AN APPROPRIATE FORECASTING TECHNIQUE FOR ELECTRICAL ENERGY DEMAND

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree

of

MASTER OF TECHNOLOGY

in

HYDRO ELECTRIC SYSTEM ENGINEERING & MANAGEMENT

By

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DECLARATION

I here declare that the dissertation titled **"AN APPROPRIATE FORECASTING TECHNIQUE FOR ELECTRICAL ENERGY DEMAND"** which is being submitted for partial fulfillment of the requirements for the award of Master's of Technology Degree in **Hydroelectric System Engineering and Management** at Water Resources Development Training Center (WRDTC), Indian Institute of Technology, Roorkee is an authentic record of my own work carried out during the period of 15.07.2002 to 04.12.2002 under the supervision and guidance of **Professor Devadutta Das**, Professor and Head of WRDTC, IIT Roorkee and **Dr. M.L.Kansal**, Associate Professor, WRDTC, IIT Roorkee.

I have not submitted the matter embodied in this dissertation previously for the award of any other Degree.

Place : Roorkee, India

Date : 04.12.2002

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HSEM

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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AN APPROPRIATE FORECASTING TECHNIQUE FOR ELECTRICAL ENERGY DEMAND

SYNOPSIS

The main objective of energy resources development for power generation is to meet the consumer's demand. Forecasting future electrical energy demand of a system is the basic requirement of system planning. It is very essential to know the magnitude and characteristics of demand in advance. Purpose of electrical energy demand forecasting is governed by the period covered. Short-term forecast (1 to 2 years) are useful for operational and budgetary purposes. Medium term forecasts (5 to 10 years) are the most important and essential for planning and design of power projects. Long-term forecasts (10 to 20 years or more) are required to prepare master plans and to study the utilization of energy resources. System and substation forecasts are essential to plan the generation, transmission and distribution requirements of the power system.

Forecasting the future electrical energy demand of a system is a challenging assignment. As the future is shrouded with uncertainties, efforts should be made to assess the demands as accurate as possible. The effects of over and under estimation on the power system and the economic development of the area served by it are to be made. It is important to study and analyse the major factors affecting the magnitude and characteristics of the electrical energy demand. Several forecasting techniques and practices used in some of the countries are to be studied. Though several forecasting methods are in use in many parts of the world, it is seen that no technique would yield full proof results because of uncertainties of the future. Each of the forecasting method has its own limitations. An appropriate forecast can only be prepared after assessing the needs by several practicable approaches and reviewing the results by applying experience, knowledge and sound judgment.

Accuracy of the forecast is also depends on the accuracy of data made use of. Use of several techniques will be often limited by the paucity of statistical information required in the field of power and economic development of the area concerned. Keeping all this in view and due to the importance of electrical energy forecasting, it is necessary to formulate an appropriate and reliable forecasting procedure, suitable for the specific place of interest. In this dissertation work attempt has been made to formulate such an appropriate procedure, which could be useful in estimating the medium and long-term requirements, and using this procedure the future electrical energy demand for the Ethiopian Power System has been estimated.

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CHAPTER 1
INTRODUCTION

1.1. FORECASTING

In a business activity, forecasting refers to the estimation of what will happen in the future. Forecasting is very essential to reach a decision or for taking action. It is indispensable to commercial and manufacturing activities. The words "Forecasting" and "Prediction" are often used to refer to the estimation of the future. However, Brown[3] makes a distinction as to the use of these words. The word " to forecast' is of Saxon origin meaning to "throw ahead". It means that there is something on hand to be thrown. For example, history of past data can be projected in to the future. It is used to denote objective computation. On the other hand " to predict" which is of Latin origin, is to say before hand. Predict is used to refer to the subjective estimates of the future, especially the anticipation of novel factors. In estimation of the future, however, the words forecasting and prediction are often used to mean the same thing.

Forecasting is essentially subjective judgment made on the basis of existing information. It is necessary to adopt alternative procedures in assessing the future and to compare results. The procedure involves statistical analysis and extrapolation of past data, and analysis of economic, technical and commercial factors which have influenced the past [7].

The future is always uncertain. Unforeseen events can upset well prepared forecast. The management has no choice but to forecast constantly. Any decisions that require an appraisal of the future involves a forecast [8].

1.2 THE INCREASING IMPORTANCE OF ELECTRICICAL ENERGY DEMAND FORECAST

During the 1950s and 1960s, interest in demand forecasts was limited[5]. The high and steady growth rate of demand lead many to think that simple extrapolation methods were largely sufficient for planning purposes. In addition, the consequences of forecast errors were limited: overestimates of future energy demand were quickly made right by demand growth, and the temporary excess capacity resulting of such errors were soon absorbed; underestimates were not critical either, because turbine generators fired by cheap oil or gas could plug the gap while new base load plants were coming on line.

This easy context began to change in the late 1960s, when it become progressively clear that the post war rapid economic growth period coming to an end. Uncertainty about future growth increased, as is attested by the first report of the club of Rome on " Zero growth". It became clear that electricity demand growth was dependent on explanatory factors like the rate of growth of the different sectors of economic activity or the accelerated evolution of social structures and behaviours, and that global extrapolation methods were not sufficient to take into account these factors.

The 1973–1974 oil crisis increased sharply the concern about an improvement of electricity demand forecasting. The uncertainty on future energy prices and on the response of the economic actors to this new and possibly evolving price structure was now added to more uncertainty on economic growth. At the same time, the consequences of forecast errors became much more important than previously. In a low growth economic context excess capacities may last several years, turning it into a financial burden for the utility. Underestimates may be even worse since it takes at least 8 years to license and build a coal fired or nuclear plant, and that low cost short term answers are no longer available.

The beginning of the 1970s has also been characterized by the development of the public concern (and thus also the concern of various political groups and administrative structures) in environmental issues. Thus, decisions on the construction of generating plants, and as a consequence the demand forecasts on which those decisions were based, were no longer "private" issues for utilities, but more or less political issues. It was then necessary not only to forecast but also to explain the forecast. This led to the necessity of more appropriate and detailed methods, making more clear the hypothesis and results, and the fact that even in the framework of voluntary energy conservation policies a growth of electricity demand, and thus the necessity of new generating plants, was likely to arise.

In response to this growing importance of electrical energy demand forecasts, the methods used have evolved rapidly during the last twenty years, and still continue to improve through important methodological work.

At the same time demand forecasts are no longer only a question of proper methodology, but also a subject of negotiation between several social or economical actors, who may have divergent interests and opinions on the more likely, or more desirable, growth of electricity demand.

1.3 PURPOSE AND PERIOD OF FORECASTING

A power system forecast is necessary to plan the future generation requirements of the entire area. Sub-station forecasts[6] indicate the power demand area wise and forms the basic for planning the sub-station expansions and subtransmission system, and together with the future generation forms the basis for planning the main transmission network.

The purpose of electrical energy demand forecasting is governed by the period of forecast. Normally the electricity demand forecasting can be classified into four categories:

(1) Long term forecasting covering 20 years or more,

(2) Medium term forecasting covering 5 – 10 years,

(3) Short term forecasting covering 1- 5 years,

(4) Very short term forecasting covering very short periods such as hourly, daily, etc.

Long term forecasting helps in formulating National Power Policies and preparation of Master Plans for power development utilizing the hydro and other resources.

The medium and short term forecasting is used for planning expansion programs of a power system. It indicates the requirement of addition or modification to be made to the generating capacity, substation capacity, transmission system and distribution networks to meet the power demands of consumers in future. In other words it is the basis on which the planning and design of power projects is carried out. It also helps in fixing the tariffs of different categories of consumers and planning the investments to the maximum advantage.

Very short term forecasting is used for operational purposes and for immediate and unforeseen developments. The main uses are for

1) Scheduling of operation and interconnection,

2) Scheduling of maintenance,

3) Load dispatching,

4) Scheduling proper and economic import of power from other sources,

5) Estimating surplus and secondary sales,

6) Arranging power factor correction.

The short term forecasts are useful in

1) Ordering fuel,
2) Forecasting revenue and fixing cost of power for operating budget,
3) Provide basis for load feasibility studies,
4) Preparation of annual budget and so on.

The Short and very short term forecasting is affected by seasonal variations, weather conditions, industrial relations etc.

The approach to the long and medium term forecasts entirely different from that used for short term forecast. While short term forecast is influenced to a good extent by the previous year's happenings, it is not given any importance for the long term forecasts. The past data for the last 10 or more years is the basis for the medium and long term forecasts.

1.4 SCOPE OF THE DISSERTAION WORK

It is seen that the medium and long-term forecasts are the important forecasts which are required at least as far ahead as the time taken to plan and install new plant. In this dissertation work, therefore, importance is given to the medium and long range system forecasts.

Several forecasting techniques and practices used in some of the countries are to be studied according to their merits and limitations. There are several forecasting techniques in use in many parts of the world. It is rather difficult to determine the measure of influence of each technique, this being dependent on the purpose of the forecast, the availability of relevant statistical data, and many factors affecting the demand, etc. Keeping all this factors in view and due to the importance of electrical energy forecasting, it is necessary to formulate a reliable forecasting procedure. To execute the same, comparison of various forecasting techniques carried out on the electrical energy demand of Ethiopia and an appropriate technique recommended with practical justification. Electrical energy forecast of the world and the status of electrical energy development in Ethiopia are included in the subsequent chapter to make the work a complete one.

Forecasting is both a science and an art and as such scientific methods and sophisticated tools are as essential as sound judgment, experience and insight. The use of digital computers have greatly helped in forecasting the future electrical energy demand with good accuracy and also likely deviation of the forecasts. The problem of forecasting is basically approached by knowledge of the past data and projecting them into future. This requires elementary knowledge of statistics. Therefore, the essential requirements of statistical knowledge, curve-fitting techniques, regression analysis are also briefly covered and are included in the appendices 'A' and 'B'. It is to be noted that as no data were available on many of the economic activities, it was not possible to analyse the demand using some of the techniques, which are based on economic factors.

CHAPTER 2

WORLD ELECTRICAL ENERGY DEMAND

2.1. WORLD ELECTRIC ENERGY DEMAND SCENARIO

The demand for electricity is on the increase through out the world. Further increase in use of electricity will be greatly enhanced by the rapid and economical substitution of the same for other forms of energy. Even in advanced countries with the highest per capita consumption like Norway, Canada, United States, etc., saturation is not yet reached. This is a useful indication that in under-developed countries where the per capita consumption is very low, the growth of electricity will be on the increase for years to come. Electricity has many advantages like simple and safe availability, exact measurement, absent of odor, no dirt and litter at all, etc [(1)]. The total energy consumption and the electricity consumed in different parts of the world are at different rates, reflecting the extent of their industrial and socio- economic development.

Per capita electricity consumption in 1999 by regions shows that regions having 72.38 % of the world population (China, Asia, Latin America and Africa) have per capita electric energy consumption of about 1500 KWh/yr or less against 7841 KWh/yr in the OECD (Organization of Economic Cooperation and Development) countries as per the WEC report [(4)] summarized on Table - 2.1.1.

2.1.1. Energy Security

More than 1.6 billion people of the world still do not have access to modern energy system with the prospect of 400 million more people during next 20 years to be added to this figure. These will be mostly in the rural areas of the developing countries where unexploited hydro development potential is very high.

Table 2.1.1. PER CAPITA ELECTRICITY CONSUMPTION IN THE WORLD

Region	Population(Million)	Per Capita Electricity Consumption (KWh/yr)
World *	5921.39	2280
OECD countries	1116.4	7841
Middle East	162.17	2437
Former USSR	290.49	3687
NON OECD Europe	58.18	2511
China	1260.32	936
Asia (excluding China & DPR Korea)	1849.63	519
Latin America**	408.97	1510
Africa	775.23	491

* Excludes Northlands Antilles and DPR of Korea

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IEA survey indicates that electricity demand grows faster than any other end use fuel. It also predicts increase in the share of electricity from about 15 % in 1999 to 20 % in 2020. Non OECD region may have highest increase in energy demand, with 19 % share in final energy demand in 2020. To meet this energy requirement almost 100 million people will have to be given access to modern energy sources every year for the next 20 years against 40 million people per year between 1970 and 1990 and 30 million per year since then.

A comparison of the per capita electricity consumption show that while Norway has 26,280 Kwh/yr, Canada has 17,635 Kwh/yr ,USA has 13,800 Kwh/yr. Some of the African countries have less than 100 Kwh/yr. But these countries have technical and economic hydro potential which, if exploited, can enhance per capita consumption drastically (this point will be discussed more, for the case of Ethiopia, in Section 2.2).

2.1.2 Capacity addition outlook

International Energy Authority sums up the electric power capacity addition outlook and the financial needs of the same as indicated below:

- Over the outlook period nearly 3,000 GW of new generating capacity are projected to be installed world – wide. About one-fifth of this new capacity will replace existing installation, and the remainder will meet new demand. Projected annual new capacity increments amount to 103GW from 1997 to 2002 and 158 GW from then to 2020.
- Slightly more than a third of the new capacity will be built in OECD(Organization of Economic Cooperation and Development) countries. Replacement capacity represents almost a third of that amount. Some older steam plants and about 30% of existing nuclear capacity could be retired over the outlook period. Although electricity demand in the OECD countries slows in the second decade, capacity additions increase because of the need to replace retired units.
- The transition economies will need new capacity mostly in the second decade. Existing capacity is under used because of low electricity demand. Thus, capacity addition in these countries are only 9 GW per year in 1997-2010, but jump to 2.5 times that in the following decade to meet rising demand.

- More than half of the projected new capacity to 2020 will be installed in developing countries. Of the 1564 GW of new capacity needs, two-third will be built in developing Asia.
- The estimated investment cost of new power plants over the outlook period, excluding the cost of new transmission and distribution lines, is nearly $ 3 trillion at today's prices. Investment in new transmission and distribution systems may be about as great, depending on the country and level of electrification. The additional cost of expanding networks is likely to be higher in developing countries, where geographic coverage is much lower. Thus, total investment could easily double the estimate.
- In the OECD, the cost of new capacity is $ 894 billion. The transition economies will need more than $ 300 billion. Many existing plants in the region will also need refurbishing because of their age and more importantly, inadequate maintenance. The cost for this is highly uncertain and not included in the projection. Developing countries around $ 1.7 trillion in new plants.
- Clearly, developing countries will need to devote significant funds over the next twenty years to the development of their electricity sectors. In the past, growth in electricity depended on public sector support. However, insufficient resources often constrained the effort and resulted in large gaps between supply and demand. Many developing countries now see private investment as an attractive option to expand their power sector. Private participation may not be the only answer to power – infrastructure expansion but, if managed correctly, it can provide significant opportunities for development. In order to generate the necessary funds for power generation expansion, many countries in the developing world will, however need to accelerate reform of their public dominated electricity sources.

2.2 ELECTRICAL ENERGY SCENARIO IN ETHIOPIA

Ethiopia is the ninth largest country in Africa in terms of land area with the population of 63 million. Out of this 88.6 % is rural and 11.4% urban with 67.3 % economically active.

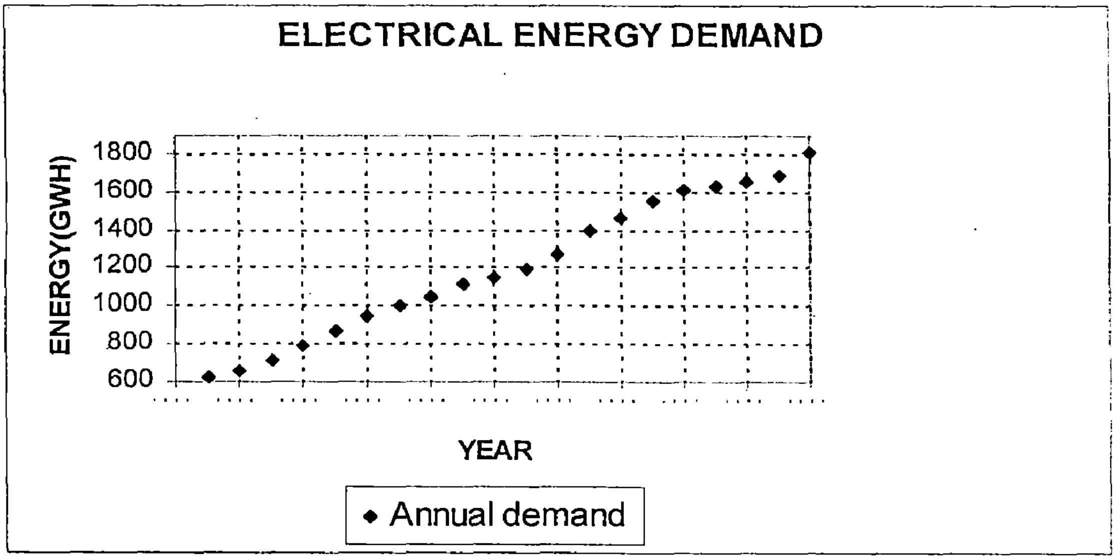
The electrical energy supply system of the country is being effected from the hydro generating plants (97 %) through the Inter-Connected System. Out of the overall population of the country, only 30 % has access to electricity. Out of the huge hydropower potential of 150,000 GWh/a, only a firm energy of 1860 GWh/a or 1.2 % of the total potential is generated. The amount of water being utilized for hydro power generation of about 3 billion m^3 out of the possible controlled discharge of 90 billion m^3 is an indication of the under-development of the water resources in Ethiopia[11]. At the same time the electrical energy demand of the country is continuously increasing at the rate of 2 to 8 % per year as can be observed from historical demand data shown in Table 2.2.1. and on Graph 2.2.1.

Considering the importance of electric power being a major infrastructural input, the present poor state of electrical energy development in the country reflects the general level of socio-economic under-development. And as the recent assessment done by the World Bank, ranked at 210th out of a total of 210 countries in GNP per capita terms and 208th in terms of GNP per capita measured at Purchasing Power Parity (PPP), Ethiopia is, by any measure, one of the poorest country on earth [2]. With this respect, the need to improve per capita power consumption in Ethiopia so as to improve the overall quality of life in the country should be the main objective of the planners. This requires effort to develop, conserve, utilize and manage the available water resources aimed to achieve sound and sustainable growth.

Table 2.2.1 ANNUAL ELECTRIC ENERGY DEMAND OF ETHIOPIA

YEAR	ENERGY (Million KWh)
1982	625
1983	654
1984	714
1985	792
1986	872
1987	945
1988	1003
1989	1049
1990	1119
1991	1152
1992	1191
1993	1278
1994	1395
1995	1465
1996	1550
1997	1614
1998	1628
1999	1653
2000	1689
2001	1811

Graph 2.2.1. HISTORICAL ELECTRICAL ENERGY DEMAND OF ETHIOPIA



CHAPTER 3

FORECASTING METHODS AND COUNTRY PRACTICES

3.1 FORECASTING METHODS

The usual major classification of consumers based on the characteristics of use of electrical energy are:

1. Residential,
2. Commercial,
3. Industrial.

The future requirements for each of these groups are analysed separately, as they are governed by different economic factors and the combined forecast is made by combining these individual forecasts together with the requirements of miscellaneous items like street lighting, agricultural use, water supply, system losses, etc.

Based on various approaches several forecasting methods have been developed for estimating the future demand of a system. These methods have been tried or in use in one or several countries all over the world. Most of the methods are suitable for component analysis such as residential, commercial, industrial, etc. However, certain methods have been developed considering only the total demand of the system. Several methods will be discussed in detail and their specific advantage and limitations shall also be discussed.

The forecasting systems can be categorized as follows:

- Mathematical – Statistical and direct extrapolation methods
- Wholesale method or method of extrapolation and comparison
- Forecasting method based on economic factors
- Input- output method
- Method of norms
- Load survey method
- Appliance and Appliance group method
- Advanced time series analysis method (Box-Jenkins method)

3.1.1 Mathematical – Statistical Methods

The mathematical – statistical method is applicable where adequate statistical data over a sufficiently long period of the electricity used in the past is available. The historical data represents the growth in the use of electricity over past years and it can reasonably be expected that the same growth is likely to continue in the near future. The future growth can be estimated by extrapolating a trend curve that can be fitted to the past date. Both peak loads and energy consumptions are often used for this purpose but the use of latter is most common. In this method the electricity demand is regarded as a function of time without taking in to account its dependence on other economic factors and as such is directly projected into future from its past history. The past data when arranged and plotted with reference to the time factor is a time series and represents the movement of a point under the influence of several economic and other factors. A time series reveals certain characteristic movements or variations some or all of which are present to varying degree [12]. The characteristic movements of time are generally classified into four main types:

i) Long-term or secular trend
ii) Seasonal variations
iii) Cyclical variations
iv) Irregular or random variations.

The statistical method of forecasting is also known as time-series analysis. The time series analysis is a technique for categorizing and studying movements in time series data. A statistical analysis of the past movement of such data enables to (a) determine the past and current pattern of these movements in a given time series and (b) obtain clues about the future pattern of these movements. These clues will be used as an aid to forecasting.

For medium and long-term forecasting purposes the concept of secular trend is used. The secular trend of a time series represents the smooth long-term pattern or sweep in the time series and is represented by a trend curve. For short-term forecasting it is necessary to study the seasonal variations.

- **Trend curve**: The isolation of secular trend is very important for forecasting purposes. The extrapolation of consumption data is appropriate when its growth is basically regular and then it can be assumed that the trend will also continue in the future[19]. The trend can be described by a time function, which averages fluctuations and exhibits a smooth curve. The trend curve is usually obtained by moving averages or by fitting a suitable mathematical function by least square method. The least square method is commonly used as it results in the sum of the squares of deviations of the actual load points from the curve fitted being a minimum. The choice of suitable curve is an important-aspect in the medium and long term forecasting.

Any type of curve can be fitted into past data but the following are some of the curves usually considered [10]:

1) Straight line	$Y = A + BX$
2) Parabola	$Y = A + BX + CX^2$
3) Log Parabola	$\text{Log } Y = A + BX + C X^2$
4) S-Curve	$Y = A + BX + C X^2 + DX^3$
5) Exponential	$Y = Ce^{DX}$
6) Modified Exponential	$Y = A + Ce^{DX}$
7) Combination	$Y = A + BX + Ce^{DX}$

In the above equations Y is the variable to be fitted, X is time in years or months, e is naperian constant, and A, B, C and D are the parameters to be computed by the method of least squares.

- **Selection of trend curve**: Any of the above curves can be fitted to the past data. It is important to estimate the standard error. This enables to make the choice of mathematically best-fitted curve. A curve with higher standard error becomes less preferable for forecasting purposes. When curves have similar standard errors, that giving the best fit to recent data is to be preferred. It is usually found that closeness of the fit is normally the same for most of the curves. There will be little to choose between the curves as representation of the actual data. But when these curves are extrapolated, they diverge and even a small extrapolation may lead to unacceptably large divergences. Therefore, it is very necessary to limit the choice of the curve to those whose slope characteristics agree with the actual data.

A systematic approach [9] using simple exponential curve fitted to data related to selected periods of not less than six years has been found to be best initial approach to an assessment of past growth. The simpler exponential function fitted by the method of least squares has an advantage over all other functions in that it provides a constant annual rate of growth. This is also justified from observing the trends in the growth in electricity demand in advanced countries. Use of simple exponential function enables to approach the problem with manual computations. The use of functions, which provide for varying rates of growth is to be confined to short-term extrapolation of one or two years ahead, unless there is clearly definable reason for their use. But advent of digital computation techniques has enabled to fit in any curve with ease. Generally a mathematically best fitted curve with least standard error and which agrees with characteristic is the curve to be considered as the trend for extrapolating purposes.

Trend curve projections into future should be taken as an aid to forecasting. The time period upon which the trend curve is based must be such that the past pattern in the series will be relevant to the future. Assumptions concerning probable future events and their effect upon the future demand are influential in selecting the final choice of trend curve. Application of human judgment is very essential. This may lead to discard of the curve that mathematically fits the best.

- **Mathematical errors and solution**: In fitting the curve, mathematical errors of prediction may arise due to the following reasons:

  i) The curve chosen may not be the one which best represents the trend or the trend may be changing, ii) The position of the curve will be subject to uncertainty since the parameters are estimated from a limited amount of data and represent only the most probable values,

iii) The actual random variations of the individual points about the fitted curve.

It is rather difficult to project the uncertainty from the first cause. It can only be based on the length of past trends and human judgment. If there is any extraordinary business activities in the recent periods of a set of past data, projecting the trend into future will result in forecasting too large a value. It is also likely that the uncertainty will increase as the period of forecasting is increased, but it is not readily possible to give a quantitative expression to this uncertainty and particular caution should be exercised when predicting beyond five years.

The second and third causes can be estimated statistically and confidence limits based on either normal distribution or student "t" distribution can be established within which the actual value may be expected to fall within a given probability. The assumption that the electricity demand is a function of time will not generally result in perfect correlation. The error of the estimate can be considered as variations due to other factors unaccounted. All such variations can be regarded as due to random errors. These random errors introduce uncertainty into the predictions, which will have to be taken care of. However, the probability distribution of deviations from the predicted value due to the random errors will be Gaussian or Normal distribution. Generally as the data handled for the analysis represents only a small sample, students "t" distribution is assumed. The probability distributions are defined by "mean" and "standard deviation" and it will be possible to predict values from the established trend which will be within the confidence limits. These limit give upper and lower envelops for the trend curve. Generally confidence limits of 90% and 95% are used.

It is essential to recognize that the confidence limit gives a range of uncertainty arising only from the estimation of the curve, and make no allowance for the uncertainty in choice of curve. Hence the doubt about the suitability of the curve will tend to increase the uncertainty of the estimates. Long term forecasting is a hazardous operation and it is difficult to expect high degree of precision. In selecting confidence limits, a balance has to be struck between reducing the risk of error in the forecast by accepting wider limits between which the true value will lie, with the attendant risk of investing capital on idle plant, and increasing the error in the forecast by accepting narrower limits with the risk of failing to meet the demand.

The effect of third cause will be felt when individual values to be expected are needed rather than the trend values. Then the confidence limits will have to be widened to allow for the additional variation of the individual figures about the fitted curve.

- **Seasonal variations**: Seasonal variations represent the recurring pattern of load swings, repeated in each 12 months period. Mostly these are related to the weather and calendar. In order to predict the future monthly values it is necessary to estimate the values of seasonal indices for different months. From the analysis of time series the seasonal indices are computed. A seasonal index is the ratio of the load for a given month to the trend value for the same month. The pattern of seasonal indices amongst the 12 months may be fixed or moving. In a fixed pattern the loads for each of the 12 months will grow at the same rate. That is the annual percentage growth of load for each month is same and each month maintains a constant percentage relationship to the trend movement. The pattern of seasonal indices are obtained by averaging out the individual values of indices for each month observed over a period of number of years. A moving seasonal pattern is one that is changing with time. This represents the rate of load growth for different months as time passes. In such cases it is necessary to establish the growth rate for each month similar to the establishment of secular trend. Only such of the months in the year where the growth rate is considerable need be considered to obtain the moving seasonal pattern. For other months where the change in growth rate is not appreciable, fixed seasonal pattern may be assumed. Predicted trend values multiplied by seasonal indices will give monthly values.

- **Cyclic variations and Random variations**: Cyclic variations refer to the movements of the series about its trend with a periodicity of more than 1 year and up to 5 years or longer. The same is obtained from the analysis of the time series. If the cyclic variation is appreciable, the same is projected into the future by superposing on to the extrapolated trend line with the appropriate periodicity.

  The random variations are due to chance events which are due to non-recurring factors like strikes, abnormal weather conditions, etc. The value of the same can be estimated. This is usually taken care of by the uncertainty expression used for prediction of future trends.

LIMITATION AND USES OF THE METHOD: Time series enables to establish trend, seasonal variation, cyclic and random variations and to project them into future. When all of these are combined, forecast on monthly basis are obtained. By considering only trend and if necessary cyclic movements long term forecast on yearly basis can be obtained.

The accuracy of forecasts from the statistical method will mainly depend upon the accuracy of the past data considered. It is necessary to edit the data before the same can be analysed meaningfully. There are many ways in which the time series may not be consistently defined for the period of time under study. If the effect of inconsistencies in definitions is relatively minor, they may be ignored. In other cases adjustments can be made to remove inconsistencies. But in each time series under study, it is better to know these inconsistencies, so that their importance can be evaluated. Statistical data of the past is an important factor in the application of this method.

The main objection to the direct method of extrapolating the past data into future is its purely formal mathematical character, the future development of the estimated quantity being regarded as a function of time. Then the future demand for electricity depends solely on the instant of time for which the estimate is made. This is not a correct assumption as the demand for electricity is dependent on several economic factors. This can be overcome to a certain extent by proper assumptions as regards to the future and selecting suitable trend curve by the application of human judgment.

The direct method yields good results when the development of electricity use is basically regular. Further the approach is simple and involves analyzing the past data, which is readily available with all the utility undertakings. This method is in common use in almost all countries of the world.

3.1.2. Wholesale Method Or Method Of Extrapolation And Comparison

Different countries may have passed through comparable situations at different periods and it is reasonable to expect that the experience of one country with a particular environment and economic structure may be used to predict the situation in another country where conditions are similar. The wholesale method essentially consists of establishing an average increase in energy consumption from the historical data comparable with the period of forecast by statistical method and comparing the same with the development of other countries with similar economic structure.

LIMITATIONS AND USES OF THE METHOD: Estimation of different stages of economic development and comparing with those of other countries is rather tedious and requires a thorough knowledge and deep study. However, though it is not suggested that the pattern of energy growth during the successive stages of economic development in one country can be directly applied to another, still the historical trends noticed in the developed countries may be regarded as useful information in attempting energy forecasts for newly developing countries. The wholesale method enables a developing country to gain knowledge and experience of a developed country, which has passed the economic development stage, which the former is undergoing presently. The rates of yearly increase for different stages corresponding to specific consumption rates are valuable guides in assessing the long range requirements without much computations.

3.1.3. Forecasting Method Based On Economic Factors

- **Economic factors**: Direct extrapolation methods relate the electricity use with time. However, the demand for electricity is dependent on many economic factors like population, gross national product (GNP), industrial production index, gross investment, raw energy consumption, etc. Of the various economic factors the GNP and the index of industrial production have been found affecting the use of electric energy to a greater extent. The output of many large business units tends to vary with the physical output of the nation's factories and mines or with the monetary value of all goods and services produced. These functions are popularly measured by the index of industrial production and gross national products. These two indicators are related to population, labor force, work-week and productivity. Though the relationships are mathematical in nature, they do not reduce the forecasting of these to a simple mathematical operation. In establishing relationships, it is necessary to make numerous key assumptions and this require knowledge of past stages of the economy and trends of certain relationship.

- **Correlation with economic factors- Simple regression analysis**: In order to correlate the economic factors to the electricity demand, it is necessary to have sufficient and accurate data of the former for the past years. It is also necessary to predict the economic factors for the future within a reasonable accuracy. Then only, it will be possible to establish relationship between electricity use and one or more economic factors and to use this relationship in preparing future forecasts.

Regression analysis by least square method is used to establish necessary relationship. The usual and common approach is towards relating the energy consumption to GNP, or industrial production or population, or national income. This involves the use of simple regression analysis involving a single variable. To fit the past data, a suitable mathematical relationship will have to be selected as in mathematical statistical methods. The relationship, which gives lesser standard error of estimate and higher correlation coefficient is to be preferred. A good correlation has been established in many countries between energy consumption and the GNP of past years. With the correlation established, it is easier to predict the electricity demand of the future by inserting appropriate value of GNP in the regression equation. Similarly good correlation has been found between the electricity demand and index of industrial production.

- **Correlation with economic factors – Multiple regression analysis:** Though relating the electricity demand to an independent economic factor like GNP and index of industrial production has given good results for the past data, it cannot be taken for granted that this tendency will continue in the future. Therefore, this requires a careful study in making long-range forecasts.

GNP can be taken as an indicator of the economic activity as it combines all the products of a nation. In a growing economy and increased national income it cannot be taken that all constituent products project the same growth rate. The economic growth is associated with considerable change in structure: some branches expand more than average and others stagnate or decline(19). For the same rate of growth in GNP, the energy consumption may vary considerably due to shift in the demand for individual goods, which may change their individual growth rates. For industrial production, the electricity demand may vary depending on the extent of human physical energy employed and also the extent of technological advances. The growth of GNP demands growth in the demand for raw energy. Many industrially well-developed countries have observed that two growth rates are not in proportion. Raw energy demand is not increasing as quickly as the GNP, but at the same time the electricity consumption is increasing at a much quicker rate. This is mainly due to progressive mechanization and mainly automation that are taking place in the industrial fields. Another factor is the substitution of various energy forms by electric energy, due to its many advantages(1).

Therefore, the demand for electricity in the future not only depends on the GNP but also on some more economic factors like industrial production, raw energy demand, investment and national income. Hence, it will be better to establish relationships between the electricity demand and more than one economic factor. When it is necessary to interrelate the electricity use with more than one variable, multiple regression analysis is used. In such cases in addition to some economic factors, the time factor may be employed as another variable, which will account for developments preceding more or less depending on time for example, technological progress and increasing use of electricity in house-holds. By mathematical statistical methods accuracy of coefficients and projection can be checked but the accuracy mainly depends on the data to be projected, which needs supplementation and confirmation by means of available technical and economic information. Using the results, future forecast can be made if the values of economic factors for the future are known or can be predicted with a high degree of probability.

LIMITATION AND USES OF THE METHOD: The accuracy of the method lies in estimating and predicting the economic factors. In using this method the economic factors are often predicted by extrapolation of the past data. Then the forecasts based on this method will not be better in any way than the forecasts estimated from direct statistical methods. When more than one economic factor is correlated with electricity demand it is necessary to study the correlation amongst the economic factors. The trend in variation in electricity demand will be affected by the fluctuations in individual economic factors. The measurement of interplay between variables is necessary in order to project the different economic factors. How the trends will be subjected to changes in future is rather difficult to estimate. It is, therefore, natural that uniformity in the nature of relationship sought is not possible. The economic structure depends on many indeterminate local factors and these factors vary with time. Therefore, such a relationship cannot be valid for long periods. The method is suitable for predicting short and medium range forecasts. The use of this method bases the quantum of electricity consumption in relationship with the economic factors, which widely affects the use of electricity. Apart from the difficulty in predicting the economic factors, the computational approach is not much complicated. Use of digital computations has enabled application of multiple regression analysis technique with ease. As almost all the countries are basing their economic and industrial development on well formed plans it may not be difficult to predict the required factors with fairly satisfactory accuracy. Experience, knowledge and judgment play an important role.

3.1.4. Input-Output Method

If it is possible to estimate the growth of different sectors which gives the estimated overall economic growth, then analysis on the sectoral basis will lead to an accurate forecast than simply relating the energy demand to overall growth. In such cases the economic development as a whole may be regarded as the resultant of all the individual processes yet the partial processes are essentially influenced and controlled by the overall development. The satisfactory procedure for this purpose comprises of a complete economic system which takes account of all important inter-relationships between the different sectors and macro-economic activities which sees that the branch forecasts and total economic forecasts are in harmony [19].

Development of economic activity essentially depends on the impact of interrelated industrial branches and the input-output method can be used for the economic analysis. The method describes the exchange relationships within an economy. In order to apply this method it is necessary to know the followings for the study area [15]:

i) The available natural resources.

ii) The expected development of basic industries, mining and manufacturing.

iii) The existing input-output pattern, including the internal mutual relations between different sections of economic activity and the boundary (import-export) conditions.

The input-output method is based on two conditions:

- There must be a definite technical relationship between the input of production factors and the output of product and this must not change in the course of time, or if so, only in a clearly comprehensible manner.
- It is assumed that the input of production is proportional to the output.

For the application of input-output method, it is necessary to determine the final demand of the economy by a separate detailed investigation and solution of this problem is an essential requirement for the successful use of this method for forecasting.

LIMITATION AND USES OF THE INPUT-OUTPUT METHOD: It is a most reliable approach for the economic analysis. But the main disadvantage lies in determination of internal mutual economic relationships which have been found to be difficult, complicated and costly even for a well defined economy with known boundary conditions. A cost of approximately one million dollar has been spent to study the U.S. national economy using input- output analysis.

The basic assumptions on which the method is based are also the limitations for the application of this method for forecasting. Even disregarding random fluctuations, constant coefficients cannot be assumed over long periods. However, if the input- output tables for a number of years are available, they can be used to check how the technical coefficients vary in the course of time, which in turn remove to a large extent limitations of this method.

A complete econometric model as is required can only be got up provided that the basic information is already well developed. In many countries, the empirical studies are still in the initial stages, data on the development mechanisms of the branches of the economy are incomplete and unsatisfactory so that there is no practicable basis for a forecast with the aid of input-output method.

Sometimes, a less perfectionist method called iterative method is used for analysis which starting from previously estimated overall economic growth, sets up a series of models for the individual sectors and branches, the individual models then being adjusted to one another within the overall frame-work by stepwise approximations.

Though at present the use of input-output analysis is rather very much limited, there is every likelihood of this method becoming powerful tool in forecasting, as additional statistical data become available.

3.1.5. Method Of Norms

The method of norms is essentially based on planned output in individual economic sectors and the specific needs per unit output. The developing countries are trying to improve their socio- economic conditions at fast rate. The rate of growth depends on the availability of natural resources, technical knowledge and capacity to exploit the same, financial resources, aids and loans from developed countries and many other factors. In order to implement the economic development, many countries have resorted to natural plans of varying periods. These plans cover the development in all field of activities all over the country and also efforts have been made to remove the regional disparities. All these may lead to the growth in different economic activities to take place at different rates than they were in the past. Under such circumstances, predicting the electricity demand from analyzing the historical data may not give good and satisfactory results and it will be necessary to forecast the demand for electricity on a comprehensive basis depending on the development of all sectors. Therefore, it is necessary to collect the data regarding the planned activities in all sectors. The electricity demand anticipated for the various industries and other energy consuming sectors is worked out, having due regard to the development in different local areas and regions in the country. These estimates are suitably integrated to give the total demand. The demand in the domestic sector and other non- industrial sectors should also be taken into account. It is also necessary to phase the demand in addition to estimating the total requirements(18).

THE NORMS: In order to prepare the estimates of future electricity requirements on the basis of planned development of different sectors, it is necessary to establish norms i.e., the specific electricity consumption per unit of production in appropriate units. The method of norms is based on the volume of productions enumerated in the plans and the average amount of power consumed in KWH per unit of physical production for various classes of consumers. The total sum of production of these two gives the total requirement of electricity. Generally the norms are expressed in terms of KW or KWH per unit of production. It is necessary to estimate the total requirements both in KW demand and total KWH consumption. This will also necessitate the prediction of either demand factors or diversity factors and the load factors of different groups of consumers. To establish proper norms a detailed investigation of different consumer groups for specific energy consumption and past development is necessary. It is also necessary to consider the latest technological progress affecting the rate of specific energy consumption in different fields of economic activities. The essential requirement for this approach is the availability of the maximum amount of basic information. The average power consumption per unit of product may increase or decrease with the changes in production technology, quality of raw materials, quality of product, extension and automation of production processes and changes in geographical conditions.

Generally the norms tend to rise as manual process in industry are replaced by mechanization and automation and improvement of working conditions and replacing other forms of energy by the use of electric energy. The norms generally tend to increase in the initial stages of development and start decreasing when the industry is well established and full production. In addition to the establishment of norms the production for separate branches of economy for the forecast period, volume of industrial production, magnitude of construction work, amount of goods to be handled by railways including electrified railways etc are required. The forecasts are made separately made for 1) industrial, 2) construction, 3)transport including electrified rail transport, 4) municipal and domestic consumption, 5) agriculture, and 6) others.

The accuracy of forecast greatly depends on the norms established and also on estimating the volume of production of each class of consumers. Therefore, detailed investigation coupled with care and proper judgment are necessary to evaluate the same. Where the norms are established on ad-hoc basis, efforts should be made to revise them as and when sufficient information based on experience is available. In a major industry the electricity requirements must be analysed in detail on itemized processes involved.

LIMITATIONS AND USES OF THE METHOD: For a country basing economic development on well laid national plans, this is a very reliable method. However for long-range forecasts the accuracy of the method is governed by the uncertainties of the nature of future planned development, for example the importance of development may be switched over from one sector to another, possibilities of substitution of one energy for another. The cost of production will also influence demand growth rate. Apart from accuracy in establishing norms for specific electricity consumption, the accuracy of the forecasts mainly depends on the extent to which the targets of the planned development are achieved.

To assess the demand of domestic sector by the method of norms is rather a difficult approach. The effect of national plans and policies on the use of energy sources is not easy to assess in the domestic sector. This is further complicated by the fact that in under-developed and developing countries the use of non-commercial energy forms a high percentage. The impact of electricity can at best be ascertained by conducting at least sample survey.

3.1.6. Load Survey Method

The method is based on the surveys made for assessing requirements of electricity for different consumer group area wise or sector wise. This involves conducting extensive field surveys covering the entire area and sectors coming under the command of the power system. The approach is to get the requirements straight from consumers. During the surveys the data for assessing the demand for electricity should be collected from various sources with particular attention to the resources of the area, possibility of exploitation of resources, transport facilities, attitude of community towards progress, possibility of substituting the electricity for other sources of energy demands, the degree of development that has taken place in the area, growth in population and standard of living, etc [13]. The survey will yield a preliminary assessment as to the electricity demand and its phasing. It is necessary to review this in order to make allowance for the future developmental plans in the area and other probable influencing factors.

Data collection by surveys: The load survey must be planned carefully and conducted completely in order to obtain useful data. Surveys should be such as to provide data of sufficient accuracy for the purposes of the study and to obtain the data as economically as possible. The object of a survey should be translated into a series of specific questions before meaningful answers can be obtained. Problems like the most effective means of obtaining the information, the extent and cost of the survey, etc. should be solved and then the questionnaires and other data collection forms must be developed. It may be necessary to train supervisors, interviewers and other personnel. The actual collection of data must be checked for accuracy and consistency and the data must be processed so that it can be effectively adopted for preparing the forecasts. Depending on the circumstances and importance, the surveys may be conducted on the basis of simple or whole. Well planned and conducted sample surveys often result in reliable collection of data at a very reasonable cost and a quicker time.

Method of obtaining the information from the respondents:

i. By personal interviews – In this procedure supervisor or trained persons interview individuals, industries, development boards and organizations, etc., on the basis of prepared questions and records the respondent's answer. This approach usually gives a high proportion of usable information as most people respond to a survey when they are approached directly. This also enables collection of relevant supplementary information

ii. Questionnaires – Individuals, industries, various boards and organizations and others are approached through well designed questionnaires, soliciting their co-operation. This approach is cheaper but generally the response is poor.

iii. Observation method – Some of the data is obtained by observing trends in the use and growth rate of electricity consumption.

iv. Registration – Necessary information is obtained by asking individuals and other agencies to register their requirements in pre designated areas.

Requirements of load survey: A load survey should be done comprehensively to assess the electricity requirements for domestic lighting, heating, small power, street lighting, industrial power – small, medium and large, irrigation pumping, etc. It is also essential to establish suitable norms for these in order to supplement the information collected by the field survey as the consumers may not be able to express their specific requirements. Before a load survey is conducted for collecting field data, it is necessary to make a study of the region to be visited as to its topography, weather conditions, rainfall, total number of towns and villages to be visited and their population, assessment of various industries run on steam and diesel driven plants which could later be converted to electric driven and such other particulars that are available from reports, etc.

LIMITATION AND USES OF THE METHOD: The method is well suited for planning the electricity supply in developed areas. The advantage of load survey is its ability to assess the demand area-wise, which is very essential for planning the distribution system and location of substations and their subsequent expansions. The forecast generally holds good for shorter periods. However, these tend to give lower values for longer periods. Load surveys are costly and unless planned and executed completely the results are subject to lot of inaccuracies and uncertainties. The person conducting interviews should be competent as otherwise, he may himself be the prime source of inaccuracy.

For any country it is a necessity that load surveys be conducted initially at least thoroughly and supplemented by periodical sample surveys as and when required.

3.1.7. Appliance And Appliance Group Method

These methods are essentially used to prepare forecasts for the residential energy requirements. For the analysis the average use per customer is made use of.

- **Appliance method:** The method represents the average residential energy use in terms of uses by the various electric appliances. The estimated saturation of each appliance multiplied by estimated annual energy provides the contribution of that appliance to total average annual residential use. The contribution of individual appliances is summed to obtain annual average use per customer.

The appliance saturations are to be obtained from appliance surveys, census reports, and study of relationships of saturation to energy use levels. The estimate of annual use for the individual appliances is to be obtained from a variety of sources like Government agencies, electric utilities, manufacturers of appliances and metering of appliances. Proper and useful judgment is to be made in selecting the use level from the data obtained from the different sources.

A historical model for the appliance method is prepared for each year for which past data is available by working out the contribution of each appliance and summing the resulting contributions to total average use. This is checked with the actual average use. Adjustments are made in the individual contributions if necessary to correlate with actual use. The saturation values and average use for past years for each of the appliances are analysed and projected to give the values for the future from which the forecasts of total average use are made.

The appliance method allows to provide for the impact of new electric appliances which may appear in the market. Historical development of markets for new appliances is a valuable tool in approaching the problem. The effect of introduction of a new appliance on total average use can be made by assuming an annual energy use and a growth rate of saturation. The forecasts of average use estimated can be reviewed in comparing with the resulting trend of historical average use.

- **Appliance group method:** This method divides the residential customers into groups according to ownership of certain electric appliances. Estimated percentages of customers in each group are multiplied by estimated average use for each group to obtain the contribution to total average use by groups. From the estimated contribution to total average use for all groups, the total residential use per customer is obtained. The customer grouping can be made as follows:

  Group I - No major appliance
  Group II - Refrigerator only
  Group III - Refrigerator and range
  Group IV - Refrigerator, range and water heater
  Group V - Refrigerator, range, water heater, house heating
  Group VI - Range only
  Group VII - Water heater only
  Group VIII - Refrigerator and water heater
  Group IX - Range and water heater.

The number of groups can be reduced by grouping the last four groups and treating them as odd combination of major appliances. The cooler loads are also to be incorporated. The necessary data will have to be obtained by conducting surveys in the area served. The method and analysis is on the lines of the appliance method.

LIMITATION AND USES OF THE METHOD: The methods are used to assess the residential requirements only. The appliance method measures total residential average use in terms of degree and extent of appliance use and is subjected to analysis in terms of individual components. However, it is a bit difficult to estimate the saturation and energy uses for the appliances, The method does not take into account other related factors like income level, trends in residential construction which have a bearing on appliance use. The appliance group method measures total residential average use in terms of customers classified ownership of major appliances. It provides an opportunity to consider energy use characteristics of all electric home and of lower power use homes. The application of the method requires detailed or sample surveys at least periodically.

3.1.8. Advanced Time Series Analysis (Box-Jenkins Method)

The model building approach method is complex, mathematically sophisticated, and expensive; but in many situations, it can be the most accurate technique for forecasting with time-series data [16][20]. It is best used where there are only a few time-series relationships important enough to justify the expense. In pursuing this approach to a time series, answers must be provided to three questions:

1. What class of models might be considered as possible generators of observed time-series?
2. How should the analyst proceed to fit a specific model from the general class to a given data set?
3. How are forecasts of future values developed from the fitted model?

In an extremely influential book, Box and Jenkins attacked the forecasting problem within this framework(14). They proposed the use of a class of models called Autoregressive Integrated Moving Average (ARIMA) models, and developed a methodology for fitting to data an appropriate member of this class.

Several years of data are necessary to apply the Box-Jenkins method. Because of its complexity, computer facilities are necessary for its application. The following describes the general mechanics of this technique.

The Box-Jenkins technique can function with complex data patterns and that the forecaster does not have to describe initially these data patterns. The Box-Jenkins model systematically eliminates inappropriate models until the most suitable one is left for the data being considered. A three-step procedure of identification, estimation, and diagnostic checking is used to arrive at a specific model. This procedure requires that the forecaster have a sound mathematical background and access to computer facilities. Because autocorrelation is an integral part of the Box-Jenkins method, it will be explained here and then the general feature of the method are presented.

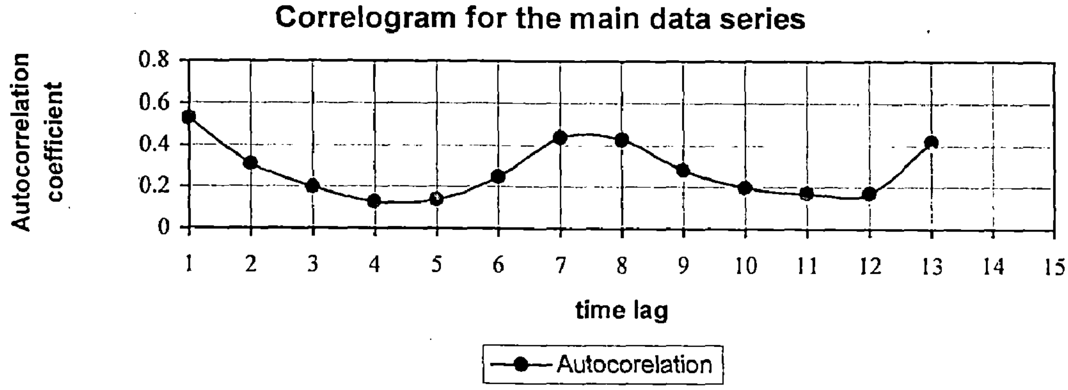
Autocorrelation can be explained best through an example. Raw data are presented in column (2) of table 3.9.1. In column (3), the same data are given, but one time-unit of phase with the data in column (2). That is, the first point in column (3) is the second point in column (2), and so on. A correlation coefficient calculated for these two columns of data is the autocorrelation coefficient for the data with a time lag of 1. Column (4) presents the data with a time lag of 2. The correlation coefficient calculated for column (2) and (4) is the autocorrelation coefficient for a time lag of 2. With seasonal data, an autocorrelation coefficient for a time lag 12 with monthly data reveals the strength of the seasonality.

Table 3.9.1 ILLUSTRATION OF LAGGED TIME SERIES DATA FOR COMPUTING AUTOCORRELATION COEFFICIENTS.

Period t (1)	Z_t (2)	Z_{t+1} (3)	Z_{t+2} (4)
1	10	11	13
2	11	13	15
3	13	15	14
4	15	14	12
5	14	12	13
6	12	13	15
7	13	15	16
8	15	16	17
9	16	17	15
10	17	15	16
11	15	16	-
12	16	-	-

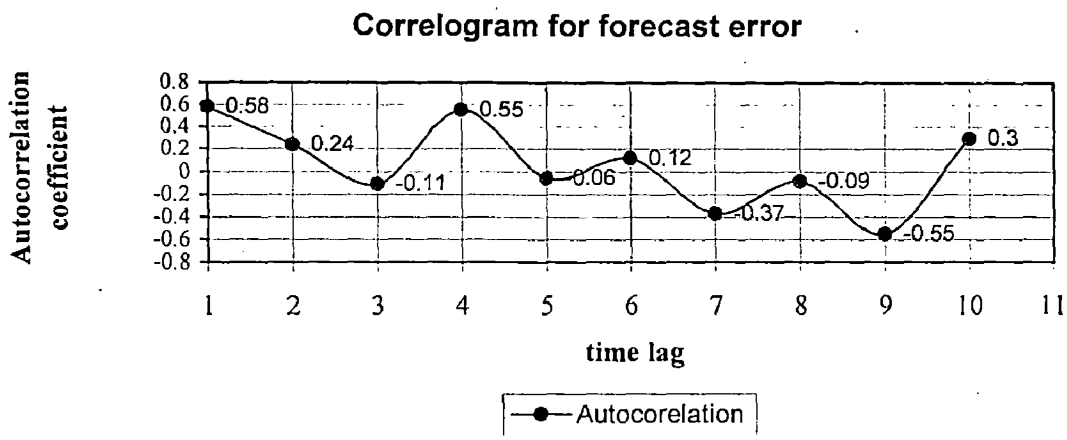
By considering time lags of 1,2,3,4... and plotting the results, as in Figure 3.1.8a, the autocorrelation in the data can be defined. The data of Figure 3.1.8a exhibit some characteristic that causes a moderately strong relationship between observations, which have been lagged by six periods. These occurrences are taken into account in the Box-Jenkins procedure. Usually the autocorrelation coefficient for a one-period lag is high, indicating the presence of serial correlation. If all autocorrelation coefficients are low, this would indicate random data.

Figure 3.1.8a CORRELOGRAM FOR THE MAIN DATA SERIES



After applying a forecasting technique, the error terms, can be analyzed by using autocorrelation analysis in exactly the same manner as the raw data were analyzed in Figure 3.1.8a. An example plot of autocorrelation coefficients for a_i values is given in Figure 3.1.8b. This indicates that forecast errors with time lags of 1, 4, 9 are related. The presence of autocorrelation in the a_i values indicates there is information in the raw data that was not utilized in making the forecast. A forecasting technique that makes the best use of available data should have random error terms.

Figure 3.1.8b CORRELOGRAM FOR THE FORECAST ERROR



The Box-Jenkins method has three basic building blocks, which are termed autoregressive, moving average, and mixed. In the autoregressive relationship, an equation such as the following is used to develop a forecast based on a linear, weighted sum of previous data:

$$Z^* = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \ldots + \phi_p Z_{t-p} + a_t \qquad (3.9.1)$$

Where Z^* is the forecast

Z_i (i= 1, 2, ….,p) is the observed value at time i

ϕ_i is the weighting coefficient for the pth previous period

a_t is the expected forecast error at time t or white noise

The weights and a_t values are determined by using multiple regression analysis, hence the name autoregressive.

The second relation ship is that of a moving average in which the forecast is a function of previous forecast errors.

$$Z^* = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots - \theta_q a_{t-q} \qquad (3.9.2)$$

where Z^* is the forecast

a_{t-i} (i= 1,2,….,q) is the white noise at time t

θ_i (i= 1,2,….,q) is a weighting coefficient for the qth previous period, which is calculated by a nonlinear least-squares method.

The third relationship is a combination of the first two, referred to as the mixed autoregressive moving-average model. The extension of the basic building block is called an integrated one and applied whenever the given time series data shows a serial correlation.

In applying the Box-Jenkins method, the analyst must examine a basic autocorrelation plot such as that of Figure 3.1.8a and decide which time lag should be considered in the model. A computer program is then used to calculate the coefficients for the equations listed above. The results of applying these equations to available data can be diagnosed to determine if the error terms are random and normally distributed. If they are not, the basic model is modified (inclusion of additional terms and/or a different relationship). Then the above procedure is repeated until a satisfactory model has been developed. Full description of the method is presented in Chapter IV and the model is practically applied for the Ethiopian Electric Energy Demand in Chapter V.

LIMITATION AND USE OF THE METHOD: Forecasts of future observations are to depend on the observed time series data; that is, the forecasts will be functions of the given data, where the particular function used in any implementation will be determined by the model that has been fitted. Now, in principle, there are infinitely many possible forecast functions from which a choice could be imposed before much progress can be made. The Box-Jenkins ARIMA model-building approach is based on the following two restrictions:

1. The forecasts are linear functions of the sample observations,

2. The aim is to find efficiently parameterized models- that is, models that provide an adequate description of the characteristics of an observed

time series as few parameters as possible, which is known as the principle of parsimony.

The restriction to linear forecast functions is not something that is a priori desirable. Rather, it is very often a practical necessity. Given a limited amount of data, it is simply impossible in any systematic way to investigate the merits of all forecasting functions. The set of linear functions still provides a very rich class of possibilities, and should prove adequate for most purposes. An excellent strategy in building models for forecasting is to seek the simplest possible model- that is, the model with the fewest parameters- that appears to provide an adequate description of the major features of the data. In this way the analyst can have some faith that the achieved model depicts genuine regularities in the data generating process, and hence that it should provide reliable forecasts.

3.2. THE MAIN METHODOLOGICAL APPROACHES AND COUNTRY PRACTICES IN ELECTRICAL ENERGY FORECASTING

3.2.1 Main Forecasting Methodology

Based on the forecasting methods discussed in section 3.1, the following four main methodological approaches are used in most of the developed countries(5)(17) and the relative advantage and disadvantage of each method is also summarized.

1. Time-series model,
2. Econometric model,
3. End-use model,
4. Hybrid models(economic/end-use model)

Table 3.2.1 **ADVANTAGES AND DISADVANTAGES OF ALTERNATIVE FORECASTING TECHNIQUES**

TIME – SERIES

Advantages

Minimal data requirements /Least data intensive/

Low cost

Can predict seasonal and daily patterns also

Disadvantages

Does not treat underlying factors explicitly

ECONOMETRIC

Advantages

Explicitly models underlying influences on energy demand

Basic on explicit theory of consumer behavior

Less data intensive than end-use models.

Disadvantages

Requires high skill level to develop models.

Difficult to address some policy issues.

Sometimes difficult or impossible to identify individual variable impacts (e.g., multi- collinearity)

END - USE

Advantages

Good policy analysis capabilities

Relatively understandable

Disadvantages

Often lacks endogenous behavioral component

Data intensive

Costly

HYBIRD

Advantages

Better behavioral component than pure end-use models

Better policy analysis capabilities than most econometric models

Disadvantages

Data intensive

Costly

Ad hoc nature can make interpretation difficult.

Can lack efficiency and elegance

3.2.2. Discription Of Country Practices

3.2.2.1- *Australia*

With regard to methods of forecasting electric energy consumption the Sydney county council in the case of domestic consumption carries out extensive survey to obtain patterns of appliance consumption to use in conjunction with appliance saturation and projected housing developments. In the case of industrial consumption, surveys are made of major users to obtain details of projected developments and anticipated energy consumption. It has been found that for commercial development the relatively short lead-time for projects renders surveys unsatisfactory.

3.2.2.2 - *Belgium*

The forecasting procedures in Belgium are very much decentralized and therefore heavily influenced by the structure of the electricity sector. There are four generating companies enjoying regional monopolies. Each makes its own forecasts. The planning of new equipments is however national, and thus based on the sum of company forecasts.

Each company has two types of customers.

- the large industrials connected in 30 kV or above
- the distribution units.

The forecasts of future load for industrial customers is based:

- for short run forecasting on estimates made by the industrials themselves. They are questioned every year on their planned growth;
- for longer run forecasting the companies estimated the growth on basis of past trends plus additional information (shut down of plants or starts of new ones.....).

The forecasts of the load for distribution customer units is subdivided into low and high voltage. Each distribution unit makes its forecasts for those two categories. The forecasts are usually made by the manager of customer services. They are based mainly on past experience. Additional knowledge derived by the local managers from recent trends in lodgings and equipments may be inserted into the forecasts on a subjective basis.

3.2.2.3- ***France***

The forecasts model used by EDF belongs clearly to the end use approach. Econometric relations are rare, and used for instance in the domestic sector to forecast a residual item in the demand related to specific uses of electricity or as a check for the total of this demand normally dealt with by end uses (6 categories of appliances lighting).

Space and water heating is treated in great detail with a breakdown into three different markets: new dwellings, first equipment and, substitutions in existing dwellings. In this last case, a further breakdown is done by kind of heating systems (direct heating and several categories of bi-energy system).

In the tertiary sector, eight subsections are distinguished. For each subsection, specific uses on one hand, space and water heating on the other hand, are treated separately. For these last uses, the modeling is similar to the one used in the domestic sector (new building interfuel substitutions, different kinds of heating system).

In industry the consummations of 29 subsections are analyzed by end uses (5 types).

3.2.2.4 - *Germany*

It exists in FRG a number of different methods, and of different forecasts, of electrical energy consumption, used or built either by institutes or by utilities. The forecasts may differ considerably from each other, which can be explained in part by the methods used, but mostly by the differences in the hypothesis used. For the eight utilities belonging to the "Deutsche verbundgesellscaft", and representing more than 90 percent of power generation, the methods used remain often very simple, relying mainly on trend extrapolation – even if more advanced methods are used by some of them, including sectoral analysis, econometric- relationships and end-uses considerations. In several cases, forecasts for the major industrial consumers are delt with separately, involving exchange of information with those consumers. Lastly, some utilities do not make directly a forecast of power consumption, but derive it from forecasts (most often based on trend extrapolations) of the maximum load and of its yearly utilization-time.

3.2.2.5 - *Italy*

The model MEDITA used is of the "Medee" type, which can be considered as an end-use approach particularly well adapted for the building of scenarios. In industry, consumptions are derived from values – added at factor costs and electrical intensities. Products with high electrical intensities (a number of raw materials) are analysed in more detail. The tertiary sector is broken down in seven subsectors. The analytic approach of the tertiary sector is compared with equations relating electrical energy consumption to the number of employees and/or economic growth.

The domestic sector consumption is analysed by kind of appliances. Use is also made of a relationship between the number of inhabitants per dwellings and the income per head. For transports, the analysis is carried on with a great detail by means of transport, and the consumption linked to the volume of transport of goods.

3.2.2.6 - *The Netherlands*

In the Netherlands the long term forecast of electrical energy is based on a mixture of end-use and econometric methods. SEP uses the economic information of the Dutch Central Planning Board, completed with information of different industrial branches.

The industrial sector is split up in 7 categories if the econometric method is applied, and in 14 categories if the end-use method is used. In the end-use method, special surveys are made about the expectations of the big individual consumers of electrical energy. The outcome of both methods has to result in a coherent and consistent forecast for each of the industrial branches.

The forecast for the residential and commercial sector is based upon an end-used method. The residential demand is not spilt up in several house types etc... because there is hardly any electric heating. The commercial sector on the contrary is divided in 15 categories.

3.2.2.7- *Sweden*

Different approaches are adopted in different consumer sectors, but the dominating method is to rely on end-use models in most of the sectors, i.e. in industry, electric heating and household electricity. In the tertiary sector (electric heating excluded) simpler form of econometric methods is used. The end-use models are complemented with consumer behaviour studies, mainly in the electric heating area.

The level of disaggregating is normally high: 10 to 30 sectors in industry, about 20 sectors in electric heating and about 20 to 30 sectors in the tertiary sector. For households electricity consumption, appreciatively 30 different types of electrical equipment are studied.

At the national level, the method used deals simultaneously with the demand for the different kinds of fuels, and thus with explicit market shares assumptions. These assumptions are based on economic comparisons between competing systems and the former development of market shares. This multi-energy approach combined with an end-use modeling framework is particularly necessary for the heating market, which is not easily dealt with by econometric models.

3.2.2.8- *United States of America*

Every conceivable type of electric energy forecasting method is employed in the U.S.A. Many of the smallest utilities still employ simple trend analysis. Small and mid-sized utilities often use on econometric-based model, which may be disaggregated into residential, commercial and industrial sectors. Among the more progressive small utilities and for many large utilities, end-use and integrated engineering-econometric end-use models are growing rapidly in popularity.

Through the work of the Electric Power Research Institute (E.P.R.I.) in modeling and analysis, the constraints toward the use of these advanced techniques have been narrowed to the availability of data. Models estimated on national and regional data are available and data collection efforts are mushrooming as many utilities move to conduct surveys of their residential and commercial customers. To date, most desegregation is at the end-use level (e.g., lighting, heating, cooling, cooking).

Current interest is in continuing work in disaggregating down to technology levels (e.g., all major type of lighting, types of heating, etc.)

An example of a time-series model that has been used by a utility in the Pacific Northwest is shown in the following equation:

$$\log (E_t) = \log(E_{t-12}) + \log(E_{t-1} / E_{t-13}) + e_t + a_2\, e_{t-1} + b_2\, e_{t-12} + a_2\, b_2\, e_{t-13} \qquad (2)$$

In this example, the period (t) is equal to month. Thus, the equation says that to forecast energy consumption in month (t), start with consumption in the same month during the previous years. To this, add the percentage change over the year in consumption for the previous month.

Finally, use the errors in predicting past growth rates to adjust the current growth rate. In the long run, this model will forecast a constant percentage growth in sales from a given month to the same month in the next year.

Time-series methods have several advantages relative to the other primary tools used to forecast energy sales and demand. They are the least data intensive of all of the primary techniques, and therefore are relatively inexpensive to develop and use.

CHAPTER 4

FORECASTING BY TIME SERIES ANALYSIS

INTRODUCTION

The forecasting methods discussed in Chapter III are of different levels of sophistication varying from simple extrapolation to complex econometric models. These models are mainly dependent on past trends as well as economic and demographic parameters. Here we come across the difficulties in models specification and unreliability of the variables involved. Forecasting of the economic parameters is a major exercise in itself and uncertainties associated with future values of these variables will lead to further errors in the forecasts, specially when the method is developed based on a very limited information. The selection of the model depends on several factors, such as quality of the forecast, availability of data, ease of the application and above all the cost involved. These factors are not necessarily compatible, calling for a search of costs effective compromises or variations. An appropriate choice in this regard is offered by the Time Series Analysis, which produces fairly accurate forecasts. The only information required is past energy data, which is readily available.

Time Series Analysis

As discussed in Chapter III, the time series is a set of observations generated sequentially in time. In analysing the time series it is treated as a realization of a stochastic process, which in turn is statistical phenomenon that evolves in time according to probabilistic law [6]. The past data available is treated as a portion of an infinite time series produced by the underlying probability mechanism of the system under study. In the analysis stress is given to data characterisation rather than investigations on cause and effects. Errors in forecasting are affected only by methods of the projection on data characteristic and not by the lack of knowledge of factors in a model or errors in model specification.

Several methods are available for the analysis such as moving average, Exponential Smoothing, Stochastic modeling etc. The commonly used time series methods are:

1. Moving average
2. Exponential Smoothing
3. Holt's Linear Trend Algorithm

4.1 MOVING AVERAGE

The demand forecast by moving average, $\hat{x}_t$, given all values up to time t-1, is

$$\hat{x}_t = \frac{1}{n}\sum_{i=1}^{n} X_{t-i}$$

where

n = number of years to be averaged (= 3 or more)

X_t = Observed data at time t

The simple moving average technique can provide useful information and insight for the forecaster by smoothing a time series so that the impact of the irregular component is damped. Forecasts of the individual components could then be amalgamated to produce forecasts of the overall time series. This notion has a good deal of intuitive appeal, but turns out to be much less attractive when one begins to explore in detail how such an approach might be put into practice. In part difficulty arises because of the arbitrary nature of the moving average scheme chosen to estimate the components, and in part it is often unclear how best to predict the trend-cycle component from these estimate.

4.2. EXPONENTIAL SMOOTHING

For the single exponential smoothing, the forecast $\hat{X}_t$ is given by

$$\hat{X}_t = \alpha X_{t-1} + (1-\alpha)\hat{X}_{t-1}$$

where

$\hat{X}_t$ - Current forecast level

$\hat{X}_{t-1}$ - Previous forecast level

X_{t-1} - Previous actual

α - a smoothing constant

The exponentially smoothing is a special kind of moving average that does not require the keeping of long historical record. The moving average technique assumes that data have no value after n periods. Some value (although possibly very little) remains in any datum, and a model that uses all the data with appropriate weightings should be superior to a model that discards data.

Like most forecasting techniques, exponential smoothing uses historical data as its prediction basis. It is a special type of moving average where past data are not given equal weight. The weight given to past data decrease geometrically with increasing age of the data. More recent data are weighted more heavily than less recent ones. The major advantage of this method is that the effect of all previous data is included in the previous forecast figure, so only one number needs to be retained to represent the demand history.

The exponential smoothing approach to forecasting is more ad hoc in character. Models are not explicitly built. Rather, a collection of intuitively plausible prediction algorithms that proved useful in practical applications has been assembled.

The better approach to the problem of forecasting by time series analysis shall involve the formal fitting to data, through efficient statistical techniques, of a model: the specific model achieved would then suggest how the components of a series should be estimated. This could be achieved by using Box and Jenkins method. Alternatively, as will be discussed in detail in the Box and Jenkins method, individual models for the constituent components might be postulated. In either case, the properties of the data will suggest an appropriate method for estimating components, rather than relying choice of some moving average scheme. As will be derived in chapter VI, the exponential smoothing can also be taken as a special case of the more general Box and Jenkins model.

4.3. WINTERS SMOOTHING METHOD

The simple exponential smoothing algorithm yields constant forecasts for all future values of a time series. In some situations, the observed data will contain information that allows the anticipation of future upward or downward movements like electrical energy demand. In this categories rather than a constant forecast function, some trending function would be preferable. Holt developed an exponential smoothing for local linear trend in a time series. Here the current level and current slope(change in level) need to be estimated. The recurrence form of this algorithm is

$$L_t = \alpha X_t + (1-\alpha)(L_{t-1} + T_{t-1}) \qquad 0 < \alpha < 1$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1} \qquad 0 < \beta < 1$$

$$\hat{x}_t = L_t + T_t \qquad \hat{x}_{t+n} = \hat{x}_t + (n+1)T_t$$

where L_t and T_t are estimates of level and trend,

α and β are smoothing constants

$\hat{x}_t$ is the forecast for time period t

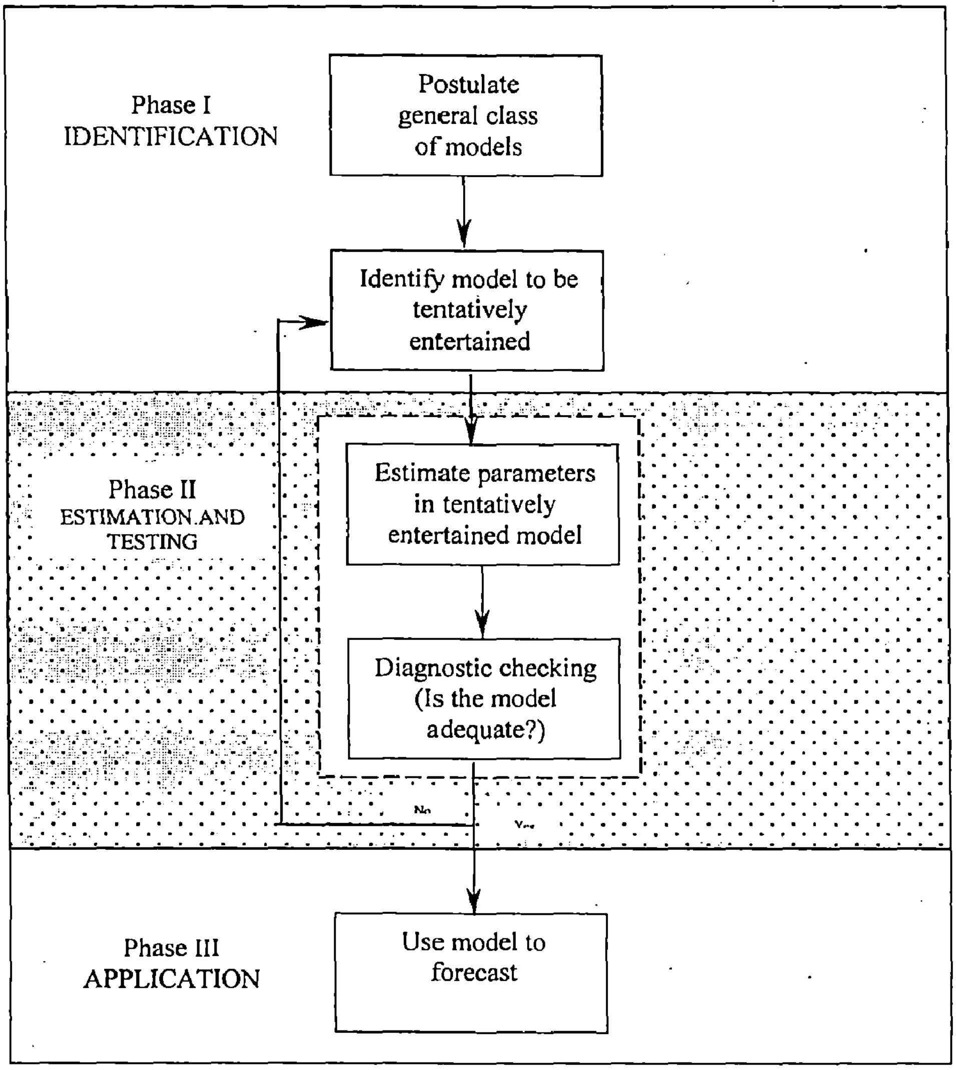
$\hat{x}_{t+n}$ is the forecast for n periods beyond the current period t

4.4. BOX-JENKIN STOCHASTIC MODELLING

G.E.P. Box and G.M.Jenkins have presented a detailed procedure in their Book 'Time Series Analysis Forecasting and Control'(6), for analysing and forecasting time series. The procedure consists of methods for building, identifying, fitting and checking stochastic models for time series and forecasting the future values. Provision is made for reviewing the forecast and updating the same periodically. Another feature of the methods is that the models developed by the method are 'parsimonious' i.e., only a minimum number of parameters are needed to describe the properties of the process.

According to Box and Jenkins 50 to 100 observations should be used for analysis by this method. However if past history of 50 or more observations are not available, the procedure is to build a preliminary model using the available data and past experience. This model may be update from time to time as more data becomes available. For clear understanding and applying of the Box and Jenkins model building method the concepts of stationarity and autocorrelation discussed in the following section.

Schematic representation of the Box-Jenkins approach



4.4.1. Stationarity and Autocorrelation

In building stochastic model for time series, we are in effect viewing an observed time series as a set of realizations of random variables. A useful way to think of this is to imagine a very large number of ordered lists of values. The particular time series that we actually see can be regarded as one of these lists, chosen at random.

When it is claimed that information on the past behavior of a time series is some value in predicting the future, the implicit assumption is that there is some regularity in the process generating the series. A frequently valuable way to view such regularity is through the concept of weak stationarity, which is the relevant variant when attention is confined to linear predictors.

The time series Z_t is said to be weakly stationary if the following three properties are satisfied:

1. The mean of the series is the same at all points in time. Then, writing this fixed mean as μ, we have

$$E(Z_t) = \mu \quad \text{for all t}$$

2. The variance of the series is the same at all points in time. Thus, if σ_z^2 denotes this fixed variance, we can write

$$\text{Variance } (Z_t) = E\left[(Z_t - \mu)^2\right] = \sigma_z^2 \quad \text{for all t}$$

3. The covariance between any two values of the series depends only on their distance apart in time, not on their absolute location in time. Then,

we can denote by γ_k the covariance between any two values separated by k time periods, so that

$$\text{Cov}(Z_t, Z_{t-k}) = E[(Z_t - \mu)(Z_{t-k} - \mu)] = \gamma_k$$

$$\text{Cov}(Z_t, Z_{t-k}) = \text{Cov}(Z_t, Z_{t+k}) \qquad \text{for all t}$$

It is still possible to relax, or modify, the assumption of weak stationarity and still make some progress in the analysis of a time series. However, some simplifying assumption of this sort is obviously essential.

The basic aim of the forecaster is to seek to explain what will happen in the future in terms of what is already known. In this sense, the mean and variance of the time series are of limited value. The most relevant information is likely to come from the covariance γ_k, defined in property (3) above.

The set of covariances γ_k are called the autocovariances of the series. The assumed stability over time of these covariances allows their exploitation in forecasting, and also allows their estimation from sample data. Given an observed time series, Z_1, Z_2, ..., Z_n, natural estimates for the mean, variance and autocovariances, given an assumption of weak stationarity, are provided by

the sample mean

$$\bar{Z} = \frac{1}{n}\sum_{t=1}^{n} Z_t$$

the sample variance

$$s_z^2 = \frac{1}{n}\sum_{t=1}^{n}(Z_t - \bar{Z})^2$$

and the sample autocovariances

$$c_k = \frac{1}{n}\sum_{t=1}^{n-k}(Z_t - \bar{Z})(Z_{t-k} - \bar{Z}) \qquad k = 1,2,\ldots K$$

Covariances are difficult to interpret since their magnitudes depend on their units of measurement of the data. It is invariably preferable to work with correlations, which provide scale-free measure of the strength of linear association. The correlations between values of a time series separated by k time periods are called the Autocorrelations of the process, and denoted by ρ_k, so that

$$\rho_k = \text{Corr}(Z_t, Z_{t-k})$$

$$\rho_k = \frac{Cov(Z_t, Z_{t-k})}{\sqrt{Var(Z_t)Var(Z_{t-k})}} = \frac{\gamma_k}{\sigma_z^2}$$

since, by property (2) of weak stationarity, Zt and Zt-k have common variance σ_z^2.

The autocorrelations of a weakly stationary time series obey the following:

1. Since the correlation between a random variable and itself is necessarily one, it follows that

$$\rho_0 = 1$$

2. Given stationarity, the correlation between Zt and Zt+k is the same as that between Zt and Zt-k, so that

$$\rho_{-k} = \rho_k$$

The autocorrelations of a time series provide the natural framework for studying and summarizing linear association between observations separated by various amounts of time.

4.4.2. Operators

Box and Jenkins have introduced certain simple operators for carrying out the modeling.

(1) Backward Shift Operators (B):

The Backward shift operator B is defined as

$$BZ_t = Z_{t-1}$$

$$B^2Z_t = Z_{t-2}$$

(2) Forward Shift Operators (F):

Performs the inverse operators of B.

$$F = B^{-1}$$

$$F Z_t = Z_{t+1}$$

and $$F^m Z_t = Z_{t+m}$$

(3) Backward Difference Operators (∇):

Can be written in terms of B as

$$\nabla Z_t = Z_t - Z_{t-1} = (1-B) Z_t$$

(4) Summation Operators (S) is the inverse operator of ∇

$$\nabla^{-1} Z_t = S_{Z\,t} = \sum_{j=0}^{\infty} Z_{t-j}$$

i.e.

$$\begin{aligned} S_{Z\,t} &= Z_t + Z_{t-1} + Z_{t-2} + \ldots \\ &= (1 + B + B^2 + \ldots) Z_t \\ &= (1 - B)^{-1} Z_t \end{aligned}$$

4.4.3. Time Series Models

The following types of stochastic time series models are generally met with.

4.4.3.1. Linear Filters Models

The linear filter model gives a weighted sum of previous observations as:

$$\begin{aligned} Z_t &= \mu + a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} + \ldots \\ &= \mu + \psi(B) a_t \end{aligned}$$

where μ is a parameter that determines the level of the process and

$$\psi(B) = 1 + \psi_1 B + \psi_2 B^2 + \ldots$$

is the linear operator that transform a_t to Z_t and is called the transfer function of filter.

a_t is the series of independent shocks which produced the time series and is known as white noise.

i.e. a_t, a_{t-1}, a_{t-2} ... are random variables from a fixed distribution with mean zero and σ_a^2 variance.

The sequence ψ_1, ψ_2 ... may be finite or infinite. If the sequence is finite or infinite and convergent the process Z_t is said to be stationary and parameter μ is mean on which process varies. Otherwise, the process is non-stationary and μ has no significance other than the reference point for the level of the process.

4.4.3.2 Autoregressive Models

In the Autoregressive Model (AR model) the current value of the process is expressed as a finite, linear aggregate of previous values of the process and a shock a_t. Let us denote the values of the process at the spaced time t, t-1, t-2,...by Z_t , Z_{t-1}, Z_{t-2} , ... and also let $\overline{Z}_t$, $\overline{Z}_{t-1}$, ... be deviations from μ for example $\overline{Z}_t = Z_t - \mu$.

Then

$$\overline{Z}_t = \phi_1 \overline{Z}_{t-1} + \phi_2 \overline{Z}_{t-2} + \ldots + \phi_p \overline{Z}_{t-p} + a_t$$

is called AR process of order p.

The Autoregressive operator of order p is given by

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p$$

or

$$\phi(B)\overline{Z}_t = a_t$$

The model contains p + 2 unknown parameters μ , ϕ_1 , ϕ_2 ..., ϕ_p , σ_a^2 which have to be estimated from data. σ_a^2 is the variance of the white noise process a_t.

4.4.3.3 Moving Average Model

A moving average process (MA process) of order q is defined by

$$\bar{Z}_t = a_t - \theta_1 a_{t-1} - \ldots - \theta_q a_{t-q}$$

or $$\bar{Z}_t = \theta(B)\, a_t$$

where, the moving average operator of order q is

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q$$

It contains q + 2 unknown parameters μ, θ_1, $\theta_2 \ldots$, θ_q, σ_a^2 to be estimated from data.

4.4.3.4. Mixed Auto Regressive Moving Average Models

To achieve greater flexibility in fitting of actual time series, both AR and MA terms are included in a model (ARMA model).

$$\bar{Z}_t = \phi_1 \bar{Z}_{t-1} + \ldots + \phi_p \bar{Z}_{t-p} + a_t - \theta_1 a_{t-1} - \ldots - \theta_q a_{t-q}$$

or $$\phi(B)\bar{Z}_t = \theta(B)\, a_t$$

There are p +q + 2 unknown parameters.

Most of the stationary time series can be accounted for in the 3 groups mentioned above, p and q are generally less than 2.

4.4.3.4. Auto Regressive Integrated Moving Average Model (ARIMA Model)

This is a general model devised to take care of both stationary as well as non- stationary processes. The non- stationary operator φ (B) can be written as

$$\varphi(B) Z_t = \phi(B)(1-B)^d$$

Where ϕ is a stationary operator. Thus the general model which can represent a homogeneous non- stationary behaviour is of the form

$$\varphi(B) Z_t = \phi(B)(1-B)^d Z_t = \theta(B)\, a_t$$

or $\quad \phi(B) W_t = \theta(B) a_t$

where $\quad W_t = \nabla^d Z_t$

Thus the ARIMA process of order (p,d,q) can be defined as

$$W_t = \phi_1 W_{t-1} + \ldots + \phi_p W_{t-p} + a_t - \theta_1 a_{t-1} - \ldots - \theta_q a_{t-q}$$

4.4.4 Model Identification

The first step in the stochastic model building is model identification. Here an appropriate sub-class of models is tentatively identified for the time series under study from the general ARIMA family:

$$\Phi_p(B) \nabla^d Z_t = \theta_o + \theta(B) a_t$$

Two steps are necessary for this.

(1) To difference Z_t as many times as necessary to produce stationarity, hopefully, reducing the process under study to the mixed ARMA process.

(2) To identify the resulting ARMA process. Autocorrelation function and partial autocorrelation function are the two main tools used to identify the model and also for preliminary estimation of parameters.

4.4.4.1. Estimation of Autocorrelation function

The theoretical Autocorrelation function ρ_k describes a conceptual stochastic process. The estimated Autocorrelation function γ_k of a time series Z_1, $Z_2, \ldots, Z_n$ of observations at lag k is estimated as described in section 4.4.1 by

$$\gamma_k = \frac{C_k}{C_o}$$

where

$$C_k = \frac{1}{N}\sum_{t=1}^{N-K}(Z_t - \overline{Z})(Z_{t+k} - \overline{Z}) \qquad k = 0,1,2, \ldots, K$$

C_k is known on the estimated autocovariance at k and $\overline{Z}$ is the mean of the time series given by

$$\frac{Z_1 + Z_2 + \ldots + Z_N}{N}$$

C_o is the estimated autocovariance of the time series at t = 0 given by

$$C_o = \sigma^2_{Z_t} = \frac{\sum (Z_t - \overline{Z})^2}{N}$$

γ_k is calculated for $K \leq N/4$

4.4.4.2 The Partial Autocorrelation function

The Partial Autocorrelation function is a device which exploits the fact whereas an AR(P) process has an Autocorrelation function which is infinite in extent, it can by its very nature be described in terms of p non-zero functions of the Autocorrelations. Denote by ϕ_{kj} the j[th] coefficient in an AR process of order k so that ϕ_{kk} is the last coefficient.

$$\rho_j = \phi_{k1}\, \rho_{j-1} + \ldots + \phi_{k(k+1)}\, \rho_{j-k+1} + \phi_{kk}\, \rho_{j-k}$$

$$j = 1, 2, \ldots, k$$

The quantity ϕ_{kk}, regarded as a function of the lag k is called partial Autocorrelation function.

For an AR process of order p, the partial Autocorrelation function ϕ_{kk} will be non-zero upto $k \leq P$ and zero for $k > P$.

ϕ_{11} is given by $\rho_1 = \gamma_1$.

Partial Autocorrelation function is calculated as given below:

$$\phi_{p+1,\, p+1} = \frac{\gamma_{p+1} - \sum_{j=1}^{p} \Phi_{pj}\, \gamma_{p+1-j}}{1 - \sum_{j=1}^{p} \Phi_{pj}\, \gamma_j}$$

$$j = 1, 2, \ldots, p$$

4.4.4.3 Identification of Model

With the Autocorrelation functions and Partial Autocorrelation function calculated and plotted, the preliminary identification of the process can be done from the behavior of the above functions with the help of the properties given in the Table 4.4.1 and 4.4.2.

It is possible to identify the same series against more than one process. In such cases all such cases should be considered for further analysis and the most suitable process may be selected.

4.4.5 Intial Estimates For The Parameters

4.4.5.1 Moving Average Process

The first q Autocorrelations of an MA(q) process are non-zero and can be written as

$$\rho_k = \frac{-\theta_k + \theta_1\theta_{k+1} + \theta_2\theta_{k+2} + \ldots + \theta_{q-k}\,\theta_q}{(1+\theta_1^2+\theta_2^2+\ldots+\theta_q^2)}$$

$$k = 1, 2, \ldots, q$$

The expressions for ρ_1 , ρ_2 , ... , ρ_q in terms of θ_1 , θ_2 , ... θ_q supplies q equations in q unknowns. Preliminary estimates of θ's can be obtained by substituting the estimates for γ_k for ρ_k and solving the equations. A preliminary estimate of σ_a^2 may then be obtained from

$$\gamma_o = \sigma_a^2(1+\theta_1^2+\ldots+\theta_q^2)$$

by substituting the preliminary estimates of θ's and replacing $\gamma_o = \sigma^2$ by its estimated C_o.

When multiple solutions are obtained for θ's, the value satisfying the condition $-1 < \theta < 1$ should be accepted.

4.4.5.1a (0, d, 1) Process

The preliminary estimates for (0, d, 1) process can be obtained from the Table 4.4.3, which relates ρ_1 to θ_1 by substituting γ_1 for ρ_1.

4.4.5.1b (0, d, 2) Process

Preliminary estimates of θ_1 and θ_2 can be obtained from chart C by substituting values of $\gamma_1(w)$ and $\gamma_2(w)$ for ρ_1 and ρ_2.

4.4.5.2 Auto- Regressive Process

For an assume AR process of order 1 or 2 initial estimates for ϕ_1 and ϕ_2 can be calculated by substituting estimates of γ_j for the theoretical Autocorrelation ρ_j in the formula given in Table 4.4.2 and solving the equations in the form given below.

For AR(1) process $\phi_{11} = \gamma_1$

For AR(2)

$$\phi_{21} = \frac{\gamma_1(1-\gamma_2)}{1-\gamma_1^2}$$

$$\phi_{22} = \frac{\gamma_2-\gamma_1^2)}{1-\gamma_1^2}$$

Chart B can be used for estimating ϕ_1 and ϕ_2 for second order AR process (2, d, 0) with γ_1 and γ_2 substituting for ρ_1 and ρ_2.

TABLE 4.4(1) : **PROPERTIES OF A.R. , M.A. AND MIXED ARMA PROCESS**

		ARP	MAP	MIX ARMAP
(1)	Model in terms of previous $\bar{Z}_t$	$\phi(B)\bar{Z}_t = a_t$	$\theta^{-1}(B)\bar{Z}_t = a_t$	$\theta^{-1}(B)\phi(B)\bar{Z}_t = a_t$
(2)	Model in terms of previous a_s	$\bar{Z}_t = \phi^{-1}(B)a_t$	$\bar{Z}_t = \theta^{-1}(B)a_t$	$\bar{Z}_t = \phi^{-1}(B)\theta(B)a_t$
(3)	π weights	Finite series	Infinite series	Infinite series
(4)	ψ weights	Infinite series	Finite series	Infinite series
(5)	Stationary condition	Roots of $\phi(B)^{-1} = 0$ lies outside the unit circle	Always Stationary	As in ARP
(6)	Invertibility	Invertible	Roots of $\theta(B) = 0$ lies outside the unit circle	As in MAP
(7)	Autocorrelation function	Infinite (Damped exponential and / or damped sine waves)	Finite	Infinite (Damped exponentials and / or damped sine waves after Ist q-p lags tails off)
(8)	Partial Autocorrelation function	Finite cuts off	Infinite (Dominated by damped exponentials and / or damped sine waves) tails off	Infinite tails off

TABLE 4.4(2) : **BEHHAVIOUR OF AUTOCORRELATION FUNCTION OF AN ARIMA PROCESS OF THE ORDER (p, d , q) AFTER THE dth DIFFERENCE**

Behhaviour of	Order (1,d,0)	Order (0,d,1)	Order (2,d,1)	Order (0,d,2)	Order (1,d,1)
Autocorrelation function	Decay exponentially	Only ϕ_{11} non-zero	Mixture of exponentials or damped sine wave	Only γ_1 and γ_2 non-zero	Decays exponentially from first lag.
Partial Autocorrelation function	Only ϕ_{11} non-zero	Exponential Dominated decay	Only ϕ_{11} and ϕ_{22} non-zero	Dominated by Mixture of exponentials or damped sine wave	Dominated by exponential decay from first lag.
Preliminary estimates from	$\phi_1 = \gamma_1$	$\gamma_1 = -\dfrac{\theta_1}{1+\theta_1^2}$	$\phi_1 = \dfrac{\gamma_1(1-\gamma_2)}{1+\gamma_1^2}$ $\phi_2 = \dfrac{\gamma_2-\gamma_1^2}{1-\gamma_1^2}$	$\gamma_1 = -\dfrac{\theta_1(1-\theta_2)}{1+\theta_1^2+\theta_2^2}$ $\gamma_2 = \dfrac{-\theta_2}{1+\theta_1^2+\theta_2^2}$	$\gamma_1 = \dfrac{(1-\theta_1\phi_1)(\phi_1-\theta_2)}{1+\theta_1^2-2\phi_1\theta_1}$ $\gamma_2 = \gamma_1\phi_1$
Admissible Reagion	$-1 < \phi_1 < 1$	$-1 < \theta_1 < 1$	$-1 < \phi_1 < 1$ $\phi_2 + \phi_1 < 1$ $\phi_2 - \phi_1 < 1$	$-1 < \theta_2 < 1$ $\theta_2 + \theta_1 < 1$ $\theta_2 - \theta_1 < 1$	$-1 < \phi_1 < 1$ $-1 < \theta_1 < 1$

Table 4.4.3: **TABLE RELATING ρ_1 TO θ_1 FOR A FIRST ORDER M.A PROCESS**

0.00	0.000	0.00	0.000
0.05	-0.050	-0.05	0.050
0.10	-0.099	-0.10	0.099
0.15	-0.147	-0.15	0.147
0.20	-0.192	-0.20	0.192
0.25	-0.235	-0.25	0.235
0.30	-0.275	-0.30	0.275
0.35	-0.315	-0.35	0.315
0.40	-0.349	-0.40	0.349
0.45	-0.374	-0.45	0.374
0.50	-0.400	-0.50	0.400
0.55	-0.422	-0.55	0.422
0.60	-0.441	-0.60	0.441
0.65	-0.457	-0.65	0.457
0.70	-0.468	-0.70	0.468
0.75	-0.480	-0.75	0.480
0.80	-0.488	-0.80	0.488
0.85	-0.493	-0.85	0.493
0.90	-0.497	-0.90	0.497
0.95	-0.499	-0.95	0.499
1.00	-0.500	-1.00	0.500

4.4.5.3 Mixed Auto- Regressive Moving Average Process

It will be often found that either initially or after suitable differencing the process

$W_t = \nabla^d Z_t$ is most economically represented by a mixed ARMA process as

$$\phi(B)\, W_t = \theta(B)\, a_t$$

i.e. $(1 - \phi_1 B)\, W_t = (1 - \theta_1 B)\, a_t$

where $W_t = \nabla^d Z_t$

Approximate values of parameters are obtained from Table 4.4.2, by substituting the estimates $\gamma_1(w)$ and $\gamma_2(w)$ for ρ_1 and ρ_2 in the expressions ρ_1 and ρ_2. Thus we get

$$\gamma_1 = \frac{(1-\theta_1\phi_1)(\phi_1-\theta_1)}{1-\theta_1^2-2\theta_1\phi_1}$$

$$\gamma_2 = \gamma_1\, \phi_1$$

4.4.5.4 (1, d, 1) Process (ARIMA Process)

Chart D relating ρ_1 and ρ_2 to γ_1 and θ_1 can be used to provide initial estimates of parameters of ARIMA (1,d,1) process, with γ_1 and γ_2 substituting ρ_1 and ρ_2.

4.4.6 Initial Estimate of Residual Variance

(1) A.R. Process

$$\sigma_a^2 = Co\,(1 - \phi_1\gamma_1 - \phi_2\gamma_2 - \dots - \phi_p\gamma_p)$$

(2) M.A. Process

$$\sigma_a^2 = \frac{Co}{1+\theta_1^2+\dots+\theta_q^2}$$

(3) Mixed Process

$$\sigma_a^2 = \frac{1-\phi_1^2}{1+\theta_1^2-2\phi_1\theta_1}Co$$

4.4.7 Approximate Standard Error for $\bar{w}$

Approximate Standard Error for $\bar{w}$

where $W_t = \nabla^d Z_t$ an ARIMA process of order (p,d,q) are given below.

Here $n = N - d$

(a) (1,d,0) Process

$$\sigma(\bar{w}) = \left\{\frac{C_o(1+\gamma_1)}{n(1-\gamma_1)}\right\}^{1/2}$$

(b) (0,d,1) Process

$$\sigma(\bar{w}) = \left\{\frac{C_o(1+2\gamma_1)}{n}\right\}^{1/2}$$

(c) (2,d,0) Process

$$\sigma(\bar{w}) = \left\{\frac{C_o(1+\gamma_1)(1-2\gamma_1^2+\gamma_2)}{n(1-\gamma_1)(1-\gamma_2)}\right\}^{1/2}$$

(d) (0,d,2) Process

$$\sigma(\bar{w}) = \left\{\frac{C_o(1+2\gamma_1+2\gamma_2)}{n}\right\}^{1/2}$$

4.4.8 Final Estimates For The Parameters

The identification process having led to a tentative formulation for model, the next step is to arrive at an accurate estimate of the parameters. After that the fitted model will be subjected to diagnostic checks and tests of goodness of fit.

The estimation of the parameters is done by the study of the sum of squares function.

In the calculation of the unconditional sum of squares, the [a]s are computed by taking conditional expectations. A preliminary back calculation provides the values w_{-j} , j = 0,1,2,... (i.e. the back forecasts) needed for starting the forward recursion.

The process can be written in forward process as:

$(1 - \phi B) W_t = (1 - \theta B)a_t$ with $W_t = \nabla^d Z_t$

or equivalently with the backward process :

$(1 - \phi F) W_t = (1 - \theta F)e_t$

it is assumed that w's have mean zero. Therefore, the expressions for $[e_t]$ and $[a_t]$ can be written as

$$[e_t] = [W_t] - \phi[W_{t+1}] + \theta [e_{t+1}] \qquad (4.4.8a)$$

$$[a_t] = [W_t] - \phi[W_{t-1}] + \theta [a_{t-1}] \qquad (4.4.8b)$$

where $[W_t] = w_t$ $(t = 1,2,\ldots,n)$, and is the back forecast of w_t for $t \leq 0$.

The layout of the calculation is shown in Table (4.4.4). The calculation is commenced by entering the table with the known values.

The known values are [w]s and zero values for $[e_o]$, $[e_{-1}]$ ... (as they are distributed independent of w). Beginning at the end of the series the backward equation is now started off by putting $e_n = 0$ and there by making the value of θ $[e_{t+1}]$ on the n-1^{th} row also equal to zero. This is necessary due to the presence of an autoregressive operator of the 1^{st} order in the model. The calculation now goes ahead using the equation for $[e_t]$. The back forecasts w_{-j} , $(j = 0,1,2)$ dies out at $j = Q$ .Value of Q depending on the accuracy of the calculation . The estimates (a_{-j}) for $j > Q$ taken as zero. The forward calculation is now started using the expression for $[a_t]$. The unconditional sum of the squares is obtained by summing the squares of all the calculated $[a_t]$'s. Thus

$$S(\phi,\theta) = \sum_{t=-Q}^{n} [a_t]^2$$

With n reasonably large, a second iteration would not be required.

TABLE 4.4.4: CALCULATION OF SUM OF SQUARES FOR ARIMA (1,d,1) MODEL

t – (Q+1)	$[a_t]$	$\theta\ [a_{t-1}]$	$-\phi[W_{t+1}]$	$[W_t]$	$-\phi[W_{t+1}]$	$\theta\ [e_{t+1}]$	$[e_t]$
-Q						0	
.						.	
.						.	
.						.	
-1						0	
0						0	
1				W_1			
2				W_2			
3				W_3			
.				.			
.				.			
.				.			
n-1				W_{n-1}		0	
n				W_n			0

4.4.9 Diagnostic Checking Of The Stochastic Model

After identification of the model and estimation of the parameters the next step is to apply diagnostic checks to the model for deciding its adequacy. If there is evidence of serious inadequacy the model should be modified suitably in the next iterative cycle. This could be achieved by applying the portmanteau lack of fit test on the residuals autocorrelation.

Consider the model

$$\phi(B)\,\omega_t = \theta(B)\,a_t$$

where $\omega_1 = \nabla^d Z_t$

The model has been fitted and M.L. estimates (ϕ,θ) obtained for the parameters. Then the quantities a_t referred to as the residuals is given by

$$a_t = \theta^{-1}(B)\,\phi(B)\omega_t$$

Suppose we have the first K.Autocorrelations (where K is sufficiently large), $\gamma_k(a)$, $(k = 1,2,\ldots,K)$ from an ARIMA (p,d,q) process, then if the fitted model is appropriate.

$Q = n\sum_{k=1}^{k}\gamma_k^2(a)$ is approximately distributed as χ^2 (k-p-q) , where n = N – d

is the number of w's used to fit the model. On the other hand if the model is inappropriate, the average values of Q will be inflated. Hence the portmanteau test of the model adequacy can be made by referring the observed value of Q to the table of the percentage points of χ^2. As stated above the degree of freedom is given by (K-p-q).

4.4.10 Forecasting From Fitted Arima Model

Once an ARIMA model has been fitted to a time series, the projection forward of that model to derive forecasts of future values is quite straightforward. The general ARIMA (p,d,q) model can be written as

$$\varphi(B)\,\overline{Z}_t = \theta(B)\,a_t \qquad (4.8.1)$$

where $\varphi(B) = \phi(B)\,\nabla^d$

$$(1 - \phi_1 B - \ldots - \phi_p B^p)(1-B)^d\,\overline{Z}_t = (1 - \theta_1 B - \ldots - \theta_q B^q)\,a_t$$

However, for forecasting, the autoregressive and differencing terms can be amalgamated, and the model is expressed as

$$(1 - \Phi_1 B - \ldots - \Phi_{p+d} B^{p+d})\,\overline{Z}_t = (1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q)\,\overline{Z}_{t-1}$$

where

$$1 - \Phi_1 B - \ldots - \Phi_{p+d} B^{p+d} = (1 - \phi_1 B - \ldots - \phi_p B^p)(1-B)^d$$

Thus, the model can be written as

$$\overline{Z}_t = \Phi_1 \overline{Z}_{t-1} + \ldots + \Phi_{p+d} \overline{Z}_{t-p-d} + a_t - \theta_1 a_{t-1} - \ldots - \theta_q a_{t-q}$$

Suppose now that, standing at time n, we want to predict the future value $\overline{Z}_{n+h}$. Setting t = n+h in the above equation, the quantity to be predicted is

$$\overline{Z}_{n+h} = \Phi_1 \overline{Z}_{n+h-1} + \ldots + \Phi_{p+d} \overline{Z}_{n+h-p-d} + a_{n+h} - \theta_1 a_{n+h-1} - \ldots - \theta_q a_{n+h-q}$$

This equation forms the basis for forecasting through the substitution on the right-hand side of estimates and forecasts for unknown quantities. This is achieved as follows:

1. The parameters $\Phi_1, \ldots, \Phi_{p+d}, \theta_1, \ldots \theta_q$ are replaced by their estimates, derived from the parameter estimation stage of the model building cycle. We will denote the estimates $\hat{\Phi}_1, \ldots, \hat{\Phi}_{p+d}, \hat{\theta}_1, \ldots, \hat{\theta}_q$.
2. For $t \le n$, $\overline{Z}_t$ will be a known observation. For t > n , $\overline{Z}_t$ is replaced by its forecast estimate made at time n.
3. For $t \le n$, the error term a_t is replaced by its estimate, the residual $\hat{a}_t$ from the fitted model. For t>n, the unknown $\hat{a}_t$ is replaced by its best forecast, zero.

Thus it follows from the above equation that the forecast of $\overline{Z}_{n+h}$ is given by

$$\hat{Z}_n(h) = \hat{\Phi}_1 \hat{Z}_n(h-1) + \ldots + \hat{\Phi}_{p+d} \hat{Z}_n(h-p-d) + \hat{a}_{n+h} - \hat{\theta}_1 \hat{a}_{n+h-1} - \ldots - \hat{\theta}_q \hat{a}_{n+h-q}; \quad h = 1, 2 \ldots$$

where

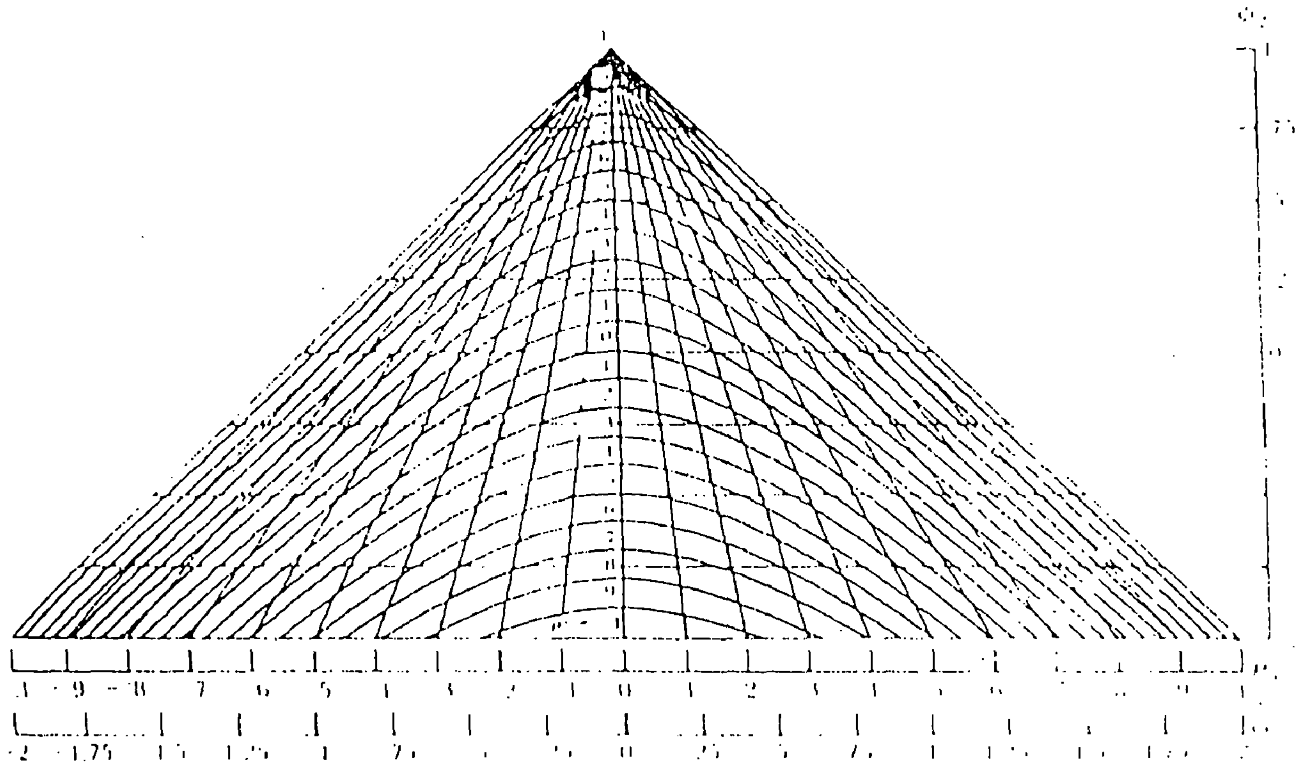
$$\hat{Z}_n(j) = \overline{Z}_{n+j} \qquad j \le 0$$

and

$$\hat{a}_{n+j} = 0 \qquad j > 0.$$

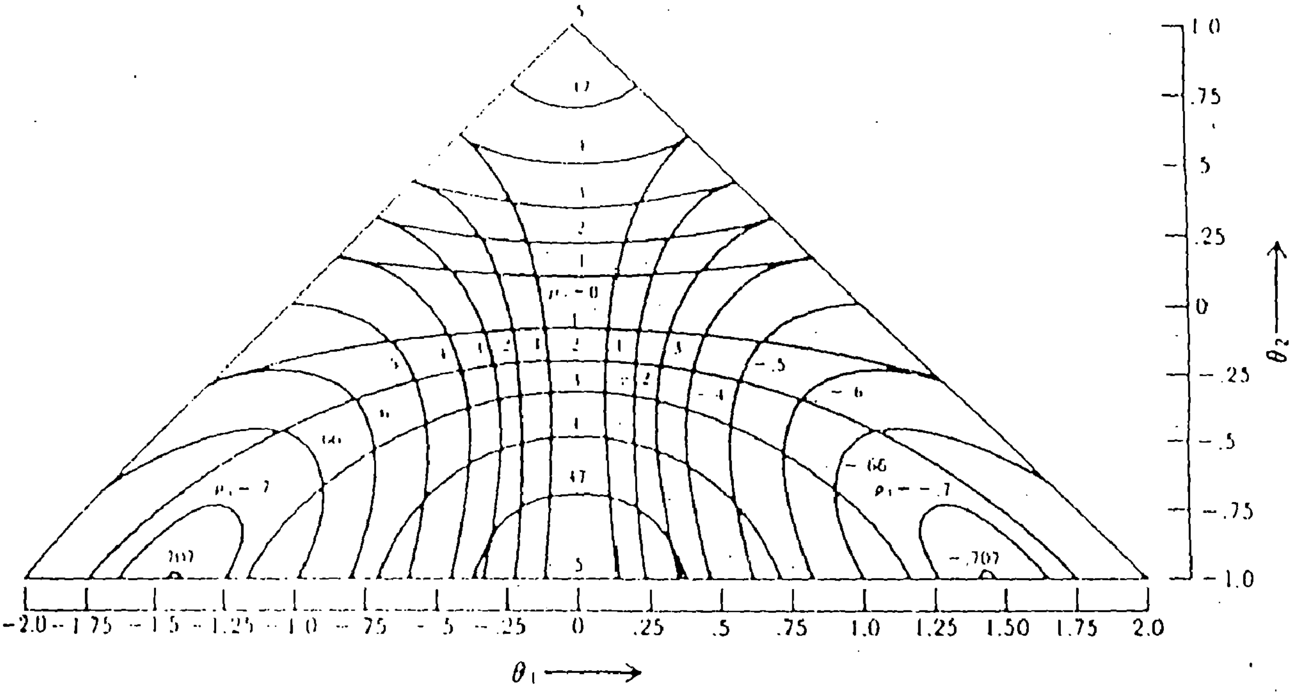
Thus using the above equation in turn for h = 1, 2, ... , forecasts as far ahead as required can easily be computed.

CHART B CHART RELATING ρ_1 AND ρ_2 TO ϕ_1 AND ϕ_2 FOR A SECOND-ORDER AUTOREGRESSIVE PROCESS



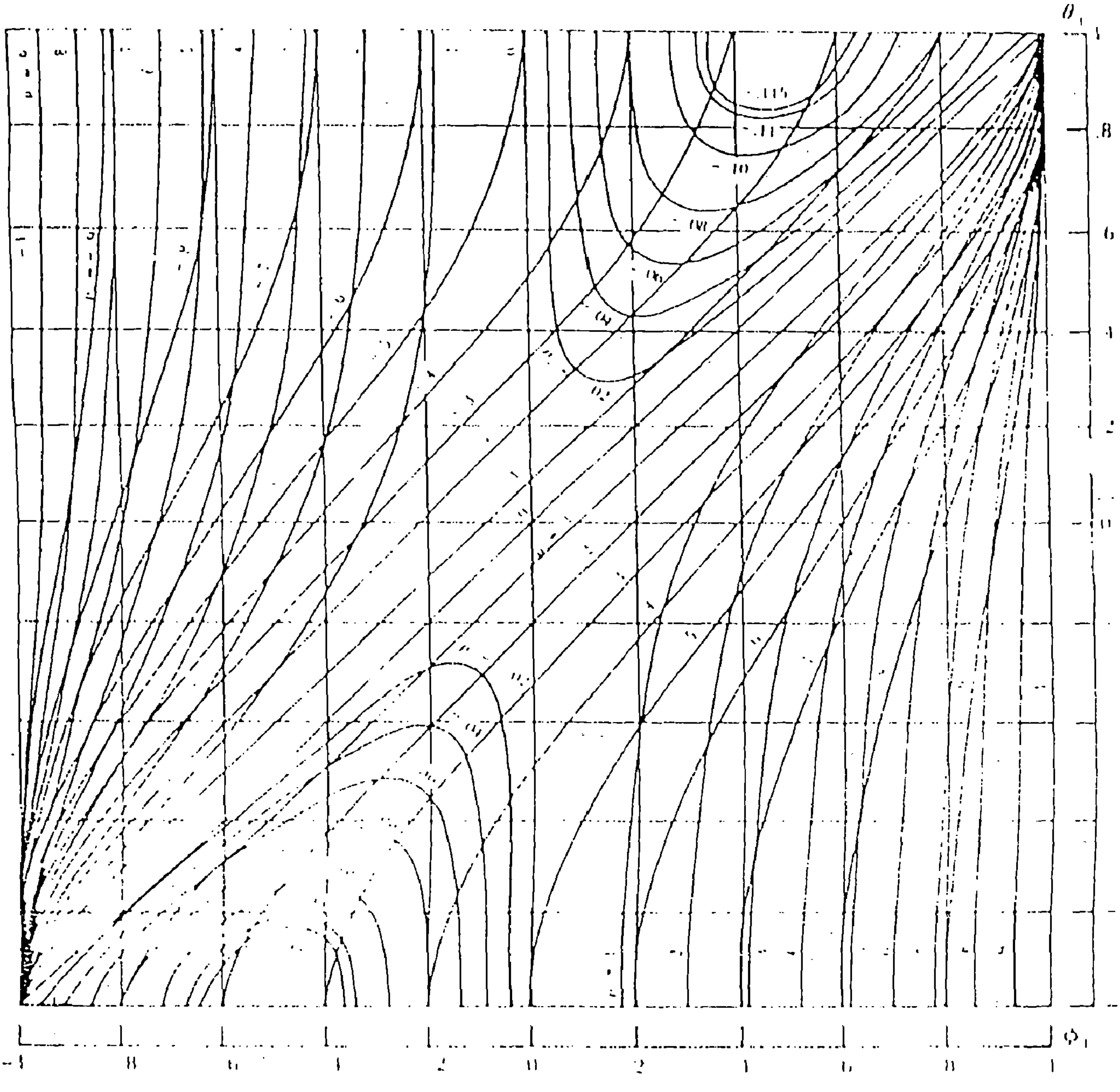
The chart may be used to obtain estimates of the parameters in the $(2, d, 0)$ process $(1 - \phi_1 B - \phi_2 B^2)w_t = a_t$, where $w_t = \nabla^d z_t$, by substituting $r_1(w)$ and $r_2(w)$ for ρ_1 and ρ_2.

Chart C CHART RELATING ρ_1 AND ρ_2 TO θ_1 AND θ_2 FOR A SECOND-ORDER MOVING AVERAGE PROCESS



The chart may be used to obtain first estimates of the parameters in the (0, d, 2) process $w_t = (1 - \theta_1 B - \theta_2 B^2)a_t$, where $w_t = \nabla^d z_t$, by substituting $r_1(w)$ and $r_2(w)$ for ρ_1 and ρ_2.

CHART D CHART RELATING ρ_1 AND ρ_2 TO ϕ AND θ FOR A MIXED FIRST-ORDER AUTOREGRESSIVE-MOVING AVERAGE PROCESS



The chart may be used to obtain first estimates of the parameters in the (1, d, 1) process $(1 - \phi B)w_t = (1 - \theta B)a_t$, where $w_t = \nabla^d z_t$, by substituting $r_1(w)$ and $r_2(w)$ for ρ_1 and ρ_2.

CHAPTER 5

APPLICATION OF BOX-JENKINS METHODS FOR ELECTRICAL ENERGY DEMAND FORECASTING

5.1. FORECASTING ELECTRICLAL ENERGY REQUIREMENT OF ETHIOPIA UPTO 2020

5.1.1. Introduction

The box – Jenkins methods of stochastic modeling has been applied to the time series formed by the actual energy requirement on the Ethiopian Power System and attempt is hereby made to forecast the future energy requirements. The actual energy requirements from 1982 to 2001 as per the historical records of the Ethiopian Electric Power Corporation are given by in Table 5.1.(1). The 1st sixteen observations (1982 – 1997) are considered for the time series analysis. The values from 1998 to 2001 (four observations) are retained for checking consistency of the forecasts. As the energy requirement figures are too large for easy handling, their logarithms are taken to form the time series and tabulated in Table 5.1.1.

Table 5.1.1 ENERGY REQUIREMENT OF ETHIOPIA (OBSERVED)

Serial Number	YEAR	Actual Energy Requirement Annual (Million KWh)	(Z_t) Log_e of the energy requirement
1	1982	625	6.4378
2	1983	654	6.4831
3	1984	714	6.5709
4	1985	792	6.6746
5	1986	872	6.7708
6	1987	945	6.8512
7	1988	1003	6.9108
8	1989	1049	6.9556
9	1990	1119	7.0202
10	1991	1152	7.0493
11	1992	1191	7.0825
12	1993	1278	7.1531
13	1994	1395	7.2406
14	1995	1465	7.2896
15	1996	1550	7.3460
16	1997	1614	7.3865
17	1998	1628	7.3951
18	1999	1653	7.4103
19	2000	1689	7.4319
20	2001	1811	7.5016

5.1.2. The Time Series

The time series under study is given by below:

t	Z_t
1	6.4378
2	6.4831
3	6.5709
4	6.6746
5	6.7708
6	6.8512
7	6.9108
8	6.9556
9	7.0202
10	7.0493
11	7.0825
12	7.1531
13	7.2406
14	7.2896
15	7.3460
16	7.3865

5.1.3. Model Estimation

The mean and variance of the series has been estimated

The mean $\overline{Z} = \dfrac{\sum_{1}^{16} Z_t}{N}$ t = 1,2, ... , N

For N = 16

$\bar{Z} = 6.9514$

The variance $\sigma^2_{zt} = \frac{1}{N}\sum_{1}^{16}(Z_t - \bar{Z})^2$

$= \frac{1}{16}\sum_{1}^{16}(Z_t - 6.9514)^2$

$= 0.0853$

Now the Autocorrelation coefficients of the series for the lags upto 6 has been estimated as per the relation

$\gamma_k = \frac{C_k}{C_0}$ where Ck is the estimated autocovariance at k given by

$C_k = \frac{1}{N}\sum_{t=1}^{N-k}(Z_t - \bar{Z})(Z_{t+k} - \bar{Z})$; k = 0,1,2, ..., K and C_0 is the estimated autocovariance at time t = 0 , given by $C_0 = \sigma^2_{zt} = 0.0853$

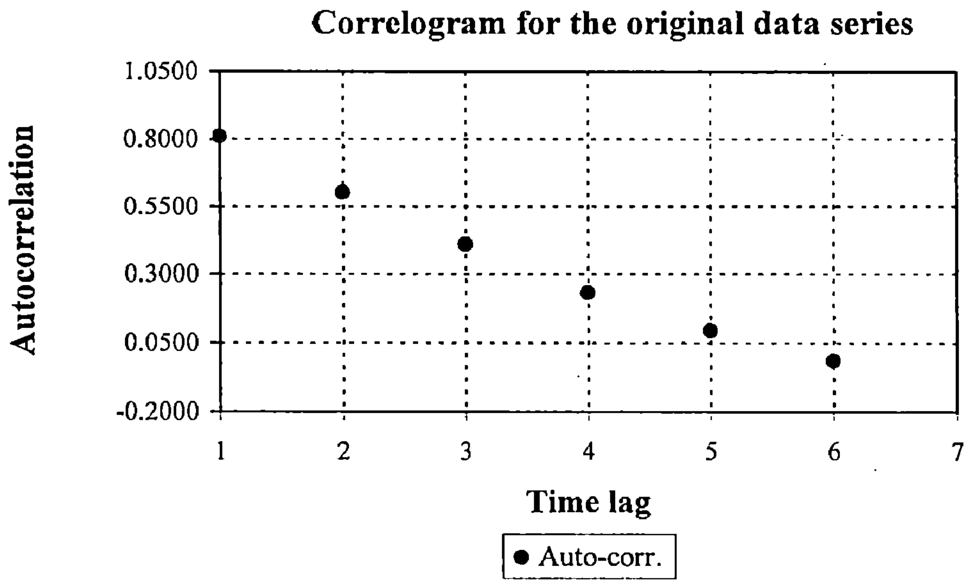
The calculated values for the estimated Autocorrelation coefficient for the various lags from 1 to 6 are given in table 5.1.(2).

Table5.1.(2). **ESTIMATION OF AUTOCORRELATION COEFFICIENT TIME SERIES (ETHIOPIAN ENERGY REQUIREMENT)**

Lag K	C_k	Co	N	$\gamma_k = \frac{C_k}{C_0}$
(a) Z_t Series			16	
1	0.0690	0.0853		0.8091
2	0.0514	0.0853		0.6028
3	0.0348	0.0853		0.4083
4	0.0200	0.0853		0.2344
5	0.0082	0.0853		0.0960
6	-0.0012	0.0853		-0.0138

As described in Chapter IV, the next step is to represent the autocorrelation of Table 5.1.2. graphically and examine its characteristic. The correlogram of the original series is represented on Graph 5.1.1

Graph 5.1.1 CORRELOGRAM FOR THE ORIGINAL DATA SERIES



The data on graph 5.1.1 show a strong trend (straight line) going from right to left as the number of time lags increase instead of drop to zero. At the same time the autocorrelation of the first three time lags are not significantly difference from zero, as per Bartlett criteria (out of the range of plus or minus 1.65 * standard error). Hence the original data shows nonstationarity.

Since trends of any kind tend to introduce spurious autocorrelations that dominate the autocorrelation pattern, it is imperative to remove it from the data before proceeding further with time series analysis. Removing trends can be routinely achieved through the method of differencing as discussed in Chapter IV.

Now the first difference of the series has found out as

$$W_t = \nabla Z_t = Z_{t-1} - Z_t$$

The observations for the W_t series (n):

$$n = N - 1$$
$$= 16 - 1 = 15$$

The W_t series obtained, as a difference of the original, is as given below:

t	W_t
1	0.0454
2	0.0878
3	0.1037
4	0.0962
5	0.0804
6	0.0596
7	0.0448
8	0.0646
9	0.0291
10	0.0333
11	0.0705
12	0.0876
13	0.0490
14	0.0564
15	0.0405

Mean of the series $\bar{w} = 0.0632$ and

$$\sigma_w^2 = 0.00052$$

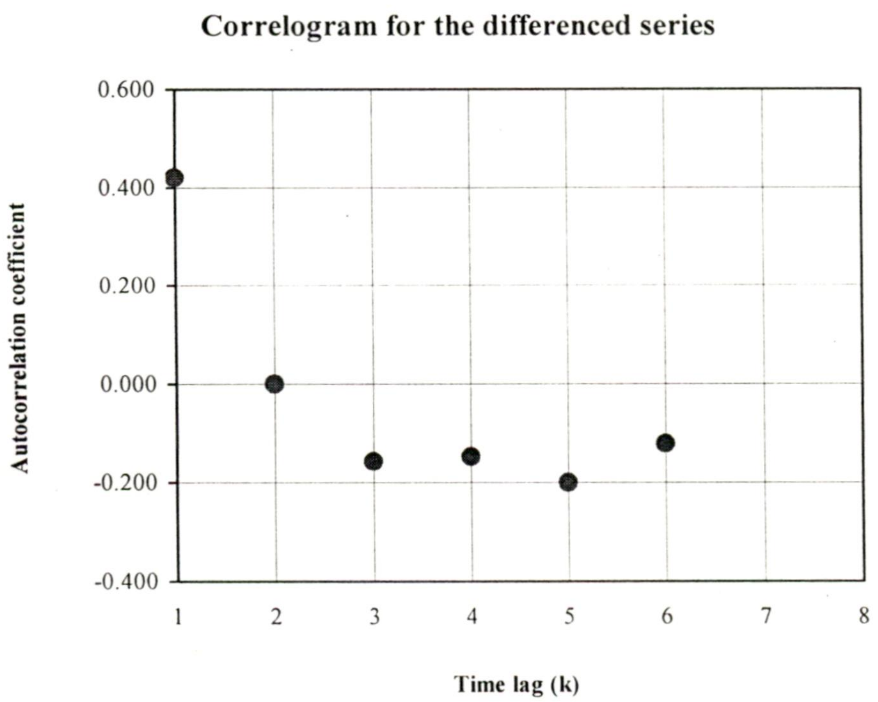
The autocorrelation coefficient of the series estimated for the first six lags are given in Table 5.13.

Table 5.1.3 ESTIMATION OF AUTOCORRELATION COEFFICIENT FOR THE FIRST DIFFERENCED DATA

Lag K	C_k	Co	N	$\gamma_k = \frac{C_k}{C_0}$
W_t Series			15	
1	0.00022	0.00052		0.421
2	0.00000	0.00052		0.000
3	-0.00008	0.00052		-0.157
4	-0.00008	0.00052		-0.147
5	-0.00010	0.00052		-0.199
6	-0.00006	0.00052		-0.121

The next step is to represent the autocorrelation of Table 5.1.3. graphically and examine its characteristic. The correlogram of the differenced series is represented below on Graph 5.1.2.

Graph 5.1.2 CORRELOGRAM OF THE FIRST DIFFERENCE



The characteristic on graph 5.1.2 shows random distribution of the data (no trend) and furthermore, after one time lag, the autocorrelations are not significantly different from zero. Where as the first lag autocorrelation is significantly different from zero. Thus, the first difference (d = 1) results a stationary series. In addition the autocorrelation function has a cut-off after the first lag (q = 1). This implies that the series can be tentatively identified as a first order Integrated Moving Average, ARIMA (0,1,1), process given by:

$Wt = (1 - \theta B)\, a_t$

Seasonal models are not considered as the annual energy requirement is not liable to seasonal changes.

5.1.4. Preliminary Estimation of Parameters

The model having tentatively identified, the next step is the preliminary estimation of parameters. As derived in Chapter IV and summarized in table 4.4.2, the parameter for the ARMA(0,1,1) can be calculated as

$$\theta = -\frac{1}{2r_1} \pm \left[\frac{1}{(2r_1)^2} - 1\right]^{1/2}$$

$$= -\frac{1}{2\times 0.412} \pm \left[\frac{1}{(2\times 0.412)^2} - 1\right]^{1/2}$$

$$= -0.55 \quad \text{or} \quad -1.826$$

As θ should lie in the region $-1 < \theta < 1$,

The values of -0.55 is acceptable.

Hence $\theta = -0.55$

$$\lambda = 1 - \theta$$

$$= 1 - (-0.55)$$

$$= 1.55$$

$$\sigma_a^2 = \frac{C_0}{1+\theta^2}$$

$$= \frac{0.00052}{1+(-0.55)^2}$$

$$= 0.000518$$

Standard error

$$= \frac{C_0\,(1+2r_1)^{1/2}}{n}$$

$$= \frac{0.00052\,(1+2\times 0.421)^{1/2}}{15}$$

$$= 0.00798$$

5.1.5. Model Estimation by Sum of Squares

The sum of squares for the residuals a_t's calculated recursively using the equations 4.4.8, again stated as

$$[e_t] = [w_t] + \theta\,[e_{t+1}] \qquad (1)$$

and $$[a_t] = [w_t] + \theta\,[a_{t+1}] \qquad (2)$$

The calculation was done using the format given below to arrive the least sum of square of a_t's as the procedure discussed in section 4.4.8. Accordingly by starting from the bottom, assuming $e_{15} = 0$, and proceeding backward the value of e_t's are calculated using equation (1) until the back forecast at t = 0 is obtained. Now the forward calculation of a_t's is started from w_1 using equation (2). After getting the a_t's for t = 1,2, ...upto 15 the sum of squares of a_t's are estimated. The process is repeated with various values of θ until the least sum of squares value is reached. The various values of θ and the corresponding sums of squares of a_t's are listed below.

θ	Sums of squares of a_t's
-0.70	0.006556
-0.61	0.006338
-0.60	0.006335
-0.59	0.006336
-0.55	0.006372
-0.50	0.006483

The value of sums of squares 0.006335 for of θ = - 0.60 is found to be the least. Hence the value of θ = - 0.60 is accepted. The calculation of a_t's for θ = - 0.60 is given in Table 5.1.4.

Table 5.1.4. CALCULATION OF SUM OF SQUARES OF RESIDUALS FOR θ = - 0.60

T	Z_t	(a_t)	$\theta\,(a_{t-1})$	(W_t)	$\theta\,(e_{t-1})$	(e_t)
-1	6.428400000	0	0	0	0	0
0	6.437751650	0.0094	0	0.0094	-0.0094	0
1	6.483107351	0.0400	-0.00563	0.0454	-0.0297	0.0156
2	6.570882962	0.0640	-0.02384	0.0878	-0.0382	0.0495
3	6.674561392	0.0650	-0.03836	0.1037	-0.0400	0.0637
4	6.770789424	0.0570	-0.03919	0.0962	-0.0296	0.0666
5	6.851184927	0.0460	-0.03422	0.0804	-0.0310	0.0494
6	6.910750788	0.0320	-0.0277	0.0596	-0.0078	0.0517
7	6.955592608	0.0260	-0.01912	0.0448	-0.0318	0.0131
8	7.020190708	0.0490	-0.01543	0.0646	-0.0116	0.0530
9	7.049254841	0.0000	-0.0295	0.0291	-0.0097	0.0194
10	7.082548569	0.034	0.00026	0.0333	-0.0172	0.0161
11	7.153051635	0.050	-0.02013	0.0705	-0.0419	0.0286
12	7.240649694	0.057	-0.03022	0.0876	-0.0178	0.0698
13	7.289610521	0.015	-0.03443	0.0490	-0.0193	0.0297
14	7.346010210	0.048	-0.00872	0.0564	-0.0243	0.0321
15	7.386470849	0.012	-0.02861	0.0405	0	0.0405

The model is then defined by

$$\nabla Z_t = (1 - (-0.6)\,B)\,a_t$$
$$= (1 + 0.6\,B)\,a_t$$

5.1.6. Diagnostic Checking of the Model

For the residual a_t's, found in section 5.1.5, the autocorrelation for time lag 1 to 6 are calculated and then tabulated below:

Table 5.1.5 AUTOCORRELATION COEFFICIENT FOR THE RESIDUAL A_T'S

Lag k	γ_k
1	0.044
2	0.019
3	-0.227
4	-0.079
5	-0.295
6	0.107

Then, the adequacy of the model fitted to the series is checked by applying the portmanteau lack of fit test to the Autocorrelation of the residuals as follows:

$$Q = n\sum_{k=1}^{K}\gamma_k^2(a) \text{ is calculated}$$

For $n = 15$

$K = 6$

$$Q = 15 \times [(0.044)^2 + (0.019)^2 + (-0.227)^2 + (-0.079)^2 + (-0.295)^2 + (0.107)^2]$$
$$= 2.38$$

TABLE 5.1.(6)

TAIL AREAS OF THE CHI-SQUARE DISTRIBUTION

p	ε = 0.995	0.99	0.975	0.95	0.9	0.75	0.5	0.25	0.1	0.05	0.025	0.01	0.005	0.001	p
1	—	—	—	—	0.016	0.102	0.455	1.32	2.71	3.84	5.02	6.63	7.88	10.8	1
2	0.010	0.020	0.051	0.103	0.211	0.575	1.39	2.77	4.61	5.99	7.38	9.21	10.6	13.8	2
3	0.072	0.115	0.216	0.352	0.584	1.21	2.37	4.11	6.25	7.81	9.35	11.3	12.8	16.3	3
4	0.207	0.297	0.484	0.711	1.06	1.92	3.36	5.39	7.78	9.49	11.1	13.3	14.9	18.5	4
5	0.412	0.554	0.831	1.15	1.61	2.67	4.35	6.63	9.24	11.1	12.8	15.1	16.7	20.5	5
6	0.676	0.872	1.24	1.64	2.20	3.45	5.35	7.84	10.6	12.6	14.4	16.8	18.5	22.5	6
7	0.989	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.0	14.1	16.0	18.5	20.3	24.3	7
8	1.34	1.65	2.18	2.73	3.49	5.07	7.34	10.2	13.4	15.5	17.5	20.1	22.0	26.1	8
9	1.73	2.09	2.70	3.33	4.17	5.90	8.34	11.4	14.7	16.9	19.0	21.7	23.6	27.9	9
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.5	16.0	18.3	20.5	23.2	25.2	29.6	10
11	2.60	3.05	3.82	4.57	5.58	7.58	10.3	13.7	17.3	19.7	21.9	24.7	26.8	31.3	11
12	3.07	3.57	4.40	5.23	6.30	8.44	11.3	14.8	18.5	21.0	23.3	26.2	28.3	32.9	12
13	3.57	4.11	5.01	5.89	7.04	9.30	12.3	16.0	19.8	22.4	24.7	27.7	29.8	34.5	13
14	4.07	4.66	5.63	6.57	7.79	10.2	13.3	17.1	21.1	23.7	26.1	29.1	31.3	36.1	14
15	4.60	5.23	6.26	7.26	8.55	11.0	14.3	18.2	22.3	25.0	27.5	30.6	32.8	37.7	15
16	5.14	5.81	6.91	7.96	9.31	11.9	15.3	19.4	23.5	26.3	28.8	32.0	34.3	39.3	16
17	5.70	6.41	7.56	8.67	10.1	12.8	16.3	20.5	24.8	27.6	30.2	33.4	35.7	40.8	17
18	6.26	7.01	8.23	9.39	10.9	13.7	17.3	21.6	26.0	28.9	31.5	34.8	37.2	42.3	18
19	6.84	7.63	8.91	10.1	11.7	14.6	18.3	22.7	27.2	30.1	32.9	36.2	38.6	43.8	19
20	7.43	8.26	9.59	10.9	12.4	15.5	19.3	23.8	28.4	31.4	34.2	37.6	40.0	45.3	20
21	8.03	8.90	10.3	11.6	13.2	16.3	20.3	24.9	29.6	32.7	35.5	38.9	41.4	46.8	21
22	8.64	9.54	11.0	12.3	14.0	17.2	21.3	26.0	30.8	33.9	36.8	40.3	42.8	48.3	22
23	9.26	10.2	11.7	13.1	14.8	18.1	22.3	27.1	32.0	35.2	38.1	41.6	44.2	49.7	23
24	9.89	10.9	12.4	13.8	15.7	19.0	23.3	28.2	33.2	36.4	39.4	43.0	45.6	51.2	24
25	10.5	11.5	13.1	14.6	16.5	19.9	24.3	29.3	34.4	37.7	40.6	44.3	46.9	52.6	25
26	11.2	12.2	13.8	15.4	17.3	20.8	25.3	30.4	35.6	38.9	41.9	45.6	48.3	54.1	26
27	11.8	12.9	14.6	16.2	18.1	21.7	26.3	31.5	36.7	40.1	43.2	47.0	49.6	55.5	27
28	12.5	13.6	15.3	16.9	18.9	22.7	27.3	32.6	37.9	41.3	44.5	48.3	51.0	56.9	28
29	13.1	14.3	16.0	17.7	19.8	23.6	28.3	33.7	39.1	42.6	45.7	49.6	52.3	58.3	29
30	13.8	15.0	16.8	18.5	20.6	24.5	29.3	34.8	40.3	43.8	47.0	50.9	53.7	59.7	30

Table of $\chi_\varepsilon^2(p)$ such that $\Pr\{\chi^2(p) > \chi_\varepsilon^2(p)\} = \varepsilon$, where p is the number of degrees of freedom

Degree of freedom (v) = K – p – q

= 6 – 0 – 1

= 5

From the Chi-squares Table (Table 5.1.6), the values for 10 percent and 5 percent points of 5 degree of freedom are found to be 9.24 and 11.21. The value of Q is below the above limits. Hence the model is quite adequate. The model

$\nabla Z_t = (1 + 0.6B)\, a_t$

or $(1 - B)\, Z_t = (1 + 0.6\, B)\, a_t$

i.e. $Z_t = Z_{t-1} + 0.6\, a_{t-1} + a_t$

$$\sigma_a^2 = \frac{C_0}{1+\theta^2} = \frac{0.00052}{1+(-0.6)^2}$$

$= 0.00038$

$$\text{Standard error} = \frac{C_0\,(1+2r_1)^{1/2}}{n}$$

$= 0.00798$

5.1.7. Forecasting

As the model is found as

$\nabla Z_t = (1 - \theta B)\, a_t$ (5.1.7.1)

and $\theta = -0.6$

The above Equation can also be written as

$(1 - B)\, Z_t = (1 - \theta B)\, a_t$ (5.1.7.2)

but $a_t = \pi(B)\, Z_t$. (5.1.7.3)

Substituting this in equation (5.1.7.2) gives

$$(1 - B) Z_t = (1 - \theta B)\, \pi(B)\, Z_t$$

$$(1 - B) = (1 - \theta B)\, \pi(B)$$

$$\pi(B) = \frac{1 - B}{1 - \theta B}$$

but 1 – B can be written as 1- θB – (1- θ)B, putting this in the above equation gives

$$\pi(B) = \frac{1 - \theta B - (1 - \theta)B}{1 - \theta B}$$

$$= \frac{1 - \theta B}{1 - \theta B} - \frac{(1 - \theta)B}{1 - \theta B}$$

$$= 1 - (1 - \theta)B\{(1 - \theta B)^{-1}\}$$

$$= 1 - (1 - \theta)B\{1 + \theta B + \theta^2 B^2 + \theta^3 B^3 \ldots\}$$

$$= 1 - (1 - \theta)\{B + \theta B^2 + \theta^2 B^3 + \theta^3 B^4 \ldots\}$$

$$= 1 - (1 - \theta)B - (1 - \theta)\theta B^2 - (1 - \theta)\theta^2 B^3 - (1 - \theta)\theta^3 B^4 \ldots$$

Let $\lambda = 1 - \theta$ and substituting in the above equation,

$$\pi(B) = 1 - \lambda B - \lambda(1 - \lambda)B^2 - \lambda(1 - \lambda)^2 B^3 - \lambda(1 - \lambda)^3 B^4 \ldots$$

From equation (5.1.7.3)

$$a_t = \pi(B)\, Z_t$$

$$a_t = (1 - \lambda B - \lambda(1 - \lambda)B^2 - \lambda(1 - \lambda)^2 B^3 - \lambda(1 - \lambda)^3 B^4 \ldots)\, Z_t$$

$$a_t = Z_t - \lambda B Z_t - \lambda(1 - \lambda)B^2 Z_t - \lambda(1 - \lambda)^2 B^3 Z_t - \lambda(1 - \lambda)^3 B^4 Z_t \ldots$$

$$Z_t = \lambda B Z_t + \lambda(1 - \lambda)B^2 Z_t + \lambda(1 - \lambda)^2 B^3 Z_t + \lambda(1 - \lambda)^3 B^4 Z_t \ldots$$

$$= \lambda Z_{t-1} + \lambda(1 - \lambda) Z_{t-2} + \lambda(1 - \lambda)^2 Z_{t-3} + \lambda(1 - \lambda)^3 Z_{t-4}$$

then for a one step ahead forecast

$$\hat{Z}_{t+1} = \lambda Z_t + \lambda(1 - \lambda) Z_{t-1} + \lambda(1 - \lambda)^2 Z_{t-2} + \ldots + \lambda(1 - \lambda)^{15} Z_{t-15}$$

and by letting $\pi_j = \lambda(1 - \lambda)^{j-1}$ for j =1,2,...

thus the forecast expressed on a weighted average of the previous observations can be written in terms of the π weight as

$$\hat{Z}_{t+1} = \pi_1 Z_t + \pi_2 Z_{t-1} + \pi_3 Z_{t-2} + \ldots$$

The π weights calculated as shown above are given in the following table.

Table 5.1.7 The calculated π weight values

j	π_j
1	1.600
2	-0.960
3	0.576
4	-0.346
5	0.207
6	-0.124
7	0.075
8	-0.044
9	0.027
10	-0.016
11	0.010
12	-0.005
13	0.004
14	-0.002
15	0.001
16	0.000

The next step forecast $(\hat{Z}_{t+1})$ is obtained by:

$$Z_{17} = 1.6 \times Z_{16} - 0.960 \times Z_{15} + 0.576 \times Z_{14} - \ldots + .001 \times Z_2 - 0.000\, Z_1$$

When Z_{17} is available, the next forecast Z_{18} can be obtained by substituting Z_{t+1} for Z_t in the formula. By this method the forecast from 1998 up to 2001 have been worked out and given in the Table 5.1.8. The corresponding actual values of demand are also tabulated for the comparison.

Table 5.1.8 Forecasted electrical energy requirement (1998 – 2001)

Year	Actual energy Requirement (GWH)	Forecast value (GWH)	Percentage Error
1998	1628	1659	1.9
1999	1653	1714	3.7
2000	1689	1772	4.9
2001	1811	1832	1.1

The forecasts done using various regression and Holt's linear trend algorithm forecasting techniques along with the Box-Jenkins method, on the same data, are given in Table 5.1.9 and plotted on Graph 5.1.3

Table5.1.9. DEMAND FORECAST USING VARIOUS TECHNIQUES FROM 1998-2001IN GWH

Year	Observed	Linear	Polynomial	Logarithmic	Exponential	Power	Holt's Algo.	ARIMA(0,1,1)
1998	**1628**	1783	1658	1463	1816	1472	1748	1659
1999	**1653**	1899	1722	1494	1945	1505	1857	1714
2000	**1689**	2022	1783	1524	2085	1537	1972	1772
2001	**1811**	2154	1841	1553	2235	1569	2094	1832

Graph 5.1.3 ACTUAL AND FORECASTED DEMAND USING VARIOUS TECHNIQUES

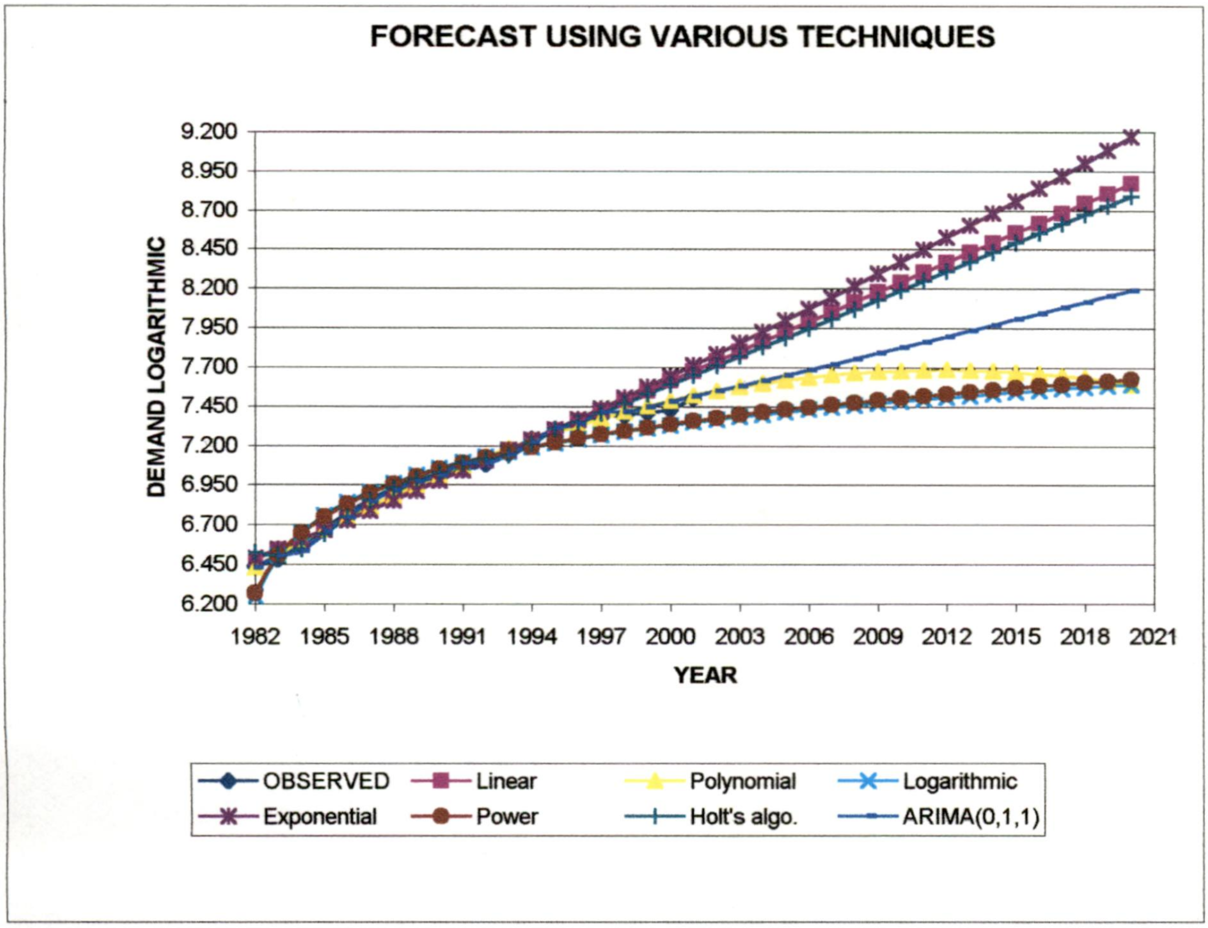


Table 5.1.10 MEAN ABSOLUTE PERCENTAGE ERRORS OF THE FORECASTING TECHNIQUES

Year	Linear	Polynomial	Logarithmic	Exponential	Power	Holt's algo.	ARIMA(0,1,1)
1998	9.5	1.8	10.1	11.6	9.6	7.4	1.9
1999	14.9	4.1	9.6	17.7	9.0	12.3	3.7
2000	19.7	5.5	9.8	23.4	9.0	16.8	4.9
2001	18.9	1.6	14.3	23.4	13.4	15.7	1.1
MAPE	15.8	3.3	11.0	19.0	10.2	13.0	2.9

It can be seen that the forecast error by Box-Jenkins method is very small (less than 3 %). Furthermore Graph 5.1.3 shows that forecasted values by Box Jenkins method is very much close to the actual values. Even though some regression forecasting techniques, like polynomial, yield forecast values close to the actual values at the beginning of the forecasted period, their characteristic tremendously changed after elapsing a certain lead-time (point of inflection).

Since the forecasting method by the selected Box-Jenkins model shows an upper hand and yields a fairly good result for the period retained for check-up purpose, the same model has been used to forecast the electrical energy demand up to 2020. The forecasted electrical energy demand values are given in Table 5.1.11. below .

Table 5.1.11. ETHIOPIAN ENERGY REQUIREMENT FORECASTED VALUES BY BOX – JENKINS METHOD

Year	Time series	Z_t (Log$_e$ of energy requirement forecast)	Forecast of energy requirement (Million KWh)
1998	17	7.4139	1659
1999	18	7.4466	1714
2000	19	7.4796	1772
2001	20	7.5129	1832
2002	21	7.5466	1894
2003	22	7.5805	1960
2004	23	7.6147	2028
2005	24	7.6491	2099
2006	25	7.6838	2173
2007	26	7.7188	2250
2008	27	7.7539	2331
2009	28	7.7893	2415
2010	29	7.8248	2502
2011	30	7.8606	2593
2012	31	7.8965	2688
2013	32	7.9325	2786
2014	33	7.9687	2889
2015	34	8.0051	2996
2016	35	8.0417	3108
2017	36	8.0784	3224
2018	37	8.1152	3345
2019	38	8.1523	3471
2020	39	8.1895	3603

5.2. PEAK DEMAND

The Annual peak demands are estimated by taking the average value observations annual load factors of the observed data, from the relation ship

$$Load\ factor = \frac{Average\ load}{Peak\ demand}$$

or

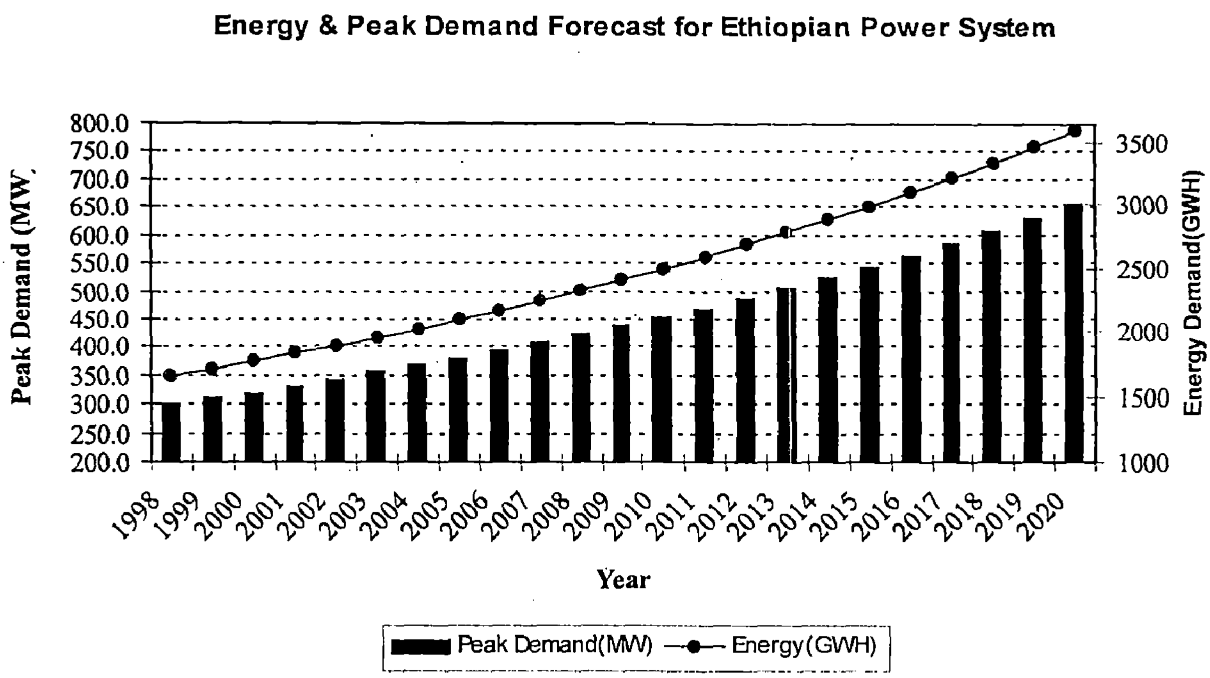
$$Peak\ demand = \frac{Annual\ Energy\ Requirement}{8760 * load\ factor}$$

The annual energy demand and peak demand are tabulated in Table 5.1.12. and plotted in Graph 5.1.4.

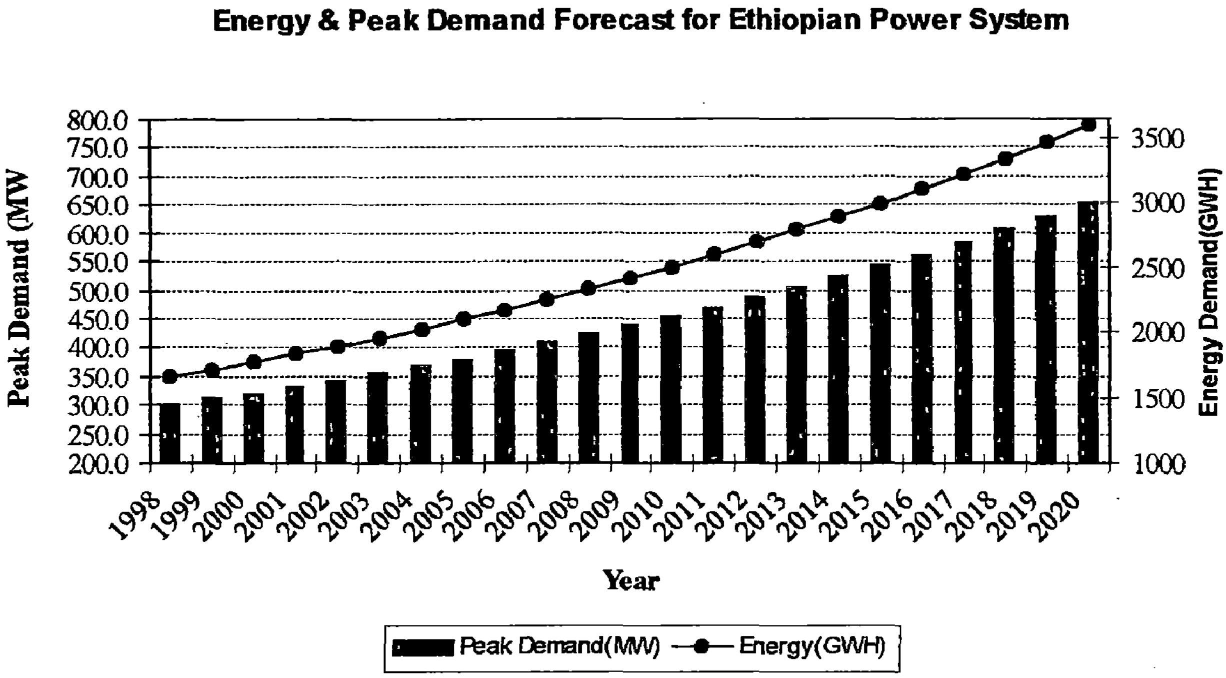
Table 5.1.12 FORECASTED ENERGY AND PEAK LOAD DEMAND

Year	Forecasted	
	Energy(GWH)	**Peak Demand (MW)**
1998	1659	300.6
1999	1714	310.6
2000	1772	321.0
2001	1832	331.9
2002	1894	343.2
2003	1960	355.1
2004	2028	367.4
2005	2099	380.3
2006	2173	393.7
2007	2250	407.7
2008	2331	422.3
2009	2415	437.5
2010	2502	453.4
2011	2593	469.8
2012	2688	487.0
2013	2786	504.9
2014	2889	523.5
2015	2996	542.9
2016	3108	563.1
2017	3224	584.2
2018	3345	606.1
2019	3471	629.0
2020	3603	652.8

Graph 5.1.4 FORECASTED ELECTRICAL ENERGY AND PEAK DEMAND



Graph 5.1.4 FORECASTED ELECTRICAL ENERGY AND PEAK DEMAND



CHAPTER 6

RELATIONSHIP AND DISTINCTION BETWEEN QUANTITATIVE FORECASTING METHODS

The three quantitative approaches to forecasting namely regression models, exponential smoothing algorithms and Box-Jenkins time series models have their own distinct features along with relation ships one to another.

In building regression models, emphasis is placed on relation ships among variables, so that forecasts of one variable will be based, in part on the values of others, where as exponential smoothing algorithms and Box-Jenkins ARIMA models use for prediction of future values of a particular variable exclusively on the basis of its own past values. Both regression and ARIMA analysis are based on the fitting to observed data of formal models. To some extent the structure of these models are determined by either the characteristics of the available data or subject matter theory, or both. The fitted models are then projected forward to derive forecasts.

In contrast, the exponential smoothing approach to forecasting is more ad hoc in character. Models are not explicitly built. Rather a collection of intuitively plausible prediction algorithms that have proved useful in practical applications has been assembled. The chief advantage of exponential smoothing is that future values of a very large number of time series can be routinely predicted with a minimum of manual intervention and expense.

6.1. RELATIONSHIP BETWEEN BOX-JENKINS ARIMA MODELS AND EXPONENTIAL SMOOTHING

The study of the relation ship between exponential smoothing and Box-Jenkins ARIMA models can be explored by using the simple exponential smoothing algorithm. Given time series Z_t, the current level of the series at time t, L_t, is found through the exponential algorithm:

$$L_t = \alpha Z_t + (1-\alpha)L_{t-1} \qquad (6.1.1)$$

Where $0 < \alpha < 1$

Standing at time t all future values Z_{t+h} of the series are then predicted by the current level, so the forecasts are

$$\hat{Z}_t(h) = L_t \qquad h = 1,2,3,\ldots$$

These forecasts, made at fixed time t, are all the same, so it is a constant prediction function.

Now let at denote the error that is made when Z_t is predicted at (t-1), so that

$$e_t = Z_t - L_{t-1} \qquad (6.1.2)$$

Thus it is now possible to transpose the simple exponential smoothing algorithm to a familiar ARIMA form. First replacing t by t-1 in equation (6.1.1) gives

$$L_{t-1} = \alpha Z_{t-1} + (1-\alpha)L_{t-2}$$

Then using equation (6.1.2) to substitute for L_{t-1} and L_{t-2} in this expression yields

$$Z_t - e_t = \alpha Z_{t-1} + (1-\alpha)(Z_{t-1} - e_{t-1})$$

so that

$$Z_t - Z_{t-1} = e_t - (1-\alpha)e_{t-1} \qquad (6.1.3)$$

Equation (6.1.3) relates the original time aeries Z_t to the one-step prediction errors e_t, from the simple exponential smoothing algorithm.

Finally setting

$$e_t = a_t \qquad \text{and} \qquad 1 - \alpha = \theta$$

in equation (6.1.3) we have

$$Z_t - Z_{t-1} = a_t - (1 - \alpha)a_{t-1}$$

This is ARIMA (0,1,1) model where the first difference of a time series obeys a first-order moving average model.

Thus forecasting through simple exponential smoothing is equivalent to prediction through a specific ARIMA model-the ARIMA (0,1,1) model with moving average parameter θ equal to one minus the smoothing constant α. It should be observed that the usual restriction in exponential smoothing, that the smoothing constant takes a value between zero and one, implies that the moving average parameter θ of the equivalent ARIMA (0,1,1) model must also be between zero and one. This is more restrictive than is required in the usual time series ARIMA model analysis where θ is permitted to take any value between -1 and 1.

Similarly prediction from a number of other exponential smoothing algorithm can be shown to be equivalent from specific ARIMA models. Moreover, if the model is indeed appropriate for the prediction of future values of a particular time series, this fact should be revealed through careful application of ARIMA model building methodology. Besides parameter estimation, the ARIMA methodology incorporates model selection and checking stages, which should allow the intelligence choice based on evidence in the data.

6.2 REGRESSION MODELS WITH ARIMA ERROR STRACTURES

Regression models are employed to forecast behavioral relationships among variables, as might be postulated by subject matter theory, while ARIMA model building provides quite a sophisticated basis for the prediction of future values of a time series exclusively from its own past history. Now, in some circumstances aspects of each of these approaches might be quite appealing. If worthwhile subject matter theory is available (say relation ship between economic indicators and electric energy demand), it is desirable to incorporate it into forecasting exercise, and the regression model framework will often provide a useful basis for completely this task. On the other hand, the forecasting must necessarily be conscious of the evolution of historical patterns, which will often be of more valuable than the understanding of contemporaneous relationships among variables. After all, the forecasting problem is essentially dynamic rather than static in character (such as relationship of electrical energy demand with time). The ARIMA model framework can be very useful for capturing such evolutionary patterns.

In fact, it is not at all necessary to choose to attack a forecasting problem through either regression model or ARIMA models. Since both are relatively simple parametric statistical models, it is quite straightforward in principle to write down a broader model incorporating each as a special case. Moreover, it is often not difficult to analyse actual data in the context of this more general model, hence to develop forecasts embodying both the regression and ARIMA components. Considering the model

$$Y_t = \alpha + \beta_1 X_{1t} + \beta_2 X_{2t} + \ldots + \beta_k X_{kt} + \in_t \qquad (6.2.1a)$$

$$\phi(B)(1-B)^d \in_t = \theta(B)a_t \qquad (6.2.1b)$$

Equation (6.2.1a) is simply the usual multiple regression model linking a dependent variable Y and k independent variables $X_1, X_2, \ldots X_k$, with an error term $\in_t$. However, extending the models of multiple regression, equation (6.2.1b) permits these errors terms to follow a general ARIMA(p,d,q) model. The notion here is that B is the back-shift operator, and the error term a_t is white noise. In other words, these errors are taken to have zero means, equal variances, and to be uncorrelated with one another.

Thus three special cases can be find from the general model (equation 6.2.1) :

i) If the error term $\in_t$ of equation (6.2.1a) is white noise, so that equation (6.2.1b) reduces to

$$\in_t = a_t$$

Then we simply have the basic multiple regression model under the standard assumptions as

- the error terms $\in_t$ all have mean zero,
- the error terms $\in_t$ have a common variance σ^2,
- the error terms $\in_t$ are not correlated with one another,
- the equation X_{it} (i= 1,...,k) are either fixed numbers or they are realizations of random variables that are uncorrelated with the error terms $\in_t$.

ii) Considering the case where the problem of autocorrelated errors in regression equations, in particular on special case where the errors obey a first order autoregressive model- that is, an ARIMA(1,0,0) process

$$\in_t - \phi \in_{t-1} = a_t$$

iii) In the special case of equation (6.2.1a)

Where

$$\beta_1 = \beta_2 = \ldots = \beta_k = 0$$

the independent variable no longer appear in equation (6.2.1a). The implication now is that the dependent variable simply follows an ARIMA model, and that the independent variables are of no value in its prediction, at least with in the framework of the model of equation (6.2.1).

CHAPTER 7
CONCLUSION

7.1 CONCLUSION

As discussed in chapter III, forecaster use a variety of techniques to develop projections of energy demand for use in utility capacity planning, revenue projections, and policy analysis. Each technique has advantages and disadvantages, which can vary from application to application. From various forecasting methods an appropriate scheme shall be selected considering, the purpose of the forecast, the type and quantity of data in hand, the time horizon to be projected and the accuracy level required.

However there is no ideal method, which can exactly reflect the requirement accurately. The forecasting of future demands involves an uncertainty factor, which makes the predictions vulnerable to random changes occurring due to indeterminate factors like social and political, climate, war, and natural calamities. However, the forecasts should be made with as high accuracy as possible using scientific methods and based on reliable data.

In the conventional extrapolation methods forecasts fail to reflect the random patterns of data fluctuation over a long period.

The correlation models provide fairly accurate results. But they have the disadvantage of requiring additional data from other fields, thus rendering the method more expensive and liable to errors if data provided are either erroneous or unreliable. Moreover the economic and other parameters on which the demand forecasts are based, have to be forecasted which is a major task by itself. Applying such methods with very limited information would result in developing erroneous or very unreliable forecast.

On the other hand, the time series analysis has no such problem. The only data required is the historical demand data, which is readily available with any electricity undertaking. The stochastic time series modeling represented by Box-Jenkins methodology can capture such data patterns over long period, which is constantly marked by random changes. As detailed in Chapter IV and illustrated by applying the method on the Ethiopian electrical energy demand in Chapter V, considerable effort is made in this modeling approach in characterizing patterns of variations of the entire set of past data and the parameters of the resulting model then provide the weightage to be placed on the past data in the ensuring forecasting scheme.

A comparative study of the forecasts of energy requirement of Ethiopia is made to observe the advantage of Box- Jenkins method. The accuracy measurement shows that the Box-Jenkins method is more accurate than the other methods. The forecasts by the Box-Jenkins method are closer to the values for the entire period available for comparison with the actual.

As described in chapter VI, the forecasting through simple exponential smoothing is equivalent to prediction through a specific ARIMA model. On the other hand, the forecasting must necessarily be conscious of the evolution of historical patterns, which will often be of more valuable than the understanding of contemporaneous relationships among variables done by regression method.

7.2 CONCLUDING REMARKS

Because of the inherent uncertainty of forecasting regardless of how sophisticated and appropriate the model is, it is useful to adopt in parallel a flexible supply-side strategy based on resources which can be brought on-line more quickly than the ten to twelve year lead time required for a large central generating unit. The popularity of conservation and load management program is, in part, due to the fact that these resources can be phased in more quickly and in much smaller components than can large generating plants. This can be achieved through the scheme of cogeneration, many renewable resource technologies, and smaller fossil fuel plants. Given these relatively short lead-time alternatives, adjustments can be made in a utilities resource plan when major deviation encountered on the appropriately developed long term forecast like forecasts by Box-Jenkins method.

APPENDIX 'A'

ELEMENTS OF STATISTICS AND PROBABILITY [12]

A.1 Importance of statistics and probability in forecasting:

Quantitative scientific data may be classified into two kinds experimental data and historical data. The experimental data are measured through experiments and can be usually determined repeatedly by experiments. The historical data on the other hand are collected from experiments. The historical data on the other hand are collected from natural phenomenon that can be observed only once and will not occur again. Historical data represents past events and will not occur again. Historical data represents past events and are often used for analyzing and predicting the future. Statistics essentially concerns with scientific methods of collecting, organizing, summarizing, presenting and analyzing data, as well as drawing valid conclusions and making reasonable decisions on the basis of each analysis. Forecasting the future is associated with uncertainty and therefore the language of probability is often used in stating conclusions.

In view of importance of statistical methods in the field of forecasting, it is appropriate to be aware of few elementary concepts of statistics and probability, which are essential for the purpose.

A. 2 Measures of Central Tendency:

Averages – An average is a value, which is typical or representative of a set of data. Since such typical values tend to be centrally within a set of data arranged according to magnitude, averages are also called measures of central tendency. The most common type of averages are mean, median, mode, geometric mean and the harmonic mean.

A.2.1 Mean or Arithmetic mean:

The mean or arithmetic mean of a set of II numbers X_1, X_2, X_3.....X_n is denoted by X and is defined as

$$\overline{X} = \frac{X_1 + X_2 + X_3 \ldots\ldots\ldots\ldots + X_N}{N} = \frac{\Sigma X}{N}$$

A.2.2 Weighted arithmetic mean:

Some times the numbers X_1, X_2....X_N are associated with certain weighting factors or weights W_1, W_2....W_N depending on significance or importance attached to the numbers. Then

$$\overline{X} = \frac{W_1 X_1 + W_2 X_2 \ldots\ldots W_N X_N}{(W_1 + W_2 \ldots\ldots W_N)} = \frac{\Sigma WX}{\Sigma W}$$

is called the weighted arithmetic mean.

A.2.3 Median:

The median of a set of numbers arranged in order or magnitude is the middle value or the arithmetic mean of the two middle values.

A.2.4. Mode:

The mode of a set of numbers is that value which occurs with the greatest frequency i.e., it is the most common value. The mode may not exist and even if it does exist it may not be unique.

A.2.5. The Geometric mean:

The Geometric mean G of a set of N numbers X_1, X_2 ...X_N is the Nth root of the product of the numbers

$$G = N\sqrt{X_1 X_2 \ldots\ldots\ldots X_N}$$

A.2.6 The Harmonic mean:

The harmonic mean H of the set of N number X_1, X_2.... X_N is the reciprocal of the arithmetic mean of the reciprocals of the numbers

$$H = \frac{N}{\Sigma \frac{1}{X}}$$

A.2.7. Relation between arithmetic, geometric and harmonic means:

The geometric mean of a set of positive numbers X_1, $X_2 \ldots X_N$ is less then or equal to their arithmetic mean but is greater or equal to their harmonic mean.

$$H \leq G \leq \overline{X}$$

The equality sign hold if all the numbers X_1, $X_2 \ldots X_N$ are identical.

A.3. Measures of Dispersion:

The degree to which numerical data tend to spread about an average value is called the variation or dispersion of the data. Some of the measures of dispersion are range, mean deviation and standard deviation.

A.3.1. Range:

The range of a set of numbers is the difference between the largest and smallest numbers in the set.

A.3.2. Mean deviation:

The mean deviation or average deviation, of a set of N numbers X_1, $X_2 \ldots X_N$ is defined by

$$\text{Mean deviation} = \text{N.D} = \frac{\Sigma\left|X - \overline{X}\right|}{N}$$

A.3.4. Standard deviation:

The standard deviation of a set of N numbers, X_1, $X_2 \ldots X_N$ is denoted by

$$S = \frac{\sqrt{\Sigma\left(X - \overline{X}\right)^2}}{N}$$

A.3.5 Variance:

The variance of a set of data is defined as the square of the standard deviation and is given by S^2.

When it is necessary to distinguish the standard deviation of a population from the standard deviation of a sample drawn from this population, the symbol S is used for the latter and σ is used for the former. Thus S^2 and σ^2 would represent the sample variance and population variance respectively.

A.3.6. Absolute and Relative Dispersion, Coefficient of variation:

The actual variation of desperation as determined from the standard deviation or other measure is called the absolute dispersion.

$$\text{The relative dispersion} = \frac{\text{Absolute dispersion}}{\text{Average}}$$

$$\text{Coefficient of variation} = \frac{\text{Standard deviation}}{\text{Arithmetic mean}} = \frac{S}{\overline{X}}$$

A.3.7. Standardized variable, Standard scores:

The variable $Z = \frac{X - \overline{X}}{S}$ which measures the deviation from the mean in units of the standard deviation is called a standardized variable and is a dimensionless quantity.

If deviations from the mean are given in units of the standard deviation, they are said to be expressed in standard units or standard scores.

A. 4 Probability:

A.4.1. Classical definition of probability:

Suppose an event E can happen in h ways out of a total of n possible equally likely ways. Then the probability of occurrence of the event (called success) is denoted by p = h/n and similarly the probability of non-occurrence of the event is denoted by

$$a = 1 - \frac{h}{n} = 1 - p \quad \text{and}\ p + q = 1$$

A.4.2 Statistical definition of probability:

The estimated probability or emphatically probability of an event is taken as the relative frequency of the event when the number of observations is large. The probability itself is the limit of the relative frequency as the number of observations increase indefinitely.

A.4.3 Normal distribution:

The most important examples of a continuous probability distribution is the normal distribution, normal curve of Gaussian distribution defined as

$$Y = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2}(x-\mu)^2 / \sigma^2$$

Where μ = mean, σ = standard deviation, π = 3.14159, θ = 2.71828. When the variable x is expressed in terms of standard units.

$$Z = (x-\mu)/\sigma \quad \text{then } Y = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

In such case Z is normally distributed with z = mean zero, and variance is equal to one.

The normal distribution is defined by mean $\overline{X}$ and standard deviation σ. The area covered by the curve which is bell shaped and the abscissa is equal to unity.

For normal distribution it turns out that:

a) 68.27% of the area of the curve is included between $\overline{X}-3\sigma$ and $\overline{X}+\sigma$

b) 95.45% of the area of the curve is included between $\overline{X}-2\sigma$ and $\overline{X}+2\sigma$

c) 99.73% of the area of the curve is included between $\overline{X}-3\sigma$ and $\overline{X}+3\sigma$

The intervals of -σ to σ, -2 σ to + 2σ and -3σ to +3σ are called as confidence intervals or confidence limits. These are also called as fidutial limits.

These limits represent the probability of the statistic lying around its mean value.

A.5 Curve fitting and the method of least squares:

A statistical data, plotted on a rectilinear co-ordinate system, usually results in a scatter diagram. However, it is possible to establish a relation between the two variables. The mathematical relationship thus established maybe linear or curvilinear. Finding the equations of approximating curves, which fit a given set of data is called curve fitting.

If a dependent variable is obtained as a series of measured or observed values, the relation between independent and dependent variable is best expressed as a line or curve. It is possible to draw an approximating line or curve to fit a set of data by free hand method. But this method is not generally accurate and it is possible that different persons draw different curves. To avoid individual Judgment in curve fitting to a set of data it is therefore necessary to define a best fitting curve.

Consider a set of data (X_1, Y_1), $(X_2, Y_2)\ldots(X_N, Y_N)$ plotted on a graph and a curve fitted for the data. For a given value of X, say X_1, there will be a difference between the value Y_1 and the corresponding values as determined from the curve. Let the same be denoted by D_1 which represents a deviation, error or residual and this may be positive, negative or zero. Then for corresponding values of X_2, $X_3\ldots X_N$ the deviations will be D_2, $D_3\ldots D_N$. a measure of the goodness of fit of the curve to the given data is provided by the quantity $D^2_1 + D^2_2 + \ldots + D^2_N$. if this is small then the fit is food, if it is large then the fit is bad.

Therefore the best fitting curve to a set of data is the one having the property that $D^2_1 + D^2_2 + \ldots\ldots + D^2_N$ i.e., sum of the squares of the deviations is a minimum. A curve having this property is said to fit the data in the least square sense and is called a least square curve.

Least square line:

The least line approximating a set of points (X_1, Y_1), $(X_2, Y_2)\ldots(X_N, Y_N)$ has the equation.

$$Y = A + BX \qquad (1)$$

The value of Y on this line corresponding to $X = X_1, X_2, \ldots X_N$ are $(A + BX_1)$, $(A + BX_2) \ldots (A+BX_N)$ while the actual values are $Y_1, Y_2 \ldots Y_N$ respectively.

Then the sum of the squares of deviation is given by $S = (A+BX_1 - Y_1)^2 + (A+RX_2-Y_2)^2 + \ldots + (A+BX_N-Y_N)^2$ and this value should be a minimum.

S is minimum when the partial derivatives of S with respect to A and B are zero i.e.,

$$\frac{\delta S}{\delta A} = s(A + BX_1 - Y_1) + (A + BX_2 - Y_2) + .. + (A + BX_N - Y_N) = 0$$

$$\frac{\delta S}{\delta B} = 2(A + BX_1 - Y_1)X_1 + (A + BX_2 - Y_2) + .. + (A + BX_N - Y_N)X_N = 0$$

Solving the above two simultaneous equations value of A and B are obtained.

$$A = \frac{(\Sigma Y)(\Sigma X)^2 - (\Sigma X)(\Sigma XY)}{N\Sigma X^2 - (\Sigma X)^2}$$

$$B = \frac{N(\Sigma XY) - (\Sigma X(\Sigma Y)}{N\Sigma\Sigma^2 - (\Sigma X)^2}$$

A.6 Regression:

For the curve fitted by least square method, if Y is dependent variable and X is the independent variable then the curve is called the regression curve of Y on X. if Y is made independent and x as dependent variable, the resulting curve is called the regression curve of X on Y. Generally the regression line or curve of Y on X is not the same as the regression line or curve of X on Y.

A.7 Time Series :

If the independent variable x is time, the data shows values of Y at various times. Data arranged according to time are called time series. The regression line or curve of Y on X in this case is called a trend line or trend curve and is often used for purposes of estimation, prediction or forecasting.

A. 8 Correlation:

The correlation is the degree of relationship which seeks to determine how well a linear or other equation describes or explains the relationship between variables (X,Y).

Whenever a curve is fitted by the method of least squares a measure of closeness of fit is obtained by the sum of squares of deviations of measured values from the curve. The measure can be used to select the best form of curve out of several trial curves.

If Y increases with X then the correlation is positive or direct correlation. If Y decreases as X increases then the correlation is negative or inverse correlation. If there is no relationship indicated between them, then they are uncorrelated.

A. 9 Standard Error of Estimate:

A measure of the scatter or variation about the line or the curve is the standard error of estimate, analogous to the standard deviation. The standard deviation measures the scatter about the arithmetic mean and is the quadratic mean of the deviations. Similarly the standard error of estimate $S_{Y.X}$ is the quadratic mean of the deviations about the line or curve of regression.

$$S_{Y.X} = \sqrt{\frac{\sum (Y - Y_{est})^2}{N}} \qquad (12)$$

If recessions of X on Y are used then the standard error is

$$S_{X.Y} = \sqrt{\frac{\Sigma (X - X_{est})^2}{N}} \qquad (13)$$

Value of Y_{est} and X_{est} represents those obtained from the respective regression equation similar to (1) and (B).

For the regression lime of the from Y = A + BX. The equation (12) can be written as

$$S^2_{Y.X} = \frac{\sum Y^2 - A\sum Y - B\sum XY}{N} \qquad (14)$$

The standard error of estimate has properties analogous to those of the standard deviation. If lines or curves parallel to the regression line or curve of Y on X are drawn at respectively vertical distances of of $S_{Y.X}$, $2S_{Y.x}$ and $3S_{Y.X}$ from it and if the value of N is large enough, then these would be included between these lines or curves about 68%, 95% and 99.7% of the data points. When the data represents a set of small samples then the modified standard deviation is given by $\hat{S} = \sqrt{\frac{N}{N-1}}$ S and a modified standard error of estimate is given by $\hat{S}_{Y.X} = \sqrt{\frac{N}{N-2}} S_{Y.X}$

Therefore some statisticians prefer to define (12) and (13) with N-2 replacing N in the denominator.

In general, $\hat{S}_{Y.X} = \sqrt{\frac{\Sigma(Y - Y_{est})^2}{N - df1}}$ (15)

Is used where df1 is the degree of freedom lost. The degree of freedom lost has value of 2 for a straight line, 3 for a simple curve, 4 for a simple line combined with a curve and 5 for S curve etc., and in general it is n if the equation is of $n\text{-}1^{th}$ order.

A.10 Coefficient of correlation:

The ratio of the explained variation to the total variation is called the coefficient of determination and the coefficient of correlation 1st the square-root of this:

$$r = \pm\sqrt{\frac{\text{explained variation}}{\text{Total variation}}} = \pm\sqrt{\frac{\Sigma(Y_{est} - V)^2}{\Sigma(Y - V)^2}} \qquad (16)$$

and the value of r varies from –1 to +1 and is a dimensionless quantity.

This is also written as $$r = \sqrt{1 - \frac{S^2 l_{Y.X}}{S^2{}_Y}} \qquad (17)$$

Where S_y is the standard deviation.

A.11 Co-variance:

Co-variance of two variables X and Y for a set of N data points is expressed by

$$S_{X.Y} = \frac{\sum(X - \bar{X})\sum(Y - \bar{Y})}{N} \qquad (18)$$

A.12 Multiple correlation:

The degree of relationship existing between three or more variables is called multiple correlations. The fundamental principles involved in problem of multiple correlations are analogous to those of simple correlation.

APPENDIX 'B'

CHOICE OF TREND CURVE

B.1 The trend curves that are frequently used for analysis of economic data are [7]:

(1) Polynomials

(2) Exponentials

(3) Modified exponentials

(4) S- curve

(5) Combination of more than one curve

B.1.1 Polynomials

Straight live – demand = a ÷ bt where a and b are constants. The slops of a straight line is constant which means that the demand is increasing by a constant amount each years. If we plot the slops against time we get a straight line and this is the required slops characteristics.

B.1.2 Parabola

Demand = a+bt +ct^2 where a, b & c are constants. Here also the slope of parabola changes uniformly with time, and the slope time relationship will be a straight line inclined at a certain angle to horizontal.

B.1.3 Exponential

Simple exponential (compound interest trend) – log (demand) = a+ bt where and b are constants. The demand increases by a constant proportion each year and the ratio of the slope of the demand to the demand is constant. The moving average gives an estimate of the demand and the ratio of slope to the moving average plotted against time results in a horizontal straight line.

Log Parabola – Log (demand) = a + bt + ct^2 where a, b and c are constant. For log parabola the ratio of slope of moving average varies linearly with time resulting in a sloping straight line.

B.1.4 Modified Exponential

The use of modified exponential curves implies the existence of an upper limit to the demand which is approached asymptotically. (i) Simple modified exponential – Here demand = a – br^t were a, b and r are positive constants and r is less than 1. The logarithm of slope against time gives a straight line sloping down to the right.

(ii) Compretz – Log (demand) = a-brt where a, b and r are positive constant sand r is less than 1. The logarithm of the ratio of the slope to the square of the moving average against time results in a straight line sloping down to the right.

(iii) Logistics: Demand = 1/(a+br+t) where a, b and r are positive constants and r is less than 1. The logarithm of the ratio of the slope to the square of the moving average against time results in a straight line sloping down to the right.

B. 2 Choice of Trend Curves

The trend curves are preferably established by means of least square method. There are two assumptions implied in fitting of a mathematical curve. The first assumption concerns the types of curve which is chosen to fit the demand figures. The second assumption is that the chosen curve when extrapolated, represent the picture of future demand.

The purpose of forecasting is to estimate the change from the current to the future position, when a trend curve is fitted to demand figures and extrapolated to provide a forecast, its reliability will be depend on the accuracy of the slope of the trend. If the trend is linear the estimates of slopes will be equal. If the trend is compound interest or modified exponential the estimates of the will be changing in a specified manner. The slope of the demand is therefore a criterion for the selection of the trend curve and forms the basis of the techniques for choosing a curve.

When trend curves are fitted to the set of data, it is usually found that the closeness of the fit is approximately the same for all the curves. There will be little to chose between the curves as representation of the actual data. When the curves are extrapolated, they diverge. Even small extrapolations may lead to unacceptable large divergence. Therefore it is necessary to find the applicability of the trend curves to be used for forecasting.

Assuming the difference between the curves may be very small which can be neglected over the range of data, the project of the curve into the future will depend on the rate of change of various curves. For any curve Y = [f(L)] the rate of change at any point of the curve can be found out by the value of its derivative dy/dt = f'(t) at that point. For the curves under consideration the difference between predictions will arise from the differences between the derivatives. It is therefore necessary to compare the curve representing the rate of change of demand with the derivatives to test the suitability of the curve for prediction. The easiest curve, which can be recognized is a straight line. Therefore, for each curve the transformation of the derivatives which will yield a straight line with 't' as independent variable is found. Each such transformation is applied to estimate rates of change of the data at each year of the period and the results are plotted against time. It is then decided by inspection whether for any transformation of the rate of change, the plotted values appear to approximate to a straight line. Where a straight line is a reasonable fit, the corresponding curve may be expected to give satisfactory predictions. The rate of change is the slope and the transformation obtained to characterize the curve are called slope characteristics. Slope characteristics of the various curves are summarized below:

Compute and plot against time	If the results oscillates about a straight line which is	Then the curve suggested is
Slope	Horizontal	Straight line
Slope	At an angle to the horizontal	Parabola
Slope/moving average	Horizontal	Simple exponential
Slope/moving average	At an angle to the horizontal.	Logarithmic parabola.
Logarithm of slope	Sloping down to the right	Simple modified exponential
Log(slope/moving Available.)	Sloping down to the right	Gompertz
Log [slope/(moving av.)^2]	Sloping down to the right	Logistic

In order to determine the slope characteristics it is necessary to calculate the slope for the set of demand data. Because of the fluctuations from year to year, the actual demand and the simple differences are not considered satisfactory for the purpose and some smoothing is done by moving averages. To obtain an estimate of the slope for a given year a short period (usually 5,7 or 9 years) is chosen with the given year at the middle. The average slope over this period is then calculated and this is used as the slope for this given year.

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