.
n	An integer, used for designating the number of fourier term.
r	Fraction of continuous beam reaction for the longitudinal beam under consideration due to four equal loads along the cross section.
W	Concentrated load.
x	Distance of the section from the support where bending moment is required.
α	Dimensionless parameter of the bridge $= \frac{12}{\pi^4}\left(\frac{L}{h}\right)^3 \frac{E_T I_T}{E I}$
β	Dimensionless parameter of the bridge $= \frac{\pi^2}{2}\left(\frac{h}{L}\right) \frac{C J}{E_T I_T}$
η	Dimensionless parameter of the bridge $= \frac{I_1}{I}$
μ	Poisson's ratio for concrete

II. METHOD OF ANALYSIS

4. Existing Methods of Analysing Gridworks

The theories, which exist for the analysis of the elastic behaviour of a system of interconnected girders, with a decking slab on the top, can be put broadly in four groups.

The first group includes the theories which treat the structure in its actual form, relating the behaviour of each member to the whole structure through a set of redundancy equations. The equations appear as linear simultaneous equations and will be large in number for a structure of any degree of complexity. Lazarides(2) suggests method of relaxation for solving these equations. Ewell, Okubo and Abrams(3) use the grid system as such in their method. Using an auxiliary force system of control for vertical deflections at joints, a moment and torque distribution process is evolved for transmission of the deflection effects.

In second group of theories, we may include those methods which reduce the actual system of discrete interconnected girders to an elastically equivalent system, uniformly distributed in both directions. Guyon(4) in his method transforms the system to an equivalent anisotropic plate. Distribution coefficients are developed as the ratio of the actual bending moment of a longitudinal section of the bridge under some loading to the bending moment of the bridge with the loading system distributed uniformly across the bridge width. The torsional stiffness of the members is assumed zero.

---

* Number in parentheses refers to the list of references at the end.

Massonet[5] has extended this method further by including the effect of torsional rigidity.

The third group of theories replace the actual transverse connection between the main girders by a simpler system of equivalent stiffness. The structural behaviour is then described by a set of simultaneous differential equations in terms of the deflections of the longitudinal beams. Pippard and de Waele[6] assumed uniformly distributed transverse stiffness as the simplification. In this method, it is further assumed that there is no rotation of the longitudinals. The solution is in the form of simultaneous fourth order linear differential equations. There are as many such equations as there are longitudinals. The arithmetical work involved is excessive in any practical problem. Leonhardt[7] replaces all the cross girders by one central cross girder of equivalent stiffness as the simplification. The effect of torsional stiffness is neglected in his method.

In the fourth group we have those methods in which the solution is obtained in series form. Usually the sine series is used to represent loading and deflection of the structure. The applied loading is resolved into sine components,each of which can be handled separately in a simple manner. The effect of the total load is found by superposition of the effect of the component loading. Based on this Hetenyi[8] has developed a method assuming no rotation of individual members at the intersections. Newmark[9] takes into account the torsional rigidity and applies the method of moment distribution to solve for individual sine components. In Hendry-Jaeger[10] method the simultaneous differential equations are solved by harmonic analysis and the amplitudes of the harmonics of bending moment curve are found for various girders.

The computations in the thesis are based upon Hendry-Jaeger method for the following reasons.

1) In a theoretical study made by Gupta(11) it was found that the results obtained by this method are in close agreement with those arrived at by moment and torque distribution method which is by far the most accurate method.

2) The validity of this method is verified experimentally by a one-fifth scale model of a 60 ft. bridge tested upto failure.

3) The derivation is comparatively simple and the results can be obtained in close algebraic form so that these may be employed directly by the designer without following the details of derivation.

4) The method has a wide field of application and has been applied to single-span bridges and continuous bridges, portal frames, skew spans, slab and a variety of other structures. Therefore, for an extension of the present work to include other structures, the same method of analysis could be used.

6. <u>Hendry-Jaeger Method</u>

Besides the usual assumptions of simple theory of bending the following two assumptions are made.

(i) The transverse medium is replaced by a continuous medium of the same total transverse moment of inertia.

(ii) The torsional stiffness of the longitudinal beams is considered but that of the transverse medium is neglected.

The first assumption is in accordance with actual fact in a beam and slab bridge. Torsion of the transverse members can be taken into account in this method[10] but its effect is usually very small. It is neglected to avoid involved computational work without appreciable loss of accuracy. Whatever error occurs on this account, tends to increase the moment in the loaded beam and thus, is an error on the safe side.

It can be proved[12] that in a system of inter-connected beams if one of the longitudinals is loaded in such a way that the load intensity at any point is proportional to the deflection at that point, all the longitudinals in the system will assume the same deflected form. In the case of simply-supported longitudinals of single span the load profile that satisfies this condition is a sine or cosine curve.

Suppose that a concentrated load 'W' is applied at a distance 'a' from the support along the length of one beam of the interconnected system giving rise to bending moment diagram for the span as shown in Fig. 2. This is termed the free bending moment diagram and the problem is to determine what proportions of this free bending moment at any section is carried by various beams of the system. To do this, the free bending moment diagram is analysed into a number of sine curves which, when superimposed, will give the shape of the free bending moment curve.

Moment 'M' at any distance 'x' from the support is given by

$$M = \frac{L - a}{L} Wx \qquad 0 \angle x \angle a$$

$$\text{or} = \frac{L - x}{L} W a \qquad a \angle x \angle L$$

$$\text{Let} \quad M = \sum_{n=1,2..\infty} A_n \operatorname{Sin} \frac{n \pi x}{L}$$

Then $A_n = \frac{2}{L}\int_0^L M \sin\frac{n\pi x}{L} d_x = \frac{2WL}{\pi^2}\frac{1}{n^2}\sin\frac{n\pi a}{L}$.

$$\therefore M = \frac{2WL}{\pi^2}\left(\sin\frac{\pi a}{L}\sin\frac{\pi x}{L} + \frac{1}{2^2}\sin\frac{2\pi a}{L}\sin\frac{2\pi x}{L} + \cdots + \frac{1}{n^2}\sin\frac{n\pi a}{L}\sin\frac{n\pi x}{L} + \cdots\right) \quad -1$$

This process of splitting a function into sine components is called harmonic analysis and the various sine components are the harmonics of the curve and designated by first, second... n^{th} harmonic, that is,

First harmonic component $\frac{2WL}{\pi^2}\sin\frac{\pi a}{L}\sin\frac{\pi x}{L}$

Second harmonic component $\frac{2WL}{\pi^2}\cdot\frac{1}{2^2}\sin\frac{2\pi a}{L}\sin\frac{2\pi x}{L}$

General, n^{th} harmonic component $\frac{2WL}{\pi^2}\frac{1}{n^2}\sin\frac{n\pi a}{L}\sin\frac{n\pi x}{L}$

In general, an infinite number of harmonics will be required to build up a given curve, but for practical purposes only the first few are sufficient to get a reasonable degree of accuracy. In Fig. 2 are shown the first three harmonics of the free bending moment diagram and their summation is drawn on the free bending moments diagram to show the closeness of result by considering only three harmonics and neglecting the reamining harmonics. If there are concentrated loads W, W', W" at distances a, a', and a" from the support on the beam, the harmonics of the free bending moment are found by superposition, that is, the first harmonic will be

$$\frac{2L}{\pi^2}\left(W \sin\frac{\pi a}{L} + W' \sin\frac{\pi a'}{L} + W'' \sin\frac{\pi a''}{L}\right)\sin\frac{\pi x}{L}$$

and so on for other harmonics.

Now $$EI \frac{d^2y}{dx^2} = -M$$

Substituting for M from Eq.1 and twice integrating,

$$y = \frac{2WL^3}{\pi^4 EI} \left(\sin\frac{\pi a}{L} \sin\frac{\pi x}{L} + \frac{1}{2^4} \sin\frac{2\pi a}{L} \sin\frac{2\pi x}{L} + \cdots + \frac{1}{n^4} \sin\frac{n\pi a}{L} \sin\frac{n\pi x}{L} + \cdots \right) \text{——} 2$$

where,

EI/L - Flexural stiffness of the longitudinal beam.

y - Free deflection of the longitudinal beam.

Thus, we see that the shape of harmonic components of deflection and bending moment are same and defined by the term $\sin \frac{n\pi x}{L}$ and the coefficients giving the maximum amplitudes of each of the harmonic component only differ in the two cases.

Hence, each harmonic component of the free bending moment diagram can be distributed amongst the longitudinals using the distribution coefficients of deflections derived below. The bending moment diagram for any beam is found by adding together the fractions of the harmonics so distributed to the beam concerned.

The method of analysis given below is for a general case of three longitudinal beam bridge having a load on the central beam. Let all the three beams be of equal flexural and transverse stiffness. EI and CJ respectively.

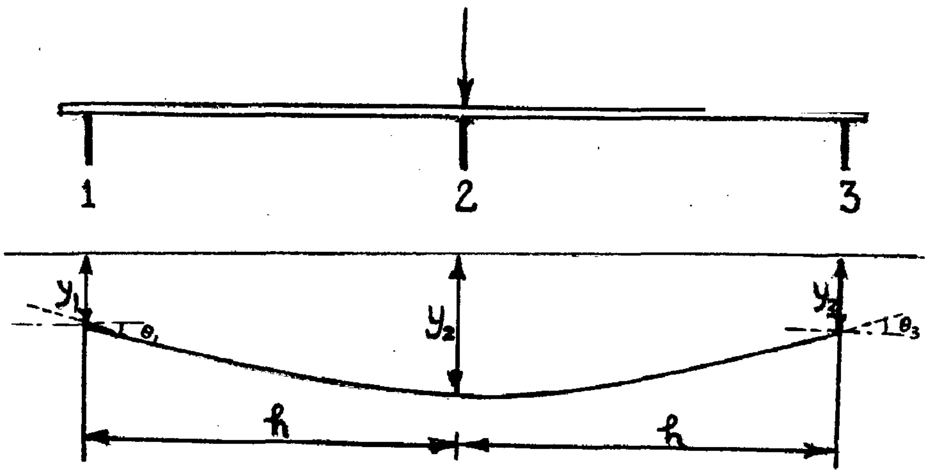


Figure shows a cross section of the bridge at a distance 'x' from the left hand support. The usual slope deflection equations give ,

$$M_{12} = \frac{6\, E_T\, I_T}{L\, h^2} \left[(y_2 - y_1) - \frac{2}{3}\, h\, \theta_1 \right] \qquad 3$$

$$V_{12} = \frac{12\, E_T\, I_T}{L\, h^3} \left[(y_2 - y_1) - \frac{1}{2}\, h\, \theta_1 \right] \qquad 4$$

where,

$\frac{E_T\, I_T}{L}$	flexural stiffness of the transverse system per unit length of the bridge.
h	spacing of the longitudinals
y_1 , y_2	first harmonic deflections of longitudinal first and second respectively, positive downward.
θ_1	first harmonic rotation of the first longitudinal positive clockwise. ($\theta_2 = 0$ by symmetry)
M_{12}	Moment per unit length in the transverse medium at the outer edge of the intercept between longitudinals I and II due to first harmonic loading.
V_{12}	shear force per unit length in transverse medium at the outer edge of the intercept between longitudinal I and II due to end moments.

Let the first harmonic deflection of the inner and outer beams be

$$y_2 = a_2 \quad \sin \frac{\pi x}{L} \qquad 5$$

$$y_1 = a_1 \ \sin \frac{\pi x}{L} \qquad 6$$

where 'a_1' and 'a_2' are the mid-span deflections due to the first harmonic of the loading. The deflection of the bridge is symmetrical about mid-span and by symmetry the torque in the outer beam is zero at mid-span because $\frac{d\theta_1}{dx} = 0$ at mid-span. Then torsional equilibrium for first beam gives;

$$\int_0^{L/2} M_{12} \ d_x = 0 \qquad 7$$

Let the first harmonic of the angle of rotation of the first beam be given by,

$$h \ \theta_1 = C_1 + \lambda_1 \sin \frac{\pi x}{L} \qquad 8$$

where C_1 and λ_1 are constants to be found.

From eqs. 3,5,6,7 and 8, we get,

$$\int_0^{L/2} M_{12} \, dx = \frac{6 \, E_T \, I_T}{L \, h^2} \int_0^{L/2} \left[(a_2 - a_1) \sin \frac{\pi x}{L} - \frac{2}{3} (C_1 + \lambda \sin \frac{\pi x}{L}) \right] dx = 0$$

From which on simplification we get,

$$\frac{3}{2} \ (a_2 - a_1) = \frac{\pi}{2} \ C_1 + \lambda_1 \qquad 9$$

The torque 'T_1' in the outer beam at this section is given by,

$$T_1 = \int_{L/2}^{L/2 - x} M_{12} \, dx$$

Integrating both the sides between the limits 0 and $\frac{L}{2}$

$$\int_0^{L/2} T_1 \ d_x = \int_0^{L/2} \int_{L/2}^{L/2-x} M_{12} \, dx \, dx \qquad 10$$

But $T_1 = C\,J\,\frac{d\theta_1}{dx} = C\,J\,\frac{\lambda_1}{h}\,\frac{\pi}{L}\,\cos\frac{\pi x}{L}$ — 11 (From Eq.8)

where CJ is the torsional rigidity of the longitudinal girder.

Integrating both the sides of Eq. 11 between the limits 0 and $\frac{L}{2}$

$$\int_0^{L/2} T_1\,dx = C\,J\,\frac{\lambda_1}{h}\,\frac{\pi}{L}\int_0^{L/2}\cos\frac{\pi x}{L}\,dx = C\,J\,\frac{\lambda_1}{h} \qquad 12$$

Substituting in Eq. 10 from Eqs. 3, 5, 6, 8 and 12

$$C\,J\,\frac{\lambda_1}{h} = \frac{6E_TI_T}{Lh^2}\int_0^{L/2}\int_{L/2}^{L/2-x}\left[(a_2 - a_1)\sin\frac{\pi x}{L} - \frac{2}{3}\left(C_1 + \lambda\sin\frac{\pi x}{L}\right)\right]dx\,dx$$

From which on simplification, we get,

$$\frac{3}{2}(a_2 - a_1) = \frac{\pi^2}{8}C_1 + (1 + \beta/2)\lambda_1 \qquad 13$$

where $$\beta = \frac{\pi^2}{2}\,\frac{h}{L}\,\frac{C\,J}{E_TI_T} \qquad 14$$

Solving Eqs. 9 and 13, we get,

$$\lambda_1 = \frac{3\left(1 - \frac{\pi}{4}\right)(a_2 - a_1)}{\left[2\left(1 - \frac{\pi}{4}\right) + \beta\right]} \qquad 15$$

and $$C_1 = \frac{3\beta(a_2 - a_1)}{\pi\left[2\left(1 - \frac{\pi}{4}\right) + \beta\right]} \qquad 16$$

Load per unit length on the outer girder is

$$EI\,\frac{d^4y_1}{dx^4} = V_{12} \qquad 17$$

Substituting in Eq. 17 from Eqs. 4, 5 and 6, we get

$$a_1\sin\frac{\pi x}{L} = \alpha\left[(a_2 - a_1)\sin\frac{\pi x}{L} - \frac{1}{2}h\,\theta_1\right] \qquad 18$$

where $$\alpha = \frac{12}{\pi^4}\left(\frac{L}{h}\right)^3\frac{E_TI_T}{EI} \qquad 19$$

(a) When the longitudinals are of zero torsional stiffness, that is,

$C\ J = 0$ or $\beta = 0$.

$h\ \theta_1 = \frac{3}{2}(a_2 - a_1)\ \text{Sin}\ \frac{\pi x}{L}$ From Eqs. 14, 15 and 16

substituting in Eq. 18, and simplifying we obtain

$$a_1 = {}^{\alpha}/_{4}\ (a_2 - a_1)$$

(b) When the longitudinals are of infinite torsional stiffness and prevented from rotation, that is, $CJ = \infty$ or $\beta = \infty$, $h\theta_1 = 0$

Substituting in Eq. 18 and simplifying we obtain,

$$a_1 = \alpha\ (a_2 - a_1)$$

(c) When the longitudinals are of infinite stiffness and rotate as rigid body, that is, $CJ = \infty$ or, $\beta = \infty$

$h\ \theta_1 = \frac{3}{\pi}(a_2 - a_1)$ From Eqs. 8, 14, 15 and 16.

Expanding the right hand side in sine series, we obtain

$$h\ \theta_1 = \frac{12}{\pi^2}(a_2 - a_1)(\text{Sin}\ \frac{\pi x}{L} + \frac{1}{3}\ \text{Sin}\ \frac{3\pi x}{L} + \frac{1}{5}\ \text{Sin}\frac{5\pi x}{L} + \dots)$$

$$= \frac{12}{\pi^2}(a_2 - a_1)\ \text{Sin}\ \frac{\pi x}{L} \quad \text{(Approximately)}$$

Substituting in Eq. 18 and simplifying, we obtain,

$$a_1 = \alpha\left(1 - \frac{6}{\pi^2}\right)(a_2 - a_1)$$

Let the amplitude of first harmonic free deflection be 'A'.

Then $A = a_1 + a_2 + a_3$

but $a_1 = a_3$ (by symmetry).

$$\therefore\ A = 2a_1 + a_2 \qquad - 20$$

Substituting Eq. 20 in the results of above three cases, the first harmonic amplitude of deflection of the various girders is reduced in the form

$$a_1 = P_{12} A \; ; \quad a_2 = P_{22} A \; ; \quad a_3 = P_{32} A \; ;$$

where P_{12}, P_{22}, P_{32} are constants and termed as distribution coefficients for the girders 1, 2 and 3 respectively for load on girder 2. The value of thse are given below for the first harmonic of the free deflection curve

	$P_{12} = P_{32}$	P_{22}
(a) Zero torsional stiffness.	$\frac{\alpha}{4 + 3\alpha}$	$\frac{4 + \alpha}{4 + 3\alpha}$
(b) Infinite torsional stiffness, no rotation.	$\frac{\alpha}{1 + 3\alpha}$	$\frac{1 + \alpha}{1 + 3\alpha}$
(c) Infinite torsional stiffness, rigid body rotation.	$\frac{\alpha(1 - \frac{6}{\pi^2})}{1 + 3\alpha(1 - \frac{6}{\pi^2})}$	$\frac{1 + \alpha(1 - \frac{6}{\pi^2})}{1 + 3\alpha(1 - \frac{6}{\pi^2})}$

If the outer and inner longitudinals are of different moment of inertia,

Let $$\eta = \frac{I_1}{I} \qquad (21)$$

where I_1 moment of inertia of outer longitudinal

I moment of inertia of inner longitudinal

Eqs. 17, 18, and 20 reduce to Eqs. 17a, 18a and 20a given below.

$$\eta E I_1 \frac{d^4 y_1}{dx^4} = V_{12} \qquad (17a)$$

$$a_1 \sin \frac{\pi x}{L} = \frac{\alpha}{\eta}\left[(a_2 - a_1)\sin\frac{\pi x}{L} - \frac{1}{2} h\,\theta_1\right] \qquad 18a$$

$$A = 2\eta a_1 + a_2 \qquad 20a$$

Using these with Eqs. 8,14,15 and 16, we get the following values of distribution coefficients for the first harmonic of free deflection, for the three cases considered ~~are obtained~~

	$P_{12} = P_{32}$	P_{22}
(a) Zero torsional stiffness.	$\dfrac{\alpha}{4\eta + \alpha(1 + 2\eta)}$	$\dfrac{4\eta + \alpha}{4\eta + \alpha(1 + 2\eta)}$
(b) Infinite torsional stiffness, no rotation.	$\dfrac{\alpha}{\eta + \alpha(1 + 2\eta)}$	$\dfrac{\eta + \alpha}{\eta + \alpha(1 + 2\eta)}$
(c) Infinite torsional stiffness, rigid body rotation.	$\dfrac{\alpha(1 - \frac{6}{\pi^2})}{\eta + \alpha(1+2\eta)(1 - \frac{6}{\pi^2})}$	$\dfrac{\eta + \alpha(1 - \frac{6}{\pi^2})}{\eta + \alpha(1+2\eta)(1 - \frac{6}{\pi^2})}$

The nth harmonic component of the free deflection curve is

$$B_n \sin \frac{n\pi x}{L}$$

where

$$B_n = \frac{2WL^3}{\pi^4 EI} \frac{1}{n^4} \sin \frac{n\pi x}{L}$$

Exactly the same analysis can be carried out for any nth harmonic component and it is easily seen that the distribution coefficient for the nth harmonic will be obtained by substituting $\frac{\alpha}{\eta^4}$ in place of α in the formulae for the first harmonic coefficients for case (a), case (b), and case (c). However, for case (c) rotation of the longitudinals is accounted almost entirely by the first harmonic bending moment component. A second harmonic would impose a system of torques on the longitudinals antisymmetrical about the mid-span producing no rotation at all, the third harmonic would result in very small unbalanced torques, hence, small rotations. Therefore, in such cases the distribution coefficients for second and higher harmonics may be computed assuming no rotation, that is, coefficient of case (b) may be used for higher harmonics of case (c).

In cases where outer and inner longitudinals are of different flexural stiffnesses, the bending moment distribution coefficients are obtained by multiplying the deflection coefficients for the outers by 'η' .

It is possible to interpolate distribution coefficients for intermediate values of β given the coefficients for $\beta = 0$ and $\beta = \infty$ by interpolation formula

$$P_\beta = P_0 + (P_\infty - P_0) \sqrt{\frac{\beta\sqrt{\alpha}}{3 + \beta\sqrt{\alpha}}}$$

where,

P_β = Required distribution coefficient for 'β'

P_∞ = Distribution coefficient for $\beta = \infty$

P_0 = Distribution coefficient for $\beta = 0$

If there are loads on more than one longitudinals, the free bending moment relative to each are distributed separately and the total effect is found by superposition. Loads applied on the deck in between the longitudinals are dealt by replacing them with equivalent loads acting on the beams. The equivalent system is obtained by treating the transverse medium as continuous over unyielding supports, provided by longitudinals, the reactions so obtained are considered as applied loads on the longitudinals and distributed as above.

6. Determination of Moments in various Girders

The free bending moment at a distance 'x' for a beam due to a unit concentrated load applied at a distance 'a' from the support is given by

$$M = \frac{2L}{\pi^2} \left(\sin\frac{\pi a}{L} \sin\frac{\pi x}{L} + \frac{1}{2^2} \sin\frac{2\pi a}{L} \sin\frac{2\pi x}{L} + \cdots + \frac{1}{n^2} \sin\frac{n\pi a}{L} \sin\frac{n\pi x}{L} + \cdots \right) \qquad 22$$

The nth harmonic term T_n is given by

$$T_n = \frac{2L}{n^2 \pi^2} \sin\frac{n\pi a}{L} \sin\frac{n\pi x}{L} \qquad 23$$

As explained in section 5, each harmonic term T_n is considered as a separate bending moment diagram and is distributed amongst the longitudinal beams using the distribution coefficients of Hendry-Jaeger given in Appendix A-1. If we denote by ${}_nP_{pq}$ * the nth harmonic distribution coefficient for pth beam load being on qth beam, the total bending moment, M_{pq} ** on any pth beam and M_{qq} on qth beam (loaded beam) will be given by

$$M_{pq} = \left(\Sigma\, {}_nP_{pq} \times T_n \right) L \qquad 24$$

$$M_{qq} = \left(\Sigma\, {}_nP_{qq} \times T_n \right) L \qquad 25$$

The value of α taken for ${}_nP_{pq}$ is $\frac{\alpha}{n^4}$.

---

* The subscript to the left (n) refers to the number of harmonic, first subscript to the right (p) refers to the number of beam under consideration and the second subscript (q) the number of beam having load.

** first subscript number (p) refers to the number of beam under consideration and the second subscript (q) the number of beam having load.

Now as n increases α decreases resulting a decrease in the value of ${}_nP_{pq}$ and corresponding increase in ${}_nP_{qq}$. For large values of n, ${}_nP_{pq}$ may be assumed as zero and ${}_nP_{qq}$ as unity. It is found that this situation arises for about n = 5 in most of the cases.

Then equation 24 reduces to

$$M_{pq} = \left(\sum_{n=1,2,\cdots 5} {}_nP_{pq} \; T_n \right) L \qquad 26$$

and can easily be evaluated.

If there are in all 'm' longitudinal beams of which say the qth beam is loaded, there will be (m - 1) equations of the type of Eq. 26 and only one equation will be of the type of eq. 25. The condition of statics at any section having a free bending moment 'M' gives

$$\sum M_{pq} + M_{qq} = M$$

or

$$M_{qq} = M - \sum M_{pq} \qquad 27$$

From Eq. 27 bending moment on the loaded girder is calculated, that is, the bending moment of the loaded girder is obtained by subtracting the bending moments of all the beams other than the loaded beam from the free bending moment of the bridge.

7. Example

To illustrate the method an example is solved here.

Example - A concentrated load 'W' acts on the model bridge of Fig. 19 at 3/8L from the left hand support midway of girder (1) and (2). Calculate the bending moment in various girders at midspan.

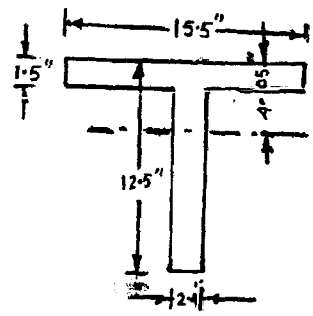
Step I

Calculate the moment of inertia of the bridge elements. In calculating moment of inertias of the bridge elements the gross cross sectional area of concrete is taken.

(i) Exterior Girder

Distance of neutral axis from the top of slab

$$= \frac{13.1 \times 1.5 \times 0.75 + 12.5 \times 2.4 \times 6.25}{13.1 \times 1.5 + 12.5 \times 2.4}$$



= 4.05 in.

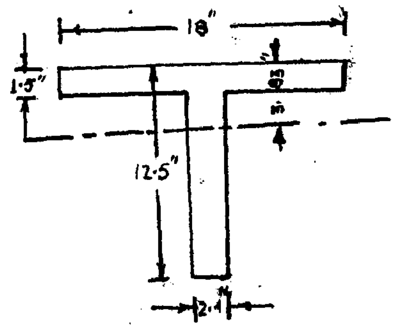
∴ Flexural moment of inertia about the neutral axis,

$$I_1 = \frac{1}{12} \times 13.1 \times 1.5^3 + \frac{1}{12} \times 2.4 \times 12.5^3 + 13.1 \times 1.5 \times 3.3^2 + 12.5 \times 2.4 \times 2.2^2 = 756 \text{ in}^4.$$

(ii) Interior Girder.

Distance of neutral axis from the top of slab

$$= \frac{15.6 \times 1.5 \times 0.75 + 12.5 \times 2.4 \times 6.25}{15.6 \times 1.5 + 12.5 \times 2.4}$$



= 3.83 in.

∴ Flexural moment of inertia about the neutral axis

$$I_2 = \frac{1}{12} \times 15.6 \times 1.5^3 + \frac{1}{12} \times 2.4 \times 12.5^3 + 15.6 \times 1.5 \times 3.08^2 + 12.5 \times 2.4 \times 2.42^2 = 796 \text{ in}^4.$$

Torsional stiffness factor,

$$J = \frac{2.4^3 \times 12.5}{3 + \frac{1.8 \times 2.4}{12.5}} + \frac{15.6 \times 1.5^3}{3 \times 1.8 \; \frac{1.5}{15.6}}$$

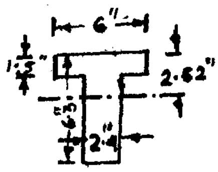
$= 68.3 \text{ in}^4.$

(iii) Cross Beams-

(a) The cross beams twoof quarter spans and one of mid span will act as T beams.

Distance of neutral axis from the top of slab

$$= \frac{3.6 \times 1.5 \times 0.75 + 2.4 \times 6.3 \times 3.15}{3.6 \times 1.5 + 2.4 \times 6.3}$$



$= 2.52$ in.

Flexural moment of inertia of the three cross beams

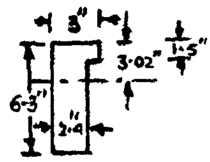
$$= 3\left[\frac{1}{12} \times 3.6 \times 1.5^3 + \frac{1}{12} \times 2.4 \times 6.3^3 + 3.6 \times 1.5 \times 1.77^2 + 2.4 \times 6.3 \times 0.63^2\right]$$

$= 222 \text{ in}^4.$

(b) The cross beams one at each end will act as L-beams.

Distance of neutral axis from the top of slab

$$= \frac{0.6 \times 1.5 \times 0.75 + 6.3 \times 2.4 \times 3.15}{0.6 \times 1.5 + 6.3 \times 2.4}$$



$= 3.02$ in.

Flexural moment of inertia of the two L-beams

$$= 2\left[\frac{1}{12} \times 0.6 \times 1.5^3 + \frac{1}{12} \times 6.3^3 \times 2.4 + 0.6 \times 1.5 \times 2.27^2 + 6.3 \times 2.4 \times 0.13^2\right]$$

$= 110 \text{ in}^4.$

(iv) Decking Slab-

Width of slab remaining after inclusion in cross beams

$= 150.8 - 6 \times 3 - 2 \times 3 \quad = 126.8$ in.

Flexural moment of inertia of the remaining slab

$= \frac{1}{12} \times 126.8 \times 1.5^3$

$= 35.6 \text{ in}^4.$

From items (iii) and (iv) total flexural moment of inertia of the transverse system, $I_T = 222 + 110 + 35.6 = 367.6 \text{ in}^4.$

Step II

Calculate the dimensionless parameters.

Transverse spacing of the girders = 1.5 ft.

Span length of the bridge = 12 ft.

Assuming Poisson's ratio of concrete 0.15, $\frac{C}{E} = \frac{1}{2.3}$

$$\alpha = \frac{12}{\pi^4} \left(\frac{12}{1.5} \right)^3 \frac{367.6}{796} = 29.1$$

$$\beta = \frac{\pi^2}{2} \times \frac{1.5}{12} \times \frac{68.3}{2.3 \times 367.6} = 0.05$$

$$\eta = \frac{756}{796} = 0.95$$

Step III

Calculate bending moment distribution coefficients.

As the bending moment is required at mid span, distribution coefficients of second and fourth harmonic will not come in the calculation. Hence, we shallfind distribution coefficients for first and third harmonics only.

(a) Distribution coefficients of bending moment for $\beta = 0, \eta = 0.95$ from graph of Appendix B or from formulae of Appendix A.

Harmonic	α	P_{21}*	P_{31}	P_{41}	P_{12}	P_{32}	P_{42}
First	29.1	0.37	0.07	-0.17	0.36	0.23	0.06
Third	0.36	0.08	-0.04	0	0.07	0.15	-0.04

(b) Distribution coefficients of bending moment for $\beta = \infty, \eta = 0.95$ from graph of Appendix B or from formulae of Appendix A.

Harmonic	α	P_{21}	P_{31}	P_{41}	P_{12}	P_{32}	P_{42}
First	29.1	0.28	0.21	0.16	0.26	0.25	0.20
Third	0.36	0.19	0.03	-0.03	0.18	0.12	0.03

(c) Distribution coefficients of bending moment for $\beta = 0.05, \eta = 0.95$ from interpolation formula $P_\beta = P_0 + (P_\infty - P_0)\sqrt{\frac{\beta\sqrt{\alpha}}{3+\beta\sqrt{\alpha}}}$.

Harmonic	α	P_{21}	P_{31}	P_{41}	P_{12}	P_{32}	P_{42}
First	29.1	0.34	0.11	-0.07	0.33	0.24	0.10
Third	0.36	0.09	-0.03	0	0.08	0.15	-0.03

Step IV

Replace the applied load by equivalent loads acting on the longitudinals.

* First subscript number (2) refers to girder number, the second (1) to the load position. Thus P_{21} is the distribution coefficient of bending moment for girder (2) with loading on girder (1).

Assuming slab to be a continuous beam resting on unyielding supports provided by longitudinals, the moment distribution gives the reactions on the girders due to a concentrated load 'W' acting midway of girder 1 and 2 as follows.

Girder	1	2	3	4
Reactions	0.40 W	0.724W	-0.148W	0.024W

Thus, we can assume, instead of single load 'W', the four loads as above acting on the respective beams. (A negative sign indicates an upward load on the girder).

Step V

Calculate bending moment on various girders at mid-span due to unit load on girder 1 and girder 2 respectively at 3/8 L.

(a) Free bending moment 'M' at mid span due to unit load at

3/8L = 0.1875 L

(b) Bending moment on various girders at mid span due to unit load on girder 1 at 3/8L.

$$M_{21}^{*} = \frac{2L}{\pi^2} \left(0.34 \times 0.924 + \frac{.09 \times 0.383}{9} \right) = 0.0645L$$

$$M_{31} = \frac{2L}{\pi^2} \left(0.11 \times 0.924 - \frac{.03 \times 0.383}{9} \right) = 0.0204L$$

$$M_{41} = \frac{2L}{\pi^2} (-0.07 \times 0.924) = -0.0131L$$

$$M_{11} = \left[0.1875 - (0.0645 + 0.0204 - 0.0131)\right] L = 0.1157\ L$$

* First subscript number refers to girder number, the second to the load position, thus M_{21} is the bending moment for girder 2 with loading on girder 1

(c) Bending moment on various girders at mid span due to unit load on girder (2) at 3/8L.

$$M_{12} = \frac{2L}{\lambda^2} \left(.33 \times .924 + \frac{.08 \times .383}{9} \right) = 0.0626L$$

$$M_{32} = \frac{2L}{\lambda^2} \left(.24 \times .924 + \frac{.15 \times .383}{9} \right) = 0.0464L$$

$$M_{42} = \frac{2L}{\lambda^2} \left(.10 \times .924 - \frac{.03 \times .383}{9} \right) = 0.0185L$$

$$M_{22} = \left[0.1875 - (0.0626 + 0.0464 + 0.0185) \right] L = 0.0600L$$

Step VI

Calculate the bending moment on the various girders due to applied loads using,

(i) Equivalent loads of step IV

(ii) Unit load bending moments of step V

(iii) Symmetry of the bridge about the longitudinal axis.

$$M_1 = (.1157 \times 0.4 + 0.0626 \times 0.724 - 0.0185 \times .148 - 0.0131 \times 0.024) WL = 0.0885 WL$$

$$M_2 = (0.0645 \times 0.4 + 0.0600 \times 0.724 - 0.0464 \times .148 + 0.0204 \times 0.024) WL = 0.0628 WL$$

$$M_3 = (0.0204 \times 0.4 + 0.0464 \times 0.724 - 0.0600 \times 0.148 + 0.0645 \times .024) WL = 0.0345 WL$$

$$M_4 = (-0.0131 \times 0.4 + 0.0185 \times 0.724 - 0.0626 \times 0.148 + .1157 \times 0.024) WL = .0017 WL$$

where M_1, M_2, M_3 and M_4 are the mid span bending moments in girders 1,2,3 and 4 respectively due to a load 'W' acting at 3/8L from left support mid way of girder (1) and (2).

III. VARIABLES AFFECTING THE BEHAVIOUR OF INTERCONNECTED-GIRDER BRIDGES

8. The Variables

The variables that enter into the analysis are as follows:

(i) Variables relating to the mechanical properties of the material.

(a)	Shearing modulus of elasticity	C.
(b)	Young's Modulus of elasticity	E.

(ii) Variables relating to the geometry of the Structure.

(a)	Number of longitudinal beams-	m
(b)	Spacing of longitudinal beams-	h
(c)	Span length of longitudinal beams.	L
(d)	Number and locations of cross-beams.	

(iii) Variables relating to the stiffness of the bridge elements.

(a)	Flexural stiffness of the longitudinal beams-	EI, EI_1
(b)	Torsional Stiffness factor-	J
(c)	Flexural Stiffness of the transverse system, $E_T I_T$	$E_T I_T$

(iv) Variables relating to the loading.

(a)	Longitudinal position of the load -	a
(b)	Number of wheel loads and transverse position.	

The above four groups of variables are considered in detail in the following sections:

9. <u>Mechanical Properties of Material C,E</u>

Shearing modulus of elasticity 'C' and Young's modulus of elasticity 'E' appear in the form of ratio $\frac{C}{E}$. In terms of Poisson's ratio 'μ',

$$\frac{C}{E} = \frac{1}{2(1+\mu)}$$

The value of 'μ' for concrete generally lies between 0 and 0.2; in this study μ = 0.15 is assumed so that $\frac{C}{E} = \frac{1}{2.3}$.

10. <u>Geometry of the Structure</u>

(a) Number of Longitudinal Beams - m

A majority of bridges can be included in two, three and four beam bridges. The actual analysis by Hendry-Jaeger method has shown that no appreciable advantage is gained of interconnections in the usual type of two beam bridges. The study has, therefore, been restricted to three and four girder bridges only where the considerations of interconnections considerably alter the design moments of the beams.

(b) <u>Spacing of Longitudinal Beams - h</u>

For two lane bridges the width of decking slab may vary from 20 to 32 ft. To include all these widths, the centre to centre spacing of 8, 9, 10 and 11 ft. of longitudinal beams in three beam bridges and 6,7 and 8 ft. in four beam bridges are considered. The design formulae finally suggested are such that any fractional value in between these widths can also be dealt with equal ease.

(c) Span Length of Longitudinal Beams - L

Only simply-supported bridges are considered here. The influence lines for moments in longitudinal girders are plotted as the coefficients of the effective span length.

(d) Location and Number of Cross Beams

The cross beam near midspan transfers the load mainly by shear and near support by torsion and as such the bridge is not sensitive to the position of transverse beam as far as the distribution of bending moment in the various beams is concerned. The number of cross beams is only important when determining the flexural stiffness of the transverse system.

11. Moment of Inertia of Bridge Elements

The cracking of concrete and presence of steel affect the absolute moment of inertia of the bridge elements but the extent of cracking is not certain and the amount of steel is not known initially and the moment of inertia appearsin the relative form. It is customary in R.C.C. structures to compute moment of inertia on the basis of gross concrete area. The same basis is adopted in this thesis.

(a) Flexural Stiffness of Longitudinal Beams EI, EI_1

The longitudinal beam acts as T or L beams, the width of the flange acting with the rib is taken as per IS: 456-1957[13] The flexural moment of inertia is calculated as usual and is denoted by I_1 and I for outer and inner longitudinal beams respectively.

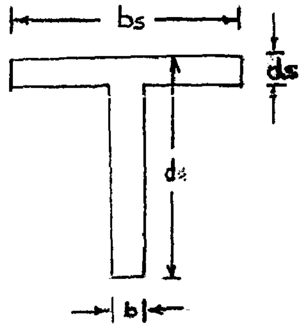
(b) Torsional Stiffness Factor - J

The torsional stiffness factor for rectangular beams is approximately given by Morley[14],

$$J = \frac{b^3 d}{3 + 1.8 \frac{b}{d}}$$

For T or L beams the value of J is found by adding to the J value of the web, the contribution of the flange. Thus,

$$J = \frac{b^3 d}{3 + 1.8 \frac{b}{d}} + \frac{(b_s - b)\, d_s^3}{3 + 1.8 \frac{d_s}{(b_s - b)}}$$



where,

d = overall depth of T or L section,

b = rib width of T or L section,

ds = depth of slab

bs = width of slab acting with the rib of T or L section.

(c) Flexural Stiffness of Transverse System EI_T

The cross beams will act as T or L beams. The flange width is taken as recommended by IS: 456-1957[13]. The flexural moments of inertia of all cross beams are added up. Let this be I_{T_1}. From thetotal length of the slab the length included with different cross beams is subtracted and the flexural moment of inertia of the remaining length of the slab is computed. Let this be denoted by I_{T_2}. Then the total flexural moment of inertia of the transverse system,

$$I_T = I_{T_1} + I_{T_2}$$

12. Loading

(a) Longitudinal position of the load - a

For the study of bending moment distribution two positions of the loading have been considered:

mid-span and quarter span, that is $a = \frac{L}{2}$ and $a = \frac{L}{4}$

(b) Number of Wheel Loads and Transverse Position

The standard load taken for finding the maximum bending moment percentage is I.R.C. Class A two lane loading; the distance between the centres of the nearest wheels of vehicles in adjacent lanes is taken 4 ft. throughout. All the four wheels along any cross-section have the same load. When considering the outer beam, the first wheel is put directly above it and to produce maximum moment in the inner beam, the second wheel is put directly above this beam.

13. Dimensionless Parameters

The conditions of the analysis are such that the variables discussed above do not appear separately but are combined into the following dimensionless parameters:

$$\eta = \frac{I_1}{I}$$

$$\beta = \frac{\pi^2}{2} \frac{h}{L} \frac{CJ}{E_T I_T}$$

$$\alpha = \frac{12}{\pi^4} \left(\frac{L}{h}\right)^3 \frac{E_T I_T}{EI}$$

A number of actual bridges were studied and the following conclusions are drawn.

(a) η - For most of the actual bridges 'η' is about one. A small variation in the value of 'η' does not cause appreciable variation in the maximum design moment of the main beams. 'η' = 1 is adopted in the theoretical analysis of this thesis throughout.

(b) The value of 'β' lies between 0 and 0.5 for the bridges in which usual amount of cross stiffness is provided by cross beams. In case of slab bridges having no cross beams 'β' values are higher. 'β' decreases as the number of girders or the span increases. The influence lines are drawn for $\beta = 0$ and $\beta = \infty$. Simplified formulae for fractions of free bending moment coming on the various beams are given for $\beta = 0$, the same formulae may be adopted for higher values of β, as the values of β are small and the variation of percentage of design bending moment is small for $\beta = \infty$ from $\beta = 0$. A table is also given for percentage of free bending moment for $\beta = \infty$.

(c) α - The value of α increases as the number of girders or the span or both increase. The values of α considered are from 0 to 100, with intermediate values 5, 10, 20 and 50 as in this range all the actual bridges are expected to lie. The values at $\alpha = 100$ may be used for α above 100 also, because for α above 100, the distribution coefficient of Hendry-Jaeger do not change appreciably and this approximation will lead to a very small error on the safe side.

IV. MAXIMUM MOMENTS AND DISTRIBUTION OF WHEEL LOADS

14. Influence Lines for Beam Moments

In order to determine accurately the maximum moment on any section of a bridge due to moving loads, an influence surface for the moment at that point is required; many such sections must be considered if a maximum moment surface is desired. However, the problem is simplified by the fact that the primary interest in design lies in finding the absolute maximum moment, which generally occurs at, or close to mid-span. To determine the maximum mid-span and quarter span moments, three types of influence lines are obtained as follows:

(a) The influence lines for moments at mid-span when the unit load moves transversely at mid-span of the bridge; that is, $x = \frac{L}{2}$, $a = \frac{L}{2}$.

(b) The influence lines for moments at midspan when the unit load moves transversely at the quarter-span; that is, $x = L/2$, $a = L/4$. Since 'a' and 'x' are interchangeable in the term $\sin \frac{n\lambda a}{L} \sin \frac{n\lambda x}{L}$, the results in this case hold also for moments at quarterspan when the unit load moves transversely at the mid-span; that is, $x = L/4$, $a = L/2$.

(c) The influence lines for moments at the quarter-span when the unit load moves transversely at the quarter-span of the bridge; that is, $x = L/4$, $a = L/4$.

From Eq. 23,

$$\frac{T_n}{L} = \frac{2}{n^2\lambda^2} \sin \frac{n\lambda a}{L} \sin \frac{n\lambda x}{L} .$$

For the above three combinations of 'a' and 'x', the values of $\frac{2}{n^2\pi^2} \sin\frac{n\pi a}{L} \sin\frac{n\pi x}{L}$ are given in the following table.

Values of $\frac{2}{n^2\pi^2} \sin\frac{n\pi a}{L} \sin\frac{n\pi x}{L}$.

n	x = L/2 a = L/2	x = L/2 a = L/4	x = L/4 a = L/4
1	0.2026	0.1433	0.1013
2	0	0	0.0507
3	0.0225	-0.0159	0.0113
4.	0	0	0
5	0.0081	-0.0057	0.0041

As we are considering only three and four girder bridges the influence ordinate for the moments on the girder 1 and girder 2 are required only as the same can be used for remaining correspon-ding girders. To do this the moments due to unit load in all the girders are obtained for load positions on 1 and 2 beam respectiv-ely. From these the influence ordinates for the moments in the beam 1 and 2 can be obtained directly for a unit load placed succ-essively on beam 1 to 3 or 1 to 4 in case of three and four girder bridges respectively.

For example, for plotting the influence line for girder 1 for x = a = L/2, we shall find the bending moment 'M' on girder 1, 2, 3 and 4 when unit load is placed on girder 1 and 2 respectively. Let these be as follows:

M_{11} M_{21} M_{31} and M_{41} unit load on beam 1

M_{12}* M_{22} M_{32} and M_{42} unit load on beam 2

*First subscript number refers to beam number, the second to the load position. Thus, M_{12} is the moment for beam 1 with loading on beam 2.

Then influence ordinates for 1 and 2 beams will be respectively

M_{11} M_{12} M_{42} M_{41} - for beam 1.

M_{21} M_{22} M_{32} M_{31} - for beam 2.

To obtain a higher degree of accuracy the ordinates of the influence diagrams midway of the beams are also calculated, say, in the case of a four beam bridge the continuous beam reactions on all the four beams due to unit load placed midway of beams 1-2 and 2-3 are calculated. Let these be as follows:

R_1 R_2 R_3 R_4 unit load midway of 1-2

S_1 S_2 S_3 S_4 unit load midway of 2-3

Then by symmetry of the problem the reactions of unit load placed midway of 3-4 will be-

R_4 R_3 R_2 R_1 unit load midway of 3-4

The values of these continuous beam reactions are given in the following table for three and four beam bridges.

<u>Continuous beam reactions for four girder bridge</u>

Beam	Unit load midway of longitudinal beam		
	1-2	2-3	3-4
1	+0.400	-0.074	+ 0.024
2	+0.724	+0.574	-0.148
3	-0.148	+0.574	+ 0.724
4	+0.024	-0.074	+ 0.400

Continuous Beam Reactions for Three-girder Bridge

	UNIT LOAD			
Beam	Over hanging beam 1 by h/2	Mid way of beam 1-2	Midway of beam 2-3	Over-hanging beam 3 by h/2
1	1.625	0.406	-0.094	0.125
2	-0.750	0.688	0.688	-0.750
3	0.125	-0.094	0.406	1.625

Then the influence diagram for the beam 1 will have the ordinates as follows:

$M_{11} R_1 + M_{12} R_2 + M_{42} R_3 + M_{41} R_4$ midway of beam 1-2

$M_{11} S_4 + M_{12} S_2 + M_{42} S_3 + M_{41} S_4.$ midway of beam 2-3

$M_{11} R_4 + M_{12} R_3 + M_{42} R_2 + M_{41} \cdot R_1$ midway of beam 3-4

Similarly the influence diagram for beam 2 will have the following ordinates:

$M_{21} R_1 + M_{22} R_2 + M_{32} R_3 + M_{31} R_4$ midway of beam 1-2

$M_{21} S_1 + M_{22} S_2 + M_{32} S_3 + M_{31} S_4$ midway of beam 2-3

$M_{21} R_4 + M_{22} R_3 + M_{32} R_2 + M_{31} R_1$ midway of beam 3-4

In this study we have assumed $\eta = 1$. For the simply supported bridge, it is found that,

(i) $M_{21} = M_{12}$ and $M_{31} = M_{42}$ for four-girder bridge.

(ii) $M_{21} = M_{12}$ for three-girder bridge

Influence coefficients for moments for the above three cases for three and four girder bridges, having $\beta = 0, \alpha = 1, .10$ and 100 are tabulated in Table 1 and 2 ; the corresponding influence lines for beam moments are given in Fig. 3, 4, 5 and 6 .

15. Maximum Moments

In order to produce maximum moments in the longitudinal beams, the position of wheel loads of two lanes of I.R.C. Class A loading has to be such that one wheel comes directly above the beam under consideration and other loads as near to it as possible. To meet this end when considering the first beam, the first wheel load is placed right above it and the remaining three as near to it as possible on one side; and when considering the second beam, the second wheel load is placed on it, first wheel on one side and third and fourth wheel on other side as shown in Fig. 8

Without considering the loads other than those acting on a cross section of the bridge and placing the loads as shown in Fig. 8 , the maximum moments in the beams are obtained from the transverse influence lines of Fig. 3, 4, 5 and 6 .

The maximum moments, thus obtained, are expressed as fractions of the corresponding static moment due to four-wheel loads across the section. A fraction thus obtained is henceforth called the "fraction of wheel loads". These fractions of wheel loads carried by the longitudinal beams have been tabulated for a spacing of 8 and 11 ft. for three-beam bridges and 6 and 8 ft. for four-beam bridges. in Tables 3 and 4 respectively. Following conclusions can be drawn from these tables.

1. The fraction of wheel loads carried by an exterior beam for a = L/2, x = L/2 is larger than for that a = L/4, x = L/4 by about

3% for three beam bridges, and by about 2% for four-beam bridges, and smaller than that for a = L/2, x = L/4 by about 4% for three-beam bridges, and by about 3% for four-beam bridges.

2. The fraction of wheel loads carried by an interior beam for a = L/2, x = L/2 is smaller than that for a = L/4, x = L/4 by about 6% for three beam bridges, and by about 2% for four-beam bridges, and larger than that for a = L/2, x = L/4 by about 9% for three-beam bridges, and by about 5% for four-beam bridges.

3. The above variation is extreme. It decreases as the spacing 'h' of the longitudinal beams decreases.

4. As the number of longitudinal beams increases, the above variation decreases.

5. As the value of α increases the fraction of wheel loads increases in an exterior beam and decreases in an interior beam.

Since the absolute maximum value of the moment in a simply supported beam occurs near mid-span and since the contribution of the wheels acting at sections other than mid-span would be less, it may be concluded that the fraction of wheel loads may be adopted on the basis of its values for a =L/2, x = L/2. Furthermore, in a case of design where thewheel loads are distributed along the longitudinal and transverse direction, the adoption of a constant fraction of wheel loads irrespective of the longitudinal position of the loads will seldom cause on error more than 1% in estimating the design moments; hence to simplify the calculation, this can be adopted.

16. Influence Line for fraction of unit load carried by longitudinal girders

In Section 14, the method of calculating the influence coefficients for bending moment influence lines, has been discussed. If these influence coefficients for $x = a = L/2$ are divided by the corresponding free moment of unit load, that is L/4, we would get the fractions of the unit load carried by the longitudinal girders. To facilitate computation of design moments of various longitudinals, influence lines for fractions of unit loads carried by various longitudinals have been drawn for girders 1 and 2 of three and four-girder bridges with $\beta = 0$ and $\beta = \infty$ for various values of α vide Tables 5 to 8 and Figs. 9 to 16 .

17. Study of the effect of α, β and 'h' on the fraction of wheel loads

From the influence lines for fractions of unit loads carried by the longitudinal girders, fraction of wheel loads have been calculated for the following cases of three and four-girder bridges.

(a) Three-girder bridge- $\beta = 0$ and $\beta = \infty$; $\alpha = 1, 5,$ 10, 20, 50 and 100 and h = 8, 9, 10 and 11 ft.

(b) Four-girder bridge $\beta = 0$ and $\beta = \infty$; $\alpha = 1, 5, 10, 20, 50$ and 100 and h = 6,7 and 8 ft.

The results have been listed in Tables 9 to 10 .

Following conclusions can be drawn from the above study:

(i) As the value of α increases, the fraction of wheel loads increases in an exterior beam and decreasesin an interior beam.

(ii) The distribution for $\beta = \infty$ is always better than $\beta = 0$. However, the value of β does not have significant effect on fraction of wheel loads for lower values of α .

For three and four-girder bridges ordinarily β is small and the form of the parameters α and β are such that as β increases α decreases. We can adopt the fraction of wheel loads, calculated on the basis of $\beta = 0$ for higher values of β also, without any appreciable error and resulting always in a conservative design.

(iii) The fraction of wheel loads increases with the spacing of the beams.

18. Author's Formulae for Design .

In an actual problem of design α and spacing 'h' may have any value in between the values considered above but not necessarily the same value. An attempt has been made by the author to suggest simple formulae to give the fraction of wheel loads for three and four-girder bridges for any general value of α and spacing 'h'.

The fractions of wheel loads for $\beta = 0$, ∞ and as calculated by the author's formulae are given given in Tables 11 and 12 , and in Figs. 17 and 18 for three and four-girder bridges respectively. A study of the results shows that the results of author's formulae are in closer agreement with those of $\beta = 0$ for various values of α and 'h'. The formulae are proposed for normal use in designing bridges for two lanes of I.R.C. Class A loading. But the fractions given by these formulae could also be used as such for I.R.C. Class B or any other loading in which the wheels of an axle are 6 ft. apart and the distance between the two trains of vehicles is 4 ft.

Number of longitudinal girders in the bridge.	Fraction of wheel loads to be used for design of longitudinal girders of the bridge.	
	Girder 1.	Girder 2.
3	$\dfrac{r + 0.031\, h\alpha}{1 + 0.76\alpha}$	$\dfrac{r + 0.096 \sqrt{h}\, \alpha}{1 + 0.79\alpha}$
4	$\dfrac{r + 0.01\, h\alpha}{1 + 0.208\,\alpha}$	$\dfrac{r + 0.013 \sqrt{h}\, \alpha}{1 + 0.12\,\alpha}$

where,

r = the fraction of the continuous beam reaction, calculated for the beam under consideration, assuming four equal loads along the cross-section as shown in the figure 8. The value of 'r', for the spacing adopted in the thesis are given in Table 13.

h = the spacing of the longitudinal beams in ft.

α = a dimensionless parameter given by $\dfrac{12}{\pi^4} \left(\dfrac{L}{h} \right)^3 \dfrac{E_T I_T}{EI}$

V. EXPERIMENTAL VERIFICATION

19. The Structure

The structure chosen was a 60 ft. span, simply supported, right, T-beam bridge, having four longitudinal and five transverse beams. This bridge was designed for class A two lane loading, adopting Hendry-Jaeger method of analysis. While considering the design loads, no allowance for impact was taken because in the test it was proposed not to impart any impact to the structure.

The actual structure tested was a model of this prototype bridge having a linear scale ratio of 1/5. All the dimensions of the prototype bridge elements were reduced to the corresponding scale and the so obtained geometrically similar model bridge was checked for the corresponding reduced loads and found to develop the adopted design stresses at these loads. The principal dimensions of the model bridge are given in Fig. 19 . The ends of the beams were supported on rollers through 1/2 inch mild steel plates giving perfect simply supported condition. A general view of the bridge at the time of testing is shown in Fig. 23-2. Besides this, several constructional, loading and testing features are illustrated in the accompanying photographs. A model of one longitudinal T-beam of bridge, henceforth called 'the equivalent beam' was also simultaneously casted having a linear scale ratio of 1/2 with respect to the model bridge.

20. Scale relationship

(a) Linear scale ratio 1/5.......................... 28

(b) $M = Qbd^2$

where,

M = Total moment of resistance of a beam

b = Width of the beam

d = depth of the beam

Q = a constant

$$\frac{M_m^{*}}{M_p^{\neq}} = \frac{b_m}{b_p} \frac{d_m^2}{d_p^2} = \frac{1}{125} \qquad \ldots 29$$

(c) $I = K\, bd^3$

where,

I = moment of inertia of the section

$$\frac{I_m}{I_p} = \frac{b_m}{b_p} \frac{d_m^3}{d_p^3} = \frac{1}{625} \qquad \ldots 30$$

(d) $A_t = \frac{M}{Jd\, f}$

where,

A_t = Total area of tensile reinforcement

jd = Lever arm of the section

f = permissible stress in tensile reinforcement

$$\frac{A_{tm}}{A_{tp}} = \frac{M_m}{M_p} \frac{d_p}{d_m} \frac{f_p}{f_m} = \frac{1}{25} \qquad \ldots 31$$

* Subscript m stands for model, thus, Mm means total moment of resistance of the model beam.

≠ Subscript p stands for prototype, thus Mp means total moment of resistance of the prototype beam.

(e) (i) In a simply supported beam of span length L, bending moment 'M' at a distance 'x' due to a concentrated load 'W' acting at a distance 'a' from the support is given by

$$M = \frac{Wx\,(L-a)}{L} \qquad x < a$$

$$\text{or} \quad W = \frac{ML}{x(L-a)}$$

$$\therefore \frac{W_m}{W_p} = \frac{M_m}{M_p} \cdot \frac{L_m}{L_p} \quad \frac{x_p\,(L-a)_p}{x_m\,(L-a)_m} = \frac{1}{25} \qquad \ldots 32$$

Same will hold good for $x > a$

(ii) In a simply supported beam of span L, bending moment 'M' at a distance 'x', due to a uniformly distributed load 'W' per ft length over the entire length, is given by

$$M = \frac{wx\,(L-x)}{2}$$

$$\text{or} \quad w = \frac{2M}{x\,(L-x)}$$

$$\therefore \frac{w_m}{w_p} = \frac{M_m}{M_p} \quad \frac{x_p\,(L-x)_p}{x_m\,(L-x)_m} = \frac{1}{5} \qquad \ldots 33$$

(iii) Let a load $\bar{w}$ per unit area be acting on the slab of area 'A' and span length 'L', the total moment 'M' on the slab is given by

$$M = K\,\bar{w}\,AL$$

$$\text{or} \quad \bar{w} = \frac{M}{KAL}$$

$$\therefore \frac{\bar{w}_m}{\bar{w}_p} = \frac{M_m}{M_p} \cdot \frac{A_p\,L_p}{A_m\,L_m} = 1 \qquad \ldots 34$$

Summary

(1) Relation 28 means that all the linear dimensions such as length, breadth, depth, deflection, longitudinal and transverse spacing of the bridge elements, longitudinal and transverse spacing of the concentrated loads are reduced to 1/5 their prototype dimensions. Further the volume of the model hence the weight of the model will be $\frac{1}{125}$ of prototype.

(2) Relations 33 and 34 give that the dead load of the model, should be 1/25 of prototype, hence a total dead load of 5 times or an additional surface load of 4 times the dead weight of model is required for true model.

(3) Relation 31 shows that the total area of tensile reinforcement is 1/25 of prototype, that is, the area of tensile reinforcement in the beams may be 1/25 of prototype and that in the slab per ft. width, 1/5 of prototype.

(4) Relation 5 means that all the wheel loads of the two lane class A loading will be reduced to 1/25 of prototype.

(5) Material used being same in prototype and model, E, C, and ρ and three dimensionless parameters α, β and η will remain the same.

21. Object and Scheme of Loading

The specific objectives of the testing programme were to obtain moment and deflection data for reinforced concrete bridges which could be corelated with the theoretical values. Strain gauges and dial gauges were used to measure moments and deflections. For each set of loading, all the gauges were read. The loading was done in the following stages.

(i) Single calculated concentrated loads were applied to develop design stresses at the loaded points at each of the points B_1 , B_2 , B_3 , B_4 , D_1 , D_2 , D_3 , D_4 , C_1 , C_2 , C_3 , C_4 , F_1 and F_2 (Fig. 19). This type of loading was required to draw the the experimental influence linesof moments.

(ii) Superimposed surface load was spread over the bridge so as to simulate the conditions of true dead load on the model. observations of this loading can be used in estimating the distribution of moment due to self load of the bridge.

(iii) Two lane class A loading is applied on the bridge. The positioning of the wheels was arranged to cause maximum moment on the second beam. This was used to compare the distribution of bending moment on various beams when all the four wheels of two lane class A loading act along a section and as found from the experimental influence diagrams drawn from individual loadings.

(iv) Finally two lane class A loading is gradually increased to failure. This was used to study the variation in distribution of moment, as the load is increased to failure.

21. Test Equipments

(a) Measuring Instruments

(i) Strain Gauges- Thirty two locally made strain gauges of Rohits & Co. Roorkee (India) were used for measuring strains. These gauges wereput on the reinforcing bars at the following locations. One strain gauge each on two bars of the lower tier of tensile steel of longitudinal beam at midspan, two nominal bars of longitudinal beam in slab at midspan, two bottom bars of the central cross beam at its junction with longitudinal beams and two top bars of the central cross beam in slab at its junction with longitudinal beams.

The strain gauges were cemented to each bar with quickfix (cement) after surfacing the bars with fine emery paper and scrupulously cleaning them with volatile solvents namely, Tolune, Acetate and Acetane. A pair of wire was soldered to each gauge and a liberal coat of wax was applied over the gauges and exposed soldered leads for water-proofing. Lead wires were taken to a central station where all readings were made. SR4 strain indicator was used to measure the strains of these strain gauges.

(ii) Dial gauges- Four-dial gauges with magnetic bases were used to measure the deflections of longitudinal beams. One dial gauge below each longitudinal beam within a few inches of mid-span and as close to the strain gauges as possible, was mounted on a steel girder supported on two independent masonry columns erected for the purpose.

(b) Loading

For applying loads, a 25-ton hydraulic jack was used. The calibration of the jack was checked in a compression testing machine. The concentrated loads were applied on an area of 2" x 4" through a rubber sponge by the hydraulic jack. It was found impracticable to apply all loads of two lane class A loading, the bridge was loaded with only eight heaviest wheels of the two lane class A loading. These eight equal loads are developed from a single jack through a grid work of simply supported beams as per details given in Fig. 23·3.

2. Testing Programme

The bridge was cast on February 17, 1962 with a number of 6" cubes and 6" x 10" cylinders. The reinforcement was kept uncovered at the strain gauge points. The wet curing was done for twenty eight days, after which the shuttering was removed and the measuring instruments were installed. The loading test started on April 25, 1962. To start with calculated concentrated loads to develop design stresses were applied at all the points without recording any observation. This loading developed hair cracks in concrete of tension zone at various points thereby bringing the concrete in a stage similar to a prototype bridge open to traffic so that the influence diagrams may be of more practical value. The sequence of loading was kept as mentioned earlier. As the bridge is symmetrical about longitudinal and transverse centre lines, the single concentrated loading at many a points were merely a repetition and this was done to get a better average influence coefficients. Each concentrated load is applied on the bridge on a width of 4" (dimension across the bridge) and length of 2" (dimension along the bridge). On the point to be loaded, first a rubber sponge of 2"x4" was kept, over it a mild steel plate of same dimension was placed and over this the jack was oriented such that the central vertical axes of all the three coincided with the loaded point. Having obtained sufficient data for transverse influence lines and deflections at 0.243L, 0.372L and 0.50L, superimposed surface load was spread over the bridge. The strains and deflections were recorded for this and then the grid work for eight heaviest wheels of two lane class A loading was arranged symmetrically about the transverse centre line of the bridge causing maximum moments on the second beam. To start with, a load of one ton was applied by the jack and readings were taken. Then the experiment continued for gradually increased load. At about 10 ton load which corresponded to three times design live load moment the readings were stopped. Because the strain gauges were observed to be slipping and to guard against any damage, dial gauges were removed. The load was gradually increased to 20 tons after which the bridge stopped taking load and started deflecting. Excessive cracks were observed on longitudinal beams 1,2,3, and in centre of central cross beam. The bridge test was concluded on May 6, 1962 with the collapse of the bridge.

The equivalent beam was tested on a five ton beam testing machine. The beam was loaded at the centre by a concentrated load which was gradually increased upto ultimate value. Strains and deflections were recorded for this loading test separately.

24. Test Results

From the test on equivalent beam, a graph was plotted with recorded strains as abscissa and applied bending moment as ordinate. The bending moment scale was multiplied by the constant factor (derived from the model analysis relationships) to use it for the longitudinal beams of the model bridge. This graph of Fig. 20. was used to determine the bending moment on the various beams from the recorded strain readings. The total bending moment on the various beams, so obtained, is adjusted in proportion of their experimental values so as to satisfy the statics. The discrepancy was never above 5% for any beam. These with theoretical results are given in Tables 14 & 15 for single concentrated and uniformly distributed loads and also on the influence diagrams in Fig. 21 . In table 19 is given the variation in the percentage of bending moment on the beams as the total live load increases from one ton to ten tons, the latter corresponds to about three times the design live load. The corresponding graph is given in Fig. 22 . The experimental influence diagrams for mid-span section gives design moments as 29.7% and 31.9% for exterior and interior beams respectively against 31.9% and 31.3% theoretical values.

Deflection Readings- In calculating the deflections, the knowledge of flexural rigidity EI is required. To get this, a number of cubes and cylinders were cast simultaneously when the bridge was constructed, and tested for crushing strengths. The crushing strength f_c is found to be 1.2 tons per sq.in. which gives the modular ratio m as 15, using the formula $m = \frac{40000}{f_c}$. Hence $E = 2 \times 10^6$ lbs/sq.in. Moment of inertia 'I' of the longitudinal beams can be calculated in either of the following ways (I.S: 456 - 1957)[13]:

(1) Compression area of the concrete section, combined with the reinforcement on the basis of the modular ratio.

(2) The entire concrete section ignoring the reinforcement.

(3) The entire concrete section including the reinforcement on the basis of the modular ratio.

Calculations were done with each of these methods. The deflections were found in closer agreement under the first assumption. Deflections under the first assumption for 'I' and the measured deflections are given in Table 17 for concentrated loads and in Table 18 for increasing loads of the two lane class A loading.

25. Discussion of Results

(a) Single-concentrated Load

The bending moment as obtained from the observed strain readings on the four beams when summed together were found to be less than the corresponding static bending moment due to a single point load on the bridge. This may be due to the use of locally made strain gauges of which the batching and workmanship was poor and a little difference in the gauge of equivalent beam and of bridge might have lead to a constant error in all readings. So, the observed readings were increased in proportion of their experimental values so as to satisfy the statics.

The adjusted experimental values in Table 14 show a little better distribution of bending moment than that obtained theoretically by Hendry-Jaeger method. This may be due to the following reasons:

(i) The concentrated loads are assumed to be acting as point loads where as they are acting on a definite contact area.

(ii) The dispersion of load along the length of the beam is neglected in the theory. This depends on the distance of the beam from the load. Hence, the error due to this approximation will be more when load is on exterior beam than when the load is on interior beam. This is also revealed by the experimental observations.

(iii) The torsional rigidity of the transverse system is neglected which also helps in the distribution of the load.

The difference in the theoreticaland experimental percentage values for a single concentrated load as given in Table 14 is not of much practical significance so far as the design of the bridge is concerned, because for design, a number of wheel loads are to be considered acting simultaneously on any transverse section. The superposition of the percentages due to individual load has the effect of equalising the percentages on the various beams so that little differences are levelled up in the case of many loads as above.

(b) Two-Lane Class A Loading

As shown in Fig. 22, two lane class A loading test on the bridge for second beam shows that there is very small change in the distribution of bending moment to various beams as the load increases, and it can be said that alteration in distribution is very small. At ultimate load it was observed that though excessive cracks developed in beams 1,2 and 3, only fine cracks were seen in the fourth beam though the eccentricity of the loading about the longitudinal centre line of the bridge was only 0.23h where 'h' is the spacing of longitudinal beams. As mentioned earlier for the second beam, the experimental and theoretical design percentages are 31.9 and 31.3 respectively differing by 0.6% only. The above two percentages are calculated from experimental and theoretical influence diagrams. But if the percentages were derived on the basis of measured strains under the action of the 2 lane loads, the second beam is found to carry 33.7% at design stresses. This difference of 2.4% against a former difference of only 0.6% indicates that the superposition of results, as is required in calculating design percentages from influence diagrams, is not quite correct and may lead to some error in arriving at the design percentages of various beams. Hence, a conservative attitude in fixing these percentages is desirable.

(c) Deflections

Measurements of deflections in tests of bridges are always of value since the deflections are of interest in themselves. Besides, these can be used to estimate fairly accurately the load distribution to the various beams. To calculate the distribution of loading in this manner, the measured deflections should be multiplied by the stiffness of the corresponding beams. Since only percentage distribution is often desired, only relative values of the rigidities 'EI' for the beams need be considered. Excepting in a few cases, there is a remarkable correspondence between the measured and theoretical deflections. The deflections are calculated by Hendry-Jaeger method and thus, the method adopted is further verified, and hence is recommended for design of interconnected bridge girders.

VI. CONCLUSIONS

1. An experimental study for the fraction of the free bending moment carried by the various longitudinals in an interconnected bridge has been made and illustrated by a one-fifth model of 60 ft. simply-supported four-girder bridge. The conclusions arrived at are:

(a) In an interconnected bridge, the results obtained by Hendry-Jaeger method which is easy to apply and less time consuming, gives results in close agreement to those obtained experimentally and this method may be used for the design of common slab and beam bridges without involving into any appreciable error.

(b) There is no marked variation in the lateral load distribution with increase of load. upto ultimate capacity of the bridge.

2. For usual interconnected bridges β may be assumed equal to zero in determining the fractions of wheel loads carried by the various girders.

3. A uniform value of the fraction of wheel loads carried by the girder, based upon its value at mid-span for load at mid-span, may be adopted for finding the moments at all points along the span. Thus, for any wheel loading and for any section of the bridge the influence lines of Figs. 9 to 16 can be used for finding design moments.

4. The following formulae are proposed by the author for calculating the fraction of wheel loads for any loading in which wheels of an axle are 6 ft. apart and the distance between

the two trains of vehicles is 4 ft.

Number of longitud-inal gird-ers in the bridge.	Fraction of wheel loads to be used for design of longitudinal girders of the bridge.	
	Girder 1	Girder 2
3	$\frac{r + 0.031\, h\alpha}{1 + 0.76\,\alpha}$	$\frac{r + 0.096\sqrt{h}\,\alpha}{1 + 0.79\,\alpha}$
4	$\frac{r + 0.01\, h\alpha}{1 + 0.208\,\alpha}$	$\frac{r + 0.013\sqrt{h}\,\alpha}{1 + 0.12\,\alpha}$

where,

r = the fraction of the continuous beam reaction, calculated for the beam under consideration, assuming four equal loads along the cross-section as shown in the figure 8. The value of r, for the spacing adopted in the thesis are given in Table 13.

h = the spacing of the longitudinal beams in ft.

α = a dimensionless parameter given by $\frac{12}{\pi^4}\left(\frac{L}{h}\right)^3 \frac{E_T I_T}{E I}$.

REFERENCES

1. I.R.C. Bridge Code Section III (under draft).

2. LAZARIDES, T.O., " The Design and Analysis of Openwork Prestressed Concrete Beam Grillages", Civil Engineering and Public Works Review, London, June 1952. Vol. 47.

3. EWELL, OKUBO AND ABRAMS, "Deflection in Grid Works and Slabs". Transactions of the American Society of Civil Engineers, Vol.117, 1952.

4. GUYON, Y., " Calcul des Ponts Larges a Poutres Multiples Solidarisees par des Entretoises." (The Calculation of Wide Multiple Beam Bridges with Diaphragm Stiffners). Annales des Ponts at Chausees, No.24, Paris, September-October, 1946.

5. MASSONNET, C., Methode de Calcul des Ponts a Poutres Multiples Tenant Compte de leur Resistance a la Torsion (A method for Calculation of Multiple Beam Bridges taking into account their Torsional Resistance.) Publications, International Association for Bridge and Structural Engineering Zurich, 1950, Vol. X.

6. Pippard, A.J.S. AND DEWAELE, J.P.A., "The Loading of Interconnected Bridge Girders", Journal of the Institution of Civil Engineers, London, 1938, Vol. 10.

7. LEONHARDT, F. AND ANDRA, W., " Die Vereinfachte Trägerrostberechnung" (The Calculation of Grillage Beams) Julius Hoffman Press, Stuttgart, 1950.

8. HETENYI, M., " A Method for Calculating Grillage Beams," S.Timoshenko 60th Anniversary Volume, New York, 1938.

9. NEWMARK, N.M., SIESS, C.P., AND OTHERS, "Studies of Slab and Beam Highway Bridges", Parts I-V, University of Illinois Engineering Experiment Station, Bulletin No.363, 375, 396, 405, and 416.

10. HENDRY-JAEGER, "The Analysis of Grid Frameworks and Related Structures". Published by Chatto and Windus Ltd., 42, William IV Street, London W.C.2.

11. GUPTA, BRAJENDRA KUMAR, M.E.THESIS, "A Critical Review of the Load Distribution Theories of Interconnected Bridge Girders", Department of CivilEngineering, University of Roorkee- India, 1962.

12. HENDRY, A.W., AND JAEGER, L.G., " A General Method for the Analysis of Grid Frameworks", Proceedings of Institution of Civil Engineers, London Part III, Vol.4, No.3 Dec. 1955.

13. Indian Standard Code of Practice for Plain and Reinforced Concrete for General Building Construction (Revised) IS:456-1957.

14. MORLEY ARTHUR ,."Strength of Materials"Published by Longmans, Green, and Co. London, New York, Toronto. Batch 1953 Page 326.

TABLE 1

THREE GIRDER BRIDGE

INFLUENCE COEFFICIENTS FOR MOMENTS IN GIRDERS AT QUARTER SPAN FOR UNIT LOAD MOVING TRANSVERSELY ON THE BRIDGE; $\beta = 0$

Influence Ordinate for Moment = Coeff. x L

α	Position of the load	Moment in Girder	Position of the unit load						
			Overhanging Girder 1 by h/2	Girder 1	Midway of Girder 1-2	Girder 2	Midway of Girder 2-3	Girder 3	Overhanging Girder 3 by h/2
1	L/4	1	.303	.180	.084	.015	-.010	-.008	-.002
		2	-.091	.015	.113	.158	.113	.015	-.091
	L/2	1	.202	.115	.062	.021	-.001	-.010	-.018
		2	-.027	.021	.064	.083	.064	.021	-.027
10	L/4	1	.301	.170	.095	.036	.001	-.018	-.035
		2	-.025	.036	.091	.116	.091	.036	-.025
	L/2	1	.198	.104	.073	.042	.011	-.021	-.053
		2	.042	.042	.042	.041	.042	.042	.042
100	L/4	1	.299	.163	.102	.049	.009	-.025	-.057
		2	.020	.049	.077	.090	.077	.049	+.020
	L/2	1	.197	.103	.072	.044	.010	-.022	-.056
		2	.050	.044	.040	.037	.040	.044	.050

TABLE 2

FOUR GIRDER BRIDGE
INFLUENCE COEFFICIENTS FOR MOMENTS IN GIRDERS AT QUARTER SPAN FOR UNIT LOAD MOVING TRANSVERSELY ON THE BRIDGE; $\beta = 0$

Influence Ordinate for Moment = Coeff. x L

α	Position of load	Moment in girder	Position of the Unit Load						
			Girder 1	Midway of girder 1 & 2	Girder 2	Midway of girder 2 & 3	Girder 3	Midway of girder 3 & 4	Girder 4
1	L/4	1	.180	.082	.014	-.008	-.005	-.001	-.001
		2	.014	.114	.156	.102	.023	-.008	-.005
	L/2	1	.112	.060	.020	-.001	-.007	-.005	0
		2	.020	.060	.079	.063	.033	.011	-.007
10	L/4	1	.164	.091	.036	.009	-.001	-.007	-.011
		2	.036	.090	.116	.085	.037	.010	-.001
	L/2	1	.095	.068	.043	.021	.003	-.008	-.016
		2	.043	.043	.043	.042	.036	.022	.003
100	L/4	1	.149	.098	.055	.023	.008	-.008	-.024
		2	.055	.079	.086	.067	.039	.019	.008
	L/2	1	.085	.070	.053	.030	.014	-.007	-.027
		2	.053	.042	.033	.028	.025	.020	.014

TABLE 3

THREE GIRDER BRIDGE
THEORETICAL FRACTION OF WHEEL LOADS
CARRIED BY LONGITUDINAL GIRDERS; $\beta = 0$

Girder	α	Spacing of the longitudinal girders 'h'					
		8 ft.			11 ft.		
		a = L/2 x = L/4	a = L/2 x = L/2	a = L/4 x = L/4	a = L/2 x = L/4	a = L/2 x = L/2	a = L/4 x = L/4
1	1	0.305	0.295	0.295	0.395	0.380	0.370
	10	0.340	0.320	0.310	0.475	0.435	0.410
	100	0.330	0.320	0.310	0.470	0.445	0.430
2	1	0.370	0.390	0.410	0.490	0.525	0.575
	10	0.335	0.360	0.375	0.335	0.420	0.475
	100	0.335	0.350	0.350	0.315	0.380	0.390

TABLE 4

FOUR GIRDER BRIDGE

Theoretical FRACTION OF WHEEL LOADS CARRIED BY THE LONGITUDINAL GIRDERS; $\beta = 0$

Girder	α	Spacing of longitudinal girders 'h' 6 ft.			8 ft.		
		a = L/2 x = L/4	a = L/2 x = L/2	a = L/4 x = L/4	a = L/2 x = L/4	a = L/2 x = L/2	a = L/4 x = L/4
1	1	0.245	0.245	0.245	0.305	0.295	0.295
	10	0.290	0.270	0.260	0.365	0.340	0.325
	100	0.300	0.285	0.280	0.400	0.375	0.360
2	1	0.300	0.310	0.315	0.400	0.430	0.435
	10	0.290	0.295	0.300	0.320	0.365	0.385
	100	0.260	0.275	0.280	0.265	0.315	0.330

TABLE 5

THREE GIRDER BRIDGE

INFLUENCE COEFFICIENTS FOR FRACTIONS OF UNIT LOAD CARRIED BY LONGITUDINAL GIRDERS ; $\beta = 0$

Moment in girder	α	Position of the Unit Load: Overhang girder 1 by h/2	Girder 1	Midway of girder 1 & 2	Girder 2	Midway of girder 2 & 3	Girder 3	Overhanging girder 3 by h/2
1	1	1.432	.940	.468	.116	-.032	-.056	-.060
	5	1.276	.892	.520	.212	.020	-.104	-.216
	10	1.236	.880	.536	.240	.036	-.120	-.264
	20	1.204	.872	.544	.260	.044	-.132	-.300
	50	1.184	.864	.552	.272	.052	-.136	-.316
	100	1.172	.860	.556	.280	.056	-.140	-.328
2	1	-.372	.116	.564	.768	.564	.116	-.372
	5	-.060	.212	.460	.576	.460	.212	-.060
	10	.032	.240	.432	.520	.432	.240	.032
	20	.096	.260	.412	.480	.412	.260	.096
	50	.136	.272	.396	.456	.396	.272	.136
	100	.160	.280	.388	.440	.388	.280	.160

TABLE 6

THREE GIRDER BRIDGE
INFLUENCE COEFFICIENTS FOR FRACTIONS OF UNIT LOAD CARRIED BY LONGITUDINAL GIRDERS; $\beta = \infty$

Moment in girder	α	Position of the Unit Load						
		Overhanging Girder 1 by h/2	Girder 1	Midway of Girder 1 & 2	Girder 2	Midway of Girder 2 & 3	Girder 3	Overhanging Girder 3 by h/2
1	1	1.296	.864	.452	.144	.016	-.008	-.012
	5	.924	.676	.436	.244	.136	.080	.032
	10	.788	.588	.396	.252	.184	.160	.144
	20	.688	.532	.384	.268	.216	.200	.192
	50	.576	.468	.364	.288	.252	.244	.240
	100	.520	.436	.356	.296	.272	.268	.268
2	1	.280	.144	.532	.712	.532	.144	-.028
	5	.040	.244	.432	.512	.432	.244	.040
	10	.064	.252	.424	.496	.424	.252	.064
	20	.120	.268	.404	.464	.404	.268	.120
	50	.188	.288	.380	.424	.380	.288	.188
	100	.208	.296	.376	.408	.376	.296	.208

TABLE 7

FOUR GIRDER BRIDGE

INFLUENCE COEFFICIENTS FOR FRACTIONS OF UNIT LOAD CARRIED BY LONGITUDINAL GIRDERS; $\beta = 0$

Moment in Girder	α	Position of the Unit Load: Girder 1	Midway of Girder 1 & 2	Girder 2	Midway of Girder 2 & 3	Girder 3	Midway of Girder 3 & 4	Girder 4
1	1	.928	.460	.112	-.028	-.040	-.024	0
	5	.856	.496	.212	.060	-.004	-.040	-.064
	10	.828	.504	.244	.092	.016	-.040	-.088
	20	.796	.520	.288	.136	.036	-.044	-.120
	50	.776	.520	.308	.164	.060	-.040	-.144
	100	.756	.524	.332	.184	.068	-.044	-.156
2	1	.112	.552	.740	.528	.188	.012	-.040
	5	.212	.448	.540	.440	.244	.100	-.004
	10	.244	.462	.524	.408	.216	.092	.016
	20	.288	.432	.476	.364	.200	.092	.036
	50	.308	.420	.444	.336	.188	.104	.060
	100	.332	.400	.404	.316	.196	.120	.068

TABLE 8

FOUR GIRDER BRIDGE

INFLUENCE COEFFICIENTS FOR FRACTIONS OF UNIT LOAD CARRIED BY LONGITUDINAL GIRDERS; $\beta = \infty$

Moment in gir-der	α	Position of the Unit Load						
		Girder 1	Midway of Girder 1 & 2	Girder 2	Midway of girder 2 & 3	Girder 3	Midway of Girder 3 & 4	Girder 4
1	1	.856	.444	.144	.020	0	0	0
	5	.668	.428	.228	.124	.080	.052	.024
	10	.584	.388	.232	.148	.112	.088	.072
	20	.504	.348	.232	.176	.152	.136	.112
	50	.432	.320	.232	.196	.184	.172	.152
	100	.384	.300	.236	.204	.196	.188	.184
2	1	.144	.524	.680	.480	.176	.032	0
	5	.228	.416	.488	.376	.204	.112	.080
	10	.232	.388	.488	.352	.208	.136	.112
	20	.232	.360	.404	.324	.212	.160	.152
	50	.232	.328	.364	.304	.220	.184	.184
	100	.236	.316	.344	.296	.224	.196	.196

TABLE 9

THREE GIRDER BRIDGE

FRACTION OF WHEEL LOADS CARRIED BY LONGITUDINAL GIRDERS

Moment in girder	α	Spacing of longitudinal girders							
		8 ft.		9 ft.		10 ft.		11 ft.	
		$\beta = 0$	$\beta = \infty$	$\beta = 0$	$\beta = \infty$	$\beta = 0$	$\beta = \infty$	$\beta = 0$	$\beta = \infty$
1	1	.295	.305	.325	.330	.355	.350	.380	.380
	5	.310	.320	.350	.340	.385	.360	.415	.380
	10	.320	.320	.365	.335	.400	.345	.435	.360
	20	.320	.325	.365	.330	.400	.340	.440	.350
	50	.320	.325	.365	.330	.405	.340	.445	.345
	100	.320	.325	.370	.330	.410	.335	.445	.345
2	1	.390	.390	.450	.435	.490	.485	.525	.515
	5	.370	.360	.410	.380	.430	.400	.450	.420
	10	.360	.355	.385	.380	.405	.400	.420	.410
	20	.360	.355	.380	.375	.395	.390	.410	.405
	50	.350	.345	.370	.360	.385	.365	.400	.375
	100	.350	.345	.360	.355	.370	.365	.390	.370

TABLE 10
FOUR GIRDER BRIDGE
FRACTION OF WHEEL LOADS CARRIED
BY LONGITUDINAL GIRDERS

Moment in girder	α	Spacing of longitudinal girders					
		6 ft.		7 ft.		8 ft.	
		$\beta = 0$	$\beta = \infty$	$\beta = 0$	$\beta = \infty$	$\beta = 0$	$\beta = \infty$
1	1	.250	.250	.265	.270	.295	.295
	5	.260	.265	.290	.285	.315	.310
	10	.270	.260	.310	.280	.340	.295
	20	.280	.255	.325	.270	.360	.285
	50	.285	.255	.330	.265	.370	.270
	100	.285	.250	.335	.260	.375	.265
2	1	.310	.305	.375	.360	.430	.400
	5	.300	.280	.340	.315	.380	.345
	10	.295	.275	.340	.305	.365	.325
	20	.285	.270	.315	.285	.340	.300
	50	.280	.260	.305	.275	.325	.290
	100	.275	.260	.295	.270	.315	.280

TABLE 11

THREE GIRDER BRIDGE

FRACTION OF WHEEL LOADS CARRIED BY LONGITUDINAL GIRDERS

Girder	α	Spacing of longitudinal girders							
		8 ft		9 ft.		10 ft.		11 ft.	
		$\beta = 0$	Author's formula	$\beta = 0$	Author's formula	$\beta = 0$	Author's formula	$\beta = 0$	Author's formula
1	1	.295	.295	.325	.320	.355	.350	.380	.380
	5	.310	.315	.350	.350	.385	.385	.415	.425
	10	.320	.320	.365	.355	.400	.395	.435	.435
	20	.320	.325	.365	.360	.400	.400	.440	.440
	50	.320	.325	.365	.365	.405	.405	.445	.446
	100	.320	.325	.370	.365	.410	.405	.445	.445
2	1	.390	.385	.450	.450	.490	.500	.525	.540
	5	.370	.360	.410	.395	.430	.425	.450	.450
	10	.360	.355	.385	.380	.405	.410	.420	.430
	20	.360	.350	.380	.375	.395	.395	.410	.415
	50	.350	.345	.370	.370	.385	.390	.400	.410
	100	.350	.345	.360	.365	.370	.385	.390	.405

TABLE 12

FOUR GIRDER BRIDGE

FRACTION OF WHEEL LOADS CARRIED BY LONGITUDINAL GIRDERS

Girder	λ	Spacing of longitudinal girders					
		6 ft.		7 ft.		8 ft.	
		$\beta = 0$	Author's formula	$\beta = 0$	Author's formula	$\beta = 0$	Author's formula
1	1	.250	.250	.265	.270	.295	.290
	5	.260	.265	.290	.300	.315	.330
	10	.270	.275	.310	.310	.340	.350
	20	.280	.280	.325	.320	.360	.365
	50	.285	.285	.330	.330	.370	.375
	100	.285	.285	.335	.335	.375	.380
2	1	.310	.305	.375	.390	.430	.445
	5	.300	.295	.340	.360	.380	.400
	10	.295	.285	.340	.340	.365	.375
	20	.285	.280	.315	.320	.340	.350
	50	.280	.270	.305	.305	.325	.330
	100	.275	.270	.295	.295	.315	.320

TABLE 13

FRACTION OF THE CONTINUOUS BEAM REACTION 'r'

Number of longitudinal girders in the bridge	Spacing of the longitudinal girders in ft.	'r' for moment in the girder	
		1	2
3	8	0.272	0.420
	9	0.285	0.516
	10	0.305	0.590
	11	0.328	0.649
4	6	0.241	0.311
	7	0.257	0.403
	8	0.272	0.460

TABLE 14

EXPERIMENTAL RESULTS FOR MOMENTS IN LONGITUDINAL GIRDERS OF MODEL BRIDGE FOR VARIOUS CONCENTRATED LOAD POSITIONS

$\alpha = 29.1$, $\beta = 0.05$, $\eta = 0.95$

Load Tons	Position of load	Total static Moment in-lb x 10^3	Girder	Strains, Micro Inch per inch: Observed		Strains, Micro Inch per inch: Mean	Mean Moments inch lb x 10^3	Moments expressed as percentage of static Moments: Observed	Adjusted	Theoretical
2.80	0.243L on 1	110	1	300 350	345 355	340	53.5	48.6	47.3	57.0
			2	235 215	265 245	240	45.5	41.4	40.3	38.0
			3	95 90	95 125	100	27.5	25.0	24.4	13.1
			4	-50 -40	-45 -35	-45	-13.5	-12.3	-12.0	-8.1
2.24	0.372L on 1	134	1	410 515	550 500	495	70.5	52.6	56.9	61.6
			2	260 280	265 260	265	48.0	35.8	38.7	34.5
			3	65 50	85 65	65	19.0	14.2	15.3	10.9
			4	-55 -50	-40 -30	-45	-13.5	-10.1	-10.9	-7.0
2.10	0.50L on 1	169.5	1	620 740	605 710	670	105	62	62.2	68.0
			2	260 280	295 305	285	50	29.5	29.5	28.4
			3	40 95	75 120	85	24	14.2	14.2	9.2
			4	-45 -45	-15 -5	-30	-10	-5.9	-5.9	-5.6
4.75	0.243L on 2	186.5	1	275 345	325 330	320	52.5	28.2	28.7	36.9
			2	260 260	285 270	270	49.0	26.3	26.8	26.6
			3	270 330	265 325	300	50.5	27.1	27.6	25.6
			4	115 70	160 180	120	31.0	16.6	16.9	11.9
3.80	0.372L on 2	227.0	1	440	420	430	61.0	26.9	30.3	33.3
			2	445	405	425	60.5	26.6	30.0	32.0
			3	295	330	315	52.0	22.9	25.8	24.8
			4	100	105	105	28.0	12.3	13.9	9.9
3.60	0.50L on 2	290	1	385 495	425 445	440	62.5	21.6	23.7	27.6
			2	755 715	765 690	730	116	40.0	43.8	43.6
			3	340 350	405 395	375	56.0	19.3	21.1	20.8
			4	125 70	160 100	115	30.0	10.4	11.4	8.0

TABLE 15

EXPERIMENTAL RESULTS FOR MOMENTS IN LONGITUDINAL GIRDERS OF MODEL BRIDGE FOR TWO LANE LOADING AND DISTRIBUTED LOADING $\alpha = 29.1$; $\beta = 0.05$; $\eta = 0.95$

Load Tons		Static Moment In. lb. x 10^3		Girder	Strains Micro inch per inch.		Mean Moments Inch LBx10^3	Moments Inch LBx10^3		Moments expressed as percentage of Static Moments.					
										Two Lane Load			Sumperimposed Distributed Load		
Two Lane Load	Super-imposed Distr-ibuted load	Super-Imposed Dist-ributed Load.	Two Lane Load		Observed	Mean		Two lane load	Super-imposed Distri-buted load.	Obser-ved	Adjus-ted	Theo-reti-cal.	Obser-ved	Adjus-ted	Theo-retical
0	5.35	216	0	1	335,345	340	54.0	0	54.0	0	0	0	25.0	25.1	24.0
				2	340,325	335	53.5	0	53.5	0	0	0	24.8	24.9	26.0
				3	325,300	315	51.5	0	51.5	0	0	0	23.8	24.9	26.0
				4	310,330	320	52.0	0	52.0	0	0	0	24.0	25.1	24.0
1	5.35	216	75.3	1	490,505	495	70.5	16.5	54.0	21.9	25.6	26.8	25.0	25.1	24.0
				2	540,510	525	76.0	22.5	53.5	29.9	34.9	30.7	24.8	24.9	26.0
				3	480,465	470	67.0	15.5	51.5	20.6	24.0	26.9	23.8	24.9	26.0
				4	395,460	430	62.0	10.0	52.0	13.3	15.5	15.8	24.0	25.1	24.0
2	5.35	216	150.6	1	615,630	620	95.0	41.0	54.0	27.2	28.0	26.8	25.0	25.1	24.0
				2	670,635	650	100.5	47.0	53.5	31.2	32.1	30.7	24.8	24.9	26.0
				3	610,595	600	90.5	39.0	51.5	25.9	26.6	26.9	23.8	24.9	26.0
				4	465,530	500	71.5	19.5	52.0	13.0	13.3	15.8	24.0	25.1	24.0
3	5.35	216	225.9	1	665,735	700	110.0	56.0	54.0	24.8	24.9	26.8	25.0	25.1	24.0
				2	845,795	820	134.0	80.5	53.5	35.6	35.7	30.7	24.8	24.9	26.0
				3	700,685	690	109.0	57.5	51.5	25.4	25.5	26.9	23.8	24.9	26.0
				4	525,605	565	83.5	31.5	52.0	13.9	13.9	15.8	24.0	25.1	24.0
4	5.35	216	301.2	1	820,920*	820	134.0	80.0	54.0	26.6	23.8	26.8	25.0	25.1	24.0
				2	990,985	985	165.0	111.5	53.5	37.0	33.2	30.7	24.8	24.9	26.0
				3	870,860	865	142.5	91.0	51.5	30.2	27.1	26.9	23.8	24.9	26.0
				4	635,720	675	105.5	53.5	52.0	17.8	15.9	15.8	24.0	25.1	24.0
6	5.35	216	451.8	1	1020,1455*	1020	172.0	118.0	54.0	26.1	22.8	26.8	25.0	25.1	24.0
				2	1290,1315	1300	226.0	172.5	53.5	38.2	33.4	30.7	24.8	24.9	26.0
				3	1125,1125	1125	192.5	141.0	51.5	31.2	27.3	26.9	23.8	24.9	26.0
				4	790,890	840	137.5	85.5	52.0	18.9	16.5	15.8	24.0	25.1	24.0
8	5.35	216	602.4	1	1250,1845*	1250,	214.0	160.0	54.0	26.6	23.1	26.8	25.0	25.1	24.0
				2	1570,1600	1585	280.0	226.5	53.5	37.6	32.6	30.7	24.8	24.9	26.0
				3	1405,1410	1405	246.5	195.0	51.5	32.4	28.1	26.9	23.8	24.9	26.0
				4	935, 1045	990	166.0	114.0	52.0	18.6	16.2	15.8	24.0	25.1	24.0
10	5.35	216	753	1	1425,2080	1425	250.0	196.0	54.0	26.0	22.7	26.8	25.0	25.1	24.0
				2	1820,1885	1850	331.0	277.5	53.5	36.8	32.1	30.7	24.8	24.9	26.0
				3	1625,1720	1670	296.5	245.0	51.5	32.5	28.4	26.9	23.8	24.9	26.0
				4	1080,1220	1150	197.0	145.0	52.0	19.2	16.8	15.8	24.0	25.1	24.0

* Erroneous gauge reading.

TABLE 16

INFLUENCE COEFFICIENTS FOR MOMENTS
IN GIRDERS AT MIDSPAN FOR UNIT LOAD MOVING
TRANSVERSALY ON THE BRIDGE; α = 29.1, β=0.05, η= 0.95
M = Coefficient x WL

Moment in girder	Load at	Influence Coefficient at mid-span of girder							
		Experimental				Theoretical			
		1	2	3	4	1	2	3	4
1	.243L	.058	.035	.021	-.015	.069	.045	.015	-.010
	.372L	.106	.056	.026	-.020	.115	.062	.018	-.013
	.50 L	.156	.059	.029	-.015	.170	.069	.020	-.014
2	.243L	.049	.032	.034	.030	.046	.031	.031	.016
	.372L	.072	.056	.048	.028	.064	.060	.046	.020
	.50 L	.074	.109	.053	.035	.071	.109	.052	.023

TABLE 17

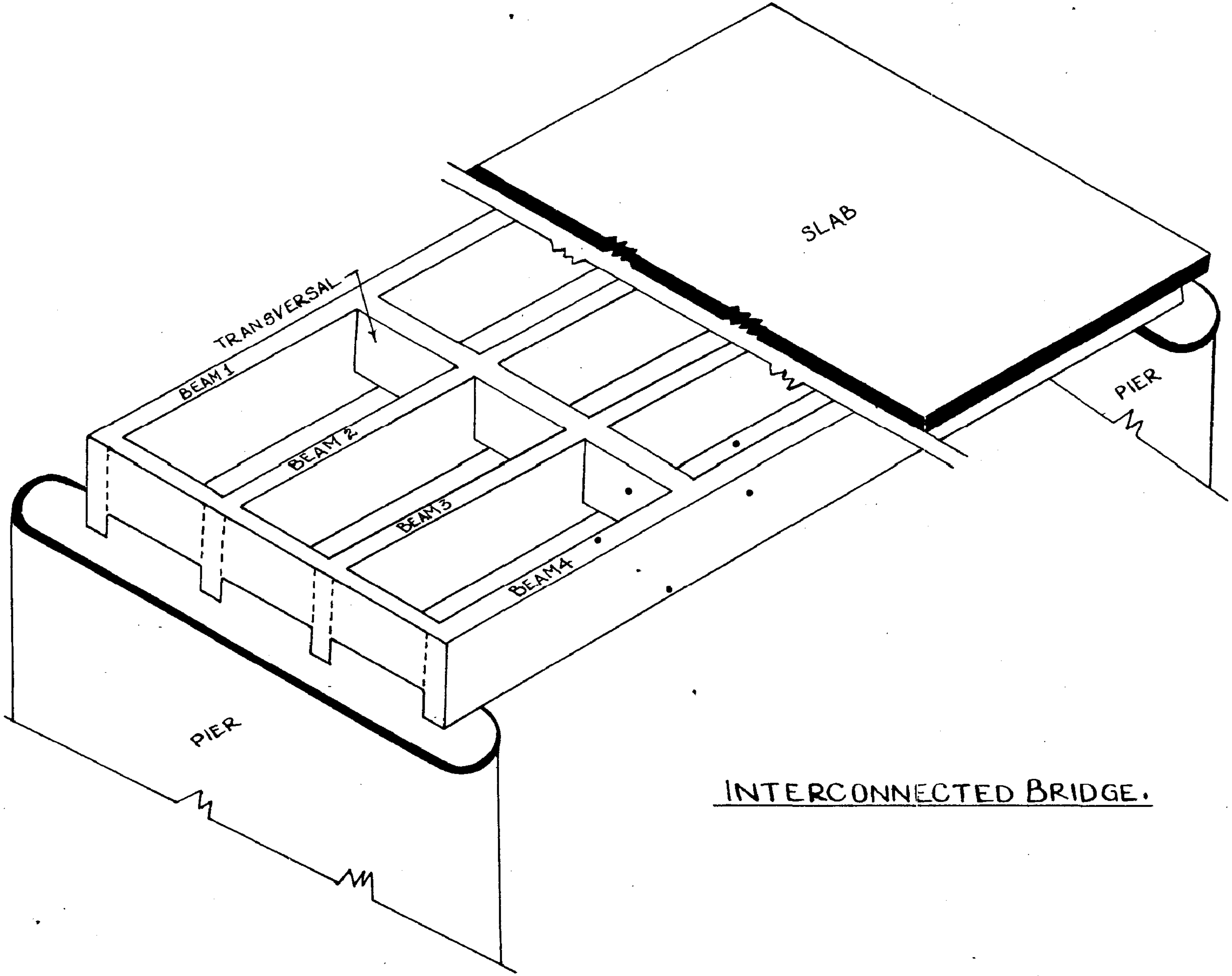
EXPERIMENTAL AND THEORETICAL DEFLECTIONS AT MID SPAN OF THE GIRDERS DUE TO CONCEN -TRATED LOADS AT VARIOUS POINTS: $\alpha = 29.1$ $\beta = 0.05$ $\eta = 0.95$ $I = 796\ in^4$.

Load Tons	Location of load	Deflection at centre of longitudinal girder in inches							
		Measured				Theoretical			
		1	2	3	4	1	2	3	4
2.80	.243L on 1	.093	.045	.006	-.026	.097	.051	.017	-.011
2.24	.372L on 1	.112	.055	.008	-.026	.104	.055	.018	-.012
2.10	.50 L on 1	.117	.054	.006	-.029	.097	.056	.018	-.011
4.75	.243L on 2	.086	.084	.059	.026	.088	.084	.061	.027
3.80	.372L on 2	.092	.090	.056	.022	.094	.090	.065	.028
3.60	.50 L on 2	.094	.101	.067	.027	.096	.095	.067	.029

TABLE 18

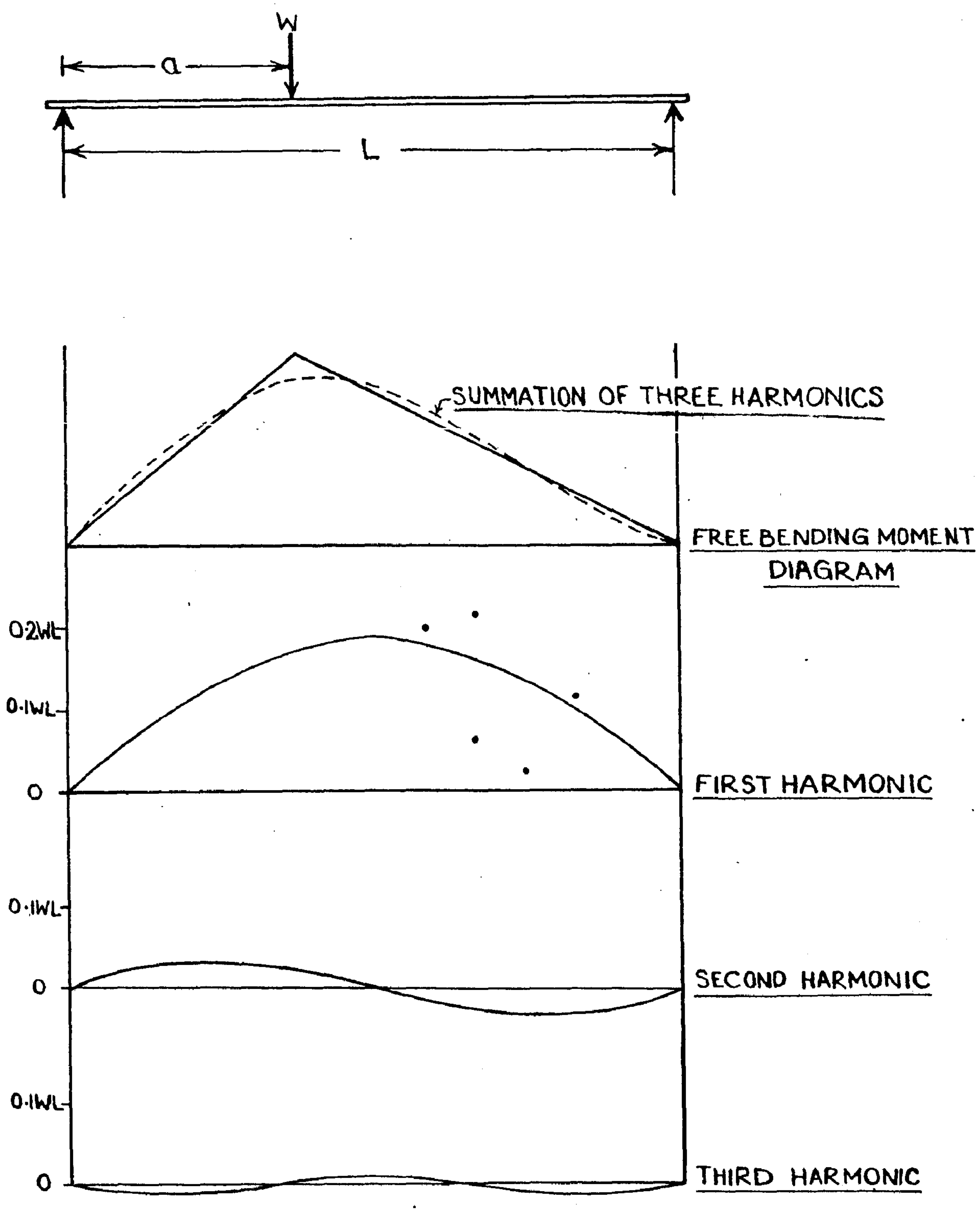
EXPERIMENTAL AND THEORETICAL DEFLECTIONS
AT MIDSPAN OF GIRDERS DUE TO TWO LANE
LOADING ARRANGED FOR GIRDER 2 OF THE BRIDGE;
$\alpha = 29.1 \quad \beta = 0.05 \quad \eta = 0.95 \quad I = 796 \text{ in}^4$

Load Tons	Deflection at centre of longitudinal girders in. INCHES							
	Measured				Theoretical			
	1	2	3	4	1	2	3	4
1	.027	.030	.026	.017	.023	.023	.020	.014
2	.046	.050	.044	.030	.046	.046	.040	.028
3	.061	.066	.059	.041	.069	.068	.060	.042
4	.084	.092	.082	.054	.092	.091	.080	.056
6	.128	.139	.123	.081	.138	.137	.120	.084
8	.169	.185	.165	.110	.184	.182	.160	.112
10	.221	.242	.212	.139	.230	.228	.200	.140



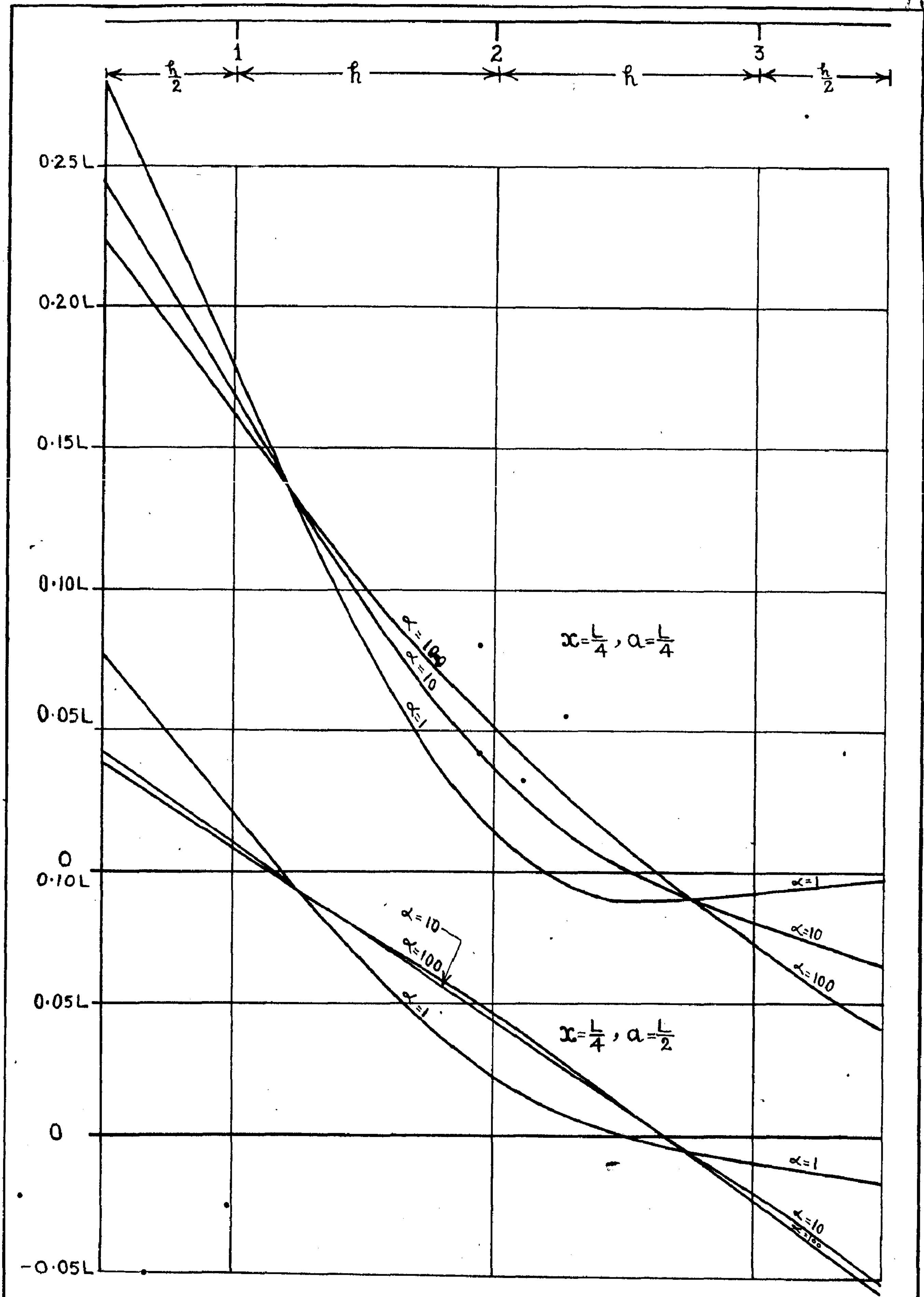
INTERCONNECTED BRIDGE.

FIG. 1.



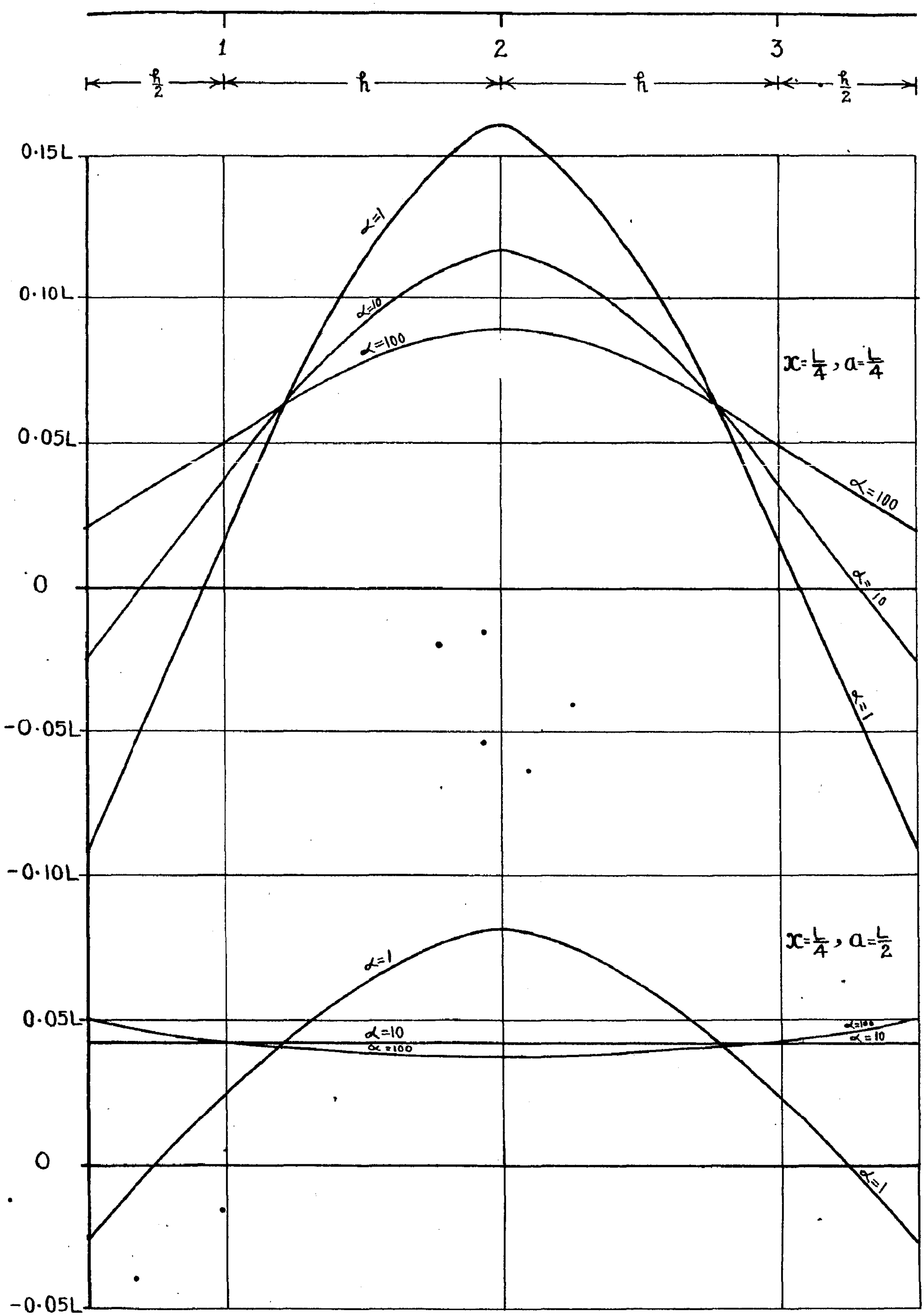
BENDING MOMENT DIAGRAM
FOR
CONCENTRATED LOAD BY HARMONICS.

FIG. 2.



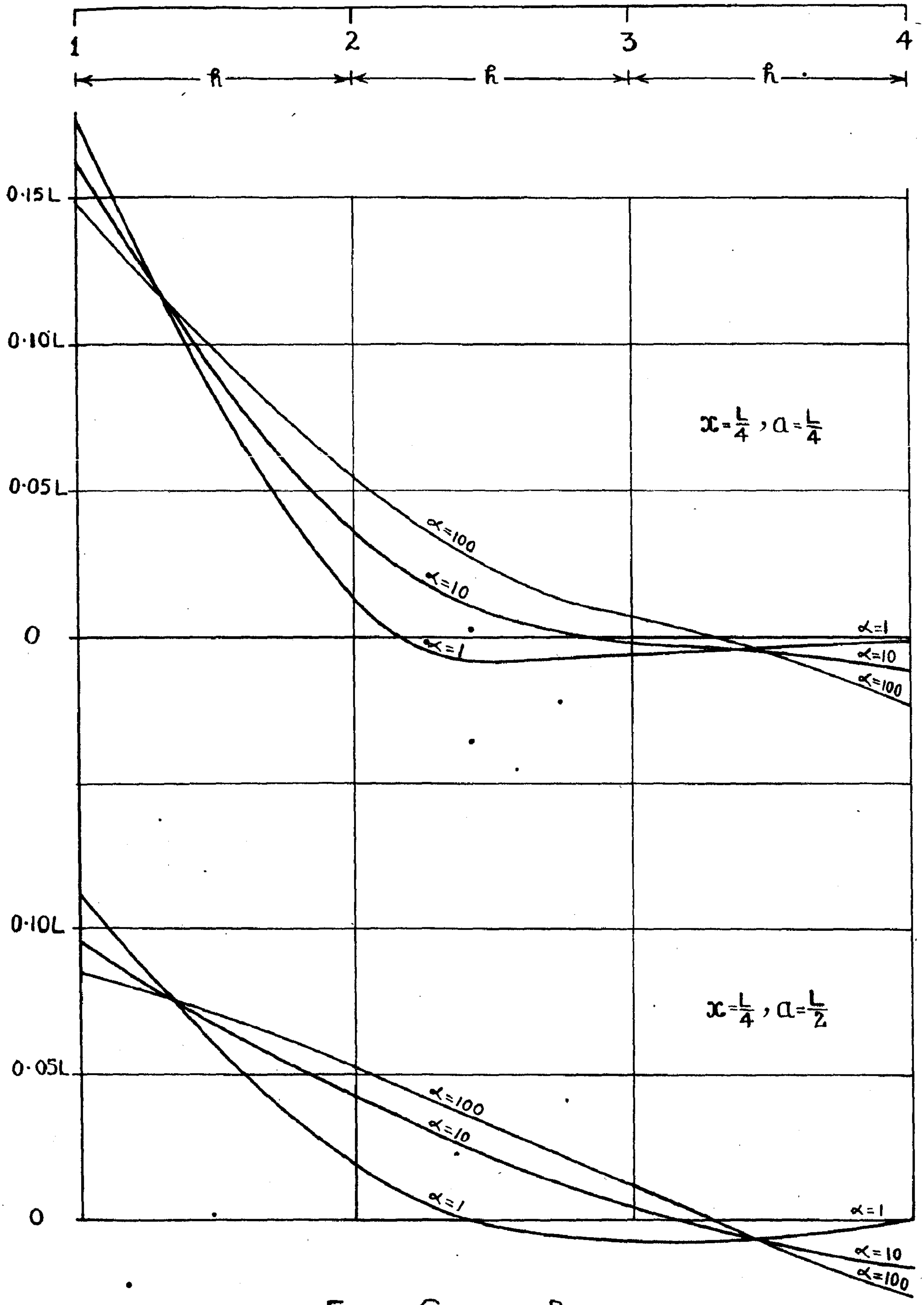
THREE GIRDER BRIDGE
INFLUENCE LINES FOR MOMENTS
IN GIRDER 1, B=0

FIG. 3



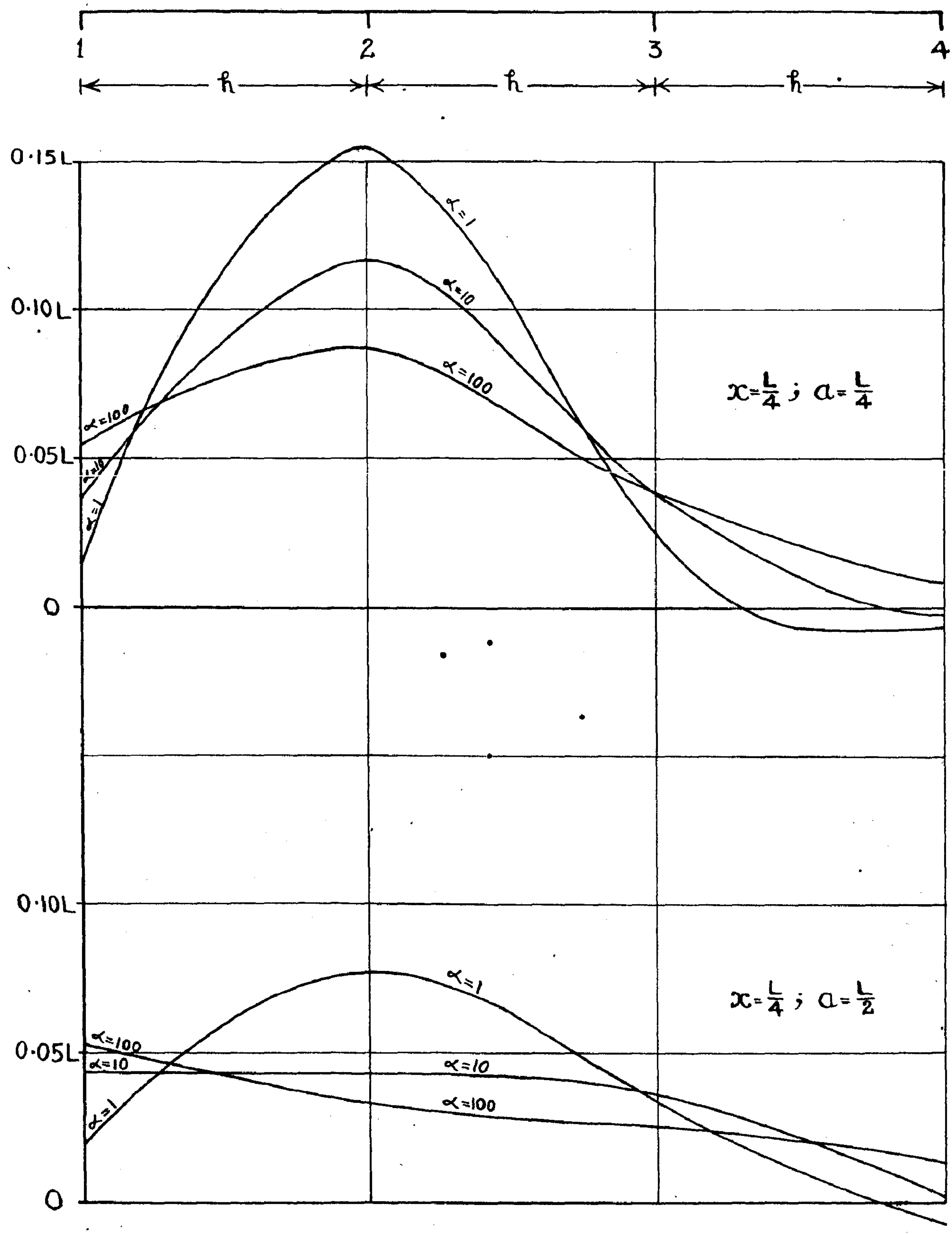
THREE GIRDER BRIDGE
INFLUENCE LINES FOR MOMENTS
IN GIRDER 2, B=0

FIG. 4



FOUR GIRDER BRIDGE
INFLUENCE LINES FOR MOMENTS
IN GIRDER 1, β=0

FIG. 5

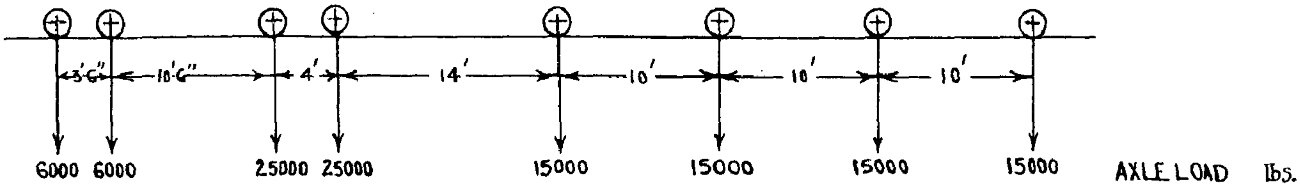


FOUR GIRDER BRIDGE

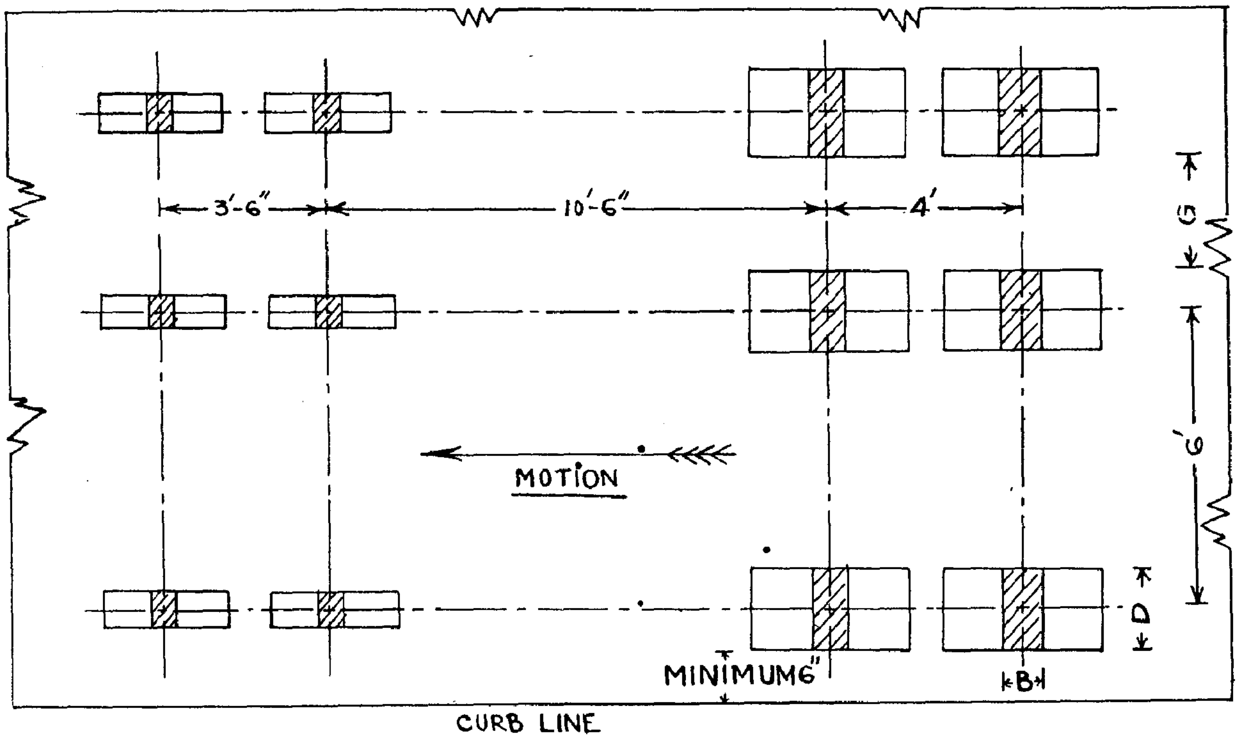
INFLUENCE LINES FOR MOMENTS

IN GIRDER 2, B=0

FIG. 6



CLASS A TRAIN OF VEHICLES.



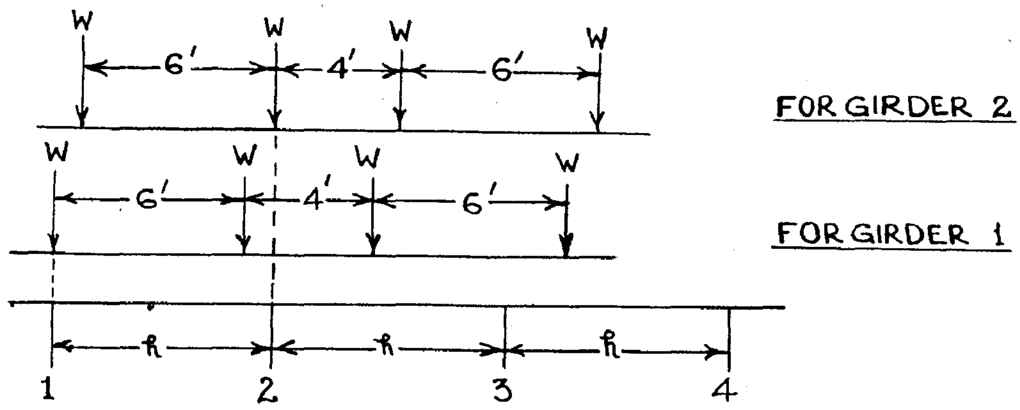
PLAN OF DRIVING VEHICLE.

AXLE LOAD lbs.	GROUND CONTACT AREA	
	B IN INCHES	D IN INCHES
25000	10	20
15000	8	15
6000	6	8

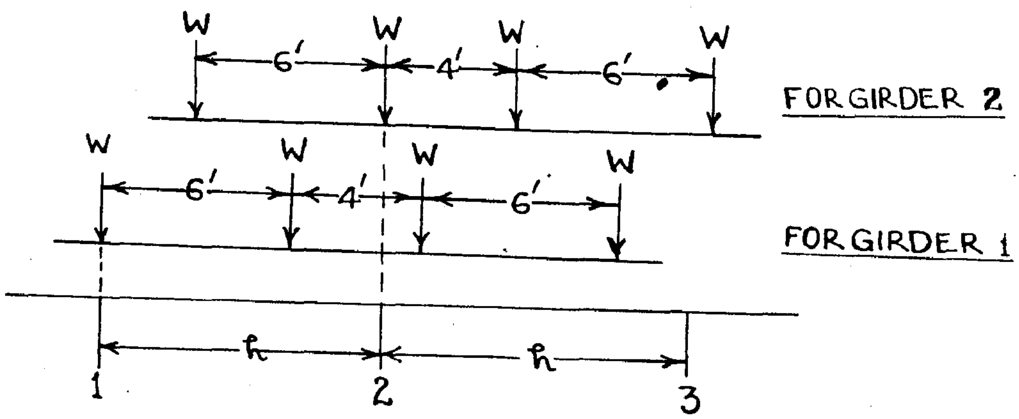
CLEAR CARRIAGE WAY WIDTH	G
18 TO 24 ft.	UNIFROMLY INCREASING 1ft.4 INCH TO 4 ft.
ABOVE 24ft.	4 ft.

FIG. 7.

62,461
CENTRAL LIBRARY UNIVERSITY OF ROORKEE.
ROORKEE.

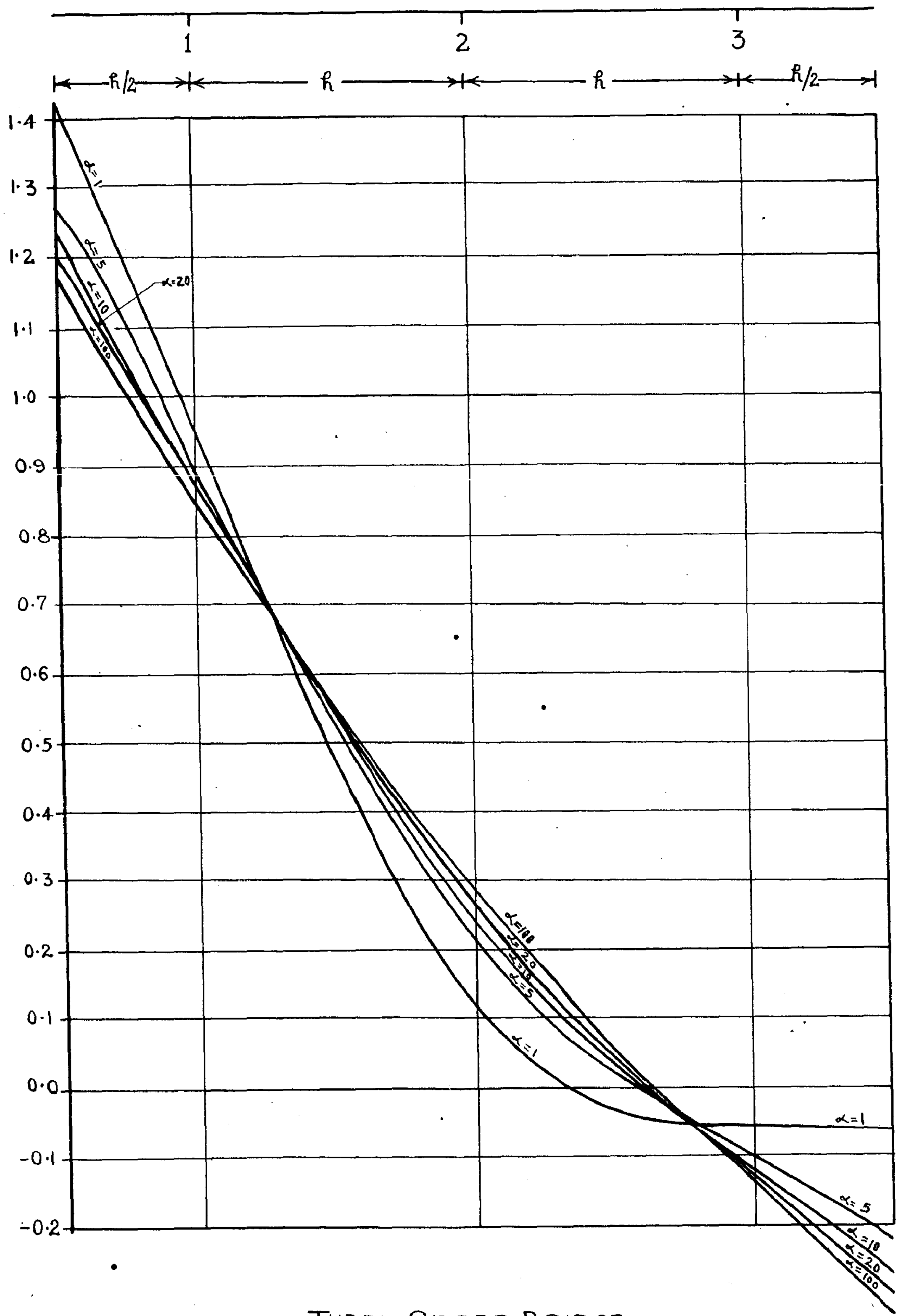


LOAD POSITION FOR MAXIMUM MOMENT IN LONGITUDINAL GIRDERS
OF
FOUR GIRDERS BRIDGE



LOAD POSITION FOR MAXIMUM MOMENT IN LONGITUDINAL GIRDERS
OF
THREE GIRDER BRIDGE

FIG. 8.

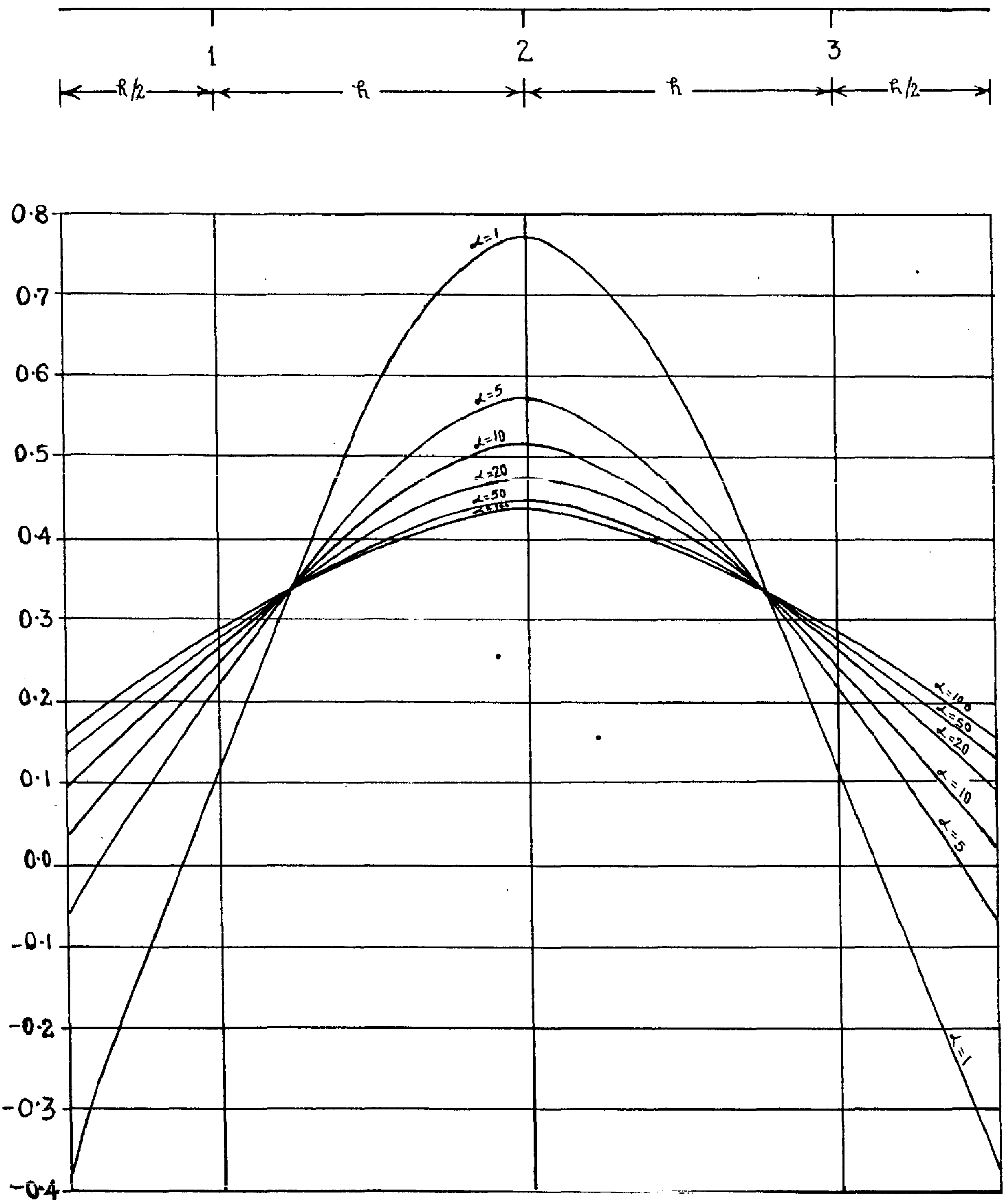


THREE GIRDER BRIDGE

INFLUENCE LINE FOR FRACTION OF UNIT LOAD

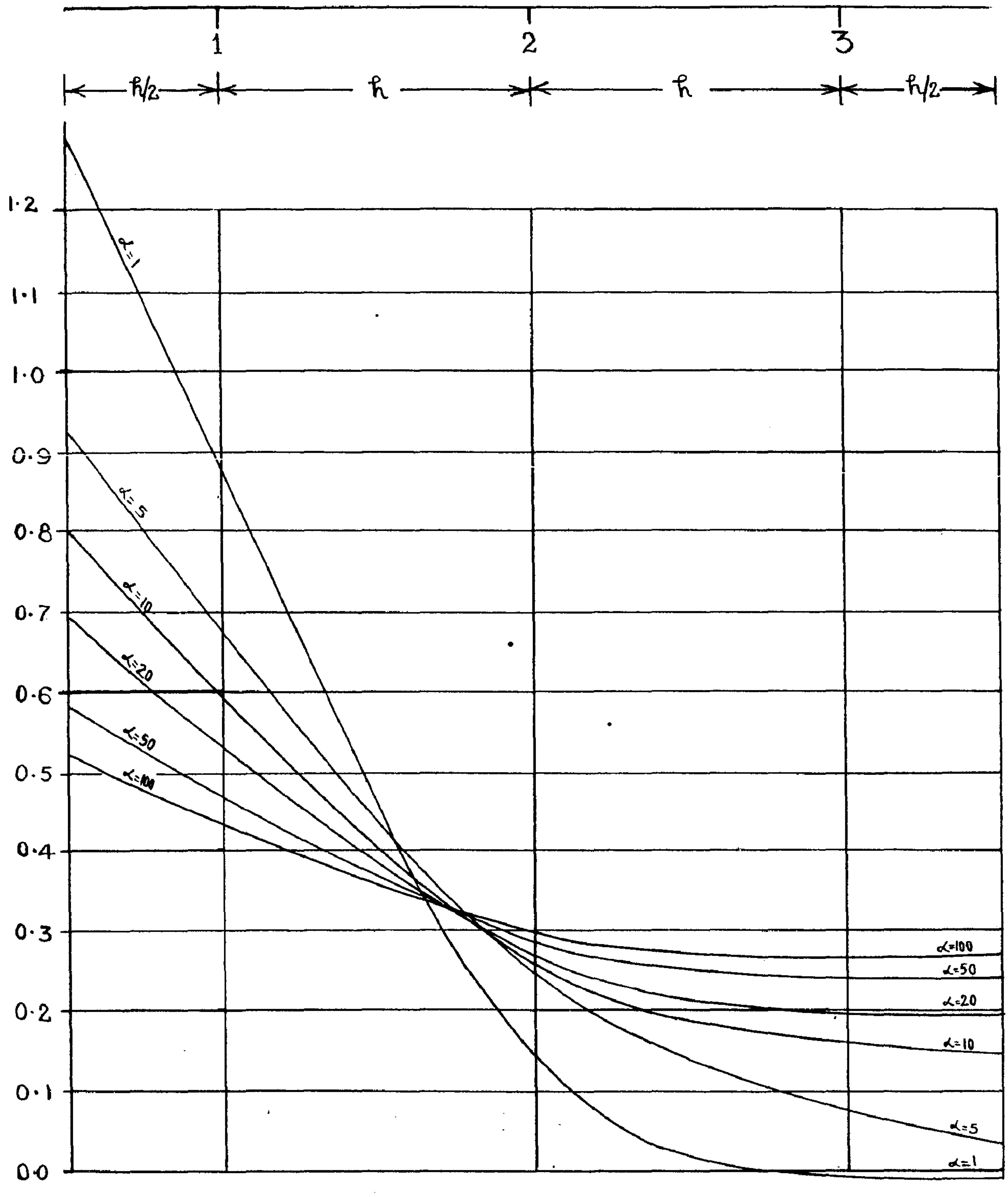
CARRIED BY LONGITUDINAL GIRDER 1; B=0.

FIG. 9.



THREE GIRDER BRIDGE
INFLUENCE LINE FOR FRACTION OF UNIT LOAD
CARRIED BY LONGITUDINAL GIRDER 2; β = 0

FIG. 10.

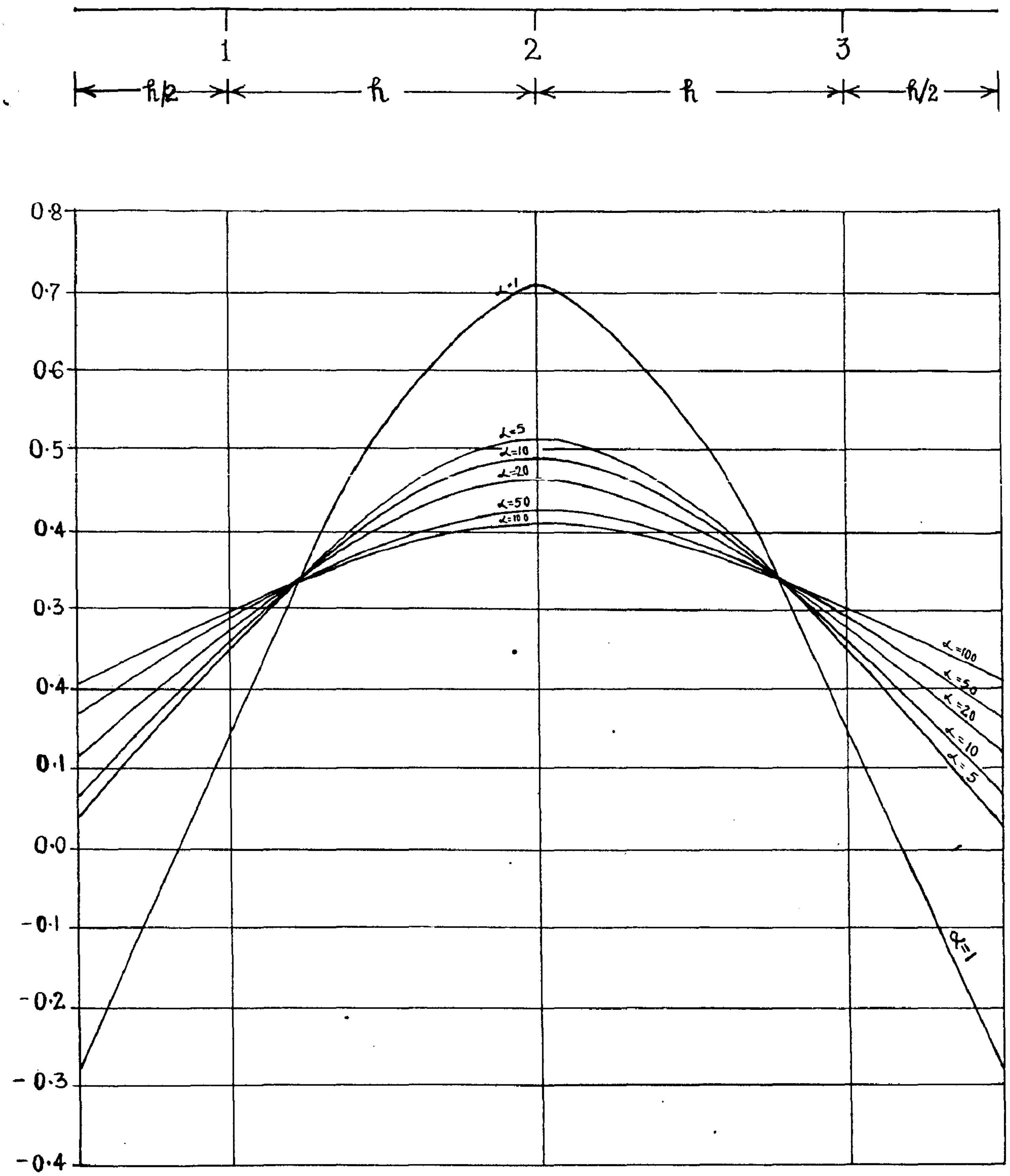


THREE GIRDER BRIDGE

INFLUENCE LINE FOR FRACTION OF UNIT LOAD

CARRIED BY LONGITUDINAL GIRDER 1; β=∞.

FIG. 11.

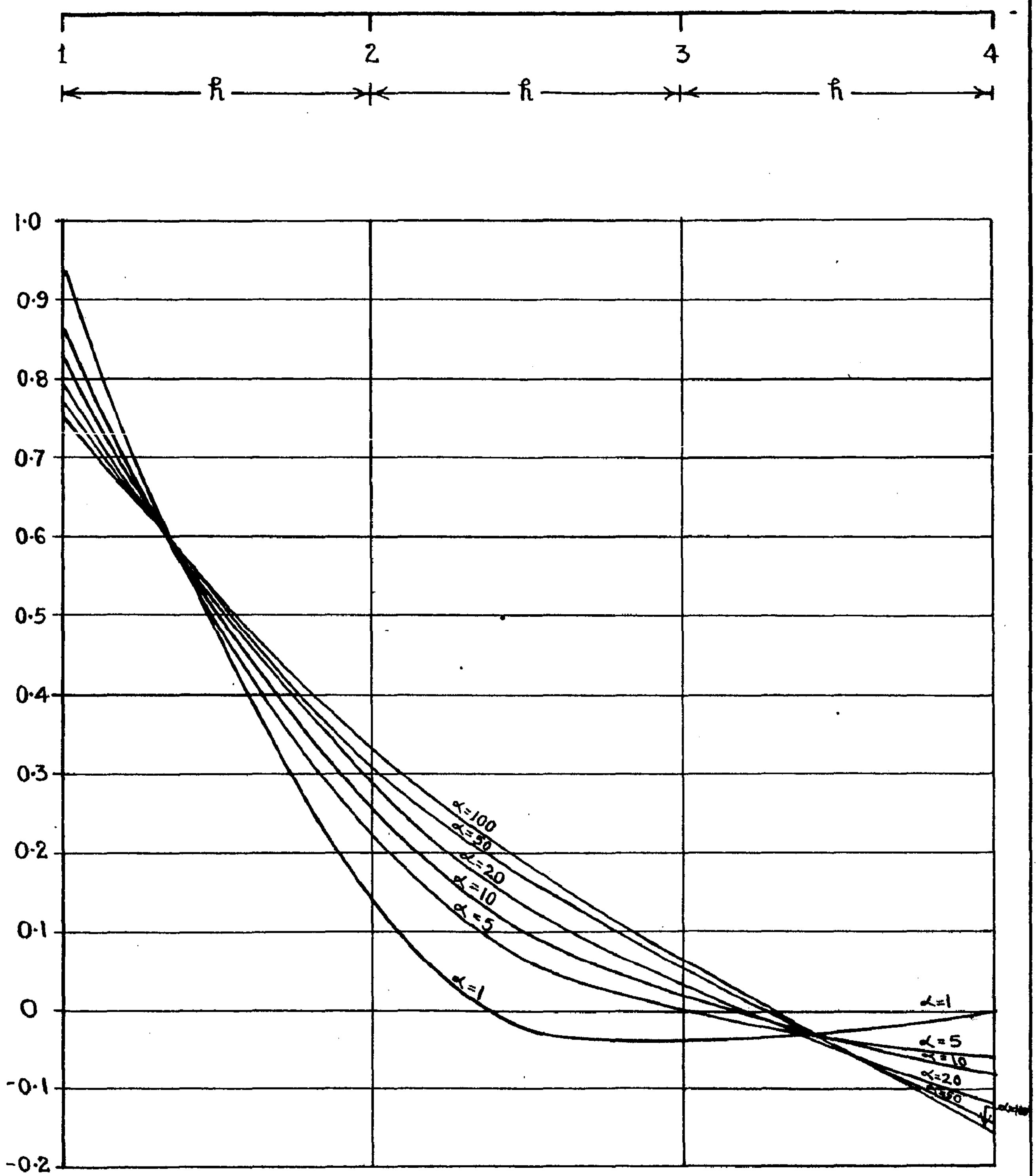


THREE GIRER BRIDGE

INFLUENCE LINE FOR FRACTION OF UNIT LOAD

CARRIED BY LONGITUDINAL GIRDER 2; $\beta=\infty$.

FIG. 12.

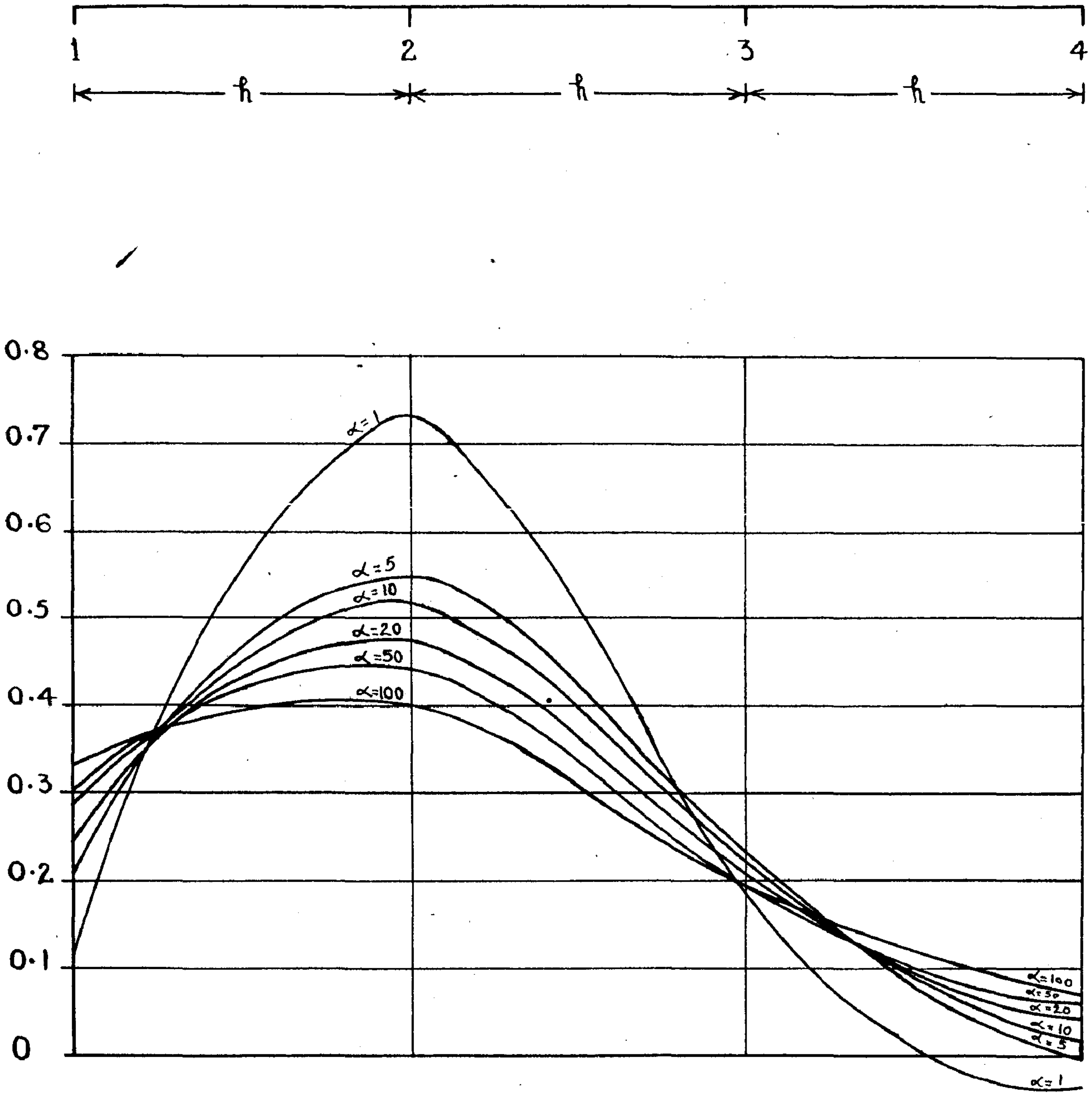


FOUR GIRDER BRIDGE

INFLUENCE LINE FOR FRACTION OF UNIT LOAD

CARRIED BY LONGITUDINAL GIRDER 1, B = 0

FIG. 13

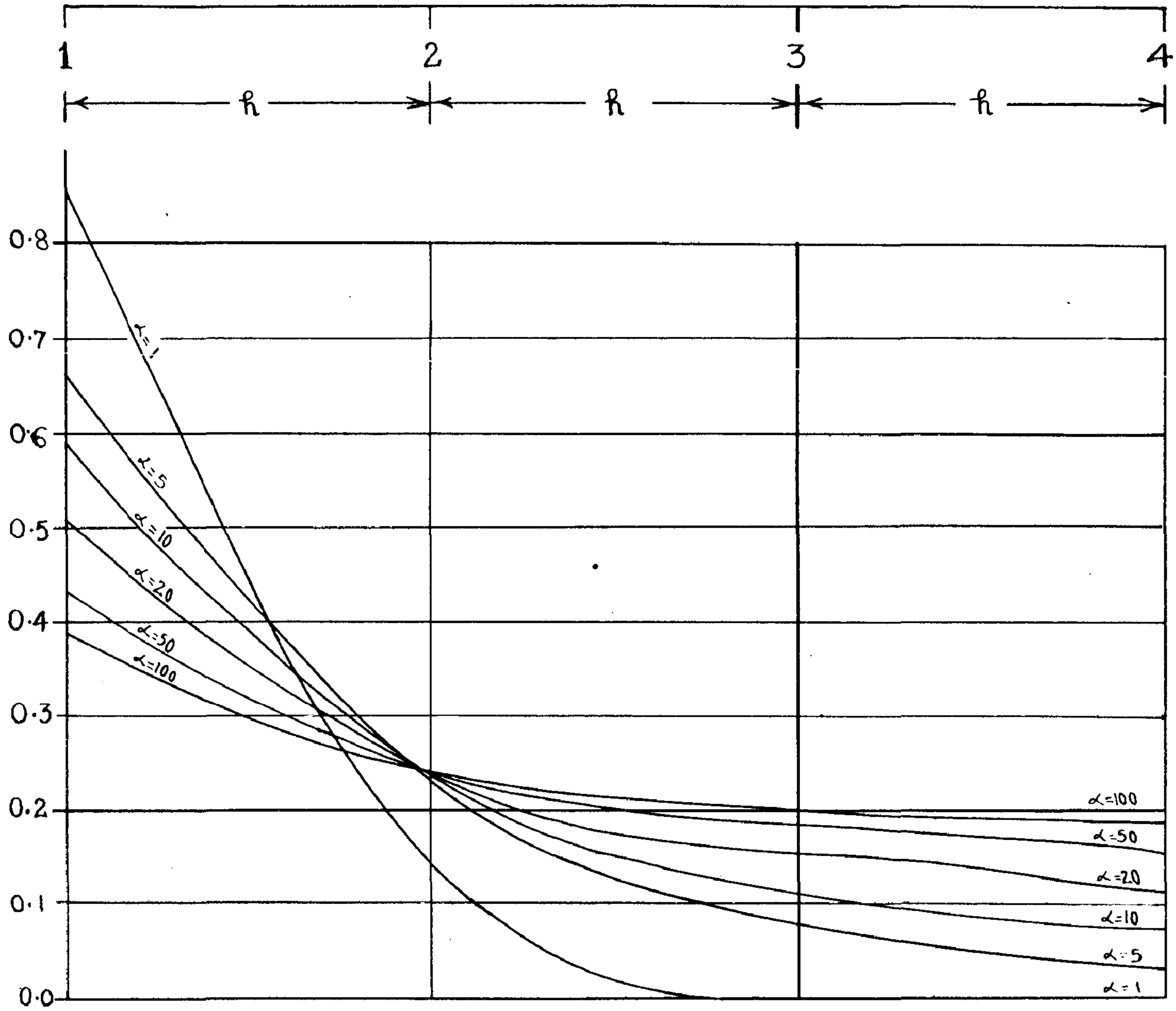


FOUR GIRDER BRIDGE

INFLUENCE LINE FOR FRACTION OF UNIT LOAD

CARRIED BY LONGITUDINAL GIRDER 2, $\beta = 0$

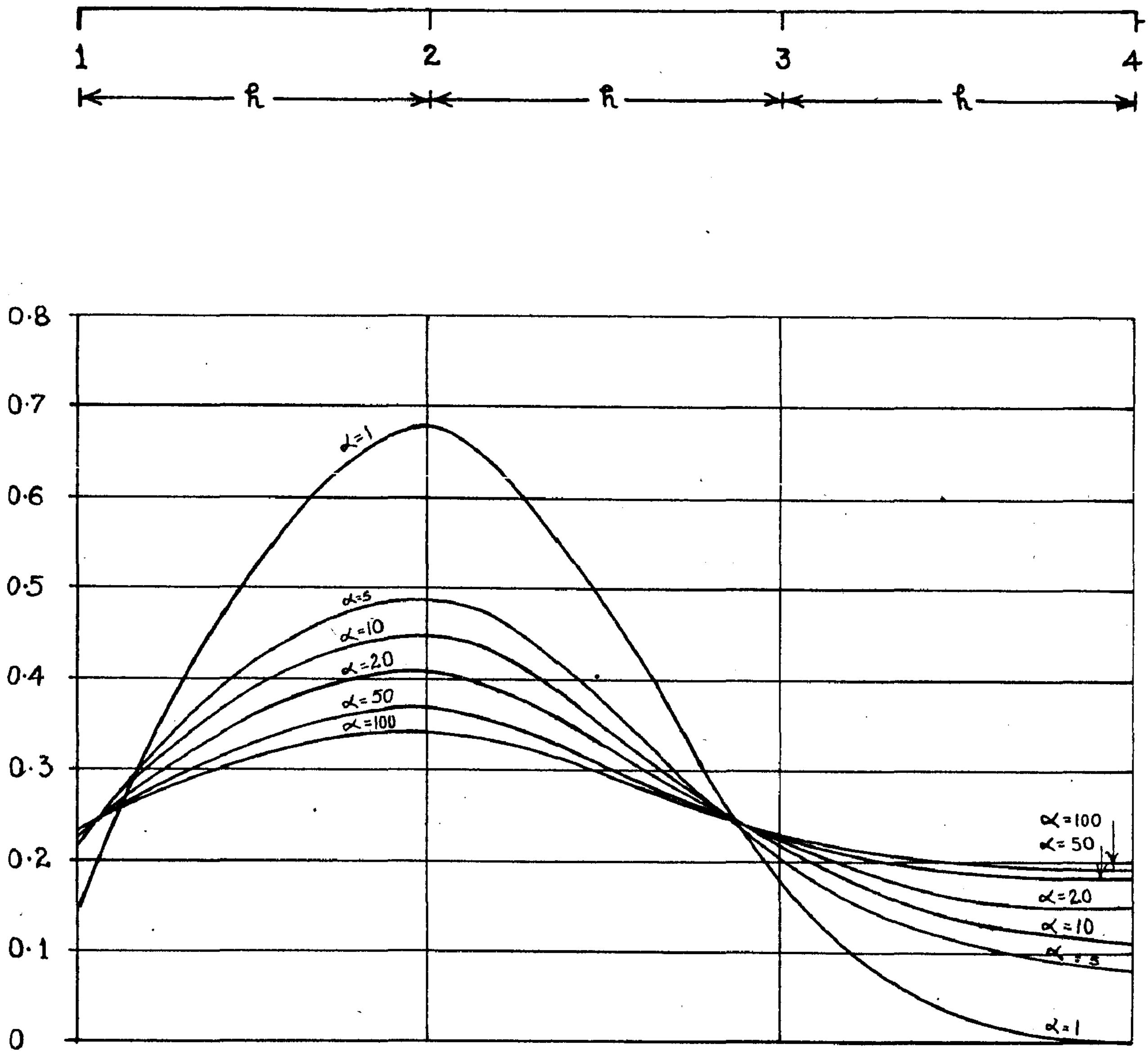
FIG. 14



FOUR GIRDER BRIDGE

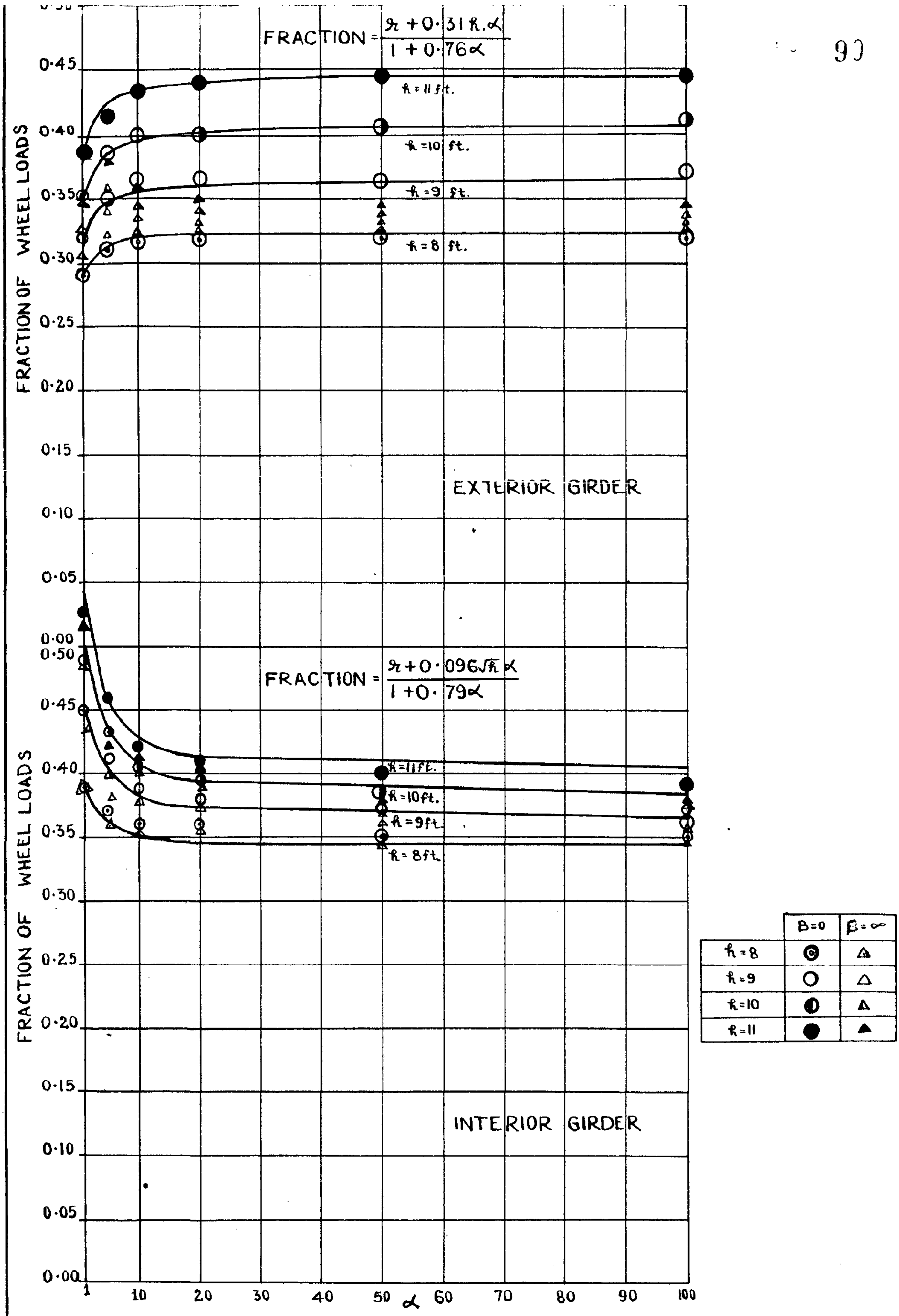
INFLUENCE LINE FOR FRACTION OF UNIT LOAD CARRIED BY LONGITUDINAL GIRDER 1; $\beta = \infty$.

FIG. 15.



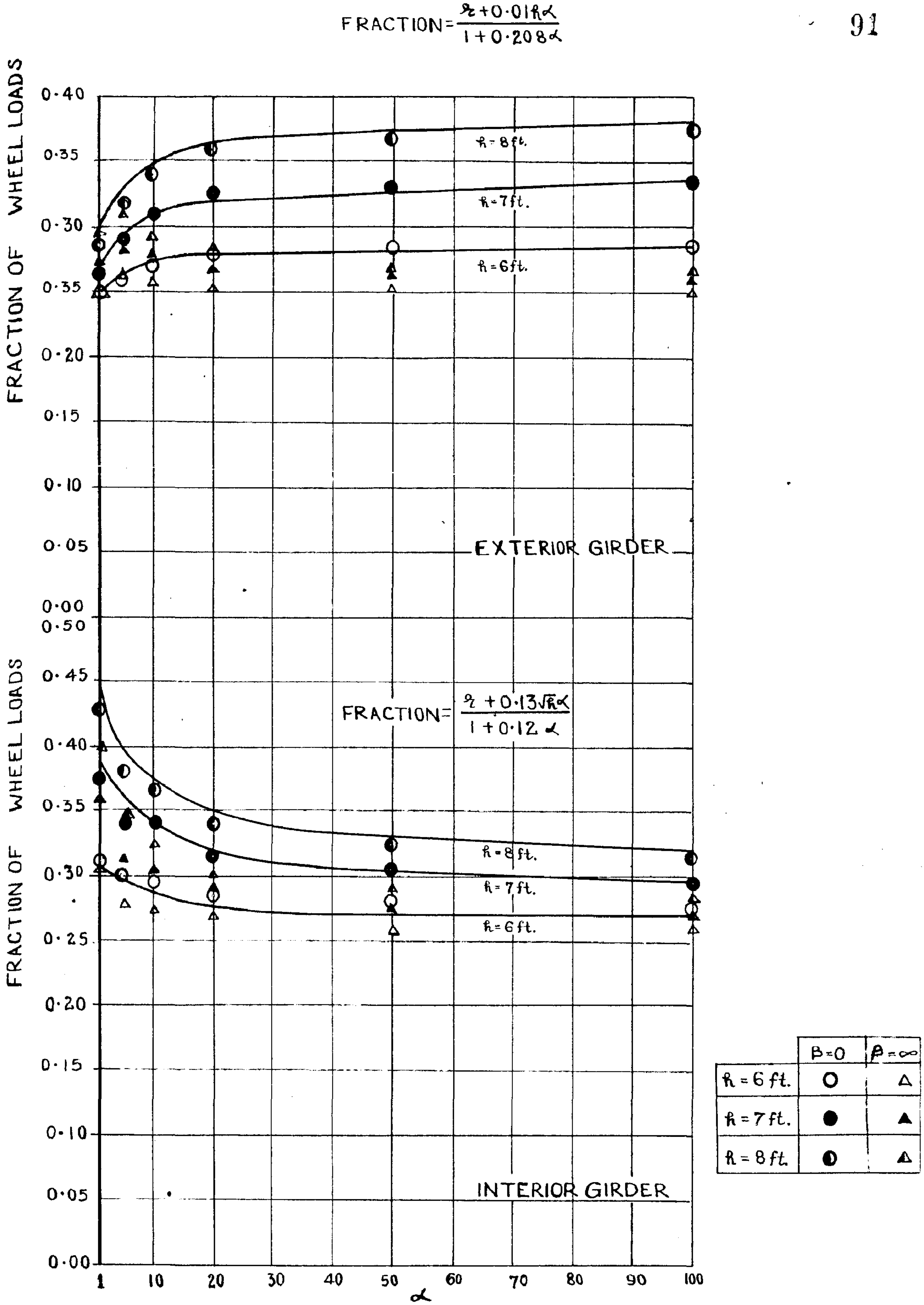
FOUR GIRDER BRIDGE
INFLUENCE LINE FOR FRACTION OF UNIT LOAD
CARRIED BY LONGITUDINAL GIRDER 2, β=∞

FIG. 16



THREE GIRDER BRIDGE

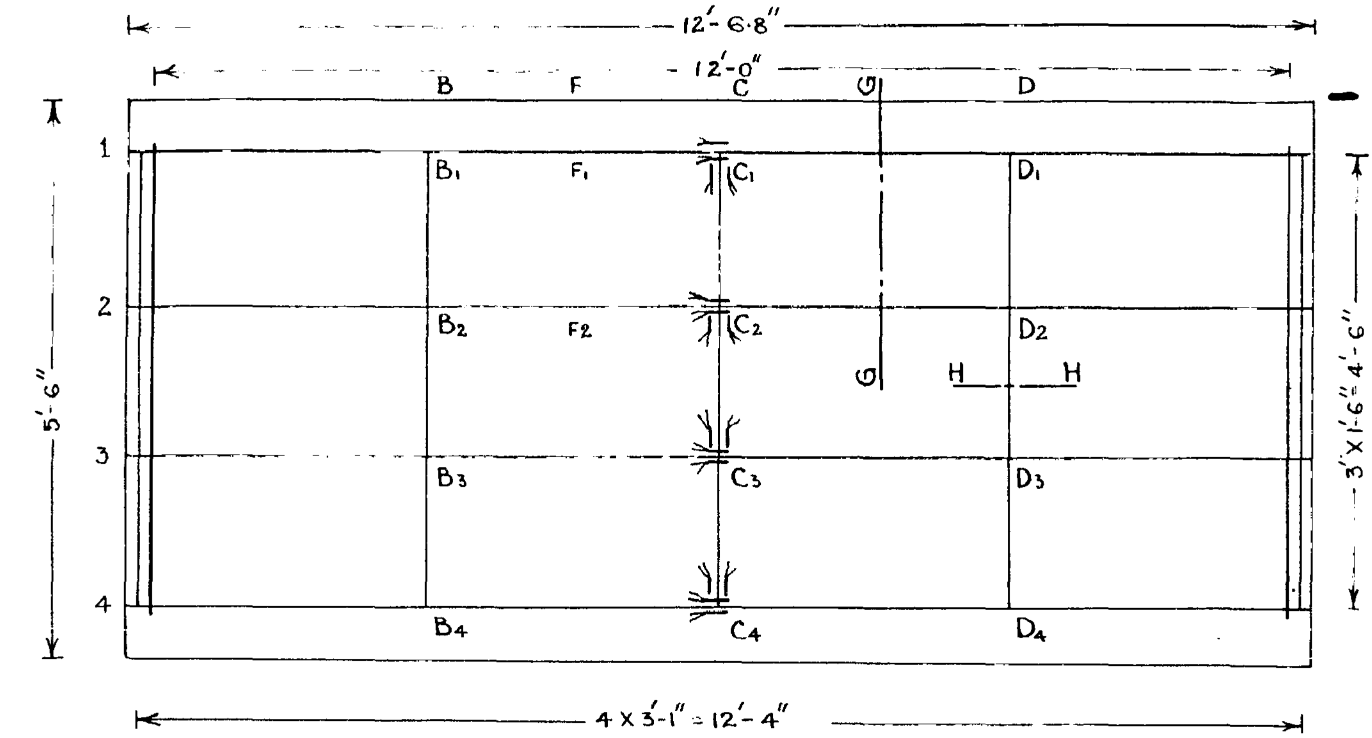
FRACTION OF WHEEL LOADS FOR TWO LANE LOADING. FIG. 17.



FOUR GIRDER BRIDGE

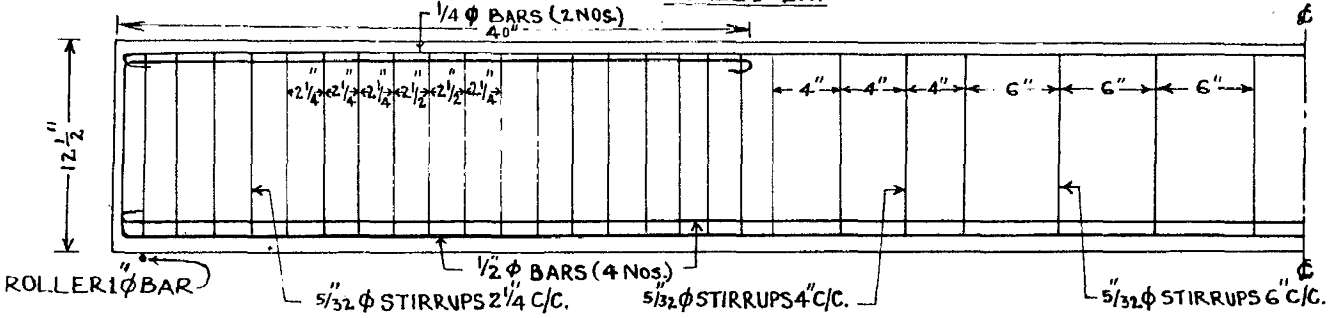
FRACTION OF WHEEL LOADS FOR TWO LANE LOADING.

FIG. 18.



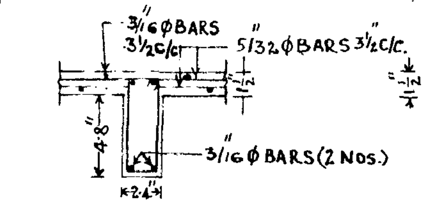
LINE DIAGRAM OF A MODEL BRIDGE SHOWING PRINCIPAL DIMENSION AND POSITION OF STRAIN GAUGES

SCALE 1" = 2ft.



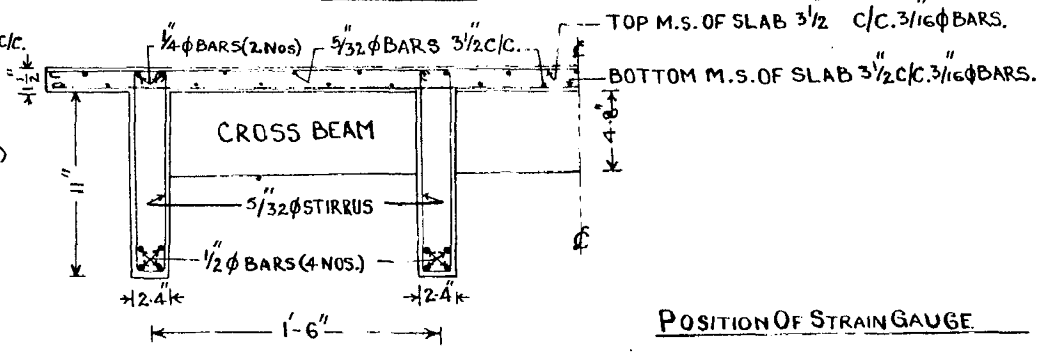
LONGITUDINAL SECTION OF THE MAIN BEAM (4 SUCH BEAMS)

SCALE 1" = 1ft.



TRANSVERSE SECTION OF CROSS-BEAM AT H.H.

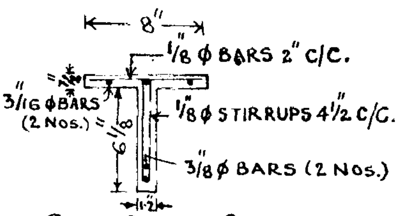
SCALE 1" = 2ft.



TRANSVERSE SECTION OF THE BRIDGE AT G. G.

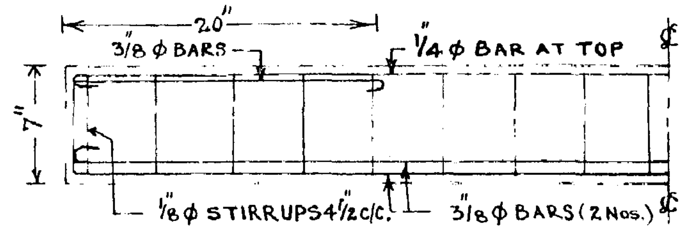
SHOWING DETAILS OF REINFORCEMENT

SCALE 1" = 1ft.



CROSS SECTION OF THE EQUIVALENT BEAM

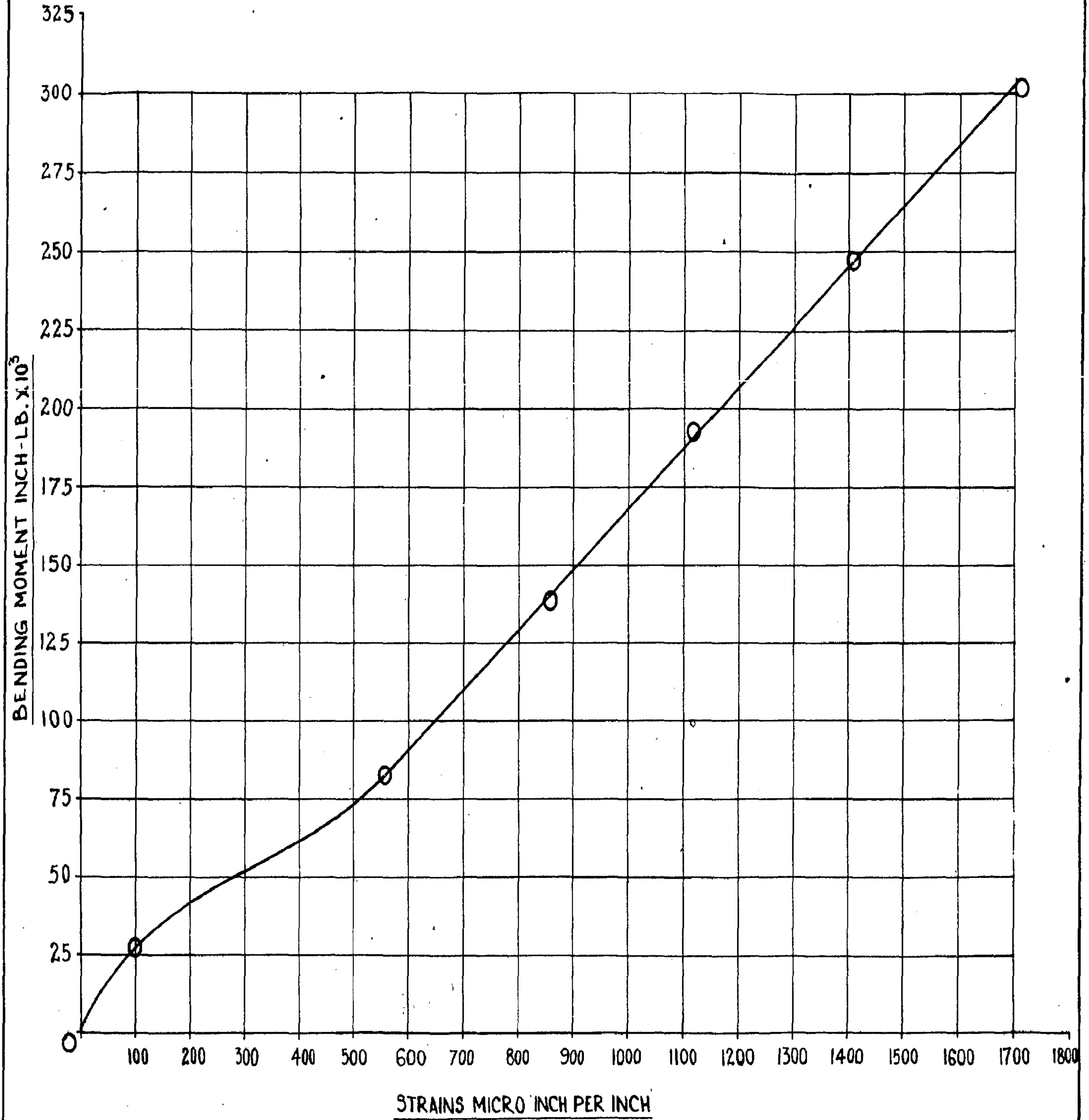
SCALE 1" = 1ft.



LONGITUDINAL SECTION OF THE EQUIVALENT BEAM

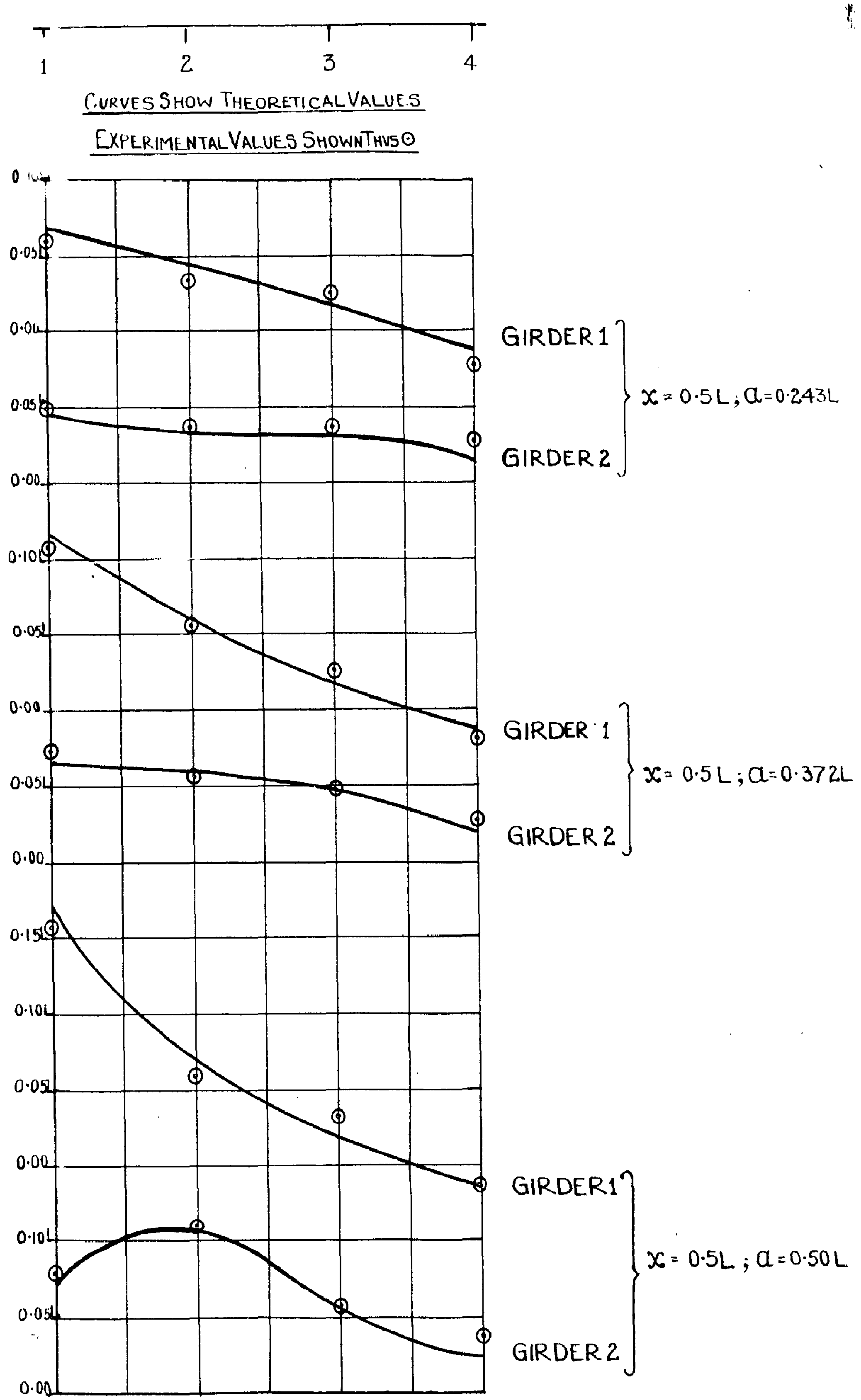
SCALE 1" = 1ft.

FIG. 19



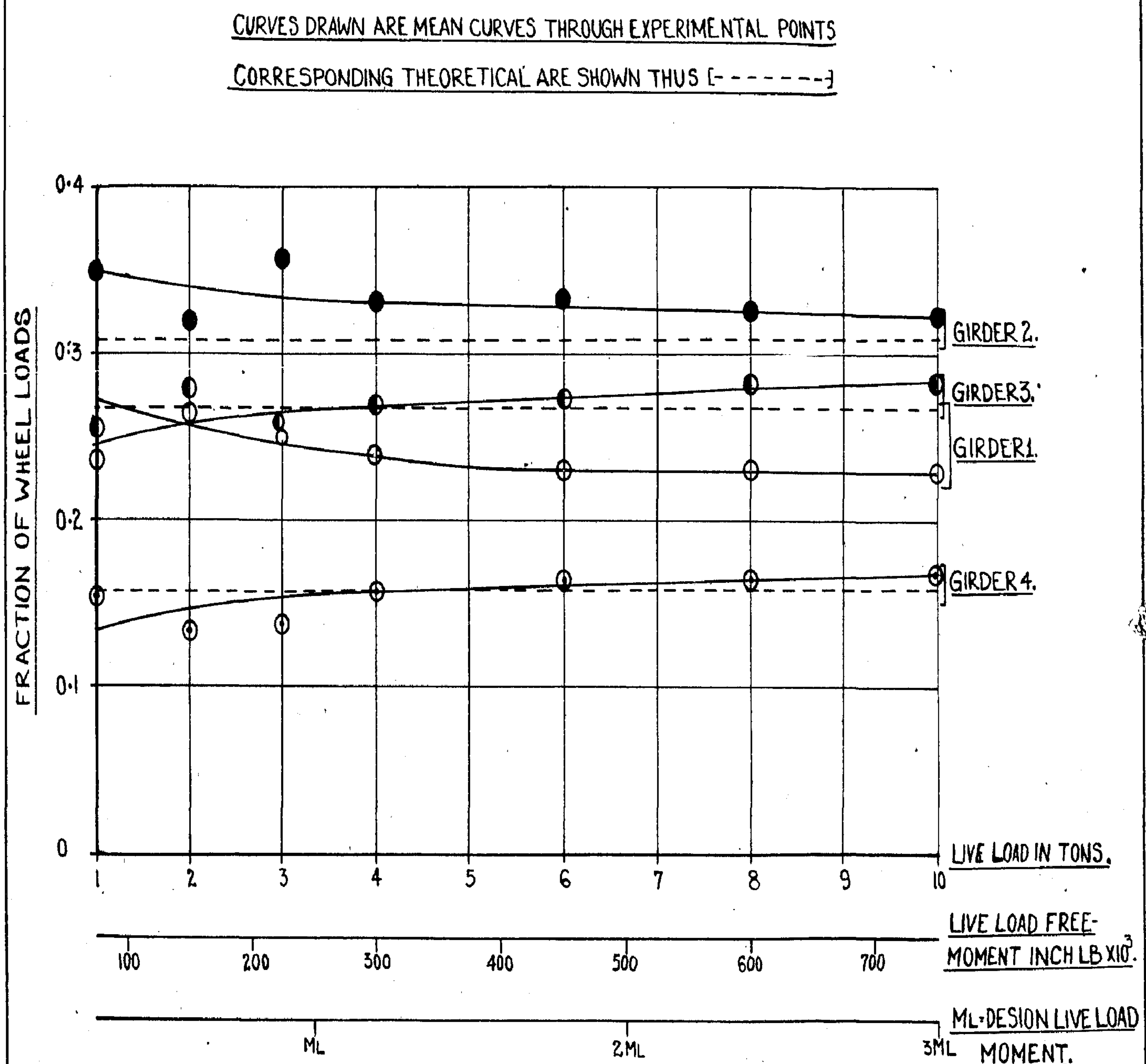
STRAIN BENDING MOMENT GRAPH FOR MODEL BRIDGE GIRDERS

FIG. 20



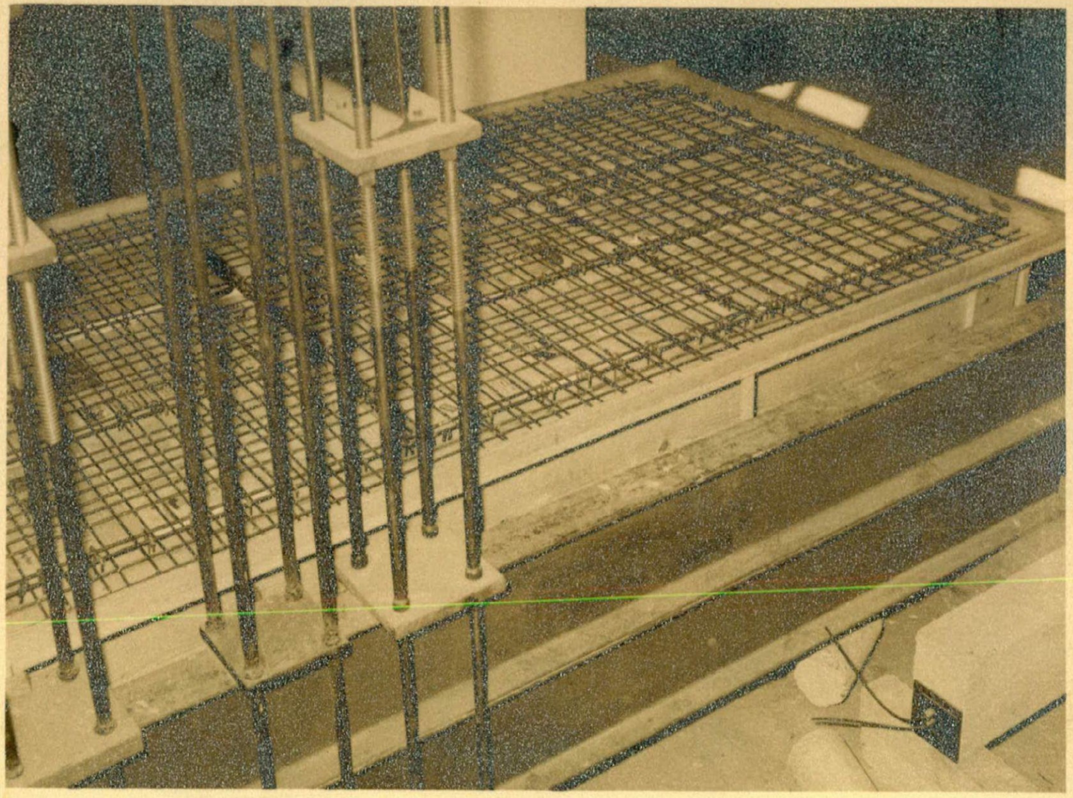
EXPERIMENTAL AND THEORETICAL INFLUENCE COEFFICIENTS OF MOMENTS FOR MODEL BRIDG S.

FIG. 21.

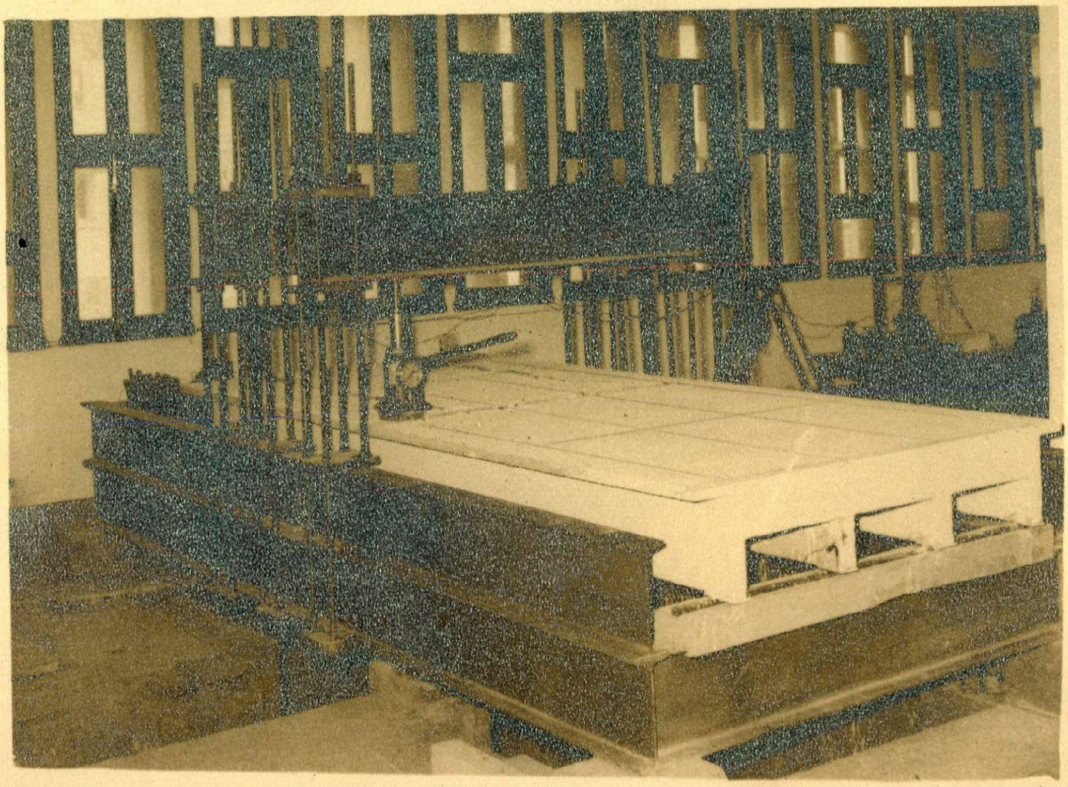


FRACTION OF WHEEL LOADS CARRIED BY VARIOUS GIRDERS OF THE MODEL BRIDGE, AS THE LIVE LOAD ON THE BRIDGE INCREASES

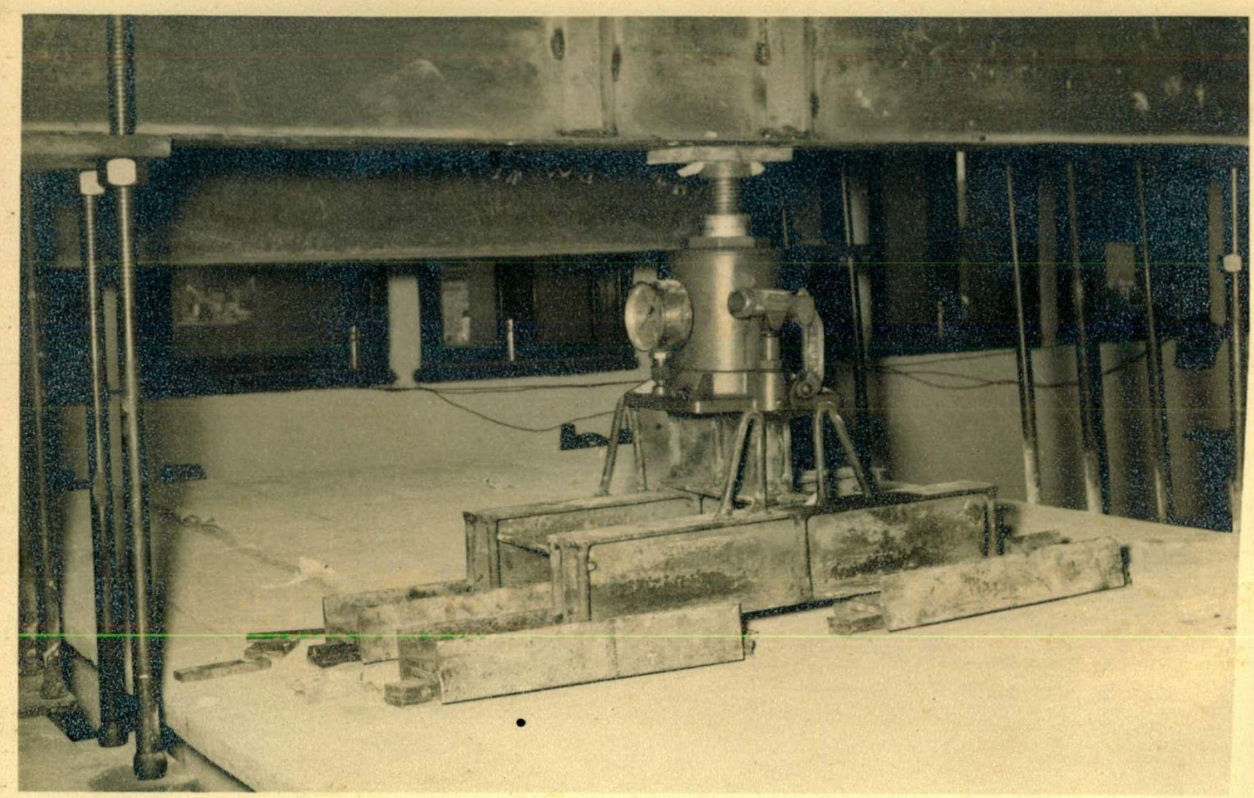
FIG. 22



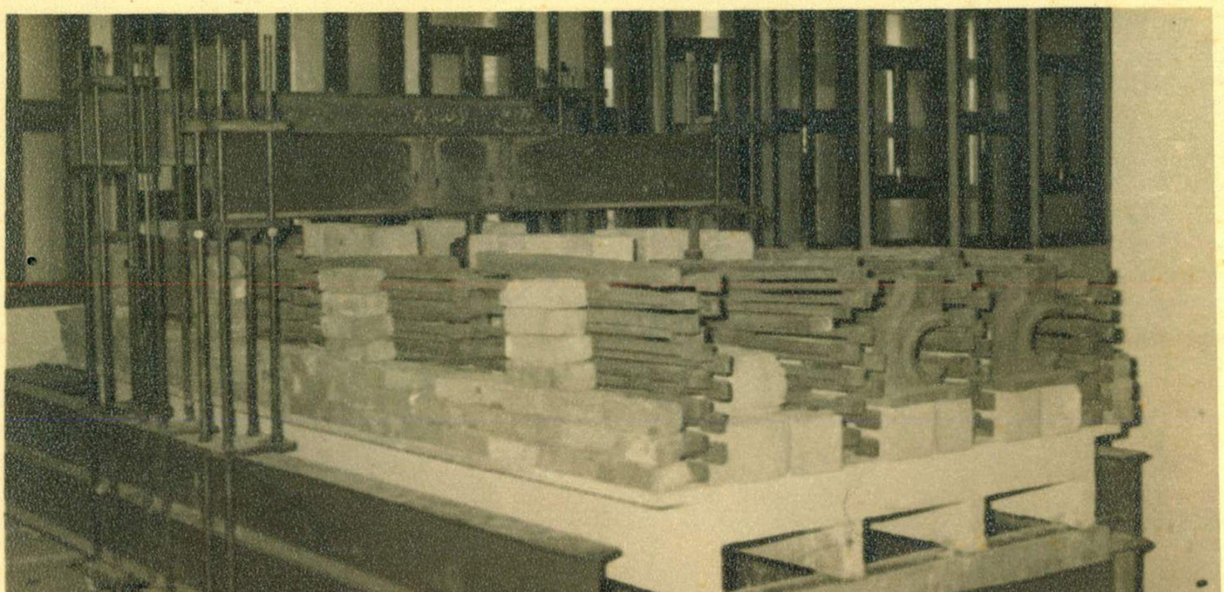
23.1. Details of Reinforcement.



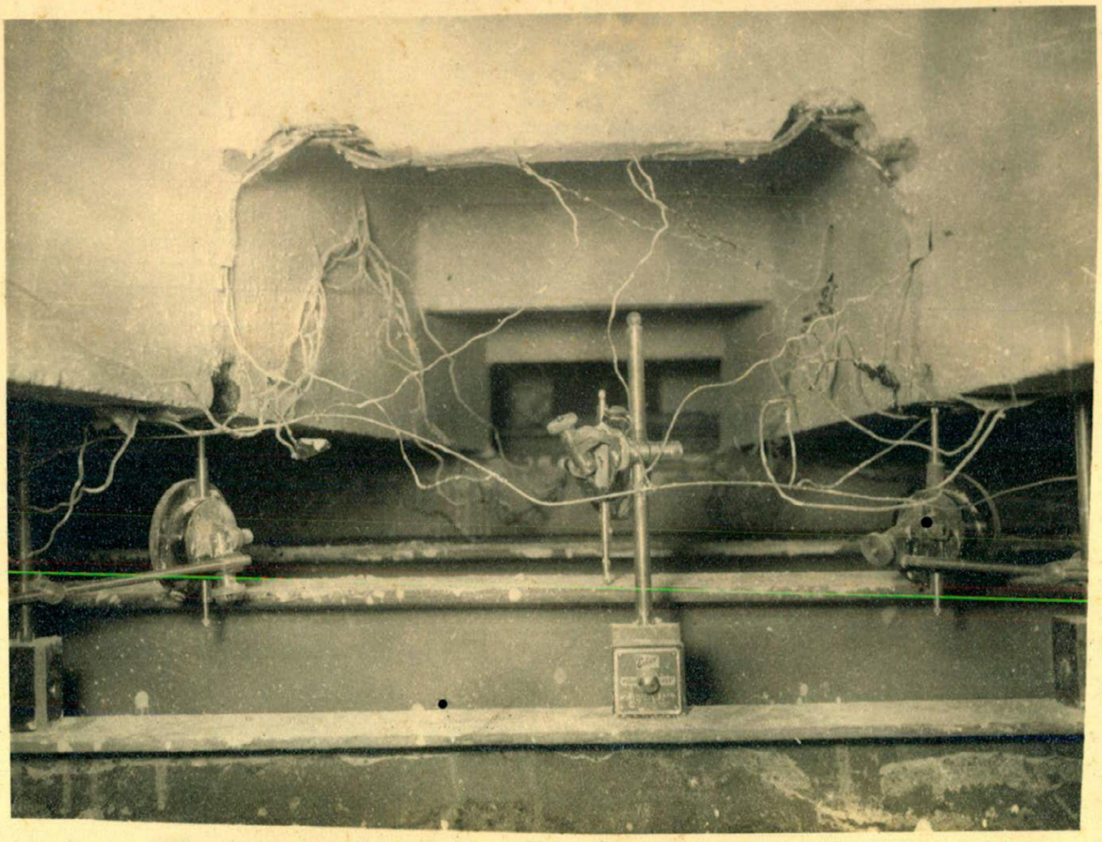
23.2. A View of the Bridge
at the time of Testing.



23.3. Two Lane Loading from a Single Jack.



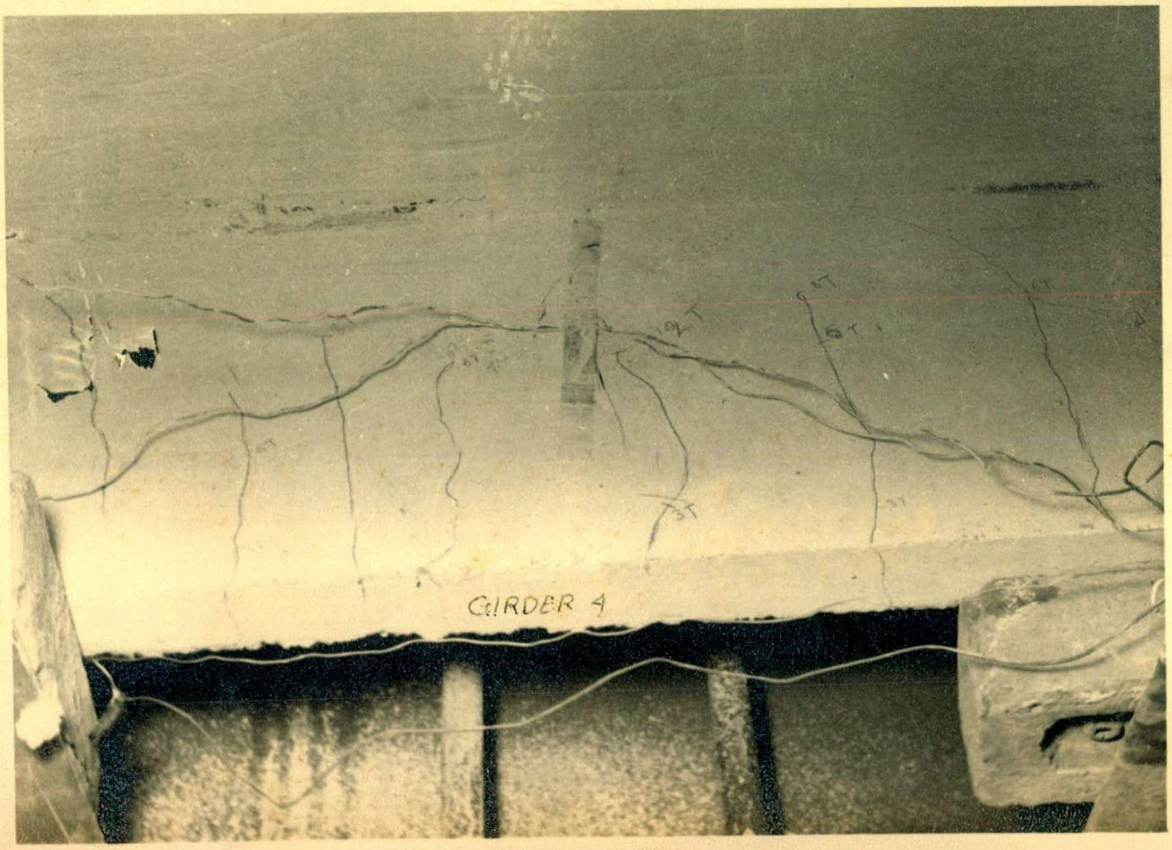
23.4. Surface Loading to Simulate the Dead Load Conditions of true model.



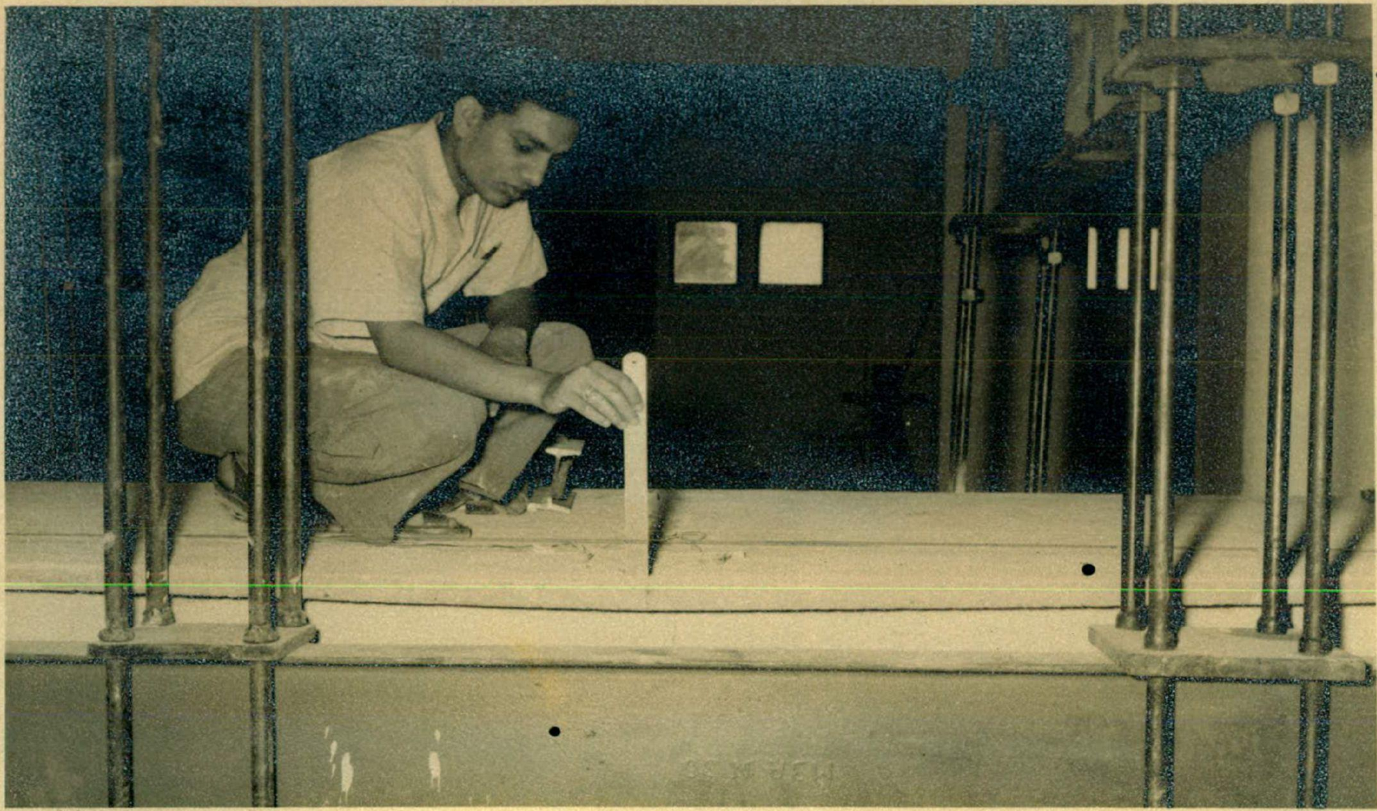
a

b

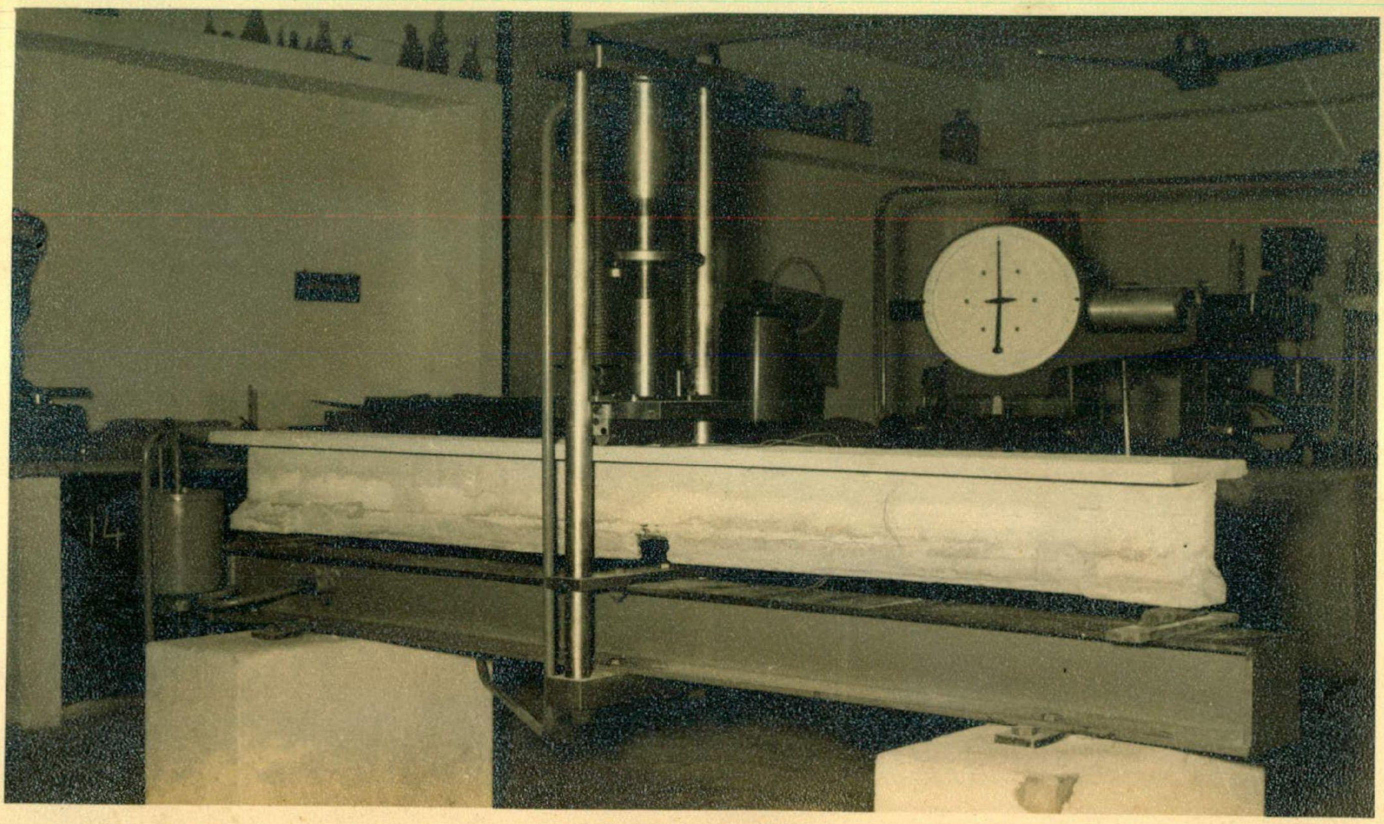
23.5. Measurement of deflections and strains
a) Dial gauges in position,
b) Reading SR_1 strain Indicator



23.6. Cracks in Longitudinal Girders
at Ultimate Load.



23.7. Persistent deflection after loads were removed.



23.8. Isolated equivalent beam under test.

APPENDIX A-1

FIRST HARMONIC BENDING MOMENT DISTRIBUTION COEFFICIENTS FOR INTERCONNECTED BRIDGE GIRDERS

$$\alpha = \frac{12}{\pi^4}\left(\frac{L}{h}\right)^3 \frac{nEI_T}{EI};\ \alpha_0 = \alpha\left(1 - \frac{8}{\pi^2}\right);\ \alpha_1 = \alpha\left(1 - \frac{6}{\pi^2}\right)$$

$$\alpha_2 = \alpha\left(1 - \frac{36}{5\pi^2}\right);\ \alpha_3 = \alpha\left(1 - \frac{20}{3\pi^2}\right);\ \alpha_4 = \alpha\left(1 - \frac{4}{\pi^2}\right)$$

For deflection coefficients divide b.m. coefficients for outers by η.

No. of Girders	Coeff-1 cient	$\beta = 0$
2 Load on (1)	P_{11}	1.0
	P_{12}	0
3 Load on (1)	P_{11}	$[8\eta + \alpha(1 + 4\eta)] / D_1$
	P_{21}	$2\alpha / D_1$
	P_{31}	$-\alpha / D_1$
		$D_1 = 8\eta + \alpha(2 + 4\eta)$
*		
3 Uniform load covering bridge	P_{10}	$(3 + 4\alpha)\eta / D_3$
	P_{20}	$(10\eta + 4\alpha) / D_3$
		$D_3 = 4[4\eta + \alpha(1 + 2\eta)]$

1 First subscript number refers to girder number, the second to the load position. Thus P_{12} is the distribution coefficient for girder (1) with loading on girder (2).

* Three girder bridge load on girder (2) See section 5, page 15.

No. of Girders	Coefficient	$\beta = 0$
4 Load on (1)	P_{11}	$\left[60\eta^2 + \alpha^2\eta(5 + 9\eta) + 8\alpha\eta(1 + 12\eta)\right] / D_4$
	P_{21}	$2\alpha\left[9\eta + \alpha + 3\alpha\eta\right] / D_4$
	P_{31}	$\alpha\left[-12\eta - \alpha + 3\alpha\eta\right] / D_4$
	P_{41}	$2\alpha\eta(1 - 2\alpha) / D_4$
		$D_4 = \left[10\eta + \alpha(1 + \eta)\right]\left[6\eta + \alpha(1 + 9\eta)\right]$
4 Load on (2)	P_{12}	$2\alpha\eta\left[9\eta + \alpha(1 + 3\eta)\right] / D_4$
	P_{22}	$\left[60\eta^2 + \alpha^2(1 + 5\eta) + 16\alpha\eta(1 + 3\eta)\right] / D_4$
	P_{32}	$2\alpha\eta(21\eta + 2\alpha) / D_4$
	P_{42}	$-\alpha\eta\left[12\eta + \alpha(1 - 3\eta)\right] / D_4$
4 Uniform load covering bridge	$P_{10} = P_{40}$	$\eta(3\alpha + 8) / D_1$
	$P_{20} = P_{30}$	$(3\alpha + 22\eta) / D_1$
		$D_1 = 6\alpha(1 + \eta) + 60\eta$

No. of Girders	Coefficient	$\beta = \infty$
2 Load on (1)	P_{11}	$(1+\alpha_0)/(1+2\alpha_0)$
	P_{12}	$\alpha_0/(1+2\alpha_0)$
3 Load on (1)	P_{11}	$\frac{\eta}{2}\left[(1+2\alpha_1)/D_2 + 1(\eta+\alpha_0)\right]$
	P_{21}	α_1/D_2
	P_{31}	$\frac{\eta}{2}\left[(1+2\alpha_1)/D_2 - 1/(\eta+\alpha_0)\right]$
		$D_2 = \eta + \alpha_1(1+2\eta)$
* 3 Uniform load covering bridge	P_{10}	$\eta(16\alpha_1+3)/[16\alpha_1(1+2\eta)+16\eta]$
	P_{20}	$(16\alpha_1+10\eta)/[16\alpha_1(1+2\eta)+16\eta]$
4 Load on (1)	P_{11}	$\frac{\eta}{2}\left[(1+\alpha_2)/D_5 + (1+3\alpha_4)/D_6\right]$
	P_{21}	$\frac{1}{2}\left[\alpha_2/D_5 + \alpha_4/D_6\right]$
	P_{31}	As P_{21} but with - sign between terms.
	P_{41}	As P_{11} but with - sign between terms.
		$D_5 = [\eta+\alpha_2(1+\eta)]$, $D_6 = [(\eta+\alpha_3)(1+3\alpha_4)-(\alpha_4)^2]$

* Three girder bridge load on girder (2) See section 5, page 15.

No. of Girders	Coefficient	$\beta = \infty$
4 Load on (2)	P_{12}	$\frac{\eta}{2}\left[\alpha_2/D_5 + \alpha_4/D_6\right]$
	P_{22}	$\frac{1}{2}\left[(\eta + \alpha_2)/D_5 + (\eta + \alpha_3)/D_6\right]$
	P_{32}	As P_{22} but with - sign between terms
	P_{42}	As P_{12} but with - sign between terms
4 Uniform load covering bridge.	$P_{10} = P_{40}$	$\eta(10\alpha_2 + 1)/D_8$
	$P_{20} = P_{30}$	$(10\alpha_2 + 4\eta)/D_8$
		$D_8 = 10\alpha_2(2 + 2\eta) + 10\eta$

APPENDIX A-II

BENDING MOMENT DISTRIBUTION COEFFICIENTS FOR HIGHER HARMONICS

For $\beta = 0$: Use coefficients of Appendix A-1 but replace α by α / p^4

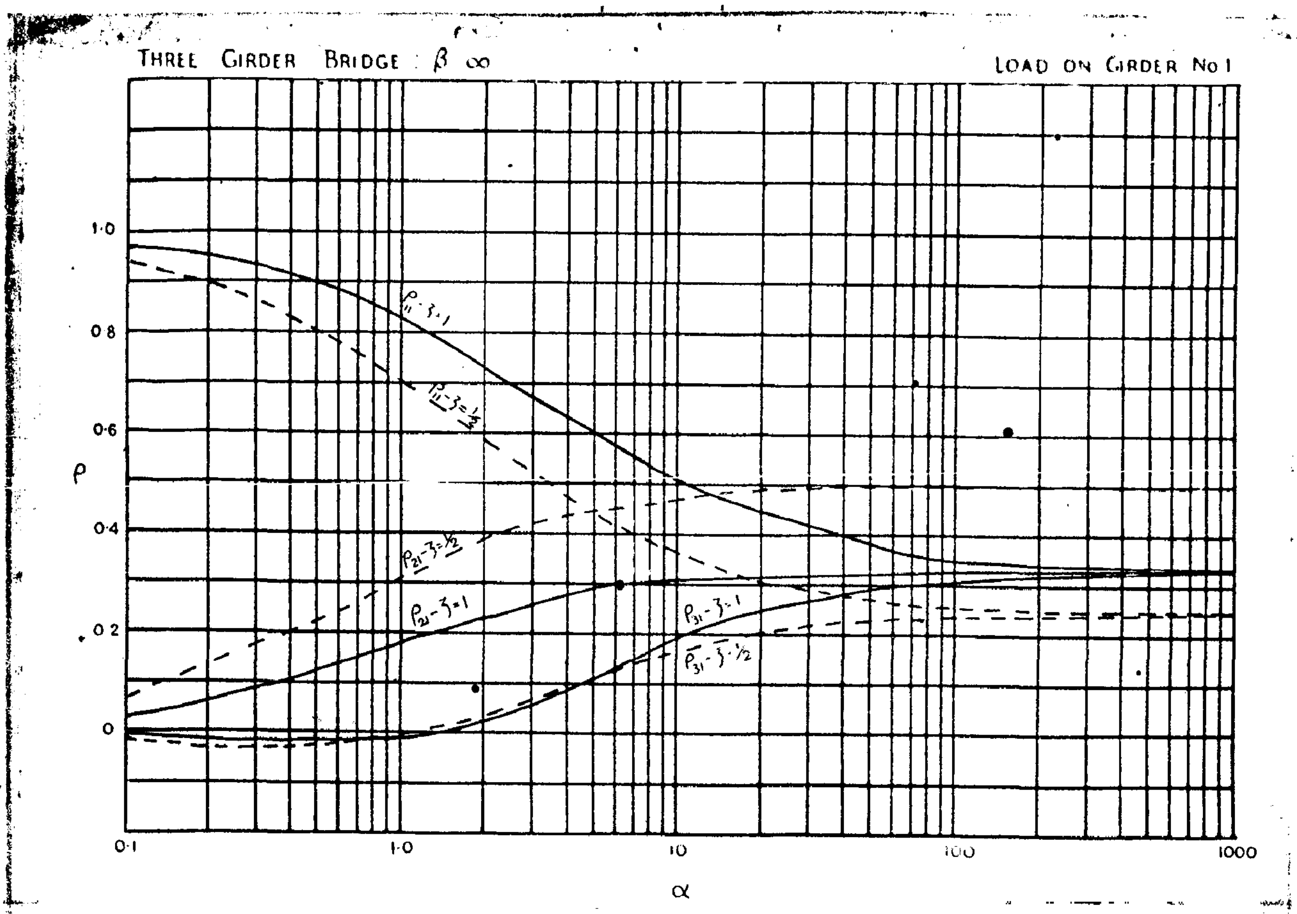
For $\beta = \infty$: Use coefficients of following Table but replace α α / p^4

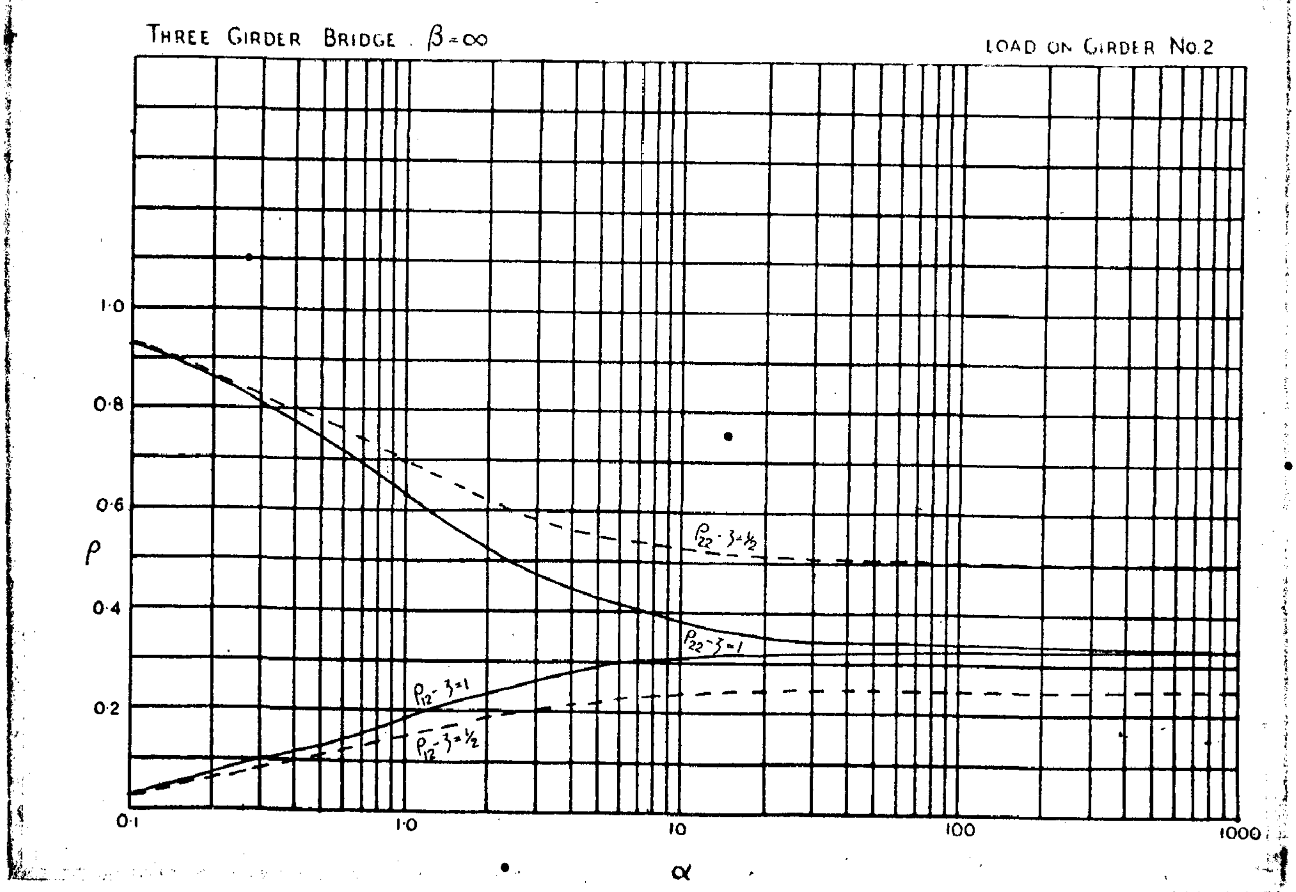
No. of Girders	Coefficient	$\beta = \infty$
2 Load on (1)	P_{11}	$(1 + \alpha) / (1 + 2\alpha)$
	P_{21}	$\alpha / (1 + 2\alpha)$
3 Load on (1)	P_{11}	$\frac{1}{2}\left[1 / D_1 + (1 + 2\alpha) / D_2\right]$
	P_{21}	α / D_2
	P_{31}	As P_{11} but with - sign between terms
		$D_1 = (\eta + \alpha); D_2 = \eta + \alpha(1 + 2\eta)$
3 Load on (2)	$P_{12} = P_{32}$	$\eta\alpha / D_2$
	P_{22}	$(\eta + \alpha) / D_2$
4 Load on (1)	P_{11}	$\frac{\eta}{2}\left[(1 + \alpha) / D_3 + (1 + 3\alpha) / D_4\right]$
	P_{21}	$\frac{1}{2}\left[\alpha / D_3 + \alpha / D_4\right]$

No. of Girders	Coefficient	$\beta = \infty$
4 LOAD ON (1)	P_{31}	As P_{21} but with - sign between terms
	P_{41}	As P_{11} but with - sign between terms
		$D_3 = \eta + \alpha(1 + \eta)$, $D_4 = \eta + \alpha(1 + 3\eta) + 2\alpha^2$
4 Load on (2)	P_{12}	$\frac{\eta}{2}\left[\alpha / D_3 + \alpha / D_4\right]$
	P_{22}	$\frac{1}{2}\left[(\eta+\alpha) / D_3 + (3\alpha+\eta) / D_4\right]$
	P_{32}	As P_{22} but with - sign between terms
	P_{42}	As P_{12} but with - sign between terms

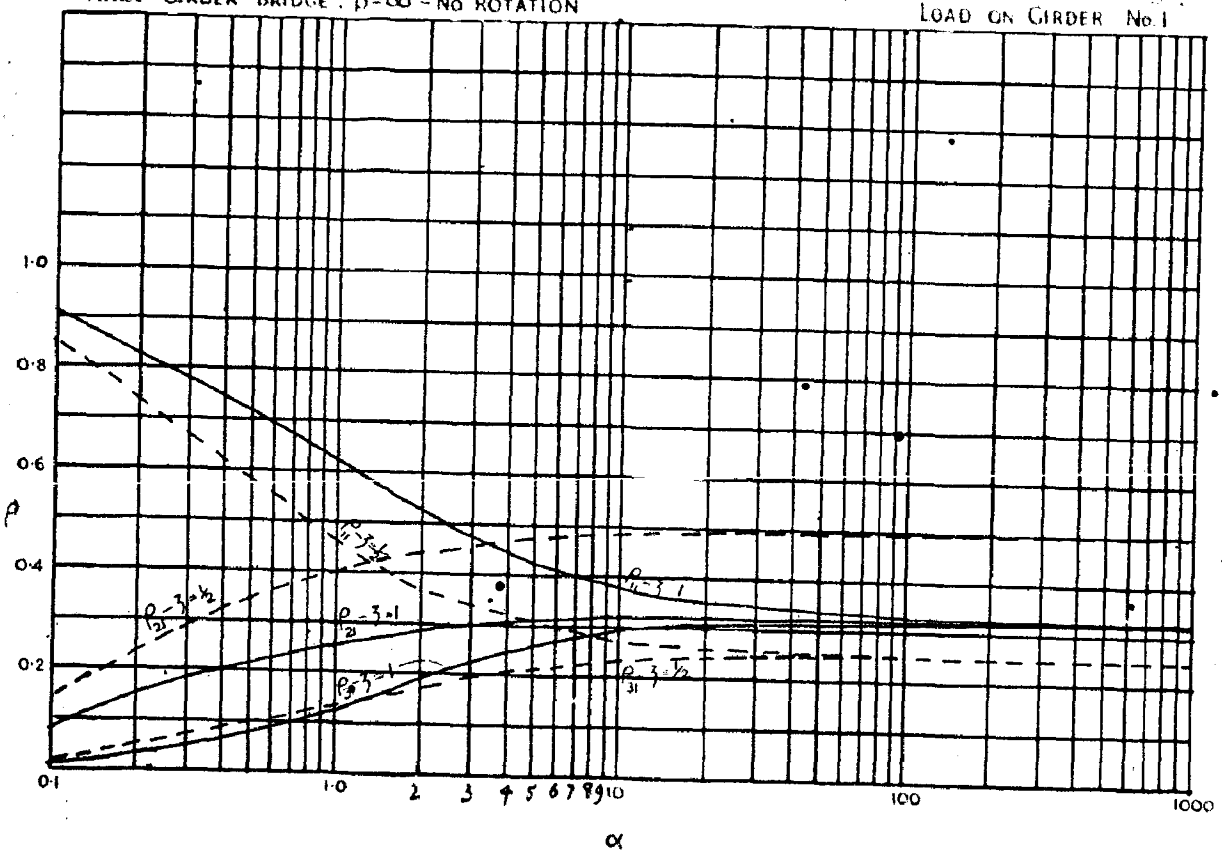
APPENDIX B

CURVES OF BENDING MOMENT DISTRIBUTION COEFFICIENTS FOR THREE AND FOUR GIRDER BRIDGES.

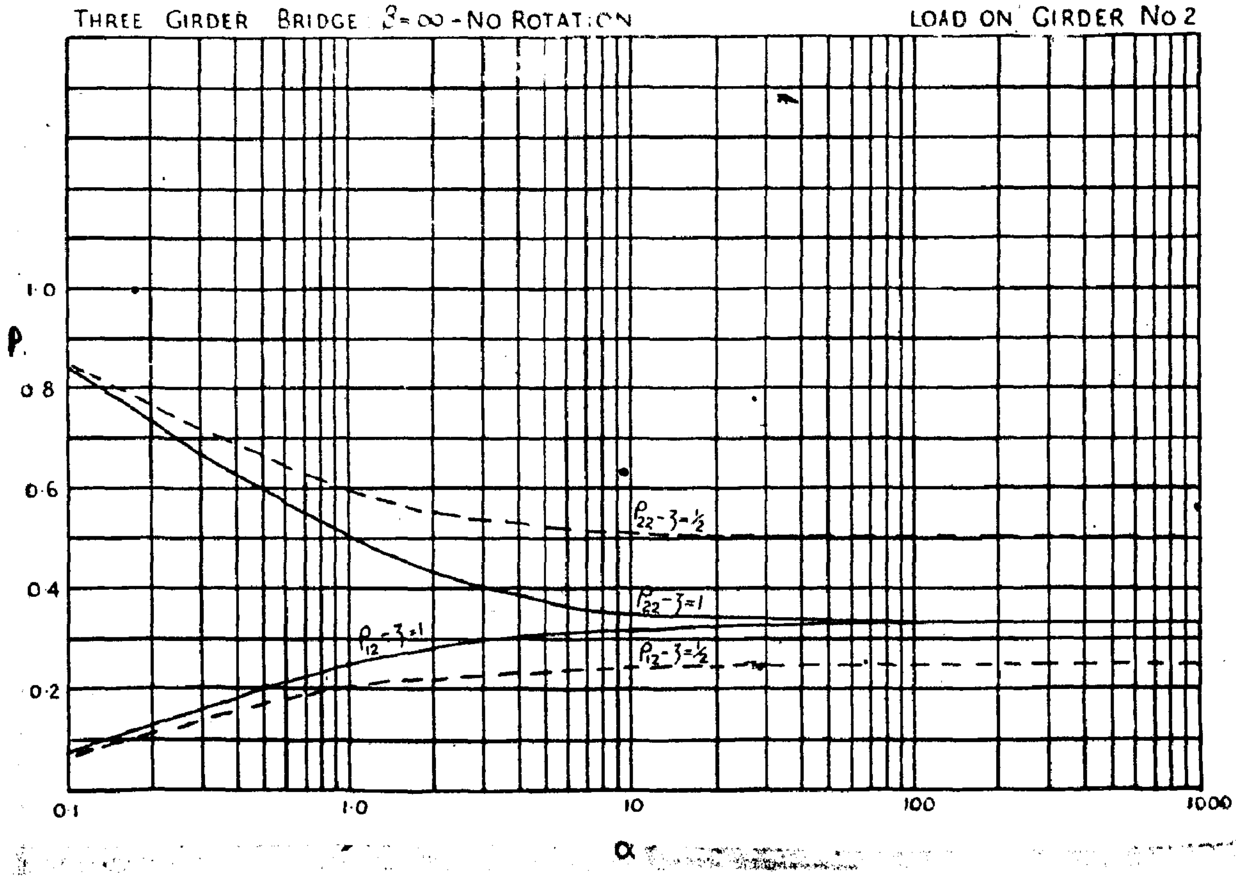

THREE GIRDER BRIDGE : β ∞
LOAD ON GIRDER No 1
1·0
0·8
0·6
ρ
0·4
0·2
0
0·1
1·0
10
100
1000
α

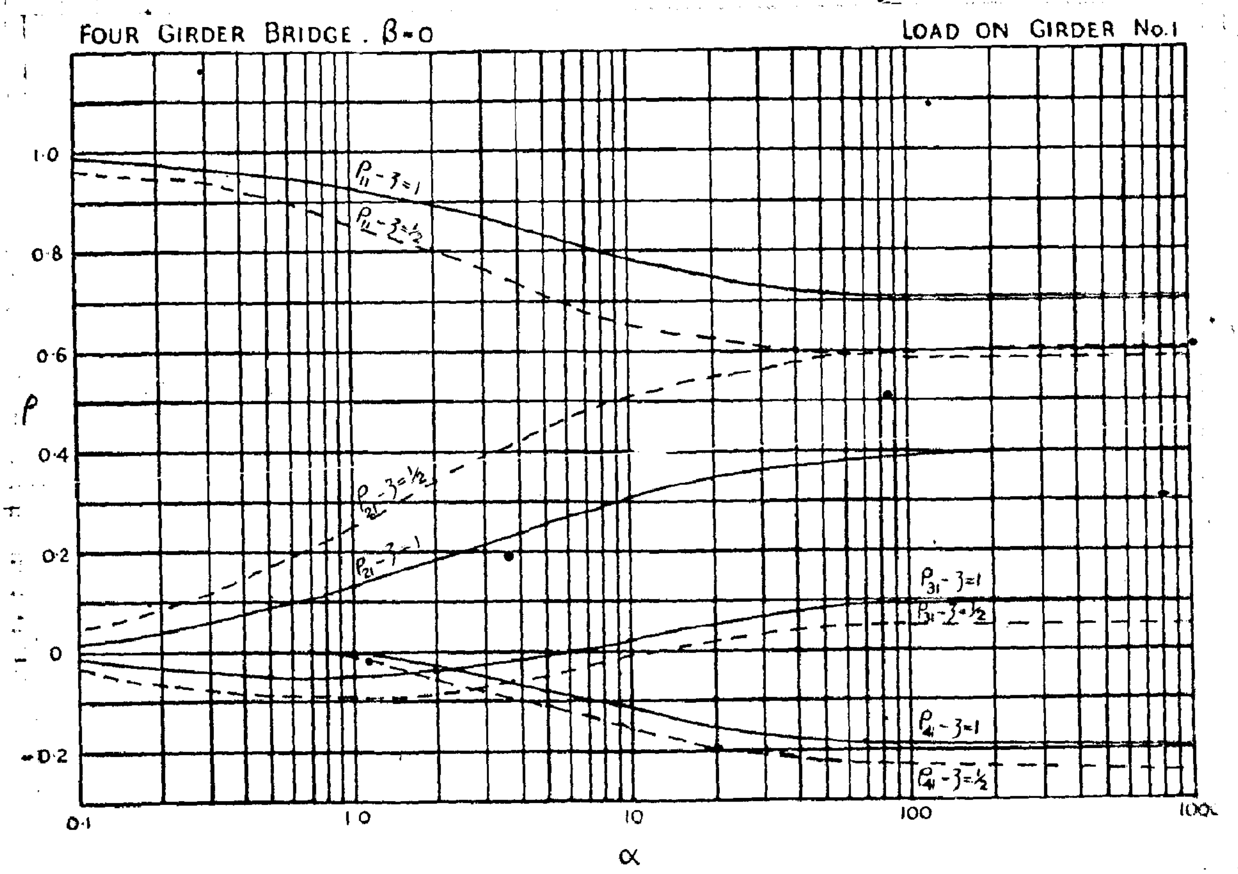

THREE GIRDER BRIDGE . β = ∞
LOAD ON GIRDER No.2
1·0
0·8
0·6
ρ
0·4
0·2
0·1
1·0
10
100
1000
α

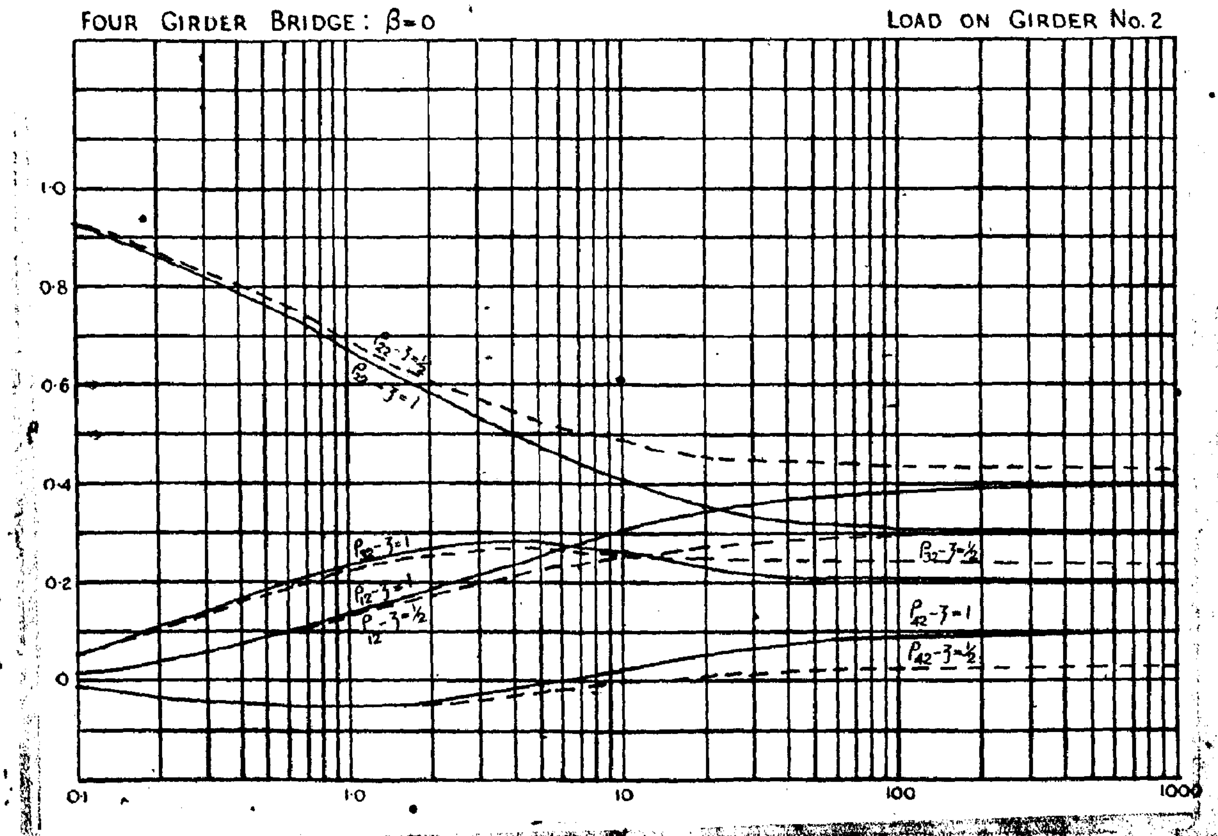
THREE GIRDER BRIDGE : $\beta = \infty$ – NO ROTATION — LOAD ON GIRDER No. 1

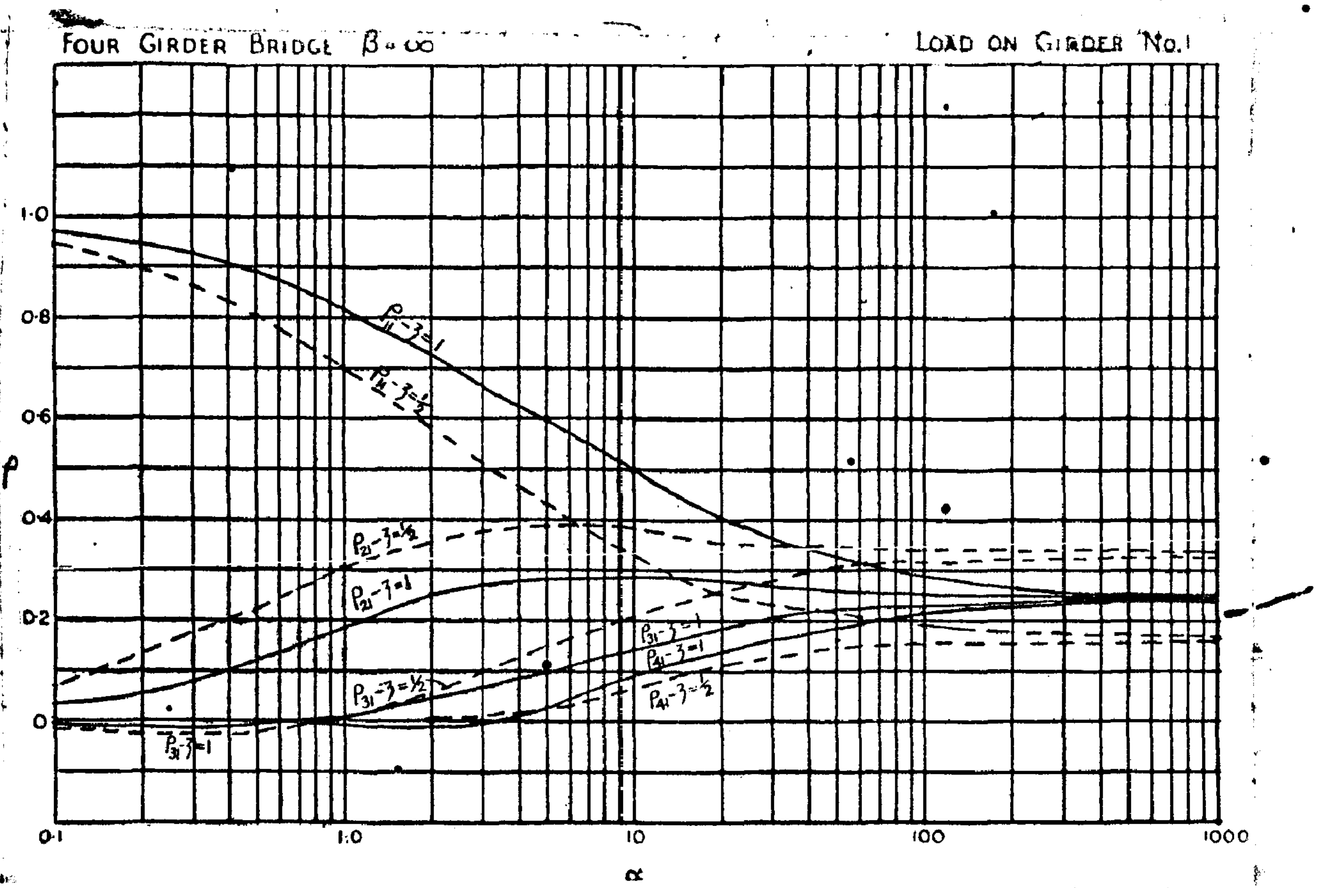


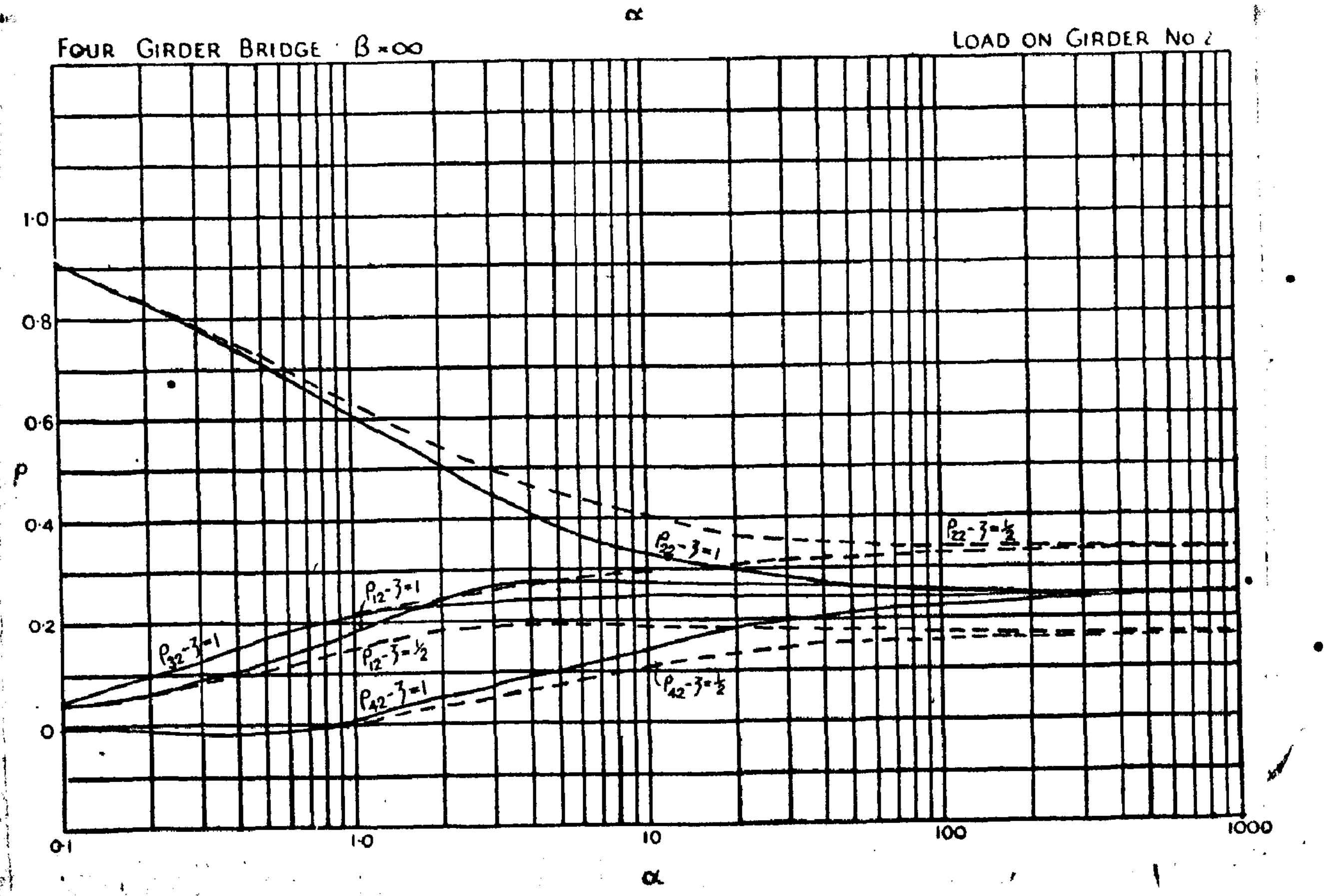
THREE GIRDER BRIDGE $\beta = \infty$ – NO ROTATION — LOAD ON GIRDER No 2

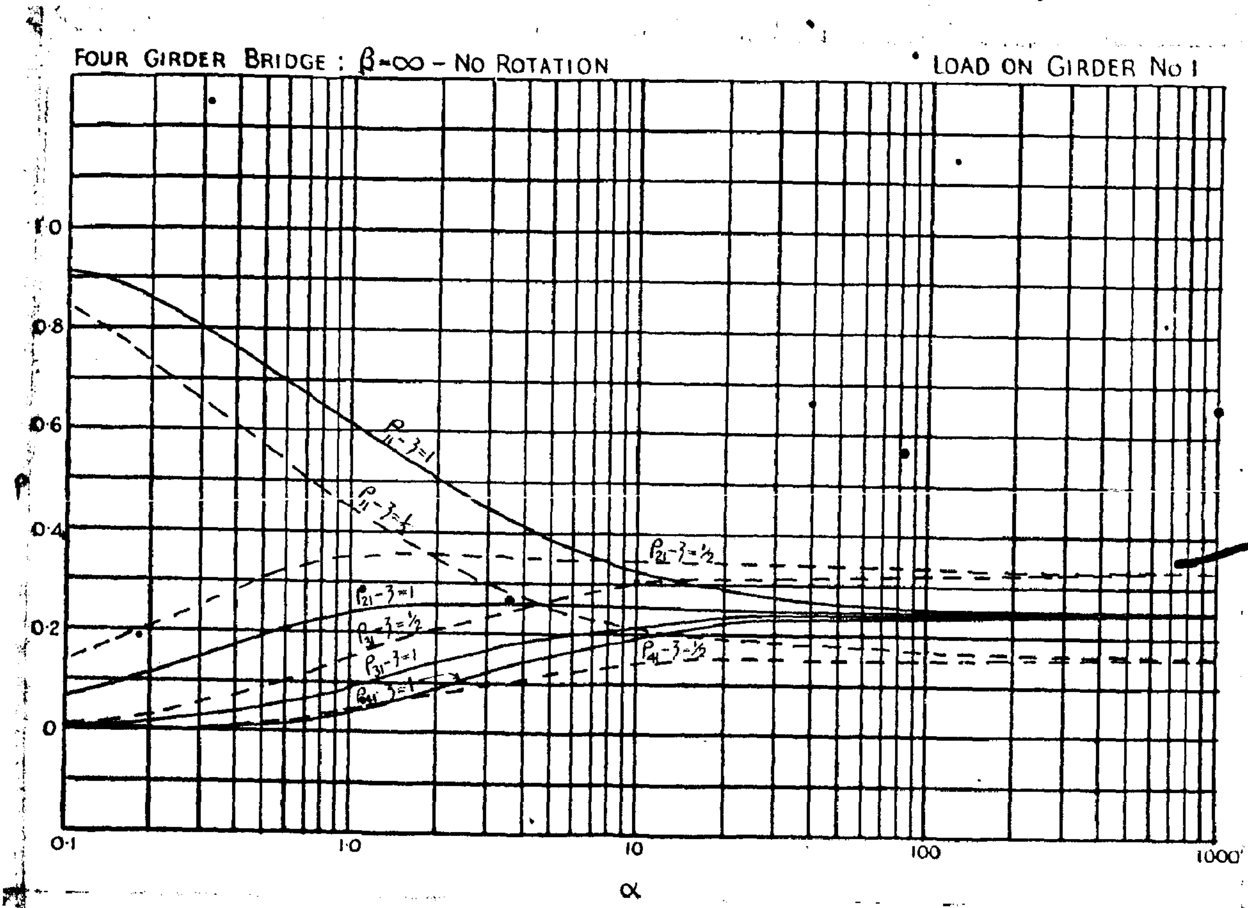


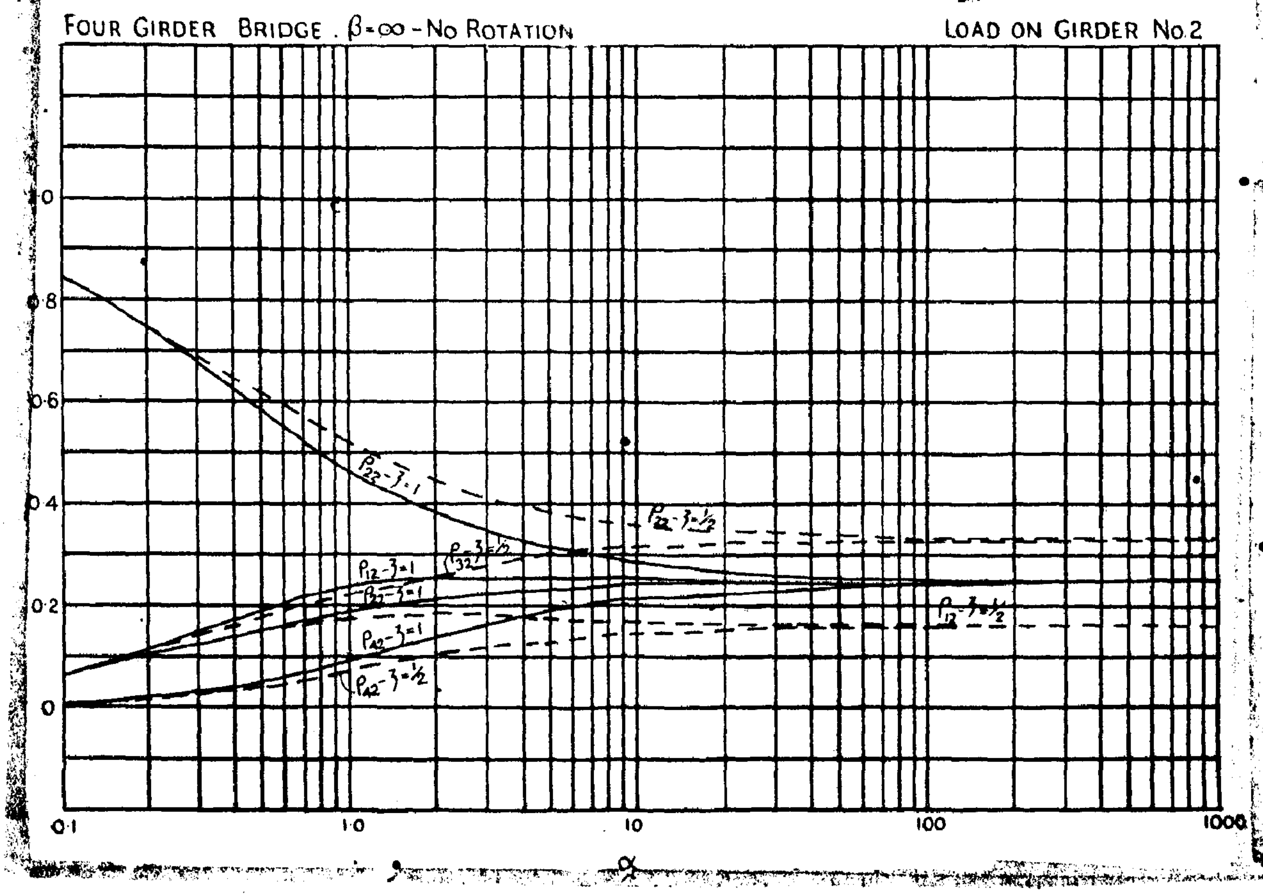

FOUR GIRDER BRIDGE . β=0
LOAD ON GIRDER No.1
1·0
0·8
0·6
0·4
0·2
0
-0·2
ρ
ρ11 - ζ=1
ρ11 - ζ=½
ρ21 - ζ=½
ρ21 - ζ=1
ρ31 - ζ=1
ρ31 - ζ=½
ρ41 - ζ=1
ρ41 - ζ=½
0·1
1·0
10
100
α


FOUR GIRDER BRIDGE: β=0
LOAD ON GIRDER No.2
1·0
0·8
0·6
0·4
0·2
0
ρ
ρ22 - ζ=½
ρ22 - ζ=1
ρ32 - ζ=1
ρ12 - ζ=1
ρ12 - ζ=½
ρ32 - ζ=½
ρ42 - ζ=1
ρ42 - ζ=½
0·1
1·0
10
100
1000
α


FOUR GIRDER BRIDGE β = ∞
LOAD ON GIRDER No. 1
1·0
0·8
0·6
P
0·4
0·2
0
0·1
1·0
10
100
1000
α


FOUR GIRDER BRIDGE β = ∞
LOAD ON GIRDER No 2
1·0
0·8
0·6
P
0·4
0·2
0
0·1
1·0
10
100
1000
α


FOUR GIRDER BRIDGE : β=∞ – NO ROTATION
LOAD ON GIRDER No 1
1·0
0·8
0·6
0·4
0·2
0
ρ
0·1
1·0
10
100
1000
α


FOUR GIRDER BRIDGE . β=∞ – NO ROTATION
LOAD ON GIRDER No.2
1·0
0·8
0·6
0·4
0·2
0
0·1
1·0
10
100
1000
α