SEEPAGE FROM A PARTIALLY PENETRATING STREAM OF FINITE WIDTH

A DISSERTATION

submitted in partial fulfillment of the
requirements for the award of the degree
of
MASTER OF ENGINEERING
in
WATER RESOURCES DEVELOPMENT

By

HERU ERMADI

WATER RESOURCES DEVELOPMENT TRAINING CENTRE
INDIAN INSTITUTE OF TECHNOLOGY, ROORKEE
ROORKEE - 247 667 (INDIA)
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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the Dissertation entitled **SEEPAGE FROM A PARTIALLY PENETRATING STREAM OF FINITE WIDTH** is being submitted in partial fulfillment of the requirements for the award of the Degree of Master of Engineering in Water Resources Development, Indian Institute of Technology, Roorkee , is an authentic record of my own work carried out from July,17, 2001 to December, 2001 under the supervision of Dr. G.C. Mishra, Proffessor, WRDTC, Indian Institute of Technology, Roorkee.

The matter embodied in this Dissertation has not submitted by me for the award of any other degree.

Dated: December 10 ,2001

(HERU ERMADI)

This is to certify that the above statements made by the candidate are correct to the best of my knowledge and belief.

Dated: December 10 ,2001

G. C. Mishra

(Dr. G.C. MISHRA)
Professor, WRDTC
Indian Institute of Technology
Roorkee – 247667
INDIA

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SYNOPSIS

For a partially penetrating stream in an unconfined aquifer, the reach transmissivity increases with increase in depth of water in the stream, decreases with increase in length of aquifer boundary and increases tending to constant value with increase in stream width.

A rigorous analytical solution for steady seepage from a trapezoidal stream/canal to an unconfined aquifer in which water table lies at a shallow depth has been derived using Zhukovsky function and Schwarz-Christoffel conformal mapping.

Steady state seepage from a stream in a confined aquifer can be expressed as $q = k F \Delta h = \Gamma_r \Delta h$ in which k is hydraulic conductivity, Δh is hydraulic head difference measured at a piezometer in the vicinity of the stream, and F is a factor which depends on location of the piezometer i.e. distance of the piezometer from the stream bank and stream geometry i.e. cross section of the stream and depth of penetration of the stream. The above linear relationship between seepage and Δh is valid for steady state and confined flow condition.

Aravin, Bouwer, Herbert, Morel-Seytoux and many other investigators have derived the factor F based on Darcy's law and Dupuit Ferchheimer flow condition at large distance from the water body.

In the present dissertation, exact relation of the parameter Γ_r/k (i.e. seepage factor F) with distance of the piezometer and stream geometry including depth of penetration has been derived.

Unsteady flow from a fully penetrating stream has been given by Carslaw and Jaeger for an analogous heat conduction problem. Partially penetrating stream, offers more resistance to flow than fully penetrating stream because of flow convergence near the stream. The sum of the resistance due to flow convergence and resistance due to fraction of the aquifer under the stream bed can be equated to the resistance of length ΔL of the aquifer for uniform flow condition. This length ΔL is known as substitute length.

In comparing the results with Herbert's formula, it is found that Herbert's formula is applicable for depth of penetration less than 30 % (the involved error < 10%) and width of the stream (B/T_2) less than 0.2.

CONTENTS

NOTATIONS

B_1	half width of the stream at bed level, (L)
B_2	half width of the stream at the water surface, (L)
Ds	depth of penetration of the stream into the aquifer, (L)
h_1	hydraulic head in the stream, (L)
h_2	hydraulic head at the boundary of the finite aquifer, (L)
h_B	hydraulic head at a point B in the aquifer, (L)
h_M	hydraulic head at a point M below the stream bed, (L)
Δh_A	draw down at point A in the aquifer, (L)
Δh_F	draw down at point F below the stream bed, (L)
k	coefficient of permeability of the aquifer, (LT^{-1})
K_{qs}	unit step response function for flow, (L^3T^{-1}/L)
L_A	aquifer length measured from the stream bank, (L)
L_B	distance of piezometer from the stream bank, (L)
q	rate of seepage per unit length of the stream, (L^2T^{-1})
Q	rate of flow, (L^3T^{-1})
R_a	aquifer resistance (T/L) $R_a = \frac{1}{k}\frac{L}{A}$
s	rise in the aquifer, (L)
t	time, (T)
T_1	thickness of the aquifer below the stream bed, (L)
T_2	thickness of the aquifer, (L)
v	velocity of flow, (LT^{-1})
$\alpha\pi$	angle of inclination of the river bank over horizontal line
β	hydraulic diffusivity of the aquifer, (L^2T^{-1})
δ	discrete a kernel for flow, (L^3T^{-1}/L)
ΔL	substitute length, (L)
Δt	size of uniform time steps, (T)
γ_w	unit weight of water

n,γ	indices denoting time-step
Φ	storage coefficient of the aquifer
σ	rise in the stream, (L)
Γ_r	reach transmissivity (L^2/T)

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CHAPTER I
INTRODUCTION

I.1 GENERAL

Streams and rivers are important geological features, which are control the occurrence, distribution and quality of surface water as well as ground water. A river rarely penetrates the entire thickness to an aquifer. If bed width of the river is more than five times the depth of aquifer below the river bed, the river can be assumed to act as boundary between the adjoining aquifers. In such case the aquifers on either sides of the river do not influence each other directly i.e. the flow from one aquifer does not enter to the other aquifer under the river bed. For steady and confined flow condition, the exchange of flow between the river and aquifer is proportional to the difference in hydraulic heads at the river and in the aquifer near the river. The constant of proportionality is known as reach transmissivity which is a function of river width, depth of aquifer below the river bed, thickness of aquifer and hydraulic conductivity of the aquifer material. The reach transmissivity for a river with large width has been derived using conformal mapping by Mishra (2001).

On this dissertation, using conformal mapping the reach transmissivity for a partially penetrating stream of finite width is derived as a function of depth of penetration of the stream, thickness of the aquifer, width of the stream, distance of piezometer from the stream bank and hydraulic conductivity of the aquifer materials. The substitute length, whose resistance is equal to the extra resistance arising due to convergence of flow, has been derived for the partially penetrating stream of finite width. Using substitute length, unsteady seepage is computed.

I.2 TWO-DIMENSIONAL STEADY FLOW OF GROUND WATER

In many cases of ground water flow the liquid particles move in planes parallel to one another. The character of the flow is the same at all points of a straight line drawn at right angles to those planes. Such a flow is a two-dimension steady flow, and the corresponding seepage problem can be solved as a two-dimensional one. Since the liquid particles move in a plane, the velocity vectors also lie in that plane. Therefore, we choose any of the planes in which the motion takes place, and obtain a solution in that plane. In the solution, the length of the flow region in the direction normal to the plane of flow, is taken to be equal to unity. The total flow for the entire flow region is then obtained by multiplying the results of the plane problem by the actual length of the region.

The assumption of two-dimensional flow means a great simplification. On the strength of it we can examine many, otherwise intractable cases, because a mathematical treatment of three-dimensional seepage flows is only feasible in few, very simple problems. Fortunately, the majority of practical problem are essentially cases of two-dimensional flow; for example, the seepage through earth dam, canal, stream, etc., where one dimension of the structure exceeds by far all the other dimensions, and the flow takes place in a plane normal to that dimension. Sometimes a flow having a three-dimensional character can be converted to a two-dimensional flow with the help of a suitable scheme.

In a steady two-dimensional seepage flow through a homogeneous and isotropic medium, all quantities depend on two coordinates only. The fundamental equations are

$$v_x = \frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y} = -k\frac{\partial h}{\partial x} \tag{1.1}$$

and

$$v_y = \frac{\partial\phi}{\partial y} = -\frac{\partial\psi}{\partial x} = -k\frac{\partial h}{\partial y} \tag{1.2}$$

where v_x and v_y are the components of discharge velocity in the direction of the coordinates axes, $\psi(x, y)$ is the stream function, and $\psi(x, y) = C$, a constant, depicts locus of a stream line, $\phi\,(x, y)$ is the velocity potential function defined as

$$\phi = -\,kh. \tag{1.3}$$

$h(x,y)$ is the hydraulic head at the point (x, y) above a chosen reference plane. For the direction of the coordinate axes being considered positive down ward.

$$h = \frac{p}{\gamma_w} - y \tag{1.4}$$

where $p(x, y)$ is the water pressure at the point (x, y), C is a constant dependent on the choice of the reference plane used in the determination of the potential function ϕ.

Since in the region of seepage the function $\phi(x, y)$ and $\psi(x, y)$ are conjugate harmonic function, we can introduce a new function, namely

$$w = \phi + i\psi \tag{1.5}$$

called the complex potential of seepage flow; in the region of seepage, which is an analytic function of the complex variable z, where

$$z = x + iy \tag{1.6}$$

i.e. a function of complex coordinate of a point in the region of seepage

$$w = \phi(x, y) + i\psi(x, y) = w(z) = w(x + iy) \tag{1.7}$$

In operations involving the complex potential w, the region of seepage is often referred to as the (z) region.

We have thus converted the solution of the seepage problem to the solution of the problem of finding in the z region an analytical function w = w(z) that will satisfy the given boundary conditions, i.e. the known values of the function ϕ and ψ on the boundaries of the region of seepage.

If we know the complex potential w = w(z), separating it into its real and imaginary parts enables us to determine the potential function $\phi(x, y)$ as well as the stream function $\psi(x, y)$

$$\phi = \text{Real } w(z) = \phi(x, y) \tag{1.8}$$

$$\psi = \text{Imaginary } w(z) = \psi(x, y) \tag{1.9}$$

On establishing the function inverse to the function w = w(z), i.e. z = z(w) and separating it into its real and imaginary parts, we obtain the relations

$$x = \text{Real } z(w) = x(\phi, \psi) \tag{1.10}$$

$$y = \text{Imaginary } z(w) = y(\phi, \psi) \tag{1.11}$$

I.3 CONFORMAL MAPPING

I.3.1 Determine of The Complex Potential

The popular method of the available methods has been that based on the use of functions of a complex variable. By its application the solution of a seepage problem is converted to that of finding the complex potential of the seepage flow according to equation 1.7 in a way that will make it satisfy the pertinent boundary conditions.

In the application, there is the task of determining a certain analytic function of the complex variable ξ

$$\omega = f(\xi) \tag{1.12}$$

under the conditions that we know the shape of the region of the values of the complex variable ξ as well as the shape of the region of the values of ω corresponding to the various values of the variable ξ, i.e. we know the shape of the boundary of the ξ and ω regions and have to find the relation (1.12) which associates the value of ω with the various value of ξ. Naturally, relation (1.12) represents different functions depending on which of the methods is being used.

I.3.2 The Schwarz-Christoffel Transformation

Theoretically, a transformation exists which will map any pair of simply connected regions conformally onto each other. This is assured by the Riemann mapping theorem; however, the determination of a general solution for the mapping problem has thus far defied discovery. At first this may appear somewhat disturbing; however, as in the case of the Zhukovsky functions, the use of appropriate auxiliary mapping techniques enables us to transform even complicated flow regions into regular geometric shapes. Generally these figures will be polygons having a finite number of vertices (one or more of which may be at infinity). Thus the method of mapping a polygon from one or more planes onto the upper half of another planes is of particular importance.

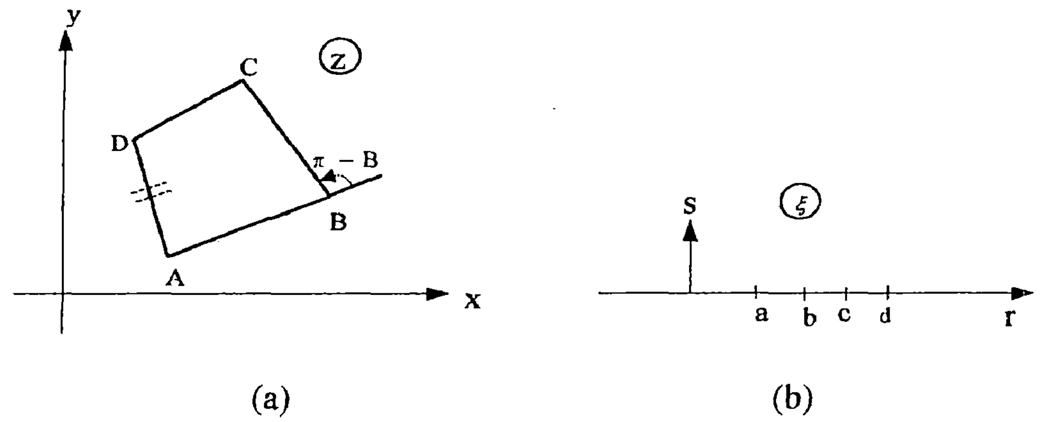

Fig. I.1 z and ξ Plane

If a polygon is located in the z plane, then the transformation that maps it conformally onto the upper half of the ξ plane ($\xi = r + is$) is

$$z = M\int \frac{d\xi}{(\xi - a)^{(1-A/\pi)}(\xi - b)^{(1-B/\pi)}\dots\dots} + N \tag{1.13}$$

where M and N are complex constants, A, B, ..., are the interior angles (in radians) of the polygon in the z plane (Fig. I.1a), and a, b, ... (a < b < ...) are points on the real axis of the ξ plane corresponding to the respective vertices A, B, ... (Fig. I.1b). We note, in particular, that the complex constant N corresponds to the point on the perimeter of the polygon that has its image at $\xi = 0$.

I.3.3 Zhukovsky Functions

A special mapping technique, of particular value when dealing with unconfined flow problems, make use of an auxiliary transformation called Zhukovsky's function.

Noting that relationship between the velocity potential and the pressure $[\phi = -k(p/\gamma_w + y)]$ can be written as $-kp/\gamma_w = \phi + ky$, if we defined as $\theta_1 = -kp/\gamma_w$, then

$$\theta 1 = \phi + ky \tag{1.14}$$

θ_1 is seen to be an harmonic function of x and y as $\nabla^2\theta_1 = \nabla^2\phi \equiv 0$. Hence, its conjugate is the function

$$\theta_2 = \psi - kx \tag{1.15}$$

Defining $\theta = \theta_1 + \theta_2$, we observe that

$$\theta = \theta_1 + \theta_2 = w - ikz \tag{1.16}$$

where $w = \phi + i\psi$, and $z = x + iy$

Definition (1.16) and any function with its real or imaginary part differing from it by a constant multiplier is called a Zhukovsky function.

I.4 OBJECTIVES OF THE STUDY

In the light of the status of the studies on the seepage from a partially penetrating stream having finite width, the objectives of the present study are :

1. Computation rate of seepage from a stream through derivation of reach transmissivity for various depth of penetration and width of the stream.
2. Study of substitute length and its application for unsteady seepage condition.
3. Study of distribution of seepage through stream bed and stream bank.

The following assumptions have been made in study:

i. The flow is two dimensional,
ii. The river forms the boundary of a single layer of aquifer,
iii. Symmetrical aquifer conditions on either sides of the aquifer,

iv. The soil is homogeneous and isotropic,

v. The stream of finite width partially penetrates the aquifer.

I.5 ORGANIZATION OF THE DISSERTATION

The presentation of the studies has been organized as follows :

In chapter 1, a general introduction to the seepage from a partially penetrating stream in single aquifer has been presented. It includes the subject matters on two dimensional flow and conformal mapping. The objectives of the study have been identified here.

Chapter 2 deals with the pertinent review of literature. It includes the subject matters on reach transmissivity, river resistance and substitute length.

In chapter 3, analytical solution for seepage from a partially penetrating stream to confined aquifer having finite width and rectangular shape with semi infinite aquifers on either sides has been obtained. Results for various of stream width and depth of penetration are presented.

In chapter 4, analytical solution for seepage from a partially penetrating stream to confined aquifer having finite width and trapezoidal shape with finite aquifer has been obtained. Results for various stream width and depth of penetration are presented.

In chapter 5, analytical solution for seepage from a partially penetrating stream to unconfined aquifer having finite width and trapezoidal shape with finite aquifer has been obtained. Results for various stream width and depth of penetration are presented.

In chapter 6, the important conclusions of the study have been summarized.

CHAPTER II
REVIEW OF LITERATURE

A literature review on reach transmissivity and substitute length has been made in this dissertation.

II.1 REACH TRANSMISSIVITY

It has been often assumed for a stream or a canal, which is hydraulically connected with an aquifer that, under steady state condition, the exchange flow rate between the stream and the aquifer is linearly dependent on the boundary potential difference causing flow (Aravin and Numerov 1965, Herbert 1970, Morel-Seytoux and Daly 1975, Besbes et al. 1978, Flug et al. 1980). Bouwer (1969) has reported that the seepage from a canal to a shallow unconfined aquifer is directly proportional to the difference in the water levels in the canal and in the aquifer in the vicinity of the canal. The constant of proportionality, which has been designated as reach transmissivity (Morel-Seytoux and Daly, 1975) depends on the hydraulic conductivity and stream cross section (Bouwer 1969). Considering an average flow path and an average flow area and using Darcy's law, Morel-Seytoux et al. (1979) have derived the following approximate expression for seepage from a partially penetrating stream of finite width in an unconfined aquifer :

$$Q = L_r k \frac{0.5w_p + e}{5w_p + 0.5e} \Delta h = \Gamma_r \Delta h \tag{2.1}$$

in which Q = seepage through a reach of the stream of length L_r, k = hydraulic conductivity, w_p = the wetted perimeter of the stream, e = the saturated thickness of the aquifer below the stream bed, and $\Delta h = (h_1 - h_B)$ = the difference in hydraulic heads in the stream reach and in an observation well which is located at a distance of $5w_p$ from the center of the stream reach and Γ_r = the reach transmissivity. It is implied that the reach transmissivity constant would vary with distance of the observation well from the stream.

Using Darcy's law for radial flow, Herbert (1970) has derived an approximate expression relating influent seepage from a partial penetrating stream with the potential difference between the stream and the aquifer below the stream bed at half the thickness of aquifer from which the following expression for reach transmissivity for unit length of a stream can be found :

$$\Gamma_r = \frac{\pi k}{ln\left(\frac{0.5(e+h_m)}{R}\right)} \tag{2.2}$$

were e = saturated thickness of the aquifer below the bed of the stream, h_m = maximum depth of water in the stream, R = radius of the equivalent semicircular section of the stream equal to w_p/π, w_p = wetted perimeter of the stream. From the logarithm relation, it is obvious that the relation is valid for $(e+h_m)/2 > R$.

The reach transmissivity parameter could be known from the expressions relating seepage with boundary potential difference derived by several investigators for different stream aquifer geometry (Numerov 1954, Bouwer 1969, Halek and Svec 1979). The various formula derived by different investigators for computation of seepage and reach transmissivity are presented in detail in appendix A.

There have been evidences that the process of stream aquifer interaction can be non-linear (Rushton and Redshaw 1979, Dillon 1983, 1984). Considering the fact that influent seepage from a canal (or a stream) is zero for zero potential difference and a finite quantity for infinite potential difference, the relationship between influent seepage and potential difference has to be non-linear in case of unconfined flow. Only in case of steady and confined flow, the relation between seepage and potential difference causing the flow can be linear.

The reach transmissivity constant which depends on the location of piezometer, in case of a partially penetrating stream of large width has been derived by Mishra (2001). A stream having a width less than five times the thickness of the aquifer under its bed can be considered to have finite width. In many ground water basins such a stream forms the hydrologic boundary of the flow domain. In this dissertation, using conformal mapping, an analytical expression for seepage from a partially penetrating stream of finite width, in a homogeneous, isotropic, and confined aquifer is derived from which the pertinent reach transmissivity parameter is obtained.

II.2 SUBSTITUTE LENGTH

The resistance of the flow domain of a partially penetrating stream of finite width up to a distance L_B from the stream bank can be decomposed into (i) the resistance of the aquifer for length L_B for rectilinear flow and (ii) an extra resistance component due to extension of the flow path resulting from curvilinear flow near the stream. The extra resistance is unevenly distributed in the aquifer. An approximate theoretical method known as the additional seepage resistance method was originally proposed by Numerov (1953) for solving complex seepage problem. Strelsova (1974) has applied the method to analyze flow to a multiple well system from a line source. In this method the distributed extra resistance is lumped at the stream bank by appending an extra length of aquifer, known as substitute length, whose resistance for rectilinear flow is equal to the extra resistance.

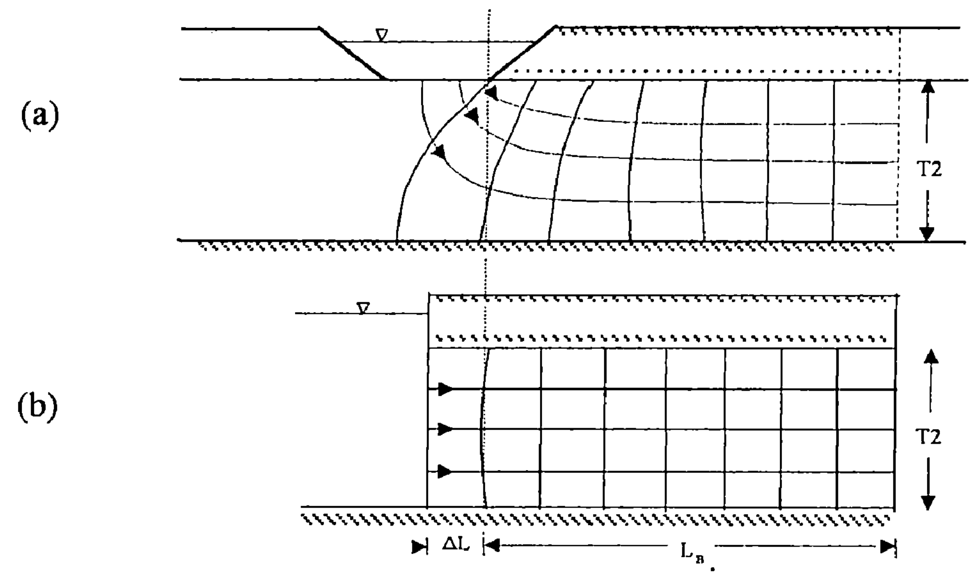

Fig. II.1 Principle of the method of substitute lengths

The fragment shown in Fig.II.1a contains a flow region near an influent reservoir. The water flows laterally into a collector system. The seepage is described by a curvilinear flow net which can be mapped conformally into a rectilinear net (Fig. II.1b). Near the dividing line between the fragments, the curvilinear net is almost rectilinear, and if we choose a suitable transformation, its shape will experience practically no change. The transformed flow region is characterized by the fact that its inlet profile is vertical and hence its shape is different from that of the original region. The main difference is seen to be in its length which has increased relative to the boundary of the original reservoir. The difference ΔL between the increased and the original length is called the substitute length. Considering electrical analogy, the resistance of the whole fragment is readily found from the relation

$$R_a = \frac{L_B + \Delta L}{T_2} \tag{2.3}$$

Using conformal mapping Numerov (1953) has analyzed the two dimensional seepage into a partially penetrating open channel having finite width draining water from either sides of a confined aquifer. A partially penetrating stream with infinite width is a particular case for which the substitute length can be obtained from the results presented by Numerov. The substitute length is derived here independently from the conformal mapping solution using electrical analogy for a partially penetrating stream of finite width.

II.3 UNSTEADY STATE FLOW FROM PARTIALLY PENETRATING STREAM

Let us consider a stream that partially penetrates a homogeneous and isotropic confined aquifer of semi infinite area extent (Fig. II.2a). By introducing substitute length, ΔL, the partially penetrating stream converts to fully penetrating stream (Fig. II.2.b). Initially, the stream and the aquifer are assumed to be at rest in which the piezometric surface of the aquifer and the stream are at the same level. Let the stream-stage be suddenly increased by σ and maintained at the new level. The partial differential equation governing the transient flow of water in the aquifer is

$$\frac{\partial h}{\partial t} = \beta \frac{\partial^2 h}{\partial x^2} \tag{2.4}$$

The initial and boundary condition are :

$$h(x, 0) = 0 \tag{2.5}$$

$$h(0, t) = \sigma \text{ and } h(\infty, t) = 0 \tag{2.6}$$

where h = h(x, t) = piezometric head in the aquifer measured from the initial piezometric surface, x = distance measured from the stream bank, β = hydraulic diffusivity of the aquifer, (L^2T^{-1}), σ = step rise in the stream stage and t = time since the step rise.

The above partial differential equation is a good approximation for an unconfined aquifer if changes in the water table are small in comparison to the average saturated depth of flow (Cooper and Rorabaugh 1963). The solution of equation (2.4) satisfying the initial and boundary conditions, has been given by Carslaw and Jaeger (1959) for an analogous heat conduction problem which is

(a)

(b)

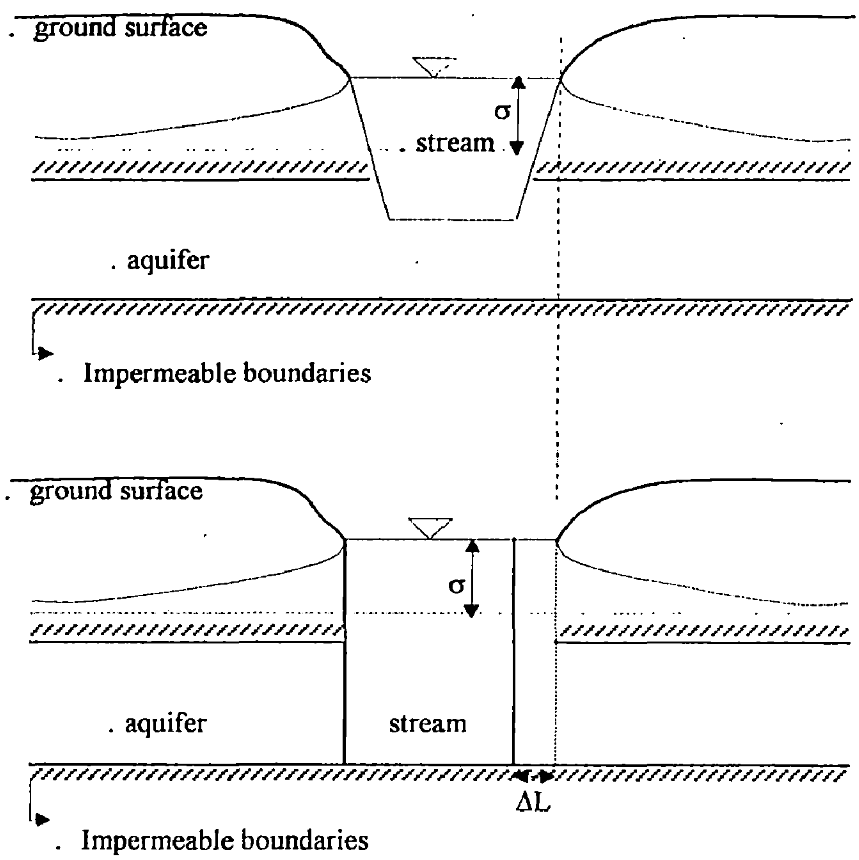

Fig. II.2 Schematic cross-section of a partially penetrating stream, converted to a fully penetrating stream

$$h = \sigma \operatorname{erfc} \frac{x}{2\sqrt{\beta t}} \tag{2.7}$$

where erfc(.) = 1 – erf(.) = complementary error function. The error function is expressed as

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du \tag{2.8}$$

A partially penetrating stream can be replaced by a fully penetrating stream by introducing substitute length. Carslaw and Jaeger solution can be applied conveniently to analyze unsteady flow from partially penetrating stream.

CHAPTER III
SEEPAGE FROM A RECTANGULAR STREAM IN A SEMI INFINITE AQUIFER

III.1 GENERAL

The section of a partially penetrating stream can be conveniently assumed as triangular, rectangular or trapezoidal for computation of seepage by analytical method. Trapezoidal section is adopted for canals conveying large discharge. For small distributory, the *Mehboob* section adopted in India can be assumed to be triangular. The mathematical complexity for computation of seepage is least for triangular section.

Approximate solution for computation of influent seepage to a partially penetrating stream having rectangular section in an unconfined aquifer has been derived by Aravin (1965). The flow domain has been decomposed into two regions; one region above the bed level and the other one below the bed level. The flow domain below the bed level has been treated as a confined flow domain and conformal mapping has been applied to compute influent seepage through bed. Dupuit Farcheimer assumptions have been used to compute part of influent seepage above bed level. Stretslove has analyzed seepage from a rectangular canal partially penetrating a confined aquifer. It has been assumed that prior to seepage water was flowing from $-\infty$ to $+\infty$.

Herbert, has considered a stream with semi circular cross section partially penetrating a confined aquifer, has derived the expression of seepage in terms of stream geometry, hydraulic conductivity and potential difference between the stream and below the stream at half depth of aquifer below the bed.

If a solution is obtained for treating the aquifer as infinite, the seepage can be computed only if the piezometric surface is measured at a piezometer near the stream. The seepage is equal to $q = \Gamma.\Delta h$; in which Δh is the potential difference coursing flow and Γ is the reach transmissivity constant. Approximate value of reach transmissivity can be obtained from the formulae given by several investigator for computation of seepage.

The reach transmissivity constant for a river of large width (width more than 5 times depth of aquifer below the river bed) has been derived by Mishra (2001) using conformal mapping.

In this chapter the analysis of steady seepage from a partially penetrating stream having finite width in a confined aquifer has been derived using conformal mapping. The study helps in checking the validity of Herbert's formula. Also the reach transmissivity for stream having finite width has been obtained.

III.2. STATEMENT OF THE PROBLEM

A partially penetrating rectangular stream in confined aquifer is shown in Fig. III.1. The flow is steady and symmetrical on either side of the stream. T_1 is thickness of the aquifer below the stream bed, T_2 is thickness of aquifer and B is half width of the stream. A piezometer is located at a distance L_B from the bank. The potential difference Δh is measured. It is aimed to find the seepage and quantify the reach transmissivity constant as a function of T_1/T_2, B/T_2, L_B/T_2 and k.

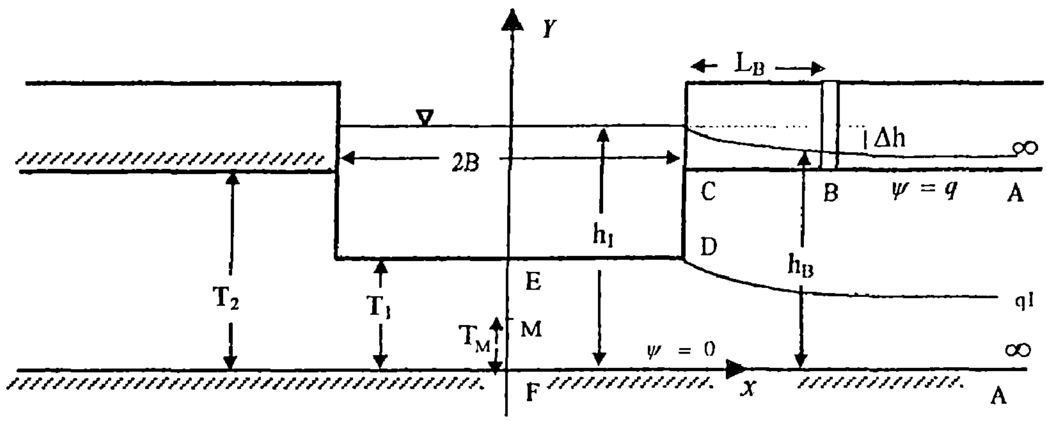

Fig. III.1 Physical flow domain in z-plane, z=x+iy

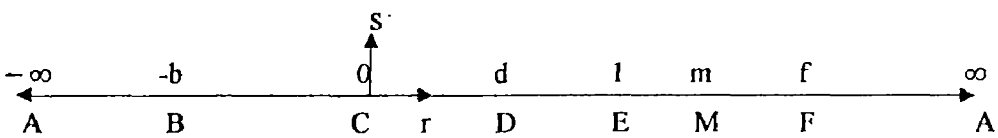

Fig.III.2 ξ-plane (ξ=r+is)

III.3 ANALYSIS

III.3.1 Mapping of The Physical Flow Domain in Z-Plane to An Auxiliary ξ-Plane

The vertices A, C, D, E and F in z plane (Fig. III.1) having been mapped onto points -∞, 0, d, 1 and f respectively of the ξ-plane (Fig. III.2). The conformal mapping of the flow domain in z plain onto the lower half an auxiliary ξ plane according Schwarz-Christoffel transformation is given by:

$$dz = M\frac{(\xi - d)^{1/2} d\xi}{\xi^{1/2}(\xi - f)^{1/2}(\xi - 1)^{1/2}} \tag{3.1}$$

Substituting $\xi = Re^{i0}$, $d\xi = iRe^{i0}\, d\theta$ and applying the condition that as one transverses in ξ-plane from $\theta = 0$ to $\theta = \pi$ along a semi circle of radius R, $R\to\infty$, the jump in z-plane = iT_2

$$iT_2 = M\int_0^{\pi}\frac{(Re^{i\theta} - d)^{\alpha}\, iRe^{i\theta}}{(Re^{i0})^{1/2}(Re^{i0} - f)^{1/2}(Re^{i0} - 1)^{1/2}} d0$$

$$Lt.R \to \infty$$

or

$$iT_2 = M\int_0^{\pi}\frac{i(Re^{i\theta})^{3/2}\left(1 - \frac{d}{Re^{i\theta}}\right)^{\alpha}}{(Re^{i0})^{3/2}\left(1 - \frac{f}{Re^{i0}}\right)^{1/2}\left(1 - \frac{1}{Re^{i0}}\right)^{1/2}} d\theta \tag{3.2}$$

$$Lt.R \to \infty$$

The constant M is found to be

$$M = \frac{T_2}{\pi} \tag{3.3}$$

The parameter of 'd' and 'f' are found as follows :

For $0 \le \xi' \le d$, z is given by:

$$z = \frac{T_2}{\pi}\int_0^{\xi'}\frac{(\xi - d)^{1/2}}{\xi^{1/2}(\xi - f)^{1/2}(\xi - 1)^{1/2}} d\xi + B + iT_2 \tag{3.4}$$

For point D, $\xi' = d$ and $Z_D = B + iT_1$; hence,

$$B + iT_1 = \frac{T_2}{\pi}\int_0^d \frac{(\xi - d)^{1/2}}{\xi^{1/2}(\xi - f)^{1/2}(\xi - 1)^{1/2}} d\xi + B + iT_2 \qquad \text{or}$$

$$\frac{\pi (T_2 - T_1)}{T_2} = \int_0^d \frac{(d - \xi)^{1/2}}{\xi^{1/2}(f - \xi)^{1/2}(1 - \xi)^{1/2}} d\xi \tag{3.5}$$

Substituting $\xi = v^2$, $d\xi = 2v\,dv$, at the lower limit $\xi = 0$, $v = 0$ and at the upper limit $\xi = d$, $v = \sqrt{d}$, where v is a dummy variable, the improper integral above is converted to the following proper integral:

$$\frac{\pi (T_2 - T_1)}{T_2} = 2\int_0^{\sqrt{d}} \frac{(d - v^2)^{1/2}}{(f - v^2)^{1/2}(1 - v^2)^{1/2}} dv \tag{3.6}$$

Substituting :

$v = \sqrt{d}\,\frac{(1+\chi)}{2}$ and $dv = \frac{\sqrt{d}}{2} d\chi$

where χ is a dummy variable, the lower and upper limit of integration above are converted to –1 and 1 respectively, and equation (3.6) reduces to

$$\frac{\pi (T_2 - T_1)}{T_2} = \sqrt{d}\int_{-1}^{1} \frac{\left[d - d\left(\frac{1+\chi}{2}\right)^2\right]^{1/2}}{\sqrt{\left[f - d\left(\frac{1+\chi}{2}\right)^2\right]\left[1 - d\left(\frac{1+\chi}{2}\right)^2\right]}} d\chi \tag{3.7}$$

For $d \le \xi' \le 1$, z is given by :

$$z = \frac{T_2}{\pi}\int_d^{\xi'} \frac{(\xi - d)^{1/2}}{\xi^{1/2}(\xi - f)^{1/2}(\xi - 1)^{1/2}} d\xi + B + iT_1 \tag{3.8}$$

For point E, $\xi' = 1$ and $Z_E = iT_1$; hence,

$$iT_1 = \frac{T_2}{\pi}\int_d^{1} \frac{(\xi - d)^{1/2}}{\xi^{1/2}(\xi - f)^{1/2}(\xi - 1)^{1/2}} d\xi + B + iT_1$$

$$B = \frac{T_2}{\pi}\int_d^{1} \frac{(\xi - d)^{1/2}}{\xi^{1/2}(f - \xi)^{1/2}(1 - \xi)^{1/2}} d\xi \tag{3.9}$$

Substituting $1-\xi = v^2$, $d\xi = -2v\,dv$, at the lower limit $\xi = d$, $v = \sqrt{(1-d)}$, and at the upper limit $\xi = 1$, $v = 0$, where v is a dummy variable, the improper integral above is converted to the following proper integral:

$$B = 2\frac{T_2}{\pi}\int_0^{\sqrt{1-d}}\frac{(1-v^2-d)^{1/2}}{(1-v^2)^{1/2}(f-1+v^2)^{1/2}}dv \qquad (3.10)$$

Substituting :

$v = \sqrt{1-d}\,\dfrac{1+\chi}{2}$ and $dv = \dfrac{\sqrt{1-d}}{2}d\chi$

where χ is a dummy variable, the lower and upper limits of integrals above are converted to -1 and 1 respectively, and equation (3.10) reduces to

$$\frac{B\pi}{T_2} = \sqrt{1-d}\int_{-1}^{1}\frac{\left[1-(1-d)\left(\frac{1+\chi}{2}\right)^2-d\right]^{1/2}}{\left[1-(1-d)\left(\frac{1+\chi}{2}\right)^2\right]^{1/2}\sqrt{f-1+(1-d)\left(\frac{1+\chi}{2}\right)^2}}d\chi \qquad (3.11)$$

The integration appearing in equations (3.7) and (3.11) are carried out numerically applying Gauss–quadrature formula. For a given value of B, T_1 and T_2, the parameters d and f are obtained by an iterative procedure. The programming in C++ has been developed to obtain these parameters.

Consider a piezometer at point B at a distance L_B from the stream bank. For $-\infty \le \xi' \le 0$, the relationship between z and ξ' is given by :

$$z = \frac{T_2}{\pi}\int_0^{\xi'}\frac{(\xi-d)^{1/2}}{\xi^{1/2}(\xi-f)^{1/2}(\xi-1)^{1/2}}d\xi + B + iT_2 \qquad (3.12)$$

For point B, $\xi' = -b$ and $Z_B = B+iT_2+L_B$; hence,

$$L_B = -\frac{T_2}{\pi}\int_0^{-b}\frac{(d-\xi)^{1/2}}{(-\xi)^{1/2}(f-\xi)^{1/2}(1-\xi)^{1/2}}d\xi \qquad (3.13)$$

Substituting $\xi = -u$, $d\xi = -du$, at the lower limit $\xi = 0$, $u = 0$ and at the upper limit $\xi = -b$, $u = b$

$$L_B = \frac{T_2}{\pi}\int_0^{b}\frac{(d+u)^{1/2}}{u^{1/2}(f+u)^{1/2}(1+u)^{1/2}}du \qquad (3.14)$$

Substituting $u = v^2$, $du = 2v\,dv$, at $u = 0$, $v = 0$ and at $u = b$, $v = \sqrt{b}$, where v is a dummy variable, the improper integral above is converted to the following proper integral :

$$L_B = 2\frac{T_2}{\pi}\int_0^{\sqrt{b}}\frac{(d+v^2)^{1/2}}{(f+v^2)^{1/2}(1+v^2)^{1/2}}dv \qquad (3.15)$$

Substituting

$$v = \sqrt{b}\,\frac{(1+\chi)}{2} \quad \text{and} \quad dv = \frac{\sqrt{b}}{2}\,d\chi$$

where χ is a dummy variable, the lower and upper limits of integral above are converted to –1 and 1 respectively, and equation (3.15) reduces to

$$\frac{L_B\pi}{T_2} = \sqrt{b}\int_{-1}^{1}\frac{\left[d+b\left(\frac{1+\chi}{2}\right)^2\right]^{1/2}}{\sqrt{f+b\left(\frac{1+\chi}{2}\right)^2}\sqrt{1+b\left(\frac{1+\chi}{2}\right)^2}}\,d\chi \tag{3.16}$$

The above integration is carried out numerically applying Gauss-quadrature formula. For a given value of L_B, the parameter b is obtained by an iterative procedure.

Consider a piezometer at point M at a distance T_M from the bottom of the aquifer. The parameter ξ lies in the range from 1 to f. For $1 \le \xi' \le f$, the relationship between z and ξ' is given by :

$$z = \frac{T_2}{\pi}\int_{1}^{\xi'}\frac{(\xi-d)^{1/2}}{\xi^{1/2}(\xi-f)^{1/2}(\xi-1)^{1/2}}\,d\xi + iT_1 \tag{3.17}$$

For point M, $\xi' = m$ and $z_M = iT_M$; hence,

$$-i(T_1 - T_M) = \frac{T_2}{\sqrt{(-1)}\pi}\int_{1}^{m}\frac{(\xi-d)^{1/2}}{\xi^{1/2}(f-\xi)^{1/2}(\xi-1)^{1/2}}\,d\xi$$

$$\frac{(T_1 - T_m)\pi}{T_2} = \int_{1}^{m}\frac{(\xi-d)^{1/2}}{\xi^{1/2}(f-\xi)^{1/2}(\xi-1)^{1/2}}\,d\xi \tag{3.18}$$

The above improper integral is converted to proper integral by removing the singularity at ξ=1. Besides, to improve the accuracy in numerical integration the range 1 to m is divided into two parts 1 to (1+m)/2 and (1+m)/2 to m

$$\frac{(T_1 - T_m)\pi}{T_2} = \int_{1}^{\frac{1+m}{2}}\frac{(\xi-d)^{1/2}}{\xi^{1/2}(f-\xi)^{1/2}(\xi-1)^{1/2}}\,d\xi + \int_{\frac{1+m}{2}}^{m}\frac{(\xi-d)^{1/2}}{\xi^{1/2}(f-\xi)^{1/2}(\xi-1)^{1/2}}\,d\xi \tag{3.19}$$

Substituting ξ-1 = v^2, $d\xi = 2v\,dv$, for first integral above, at the lower limit ξ = 1, v = 0 and at the upper limit ξ = (1+m)/2, $v = \sqrt{\{[m-1]/2\}}$ and substituting also $f - \xi = v^2$,

$d\xi = -2v\,dv$, for second integral, at the lower limit $\xi = (1+m)/2$, $v = \sqrt{\{[2f-m-1]/2\}}$ and at the upper limit $\xi = m$, $v = \sqrt{(f-m)}$. Where v is a dummy variable, the improper integral above is converted to the following proper integral :

$$\frac{(T_1 - T_m)\pi}{T_2} = 2\int_0^{\sqrt{\frac{m-1}{2}}} \frac{(v^2+1-d)^{1/2}}{(v^2+1)^{1/2}(f-v^2-1)^{1/2}}dv + 2\int_{\sqrt{f-m}}^{\sqrt{\frac{2f-m-1}{2}}} \frac{(f-v^2-d)^{1/2}}{(f-v^2)^{1/2}(f-v^2-1)^{1/2}}dv \quad (3.20)$$

Making further substitution

$$v = \sqrt{\frac{m-1}{2}}\left(\frac{1+\chi}{2}\right) = f_1(\chi);\quad dv = \sqrt{\frac{m-1}{2}}\frac{d\chi}{2}\quad \text{for the first integral}$$

and

$$v = \frac{\sqrt{\frac{2f-m-1}{2}} - \sqrt{f-m}}{2}\chi + \frac{\sqrt{\frac{2f-m-1}{2}} + \sqrt{f-m}}{2} = f_2(\chi);$$

$$dv = \frac{\sqrt{\frac{2f-m-1}{2}} - \sqrt{f-m}}{2}d\chi \quad \text{for the second integral above}$$

where χ is a dummy variable, the lower and upper limits of above integration are converted to –1 and 1 respectively, and equation (3.20) reduces to :

$$\frac{(T_1 - T_M)\pi}{T_2} = \sqrt{\frac{m-1}{2}}\int_{-1}^{1}\frac{\sqrt{\{f_1^2(\chi)+1-d\}}}{\sqrt{\{f_1^2(\chi)+1\}\{f-f_1^2(\chi)-1\}}}d\chi +$$

$$\left(\sqrt{\frac{2f-m-1}{2}} - \sqrt{f-m}\right)\int_{-1}^{1}\frac{\sqrt{\{f-f_2^2(\chi)-d\}}}{\sqrt{\{f-f_2^2(\chi)\}\{f-f_2^2(\chi)-1\}}}d\chi \quad (3.21)$$

The above integration is carried out numerically applying Gauss-quadrature formula. For a given value of T_M, the parameter m is obtained by an iterative procedure.

III.3.2 Mapping of The Complex Potential w Plane to The Auxiliary ξ Plane

The complex potential w corresponding to the flow domain is shown in Fig. III.3. $w = \phi + i\psi$, where ψ is the stream function and ϕ is the velocity potential function, defined as $\phi = -k(p/\gamma_w + y) + c$. Constant c has been assumed to be zero.

The conformal mapping of the w-plane onto the lower half of the ξ-plane is given by :

$$\frac{dw}{d\xi}=\frac{M}{\xi^{1/2}(\xi-1)^{1/2}} \quad (3.22)$$

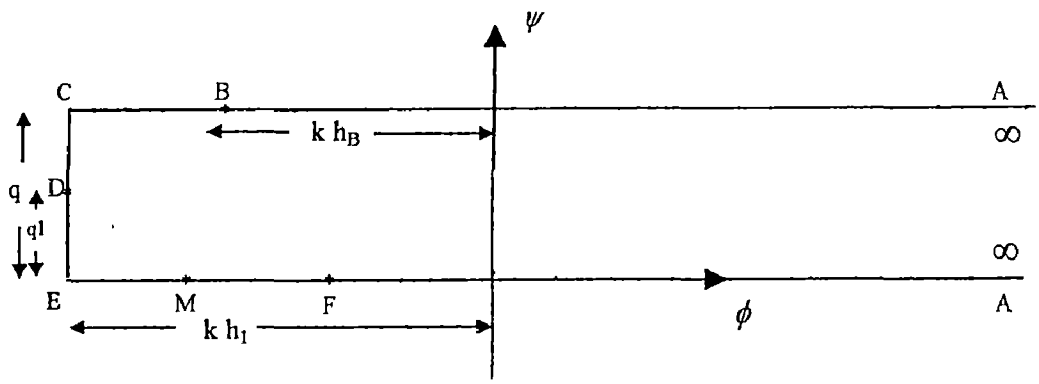

Fig. III.3 w-plane (w=φ+iψ)

For $0 \le \xi' \le 1$, the corresponding w is given by :

$$w=\frac{M}{i}\int_0^{\xi'}\xi^{(1/2-1)}(1-\xi)^{(1/2-1)}d\xi-kh_1+iq$$

$$=\frac{M}{i}B_{\xi'}(1/2,1/2)\quad -kh_1+iq \quad (3.23)$$

in which $B_{\xi'}(m,n)$ is incomplete Beta function. For point E, $\xi'=1$ and $w=-kh_1$, hence

$$-kh_1 = M\,B_{\xi'(1/2,1/2)}/i - kh_1 + iq \quad (3.24a)$$

in which $B_{(1/2,1/2)}$ is complete beta function, hence,

$$M=\frac{q}{\pi} \quad (3.24b)$$

For $-b \le \xi' \le 0$, the corresponding w is given by

$$w=\frac{q}{\pi}\int_0^{\xi'}\frac{d\xi}{\xi^{1/2}(\xi-1)^{1/2}}-kh_1+iq \quad (3.25)$$

For point B, $\xi'=-b$ and $w=-kh_B+iq$; hence,

$$-kh_B+iq=\frac{q}{\pi}\int_0^{-b}\frac{d\xi}{\xi^{1/2}(\xi-1)^{1/2}}-kh_1+iq \quad (3.26)$$

Substituting ξ = -v, dξ = -dv, at ξ = 0, v = 0 and at ξ = -b, v = b, and re-arranging the equation (3.26)

$$k(h_l - h_B) = \frac{q}{\pi}\int_0^b \frac{dv}{v^{1/2}(1+v)^{1/2}} \tag{3.27}$$

Substituting 1+v = u^2, dv = 2u du, at v = 0, u = 1 and at v = b, u = √(1+b), where v is a dummy variable, the improper integral above is converted to the following proper integral

$$k(h_l - h_B) = \frac{2q}{\pi}\int_1^{\sqrt{1+b}} \frac{du}{\sqrt{u^2-1}} \quad k(h_l - h_B) = \frac{2q}{\pi}\ln\left[u + \sqrt{u^2-1}\right]_1^{\sqrt{1+b}} \tag{3.28}$$

Hence,

$$q = \frac{\pi k(h_l - h_B)}{2\ln\left(\sqrt{1+b} + \sqrt{b}\right)} \tag{3.29}$$

in which h_l is head in the stream and h_B is piezometric head at a distance L_B from the stream bank and q is rate of seepage for half section of the stream.

For domain E to F, i.e. 1 ≤ ξ' ≤ f, the corresponding w is given by

$$w = \frac{q}{\pi}\int_1^{\xi'} \frac{d\xi}{\xi^{1/2}(\xi-1)^{1/2}} - kh_l \tag{3.30}$$

For point M, ξ' = m and w = - kh_M; hence,

$$k(h_l - h_M) = \frac{q}{\pi}\int_1^m \frac{d\xi}{\xi^{1/2}(\xi-1)^{1/2}} \tag{3.31}$$

Substituting $\xi = v^2$, dξ = 2v dv, at ξ = 1, v = 1 and at ξ = m, v = √m, where v is a dummy variable, the integration leads to

$$k(h_l - h_M) = \frac{2q}{\pi}\int_1^{\sqrt{m}} \frac{dv}{\sqrt{v^2-1}} = \frac{2q}{\pi}\ln\left[v + \sqrt{v^2-1}\right]_1^{\sqrt{m}} \tag{3.32}$$

Hence,

$$q = \frac{\pi\, k(h_l - h_M)}{2\ln\left(\sqrt{m} + \sqrt{m-1}\right)} \tag{3.33}$$

in which h_M is the head at a point located at a distance T_M from the bottom of the aquifer and q is rate of seepage for half section of the stream.

For domain D to E, i.e. $1 \le \xi' \le d$, the corresponding w is given by :

$$w = \frac{q}{i\pi}\int_{1}^{\xi'} \frac{d\xi}{\xi^{1/2}(1-\xi)^{1/2}} - kh_1$$

$$w = \frac{-q}{i\pi}\int_{\xi'}^{1} \frac{d\xi}{\xi^{1/2}(1-\xi)^{1/2}} - kh_1 \tag{3.34}$$

$$w = \frac{-q}{i\pi}\left[\int_{0}^{1} \frac{d\xi}{\xi^{1/2}(1-\xi)^{1/2}} - \int_{0}^{\xi'} \frac{d\xi}{\xi^{1/2}(1-\xi)^{1/2}}\right] - kh_1$$

$$w = \frac{-q}{i\pi}\left[\pi - \beta_{\xi'}(1/2,1/2)\right] - kh_1 \tag{3.35}$$

For point D, $\xi' = d$ and $w = -kh_1 + i\psi$; hence,

$$\frac{\psi(\xi')}{q} = 1 - \frac{2}{\pi}\sin^{-1}\sqrt{\xi'} \tag{3.36}$$

$$\frac{q_1}{q} = 1 - \frac{2}{\pi}\sin^{-1}\sqrt{d} \tag{3.37}$$

in which q_1 is seepage through the stream bed for half section of the stream.

III.4 SUBSTITUTE LENGTH

The resistance of one half of the flow domain of a partially penetrating stream up to a distance L_B from the stream bank can be decomposed into (i) the resistance of the aquifer for length L_B for rectilinear flow and (ii) resistance pertaining to the curvilinear flow near the stream. The resistance pertaining to curve linear flow is unevenly distributed in the aquifer. An approximate theoretical method known as the additional seepage resistance method was originally proposed by Numerov (1953) for solving complex seepage problem. In this method the distributed extra resistance is lumped at the stream bank by appending an extra length of aquifer, known as substitute length, whose resistance for rectilinear flow is equal to the extra resistance. Using conformal mapping Numerov (1953) has analyzed the two dimensional seepage into a partially penetrating open channel having finite width draining water from either sides of a confined aquifer. Numerov has considered the case in which steady flow occurs from left side of the confined aquifer to the right side and a partially penetrating stream interferes the flow. The substitute length is derived here independently from the conformal mapping solution using electrical analogy. The flow is symmetrical on either side of the stream.

Let us consider the location of a piezometer at a distance L_B form the stream bank. The combined aquifer and stream resistance R_r, up to length L_B from (3.29) is given by :

$$R_r = \frac{2\ln\left(\sqrt{1+b}+\sqrt{b}\right)}{\pi k} \tag{3.38}$$

Let ΔL be the extra length, whose resistance is equal to the extra resistance owing to flow convergence within length L_B. For uniform rectilinear flow, the aquifer resistance R_a of length $L_B+\Delta L$ is

$$R_a = \frac{L_B + \Delta L}{kT_2} \tag{3.39}$$

Since $R_r = R_a$, we get

$$\frac{\Delta L}{T_2} = \frac{2\ln\left(\sqrt{1+b}+\sqrt{b}\right)}{\pi} - \frac{L_B}{T_2} \tag{3.40}$$

The limiting value of ΔL, $L_B \rightarrow \infty$, is the substitute length.

The substitute length is a measure of stream resistance to flow. The various of substitute length with distance from the stream bank for different width of the stream are presented in Fig. 3.10a through 3.10f. Since substitute length pertains to the curve linear flow near the stream bed and bank, and flow paths are extended only within a limited distance in the aquifer, $\Delta L/T_2$ converges to a finite value as L_B/T_2 increases. With increasing depth of penetration the curve linear flow tends to linear flow. Therefore stream having higher depth of penetration will have lower substitute length. The stream resistance is higher for stream having less width. Therefore as B/T_2 increases, the substitute length decreases.

III.5 UNSTEADY STATE FLOW

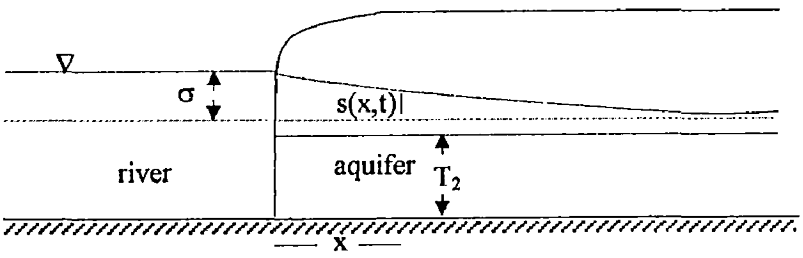

Fig. III.4 Step rise in a river

The substitute length can be used to convert a partially penetrating stream into a fully penetrating one by appending the substitute length to the aquifer at the interface of the stream and aquifer. The solution of unsteady flow from fully penetrating stream to aquifer derived earlier by Carslaw and Jaeger for an analogous heat conduction problem can be conveniently used. The solution for unsteady flow from a partially penetrating stream is derived in the following paragraphs.

Let us consider a step rise in the stream stage σ, (Fig. III.4). The rise at a distance x from the stream bank at a time t after onset of change in stream stage is given by:

$$s(x,t) = \sigma\left[1 - \text{erf}\left(\frac{x}{\sqrt{4\beta t}}\right)\right] \tag{3.41}$$

in which

The error function $\text{erf}(X) = \dfrac{2}{\sqrt{\pi}}\displaystyle\int_0^X e^{-v^2}\,dv$

β = hydraulic diffusivity = T/Φ

Φ = storativity

T = transmissivity = kT_2

k = hydraulic conductivity

The hydraulic gradient is given by:

$$\frac{\partial s}{\partial x} = \frac{\partial}{\partial x}\left\{\sigma\left[1 - \frac{2}{\sqrt{\pi}}\int_0^{\frac{x}{\sqrt{4\beta t}}} e^{-u^2}\,du\right]\right\} \tag{3.42}$$

Multiplication of hydraulic gradient and coefficient of permeability k, gives the Darcy velocity

$$v_x = (-k)\left\{\frac{-2}{\sqrt{\pi}}\,\sigma\, e^{-\frac{x^2}{\sqrt{4\beta t}}}\,\frac{1}{\sqrt{4\beta\, t}}\right\} \tag{3.43}$$

Multiplication of hydraulic gradient and transmissivity T, gives the rate of flow at section x in the aquifer

$$Q_x = T\frac{2}{\sqrt{\pi}}\sigma e^{-\frac{x^2}{\sqrt{4\beta t}}}\frac{1}{\sqrt{4\beta\, t}} \tag{3.44}$$

At a point x = 0, i.e. interface between the river and the aquifer, the rate of flow is

$$Q_0(t) = T\frac{2}{\sqrt{\pi}}\sigma\frac{1}{\sqrt{4\beta\, t}} \tag{3.45}$$

Let the step rise σ be equal to 1 and the corresponding flow be designated as $K_{qs}(t)$

$$K_{qs}(t) = \sqrt{\frac{T\Phi}{\pi}}\,\frac{1}{\sqrt{t}} \tag{3.46}$$

Let the change in stream stage follow a ramp instead of a step i.e. let the step rise linearly from zero at t = 0 and attain a unit height at t = Δt after which let the stream stage remain unchanged (Fig. III.5)

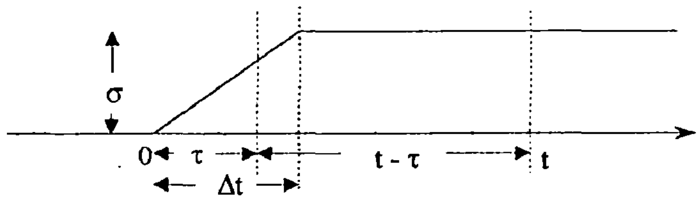

Fig. III.5 One rump rise

The response of an aquifer to a ramp perturbation, $\delta_{q\gamma}(t)$ can be derived from the response to a unit step perturbation using convolution technique.

$$\delta_{q\gamma}(t) = \int_0^t \frac{d\sigma}{d\tau}K_{qs}(t-\tau)d\tau$$

$$= \int_0^{\Delta t}\frac{d\sigma}{d\tau}\sqrt{\frac{T\Phi}{\pi}}\frac{1}{\sqrt{t-\tau}}d\tau + \int_{\Delta t}^{\infty}\frac{d\sigma}{d\tau}\sqrt{\frac{T\Phi}{\pi}}\frac{1}{\sqrt{t-\tau}}d\tau \tag{3.47}$$

Beyond time Δt, dσ/dτ = 0, and within Δt dσ/dτ is constant and is equal to 1/Δt. Let t = nΔt, where n is an integer

$$\delta_{q\gamma}(n,\Delta t) = \frac{1}{\Delta t}\sqrt{\frac{T\Phi}{\pi}}\int_0^{\Delta t}\frac{d\tau}{\sqrt{n\Delta t-\tau}} \tag{3.48}$$

Integrating

$$\delta_{q\gamma}(n,\Delta t)=\frac{1}{\Delta t}\sqrt{\frac{T\Phi}{\pi}}\int_0^{\Delta t}\frac{d\tau}{\sqrt{n\Delta t-\tau}}$$

$$=\frac{1}{\Delta t}\sqrt{\frac{T\Phi}{\pi}}\left[-2\sqrt{n\Delta t-\tau}\right]_0^{\Delta t}$$

$$=\frac{1}{\Delta t}\sqrt{\frac{T\Phi}{\pi}}\left[-2\sqrt{n\Delta t-\Delta t}+2\sqrt{n\Delta t}\right]$$

$$=\sqrt{\frac{T\Phi}{\pi\,\Delta t}}2\left[\sqrt{n}-\sqrt{n-1}\right]$$

$$\delta(n,\ \Delta t)=2\sqrt{\frac{T\Phi}{\Delta t\pi}}\left[\sqrt{n}-\sqrt{n-1}\right] \tag{3.49}$$

For variable stream stage the return flow at the end of n^{th} time step is given by

$$Q_\gamma(n\Delta t)=\int_0^t\frac{d\sigma}{d\tau}\sqrt{\frac{T\Phi}{\pi}}\frac{d\tau}{\sqrt{t-\tau}} \tag{3.50}$$

Discretising the time domain into n steps and assuming that within each time step dσ/dτ remains constant but changes from time step to step

$$Q_\gamma(n,\Delta t)=\int_0^{\Delta t}\left\{\frac{\sigma(\Delta t)-\sigma(0)}{\Delta t}\right\}\sqrt{\frac{T\Phi}{\pi}}\frac{d\tau}{\sqrt{n\Delta t-\tau}}$$

$$+\ldots\ldots\ldots+\int_{(\gamma-1)\Delta t}^{\gamma\Delta t}\frac{\sigma(\gamma\,\Delta t)-\sigma((\gamma-1)\Delta t)}{\Delta t}\sqrt{\frac{T\Phi}{\pi}}\frac{d\tau}{\sqrt{n\Delta t-\tau}}$$

$$+\ldots\ldots\ldots+\int_{(n-1)\Delta t}^{n\Delta t}\frac{\sigma(n\Delta t)-\sigma((n-1)\Delta t)}{\Delta t}\sqrt{\frac{T\Phi}{\pi}}\frac{d\tau}{\sqrt{n\Delta t-\tau}} \tag{3.51}$$

Substituting $\tau=u+(\gamma-1)\Delta t$ and $u=\tau-(\gamma-1)\Delta t$

$$\frac{1}{\Delta t}\sqrt{\frac{T\Phi}{\pi}}\int_{(\gamma-1)\Delta t}^{\gamma\Delta t}\frac{d\tau}{\sqrt{n\Delta t-\tau}}=\frac{1}{\Delta t}\sqrt{\frac{T\Phi}{\pi}}\int_0^{\Delta t}\frac{du}{\sqrt{n\Delta t-(\gamma-1)\Delta t-u}}$$

$$=\frac{1}{\Delta t}\sqrt{\frac{T\Phi}{\pi}}\int_0^{\Delta t}\frac{du}{\sqrt{(n-\gamma+1)\Delta t-u}}$$

$$=\delta((n-\gamma+1),\Delta t)$$

$$=2\sqrt{\frac{T\Phi}{\Delta t\pi}}\left[\sqrt{n-\gamma+1}-\sqrt{n-\gamma}\right] \tag{3.52}$$

Thus

$$Q_\gamma(n,\Delta t)=\sum_{\gamma=1}^{n}\delta(n-\gamma+1,\Delta t)\{\sigma(\gamma)-\sigma(\gamma-1)\} \tag{3.53}$$

It may be noted that the substitute length has no storage effect. The flow through substitute length takes place similar to that in pipe.

Let the unknown rise at the interface of substitute length and aquifer at the end of the first time step be $\Delta h_a(1)$. The rise in the river stage be $\Delta h_r(1)$. Applying mass balance at the end of the first step i.e. the flow rate leaving the substitute length enters to the aquifer

$$T\left\{\frac{\Delta h_r-\Delta h_a(1)}{\Delta L}\right\}=\Delta h_a(1)\delta(1,\Delta t) \tag{3.54a}$$

or

$$\Delta h_a(1)=\frac{T}{1+\frac{\Delta L}{T}\delta(1,\Delta t)} \tag{3.54b}$$

or

$$\sigma_a(1)=\frac{\sigma_r(1)-\sigma_r(0)}{1+\frac{\Delta L}{T}\delta(1,\Delta t)} \tag{3.54c}$$

Similarly applying mass balance at the end of $n\Delta t$

$$T\frac{(\sigma_r(n)-\sigma_a(n))}{\Delta L}=\sum_{\gamma=1}^{n}\{\sigma_a(\gamma)-\sigma_a(\gamma-1)\}\delta(n-\gamma+1,\Delta t)$$

$$=\left\{\sum_{\gamma=1}^{n-1}\{\sigma_a(\gamma)-\sigma_a(\gamma-1)\}\delta(n-\gamma+1,\Delta t)\right\}+\{\sigma_a(n)-\sigma_a(n-1)\}\delta(1,\Delta t)$$

$$\sigma_r(n)-\sigma_a(n)=\frac{\Delta L}{T}\left\{\sum_{\gamma=1}^{n-1}\{\sigma_a(\gamma)-\sigma_a(\gamma-1)\}\delta(n-\gamma+1,\Delta t)\right\}+\frac{\Delta L}{T}\{\sigma_a(n)-\sigma_a(n-1)\}\delta(1,\Delta t)$$

$$\sigma_a(n)=\frac{\sigma_r(n)-\frac{\Delta L}{T}\left\{\sum_{\gamma=1}^{n-1}\{\sigma_a(\gamma)-\sigma_a(\gamma-1)\}\delta(n-\gamma+1,\Delta t)\right\}+\frac{\Delta L}{T}\sigma_a(n-1)\,\delta(1,\Delta t)}{1+\frac{\Delta L}{T}\delta(1,\Delta t)}$$

......(3.55)

$\sigma_a(n)$ can be solved in succession starting from time step 1. Once $\sigma_a(n)$ are found. $Q_\gamma(n)$ can be computed using equation (3.53). For a unit step rise the seepage from a stream for

$B/T_2 = 0.5$ and $T_1/T_2 = 0.25$, 0.5, and 0.999 is shown in Fig. III.6. Also in the graph the influent seepage for a fully penetrating stream is shown for the purpose of comparison.

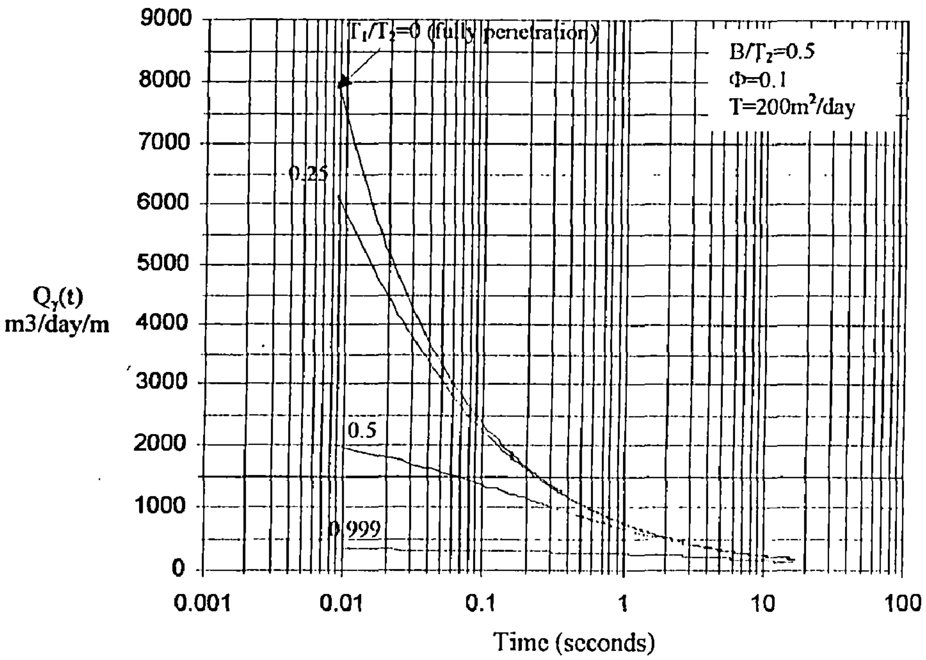

Fig. III.6 Rate of seepage with time for $B/T_2=0.5$

III.6 RESULTS AND DISCUSSION

For computing steady seepage from a stream or canal, whose section conforms to a rectangular one, the parameters and data required are :

(i) the hydraulic conductivity, k,

(ii) the difference in piezometric level recorded at a piezometer in the vicinity of the stream and water surface level in the stream,

(iii) distance of the piezometer from the stream bank,

(iv) thickness of aquifer below the stream bed,

(v) thickness of aquifer beyond the stream bed and

(vi) width of the stream

The seepage is given by

$$Q = F.k.\Delta h \qquad (3.56)$$

where F depends on the seepage factor stream geometry and distance of the piezometer from the bank. The seepage, q, has been expressed by Morel Seytoux as :

$$q = \Gamma_r.\Delta h \qquad (3.57)$$

From equation (3.29) the reach transmissivity per unit length of stream is given by

$$\Gamma_r = \frac{\pi k}{2\ln\left(\sqrt{1+b}+\sqrt{b}\right)} \tag{3.58}$$

Therefore the dimensionless factor

$$F = \frac{q}{k\Delta h} = \frac{\Gamma_r}{k} \tag{3.59}$$

Γ_r is a function of the distance of the piezometer from the stream bank for a particular stream. This factor would change with charge in depth of penetration and width of the stream. The relationship of seepage factor F or $q/(k\Delta h)$ or Γ_r/k with L_B/T_2 for different T_1/T_2 and B/T_2 are presented in Fig. III.7a through Fig. III.7f. From the figures it could be seen that for stream having comparatively large width ($B/T_2 \geq 1$), the seepage factor is independent of the depth of penetration only if the piezometer is located beyond 5 T_2. The reach is always dependent on L_B, the distance of the piezometer where Δh is observed. Γ_r/k increases as depth of penetration of the stream increases i.e. lower the T_1/T_2, higher the Γ_r/k. In accordance to law of resistance (Resistance is directly proportional to length of the conductor and inversely proportional to area of the conductor) Γ_r/k decreases with L_B/T_2. As B/T_2 increases i.e. stream cross section increases the reach transmissivity increases.

The fraction of seepage through bed decreases as depth of penetration of the stream increases. Incase of a canal running in a porous medium of large depth, seepage increases with increasing width of the canal when water table lies at infinite. From the Fig. III.9, it is seen that when the aquifer is confined, the seepage from the stream bed tends to a limiting value. For $T_1/T_2 = 0.9$ the fraction of seepage through bed does not increase for $B/T_2>1$.

In ground water modeling, some times the seepage from a stream is linked to the potential with the aquifer below the stream bed. The relationship of seepage with potential difference are shown in Fig. III.11a through III.11c for $T_1/T_2 = 0.1$, 0.5 and 0.999 for various location of the piezometer below the stream.

Treating the stream cross section as semi circular one, Herbert has applied Darcy law and obtained a logarithmic relationship between influent seepage and potential at middle of the aquifer below the stream bed. Preserving the method perimeter stream of any other shape can be converted to equivalent semi circular stream. The computation of seepage by Herbert method is compared with the seepage estimated rigorously by conformal mapping. The results are compared in Fig. III.12 and Fig. III.13. It could be seen that for 10 % penetration, Herbert formula is only applicable up to $B/T_2 = 0.2$. The difference between seepage computed from Herbert formula and conformal mapping for depth of penetration equal to half width of stream (i.e. Ds = B) is shown as a function of Ds/T_2, $Ds/T_2 < 0.5$. The discrepancy of Herbert formula increases rapidly for $Ds/T_2 > 0.3$. The error involved in Herbert formula is more than 10 %.

The piezometric surface in the aquifer near the top impervious layer is shown in Fig. III.14, for $T_1/T_2 = 0.9$, $B/T_2 = 0.1$, $h_1/T_2 = 1.1$ and $\Delta h = 0.025$ at a distance $L_B/T_2 = 1$. The piezometric surface falls below the impervious layer beyond $L_B/T_2 > 5$. The confined condition imposed on the aquifer is no longer valid for $L_B/T_2 > 5$.

For unsteady state flow, the rise in the piezometric surface at the interface of substitute length and aquifer due to a step rise (1 m) in the stream, for T_1/T_2=0.5; B/T_2=0.5; Φ=0.1 and T=200 m^2/day is shown in Fig. III.15. It is seen that the piezometric surface does not tend to 1 because of head loss in the substitute length. The rise at the interface at near steady state conditions will be less than unit. Therefore the rise as in the case of a fully penetrating stream does not converse to the rise in case of a partially penetrating stream.

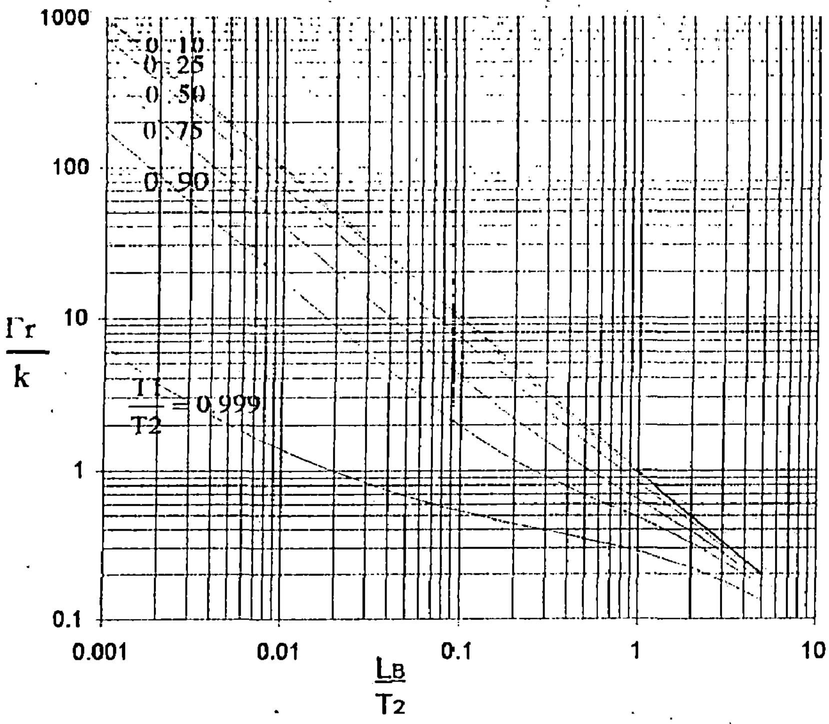

Fig. III.7a Variation of $\frac{\Gamma r}{k}$ or $\frac{q}{k\Delta h}$ or F with distance of piezometer from stream bank for different depth of penetration of the stream, for B/T2=0.01

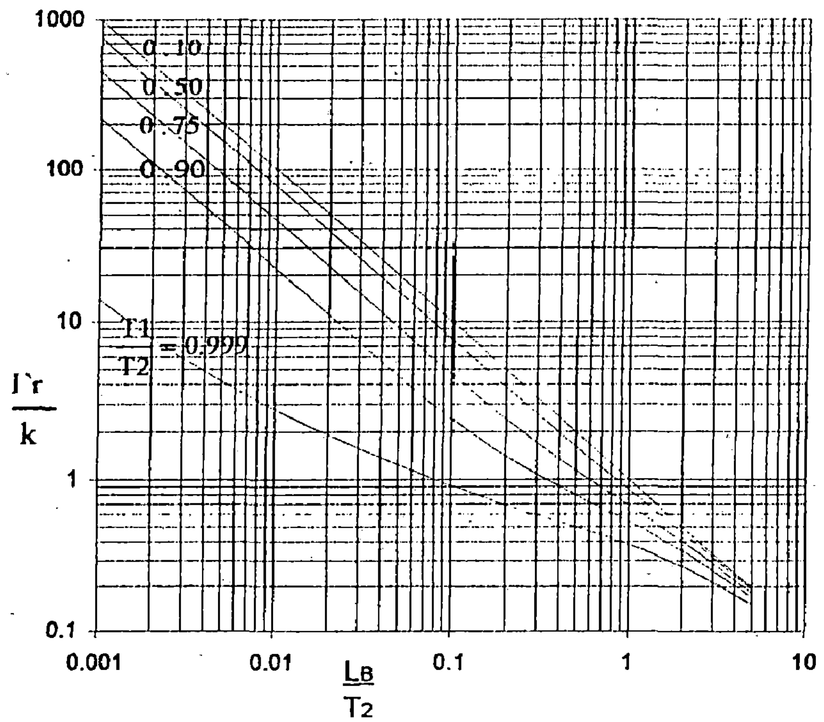

Fig. III.7b Variation of $\frac{\Gamma r}{k}$ or $\frac{q}{k\Delta h}$ or F with distance of piezometer from stream bank for different depth of penetration of the stream, for B/T2=0.05

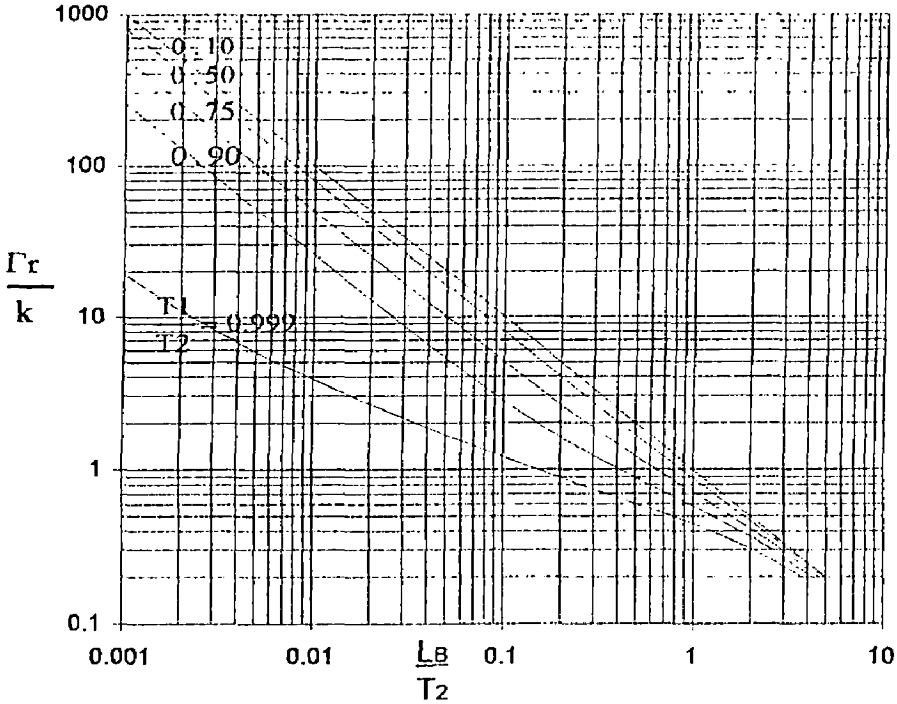

Fig. III.7c Variation of $\frac{\Gamma r}{k}$ or $\frac{q}{k\Delta h}$ or F with distance of piezometer from stream bank for different depth of penetration of the stream, for B/T2=0.1

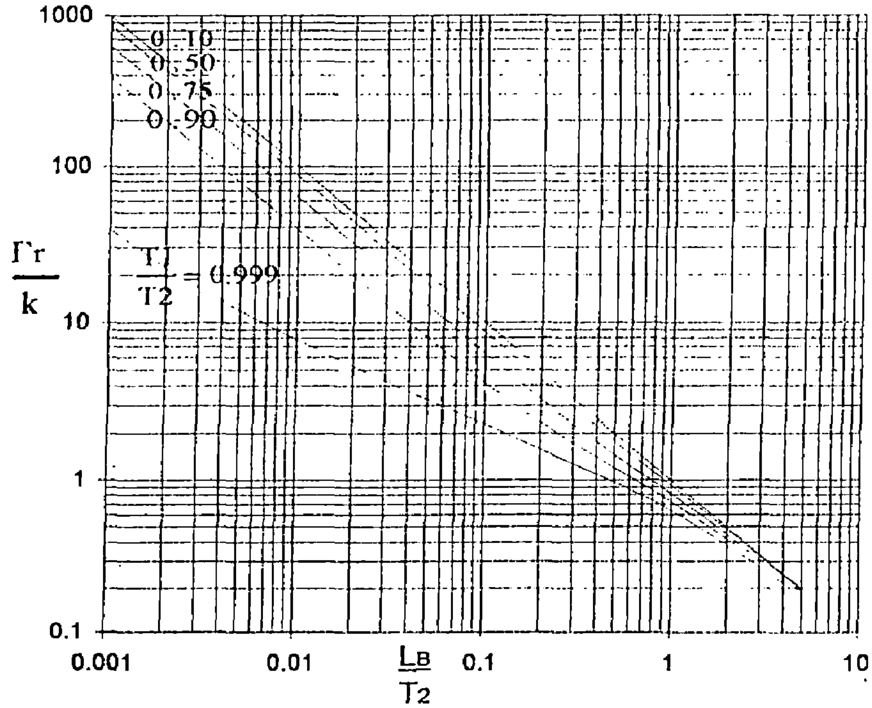

Fig. III.7d Variation of $\frac{\Gamma r}{k}$ or $\frac{q}{k\Delta h}$ or F with distance of piezometer from stream bank for different depth of penetration of the stream, for B/T2=0.5

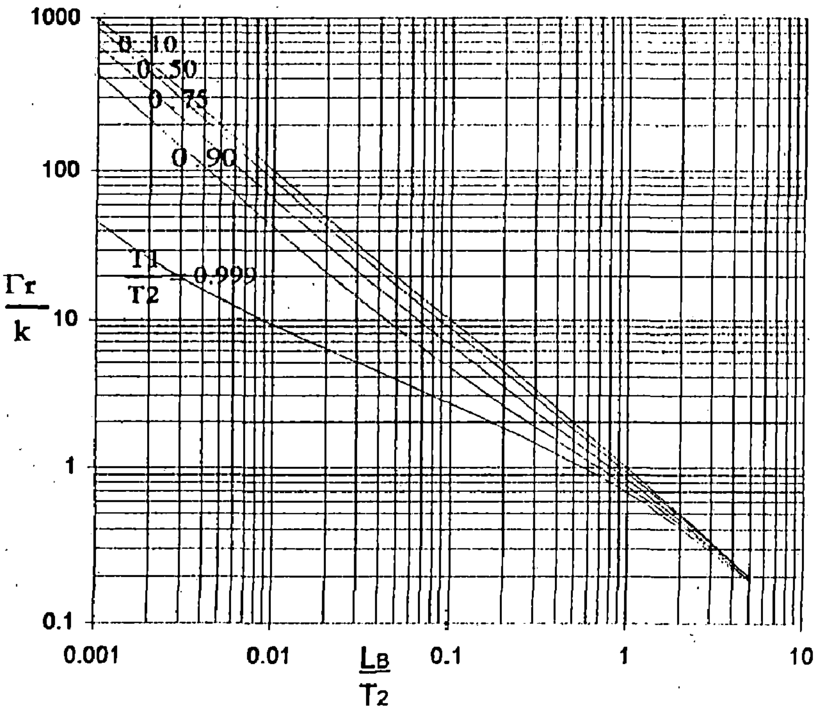

Fig. III.7e Variation of $\frac{\Gamma r}{k}$ or $\frac{q}{k\Delta h}$ or F with distance of piezometer from stream bank for different depth of penetration of the stream, for B/T2=1

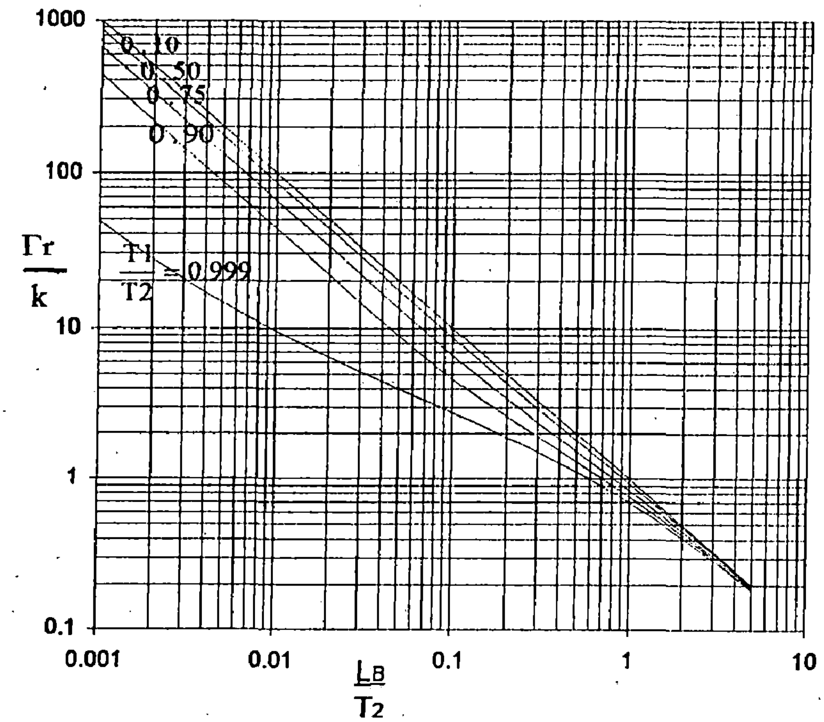

Fig. III.7f Variation of $\frac{\Gamma r}{k}$ or $\frac{q}{k\Delta h}$ or F with distance of piezometer from stream bank for different depth of penetration of the stream, for B/T2=2

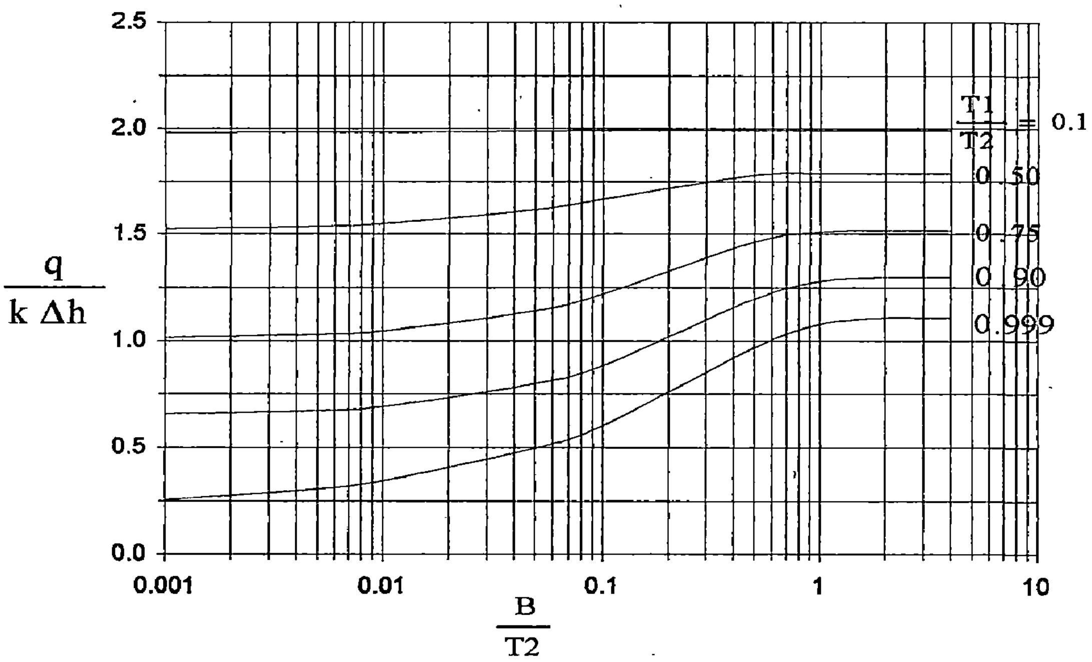

Fig. III.8 Variation of $\frac{\Gamma r}{k}$ or $\frac{q}{k\Delta h}$ or F with width of the stream for distance of piezometer from the stream bank, L/T2=0.5

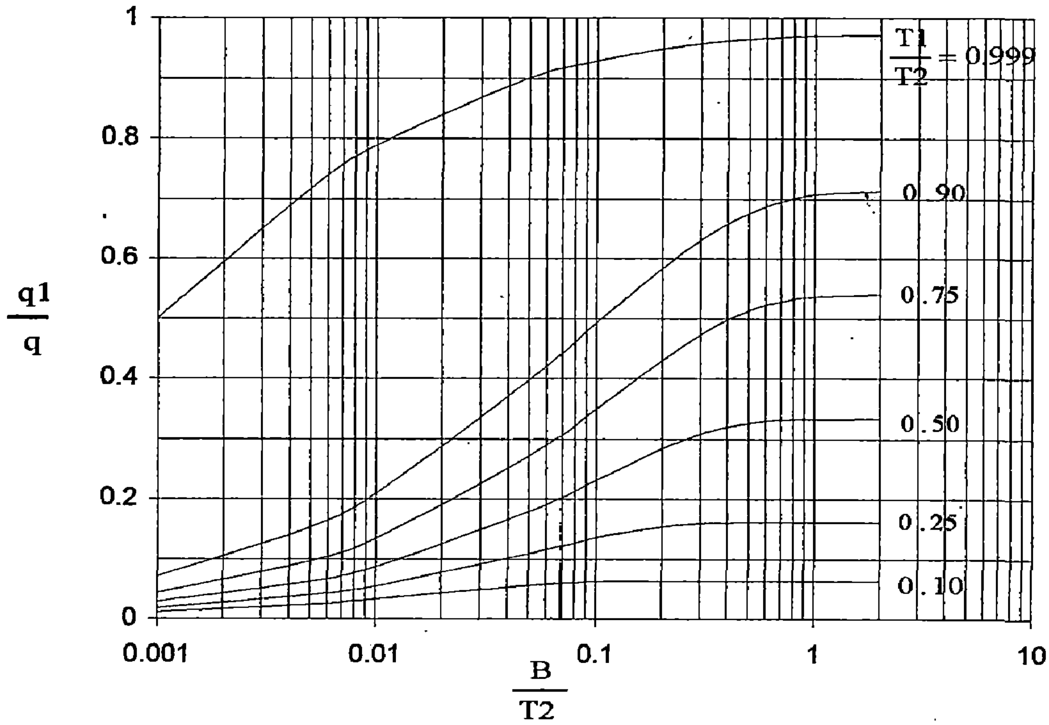

Fig. III.9 Seepage through stream bed for different width and penetration of the stream

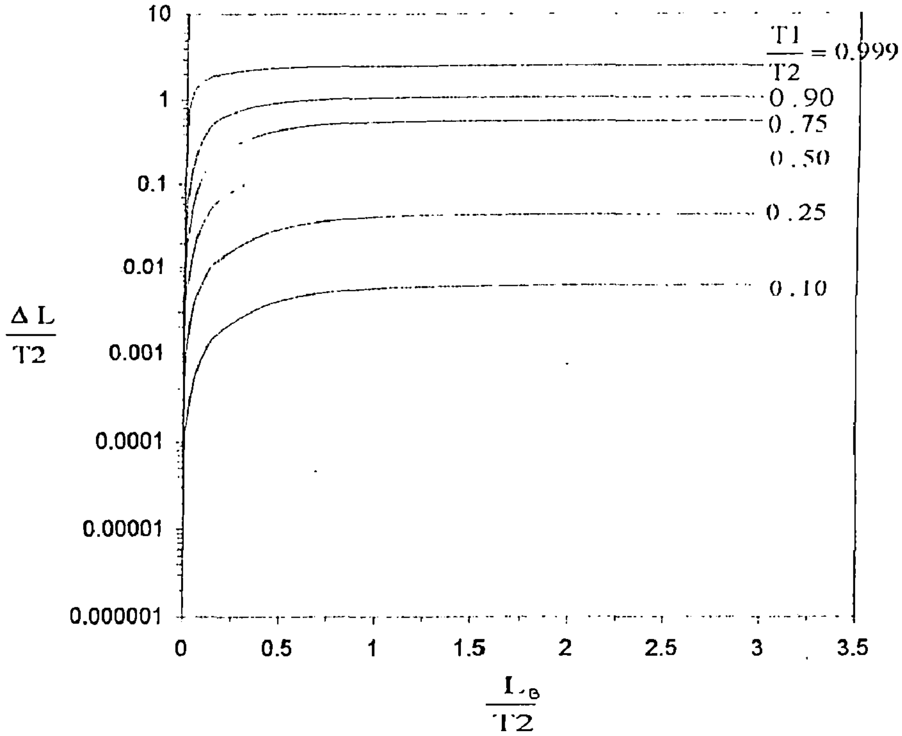

Fig. III.10a Variation of substitute length with distance from the stream bank for different depth of penetration of the stream, for B/T2=0.01

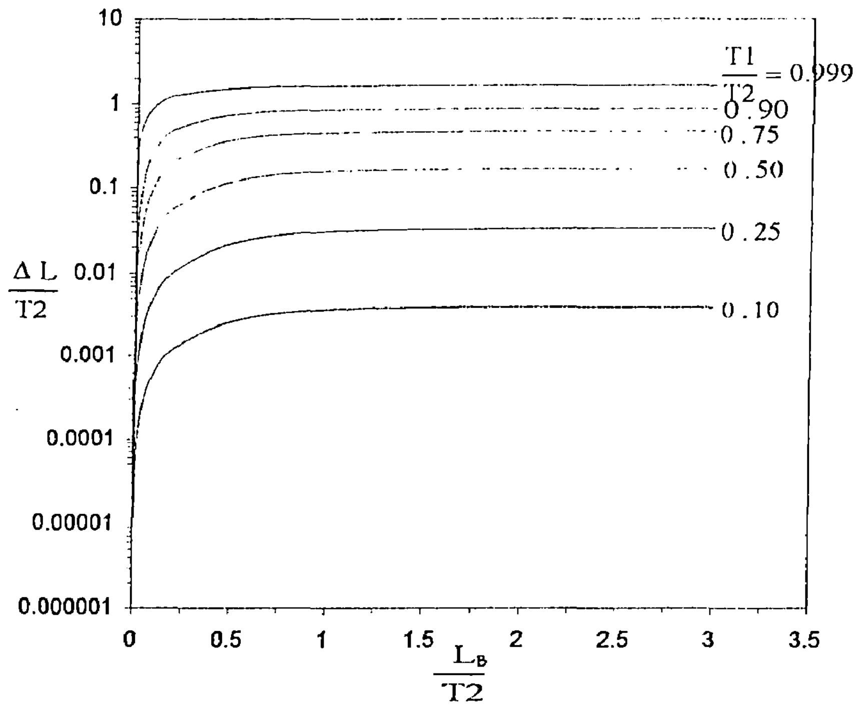

Fig. III.10b Variation of substitute length with distance from the stream bank for different depth of penetration of the stream, for B/T2=0.05

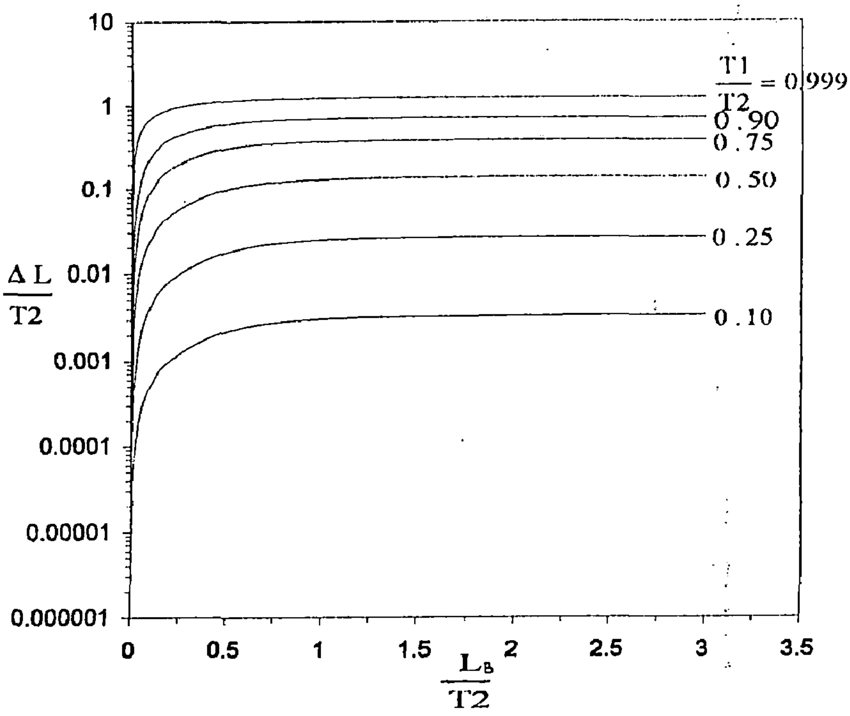

Fig. III.10c **Variation of substitute length with distance from the stream bank for different depth of penetration of the stream, for B/T2=0.1**

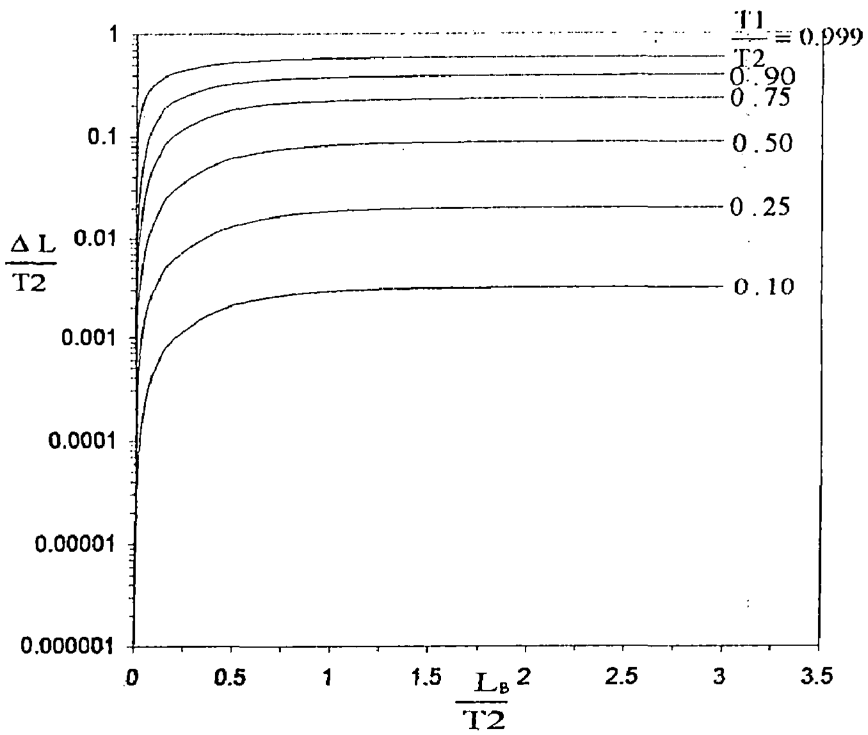

Fig. III.10d **Variation of substitute length with distance from the stream bank for different depth of penetration of the stream, for B/T2=0.5**

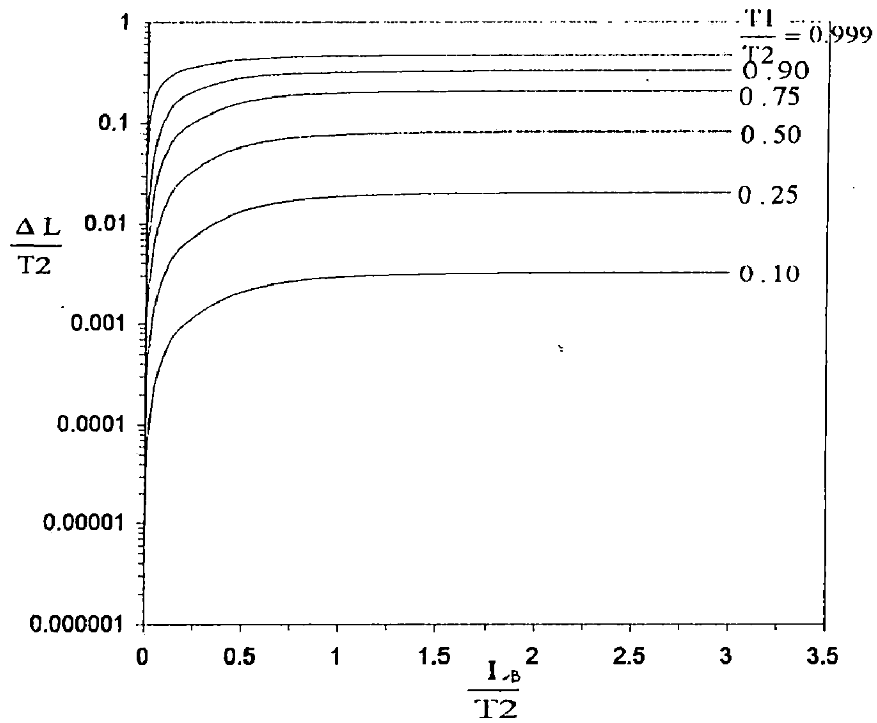

Fig. III.10e Variation of substitute length with distance from the stream bank for different depth of penetration of the stream, for B/T2=1.0

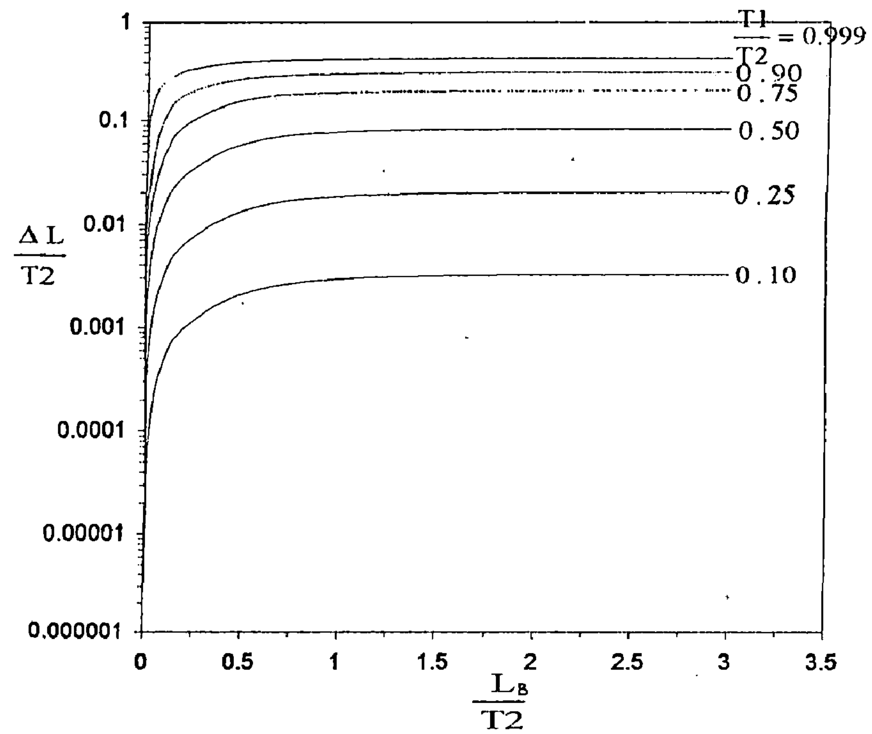

Fig. III.10f Variation of substitute length with distance from the stream bank for different depth of penetration of the stream, for B/T2=2.0

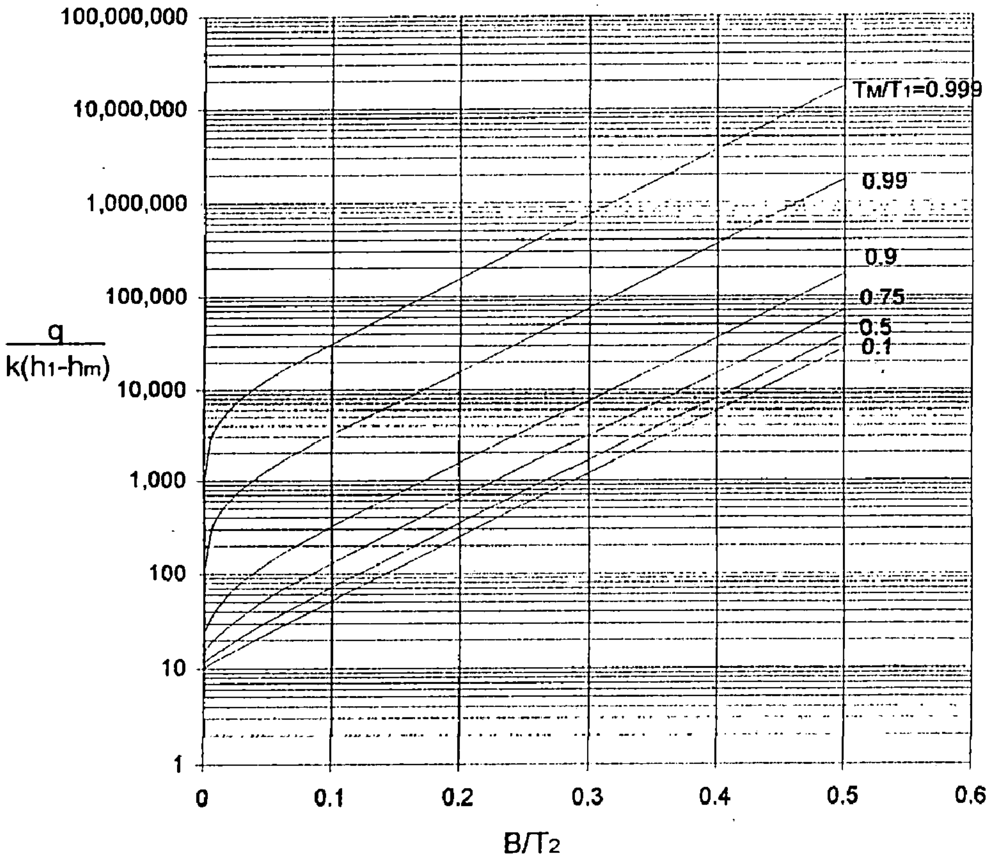

Fig. III.11a **Variation of $\frac{q}{k\ \Delta\ h}$ with width of the stream for different point below stream bed, for T1/T2 = 0.1**

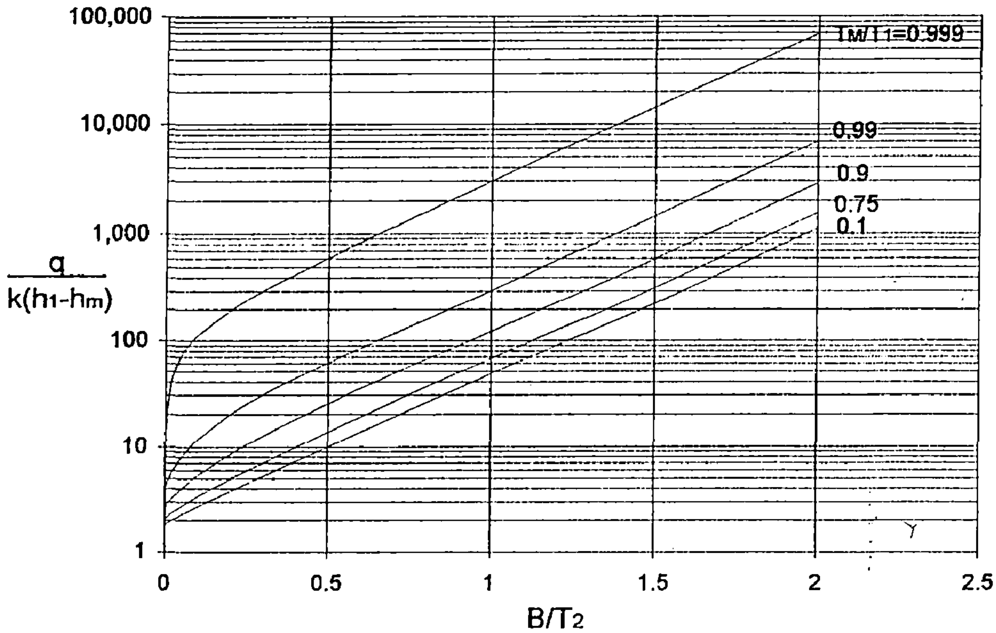

Fig. III.11b **Variation of $\frac{q}{k\ \Delta\ h}$ with width of the stream for different point below stream bed, for T1/T2 = 0.5**

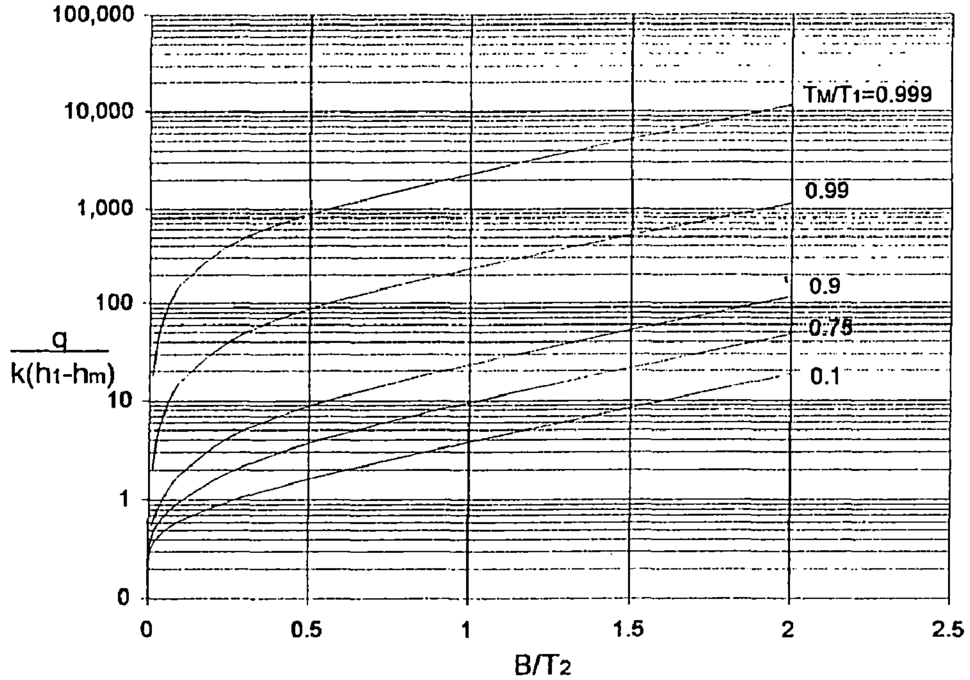

Fig. III.11c **Variation of $\frac{q}{k\,\Delta\,h}$ with width of the stream for different point below stream bed, for T1/T2 = 0.999**

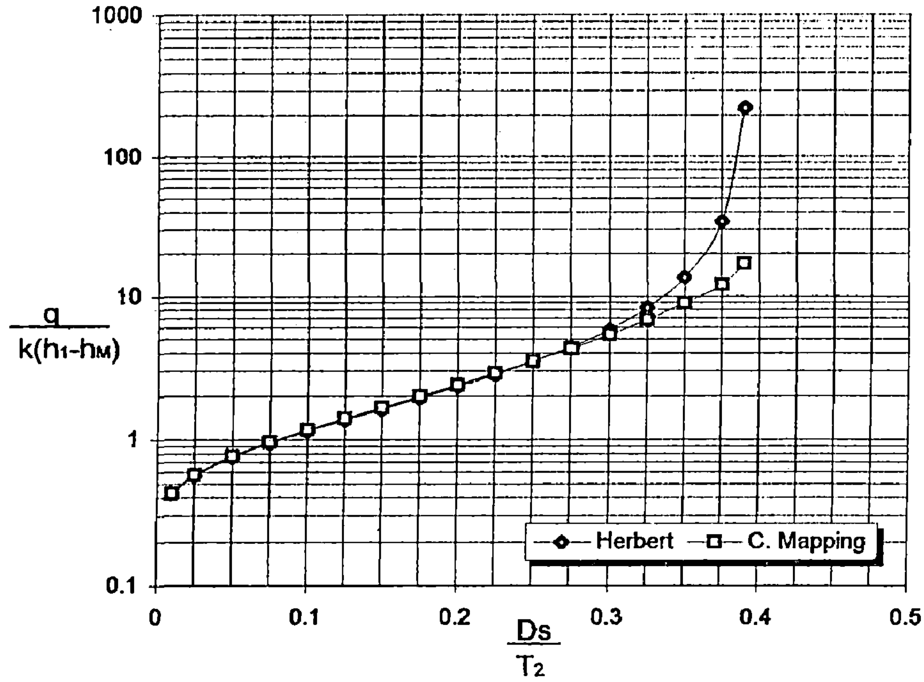

Fig. III.12a Comparison of $\frac{q}{k(h_1-h_M)}$ between Conformal Mapping and Herbert for rectangular type of the stream (Ds = B)

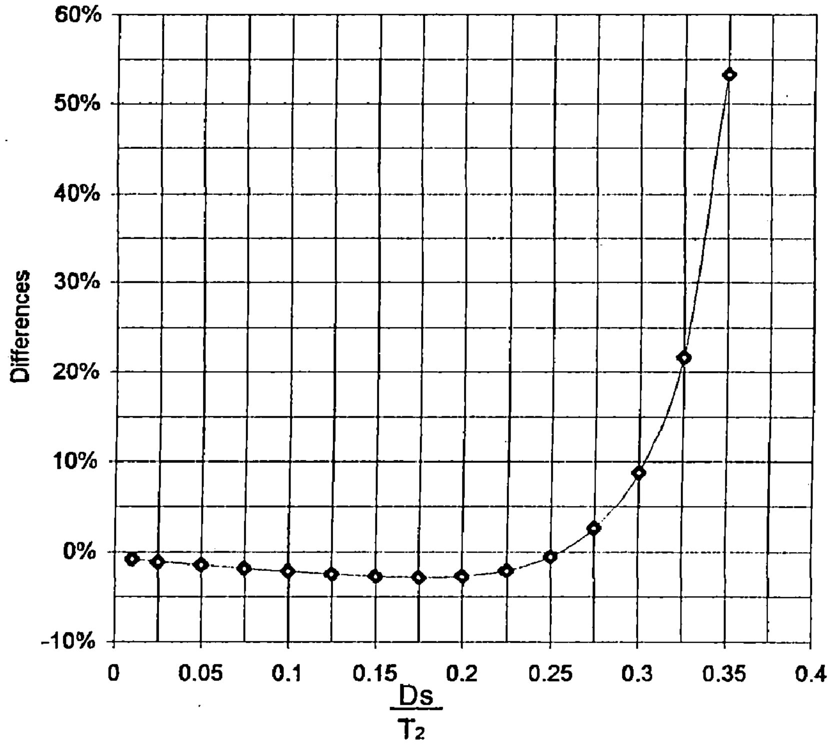

Fig. III.12b Differences of $\frac{q}{k(h_1-h_M)}$ between Conformal Mapping and Herbert for rectangular type of the stream (Ds = B)

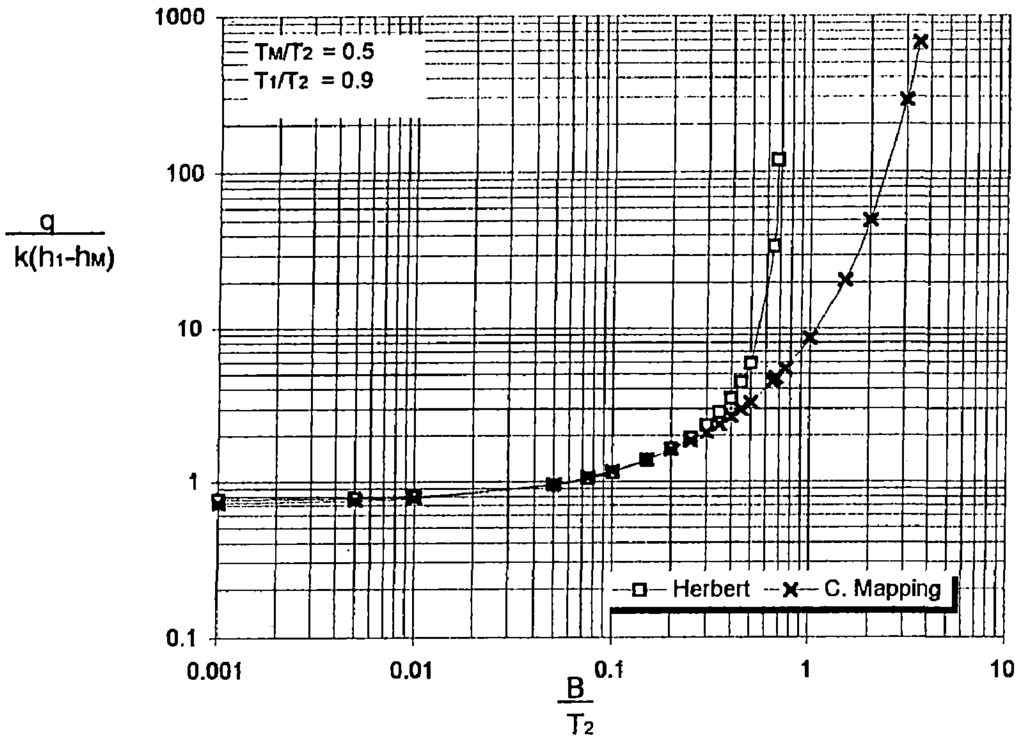

Fig. III.13a Comparison of $\frac{q}{k(h_1-h_M)}$ between Conformal Mapping and Herbert for 10 % depth of penetration of the stream ($T_1/T_2 = 0.9$)

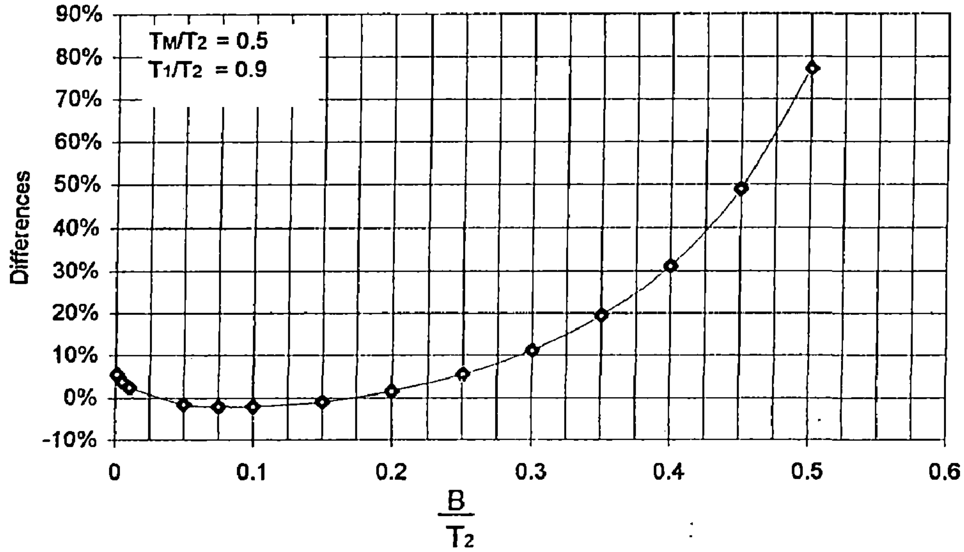

Fig. III.13b Differences of $\frac{q}{k(h_1-h_M)}$ between Conformal Mapping and Herbert for 10 % depth of penetration of the stream ($T_1/T_2 = 0.9$)

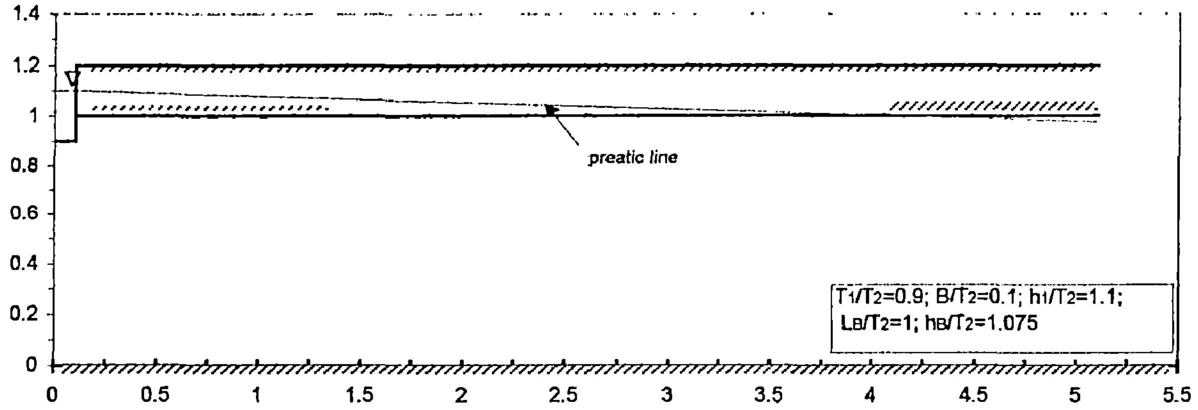

Fig. III.14 Locus of the phreatic line

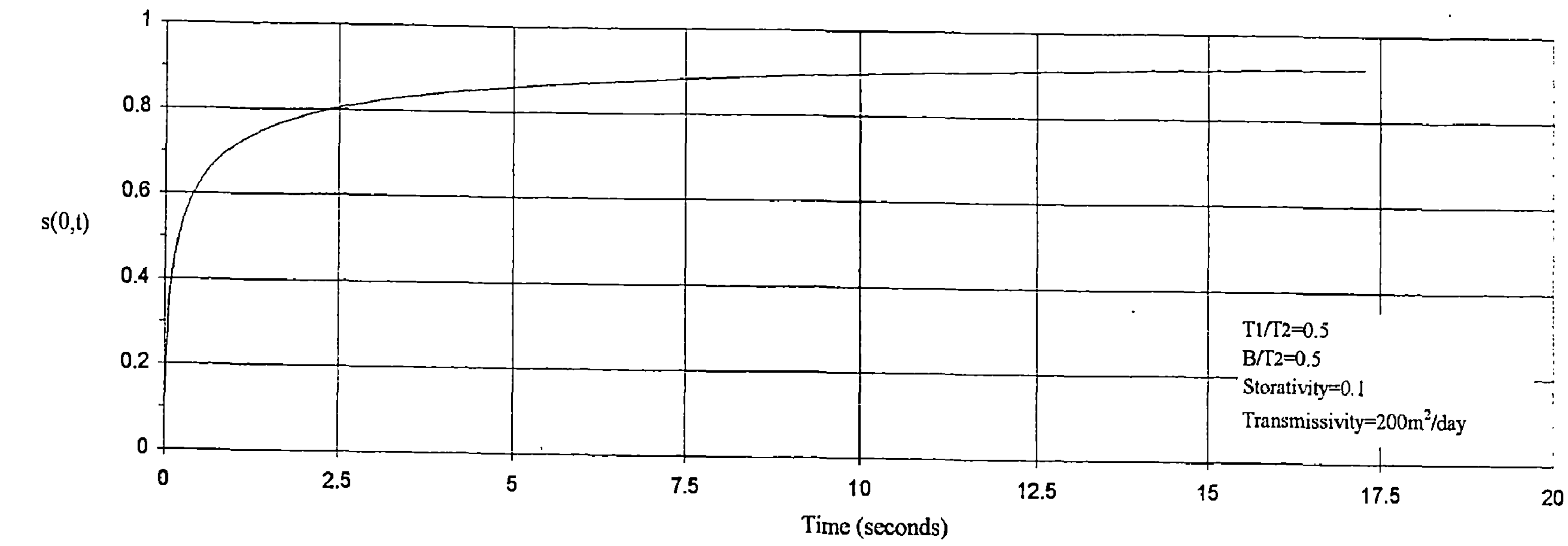

Fig. III.15 **Rise in piezometeric surface at the interface of substitute length and aquifer due to a step rise (1 m) in the stream**

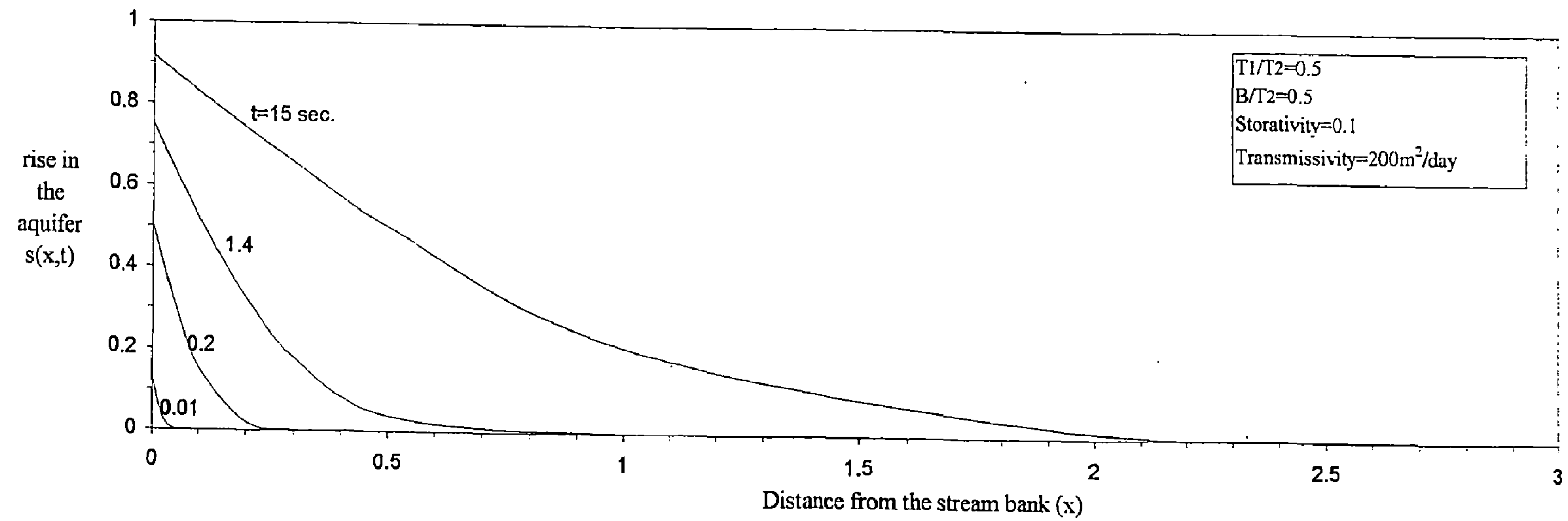

Fig. III.16 **Piezometric surface in the aquifer s(x,t) corresponding to a step rise in the stream**

CHAPTER IV

SEEPAGE FROM A STREAM IN A FINITE AQUIFER

IV.1 GENERAL

Seepage from a canal in a semi-infinite aquifer has been discussed in chapter III. For a piezometer located at a distance beyond 5 times of thickness of the aquifer from center of the stream, the parameter b in ξ plane is found to attend very high value. That height of the piezometer surface decreases with distance from the stream and falls below the upper confining layer. Beyond this point, the aquifer would be unconfined. It is thus physically not possible that steady flow takes place from a stream to a confined aquifer of infinite length. For steady state flow, the flow at any section in the aquifer is constant, for the flow to take place the hydraulic head has to decrease which will lead the piezometer surface to fall below the upper boundary of the confined aquifer. In this chapter, steady flow from a stream with more generalized section in a confined aquifer of finite length has been analyzed using potential theory. The flow is assumed to be identical on either side of the stream.

IV.2 ANALYSIS

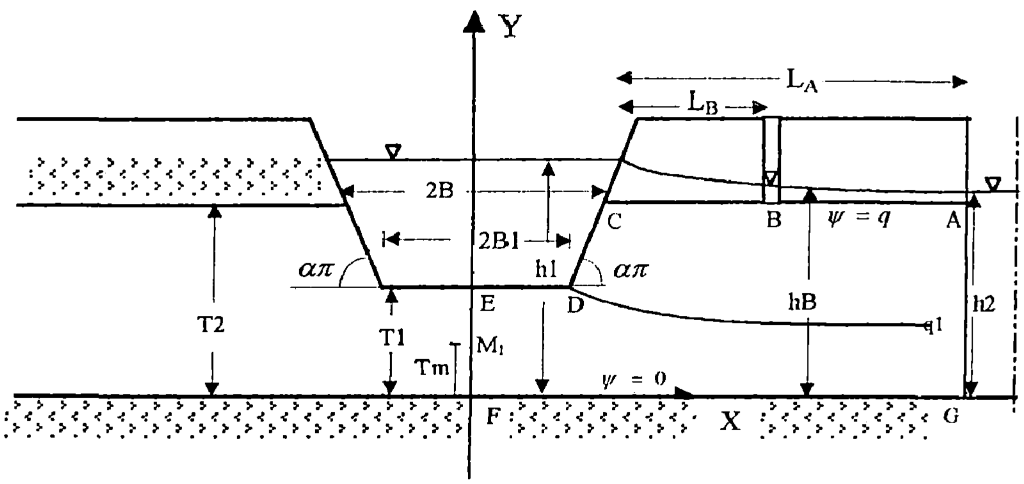

Fig. IV.1 Physical flow domain in z-plane (z=x+iy)

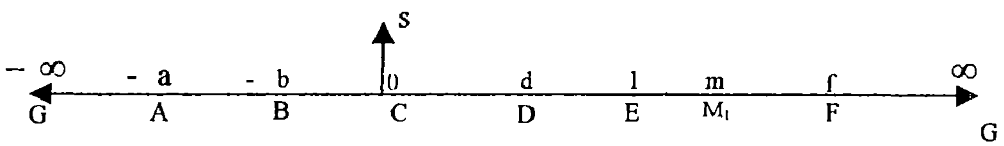

Fig. IV.2 ξ-plane (ξ=r+is)

IV.2.1 Mapping of The Physical Flow Domain in Z-Plane to An Auxiliary ξ-Plane

The stream bank is inclined of angle $\alpha\pi$ with horizontal. According to the Schwarz–Christoffel transformation, the conformal mapping of the flow domain in z plane onto the lower half an auxiliary ξ plane is given by :

$$\frac{dz}{d\xi} = M\frac{(\xi - d)^{\alpha}}{(\xi + a)^{1/2}\xi^{\alpha}(\xi - 1)^{1/2}(\xi - f)^{1/2}} \tag{4.1}$$

in which

$$\alpha = \frac{\tan^{-1}\dfrac{T_2 - T_1}{B - B_1}}{\pi} \tag{4.2}$$

B is half width of the stream surface at the bottom of upper confining layer, B_1 is half of the bottom width of the stream, T_1 and T_2 are thickness of aquifer below the stream bed and thickness of aquifer beyond the stream bank. The vertices G, A, C, D, E and F in z plane (Fig. IV.1) have been mapped onto points -∞, -a, 0, d, 1 and f respectively of the ξ-plane (Fig. IV.2). The parameters a, d and f are found as follows:

For $-a \le \xi' \le 0$; the corresponding z is given by :

$$z = M\int_0^{\xi'}\frac{(\xi - d)^{\alpha}}{(\xi + a)^{1/2}(\xi)^{\alpha}(\xi - 1)^{1/2}(\xi - f)^{1/2}}d\xi + B + iT_2 \tag{4.3}$$

For point A, $\xi' = -a$ and $Z_A = B + L_A + iT_2$; hence,

$$B + L_A + iT_2 = -M\int_0^{-a}\frac{(d - \xi)^{\alpha}}{(\xi + a)^{1/2}(-\xi)^{\alpha}(1 - \xi)^{1/2}(f - \xi)^{1/2}}d\xi + B + iT_2 \tag{4.4}$$

where L_A is distance of the aquifer boundary from the stream bank. Substituting ξ=-u, hence,

$$L_A = M\int_0^{a}\frac{(d + u)^{\alpha}}{(a - u)^{1/2}u^{\alpha}(1 + u)^{1/2}(f + u)^{1/2}}du \tag{4.5a}$$

or

$$L_A = M\left\{\int_0^{a/2}\frac{(d + u)^{\alpha}}{(a - u)^{1/2}u^{\alpha}(1 + u)^{1/2}(f + u)^{1/2}}du + \int_{a/2}^{a}\frac{(d + u)^{\alpha}}{(a - u)^{1/2}u^{\alpha}(1 + u)^{1/2}(f + u)^{1/2}}du\right\} \tag{4.5b}$$

Substituting $u = v^2$ for the first integral and $a - u = v^2$ for the second integral above, where v is a dummy variable, the improper integral 4.5b is converted to the following proper integral :

$$L_A = M\left\{2\int_0^{\sqrt{a/2}} \frac{(d+v^2)^{\alpha}\, v^{(1-2\alpha)}}{(a-v^2)^{1/2}(1+v^2)^{1/2}(f+v^2)^{1/2}}\,dv + 2\int_0^{\sqrt{a/2}} \frac{(d+a-v^2)^{\alpha}}{(a-v^2)^{\alpha}(1+a-v^2)^{1/2}(f+a-v^2)^{1/2}}\,dv\right\}$$

......(4.6)

Substituting :

$$v = \sqrt{a/2}\,\frac{(1+\chi)}{2} \quad \text{and} \quad dv = \frac{\sqrt{a/2}}{2}\,d\chi$$

where χ is a dummy variable, the lower and upper limits of integral 4.6 are converted to -1 and 1 respectively

$$L_A = M\sqrt{a/2}\left\{\int_{-1}^{1} \frac{\left[\sqrt{a/2}\,\frac{1+\chi}{2}\right]^{(1-2\alpha)}\left[d+(a/2)\left(\frac{1+\chi}{2}\right)^2\right]^{\alpha}}{\sqrt{\left[a-(a/2)\left(\frac{1+\chi}{2}\right)^2\right]\left[1+(a/2)\left(\frac{1+\chi}{2}\right)^2\right]\left[f+(a/2)\left(\frac{1+\chi}{2}\right)^2\right]}}\,d\chi\right\}+$$

$$M\sqrt{a/2}\left\{\int_{-1}^{1} \frac{\left[d+a-(a/2)\left(\frac{1+\chi}{2}\right)^2\right]^{\alpha}}{\left[a-(a/2)\left(\frac{1+\chi}{2}\right)^2\right]^{\alpha}\sqrt{\left[1+a-(a/2)\left(\frac{1+\chi}{2}\right)^2\right]\left[f+a-(a/2)\left(\frac{1+\chi}{2}\right)^2\right]}}\,d\chi\right\}$$

......(4.7)

For region $0 \le \xi' \le d$; the corresponding z is given by :

$$z = M\int_0^{\xi'} \frac{(\xi-d)^{\alpha}}{(\xi+a)^{1/2}\xi^{\alpha}(\xi-1)^{1/2}(\xi-f)^{1/2}}\,d\xi + B + iT_2 \tag{4.8}$$

For point D, $\xi' = d$ and $Z_D = B_1 + iT_1$; hence,

$$(B_1 - B) + (iT_1 - iT_2) = \frac{M}{i^2}\int_0^{d} \frac{(-1)^{\alpha}(d-\xi)^{\alpha}}{(\xi+a)^{1/2}(\xi)^{\alpha}(1-\xi)^{1/2}(f-\xi)^{1/2}}\,d\xi \tag{4.9}$$

Equating the modulus of either side

$$\sqrt{(B-B_1)^2+(T_2-T_1)^2} = |M|\int_0^{d} \frac{(d-\xi)^{\alpha}}{(\xi+a)^{1/2}(\xi)^{\alpha}(1-\xi)^{1/2}(f-\xi)^{1/2}}\,d\xi \tag{4.10}$$

Substituting $\xi = v^2$, $d\xi = 2v\,dv$, at the lower limit $\xi = 0$, $v = 0$ and at the upper limit $\xi = d$, $v = \sqrt{d}$, where v is a dummy variable, the improper integral 4.10 is converted to the following proper integral

$$\sqrt{(B-B_1)^2+(T_2-T_1)^2} = 2\,|M|\int_0^{\sqrt{d}} \frac{(d-v^2)^{\gamma}\,(v)^{1-2\alpha}}{(v^2+a)^{1/2}(1-v^2)^{1/2}(f-v^2)^{1/2}}\,dv \qquad (4.11)$$

Substituting :

$$v=\sqrt{d}\,\frac{(1+\chi)}{2} \quad \text{and} \quad dv=\frac{\sqrt{d}}{2}\,d\chi$$

where χ is a dummy variable, the lower and upper limits of integral 4.11 are converted to -1 and 1 respectively and equation 4.11 reduces to

$$\sqrt{(B-B_1)^2+(T_2-T_1)^2} = M\sqrt{d}\int_{-1}^{1} \frac{\left[\sqrt{d}\,\frac{1+\chi}{2}\right]^{(1-2\alpha)}\left[d-d\left(\frac{1+\chi}{2}\right)^2\right]^{\gamma}}{\sqrt{\left[d\left(\frac{1+\chi}{2}\right)^2+a\right]\left[1-d\left(\frac{1+\chi}{2}\right)^2\right]\left[f-d\left(\frac{1+\chi}{2}\right)^2\right]}}\,d\chi$$

......(4.12)

For domain $d \le \xi' \le 1$; the corresponding z is given by :

$$z = M\int_d^{\xi'} \frac{(\xi-d)^{\gamma}}{(\xi+a)^{1/2}\xi^{\alpha}(\xi-1)^{1/2}(\xi-f)^{1/2}}\,d\xi + B_1 + iT_1 \qquad (4.13)$$

For point E, $\xi' = 1$ and $Z_E = iT_1$; hence,

$$iT_1 = -M\int_d^{1} \frac{(\xi-d)^{\gamma}}{(\xi+a)^{1/2}\xi^{\alpha}(1-\xi)^{1/2}(f-\xi)^{1/2}}\,d\xi + B_1 + iT_1$$

or

$$B_1 = M\int_d^{1} \frac{(\xi-d)^{\gamma}}{(\xi+a)^{1/2}\xi^{\alpha}(1-\xi)^{1/2}(f-\xi)^{1/2}}\,d\xi \qquad (4.14)$$

Substituting $1-\xi = v^2$, $d\xi = -2v\,dv$, at the lower limit $\xi = d$, $v = \sqrt{(1-d)}$, and at the upper limit $\xi = 1$, $v = 0$, where v is a dummy variable, the improper integral 4.14 is converted to the following proper integral

$$B_1 = 2M\int_0^{\sqrt{1-d}} \frac{(1-v^2-d)^{\gamma}}{(1-v^2+a)^{1/2}(1-v^2)^{\alpha}(f-1+v^2)^{1/2}}\,dv \qquad (4.15)$$

Substituting

$$v=\sqrt{1-d}\,\frac{1+\chi}{2} \quad \text{and} \quad dv=\frac{\sqrt{1-d}}{2}d\chi$$

where χ is a dummy variable, the lower and upper limits of integral above are converted to –1 and 1 respectively and equation 4.15 reduces to

$$B_1=M\sqrt{1-d}\int_{-1}^{1}\frac{\left[1-(1-d)\left(\frac{1+\chi}{2}\right)^2-d\right]^{\alpha}}{\left[1-(1-d)\left(\frac{1+\chi}{2}\right)^2\right]^{\alpha}\sqrt{\left[1-(1-d)\left(\frac{1+\chi}{2}\right)^2+a\right]\left[f-1+(1-d)\left(\frac{1+\chi}{2}\right)^2\right]}}d\chi$$

......(4.16)

For domain $1\le\xi'\le f$; the corresponding z is given by

$$z=M\int_{1}^{\xi'}\frac{(\xi-d)^{\alpha}}{(\xi+a)^{1/2}\xi^{\alpha}(\xi-1)^{1/2}(\xi-f)^{1/2}}d\xi+iT_1 \tag{4.17}$$

For point F, $\xi'=f$ and $Z_F=0$; hence,

$$T_1=M\int_{1}^{f}\frac{(\xi-d)^{\alpha}}{(\xi+a)^{1/2}\xi^{\alpha}(\xi-1)^{1/2}(f-\xi)^{1/2}}d\xi \tag{4.18}$$

Re-writing equation 4.18 to convert the improper integral to the proper integral

$$T_1=M\left\{\int_{1}^{\frac{1+f}{2}}\frac{(\xi-d)^{\alpha}}{(\xi+a)^{1/2}\xi^{\alpha}(\xi-1)^{1/2}(f-\xi)^{1/2}}d\xi+\int_{\frac{1+f}{2}}^{f}\frac{(\xi-d)^{\alpha}}{(\xi+a)^{1/2}\xi^{\alpha}(\xi-1)^{1/2}(f-\xi)^{1/2}}d\xi\right\} \tag{4.19}$$

Substituting $\xi-1=v^2$ for the first integral and $f-\xi=v^2$ for the second integral above, where v is a dummy variable, the improper integral in equation 4.19 is converted to the following proper integral :

$$T_1=M\left\{2\int_{0}^{\sqrt{\frac{f-1}{2}}}\frac{(v^2+1-d)^{\alpha}}{(v^2+1+a)^{1/2}(v^2+1)^{\alpha}(f-v^2-1)^{1/2}}dv+2\int_{0}^{\sqrt{\frac{f-1}{2}}}\frac{(f-v^2-d)^{\alpha}}{(f-v^2+a)^{1/2}(f-v^2)^{\alpha}(f-v^2-1)^{1/2}}dv\right\} \tag{4.20}$$

Substituting :

$$v=\sqrt{(f-1)/2}\left[\frac{1+\chi}{2}\right] \quad \text{and} \quad dv=\frac{\sqrt{(f-1)/2}}{2}d\chi$$

where χ is a dummy variable, the lower and upper limits of integral above are converted to –1 and 1 respectively and equation 4.20 reduces to

$$T_1 = M\sqrt{\frac{f-1}{2}}\left\{\int_{-1}^{1}\frac{\left[\left(\frac{f-1}{2}\right)\left(\frac{1+\chi}{2}\right)^2+1-d\right]^{\alpha}}{\left[\left(\frac{f-1}{2}\right)\left(\frac{1+\chi}{2}\right)^2+1\right]^{\alpha}\sqrt{\left[\left(\frac{f-1}{2}\right)\left(\frac{1+\chi}{2}\right)^2+1+a\right]\left[f-\left(\frac{f-1}{2}\right)\left(\frac{1+\chi}{2}\right)^2-1\right]}}d\chi\right\}+$$

$$M\sqrt{\frac{f-1}{2}}\left\{\int_{-1}^{1}\frac{\left[f-\left(\frac{f-1}{2}\right)\left(\frac{1+\chi}{2}\right)^2-d\right]^{\alpha}}{\left[f-\left(\frac{f-1}{2}\right)\left(\frac{1+\chi}{2}\right)^2\right]^{\alpha}\sqrt{\left[f-\left(\frac{f-1}{2}\right)\left(\frac{1+\chi}{2}\right)^2+a\right]\left[f-\left(\frac{f-1}{2}\right)\left(\frac{1+\chi}{2}\right)^2-1\right]}}d\chi\right\}$$

......(4.21)

The parameter a, d, f and constant M are solved from equation 4.7, 4.12, 4.16 and 4.21 using iteration procedure. The integration are carried out numerically applying Gauss-quadrature formula.

For region $-b \le \xi' \le 0$; the corresponding z is given by :

$$z = M\int_0^{\xi'}\frac{(\xi-d)^{\alpha}}{(\xi+a)^{1/2}\xi^{\alpha}(\xi-1)^{1/2}(\xi-f)^{1/2}}d\xi + B + iT_2 \qquad (4.22)$$

For point B, $\xi' = -b$ and $Z_B = B + L_B + iT_2$; hence,

$$B + L_B + iT_2 = -M\int_0^{-b}\frac{(d-\xi)^{\alpha}}{(\xi+a)^{1/2}(-\xi)^{\alpha}(1-\xi)^{1/2}(f-\xi)^{1/2}}d\xi + B + iT_2 \qquad (4.23)$$

Substituting $\xi = -u$,

$$L_B = M\int_0^{b}\frac{(d+u)^{\alpha}}{(a-u)^{1/2}u^{\alpha}(1+u)^{1/2}(f+u)^{1/2}}du \qquad (4.24)$$

Splitting the limit into two parts, equation 4.24 is written as :

$$L_B = M\left\{\int_0^{b/2}\frac{(d+u)^{\alpha}}{(a-u)^{1/2}u^{\alpha}(1+u)^{1/2}(f+u)^{1/2}}du + \int_{b/2}^{b}\frac{(d+u)^{\alpha}}{(a-u)^{1/2}u^{\alpha}(1+u)^{1/2}(f+u)^{1/2}}du\right\} \qquad (4.25)$$

Substituting $u = v^2$ for the first integral and $a - u = v^2$ for the second integral above, where v is a dummy variable, the improper integral is converted to the following proper integral

$$L_B = M\left\{2\int_0^{\sqrt{b/2}} \frac{(d+v^2)^{\alpha}\, v^{(1-2\alpha)}}{(a-v^2)^{1/2}(1+v^2)^{1/2}(f+v^2)^{1/2}}dv + 2\int_{\sqrt{a-b}}^{\sqrt{a-b/2}} \frac{(d+a-v^2)^{\alpha}}{(a-v^2)^{\alpha}(1+a-v^2)^{1/2}(f+a-v^2)^{1/2}}dv\right\}$$

......(4.26)

Substituting :

$$v = \sqrt{b/2}\,\frac{(1+\chi)}{2} \quad \text{and} \quad dv = \frac{\sqrt{b/2}}{2}d\chi$$

for the first part integral and substituting

$$v = \frac{\sqrt{a-b/2}-\sqrt{a-b}}{2}\chi + \frac{\sqrt{a-b/2}+\sqrt{a-b}}{2} = f(\chi) \text{ and } dv = \frac{\sqrt{a-b/2}-\sqrt{a-b}}{2}d\chi$$

for the second integral, where χ is a dummy variable, the lower and upper limits of integral 4.26 are converted to –1 and 1 respectively resulting in

$$L_B = M\sqrt{b/2}\left\{\int_{-1}^{1} \frac{\left[\sqrt{b/2}\,\frac{1+\chi}{2}\right]^{(1-2\alpha)}\left[d+(b/2)\left(\frac{1+\chi}{2}\right)^2\right]^{\alpha}}{\left[a-(b/2)\left(\frac{1+\chi}{2}\right)^2\right]^{1/2}\left[1+(b/2)\left(\frac{1+\chi}{2}\right)^2\right]^{1/2}\left[f+(b/2)\left(\frac{1+\chi}{2}\right)^2\right]^{1/2}}d\chi\right\} +$$

$$M\left(\sqrt{a-b/2}-\sqrt{a-b}\right)\left\{\int_{-1}^{1} \frac{\left[d+a-f^2(\chi)\right]^{\alpha}}{\left[a-f^2(\chi)\right]^{\alpha}\left[1+a-f^2(\chi)\right]^{1/2}\left[f+a-f^2(\chi)\right]^{1/2}}d\chi\right\}.$$

......(4.27)

For point M_I, which lies between E and F, the corresponding z is given by equation 4.17. For point M_1, $\xi' =$ m and $Z_M = iT_M$; hence,

$$T_I - T_M = M\int_1^m \frac{(\xi-d)^{\alpha}}{(\xi+a)^{1/2}(\xi)^{\alpha}(\xi-1)^{1/2}(f-\xi)^{1/2}}d\xi \qquad (4.28)$$

Splitting the integration into two parts

$$T_I - T_M = M\left\{\int_1^{\frac{1+m}{2}} \frac{(\xi-d)^{\alpha}}{(\xi+a)^{1/2}\xi^{\alpha}(\xi-1)^{1/2}(f-\xi)^{1/2}}d\xi + \int_{\frac{1+m}{2}}^{m} \frac{(\xi-d)^{\alpha}}{(\xi+a)^{1/2}\xi^{\alpha}(\xi-1)^{1/2}(f-\xi)^{1/2}}d\xi\right\}$$

......(4.29)

Substituting $\xi - 1 = v^2$ for the first integral and $f - \xi = v^2$ for the second integral above, where v is a dummy variable, the improper integral in equation 4.29 is converted to the following proper integral :

$$T_1 - T_M = M\left\{2\int_0^{\sqrt{\frac{m-1}{2}}} \frac{(v^2+1-d)^{\pi}}{(v^2+1+a)^{1/2}(v^2+1)^{\pi}(f-v^2-1)^{1/2}}dv + 2\int_{\sqrt{f-m}}^{\sqrt{\frac{2f-1-m}{2}}} \frac{(f-v^2-d)^{\pi}}{(f-v^2+a)^{1/2}(f-v^2)^{\pi}(f-v^2-1)^{1/2}}dv\right\}$$

......(4.30)

Further substituting :

$$v = \sqrt{(m-1)/2}\left[\frac{1+\chi}{2}\right] \text{ and } dv = \frac{\sqrt{(m-1)/2}}{2}d\chi \quad \text{for the first integral,}$$

$$v = \frac{\sqrt{\frac{2f-1-m}{2}} - \sqrt{f-m}}{2}\chi + \frac{\sqrt{\frac{2f-1-m}{2}} + \sqrt{f-m}}{2}$$

$$\text{and } dv = \frac{\sqrt{\frac{2f-1-m}{2}} - \sqrt{f-m}}{2}d\chi = f(\chi) \quad \text{for the second integral}$$

where χ is a dummy variable, the lower and upper limits of integration in equation 4.30 are converted to –1 and 1 respectively and it reduces to

$$T_1 - T_M = M\sqrt{\frac{m-1}{2}}\left\{\int_{-1}^{1} \frac{\left[\left(\frac{m-1}{2}\right)\left(\frac{1+\chi}{2}\right)^2 + 1 - d\right]^{\pi}}{\left[\left(\frac{m-1}{2}\right)\left(\frac{1+\chi}{2}\right)^2 + 1\right]^{\pi}\left[\left(\frac{m-1}{2}\right)\left(\frac{1+\chi}{2}\right)^2 + 1 + a\right]^{1/2}\left[f - \left(\frac{m-1}{2}\right)\left(\frac{1+\chi}{2}\right)^2 - 1\right]^{1/2}}d\chi\right\} +$$

$$M\left(\sqrt{\frac{2f-1-m}{2}} - \sqrt{f-m}\right)\left\{\int_{-1}^{1} \frac{\left[f - f^2(\chi) - d\right]^{\pi}}{\left[f - f^2(\chi)\right]^{\pi}\left[f - f^2(\chi) + a\right]^{1/2}\left[f - f^2(\chi) - 1\right]^{1/2}}d\chi\right\}$$

......(4.31)

IV.2.2 Mapping of The Complex Potential w-Plane to The Auxiliary ξ-Plane

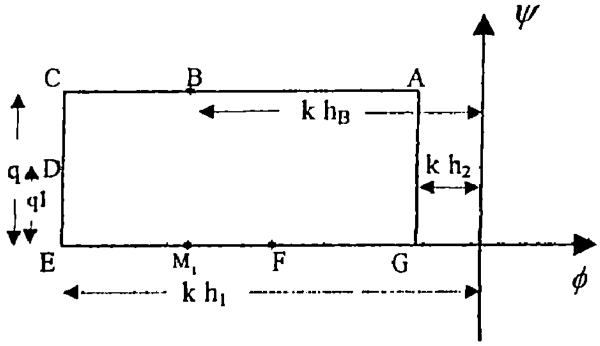

Fig. IV.3 w-plane (w=φ+iψ)

The conformal mapping of the w-plane onto the lower half of the ξ-plane is given by :

$$\frac{dw}{d\xi}=\frac{M_2}{(\xi+a)^{1/2}(\xi)^{1/2}(\xi-1)^{1/2}} \tag{4.32}$$

For points C to E, ξ=ξ' and $0 \le \xi' \le 1$, the corresponding w is given by :

$$w=M_2\int_0^{\xi'}\frac{d\xi}{(\xi+a)^{1/2}(\xi)^{1/2}(\xi-1)^{1/2}}-kh_1+iq \tag{4.33}$$

where h_1 is head at the stream and q is rate of seepage from half section of the stream. For Point E, ξ'= 1 and $w_E=-kh_1$; hence,

$$-kh_1=M_2\int_0^{1}\frac{d\xi}{(\xi+a)^{1/2}(\xi)^{1/2}\sqrt{-1}(1-\xi)^{1/2}}-kh_1+iq$$

or

$$q=M_2\int_0^{1}\frac{d\xi}{(\xi+a)^{1/2}(\xi)^{1/2}(1-\xi)^{1/2}}$$

$$=M_2\frac{2}{\sqrt{1+a}}F\left(\sin^{-1}\sqrt{\frac{(1+a)\xi}{\xi+a}},\frac{1}{\sqrt{1+a}}\right)\Bigg|_0^1 \tag{4.34}$$

Applying the limit, hence,

$$q=M_2\frac{2}{\sqrt{1+a}}F\left(\frac{\pi}{2},\frac{1}{\sqrt{1+a}}\right) \tag{4.35}$$

and

$$M_2 = \frac{q\sqrt{1+a}}{2F\left(\frac{\pi}{2}, \frac{1}{\sqrt{1+a}}\right)} \tag{4.36}$$

where

$$F\left(\frac{\pi}{2}, \frac{1}{\sqrt{1+a}}\right) = F\left(\frac{\pi}{2}, m_1\right) \tag{4.37}$$

is complete elliptic integral of first kind, i.e.

$$F\left(\frac{\pi}{2}, m_1\right) = \int_0^{\pi/2} \frac{d\varphi}{\sqrt{1 - m_1^2 \sin^2\varphi}} \tag{4.38}$$

The complete elliptic integral of the first kind is evaluated using Gauss quadrature as described below:

Substituting $\varphi=\pi/4\ (1+\chi)$, where χ is a dummy variable, the lower and upper limits of the elliptic integral are converted to –1 and 1 respectively and it reduces to:

$$F\left(\frac{\pi}{2}, m_1\right) = \frac{\pi}{4} \int_{-1}^{1} \frac{d\chi}{\sqrt{1 - m_1^2 \sin^2\left(\frac{\pi(1+\chi)}{4}\right)}} \tag{4.39}$$

For point B to C, $\xi=\xi'$ and $-b \le \xi' \le 0$, the corresponding w is given by:

$$w = M_2 \int_{-b}^{\xi'} \frac{d\xi}{(\xi + a)^{1/2} (\xi)^{1/2} (\xi - 1)^{1/2}} - kh_B + iq \tag{4.40}$$

For point C, $\xi' = 0$ and $w_C = -kh_1+iq$; hence,

$$-kh_1 + iq = M_2 \int_{-b}^{0} \frac{d\xi}{(\xi + a)^{1/2} (\xi)^{1/2} (\xi - 1)^{1/2}} - kh_B + iq$$

or

$$k(h_1 - h_B) = M_2 \int_{-b}^{0} \frac{d\xi}{(\xi + a)^{1/2} (0 - \xi)^{1/2} (1 - \xi)^{1/2}}$$

$$= M_2 \frac{2}{\sqrt{1+a}} F(\vartheta, m_1)\Big|_{-b}^{0} \tag{4.41}$$

in which

$$\vartheta = \sin^{-1}\sqrt{\frac{(1+a)(-\xi)}{a(1-\xi)}} \tag{4.42}$$

$$m_1^2 = \frac{a}{1+a} \tag{4.43}$$

For point B, ξ = -b and

$$\vartheta = \sin^{-1}\sqrt{\frac{(1+a)b}{a(1+b)}}$$

hence,

$$q = \frac{k(h_1 - h_B)F\left(\pi/2, 1/\sqrt{(1+a)}\right)}{F\left(\sin^{-1}\sqrt{(1+a)b/a(1+b)}, \sqrt{a/(1+a)}\right)} \tag{4.44}$$

in which q is seepage rate for half section of the stream.

Applying the boundary condition at A. We derive in similar manner, the relation

$$q = \frac{k(h_1 - h_2)F\left(\pi/2, 1/\sqrt{(1+a)}\right)}{F\left(\pi/2, \sqrt{a/(1+a)}\right)}$$

or

$$\frac{q}{k(h_1 - h_2)} = \frac{\Gamma_r}{k} = \frac{F\left(\pi/2, 1/\sqrt{(1+a)}\right)}{F\left(\pi/2, \sqrt{a/(1+a)}\right)} \tag{4.45}$$

For point D to E, ξ=ξ' and d ≤ ξ' ≤ 1, the corresponding w is given by

$$w = M_2\int_d^{\xi'} \frac{d\xi}{(\xi + a)^{1/2}(\xi)^{1/2}(\xi - 1)^{1/2}} - kh_1 + iq_1 \tag{4.46}$$

For point E, ξ' = 1 and $w_E = -kh_1$; hence,

$$-kh_1 = M_2\int_d^{1} \frac{d\xi}{(\xi + a)^{1/2}(\xi)^{1/2}\sqrt{-1}(1-\xi)^{1/2}} - kh_1 + iq_1$$

or

$$q_1 = M_2\int_d^{1} \frac{d\xi}{(\xi + a)^{1/2}(\xi)^{1/2}(1-\xi)^{1/2}}$$

$$= M_2 \frac{2}{\sqrt{1+a}} F(\vartheta, m_1)\Big|_d^1 \tag{4.47}$$

in which

$$\vartheta = \sin^{-1}\sqrt{1-\xi} \qquad (4.48)$$

$$m_1^2 = \frac{1}{1+a} \qquad (4.49)$$

Applying the limit, hence,

$$\frac{q_1}{q} = \frac{F\left(\sin^{-1}\sqrt{1-d},\ 1\Big/\sqrt{(1+a)}\right)}{F\left(\pi\big/2,\ 1\Big/\sqrt{(1+a)}\right)} \qquad (4.50)$$

in which q_1 is seepage through the stream bed.

For point E to F, $\xi = \xi'$ and $1 \le \xi' \le f$, the corresponding w is given by

$$w = M_2 \int_1^{\xi'} \frac{d\xi}{(\xi + a)^{1/2}(\xi)^{1/2}(\xi - 1)^{1/2}} - kh_1 \qquad (4.51)$$

For point M_1, $\xi' = m$, and $w_M = -kh_M$, where h_M is head at point M_1; hence,

$$-kh_M = M_2 \int_1^{m} \frac{d\xi}{(\xi + a)^{1/2}(\xi)^{1/2}(\xi - 1)^{1/2}} - kh_1$$

or

$$k(h_1 - h_M) = M_2 \int_1^{m} \frac{d\xi}{(\xi + a)^{1/2}(\xi)^{1/2}(\xi - 1)^{1/2}}$$

$$= M_2 \frac{2}{\sqrt{1+a}} F(\vartheta, m_1)\Big|_1^m \qquad (4.52)$$

in which

$$\vartheta = \sin^{-1}\sqrt{\frac{\xi - 1}{\xi}} \qquad (4.53)$$

$$m_1^2 = \frac{a}{1+a} \qquad (4.54)$$

Applying the limit

$$\frac{q}{k(h_1 - h_m)} = \frac{F\left(\pi\big/2, \sqrt{1\big/(1+a)}\right)}{F\left(\sin^{-1}\sqrt{(m-1)\big/m}, \sqrt{a\big/(1+a)}\right)} \qquad (4.55)$$

IV.3 SUBSTITUTE LENGTH

Let us consider the location of a piezometer at a distance L_B form the stream bank. The combined aquifer and stream resistance R_r, up to length L_B from equation 4.44 is given by :

$$R_r = \frac{F\left(\sin^{-1}\sqrt{\frac{(1+a)b}{a(1+b)}}, \sqrt{\frac{a}{(1+a)}}\right)}{k\, F\left(\frac{\pi}{2}, \sqrt{\frac{1}{(1+a)}}\right)} \tag{4.56}$$

Let ΔL be the extra length, whose resistance is equal to the extra resistance owing to flow convergence within length L_B. For uniform rectilinear flow, the aquifer resistance R_a of length $L_B+\Delta L$ is

$$R_a = \frac{L_B + \Delta L}{kT_2} \tag{4.57}$$

Since $R_r = R_a$, we get

$$\frac{\Delta L}{T_2} = \frac{F\left(\sin^{-1}\sqrt{\frac{(1+a)b}{a(1+b)}}, \sqrt{\frac{a}{1+a}}\right)}{T_2 F\left(\frac{\pi}{2}, \sqrt{\frac{1}{1+a}}\right)} - \frac{L_B}{T_2} \tag{4.58}$$

IV.4 RESULTS AND DISCUSSION

Influent seepage, reach transmissivity, and substitute length for stream in a finite length of aquifer are presented. For length of aquifer greater than five times aquifer thickness measured from center of the stream, the flow characteristic remain same as that of semi-infinite aquifer.

From the relationship of reach transmissivity with distance of the point of observation of piezometric head, it is seen that reach transmissivity increases with depth of penetration of the stream bed and with increase in width of the stream.

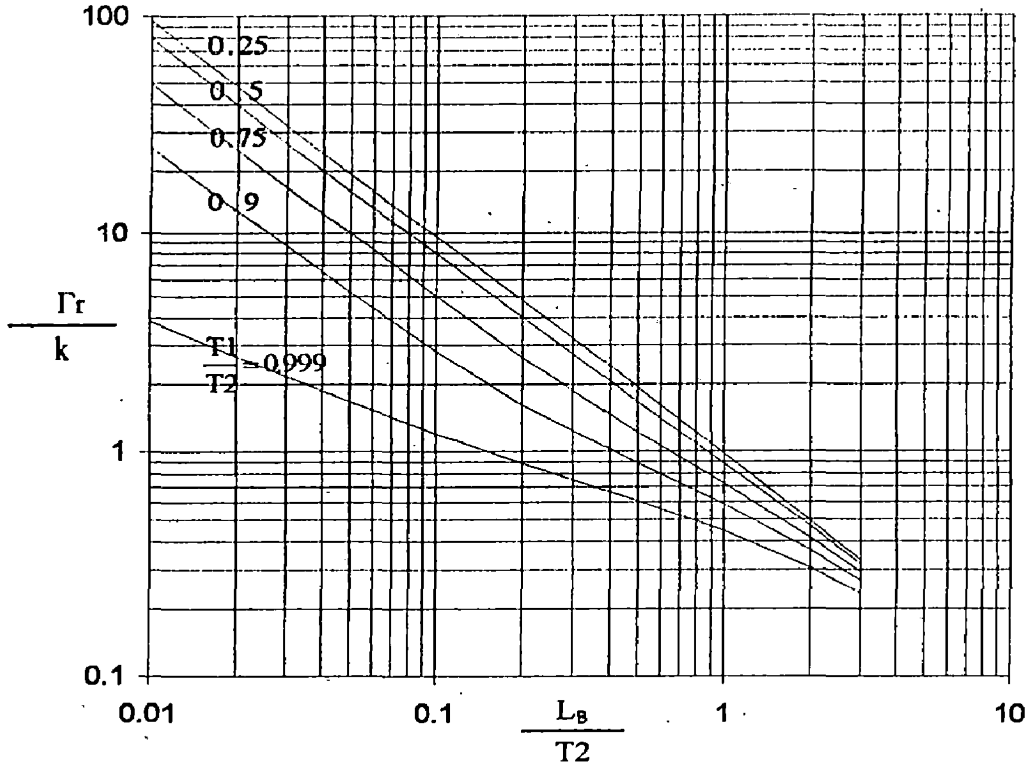

Fig. IV.4a **Variation of** $\frac{\Gamma r}{k}$ *or* $\frac{q}{k\Delta h}$ **with distance of piezometer from stream bank for different depth of penetration of the stream, for B/T2=0.10**

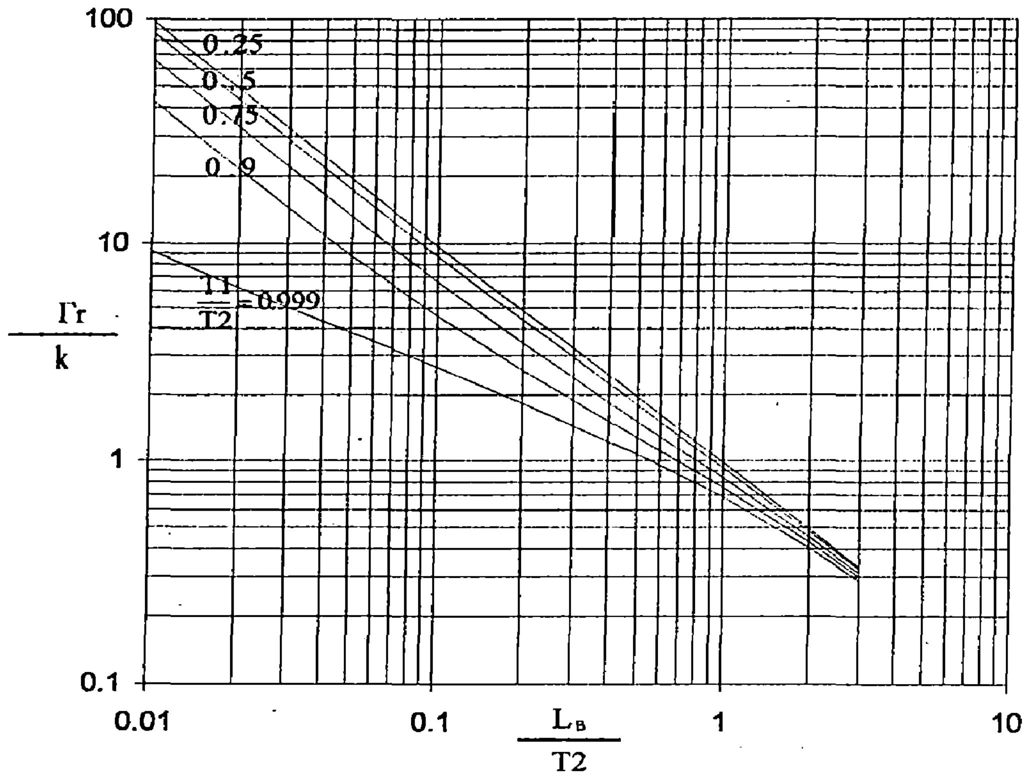

Fig. IV.4b ***Variation of*** $\frac{\Gamma r}{k}$ *or* $\frac{q}{k\Delta h}$ ***with distance of piezometer from stream*** **bank for different depth of penetration of the stream, for B/T2=1**

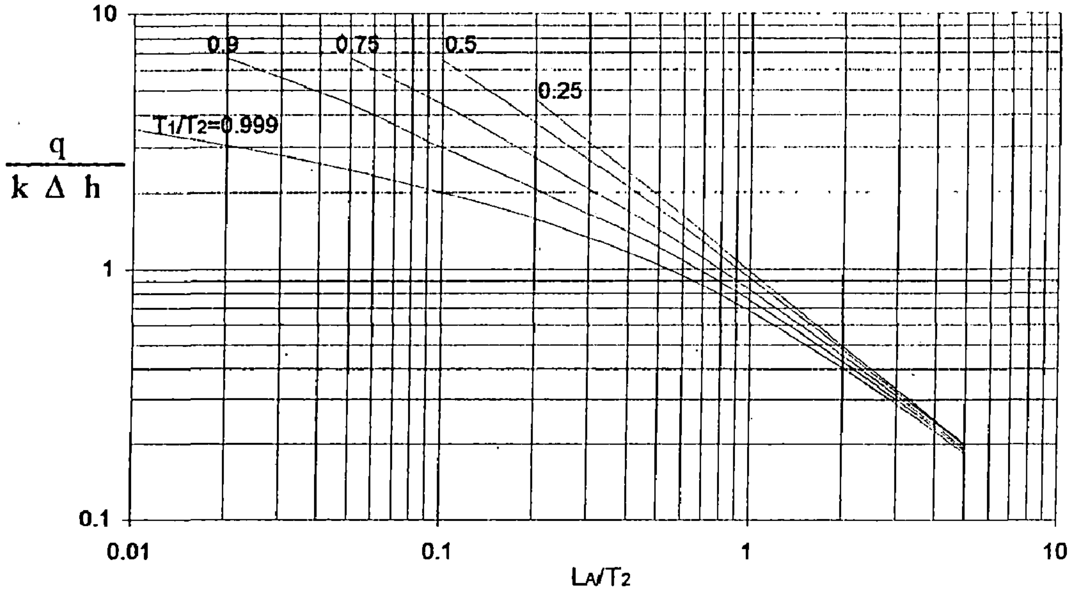

Fig. IV.5 Variation of $\frac{q}{k\,\Delta\,h}$ with distance of aquifer boundary from stream bank for B/T2=1

CHAPTER V
SEEPAGE FROM A STREAM IN AN UNCONFINED AQUIFER

V.1 GENERAL

A river, comprising a boundary of flow, is encountered in regional ground water flow modeling. The river reach can approximate a boundary of prescribed head, only where it fully penetrates an aquifer and has a large discharge as compared to the exchange of flow between the river reach and the aquifer. However, a situation is rarely seen where a river completely penetrates the aquifer. In the case of a partially penetrating river of large discharge, the exchange of flow between the river and the aquifer, which acts in similar manner to leakage through an overlying stratum, has to be taken into account besides treating the river as a boundary of prescribed head (Rushton and Redshaw, 1972). Mishra and Seth has analyzed seepage from a river of large width.

In the present using Zhukovsky's function and Schwarz-Christoffel conformal mapping technique, unconfined seepage from a stream of finite width has been analyzed for a steady state condition.

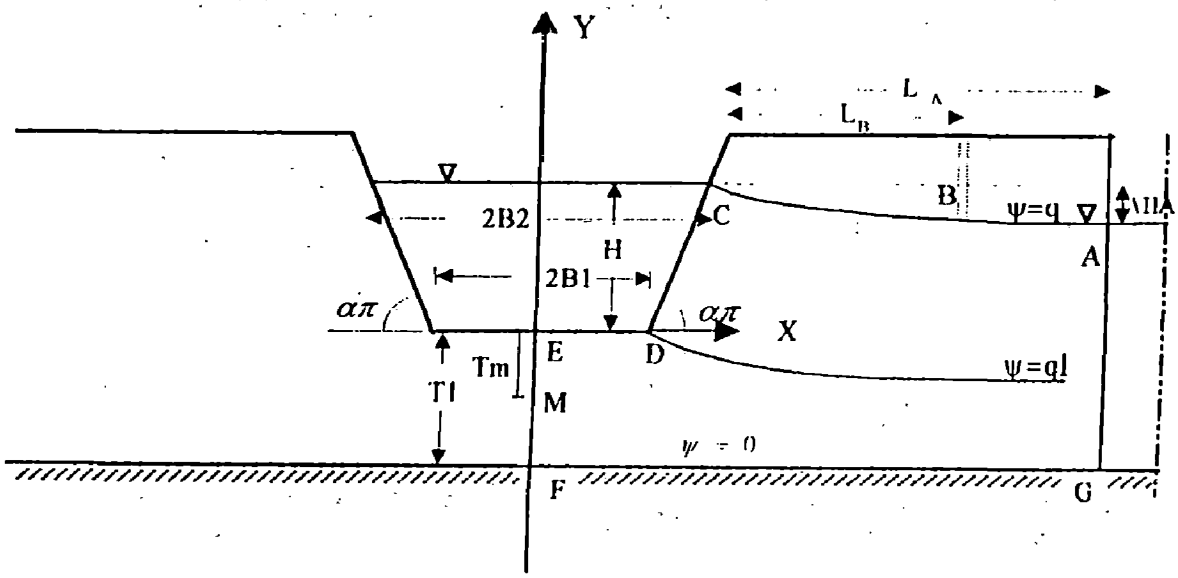

Fig. V.1 Physical flow domain in z - plane

Figure V.1 shows as a schematic cross section of stream in Z plane. The stream is partially penetrating and has finite width. An impervious stratum is underlying at a depth T_l below the streambed. If the width of streambed is less than $4T_l$, the stream can be regarded to have finite width. This specification of finite width is based on the empirical rule (Aravin and Numerov, 1965) followed in preparing the scale model of a prototype for seepage study in homogeneous soil. The depth of water in the stream is H. At a distance L_A from the stream bank, the water table in the aquifer is at a depth ΔH_A below the level of water in the stream. It is required to find the quantity of water recharged by stream to the aquifer.

V.2 ANALYSIS

The pertinent complex potential plane w, where $w = \phi + i\psi$, is shown in figure V.2, in which ψ is the stream function and ϕ is the velocity potential function defined as (Harr, 1962)

$$\phi = -k\left(\frac{p}{\gamma_w} + y\right) + c \tag{5.1}$$

where k is the coefficient of permeability, p is the pressure, γ_w is the unit weight of water, y is the elevation head, and c is an arbitrary constant which has been assumed to be zero.

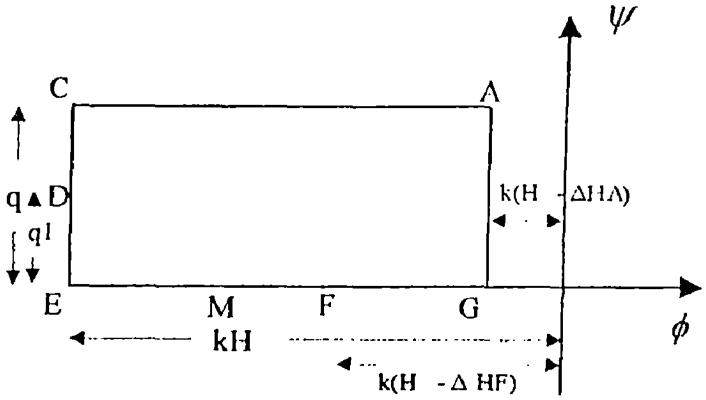

Fig. V.2 w - plane

V.2.1 Mapping of Zhukovsky's θ-Plane onto an auxiliary ξ-Plane

The flow domain consists of a phreatic line which is curvilinear and unknown a priori. Conformal mapping can be applied to analyze the unconfined flow after transforming the flow domain to Zhukovsky's θ plane (Zhukovsky, 1949). The pertinent θ plane, in which

$$\theta = z + \frac{iw}{k}$$

$$= \left(x - \frac{\psi}{k}\right) + i\left(y + \frac{\phi}{k}\right) \qquad (5.2)$$

is shown in Figure V.3. The loci of CD and FG are not known. CD and FG are idealized as straight lines as shown.

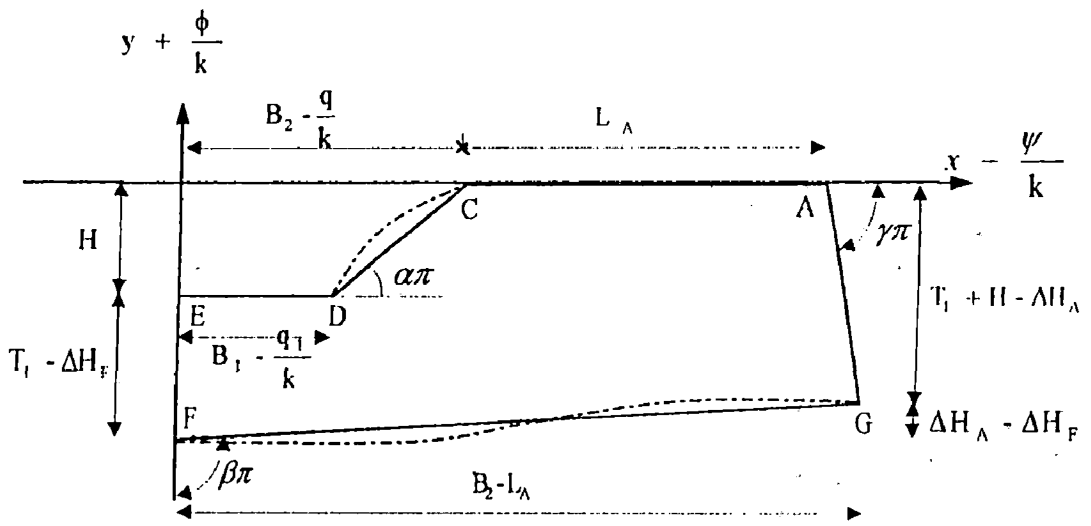

Fig. V.3 θ - plane

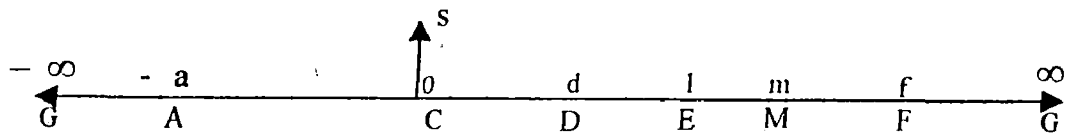

Fig. V.4 ξ - plane

According to Schwarz-Christoffel transformation, the conformal mapping of the polygon θ plane onto upper the half of the auxiliary ξ plane is given by (Harr, 1962).

$$\theta = M\int \frac{(\xi - d)^{\alpha}}{(\xi + a)^{\gamma}\,\xi^{\alpha}\,(\xi - 1)^{1/2}(\xi - f)^{\beta}}\,d\xi + N \qquad (5.3)$$

The vertices A, C, D, E, F, and G are mapped onto points –a, 0, d, 1, f and ∞ respectively on the real axis of the ξ plane. M is complex constant to be evaluated.

Values of angle α, β and γ in equation 5.3 refer to Figure V.3 and they are found to be

$$\alpha = \frac{\tan^{-1}\left(\dfrac{H}{\left(B_2 - \dfrac{q}{k}\right) - \left(B_1 - \dfrac{q_1}{k}\right)}\right)}{\pi} \tag{5.4}$$

$$\beta = \frac{\tan^{-1}\left(\dfrac{\Delta H_A - \Delta H_F}{B_2 + L_A}\right) + \dfrac{\pi}{2}}{\pi} \tag{5.5}$$

$$\gamma = \frac{\tan^{-1}\left(\dfrac{T_1 + H - \Delta H_A}{\dfrac{q}{k}}\right)}{\pi} \tag{5.6}$$

For a point between A to C, $\xi = \xi'$ and $-\infty \le \xi' \le 0$

At point A, $\theta = \theta_A = \left(B_2 + L_A - \dfrac{q}{k}\right)$ and $\xi' = -a$, and

at point C, $\theta = \theta_C = \left(B_2 - \dfrac{q}{k}\right)$ and $\xi' = 0$.

Hence,

$$B_2 - \frac{q}{k} = M\int_{-a}^{0} \frac{(\xi - d)^{\alpha}}{(\xi + a)^{\gamma}\xi^{\alpha}(\xi - 1)^{1/2}(\xi - f)^{\beta}} d\xi + B_2 - \frac{q}{k} + L_A$$

or

$$L_A = -M\int_{0}^{-a} \frac{(d - \xi)^{\alpha}}{(\xi + a)^{\gamma}(-\xi)^{\alpha}(1 - \xi)^{1/2}(f - \xi)^{\beta}} d\xi \tag{5.7}$$

Substituting $\xi = -u$, and $d\xi = -du$, where u is a dummy variable

$$L_A = M\int_{0}^{a} \frac{(d + u)^{\alpha}}{(a - u)^{\gamma}\, u^{\alpha}\, (1 + u)^{1/2}(f + u)^{\beta}} du$$

u = 0 and u = a are singular points. Splitting the limit 0 to a into 0 to ½ a, and ½ a to a

$$L_A = M\left\{\int_0^{a/2}\frac{(d+u)^{\alpha}}{(a-u)^{\gamma}\,u^{\alpha}\,(1+u)^{1/2}(f+u)^{\beta}}du + \int_{a/2}^{a}\frac{(d+u)^{\alpha}}{(a-u)^{\gamma}\,u^{\alpha}\,(1+u)^{1/2}(f+u)^{\beta}}du\right\}$$

......(5.8)

Substituting u = v^2 for the first integral and a-u = v^2 for the second integral, where v is a dummy variable, the improper integral above is converted to the following proper integral:

$$L_A = M\left\{\int_0^{\sqrt{a/2}}\frac{2v^{1-2\alpha}(d+v^2)^{\alpha}}{(a-v^2)^{\gamma}(1+v^2)^{1/2}(f+v^2)^{\beta}}dv + \int_0^{\sqrt{a/2}}\frac{2v^{1-2\gamma}(d+a-v^2)^{\alpha}}{(a-v^2)^{\alpha}(1+a-v^2)^{1/2}(f+a-v^2)^{\beta}}dv\right\}$$

......(5.9)

substituting $v = \sqrt{\frac{a}{2}}\left(\frac{1+\chi}{2}\right)$ and $dv = \frac{\sqrt{a/2}}{2}d\chi$,

where χ is a dummy variable, the lower and upper limits of integrals above are converted to –1 and 1 respectively and it reduces to

$$L_A = M\sqrt{\frac{a}{2}}\left\{\int_{-1}^{1}\frac{\left[\sqrt{\frac{a}{2}}\left(\frac{1+\chi}{2}\right)\right]^{1-2\alpha}\left[d+\frac{a}{2}\left(\frac{1+\chi}{2}\right)^2\right]^{\alpha}}{\left[a-\frac{a}{2}\left(\frac{1+\chi}{2}\right)^2\right]^{\gamma}\left[1+\frac{a}{2}\left(\frac{1+\chi}{2}\right)^2\right]^{1/2}\left[f+\frac{a}{2}\left(\frac{1+\chi}{2}\right)^2\right]^{\beta}}d\chi\right\}$$

$$+M\sqrt{\frac{a}{2}}\left\{\int_{-1}^{1}\frac{\left[\sqrt{\frac{a}{2}}\left(\frac{1+\chi}{2}\right)\right]^{1-2\gamma}\left[d+a-\frac{a}{2}\left(\frac{1+\chi}{2}\right)^2\right]^{\alpha}}{\left[a-\frac{a}{2}\left(\frac{1+\chi}{2}\right)^2\right]^{\alpha}\left[1+a-\frac{a}{2}\left(\frac{1+\chi}{2}\right)^2\right]^{1/2}\left[f+a-\frac{a}{2}\left(\frac{1+\chi}{2}\right)^2\right]^{\beta}}d\chi\right\}$$

......(5.10)

or $L_A = M\sqrt{(a/2)}\ \{I_1 + I_2\}$

The constant M in equation 5.7 is found to be

$$M = \frac{L_A}{\sqrt{a/2}\{I_1 + I_2\}} \qquad (5.11)$$

For point between B and C, $\xi = \xi'$, $-b \le \xi' \le 0$

For pint B, $\theta = \theta_B = \left(B_2 + L_B - \frac{q}{k}\right)$ and $\xi' = -b$, and for point C, $\theta = \theta_C = \left(B_2 - \frac{q}{k}\right)$ and $\xi' = 0$

Hence,

$$B_2 - \frac{q}{k} = M\int_{-b}^{0} \frac{(\xi - d)^\alpha}{(\xi + a)^\gamma \xi^\alpha (\xi - 1)^{1/2} (\xi - f)^\beta} d\xi + B_2 - \frac{q}{k} + L_B$$

$$L_B = -M\int_{0}^{-b} \frac{(d - \xi)^\alpha}{(\xi + a)^\gamma (-\xi)^\alpha (1 - \xi)^{1/2} (f - \xi)^\beta} d\xi \qquad (5.12)$$

Substituting $\xi = -u$, and $d\xi = -du$

$$L_B = M\int_{0}^{b} \frac{(d + u)^\alpha}{(a - u)^\gamma u^\alpha (1 + u)^{1/2} (f + u)^\beta} du$$

Dividing the integration into two parts

$$L_B = M\left\{\int_{0}^{b/2} \frac{(d + u)^\alpha}{(a - u)^\gamma u^\alpha (1 + u)^{1/2} (f + u)^\beta} du + \int_{b/2}^{b} \frac{(d + u)^\alpha}{(a - u)^\gamma u^\alpha (1 + u)^{1/2} (f + u)^\beta} du\right\} \qquad (5.13)$$

Substituting $u = v^2$ for the first integral and $a - u = v^2$ for the second integral, where v is a dummy variable, the improper integral above is converted to the following proper integral:

$$L_B = M\left\{\int_{0}^{\sqrt{b/2}} \frac{2v^{1-2\alpha} (d + v^2)^\alpha}{(a - v^2)^\gamma (1 + v^2)^{1/2} (f + v^2)^\beta} dv + \int_{\sqrt{a-b}}^{\sqrt{a-\frac{b}{2}}} \frac{2v^{1-2\gamma} (d + a - v^2)^\alpha}{(a - v^2)^\alpha (1 + a - v^2)^{1/2} (f + a - v^2)^\beta} dv\right\}$$

......(5.14)

substituting $v = \sqrt{\frac{b}{2}}\left(\frac{1+\chi}{2}\right) = f_1(\chi)$ and $dv = \frac{\sqrt{b/2}}{2} d\chi$ for first integral

substituting $v = \left(\frac{\sqrt{a - \frac{b}{2}} - \sqrt{a - b}}{2}\right)\chi + \left(\frac{\sqrt{a - \frac{b}{2}} + \sqrt{a - b}}{2}\right) = f_2(\chi)$

and $dv = \left(\frac{\sqrt{a - \frac{b}{2}} - \sqrt{a - b}}{2}\right) d\chi$ for second integral,

where χ is a dummy variable, the lower and upper limits of integrals above are converted to -1 and 1 respectively and it reduces to

$$L_B = M\sqrt{\frac{b}{2}}\left\{\int_{-1}^{1}\frac{f_1(\chi)^{1-2\alpha}(d+f_1^2(\chi))^{\alpha}}{(a-f_1^2(\chi))^{\gamma}(1+f_1^2(\chi))^{1/2}(f+f_1^2(\chi))^{\beta}}d\chi\right\}$$

$$+M\left(\sqrt{a-\frac{b}{2}}-\sqrt{a-b}\right)\left\{\int_{-1}^{1}\frac{f_2(\chi)^{1-2\gamma}(d+a-f_2^2(\chi))^{\alpha}}{(a-f_2^2(\chi))^{\alpha}(1+a-f_2^2(\chi))^{1/2}(f+a-f_2^2(\chi))^{\beta}}d\chi\right\} \quad (5.15)$$

For point C to D, $\xi = \xi'$ and $0 \le \xi' \le d$

For point C, $\theta = \theta_C = \left(B_2 - \frac{q}{k}\right)$, and $\xi' = 0$ and

for point D, $\theta = \theta_D = \left(B_1 - \frac{q_1}{k}\right) - iH$, and $\xi' = d$

Applying these conditions

$$B_1 - \frac{q_1}{k} - iH = M\int_0^d \frac{(\xi - d)^{\alpha}}{(\xi + a)^{\gamma}\xi^{\alpha}(\xi - 1)^{1/2}(\xi - f)^{\beta}}d\xi + B_2 - \frac{q}{k}$$

or

$$\left(B_1 - B_2 - \frac{q_1}{k} + \frac{q}{k}\right)i + H = M\int_0^d \frac{(d-\xi)^{\alpha}}{(\xi + a)^{\gamma}\xi^{\alpha}(1-\xi)^{1/2}(f-\xi)^{\beta}}d\xi$$

Equating the moduli on either side

$$\sqrt{\left((B_2 - B_1) - (\frac{q}{k} - \frac{q_1}{k})\right)^2 + H^2} = M\int_0^d \frac{(d-\xi)^{\alpha}}{(\xi + a)^{\gamma}\xi^{\alpha}(1-\xi)^{1/2}(f-\xi)^{\beta}}d\xi \quad (5.16)$$

Substituting $\xi = v^2$, $d\xi = 2v\,dv$, where v is a dummy variable, the improper integral above is converted to the following proper integral:

$$\sqrt{\left((B_2 - B_1) - (\frac{q}{k} - \frac{q_1}{k})\right)^2 + H^2} = 2M\int_0^{\sqrt{d}} \frac{v^{1-2\alpha}(d - v^2)^{\alpha}}{(v^2 + a)^{\gamma}(1 - v^2)^{1/2}(f - v^2)^{\beta}}dv \quad (5.17)$$

substituting

$$v = \sqrt{d}\frac{1+\chi}{2} \text{ and } dv = \frac{\sqrt{d}}{2}d\chi$$

where χ is a dummy variable, the lower and upper limits of integrals above are converted to -1 and 1 respectively and it reduces to

$$\sqrt{\left((B_2 - B_1) - (\frac{q}{k} - \frac{q_1}{k})\right)^2 + H^2} = M\sqrt{d}\int_{-1}^{1} \frac{\left(\sqrt{d}\,\frac{1+\chi}{2}\right)^{1-2\alpha}\left(d - d\left(\frac{1+\chi}{2}\right)^2\right)^{\alpha}}{\left(d\left(\frac{1+\chi}{2}\right)^2 + a\right)^{\gamma}\left(1 - d\left(\frac{1+\chi}{2}\right)^2\right)^{1/2}\left(f - d\left(\frac{1+\chi}{2}\right)^2\right)^{\beta}} d\chi \qquad \ldots\ldots(5.18)$$

For point D to E, $\xi = \xi'$ and $d \le \xi' \le 1$

For point D, $\theta = \theta_D = \left(B_1 - \frac{q_1}{k}\right) - iH$, and $\xi' = d$; and

for point E, $= \theta_E = -iH$, and $\xi' = 1$; hence,

$$-iH = M\int_{d}^{1} \frac{(\xi - d)^{\alpha}}{(\xi + a)^{\gamma}\xi^{\alpha}(\xi - 1)^{1/2}(\xi - f)^{\beta}} d\xi + B_1 - \frac{q_1}{k} - iH$$

$$B_1 - \frac{q_1}{k} = M\int_{d}^{1} \frac{(\xi - d)^{\alpha}}{(\xi + a)^{\gamma}\xi^{\alpha}(1-\xi)^{1/2}(f-\xi)^{\beta}} d\xi \qquad (5.19)$$

Substituting $1-\xi = v^2$, $d\xi = -2v\,dv$, at $\xi=d$, $v=\sqrt{(1-d)}$ and at $\xi=1$, $v=0$, where v is a dummy variable, the improper integral above is converted to the following proper integral:

$$B_1 - \frac{q_1}{k} = 2M\int_{0}^{\sqrt{1-d}} \frac{(1 - v^2 - d)^{\alpha}}{(1 - v^2 + a)^{\gamma}(1 - v^2)^{\alpha}(f - 1 + v^2)^{\beta}} dv \qquad (5.20)$$

substituting

$$v = \sqrt{1-d}\,\frac{1+\chi}{2} \quad \text{and} \quad dv = \frac{\sqrt{1-d}}{2} d\chi$$

where χ is a dummy variable, the lower and upper limits of integrals above are converted to -1 and 1 respectively and it reduces to

$$B_1 - \frac{q_1}{k} = M\sqrt{1-d}\int_{-1}^{1} \frac{\left(1 - (1-d)\left(\frac{1+\chi}{2}\right)^2 - d\right)^{\alpha}}{\left(1 - (1-d)\left(\frac{1+\chi}{2}\right)^2 + a\right)^{\gamma}\left(1 - (1-d)\left(\frac{1+\chi}{2}\right)^2\right)^{\alpha}\left(f - 1 + (1-d)\left(\frac{1+\chi}{2}\right)^2\right)^{\beta}} d\chi \qquad \ldots\ldots(5.21)$$

For point E to F, $\xi = \xi'$ and $1 \le \xi' \le f$

For point E, $\theta = \theta_E = -iH$, and $\xi' = 1$ and for point F, $\theta = \theta_F = -i(T_1+H-\Delta H_F)$, and $\xi' = f$, hence,

$$-i(T_1 + H - \Delta H_F) = M\int_1^f \frac{(\xi - d)^\alpha}{(\xi + a)^\gamma \xi^\alpha (\xi - 1)^{1/2} (\xi - f)^\beta} d\xi - iH$$

or

$$T_1 - \Delta H_F = M\int_1^f \frac{(\xi - d)^\alpha}{(\xi + a)^\gamma \xi^\alpha (\xi - 1)^{1/2} (f - \xi)^\beta} d\xi$$

Splitting the integration into two parts

$$T_1 - \Delta H_F = M \int_1^{\frac{1+f}{2}} \frac{(\xi - d)^\alpha}{(\xi + a)^\gamma \xi^\alpha (\xi - 1)^{1/2} (f - \xi)^\beta} d\xi + M \int_{\frac{1+f}{2}}^{f} \frac{(\xi - d)^\alpha}{(\xi + a)^\gamma \xi^\alpha (\xi - 1)^{1/2} (f - \xi)^\beta} d\xi$$

......(5.22)

Substituting $\xi-1 = v^2$ in the first integral and $f-\xi = v^{10}$ in the second integral, where v is a dummy variable, the improper integral above is converted to the following proper integral:

$$T_1 - \Delta H_F = 2M \int_0^{\sqrt{\frac{f-1}{2}}} \frac{(v^2 + 1 - d)^\alpha}{(v^2 + 1 + a)^\gamma (v^2 + 1)^\alpha (f - v^2 - 1)^\beta} dv$$

$$+ 10M \int_0^{\sqrt[10]{\frac{f-1}{2}}} \frac{v^{9-10\beta} (f - v^{10} - d)^\alpha}{(f - v^{10} + a)^\gamma (f - v^{10})^\alpha (f - v^{10} - 1)^{1/2}} dv \qquad (5.23)$$

The substitution is valid for $\beta \le 9/10$.

Substituting

$v = \sqrt{\frac{f-1}{2}\left(\frac{1+\chi}{2}\right)}$ and $dv = \frac{\sqrt{\frac{f-1}{2}}}{2} d\chi$ in the first integral and

$v = \sqrt[10]{\frac{f-1}{2}\left(\frac{1+\chi}{2}\right)}$ and $dv = \frac{\sqrt[10]{\frac{f-1}{2}}}{2} d\chi$ in the second integral,

where χ is a dummy variable, the lower and upper limits of integrals above are converted to -1 and 1 respectively and equation 5.23 reduces to

$$T_I - \Delta H_F = M\sqrt{\frac{f-1}{2}}\int_{-1}^{1}\frac{\left(\frac{f-1}{2}\left(\frac{1+\chi}{2}\right)^2+1-d\right)^{\alpha}}{\left(\frac{f-1}{2}\left(\frac{1+\chi}{2}\right)^2+1+a\right)^{\gamma}\left(\frac{f-1}{2}\left(\frac{1+\chi}{2}\right)^2+1\right)^{\alpha}\left(f-\frac{f-1}{2}\left(\frac{1+\chi}{2}\right)^2-1\right)^{\beta}}d\chi$$

$$+5M\sqrt[10]{\frac{f-1}{2}}\int_{-1}^{1}\frac{\left(\sqrt[10]{\frac{f-1}{2}}\left(\frac{1+\chi}{2}\right)\right)^{9-10\beta}\left(f-\frac{f-1}{2}\left(\frac{1+\chi}{2}\right)^{10}-d\right)^{\alpha}}{\left(f-\frac{f-1}{2}\left(\frac{1+\chi}{2}\right)^{10}+a\right)^{\gamma}\left(f-\frac{f-1}{2}\left(\frac{1+\chi}{2}\right)^{10}\right)^{\alpha}\left(f-\frac{f-1}{2}\left(\frac{1+\chi}{2}\right)^{10}-1\right)^{1/2}}d\chi$$

......(5.24)

For point E to M, $\xi = \xi'$ and $1 \le \xi' \le m \le f$

For point E, $\theta = \theta_E = -iH$, and $\xi' = 1$

For point M, $\theta = \theta_M = -i(T_M+H-\Delta H_M)$, and $\xi' = m$

$$-i(T_M + H - \Delta H_M) = M\int_1^m \frac{(\xi - d)^{\alpha}}{(\xi + a)^{\gamma}\xi^{\alpha}(\xi - 1)^{1/2}(\xi - f)^{\beta}}d\xi - iH$$

$$T_M - \Delta H_M = M\int_1^m \frac{(\xi - d)^{\alpha}}{(\xi + a)^{\gamma}\xi^{\alpha}(\xi - 1)^{1/2}(f - \xi)^{\beta}}d\xi$$

Splitting the integration into two parts

$$T_M - \Delta H_M = M\int_1^{\frac{1+m}{2}} \frac{(\xi - d)^{\alpha}}{(\xi + a)^{\gamma}\xi^{\alpha}(\xi - 1)^{1/2}(f - \xi)^{\beta}}d\xi + M\int_{\frac{1+m}{2}}^{m} \frac{(\xi - d)^{\alpha}}{(\xi + a)^{\gamma}\xi^{\alpha}(\xi - 1)^{1/2}(f - \xi)^{\beta}}d\xi$$

......(5.25)

Substituting ξ-1 = v^2 for the first integral and f-ξ = v^{10} for the second integral, where v is a dummy variable, the improper integral above is converted to the following proper integral:

$$T_M - \Delta H_M = 2M\int_0^{\sqrt{\frac{m-1}{2}}} \frac{(v^2+1-d)^{\alpha}}{(v^2+1+a)^{\gamma}(v^2+1)^{\alpha}(f-v^2-1)^{\beta}}dv$$

$$+10M\int_{\sqrt[10]{f-m}}^{\sqrt[10]{\frac{2f-m-1}{2}}} \frac{v^{9-10\beta}(f-v^{10}-d)^{\alpha}}{(f-v^{10}+a)^{\gamma}(f-v^{10})^{\gamma}(f-v^{10}-1)^{1/2}}dv \qquad (5.26)$$

Substituting

$$v=\sqrt{\frac{m-1}{2}}\left(\frac{1+\chi}{2}\right) \text{ and } dv=\frac{\sqrt{\frac{m-1}{2}}}{2}d\chi \quad \text{for the first integral and}$$

$$v=\frac{\left(\sqrt[10]{\frac{2f-m-1}{2}}-\sqrt[10]{f-m}\right)}{2}\chi+\frac{\left(\sqrt[10]{\frac{2f-m-1}{2}}+\sqrt[10]{f-m}\right)}{2}=f(\chi) \text{ and}$$

$$dv=\frac{\left(\sqrt[10]{\frac{2f-m-1}{2}}-\sqrt[10]{f-m}\right)}{2}d\chi \quad \text{for the second integral}$$

$$T_M-\Delta H_M=M\sqrt{\frac{m-1}{2}}\int_{-1}^{1}\frac{\left(\frac{m-1}{2}\left(\frac{1+\chi}{2}\right)^2+1-d\right)^{\sigma}}{\left(\frac{m-1}{2}\left(\frac{1+\chi}{2}\right)^2+1+a\right)^{\gamma}\left(\frac{m-1}{2}\left(\frac{1+\chi}{2}\right)^2+1\right)^{\alpha}\left(f-\frac{m-1}{2}\left(\frac{1+\chi}{2}\right)^2-1\right)^{\beta}}d\chi$$

$$+5M\left(\sqrt[10]{\frac{2f-m-1}{2}}-\sqrt[10]{f-m}\right)\int_{-1}^{1}\frac{f^{9-10\beta}(\chi)\left(f-f^{10}(\chi)-d\right)^{\sigma}}{\left(f-f^{10}(\chi)+a\right)^{\gamma}\left(f-f^{10}(\chi)\right)^{\alpha}\left(f-f^{10}(\chi)-1\right)^{1/2}}d\chi \qquad (5.27)$$

V.2.2 Mapping of The Complex Potential w-Plane onto The Auxiliary ξ-Plane

The conformal mapping of the w-plane onto the lower half of the ξ-plane is given by :

$$\frac{dw}{d\xi}=\frac{M_2}{(\xi+a)^{1/2}(\xi)^{1/2}(\xi-1)^{1/2}} \qquad (5.28)$$

The complex potential for the confined flow domain dealt in chapter IV, and the potential for the unconfined flow domain is similar with that confined flow.

Using conditions at points C and E, constant M_2 is found to be :

$$M_2=\frac{q\sqrt{1+a}}{2F\left(\frac{\pi}{2},\frac{1}{\sqrt{1+a}}\right)} \qquad (5.29)$$

Using conditions at points A and C

$$q = \frac{k\Delta H_A F\left(\pi/2, \sqrt{1/(1+a)}\right)}{F\left(\pi/2, \sqrt{a/(1+a)}\right)} \tag{5.30}$$

in which q is seepage rate for half section of the stream.

Using conditions at points B and C

$$q = \frac{k\,\Delta H_B\, F\left(\pi/2, \sqrt{1/(1+a)}\right)}{F\left(\sin^{-1}\sqrt{(1+a)b/a(1+b)}, \sqrt{a/(1+a)}\right)} \tag{5.31}$$

Using the relationship at points D and E

$$\frac{q_1}{q} = \frac{F\left(\sin^{-1}\sqrt{1-d}, \sqrt{1/(1+a)}\right)}{F\left(\pi/2, \sqrt{1/(1+a)}\right)} \tag{5.32}$$

in which q_1 is seepage through the streambed.

Using the relationship at points E and F

$$\frac{q}{k\Delta H_F} = \frac{F\left(\sqrt{\pi/2}, \sqrt{1/(1+a)}\right)}{F\left(\sin^{-1}\sqrt{(f-1)/f}, \sqrt{a/(1+a)}\right)}$$

or

$$\Delta H_F = \frac{q}{k}\frac{F\left(\sin^{-1}\sqrt{(f-1)/f}, \sqrt{a/(1+a)}\right)}{F\left(\sqrt{\pi/2}, \sqrt{1/(1+a)}\right)} \tag{5.33}$$

Using the relationship at points E and M

$$\frac{q}{k\Delta H_M} = \frac{F\left(\sqrt{\pi/2}, \sqrt{1/(1+a)}\right)}{F\left(\sin^{-1}\sqrt{(m-1)/m}, \sqrt{a/(1+a)}\right)}$$

or

$$\Delta H_M = \frac{q}{k}\frac{F\left(\sin^{-1}\sqrt{(m-1)/m},\sqrt{a/(1+a)}\right)}{F\left(\sqrt{\pi/2},\sqrt{1/(1+a)}\right)} \tag{5.34}$$

The ten unknowns M, a, d, f, q, q_1, ΔH_F, α, β, and γ can be found from equation (5.4), (5.5), (5.6), (5.11), (5.18), (5.21), (5.24), (5.32), (5.34), and (5.35).

V.3 SUBSTITUTE LENGTH

The equation of seepage discharge is

$$q = \frac{k\,F\left(\pi/2,\sqrt{1/(1+a)}\right)}{F\left(\pi/2,\sqrt{a/(1+a)}\right)}\Delta H_A$$

Hence the resistance of the stream aquifer system is found to be

$$R_r = \frac{F\left(\pi/2,\sqrt{a/(1+a)}\right)}{k\,F\left(\pi/2,\sqrt{1/(1+a)}\right)} \tag{5.35}$$

The equivalent resistance of substitute length and aquifer

$$R_a = \frac{(L_A + \Delta L)}{k\left(T_1 + H - 0.5\Delta H_A\right)} \tag{5.36}$$

Since $R_r = R_a$, we get

$$\Delta L = \left[\frac{F\left(\pi/2,\sqrt{a/(1+a)}\right)}{F\left(\pi/2,\sqrt{1/(1+a)}\right)}\right]\left[H + T_1 - 0.5\Delta H_A\right] - L_A \tag{5.37}$$

in which ΔL is the substitute length.

V.4 RESULTS AND DISCUSSION

Numerical values for the stream and aquifer dimensions B_1, B_2, T_1, L_A and depth of water in the stream H and the head difference ΔH_A are assumed. The parameter 'a', 'd' and 'f' are assumed considering in which these parameter are located in the auxiliary ξ plane (i.e. a> 0; 0<d<1; f>1). q/k is computed from equation (5.30). q_1/k is estimated from

(5.32) and ΔH_F is found from (5.33). α, β and γ are computed from equations (5.4), (5.5) and (5.6) respectively after computing q/k, q_1/k and ΔH_F. Constant M is computed from equation (5.11). If parameters a, d and f have been correctly chosen they should satisfy equations (5.18), (5.21) and (5.24). Using Newton Raphson iteration procedure a, d and f are searched which satisfy equations (5.18), (5.21) and (5.24) with reasonable accuracy.

Variation of $q/(k\Delta H_A)$ with distance of aquifer boundary from stream bank, L_A/T_1 for different B_1/T_1 is presented in Fig. V.5. $q/(k\Delta H_A)$ or Γ_r/k decreases with increasing L_A/T_1. Γ_r/k is higher for a stream with large width. However when $B_1/T_1 \geq 2$, width of stream has little influence on reach transmissivity. In other word, all other parameter remaining unchanged, the seepage does not increase as B_1/T_1 increases beyond 2.0.

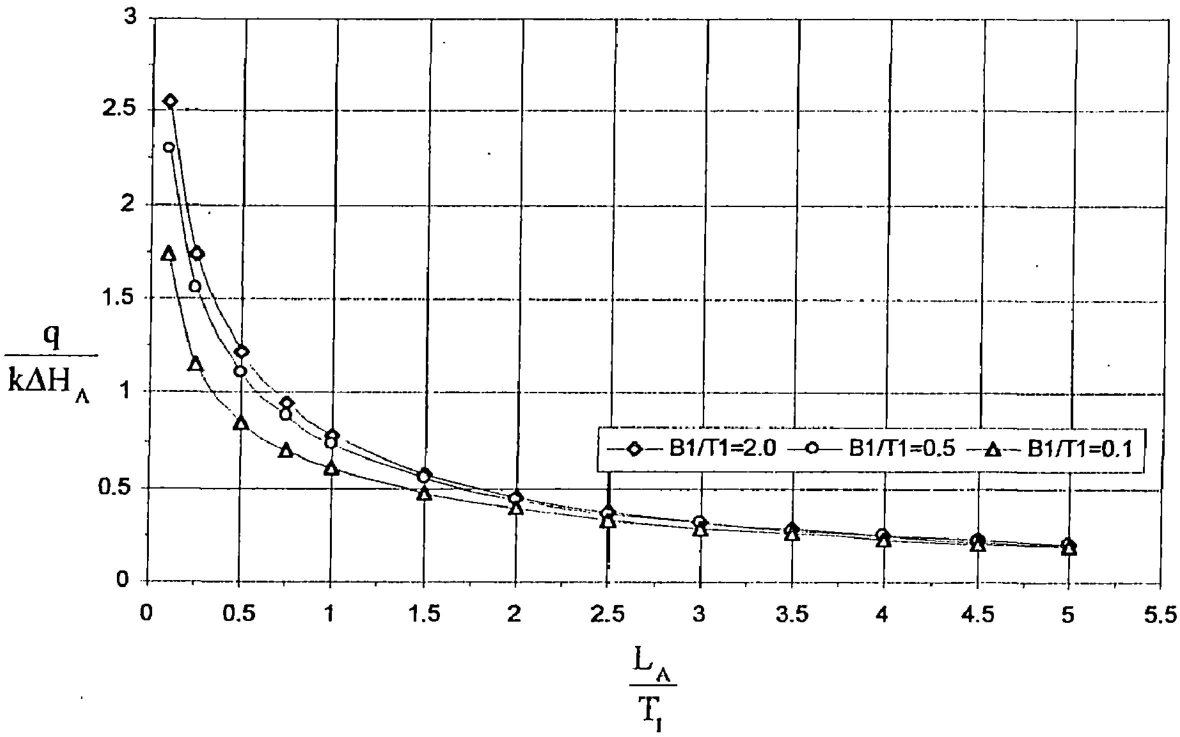

Fig. V.5 Variation of $q/(k\Delta H_A)$ with distance of aquifer boundary from the stream bank (L_A/T_1), for $H/T_1=0.1$; $\Delta H_A/T_1=0.01$ and $B_2=B_1+0.1$

The variations of non dimensional seepage from the stream and seepage through stream bed with L_A/T_1 for a particular value of $\Delta H_A/T_1$ (=0.01 and 0.1) are presented in Fig. V.6a through V.6c. $q/(k\Delta H_A)$ or Γ_r/k is the dimensionless reach transmissivity corresponding to length L_A and the dimension of the stream cross section. Reach transmissivity being inverse of the resistance of stream aquifer system, it decreases with increase in L_A. The decrease is monotonic beyond LA/T1 > 4. The results are presented in table V.1 and V.2. The computed α, β, γ and parameters a, d, f are presented including $\Delta H_F/T_1$, $q/(k\Delta H_A)$ and $q_1/(k\Delta H_A)$ for given B_1/T_1, B_2/T_1, H/T_1, $\Delta H_A/T_1$ and L_A/T_1.

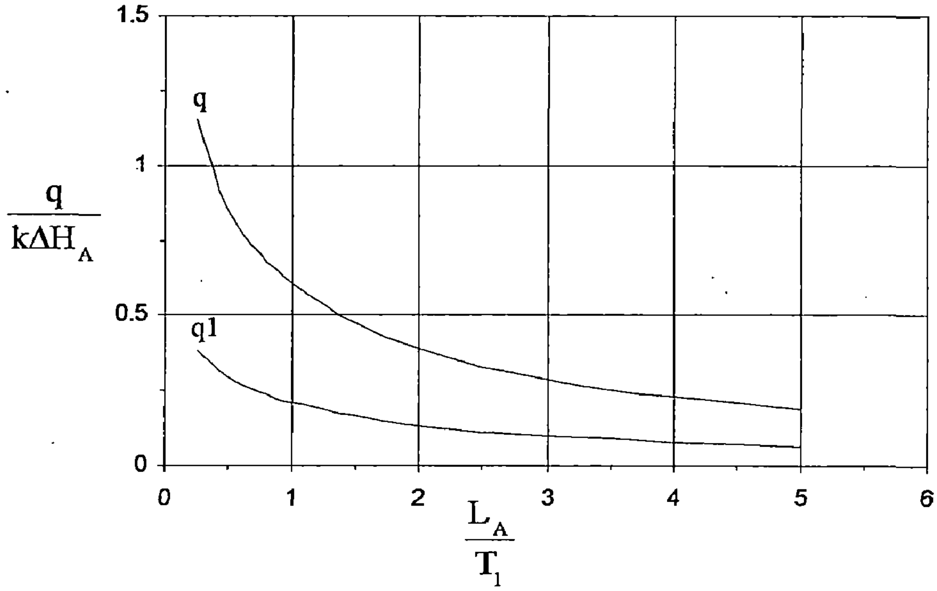

Fig. V.6a Variations of $q/(k\Delta H_A)$ and $q_1/(k\Delta H_A)$ with L_A/T_1 for B_1/T_1=0.1; B_2/T_1=0.2; H/T_1=0.1 and $\Delta H_A/T_1$=0.01

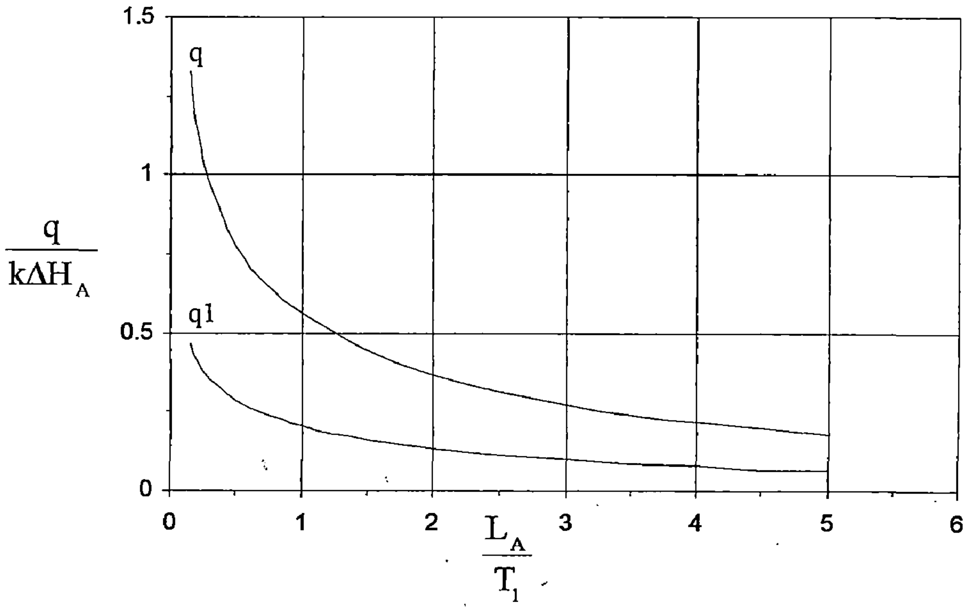

Fig. V.6b Variations of $q/(k\Delta H_A)$ and $q_1/(k\Delta H_A)$ with L_A/T_1 for B_1/T_1=0.1; B_2/T_1=0.2; H/T_1=0.1 and $\Delta H_A/T_1$=0.1

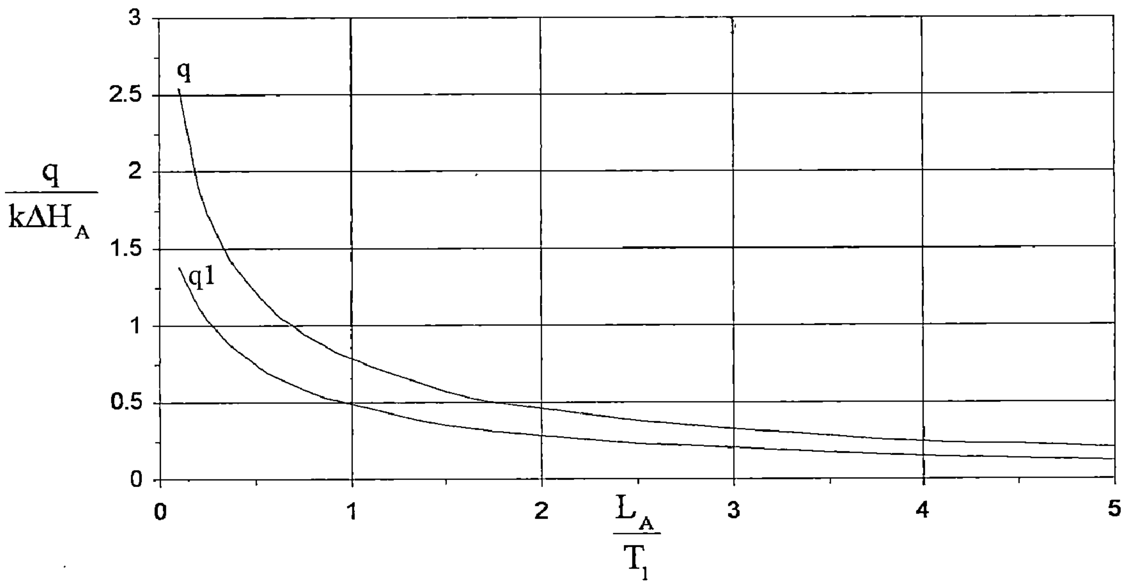

Fig. V.6c Variations of $q/(k\Delta H_A)$ and $q_1/(k\Delta H_A)$ with L_A/T_1 for $B_1/T_1=2$; $B_2/T_1=2.1$; $H/T_1=0.1$ and $\Delta H_A/T_1=0.01$

The variation of $q/(k\Delta H_A)$ and $q_1/(k\Delta H_A)$ with B_1/T_1 for $\Delta H_A/T_1=0.1$, $H/T_1=0.2$ and $L_A/T_1=5$ are shown in Fig. V.7. As B_1 increases seepage through bed and total seepage increase. For $B_1/T_1 > 1$, the increase in seepage is in significant.

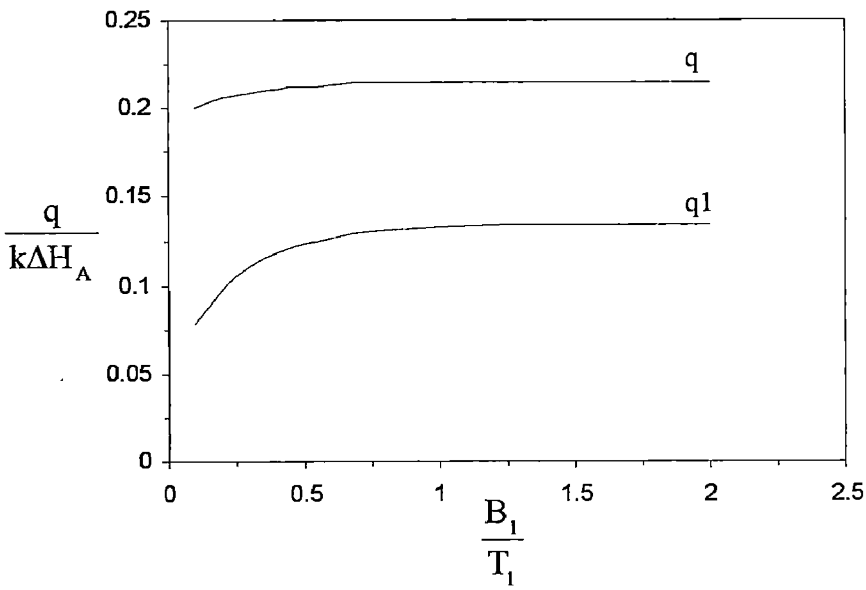

Fig. V.7 Variations of $q/(k\Delta H_A)$ and $q_1/(k\Delta H_A)$ with B_1/T_1 for $\Delta H_A=0.1$; $H/T_1=0.2$; $B_2=B_1$; and $L_A/T_1=5$

The variation of $q/(k\Delta H_A)$ with H/T_1 for different L_A/T_1 and particular values of B_1, B_2 and ΔH_A are presented in Fig. V.8. It is seen that reach transmissivity increases with increase in depth of water in the stream. The increase is linear for $L_A/T_1 > 1$.

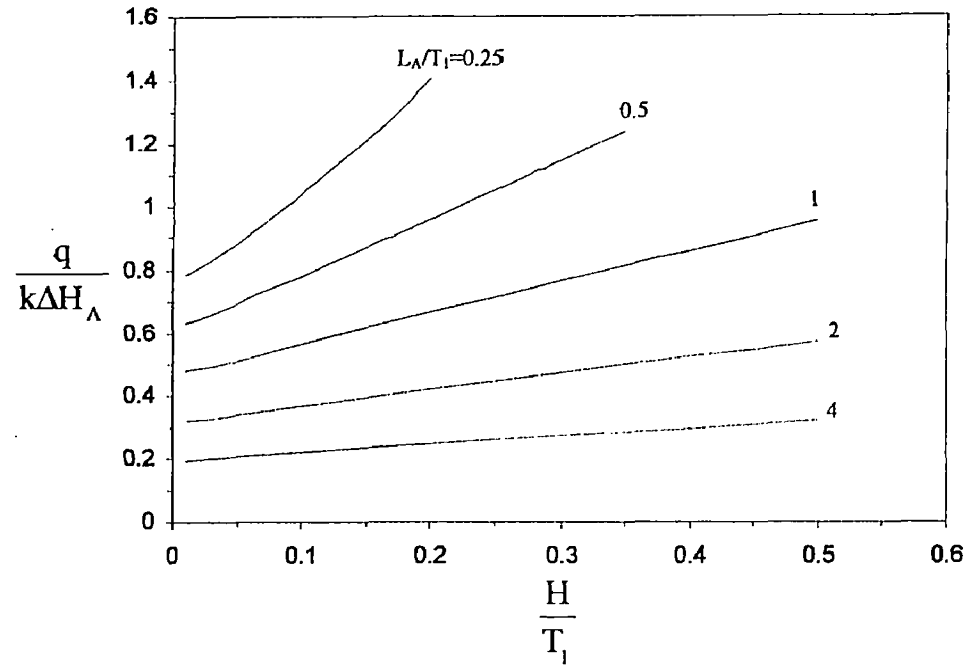

Fig. V.8 Variation of $q/(k\Delta H_A)$ with H/T_1 for B_1/T_1=0.1; B_2/T_1=0.2 and $\Delta H_A/T_1$=0.1

The variation of $q/(k\Delta H_A)$ with $\Delta H_A/T_1$ for different L_A/T_1 are presented in fig. V.9. It is seen that as $L_A/T_1 \geq 4$ the reach transmissivity is independent of draw down ΔH_A.

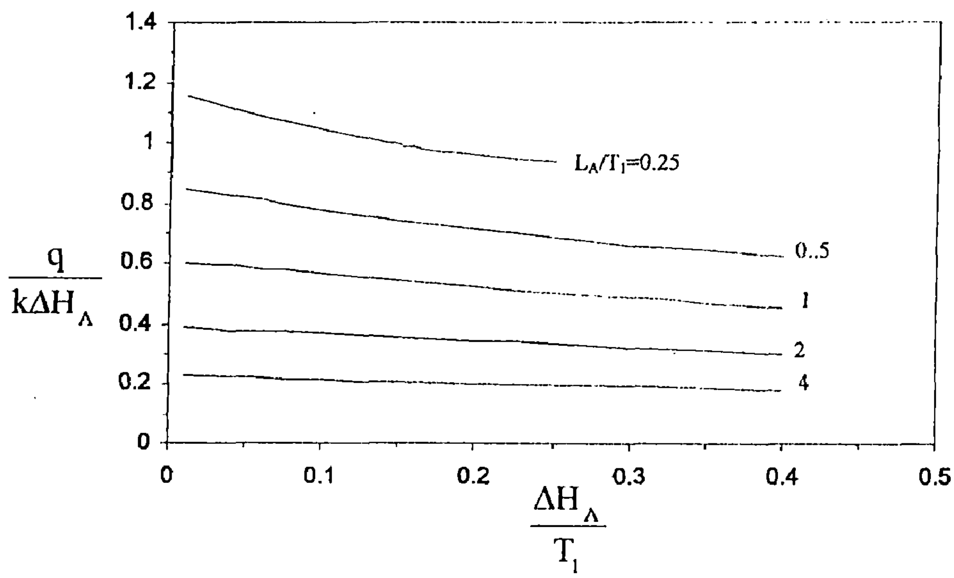

Fig. V.9 Variation of $q/(k\Delta H_A)$ with $\Delta H_A/T_1$ for B_1/T_1=0.1; B_2/T_1=0.2 and H/T_1=0.1

The variation of substitute length with width of rectangular stream is shown in Fig. V.10. The substitute length decreases with increasing stream width since the curvature of the flow lines will reduce with increase in bed width. Beyond B ≥ T_1, there is no further of reduction.

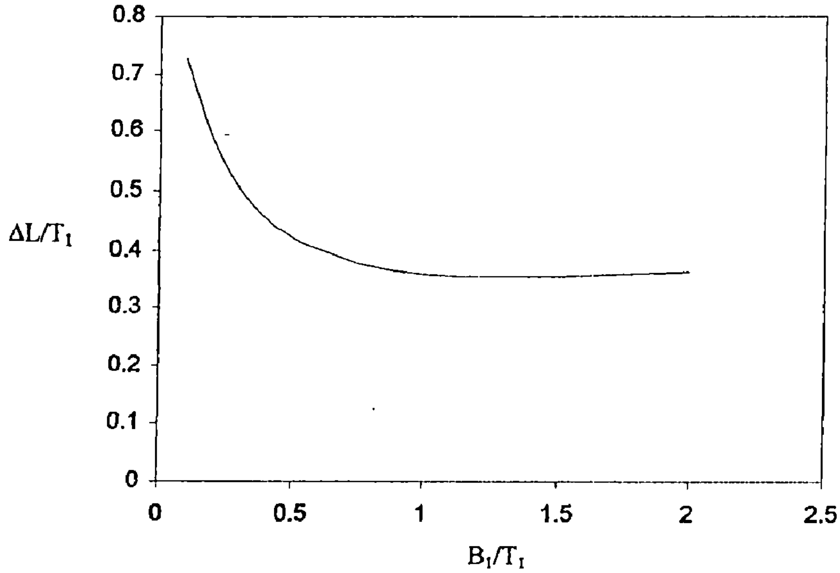

Fig. V.10 Variation of substitute length with width of stream for $\Delta H_A/T_1$=0.1; H/T_1=0.2 ; B_2=B_1 and L_A/T_1=5

The distribution of vertical down ward velocity with depth from the stream bed is presented in Fig. V.11. As the fluid approaches the lower impervious bed, the velocity decreases and tends to zero at y/T_1=1 as expected.

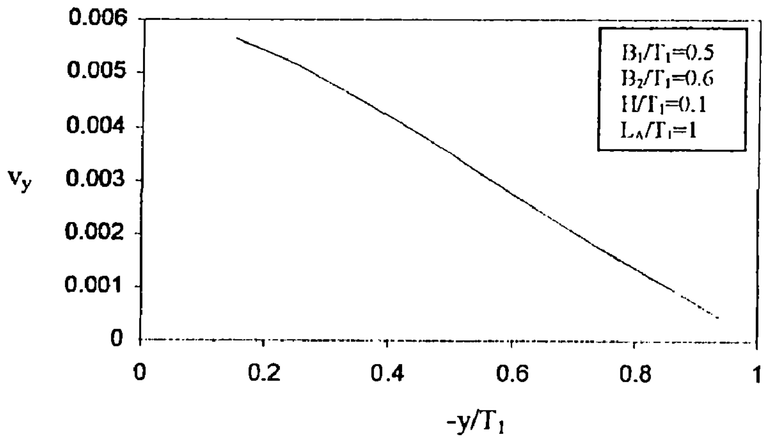

Fig. V.11 Distribution of velocity down ward

The locus of phreatic line at the entry through bank is magnified and shown in Fig. V.12a and d V.12b. As seen from the figure, the phreatic line which is a stream line and the stream bank which is an equipotential line and orthogonal.

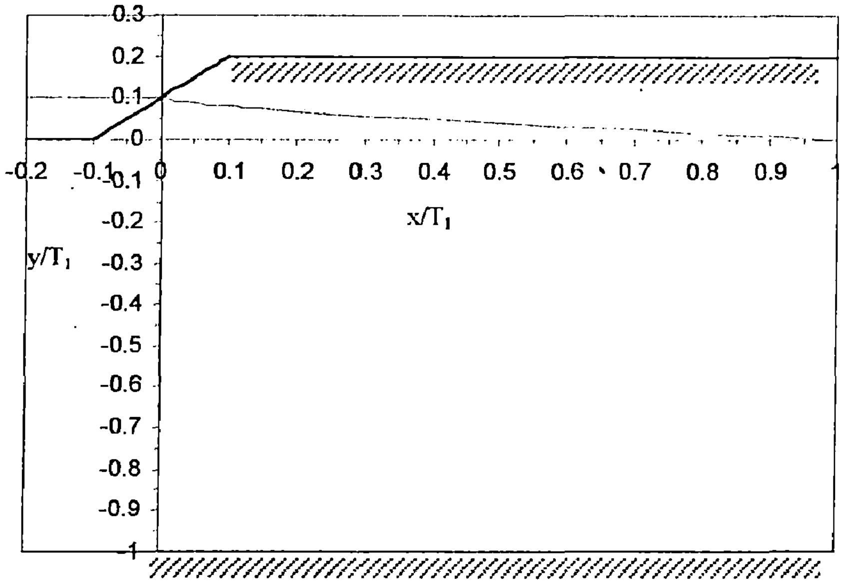

Fig. V.12a Locus of the phreatic line, for $B_1/T_1=2$; $B_2/T_1=2.1$; $H/T_1=0.1$; $\Delta H_A/T_1=0.1$; $L_A/T_1=1$

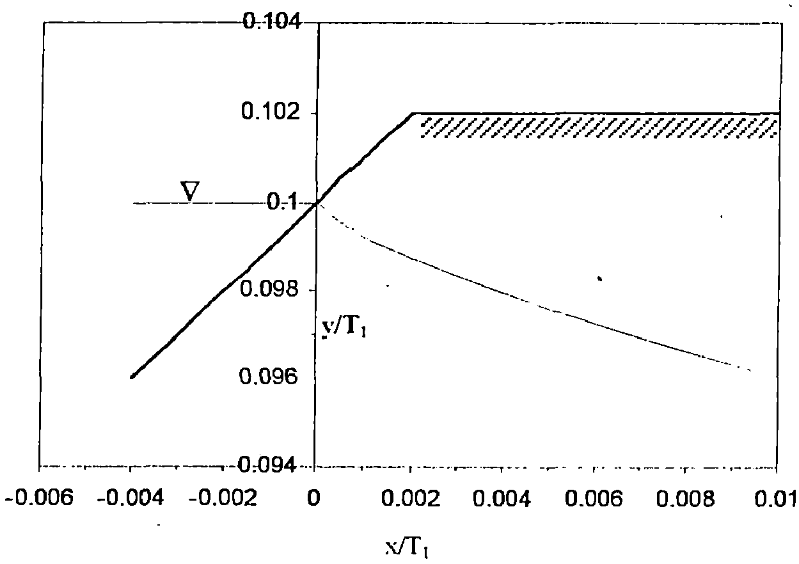

Fig. V.12b Locus of the phreatic line at entry through stream bank

Table V.1 Variation of $q/(k\Delta H_A)$; $q1/(k\ \Delta H_A)$ and $\Delta H_F/T_1$ for B_1/T_1=0.1; B_2/T_1=0.2 and H/T_1=0.1

$\Delta H_A/T_1$	La/T1	a	d	f	alpha	beta	gama	$\Delta H_F/T_1$	$q/(k\Delta H_A)$	$q1/(k\ \Delta H_A)$
0.01										
	0.10	0.0715	0.53511	3,956.8455	0.27060	0.50011	0.49494	0.0099	1.7335	0.5165
	0.25	0.5334	0.64351	150.3284	0.26273	0.50041	0.49663	0.0094	1.1534	0.3840
	0.50	2.0444	0.69294	22.0947	0.25911	0.50082	0.49752	0.0082	0.8504	0.2939
	0.75	4.9774	0.71154	11.1194	0.25746	0.50100	0.49795	0.0070	0.7027	0.2445
	1.00	10.8468	0.72063	8.3089	0.25639	0.50104	0.49824	0.0061	0.6040	0.2104
	1.50	47.1998	0.72769	6.8207	0.25499	0.50098	0.49862	0.0048	0.4733	0.1647
	2.00	199.4707	0.72966	6.5085	0.25409	0.50088	0.49886	0.0039	0.3893	0.1354
	2.50	838.0587	0.73032	6.4287	0.25347	0.50079	0.49904	0.0033	0.3305	0.1149
	3.00	3,516.8616	0.73062	6.4037	0.25301	0.50071	0.49916	0.0029	0.2872	0.0998
	3.50	14,755.4510	0.73080	6.3930	0.25266	0.50064	0.49926	0.0026	0.2539	0.0882
	4.00	61,905.5304	0.73093	6.3866	0.25238	0.50059	0.49934	0.0023	0.2276	0.0790
	4.50	259,671.8015	0.73104	6.3820	0.25216	0.50054	0.49940	0.0021	0.2061	0.0716
	5.00	1,088,498.0971	0.73113	6.3784	0.25197	0.50050	0.49945	0.0019	0.1885	0.0654
0.1	0.15	0.2811	0.56478	2,951.1954	0.45692	0.50114	0.45801	0.0988	1.3269	0.4631
	0.25	0.8410	0.62762	225.9040	0.39718	0.50351	0.46700	0.0950	1.0403	0.3751
	0.50	3.0488	0.67446	25.8460	0.35090	0.50822	0.47525	0.0819	0.7790	0.2850
	0.75	7.2859	0.69217	12.7853	0.33116	0.51013	0.47930	0.0698	0.6513	0.2378
	1.00	15.9162	0.70163	9.5167	0.31855	0.51055	0.48207	0.0602	0.5640	0.2050
	1.50	71.4994	0.71076	7.7085	0.30242	0.50992	0.48582	0.0470	0.4457	0.1607
	2.00	316.6220	0.71498	7.2331	0.29239	0.50890	0.48829	0.0385	0.3682	0.1319
	2.50	1,402.4832	0.71754	7.0381	0.28556	0.50795	0.49002	0.0325	0.3136	0.1118
	3.00	6,223.8484	0.71936	6.9267	0.28061	0.50714	0.49131	0.0282	0.2730	0.0970
	3.50	27,662.0425	0.72076	6.8499	0.27687	0.50646	0.49231	0.0249	0.2417	0.0857
	4.00	123,065.5590	0.72187	6.7918	0.27393	0.50589	0.49310	0.0222	0.2168	0.0767
	4.50	547,689.2523	0.72279	6.7460	0.27158	0.50541	0.49375	0.0201	0.1965	0.0694
	5.00	2,434,443.5431	0.72356	6.7084	0.26964	0.50500	0.49428	0.0184	0.1797	0.0633

Table V.2 Variation of $q/(k\Delta H_A)$; $ql/(k\Delta H_A)$ and $\Delta H_F/T_1$ for $B_1/T_1=2$; $B_2/T_1=2.1$ and $H/T_1=0.1$

$\Delta H_A/T_1$	La	a	d	f	alpha	beta	gama	$\Delta H_F/T_1$	$q/(k\Delta H_A)$	$ql/(k\Delta H_A)$
0.01	0.1	0.00540	0.04915	1.01660	0.26974	0.50133	0.49257	0.00082	2.54550	1.37600
	0.25	0.07104	0.11515	1.01130	0.26154	0.50127	0.49493	0.00066	1.73530	1.03540
	0.5	0.42410	0.20244	1.00725	0.25758	0.50116	0.49646	0.00050	1.21370	0.74830
	0.75	1.27236	0.25593	1.00562	0.25583	0.50107	0.49724	0.00039	0.94650	0.58700
	1	3.07352	0.28529	1.00490	0.25477	0.50099	0.49773	0.00032	0.77760	0.48260
	1.5	14.44224	0.30838	1.00441	0.25351	0.50086	0.49833	0.00024	0.57370	0.35580
	2	62.26781	0.31439	1.00430	0.25277	0.50076	0.49867	0.00019	0.45450	0.28170
	2.5	263.21497	0.31605	1.00428	0.25230	0.50068	0.49890	0.00016	0.37640	0.23320
	3	1,107.37508	0.31659	1.00427	0.25196	0.50062	0.49906	0.00013	0.32110	0.19890
	3.5	4,653.13890	0.31683	1.00427	0.25171	0.50056	0.49918	0.00012	0.28000	0.17350
	4	19,543.95067	0.31697	1.00427	0.25151	0.50052	0.49928	0.00010	0.24830	0.15380
	4.5	82,058.51802	0.31707	1.00427	0.25136	0.50048	0.49935	0.00009	0.22300	0.13810
	5	344,295.18747	0.31714	1.00428	0.25123	0.50045	0.49941	0.00008	0.20240	0.12530

CHAPTER VI
CONCLUSIONS

Using conformal mapping and Zhukovsky function, seepage from a partially penetrating stream has been obtained for the following hydro geological conditions :

(i) a partially penetrating rectangular stream in a semi infinite confined aquifer,

(ii) a partially penetrating stream with trapezoidal section in a finite confined aquifer, and

(iii) a partially penetrating stream with trapezoidal section in a finite unconfined aquifer.

Steady state seepage from a stream in a confined aquifer can be expressed as :

$q = k\ F\ \Delta h = \Gamma_r\ \Delta h$

in which :

k = hydraulic conductivity,

Δh = hydraulic head difference measured at a piezometer in the vicinity of the stream,

and F is a factor which depends on location of the piezometer i.e. distance of the piezometer from the stream bank and stream geometry i.e. cross section of the stream and depth of penetration of the stream. The above linear relationship between seepage and Δh is valid for steady state and confined flow condition.

Aravin, Bouwer, Herbert, Morel-Seytoux and many other investigators have derived the factor F based on Darcy's law and Dupuit Ferchheimer flow condition at large distance from the water body.

In the present thesis, exact relation of the parameter Γ_r/k (i.e. seepage factor F) with distance of the piezometer and stream geometry including depth of penetration has been derived. It is found that the reach transmissivity increases with increase in stream width, depth of penetration and hydraulic conductivity and it decreases with increase in distance of observation point from the stream bank. Unlike seepage from a trapezoidal canal in an unconfined aquifer of infinite depth, the total seepage and seepage through bed of a stream in a confined or unconfined aquifer of finite depth tend to constant value for B/T_2 greater than 1. The fraction of seepage through bed decreases as depth of penetration increases.

Unsteady flow from a fully penetrating stream has been given by Carslaw and Jaeger for an analogous heat conduction problem. Partially penetrating stream, offers more resistance to flow than fully penetrating stream because of flow convergence near the stream. The sum of the resistance due to flow convergence and resistance due to fraction of the aquifer under the stream bed can be equated to the resistance of length ΔL of the aquifer for uniform flow condition. This length ΔL is known as substitute length. The substitute length increases with increase in distance of observation well from the stream bank and decreases with increase of width of the stream and depth of penetration. The substitute length tends to a finite value as distance of observation well increases. In the application substitute length for unsteady flow, it is seen that the rise piezometric surface in the aquifer for a unit step rise in the stream, is less than 1 due to the head loss along substitute length.

In comparing the results with Herbert's formula, it is found that Herbert's formula is applicable for depth of penetration less than 30 % (the involved error < 10%) and width of the stream (B/T_2) less than 0.2.

For a partially penetrating stream in an unconfined aquifer, the reach transmissivity increases with increase in depth of water in the stream, decreases with increase in length of aquifer boundary and increases tending to constant value with increase in stream width.

A rigorous analytical solution for steady seepage from a trapezoidal stream/canal to an unconfined aquifer in which water table lies at a shallow depth has been derived using Zhukovsky function and Schwarz-Christoffel conformal mapping.

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APPENDIX A
REACH TRANSMISSIVITY

The use of reach transmissivity has been introduced by Morel-Seytoux and Daly (1975), for solving unsteady state stream-aquifer interaction problem. The reach transmissivity has been defined as the constant of proportionality between the return flow from a river and the difference of potentials at the periphery of the river and in the aquifer in the vicinity of the river. The constant of proportionality has been obtained analytically by various investigators, e.g., Hammad (1959), Ernst (1962) Aravin and Numerov (1965), Bouwer (1965), Herbert (1970) and Streltsova (1974), for different aquifer and river geometry. According to Muskat (1946), and Bouwer (1969), an unsteady state can be treated as a succession of steady states. The validity of this assumption has been reasoned out by Muskat in detail [Muskat (1946), pp.621-625]. Based on the above principle, the reach transmissivity constant, though has been derived on the assumption of steady flow condition, has been used for analysis of unsteady state problems by Morel-Seytoux (1975). The reach transmissivity constant derived by various investigators for different canal and aquifer geometry has been reviewed in the following paragraphs :

The geometry of a channel constructed in an aquifer on finite depth, which is underlain by impermeable layer is shown in Figure A.1. The channel is hydraulically connected with the aquifer. For a specific case in which the channel is rectangular and the bottom of the channel extends to the impermeable layer, the seepage loss is given by (Bouwer, 1965).

$$Q = \frac{2k(H_W - 0.5D_W)D_W}{(L - 0.5W_b)} \tag{A.1}$$

The reach transmissivity for a fully penetrating canal of reach length L_r , therefore, is given by :

$$\Gamma_r = \frac{2kL_r(H_W - 0.5D_W)}{(L - 0.5W_b)} \tag{A.2}$$

L can be regarded as the distance of the observation well where the draw down D_W is observed.

Approximate expression for seepage from a partially penetrating channel shown in Figure A.1 is given by (vide Bouwer, 1969).

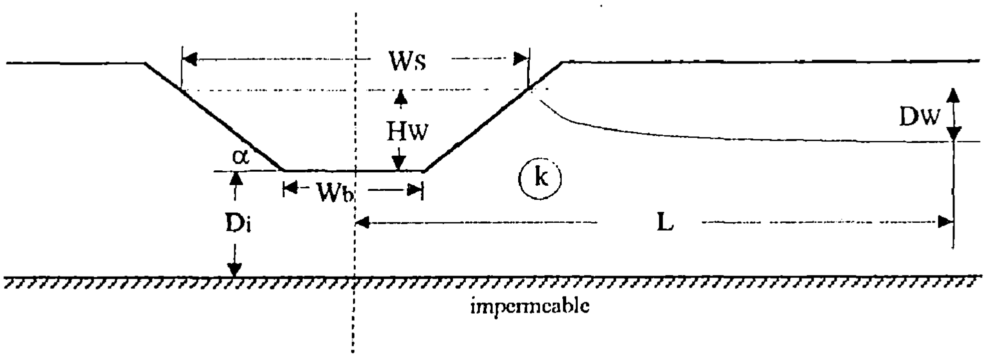

Fig. A.1 Geometry for channels in soil underlain by impermeable material

$$Q = \frac{2k(H_W + D_i - 0.5D_W)D_W}{(L - 0.25W_b - 0.25W_S)} \tag{A.3}$$

Hence, the approximate expression for reach transmissivity for a canal conforming to the configuration depicted in Figure A.1 is,

$$\Gamma_r = \frac{2kL_r(H_W + D_i - 0.5D_W)}{(L - 0.25W_b - 0.25W_S)} \tag{A.4}$$

According to the Bouwer (1969), the above expression is not exact and the error in Γ_r will increase with increasing D_i. The error in equation A.4 is due to the curvature and divergence of the streamlines in the vicinity of the channel.

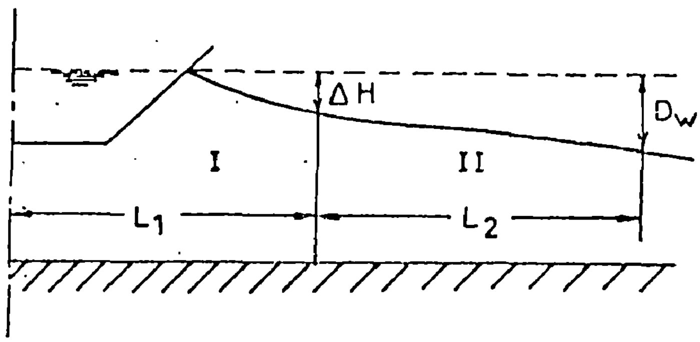

Fig. A.2 Division of flow system in regions I and II for Dachler's analysis

Dachler (1936) had divided the flow system on the basis of model studies into a region with curvilinear flow (region I) and the other with Dupuit Forchheimer flow (region II) [Fig. A.2], the dividing line being at a distance, L_1, from the center of the canal, where

$$L1 = \frac{W_S + H_W + D_i}{2} \tag{A.5}$$

The flow in region I was analyzed with an approximate equation for the potential and the stream line distribution under a plain source of finite width. A factor 'F' has been determined to estimate flow in region I as :

$$Q_I = 2\,F\,k\,\Delta H \qquad (A.6)$$

where ΔH is the vertical distance between the water surface in the canal and the ground water table at the dividing line between the two flow regions. Values of F given by Dachler are presented in Figure A.3.

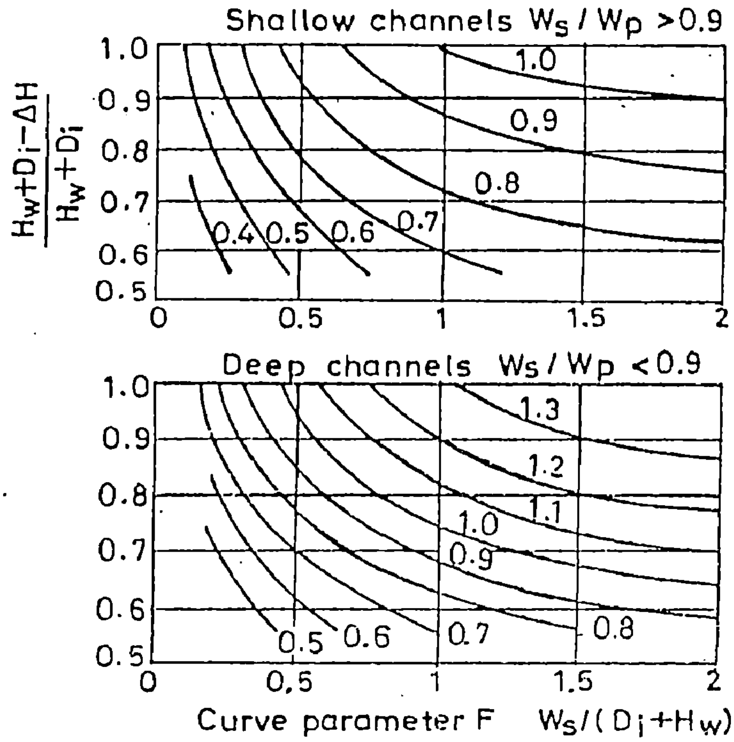

Fig. A.3 Dachler's values of F for shallow and for deep channels

The flow in region II has been expressed with Dupuit Forchheimer theory as :

$$Q_{II} = \frac{2k(D_W - \Delta H)}{L_2}[D_i + H_W - 0.5\Delta H - 0.5D_W] \qquad (A.7)$$

Since it is required to calculate the seepage for a given value of D_W at a distance (L_1+L_2) from the channel center, ΔH will not be known initially. ΔH is found by trial and error which satisfies the condition Q_I=Q_{II}. The reach transmissivity for a canal reach of length L_r will be given by:

$$\Gamma_r = \frac{2kL_r}{L_2}\left(1 - \frac{\Delta H}{D_W}\right)(D_i + H_W - 0.5\Delta H - 0.5D_W) \qquad (A.8)$$

Bouwer (1969) has applied Ernst's approach to analyze seepage from a canal constructed in a porous medium of finite depth underlain by an impervious layer. Following Ernst's approximate solution for potential distribution pertaining to flow to a line sink, the head loss, h_r, due to radial flow in the vicinity of the canal, has been expressed by Bouwer as:

$$hr = \frac{Q}{\pi k}\log_e\left(\frac{D_i + H_W}{W_p}\right) \quad \text{(A.9)}$$

Hence, reach transmissivity for a canal reach of length L_r is given by

$$\Gamma_r = \frac{\pi k \; L_r}{\log_e\left(\frac{D_i + H_W}{W_p}\right)} \quad \text{(A.10)}$$

The head loss, h_h, due to horizontal flow in the region away from the canal has been expressed by Bouwer as

$$h_h = \frac{Q}{2k}\frac{L}{(D_i + H_W - 0.5D_W)} \quad \text{(A.11)}$$

Since $D_W = h_r + h_h$, Bouwer has combined equations A.9 and A.11 to obtain the relation :

$$Q = \frac{kD_W}{\frac{1}{\pi}\log_e\left(\frac{D_i + H_W}{W_p}\right) + \frac{0.5L}{D_i + H_W - 0.5D_W}} \quad \text{(A.12)}$$

The reach transmissivity for a canal reach of length L_r from equation A.12 can be obtained as :

$$\Gamma_r = \frac{kL_r}{\frac{1}{\pi}\log_e\left(\frac{D_i + H_W}{W_p}\right) + \frac{0.5L}{D_i + H_W - 0.5D_W}} \quad \text{(A.13)}$$

Equation A.9 was developed for semi circular channels of radius r, where the wetted perimeter W_p is πr. The equation according to Bouwer (1969) can be used for channels of other shapes by substituting the actual wetted perimeter as shown in the above equation. For shallow channels ($W_S >> H_W$), the seepage rate can be more accurately estimated by the following expression :

$$Q = \frac{k\pi}{\log_e\left(\frac{4D_i + H_W}{\pi W_S}\right)} . h_r \tag{A.14}$$

Hence, the reach transmissivity for a canal reach of length L_r by Ernst modified formula would be given by :

$$\Gamma_r = \frac{k\pi L_r}{\log_e\left(\frac{4D_i + H_W}{\pi W_S}\right)} \tag{A.15}$$

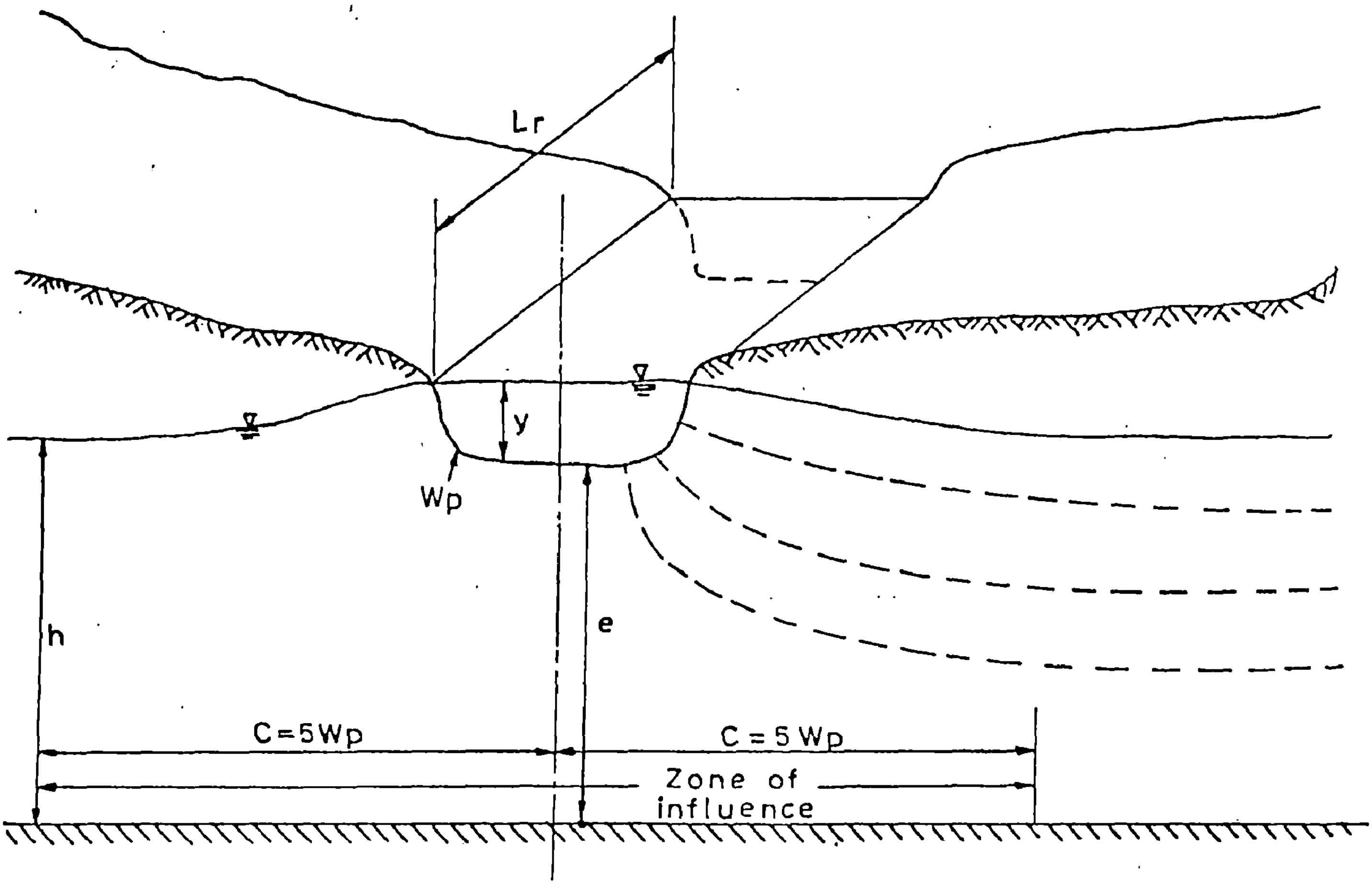

Fig. A.4 Schematic view of a s tream in hydraulic connection with an aquifer and definition of teminology

Using a simple potential theory Morel-Seytoux et al (1979) have derived the following expression of reach transmissivity for a canal in a porous medium underlain by an impervious layer (Fig. A.4):

$$\Gamma_r = \frac{TL_r}{e} \frac{0.5W_p + e}{5W_p + 0.5e} \tag{A.16}$$

in which,

L_r = length of canal reach,

T = transmissivity of the aquifer,

W_p = wetted perimeter of the canal, and

e = saturated thickness below the canal bed.

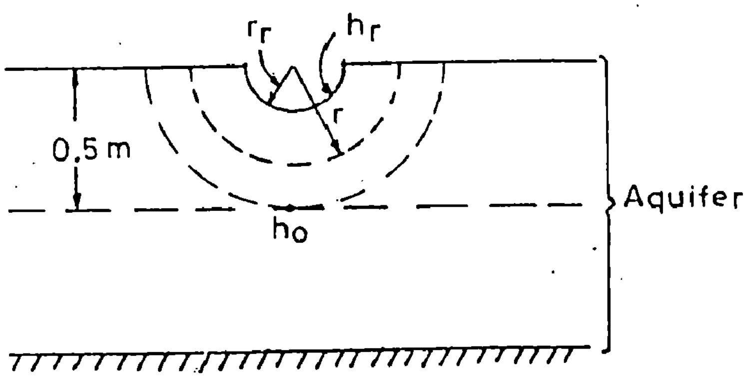

Fig. A.5 Representation of partially penetrating river

Herbert (1970) has the related the flow from a partially penetrating river, having semicircular cross section (Fig. A.5), to the potential difference between the river and in the aquifer below the river bed. The expression is given by :

$$Q_r = \frac{\pi L_r k(h_r - h_o)}{\log_e\left(\frac{0.5m}{r_r}\right)} \tag{A.17}$$

in which,

L_r = length of river reach,

h_r = potential at the river boundary,

h_0 = potential in the aquifer below the river bed,

m = saturated thickness of the aquifer, and

r_r = radius of the semicircular river cross section.

The reach transmissivity, which could be obtained from equation A.17, is

$$\Gamma_r = \frac{\pi L_r k}{\log_e\left(\frac{0.5m}{r_r}\right)} \tag{A.18}$$

For a rectangular channel shown in Fig. A.6, Aravin (1965) has derived the following expression for flow to the channel :

$$Q = \frac{k(H+h)(H-h)}{L-\frac{B}{2}} + \frac{k(H-h)}{\frac{L}{2T} - \frac{1}{\pi}\log_e\left[\sinh\left(\frac{\pi B}{4T}\right)\right]} \quad (A.19)$$

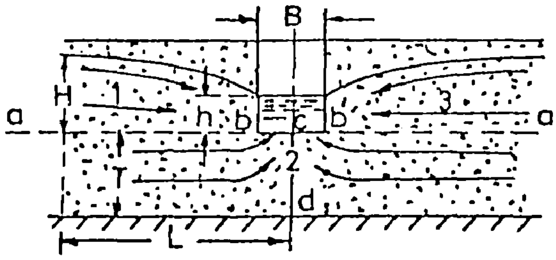

Fig. A.6 Flow to a rectangular ditch

The reach transmissivity for a canal reach of length L_r could be written as :

$$\Gamma_r = \frac{kL_r(H+h)}{L-0.5B} + \frac{kL_r}{0.5\frac{L}{T} - \frac{1}{\pi}\log_e\left[\sinh\left(\frac{\pi B}{4T}\right)\right]} \quad (A.20)$$

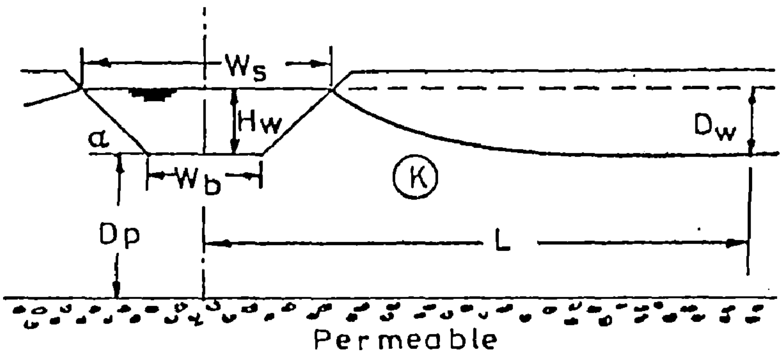

Fig. A.7 Geometry and symbols for channels in soil underlain by permeable material

Seepage flow from a canal embedded in a porous medium of finite depth, underlain by a highly pervious layer, Fig.A.7, has been analyzed for simplified canal geometry by Hammad (1959). The analysis is valid for the situation in which the piezometric head in the underlying highly pervious layer is very near the canal water level. According to Hammad,

$$Q = kD_W \frac{2K_1}{K_1' - C} \quad (A.21)$$

in which, K_1 and K'_1 are the complete elliptic integral of the first kind corresponding to modulus K_1 and complementary modulus K'_1 respectively. The moduli are defined as :

$$K_1 = 0.5\left[\frac{W'_S}{2} + \left(\frac{W'^2_S}{4} - 2H'^2_W\right)^{1/2}\right]$$

$$K'_1 = \left(1 - K_1^2\right)^{1/2}$$

The other constants are

$$C = \frac{H'_W}{K_1}$$

$$H'_W = \tan\left(\frac{\pi\, H_W}{2(H_W + D_p)}\right), \quad \text{for } H_W < D_p$$

and

$$W'_S = 2\tanh\left(\frac{\pi\, W_S}{4(H_W + D_p)}\right), \quad \text{for } H_W < D_p$$

The reach transmissivity for a canal reach of length L_r can be written as :

$$\Gamma_r = kL_r\left(\frac{2K_1}{K'_1 - C}\right) \tag{A.22}$$

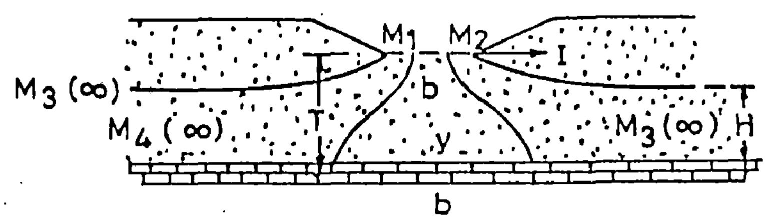

Fig. A.8. Seepage from a canal with shallow water depth embedded in a porous medium underlain by a highly permeable layer

Aravin (1965) has analyzed the seepage from a canal which has very shallow water depth in it. The water table lies above the highly permeable layer as shown in Fig. A.8. The analysis has been carried out using zhukovsky's function and conformal mapping. The seepage quantity is given by,

$$Q = k(T - H)\frac{K'_1}{K_1} \tag{A.23}$$

in which, K_1 is the complete elliptical integral of first kind with modulus

$$K = \exp\left[\frac{-\left(b+\frac{Q}{k}\right)}{2H}\right]$$

K'_1 is complete elliptic integral of first kind with modulus K', where K' is given by

$$K' = \sqrt{1-K^2}$$

when K is very near to zero, the seepage rate is given by :

$$Q = \frac{k(T-H)(b+0.882H)}{T}$$

Thus,

$$\Gamma_r = \frac{k\,L_r(b+0.882H)}{T} \qquad (A.24)$$

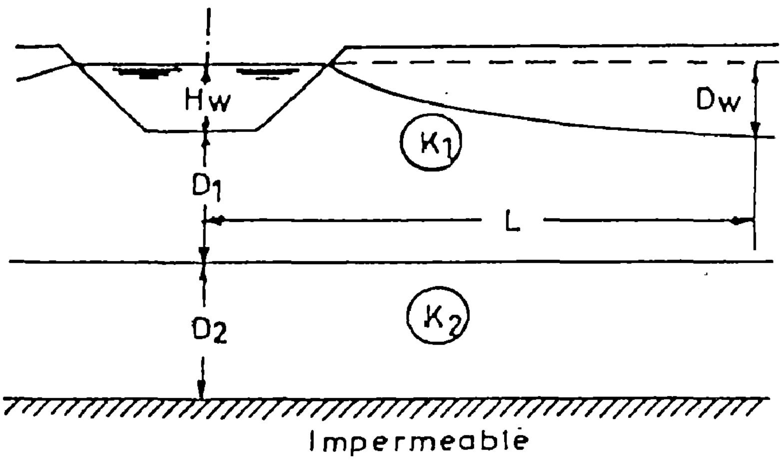

Fig. A.9 Canal in a two layered soil system

The case of seepage from a canal in two layered soil Fig. A.9, underlain by an impermeable layer, has been analyzed by Ernst (vide Bouwer, 1969). Following Ernst's solution, the reach transmissivity pertaining to a two layered soil system can be written as:

$$\Gamma_r = \frac{k_1 L_r}{\dfrac{0.5k_1 L}{k_1(D_1 + H_W - 0.5D_W) + k_2 D_2} + \dfrac{1}{\pi}\ln\left(\dfrac{\alpha\,(H_W + D_1)}{W_p}\right)} \qquad (A.25)$$

in which k_1 and k_2 are permeabilities of the top and bottom layer respectively. The parameter α given by Van Beer (vide Bouwer, 1969), is shown in Fig. A.10.

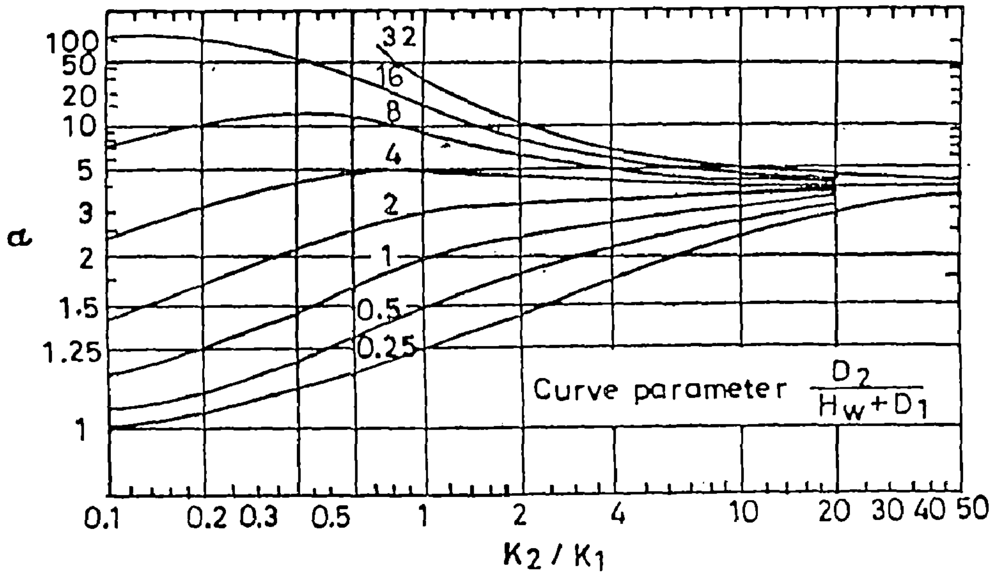

Fig. A.10 Parameter α for calculating seepage loss from a canal in a two layered soil system

APPENDIX B

B.1 Computer Programming for Seepage in Confined Aquifer

```
#include<iostream.h>
#include<math.h>
#include<iomanip.h>
#include<process.h>
#include<conio.h>
const int n=48;
const int m=48;
const int u=2;
const int v=2;
const int w=2;
const int x=1;

void main()
{
clrscr();
long double d,f,dcd,dde,gcd,gcd1,gcd2,gde,gde1,gde2,gca,gca1,gca2,sel,self,seld,f1,d1,f2,d2;
long double ycd1,yde1,yca1,ycd2,yde2,yca2,b,bca,e,z,d0,f0,dd,df,Fd0f0,Gd0f0;
long double Fdf0,Fd0f,Gdf0,Gd0f,gcda,ycd1a,ycd2a,gcd2a,gcd1a,gdea,yde1a;
long double yde2a,gde1a,gde2a,gcdb,ycd1b,ycd2b,gcd2b,gcd1b,gdeb,yde1b,yde2b;
long double gde1b,gde2b,dfpdd,dfpdf,dgpdd,dgpdf,ap,iap,ap2,iap2;
float t1,t2,ts,wb,wb1,lb,pi,q_by_kdh;
int i,q,j,k,r,s,t;
long double ss,s0,s1,s2,gefa,gefa1,gefa2,gefb,gefb1,gefb2,gefc,gefc1,gefc2,q_by_kdh2;
long double yef1,yef2,yef01,yef02,yef11,yef12,e2,ds,fs,fs0,fs1,dfpds,tes1,ts_by_t1;
char cont_q;int cont_s;

long double
xi[m]={.01627674484960296957 9,.0488129851360497311 12,.08129749546442555899 4,.1136958501106659209 11,.145
97371465489694198 9,.17809688236761860275 9,
.21003131046056720360 3,.24174315616384001232 8,.27319881259104914148 7,.30436494435449635302 4,.33520852
2892625422616,.36569686147231363503 1,
.39579764982890860328 5,.42547898840730054536 5,.45470942216774300863 6,.48345797392059635976 8,.51169417
7154667673586,.53938810832435743622 7,
.56651041856139716840 4,.59303236477757208068 4,.61892584012546857038 6,.64416340378496710679 8,.66871831
0043916153953,.69256453664217156134 4,
.71567681234896762622 5,.73803064374440013285 1,.75960234117664749870 3,.78036904386743321760 4,.80030874
4´39140817229,.81940031073793167553 9,
.83762351122818712149 4,.85495903343460145546 3,.87138850590929650287 4,.88689451740242041605 7,.90146063
5315852341319,.91507142312089807420 6,
.92771245672230869096 5,.93937033975275521693 2,.95003271778443763575 6,.95968829144874253930 0,.96832682
8463264212174,.97593917458513646645 3,
.98251726356301467744 7,.98805412632962379948 1,.99254390032376262457 2,.99598184298720929065 0,.99836437
5863181677724,.99968950388323076682 8};
long double
wi[n]={.03255061449236316624 2,.03251611871386883598 7,.03244716371406426936 4,.03234382256857592842 9,.032
206204794030250669,.03203445623199266321 8,
.03182875889441100653 5,.03158933077072716855 8,.03131642559686135581 3,.03101033258631383742 3,.03067137
6123669149014,.03029991542082759379 4,
.02989634413632838598 4,.02946108995816790597 0,.02899461415055523654 3,.02849741106508538564 6,.02797000
7616848334440,.02741296272602924282 3,
.02682686672559176219 8,.02621234073567241391 3,.02557003600534936149 9,.02490063322248361028 8,.02420484
1792364691282,.02348339908592621984 2,
.02273706965832937400 1,.02196664443874434919 5,.02117293989219129898 8,.02035679715433332459 5,.01951908
1140145022410,.01866067962741146738 5,
.01778250231604526083 8,.01688547986424517245 0,.01597056290256229138 1,.01503872102699493800 6,.01409094
1772314860916,.01312822956696157263 7,
.01215160467108831963 5,.01116210209983849859 1,.01016077053500841575 8,.00914867123078338663 3,.00812687
6925698759217,.00709647079115386526 9,
.00605854550423596168 3,.00501420274292751769 3,.00396455433844468667 4,.00291073181793494640 8,.00185396
0788946921732,.00079679206555201242 9};

clrscr();
cout<<"    SEEPAGE FROM PARTIALLY PENETRATING STREAM IN CONFINED AQUIFER"<<endl<<endl;
do{//for total
do{//for case 1
clrscr();
cout<<" Input thickness of aquifer              T2  = ";cin>>t2;cout<<endl;
cout<<" Input half width of top level of aquifer  B  = ";cin>>wb;cout<<endl;
```

```
cout<<" Input half width of stream bed          B1  = ";cin>>wb1;cout<<endl;
cout<<" Input thickness of bed                 T1  = ";cin>>t1;cout<<endl;
cout<<" Input distance of piezometric           L  = ";cin>>lb;cout<<endl<<endl;
cout<<" Input aproximate value of             0<d<1  = ";cin>>d2;cout<<endl;
cout<<" Input aproximate value of               f>1  = ";cin>>f2;cout<<endl;
dd=0.0000000001;df=(f2-1)/100000000000;
pi=3.141592654;
if((wb-wb1)<0.000001) ap=0.5;else ap=(atan((t2-t1)/(wb-wb1)))/pi;
ap2=1.-2.*ap;
do
        {
        d0=d2;
        f0=f2;
        d =d2+dd;
        f =f2+df;
        gcd=0;
        for(i=0;i<n;i++) //coresponding F (d0,f0)
                {
                ycd1=sqrt(sqrt(pow(d0,2)))*(1+xi[i])/2;
                ycd2=sqrt(sqrt(pow(d0,2)))*(1-xi[i])/2;
                gcd1=wi[i]*pow(ycd1,ap2)*pow((d0-pow(ycd1,2)),ap)/((sqrt(sqrt(pow((f0-
                pow(ycd1,2)),2))))*(sqrt(sqrt(pow((1-pow(ycd1,2)),2)))));
                gcd2=wi[i]*pow(ycd2,ap2)*pow((d0-pow(ycd2,2)),ap)/((sqrt(sqrt(pow((f0-
                pow(ycd2,2)),2))))*(sqrt(sqrt(pow((1-pow(ycd2,2)),2)))));
                gcd=gcd+(gcd1+gcd2);
                }
        Fd0f0=sqrt(sqrt(pow(d0,2)))*gcd-pi*sqrt(pow((wb-wb1),2)+pow((t2-t1),2))/t2;

        gde=0;
        for(q=0;q<m;q++) //corresponding G (d0,f0)
                {
                yde1=(sqrt(sqrt(pow((1-d0),2))))/2*(1+xi[q]);
                yde2=(sqrt(sqrt(pow((1-d0),2))))/2*(1-xi[q]);
                gde1=wi[q]*pow((1-pow(yde1,2)-d0),ap)/((pow((1-pow(yde1,2)),ap))*(sqrt(sqrt(pow((f0-
                1+pow(yde1,2)),2)))));
                gde2=wi[q]*pow((1-pow(yde2,2)-d0),ap)/((pow((1-pow(yde2,2)),ap))*(sqrt(sqrt(pow((f0-
                1+pow(yde2,2)),2)))));
                gde=gde+gde1+gde2;
                }
        Gd0f0=sqrt(sqrt(pow((1-d0),2)))*gde-wb1*pi/t2;//1-pow((wb*pi/(t2*gde)),2)-d0;

        gcda=0;
        for(j=0;j<n;j++) //Corresponding F(d0+dd,f0)
                {
                ycd1a=sqrt(sqrt(pow(d,2)))*(1+xi[j])/2;
                ycd2a=sqrt(sqrt(pow(d,2)))*(1-xi[j])/2;
                gcd1a=wi[j]*pow(ycd1a,ap2)*pow((d-pow(ycd1a,2)),ap)/((sqrt(sqrt(pow((f0-
                pow(ycd1a,2)),2))))*(sqrt(sqrt(pow((1-pow(ycd1a,2)),2)))));
                gcd2a=wi[j]*pow(ycd2a,ap2)*pow((d-pow(ycd2a,2)),ap)/((sqrt(sqrt(pow((f0-
                pow(ycd2a,2)),2))))*(sqrt(sqrt(pow((1-pow(ycd2a,2)),2)))));
                gcda=gcda+(gcd1a+gcd2a);
                }
        Fdf0=sqrt(sqrt(pow(d,2)))*gcda-pi*sqrt(pow((wb-wb1),2)+pow((t2-t1),2))/t2;

        gdea=0;
        for(k=0;k<m;k++)//Corresponding G(d0+dd,f0)
                {
                yde1a=(sqrt(sqrt(pow((1-d),2))))/2*(1+xi[k]);
                yde2a=(sqrt(sqrt(pow((1-d),2))))/2*(1-xi[k]);
                gde1a=wi[k]*pow((1-pow(yde1a,2)-d),ap)/((pow((1-pow(yde1a,2)),ap))*(sqrt(sqrt(pow((f0-
                1+pow(yde1a,2)),2)))));
                gde2a=wi[k]*pow((1-pow(yde2a,2)-d),ap)/((pow((1-pow(yde2a,2)),ap))*(sqrt(sqrt(pow((f0-
                1+pow(yde2a,2)),2)))));
                gdea=gdea+gde1a+gde2a;
                }
        Gdf0=sqrt(sqrt(pow((1-d),2)))*gdea-wb1*pi/t2;//1-pow((wb*pi/(t2*gdea)),2)-d;

        gcdb=0;
        for(s=0;s<n;s++) //Corresponding F(d0,f0+df)
                {
                ycd1b=sqrt(sqrt(pow(d0,2)))*(1+xi[s])/2;
                ycd2b=sqrt(sqrt(pow(d0,2)))*(1-xi[s])/2;
```

```
cout<<endl<<" ***********************************************************"<<endl;}
else {clrscr(); cout<<endl<<"   APROXIMATION OF VALUE OF 'f' or 'd' HAVE TO BE CHANGED"<<endl;getch();exit(0);}

cout<<endl<<" Repeat for different value of B, T1, L ? (PRESS : 1)";
cout<<endl<<" Continue for different potential head  ? (PRESS : 2)";
cout<<endl<<" Terminate this programe ...............? (PRESS : 3)"<<endl;
cout<<endl<<"                        YOUR CHOICE NUMBER : ";cin>>cont_s;}
while (cont_s==1);

if (cont_s==3) exit(0);
if (cont_s==2)
{
//Calculation of seepage for different potential head
cout<<" Value of Ts (maximum Ts=T1) = ";cin>>ts;cout<<endl;
cout<<" Aproximate value of s      = "<<f-ts*(f-1)/t1<<endl;
cout<<" Input aproximate value of s = ";cin>>ss;
cout<<endl;
ds=(ss-1)/100;
do
        {
        s0=ss-ds;
        s1=ss+ds;
        gefa=0;
        for(r=0;r<n;r++)//corresponding s0
                {
                yef01=sqrt(sqrt(pow((s0-1),2)))*(1+xi[r])/2;
                yef02=sqrt(sqrt(pow((s0-1),2)))*(1-xi[r])/2;
                gefa1=wi[r]*pow((1-d+pow(yef01,2)),ap)/(sqrt(sqrt(pow((f-1-
                pow(yef01,2)),2)))*pow((1+pow(yef01,2)),ap));
                gefa2=wi[r]*pow((1-d+pow(yef02,2)),ap)/(sqrt(sqrt(pow((f-1-
                pow(yef02,2)),2)))*pow((1+pow(yef02,2)),ap));

                gefa =gefa+(gefa1+gefa2);
                }
        fs0=sqrt(sqrt(pow((s0-1),2)))*gefa-(t1-ts)*pi/t2;

        gefb=0;
        for(r=0;r<n;r++)//corresponding s1
                {
                yef11=sqrt(sqrt(pow((s1-1),2)))*(1+xi[r])/2;
                yef12=sqrt(sqrt(pow((s1-1),2)))*(1-xi[r])/2;
                gefb1=wi[r]*pow((1-d+pow(yef11,2)),ap)/(sqrt(sqrt(pow((f-1-
                pow(yef11,2)),2)))*pow((1+pow(yef11,2)),ap));
                gefb2=wi[r]*pow((1-d+pow(yef12,2)),ap)/(sqrt(sqrt(pow((f-1-
                pow(yef12,2)),2)))*pow((1+pow(yef12,2)),ap));

                gefb =gefb+(gefb1+gefb2);
                }
        fs1=sqrt(sqrt(pow((s1-1),2)))*gefb-(t1-ts)*pi/t2;

        gefc=0;
        for(r=0;r<n;r++)//corresponding s
                {
                yef1=sqrt(sqrt(pow((ss-1),2)))*(1+xi[r])/2;
                yef2=sqrt(sqrt(pow((ss-1),2)))*(1-xi[r])/2;
                gefc1=wi[r]*pow((1-d+pow(yef1,2)),ap)/(sqrt(sqrt(pow((f-1-pow(yef1,2)),2)))*pow((1+pow(yef1,2)),ap));
                gefc2=wi[r]*pow((1-d+pow(yef2,2)),ap)/(sqrt(sqrt(pow((f-1-pow(yef2,2)),2)))*pow((1+pow(yef2,2)),ap));

                gefc =gefc+(gefc1+gefc2);
                }
        fs=sqrt(sqrt(pow((ss-1),2)))*gefc-(t1-ts)*pi/t2;

        dfpds=(fs1-fs0)/(ds*2.);cout<<" s0 = "<<s0<<" s1 = "<<s1;tes1=sqrt(pow(dfpds,2));
        if (tes1>3.4e-4900) {ss=ss-fs/dfpds;ds=-fs/dfpds;}
        else if (ss<1) ss=1;
        else {ss=ss;ds=0;}

        cout<<" ds = "<<ds<<" s= "<<ss;

        if(ds>0)
        e2=ds;else
        e2=-ds;
        }
```

```
                    gcd1b=wi[s]*pow(ycd1b,ap2)*pow((d0-pow(ycd1b,2)),ap)/((sqrt(sqrt(pow((f-
                    pow(ycd1b,2)),2))))*(sqrt(sqrt(pow((1-pow(ycd1b,2)),2)))));
                    gcd2b=wi[s]*pow(ycd2b,ap2)*pow((d0-pow(ycd2b,2)),ap)/((sqrt(sqrt(pow((f-
                    pow(ycd2b,2)),2))))*(sqrt(sqrt(pow((1-pow(ycd2b,2)),2)))));
                    gcdb=gcdb+(gcd1b+gcd2b);
                    }
                    Fd0f=sqrt(sqrt(pow(d0,2)))*gcdb-pi*sqrt(pow((wb-wb1),2)+pow((t2-t1),2))/t2;

          gdeb=0;
          for(t=0;t<m;t++)//Corresponding G(d0,f0+df)
                    {
                    yde1b=(sqrt(sqrt(pow((1-d0),2))))/2*(1+xi[t]);
                    yde2b=(sqrt(sqrt(pow((1-d0),2))))/2*(1-xi[t]);
                    gde1b=wi[t]*pow((1-pow(yde1b,2)-d0),ap)/((pow((1-pow(yde1b,2)),ap))*(sqrt(sqrt(pow((f-
                    1+pow(yde1b,2)),2)))));
                    gde2b=wi[t]*pow((1-pow(yde2b,2)-d0),ap)/((pow((1-pow(yde2b,2)),ap))*(sqrt(sqrt(pow((f-
                    1+pow(yde2b,2)),2)))));
                    gdeb=gdeb+gde1b+gde2b;
                    }
          Gd0f=sqrt(sqrt(pow((1-d0),2)))*gdeb-wb1*pi/t2;//1-pow((wb*pi/(t2*gdeb)),2)-d0;

          dfpdd=(Fdf0-Fd0f0)/dd;
          dfpdf=(Fd0f-Fd0f0)/df;
          dgpdd=(Gdf0-Gd0f0)/dd;
          dgpdf=(Gd0f-Gd0f0)/df;

          long double det=1./(dfpdd*dgpdf-dfpdf*dgpdd);
          d1=d-det*(Fd0f0*dgpdf-dfpdf*Gd0f0);
          f1=f-det*(dfpdd*Gd0f0-Fd0f0*dgpdd);
          d2=(d1+d)/2;
          f2=(f1+f)/2;
          dd=d2-d;
          df=f2-f;
          cout.precision(12);
          cout<<" d = "<<d<<" f = "<<f<<" F(d0,F0) = "<<Fd0f0<<" G(d0,f0) = "<<Gd0f0<<endl;
          cout<<" F(d,f0) = "<<Fdf0<<" G(d,f0) = "<<Gdf0<<" F(d0,f) = "<<Fd0f<<" G(d0,f) = "<<Gd0f<<endl;
          cout<<" dF/dd = "<<dfpdd<<" dF/df = "<<dfpdf<<" dG/dd = "<<dgpdd<<" dG/df = "<<dgpdf<<endl;
          cout<<" df = "<<df<<" dd = "<<dd<<" d0 = "<<d0<<" f0 = "<<f0<<endl;

          if (df<0) self=-df;else self = df;
          if (dd<0) seld=-dd;else seld = dd;
          }
while(self>1e-18&&seld>1e-18);

bca=.1;
do
          {
          gca=0;
          b=bca;
          for(r=0;r<n;r++)
                    {
                    yca1=sqrt(b)*(1+xi[r])/2;
                    yca2=sqrt(b)*(1-xi[r])/2;
                    gca1=wi[r]*pow(yca1,ap2)*pow((d+pow(yca1,2)),ap)/(sqrt(f+pow(yca1,2))*sqrt(1+pow(yca1,2)));
                    gca2=wi[r]*pow(yca2,ap2)*pow((d+pow(yca2,2)),ap)/(sqrt(f+pow(yca2,2))*sqrt(1+pow(yca2,2)));
                    gca=gca+(gca1+gca2);
                    }
          bca=pow((lb*pi/(t2*gca)),2);
          if (b>bca) e=b-bca;else e=bca-b;
          }
while(e>1e-12);

clrscr();cout<<endl;
cout.precision(4);cout.precision(4);
cout<<" T2 : "<<t2<<"   T1 : "<<t1<<"   B : "<<wb<<"   B1 = "<<wb1<<"   L : "<<lb<<endl;
q_by_kdh=pi/(2*log(sqrt(1+b)+sqrt(b)));cout.precision(12);if (0<=d&&d<1&&f>=1) {
cout<<endl<<" ******************************************************************"<<endl
<<endl<<endl<<setw(6)<<" d = "<<setw(20)<<d<<setw(14)<<" delta dd = "<<setw(20)<<dd<<endl<<setw(6)<<" f =
"<<setw(20)<<f<<setw(14)<<" delta df = "<<setw(20)<<df<<endl;
cout<<setw(6)<<" b = "<<setw(20)<<b<<setw(14)<<" delta db = "<<setw(20)<<e<<endl<<endl;
cout<<setw(20)<<" q/(k dh)       = "<<setw(20)<<setprecision(6)<<q_by_kdh<<endl;
cout<<setw(20)<<" q(bed)/q(total) = "<<setw(20)<<setprecision(6)<<2*asin(sqrt(1-d))/pi<<endl;
cout<<setw(20)<<" dL/T2          = "<<setw(20)<<setprecision(6)<<2.*log(sqrt(1+b)+sqrt(b))/pi-lb/t2<<endl<<endl;
```

```
while(e2>1e-14);

q_by_kdh2=pi/(2.*log(sqrt(ss-1)+sqrt(ss)));
clrscr();
cout.precision(4);
cout<<endl<<endl<<"  ****************************************** "<<endl<<endl;
cout<<"  T2 = "<<t2<<"  B = "<<wb<<"  T1 = "<<t1<<"   Ts = "<<ts<<endl;
cout.precision(12);
cout<<"   s = "<<ss<<" ds = "<<ds<<endl<<endl<<endl
<<"   q/[k(h1-hs)] = "<<q_by_kdh2<<endl<<endl;
cout<<"  ****************************************** "<<endl<<endl;
getch();
}
else{cout<<endl<<" Your choice is beyond this program ";getch();exit(0);}
cout<<"   Continue for another data (Y/N) : ";cin>>cont_q;
}
while (cont_q=='y'||cont_q=='Y');
}
```

B.2 Computer Programming for Seepage in Unconfined Aquifer

```
#include<iostream.h>
#include<math.h>
#include<iomanip.h>
#include<process.h>
#include<conio.h>
#include<string.h>
#include<fstream.h>

const int m=48;//number of x-gauss coefficient
const int n=48;//number of wi-gauss coefficient
long double fkce(long double);
long double fkie(long double ,long double );
long double qbyk(float ,long double );
long double q1byk(float ,long double , long double );
long double dhf(float ,long double ,long double );
long double m1(long double ,long double ,long double ,float ,long double ,long double ,long double );
long double cd(long double ,long double ,long double ,float ,long double ,long double ,long double ,float ,float ,float ,float );
long double de(long double ,long double ,long double ,float ,long double ,long double ,long double ,float ,float );
long double ef(long double ,long double ,long double ,float ,long double ,long double ,long double ,float ,float );
long double dhb(float ,long double ,long double );
long double bc(long double ,long double ,long double ,float ,float ,long double ,long double ,long double ,long double );
long double dhm(float ,long double ,long double );
long double em(long double ,long double ,long double ,float ,long double ,long double ,long double ,float ,float ,long double
);

void main()
{
ofstream outfile ("cs3-m01.cpp");
float wb1,wb2,la,lb,t1,dha,pi,h,tm;
long double a,a0,a1,b,b0,b1,d,d0,d1,f,f0,f1,da,db,dd,df,mm,mm0,mm1,dm;
long double ap,bt,gm,conap;
long double fcd,fcda0,fcda1,fcdb0,fcdb1,fcdd0,fcdd1,fcdf0,fcdf1;
long double fde,fdea0,fdea1,fdeb0,fdeb1,fded0,fded1,fdef0,fdef1;
long double fef,fefa0,fefa1,fefb0,fefb1,fefd0,fefd1,feff0,feff1;
long double fbc,fbcb0,fbcb1,dbcpdb,corb;
long double fem,femm0,femm1,dempdm,corm;
long double det,co_a,co_d,co_f,er_a,er_d,er_f,sum_er;
long double dcdpda,dcdpdd,dcdpdf,ddepda,ddepdd,ddepdf,defpda,defpdd,defpdf;
long double qperk,q1perk,delhf,delhb,delhm;
long double ksc,fk1,fk2,dl;
char lanjut,preatic,finis1,finis2,potential;
outfile<<" T1 "<<" B1 "<<" B2 "<<" H "<<" dha "<<" La "<<" a "<<" d "<<" f "<<" dhf "<<" alpha "<<" betha "<<" gamma "<<"
q/(k*dha) "<<" q1/(k*dha) "<<" dL "<<endl;

do
    { //another data
    clrscr();
    cout<<endl<<"    SEEPAGE FROM PARTIALLY PENETRATING STREAM IN UN-CONFINED
    AQUIFER"<<endl<<endl;
    pi=3.141592654;
    cout<<"  Input thickness of aquifer below stream bed T1        = ";cin>>t1;
    cout<<"  Input half width of streambed                   B1  = ";cin>>wb1;
    cout<<"  Input half width of top of water level          B2  = ";cin>>wb2;
    cout<<"  Input depth  of water in the stream              H   = ";cin>>h;
    cout<<"  Input drawdown in observation well              dha = ";cin>>dha;
    cout<<"  Input distance of piezometer from the stream bank La  = ";cin>>la;
    cout<<endl;
    cout<<"  Input approximate value of a (a>0)    = ";cin>>a;
    cout<<"  Input approximate value of d (d0<d<1) = ";cin>>d;
    cout<<"  Input approximate value of f (f>1)    = ";cin>>f;

    da=a/1000;
    dd=d/1000;
    df=f/1000;
    do
        { //iteration
        a0=a-da;a1=a+da;
        d0=d-dd;d1=d+dd;
        f0=f-df;f1=f+df;
        conap=(wb2-qbyk(dha,a))-(wb1-q1byk(dha,a,d));
        if (conap<0) conap=-conap; else {conap=conap;}
```

```
if (conap<0.00000001) ap=0.5;else ap=atan(h/((wb2-qbyk(dha,a))-(wb1-q1byk(dha,a,d))))/pi;
if (((dha-dhf(dha,a,f))/(wb2+la))<0.00000001) bt=0.5;else bt=(atan((dha-dhf(dha,a,f))/(wb2+la))+pi/2)/pi;
if(qbyk(dha,a)<0.0000001) gm=0.5;else gm=atan((t1+h-dha)/qbyk(dha,a))/pi;
fcd=cd(ap,bt,gm,la,a,d,f,wb2,wb1,h,dha);
fcda0=cd(ap,bt,gm,la,a0,d,f,wb2,wb1,h,dha);
fcda1=cd(ap,bt,gm,la,a1,d,f,wb2,wb1,h,dha);
fcdd0=cd(ap,bt,gm,la,a,d0,f,wb2,wb1,h,dha);
fcdd1=cd(ap,bt,gm,la,a,d1,f,wb2,wb1,h,dha);
fcdf0=cd(ap,bt,gm,la,a,d,f0,wb2,wb1,h,dha);
fcdf1=cd(ap,bt,gm,la,a,d,f1,wb2,wb1,h,dha);

fde=de(ap,bt,gm,la,a,d,f,wb1,dha);
fdea0=de(ap,bt,gm,la,a0,d,f,wb1,dha);
fdea1=de(ap,bt,gm,la,a1,d,f,wb1,dha);
fded0=de(ap,bt,gm,la,a,d0,f,wb1,dha);
fded1=de(ap,bt,gm,la,a,d1,f,wb1,dha);
fdef0=de(ap,bt,gm,la,a,d,f0,wb1,dha);
fdef1=de(ap,bt,gm,la,a,d,f1,wb1,dha);

fef=ef(ap,bt,gm,la,a,d,f,t1,dha);
fefa0=ef(ap,bt,gm,la,a0,d,f,t1,dha);
fefa1=ef(ap,bt,gm,la,a1,d,f,t1,dha);
fefd0=ef(ap,bt,gm,la,a,d0,f,t1,dha);
fefd1=ef(ap,bt,gm,la,a,d1,f,t1,dha);
feff0=ef(ap,bt,gm,la,a,d,f0,t1,dha);
feff1=ef(ap,bt,gm,la,a,d,f1,t1,dha);

dcdpda=(fcda1-fcda0)/(2*da);
dcdpdd=(fcdd1-fcdd0)/(2*dd);
dcdpdf=(fcdf1-fcdf0)/(2*df);

ddepda=(fdea1-fdea0)/(2*da);
ddepdd=(fded1-fded0)/(2*dd);
ddepdf=(fdef1-fdef0)/(2*df);

defpda=(fefa1-fefa0)/(2*da);
defpdd=(fefd1-fefd0)/(2*dd);
defpdf=(feff1-feff0)/(2*df);

det=1./((dcdpda*ddepdd*defpdf+dcdpdd*ddepdf*defpda+dcdpdf*ddepda*defpdd)-
(dcdpdf*ddepdd*defpda+ddepdf*defpdd*dcdpda+defpdf*dcdpdd*ddepda));
co_a=(fcd*ddepdd*defpdf+dcdpdd*ddepdf*fef+dcdpdf*fde*defpdd)-
(dcdpdf*ddepdd*fef+ddepdf*defpdd*fcd+defpdf*dcdpdd*fde);
co_d=(dcdpda*fde*defpdf+fcd*ddepdf*defpda+dcdpdf*ddepda*fef)-
(dcdpdf*fde*defpda+ddepdf*fef*dcdpda+defpdf*fcd*ddepda);
co_f=(dcdpda*ddepdd*fef+dcdpdd*fde*defpda+fcd*ddepda*defpdd)-
(fcd*ddepdd*defpda+fde*defpdd*dcdpda+fef*dcdpdd*ddepda);

a=a-det*co_a;
d=d-det*co_d;
f=f-det*co_f;

da=-det*co_a;
dd=-det*co_d;
df=-det*co_f;

if(da>0) er_a=da;else er_a=-da;
if(dd>0) er_d=dd;else er_d=-dd;
if(df>0) er_f=df;else er_f=-df;
sum_er=er_a+er_d+er_f;

cout<<" "<<da;
}//end do iteration
while (sum_er>1e-10);

qperk=qbyk(dha,a);
q1perk=q1byk(dha,a,d);
delhf=dhf(dha,a,f);
ksc=1./(1+a);
fk1=fkce(ksc);
ksc=a/(1+a);
fk2=fkce(ksc);
dl=fk2*(t1+h-0.5*dha)/fk1-la;
```

```
cout<<endl<<endl<<" a= "<<a<<" d= "<<d<<" f= "<<f<<endl;
cout<<" ap= "<<ap<<" bt= "<<bt<<" gm= "<<gm<<endl;
cout<<" q/k= "<<qperk<<" q1/k= "<<q1perk<<" dhf= "<<delhf<<endl<<endl;
outfile<<" "<<t1<<" "<<wb1<<" "<<wb2<<" "<<h<<" "<<dha<<" "<<la<<" "<<a<<" "<<d<<" "<<f<<" "<<delhf<<"
"<<ap<<" "<<bt<<" "<<gm<<" "<<qperk/dha<<" "<<q1perk/dha<<" "<<dl<<endl;

//starting of preatic line (b)
cout<<" Continue for preatic line (Y/N)...? ";cin>>preatic;
if (preatic=='y'||preatic=='Y')
{
do
{//star do preatic line
cout<<" Input diatance of a point from stream bank Lb = ";cin>>lb;
cout<<" Input approximate value of b  (0<b<a)      b  = ";cin>>b;
db=b/1000;
   do
   {//star do iteration preatic
   b0=b-db;
   b1=b+db;

   fbc=bc(ap,bt,gm,la,lb,a,d,f,b);
   fbcb0=bc(ap,bt,gm,la,lb,a,d,f,b0);
   fbcb1=bc(ap,bt,gm,la,lb,a,d,f,b1);

   dbcpdb=(fbcb1-fbcb0)/(2*db);
   b=b-fbc/dbcpdb;
   db=-fbc/dbcpdb;
   corb=sqrt(pow(db,2));
   cout<<" "<<db;

   }//end do iteration preatic
   while(corb>1e-10);
delhb=dhb(dha,a,b);
cout<<endl<<endl<<" Lb = "<<lb<<" b = "<<b<<" dhb = "<<delhb<<endl<<endl;
outfile<<" Lb= "<<lb<<" b= "<<b<<" dhb= "<<delhb<<endl;
cout<<" Continue for different Lb (Y/N)..............? ";cin>>finis1;
}//end do preatic line
while (finis1=='y'||finis1=='Y');
}
//ending of preatic line

//9999
//starting of potential head (m)
cout<<" Continue for different potential head (Y/N)...? ";cin>>potential;
if (potential=='y'||potential=='Y')
{
do
{//star do potential
cout<<" Input distance of a point below stream bed Tm = ";cin>>tm;
cout<<" Input approximate value of m   (1<m<f)       = ";cin>>mm;
dm=mm/1000;
   do
   {//star do iteration potential
   mm0=mm-dm;
   mm1=mm+dm;

   fem=em(ap,bt,gm,la,a,d,f,tm,dha,mm);
   femm0=em(ap,bt,gm,la,a,d,f,tm,dha,mm0);
   femm1=em(ap,bt,gm,la,a,d,f,tm,dha,mm1);

   dempdm=(femm1-femm0)/(2*dm);
   mm=mm-fem/dempdm;
   dm=-fem/dempdm;
   corm=sqrt(pow(dm,2));
   cout<<" "<<dm;

   }//end do iteration potential
   while(corm>1e-10);
delhm=dhm(dha,a,mm);
cout<<endl<<endl<<" Tm = "<<tm<<" m = "<<mm<<" dhm = "<<delhm<<endl<<endl;
outfile<<" Tm= "<<tm<<" m= "<<mm<<" dhm= "<<delhm<<endl;
cout<<" Continue for different Tm (Y/N)..............? ";cin>>finis2;
}//end do potential head
```

```
    while (finis2=='y'||finis2=='Y');
    }
    //ending of potential
    cout<<" Continue for another data (Y/N)..............? ";cin>>lanjut;
    }//end do another data
while (lanjut=='y'||lanjut=='Y');
}//end void main

long double
xi[m]={.0162767448496029695 79,.0488129851360497311 12,.0812974954644255589 94,.1136958501106659209 11,.145
97371465489694 1989,.1780968823676186027 59,
.2100313104605672036 03,.2417431561638400123 28,.2731988125910491414 87,.3043649443544963530 24,.33520852
2892625422616,.3656968614723136350 31,
.3957976498289086032 85,.4254789884073005453 65,.4547094221677430086 36,.4834579739205963597 68,.51169417
7154667673586,.5393881083243574362 27,
.5665104185613971684 04,.5930323647775720806 84,.6189258401254685703 86,.6441634037849671067 98,.66871831
0043916153953,.6925645366421715613 44,
.7156768123489676262 25,.7380306437444001328 51,.7596023411766474987 03,.7803690438674332176 04,.80030874
4139140817229,.8194003107379316755 39,
.8376235112281871214 94,.8549590334346014554 63,.8713885059092965028 74,.8868945174024204160 57,.90146063
5315852341319,.9150714231208980742 06,
.9277124567223086909 65,.9393703397527552169 32,.9500327177844376357 56,.9596882914487425393 00,.96832682
8463264212174,.9759391745851364664 53,
.9825172635630146774 47,.9880541263296237994 81,.9925439003237626245 72,.9959818429872092906 50,.99836437
5863181677724,.9996895038832307668 28};

long double
wi[n]={.0325506144923631662 42,.0325161187138688359 87,.0324471637140642693 64,.0323438225685759284 29,.032
2062047940302506 69,.0320344562319926632 18,
.0318287588944110065 35,.0315893307707271685 58,.0313164255968613558 13,.0310103325863138374 23,.03067137
6123669149014,.0302999154208275937 94,
.0298963441363283859 84,.0294610899581679059 70,.0289946141505552365 43,.0284974110650853856 46,.02797000
7616848334440,.0274129627260292428 23,
.0268268667255917621 98,.0262123407356724139 13,.0255700360053493614 99,.0249006332224836102 88,.02420484
1792364691282,.0234833990859262198 42,
.0227370696583293740 01,.0219666444387443491 95,.0211729398921912989 88,.0203567971543333245 95,.01951908
1140145022410,.0186606796274114673 85,
.0177825023160452608 38,.0168854798642451724 50,.0159705629025622913 81,.0150387210269949380 06,.01409094
1772314860916,.0131282295669615726 37,
.0121516046710883196 35,.0111621020998384985 91,.0101607705350084157 58,.0091486712307833866 33,.00812687
6925698759217,:0070964707911538652 69,
.0060585455042359616 83,.0050142027429275176 93,.0039645543384446866 74,.0029107318179349464 08,.00185396
0788946921732,.0007967920655520124 29};

float pi=3.141592654;

//first kind complete elliptic integral
long double fkce(long double ksc)
{
long double yfk,yfkp,yfkn,fk;
yfk=0;
   for (int ifk=0;ifk<m;ifk++)
   {
   yfkp=wi[ifk]/sqrt((1-pow((sin(pi*(1+xi[ifk])/4)),2)*ksc));
   yfkn=wi[ifk]/sqrt((1-pow((sin(pi*(1-xi[ifk])/4)),2)*ksc));
   yfk=yfk+(yfkp+yfkn);
   }
   fk=pi*yfk/4;
   return fk;
}

//first kind in-complete elliptic integral
long double fkie(long double pai,long double ks)
{
long double yfk,yfkp,yfkn,fk;
yfk=0;
   for (int ifk=0;ifk<m;ifk++)
   {
   yfkp=wi[ifk]/sqrt(1-ks*pow((sin(pai*(1+xi[ifk])/2)),2));
   yfkn=wi[ifk]/sqrt(1-ks*pow((sin(pai*(1-xi[ifk])/2)),2));
   yfk=yfk+(yfkp+yfkn);
   }
fk=pai*yfk/2;
```

```
return fk;
}

//w-plane A to C
long double qbyk(float dha,long double a)
{
long double ks,pai,qperk,ksc,fk1,fk2;
ksc=1/(1+a);
fk1=fkce(ksc);
ksc=a/(1+a);
fk2=fkce(ksc);
qperk=dha*fk1/fk2;
return qperk;
}

//w-plane C to B - preatic line
long double dhb(float dha,long double a,long double b)
{
long double ks,pai,qperk,ksc,fk1,fk2,delhb;
ksc=1/(1+a);
fk1=fkce(ksc);
ks=a/(1+a);
pai=asin(sqrt(((1+a)*b)/(a*(1+b))));
fk2=fkie(pai,ks);
delhb=qbyk(dha,a)*fk2/fk1;
return delhb;
}

//w-plane D to E
long double q1byk(float dha,long double a, long double d)
{
long double ks,pai,q1perk,ksc;
pai=asin(sqrt(1-d));
ks=1./(1+a);
ksc=1./(1+a);
q1perk=qbyk(dha,a)*fkie(pai,ks)/fkce(ksc);
return q1perk;
}

//w-plane E to F
long double dhf(float dha,long double a,long double f)
{
long double ks,pai,delhf,ksc;
pai=asin(sqrt((f-1)/f));
ks=a/(1+a);
ksc=1./(1+a);
delhf=qbyk(dha,a)*fkie(pai,ks)/fkce(ksc);
return delhf;
}

//w-plane E to M
long double dhm(float dha,long double a,long double mm)
{
long double ks,pai,delhm,ksc;
pai=asin(sqrt((mm-1)/mm));
ks=a/(1+a);
ksc=1./(1+a);
delhm=qbyk(dha,a)*fkie(pai,ks)/fkce(ksc);
return delhm;
}

//z-plane A to C ... constant m
long double m1(long double ap,long double bt,long double gm,float la,long double a,long double d,long double f)
{
long double yac,yacp,yacn,fxp,fxn,mz;
yac=0;
for (int iac=0;iac<m;iac++)
   {
   fxp=sqrt(a/2)*(1+xi[iac])/2;
   fxn=sqrt(a/2)*(1-xi[iac])/2;
   yacp=wi[iac]*pow(fxp,(1-2*ap))*pow((d+pow(fxp,2)),ap)/(pow((a-
pow(fxp,2)),gm)*sqrt(1+pow(fxp,2))*pow((f+pow(fxp,2)),bt))+
```

```
 wi[iac]*pow(fxp,(1-2*gm))*pow((d+a-pow(fxp,2)),ap)/(pow((a-pow(fxp,2)),ap)*sqrt(1+a-pow(fxp,2))*pow((f+a-
pow(fxp,2)),bt));
yacn=wi[iac]*pow(fxn,(1-2*ap))*pow((d+pow(fxn,2)),ap)/(pow((a-
pow(fxn,2)),gm)*sqrt(1+pow(fxn,2))*pow((f+pow(fxn,2)),bt))+
 wi[iac]*pow(fxn,(1-2*gm))*pow((d+a-pow(fxn,2)),ap)/(pow((a-pow(fxn,2)),ap)*sqrt(1+a-pow(fxn,2))*pow((f+a-
pow(fxn,2)),bt));
   yac=yac+(yacp+yacn);
   }
mz=la/(sqrt(a/2)*yac);
return mz;
}

//z-plane from C to D
long double cd(long double ap,long double bt,long double gm,float la,long double a,long double d,long double f,float
wb2,float wb1,float h,float dha)
{
long double fxp,fxn,ycd,ycdp,ycdn,fcd,fxps,fxns;
ycd=0;
for (int icd=0;icd<m;icd++)
   {
   fxp=sqrt(d)*(1+xi[icd])/2; fxps=pow(fxp,2);
   fxn=sqrt(d)*(1-xi[icd])/2; fxns=pow(fxn,2);
   ycdp=wi[icd]*pow(fxp,(1-2*ap))*pow((d-fxps),ap)/(pow((fxps+a),gm)*sqrt(1-fxps)*pow((f-fxps),bt));
   ycdn=wi[icd]*pow(fxn,(1-2*ap))*pow((d-fxns),ap)/(pow((fxns+a),gm)*sqrt(1-fxns)*pow((f-fxns),bt));
   ycd=ycd+(ycdp+ycdn);
   }
fcd=m1(ap,bt,gm,la,a,d,f)*sqrt(d)*ycd-sqrt(pow((wb2-wb1-qbyk(dha,a)+q1byk(dha,a,d)),2)+pow(h,2));
return fcd;
}

//z-plane D to E
long double de(long double ap,long double bt,long double gm,float la,long double a,long double d,long double f,float
wb1,float dha)
{
long double fxp,fxn,yde,ydep,yden,fde;
yde=0;
for (int ide=0;ide<m;ide++)
   {
   fxp=sqrt(1-d)*(1+xi[ide])/2;
   fxn=sqrt(1-d)*(1-xi[ide])/2;
   ydep=wi[ide]*pow((1-pow(fxp,2)-d),ap)/(pow((1-pow(fxp,2)+a),gm)*pow((1-pow(fxp,2)),ap)*pow((f-1+pow(fxp,2)),bt));
   yden=wi[ide]*pow((1-pow(fxn,2)-d),ap)/(pow((1-pow(fxn,2)+a),gm)*pow((1-pow(fxn,2)),ap)*pow((f-1+pow(fxn,2)),bt));
   yde=yde+(ydep+yden);
   }
fde=m1(ap,bt,gm,la,a,d,f)*sqrt(1-d)*yde-wb1+q1byk(dha,a,d);
return fde;
}

//z-plane E to F
long double ef(long double ap,long double bt,long double gm,float la,long double a,long double d,long double f,float t1,float
dha)
{
long double fef,yef1,yef2,yefp1,yefp2,yefn1,yefn2,fxp1,fxp2,fxn1,fxn2;
yef1=0;yef2=0;
for (int ief=0;ief<m;ief++)
   {
   fxp1=sqrt((f-1)/2)*(1+xi[ief])/2;
   fxn1=sqrt((f-1)/2)*(1-xi[ief])/2;
   fxp2=pow(((f-1)/2),0.1)*(1+xi[ief])/2;
   fxn2=pow(((f-1)/2),0.1)*(1-xi[ief])/2;
   yefp1=wi[ief]*pow((pow(fxp1,2)+1-d),ap)/(pow((pow(fxp1,2)+1+a),gm)*pow((pow(fxp1,2)+1),ap)*pow((f-pow(fxp1,2)-
1),bt));
   yefp2=wi[ief]*pow(fxp2,(9-10*bt))*pow((f-pow(fxp2,10)-d),ap)/(pow((f-pow(fxp2,10)+a),gm)*pow((f-
pow(fxp2,10)),ap)*sqrt(f-pow(fxp2,10)-1));
   yefn1=wi[ief]*pow((pow(fxn1,2)+1-d),ap)/(pow((pow(fxn1,2)+1+a),gm)*pow((pow(fxn1,2)+1),ap)*pow((f-pow(fxn1,2)-
1),bt));
   yefn2=wi[ief]*pow(fxn2,(9-10*bt))*pow((f-pow(fxn2,10)-d),ap)/(pow((f-pow(fxn2,10)+a),gm)*pow((f-
pow(fxn2,10)),ap)*sqrt(f-pow(fxn2,10)-1));
   yef1=yef1+(yefp1+yefn1);
   yef2=yef2+(yefp2+yefn2);
   }
fef=m1(ap,bt,gm,la,a,d,f)*sqrt((f-1)/2)*yef1+5*m1(ap,bt,gm,la,a,d,f)*pow(((f-1)/2),.1)*yef2-t1+dhf(dha,a,f);
return fef;
```

```
}

//z-plane B to C
long double bc(long double ap,long double bt,long double gm,float la,float lb,long double a,long double d,long double
f,long double b)
{
long double fbc,ybc1,ybc2,ybcp1,ybcp2,ybcn1,ybcn2,fxp1,fxp2,fxn1,fxn2;
ybc1=0;ybc2=0;
for (int ibc=0;ibc<m;ibc++)
   {
   fxp1=sqrt(b/2)*(1+xi[ibc])/2;
   fxn1=sqrt(b/2)*(1-xi[ibc])/2;
   fxp2=(sqrt(a-b/2)-sqrt(a-b))/2*xi[ibc]+(sqrt(a-b/2)+sqrt(a-b))/2;
   fxn2=-(sqrt(a-b/2)-sqrt(a-b))/2*xi[ibc]+(sqrt(a-b/2)+sqrt(a-b))/2;
   ybcp1=wi[ibc]*pow(fxp1,(1-2*ap))*pow((d+pow(fxp1,2)),ap)/(pow((a-
pow(fxp1,2)),gm)*sqrt(1+pow(fxp1,2))*pow((f+pow(fxp1,2)),bt));
   ybcp2=wi[ibc]*pow(fxp2,(1-2*gm))*pow((d+a-pow(fxp2,2)),ap)/(pow((a-pow(fxp2,2)),ap)*sqrt(1+a-
pow(fxp2,2))*pow((f+a-pow(fxp2,2)),bt));
   ybcn1=wi[ibc]*pow(fxn1,(1-2*ap))*pow((d+pow(fxn1,2)),ap)/(pow((a-
pow(fxn1,2)),gm)*sqrt(1+pow(fxn1,2))*pow((f+pow(fxn1,2)),bt));
   ybcn2=wi[ibc]*pow(fxn2,(1-2*gm))*pow((d+a-pow(fxn2,2)),ap)/(pow((a-pow(fxn2,2)),ap)*sqrt(1+a-
pow(fxn2,2))*pow((f+a-pow(fxn2,2)),bt));
   ybc1=ybc1+(ybcp1+ybcn1);
   ybc2=ybc2+(ybcp2+ybcn2);
   }
fbc=m1(ap,bt,gm,la,a,d,f)*sqrt(b/2)*ybc1+m1(ap,bt,gm,la,a,d,f)*(sqrt(a-b/2)-sqrt(a-b))*ybc2-lb;
return fbc;
}

//z-plane E to M
long double em(long double ap,long double bt,long double gm,float la,long double a,long double d,long double f,float
tm,float dha,long double mm)
{
long double fem,yem1,yem2,yemp1,yemp2,yemn1,yemn2,fxp1,fxp2,fxn1,fxn2;
yem1=0;yem2=0;
for (int iem=0;iem<m;iem++)
   {
   fxp1=sqrt((mm-1)/2)*(1+xi[iem])/2;
   fxn1=sqrt((mm-1)/2)*(1-xi[iem])/2;
   fxp2=(pow(((2*f-mm-1)/2),0.1)-pow((f-mm),0.1))*xi[iem]/2+(pow(((2*f-mm-1)/2),0.1)+pow((f-mm),0.1))/2;
   fxn2=-(pow(((2*f-mm-1)/2),0.1)-pow((f-mm),0.1))*xi[iem]/2+(pow(((2*f-mm-1)/2),0.1)+pow((f-mm),0.1))/2;
   yemp1=wi[iem]*pow((pow(fxp1,2)+1-d),ap)/(pow((pow(fxp1,2)+1+a),gm)*pow((pow(fxp1,2)+1),ap)*pow((f-pow(fxp1,2)-
1),bt));
   yemp2=wi[iem]*pow(fxp2,(9-10*bt))*pow((f-pow(fxp2,10)-d),ap)/(pow((f-pow(fxp2,10)+a),gm)*pow((f-
pow(fxp2,10)),ap)*sqrt(f-pow(fxp2,10)-1));
   yemn1=wi[iem]*pow((pow(fxn1,2)+1-d),ap)/(pow((pow(fxn1,2)+1+a),gm)*pow((pow(fxn1,2)+1),ap)*pow((f-pow(fxn1,2)-
1),bt));
   yemn2=wi[iem]*pow(fxn2,(9-10*bt))*pow((f-pow(fxn2,10)-d),ap)/(pow((f-pow(fxn2,10)+a),gm)*pow((f-
pow(fxn2,10)),ap)*sqrt(f-pow(fxn2,10)-1));
   yem1=yem1+(yemp1+yemn1);
   yem2=yem2+(yemp2+yemn2);
   }
fem=m1(ap,bt,gm,la,a,d,f)*sqrt((mm-1)/2)*yem1+5*m1(ap,bt,gm,la,a,d,f)*(pow(((2*f-mm-1)/2),.1)-pow((f-mm),.1))*yem2-
tm+dhm(dha,a,mm);
return fem;
}
```

B.3 Computer Programming for Unsteady State Flow

```
#include<iostream.h>
#include<math.h>
#include<iomanip.h>
#include<process.h>
#include<conio.h>
#include<graphics.h>
#include<string.h>
#include<fstream.h>

const int m=1000;

void main ()
{
ofstream outfile ("uns03.cpp");
clrscr();
float dL,T,Zr,k,St,dt,beta,t1,t2,b,b1,pi;
float Za[m],d[m],q[m];
long double sum,sum1;
int i,n;

cout<<endl;
cout<<" Input step rise in the river   : ";cin>>Zr;
cout<<" Coefficient of permeability    : ";cin>>k;
cout<<" Coefficient of storage         : ";cin>>St;
cout<<" Thickness of aquifer below stream bed : ";cin>>t1;
cout<<" Thickness of aquifer           : ";cin>>t2;
cout<<" Half width of stream           : ";cin>>b;
cout<<" Substitute Length dL           : ";cin>>dL;
cout<<" Delta time dt                  : ";cin>>dt;

pi=3.141592654;
outfile<<" B= "<<b<<" Zr= "<<Zr<<endl<<" T1= "<<t1<<" T= "<<T<<endl<<" T2= "<<t2<<" dL= "<<dL<<endl<<" dt=
"<<dt<<" St= "<<St<<endl<<endl;
outfile<<" n"<<" nxdt"<<" Za[n]"<<" q[n]"<<endl;

for (n=1;n<=m;n++)
{
sum=0;
Za[-1]=0;
Za[0]=0;
for (i=1;i<=n-1;i++)
  {
  d[n-i+1]=2*sqrt((T*St)/(dt*pi))*(sqrt(n-i+1)-sqrt(n-i));
  sum1=(Za[i]-Za[i-1])*d[n-i+1];
  sum=sum+sum1;
  }
d[1]=2*sqrt((T*St)/(dt*pi));
Za[n]=(Zr-dL/T*sum+dL/T*Za[n-1]*d[1])/(1+dL/T*d[1]);
q[n]=T*(Zr-Za[n])/dL;

cout<<" t: "<<dt*n<<"   Za["<<n<<"]: "<<Za[n]<<"   d["<<n<<"]: "<<d[n]<<endl;
outfile<<" "<<n<<" "<<n*dt<<" "<<Za[n]<<" "<<q[n]<<endl;

if (Za[n]>(0.99*Zr)){getch(); exit(0);}

}//looping za[n]

getch();
}//end
```