COMPUTER AIDED DESIGN OF WEIRS ON PERMEABLE FOUNDATION

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree

of

MASTER OF TECHNOLOGY

in

WATER RESOURCES DEVELOPMENT

By

NGUYEN VAN KIEN

DEPARTMENT OF WATER RESOURCES DEVELOPMENT AND MANAGEMENT
INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
ROORKEE -247 667 (INDIA)

JUNE, 2007

DEPARTMENT OF WATER RESOURCES

DEVELOPMENT & MANAGERMENT

INDIAN INSTITUDE OF TECHNOLOGY ROORKEE

Candidate's Declaration

I hereby declare that the dissertation titled **"Computer Aided Design of Weirs on Permeable Foundation"** which is being submitted in partial fulfillment of the requirements for the award of Degree of Master of Technology in **Water Resources Development (Civil)** at Department of Water Resources Development and Management (WRD&M), Indian Institute of Technology, Roorkee is an authentic record of my own work carried out during the period of July, 2006 to June, 2007 under the supervision and guidance of **Professor G.C Mishra,** WRD&M, IIT, Roorkee.

I have not submitted the matter embodied in this dissertation for the award of any other degree.

Place: Roorkee

Dated: June, 2007

……………………

Nguyen Van Kien

This is to certify that the above statement made by the candidature is correct to the best of my knowledge.

……………………

Emeritus Fellow

Prof. G.C Mishra

Ex-Professor

WRD&M, IIT, Roorkee

Roorkee-247667

(India)

ACKNOWLEDGEMENT

I take this opportunity to express my profound sense of gratitude and grateful regards to **Professor G.C Mishra**, Department of Water Resources Development & Management, IIT Roorkee for his valuable suggestion and guidance during this dissertation work.

I would also like to express my gratitude to Dr. S. K. Tripathi, Professor& Head and all the faculty members of WRD&M, IIT, Roorkee for their valuable suggestions during this dissertation work.

I wish to express my thanks to Center for Irrigation and Water Supply Research, Vietnam Institute for Water Resource Research, Ministry of Agriculture & Rural Development, Ministry of Education and Training, Government of Viet Nam and Government of India for giving me an opportunity to undergo M. Tech course at IIT Roorkee. Financial assistance provided by ITEC during the M. Tech course in Water Resources Development (Civil) is also highly acknowledged.

I would like to thank my friends and colleagues who presented me an advice or assistance during this work.

I am also grateful to the staff of the WRD&M who extended all cooperation whenever required.

Last but not the least, I would like to extend my gratitude to my parents, my brothers, sisters and my girl friend for their patients and good understanding throughout the course of my study.

Date
June, 2007

NGUYEN VAN KIEN

CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS

$A_{1,2,3,..}$	Geometry factors
C	Constant
ds	Depth of step
$F(\varphi,m)$	Elliptic integral first kind
I_E	Exit gradient
i	The imaginary unit
k	Coefficient of permeable
L	Length of weir floor
L_1,L_2,L_3	Length of fragments
M	Complex constants
N	Complex constants
P	Uplift pressure
q	Quantity of seepage
S, S_1,S_{21},S_{22},S_3	Depth of sheet pile
$T_1,T_2,T_{21},T_{22},T_3$	Depths of flow domain under structure
t	Transformation plane
u	Velocity in x-direction
v	Velocity in y-direction
x	Horizontal co-ordinate;
y	Vertical co-ordinate;
z	Complex variable
w	Complex variable
α	Coefficient angle of impervious floor boundary
$\alpha_1, \alpha_2, \alpha_3$	Head loss coefficient through fragments 1,2,3…
β	Coefficient angle of impervious boundary
Ø	Velocity potential function
Ψ	Stream function

SYNOPSIS

In irrigation engineering, weir is the most extensively used hydraulic structure for diversion of river flow. A diversion of water may be required for the purposes of irrigation, hydro electric power generation, drinking water supply or industrial, navigation, inter basin transfer of water or combination of any of these purposes.

The problems involved in hydraulic design of structures are linked to surface and subsurface flow conditions. In general design of weir on permeable foundation, for a given surface flow criteria, the cost of the apron can be minimized with respect to subsurface flow consideration to safeguard against undermining, the exit gradient must not be allowed to exceed a certain safe limit, the uplift force must not exceed the weight of the structures but the floor thickness should be at minimum from economic consideration. The provision of sheet piles or cutoffs play an important role in the distribution of the uplift pressure. The stability of a weir demands provision of an upstream sheet pile and downstream sheet pile to prevent slipping of the soil under the weir to anticipated scour holes at the upstream and downstream reaches. Further more, the downstream sheet pile prevents undermining but uplift pressures at pucca floor is increased with length of downstream sheet pile. Therefore, some intermediate sheet piles or cut off walls should be built for decreasing uplift pressure and exit gradient. However, the mapping the flow region onto the half-plane becomes complicated when the weir has several cut-offs. The number of parameters to be determined increases, since each cut off adds three new vertices to flow region.

The present study is primarily concerned with the analysis of two dimensional steady confined flow through isotropic porous media and the evaluation of quantity of seepage, pressure and exit gradient distribution for the hydraulic structures with multi cut-offs generally constructed in the field. The analysis for confined flows is done by applying Schwarz – Christoffel transformation. Analytical method of fragments was furnished by Pavlovsky (vide Harr 1962) to analysis uplift pressure distribution on the base of structure and exit gradient with composite cut offs. In this study various geometry of weir floor and slope of underlying impervious boundary has been considered. With known geometry, for any complex hydraulic structure, the uplift pressure and exit gradient can be computed using developed software.

CHAPTER 1

INTRODUCTION

1.1 General

In irrigation engineering, weir is the most extensively used hydraulic structure for diversion of river flow for the purposes of irrigation, hydro electric power generation, drinking water supply or industrial use, navigation, inter basin transfer of water or combination of any of these purposes. Type and shape of weir differ from one place to other, depend on available materials of construction, type of soil foundation and hydrology of the river.

The designs of the diversion structures are to be carried out in two parts, namely hydraulic and structural. In the hydraulic designs, overall dimensions and profiles of main structure and a few of the components are worked out so that satisfactory hydraulic performance of the structure can be ensured. In structural designs, the various sections and reinforcement wherever needed are worked out. Details are then worked out to have a structure which will be safer under any possible and probable combination of loading. In both the cases, the diversion structure has to be properly designed for both the surface and sub-surface flow condition. The surface designs will include the fixing up of waterway, top profile of various structures energy dissipation arrangements, protection works, safeguard against scour, length and protection of divide walls alignment, levels and protection of guide bunds, afflux bunds, etc. The subsurface designs includes fixing of the depth and section of cutoffs, uplift pressure calculation and computation of exit gradient, etc. Many structures failed in the past by undermining through piping due to excessive exit gradient, eruption of floor caused by uplift pressure exceeding gravity force, deep scour in the immediate vicinity on upstream or downstream of the impervious floor etc. In fact the modern designs of diversion structures have been developed from analysis of these failures.

As a weir is founded on porous medium, therefore, in addition to drag forces on account of the surface flow it is subjected to lift forces due to seeping of water under the weir foundation. The provision of sheet piles plays an important role in the distribution of the uplift pressure under the floor. The sheet piles at upstream and downstream ends of the impervious floor should be designed against scour due to surface flow conditions. The maximum depth of scour at a particular location depends upon the type of structure, and the curvature of the river. For given surface flow criteria, the pressure distribution under the floor and exit gradient, which depend on the length and geometry of the floor, depth and number of sheet-piles, depth of porous medium and others. However, calculation uplift pressure and exit gradient become complicated when the weir has several cutoffs, as is apparent from paragraph the number of parameters to be increases, since each cutoff add two or three new vertices to the law of transformation. An approximate analytical method of solution for any confined flow system of finite depth, directly applicable to design, was furnished by Pavlovsky in 1935 (vide Harr 1962). The fundamental assumption of this method, called the method of fragment, is equipotential lines at various critical parts of the flow region can be approximated by straight vertical lines that divide the region into fragments or sections. By method of fragment the complexity of transformation is simplified and, the uplift pressure distribution under weir floor and exit gradient can be easily obtained.

1.2. Objective of Study

The present investigation is primarily concerned with two – dimensional steady confined flow through foundation of a weir resting on a pervious foundation with finite depth, the impervious boundary may dip upstream or downstream or it may be horizontal. The Schwarz – christoffel, transformation, and method of fragment are used for analyses.

1.3. Scope of Study

In this study, an attempt has been made to study the uplift pressure distribution under weir floor and exit gradient. The following hydraulic structures have been studied:

(i) A depressed weir resting on porous medium of finite depth.

(ii) A weir having multi vertical cutoffs and resting on porous medium of finite depth. The underlying impervious boundary is not horizontal, either the upstream flow domain or the downstream flow domain has a triangular shape.

The solutions of the above cases have been obtained with the help of conformal mapping and numerical integration, King method (vide Zhang Shanjie) have been used to carry out the elliptic integral. A computer programming in C++ has been developed.

CHAPTER 2

REVIEW OF LITERATURE

1. GENERAL

The subject of flow of water through soil is of prime importance in the design, construction and operation of hydro-engineering installations. From the time Darcy established the linear relationship between discharge velocity and hydraulic gradient, this field has been enriched by the contribution of several investigation and many field problems have been analysed.

Confined flow under a weir having a vertical sheet piles and resting on a horizontal layer of porous medium underlain by either a horizontal impervious stratum or a draining layer was investigated by Muskat, 1937; Pavlovsky (vide Harr, 1962)

Chawla (1967) computed the exit gradient and uplift pressures at the key points of hydraulic structures by changing the length of both upstream and downstream pervious reaches.

The study of uplift pressures on depressed weir floors with downstream vertical cutoff was conducted by Surendra Kumar (1968) by electrical analogy. He has studied the effect of different lengths of pervious medium upstream and downstream of the structure.

Verigin (vide Harr 1962), Mishra (1972), Basu (1976) and many other have analysed the confined flow under a weir laid on permeable foundation for various shape of the foundation floor by using Schwarz – Christoffel conformal mapping technique.

Several methods are applicable to calculate seepage characteristics of various confined flow systems. Although closed-form solutions exist for special structures with two, three, and even four sheet-piles, the resulting expressions are generally too complicated for engineering use. Theoretically the solution exists for any configuration; however, with each additional sheetpile or alterlation in contour of the structure the flow domain adds two or three new vertices to the Schwarz – Christoffel transformation and hence necessitates the evaluation of hyper elliptic integrals. Some approximate method of solution for confined flow problems are: Graphical Flow Net, Electrical Analogue, Viscous Flow Models (Hele – Shaw Model), Relation Method, Method of Fragment etc.

In this study we apply an approximate analytical method of solution for any confined flow system of finite depth, directly applicable to design, which was proposed by Pavlovsky (section 2.4) to analysis seepage flow under weir structure.

2. THEORETICAL ANALYSES

2.1 Two dimensional flow:

Physically, all flow system extend in three dimensions. However, in many problems the features of groundwater motion are essentially planar, with the motion being substantially the same in parallel planes. For this problem, the flow system can be simplified as two-dimensional flow.

The equation of continuity for a two dimensional steady flow is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2.1.1)$$

in which

u, v = discharge velocities in x and y directions, respectively. From generalized Darcy's law,

$$u = -k_x \frac{\partial h}{\partial x} = \frac{\partial \emptyset}{\partial x} = \frac{\partial \psi}{\partial y} \qquad (2.1.2)$$

$$v = -k_y \frac{\partial h}{\partial y} = \frac{\partial \emptyset}{\partial y} = -\frac{\partial \psi}{\partial x} \qquad (2.1.3)$$

k_x, k_y = Principle coefficients of permeability in x and y directions, respectively. In case of isotropic soil k_x=k_y=k

h = Total head = $\left(\frac{p}{\gamma_w} + y\right)$

Φ= velocity potential $= -kh + c$

Then the continuity equation becomes

$$\frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \emptyset}{\partial y^2} = 0 \qquad (2.1.4)$$

A combination of the function Φ and Ψ is called complex potential and defined by $w = \phi + i\psi$

Much of the analytical method for the solution of two-dimensional ground water problems is concerned with the determination of a function which will transform a problem from a geometrical domain within which a solution is sought into one within which the solution is known. We shall consider the more general method of finding a functional relationship that provide a specific transformation.

Let $\omega = \Phi + i\psi$ be an analytic function of $z = x + iy$ and a geometrical representation of $\omega = f(z)$.

2.2. The Schwarz – Christoffel Transformation:

In ground water problem, where it is often necessary to determine the seepage characteristics within complicated boundaries, solving the flow problem by drawing flow net is far from satisfactory. Theoretically, a transformation exists which will map any pair of simply connected regions conformally onto each other. This is assured by the Riemann mapping theorem.

If a polygon is located in the z plane, then the transformation that maps it conformally onto the upper half of the t plane (t = r + is) is:

$$z = M \int \frac{dt}{(t-a)^{1-\frac{A}{\pi}}(t-b)^{1-\frac{B}{\pi}}(t-c)^{1-\frac{C}{\pi}} \ldots} + N \tag{2.1.5}$$

where M and N are complex constants A, B, C,....., are the interior angles (in radians) of the polygon in the z plane and a,b,c...(a < b < c <...) are points on the real axis of the t plane corresponding to the respective vertices A, B, C, ... The parameters a, b, c, and constants M and N are determined from the geometry of the flow domain.

2.3. Exit gradient:

The exit gradient is the hydraulic gradient of the seepage flow under the base of the weir floor. As the rate of the seepage increases in exit gradient, and such an increase would cause boiling of the surface soil and the soil to be washed away by the percolating water The flow concentrates into the resulting depression thus removing more soil and creating progressive scour backwards. This phenomenon is called piping and eventually undermines the weir foundations.

Criteria for safety against piping is not the average hydraulic gradient as enunciated by Blight (1916). Water has a certain residual force at each part a long its flow through the subsoil which acts in the direction of flow and is proportional to the hydraulic gradient at that point. At the tail end, this force is obviously upwards and will tend to lift up the soil particles if it is more than the submerged weight of the latter. The frictional resistance, cohesion, etc., of the adjacent soil will have to be considered in certain cases. Once the surface particles are disturbed the resistance against upward pressure of water will be further reduced, tending to progressive disruption of the subsoil. The flow gathers into a series of pipes in the latter and dislocation of particles is accelerated. The subsoil is thus progressively undermined. Soil erosion can also occur through natural pipes or faults in subsoil. The factor of safety has to take note of the class of material, specific weight and pore space, angle of faint in the subsoil.

The seepage force per unit volume is proportional to the hydraulic gradient as

$$F = \gamma_w \left(\frac{\partial h}{\partial s}\right) \tag{2.1.6}$$

The submerged unit weight of the soil (W_S) is

$$Ws = \left(\frac{Gs - 1}{1 + e}\right)\gamma_w = \gamma_w(1 - n)(Gs - 1) \tag{2.1.7}$$

where: Gs = the specific gravity of soil particles

n = the porosity

e = the void ratio

γ_w = the specific weight of water.

In the critical condition, the upward force will be just balanced by the submerged unit weight of soil. Thus the critical gradient I_{cr} will be found as follows:

$$F = Ws$$

$$\gamma_w \left(\frac{\partial h}{\partial s}\right) = \gamma_w(1 - n)(Gs - 1)$$

$$\left(\frac{\partial h}{\partial s}\right) = (1 - n)(Gs - 1)$$

$$I_{cr} = (1 - n)(Gs - 1) \tag{2.1.8}$$

2.4. Method of Fragment:

For solving Laplace equation by conformal mapping method Schwarz – Christoffel transformation is used. However, for the complicated apron profiles with multiple sheet piles, solution by Schwarz – Christoffel transformation has been found to be difficult. An approximate analytical method of solution for any confined flow system of finite depth, directly applicable to design, was furnished by Pavlovsky in 1935. The fundamental assumption of this method called the method of fragment, is that equipotential lines at various critical parts of the flow region can be approximated by straight vertical lines that divide the region into fragment or section

Suppose, now that we can compute the discharge in the m^{th} fragment as

$$q = \frac{kh_m}{\emptyset_m} \qquad m = 1, 2, \ldots n \tag{2.1.9}$$

Where

h_m=head loss through fragment,

ϕ_m = dimensionless form factor

Then, since the discharge through all fragment must be the same

$$q = \frac{kh_1}{\emptyset_1} = \frac{kh_2}{\emptyset_2} = \frac{kh_m}{\emptyset_m} = \cdots = \frac{kh_n}{\emptyset_n} \tag{2.1.10}$$

$$q = k \sum \frac{h_m}{\emptyset_m} = \frac{kh}{\sum_{m=1}^{n} \emptyset_m} \tag{2.1.11}$$

Where h (with out subscript) is the total loss through the section. By similar reasoning we find that the head loss in the m^{th} fragment can be calculated from

$$h_m = \frac{h\emptyset_m}{\sum_{m=1}^{n} \emptyset_m} \tag{2.1.12}$$

Once the head loss for any fragment has been determined the pressure distribution on the base of the structure and the exit gradient can be easily obtained. Thus the primary task is to implement this method by establishing a catalogue of typical form factor.

CHAPTER 3

A STEPPED WEIR ON A POROUS MEDIUM OF FINITE DEPTH

3.1 INTRODUCTION:

Analysis of flow under a stepped weir with a sheet pile resting on a porous medium of finite depth has been presented by Khosla, Bose, and Taylor. In nature, the thickness of subsoil porous layer is always finite. The uplift pressure acting on the base of the structure is governed by the thickness of this layer. We analyses the flow under weir resting on a porous medium of finite depth. The flow domain is decomposed into two fragments and conformal mapping technique is applied.

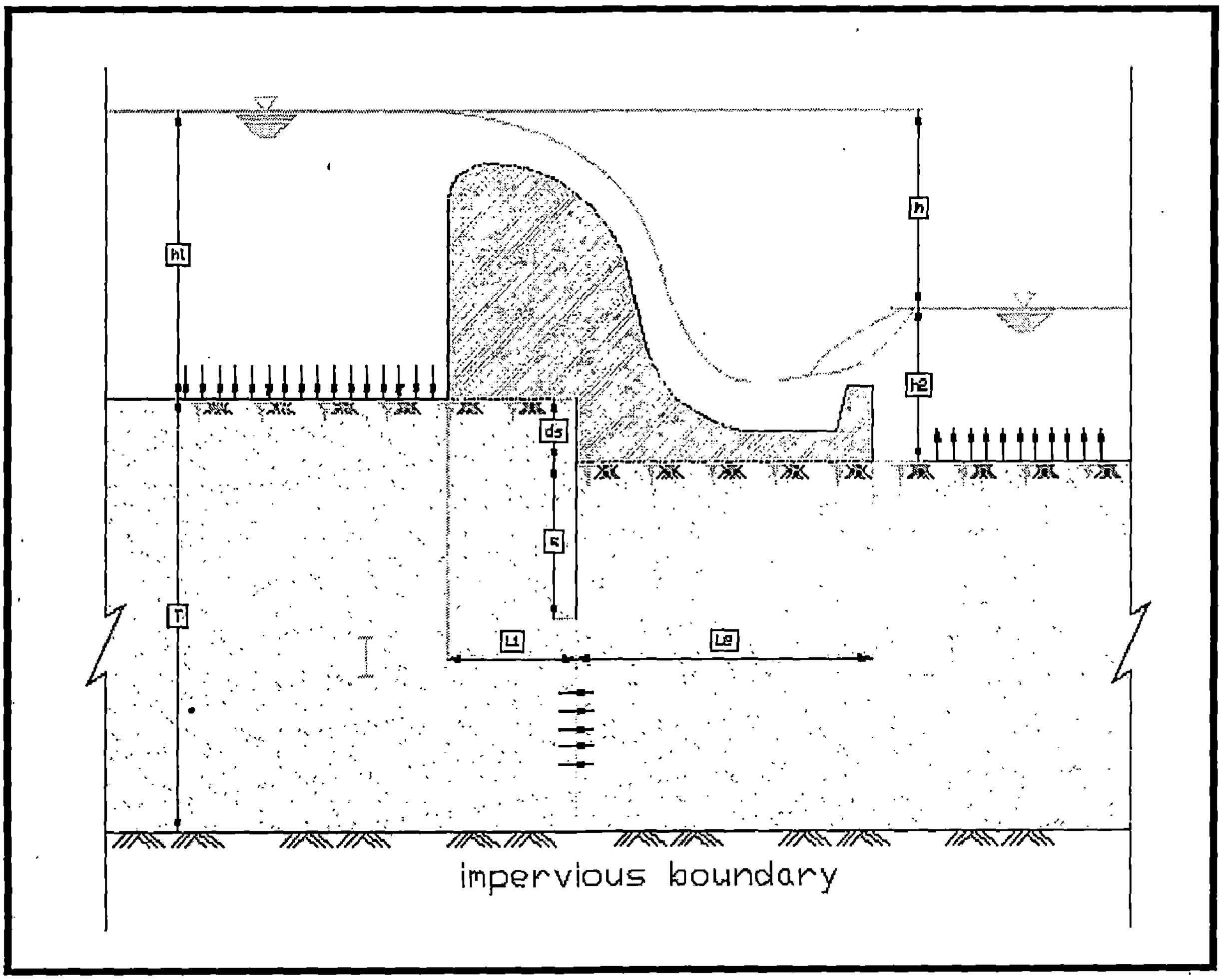


FIG 3.1 A STEPPED WEIR ON PERMEABLE FOUNDATION OF FINITE DEPTH

3.2 STATEMENT OF THE PROBLEM:

The weir resting on permeable foundation of finite depth hasthe following dimension:

- Height of water at upstream of weir	h_1
- Height of water at downstream of weir	h_2
- Difference of hydraulic head between upstream and downstream	h
- Length of weir floor of fragment 1	L_1
- Length of weir floor of fragment 2	L_2
- Total length of weir floor	$L=L_1+L_2$
- Depth of step	ds
- Depth of sheet pile	s
- Discharge per unit width of weir	q
- Coefficient of permeability of the homogeneous isotropic soil	k

3.3 SEEPAGE FLOW ANALYSIS

3.3.1 FRAGMENT 1:

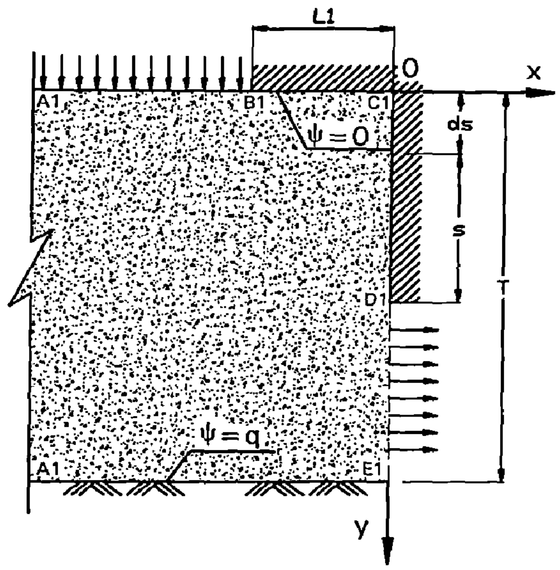
3.3.1.1 Mapping of the flow domain in z plane onto t plane z = $f_1(t)$ (Fig 3.2a)

According to Schwarz – Christoffel transformation, the mapping function is

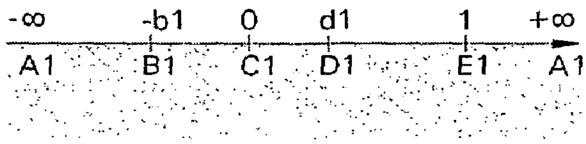
$$\frac{dz}{dt} = \frac{M}{t^{\frac{1}{2}}(1-t)^{\frac{1}{2}}}$$

$$or \quad z(t') = M\int_0^t \frac{dt}{t^{\frac{1}{2}}(1-t)^{\frac{1}{2}}} + N = 2M\sin^{-1}\sqrt{t} + N \qquad (3.1.1.1)$$

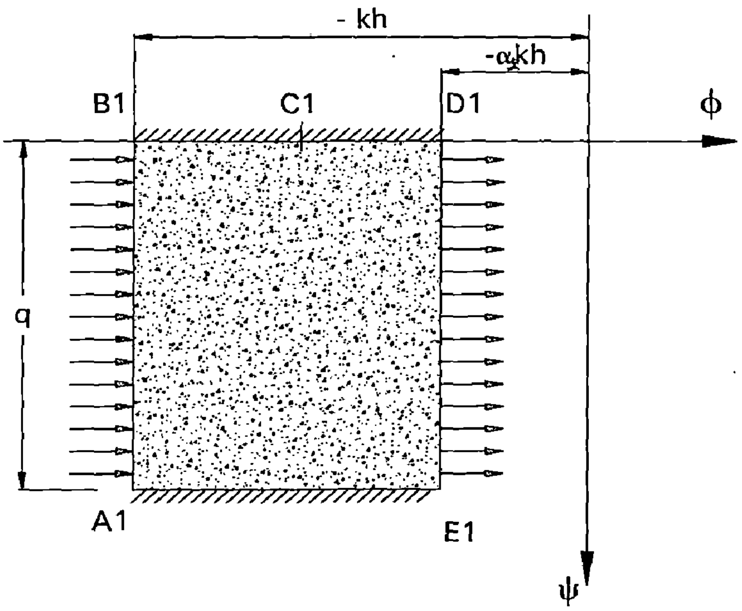
where M, N are constants



(a) z – Plane (z = x + iy)



(b) t –Plane (t = r + is)



(c) w –Plane (w = Ø + iΨ)

FIG. 3.2 STEP OF MAPPING FOR FRAGMENT 1 (CASE 1)

(i) For point C_1, t=0, and $z(t) = z_{C1}$; hence N=0

Hence $z(t') = 2M \sin^{-1}\sqrt{t}$

(ii) For point E_1, t = 1, Z(t) = Z_E = iT

hence, $iT = 2M \sin^{-1}\sqrt{1}$ *or,* $Z_{E1} = M\pi$

$$then \;\; M = \frac{iT}{\pi} \tag{3.1.1.2}$$

(iii) For point D_1, t = d_1, and $Z_{D1} = i(ds + s)$

Hence, $i(d_s + s) = \frac{2iT}{\pi} \sin^{-1}\sqrt{d_1}$

$$or, \;\; d_1 = \left\{\sin\left[\frac{\pi(d_s + s)}{2T}\right]\right\}^2 \tag{3.1.1.3}$$

(iv) For point B_1, t = -b_1, Z(t) =Z_{B1}= -L_1

Hence, $-L_1 = \frac{2iT}{\pi} \sin^{-1}\sqrt{-b_1}$

$\left[Note \;\; i^2 = -1, \qquad \text{Sin}(ix) = isinh(x), \qquad \sinh(x) = \frac{e^x - e^{-x}}{2}\right]$

Simplifying, we get

$$b_1 = \left[\sinh\left(\frac{L_1\pi}{2T}\right)\right]^2 \tag{3.1.1.4}$$

3.3.1.2 Complex potential plane $w = f_2(t)$

The transformation of the polygon in w plane onto t plane (Figure 3.2 c) is given by

$$\frac{dw}{dt} = \frac{M_1}{(-b_1 - t)^{\frac{1}{2}}(d_1 - t)^{\frac{1}{2}}(1 - t)^{\frac{1}{2}}}$$

$$or, w = \int_{-\infty}^{t} \frac{M_1}{(-b_1 - t)^{\frac{1}{2}}(d_1 - t)^{\frac{1}{2}}(1 - t)^{\frac{1}{2}}} + N_1$$

where M_1 and N_1 are constants

(a) Integration along flow boundary A_1B_1 $(-\infty \leq t \leq -b_1)$

(i) For point A_1, t = -∞, $w = iq - kh$

(ii) At point B_1 , $t = -b1, w = -kh$, hence

$$-kh = \int_{-\infty}^{-b1} \frac{M_1}{(-b_1 - t)^{\frac{1}{2}}(d_1 - t)^{\frac{1}{2}}(1 - t)^{\frac{1}{2}}} + iq - kh$$

$$q = -\frac{M_1}{i}\int_{-\infty}^{-b1} \frac{dt}{(-b_1 - t)^{\frac{1}{2}}(d_1 - t)^{\frac{1}{2}}(1 - t)^{\frac{1}{2}}}$$

Performing the integration (Byrd and Friedman, 1971)

$q = -\frac{M_1}{i} gF(\varphi, m)$

$$where\ , g = \frac{2}{\sqrt{1 + b_1}}, \quad \varphi = \sin^{-1}\sqrt{\frac{1 + b_1}{1 + b_1}} = \frac{\pi}{2}, \quad m = \sqrt{\frac{1 - d_1}{1 + b_1}}$$

Hence,

$$q = \frac{2M_1 i}{\sqrt{1 + b_1}} F\left(\frac{\pi}{2}, \sqrt{\frac{1 - d_1}{1 + b_1}}\right) \qquad (3.1.2.1)$$

(b) Integration along flow boundary B_1D_1 ($-b_1 \leq t \leq d_1$)

(i) At point B1, t = $-b_1$, $w = -kh$

(ii) At point D1, t = 1, $w = -\alpha_1 kh$

Hence

$$-\alpha_1\ kh = M_1 \int_{-b1}^{d1} \frac{dt}{(1 - t)^{\frac{1}{2}}(d_1 - t)^{\frac{1}{2}}(-b_1 - t)^{\frac{1}{2}}} - kh$$

$$or, \quad -\alpha_1\ kh = \frac{M_1}{\sqrt{i^2}} \int_{-b}^{d} \frac{dt}{(1 - t)^{\frac{1}{2}}(d_1 - t)^{\frac{1}{2}}[t - (-b_1)]^{\frac{1}{2}}} - kh$$

Performing the integration (Byrd and Friedman, 1971)

$$-\alpha_1\ kh = \frac{M_1}{\sqrt{i^2}} gF(\varphi, m) - kh$$

$$g = \frac{2}{\sqrt{1 + b_1}}, \quad \varphi = \sin^{-1}\sqrt{\frac{1 + b_1}{1 + b_1}} = \frac{\pi}{2}, \quad m = \sqrt{\frac{d_1 + b_1}{1 + b_1}}$$

Therefore,

$$-\propto_1 kh = \frac{M_1}{\sqrt{i^2}}\frac{2}{\sqrt{1+b_1}}F\left(\frac{\pi}{2},\sqrt{\frac{d_1+b_1}{1+b_1}}\right) - kh$$

$$i.e. \quad M_1 = \frac{\sqrt{i^2}(1-\propto_1)kh\sqrt{1+b_1}}{2F\left(\frac{\pi}{2},\sqrt{\frac{d_1+b_1}{1+b_1}}\right)} \tag{3.1.2.2}$$

Seepage discharge is positive then, substituting value of M_1 into (3.1.2.1) we have

$$q = (1-\propto_1)kh\frac{F\left(\frac{\pi}{2},\sqrt{\frac{1-d_1}{1+b_1}}\right)}{F\left(\frac{\pi}{2},\sqrt{\frac{d_1+b_1}{1+b_1}}\right)} \tag{3.1.2.3}$$

$$letting, A_1 = \frac{F\left(\frac{\pi}{2},\sqrt{\frac{1-d_1}{1+b_1}}\right)}{F\left(\frac{\pi}{2},\sqrt{\frac{d_1+b_1}{1+b_1}}\right)} \qquad then, \qquad q = (1-\propto_1)khA_1 \tag{3.1.2.4}$$

(c) Integration along floor boundary B_1C_1 $(-b_1 \le t \le 0)$

(i) At point B_1, $t = -b_1$; $w(t) = -kh$

(ii) At point C_1, $t = 0$, $w = w(C_1)$

$$w(C_1) = M_1\int_{-b1}^{0}\frac{dt}{(1-t)^{\frac{1}{2}}(d_1-t)^{\frac{1}{2}}(-b_1-t)^{\frac{1}{2}}} - kh$$

$$or \quad w(C_1) = \frac{M_1}{\sqrt{-1}}\int_{-b}^{0}\frac{dt}{(1-t)^{\frac{1}{2}}(d_1-t)^{\frac{1}{2}}[t-(-b_1)]^{\frac{1}{2}}} - kh \tag{3.1.2.5}$$

Performing the integration (Bryd and Fried man, 1971)

$$w(C_1) = \frac{M_1}{\sqrt{-1}}gF(\varphi, m) - kh$$

$$g = \frac{2}{\sqrt{1+b_1}}, \qquad \varphi = \sin^{-1}\sqrt{\frac{b_1}{d_1+b_1}}, m = \sqrt{\frac{d_1+b_1}{1+b_1}}$$

$$or, w(C_1) = \frac{M_1}{\sqrt{-1}}\frac{2}{\sqrt{1+b_1}}F\left(\sin^{-1}\sqrt{\frac{b_1}{d_1+b_1}},\sqrt{\frac{d_1+b_1}{1+b_1}}\right) - kh \tag{3.1.2.6}$$

Substituting M_1 from (3.1.2.2) into (3.1.2.6) we have

$$w(C_1) = (1 - \propto_1)kh\frac{F\left(\sin^{-1}\sqrt{\frac{b_1}{d_1+b_1}}, \sqrt{\frac{d_1+b_1}{1+b_1}}\right)}{F\left(\frac{\pi}{2}, \sqrt{\frac{d_1+b_1}{1+b_1}}\right)} - kh \qquad (3.1.2.7)$$

3.3.2. FRAGMENT 2:

3.1.1.1 Mapping of the flow domain in z plane onto t plane $z = f_1(t)$ (Fig 3.3a)

According to Schwarz – Christoffel transformation the conformal mapping of fragment 2 in Z plane onto the auxiliary t plane is given by

$$\frac{dz}{dt} = \frac{M}{t^{\frac{1}{2}}(t - c_2)^{1/2}}$$

$$or, z = M\int_0^t \frac{dt}{t^{\frac{1}{2}}(t - c_2)^{1/2}} + N \qquad (3.2.1.1)$$

Where M, N are constants.

(a) Integration along boundary A_2B_2 and A_2C_2 $(0 \le t \le b_2 \; and \; 0 \le t \le c_2)$

(i) For point A_2, $t = 0$, $z = z_{A2}$, hence $N = z_{A2}$

(ii) For point B_2, $t = b_2$, $z = z_{B_2}$

(iii) For point C_2, $t = c_2$, $z = z_{C_2}$

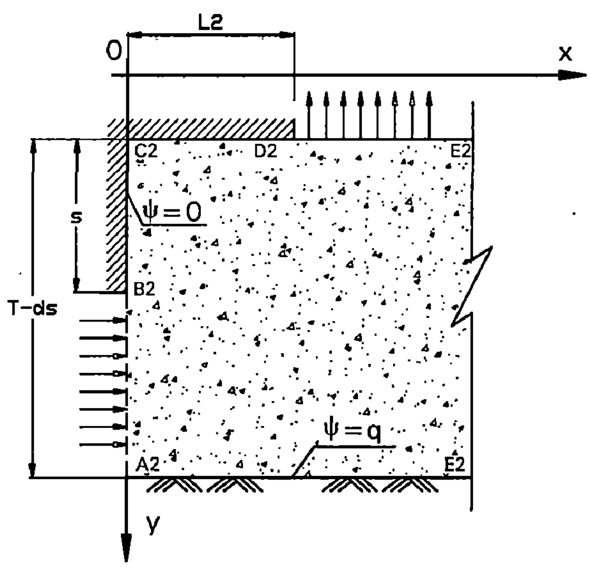
$$z(t) = M\int_0^t \frac{dt}{t^{\frac{1}{2}}(t - c_2)^{1/2}} + Z_A$$

$$or \qquad z(t') = M\int_0^t \frac{dt}{t^{\frac{1}{2}}(-1)^{1/2}(t - c_2)^{1/2}} + Z_A \qquad (3.2.1.2)$$

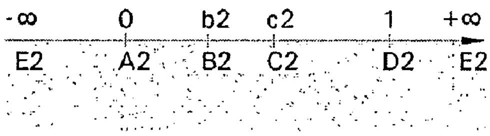
Setting $t = c_2 \, sin^2 r, dt = 2c_2 \, \sin r \cos r \; dr$

Substituting to the equation (3.2.1.2) we have

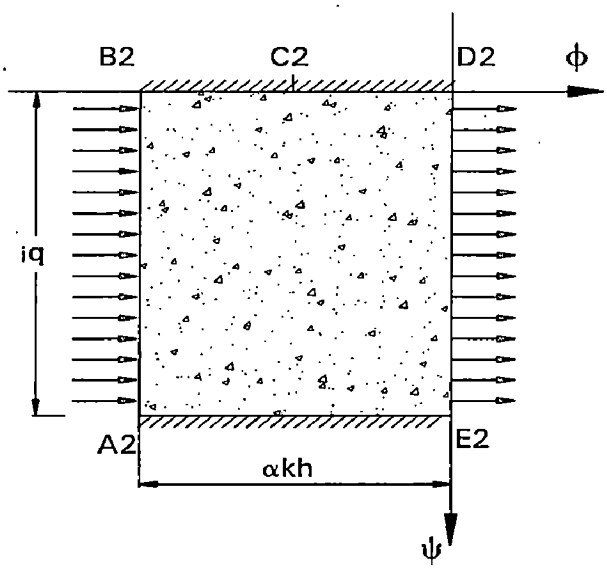
$$z(t) = \frac{M}{i}\int_0^t \frac{2c_2 \sin r \cos r \, dr}{(\sqrt{c_2})^2 \sin r \cos r} + Z_{A2} = 2\frac{M}{i}\int_0^{\sin^{-1}\sqrt{\frac{t'}{c2}}} dr$$



(a) z – Plane (z = x + iy)



(b) t –Plane (t = r + is)



(c) w –Plane (w = Φ + iψ)

FIG. 3.3 STEP OF MAPPING FOR FRAGMENT 2 (CASE 1)

$$z(t) = \frac{2M}{i}\sin^{-1}\sqrt{\frac{t}{c_2}} + z_A \qquad (3.2.1.3)$$

Replacing values of $t = b_2$ and $t = c_2$ to (3.2.1.3) we have

$$z_{B2} = \frac{2M}{i} \sin^{-1} \sqrt{\frac{b_2}{c_2}} + z_{A2} \tag{3.2.1.4}$$

$$z_{C2} = \frac{2M}{i} \sin^{-1} \sqrt{\frac{c_2}{c_2}} + z_{A2} = \frac{M\pi}{i} + z_{A2} \tag{3.2.1.5}$$

$$z_{C2} - z_{A2} = \frac{M\pi}{i} \text{ or } M = \frac{i(z_{C2} - z_{A2})}{\pi} = \frac{T - d_s}{\pi}$$

Therefore,

$$z_{C2} - z_{B2} = \frac{2M}{i}\left(\pi - \sin^{-1}\sqrt{\frac{b_2}{c_2}}\right) \text{ and } z_{B2} - z_{A2} = \frac{2M}{i} \sin^{-1}\sqrt{\frac{b_2}{c_2}} \tag{3.2.1.6}$$

From z-Plane, we have

$$z_{B2} - z_{A2} = -i(T - d_s - s) \quad \text{and} \quad z_{C2} - z_{B2} = -iS \tag{3.2.1.7}$$

From the equations (3.2.1.6) and (3.2.1.7) we have the ratio

$$\frac{z_{C2} - z_{B2}}{z_{B2} - z_{A2}} = \frac{\pi - 2\sin^{-1}\sqrt{\frac{b_2}{c_2}}}{2\sin^{-1}\sqrt{\frac{b_2}{c_2}}} \qquad \text{simplifying,} \qquad \frac{\pi(T - d_s - s)}{2(T - d_s)} = \sin^{-1}\sqrt{\frac{b_2}{c_2}}$$

therefore,

$$\frac{b_2}{c_2} = sin^2\left[\frac{\pi(T - d_s - s)}{2(T - d_s)}\right] \tag{3.2.1.8}$$

(b) Integration along weir floor boundary C_2D_2 $(c \leq t \leq 1)$

(i) For point C_2, $t = c_2$, $z = z_{C2}$, hence $N = z_{C2}$

(ii) For point D_2, $t = 1$, $z = z_{D2}$

$$z(t) = M \int_{c}^{t'} \frac{dt}{t^{\frac{1}{2}}(t - c_2)^{\frac{1}{2}}} + z_{C2} \tag{3.2.1.9}$$

$$z(t) = 2M \cosh^{-1}\sqrt{\frac{t}{c_2}} + z_{C2} \tag{3.2.1.10}$$

$$z_{D2} = 2M \cosh^{-1} \sqrt{\frac{1}{c_2}} + z_C \quad or \quad z_{D2} - z_{C2} = 2M \cosh^{-1} \sqrt{\frac{1}{c_2}} \tag{3.2.1.11}$$

$$[\text{note } \cosh^{-1} x = \ln (x \pm \sqrt{x^2 - 1})]$$

From z-plane, we have $z_{D2} - z_{C2} = L_2$ hence,

$$L_2 = 2M \cosh^{-1} \sqrt{\frac{1}{c_2}}$$

$$or, \; L_2 = \frac{2(T - d_s)}{\pi} \cosh^{-1} \sqrt{\frac{1}{c_2}} \tag{3.2.1.12}$$

$$\cosh^{-1} \sqrt{\frac{1}{c_2}} = \ln \left(\sqrt{\frac{1}{c_2}} \pm \sqrt{\frac{1}{c_2} - 1} \right) \; \textit{substituting to the equation } (2.1.12) \textit{we have}$$

$$\frac{\pi L_2}{2(T - d_s)} = \ln \left(\sqrt{\frac{1}{c_2}} + \sqrt{\frac{1}{c_2} - 1} \right) \tag{3.2.1.13}$$

Therefore,

$$e^{\frac{\pi L_2}{2(T - d_s)}} = \sqrt{\frac{1}{c_2}} + \sqrt{\frac{1}{c_2} - 1} \tag{3.2.1.14}$$

By an iteration we get the value of c from (3.2.1.4) and substituting the value of c in equation (3.2.1.8) we get the value of b.

3.3.2.2 Complex potential plane $w = f_2(t)$

The transformation of the polygon in the mapping of w plane onto t plane (Fig 3.3 c) is given by

$$\frac{dw}{dt} = \frac{M_1 dt}{(t-1)^{\frac{1}{2}} (t - b_2)^{\frac{1}{2}} t^{\frac{1}{2}}}$$

$$or, w(t) = \int_0^t \frac{M_1 dt}{(t-1)^{\frac{1}{2}} (t - b_2)^{\frac{1}{2}} t^{\frac{1}{2}}} + N_1$$

where M_1 and N_1 are constants

(a) Integration along boundary A_2B_2 ($0 \leq t \leq b_2$)

(i) At point A_2, $t = 0$, $w(t) = -\propto_1 kh + iq,$

(ii) At point B_2, $t = b_2$, $w(t) = -\propto_1 kh$

Applying these condition in above equation

$$-\propto_1 kh = \int_0^{b_2} \frac{M_1 dt}{(1-t)^{\frac{1}{2}}(b_2-t)^{\frac{1}{2}}t^{\frac{1}{2}}} - \alpha kh + iq \tag{3.2.2.1}$$

$$q = M_1 i \int_0^{b_2} \frac{dt}{(1-t)^{\frac{1}{2}}(b_2-t)^{\frac{1}{2}}t^{\frac{1}{2}}} \tag{3.2.2.2}$$

Performing the integration (Bryd and Fried man, 1971)

$$q = M_1 i g F(\varphi, m)$$

Where $g = 2$, $\varphi = \frac{\pi}{2}$, $m = \sqrt{b_2}$ (3.2.2.3)

Hence,

$$q = 2M_1 i\, F\left(\frac{\pi}{2}, \sqrt{b_2}\right) \tag{3.2.2.4}$$

(b) Integration along floor boundary B_2D_2 ($b_2 \leq t \leq 1$)

(i) At point B_2, , $t = b_2, w = -\propto_1 kh$

(ii) At point D_2, $t = d_2, w = 0$

Applying these conditions

$$0 = \int_b^{t\prime} \frac{M_1 dt}{(t-1)^{\frac{1}{2}}(t-b_2)^{\frac{1}{2}}t^{\frac{1}{2}}} - \propto_1 kh \tag{3.2.2.5}$$

$$or, \quad 0 = \frac{M_1}{\sqrt{-1}} \int_b^{t\prime} \frac{dt}{(1-t)^{\frac{1}{2}}(t-b_2)^{\frac{1}{2}}t^{\frac{1}{2}}} - \propto_1 kh \tag{3.2.2.6}$$

$$or, \quad \propto_1 kh = \frac{M_1}{\sqrt{-1}} \int_b^{t\prime} \frac{dt}{(1-t)^{\frac{1}{2}}(t-b_2)^{\frac{1}{2}}t^{\frac{1}{2}}} \tag{3.2.2.7}$$

Performing the integration (Bryd and Fried man, 1971)

$$\propto_1 kh = \frac{M_1}{\sqrt{-1}} g F(\varphi, m) \tag{3.2.2.8}$$

$$\text{where} \quad \begin{cases} g = 2 \\ \varphi = \frac{\pi}{2} \\ m = \sqrt{1-b_2} \end{cases}$$

$$i.e.\, M_1 = \propto_1 kh \frac{\sqrt{-1}}{2F\left(\frac{\pi}{2}, \sqrt{1-b_2}\right)} \tag{3.2.2.9}$$

Substituting the value of M_1 from (3.2.2.9) to (3.2.2.4) we get the value of q

$$q = \propto_1 kh \frac{F\left(\frac{\pi}{2}, \sqrt{b_2}\right)}{F\left(\frac{\pi}{2}, \sqrt{1-b_2}\right)} \tag{3.2.2.10}$$

$$letting, A_2 = \frac{F\left(\frac{\pi}{2}, \sqrt{b_2}\right)}{F\left(\frac{\pi}{2}, \sqrt{1-b_2}\right)} \quad therefore, \quad q = \propto_1 khA_2 \tag{3.2.2.11}$$

(c) Integration along boundary B_2C_2 ($b_2 \leq t \leq c_2$)

(i) At point B2, $t = b_2, w = -\propto_1 kh$

(ii) At point C_2, $t = c_2, w = w_{C2}$

Applying these conditions in w(t) plane

$$w(C_2) = \int_{b2}^{c2} \frac{M_1 dt}{(t-1)^{\frac{1}{2}}(t-b_2)^{\frac{1}{2}}t^{\frac{1}{2}}} - \alpha kh$$

$$w(C_2) = \frac{M_1}{\sqrt{-1}} \int_{b2}^{c2} \frac{dt}{(1-t)^{\frac{1}{2}}(t-b_2)^{\frac{1}{2}}\, t^{\frac{1}{2}}} - \alpha kh$$

Performing the integration (Bryd and Fried man, 1971)

$$w(C_2) = \frac{M_1}{\sqrt{-1}} gF(\varphi_1, m_1) - \propto_1 kh$$

where $g = 2, \quad \varphi = \sin^{-1}\sqrt{\frac{c_2 - b_2}{1 - b_2}}, \quad m_1 = \sqrt{1 - b_2}$

hence,

$$w(C_2) = \frac{2M_1}{\sqrt{-1}} F\left(\sin^{-1}\sqrt{\frac{c_2 - b_2}{1 - b_2}}, \sqrt{1 - b_2}\right) - \alpha kh \tag{3.2.2.12}$$

Substituting value of M_1 from (3.2.2.9) to (3.2.2.12) we have

$$w(C_2) = \propto_1 kh \frac{F\left(\sin^{-1}\sqrt{\frac{c_2 - b_2}{1 - b_2}}, \sqrt{1 - b_2}\right)}{F\left(\frac{\pi}{2}, \sqrt{1 - b_2}\right)} - \alpha kh \qquad (3.2.2.13)$$

3.3.3. CALCULATING HEAD LOSS FACTOR THROUGH FRAGMENTS:

From equation 3.1.2.4 and 3.2.2.11 we have the value of q

$$q = \propto_1 \ kh\, A_2 \ where \ A_2 = \frac{F\left(\frac{\pi}{2}, \sqrt{b_2}\right)}{F\left(\frac{\pi}{2}, \sqrt{1 - b_2}\right)} \qquad (3.3.3.1)$$

$$q = (1 - \propto_1) kh A_1 \ where \ A_1 = \frac{F\left(\frac{\pi}{2}, \sqrt{\frac{1 - d_1}{1 + b_1}}\right)}{F\left(\frac{\pi}{2}, \sqrt{\frac{d_1 + b_1}{1 + b_1}}\right)} \qquad (3.3.3.2)$$

Dividing (3.3.3.1) by (3.3.3.2) we have

$$1 = \frac{\propto_1}{1 - \propto_1} \frac{A_2}{A_1}$$

$$or, \quad 1 - \alpha = \propto_1 \frac{A_2}{A_1}$$

$$\propto_1 = \frac{A_1}{A_1 + A_2} \qquad (3.3.3.3)$$

$$q = \frac{A_1 A_2}{A_1 + A_2} kh \qquad (3.3.3.4)$$

With given characteristics of weir foundation profile, characteristic of soil and impervious boundary, total head of water . We can easy find out head loss factors and discharge per unit width of weir by computer programming.

3.3.4. CALCULATING UPLIFT PRESSURE AT WEIR FLOOR:

For design purposes, we need to know the pressure distribution acting along the various sections of the structure and the magnitude of the exit gradient. Along the contour of the structure $\psi = 0$, and $w = \emptyset$, where the velocity potential function $\emptyset$ is given by

$$\phi = -k\left(\frac{p}{\gamma_w} - y\right) + C \qquad (3.4.1)$$

Let an origin be chosen at the step (point C_1)

let $C = k(h_2 - d_s)$, in that case Ø along downstream base $D_2E_2 = 0$ and Ø along the upstream base $A_1B_1 = -kh$, the uplift pressure acting on the impervious floor is given by

$$\frac{p}{\gamma_w} = -\frac{\phi}{k} + (h_2 - ds) + y \qquad (3.4.2)$$

We have the values of w at points C_1 (eq.3.2.1.7), D_1 (or B_2) and C_2 (eq. 3.2.2.13) therefore,

$$\phi(C_1) = (1 - \propto_1)kh\frac{F\left(\sin^{-1}\sqrt{\frac{b_1}{d_1 + b_1}}, \sqrt{\frac{d_1 + b_1}{1 + b_1}}\right)}{F\left(\frac{\pi}{2}, \sqrt{\frac{d_1 + b_1}{1 + b_1}}\right)} - kh \qquad (3.4.3)$$

$$\phi(D_1) = \phi(B_2) = -\alpha_1 kh \qquad (3.4.4)$$

$$\phi(C_2) = \propto_1 kh\frac{F\left(\sin^{-1}\sqrt{\frac{c_2 - b_2}{1 - b_2}}, \sqrt{1 - b_2}\right)}{F\left(\frac{\pi}{2}, \sqrt{1 - b_2}\right)} - \propto_1 kh \qquad (3.4.5)$$

Substituting (3.3.3), (3.3.4) and (3.3.5) to equation (3.4.2) we have value of uplift pressures at the particulars points

$$\frac{p_{C1}}{\gamma_w} = h - (1 - \propto_1)h\frac{F\left(\sin^{-1}\sqrt{\frac{b_1}{d_1 + b_1}}, \sqrt{\frac{d_1 + b_1}{1 + b_1}}\right)}{F\left(\frac{\pi}{2}, \sqrt{\frac{d_1 + b_1}{1 + b_1}}\right)} + (h_2 - d_s) \qquad (3.4.6)$$

$$y_{C1} = 0$$

$$\frac{p_{D1}}{\gamma_w} = \alpha_1 kh + (h_2 - d_s) + (s + d_s) = \alpha_1 kh + h_2 + S \qquad (3.4.7)$$

$$y_{D1} = S + d_s$$

$$\frac{p_{C2}}{\gamma_w} = \propto_1 h - \propto_1 h\left[1 - \frac{F\left(\sin^{-1}\sqrt{\frac{c_2 - b_2}{1 - b_2}}, \sqrt{1 - b_2}\right)}{F\left(\frac{\pi}{2}, \sqrt{1 - b_2}\right)}\right] + h_2 \qquad (3.4.8)$$

$$y_{C2} = d_s$$

3.3.5. CALCULATING EXIT GRADIENT:

Let the complex potential $w = \phi + i\psi$ be analytic function of the complex variable z

$$then, \frac{dw}{dz} = \frac{\partial \phi}{\partial x} + i\frac{\partial \psi}{\partial x} \tag{3.5.1}$$

Substituting the velocity components, yields the complex velocity

$$\frac{dw}{dz} = u - iv$$

Along the downstream horizontal boundary u = 0, hence

$$\frac{dw}{dz} = -iv \tag{3.5.2}$$

From Darcy's Law

$$v = -I_E k \tag{3.5.3}$$

Substituting (3.5.3) in (3.5.2)

$$\frac{dw}{dz} = iI_E k \quad therefore\ I_E = \frac{1}{ik}\frac{dw}{dz} = \frac{1}{ik}\frac{dw}{dt}\frac{dt}{dz} \tag{3.5.4}$$

The exit gradient is computed from Fragment 2

From analysis of fragment 2

$$\frac{dz}{dt} = \frac{T - ds}{\pi}\frac{1}{t^{\frac{1}{2}}(t-c)^{1/2}} \tag{2.5.5}$$

$$\frac{dw}{dt} = \frac{M_1}{(t-1)^{\frac{1}{2}}(t-b)^{\frac{1}{2}}\, t^{\frac{1}{2}}}$$

$$M_1 = \propto_1 kh\frac{i}{2F\left(\frac{\pi}{2}, \sqrt{1-b_2}\right)}\ hence$$

$$\frac{dw}{dt} = \propto_1 kh\frac{i}{2F\left(\frac{\pi}{2}, \sqrt{1-b_2}\right)}\frac{1}{(t-1)^{\frac{1}{2}}(t-b)^{\frac{1}{2}}\, t^{\frac{1}{2}}} \tag{2.5.6}$$

Substituting eq. (3.5.6)and eq. (3.5.5) in eq.(3.5.2) we have,

$$I_E = \frac{1}{ik}\propto_1 kh\frac{i}{2F\left(\frac{\pi}{2}, \sqrt{1-b_2}\right)}\frac{1}{(t-1)^{\frac{1}{2}}(t-b)^{\frac{1}{2}}\, t^{\frac{1}{2}}}\frac{\pi t^{\frac{1}{2}}(t-c)^{1/2}}{(T-ds)}$$

$$I_E = \frac{\pi \propto_1 h}{2(T-ds)F\left(\frac{\pi}{2}, \sqrt{1-b_2}\right)}\frac{(t-c)^{1/2}}{(t-1)^{\frac{1}{2}}(t-b)^{\frac{1}{2}}} \tag{2.5.7}$$

$$or, \frac{I_E}{h} = \frac{\pi \propto_1 h}{2(T-ds)F\left(\frac{\pi}{2}, \sqrt{1-b_2}\right)}\frac{(t-c)^{1/2}}{(t-1)^{\frac{1}{2}}(t-b)^{\frac{1}{2}}} \tag{2.5.8}$$

Substituting some value of $t > 1$, we can have values of exit gradient through foundation structure.

3.4 RESULT AND DISCUSSION:

Variation of q/kh with S/T is presented in the Figure 3.4. As a length of sheet pile increase, the seepage quantity decreases and at S/T=1, q/kh = 0. This result is in agreement with those given by Pavlopsky. The quantity of seepage decreases as either the depth of sheet pile or the weir increases; How ever the benefit to be gained by increasing the depth of sheet pile embedment is seen to decrease sharply as the ratio of width of the weir and thickness of the permeable foundation layer. Thus little or no material advantage is to be gained by increasing the piling depth for ratio $L/2T \geq 1$ unless the piling can be driven into the impervious base. This is particularly noteworthy, as with increasing depths of driving the risk of faulty connections between the individual piling sections is also likely to increase.

For different floor length and position of cut off, the seepage variation with L_1/L is presented in Figure 3.5. At the position of cut off has little influence on the magnitude at seepage loss. However, cut off position is important for controlling the exit gradient. If a cut off is not provided at the downstream end, the exit gradient becomes infinite which is not safe for the hydraulic structure.

Table 3.1 Discharge (q/kh) for symmetrically placed pilings as a function of S/T and L/2T

S/T	b/T=0	b/T=0.25	b/T=0.5	b/T=0.75	b/T=1	b/T=1.25	b/T=1.5
0.00		0.7428	0.5332	0.4200	0.3470	0.2956	0.2576
0.05	1.2509	0.7364	0.5314	0.4191	0.3464	0.2953	0.2573
0.10	1.0298	0.7185	0.5263	0.4166	0.3449	0.2942	0.2565
0.15	0.8999	0.6924	0.5181	0.4125	0.3423	0.2924	0.2552
0.20	0.8072	0.6615	0.5071	0.4069	0.3388	0.2900	0.2533
0.25	0.7346	0.6284	0.4939	0.3998	0.3343	0.2868	0.2509
0.30	0.6747	0.5949	0.4787	0.3914	0.3288	0.2829	0.2480
0.35	0.6233	0.5619	0.4621	0.3817	0.3225	0.2784	0.2446
0.40	0.5780	0.5298	0.4443	0.3710	0.3153	0.2731	0.2406
0.45	0.5373	0.4988	0.4257	0.3593	0.3072	0.2672	0.2361
0.50	0.5000	0.4689	0.4064	0.3466	0.2984	0.2607	0.2310
0.55	0.4653	0.4400	0.3865	0.3331	0.2887	0.2534	0.2253
0.60	0.4325	0.4117	0.3661	0.3187	0.2782	0.2454	0.2190
0.65	0.4011	0.3840	0.3452	0.3034	0.2667	0.2366	0.2120
0.70	0.3706	0.3565	0.3237	0.2872	0.2544	0.2269	0.2042
0.75	0.3403	0.3288	0.3013	0.2698	0.2409	0.2161	0.1955
0.80	0.3097	0.3004	0.2777	0.2510	0.2259	0.2041	0.1856
0.85	0.2778	0.2704	0.2522	0.2302	0.2089	0.1902	0.1740
0.90	0.2428	0.2372	0.2232	0.2059	0.1888	0.1733	0.1599
0.95	0.1999	0.1961	0.1865	0.1743	0.1619	0.1504	0.1402
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

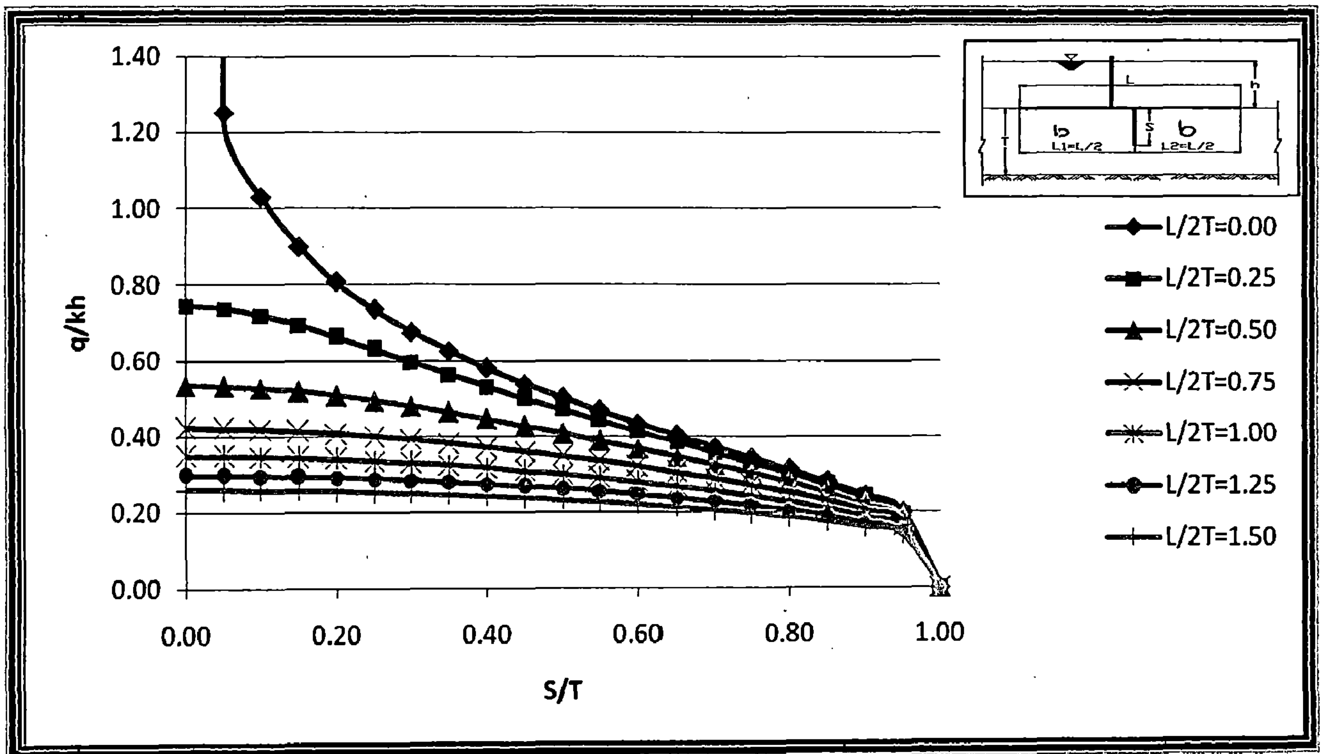


Figure 3.4: Discharge (q/kh) for symmetrically placed pilings as a function of S/T and L/2T

Table 3.2 Influence of the position of the piling on the discharge for various combinations of depth of embedment and size of structure

L1/L	L/2T=1/4			L/2T=1/2	
	S/T=1/4	S/T=1/3	S/T=1/2	S/T=1/4	S/T=1/2
0.00	0.59064	0.54035	0.44834	0.45948	0.37372
0.05	0.59830	0.54646	0.45210	0.46888	0.38009
0.10	0.60505	0.55197	0.45552	0.47623	0.38579
0.15	0.61088	0.55687	0.45859	0.48173	0.39078
0.20	0.61578	0.56111	0.46129	0.48576	0.39504
0.25	0.61980	0.56470	0.46360	0.48866	0.39859
0.30	0.62298	0.56762	0.46551	0.49074	0.40144
0.35	0.62539	0.56989	0.46700	0.49219	0.40362
0.40	0.62707	0.57150	0.46807	0.49314	0.40515
0.45	0.62806	0.57247	0.46872	0.49368	0.40606
0.50	0.62839	0.57279	0.46894	0.49385	0.40636
0.55	0.62806	0.57247	0.46872	0.49368	0.40606
0.60	0.62707	0.57150	0.46807	0.49314	0.40515
0.65	0.62539	0.56989	0.46700	0.49219	0.40362
0.70	0.62298	0.56762	0.46551	0.49074	0.40144
0.75	0.61980	0.56470	0.46360	0.48866	0.39859
0.80	0.61578	0.56111	0.46129	0.48576	0.39504
0.85	0.61088	0.55687	0.45859	0.48173	0.39078
0.90	0.60505	0.55197	0.45552	0.47623	0.38579
0.95	0.59830	0.54646	0.45210	0.46888	0.38009
1.00	0.59064	0.54035	0.44834	0.45948	0.37372

Table 3.3 Influence of saturation depth (T) on potential at point C_1 and C_2 evaluated for L_1,L_2

T	s=3	ds=0	L1=10	L2=10	T	s=3	ds=0	L1=7	L2=7
	α	q/kh	ϕ_{C1}	ϕ_{C2}		α	q/kh	ϕ_{C1}	ϕ_{C2}
8	0.5	0.27584	-0.54149	-0.47345	8	0.5	0.34563	-0.61071	-0.37767
9	0.5	0.30324	-0.51994	-0.44041	9	0.5	0.37739	-0.57386	-0.57540
10	0.5	0.32884	-0.50353	-0.50510	10	0.5	0.40654	-0.46188	-0.44449
11	0.5	0.35284	-0.49268	-0.46299	11	0.5	0.43346	-0.64733	-0.41938
12	0.5	0.37540	-0.47931	-0.45094	12	0.5	0.45845	-0.57464	-0.40582
13	0.5	0.39667	-0.46933	-0.44223	13	0.5	0.48175	-0.55610	-0.39591
14	0.5	0.41677	-0.45515	-0.43082	14	0.5	0.50356	-0.54687	-0.38760
15	0.5	0.43581	-0.43252	-0.41541	15	0.5	0.52406	-0.54113	-0.38012

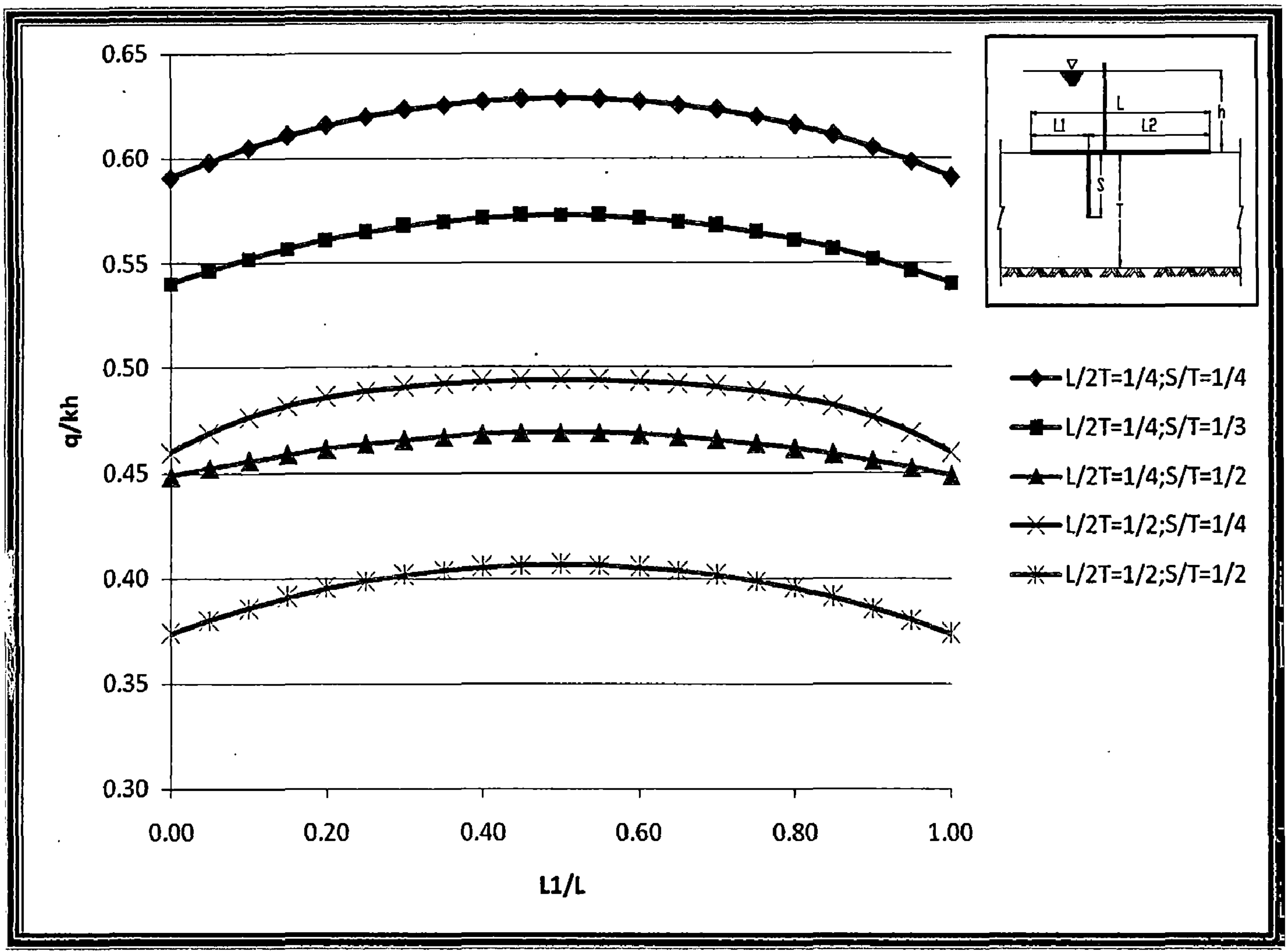


Figure 3.5 Influence of the position of the piling on the discharge for various combinations of depth of embedment and size of structure

CHAPTER 4

A WEIR WITH MULTI CUT - OFFS RESTING ON A POROUS MEDIUM OF FINITE DEPTH

4.1. INTRODUCTION:

In chapter 3 we analyzed flow under a depressed weir resting on a porous medium of finite depth. In practice, we have to provide some cut off walls or sheet piles for the safety of weir and economic considerations. The provision of sheet piles plays an important role in the distribution of the uplift pressure under the floor. The sheet piles at up stream and downstream ends of the impervious floor should be designed against scour due to surface flow condition. The maximum depth of scour at a particular location depends upon the type of structure, and the curvature of the river. For given surface flow criteria, the pressure distribution under the floor and exit gradient depend on the length and geometry of the floor, depth and number of sheet-piles, depth of porous medium. However, calculation uplift pressure and exit gradient become complicated when the weir has several cutoffs, as the number of parameters involved increases, since each cutoff add three new vertices to the flow region. In this chapter, we analysis a weir with multi cut offs, resting on permeable foundation of finite depth. The uplift pressure and exit gradient distribution have been analyzed using Schwartz and Christoffel, conformal mapping, and method of fragment.

4.2. STATEMENT OF THE PROBLEM:

A weir is embedded in a permeable foundation of finite depth. The underlying impervious boundary. The angle between impervious boundary and vertical axis is $\beta\pi$. We decompose the flow domain into 5 fragments (fragment I,II, III, IV and V) division. We can group the fragment into three types.

First type: Fragment I, Second type: Fragment V, and third type: Fragment II, III, and IV. Which differ only in depth of sheet piles, slope of impervious floor. The number of vertices for each fragment II, III, IV are identical.

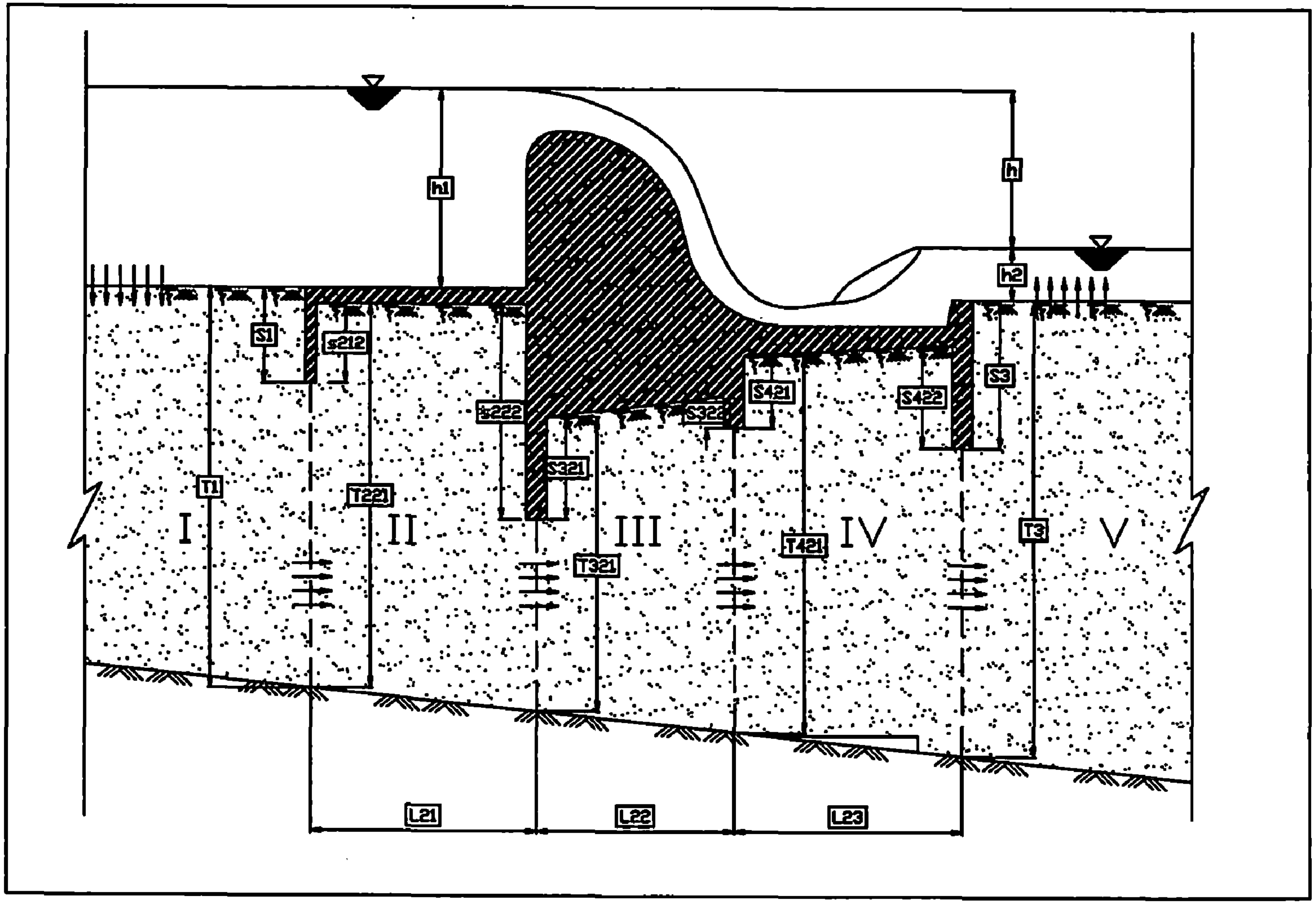


Fig 4.1 WEIR WITH MULTI CUT OFFS RESTING ON POROUS MEDIUM OF FINITE DEPTH

4.3. ANALYSIS

4.3.1 FRAGMENT TYPE 1:

4.3.1.1 Mapping of the flow domain (see Fig 4.2)in z plane to t plane $z = f_{11}(t)$

The Schwarz – Christoffel transformation that gives the afore mentioned mapping is

$$\frac{dz}{dt} = \frac{M}{t^{\frac{1}{2}}(1-t)^{(1-\beta)}} \tag{4.1.1}$$

$$or, \qquad z(t) = \int_0^t \frac{Mdt}{t^{\frac{1}{2}}(1-t)^{(1-\beta)}} + N$$

M, N are constants.

$$z(t) = M\int_0^t t^{-\frac{1}{2}}(1-t)^{\beta-1}dt + N$$

$$z(t) = MB_t\left(\frac{1}{2},\beta\right) + N$$

$$B_t\left(\frac{1}{2},\beta\right) \text{ is incomplete Beta function, its value depends on the value of } t$$

(i) For point B, $t = 0$, $z(t)=z_B$.

From Z plane we have $z = z_B$ therefore $N = z_B$

$$z(t) = MB_t\left(\frac{1}{2},\beta\right) + z_B$$

(ii) For point C, $t = c, Z(t) = z_C$

$$Hence, \qquad Z_C = MB_c\left(\frac{1}{2},\beta\right) + Z_B \tag{4.1.1.2}$$

$$where\ B_c\left(\frac{1}{2},\beta\right) \text{ is incomplete Beta function}$$

(iii) At $t = 1, Z(t) = z_D$

$$Hence, Z_D = MB\left(\frac{1}{2},\beta\right) + Z_B \tag{4.1.1.3}$$

Where $B\left(\frac{1}{2},\beta\right)$ *is complete Beta function*

From (4.1.1.2) and (4.1.1.3) we have

$$Z_C - Z_B = MB_c\left(\frac{1}{2},\beta\right), \quad and \quad Z_D - Z_B = MB\left(\frac{1}{2},\beta\right) \tag{4.1.1.4}$$

From geometry, we have

$$Z_C - Z_B = iS_1, \qquad Z_D - Z_B = i\,T_1 \tag{4.1.1.5}$$

Combination two equations (4.1.1.4) and (4.1.1.5) results in

$$\left.\begin{aligned} \frac{Z_C - Z_B}{Z_C - Z_B} &= \frac{B_c\left(\frac{1}{2},\beta\right)}{B\left(\frac{1}{2},\beta\right)} \\ \frac{Z_C - Z_B}{Z_C - Z_B} &= \frac{S_1}{T_1} \end{aligned}\right\} \rightarrow \frac{S_1}{T_1} = \frac{B_c\left(\frac{1}{2},\beta\right)}{B\left(\frac{1}{2},\beta\right)} \tag{4.1.1.6}$$

From equation (4.1.1.6) with one variable c. We can get the value of c by an interpolation.

4.3.1.2. Complex potential plane $w = f_{12}(t)$:

The transformation of the polygon in w – plane onto t- plane (Fig 4.2c)

$$\frac{dw}{dt} = \frac{M_1}{(-t)^{\frac{1}{2}}(c-t)^{\frac{1}{2}}(1-t)^{\frac{1}{2}}}$$

$$or, w(t) = \int_{-\infty}^{t} \frac{M_1 dt}{(-t)^{\frac{1}{2}}(c-t)^{\frac{1}{2}}(1-t)^{\frac{1}{2}}} + N_1 \tag{4.1.2.1}$$

(a) Integration along flow boundary A B $(-\infty \le t \le 0)$

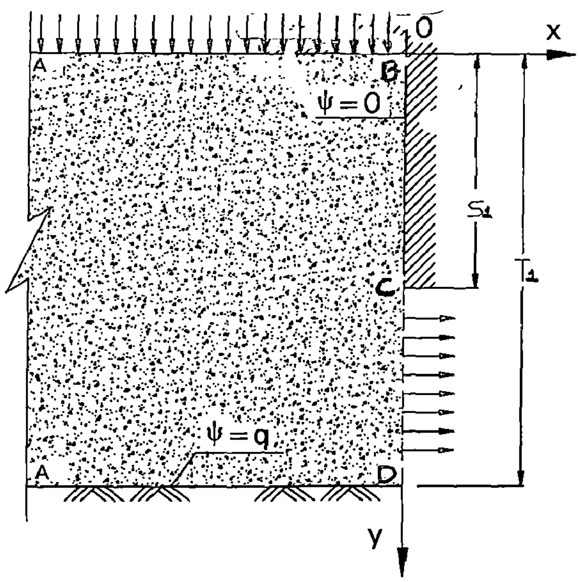
(i) At point A, $t = -\infty, w = +iq - kh\ hence, N1 = iq - kh$

(ii) At point B, $t = 0, w = -kh$

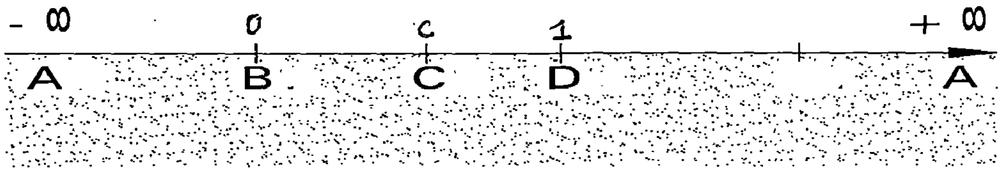
Therefore

$$-kh = \int_{-\infty}^{0} \frac{M_1 dt}{(-t)^{1/2}(c-t)^{1/2}(1-t)^{1/2}} + iq - kh$$

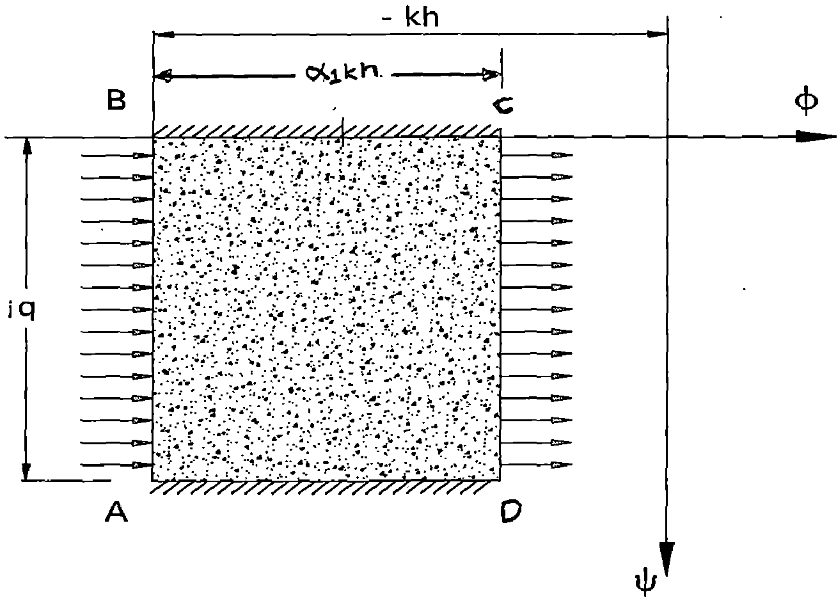
Performing the integration (Byrd & Friedman, 1971)



(a) Z-Plane (z= x + iy)



(b) t – Plane ($t = r + is$)



(c) w – Plane ($w = \phi + i\Psi$)

FIG 4.2 – TRANSFORMATION LAYOUT (FRAGMENT TYPE 1 – CASE 2)

$$-kh = M_1 g\, F(\varphi, m) + iq - kh \qquad \begin{cases} g = \dfrac{2}{\sqrt{1-0}} = 2 \\ \varphi = \sin^{-1}\sqrt{\dfrac{1-0}{1-0}} = \dfrac{\pi}{2} \\ m = \sqrt{\dfrac{1-c}{1-0}} = \sqrt{1-c} \end{cases}$$

$$-kh = 2M_1\, F\left(\frac{\pi}{2}, \sqrt{1-c}\right) + iq - kh$$

$$or, \qquad q = \frac{2M_1}{i} F\left(\frac{\pi}{2}, \sqrt{1-c}\right) \tag{4.1.2.3}$$

(b) Integration along floor boundary BC ($0 \le t \le c$)

(i) At point B, $t = 0, w(t) = -kh, hence\ N = -kh$

(ii) At point C, $t = 1, w(t) = -kh + \alpha_1 kh = -(1-\alpha_1)kh$

$$w(t) = \int_0^c \frac{M_1 dt}{(-t)^{1/2}(c-t)^{1/2}(1-t)^{1/2}} - kh$$

$$w(t) = \frac{M_1}{\sqrt{-1}}\int_0^c \frac{dt}{(t)^{\frac{1}{2}}(c-t)^{\frac{1}{2}}(1-t)^{\frac{1}{2}}} - kh = \frac{M_1}{\sqrt{-1}}\int_0^c \frac{dt}{\sqrt{(1-t)(c-t)t}} - kh$$

Performing the integration (Bryrd and Friedman, 1971)

$$w(t) = \frac{M_1}{\sqrt{-1}} g\, F(\varphi, m) - kh \qquad \begin{cases} g = \dfrac{2}{\sqrt{1-0}} = 2 \\ \varphi = \sin^{-1}\sqrt{\dfrac{c-0}{c_1-0}} = \dfrac{\pi}{2} \\ m = \sqrt{\dfrac{c-0}{1-0}} = \sqrt{c} \end{cases}$$

$$w(t) = \frac{2M_1}{\sqrt{-1}} F\left(\frac{\pi}{2}, \sqrt{c}\right) - kh$$

$$\text{or,} -(1 - \alpha_1)\text{kh} = \frac{2M_1}{\sqrt{-1}} F\left(\frac{\pi}{2}, \sqrt{c}\right) - kh$$

or, $$M_1 = \frac{\sqrt{-1}\alpha_1 kh}{2F\left(\frac{\pi}{2},\sqrt{c}\right)} \quad (4.1.2.5)$$

Substituting the value of M_1 from the equation (4.1.2.5) to the equation (4.1.2.3) we get the value of q

$$q = \alpha_1 kh \frac{F\left(\frac{\pi}{2},\sqrt{1-c}\right)}{F\left(\frac{\pi}{2},\sqrt{c}\right)} \quad (4.1.2.6)$$

$$Letting\ A_1 = \frac{F\left(\frac{\pi}{2},\sqrt{1-c}\right)}{F\left(\frac{\pi}{2},\sqrt{c}\right)}\ therefore, \quad q = \alpha_1 kh A_1 \quad (4.1.2.7)$$

4.3.2.FRAGMENT TYPE 2:

4.3.2.1. Mapping of the flow domain into z - plane onto t -plane $z = f_2(t)$ see Fig 4.3

The Schwarz – Christoffel transformation that gives the afore mentioned mapping is

$$\frac{dz}{dt} = \frac{M}{t^\beta\,(c-t)^\alpha (1-t)^{1-\alpha}} \quad (4.2.1.1a)$$

$$z = \int_0^t \frac{Mdt}{t^\beta\,(c-t)^\alpha (1-t)^{1-\alpha}} + N \quad (4.2.1.b)$$

(a) For point A, t = 0, z = z_A, hence N = z_A

Integration along the boundary AB ($0 \le t \le b$) and AC ($0 \le t \le c$)

$$z(t) = M\int_0^t \frac{Mdt}{t^\beta\,(c-t)^\alpha (1-t)^{1-\alpha}} + z_A = M\int_0^t t^{-\beta}(c-t)^{-\alpha}(1-t)^{\alpha-1}\,dt + z_A$$

$$z(t) = Mc^{-\alpha}\int_0^t t^{-\beta}(1-t)^{\alpha-1}(1-\tfrac{t}{c})^{-\alpha}\,dt + z_A$$

Expanding the term $\left(1-\frac{t}{c}\right)^{-\alpha}$ according to Binomial theorem

$$(1-\frac{t}{c})^{-\alpha} = 1 + \frac{\alpha\, t}{1!\,c} + \frac{\alpha\,(\alpha+1)t^2}{2!\,c^2} + \frac{\alpha\,(\alpha+1)(\alpha+2)t^3}{3!\,c^3} + \cdots$$

$$z(t) = Mc^{-\alpha}\int_0^t \left\{ t^{-\beta}(1-t)^{\alpha-1}\left[1 + \frac{\propto t}{1!\,c} + \frac{\propto(\propto+1)t^2}{2!\,c^2} + \frac{\propto(\propto+1)(\propto+2)t^3}{3!\,c^3} + \frac{\propto(\propto+1)(\propto+2)(\propto+3)t^4}{3!\,c^4}\ldots\right]\right\} dt + z_A \quad (4.2.1.1c)$$

integrating term by term the following relationship between z and t is obtained

$$z(t) = Mc^{-\alpha}\left\{B_t(1-\beta,\alpha) + \frac{\propto}{1!\,c}B_t(2-\beta,\alpha) + \frac{\propto(\propto+1)}{2!\,c^2}B_t(3-\beta,\alpha) + \frac{\propto(\propto+1)(\propto+2)}{3!\,c^3}B_t(4-\beta,\alpha) + \cdots\right\} + z_A$$

$$z = Mc^{-\propto}\sum_{r=0}^{+\infty}\frac{(\propto)_r}{r!\,c^r}B_t(r+1-\beta,\alpha) + z_A$$

where $(\propto)_r = \propto(\propto+1)(\propto+2)\ldots(\propto+r-1) = \frac{\Gamma(\propto+r)}{\Gamma(\propto)}$

$(\propto)_r$ Poch hammer symbol

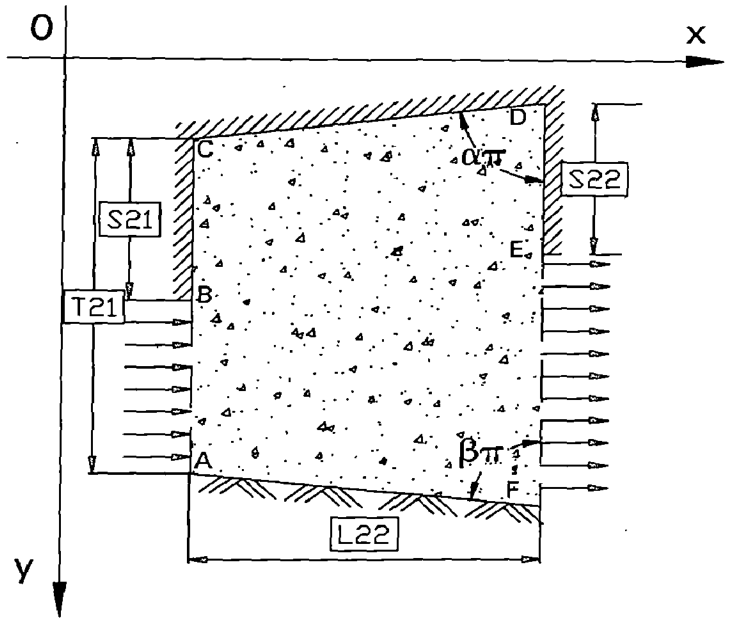
$\Gamma(\propto+r)$ = Gamma function

where $B_t(r+1-\beta,\alpha)$ is an incomplete Beta function

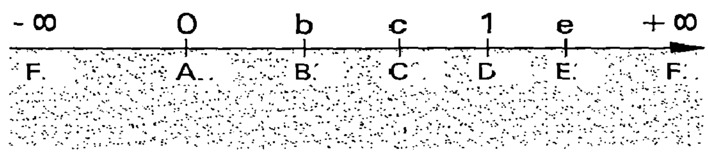
(i) For point B, t = b, $z(t) = z_B$

$$z_B = Mc^{-\alpha}\{B_b(1-\beta,\alpha) + \frac{\propto}{1!\,c}B_b(2-\beta,\alpha) + \frac{\propto(\propto+1)}{2!\,c^2}B_b(3-\beta,\alpha) + \frac{\propto(\propto+1)(\propto+2)}{3!\,c^3}B_b(4-\beta,\alpha) + \cdots\} + z_A$$

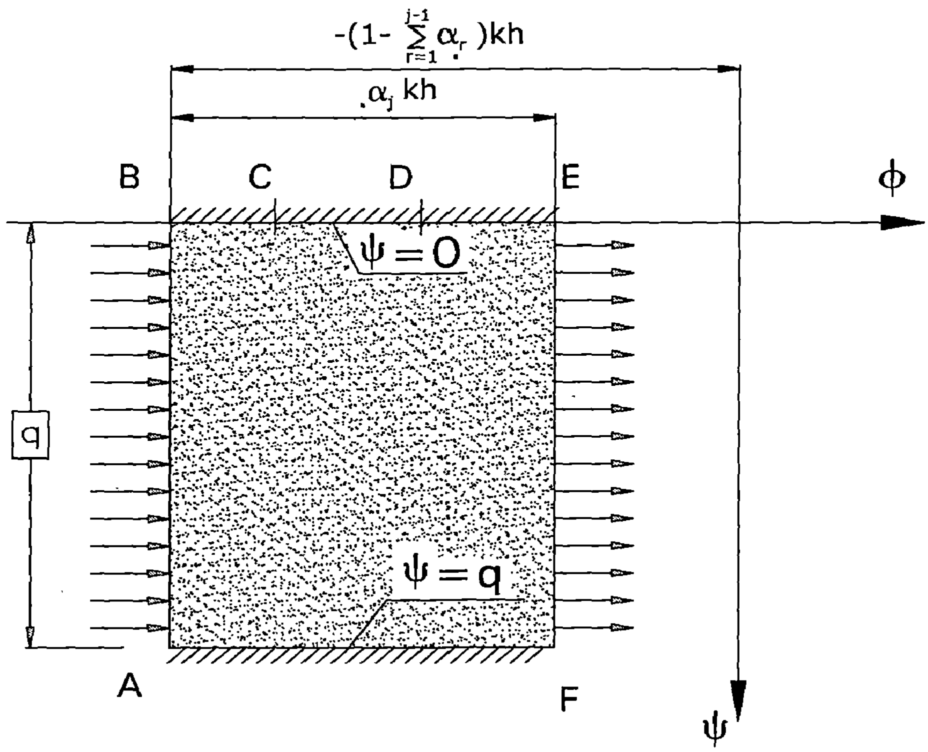
$$or, z_B = Mc^{-\propto}\sum_{r=0}^{+\infty}\frac{(\propto)_r}{r!\,c^r}B_b(r+1-\beta,\alpha) + z_A$$



(a) Z-Plane (z= x + iy)



(b) t – Plane (t = r + is)



(c) w – Plane (w = $\phi + i\Psi$)

Fig 4.3 – Transformation layout (Fragment type 2) – Case 2

$$letting, \qquad I_{21} = \sum_{r=0}^{+\infty} \frac{(\propto)_r}{r!\, c^r} B_b(r+1-\beta, \alpha)$$

For the parameter c is less than 1and parameter **b** is less than c. The term reduces to zero and the series disconverged. The series I_{21}, is evaluated as follows:

(note we can express in hypergeometric series as follow

$$B_x(p,q) = \frac{x^p}{p} \quad {}_2F_1(p, 1-q, 1+p, x) \quad (Bateman, 1953)$$

In which ${}_2F_1(p, 1-q, 1+p, x) = Gauss\ hypergeometric\ series$

$$\begin{aligned} {}_2F_1(p, 1-q, 1+p, x) \\ &= 1 + \frac{p(1-q)}{(1+p)1!}x + \frac{p(p+1)(1-q)(1-q+1)}{(1+p)(1+p+1)2!}x^2 \\ &+ \frac{p(p+1)(p+2)(1-q)(1-q+1)(1-q+2)}{(1+p)(1+p+1)(1+p+2)3!}x^3 + \cdots \end{aligned}$$

Hence,

$$B_b(1-\beta, \alpha) = \frac{b^{1-\beta}}{1-\beta} \quad {}_2F_1(1-\beta, 1-\alpha, 2-\beta, b)$$

$$\begin{aligned} \frac{1}{c} B_b(2-\beta, \alpha) &= \frac{1}{c}\frac{b^{2-\beta}}{2-\beta} \quad {}_2F_1(2-\beta, 1-\alpha, 3-\beta, b) \\ &= \frac{b}{c}\frac{b^{1-\beta}}{2-\beta} \quad {}_2F_1(2-\beta, 1-\alpha, 3-\beta, b) \end{aligned}$$

$$\begin{aligned} \frac{1}{c^2} B_b(3-\beta, \alpha) &= \frac{1}{c^2}\frac{b^{3-\beta}}{3-\beta} \quad {}_2F_1(3-\beta, 1-\alpha, 4-\beta, b) \\ &= \left(\frac{b}{c}\right)^2 \frac{b^{1-\beta}}{3-\beta} \quad {}_2F_1(2-\beta, 1-\alpha, 3-\beta, b) \end{aligned}$$

$$\begin{aligned} \frac{1}{c^3} B_b(4-\beta, \alpha) &= \frac{1}{c^3}\frac{b^{4-\beta}}{4-\beta} \quad {}_2F_1(4-\beta, 1-\alpha, 5-\beta, b) \\ &= \left(\frac{b}{c}\right)^3 \frac{b^{1-\beta}}{4-\beta} \quad {}_2F_1(4-\beta, 1-\alpha, 5-\beta, b) \end{aligned}$$

$$I_{21} = b^{1-\beta}\left\{\frac{1}{1-\beta} \quad {}_2F_1(1-\beta, 1-\alpha, 2-\beta, b)\right.$$

$$+\frac{\propto}{1!\,(2-\beta)}\left(\frac{b}{c}\right) \quad {}_2F_1(2-\beta, 1-\alpha, 3-\beta, b)$$

$$+\frac{\propto(\propto+1)}{2!\,(3-\beta)}\left(\frac{b}{c}\right)^2 \quad {}_2F_1(2-\beta, 1-\alpha, 3-\beta, b)$$

$$\left.+\frac{\propto(\propto+1)(\propto+2)}{3!\,(4-\beta)}\left(\frac{b}{c}\right)^3 \quad {}_2F_1(4-\beta, 1-\alpha, 5-\beta, b)+\cdots\right\} \qquad (4.2.1.2a)$$

$$I_{21} = b^{1-\beta}\sum_{r=0}^{+\infty}\frac{(\propto)_r}{r!\,(r+1-\beta)}\left(\frac{b}{c}\right)^r \quad {}_2F_1(r+1-\beta, 1-\alpha, r+2-\beta, b) \qquad (4.2.1.2b)$$

Thus, I_{21} has a finite value as the term b/c<1 series converges.

$$Z_B = Mc^{-\alpha}I_{21} + Z_A \qquad or \qquad Z_B - Z_A = Mc^{-\alpha}I_{21} \qquad (4.2.1.2c)$$

(ii) For point C, At t = c we have Z(t) = Z_C

$$z_C = Mc^{-\alpha}\left\{B_c(1-\beta,\alpha)+\frac{\propto}{1!\,c}B_c(2-\beta,\alpha)+\frac{\propto(\propto+1)}{2!\,c^2}B_c(3-\beta,\alpha)\right.$$

$$\left.+\frac{\propto(\propto+1)(\propto+2)}{3!\,c^3}B_c(4-\beta,\alpha)+\cdots\right\}+z_A$$

$$Letting, I_{22} = \left\{B_c(1-\beta,\alpha)+\frac{\propto}{1!\,c}B_c(2-\beta,\alpha)+\frac{\propto(\propto+1)}{2!\,c^2}B_c(3-\beta,\alpha)\right.$$

$$\left.+\frac{\propto(\propto+1)(\propto+2)}{3!\,c^3}B_c(4-\beta,\alpha)+\cdots\right\}$$

I_{22} is evaluated as follow :

$$B_c(1-\beta,\alpha) = \frac{c^{1-\beta}}{1-\beta} \quad {}_2F_1(1-\beta, 1-\alpha, 2-\beta, c)$$

$$\frac{1}{c}B_c(2-\beta,\alpha) = \frac{1}{c}\frac{c^{2-\beta}}{2-\beta} \quad {}_2F_1(2-\beta, 1-\alpha, 3-\beta, c)$$

$$= \frac{c^{1-\beta}}{2-\beta} \quad {}_2F_1(2-\beta, 1-\alpha, 3-\beta, c)$$

$$\frac{1}{c^2}B_c(3-\beta,\alpha) = \frac{1}{c^2}\frac{c^{3-\beta}}{3-\beta} \quad {}_2F_1(3-\beta, 1-\alpha, 4-\beta, c)$$

$$= \frac{c^{1-\beta}}{3-\beta} \quad {}_2F_1(2-\beta, 1-\alpha, 3-\beta, c)$$

$$\frac{1}{c^3}B_c(4-\beta,\alpha) = \frac{1}{c^3}\frac{c^{4-\beta}}{4-\beta} \quad {}_2F_1(4-\beta,1-\alpha,5-\beta,c)$$

$$= \frac{c^{1-\beta}}{3-\beta} \quad {}_2F_1(4-\beta,1-\alpha,5-\beta,c)$$

Therefore,

$$I_{22} = c^{1-\beta}\left\{\frac{1}{1-\beta} \quad {}_2F_1(1-\beta,1-\alpha,2-\beta,c) + \frac{\propto}{1!\,(2-\beta)} \quad {}_2F_1(2-\beta,1-\alpha,3-\beta,c) + \frac{\propto(\propto+1)}{2!\,(3-\beta)} \quad {}_2F_1(2-\beta,1-\alpha,3-\beta,c) + \frac{\propto(\propto+1)(\propto+2)}{3!\,(4-\beta)} \quad {}_2F_1(4-\beta,1-\alpha,5-\beta,c) + \cdots\right\}$$

$$or, I_{22} = c^{1-\beta}\sum_{r=0}^{+\infty}\frac{(\propto)_r}{r!\,(r+1-\beta)} \quad {}_2F_1(r+1-\beta,1-\alpha,r+2-\beta,c) \qquad (4.2.1.3a)$$

$where\ (\propto)_r$ = Poch hammer symbol

${}_2F_1(r+1-\beta,1-\alpha,r+2-\beta,c) = hypergeometric\ function$

I_{22} is a converging series as r →∞, ${}_2F_1(r+1-\beta,1-\alpha,r+2-\beta,c)$ converges to a finite value and $\frac{(\propto)_r}{r!(r+1-\beta)}$ also converges to a finite value and $\sum_{r=0}^{+\infty}\frac{(\propto)_r}{r!(r+1-\beta)} \quad {}_2F_1(r+1-\beta,1-\alpha,r+2-\beta,c)$→0.

$$z_C = Mc^{-\alpha}I_{22} + z_A \quad \text{or,} \quad z_C - z_A = Mc^{-\alpha}I_{22} \qquad (4.2.1.3b)$$

(b) Integrating along weir floor CD $(c \le t \le 1)$

(i) At $t = c, z = z_C$

(ii) At $t = 1, z = z_D$

$$z_D = M\int_c^1 t^{-\beta}(c-t)^{-\alpha}(1-t)^{\alpha-1}\,dt + z_C$$

$$z_D = M(-1)^{-\alpha}\int_c^1 \{[1-(1-t)]^{-\beta}(t-c)^{-\alpha}(1-t)^{\alpha-1}\}\,dt + z_C \qquad (4.2.1.4)$$

To evaluate the integral, we expand $[1-(1-t)]^{-\beta}$ according to Binomial Theorem and integrating term by term

$$[1-(1-t)]^{-\beta} = 1 + \frac{\beta}{1!}(1-t) + \frac{\beta(\beta+1)}{2!}(1-t)^2 + \frac{\beta(\beta+1)(\beta+3)}{3!}(1-t)^3 + \cdots$$

Substituting the expansion the equation (4.2.1.4) we get

$$z_D = M(-1)^{-\alpha}\int_c^1 \left\{(t-c)^{-\alpha}(1-t)^{\alpha-1}\left[1 + \frac{\beta}{1!}(1-t) + \frac{\beta(\beta+1)}{2!}(1-t)^2 + \frac{\beta(\beta+1)(\beta+3)}{3!}(1-t)^3 + \cdots\right]\right\} dt + z_C$$

setting, $r = \frac{1-t}{1-c}$ (Grobner and Hofreiter, 1961)

$$or, z_D = M(-1)^{-\alpha}\int_0^1 \left\{r^{\alpha-1}(1-r)^{-\alpha}\left[1 + \frac{\beta}{1!}(1-c)r + \frac{\beta(\beta+1)}{2!}(1-c)^2r^2 + \frac{\beta(\beta+1)(\beta+3)}{3!}(1-c)^3r^3 + \cdots\right]\right\} dr + z_C$$

$$or, z_D = M(-1)^{-\alpha}\left\{B(\alpha,1-\alpha) + \frac{\propto}{1!}(1-c)B(\alpha+1,1-\alpha) + \frac{\propto(\propto+1)}{2!}(1-c)^2B(\alpha+2,1-\alpha) + \frac{\propto(\propto+1)(\propto+2)}{3!}(1-c)^3B(\alpha+3,1-\alpha) + \cdots\right\} + z_C$$

$$Letting\ I_{23} = B(\alpha,1-\alpha) + \frac{\propto}{1!}(1-c)B(\alpha+1,1-\alpha) + \frac{\propto(\propto+1)}{2!}(1-c)^2B(\alpha+2,1\alpha) + \frac{\propto(\propto+1)(\propto+2)}{3!}(1-c)^3B(\alpha+3,1-\alpha) + \cdots$$

$$or, I_{23} = \sum_{r=0}^{+\infty}\frac{(\propto)_r}{r!}(1-c)^r\, B(\alpha+r,1-\alpha)$$

$z_D = M(-1)^{-\alpha}I_{23} + z_C$ or $z_D - z_C = M(-1)^{-\alpha}I_{23}$ (4.2.1.5)

(c) Integrating along DE $(1 \le t \le e)$

$$z_E = M\int_1^e t^{-\beta}(c-t)^{-\alpha}(1-t)^{\alpha-1}\,dt + Z_D$$

$$= M\int_1^e t^{-\beta}(-1)^{-\alpha}(t-c)^{-\alpha}(-1)^{\alpha-1}(t-1)^{\alpha-1}\,dt + Z_D$$

$$= -M\int_1^e t^{-\beta}(t-c)^{-\alpha}(t-1)^{\alpha-1}\,dt + z_D$$

Setting $t = \frac{1}{r}$ *we have* $dt = \frac{-dr}{r^2}$

$$z_E = -M\int_{1/e}^1 r^{\beta-1}(1-r)^{\alpha-1}(1-cr)^{-\alpha}\,dr + z_D$$

$$z_E = M\int_0^{\frac{1}{e}} r^{\beta-1}(1-r)^{\alpha-1}(1-cr)^{-\alpha}\,dr$$

$$- M\int_0^1 r^{\beta-1}(1-r)^{\alpha-1}(1-cr)^{-\alpha}\,dr + z_D \tag{4.2.1.6}$$

$$z_E = MI_{24} - MI_{25} + z_D \tag{4.2.1.7}$$

$$letting, I_{24} = M\int_0^{\frac{1}{e}} r^{\beta-1}(1-r)^{\alpha-1}(1-cr)^{-\alpha}\,dr \tag{4.2.1.8}$$

$$letting, I_{25} = M\int_0^1 r^{\beta-1}(1-r)^{\alpha-1}(1-cr)^{-\alpha}\,dr \tag{4.2.1.9}$$

We expand the term $(1-cr)^{-\alpha}$according to Binomial theorem and integrating term by term Eq (4.2.1.8) and (4.2.1.9) reduces

$$(1-cr)^{-\alpha} = 1 + c\frac{\propto r}{1!} + c^2\frac{\propto(\propto+1)r^2}{2!} + c^3\frac{\propto(\propto+1)(\propto+2)r^3}{3!}\ldots$$

$$I_{24} = \int_0^{\frac{1}{e}} r^{\beta-1}(1-r)^{\alpha-1}\,\{1 + c\frac{\propto r}{1!} + c^2\frac{\propto(\propto+1)r^2}{2!} + c^3\frac{\propto(\propto+1)(\propto+2)r^3}{3!}\ldots\}dr$$

$$I_{25} = \int_0^1 r^{\beta-1}(1-r)^{\alpha-1}\,\{1 + c\frac{\propto r}{1!} + c^2\frac{\propto(\propto+1)r^2}{2!} + c^3\frac{\propto(\propto+1)(\propto+2)r^3}{3!}\ldots\}dr$$

Performing the integration

$$I_{24} = B_{\frac{1}{e}}(\beta,\alpha) + c\frac{\propto}{1!}B_{\frac{1}{e}}(\beta+1,\alpha) + c^2\frac{\propto(\propto+1)}{2!}B_{\frac{1}{e}}(\beta+2,\alpha)$$
$$+ c^3\frac{\propto(\propto+1)(\propto+2)r^3}{3!}B_{\frac{1}{e}}(\beta+3,\alpha) + \cdots \tag{4.2.1.10}$$

$$I_{25} = B(\beta\,,\alpha) + c\frac{\propto}{1!}B(\beta+1\,,\alpha) + c^2\frac{\propto(\propto+1)}{2!}B(\beta+2\,,\alpha) + c^3\frac{\propto(\propto+1)(\propto+2)r^3}{3!}B(\beta+3\,,\alpha) + \cdots \tag{4.2.1.11}$$

From z plane and t plane we have

$Z_B - Z_A = -i(T_{21} - S_{21})$ therefore, $-i(T_{21} - S_{21}) = Mc^{-\alpha}I_{21}$ (4.2.1.12)

$Z_C - Z_A = -iT_{21}$ therefore, $-iT_{21} = Mc^{-\alpha}I_{22}$ (4.2.1.13)

$Z_D - Zc = L_2 - iL_2Cotan\,(\alpha\pi)$

Therefore, $L_2 - iL_2Cotan\,(\alpha\pi) = M(-1)^{-\alpha}I_{23}$ (4.2.1.14)

$Z_E - Z_D = iS_{22}$ *or* $iS_{22} = MI_{24} - MI_{25}$ (4.2.1.15)

$I_{21} = F_1(b,c)\,,\ I_{22} = F_2(c),\ I_{23} = F_3(c), I_{24} = F(c), I_{25} = F_4(c,e)$

$$\frac{Z_D - Z_C}{Z_C - Z_A} = \frac{L_2 - iL_2Cotan\,(\alpha\pi)}{-iT_{21}} = \frac{L_2Cotan\,(\alpha\pi)}{T_{21}} + \frac{L_2}{T_{21}}i$$

The right part is a complex number and we can express the complex number in the form $Re^{i\theta}$

The modulus is given by

$$R = \sqrt{(\frac{L_2Cotan\,(\alpha\pi)}{T_{21}})^2 + (\frac{L_2}{T_{21}})^2} = \left(\frac{L_2}{T_{21}}\right)\sqrt{[Cotan\,(\alpha\pi)]^2 + 1}$$

$$R = \frac{L_2}{T_{21}sin(\alpha\pi)},$$

The argument is given by

$$\theta = tan^{-1}\left(\frac{\frac{L_2}{T_{21}}}{\frac{L_2Cotan\,(\alpha\pi)}{T_{21}}}\right) = tan^{-1}\,(\frac{1}{Cotan\,(\alpha\pi)}) = \propto\pi$$

Therefore,

$$\frac{Z_D - Z_C}{Z_C - Z_A} = \frac{L_2}{sin(\alpha\pi)T_{21}}e^{i\propto\pi} \tag{4.2.1.16}$$

$$\frac{Z_D - Z_C}{Z_C - Z_A} = \frac{M(-1)^{-\alpha} I_{23}}{Mc^{-\alpha} I_{22}} = \frac{(-1)^{-\alpha} I_{23}}{c^{-\alpha} I_{22}} \tag{4.2.1.17}$$

We can express $(-1) = [\cos(\pm\pi) + i sin(\pm\pi)]$ and applying De Moivre's Theorem (the law of multiplication of complex numbers in preceding section).

$$(-1)^{-\alpha} = [\cos(\pm\pi) + i sin(\pm\pi)]^{-\alpha} = e^{\pm i\alpha\pi}$$

Substituting above in equation (4.2.1.17) we get

$$\frac{Z_D - Z_C}{Z_C - Z_A} = \frac{e^{\pm i\alpha\pi} I_{23}}{c^{-\alpha} I_{22}} \tag{4.2.1.18}$$

From (4.2.1.16) and (4.2.1.18) we have

$$\frac{L_2}{sin(\alpha\pi) T_{21}} e^{i\alpha\pi} = \frac{I_{23}}{c^{-\alpha} I_{22}} e^{\pm i\alpha\pi}$$

$$or, \left[\frac{L_2}{sin(\alpha\pi) T_{21}} - \frac{I_{23}}{I_{22}} c^{\alpha}\right] e^{i\alpha\pi} = 0 \quad because\ of\ e^{i\alpha\pi} \neq 0 \quad then,$$

$$\left[\frac{L_2}{sin(\alpha\pi) T_{21}} - \frac{I_{23}}{I_{22}} c^{\alpha}\right] = 0 \tag{4.2.1.19}$$

From equation (4.2.1.19) the only unknown c can be found using an iteration for given L_2, T_{21}, and α.

From equations (4.2.1.12) and (4.2.1.13) we have

$$\frac{Z_B - Z_A}{Z_C - Z_A} = \frac{-i(T_{21} - S_{21})}{-iT_{21}} = \frac{(T_{21} - S_{21})}{T_{21}} \tag{4.2.1.20}$$

$$\frac{Z_B - Z_A}{Z_C - Z_A} = \frac{Mc^{-\alpha} I_{21}}{Mc^{-\alpha} I_{22}} = \frac{I_{21}}{I_{22}} \tag{4.2.1.21}$$

From (4.2.1.20) and (4.2.1.21) we have

$$\frac{(T_{21} - S_{21})}{T_{21}} = \frac{I_{21}}{I_{22}} \tag{4.2.1.22}$$

From the equation (4.2.1.22) with two unknown variable b and c but we have the value of c from the equation (4.2.1.19) so that only unknown variable b. To find out the value of b, by interpolation we can get it.

From the equation (4.2.1.13) and (4.2.1.15) we have

$$\frac{Z_E - Z_D}{Z_C - Z_A} = \frac{iS_{22}}{-iT_{21}} = -\frac{S_{22}}{T_{21}} \tag{4.2.1.23}$$

$$\frac{Z_E - Z_D}{Z_C - Z_A} = \frac{MI_{24} - MI_{25}}{Mc^{-\alpha}I_{22}} = -c^{\alpha}\frac{I_{25} - I_{24}}{I_{22}} \tag{4.2.1.24}$$

From the equation (4.2.1.23) and (4.2.1.24) we have

$$\frac{S_{22}}{T_{21}} = c^{\alpha}\left(\frac{I_{25} - I_{24}}{I_{22}}\right) \tag{4.2.1.25}$$

With two unknown variables in the equation (4.2.1.25) are c and e. But variable c we have got from (4.2.1.19) so that only unknown variable e we need to find out. By interpolation we can get the value of e.

4.3.2.2. Mapping of complex potential on w plane w=f$_2$(t) see (Fig 3.2c)

The transformation of the polygon in w-plane onto the t plane is given by

$$\frac{dw}{dt} = \frac{M_1}{t^{\frac{1}{2}}(t-b)^{\frac{1}{2}}(t-e)^{\frac{1}{2}}}$$

$$or, w = \int_0^t \frac{M_1}{t^{\frac{1}{2}}(t-b)^{\frac{1}{2}}(t-e)^{\frac{1}{2}}} + N_1 \tag{4.2.2.1}$$

Where M_1, N_1 are constants

(a) Integration along boundary AB ($0 \leq t \leq b$)

(i) At point A, t = 0, $w = -\left(1 - \sum_{r=1}^{j-1}\alpha_r\right)kh + iq$

where α_r=head loss coefficient through fragment r, j is the fragment number.

(ii) At point B, t = b, $w = -(1 - \sum_{r=1}^{j-1}\alpha_r)kh$

Therefore,

$$-\left(1 - \sum_{r=1}^{j-1}\alpha_r\right)kh = \int_0^b \frac{M1dt}{t^{\frac{1}{2}}(t-b)^{\frac{1}{2}}(t-e)^{\frac{1}{2}}} - \left(1 - \sum_{r=1}^{j-1}\alpha_r\right)kh + iq$$

$$or, \quad q = iM_1\int_0^b \frac{dt}{t^{1/2}(b-t)^{1/2}(e-t)^{1/2}}$$

Performing the integration (Bryd and Fried man, 1971)

$q = iM_1 gF(\varphi, m)$

In this case $g = \frac{2}{\sqrt{e}}, \vartheta_1 = sin^{-1}(1) = \frac{\pi}{2}, m = \sqrt{\frac{b}{e}},$

$$therefore, \quad q = iM_1 \frac{2}{\sqrt{e}} F\left(\frac{\pi}{2}, \sqrt{\frac{b}{e}}\right) \qquad (4.2.2.2)$$

(b) Integration along boundary BCDE ($b \leq t \leq e$)

(i) At point B, t = b, $w = -(1 - \sum_{r=1}^{j-1} \alpha_r)kh$

Therefore

$$w(t) = \int_b^t \frac{M_1 dt}{t^{1/2} (t-b)^{1/2} (t-e)^{1/2}} - (1 - \sum_{r=1}^{j-1} \alpha_r)kh$$

$$w(t) = \frac{M_1}{\sqrt{-1}} \int_b^t \frac{dt}{t^{1/2} (t-b)^{1/2} (e-t)^{1/2}} - (1 - \sum_{r=1}^{j-1} \alpha_r)kh$$

Performing the integration (Bryd and Fried man, 1971)

$$w(t) = \frac{M_1}{\sqrt{-1}} gF(\varphi, m) - \left(1 - \sum_{r=1}^{j-1} \alpha_r\right) kh \qquad (4.2.2.3)$$

In this case

$$g = \frac{2}{\sqrt{e}}, \varphi = sin^{-1}\left(\sqrt{\frac{e(t-b)}{(e-b) * t}}\right), m = \sqrt{\frac{e-b}{e}}$$

(ii) At point C, t = c, $w = w_C$

$$w(C) = \frac{M_1}{\sqrt{-1}} \frac{2}{\sqrt{e}} F\left(sin^{-1} \sqrt{\frac{e(c-b)}{c(e-b)}}, \sqrt{\frac{e-b}{e}}\right) - \left(1 - \sum_{r=1}^{j-1} \alpha_r\right) kh \qquad (4.2.2.4)$$

(iii) At point D, t = 1, $w = w_D$

$$w(D) = \frac{M_1}{\sqrt{-1}}\frac{2}{\sqrt{e}}F\left(sin^{-1}\sqrt{\frac{e(1-b)}{(e-b)}}, \sqrt{\frac{e-b}{e}}\right) - \left(1 - \sum_{r=1}^{j-1}\alpha_r\right)kh \qquad (4.2.2.5)$$

(iv) At point E, t = e, $w = -\left(1 - \sum_{r=1}^{j-1}\alpha_r\right)kh$

$$w(E) = \frac{M_1}{\sqrt{-1}}\frac{2}{\sqrt{e}}F\left(\frac{\pi}{2}, \sqrt{\frac{e-b}{e}}\right) - \left(1 - \sum_{r=1}^{j-1}\alpha_r\right)kh \qquad (4.2.2.6)$$

Therefore

$$-\left(1 - \sum_{r=1}^{j}\alpha_r\right)kh = \frac{M_1}{\sqrt{-1}}\frac{2}{\sqrt{e}}F\left(\frac{\pi}{2}, \sqrt{\frac{e-b}{e}}\right) - \left(1 - \sum_{r=1}^{j-1}\alpha_r\right)kh$$

$$or, \qquad \propto_j kh = \frac{M_1}{\sqrt{-1}}\frac{2}{\sqrt{e}}F\left(\frac{\pi}{2}, \sqrt{\frac{e-b}{e}}\right)$$

$$then, \qquad M_1 = \frac{\sqrt{-1}\sqrt{e} \propto_j kh}{2F\left(\frac{\pi}{2}, \sqrt{\frac{e-b}{e}}\right)} \qquad (4.2.2.7)$$

Substituting the value of M_1 from (4.2.2.7) in (4.2.2.4), (4.2.2.5) (4.2.2.6) and (4.2.2.2) we have

$$w(C) = \alpha_j kh\frac{F\left(sin^{-1}\sqrt{\frac{e(c-b)}{c(e-b)}}, \sqrt{\frac{e-b}{e}}\right)}{F\left(\frac{\pi}{2}, \sqrt{\frac{e-b}{e}}\right)} - \left(1 - \sum_{r=1}^{j-1}\alpha_r\right)kh \qquad (4.2.2.8)$$

$$w(D) = \alpha_j kh\frac{F\left(sin^{-1}\sqrt{\frac{e(1-b)}{(e-b)}}, \sqrt{\frac{e-b}{e}}\right)}{F\left(\frac{\pi}{2}, \sqrt{\frac{e-b}{e}}\right)} - \left(1 - \sum_{r=1}^{j-1}\alpha_r\right)kh \qquad (4.2.2.9)$$

$$q = \propto_j kh\frac{F\left(\frac{\pi}{2}, \sqrt{\frac{b}{e}}\right)}{F\left(\frac{\pi}{2}, \sqrt{\frac{e-b}{e}}\right)} \qquad (4.2.2.10)$$

$$Let\ A_{2j} = \frac{F\left(\frac{\pi}{2}, \sqrt{\frac{b}{e}}\right)}{F\left(\frac{\pi}{2}, \sqrt{\frac{e-b}{e}}\right)}$$

$$substituting\ for\ A_{2j}\ and\ simplifying \quad q = \propto_j khA_{j2} \tag{4.2.2.11}$$

4.3.3 FRAGMENT TYPE 3:

4.3.3.1. Mapping of the flow domain in to z plane onto t plane z=f₁(t) (see Fig 4.3a)

The schwarz – Christoffel transformation that given the afore mentioned mapping is given by

$$\frac{dz}{dt} = \frac{M}{t^{\beta}(t-1)^{\frac{1}{2}}}$$

$$z(t) = \int_0^t \frac{Mdt}{t^{\beta}(t-1)^{\frac{1}{2}}} + N = \frac{M}{i}\int_0^t t^{-\beta}(1-t)^{-\frac{1}{2}}dt + N \quad = \frac{M}{i}B_t\left(1-\beta, \frac{1}{2}\right) + N$$

$B_t\left(1-\beta, \frac{1}{2}\right)$ *is complete or incomplete Beta function and depends on* t

(i) At point A, $t = 0$, $z(t) = z_A$ so that $N = z_A$

(ii) At point B, $t = b$

$$z_B = \frac{M}{i}B_b\left(1-\beta, \frac{1}{2}\right) + z_A \ or, \qquad z_B - z_A = \frac{M}{i}B_b\left(1-\beta, \frac{1}{2}\right) \tag{4.3.1.1}$$

(iii) At point C, $t = 1$, $z = z_C$

$$z_C = \frac{M}{i}B\left(1-\beta, \frac{1}{2}\right) + z_A \ or, \qquad z_C - z_A = \frac{M}{i}B\left(1-\beta, \frac{1}{2}\right) \tag{4.3.1.2}$$

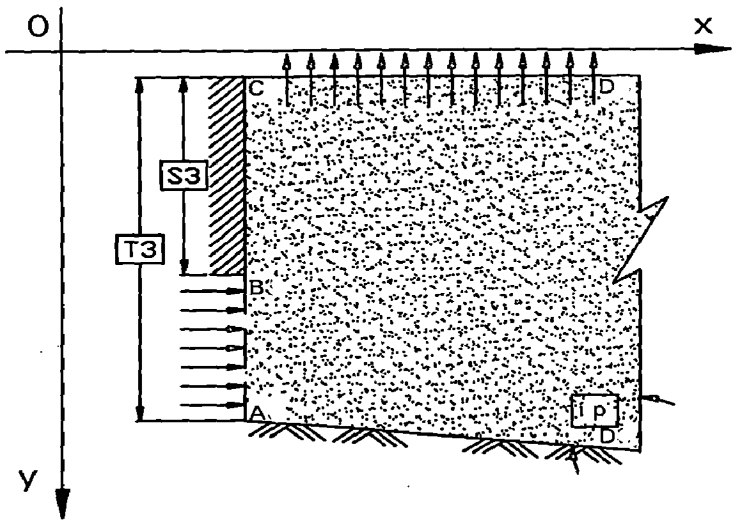
From z-plane

$$Z_C - Z_A = -iT_3 \qquad and \qquad Z_B - Z_A = -i(T_3 - S_3) \tag{4.3.1.3}$$

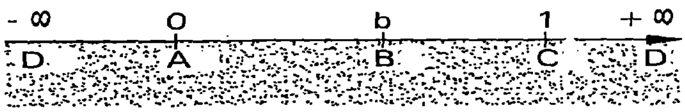
From (4.3.1.1), (4.3.1.2) and (4.3.1.3), we have

$$\left.\begin{array}{l} \dfrac{Z_C - Z_A}{Z_B - Z_A} = \dfrac{T_3}{(T_3 - S_3)} \\ \dfrac{Z_C - Z_A}{Z_B - Z_A} = \dfrac{B\left(1-\beta, \frac{1}{2}\right)}{B_b\left(1-\beta, \frac{1}{2}\right)} \end{array}\right\} therefore \quad \frac{(T_3 - S_3)}{T_3} = \frac{B_b\left(1-\beta, \frac{1}{2}\right)}{B\left(1-\beta, \frac{1}{2}\right)} \tag{4.3.1.4}$$

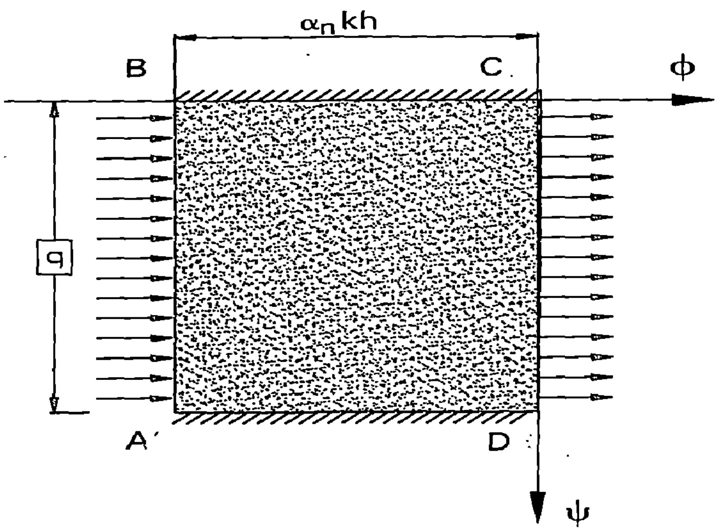
Equation (4.1.3.4) the unknown variable b can be solved by iteration.



(a) z-Plane (z= x + iy)



(b) t – Plane (t = r + is)



(c) w – Plane (w = ϕ + iΨ)

FIG 4.4 – TRANSFORMATION LAYOUT TYPE 3 – CASE 2

4.3.3.2.Complex potential plane w = $f_2(t)$

The transformation of th polygon in w plane onto the t plane (Fig 4.3 c)

$$\frac{dw}{dt} = \frac{M_1 dt}{t^{\frac{1}{2}}(t-b)^{\frac{1}{2}}(t-1)^{\frac{1}{2}}}$$

Hence,

$$w(t) = \int_0^t \frac{M_1 dt}{t^{\frac{1}{2}}t(t-b)^{\frac{1}{2}}(t-1)^{\frac{1}{2}}} + N_1 \qquad (4.3.2.1)$$

Where M_1 and N_1 are constants.

(a) Integration along flow boundary AB ($0 \leq t \leq b$)

(i) At point A, t = 0, $w(t) = -\left(1 - \sum_{r=1}^{n-1} \alpha_r\right)kh - iq$

(ii) At point B, t = b, $w(t) = -\alpha_3\ kh$ therefore,

$$-\left(1 - \sum_{r=1}^{n-1} \alpha_r\right)kh = \int_0^t \frac{M_1 dt}{t^{\frac{1}{2}}(t-b)^{\frac{1}{2}}(t-1)^{\frac{1}{2}}} + iq - \left(1 - \sum_{r=1}^{n-1} \alpha_r\right)kh \qquad (4.3.2.2)$$

Performing the integration (Bryd and Fried man, 1971)

$$-\left(1 - \sum_{r=1}^{n-1} \alpha_r\right)kh = M_1 g\ F(\varphi, m) + \ iq - \left(1 - \sum_{r=1}^{n-1} \alpha_r\right)kh \quad where \begin{cases} g = 2 \\ \varphi = \frac{\pi}{2} \\ m = \sqrt{b} \end{cases}$$

Then

$$q = 2iM_1 F\left(\frac{\pi}{2}, \sqrt{b}\right) \qquad (4.3.2.3)$$

(b) Integration along sheet pile BC ($b \leq t \leq 1$)

(i) At point B, t = b, $w(t) = -\left(1 - \sum_{r=1}^{n-1} \alpha_r\right)kh$

(ii) At point C, t = 1, $w(t) = 0$

Therefore

$$0 = \int_b^1 \frac{M_1 dt}{t^{\frac{1}{2}}(t-b)^{\frac{1}{2}}(-1)^{\frac{1}{2}}(1-t)^{\frac{1}{2}}} - \left(1 - \sum_{r=1}^{n-1} \alpha_r\right)kh \qquad (4.3.2.4)$$

Performing the integration (Bryd and Fried man, 1971)

$$0 = \frac{M_1}{\sqrt{-1}} g\, F(\varphi, m) - \left(1 - \sum_{r=1}^{n-1} \alpha_r\right) kh \qquad \begin{cases} g = \dfrac{2}{\sqrt{1-0}} = 2 \\ \varphi = \sin^{-1}\sqrt{\dfrac{(1-0)(1-b)}{(1-b)(1-0)}} = \dfrac{\pi}{2} \\ m = \sqrt{\dfrac{1-b}{1-0}} = \sqrt{1-b} \end{cases}$$

Therefore,

$$M_1 = \frac{\sqrt{-1}(1 - \sum_{r=1}^{n-1} \alpha_r) kh}{2\, F\left(\frac{\pi}{2}, \sqrt{1-b}\right)} \tag{4.3.2.5}$$

Substitute the value of M_1 from the eq (4.3.2.5) to the equation (4.3.2.3) we have

$$q = \left(1 - \sum_{r=1}^{n-1} \alpha_r\right) kh \frac{F\left(\frac{\pi}{2}, \sqrt{b}\right)}{F\left(\frac{\pi}{2}, \sqrt{1-b}\right)} \tag{4.3.2.6}$$

$$letting, A_{n3} = \frac{F\left(\frac{\pi}{2}, \sqrt{b}\right)}{F\left(\frac{\pi}{2}, \sqrt{1-b}\right)} \; and \; \alpha_n = \left(1 - \sum_{r=1}^{n-1} \alpha_r\right)$$

$$therefore, \quad q = \alpha_n kh A_{n3} \tag{4.3.2.7}$$

4.3.4. CALCULATING HEAD LOSS COEFFICIENTS THROUGH 5 FRAGMENTS:

Discharge through each fragment is the same so:

$$q = \propto_1 kh\, A_1 = \propto_2 kh\, A_2 = \propto_3 kh\, A_3 \ldots\ldots = \propto_n kh\, A_n \tag{4.4.1}$$

where n = number of fragments.

From eq (4.4.1) we can calculate headloss coefiction through any fragment

$$\propto_1 = \frac{q}{kh}\frac{1}{A_1};\ \propto_2 = \frac{q}{kh}\frac{1}{A_2};\ \propto_3 = \frac{q}{kh}\frac{1}{A_3};\ \ldots;\propto_n = \frac{q}{kh}\frac{1}{A_n} \tag{4.4.2}$$

$$therefore, \quad \propto_1 + \propto_2 + \propto_3 + \cdots . + \propto_n = \frac{q}{kh}\left(\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \cdots + \frac{1}{A_n}\right)$$

$$or \quad , \frac{q}{kh} = \frac{\propto_1 + \propto_2 + \propto_3 + \cdots . + \propto_n}{\left(\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \cdots + \frac{1}{A_n}\right)}$$

But, $(\propto_1+\propto_2+\propto_3+\cdots.+\propto_n)=1\ then,$

$$\frac{q}{kh}=\frac{1}{\left(\frac{1}{A_1}+\frac{1}{A_2}+\frac{1}{A_3}+\cdots+\frac{1}{A_n}\right)}\ or, q=\frac{kh}{\left(\frac{1}{A_1}+\frac{1}{A_2}+\frac{1}{A_3}+\cdots+\frac{1}{A_n}\right)} \tag{4.4.3}$$

Now we substituting value of q from eqtion (4.4.2) and get

$$\propto_1=\frac{\left(\frac{1}{A_1}\right)}{\left(\frac{1}{A_1}+\frac{1}{A_2}+\frac{1}{A_3}+\cdots+\frac{1}{A_n}\right)};\ \propto_2=\frac{\left(\frac{1}{A_2}\right)}{\left(\frac{1}{A_1}+\frac{1}{A_2}+\frac{1}{A_3}+\cdots+\frac{1}{A_n}\right)};$$

$$\ldots;\propto_{n-1}=\frac{\left(\frac{1}{A_{n-1}}\right)}{\left(\frac{1}{A_1}+\frac{1}{A_2}+\frac{1}{A_3}+\cdots+\frac{1}{A_n}\right)};\ \propto_n=\frac{\left(\frac{1}{A_n}\right)}{\left(\frac{1}{A_1}+\frac{1}{A_2}+\frac{1}{A_3}+\cdots+\frac{1}{A_n}\right)} \tag{4.4.4}$$

From eq(4.4.4) one can easily find head loss coefficient through any fragment if we know the geometry of weir foundation. For present study, we find head loss coefficient through five fragments from eq (4.4.4)

$$A_1=\frac{F\left(\frac{\pi}{2},\sqrt{1-c_1}\right)}{F\left(\frac{\pi}{2},\sqrt{c_1}\right)};\ A_2=\frac{F\left(\frac{\pi}{2},\sqrt{\frac{b_{22}}{e_{22}}}\right)}{F\left(\frac{\pi}{2},\sqrt{\frac{e_{22}-b_{22}}{e_{22}}}\right)};A_3=\frac{F\left(\frac{\pi}{2},\sqrt{\frac{b_{32}}{e_{32}}}\right)}{F\left(\frac{\pi}{2},\sqrt{\frac{e_{32}-b_{32}}{e_{32}}}\right)}$$

$$A_4=\frac{F\left(\frac{\pi}{2},\sqrt{\frac{b_{42}}{e_{32}}}\right)}{F\left(\frac{\pi}{2},\sqrt{\frac{e_{42}-b_{42}}{e_{42}}}\right)};A_5=\frac{F\left(\frac{\pi}{2},\sqrt{b_{53}}\right)}{F\left(\frac{\pi}{2},\sqrt{1-b_{53}}\right)}$$

Where, b_{n2}, c_{n2} mean values of b and c at the fragment nth by type 2
b_{53}means value of b at the fragment 5 by type 3

With the given geometry parameters, depth of sheet piles, cut offs, length of fragment, depth of impervious boundary etc. by iteration we can find the value of b, c, e at any fragment and calculate elliptic integral in this study we can apply numerical method given byKing L.V which to be introduced in the appendix 1. After calculating the values of A_1, A_2, A_3, A_4, A_5. We will substituting in the equation (4.4.4) and gain head loss coefficients through each of the fragments.

4.3.5. CALCULATING UPLIFT PRESSURE ACTING ON WEIR FLOOR:

For design purposes, we need to know the pressure distribution acting along the various sections of the structure and the magnitude of the exit gradient . A long the contour of the structure $\psi=0\ and\ \emptyset=w$

$$\phi=-k\left(\frac{p}{\gamma_w}-y\right)+C \tag{4.5.1}$$

Choosing an origin at 'O' let the constant C
At $y = y_C, \Phi = 0$ and $\frac{p}{\gamma_w} = h_2$ $or, C = k(h_2 - y_{C_n})$

$$\frac{p}{\gamma_w} = y + (h_2 - y_{C_n}) - \frac{\phi}{k} \tag{4.5.2}$$

From w plane analytic of

(a) Calculating uplift pressure at fragment 1:

At fragment 1, we calculate the uplift pressure at the deepest of sheet pile (cut offs). Note that along the sheet pile, $w = \emptyset = -(1-\alpha_1)kh, y = s_1$
Substituting value of $\emptyset$ in equation (3.4.2) we have

$$\frac{p_B}{\gamma_w} = s_1 + (h_2 - y_{C_n}) - \frac{-(1-\alpha_1)kh}{k} = s_1 + (h_2 - y_{C_n}) + (1-\alpha_1)kh \tag{4.5.3}$$

(b) Calculating uplift pressure at fragment 2:

At fragment 2,(type 2) we calculate uplift pressure at the points C,D and E. At w-plane analysis on fragment type 2, value of w(C)&w(D) in the equation (4.2.2.8)&(4.2.2.9) Replacing value of n = 2 we have value of $\emptyset_C$ and substituting in (4.5.2) to get value of uplift pressure.

$$\frac{p_C}{\gamma_w} = y_C + (h_2 - y_{C_n}) - \frac{\phi_C}{k} \tag{4.5.4}$$

$$where\ \emptyset_C = w(C) = \alpha_2 kh \frac{F\left(sin^{-1}\sqrt{\frac{e_{22}(c_{22}-b_{22})}{c_{22}(e_{22}-b_{22})}}, \sqrt{\frac{e_{22}-b_{22}}{e_{22}}}\right)}{F\left(\frac{\pi}{2}, \sqrt{\frac{e_{22}-b_{22}}{e_{22}}}\right)} - (1-\alpha_1)kh$$

$$\frac{p_D}{\gamma_w} = y_D + (h_2 - y_{C_n}) - \frac{\phi_D}{k} \tag{4.5.5}$$

$$where, \emptyset_D = w(D) = \alpha_2 kh \frac{F\left(sin^{-1}\sqrt{\frac{e_{22}(1-b_{22})}{(e_{22}-b_{22})}}, \sqrt{\frac{e_{22}-b_{22}}{e_{22}}}\right)}{F\left(\frac{\pi}{2}, \sqrt{\frac{e_{22}-b_{22}}{e_{22}}}\right)} - (1-\alpha_1)kh$$

$$\frac{p_E}{\gamma_w} = y_E - (h_2 - y_{C_n}) - \frac{\phi_E}{k} \tag{4.5.6}$$

where, $\phi_E = -(1-\alpha_1-\alpha_2)kh$

(c) Calculating uplift pressure at fragment 3:

At fragment 3,(type 2) we calculate uplift pressure at the points F,G and H. At w-plane analysis on fragment type 2, value of w in the equation (4.2.2.8)& (4.2.2.9). Replacing value of j=3 we have value of $\emptyset_F, \emptyset_G, \emptyset_H$ and substituting in (4.5.2) to get value of uplift pressure.

$$\frac{p_F}{\gamma_w} = y_F + (h_2 - y_{C_n}) - \frac{\phi_F}{k} \tag{4.5.7}$$

$$where\ \emptyset_F = w(F) = \alpha_3 kh \frac{F\left(sin^{-1}\sqrt{\frac{e_{32}(c_{32}-b_{32})}{c_{32}(e_{32}-b_{32})}}, \sqrt{\frac{e_{32}-b_{32}}{e_{32}}}\right)}{F\left(\frac{\pi}{2}, \sqrt{\frac{e_{32}-b_{32}}{e_{32}}}\right)} - (1-\alpha_1-\alpha_2)kh$$

$$\frac{p_G}{\gamma_w} = y_G + (h_2 - y_{C_n}) - \frac{\phi_G}{k} \tag{4.5.8}$$

$$where\ \emptyset_F = w(F) = \alpha_3 kh \frac{F\left(sin^{-1}\sqrt{\frac{e_{32}(1-b_{32})}{(e_{32}-b_{32})}}, \sqrt{\frac{e_{32}-b_{32}}{e_{32}}}\right)}{F\left(\frac{\pi}{2}, \sqrt{\frac{e_{32}-b_{32}}{e_{32}}}\right)} - (1-\alpha_1-\alpha_2)kh$$

$$\frac{p_H}{\gamma_w} = y_E + (h_2 - y_{C_n}) - \frac{\phi_H}{k} \tag{4.5.9}$$

$$where, \phi_E = -(1-\alpha_1-\alpha_2-\alpha_3)kh$$

(d) Calculating uplift pressure at fragment 4:

At fragment 4,(type 2) we calculate uplift pressure at the points Iand J. At the point I and point J, from w-plane analysis on fragment type 2, value of w in the equation (4.2.2.8)& (4.2.2.9). Replacing value of n = 4 we have value of $\emptyset_I, \emptyset_J$ and substituting in (4.5.2) to get value of uplift pressure.

$$\frac{p_I}{\gamma_w} = y_I + (h_2 - y_{C_n}) - \frac{\phi_I}{k} \tag{4.5.10}$$

$$where, \emptyset_I = w(I) = \alpha_4 kh \frac{F\left(sin^{-1}\sqrt{\frac{e_{42}(1-b_{42})}{(e_{42}-b_{42})}}, \sqrt{\frac{e_{42}-b_{42}}{e_{42}}}\right)}{F\left(\frac{\pi}{2}, \sqrt{\frac{e_{42}-b_{42}}{e_{42}}}\right)} - \left(1-\sum_{r=1}^{3}\alpha_r\right)kh$$

$$\frac{p_J}{\gamma_w} = y_J + (h_2 - y_{C_n}) - \frac{\phi_J}{k} \tag{4.5.11}$$

$$where, \emptyset_J = w(J) = \alpha_4 kh \frac{F\left(sin^{-1}\sqrt{\frac{e_{42}(1-b_{42})}{(e_{42}-b_{42})}}, \sqrt{\frac{e_{42}-b_{42}}{e_{42}}}\right)}{F\left(\frac{\pi}{2}, \sqrt{\frac{e_{42}-b_{42}}{e_{42}}}\right)} - \left(1-\sum_{r=1}^{3}\alpha_r\right)kh$$

(e) Calculating uplift pressure at fragment 5:

Uplift pressure at the point K

$$\frac{p_K}{\gamma_w} = y_J + (h_2 - y_{C_n}) - \frac{\phi_K}{k} \tag{4.5.12}$$

$$where, \phi_K = -(1 - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_4)kh = \alpha_5 kh$$

From equations (4.5.3), (4.5.4), (4.5.5), (4.5.6), (4.5.7), (4.5.8), (4.5.9), (4.5.10), (4.5.11), (4.5.12) by numerical calculation, we can find the value of uplift pressure at some special points of weir foundation and analysis the distribution of uplift pressure along the base of structure with variable of geometry characteristics.

4.3.6. CALCULATING EXIT GRADIENT:

The exit gradient is computed by fragment 5.
Let the complex potential $w = \phi + i\psi$ be analytic function of the complex variable z

$$then, \frac{dw}{dz} = \frac{\partial\phi}{\partial x} + i\frac{\partial\psi}{\partial x} \tag{4.6.1}$$

Which, substituting the velocity components, yields the complex velocity

$$\frac{dw}{dz} = u - iv$$

Along the downstream horizontal boundary u = 0, hence

$$\frac{dw}{dz} = -iv \tag{4.6.2}$$

From Darcy's Law

$$v = -I_E k \tag{4.6.3}$$

Substituting (3.5.3) in (3.5.2)

$$\frac{dw}{dz} = iI_E k \quad therefore\ I_E = \frac{1}{ik}\frac{dw}{dz} = \frac{1}{ik}\frac{dw}{dt}\frac{dt}{dz} \tag{4.6.4}$$

The exit gradient is computed from Fragment 2
From analysis of fragment 2

$$\frac{dz}{dt} = \frac{M}{t^{\beta}(t-1)^{\frac{1}{2}}} = \frac{B\left(1-\beta, \frac{1}{2}\right)}{T_3}\frac{1}{t^{\beta}(t-1)^{\frac{1}{2}}} \tag{4.6.5}$$

$$\frac{dw}{dt} = \frac{M_1 dt}{t^{\frac{1}{2}}(t-b)^{\frac{1}{2}}(t-1)^{\frac{1}{2}}}$$

$$where, M_1 = \frac{i\alpha_n kh}{2\,F\left(\frac{\pi}{2}, \sqrt{1-b}\right)} \quad hence,$$

$$\frac{dw}{dt} = \frac{i\alpha_n kh}{2\,F\left(\frac{\pi}{2}, \sqrt{1-b}\right)}\frac{1}{t^{\frac{1}{2}}(t-b)^{\frac{1}{2}}(t-1)^{\frac{1}{2}}} \tag{4.6.6}$$

Substituting eq. (3.5.6)and eq. (3.5.5) in eq.(3.5.2) we have,

$$I_E = \frac{1}{ik} \frac{i\alpha_n kh}{2\,F\left(\frac{\pi}{2}, \sqrt{1-b}\right)} \frac{1}{t^{\frac{1}{2}}(t-b)^{\frac{1}{2}}(t-1)^{\frac{1}{2}}} \frac{T_3 t^{\beta}(t-1)^{\frac{1}{2}}}{B\left(1-\beta, \frac{1}{2}\right)}$$

$$I_E = \frac{\alpha_n h T_3}{2\,B\left(1-\beta, \frac{1}{2}\right) F\left(\frac{\pi}{2}, \sqrt{1-b}\right)} \frac{t^{\beta}}{t^{\frac{1}{2}}(t-b)^{\frac{1}{2}}} \qquad (4.6.7)$$

$$or, \frac{I_E}{h} = \frac{\alpha_n T_3}{2\,B\left(1-\beta, \frac{1}{2}\right) F\left(\frac{\pi}{2}, \sqrt{1-b}\right)} \frac{t^{\beta}}{t^{\frac{1}{2}}(t-b)^{\frac{1}{2}}} \qquad (4.6.8)$$

Substituting some value of $t > 1$, we can have values of exit gradient through foundation structure.

4.4 RESULT AND DISCUSSION:

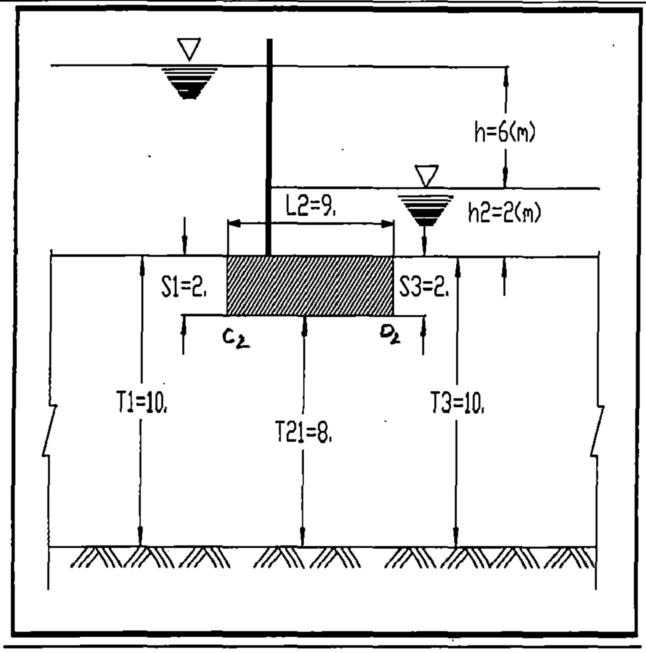
From example, exit gradient is at maximum value if we didn't provide any sheet pile at upstream and downstream of weir floor. Providing upstream sheet pile more effective decreasing up lift pressure but exit gradient is high. providing the sheet pile at downstream end of impervious floor more effective to decrease exit gradient but increasing at uplift pressure at weir floor (P_{D2}). In case of providing two sheet piles at upstream and downstream of weir floor exit gradient is minimum and uplift pressures is in medium.

For weir design required, we should find the good combination of weir foundation profile for safety and economic consideration.

ILLUSTRATE EXAMPLE:

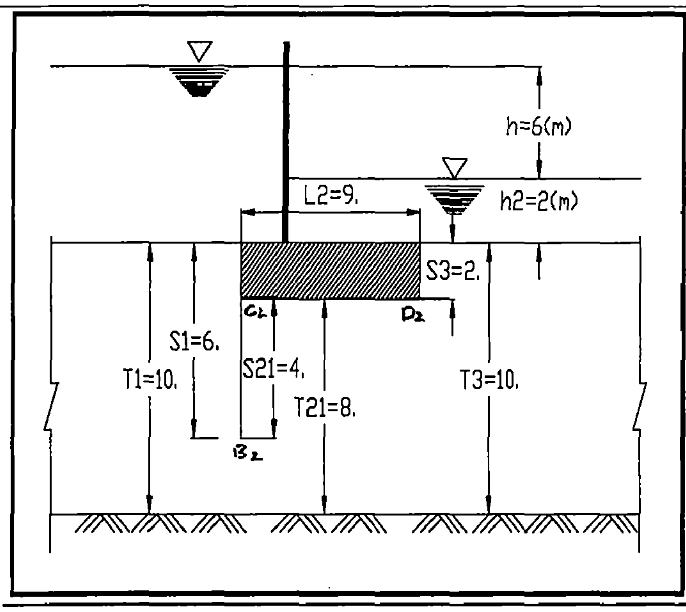
CASE1:

h=6(m) h2=2(m),Slope of impervious boundary =0.5
Number of fragment n =3
********Fragment type 1********
S1=2,T1=10,A1=2.599531
Fragment Type 2 [1]
Angle alpha=0.5
L2[2]=9, T21[2]=8, S21[2]=0,S22[2]=0
A[2]=1.165983
********Fragment type 3 ********
S3[3]= 2, T3[3]= 10
A[3]=2.036505
Head loss coefficient Alpha[1]=0.221929
Head loss coefficient Alpha[2]=0.494786
Head loss coefficient Alpha[3]=0.283285
exit gradient Ie/h =1.758494
PB2[2]=7.668425,
PC2[2]=7.668425,
PD2[2]=4.699711,
PE2[2]=4.699711



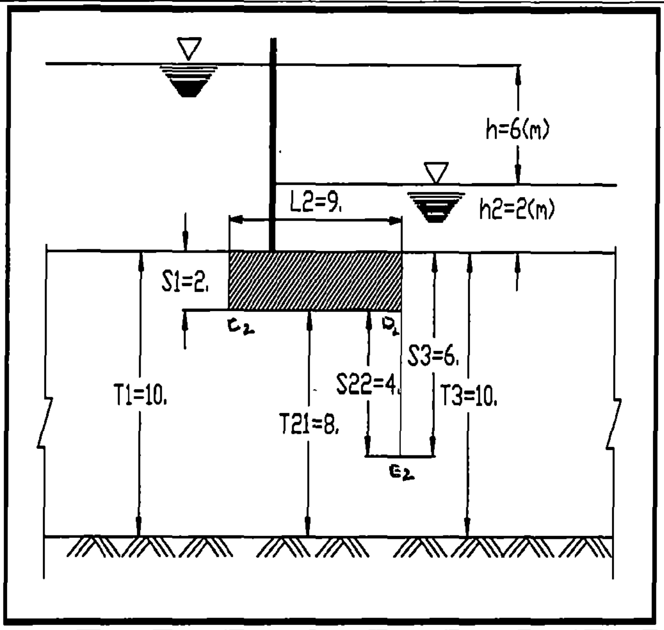
CASE 2:

h=6(m) h2=2(m),Slope of impervious boundary =0.5
Number of fragment n =3
********Fragment type 1********
S1=6,T1=10, A1=1.74141
Fragment Type 2 [1]
Angle alpha=0.5,
L2[2]=9,T21[2]=8,S21[2]=4,S22[2]=0,
A[2]=0.819214
********Fragment type 3 ********
S3[3]= 2,T3[3]= 10, A[3]=2.036505
Head loss coefficient Alpha[1]=0.251205
Head loss coefficient Alpha[2]=0.533989
Head loss coefficient Alpha[3]=0.214805
exit gradient Ie/h =1.333404
Uplift pressure at fragment type 2 are
PB2[2]=11.492767
PC2[2]=5.509961
PD2[2]=4.288831
PE2[2]=4.288831



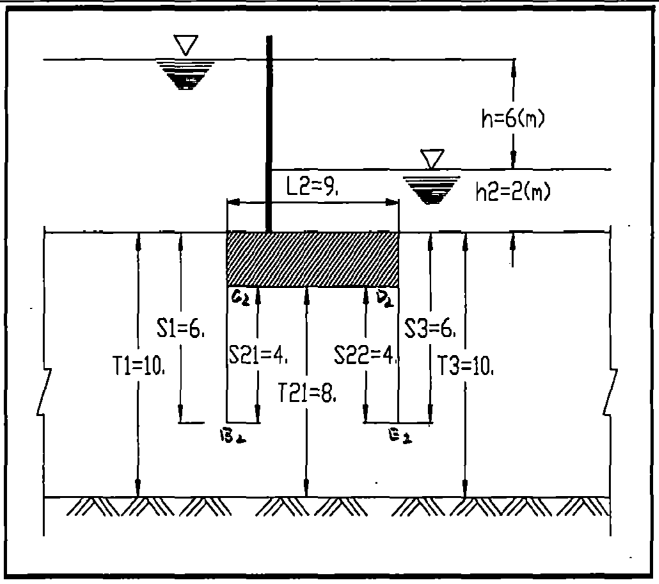
CASE 3:

h=6(m) h2=2(m),Slope of impervious boundary =0.5
Number of fragment n =3
********Fragment type 1********
S1=2, T1=10, A1=2.599531
Fragment Type 2 [1]
Angle alpha=0.5, L2[2]=9, T21[2]=8, S21[2]=0, S22[2]=4
A[2]=1.018869
********Fragment type 3 ********
S3[3]= 6, T3[3]= 10
A[3]=0.864205
Head loss coefficient Alpha[1]=0.152453
Head loss coefficient Alpha[2]=0.388967
Head loss coefficient Alpha[3]=0.45858
exit gradient Ie/h =0.447514
Uplift pressure at fragment type 2 are
PB2[2]=8.085281
PC2[2]=8.085281
PD2[2]=6.91116
PE2[2]=9.751478



CASE 4:

h=6(m) h2=2(m),Slope of impervious boundary =0.5
Number of fragment n =3
********Fragment type 1********
S1=6,T1=10, A1=1.74141
Fragment Type 2 [1]
Angle alpha=0.5, L2[2]=9, T21[2]=8, S21[2]=4, S22[2]=4,
A[2]=0.763024
********Fragment type 3 ********
S3[3]= 6, T3[3]= 10,
A[3]=0.864205,
Head loss coefficient Alpha[1]=0.188776
Head loss coefficient Alpha[2]=0.430833
Head loss coefficient Alpha[3]=0.380391
exit gradient Ie/h =0.371212
Uplift pressure at fragment type 2 are
PB2[2]=11.867346
PC2[2]=7.530658
PD2[2]=5.645727
PE2[2]=9.282347



CHAPTER 5

RESULT AND DISCUSSIONS

Two dimensional flow under a hydraulic structure founded on a porous medium of finite depth can be solved by method of fragments. The method of fragment is applicable if the porous medium is underlain by an impervious boundary. Conformal mapping technique when applied to the whole flow domain, the solution becomes cumbersome and intractable. In method of fragments, the flow domain is decomposed to several fragments; through each fragment, the seepage quantity is same; the boundary of the fragments are straight line, which is either an equipotential line or a stream line.

A fragment can have maximum four vertices, and a maximum of three vertices take part in transformation, the solution is tractable and simple. Khosla, Bose and Taylor have analyzed flow under a stepped weir resting on a porous medium of infinite depth. In nature, the foundation soil layer is always finite. In chapter 3, using method of fragments, the two dimensional flow have been analysed. The potentials at salient points have been determined. If the porous medium is assumed to be of infinite depth, the up lift pressure is under estimated. In table 3.3 we present potential for different thickness of the soil layer. It could be observed from this table, as thickness decreases, the potential increases, hence, the up lift pressure increases. The exit gradient is also under estimated with assumption of soil layer of large depth.

In chapter 4, a structure generally constructed have been dealt. The flow domain is decomposed into 5 fragments. The solution is presented for a structure that can have any number of fragments. An example has been presented for computation of the flow characteristics, uplift pressure and exit gradient are presented for a weir with two cut offs.

A soft ware ha been developed. The elliptic integrals have been evaluated using numerical method, all other integrations have been evaluated analytically.

CONCLUSION

Method of fragment considerably simplifies the mapping technique. Flow under hydraulic structure having large number of vertices, can be analyzed easily. Assumption of infinite depth of soil layer under estimates up lift pressure and exit gradient, which may endanger the safety of the structure. The only limitation in method of fragment is that the soil layer is underlain by an impervious layer. Method of fragment is not applicable if the underlying layer is a draining layer.

The soft ware developed can be incorporated in developing a soft ware for designing hydraulic structure considering both surface and subsurface flow conditions.

REFERENCES

1	Arora K.R, " Irrigation Water Power And Water Resources Engineering", Standard Publishers Distributors, Delhi, Fourth Edition (2002), Pages (17-18), Pages (107-132)
2	Central Board Of Irrigation And Power (October 1985) " Manual On Barrages And Weirs On Permeable Foundation"
3	Chawla, A.S "Effect Of Boundary Conditions On Uplift Pressure And Exit Gradients Due To Subsurface Flow Under Hydraulic Structures" M.E Thesis University Of Roorkee (1967)
4	Harr M.E, "Ground Water And Seepage" Mc Graw-Hill Book Company (1962).
5	IS 6966 - 1973, " Criteria For Hydraulic Design Of Barrages And Weirs".
6	IS 7720 – 1975, "Criteria For Investigation, Planning And Layout Of Barrages And Weirs"
7	IS 10084 – 1982, " Criteria For Design Of Diversion Head-Works".
8	Khosla, A.N., Bose And Taylor, "Design Of Weirs On Permeable Foundations. General Board Of Irrigation And Power, Puplication N012, (1954)
9	Lafore, R., "Object – Oriented Programming In Turbo C++", Galgotia Publications Pvt.Ltd, (1999)
10	Mishra G.C, "Confined And Unconfined Flows Through Anisotropic Porous Media" Phd Thesis, Indian Institute Of Science Bangalore (1972)
11	Paul, F.B., And Morris, D.F. , "Hand Book Of Elliptic Integral For Engineers And Scientist", Second Edition, Springer – Verlag Berlin Heidelberg New York, USA (1971)
12	Polubarinova P.YA – Kochina, "Theory Of Ground Water Movement", Princeton University Press, New Jersey (1962).
13	Ranga Raju K.G, " Flow Through Open Channels", Second Edition, Tata Mc Graw-Hill Publishing Company (2002), Pages (58-79), (181-209).
14	Snedden I.N, "Special Functions Of Mathematical Physics And Chemistry", New York: Interscience Publishers, Inc, (1961)
15	Varshney, Gupta S.C, Gupta R.L, "Theory And Design Of Irrigation Structures", Nem Chand And Bros Roorkee (U.P) (1993), Part 1
16	Zhang S., Jin J. "Computation Of Special Function" A Wiley – Interscience Publication (1996)

INDEX 1

ELLIPTIC INTEGRAL BY KING'S METHOD

1. Elliptic Integrals

The elliptic integral of the first kind is defined by

$$F(\varphi, m) = \int_0^{\varphi} \frac{d\theta}{\sqrt{(1 - m^2 \sin\theta)}} \ (0 \leq m \leq 1)$$

where m is a parameter. Letting t = sin θ, can be written as

$$F(x, m) = \int_0^{x} \frac{dt}{\sqrt{(1-t)(1-m^2 t)}} (0 \leq m \leq 1)$$

$when\ \varphi = \frac{\pi}{2}, F(\varphi, m)\ is\ complete\ elliptic\ integral$

2. Arithmetic – Geometric Mean Method

This method was described by King and its basic steps are as follows:

Given a set of initial values (a_0, b_0, c_0, φ_0), calculate iteratively a new set of values ((a_1, b_1, c_1, φ_1), (a_2, b_2, c_2, φ_2),…, (a_n, b_n, c_n, φ_n) according to the follow scheme:

$$a_0 = 1 \qquad b_0 = m' \qquad c_0 = m \qquad \varphi_0 = \varphi$$

$$a_1 = \frac{a_0 + b_0}{2} \qquad b_1 = \sqrt{a_0 b_0} \qquad c_1 = \frac{a_0 + a_0}{2} \qquad \varphi_1 = \varphi_0 + x_0$$

$$a_2 = \frac{a_1 + b_1}{2} \qquad b_2 = \sqrt{a_1 b_1} \qquad c_2 = \frac{a_1 + a_1}{2} \qquad \varphi_2 = \varphi_1 + x_1$$

………

$$a_n = \frac{a_{n-1} + b_{n-1}}{2} \qquad b_n = \sqrt{a_{n-1} b_{n-1}} \qquad c_n = \frac{a_{n-1} + a_{n-1}}{2} \qquad \varphi_n = \varphi_{n-1} + x_{n-1}$$

where

$$x_i = \tan^{-1}\left(\frac{b_i}{a_i} \tan \varphi_i\right) \qquad (i = 0,1,\ldots,n-1)$$

The terminate the iteraction when c_n is less than a given small number such as 10^{-7}. The value of the elliptic integrals can then be determined as

$$K(m) = \frac{\pi}{2a_n}$$

$$E(m) = \frac{\pi}{4a_n}(2 - c_0^2 - 2c_0^2 - \dots - 2^n c_n^2)$$

$$F(\varphi, m) = \frac{\varphi n}{2n\, c_n} \quad (k \neq 1)$$

F(φ,1) = 1/2 ln (1+sinφ)/(1-sinφ)

$$F(\varphi, m) = \frac{\varphi n}{2n\, c_n} + \sum_{j=1}^{n} cj \sin \varphi j - \frac{\varphi n}{2n+1\, a_n} \quad \sum_{j=0}^{n} 2^j c_j^2 \qquad (k \neq 1)$$

$$F(\varphi, 1) = \sin \varphi$$

If we need to calculate the complementary elliptic integrals, we can choose the initial values as $(a_0', b_0', c_0', \varphi_0') = (1, k, k', \frac{\pi}{2})$ and then proceed with , until $((a_n', b_n', c_n', \varphi_n')$ with $c_n' < 10^{-7}$.

COMPUTER PROGRAMMING FOR DEPRESSED WEIR RESTING ON POROUS MEDIA OF FINITE DEPTH

```
#include<iostream.h> #include<math.h> #include<conio.h>#include<stdio.h> #include<iomanip.h>
#include<fstream.h>
const float PAI=3.141592653589793;
double ellip(double angle, double hk);//Elliptic integral first kind
void  fra1(double l1,double t, double ds,double s,   double&b1,double&d1,double&A1);
void fra22(double l2,double t, double ds,double s,   double&b2,double&c2,double&A2);
void main()
{clrscr();
ofstream outfile("bt1.text");
double l1,ds,s,d1,b1,t,A1;  //Fragment 1;
double l2,b2,c2,A2;        //Fragment 2;
double qkh,alpha,b;
double phic1,u1,v1;
double phic2,u2,v2;
double aqkh[50],aalpha[50],pbc1[50],pbc2[50],T[50];
int i=1,n=20,k=0;
b=10.;l1=b;l2=10;ds=0.;s=3.;t=8.;
outfile<<"\nL1="<<l1<<";\tS="<<s<<"\n";
outfile<<"t"<<"\t"<<"alpha"<<"\t"<<"  q/kh"<<"\t\t"<<"PhiC1"<<"\t\t"<<"PhiC2"<<"\n";
for(i=0; i<n;i++)
{fra1(l1,t,ds,s,b1,d1,A1);fra22(l2,t,ds,s,b2,c2,A2);
 alpha=A1/(A1+A2);
 aalpha[i]=alpha;
 aqkh[i]=A1*A2/(A1+A2);
 T[i]=t;
 u1=asin(sqrt(b1/(d1+b1)));
 v1=sqrt((d1+b1)/(1.+b1));
 phic1=(1.-alpha)*ellip(u1,v1)/ellip(.5*PAI,v1)-1.;
 u2=asin(sqrt((c2-b2)/(1.-b2)));
 v2=sqrt(1.-b2);
 phic2=alpha*ellip(u2,v2)/ellip(.5*PAI,v2)-alpha;
 pbc1[i]=phic1;pbc2[i]=phic2;
 k=k+1;
 if(phic2<phic1||phic2<-1.||phic1<-1.)break;
 cout<<"\nT="<<t<<"\tAlpha = "<<alpha;
 cout<<"\tq/kh="<<A1*A2/(A1+A2)<<endl;
 cout<<"\tPhiC1/kh ="<<phic1;
 cout<<"\n\tPhiC2/kh ="<<phic2<<endl
 outfile<<t<<"\t"<<alpha<<"\t"<<A1*A2/(A1+A2)<<"\t"<<phic1<<"\t"<<phic2<<"\n";
t=t+.5;
}
cout<<endl<<endl;
cout<<" T\t   q/kh\talpha\t   PhiC1\tPhiC2"<<endl;
for(i=1;i<k;i++)
cout<<setiosflags(ios::showpoint)
  <<setw(5)<<setprecision(1)<<T[i]   <<setw(12)<<setprecision(4)<<aqkh[i]  <<setw(12)<<setprecision(2)<<aalpha[i]
<<setw(12)<<setprecision(4)<<pbc1[i] <<setw(12)<<setprecision(4)<<pbc2[i]<<endl;
getch();

}
void fra1(double l1,double t, double ds, double s,
                   double& b1,double& d1,double&A1)
          {double temp=ds+s,temp1, temp2,temp3;
           if  (temp==0) d1=0.;
           else {temp1=.5*PAI*(ds+s)/t; d1=pow(sin(temp1),2.); }
           if   (l1==0)b1=0;
```

```
            else        b1=sinh(l1*PAI*.5/t)*sinh(l1*PAI*.5/t);
            temp2=sqrt((1.-d1)/(1.+b1));
            temp3=sqrt((d1+b1)/(1.+b1));
            A1=ellip(.5*PAI,temp2)/ellip(.5*PAI,temp3);
             }
void fra22(double l2, double t, double ds,double s, double&b2,
                        double&c2, double&A2)
 {double temp1,temp2;
  if(l2==0) c2=1;
  else {    temp1=PAI*l2*.5/(t-ds);       c2=1./cosh(temp1)/cosh(temp1);}
  if(s==0.) b2=c2;
  else   {temp2=sin(.5*PAI*(t-ds-s)/(t-ds));b2=c2*pow(temp2,2.);}
  A2=ellip(.5*PAI,sqrt(b2))/ellip(.5*PAI,sqrt(1.-b2));
  }
double ellip(double angle, double hk)
   {double phi;
    phi=angle*180/PAI;
    int i;
    double fe,d0,r,fac,ck,a,b,c,d,g=0.,a0=1.;
    double b0=sqrt(1.-hk*hk); d0=(PAI/180.)*phi; r=hk*hk;
    if(hk==1.&&phi==90.)  fe=1.+30000.;
    else
     { if (hk==1.) fe=log((1.+sin(d0))/cos(d0));
            else
            { fac=1.;
              for(i=1;i<=40;i++)
              { a=(a0+b0)/2.; b=sqrt(a0*b0); c=(a0-b0)/2.;
                fac=2.*fac; r=r+fac*c*c;
                if(phi<=90) {d=d0+atan((b0/a0))*tan(d0);g=g+c*sin(d);
                d0=d+PAI*floor(d/PAI+0.5);}
                a0=a;b0=b;
              if(c<0.000001) break;
               }
             ck=PAI/(2.*a); if (phi==90) fe=ck; else fe=d/(fac*a);
               }
            }
  return fe;}
```

COMPUTER PROGRAMMING FOR COMPUTATION UPLIFT PRESSURE AND EXIT GRADIENT FOR WEIR WITH MULTI CUT OFFS RESTING ON FINITE PERMEABLE FOUNDATION OF FINITE DEPTH

```
#include<iostream.h>
#include<math.h>
#include<conio.h>
#include<stdio.h>
#include<iomanip.h>
#include<fstream.h>
const float PAI=3.141592653589793;
long double gamma(double x);
double beta(float p, float q);
double inbeta(float p, float q, float x);
double betain1(float a, float b, float x);
double comel(float k);
double betain3 (float a, float b, float x);
double ellip(float angle, float k);          int factor(int );
long double F21(double a,double b,double c, double z);
long double I21(double aa,double bb,double b, double c);
long double I22(double aa,double bb,double c);
long double I23(double aa,double c);
long double I24(double aa,double bb,double c, double e);
long double I25(double aa,double bb,double c);
void fra21(double abeta, double s1, double t1,double& c1,double& form1);
void fra22(double aa, double bb, double l2,double t21, double s21,
                    double s22,double&b, double &c, double&e,double&form2) ;
void fra32(double bb, double t3, double s3, double &b3,double &form3 );
double gra(double an,double bb,double b, double t3);

void main()

{clrscr();ofstream fout("b43.text");
double bb=.5;        //bb=Angle beta (slope of impervious boundaries;
double h,h1,h2,sum1;
double s1,t1,c1,form1;            //for fragment 1;
double aa,l2,t21,s21,s22,b,c,e,form2;  //for fragment 2;
double yb2[10],yc2[10],yd2[10],ye2[10],pb2[10],pc2[10],pd2[10],pe2[10];
double aaa[10],l22[10],t212[10],s212[10],s222[10] ;
double u21,u22,u23;
double t3,s3,b3,form3;            //for fragment 3;
double total,form[20],an[20],sum=0,ycn;
int i,n;
h=6.; h2=2.;
fout<<"\nh="<<h<<"(m)\th2="<<h2<<"(m)\n";
fout<<"Slope of impervious boundary ="<<bb<<"\n";

cout<<"\nEnter number of fragment n =";  cin>>n;
fout<<"\nNumber of fragment n =";  fout<<n;
cout<<"\n\n********Fragment type 1********" <<endl;
fout<<"\n********Fragment type 1********\n";
        cout<<"\nS1=";cin>>s1;  fout<<"\nS1="<<s1<<"\t";
        cout<<"\nT1=";cin>>t1;  fout<<"\nT1="<<t1<<"\t";
        fra21(bb,s1,t1,c1,form1);fout<<"\nA1="<<form1<<"\n";
        form[1]=form1;cout<<"\nForm1="<<form[1];
cout<<"\n\n********Fragment type 2 ********" <<endl;
        for(i=2;i<n;i++)
```

```
            {cout<<"\nFragment Type 2 ["<<i-1<<"]";                fout<<"Fragment Type 2 ["<<i-1<<"]\n";
             cout<<"\naa["<<i<<"]="; cin>>aa; aaa[i]=aa;        fout<<"\nAngle alpha="<<aa<<"\n";
             cout<<"\nL2["<<i<<"]="; cin>>l2; l22[i]=l2;          fout<<"L2["<<i<<"]="<<l2<<"\n";
             cout<<"\nT21["<<i<<"]=";cin>>t21;t212[i]=t21;  fout<<"T21["<<i<<"]="<<t21<<"\n";
             cout<<"\nS21["<<i<<"]=";cin>>s21;s212[i]=s21;  fout<<"S21["<<"]="<<s21<<"\n";
             cout<<"\nS22["<<i<<"]=";cin>>s22;s222[i]=s22;  fout<<"S22["<<i<<"[="<<s22<<"\n";
             fra22(aa,bb,l2,t21,s21,s22,b,c,e,form2);
             form[i]=form2;cout<<"\nForm["<<i<<"]="<<form[i]<<endl;      fout<<"\nA["<<i<<"]="<<form2<<"\n";
             }
cout<<"\n\n********Fragment type 3 ********\n" <<endl;
            cout<<"\nEnter value of S3 = ";cin>>s3;                fout<<"\nS3["<<n<<"]= "<<s3<<"\n";
            cout<<"\nEnter value of T3 = ";cin>>t3;                fout<<"T3["<<n<<"]= "<<t3<<"\n";
            fra32(bb,t3,s3,b3,form3); form[n]=form3;            fout<<"\nA["<<i<<"]="<<form3<<"\n";
for(i=1;i<=n;i++) cout<<"\nform["<<i<<"]="<<form[i]<<endl;
sum=0; for(i=1;i<=n;i++) sum=sum+1./form[i];total=sum;

for(int k=1;k<=n;k++)
{ an[k]=1./(total*form[k]);cout<<"\nan["<<k<<"]="<<an[k];  fout<<"\nHead loss coefficient at fragment
Alpha["<<k<<"]="<<an[k]<<"\n";
}

//calculate exit gradient at distance x:

double u;
u=an[n];
fra32(bb,t3,s3,b3,form3);
cout<<"\nexit gradient Ie/h =" <<gra(u,bb,b3,t3);fout<<"\nexit gradient Ie/h =" <<gra(u,bb,b3,t3);

//calculate uplift pressure

double pp2,pp3,pp4,pp5,pp6,el1,el2,el3;
cout<<"\nEnter value of Ycn=";cin>>ycn;
for (i=2;i<n;i++)
{  aa=aaa[i];l2=l22[i];t21=t212[i];s21=s212[i];s22=s222[i];
   fra22(aa,bb,l2,t21,s21,s22,b,c,e,form2);
   pp2=sqrt(e*(c-b)/c/(e-b));   pp3=asin(pp2);   pp4=sqrt((e-b)/e);
   pp5=sqrt(e*(1.-b)/(e-b));    pp6=asin(pp5);
   el1=ellip(pp3,pp4);   el2=ellip(pp6,pp4);   el3=ellip(.5*PAI,pp4);
   sum=0;sum1=0;
   for(k=1;k<i;k++) sum=sum+an[k];
   for(k=1;k<=i;k++) sum1=sum1+an[k];
   cout<<"\nEnter value of y C2=";cin>>yc2[i];
   yb2[i]=yc2[i]+s21;
   cout<<"\nEnter value of y D2=";cin>>yd2[i];
   ye2[i]=yd2[i]+s22;
   pb2[i]=yb2[i]+(h2-ycn)+h*(1-sum);
   if(s21==0) pc2[i]=pb2[i];
   else  pc2[i]=yc2[i]+(h2-ycn)+h*(1.-sum-an[i]*el1/el3);
   pe2[i]=ye2[i]+(h2-ycn)+h*(1-sum1);
   if(s22==0)pd2[i]=pe2[i];
   else  pd2[i]=yd2[i]+(h2-ycn)+h*(1.-sum-an[i]*el2/el3);
   fout<<"\nUplift pressure at fragment type 2 are\n"
            <<"\nPB2["<<i<<"]="<<pb2[i]
            <<"\nPC2["<<i<<"]="<<pc2[i]
            <<"\nPD2["<<i<<"]="<<pd2[i]
            <<"\nPE2["<<i<<"]="<<pe2[i];
}
```

```
for (i=2;i<n;i++)
          {cout<<"\nPB2["<<i<<"]="<<pb2[i]<<endl;
           cout<<"\nPC2["<<i<<"]="<<pc2[i]<<endl;
           cout<<"\nPD2["<<i<<"]="<<pd2[i]<<endl;
           cout<<"\nPE2["<<i<<"]="<<pe2[i]<<endl;
          }

getch();

}
```

```
double gra(double an,double bb,double b, double t3)
{          double t=1.0;
           double te1,te2,te3;
           te1=2.*beta(1.-bb,.5);          te2=ellip(.5*PAI,sqrt(1.-b));
           te3=1./pow(t-b,.5); double ieh;
           ieh=an*t3*te3/te1/te2;
return ieh;
}

void fra21(double abeta, double s1, double t1,double& c1,double& form1)
{          double fc1,cr,cl;
           double b,x,u;
           if (s1/t1<.5)c1=.3;else c1=.7;
           u=inbeta(.5,abeta,c1);
           fc1=s1/t1-u/beta(0.5,abeta);
           while(fabs(fc1)>0.0001)
            { if(fc1>0.) c1=c1+0.0001;else { cr=c1;cl=c1-0.0001;c1=(cr+cl)*0.5;}
              u=inbeta(.5,abeta,c1); fc1=s1/t1-u/beta(0.5,abeta);
             }
           form1=ellip(.5*PAI,sqrt(1.-c1));
}

void fra22(double aa, double bb, double l2,double t21, double s21,
            double s22,double&b, double &c, double&e,double&form2)
{          long double fc3,fc2,fc1,cr,cl;
    e=1.000001,b=.0001;c=.3;
    fc1=I23(aa,c)*pow(c,aa)/I22(aa,bb,c)-l2/t21/sin(aa*PAI);
           while(fabs(fc1)>0.0001)
            { if(fc1>0.) c=c+0.0001; else  { cr=c;cl=c-0.0001;c=(cr+cl)*0.5;}
              fc1=I23(aa,c)*pow(c,aa)/I22(aa,bb,c)-l2/t21/sin(aa*PAI);
              if(c>1.) {cout<<"\nerror, c>1";break;}
             }
           if(fc1>0) c=c-.0001; else c=c+.0001;
           double c1=c;
           if(s21==0) b=c1;
           else
           {          fc2=(t21-s21)/t21-I21(aa,bb,b,c1)/I22(aa,bb,c1);
                      while(fabs(fc2)>0.001)
                      { if(fc2>0.) b=b+0.0001; else  { cr=c1;cl=b-0.0001;b=(cr+cl)*0.5;}
                      fc2=(t21-s21)/t21-I21(aa,bb,b,c1)/I22(aa,bb,c1);
                      if(b<0.) {cout<<"error b1<0";break;}
                      }
                      if(fc2>0) b=b-.0001; else b=b+.0001;
           }
           double b1=b;
           if(s22==0) e=1;
           else
```

```
{
fc3=(I25(aa,bb,c1)-I24(aa,bb,c1,e))/I23(aa,c1)-s22*sin(aa*PAI)/I2;
while(fabs(fc3)>0.0001)
 { if(fc3<0.) e=e+0.0001;else { cr=e;cl=e-0.0001;e=(cr+cl)*0.5;}
  fc3=(I25(aa,bb,c1)-I24(aa,bb,c1,e))/I23(aa,c1)-s22*sin(aa*PAI)/I2;
  }
if(fc3>0) e=e-.0001; else e=e+.0001;
if(e<1.) e=1.00001;
}

double u2,v2;
u2=asin(sqrt(e*(c1-b1)/(e-b1)));
v2=sqrt((e-b1)/e);
form2=ellip(.5*PAI,sqrt(b1/e))/ellip(.5*PAI,sqrt((e-b1)/e));
}

void fra32(double bb, double t3, double s3, double &b3,double &form3 )
{          double cr,cl;
           long double u,fc1,b1;
           if(s3/t3>.5) b1=.3;   else b1=.8;
           u=pow(b1,1.-bb)*F21(1.-bb,.5,2.-bb,b1)/(1.-bb);
           fc1=u/beta(1.-bb,.5)-(t3-s3)/t3;
           while(fabs(fc1)>0.001)
                    { if(fc1<0.)   b1=b1+0.00001;
                      else { cr=b1;cl=b1-0.00001;b1=(cr+cl)*0.5;}
                      u=pow(b1,1.-bb)*F21(1.-bb,.5,2.-bb,b1)/(1.-bb);
                      fc1=u/beta(1.-bb,.5)-(t3-s3)/t3;
                     if (b1>1.) break;
                    }
           if(fc1>0) b1=b1+.00001;else b1=b1-.00001;
           b3=b1;
           form3=ellip(PAI/2.,sqrt(b3))/ellip(PAI/2.,sqrt(1.-b3));
}

long double F21(double a,double b,double c, double z)
{          long double sum=0.,sum2;
   for (int i=0;i<8;i++)
   { sum2=gamma(a+i)*gamma(b+i)*gamma(c)*pow(z,float(i))/
          (gamma(a)*gamma(b)*gamma(i+c)*factor(i));
    sum=sum+sum2;
    } return sum;
}

long double I21(double aa,double bb,double b, double c)
{          double sum=0.,sum1,t;
           for (int i=0;i<7;i++)
           { sum1=gamma(aa+i)*pow(b/c,float(i))*F21(i+1-bb,1.-aa,i+2-bb,b)/
            (gamma(aa)*factor(i)*(i+1.-bb));
     sum=sum+sum1;
    }
    t=pow(b,1.-bb)*sum;
return t;
}

long double I22(double aa,double bb,double c)
           {long double sum=0.,sum1,t;
           for (int i=0;i<7;i++)
           { sum1=gamma(aa+i)*F21(i+1-bb,1.-aa,i+2-bb,c)/
```

```
            (gamma(aa)*factor(i)*(i+1.-bb));
           sum=sum+sum1;
           }
           t=pow(c,1.-bb)*sum;
return t;
}

long double I23(double aa,double c)
           {long double sum=0.,sum2,t;
           for (int i=0;i<8;i++)
           { sum2=gamma(aa+i)*pow(1.-c,float(i))*gamma(aa+i)*gamma(1.-aa)/
             (factor(i)*gamma(aa)*gamma(i+1.));
           sum=sum+sum2;
           }
return sum;
}


long double I24(double aa,double bb,double c, double e)
{          double sum=0.,sum2,u;  u=1./e;
           for (int i=0;i<8;i++)
           {sum2=gamma(aa+i)*pow(c,float(i))*pow(u,bb+float(i))
            *F21(i+bb,1.-aa,i+bb+1.,u) /(factor(i)*gamma(aa)*(bb+i));
           sum=sum+sum2;
           }
return sum;
}
long double I25(double aa,double bb,double c)
{          long double sum=0.,sum2,t;
           for (int i=0;i<7;i++)
           { sum2=gamma(aa+i) * pow(c,float(i))* beta(bb+i,aa)
            /(factor(i) * gamma(aa));
           sum=sum+sum2;
  }
return sum;
}

long double gamma(double x)
           {long double ga;double m1,z,m,r;          int k;
           double gr,G[26]={0.,1.,.5772156649015329,-.6558780715202538,
    -.0420026350340952,.1665386113822915,-.0421977345555443,
    -.0096219715278770,.0072189432466630,-.0011651675918591,
    -.0002152416741149,.0001280502823882,-.0000201348547807,
    -.0000012504934821,.0000011330272320,-.0000002056338417,
           .0000000061160950,.0000000050020075,-.0000000011812746,
           .0000000001043427,.0000000000077823,-.0000000000036968,
           .0000000000005100,-.0000000000000206,-.0000000000000054,
           .0000000000000014};
           if(x==floor(x))
            { if (x>0)
                     { ga=1.;m1=x-1.;
                     for(k=2;k<=m1;k++) ga=ga*k;
                     }
              else ga=1.0+300.0;
             }
           else
           {
             if (fabs(x)>1.0)
```

```
        { z=fabs(x);
            m=floor(z);
            r=1.0;
            for(k=1;k<=m;k++) r=r*(z-k);
            z=z-m;
        }
        else z=x;
        gr=G[25];
        for (k=24;k>=1;k--) gr=gr*z+G[k];
        ga=1.0/(gr*z);
        if(fabs(x)>1.)
        {ga=ga*r;
        if(x<0.) ga=-PAI/(x*ga*sin(PAI*x));}
    }
return ga;
}
```

double beta(float pp, float qq)

```
{       float xx,yy,zz;
        double tt;      xx=gamma(pp);     yy=gamma(qq);
zz=gamma(pp+qq);        tt=(xx*yy)/zz;
return tt;
}
```

double betain1(float a, float b, float x)

```
  {float dk[55],fk[55],ta,tb,t1=0.;
        double bix;
        float so=(a+1.)/(a+b+2.);
        double bt=beta(a,b);
        int k;
        if (x<=so)
         {for(k=1;k<=20;k++)
           dk[2*k]=k*(b-k)*x/(a+2.*k-1.)/(a+2.*k);
          for(k=0;k<=20;k++)
           dk[2*k+1]=-(a+k)*(a+b+k)*x/(a+2.*k)/(a+2.*k+1.);
          t1=0.;
          for (k=20;k>=1;k--) t1=dk[k]/(1.+t1);
          ta=1./(1.+t1);
          bix=pow(a,x)*pow((1.-x),b)/(a*bt)*ta;
          }
        else
         {
          for(k=1;k<=20;k++)
           fk[2*k]=k*(a-k)*(1.-x)/(b+2.*k-1.)/(b+2.*k);
          for(k=0;k<=20;k++)
           fk[2*k+1]=-(b+k)*(a+b+k)*(1.-x)/(b+2.*k)/(b+2.*k+1.);
          float t2=0.;
          for(k=20;k>=1;k--)
           t2=fk[k]/(1.+t2);
           tb=1./(1.+t2);
           bix=1.-pow(x,a)*pow((1.-x),b)/(b*bt)*tb;
          }
   float t2=bix*bt;
return t2;
}
```

double betain3 (float a, float b, float x)

```
{       double  sum=1.,t=1.,u; int k=0;
        for(k=0;k<20;k++)
```

```
        {t=pow(x,float(k+1.))*beta(a+1.,float(k+1.))/beta(a+b,float(k+1.));
        sum=sum+t; }
        u=sum*pow(x,a)*pow(1.-x,b)/a;
return u;
}
```

double ellip(float angle, float hk)

```
{       double phi;
        phi=angle*180/PAI;
        int i;
        double fe,d0,r,fac,ck,a,b,c,d,g=0.,a0=1.;
        double b0=sqrt(1.-hk*hk); d0=(PAI/180.)*phi; r=hk*hk;
        if(hk==1.&&phi==90.)  fe=1.+30000.;
        else
        { if (hk==1.) fe=log((1.+sin(d0))/cos(d0));
         else
          { fac=1.;
           for(i=1;i<=40;i++)
            { a=(a0+b0)/2.; b=sqrt(a0*b0); c=(a0-b0)/2.;
             fac=2.*fac; r=r+fac*c*c;
             if(phi<=90) {d=d0+atan((b0/a0))*tan(d0);g=g+c*sin(d);
             d0=d+PAI*floor(d/PAI+0.5);}
             a0=a;b0=b;
            if(c<0.000001) break;
             }
          ck=PAI/(2.*a); if (phi==90) fe=ck; else fe=d/(fac*a);
            }
        }
return fe;
}
```

double inbeta(float pp, float qq,float xx)

```
{       double betain1(float a, float b, float x);
         double betain3(float a, float b, float x);
        double inbeta;
        if (pp>=qq) inbeta=betain3(pp,qq,xx);
        else
          {if(xx<=.5) inbeta=betain3(pp,qq,xx);
           else inbeta=betain1(pp,qq,xx);
          }
return inbeta;
}
```