ANALYSIS OF SEEPAGE UNDER A DEPRESSED STEPPED WEIR WITH A SHEET PILE

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree

of

MASTER OF TECHNOLOGY

in

WATER RESOURCES DEVELOPMENT

By

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DECLARATION

I hereby declare that the dissertation titled **"Analysis of Seepage Under a Depressed Stepped Weir with a Sheet Pile"** which is being submitted for partial fulfillment of the requirements for the award of Master's of technology in **Water Resources Development (civil)** at Water Resources Development Training Center (WRDTC), Indian Institute of Technology, Roorkee is an authentic record of my own work carried out during the period of 16.07.2002 to 30.11.2002 under the supervision and guidance of **Dr. B.N.Asthana**, Professor Emeritus, WRDTC, IIT Roorkee and **Dr.G.C.Mishra.**, Professor ,WRDTC,IIT Roorkee.

I have not submitted the matter embodied in this dissertation previously for the award of any other degree.

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Place: Roorkee

Dated: 30.11.2002

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This is to certify that the above statement made by the candidate is correct to the best of my knowledge and belief.

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Place: Roorkee B.Shyam Sundar Patro.

Dated: 30.11.2002

Abstract

The movement of ground water is a basic part of soil mechanics. Its influence can be found in almost every area of civil engineering, including irrigation and reclamation. In addition, the elegance and logical structure of its theory renders it of interest to engineering scientists. It plays a vital role in Irrigation engineering for an irrigation engineer.

Since ancient times in irrigation engineering, weirs remain as the most extensively used control structures for the diversion of flow and for measurement of flow. Though the type and shape of weirs differ from place to place, depending on the available materials for construction, sub-soil condition and hydrology of the river, they are provided with one or more sheet piles when constructed in alluvial soils. Weirs are designed to satisfy the surface and sub-surface flow considerations. Where as the surface flow considerations decide the crest level, down stream floor length and minimum depths of upstream and downstream sheet-pile/cut-off, the sub-surface flow considerations at the maximum ponding condition requires more attention to protect the structure against heaving, roofing, piping and uplift. The parameters i.e. sheet-pile depth and floor length influence the uplift pressure at different points under the floor. The uplift pressures are counteracted by the weight of the floor. The weir generally consists of either a horizontal or sloping floor with sheet pile.

Khosla et.al. have analysed the flow under a stepped weir considering it to be resting on the surface of a porous medium of infinite depth. They have presented design charts, which are extensively used by the field engineers. Khosla's concept of barrage or weir design for subsurface flow (Khosla et.al.1936) is based on the assumption that the thickness of floor is negligible and it is resting on the surface, the values of uplift pressure thus obtained refer to the bottom level of the floor, where in practical, structures are somewhat depressed into, acting as foundation. To remove the error in pressure distribution

for neglecting floor thickness, a correction is being applied to the uplift pressure obtained according to Khosla's theory. This factor is being computed by interpolation assuming that there occurs a linear variation in the pressure along the depth of sheet-pile and the variation is equal to the variation in pressure distribution along the depth of depression. In fact, in order to achieve a tractable analytical solution, the depression of the hydraulic structure has been neglected. With such assumptions, the number of vertices taking part in the conformal transformation is reduced.

Hence, the present study was undertaken to analyse the flow under a depressed-stepped weir using the conformal mapping technique to compare the solution with that of Khosla et.al and to develop an analytical solution using numerical methods for computation of pressure distribution which can be directly used as the equation for anticipated uplift pressure and there will be no need of applying a correction factor.

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CONTENTS

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LIST OF FIGURES:

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List of Symbols:

b_1 = length of floor upstream of sheet-pile in m

b_2 = length of floor downstream of sheet-pile in m

C = constant of integration

d_1 = depression of floor upstream of sheet-pile in m

d_2 = depression of floor at junction of sheet-pile in m

d_3 = depression of floor downstream of sheet-pile in m

F_1 through F_5 : residues of non-linear equations

h = hydraulic head difference causing seepage in m

h_1 = water surface height upstream of weir in m

h_2 = tail water depth in m

i = imaginary unit ($\sqrt{-1}$)

I_1 through I_6 : integrals

k = coefficient of permeability

M = modulus of complex variable

M_1 = complex constant

N = complex constant

N_1 = complex constant

P = uplift pressure

p = pressure

S = depth of sheet-pile in m

t = transformation plane

u = streamline function

x = Cartesian horizontal co-ordinate

y = Cartesian vertical co-ordinate

z = complex variable (x + iy)

α_1 through α_7 : ordinates of vertices on t – plane

$\beta, \gamma, \lambda, m, \mu$: parameters

ϕ = velocity potential function

ϕ_d = velocity potential at downstream bed

ϕ_u = velocity potential at upstream bed

γ_w = unit weight of water gm/cc

η = parameter

λ_1 through λ_7 : parameters

ω = complex variable

ψ = stream function

ζ = parameter

Note : additional notations are defined locally wherever they occur.

Chapter 1

INTRODUCTION

1.1 General

Since ancient times in irrigation engineering, weirs remain as the most extensively used control structures for the diversion of flow and flow measurement. Though the types and shapes of weirs differ from place to place, depending on the available materials for construction, sub-soil condition and hydrology of the river, they are provided with one or more sheet piles when constructed in alluvial soils. Weirs are designed to satisfy the surface and sub-surface flow considerations. Where as the surface flow considerations decide the crest level, down stream floor length and minimum depths of upstream and downstream sheet-pile/cut-off, the sub-surface flow considerations at the maximum ponding condition require more attention to protect the structure against heaving, roofing, piping and uplift. The parameters i.e. sheet-pile depth and floor length influence the uplift pressure at different points under the floor. The uplift pressures are counteracted by the floor thickness. A weir generally consists of either a horizontal or sloped floor with sheet piles. The sheet-pile in the upstream is provided to reduce the uplift pressures under the floor and to cut-off the seepage-lines through permeable upper layers where as the provision of a down stream sheet-pile raises the uplift pressures under the floor. A down-stream sheet-pile is necessary from scour consideration as well as to keep the exit gradient below the safe limit. This helps in mitigating the piping below the floor. The depression of the floor can replace the need of a sheet pile to certain extent.

1.2 Back ground

The sub-soil flow below weirs along with the hydraulic gradients and uplift-pressures has been widely recognised as the determining factor in design of a weir on permeable foundation after the classic experiments that has been carried out by Col.Clibborns, the then Principal of Thomson Civil Engineering College, Roorkee in connection with the failure of Khanki Weir, in India during 1895-97. It was then concluded and accepted eventually by all over that the subject of sub-surface flow is more complex than what the Bligh's creep theory indicated then.

In 1934 Rai Bahadur A.N.Khosla,ISE presented a note on the observations and records of pressures below works on permeable foundations in publication No.8 of Central Board of Irrigation and Power.

Khosla et.al have analysed the flow under a stepped weir considering it to be resting on the surface of a porous medium of infinite depth. They have presented design charts, which are extensively used by the field engineers.

1.3 Need for further studies

As Khosla's concept of barrage or weir design for subsurface flow (Khosla et.al.1936)[1] is based on the assumption that the thickness of floor is negligible and it is resting on the surface, the values of uplift pressure thus obtained refer to the bottom level of the floor, where in practice; structures are somewhat depressed into, acting as foundation. In fact, in order to achieve a tractable analytical solution, the depression of the hydraulic structure has been neglected. With such assumptions, four extra vertices, which should take part in the conformal transformation, are reduced and some part of the seepage head is lost through the foundation depth. To remove the difference due to floor thickness, a correction factor is applied to the uplift pressure obtained from Khosla's equation. This factor is being computed by interpolation assuming that, there occurs a linear variation in the pressure along the sheet-pile length.

1.4 Scope of present study

The present study was undertaken to analyse the flow under a depressed stepped weir, using the conformal mapping technique to compare the solution with that of Khosla et.al. The results so obtained can be directly used as the anticipated uplift pressure and there will be no need of applying a correction factor.

1.5 Objectives of Present Study

Present study was undertaken to find an analytical solution which can quantify uplift pressure below the floor of depressed weir and to prepare a comprehensive comparison of the values of uplift pressure with that obtained, by using the equation of Khosla et.al.(1936). The comparison is to be carried for weirs with depression and with a sheet-pile at various positions.

It is proposed to compare for the following depressed hydraulic structures:

I. Depressed weir with sheet-pile positioned at various options. (**Figure 1.1**)

II. Depressed-stepped weir without sheet-pile. (**Figure 1.2**)

III. Depressed-stepped weir with a sheet-pile at the step. (**Figure 1.3**)

Use of conformal mapping technique generally results in non-linear equations containing multivariable. The non-linear equations are not easily solvable. It is proposed to solve the set of non-linear equations by Newton-Raphson technique[2]. The uplift pressure distribution and exit gradients are then determined.

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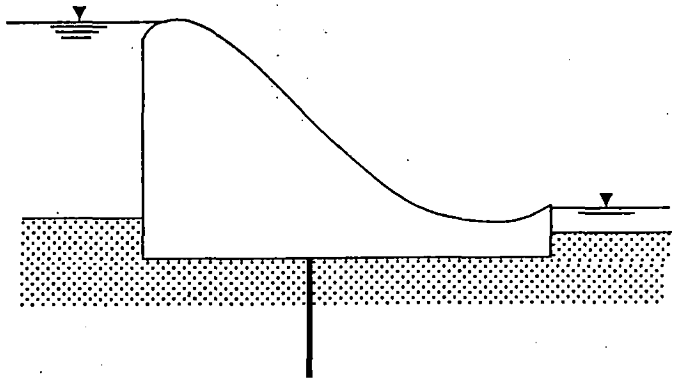

Figure 1.1 Case I. Depressed Floor with a Sheet Pile.

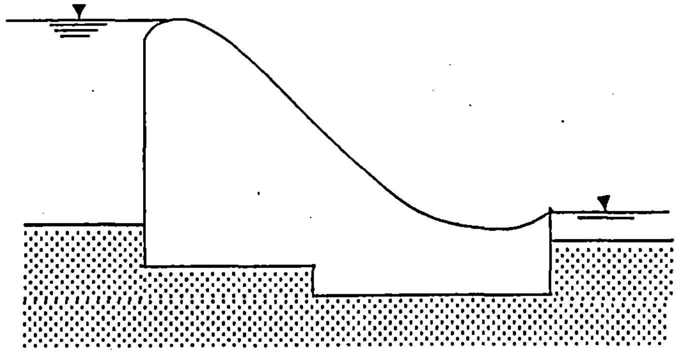

Figure 1.2 Case II. Depressed Stepped Floor without Pile

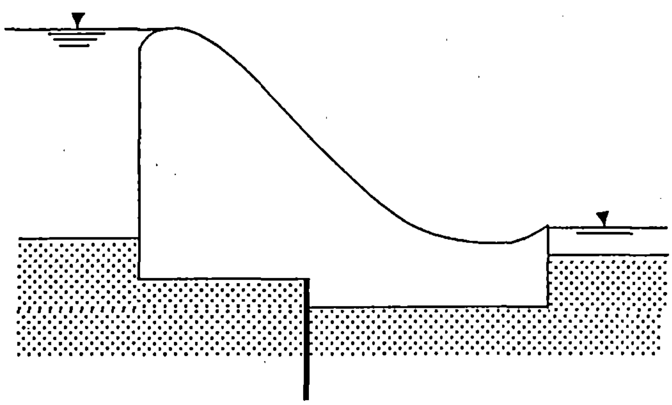

Figure 1.3 Case III. Depressed Stepped Floor with Pile at the Step

Chapter 2

LITERATURE REVIEW

2.1 General

Khosla et.al. (1936)[1] found solutions to two-dimensional steady flow under a number of simple profiles of weirs resting on a homogeneous and isotropic soil of infinite depth using the Scwarz-Christoffel conformal transformation technique[3]. Pressure heads; at key points (C, D, and E as shown in Figure.2.1) in excess of the hydrostatic head at the downstream boundary have been presented as a percentage of the seepage head in the form of charts, which are widely in use for the sub surface design of hydraulic structure. Khosla et.al. have neglected the depth of depression to reduce the number of vertices taking part in the conformal mapping. By reducing the number of vertices it was possible to carryout the integration required in solving the transformation. Numerical integration is necessary in case of structures having vertices more than three.

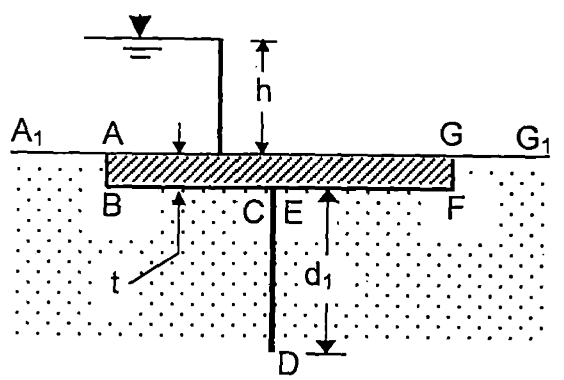

Figure. 2.1

2.2 Approximate Method for Accounting Depression:

In Khosla's method of analysis, the excess pressure head has been derived, assuming that the thickness of floor is negligible and the structure is resting on the surface. As the foundation has some thickness, a part of the seepage head is lost along the foundation depth, which has to be accounted for.

To account for the head lost along the floor thickness, Khosla et.al. has suggested a correction. This is being computed by interpolation under the assumption that, the variation of hydraulic head is linear along the sheet-pile depth and the rate of variation is equal to the variation along the depth of depression. The correction for accounting depression for a flat-based weir proposed by them is as follows:

The correction for pressure head[4] at point C in Figure.2.1 is $\left(\frac{\phi_C - \phi_D}{d_1}\right)t_{min}$ which is subtracted from the value of ϕ_C. The correction for pressure head at the point 'E' is $\left(\frac{\phi_D - \phi_E}{d_1}\right)t_{min}$ which is added to the value of ϕ_E, where ϕ_C, ϕ_D and ϕ_E are the pressure heads at points C, D and E respectively which have been obtained by neglecting the depression and using conformal mapping.

It may be noted here that the nature of dissipation of head along the depth of depression and sheet-pile are not similar. Because, at point A, the flow velocity is finite, where as, at point C the velocity is zero. Therefore, the corrections proposed by Khosla needs an investgation.

In the present scientifically developed era, there is an advantage to the present day researchers which the yester decades researchers did not have. Now a days, it is possible to carryout numerical integration and solve non-linear equations easily using high speed computers. So instead of applying a correction factor as proposed by Khosla, in this thesis, a solution has been given accounting floor thickness below the ground level for direct computation of the uplift pressure.

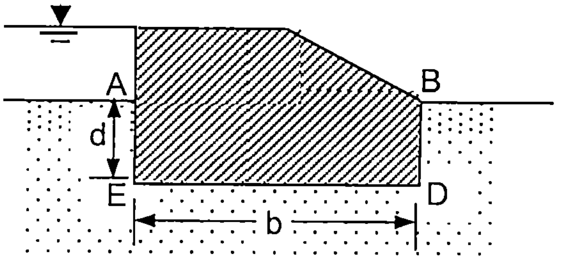

Figure. 2.2

Khosla has also suggested an emprical formula[1] for computation of uplift pressure under a flat bottom depressed weir, the type shown in Fig.2.2. The formula is based on tests conducted on a scale model. The empirical formula is

$$\phi_D' = \phi_D - \frac{2}{3}(\phi_C - \phi_D) + \frac{3}{\alpha^2}$$

in which ϕ_D and ϕ_C are pressures at D and C corresponding to figure.2.1 for which Khosla et.al. have given analytical solution.The parameter α is equal to b/d. ϕ'_D is the pressure at point D in figure 2.2.

Using the conformal mapping technique, Malhotra (1962)[1] has given solution for flow under a depressed hydraulic structure having two sheet-piles one at each end.

Safety against piping for depressed structure can be investigated using Lane's weighted creep theory (Lane,1935)[5].

However no analytical solution are available for stepped-depressed weir.

2.3 Analytical Method for Accounting Depression:

Pavlovsky (1922)[6] has given solution to a flat bottomed depressed weir using Scwartz-christoffel transformation. Analytical solutions for the uplift pressure under the floor and the maximum exit gradient have been given.

Confomal mapping technique has been applied to compute uplift pressure and exit gradient for a flat depressed structure with two symmetrical row of piling on a permeable soil of infinite depth (Harr,1962)[3]. The solution has been given for structure on foundation of finite depth by Filchakov (Polubarinova-Kochina,1962)[5]. The analytical solution is not tractable as it contains elliptic integral of third kind[6,7].

2.4 Conclusion

Analytical solution for a stepped-depressed weir is not available. In current practice corrections are applied to the solution that has been obtained neglecting depression. Analytical solution for flat bottomed depressed floor resting on a soil of finite depth is available. However uplift pressure, exit gradient can not be computed easily as the derived equations are highly non-linear and contain elliptic integral of third kind. Solution to flow under structure having vertices more than three can be obtained using conformal mapping and applying Newton-Raphson technique for solving the non-linear equation.

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ANALYSIS

3.1 General

Weirs on permeable foundation are designed to safeguard against uplift pressure and piping. The flow characteristics are determined assuming the flow to be two dimensional and steady. For non-homogeneous sub-soil, numerical method is used to solve the two dimensional equation

$$\frac{\partial}{\partial x}\left\{-k(x,y)\frac{\partial h}{\partial x}\right\}+\frac{\partial}{\partial y}\left\{-k(x,y)\frac{\partial h}{\partial y}\right\}=0$$

satisfying the boundary conditions. For homogeneous, isotropic soil, the governing equation is the Laplace equation, which can be solved analytically using conformal mapping.

Using the Scwartz-Christoffel conformal mapping technique, Khosla et.al. (1936) have obtained analytical solutions for a stepped weir with a sheet pile provided at the step, resting on a homogeneous, isotropic porous medium of infinite depth. They have neglected the depression so as to reduce the number of vertices to arrive at a simple solution and suggested a correction factor to account for the depression. In this thesis, an analytical solution for the flow around a depressed-stepped weir with a sheet-pile at the step has been obtained using the Schwartz-Christoffel conformal mapping technique.

3.2 Statement of the Problem

The depressed-stepped weir with a sheet-pile at the step is shown in **Figure.3.1 (a)**. The width of floor upstream to the sheet-pile is 'b_1'. The width of down stream floor is 'b_2'. The depth of the vertical sheet-pile is 's'. The depths of depression of the floor at the upstream floor, at the junction of sheet-pile and floor, and at the down stream floor are d_1, d_2, and d_3 respectively. The heights of water above the upstream and downstream bed are h_1 and h_2 respectively and the difference in the total heads between the upstream and downstream

z-plane

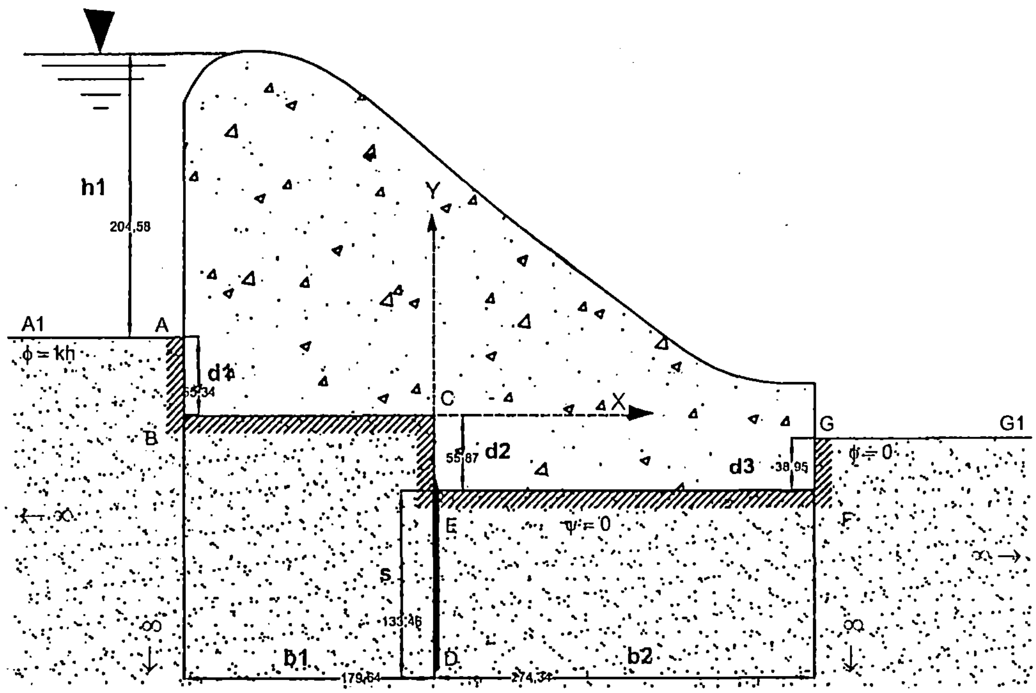

Figure. 3.1(a) : Physical Domain in z-plane

t-plane

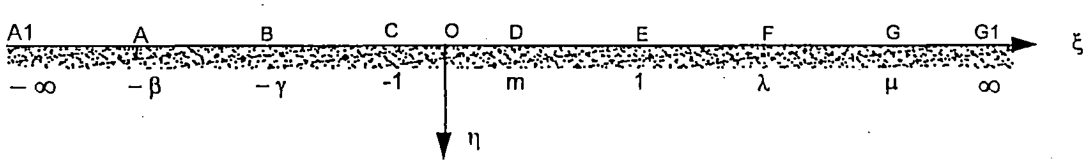

Figure.3.1(b) : Physical Domain Mapped onto t-plane

boundaries is 'h'. It is required to find the pressure distribution along the impervious base BCDEF of the structure and exit gradient along the downstream boundary GG_1.

3.3 Analysis

3.3.1 Mapping of z- plane onto t- plane: z = f (t)

The conformal mapping of the flow domain in z-plane onto the lower half of the auxiliary t-plane is given by:

$$Z = M\int \frac{(m - t)\sqrt{(t + \gamma)(\lambda - t)}}{\sqrt{(t + \beta)(1 - t^2)(\mu - t)}}dt + N \qquad (3.3.1)$$

the vertices A_1, A, B, C, D, E, F, G, G_1 being mapped onto $-\infty$, β, γ, -1, m, 1, λ, μ and ∞ respectively in the t-plane. M and N are complex constants to be determined. The constant N is governed by the lower limit of integration. To find the constants M and N, and the relationship between the transformation parameters and dimension of the structure we carryout integration between consecutive vertices.

i). Integration between vertices **D** and **E** $(m \leq t \leq 1)$

Applying the conditions

at point D: $t = m$, for which $z = -i(s + d_2)$ and

at point E: $t = 1$, for which $z = -id_2$

we obtain:

$$-id_2 = M\int_m^1 \frac{(t - m)\sqrt{(t + \gamma)(\lambda - t)}}{\sqrt{(t + \beta)(1 - t^2)(\mu - t)}}dt - i(s + d_2)$$

or $$-id_2 = M\int_m^1 f_1(t)dt - i(s + d_2)$$

or $$is = M\,I_1 \quad \text{where } I_1 = \int_m^1 f_1(t)dt$$

or $$M = \frac{is}{I_1} \qquad (3.3.2)$$

ii). Integration between vertices **E** and **F** $(1 \le t \le \lambda)$

Applying the conditions

at point E: $t = 1$, for which $z = -id_2$ and

at point F: $t = \lambda$, for which $z = b_2 - id_2$

we obtain:

$$b_2 - id_2 = M\int_1^{\lambda} \frac{(t-m)\sqrt{(t+\gamma)(\lambda - t)}}{\sqrt{(t+\beta)(t^2-1)(\mu - t)}} \frac{dt}{\sqrt{-1}} - id_2$$

or $$b_2 = M\frac{I_2}{\sqrt{-1}} = \frac{is}{I_1}\cdot\frac{I_2}{i}$$

or $$\frac{b_2}{s} = \frac{I_2}{I_1}$$

or $$\mathbf{F_1} = \frac{\mathbf{b_2}}{\mathbf{s}} - \frac{\mathbf{I_2}}{\mathbf{I_1}} \qquad (3.3.3)$$

iii). Integration between vertices **F** and **G** $(\lambda \le t \le \mu)$

Applying the conditions

at point F: $t = \lambda$, for which $z = b_2 - id_2$ and

at point G: $t = \mu$, for which $z = b_2 - i(d_2 - d_3)$

we obtain

$$b_2 - i(d_2 - d_3) = M\int_{\lambda}^{\mu} \frac{(t-m)\sqrt{(t+\gamma)(t-\lambda)}}{\sqrt{(t+\beta)(t^2-1)(\mu - t)}} dt + b_2 - id_2$$

or $$b_2 - i(d_2 - d_3) = M I_3 + b_2 - id_2$$

or $$\frac{d_3}{s} = \frac{I_3}{I_1}$$

or $$\mathbf{F_2} = \frac{\mathbf{d_3}}{\mathbf{s}} - \frac{\mathbf{I_3}}{\mathbf{I_1}} \qquad (3.3.4)$$

iv). Integration between vertices **D** and **C** $(-1 \le t \le m)$

Applying the conditions

at point D: $t = m$, for which $z = -i(s + d_2)$ and

at point C: $t = -1$, for which $z = 0$

we obtain:

$$i(d_2 + s) = M \int_{-1}^{m} \frac{(m - t)\sqrt{(t + \gamma)(\lambda - t)}}{\sqrt{(t + \beta)(1 - t^2)(\mu - t)}} dt + 0$$

for integration with values of $t \leq 0$, we replace t with $-T$; then the above equation becomes:

or $$i(d_2 + s) = M \int_{m}^{1} \frac{(m + T)\sqrt{(\gamma - T)(\lambda + T)}}{\sqrt{(\beta - T)(1 - T^2)(\mu + T)}} dT$$

$$i(d_2 + s) = MI_4 = \frac{is}{I_1} I_4$$

or $$\frac{d_2 + s}{s} = \frac{I_4}{I_1}$$

or $$\mathbf{F_3 = \frac{d_2 + s}{s} - \frac{I_4}{I_1}} \qquad (3.3.5)$$

v). Integration between vertices **C** and **B** $(-\gamma \leq t \leq -1)$

Applying the conditions

at point C: $t = -1$, for which $z = 0$ and

at point B: $t = -\gamma$, for which $z = -b_1$

we obtain:

$$0 = M \int_{-\gamma}^{-1} \frac{(m - t)\sqrt{(t + \gamma)(\lambda - t)}}{\sqrt{(t + \beta)(1 - t^2)(\mu - t)}} dt - b_1$$

Replacing t with -T;

$$0 = M \int_{1}^{\gamma} \frac{(m + T)\sqrt{(\gamma - T)(\lambda + T)}}{\sqrt{(\beta - T)(T^2 - 1)(\mu + T)}} \frac{dT}{i} - b_1$$

or $$0 = \frac{MI_5}{i} - b_1 = \frac{is}{I_1} \cdot \frac{I_5}{i} - b_1$$

or $$\frac{b_1}{s} = \frac{I_5}{I_1}$$

or $$\mathbf{F_4 = \frac{b_1}{s} - \frac{I_5}{I_1}} \qquad (3.3.6)$$

vi). Integration between vertices **B** and **A** $(-\beta \leq t \leq -\gamma)$

Applying the conditions

at point B: $t = -\gamma$, for which $z = -b_1$ and

at point A: $t = -\beta$, for which $z = -b_1 + id_1$

we obtain :

$$-b_1 = M\int_{-\beta}^{-\gamma} \frac{(m-t)\sqrt{(t+\gamma)(\lambda-t)}}{\sqrt{(t+\beta)(1-t^2)(\mu-t)}}\,dt - b_1 + id_1$$

Replacing t with -T;

$$0 = M\int_{\gamma}^{\beta} \frac{(m+T)\sqrt{(\gamma-T)(\lambda+T)}}{\sqrt{(\beta-T)(1-T^2)(\mu+T)}}\,dT + id_1$$

$$id_1 = M\int_{\beta}^{\gamma} \frac{(m+T)\sqrt{(T-\gamma)(\lambda+T)}}{\sqrt{(\beta-T)(T^2-1)(\mu+T)}}\,dT$$

or $\quad id_1 = MI_6 = \frac{is}{I_1}.I_6 \qquad$ or $\quad \frac{d_1}{s} = \frac{I_6}{I_1}$

or $$\mathbf{F_5} = \frac{\mathbf{d_1}}{\mathbf{s}} - \frac{\mathbf{I_6}}{\mathbf{I_1}} \qquad (3.3.7)$$

The parameters β, γ, m, λ and μ are to be found for known values of $\frac{d_1}{s}, \frac{d_3}{s}, \frac{d_2+s}{s}, \frac{b_1}{s}, \frac{b_2}{s}$ from Eqs. (3.3.3) to (3.3.7). The equations are non-linear. Newton-Raphson technique has been used to find the solution and this has been explained in Appendix-II. The solution is given by the Jacobian matrix:

$$\begin{bmatrix} \frac{\partial F_1^*}{\partial\beta} & \frac{\partial F_1^*}{\partial\gamma} & \frac{\partial F_1^*}{\partial m} & \frac{\partial F_1^*}{\partial\lambda} & \frac{\partial F_1^*}{\partial\mu} \\ \frac{\partial F_2^*}{\partial\beta} & \frac{\partial F_2^*}{\partial\gamma} & \frac{\partial F_2^*}{\partial m} & \frac{\partial F_2^*}{\partial\lambda} & \frac{\partial F_2^*}{\partial\mu} \\ \frac{\partial F_3^*}{\partial\beta} & \frac{\partial F_3^*}{\partial\gamma} & \frac{\partial F_3^*}{\partial m} & \frac{\partial F_3^*}{\partial\lambda} & \frac{\partial F_3^*}{\partial\mu} \\ \frac{\partial F_4^*}{\partial\beta} & \frac{\partial F_4^*}{\partial\gamma} & \frac{\partial F_4^*}{\partial m} & \frac{\partial F_4^*}{\partial\lambda} & \frac{\partial F_4^*}{\partial\mu} \\ \frac{\partial F_5^*}{\partial\beta} & \frac{\partial F_5^*}{\partial\gamma} & \frac{\partial F_5^*}{\partial m} & \frac{\partial F_5^*}{\partial\lambda} & \frac{\partial F_5^*}{\partial\mu} \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta\gamma \\ \Delta m \\ \Delta\lambda \\ \Delta\mu \end{bmatrix} = \begin{bmatrix} F_1(\beta^*,\gamma^*,m^*,\lambda^*,\mu^*) \\ F_2(\beta^*,\gamma^*,m^*,\lambda^*,\mu^*) \\ F_3(\beta^*,\gamma^*,m^*,\lambda^*,\mu^*) \\ F_4(\beta^*,\gamma^*,m^*,\lambda^*,\mu^*) \\ F_5(\beta^*,\gamma^*,m^*,\lambda^*,\mu^*) \end{bmatrix}$$

The integrals are improper and the singularities have been removed by using the Gaussian-Quadrature method of substitution. The Solution of the Jacobian matrix is done by using FORTRAN program. The FORTRAN program has been listed in Appendix-III.

3.3.2 Mapping of ω - plane onto t - plane:

The complex potential ω is defined as $\omega = \phi + i\psi$ in which

ϕ = velocity potential and ψ = stream function.

The velocity potential function ϕ is defined as $\phi = -k\left(\frac{p}{\gamma_w} + y\right) + C$ (3.3.8)

where C is a constant.

The ω-plane for the flow domain of Figure.3.1(a) is shown in Figure.3.1(c). If $C = k\,(h_2 - d_2 + d_3)$, then the velocity potential at down-stream bed $\phi_d = 0$, and the velocity potential at up-stream bed $\phi_u = -kh$

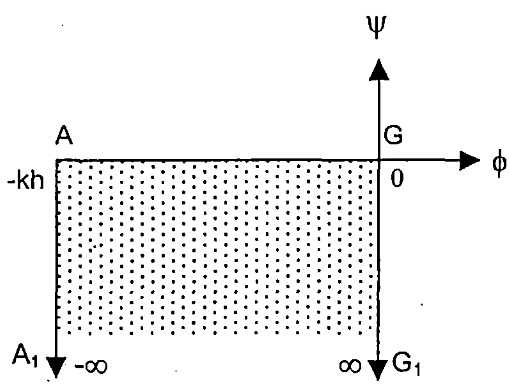

Figure .3.1(c) ω - plane

Mapping of the complex potential plane onto the t –plane is given by

$$\omega = M_1 \int (t+\beta)^{-0.5}(\mu - t)^{-0.5}\,dt + N_1 \quad (3.3.9)$$

or
$$\omega = M_1.\mathrm{Sin}^{-1}\left(\frac{2t+\beta-\mu}{\beta+\mu}\right) + N_1 \quad (3.3.10)$$

where M_1 and N_1 are complex constants.

Applying the conditions at the points A, G in ω -plane and t -plane, the constants M_1, N_1 is computed as under:

i. At point G ($\omega = 0, t = \mu$)

Substituting ω and t in the Eq. (3.3.10) we have:

$$0 = M_1 \text{Sin}^{-1}\left(\frac{2\mu + \beta - \mu}{\beta + \mu}\right) + N_1$$

$$= M_1 \text{Sin}^{-1}(1.0) + N_1$$

$$= M_1 \frac{\pi}{2} + N_1$$

or $N_1 = -M_1 \frac{\pi}{2}$

ii. At point A ($\omega = -kh,\ t = -\beta$)

Substituting ω and t in Eq. (3.3.10) we have:

$$-kh = M_1 \text{Sin}^{-1}\left(\frac{-2\beta + \beta - \mu}{\beta + \mu}\right) - M_1 \frac{\pi}{2}$$

$$= -M_1 \frac{\pi}{2} - M_1 \frac{\pi}{2}$$

or $\mathbf{M_1 = \frac{kh}{\pi}}$ (3.3.11)

and $\mathbf{N_1 = -\frac{Kh}{2}}$ (3.3.12)

Putting the value of M_1 and N_1 in Eq. (3.3.10) we have;

$$\omega = \frac{kh}{\pi} \text{Sin}^{-1}\left(\frac{2t + \beta - \mu}{\beta + \mu}\right) - \frac{kh}{2} \quad (3.3.13)$$

The Eq. (3.3.13) is the general equation, which provides relation between ω-plane and t-plane. For boundary BCDEF, $\psi = 0$ and hence;

$\omega = \phi$, so the Eq. (3.3.13) now becomes

$$\phi = \frac{kh}{2}\left\{\frac{2}{\pi} \text{Sin}^{-1}\left(\frac{2t + \beta - \mu}{\beta + \mu}\right) - 1\right\} \quad (3.3.14)$$

Now equating the value of ϕ from Eq. (3.3.8),we have;

$$-k\left(\frac{p}{\gamma_w}+y\right)+k(h_2-d_2+d_3)=\frac{kh}{2}\left\{\frac{2}{\pi}\text{Sin}^{-1}\left(\frac{2t+\beta-\mu}{\beta+\mu}\right)-1\right\}$$

which yields,

$$\mathbf{p}=\gamma_w\left[\mathbf{h_2-d_2+d_3-y-\frac{h}{2}\left\{\frac{2}{\pi}Sin^{-1}\left(\frac{2t+\beta-\mu}{\beta+\mu}\right)-1\right\}}\right] \quad (3.3.15)$$

3.4 The Pressure Distribution

Eq. (3.3.15) is the general equation for seepage pressure under the floor. To find the pressure at various points below the floor, the ordinate of 'y' from z-plane and the corresponding 't' from t-plane has to be entered in Eq. (3.3.15):

i. At point **A** ($y = d_1$, $t = -\beta$)

$$\mathbf{P_A} = \gamma_w.h_1 \quad (3.3.16)$$

ii. At point **B** ($y = 0$, $t = -\gamma$)

$$P_B=\gamma_w\left[h_2-d_2+d_3-\frac{h}{2}\left\{\frac{2}{\pi}\text{Sin}^{-1}\left(\frac{-2\gamma+\beta-\mu}{\beta+\mu}\right)-1\right\}\right] \quad (3.3.17)$$

iii. At point **C** ($y = 0$, $t = -1$)

$$P_C=\gamma_w\left[h_2-d_2+d_3-\frac{h}{2}\left\{\frac{2}{\pi}\text{Sin}^{-1}\left(\frac{\beta-\mu-2}{\beta+\mu}\right)-1\right\}\right] \quad (3.3.18)$$

iv. At point **C** ($y = -(d_2+s)$, $t = m$)

$$P_D=\gamma_w\left[h_2+d_3+s-\frac{h}{2}\left\{\frac{2}{\pi}\text{Sin}^{-1}\left(\frac{\beta-\mu-2}{\beta+\mu}\right)-1\right\}\right] \quad (3.3.19)$$

v. At point **E** ($y = -d_2$, $t = 1$)

$$P_E=\gamma_w\left[h_2+d_3-\frac{h}{2}\left\{\frac{2}{\pi}\text{Sin}^{-1}\left(\frac{2+\beta-\mu}{\beta+\mu}\right)-1\right\}\right] \quad (3.3.20)$$

vi. At point **F** ($y = -d_2$, $t = \lambda$)

$$P_F=\gamma_w\left[h_2+d_3-\frac{h}{2}\left\{\frac{2}{\pi}\text{Sin}^{-1}\left(\frac{2\lambda+\beta-\mu}{\beta+\mu}\right)-1\right\}\right] \quad (3.3.21)$$

vii. At point **G** $(y = -(d_2 - d_3), t = \mu)$

$$P_G = \gamma_w h_2 \qquad (3.3.22)$$

3.5 The Exit Gradient

The exit gradient (Harr 1962) can be expressed as:

$$I_E = \frac{i}{k}\left(\frac{dw}{dt}\cdot\frac{dt}{dz}\right)$$

Using Eqs. (3.3.1),(3.3.2) and (3.3.13) in the above equation we have;

$$I_E = \frac{hl_1}{\pi s}\left\{\frac{\sqrt{(t^2-1)}}{(t-m)\sqrt{(t+\gamma)(t-\lambda)}}\right\}$$

Maximum exit gradient occurs at 'G', where $t = \mu$;

$$I_{Emax} = \frac{hl_1}{\pi s}\left\{\frac{\sqrt{(\mu^2-1)}}{(\mu-m)\sqrt{(\mu+\gamma)(\mu-\lambda)}}\right\} \qquad (3.3.23)$$

3.6 Results and Discussions

Numerical results for pressure distribution and exit gradient are obtained for the following structures:

i. flat based weir with a sheet-pile resting on the surface

ii. stepped weir with a sheet-pile at the step resting on the surface

iii. depressed weir with a sheet-pile

iv. depressed-stepped weir with a sheet-pile at the step

Case iv being the general one, the results for other cases can be obtained by manipulating the structure parameters appearing in the solution of case iv.

The results of case I and case II obtained using the present method are compared with the analytical solution that have been given by Khosla.et.al.

The comparison is given in Table3.1.(a) and Table3.1.(b) and shown graphically in Figs.3.6.(a) to Fig: 3.6.(e). The present numerical method is free from error. The Newton-Raphson method therefore can be used conveniently in solving the non-linear equations appearing in conformal mapping technique, which involves more than three variables.

The correction suggested by Khosla.et.al for accounting depression is checked for its accuracy by comparing the pressure at key points computed by the present method with those obtained using the method given by Khosla.et.al. The comparison is given in Table3.2.(a) through Table3.2.(b) and shown graphically in Figs.3.7 (a) through Fig: 3.7(d). The pressure distribution obtained by the approximate method suggested by Khosla.et.al differs by 5.5% -8.4% from the results obtained by present rigorous method for the ratio, depth of depression to total base width of weir (d/b) =0.125 and s/b=0.5 for flat based depressed weir. For depressed stepped weir the deviation is of the order of 3.8% -34.5% in the down stream side of the sheet-pile and 2.8% - 3.9% in upstream side for the ratio d/b_1= 0.25 and s/b_1 = 1.0 and d/b=0.125 to 1.0.

The variation of exit gradient has been shown in Figure 3.6(f). The variation of maximum exit gradient with the ratio, depth of depression to base width of weir (d/b) varies rapidly with the decrease in the ratio of d/b.

Khosla's approximate method predicts higher value of pressure distribution for points down streamside and lower value for upstream side of sheet-pile or step as compared to the present method.

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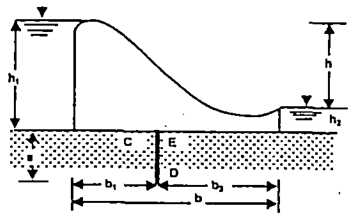
Comparison of Khosla's Approximate Method for flat based weir with Present Exact Analytical Solution.

Table No: 3.1(a)

Excess Hydrostatic Pressure $\delta p/(\gamma_\omega h)$ in % of h at key points.



h1= 3
h2= 1
S = 4
b = 8
d1= 0
d2= 0
d3= 0

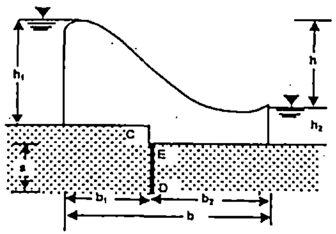
Case I (A) : Flat Base Weir with a Sheet-pile with no Depression

Weir Parameters				As per approximate Khosla's Equation		As per present exact solution		Deviation in % from present solution	
$\alpha = b/s$	b1	b2	b1/b	C	E	C	E	C	E
2	0.00	8.00	0.00	100.00	42.41	99.99	42.42	0.00	0.00
	1.00	7.00	0.13	93.59	39.19	93.58	39.20	0.00	0.00
	2.00	6.00	0.25	87.11	35.13	87.10	35.14	0.00	0.00
	3.00	5.00	0.38	80.86	30.36	80.85	30.37	0.00	0.00
	4.00	4.00	0.50	75.00	25.00	74.99	25.01	0.00	0.00
	5.00	3.00	0.63	69.64	19.14	69.63	19.15	0.00	0.00
	6.00	2.00	0.75	64.87	12.89	64.86	12.90	0.00	0.00
	7.00	1.00	0.88	60.81	6.41	60.80	6.42	0.00	0.00
	8.00	0.00	1.00	57.59	0.00	57.58	0.00	0.00	0.00

Comparison of Khosla's Approximate Method for stepped weir with Present Exact Analytical Solution.

Table No: 3.1(b)

Excess Hydrostatic Pressure $\delta p/(\gamma_\omega h)$ in % of h at key points.



h1= 3
h2= 1
S = 4
b1= 4
d1= 0
d2= 1
d3= 0

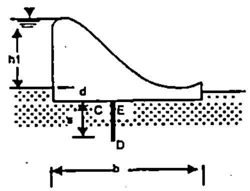
Case III (A): Stepped Weir with a Sheet-pile without Depression

Weir Parameters		$\delta p_C/(\gamma_\omega h)$			$\delta p_D/(\gamma_\omega h)$			$\delta p_E/(\gamma_\omega h)$		
		As per approximate Khosla's Equation		Present exact solution	As per approximate Khosla's Equation		Present exact solution	As per approximate Khosla's Equation		Present exact solution
b_2	b_2/b_1	Expmt.	Theort.		Expmt.	Theort.		Expmt.	Theort.	
0	0.00	77.00	76.30	76.33	43.40	43.80	43.75	0.00	0.00	0.00
1	0.25	77.10	76.50	76.52	44.00	44.30	44.28	6.80	6.80	6.76
2	0.50	77.40	76.90	76.88	45.60	45.60	45.55	12.40	13.70	13.62
3	0.75	78.20	77.40	77.46	47.10	47.30	47.30	18.70	19.40	19.48
4	1.00	78.60	78.10	78.13	48.90	49.20	49.24	24.20	24.60	24.57
5	1.25	79.30	78.80	78.84	50.50	51.20	51.15	27.90	28.90	28.66
6	1.50	79.90	79.50	79.49	53.30	53.10	53.05	31.90	32.70	32.74
7	1.75	80.70	80.00	80.14	55.40	54.80	54.79	37.10	36.00	36.02
8	2.00	81.70	80.80	80.75	56.50	56.40	56.40	39.40	38.90	38.90

Comparison of Khosla's Approximate Method for depressed weir with exact Analytical Solution.

Table No: **3.2(a)**

Excess Hydrostatic Pressure $\delta p/(\gamma_\omega h)$ in % of h at key points.



h1=	2
h2=	0
S =	4
b =	8
d1=	1
d2=	0
d3=	1

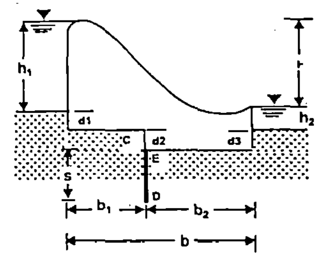
Case I (B): Flat Base Weir with a sheet-pile with Depression

Weir Parameters				As per Khosla's approximate equation for no depression			As per Khosla's correction for Deprn.		As per present exact solution		Deviation in % from present solution	
$\alpha = b/s$	b_1	b_2	b_1/b	C	D	E	C	E	C	E	C	E
2	0	8	0.000	100.00	62.48	42.41	90.62	47.43	**95.93**	**45.14**	5.54	-5.07
	1	7	0.125	93.59	60.48	39.19	85.31	44.51	**89.74**	**42.40**	4.94	-4.98
	2	6	0.250	87.11	57.53	35.13	79.72	40.73	**83.76**	**38.73**	4.82	-5.16
	3	5	0.375	80.86	53.93	30.36	74.13	36.32	**78.25**	**34.52**	5.27	-5.21
	4	4	0.500	75.00	50.00	25.00	68.75	31.25	**73.15**	**29.86**	6.02	-4.66
	5	3	0.625	69.64	46.07	19.14	63.75	25.87	**68.48**	**24.76**	6.91	-4.48
	6	2	0.750	64.87	42.47	12.89	59.27	20.28	**64.27**	**19.24**	7.78	-5.41
	7	1	0.875	60.81	39.52	6.41	55.49	14.69	**60.60**	**13.26**	8.43	-10.78
	8	0	1.000	57.59	37.52	0.00	52.57	9.38	**57.39**	**7.10**	8.40	-32.11

Comparison of Khosla's Approximate Method for depressed stepped weir with exact Analytical Solution.

Table No: 3.3

Excess Hydrostatic Pressure $\delta p/(\gamma_\omega h)$ in % of h at key points.



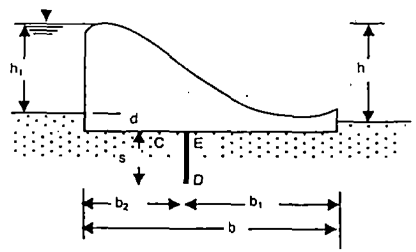
h1= 3
h2= 1
S = 4
b1= 4
d1= 1
d2= 1
d3= 1

Case III (B): Depressed-Stepped Weir with a sheet-pile

Weir Parameters		$\delta p_C/(\gamma_\omega h)$				$\delta p_D/(\gamma_\omega h)$	$\delta p_E/(\gamma_\omega h)$			
		As per approximate Khosla's Equation		As per Present exact solution	Deviation in % from present solution	As per Khosla	As per approximate Khosla's Equation		As per Present exact solution	Deviation in % from present solution
b_2	b_2/b_1	Theort. values	Corrected for deprn.			Theort. values	Theort. values	Corrected for deprn.		
0	0.00	76.30	69.8	72.52	3.89	43.80	0.00	10.95	7.16	34.57
1	0.25	76.50	70.06	73.23	4.52	44.30	6.80	16.18	14.37	11.19
2	0.50	76.90	70.64	73.80	4.47	45.60	13.70	21.68	20.24	6.64
3	0.75	77.10	71.14	74.49	4.71	47.30	19.40	26.38	25.24	4.32
4	1.00	78.10	72.32	75.24	4.04	49.20	24.60	30.75	29.52	4.00
5	1.25	78.80	73.28	75.99	3.70	51.20	28.90	34.48	33.20	3.71
6	1.50	79.50	74.22	76.71	3.35	53.10	32.70	37.80	36.39	3.73
7	1.75	80.00	74.96	77.40	3.26	54.80	36.00	40.70	39.17	3.76
8	2.00	80.80	75.92	78.05	2.81	56.40	38.90	43.28	41.63	3.81

Table No.3.4

Variation of Exit-gradient with the ratio of Depth of depression to Base width of weir (d/b) for a Flat Based Weir with a sheet pile.



$h_1 = 3$

$h_2 = 1$

$b = 8$

$b_1 = 2, 7.5$

$b_2 = 6, 0.5$

Depth of depression	d/b	Exit Gradient		
		$b_1/b=0.25$, $s/b = 0.5$	$b_1/b=0.9375$, $s/b = 0.5$	$b_1/b=0.9375$, $s/b = 0.175$
0.001	0.000	3.638	2.986	4.860
0.010	0.001	1.140	0.987	1.560
0.020	0.003	0.809	0.670	1.120
0.030	0.004	0.662	0.532	0.960
0.040	0.005	0.574	0.470	0.810
0.050	0.006	0.515	0.423	0.730
0.100	0.013	0.367	0.290	0.530
0.200	0.025	0.287	0.223	0.390
0.300	0.038	0.240	0.189	0.330
0.400	0.050	0.205	0.174	0.285
0.500	0.063	0.173	0.158	0.265
0.600	0.075	0.164	0.149	0.240
0.700	0.088	0.155	0.141	0.225
0.800	0.100	0.146	0.136	0.210
0.900	0.113	0.137	0.130	0.200
1.000	0.125	0.128	0.125	0.190
2.000	0.250	0.100	0.100	0.124

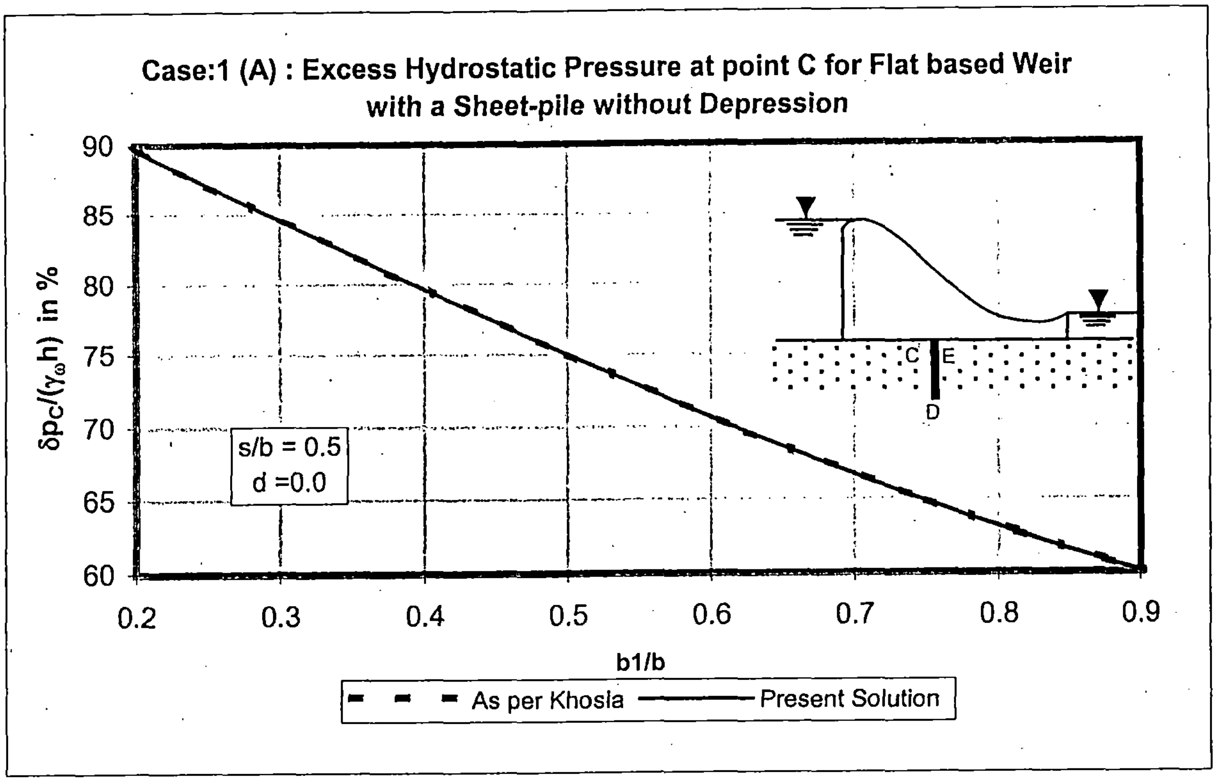


Figure: 3.6(a)

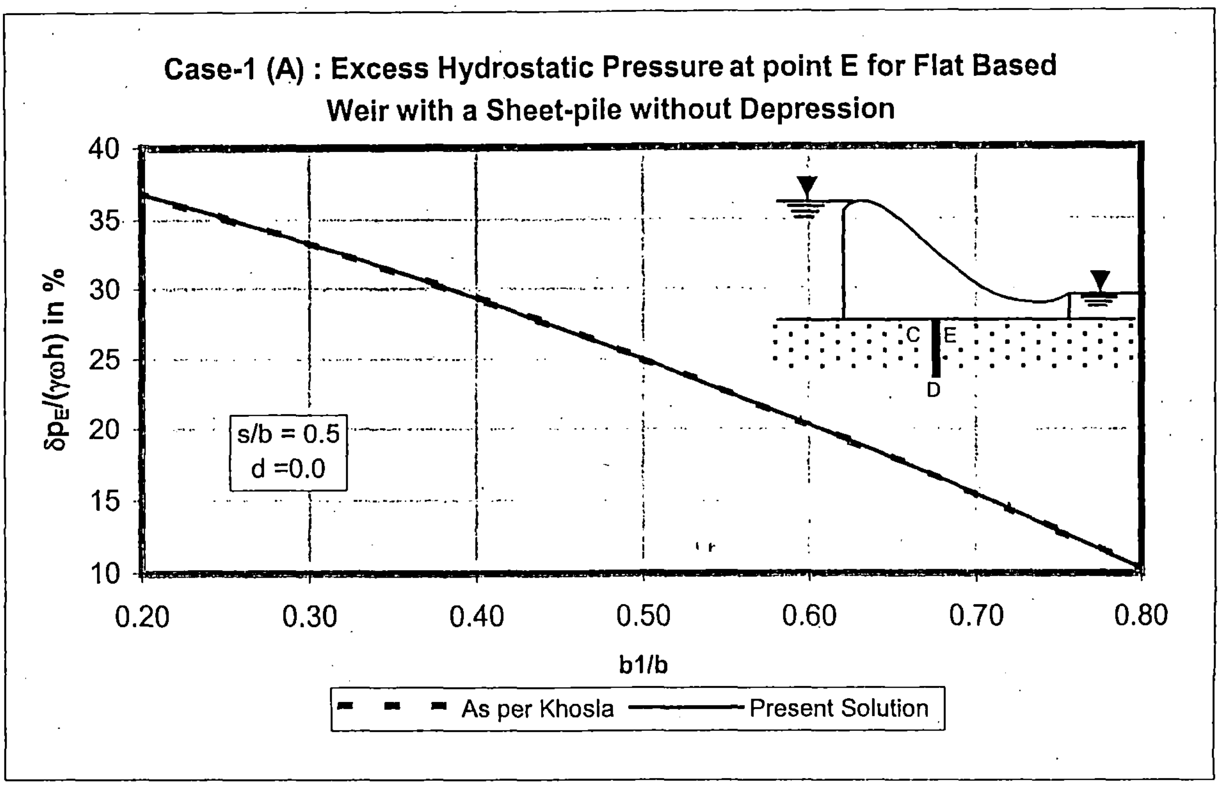


Figure: 3.6.(b)

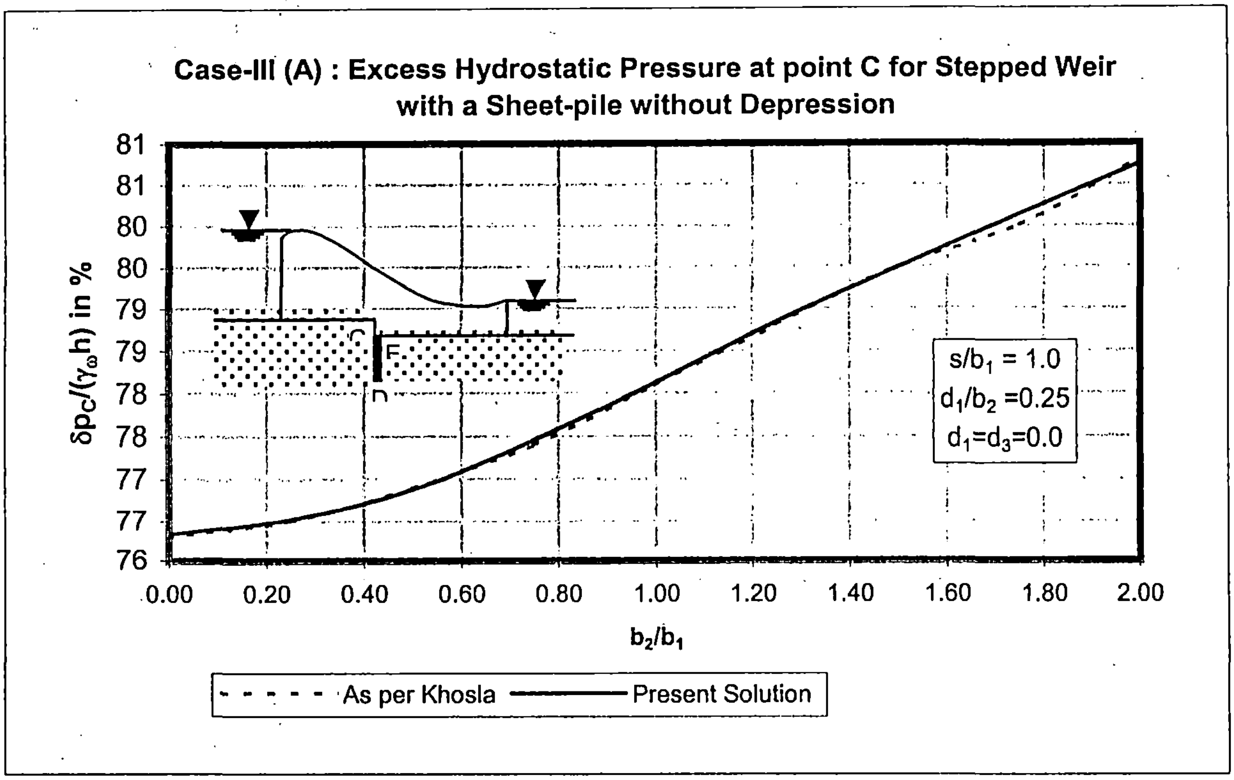


Figure: 3.6(c)

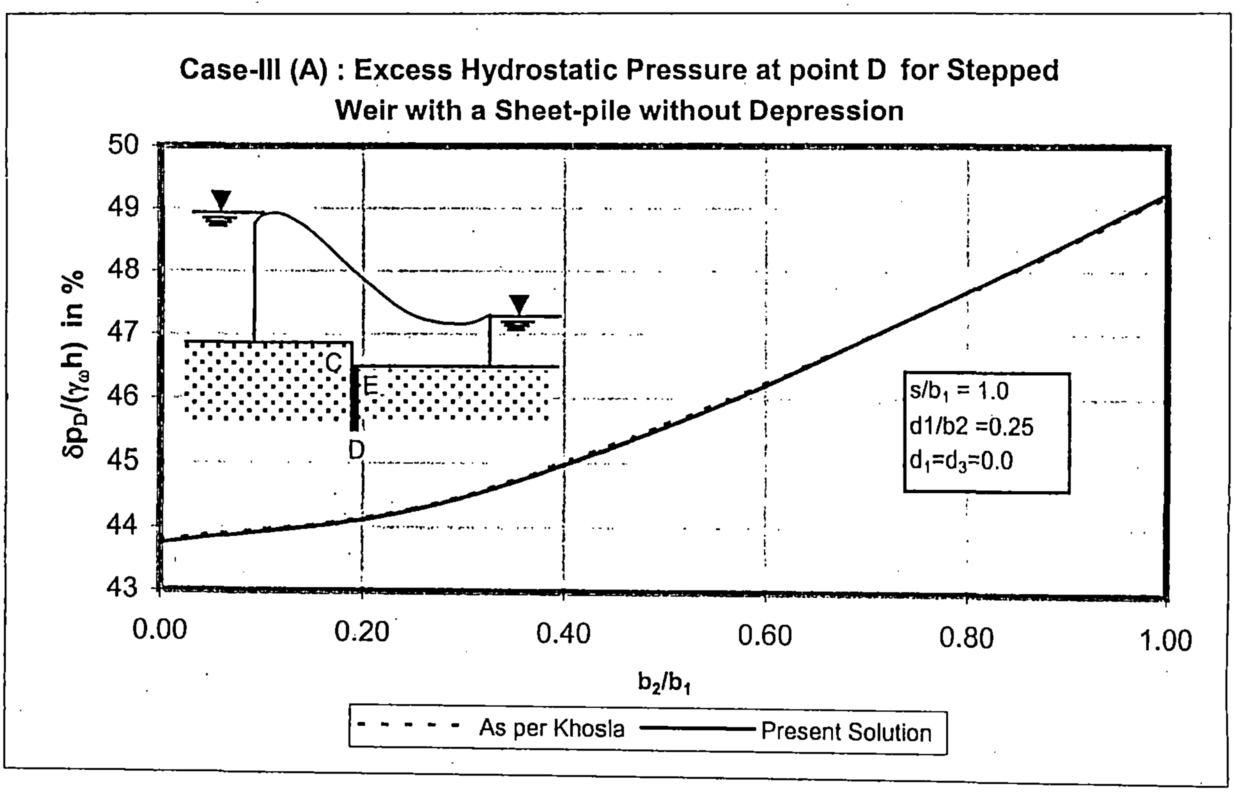


Figure: 3.6(d)

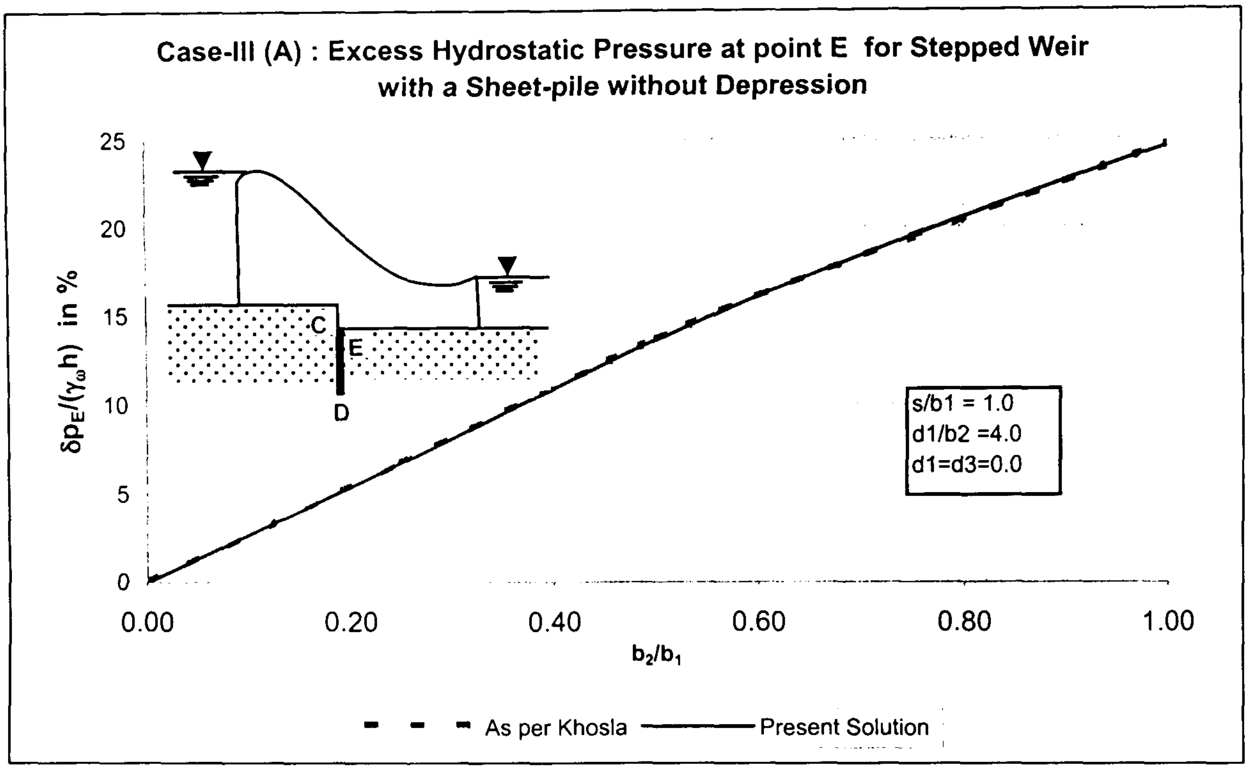


Figure: 3.6(e)

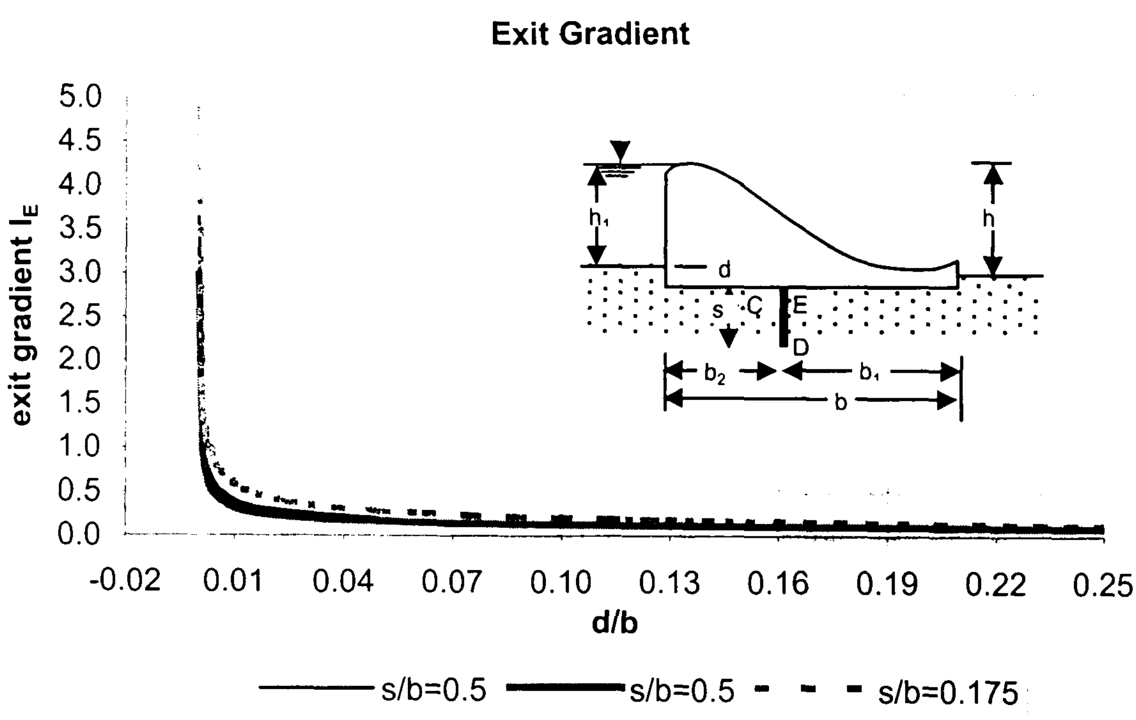


Figure: 3.6(f)

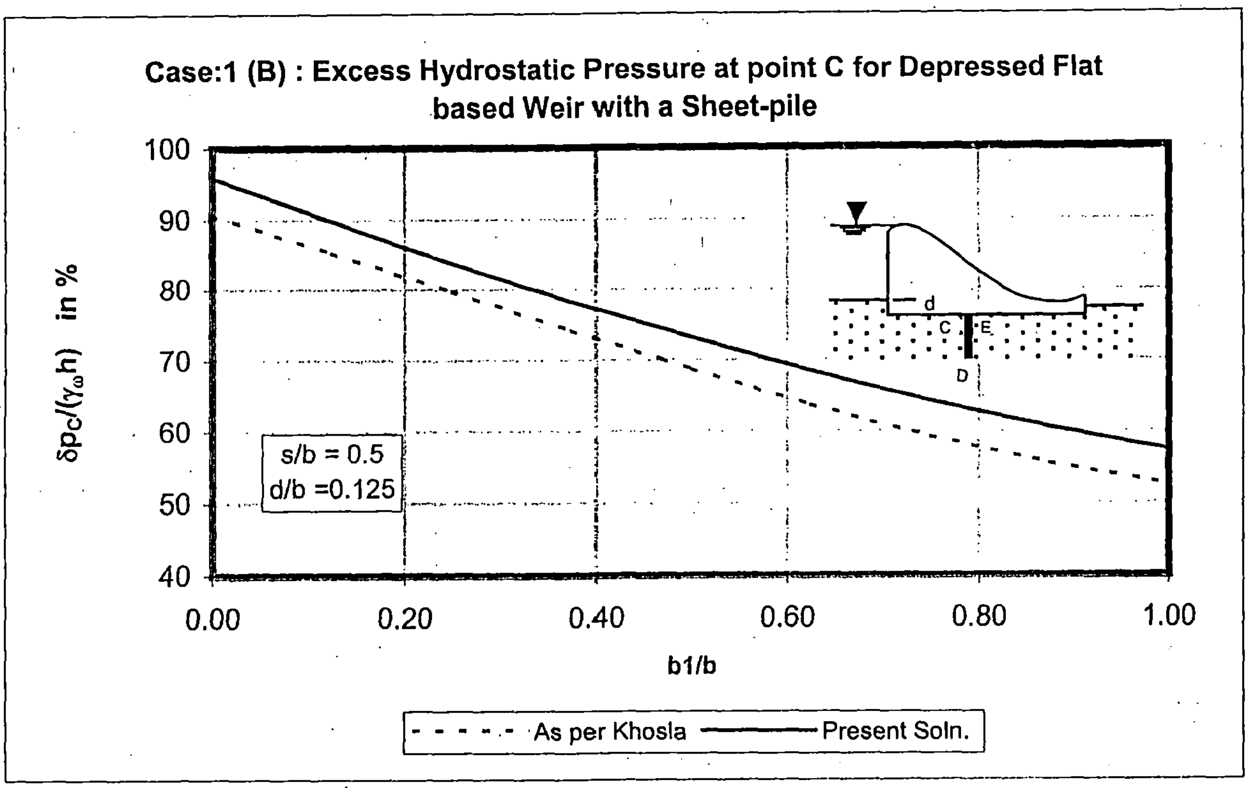


Figure: 3.7(a)

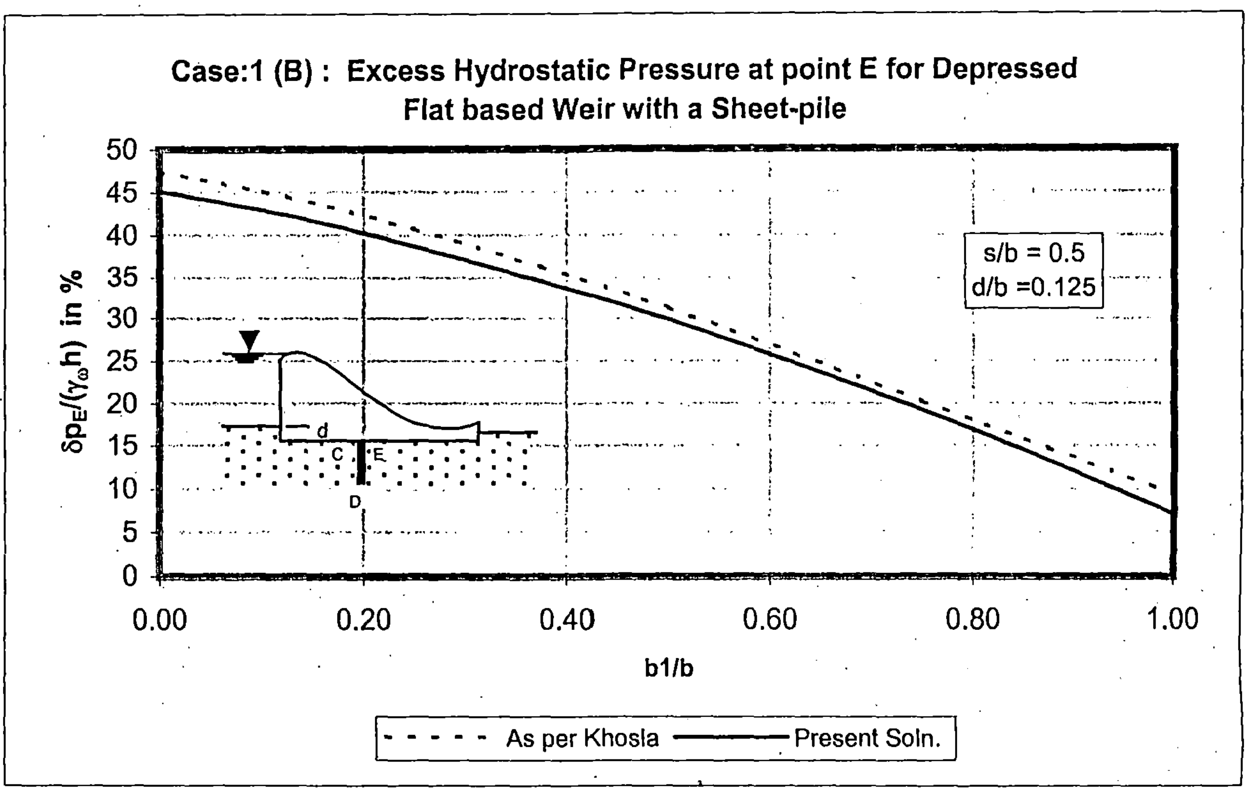


Figure: 3.7(b)

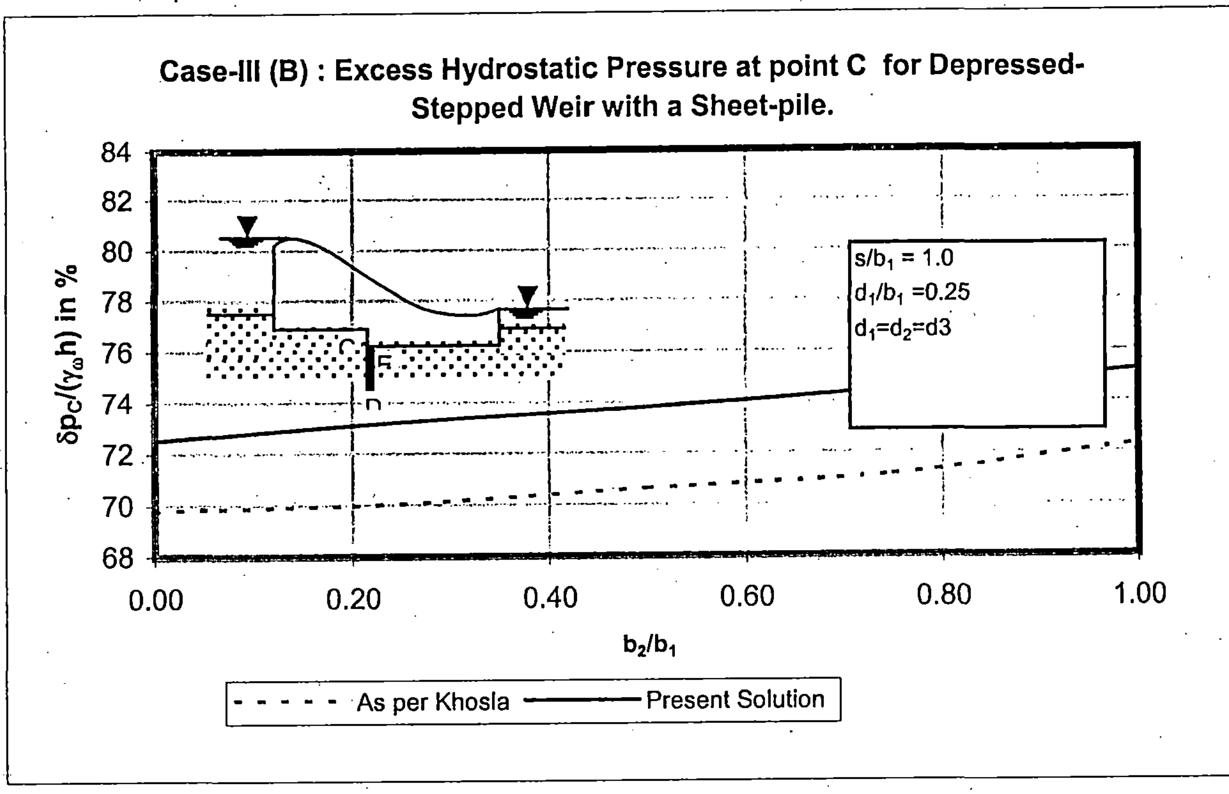


Figure: 3.7(c)

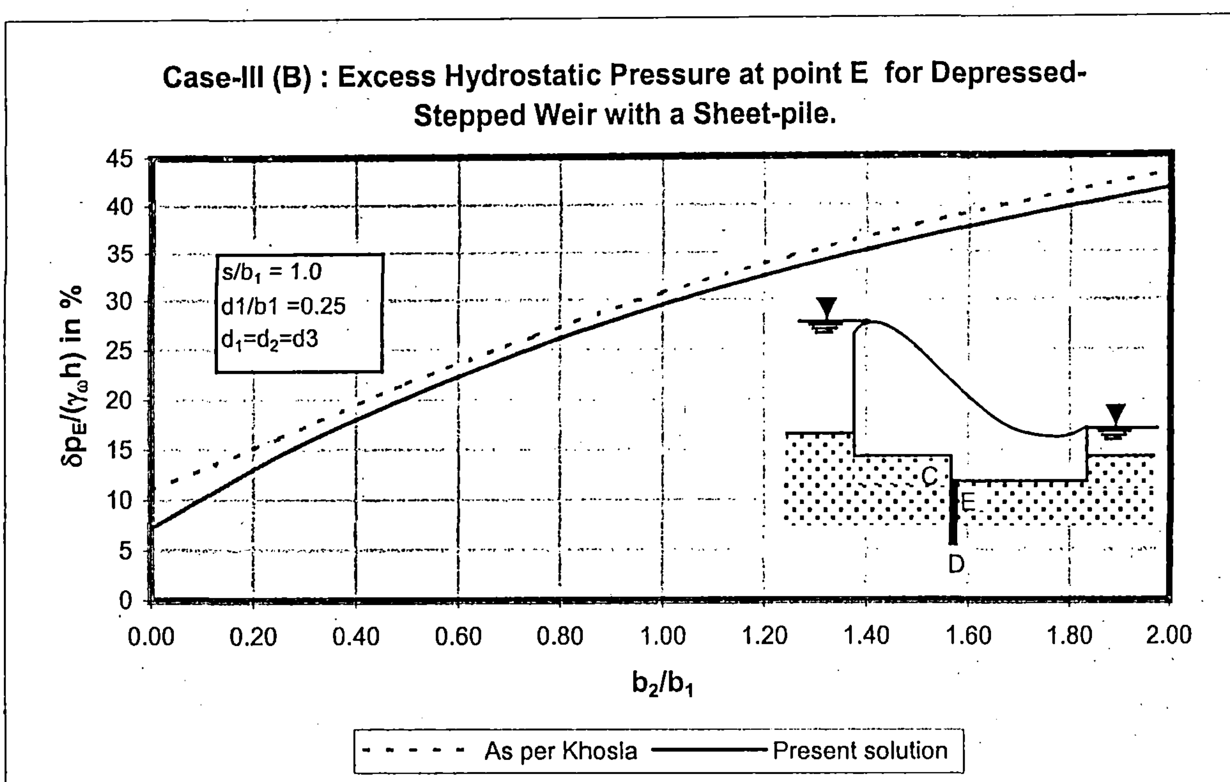


Figure: 3.7(d)

Conclusion

An analytical solution using Scwarz-christoffel conformal mapping technique has been obtained for computing uplift pressure at key points for a depressed stepped weir with a sheet pile at the step. From the general solution result for stepped weir without depression can be obtained. It is found that the solution of non-linear equations relating the parameters of transformation and the dimensions of the structure can be determined applying Newton-Raphson technique.

The pressure computed at salient points using the Scwarz-Christoffel transformation and Newton-Raphson technique compares well that of Khosla's solution for a stepped weir without depression.

Khosla's approximate method underestimates the pressure on the upstream side and over estimates on the downstream side. Therefore Khosla's solution can be applied safely. The deviation from true value in Khosla's method on the upstream side varies from 2.8% to 3.9% and in the downstream side it varies from 3.8% to 34.5% for d/b ranging from 0.083 to 0.25.

Depression should not be neglected. The difference in result for without depression and with depression varies from 3.5% to 5.25% for d/b changing from 0.083 to 0.25 and the difference is on the positive side i.e. with depression the pressure is lower than that without depression. In the down stream side the difference in pressure at the step without and with depression varies from 6.5% to 100% for d/b changing from 0.125 to 0.25.

The depressed part of hydraulic structure functions as a downstream sheet pile, which reduces the exit gradient. The variation of maximum exit gradient with the ratio d/b (Fig:3.6(f)) shows that exit gradient varies rapidly with the decrease in d/b. From table 3.4 it is seen that, a depressed floor of 0.5m thick along with 1.4m deep sheet pile can replace a 4.0m deep sheet pile. This gives the idea of the contribution of depressed floor on exit gradient.

Using the present solution a software is written in FORTRAN, which can be used in the computation of uplift pressure directly and can be further developed as per the requirement.

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Appendix - I

General

Most of the analytical method for the solution of two-dimensional groundwater problems is concerned with the determination of a function, which will transform a problem from a geometrical domain within which a solution is sought for into the one within which the solution is known. This chapter deals with the study of elementary functions and the manner in which these functions transform geometric figures from one complex plane to another.

Conformal Mapping technique is a powerful tool for solving two-dimensional Laplace equations. The method is used for solving the problems of flow under hydraulic structures.

Conformal Mapping Technique

It is generally known that for a weir with flat base and resting on the surface of ground, the streamlines or lines of flow are confocal ellipses with their focci at 'O' as shown in the **Figure: A.1**. The equation to these ellipses are given by :

$$\frac{x^2}{\left(\frac{b}{2}\cosh\ u\right)^2} + \frac{y^2}{\left(\frac{b}{2}\sinh\ u\right)^2} = 1$$

where **u** is *streamline function.*

Consider the physical domain in the Z-plane (**Figure: A.2**). When a vertical obstruction like a sheet pile or the stepped depression is introduced, the configurations of the streamlines or the flow lines are distorted.

By applying the Scwartz-Christoffel transformation technique, the distortion can be brought back to normal configuration. The streamlines that will be formed after the transformation are smooth ellipses with confocal points.

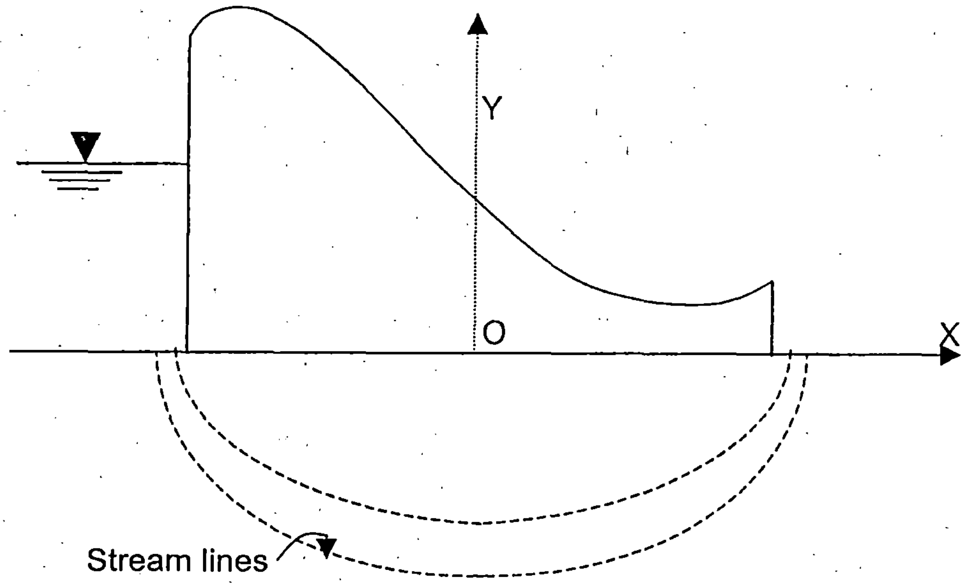

Figure: A1 Streamlines for flat base weirs on surface.

Assuming the physical domain to be on the Z-plane where any point on this is given by the relation Z = x + iy, the transformed plane is known as t-plane where any point on this is described by $\zeta = \xi + i\eta$.

In this process, the physical flow domains in z -plane as well as the complex potential domain ω are transformed onto a common platform known as the auxiliary **t-plane** from which a direct relation between **z-plane** and **ω-plane** are obtained. As such the flow region in the z-plane is first mapped onto the lower half of an auxiliary t-plane. Then the complex potential plane is also mapped onto the lower half of the t-plane. From these two conformal mappings, the relationship between z and ω is obtained.

This transformation is given by the relation:

$$Z = M\int \frac{dt}{(t-\alpha_1)^{\lambda_1}(t-\alpha_2)^{\lambda_2}(t-\alpha_3)^{\lambda_3}(t-\alpha_4)^{\lambda_4}(t-\alpha_5)^{\lambda_5}(t-\alpha_6)^{\lambda_6}(t-\alpha_7)^{\lambda_7}} \quad ......(1)$$

where $\lambda_1\pi$, $\lambda_2\pi$, $\lambda_3\pi$, $\lambda_4\pi$, $\lambda_5\pi$, $\lambda_6\pi$, $\lambda_7\pi$ are the changes in the angles at vertices at A ,B,C,D,E,F,G in the positive sense and α_1, α_2, α_3, α_4 ,α_5, α_6, α_7 are the ordinates at the points A, B, C, D, E, F, G in the t – plane on which the points A ,B,C,D,E,F,G of the Z – plane are mapped.

As seen (Figure.A.2) on Z – plane, the angles of turning at A, B, C, D, E, F, G are $\frac{\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, -\pi, \frac{\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}$ respectively so that;

$$\lambda_1\pi = \frac{\pi}{2} \quad \text{or} \quad \lambda_1 = \frac{1}{2}$$

$$\lambda_2\pi = -\frac{\pi}{2} \quad \text{or} \quad \lambda_2 = -\frac{1}{2}$$

$$\lambda_3\pi = \frac{\pi}{2} \quad \text{or} \quad \lambda_3 = \frac{1}{2} \quad \text{and so on.}$$

The origin in the figure in Z –plane is at C, which has been chosen at a point midway between CE in t – plane.

Now assuming $\alpha_1 = -\beta$, $\alpha_2 = -\gamma$, $\alpha_3 = -1$, $\alpha_4 = m$

$\alpha_5 = +1$, $\alpha_6 = \lambda$, $\alpha_7 = \mu$

the equation of transformation reduces to ;

$$Z = M\int \frac{dt}{(t+\beta)^{\frac{1}{2}}(t+\gamma)^{-\frac{1}{2}}(t+1)^{\frac{1}{2}}(m-t)^{-1}(1-t)^{\frac{1}{2}}(\lambda-t)^{-\frac{1}{2}}(\mu-t)^{\frac{1}{2}}} + N$$

$$\text{or} \quad Z = M\int \frac{(m-t)\sqrt{(t+\gamma)(\lambda-t)}}{\sqrt{(t+\beta)(1-t^2)(\mu-t)}}dt + N \qquad \text{..........(2)}$$

The equation above is the general equation for relation between Z-plane and t-plane obtained by Scwartz-Christoffel transformation technique for the physical domain shown in **Figure.A.2**

Similarly by applying the same transformation technique the relation between ω - plane and t - plane can be obtained as explained in **Chapter 3** (**Figure .3.1(c)**). The equation is read as:

$$\text{or} \quad \omega = M_1.\mathrm{Sin}^{-1}\left(\frac{2t+\beta-\mu}{\beta+\mu}\right) + N_1 \qquad \text{.........(3)}$$

By equating equations (2) and (3) the parameter 't' can be eliminated and direct relation between Z – plane and ω– plane can be obtained.

Z-Plane

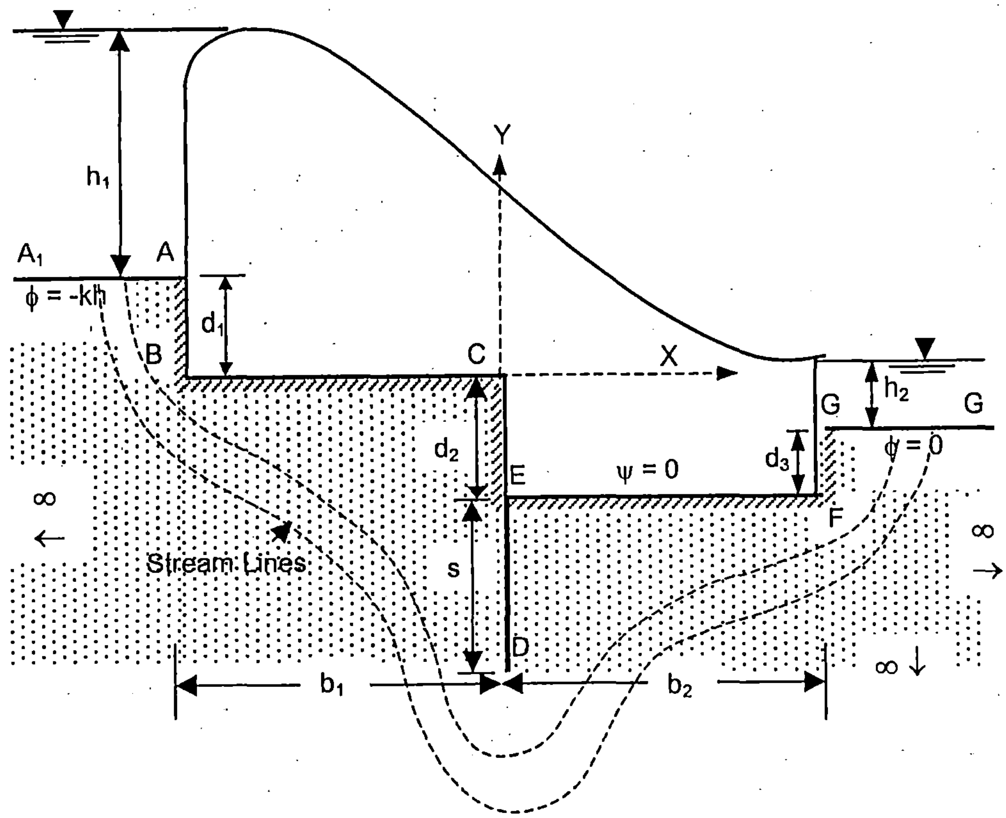

Figure .A.2 : Physical Domain in Z-plane

t- Plane

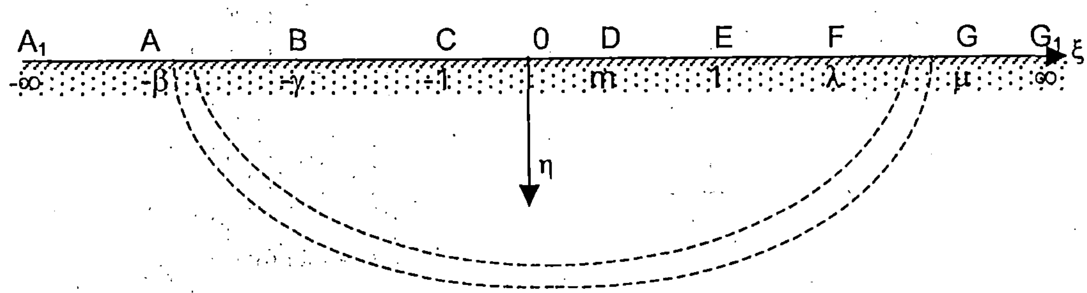

Figure .A.3 : Physical Domain Mapped on t-plane

Appendix - II

General

Often mapping steps result in a set of non-linear equations, which require a suitable technique to compute the unknown parameters. The implicit nature of the non-linear equations restricts the range of its applicability. In this appendix a methodology for solving a set of highly non-linear equations is described which can be used for solving two-dimensional flow problems in a complex domain with a great accuracy. The method described here is an iterative type popularly known as "Newton-Raphson Method for Non-linear systems of Equations".

Newton-Raphson Method

Chapter- 4 reveals that the problem consists of highly non-linear objective functions involving multivariable, which makes it difficult to solve by analytically. The process of numerical application is explained below:

The non-linear equations from (3.3.2) to (3.3.7) as in **chapter- 3** are represented by: $F_i\,(X_1, X_2, \ldots\ldots X_n) = 0$, where i = 1,2,3,.......n constitute the variables $X_1, X_2, \ldots\ldots X_n$.

Let '**X**' denote the entire vector of values x_i and **F** denote the entire vector of functions F_i . In the neighbourhood of X ,each of the functions F_i can be expanded in Taylor series.

$$F_i(X + \delta x) = F_i(X) + \sum_{j=1}^{n} \frac{\partial F_i}{\partial x_j} \Delta x_j + 0\delta x^2$$

In matrix notation, the above equation can be written as:

$$F_i\,(X+\delta x) = F_i(X) + \mathbf{J}.\,\Delta x_j + 0\delta x^2$$

Now neglecting the terms of the order δx^2 and higher and setting

$F_i(\mathbf{X} + \delta x) = 0,$

we have : $\mathbf{J}.\Delta x = - F(X)$ is an equation of matrix of a set of non-linear equations.

This matrix equation can be solved by LU decomposition and corrections are then added to the solution vector as $X_{new} = X_{old} + \Delta x$

where **J** is known as the Jacobian matrix and is represented as:

$$J = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \cdots & \frac{\partial F_1}{\partial x_n} \\ \frac{\partial F_2}{\partial x_1} & & & \\ \vdots & & & \\ \frac{\partial F_n}{\partial x_1} & \cdots & \cdots & \frac{\partial F_n}{\partial x_n} \end{bmatrix}$$

where

$$\frac{\partial F_i}{\partial x_j} = \frac{F_i(x_1, x_2, \ldots x_j + \Delta h, \ldots, x_n) - F_i(x_1, x_2, \ldots x_j, \ldots, X_n)}{\Delta h}$$

and $\Delta x_i = -\mathbf{F}.[\, \mathbf{J}\,]^{-1}$

or $\mathbf{X_i = X_i + \Delta x_i}$

This X_i represents the variables in the non-linear equations.

****

Appendix III

FORTRAN PROGRAM

```
*****************************************************************************
! This PROGRAM is a part of the M.Tech thesis for WRDTC,I.I.T.Roorkee,       !
! developed by B.Shyam Sundar Patro,M.Tech,WRD(civil)2002.                   !
! This source code is only intended as a supplement to the thesis            !
! "Analysis of seepage under a stepped depressed weir with a sheet pile"     !
! See these sources for detailed information regarding the input files      !
! and dependencies.                                                          !
!*****************************************************************************

!B1=BASE1,B2=BASE2,D1=DEPTH1,D2=DEPTH2,D3=DEPTH3,S=PILEDEPTH
    PROGRAM WEIRP
    DIMENSION WW(96),XX(96)

        OPEN (Unit=1,file='WEIRP.dat',STATUS='old')
        OPEN (Unit=2,file='WEIRP.out',STATUS='Unknown')
        OPEN (Unit=3,file='GAUSS.dat',STATUS='old')

        READ (3,*) (WW(I),I=1,96)
        READ (3,*) (XX(I),I=1,96)
        READ (1,*) B1,B2,D1,D2,D3,S,H1,H2

        WRITE(2,*)'PROGRAM RESULT FOR UPLIFT PRESSURE'
        WRITE(2,*)'*****************************************************'
5       FORMAT(8F7.2)
        WRITE(2,*)'  B1   B2   D1   D2   D3   S   H1   H2'
        WRITE(2,5)B1,B2,D1,D2,D3,S,H1,H2
        WRITE(2,*)'*****************************************************'

6       FORMAT(8F7.3)

        INDEX=1
        B=B1+B2

        SM0=0.1
        GAMA0=1.1+b1/b
        BETA0=GAMA0+0.1
        CLMDA0=1.1+b2/b
        CMU0=CLMDA0+0.1

10      CONTINUE
        WRITE(2,*)' BETA0  GAMA0   SM0 CLMDA0  CMU0  B1  B2  B1/B '
        WRITE(2,6)BETA0,GAMA0,SM0,CLMDA0,CMU0,B1,B2,(B1/B)


        CALL MAIN(WW,XX,BETA0,GAMA0,SM0,CLMDA0,CMU0,
     1  Res1,Res2,Res3,Res4,Res5,Res6,B1,B2,D1,D2,D3,S,
     2  FA,FB,FC,FD,FE,FF1,FF2,FF3,FF4,FF5,
     3  DBETA0,DGAMA0,DELSM0,DLMDA0,DELMU0)
```

```
      Write(2,*)'value of Res1=',res1
      write(2,*)'*******************************************'

      CALL PHI(D1,D2,D3,H1,H2,S,
     1      BETA0,GAMA0,SM0,CLMDA0,CMU0,PC,PD,PE,PF)

      WRITE(2,*)'   PC      PD     PE      PF    '
      WRITE(2,*)'                  '
      WRITE(2,36)PC,PD,PE,PF
36    FORMAT(7F8.2)

      WRITE(2,*)'                  '
      WRITE(2,*)'***********  END OF RESULT  ***********'

      STOP
      END PROGRAM WEIRP


*****************************************************************
!     SUBROUTINE MAIN     (Solution of Jacobian Matrix)
*****************************************************************
      SUBROUTINE MAIN(WW,XX,BETA0,GAMA0,SM0,CLMDA0,CMU0,
   1  Res1,Res2,Res3,Res4,Res5,Res6,B1,B2,D1,D2,D3,S,
   2  FA,FB,FC,FD,FE,FF1,FF2,FF3,FF4,FF5,
   3  DBETA0,DGAMA0,DELSM0,DLMDA0,DELMU0)
      DIMENSION WW(96),XX(96)
      DIMENSION AA(5,5),CC(5)

   EPSILON=0.00001

5     FORMAT(5F8.5)
10    CONTINUE
      CALL BX(WW,XX,BETA0,GAMA0,SM0,CLMDA0,CMU0,
   1  Res1,Res2,Res3,Res4,Res5,Res6,B1,B2,D1,D2,D3,S,
   2  FA,FB,FC,FD,FE,FF1,FF2,FF3,FF4,FF5)
      CC(1)=-FF1
      CC(2)=-FF2
      CC(3)=-FF3
      CC(4)=-FF4
      CC(5)=-FF5
C     ************

      DBETA=EPSILON
      DGAMA=EPSILON
      DELSM=EPSILON
      DLMDA=EPSILON
      DELMU=EPSILON

C     ************
      BETA1=BETA0+DBETA
      CALL BX(WW,XX,BETA1,GAMA0,SM0,CLMDA0,CMU0,
   1  Res1,Res2,Res3,Res4,Res5,Res6,B1,B2,D1,D2,D3,S,
   2  FA,FB,FC,FD,FE,FF11,FF22,FF33,FF44,FF55)

      AA(1,1)=(FF11-FF1)/DBETA
```

```
      AA(2,1)=(FF22-FF2)/DBETA
      AA(3,1)=(FF33-FF3)/DBETA
      AA(4,1)=(FF44-FF4)/DBETA
      AA(5,1)=(FF55-FF5)/DBETA
C     ************************
      GAMA1=GAMA0+DGAMA
      CALL BX(WW,XX,BETA0,GAMA1,SM0,CLMDA0,CMU0,
     1 Res1,Res2,Res3,Res4,Res5,Res6,B1,B2,D1,D2,D3,S,
     2 FA,FB,FC,FD,FE,FF11,FF22,FF33,FF44,FF55)

      AA(1,2)=(FF11-FF1)/DGAMA
      AA(2,2)=(FF22-FF2)/DGAMA
      AA(3,2)=(FF33-FF3)/DGAMA
      AA(4,2)=(FF44-FF4)/DGAMA
      AA(5,2)=(FF55-FF5)/DGAMA
C     ************************
      SM1=SM0+DELSM
      CALL BX(WW,XX,BETA0,GAMA0,SM1,CLMDA0,CMU0,
     1 Res1,Res2,Res3,Res4,Res5,Res6,B1,B2,D1,D2,D3,S,
     2 FA,FB,FC,FD,FE,FF11,FF22,FF33,FF44,FF55)

      AA(1,3)=(FF11-FF1)/DELSM
      AA(2,3)=(FF22-FF2)/DELSM
      AA(3,3)=(FF33-FF3)/DELSM
      AA(4,3)=(FF44-FF4)/DELSM
      AA(5,3)=(FF55-FF5)/DELSM
C     ************************
      CLMDA1=CLMDA0+DLMDA
       CALL BX(WW,XX,BETA0,GAMA0,SM0,CLMDA1,CMU0,
     1 Res1,Res2,Res3,Res4,Res5,Res6,B1,B2,D1,D2,D3,S,
     2 FA,FB,FC,FD,FE,FF11,FF22,FF33,FF44,FF55)

      AA(1,4)=(FF11-FF1)/DLMDA
      AA(2,4)=(FF22-FF2)/DLMDA
      AA(3,4)=(FF33-FF3)/DLMDA
      AA(4,4)=(FF44-FF4)/DLMDA
      AA(5,4)=(FF55-FF5)/DLMDA
C     ************************
      CMU1=CMU0+DELMU
       CALL BX(WW,XX,BETA0,GAMA0,SM0,CLMDA0,CMU1,
     1 Res1,Res2,Res3,Res4,Res5,Res6,B1,B2,D1,D2,D3,S,
     2 FA,FB,FC,FD,FE,FF11,FF22,FF33,FF44,FF55)

      AA(1,5)=(FF11-FF1)/DELMU
      AA(2,5)=(FF22-FF2)/DELMU
      AA(3,5)=(FF33-FF3)/DELMU
      AA(4,5)=(FF44-FF4)/DELMU
      AA(5,5)=(FF55-FF5)/DELMU
C     ************************
      MM=5
       CALL MATRIXIN(AA,MM)
C     ************************
              SUM=0
              DO J=1,5
              SUM=SUM+AA(1,J)*CC(J)
```

```
      ENDDO
      DBETA0=SUM

      SUM=0
      DO J=1,5
      SUM=SUM+AA(2,J)*CC(J)
      ENDDO
      DGAMA0=SUM

      SUM=0
      DO J=1,5
      SUM=SUM+AA(3,J)*CC(J)
      ENDDO
      DELSM0=SUM

      SUM=0
      DO J=1,5
      SUM=SUM+AA(4,J)*CC(J)
      ENDDO
      DLMDA0=SUM

      SUM=0
      DO J=1,5
      SUM=SUM+AA(5,J)*CC(J)
      ENDDO
      DELMU0=SUM
C     ************************
      BETA0=DBETA0+BETA0
      GAMA0=DGAMA0+GAMA0
      SM0=DELSM0+SM0
      CLMDA0=DLMDA0+CLMDA0
      CMU0=DELMU0+CMU0
C     ************************
      INDEX=INDEX+1
      IF(INDEX.GT.10)GOTO 20

      IF(ABS(DBETA0).GT.0.000001)GOTO 10
      IF(ABS(DGAMA0).GT.0.000001)GOTO 10
      IF(ABS(DELSM0).GT.0.000001)GOTO 10
      IF(ABS(DLMDA0).GT.0.000001)GOTO 10
      IF(ABS(DELMU0).GT.0.000001)GOTO 10
      GOTO 30
20    CONTINUE
      WRITE(2,*)'ITERATION HAS FAILED'
      GOTO 40
30    CONTINUE

      WRITE(2,*)'          '
      WRITE(2,*)'NUMBER OF ITERATIONS=',INDEX
      WRITE(2,*)'          '
      WRITE(2,*)'VALUES OF THE FUNCTIONS AFTER ITERATIONS'
      WRITE(2,*)'          '
      write(2,5)cc(1),cc(2),cc(3),cc(4),cc(5)
      WRITE(2,*)'******************************************'
      WRITE(2,*)"  BETA   GAMA    SM  CLMDA    CMU"
```

```
      WRITE(2,5)BETA0,GAMA0,SM0,CLMDA0,CMU0
      WRITE(2,*)'****************************************'
40    CONTINUE

      RETURN
      END

*************************************************************
!     SUBROUTINE MATRIXINV (LU decomposition)
*************************************************************
      SUBROUTINE MATRIXIN(AA,MM)

      DIMENSION AA(5,5),B(5),C(5)

    NN=MM-1
      AA(1,1)=1./AA(1,1)
      DO 8  M=1,NN
      K=M+1
      DO 3 I=1,M
      B(I)=0.0
      DO 3 J=1,M
3     B(I)=B(I)+AA(I,J)*AA(J,K)
      D=0.0
      DO 4 I=1,M
4     D=D+AA(K,I)*B(I)
      D=-D+AA(K,K)
      AA(K,K)=1./D
      DO 5 I=1,M
5     AA(I,K)=-B(I)*AA(K,K)
      DO 6 J=1,M
      C(J)=0.0
      DO 6 I=1,M
6     C(J)=C(J)+AA(K,I)*AA(I,J)
      DO 7 J=1,M
7     AA(K,J)=-C(J)*AA(K,K)
      DO 8 I=1,M
      DO 8 J=1,M
8     AA(I,J)=AA(I,J)-B(I)*AA(K,J)

    RETURN
      END

*************************************************************
!     SUBROUTINE PRESSURE (Calculates Uplift pressure)
*************************************************************
      SUBROUTINE PHI(D1,D2,D3,H1,H2,S,
      1     BETA0,GAMA0,SM0,CLMDA0,CMU0,PC,PD,PE,PF)
      PI=3.141592654

      H=H1+D1+D2-D3-H2

      TERM2=ASIN((BETA0-CMU0-2.)/(BETA0+CMU0))*(2./PI)
      TERM22=(H*0.5*(TERM2-1.))
```

```
      PC=H2-D2+D3-TERM22*100./H

      TERM3=ASIN(((2.0*SM0)+BETA0-CMU0)/(BETA0+CMU0))*(2./PI)
      TERM33=(H*0.5*(TERM3-1.))
      PD=-4.98+H2+D3+S-TERM33*100./H

      TERM4=ASIN((BETA0-CMU0+2.)/(BETA0+CMU0))*(2./PI)
      TERM44=(H*0.5*(TERM4-1.))
      PE=H2+D3-1.-TERM44*100./H

      TERM5=ASIN((BETA0-CMU0+(2.0*CLMDA0))/(BETA0+CMU0))*(2./PI)
      TERM55=(H*0.5*(TERM5-1.))
      PF=H2+D3-TERM55*100./H

      RETURN
      END

*******************************************
!     SUBROUTINE  BX (Grouping of Subroutines)
*******************************************

      SUBROUTINE BX(WW,XX,BETA0,GAMA0,SM0,CLMDA0,CMU0,
     1            Res1,Res2,Res3,Res4,Res5,Res6,B1,B2,D1,D2,D3,S,
     2            FA,FB,FC,FD,FE,FF1,FF2,FF3,FF4,FF5)

      CALL Fx1(WW,XX,BETA0,GAMA0,SM0,CLMDA0,CMU0,Res1)
      CALL Fx2(WW,XX,BETA0,GAMA0,SM0,CLMDA0,CMU0,Res2)
      CALL Fx3(WW,XX,BETA0,GAMA0,SM0,CLMDA0,CMU0,Res3)
      CALL Fx4(WW,XX,BETA0,GAMA0,SM0,CLMDA0,CMU0,Res4)
      CALL Fx5(WW,XX,BETA0,GAMA0,SM0,CLMDA0,CMU0,Res5)
      CALL Fx6(WW,XX,BETA0,GAMA0,SM0,CLMDA0,CMU0,Res6)

      FA=RES2/RES1
      FB=RES3/RES1
      FC=RES4/RES1
      FD=RES5/RES1
      FE=RES6/RES1

      FF1=(B2/S)-FA
      FF2=(D3/S)-FB
      FF3=((D2+S)/S)-FC
      FF4=(B1/S)-FD
      FF5=(D1/S)-FE

      RETURN
      END

!     SUBROUTINE Fx1
      SUBROUTINE Fx1(WW,XX,BETA0,GAMA0,SM0,CLMDA0,CMU0,Res1)
      DIMENSION WW(96),XX(96)

    SUM=0
      DO I=1,96
      U=XX(I)
      V=(U+1.)*(SQRT(1.-SM0))/2.
      F1N=(1-V**2-SM0)*SQRT((1-V**2+GAMA0)*(CLMDA0-1+V**2))
```

```
      F1D=SQRT((1-V**2+BETA0)*(2-V**2)*(CMU0-1+V**2))
      F1=F1N/F1D
      SUM=SUM+WW(I)*F1
      ENDDO
      Res1=SUM*SQRT(1.-SM0)

      RETURN
      END

!     SUBROUTINE Fx2
      SUBROUTINE Fx2(WW,XX,BETA0,GAMA0,SM0,CLMDA0,CMU0 ,Res2)
      DIMENSION WW(96),XX(96)

   SUM=0
      DO I=1,96
      U=XX(I)
      V=(U+1.)*(SQRT(CLMDA0-1.))/2.
      F2N=(1+V**2-SM0)*SQRT((1+V**2+GAMA0)*(CLMDA0-1-V**2))
      F2D=SQRT((1+V**2+BETA0)*(2+V**2)*(CMU0-1-V**2))
      F2=F2N/F2D
      SUM=SUM+WW(I)*F2
      ENDDO
      Res2=SUM*SQRT(CLMDA0-1.)

      RETURN
      END

!     SUBROUTINE Fx3
      SUBROUTINE Fx3(WW,XX,BETA0,GAMA0,SM0,CLMDA0,CMU0 ,Res3)
      DIMENSION WW(96),XX(96)

   SUM=0
      DO I=1,96
      U=XX(I)
      V=(U+1.)*(SQRT(CMU0-CLMDA0))*(0.5)
  F3N=(CMU0-V**2-SM0)*SQRT((CMU0-V**2+GAMA0)*(CMU0-V**2-CLMDA0))
      F3D=SQRT((CMU0-V**2+BETA0)*(1+CMU0-V**2)*(CMU0-V**2-1))
      F3=F3N/F3D
      SUM=SUM+WW(I)*F3
      ENDDO
      Res3=SUM*SQRT(CMU0-CLMDA0)

      RETURN
      END

!     SUBROUTINE Fx4
      SUBROUTINE Fx4(WW,XX,BETA0,GAMA0,SM0,CLMDA0,CMU0 ,Res4)
      DIMENSION WW(96),XX(96)

   SUM=0
      DO I=1,96
      U=XX(I)
      V=(U+1.)*(SQRT(1.+SM0))/2.
      F4N=(SM0+1-V**2)*SQRT((V**2-1+GAMA0)*(CLMDA0+1-V**2))
      F4D=SQRT((V**2-1+BETA0)*(2-V**2)*(CMU0-V**2+1))
```

```
        F4=F4N/F4D
        SUM=SUM+WW(I)*F4
        ENDDO
        Res4=SUM*SQRT(1.+SM0)

        RETURN
        END

!       SUBROUTINE Fx5
        SUBROUTINE Fx5(WW,XX,BETA0,GAMA0,SM0,CLMDA0,CMU0 ,Res5)
        DIMENSION WW(96),XX(96)

    SUM=0
        DO I=1,96
        U=XX(I)
    V=(U+1.)*(SQRT(GAMA0-1.))/2.
        F5N=(1+V**2+SM0)*SQRT((GAMA0-1-V**2)*(1+V**2+CLMDA0))
        F5D=SQRT((BETA0-1-V**2)*(2+V**2)*(CMU0+1+V**2))
        F5=F5N/F5D
        SUM=SUM+WW(I)*F5
        ENDDO
        Res5=SUM*SQRT(GAMA0-1.)

        RETURN
        END

!       SUBROUTINE Fx6
        SUBROUTINE Fx6(WW,XX,BETA0,GAMA0,SM0,CLMDA0,CMU0 ,Res6)
        DIMENSION WW(96),XX(96)

    SUM=0
        DO I=1,96
        U=XX(I)
    V=(U+1.)*(SQRT(BETA0-GAMA0))*(0.5)
F6N=(SM0+BETA0-V**2)*SQRT((BETA0-V**2-GAMA0)*(CLMDA0+BETA0-V**2))
        F6D=SQRT((BETA0-V**2-1)*(BETA0-V**2+1)*(CMU0+BETA0-V**2))
        F6=F6N/F6D
        SUM=SUM+WW(I)*F6
        ENDDO
        Res6=SUM*SQRT(BETA0-GAMA0)

        RETURN
        END
```

****

****

Data Entry Procedures: (weir parameters to be entered as per below)

****

b1 b2 d1 d2 d3 s h1 h2

****

Sample Result Output:

PROGRAM RESULT FOR UPLIFT PRESSURE

****

B1	B2	D1	D2	D3	S	H1	H2
2.00	6.00	.04	.00	.04	4.00	3.00	1.00

****

BETA0	GAMA0	SM0	CLMDA0	CMU0	B1	B2	B1/B
1.450	1.350	.100	1.850	1.950	2.000	6.000	.250

NUMBER OF ITERATIONS= 6

VALUES OF THE FUNCTIONS AFTER ITERATIONS

.00000 .00000 .00000 .00000 .00000

****

BETA	GAMA	SM	CLMDA	CMU
1.12739	1.12447	.00119	1.81432	1.81962

****

value of Res1= 9.946941E-01

****

PC	PD	PE	PF
87.71	57.58	35.40	3.74

*********** END OF RESULT ***********

****

Data Entry Procedures: (*weir parameters to be entered as per below)*

****

b1 b2 d1 d2 d3 s h1 h2

****

Sample Result Output:

PROGRAM RESULT FOR UPLIFT PRESSURE

```
*********************************************************
  B1    B2    D1    D2    D3    S     H1    H2
 2.00  6.00   .04   .00   .04  4.00  3.00  1.00
*********************************************************
 BETA0 GAMA0  SM0 CLMDA0  CMU0   B1    B2   B1/B
 1.450 1.350  .100 1.850 1.950 2.000 6.000  .250

NUMBER OF ITERATIONS=        6

VALUES OF THE FUNCTIONS AFTER ITERATIONS

 .00000 .00000 .00000 .00000 .00000
*****************************************
  BETA   GAMA    SM   CLMDA    CMU
1.12739 1.12447 .00119 1.81432 1.81962
*****************************************
value of Res1=  9.946941E-01
*****************************************
   PC     PD     PE     PF

 87.71  57.58  35.40   3.74

*********** END OF RESULT ***********
```