# EFFECT OF DRAINAGE ARRANGEMENT ON UPLIFT PRESSURE ON THE FLOOR OF A DEPRESSED WEIR

## A DISSERTATION

submitted in partial fulfillment of the
requirements for the award of the degree
of
MASTER OF ENGINEERING
in
WATER RESOURCES DEVELOPMENT

By
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## CANDIDATE'S DECLARATION

I hereby declare that work which is presented in this Dissertation entitled, **"EFFECT OF DRAINAGE ARRANGEMENT ON UPLIFT PRESSURE ON THE FLOOR OF A DEPRESSED WEIR "**, in partial fulfilment of the requirement for the award of the degree of MASTER OF ENGINEERING IN WATER RESOURCES DEVELOPMENT, submitted in Water Resources Development Training Centre, Indian Institute of Technology, Roorkee, is a record of my own work carried out during the period from July 16th, 2001 to December, 2001 under the supervision of **Dr. G.C. Mishra**, Professor, WRDTC, Indian Institute of Technology, Roorkee (India).

The matter embodied in this dissertation has not been submitted by me for the award of any other degree or diploma.

**(FAKIH USMAN)**

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This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

Place : Roorkee
Dated : December 12, 2001

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**(FAKIH USMAN)**

Place : Roorkee

Dated: December 12 , 2001

# CONTENTS

## LIST OF SYMBOLS

The following symbols are used in this dissertation

| | |
|---|---|
| C | = constant; |
| $d_1$ | = depth of upstream depressed ; |
| $d_2$ | = depth of middle stepped |
| $d_3$ | = depth of down stream depressed |
| $F(\phi, m)$ | = elliptic Integral First Kind |
| $I_E$ | = exit gradient; |
| i | = the imaginary unit; |
| k | = coefficient of permeability; |
| $L_1, L_2, L_3 L_4$ | = floor length; |
| $T_1$ , $T_2$ ,$T_3$, | = depth of flow domain under structure ; |
| M | = complex constants; |
| N | = complex constants; |
| P | = uplift pressure |
| $s_1$ | = upstream sheet pile; |
| $s_2$ | = downstream sheet pile; |
| w | = complex variable; |
| t | = transformation plane; |
| z | = complex variable; |
| x | = horizontal co-ordinate; |
| y | = vertical co-ordinate; |

| | |
|---|---|
| $\gamma_w$ | = the unit weight of water; |
| u | = velocity in x-direction; |
| v | = velocity in y-direction; |
| q | = quantity of seepage; |
| $q_1$ | = quantity of water through the drain |
| b | = clear distance between upstream and downstream floor |
| $\phi$ | = velocity potential function; and |
| $\psi$ | = stream function; |

# SYNOPSIS

A weir is designed against uplift pressure. Higher the uplift pressure means provision of more height of hydraulic structure. This leads to high cost of the structure. The uplift pressure can be reduced by providing drain at the base of structure.

Analytical solution for finding pressure acting on the base of a depressed weir with a drain is not available.

In the thesis, it is aimed to derive an analytical solution for the seepage problem using conformal mapping method of fragments. The hydraulic structure generally used in practice by providing several cut-offs, therefore we apply method of fragments which was proposed by Pavlovsky (vide Harr, 1962). This method divides flow region into fragment by means of vertical lines drawn through the ends of the cut-offs. The dividing lines are taken for lines of constant potential. Actually they differ from equipotential lines, but they approach the latter as the depth of the pervious stratum become smaller.

The solution to this problem is obtained with the help of conformal mapping and numerical integration. Gauss quadrature has been used for numerical integration.

# INTRODUCTION

The design of hydraulic structure founded on a permeable foundation presents problems of complex nature. Besides testing such structures for forces due to surface flow, their stability against forces caused by uplift pressure has to be ensured. Uplift pressure and exit gradient determine the final design and dimension of hydraulic structure.

Excessive uplift pressures and piping are often the cause of damage to structure. A study of the causes of failures of structures founded on permeable foundation would indicate that most of failure may be attributed to the destructive effect of uplift pressure.

For safe design of structure founded on permeable foundation, knowledge of behavior of flow pattern becomes essential in order to design the thickness of floors. Numerous studies conducted by Bligh, Lane, Khosla and others for estimating the sub soil pressure below floors have indicated that massive thickness of concrete has to be provided on the downstream side to account for excessive uplift that develops near the junction of the upstream and downstream floor of hydraulic structures.

Intermediate drain or filter can be provided below hydraulic structure founded on permeable foundation to reduce uplift pressure resulting in appreciable saving. Melenshcentko (1936) and Numerov (1948) determined the effect of drainage hole below an impervious floor. Zemerin (1931) obtained the effect of plane drainage on the floor below flat floor or a single over-fall founded on infinite depth of permeable soil. Chawla ((1973) has investigated effect of intermediate drains on uplift pressure for flat bottom weir resting on permeable foundation of infinite depth Chawla Kumar (1985) has investigated effect of intermediate drains on uplift pressure and exit gradient for flat bottom weir resting on permeable foundation of finite depth.

An approximate analytical method of solution for any confined flow system of finite depth, directly applicable to design, was furnished by Pavlovsky in 1935. The fundamental assumption of this method called the *method of fragments,* is that equipotential lines at various critical parts of the flow region can be approximated by straight vertical lines that divide the region into section or fragments, in fig.1.1

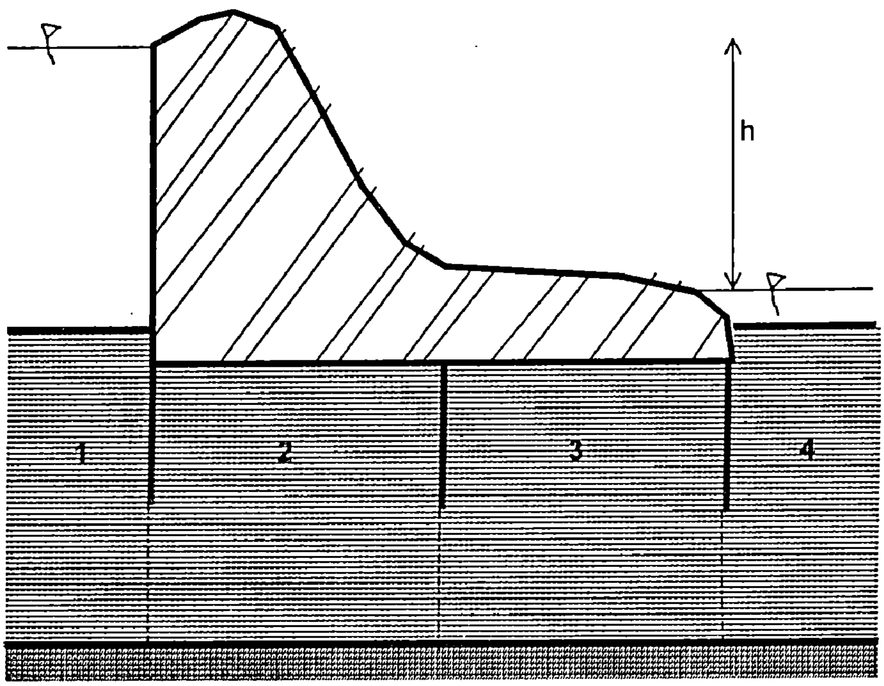



**Fig. 1.1 Division of the Confined Flow Domain into Different Fragments**

Suppose, now that we can compute the discharge in the m[th] fragment as

$$q = \frac{kh_m}{\Phi_m} \qquad m = 1,2.........,n \qquad (1.1)$$

where $h_m$ = head loss through fragment,

$\Phi_m$ = dimensionless from factor

then, since the discharge through all fragments must be the same

$$q = \frac{kh_1}{\Phi_1} = \frac{kh_2}{\Phi 2} = \frac{kh_m}{\Phi m} = ..... = \frac{kh_n}{\Phi_n}$$

$$q = k\frac{\sum h_m}{\sum \Phi} = \frac{kh}{\sum_{m=1}^{n} \Phi_m} \qquad (1.2)$$

where h (without subscript) is the total loss through the section. By similar reasoning we find that the he d loss in the m[th] fragment can be calculated from

$$h_m = \frac{h\Phi_m}{\sum \Phi} \qquad (1.3)$$

Once the head loss for any fragment has been determined the pressure distribution on the base of the structure and the exit gradient can be easily obtained. Thus the primary task is to implement this method by establishing a catalogue of typical from factors.

In this thesis an attempt has been made to study the effects of intermediate drain on uplift pressure for step depressed weirs. In addition, the maximum exit gradient has been obtained by analytical method. Method of fragments has been used for solving the flow under composite hydraulic structure. The reduction in uplift pressure has been computed for various position of the drain.

# REVIEW OF LITERATURE

A literature review has been made on study of performance of drain in hydraulic structure.

Meleshchenko (1936) and Numerov (1948) have provided solution for hydraulic structure with drainage holes, wherein they studied in effect of one or two drainage holes in the otherwise impervious floor. The effect of plane drainage connected to downstream bed in case of seepage below a flat apron or a single overfall founded on infinite depth of permeable soil was obtained by Zamarin (1931). Sangal (1964)determined the extent of reduction in pressure affected by a flat and deep filter of particular dimensions below the foundation of a barrage with the help of electrical analogy model. No solution was till now available to determine the effect of plane drainage located anywhere between the two cutoffs.

Using electrical analogue the effect of intermediate drainage filter on seepage pressure has been studied by Arumugam (1971). A case of a flat bottom weir resting on a porous medium of infinite depth has been considered.

Chawla (1973) has used conformal mapping to find the performance of intermediate drain provided at the base of a hydraulic structure with two end sheet pile resting on a permeable foundation of infinite depth. Conformal mapping technique has been used.

Kumar has studied the effect of intermediate filter for the following boundary conditions (1985)

(i) Flow under a weir with unequal partial cut-offs at both ends of the floor and intermediate filter founded on permeable soil of finite depth .

(ii) Flow under a weir with a partial cut-off at upstream end, a complete cut-off at downstream end of the floor and an intermediate filter founded on permeable soil of finite depth.

(iii) Flow under a weir with unequal partial cut-offs at both ends of the floor and an intermediate filter founded on permeable soil underlain by a sloping impervious stratum.

The solution of the problems of first two cases has been obtained with the help of conformal mapping. The transformation equations have been integrated numerically using Simpson's formula The solution of the problem in the third case was obtained by solving the Laplace equation using finite element method. The factor of safety against have below the filter has also been determined to and the safety against piping below the filter has been investigated.

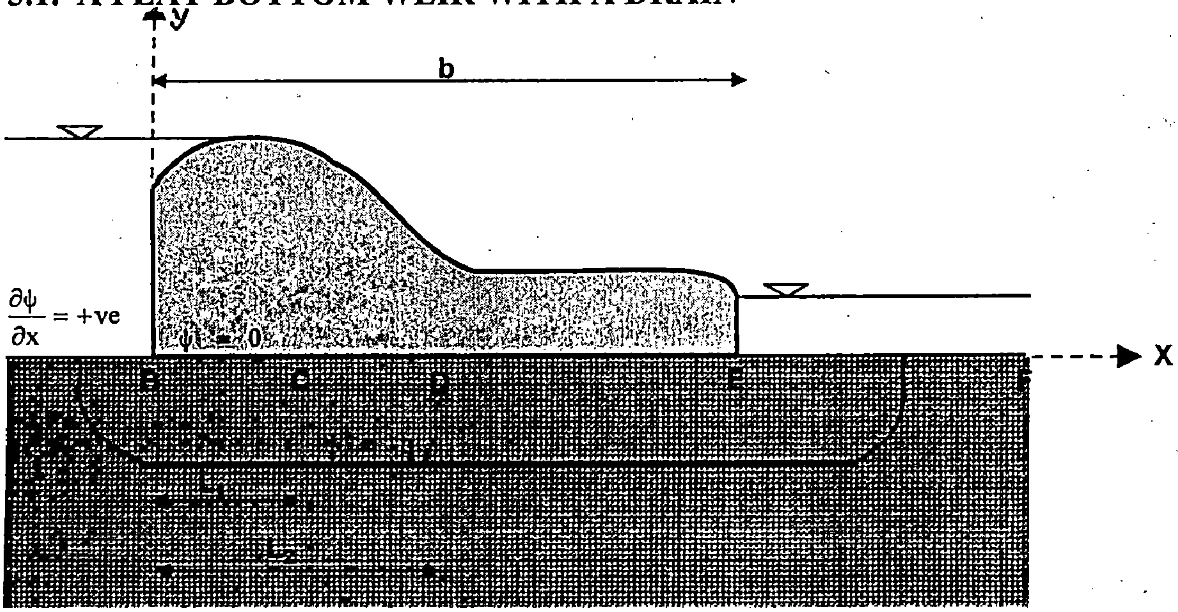
The finite depth of soil would cause more uplift pressure and provision of drain along the base of the structure would influence the pressure distribution. In this thesis the effect intermediate drain has been studied for weir resting on porous medium of finite death suing method of fragments.

# A FLAT BOTTOM WEIR WITH A DRAIN RESTING ON A PERMEABLE FOUNDATION OF INFINITE DEPTH

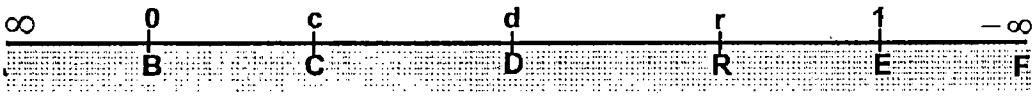
## 3.0 INTRODUCTION

Depending on the potential prevalent at the drain, a drain would perform either as a sink, or partly as a sink and partly as a source or as a source. When the head along he drain is zero, it acts as a sink only ; water flows from the foundation soil to the sink. In order to study the relationship between potential at the drain and performance of drain (whether the drain would act as a source or sink or as a composite source and sink flow under a flat bottom weir with a drain resting on a porous medium of infinite depth has been analysed.

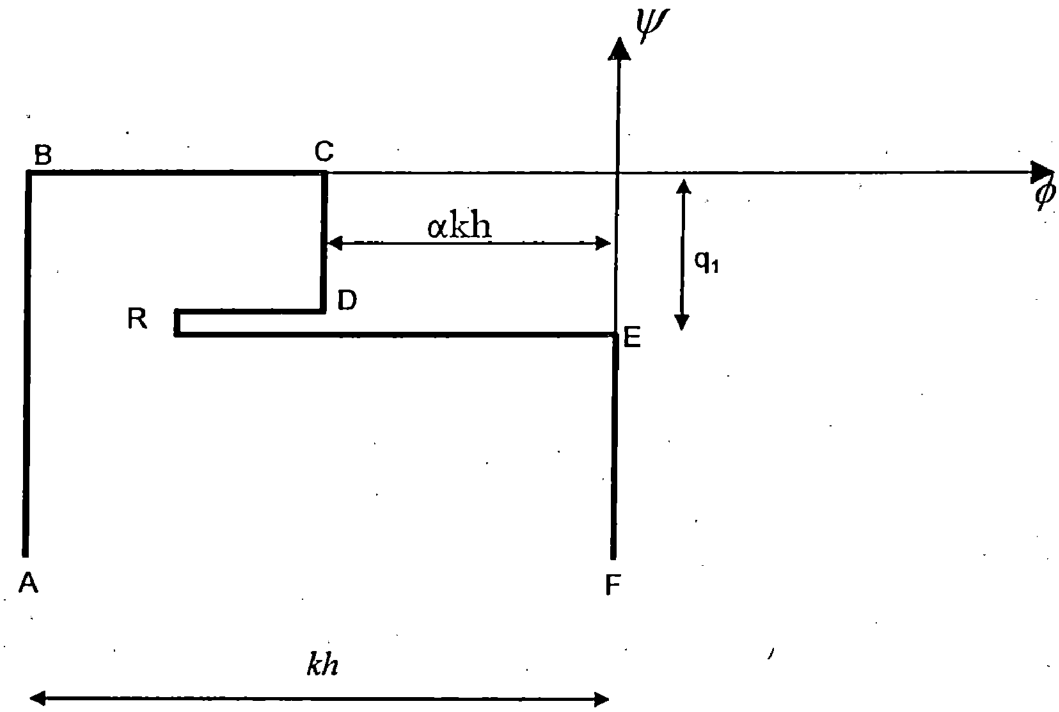
## 3.1. A FLAT BOTTOM WEIR WITH A DRAIN




Physical flow domain
Z ( = X+iY ) plane




t ( = r + is ) plane




The complex potential w (=ϕ+iψ) plane
Flow domain consider a drain along CD
**Figure 3.1flow Domain, Ausiliary Plane And Complex Potential Plane**

**Determination of Constant $M_1$**

Substituting $t = Re^{i\theta}$, and $dt = Re^{i\theta} i\, d\theta$ in equation (3.2) and applying the condition that as one travels along a semicircle of radius R, R tending to infinite, there is a jump kh in w plane the constant $M_1$ is found.

$$\lim_{R \to \infty} \int_{\pi}^{2\pi} \frac{\left(r - Re^{i\theta}\right) Re^{i\theta} i\, d\theta}{\left(Re^{i\theta}\right)^{1/2} \left(c - Re^{i\theta}\right)^{1/2} \left(d - Re^{i\theta}\right)^{1/2} \left(1 - Re^{i\theta}\right)^{1/2}}$$

$$= \int_{\pi}^{2\pi} \frac{R\left[\frac{r}{R} - e^{i\theta}\right] \quad Re^{i\theta} i\, d\theta}{R^2 \frac{e^{i\theta/2}}{e} \left(\frac{c}{R} - e^{i\theta}\right)^{1/2} \left(\frac{d}{R} - e^{i\theta}\right)^{1/2} \left(\frac{1}{R} - e^{i\theta}\right)^{1/2}}$$

$$= \int_{\pi}^{2\pi} \frac{-id\theta}{-i} \qquad = \pi$$

Hence, $M_1 = \dfrac{kh}{\pi}$ (3.3)

Starting from equation (3.2)

$$\frac{dw}{dt} = \frac{M_1 r}{t^{1/2}(c-t)^{1/2}(d-t)^{1/2}(1-t)^{1/2}} - \frac{M_1 t^{1/2}}{(c-t)^{1/2}(d-t)^{1/2}(1-t)^{1/2}}$$

or

$$w = M_1 r \int_0^{t'} \frac{dt}{t^{1/2}(c-t)^{1/2}(d-t)^{1/2}(1-t)^{1/2}} - M_1 \int_0^{t'} \frac{t^{1/2} dt}{(c-t)^{1/2}(d-t)^{1/2}(1-t)^{1/2}} - kh$$

At $t' = c$, $w = -\alpha\, kh$. Hence,

$$(1\text{-}\alpha)\, kh = M_1 r \int_0^c \frac{dt}{t^{1/2}(c-t)^{1/2}(d-t)^{1/2}(1-t)^{1/2}} - M_1 \int_0^c \frac{t^{1/2}dt}{(c-t)^{1/2}(d-t)^{1/2}(1-t)^{1/2}} \tag{3.4}$$

Substituting for $M_1$ and solving for the unknown parameter, r

$$\pi(1\text{-}\alpha) = rI_1 - I_2$$

$$r = \frac{I_2 + \pi\left(1-\alpha_d\right)}{I_1} \tag{3.5}$$

in which

$$I_1 = \int_0^c \frac{dt}{t^{1/2}(c-t)^{1/2}(d-t)^{1/2}(1-t)^{1/2}}$$

and

$$I_2 = \int_0^c \frac{t^{1/2}dt}{(c-t)^{1/2}(d-t)^{1/2}(1-t)^{1/2}}$$

The integration $I_1$ and $I_2$ are computed using Gauss quadrature after removing the similarity.

$$I_1 = \int_0^{\frac{c}{2}} \frac{dt}{t^{1/2}(c-t)^{1/2}(d-t)^{1/2}(1-t)^{1/2}} + \int_{c/2}^{c} \frac{dt}{t^{1/2}(c-t)^{1/2}(d-t)^{1/2}(1-t)^{1/2}}$$

$$= I_{11} + I_{12}$$

Substituting :$t = u^2$, and $dt = 2u\ du$ for evaluation of $I_{11}$

$$I_{11} = \int_0^{\sqrt{\frac{c}{2}}} \frac{2udu}{u\left(c-u^2\right)^{1/2}\left(d-u^2\right)^{1/2}\left(1-u^2\right)^{1/2}} \tag{3.6}$$

Further Substituting

$$u = \sqrt{\frac{c}{2}}\left(\frac{1}{2}+\frac{v}{2}\right) \qquad \text{and} \qquad du = \frac{1}{2}\sqrt{\frac{c}{2}}\ dv$$

where v is a dummy variable, the lower and upper limits of integration (3.6) are converted to -1and 1 receptively and (3.6) reduces to.

$$I_{11} = \sqrt{\frac{c}{2}}\int_{-1}^{1}\frac{dv}{\sqrt{c-\frac{c}{2}\left(\frac{1}{2}+\frac{v}{2}\right)^2}\cdot\sqrt{d-\frac{c}{2}\left(\frac{1}{2}+\frac{v}{2}\right)^2}\cdot\sqrt{1-\frac{c}{2}\left(\frac{1}{2}+\frac{v}{2}\right)^2}}$$

$$= \sqrt{\frac{c}{2}}\int_{-1}^{1}\frac{dv}{f_{1.1}(v)} \qquad (3.7)$$

$$I_{1.2} = \int_{c/2}^{c}\frac{dt}{t^{1/2}(c-t)^{1/2}(d-t)^{1/2}(1-t)^{1/2}}$$

Substituting $c - t = u^2$, $c - u^2 = t$, $-dt = 2u\ du$

$$I_{12} = 2\int_{0}^{\sqrt{c/2}}\frac{du}{\sqrt{(c-u^2)\ (d-c+u^2)\ (1-c+u^2)}}$$

$$= \sqrt{\frac{c}{2}}\int_{-1}^{1}\frac{dv}{\sqrt{\left(c-\frac{c}{2}\left(\frac{1}{2}+\frac{v}{2}\right)^2\right)\left((d-c)\frac{c}{2}\left(\frac{1}{2}+\frac{v}{2}\right)^2\right)\left(1-c+\frac{c}{2}\left(\frac{1}{2}+\frac{v}{2}\right)^2\right)}} \qquad (3.8)$$

$$= \sqrt{\frac{c}{2}}\int_{-1}^{1}\frac{dv}{f_{1.2}(v)}$$

$$I_2 = \int_{0}^{c}\frac{t^{1/2}dt}{(c-t)^{1/2}(d-t)^{1/2}(1-t)^{1/2}}$$

Substituting

$c - t = u^2$, $dt = -2u\,du$, $I_2$ reduces to

$$I_2 = 2\int_0^{\sqrt{c}} \frac{\left(c-u^2\right)^{1/2} du}{\sqrt{d-c+u^2}\sqrt{1-c+u^2}}$$

Further subtitling

$$u = \sqrt{c}\left(\frac{1}{2}+\frac{v}{2}\right)$$

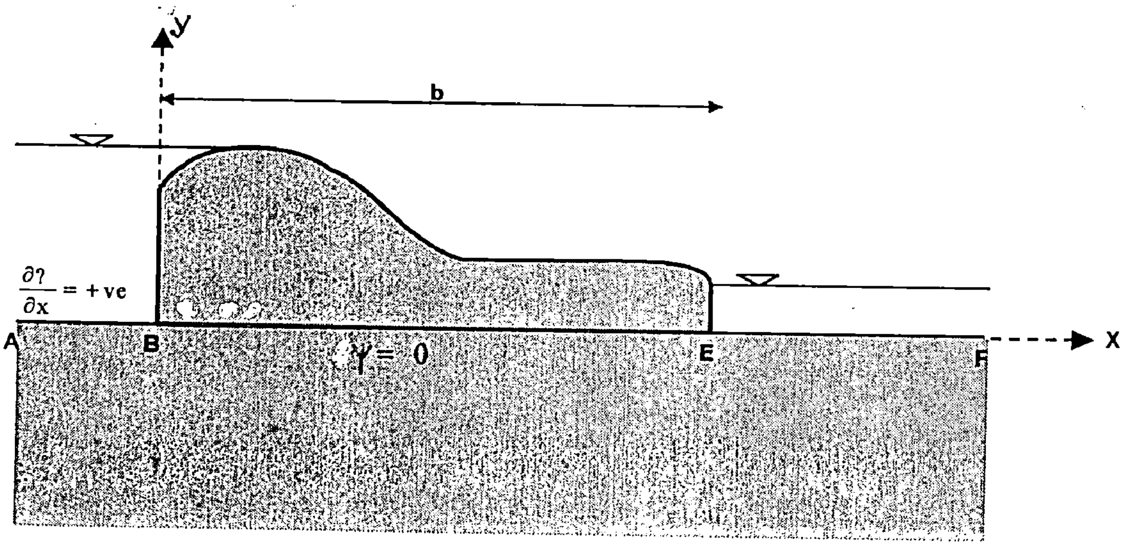
$$du = \frac{\sqrt{2}}{2} dv$$

$$\sqrt{c}\int_{-1}^{1} \frac{\sqrt{c - c\left(\frac{1}{2}+\frac{v}{2}\right)^2}\, dv}{\sqrt{\left(d-c+c\left(\frac{1}{2}+\frac{v}{2}\right)^2\right)\left(1-c+c\left(\frac{1}{2}+\frac{v}{2}\right)^2\right)}}$$

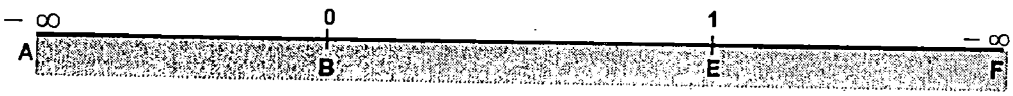
## 3.2 FLAT BOTTOM WEIR WITHOUT DRAIN

A flat bottom weir without drain and the corresponding complex potential are shown in figure 3.2

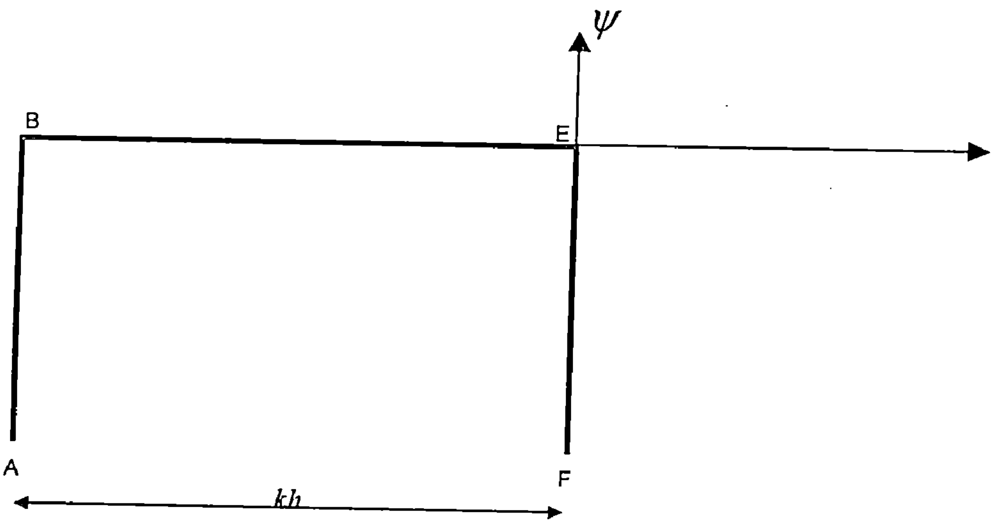
**III.2.1 Analysis** : The conformal mapping of the w plane onto auxiliary t plane is given by:




Physical flow domain
Z ( = X+iY ) plane




t ( = r + is ) plane




Flow domain without drain along BE

$$\frac{dw}{dt} = \frac{M_1}{t^{1/2}(1-t)^{1/2}}$$

$$w = M_1 \int_0^t \frac{dt}{t^{1/2}(1-t)^{1/2}} - kh$$

Substituting

$$t = \sin^2\theta$$

$$dt = 2\sin^2\theta \cos\theta \, d\theta$$

$$w = \int \frac{2\sin\theta\sin\theta c \, d\theta}{\sin\theta in\theta co} = 2\theta - kh$$

$$w = 2M_1 \ \sin^{-1}\sqrt{t'} - kh$$

at $t' = 1$, $w = 0$. Hence,

$$0 = 2M_1 \ \frac{\pi}{2} - kh,$$

$$M_1 = \frac{kh}{\pi}$$

and

$$w = \frac{2kh}{\pi}\sin^{-1}\sqrt{t} - kh$$

At $t' = c$ , $\phi = \phi_c = \alpha_c kh$; Hence

$$\alpha_c = 1 - \frac{2}{\pi}\sin^{-1}\sqrt{c} \qquad (3.9)$$

Similarly $\alpha_d = 1 - \frac{2}{\pi}\sin^{-1}\sqrt{d}$

For the filter zone to act as a drain the potential $\alpha_h$ along the drain at the location envisaged has to be less then $\alpha_{dh}$

## 3.3 RESULT AND DISCUSSION

In figure 3.3a the variation of potential along the base of the weir is shown. In Fig 3.3b the variation of potential when the potential at the drain = 0 is presented. If potential other then zero is to be maintained along the drain it should be less then the potential that would prevail without the drain. Other wise the drain would not function as desired

The distribution of potential / (-kh) along the base of a flat bottom weir, is shown in Fig. 3.3. If at x/B = 0.8 a filter to function as a drain, the value of $\alpha_d$ should be less than 0.2. Otherwise it will not act as a drain.

The function of a drain located at 0.5 to 0.65 is shown in Fig. 3.3. The drain reduces uplift pressure at along the base.

The distribution of potential (-kh) along the base of a flat bottom weir with a drain and without the drain is shown in Fig. 3.3a. The potential imposed on the drain is zero. As shown from the figure a drain considerably reduces the head hence, the pressure.

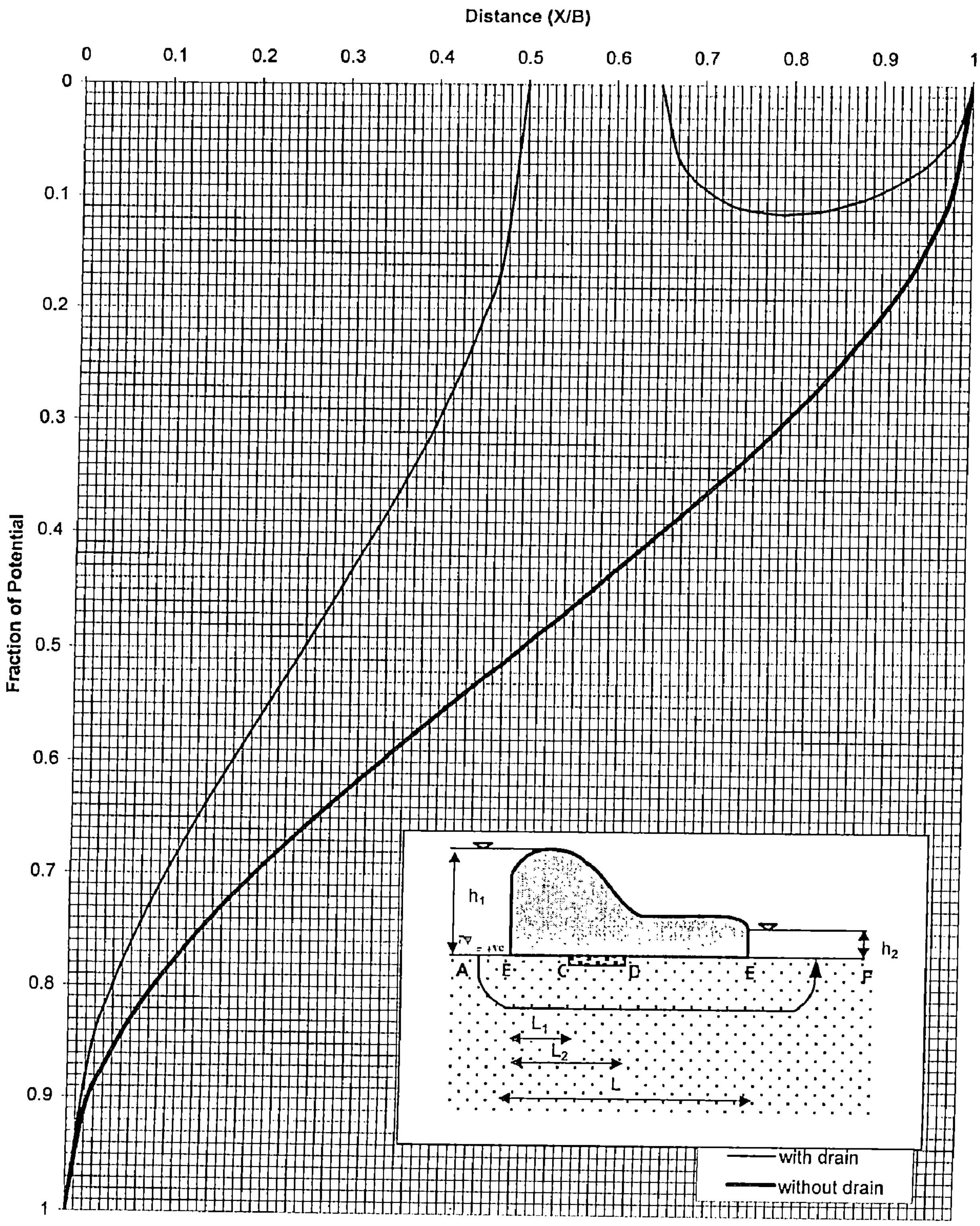



**Figure 3.3**
**Variation of Potential along the Base of A Flat Bottom Weir**

b

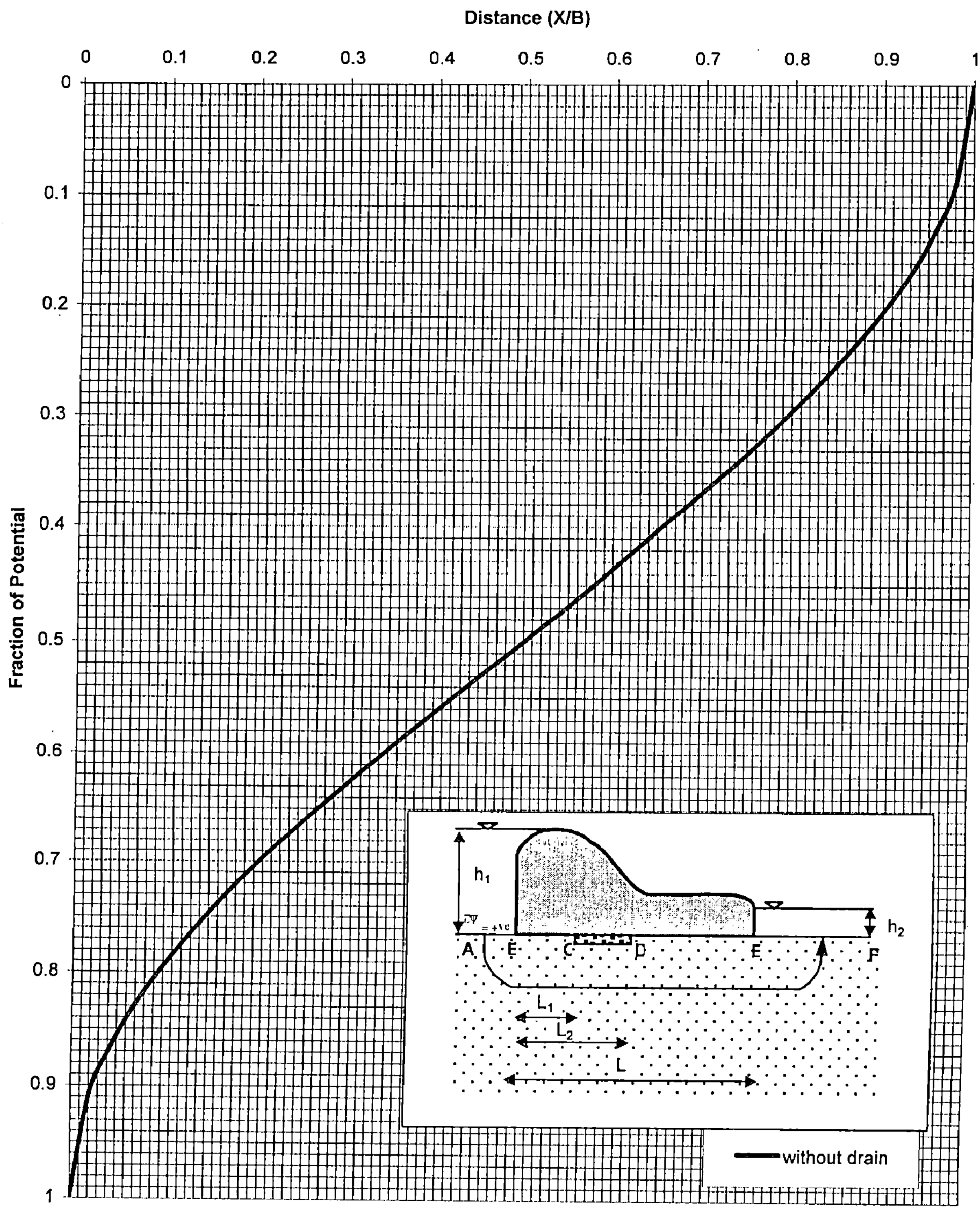



**Figure 3.3a**
**Variation of Potential along the Base of A Flat Bottom Weir without Drain**

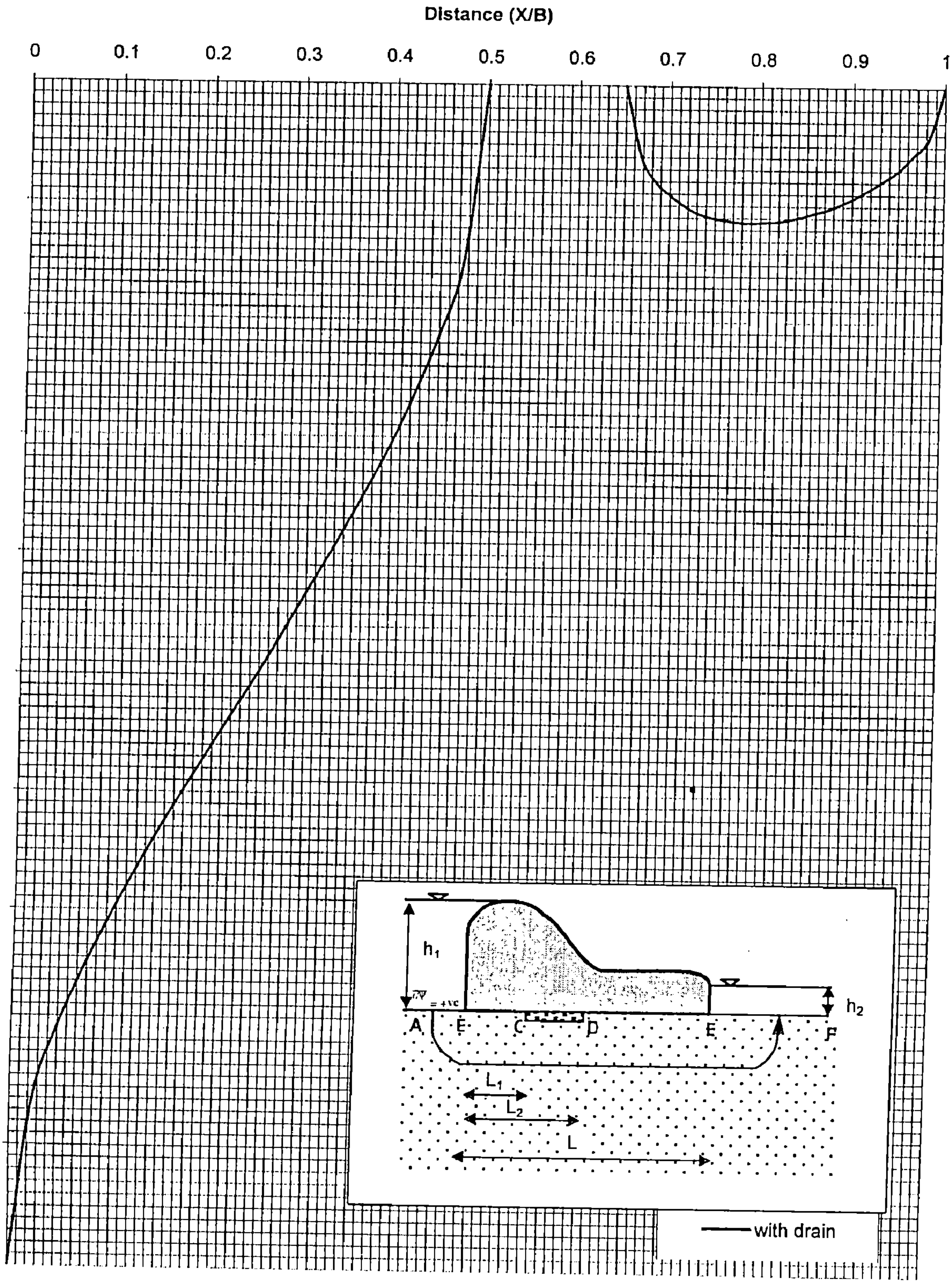



**Figure 3.3b**
**Variation of Potential along the Base of A Flat Bottom Weir**

# FLOW UNDER A DEPRESSED WEIR WITH A DRAIN RESTING ON A POROUS FOUNDATION OF FINITE DEPTH

## 4.0 INTRODUCTION

A depressed weir is shown in Fig 4.0. The soil under the weir is homogeneous tropic and of finite depth. The flow domain is decomposed into four fragments. The flow through each fragment is analysed

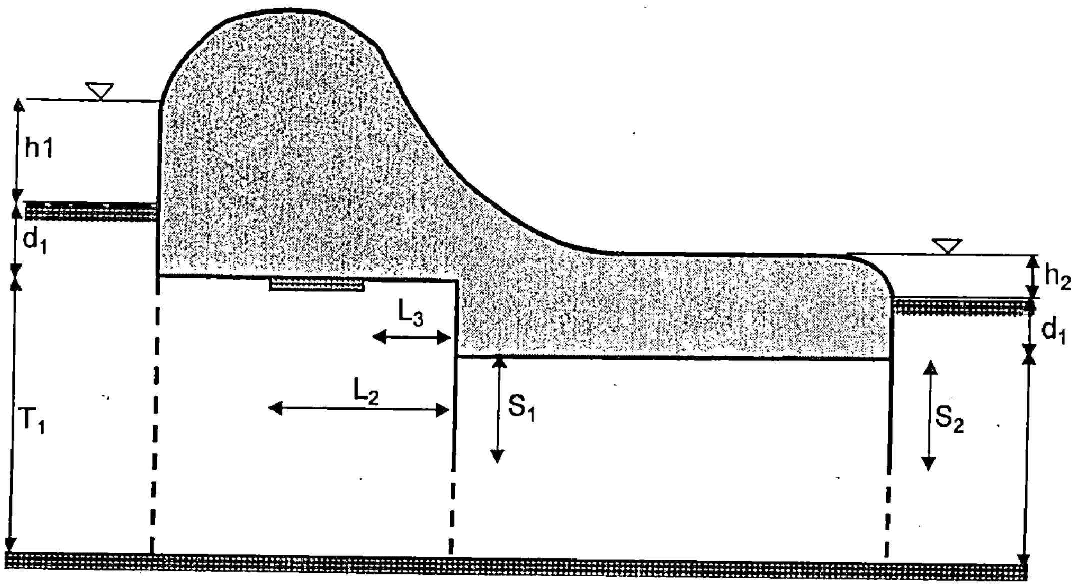



**Figure 4.0 A Depressed Weir with A Drain**

## 4.1 FRAGMENT I

The flow domain pertaining to fragment I and the corresponding w plane are shown in Fig 4.1

**A. FRAGMENT I**

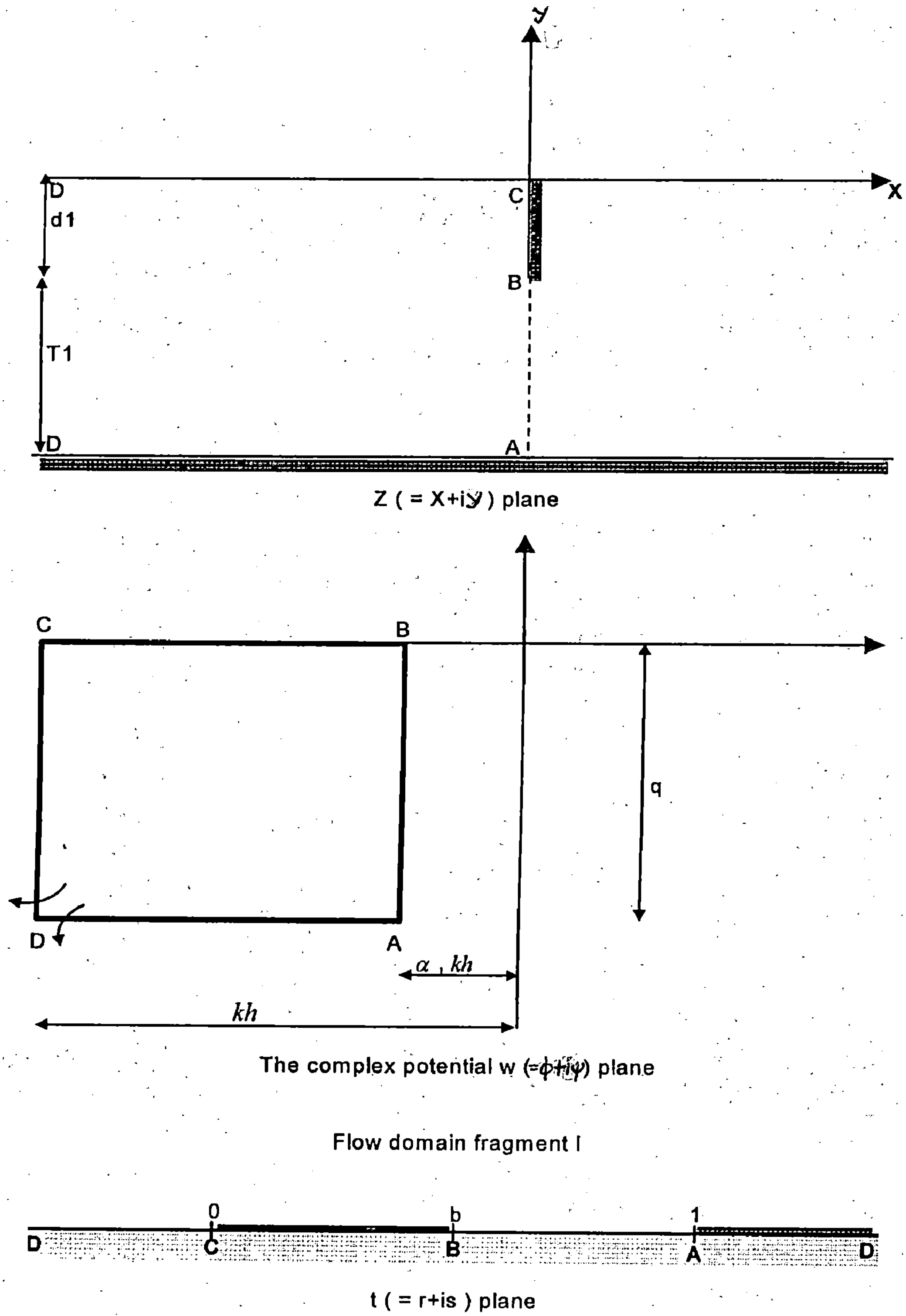



**Figure 4.1 Flow Domain of Segment 1, Complex Potential Plane, w and Auxiliary , t plane**

Applying Schwarz – Christoffel transformation, the conformal mapping of the fragment I in z plane onto the auxiliary t plane is given by

$$z = M\int \frac{dt}{t^{1/2}(1-t)^{1/2}} + N$$

Integrating

$$z = 2M\sin^{-1}\sqrt{t} + N \tag{4.1.1}$$

For point C, t = 0, and z = 0; hence N = 0.

For point A, t = 1, and z = $-i(d_1 + T_1)$

Applying these condition in equation (4.1.1) the constant M is found to be

$$M = \frac{-i(d_1 + T_1)}{\pi} \tag{4.1.2}$$

Hence

$$z = -2i\left(\frac{d_1 + T_1}{\pi}\right)\sin^{-1}\sqrt{t} \tag{4.1.3}$$

For point B, t = b, z = $-id_4$

Hence,

$$b = \left[\sin\frac{\pi}{2}\left(\frac{d_1}{d_1 + T_1}\right)\right]^2 \tag{4.1.4}$$

The Schwarz-Christoffel conformal mapping of the complex potential plane onto the t plane is given by:

$$w(t^1) = M_1\int_{-\infty}^{t'} \frac{dt}{(-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} - iq - kh$$

$$= M_1 2F(\vartheta, m) - iq - kh \qquad (\text{for } -\infty < t' \le 0)$$

in which $\vartheta = \sin^{-1}\sqrt{\frac{1}{1-t'}}$

$m = \sqrt{1-b}$

F ( $\vartheta$, m) = elliptic integral of the first kind with amplitude $\vartheta$ and modulus m

For point c, $t' = 0$, and w = -kh

Hence,

$$M_1 = \frac{iq}{2F\left(\frac{\pi}{2}, \sqrt{1-b}\right)} \tag{4.1.5}$$

For $0 \le t' \le b$,

the relation between w and t plane is given by

$$w(t') = M_1 \int_0^{t'} \frac{dt}{(-1)^{1/2}(t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} - k\,h$$

$$= \frac{M_1}{i} 2F(\vartheta, m) - kh$$

in which $\vartheta = \sin^{-1}\sqrt{\frac{t'}{b}}$

and $m = \sqrt{b}$

For point B, t' = b, and w = - $\alpha_1$ kh

Hence,

$$-\alpha_1 kh = \frac{2M_1}{i} F\left(\frac{\pi}{2}, \sqrt{b}\right) - kh \tag{4.1.6}$$

Substituting $M_1$ from equation (4.1.5) into equation (4.1.6)

$$q = kh(1-\alpha_1)\frac{F\left(\frac{\pi}{2}, \sqrt{1-b}\right)}{F\left(\frac{\pi}{2}, \sqrt{b}\right)} \tag{4.1.7}$$

Substituting b from (4.1.4) into (4.1.7)

$$q = kh(1-a_1)\frac{F\left[\frac{\pi}{2},\sqrt{1-\left(\sin\frac{\pi}{2}\left(.\frac{d_1}{d_1+T_1}\right)\right)^2}\right]}{F\left[\frac{\pi}{2},\left(\sin\frac{\pi}{2}\left(.\frac{d_1}{d_1+T_1}\right)\right)\right]}$$

or

$$\frac{q}{kh} = (1-a_1)\frac{F\left(\frac{\pi}{2},\cos\frac{\pi}{2}\left(\frac{d_1}{d_1+T_1}\right)\right)}{F\left(\frac{\pi}{2},\sin\frac{\pi}{2}\left(\frac{d_1}{d_1+T_1}\right)\right)} \tag{4.1.8}$$

Let us assume $A = \frac{F\left(\frac{\pi}{2},\cos\frac{\pi}{2}\left(\frac{d_1}{d_1+T_1}\right)\right)}{F\left(\frac{\pi}{2},\sin\frac{\pi}{2}\left(\frac{d_1}{d_1+T_1}\right)\right)}$

Hence,

$$\frac{q}{kh} + \alpha_1 A = A \tag{4.1.9}$$

## 4.2 FRAGEMENT II

The fragment II which contains the drain is shown is Fig.4.2

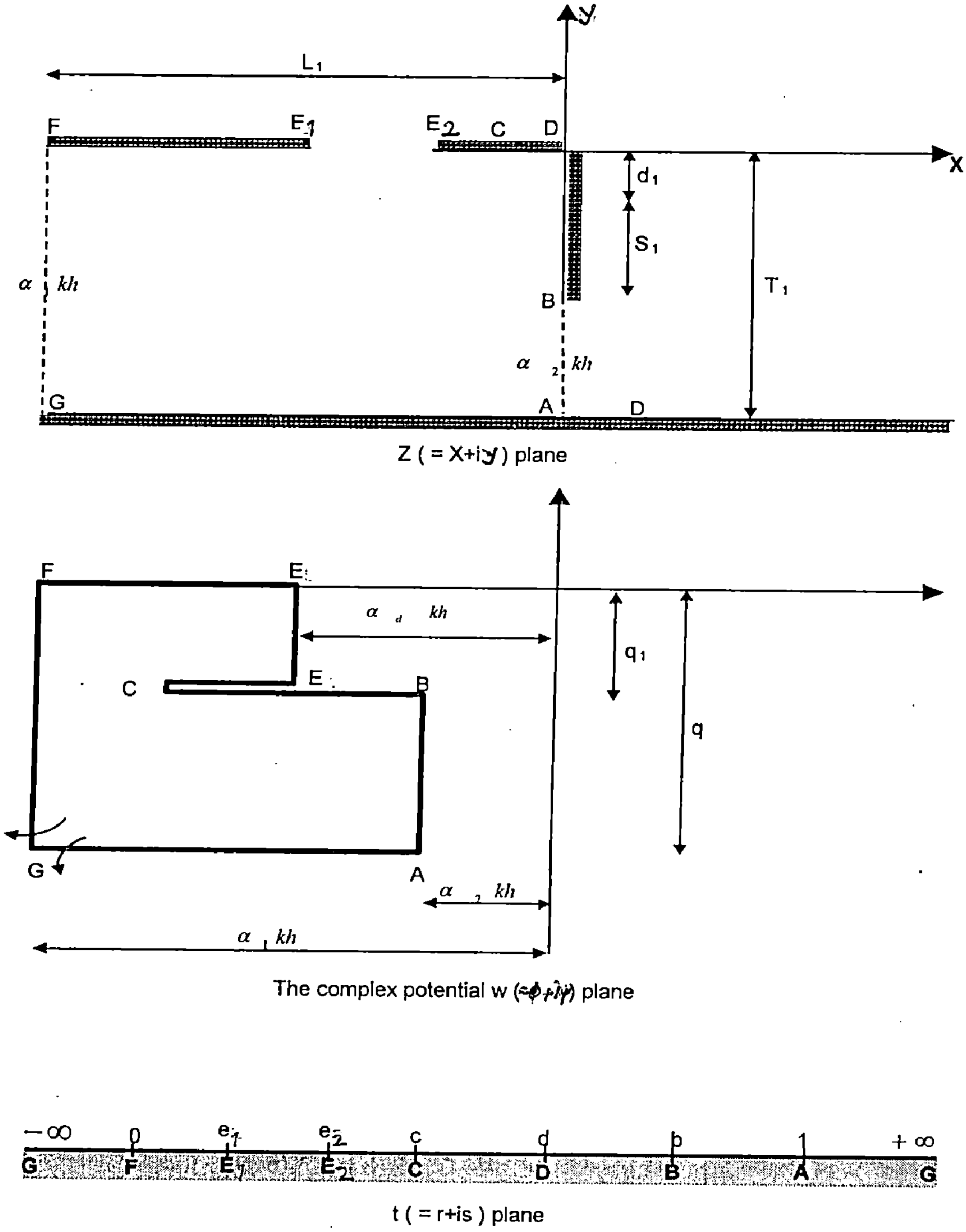


**Figure 4.2 Flow Domain of Fragment II , the Pertinent Complex Potential Plane and the Auxiliary t Plane**

Applying the Schwarz-Christoffel transformation the conformal mapping of Fragment II in z plane onto auxiliary t plane is given by:

$$z = M\int_{-\infty}^{t'}\frac{dt}{(-t)^{1/2}(d-t)^{1/2}(1-t)^{1/2}} - L_l - iT_1 \qquad (4.2.1)$$

For $-\infty < t' < 0$ we get

$$z = M\,2F(\varphi, m) - L_l - iT_1 \qquad (4.2.2)$$

in which,

$$\varphi = \sin^{-1}\sqrt{\frac{1}{1-t'}}$$

and

$$m = \sqrt{1-d}$$

For point F, $t' = 0$, $z = -L_l$ $\varphi = \frac{\pi}{2}$

Hence,

$$-L_l = M\,2F\left(\frac{\pi}{2}, \sqrt{1-d}\right) - L_1 - iT_1$$

$$M = \frac{iT_1}{2F\left(\frac{\pi}{2}, \sqrt{1-d}\right)} \qquad (4.2.3)$$

For $0>t'<d$, the corresponding z is given by :

$$z = M\int_0^{t'} \frac{dt}{(-t)^{1/2}t^{1/2}(d-t)^{1/2}(1-t)^{1/2}} - L_1$$

or

$$z = \frac{M}{\sqrt{-1}} 2\,F\,(\varphi, m) - L_1 \tag{4.2.4}$$

in which,

$$\varphi = \sin^{-1}\sqrt{\frac{t'}{d}}$$

$$m = \sqrt{d}$$

For point D, $t' = d$, $z = 0$ and $\varphi = \frac{\pi}{2}$

Hence,

$$0 = \frac{M}{i} 2F\left(\frac{\pi}{2}\sqrt{d}\right) - L_1$$

$$L_1 = \frac{M}{i} 2F\left(\frac{\pi}{2}\sqrt{d}\right) \tag{4.2.5}$$

Substituting M from (4.2.3) in (4.2.5)

$$\frac{L_1}{T_1} = \frac{F\left(\frac{\pi}{2}, \sqrt{d}\right)}{F\left(\frac{\pi}{2}, \sqrt{1-d}\right)} \tag{4.2.6}$$

the parameter d can be obtained by an iteration.

For point $E_L$, $t' = e_1$, $z = -L_2$, $\varphi = \sin^{-1}\sqrt{\frac{e_1}{d}}$

Hence,

$$-L_2 = \frac{M}{i}2F\left(\sin^{-1}\sqrt{\frac{e_1}{d}},\sqrt{d}\right) - L_1$$

or

$$L_1\text{-}L_2 = \frac{M}{i}2F\left(\sin^{-1}\sqrt{\frac{e_1}{d}},\sqrt{d}\right) \qquad (4.2.7)$$

Substituting M

$$\frac{L_1 - L_2}{T_1} = \frac{F\left(\sin^{-1}\sqrt{\frac{e_1}{d}},\sqrt{d}\right)}{F\left(\frac{\pi}{2},\sqrt{1-d}\right)} \qquad (4.2.8)$$

the parameter $e_1$ can be obtained by an iteration

For point $E_R$ , $t' = e_2$, $z = -L_3$, $\varphi = \sin^{-1}\sqrt{\frac{e_2}{d}}$

$$-L_3 = \frac{M}{i}2F\left(\sin^{-1}\sqrt{\frac{e_2}{d}},\ \sqrt{d}\right)\text{-}L_1$$

Substituting M

$$\frac{L_1 - L_3}{T_1} = \frac{F\left(\sin^{-1}\sqrt{\frac{e_2}{d}},\sqrt{d}\right)}{F\left(\frac{\pi}{2},\sqrt{1-d}\right)} \qquad (4.2.9)$$

The parameter $e_2$ can be obtained by an iteration.

For $d < t' \le 1$ , the corresponding z is given by :

$$z = M\int_d^t \frac{dt}{(-1)^{1/2}t^{1/2}(-1)^{1/2}(t-d)^{1/2}(1-t)} \qquad (4.2.10)$$

or

$$z = -M\, 2\, F(\varphi, m)$$

in which,

$$\vartheta = \sin^{-1}\sqrt{\frac{(t'-d)}{(1-d)t'}}$$

and

$$m = \sqrt{1-d}$$

For point B, $t' = b$, $z = -i(d_2 + S_1)$, and $\vartheta = \sin^{-1}\sqrt{\frac{(b-d)}{(1-d)b}}$

Hence,

$$-i\,(d_2 + S_1) = -\,M\, 2\, F\left(\sin^{-1}, \sqrt{\frac{(b-d)}{(1-d)b}}, \sqrt{1\text{-d}}\right) \qquad (4.2.11)$$

Substituting M

$$i(d_1+S_1) = \frac{iT_1}{2F\left(\frac{\pi}{2}, \sqrt{1-d}\right)} 2F\left(\sin^{-1}\sqrt{\frac{(b-d)}{(1-d)}}, \sqrt{1-d}\right)$$

or

$$\frac{d_2 + S_1}{T_1} = \frac{F\left(\sin^{-1}\sqrt{\frac{(b-d)}{(1-d)b}}, \sqrt{1-d}\right)}{F\left(\frac{\pi}{2}, \sqrt{1-d}\right)} \qquad (4.2.12)$$

The parameter b can be found by iteration

For point A, $t' = 1$, $z = -iT_1$, $\vartheta = \frac{\pi}{2}$

Hence,

$$-iT_1 = -M2F\left(\frac{\pi}{2}, \sqrt{1-d}\right)$$

This equation is same with (4.2.3)

For $1 \le t' < \infty$, the relationship between z and t' does not yield any in dependent equation.

The Schwarz-Christoffel conformal mapping of the complex potential onto the t plane for $-\infty < t' < 0$ is given by:

$$w(t') = (Mi) \int_{-\infty}^{t'} \frac{(c-t)dt}{(-t)^{1/2}(e_1)^{1/2}(e_2-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} - \alpha_1 kh - iq \quad (4.2.13)$$

or

$$w(t') = (Mi) \int_{-\infty}^{0} \frac{c\,dt}{f(t)} + (Mi) \int_{-\infty}^{0} \frac{(-t)dt}{f(t)} - \alpha_1 kh - iq$$

In which $f(t) = (-t)^{1/2} (e_1 -t)^{1/2} (e_2 -t)^{1/2} (b-t)^{1/2} (1-t)^{1/2}$

For point F, $t = 0$, $w = -\alpha_1 kh$.

Hence

$$-\alpha_1 kh = Mci \int_{-\infty}^{0} \frac{dt}{f(t)} + (Mi) \int_{-\infty}^{0} \frac{(-t)dt}{f(t)} - \alpha_1 kh - iq$$

or

$$q = M.c \,.\, I_1 + M\, I_2 \qquad (4.2.14)$$

$$I_1 = \int_{-\infty}^{0} \frac{dt}{(-t)^{1/2}(e_1-t)^{1/2}(e_2-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}}$$

Let us assume -t =v , -dt = dv

Hence

$$I_1 = \int_{0}^{\infty} \frac{dv}{(v)^{1/2}(e_1+v)^{1/2}(e_2+v)^{1/2}(b+v)^{1/2}(1+v)^{1/2}}$$

$I_1$ is an improper integral because of singularity at v = 0 and the upper limit which tends to ∞. The improper inertial is converted to proper integral as follows.

Let us assume $v = u^2$ , dv = 2u du

Hence,

$$I_1 = \int_{0}^{\infty} \frac{2udu}{u\,(e_1+u^2)^{1/2}(e_2+u^2)^{1/2}(b+u^2)^{1/2}(1+u^2)^{1/2}}$$

Splitting the limit of integration into two parts

$$I_1 = 2\int_{0}^{1} \frac{du}{f(u)} + 2\int_{1}^{\infty} \frac{du}{f(u)}$$

in which $f(u) = (e + u^2)^{1/2} (d + u^2)^{1/2} (b + u^2)^{1/2} (1+ u^2)^{1/2}$

Thus

$$I_1 = I_{1.1} + I_{1.2}$$

$$I_{11} = 2\int_{0}^{1} \frac{du}{(e_1+u^2)^{1/2}(e_2+u^2)^{1/2}(b+u^2)^{1/2}(1+u^2)^{1/2}}$$

$I_{11}$ is a proper integral. To evaluate the integration by Gauss quadrature let us assume

$$u = \frac{1}{2} + \frac{1}{2}X, \quad du = dx/2$$

At u = 0, X = -1. At u = 1, X = 1

Hence,

$$I_{11} = \int_{-1}^{1} \frac{dx}{\left\{e_1 + \left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2} \left\{e_2 + \left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2} \left\{b + \left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2} \left\{1 + \left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2}}$$

$$I_{1.2} = 2\int_{1}^{\infty} \frac{du}{(e_1+u^2)^{1/2}(e_2+u^2)^{1/2}(b+u^2)^{1/2}(1+u)^{1/2}}$$

$I_{12}$ is an improper integral because of upper limit.

Let us assume $u = \frac{1}{v}$, $du =- \frac{1}{v^2}dv$.

At $u = 1$ $v = 1$, $u = \infty$, $v = 0$

$$I_{12} = 2\int_{0}^{1} \frac{v^2 dv}{(e_1 v^2+1)^{1/2}(e_2 v^2+1)^{1/2}(bv^2+1)^{1/2}(v^2+1)^{1/2}}$$

let us assume v = 1/2 x 1/2 , and dv = d x/2.

At v = 1, x = 1, and at v = 0, x = -1.

$$I_{12} = \int_{-1}^{1} \frac{\left(\frac{1}{2}+\frac{1}{2}x\right)^2 dx}{\left\{e_1\left(\frac{1}{2}+\frac{1}{2}x\right)^2+1\right\}^{1/2} \left\{e_2\left(\frac{1}{2}+\frac{1}{2}x\right)^2+1\right\}^{1/2} \left\{b\left(\frac{1}{2}+\frac{1}{2}x\right)^2+1\right\}^{1/2} \left\{\left(\frac{1}{2}+\frac{1}{2}x\right)^2+1\right\}^{1/2}}$$

$$I_2 = \int_{-\infty}^{0} \frac{(-t)dt}{(-t)^{1/2}(e_1-t)^{1/2}(e_2-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}}$$

$$= \int_{-\infty}^{0} \frac{(-t)^{1/2}\,dt}{(e_1-t)^{1/2}(e_2-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}}$$

Let as assume -t = v , -dt = dv

Hence,

$$I_2 = \int_{0}^{\infty} \frac{v^{1/2}dv}{(e_1+v)^{1/2}(e_2+v)^{1/2}(b+v)^{1/2}(1+v)^{1/2}}$$

$$= \int_{0}^{1} \frac{v^{1/2}dv}{f(v)} + \int_{1}^{\infty} \frac{v^{1/2}dv}{f(v)}$$

$$= I_{2.1} + I_{2.2}$$

in which $f(v) = (e_1 + v)^{1/2}(e_2 + v)^{1/2}(b + v)^{1/2}(1 + v)^{1/2}$.

$$I_{2.1} = \int_{0}^{1} \frac{v^{1/2}dv}{(e_1+v)^{1/2}(e_2+v)^{1/2}(b+v)^{1/2}(1+v)^{1/2}}$$

Let us assume $v = u^2$

$$dv = 2u\,du$$

$$= \int_{0}^{1} \frac{u(2u\ du)}{(e_1+u^2)^{1/2}(e_2+u^2)^{1/2}(b+u^2)^{1/2}(1+u^2)^{1/2}}$$

$$=2\int_{0}^{1} \frac{u^2\ du}{(e_1+u^2)^{1/2}(e_2+u^2)^{1/2}(b+u^2)^{1/2}(1+u^2)^{1/2}}$$

Let us assume u = 1/2 + 1/2X, du = dX /2. At u = 0 , X = -1, at u = 1., X = 1

$$I_{21}=2\int_{-1}^{1} \frac{\left(\frac{1}{2}+\frac{1}{2}X\right)^2 dX/2}{\left\{e_1+\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2}\left\{e_2+\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2}\left\{b+\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2}\left\{1+\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2}}$$

$$I_{2.2} = \int_1^{\infty} \frac{v^{1/2} dv}{(e_1+v)^{1/2}(e_2+v)^{1/2}(b+v)^{1/2}(1+v)^{1/2}}$$

Let us assume $v = u^2$, $dv = 2\,u\,du$

$$I_{22} = \int_1^{\infty} \frac{zu\,2u\;du}{(e_1+u^2)^{1/2}(e_2+u^2)^{1/2}(b+u^2)^{1/2}(1+u^2)^{1/2}}$$

$$= 2\int_1^{\infty} \frac{u^2\;du}{(e_1+u^2)^{1/2}(e_2+u^2)^{1/2}(b+u^2)^{1/2}(1+u^2)^{1/2}}$$

Let us assume $u = \frac{1}{v}$, $du = -\frac{1}{v^2}dv$

At $u = 1$, $v = 1$; at $u = \infty$ $v = 0$.

Hence,

$$I_{22} = 2\int_1^{0} \frac{\left(\frac{1}{v}\right)^2\left(-\frac{1}{v^2}\right)dv}{\left(e_1+\frac{1}{v^2}\right)^{1/2}\left(e_2+\frac{1}{v^2}\right)^{1/2}\left(b+\frac{1}{v^2}\right)^{1/2}\left(1+\frac{1}{v^2}\right)^{1/2}}$$

$$= 2\int_0^{1} \frac{dv}{(e_1v^2+1)^{1/2}(e_2v^2+1)^{1/2}(bv^2+1)^{1/2}(v^2+1)^{1/2}}$$

Let us assume $v = 1/2 + 1/2\,X$, $dv = dX/2$

*At* $v = 0$, $X = -1$, *at* $v = 1$, $X = 1$.

$$I_{22} = \int_{-1}^{1} \frac{dX}{\left\{e_1\left(\frac{1}{2}+\frac{1}{2}X\right)^2+1\right\}^{1/2}\left\{e_2\left(\frac{1}{2}+\frac{1}{2}x\right)^2+1\right\}^{1/2}\left\{b\left(\frac{1}{2}+\frac{1}{2}X\right)^2+1\right\}^{1/2}\left\{\left(\frac{1}{2}+\frac{1}{2}X\right)^2+1\right\}^{1/2}}$$

For $0 \le t' < e_1$

$$w(t') = M\int_0^{t'}\frac{(c-t)dt}{(-1)^{1/2}(t)^{1/2}(e_1-t)^{1/2}(e_2-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} - \alpha_1 kh \qquad (4.2.15)$$

For point *E*, $t = e_1$, $w = -\alpha_d kh$

Hence,

$$\alpha_1 kh - \alpha dkh = \frac{M}{\sqrt{-1}}c\int_0^{e_1}\frac{dt}{f(t)} - \frac{M}{\sqrt{-1}}\int_0^{e_1}\frac{tdt}{f(t)}$$

In which

$$f(t) = (t)^{1/2}(e_1-t)^{1/2}(e_2-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}$$

$$kh(\alpha_1 - \alpha_d) = \frac{M}{\sqrt{-1}}cI_3 - \frac{M}{\sqrt{-1}}I_4 \qquad (4.2.16)$$

$$I_3 = \int_0^{e_1}\frac{dt}{f(t)} = \int_0^{e_1/2}\frac{dt}{f(t)} + \int_{e_1/2}^{0}\frac{dt}{f(t)}$$

$= I_{3.1} + I_{3.2}$

$$I_{3.1} = \int_0^{e_1/2}\frac{dt}{(t)^{1/2}(e_1-t)^{1/2}(e_2-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}}$$

Let us assume $t = v^2$, $dt = 2v\,dv$

$At\ t = 0$, $v = 0$; at $t = e_1/2$, $v = \sqrt{\frac{e_1}{2}}$

Hence,

$$I_{31} = 2\int_0^{\sqrt{\frac{e_1}{2}}}\frac{dv}{(e_1-v^2)^{1/2}(e_2-v^2)^{1/2}(b-v^2)^{1/2}(1-v^2)^{1/2}}$$

Substituting $v = \sqrt{\frac{e_1}{2}}\left(\frac{1}{2}+\frac{1}{2}X\right)$, $dv = \sqrt{\frac{e_1}{2}}dX/2$

$$I_{31}=\sqrt{\frac{e_1}{2}}\int_{-1}^{1}\frac{dX}{\left\{e_1-\frac{e_1}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2}\left\{e_2-\frac{e_2}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2}\left\{b-\frac{b}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2}\left\{1-\frac{e_1}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2}}$$

$$I_{3.2}=\int_{\frac{e_1}{2}}^{e_1}\frac{dt}{t^{1/2}(e_1-t)^{1/2}(e_2-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}}$$

Let us assume $e_1 - t = v^2, \quad t = e_1 - v^2, \quad dt = -2v\,dv.$

At $t=\frac{e_1}{2}, v=\sqrt{\frac{e_1}{2}}$ , and at $t=e_1, \quad v=0$

Hence,

$$I_{32}=2\int_{0}^{\sqrt{\frac{e_1}{2}}}\frac{dv}{(e_1-v^2)^{1/2}(e_2-e_1+v^2)^{1/2}(b-e_1+v^2)^{1/2}(1-e_1+v^2)^{1/2}}$$

Let assume $v=\sqrt{\frac{e_1}{2}}\left(\frac{1}{2}+\frac{1}{2}x\right), \qquad dv=\sqrt{\frac{e_1}{2}}dx/2$

At $v=0, \quad x=-1,$ at $v=\sqrt{\frac{e_1}{2}}, \quad x=1$ .

Hence,

$$I_{32}=\sqrt{\frac{e_1}{2}}\int_{-1}^{1}\frac{dX}{\left\{e_1-\frac{e_1}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2}\left\{e_2-e_1+\frac{e_1}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2}}$$

$$\frac{1}{\left\{b-e_1+\frac{e_1}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2}\left\{1-e_1+\frac{e_1}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2}}$$

$$I_4 = \int_0^{e_1} \frac{t\ dt}{f(t)} = \int_0^{\frac{e_1}{2}} \frac{t\,dt}{f(t)} + \int_{\frac{e}{2}}^{e} \frac{t\,dt}{f(t)}$$

$= I_{4.1} + I_{4.2}$

$$I_{4.1} = \int_0^{\frac{e_1}{2}} \frac{t^{1/2}\ dt}{(e_1-t)^{1/2}(e_2-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}}$$

Assuming $t = v^2$, $dt = 2v\,dv$

At $t = 0$, $v = 0$ : at $t = e/2 \rightarrow v = \sqrt{\frac{e}{2}}$

Hence,

$$I_{41} = \int_0^{\sqrt{\frac{e_1}{2}}} \frac{v.2v\,dv}{(e_1-v^2)^{1/2}(e_2-v^2)^{1/2}(b-v^2)^{1/2}(1-v^2)^{1/2}}$$

$$= 2\int_0^{\sqrt{\frac{e_1}{2}}} \frac{v^2\,dv}{(e_1-v^2)^{1/2}(e_2-v^2)^{1/2}(b-v^2)^{1/2}(1-v^2)^{1/2}}$$

Let assume $v = \sqrt{\frac{e_1}{2}}\left(\frac{1}{2}+\frac{1}{2}X\right)$, $dv = \sqrt{\frac{e_1}{2}}dX/2$

At v = 0, X = -1 : at $v = \sqrt{\frac{e_1}{2}}$, $X = 1$

$$I_{41} = \sqrt{\frac{e_1}{2}} \int_{-1}^{1} \frac{\left\{ \frac{e_1}{2}\left(\frac{1}{2}+\frac{1}{2}x\right)^2 \right\} dX}{\left\{ e_1 - \frac{e_1}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2 \right\}^{1/2} \left\{ e_2 - \frac{e_1}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2 \right\}^{1/2} \left\{ b - \frac{e_1}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2 \right\}^{1/2} \left\{ 1 - \frac{e_1}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2 \right\}^{1/2}}$$

$$I_{4.2} = \int_{e_1/2}^{e_1} \frac{t^{1/2} dt}{(e-t)^{1/2}(e_2-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}}$$

Let us assume $e - t = u^2, \quad t = e - u^2, dt = -2u\, du$

$$At\ t = e_1/2,\ u = \sqrt{\frac{e_1}{2}} \qquad at\ t = e,\quad u = 0$$

Hence

$$I_{42} = 2 \int_{0}^{\sqrt{\frac{e}{2}}} \frac{(e_1 - u^2)^{1/2} du}{\left\{ e_2 - e_1 + u^2 \right\}^{1/2} \left\{ b - e_1 + u^2 \right\}^{1/2} \left\{ 1 - e_1 + u^2 \right\}^{1/2}}$$

Assuming $u = \sqrt{\frac{e_1}{2}}\left(\frac{1}{2}+\frac{1}{2}X\right)$

$$I_{42} = \sqrt{\frac{e_1}{2}} \int_{-1}^{1} \frac{\left\{ e_1 - \frac{e_1}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2 \right\}^{1/2} dX}{\left\{ e_2 - e_1 + \frac{e_1}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2 \right\}^{1/2} \left\{ b - e_1 + \frac{e_1}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2 \right\}^{1/2} \left\{ 1 - e_1 + \frac{e_1}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2 \right\}^{1/2}}$$

For $e_1 \le t' \le e_2$

$$w(t') = M \int_{e_1}^{t'} \frac{(c-t)dt}{(-1)^{1/2}(t)^{1/2}(-1)^{1/2}(t-e_1)^{1/2}(e_1-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} - \alpha_d kh \qquad (4.1.17)$$

For point $E_R$ $\quad t' = e_2$ . $w = -iq_1 - \alpha_d kh$

Hence ,

$$iq_1 = Mc \int_{e_1}^{e_2} \frac{dt}{f(t)} - M \int_{e_1}^{e_2} \frac{tdt}{f(t)}$$

in which

$$f(t) = t^{1/2} (t-e_1)^{1/2} (e_2-t)^{1/2} (b-t)^{1/2} (1-t)^{1/2}$$

$$iq_1 = Mc\, I_5 - M.\, I_6 \qquad (4.2.18)$$

$$I_5 = \int_{e_1}^{e_2} \frac{dt}{f(t)} = \int_{e_1}^{\frac{e_2+e_1}{2}} \frac{dt}{f(t)} \cdot \int_{\frac{e_2+e_1}{2}}^{e_2} \frac{dt}{f(t)}$$

$$= I_{5.1} + I_{5.2}$$

$$I_{5.1} = \int_{e_1}^{\frac{e_2+e_1}{2}} \frac{dt}{t^{1/2}(t-e_1)^{1/2}(e_2-t)^{1/2} b-t)^{1/2}(1-t)^{1/2}}$$

Let us assume

$$t - e_1 = u^2 \qquad t = u^2 + e_1 \qquad d_t = 2\, u\, du$$

At $t = e_1$, $u = 0$, at $t = \frac{e_2+e_1}{2}$, $u = \sqrt{\frac{e_2 - e_1}{2}}$

$$I_{5.1} = 2 \int_0^{\sqrt{\frac{e_2-e_1}{2}}} \frac{du}{(u^2+e_1)^{1/2}(b-u^2-e_1)^{1/2}(1-u^2-e_1)^{1/2}}$$

Assuming

$$u = \sqrt{\frac{e_2 - e_1}{2}} \left( \frac{1}{2} + \frac{1}{2} X \right)$$

$$I_{51}=\sqrt{\frac{e_2-e_1}{2}}\int_{-1}^{1}\frac{dX}{\left\{\frac{e_2-e_1}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2+e_1\right\}^{1/2}\left\{b-\frac{e_2-e_1}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2+e_1\right\}^{1/2}\left\{1-\frac{e_2-e_1}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2-e_1\right\}^{1/2}}$$

$$I_{5.2}=\int_{\frac{e_2+e_1}{2}}^{e_2}\frac{dt}{t^{1/2}(t-e_1)^{1/2}(e_2-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}}$$

Let us assume

$e_2 - t = u^2 \qquad t = e_2 + u^2 \qquad d_t = 2u\,du$

At $t = \frac{e_2+e_1}{2}$, $u=\sqrt{\frac{e_2-e_1}{2}}$ at $t =$ $t = e_2$, $u = 0$

$$I_{52}=2\int_{0}^{\sqrt{\frac{e_2-e_1}{2}}}\frac{du}{(e_2+u^2)^{1/2}(e_2+u^2-e_1)^{1/2}(b-e_2-u^2)^{1/2}(1-e_2-u^2)^{1/2}}$$

Assuming

$$u=\sqrt{\frac{e_2-e_1}{2}}\left(\frac{1}{2}+\frac{1}{2}X\right)$$

$$I_{5.2}=\sqrt{\frac{e_2-e_1}{2}}\int_{-1}^{1}\frac{dx}{\left\{\frac{e_2-e_1}{2}\left(\frac{1}{2}+\frac{1}{2}x\right)^2+e_2\right\}^{1/2}\left\{e_2-\frac{e_2-e_1}{2}\left(\frac{1}{2}+\frac{1}{2}x\right)^2-e_1\right\}^{1/2}}$$

$$\frac{1}{\left\{b+\frac{e_2-e_1}{2}\left(\frac{1}{2}+\frac{1}{2}\right)^2-e_2\right\}^{1/2}\left\{1-\frac{e_2-e_1}{2}\left(\frac{1}{2}+\frac{1}{2}\right)^2-e_2\right\}^{1/2}}$$

$$I_6=\int_{e_1}^{e_2}\frac{t\,dt}{f(t)}=\int_{e_1}^{\frac{e_2+e_1}{2}}\frac{t\,dt}{f(t)}+\int_{\frac{e_2+e_1}{2}}^{e_2}\frac{t\,dt}{f(t)}$$

$$= I_{6.1} + I_{6.2}$$

$$I_{6.1} = \int_{e_1}^{\frac{e_2+e_1}{2}} \frac{t\,dt}{t^{1/2}(t-e_1)^{1/2}(e_2-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}}$$

$$= \int_{e_1}^{\frac{e_2+e_1}{2}} \frac{t^{1/2}dt}{(t-e_1)^{1/2}(e_2-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}}$$

Substituting

$$t - e_1 = u^2, \qquad t = u^2 + e_1, \qquad dt = 2u\,du.$$

$$\text{At } t = e_1, \quad u = 0 \qquad \text{and} \qquad \text{at } t = \frac{e_2+e_1}{2}, u = \sqrt{\frac{e_2-e_1}{2}}$$

$$I_{61} = \int_0^{\sqrt{\frac{e_2-e_1}{2}}} \frac{(u^2+e_1)^{1/2}2.du}{(e_2-u^2-e_1)^{1/2}(b+u^2-e_1)^{1/2}(1-e_2+u^2)^{1/2}}$$

Let us assume

$$u = \sqrt{\frac{e_2-e_1}{2}}\left(\frac{1}{2}+\frac{1}{2}X\right)$$

$$I_{61} = \sqrt{\frac{e_2-e_1}{2}}z\int_{-1}^{1} \frac{\left\{\frac{e_2.e_1}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2+e_1\right\}^{1/2} dX}{\left\{e_2-\frac{e_2-e_1}{2}\left(\frac{1}{2}+\frac{1}{2}x\right)^2-e_1\right\}^{1/2}\left\{b-\frac{e_2-e_1}{2}\left(\frac{1}{2}+\frac{1}{2}x\right)^2-e_1\right\}^{1/2}\left\{1-\frac{e_2-e_1}{2}\left(\frac{1}{2}+\frac{1}{2}x\right)^2-e_1\right\}^{1/2}}$$

$$I_{6.2} = \int_{\frac{e_2+e_1}{2}}^{e_2} \frac{t^{1/2}dt}{(t-e_1)^{1/2}(e_2-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}}$$

Let us assume

$$e_2 - t = u^2, \quad t = e_2 - u^2, \quad dt = -2u\,du$$

At $t = \frac{e_2 + e_1}{2}$, $u = \sqrt{\frac{e_2 - e_1}{2}}$, at $t = e_2$, $u = 0$

$$I_{62} = 2 \int_0^{\frac{\sqrt{e_2 - e_1}}{2}} \frac{(e_2 - u^2)^{1/2} du}{(e_2 - u^2 - e_1)^{1/2}(b - e_2 + u^2)^{1/2}(1 - e_2 + u^2)^{1/2}}$$

Let us assume

$$u = \sqrt{\frac{e_2 - e_1}{2}}\left(\frac{1}{2} + \frac{1}{2}X\right)$$

$$I_{62} = \sqrt{\frac{e_2 - e_1}{2}} \int_{-1}^{1} \frac{\left\{e_2 - \frac{e_2 . e_1}{2}\left(\frac{1}{2} + \frac{1}{2}X\right)^2\right\}^{1/2} dX}{\left\{e_2 - \frac{e_2 - e_1}{2}\left(\frac{1}{2} + \frac{1}{2}X\right)^2 - e_1\right\}^{1/2}\left\{b + \frac{e_2 - e_1}{2}\left(\frac{1}{2} + \frac{1}{2}X\right)^2 - e_1\right\}^{1/2}\left\{1 + \frac{e_2 - e_1}{2}\left(\frac{1}{2} + \frac{1}{2}x\right)^2 - e_2\right\}^{1/2}}$$

For $e_2 < t' < b$

$$w(t') = M \int_{e_2}^{t'} \frac{(c - t).dt}{\sqrt{(-1)(t)(-1)(t - e_1)(-1)(t - e_2)(b - t)(1 - t)}} - iq_1 - \alpha_d kh \qquad (4.2.19)$$

For point B, $t = b$ $w = \alpha_2 kh - iq_{-1}$

Hence,

$$\alpha_2 kh - iq_1 = \frac{M}{\sqrt{-1}} c \int_{e_2}^{b} \frac{dt}{f(t)} - \frac{M}{\sqrt{-1}} \int_{e_2}^{b} \frac{tdt}{f(t)} - iq_1 - \alpha_d kh$$

in which,

$$f(t) = t^{1/2} (t-e_1)^{1/2} (t-e_2)^{1/2} (b-t)^{1/2} (1-t)^{1/2}$$

or

$$kh(\alpha_d - \alpha_2) = \frac{M}{\sqrt{-1}} c \int_{e_2}^{b} \frac{dt}{f(t)} - \frac{M}{\sqrt{-1}} \int_{e_2}^{b} \frac{tdt}{f(t)}$$

$$kh(a_d - \alpha_2) = -\frac{M}{\sqrt{-1}} e \ \mathrm{I}_7 + \frac{M}{\sqrt{-1}} . I_8 \qquad (4.2.20)$$

$$\mathrm{I}_7 = \int_{e_2}^{b} \frac{dt}{f(t)} = \int_{e_2}^{\frac{b+e_2}{2}} \frac{dt}{f(t)} + \int_{\frac{b+e_2}{2}}^{b} \frac{dt}{f(t)}$$

$$= \mathrm{I}_{7.1} + \mathrm{I}_{7.2}$$

$$\mathrm{I}_{7.1} = \int_{e_2}^{\frac{b+e_2}{2}} \frac{dt}{t^{1/2}(t-e_1)^{1/2}(t-e_2)^{1/2}(b-t)^{1/2}(1-t)^{1/2}}$$

Let us assume

$$t - e_2 = u^2 \qquad t = u^2 + e_2$$

At $t = e_2$, $u = 0$, at $t = \frac{b+e_2}{2}$, $u = \sqrt{\frac{b-e_2}{2}}$

$$I_{7.1} = \int_{e_2}^{\frac{b+e_2}{2}} \frac{dt}{t^{1/2}(t-e_1)^{1/2}(t-e_2)^{1/2}(b-t)^{1/2}(1-t)^{1/2}}$$

Let us assume

$$t - e_2 = u^2 \qquad t = u^2 + e_2$$

At $t = e_2$, $u = 0$; at $t = \frac{b+e_2}{2}$, $u = \sqrt{\frac{b-e_2}{2}}$.

$$I_{71} = \int_0^{\sqrt{\frac{b+e_2}{2}}} \frac{2udu}{(u^2+e_2)^{1/2}(u^2+e_2-e_1)^{1/2}(b-u^2-e_2)^{1/2}(1-u^2-e_2)^{1/2}}$$

$$= 2\int_0^{\sqrt{\frac{b+e_2}{2}}} \frac{2u\,du}{(u^2+e_2)^{1/2}(u^2+e_2-e_1)^{1/2}(b-u^2-e_2)^{1/2}(1-u^2-e_2)^{1/2}}$$

Let us assume

$$u = \sqrt{\frac{b-e_2}{2}}\left(\frac{1}{2}+\frac{1}{2}X\right) \quad ,$$

$$I_{71} = \sqrt{\frac{b-e_2}{2}}\int_{-1}^{1} \frac{dX}{\left\{\frac{b-e_2}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2+e_2\right\}^{1/2}\left\{\frac{b-e_2}{2}\left(\frac{1}{2}+\frac{1}{2}x\right)^2+e_2-e_1\right\}^{1/2}}$$

$$\frac{1}{\left\{b-\frac{b-e_2}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2-e_2\right\}^{1/2}\left\{1-\frac{b-e_2}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2-e_2\right\}^{1/2}}$$

$$\mathrm{I}_{7.2} = \int_{\frac{b+e_2}{2}}^{b} \frac{dt}{t^{1/2}(t-e_1)^{1/2}(t-e_2)^{1/2}(b-t)^{1/2}(1-t)^{1/2}}$$

Let us assume b-t = $u^2$, t = b $-u^2$ dt = - 2u du

At t = b, u = 0 at t = $\frac{b+e_2}{2}$ , z = $\frac{\sqrt{b-e_2}}{2}$

$$\mathrm{I}_{72} = 2\int_{0}^{\sqrt{\frac{b-e_2}{2}}} \frac{du}{(b-u^2)^{1/2}(b-u^2-e_1)^{1/2}(b-u^2-e_2)^{1/2}(1-b+u^2)^{1/2}}$$

Let us assume

$$\mathrm{u} = \sqrt{\frac{b-e_2}{2}}\left(\frac{1}{2}+\frac{1}{2}X\right)$$

$$\mathrm{I}_{72} = \sqrt{\frac{b-e_2}{2}}\int_{-1}^{1} \frac{dX}{\left\{b-\frac{b-e_2}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2}\left\{b-\frac{b-e_2}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2-e_1\right\}^{1/2}}$$

$$\frac{1}{\left\{b-\frac{b-e_2}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2-e_2\right\}^{1/2}\left\{1-b+\frac{b-e_2}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2}}$$

$$\mathrm{I}_8 = \int_{e_2}^{b}\frac{t\,dt}{f(t)} = \int_{e_2}^{\frac{b+e_2}{2}}\frac{t\,dt}{f(t)} + \int_{\frac{b+e_2}{2}}^{b}\frac{t\,dt}{f(t)}$$

$$= \mathrm{I}_{8.1} + \mathrm{I}_{8.2}$$

$$\mathrm{I}_{8.1} = \int_{e_2}^{\frac{b+e_2}{2}} \frac{t\,dt}{t^{1/2}(t-e)^{1/2}(t-e_2)^{1/2}(b-t)^{1/2}(1-t)^{1/2}}$$

$$= \int_{e_2}^{\frac{b+e_2}{2}} \frac{t^{1/2}dt}{(t-e_1)^{1/2}(t-e_2)^{1/2}(b-t)^{1/2}(1-t)^{1/2}}$$

Let us assume

$t - e_2 = u^2$ $\qquad\qquad$ $t = u^2 + e_2$

At $t = \dfrac{b+e_2}{2}$, $\quad u = \sqrt{\dfrac{b-e_2}{2}}$, $\qquad$ at $t = e_2$, $u = 0$

$$I_{81} = 2 \int_0^{\sqrt{\frac{b-e_2}{2}}} \frac{(u^2+e_2)^{1/2}du}{(u^2+e_2-e_1)^{1/2}(b-u^2-e_2)^{1/2}(1-u^2-e_2)^{1/2}}$$

Let us assume

$$u = \sqrt{\frac{b-e_2}{2}}\left(\frac{1}{2}+\frac{1}{2}X\right),$$

$$I_{81} = \sqrt{\frac{b-e_2}{2}} \int_{-1}^{1} \frac{\left\{\frac{b-e_2}{2}\left(\frac{1}{2}+\frac{1}{2}x\right)^2+e_2\right\}^{1/2} dx}{\left\{\frac{b-e_2}{2}\left(\frac{1}{2}+\frac{1}{2}x\right)^2+e_2-e_1\right\}^{1/2}\left\{b-\frac{b-e_2}{2}\left(\frac{1}{2}+\frac{1}{2}x\right)^2-e_2\right\}^{1/2}}$$

$$\frac{1}{\left\{1-\frac{b-e_2}{2}\left(\frac{1}{2}+\frac{1}{2}x\right)^2-e_2\right\}^{1/2}}$$

$$I_{8.2} = \int_{\frac{b+e_2}{2}}^{b} \frac{t^{1/2}dt}{(t-e_1)^{1/2}(t-e_2)^{1/2}(b-t)^{1/2}(1-t)^{1/2}}$$

let us assume $b - t = u^2$, $At = b - u^2$, $dt = -2\,udu$

at $t = b$, $u = 0$, $t = \dfrac{b+e_2}{2}$, $X = \sqrt{\dfrac{b-e_2}{2}}$

$$I_{82} = 2\int_0^{\sqrt{\frac{b-e_2}{2}}} \frac{(b-u^2)^{1/2}du}{(b-u^2-e_1)^{1/2}(b-u^2-e_2)^{1/2}(1-b+u^2)^{1/2}}$$

Let us assume

$$u = \sqrt{\frac{b-e_2}{2}}\left(\frac{1}{2}+\frac{1}{2}X\right)$$

$$I_{82} = \sqrt{\frac{b-e_2}{2}}\int_{-1}^{1}\frac{\left\{b-\frac{b-e_2}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2} dX}{\left\{b-\frac{b-e_2}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2-e_1\right\}^{1/2}\left\{b-\frac{b-e_2}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2-e_2\right\}^{1/2}\left\{1-b+\frac{b-e_2}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2}}$$

For $b \le t' \le 1$

$$w(t') = M\int_b^{t'}\frac{(c-t).dt}{\sqrt{(-1)(t)(-1)(t-e_1)(-1)(t-e_2)(-1)(b-t)(1-t)}} - iq_1 - \alpha_2 kh \qquad (4.2.21)$$

For point A, $t' = 1$, $w = -\alpha_2 kh - iq$

Hence,

$$-\alpha_2 kh - iq = M\int_b^1\frac{(c-t).dt}{(t)^{1/2}(t-e_1)^{1/2}(t-e_2)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} - iq_1 - \alpha_2 kh$$

or

$$i(q_1 - q) = Mc\int_b^1\frac{dt}{f(t)} - M\int_b^1\frac{tdt}{f(t)}$$

in which,

$$f(t) = t^{1/2} (t - e_1)^{1/2} (t - e_2)^{1/2} (t - b)^{1/2} (1 - t)^{1/2}$$

$$i(q_1 - q) = Mc\ I_9 - M.I_{10} \qquad (4.2.22)$$

in which,

$$I_9 = \int_b^1 \frac{dt}{f(t)} = \int_b^{\frac{1+b}{2}} \frac{dt}{f(t)} + \int_{\frac{1+b}{2}}^1 \frac{dt}{f(t)}$$

$$= I_{9.1} + I_{9.2}$$

$$I_{9.1} = \int_b^{\frac{1+b}{2}} \frac{dt}{t^{1/2}(t-e_1)^{1/2}(t-e_2)^{1/2}(t-b)^{1/2}(1-t)^{1/2}}$$

Let us assume

$$t - b = u^2, \qquad t = u^2 + b, \qquad dt = 2u\,du$$

Hence,

$$I_{91} = \int_0^{\sqrt{\frac{1-b}{2}}} \frac{2z\,du}{\left(u^2+b\right)^{1/2}\left(u^2+b-a\right)^{1/2}\left(u^2+b-e_2\right)^{1/2}\left(u^2+b-b\right)^{1/2}\left(1-u^2-b\right)^{1/2}}$$

Assuming $u = \sqrt{\frac{1-b}{2}}\left(\frac{1}{2}+\frac{1}{2}X\right)$

$$I_{91} = \sqrt{\frac{1-b}{2}} \int_{-1}^{1} \frac{dX}{\left\{\frac{1-b}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2 + b\right\}^{1/2} \left\{\frac{1-b}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2 + b - e_1\right\}^{1/2}}$$

$$\frac{1}{\left\{\frac{1-b}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2 + b - e_2\right\}^{1/2} \left\{1 - \frac{1-b}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2 - b\right\}^{1/2}}$$

$$I_{9.2} = \int_{\frac{1+b}{2}}^{1} \frac{dt}{t^{1/2}(t-e_1)^{1/2}(t-e_2)^{1/2}(t-b)^{1/2}(1-t)^{1/2}}$$

Let us assume $1-t = u^2$, $t = 1-u^2$, $dt = -2u\,du$, $u = \sqrt{\frac{1-b}{2}}$

At $t = 1$, $u = 0$, at $t = \frac{1+b}{2}$,

$$I_{92} = 2\int_{0}^{\sqrt{\frac{1-b}{2}}} \frac{-2du}{(1-u^2)^{1/2}(1-u^2-e_1)^{1/2}(1-u^2-e_2)^{1/2}(1-u^2-b)^{1/2}}$$

Assuming $u = \sqrt{\frac{1-b}{2}}\left(\frac{1}{2}+\frac{1}{2}X\right)$,

$$I_{92} = \sqrt{\frac{1-b}{2}}\int_{-1}^{1} \frac{dx}{\left\{1-\frac{1-b}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2}\left\{1-\frac{1-b}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2-e_1\right\}^{1/2}}$$

$$\frac{1}{\left\{1-\frac{1-b}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2-e_2\right\}^{1/2}\left\{1-\frac{1-b}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2-b\right\}^{1/2}}$$

$$I_{10} = \int_{b}^{1}\frac{tdt}{f(t)} = \int_{b}^{\frac{1+b}{2}}\frac{tdt}{f(t)} + \int_{\frac{1+b}{2}}^{b}\frac{tdt}{f(t)}$$

$= I_{10.1} + I_{10.2}$

$$I_{101} = \int_{b}^{\frac{1+b}{2}} \frac{t^{1/2}dt}{(t-e_1)^{1/2}(t-e_2)^{1/2}(t-b)^{1/2}(1-t)^{1/2}}$$

= let us assume $t - b = u^2$, $t = u^2 + b$, $dt = 2u\,du$

At t = b , u = 0, at $t = \frac{1+b}{2}$, $u = \sqrt{\frac{1-b}{2}}$

$$I_{10.1} = 2\int_0^{\sqrt{\frac{1-b}{2}}} \frac{\left(u^2+b\right)^{1/2} du}{\left(u^2+b-e_1\right)^{1/2}\left(u^2+b-e_2\right)^{1/2}\left(1-u^2-b\right)^{1/2}}$$

Assuming $u = \sqrt{\frac{1-b}{2}}\left(\frac{1}{2}+\frac{1}{2}X\right)$

$$I_{10.1} = \sqrt{\frac{1-b}{2}}\int_{-1}^{1} \frac{\left\{\frac{1-b}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2+b\right\}^{1/2} dX}{\left\{\frac{1-b}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2+b-e_1\right\}^{1/2}\left\{\frac{1-b}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2+b-e_2\right\}^{1/2}\left\{1-\frac{1-b}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2-b\right\}^{1/2}}$$

$$I_{102} = \int_{\frac{1+b}{2}}^{1} \frac{tdt}{(t-e_1)^{1/2}(t-e_2)^{1/2}(t-b)^{1/2}(1-t)^{1/2}}$$

let us assume $1 - t = u^2$ $\quad t = 1 - u^2$, $\quad dt = -2u\,du$

At t = 1, u = 0, $\quad$ at $t = \frac{1+b}{2}$, $u = \sqrt{\frac{1-b}{2}}$

$$I_{10.2} = \int_0^{\frac{1-b}{2}} \frac{(1-u^2)^{1/2}\mathrm{du}}{(1-u^2-e_1)^{1/2}(1-u^2-e_2)^{1/2}(1-u^2-b)^{1/2}}$$

Assuming $u = \sqrt{\frac{1-b}{2}}\left(\frac{1}{2}+\frac{1}{2}x\right)$

$$I_{10.2} = \sqrt{\frac{1-b}{2}}\int_{-1}^{1} \frac{\left\{1-\frac{1-b}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2} dX}{\left\{1-\frac{1-b}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2-e_1\right\}^{1/2}\left\{1-\frac{1-b}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2-e_2\right\}^{1/2}\left\{1-\frac{1-b}{2}\left(\frac{1}{2}+\frac{1}{2}X\right)^2-b\right\}^{1/2}}$$

For $1 \leq t' \leq \infty$

$$w(t) = M\int_1^{\infty}\frac{(c-t)dt}{\sqrt{(-1)(t)(-1)(t-e_1)(-1)(t-b)(-1)(-1)(t-1)}} - \alpha_2 kh - iq \qquad (4.2.23)$$

For point G $t' = \infty$, $w = -\alpha_1 kh - iq$

$$-\alpha_1 kh - iq = \frac{M}{\sqrt{1}}c\int_1^{\infty}\frac{dt}{f(t)} - \frac{M}{\sqrt{-1}}\int_1^{\infty}\frac{tdt}{f(t)} - \alpha_2 kh - iq$$

Hence,

$$-kh(\alpha_1 - \alpha_2) = \frac{M}{\sqrt{-1}}cI_{11} - \frac{M}{\sqrt{-1}}I_{12} \qquad (4.2.24)$$

$$I_{11} = \int_1^{\infty}\frac{dt}{t^{1/2}(t-e_1)^{1/2}(t-e_2)^{1/2}(t-b)^{1/2}(t-1)^{1/2}}$$

let us assume $t-1 = u^2$, $dt = 2u\,du$

At $t = 1$, $u = 0$, at $t = \infty$, $u = \infty$

$$I_{11} = 2\int_0^{\infty}\frac{du}{(u^2+1)^{1/2}(u^2+1-e_1)^{1/2}(u^2+1-e_2)^{1/2}(u^2+1-b)^{1/2}}$$

$$= 2\int_0^{1}\frac{dz}{f(z)} + 2\int_1^{\infty}\frac{dz}{f(z)}$$

$$= I_{11.1} + I_{11.2}$$

$$I_{11.1} = 2\int_0^{1}\frac{du}{(u^2+1)^{1/2}(u^2+1-e_1)^{1/2}(u^2+1-e_2)^{1/2}(u^2+1-b)^{1/2}}$$

Let as assume $u = \frac{1}{2} + \frac{1}{2}X$,

$$I_{111} = \int_{-1}^{1} \frac{dX}{\left\{\left(\frac{1}{2}+\frac{1}{2}X\right)^2 +1\right\}^{1/2} \left\{\left(\frac{1}{2}+\frac{1}{2}X\right)^2 +1-e_1\right\}^{1/2} \left\{\left(\frac{1}{2}+\frac{1}{2}X\right)^2 +1-e_2\right\}^{1/2} \left\{\left(\frac{1}{2}+\frac{1}{2}X\right)^2 +1-b\right\}^{1/2}}$$

$$I_{11.2} = 2\int_{1}^{\infty} \frac{du}{(u^2+1)^{1/2}(u^2+1-e_1)^{1/2}(u^2+1-e_2)^{1/2}(u^2+1-b)^{1/2}}$$

Let as assume $u = \frac{1}{v}$, $du = -\frac{1}{v^2}dv$

at $u = 1, v = 1$, at $u = \infty$ , $v = 0$

Hence,

$$I_{112} = 2\int_{0}^{1} \frac{v^2.dv}{\left(1+v^2\right)^{1/2}\left(1+v^2-e_1v^2\right)^{1/2}\left(1+v^2-e_2v^2\right)^{1/2}\left(1+v^2-bv^2\right)^{1/2}}$$

let's assume $v = \frac{1}{2}+\frac{1}{2}X$

$$I_{112} = \int_{-1}^{1} \frac{\left\{\frac{1}{2}+\frac{1}{2}x\right\}^2 dx}{\left\{1+\left(\frac{1}{2}+\frac{1}{2}x\right)^2\right\}^{1/2} \left\{1+\left(\frac{1}{2}+\frac{1}{2}x\right)^2 - e_1\left(\frac{1}{2}+\frac{1}{2}x\right)^2\right\}^{1/2}}$$

$$\frac{1}{\left\{1+\left(\frac{1}{2}++\frac{1}{2}x\right)^2 - e_2\left(+\frac{1}{2}++\frac{1}{2}x\right)^2\right\}^{1/2} \left\{\left(2+\frac{1}{2}+\frac{1}{2}x\right)^2 - b\left(\frac{1}{2}+\frac{1}{2}x\right)^2\right\}^{1/2}}$$

$$I_{12} = \int_{1}^{\infty} \frac{t^{1/2}\,\mathrm{dt}}{(t-e)^{1/2}(t-e_2)^{1/2}(t-b)^{1/2}(t-1)^{1/2}}$$

Let us assume

$t - 1 = u^2$, $\quad t = u^2+1$, $\quad dt = 2u\,du$

At $t = \infty$, $u = \infty$, at $t = 1$, $u = 0$

$$I_{12} = 2\int_0^\infty \frac{(u^2+1)^{1/2}\,du}{(u^2+1-e_1)^{1/2}(u^2+1-e_2)^{1/2}(u^2+1-b)^{1/2}}$$

$$= 2\int_0^1 \frac{(zu^2+1)^{1/2}\,du}{f(u)} + 2\int_1^\infty \frac{(u^2+1)^{1/2}\,du}{f(u)}$$

$$= I_{12.1} + I_{12.2}$$

in which

$$f(u) = (u^2 + 1-e_1)^{1/2}\,(u^2+1-e_2)^{1/2}\,(u^2+1-b)$$

$$I_{12..1} = 2\int_0^1 \frac{(u^2+1)^{1/2}\,du}{(u^2+1-e_1)^{1/2}(u^2+1-e_2)^{1/2}(u^2+1-b)^{1/2}}$$

Let us assume $u = \dfrac{1}{2} + \dfrac{1}{2}X$

$$I_{12.1} = \int_{-1}^{1} \frac{\left\{\left(\frac{1}{2}+\frac{1}{2}X\right)^2+1\right\}^{1/2}}{\left\{\left(\frac{1}{2}+\frac{1}{2}X\right)^2+1-e_1\right\}^{1/2}\left\{\left(\frac{1}{2}+\frac{1}{2}X\right)^2+1-e_2\right\}^{1/2}\left\{\left(\frac{1}{2}+\frac{1}{2}X\right)^2+1-b\right\}^{1/2}}$$

$$I_{12..2} = 2\int_1^\infty \frac{(u^2+1)^{1/2}\,du}{(u^2+1-e_1)^{1/2}(u^2+1-e_2)^{1/2}(u^2+1-b)^{1/2}}$$

Let as assume $u = \dfrac{1}{v}$, $\quad du = -\dfrac{1}{v^2}dv$

At $u = \infty$, $v = 0$ at $u = 1$, $v = 1$

$$I_{12.1} = 2\int_0^1 \frac{\left(1+v^2\right)^2 .dv}{\left(1+v^2-e_1v^2\right)^{1/2}\left(1+v^2-e_2v^2\right)^{1/2}\left(1+v^2-bv^2\right)^{1/2}}$$

Assuming $u = \frac{1}{2}+\frac{1}{2}X$

$$I_{12.2} = \int_{-1}^{1} \frac{\left\{1+\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2} dx}{\left\{1+\left(\frac{1}{2}+\frac{1}{2}X\right)^2-e_1\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2}\left\{1+\left(\frac{1}{2}+\frac{1}{2}X\right)^2-e_2\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2}}.$$

$$\frac{1}{\left\{1+\left(\frac{1}{2}+\frac{1}{2}X\right)^2-b\left(\frac{1}{2}+\frac{1}{2}X\right)^2\right\}^{1/2}}$$

## 4.3 FRAGMENT III

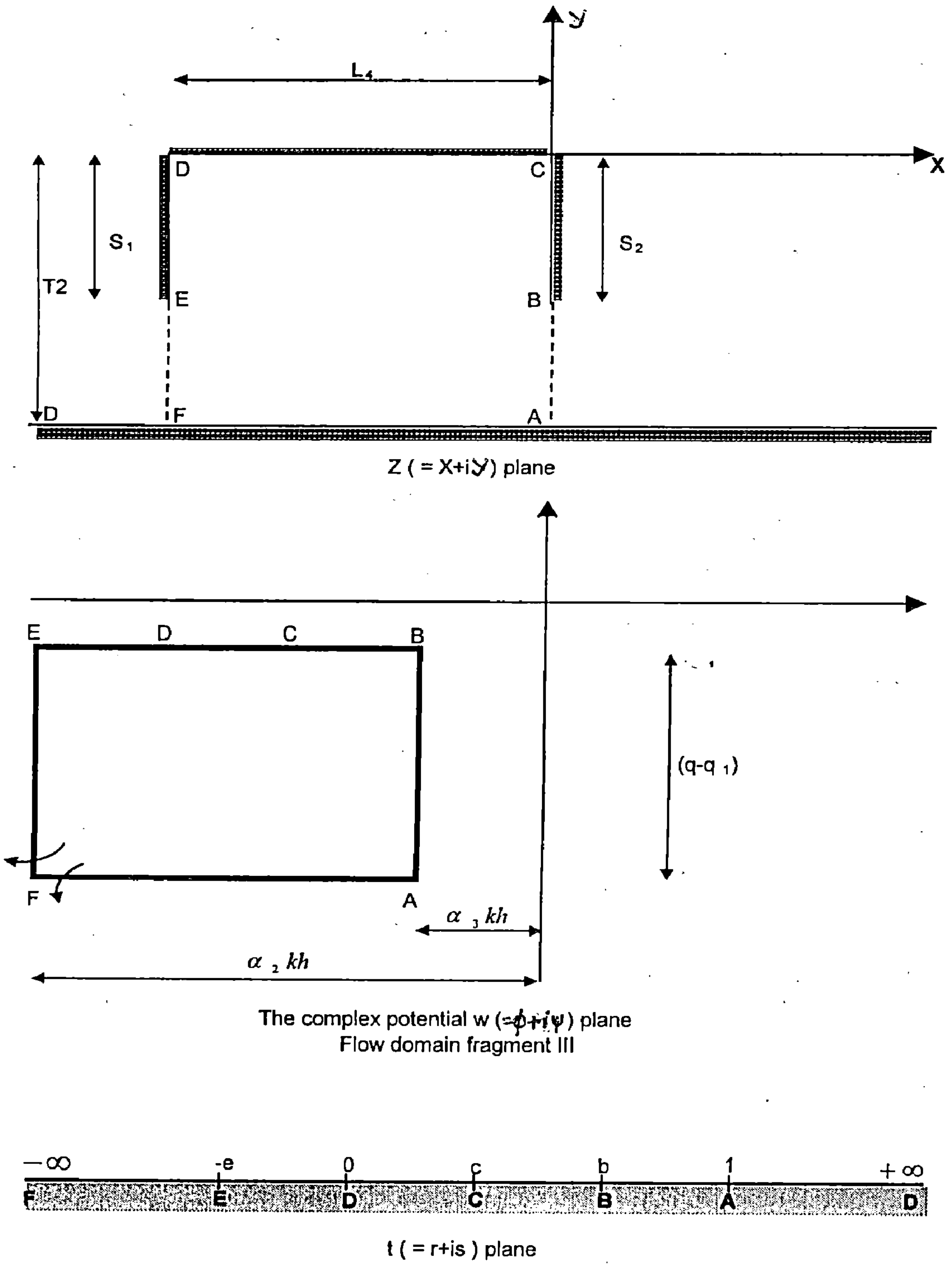



**Figure 4.3 Flow Domain of Fragment III, the Complex Potential Plane and t Plane**

Applying the Schwarz-Christoffel transformation, the conformal mapping of flow domain in fragment III in z-plane onto the auxiliary t-plane is given by

$$z = M\int_{-\infty}^{t'} \frac{dt}{(-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} - L_4 - iT_2 \qquad (4.3.1)$$

$$z = M2F(\vartheta, m) - L_4 - iT_2$$

in which

$$\vartheta = \sin^{-1}\sqrt{\frac{1}{1-t'}}$$

$$m = \sqrt{1-c}$$

For point E, t = -e, z = $-L_4 - iS_1$.

Hence,

$$-L_4 - iS_1 = M2F\left(\sin^{-1}\sqrt{\frac{1}{1+e}}, \sqrt{1-c}\right) - L_4 - iT_2$$

$$i(T_2 - S_1) = M2F\left(\sin^{-1}\sqrt{\frac{1}{1+e}}, \sqrt{1-c}\right) \qquad (4.3.2)$$

For point D, t = 0, Z = $-L_4$, hence

$$-L_4 = M\,2F\left(\pi/2, \sqrt{1-c}\right) - L_4 - iT_2$$

$$M = \frac{iT_2}{2F\left(\frac{\pi}{2}, \sqrt{1-c}\right)} \qquad (4.3.3)$$

Substituting M in (4.3.2)

$$\frac{T_2 - S_1}{T_2} = \frac{F\left(\sin^{-1}\sqrt{\dfrac{1}{1+e}}, \sqrt{1-c}\right)}{F\left(\dfrac{\pi}{2}, \sqrt{1-c}\right)} \qquad (4.3.4)$$

For $0 \le t' \le c$, the corresponding z is given by:

$$z = M\int_0^{t'} \frac{dt}{(-1)^{1/2}(t)^{1/2}(c-t)^{1/2}(1-t)^{1/2}} - L_4$$

$$= \frac{M}{i} 2MF(\vartheta, m) - L_4$$

in which,

$$\vartheta = \sin^{-1}\sqrt{\frac{t'}{c}}$$

and

$$m = \sqrt{c}$$

For pint C, $t' = c$ , $z = 0$

Hence,

$$0 = \frac{M}{i} 2F(\frac{\pi}{2}, \sqrt{c}) - L_4$$

or

$$L_4 = \frac{M}{i} 2F(\frac{\pi}{2}, \sqrt{c}) \qquad (4.3.5)$$

Substituting M

$$\frac{L_4}{T_2} = \frac{F\left(\frac{\pi}{2}, \sqrt{c}\right)}{F\left(\frac{\pi}{2}, \sqrt{1-c}\right)} \tag{4.3.6}$$

the parameter c is obtained through iteration.

For $c < t' < 1$, the corresponding z is given by:

$$z = M\int_0^{t'} \frac{dt}{(-1)^{1/2}(t)^{1/2}(-1)^{1/2}(t-c)^{1/2}(1-t)^{1/2}} + N, \tag{4.3.7}$$

$$= -M2F(\vartheta, m) + N$$

in which

$$\vartheta = \sin^{-1}\sqrt{\frac{t'-c}{(1-c)t'}}$$

$$m = \sqrt{1-c}$$

since for $t' = 0$, $z = 0$, $N = 0$

For pint A, $t' = 1$, $z = i\,T_2$

Hence,

$$-iT_2 = -M2F\left(\frac{\pi}{2}, \sqrt{1-c}\right) \tag{4.3.8}$$

Eq (4.3.8) is same as (4.3.3).

For point B, $t' = b$ and $z = i\,S_2$

Hence

$\sqrt{1-c}$

$$-i\,S_2 = M2F\left(\sin^{-1}\sqrt{\frac{(b-c)}{b(1-c)}}, \sqrt{1-c}\right) \tag{4.3.9}$$

Substituting M

$$-i\ S_2 = -\frac{iT_2}{2F\left(\frac{\pi}{2},\sqrt{1-c}\right)}2F\left(\sin^{-1}\sqrt{\frac{(b-c}{b(1-c)}},\sqrt{1-c}\right) \tag{4.3.10}$$

Hence,

$$\frac{S_2}{T_2} = \frac{F\left(\sin^{-1}\sqrt{\frac{b-c}{b(1-c)}},\sqrt{1-c}\right)}{F\left(\frac{\pi}{2},\sqrt{1-c}\right)} \tag{4.3.11}$$

The parameter c has already been known. The parameter 'b' is obtained following iteration procedure. We do not get any independent equation when we consider. The domain from 1 to ∞.

The Schwarz-Chritaffel conformal mapping of the complex potential plane pertaining to flow domain fragment III onto t-plane is given by

$$w\ (t') = M_1 \int_{-\infty}^{t'} \frac{dt}{(-e-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} - \alpha_2 kh - iq \tag{4.3.12}$$

For $-\infty < t' < -e$

$$w(t') = M_1\, 2F\,(\vartheta, m) - \alpha_2 kh - iq$$

in which,

$$\vartheta = \sin^{-1}\sqrt{\frac{1+e}{1-t'}}$$

$$m = \sqrt{\frac{1+b}{1+e}}$$

For point E , $t' = -e$ , $w = -\alpha_2 kh - iq_1$

Hence,

$$i\ (q-q_1) = 2\,M_1F\left(\frac{\pi}{2},\sqrt{\frac{1-b}{1+e}}\right)$$

and

$$M_1 = \frac{i(q-q_1)}{2F\left(\frac{\pi}{2},\sqrt{\frac{1-b}{1+e}}\right)} \qquad (4.3.13)$$

For $-e < t' < -b$, the corresponding w is given by:

$$w(t') = M_1 \int_{-e}^{t'} \frac{dt}{(-1)^{1/2}(t-e)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} - \alpha_2 kh - iq_1 \qquad (4.3.14)$$

$$= \frac{M_1}{i} 2F(\vartheta, m) - \alpha_2 kh - iq_1$$

in which

$$\vartheta = \sin^{-1}\sqrt{\frac{t'+e}{b+e}}$$

and

$$m = \sqrt{\frac{b+e}{1+e}}$$

For point B, $t' = b$ and $\omega = -\alpha_3 kh - iq_1$

Hence,

$$ikh(\alpha_2 - \alpha_3) = M_1 2F\left(\frac{\pi}{2},\sqrt{\frac{b+e}{1+e}}\right) \qquad (4.3.15)$$

Substituting $M_1$ from equation (4.3.13) into (4.3.15)

$$(q-q_1)=kh(\alpha_2-\alpha_3)=\frac{F\left(\frac{\pi}{2},\sqrt{\frac{1-b}{1+e}}\right)}{F\left(\frac{\pi}{2},\sqrt{\frac{b+e}{1+e}}\right)} \tag{4.3.16}$$

Let us assume:

$$\frac{F\left(\frac{\pi}{2},\sqrt{\frac{1-b}{1+e}}\right)}{F\left(\frac{\pi}{2},\sqrt{\frac{b+e}{1+e}}\right)}=C$$

$$\frac{q}{kh}-\frac{q_1}{kh}=\alpha_2 C-\alpha_3 C$$

$$\frac{q}{kh}-\frac{q_1}{kh}-\alpha_2 C+\alpha_3 C=o \tag{4.3.17}$$

## 4.4 FRAGMENT IV

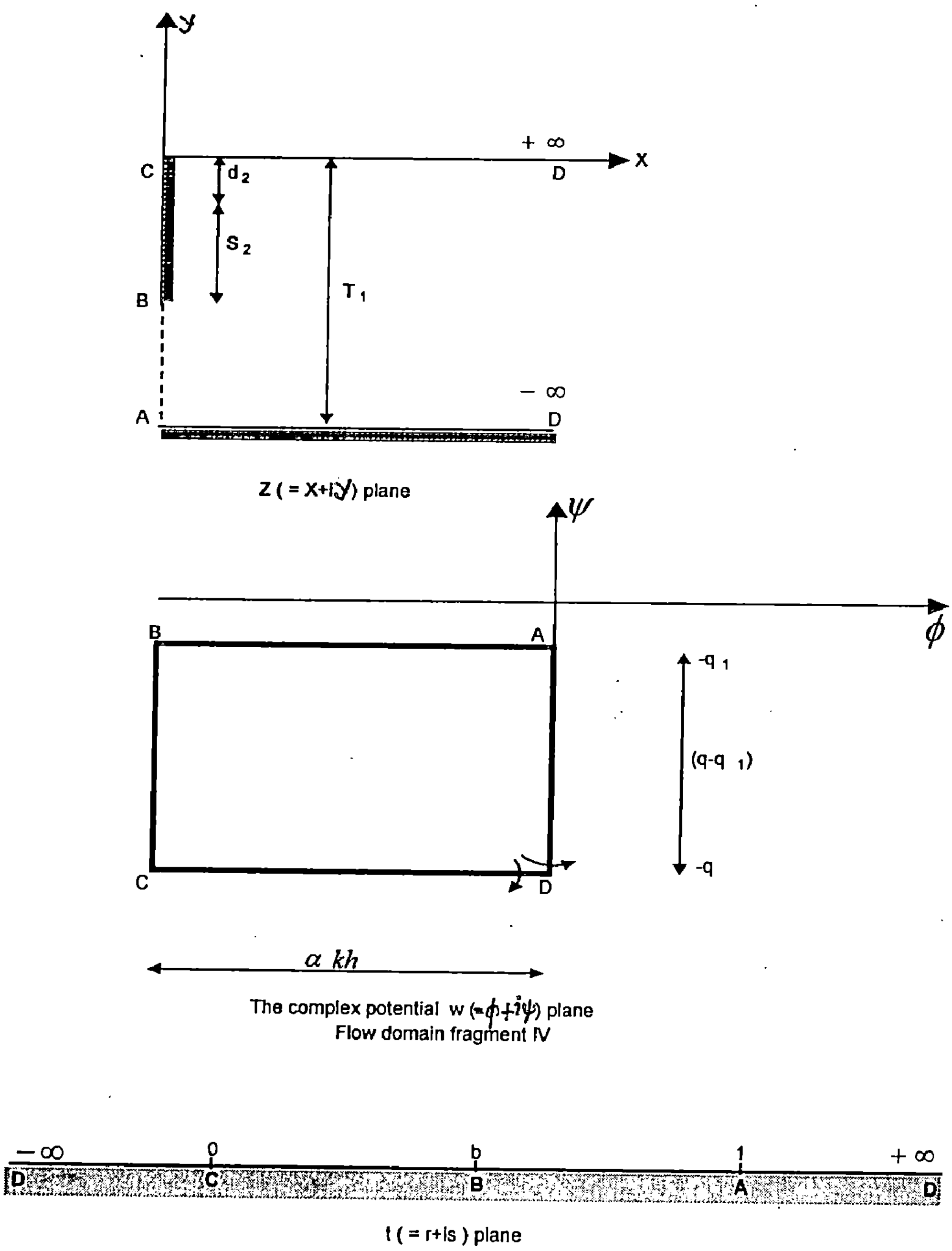



**Figure 4.4 Flow Domain of Fragment IV and its Complex Potential Plane and the Auxiliary t plane**

Applying the Schwarz –Christoffel transformation, the conformal mapping of the fragment IV in z plane on to the auxiliary t plane is given by

$$z = M \int \frac{dt}{t^{1/2}(1-t)^{1/2}} + N \qquad (4.4.1)$$

Integrating

$$z = 2\,M\,\sin^{-1}\sqrt{t} + N$$

For point C, t = 0 ,z = $-iT_3$

Hence $N = -iT_3$

For point A, t = 1 and z = 0 hence,

$$o = 2\,M\,.\frac{\pi}{2} + (-iT_3)$$

and

$$M = \frac{iT_3}{\pi}$$

The relation between z and t plane is

$$z = 2\,\frac{iT_3}{\pi}\sin^{-1}\sqrt{t} - iT_3 \qquad (4.4.2)$$

For point B, t = b, z = $-i(d_3 + S_2)$

$$-i(d_3 + S_2) = 2\,\frac{iT_3}{\pi}\sin^{-1}\sqrt{b} - iT_3$$

or

$$b = \left[\sin\frac{\pi}{2}\left\{1 - \left(1 - \frac{d_3 + S_2}{T_3}\right)\right\}\right]^2 \qquad (4.4.3)$$

The Schwarz –Christoffel conformal mapping of complex potential onto t-plane is given by:

For $-\infty < t' \leq c$, the corresponding w is given by:

$$w(t') = M_1 \int_{-\infty}^{t'} \frac{dt}{(-t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} - iq$$

or

$$w(t') = M_1\, 2F(\vartheta, m) - iq$$

in which

$$\vartheta = \sqrt{\frac{1}{1-t'}}$$

and

$$m = \sqrt{1-b}$$

For point C, $t' = 0$, $w = -\alpha_3 kh - iq$

Hence,

$$-\alpha_3 kh - iq = M_1 2F\left(\frac{\pi}{2}, \sqrt{1-b}\right) - iq$$

from which

$$M_1 = \frac{-\alpha_3 kh}{2F\left(\frac{\pi}{2}, \sqrt{1-b}\right)} \qquad (4.4.4)$$

For $0 < t' < b$, the relation between w and t plane is given by

$$w(t) = M_1 \int_0^{t'} \frac{dt}{(-1)^{1/2}(t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} - \alpha_3 kh - iq$$

$$= \frac{M_1}{i} 2F(\vartheta, m) - \alpha_3 kh - iq$$

in which

$$\vartheta = \sqrt{\frac{t'}{b}}$$

$$m = \sqrt{b}$$

For point B , $t' = b$ and $w = -\alpha_3 kh - iq_1$

Hence,

$$-\alpha_3 kh - iq_1 = 2\frac{M_1}{i} F\left(\frac{\pi}{2}, \sqrt{b}\right) - \alpha_3 kh - iq$$

or

$$q - q_1 = -2M_1 F\left(\frac{\pi}{2}, \sqrt{b}\right) \tag{4.4.5}$$

Substituting $M_1$

$$q - q_1 = \alpha_3 kh \frac{F\left(\frac{\pi}{2}, \sqrt{b}\right)}{F\left(\frac{\pi}{2}, \sqrt{1-b}\right)} \tag{4.4.6}$$

Substituting the parameter be from (4.4.3)

Hence

$$\frac{q - q_1}{kh} = \alpha_3 \frac{F\left[\frac{\pi}{2}, \left\{\sin\frac{\pi}{2}\left(1 - \frac{d_3 + S_2}{T_3}\right)\right\}\right]}{F\left[\frac{\pi}{2}, \sqrt{1 - \left\{\sin\frac{\pi}{2}\left(1 - \frac{d_3 + S_2}{T_3}\right)\right\}^2}\right]}$$

Let us assume

$$\frac{F\left[\frac{\pi}{2}, \left\{\sin\frac{\pi}{2}\left(1 - \frac{d_3 + S_2}{T_3}\right)\right\}\right]}{F\left[\frac{\pi}{2}, \sqrt{1 - \left\{\sin\frac{\pi}{2}\left(1 - \frac{d_3 + S_2}{T_3}\right)\right\}^2}\right]} = D$$

Hence

$$\frac{q}{kh} - \frac{q_1}{kh} - \alpha_3 D = 0 \qquad (4.4.7)$$

## 4.5 RESULT AND DISCUSION

The following relationships have been derived using w and t planes in fragment II.

$$q = Mc\, I_1 + MI_2 \qquad (4.2.14)$$

$$kh(\alpha_1 - \alpha_d) = \frac{M}{\sqrt{-1}} cI_3 - \frac{M}{\sqrt{-1}} I_4 \qquad (4.2.16)$$

$$iq_1 = Mc\, I_5 - MI_6 \qquad (4.2.18)$$

$$kh(\alpha_d - \alpha_2) = \frac{M}{\sqrt{-1}} cI_7 + \frac{M}{\sqrt{-1}} I_8 \qquad (4.2.20)$$

$$i(q - q_1) = Mc\, I_9 - MI_{10} \qquad (4.2.22)$$

$$kh(\alpha_1 - \alpha_2) = -\frac{M}{\sqrt{-1}} cI_{11} + \frac{M}{\sqrt{-1}} dI_{12} \qquad (4.2.24)$$

To solve the unknown $\frac{q}{kh}, \frac{q_1}{kh}, \alpha_1, \alpha_2, \alpha_3$, the parameter c and the constant M that relates w

and t plane in Fragment II, the following procedure is followed,:

We assume a value of $c > d$ and predict the value of $\alpha_d$

From equation (4.2.22) and (4.2.24) we get

$$\frac{q - q_1}{kh(k_1 - k_2)} = \frac{-cI_9 + I_{10}}{-cI_{11} + I_{12}}$$

Let $$V = \frac{-cI_9 + I_{10}}{-cI_{11} + I_{12}}$$

Hence,

$$\frac{q}{kh}-\frac{q_1}{kh}-\alpha_1 V+\alpha_2 V=0 \tag{4.2.25}$$

Using eq. (4.2.16) and (4.2.24) we get

$$\frac{(\alpha_1-\alpha_d)}{(\alpha_1-\alpha_2)}=\frac{cI_3-I_4}{-cI_{11}+I_{12}}$$

Let $$\mathrm{U}=\frac{cI_3-I_4}{-cI_{11}+I_{12}}$$

Hence,

$$\alpha_1(1-u)+u\alpha_2-\alpha_d=0 \tag{4.2.26}$$

using eq. (4.2.14)and (4.2.20) we get

$$\frac{q}{kh(\alpha_d-\alpha_2)}=\frac{cI_1+I_2}{-cI_7+I_8}$$

Let $$\mathrm{W}=\frac{cI_1+I_2}{-cI_7+I_8}$$

Hence,

$$\frac{q}{kh}+W\alpha_2-W\alpha_d=0 \tag{4.2.27}$$

From segment I from eq.(4.1.19)

$$Aa_1+\frac{q}{kh}=A$$

From Fragment III from eq. (4.3.17).

$$-Ca_2+C\alpha_3+\frac{q}{kh}-\frac{q_1}{kh}=0$$

From Fragment IV from eq. (4..4.7).

$$-D\alpha_3+\frac{q}{kh}-\frac{q_1}{kh}=0$$

The 6 equations can be written in the following matrix form:

$$\begin{bmatrix} A & 0 & 0 & 0 & 1 & 0 \\ 0 & -C & C & 0 & 1 & -1 \\ 0 & 0 & -D & 0 & 1 & -1 \\ 1-U & U & 0 & -1 & 0 & 0 \\ -V & V & 0 & 0 & 1 & -1 \\ 0 & W & 0 & -W & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_d \\ q/kh \\ q_1/kh \end{bmatrix} = \begin{bmatrix} A \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence,

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_d \\ q/kh \\ q_1/kh \end{bmatrix} = \begin{bmatrix} A & 0 & 0 & 0 & 1 & 0 \\ 0 & -C & C & 0 & 1 & -1 \\ 0 & 0 & -D & 0 & 1 & -1 \\ 1-U & U & 0 & -1 & 0 & 0 \\ -V & V & 0 & 0 & 1 & -1 \\ 0 & W & 0 & -W & 1 & 0 \end{bmatrix}^{-1} \cdot \begin{bmatrix} A \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

## 4.6 COMPUTATION OF POTENTIAL ALONG THE BASE

After solving $\alpha_1, \alpha_2, \alpha_3, \alpha_d$, $q_1$ / (kh) and $q_1$ / (kh ), the potential can be computed in different fragments. Let us consider computation of potential in the second fragment beyond the drain.

From segments II for $e_2 < t' < b$

$$\phi(t') = \frac{-M}{\sqrt{-1}} \int_{2}^{t'} \frac{(c-t)dt}{(t)^{1/2}(t-e_1)^{1/2}(t-e_2)^{1/2}(b-t)^{1/2}(1-t)^{1/2}} - \alpha_d kh$$

substituting $( t - e_2) = u^2$, $t = e_2 + u^2$ , $dt = 2u\, du$

and further submitting

$$u = \frac{\sqrt{t'-e_2}}{2}(1+v), du = \frac{\sqrt{t'-e_2}}{2} dv$$

$$\phi(t') = \frac{-M}{\sqrt{-1}} \int_{-1}^{1} \frac{\left\{c - e_2 - \frac{t'-e_2}{4}(1+v)^2\right\} dv}{\left\{e_2 + \frac{t'-e_2}{4}(1+v)^2\right\}^{1/2} \left\{e_2 - \frac{t'-e_2}{4}(1+v)^2 - e_1\right\}^{1/2} \left\{b - e_2 - \frac{t'-e_R}{4}(1+v)^2\right\}^{1/2} \left\{1 - e_1 - \frac{t'-e_2}{4}(1+v)^2\right\}^{1/2}} - \alpha_d kh$$

$\phi(t') = \phi_C$ for $t' = c$.

$\phi(t') = \phi_C$ for $t' = d$.

Similar expression can be obtained for computing potential at any desired point in the fragments.

## 4.7 COMPUTATION OF EXIT GRADIENT

The exit gradient can be obtained from the analysis of flow in segment IV.

$$\frac{dw}{dz} = \frac{dw}{dt.}\frac{dt}{dz} = u - iv \; = \text{u-i( k } I_E) .$$

As the downstream boundary is horizontal velocity u = 0,

**Hence,**

$$\frac{dw}{dz} = -ivkI_E$$

From analysis of segment II

$$\frac{dz}{dt} = \frac{iT_3}{\pi} . \frac{1}{t^{1/2}(1-t)^{1/2}}$$

and

$$\frac{dw}{dt} = \frac{-\alpha_3 kh}{2F\left(\frac{\pi}{2}, \sqrt{1-b}\right)} . \frac{1}{(-1)^{1/2}(t)^{1/2}(b-t)^{1/2}(1-t)^{1/2}}$$

Hence,

$$-ikI_E = \frac{-\alpha_3 kh}{2F\left(\frac{\pi}{2}, \sqrt{1-b}\right)(-1)^{1/2} t^{1/2}(b-t)^{1/2}(1-t)^{1/2}} . \frac{\pi t^{1/2}(1-t)^{1/2}}{iT_3}$$

Substituting t = 1

$$I_E /h = \frac{-\alpha_3}{2F\left(\frac{\pi}{2}, \sqrt{1-b}\right) . \sqrt{1-b} . T_3}$$

By computer program we will find out the value of maximum exit gradient

## 4.8 RESULTS AND DISCUSSION

### 4.8.1 Computation of Complete Elliptic Integral of First Kind

The complete elliptic integral F (π/2,m) of the first kind is defined as

$$F\left(\frac{\pi}{2}, m\right) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1-m^2\sin^2\theta}}$$

Substituting:

$$\theta = \frac{\pi}{2}\left(\frac{1}{2}+\frac{1}{2}X\right), \quad d\theta = \frac{\pi}{4}dx$$

$$F\left(\frac{\pi}{2},m\right) = \int_{-1}^{1} \frac{\frac{\pi}{4}dx}{\sqrt{1-\sin^2\left(\frac{\pi}{4}+\frac{\pi}{4}X\right)}} = \int_{-1}^{1} F(X)dx$$

$$= \sum_{i=1}^{96} W(i)\,F(X_i)$$

$$F\left(\frac{\pi}{2},m\right) = \sum_{i=1}^{96} W(i)\left[\frac{\pi/4}{\sqrt{1-m^2\sin^2\left(\frac{\pi}{2}+\frac{\pi}{2}X\right)}}\right]$$

$F\left(\frac{\pi}{2},m\right)$ can be evaluated using Gauss qudratue formala.

### 4.8.2 Computation of Incomplete Elliptic Integral of First Kind

The incomplete elliptic integral F(ϑ,m)of the first kind is defined as:

$$F(\vartheta,m) = \int_0^{\vartheta_1} \frac{dt}{\sqrt{1-m^2\sin^2\theta}}$$

Substituting

$$\theta = \frac{1}{2}\vartheta_1 + \frac{1}{2}\vartheta_1 X, \qquad d\theta = \frac{1}{2}\varphi_1 dX$$

hence,

$$F(\vartheta,m) = \int_{-1}^{1} \frac{\frac{1}{2}\vartheta_1 dx}{\sqrt{1 - m^2 \sin^2\left(\frac{\vartheta_1}{2} + \frac{\vartheta_1}{2}X\right)}} = \int_{-1}^{1} f(X)dX$$

$$= \sum_{1}^{96} W(i)F(x_i)$$

$$= \sum_{-1}^{96} W(i)\left[\frac{\frac{1}{2}\vartheta_1}{\sqrt{1 - m^2 \sin^2\left(\frac{\vartheta_1}{2} + \frac{\vartheta_1}{2}X_i\right)}}\right]$$

incomplete elliptic integral can be evaluated using Gauss quadrature formla.

In this analysis we have assumed c and determined the corresponding $\alpha_d$. Point c is the point along the impervious boundary of the structure at which reversal of flow takes place. If a drain is maintained at $-\alpha_d h$ potential, the corresponding parameter c can be known from figures 4.8.3.1(a) to (c). We have maintained zero potential along the drain, therefore $\alpha_d = 0$, for which the corresponding parameter c can be known from these figures.

The performance of the drain is studied considering its position at three different locations on the first floor of the depressed weir. The length of the drain has been taken as 2 m. The corresponding parameter $\alpha_d$. for the three locations considered is presented in Fig. 4.8.3.2.

The distribution of potential / (-kh) along the base is shown in figs. 4.8.3.3(a) to (c) for different location of the drain. It could be seen that a perfect drain reduces the uplift pressure considerably. The reduction is more near the drain.

The reductions in potential in the $2^{nd}$ floor are shown in figs. 4.8.3.4(a) to (c). Though the drain is located along the first floor, still it reduces the potential under the second floor and hence uplift pressure on the base of the $2^{nd}$ floor.

**FLOW UNDER A DEPRESSED WEIR WITHOUT A DRAIN RESTING ON A POROUS FOUNDATION OF FINITE DEPTH**

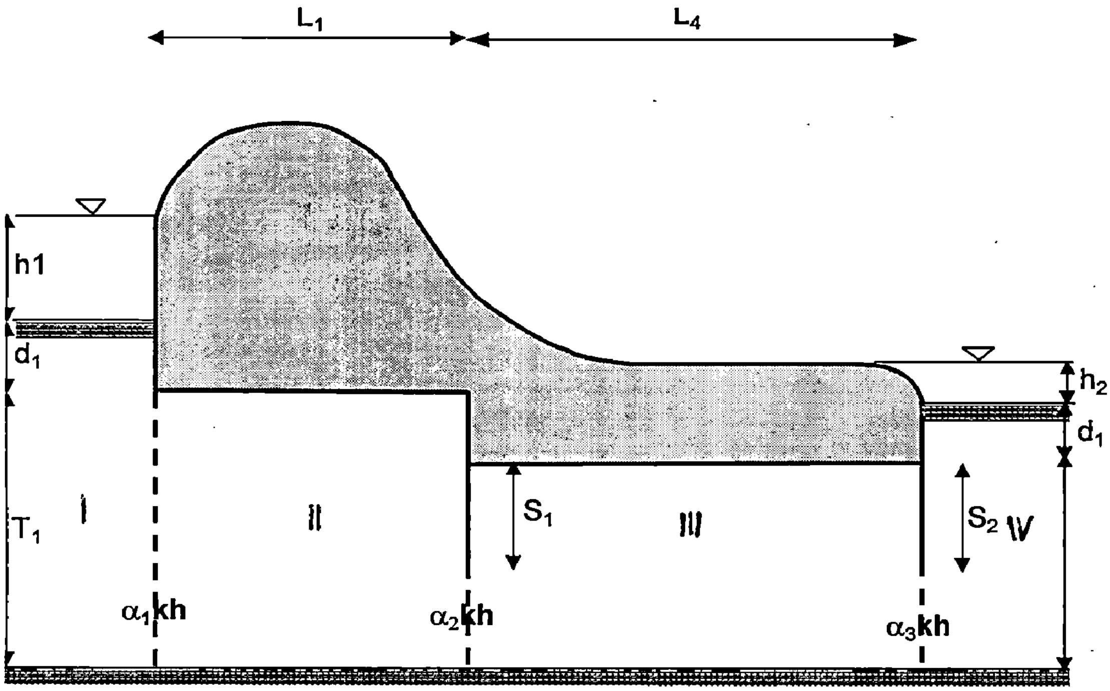



Figure 1

By same manner we get equation for depressed weir without drain as follow:

For fragment I

$$\alpha_1 A + \frac{q}{kh} = A \tag{4.2.28}$$

From fragment II

$$-\alpha_1 B + \alpha_2 B + \frac{q}{kh} = 0 \tag{4.2.29}$$

From fragment III

$$\alpha_2 C + \alpha_3 C + \frac{q}{kh} = 0 \tag{4.2.30}$$

From fragment IV

$$-\alpha_3 D + \frac{q}{kh} = 0 \tag{4.2.31}$$

The 4 equations can be written in following matrix form:

$$\begin{bmatrix} A & 0 & 0 & 1 \\ -B & B & 0 & 1 \\ 0 & -C & C & 1 \\ 0 & 0 & -D & 1 \end{bmatrix} \cdot \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \frac{q}{kh} \end{bmatrix} = \begin{bmatrix} A \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence,

$$\begin{bmatrix} A & 0 & 0 & 1 \\ -B & B & 0 & 1 \\ 0 & -C & C & 1 \\ 0 & 0 & -D & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \frac{q}{kh} \end{bmatrix} = \begin{bmatrix} A \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

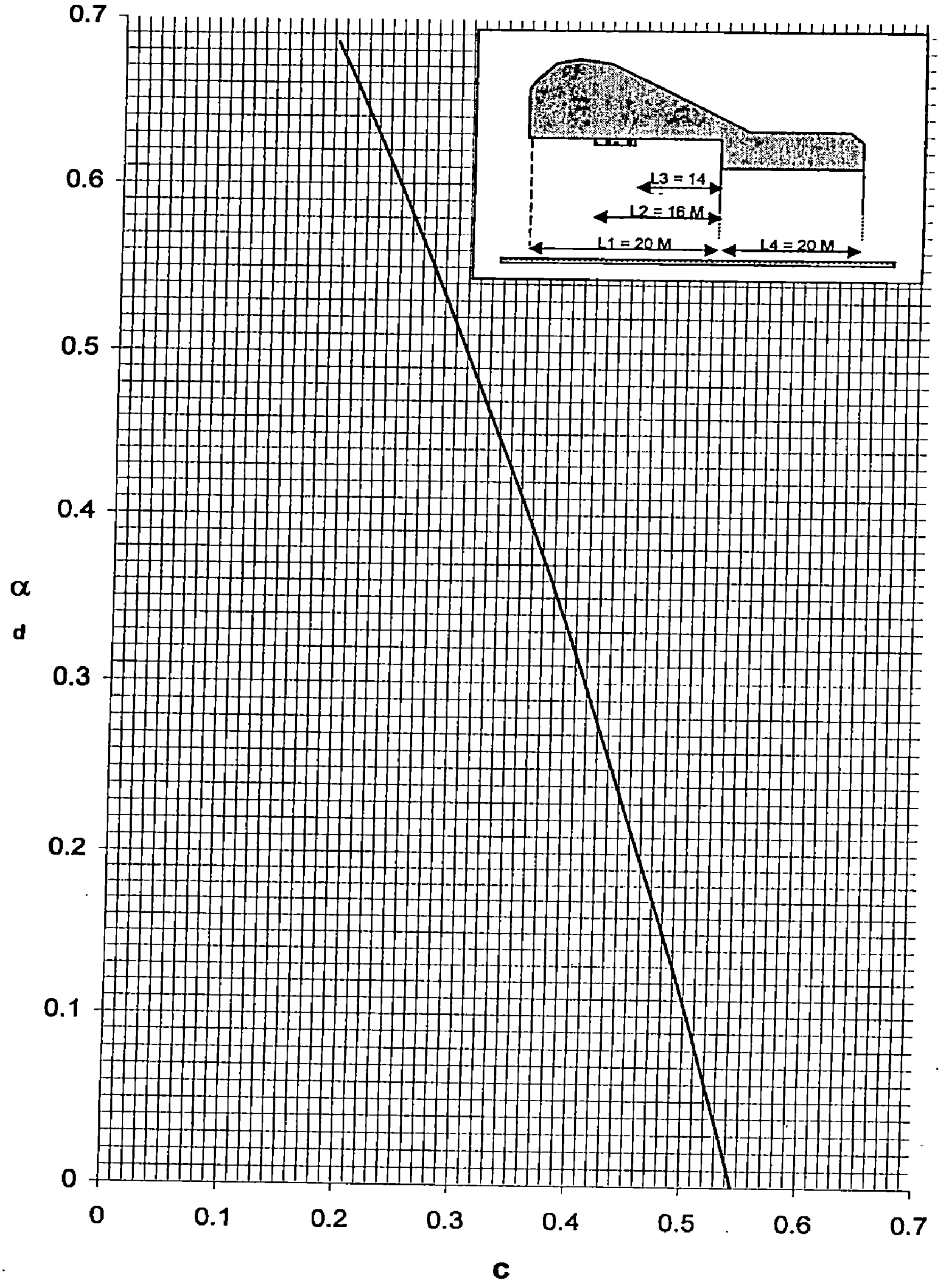



**Figure 4.8.3.1a**
**Relationship between $\alpha_d$ and Parameter C**
**for the Spesific Location of Drain Shown**

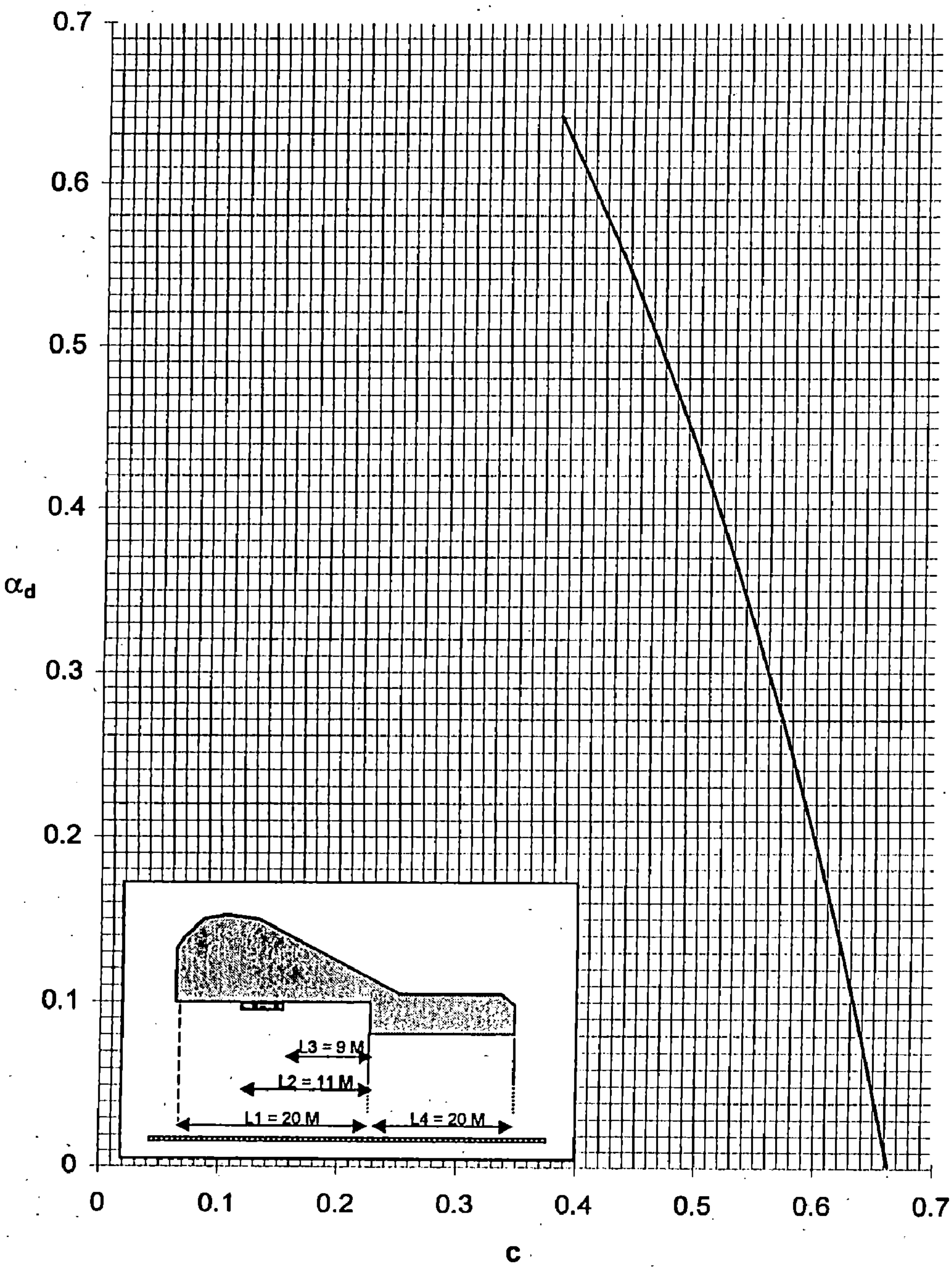



**figure 4.8.31b**
**Relationship between $\alpha_d$ and Parameter c**
**for the Spesific Location of Drain Shown**

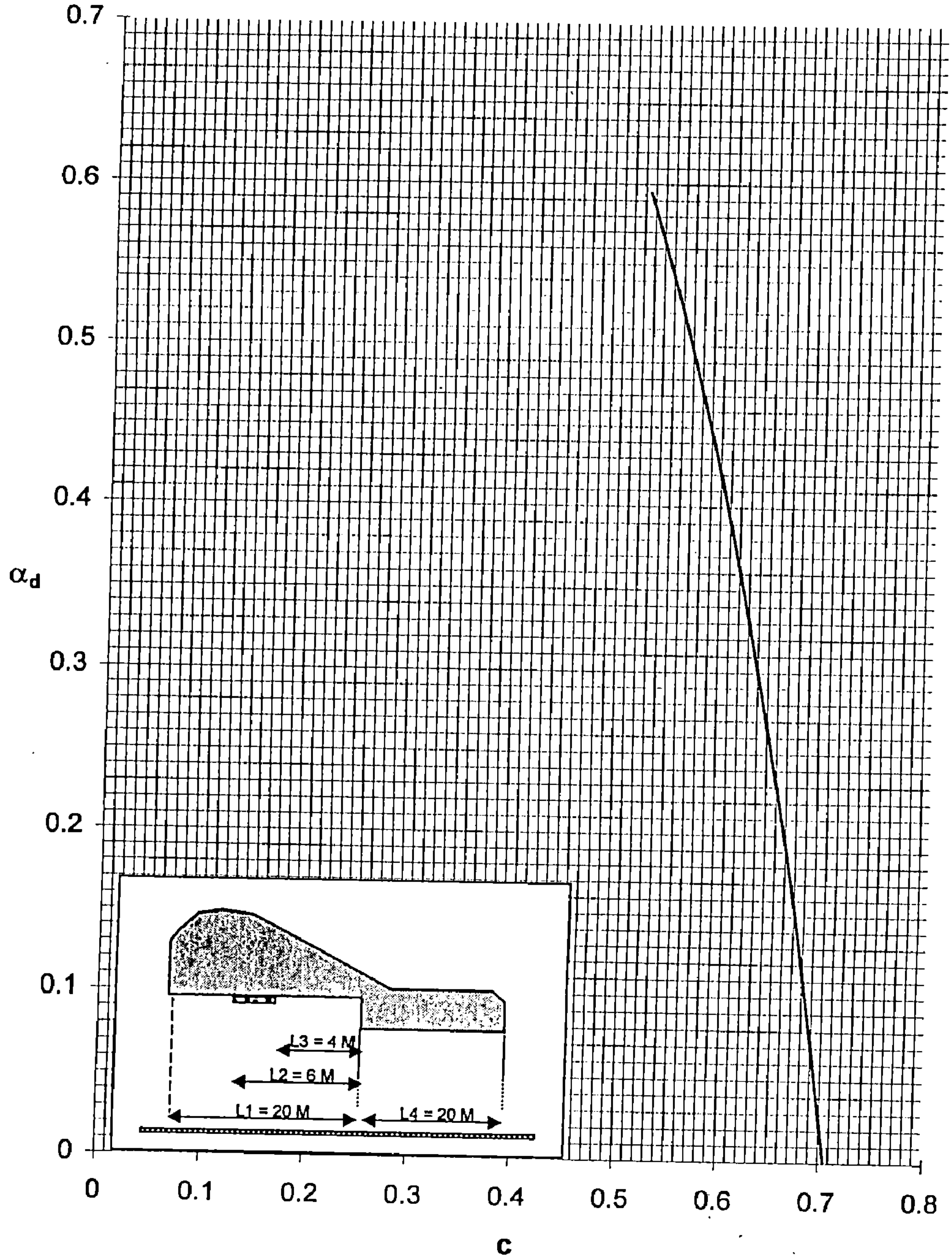



**Figure 4.8.3.1c**
**Relationship between $\alpha_d$ and Parameter c for the Spesific Location of Drain Shown**

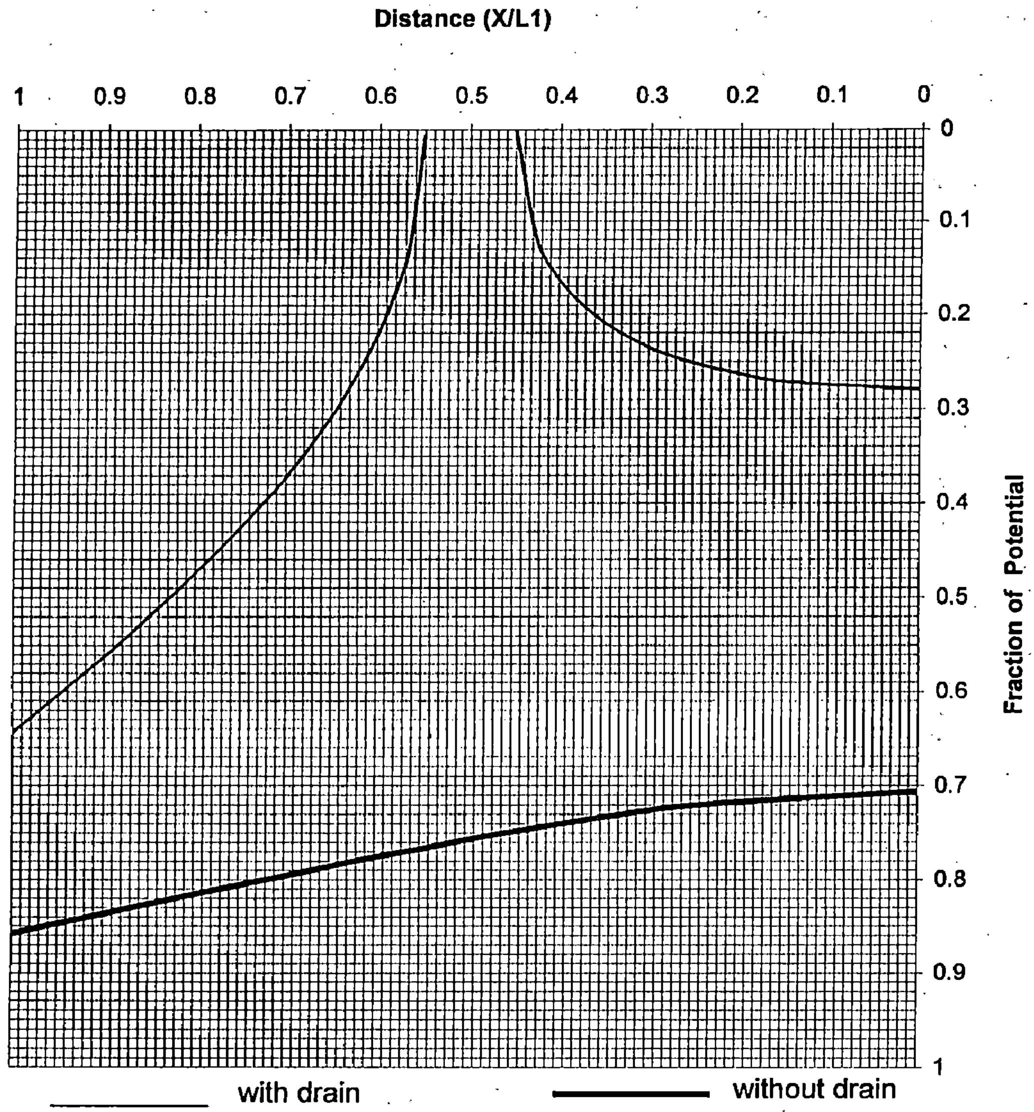



**Figure 4.8.3.3b**
**Variation of Potential along the Base of Fragment 2**
**for $L_1$ = 20 M, $L_2$ = 11 M and $L_3$ = 9 M**

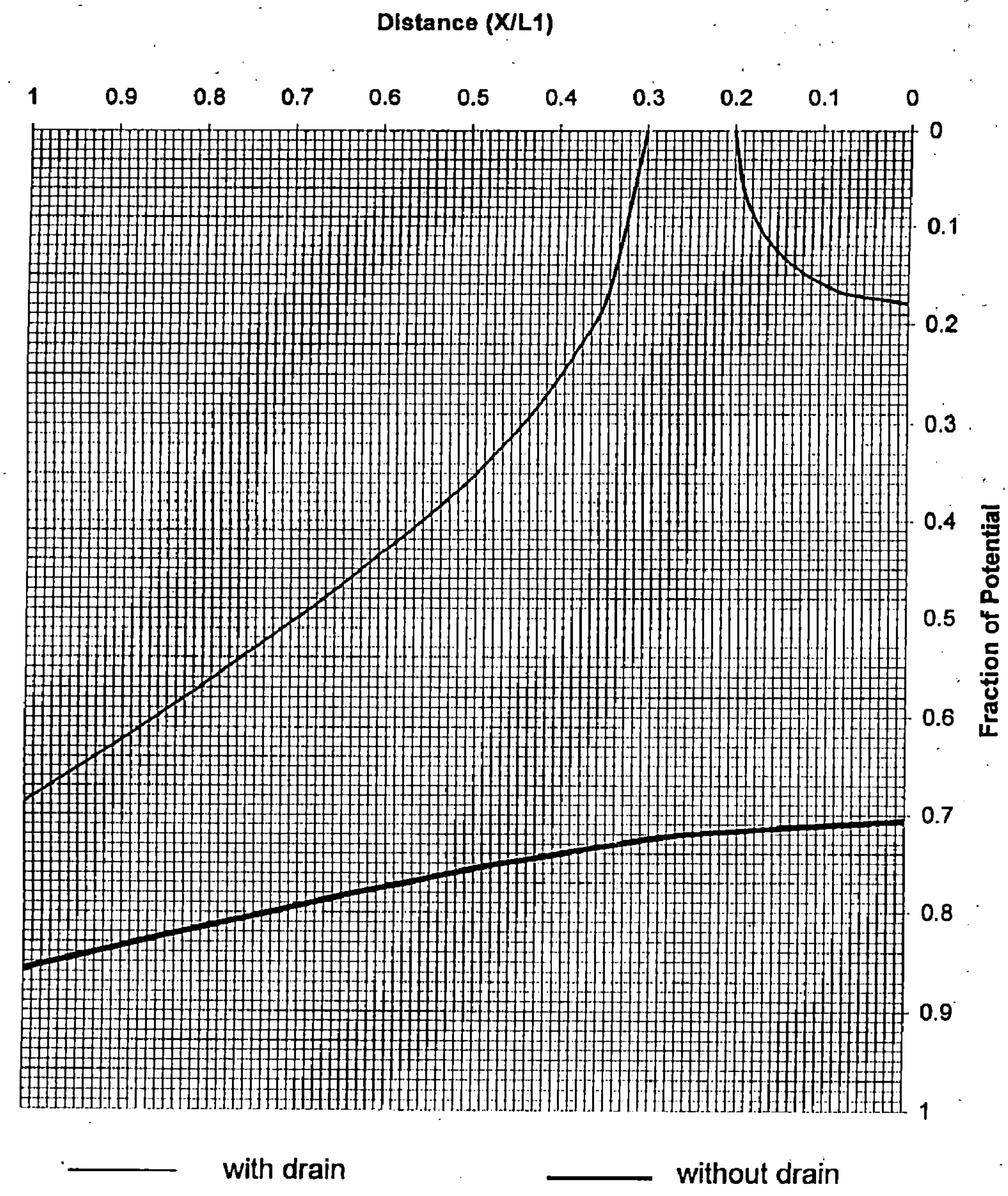



**Figure 4.8.3.3c**
**Variation of Potential along the Base of Fragment 2**
**for $L_1$ = 20 M, $L_2$ = 6 M, $L_3$ = 4 M**

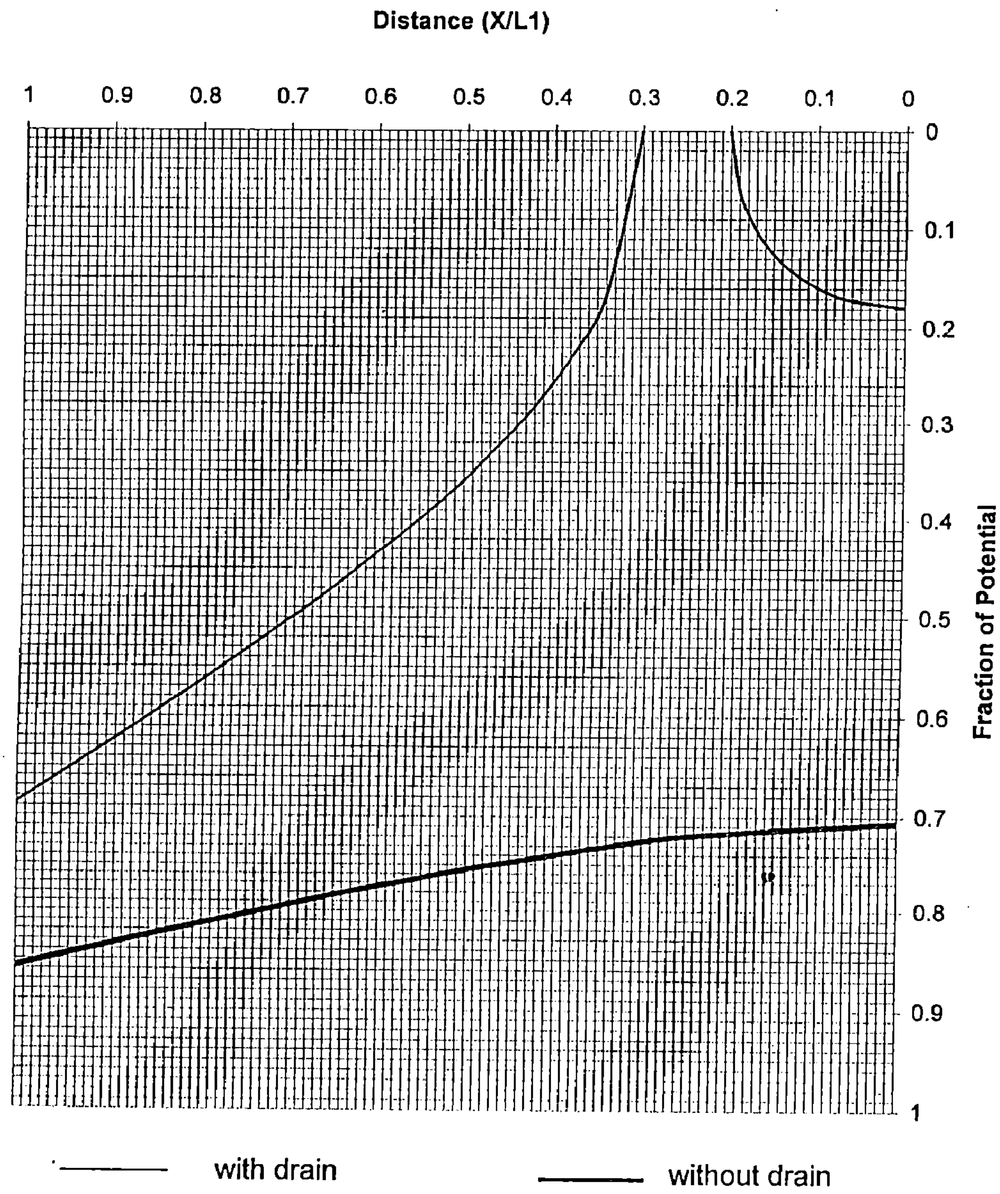



**Figure 4.8.3.3c**
**Variation of Potential along the Base of Fragment 2**
**for $L_1 = 20$ M, $L_2 = 6$ M, $L_3 = 4$ M**

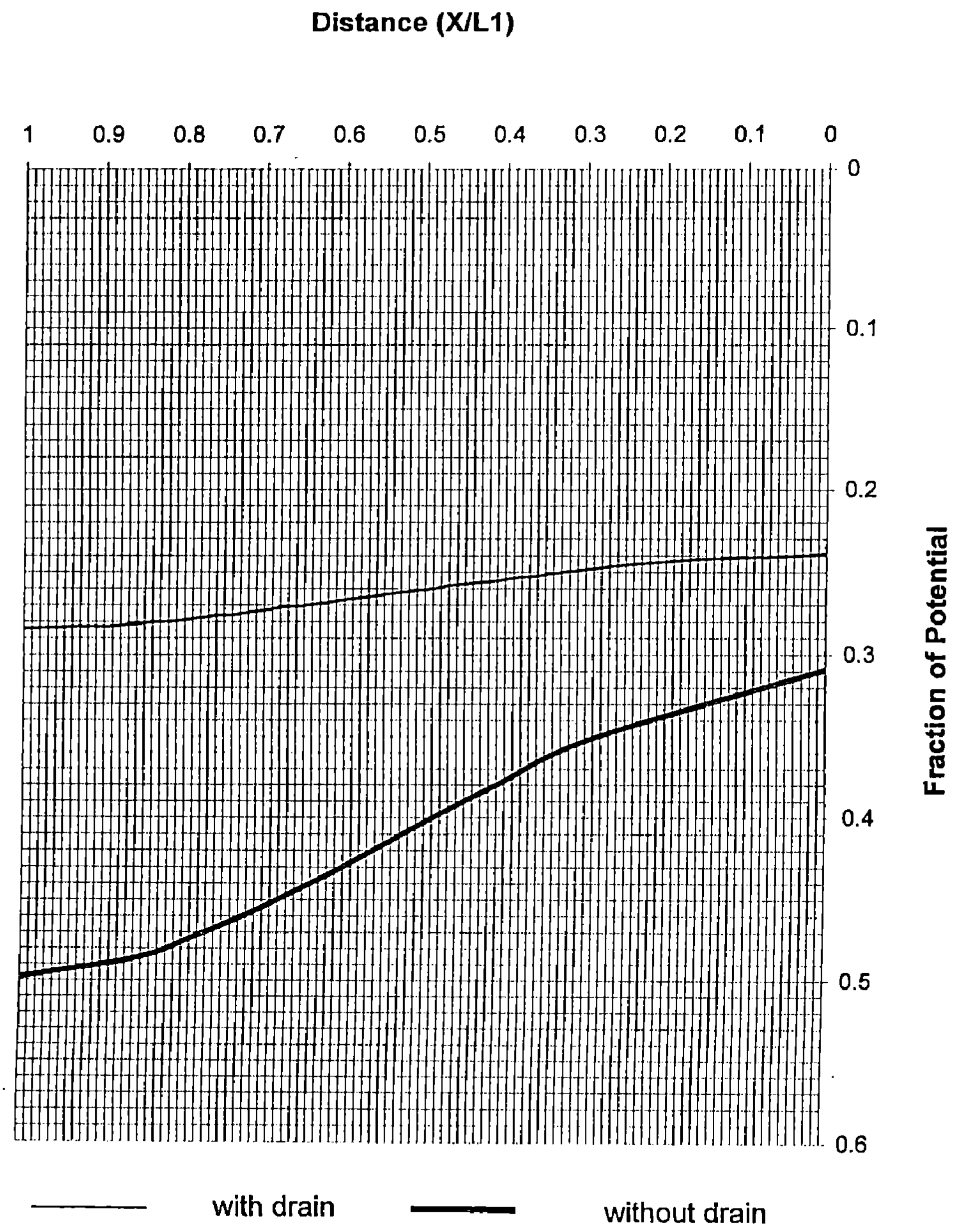



**Figure 4.8.3.4a**
**Variation of Potential Along the Base of Fragment 3 for $L_1$= 20 M, $L_2$= 16 M, $L_3$ = 14 M and $L_4$ = 20 M**

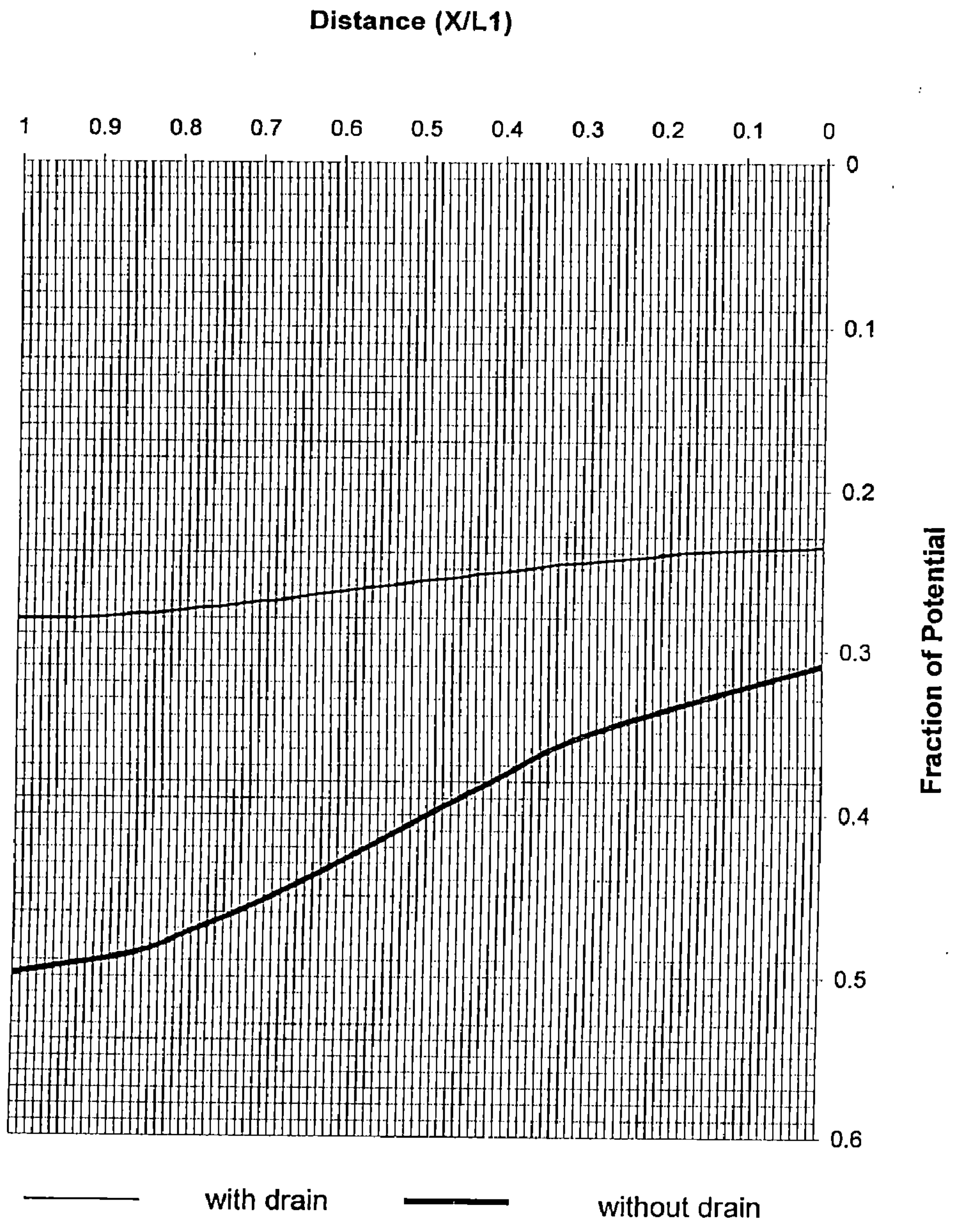



**Figure 4.8.3.4b**
**Variation of Potential Along the Base of Fragment 3**
**for $L_1$= 20 M, $L_2$= 11 M, $L_3$ = 9 M and $L_4$ = 20 M**

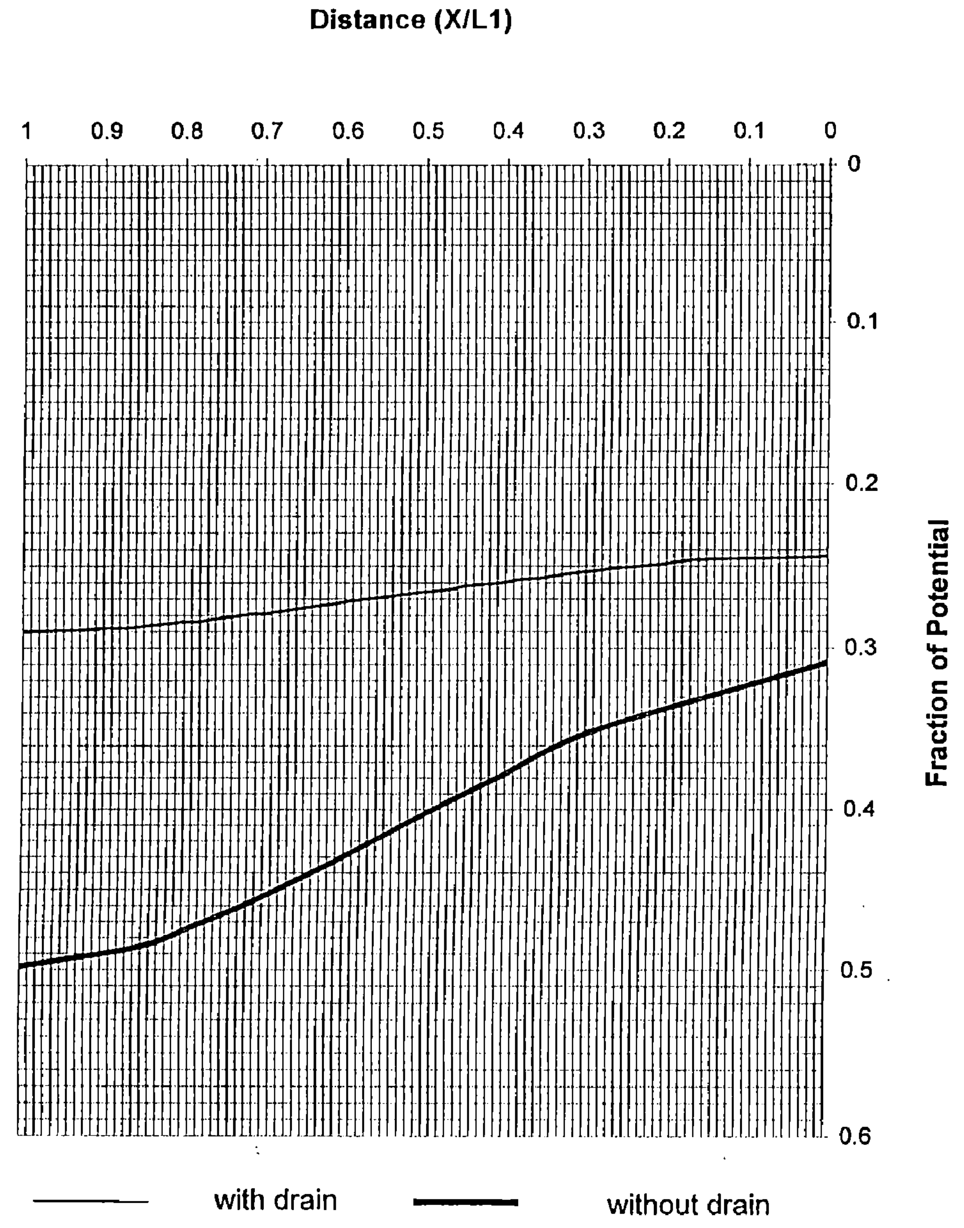



**Figure 4.8.3.4c**
**Variation of Potential Along the Base of Fragment 3**
**for $L_1$= 20 M, $L_2$= 6 M, $L_3$ = 4 M and $L_4$ = 20 M**

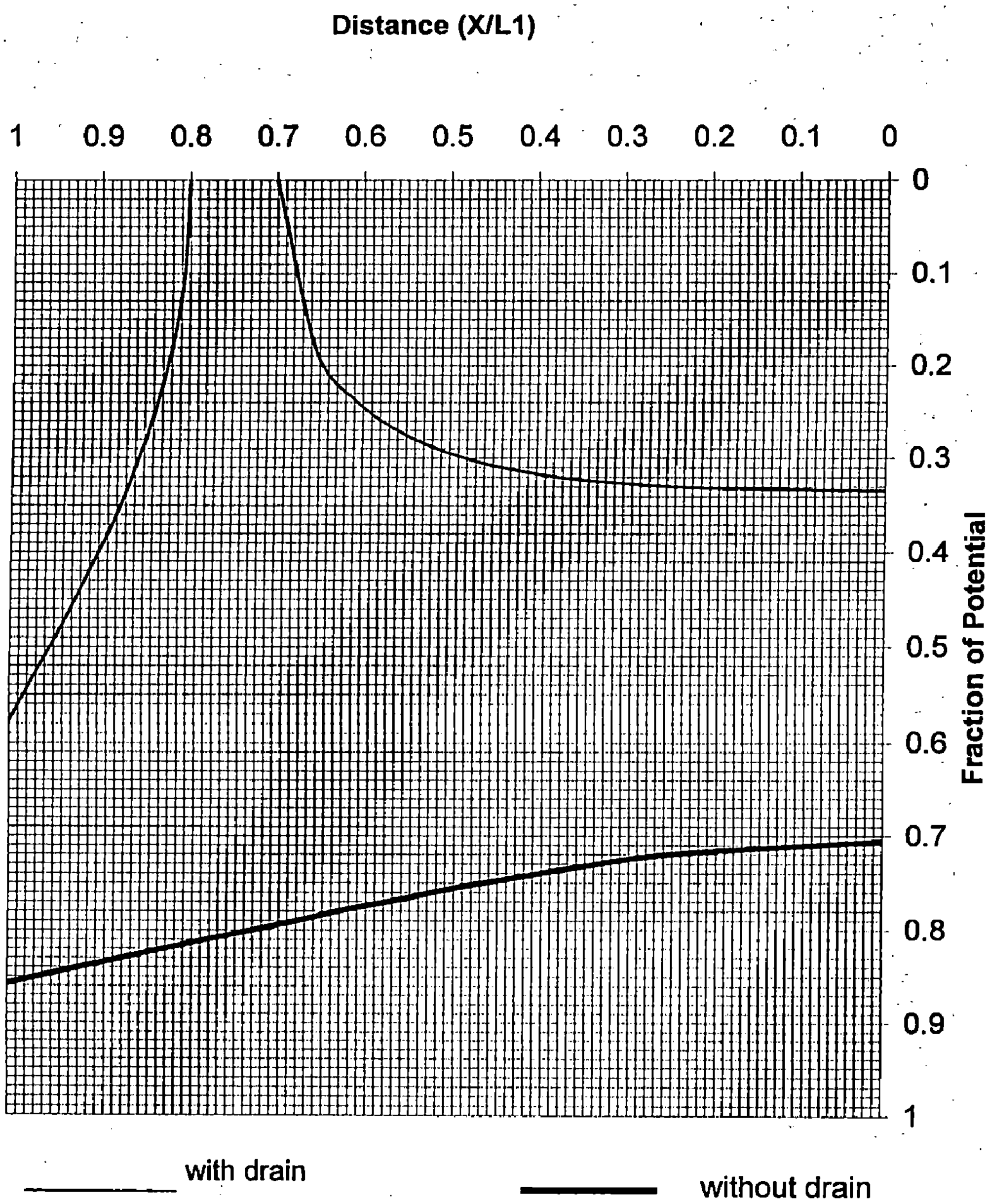



**Figure 4.8.3.3a**
**Variation of Potential Along the Base of fragment 2**
**for $L_1 = 20$ M, $L_2 = 16$ M and $L_3 = 14$ M**

# CONCLUSIONS

Using the method of fragments two dimensional flow under a stepped depressed weir having two sheet pile has been analysed. The involved integrations have been carried out using Gauss quadrature formula. The complete and incomplete elliptic integrals have also been computed using Gauss quadrature. From the analysis of flow under a flat bottom weir, it is found that unless the potential along the drain is much less than the potential under a floor without drain, the drain would not perform as a drain i.e. it will not act as a sink. Using this conclusion the parameter c of segment II which has a drain is assumed and the corresponding potential along the drain is computed. However, the drain will function at its best when the potential along it is zero. From the analysis and the computer program developed, the uplift pressure and the maximum exit gradient can be computed. The analysis also includes the situation where there is no drain.

The solution presented is very useful for field engineers to design weirs on permeable foundation of finite depth. The depression of structure has been considered in the analysis. Methods of fragment is found to be most versatile in analyzing flow under hydraulic structure having several sheet pile and floors.

# REFERENCES


1. Aravin, V.I. and Numerov, S.N. "Theory of Fluid Flow in Underformeable Porous Media", Translated from Russian by Israel Program for Scientific Translations, Jerusalem 1965.
2. Byrd, P.F. and Friedman, M.D., "Handbook of Elliptic Integrals for Engineers and Scientists", Springer - Verlag Berlin Heidelberg, New York, 1971.
3. Chawla, A.S., "Effect of Intermediate Drains and Leakage Through Sheet Pile Line on the Uplift Pressures", Ph.D., WRDTC, 1973.
4. Churchil, Ruel V. and Brown, James Ward : "Complex Variables and Applications", Mc Graw-Hill Book Company, Inc., New York, 1996 (Sixth Edition)
5. De Wiest, Roger J.M.; "Flow Through Porous Media", (Professor Princeton University, Princeton, New Jersey) Academic Press, New York and London 1969.
6. Harr, M.E. "Ground Water and Seepage", Mc.Graw-Hill Book Company, Inc. New York, 1962.
7. Khosla, R.B.A.N., Bose N.K. and Taylor, E.M.K., (1954), "Design of Weirs on Permeable Foundations", Central Board of Irrigation, New Delhi, India.
8. Pollubarinova - Kochina, P. Ya : "Theory of Groundwater Movement", Translated Russian Book by R.De Wiest, 1962 Princeton, New Jersey; Princeton University Press.

9. Seymour Lipschutz and Arthw Roe , "Theory and Problems of Programming with FORTRAN including Structured Fortran", SCHAUM's Outline Sevier, McGraw-Hill Book Company, Singapore, Copyright © 1982.

10. U.S. Department of Commerce, National Bureau of Standards Applied Mathematic Series 55 : "Handbook of Mathematical Functions with Formulae, Graphs, and Mathematical Tables", Edited by : Milton Abramowits and Irene A. Stegun; Issued June 1964, New Printing, Nov. 1970, with Correction.

11. Vaclav Halek, (Ass. Prof. Technical University, Bros.) , Jain Suec (Institute of Hydrodynamics of the Czechoslovak Academy of Sciences, Prague), "Groundwater Hydraulics", Elsevier Scientific Publishing Company, Amstrendam - Oxford - New York, 1979.

12. Varshney, R.S., Gupta, S.C.and Gupta, R.L. : "Theory and Design of Irrigation Structures", Nem Chand & Bros, Civil Lines, Roorkee – 247 667, India, 1993.

# FBWROPFIFD

```
      DIMENSION W(96),XX(96)
      OPEN (1, status='old',FILE='USMANC.DAT')
      OPEN (3, status='old',FILE='GAUSS.DAT')
      OPEN (2, status='unknown',FILE='USMANC.OUT')
      READ (3,*) (W(I),I=1,96)
      READ (3,*) (XX(I),I=1,96)
      READ(1,*)AL1,AL2,AL
      PAI=3.141592654

      WRITE(2,*)'HEAD DISTRIBUTION UNDER A FLAT BOTTOM WEIR WITHOUT D
     1DRAIN'

      T=0.0
      DELT=0.020
      WRITE(2,6)
 6    FORMAT(6X,'X/B',7X,'H(X)/H')

 100  CONTINUE
      FACTOR=1.-2.*ASIN(SQRT(T))/PAI
      WRITE(2,7)T,FACTOR
      T=T+DELT
 7    FORMAT(2F12.3)
      IF(T.LE.0.9999999) GO TO 100

      C=AL1/AL
      D=AL2/AL
      WRITE(2,*)'LOCATION OF DRAIN IN t PLANE'
      WRITE(2,*)'C=',C
      WRITE(2,*)'D=',D
      ALPHAC=1.-2./PAI*ASIN(SQRT(C))
      ALPHAD=1.-2./PAI*ASIN(SQRT(D))
      WRITE(2,4)
 4    FORMAT(6X,'ALPHAC',3X,'ALPHAD',4X, 'WITHOUD DRAIN')
      WRITE(2,5)ALPHAC,ALPHAD
 5    FORMAT(2F10.3)
C     COMPUTATION OF AI1
      SFX1=0.
      SFX2=0.
      DO I=1,96
      X=XX(I)
      CALL EQUA1(C,D,X,FX1,FX2)
      SFX1=SFX1+W(I)*FX1
      SFX2=SFX2+W(I)*FX2
      END DO

            AI11=SQRT(0.5*C)*SFX1
      AI12=SQRT(0.5*C)*SFX2
      AI1=AI11+AI12

C     COMPUTATION OF AI2
```

```
      SFX1=0.

      DO I=1,96
      X=XX(I)
        CALL EQUA2(C,D,X,FX1)
      SFX1=SFX1+W(I)*FX1
      END DO

        AI2=SQRT(C)*SFX1
      ALPHA=ALPHAD-0.05
      WRITE(2,8)
 8    FORMAT(6X,'ALPHA ASSUMED',7X,'CORRESPONDING R')
 200  CONTINUE

      R=(AI2+PAI*(1.-ALPHA))/AI1

      WRITE(2,9)ALPHA,R
 9    FORMAT(3F10.4)
      ALPHA=ALPHA-0.05
      IF(ALPHA.GT.0.0) GO TO 200

      WRITE(2,*)'RESULT FOR ALPHA=0. ARE GIVEN HERE AFTER'
      ALPHA=0.
      R=(AI2+PAI)/AI1
      WRITE(2,*)'VALUE OF R=',R
      PX=AL1/20.
      TP=PX/AL
      WRITE(2,6)
 300  CONTINUE
      SFX=0.
      DO I=1,96
      X=XX(I)
      CALL EQUA3(C,D,R,TP,X,FX)
      SFX=SFX+W(I)*FX
      END DO
            AI3=SQRT(TP)*SFX

      PHAI=AI3/PAI-1.
      PX1=PX/AL
      WRITE(2,203)PX1,PHAI
 203  FORMAT(2F10.3)
      PX=PX+AL1/20.
      TP=PX/AL
      IF(TP.LT.C) GO TO 300

      DELX=(AL-AL2)/20.
      PX=AL2+DELX
 204  CONTINUE
      TP=PX/AL
      SFX=0.
      DO I=1,96
      X=XX(I)
      CALL EQUA4(C,D,R,TP,X,FX)
      SFX=SFX+W(I)*FX
      END DO
            AI4=SQRT(TP-D)*SFX
      PHAI=-AI4/PAI
```

```
      WRITE(2,203)TP,PHAI
      PX=PX+DELX
      IF(PX.LT.AL) GO TO 204
      STOP
      END

      SUBROUTINE EQUA4(C,D,R,TP,X,FX)
      U=SQRT(TP-D)*0.5*(1.+X)
      V=U*U
      FX=(R-D-V)/SQRT ( (D+V)*(D+V-C)*(1.-D-V))
      RETURN
      END

      SUBROUTINE EQUA3(C,D,R,TP,X,FX)
      U=SQRT(TP)*0.5*(1.+X)
      V=U*U
      FX=(R-V)/SQRT ( (C-V)*(D-V)*(1.-V))
      RETURN
      END

      SUBROUTINE EQUA1(C,D,X,FX1,FX2)
      U=0.125*C*(1.+X)**2
      FX1=1./( ( (C-U)*(D-U  )*(1.-U  ) )**0.5)
      FX2=1./( ( (C-U)*(D-C+U)*(1.-C+U) )**0.5)
      RETURN
      END

      SUBROUTINE EQUA2(C,D,X,FX1)
      U=C-0.25*C*(1.+X)**2
        FX1=SQRT(U)/( ( (D-U)*(1.-U) )**0.5)
      RETURN
      END


      SUBROUTINE CEF(W,XX,AKS,CEF1)
      DIMENSION W(96), XX(96)
      PAI=3.141592654
      SUM=0.0
      DO 10 I=1,96
      THETA=PAI/4.*(1.+XX(I))
      TERM=0.25*PAI/SQRT(1.-AKS*SIN(THETA)**2)
      SUM=SUM+TERM*W(I)
10          CONTINUE
      CEF1=SUM
      RETURN
      END

      SUBROUTINE CIEF(W,XX,AKS,PHAI,CIEF1)
      DIMENSION W(96), XX(96)
      PAI=3.141592654
      SUM=0.0
      DO 10 I=1,96
      THETA=PHAI/2.*(1.+XX(I))
      TERM=0.5*PHAI/SQRT(1.-AKS*SIN(THETA)**2)
      SUM=SUM+TERM*W(I)
10          CONTINUE
      CIEF1=SUM
```

```
      RETURN
      END

      SUBROUTINE RAPH21(W,XX,AL1,T1,D,RESIDUE)
      DIMENSION W(96),XX(96)
      D=0.01
      DELD=0.00045
 100  CONTINUE
      AKS=D
      CALL CEF(W,XX,AKS,CEF1)
      TERM1=CEF1
      AKS=1.-D
      CALL CEF(W,XX,AKS,CEF1)
      TERM2=CEF1
      TERM3=TERM1/TERM2
      RESIDUE=AL1/T1-TERM3
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      D=D+DELD
      IF (RESIDUE.GT.0.0) GO TO 100
      DR=D-DELD
      DL=DR-DELD
 200  D=(DL+DR)/2.
      AKS=D
      CALL CEF(W,XX,AKS,CEF1)
      TERM1=CEF1
      AKS=1.-D
      CALL CEF(W,XX,AKS,CEF1)
      TERM2=CEF1
      TERM3=TERM1/TERM2
      RESIDUE=AL1/T1-TERM3
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      IF(RESIDUE.GT.0.) GO TO 300
      IF(RESIDUE.LT.0.) GO TO 400
 300  DL=D
      GO TO 200
 400  DR=D
      GO TO 200
 500  CONTINUE
      RETURN
      END


      SUBROUTINE RAPH22(W,XX,AL1,AL2,T1,D,E1,RESIDUE)
      DIMENSION W(96),XX(96)
      AKS=1.-D
      CALL CEF(W,XX,AKS,CEF1)
      E1=0.0001
      DELE1=0.01
      AKS=D
 100  CONTINUE
      PHAI=ASIN (SQRT(E1/D))
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      RESIDUE= (AL1-AL2)/T1-CIEF1/CEF1
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      E1=E1+DELE1
      IF (RESIDUE.GT.0.0) GO TO 100
      E1R=E1-DELE1
```

```
      E1L=E1R-DELE1
 200  E1=(E1L+E1R)/2.
      PHAI=ASIN(SQRT(E1/D))
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      RESIDUE=(AL1-AL2)/T1-CIEF1/CEF1
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      IF(RESIDUE.GT.0.) GO TO 300
      IF(RESIDUE.LT.0.) GO TO 400
 300  E1L=E1
      GO TO 200
 400  E1R=E1
      GO TO 200
 500  CONTINUE
      RETURN
      END
C
      SUBROUTINE RAPH23(W,XX,AL1,AL3,T1,D,E1,E2,RESIDUE)
      DIMENSION W(96),XX(96)
      AKS=1.-D
      CALL CEF(W,XX,AKS,CEF1)
      TERM=(AL1-AL3)/T1
      E2=E1+0.0001
      DELE2=0.01
      AKS=D
 100  CONTINUE
      PHAI=ASIN(SQRT(E2/D))
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      RESIDUE=TERM-CIEF1/CEF1
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      E2=E2+DELE2
      IF (RESIDUE.GT.0.0) GO TO 100
      E2R=E2-DELE2
      E2L=E2R-DELE2
 200  E2=(E2L+E2R)/2.
      PHAI=ASIN(SQRT(E2/D))
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      RESIDUE=TERM-CIEF1/CEF1
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      IF(RESIDUE.GT.0.) GO TO 300
      IF(RESIDUE.LT.0.) GO TO 400
 300  E2L=E2
      GO TO 200
 400  E2R=E2
      GO TO 200
 500  CONTINUE
      RETURN
      END

      SUBROUTINE RAPH24(W,XX,D2,S1,T1,D,B,RESIDUE)
      DIMENSION W(96),XX(96)
      AKS=1.-D
      CALL CEF(W,XX,AKS,CEF1)
      TERM=(D2+S1)/T1
      B=D+0.0001
      DELB=0.01
      AKS=1.-D
```

```
 100   CONTINUE
       PHAI=ASIN(SQRT((B-D)/(B*(1.-D))))
       CALL CIEF(W,XX,AKS,PHAI,CIEF1)
       RESIDUE=TERM-CIEF1/CEF1
       IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
       B=B+DELB
       IF (RESIDUE.GT.0.0) GO TO 100
       BR=B-DELB
       BL=BR-DELB
 200   B=(BL+BR)/2.
       PHAI=ASIN(SQRT((B-D)/(B*(1.-D))))
       CALL CIEF(W,XX,AKS,PHAI,CIEF1)
       RESIDUE=TERM-CIEF1/CEF1
       IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
       IF(RESIDUE.GT.0.) GO TO 300
       IF(RESIDUE.LT.0.) GO TO 400
 300   BL=B
       GO TO 200
 400   BR=B
       GO TO 200
 500   CONTINUE
       RETURN
       END

       SUBROUTINE RAPH31(W,XX,AL4,T2,C,RESIDUE)
       DIMENSION W(96),XX(96)
       C=0.00001
       DELC=0.0001
 100   CONTINUE
       AKS=C
       CALL CEF(W,XX,AKS,CEF1)
       TERM1=CEF1
       AKS=1.-C
       CALL CEF(W,XX,AKS,CEF1)
       TERM2=CEF1
       TERM3=TERM1/TERM2
       RESIDUE=AL4/T2-TERM3
       IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
       C=C+DELC
       IF (RESIDUE.GT.0.0) GO TO 100
       CR=C-DELC
       CL=CR-DELC
 200   C=(CL+CR)/2.
       AKS=C
       CALL CEF(W,XX,AKS,CEF1)
       TERM1=CEF1
       AKS=1.-C
       CALL CEF(W,XX,AKS,CEF1)
       TERM2=CEF1
       TERM3=TERM1/TERM2
       RESIDUE=AL4/T2-TERM3
       IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
       IF(RESIDUE.GT.0.) GO TO 300
       IF(RESIDUE.LT.0.) GO TO 400
 300   CL=C
       GO TO 200
 400   CR=C
```

```
      GO TO 200
 500  CONTINUE
      RETURN
      END

      SUBROUTINE RAPH32(W,XX,T2,S1,C,E,RESIDUE)
      DIMENSION W(96),XX(96)
      AKS=1.-C
      CALL CEF(W,XX,AKS,CEF1)
      E=0.0001
      DELE=0.01
      TERM=(T2-S1)/T2
 100  CONTINUE
      PHAI=ASIN(SQRT(1./(1.+E)))
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      RESIDUE=CIEF1/CEF1-TERM
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      E=E+DELE
      IF (RESIDUE.GT.0.0) GO TO 100
      ER=E-DELE
      EL=ER-DELE
 200  E=(EL+ER)/2.
      PHAI=ASIN(SQRT(1./(1.+E)))
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      RESIDUE=CIEF1/CEF1-TERM
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      IF(RESIDUE.GT.0.) GO TO 300
      IF(RESIDUE.LT.0.) GO TO 400
 300  EL=E
      GO TO 200
 400  ER=E
      GO TO 200
 500  CONTINUE
      RETURN
      END

      SUBROUTINE RAPH33(W,XX,T2,S2,C,B,RESIDUE)
      DIMENSION W(96),XX(96)
      AKS=1.-C
      CALL CEF(W,XX,AKS,CEF1)
      B=C+0.0001
      DELB=0.0001
 100  CONTINUE
      PHAI=ASIN (SQRT((B-C)/(B*(1.-C))))
C     WRITE(2,*)'PHAI=',PHAI,'OVER'
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      RESIDUE= S2/T2-CIEF1/CEF1
C     WRITE(2,*)'RESIDUE=',RESIDUE
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      B=B+DELB
      CHECK=(SQRT((B-C)/(B*(1.-C))))
C     WRITE(2,*)'CHECK=',CHECK
      IF (RESIDUE.GT.0.0) GO TO 100
      BR=B-DELB
      BL=BR-DELB
 200  B=(BL+BR)/2.
      PHAI=ASIN(SQRT((B-C)/(B*(1.-C))))
```

```
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      RESIDUE=S2/T2-CIEF1/CEF1
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      IF(RESIDUE.GT.0.) GO TO 300
      IF(RESIDUE.LT.0.) GO TO 400
300   BL=B
      GO TO 200
400   BR=B
      GO TO 200
500   CONTINUE
      RETURN
      END
```

## DSWWD

```
      DIMENSION W(96),XX(96),AAA(6,6),BBB(6),CCC(6),AAT(6,6),AAAT(6,6)
      OPEN (1, status='old',FILE='USMAN5.DAT')
      OPEN (3, status='old',FILE='GAUSS.DAT')
      OPEN (2, status='unknown',FILE='USMAN5.OUT')
      READ (3,*) (W(I),I=1,96)
      READ (3,*) (XX(I),I=1,96)
      READ(1,*)D1,T1
      READ(1,*)AL1,AL2,AL3,D2,S1
      READ(1,*)AL4,S2,T2
      READ(1,*)D3,T3
      PAI=3.141592654
C
C     COMPUTATION FOR SEGMENT1
      TERM=SIN(D1/(D1+T1)*PAI/2.)
      B1=TERM**2
      AKS=B1
      CALL CEF(W,XX,AKS,CEF1)
      TERM1=CEF1
      AKS=1.-B1
      CALL CEF(W,XX,AKS,CEF1)
      TERM2=CEF1
      A=TERM2/TERM1
      CHECKA=A
C
C     GENERATING ELEMENT OF MATRIX
      DO I=1,6
      DO J=1,6
      AAA(I,J)=0.
      END DO
      END DO
C     UNKNOWN(1)=ALPHA1=BBB(1)
C     UNKNOWN(2)=ALPHA2=BBB(2)
C     UNKNOWN(3)=ALPHA3=BBB(3)
C     UNKNOWN(4)=ALPHAD=BBB(4)
C     UNKNOWN(5)=Q/KH   =BBB(5)
C     UNKNOWN(6)=Q1/KH =BBB(6)

      AAA(1,1)=A
      AAA(1,5)=1.
      CCC(1)=A
C
C     COMPUTATION FOR SEGMENT 3
      CALL RAPH31 (W,XX,AL4,T2,C,RESIDUE)
      WRITE(2,*)'C=',C
      WRITE(2,*)'RESIDUE=',RESIDUE
      CINS3=C
      DINS3=0.
C     COMPUTATION OF E
      CALL RAPH32(W,XX,T2,S1,C,E,RESIDUE)
      WRITE(2,*)'E=',E
      WRITE(2,*)'RESIDUE=',RESIDUE
      EINS3=E
C     COMPUTATION OF B
```

```
      CALL RAPH33(W,XX,T2,S2,C,B,RESIDUE)
      WRITE(2,*)'B=',B
      WRITE(2,*)'RESIDUE=',RESIDUE
      BINS3=B
      AKS=(1.-B)/(1.+E)
      CALL CEF(W,XX,AKS,CEF1)
      TERM1=CEF1
      AKS=(B+E)/(1.+E)
      CALL CEF(W,XX,AKS,CEF1)
      TERM2=CEF1
      TERM=TERM1/TERM2
      CHECKB=TERM
      AAA(2,2)=-TERM
      AAA(2,3)=TERM
      AAA(2,5)=1.
      AAA(2,6)=-1.
      CCC(2)=0.
C
C     COMPUTATION FOR SEGMENT IV
C
      B=(SIN(PAI/2.*(1.-(D3+S2)/T3)) )**2
      BINS4=B
      AKS=B
      CALL CEF(W,XX,AKS,CEF1)
      TERM1=CEF1
      AKS=1.-B
      CALL CEF(W,XX,AKS,CEF1)
      TERM2=CEF1
      TERM=TERM1/TERM2
      CHECKC=TERM
      AAA(3,3)=-TERM
      AAA(3,5)=1.
      AAA(3,6)=-1.
      CCC(3)=0.
C
C     COMPUTATION FOR SEGMENT II

C     COMPUTATION OF D
      CALL RAPH21(W,XX,AL1,T1,D,RESIDUE)
      WRITE(2,*)'D=',D
      WRITE(2,*)'RESIDUE=',RESIDUE
      DINS2=D
C     COMPUTATION OF E1 AT LEFT
      CALL RAPH22(W,XX,AL1,AL2,T1,D,E1,RESIDUE)
      WRITE(2,*)'E1=',E1
      WRITE(2,*)'RESIDUE=',RESIDUE
C     COMPUTATION OF E2 AT RIGHT
C
      CALL RAPH23(W,XX,AL1,AL3,T1,D,E1,E2,RESIDUE)
      WRITE(2,*)'E2=',E2
      WRITE(2,*)'RESIDUE=',RESIDUE
C     COMPUTATION OF B
      CALL RAPH24(W,XX,D2,S1,T1,D,B,RESIDUE)
      WRITE(2,*)'B=',B
      WRITE(2,*)'RESIDUE=',RESIDUE
      BINS2=B
C     COMPUTATION OF EQUATION IV
```

```
      SFX1=0.
      SFX2=0.
      SFX3=0.
      SFX4=0.

      DO I=1,96
      X=XX(I)
      CALL EQUAIV(E2,E1,B,X,FX1,FX2,FX3,FX4)
      SFX1=SFX1+W(I)*FX1
      SFX2=SFX2+W(I)*FX2
      SFX3=SFX3+W(I)*FX3
      SFX4=SFX4+W(I)*FX4
      END DO

      AI11=SFX1
      AI12=SFX2
      AI21=SFX3
      AI22=SFX4

      AI1=AI11+AI12
      AI2=AI21+AI22

C     COMPUTATION OF EQUATION V
      SFX1=0.
      SFX2=0.
      SFX3=0.
      SFX4=0.

      DO I=1,96
      X=XX(I)
      CALL EQUAV(E2,E1,B,X,FX1,FX2,FX3,FX4)
      SFX1=SFX1+W(I)*FX1
      SFX2=SFX2+W(I)*FX2
      SFX3=SFX3+W(I)*FX3
      SFX4=SFX4+W(I)*FX4
      END DO

      AI31=SQRT(0.5*E1)*SFX1
      AI32=SQRT(0.5*E1)*SFX2
      AI41=SQRT(0.5*E1)*SFX3
      AI42=SQRT(0.5*E1)*SFX4

      AI3=AI31+AI32
      AI4=AI41+AI42

C     COMPUTATION OF EQUATION VI
      SFX1=0.
      SFX2=0.
      SFX3=0.
      SFX4=0.

      DO I=1,96
      X=XX(I)
      CALL EQUAVI(E2,E1,B,X,FX1,FX2,FX3,FX4)
      SFX1=SFX1+W(I)*FX1
      SFX2=SFX2+W(I)*FX2
      SFX3=SFX3+W(I)*FX3
```

```
      SFX4=SFX4+W(I)*FX4
      END DO

             AI51=SQRT(0.5*(E2-E1))*SFX1
      AI52=SQRT(0.5*(E2-E1))*SFX2
      AI61=SQRT(0.5*(E2-E1))*SFX3
      AI62=SQRT(0.5*(E2-E1))*SFX4

      AI5=AI51+AI52
      AI6=AI61+AI62

C     COMPUTATION OF EQUATION VII
      SFX1=0.
      SFX2=0.
      SFX3=0.
      SFX4=0.

      DO I=1,96
      X=XX(I)
        CALL EQUAVII(E2,E1,B,X,FX1,FX2,FX3,FX4)
      SFX1=SFX1+W(I)*FX1
      SFX2=SFX2+W(I)*FX2
      SFX3=SFX3+W(I)*FX3
      SFX4=SFX4+W(I)*FX4
      END DO

        AI71=SQRT(0.5*(B-E2))*SFX1
        AI72=SQRT(0.5*(B-E2))*SFX2
      AI81=SQRT(0.5*(B-E2))*SFX3
      AI82=SQRT(0.5*(B-E2))*SFX4

      AI7=AI71+AI72
      AI8=AI81+AI82

C     COMPUTATION OF EQUATION VIII
      SFX1=0.
      SFX2=0.
      SFX3=0.
      SFX4=0.

      DO I=1,96
      X=XX(I)
        CALL EQUAVIII(E2,E1,B,X,FX1,FX2,FX3,FX4)
      SFX1=SFX1+W(I)*FX1
      SFX2=SFX2+W(I)*FX2
      SFX3=SFX3+W(I)*FX3
      SFX4=SFX4+W(I)*FX4
      END DO

        AI91=SQRT(0.5*(1.-B))*SFX1
        AI92=SQRT(0.5*(1.-B))*SFX2
      AI101=SQRT(0.5*(1.-B))*SFX3
      AI102=SQRT(0.5*(1.-B))*SFX4

      AI9=AI91+AI92
      AI10=AI101+AI102
```

```
C      COMPUTATION OF EQUATION IX
       SFX1=0.
       SFX2=0.
       SFX3=0.
       SFX4=0.

       DO I=1,96
       X=XX(I)
         CALL EQUAIX(E2,E1,B,X,FX1,FX2,FX3,FX4)
       SFX1=SFX1+W(I)*FX1
       SFX2=SFX2+W(I)*FX2
       SFX3=SFX3+W(I)*FX3
       SFX4=SFX4+W(I)*FX4
       END DO

         AI111=SFX1
         AI112=SFX2
       AI121=SFX3
       AI122=SFX4

       AI11=AI111+AI112
       AI12=AI121+AI122

       TERM1=(AI3-AI7)+AI11
       TERM2=AI8-AI4-AI12
       WRITE(2,*)'CHECKING THAT, TERM1=TERM2=0',TERM1,TERM2
       TERM1=AI1-AI5+AI9
       TERM2=AI2+AI6-AI10
       WRITE(2,*)'CHECKING THAT, TERM1=TERM2=0.',TERM1,TERM2

       C=D+0.05
       CINS2=C
       WRITE(2,*)'C=',C,'B=',B
       IF(C.GT.B) GO TO 800

       U=(C*AI3-AI4)/(-C*AI11+AI12)
       V=(-C*AI9+AI10)/(-C*AI11+AI12)
       AAA(4,1)=1.-U
       AAA(4,2)=U
       AAA(4,4)=-1.
       CCC(4)=0.
       AAA(5,1)=-V
       AAA(5,2)=V
       AAA(5,5)=1.
       AAA(5,6)=-1.
       CCC(5)=0.
       W1=(C*AI1+AI2)/(-C*AI7+AI8)
       AAA(6,2)=W1
       AAA(6,4)=-W1
       AAA(6,5)=1.
       CCC(6)=0.
       WRITE(2,*)'MATRIX ELEMENT'
       DO I=1,6
       WRITE(2,222)(AAA(I,J), J=1,6)
222    FORMAT(6E12.4)
       END DO
```

```
      DO I=1,6
      DO J=1,6
      AAT(I,J)=AAA(I,J)
      END DO
      END DO
      MMM=6
        CALL MATIN(AAA,MMM)

      WRITE(2,*)'INVERSE MATRIX ELEMENT'

      DO I=1,6
      WRITE(2,222)(AAA(I,J),J=1,6)
      END DO

      WRITE(2,*)'CHECKING INVERSION'

      DO I=1,6
      DO K=1,6
      SUM=0.
      DO J=1,6
      SUM=SUM+AAA(I,J)*AAT(J,K)
      END DO
      AAAT(I,K)=SUM
      END DO
      END DO

      DO I=1,6
      WRITE(2,199)(AAAT(I,J),J=1,6)
      END DO
199   FORMAT(6E10.3)

      DO I=1,6
      SUM=0.
      DO J=1,6
      SUM=SUM+AAA(I,J)*CCC(J)
      END DO
      BBB(I)=SUM
      END DO

      WRITE(2,198)
198   FORMAT(4X,'ALPHA1',4X,'ALPHA2',4X,'ALPHA3',4X,'ALPHAD')
      WRITE(2,197)BBB(1),BBB(2),BBB(3),BBB(4)
197   FORMAT(4F10.5)
      WRITE(2,196)
196   FORMAT(4X,'q/kh',5x,'q1/kh')
      WRITE(2,195)BBB(5),BBB(6)
195   FORMAT(2F10.5)
      WRITE(2,*)'FINAL CHECKING'
      AMBKH=BBB(5)/(C*AI1+AI2)
      RESD1=BBB(6)-AMBKH*(C*AI5-AI6)
      RESD2=BBB(5)-BBB(6)-AMBKH*(-C*AI9+AI10)
      RESD3=BBB(1)-BBB(4)-AMBKH*(C*AI3-AI4)
      RESD4=BBB(4)-BBB(2)-AMBKH*(-C*AI7+AI8)
      RESD5=BBB(1)-BBB(2)-AMBKH*(-C*AI11+AI12)
      RESD6=BBB(5)-BBB(6)-CHECKB*(BBB(2)-BBB(3))
      RESD7=BBB(5)+BBB(1)*CHECKA-CHECKA
      RESD8=BBB(5)-BBB(6)-BBB(3)*CHECKC
```

```
      WRITE(2,*)RESD1,RESD2,RESD3,RESD4
      WRITE(2,*)RESD5,RESD6,RESD7,RESD8
C     COMPUTATION OF PHI AT  POINT D
      TP=DINS2
      WRITE(2,*)'E2=',E2
      WRITE(2,*)'CINS2=',CINS2,C
      write(2,*)'DINS2=',DINS2,D
      WRITE(2,*)'BINS2=',BINS2,B
      SFXTP=0.
      DO I=1,96
      X=XX(I)
      CALL EQUATP(CINS2,E2,E1,BINS2,TP,X,FX)
      SFXTP=SFXTP+W(I)*FX
      END DO
        AITP=SFXTP
      CONM=BBB(5)/(CINS2*AI1+AI2)
      RES=BBB(6)-CONM*(CINS2*AI5-AI6)
      WRITE(2,*)'RES=',RES
      PHITP=-CONM*AITP-BBB(4)
      PHITPD=PHITP
      WRITE(2,*)'PHI AT D IN SEGMENT 2=',PHITP


C     COMPUTATION OF PHI AT  POINT C
      TP=CINS2
      SFXTP=0.
      DO I=1,96
      X=XX(I)
      CALL EQUATP(CINS2,E2,E1,BINS2,TP,X,FX)
      SFXTP=SFXTP+W(I)*FX
      END DO
        AITP=SFXTP
      CONM=BBB(5)/(CINS2*AI1+AI2)
      RES=BBB(6)-CONM*(CINS2*AI5-AI6)
      WRITE(2,*)'RES=',RES
      PHITP=-CONM*AITP-BBB(4)
      PHITPC=PHITP
      WRITE(2,*)'PHI AT C IN SEGMENT 2=',PHITP

C     COMPUTATION OF PHI IN SEGMENT 3

      AKS=(BINS3+EINS3)/(1.+EINS3)
      PHAI=ASIN( SQRT((DINS3+EINS3)/(BINS3+EINS3)))
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      CIEF1D=CIEF1
      PHAI=ASIN( SQRT((CINS3+EINS3)/(BINS3+EINS3)))
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      CIEF1C=CIEF1
      AKS=(1.-BINS3)/(1.+EINS3)
      CALL CEF(W,XX,AKS,CEF1)
      CONM=0.5*(BBB(5)-BBB(6))/CEF1
      PHAID=CONM*CIEF1D-BBB(2)
      PHAIC=CONM*CIEF1C-BBB(2)
      WRITE(2,*)'PHI AT D IN SEGMENT 3=',PHAID
      WRITE(2,*)'PHI AT C IN SEGMENT 3=',PHAIC

C     COMPUTATION OF EXIT GRADIEN
```

```
      AKS=1.-BINS4
      CALL CEF(W,XX,AKS,CEF1)
      EXITG=BBB(3)*PAI*0.5/(T3*CEF1*SQRT(1.-BINS4))
      WRITE(2,*)'MAXIMUM EXIT GRADIENT/h=',EXITG

C     COMPUTATION OF PRESSURE FROM PHI
C     FIRST  CHOOSE AN ORIGIN AND
C       FIND CONSTANT C SO THAT PHI=O AT DOWNSTREAM BOUNDARY
      H1=12.
      H2=2.
      H=15.
      H=H1+D1+D2-D3-H2
      WRITE(2,*)'H=',H
      CONSTC=H2+(-D1-D2+D3)
      WRITE(2,*)'CONSTC=',CONSTC
      Y=-D1
      P0=-(-BBB(1)*H-CONSTC+Y)
      WRITE(2,*)'PRESSURE AT F IN SEGMENT 2=',P0
      P1=-(PHITPD*H-CONSTC+Y)
      WRITE(2,*)'PRESSURE AT D IN SEGMENT 2=',P1
      Y=-D1-(0.05/(BINS2-DINS2))*(D2+S1)
      P2=-(PHITPC*H-CONSTC+Y)
      WRITE(2,*)'PRESSURE AT C IN SEGMENT 2=',P2

      Y=-D1-D2
      P3=-(PHAID*H-CONSTC+Y)
      WRITE(2,*)'PRESSURE AT D IN SEGMENT 3=',P3
      P4=-(PHAIC*H-CONSTC+Y)
      WRITE(2,*)'PRESSURE AT C IN SEGMENT 3=',P4

      GO TO 701
800   WRITE(2,*)'C VALUE ASSUMED WRONGLY'
701   CONTINUE
      STOP
      END

      SUBROUTINE EQUATP(CINS2,E2,E1,BINS2,TP,X,FX)
      USQ=0.25*(TP-E2)*(1.+X)**2
      TERM1=SQRT((E2+USQ)*(E2+USQ-E1)*(BINS2-E2-USQ)*(1.-E2-USQ))
      FX=(CINS2-E2-USQ)/TERM1*SQRT(TP-E2)
      RETURN
      END



      SUBROUTINE EQUAIV(E2,E1,B,X,FX1,FX2,FX3,FX4)
      U=0.25*(1.+X)**2
      FX1=1./((E1+U)*(E2+U)*(B+U)*(1.+U))**0.5
      FX2=U/((E1*U+1.)*(E2*U+1.)*(B*U+1.)*(U+1.))**0.5
      FX3=FX1*U
      FX4=FX2/U
      RETURN
      END
      SUBROUTINE EQUAV(E2,E1,B,X,FX1,FX2,FX3,FX4)
      U=0.125*E1*(1.+X)**2
        FX1=1./((E1-U)*(E2-U)*(B-U)*(1.-U))**0.5
      FX2=1./((E1-U)*(E2-E1+U)*(B-E1+U)*(1.-E1+U))**0.5
```

```
      TERM=((E1-U)*(E2-U)*(B-U)*(1.-U))**0.5
      FX3=U/TERM
     TERM=((E2-E1+U)*(B-E1+U)*(1.-E1+U))**0.5
     FX4=SQRT(E1-U)/TERM
     RETURN
     END

     SUBROUTINE EQUAVI(E2,E1,B,X,FX1,FX2,FX3,FX4)
     U=0.125*(E2-E1)*(1.+X)**2
     TERM=((U+E1)*(E2-U-E1)*(B-U-E1)*(1.-U-E1))**0.5
     FX1=1./TERM
     TERM=((E2-U)*(E2-U-E1)*(B-E2+U)*(1.-E2+U))**0.5
     FX2=1./TERM
     TERM=((E2-U-E1)*(B-U-E1)*(1.-U-E1))**0.5
     FX3=SQRT(U+E1)/TERM
     TERM=((E2-U-E1)*(B-E2+U)*(1.-E2+U))**0.5
     FX4=SQRT(E2-U)/TERM
     RETURN
     END

     SUBROUTINE EQUAVII(E2,E1,B,X,FX1,FX2,FX3,FX4)
     U=0.125*(B-E2)*(1.+X)**2
     TERM=((U+E2)*(U+E2-E1)*(B-U-E2)*(1.-U-E2))**0.5
     FX1=1./TERM
     TERM=((B-U)*(B-U-E1)*(B-U-E2)*(1.-B+U))**0.5
     FX2=1./TERM
     TERM=((U+E2-E1)*(B-U-E2)*(1.-U-E2))**0.5
     FX3=SQRT(U+E2)/TERM
     TERM=((B-U-E1)*(B-U-E2)*(1.-B+U))**0.5
     FX4=SQRT(B-U)/TERM
     RETURN
     END


     SUBROUTINE EQUAVIII(E2,E1,B,X,FX1,FX2,FX3,FX4)
     U=0.125*(1.-B)*(1.+X)**2
     FX1=1./((U+B)*(U+B-E1)*(U+B-E2)*(1.-U-B))**0.5
     FX2=1./((1.-U)*(1.-U-E1)*(1.-U-E2)*(1.-U-B))**0.5
     TERM=((U+B-E1)*(U+B-E2)*(1.-U-B))**0.5
     FX3=SQRT(U+B)/TERM
     TERM=((1.-U-E1)*(1.-U-E2)*(1.-U-B))**0.5
     FX4=SQRT(1.-U)/TERM
     RETURN
     END

     SUBROUTINE EQUAIX(E2,E1,B,X,FX1,FX2,FX3,FX4)
     U=0.25*(1.+X)**2
     TERM=((U+1.)*(U+1.-E1)*(U+1.-E2)*(U+1.-B))**0.5
     FX1=1./TERM
     TERM=((1.+U)*(1.+U-E1*U)*(1.+U-E2*U)*(1.+U-B*U))**0.5
     FX2=U/TERM
     TERM=((U+1.-E1)*(U+1.-E2)*(U+1.-B))**0.5
     FX3=SQRT(U+1.)/TERM
     TERM=((1.+U-E1*U)*(1.+U-E2*U)*(1.+U-B*U))**0.5
     FX4=SQRT(1.+U)/TERM
     RETURN
     END
```

```
      SUBROUTINE CEF(W,XX,AKS,CEF1)
      DIMENSION W(96), XX(96)
      PAI=3.141592654
      SUM=0.0
      DO 10 I=1,96
      THETA=PAI/4.*(1.+XX(I))
      TERM=0.25*PAI/SQRT(1.-AKS*SIN(THETA)**2)
      SUM=SUM+TERM*W(I)
 10         CONTINUE
      CEF1=SUM
      RETURN
      END

      SUBROUTINE CIEF(W,XX,AKS,PHAI,CIEF1)
      DIMENSION W(96), XX(96)
      PAI=3.141592654
      SUM=0.0
      DO 10 I=1,96
      THETA=PHAI/2.*(1.+XX(I))
      TERM=0.5*PHAI/SQRT(1.-AKS*SIN(THETA)**2)
      SUM=SUM+TERM*W(I)
 10         CONTINUE
      CIEF1=SUM
      RETURN
      END

      SUBROUTINE RAPH21(W,XX,AL1,T1,D,RESIDUE)
      DIMENSION W(96),XX(96)
      D=0.01
      DELD=0.00045
100   CONTINUE
      AKS=D
      CALL CEF(W,XX,AKS,CEF1)
      TERM1=CEF1
      AKS=1.-D
      CALL CEF(W,XX,AKS,CEF1)
      TERM2=CEF1
      TERM3=TERM1/TERM2
      RESIDUE=AL1/T1-TERM3
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      D=D+DELD
      IF (RESIDUE.GT.0.0) GO TO 100
      DR=D-DELD
      DL=DR-DELD
200   D=(DL+DR)/2.
      AKS=D
      CALL CEF(W,XX,AKS,CEF1)
      TERM1=CEF1
      AKS=1.-D
      CALL CEF(W,XX,AKS,CEF1)
      TERM2=CEF1
      TERM3=TERM1/TERM2
      RESIDUE=AL1/T1-TERM3
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      IF(RESIDUE.GT.0.) GO TO 300
      IF(RESIDUE.LT.0.) GO TO 400
```

```
  300   DL=D
        GO TO 200
  400   DR=D
        GO TO 200
  500   CONTINUE
        RETURN
        END

        SUBROUTINE RAPH22(W,XX,AL1,AL2,T1,D,E1,RESIDUE)
        DIMENSION W(96),XX(96)
        AKS=1.-D
        CALL CEF(W,XX,AKS,CEF1)
        E1=0.0001
        DELE1=0.01
        AKS=D
  100   CONTINUE
        PHAI=ASIN (SQRT(E1/D))
        CALL CIEF(W,XX,AKS,PHAI,CIEF1)
        RESIDUE= (AL1-AL2)/T1-CIEF1/CEF1
        IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
        E1=E1+DELE1
        IF (RESIDUE.GT.0.0) GO TO 100
        E1R=E1-DELE1
        E1L=E1R-DELE1
  200   E1=(E1L+E1R)/2.
        PHAI=ASIN(SQRT(E1/D))
        CALL CIEF(W,XX,AKS,PHAI,CIEF1)
        RESIDUE=(AL1-AL2)/T1-CIEF1/CEF1
        IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
        IF(RESIDUE.GT.0.) GO TO 300
        IF(RESIDUE.LT.0.) GO TO 400
  300   E1L=E1
        GO TO 200
  400   E1R=E1
        GO TO 200
  500   CONTINUE
        RETURN
        END
C
        SUBROUTINE RAPH23(W,XX,AL1,AL3,T1,D,E1,E2,RESIDUE)
        DIMENSION W(96),XX(96)
        AKS=1.-D
        CALL CEF(W,XX,AKS,CEF1)
        TERM=(AL1-AL3)/T1
        E2=E1+0.0001
        DELE2=0.01
        AKS=D
  100   CONTINUE
        PHAI=ASIN(SQRT(E2/D))
        CALL CIEF(W,XX,AKS,PHAI,CIEF1)
        RESIDUE=TERM-CIEF1/CEF1
        IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
        E2=E2+DELE2
        IF (RESIDUE.GT.0.0) GO TO 100
        E2R=E2-DELE2
        E2L=E2R-DELE2
  200   E2=(E2L+E2R)/2.
```

```
      PHAI=ASIN(SQRT(E2/D))
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      RESIDUE=TERM-CIEF1/CEF1
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      IF(RESIDUE.GT.0.) GO TO 300
      IF(RESIDUE.LT.0.) GO TO 400
  300 E2L=E2
      GO TO 200
  400 E2R=E2
      GO TO 200
  500 CONTINUE
      RETURN
      END


      SUBROUTINE RAPH24(W,XX,D2,S1,T1,D,B,RESIDUE)
      DIMENSION W(96),XX(96)
      AKS=1.-D
      CALL CEF(W,XX,AKS,CEF1)
      TERM=(D2+S1)/T1
      B=D+0.0001
      DELB=0.01
      AKS=1.-D
  100 CONTINUE
      PHAI=ASIN(SQRT((B-D)/(B*(1.-D))))
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      RESIDUE=TERM-CIEF1/CEF1
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      B=B+DELB
      IF (RESIDUE.GT.0.0) GO TO 100
      BR=B-DELB
      BL=BR-DELB
  200 B=(BL+BR)/2.
      PHAI=ASIN(SQRT((B-D)/(B*(1.-D))))
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      RESIDUE=TERM-CIEF1/CEF1
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      IF(RESIDUE.GT.0.) GO TO 300
      IF(RESIDUE.LT.0.) GO TO 400
  300 BL=B
      GO TO 200
  400 BR=B
      GO TO 200
  500 CONTINUE
      RETURN
      END

      SUBROUTINE RAPH31(W,XX,AL4,T2,C,RESIDUE)
      DIMENSION W(96),XX(96)
      C=0.00001
      I=1
      DELC=0.0001
  100 CONTINUE
      AKS=C
      CALL CEF(W,XX,AKS,CEF1)
      TERM1=CEF1
      AKS=1.-C
```

```
      CALL CEF(W,XX,AKS,CEF1)
      TERM2=CEF1
      TERM3=TERM1/TERM2
      RESIDUE=AL4/T2-TERM3
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      C=C+DELC
      IF (RESIDUE.GT.0.0) GO TO 100
      CR=C-DELC
      CL=CR-DELC
 200  C=(CL+CR)/2.
      AKS=C
      CALL CEF(W,XX,AKS,CEF1)
      TERM1=CEF1
      AKS=1.-C
      CALL CEF(W,XX,AKS,CEF1)
      TERM2=CEF1
      TERM3=TERM1/TERM2
      RESIDUE=AL4/T2-TERM3
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      IF(RESIDUE.GT.0.) GO TO 300
      IF(RESIDUE.LT.0.) GO TO 400
 300  CL=C
      GO TO 200
 400  CR=C
      GO TO 200
 500  CONTINUE
      RETURN
      END

      SUBROUTINE RAPH32(W,XX,T2,S1,C,E,RESIDUE)
      DIMENSION W(96),XX(96)
      AKS=1.-C
      CALL CEF(W,XX,AKS,CEF1)
      E=0.0001
      DELE=0.01
      TERM=(T2-S1)/T2
 100  CONTINUE
      PHAI=ASIN(SQRT(1./(1.+E)))
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      RESIDUE=CIEF1/CEF1-TERM
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      E=E+DELE
      IF (RESIDUE.GT.0.0) GO TO 100
      ER=E-DELE
      EL=ER-DELE
 200  E=(EL+ER)/2.
      PHAI=ASIN(SQRT(1./(1.+E)))
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      RESIDUE=CIEF1/CEF1-TERM
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      IF(RESIDUE.GT.0.) GO TO 300
      IF(RESIDUE.LT.0.) GO TO 400
 300  EL=E
      GO TO 200
 400  ER=E
      GO TO 200
 500  CONTINUE
```

```
      RETURN
      END

      SUBROUTINE RAPH33(W,XX,T2,S2,C,B,RESIDUE)
      DIMENSION W(96),XX(96)
      AKS=1.-C
      CALL CEF(W,XX,AKS,CEF1)
      B=C+0.0001
      DELB=0.0001
 100  CONTINUE
      PHAI=ASIN (SQRT((B-C)/(B*(1.-C))))
C     WRITE(2,*)'PHAI=',PHAI,'OVER'
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      RESIDUE= S2/T2-CIEF1/CEF1
C     WRITE(2,*)'RESIDUE=',RESIDUE
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      B=B+DELB
      CHECK=(SQRT((B-C)/(B*(1.-C))))
C     WRITE(2,*)'CHECK=',CHECK
      IF (RESIDUE.GT.0.0) GO TO 100
      BR=B-DELB
      BL=BR-DELB
 200  B=(BL+BR)/2.
      PHAI=ASIN(SQRT((B-C)/(B*(1.-C))))
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      RESIDUE=S2/T2-CIEF1/CEF1
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      IF(RESIDUE.GT.0.) GO TO 300
      IF(RESIDUE.LT.0.) GO TO 400
 300  BL=B
      GO TO 200
 400  BR=B
      GO TO 200
 500  CONTINUE
      RETURN
      END

      SUBROUTINE MATIN (AAA,MMM)
      DIMENSION AAA(6,6),B(6),C(6)
        NN=MMM-1
        AAA(1,1)=1./AAA(1,1)
        DO 8 M=1,NN
        K=M+1
        DO 3 I=1,M
        B(I)=0.0
        DO 3 J=1,M
 3      B(I)=B(I)+AAA(I,J)*AAA(J,K)
        D=0.0
        DO 4 I=1,M
 4      D=D+AAA(K,I)*B(I)
        D=-D+AAA(K,K)
        AAA(K,K)=1./D
        DO 5 I=1,M
 5      AAA(I,K)=-B(I)*AAA(K,K)
        DO 6 J=1,M
        C(J)=0.0
        DO 6 I=1,M
```

# DSWND

```
      DIMENSION W(96),XX(96),AAA(4,4),BBB(4),CCC(4),AAT(4,4),AAAT(4,4)
      OPEN (1, status='old',FILE='USMAN11.DAT')
      OPEN (3, status='old',FILE='GAUSS.DAT')
      OPEN (2, status='unknown',FILE='USMAN11.OUT')
      READ (3,*) (W(I),I=1,96)
      READ (3,*) (XX(I),I=1,96)
      READ(1,*)D1,T1
      READ(1,*)AL1,D2,S1
      READ(1,*)AL4,S2,T2
      READ(1,*)D3,T3
      PAI=3.141592654
C     USMAN6 DEALS WITH WEIR WITHOUT DRAIN
444   FORMAT(2F10.3)
C     COMPUTATION FOR SEGMENT1
      TERM=SIN(D1/(D1+T1)*PAI/2.)
      B1=TERM**2
      AKS=B1
      CALL CEF(W,XX,AKS,CEF1)
      TERM1=CEF1
      AKS=1.-B1
      CALL CEF(W,XX,AKS,CEF1)
      TERM2=CEF1
C
C     GENERATING ELEMENT OF MATRIX
      DO I=1,4
      DO J=1,4
      AAA(I,J)=0.
      END DO
      END DO
C     UNKNOWN(1)=ALPHA1=BBB(1)
C     UNKNOWN(2)=ALPHA2=BBB(2)
C     UNKNOWN(3)=ALPHA3=BBB(3)
C     UNKNOWN(4)=Q/KH   =BBB(4)

      AAA(1,1)=TERM2/TERM1
      AAA(1,4)=1.
      CCC(1)=TERM2/TERM1


C
C     COMPUTATION FOR SEGMENT II

C     COMPUTATION OF D
      CALL RAPH21(W,XX,AL1,T1,D,RESIDUE)
      WRITE(2,*)'D=',D
      WRITE(2,*)'RESIDUE=',RESIDUE
C     COMPUTATION OF B
      CALL RAPH24(W,XX,D2,S1,T1,D,B,RESIDUE)
      WRITE(2,*)'B=',B
```

```
      WRITE(2,*)'RESIDUE=',RESIDUE

      PBINS2=B
      PDINS2=D
      AKS=B
      CALL CEF(W,XX,AKS,CEF1)
      TERM1=CEF1
      AKS=1.-B
      CALL CEF(W,XX,AKS,CEF1)
      TERM2=CEF1

      AAA(2,1)=-TERM2/TERM1
      AAA(2,2)=TERM2/TERM1
      AAA(2,4)=1.
      CCC(2)=0.
C
C     COMPUTATION FOR SEGMENT 3
      CALL RAPH31 (W,XX,AL4,T2,C,RESIDUE)
      WRITE(2,*)'C=',C
      WRITE(2,*)'RESIDUE=',RESIDUE
C     COMPUTATION OF E
      CALL RAPH32(W,XX,T2,S1,C,E,RESIDUE)
      WRITE(2,*)'E=',E
      WRITE(2,*)'RESIDUE=',RESIDUE
C     COMPUTATION OF B
      CALL RAPH33(W,XX,T2,S2,C,B,RESIDUE)
      WRITE(2,*)'B=',B
      WRITE(2,*)'RESIDUE=',RESIDUE
      PCINS3=C
      PEINS3=E
      PBINS3=B
      AKS=(1.-B)/(1.+E)
      CALL CEF(W,XX,AKS,CEF1)
      TERM1=CEF1
      AKS=(B+E)/(1.+E)
      CALL CEF(W,XX,AKS,CEF1)
      TERM2=CEF1
      AAA(3,2)=-TERM1/TERM2
      AAA(3,3)=TERM1/TERM2
      AAA(3,4)=1.
      CCC(3)=0.
C
C     COMPUTATION FOR SEGMENT IV
C
      B=(SIN(PAI/2.*(1.-(D3+S2)/T3)) )**2
      PBINS4=B
      AKS=B
      CALL CEF(W,XX,AKS,CEF1)
      TERM1=CEF1
      AKS=1.-B
      CALL CEF(W,XX,AKS,CEF1)
      TERM2=CEF1
      AAA(4,3)=-TERM1/TERM2
      AAA(4,4)=1.
      CCC(4)=0.

      WRITE(2,*)'MATRIX ELEMENT'
```

```
      DO I=1,4
      WRITE(2,222)(AAA(I,J), J=1,4)
 222  FORMAT(4E12.4)
      END DO

      DO I=1,4
      DO J=1,4
      AAT(I,J)=AAA(I,J)
      END DO
      END DO

      MMM=4
        CALL MATIN(AAA,MMM)

      WRITE(2,*)'INVERSE MATRIX ELEMENT'

      DO I=1,4
      WRITE(2,222)(AAA(I,J),J=1,4)
      END DO

      WRITE(2,*)'CHECKING INVERSION'

      DO I=1,4
      DO K=1,4
      SUM=0.
      DO J=1,4
      SUM=SUM+AAA(I,J)*AAT(J,K)
      END DO
      AAAT(I,K)=SUM
      END DO
      END DO

      DO I=1,4
      WRITE(2,199)(AAAT(I,J),J=1,4)
      END DO
 199  FORMAT(4F10.3)

      DO I=1,4
      SUM=0.
      DO J=1,4
      SUM=SUM+AAA(I,J)*CCC(J)
      END DO
      BBB(I)=SUM
      END DO

      WRITE(2,198)
 198  FORMAT(4X,'ALPHA1',4X,'ALPHA2',4X,'ALPHA3',4X,'q/kh')
      WRITE(2,197)BBB(1),BBB(2),BBB(3),BBB(4)
 197  FORMAT(4F10.5)

      AKS=1.-PBINS2
      CALL CEF(W,XX,AKS,CEF1)
      CONM1=0.5*BBB(4)/CEF1
      AKS=PBINS2
      PHAI=ASIN( (PDINS2/PBINS2)**0.5)
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      PHIBYKH=CONM1*2.*CIEF1-BBB(1)
```

```
      PHI1=PHIBYKH
      WRITE(2,*)'PHIBYKH AT D IN SEGMENT 2=', PHIBYKH

      WRITE(2,*)'PHAI IN SEGMENT 2'
      DTP=PDINS2/10.
      TP=DTP
      AKS=1.-PDINS2
      CALL CEF(W,XX,AKS,CEF1)
      CONMZ=T1/(2.*CEF1)

      AKS=1.-PBINS2
      CALL CEF(W,XX,AKS,CEF1)
      CONMW=BBB(4)/(2.*CEF1)
      WRITE(2,*)CONMZ,CONMW
777   CONTINUE
      AKS=PDINS2
      PHAI=ASIN((TP/PDINS2)**0.5)
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      ZTP=(CONMZ*2.*CIEF1-AL1)/AL1

      PHAI=ASIN( (TP/PBINS2)**0.5)
      AKS=PBINS2
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      PHAITP=CONMW*2.*CIEF1-BBB(1)
      TP=TP+DTP
      WRITE(2,444)ZTP,PHAITP
      IF(TP.LT.PDINS2) GO TO 777

C     COMPUTING PHAI AT D FOR WHICH PARAMETER T=0. IN SEGMENT 3
      AKS=(1.-PBINS3)/(1.+PEINS3)
      CALL CEF(W,XX,AKS,CEF1)
      AKS=(PBINS3+PEINS3)/(1.+PEINS3)
      PHAI=ASIN( (PEINS3/(PBINS3+PEINS3))**0.5)
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      CONM1=0.5*BBB(4)/CEF1
      PHIBYKH=CONM1*2.*CIEF1-BBB(2)
      PHI2=PHIBYKH
      WRITE(2,*)'PHIBYKH AT D IN SEGMENT 3=',PHIBYKH
      PHAI=ASIN( ( (PCINS3+PEINS3) / (PBINS3+PEINS3) )**0.5)
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      PHIBYKH=CONM1*2.*CIEF1-BBB(2)
      PHI3=PHIBYKH
      WRITE(2,*)'PHIBYKH AT C IN SEGMENT 3=',PHIBYKH

      PDINS3=0.
      AKS=1.-PCINS3
      CALL CEF(W,XX,AKS,CEF1)
      CONMZ=T2/(2.*CEF1)

      AKS=(1.-PBINS3)/(1.+PEINS3)
      CALL CEF(W,XX,AKS,CEF1)
      CONMW=BBB(4)/(2.*CEF1)

      DTP=(PCINS3-PDINS3)/10.
      TP=PDINS3+DTP
888   CONTINUE
      AKS=PCINS3
```

```
      PHAI=ASIN((TP/PCINS3)**0.5)
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      ZTP=(CONMZ*2.*CIEF1-AL4)/AL4
      AKS=(PBINS3+PEINS3)/(1.+PEINS3)
      PHAI=ASIN( ((TP+PEINS3)/(PBINS3+PEINS3))**0.5)
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      PHAITP=CONMW*2.*CIEF1-BBB(2)
      WRITE(2,444)ZTP,PHAITP
      TP=TP+DTP
      IF(TP.LT.PCINS3) GO TO 888

      AKS=1.-PBINS4
      CALL CEF(W,XX,AKS,CEF1)
      EXITG=BBB(3)*PAI*0.5/(T3*CEF1*SQRT(1.-PBINS4))
      WRITE(2,*)'MAXIMUM EXIT GRADIENT/h=',EXITG
C     COMPUTATION OF PRESSURE FROM PHI
C     FIRST  CHOOSE AN ORIGIN AND
C        FIND CONSTANT C SO THAT PHI=O AT DOWNSTREAM BOUNDARY
      H1=12.
      H2=2.
      H=15.
      H=H1+D1+D2-D3-H2
      WRITE(2,*)'H=',H
      CONSTC=H2+(-D1-D2+D3)
      WRITE(2,*)'CONSTC=',CONSTC
      PU=(-H-CONSTC)*(-1.)
      WRITE(2,*)'PRESSURE AT UPSTREAM=',PU
      Y=-D1
      P0=-(-BBB(1)*H-CONSTC+Y)
      WRITE(2,*)'PRESSURE AT END OF FIRST STEP=',P0
      P1=-(PHI1*H-CONSTC+Y)
      WRITE(2,*)'PRESSURE AT 1=',P1
      Y=-D1-D2
      P2=-(PHI2*H-CONSTC+Y)
      WRITE(2,*)'PRESSURE AT 2=',P2
      P3=-(PHI3*H-CONSTC+Y)
      WRITE(2,*)'PRESSURE AT 3=',P3
      PHID=-(H2-D1-D2+D3)+CONSTC
      PHIU=-H1+CONSTC
      WRITE(2,*)PHIU,PHID
      STOP
      END

      SUBROUTINE CEF(W,XX,AKS,CEF1)
      DIMENSION W(96), XX(96)
      PAI=3.141592654
      SUM=0.0
      DO 10 I=1,96
      THETA=PAI/4.*(1.+XX(I))
      TERM=0.25*PAI/SQRT(1.-AKS*SIN(THETA)**2)
      SUM=SUM+TERM*W(I)
 10         CONTINUE
      CEF1=SUM
      RETURN
      END

      SUBROUTINE CIEF(W,XX,AKS,PHAI,CIEF1)
```

```
      DIMENSION W(96), XX(96)
      PAI=3.141592654
      SUM=0.0
      DO 10 I=1,96
      THETA=PHAI/2.*(1.+XX(I))
      TERM=0.5*PHAI/SQRT(1.-AKS*SIN(THETA)**2)
      SUM=SUM+TERM*W(I)
   10         CONTINUE
      CIEF1=SUM
      RETURN
      END

      SUBROUTINE RAPH21(W,XX,AL1,T1,D,RESIDUE)
      DIMENSION W(96),XX(96)
      D=0.01
      DELD=0.00045
  100 CONTINUE
      AKS=D
      CALL CEF(W,XX,AKS,CEF1)
      TERM1=CEF1
      AKS=1.-D
      CALL CEF(W,XX,AKS,CEF1)
      TERM2=CEF1
      TERM3=TERM1/TERM2
      RESIDUE=AL1/T1-TERM3
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      D=D+DELD
      IF (RESIDUE.GT.0.0) GO TO 100
      DR=D-DELD
      DL=DR-DELD
  200 D=(DL+DR)/2.
      AKS=D
      CALL CEF(W,XX,AKS,CEF1)
      TERM1=CEF1
      AKS=1.-D
      CALL CEF(W,XX,AKS,CEF1)
      TERM2=CEF1
      TERM3=TERM1/TERM2
      RESIDUE=AL1/T1-TERM3
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      IF(RESIDUE.GT.0.) GO TO 300
      IF(RESIDUE.LT.0.) GO TO 400
  300 DL=D
      GO TO 200
  400 DR=D
      GO TO 200
  500 CONTINUE
      RETURN
      END

      SUBROUTINE RAPH24(W,XX,D2,S1,T1,D,B,RESIDUE)
      DIMENSION W(96),XX(96)
      AKS=1.-D
      CALL CEF(W,XX,AKS,CEF1)
      TERM=(D2+S1)/T1
      B=D+0.0001
```

```
      DELB=0.01
      AKS=1.-D
 100  CONTINUE
      PHAI=ASIN(SQRT((B-D)/(B*(1.-D))))
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      RESIDUE=TERM-CIEF1/CEF1
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      B=B+DELB
      IF (RESIDUE.GT.0.0) GO TO 100
      BR=B-DELB
      BL=BR-DELB
 200  B=(BL+BR)/2.
      PHAI=ASIN(SQRT((B-D)/(B*(1.-D))))
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      RESIDUE=TERM-CIEF1/CEF1
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      IF(RESIDUE.GT.0.) GO TO 300
      IF(RESIDUE.LT.0.) GO TO 400
 300  BL=B
      GO TO 200
 400  BR=B
      GO TO 200
 500  CONTINUE
      RETURN
      END

      SUBROUTINE RAPH31(W,XX,AL4,T2,C,RESIDUE)
      DIMENSION W(96),XX(96)
      C=0.00001
      I=1
      DELC=0.0001
 100  CONTINUE
      AKS=C
      CALL CEF(W,XX,AKS,CEF1)
      TERM1=CEF1
      AKS=1.-C
      CALL CEF(W,XX,AKS,CEF1)
      TERM2=CEF1
      TERM3=TERM1/TERM2
      RESIDUE=AL4/T2-TERM3
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      C=C+DELC
      IF (RESIDUE.GT.0.0) GO TO 100
      CR=C-DELC
      CL=CR-DELC
 200  C=(CL+CR)/2.
      AKS=C
      CALL CEF(W,XX,AKS,CEF1)
      TERM1=CEF1
      AKS=1.-C
      CALL CEF(W,XX,AKS,CEF1)
      TERM2=CEF1
      TERM3=TERM1/TERM2
      RESIDUE=AL4/T2-TERM3
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      IF(RESIDUE.GT.0.) GO TO 300
      IF(RESIDUE.LT.0.) GO TO 400
```

```
 300   CL=C
       GO TO 200
 400   CR=C
       GO TO 200
 500   CONTINUE
       RETURN
       END

       SUBROUTINE RAPH32(W,XX,T2,S1,C,E,RESIDUE)
       DIMENSION W(96),XX(96)
       AKS=1.-C
       CALL CEF(W,XX,AKS,CEF1)
       E=0.0001
       DELE=0.01
       TERM=(T2-S1)/T2
 100   CONTINUE
       PHAI=ASIN(SQRT(1./(1.+E)))
       CALL CIEF(W,XX,AKS,PHAI,CIEF1)
       RESIDUE=CIEF1/CEF1-TERM
       IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
       E=E+DELE
       IF (RESIDUE.GT.0.0) GO TO 100
       ER=E-DELE
       EL=ER-DELE
 200   E=(EL+ER)/2.
       PHAI=ASIN(SQRT(1./(1.+E)))
       CALL CIEF(W,XX,AKS,PHAI,CIEF1)
       RESIDUE=CIEF1/CEF1-TERM
       IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
       IF(RESIDUE.GT.0.) GO TO 300
       IF(RESIDUE.LT.0.) GO TO 400
 300   EL=E
       GO TO 200
 400   ER=E
       GO TO 200
 500   CONTINUE
       RETURN
       END

       SUBROUTINE RAPH33(W,XX,T2,S2,C,B,RESIDUE)
       DIMENSION W(96),XX(96)
       AKS=1.-C
       CALL CEF(W,XX,AKS,CEF1)
       B=C+0.0001
       DELB=0.0001
 100   CONTINUE
       PHAI=ASIN (SQRT((B-C)/(B*(1.-C))))
C      WRITE(2,*)'PHAI=',PHAI,'OVER'
       CALL CIEF(W,XX,AKS,PHAI,CIEF1)
       RESIDUE= S2/T2-CIEF1/CEF1
C      WRITE(2,*)'RESIDUE=',RESIDUE
       IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
       B=B+DELB
       CHECK=(SQRT((B-C)/(B*(1.-C))))
C      WRITE(2,*)'CHECK=',CHECK
       IF (RESIDUE.GT.0.0) GO TO 100
       BR=B-DELB
```

```
      BL=BR-DELB
 200  B=(BL+BR)/2.
      PHAI=ASIN(SQRT((B-C)/(B*(1.-C))))
      CALL CIEF(W,XX,AKS,PHAI,CIEF1)
      RESIDUE=S2/T2-CIEF1/CEF1
      IF ( ABS(RESIDUE).LT.0.00001) GO TO 500
      IF(RESIDUE.GT.0.) GO TO 300
      IF(RESIDUE.LT.0.) GO TO 400
 300  BL=B
      GO TO 200
 400  BR=B
      GO TO 200
 500  CONTINUE
      RETURN
      END

      SUBROUTINE MATIN (AAA,MMM)
      DIMENSION AAA(4,4),B(4),C(4)
        NN=MMM-1
        AAA(1,1)=1./AAA(1,1)
        DO 8 M=1,NN
        K=M+1
        DO 3 I=1,M
        B(I)=0.0
        DO 3 J=1,M
3       B(I)=B(I)+AAA(I,J)*AAA(J,K)
        D=0.0
        DO 4 I=1,M
4       D=D+AAA(K,I)*B(I)
        D=-D+AAA(K,K)
        AAA(K,K)=1./D
        DO 5 I=1,M
5       AAA(I,K)=-B(I)*AAA(K,K)
        DO 6 J=1,M
        C(J)=0.0
        DO 6 I=1,M
6       C(J)=C(J)+AAA(K,I)*AAA(I,J)
        DO 7 J=1,M
7       AAA(K,J)=-C(J)*AAA(K,K)
        DO 8 I=1,M
        DO 8 J=1,M
8       AAA(I,J)=AAA(I,J)-B(I)*AAA(K,J)
        RETURN
        END
```

## EXAMPLE INPUT DATA

```
5.    20.
20.     16. 14.   5.0         5.0
20.   5.0   15.
5.    20.
```

## EXAMPLE OUTPUT DATA

```
 D=    4.999997E-01
 RESIDUE=    2.980232E-07
 B=    7.071112E-01
 RESIDUE=   -6.011659E-06
 C=    7.845029E-01
 RESIDUE=   -4.212061E-06
 E=    2.949828E-01
 RESIDUE=   -6.120251E-06
 B=    8.335892E-01
 RESIDUE=   -8.816961E-07
 MATRIX ELEMENT
   .1614E+01    .0000E+00    .0000E+00    .1000E+01
  -.8196E+00    .8196E+00    .0000E+00    .1000E+01
   .0000E+00   -.6605E+00    .6605E+00    .1000E+01
   .0000E+00    .0000E+00   -.1000E+01    .1000E+01
 INVERSE MATRIX ELEMENT
   .5313E+00   -.1736E+00   -.2154E+00   -.1423E+00
   .3577E+00    .7045E+00   -.6397E+00   -.4225E+00
   .1423E+00    .2802E+00    .3478E+00   -.7703E+00
   .1423E+00    .2802E+00    .3478E+00    .2297E+00
 CHECKING INVERSION
     1.000        .000        .000        .000
      .000       1.000        .000        .000
      .000        .000       1.000        .000
      .000        .000        .000       1.000
    ALPHA1      ALPHA2      ALPHA3      q/kh
    .85771      .57747      .22970      .22970
 PHIBYKH AT D IN SEGMENT 2=  -7.059705E-01
 PHIBYKH AT D IN SEGMENT 3=  -4.982702E-01
 PHIBYKH AT C IN SEGMENT 3=  -3.088977E-01
 MAXIMUM EXIT GRADIENT/h=   1.376054E-02
 H=        15.000000
 CONSTC=         -3.000000
 PRESSURE AT UPSTREAM=        12.000000
 PRESSURE AT F IN SEGMENT 2=        14.865710
 PRESSURE AT D IN SEGMENT 2=        12.589560
 PRESSURE AT D IN SEGMRNT 3=        14.474050
 PRESSURE AT C IN SEGMENT 3=        11.633460
      -15.000000     0.000000E+00
```