PERFORMANCE ANALYSIS OF FILTER IN EARTH DAMS

A DISSERTATION

submitted in partial fulfilment of the
requirements for the award of the degree
of
MASTER OF TECHNOLOGY
in
WATER RESOURCES DEVELOPMENT

By

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CANDIDATE'S DECLARATION

I hereby certify that the work presented in the thesis report entitled, **"PERFORMANCE ANALYSIS OF FILTER IN EARTH DAMS",** in partial fulfillment of the requirements for the award of the degree of **MASTER OF TECHNOLOGY** in **WATER RESOURCES DEVELOPMENT**, submitted in Water Resources Development Training Centre, Indian Institute of Technology, Roorkee, is an authentic record of my own work carried out during the period from July16th, 2001 to February, 2002 under the supervision of **Dr. G.C.Mishra**, *Professor, WRDTC* , and **Dr. N.K.Samadhiya**, *Assistant Professor, Department of Civil Engineering*, Indian Institute of Technology, Roorkee, India.

The matter embodied in this thesis has not been submitted by me for the award of any other degree or diploma.

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Date: ***February 1, 2002***

Place: Roorkee

CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of my knowledge and belief.

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SYNOPSIS

The water seeping through the body of the earth dam and through the foundation of the earth dam may prove harmful to the stability of the dam by causing softening and sloughing of the slopes due to development of pore pressures. It may also cause piping either through the body or through the foundation, and thus resulting in the failure of the dam.

For a homogeneous dam founded on a pervious foundation, seepage is expected to appear on the downstream face unless a cut off has been constructed through the pervious foundation, thus permitting the downstream portion of pervious foundation to act as a drain. Of course it is a simple matter to provide drainage so that the seepage does not reach the downstream face.

The problem of seepage through an earth dam resting on a pervious foundation has been analyzed in the thesis. In addition, a horizontal toe drain (under filter) was located at the downstream portion of the dam, and the performance of filter in earth dam has been studied. The purpose of the drain is to control seepage through the dam and reduce the exit gradient in the downstream of the earth dam.

The seepage flow to a filter of finite width in a homogeneous earth dam resting on a porous medium of finite thickness has been computed using potential theory and conformal mapping. The seepage water to the filter of finite width is drained by parallel pipes to the downstream. Depending on the hydraulic conductivity of the porous material in the drain pipe, spacing of the pipes and width of the filter, pore water pressure develops along the filter, which influences seepage through the foundation and exit gradient downstream. Provision of filter increases seepage but reduces the exit gradient hence reduces the harmful effect of the seepage force.

Further the performance of upstream blanket in reducing seepage has been analyzed.

CONTENTS

NOTATIONS

A_d = area of the drain

b = thickness of upstream blanket

b_1 = distance from upstream to the starting point of filter

b_2 = distance from upstream to the end of filter

b_3 = base width of the dam.

C = constant

$c,d,f,$ = value of t corresponding to C,D,F respectively

$F(\phi,m)$ = elliptic integral of the first kind

H_1,H_2 = height of upstream and downstream water level

H = hydraulic head different

I_E = exit gradient at the down stream

i = imaginary unit

k = coefficient of permeability of pervious foundation

k_b = coefficient of permeability of blanket material

k_f = coefficient of permeability of filter material

L_b = length of the upstream blanket

L_d = length of the cross drain

m = modulus of elliptic integral

M, N = complex constants

p = water pressure

q_1 = quality of seepage

S_d = spacing of the cross drains

T = depth of pervious stratum

t = $r+is$ = complex variable representing semi in finite plane.

u = discharge velocity in x direction

v = discharge velocity in f direction

w_f = width of the filter

w = $\phi + i\psi$ = complex potential

z = $x+iy$ = complex variable representing physical plane.

Greek Symbols

γ = the unit weight of water

ϕ = velocity potential function

ψ = stream function

CHAPTER – 1

INTRODUCTION

1.1 GENERAL

The seepage of liquid through porous media has many practical applications in hydrologic, irrigation, sanitary, civil and petroleum engineering. Seepage takes place through earth dam, as all soil materials are pervious to a smaller or larger degree. Seepage is one of the major causes of the Earth dam failure. It is of fundamental importance to control the seepage through Earth dams not only to keep the water loss well within economic limits but also to take adequate measure to ensure the safety of the Dam. The understanding of the basic principle of seepage flow is essential for design and analysis of earth dam safety.

Seepage through the Embankment as well as the foundation is controlled by two approaches, generally used in combination: The first approach involves reduction of the quantity of seepage, or keeping the water out as far as feasible. In the embankment, this requires provision of an impervious zone or impervious membrane of manufactured material. The second approach involves providing a safe outlet to water, which still enters the embankment or the foundation, in spite of measures taken in the first category. This requires provision of drainage arrangements downstream of the seepage barrier such that the seepage forces are not able to cause soil migration, and their magnitude and direction are such that they do not cause embankment sliding or sloughing or foundation blow out.

1.2 OBJECTIVE OF STUDY

The present investigation is primarily concerned with two-dimensional steady unconfined flow through foundation of an earth dam resting on a pervious foundation. In addition a horizontal toe drain filter is located at the downstream portion of the dam the Schwarz-Christoffel transformation is used for finding the solution to the problem.

A drain system comprised of a filter not extending to the toe of the dam and provided with parallel pipe drains is economical. The performance of such drain system has not been studied yet. In this thesis the effect of width of the filter, location of the filter, the influence of the spacing area, and filing material of the drain pipe on seepage and exit gradient has been analyzed

1.3 SCOPE OF STUDY

In this study, an attempt has been made to study the effect of horizontal drain with different boundaries conditions. The cases with the following boundaries conditions have been studied

(i) Flow under an earth dam without drain founded on permeable soil of finite depth (Chapter 3).

(ii) Flow under an earth dam with drain founded on permeable soil of finite depth (Chapter 4).

(iii) Flow through foundation of an earth dam with an upstream blanket and a filter drain system (Chapter 5).

The solutions to the above cases have been obtained with the help of conformal mapping and numerical integration, Gaussian Quadrature method has been used to carry out the integration. A computer programming in 'Fortran' has been developed.

CHAPTER – 2

REVIEW OF LITERATURE

A literature review has been made on study of performance of drain in hydraulic structure.

Meleshchenko (1936) and Numerov (1948) have provided solution for hydraulic structure with drainage holes, wherein they studied in effect of one or two drainage holes in the otherwise impervious floor. The effect of plane drainage connected to downstream bed in case of seepage below a flat apron or a single overfall founded on infinite depth of permeable soil was obtained by Zamarin (1931). Sangal (1964) determined the extent of reduction in pressure affected by a flat and deep filter of particular dimensions below the foundation of a barrage with the help of electrical analogy model. Using electrical analogue the effect of intermediate drainage filter on seepage pressure has been studied by Arumugam (1971). A case of a flat bottom weir resting on a porous medium of infinite depth has been considered.

Chawla (1973) has used conformal mapping to find the performance of intermediate drain provided at the base of a hydraulic structure with two end sheet piles resting on a permeable foundation of infinite depth. Conformal mapping technique has been used.

Kumar has studied the effect of intermediate filter for the following boundary conditions (1995).

(i) Flow under a weir with unequal partial cut-offs at both end of the floor and intermediate filter founded on permeable soil of finite depth.

(ii) Flow under a weir with a partial cut-off at upstream end, a complete cut-off at downstream end of the floor and an intermediate filter founded on permeable soil of finite depth.

(iii) Flow under a weir with unequal partial cut-offs at both end of the floor and an intermediate filter founded on permeable soil underlain by a sloping impervious stratum.

The solution of the problems of first two cases has been obtained with the help of conformal mapping. The transformation equations have been integrated numerically using Simpson's formula. The solution of the problem in the third case is obtained by solving the Laplace equation by finite element method. A computer program has been developed to compute the uplift pressure all along the floor and exit gradient at the end of the floor. The factor of safety against heave below the filter is also determined to study the safety against piping below the filter.

CHAPTER – 3

FLOW UNDER AN EARTH DAM WITHOUT FILTER FOUNDED ON PERMEABLE SOIL OF FINITE DEPTH

3.1 INTRODUCTION

In order to study the performance of filter in earth dam first we study the flow under the structure in the absence of the filter. We assume that the hydraulic conductivity of the compacted material is much less than the hydraulic conductivity of the soil foundation. For analyzing the flow through foundation, the body of the earth dam has been assumed to the impervious.

3.2 STATEMENT OF THE PROBLEM

A flat bottom earth dam resting on a homogeneous isotropic porous medium of finite depth is shown in Fig. 3.1. The flow through the body of the earth dam is neglected. The bottom width of the earth dam is b_3, the thickness of the foundation soil layer is T. Seepage through foundation soil occurs because of seepage head, h (hydraulic head difference). We intend to find the seepage flow rate as a function of b and T.

3.3 ANALYSIS

3.3.1 Mapping of the Flow Domain in z- Plane onto t- Plane: z= $f_1(t)$

The Schwarz–Christoffel transformation that gives the afore-mentioned mapping is

$$\frac{dz}{dt} = \frac{M}{t-f}$$

or,

$$z = M\int \frac{dt}{t-f} + N \tag{3.1}$$

As one traverses in t - plane along a small circle of radius - r around point F (t=f) from $\theta = \pi$ to $\theta = 2\pi$, there is a change of (-iT) in z plane.

Putting : $t\text{-}f = r.e.^{i\theta}$; $dt = r.e.^{i\theta} \,.\, id\theta$, we have:

$$-iT = M\int_{\pi}^{2\pi}\frac{r.e.^{i\theta}.id\theta}{r.e.^{i\theta}} = M\pi i$$

or, $$M = -\frac{T}{\pi} \tag{3.2}$$

Inserting value of M into Eq. 3.1, we have

$$z = -\frac{T}{\pi}\int\frac{dt}{t-f} + N$$

or, $$z = -\frac{T}{\pi}\int(-)\frac{dt}{t-f} + N$$

Hence, $$t = -\frac{T}{\pi}\ln\left(f-t\right)+N \tag{3.3}$$

(i) *At point B*: t= 0, z = 0; hence, $0 = -\frac{T}{\pi}\ln\left(f-0\right)+N$

and, $$N = \frac{T}{\pi}\ln f \tag{3.4}$$

Inserting value of N into Eq. 3.3, we have

$$z = -\frac{T}{\pi}\ln\left(f-t\right)+\frac{T}{\pi}\ln f$$

or, $$z = -\frac{T}{\pi}\ln\frac{f-t}{f}$$

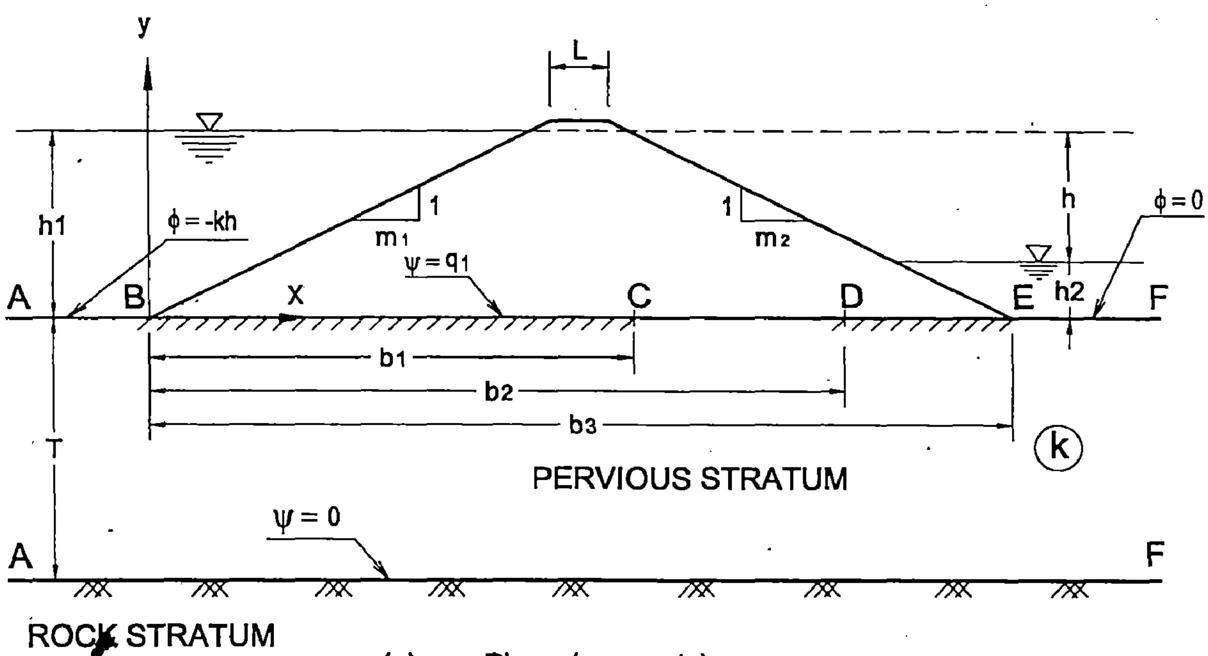
and, $$e^{-\frac{\pi}{T}z} = \frac{f-t}{f}$$

Hence, $$t = f\left[1-e^{-\frac{\pi}{T}z}\right] \tag{3.5}$$

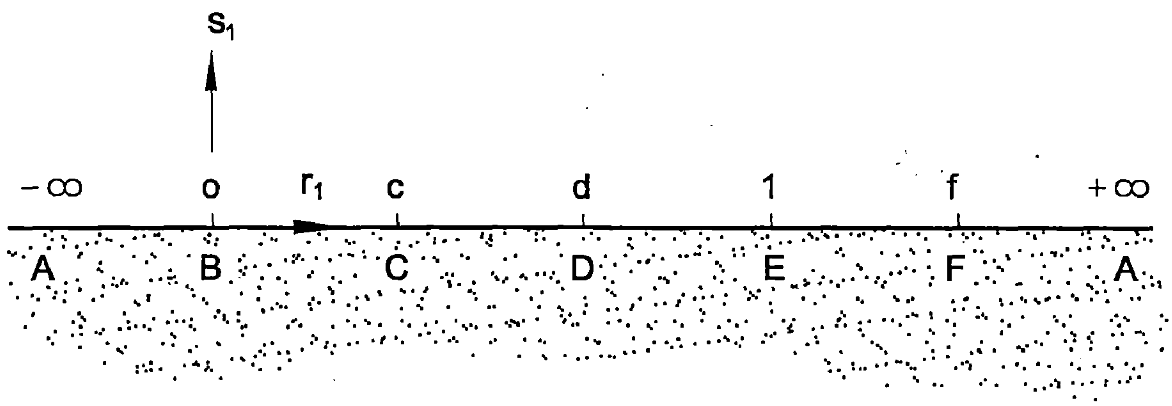
Therefore, $$f = \frac{1}{1-e^{-\frac{\pi}{T}L}} \tag{3.6}$$

$$c = f\left[1-e^{-\frac{\pi}{T}L}\right] \tag{3.7}$$

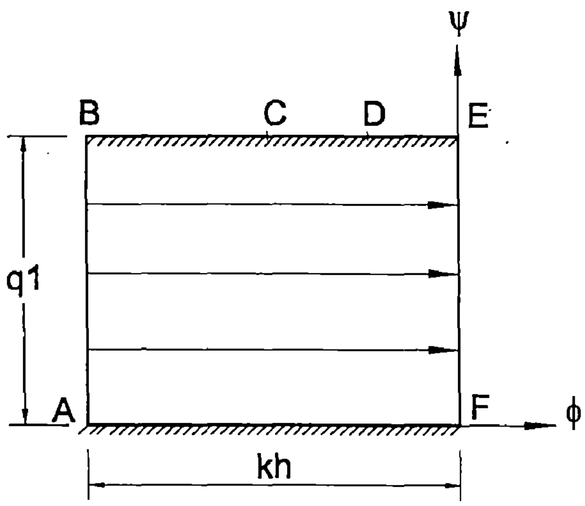
$$d = \left[1-e^{-\frac{\pi}{T}L_d}\right] \tag{3.8}$$



(a) z - Plane (z = x + iy)



(b) t - Plane (t = r1 + is1)



(c) w - Plane (w = φ + iψ)

Fig. 3.1 - TRASFORMATION LAYOUT

3.3.2 Complex Potential Plane: $w = f_2(t)$

The complex potential plane w, where $w = \phi + i\psi$ pertinent to the earth dam is shown in Fig. 3.1c. ϕ is the velocity potential function defined as: $\phi = -k (P/\gamma_w + y) + C$. Let us assume $C = kh_2$, accordingly ϕ along AB = -kh and ϕ along EF = 0. We assume that BE is a streamline defined by $\psi = q_1$. The imperious base AF is also a stream line defined by $\psi = 0$

In the following operation the flow field in the w-plane, shown in Fig.3.1c, is transformed on to the semi-infinite t – plane, shown in fig.3.1b; the transformation of the polygon in w-plane on to the t-plane is given by:

$$\frac{dw}{dt} = \frac{M}{\sqrt{(-t)(1-t)(f-t)}}$$

or,
$$w = M_1 \int \frac{dt}{\sqrt{(-t)(1-t)(f-t)}} + N_1 \tag{3.9}$$

This integration has been performed in various portions of the seepage boundaries:

(a) Integration along the upstream floor AB ($-\infty < t < 0$):

(i) At point A: $t = -\infty$, $w = -kh$; hence : $N_1 = -kh$

(ii) At point B: $t = 0$, $w = -kh + iq_1$

Therefore, Eq. 3.9 becomes

$$-kh + iq_1 = M \int_{-\infty}^{0} \frac{dt}{\sqrt{(-t)(1-t)(f-t)}} - kh$$

or,
$$iq_1 = M \int_{-\infty}^{0} \frac{dt}{\sqrt{(-t)(1-t)(f-t)}} \tag{3.10}$$

Performing the integration (Byrd & Fried Man, 1971) Eq. 3.10 becomes:

$$iq_1 = M \frac{2}{\sqrt{f}} F\left(\frac{\pi}{2}, \sqrt{\frac{f-1}{f}}\right) \tag{3.11a}$$

where, $F\left(\frac{\pi}{2}, \sqrt{\frac{f-1}{f}}\right)$ is elliptic Integral of the first kind, with modulus $\sqrt{\frac{f-1}{f}}$

$$F\left(\frac{\pi}{2}, \sqrt{\frac{f-1}{f}}\right) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1 - m^2 \sin^2\theta}} \; ; \; m = \sqrt{\frac{f-1}{f}}$$

The elliptic integral is computed using Gaussian Quadrature as explained below:

Putting : $\theta = \frac{1}{2}\left(\frac{\pi}{2}v + \frac{\pi}{2}\right) = \frac{\pi}{4}(v+1)$ or $d\theta = \frac{\pi}{4}dv$

so,

$$F\left(\frac{\pi}{2}, \sqrt{\frac{f-1}{f}}\right) = \int_{-1}^{1} \frac{\frac{\pi}{4}dv}{\sqrt{1 - m^2 \sin^2\left[\frac{\pi}{4}(v+1)\right]}}$$

Therefore Eq. 3.11a becomes:

$$iq_1 = M\frac{2}{\sqrt{f}}\int_{-1}^{1} \frac{\frac{\pi}{4}dv}{\sqrt{1 - m^2 \sin^2\left[\frac{\pi}{4}(v+1)\right]}} \qquad (3.11b)$$

(b) Integration along floor BE (0< t < 1):

(i) At point B: $t = 0$, $w = -kh + iq_1 = N$

(ii) At point E: $t = 1$, $w = iq_1$

Therefore, Eq. 3.9 becomes:

$$iq_1 = M\int_{0}^{1} \frac{dt}{\sqrt{(-t)(1-t)(f-t)}} - kh + iq_1$$

or,

$$kh = \frac{M}{\sqrt{-1}}\int_{0}^{1} \frac{dt}{\sqrt{t(1-t)(f-t)}} \qquad (3.12)$$

For the integrand the square root of the cubic, the integration reduces to elliptic integration of the first kind (Byrd and Friend Man, 1971) and Eq. (3.12) become:

$$kh = \frac{M}{\sqrt{-1}}\frac{2}{\sqrt{f}}F\left(\frac{\pi}{2}, \sqrt{\frac{1}{f}}\right) \qquad (3.13)$$

or,

$$kh = \frac{M}{\sqrt{-1}}\frac{2}{\sqrt{f}}\int_{-1}^{1} \frac{\frac{\pi}{4}dv}{\sqrt{1 - m^2 \sin^2\left[\frac{\pi}{4}(v+1)\right]}} \qquad (3.13b)$$

in which: $m = \sqrt{\frac{1}{f}}$

(c) Integration along floor EF (1< t< f):

(i) At point E: t = 1, w = iq_1 ; Hence , $N_1 = iq_1$.

(ii) At point F: t = f, w = 0.

Therefore, Eq. 3.9 becomes:

$$0 = M\int_1^f \frac{dt}{\sqrt{(-t)(1-t)(f-t)}} + iq_1$$

or,

$$iq_1 = M\int_1^f \frac{dt}{\sqrt{t(t-1)(f-t)}} \tag{3.14}$$

Performing the integration (Byrd and friend man, 1971) Eq. 3.14 reduces to :

$$iq_1 = M\frac{2}{\sqrt{f}} . F\left(\frac{\pi}{2}, \sqrt{\frac{f-1}{f}}\right) \tag{3.15}$$

Thus the integration along the floor EF (i.e. 1< t < f) does not yield and independent equation.

Let M be equal to $M_1 i$, from Eq. 3.12

$$M_1 = \frac{kh\sqrt{f}}{2F\left(\frac{\pi}{2}, \sqrt{\frac{1}{f}}\right)}$$

Substituting M in 3.11a.

$$q_1 = kh\frac{F\left(\frac{\pi}{2}, \sqrt{1-\frac{1}{f}}\right)}{F\left(\frac{\pi}{2}, \sqrt{\frac{1}{f}}\right)}$$

3.4 COMPUTATION OF POTENTIAL ALONG THE BASE:

For floor base BE ($0 \le t' \le 1$)

$$w(t') = \frac{M}{\sqrt{-1}}\int_0^{t'} \frac{dt}{\sqrt{t(1-t)(f-t)}} - h + iq$$

Performing the integration (Byrd and Fried Man, 1971)

$$w(t') = \phi(t') + iq_1 = \frac{M}{\sqrt{-1}} F\left(\sin^{-1}\sqrt{t'}, \frac{1}{\sqrt{f}}\right) - kh + iq_1$$

$$\phi(t') = \frac{kh\sqrt{f}}{2F\left(\frac{\pi}{2},\sqrt{\frac{1}{f}}\right)} F\left(\sin^{-1}\sqrt{t'},\frac{1}{\sqrt{f}}\right) - kh$$

$$\frac{-\phi(t')}{kh} = 1 - \frac{\sqrt{f}}{2} \frac{F\left(\sin^{-1}\sqrt{t'},\frac{1}{\sqrt{f}}\right)}{F\left(\frac{\pi}{2},\sqrt{\frac{1}{f}}\right)}$$

where $F\left(\sin^{-1}\sqrt{t'},\frac{1}{\sqrt{f}}\right)$ is incomplete elliptic integral of the first kind which is evaluated by Gauss quadrature as explained below:

Let $\sin^{-1}\sqrt{t'} = \theta_{t'}$

$$F\left(\sin^{-1}\sqrt{t'},\frac{1}{\sqrt{f}}\right) = \int_0^{\theta_{t'}} \frac{d\theta}{\sqrt{1-m^2\sin^2\theta}}, \quad m = \frac{1}{\sqrt{f}}$$

or,

$$F\left(\sin^{-1}\sqrt{t'},\frac{1}{\sqrt{f}}\right) = \int_{-1}^{1} \frac{\frac{\theta_{t'}}{2}dv}{\sqrt{1-m^2\sin^2\left(\frac{\theta_{t'}(1+v)}{2}\right)}}$$

3.5 EXIT GRADIENT:

It is important to know the hydraulic gradient at the downstream end of the floor i.e. at point E and beyond we note that the gradient at any point in an isotropic flow region is

$$I = dh/ds \qquad (3.5.1)$$

in which h = the hydraulic head at any point along the floor and s= distance measured along the streamline passing that point. Eq. 3.5.1 can be written as

$$I = \frac{1}{k}\frac{d\phi}{ds} = \frac{1}{k}.\frac{d\phi}{dt}\frac{dt}{dz}\frac{dz}{ds} \qquad (3.5.2)$$

Defining the angle between the direction of the streamline and the x axis as θ, we have dz/ds = cosθ + isinθ. Since the stream line at the critical exit point (point E in Fig. 3.1a) generally represent ψ = constant (hence dϕ/dt = dw/dt) and intersects the tail water equi-potential boundary at 90^0 ($\theta = 90^0$), Eq. 3.5.2 will reduce to

$$I_E = \frac{i}{k}\left(\frac{dw}{dt}\,\frac{dt}{dz}\right)$$

In other words: $\frac{dw}{dz} = \frac{dw}{dt}\,\frac{dt}{dz}$ = u-iv = u-I(kI_E)

As the downstream boundary is horizontal velocity u = 0

Hence, $\frac{dw}{dz} = ikI_E$

From analysis we have

$$\frac{dz}{dt} = -\frac{T}{\pi}\frac{1}{1-f}$$

$$\frac{dw}{dt} = \frac{M}{\sqrt{(-t)(1-t)(f-t)}}$$

so, $I_E = \frac{1}{ik}\frac{dw}{dt}.\frac{dt}{dz} = \frac{M_1}{\sqrt{(-t)(1-t)}}\frac{\pi}{T}\sqrt{(f-t)}$

Hence, $I_E = \frac{h\sqrt{f}}{2}\,F\left(\frac{\pi}{2},\sqrt{\frac{1}{f}}\right)\frac{\pi}{t}\frac{\sqrt{f-t}}{\sqrt{t(t-1)}}$

3.6 RESULTS AND DISCUSSION

A computer program was developed for computation of potential along the base and exit gradient at the downstream side for various combination of the values of the variables involved. The calculations involve the use of elliptic function of first find, which were computed by a subroutine CEF and CIEF.

The variation of q_1/kh with b_3/T, the bottom width of the dam is shown in Fig. 3.2. The distribution of potential along the base of the dam is shown in Fig. 3.3. The exit gradient in the downstream of the dam shown in Fig. 3.4,

The exit gradient is infinite at the toe of the dam. Therefore an invested filter has to be provided at this location. The exit gradient decreases with distance from the toe of the dam. The zone in which the exit gradient exceeds the critical value (critical exit gradient ≈1), the invested filter is to be provided in this zone.

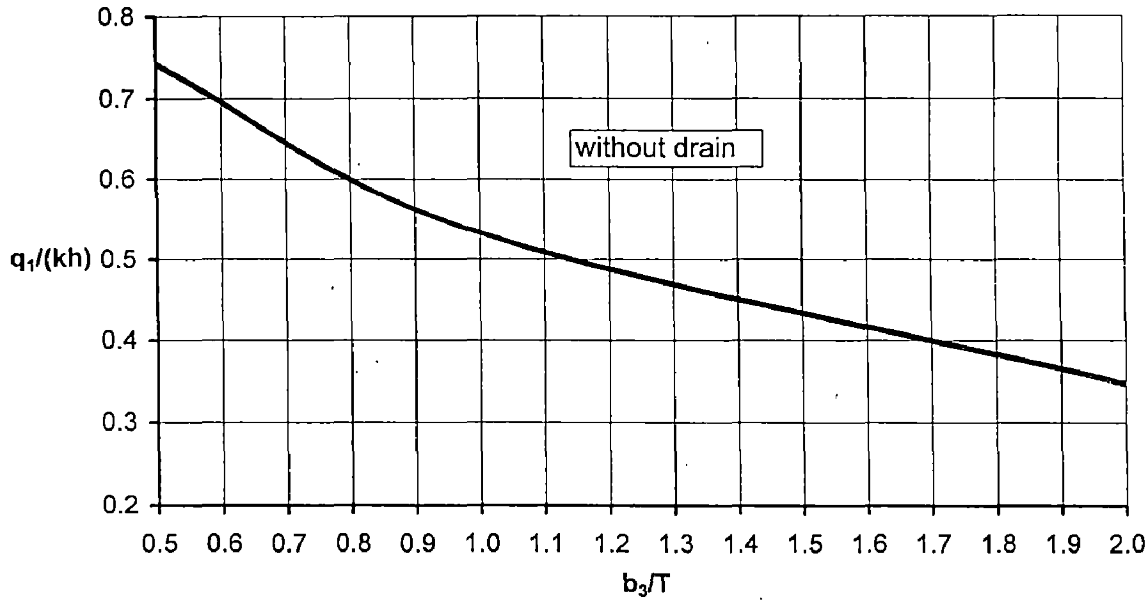


Fig.3.2- Variation of seepage through foundation with bottom width of the dam

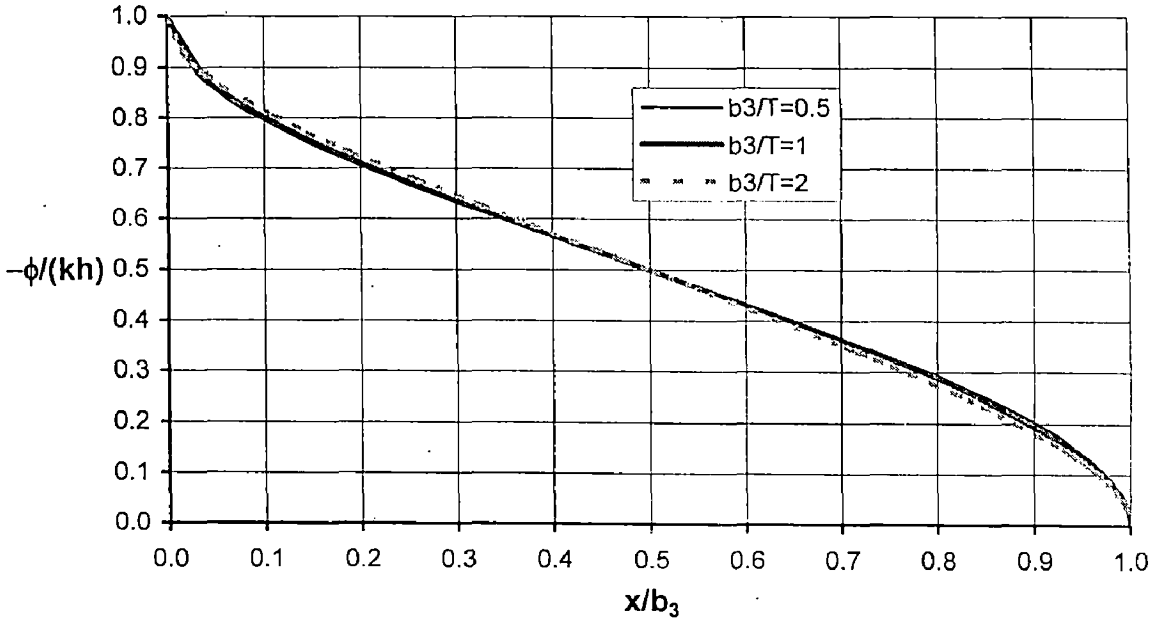


Fig.3.3- Distribution of potential along the base of the dam

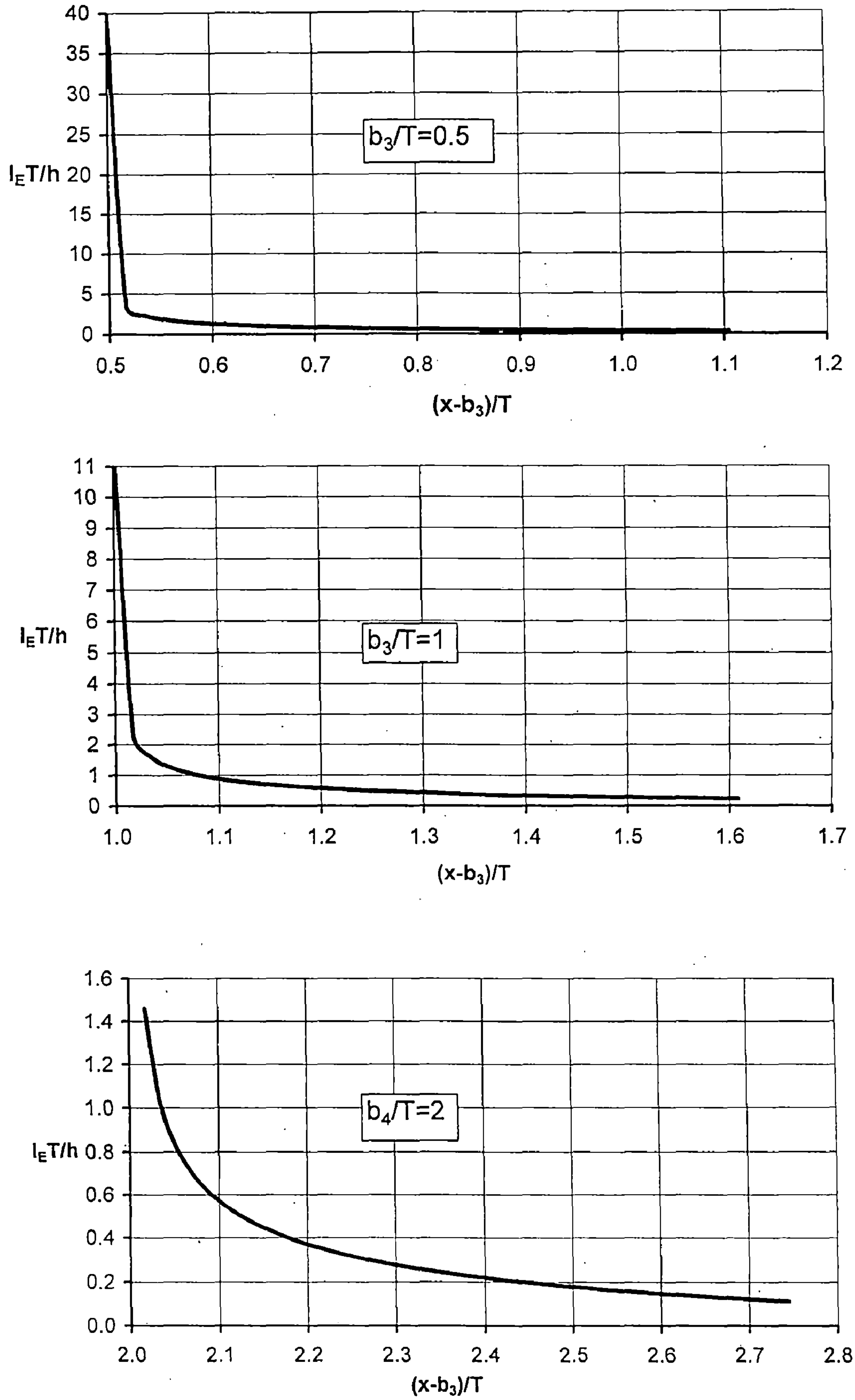


Fig3.4- Distribution of exit gradient in the downstream of the Dam

CHAPTER - 4

FLOW UNDER AN EARTH DAM WITH FILTER FOUNDED ON PERMEABLE SOIL OF FINITE DEPTH

4.1 INTRODUCTION

The stability of hydraulic structure founded on permeable soil has to be ensured for safety against uplift pressure and piping. Intermediate filters or drains are provided below hydraulic structures founded on permeable soil to reduce uplift pressure resulting in appreciable savings. Some times because of non availability of filter material (coarse grained soil) and for economy filter of finite width which does not extend up to the toe of the dam is provided. Parallel drain pipes are used to dispose the water that seeps into the filter. Solution to the seepage problem pertaining to such drainage system, shown in Fig. 4.1, is not yet available. In this chapter using conformal mapping and potential theory, the flow characteristics (i.e. quantity of seepage to the drain through filter, seepage to the downstream through foundation and exit gradient on the down stream side) have been quantified.

4.2 LAYOUT OF BOUNDARY CONDITIONS AND METHOD OF SOLUTION

Consider an impervious floor BE of length b_3 founded on a homogeneous permeable soil of depth T. An intermediate filter CD of width w_f is located at a distance b_1 from the heel of the dam (from point B). The structure is founded on a permeable soil of depth T underlined by an impervious stratum AF. The profile is presented in the z plane (z = x + iy) as shown in Fig. 4.2a.

The steady seepage through the previous foundation of the structure that causes uplift is governed by the Laplace equation:

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \qquad (4.1)$$

in which: ϕ = velocity potential function defined as:

ϕ = -k (γ_w -+y) + C

where: C = constant, and k = hydraulic conductivity.

Assuming C = kh_2, along the upstream bed AB, ϕ = -kh (h = h_1-h_2) and along the downstream bed EF, ϕ = 0. Starting from some where at the upstream, a streamline ψ= q_2, would meet some where the floor DE, at an unknown point R where it would yet divided into two stream lines, one along RD emerging at D and other along RE emerging at E. The potential along the floor DE would be maximum at R. The impervious boundary BC forms another streamline ψ = q_1 in which q_1 = total discharge seeping below the foundation. Along impervious stratum, ψ = 0. The complex potential is represented by $w=\phi+i\psi$. The layout of various boundaries in the w-plane is shown in Fig. 4.2c. The seepage domain in the complex potential plane w is the area between the vertical lines AB (ϕ= -kh), CD (ϕ= -kα_d h) and EF (ϕ=0), and horizontal lines AF (ψ = 0), DRF (ψ = q_2) and BC (ψ = q_1). Depending upon the head that would develop along the filter, an upstream part of the filter may act as a sink and the remaining down stream part of the filter would then act as a source. The corresponding complex potential plane is shown in Fig. 4.3.

To obtain the solution, both the profiles of structure in the z-plane and in the complex potential w-plane have been transformed onto lower half of an auxiliary semi-infinite t-plane (t = r_1 + is_1) using the Schwarts-Christoffel transformation. The following relations are thus obtained:

$$z = f_1(t) \quad (4.2)$$

$$w = F(t) \quad (4.3)$$

combining Eqs. 4.2 and 4.3

$$z = f(t) = f_1(F^{-1}(w)) \quad (4.4)$$

and

$$w = F(t) = F(f^{-1}(z)) \quad (4.5)$$

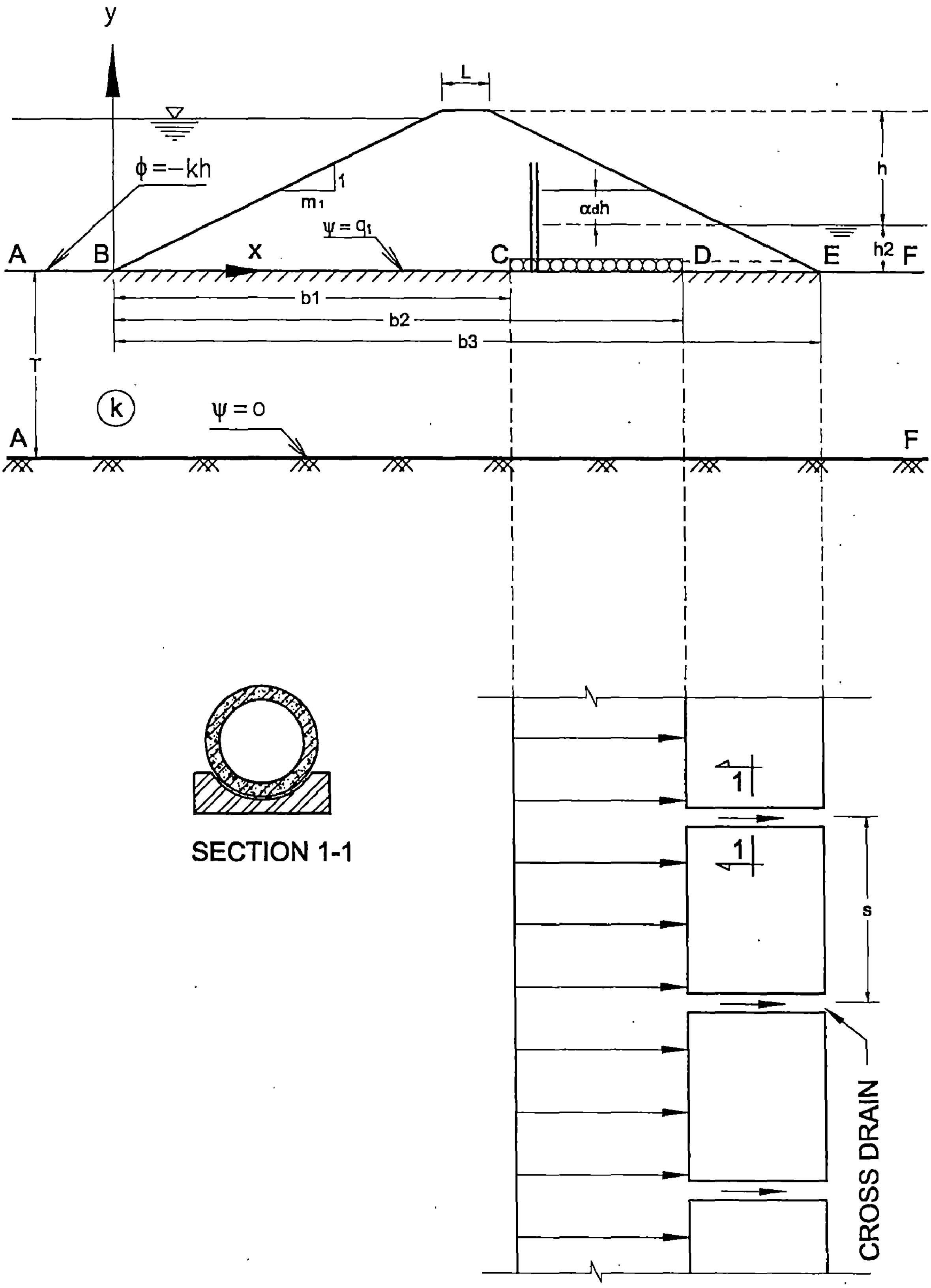
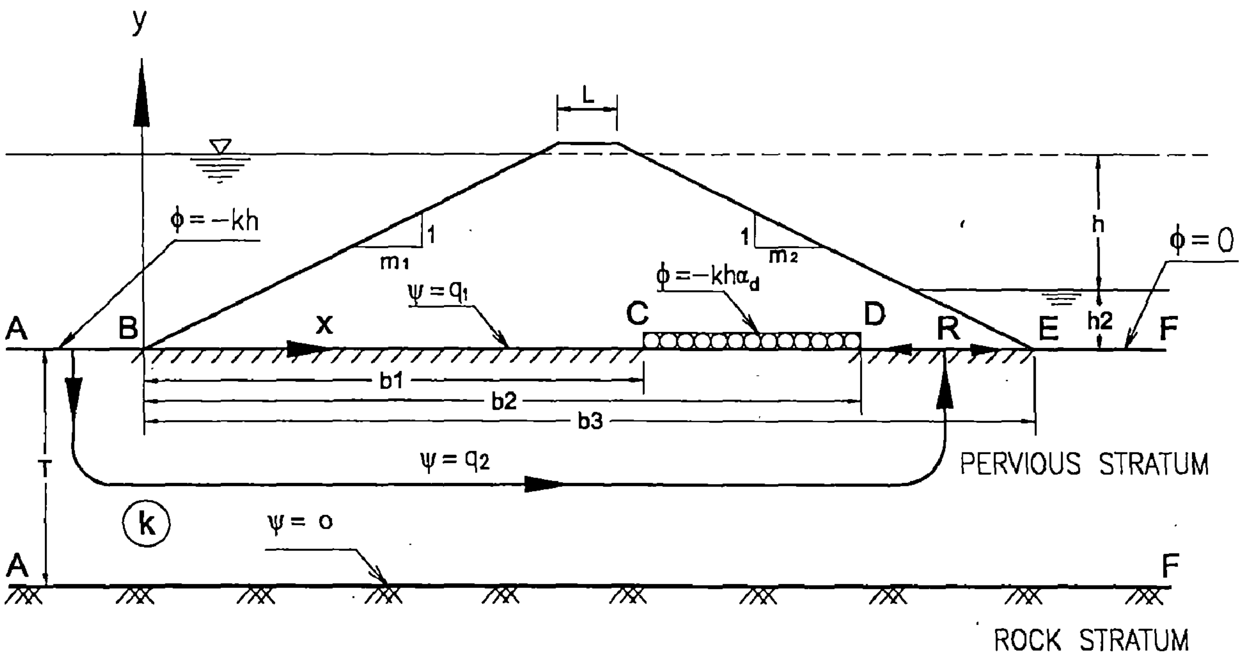
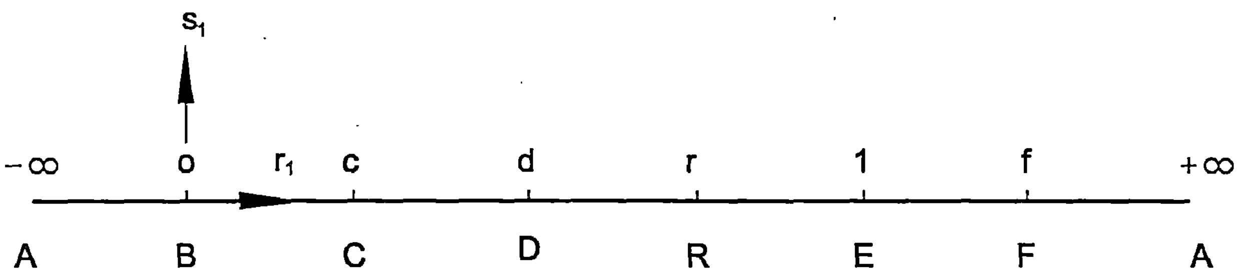


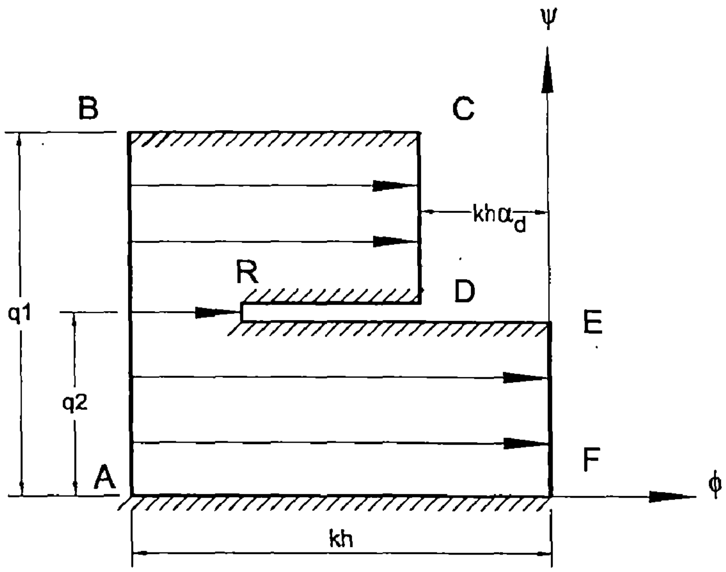
Fig. 4.1 - DRAIN SYSTEM



(a) z - Plane (z = x + iy)

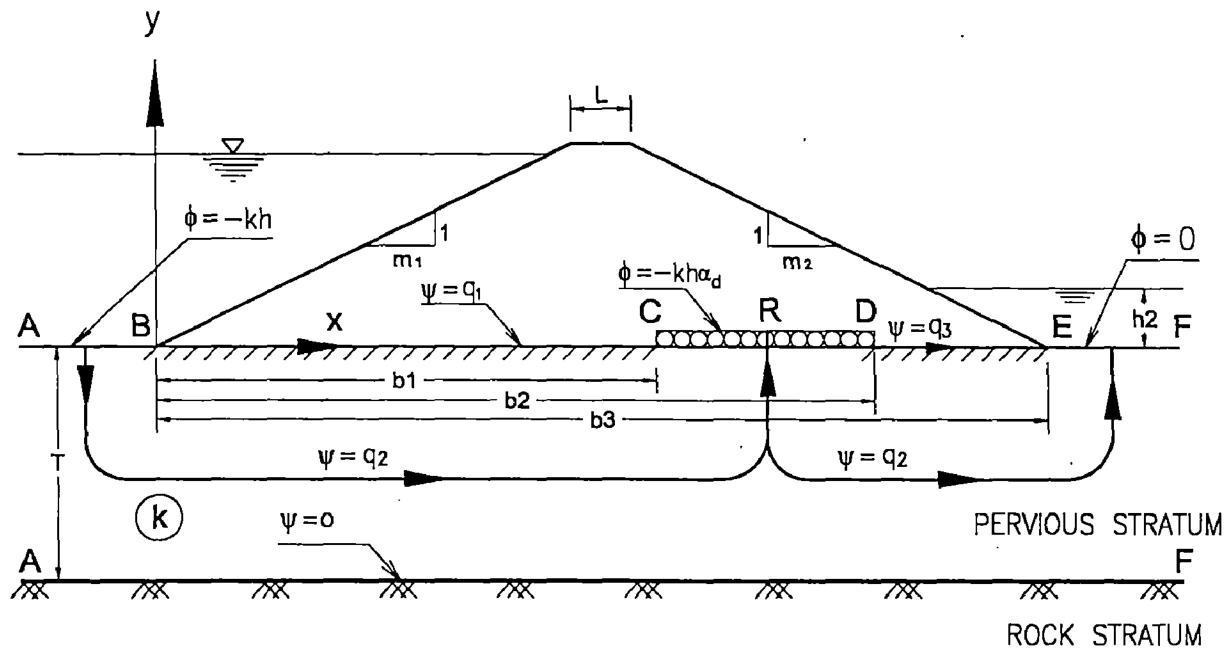


(b) t - Plane (t = r1 + is1)

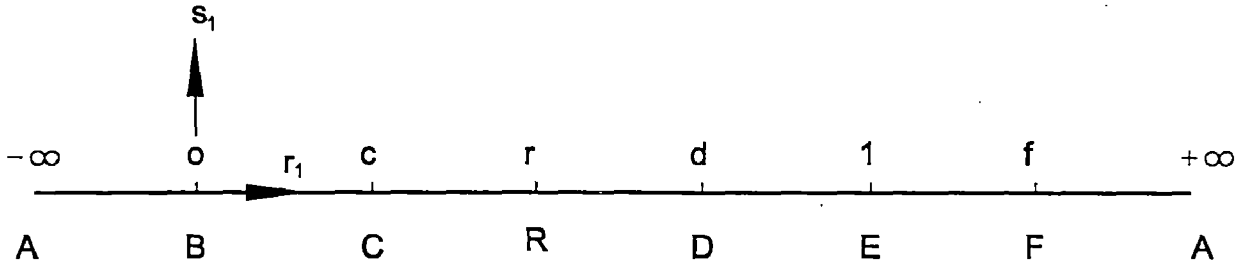


(c) w - Plane (w = φ + iψ)

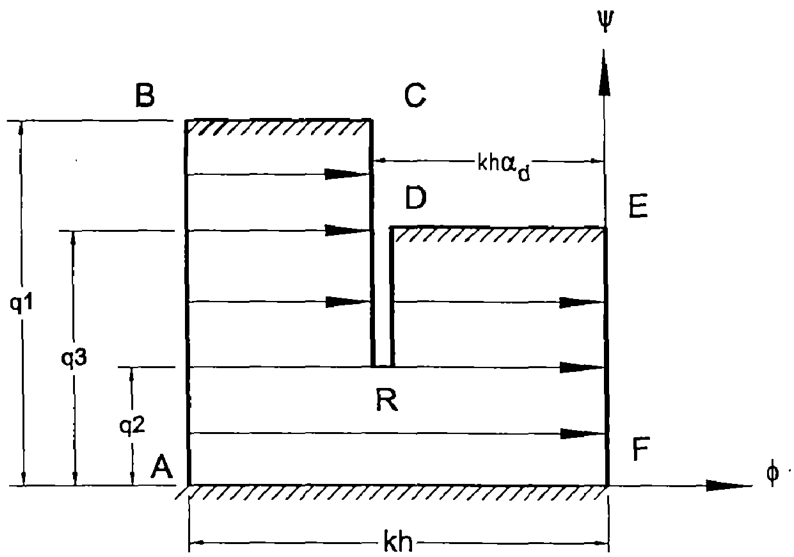
Fig. 4.2 - TRANSFORMATION LAYOUT



(a) z - Plane (z = x + iy)



(b) t - Plane ($t = r_1 + is_1$)



(c) w - Plane (w = ϕ + iψ)

Fig. 4.3 - TRANSFORMATION LAYOUT

4.3 THEORETICAL SOLUTION

4.3.1 First Operation z = $f_1(t)$

In this operation the profile of the hydraulic structure in the z-plane is transformed onto the real axis of the t-plane. On the t-plane, the points B and E are located at 0 and 1 and the points C, D, N and F are mapped onto points at c, d, n and f, respectively. The values of these parameters are to be determined. The Schwars-Christoffel transformation that gives the afore-mentioned mapping is

$$\frac{dz}{dt} = \frac{M}{(1-t)} \tag{4.6}$$

As derived in Chapter 3.

$$M = -T/\pi \tag{4.7}$$

$$t = f[1 - e^{\frac{-\pi z}{T}}] \tag{4.8}$$

$$f = \frac{1}{1 - e^{-\frac{\pi}{T}b_3}} \tag{4.9}$$

$$c = f\left[1 - e^{-\frac{\pi}{T}b_1}\right] \tag{4.10}$$

$$d = f\left[1 - e^{-\frac{\pi}{T}b_2}\right] \tag{4.11}$$

4.3.2 Second Operation w = F(t)

In this operation, the flow field in the w-plane, shown in Fig. 4.2c is transformed onto the semi -infinite t-plane as shown Fig. 4.2b. The transformation of the polygon in w-plane onto the t-plane is given by :

$$\frac{dw}{dt} = \frac{M_1(r-t)}{\sqrt{(-t)(c-t)(d-t)(1-t)(f-t)}} \tag{4.12}$$

or,

$$w(t') = M_1 \int_{-\infty}^{t} \frac{(r-t)dt}{\sqrt{(-t)(c-t)(d-t)(1-t)(f-t)}} + N$$

To find the parameters c, d, f, and r; constants M_1, and N we carry out integration between consecutive vertices and find the required equations.

(a) Integration between vertices A and B $(-\infty < t < 0)$

(i) At point A : $t = -\infty$, $w = -kh$; hence, $N = -kh$.

(ii) At point B : $t = 0$; $w = -kh + iq_1$

Applying these conditions we obtain:

$$iq_1 = M_1 \int_{-\infty}^{0} \frac{(r-t)dt}{\sqrt{(-t)(c-t)(d-t)(1-t)(f-t)}}$$

$$= M_1 r \int_{-\infty}^{0} \frac{dt}{\sqrt{(-t)(c-t)(d-t)(1-t)(f-t)}} + M_1 \int_{-\infty}^{0} \frac{\sqrt{(-t)}\ dt}{\sqrt{(c-t)(d-t)(1-t)(f-t)}}$$

$$= M_1 [r.F_1 + F_2];$$

where:

$$F_1 = \int_{-\infty}^{0} \frac{dt}{\sqrt{(-t)(c-t)(d-t)(1-t)(f-t)}}$$

$$F_2 = \int_{-\infty}^{0} \frac{\sqrt{(-t)}\ dt}{\sqrt{(c-t)(d-t)(1-t)(f-t)}}$$

Let us assume $M_1 = M_2 i$

so, $q_1 = M_2 [rF_1 + F_2]$ (4.13)

Evaluation of F_1 and F_2 are given in appendix 1 and 2.

(b) Integration between vertices B and C $(0 < t < c)$

(i) At point B: $t = 0$, $w = -kh + iq_1$, hence: $N = -kh + iq_1$

(ii) At point C: $t = c$, $w = k\alpha_d h + iq_1$

Therefore,

$$kh(1 - \alpha_d) = \int_{0}^{c} \frac{M_1(r-t)dt}{\sqrt{(-t)(c-t)(d-t)(1-t)(f-t)}}$$

$$= M_1 r \int_{0}^{c} \frac{dt}{\sqrt{(-t)\ (c-t)(d-t)(1-t)(f-t)}} - M_1 \int_{0}^{c} \frac{t\ dt}{\sqrt{(-t)(c-t)(d-t)(1-t)(f-t)}}$$

$$= \frac{M_1 r}{\sqrt{-1}} \int_{0}^{c} \frac{dt}{\sqrt{t(c-t)(d-t)(1-t)(f-t)}} - \frac{M_1}{\sqrt{-1}} \int_{0}^{c} \frac{\sqrt{t}\ dt}{\sqrt{(c-t)(d-t)(1-t)(f-t)}}$$

$$= \frac{M_1}{\sqrt{-1}} [r.F_3 - F_4]$$

or,

$kh(1 - \alpha_d) = M_2\ (r.\ F_3 - F_4)$ (4.14)

In which,

$$F_3 = \int_0^c \frac{dt}{\sqrt{t(c-t)(d-t)(1-t)(f-t)}}$$

$$F_4 = \int_0^c \frac{\sqrt{t}\ dt}{\sqrt{(c-t)(d-t)(1-t)(f-t)}}$$

Evaluation of F3 and F4 are given in Appendix 3 and 4.

(c) ***Integration between vertices C and D (c< t < d)***

(i) *At point C:* $t = c$, $w = -k\alpha_d h + iq_1$; hence, $N = -k\alpha_d h + iq_1$

(ii) *At point D:* $t = d$; $w = -k\alpha_d h + iq_2$

Therefore,

$$i(q_2 - q_1) = \int_c^d \frac{M_1(r-t)dt}{\sqrt{(-t)(c-t)(d-t)(1-t)(f-t)}}$$

$$= \int_c^d \frac{M_1(r-t)dt}{\sqrt{t(t-c)(d-t)(1-t)(f-t)}}$$

$$= M_1 r \int_c^d \frac{dt}{\sqrt{t(t-c)(d-t)(1-t)(f-t)}} - M_1 \int_c^d \frac{\sqrt{t}\,dt}{\sqrt{(t-c)(d-t)(1-t)(f-t)}}$$

$$= M_1 [r.F_5 - F_6]$$

or,

$$q_2 - q_1 = M_2 [rF_5 - F_6] \qquad (4.15)$$

where,

$$F_5 = \int_c^d \frac{dt}{\sqrt{t(t-c)(d-t)(1-t)(f-t)}}$$

$$F_6 = \int_c^d \frac{\sqrt{t}\,dt}{\sqrt{(t-c)(d-t)(1-t)(f-t)}}$$

Evaluation of F5 and F6 are given in Appendix 5 and 6.

(d) Integration between vertices D and E (d<t<1)

(i) *At point D:* t= d; w = -kα_d h + iq_2, hence N = -kα_d h + iq_2,

(ii) *At point E:* t = 1, w = iq_2

Therefore,

$$k\alpha_d\, h = \int_d^1 \frac{M_1(r-dt)}{\sqrt{(-t)(c-t)(d-t)(1-t)(f-t)}}$$

$$= \frac{1}{\sqrt{-1}}\int_d^1 \frac{M_1(r-dt)}{\sqrt{t(t-c)(t-d)(1-t)(f-t)}}$$

$$= \frac{M_1 r}{\sqrt{-1}}\int_d^1 \frac{dt}{\sqrt{t(t-c)(t-d)(1-t)(f-t)}} - \frac{M_1}{\sqrt{-1}}\int_d^1 \frac{\sqrt{t}\,dt}{\sqrt{(t-c)(t-d)(1-t)(f-t)}}$$

$$= \frac{M_1}{\sqrt{-1}}\left[r.F_7 - F_8\right]$$

or,

$$k\alpha_d\, h = M_2\left[-r.F_7 + F_8\right] \qquad (4.16)$$

in which,

$$F_7 = \int_d^1 \frac{dt}{\sqrt{t(t-c)(t-d)(1-t)(f-t)}}$$

$$F_8 = \int_d^1 \frac{\sqrt{t}\,dt}{\sqrt{(t-c)(t-d)(1-t)(f-t)}}$$

Evaluation of F_7 and F_8 are given in Appendix 7 and 8.

(e) Integration between vertices E and F (1<t<f)

(i) At point E : t = 1 ; w = iq_2 ; hence N = iq_2

(ii) At point F : t = f ; w = 0

Therefore,

$$-iq_2 = \int_1^f \frac{M_1(r-t)dt}{\sqrt{(-t)(c-t)(d-t)(1-t)(f-t)}}$$

$$= \int_1^f \frac{M_1(r-t)dt}{\sqrt{t(t-c)(t-d)(t-1)(f-t)}}$$

$$= M_1 r\int_1^f \frac{dt}{\sqrt{t(t-c)(t-d)(t-1)(f-t)}} - M_1\int_1^f \frac{\sqrt{t}dt}{\sqrt{(t-c)(t-d)(t-1)(f-t)}}$$

$$= M_1\left[r.\,F_9 - F_{10}\right]$$

or $q_2 = M_2 [F_{10} - r. F_9]$ (4.17)

where,

$$F_9 = \int_1^f \frac{dt}{\sqrt{t(t-c)(t-d)(t-1)(f-t)}}$$

$$F_{10} = \int_1^f \frac{\sqrt{t}\,dt}{\sqrt{(t-c)(t-d)(t-1)(f-t)}}$$

Evaluation of F9 and F10 are given in Appendix 9 and 10.

(f) Integration between vertices F and A (f<t< ∞):

(i) *At point F*: t = f; w = N = 0

(ii) *At point A*: t = ∞ ; w = -kh.

Therefore,

$$-kh = M_1 \int_f^\infty \frac{(r-t)dt}{\sqrt{(-t)(c-t)(d-t)(1-t)(f-t)}}$$

$$= M_1 \int_f^\infty \frac{(r-t)dt}{\sqrt{t(t-c)(t-d)(t-1)(t-f)}}$$

$$= M_1\, r \int_f^\infty \frac{dt}{\sqrt{t(t-c)(t-d)(t-1)(t-f)}} - M_1 \int_f^\infty \frac{\sqrt{t}\,dt}{\sqrt{(t-c)(t-d)(t-1)(t-f)}}$$

$$= M_1 [rI_1 - I_2]$$

or $kh = M_2 [I_2 - rI_1]$ (4.18)

in which: $$I_1 = \int_f^\infty \frac{dt}{\sqrt{t(t-c)(t-d)(t-1)(t-f)}}$$

$$I_2 = \int_f^\infty \frac{\sqrt{t}\,dt}{\sqrt{(t-c)(t-d)(t-1)(t-f)}}$$

Evaluation of I_1 and I_2 are given in Appendix 11 and 12.

The following relationships have been derived using w and t plane:

q_1	$= M_2 [rF_1 + F_2]$
$kh(1-\alpha_d]$	$= M_2 [rF_3 - F_4]$
$q_1 - q_2$	$= M_2 [rF_5 - F_6]$
$kh\alpha_d$	$= M_2 [F_8 - r.F_7]$
q_2	$= M_2 [rF_{10} - rF_9]$
kh	$= M_2 [I_2 - r\, I_1]$

The value of α_d is unknown. To solve the unknowns q_1/kh, q_2/kh, α_1, r, α_d, and constant M_2, the following procedure is followed:

From Eqs. 4.14 and 4.16,

$$\frac{1-\alpha_d}{\alpha_d} = \frac{rF_3 - F_4}{F_8 - rF_7}$$

hence, $$r = \frac{F_8 + \alpha_d(F_4 - F_8)}{\alpha_d(F_3 - F_7) + F_7} \quad (4.19)$$

We assume value of α_d and get value of r from Eq. 4.19. If the assumed value of α_d is correct then it would satisfy the following continuity equations, i.e. the quantity of seepage entering into the filter is drained out through cross drain.

$$s\,(q_1 - q_2) = k_f \frac{\alpha_d h}{L_d} A_d$$

or, $$\frac{kh(1-\alpha_d)}{rF_3 - F_4}\{rF_1 + F_2 - F_{10} + rF_9\} = k_f \frac{\alpha_d h}{L_d} A_d$$

$$M_2 = \frac{kh(1-\alpha_d)}{rF_3 - F_4}$$

$$q_1 = M_2 [r\, F_1 + F_2]$$

$$q_2 = M_2 [F_{10} - r.F_9]$$

4.4 EXIT GRADIENT

It is important to know the hydraulic gradient at the downstream end of the floor i.e. at point E. we note that the gradient at any point in an isotropic flow region is

$$I = dh/ds \tag{4.20}$$

in which h = the head at any point along the floor and s= distance measure along the streamline passing that point Eq. 4.20 can be written as:

$$I = \frac{1}{k}\frac{d\phi}{ds} = \frac{1}{k}.\frac{d\phi}{dt}\frac{dt}{dz}\frac{dz}{ds} \tag{4.21}$$

Defining the angle between the direction of the streamline and the x axis as θ, we have dz/ds = cosθ + isinθ. Since the stream line at the critical exit point (point E in Fig. 4.2) generally represent ψ = constant (hence dϕ/dt = dw/dt) and interests the tail water equipotential boundary at 90^0 ($\theta = 90^0$), Eq. 4.21 will reduce to

$$I_E = \frac{i}{k}\left(\frac{dw}{dt}\frac{dt}{dz}\right)$$

Other words: $\frac{dw}{dz} = \frac{dw}{dt}\frac{dt}{dz}$ = u-iv = u-I(kI_E)

As the downstream boundary is horizontal velocity u = 0.

Hence, $\frac{dw}{dz} = ikI_E$

From analysis we have

$$\frac{dz}{dt} = -\frac{T}{\pi}\frac{1}{t-1} \quad or \quad \frac{dt}{dz} = \frac{\pi}{T}(1-t)$$

and $$\frac{dw}{dt} = \frac{M_1(r-t)}{\sqrt{(-t)(c-t)(d-t)(1-t)(f-t)}}$$

Hence,

$$I_E = \frac{\pi M_1(r-t)}{ik\sqrt{t(t-c)(t-d)(t-1)(f-t)}} = \frac{h(1-\alpha_d)}{rF_3 - F_4}\frac{\pi(r-t)}{\sqrt{t(t-c)(t-d)(t-1)(f-t)}}$$

4.5 RESULT AND DISCUSSION

The hydraulic head that develops in the filter is governed by

i) location of the filter,

ii) width of the filter,

iii) thickness of the foundation soil,

iv) hydraulic conductivity of the foundation soil,

v) spacing of cross draining pipe,

vi) area of the pipe, and

vii) hydraulic conductivity of the material in the pipe.

A set of sample results is presented in Fig. 4.4a showing variation of α_d width b_1/T for different bottom widths of the dam. It could be seen that α_d decreases as location of filter shifts towards downstream.

From fig. 4.4b it is found that as width w increases α_d decreases.

It can be seen from Fig. 4.4c that as hydraulic conductivity of the filling material increases, α_d decreases.

Variations of q_2/kh (flow to the down stream side) is shown in Fig. 4.5a with different location of drain width of the dam.

The variation of q_1/kh (the seepage through foundation prior to interception by drain) with location of the filter is shown in Fig. 4.5a. Without filter, for $b_3/T = 2$, the seepage, $q_1 / (kh)$, is 0.35 (refer fig. 3.2). Thus a filter induces more seepage to occur. As the location of filter approaches towards the upstream end, seepage through foundation layer increases. Also as width of the filter increases, $q_1/(kh)$ increases.

Variation of seepage emerging at the downstream through foundation soil (q_2/kh) is shown in Fig. 4.5b, which indicates that q_2/kh decreases rapidly as b_1/T increases.

The distribution of exit gradient is shown in Fig. 4.6. Provision of filter reduces the distribution of exit gradient. However the exit gradient at the toe of the structure is infinite. Therefore, a zone near the toe is vulnerable to piping. Hence an inverted filter should be provided within the zone upto, which I_E is greater than or equal to 1.

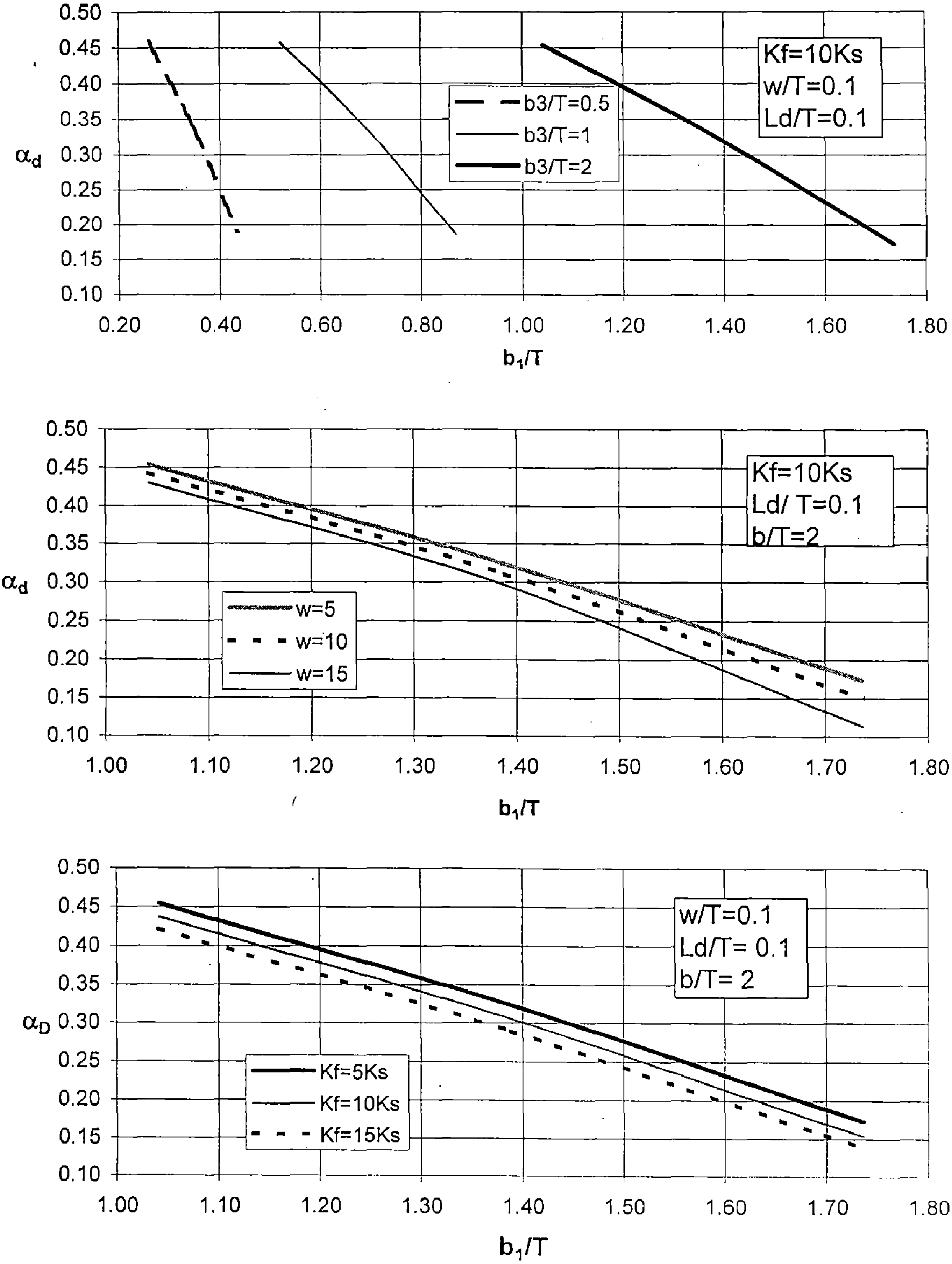


Fig.4.4- Influence location of filter on head developed at the filter

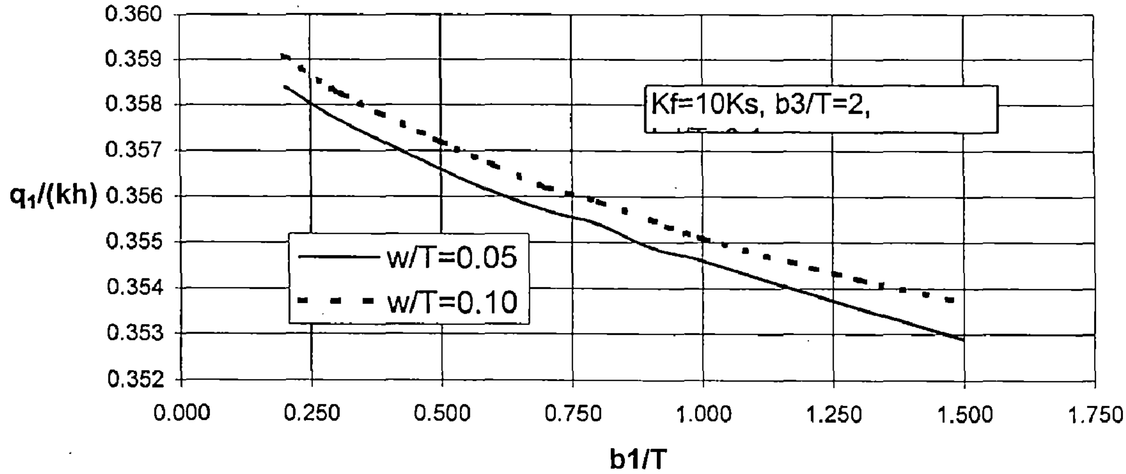


Fig.4.5a- Variation of q1/(kh) with location of the filter

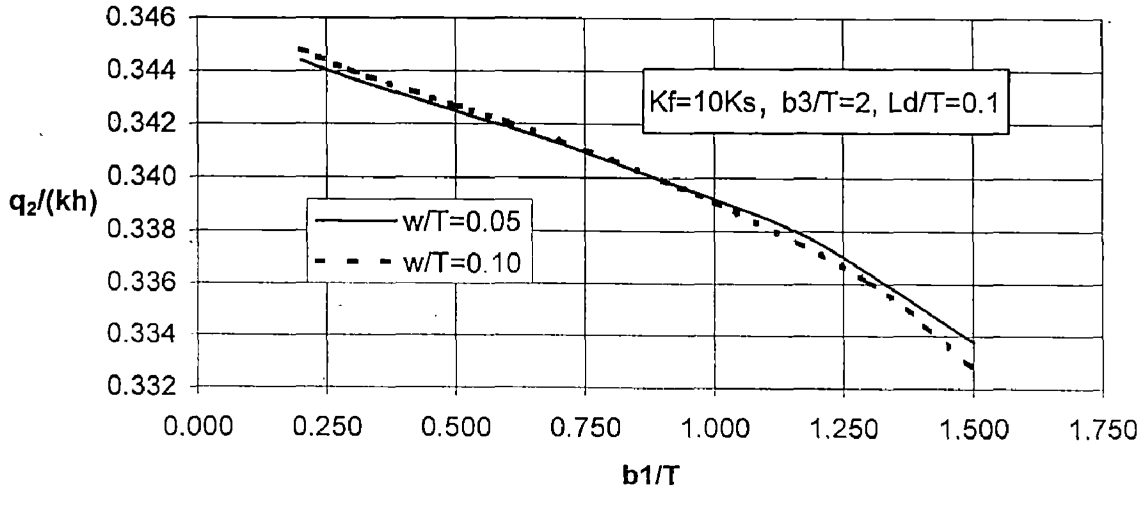
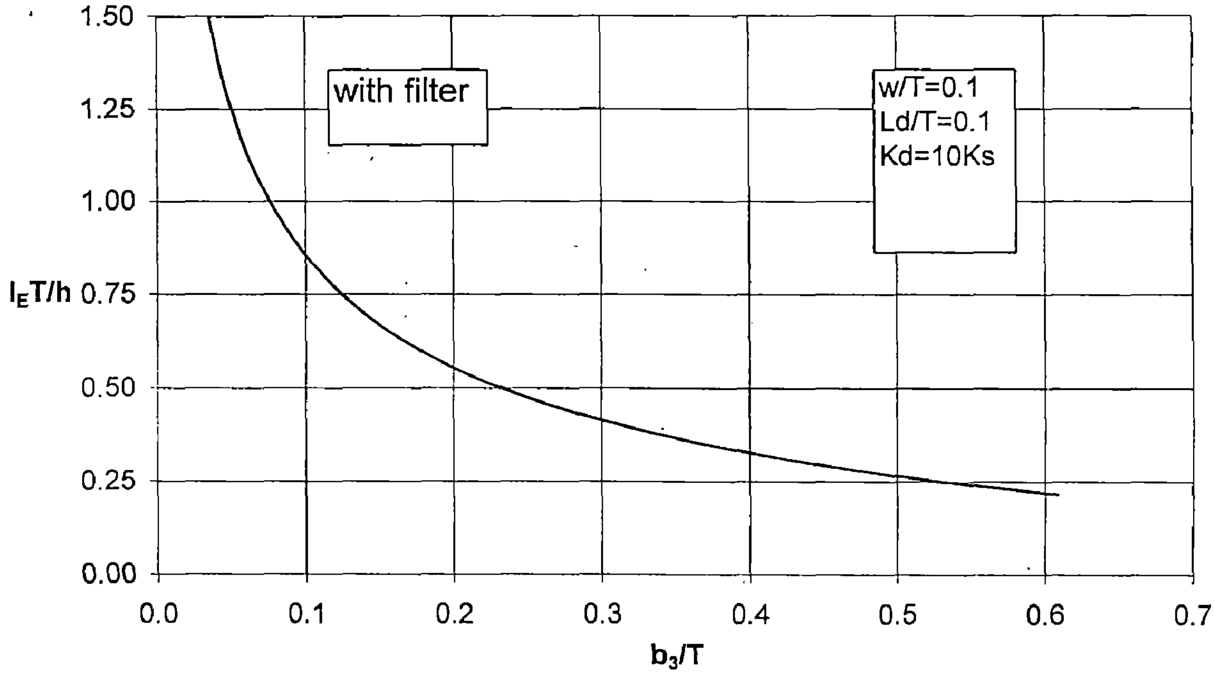
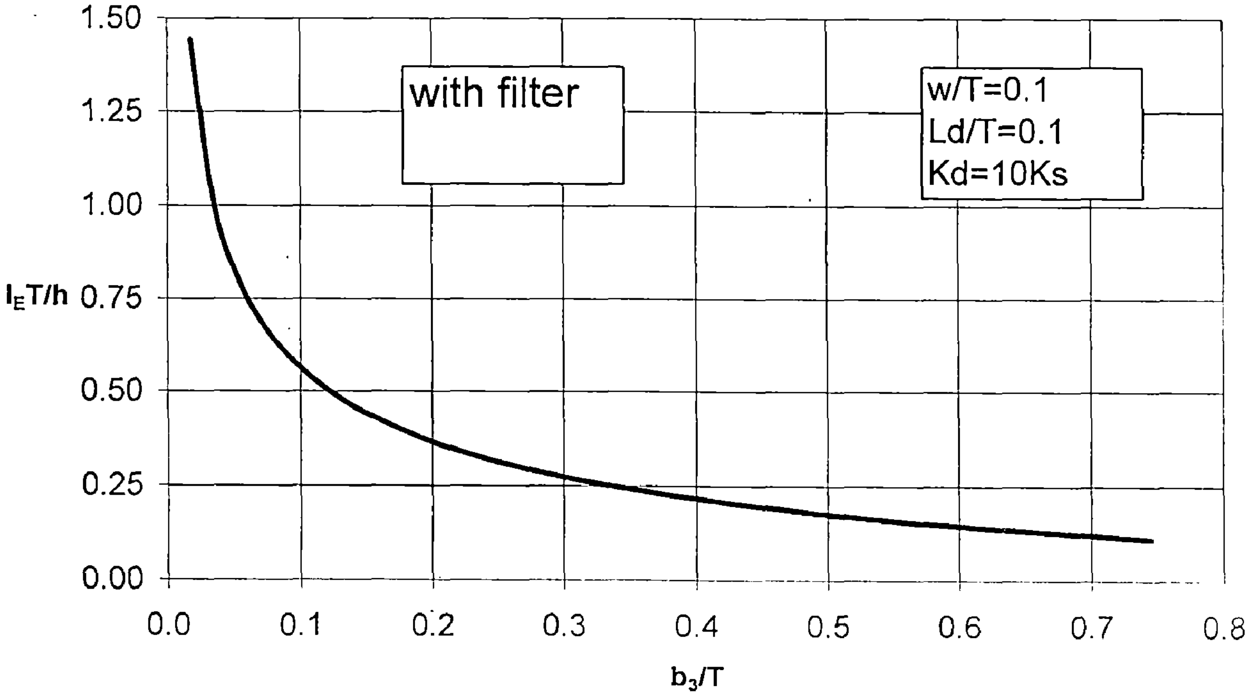


Fig.4.5b- Variation of the q2/(kh) with location of the filter



a- Incase (x-b3)/T = 1



b- Incase (x-b3)/T = 2

Fig.4.6- Distribution of exit gradient in the D/s of the dam

CHAPTER 5

FLOW THROUGH FOUNDATION OF AN EARTHDAM WITH AN UPSTREAM BLANKET AND A FILTER DRAIN SYSTEM

5.1 INTRODUCTION

A horizontal upstream impervious blanket, which increase the horizontal length of the average flow path of under seepage, is more effective in controlling seepage through a homogeneous soil foundation than a partial vertical cut-off. If the blanket is very impervious compared to the natural foundation so that relatively little seepage occurs through the blanket, the reduction in the seepage quantities and pressure at the downstream toe is directly related to the length of the blanket.

5.2 STATEMENT OF THE PROBLEM

An earth dam with an upstream blanket and a filter drain system is shown in Fig. 5.1. The soil under the earth dam structure is homogeneous isotropic and is of finite depth. For the purpose of analysis the flow domain is decomposed into three fragments. The flow through each fragment is analysed.

5.3 ANALYSIS

5.3.1 Fragment -1

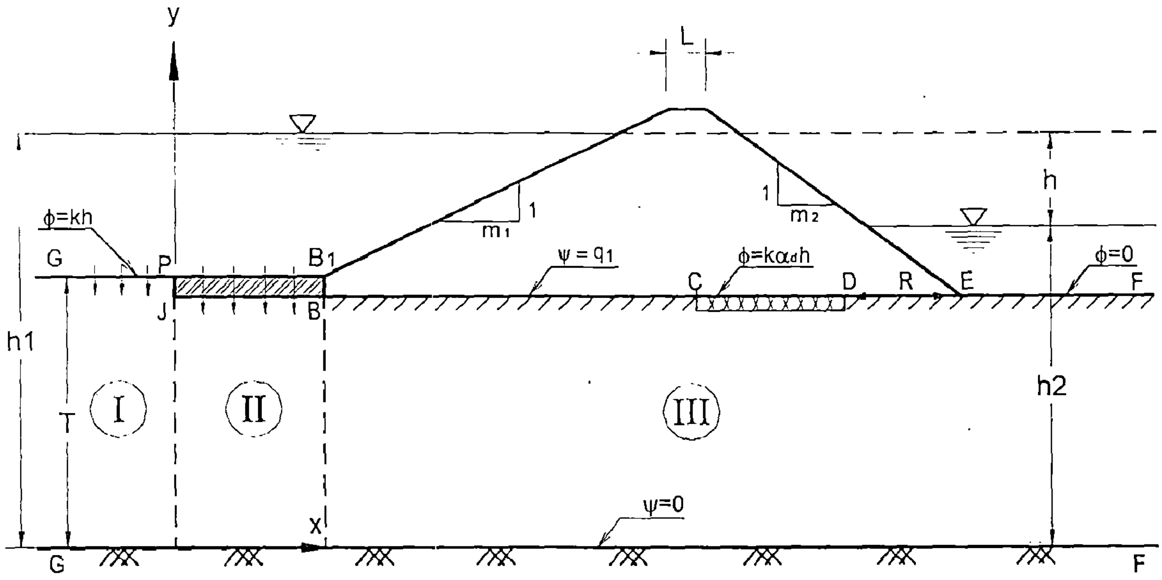
5.3.1.1 Mapping of the Flow domain in z-plane onto t plane: $z = f_1(t)$

The Schwarz-Christoffel transformation that gives the afore mentioned mapping is:

$$\frac{dz}{dt} = \frac{M}{(t+1)^{\frac{1}{2}}(1-t)^{\frac{1}{2}}} = \frac{M}{\sqrt{1-t^2}}$$

or, $$Z = M \int \frac{dt}{\sqrt{1-t^2}} + N$$

$$= M \sin^{-1} t + N \qquad (5.1)$$



(a) z - Plane (z = x + iy)

Fig. 5.1 - FLOW DOMAIN

(i) *For point L:* $t = 1;\ z = -iT$

From Eq. 5.1 $$-iT = M\frac{\pi}{2} + N \qquad (5.2)$$

(ii) *For point P;* $t = -1\ ;\ z_p = 0$

From Eq. 5.1 $$0 = M\left(-\frac{\pi}{2}\right) + N$$

Here, $$M = -\frac{iT}{\pi} \text{ and,}$$

$$N = -\frac{iT}{\pi}\frac{\pi}{2} = -\frac{iT}{2}$$

Therefore,

$$z = -\frac{iT}{\pi}\sin^{-1}(t) - \frac{iT}{2} \qquad (5.3)$$

or, $$t = \sin\left(-\frac{z\pi}{iT} - \frac{\pi}{2}\right) = -\sin\left(\frac{z\pi}{iT} + \frac{\pi}{2}\right) = -\cos\left(-\frac{\pi z}{iT}\right) \qquad (5.4)$$

(iii) *For point J:* t = j, and z_j = -ib

so, $j = -\cos\frac{b\pi}{T}$

5.3.1.2 Complex Potential Plane w = f2(t)

The transformation of the polygon in w-plane onto the t-plane (Fig. 5.2) is given by:

$$\frac{dw}{dt} = \frac{M}{(-1-t)^{1-1/2}(j-t)1^{-1/2}(1-t)^{1/2}}$$

so $$w = M\int_{-\infty}^{t}\frac{dt}{\sqrt{(-1-t)(j-t)(1-t)}} + N_1 \qquad (5.5)$$

(a) *Integration along flow boundary GP (-∞ < t< -1):*

(i) *At point G :* t = -∞ , w = -kh ; Hence N_1 = -kh

(ii) *At point P :* t = -1 , w = -kh + iq_3

Therefore :

$$iq_3 = M\int_{-\infty}^{-1}\frac{dt}{\sqrt{(-1-t)\,(j-t)(1-t)}} \qquad (5.6a)$$

Performing the integration (Byrd & Fried man, 1971)

$$iq_3 = M\sqrt{2}\,F\left(\frac{\pi}{2},\sqrt{\frac{1-j}{2}}\right)$$

or, $$iq_3 = M\sqrt{2}\,F\left(\frac{\pi}{2},\sqrt{\frac{1+\cos\pi\frac{b}{T}}{2}}\right) \qquad (5.6.b)$$

(b) Integration along floor boundary PJ (-1 < t < j)

(i) *At point P :* t = 1 , w = -kh + iq_3

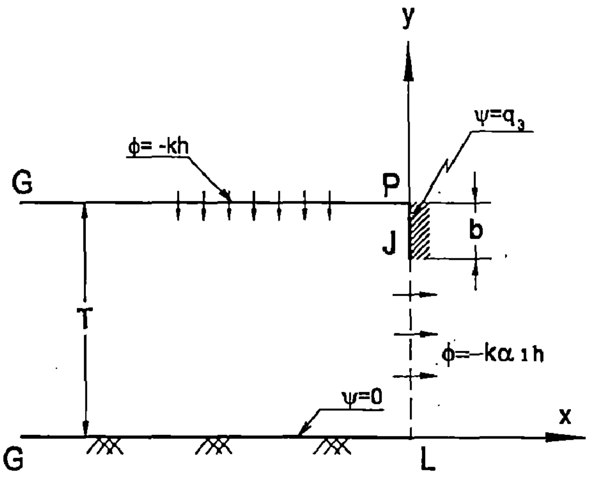
Hence, N_1 = -kh+ iq_3

(ii) *At point J :* t = j , w = -khα_1 + iq_3

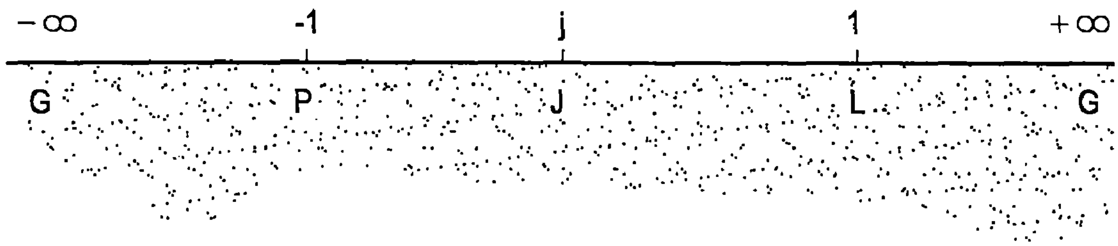
Therefore,

$$kh(1-\alpha_1) = M\int_{-1}^{j}\frac{dt}{\sqrt{(-1-t)\,(j-t)(1-t)}}$$

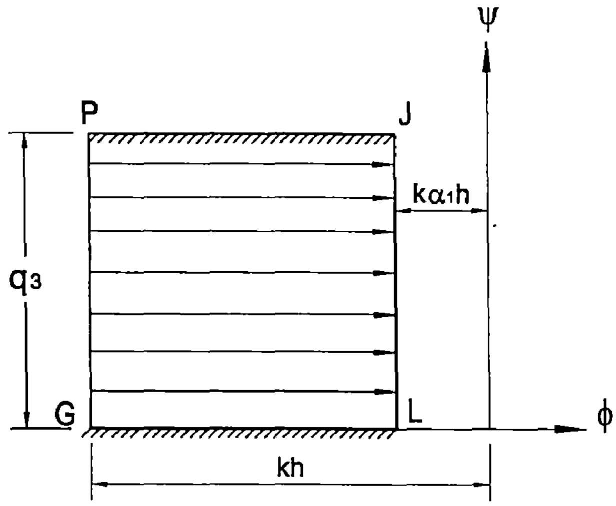
$$= \frac{M}{\sqrt{-1}}M\int_{-1}^{j}\frac{dt}{\sqrt{(1+t)\,(j-t)(1-t)}} \qquad (5.7a)$$



(a) z - Plane (z = x + iy)



(b) t - Plane (t = r1 + is1)



(c) w - Plane (w = φ + iψ)

Fig. 5.2 - TRASFORMATION LAYOUT (Fragment I)

Performing the integration (Byrd & Fried man - 1971)

$$Kh\ (1-\alpha_1) = \sqrt{2}\ F\left(\frac{\pi}{2}, \sqrt{\frac{1+j}{2}}\right)$$

$$= \frac{M}{\sqrt{-1}}\sqrt{2}\ F\left(\frac{\pi}{2}, \sqrt{\frac{1-\cos\pi b/T}{2}}\right) \qquad (5.7b)$$

From Eq. 5.6b and 5.7b, we have

$$M_1 = \frac{Kh\ (1-\alpha_1)}{\sqrt{2}.F\left(\frac{\pi}{2}, \sqrt{\frac{1-\cos\pi b/T}{2}}\right)}$$

and,

$$q_3 = Kh\ (1-\alpha_1)\ \frac{F\left(\frac{\pi}{2}, \sqrt{\frac{1+\cos\pi b/T}{2}}\right)}{F\left(\frac{\pi}{2}, \sqrt{\frac{1-\cos\pi b/T}{2}}\right)} \qquad (5.8a)$$

or,

$$q_3 = Kh\ (1-\alpha_1)\ I_3 \qquad (5.8b)$$

5.3.2 Fragment –II

In fragment II we assume that the flow through the blanket is vertical and in the foundation it is horizontal. Since the hydraulic conductivity of the blanket material is very much less than that of foundation soil, such assumption can be made.

Definition of boundary conditions:

Let the hydraulic head h_a be defined as:

$$h_a = p/\gamma_w + y.$$

$$\phi = -k(P/\gamma_w + y) + C\ ; \quad C = kh_2$$

Let us choose the impervious bed as the datum and y is the measure from this datum.

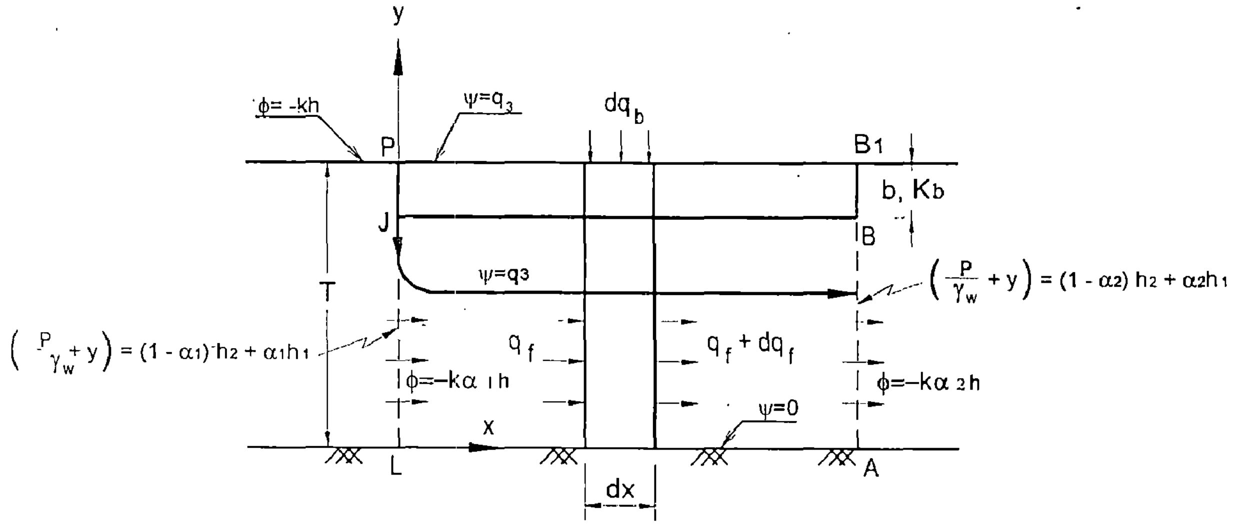


Fig. 5.3 - (Fragment II)

(i) ***Along floor JL :***

$\phi = -k\alpha_1 h \;;\; h = h_1 - h_2$

or

$$-k\alpha_1 h = -k\,(p/\gamma_w + y)\,|_{JL} + kh_2$$

$$(p/\gamma_w + y)|_{JL} = \alpha_1 (h_1 - h_2) + h_2 = \alpha_1 h_1 + (1-\alpha_1)\,h_2$$

so,

$$\phi_{JL} = -k(p/\gamma_w + y)|_{JL} + C$$

or

$$\phi_{JL} = -k[\alpha_1 h_1 + (1-\alpha_1)\,h_2] + kh_2 = -k\alpha_1 h_1 - kh_2\,(1-\alpha_1) + kh_2$$

$$= -k\alpha_1\,(h_1 - h_2) = -k\alpha_1 h$$

(ii) ***Along floor BA:***

$\phi = -k\alpha_2 h$

Since,

$$\phi_{BA} = -k(p/\gamma_w + y)\,|_{BA} + kh_2$$

or $-k\alpha_2 h = -\,k\,(p/\gamma_w + y)|_{BA} + kh_2$

or $(p/\gamma_w + y)|_{BA} = \alpha_2\,(h_1 - h_2) + h_2 = \alpha_2 h_1 + (1-\alpha_2)\,h_2$

Referring to Fig. 5.2, showing the seepage flow through the pervious foundation of the earth dam with upstream blanket of material which permits some leakage, the horizontal flow q_f in the pervious stratum under an elemental length dx of the blanket is increased by an amount dq_f equal to the vertical inflow dx $q_b(x)$ through the length dx of the blanket, so that:

$$\frac{dq_f(x)}{dx} = q_b(x) \tag{5.9}$$

or,

$$\frac{d}{dx}\left[-k(T-b)\,\frac{dhu}{dx}\right] = k_b\,\frac{(h_1 - h_u)}{b} \tag{5.10a}$$

In which h_u is the hydraulic head under the blanket.

Hence,

$$-k(T-b)\frac{d^2hu}{dx^2} = \frac{k_b(h_1 - h_u)}{b} \tag{5.10b}$$

Putting $h_1 - h_u = H$

or,

$$-\frac{d^2h_u}{dx^2} = \frac{d^2H}{dx^2}$$

in Eq. 5.10b:

$$k\,(T\text{-}b)\,\frac{d^2H}{dx^2} = \frac{k_b}{b}.H$$

or

$$\frac{d^2H}{dx^2} - \beta^2 H = 0 \tag{5.10c}$$

Here: $\beta^2 = \dfrac{kb}{b.k(T-b)}$

$$H = C_1e^{\beta x} + C_2e^{-\beta x} \tag{5.11}$$

C_1 and C_2 are to be evaluated applying condition at section P, and J

(i) *At point J* : $x = 0$, $h_u|_{x=0} = \alpha_1 h_1 + (1\text{-}\alpha_1)\, h_2$

Hence: $H|_{x=0} = h_1 - h_u|_{x=0} = h_1 - \alpha_1 h_1 - (1\text{-}\alpha_1)\, h_2$

or, $H|_{x=0} = (1\text{-}\alpha_1)\,(h_1 - h_2)$

So: $$(1\text{-}\alpha_1)\,(h_1 - h_2) = C_1 + C_2 \tag{5.12}$$

(ii) *At point B* : $x = l$; $h_u|_{x=l} = \alpha_2 h_1 + (1\text{-}\alpha_2)\, h_2$

or,

$H|_{x=l} = h_1 - h_u|_{x=l} = h_1 - \alpha_2 h_1 - (1\text{-}\alpha_2)\, h_2$

So, $(1\text{-}\alpha_2)\,(h_1 - h_2) = C_1e^{\beta l} + C_2\, e^{-\beta l}$

and, $$(1\text{-}\alpha_2)\,(h_1 - h_2)\, e^{-\beta l} = C_1 + C_2e^{-\beta l} \tag{5.13}$$

From Eq. 5.12 and Eq. 5.13 :

$$C_1 = \frac{(1-\alpha_1)h(1-e^{-2\beta l}) - (1-\alpha_1)h + (1-\alpha_2)e^{\beta l}.h}{(1-e^{-2\beta l})} \tag{5.14}$$

and
$$C_2 = \frac{(h_1-h_2)e^{-\beta l}\left[(1-\alpha_2)-(1-\alpha_1)e^{-\beta l}\right]}{(1-e^{-2\beta l})} \tag{5.15}$$

We have : $h_u = h_1 - H = h_1 - C_1\, e^{\beta x} - C_2\, e^{-\beta x}$

$$\frac{\partial h_u}{\partial x} = -C_1.\beta.e^{\beta x} + C_2.\beta.e^{-\beta x} \tag{5.16}$$

and, $q_f(x) = -k\,(T-b)\,[C_2.\beta.e^{-\beta x} - C_1.\beta.e^{\beta x}]$

At x = 0, $q_f(0) = k\,(T-b)\,\beta\,(C_2 - C_1)$

Since, $q_f(0) = q_3$

$$k\,(T-b)\,\beta\,(C_2 - C_1) = kh\,(1-\alpha_1)\,l_3 \tag{5.17}$$

Flow through the blanket,

From Eq. 5.9 :

$$Q_b = \int_0^l dq_b(x) = \int_0^l k_b \frac{H}{b}\,dx$$

or,
$$Q_b = \frac{k_b}{b}\int_0^l \left(C_1 e^{\beta x} + C_2 e^{-\beta x}\right)dx = \frac{k_b}{b}\left(\frac{C_1 e^{\beta x}}{\beta} - \frac{C_2 e^{-\beta x}}{\beta}\right)\Bigg|_0^l$$

Hence,
$$Q_b = \frac{k_b}{\beta.b}\left[\left(C_1 e^{\beta l} - C_2 e^{-\beta l}\right) - (C_1 - C_2)\right]$$

and,
$$Q_b = \frac{k_b}{\beta.b}\left[\left(1-e^{-\beta l}\right)C_2 - \left(1-e^{\beta l}\right)C_1\right]$$

Substituting C_1, and C_2, we have

$$Q_b = \frac{k_b}{\beta b}\left\{(1-e^{-\beta l})\frac{(h_1-h_2)\left[(1-\alpha_1)-(1-\alpha_2)e^{-\beta l}\right]}{(1-e^{-2\beta l})} - (1-e^{\beta l})\frac{(h_1-h_2)\,e^{-\beta l}\left[(1-\alpha_2)-(1-\alpha_1)e^{-\beta l}\right]}{(1-e^{-2\beta l})}\right\}$$

$$= \frac{k_b(1-e^{-\beta l})(h_1-h_2)}{\beta.b(1-e^{-2\beta l})}\left\{\left[(1-\alpha_1)-(1-\alpha_2)e^{-\beta l}\right] + \left[(1-\alpha_2)-(1-\alpha_1)\right]e^{-\beta l}\right\}$$

$$= \frac{k_b(h_1-h_2)(1-e^{-\beta l})}{\beta.b(1-e^{-2\beta l})}\left\{2-(\alpha_1+\alpha_2) - \left[2-(\alpha_1+\alpha_2)\right]e^{-\beta l}\right\}$$

$$= \frac{k_b(h_1-h_2)(1-e^{-\beta l})\left[2-(\alpha_1+\alpha_2)\right](1-e^{-\beta l})}{\beta.b(1-e^{-2\beta l})}$$

or,

$$Q_b = \frac{k_b(h_1 - h_2)(1-e^{-\beta l})[2-(\alpha_1+\alpha_2)]}{\beta.b(1+e^{-\beta l})} \qquad (5.18)$$

5.3.3 Fragment - III

5.3.3.1 Mapping of the flow Domain in z- plane onto t-plane : $z = f_1(t)$

The Schwarz-Christoffel transformation that gives the afore-mentioned mapping is

$$\frac{dz}{dt} = \frac{M}{t^{1/2}(t-f)}$$

or

$$z = M\int \frac{dt}{t^{1/2}(t-f)} + N \qquad (5.19)$$

As one traverses in t- plane along a small circle of radius - r around point F (t= f) from $\theta = \pi$ to $\theta = 2\pi$, there is a change of (-iT) in z - plane.

Putting: $t - f = r e^{i\theta}$, or $t = f + r e^{i\theta}$

or $dt = r e^{i\theta}\, i d\theta$

we have $$\int dz = M \int_{\pi}^{2\pi} \frac{re^{i\theta} i d\theta}{(f + re^{i\theta})^{1/2}\ r.e^{i\theta}}$$

or $$-iT = \lim_{r\to 0} M \int_{\pi}^{2\pi} \frac{i d\theta}{(f+re^{i\theta})^{\frac{1}{2}}} = \frac{Mi\pi}{\sqrt{f}}$$

or $$M = -\frac{T.f^{1/2}}{\pi} \qquad (5.20)$$

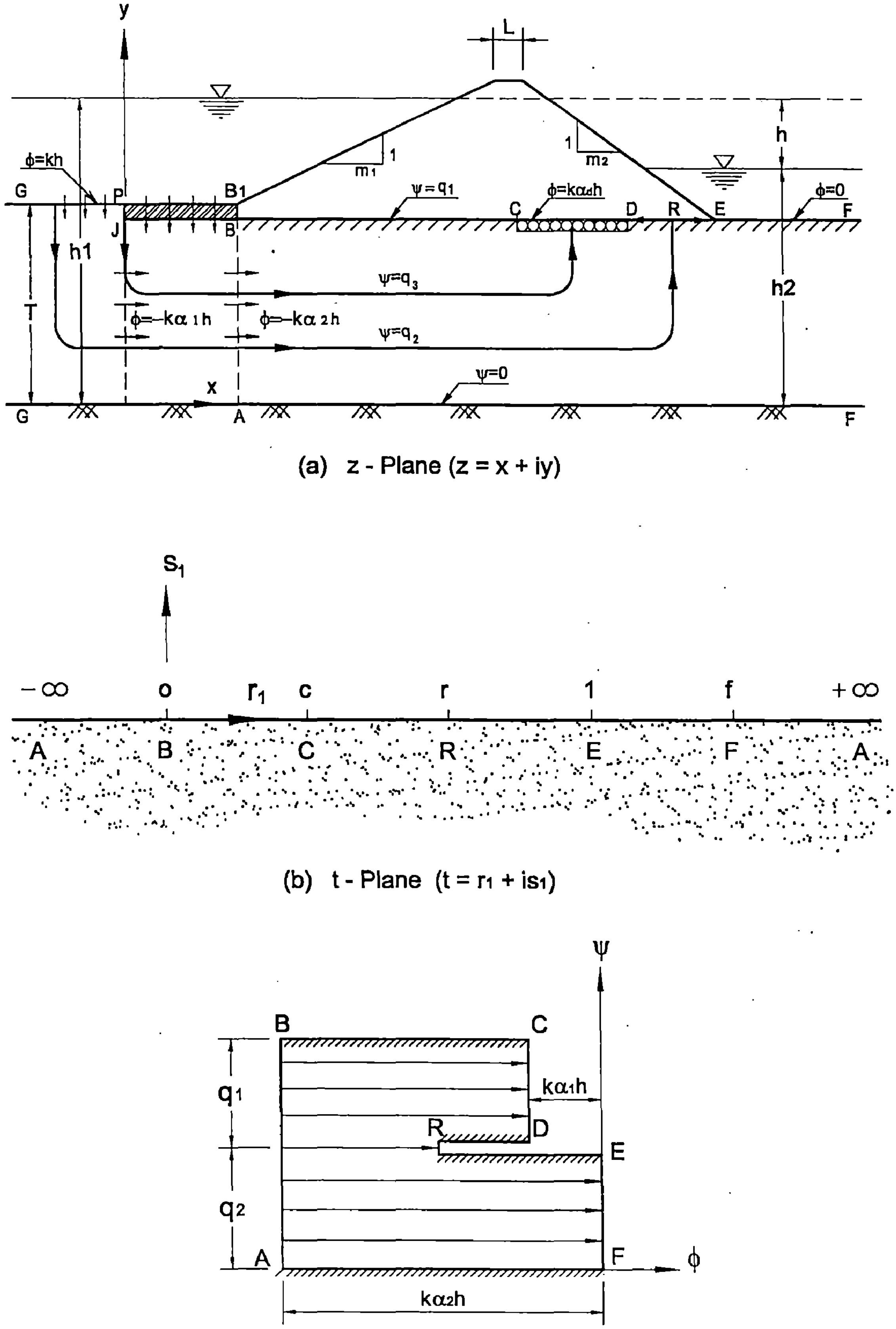
Inserting value M into the Eq. 5.19, we have

$$z = -\frac{T.f^{1/2}}{\pi}\int \frac{dt}{t^{1/2}(t-f)} + N$$

Since,

$$\int \frac{dt}{t^{1/2}(t-f)} = \frac{1}{f^{1/2}} \log_e \frac{t^{1/2} - f^{1/2}}{t^{1/2} + f^{1/2}}$$

$$z = -\frac{T}{\pi}\log_e \frac{t^{1/2} - f^{1/2}}{t^{1/2} + f^{1/2}} + N \qquad (5.21a)$$

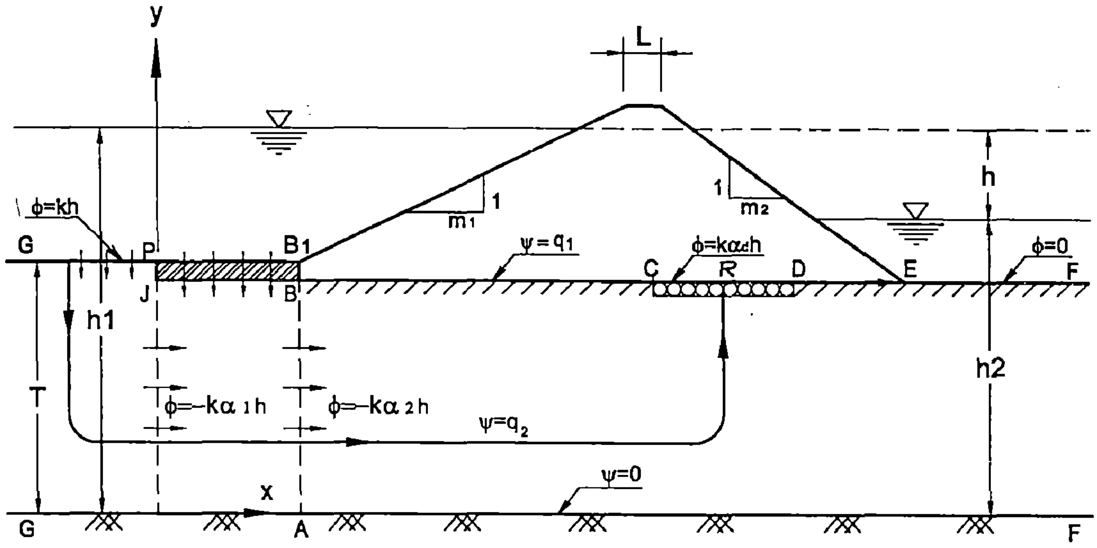


(a) z - Plane (z = x + iy)

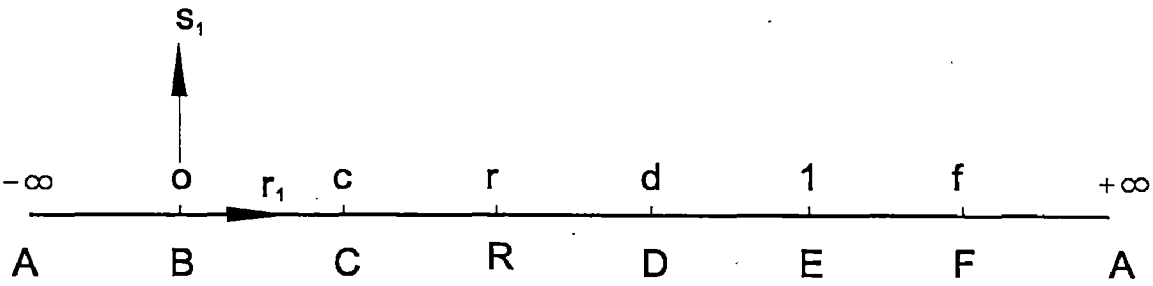
(b) t - Plane (t = r1 + is1)

(c) w - Plane (w = ϕ + iψ)

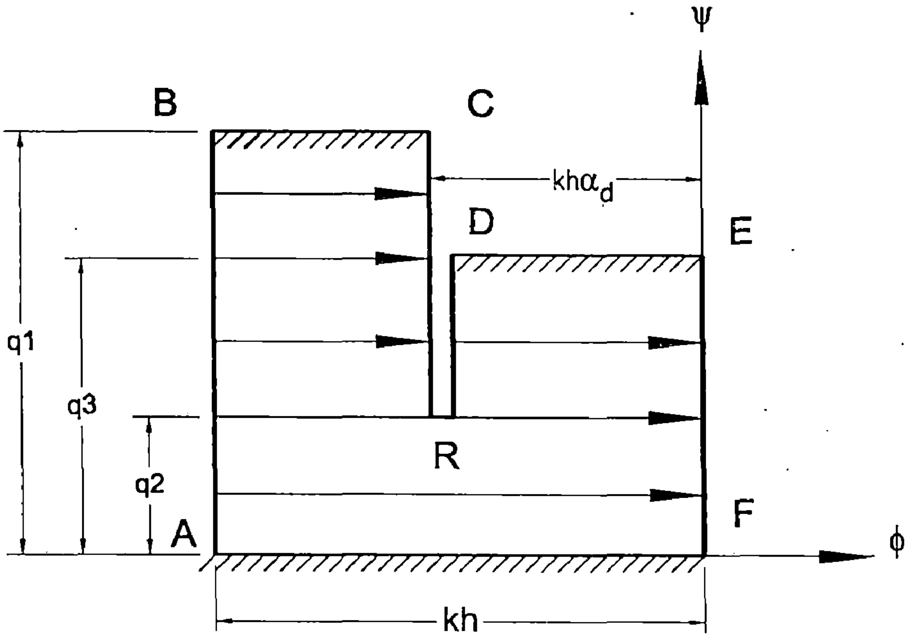
Fig. 5.4 - TRASFORMATION LAYOUT (Fragment III) - CASE 1



(a) z - Plane (z = x + iy)



(b) t - Plane (t = r1 + is1)



(c) w - Plane (w = ϕ + iψ)

Fig. 5.5 - TRASFORMATION LAYOUT (FragmentIII) - CASE 2

(i) *At point B:* $t = 0$; $z_B = 0$. Hence,

$$0 = -\frac{T}{\pi}\log_e(-1) + N = -\frac{T}{\pi}\log_e e^{i\pi} + N$$

or $N = iT$

Setting value of N into Eq. 5.21a we have

$$z = -\frac{T}{\pi}\log_e \frac{t^{1/2} - f^{1/2}}{t^{1/2} + f^{1/2}} + iT \qquad (5.21b)$$

or,

$$z = -\frac{T}{\pi}\log\left\{(-)\frac{f^{1/2} - t^{1/2}}{t^{1/2} + f^{1/2}}\right\} + iT$$

$$= -\frac{T}{\pi}\log_e\left\{e^{\pm i\pi}\frac{f^{1/2} - t^{1/2}}{t^{1/2} + f^{1/2}}\right\} + iT$$

$$= -\frac{T}{\pi}\log_e e^{\pm i\pi} - \frac{T}{\pi}\text{long}_e \frac{f^{1/2} - t^{1/2}}{t^{1/2} + f^{1/2}} + i\pi$$

Hence,

$$z = -\frac{T}{\pi}\log_e \frac{f^{1/2} - t^{1/2}}{t^{1/2} + f^{1/2}} \qquad (5.21c)$$

Let us find t as a function of z from Eq. 5.21c

$$e^{-\frac{\pi}{T}z} = \frac{f^{1/2} - t^{1/2}}{t^{1/2} + f^{1/2}}$$

or, $$e^{-\frac{\pi}{T}z}\left(t^{1/2} + f^{1/2}\right) = f^{1/2} - t^{1/2}$$

or, $$t^{1/2}\left(1 + e^{-\frac{\pi}{T}z}\right) = f^{1/2}\left(1 - e^{-\frac{\pi}{T}z}\right)$$

Hence, $$t = \left[\frac{f^{1/2}\left(1 - e^{-\frac{\pi}{T}z}\right)}{1 + e^{-\frac{\pi}{T}z}}\right]^2 \qquad (5.22)$$

(ii) *At point E :* t = 1, z = L ; Hence

$$1 = \left[\frac{f^{1/2}\left(1 - e^{-\frac{\pi}{T}L}\right)}{1 + e^{-\frac{\pi}{T}L}} \right]^2$$

or

$$f = \left[\frac{1 + e^{-\frac{\pi}{T}L}}{1 - e^{-\frac{\pi}{T}L}} \right]^2 \qquad (5.23a)$$

(iii) *At point C :* t = c, z = Lc, Hence

$$c = \left[\frac{f^{1/2}\left(1 - e^{-\frac{\pi}{T}Lc}\right)}{1 + e^{-\frac{\pi}{T}Lc}} \right]^2 \qquad (5.23b)$$

(iv) *At point d* ; t = d , z = Ld; Hence

$$d = \left[\frac{f^{1/2}\left(1 - e^{-\frac{\pi}{T}Ld}\right)}{1 + e^{-\frac{\pi}{T}Ld}} \right]^2 \qquad (5.23c)$$

5.3.3.2 Complex Potential Plane : $w = f_2(t)$

In the following operation the flow field in the w-plane is transformed onto the semi-infinite t- plane, which are shown in Fig.5.1. The transformation of the polygon in w-plane onto the t-plane is given by:

$$\frac{dw}{dt} = \frac{M_1(r - t)}{\sqrt{(-t)(c - t)(d - t)(1 - t)(f - t)}}$$

or

$$w = M_1 \int_{-\infty}^{t'} \frac{(r - t)dt}{\sqrt{(-t)(c - t)(d - t)(1 - t)(f - t)}} + N_1$$

we have equation systems as:

$$q_1 = M_2\,[rF_1 + F_2] \quad (a)$$

$$kh(\alpha_2 - \alpha_d) = M_2\,[\,r.F_3 - F_4] \quad (b)$$

$$q_1 - q_2 = M_2\,[r.F_5 - F_6] \quad (c)$$

$$kh\alpha_d = M_2\,[F_8 - r.F_7] \quad (d)$$

$$q_2 = M_2\,[\,F_{10} - r.F_9] \quad (e)$$

$$kh\alpha_2 = M_2[I_2 - r.I_1] \quad (f)$$

$$q_3 = kh\,(1-\alpha_1)\,I_3 \quad (g)$$

$$Q_b = \frac{k_b\left(1-e^{-\beta l}\right)\left[2-\left(\alpha_1+\alpha_2\right)\right]}{1+e^{-\beta l}} \quad (h)$$

The magnitude of F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, I_1, I_2 are the same as derived in Chapter 4.

The unknowns are the complex constant M and the parameters r, α_1, α_2, α_d. Since the equations involve multiplication of unknown parameter r, and complex constant M, the equations (a) to (f) are non-linear. We find the parameter following an iteration procedure. We assume values for α_2 and α_d.

From (d) and (f).

$$\frac{\alpha_d}{\alpha_2} = f_1 = \frac{F_8 - rF_7}{I_2 - rI_1}$$

or

$$r = \frac{f_1 I_2 - F_8}{f_1 I_1 - F_7}$$

Hence, once α_2 and α_d are assumed, r is fixed. From equation (f) the constant M is fixed i.e.

$$M_2 = \frac{\alpha_2}{I_2 - rI_1} kh$$

Once $\left(\frac{M_2}{kh}\right)$ and r are fixed by assuming α_2 and α_d, the seepage quantity q_1/kh and q_2/kh are fixed i.e.

$$\frac{q_1}{kh} = \frac{\alpha_2}{I_2 - rI_1}\left[rF_1 + F_2\right] \quad \text{(i)}$$

$$\frac{q_2}{kh} = \frac{\alpha_2}{I_2 - rI_1}\left[F_{10} - rF_9\right] \quad \text{(j)}$$

$$\frac{q_1 - q_2}{kh} = \frac{\alpha_2}{I_2 - rI_1}\left[rF_5 - F_6\right] \quad \text{(k)}$$

It is found that equation (k) is not an independent equation since equation (k) can be obtained by subtracting equation (j) from equation (i). Since equation (b) can be obtained subtracting equation (d) from Eq. (f), equation (b) is also not an independent equation.

Performing the mass balance for a steady flow condition, the inflow to the filter is equal to the out flow from the filter, which is in turn equal to inflow to the drain.

Hence,

$$S_d\left[q_1 - q_2\right] = k_f \frac{\alpha_d h}{L_d} A_d$$

or $$\frac{q_1}{kh} - \frac{q_2}{kh} = \left(\frac{k_f}{k}\right)\left(\frac{A_d}{L_d S_d}\right)\alpha_d$$

or $$\frac{\alpha_2}{I_2 - rI_1}\left[rF_5 - F_6\right] = \left(\frac{k_f}{k}\right)\left(\frac{A_d}{L_d S_d}\right)\alpha_d \quad \text{(l)}$$

The assumed α_2 and α_d should satisfy Eq. (l).

Using the relation $q_3 + Q_b = q_1$, we obtain the following relation between α_1 and α_2 and α_d (vide r).

$$\alpha_1\left[I_3+\frac{K_b}{k}\left\{\frac{1-e^{\beta l}}{1+e^{-\beta l}}\right\}\right]=I_3+\left(\frac{K_b}{k}\right)\left(\frac{1-e^{-\beta l}}{1+e^{-\beta l}}\right)(2-\alpha_2)-\alpha_2\left[\frac{rF_1+F_2}{I_2-rI_1}\right] \quad \text{(m)}$$

Since, α_2 is assumed, α_1 is known.

Also the flow into the aquifer at the upstream end of the blanket (i.e. x = 0 in fragment II) is equal to the out flow from fragment I.

Hence,

$$K(T-b)\,\beta\,(C_2-C_1)=Kh\,(1-\alpha_1)$$

Substituting C_2 and C_1.

$$\left\{[1-\alpha_1]\,[h_1-h_2]-\frac{(h_1-h_2}{1-e^{-2\beta l}}2(1-\alpha_1)+\frac{(h_1-h_2}{1-e^{-2\beta l}}2[1-\alpha_2]e^{-\beta l}\right\}\beta k(T-b)=k(1-\alpha_1)hI_3 \quad \text{(n)}$$

α_1 evaluated from (m) and (n) should match.

5.4 RESULT AND DISCUSSION

Numerical results depicting variation of total seepage through foundation soil before intercepted by filter is shown in Figs. 5.6 – 5.9. It could be seen that total seepage decreases with increasing length of blanket. The flow to the down stream side also decreases with increasing length of blanket. As blanket length increases total seepage through blanket would increase. The seepage through blanket is insignificant in comparison to the seepage through upstream bed beyond the blanket.

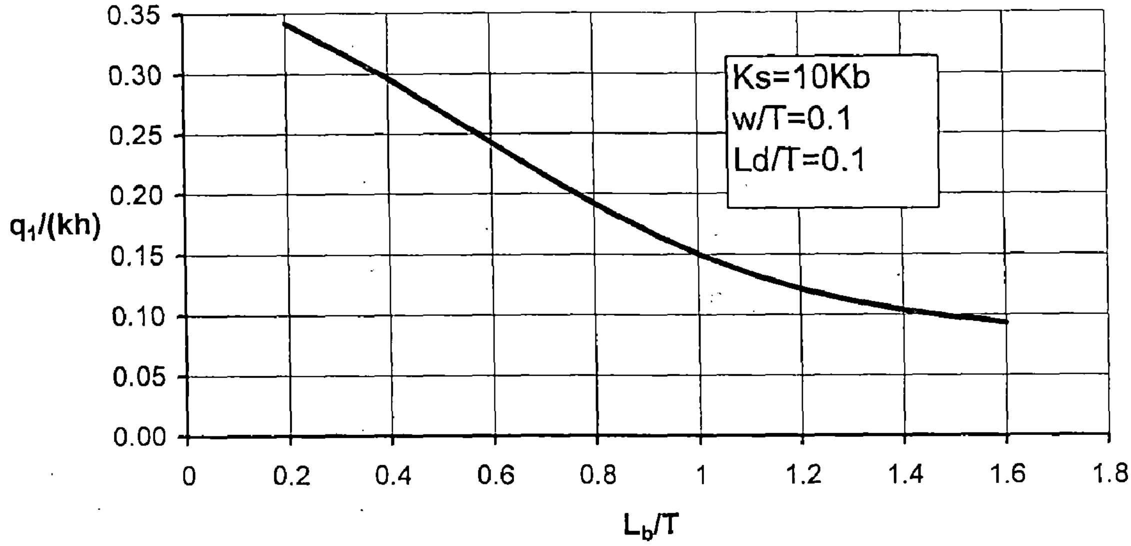


Fig.5.6 - Distribution of seepage

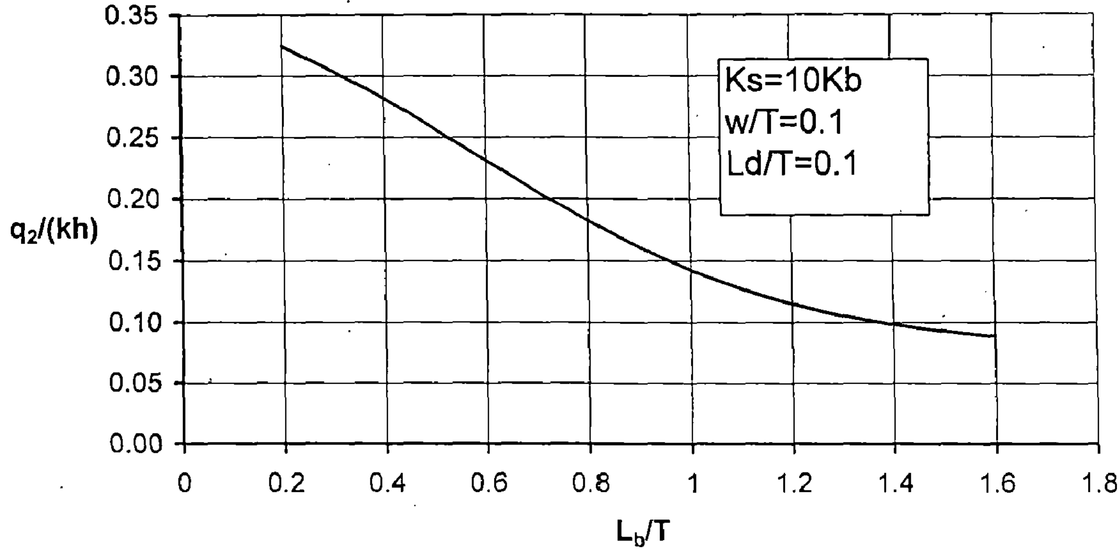


Fig.5.7 - Distribution of seepage

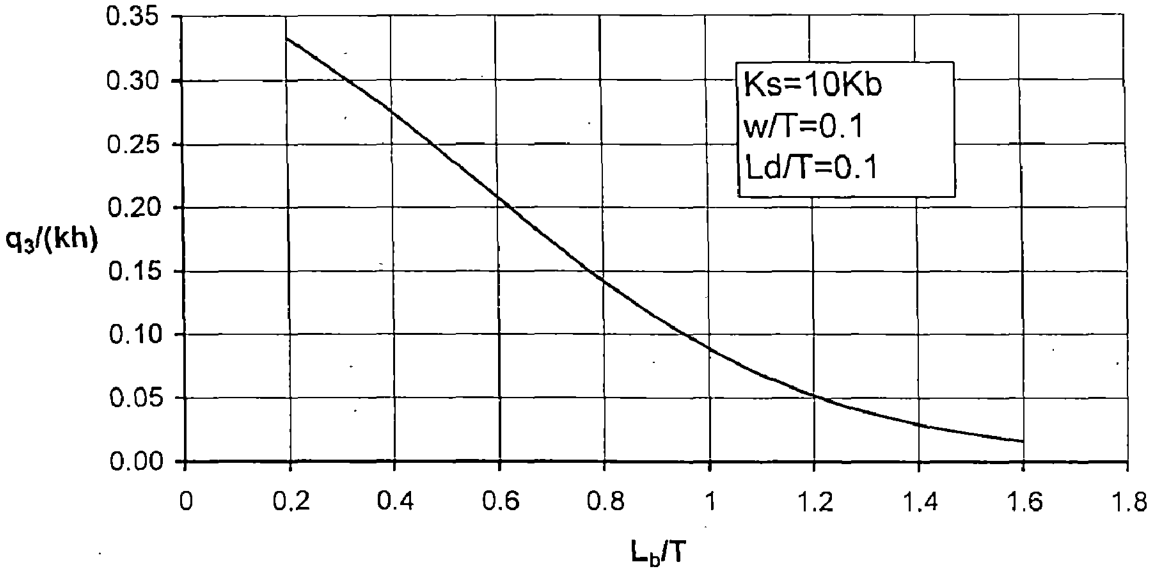


Fig.5.8 - Distribution of seepage

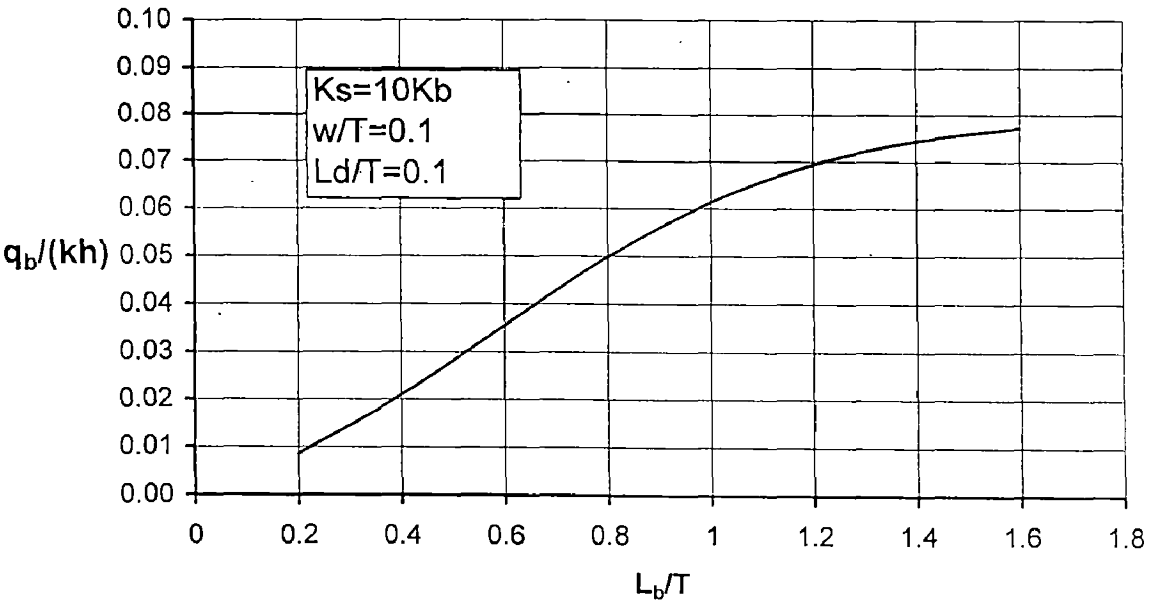


Fig.5.9 - Distribution of seepage

CHAPTER –6

CONCLUSIONS

6.1 GENERAL

Using potential theory, Kozeny (vide Har, 1962) has derived that the minimum required width of a filter w_d which would control the saturated zone in a homogenous earth dam resting on an impervious stratum is given by:

$$w_d = 0.5\sqrt{d^2 + h^2} - d/2$$

in which, d is the horizontal distance of the upstream end of the filter from a vertical line through the intersection of upstream slope of embankment, and reservoir water table, and h is the depth of water in the reservoir.

In practice, the width of the filter is extended to the down stream side. However because of economy and paucity of filter material, a filter of finite width with cross draining pipes is recommended. In many situations an earth dam is constructed under laying a porous material of finite soil layer.

So far analytical solution for seepage through an earth dam with finite filter and cross draining pipes is not available. Also rigorous solution of seepage through earth dam with less pervious blanket is not there. In this thesis, solutions have been given for these two problems.

6.2 CONCLUSIONS

Analytical solutions have been obtained for the problem of two dimensional seepage flows below an earth dam structure founded on permeable soil of finite depth with the help of conformal mapping for the following boundary conditions:

(i) An earth dam with a horizontal filter of finite width located at the base of the dam with cross draining pipes,

(ii) An earth dam having a horizontal filter of finite width with cross draining pipes and a less porous upstream blanket

The equations derived have been used for the computation of potential distribution, quantity of seepage and distribution of exit gradient. It is found that the head that develops in the filter depends on:

i) location of the filter

ii) width of the filter

iii) length ,area and filling material of the cross draining pipes.

iv) thickness and hydraulic conductivity of foundation soil.

The head that develops in the filter is unknown and it has been quantified in this thesis.

The effect of various parameters on the potential, quantity of seepage and exit gradient have been studied.

A filter increases the quantity of seepage. Since it collects part of the seepage the exit gradient is reduced. The variation of exit gradient in the downstream side, that has been presented in thesis, would help in deciding the width of invested filter.

Bennett (1946) has provided an approximate solution to analyze performance of upstream blanket in reducing seepage. In the thesis, a more rigorous solution has been obtained using method of fragments and conformal mapping. The upstream blanket is effective in reducing the seepage quantities.

Future Scope of Study

While deriving the solution for seepage through foundation of an earth dam, it was assumed that the hydraulic conductivity of the embankment material is small in comparison to that of the foundation soil. Such assumption has made it possible to decompose the unconfined seepage through the body of the earth dam from confined seepage through foundation soil and apply conformal mapping. The composite flow problem (flow through and below the earth dam) can only be solved using numerical methods, such as finite element method, for various combinations of parameters involved.

6.3 RECOMMENDATIONS

Intermediate filter of finite width can be provided below the hydraulic structures to reduce the exit gradient and hence to reduces the harmful effect of the seepage forces.

A horizontal blanket of impervious soil can be provided on the river bed on the upstream side to reduce the quantity of seepage through the pervious foundation under an earth dam.

The impervious blanket increases the length of the path of seepage under the dam and thus reduces the velocity and quantity of seepage. However, it is necessary to provide a relief well near the downstream toe of the dam to collect water seeping through the foundation and to control the exit gradient which other wise is infinite.

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APPENDIX - 1

At each of the vertices, the integrand tends to ∞. Therefore all the integral is an improper integral. For their evaluation the improper integrals to proper integral are converted by splitting the limits of integration removing the singularity by substitution.

$$F_1 = \int_{-\infty}^{-1} \frac{dt}{\sqrt{(-t)(c-t)(d-t)(1-t)(f-t)}} + \int_{-1}^{0} \frac{dt}{\sqrt{(-t)(c-t)(d-t)(1-t)(f-t)}}$$

Setting : u = -t or du = - dt

we obtain

$$F_1 = \int_{\infty}^{1} \frac{-du}{\sqrt{u(c+u)(d+u)(1+u)(f+u)}} + \int_{1}^{0} \frac{-du}{\sqrt{u(c+u)(d+u)(1+u)(f+u)}}$$

Designating: $F_{1.1} = \int_{\infty}^{1} \frac{-du}{\sqrt{u(c+u)(d+u)(1+u)(f+u)}}$

$$F_{1.2} = \int_{1}^{0} \frac{-du}{\sqrt{u(c+u)(d+u)(1+u)(f+u)}}$$

Now, setting : u = 1/v or du = $-1/v^2$ dv

$$F_{1.1} = \int_{0}^{1} \frac{\frac{1}{v^2}dv}{\sqrt{\frac{1}{v}\left(c+\frac{1}{v}\right)\left(d+\frac{1}{v}\right)\left(1+\frac{1}{v}\right)\left(f+\frac{1}{v}\right)}}$$

$$= \int_{0}^{1} \frac{\sqrt{v}\,dv}{\sqrt{(1+cv)(1+dv)(1+v)(1+fv)}}$$

Substituting: $v = \frac{1}{2}(x+1)$ and $dv = \frac{1}{2}dx$

$$F_{1.1} = \int_{-1}^{1} \frac{\frac{1}{2\sqrt{2}}(x+1)^{\frac{1}{2}}dx}{\sqrt{\left\{1+\frac{c}{2}(x+1)\right\}\left\{1+\frac{d}{2}(x+1)\right\}\left\{1+\frac{1}{2}(x+1)\right\}\left\{1+\frac{f}{2}(x+1)\right\}}}$$

$$F_{1.1} = \sqrt{2}\int_{-1}^{1} \frac{(x+1)^{\frac{1}{2}}dx}{\sqrt{\{2+c(x+1)\}\{2+d(x+1)\}\{2+f(x+1)\}\{3+x\}}}$$

and,

$$F_{1.2} = \int_0^1 \frac{du}{\sqrt{u(c+u)(d+u)(1+u)(f+u)}}$$

Setting : $u = v^2$ and $du = 2vdv$

So that

$$F_{1.2} = \int_0^1 \frac{2vdv}{\sqrt{v^2(c+v^2)(d+v^2)(1+v^2)(f+v^2)}}$$

$$= \int_0^1 \frac{2dv}{\sqrt{(c+v^2)(d+v^2)(1+v^2)(f+v^2)}}$$

Substituting: $v = \frac{1}{2}(x+1)$ *and* $dv = \frac{1}{2}dx$

So that

$$F_{1.2} = \int_{-1}^1 \frac{dx}{\sqrt{\left\{c+\frac{1}{4}(x+1)^2\right\}\left\{d+\frac{1}{4}(x+1)^2\right\}\left\{1+\frac{1}{4}(x+1)^2\right\}\left\{f+\frac{1}{4}(x+1)^2\right\}}}$$

$$F_{1.2} = 16\int_{-1}^1 \frac{dx}{\sqrt{\{4c+(x+1)^2\}\{4d+(x+1)^2\}\{4+(x+1)^2\}\{4f+(x+1)^2\}}}$$

APPENDIX - 2

$$F_2 = \int_{-\infty}^{-1} \frac{(-t)^{\frac{1}{2}}dt}{\sqrt{(c-t)(d-t)(1-t)(f-t)}} + \int_{-1}^{0} \frac{(-t)^{\frac{1}{2}}dt}{\sqrt{(c-t)(d-t)(1-t)(f-t)}}$$

Putting: u = -t and du = -dt

So $$F_2 = \int_{\infty}^{1} \frac{(-)\,u^{\frac{1}{2}}du}{\sqrt{(c+u)(d+u)(1+u)(f+u)}} + \int_{1}^{0} \frac{(-)\,u^{\frac{1}{2}}du}{\sqrt{(c+u)(d+u)(1+u)(f+u)}}$$

Setting

$$F_{2.1} = \int_{\infty}^{1} \frac{(-)\,u^{\frac{1}{2}}du}{\sqrt{(c+u)(d+u)(1+u)(f+u)}}$$

$$F_{2.2} = \int_{1}^{0} \frac{(-)\,u^{\frac{1}{2}}du}{\sqrt{(c+u)(d+u)(1+u)(f+u)}}$$

Substituting : $u = \frac{1}{v}$ *or* $du = -\frac{1}{v^2}dv$

$$F_{2.1} = \int_{0}^{1} \frac{\frac{1}{\sqrt{v}} \cdot \frac{1}{v^2}dv}{\sqrt{\left(c+\frac{1}{v}\right)\left(d+\frac{1}{v}\right)\left(1+\frac{1}{v}\right)\left(f+\frac{1}{v}\right)}}$$

$$= \int_{0}^{1} \frac{dv}{\sqrt{v(1+cv)(1+dv)(1+fv)(1+v)}}$$

Putting $v = u^2$ and $dv = 2udu$

$$F_{2.1} = \int_{0}^{1} \frac{2du}{\sqrt{(1+cu^2)(1+du^2)(1+fu^2)(1+u^2)}}$$

Putting : $u = \frac{1}{2}x + \frac{1}{2} = \frac{1}{2}(x+1)$ or $du = \frac{1}{2}dx$

$$F_{2.1} = \int_{-1}^{1} \frac{2\frac{1}{2}dx}{\sqrt{\left\{1+\frac{c}{4}(x+1)^2\right\}\left\{1+\frac{d}{4}(x+1)^2\right\}\left\{1+\frac{1}{4}(x+1)^2\right\}\left\{1+\frac{f}{4}(x+1)^2\right\}}}$$

$$= 16 \int_{-1}^{1} \frac{dx}{\sqrt{\{4+c(x+1)^2\}\{4+d(x+1)^2\}\{4+f(x+1)^2\}\{4+(x+1)^2\}}}$$

$$= \int_{0}^{1} \frac{u^{\frac{1}{2}} du}{\sqrt{(c+u)(d+u)(1+u)(f+u)}}$$

Substituting: $u = \frac{1}{2}(x+1)$ and $du = \frac{1}{2}dx$

$$F_{2.2} = \int_{-1}^{1} \frac{\frac{1}{2\sqrt{2}}(x+1)^{\frac{1}{2}} dx}{\sqrt{\left\{c+\frac{1}{2}(x+1)\right\}\left\{d+\frac{1}{2}(x+1)\right\}\left\{1+\frac{1}{2}(x+1)\right\}\left\{f+\frac{1}{2}(x+1)\right\}}}$$

$$= \sqrt{2} \int_{-1}^{1} \frac{(x+1)^{\frac{1}{2}} dx}{\sqrt{\{2c+(x+1)\}\{2d+(x+1)\}\{2f+(x+1)\}\{2+(x+1)\}}}$$

APPENDIX – 3

$$F_3 = \int_0^{\frac{c}{2}} \frac{dt}{\sqrt{t(c-t)(d-t)(1-t)(f-t)}} + \int_{\frac{c}{2}}^{c} \frac{dt}{\sqrt{t(c-t)(d-t)(1-t)(f-t)}}$$

$$F_3 = F_{3.1} + F_{3.2}$$

We have $F_{3.1} = \int_0^{\frac{c}{2}} \frac{dt}{\sqrt{t(c-t)(d-t)(1-t)(f-t)}}$

Setting : $t = u^2$ and $dt = 2udu$

$$F_{3.1} = \int_0^{\sqrt{\frac{c}{2}}} \frac{2udu}{\sqrt{u^2(c-u^2)(d-u^2)(1-u^2)(f-u^2)}}$$

$$= \int_0^{\sqrt{\frac{c}{2}}} \frac{2du}{\sqrt{(c-u^2)(d-u^2)(1-u^2)(f-u^2)}}$$

Substituting : $u = \frac{1}{2}\sqrt{\frac{c}{2}}(x+1)$ and $du = \frac{1}{2}\sqrt{\frac{c}{2}}\,dx$

$$u^2 = \frac{1}{8}c(x+1)^2 = \frac{1}{8}P_1$$

Therefore

$$F_{3.1} = \int_{-1}^{1} \frac{\sqrt{\frac{c}{2}}dx}{\left[c-\frac{1}{8}P_1\right]\left[d-\frac{1}{8}P_1\right]\left[1-\frac{1}{8}P_1\right]\left[f-\frac{1}{8}P_1\right]}$$

$$= 64\sqrt{\frac{c}{2}}\int_{-1}^{1} \frac{dx}{\sqrt{[8c-P_1][8d-P_1][8f-P_1][8-P_1]}}$$

and

$$F_{3.2} = \int_{\frac{c}{2}}^{c} \frac{dt}{\sqrt{t(c-t)(d-t)(1-t)(f-t)}}$$

Putting : $c - t = u^2$ and $t = c - u^2$; hence, $dt = -2udu$

$$F_{3.2} = \int_{\frac{c}{2}}^{0} \frac{-2udu}{\sqrt{u^2(c-u^2)\left[d-(c-u^2)\right]\left[1-(c-u^2)\right]\left[f-(c-u^2)\right]}}$$

$$= \int_{0}^{\sqrt{\frac{c}{2}}} \frac{2du}{\left[c-u^2\right]\left[d-(c-u^2)\right]\left[1-(c-u^2)\right]\left[f-(c-u^2)\right]}$$

Substituting : $u = \frac{1}{2}\sqrt{\frac{c}{2}}(x+1)$ and $du = \frac{1}{2}\sqrt{\frac{c}{2}}\,dx$

$$c-u^2 = c - \frac{1}{8}c(x+1)^2 = \frac{1}{8}\left[8c - c(x+1)^2\right] = \frac{1}{8}P_2$$

So, $$F_{3.2} = \int_{-1}^{1} \frac{\sqrt{\frac{c}{2}}dx}{\left(\frac{1}{8}P_2\right)\left(d-\frac{1}{8}P_2\right)\left(1-\frac{1}{8}P_2\right)\left(f-\frac{1}{8}P_2\right)}$$

$$= 64\sqrt{\frac{c}{2}} \int_{-1}^{1} \frac{dx}{\sqrt{P_2(8d-P_2)(8-P_2)(8f-P_2)}}$$

APPENDIX - 4

$$F_4 = \int_0^c \frac{\sqrt{t}\,dt}{\sqrt{(c-t)(d-t)(1-t)(f-t)}}$$

Putting: c-t $= u^2$ and $t = c - u^2$; hence, dt $= -2u\,du$

So, $$F_4 = \int_{\sqrt{c}}^{0} \frac{(-)\sqrt{c-u^2}\,2u\,du}{\sqrt{u^2[d-(c-u^2)][1-(c-u^2)][f-(c-u^2)]}}$$

$$= \int_{\sqrt{c}}^{0} \frac{2\sqrt{c-u^2}\,du}{\sqrt{[d-(c-u^2)][1-(c-u^2)][f-(c-u^2)]}}$$

Substituting: u = $= \frac{1}{2}\sqrt{c}(x+1)$ and du = $\frac{1}{2}\sqrt{c}\,dx$

$$c-u^2 = c - \frac{1}{4}c(x+1)^2 = \frac{1}{4}\left[4c - c(x+1)^2\right] = \frac{1}{4}P_3$$

So, $$F_4 = \int_{-1}^{1} \frac{\sqrt{\frac{1}{4}P_3}\ \sqrt{c}\,dx}{\sqrt{\left\{d-\frac{1}{4}P_3\right\}\left\{1-\frac{1}{4}P_3\right\}\left\{f-\frac{1}{4}P_3\right\}}}$$

$$= 4\sqrt{c}\int_{-1}^{1} \frac{\sqrt{P_3}\ \ dx}{\sqrt{\{4d-P_3\}\{4-P_3\}\{4f-P_3\}}}$$

APPENDIX – 5

$$F_5 = \int_c^{\frac{d+c}{2}} \frac{dt}{\sqrt{t\,(t-c)\,(d-t)\,(1-t)(f-t)}} + \int_{\frac{d+c}{2}}^{d} \frac{dt}{\sqrt{t\,(t-c)\,(d-t)\,(1-t)(f-t)}}$$

$$= F_{5\,1} + F_{5\,2}$$

We have:

$$F_{5\,1} = \int_c^{\frac{d+c}{2}} \frac{dt}{\sqrt{t\,(t-c)\,(d-t)\,(1-t)(f-t)}}$$

Putting: $t\text{-}c = u^2$ and $t = c+u^2$; hence, $dt = 2udu$

$$F_{5\,1} = \int_o^{\sqrt{\frac{d-c}{2}}} \frac{2udu}{\sqrt{[c+u^2][u^2]\,[d-(c+u^2)]\,[1-(c+u^2)][f-(c+u^2)]}}$$

$$= \int_o^{\sqrt{\frac{d-c}{2}}} \frac{2du}{\sqrt{[c+u^2]\,[d-(c+u^2)]\,[1-(c+u^2)][f-(c+u^2)]}}$$

Substituting: $u = \frac{1}{2}\sqrt{\frac{d-c}{2}}\,(x+1)$ and $du = \frac{1}{2}\sqrt{\frac{d-c}{2}}\,dx$

$$c+u^2 = c+\frac{1}{8}(d-c)(x+1)^2 = \frac{1}{8}\left[8c+(d-c)\,(x+1)^2\right] = \frac{1}{8}P_4$$

Therefore,

$$F_{5\,1} = \int_{-1}^{1} \frac{\sqrt{\frac{d-c}{2}}\,dx}{\sqrt{\frac{1}{8}P_4\left[d-\frac{1}{8}P_4\right]\left[1-\frac{1}{8}P_4\right]\left[f-\frac{1}{8}P_4\right]}}$$

$$= 64\sqrt{\frac{d-c}{2}}\int_{-1}^{1}\frac{dx}{\sqrt{P_4\,[8d-P_4][8-P_4][8f-P_4]}}$$

$$F_{5\,2} = \int_{\frac{d+c}{2}}^{d} \frac{dt}{\sqrt{t\,(t-c)\,(d-t)\,(1-t)(f-t)}}$$

Putting $d\text{-}t = u^2$ and $t = d\text{-}u^2$; hence $dt = -2udu$

$$F_{5\ 2} = \int_{\sqrt{\frac{d-c}{2}}}^{0} \frac{-2udu}{\sqrt{[d-u^2]\,[(d-u^2-c)]\,[u^2]\,[1-(d-u^2)]\,[f-(d-u^2)]}}$$

$$= \int_{c}^{\sqrt{\frac{d-c}{2}}} \frac{2du}{\sqrt{[d-u^2][(d-u^2)-c][1-(d-u^2)][f-(d-u^2)]}}$$

Substituting $u = = \frac{1}{2}\sqrt{\frac{d-c}{2}}\,(x+1)$ and $du = \frac{1}{2}\sqrt{\frac{d-c}{2}}\,dx$

$$d-u^2 = d-\frac{1}{8}(d-c)(x+1)^2 = \frac{1}{8}\left[8d-(d-c)\,(x+1)^2\right] = \frac{1}{8}P_5$$

Therefore,

$$F_{5.2} = \int_{-1}^{1} \frac{\sqrt{\frac{d-c}{2}}\,dx}{\sqrt{\frac{1}{8}P_5\left[\frac{1}{8}P_5-c\right]\left[1-\frac{1}{8}P_5\right]\left[f-\frac{1}{8}P_5\right]}}$$

$$= 64\sqrt{\frac{d-c}{2}}\int_{-1}^{1}\frac{dx}{\sqrt{P_5\,[P_5-8c][8-P_5][8f-P_5]}}$$

APPENDIX - 6

$$F_6 = \int_{c}^{\frac{d+c}{2}} \frac{\sqrt{t}\,dt}{\sqrt{(t-c)(d-t)(1-t)(f-t)}} + \int_{\frac{d+c}{2}}^{d} \frac{\sqrt{t}\,dt}{\sqrt{(t-c)(d-t)(1-t)(f-t)}}$$

$= F_{6\,1} + F_{6\,2}$

Putting $t-c = u^2$ and $t = c+u^2$, hence $dt = 2udu$

$$F_{6\,1} = \int_{c}^{\sqrt{\frac{d-c}{2}}} \frac{2\sqrt{c+u^2}\,udu}{\sqrt{[u^2][d-(c+u^2)][1-(c+u^2)][f-(c+u^2)]}}$$

$$= \int_{c}^{\sqrt{\frac{d-c}{2}}} \frac{2\sqrt{c+u^2}\,du}{\sqrt{[d-(c+u^2)][1-(c+u^2)][f-(c+u^2)]}}$$

Substituting : $u = \frac{1}{2}\sqrt{\frac{d-c}{.2}}(x+1)$ and $du = \frac{1}{2}\sqrt{\frac{d-c}{2}}dx$

$$c+u^2 = c+\frac{1}{8}(d-c)(x+1)^2 = \frac{1}{8}\left[8c+(d-c)(x+1)^2\right] = \frac{1}{8}P_6$$

Therefore,

$$F_{6\,1} = \int_{-1}^{1} \frac{\sqrt{\frac{1}{8}P_6}\sqrt{\frac{d-c}{2}}\,dx}{\sqrt{\left[d-\frac{1}{8}P_6\right]\left[1-\frac{1}{8}P_6\right]\left[f-\frac{1}{8}P_6\right]}}$$

$$= 4\sqrt{2}.\sqrt{d-c}\int_{-1}^{1} \frac{\sqrt{P_6}\,dx}{\sqrt{[8d-P_6][8-P_6][8f-P_6]}}$$

and

$$F_{6\,2} = \int_{\frac{d+c}{2}}^{d} \frac{\sqrt{t}\,dt}{\sqrt{(t-c)(d-t)(1-t)(f-t)}}$$

Setting

$d-t = u^2$ and $t = d-u^2$; hence, $dt = -2\,udu$

$$F_{6\,2} = \int_{\sqrt{\frac{d-c}{2}}}^{0} \frac{(-)2\sqrt{d-u^2}\;udu}{\sqrt{[(d-u^2)-c]\,[u^2]\,[(1-(d-u^2))]\,[f-(d-u^2)]}}$$

Or, $$F_{62} = \int_0^{\sqrt{\frac{d-c}{2}}} \frac{2\sqrt{d-u^2}\ du}{\sqrt{\left[\left(d-u^2\right)-c\right]\ \left[\left(1-\left(d-u^2\right)\right)\right]\ \left[f-\left(d-u^2\right)\right]}}$$

Substituting

$$u = \frac{1}{2}\sqrt{\frac{d-c}{2}}\,(x+1) \quad \text{and} \quad du = \frac{1}{2}\sqrt{\frac{d-c}{2}}\,dx$$

$$d-u^2 = c - \frac{1}{8}(d-c)(x+1)^2 = \frac{1}{8}\left[8d-(d-c)\,(x+1)^2\right] = \frac{1}{8}P_7$$

Therefore,

$$F_{62} = \int_{-1}^{1} \frac{\sqrt{\frac{1}{8}P_7}\sqrt{\frac{d-c}{2}}\ dx}{\sqrt{\left[\frac{1}{8}P_7 - c\right]\left[1-\frac{1}{8}P_7\right]\ \left[f-\frac{1}{8}P_7\right]}}$$

$$= 4\sqrt{2}.\sqrt{d-c}\int_{-1}^{1} \frac{\sqrt{P_7}\ .dx}{\sqrt{\left[P_7 - 8c\right]\left[8-P_7\right]\left[8f-P_7\right]}}$$

APPENDIX – 7

$$F_7 = \int_d^{\frac{d+1}{2}} \frac{dt}{\sqrt{t(t-c)(t-d)(1-t)(f-t)}} + \int_{\frac{d+1}{2}}^{1} \frac{dt}{\sqrt{t(t-c)(t-d)(1-t)(f-t)}}$$

$$= F_{71} + F_{72}$$

We obtain:

$$F_{71} = \int_d^{\frac{d+1}{2}} \frac{dt}{\sqrt{t(t-c)(t-d)(1-t)(f-t)}}$$

Putting: $t - d = u^2$ and $t = d + u^2$; hence, $dt = 2udu$

So,

$$F_{71} = \int_d^{\sqrt{\frac{1-d}{2}}} \frac{2udu}{\sqrt{(d+u^2)[(d+u^2)-c][u^2][1-(d+u^2)][f-(d+u^2)]}}$$

$$= \int_d^{\sqrt{\frac{1-d}{2}}} \frac{2du}{\sqrt{(d+u^2)[(d+u^2)-c][1-(d+u^2)][f-(d+u^2)]}}$$

Substituting,

$$u = \frac{1}{2}\sqrt{\frac{1-d}{2}}(x+1) \quad \text{and} \quad du = \frac{1}{2}\sqrt{\frac{1-d}{2}}dx$$

$$d + u^2 = d + \frac{1}{8}(1-d)(x+1)^2 = \frac{1}{8}\left[8d + (1-d)(x+1)^2\right] = \frac{1}{8}P_8$$

Therefore,

$$F_{71} = \int_{-1}^{1} \frac{\sqrt{\frac{1-d}{2}}\,dx}{\sqrt{\left[\frac{1}{8}P_8\right]\left[\frac{1}{8}P_8 - c\right]\left[1-\frac{1}{8}P_8\right]\left[f-\frac{1}{8}P_8\right]}}$$

$$= 64\sqrt{\frac{1-d}{2}}\int_{-1}^{1} \frac{dx}{\sqrt{P_8[P_8-8c][8-P_8][8f-P_8]}}$$

and

$$F_{72} = \int_{\frac{d+1}{2}}^{1} \frac{dt}{\sqrt{t(t-c)(t-d)(1-t)(f-t)}}$$

Putting: $1 - t = u^2$ or $t = 1 - u^2$ and $dt = -2udu$

So,

$$F_{72} = \int_{\sqrt{\frac{1-}{2}}}^{0} \frac{-2udu}{\sqrt{\left(1+u^2\right)\left[\left(1+u^2\right)-c\right]\left[\overline{1}+u^2\right]\left[u^2\right]\left[f-\left(1+u^2\right)\right]}}$$

$$= \int_{0}^{\sqrt{\frac{1-d}{2}}} \frac{2du}{\sqrt{\left(1+u^2\right)\left[\left(1+u^2\right)-c\right]\left[\overline{1}+u^2\right]\left[f-\left(1+u^2\right)\right]}}$$

Substituting: $u = \frac{1}{2}\sqrt{\frac{1-\mathrm{d}}{2}}(\mathrm{x}+1)$ and $du = \frac{1}{2}\sqrt{\frac{1-d}{2}}dx$

Therefore,

$$F_{72} = \int_{-1}^{1} \frac{\sqrt{\frac{1-d}{2}}\,dx}{\sqrt{\left[\frac{1}{8}P_9\right]\left[\frac{1}{8}P_9-c\right]\left[\frac{1}{8}P_9-d\right]\left[f-\frac{1}{8}P_9\right]}}$$

$$= 64\sqrt{\frac{1-d}{2}}\int_{-1}^{1} \frac{dx}{\sqrt{P_9\left[P_9-8c\right]\left[P_9-8d\right]\left[8f-P_9\right]}}$$

APPENDIX – 8

$$F_8 = \int_d^{\frac{d+1}{2}} \frac{\sqrt{t}\,dt}{\sqrt{(t-c)(t-d)(1-t)(f-t)}} + \int_{\frac{d+1}{2}}^{1} \frac{\sqrt{t}\,dt}{\sqrt{t(t-c)(t-d)(1-t)(f-t)}}$$

$$= F_{8.1} + F_{8.2}$$

$$= \int_d^{\frac{d+1}{2}} \frac{\sqrt{t}\,dt}{\sqrt{(t-c)(t-d)(1-t)(f-t)}}$$

Putting $t - d = u^2$ and $t = d - u^2$; hence, $dt = 2udu$

$$F_{8.1} = \int_0^{\sqrt{\frac{1-d}{2}}} \frac{2\sqrt{d+u^2}\,udu}{\sqrt{\left[(d+u^2)-c\right]\left[u^2\right]\left[1-(d+u^2)\right]\left[f-(d+u^2)\right]}}$$

$$= \int_0^{\sqrt{\frac{1-d}{2}}} \frac{2\sqrt{d+u}^{2}\,du}{\sqrt{\left[(d+u^2)-c\right]\left[1-(d+u^2)\right]\left[f-(d+u^2)\right]}}$$

Substituting:

$$u = \; = \frac{1}{2}\sqrt{\frac{1-\mathrm{d}}{2}}(\mathrm{x}+1) \qquad \text{and} \qquad du = \frac{1}{2}\sqrt{\frac{1-d}{2}}dx$$

$$d+u^2 = d + \frac{1}{8}(1-d)(x+1)^2 = \frac{1}{8}\left[8d + (1-d)(x+1)^2\right] = \frac{1}{8}P_{10}$$

Therefore,

$$F_{8.1} = \int_{-1}^{1} \frac{\sqrt{\frac{1}{8}p_{10}}\sqrt{\frac{1-d}{2}}\,dx}{\sqrt{\left[\frac{1}{8}P_{10}-c\right]\left[1-\frac{1}{8}P_{10}\right]\left[f-\frac{1}{8}P_{10}\right]}}$$

$$= 4\sqrt{2}\sqrt{(1-d)}\int_{-1}^{1} \frac{\sqrt{p_{10}}\,dx}{\sqrt{\left[P_{10}-8c\right]\left[8-P_{10}\right]\left[8f-P_{10}\right]}}$$

and

$$F_{8.2} = \int_{\frac{d+1}{2}}^{1} \frac{\sqrt{t}\,dt}{\sqrt{(t-c)(t-d)(1-t)(f-t)}}$$

Putting: $1-t = u^2$ and or $t = 1-u^2$; hence, $dt = -2udu$

So, $F_{8.2} = \int_{\sqrt{\frac{1-d}{2}}}^{0} \frac{-2u\sqrt{1-u^2}\,du}{\sqrt{\left[\left(1-u^2\right)-c\right]\left[\left(1-u^2\right)-d\right]\left[u^2\right]\left[f-\left(1+u^2\right)\right]}}$

$$= \int_{0}^{\sqrt{\frac{1-d}{2}}} \frac{2\sqrt{1-u^2}\,du}{\sqrt{\left[\left(1--u^2\right)-c\right]\left[\left(1-u^2\right)-d\right]\left[f-\left(1+u^2\right)\right]}}$$

Substituting, $u = \frac{1}{2}\sqrt{\frac{1-d}{2}}(x+1)$ or $du = \frac{1}{2}\sqrt{\frac{1-d}{2}}dx$

$$1-u^2 = 1-\frac{1}{8}(1-d)(x+1)^2 = \frac{1}{8}\left[8-(1-d)(x+1)^2\right] = \frac{1}{8}P_{11}$$

Therefore:

$$F_{8.2} = \int_{-1}^{1} \frac{\sqrt{\frac{1-d}{2}}\sqrt{\frac{1}{8}p_{11}}\,dx}{\sqrt{\left[\frac{1}{8}P_{11}-c\right]\left[\frac{1}{8}P_{11}-d\right]\left[f-\frac{1}{8}P_{11}\right]}}$$

$$= 4\sqrt{2}\sqrt{(1-d)}\int_{-1}^{1} \frac{\sqrt{p_{11}}\,dx}{\sqrt{\left[P_{11}-8c\right]\left[p_{11}-8d\right]\left[8f-P_{11}\right]}}$$

APPENDIX – 9

$$F_9 = \int_1^{\frac{1+f}{2}} \frac{dt}{\sqrt{t(t-c)(t-d)(t-1)(f-t)}} + \int_{\frac{1+f}{2}}^{1} \frac{dt}{\sqrt{t(t-c)(t-d)(t-1)(f-t)}}$$

$= F_{9.1} + F_{9.2}$

Putting: $t-1 = u^2$ and $t = 1 + u^2$; hence, $dt = 2udu$

So, $$F_{9.1} = \int_o^{\sqrt{\frac{f-1}{2}}} \frac{2udu}{\sqrt{[1+u^2][(1+u^2)-c][(1+u^2)-d][u^2][f-(1+u^2)]}}$$

$$= \int_o^{\sqrt{\frac{f-1}{2}}} \frac{2du}{\sqrt{[1+u^2][1(1+u^2)-c][(1+u^2)-d][f-(1+u^2)]}}$$

Substituting: $u = \frac{1}{2}\sqrt{\frac{f-1}{2}}(x+1)$ and $du = \frac{1}{2}\sqrt{\frac{f-1}{2}}dx$

$$1+u^2 = 1+\frac{1}{8}(f-1)(x+1)^2 = \frac{1}{8}\left[8+(f-1)(x+1)^2\right] = \frac{1}{8}P_{12}$$

Therefore

$$F_{9.1} = \int_{-1}^{1} \frac{\sqrt{\frac{f-1}{2}}dx}{\sqrt{\left[\frac{1}{8}P_{12}\right]\left[\frac{1}{8}P_{12}-c\right]\left[\frac{1}{8}P_{12}-d\right]\left[f-\frac{1}{8}P_{12}\right]}}$$

$$= 64\sqrt{\frac{f-1}{2}}\int_{-1}^{1} \frac{dx}{\sqrt{P_{12}[P_{12}-8c][p_{12}-8d][8f-P_{12}]}}$$

and

$$F_{9.2} = \int_{\frac{1+f}{2}}^{f} \frac{dt}{\sqrt{t(t-c)(t-d)(t-1)(f-t)}}$$

Putting: $f - t = u^2$ and $t = f - u^2$; hence, $dt = -2udu$

So,

$$F_{9.2} = \int_{\sqrt{\frac{f-1}{2}}}^{0} \frac{-2udu}{\sqrt{(f-u^2)[(f-u^2)-c][(f-u^2)-d][(f-u^2)-1][u^2]}}$$

Or

$$F\ 9.2 = \int_0^{\sqrt{\frac{f-1}{2}}} \frac{2du}{\sqrt{(f-u^2)[(f-u^2)-c][(f-u^2)-d][(f-u^2)-1]}}$$

Substituting: $u=\frac{1}{2}\sqrt{\frac{f-1}{2}}(x+1)$ and $du = \frac{1}{2}\sqrt{\frac{f-1}{2}}dx$

$$f-u^2 = f-\frac{1}{8}(f-1)(x+1)^2 = \frac{1}{8}\left[8f-(f-1)(x+1)^2\right] = \frac{1}{8}p_{13}$$

Therefore :

$$F\ 9.2 = \int_{-1}^{1} \frac{\sqrt{\frac{f-1}{2}}dx}{\sqrt{\left(\frac{1}{8}p_{13}\right)\left[\frac{1}{8}p_{13}-c\right]\left[\frac{1}{8}p_{13}-d\right]\left[\frac{1}{8}p_{13}-1\right]}}$$

$$= 64\sqrt{\frac{f-1}{2}}\int_{-1}^{1}\frac{dx}{\sqrt{(p_{13})[p_{13}-8c][p_{13}-8d][p_{13}-8]}}$$

APPENDIX - 10

$$F\,10 = \int_{1}^{\frac{1+f}{2}} \frac{dt}{\sqrt{(t-c)(t-d)(t-1)(f-t)}} + \int_{\frac{1+f}{2}}^{f} \frac{dt}{\sqrt{(t-c)(t-d)(t-1)(f-t)}}$$

$$= F_{10.1} + F_{10.2}$$

Putting: $t-1 = u^2$ and $t = 1+u^2$; hence, $dt = 2udu$

So,

$$F\,10.1 = \int_{0}^{\sqrt{\frac{f-1}{2}}} \frac{\sqrt{1+u^2}\,2du u}{\sqrt{\left[\left(1+u^2\right)-c\right]\left[\left(1+u^2\right)-d\right]u^2\left[f-\left(1+u^2\right)\right]}}$$

$$= \int_{0}^{\sqrt{\frac{f-1}{2}}} \frac{2\sqrt{1+u^2}\,du}{\sqrt{\left[\left(1+u^2\right)-c\right]\left[\left(1+u^2\right)-d\right]\left[f-\left(1+u^2\right)\right]}}$$

Substituting: $u = \frac{1}{2}\sqrt{\frac{f-1}{2}}(x+1)$ and $du = \frac{1}{2}\sqrt{\frac{f-1}{2}}dx$

$$1+u^2 = 1+\frac{1}{8}(f-1)(x+1)^2 = \frac{1}{8}\left(8+(f-1)(x+1)^2\right) = \frac{1}{8}p_{14}$$

Therefore

$$F\,10.1 = \int_{-1}^{1} \frac{\sqrt{\frac{1}{8}p_{14}}\sqrt{\frac{f-1}{2}}dx}{\sqrt{\left[\frac{1}{8}p_{14}-c\right]\left[\frac{1}{8}p_{14}-d\right]\left[f-\frac{1}{8}p_{14}\right]}}$$

$$= 4\sqrt{2}\sqrt{f-1}\int_{-1}^{1} \frac{\sqrt{p_{14}}\,dx}{\sqrt{\left[p_{14}-8c\right]\left[p_{14}-8d\right]\left[8f-p_{14}\right]}}$$

$$F\,10.2 = \int_{\frac{1+f}{2}}^{f} \frac{\sqrt{t}\;dt}{\sqrt{(t-c)(t-d)(t-1)(f-t)}}$$

Putting: $f - t = u^2$ and $t = f - u^2$; hence, $dt = -2udu$

So,

$$F\ 10.2 = \int_{\sqrt{\frac{f-1}{2}}}^{0} \frac{(-)\sqrt{f-u^2}\ 2u\,du}{\sqrt{\left[\left(f-u^2\right)-c\right]\left[\left(f-u^2\right)-d\right]\left[\left(f-u^2\right)-1\right]\left[u^2\right]}}$$

$$= \int_{0}^{\sqrt{\frac{f-1}{2}}} \frac{2\ \sqrt{f-u^2}\ du}{\sqrt{\left[\left(f-u^2\right)-c\right]\left[\left(f-u^2\right)-d\right]\left[\left(f-u^2\right)-1\right]}}$$

Substituting: $u = \frac{1}{2}\sqrt{\frac{f-1}{2}}(x+1)\frac{1}{2}\sqrt{\frac{f-1}{2}}x$ and $du = \frac{1}{2}\sqrt{\frac{f-1}{2}}\ dx$

$$f-u^2 = f - \frac{1}{8}(f-1)(x+1)^2 = \frac{1}{8}\left(8f-(f-1)(x+1)^2\right) = \frac{1}{8}p_{15}$$

Therefore,

$$F\ 10.2 = \int_{-1}^{1} \frac{\sqrt{\frac{1}{8}p_{15}}\sqrt{\frac{f-1}{2}}dx}{\sqrt{\left[\frac{1}{8}p_{15}-c\right]\left[\frac{1}{8}p_{15}-d\right]\left[\frac{1}{8}p_{15}-1\right]}}$$

$$= 4\sqrt{2}\sqrt{f-1}\int_{-1}^{1} \frac{\sqrt{p_{15}}dx}{\sqrt{\left[p_{15}-8c\right]\left[p_{15}-8d\right]\left[P_{15}-8\right]}}$$

APPENDIX – 11

$$I_1 = \int_f^{\infty} \frac{dt}{\sqrt{t(t-c)(t-d)(t-1)(t-f)}}$$

Putting: t- f = u^2 and t = = f+u^2 ; hence, dt = 2udu

$$I_1 = \int_0^{\infty} \frac{2udu}{\sqrt{\{f+u^2\}\{f+u^2-c\}\{f+u^2-d\}\{f+u^2-1\}\{u^2\}}}$$

$$= 2\int_0^{\infty} \frac{du}{\sqrt{\{f+u^2\}\{f+u^2-c\}\{f+u^2-d\}\{f+u^2-1\}}}$$

$$= 2$$

$$\left[\int_0^{\infty} \frac{du}{\sqrt{\{f+u^2\}\{f+u^2-c\}\{f+u^2-d\}\{f+u^2-1\}}} + \int_1^{\infty} \frac{du}{\sqrt{\{f+u^2\}\{f+u^2-c\}\{f+u^2-d\}\{f+u^2-1\}}}\right]$$

$$= 2[I_{11} + I_{12}]$$

Where:

$$I_{11} = \int_0^{1} \frac{du}{\sqrt{\{f+u^2\}\{f+u^2-c\}\{f+u^2-d\}\{f+u^2-1\}}}$$

$$I_{12} = \int_1^{\infty} \frac{du}{\sqrt{\{f+u^2\}\{f+u^2-c\}\{f+u^2-d\}\{f+u^2-1\}}}$$

Substituting: $u= \frac{1}{2}x + \frac{1}{2} = \frac{1}{2}(x+1)$ and $du = \frac{1}{2}dx$

$$f+u^2= f + \frac{1}{4}(x+1)^2 = \frac{1}{4}\left(4f + (x+1)^2\right) = \frac{1}{4}p_{16}$$

So,

$$I_{11} = \int_{-1}^{1} \frac{\frac{1}{2}x}{\sqrt{\frac{1}{4}P_{16}\left(\frac{1}{4}P_{16} - c\right)\left(\frac{1}{4}P_{16} - d\right)\left(\frac{1}{4}P_{16} - 1\right)}}$$

$$= 8 \int_{-1}^{1} \frac{dx}{\sqrt{P_{16}(P_{16} - 4c)(P_{16} - 4d)(P_{16} - 4)}}$$

and

$$I_{1.2} = \int_{1}^{\infty} \frac{du}{\sqrt{(f + u^2)(f - c + u^2)(f - d + u^2)(f - 1 + u^2)}}$$

Putting: $u = \frac{1}{v}$ and $du = -\frac{1}{v^2}dv$

$$I_{12} = \int_{1}^{0} \frac{-\frac{1}{v^2}dv}{\sqrt{\left(f + \frac{1}{v^2}\right)\left(f - c + \frac{1}{v^2}\right)\left(f - d + \frac{1}{v^2}\right)\left(f - 1 + \frac{1}{v^2}\right)}}$$

$$= \int_{1}^{\infty} \frac{v^2 du}{\sqrt{(fv^2 + 1)((f - c)v^2 + 1)((f - d)v^2 + 1)((f - 1)v^2 + 1)}}$$

Substituting : $v = \frac{1}{2}x + \frac{1}{2} = \frac{1}{2}(x - 1)$ and $du = \frac{1}{2}dx$

$$v^2 = \frac{1}{4}(x - 1)^2 = \frac{1}{4}p$$

$$I_{12} = \int_{-1}^{1} \frac{\frac{1}{4}p\frac{1}{2}dx}{\sqrt{\left(\frac{f}{4}p + 1\right)\left((f - c)\frac{1}{4}p + 1\right)\left((f - d)\frac{1}{4}p + 1\right)\left((f - 1)\frac{1}{4}p + 1\right)}}$$

$$= 2 \int_{-1}^{1} \frac{p.dx}{\sqrt{(fp + 4)((f - c)p + 4)((f - d)p + 4)((f - 1)p + 4)}}$$

APPENDIX – 12

$$I_2 = \int_f^{\infty} \frac{\sqrt{t}.dt}{\sqrt{(t-c)(t-d)(t-1)(t-f)}}$$

Putting: t - f = u^2 and t = f+ u^2 ; hence, dt = 2udu

So,

$$I_2 = \int_o^{\infty} \frac{\sqrt{f+u^2}.2udu}{\sqrt{(f+u^2-c)(f+u^2-d)(f+u^2-1)(u^2)}} + \int_o^{\infty} \frac{2\sqrt{f+u^2}.du}{\sqrt{(f+u^2-c)(f+u^2-d)(f+u^2-1)}}$$

$$= 2\left[\int_o^1 \frac{\sqrt{f+u^2}.du}{\sqrt{(f+u^2-c)(f+u^2-d)(f+u^2-1)}} + \int_1^{\infty} \frac{\sqrt{f+u^2}.du}{\sqrt{(f+u^2-c)(f+u^2-d)(f+u^2-1)}}\right]$$

$$= 2\,[I_{21} + I_{22}]$$

Where:

$$I_{21} = \int_0^1 \frac{\sqrt{f+u^2}.du}{\sqrt{(f+u^2-c)(f+u^2-d)(f+u^2-1)}}$$

$$I_{22} = \int_0^1 \frac{\sqrt{f+u^2}.du}{\sqrt{(f+u^2-c)(f+u^2-d)(f+u^2-1)}}$$

Substituting: $u = \frac{1}{2}x + \frac{1}{2} = \frac{1}{2}(x+1) \quad or \quad du = \frac{1}{2}dx$

$$u^2 = \frac{1}{4}(x+1)^2 \ or \ u^2 = \frac{1}{4}P$$

so $$I_{.2.1} = \int_{-1}^{1} \frac{\sqrt{f+\frac{1}{4}p}\,\frac{1}{2}dx}{\sqrt{\left((f-c)+\frac{1}{4}p\right)\left((f-d)+\frac{1}{4}p\right)\left((f-1)+\frac{1}{4}p\right)}}$$

$$= 2\int_{-1}^{1} \frac{\sqrt{4f+p}\ dx}{\sqrt{(4(f-c)+p)(4(f-d)+p)(4(f-1)+p)}}$$

Putting : $u = \frac{1}{v} \quad or \quad du = -\frac{1}{v^2}dv$

And

$$I_{2.2} = \int_{1}^{0} \frac{(-)\sqrt{f + \frac{1}{v^2}}\,\frac{1}{v^2}\,dv}{\sqrt{\left((f-c)+\frac{1}{v^2}\right)\left((f-d)+\frac{1}{v^2}\right)\left((f-1)+\frac{1}{v^2}\right)}}$$

$$= \int_{0}^{1} \frac{\sqrt{fv^2+1}\ dv}{\sqrt{\left((f-c)v^2+1\right)\left((f-d)v^2+1\right)\left((f-1)v^2+1\right)}}$$

substituting : $v = \frac{1}{2}x + \frac{1}{2} = \frac{1}{2}(x+1)$ *or* $dv = \frac{1}{2}dx$

$$v^2 = \frac{1}{4}(x+1)^2 \quad or \quad v^2 = \frac{1}{4}P$$

Therefore

$$I_{2.2} = \int_{-1}^{1} \frac{\sqrt{\frac{f}{4}p+1}\ \frac{1}{2}dx}{\sqrt{\left((f-c)\frac{1}{4}p+1\right)\left((f-d)\frac{1}{4}p+1\right)\left((f-1)\frac{1}{4}p+1\right)}}$$

$$= 2\int_{-1}^{1} \frac{\sqrt{fp+4}\ \ dx}{\sqrt{\left((f-c)p+4\right)\left((f-d)p+4\right)\left((f-1)p+4\right)}}$$

APPENDIX – 13

INPUT DATA

```
125.    135.      200.    100.     12.       2
3.14    0.1   0.01  10.
160.   1.0    0.001
```

OUTPUT:

```
               PROGRAM FOR COMPUTATION OF
           PERFORMANCE OF FILTER IN EARTH DAMS

 BASE WIDTH OF THE DAM                      =     200.00
 THICKNESS OF THE POROUS FOUNDATION =     100.00
 DISTANCE OF THE DRAIN FROM UPSTREAM=     125.00
 DRAIN WIDTH                                =      10.00
 K of filter/ K of foundation soil    =      10.00

 b/T   =       2.00
 11/T =       1.25
 w/T   =       0.1000
   f   =    1.007498
   c   =    9.311342E-01
   d   =    9.511359E-01

     1.FROM POINT A TO B
       F1 =  1.639
       F2 =  1.593

     2.FROM POINT B TO C
       F3 =        43.68323
       F4 =        37.01600

     3.FROM POINT C TO D
       F5 =        52.47112
       F6 =        49.42336

     4.FROM POINT  D TO E
       F7 =        98.47589
       F8 =        96.27065

     5.FROM POINT  E TO F
       F9  =        50.83193
       F10 =        51.01646

     6.FROM POINT  F TO A
       I1 =        54.79265
       I2 =        59.25465

 THE FIRST CYCLE STARTS
 RESULT AFTER THE FIRST CYCLE
   9.957678E-01     9.954484E-01     2.200006E-01     9.475494E-02
  -7.599691E-06    -3.193971E-04
   9.448821E-01
   9.448821E-01     4.307031E-01
```

```
THE SECOND CYCLE STARTS
RESULT AFTER THE SECOND  CYCLE
   9.954816E-01    9.954601E-01    2.220006E-01    9.561554E-02
   4.912050E-06   -2.150498E-05
   9.448825E-01
   9.448825E-01    4.306995E-01
THE THIRD CYCLE STARTS
RESULT AFTER THIRD CYCLE
   9.954674E-01    9.954607E-01    2.221006E-01    9.565903E-02
  -1.414587E-06   -6.676010E-06
   9.448823E-01
   9.448823E-01    4.307015E-01
LENGTH OF BLANKET     160.000000
THICKNESS OF BLANKET        1.000000
CONDUCTIVITY OF THE BLANKET   1.000000E-03
CONDUCTIVITY OF THE FOUNDATION SOIL   1.000000E-02
BK/AK   1.000000E-01
FLOW THROUGH BLANKET Qb/kh   7.728115E-02
Q3BYKH   1.599751E-02
QBBYKH   7.728115E-02
Q1BYKH   9.326667E-02
Q2BYKH   8.864567E-02
BLBYT       1.600000
```

APPENDIX - 14

```
C   *****************************************************************
C
C      PROGRAMME FOR PERFORMANCE ANALYSIS OF FILTER IN EARTH DAMS
C
C   *****************************************************************
C
      DIMENSION W(96),XX(96)
      CHARACTER*12 input,output
C         WRITE (*,*) ' Please enter name of input file'
C         READ(*,100) input
C         WRITE (*,*) ' Please enter name of output file'
C         READ(*,100) output
C         OPEN (1, status='old',FILE=input)
C         OPEN (3, status='old',FILE='HUNGNEW.DAT')
C         OPEN (2, status='unknown',FILE=output)
          OPEN (1, status='old',FILE='AHUNGFN.DAT')
          OPEN (3, status='old',FILE='GAUSS.DAT')
          OPEN (2, status='unknown',FILE='AFN.OUT')
          READ (3,*) (W(I),I=1,96)
          READ (3,*) (XX(I),I=1,96)
   90 FORMAT(15X,'PROGRAM FOR COMPUTATION OF')
   91 FORMAT(10X,' PERFORMANCE OF FILTER IN EARTH DAMS')
    1 FORMAT('W1=',F7.3,5x,'W96=',F7.3,5x,'XX1=',F7.3,5x,'XX96=',F7.3)
    2 FORMAT(' C=',F7.3,5x,'  D=',F7.3,5x,'  F=',F7.3)
    3 FORMAT(5X'1.FROM POINT A TO B')
    4 FORMAT(7X'F1 =',F7.3)
    5 FORMAT(7X'F2 =',F7.3)
    6 FORMAT(5X'2.FROM POINT B TO C')
    7 FORMAT(7X'F3 =',F15.5)
    8 FORMAT(7X'F4 =',F15.5)
    9 FORMAT(5X'3.FROM POINT C TO D')
   10 FORMAT(7X'F5 =',F15.5)
   11 FORMAT(7X'F6 =',F15.5)
   12 FORMAT(5X'4.FROM POINT  D TO E')
   13 FORMAT(7X'F7 =',F15.5)
   14 FORMAT(7X'F8 =',F15.5)
   15 FORMAT(5X'5.FROM POINT  E TO F')
   16 FORMAT(7X'F9  =',F15.5)
   17 FORMAT(7X'F10 =',F15.5)
   18 FORMAT(5X'6.FROM POINT  F TO A')
   19 FORMAT(7X'I1 =',F15.5)
   20 FORMAT(7X'I2 =',F15.5)
  100 FORMAT (a)
      WRITE(2,90)
      WRITE(2,91)
      WRITE(2,*)
c     WRITE(2,*)'W1 =',W(1), '      W96 =',W(96)
c     WRITE(2,*)'XX1=',XX(1),'      XX96=',XX(96)
      READ(1,*)AL1,AL2,AL,T,H1,H2
      READ(1,*)DAREA, DK,AK, DSPAC
      READ(1,*)BL,THICKB,BK
      DWIDTH=AL2-AL1
      DLENGTH=AL-AL2
      AKRATIO=DK/AK
      WRITE(2,*)'BASE WIDTH OF THE DAM='
```

```
        WRITE(2,600)AL
      WRITE(2,*)'THICKNESS OF THE POROUS FOUNDATION='
        WRITE(2,600)T
      WRITE(2,*)'DISTANCE OF THE DRAIN FROM UPSTREAM='
      WRITE(2,600)AL1
      WRITE(2,*)'DRAIN WIDTH='
        WRITE(2,600)DWIDTH
      WRITE(2,*)'K of filter/ K of foundation soil='
      WRITE(2,600)akratio
      BBYT=AL/T
      AL1BYT=AL1/T
      DWBYT=DWIDTH/T
      WRITE(2,*)'b/T='
      WRITE(2,600)BBYT
        WRITE(2,*)'l1/T='
      WRITE(2,600)AL1BYT
600   FORMAT(F10.2)
      WRITE(2,*) 'w/T='
      WRITE(2,21) DWBYT
21    FORMAT(F10.4)
      DALPHA=0.05
      H=H1-H2
C
C     COMPUTATION FOR C, D, F
C
      PAI=3.14159265
      TERM=EXP(-PAI*AL/T)
      F=((1.+TERM)/(1.-TERM))**2
      V1=EXP(-PAI*AL1/T)
      V2=EXP(-PAI*AL2/T)
      C=F*((1.-V1)/(1.+V1))**2
      D=F*((1.-V2)/(1.+V2))**2
      WRITE(2,*)'  f=',F
        WRITE(2,*)'  c=',C ,'          d=',D
      CONM=-T/PAI

C     COMPUTATION OF F1=F11+F12
C     COMPUTATION OF F11
      F11=0.
      DO I=1,96
      X=XX(I)
      CALL CF11(C,D,F,X,FX11)
      F11=F11+W(I)*FX11
      END DO
      F11=SQRT(2.)*F11

C     COMPUTATION OF F12
      F12=0.
      DO I=1,96
      X=XX(I)
      CALL CF12(C,D,F,X,FX12)
      F12=F12+W(I)*FX12
      END DO
      F12=16*F12
      F1=F11+F12
      WRITE(2,*)
      WRITE(2,3)
      WRITE(2,4)F1
C
C     COMPUTATION OF F2,F3,F4,F5,F6,F7,F8,F9,F10
```

```
F21=0.
F22=0.
F31=0.
F32=0.
F4 =0.
F51=0.
F52=0.
F61=0.
F62=0.
F71=0.
F72=0.
F81=0.
F82=0.
F91=0.
F92=0.
  F101=0.
F102=0.
FI11=0.
FI12=0.
FI21=0.
FI22=0.

DO I=1,96
X=XX(I)

CALL CF21(C,D,F,X,FX21)
F21=F21+W(I)*FX21

CALL CF22(C,D,F,X,FX22)
F22=F22+W(I)*FX22

CALL CF31(C,D,F,X,FX31)
F31=F31+W(I)*FX31

CALL CF32(C,D,F,X,FX32)
F32=F32+W(I)*FX32

CALL CF4(C,D,F,X,FX4)
F4=F4+W(I)*FX4

CALL CF51(C,D,F,X,FX51)
F51=F51+W(I)*FX51

CALL CF52(C,D,F,X,FX52)
F52=F52+W(I)*FX52

CALL CF61(C,D,F,X,FX61)
F61=F61+W(I)*FX61

CALL CF62(C,D,F,X,FX62)
F62=F62+W(I)*FX62

CALL CF71(C,D,F,X,FX71)
F71=F71+W(I)*FX71

CALL CF72(C,D,F,X,FX72)
F72=F72+W(I)*FX72

CALL CF81(C,D,F,X,FX81)
F81=F81+W(I)*FX81
```

```
CALL CF82(C,D,F,X,FX82)
F82=F82+W(I)*FX82

CALL CF91(C,D,F,X,FX91)
F91=F91+W(I)*FX91

CALL CF92(C,D,F,X,FX92)
F92=F92+W(I)*FX92

CALL CF101(C,D,F,X,FX101)
F101=F101+W(I)*FX101

CALL CF102(C,D,F,X,FX102)
F102=F102+W(I)*FX102

CALL CFI11(C,D,F,X,FIX11)
FI11=FI11+W(I)*FIX11

CALL CFI12(C,D,F,X,FIX12)
FI12=FI12+W(I)*FIX12

CALL CFI21(C,D,F,X,FIX21)
FI21=FI21+W(I)*FIX21

CALL CFI22(C,D,F,X,FIX22)
FI22=FI22+W(I)*FIX22

END DO

F21=16*F21
F22=SQRT(2.)*F22
F2=F21+F22

F31=64*SQRT(C/2.)*F31
F32=64*SQRT(C/2.)*F32
F3=F31+F32

F4=4*SQRT(C)*F4

F51=64*SQRT((D-C)/2.)*F51
F52=64*SQRT((D-C)/2.)*F52
F5=F51+F52

F61=4*SQRT(2.*(D-C)) *F61
F62=4.*SQRT(2.*(D-C))*F62
F6=F61+F62

F71=64*SQRT((1-D)/2.)*F71
F72=64*SQRT((1-D)/2.)*F72
F7=F71+F72

F81=4*SQRT(2.*(1-D)) *F81
F82=4.*SQRT(2.*(1-D))*F82
F8=F81+F82

F91=64*SQRT((F-1)/2.)*F91
F92=64*SQRT((F-1)/2.)*F92
F9=F91+F92

F101=4*SQRT(2.*(F-1)) *F101
F102=4.*SQRT(2.*(F-1))*F102
```

```
      F10=F101+F102

      FI11=8*FI11
      FI12=2*FI12
      FI1=2*(FI11+FI12)

      FI21=2*FI21
      FI22=2*FI22
      FI2=2*(FI21+FI22)

      WRITE(2,5)F2
      WRITE(2,*)
      WRITE(2,6)
      WRITE(2,7)F3
      WRITE(2,8)F4
      WRITE(2,*)
      WRITE(2,9)
      WRITE(2,10)F5
      WRITE(2,11)F6
      WRITE(2,*)
      WRITE(2,12)
      WRITE(2,13)F7
      WRITE(2,14)F8
      WRITE(2,*)
      WRITE(2,15)
      WRITE(2,16)F9
      WRITE(2,17)F10

      WRITE(2,*)
      WRITE(2,18)
      WRITE(2,19)FI1
      WRITE(2,20)FI2

      AKS=0.5*(1.+COS(PAI*THICKB/T))
        CALL CEF(W,XX,AKS,CEF1)
      TERM1=CEF1
      AKS=0.5*(1.-COS(PAI*THICKB/T))
        CALL CEF(W,XX,AKS,CEF1)
      FI3=TERM1/CEF1

      BETA=SQRT(BK/(THICKB*AK*(T-THICKB)))
      AF3=1-2./(1.-EXP(-2.*BETA*BL))-FI3/(BETA*(T-THICKB))
      AF4=2.*EXP(-BETA*BL)/(1.-EXP(-2.*BETA*BL))

      WRITE(2,*)'THE FIRST CYCLE STARTS'
      ALPHA2=0.9
22    CONTINUE

      ALPHAD=0.01
      DALPHAD=0.05
      INDEX=1
23    CONTINUE

            ff1=ALPHAD/ALPHA2
            R=(ff1*FI2-F8)/(ff1*FI1-F7)
            CM2BYKH=ALPHA2/(FI2-R*FI1)
            Q1BYKH=CM2BYKH*(R*F1+F2)
            Q2BYKH=CM2BYKH*(F10-R*F9)
      ALPHADN=(Q1BYKH-Q2BYKH)*(AK/DK)*(DLENGTH*DSPAC)/DAREA

      RESIDUE1=ALPHADN-ALPHAD
```

```
      index=index+1
      if(index.gt.200) go to 999
      IF(ABS(RESIDUE1).LT.0.00001) GO TO 25
      ALPHAD=ALPHAD+DALPHA
      IF(RESIDUE1.GT.0.0) GO TO 23

      ALPHADR=ALPHAD-DALPHA
      ALPHADL=ALPHADR-DALPHA

26    ALPHAD=(ALPHADR+ALPHADL)*0.5

      ff1=ALPHAD/ALPHA2
      R=(ff1*FI2-F8)/(ff1*FI1-F7)
      CM2BYKH=ALPHA2/(FI2-R*FI1)
      Q1BYKH=CM2BYKH*(R*F1+F2)
      Q2BYKH=CM2BYKH*(F10-R*F9)
      ALPHADN=(Q1BYKH-Q2BYKH)*(AK/DK)*(DLENGTH*DSPAC)/DAREA

      RESIDUE1=ALPHADN-ALPHAD
      IF(ABS(RESIDUE1).LT.0.00001) GO TO 25
      index=index+1
      if(index.gt.200) go to 999
      IF(RESIDUE1.GT.0.0) GO TO 32
      IF(RESIDUE1.LT.0.0) GO TO 33
32    ALPHADL=ALPHAD
      GO TO 26

33    ALPHADR=ALPHAD
      GO TO 26

25    CONTINUE

      RIGHT=ALPHA2/(FI2-R*FI1)*(R*F1+F2)
      TERM1=(BK/AK)*(1.-EXP(-BETA*BL))/(1.+EXP(-BETA*BL))
      ALPHA11=(FI3+TERM1*(2.-ALPHA2)-RIGHT)/(FI3+TERM1)
      ALPHA12=(AF3+AF4-AF4*ALPHA2)/AF3

      RESIDUE2=ALPHA12-ALPHA11

      ALPHA2=ALPHA2-0.01
      IF(RESIDUE2.GT.0.0) GO TO 22
      ALPHA2=ALPHA2+0.01

      WRITE(2,*)'RESULT AFTER THE FIRST CYCLE'
      WRITE(2,*)ALPHA11,ALPHA12,ALPHA2,ALPHAD
      WRITE(2,*)RESIDUE1,RESIDUE2
      WRITE(2,*)R
      ff1=ALPHAD/ALPHA2
      R=(ff1*FI2-F8)/(ff1*FI1-F7)
      WRITE(2,*)R,FF1

      WRITE(2,*)'THE SECOND CYCLE STARTS'

      ALPHA2=ALPHA2+0.01

122   CONTINUE

      ALPHAD=0.01
      DALPHAD=0.05
      INDEX=1
```

```
123   CONTINUE

            ff1=ALPHAD/ALPHA2
            R=(ff1*FI2-F8)/(ff1*FI1-F7)
            CM2BYKH=ALPHA2/(FI2-R*FI1)
            Q1BYKH=CM2BYKH*(R*F1+F2)
            Q2BYKH=CM2BYKH*(F10-R*F9)

      ALPHADN=(Q1BYKH-Q2BYKH)*(AK/DK)*(DLENGTH*DSPAC)/DAREA

      RESIDUE1=ALPHADN-ALPHAD
      index=index+1
      if(index.gt.200) go to 999
      IF(ABS(RESIDUE1).LT.0.00001) GO TO 125
      ALPHAD=ALPHAD+DALPHA
      IF(RESIDUE1.GT.0.0) GO TO 123

      ALPHADR=ALPHAD-DALPHA
      ALPHADL=ALPHADR-DALPHA

126   ALPHAD=(ALPHADR+ALPHADL)*0.5

      ff1=ALPHAD/ALPHA2
      R=(ff1*FI2-F8)/(ff1*FI1-F7)
      CM2BYKH=ALPHA2/(FI2-R*FI1)
      Q1BYKH=CM2BYKH*(R*F1+F2)
      Q2BYKH=CM2BYKH*(F10-R*F9)
      ALPHADN=(Q1BYKH-Q2BYKH)*(AK/DK)*(DLENGTH*DSPAC)/DAREA

      RESIDUE1=ALPHADN-ALPHAD
      IF(ABS(RESIDUE1).LT.0.00001) GO TO 125
      index=index+1
      if(index.gt.200) go to 999
      IF(RESIDUE1.GT.0.0) GO TO 132
      IF(RESIDUE1.LT.0.0) GO TO 133
132   ALPHADL=ALPHAD
      GO TO 126

133   ALPHADR=ALPHAD
      GO TO 126

125   CONTINUE

      RIGHT=ALPHA2/(FI2-R*FI1)*(R*F1+F2)
      TERM1=(BK/AK)*(1.-EXP(-BETA*BL))/(1.+EXP(-BETA*BL))
      ALPHA11=(FI3+TERM1*(2.-ALPHA2)-RIGHT)/(FI3+TERM1)
      ALPHA12=(AF3+AF4-AF4*ALPHA2)/AF3

      RESIDUE2=ALPHA12-ALPHA11

      ALPHA2=ALPHA2-0.001
      IF(RESIDUE2.GT.0.0) GO TO 122

      ALPHA2=ALPHA2+0.001
      WRITE(2,*)'RESULT AFTER THE SECOND  CYCLE'
      WRITE(2,*)ALPHA11,ALPHA12,ALPHA2,ALPHAD
      WRITE(2,*)RESIDUE1,RESIDUE2
      WRITE(2,*)R

            ff1=ALPHAD/ALPHA2
            R=(ff1*FI2-F8)/(ff1*FI1-F7)
```

```
      WRITE(2,*)R,FF1
WRITE(2,*)'THE THIRD CYCLE STARTS'

ALPHA2=ALPHA2+0.001

1122  CONTINUE

ALPHAD=0.01
DALPHAD=0.05
INDEX=1

1123  CONTINUE

      ff1=ALPHAD/ALPHA2
      R=(ff1*FI2-F8)/(ff1*FI1-F7)
      CM2BYKH=ALPHA2/(FI2-R*FI1)
      Q1BYKH=CM2BYKH*(R*F1+F2)
      Q2BYKH=CM2BYKH*(F10-R*F9)

ALPHADN=(Q1BYKH-Q2BYKH)*(AK/DK)*(DLENGTH*DSPAC)/DAREA

RESIDUE1=ALPHADN-ALPHAD
index=index+1
if(index.gt.200) go to 999
IF(ABS(RESIDUE1).LT.0.00001) GO TO 1125
ALPHAD=ALPHAD+DALPHA
IF(RESIDUE1.GT.0.0) GO TO 1123

ALPHADR=ALPHAD-DALPHA
ALPHADL=ALPHADR-DALPHA

1126  ALPHAD=(ALPHADR+ALPHADL)*0.5

ff1=ALPHAD/ALPHA2
R=(ff1*FI2-F8)/(ff1*FI1-F7)
CM2BYKH=ALPHA2/(FI2-R*FI1)
Q1BYKH=CM2BYKH*(R*F1+F2)
Q2BYKH=CM2BYKH*(F10-R*F9)
ALPHADN=(Q1BYKH-Q2BYKH)*(AK/DK)*(DLENGTH*DSPAC)/DAREA

RESIDUE1=ALPHADN-ALPHAD
IF(ABS(RESIDUE1).LT.0.00001) GO TO 1125
index=index+1
if(index.gt.200) go to 999
IF(RESIDUE1.GT.0.0) GO TO 1132
IF(RESIDUE1.LT.0.0) GO TO 1133
1132  ALPHADL=ALPHAD
GO TO 1126
1133  ALPHADR=ALPHAD
GO TO 1126

1125  CONTINUE

RIGHT=ALPHA2/(FI2-R*FI1)*(R*F1+F2)
TERM1=(BK/AK)*(1.-EXP(-BETA*BL))/(1.+EXP(-BETA*BL))
ALPHA11=(FI3+TERM1*(2.-ALPHA2)-RIGHT)/(FI3+TERM1)
ALPHA12=(AF3+AF4-AF4*ALPHA2)/AF3

RESIDUE2=ALPHA12-ALPHA11

ALPHA2=ALPHA2-0.0001
```

```
      IF(RESIDUE2.GT.0.0) GO TO 1122

      ALPHA2=ALPHA2+0.0001
      WRITE(2,*)'RESULT AFTER THIRD CYCLE'
      WRITE(2,*)ALPHA11,ALPHA12,ALPHA2,ALPHAD
      WRITE(2,*)RESIDUE1,RESIDUE2
      WRITE(2,*)R
      ff1=ALPHAD/ALPHA2
      R=(ff1*FI2-F8)/(ff1*FI1-F7)
      WRITE(2,*)R,FF1
      ALPHA1=(ALPHA11+ALPHA12)*0.5

      WRITE(2,*)'LENGTH OF BLANKET',BL
      WRITE(2,*)'THICKNESS OF BLANKET',THICKB
      WRITE(2,*)'CONDUCTIVITY OF THE BLANKET',BK
      WRITE(2,*)'CONDUCTIVITY OF THE FOUNDATION SOIL',AK
      RATIO=BK/AK
      WRITE(2,*)'BK/AK', RATIO
      QBBYKH=BK/AK*( (1.-EXP(-BETA*BL)) /( (1.+EXP(-BETA*BL))))*
     1      (2.-ALPHA1-ALPHA2)
      WRITE(2,*)'FLOW THROUGH BLANKET Qb/kh',QBBYKH
      Q1BYKH=CM2BYKH*(R*F1+F2)
      Q2BYKH=CM2BYKH*(F10-R*F9)
      Q3BYKH=(1.-ALPHA1)*FI3
      WRITE(2,*)'Q3BYKH', Q3BYKH
      WRITE(2,*)'QBBYKH', QBBYKH
      WRITE(2,*)'Q1BYKH', Q1BYKH
      WRITE(2,*)'Q2BYKH', Q2BYKH
      BLBYT=BL/T
      WRITE(2,*)'BLBYT', BLBYT

      GO TO 1000
999   CONTINUE
      WRITE(2,*)'ITERATION FAILED'
1000  CONTINUE
      STOP
      END

      SUBROUTINE CF11(C,D,F,X,FX11)
      U=1.+X
      TERM=SQRT((2+U*C)*(2+U*D)*(2+U*F)*(2+U))
      FX11=(U**0.5)/TERM
      RETURN
      END

      SUBROUTINE CF12(C,D,F,X,FX12)
      U=1.+X
      TERM=SQRT((4*C+U**2)*(4*D+U**2)*(4*F+U**2)*(4+U**2))
      FX12=1/TERM
      RETURN
      END

      SUBROUTINE CF21(C,D,F,X,FX21)
      U=(1.+X)**2
      TERM=SQRT((4.+C*U)*(4.+D*U)*(4.+F*U)*(4.+U))
      FX21=1/TERM
      RETURN
      END

      SUBROUTINE CF22(C,D,F,X,FX22)
      U=1.+X
```

```
TERM=SQRT(  (2*C+U)*(2*D+U)*(2*F+U)*(2+U)  )
FX22=(U**0.5)/TERM
RETURN
END

SUBROUTINE CF31(C,D,F,X,FX31)
U=(1.+X)**2
P1=C*U
TERM=SQRT((8.*C-P1)*(8.*D-P1)*(8.-P1)*(8.*F-P1)  )
FX31=1/TERM
RETURN
END

SUBROUTINE CF32(C,D,F,X,FX32)
U=1.+X
P2=8.*C-C*U**2
TERM=SQRT(P2*(8*D-P2)*(8-P2)*(8*F-P2))
FX32=1/TERM
RETURN
END

SUBROUTINE CF4(C,D,F,X,FX4)
U=1.+X
P3=4*C-C*U**2
TERM=(4*D-P3)*(4-P3)*(4*F-P3)
FX4=SQRT(P3/TERM)
RETURN
END

SUBROUTINE CF51(C,D,F,X,FX51)
U=X+1.
P4=(8*C+(D-C)*U**2)
TERM=SQRT(P4*(8*D-P4)*(8-P4)*(8*F-P4)  )
FX51=1/TERM
RETURN
END

SUBROUTINE CF52(C,D,F,X,FX52)
U=1.+X
P5=(8*D-(D-C)*U**2)
TERM=SQRT(P5*(P5-8*C)*(8*F-P5)*(8-P5)  )
FX52=1/TERM
RETURN
END

SUBROUTINE CF61(C,D,F,X,FX61)
U=X+1.
P6=(8*C+(D-C)*U**2)
TERM=(8*D-P6)*(8*F-P6)*(8-P6)
FX61=SQRT(P6/TERM)
RETURN
END

SUBROUTINE CF62(C,D,F,X,FX62)
U=1.+X
P7=(8*D-(D-C)*U**2)
TERM=SQRT((P7-8*C)*(8*F-P7)*(8-P7))
FX62=SQRT(P7)/TERM
RETURN

END
```

```
SUBROUTINE CF71(C,D,F,X,FX71)
U=X+1.
P8=(8*D+(1-D)*U**2)
TERM=SQRT(P8*(P8-8*C)*(8-P8)*(8*F-P8) )
FX71=1/TERM
RETURN
END

SUBROUTINE CF72(C,D,F,X,FX72)
U=1.+X
P9=(8-(1-D)*U**2)
TERM=SQRT(P9*(P9-8*C)*(P9-8*D)*(8*F-P9) )
FX72=1/TERM
RETURN
END

SUBROUTINE CF81(C,D,F,X,FX81)
U=X+1.
P10=(8*D+(1-D)*U**2)
TERM=SQRT((P10-8*C)*(8-P10)*(8*F-P10) )
FX81=SQRT(P10)/TERM
RETURN
END

SUBROUTINE CF82(C,D,F,X,FX82)
U=1.+X
P11=(8-(1-D)*U**2)
TERM=(P11-8*C)*(P11-8*D)*(8*F-P11)
FX82=SQRT(P11/TERM)
RETURN
END

SUBROUTINE CF91(C,D,F,X,FX91)
U=X+1.
P12=(8+(F-1)*U**2)
TERM=SQRT(P12*(P12-8*C)*(P12-8*D)*(8*F-P12) )
FX91=1/TERM
RETURN
END

SUBROUTINE CF92(C,D,F,X,FX92)
U=1.+X
P13=(8*F-(F-1)*U**2)
TERM=SQRT(P13*(P13-8*C)*(P13-8*D)*(P13-8) )
FX92=1/TERM
RETURN
END

SUBROUTINE CF101(C,D,F,X,FX101)
U=X+1.
P14=(8+(F-1)*U**2)
TERM=(P14-8*C)*(P14-8*D)*(8*F-P14)
FX101=SQRT(P14/TERM)
RETURN
END

SUBROUTINE CF102(C,D,F,X,FX102)
U=1.+X
P15=(8*F-(F-1)*U**2)
TERM=SQRT((P15-8*C)*(P15-8*D)*(P15-8))
```

```
      FX102=SQRT(P15)/TERM
      RETURN
      END

      SUBROUTINE CFI11(C,D,F,X,FIX11)
      U=(X+1.)**2
      P16=(4*F+U)
      TERM=SQRT(P16*(P16-4*C)*(P16-4*D)*(P16-4))
      FIX11=1/TERM
      RETURN
      END

      SUBROUTINE CFI12(C,D,F,X,FIX12)
      P=(X+1.)**2.
      TERM=SQRT((F*P+4)*((F-C)*P+4)*((F-D)*P+4)*((F-1)*P+4))
      FIX12=P/TERM
      RETURN
      END

      SUBROUTINE CFI21(C,D,F,X,FIX21)
      P=(X+1.)**2.
      TERM=SQRT((4*(F-C)+P)*(4*(F-D)+P)*(4*(F-1)+P))
      FIX21=SQRT(4*F+P)/TERM
      RETURN
      END

      SUBROUTINE CFI22(C,D,F,X,FIX22)
      P=(X+1.)**2.
      TERM=SQRT(((F-C)*P+4)*((F-D)*P+4)*((F-1)*P+4))
      FIX22=SQRT(F*P+4)/TERM
      RETURN
      END

      SUBROUTINE CEF(W,XX,AKS,CEF1)
      DIMENSION W(96), XX(96)
      PAI=3.141592654
      SUM=0.0
      DO 10 I=1,96
      THETA=PAI/4.*(1.+XX(I))
      TERM=0.25*PAI/SQRT(1.-AKS*SIN(THETA)**2)
      SUM=SUM+TERM*W(I)
10          CONTINUE
      CEF1=SUM
      RETURN
      END

      SUBROUTINE CIEF(W,XX,AKS,PHAI,CIEF1)
      DIMENSION W(96), XX(96)
      PAI=3.141592654
      SUM=0.0
      DO 10 I=1,96
      THETA=PHAI/2.*(1.+XX(I))
      TERM=0.5*PHAI/SQRT(1.-AKS*SIN(THETA)**2)
      SUM=SUM+TERM*W(I)
10          CONTINUE
      CIEF1=SUM
      RETURN
      END
```

APPENDIX – 15

GAU.DAT

0.0325506144	0.0325161187	0.0324471637	0.0323438225	0.0322062047
0.0320344562	0.0318287588	0.0315893307	0.0313164255	0.0310103325
0.0306713761	0.0302999154	0.0298963441	0.0294610899	0.0289946141
0.0284974110	0.0279700076	0.0274129627	0.0268268667	0.0262123407
0.0255700360	0.0249006332	0.0242048417	0.0234833990	0.0227370696
0.0219666444	0.0211729398	0.0203567971	0.0195190811	0.0186606796
0.0177825023	0.0168854798	0.0159705629	0.0150387210	0.0140909417
0.0131282295	0.0121516046	0.0111621020	0.0101607705	0.0091486712
0.0081268769	0.0070964707	0.0060585455	0.0050142027	0.0039645543
0.0029107318	0.0018539607	0.0007967920		
0.0325506144	0.0325161187	0.0324471637	0.0323438225	0.0322062047
0.0320344562	0.0318287588	0.0315893307	0.0313164255	0.0310103325
0.0306713761	0.0302999154	0.0298963441	0.0294610899	0.0289946141
0.0284974110	0.0279700076	0.0274129627	0.0268268667	0.0262123407
0.0255700360	0.0249006332	0.0242048417	0.0234833990	0.0227370696
0.0219666444	0.0211729398	0.0203567971	0.0195190811	0.0186606796
0.0177825023	0.0168854798	0.0159705629	0.0150387210	0.0140909417
0.0131282295	0.0121516046	0.0111621020	0.0101607705	0.0091486712
0.0081268769	0.0070964707	0.0060585455	0.0050142027	0.0039645543
0.0029107318	0.0018539607	0.0007967920	0.0162767448	0.0488129851
0.0812974954	0.1136958501	0.1459737146		
0.1780968823	0.2100313104	0.2417431561	0.2731988125	0.3043649443
0.3352085228	0.3656968614	0.3957976498	0.4254789884	0.4547094221
0.4834579739	0.5116941771	0.5393881083	0.5665104185	0.5930323647
0.6189258401	0.6441634037	0.6687183100	0.6925645366	0.7156768123
0.7380306437	0.7596023411	0.7803690438	0.8003087441	0.8194003107
0.8376235112	0.8549590334	0.8713885059	0.8868945174	0.9014606353
0.9150714231	0.9277124567	0.9393703397	0.9500327177	0.9596882914
0.9683268284	0.9759391745	0.9825172635	0.9880541263	0.9925439003
0.9959818429	0.9983643758	0.9996895038		
-0.0162767448	-0.0488129851	-0.0812974954	-0.1136958501	0.1459737146
-0.1780968823	-0.2100313104	-0.2417431561	-0.2731988125	-0.3043649443
-0.3352085228	-0.3656968614	-0.3957976498	-0.4254789884	-0.4547094221
-0.4834579739	-0.5116941771	-0.5393881083	-0.5665104185	-0.5930323647
-0.6189258401	-0.6441634037	-0.6687183100	-0.6925645366	-0.7156768123
-0.7380306437	-0.7596023411	-0.7803690438	-0.8003087441	-0.8194003107
-0.8376235112	-0.8549590334	-0.8713885059	-0.8868945174	-0.9014606353
-0.9150714231	-0.9277124567	-0.9393703397	-0.9500327177	-0.9596882914
-0.9683268284	-0.9759391745	-0.9825172635	-0.9880541263	-0.9925439003
-0.9959818429	-0.9983643758	-0.9996895038		