A STUDY ON METHOD OF FRAGMENTS FOR SEEPAGE ANALYSIS UNDER HYDRAULIC STRUCTURE

A DISSERTATION

submitted in partial fulfillment of the
requirements for the award of the degree
of
MASTER OF TECHNOLOGY
in
WATER RESOURCES DEVELOPMENT

By

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CANDIDATE'S DECLARATION

I hereby declare that work which is presented in this Dissertation entitled, **"A STUDY ON METHOD OF FRAGMENTS FOR SEEPAGE ANALYSIS UNDER HYDRAULIC STRUCTURE"**, in partial fulfilment of the requirement for the award of the degree of MASTER OF TECHNOLOGY IN WATER RESOURCES DEVELOPMENT, submitted in Water Resources Development Training Centre, Indian Institute of Technology, Roorkee, is a record of my own work carried out during the period from July 16th, 2001 to December 10th, 2001 under the supervision of **Dr. G.C. Mishra**, Professor, WRDTC, Indian Institute of Technology Roorkee, India.

The matter embodied in this dissertation has not been submitted by me for the award of any other degree or diploma.

(MUHAMAD SABRUDIN)

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

Place : Roorkee
Dated : December, 10, 2001

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CONTENTS

LIST OF SYMBOLS

The following symbols are used in this dissertation

C	= constant;
e	= exponential
d_1	= downstream sheet piles;
d_2	= upstream sheet piles;
$F(\vartheta, m)$	= incomplete elliptic integral of first kind with amplitude ϑ and modulus m
$F(\pi/2, m)$	= complete elliptic integral of first kind with amplitude ϑ and modulus m
I_E	= exit gradient;
g	= the gravitational acceleration;
h	= difference in total heads at upstream and downstream boundaries;
i	= the imaginary unit($\sqrt{-1}$);
k	= coefficient of permeability;
k_x, k_y	= principal coefficient of permeability in the directions x and y,
L_1,L_2,L_3	= floor length;
M	= complex constants;
N	= complex constants;
P	= uplift pressure
R	= scour depth
s_1	= upstream sheet pile;
s_2	= downstream sheet pile;
w	= complex variable;
t	= transformation plane;

z = complex variable = x + iy;

Z_A, Z_B, Z_C = Z coordinate corresponding to point A, B, C respectively

x = horizontal co-ordinate;

y = vertical co-ordinate;

α = sloping floor angle in units of π; in chapter iv, inclined sheet pile angle in units π in chapter III

$\alpha_1, \alpha_2, \alpha_3, \alpha_4$ = factor of velocity potential

B = complete beta function;

B_t = incomplete beta function;

Γ = complete gamma function;

γ = angle made by embedded length of sheet pile with downstream base, in unit of π in chapter III;

γ_w = the unit weight of water;

u = velocity in x-direction;

v = velocity in y-direction;

q = quantity of seepage;

b = clear distance between upstream and downstream floor

ϕ = velocity potential function; and

ψ = stream function;

λ, μ = Cartesian coordinates;

η = $\dfrac{\lambda}{\left(\dfrac{k\lambda}{k\mu}\right)^{1/2}}$

SYNOPSIS

The techniques in the investigation of problem of flow of subsoil water below hydraulic structure are commonly employed for the solution of Laplace's equation. The technique consists of:

1. Graphical method
2. Hydraulic models
3. Electrical analogy models
4. Analytical method.

The analytical method aims at the solution of Laplace's equation mathematically by Conformal Mapping transformation. In this method the parameters of transformation are obtained from the boundary conditions.

An approximate analytical method of solution for any confined flow system with finite depth of porous medium, directly applicable to design, known as "METHOD OF FRAGMENTS" has been suggested by Pavlovsky, vide Harr, 1962 to solve the computation of seepage under weir with multiple pilling.

The fundamental assumption of method of fragment is that equipotential lines at various critical parts of flow region can be approximated by straight vertical lines that divide the region into section or fragment.

The accuracy of method of fragment depends on the shape of seepage region involved in the manner of fragmentation. Many of the practical working formulae available in the literature are worked out by the Method of Fragment.

The accuracy of method of fragment in computation of seepage under hydraulic structure resting on finite depth is investigated with many types of conditions and the result of designing of hydraulic structure is compared with solution obtained by vigorous method such as stepped weir on permeable foundation of infinite depth, case of a sheet pile in finite depth, case of a weir with two sheet pile.

It is found that the method of fragments is accurate and can be used conveniently for solving complex confined seepage under hydraulic structure founded on porous medium of finite depth.

CHAPTER – I

INTRODUCTION

Design weir on permeable foundation involves diverse fields such as sub-surface flow, surface flow and economic consideration. The stability of weir demands an upstream sheet pile and downstream sheet pile, to prevent slipping of the soil under the weir to the anticipated scour holes at the upstream and downstream reaches.

The present study was undertaken to develop an algorithm for optimal design of a weir using method of fragments for seepage analysis under hydraulic structure such as weir with a slopping floor and having a number of sheet piles in various combinations.

The safety requirement such as length of structure in Lane's theory and depth of sheet pile in scour depth calculation were expressed in the form of constraints. The uplift pressure and the maximum exit gradient have been obtained using the Schwarz-Christoffel conformal mapping. The transformation parameters have been evaluated solving the implicit transformation equation using method of fragments.

The accuracy of method of fragments in computation of seepage under hydraulic structure resting on finite depth is investigated with many types of conditions and the result of designing of hydraulics structure is compared with solution obtained by rigorous method, such as: a stepped weir on permeable foundation on finite depth with two sheet pile and weir with an inclined sheet pile.

In the process of study of method of fragments, we cannot avoid what we call 'trial and error'. Due to many computations that must be done, it will consume long time if it is done manually. In this present study, based on the analytical solution using conformal mapping technique, FORTRAN Computer Programs aided for computation of seepage have been developed.

REVIEW OF LITERATURE

2.1 THEORY OF SEEPAGE

The discharge velocity is a quantity of fluid that percolates through a unit of total area of the porous medium in a unit time

$$Q = m\overline{V}A \qquad \text{(II.1.1)}$$

where:

Q = quantity of seepage

$\overline{V}$ = the seepage velocity

$m = A_p/A$ = effective ratio of the area of pores, A_p to the total area

$m\overline{v}$ = discharge velocity

From fluid mechanics, for steady flow of non-viscous incompressible fluids, Bernoulli's equation is:

$$\frac{P}{\gamma_w} + z + \frac{\overline{V}^2}{2g} = \text{Constant} = h \qquad \text{(II.1.2)}$$

where:

h = total head or hydraulic head

$\overline{V}$ = seepage velocity

p = pressure

In ground water flow, to account for the loss of energy due to the viscous resistance within the individual pores Bernoulli's equation is taken as:

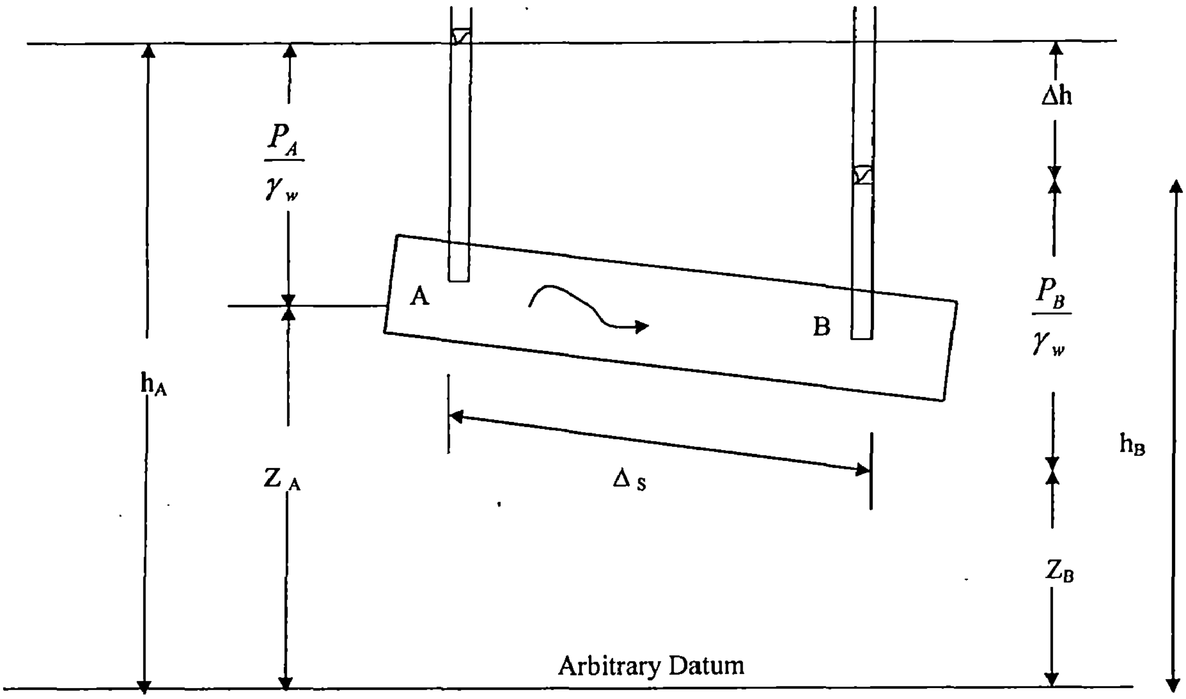


Fig. (2.1) Ground Water Flow

$$\frac{P_A}{\gamma_w} + Z_A + \frac{V_A^2}{2g} = \frac{P_B}{\gamma_w} + Z_B + \frac{V_B^2}{2g} + \Delta h \qquad \text{(II.1.3)}$$

where :

Δ h = the total head loss (energy per unit weight of fluid)

The ratio : $i = -\lim_{\Delta P \to 0} \frac{\Delta h}{\Delta s}$

$$i = -\frac{dh}{ds}$$

i = hydraulic gradient

In most ground water problems, the velocity heads (kinetic energy) are so small that, they can be neglected. Bernoulli's equation become:

$$\frac{P_A}{r_w} + z_A = \frac{P_B}{r_w} + z_B + \Delta h \qquad \text{(II.1.4)}$$

and total head at any point in the flow domain, is

$$h = \frac{P}{\gamma_w} + z \qquad \text{(II.1.5)}$$

Darcy's Law

The flow through porous media can be represent with equation:

$$V = ki = -k\frac{dh}{ds} \qquad \text{(II.1.6)}$$

This law is called Darcy's law.

If demonstrates a linear dependency between the hydraulic gradient and discharge velocity, k = coefficient of permeability.

The Darcy's law is applicable to laminar flow only.

General Hydrodynamic Equation, Velocity Potential:

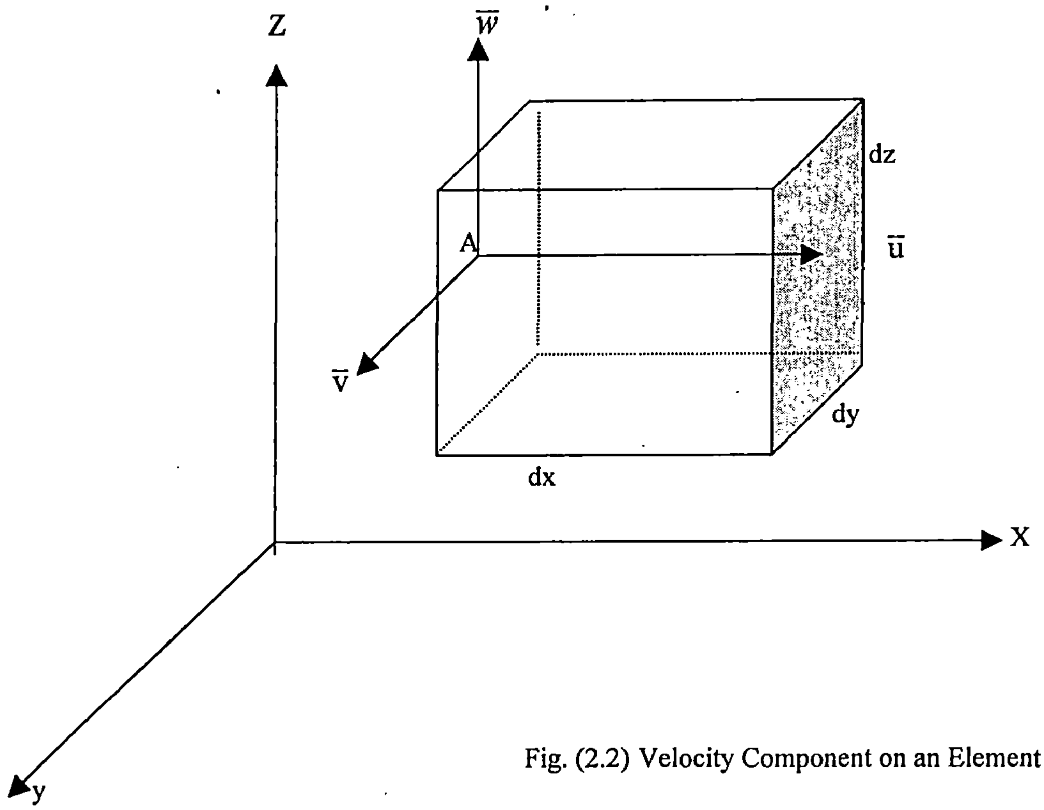


Fig. (2.2) Velocity Component on an Element of Soil

u, v *and* w are the components of the discharge velocity at the point in the fluid A (x, *y*, *z*) at time t.

From, Darcy's law : $V = -k\dfrac{dh}{ds}$, we obtain

$$u = -k\frac{dh}{dx}; \qquad v = -k\frac{dh}{dy}; \qquad \text{and} \qquad w = -k\frac{dh}{dz}$$

If the fluid and flow medium are both incompressible, the total gain of fluid per unit must be indentically zero. Hence,

$$\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0 \tag{II.1.7}$$

This equation is the equation of continuity in three dimension.

It is of the almost convenience in groundwater flow in to introduce the velocity potential (ϕ), defined as:

$$\phi(x, y, z) = -k\left(\frac{P}{\gamma_w} + z\right) + c = -kh + c \tag{II.1.8}$$

where c is an arbitrary constant. If $c = 0$,

$$\phi = -kh$$

$$u = -k\frac{dh}{dx}, \qquad v = -k\frac{dh}{dy} \qquad \text{and} \qquad w = -k\frac{dh}{dz}$$

become

$$u = \frac{\partial\phi}{\partial v}, \qquad v = \frac{\partial\phi}{\partial y} \text{ and } \qquad w = \frac{\partial\phi}{\partial z} \tag{II.1.9}$$

Equation (II.1.1) and (II.1.9) represent the generalized Darcy's law which provide the dynamical framework for all investigation into groundwater flow.

Substituting (II.1.9)in (II.1.7) we obtain, the Laplace equation

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial v^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \qquad \text{(II.1.10)}$$

Equation (II.1.10) indicates that for conditions of steady-state, laminar flow, the form of the groundwater motion can be completely determined by solving one equation, subject to the boundary condition of the flow domain for two dimensional flow equation:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \qquad \text{(II.1.11)}$$

Graphically, equation (II.1.11) can be represented by two sets of curves that intersect at right angles. The curves are flowlines / streamlines and equipotential lines. The combine representation of two sets of line is called a flow net.

Stream Function (ψ):

Continuity equation (II.1.11) becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{(II.1.12)}$$

In groundwater flow literature the function ψ (x, y) is called the stream function, and is defined as

$$u = \frac{\partial\psi}{\partial y} \qquad \text{v} = -\frac{\partial\psi}{\partial x} \qquad \text{(II.1.13)}$$

Combining equation (II.1.12) and (II.1.13), we obtain

$$\frac{\partial^2\psi}{\partial x\ \partial y} - \frac{\partial^2\psi}{\partial y\ \partial x} = 0 \qquad \text{(II.1.14)}$$

Equating the respective potential and stream functions of ϕ & ψ

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \qquad \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

Hence,

$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \qquad \text{(II.1.15)}$$

An important distinction between the ϕ and ψ functions lies in the fact that the ϕ functions exist only for irrotational flow. A particle of fluid is said to have zero net rotation or to be irrotational if the circulation, the line integral of the tangential velocity taken around the particle, is zero.

If should be noted that within a given region of flow the streamlines and equipotential lines are unique. That is, considering the total differential.

$$d\psi = \frac{\partial \psi}{\partial x}\, dv + \frac{\partial \psi}{\partial y} dy \qquad \text{(II.1.16)}$$

From the Cauchy - Rieman equations

$$\psi = \int\left(\frac{\partial \psi}{dx} dy - \frac{\partial \psi}{\partial y} dx\right)$$

and $$\phi = \int\left(\frac{\partial \psi}{\partial y} dx - \frac{\partial \psi}{\partial x} dy\right)$$

A combination of the function ϕ and ψ, called the complex potential and defined by

$$w = \phi + i\psi \qquad \text{(II.1.17)}$$

BOUNDARY CONDITIONS

In steady -state flow of groundwater through homogeneous soil, four types of boundaries are encountered. These are:

1) Impervious Boundary

- Upper surface of a soil stratum or rock (the lowest stream lines) such as: A-B in figure 2.3
- The bottom contour of the impervious structure (Upper flow line) such as: 1-4 in figure 2.3

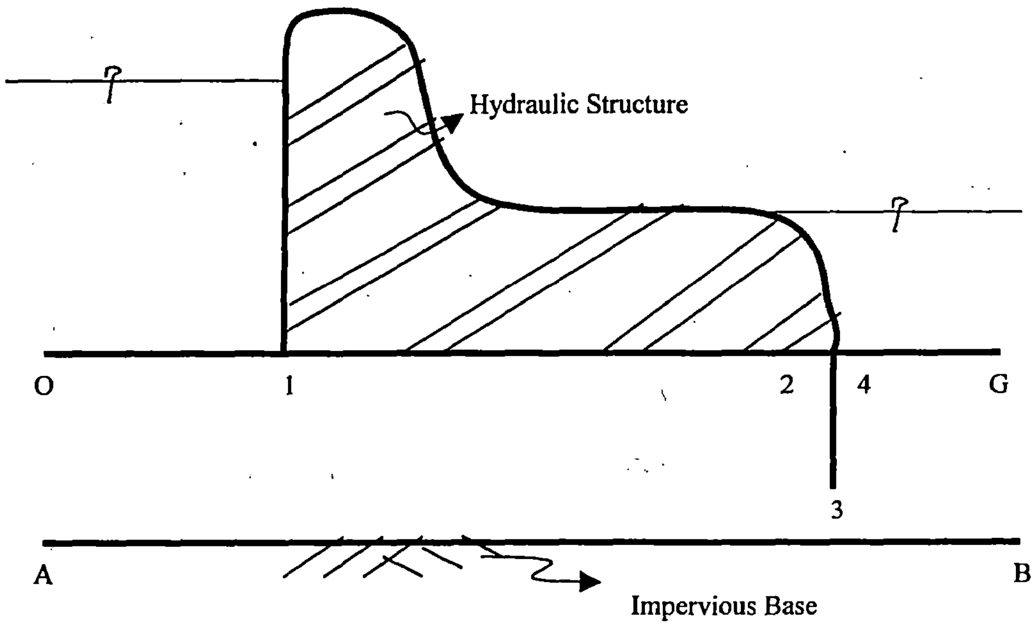


Fig. (2.3) Boundary Conditions in Weir with Vertical Sheet Pile

2) Boundaries of the reservoirs

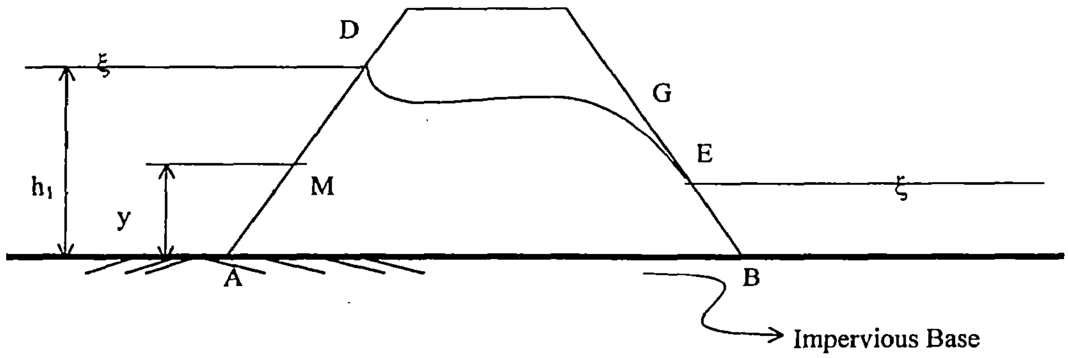


Fig. (2.4) Boundary Condition in Earth Dam

01 in figure (2.3) and AG in figure (2.4) and

AD and EB are equipotential lines.

Along boundary AD pressure is:

$$p = \gamma_w (h_1 - y)$$

and potential is:

$$\phi = -kh_1 + c$$

3) Surface of seepage
GE in figure (2.4) is surface of seepage.
If represent a boundary where the seepage leaving the flow region enters a zone free of both liquid and soil.
The pressure on this surface is both constant and atmospheric and the surface is neither an equipotential live nor a streamline.

4) Line of seepage (free surface, depression curve) is a stream line: along this line. $\phi + ky = \text{constant}$, DG is a phreatic line.

Flow net

Flow not is graphical representation of the family of streamlines and their corresponding equipotential lines within a flow region. If
Nf = the number of flow channels,
Ne = the number of equipotential drops along each of the channels, the quantity of seepage is given by $q = \frac{Nf}{Ne}.kh$ (II.1.18)

where h = total loss in head

$$h = Ne.\, \Delta h$$

2.2 Creep Theory

- **Bligh's Creep Theory**

Consider a horizontal floor of length (L) metre in fig. (2.5), impounding a depth H metre of water. The loss of head per metre of floor, H/L is called the hydraulic gradient. Bligh called it percolation coefficient. Mathematically this value is written as bellow.

$$C = \frac{H}{b1 + 2d1 + b2 + 2d2 + b3} \qquad (II.2.1)$$

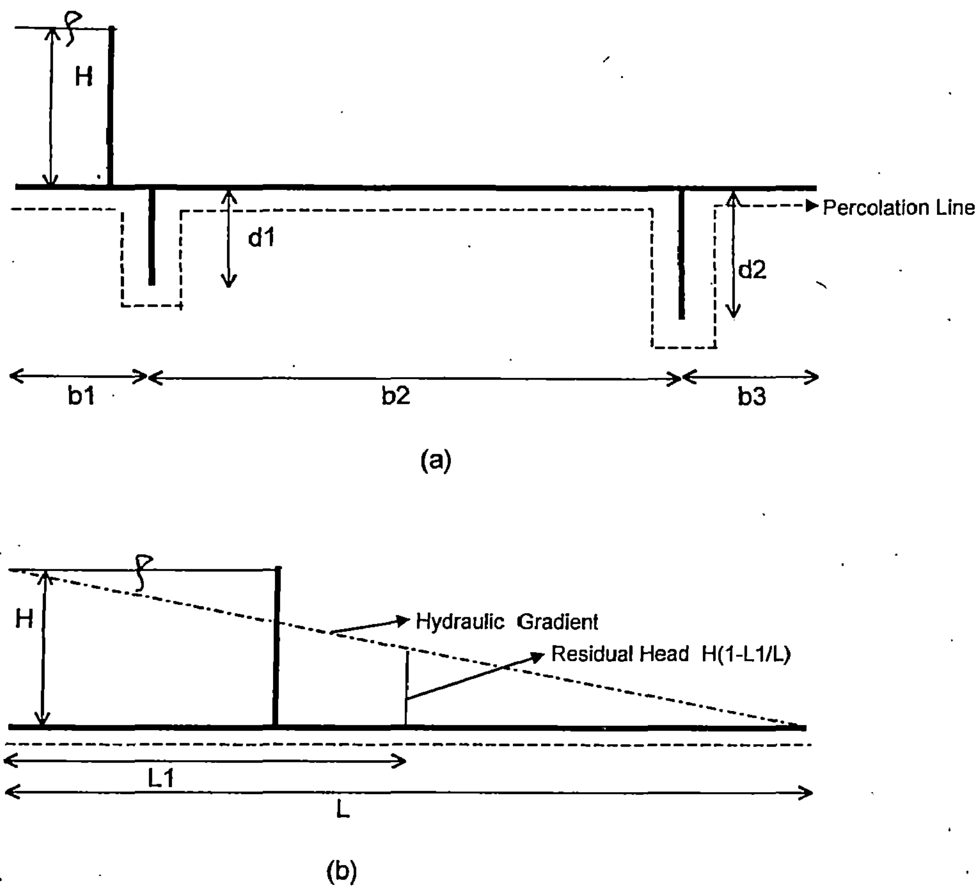


Fig (2.5) Bligh 's Creep Theory- Definition Sketch

The dissipation of the head at any point is supposed to be proportional to the length traveled. At a distance L1 the residual head would be H-(H/L) L1.

Bligh presumed that percolation water creeps along the contact of the base profile of the structure with sub soil and losses head in proportion to the length of its travel. No discrimination was made by him in horizontal and vertical creeps in assessing their effectiveness against undermining or piping. Because of its simplicity, Bligh's theory found general acceptance.

- **Lane's Weighted Creep Theory**

Lane approached the problem by making a statistical examination of large number of structure on pervious foundations. He developed the weighted creep theory which in effect may be called "Bligh's Creep Theory corrected for vertical contacts"

According to Lane's Weighted Creep Theory, the weighted creep length (L_W) is given as

$$L_W = 1/3\, N + V \qquad \text{(II.2.2)}$$

where;

N is the sum of the horizontal contacts and all the sloping contacts less than 45 degree.

V is the sum of all the vertical contacts plus the sloping contacts greater than 45 degree.

While Lane's weighted creep theory is an improvement over Bligh's creep theory, it too suffers from the limitations of an empirical approach.

2.3 Design Sheet Pile with Consideration of Scour Depth

The depth of scour (R) in metre in relation to the discharge in cum per metre run 'q' and silt factor 'f' is

$$R = 1.35\,(q^2/f)^{1/3} \qquad \text{(II.3.1)}$$

The relation between R and q for different values of 'f' are given in Figure (2.6)

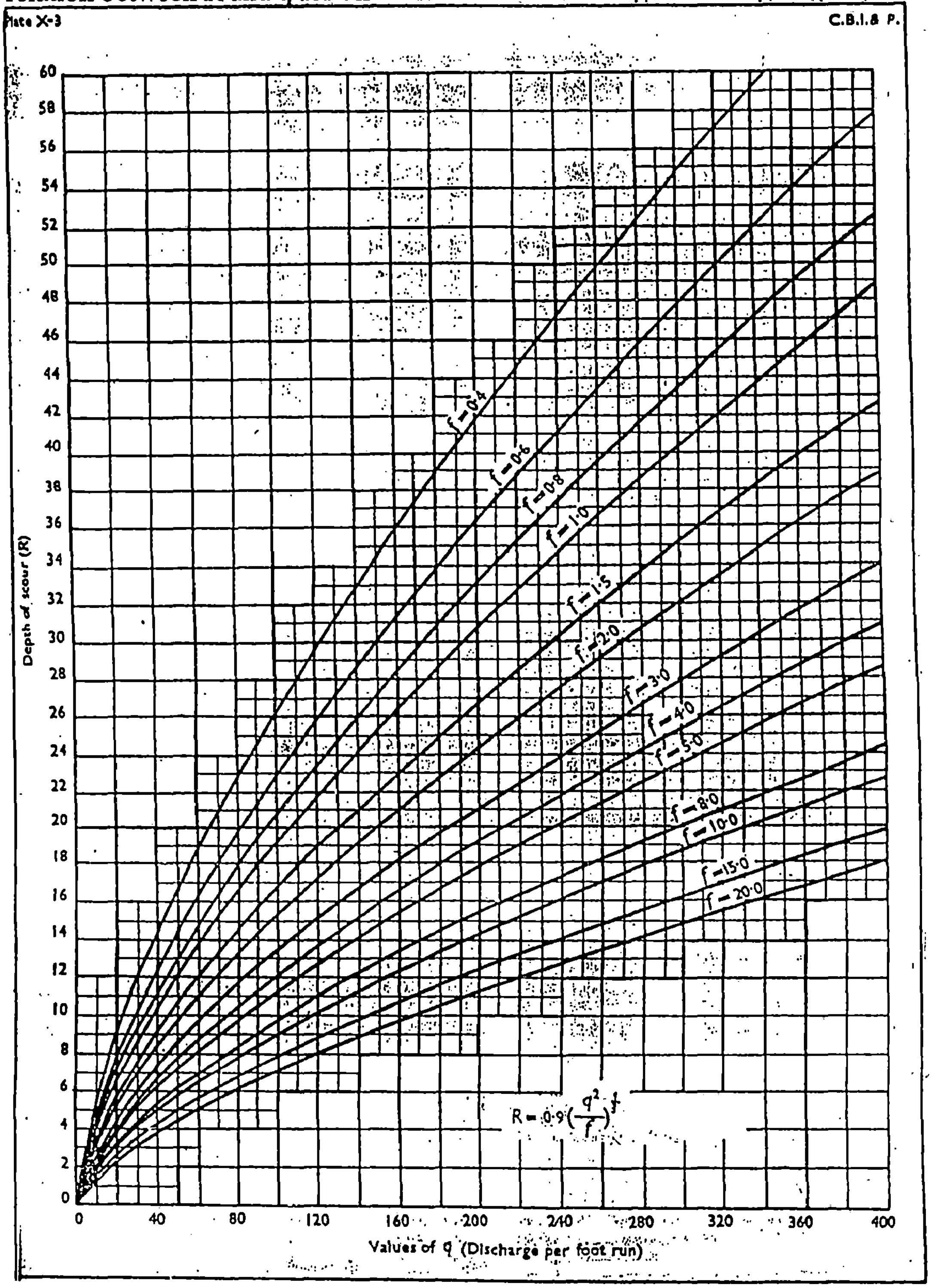


Figure 2.6 The relation between R and q for different values of silt factor 'f'

According to Lacey, the scour can be divided in four classes as given in Table 2.1.

Class A	Straight reach	1.25 R
Class B	Moderate bend	1.50 R
Class C	Severe bend	1.75 R
Class D	Right angled bend	2.00 R

Table (2.1). Recommended Scour Depths

Class 'A' is likely to occur any where just below the loose aprons. Class 'B' is likely to occur anywhere along the aprons of guide banks in the straight reach. Class 'C' and 'D' may occur at or below the noses of guide banks of loose aprons.

For the design of piles it is sufficient to provide the upstream pile line to a depth from 1.0 R to 1.25 R and the downstream piles line to a depth from 1.25 R to 1.50R.

2.4 ANISOTROPIC POROUS MEDIA

The theory of flow of fluids through anisotropic porous media is presented by Ferrandon (1948), Scheidegger (1957), Polubarinova-Kochina (1962)[9], Harr (1962)[6] and Marcus (1962).

Problems concerned with anisotropic porous media may be solved by transforming the actual anisotropic flow region into a fictitious isotropoc region by an appropriate co-ordinate transformation. The required scale for transformation of a two-dimensional flow problem is obtained from the equation of continuity as follows (Harr,1962);

The equation of continuity for a two-dimensional steady flow is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \text{(II.4.1)}$$

in which

u, v = discharge velocities in x and y directions, respectively. From the generalized

Darcy's law,

$$u = -k_x \frac{\partial h}{\partial x} \tag{II.4.2}$$

and

$$v = -k_y \frac{\partial h}{\partial y} \tag{II.4.3}$$

in which

k_x, k_y = principal coefficients of permeability in x and y directions, respectively ,

$h = \frac{p}{\gamma_w} + y$;

p = pressure ;

γ_w = unit weight of water ;

x,y = co-ordinates.

Substituting the values of u and v in Eq. II.4.3,

$$kx \frac{\partial^2 h}{\partial\left[\left(\frac{k_y}{k_x}\right)x^2\right]} + ky \frac{\partial^2 h}{\partial y^2} = 0 \tag{II.4.4}$$

Substitution of $X = x\left(\frac{k_y}{k_x}\right)^{1/2}$ reduce Eq. (II.4.4)

$$\frac{\partial^2 h}{\partial X^2} + \frac{\partial^2 h}{\partial y^2} = 0 \tag{II.4.5}$$

In a similar manner, substituting $Y = y\left(\frac{k_x}{k_y}\right)^{1/2}$, it is found that

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \tag{II.4.6}$$

Thus, by choosing one of the above two scales of transformation, a homogeneous anisotropic region can be converted into a fictitious isotropic region for which the Laplace equation is valid.

Examples of transformation of an anisotropic flow domain to an isotropic domain can be found in the books by Harr (1962)[6], Polubarinova-Kochina (1962)[9], and De Wiest (1965)[5]. The mathematical procedure for transformation follows the example of Polubarinova-Kochina and is given below.

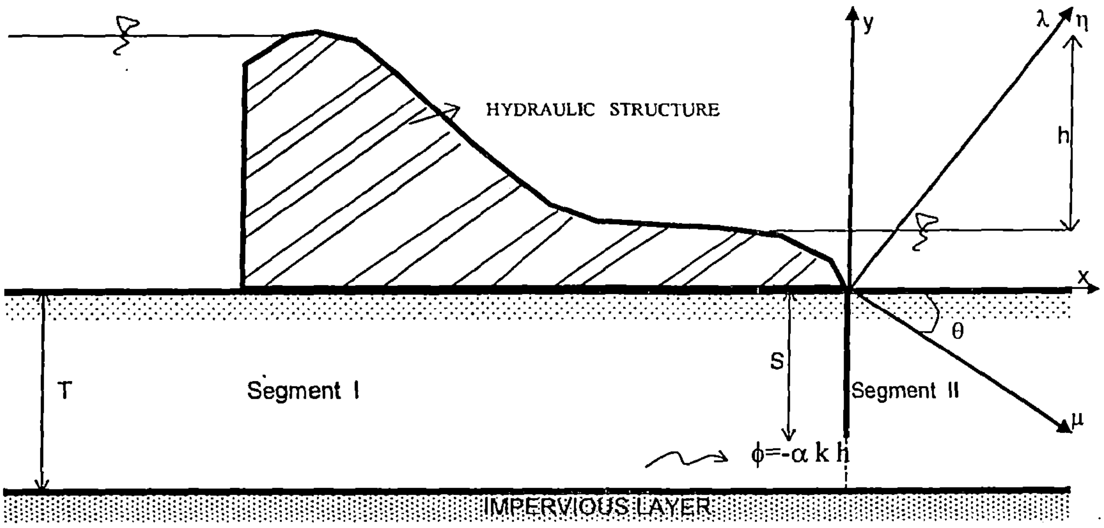


Fig. 2.7 Flat Bottomed Weir an Anisotropic Medium

Fig. (2.7) represents in $\bar{x}, \bar{y}$ plane a flat bottomed weir having a vertical sheet pile and resting on an anisotropoic porous medium. The direction of maximum coefficient of permeability makes an angle θ with horizontal as shown. The direction of co-ordinate axes, μ and λ, are chosen to coincide with the directions of maximum and minimum coefficients of permeability, respectively. The correspondence between these co-ordinate systems in given by

$$\mu = \bar{x} \cos\theta - \bar{y} \sin\theta \qquad \text{(II.4.7)}$$

$$\lambda = \bar{x} \sin\theta - \bar{y} \cos\theta \qquad \text{(II.4.8)}$$

In order to transform the anisotropic flow region to isotropic one, an expansion in the direction of λ is necessary. As stated earlier the co-ordinate in the direction of λ should be expanded using the multiplying factor $\left(\frac{k_\mu}{k_\lambda}\right)^{1/2}$

in which

k_μ and k_λ = principal coefficients of permeability in the directions of μ and λ, respectively.

Designating $\eta = \lambda\left(\frac{k_\mu}{k_\lambda}\right)^{1/2}$ and replacing the value of λ from Eq.(II.4.8)

$$\eta = \left(\frac{k_\mu}{k_\lambda}\right)^{1/2} (\bar{x} \sin\theta + \bar{y} \cos\theta) \qquad \text{(II.4.9)}$$

Thus, the physical anisotropic flow domain in $\bar{x}$, $\bar{y}$ plane is to be transformed form $\bar{x}$, $\bar{y}$ plane to fictitious isotropic flow domain in μ, η plane by using Eqs. II.4.7 and II.4.9. The straight line $\bar{y} = 0$ is transformed to a straight line in μ, η plane, governed by the equation

$$\frac{\eta}{\mu} = \left(\frac{k_\mu}{k_\lambda}\right)^{1/2} \tan\theta \qquad \text{(II.4.10)}$$

and the straight line $\bar{x} = 0$ is transformed to a straight line given by the equation

$$\frac{\eta}{\mu} = -\left(\frac{k_\mu}{k_\lambda}\right)^{1/2} \cot\theta \qquad \text{(II.4.11)}$$

if $\bar{s}$ is the length of a vertical sheet pile in anisotropic medium, its new length, s, in fictitious isotropic medium is given by

$$s = \bar{s}\left(\sin^2\theta + \frac{k_\mu}{k_\lambda}\sin^2\theta\right)^{1/2} \qquad \text{(II.4.12)}$$

A horizontal blanket of width $\bar{b}$ in the physical flow domain is transformed in the fictitious isotropic flow domain to a blanket of width b which is given by

$$b = \bar{b}\left(\cos^2\theta + \frac{k_\mu}{k_\lambda}\sin^2\theta \right)^{1/2} \quad \text{(II.4.13)}$$

A mathematical solution which considers the general anisotropic nature of the porous medium is available for the case of a single sheet pile embedded in a semi-infinite horizontal stratum. A vertical sheet pile in an anisotropic porous medium becomes inclined in the fictitious flow domain when the directions of the principal coefficients of permeability are other than vertical and horizontal. The solution to the problem of inclined sheet pile was obtained by Verigin (vide Harr, 1962). The mapping was achieved by making use of the transformation

$$Z = C\,e^{i\pi\alpha}n\,(a_1 - t)^{\alpha_n-\alpha_1}(a_2 - t)^{\alpha_1-\alpha_2}\ldots\ldots\ldots\ldots(a_n - t)^{\alpha_{n-1}-\alpha_n} \quad \text{(II.4.14)}$$

which maps a region of radial slits in z plane to the upper half of t plane in fig. 2.8b

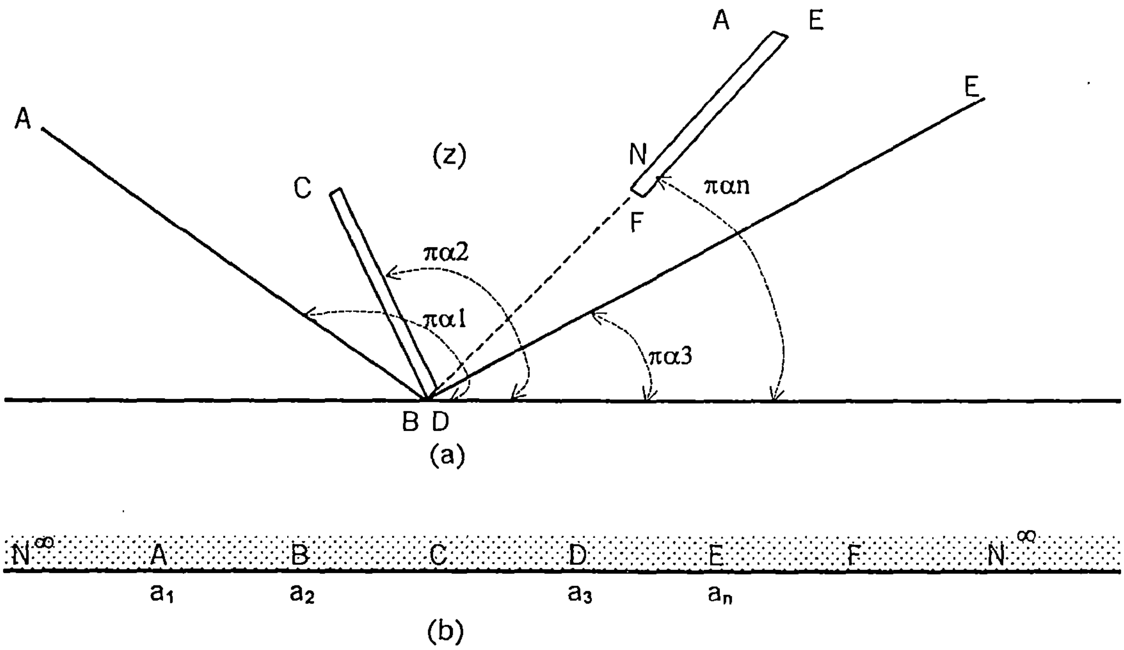


Fig.2.8. Mapping of a Region with Radial Slits

C = constant ;

$\pi\alpha_1, \pi\alpha_2, \pi\alpha_3$...... = angles that the sides of the slits make with the abscissa ;

$\pi\alpha_n$ = particular angle of the slit ANFE (see in fig. 2.8a)

$a_1, a_2, a_3, \ldots\ldots, a_n$ = the image points on the real axis of t plane.

The flow domain with a single inclined sheet pile shown in fig. 2.9a is a particular case of the generalized case discussed above and the governing relationship for conformal transformation is given by.

$$Z = C\,(1+t)^{1-\gamma}\,(1-t)^{\gamma} \qquad \text{(II.4.15)}$$

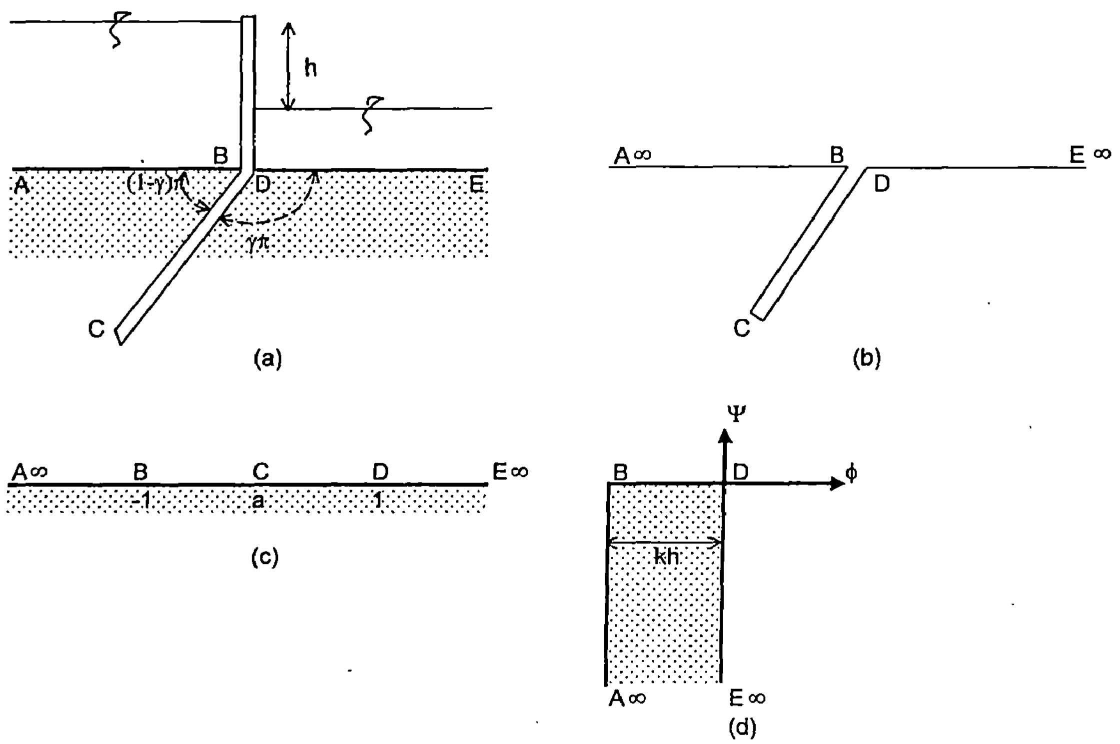


Fig. 2.9 Steps of Mapping for Flow Domain with Inclined Sheet Pile

According to the Schwarz-Christoffel transformation, the mapping of the region $A_\infty BCDE_\infty$ onto the lower half of the t plane is given by

$$\frac{dz}{dt} = M_1 (1+t)^{-\gamma} (t-a)(1-t)^{\gamma-1} \qquad \text{(II.4.16)}$$

the vertices B, C and D being mapped onto -1, a and l, receptively. Differentiating Eq. (II.4.15) and comparing it with Eq. (II.4.16), since they have to be identical, a is found to be equal to 1-2γ. Applying the condition at the tip of the sheet pile for which t = a, and $z = s\,e^{i\gamma\pi}$,

$$C = \frac{s\,e^{-\pi\gamma i}}{(1+a)^{1-\gamma}(1-a)^{\gamma}} \qquad \text{(II.4.17)}$$

Hence,

$$Z = s\,e^{-\gamma\pi I}\left(\frac{1+t}{1+a}\right)^{1-\lambda}\left(\frac{1-t}{1-a}\right)^{\gamma} \qquad \text{(II.4.18)}$$

The relationship between w and t planes is given by

$$t = \cos\frac{\pi w}{kh} \qquad \text{(II.4.19)}$$

in which

$w = \phi + i\psi$;

$\phi = -k\,(p/\gamma_W + y)$; and

ψ = Stream function

from Eqs. II.4.18 and II.4.19 the pressure distribution along the sheet pile and exit gradient can be determined.

FLOW UNDER WEIR WITH INCLINED SHEET PILE

3.1. ANALYSIS

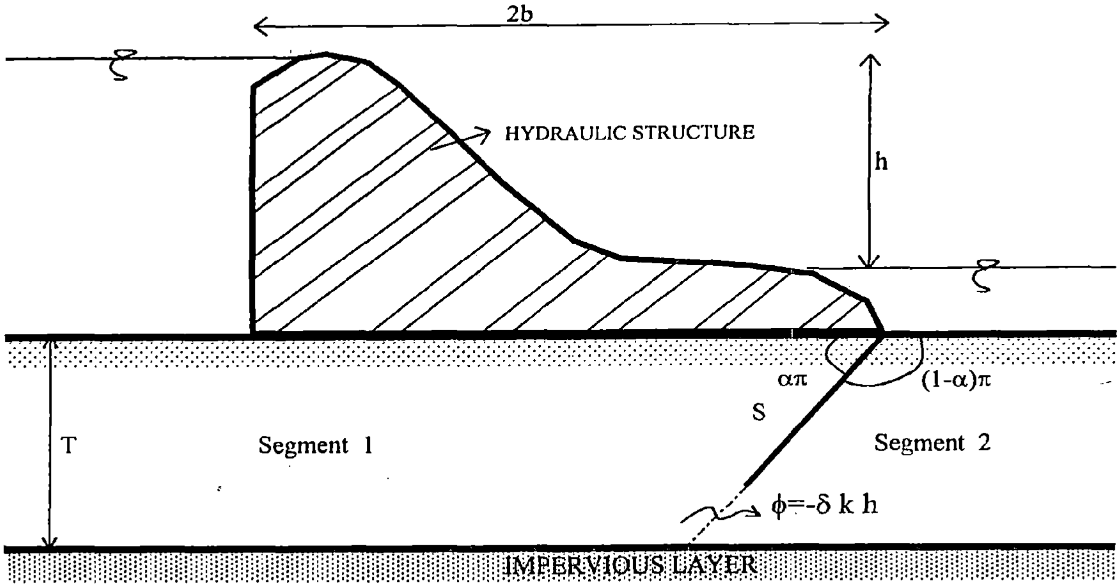
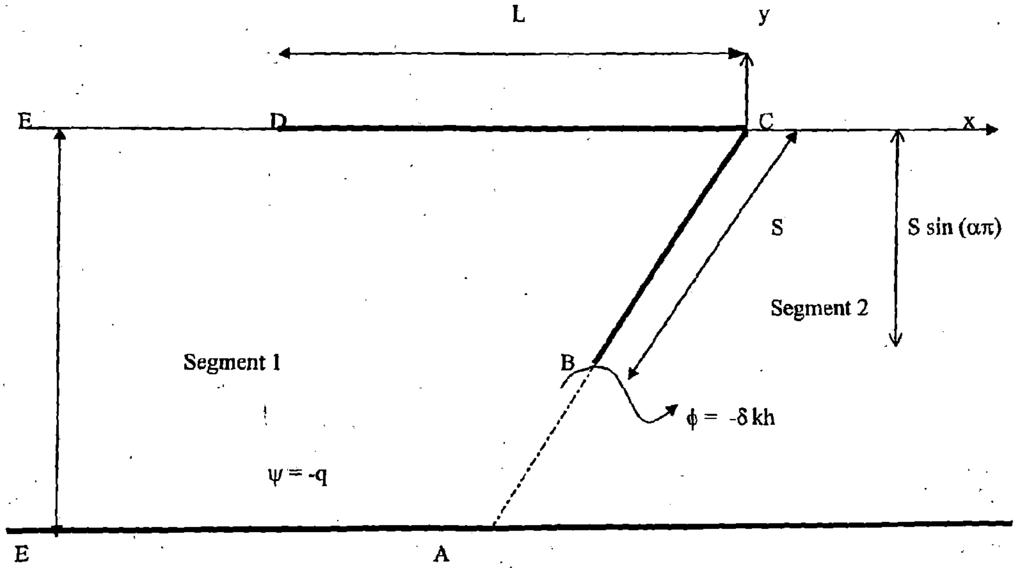


FIG. 3.1 WEIR WITH AN INCLINED SHEET PILE

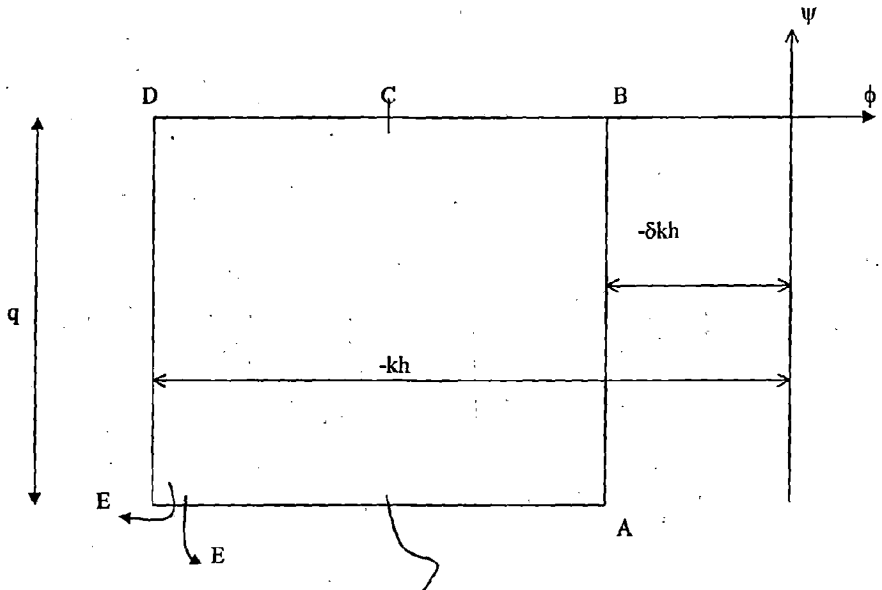
A weir with a vertical sheet pile in an anisotropic domain is converted to a weir with an inclined sheet pile in the corresponding fictitious isotropic domain. An analytical solution based on conformal mapping for flow under hydraulic structure with inclined sheet pile has been given by Reddy et. al (1971)[(10)].

The same problem has been solved here using the concept of method of fragments. The flow domain is decomposed into two parts by a sleet passing through the tip of the sheet pile and parallel to the sheet pile. The sleet divides the flow domain into two regions. Along the sleet boundary, the potential is assumed to be a constant ie. $\phi = -\delta\ k\ h$. By such conceptualisation the number of vertices has been decreased from 5 to 3. When the whole flow domain is considered the number of vertices = 5. When it is decomposed into two fragments the vertices for each fragments are 3. The complexity of the mapping function is considerably reduced when the problem is solved by method of fragments. The accuracy of the method of fragments is checked by comparing the flow characteristics obtained using method of fragments with exact solution.

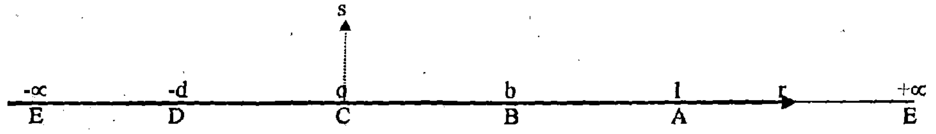
A. SEGMENT 1



z (= x + iy) plane



Complex Potential w (= ϕ + iψ) plane



t (= r + is) plane

Applying Schwarz - Christoffel transformation, the conformal mapping of the segment 1 in z -plane onto the auxiliary t - plane is given by

$$Z = M \int_0^{t'} \frac{dt}{(t)^{1-\alpha}(1-t)^{\alpha}} + N$$

$$= M\, B_{t'}(\alpha, 1-\alpha) + N \qquad \text{(III.1.1)}$$

in which $B_{t'}(\alpha, 1-\alpha)$ is complete Beta function.

For point C, $t' = 0$ and $Z = Z_C = 0$. Hence, $N = 0$.
For point A, $t' = 1$ and $Z = Z_A = T \operatorname{cosec} \alpha\pi \; e^{i(1+\alpha)\pi}$

Hence,

$$T \operatorname{cosec} \alpha\pi \; e^{i(1+\alpha)\pi} = M\, B(\alpha, 1-\alpha)$$
$$= M\, \Gamma(\alpha)\Gamma(1-\alpha)$$
$$= M \frac{\pi}{\sin \alpha\pi}$$

Hence, $$M = \frac{T}{\pi} e^{i(1+\alpha)\pi} \qquad \text{(III.1.2)}$$

Thus the relationship between z and t plane is

$$Z = \frac{T}{\pi} e^{i(1+\alpha)\pi}\, B_t(\alpha, 1-\alpha) \qquad \text{(III.1.3)}$$

For point B, $t' = b$ and $Z = Z_B = S\, e^{i(1+\alpha)\pi}$

$$\frac{S\pi}{T} = B_b(\alpha, \; 1-\alpha) \qquad \text{(III.1.4)}$$

The parameter b is found by an iteration.

For point D $t = -d$ and $Z = Z_D = -L$

Hence,

$$-L = M \int_0^{-d} \frac{dt}{(1-t)^{\alpha}(t)^{1-\alpha}}$$

Assuming $-t = u$ and $dt = -du$

$$-L = M \int_0^{d} \frac{-du}{(1+u)^{\alpha}(-u)^{1-\alpha}}$$

or $$L = \left[\frac{T}{\pi}\right]\left[\frac{e^{i(1+\alpha)\pi}}{(-1)^{1-\alpha}}\right]\int_0^d \frac{du}{(1+u)^{\alpha}(u)^{1-\alpha}}$$

Noting that

$$\frac{e^{i(1+\alpha)\pi}}{(-1)^{1-\alpha}} = \frac{e^{i\pi}.e^{i\alpha\pi}}{(-1)^{(1-\alpha)}}$$

$$= \frac{e^{i\pi}.e^{i\alpha\pi}}{e^{(-i\pi)(1-\alpha)}}$$

$$= \frac{e^{i\pi}.e^{i\alpha\pi}}{e^{-i\pi}.e^{i\alpha\pi}} = 1$$

$$L = \frac{T}{\pi}\int_0^d \frac{du}{(1+u)^{\alpha}(u)^{1-\alpha}}$$

The improper integral is converted to proper integral substituting $u = v^8$ and $du = 8\,v^7\,dv$.

$$L = \frac{T}{\pi}\int_0^{d^{1/8}} \frac{8v^{8\alpha-1}dv}{(1+v^8)^{\alpha}}$$

The above substitution is applicable for $\alpha > 1/8$

Further substituting

$$v = \frac{1}{2}d^{1/8}(1+u) \qquad \text{a lower limit } v = 0,\ u = -1$$

$$dv = \frac{1}{2}d^{1/8}du \qquad \text{and upper limit } v = d,\ u = 1$$

$$\frac{\pi L}{T} = \int_{-1}^{1} \frac{8\left\{\frac{1}{2}d^{1/8}(1+u)\right\}^{8\alpha-1}\left[\frac{1}{2}d^{1/8}\right]du}{\left[1+\left\{\frac{1}{2}d^{1/8}(1+u)\right\}^{8}\right]^{\alpha}}$$

we obtain the parameter 'd' by an iteration procedure.

Assuming that the common boundary is an equipotential boundary, and the potential $\phi = -\delta$ kh at this boundary, the complex potential plane, w, pertaining to segment 1 is drawn (Fig 3.1)

Applying Schwarz-Christoffel transformation, the conformal mapping of the in w-plane onto the auxiliary t-plane is given by

$$w(t) = M\int_{-\infty}^{t'} \frac{dt}{(-d-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} - kh - iq$$

$$= M\,2\,F(\vartheta, m) - kh - iq$$

in which, $F(\vartheta, m) = \int_0^{\vartheta} \frac{dv}{\sqrt{1-m^2\sin^2 v}}$

$$\vartheta = \sin^{-1}\sqrt{\frac{1+d}{1-t'}}$$

$$m = \frac{1-b}{1+d}$$

For point D, $t' = -d$, $w = -kh$

$$iq = M\left(\frac{2}{\sqrt{1+d}}\right)F\left(\frac{\pi}{2}, \frac{1-b}{1+d}\right)$$

$$M = \frac{iq}{\left(\frac{2}{\sqrt{1+d}}\right)F\left(\pi/2, \frac{1-b}{1+d}\right)} \qquad \text{(III.1.5)}$$

For $-d < t' < b$

$$w(t') = \frac{M}{i}\left(\frac{2}{\sqrt{1+d}}\right)F(\vartheta, m) - kh$$

$$\vartheta = \sin^{-1}\sqrt{\frac{t'+b}{b+d}}$$

and $m = \frac{b+d}{1+d}$

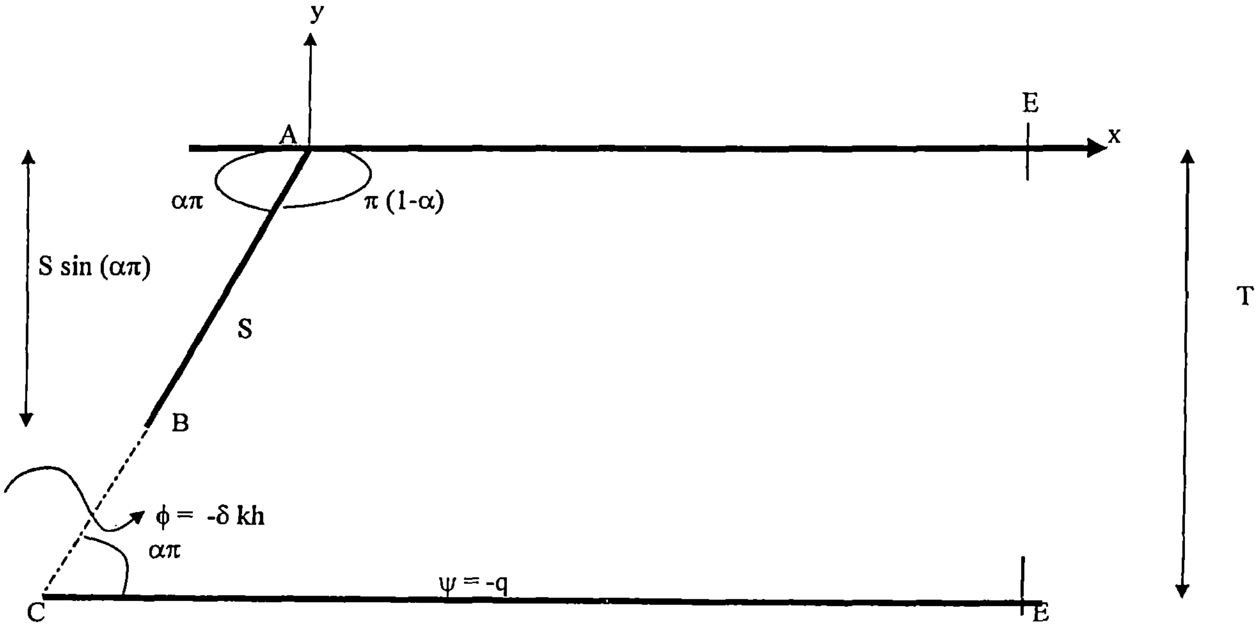
For point B, $t' = b$ and $w = -\delta kh$

$$-\delta kh = \frac{M}{i}\left(\frac{2}{\sqrt{1+d}}\right)F\left(\frac{\pi}{2}, \frac{b+d}{1+d}\right) - kh \qquad \text{(III.1.6)}$$

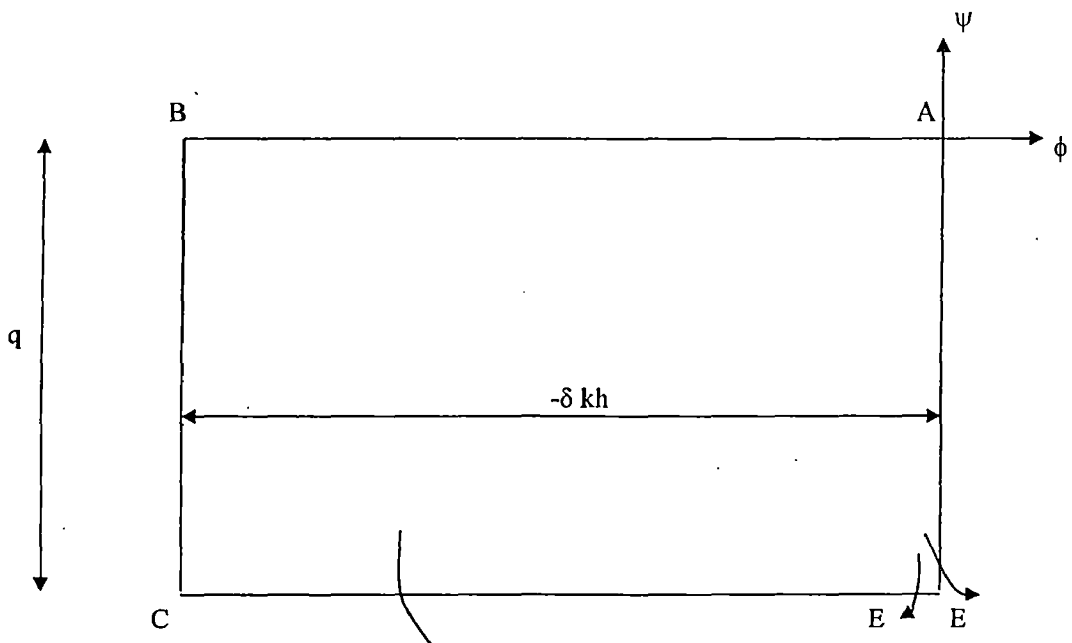
Substituting M, from eq (III. 1.4) in (III.1.5)

$$q = kh\,(1-\delta)\frac{F\left(\frac{\pi}{2}, \frac{1-b}{1+d}\right)}{F\left(\frac{\pi}{2}, \frac{b+d}{1+d}\right)} \qquad \text{(III.1.7)}$$

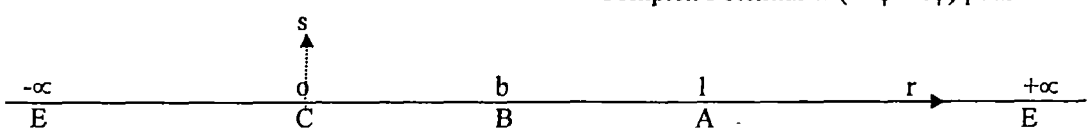
B. SEGMENT 2



z(= x + iy) plane



Complex Potential w (= ϕ + iψ) plane



t (= r + is) plane

The conformal mapping of segment 2 in z-plane onto t-plane is given by:

$$z = M \int_0^{t'} \frac{dt}{(1-t)^{\alpha}(t)^{1-\alpha}} + N \qquad \text{(III.1.8)}$$

For t' = 0, $Z_C = (T \text{ Cosec } \alpha\pi)e^{i(1+\alpha)\pi}$

For t' = 1, $Z_A = 0$

Applying these conditions

$$0 = M \int_0^1 \frac{dt}{(1-t)^{\alpha}(t)^{1-\alpha}} + (T \text{ Cosec } \alpha\,\pi)\, e^{i(1+\alpha)\pi}$$

$$-[T \text{ Cosec } \alpha\pi]\, e^{i(1+\alpha)\pi} = M\, B(\alpha, 1-\alpha) = M \frac{\pi}{\sin \alpha\pi}$$

$$M = \frac{-T\, e^{i(1+\alpha)\pi}}{\pi} \qquad \text{(III.1.9)}$$

For $t = b, \quad Z_B = S\, e^{i(1+\alpha)\pi}$

$$S\, e^{i(1+\alpha)\pi} = M\, B_b(\alpha, 1-\alpha) + (T \text{ cosec } \alpha\,\pi)\, e^{i(1+\alpha)\pi}$$

$$-(T \text{ cosec } \alpha\pi - S)\, e^{i(1+\alpha)\pi} = M\, B_b(\alpha, 1-\alpha)$$

Substituting M,

$$\pi\left(\operatorname{cosec}\alpha\pi - \frac{s}{T}\right) = B_b(\alpha, 1-\alpha) \qquad \text{(III.1.10)}$$

By computer program we will get 'b' value

The complex potential for the second segment is shown in Fig. 3.1. Applying Schwarz-Christoffel transformation, the conformal mapping of the complex potential onto t-plane is given by

$$w(t') = M \int_{\infty}^{t'} \frac{dt}{(-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} - iq$$

$$= M.\,2\,F(\vartheta, m) - iq \qquad (\text{for } \infty < t' \le 0)$$

in which $\vartheta = \sin^{-1}\sqrt{\frac{1}{1-t'}}$

$m = 1 - b$

For point C, t' = 0 and w = - $\delta kh - iq$

Hence,

$$M = \frac{kh(-\delta)}{2F\left(\frac{\pi}{2}, 1-b\right)} \qquad (III.1.11)$$

For $0 \le t' \le b$, the relation between w and t-plane is given by

$$w(t') = M \int_{0}^{t'} \frac{dt}{(-1)^{1/2}(t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} - iq - \delta kh$$

$$= \frac{M}{i} 2F(\vartheta, m) - iq - \delta kh$$

in which $\vartheta = \sin^{-1}\sqrt{\frac{t'}{b}}$

$m = b$

For point B, t' = b and w = -δ kh

Hence,

$$q = -M\,2\,F(\pi/2, b) \qquad (III.1.12)$$

Substituting M_1 from (III.1.11) in (III.1.12)

$$q = \delta kh \frac{F(\pi/2, b)}{F(\pi/2, 1-b)} \qquad (III.\ 1.13)$$

Since out flow from fragment 1 in flow to fragment 2

$$kh\,(1-\delta)\frac{F\left(\frac{\pi}{2}, \frac{1-b}{1+d}\right)}{F\left(\frac{\pi}{2}, \frac{b+d}{1+d}\right)} = \delta kh \frac{F(\pi/2, b)}{F(\pi/2, 1-b)}$$

Hence, δ value we will get by computer program .

3.2 RESULT

3.2.1 Variation of Quantity of Seepage with Length of Sheet Pile for a Weir with Upstream Blanket.

A structure with an upstream blanket an inclined sheet pile is shown in Fig. 3.1. The seepage would reduce with increase with in length of sheet pile. The seepage would also decrease with increase in length of upstream blanket.

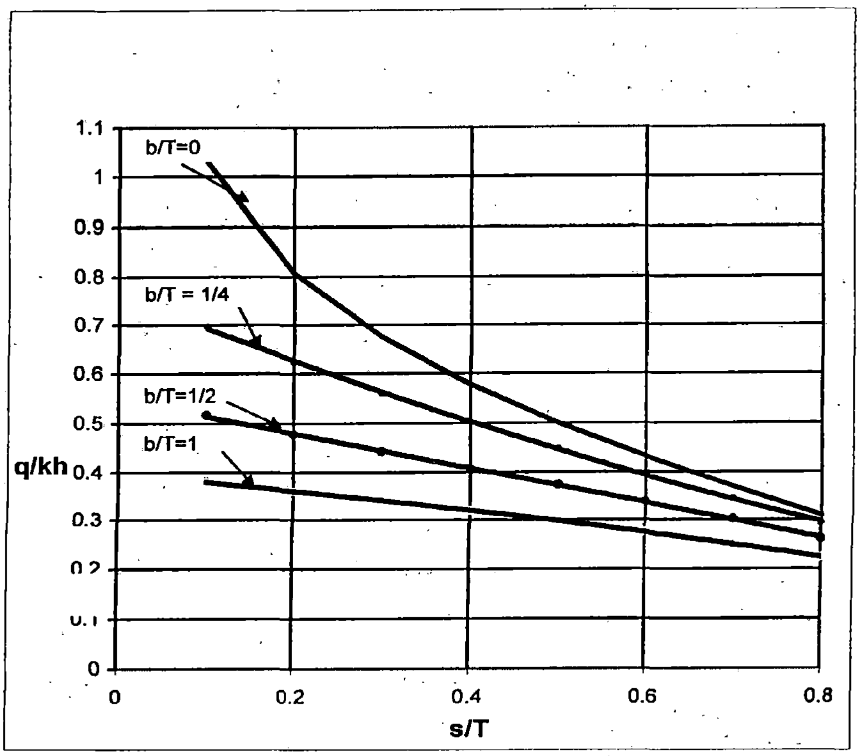


Fig.3.2. Effect of Length of Sheet Pile and width of Blanket on Quantity of Seepage for $\alpha = 1/2$ (ie. Vertical Sheet Pile)

The results obtained for a weir having a vertical sheet pile using the method of fragments are shown in Fig. 3.2. These results compare with the results given by Polubarinova-Kochina.

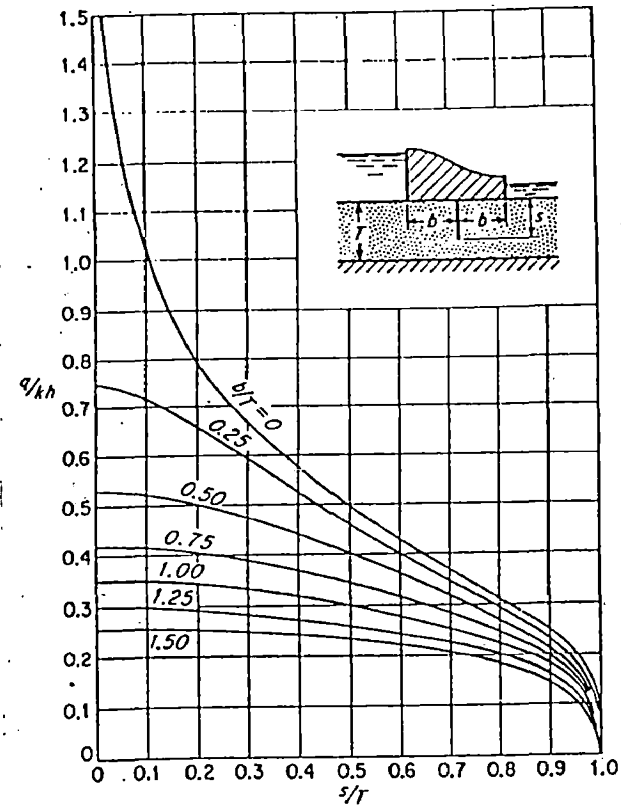
Table 3.1. q/kh for different b/T and s/T

α=1/2

s/T	q/kh			
	b/T=0	b/T=1/4	b/T=1/2	b/T=1
0.1	1.0297	0.6966	0.5167	0.3811
0.2	0.8072	0.6229	0.4773	0.3595
0.3	0.6747	0.5600	0.4422	0.3395
0.4	0.5780	0.5024	0.4080	0.3194
0.5	0.5000	0.4483	0.3737	0.2983
0.6	0.4325	0.3966	0.3386	0.2757
0.7	0.3705	0.3455	0.3017	0.2509
0.8	0.3097	0.2929	0.2613	0.2225

The q/kh obtained from method of fragments are presented in Table 3.1. The results given by Pollubarinova-Kochina are shown Fig 3.2 and Fig 3.3. The results obtained by method of fragments are same as that which have been obtained by Pollubarinova- Kochina.

The variation of q/kh with s/T for b/T = 0 ie the simple case of a vertical sheet pile embedded in a finite layer of porous medium, is shown in Fig. 3.3. The variation of seepage an upstream blanket is also shown is Fig 3.3. These graphs are given by Pollubarinova-Kochina. From the present analysis by putting $\alpha = 1/2$, the obtain these graphs from method of fragments.



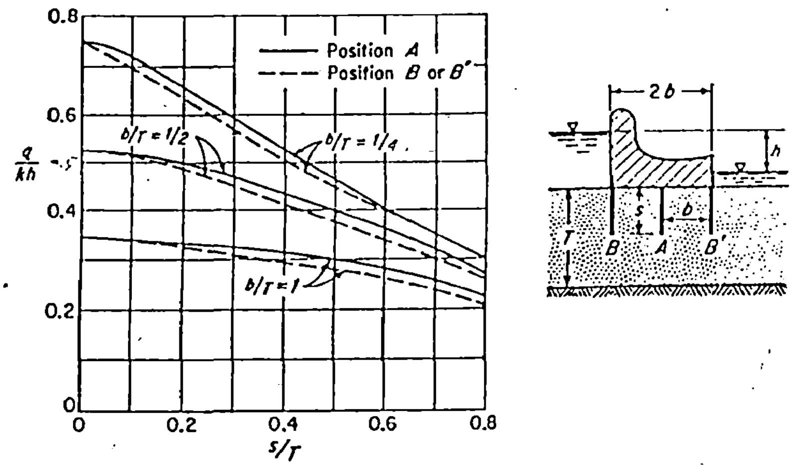


Fig. 3.3 Variation in Quantity of Seepage with length of sheet pile

3.2.2 RESULT INCLINED SHEET PILE WITH VARIATION OF ANGLE

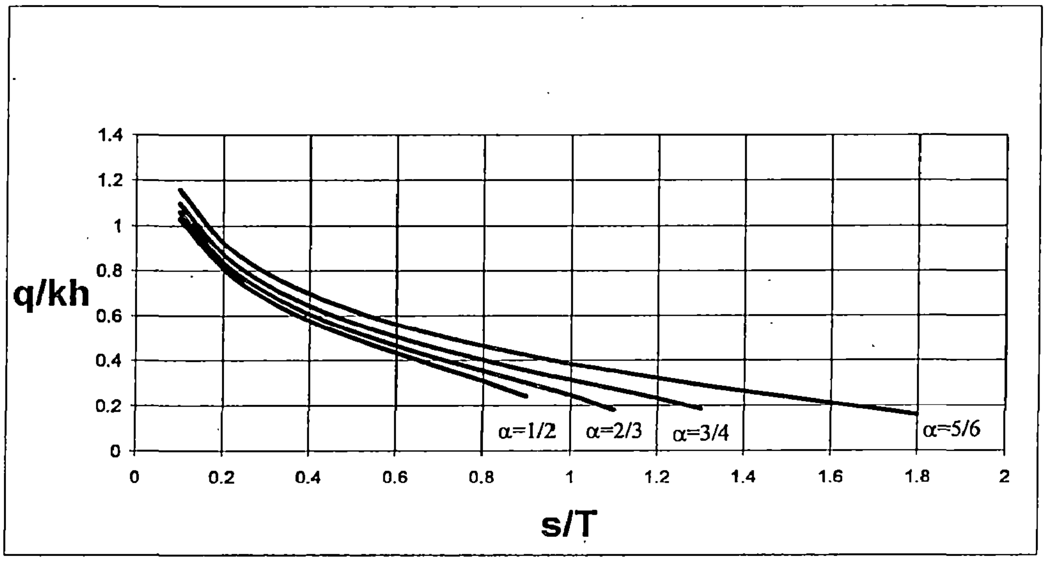


Fig.3.4 Effect of Length of Inclined Sheet pile with Blanket on Quantity of Seepage

Variation of q/kh with length of an inclined sheet pile is shown in Fig 3.4. The results have been compared with the results obtained by analytical method given by Reddy et al (1971)[10]. It is found that the variation of seepage with s/T is same as that given by Reddy et al(1971)[10].

Table 3.2 Seepage for Different Length of Sheet Pile

s/T	q/kh			
	α = ½=0.5	α = 2/3=0.667 or 1/3=0.333	α = 3/4=0.75 or 1/4 =0.25	α = 5/6=0.833 or 1/6
0.1	1.0297	1.0589	1.0958	1.1569
0.2	0.8072	0.8355	0.8721	0.9235
0.3	0.6747	0.7027	0.7395	0.7914
0.4	0.5780	0.6064	0.6438	0.6969
0.5	0.5000	0.5297	0.5682	0.6227
0.6	0.4325	0.4646	0.5052	0.5616
0.7	0.3705	0.4069	0.4504	0.5095
0.8	0.3097	0.3534	0.4015	0.4637
0.9	0.2428	0.3015	0.3566	0.4231
1.0		0.2473	0.3144	0.3836
1.1		0.1798	0.2734	0.3526
1.2			0.2317	0.3214
1.3			0.1856	0.2922
1.4				0.2645
1.5				0.2379
1.6				0.2119
1.7				0.1857
1.8				0.1583

The quantity of seepage corresponding to different length of sheet pile for b/T =0 are presented in Table 3.2. The seepage quantity computed by method of fragments compare exactly with those obtained by rigorous Conformal Mapping method.

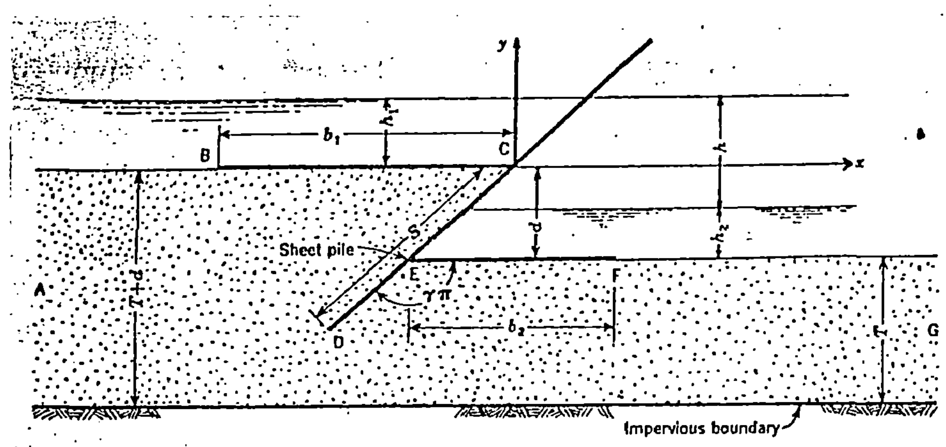


Fig.3.5 Weir with Inclined Sheet Pile

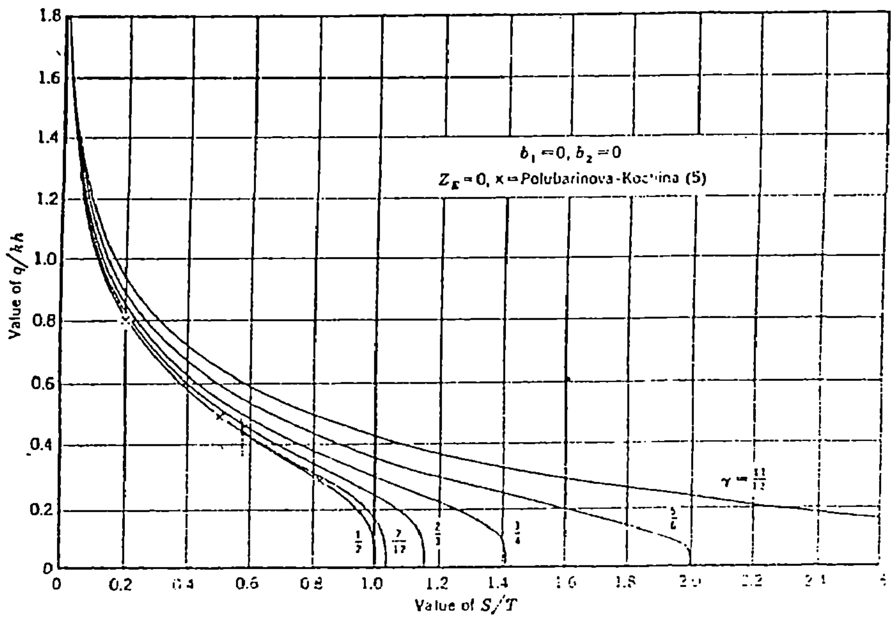


Fig. 3.6 Effect of Length of Inclined Sheet Pile on Quantity of Seepage From Reddy et al (1971)[10].

CHAPTER - IV

APPLICATION OF METHOD OF FRAGMENTS ON FLOW UNDER A WEIR WITH TWO SHEET PILES

4.1 INTRODUCTION

The accuracy of method of fragments is checked in chapter - 3. The method of fragment reduces the complexity of application of conformal mapping to practical weir design.

4.2 STATEMENT OF PROBLEM

A weir resting on a permeable foundation of finite depth is shown in Fig. 4.1. The depressed weir has depressed floor as well as has slopping base. There are two sheet piles, one at the upstream and another at the downstream end. It is required to compute the uplift pressure and the maximum exit gradient.

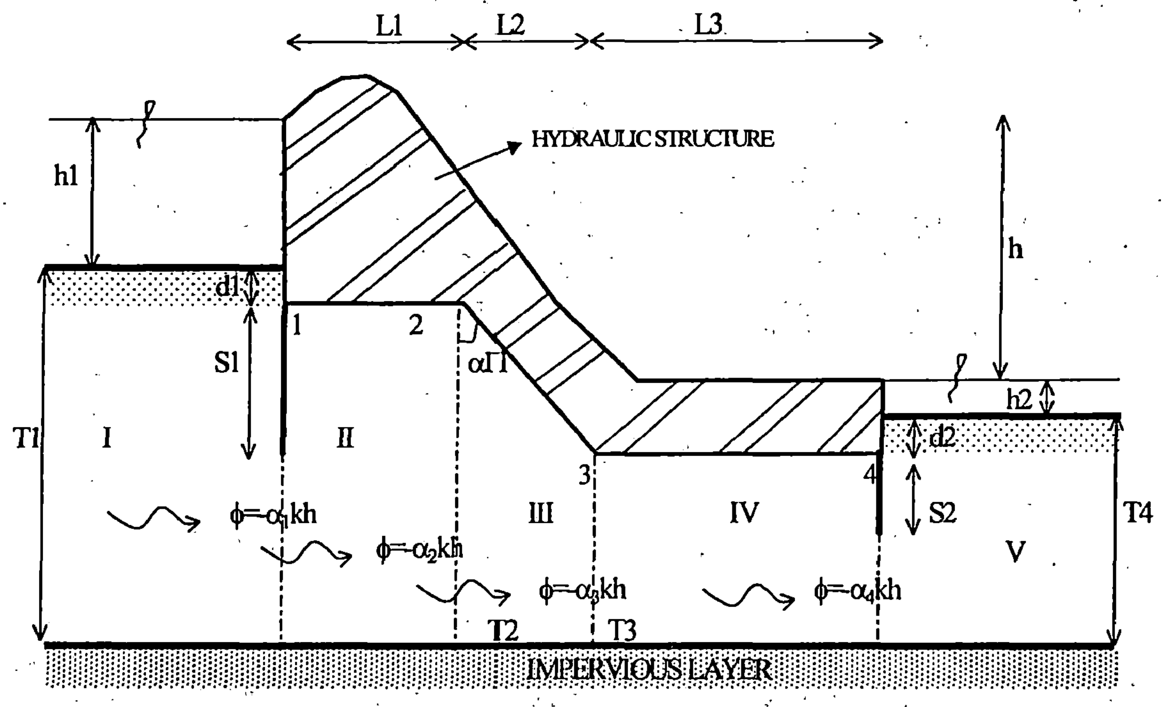


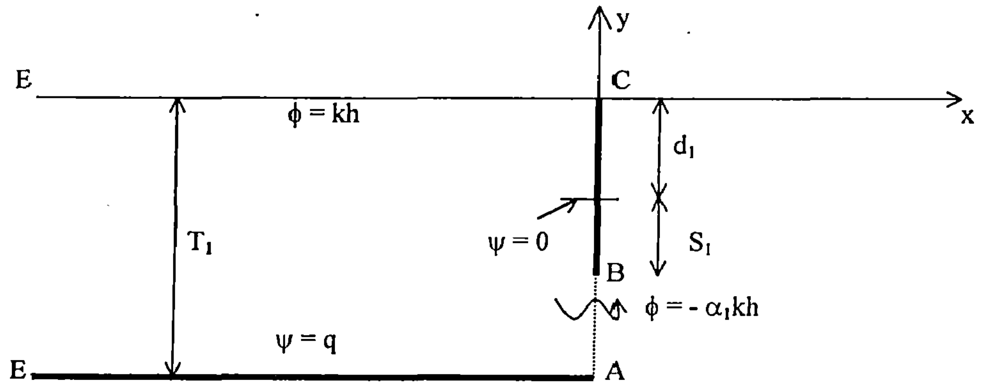
Fig. 4.1 A Weir with Two Sheet Piles

4.3 ANALYSIS

The flow domain is decomposed into five fragments as shown in the Fig. 4.1. The boundary between adjacent segment is assumed to be an equipotential boundary of unknown potential. The flow in each segment is analysed using conformal mapping.

4.3.1 SEGMENT I

We assume a convenient origin for each segment.



z (= x + iy) plane

The velocity potential function ϕ is defined as:

$$\phi = -k\left(\frac{p}{\gamma_w} + y\right) + c$$

in which

k = coefficient of permeability,

p = water pressure;

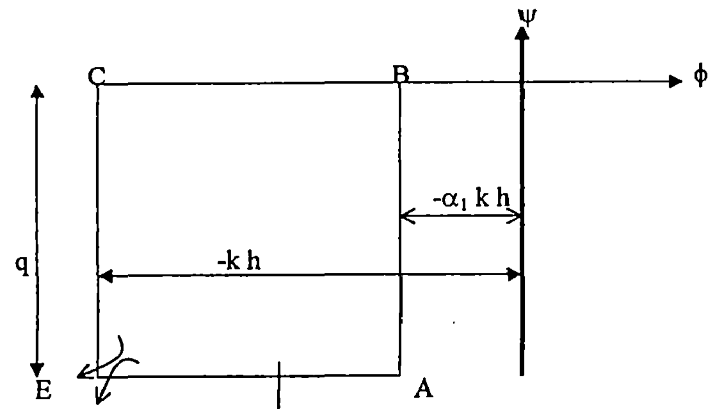
γ_w = unit weight of water;

y = elevation head, and

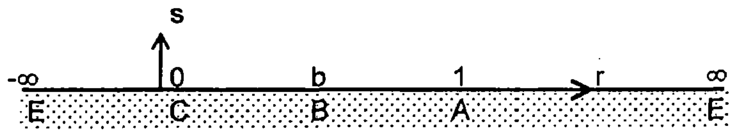
c = a constant

The complex potential plane w (= $\phi + i\psi$) for segment I is shown below:

The constant c is assumed such that ϕ = -kh for the upstream boundary and ϕ = 0 for the downstream boundary, h is hydraulic head difference that causes seepage. It is assumed that ϕ along AB is $-\alpha_1$ kh and α_1 is unknown.



Complex Potential w(= φ + iψ) plane



t(=r+is) plane

Applying Schwarz – Christoffel transformation, the conformal mapping of the segment I in z plane onto the auxiliary t plane is given by

$$z = M\int \frac{dt}{t^{1/2}(1-t)^{1/2}} + N$$

Integrating

$$z = 2M\sin^{-1}\sqrt{t} + N \qquad (4.1.1)$$

For point C, t = 0, and z = 0; hence N = 0

For point A, t = 1, and z = $-iT_1$

Applying these condition in equation (4.1.1) the constant M is found to be

$$M = \frac{-iT_1}{\pi} \qquad (4.1.2)$$

Hence

$$z = \frac{-2iT_1}{\pi}\sin^{-1}\sqrt{t} \qquad (4.1.3)$$

For point B, z = $-i\,(d_1 + s_1)$, and t = b

Hence,

$$b = \left[\sin\frac{\pi}{2}\left(\frac{d_1 + s_1}{T_1}\right)\right]^2 \qquad (4.1.4)$$

The Schwarz-Christoffel conformal mapping of the complex potential onto the t plane is given by:

$$w(t') = M_1 \int_{-\infty}^{t'} \frac{dt}{(-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} - iq - kh$$

$$= M_1 2F(\vartheta_1, m_1) - iq - kh \qquad (\text{for } -\infty < t' \leq 0)$$

in which $\vartheta = \sin^{-1}\sqrt{\frac{1}{1-t}}$

$m_1 = 1 - b$

F (ϑ, m) = elliptic integral of the first kind with amplitude ϑ and modulus m (Byrd and Friedman, 1971)

$$F(\vartheta, m) = \int_0^{\theta} \frac{du}{\sqrt{1 - m\sin^2 u}}$$

For point C, $t' = 0$ and $w = -kh$
Hence,

$$M_1 = \frac{iq}{2F(\pi/2, 1-b)} \qquad (4.1.5)$$

For $0 \leq t' \leq b$,

the relation between w and t plane is given by

$$w(t') = M_1 \int_0^{t'} \frac{dt}{(-1)^{1/2}(t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} - kh$$

$$= \frac{M_1}{i} 2F(\vartheta_2, m_2) - kh$$

in which $\vartheta_2 = \sin^{-1}\sqrt{\frac{t'}{b}}$

and $m_2 = b$

For point B, $t' = b$ and $w = -\alpha_1 kh$

Hence,

$$-\alpha_1 kh = \frac{2M_1}{i} F(\pi/2, b) - kh \qquad (4.1.6)$$

Substituting M_1 from (4.1.5) in (4.1.6)

$$q = kh(1-\alpha_1)\frac{F\left(\frac{\pi}{2},1-b\right)}{F\left(\frac{\pi}{2},b\right)} \tag{4.1.7}$$

For checking

For $b \le t' \le 1$

the relation between w and t plane is given by

$$w(t') = M_1 \int_b^{t'} \frac{dt}{(1-t)^{1/2}(-1)^{1/2}(t-b)^{1/2}(-1)^{1/2}(t-c)^{1/2}} - \alpha_1 kh$$

$$= -M\, g\, F(\vartheta_1, m_1) - \alpha_1 kh$$

in which $\vartheta = \sin^{-1}\sqrt{\dfrac{(t'-b)}{(1-b)(t')}}$

and $m_1 = 1-b$

For point A, $w = -iq - \alpha_1 kh$, $t' = 1$ and $\vartheta = \pi/2$

$-iq - \alpha_1 kh = -M_1\, 2\, F(\pi/2, 1-b) - \alpha_1 kh$

$iq = M_1\, 2\, F(\pi/2, 1-b)$ (This equation is same as 4.1.5)

For $a \le t' \le \infty$

$$w(t') = M_1 \int^{t'} \frac{dt}{(-1)^{1/2}(t-1)^{1/2}(-1)^{1/2}(t-b)^{1/2}(-1)^{1/2}(t-0)^{1/2}} + N$$

$$= -\frac{M}{i}\, g\, F(\vartheta_1, m_1) - \alpha_1 kh - iq$$

in which $\vartheta_2 = \sin^{-1}\sqrt{\dfrac{1}{t'}}$

and $m_2 = b$

For point E, $w = -iq - kh$, $t' = \infty$ and $\vartheta = \pi/2$

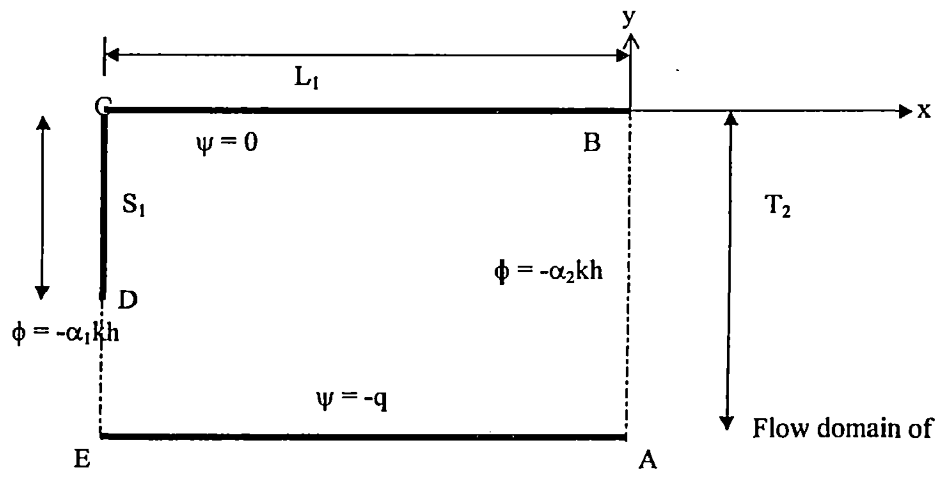
Hence,

$$kh = \frac{M_1}{i}\, 2F(\pi/2, b) + \alpha_1 kh$$

$2\, M_1\, F(\pi/2, b) = i\, kh\, (1-\alpha_1)$ (This equation is same as 4.1.6)

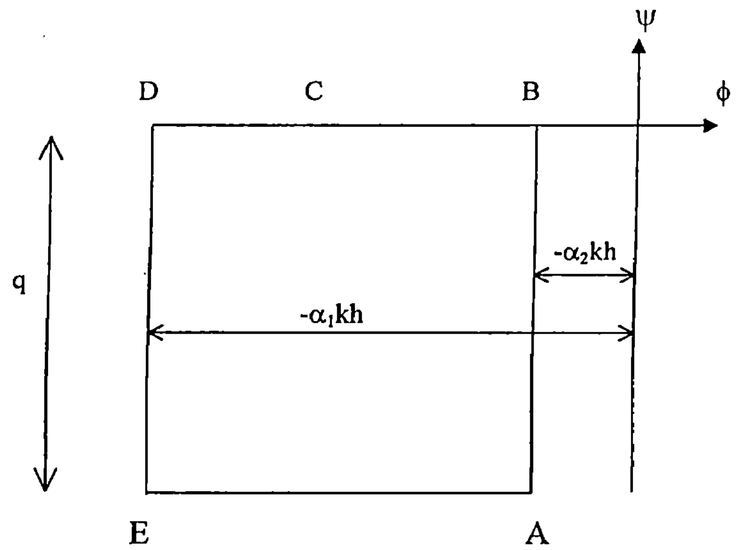
Hence further the duplicate equations have been omitted.

4.3.2 SEGMENT II

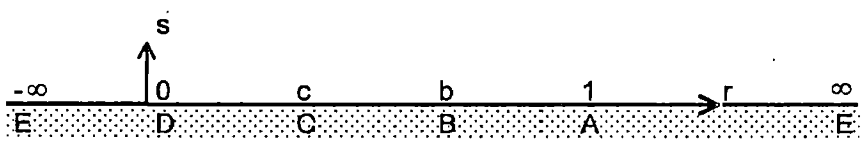


Flow domain of segment II

z(=x+iy) plane



Complex potential plane w (= φ + i ψ)



t(=r+is) plane

Applying the Schwarz-Christoffel transformation, the conformal mapping of the segment II in z-plane onto the auxiliary t-plane is given by

$$z = M\int_{-\infty}^{t'} \frac{dt}{(-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} - L_1 - iT_2$$

$$z = M2F(\vartheta_1, m_1) - L_1 - iT_2$$

in which

$$\vartheta_1 = \sin^{-1}\sqrt{\frac{1}{1-t'}}$$

$$m_1 = 1-b$$

For point D, z = $-L_1 - iS_1$, t' = -d

$$-L_1 - iS_1 = M2F\left(\sin^{-1}\sqrt{\frac{1}{1+d}}, 1-b\right) - L_1 - iT_2$$

or $$i(T_2 - S_1) = M2F\left(\sin^{-1}\sqrt{\frac{1}{1+d}}, 1-b\right) \qquad (4.2.1)$$

For point C, t = $-L_1$, t' = 0

- L_1 = M 2F $(\pi/2, 1-b)$-L_1 $-iT_2$

$$M = \frac{iT_2}{2F(\pi/2, 1-b)} \qquad (4.2.2)$$

Substituting equation (4.2.1)) into (4.2.2)

$$\frac{T_2 - S_1}{T_2} = \frac{F\left(\sin^{-1}\sqrt{\frac{1}{1+d}}, 1-b\right)}{F(\pi/2, 1-b)} \qquad (4.2.3)$$

For $0 \le t \le b$

$$z = \frac{M}{i}2F(\vartheta_2, m_2) - L_1$$

in which

$$\vartheta_2 = \sin^{-1}\sqrt{\frac{t'}{b}}$$

$$m_2 = b$$

For pint B, z = 0, t' = b

For pint C, z = $-L_1$

$$L_1 = \frac{M}{i}2F(\pi/2, b) \qquad (4.2.4)$$

Substituting (4.2.2) in (4.2.4)

$$\frac{L_1}{T_2} = \frac{F(\pi/2, b)}{F(\pi/2, 1-b)} \tag{4.2.5}$$

The parameter be in eq.(4.2.5) in found by an iteration. Knowing b, d is found form eq (4.2.3) by iteration. The relationship between z and t plane for the second segment is completely defined after we have found constant M and parameters b and d.

The Schwarz-Christoffel transformation of the w-plane onto the t-plane is given by

$$w(t') = M_1 \int_{-\infty}^{t'} \frac{dt}{(t+d)^{1/2}(t-b)^{1/2}(t-1)^{1/2}} - \alpha_1 kh - iq$$

For $-\infty < t' \leq -d$

$$w(t') = M_1 2F(\vartheta, m)$$

in which, $\vartheta = \sin^{-1}\sqrt{\frac{1+d}{1-t'}}$

and $m = \frac{1-b}{1+d}$

For point D, $t' = -d$, $w = \alpha_1 kh$

Hence,

$$-\alpha_1 kh = 2M_1 F\left(\pi/2, \frac{1-b}{1+d}\right) - \alpha_1 kh - iq$$

$$M_1 = \frac{iq}{2F\left(\pi/2, \frac{1-b}{1+d}\right)} \tag{4.2.6}$$

For $-d < t' \leq b$

$$w(t') = \frac{M_1}{i} 2F(\vartheta, m) - \alpha_1 kh$$

in which

$$\vartheta = \sin^{-1}\sqrt{\frac{t'+d}{b+d}}$$

$$m = \frac{b+d}{1+d}$$

For point B, $t' = b$ and $w = -\alpha_2 kh$

$$-\alpha_2 kh = \frac{M_1}{i} 2F\left(\pi/2, \frac{b+d}{1+d}\right) - \alpha_1 kh$$

$$ikh(\alpha_1 - \alpha_2) = M2F\left(\pi/2, \frac{b+d}{1+d}\right)$$

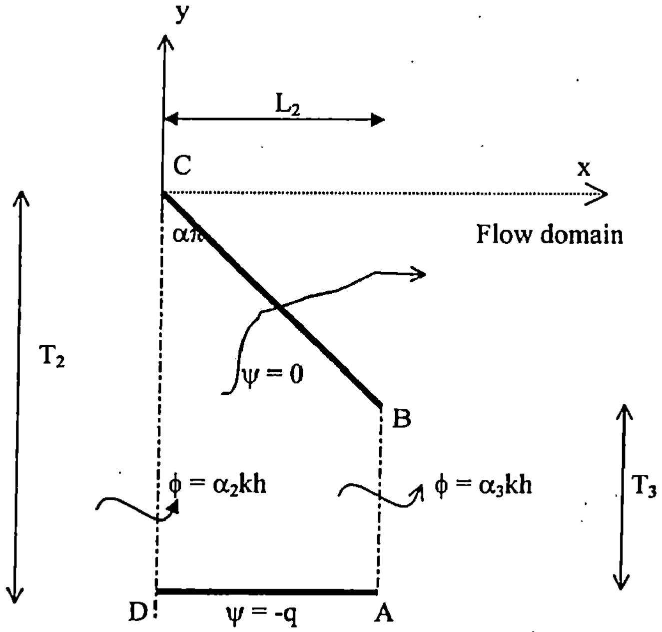
Substituting M_1

$$ikh(\alpha_1 - \alpha_2) = \frac{iq\,2F\left(\pi/2, \frac{b+d}{1+d}\right)}{2F\left(\pi/2, \frac{1-b}{1+d}\right)}$$

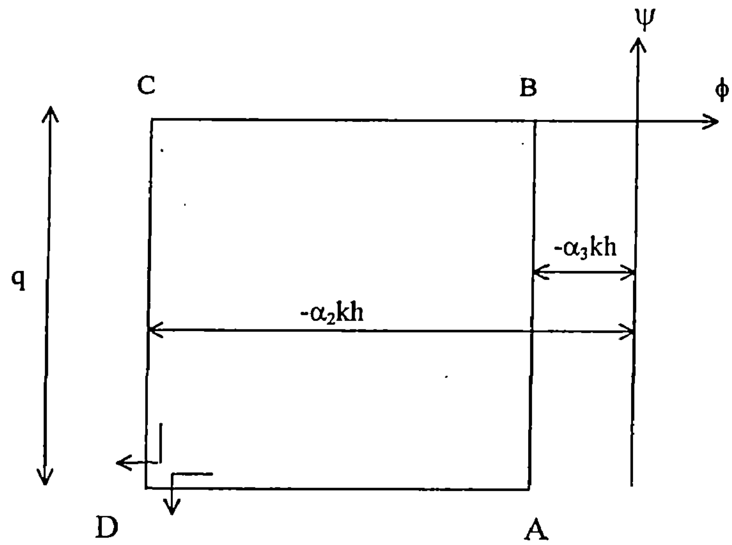
Hence,

$$q = kh(\alpha_1 - \alpha_2)\frac{F\left(\pi/2, \frac{1-b}{1+d}\right)}{F\left(\pi/2, \frac{b+d}{1+d}\right)} \tag{4.2.7}$$

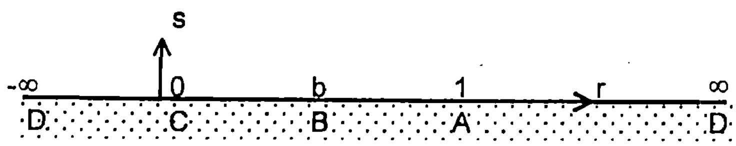
4.3.3 SEGMENT III



Z(=x+iy) plane



Complex potential plane w (= $\phi + i\psi$)



t(=r+is) plane

Applying Schwarz-Christoffel transformation, the conformal mapping of the segment III in z plane onto t-plane is given by

$$z = M\int \frac{dt}{(t-1)^{1/2}(t-b)^{\alpha}(t)^{1-\alpha}} + N$$

At point C, t = 0, $Z = Z_C = 0$

Point B, t = b, $Z = Z_B = L_2 \operatorname{cosec} \alpha\pi \, e^{i(3/2+\alpha)\pi} = L_2 - i(T_2 - T_3)$

$$L_2 - i(T_2 - T_3) = M\int_0^b \frac{dt}{(-1)^{1/2}(-1)^{\alpha}(1-t)^{1/2}(b-t)^{\alpha}(t)^{1-\alpha}}$$

$$L_2 \operatorname{cosec} \alpha\pi = M\int_0^{b/2} \frac{dt}{(1-t)^{1/2}(b-t)^{\alpha}(t)^{1-\alpha}} + M\int_{b/2}^{b} \frac{dt}{(1-t)^{1/2}(b-t)^{\alpha}(t)^{1-\alpha}} = M(I_1 + I_2)$$

The integration has been decomposed into two parts to convert the improper integral to proper one.

Let us assumed $t = v^8$ to evaluate I_1, $dt = 8v^7\,dv$

At the lower limit t = 0, v = 0

And upper limit t = b/2, $v = (b/2)^{1/8}$

$$I_1 = \int_0^{b/2^{1/8}} \frac{8v^{8\alpha-1}dv}{(1-v^8)^{1/2}(b-v^8)^{\alpha}}$$

Further are assume

$v = 1/2\,(b/2)^{1/8}\,(1+u)$ v = 0 → u = -1

$dv = 1/2\,(b/2)^{1/8}\,du$ $v = (b/2)^{1/8}$ → u = 1

Hence,

$$I_1 = \int_{-1}^{1} \frac{8\left[\frac{1}{2}(b/2)^{1/8}(1+u)\right]^{8\alpha-1}\left(\frac{1}{2}(b/2)^{1/8}\right)du}{\left[1-\left(\frac{1}{2}(b/2)^{1/8}(1+u)\right)^8\right]^{1/2}\left[\left(\frac{1}{2}(b/2)^{1/8}(1+u)\right)^8\right]^{\alpha}}$$

$$I_2 = \int_{b/2}^{b} \frac{dt}{(1-t)^{1/2}(b-t)^{\alpha}(t)^{1-\alpha}}$$

Let us assume $b - t = z^2$; $t = b/2 \rightarrow z = \sqrt{b/2}$; $t = b \rightarrow z = 0$

$t = b - z^2$

$dt = -2z\, dz$

$$I_2 = \int_0^{\sqrt{b/2}} \frac{2z^{1-2\alpha} dz}{(1-b+z^2)^{1/2}(b-z^2)^{1-\alpha}}$$

Further we assume $v = 1/2\,(b/2)^{1/2}\,(1+u)$; $v = 0$ $u = -1$; $v = (b/2)^{1/2}$ $u = 1$

$dv = 1/2\,(b/2)^{1/2}\,du$

$$I_2 = \int_{-1}^{1} \frac{2\left[\frac{1}{2}(b/2)^{1/2}(1+u)\right]^{1-2\alpha} \frac{1}{2}(b/2)^{1/2} du}{\left[1-b+\left(\frac{1}{2}(b/2)^{1/2}(1+u)\right)^2\right]^{1/2}\left[b-\left(\frac{1}{2}(b/2)^{1/2}(1+u)\right)^2\right]^{1-\alpha}}$$

$$M = \frac{L_2 \operatorname{cosec} \alpha\pi}{I_1 + I_2} \tag{4.3.1}$$

For $b \le t' < 1$

$$Z(t') = M \int_b^{t'} \frac{dt}{(-1)^{1/2}(1-t)^{1/2}(t-b)^{1/2}(t)^{1-\alpha}} + Z_B$$

At $t' = 1$

$Z = Z_A = L_2 - iT_2$

Substituting these is above and simplifying

$$T_3 = M \int_b^1 \frac{dt}{(1-t)^{1/2}(t-b)^{\alpha} t^{1-\alpha}}$$

$$= M \int_b^{\frac{1+b}{2}} \frac{dt}{(1-t)^{1/2}(t-b)^{\alpha} t^{1-\alpha}} + M \int_{\frac{1+b}{2}}^{1} \frac{dt}{(1-t)^{1/2}(t-b)^{\alpha} t^{1-\alpha}}$$

$= M\, I_3 + M\, I_4$

Let us assume

$t - b = u^2$ $\qquad t = b \rightarrow u = 0$

$t = u^2 + b$ $\qquad t = \frac{1+b}{2} \rightarrow u = \sqrt{\frac{1-b}{2}}$

$dt = 2u\, du$

$$I_3 = \int_0^{\frac{1-b}{2}} \frac{2u^{1-2\alpha}du}{(1-u^2-b)^{1/2}(u^2+b)^{1-\alpha}}$$

Further assumed

$$v = 1/2\left(\frac{1-b}{2}\right)^{1/2}(1+u) \qquad v = 0 \qquad \rightarrow u = -1$$

$$dv = 1/2\left(\frac{1-b}{2}\right)^{1/2} du \qquad v = \left(\frac{1-b}{2}\right)^{1/2} \qquad \rightarrow u = 1$$

$$I_3 = \int_{-1}^{1} \frac{2\left[\frac{1}{2}\left({}^{1/2}\frac{1-b}{2}\right)^{1/2}\right]^{1-2\alpha}\frac{1}{2}(1/2-b)^{1/2}du}{\left[1-\left(\frac{1}{2}(1/2-b)^{1/2}(1+u)\right)^2-b\right]^{1/2}\left[\left(\frac{1}{2}(1/2-b)^{1/2}(1+u)\right)^2+b\right]^{1-\alpha}}$$

$$I_4 = \int_{1/2}^{1} \frac{dt}{(1-t)^{1/2}(t-b)^{\alpha}(t)^{1-\alpha}}$$

Assumed, $1 - t = u^2$ at $t = 1/2 \rightarrow u = \sqrt{\frac{1}{2}}$

$t = 1 - u^2$ $t = 1 \rightarrow u = 0$

$dt = -2\ z\ dz$

$$I_4 = \int_0^{\sqrt{\frac{1-b}{2}}} \frac{2du}{(1-u^2-b)^{\alpha}(1-u^2)^{1-\alpha}}$$

Assumed, $v = 1/2\sqrt{\frac{1+b}{2}}\ (1+u)$ $\qquad v = 0 \qquad \rightarrow u = -1$

$dv = 1/2\sqrt{\frac{1+b}{2}}\ du$ $\qquad v = \sqrt{\frac{1-b}{2}} \qquad \rightarrow u = 1$

$$I_4 = \int_{-1}^{1} \frac{2.\frac{1}{2}\sqrt{\frac{1+b}{2}}du}{\left[1-\left(\frac{1}{2}\sqrt{\frac{1+b}{2}}(1+u)\right)^2-b\right]^{\alpha}\left[1-\left(\frac{1}{2}\sqrt{\frac{1+b}{2}}(1+u)\right)^2\right]^{1-\alpha}}$$

Hence, $T_3 = M(I_3 + I_4)$

Substituting M,

$$\frac{T_3 \sin\alpha\pi}{L_2} = \frac{(I_3 + I_4)}{(I_1 + I_2)} \tag{4.3.2}$$

The Schwarz-Christoffel conformal mapping of the complex potential onto the t-plane is given by:

$$w(t') = M \int_{-\infty}^{t'} \frac{dt}{(-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} + N$$

$= M\, 2F(\vartheta, m)$ in which $\vartheta = \sin^{-1}\sqrt{\dfrac{1}{1-t'}}$

$m = 1 - b$

at $t' = -\infty$, $w = -\alpha_2 kh - iq$

Hence, $N = -\alpha_2 kh - iq$

At $t' = 0$, $w = -\alpha_2 kh$

Applying these conditions

$$M = \frac{iq}{2F\left(\pi/2, \sqrt{1-b}\right)} \tag{4.3.3}$$

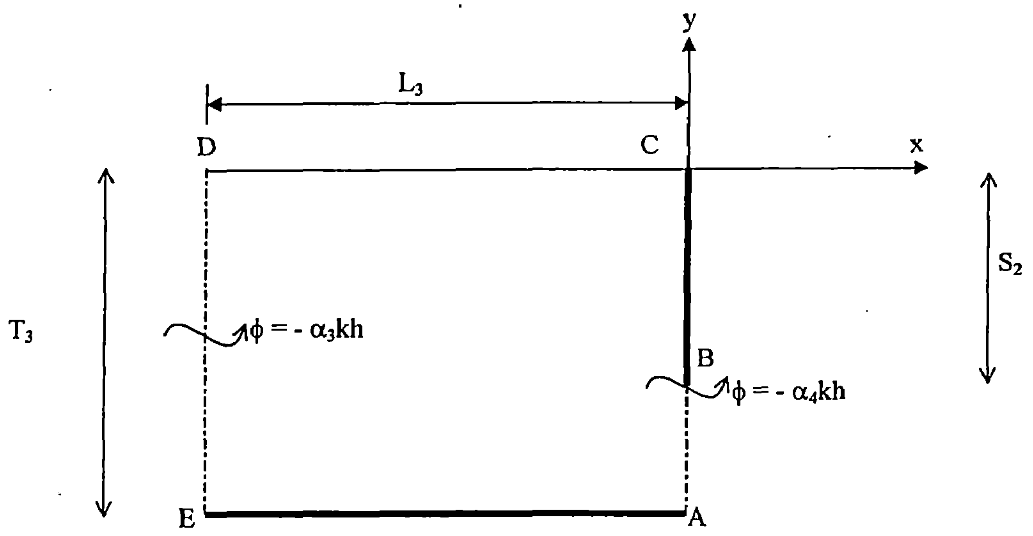
For $0 \le t' \le b$

$$-\alpha_3 kh = \frac{2M}{i} F\left(\pi/2, \sqrt{b}\right) - \alpha_2 kh$$

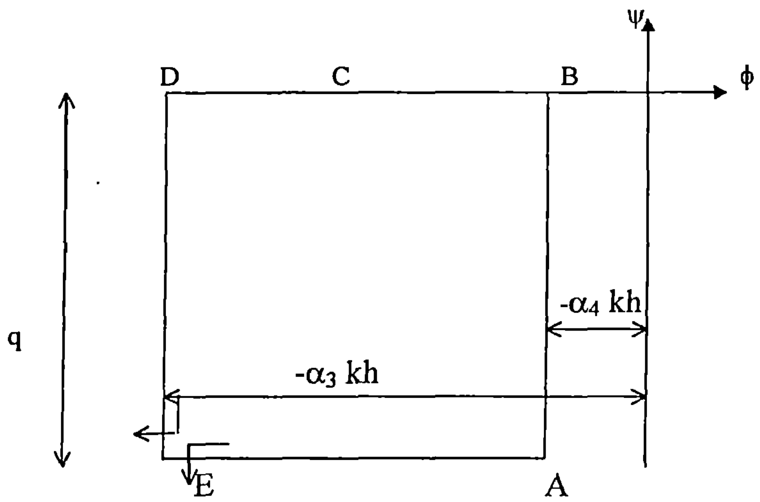
Substituting M, from eq. (4.3.3) in above

$$q = \frac{kh(\alpha_2 - \alpha_3) F\left(\pi/2, \sqrt{1-b}\right)}{F\left(\pi/2, \sqrt{b}\right)} \tag{4.3.4}$$

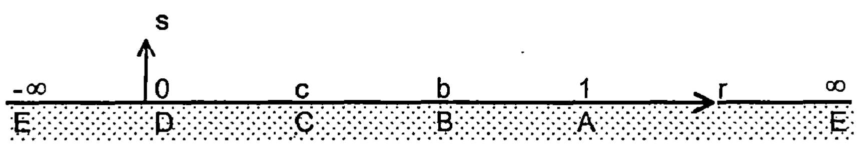
4.3.4 SEGMENT IV



z (= x+ iy) plane



w (= φ + iψ) plane



t(=r+is) plane

Applying Schwarz-Christoffel transformation, the conformal mapping of the segment IV in z plane onto the auxiliary t plane is given by

$$z = M\int_{-\infty}^{t'} \frac{dt}{(0-t)^{1/2}(c-t)^{1/2}(1-t)^{1/2}} - L_3 - iT_3$$

$$= 2MF(\vartheta, m) - L_3 - iT_3$$

in which $\vartheta = \sin^{-1}\sqrt{\frac{1}{1-t'}}$

$m = 1 - c$

For point D, $z = -L_3$, $t' = 0$

$$iT_3 = 2M \quad F(\pi/2, 1-c)$$

$$M = \frac{iT_3}{2F(\pi/2, 1-c)} \qquad (4.4.1)$$

For $0 < t \leq c$

$$Z = M/i \; 2F(\vartheta, m) - L_3$$

in which $\vartheta = \sin^{-1}\sqrt{\frac{t'}{c}}$

$m = c$

at $t' = c$

$$L_3 = \frac{M}{i} 2F(\pi/2, c)$$

substituting M,

$$\frac{L_3}{T_3} = \frac{F(\pi/2, c)}{F(\pi/2, 1-c)} \qquad (4.4.2)$$

The parameter c is obtain by and iteration procedure.

For $c < t \leq 1$

$$Z = M/i \; 2F(\vartheta, m)$$

In which $\vartheta = \sin^{-1}\sqrt{\dfrac{t'-c}{t'(1-c)}}$

$$m = 1-c$$

At $t=1$, $Z=-iT_1$

$$-iT_3 = \frac{M}{i}2F(\pi/2, 1-c) + 0$$

$$M = \frac{T_3}{2F(\pi/2, 1-c)}$$

For $t' = c$

$$-iS_2 = \frac{M}{i}2F\left(\sin^{-1}\sqrt{\frac{(b-c)}{b(1-c)}}, 1-c\right)$$

Substituting M

$$\frac{S_2}{T_3} = \frac{2F\left(\sin^{-1}\sqrt{\dfrac{(b-c)}{b(1-c)}}, 1-c\right)}{2F(\pi/2, 1-c)} \qquad (4.4.3)$$

The parameter b is obtained by an iteration after parameter c is obtained by iteration from eq. (4.4.2)

The Schwartz-Christoffel conformal mapping of the complex potential onto the t plane is given by

$$w(t') = M_1 \int_{\infty}^{t'} \frac{dt}{(t-0)^{1/2}(t-b)^{1/2}(t-1)^{1/2}} - \alpha_3 kh - iq$$

$$= M_1 2F(\vartheta, m) - \alpha_3 kh - iq$$

in which $\vartheta = \sin^{-1}\sqrt{\frac{1}{1-t'}}$

m = 1- b

For point D, t' = 0, w = - α_3 kh

$$iq = M_1 2F(\pi/2, 1-b)$$

Hence, $M = \frac{iq}{2F(\pi/2, \sqrt{1-b})}$ (4.4.4)

For 0 ≤ t' ≤ b

$$w(t') = \frac{M_1}{i} 2F(\vartheta, m) - \alpha_3 kh$$

in which $\vartheta = \sin^{-1}\sqrt{\frac{t'}{b}}$

m = b

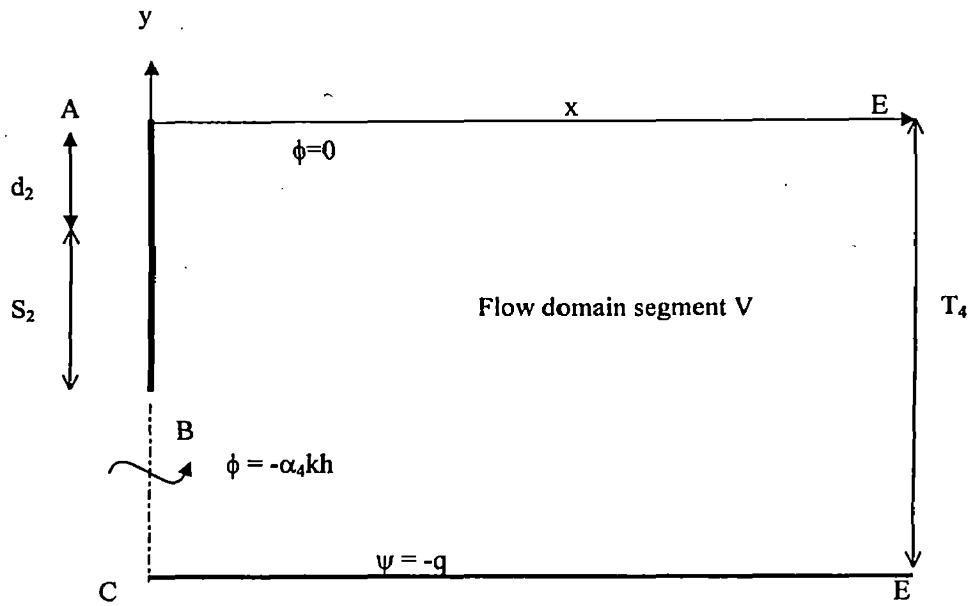
For point B, t' = b, w = -α_4 kh

Hence, $-\alpha_4 kh = \frac{M_1}{i} 2F(\pi/2, b) - \alpha_3 kh$

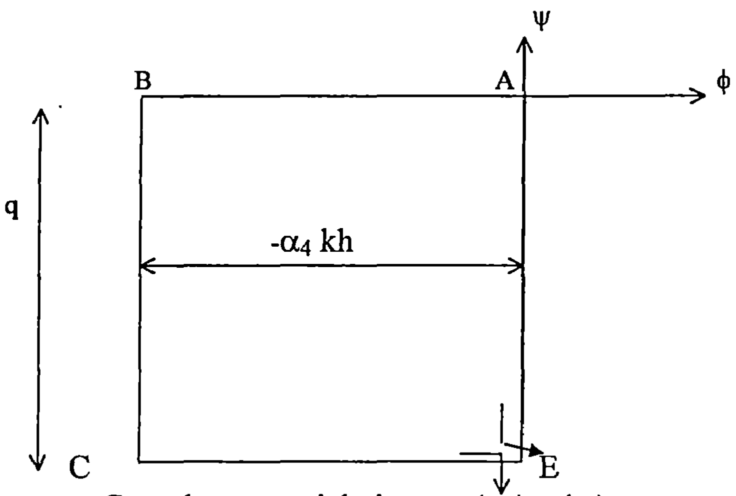
Substituting M,

$$q = kh(\alpha_3 - \alpha_4)\frac{F(\pi/2, 1-b)}{F(\pi/2, b)} \quad (4.4.5)$$

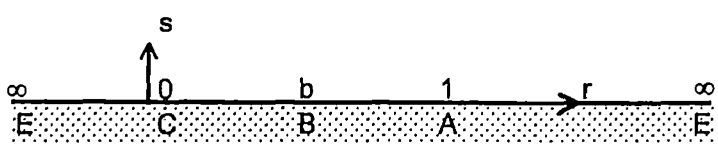
4.3.5 SEGMENT V



z(= x + iy) plane



Complex potential plane w (= ϕ + iψ)



t(=r+is) plane

Applying Schwarz-Christoffel transformation, the conformal mapping of the segment V in z plane on to the auxiliary plane is given by

$$z = M\int \frac{dt}{t^{1/2}(1-t)^{1/2}} + N$$

Integrating

$$z = 2M \sin^{-1}\sqrt{t} + N \qquad (4.5.1)$$

For point A, t = 1 and z = 0 hence N = - π M

For point C, t = 0 and t = -i T_4

Hence, $M = \dfrac{iT_4}{\pi}$ (4.5.2)

$$N = iT_4 \qquad (4.5.3)$$

and $z = 2\dfrac{iT_4}{\pi}\sin^{-1}\sqrt{t} - iT_4$ (4.5.4)

For point B, z = -i(d_2 + S_2) and t = b

Hence,

$$b = \left[\sin\frac{\pi}{2}\left(1-\left(\frac{d_2+S_2}{T_4}\right)\right)\right]^2 \qquad (4.5.5)$$

The Schwartz-Christoffel conformal mapping of the complex potential onto t-plane is given by:

$$w(t') = M_1 \int_{\infty}^{t'} \frac{dt}{(-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} - iq$$

$$= M_1 2F(\vartheta, m) - iq \qquad \text{(for } -\infty < t' \leq 0\text{)}$$

in which $\vartheta = \sin^{-1}\sqrt{\frac{1}{1-t'}}$

$m = 1-b$

For point C, $t' = 0$, and $w = -\alpha_4 kh - iq$

Hence,

$$M_1 = \frac{\alpha_4 kh}{2F\left(\pi/2, \sqrt{1-b}\right)} \qquad (4.5.6)$$

For $0 \leq t' \leq b$, the relation between w and t plane is given by

$$w(t') = M_1 \int_0^{t'} \frac{dt}{(-1)^{1/2}(t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} - iq - \alpha_4 kh$$

$$= \frac{M_1}{i} 2F(\vartheta, m) - iq - \alpha_4 kh$$

in which $\vartheta = \sin^{-1}\sqrt{\frac{t'}{b}}$

$m = b$

For point B, $t' = b$, $w = -\alpha_4\, kh$

Hence,

$$q = -M_1 2F(\pi/2, b)$$

Substituting M_1

$$q = kh(\alpha_4)\frac{F(\pi/2, b)}{F(\pi/2, 1-b)} \qquad (4.5.7)$$

4.3.6 FORMULATION OF EQUATION FOR SOLVING α_1, α_2, α_3, α_4

The Following Equations have been derived considering the flow in each of the five fragments.

From Segment I

$$q/kh = (1-\alpha_1)\frac{F\left[\frac{\pi}{2},1-b_1\right]}{F\left[\frac{\pi}{2},b_1\right]} = (1-\alpha_1)A \tag{4.6.1}$$

From Segment II

$$q/kh = (\alpha_1-\alpha_2)\frac{F\left[\frac{\pi}{2},\frac{1-b_2}{1+d_2}\right]}{F\left[\frac{\pi}{2},\frac{b_2+d_2}{1+d_2}\right]} = (\alpha_1-\alpha_2)B \tag{4.6.2}$$

From Segment III

$$q/kh = (\alpha_2-\alpha_3)\frac{F(\pi/2,1-b_3)}{F(\pi/2,b_3)} = (\alpha_2-\alpha_3)C \tag{4.6.3}$$

From Segment IV

$$q/kh = (\alpha_3-\alpha_4)\frac{F(\pi/2,1-b)}{F(\pi/2,b)} = (\alpha_3-\alpha_4)D \tag{4.6.4}$$

From Segment V

$$q/kh = \alpha_4\frac{F\left[\frac{\pi}{2},b_5\right]}{F\left[\frac{\pi}{2},1-b_5\right]} = \alpha_4 E \tag{4.6.5}$$

(b_1 is parameter b of segment 1 and b_5 is parameter b of segment 5 and so for other parameters).

Equating (4.6.1) and (4.6.2)

$$(A+B)\alpha_1 - B\alpha_2 = A \tag{4.6.6}$$

Equating (4.6.2) and (4.6.3)

$$B\alpha_1 - (B+C)\alpha_2 + C\alpha_3 = 0 \tag{4.6.7}$$

Equating (4.6.3) and (4.6.4)

$$\alpha_2 C - \alpha_3 (C + D) + \alpha_4 D = 0 \tag{4.6.8}$$

Equating (4.6.4) and (4.6.5)

$$\alpha_3 D - (D - E)\alpha_4 = 0 \tag{4.6.9}$$

In matrix notation

$$\begin{bmatrix} (A+B) & \text{-B} & 0 & 0 \\ B & \text{-(B+C)} & \text{C} & 0 \\ 0 & \text{C} & \text{(-C+D)} & \text{D} \\ 0 & 0 & \text{D} & \text{-(D+E)} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} = \begin{bmatrix} \text{A} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence,

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} = \begin{bmatrix} (A+B) & \text{-B} & 0 & 0 \\ B & \text{-(B+C)} & \text{C} & 0 \\ 0 & \text{C} & \text{(-C+D)} & \text{D} \\ 0 & 0 & \text{D} & \text{-(D+E)} \end{bmatrix}^{-1} \begin{bmatrix} \text{A} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

4.3.7 COMPUTATION OF UPLIFT PRESSURE

The uplift pressure can be computed using origin for the composite structure at a suitable point. Let us measure, the y-coordinate from the impervious stratum. The potential ϕ is defined as

$$\phi = -k\,(P/\gamma w + y) + C$$

If

$C = k\,(h_2 + T_4)$

ϕ along the downstream boundary = 0 and ϕ along the upstream boundary = -kh.

Where h is the hydraulic head difference which causes the seepage to occur

$$h = T_1 + h_1 - T_4 - h_2.$$

Let us compute uplift pressure at point 2.

At location 2

$$\phi = -\alpha_2 kh, \; Y = T_1 - d_1$$

$$-\alpha_2 kh = -k\left(\frac{p_2}{\gamma_w} + T_1 - d_1\right) + k(T_4 + h_2)$$

$$p_2 / \gamma_w = \alpha_2 h + h_2 + T_4 + d_1 - T_1$$

Similarly uplift pressure can be computed knowing ϕ at the desired point.

4.3.8 COMPUTATION OF MAXIMUM EXIT GRADIENT

The maximum exit gradient can computed from segment V.

$$\frac{dw}{dz} = u - i\,v$$

Along the downstream horizontal boundary u = 0

$$\text{Hence,} \quad \frac{dw}{dz} = -i\,v$$

From Darcy's law

$$V = -k\,I_E$$

$$\text{and} \quad \frac{dw}{dz} = \frac{dw}{dt}\cdot\frac{dt}{dz} = -iv$$

$$\frac{dz}{dt} = \frac{iT_4}{\pi}\frac{1}{t^{1/2}(1-t)^{1/2}}$$

$$\frac{dw}{dt} = \frac{-\alpha_4 kh}{2F(\pi/2, \sqrt{1-b})}\frac{1}{(-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}}$$

Hence,

$$i\,k\,I_E = \frac{-\alpha_4 kh}{2F(\pi/2, \sqrt{1-b})}\frac{1}{(-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}}\frac{\pi t^{1/2}(1-t)^{1/2}}{i\,T_4}$$

Exit gradient is maximum at point A in segment V for which t = 1.

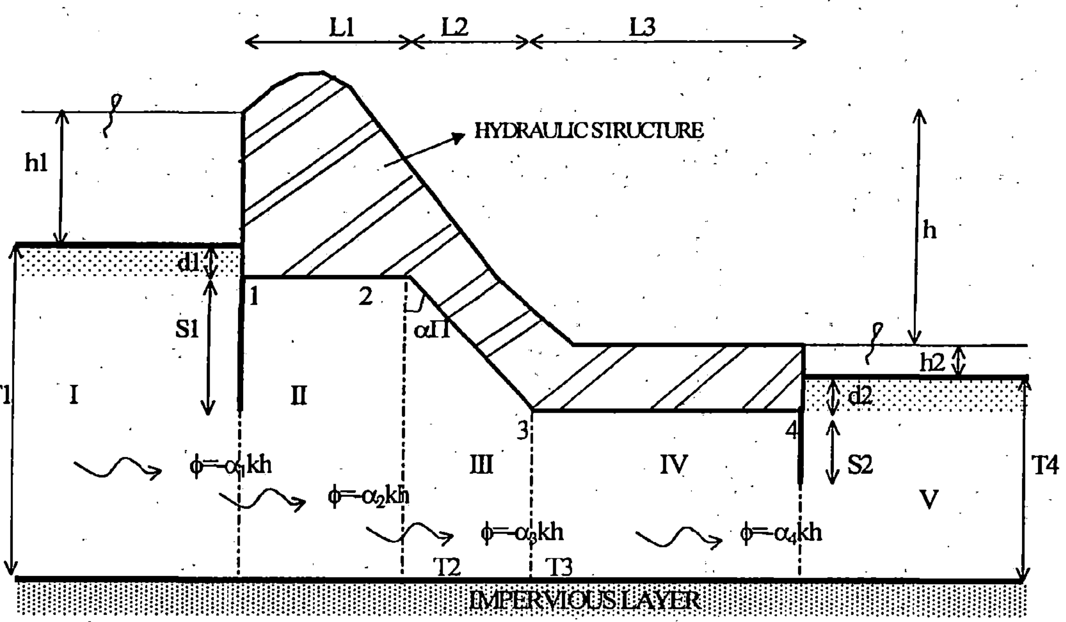
Substituting t = 1

$$IE = \frac{\pi\alpha_4 h}{(1-b_5)^{1/2}\,2T_4 F(\pi/2, 1-b_5)}$$

4.4 RESULT

Uplift pressure is computed for the following dimension of the hydraulic structure:

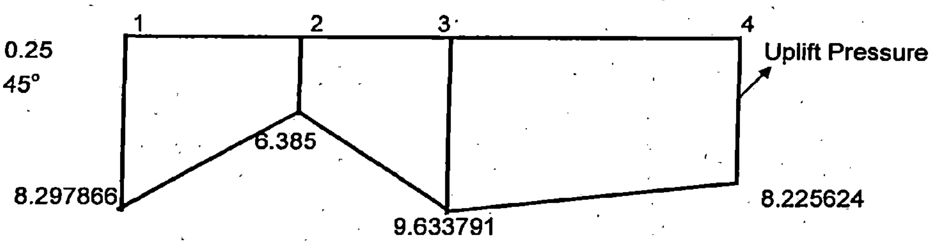
Case - I



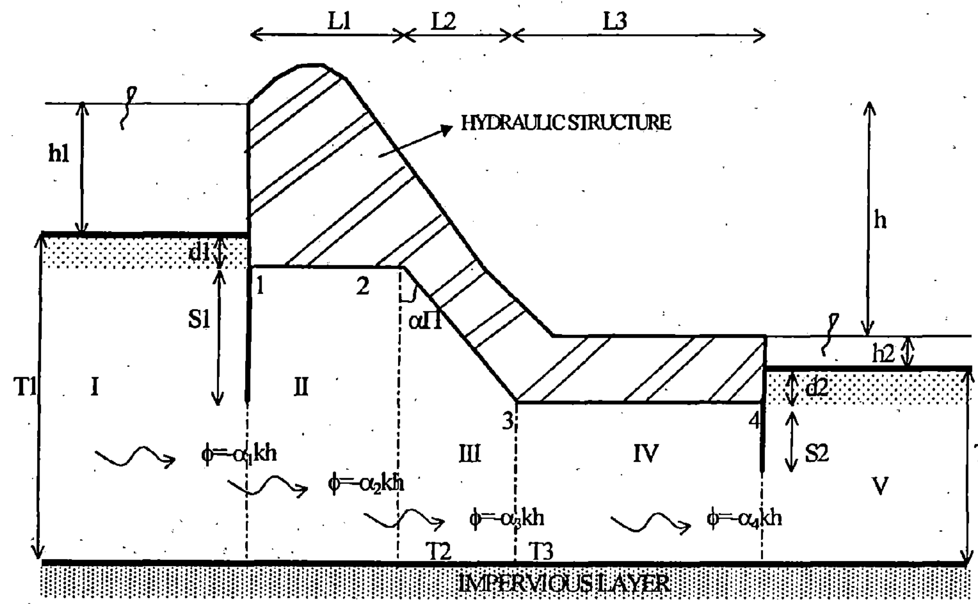
L1=10 L2=4 L3=10
T1=10 T2=9 T3=5 T4=6
d1=1 d2=1
s1=4 s2=4
h1=10 h2=2

Hence, the result :

For α = 0.25
$\alpha\Pi$ = 45°



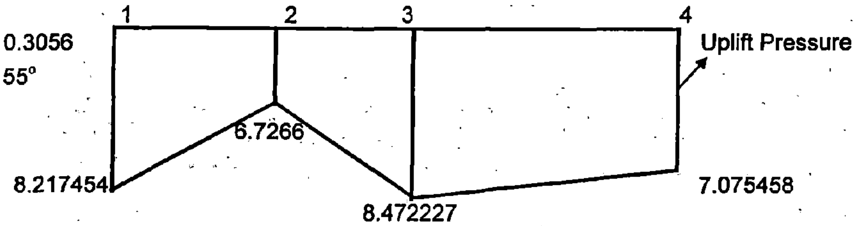
Case II :



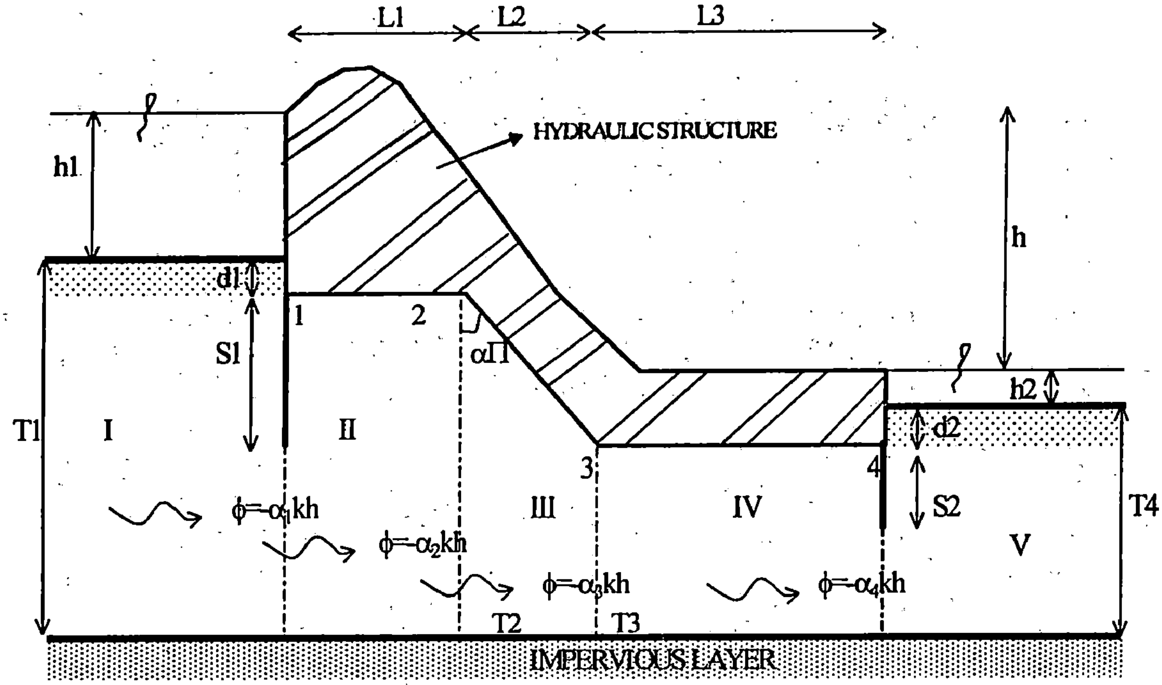
L1=10	L2=4	L3=10
T1=10	T2=9	T3=6.199 T4=7.199
d1=1	d2=1	
s1=4	s2=4	
h1=10	h2=2	

Hence, the result :

For α = 0.3056
$\alpha\Pi$ = 55°



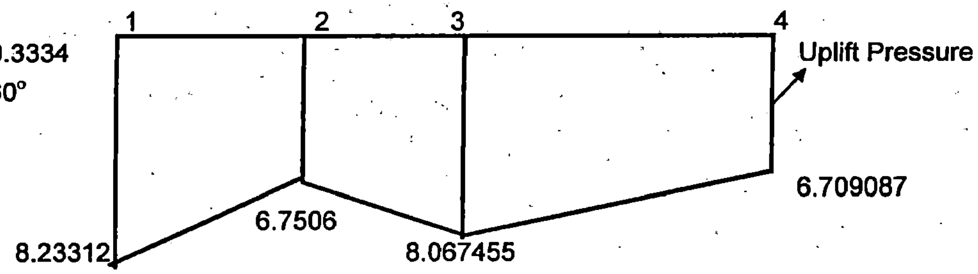
Case III



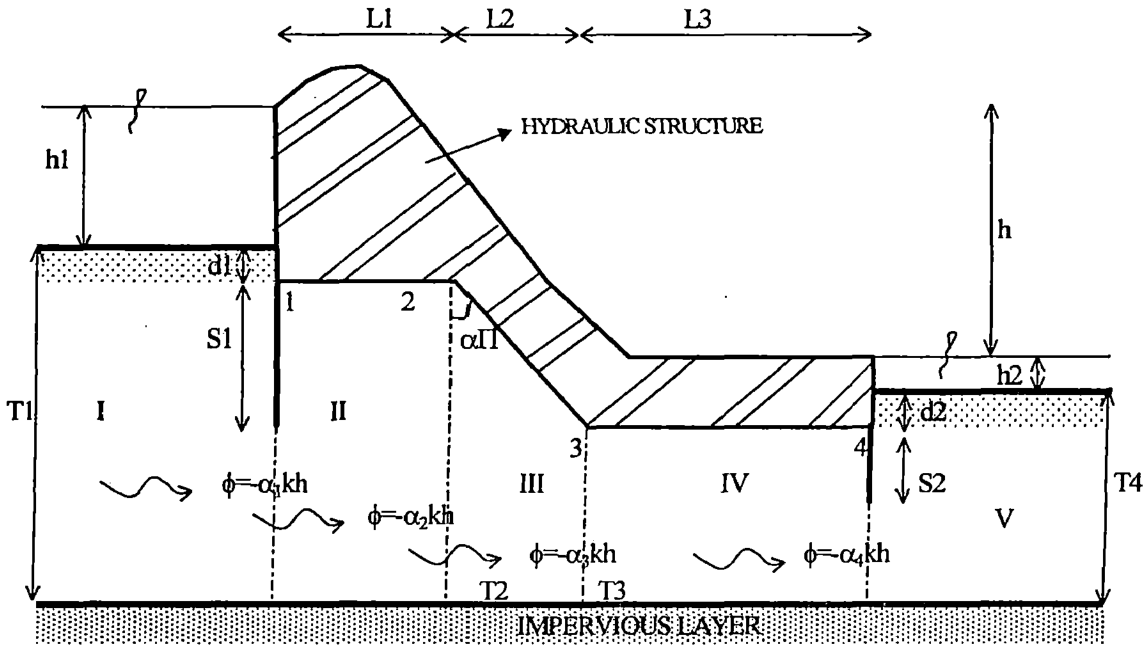
L1=10	L2=4	L3=10	
T1=10	T2=9	T3=6.69	T4=7.69
d1=1	d2=1		
s1=4	s2=4		
h1=10	h2=2		

Hence, the result :

For α = 0.3334
$\alpha\Pi$ = 60°



Case IV

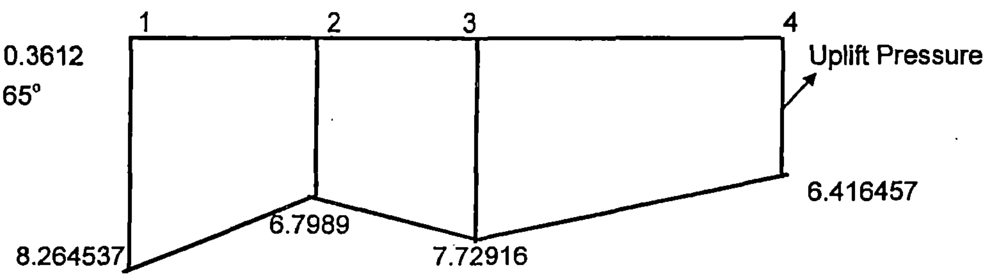


L1=10	L2=4	L3=10	
T1=10	T2=9	T3=7.13	T4=8.13
d1=1	d2=1		
s1=4	s2=4		
h1=10	h2=2		

Hence, the result :

For α = 0.3612

$\alpha\Pi$ = 65°



Result of quantity of seepage and exit gradient:

Hydraulic Structure	Case 1 45°	Case 2 55°	Case 3 60°	Case 4 65°
q/kh	0.1516	0.1734815	0.18072	0.1866
Exit Gradient	0.2057	0.20487	0.2039	0.202962

Table 4.1. Variation of Angle on Quantity of Seepage and Exit Gradient

CHAPTER - V

GENERAL DISCUSSION AND CONCLUSION

5.1. DISCUSSION

This chapter deals with the critical examination of the investigation reported in this thesis and the important conclusion derived from these investigations.

The conformal mapping technique is a powerful tool for solving two-dimensional Laplace's equation. The method is used for solving the problem of flow under hydraulic structures. Often the mapping steps result in a set of non-linear equation, which require a suitable technique to find unknown parameters. The implicit nature of the non-linear equation restricts the range of the applicability of conformal mapping. A methodology for solving a set of highly non-linear equation is described which can be used for solving two-dimensional flow in a complex flow domain with a great accuracy.

The aim of the present study is to investigate the accuracy on method of fragments as part of Analytical Method by Conformal Mapping for seepage analysis in two-dimensional steady confined through permeable foundation for various type of hydraulic structure.

Analysis of flow through anisotropic porous medium can be carried out by transforming the actual anisotropic flow domain with suitable coordinate transformation into fictitious isotropic flow domain for which the Laplace's equation is valid and conformal mapping technique are applicable. We can compute the discharge of seepage through anisotropic porous medium below hydraulic structure with vertical sheet pile.

After transformation into a fictitious isotropic flow region, the vertical sheet pile become inclined sheet pile. The method of fragments which is found to be accurate can be used to analyze flow under complex hydraulic structure.

Applying for seepage analysis under hydraulic structure with two sheet piles using method of fragments has been done in chapter - IV.

5.2 CONCLUSIONS

Investigations were done in seepage analysis under hydraulic structure by conformal mapping using Method of Fragments. The agreement between the result obtained from computer program and those derived mathematically in all cases was very satisfactory.

The flow characteristics (seepage quantity) computed using method of fragments match with those reported by Polubarinova-Kochina [9] and Reddy et.al (1971)[10].

The calculation of seepage analysis using Method of Fragments has some advantages:

1. The method is suitable for permeable foundation in finite depth.
2. The flow region is divided into number of tractable fragments, we can compute the seepage through each fragment which must be the same and this continuity equation enable solving composite complex structure with ease.
3. After calculation of potential, we can compute uplift the seepage and uplift pressure in each segment and exit gradient automatically.

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APPENDIX - A

CONFORMAL MAPPING

The transformation $z = w^2$ provides a certain geometrical similarity between the z plane and the w plane; that is, the mapping function preserves the angles of intersection and the approximate geometric shapes between planes except at the origin. A transformation that possesses the property of preserving angles of inter section and the approximate image of small shapes is said to be conformal. In the present section we shall explain the behavior of conformal transformation and the conditions under which they fail.

If $w = \phi + i\,\psi = f(z)$ is analytic within a region R its derivative $f'(z)$ is single-valued; that is, $f'(z)$ has only one value one value point in z. However, as z varies from point to point, $f'(z)$ will in general be a function of z. In Fig. A-1, let C be a smooth

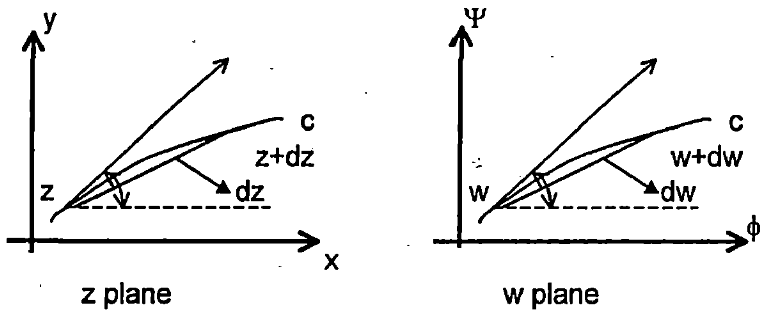


Fig. A-1 Conformal Mapping

curve through a point z, and let C_1 be its image through point ω under the transformation w = f (z) when f(z) is analytic at z and f' (z) ≠ 0. As f' (z) must be a complex number, say f' (z) = A exp iα, then from the definition of a derivative

$$f'(z) = \frac{dw}{dz} = \lim_{\Delta z \to 0} \frac{\Delta w}{\Delta z}$$

we obtain the two equations

$$\alpha = \arg f'(z) = \lim_{\Delta z \to 0} \left(\arg \frac{\Delta \omega}{\Delta z} \right) \tag{A.1.1}$$

$$A = |f'(z)| = \lim_{\Delta z \to 0} \left| \frac{\Delta \omega}{\Delta z} \right| \tag{A.1.2}$$

Now, as Δz→0, the limit of the argument of Δz approaches the angle θ_1. In a similar manner, as Δω is the image of Δz, the argument of Δω approaches the angle θ_2 as Δz→0. Hence from Eq. (A.1.1)

$$\alpha = \arg f'(z) = \theta_2 - \theta_1$$

or $$\theta_2 = \alpha + \theta_1 \tag{A.1.3}$$

Thus in the transformation from the z plane to the w plane the direction tangent to a curve at point z is rotated through the angle α = arg f' (z). Now, as f' (z) has only one value at any point z, any two curves intersection at a particular angle at point z will, even after transformation, intersect at the same angle at w (the image of z); that is, the sides of the angle at w are rotated in the same direction by the same amount.

Similarly, as Δz→0, we conclude form Eq. (A.1.2) that in the transformation from the z plane to the w plane, infinitesimal lengths in z are magnified at w by the factor A = mod f' (z). Now as Δz→ε > 0, Eq. (A.1.2) becomes only approximate and hence there is some distortion in the length Δw, with the degree of distortion depending on the magnitude of ε. Thus large figures in the z plane may transform into shapes bearing little resemblance to the original but, it should be emphasized, the angles formed by corresponding intersecting curves in these planes are preserved exactly even for large figures (except where f' (z) = 0).

Points at which f' (z) = 0 are said to be critical points of the transformation; that is, they represent points where angles are not preserved conformally. For example, the transformation $w = z^2$ demonstrates that angles at the origin where f' (z) = 0 are doubled. For the function $w = z^3$, angles at the origin are tripled. Indeed, it can be shown that for $w = z^n$, angles at z = 0 are multiplied nfold. A function f'(z) is said to have nfold zeros when n is the number of derivatives which are zero for a particular z. In other words, for $w = z^2$ the derivative f' (z) has one zero for z = 0, $w = z^3$ has a double zero at the origin. Thus we can generalize that if f' (z) has nfold zeros, angles are not preserved at the critical points but are multiplied (n+1) times (one greater that the number of zero derivatives).

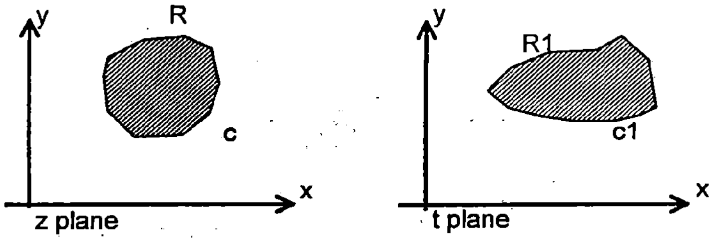
A.2. FUNDAMENTALS OF SOLUTIONS OF TWO-DIMENSIONAL FLOW PROBLEMS BY CONFORMAL MAPPING

The usefulness of conformal mapping in two-dimensional flow problems stems form the fact that solutions of Laplace's equation remain solutions when subjected to conformal transformations.

Let $\omega = \phi + i\psi = f(z)$ be the complex potential and let its real and imaginary parts satisfy Laplace's equation in the region R of the z plane (Fig. A.2), so that

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

Now suppose that there is a second analytic function z = F(t), with t = r + is, which maps the interior of the curve C into the interior of the curve C_1 (Fig. A.2b). The



(a) (b)

Fig. A.2 Curve Transformation

function of an analytic function, which in turn is also analytic and hence

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{\partial^2 \phi}{\partial s^2} = 0 \qquad \frac{\partial^2 \psi}{\partial r^2} + \frac{\partial^2 \psi}{\partial s^2} = 0$$

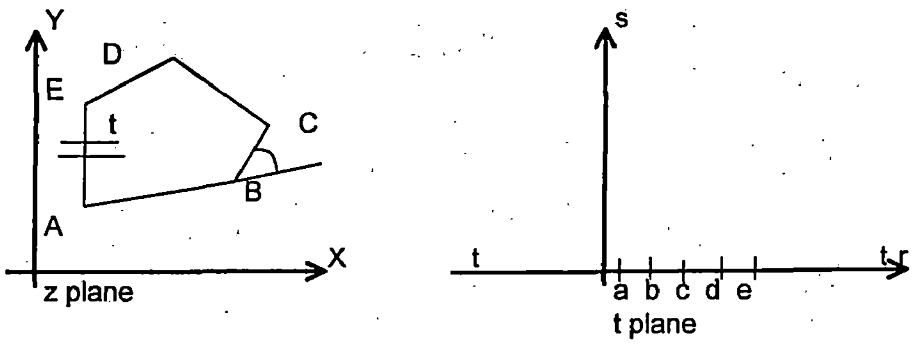
The solution of a two-dimensional ground water problem could be reduced to one of seeking the solution of Laplace's equation subject to certain boundary conditions with a region R in the z plane. A more or less direct attack was provided by the graphical construction of flow nets. From the standpoint of an analytical solution to Laplace's equation, unless the region R is of a very simple shape a direct approach to the problem is generally ver difficult. However, by means of conformal mapping, it is often possible to transform the region R into a simpler region R_1 wherein Laplace's equation can be solved subject to the transformed boundary conditions. Once the solution has been obtained in region R_1, it can be carried back by the inverse transformation to the region R, the original problem. Hence the crux of the problem is finding a transformation (or series of transformations) that will map a region R conformally into a region R_1 so that R_1 will be of a simple shape, such as a rectangle (whose sides may even extend to infinity) or a circle.

THE SCHWARZ-CHRISTOFFEL TRANSFORMATION

If a polygon is located in the z plane, then the transformation that maps it conformally onto the upper half of the t plane (t = r + is) is

$$z = M\int \frac{dt}{(t-a)^{1-A/\pi}(t-b)^{1-B/\pi}(t-c)^{1-C/\pi}\dots} + N \qquad \text{(B.1)}$$

where M and N are complex constants, A, B, C........, are the interior angels (in radians) of the polygon in the z plane (Fig. B-1.a), and a, b, c,.......(a<b<c<.....)are



points on the real axis of the t plane corresponding to the respective vertices A, B, C,......(Fig. B-1. b). We responding o the respective vertices A, B, C, ...(Fig. B-1).

(a) (b)

Fig. B-1. Z-plane and t-plane

We note, in particular, that the complex constant N corresponds to the point on the perimeter of the polygon that has its image at t = 0. Equation (B-1) is called the *Schwarz-Christoffel transformation* in honor of the two mathematicians, the German H.

A. Schwarz (1843-1921) and the Swiss E. B. Christoffel (1829-1900), who discovered it independently.

The transformation can be considered as the mapping of a polygon form the z plane onto a similar polygon in the t plane in such a manner that the sides of the polygon in the z plane extend through the real axis of the t plane. This is accomplished by opening the polygon at some convenient point, say between A and E of Fig.(B-1) a, and extending one side to t = - ∞ and the other to t = + ∞ (Fig. 4-13 b). In this operation the sides of the polygon are bent into a straight line extending from t = -∞ to t = + ∞ and are placed along the real axis of the t plane. The interior angle at the point of opening may be regarded as π (in the z plane) and, as noted in Eq. (B-1), takes no part in the transformation. The point of opening in the z plane is represented in the upper half of the t plane by a semicircle with a radius of infinity. Thus the Schwarz Christoffel transformation, in effect, maps conformally the region interior to the polygon ABC..... of the z plane into the interior of the polygon bounded by the sides ab, ac .....and a semicircle with a radius of infinity in the upper half of the t plane, or, more simply, into the entire upper half of the t plane.

To demonstrate the mechanism of the Schwarz-Christoffel transformation we recall that a derivative of the form dz/dt could be considered as a complex operator that transforms an element of t, by rotation and magnification, into a corresponding element in z. Thus, writing eq. (B-1) as

$$\frac{dz}{dt} = M(t-a)^{A/\pi-1}(t-b)^{B/\pi-1}(t-c)^{C/\pi-1}....... \qquad (B.2)$$

and taking the arguments of both sides, we find that any section of the real axis of t (where arg dt = 0) will be rotated in z by

$$\arg dz = L + \left(\frac{A}{\pi} - 1\right)\arg(t-a) + \left(\frac{B}{\pi} - 1\right)\arg(t-b) + \left(\frac{C}{\pi} - 1\right)\arg(t-c) + \ldots\ldots \text{(B.3)}$$

where L = arg M is a constant. Let us now consider the rotation of elements along the real axis of t, excluding the terminal points a, b, c, ……For $a < t < b$, arg (t-a) = 0 since (t-a) is real and positive, and arg (t-b) = arg (t-c) = ……. = π since (t-b), (t-c) ,…….., are all real and negative. Thus, for $a < t < b$, arg dz is a constant equal to

$$\arg dz = L + B - \pi + C - \pi + \ldots\ldots$$

which shows that this section of the t plane has its image in the z plane along a straight line (AB in Fig. B-1. a). In like manner, for $b < t < c$, we have

$$\arg dz = L + C - \pi + \ldots\ldots\ldots$$

which will have its image along the straight line BC; that is, arg dz for $b < t < c$ exceeds arg dz for $a < t < b$ by the positive angle π-B, which is precisely the deflection angle at point B. as t moves form $-\infty$ to $+\infty$ along the real axis, it is seen that z completes its circuit through the total external angular change of 2π radians and hence encloses the polygon ABC…. The complex constants M and N of eg. (B.1) merely control the size and position of the polygon.

The validity of the transformation at the points t = a, t = b, ….remains to be investigated. As t moves along the real axis through the point b, (t-b) changes from a negative to a positive number and arg (t-b) decreases from decreases from π to 0. Hence the third term in eq. (3) changes by $(B/\pi-1)(-\pi) = \pi - B$, which, as was noted previously, is the value of the positive deflection angle at the vertex B (Fig. B-2.a). In effect,

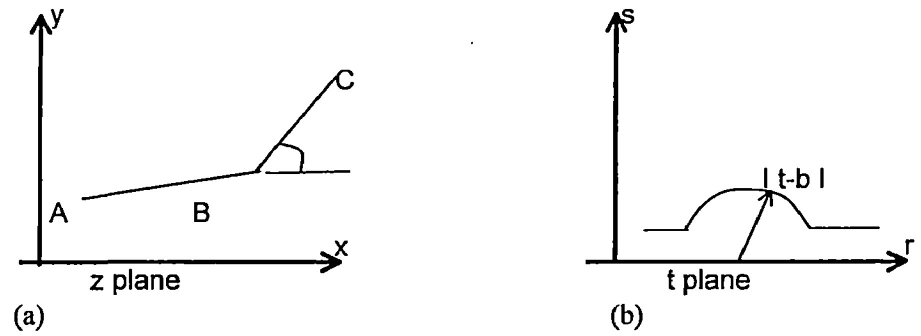


Fig. B-2 Deflection Angle

as z passes through the vertex B, its image in the t plane passes around an indented semicircle at point b (Fig. B-2.b), the radius of which $(|t-b|)$, can be made as small as we wish by adjusting $|M|$.

In many problems we shall place one or more vertices of the polygon in the z plane at infinity in the t plane. If, for example, a→-∞, we can take the complex constant M to be of argument L and modulus C $(-a)^{-A/\pi+1}$, so that eq. (B.2) becomes

$$\frac{dz}{dt} = Ce^{iL}(-a)^{1-A/\pi}(t-a)^{A/\pi-1}(t-b)^{B/\pi-1}....$$

or $$\frac{dz}{dt} = Ce^{iL}\left(\frac{t-a}{a}\right)^{A/\pi-1}(t-b)^{B/\pi-1}........$$

Now, as a→-∞, $[(t-a)/-a]^{A/\pi-1} \to 1$, and hence we see that factors corresponding to verities at infinity in the t plane do not appear in the transformation.

On the basis of the foregoing it follows that corresponding values of a, b, c, … and A, B, C, … can be chooses so that the polygons in their respective planes are similar. It can be shown [101] that any three of the values a, b, c, .. can be chosen arbitrarily to correspond to three of the vertices of the given polygon A, B, C,…… The (n-3) remaining values must then be determined so as to satisfy conditions of similarity. Whereas we shall often choose to map a vertex of the flow region (z plane) into one at infinity in the t plane, it is important to note that not only is this factor omitted from the transformation but the number of arbitrary values is reduced by 1.

METHOD OF FRAGMENTS

An approximate analytical method of solution for any confined flow system of finite depth, directly applicable to design, was furnished by Pavlovsky in 1935. The fundamental assumption of this method called the *method of fragments,* is that equipotential lines at various critical parts of the flow region can be approximated by straight vertical lines that divide the region into section or fragments,in fig.C.1.

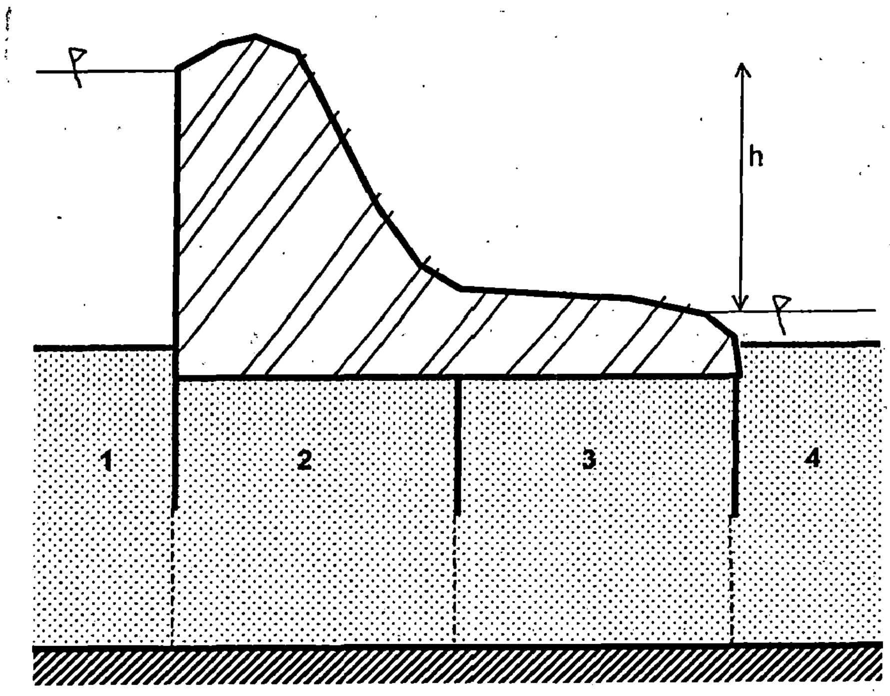


Fig. C.1 Weir with Three Sheet Piles

Suppose, now that we can compute the discharge in the mth fragment as

$$q = \frac{kh_m}{\Phi_m} \; m = 1,2.........,n \qquad (C.1)$$

Where h_m = head loss through fragment

Φ_m = dimensionless from factor*

Then, since the discharge through all fragments must be the same

$$q = \frac{kh_1}{\Phi_1} = \frac{kh_2}{\Phi 2} = \frac{kh_m}{\Phi m} = \ldots\ldots = \frac{kh_n}{\Phi_n}$$

$$q = k\frac{\sum h_m}{\sum \Phi} = \frac{kh}{\sum_{m=1}^{n} \Phi_m} \qquad (C.2)$$

Where h (without subscript) is the total loss through the section. By similar reasoning we find that the head loss in the m^{th} fragment can be calculated from

$$h_m = \frac{h\Phi_m}{\sum \Phi} \qquad (C.3)$$

Once the head loss for any fragment has been determined the pressure distribution on the base of the structure and the exit gradient can be easily obtained. Thus the primary task is to implement this method by establishing a catalogue of typical from factors will be divided into types, and the characteristics of each type will be studied. Finally, the results will be summarized in tabular from for easy reference.

Type I (Fig. C-1). The fragment of type I is a region of parallel horizontal flow between imperious boundaries. From Darcy's law, we have simply q = kah/L and hence the form factor is

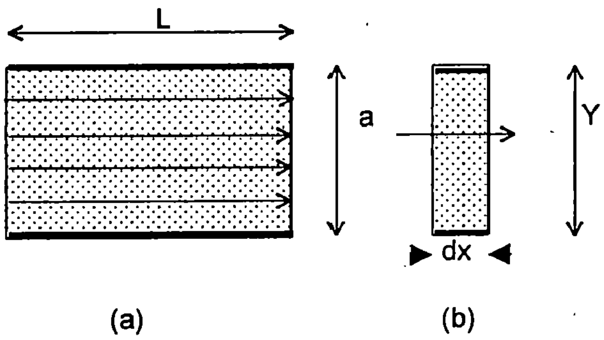


Fig. C-2 Type I

$$\Phi = \frac{L}{a} \qquad \text{(C.4a)}$$

For an elemental section (Fig. C-2.b),

$$d\Phi = \frac{dx}{y} \qquad \text{(C.4b)}$$

obviously, the pressure distribution for the type I fragment is linear.

Type II (fig. C.3), we find that the discharge for flow around a single sheetpile of embedments in a layer of thickness T (Fig. C.3a) is

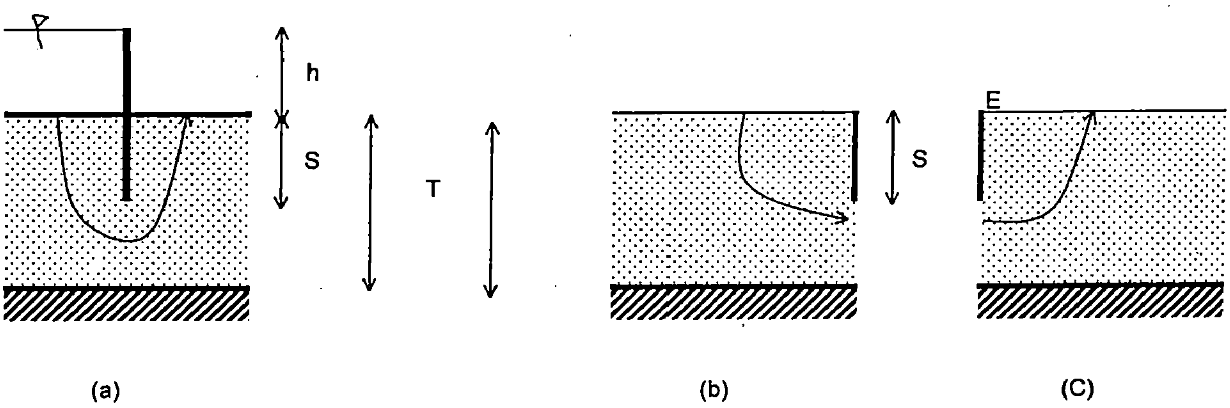


Fig. C-3 Type II

$$q = \frac{khK'}{2K}$$

with modulus m = sin (πs /2T). considering the type II fragment ads either of the section in Fig. C-3b or c (b is an entrance condition , c an exit condition), we have for these cases q = k h K'/K, where h is taken as the head loss through the fragment. The modulus as given above can be obtained directly From fig 5-15, Page 118, ME. Harr . Hence the form factor is

$$\phi = \frac{K}{K'} \quad m = \sin\frac{\pi 8}{2T} \tag{C.5a}$$

$$I_E = \frac{h\pi}{2KTm} \qquad m = \sin\frac{\pi 8}{2T} \tag{C.5b}$$

where h is again the head loss through the fragment.

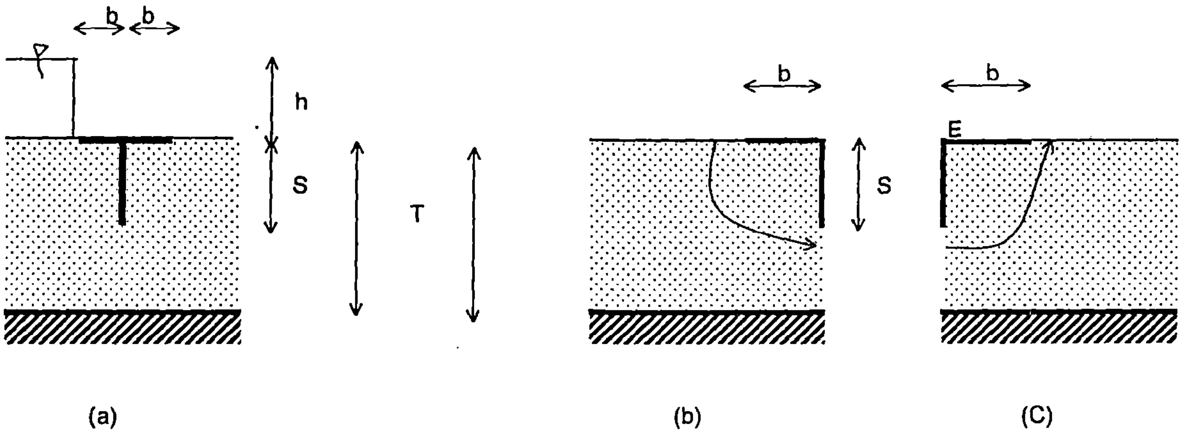


Fig. C-4 Type III

Type III (Fig. C-4). From Eq. (14c).(ME Harr) Sec. 5-5, the discharge for Fig. C-3a is q = khK'/2K, where the modulus is given by Eq. (14b)(ME HArr), Sec. 5-5. Hence for either of the fragments of Fig (C-4.b) or c

$$\Phi = \frac{K}{K'} \tag{C.6.a}$$

where the modulus

$$m = \cos\frac{\pi\delta}{2T}\sqrt{ta^2\frac{\pi b}{2T} + \tan^2 F\frac{\pi\delta}{2T}} \qquad \text{(C.6.b)}$$

can be obtained directly from Fig. 5-15. (M.E.Harr)

Type IV (Fig. C-5). Pavlovsky considers the sessions shown in (Fig. C.5.a) as his type IV fragments. The exact solution for this fragment is

$$\Phi = \frac{K}{K'}$$

with the modulus

$$m = \lambda \operatorname{sn}\left(\frac{a}{T}\Lambda, \lambda\right)$$

where Λ = complete elliptic integral of first kind of modulus λ

Λ' = complete elliptic integral of first kind of complementary modulus λ'

$$\frac{\Lambda}{\Lambda'} = \frac{T}{b}$$

The method of solution will now be demonstrated by an example.

To simplify the solution, Pavlovsky noted from his electrical analogue that the quantity of seepage above the streamline AB of (Fig. C-5b) was of small order and could be neglected. Hence he divided the flow region into two parts, labeled active an passive in Fig. C.5e, with the dividing

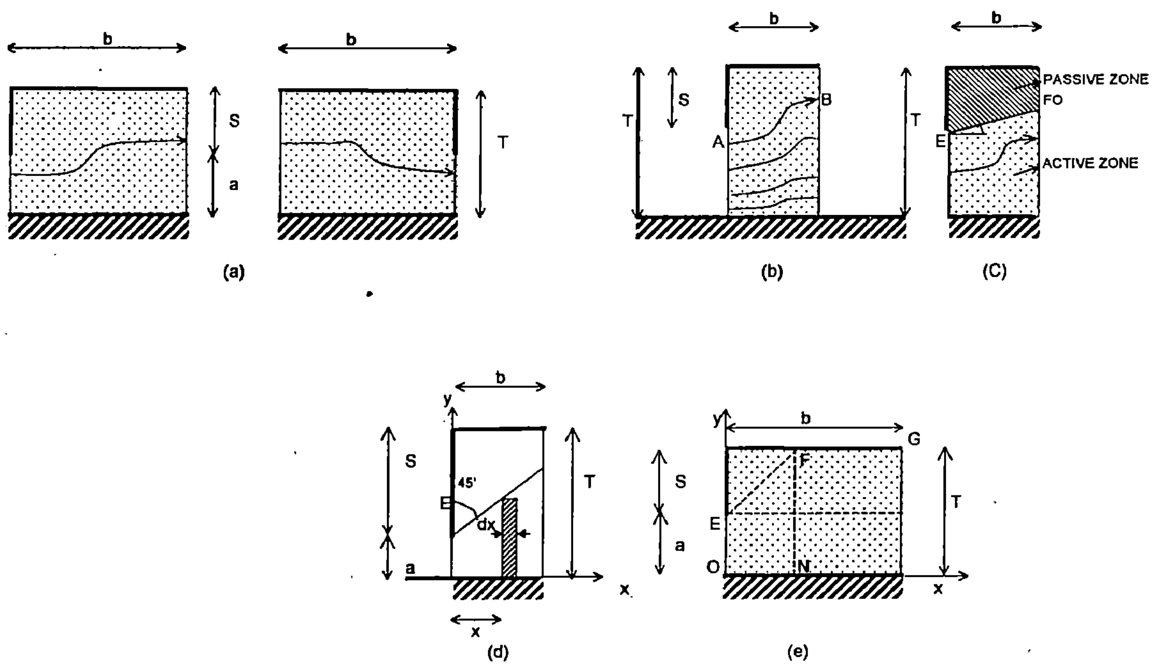


Fig. C.5 Type IV

Line EF_0 at an angle θ. On the basis of his analogue studies, Pavlovsky assumed $\theta = 45^0$. With this assumption, two conditions need to be considered for type IV fragments, depending in the ratio of b to s.

1. $b \leq s$. For this case (Fig. C-5d), following Pavlovsky, we shall consider the active zone to be composed of element s of type I fragments of width dx.

   Hence

$$\Phi = \int_0^b \frac{dx}{y} = \int_0^b \frac{dx}{a+x}$$

and the form factor is

$$\Phi = \ln\left(1 + \frac{b}{a}\right) \qquad \text{(C.7a)}$$

2. $b \geq s$. For this case (Fig. C-5e)

$$\Phi = \int_0^s \frac{ds}{a+x} + \int_s^b \frac{dx}{T}$$

and the form factor is

$$\Phi = \ln\left(1+\frac{s}{a}\right)+\frac{b-s}{T} \tag{C.7b}$$

Type V (Fig. C.6). We see from Fig. C-6 that the form factor for the type V fragment is twice that of the Type IV fragment; hence for L ≤ 2s,

$$\Phi = 2\ln\left(1+\frac{L}{2a}\right) \tag{C.8a}$$

and for L ≥ 2s

$$\Phi = 2\ln\left(1+\frac{S}{a}\right)+\frac{L-2s}{T} \tag{C.8b}$$

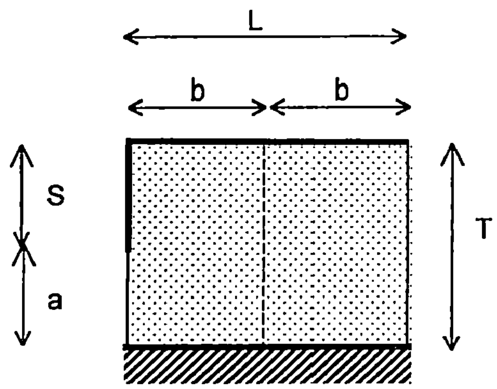


Fig.C.6 Type V

Type VI (Fig. C-7). Using the same approximations as for the type IV fragments, we see that two cases are to be considered.

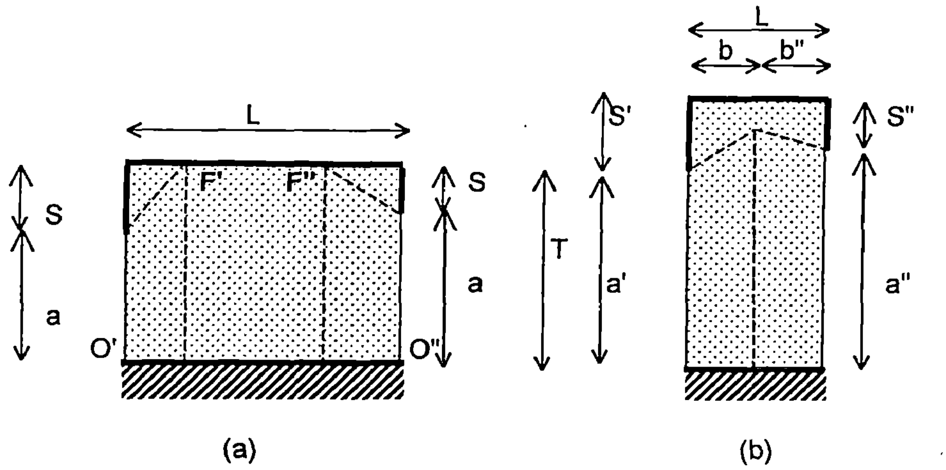


Fig. C-7 Type VI

1. $L \geq s' + s''$. Noting in this case (Fig. C-7a) that

$$\Phi = \int_0^{a'} \frac{dx}{a'+x} + \int_{s'}^{L-s''} \frac{dx}{T} + \int_{L-s''}^{L} \frac{dx}{a''+L-x}$$

We obtain

$$\Phi = \ln\left[\left(1+\frac{s'}{a'}\right)\left(1+\frac{s''}{a''}\right)\right] + \frac{L-(s'+s'')}{T} \qquad \text{(C-9a)}$$

2. $L \leq s' + s''$. For this case (Fig. C-7b), we have

$$\Phi = \int_0^{b_1} \frac{dx}{a'+x} + \int_{b_1}^{L} \frac{dx}{a''+L-x}$$

Hence

$$\Phi = \ln\left[\left(1+\frac{b'}{a'}\right)\left(1+\frac{b''}{a''}\right)\right] \qquad \text{(C.9b)}$$

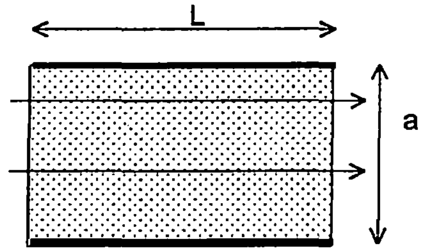
where

$$b' = \frac{L-(s'-s'')}{2} \qquad b'' = \frac{L-(s'-s'')}{2}$$

Table –C : Summary of Fragment Types and Form Factors

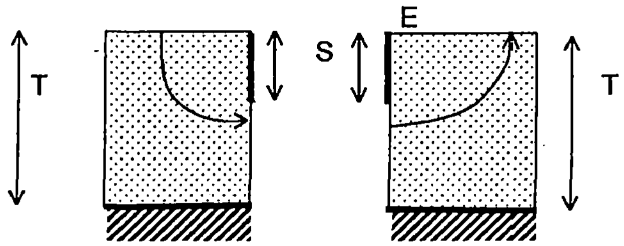
Fragment Type | **ϕ- Form Factor (h is head loss through fragment)**

I. $\Phi = \dfrac{L}{a}$



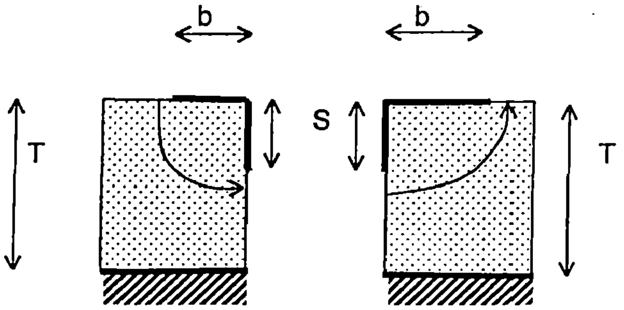
II. $\Phi = \dfrac{K}{K'}; m = \sin\dfrac{\pi s}{2T}$

$I_E = \dfrac{h\pi}{2kTm}$ (See Fig. 5-22,Harr)



III. $\Phi = \dfrac{K}{K'}$

$m = \cos\dfrac{\pi s}{2T}\sqrt{\tanh^2\dfrac{\pi b}{2T} + \tan^2\dfrac{\pi s}{2T}}$ (See Fig. 5-15, Harr)



IV.

Exact solution (see Example 6-1,Harr) :

$$\frac{\Lambda}{\Lambda'} = \frac{T}{b}; \text{modulus} = \lambda$$

$$\Phi = \frac{K'(m)}{K(m)}; m = \lambda \, sn\left(\frac{a}{T}\Lambda, \lambda\right)$$

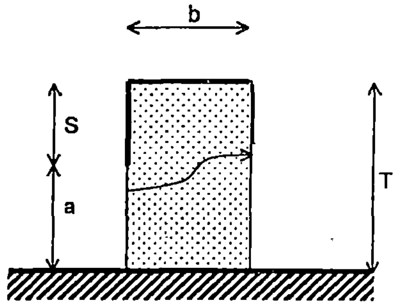
Approximate solution:

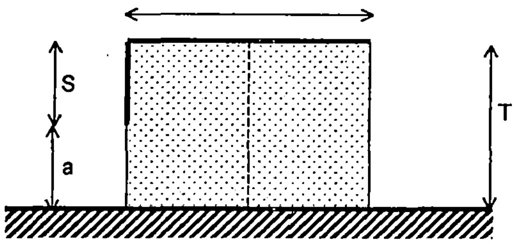
S ≥ b:

$$\Phi = \ln\left(1 + \frac{b}{a}\right)$$

S ≥ b:

$$\Phi = \ln\left(1 + \frac{s}{a}\right) + \frac{b-s}{T}$$





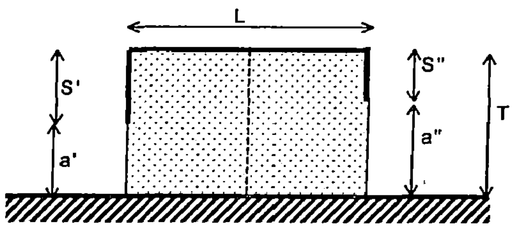
V.

L ≤ 2s:

$$\Phi = 2\ln\left(1 + \frac{L}{2a}\right)$$

L ≤ 2s:

$$\Phi = 2\ln\left(1 + \frac{s}{a}\right) + \frac{L-2s}{T}$$



VI.

L > s' + s":

$$\Phi = \ln\left[\left(1+\frac{s'}{a'}\right)\left(1+\frac{s''}{a''}\right)\right] + \frac{L-(s'+s'')}{T}$$

L = s' + s":

$$\Phi = \ln\left[\left(1+\frac{s'}{a'}\right)\left(1+\frac{s''}{a''}\right)\right]$$

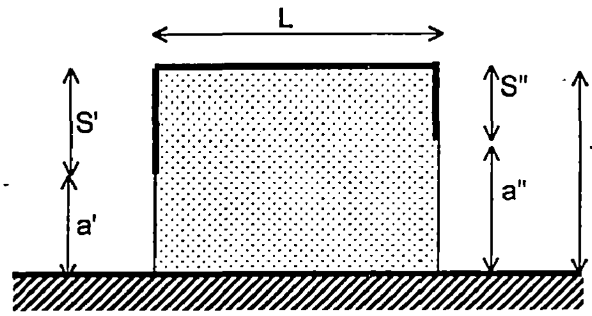
L < s' + s":

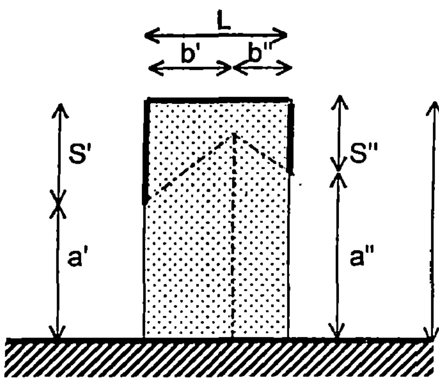
$$\Phi = \ln\left[\left(1+\frac{b'}{a'}\right)\left(1+\frac{b''}{a''}\right)\right]$$

where

$$b' = \frac{L+(s'-s'')}{2}$$

$$b'' = \frac{L-(s'-s'')}{2}$$





The various fragment types and pertinent relationships are presented in Table C for easy reference. To determine the pressure distribution on the base of a structure (such as that along C′CC″. In Fig. (C-8), we shall assume that the head loss within the fragment is linearly distributed along the impervious boundary. Thus, in Fig. C-8 if h_m is the head loss within the fragment, the rate of loss along E′ C′CC″ E″ will be

$$R = \frac{h_m}{L + s' + s''} \qquad \text{(C.10)}$$

Once the total head is known at any point, the pressure can easily be determined.

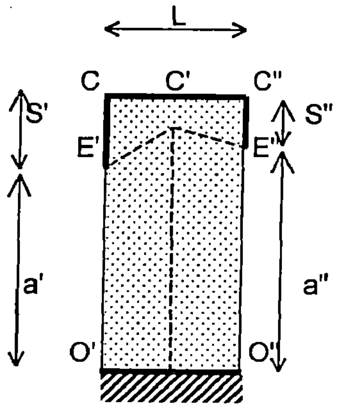


Fig C-8

APPENDIX - D

COMPUTER PROGRAM OF FLOW UNDER A WEIR WITH INCLINED SHEET PILE

```
      DIMENSION W(96), XX(96)
      OPEN(UNIT=1,STATUS='OLD',FILE='SABL5.DAT')
      OPEN(UNIT=2,STATUS='UNKNOWN',FILE='SABL5.OUT')
      OPEN(UNIT=3,STATUS='OLD',FILE='GAUSS.DAT')
C     THIS PROGRAMME IS FOR S/T>1
      READ(1,*) S,AL,T,ALPHA
      READ(3,*)(W(I),I=1,96)
      READ(3,*)(XX(I),I=1,96)
C     SEGMENT I
      WRITE(2,200)
200   FORMAT(5X,'AL',8X,'S',9X,'T',10X,'ALPHA')
      WRITE(2,201)AL,S,T,ALPHA
201   FORMAT(4F10.4)
C     COMPUTATION  OF B
      CALL RAPHB1(ALPHA,S,T,B,RESIDUE)
      WRITE(2,*) 'B=', B
      WRITE(2,*)'RESIDUE=',RESIDUE
C     COMPUTATION OF D
      CALL RAPHD(W,XX,ALPHA,AL,T,D,RESIDUE)
      WRITE(2,*) 'D=', D
      WRITE(2,*)'RESIDUE=',RESIDUE
C     COMPUTATION FOR SEGM1
C     FOR AL=0,D=0.
C     D=0.
      AKS=(1.-B)/(1.+D)
      CALL CEF(W,XX,AKS,CEF1)
      TERM1=CEF1
      AKS=(B+D)/(1.+D)
      WRITE(2,*)'AKS=',AKS
      CALL CEF(W,XX,AKS,CEF1)
      TERM2=CEF1
      SEGM1=TERM1/TERM2
      WRITE (2,*)'SEGM1=',SEGM1
C     SEGMENT II
C     COMPUTATION OF B
      CALL RAPHB2(ALPHA,S,T,B,RESIDUE)
      WRITE(2,*) 'B=', B
      WRITE(2,*)'RESIDUE=',RESIDUE
```

```
C      COMPUTATION FOR SEGM2
       AKS=B
       CALL CEF(W,XX,AKS,CEF1)
       TERM1=CEF1
       AKS=1.-B
       CALL CEF(W,XX,AKS,CEF1)
       TERM2=CEF1
       SEGM2=TERM1/TERM2
       WRITE (2,*)'SEGM2=',SEGM2
       ALPHA1=SEGM1/(SEGM1+SEGM2)
       QBYKH1=ALPHA1*SEGM2
       QBYKH2=(1.-ALPHA1)*SEGM1
       SBYT=S/T
       ALBYT=AL/T
       WRITE(2,202)
202    FORMAT(8X,'AL/T',10X,'S/T',10X,'Q/KH')
       WRITE(2,203)ALBYT,SBYT,QBYKH1
203    FORMAT(F10.2,5X,F10.2,5X,F10.4)
       STOP
       END

       SUBROUTINE RAPHB1(ALPHA,S,T,B,RESIDUE)
       PAI=3.14159265
       BETAC=PAI/(SIN(PAI*ALPHA))
       TERM1=S/T*SIN(ALPHA*PAI)
       P=ALPHA
       Q=1.-ALPHA
       I=1
       B=0.000001
       DELB=(0.9999999-B)/100.
100    CONTINUE
       CALL BETAIN(P,Q,B,BETAI)
       RESIDUE=TERM1-BETAI/BETAC
       IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
       IF(B.GE.1.0000000) GO TO 600
       B=B+DELB
       I=I+1
       IF(I.GT.500) GO TO 600
       IF (RESIDUE.GT.0.0) GO TO 100
       BR=B-DELB
       BL=BR-DELB
200    B=(BL+BR)/2.
       CALL BETAIN(P,Q,B,BETAI)
       RESIDUE=TERM1-BETAI/BETAC
       IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
```

```
      I=I+1
      IF(I.GT.500) GO TO 600
      IF(RESIDUE.GT.0.) GO TO 300
      IF(RESIDUE.LT.0.) GO TO 400
 300  BL=B
      GO TO 200
 400  BR=B
      GO TO 200
 600  CONTINUE
      WRITE(2,*)'ITERATION HAS FAILED IN COMPUTING B IN SEGMENT I'
 500  CONTINUE
      RETURN
      END

      SUBROUTINE RAPHB2(ALPHA,S,T,B,RESIDUE)
      PAI=3.14159265
      BETAC=PAI/(SIN(PAI*ALPHA))
      TERM1=1.-S/T*SIN(ALPHA*PAI)
C     write(2,*)'term1=',term1
      P=ALPHA
      Q=1.-ALPHA
      B=0.000001
      DBETA=(0.999999-0.000001)/10.
      I=1
 100  CONTINUE
      IF(B.GE.1.00000000) GO TO 600
      CALL BETAIN(P,Q,B,BETAI)
      RESIDUE=TERM1-BETAI/BETAC
c     term2=betai/betac
c     write(2,*)i,b,term1,term2,residue
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      I=I+1
      IF(I.GT.500) GO TO 600
      B=B+DBETA
      IF (RESIDUE.GT.0.0) GO TO 100
      BR=B-DBETA
      write(2,*)'this point is crossed'
c     CALL BETAIN(P,Q,BR,BETAI)
c     RESIDUE=TERM1-BETAI/BETAC
c     WRITE(2,*)'BR=',BR,'RESIDUE=',RESIDUE
      BL=BR-DBETA
c     CALL BETAIN(P,Q,BL,BETAI)
c     RESIDUE=TERM1-BETAI/BETAC
c     WRITE(2,*)'BL=',BL,'RESIDUE=',RESIDUE
```

```
200   B=(BL+BR)/2.
      CALL BETAIN(P,Q,B,BETAI)
      RESIDUE=TERM1-BETAI/BETAC
      IF ( ABS(RESIDUE).LT.0.0001) GO TO 500
      I=I+1
      IF(I.GT.200) GO TO 600
      IF(RESIDUE.GT.0.) GO TO 300
      IF(RESIDUE.LT.0.) GO TO 400
300   BL=B
c     write(2,*)'iteration entered 300'
      GO TO 200
400   BR=B
c     write(2,*)'iteration entered 400'
      GO TO 200
600   CONTINUE
      WRITE(2,*)'ITERATION HAS FAILED COMPUTING B IN SEGMENT II'
500   CONTINUE
      RETURN
      END
      SUBROUTINE RAPHD(W,XX,ALPHA,AL,T,D,RESIDUE)
      DIMENSION W(96),XX(96)
      PAI=3.14159265
      BETAC=PAI/(SIN(PAI*ALPHA))
      I=1
      D=0.00001
      DELD=0.1
      TERM=AL/T*SIN(ALPHA*PAI)
100   CONTINUE
      CALL AINT(W,XX,ALPHA,D,G)
      RESIDUE=TERM-G/BETAC
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      I=I+1
      IF(I.GT.500) GO TO 600
      D=D+DELD
      IF(RESIDUE.GT.0.0) GO TO 100
      DR=D-DELD
      DL=DR-DELD
200   D=(DL+DR)/2.
      CALL AINT(W,XX,ALPHA,D,G)
      RESIDUE=TERM-G/BETAC
      IF(ABS(RESIDUE).LT.0.00001) GO TO 500
      I=I+1
      IF(I.GT.500) GO TO 600
      IF(RESIDUE.GT.0.) GO TO 300
      IF(RESIDUE.LT.0.) GO TO 400
```

```
300   DL=D
      GO TO 200
400   DR=D
      GO TO 200
600   CONTINUE
      WRITE(2,*)'ITERATION HAS FAILED IN COMPUTING D'
500   CONTINUE
      RETURN
      END

      SUBROUTINE AINT(W,XX,ALPHA,D,G)
      DIMENSION W(96), XX(96)
      SUM=0.0
      TERM=0.5*D**(1./8.)

      DO 10 I=1,96
      TERM1=TERM*(1.+XX(I))
      TERM2=TERM1**(8.*ALPHA-1.)
      TERM3=(1.+TERM1**8)**ALPHA
      TERM4=TERM2/TERM3
      SUM=SUM+W(I)*TERM4
10    CONTINUE
      G=SUM*TERM*8.
      RETURN
      END
      SUBROUTINE CEF(W,XX,AKS,CEF1)
      DIMENSION W(96), XX(96)
      PAI=3.141592654
      SUM=0.0
      DO 10 I=1,96
      THETA=PAI/4.*(1.+XX(I))
      TERM=0.25*PAI/SQRT(1.-AKS*SIN(THETA)*SIN(THETA))
      SUM=SUM+TERM*W(I)
10    CONTINUE
      CEF1=SUM
      RETURN
      END
      SUBROUTINE BETAIN(P,Q,X,BETAI)
      IF(Q.GT.1.) GO TO 100
      I=1
      C4=P
      C5=1.0-Q
      C6=1.0+P
      C7=1.0
      C9=1.0
      C10=1.0
```

```
7       C9=C9*X*(C4/C6)*C5/C7
        C10=C10+C9
        C4=C4+1.0
        C5=C5+1.0
        C6=C6+1.0
        C7=C7+1.0
        I=I+1
        A=ABS(C9)
        IF(A.GT.0.0000001) GO TO 7
        BETAI=X**P*C10/P
        RETURN

100     X=1.-X
        P1=Q
        Q1=P
        C4=P1
        C5=1.0-Q1
        C6=1.0+P1
        C7=1.0
        C9=1.0
        C10=1.0
77      C9=C9*X*(C4/C6)*C5/C7
        C10=C10+C9
        C4=C4+1.0
        C5=C5+1.0
        C6=C6+1.0
        C7=C7+1.0
        A=DABS(C9)
        IF(A.GT.0.0000001) GO TO 77
        BETAI=X**P*C10/P
        RETURN
        END
```

COMPUTER PROGRAM OF FLOW UNDER A WEIR WITH TWO SHEET PILES

```
      DIMENSION W(96),X(96)
      DIMENSION A(4,4),RCOL(4)

      OPEN (1,STATUS='OLD',FILE='GAUSS.DAT')
      OPEN (2,STATUS='UNKNOWN',FILE='SABM8.OUT')
      OPEN (3,STATUS='OLD',FILE='SABM8.DAT')

      READ(1,*)(W(I),I=1,96)
      READ(1,*)(X(I),I=1,96)
      READ(3,*)AL1,AL2,AL3
      READ(3,*)T1,T2,T3,T4
      READ(3,*)DT1,DT2
      READ(3,*)S1,S2
      READ(3,*)ALPA
      READ(3,*)h1,h2
      PAI=3.141592654
C     COMPUTATION OF DISCHARGE OF SEEPAGE IN SEGMENT I
      WRITE(2,*)'OUTPUT OF COMPUTER PROGRAM IN SEGMENT I'
C     COMPUTATION BT1
      BT1=(SIN((PAI/2)*((DT1+S1)/T1)))**2
      WRITE(2,*)'BT1= ',BT1
C     COMPUTATION SEGM1
      AKS=1.-BT1
      CALL CEF(W,X,AKS,CEF1)
      TER1=CEF1
      AKS1=BT1
      CALL CEF(W,X,AKS1,CEF2)
      TER2=CEF2
      SEGM1=TER1/TER2
      WRITE(2,*)'SEGM1=',SEGM1

C     COMPUTATION OF DISCHARGE OF SEEPAGE IN SEGMENT II
      WRITE(2,*)'OUTPUT OF COMPUTER PROGRAM IN SEGMENT II'
C     COMPUTATION B2
      CALL RAPHB2(W,X,AL1,T2,B2,RESIDUE)
      WRITE(2,*)'B2=', B2
C     COMPUTATION D2
      CALL RAPHD2(W,X,T2,S1,B2,D2,RESIDUE)
      WRITE(2,*)'D2=',D2
```

```
C     COMPUTATION SEGM2
      AKS=(1.0-B2)/(1.0+D2)
      CALL CEF(W,X,AKS,CEF1)
      TERM1=CEF1
      AKS1=(B2+D2)/(1.0+D2)
      CALL CEF(W,X,AKS1,CEF2)
      TERM2=CEF2
      SEGM2=TERM1/TERM2
      WRITE(2,*)'SEGM2=',SEGM2

C     COMPUTATION OF DISCHARGE OF SEEPAGE IN SEGMENT III
      WRITE(2,*)'OUTPUT OF COMPUTER PROGRAM IN SEGMENT III'
C     COMPUTATION B3
      CALL RAPHB3(W,X,ALPA,AL2,T3,B3,RESIDUE)
      WRITE(2,*)'B3=',B3
C     COMPUTATION SEGM3
      AKS=1.0-B3
      CALL CEF(W,X,AKS,CEF1)
      TERM1=CEF1
      AKS1=B3
      CALL CEF(W,X,AKS1,CEF2)
      TERM2=CEF2
      SEGM3=TERM1/TERM2
      WRITE (2,*)'SEGM3=',SEGM3

C     COMPUTATION OF DISCHARGE OF SEEPAGE IN SEGMENT IV
      WRITE(2,*)'OUTPUT OF COMPUTER PROGRAM IN SEGMENT IV'
C     COMPUTATION C4
      CALL RAPHC4(W,X,AL3,T3,C4,RESIDUE)
      WRITE(2,*)'C4=',C4
C     COMPUTATION B4
      CALL RAPHB4(W,X,S2,T3,B4,C4,RESIDUE)
      WRITE(2,*)'B4=', B4
C     COMPUTATION SEGM4
      AKS=1.0-B4
      CALL CEF(W,X,AKS,CEF1)
      TERM1=CEF1
      AKS1=B4
      CALL CEF(W,X,AKS1,CEF2)
      TERM2=CEF2
      SEGM4=TERM1/TERM2
      WRITE(2,*)'SEGM4=',SEGM4

C     COMPUTATION OF DISCHARGE OF SEEPAGE IN SEGMENT V
      B5=(SIN((PAI/2)*(1-((DT2+S2)/T4))))**2
      WRITE(2,*)'OUTPUT OF COMPUTER PROGRAM IN SEGMENT V'
      WRITE(2,*) 'B5= ', B5
```

```
C     COMPUTATION SEGM5
      AKS=B5
      CALL CEF(W,X,AKS,CEF1)
      TERM1=CEF1
      AKS1=1.0-B5
      CALL CEF(W,X,AKS1,CEF2)
      TERM2=CEF2
      SEGM5=TERM1/TERM2
      WRITE(2,*)'SEGM5=',SEGM5

C     COMPUTATION OF MATRIX OF ALPHA(1,2,3,4,)
      A1=SEGM1
      B1=SEGM2
      C1=SEGM3
      D1=SEGM4
      E1=SEGM5

      RCOL(1)=A1
      RCOL(2)=0.
      RCOL(3)=0.
      RCOL(4)=0.

      DO I=1,4
      DO J=1,4
      A(I,J)=0.
      END DO
      END DO
      A(1,1)=A1+B1
      A(1,2)=-B1

      A(2,1)=B1
      A(2,2)=-(B1+C1)
      A(2,3)=C1

      A(3,2)=C1
      A(3,3)=-(C1+D1)
      A(3,4)=D1

      A(4,3)=D1
      A(4,4)=-(D1+E1)

      MMM=4
      CALL MATIN(A,MMM)

      SUM1=0.
      SUM2=0.
      SUM3=0.
```

```
      SUM4=0.
      DO    J=1,4
      SUM1=SUM1+A(1,J)*RCOL(J)
      SUM2=SUM2+A(2,J)*RCOL(J)
      SUM3=SUM3+A(3,J)*RCOL(J)
      SUM4=SUM4+A(4,J)*RCOL(J)
      END DO
      ALPHA1=SUM1
      ALPHA2=SUM2
      ALPHA3=SUM3
      ALPHA4=SUM4

      WRITE(2,*)'OUTPUT OF COMPUTER PROGRAM OF ALPHA'
      WRITE(2,*)'ALPHA1=',ALPHA1
      WRITE(2,*)'ALPHA2=',ALPHA2
      WRITE(2,*)'ALPHA3=',ALPHA3
      WRITE(2,*)'ALPHA4=',ALPHA4

C     COMPUTATION OF QUANTITY OF SEEPAGE
      QBKH=(1.-ALPHA1)*A1
      WRITE(2,*)'q/kh=',QBKH

C     COMPUTATION OF UPLIFT PRESSURE
C     SEGMENT II
      h=(h1+T1)-(T4+h2)
      AKS=(B2+D2)/(1.+D2)
      PHAI=ASIN(SQRT(D2/(B2+D2)))
      CALL CIEF(W,X,AKS,PHAI,CIEF1)
      TIM1=CIEF1
      AKS=(1.-B2)/(1.+D2)
      CALL CEF(W,X,AKS,CEF1)
      TIM2=CEF1
      TIM3=TIM1/TIM2
      PHIBKH=QBKH*TIM3-ALPHA1
      WRITE(2,*)'PHIBKH2=',PHIBKH
      P1BGW=-PHIBKH*h-T2+h2+T4
      WRITE(2,*)'(P1/GW)=',P1BGW

      P2BGW=ALPHA2*h-T2+h2+T4
      WRITE(2,*)'(P2/GW)=',P2BGW

C     SEGMENT III
      P3BGW=ALPHA3*h-T3+h2+T4
      WRITE(2,*)'(P3/GW)=',P3BGW

C     SEGMENT IV
      AKS=B4
```

```
      PHAI=C4/B4
      CALL CIEF(W,X,AKS,PHAI,CIEF1)
      TEM1=CIEF1
      AKS=(1.-B4)
      CALL CEF(W,X,AKS,CEF1)
      TEM2=CEF1
      TEM3=TEM1/TEM2
      PHIBKH=QBKH*TEM3-ALPHA3
      WRITE(2,*)'PHIBKH4=',PHIBKH
      P4BGW=-PHIBKH*h-T3+h2+T4
      WRITE(2,*)'(P4/GW)=',P4BGW

C     COMPUTATION OF EXIT GRADIENT
C     SEGMENT V
      AKS=1.0-B5
      CALL CEF(W,X,AKS,CEF1)
      TERM5=CEF1
      h=h1-h2
      AIE=ALPHA4*(PAI*h/(2.*T4))/(TERM5*(1.-B5)**0.5)
      WRITE(2,*)'IE=',AIE
      STOP
      END


      SUBROUTINE RAPHB2(W,X,AL1,T2,B2,RESIDUE)
      U2=AL1/T2
      BINI=0.01
      DELB=0.01
      B2=BINI
    1 B2=B2+DELB
      AKS=B2
      CALL CEF(W,X,AKS,CEF1)
      TEM1=CEF1
      AKS1=1.0-B2
      CALL CEF(W,X,AKS1,CEF2)
      TEM2=CEF2
      RESIDUE=U2-(TEM1/TEM2)
      IF(RESIDUE.GT.0.00) GO TO 1
      BR=B2
      BL=BR-DELB
    2 B2=(BR+BL)/2.0
      AKS=B2
      CALL CEF(W,X,AKS,CEF1)
      TEM1=CEF1
      AKS1=1.-B2
      CALL CEF(W,X,AKS1,CEF2)
      TEM2=CEF2
```

```
      RESIDUE=U2-(TEM1/TEM2)
      IF(ABS(RESIDUE).LT.0.0001) GO TO 5
      IF(RESIDUE.GT.0.0) GO TO 4
      IF(RESIDUE.LT.0.0) GO TO 3
    3 BR=B2
      GO TO 2
    4 BL=B2
      GO TO 2
    5 CONTINUE
      RETURN
      END


      SUBROUTINE RAPHD2(W,X,T2,S1,B2,D2,RESIDUE)
      I=1
      V2=(T2-S1)/T2
      D2=0.0001
      DELD=0.01
    6 CONTINUE
      I=I+1
      AKS1=1.-B2
      CALL AIEF(W,X,B2,D2,AIEFK)
      TIM1=AIEFK
      AKS1=1.-B2
      CALL CEF(W,X,AKS1,CEF2)
      TIM2=CEF2
      RESIDUE=(TIM1/TIM2)-V2
      IF(ABS(RESIDUE).LT.0.00001)GO TO 10
      D2=D2+DELD
      IF(RESIDUE.GT.0.00) GO TO 6
      DR=D2-DELD
      DL=DR-DELD
    7 D2=(DR+DL)/2.0
      I=I+1
      AKS1=1.-B2
      CALL AIEF(W,X,B2,D2,AIEFK)
      TIM1=AIEFK
      AKS1=1.-B2
      CALL CEF(W,X,AKS1,CEF2)
      TIM2=CEF2
      RESIDUE=(TIM1/TIM2)-V2
      IF(ABS(RESIDUE).LT.0.0001) GO TO 10
      IF(I.GT.100) GO TO 10
      IF(RESIDUE.GT.0.0) GO TO 9
      IF(RESIDUE.LT.0.0) GO TO 8
    8 DR=D2
      GO TO 7
```

```
  9 DL=D2
    GO TO 7
 10 CONTINUE
    RETURN
    END

    SUBROUTINE RAPHB3(W,X,ALPA,AL2,T3,B3,RESIDUE)
    PAI=3.141592654
    U3=T3/AL2*SIN(ALPA*PAI)
    BINI=0.01
    DELB=0.01
    B3=BINI
 11 B3=B3+DELB
    CALL AINA(W,X,ALPA,B3,AINA1)
    TEM1=AINA1
    CALL AINB(W,X,ALPA,B3,AINB1)
    TEM2=AINB1
    CALL AINC(W,X,ALPA,B3,AINC1)
    TEM3=AINC1
    CALL AIND(W,X,ALPA,B3,AIND1)
    TEM4=AIND1
    TEM5=(TEM3+TEM4)/(TEM1+TEM2)
    RESIDUE=TEM5-U3
    IF(RESIDUE.GT.0.00) GO TO 11
    BR=B3
    BL=BR-DELB
 12 B3=(BR+BL)/2.0
    CALL AINA(W,X,ALPA,B3,AINA1)
    TEM1=AINA1
    CALL AINB(W,X,ALPA,B3,AINB1)
    TEM2=AINB1
    CALL AINC(W,X,ALPA,B3,AINC1)
    TEM3=AINC1
    CALL AIND(W,X,ALPA,B3,AIND1)
    TEM4=AIND1
    TEM5=(TEM3+TEM4)/(TEM1+TEM2)
    RESIDUE=TEM5-U3
    IF(ABS(RESIDUE).LT.0.0001) GO TO 15
    IF(RESIDUE.GT.0.0) GO TO 14
    IF(RESIDUE.LT.0.0) GO TO 13
 13 BR=B3
    GO TO 12
 14 BL=B3
    GO TO 12
 15 CONTINUE
    RETURN
    END
```

```
      SUBROUTINE RAPHC4(W,X,AL3,T3,C4,RESIDUE)
      V4=AL3/T3
      CINI=0.01
      DELC=0.01
      C4=CINI
   16 C4=C4+DELC
      AKS=C4
      CALL CEF(W,X,AKS,CEF1)
      TIM1=CEF1
      AKS1=1.-C4
      CALL CEF(W,X,AKS1,CEF2)
      TIM2=CEF2
      RESIDUE=V4-(TIM1/TIM2)
      IF(RESIDUE.GT.0.00) GO TO 16
      CR=C4
      CL=CR-DELC
   17 C4=(CR+CL)/2.0
      AKS=C4
      CALL CEF(W,X,AKS,CEF1)
      TIM1=CEF1
      AKS1=1.-C4
      CALL CEF(W,X,AKS1,CEF2)
      TIM2=CEF2
      RESIDUE=V4-(TIM1/TIM2)
      IF(ABS(RESIDUE).LT.0.0001) GO TO 20
      IF(RESIDUE.GT.0.0) GO TO 19
      IF(RESIDUE.LT.0.0) GO TO 18
   18 CR=C4
      GO TO 17
   19 CL=C4
      GO TO 17
   20 CONTINUE
      RETURN
      END

      SUBROUTINE RAPHB4(W,X,S2,T3,B4,C4,RESIDUE)
      I=1.
      U4=S2/T3
      B4=C4+0.001
      DELB=0.01
   21 CONTINUE
      I=I+1.
      AKS=1.-C4
      PHAI=(B4-C4)/(B4*(1.-C4))
      CALL CIEF(W,X,AKS,PHAI,CIEF1)
      TEM1=CIEF1
      AKS=1.0-C4
```

```
      CALL CEF(W,X,AKS,CEF1)
      TEM2=CEF1
      RESIDUE=(TEM1/TEM2)-U4
      IF(ABS(RESIDUE).LT.0.00001) GO TO 25
      B4=B4+DELB
      IF(RESIDUE.GT.0.00) GO TO 21
      BR=B4-DELB
      BL=BR-DELB
   22 B4=(BR+BL)/2
      I=I+1.
      AKS=1.-C4
      PHAI=(B4-C4)/(B4*(1.-C4))
      CALL CIEF(W,X,AKS,PHAI,CIEF1)
      TEM1=CIEF1
      AKS=1.0-C4
      CALL CEF(W,X,AKS,CEF1)
      TEM2=CEF1
      RESIDUE=(TEM1/TEM2)-U4
      IF(ABS(RESIDUE).LT.0.00001) GO TO 25
      IF(I.GT.100)GO TO 25
      IF(RESIDUE.GT.0.0) GO TO 24
      IF(RESIDUE.LT.0.0) GO TO 23
   23 BR=B4
      GO TO 22
   24 BL=B4
      GO TO 22
   25 CONTINUE
      RETURN
      END

      SUBROUTINE MATIN (A,MMM)
      DIMENSION A(4,4),B(4),C(4)
      NN=MMM-1
      A(1,1)=1./A(1,1)
      DO 8 M=1,NN
      K=M+1
      DO 3 I=1,M
      B(I)=0.0
      DO 3 J=1,M
3     B(I)=B(I)+A(I,J)*A(J,K)
      D=0.0
      DO 4 I=1,M
4     D=D+A(K,I)*B(I)
      D=-D+A(K,K)
      A(K,K)=1./D
      DO 5 I=1,M
```

```
5     A(I,K)=-B(I)*A(K,K)
      DO 6 J=1,M
      C(J)=0.0
      DO 6 I=1,M
6     C(J)=C(J)+A(K,I)*A(I,J)
      DO 7 J=1,M
7     A(K,J)=-C(J)*A(K,K)
      DO 8 I=1,M
      DO 8 J=1,M
8     A(I,J)=A(I,J)-B(I)*A(K,J)
      RETURN
      END

      SUBROUTINE CEF(W,X,AKS,CEF1)
      DIMENSION W(96), X(96)
      PAI=3.141592654
      SUM=0.0
      DO 10 I=1,96
      THETA=PAI/4.*(1.+X(I))
      TERM=0.25*PAI/SQRT(1.-AKS*SIN(THETA)**2)
      SUM=SUM+TERM*W(I)
   10 CONTINUE
      CEF1=SUM
      RETURN
      END

      SUBROUTINE AIEF(W,X,B2,D2,AIEFK)
      DIMENSION W(96),X(96)
      AKS1=1.-B2
      SUM=0.0
      PHI1=ASIN(SQRT(1./(1.+D2)))
      DO 10 I=1,96
      TERM1=(1.0-AKS1*SIN(0.5*PHI1*(1.0+X(I)))**2.0)**0.5
      TERM2=W(I)*0.5*PHI1/TERM1
      SUM=SUM+TERM2
   10 CONTINUE
      AIEFK=SUM
      RETURN
      END

      SUBROUTINE AINB(W,X,ALPA,B3,AINB1)
      DIMENSION W(96), X(96)
      SUM=0.0
      DO 10 I=1,96
      TERM=0.5*(B3*0.5)**0.5*(1.+X(I))
      TERM1=(TERM)**(1.-2.*ALPA)
      TERM2=(1.-B3+(TERM)**2.)**0.5
      TERM3=(B3-(TERM)**2.)**(1.-ALPA)
```

```
      TERM4=TERM1/(TERM2*TERM3)
      SUM=SUM+W(I)*TERM4
   10 CONTINUE
      AINB1=SUM*(B3*0.5)**0.5
      RETURN
      END
      SUBROUTINE AINA(W,X,ALPA,B3,AINA1)
      DIMENSION W(96), X(96)
      SUM=0.0
      DO 10 I=1,96
      TERM=0.5*(B3*0.5)**(1./8.)*(1.+X(I))
      TERM1=(TERM)**(8.*ALPA-1.)
      TERM2=(1.-(TERM)**8.)**0.5
      TERM3=(B3-(TERM)**8.)**ALPA
      TERM4=TERM1/(TERM2*TERM3)
      SUM=SUM+W(I)*TERM4
   10 CONTINUE
      AINA1=SUM*4.*(B3*0.5)**(1./8.)
      RETURN
      END
      SUBROUTINE AINC(W,X,ALPA,B3,AINC1)
      DIMENSION W(96), X(96)
      SUM=0.0
      DO 10 I=1,96
      TERM=0.5*(0.5*(1.-B3))**0.5*(1.+X(I))
      TERM1=(TERM)**(1.-2.*ALPA)
      TERM2=(1.-(TERM)**2.-B3)**0.5
      TERM3=((TERM)**2.+B3)**(1.0-ALPA)
      TERM4=TERM1/(TERM2*TERM3)
      SUM=SUM+W(I)*TERM4
    ) CONTINUE
      AINC1=SUM*(0.5-B3)**0.5
      RETURN
      END
      SUBROUTINE AIND(W,X,ALPA,B3,AIND1)
      DIMENSION W(96), X(96)
      SUM=0.0
      DO 10 I=1,96
      TERM=0.5*(0.5*(1.+B3))**0.5*(1.+X(I))
      TERM1=(1.-(TERM)**2.-B3)**ALPA
      TERM2=(1.-(TERM)**2.)**(1.0-ALPA)
      SUM=SUM+W(I)/(TERM1*TERM2)
   10 CONTINUE
      AIND1=SUM*(0.5)**0.5
      RETURN
      END
```

```
      SUBROUTINE CIEF(W,X,AKS,PHAI,CIEF1)
      DIMENSION W(96), X(96)
      PAI=3.141592654
      SUM=0.0
      DO 10 I=1,96
      THETA=PHAI/2.*(1.+X(I))
      TERM=0.5*PHAI/SQRT(1.-AKS*SIN(THETA)**2.)
      SUM=SUM+TERM*W(I)
   10 CONTINUE
      CIEF1=SUM
      RETURN
      END
```