GROUNDWATER MODELLING USING AN IMPROVED FINITE DIFFERENCE SCHEME

A DISSERTATION

submitted in partial fulfilment of the
requirements for the award of the degree
of
MASTER OF ENGINEERING
in
IRRIGATION WATER MANAGEMENT

By
MANI KUMAR

WATER RESOURCES DEVELOPMENT TRAINING CENTRE
UNIVERSITY OF ROORKEE
ROORKEE-247 667 (INDIA)
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CANDIDATE'S DECLARATION

I hereby declare that the work, which is being presented in this dissertation entitled **"GROUNDWATER MODELLING USING AN IMPROVED FINITE DIFFERENCE SCHEME "** in the partial fulfillment of the requirement for the award of Degree of **Master of Engineering** in **Irrigation Water Management** and submitted in the department of Water Resources Development training center of the University, is an authentic record of my own work carried out during a period of July 2000 to Feb. 2001 under the supervision of **Dr. G.C. Mishra**, Professor, Water Resources Development Training Center, University of Roorkee, Roorkee.

The matter presented in this dissertation has not been submitted by me for award of any other degree.

Dated: February, 2001

Place: Roorkee

(Mani Kumar)

This is to certify that the statement made by the candidate is correct to the best of my knowledge.

Place: Roorkee

Date:

(Dr. G. C. Mishra)
Professor, WRDTC
University of Roorkee
Roorkee-247667

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(Mani Kumar)
Trainee officer,
20th IWM batch

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NOTATIONS

D	thickness of the aquifer, (L);
$h(j, k)$	piezometric head at node (j) during k^{th} time step, (L);
K	hydraulic conductivity of the aquifer, (LT^{-1});
t	time, (T);
T	transmissivity of the aquifer, (L^2T^{-1});
$T_{HM2}(j)$	Harmonic mean transmissivity at interface 2 of node (j), (L^2T^{-1});
$T_{HM4}(j)$	Harmonic mean transmissivity at interface 4 of node (j),(L^2T^{-1});
x	distance measured from stream aquifer interface, (L);
S	storage coefficient of the aquifer, (dimensionless);
β	hydraulic diffusivity of the aquifer, (L^2T^{-1});
Γ	reach transimissivity of the stream, (L^2T^{-1});
$f(t)$	time varying boundary perturbation;
$f'(t)$	time derivative of the input function;
$\propto(x, \Delta t, m)$	discrete pulse kernel (fully penetrating stream);
$\delta(x, \Delta t, m)$	discrete ramp kernel (fully penetrating stream);
Δt	size of uniform time step, (T);
Δv	volume of water entering during time step, (L^3);

SYNOPSIS

In the existing finite difference method, the flow (inflow/outflow) is assumed to follow a linear variation with time where as the spatial variation is non-linear and quadratic in nature. In case of a non-homogeneous, semi-infinite aquifer bounded by a fully penetrating stream, the flow rate at the boundary subsequent to a sudden change in boundary head varies inversely to a square root of time. Therefore the variation of flow at any section can be presumed to be non-linear with time. In improved finite difference scheme, non-linear temporal variation i.e. quadratic in nature has been taken in account. Further in a non-homogeneous case the head variation is sharp within small distance. Therefore a spatial variation may follow a polynomial equation of higher degree. This spatial variation is also accounted in Improved finite difference method. Thus proposed finite difference method that includes both temporal and spatial variation in consideration may give efficient results.

All numerical methods exhibit oscillations in computed head. This oscillation is prominent at grids adjacent to the boundary. In all these methods, inflow is not accounted truly. The oscillation can be minimized if the inflow is properly accounted for.

CHAPTER 1

INTRODUCTION

1.1 GENERAL

With the advancement of high-speed computers, numerical methods are exclusively used in analyzing groundwater flow problems. For ideal situations, groundwater flow problems are solved by combining Darcy's equation and continuity equation. The resulting differential equations describe the hydraulic relations within an aquifer. To solve the equations, the aquifer geometry, hydraulic characteristics, initial and boundary conditions should be known. An exact analytical solution can be obtained only if the equations, characteristic and conditions are simple. There are many groundwater flow problems for which analytic solutions are difficult. The reason is that these problems are complex, possessing non-linear features that cannot be included in analytical solutions. Owing to these difficulties, there has been a need for some techniques that enable meaningful solution to the above problems. Such technique exists in numerical modeling.

In most numerical techniques, an approximate solution is obtained by replacing the basic differential equations by another sets of equations that can be solved easily by a computer. Finite difference method is one of them.

1.2 FINITE DIFFERENCE METHOD

In finite difference method space are divided into small but finite intervals. The small areas thus formed are called nodal areas. There is a point called node in each small area, which is considered to be the representative of its nodal area. A certain storage coefficient and tranmissivity is assigned to each node. Now for each node point, an equation is written combining Darcy's and continuity equation. The resulting equation thus obtained are classified into explicit, implicit and crank nicholson approximation based on the assumption of variables at different space and time. In case of implicit and crank nicholson scheme, a set of equations are obtained which can be solved either by

Gauss elimination or Gauss-Siedel iteration method. In this thesis both schemes are used to analyze interaction of a fully penetrating stream and a semi-infinite aquifer. A comparative study is done for step input and ramp input.

1.3 IMPROVED FINITE DIFFERENCE METHOD

In existing finite difference method the flow (inflow/outflow) is assumed to vary linearly with time. However flow may vary non-linearly with time. The hydraulic head is also assumed to vary linearly with distance between the grid while mass balance is carried out or when the higher order term are neglected to write the finite difference of the governing differential equation.

In improved finite difference method, it has been assumed that:

i)flow variation with time at a particular section follows a quadratic distribution.

i.e. $h(x,t) = f(t) = a_1 + b_1 t + c_1 t^2$

ii) head between nodes at a particular time follows a polynomial equation as given below:

$h(x,t) = a + b\,x + c\,x^2 + d\,x^3$

Where a_1, b_1, c_1 are derived in terms of $h(j, t-\Delta t)$, $h(j, t)$ $h(j, t+\Delta t)$ and a, b, c, d are derived in terms of $h(j-1, t)$, $h(j, t)$, $h(j+1, t)$ and $h(j+2, t)$.

1.4 ANALYTIC METHOD

The unit step response function that relates rise in piezometric surface in an initially steady-state semiinfinite homogeneous and isotropic confined aquifer, bounded by a fully penetrating straight stream, to a step rise in stream stage has been derived by Carslaw and Jaegar(1959) for an analogous heat conduction problem. The unit step response function is:

$$K(x,t) = erfc\left\{\frac{x}{\sqrt{(4\beta\, t)}}\right\} \qquad \ldots(1.1)$$

Where x = distance from the bank of the stream,

t = time measured since the onset of change in stream stage,

β = hydraulic diffusivity of the aquifer defined as ratio of transmissivity to storage coefficient,

$erfc\{\cdot\}$ = complementary error function.

For varying stream stage, σ(t), the rise in piezometric surface, s (x, t), according to Duhamel's integral (Thomson1950) is given by

$$s(x,t) = \sigma_0 K(x,t) + \int_0^t K(x,t-\tau)\frac{d\sigma(\tau)}{d\tau}d\tau \qquad \ldots (1.2)$$

Where σ_0 = initial sudden rise in the stream stage,

τ = time variable.

Duhamel's integral, which can be expressed in two different forms (Thomson, 1950), has been used extensively for solving stream-aquifer interaction problems (Pinder et al. 1969; Venetis 1970; Hall and Moench 1972; Abdulrazzak and Morel- Seytoux 1983; Morel - Seytoux1988). The time span is discretized into uniform time steps of size Δt. The rate of change of stream stage, dσ(τ)/dτ, is assumed to be a constant within a time step which may vary from one time step to the next. Equation 2 is rewritten in terms of discrete kernel coefficient as:

$$s(x,n\Delta t) = \sigma_0\ erfc\left\{\frac{x}{\sqrt{(4\beta\, n\Delta t)}}\right\} + \sum_{\gamma=1}^{n}\left[(\sigma_\gamma - \sigma_{\gamma-1})\delta_\gamma(x,\Delta t, n-\gamma+1)\right] \qquad \ldots (1.3)$$

1.5 THE SCOPE OF STUDY

CASE1:

In numerical method the head at node near the boundary sometimes exceeds the boundary head even though there is no external recharge other than the stream, and the head at the node near the boundary oscillate exhibiting unacceptable values. It is intended to study whether oscillation is caused because of the boundary condition i.e. sudden change in stream stage causing infinite rate in flow or due to course time and space grid. The analysis has been considering a step input and a ramp input. Crank Nicholson method, completely implicit scheme and non-linear variation in flow rate have been considered separately to study the oscillation and convergence encounted in numerical method.

CASE2:

Polynomial spatial variation in head at a particular time has been considered and aquifer response to step and ramp variation has been obtained. The results obtained using numerical method have been compared with analytical results. The performance of the methods has been evaluated. The stream aquifer interaction has been used to study the performance.

CHAPTER 2

REVIEW OF LITERATURE

2.1 FINITE DIFFERENCE METHOD

Groundwater flow problem is solved by numerical methods. Finite difference method is one of them which is fairly simple and straight forward but the application of the model to a given physical system can be complex and requires considerable judgement and skill in setting up the problem and in interpretation of the result. Thus it should not be viewed as a replacement for the analytic method, but rather as a tool for the evaluation of complex groundwater flows. Here we consider a simple one-dimensional flow in homogeneous, isotropic and confined aquifer.

2.2 ONE DIMENSIONAL FLOW IN A CONFINED AQUIFER

Let us consider the nonsteady, one-dimensional flow in a confined aquifer.

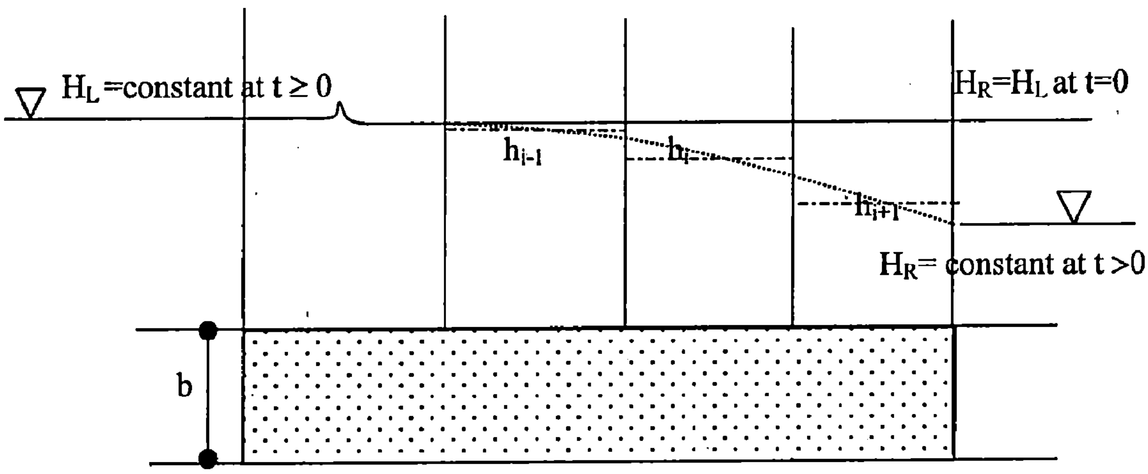

Fig. 2.1: Sectional view of a one-dimensional flow domain

Schematic diagrams showing sectional view and plan of a one- dimensional confined flow domain are shown in fig.2.1 and fig.2.2 respectively.

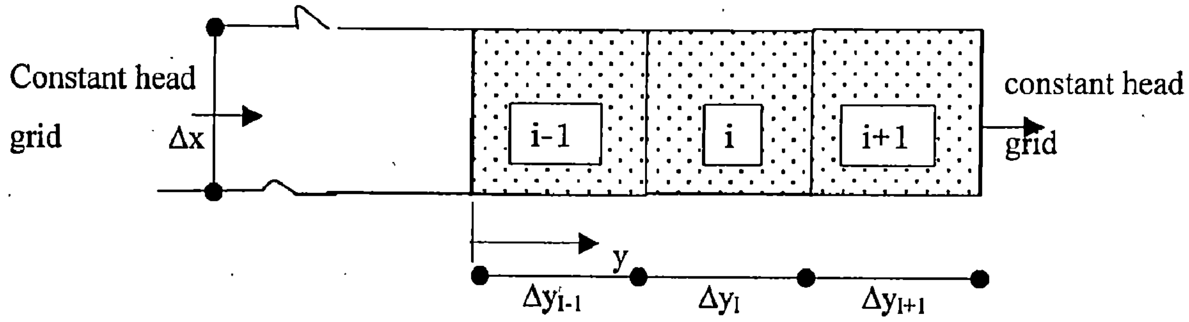

Fig. 2.2: Plan of the one dimensional aquifer

Let for grid (i), K_i be the hydraulic conductivity, Δy_i be the grid dimension, b be the aquifer thickness, S be the storage coefficient and h_i be the initial value of head . Let us consider a homogeneous and isotropic aquifer of constant thickness.

Let Δx=1; $\Delta y_{i-1} = \Delta y_i = \Delta y_{i+1} = \Delta y$;

Now for grid (i), inflow ($Q_{I-1 \to I}$) is given as:

$$Q_{i-1 \to i} = -kA \frac{h(i,n) - h(i-1,n)}{\Delta y}$$

where in h(i,n), i refers to the grid number and n to the selected time.

Now the area A is the product of Δx and b, equation becomes:

$$Q_{i-1 \to i} = -T \frac{h(i,n) - h(i-1,n)}{\Delta y} \qquad \text{...(2.1)}$$

where T be the transmissivity of the aquifer.

Likewise for grid (i), outflow ($Q_{i \to I+1}$) is given by:

$$Q_{i \to i+1} = -T \frac{h(i+1,n) - h(i,n)}{\Delta y} \qquad \text{...(2.2)}$$

Performing the mass balance over a time period Δt

Inflow-outflow = change in storage

$$(Q_{i-1 \to i} - Q_{i \to i+1})\Delta t = \Delta v_i$$

where $\Delta v_i = S \Delta y \{h(i, t + \Delta t) - h(i, t)\}$ ...(2.3)

Introducing (2.1) & (2.2) in (2.3) and simplifying

$$\left[-T\left\{\frac{h(i,n)-h(i-1,n)}{\Delta y}\right\}\right]-\left[-T\left\{\frac{h(i+1,n)-h(i,n)}{\Delta y}\right\}\right]=S\,\Delta y\left\{\frac{h(i,t+\Delta t)-h(i,t)}{\Delta t}\right\}$$

This can be rearranged to yield

$$h(i+1,n) - 2h(i,n) + h(i-1,n) = \frac{S}{T}\frac{(\Delta y)^2}{\Delta t}\{h(i,t+\Delta t) - h(i,t)\} \qquad \text{...(2.4)}$$

If n = t (the present known value), then *the explicit or forward difference equation* is obtained. On the other hand if n = t+Δt, then it becomes an *implicit or backward equation.*

2.2.1 Forward Difference Equation or Explicit Solution

The forward difference equation is expressed as:

$$h(i,t+\Delta t) = \frac{T\Delta t}{S(\Delta y)^2}\{h(i+1,t) + h(i-1,t)\} + h(i,t)\left(1 - \frac{2T\Delta t}{S\Delta y^2}\right) \qquad \text{...(2.5)}$$

The space derivative are centered at the beginning of time step and the only unknown is h (i, t+Δt) which results from time derivatives. The solution represented by above equation is only an approximation of the exact solution. The degree to which the equation approximates to the exact solution depends upon the selection of Δy and Δt. For a particular value of Δy and Δt, the difference between the approximate and exact solution may grow as t increases. The approximate solution is said to be unstable in this case.

In a one-dimensional case a stable solution is ensured, if

$$\frac{T\Delta t}{S(\Delta y)^2} \angle \frac{1}{2} \qquad \ldots(2.6)$$

Consequently the time increment cannot be selected independently of the space increment.

2.2.2 Backward Difference Equation or Implicit Solution

The backward difference equation is obtained by putting n = t + Δt.

$$h(i+1,t+\Delta t) - 2h(i,t+\Delta t) + h(i-1,t+\Delta t) = \frac{S\Delta y^2}{T\Delta t}\{h(i,t+\Delta t) - h(i,t)\}$$

Rearranging the above equation, we have

$$h(i+1,t+\Delta t) \quad -\left(2+\frac{S\Delta y^2}{T\Delta t}\right) h(i,t+\Delta t) \;+\; h(i-1,t+\Delta t) \quad = -\left(\frac{S\Delta y^2}{T\Delta t}\right) h(i,t) \quad \ldots(2.7)$$

Here the head in grid (i) at time t+Δt depends upon the value of head in the adjacent grid (i+1) and (i-1). Thus equations represent a set of algebraic equations that can be solved simultaneously. By analysis it is seen that stability is not a problem with the backward difference equation and space increment (Δy) and (Δt) can be selected independent of each other. The selection of the values for space and time increment depends upon the user's requirement, for accuracy, detail of analysis and availability of data.

2.2.3 Crank Nicholson Approximation

The Crank Nicholson equation for one-dimensional flow in a homogeneous, isotropic and confined aquifer is expressed as:

$$w\left\{\frac{h(i+1,t+\Delta t)-2h(i,t+\Delta t)+h(i-1,t+\Delta t)}{(\Delta y)^2}\right\}+(1-w)\left\{\frac{h(i+1,t)-2h(i,t)+h(i-1,t)}{(\Delta y)^2}\right\}=$$
$$\frac{S}{T\Delta t}\{h(i,t+\Delta t)-h(i,t)\}$$

In Crank Nicholson Scheme, w = 1/2.Above equation shows that Crank Nicholson approximation is the result of successive application of the explicit and implicit method. It can be rearranged as:

$$h(i+1,t+\Delta t)-\left\{2+2\frac{S(\Delta y)^2}{T\Delta t}\right\}h(i,t+\Delta t)+h(i-1,t+\Delta t)=$$
$$-\frac{S\Delta y^2}{T\Delta t}h(i,t)-\left\{h(i+1,t)-(2+\frac{S\Delta y^2}{T\Delta t})h(i,t)+h(i-1,t)\right\} \quad ...(2.8)$$

The equation is identical to the implicit equation and its solution procedure is also same.

The Crank Nicholson equation, although stable for all values of Δy and Δt, has the potential for severe oscillations of the computed heads. The solution oscillates about the true value and for some cases, the oscillations become insignificant after sufficiently long period of time (Rushton 1973). The oscillation cannot be predicted in advance but the severity depends upon the boundary and initial conditions. Small values of Δt tend to reduce the severity of oscillations.

2.3 ANALYTIC SOLUTION

To find the aquifer responses (piezometric head, flow, volume etc.) for any arbitrary variation in the stream stage, convolution principle (also known as Duhamel's theorem), which utilizes the respective unit response function and the boundary perturbation is applicable. The response of a linear stream- aquifer system to a unit step perturbation is a basic property of the system. Many investigators (Hall 1968, 1969, Venetis 1970; Moench and Kisiel 1970; Morel -Seytoux 1975; Morel-Seytoux and Daly 1975) have applied convolution principles for solving groundwater flow problems. Response of a linear system to a time varying input using convolution integral may be written (Thomson 1957, Venetis 1970) in two ways as;

$$y(x,t)=\int_0^t f(\tau)u(x,t-\tau)d\tau \qquad ...(2.9)$$

$$\text{or } y(x,t)=\int_0^t f'(\tau)U(x,t-\tau)d\tau \qquad ...(2.10)$$

where y(x,t) = Response of the system after time t since the beginning of the system input(boundary perturbation).

f(t) = time varying boundary perturbation.

u(x,t) = instantaneous response function of the system.

U(x,t) = unit step response function of the system.

f'(x,t) = Time derivative of the input function.

If the system response is not zero at t = 0, equation 2 gets modified as:

$$y(x,t)=f(0)U(t)+\int_0^t f'(\tau)U(x,t-\tau)d\tau \qquad ...(2.11)$$

The convolution integrals appearing in equation 2.9 & 2.11 may be solved by using two discrete kernels approach i.e. discrete pulse kernel and discrete ramp kernel approach.

2.3.1 Discrete Pulse Kernel

Discretizing the time step into uniform time steps and assuming the input to be invariant during a time step equation 2.9 can be written as:

$$y(x,n\Delta t)=\sum_{\gamma=1}^{n} f(\gamma\,\Delta t)\int_{(\gamma-1)\Delta t}^{\gamma\Delta t} u(x,n\Delta t-\tau)d\tau$$

$$\text{or } y(x,n\Delta t)=\sum_{\gamma=1}^{n} f(\gamma)\alpha(x,\Delta t,n-\gamma+1) \qquad ...(2.12)$$

where α(.) is the discrete pulse kernel expressed as:

$$\alpha(x,\Delta t,m)=\int_{(m-1)\Delta t}^{m\Delta t} u(x,\tau)d\tau=U(x,m\Delta t)-U(x,(m-1)\Delta t) \qquad ...(2.13)$$

In case of a stream, the discrete pulse kernel is the response of the aquifer to a unit step rise in stream stage which continues only for a period Δt. The application of this kernel requires the time variation of input to be discretized as a train of pulses of uniform duration Δt and input is considered uniform over a time step.

$$w\left\{\frac{h(i+1,t+\Delta t)-2h(i,t+\Delta t)+h(i-1,t+\Delta t)}{(\Delta y)^2}\right\}+(1-w)\left\{\frac{h(i+1,t)-2h(i,t)+h(i-1,t)}{(\Delta y)^2}\right\}=$$
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The convolution integrals appearing in equation 2.9&2.11 may be solved by using two discrete kernels approach i.e. discrete pulse kernel and discrete ramp kernel approach.

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$$y(x,n\Delta t)=\sum_{\gamma=1}^{n} f(\gamma\,\Delta t)\int_{(\gamma-1)\Delta t}^{\gamma\Delta t} u(x,n\Delta t-\tau)d\tau$$

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where α(.) is the discrete pulse kernel expressed as:

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In case of a stream, the discrete pulse kernel is the response of the aquifer to a unit step rise in stream stage which continues only for a period Δt. The application of this kernel requires the time variation of input to be discretized as a train of pulses of uniform duration Δt and input is considered uniform over a time step.

2.3.2 Discrete Ramp Kernel

Assuming slope of the input to be invariant during a time step, equation2.11may be written in discretized form as:

$$y(x,n\Delta t)=\sum_{\gamma=1}^{n}\frac{f(\gamma\,\Delta t)-f(\gamma-1)\Delta t}{\Delta t}\int_{(\gamma-1)\Delta t}^{\gamma\Delta t}U(x,n\Delta t-\tau)d\tau$$

$$y(x,n\Delta t)=\sum_{\gamma=1}^{n}\frac{f(\gamma\Delta t)-f(\gamma-1)\Delta t}{\Delta t}\delta(x,\Delta t,n-\gamma+1) \qquad \ldots(2.14)$$

where δ(.) is defined as discrete ramp kernel.

The application of this kernel requires the time variation of input to be approximated as a series of incremental discrete ramps. This approach assumes that the slope of the perturbation (input) is constant during a time step. Therefore the use of discrete ramp kernel logically, should give superior results as compared to that obtained during discrete pulse kernel.

CHAPTER 3

AN IMPROVED FINITE DIFFERNCE SCHEME ACCOUNTING TEMPORAL VARIATION

3.1 INTRODUCTION

In Crank Nicholson finite difference scheme it is implied that the variation of flow with time during t_0 to $t_0+\Delta t$ is linear. In completely implicit scheme it is implied that flow rate is constant within the time step Δt and is equal to the rate that prevails at $t_0+\Delta t$. In the explicit scheme it is implied that the flow rate during the time interval is also constant but equal to the flow rate that prevails at the initial time t_0. The Crank Nicholson technique though rigorously accounts for mass balance, it is found that piezometric head computed by Crank Nicholson scheme exhibits oscillation. The oscillation may be due to sudden step rise in boundary head causing mathematically infinite rate of inflow at the boundary at time t_0. It is known that the flow rate at the boundary subsequent to a sudden change in boundary head varies inversely to square root of time. Therefore the variation of flow with time at any section can be presumed to be non-linear. A finite difference scheme is developed that accounts for non-linear variation of flow at any section.

3.2 STATEMENT OF THE PROBLEM

In the proposed finite difference scheme, it is assumed that during a time interval $(t_0-\Delta t_2)$ to $(t_0+\Delta t_1)$, the flow at a section follows a quadratic distribution. The flow, q(t), is expressed as:

$q(t) = a_1 + b_1(t-t_0) + c_1(t-t_0)^2$

in which a_1, b_1, c_1 are constants to be determined.

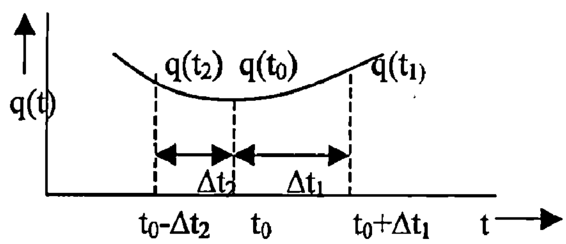

Fig.3.1 Variation of flow with time at a particular section

Since at t = t_0, q(t) = q(t_0)

$a_1 = q(t_0)$ ...(3.1)

At $t = t_0+\Delta t_1$, $q(t) = q(t_1)$

$= a_1 + b_1\Delta t_1 + c_1\Delta t_1^2$ ...(3.2)

At $t = t_0-\Delta t_2$, $q(t) = q(t_2)$

$= a_1 - b_1 \Delta t_2 + c_1 \Delta t_2^2$ ...(3.3)

From equation (3.1), (3.2), & (3.3)

$a_1 = q(t_0)$

$$b_1 = \frac{q(t_1)-q(t_0)}{\Delta t_1} - \frac{\Delta t_1}{\Delta t_1+\Delta t_2}\left\{\frac{q(t_1)-q(t_0)}{\Delta t_1} + \frac{q(t_2)-q(t_0)}{\Delta t_2}\right\}$$

$$c_1 = \frac{1}{\Delta t_1+\Delta t_2}\left\{\frac{q(t_1)-q(t_0)}{\Delta t_1} + \frac{q(t_2)-q(t_0)}{\Delta t_2}\right\}$$

The volume of flow $V(\Delta t_1)$ passing through the section during time interval t_0 to $t_0+\Delta t_1$

$$= \int_{t_0}^{t_0+\Delta t_1} q(t)dt$$

$$= \int_{t_0}^{t_0+\Delta t_1} \left\{a_1 + b_1(t-t_0) + c_1(t-t_0)^2\right\}dt$$

$$= \int_0^{\Delta t_1} (a_1 + b_1\tau + c_1\tau^2)d\tau$$

$$= a_1\,\Delta t_1 + b_1\frac{\Delta t_1^2}{2} + c_1\frac{\Delta t_1^3}{3}$$

Putting the values of a_1, b_1, c_1 we get

$$V(\Delta t_1) = q(t_0)\Delta t_1 + \left[\frac{q(t_1)-q(t_0)}{\Delta t_1} - \frac{\Delta t_1}{\Delta t_1+\Delta t_2}\left\{\frac{q(t_1)-q(t_0)}{\Delta t_1} + \frac{q(t_2)-q(t_0)}{\Delta t_2}\right\}\right]\frac{\Delta t_1^2}{2}$$

$$+ \frac{1}{\Delta t_1+\Delta t_2}\left\{\frac{q(t_1)-q(t_0)}{\Delta t_1} + \frac{q(t_2)-q(t_0)}{\Delta t_2}\right\}\frac{\Delta t_1^3}{3}$$

$$= q(t_0)\left\{\Delta t_1 - \frac{\Delta t_1}{2} + \frac{\Delta t_1^2}{2(\Delta t_1+\Delta t_2)} + \frac{\Delta t_1^3}{2\Delta t_2(\Delta t_1+\Delta t_2)} - \frac{\Delta t_1^2}{3(\Delta t_1+\Delta t_2)} - \frac{\Delta t_1^3}{3\Delta t_2(\Delta t_1+\Delta t_2)}\right\}$$

$$+ q(t_1)\left\{\frac{\Delta t_1}{2} - \frac{\Delta t_1^2}{2(\Delta t_1+\Delta t_2)} + \frac{\Delta t_1^2}{3(\Delta t_1+\Delta t_2)}\right\} - q(t_2)\left\{\frac{\Delta t_1^3}{2(\Delta t_1+\Delta t_2)\Delta t_2} - \frac{\Delta t_1^3}{3(\Delta t_1+\Delta t_2)\Delta t_2}\right\}$$

If $\Delta t_1 = \Delta t_2 = \Delta t$

$$v(\Delta t) = \left(\Delta t - \frac{\Delta t}{2} + \frac{\Delta t}{4} + \frac{\Delta t}{4} - \frac{\Delta t}{6} - \frac{\Delta t}{6}\right)q(t_0) + \left(\frac{\Delta t}{2} - \frac{\Delta t}{4} + \frac{\Delta t}{6}\right)q(t_0 + \Delta t)$$
$$- \left(\frac{\Delta t}{4} - \frac{\Delta t}{6}\right)q(t_0 - \Delta t)$$

$$= \frac{2}{3}q(t_0)\Delta t + \frac{5}{12}q(t_0 + \Delta t)\Delta t_1 - \frac{1}{12}q(t_0 - \Delta t)\Delta t$$

$$= -\frac{1}{12}q(t_0 - \Delta t)\Delta t + \frac{2}{3}q(t_0)\Delta t + \frac{5}{12}q(t_0 + \Delta t)$$

Thus the inflow or outflow during Δt is given by

$$\int_t^{t+\Delta t} q(t)dt = \{w1q(t - \Delta t) + w2q(t) + w3q(t + \Delta t)\}\Delta t$$

where w1= -1/12 , w2 = 2/3, w3 = 5/12

3.3 A CASE STUDY

The proposed finite difference scheme is applied to solve an unsteady flow in a finite isotropic confined aquifer bounded by fully penetrating streams consequent to a step or ramp change in stream stage. The aquifer is non-homogeneous.

3.3.1 Need For Considering Harmonic Mean Transimissivity

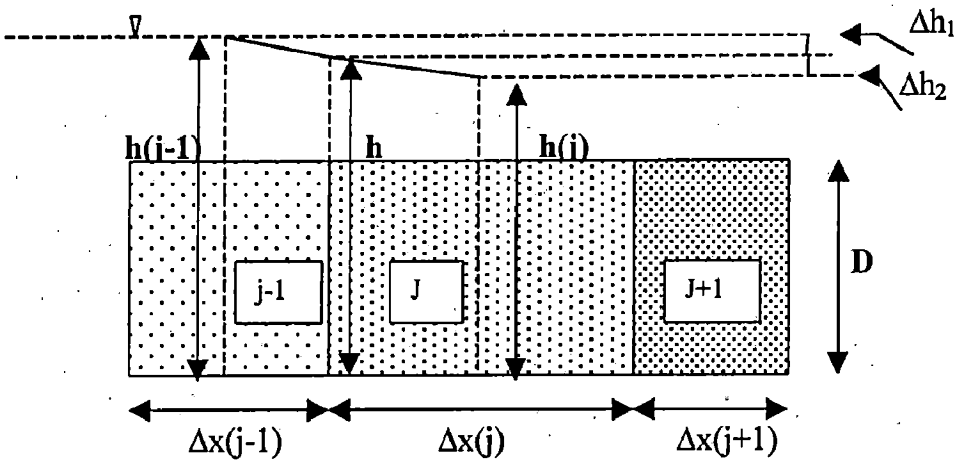

Fig.3.2 Flow in a non-homogeneous & isotropic aquifer

Let us consider the zone between the two adjacent nodes.

Let Δx(j-1), Δx(j) be the length , T_1,T_2 be the transimissivity and K_1,K_2 be hydraulic conductivity of the two adjacent zones respectively.

Let the flow be in steady state condition. Let h(j-1) and h(j) be the hydraulic head at (j-1)th & jth node respectively and let h be the head at the interface of the grids. The flow being steady

$$-K_1 D\frac{h-h(j-1)}{0.5\Delta x(j-1)}=-K_2 D\frac{h(j)-h}{0.5\Delta x(j)}$$

Solving for h,

$$h=\frac{T_1\Delta x(j)h(j-1)+T_2\Delta x(j-1)h(j)}{T_1\Delta x(j)+T_2\Delta x(j-1)}$$

The hydraulic gradient in zone 1 is

$$i_1=\frac{h-h(j-1)}{0.5\Delta x(j-1)}$$

$$=\frac{2T_2\{h(j)-h(j-1)\}}{T_1\Delta x(j)+T_2\Delta x(j-1)}$$

The hydraulic gradient in zone 2 is

$$i_2=\frac{h(j)-h}{0.5\Delta x(j)}$$

$$=\frac{2T_1\{h(j)-h(j-1)\}}{T_1\Delta x(j)+T_2\Delta x(j-1)}$$

Thus $i_1 \neq i_2$, and the graph of the head vs. distance is not differentiable at the interface of the zones of different conductivity. In numerical method for computing inflow (outflow) to carry out mass balance in a grid area, it is necessary to convert the two zones between node j-1 and j to a fictitious zone so that the head is differentiable at each point within two consecutive grid nodes.

The gradient in fictitious zone between grid (j-1) and j is

$$i=\frac{h(j)-h(j-1)}{\dfrac{\Delta x(j-1)+\Delta x(j)}{2}}$$

Let T_e be the effective transmissivity of the zone between grid j-1 & j.

For conservation of mass

$-T_1 i_1 = - T_2 i_2 = - T_e i$

$$\frac{T_1 2T_2 \{h(j)-h(j-1)\}}{T_1 \Delta x(j) + T_2 \Delta x(j-1)} = T_e \frac{h(j)-h(j-1)}{\frac{\Delta x(j-1)+\Delta x(j)}{2}}$$

$$\text{or } T_e = \frac{T_1 T_2 \{\Delta x(j-1)+\Delta x(j)\}}{T_1 \Delta x(j) + T_2 \Delta x(j-1)} = \frac{\Delta x(j-1)+\Delta x(j)}{\frac{\Delta x(j-1)}{T_1}+\frac{\Delta x(j)}{T_2}}$$

T_e is the harmonic mean transmissivity, which should be used in formulating the numerical scheme.

3.4 DEVELOPMENT OF THE FINITE DIFFERENCE SCHEME

Carrying out water balance for the control volume containing node (j) for a time period of Δt during $(k-1)^{th}$ to $(k)^{th}$ time step and applying Darcy's law, the finite difference equations are derived. The equations are written for three zones containing

a) Interior node (j) where $j = 2, j_{max}-1$
b) Boundary node(left) i.e. $j = 1$
c) Boundary node (right) i.e. $j = j_{max}$

3.4.1 Equation For Interior Node($j = 2, j_{max}-1$)

The water balance i.e. Inflow – outflow = change of storage, can be written as:

$$w1\left[T_{HM4}(j)\left\{\frac{h(j-1,k-2)-h(j,k-2)}{\frac{\Delta X(j-1)+\Delta X(j)}{2}}\right\}+T_{HM2}(j)\left\{\frac{h(j+1,k-2)-h(j,k-2)}{\frac{\Delta X(j)+\Delta X(j+1)}{2}}\right\}\right]\Delta t$$

$$+w2\left[T_{HM4}(j)\left\{\frac{h(j-1,k-1)-h(j,k-1)}{\frac{\Delta X(j-1)+\Delta X(j)}{2}}\right\}+T_{HM2}(j)\left\{\frac{h(j+1,k-1)-h(j,k-1)}{\frac{\Delta X(j)+\Delta X(j+1)}{2}}\right\}\right]\Delta t$$

$$+w3\left[T_{HM4}(j)\left\{\frac{h\ (j-1,k)-h(j,k)}{\frac{\Delta X(j-1)+\Delta X(j)}{2}}\right\}+T_{HM2}(j)\left\{\frac{h(j+1,k)-h(j,k)}{\frac{\Delta X(j)+\Delta X(j+1)}{2}}\right\}\right]\Delta t$$

$$= S(j)\{h(j,k)-h(j,k-1)\}\Delta X(j)$$

Dividing both sides by ΔX(j). Δt we get

$$w1\left[T_{HM4}(j)\left\{\frac{h(j-1,k-2)-h(j,k-2)}{\frac{\Delta X(j-1)+\Delta X(j)}{2}\Delta X(j)}\right\}+T_{HM2}(j)\left\{\frac{h(j+1,k-2)-h(j,k-2)}{\frac{\Delta X(j)+\Delta X(j+1)}{2}\Delta X(j)}\right\}\right]$$

$$+w2\left[T_{HM4}(j)\left\{\frac{h(j-1,k-1)-h(j,k-1)}{\frac{\Delta X(j-1)+\Delta X(j)}{2}\Delta X(j)}\right\}+T_{HM2}(j)\left\{\frac{h(j+1,k-1)-h(j,k-1)}{\frac{\Delta X(j)+\Delta X(j+1)}{2}\Delta X(j)}\right\}\right]$$

$$+w3\left[T_{HM4}(j)\left\{\frac{h(j-1,k)-h(j,k)}{\frac{\Delta X(j-1)+\Delta X(j)}{2}\Delta X(j)}\right\}+T_{HM2}(j)\left\{\frac{h(j+1,k)-h(j,k)}{\frac{\Delta X(j)+\Delta X(j+1)}{2}\Delta X(j)}\right\}\right]$$

$$=S(j)\{h(j,k)-h(j,k-1)\}\frac{1}{\Delta t}$$

where S(j) be the storage coefficient.

Designating $w3\ \frac{T_{HM2}(j)}{\frac{(\Delta X(j)+\Delta X(j+1))}{2}\Delta X(j)} = F_2(j)$

$$w3\ \frac{T_{HM4}(j)}{\frac{(\Delta X(j)+\Delta X(j-1))}{2}\Delta X(j)} = F_4(j)$$

$$-F_2(j)-F_4(j)-\frac{S(j)}{\Delta t}=F_1(j)$$

The above equation can be written as:

$$w1\left[F_4(j)\frac{1}{w3}\{h(j-1,k-2)-h(j,k-2)\}+F_2(j)\frac{1}{w3}\{h(j+1,k-2)-h(j,k-2)\}\right]$$

$$+w2\left[F_4(j)\frac{1}{w3}\{h(j-1,k-1)-h(j,k-1)\}+F_2(j)\frac{1}{w3}\{h(j+1,k-1)-h(j,k-1)\}\right]$$

$$+\left[F_4(j)\{h(j-1,k)-h(j,k)\}+F_2(j)\{h(j+1,k)-h(j,k)\}\right]$$

$$=S(j)\{h(j,k)-h(j,k-1)\}\frac{1}{\Delta t}$$

Or, $F_4(j)h(j-1,k)+F_2(j)h(j+1,k)+\left\{-F_4(j)-F_2(j)-\frac{S(j)}{\Delta t}\right\}h(j,k)$

$$+w1\left[F_4(j)\frac{1}{w3}\{h(j-1,k-2)-h(j,k-2)\}+F_2(j)\frac{1}{w3}\{h(j+1,k-2)-h(j,k-2)\}\right]$$

$$+w2\left[F_4(j)\frac{1}{w3}\{h(j-1,k-1)-h(j,k-1)\}+F_2(j)\frac{1}{w3}\{h(j+1,k-1)-h(j,k-1)\}\right]$$

$$+S(j)h(j,k-1)\frac{1}{\Delta t}=0$$

The equation is finally be written as:

$$F_4(j)\ h(j-1,k)+F_1(j)\ h(j,k)+F_2(j)h(j+1,k)+F_6(j,k)=0$$

where

$$F_6(j,k)=w1\left[F_4(j)\frac{1}{w3}\{h(j-1,k-2)-h(j,k-2)\}+F_5(j)\frac{1}{w3}\{h(j+1,k-2)-h(j,k-2)\}\right]$$

$$+w2\left[F_4(j)\frac{1}{w3}\{h(j-1,k-1)-h(j,k-1)\}+F_5(j)\frac{1}{w3}\{h(j+1,k-1)-h(j,k-1)\}\right]$$

$$+S(j)\,h(j,k-1)\frac{1}{\Delta t}$$

3.4.2 Equation For Boundary Node (left) i.e. j = 1

Let Γ_r be the reach transmissivity of the stream.

Let $h_r(k-2)$, $h_r(k-1)$, $h_r(k)$ be the head in the stream at (k-2),(k-1) & k^{th} time step respectively.

Let h(j,k-2), h(j,k-1), h(j,k) be the head of node(j) at (k-2), (k-1) & k^{th} time step respectively.

Inflow from stream to the aquifer will be equal to the multiplication of reach transmissivity and the head difference between the stream and left node of the aquifer.

The water balance equation is given by:

$$w1\left[\Gamma_r\{h_r(k-2)-h(j,k-2)\}+T_{HM\,2}(j)\left\{\frac{h(j+1,k-2)-h(j,k-2)}{\frac{\Delta x(j)+\Delta x(j+1)}{2}}\right\}\right]\Delta t$$

$$+w2\left[\Gamma_r\{h_r(k-1)-h(j,k-1)\}+T_{HM\,2}(j)\left\{\frac{h(j+1,k-1)-h(j,k-1)}{\frac{\Delta x(j)+\Delta x(j+1)}{2}}\right\}\right]\Delta t$$

$$+w3\left[\Gamma_r\{h_r(k)-h(j,k)\}+T_{HM\,2}(j)\left\{\frac{h(j+1,k)-h(j,k)}{\frac{\Delta x(j)+\Delta x(j+1)}{2}}\right\}\right]\Delta t$$

$$=S(j)\{h(j,k)-h(j,k-1)\}\,\Delta X(j)$$

Dividing both sides by $\Delta X(j)\,\Delta t$

$$w1\left[\Gamma_r\frac{1}{\Delta X(j)}\{h_r(k-2)-h(j,k-2)\}+T_{HM\,2}(j)\left\{\frac{h(j+1,k-2)-h(j,k-2)}{\frac{\Delta x(j)+\Delta x(j+1)}{2}\Delta X(j)}\right\}\right]$$

$$+w2\left[\Gamma_r\frac{1}{\Delta X(j)}\{h_r(k-1)-h(j,k-1)\}+T_{HM\,2}(j)\left\{\frac{h(j+1,k-1)-h(j,k-1)}{\frac{\Delta x(j)+\Delta x(j+1)}{2}\Delta X(j)}\right\}\right]$$

$$+ w3\left[\Gamma_r \frac{1}{\Delta X(j)}\{h_r(k) - h(j,k)\} + T_{HM\,2}(j)\left\{\frac{h(j+1,k) - h(j,k)}{\frac{\Delta x(j) + \Delta x(j+1)}{2}\Delta X(j)}\right\}\right]$$

$$= S(j)\frac{1}{\Delta t}\{h(j,k) - h(j,k-1)\}$$

Designating, $w3 \frac{T_{HM\,2}(j)}{\frac{\Delta x(j) + \Delta x(j+1)}{2}\Delta X(j)} = F_2(j)$ ·

$$- F_2(j) - w3\frac{\Gamma_r}{\Delta X(j)} - \frac{S(j)}{\Delta t} = F_1(j)$$

The above equation reduces to:

$$w1\left[\Gamma_r \frac{1}{\Delta X(j)}\{h_r(k-2) - h(j,k-2)\} + \frac{F_2(j)}{w3}\{h(j+1,k-2) - h(j,k-2)\}\right]$$

$$+ w2\left[\Gamma_r \frac{1}{\Delta X(j)}\{h_r(k-1) - h(j,k-1)\} + \frac{F_2(j)}{w3}\{h(j+1,k-1) - h(j,k-1)\}\right]$$

$$+ \left[w3\frac{\Gamma_r}{\Delta X(j)}\{h_r(k) - h(j,k)\} + F_2(j)\{h(j+1,k) - h(j,k)\}\right]$$

$$= S(j)\frac{1}{\Delta t}\{h(j,k) - h(j,k-1)\}$$

Or $\left\{- w3\frac{\Gamma_r}{\Delta X(j)} - F_2(j) - \frac{S(j)}{\Delta t}\right\}h(j,k) + F_2(j)\; h(j+1,k)$

$$+ w1\left[\Gamma_r \frac{1}{\Delta X(j)}\{h_r(k-2) - h(j,k-2)\} + \frac{F_2(j)}{w3}\{h(j+1,k-2) - h(j,k-2)\}\right]$$

$$+ w2\left[\Gamma_r \frac{1}{\Delta X(j)}\{h_r(k-1) - h(j,k-1)\} + \frac{F_2(j)}{w3}\{h(j+1,k-1) - h(j,k-1)\}\right]$$

$$+ S(j)\frac{1}{\Delta t}h(j,k-1) + w3\frac{\Gamma_r}{\Delta X(j)}h_r(k) = 0$$

The equation is finally written as:

$$F_1(j)\; h(j,k) + F_2(j)\,h(j+1,k) + F_6(j,k) = 0$$

where

$$F_6(j,k) = w1\left[\Gamma_r \frac{1}{\Delta X(j)}\{h_r(k-2) - h(j,k-2)\} + \frac{F_2(j)}{w3}\{h(j+1,k-2) - h(j,k-2)\}\right]$$

$$+ w2\left[\Gamma_r \frac{1}{\Delta X(j)}\{h_r(k-1) - h(j,k-1)\} + \frac{F_2(j)}{w3}\{h(j+1,k-1) - h(j,k-1)\}\right]$$

$$+ S(j)\frac{1}{\Delta t}h(j,k-1) + w3\frac{\Gamma_r}{\Delta X(j)}h_r(k)$$

3.4.3 Equation For Boundary Node (right) i.e. $j = j_{max}$

Let Γ_r be the reach transmissivity of the stream.

Let h(j,k-2), h(j,k-1), h(j,k) be the head in the right boundary node of the aquifer at (k-2), (k-1), k_{th} time step respectively.

Let $h_r(k-2)$, $h_r(k-1)$, $h_r(k)$ be the head in the right stream at (k-2), (k-1), k^{th} time step respectively.

S(j) be the storage coefficient of the aquifer.

Now the water balance equation can be written as:

$$w1\left[T_{HM\,4}(j)\frac{h(j-1,k-2)-h(j,k-2)}{\frac{\Delta X(j-1)+\Delta X(j)}{2}}+\Gamma_r\{h_r(k-2)-h(j,k-2)\}\right]\Delta t$$

$$+w2\left[T_{HM\,4}(j)\frac{h(j-1,k-1)-h(j,k-1)}{\frac{\Delta X(j-1)+\Delta X(j)}{2}}+\Gamma_r\{h_r(k-1)-h(j,k-1)\}\right]\Delta t$$

$$+w3\left[T_{HM\,4}(j)\frac{h(j-1,k)-h(j,k)}{\frac{\Delta X(j-1)+\Delta X(j)}{2}}+\Gamma_r\{h_r(k)-h(j,k)\}\right]\Delta t$$

$$=S(j)\{h(j,k)-h(j,k-1)\}\Delta X(j)$$

Dividing both sides by $\Delta X(j)\,\Delta t$ we get

$$w1\left[T_{HM\,4}(j)\frac{h(j-1,k-2)-h(j,k-2)}{\frac{\Delta X(j)+\Delta X(j-1)}{2}\Delta X(j)}+\Gamma_r\frac{1}{\Delta X(j)}\{h_r(k-2)-h(j,k-2)\}\right]$$

$$+w2\left[T_{HM\,4}(j)\frac{h(j-1,k-1)-h(j,k-1)}{\frac{\Delta X(j)+\Delta X(j-1)}{2}\Delta X(j)}+\Gamma_r\frac{1}{\Delta X(j)}\{h_r(k-1)-h(j,k-1)\}\right]$$

$$+w3\left[T_{HM\,4}(j)\frac{h(j-1,k)-h(j,k)}{\frac{\Delta X(j)+\Delta X(j-1)}{2}\Delta X(j)}+\Gamma_r\frac{1}{\Delta X(j)}\{h_r(k)-h(j,k)\}\right]$$

$$=S(j)\frac{1}{\Delta t}\{h(j,k)-h(j,k-1)\}$$

Designate $w3\dfrac{T_{HM}(j)}{\frac{\{\Delta X(j)+\Delta X(j-1)\}}{2}\Delta X(j)}=F_4(j)$

$$-F_4(j)-\frac{\Gamma_r}{\Delta X(j)}w3-\frac{S(j)}{\Delta t}=F_1(j)$$

Now the equation can be written as:

$$w1\left[F_4(j)\frac{1}{w3}\{h(j-1,k-2)-h(j,k-2)\}+\Gamma_r\frac{1}{\Delta X(j)}\{h_r(k-2)-h(j,k-2)\}\right]$$

$$+w2\left[F_4(j)\frac{1}{w3}\{h(j-1,k-1)-h(j,k-1)\}+\Gamma_r\frac{1}{\Delta X(j)}\{h_r(k-1)-h(j,k-1)\}\right]$$

$$+\left[F_4(j)\{h(j-1,k)-h(j,k)\}+\Gamma_r\frac{w3}{\Delta X(j)}\{h_r(k)-h(j,k)\}\right]$$

$$=S(j)\frac{1}{\Delta t}\{h(j,k)-h(j,k-1)\}$$

Or $F_4(j)h(j-1,k)+\left\{-F_4(j)-\Gamma_r\frac{w3}{\Delta X(j)}-S(j)\frac{1}{\Delta t}\right\}h(j,k)$

$$+w1\left[F_4(j)\frac{1}{w3}\{h(j-1,k-2)-h(j,k-2)\}+\Gamma_r\frac{1}{\Delta X(j)}\{h_r(k-2)-h(j,k-2)\}\right]$$

$$+w2\left[F_4(j)\frac{1}{w3}\{h(j-1,k-1)-h(j,k-1)\}+\Gamma_r\frac{1}{\Delta X(j)}\{h_r(k-1)-h(j,k-1)\}\right]$$

$$+S(j)\frac{1}{\Delta t}h(j,k-1)+w3\frac{\Gamma_r}{\Delta X(j)}h_r(k)=0$$

Finally the equation can be written as:

$$F_4(j)\,h(j-1,k)+F_1(j)\,h(j,k)+F_6(j,k)=0$$

Where $F_6(j,k)=\frac{S(j)}{\Delta t}h(j,k-1)+w3\frac{\Gamma_r}{\Delta X(j)}h_r(k)$

$$+w1\left[F_4(j)\frac{1}{w3}\{h(j-1,k-2)-h(j,k-2)\}+\Gamma_r\frac{1}{\Delta X(j)}\{h_r(k-2)-h(j,k-2)\}\right]$$

$$+w2\left[F_4(j)\frac{1}{w3}\{h(j-1,k-1)-h(j,k-1)\}+\Gamma_r\frac{1}{\Delta X(j)}\{h_r(k-1)-h(j,k-1)\}\right]$$

The equation for each node can be written in matrix form as:

$$\begin{bmatrix} F_1(1) & F_2(I) & \cdots & \cdots & \cdots & \cdots \\ F_4(2) & F_1(2) & F_2(2) & \cdots & \cdots & \cdots \\ \cdots & F_4(3) & F_1(3) & F_2(3) & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & F_4(J_{\max-1}) & F_1(J_{\max-1}) & F_2(J_{\max-1}) & \\ \cdots & \cdots & \cdots & & F_4(J_{\max}) & F_1(J_{\max}) \end{bmatrix} \begin{bmatrix} h(1,k) \\ h(2,k) \\ h(3,k) \\ \cdots \\ h(J_{\max-1},k) \\ h(J_{\max},k) \end{bmatrix} = - \begin{bmatrix} F_6(1,k) \\ F_6(2,k) \\ F_6(3,k) \\ \cdots \\ F_6(J_{\max-1},K) \\ F_6(J_{\max},K) \end{bmatrix}$$

Or

$[h(1,k),h(2,k),\cdots h(j_{\max},k)]^T =$

$$\begin{bmatrix} F_1(1) & F_2(1) & \cdots & \cdots & \cdots & \cdots \\ F_4(2) & F_1(2) & F_2(2) & \cdots & \cdots & \cdots \\ \cdots & F_4(3) & F_1(3) & F_2(3) & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & F_4(j_{\max-1}) & F_1(j_{\max-1}) & F_2(j_{\max-1}) & \\ \cdots & \cdots & \cdots & & F_4(j_{\max}) & F_1(j_{\max}) \end{bmatrix}^{-1} \begin{bmatrix} -F_6(1,k) \\ -F_6(2,k) \\ -F_6(3,k) \\ \cdots \\ -F_6(j_{\max-1},k) \\ -F_6(j_{\max},k) \end{bmatrix}$$

CHAPTER 4

AN IMPROVED FINITE DIFFERENCE SCHEME ACCOUNTING SPATIAL VARIATION

4.1 INTRODUCTION

In the existing finite difference scheme, it is implied that the spatial head distribution at a particular time follows a second-degree polynomial (see Appendix-1). In the improved finite difference scheme, the spatial variation of hydraulic head is assumed to follow a three-degree polynomial, the scheme has been written for uniform grid size as well as non-uniform grid size.

4.2 FINITE DIFFERENCE SCHEME FOR UNIFORM GRID SIZE

For the finite difference method, following two assumptions are made:

- Flow variation with time at a particular section follows a quadratic distribution;
- Variation of head between nodes at a particular time follows a polynomial equation as given below:

$h(x)= a + bx + cx^2 + dx^3$,where a, b ,c, d are the unknown constants to be derived in terms of h(j-1), h(j), h(j+1), h(j+2).

Let us consider a non-homogeneous, isotropic, one-dimensional confined aquifer system bounded by fully penetrating stream. Let us discretise the aquifer into grids of equal sizes. Four types of grids for which the equations are to be written are:

a. All interior nodes i.e. $j = 2, j_{max}-2$

b. The boundary node (left) i.e. $j = 1$

c. The interior node i.e. $j = j_{max}-1$

d. The boundary node (right) i.e. $j = j_{max}$

4.2.1. Equation For Interior Nodes i.e. $j = 2, j_{max}-2$:

Selecting the origin at j^{th}node, the head h(x), $j-1 \leq x \leq j+2$ is expressed as:

$h(x)=a + bx + cx^2 + dx^3$

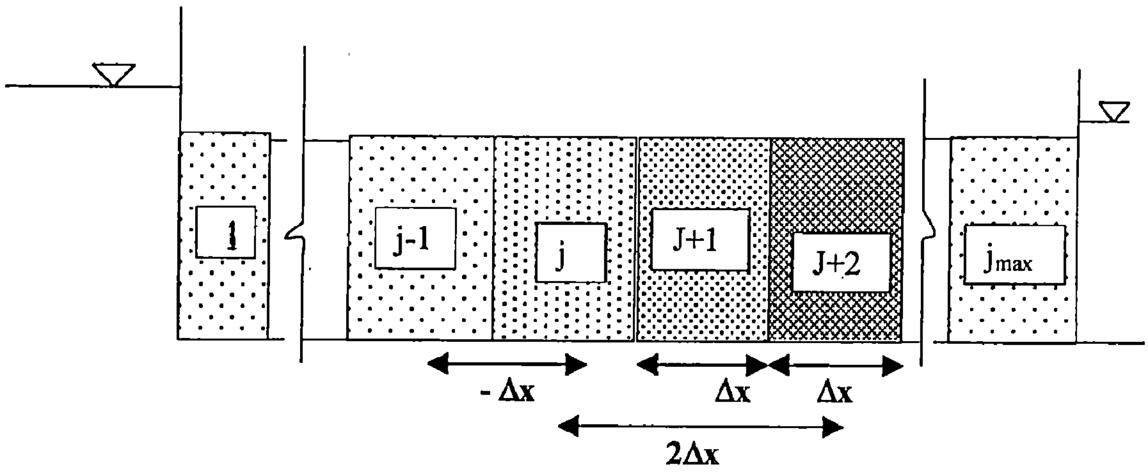

Fig.4.1: Discretisation of flow domain, origin at j ($2 \leq j \leq j_{max}-2$)

At x = 0, h(0) = h(j) = a

At x = -Δx, $h(-\Delta x) = h(j-1) = a - b\,\Delta x + c\,\Delta x^2 - d\,\Delta x^3$

At x = Δx, $h(\Delta x) = h(j+1) = a + b\,\Delta x + c\,\Delta x^2 + d\,\Delta x^3$

At x = 2Δx, $h(2\Delta x) = h(j+2) = a + 2b\,\Delta x + 4c\,\Delta x^2 + 8d\,\Delta x^3$

Solving above equations we get,

a = h(j)

b = $\{-h(j+2)+6h(j+1)-3h(j)-2h(j-1)\}/(6\Delta x)$

c = $\{h(j+1)-2h(j)+h(j-1)\}/(2\Delta x^2)$

d = $\{h(j+2)-3h(j+1)+3h(j)-h(j-1)\}/(6\Delta x^3)$

The derivative $dh/dx = b + 2cx + 3dx^2$

$$\frac{dh}{dx}_{at\,x=-\Delta x/2} = \{-h(j+2)+3h(j+1)+21h(j)-23h(j-1)\}\frac{1}{24\Delta x}$$

$$\frac{dh}{dx}_{at\,x=\Delta x/2} = \{-h(j+2)+27h(j+1)-27h(j)+h(j-1)\}\frac{1}{24\Delta x}$$

The water balance equation is written as:

Inflow-outflow = change in storage

$$\text{Or}\left\{-T_{HM4}(j)\frac{dh}{dx}_{at\,x=-\Delta x/2}\right\}-\left\{-T_{HM2}(j)\frac{dh}{dx}_{at\,x=\Delta x/2}\right\}=S\{h(j,k)-h(j,k-1)\}\frac{\Delta x}{\Delta t}$$

$$-\left\{T_{HM4}(j)\frac{dh}{dx}_{at\,x=-\Delta x/2}\right\}+\left\{T_{HM2}(j)\frac{dh}{dx}_{at\,x=\Delta x/2}\right\}=S\{h(j,k)-h(j,k-1)\}\frac{\Delta x}{\Delta t}$$

Taking into account of the variation of flow during different time steps i.e. (k-2), (k-1) & k^{th} time steps, the water balance equation is written as:

$$-w1T_{HM}(j)\ \{-h(j+2,k-2)+3h(j+1,k-2)+21h(j,k-2)-23h(j-1,k-2)\}$$
$$+w1T_{HM2}(j)\ \{-h(j+2,k-2)+27h(j+1,k-2)-27h(j,k-2)+h(j-1,k-2)\}$$
$$-w2T_{HM4}(j)\ \{-h(j+2,k-1)+3h(j+1,k-1)+21h(j,k-1)-23h(j-1,k-1)\}$$
$$+w2T_{HM2}(j)\ \{-h(j+2,k-1)+27h(j+1,k-1)-27h(j,k-1)+h(j-1,k-1)\}$$
$$-w3T_{HM4}(j)\ \{-h(j+2,k)+3h(j+1,k)+21h(j,k)-23h(j-1,k)\}$$
$$+w3T_{HM2}(j)\ \{-h(j+2,k)+27h(j+1,k)-27h(j,k)+h(j-1,k)\}$$
$$=S\{h(j,k)-h(j,k-1)\}\frac{24\Delta x^2}{\Delta t}$$

The above equation is rearranged as:

$$-w3T_{HM4}(j)\{-h(j+2,k)+3h(j+1,k)+21h(j,k)-23h(j-1,k)\}$$
$$+w3T_{HM2}(j)\{-h(j+2,k)+27h(j+1,k)-27h(j,k)+h(j-1,k)\}$$
$$-w1T_{HM4}(j)\{-h(j+2,k-2)+3h(j+1,k-2)+21h(j,k-2)-23h(j-1,k-2)\}$$
$$+w1T_{HM2}(j)\{-h(j+2,k-2)+27h(j+1,k-2)-27h(j,k-2)+h(j-1,k-2)\}$$
$$-w2T_{HM4}(j)\{-h(j+2,k-1)+3h(j+1,k-1)+21h(j,k-1)-23h(j-1,k-1)\}$$
$$+w2T_{HM2}(j)\{-h(j+2,k-1)+27h(j+1,k-1)-27h(j,k-1)+h(j-1,k-1)\}$$
$$-S\{h(j,k)-h(j,k-1)\}\frac{24\Delta x^2}{\Delta t}=0$$

Designating, $w3\{T_{HM4}(j)-T_{HM2}(j)\}=F_{22}(j)$

$$w3\{-3T_{HM4}(j)+27T_{HM2}(j)\}=F_2(j)$$

$$-24S\frac{\Delta x^2}{\Delta t}+w3\{-21T_{HM4}(j)-27T_{HM2}(j)\}=F_1(j)$$

$$w3\{23T_{HM4}(j)+T_{HM2}(j)\}=F_4(j)$$

$$F_6(j,k)=-w1T_{HM4}(j)\{-h(j+2,k-2)+3h(j+1,k-2)+21h(j,k-2)-23h(j-1,k-2)\}$$
$$+w1T_{HM2}(j)\{-h(j+2,k-2)+27h(j+1,k-2)-27h(j,k-2)+h(j-1,k-2)\}$$
$$-w2T_{HM4}(j)\{-h(j+2,k-1)+3h(j+1,k-1)+21h(j,k-1)-23h(j-1,k-1)\}$$
$$+w2T_{HM4}(j)\{-h(j+2,k-1)+3h(j+1,k-1)+21h(j,k-1)-23h(j-1,k-1)\}+S\,h(j,k-1)\frac{24\Delta x^2}{\Delta t}$$

The water balance equation for interior node is written as:

$F_{22}(j)h(j+2,k)+F_2(j)h(j+1,k)+F_1(j)h(j,k)+F_4(j)h(j-1,k)+F_6(j,k)=0$

4.2.2. Equation For Boundary Node (left) i.e. j = 1

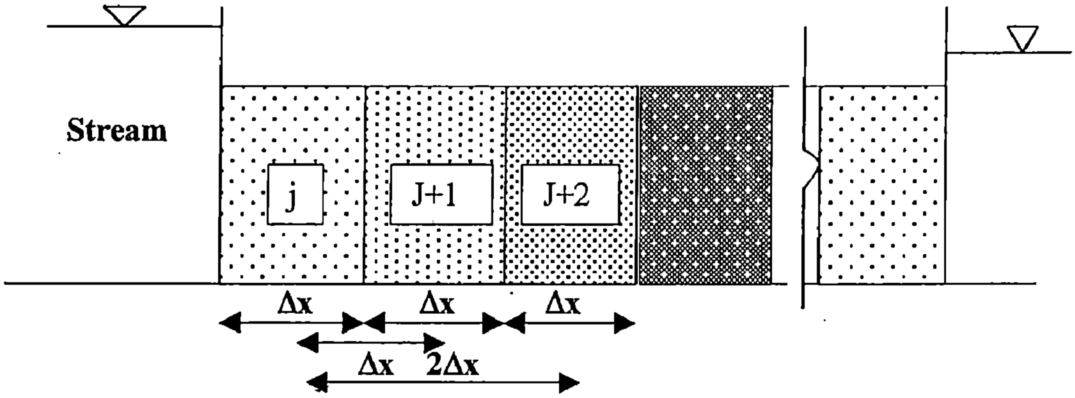

Fig.4.2: Discretisation of flow domain origin at node 1

Let head for $-\Delta x/2 \le x \le 2\Delta x$ is expressed as:

$h(x) = a + bx + cx^2 + dx^3$, constants a, b, c, d are derived in terms of h(j-1), h(j), h(j+1), h(j+2).

$$At\ x = o \qquad h(0) = h(j) = a$$

$$At\ x = -\frac{\Delta x}{2} \qquad h(-\Delta x/2) = h(j-1) = h(j) - \frac{b}{2}\Delta x + \frac{c}{4}\Delta x^2 - \frac{d}{8}\Delta x^3$$

$$At\ x = \Delta x \qquad h(\Delta x) = h(j+1) = h(j) + b\Delta x + c\Delta x^2 + d\Delta x^3$$

$$At\ x = 2\Delta x \qquad h(2\Delta x) = h(j+2) = h(j) + 2b\Delta x + 4c\Delta x^2 + 8d\Delta x^3$$

Solving above we get the value of a, b, c, d:

a = h(j)

$$b = \frac{-3h(j+2) + 20h(j+1) + 15h(j) - 32h(j-1)}{30\Delta x}$$

$$c = \frac{-h(j+2) + 10h(j+1) - 25h(j) + 16h(j-1)}{10\Delta x^2}$$

$$d = \frac{6h(j+2) - 20h(j+1) + 30h(j) - 16h(j-1)}{30\Delta x^3}$$

The gradient $\frac{dh}{dx} = b + 2cx + 3dx^2$,

$$\left.\frac{dh}{dx}\right|_{x=-\Delta x/2} = \frac{18h(j+2) - 100h(j+1) + 450h(j) - 368h(j-1)}{120\Delta x}$$

$$\left.\frac{dh}{dx}\right|_{x=\Delta x/2} = \frac{-6h(j+2) + 140h(j+1) - 150h(j) + 16h(j-1)}{120\Delta x}$$

The water balance equation is written as:

Inflow-outflow = change in storage

$$\left\{-T(j)\frac{dh}{dx}_{at x=-\Delta x/2}\right\}-\left\{-T_{HM2}(j)\frac{dh}{dx}_{at x=\Delta x/2}\right\}=S\{h(j,k)-h(j,k-1)\}\frac{\Delta x}{\Delta t}$$

$$\left\{-T(j)\frac{dh}{dx}_{at x=-\Delta x/2}\right\}+\left\{T_{HM2}(j)\frac{dh}{dx}_{at x=\Delta x/2}\right\}=S\{h(j,k)-h(j,k-1)\}\frac{\Delta x}{\Delta t}$$

Taking into account the variation of flow during time step (k-2), (k-1), and kth, the water balance equation can be written as:

$$-w1T(j)\{18h(j+2,k-2)-100h(j+1,k-2)+450h(j,k-2)-368h(j-1,k-2)\}\frac{1}{120\Delta x}$$

$$+w1T_{Hm2}(j)\{-6h(j+2,k-2)+140h(j+1,k-2)-150h(j,k-2)+16h(j-1,k-2)\}\frac{1}{120\Delta x}$$

$$-w2T(j)\{18h(j+2,k-1)-100h(j+1,k-1)+450h(j,k-1)-368h(j-1,k-1)\}\frac{1}{120\Delta x}$$

$$+w2T_{Hm2}(j)\{-6h(j+2,k-1)+140h(j+1,k-1)-150h(j,k-1)+16h(j-1,k-1)\}\frac{1}{120\Delta x}$$

$$-w3T(j)\{18h(j+2,k)-100h(j+1,k)+450h(j,k)-368h(j-1,k)\}\frac{1}{120\Delta x}$$

$$+w3T_{Hm2}(j)\{-6h(j+2,k)+140h(j+1,k)-150h(j,k)+16h(j-1,k)\}\frac{1}{120\Delta x}$$

$$=S\{h(j,k)-h(j,k-1)\}\frac{\Delta x}{\Delta t}$$

The above equation is rearranged as:

$$-w3T(j)\{18h(j+2,k)-100h(j+1,k)+450h(j,k)-368h(j-1,k)\}$$

$$+w3T_{Hm2}(j)\{-6h(j+2,k)+140h(j+1,k)-150h(j,k)+16h(j-1,k)\}$$

$$-120\frac{\Delta x^2}{\Delta t}Sh(j,k) \quad = \quad -120\frac{\Delta x^2}{\Delta t}Sh(j,k-1)$$

$$+w1T(j)\{18h(j+2,k-2)-100h(j+1,k-2)+450h(j,k-2)-368h(j-1,k-2)\}$$

$$-w1T_{Hm2}(j)\{-6h(j+2,k-2)+140h(j+1,k-2)-150h(j,k-2)+16h(j-1,k-2)\}$$

$$+w2T(j)\{18h(j+2,k-1)-100h(j+1,k-1)+450h(j,k-1)-368h(j-1,k-1)\}$$

$$-w2T_{Hm2}(j)\{-6h(j+2,k-1)+140h(j+1,k-1)-150h(j,k-1)+16h(j-1,k-1)\}$$

Designating, $w3\{-18T(j)-6T_{HM2}(j)\} \quad = \quad F_{22}(j)$

$w3\{100T(j)+140T_{HM2}(j)\} \quad = \quad F_2(j)$

$$-120\frac{\Delta x^2}{\Delta t}S + w3\{-450T(j) - 150T_{HM2}(j)\} \quad = \quad F_1(j)$$

$$F_6(j,k) = \quad 120\frac{\Delta x^2}{\Delta t}Sh(j,k-1) + w3\{368T(j) + 16T_{HM2}(j)\}$$

$$-w1T(j)\{18h(j+2,k-2) - 100h(j+1,k-2) + 450h(j,k-2) - 368h(j-1,k-2)\}$$

$$+w1T_{Hm2}(j)\{-6h(j+2,k-2) + 140h(j+1,k-2) - 150h(j,k-2) + 16h(j-1,k-2)\}$$

$$-w2T(j)\{18h(j+2,k-1) - 100h(j+1,k-1) + 450h(j,k-1) - 368h(j-1,k-1)\}$$

$$+w2T_{Hm2}(j)\{-6h(j+2,k-1) + 140h(j+1,k-1) - 150h(j,k-1) + 16h(j-1,k-1)\}$$

The water balance equation is written as:

$F_{22}(j)\,h(j+2,k) + F_2(j)\,h(j+1,k) + F_1(j)\,h(j,k) + F_6(j,k) = 0$

4.2.3. Equation For Interior Node i.e. $j = j_{max}-1$

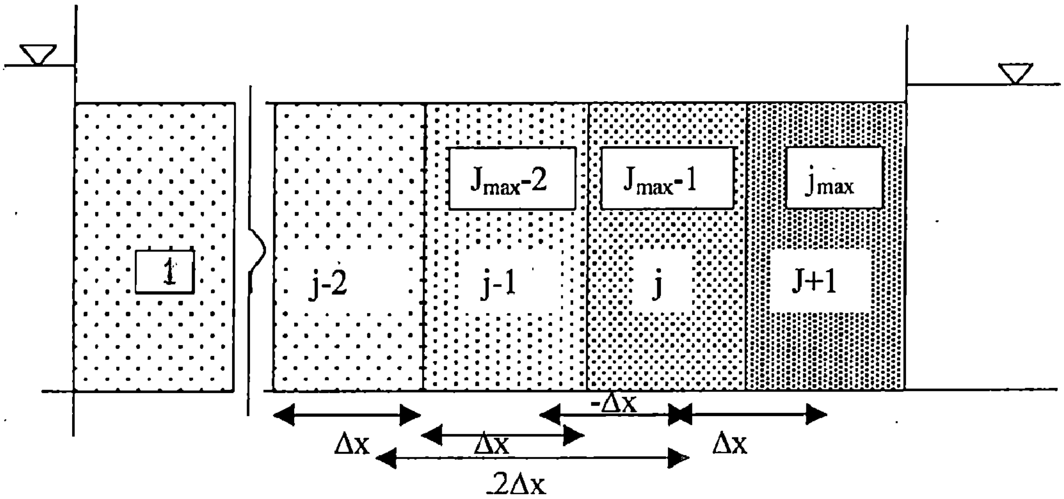

Fig.4.3: Discretisation of flow domain, origin at J_{max}-1

Let node j be origin. Let in the region $(j_{max}-3) \le x \le j_{max}$ the head be expressed as:

$h(x) = a + bx + cx^2 + dx^3$

where a, b, c, d are derived in terms of h(j-2), h(j-1), h(j), h(j-1).

At x = 0, $\quad h(0) = h(j) = a$

At x = Δx, $\quad h(\Delta x) = h(j+1) = a + b\,\Delta x + c\,\Delta x^2 + d\,\Delta x^3$

At x = -Δx, $\quad h(-\Delta x) = h(j-1) = a - b\,\Delta x + c\,\Delta x^2 - d\,\Delta x^3$

At x = -2Δx, $\quad h(-2\Delta x) = h(j-2) = a - 2b\,\Delta x + 4c\,\Delta x^2 - 8d\,\Delta x^3$

Solving above equation we get,

a = h(j)

b = {h(j-2)-6h(j-1)+3h(j)+2h(j+1)}/(6Δx)

c = {h(j+1)-2h(j)+h(j-1)}/($2\Delta x^2$)

d = {-h(j-2)+3h(j-1)-3h(j)+h(j+1)}/($6\Delta x^3$)

The differential dh/dx = b+ 2cx + $3dx^2$

$$\frac{dh}{dx}_{at\,x=-\Delta x/2} = \{h(j-2)-27h(j-1)+27h(j)-h(j+1)\}\frac{1}{24\Delta x}$$

$$\frac{dh}{dx}_{at\,x=\Delta x/2} = \{h(j-2)-3h(j-1)-21h(j)+23h(j+1)\}\frac{1}{24\Delta x}$$

The water balance equation is written as:

Inflow-outflow = change in storage

$$\text{Or}\left\{-T_{HM4}(j)\frac{dh}{dx}_{at\,x=-\Delta x/2}\right\}-\left\{-T_{HM2}(j)\frac{dh}{dx}_{at\,x=\Delta x/2}\right\}=S\{h(j,k)-h(j,k-1)\}\frac{\Delta x}{\Delta t}$$

$$-\left\{T_{HM4}(j)\frac{dh}{dx}_{at\,x=-\Delta x/2}\right\}+\left\{T_{HM2}(j)\frac{dh}{dx}_{at\,x=\Delta x/2}\right\}=S\{h(j,k)-h(j,k-1)\}\frac{\Delta x}{\Delta t}$$

Taking into account of the variation in flow during time steps (k-2), (k-1) & k[th], the water balance equation is written as:

$$-w1T_{HM4}(j)\{h(j-2,k-2)-27h(j-1,k-2)+27h(j,k-2)-h(j+1,k-2)\}$$
$$+w1T_{HM2}(j)\{h(j-2,k-2)-3h(j-1,k-2)-21h(j,k-2)+23h(j+1,k-2)\}$$
$$-w2T_{HM4}(j)\{h(j-2,k-1)-27h(j-1,k-1)+27h(j,k-1)-h(j+1,k-1)\}$$
$$+w2T_{HM2}(j)\{h(j-2,k-1)-3h(j-1,k-1)-21h(j,k-1)+23h(j+1,k-1)\}$$
$$-w3T_{HM4}(j)\{h(j-2,k)-27h(j-1,k)+27h(j,k)-h(j+1,k)\}$$
$$+w3T_{HM2}(j)\{h(j-2,k)-3h(j-1,k)-21h(j,k)+23h(j+1,k)\}$$
$$=S\{h(j,k)-h(j,k-1)\}\frac{24\Delta x^2}{\Delta t}$$

Designating,

$$w3\{-T_{HM4}(j)+T_{HM2}(j)\}=F_{44}(j)$$

$$w3\{27T_{HM4}(j)-3T_{HM2}(j)\}=F_4(j)$$

$$-24S\frac{\Delta x^2}{\Delta t}+w3\{-27T_{HM4}(j)-21T_{HM2}(j)\}=F_1(j)$$

$$w3\{T_{HM4}(j)+23T_{HM2}(j)\}=F_2(j)$$

$$F_6(j,k)=-w1T_{HM4}(j)\{h(j-2,k-2)-27h(j-1,k-2)+27h(j,k-2)-h(j+1,k-2)\}$$
$$+w1T_{HM2}(j)\{h(j-2,k-2)-3h(j-1,k-2)-21h(j,k-2)+23h(j+1,k-2)\}$$
$$-w2T_{HM4}(j)\{h(j-2,k-1)-27h(j-1,k-1)+27h(j,k-1)-h(j+1,k-1)\}$$
$$+w2T_{HM2}(j)\{h(j-2,k-1)-3h(j-1,k-1)-21h(j,k-1)+23h(j+1,k-1)\}$$
$$+Sh(j,k-1)\frac{24\Delta x^2}{\Delta t}$$

The water balance equation is written as:

$$F_{44}(j)h(j-2,k)+F_4(j)h(j-1,k)+F_1(j)h(j,k)+F_2(j)h(j+1,k)+F_6(j,k)=0$$

4.2.4. Equation For Boundary Node (right) i.e. = j_{max}

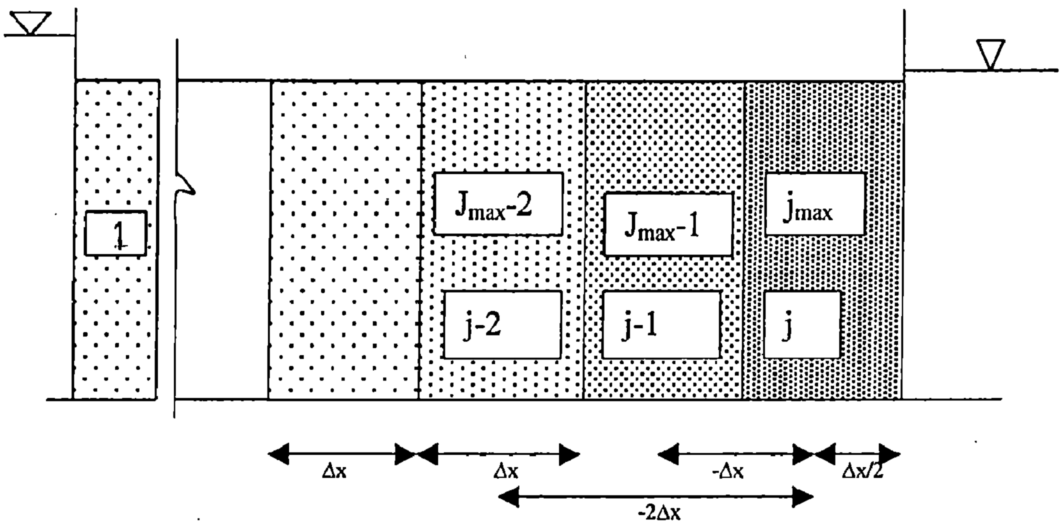

Fig.4.4: Discretisation of flow domain, origin at j_{max}

Let the head for $-2\Delta x \le x \le \Delta x/2$ is expressed as:

$h(x) = a + bx + cx^2 + dx^3$, constants a, b, c, d are derived in terms of h(j-2), h(j-1), h(j), h(j+1).

At $x = 0$, $h(0) = h(j) = a$

At $x = \Delta x/2$, $h(\Delta x/2) = h(j+1) = a + b\,\Delta x/2 + c\,\Delta x^2/4 + d\,\Delta x^3/8$

At $x = -\Delta x$, $h(-\Delta x) = h(j-1) = a - b\,\Delta x + c\,\Delta x^2 - d\,\Delta x^3$

At $x = -2\Delta x$, $h(-2\Delta x) = h(j-2) = a - 2b\,\Delta x + 4c\,\Delta x^2 - 8d\,\Delta x^3$

Solving above equation we get,

$a = h(j)$

$b = \{32h(j+1)-15h(j)-20h(j-1)+3h(j-2)\}/(30\Delta x)$

$c = \{16h(j+1)-25h(j)+10h(j-1)-h(J-2)\}/(10\Delta x^2)$

$d = \{16h(j+1)-30h(j)+20h(j-1)-6h(j-2)\}/(30\Delta x^3)$

The gradient $dh/dx = b + 2cx + 3dx^2$

$$\frac{dh}{dx}_{at\,x=-\Delta x/2} = \{-16h(j+1)+150h(j)-140h(j-1)+6h(j-2)\}\frac{1}{120\Delta x}$$

$$\frac{dh}{dx}_{at\,x=\Delta x/2} = \{368h(j+1)-450h(j)+100h(j-1)-18h(j-2)\}\frac{1}{120\Delta x}$$

The water balance equation can be written as:

Inflow-outflow = change in storage

$$\left\{-T_{HM4}(j)\frac{dh}{dx}_{at\,x=-\Delta x/2}\right\}-\left\{-T(j)\frac{dh}{dx}_{atx=\Delta x/2}\right\}=S\{h(j,k)-h(j,k-1)\}\frac{\Delta x}{\Delta t}$$

$$-\left\{T_{HM4}(j)\frac{dh}{dx}_{at\,x=-\Delta x/2}\right\}+\left\{T(j)\frac{dh}{dx}_{atx=\Delta x/2}\right\}=S\{h(j,k)-h(j,k-1)\}\frac{\Delta x}{\Delta t}$$

Taking into account the variation of flow during different time steps (k-2), (k-1), k[th], the water balance equation will be written as:

$$-w1T_{Hm4}(j)\{-16h(j+1,k-2)+150h(j,k-2)-140h(j-1,k-2)+6h(j-2,k-2)\}\frac{1}{120\Delta x}$$

$$+w1T(j)\{368h(j+1,k-2)-450h(j,k-2)+100h(j-1,k-2)-18h(j-2,k-2)\}\frac{1}{120\Delta x}$$

$$-w2T_{Hm4}(j)\{-16h(j+1,k-1)+150h(j,k-1)-140h(j-1,k-1)+6h(j-2,k-1)\}\frac{1}{120\Delta x}$$

$$+w2T(j)\{368h(j+1,k-1)-450h(j,k-1)+100h(j-1,k-1)-18h(j-2,k-1)\}\frac{1}{120\Delta x}$$

$$-w3T_{Hm4}(j)\{-16h(j+1,k)+150h(j,k)-140h(j-1,k)+6h(j-2,k)\}\frac{1}{120\Delta x}$$

$$+w3T(j)\{368h(j+1,k)-450h(j,k)+100h(j-1,k)-18h(j-2,k)\}\frac{1}{120\Delta x}$$

$$=S\{h(j,k)-h(j,k-1)\}\frac{\Delta x}{\Delta t}$$

The equation is rearranged as:

$$-w3T_{Hm4}(j)\{-16h(j+1,k)+150h(j,k)-140h(j-1,k)+6h(j-2,k)\}$$

$$+w3T(j)\{368h(j+1,k)-450h(j,k)+100h(j-1,k)-18h(j-2,k)\}$$

$$-120S\frac{\Delta x^2}{\Delta t}h(j,k)=-120S\frac{\Delta x^2}{\Delta t}h(j,k-1)$$

$$+w1T_{Hm4}(j)\{-16h(j+1,k-2)+150h(j,k-2)-140h(j-1,k-2)+6h(j-2,k-2)\}$$

$$-w1T(j)\{368h(j+1,k-2)-450h(j,k-2)+100h(j-1,k-2)-18h(j-2,k-2)\}$$

$$+w2T_{Hm4}(j)\{-16h(j+1,k-1)+150h(j,k-1)-140h(j-1,k-1)+6h(j-2,k-1)\}$$

$$-w2T(j)\{368h(j+1,k-1)-450h(j,k-1)+100h(j-1,k-1)-18h(j-2,k-1)\}$$

Designating,

$$-120S\frac{\Delta x^2}{\Delta t}+\{-150T_{HM4}(j)-450T(j)\}w3=F_1(j)$$

$$\{140T_{HM4}(j)+100T(j)\}w3=F_4(j)$$

$$\{-6T_{HM4}(j)-18T(j)\}w3=F_{44}(j)$$

$$F_6(j,k)=120S\frac{\Delta x^2}{\Delta t}h(j,k-1)+w3\{16T_{HM4}(j)+368T(j)\}h(j+1,k)$$

$$-w1T_{Hm4}(j)\{-16h(j+1,k-2)+150h(j,k-2)-140h(j-1,k-2)+6h(j-2,k-2)\}$$

$$+w1T(j)\{368h(j+1,k-2)-450h(j,k-2)+100h(j-1,k-2)-18h(j-2,k-2)\}$$

$$-w2T_{Hm4}(j)\{-16h(j+1,k-1)+150h(j,k-1)-140h(j-1,k-1)+6h(j-2,k-1)\}$$

$$+w2T(j)\{368h(j+1,k-1)-450h(j,k-1)+100h(j-1,k-1)-18h(j-2,k-1)\}$$

The water balance equation is written as:

$$F_1(j)h(j,k)+F_4(j)h(j-1,k)+F_{44}(j)h(j-2,k)+F_6(j,k)=0$$

4.3 FINITE DIFFERENCE SCHEME FOR NON-UNIFORM GRID SIZE

Let us consider a non-homogeneous, isotropic, one-dimensional confined aquifer bounded by fully penetrating stream. Let us discretise the aquifer into grids of unequal sizes. Four types of grid for which equations are to be written:

a. All interior nodes i.e. $j = 2, j_{max}-2$

b. The boundary node (left) i.e. $j = 1$

c. The interior node i.e. $j = j_{max}-1$

d. The boundary node right i.e. $j = j_{max}$

4.3.1. Equation For Interior Nodes i.e. $j = 2, j_{max}-2$

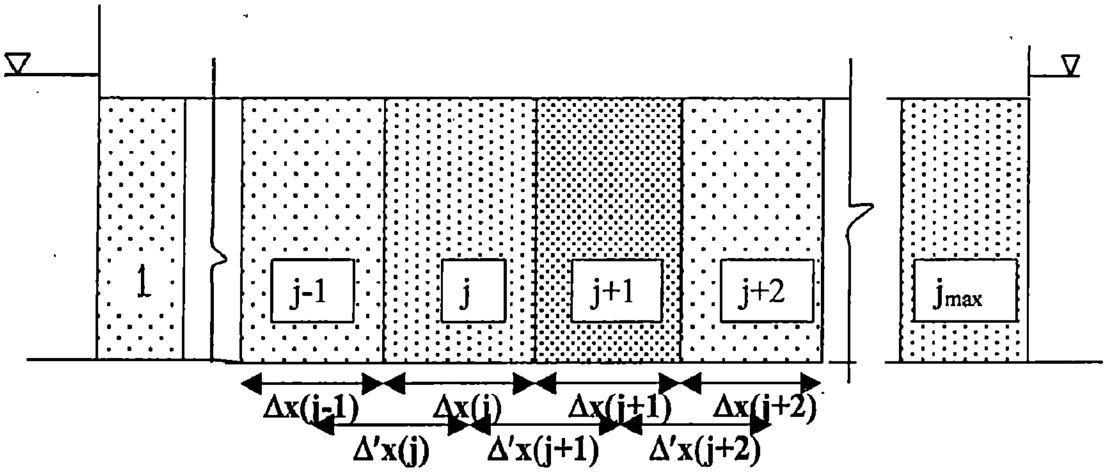

Fig.4.5: Discretisation of flow domain, origin at $j, 2 \le j \le j_{max}-2$

Selecting the origin at j^{th} node, the head h(x), $j-1 \le x \le j+2$ is expressed as:

$h(x) = a + bx + cx^2 + dx^3$

where constants a, b, c, d are derived in terms of h(j-1), h(j), h(j+1), h(j+2).

Designate, $\Delta' x(j) = \{\Delta x(j-1) + \Delta x(j)\}/2$

$\Delta' x(j+1) = \{\Delta x(j) + \Delta x(j+1)\}/2$

$\Delta' x(j+2) = \{\Delta x(j+1) + \Delta x(j+2)\}/2$

Hence at $x = 0$, $\quad h(0) = h(j) = a$

At $x = -\Delta' x(j)$, $\quad h\{-\Delta' x(j)\} = h(j-1)$,

$$h(j-1)-h(j)= -b\Delta'x(j)+ c\Delta'x^2(j)- d\Delta'x^3(j) \qquad \ldots(4.1)$$

At $x= \Delta'x(j+1)$, $\quad h\{\Delta'x(j+1)\}=h(j+1)$

$$h(j+1)-h(j) = b\Delta'x(j+1)+ c\Delta'x^2(j+1)+ d\Delta'x^3(j+1) \qquad \ldots(4.2)$$

At $x= \Delta'x(j+1)+ \Delta'x(j+2)$, $\quad h\{\Delta'x(j+1)+ \Delta'x(j+2)\} = h(j+2)$

$$h(j+2)-h(j)=b\{\Delta'x(j+1)+ \Delta'x(j+2)\}+ c\{\Delta'x(j+1)+ \Delta'x(j+2)\}^2+d\{\Delta'x(j+1)+ \Delta'x(j+2)\}^3 \qquad \ldots(4.3)$$

Above equations (4.1), (4.2), (4.3) are solved for getting b, c, d :

Designating, $D=\begin{vmatrix} -\Delta'x(j) & \Delta'x^2(j) & -\Delta'x^3(j) \\ \Delta'x(j+1) & \Delta'x^2(j+1) & \Delta'x^3(j+1) \\ \{\Delta'x(j+1)+\Delta'x(j+2)\} & \{\Delta'x(j+1)+\Delta'x(j+2)\}^2 & \{\Delta'x(j+1)+\Delta'x(j+2)\}^3 \end{vmatrix}$

$$\therefore b=\begin{vmatrix} h(j-1)-h(j) & \Delta'x^2(j) & -\Delta'x^3(j) \\ h(j+1)-h(j) & \Delta'x^2(j+1) & \Delta'x^3(j+1) \\ h(j+2)-h(j) & \{\Delta'x(j+1)+\Delta'x(j+2)\}^2 & \{\Delta'x(j+1)+\Delta'x(j+2)\}^3 \end{vmatrix} / D$$

$$\text{Or } b=\begin{vmatrix} h(j-1)-h(j) & h(j+1)-h(j) & h(j+2)-h(j) \\ \Delta'x^2(j) & \Delta'x^2(j+1) & \{\Delta'x(j+1)+\Delta'x(j+2)\}^2 \\ -\Delta'x^3(j) & \Delta'x^3(j+1) & \{\Delta'x(j+1)+\Delta'x(j+2)\}^3 \end{vmatrix} / D$$

Designating, $\begin{vmatrix} \Delta'x^2(j+1) & \{\Delta'x(j+1)+\Delta'x(j+2)\}^2 \\ \Delta'x^3(j+1) & \{\Delta'x(j+1)+\Delta'x(j+2)\}^3 \end{vmatrix} / D = D_1$

$$\begin{vmatrix} \Delta'x^2(j) & \{\Delta'x(j+1)+\Delta'x(j+2)\}^2 \\ -\Delta'x^3(j) & \{\Delta'x(j+1)+\Delta'x(j+2)\}^3 \end{vmatrix} / D = D_2$$

$$\begin{vmatrix} \Delta'x^2(j) & \Delta'x^2(j+1) \\ -\Delta'x^3(j) & \Delta'x^3(j+1) \end{vmatrix} / D = D_3$$

Hence $b = \{h(j-1)-h(j)\}D_1 - \{h(j+1)-h(j)\}D_2 + \{h(j+2)-h(j)\}D_3$

$$= D_1 h(j-1) + (-D_1 + D_2 - D_3)h(j) - D_2 h(j+1) + D_3 h(j+2) \qquad \ldots(4.4)$$

$$\text{Now } c = \begin{vmatrix} -\Delta' x(j) & h(j-1)-h(j) & -\Delta' x^3(j) \\ \Delta' x(j+1) & h(j+1)-h(j) & \Delta' x^3(j+1) \\ \{\Delta' x(j+1)+\Delta' x(j+2)\} & h(j+2)-h(j) & \{\Delta' x(j+1)+\Delta' x(j+2)\}^3 \end{vmatrix} / D$$

$$\text{Or } c = -\begin{vmatrix} h(j-1)-h(j) & h(j+1)-h(j) & h(j+2)-h(j) \\ -\Delta' x(j) & \Delta' x(j+1) & \{\Delta' x(j+1)+\Delta' x(j+2)\} \\ -\Delta' x^3(j) & \Delta' x^3(j+1) & \{\Delta' x(j+1)+\Delta' x(j+2)\}^3 \end{vmatrix} / D$$

Designating, $\begin{vmatrix} \Delta' x(j+1) & \{\Delta' x(j+1)+\Delta' x(j+2)\} \\ \Delta' x^3(j+1) & \{\Delta' x(j+1)+\Delta' x(j+2)\}^3 \end{vmatrix} / D = D_4$

$$\begin{vmatrix} -\Delta' x(j) & \{\Delta' x(j+1)+\Delta' x(j+2)\} \\ -\Delta' x^3(j) & \{\Delta' x(j+1)+\Delta' x(j+2)\}^3 \end{vmatrix} / D = D_5$$

$$\begin{vmatrix} -\Delta' x(j) & \Delta' x(j+1) \\ -\Delta' x^3(j) & \Delta' x^3(j+1) \end{vmatrix} / D = D_6$$

Hence $c = -\{h(j-1)-h(j)\}D_4 + \{h(j+1)-h(j)\}D_5 - \{h(j+2)-h(j)\}D_6$

$$= -D_4 h(j-1) + (D_4 - D_5 + D_6)h(j) + D_5 h(j+1) - D_6 h(j+2) \qquad \ldots(4.5)$$

$$d = \begin{vmatrix} -\Delta' x(j) & \Delta' x^2(j) & h(j-1)-h(j) \\ \Delta' x(j+1) & \Delta' x^2(j+1) & h(j+1)-h(j) \\ \{\Delta' x(j+1)+\Delta' x(j+2)\} & \{\Delta' x(j+1)+\Delta' x(j+2)\}^2 & h(j+2)-h(j) \end{vmatrix} / D$$

$$d = \begin{vmatrix} h(j-1)-h(j) & h(j+1)-h(j) & h(j+2)-h(j) \\ -\Delta' x(j) & \Delta' x(j+1) & \{\Delta' x(j+1)+\Delta' x(j+2)\} \\ \Delta' x^2(j) & \Delta' x^2(j+1) & \{\Delta' x(j+1)+\Delta' x(j+2)\}^2 \end{vmatrix} / D$$

Designating, $\begin{vmatrix} \Delta' x(j+1) & \{\Delta' x(j+1)+\Delta' x(j+2)\} \\ \Delta' x^2(j+1) & \{\Delta' x(j+1)+\Delta' x(j+2)\}^2 \end{vmatrix} / D = D_7$

$$\begin{vmatrix} -\Delta' x(j) & \{\Delta' x(j+1)+\Delta' x(j+2)\} \\ \Delta' x^2(j) & \{\Delta' x(j+1)+\Delta' x(j+2)\}^2 \end{vmatrix} / D = D_8$$

$$\begin{vmatrix} -\Delta' x(j) & \Delta' x(j+1) \\ \Delta' x^2(j) & \Delta' x^2(j+1) \end{vmatrix} / D = D_9$$

Hence $d = \{h(j-1)-h(j)\}D_7 - \{(h(j+1)-h(j)\}D_8 + \{h(j+2)-h(j)\}D_9$

$$= D_7 h(j-1) + (-D_7 + D_8 - D_9)h(j) - D_8 h(j+1) + D_9 h(j+2) \quad \ldots(4.6)$$

The gradient, dh/dx = b +2cx +3dx^2

$$\left.\frac{dh}{dx}\right|_{at x=-\Delta x(j)/2} = D_1 h(j-1) + (-D_1 + D_2 - D_3)h(j) - D_2 h(j+1) + D_3 h(j+2)$$

$$-\{-D_4 h(j-1) + (D_4 - D_5 + D_6)h(j) + D_5 h(j+1) - D_6 h(j+2)\}\Delta x(j)$$

$$+\{D_7 h(j-1) + (-D_7 + D_8 - D_9)h(j) - D_8 h(j+1) + D_9 h(j+2)\}\frac{3}{4}\Delta x^2(j)$$

dh/dx$_{atx=-\Delta x(j)/2}$ =

$$h(j-1)\left\{D_1 + D_4\Delta x(j) + D_7\frac{3}{4}\Delta x^2(j)\right\}$$

$$+h(j)\left\{-D_1 + D_2 - D_3 - (D_4 - D_5 + D_6)\Delta x(j) + (-D_7 + D_8 - D_9)\frac{3}{4}\Delta x^2(j)\right\}$$

$$+h(j+1)\left\{-D_2 - D_5\Delta x(j) - D_8\frac{3}{4}\Delta x^2(j)\right\} + h(j+2)\left\{D_3 + D_6\Delta x(j) + D_9\frac{3}{4}\Delta x^2(j)\right\}$$

$$\left.\frac{dh}{dx}\right|_{at x=\Delta x(j)/2} = D_1 h(j-1) + (-D_1 + D_2 - D_3)h(j) - D_2 h(j+1) + D_3 h(j+2)$$

$$+\{-D_4 h(j-1) + (D_4 - D_5 + D_6)h(j) + D_5 h(j+1) - D_6 h(j+2)\}\Delta x(j)$$

$$+\{D_7 h(j-1) + (-D_7 + D_8 - D_9)h(j) - D_8 h(j+1) + D_9 h(j+2)\}\frac{3}{4}\Delta x^2(j)$$

dh/dx$_{at\ x=\Delta x(j)/2}$ =

$$h(j-1)\left\{D_1 - D_4\Delta x(j) + D_7\frac{3}{4}\Delta x^2(j)\right\}$$

$$+h(j)\left\{-D_1 + D_2 - D_3 + (D_4 - D_5 + D_6)\Delta x(j) + (-D_7 + D_8 - D_9)\frac{3}{4}\Delta x^2(j)\right\}$$

$$+h(j+1)\left\{-D_2 + D_5\Delta x(j) - D_8\frac{3}{4}\Delta x^2(j)\right\} + h(j+2)\left\{D_3 - D_6\Delta x(j) + D_9\frac{3}{4}\Delta x^2(j)\right\}$$

The water balance equation is given by:

Inflow-outflow = change in storage

$$-T_{HM4}(j)\left.\frac{dh}{dx}\right|_{at x=-\Delta x(j)/2} - \left\{-T_{HM2}(j)\left.\frac{dh}{dx}\right|_{at x=\Delta x(j)/2}\right\} = S\{h(j,k) - h(j,k-1)\}\frac{\Delta x(j)}{\Delta t}$$

$$-T_{HM4}(j)\left.\frac{dh}{dx}\right|_{at x=-\Delta x(j)/2} + \left\{T_{HM2}(j)\left.\frac{dh}{dx}\right|_{at x=\Delta x(j)/2}\right\} = S\{h(j,k) - h(j,k-1)\}\frac{\Delta x(j)}{\Delta t}$$

Taking into account of the variation of flow during time step (k-2), (k-1), k^{th}, the water balance equation is written as:

$$w1\left[-T_{HM4}(j)\frac{dh}{dx}_{atx=-\Delta x/2}+T_{HM2}(j)\frac{dh}{dx}_{atx=\Delta x/2}\right]+w2\left[-T_{HM4}(j)\frac{dh}{dx}_{atx=-\Delta x/2}+T_{HM2}(j)\frac{dh}{dx}_{atx=\Delta x/2}\right]$$

$$+w3\left[-T_{HM4}(j)\frac{dh}{dx}_{atx=-\Delta x/2}+T_{HM2}(j)\frac{dh}{dx}_{atx=\Delta x/2}\right]=S\{h(j,k)-h(j,k-1)\}\frac{\Delta x}{\Delta t}$$

Designating, $F_4(j)=-w3T_{Hm4}(j)\left\{D_1+D_4\Delta x(j)+D_7\frac{3}{4}\Delta x^2(j)\right\}$

$$+w3T_{Hm2}(j)\left\{D_1-D_4\Delta x(j)+D_7\frac{3}{4}\Delta x^2(j)\right\}$$

$$F_1(j)=-w3T_{Hm4}(j)\left\{-D_1+D_2-D_3-(D_4-D_5+D_6)\Delta x(j)+(-D_7+D_8-D_9)\frac{3}{4}\Delta x^2(j)\right\}$$

$$+w3T_{Hm2}(j)\left\{-D_1+D_2-D_3+(D_4-D_5+D_6)\Delta x(j)+(-D_7+D_8-D_9)\frac{3}{4}\Delta x^2(j)\right\}-S\frac{\Delta x(j)}{\Delta t}$$

$$F_2(j)=-w3T_{HM4}(j)\left\{-D_2-D_5\Delta x(j)-D_8\frac{3}{4}\Delta x\ \ (j)\right\}+w3T_{HM2}(j)\left\{-D_2+D_5\Delta x(j)-D_8\frac{3}{4}\Delta x^2(j)\right\}$$

$$F_{22}(j)=-w3T_{HM4}(j)\left\{D_3+D_6\Delta x(j)+D_9\frac{3}{4}\Delta x^2(j)\right\}+w3T_{HM2}(j)\left\{D_3-D_6\Delta x(j)+D_9\frac{3}{4}\Delta x^2(j)\right\}$$

$$F_6(j,k)=-S\frac{\Delta x}{\Delta t}h(j,k)-w1T_{HM4}(j)\left\{\frac{dh}{dx}_{atx=(-\Delta x/2)}\right\}+w1T_{HM2}(j)\left\{\frac{dh}{dx}_{atx=(\Delta x/2)}\right\}$$

$$-w2T_{HM4}(j)\left\{\frac{dh}{dx}_{atx=(-\Delta x/2)}\right\}+w2T_{HM2}(j)\left\{\frac{dh}{dx}_{atx=(\Delta x/2)}\right\}$$

Finally the water balance equation becomes:

$$F_4(j)h(j-1,k)+F_1(j)h(j,k)+F_2(j)h(j+1,k)+F_{22}(j)h(j+2,k)+F_6(j,k)=0$$

4.3.2 Equation for Boundary Node (left) i.e.j=1:

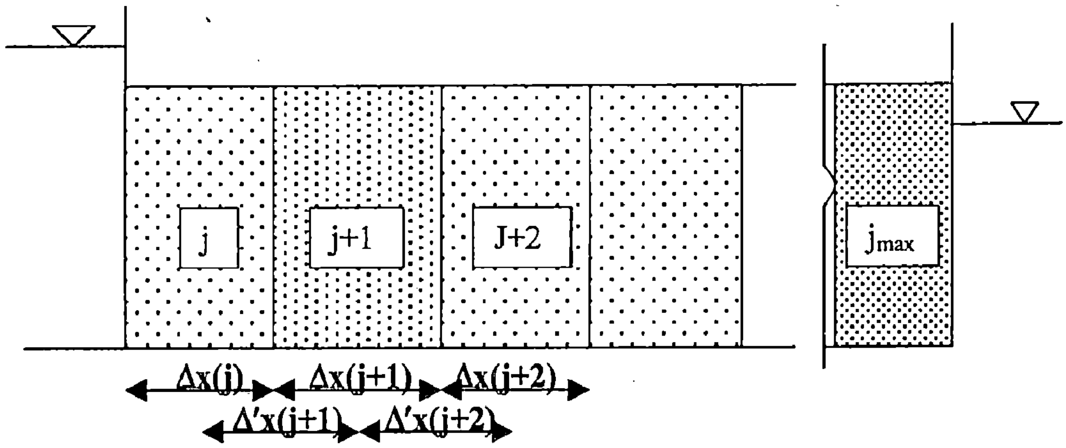

Fig.4.6: Discretisation of flow domain origin at node 1

Let head for $-\Delta x/2 \le x \le 2\Delta x$ is expressed as:

$h(x)=a+ b x +c x^2 + dx^3$

Where constants a, b, c, d are derived in terms of h (j-1), h(j), h(j+1), h(j+2).

Let node (j) be the origin.

Designating, $\Delta'x(j+1)=\{\Delta x(j)+\Delta x(j+1)\}/2$

$\Delta'x(j+2)=\{\Delta x(j+1)+\Delta x(j+2)\}/2$

Hence at x=0, $h(0) = h(j) = a$,

At $x=\Delta'x(j+1)$, $h\{\Delta'x(j+1)\} = h(j+1)$,

Or $h(j+1)-h(j) = b\Delta'x(j+1)+c\Delta'x^2(j+1)+d\Delta'x^3(j+1)$

At $x=\Delta'x(j+1)+\Delta'x(j+2)$, $h\{\Delta'x(j+1)+\Delta'x(j+2)\} = h(j+2)$

Or $h(j+2)-h(j) = b\{\Delta'x(j+1)+\Delta'x(j+2)\}+c\{\Delta'x(j+1)+\Delta'x(j+2)\}^2 +d\{\Delta'x(j+1)+\Delta'x(j+2)\}^3$

At $x=-\Delta x(j)/2$, $h\{-\Delta x(j)/2\} = h(j-1)$

Or $h(j-1)-h(j) = -b\ \{\Delta x(j)/2\}+c\{\Delta x(j)/2\}^2-d\{\Delta x(j)/2\}^3$

$= -b\ \{\Delta x(j)/2\}+c\{\Delta x^2(j)/4\}-d\{\Delta^3 x(j)/8\}$

Now the above equations are rearranged as:

$h(j-1)-h(j) = -b\ \{\Delta x(j)/2\}+c\{\Delta x^2(j)/4\}+d\{\Delta^3 x(j)/8\}$ ...(4.7)

$$h(j+1)-h(j) = b\Delta'x(j+1)+c\Delta'x^2(j+1)+d\Delta'x^3(j+1) \qquad \ldots(4.8)$$

$$h(j+2)-h(j) = b\{\Delta'x(j+1)+\Delta'x(j+2)\}+c\{\Delta'x(j+1)+\Delta'x(j+2)\}^2+d\{\Delta'x(j+1)+\Delta'x(j+2)\}^3 \qquad \ldots(4.9)$$

Designating, $D=\begin{vmatrix} \frac{-\Delta x(j)}{2} & \frac{\Delta x^2(j)}{4} & -\frac{\Delta x^3(j)}{8} \\ \Delta' x(j+1) & \Delta' x^2(j+1) & \Delta' x^3(j+1) \\ \{\Delta' x(j+1)+\Delta' x(j+2)\} & \{\Delta' x(j+1)+\Delta' x(j+2)\}^2 & \{\Delta' x(j+1)+\Delta' x(j+2)\}^3 \end{vmatrix}$

Now $b=\begin{vmatrix} h(j-1)-h(j) & \frac{\Delta x^2(j)}{4} & -\frac{\Delta x^3(j)}{8} \\ h(j+1)-h(j) & \Delta' x^2(j+1) & \Delta' x^3(j+1) \\ h(j+2)-h(j) & \{\Delta' x(j+1)+\Delta' x(j+2)\}^2 & \{\Delta' x(j+1)+\Delta' x(j+2)\}^3 \end{vmatrix} / D$

Or $b=\begin{vmatrix} h(j-1)-h(j) & h(j+1)-h(j) & h(j+2)-h(j) \\ \frac{\Delta x^2(j)}{4} & \Delta' x^2(j+1) & \{\Delta' x(j+1)+\Delta' x(j+2)\}^2 \\ -\frac{\Delta x^3(j)}{8} & \Delta' x^3(j+1) & \{\Delta' x(j+1)+\Delta' x(j+2)\}^3 \end{vmatrix} / D$

Designating, $\begin{vmatrix} \Delta' x^2(j+1) & \{\Delta' x(j+1)+\Delta' x(j+2)\}^2 \\ \Delta' x^3(j+1) & \{\Delta' x(j+1)+\Delta' x(j+2)\}^3 \end{vmatrix} / D = D_{41}$

$\begin{vmatrix} \frac{\Delta x^2(j)}{4} & \{\Delta' x(j+1)+\Delta' x(j+2)\}^2 \\ -\frac{\Delta x^3(j)}{8} & \{\Delta' x(j+1)+\Delta' x(j+2)\}^3 \end{vmatrix} / D = D_{42}$

$\begin{vmatrix} \frac{\Delta x^2(j)}{4} & \Delta' x^2(j+1) \\ -\frac{\Delta x^3(j)}{8} & \Delta' x^3(j+1) \end{vmatrix} / D = D_{43}$

Hence $b=\{h(j-1)-h(j)\}D_{41}-\{h(j+1)-h(j)\}D_{42}+\{h(j+2)-h(j)\}D_{43}$

$$= D_{41}h(j-1)+(-D_{41}+D_{42}-D_{43})h(j)-D_{42}h(j+1)+D_{43}h(j+2) \qquad \ldots(4.10)$$

$$\text{Now } c=\begin{vmatrix} \frac{-\Delta x(j)}{2} & h(j-1)-h(j) & -\frac{\Delta x^3(j)}{8} \\ \Delta' x(j+1) & h(j+1)-h(j) & \Delta' x^3(j+1) \\ \{\Delta' x(j+1)+\Delta' x(j+2)\} & h(j+2)-h(j) & \{\Delta' x(j+1)+\Delta' x(j+2)\}^3 \end{vmatrix} \Big/ D$$

$$\text{Or } c=-\begin{vmatrix} h(j-1)-h(j) & \frac{-\Delta x(j)}{2} & -\frac{\Delta x^3(j)}{8} \\ h(j+1)-h(j) & \Delta' x(j+1) & \Delta' x^3(j+1) \\ h(j+2)-h(j) & \{\Delta' x(j+1)+\Delta' x(j+2)\} & \{\Delta' x(j+1)+\Delta' x(j+2)\}^3 \end{vmatrix} \Big/ D$$

$$\text{Or } c=-\begin{vmatrix} h(j-1)-h(j) & h(j+1)-h(j) & h(j+2)-h(j) \\ -\frac{\Delta x(j)}{2} & \Delta' x(j+1) & \{\Delta' x(j+1)+\Delta' x(j+2)\} \\ -\frac{\Delta x^3(j)}{8} & \Delta' x^3(j+1) & \{\Delta' x(j+1)+\Delta' x(j+2\}^3 \end{vmatrix} \Big/ D$$

$$\text{Designating, } \begin{vmatrix} \Delta' x(j+1) & \{\Delta' x(j+1)+\Delta' x(j+2)\} \\ \Delta' x^3(j+1) & \{\Delta' x(j+1)+\Delta' x(j+2\}^3 \end{vmatrix} \Big/ D = D_{44}$$

$$\begin{vmatrix} -\frac{\Delta x(j)}{2} & \{\Delta' x(j+1)+\Delta' x(j+2)\} \\ -\frac{\Delta x^3(j)}{8} & \{\Delta' x(j+1)+\Delta' x(j+2\}^3 \end{vmatrix} \Big/ D = D_{45}$$

$$\begin{vmatrix} -\frac{\Delta x(j)}{2} & \Delta' x(j+1) \\ -\frac{\Delta x^3(j)}{8} & \Delta' x^3(j+1) \end{vmatrix} \Big/ D = D_{46}$$

$$\text{Hence } c=-\{h(j-1)-h(j)\}D_{44}+\{h(j+1)-h(j)\}D_{45}-\{h(j+2)-h(j)\}D_{46}$$

$$=-D_{44}h(j-1)+(D_{44}-D_{45}+D_{46})h(j)+D_{45}h(j+1)-D_{46}h(j+2) \qquad \ldots(4.11)$$

$$\text{Now } d=\begin{vmatrix} \frac{-\Delta x(j)}{2} & \frac{\Delta x^2(j)}{4} & h(j-1)-h(j) \\ \Delta' x(j+1) & \Delta' x^2(j+1) & h(j+1)-h(j) \\ \{\Delta' x(j+1)+\Delta' x(j+2)\} & \{\Delta' x(j+1)+\Delta' x(j+2)\}^2 & h(j+2)-h(j) \end{vmatrix} \Big/ D$$

$$\text{Or } d = \begin{vmatrix} h(j-1)-h(j) & \frac{-\Delta x(j)}{2} & \frac{\Delta x^2(j)}{4} \\ h(j+1)-h(j) & \Delta' x(j+1) & \Delta' x^2(j+1) \\ h(j+2)-h(j) & \{\Delta' x(j+1)+\Delta' x(j+2)\} & \{\Delta' x(j+1)+\Delta' x(j+2)\}^2 \end{vmatrix} / D$$

$$\text{Or d } = \begin{vmatrix} h(j-1)-h(j) & h(j+1)-h(j) & h(j+2)-h(j) \\ -\frac{\Delta x(j)}{2} & \Delta' x(j+1) & \{\Delta' x(j+1)+\Delta' x(j+2)\} \\ \frac{\Delta x^2(j)}{4} & \Delta' x^2(j+1) & \{\Delta' x(j+1)+\Delta' x(j+2)\}^2 \end{vmatrix} / D$$

Designating, $\begin{vmatrix} \Delta' x(j+1) & \{\Delta' x(j+1)+\Delta' x(j+2)\} \\ \Delta' x^2(j+1) & \{\Delta' x(j+1)+\Delta' x(j+2)\}^2 \end{vmatrix} / D = D_{47}$

$$\begin{vmatrix} -\frac{\Delta x(j)}{2} & \{\Delta' x(j+1)+\Delta' x(j+2)\} \\ \frac{\Delta x^2(j)}{4} & \{\Delta' x(j+1)+\Delta' x(j+2)\}^2 \end{vmatrix} / D = D_{48}$$

$$\begin{vmatrix} -\frac{\Delta x(j)}{2} & \Delta' x(j+1) \\ \frac{\Delta x^2(j)}{4} & \Delta' x^2(j+1) \end{vmatrix} / D = D_{49}$$

hence $d = \{h(j-1)-h(j)\}D_{47} - \{h(j+1)-h(j)\}D_{48} + \{h(j+2)-h(j)\}D_{49}$

$$= D_{47}h(j-1) + (-D_{47} + D_{48} - D_{49})h(J) - D_{48}h(j+1) + D_{49}h(j+2) \qquad \text{...(4.12)}$$

$$\frac{dh}{dx} = b + 2cx + 3dx^2$$

$$\left.\frac{dh}{dx}\right|_{at x=-\Delta x(j)/2} = b - c\Delta x(j) + \frac{3}{4}d\Delta x^2(j)$$

$$= D_{41}h(j-1) + (-D_{41} + D_{42} - D_{43})h(j) - D_{42}h(j+1) + D_{43}h(j+2)$$

$$-\{ -D_{44}h(j-1) + (D_{44} - D_{45} + D_{46})h(j) + D_{45}h(j+1) - D_{46}h(j+2) \} \ \Delta x(j)$$

$$+\{ D_{47}h(j-1) + (-D_{47} + D_{48} - D_{49})h(J) - D_{48}h(j+1) + D_{49}h(j+2) \} \ \tfrac{3}{4}\Delta x^2(j)$$

$$\left.\frac{dh}{dx}\right|_{at x=-\Delta x(j)/2} = h(j-1)\left\{D_{41} + D_{44}\Delta x(j) + D_{47}\frac{3}{4}\Delta x^2(j)\right\}$$

$$+ h(j)\left\{-D_{41} + D_{42} - D_{43} - (D_{44} - D_{45} + D_{46})\Delta x(j) + (-D_{47} + D_{48} - D_{49})\frac{3}{4}\Delta x^2(j)\right\}$$

$$+h(j+1)\left\{-D_{42}-D_{45}\Delta x(j)-D_{48}\frac{3}{4}\Delta x^2(j)\right\}$$

$$+h(j+2)\left\{D_{43}+D_{46}\Delta x(j)+D_{49}\frac{3}{4}\Delta x^2(j)\right\}$$

$$\frac{dh}{dx}_{atx=\Delta x(j)/2}=b+c\Delta x(j)+\frac{3}{4}d\Delta x^2(j)$$

$$=D_{41}h(j-1)+(-D_{41}+D_{42}-D_{43})h(j)-D_{42}h(j+1)+D_{43}h(j+2)$$

$$+\{-D_{44}h(j-1)+(D_{44}-D_{45}+D_{46})h(j)+D_{45}h(j+1)-D_{46}h(j+2)\}\,\Delta x(j)$$

$$+\{D_{47}h(j-1)+(-D_{47}+D_{48}-D_{49})h(J)-D_{48}h(j+1)+D_{49}h(j+2)\}\,\tfrac{3}{4}\Delta x^2(j)$$

$$\frac{dh}{dx}_{atx=\Delta x(j)/2}=h(j-1)\left\{D_{41}-D_{44}\Delta x(j)+D_{47}\frac{3}{4}\Delta x^2(j)\right\}$$

$$+h(j)\left\{-D_{41}+D_{42}-D_{43}+(D_{44}-D_{45}+D_{46})\Delta x(j)+(-D_{47}+D_{48}-D_{49})\frac{3}{4}\Delta x^2(j)\right\}$$

$$+h(j+1)\left\{-D_{42}+D_{45}\Delta x(j)-D_{48}\frac{3}{4}\Delta x^2(j)\right\}$$

$$+h(j+2)\left\{D_{43}-D_{46}\Delta x(j)+D_{49}\frac{3}{4}\Delta x^2(j)\right\}$$

Taking into account the variation of flow during time steps (k-2), (k-1), & kth, the water balance equation is written as:

Inflow-outflow = change in storage

$$w1\left\{-T(j)\frac{dh}{dx}_{atx=-\Delta x(j)/2}+T_{HM2}(j)\frac{dh}{dx}_{atx=\Delta x(j)/2}\right\}+w2\left\{-T(j)\frac{dh}{dx}_{atx=-\Delta x(j)/2}+T_{HM2}(j)\frac{dh}{dx}_{atx=\Delta x(j)/2}\right\}$$

$$+w3\left\{-T(j)\frac{dh}{dx}_{atx=-\Delta x(j)/2}+T_{HM2}(j)\frac{dh}{dx}_{atx=\Delta x(j)/2}\right\}=S\frac{\Delta x(j)}{\Delta t}\{h(j,k)-h(j,k-1)\}$$

Equation is rearranged as:

$$-w3T(j)\Big[\,h(j-1,k)\left\{D_{41}+D_{44}\Delta x(j)+D_{47}\frac{3}{4}\Delta x^2(j)\right\}$$

$$+h(j,k)\left\{-D_{41}+D_{42}-D_{43}-(D_{44}-D_{45}+D_{46})\Delta x(j)+(-D_{47}+D_{48}-D_{49})\frac{3}{4}\Delta x^2(j)\right\}$$

$$+h(j+1,k)\left\{-D_{42}-D_{45}\Delta x(j)-D_{48}\frac{3}{4}\Delta x^2(j)\right\}+h(j+2,k)\left\{D_{43}+D_{46}\Delta x(j)+D_{49}\frac{3}{4}\Delta x^2(j)\right\}\Big]$$

$$+w3T_{HM2}(j)\ \Big[h(j-1,k)\Big\{D_{41}-D_{44}\Delta x(j)+D_{47}\frac{3}{4}\Delta x^2(j)\Big\}$$

$$+h(j,k)\Big\{-D_{41}+D_{42}-D_{43}+(D_{44}-D_{45}+D_{46})\Delta x(j)+(-D_{47}+D_{48}-D_{49})\frac{3}{4}\Delta x^2(j)\Big\}$$

$$+h(j+1,k)\Big\{-D_{42}+D_{45}\Delta x(j)-D_{48}\frac{3}{4}\Delta x^2(j)\Big\}+\ h(j+2,k)\Big\{D_{43}-D_{46}\Delta x(j)+D_{49}\frac{3}{4}\Delta x^2(j)\Big\}\Big]$$

$$+w1\Big\{-T(j)\frac{dh}{dx}_{at x=-\Delta x(j)/2}+T_{HM2}(j)\frac{dh}{dx}_{at x=\Delta x(j)/2}\Big\}+\ w2\Big\{-T(j)\frac{dh}{dx}_{at x=-\Delta x(j)/2}+T_{HM2}(j)\frac{dh}{dx}_{at x=\Delta x(j)/2}\Big\}$$

$$=S\frac{\Delta x(j)}{\Delta t}\{h(j,k)-h(j,k-1)\}$$

Designating,

$$F_1(j)=-w3T(j)\ \Big\{-D_{41}+D_{42}-D_{43}-(D_{44}-D_{45}+D_{46})\Delta x(j)+(-D_{47}+D_{48}-D_{49})\frac{3}{4}\Delta x^2(j)\Big\}$$

$$+w3T_{HM2}(j)\ \Big\{-D_{41}+D_{42}-D_{43}+(D_{44}-D_{45}+D_{46})\Delta x(j)+(-D_{47}+D_{48}-D_{49})\frac{3}{4}\Delta x^2(j)\Big\}-S\frac{\Delta x(j)}{\Delta t}$$

$$F_2(j)=-w3T(j)\ \Big\{-D_{42}-D_{45}\Delta x(j)-D_{48}\frac{3}{4}\Delta x^2(j)\Big\}+\ w3T_{HM2}(j)\ \Big\{-D_{42}+D_{45}\Delta x(j)-D_{48}\frac{3}{4}\Delta x^2(j)\Big\}$$

$$F_{22}(j)=-w3T(j)\ \Big\{D_{43}+D_{46}\Delta x(j)+D_{49}\frac{3}{4}\Delta x^2(j)\Big\}\ +w3T_{HM2}(j)\ \Big\{D_{43}-D_{46}\Delta x(j)+D_{49}\frac{3}{4}\Delta x^2(j)\Big\}$$

$$F_6(j,k)=S\frac{\Delta x(j)}{\Delta t}h(j,k-1)-T(j)w3\ \Big\{D_{41}+D_{44}\Delta x(j)+D_{47}\frac{3}{4}\Delta x^2(j)\Big\}$$

$$+w3T_{HM2}(j)\Big\{D_{41}-D_{44}\Delta x(j)+D_{47}\frac{3}{4}\Delta x^2(j)\Big\}$$

$$+w1\Big\{-T(j)\frac{dh}{dx}_{at x=-\Delta x(j)/2}+T_{HM2}(j)\frac{dh}{dx}_{at x=\Delta x(j)/2}\Big\}$$

$$+w2\Big\{-T(j)\frac{dh}{dx}_{at x=-\Delta x(j)/2}+T_{HM2}(j)\frac{dh}{dx}_{at x=\Delta x(j)/2}\Big\}$$

Finally the equation becomes:

$$F_1(j)h(j,k)+F_2(j)h(j+1,k)+F_{22}(j)h(j+2,k)+F_6(j,k)=0$$

4.3.3 Equation For The Interior Node i.e. $j = j_{max}-1$

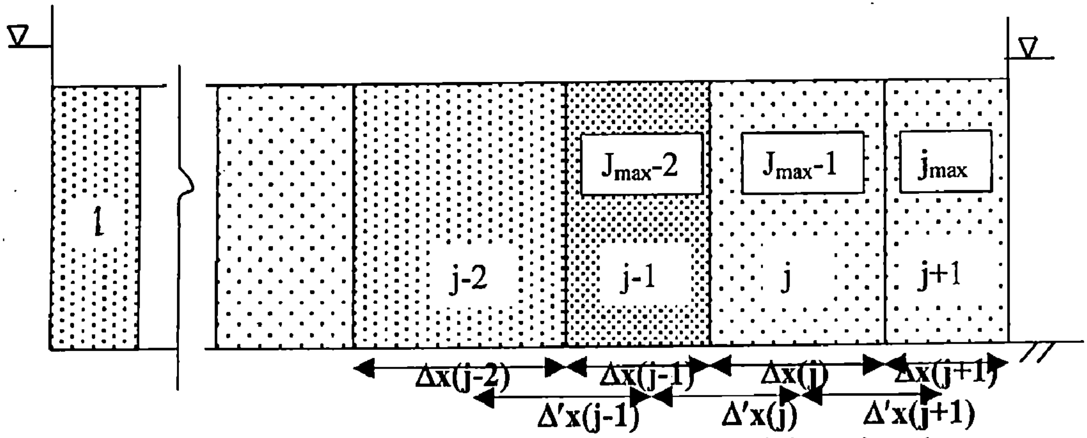

Fig.4.7: Discretisation of flow domain origin at $j_{max}-1$

Let the head in the region $(j_{max}-3) \le x \le j_{max}$ be expressed as:

$h(x)= a +bx+cx^2+dx^3$

where constants a, b, c, d are derived in terms of h(j+1), h(j), h(j-1),h(j-2).

Designating, $\Delta'x(j-1) = \{\Delta x(j-2)+\Delta x(j-1)\}/2$

$\Delta'x(j) = \{\Delta x(j-1)+\Delta x(j)\}/2$

$\Delta'x(j+1) = \{\Delta x(j)+\Delta x(j+1)\}/2$

Let nodej be the origin.

Hence at x =0, $h(0) = h(j) = a$

At $x=- \{\Delta'x(j)+ \Delta'x(j-1)\}$, $h\{-\Delta'x(j)- \Delta'x(j-1)\} = h(j-2)$

$$h(j-2)-h(j)= - b\{\Delta'x(j)+ \Delta'x(j-1)\}+ c\{\Delta'x(j)+ \Delta'x(j-1)\}^2-d\{\Delta'x(j)+ \Delta'x(j-1)\}^3 \quad ...(4.13)$$

At $x= -\Delta'x(j)$, $h\{-\Delta'x(j)\} = h(j-1)$

$$h(j-1)-h(j)= -b\Delta'x(j)+ c\Delta'x^2(j)- d\Delta'x^3(j) \quad ...(4.14)$$

At $x= \Delta'x(j+1)$, $h\{\Delta'x(j+1)\} = h(j+1)$

$$h(j+1)-h(j)= b\Delta'x(j+1)+ c\Delta'x^2(j+1)+ d\Delta'x^3(j+1) \quad ...(4.15)$$

Above equations are solved for getting b, c, d:

Designating, D= $\begin{vmatrix} -\{\Delta'x(j)+\Delta'x(j-1)\} & \{\Delta'x(j)+\Delta'x(j-1)\}^2 & -\{\Delta'x(j)+\Delta'x(j-1)\}^3 \\ -\Delta'x(j) & \Delta'x^2(j) & -\Delta'x^3(j) \\ \Delta'x(j+1) & \Delta'x^2(j+1) & \Delta'x^3(j+1) \end{vmatrix}$

$\therefore$ b= $\begin{vmatrix} h(J-2)-h(J) & \{\Delta'x(j)+\Delta'x(j-1)\}^2 & -\{\Delta'x(j)+\Delta'x(j-1)\}^3 \\ h(j-1)-h(j) & \Delta'x^2(j) & -\Delta'x^3(j) \\ h(j+1)-h(j) & \Delta'x^2(j+1) & \Delta'x^3(j+1) \end{vmatrix} / D$

Or b = $\begin{vmatrix} h(j-2)-h(j) & h(j-1)-h(j) & h(j+1)-h(j) \\ \{\Delta'x(j)+\Delta'x(j-1)\}^2 & \Delta'x^2(j) & \Delta'x^2(j+1) \\ -\{\Delta'x(j)+\Delta'x(j-1)\}^3 & -\Delta'x^3(j) & \Delta'x^3(j+1) \end{vmatrix} / D$

Designating, $\begin{vmatrix} \Delta'x^2(j) & \Delta'x^2(j+1) \\ -\Delta'x^3(j) & \Delta'x^3(j+1) \end{vmatrix} / D = D_{31}$

$\begin{vmatrix} \{\Delta'x(j)+\Delta'x(j-1)\}^2 & \Delta'x^2(j+1) \\ -\{\Delta'x(j)+\Delta'x(j-1)\}^3 & \Delta'x^3(j+1) \end{vmatrix} / D = D_{32}$

$\begin{vmatrix} \{\Delta'x(j)+\Delta'x(j-1)\}^2 & \Delta'x^2(j) \\ -\{\Delta'x(j)+\Delta'x(j-1)\}^3 & -\Delta'x^3(j) \end{vmatrix} /D = D_{33}$

Hence $b = \{h(j-2)-h(j)\}D_{31} - \{h(j-1)-h(j)\}D_{32} + \{h(j+1)-h(j)\}D_{33}$

$= D_{31}h(j-2) - D_{32}h(j-1) + (-D_{31}+D_{32}-D_{33})h(j) + D_{33}h(j+1)$ ...(4.16)

$\therefore$ c= $\begin{vmatrix} -\{\Delta'x(j)+\Delta'x(j-1)\} & h(j-2)-h(j) & -\{\Delta'x(j)+\Delta'x(j-1)\}^3 \\ -\Delta'x(j) & h(j-1)-h(j) & -\Delta'x^3(j) \\ \Delta'x(j+1) & h(j+1)-h(j) & \Delta'x^3(j+1) \end{vmatrix} / D$

Or $c = -\begin{vmatrix} h(j-2)-h(j) & h(j-1)-h(j) & h(j+1)-h(j) \\ -\{\Delta'x(j)+\Delta'x(j-1)\} & -\Delta'x(j) & \Delta'x(j+1) \\ -\{\Delta'x(j)+\Delta'x(j-1)\}^3 & -\Delta'x^3(j) & \Delta'x^3(j+1) \end{vmatrix} /D$

Designating, $\begin{vmatrix} -\Delta'x(j) & \Delta'x(j+1) \\ -\Delta'x^3(j) & \Delta'x^3(j+1) \end{vmatrix} /D = D_{34}$

$\begin{vmatrix} -\{\Delta'x(j)+\Delta'x(j-1)\} & \Delta'x(j+1) \\ -\{\Delta'x(j)+\Delta'x(j-1)\}^3 & \Delta'x^3(j+1) \end{vmatrix} /D = D_{35}$

$$\begin{vmatrix} -\{\Delta'x(j)+\Delta'x(j-1)\} & -\Delta'x(j) \\ -\{\Delta'x(j)+\Delta'x(j-1)\}^3 & -\Delta'x^3(j) \end{vmatrix} / \mathrm{D} = \mathrm{D}_{36}$$

$$c = -\{h(j-2)-h(j)\}D_{34} + \{h(j-1)-h(j)\}D_{35} - \{h(j+1)-h(j)\}D_{36}$$

$$= -D_{34}h(j-2) + D_{35}h(j-1) + (D_{34} - D_{35} + D_{36})h(j) - D_{36}h(j+1) \qquad \ldots(4.17)$$

$$\text{Now } d = \begin{vmatrix} -\{\Delta'x(j)+\Delta'x(j-1)\} & \{\Delta'x(j)+\Delta'x(j-1)\}^2 & h(j-2)-h(j) \\ -\Delta'x(j) & \Delta'x^2(j) & h(j-1)-h(j) \\ \Delta'x(j+1) & \Delta'x^2(j+1) & h(j+1)-h(j) \end{vmatrix} / \mathrm{D}$$

$$\text{Or d} = \begin{vmatrix} h(j-2)-h(j) & h(j-1)-h(j) & h(j+1)-h(j) \\ -\{\Delta'x(j)+\Delta'x(j-1)\} & -\Delta'x(j) & \Delta'x(j+1) \\ \{\Delta'x(j)+\Delta'x(j-1)\}^2 & \Delta'x^2(j) & \Delta'x^2(j+1) \end{vmatrix} / \mathrm{D}$$

$$\text{Designating, } \begin{vmatrix} -\Delta'x(j) & \Delta'x(j+1) \\ \Delta'x^2(j) & \Delta'x^2(j+1) \end{vmatrix} / \mathrm{D} = \mathrm{D}_{37}$$

$$\begin{vmatrix} -\{\Delta'x(j)+\Delta'x(j-1)\} & \Delta'x(j+1) \\ \{\Delta'x(j)+\Delta'x(j-1)\}^2 & \Delta'x^2(j+1) \end{vmatrix} / \mathrm{D} = \mathrm{D}_{38}$$

$$\begin{vmatrix} -\{\Delta'x(j)+\Delta'x(j-1)\} & -\Delta'x(j) \\ \{\Delta'x(j)+\Delta'x(j-1)\}^2 & \Delta'x^2(j) \end{vmatrix} / \mathrm{D} = \mathrm{D}_{39}$$

$$d = \{h(j-2)-h(j)\}D_{37} - \{h(j-1)-h(j)\}D_{38} + \{h(j+1)-h(j)\}D_{39}$$

$$= D_{37}h(j-2) - D_{38}h(j-1) + (-D_{37} + D_{38} - D_{39})h(j) + D_{39}h(j+1) \qquad \ldots(4.18)$$

$$\frac{dh}{dx}_{at x=-\Delta x(j)/2} = b - c\Delta x(j) + d\frac{3}{4}\Delta x^2(j)$$

$$= D_{31}h(j-2) - D_{32}h(j-1) + (-D_{31} + D_{32} - D_{33})h(j) + D_{33}h(j+1)$$

$$-\{-D_{34}h(j-2) + D_{35}h(j-1) + (D_{34} - D_{35} + D_{36})h(j) - D_{36}h(j+1)\}\Delta x(j)$$

$$+\{D_{37}h(j-2) - D_{38}h(j-1) + (-D_{37} + D_{38} - D_{39})h(j) + D_{39}h(j+1)\}\frac{3}{4}\Delta x^2(j)$$

$$= h(j-2)\left\{D_{31} + D_{34}\Delta x(j) + D_{37}\frac{3}{4}\Delta x^2(j)\right\} + h(j-1)\left\{-D_{32} - D_{35}\Delta x(j) - D_{38}\frac{3}{4}\Delta x^2(j)\right\}$$

$$+ h(j)\left\{-D_{31} + D_{32} - D_{33} - (D_{34} - D_{35} + D_{36})\Delta x(j) + (-D_{37} + D_{38} - D_{39})\frac{3}{4}\Delta x^2(j)\right\}$$

$$+ h(j+1)\left\{D_{33} + D_{36}\Delta x(j) + D_{39}\frac{3}{4}\Delta x^2(j)\right\}$$

$$\frac{dh}{dx}_{at x=\Delta x(j)/2} = b + c\Delta x(j) + d\frac{3}{4}\Delta x^2(j)$$

$$= D_{31}h(j-2) - D_{32}h(j-1) + (-D_{31} + D_{32} - D_{33})h(j) + D_{33}h(j+1)$$

$$+\{-D_{34}h(j-2) + D_{35}h(j-1) + (D_{34} - D_{35} + D_{36})h(j) - D_{36}h(j+1)\}\Delta x(j)$$

$$+\{D_{37}h(j-2)-D_{38}h(j-1)+(-D_{37}+D_{38}-D_{39})h(j)+D_{39}h(j+1)\}\frac{3}{4}\Delta x^2(j)$$

$$=h(j-2)\left\{D_{31}-D_{34}\Delta x(j)+D_{37}\frac{3}{4}\Delta x^2(j)\right\}+h(j-1)\left\{-D_{32}+D_{35}\Delta x(j)-D_{38}\frac{3}{4}\Delta x^2(j)\right\}$$

$$+h(j)\left\{-D_{31}+D_{32}-D_{33}+(D_{34}-D_{35}+D_{36})\Delta x(j)+(-D_{37}+D_{38}-D_{39})\frac{3}{4}\Delta x^2(j)\right\}$$

$$+h(j+1)\left\{D_{33}-D_{36}\Delta x(j)+D_{39}\frac{3}{4}\Delta x^2(j)\right\}$$

Taking into account of the variation in flow during time steps (k-2), (k-1) & k[th], the water balance equation is written as:

$$w1\left\{-T_{HM4}(j)\frac{dh}{dx}_{atx=-\Delta x(j)/2}+T_{HM2}(j)\frac{dh}{dx}_{atx=\Delta x(j)/2}\right\}$$

$$+w2\left\{-T_{HM4}(j)\frac{dh}{dx}_{atx=-\Delta x(j)/2}+T_{HM2}(j)\frac{dh}{dx}_{atx=\Delta x(j)/2}\right\}$$

$$+w3\left\{-T_{HM4}(j)\frac{dh}{dx}_{atx=-\Delta x(j)/2}+T_{HM2}(j)\frac{dh}{dx}_{atx=\Delta x(j)/2}\right\}=S\frac{\Delta x(j)}{\Delta t}\{h(j,k)-h(j,k-1)\}$$

Now rearranging the above equation becomes :

$$-w3T_{HM4}(j)\,[h(j-2)\left\{D_{31}+D_{34}\Delta x(j)+D_{37}\frac{3}{4}\Delta x^2(j)\right\}+h(j-1)\left\{-D_{32}-D_{35}\Delta x(j)-D_{38}\frac{3}{4}\Delta x^2(j)\right\}$$

$$+h(j)\left\{-D_{31}+D_{32}-D_{33}-(D_{34}-D_{35}+D_{36})\Delta x(j)+(-D_{37}+D_{38}-D_{39})\frac{3}{4}\Delta x^2(j)\right\}$$

$$+h(j+1)\left\{D_{33}+D_{36}\Delta x(j)+D_{39}\frac{3}{4}\Delta x^2(j)\right\}]$$

$$+w3T_{HM2}(j)\,[h(j-2)\left\{D_{31}-D_{34}\Delta x(j)+D_{37}\frac{3}{4}\Delta x^2(j)\right\}+h(j-1)\left\{-D_{32}+D_{35}\Delta x(j)-D_{38}\frac{3}{4}\Delta x^2(j)\right\}$$

$$+h(j)\left\{-D_{31}+D_{32}-D_{33}+(D_{34}-D_{35}+D_{36})\Delta x(j)+(-D_{37}+D_{38}-D_{39})\frac{3}{4}\Delta x^2(j)\right\}$$

$$+h(j+1)\left\{D_{33}-D_{36}\Delta x(j)+D_{39}\frac{3}{4}\Delta x^2(j)\right\}]+w1\left\{-T_{HM4}(j)\frac{dh}{dx}_{atx=-\Delta x(j)/2}+T_{HM2}(j)\frac{dh}{dx}_{atx=\Delta x(j)/2}\right\}$$

$$+w2\left\{-T_{HM4}(j)\frac{dh}{dx}_{atx=-\Delta x(j)/2}+T_{HM2}(j)\frac{dh}{dx}_{atx=\Delta x(j)/2}\right\}=S\frac{\Delta x(j)}{\Delta t}\{h(j,k)-h(j,k-1)\}$$

Designating, $F_{44}(j)=-w3T_{HM4}(j)\left\{D_{31}+D_{34}\Delta x(j)+D_{37}\frac{3}{4}\Delta x^2(j)\right\}$

$$+w3T_{HM2}(j)\left\{D_{31}-D_{34}\Delta x(j)+D_{37}\frac{3}{4}\Delta x^2(j)\right\}$$

$$F_4(j)=-w3T_{HM4}(j)\left\{-D_{32}-D_{35}\Delta x(j)-D_{38}\frac{3}{4}\Delta x^2(j)\right\}$$

$$+w3T_{HM2}(j)\left\{-D_{32}+D_{35}\Delta x(j)-D_{38}\frac{3}{4}\Delta x^2(j)\right\}$$

$$F_1(j)=-w3T_{HM4}(j)\left\{-D_{31}+D_{32}-D_{33}-(D_{34}-D_{35}+D_{36})\Delta x(j)+(-D_{37}+D_{38}-D_{39})\frac{3}{4}\Delta x^2(j)\right\}$$

$$+w3T_{HM2}(j)\left\{-D_{31}+D_{32}-D_{33}+(D_{34}-D_{35}+D_{36})\Delta x(j)+(-D_{37}+D_{38}-D_{39})\frac{3}{4}\Delta x^2(j)\right\}-S\frac{\Delta x(j)}{\Delta t}$$

$$F_2(j)=-w3T_{HM4}(j)\left\{D_{33}+D_{36}\Delta x(j)+D_{39}\frac{3}{4}\Delta x^2(j)\right\}+w3T_{HM2}(j)\left\{D_{33}-D_{36}\Delta x(j)+D_{39}\frac{3}{4}\Delta x^2(j)\right\}$$

$$F_6(j,k)=S\frac{\Delta x(j)}{\Delta t}h(j,k-1)+w1\left\{-T_{HM4}(j)\frac{dh}{dx}_{at x=-\Delta x(j)/2}+T_{HM2}(j)\frac{dh}{dx}_{at x=\Delta x(j)/2}\right\}$$

$$+w2\left\{-T_{HM4}(j)\frac{dh}{dx}_{at x=-\Delta x(j)/2}+T_{HM2}(j)\frac{dh}{dx}_{at x=\Delta x(j)/2}\right\}$$

Finally the equation can be written as:

$$F_{44}(j)h(j-2,k)+F_4(j)h(j-1,k)+F_1(j)h(j,k)+F_2(j)h(j+1.k)+F_6(j,k)=0$$

4.3.4 Equation for Boundary Node (right) i.e. $j = j_{max}$

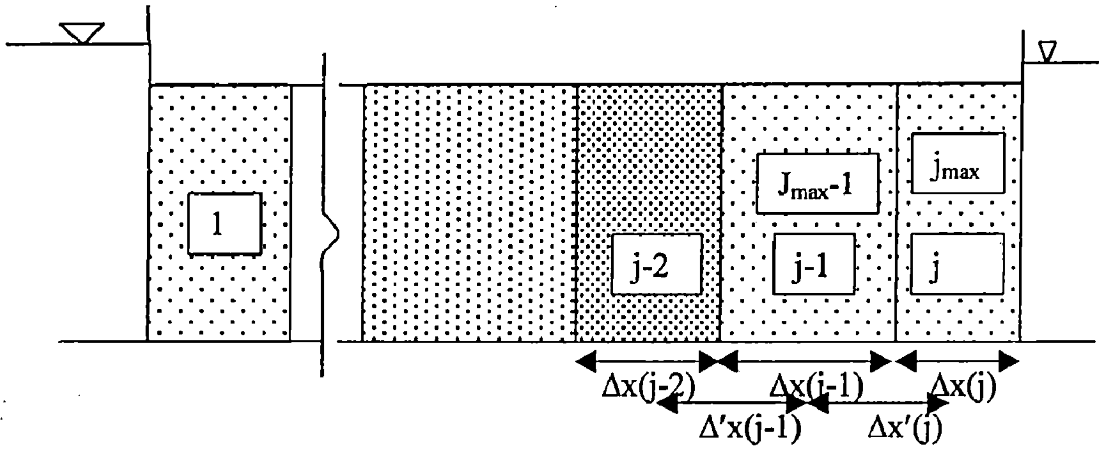

Fig.4.8 Discretisation of flow domain origin at node j_{max}

Selecting the origin at jth node. Let the head h(x)at $j_{max}-2 \le x \le \Delta x(j)/2$, be expressed as:

$h(x)=a + bx + cx^2 + dx^3$, where constants a, b, c, d are derived in terms of h(j-2), h(j-1), h(j) & h(j+1).

Designating, $\Delta x'(j)= \{\Delta x(j-1)+\Delta x(j)\}/2$

$\Delta x'(j-1)= \{\Delta x(j-2)+\Delta x(j-1)\}/2$

Now at $x=0$, $h(x)=h(j)=a$;

At $x=-\{\Delta' x(j)+\Delta' x(j-1)\}$, $h\{-\Delta' x(j)-\Delta' x(j-1)\}=h(j-2)$

$$h(j-2)-h(j)=-b\{\Delta' x(j)+\Delta' x(j-1)\}+c\{\Delta' x(j)+\Delta' x(j-1)\}^2-d\{\Delta' x(j)+\Delta' x(j-1)\}^3 \quad \ldots(4.19)$$

$x = -\Delta' x(j), \qquad h\{-\Delta' x(j)\} = h(j-1)$

$$h(j-1) - h(j) = -b\Delta' x(j) + c\Delta' x^2(j) - d\Delta' x^3(j) \qquad \ldots(4.20)$$

$x = \dfrac{\Delta x(j)}{2}, \qquad h\{\Delta x(j)/2\} = h(j+1)$

$$h(j+1) - h(j) = b\frac{\Delta x(j)}{2} + c\frac{\Delta x^2(j)}{4} + d\frac{\Delta x^3(j)}{8} \qquad \ldots(4.21)$$

Above equations are solved for getting b, c, d:

Designating, $\begin{vmatrix} -\{\Delta' x(j) + \Delta' x(j-1)\} & \{\Delta' x(j) + \Delta' x(j-1)\}^2 & -\{\Delta' x(j) + \Delta' x(j-1)\}^3 \\ -\Delta' x(j) & \Delta' x^2(j) & -\Delta' x^3(j) \\ \dfrac{\Delta x(j)}{2} & \dfrac{\Delta x^2(j)}{4} & \dfrac{\Delta x^3(j)}{8} \end{vmatrix} = D$

$\therefore$ b= $\begin{vmatrix} h(j-2) - h(j) & \{\Delta' x(j) + \Delta' x(j-1)\}^2 & -\{\Delta' x(j) + \Delta' x(j-1)\}^3 \\ h(j-1) - h(j) & \Delta' x^2(j) & -\Delta' x^3(j) \\ h(j+1) - h(j) & \dfrac{\Delta x^2(j)}{4} & \dfrac{\Delta x^3(j)}{8} \end{vmatrix} / D$

Or b = $\begin{vmatrix} h(j-2) - h(j) & h(j-1) - h(j) & h(j+1) - h(j) \\ \{\Delta' x(j) + \Delta' x(j-1)\}^2 & \Delta' x^2(j) & \dfrac{\Delta x^2(j)}{4} \\ -\{\Delta' x(j) + \Delta' x(j-1)\}^3 & -\Delta' x^3(j) & \dfrac{\Delta x^3(j)}{8} \end{vmatrix} / D$

Designating, $\begin{vmatrix} \Delta' x^2(j) & \dfrac{\Delta x^2(j)}{4} \\ -\Delta' x^3(j) & \dfrac{\Delta x^3(j)}{8} \end{vmatrix} / D = D_{51}$

$\begin{vmatrix} \{\Delta' x(j) + \Delta' x(j-1)\}^2 & \dfrac{\Delta x^2(j)}{4} \\ -\{\Delta' x(j) + \Delta' x(j-1)\}^3 & \dfrac{\Delta x^3(j)}{8} \end{vmatrix} / D = D_{52}$

$\begin{vmatrix} \{\Delta' x(j) + \Delta' x(j-1)\}^2 & \Delta' x^2(j) \\ -\{\Delta' x(j) + \Delta' x(j-1)\}^3 & -\Delta' x^3(j) \end{vmatrix} / D = D_{53}$

Hence $b = \{h(j-2) - h(j)\}D_{51} - \{h(j-1) - h(j)\}D_{52} - \{h(j+1) - h(j)\}D_{53}$

$$= D_{51}h(j-2) - D_{52}h(j-1) + (-D_{51} + D_{52} - D_{53})h(j) + D_{53}h(j+1) \qquad \ldots(4.22)$$

$$\text{Now } c = \begin{vmatrix} -\{\Delta' x(j) + \Delta' x(j-1)\} & h(j-2) - h(j) & -\{\Delta' x(j) + \Delta' x(j-1)\}^3 \\ -\Delta' x(j) & h(j-1) - h(j) & -\Delta' x^3(j) \\ \dfrac{\Delta x(j)}{2} & h(j+1) - h(j) & \dfrac{\Delta x^3(j)}{8} \end{vmatrix} / D$$

$$\text{Or c } = -\begin{vmatrix} h(j-2) - h(j) & h(j-1) - h(j) & h(j+1) - h(J) \\ -\{\Delta' x(j) + \Delta' x(j-1)\} & -\Delta' x(j) & \dfrac{\Delta x(j)}{2} \\ -\{\Delta' x(j) + \Delta' x(j-1)\}^3 & -\Delta' x^3(j) & \dfrac{\Delta x^3(j)}{8} \end{vmatrix} / D$$

$$\text{Designating, } \begin{vmatrix} -\Delta' x(j) & \dfrac{\Delta x(j)}{2} \\ -\Delta' x^3(j) & \dfrac{\Delta x^3(j)}{8} \end{vmatrix} / D = D_{54}$$

$$\begin{vmatrix} -\{\Delta' x(j) + \Delta' x(j-1)\} & \dfrac{\Delta x(j)}{2} \\ -\{\Delta' x(j) + \Delta' x(j-1)\}^3 & \dfrac{\Delta x^3(j)}{8} \end{vmatrix} / D = D_{55}$$

$$\begin{vmatrix} -\{\Delta' x(j) + \Delta' x(j-1)\} & -\Delta' x(j) \\ -\{\Delta' x(j) + \Delta' x(j-1)\}^3 & -\Delta' x^3(j) \end{vmatrix} / D = D_{56}$$

Hence $c = -\{h(j-2) - h(j)\}D_{54} + \{h(j-1) - h(j)\}D_{55} - \{h(j+1) - h(j)\}D_{56}$

$$= -D_{54}h(j-2) + D_{55}h(j-1) + (D_{54} - D_{55} + D_{56})h(j) - D_{56}h(j+1) \qquad \ldots(4.23)$$

$$\therefore d = \begin{vmatrix} -\{\Delta' x(j) + \Delta' x(j-1)\} & \{\Delta' x(j) + \Delta' x(j-1)\}^2 & h(j-2) - h(j) \\ -\Delta' x(j) & \Delta' x^2(j) & h(j-1) - h(j) \\ \dfrac{\Delta x(j)}{2} & \dfrac{\Delta x^2(j)}{4} & h(j+1) - h(j) \end{vmatrix} / D$$

$$\text{Or d } = \begin{vmatrix} h(j-2) - h(j) & h(j-1) - h(j) & h(j+1) - h(j) \\ -\{\Delta' x(j) + \Delta' x(j-1)\} & -\Delta' x(j) & \dfrac{\Delta x(j)}{2} \\ \{\Delta' x(j) + \Delta' x(j-1)\}^2 & \Delta' x^2(j) & \dfrac{\Delta x^2(j)}{4} \end{vmatrix} / D$$

$$\begin{vmatrix} -\Delta' x(j) & \dfrac{\Delta x(j)}{2} \\ \Delta' x^2(j) & \dfrac{\Delta x^2(j)}{4} \end{vmatrix} / D = D_{57}$$

$$\begin{vmatrix} -\{\Delta' x(j) + \Delta' x(j-1)\} & \dfrac{\Delta x(j)}{2} \\ \{\Delta' x(j) + \Delta' x(j-1)\}^2 & \dfrac{\Delta x^2(j)}{4} \end{vmatrix} / D = D_{58}$$

$$\begin{vmatrix} -\{\Delta' x(j)+\Delta' x(j-1)\} & -\Delta' x(j) \\ \{\Delta' x(j)+\Delta' x(j-1)\}^2 & \Delta' x^2(j) \end{vmatrix} \; / \, D = D_{59}$$

$$d = \{h(j-2)-h(j)\}D_{57} - \{h(j-1)-h(j)\}D_{58} + \{h(j+1)-h(j)\}D_{59}$$

$$= D_{57}h(j-2) - D_{58}h(j-1) + (-D_{57}+D_{58}-D_{59})h(j) + D_{59}h(j+1) \qquad \ldots(4.24)$$

The gradient, $\dfrac{dh}{dx}_{at x=-\Delta x(j)/2} = b - c\Delta x(j) + d\dfrac{3}{4}\Delta x^2(j)$

$$= D_{51}h(j-2) - D_{52}h(j-1) + (-D_{51}+D_{52}-D_{53})h(j) + D_{53}h(j+1)$$

$$-[-D_{54}h(j-2) + D_{55}h(j-1) + (D_{54}-D_{55}+D_{56})h(j) - D_{56}h(j+1)]\Delta x(j)$$

$$+[D_{57}h(j-2) - D_{58}h(j-1) + (-D_{57}+D_{58}-D_{59})h(j) + D_{59}h(j+1)]\frac{3}{4}\Delta x^2(j)$$

$$= h(j-2)\left\{D_{51} + D_{54}\Delta x(j) + D_{57}\frac{3}{4}\Delta x^2(j)\right\} + h(j-1)\left\{-D_{52} - D_{55}\Delta x(j) - D_{58}\frac{3}{4}\Delta x^2(j)\right\}$$

$$+ h(j)\left\{-D_{51} + D_{52} - D_{53} - (D_{54}-D_{55}+D_{56})\Delta x(j) + (-D_{57}+D_{58}-D_{59})\frac{3}{4}\Delta x^2(j)\right\}$$

$$+ h(j+1)\left\{D_{53} + D_{56}\Delta x(j) + D_{59}\frac{3}{4}\Delta x^2(j)\right\}$$

$$\frac{dh}{dx}_{at x=\Delta x(j)/2} = b + c\Delta x(j) + d\frac{3}{4}\Delta x^2(j)$$

$$= D_{51}h(j-2) - D_{52}h(j-1) + (-D_{51}+D_{52}-D_{53})h(j) + D_{53}h(j+1)$$

$$+[-D_{54}h(j-2) + D_{55}h(j-1) + (D_{54}-D_{55}+D_{56})h(j) - D_{56}h(j+1)]\Delta x(j)$$

$$+[D_{57}h(j-2) - D_{58}h(j-1) + (-D_{57}+D_{58}-D_{59})h(j) + D_{59}h(j+1)]\frac{3}{4}\Delta x^2(j)$$

This can be rearranged as:

$$\frac{dh}{dx}_{at x=\Delta x(j)/2} = h(j-2)\left\{D_{51} - D_{54}\Delta x(j) + D_{57}\frac{3}{4}\Delta x^2(j)\right\} + h(j-1)\left\{-D_{52} + D_{55}\Delta x(j) - D_{58}\frac{3}{4}\Delta x^2(j)\right\}$$

$$+ h(j)\left\{-D_{51} + D_{52} - D_{53} + (D_{54}-D_{55}+D_{56})\Delta x(j) + (-D_{57}+D_{58}-D_{59})\frac{3}{4}\Delta x^2(j)\right\}$$

$$+ h(j+1)\left\{D_{53} - D_{56}\Delta x(j) + D_{59}\frac{3}{4}\Delta x^2(j)\right\}$$

Taking into account of the variation of flow during time steps (k-2), (k-1), kth time steps, the water balance equation is written as:

$$w1\left\{-T_{HM4}(j)\frac{dh}{dx}_{at x=-\Delta x(j)/2}\right\} + \left\{T(j)\frac{dh}{dx}_{at x=\Delta x(j)/2}\right\}$$

$$+ w2\left\{-T_{HM4}(j)\frac{dh}{dx}_{at x=-\Delta x(j)/2}\right\} + \left\{T(j)\frac{dh}{dx}_{at x=\Delta x(j)/2}\right\}$$

$$+ w3\left\{-T_{HM4}(j)\frac{dh}{dx}_{at x=-\Delta x(j)/2}\right\} + \left\{T(j)\frac{dh}{dx}_{at x=\Delta x(j)/2}\right\} = S\frac{\Delta x(j)}{\Delta t} h(j,k) - h(j,k-1)$$

Above equation is rearranged as:

$$-w3T_{HM4}(j)\left[h(j-2)\left\{D_{51}+D_{54}\Delta x(j)+D_{57}\frac{3}{4}\Delta x^2(j)\right\}+h(j-1)\left\{-D_{52}-D_{55}\Delta x(j)-D_{58}\frac{3}{4}\Delta x^2(j)\right\}\right.$$

$$+h(j)\left\{-D_{51}+D_{52}-D_{53}-(D_{54}-D_{55}+D_{56})\Delta x(j)+(-D_{57}+D_{58}-D_{59})\frac{3}{4}\Delta x^2(j)\right\}$$

$$\left.+h(j+1)\left\{D_{53}+D_{56}\Delta x(j)+D_{59}\frac{3}{4}\Delta x^2(j)\right\}\right]+w3T(j)\left[h(j-2)\left\{D_{51}-D_{54}\Delta x(j)+D_{57}\frac{3}{4}\Delta x^2(j)\right\}\right.$$

$$+h(j-1)\left\{-D_{52}+D_{55}\Delta x(j)-D_{58}\frac{3}{4}\Delta x^2(j)\right\}$$

$$+h(j)\left\{-D_{51}+D_{52}-D_{53}+(D_{54}-D_{55}+D_{56})\Delta x(j)+(-D_{57}+D_{58}-D_{59})\frac{3}{4}\Delta x^2(j)\right\}$$

$$\left.+h(j+1)\left\{D_{53}-D_{56}\Delta x(j)+D_{59}\frac{3}{4}\Delta x^2(j)\right\}\right]+w1\left\{-T_{HM4}(j)\frac{dh}{dx}_{atx=-\Delta x(j)/2}\right\}+w1\left\{T(j)\frac{dh}{dx}_{atx=\Delta x(j)/2}\right\}$$

$$+w2\left\{-T_{HM4}(j)\frac{dh}{dx}_{atx=-\Delta x(j)/2}\right\}+w2\left\{T(j)\frac{dh}{dx}_{atx=\Delta x(j)/2}\right\}=S\frac{\Delta x(j)}{\Delta t}h(j,k)-h(j,k-1)$$

Designating,

$$F_{44}(j)=-w3T_{HM4}(j)\left\{D_{51}+D_{54}\Delta x(j)+D_{57}\frac{3}{4}\Delta x(j)\right\}+w3T(j)\left\{D_{51}-D_{54}\Delta x(j)+D_{57}\frac{3}{4}\Delta x(j)\right\}$$

$$F_4(j)=-w3T_{HM4}(j)\left\{-D_{52}-D_{55}\Delta x(j)-D_{58}\frac{3}{4}\Delta x(j)\right\}+w3T(j)\left\{-D_{52}+D_{55}\Delta x(j)-D_{58}\frac{3}{4}\Delta x(j)\right\}$$

$$F_1(j)=-w3T_{HM4}(j)\left\{-D_{51}+D_{52}-D_{53}-(D_{54}-D_{55}+D_{56})\Delta x(j)+(-D_{57}+D_{58}-D_{59})\frac{3}{4}\Delta x^2(j)\right\}$$

$$+w3T(j)\left\{-D_{51}+D_{52}-D_{53}+(D_{54}-D_{55}+D_{56})\Delta x(j)+(-D_{57}+D_{58}-D_{59})\frac{3}{4}\Delta x^2(j)\right\}-\frac{\Delta x(j)}{\Delta t}S$$

$$F_6(j,k)=S\frac{\Delta x(j)}{\Delta t}h(j,k-1)-w3T_{HM4}(j)\left\{D_{53}+D_{56}\Delta x(j)+D_{59}\frac{3}{4}\Delta x^2(j)\right\}h(j+1,k)$$

$$+w_3T(j)\left\{D_{53}-D_{56}\Delta x(j)+D_{59}\frac{3}{4}\Delta x^2(j)\right\}h(j+1)$$

$$+w1\left\{-T_{HM4}(j)\frac{dh}{dx}_{atx=-\Delta x(j)/2}\right\}+w1\left\{T(j)\frac{dh}{dx}_{atx=\Delta x(j)/2}\right\}$$

$$+w2\left\{-T_{HM4}(j)\frac{dh}{dx}_{atx=-\Delta x(j)/2}\right\}+w2\left\{T(j)\frac{dh}{dx}_{atx=\Delta x(j)/2}\right\}$$

Finally the equation becomes:

$$F_{44}(j)h(j-2,k)+F_4(j)h(j-1,k)+F_1(j)h(j,k)+F_6(j,k)=0$$

CHAPTER-5
RESULT AND DISCUSSION

In Appendix-2, a computer programme has been written which calculates heads at nodes using the improved finite difference scheme. In order to compare the improved finite difference scheme with the existing finite difference schemes, numerical results are obtained for the following set of data:

$T = 100m^2/day$, $S = 0.02$, initial head = 10m, duration of simulation = 10days,

$\Delta x = 10m, 50m, 100m$, $\Delta t = 0.01, 0.1, 1$

Two types of variation in boundary head is assumed:

i) A step rise in stream stage at $x = 0$;

ii) A ramp rise in stream stage at $x = 0$;

There is no rise in stream stage at $x = L$.

For small time step i.e. $\Delta t = 0.01$, the boundary condition at $x = L$ will not be effective and the aquifer will behave like a semi- infinite aquifer bounded by the stream at $x = 0$, for which the analytical solution is given by Carslaw and Jaegar for an analogous heat conduction problem. The numerical results obtained by the finite difference scheme are compared with the analytical solution.

The hydraulic heads computed by improved finite difference scheme, completely implicit scheme and Crank Nicholson Scheme for $\Delta t = 0.1, 0.01$ are presented in Table-5.1, 5.2, 5.3, 5.4, 5.5, 5.6 respectively. It could be seen that the computed heads tally with the analytical results as expected.

Oscillation in the computed head at a grid corresponding to a step rise in stream stage is exhibited in numerical method. The oscillation is very prominent for course time step at grid adjacent to the boundary. Oscillation is caused when the actual boundary inflow is not accounted truly. For a step rise in stream stage the influent seepage rate to the grid adjacent to the boundary is infinite.

It is implied that existing finite difference schemes consider the flow rate during any time step and the actual volume of flow that enters to the grid during the first time step is not accounted correctly leading to oscillation. Oscillation decreases with decreasing time step. The oscillation when a time step sizes are 0.1, 0.01, are presented in Fig.5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 5.7, 5.8 respectively.

Results for ramp input are presented in Table- 5.7, 5.8 and 5.9.

It is seen that for small time step size, as expected all finite difference schemes, the existing one and the proposed one perform equally.

For non- homogeneous case the head variation will be sharp within small distance and the proposed finite difference scheme which takes care of variation of head with time and space will be more efficient.

Table-5.1

Results: by Improved Finite Difference Scheme for Step Input
(For Delt=0.1)

TRANS= 100.000000
STORC= 2.000000E-02
JMAX DELTAX
8 10.00
NTIME DELT
10 .10
BHL BHR HI
15.00 10.00 10.00
THE RESULTS ARE FOR A STEP INPUT
FIRST TIME STEP
W2= 0.000000E+00
W3= 1.000000

HEAD(METRE) AT DIFFERENT TIMES AND NODES

□

13.999	12.562	11.637	11.039	10.649	10.388	10.206	10.064
14.849	13.987	12.988	12.114	11.423	10.897	10.491	10.156
14.269	13.623	13.123	12.553	11.942	11.344	10.783	10.257
15.106	14.185	13.274	12.571	11.979	11.417	10.855	10.286
14.066	13.841	13.466	12.835	12.145	11.488	10.875	10.289
15.528	14.189	13.259	12.681	12.145	11.562	10.944	10.316
13.474	13.955	13.636	12.886	12.155	11.510	10.904	10.302
16.438	14.080	13.144	12.731	12.223	11.599	10.950	10.314
12.124	14.164	13.811	12.853	12.111	11.516	10.930	10.315
18.462	13.750	12.952	12.813	12.292	11.599	10.926	10.302

RISE IN PIEZOMETRIC SURFACE COMPUTED NUMERICALLY

□

3.9993	2.5618	1.6366	1.0388	.6487	.3884	.2057	.0642

□

4.8494	3.9865	2.9877	2.1137	1.4233	.8972	.4909	.1558
4.2692	3.6233	3.1229	2.5532	1.9423	1.3436	.7828	.2567
5.1060	4.1850	3.2737	2.5707	1.9787	1.4174	.8549	.2858
4.0661	3.8415	3.4657	2.8349	2.1450	1.4882	.8749	.2887
5.5283	4.1886	3.2593	2.6806	2.1447	1.5618	.9443	.3156
3.4741	3.9546	3.6359	2.8860	2.1546	1.5096	.9036	.3016
6.4382	4.0804	3.1438	2.7309	2.2227	1.5994	.9499	.3141
2.1241	4.1642	3.8107	2.8532	2.1106	1.5158	.9304	.3146
8.4623	3.7497	2.9519	2.8132	2.2924	1.5995	.9262	.3023

RISE COMPUTED ANALYTICALLY (NODE (1 TO 6)

4.3718	3.1763	2.1460	1.3419	0.7736	0.4100
4.5549	3.6866	2.8808	2.1692	1.5715	1.0938
4.6363	3.9210	3.2404	2.6141	2.0566	1.5765
4.6849	4.0626	3.4632	2.9000	2.3838	1.9225
4.7181	4.1600	3.6184	3.1031	2.6226	2.1834
4.7427	4.2323	3.7344	3.2569	2.8064	2.3884
4.7617	4.2886	3.8254	3.3785	2.9534	2.5547
4.7771	4.3341	3.8993	3.4778	3.0744	2.6930
4.7898	4.3718	3.9607	3.5609	3.1763	2.8104
4.8006	4.4038	4.0129	3.6317	3.2636	2.9116

Table-5.2

Results: by Improved Finite Difference Scheme for Step Input
(for Delt=0.01)

```
TRANS=     100.000000
STORC=   2.000000E-02
JMAX     DELTAX
  8       10.00
NTIME      DELT
 10         .01
    BHL        BHR         HI
    15.00      10.00      10.00
THE RESULTS ARE FOR A STEP INPUT
```

FIRST TIME STEP

W2= 0.000000E+00

W3= 1.000000

HEAD(METRE) AT DIFFERENT TIMES AND NODES

12.470	10.662	10.177	10.048	10.013	10.003	10.001	10.000
13.930	11.421	10.436	10.125	10.035	10.009	10.003	10.000
13.699	11.982	10.786	10.265	10.082	10.024	10.007	10.001
14.112	12.242	11.089	10.443	10.158	10.051	10.015	10.003
14.061	12.529	11.331	10.621	10.255	10.094	10.031	10.007
14.225	12.692	11.552	10.790	10.361	10.148	10.054	10.013
14.232	12.860	11.730	10.949	10.470	10.211	10.083	10.021
14.310	12.976	11.891	11.095	10.579	10.279	10.118	10.032
14.332	13.087	12.027	11.229	10.685	10.350	10.157	10.044
14.376	13.175	12.150	11.351	10.787	10.421	10.198	10.057

RISE IN PIEZOMETRIC SURFACE COMPUTED NUMERICALLY

2.4697	.6618	.1773	.0475	.0127	.0034	.0009	.0002
3.9298	1.4206	.4359	.1251	.0348	.0095	.0025	.0004
3.6990	1.9821	.7863	.2650	.0816	.0238	.0065	.0012
4.1122	2.2418	1.0888	.4434	.1580	.0513	.0153	.0030
4.0607	2.5289	1.3314	.6211	.2550	.0937	.0308	.0066
4.2250	2.6922	1.5516	.7900	.3609	.1482	.0538	.0127
4.2320	2.8596	1.7302	.9494	.4703	.2110	.0835	.0211
4.3101	2.9761	1.8908	1.0946	.5793	.2791	.1184	.0316
4.3319	3.0873	2.0272	1.2288	.6852	.3498	.1569	.0437
4.3759	3.1750	2.1496	1.3509	.7869	.4211	.1976	.0568

ANALYTICAL SOLUTION

RISE COMPUTED ANALYTICALLY FOR NODE (1 TO 6)

3.0854	0.6681	0.0621	0.0023	0.0000	0.0000
3.6184	1.4442	0.3855	0.0666	0.0073	0.0005

3.8642	1.9324	0.7446	0.2165	0.0469	0.0075
4.0129	2.2663	1.0565	0.4006	0.1222	0.0298
4.1153	2.5117	1.3178	0.5876	0.2209	0.0695
4.1913	2.7015	1.5372	0.7652	0.3310	0.1237
4.2505	2.8538	1.7235	0.9294	0.4449	0.1882
4.2984	2.9794	1.8838	1.0796	0.5581	0.2591
4.3382	3.0854	2.0233	1.2167	0.6681	0.3338
4.3718	3.1763	2.1460	1.3419	0.7736	0.4100

Table-5.3

Results: by Implicit Scheme for Step Input (for Delt=0.1)

THE RESULTS ARE FOR A STEP INPUT
THE INFLOW AT X=0 IS OBTAINED NUMERICALLY

DELT
.100

TRANSMISSIVITY	STORATIVITY	DELXJ	INITIALHEAD	HEADL	HEADR
100.0	.020	10.0	10.0	15.00	10.00

BOUNDARY CONDITION AT X=0; SRIVL(K)

15.00	15.00	15.00	15.00	15.00
15.00	15.00	15.00	15.00	15.00

W2= 0.000000E+00
W3= 1.000000
W1= 0.000000E+00
W2= 0.000000E+00
W3= 1.000000

HEAD (METRE) AT DIFFERENT TIMES AND NODES

13.91	12.50	11.60	11.01	10.63	10.38	10.20	10.06
14.42	13.37	12.48	11.78	11.23	10.79	10.44	10.14
14.56	13.71	12.93	12.23	11.62	11.10	10.63	10.21
14.62	13.87	13.15	12.48	11.86	11.29	10.76	10.25
14.65	13.96	13.28	12.63	12.00	11.41	10.83	10.28
14.67	14.00	13.35	12.71	12.08	11.47	10.88	10.29
14.68	14.03	13.39	12.75	12.13	11.51	10.90	10.30
14.68	14.04	13.41	12.78	12.15	11.53	10.92	10.31
14.68	14.05	13.42	12.79	12.17	11.55	10.93	10.31
14.69	14.06	13.43	12.80	12.18	11.55	10.93	10.31

Table-5.4

Results: by Implicit Scheme for Step Input (for Delt=0.01)

THE RESULTS ARE FOR A STEP INPUT

THE INFLOW AT X=0 IS OBTAINED NUMERICALLY

DELT

.010

TRANSMISSIVITY	STORATIVITY	DELXJ	INITIALHEAD	HEADL	HEADR
100.0	.020	10.0	10.0	15.00	10.00

BOUNDARY CONDITION AT X=0; SRIVL(K)

15.00	15.00	15.00	15.00	15.00
15.00	15.00	15.00	15.00	15.00

W2= 0.000000E+00

W3= 1.000000

W1= 0.000000E+00

W2= 0.000000E+00

W3= 1.000000

HEAD (METRE) AT DIFFERENT TIMES AND NODES

12.11	10.57	10.15	10.04	10.01	10.00	10.00	10.00
13.08	11.15	10.40	10.13	10.04	10.01	10.00	10.00
13.56	11.63	10.67	10.26	10.09	10.03	10.01	10.00
13.82	12.01	10.94	10.40	10.16	10.06	10.02	10.01
13.99	12.30	11.18	10.56	10.25	10.10	10.04	10.01
14.10	12.52	11.40	10.71	10.34	10.15	10.06	10.02
14.18	12.70	11.59	10.86	10.44	10.21	10.09	10.02
14.24	12.85	11.76	11.01	10.54	10.27	10.12	10.03
14.29	12.98	11.91	11.14	10.64	10.33	10.15	10.04
14.33	13.08	12.04	11.26	10.73	10.40	10.19	10.06

Table-5.5

Results: by Crank Nicholson Scheme for Step Input

(for Delt=0.1)

THE RESULTS ARE FOR A STEP INPUT

THE INFLOW AT X=0 IS OBTAINED NUMERICALLY

DELT
.100

TRANSMISSIVITY	STORATIVITY	DELXJ	INITIALHEAD	HEADL	HEADR
100.0	.020	10.0	10.0	15.00	10.00

BOUNDARY CONDITION AT X=0; SRIVL(K)

15.00	15.00	15.00	15.00	15.00
15.00	15.00	15.00	15.00	15.00

W2= 5.000000E-01

W3= 5.000000E-01

W1= 0.000000E+00

W2= 5.000000E-01

W3= 5.000000E-01

HEAD (METRE) AT DIFFERENT TIMES AND NODES

16.98	13.75	12.01	11.08	10.57	10.30	10.14	10.04
12.74	13.72	13.20	12.35	11.57	10.97	10.52	10.16
15.98	13.72	12.98	12.47	11.93	11.36	10.81	10.27
13.65	14.22	13.47	12.69	12.03	11.43	10.86	10.29
15.45	13.80	13.30	12.77	12.16	11.52	10.90	10.30
14.08	14.26	13.46	12.76	12.14	11.54	10.93	10.31
15.15	13.87	13.41	12.83	12.19	11.55	10.93	10.31
14.32	14.22	13.44	12.79	12.18	11.56	10.94	10.31
14.97	13.93	13.44	12.83	12.19	11.56	10.93	10.31
14.46	14.17	13.43	12.80	12.19	11.56	10.94	10.31

Table-5.6

Results: by Crank Nicholson Scheme for Step Input

(for Delt=0.01)

```
THE RESULTS ARE FOR A STEP INPUT
THE INFLOW AT X=0 IS OBTAINED NUMERICALLY
    DELT
   .010
TRANSMISSIVITY   STORATIVITY     DELXJ     INITIALHEAD    HEADL       HEADR
  100.0           .020           10.0         10.0        15.00       10.00
BOUNDARY CONDITION AT X=0; SRIVL(K)
  15.00     15.00     15.00     15.00     15.00
  15.00     15.00     15.00     15.00     15.00
W2=   5.000000E-01
W3=   5.000000E-01
W1=   0.000000E+00
W2=   5.000000E-01
W3=   5.000000E-01
HEAD (METRE) AT DIFFERENT TIMES AND NODES
12.93   10.50   10.09   10.01   10.00   10.00   10.00   10.00
13.54   11.32   10.35   10.08   10.02   10.00   10.00   10.00
13.81   11.84   10.69   10.21   10.06   10.01   10.00   10.00
13.98   12.19   11.00   10.38   10.13   10.04   10.01   10.00
14.09   12.45   11.27   10.56   10.22   10.08   10.02   10.01
14.17   12.65   11.49   10.74   10.32   10.13   10.04   10.01
14.23   12.81   11.68   10.90   10.43   10.19   10.07   10.02
14.28   12.95   11.85   11.05   10.54   10.26   10.11   10.03
14.33   13.06   11.99   11.19   10.65   10.33   10.14   10.04
14.36   13.15   12.11   11.31   10.75   10.40   10.18   10.05
```

Table-5.7

Results: by Improved Finite Difference Scheme & by Analytical Solution for Ramp Input(Delt=0.1)

Solution by Improved Finite Difference Scheme:

```
TRANS=     100.000000
STORC=   2.000000E-02
JMAX      DELTAX
 8         10.00
NTIME   DELT
 10     .10
  BHL        BHR        HI
  15.00     10.00     10.00
FIRST TIME STEP
W2=   0.000000E+00
W3=        1.000000
HEAD(METRE) AT DIFFERENT TIMES AND NODES
10.400  10.256  10.164  10.104  10.065  10.039  10.021  10.006
10.839  10.578  10.390  10.257  10.165  10.101  10.054  10.017
11.294  10.954  10.689  10.485  10.329  10.209  10.116  10.037
11.761  11.343  11.007  10.739  10.522  10.345  10.196  10.063
12.221  11.744  11.343  11.008  10.727  10.490  10.282  10.092
12.699  12.145  11.679  11.283  10.941  10.642  10.373  10.122
13.151  12.552  12.023  11.561  11.156  10.795  10.465  10.153
13.643  12.954  12.363  11.842  11.374  10.950  10.558  10.184
14.076  13.366  12.710  12.121  11.591  11.106  10.651  10.215
14.598  13.762  13.048  12.404  11.811  11.262  10.744  10.246

RISE IN PIEZOMETRIC SURFACE COMPUTED NUMERICALLY

.3999  .2562  .1637  .1039  .0649  .0388  .0206  .0064

.8391  .5781  .3897  .2574  .1655  .1012  .0543  .0171
1.2937  .9541  .6891  .4846  .3286  .2094  .1157  .0369
1.7614 1.3434 1.0074  .7385  .5221  .3451  .1960  .0635
2.2207 1.7438 1.3429 1.0076  .7273  .4897  .2820  .0921
2.6989 2.1452 1.6791 1.2828  .9411  .6416  .3726  .1221
3.1511 2.5519 2.0232 1.5609 1.1559  .7950  .4649  .1529
3.6434 2.9541 2.3626 1.8415 1.3745  .9503  .5575  .1837
4.0764 3.3655 2.7096 2.1209 1.5913 1.1060  .6514  .2151
4.5984 3.7624 3.0485 2.4039 1.8112 1.2618  .7443  .2460
```

Analytical Solution:

```
PIEZOMETRIC RISE OBTAINED ANALLYTICALLY
RISE AT FIRST NODE
.3858  .8337 1.2937 1.7599 2.2302 2.7033 3.1785 3.6555 4.1339 4.6134
RISE AT SECOND NODE
.2200  .5673  .9488 1.3485 1.7599 2.1797 2.6058 3.0370 3.4724 3.9112
RISE AT THIRD NODE
.1180  .3747  .6824 1.0183 1.3728 1.7407 2.1189 2.5052 2.8983 3.2971
RISE AT FORTH NODE
.0593  .2399  .4809  .7575 1.0582 1.3765 1.7085 2.0515 2.4035 2.7632
RISE AT FIFTH NODE
.0278  .1486  .3318  .5548  .8056 1.0775 1.3657 1.6673 1.9800 2.3021
```

Table-5.8

Results: by Implicit Scheme for Ramp Input

(for Delt=0.1)

THE RESULTS ARE FOR A RAMP INPUT

THE INFLOW AT X=0 IS OBTAINED NUMERICALLY

DELT

.100

TRANSMISSIVITY	STORATIVITY	DELXJ	INITIALHEAD	HEADL	HEADR
100.0	.020	10.0	10.0	15.00	10.00

BOUNDARY CONDITION AT X=0; SRIVL(K)

10.50	11.00	11.50	12.00	12.50
13.00	13.50	14.00	14.50	15.00

W2= 0.000000E+00
W3= 1.000000
W1= 0.000000E+00
W2= 0.000000E+00
W3= 1.000000

HEAD (METRE) AT DIFFERENT TIMES AND NODES

10.39	10.25	10.16	10.10	10.06	10.04	10.02	10.01
10.83	10.59	10.41	10.28	10.19	10.12	10.06	10.02
11.29	10.96	10.70	10.50	10.35	10.23	10.13	10.04
11.75	11.34	11.02	10.75	10.53	10.36	10.20	10.07
12.22	11.74	11.34	11.01	10.74	10.50	10.29	10.09
12.68	12.14	11.68	11.28	10.94	10.64	10.38	10.12
13.15	12.54	12.02	11.56	11.16	10.80	10.47	10.15
13.62	12.95	12.36	11.84	11.37	10.95	10.56	10.18
14.09	13.35	12.70	12.12	11.59	11.10	10.65	10.21
14.55	13.76	13.04	12.40	11.81	11.26	10.74	10.25

Table-5.9

Results: by Crank Nicholson Scheme for Ramp Input
(for Delt=0.1)

THE RESULTS ARE FOR A RAMP INPUT
THE INFLOW AT X=0 IS OBTAINED NUMERICALLY

DELT
.100

TRANSMISSIVITY	STORATIVITY	DELXJ	INITIALHEAD	HEADL	HEADR
100.0	.020	10.0	10.0	15.00	10.00

BOUNDARY CONDITION AT X=0; SRIVL(K)

10.50	11.00	11.50	12.00	12.50
13.00	13.50	14.00	14.50	15.00

W2= 5.000000E-01
W3= 5.000000E-01
W1= 0.000000E+00
W2= 5.000000E-01
W3= 5.000000E-01

HEAD (METRE) AT DIFFERENT TIMES AND NODES

10.35	10.19	10.10	10.05	10.03	10.01	10.01	10.00
10.84	10.56	10.36	10.22	10.14	10.08	10.04	10.01
11.27	10.93	10.67	10.47	10.31	10.20	10.11	10.03
11.75	11.33	10.99	10.72	10.51	10.33	10.19	10.06
12.21	11.73	11.33	11.00	10.72	10.48	10.28	10.09
12.68	12.13	11.67	11.27	10.93	10.64	10.37	10.12
13.15	12.54	12.01	11.55	11.15	10.79	10.46	10.15
13.62	12.95	12.35	11.83	11.37	10.95	10.56	10.18
14.08	13.35	12.70	12.11	11.59	11.10	10.65	10.21
14.56	13.76	13.04	12.40	11.80	11.26	10.74	10.25

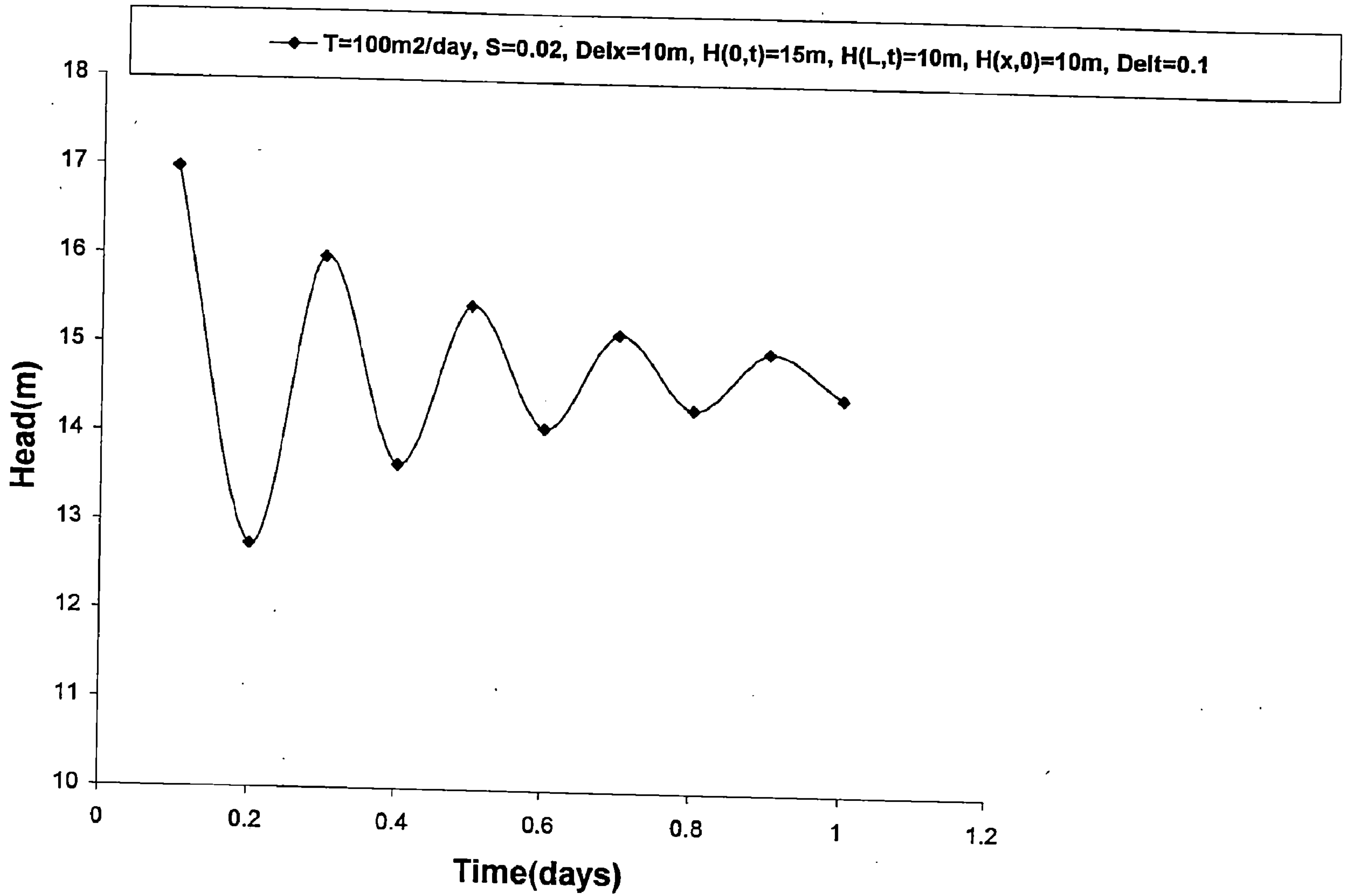

Fig.5.1: Head vs. Time curve(x=0m, t=0.1day), by Crank Nicholson Scheme

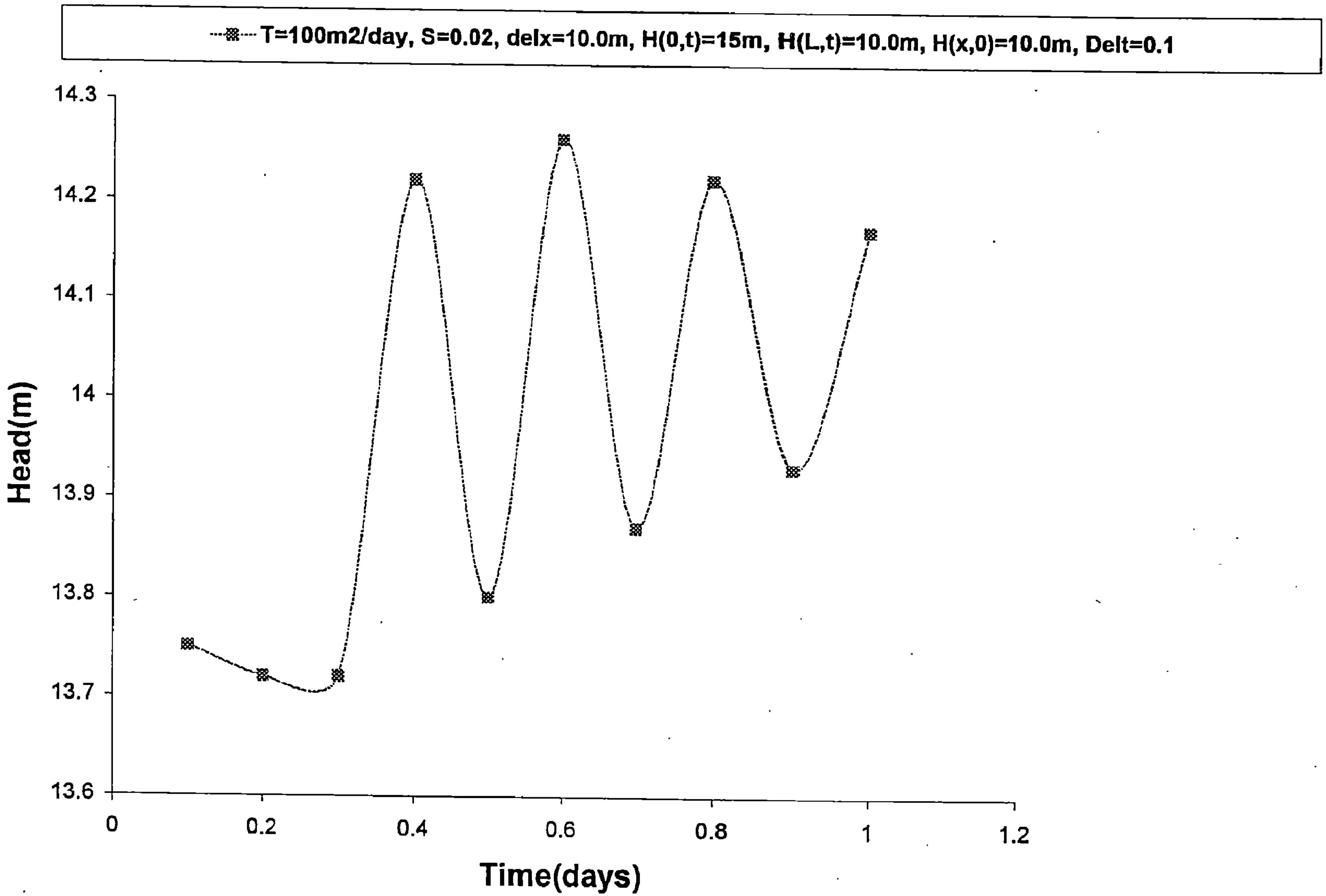

Fig.5.2 : Head vs. Time curve (x=10.0m, t=0.1day), by Crank Nicholson scheme

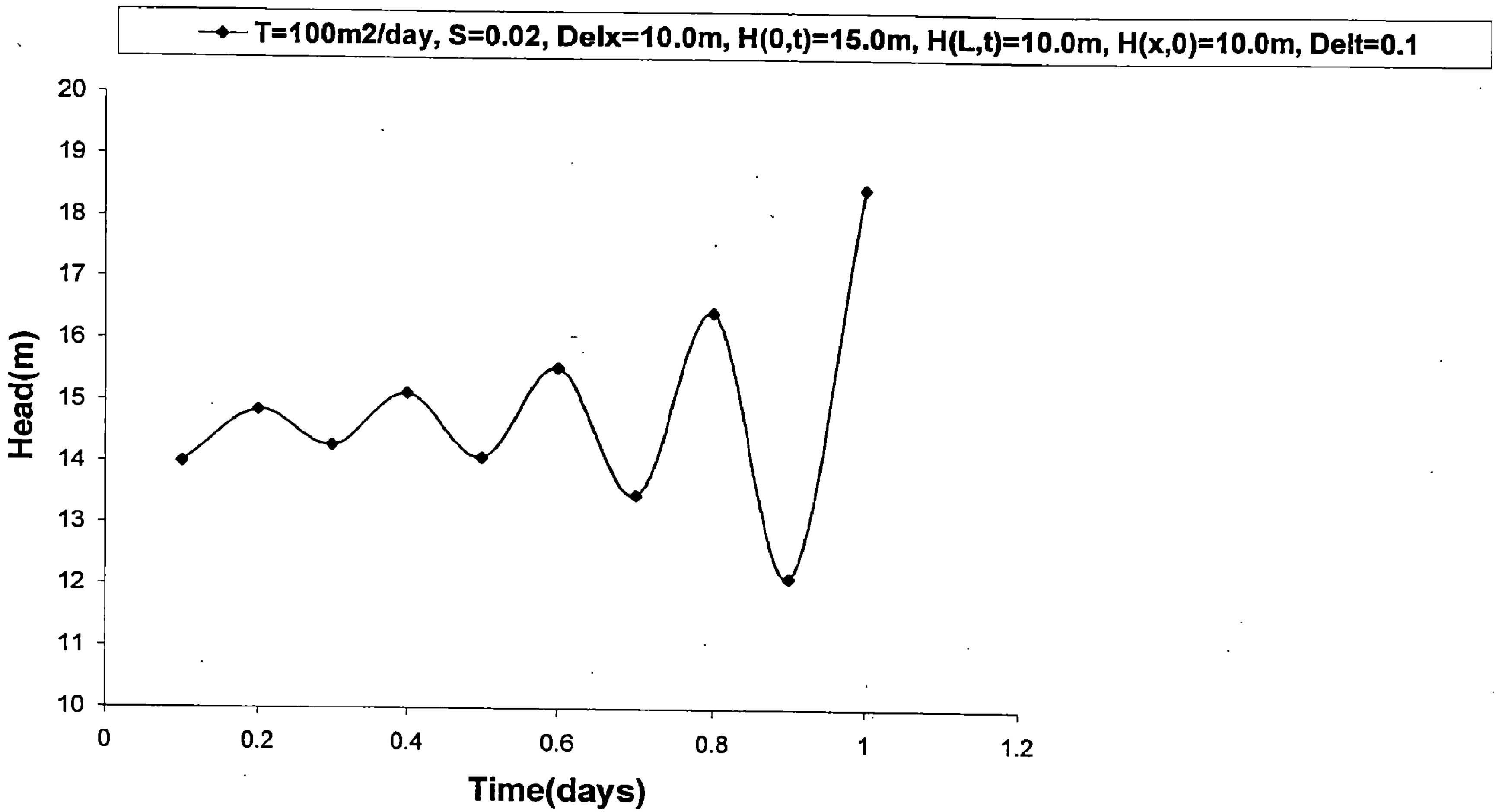

Fig. 5.3 : Head vs. Time curve (x=0m, t=0.1 day), by Improved Finite Difference Scheme.

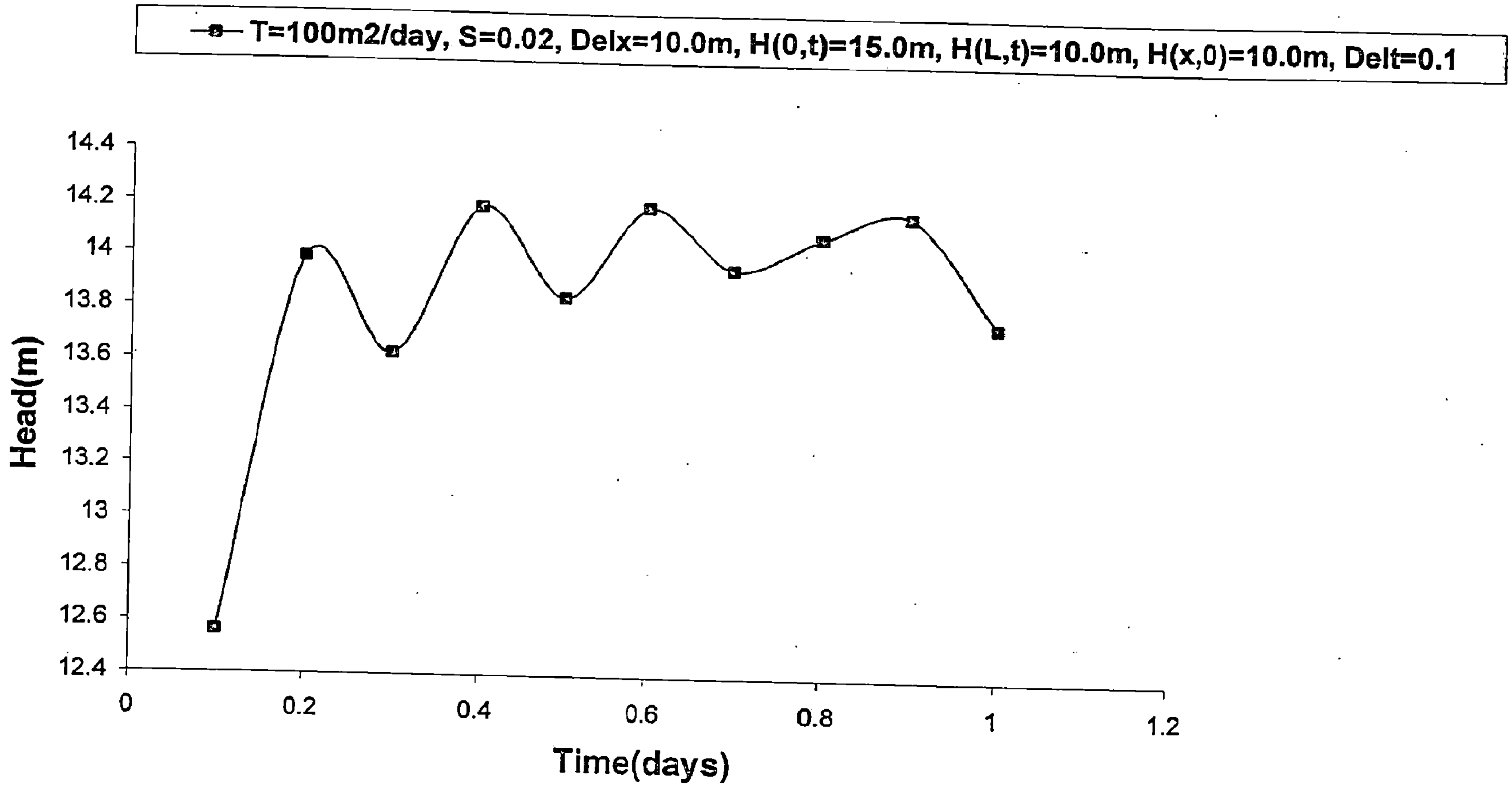

Fig. 5.4 : Head vs. Time curve (x=10m, t=0.1 day), by Improved Finite Difference Scheme.

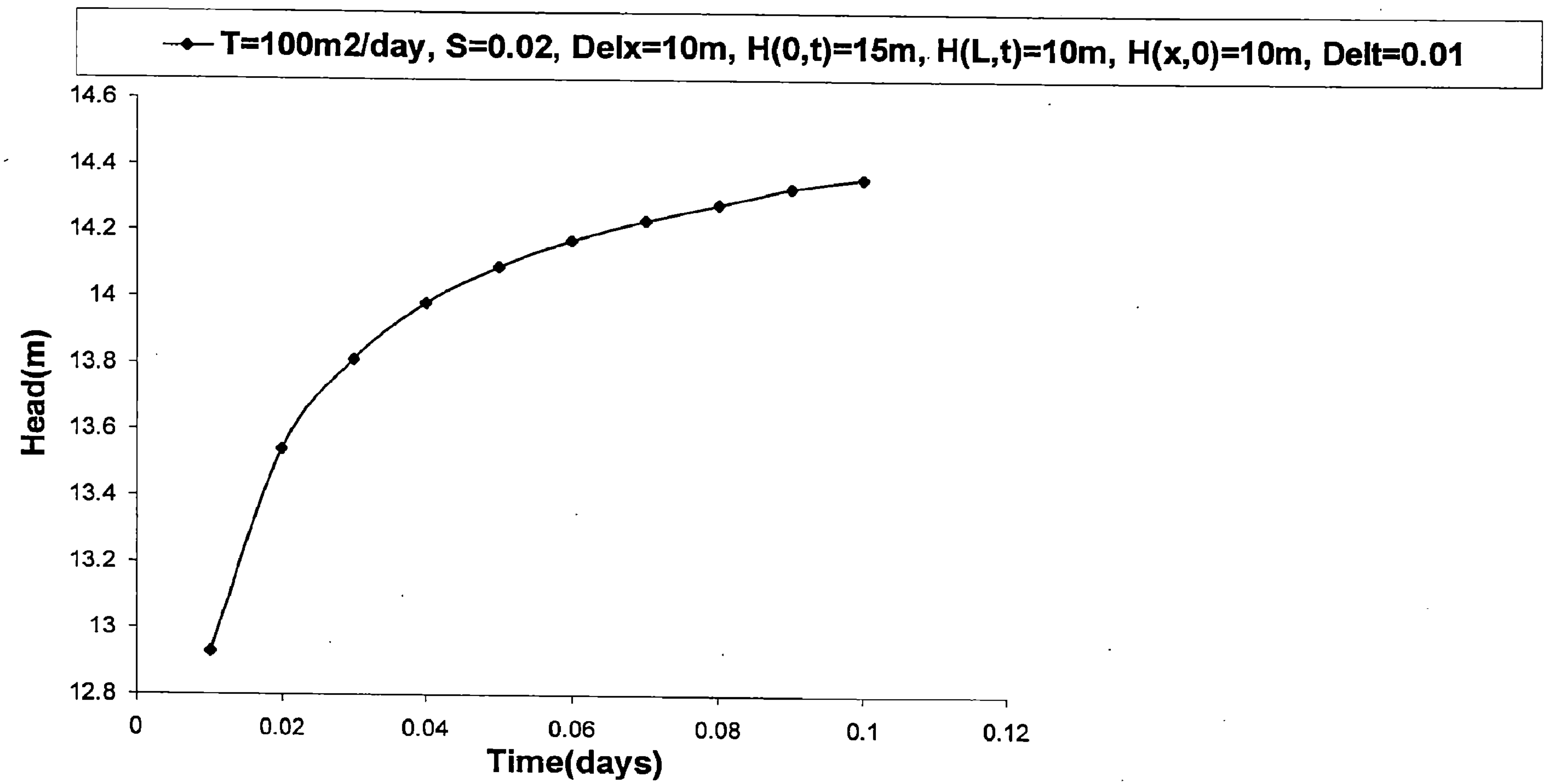

Fig. 5.5 : Head vs. Time curve (x=0m, t=0.01 day), by Crank Nicholson Scheme.

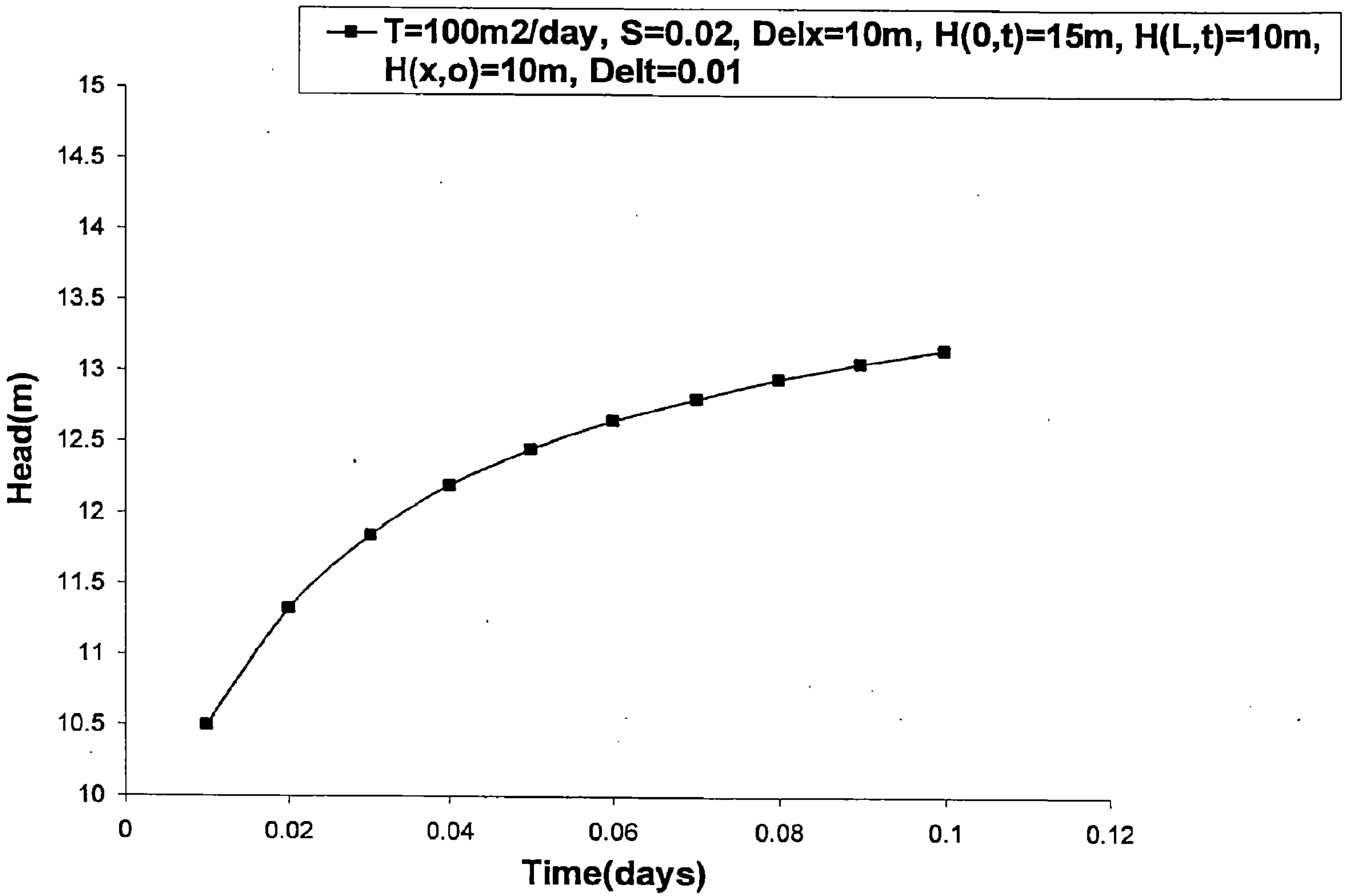

Fig. 5.6 : Head vs. Time curve (x=10m, t=0.01 day), by Crank Nicholson Scheme.

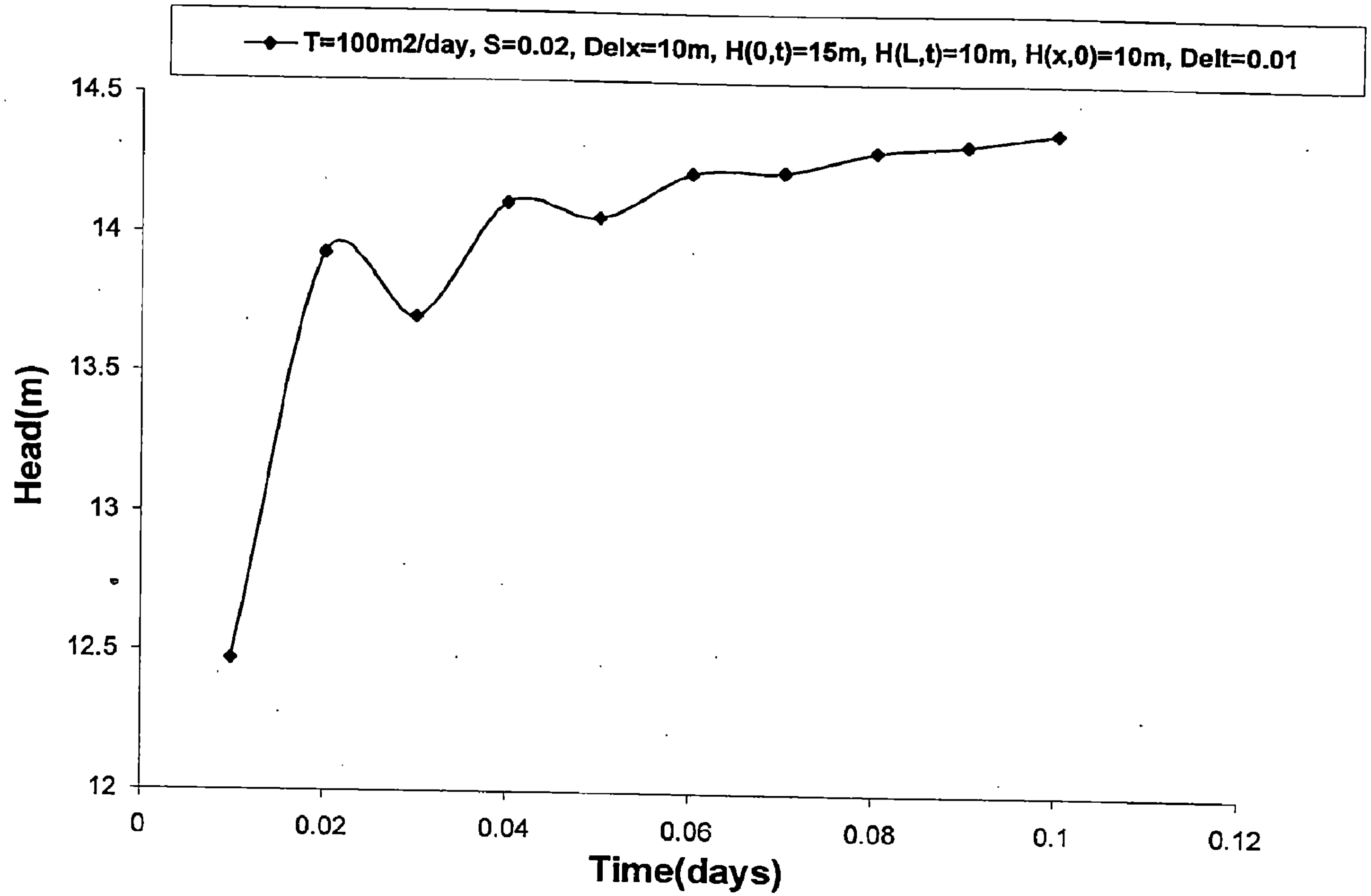

Fig. 5.7 : Head vs. Time curve (x=0m, t=0.01 day), by Improved Finite Difference Scheme.

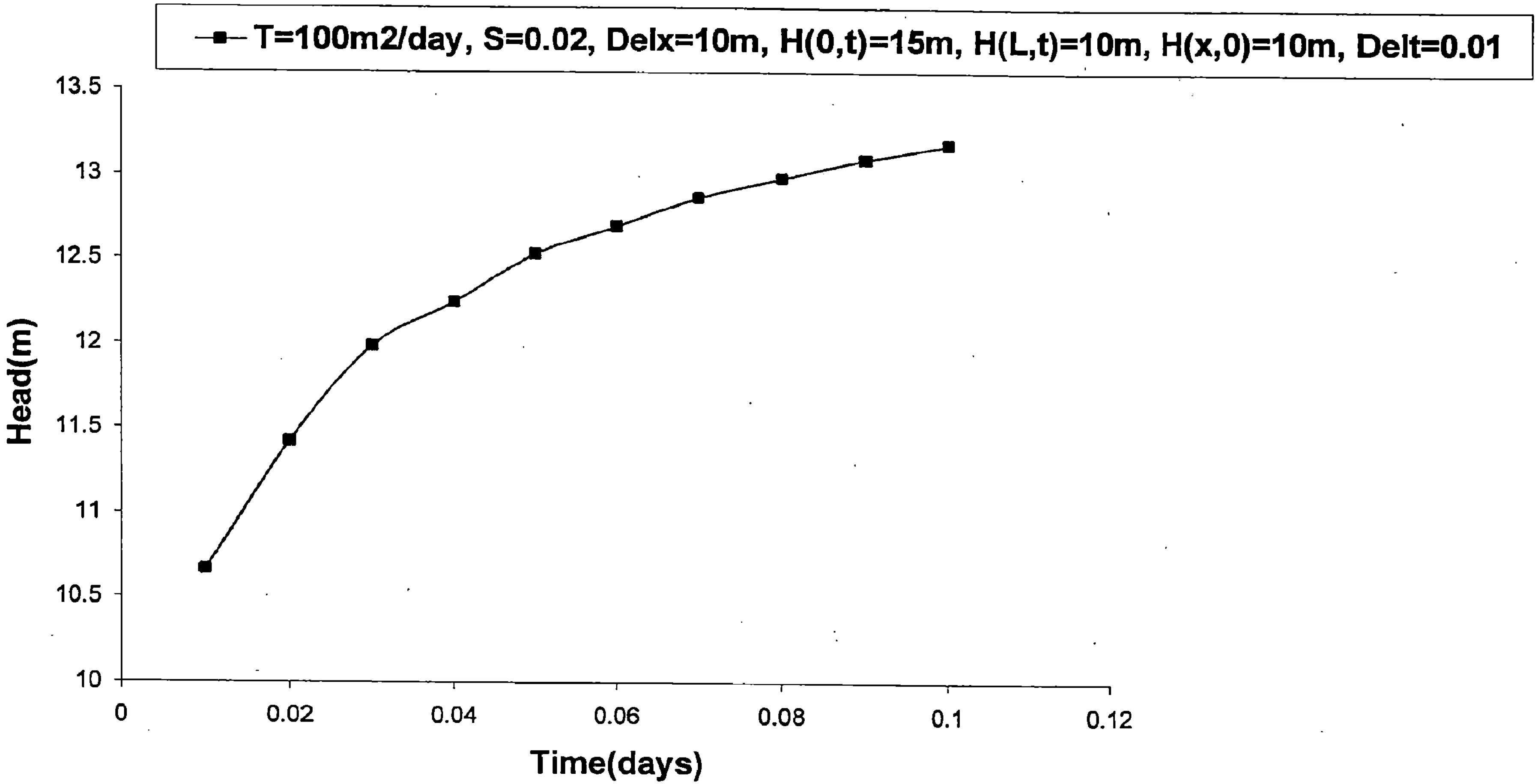

Fig. 5.8 : Head vs. Time curve (x=10m, t=0.01 day), by Improved Finite Difference Scheme.

CHAPTER -6

CONCLUSION

Following conclusions are made:

-In existing finite difference method, the spatial variation of flow is assumed non-linear which follows a second-degree equation.

-For small time step, the existing finite difference scheme and the improved finite difference scheme performs equally and the result tallies with analytical solution.

-All numerical methods exhibit oscillations in computed head. The oscillation is prominent for course time step at grid adjacent to the boundary.

-Oscillation is caused when the actual boundary inflow is not accounted truly.

- For non-homogeneous case, the improved finite difference scheme which takes care of variation of head with time and space will be more efficient.

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APPENDICES

APPENDIX- 1

Let us consider that the spatial variation of head in the vicinity of x_0 follows an equation of the form $h = a_1 + b_1(x-x_0) + c_1(x-x_0)^2$

$$h = h(x) + \left[\frac{h(x_0+\Delta x_1)-h(x_0)}{\Delta x_1} - \frac{\Delta x_1}{\Delta x_1+\Delta x_2}\left\{\frac{h(x_0+\Delta x_1)-h(x_0)}{\Delta x_1} + \frac{h(x_0-\Delta x_2)-h(x_0)}{\Delta x_2}\right\}\right](x-x_0)$$

$$+\frac{1}{\Delta x_1+\Delta x_2}\left\{\frac{h(x_0+\Delta x_1)-h(x_0)}{\Delta x_1} + \frac{h(x_0-\Delta x_2)-h(x_0)}{\Delta x_2}\right\}(x-x_0)^2$$

$$\Delta x_1 = \frac{\Delta x(j)+\Delta x(j+1)}{2};\ \Delta x_2 = \frac{\Delta x(j-1)+\Delta x(j)}{2}$$

The gradient at any point within $(x_0-\Delta x_2) \le x \le (x_0+\Delta x_1)$

$$\frac{\partial h}{\partial x} = b_1 + 2c_1(x-x_0)$$

Substituting the value of b_1 and c_1

$$\frac{\partial h}{\partial x} = \left[\frac{h(x_0+\Delta x_1)-h(x_0)}{\Delta x_1} - \frac{\Delta x_1}{\Delta x_1+\Delta x_2}\left\{\frac{h(x_0+\Delta x_1)-h(x_0)}{\Delta x_1} + \frac{h(x_0-\Delta x_2)-h(x_0)}{\Delta x_2}\right\}\right]$$

$$+\frac{2}{\Delta x_1+\Delta x_2}\left\{\frac{h(x_0+\Delta x_1)-h(x_0)}{\Delta x_1} + \frac{h(x_0-\Delta x_2)-h(x_0)}{\Delta x_2}\right\}(x-x_0)$$

$$\frac{\partial h}{\partial x}_{at x_0-\Delta x_2/2} = \left[\frac{h(x_0+\Delta x_1)-h(x_0)}{\Delta x_1} - \frac{\Delta x_1}{\Delta x_1+\Delta x_2}\left\{\frac{h(x_0+\Delta x_1)-h(x_0)}{\Delta x_1} + \frac{h(x_0-\Delta x_2)-h(x_0)}{\Delta x_2}\right\}\right]$$

$$+\frac{2}{\Delta x_1+\Delta x_2}\left\{\frac{h(x_0+\Delta x_1)-h(x_0)}{\Delta x_1} + \frac{h(x_0-\Delta x_2)-h(x_0)}{\Delta x_2}\right\}(x_0 - \frac{\Delta x_2}{2} - x_0)$$

$$= \frac{h(x_0+\Delta x_1)-h(x_0)}{\Delta x_1} - \frac{h(x_0+\Delta x_1)-h(x_0)}{\Delta x_1+\Delta x_2} - \frac{\Delta x_1}{\Delta x_2(\Delta x_1+\Delta x_2)}\{h(x_0-\Delta x_2)-h(x_0)\}$$

$$-\frac{\Delta x_2}{\Delta x_1+\Delta x_2}\frac{h(x_0+\Delta x_1)-h(x_0)}{\Delta x_1} - \frac{h(x_0-\Delta x_2)-h(x_0)}{\Delta x_1+\Delta x_2}$$

$$= h(x_0+\Delta x_1)\left\{\frac{1}{\Delta x_1} - \frac{1}{\Delta x_1+\Delta x_2} - \frac{\Delta x_2}{\Delta x_1(\Delta x_1+\Delta x_2)}\right\}$$

$$+h(x_0)\left\{-\frac{1}{\Delta x_1} + \frac{1}{\Delta x_1+\Delta x_2} + \frac{\Delta x_1}{\Delta x_2(\Delta x_1+\Delta x_2)} + \frac{\Delta x_2}{\Delta x_1(\Delta x_1+\Delta x_2)} + \frac{1}{\Delta x_1+\Delta x_2}\right\} - \frac{h(x_0-\Delta x_2)}{\Delta x_1+\Delta x_2}$$

$$-\frac{\Delta x_1 h(x_0-\Delta x_2)}{\Delta x_2(\Delta x_1+\Delta x_2)}$$

$$= \frac{h(x_0) - h(x_0 - \Delta x_2)}{\Delta x_2}$$

$$= \frac{h(j) - h(j-1)}{\frac{\Delta x(j-1) + \Delta x(j)}{2}}$$

In finite difference the gradient between two consecutive grid points is also given by the above equation.

APPENDIX - 2
COMPUTER PROGRAMME

APPENDIX - 2A

```
C     THIS PROGRAMME COMPUTES HEAD BY THE NEW FINITE DIFFERENCE METHOD
C     THE GRID SIZES ARE EQUAL,THE AQUIFER IS NON HOMOGENEOUS
      DIMENSION DELX(25),T(25),S(25),TRISE(25,100)
      DIMENSION SRIVR(0:100),H(25,0:100),SRIVL(0:100),RISE(25,100)
      DIMENSION FC1(25),FC2(25),FC4(25),FC22(25),
     1          FC44(25),FC6(25,100),HSOL(25),
     2          A(25,25),RIGHT(25),DKER(100),SIGMA(0:100)
C     DIMENSION AA(25,25),AAAA(25,25)

      OPEN(1,FILE='SREST.DAT',STATUS='OLD')
      OPEN(2,FILE='SREST.OUT',STATUS='UNKNOWN')
C
      READ(1,*)JMAX,DELTAX
      READ(1,*)TRANS,STORC
      READ(1,*) DELT,NTIME
      READ(1,*)BHL,BHR,HI
      WRITE(2,*)'TRANS=',TRANS
      WRITE(2,*)'STORC=',STORC
      WRITE(2,60)
60    FORMAT(2X,'JMAX',5X,'DELTAX')
      WRITE(2,51)JMAX,DELTAX
51    FORMAT(2X,I2,4X,F8.2)

      DO J=1,JMAX
      DELX(J)       =DELTAX
      END DO
      WRITE(2,70)
70    FORMAT(2X,'NTIME',2X,'DELT')
      WRITE(2,51)NTIME,DELT
C
      WRITE(2,61)
61    FORMAT(5X,'BHL', 5X,  'BHR', 5X, 'HI')
      WRITE(2,52)BHL,BHR,HI
52    FORMAT(3F10.2)

      DO J=1,JMAX
      H(J,0)=HI
      END DO
      SRIVR(0)=BHR
C     SRIVL(0)=BHL
      SRIVL(0)=HI
      SIGMA(0)=HI
      WRITE(2,*)'THE RESULTS ARE FOR A RAMP INPUT'
      DO K=1,NTIME
      AK=K
      SRIVR(K)=BHR
      SRIVL(K)=HI+(BHL-HI)*DELT*AK
      SIGMA(K)=SRIVL(K)
      END DO
C
C     TRANSMISSIVITY VALUES ARE BEING READ
C
C     READ(1,*)  (T(J),J=1,JMAX)
C     WRITE(2,54)(T(J),J=1,JMAX)
C54   FORMAT(6F8.2)
```

```
C
      DO J=1,JMAX
      T(J)=TRANS
      END DO
C
C     STORAGE COEFFICIENT VALUES ARE BEING READ
C
C     READ(1,*)  (S(J),J=1,JMAX)
C     WRITE(2,54)(S(J),J=1,JMAX)
C
      DO J=1,JMAX
      S(J)=STORC
      END DO
C
      M=JMAX
      DO I=1,M
      DO J=1,M
      A(I,J)=0.
      END DO
      END DO
      K=1
C     W2=0.5
C     W3=0.5
      W2=0.
      W3=1.
      WRITE(2,*)'FIRST TIME STEP'
      WRITE(2,*)'W2=',W2
      WRITE(2,*)'W3=',W3
      J=1
      HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
      FC22(J)=W3*(-18.*T(J)-6.*HMT2)
      FC2(J)=W3*(100.*T(J)+140.*HMT2)
      FC1(J)=-120*S(J)*DELX(J)**2/DELT+W3*(-450.*T(J)-150.*HMT2)
      DO J=2,JMAX-2
      HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
      HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))
      FC22(J)=W3*(HMT4-HMT2)
      FC2(J)=W3*(-3.*HMT4+27.*HMT2)
      FC1(J)=-24.*S(J)*DELX(J)**2/DELT+W3*(-21.*HMT4-27.*HMT2)
      FC4(J)=W3*(23.*HMT4+HMT2)
      END DO

      J=JMAX-1
      HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
      HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))
      FC2(J)=W3*(HMT4+23.*HMT2)
      FC1(J)=-24.*S(J)*DELX(J)**2/DELT+W3*(-27.*HMT4-21.*HMT2)
      FC4(J)=W3*(27.*HMT4-3.*HMT2)
      FC44(J)=W3*(-HMT4+HMT2)

      J=JMAX
      HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))
      FC1(J)=-120.*S(J)*DELX(J)**2/DELT+W3*(-150.*HMT4-450.*T(J))
      FC4(J)=w3*(140.*HMT4+100.*T(j))
      FC44(J)=w3*(-6.*HMT4-18.*T(j))

      I=1
      A(I,I)=FC1(I)
      A(I,I+1)=FC2(I)
      A(I,I+2)=FC22(I)
```

```
      DO I=2,JMAX-2
      A(I,I-1)=FC4(I)
      A(I,I)=FC1(I)
      A(I,I+1)=FC2(I)
      A(I,I+2)=FC22(I)
      END DO

      I=JMAX-1

      A(I,I+1)=FC2(I)
      A(I,I)=FC1(I)
      A(I,I-1)=FC4(I)
      A(I,I-2)=FC44(I)

      I=JMAX
      A(I,I)=FC1(I)
      A(I,I-1)=FC4(I)
      A(I,I-2)=FC44(I)

C     WRITE(2,*)'ELEMENT OF THE MATRIX'
C     DO I=1,M
C     WRITE(2,197)(A(I,J),J=1,M)
C     END DO

C     STORING OF THE MATRIX ELEMENT
C     DO I=1,M
C     DO J=1,M
C     AA(I,J)=A(I,J)
C     END DO
C     END DO
C     DO I=1,M
C     WRITE(2,197)( A(I,J),J=1,M)
C     END DO
197   FORMAT(7E10.4)
C
      CALL MATIN (A,M)
C     WRITE(2,*)'MATRIX ELEMENT AFTER INVERSION'
C     DO I=1,M
C     WRITE(2,197)(A(I,J),J=1,M)
C     END DO

C     WRITE(2,*)'CHECKING THE MATRIX INVERSION'
C     KK=0
C     DO I=1,M
C     SUM=0
C     KK=KK+1
C     DO J=1,M
C     SUM=SUM+AA(I,J)*A(J,I)
C     END DO
C     AAAA(I,KK)=SUM
C     END DO
C     DO I=1,M
C     WRITE(2,197)(AAAA(I,J),J=1,M)
C     END DO
C     SOLUTION FOR THE FIRST TIME STEP
C     FC6(J,1)  AT  INTERIOR NODES
      K=1
      J=1
      HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
      FC6(J,K)=(120.*S(J)*DELX(J)**2/DELT)*H(J,K-1)-w2*T(J)*
     1(18.*h(j+2,k-1)-100.*h(j+1,k-1)+450.*h(j,k-1)-368.*srivl(k-1))+
```

```
     2 w2*HMT2*(-6.*h(j+2,k-1)+140.*h(j+1,k-1)-150.*h(j,k-1)+
     3 16.*srivl(k-1))+
     4 W3*(368.*T(J)*srivl(k)+16.*HMT2*srivl(k))


      DO J=2,JMAX-2
      HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
      HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))

      FC6(J,K)=(24*S(J)*DELX(J)**2/DELT)*H(J,K-1)
     1 -w2*HMT4*(-h(j+2,k-1)+3.*h(j+1,k-1)+21.*h(j,k-1)-23.*h(j-1,k-1))
     2 +w2*HMT2*(-h(j+2,k-1)+27.*h(j+1,k-1)-27.*h(j,k-1)+h(j-1,k-1))
      END DO

      J=JMAX-1
      HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
      HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))

      FC6(J,K)=(24.*S(J)*DELX(J)**2/DELT)*H(J,K-1)
     1 -w2*HMT4*(h(j-2,k-1)-27.*h(j-1,k-1)+27.*h(j,k-1)-h(j+1,k-1))
     2 +w2*HMT2*(h(j-2,k-1)-3.*h(j-1,k-1)-21.*h(j,k-1)+23.*h(j+1,k-1))

      J=JMAX
      HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))

      FC6(J,K)=(120.*S(J)*DELX(J)**2/DELT)*H(J,K-1)
     1 - w2*HMT4*(-16.*srivr(k-1)+150.*h(j,k-1)-140.*h(j-1,k-1)
     2 +6.*h(j-2,k-1))+w2*T(j)*
     3 (368.*srivr(k-1)-450.*h(j,k-1)+100.*h(j-1,k-1)-18.*h(j-2,k-1))
     4 +w3*(16.*HMT4*srivr(k)+368.*T(j)*srivr(k))

C
      INDEX=0
      DO J=1,JMAX
      INDEX=INDEX+1
      RIGHT(INDEX)=-FC6(J,K)
      END DO

C     WRITE(2,*)'K=',K
C     WRITE(2,*)'RIGHTHAND COLUMN MATRIX'
C     WRITE(2,197)(RIGHT(J),J=1,JMAX)

      DO I=1,M
      HSOL(I)=0.
      DO J=1,M
      HSOL(I)=HSOL(I)+A(I,J)*RIGHT(J)
      END DO
      END DO
C
      INDEX=0
      DO J=1,JMAX
      INDEX=INDEX+1
      H(J,K)=HSOL(INDEX)
      END DO
C     SOLUTION FOR K=2

      M=JMAX

      DO I=1,M
      DO J=1,M
      A(I,J)=0.
```

```
END DO
END DO

K=2

W1=-1./12.
W2=2./3.
W3=5./12.

J=1

HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
FC22(J)=W3*(-18.*T(J)-6.*HMT2)
FC2(J)=W3*(100.*T(J)+140.*HMT2)
FC1(J)=-120*S(J)*DELX(J)**2/DELT+W3*(-450.*T(J)-150.*HMT2)

DO J=2,JMAX-2
HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))
FC22(J)=W3*(HMT4-HMT2)
FC2(J)=W3*(-3.*HMT4+27.*HMT2)
FC1(J)=-24.*S(J)*DELX(J)**2/DELT+W3*(-21.*HMT4-27.*HMT2)
FC4(J)=W3*(23.*HMT4+HMT2)
END DO
J=JMAX-1
HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))
FC2(J)=W3*(HMT4+23.*HMT2)
FC1(J)=-24.*S(J)*DELX(J)**2/DELT+W3*(-27.*HMT4-21.*HMT2)
FC4(J)=W3*(27.*HMT4-3.*HMT2)
FC44(J)=W3*(-HMT4+HMT2)

J=JMAX
HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))
FC1(J)=-120.*S(J)*DELX(J)**2/DELT+W3*(-150.*HMT4-450.*T(J))
FC4(J)=w3*(140.*HMT4+100.*T(j))
FC44(J)=w3*(-6.*HMT4-18.*T(j))

I=1
A(I,I)=FC1(I)
A(I,I+1)=FC2(I)
A(I,I+2)=FC22(I)

DO I=2,JMAX-2
A(I,I-1)=FC4(I)
A(I,I)=FC1(I)
A(I,I+1)=FC2(I)
A(I,I+2)=FC22(I)
END DO

I=JMAX-1

A(I,I+1)=FC2(I)
A(I,I)=FC1(I)
A(I,I-1)=FC4(I)
A(I,I-2)=FC44(I)

I=JMAX
A(I,I)=FC1(I)
A(I,I-1)=FC4(I)
A(I,I-2)=FC44(I)
```

```
C     WRITE(2,*)'ELEMENT OF THE MATRIX FOR K=2'
C     DO I=1,M
C     WRITE(2,197)(A(I,J),J=1,M)
C     END DO
C     STORING OF THE MATRIX ELEMENT
C     DO I=1,M
C     DO J=1,M
C     AA(I,J)=A(I,J)
C     END DO
C     END DO
C     CALL MATRIX INVERSE
C     WRITE(2,*)'WRITING THE MATRIX ELEMENT BEFORE INVERSION'
C     DO I=1,M
C     WRITE(2,197)( A(I,J),J=1,M)
C     END DO

      CALL MATIN (A,M)

C     WRITE(2,*)'MATRIX ELEMENT AFTER INVERSION'
C     DO I=1,M
C     WRITE(2,197)(A(I,J),J=1,M)
C     END DO

C     WRITE(2,*)'CHECKING THE MATRIX INVERSION'
C     KK=0
C     DO I=1,M
C     SUM=0
C     KK=KK+1
C     DO J=1,M
C     SUM=SUM+AA(I,J)*A(J,I)
C     END DO
C     AAAA(I,KK)=SUM
C     END DO

C     DO I=1,M
C     WRITE(2,197)(AAAA(I,J),J=1,M)
C     END DO

C     SOLUTION FOR THE 2ND TIME STEP ONWARD

1000  CONTINUE
      J=1
      FC6(J,K)=(120.*S(J)*DELX(J)**2/DELT)*H(J,K-1)-w1*T(J)*
     1(18.*h(j+2,k-2)-100.*h(j+1,k-2)+450.*h(j,k-2)-368.*srivl(k-2))+
     2w1*HMT2*(-6.*h(j+2,k-2)+140.*h(j+1,k-2)-150.*h(j,k-2)+
     316.*srivl(k-2))-w2*T(J)*
     4(18.*h(j+2,k-1)-100.*h(j+1,k-1)+450.*h(j,k-1)-368.*srivl(k-1))+
     6w2*HMT2*(-6.*h(j+2,k-1)+140.*h(j+1,k-1)-150.*h(j,k-1)
     7+16.*srivl(k-1))+ W3*(368.*T(J)*srivl(k)+16.*HMT2*srivl(k))

      DO J=2,JMAX-2

      FC6(J,K)=(24*S(J)*DELX(J)**2/DELT)*H(J,K-1)
     1-w1*HMT4*(-h(j+2,k-2)+3.*h(j+1,k-2)+21.*h(j,k-2)-23.*h(j-1,k-2))
     2-w2*HMT4*(-h(j+2,k-1)+3.*h(j+1,k-1)+21.*h(j,k-1)-23.*h(j-1,k-1))
     3+w1*HMT2*(-h(j+2,k-2)+27.*h(j+1,k-2)-27.*h(j,k-2)+h(j-1,k-2))
     4+w2*HMT2*(-h(j+2,k-1)+27.*h(j+1,k-1)-27.*h(j,k-1)+h(j-1,k-1))

      END DO
```

```
      J=JMAX-1
      FC6(J,K)=(24*S(J)*DELX(J)**2/DELT)*H(J,K-1)
     1-w1*HMT4*(h(j-2,k-2)-27.*h(j-1,k-2)+27.*h(j,k-2)-h(j+1,k-2))
     2-w2*HMT4*(h(j-2,k-1)-27.*h(j-1,k-1)+27.*h(j,k-1)-h(j+1,k-1))
     3+w1*HMT2*(h(j-2,k-2)-3.*h(j-1,k-2)-21.*h(j,k-2)+23.*h(j+1,k-2))
     4+w2*HMT2*(h(j-2,k-1)-3.*h(j-1,k-1)-21.*h(j,k-1)+23.*h(j+1,k-1))
      J=JMAX
      FC6(J,K)=(120.*S(J)*DELX(J)**2/DELT)*H(J,K-1)-w1*HMT4*
     1(-16.*srivr(k-2)+150.*h(j,k-2)-140.*h(j-1,k-2)+6.*h(j-2,k-2))
     3+w1*T(j)*(368.*srivr(k-2)-450.*h(j,k-2)+100.*h(j-1,k-2)
     4-18.*h(j-2,k-2))-w2*HMT4*
     5(-16.*srivr(k-1)+150.*h(j,k-1)-140.*h(j-1,k-1)+6.*h(j-2,k-1))
     6+w2*T(j)*(368.*srivr(k-1)-450.*h(j,k-1)+100.*h(j-1,k-1)-
     718.*h(j-2,k-1))+ w3*(16.*HMT4*srivr(k)+368.*T(j)*srivr(k))
      INDEX=0
      DO J=1,JMAX
      INDEX=INDEX+1
      RIGHT(INDEX)=-FC6(J,K)
      END DO
C     WRITE(2,*)'K=',K
C     WRITE(2,*)'RIGHTHAND COLUMN MATRIX'
C     WRITE(2,*)(RIGHT(J),J=1,JMAX)
      DO I=1,M
      HSOL(I)=0.
      DO J=1,M
      HSOL(I)=HSOL(I)+A(I,J)*RIGHT(J)
      END DO
      END DO
C
      INDEX=0
      DO J=1,JMAX
      INDEX=INDEX+1
      H(J,K)=HSOL(INDEX)
      END DO



      K=K+1
      IF(K.LE.NTIME)GO TO 1000
C     WRITE(2,*)'COMPUTATION OVER'
C     WRITE(2,*)'K=',K
C     WRITE(2,*)'K=',K
C     WRITE(2,*) 'HEAD VALUES ARE WRITTEN HERE'

      DO K=1,NTIME
C     WRITE(2,*)'TIME STEP=',K
      WRITE(2,222)(H(J,K), J=1,JMAX)
      END DO
222   FORMAT(10F8.3)

      DO K=1,NTIME
      DO J=1,JMAX
      RISE(J,K)=H(J,K)-HI
      END DO
      END DO
      WRITE(2,*)'RISE IN PIEZOMETRIC SURFACE COMPUTED NUMERICALLY'

      DO K=1,NTIME
      WRITE(2,333)(RISE(J,K),J=1,JMAX)
      END DO
C     WRITE(2,*)'PIEZOMETRIC RISE OBTAINED ANALLYTICALLY'
```

```
      DISTX=DELX(1)/2.
      T=TRANS*DELT
      PHI=STORC
      DO J=1,NTIME
      CALL  DISCK(DISTX,T,PHI,J,RES)
      DKER(J)=RES
      END DO

      DO N=1,NTIME
      SUM=0.
      DO NGAMA=1,N
      SUM=SUM+(SIGMA(NGAMA)-SIGMA(NGAMA-1))*DKER(N-NGAMA+1)
      END DO
      TRISE(1,N)=SUM
      END DO

333   format(10f7.4)


      DISTX=DELX(1)+DELX(2)/2.
      DO J=1,NTIME
      CALL  DISCK(DISTX,T,PHI,J,RES)
      DKER(J)=RES
      END DO

      DO N=1,NTIME
      SUM=0.
      DO NGAMA=1,N
      SUM=SUM+(SIGMA(NGAMA)-SIGMA(NGAMA-1))*DKER(N-NGAMA+1)
      END DO
      TRISE(2,N)=SUM
      END DO

      write(2,333)(trise(1,k),k=1,ntime)
      WRITE(2,*)'RISE AT SECOND NODE'
      write(2,333)(trise(2,k),k=1,ntime)
      STOP
      END


      SUBROUTINE MATIN (AAA,MMM)
      DIMENSION AAA(25,25),B(25),C(25)
      NN=MMM-1
      AAA(1,1)=1./AAA(1,1)
      DO 8 M=1,NN
      K=M+1
      DO 3 I=1,M
      B(I)=0.0
      DO 3 J=1,M
3     B(I)=B(I)+AAA(I,J)*AAA(J,K)
      D=0.0
      DO 4 I=1,M
4     D=D+AAA(K,I)*B(I)
      D=-D+AAA(K,K)
      AAA(K,K)=1./D
      DO 5 I=1,M
5     AAA(I,K)=-B(I)*AAA(K,K)
      DO 6 J=1,M
      C(J)=0.0
      DO 6 I=1,M
6     C(J)=C(J)+AAA(K,I)*AAA(I,J)
```

```
      DO 7 J=1,M
 7    AAA(K,J)=-C(J)*AAA(K,K)
      DO 8 I=1,M
      DO 8 J=1,M
 8    AAA(I,J)=AAA(I,J)-B(I)*AAA(K,J)
      RETURN
      END


      SUBROUTINE ERF(X,ERFX)
      XINDEX=X
      X1=X
      IF(X)4,5,5
 4    X1=-X
 5    CONTINUE
      IF(X1-15.)1,2,2
 1    CONTINUE
      T=1.0/(1.0+0.3275911*X1)
      ERFX=1.0-(0.25482959*T-0.28449673*T**2+1.42141374*T**3-1.
     1 45315202*T**4+1.06140542*T**5)*EXP(-X1**2)
      GO TO3
 2    ERFX=1.
 3    CONTINUE
      IF(XINDEX)6,7,7
 6    ERFX=-ERFX
 7    CONTINUE
C     WRITE(2,52)X,ERFX
C52   FORMAT(2F10.5)
      RETURN
      END

      SUBROUTINE DISCK(DISTX,T,PHI,J,RES)
      BETA=T/PHI
      PAI=3.14159265
      C1=2.*SQRT(BETA)
      AJ=J
      IF (J.EQ.1)GO TO 100
      X=DISTX/( C1*SQRT(AJ-1.) )
      TERM5=EXP(-X*X)
      CALL ERF(X,ERFX)
      TERM1=ERFX
      X=DISTX/( C1*SQRT(AJ) )
      TERM6=EXP(-X*X)
      CALL ERF(X,ERFX)
      TERM2=ERFX
      TERM3=DISTX*SQRT( (AJ-1.) / (BETA*PAI) )
      TERM4=DISTX*SQRT( (AJ)/ (BETA*PAI) )
      RES=1.+( (AJ-1.)+DISTX**2/(2.*BETA) )*TERM1
     1 -( AJ+DISTX**2/(2.*BETA) )*TERM2+TERM3*TERM5-TERM4*TERM6
      IF(RES.LT.0.0)RES=0.0
      IF(abs(RES).LT.0.00001)RES=0.0
      GO TO 200
100   X=DISTX/C1
      E=EXP(-X*X)
      CALL ERF(X,ERFX)
      RES=1.-ERFX+DISTX**2/(2.*BETA)*(1.-ERFX)-DISTX/SQRT(BETA*PAI)*E
      IF(RES.LT.0.0)RES=0.0
200   CONTINUE
      RETURN
      END
```

APPENDIX - 2B

```
C     THIS PROGRAMME COMPUTES HEADS BY THE IMPROVED FINITE DIFFERENCE
C     METHOD AT VARIOUS NODES

C     THE GRIDS ARE UNEQUAL, THE AQUIFER IS NON-HOMOGENEOUS

      DIMENSION  DELX(25),DELXD(25),T(25),S(25),TRISE(25,100)

      DIMENSION SRIVR(0:100),H(25,0:100),SRIVL(0:100),RISE(25,100)

      DIMENSION  FC1(25),FC2(25),FC4(25),FC22(25),
     1             FC44(25),FC6(25,100),HSOL(25),
     2               A(25,25),RIGHT(25)
      DIMENSION   AA(25,25),AAAA(25,25)

      OPEN(1,FILE='SREST1.DAT',STATUS='OLD')
      OPEN(2,FILE='SREST1.OUT',STATUS='UNKNOWN')


      READ(1,*)JMAX,DELTAX
c     READ(1,*)TRANS,STORC
      READ(1,*) DELT,NTIME
      READ(1,*)BOUNHL,BOUNHR,HEADI

C
C     TRANSMISSIVITY VALUES ARE BEING READ
C
      READ(1,*)  (T(J),J=1,JMAX)
      WRITE(2,54)(T(J),J=1,JMAX)
54    FORMAT(6F8.2)


c     DO J=1,JMAX
c     T(J)=TRANS
c     END DO

C
C
C     STORAGE COEFFICIENT VALUES ARE BEING READ
C
      READ(1,*)  (S(J),J=1,JMAX)
      WRITE(2,54)(S(J),J=1,JMAX)
C
c     DO J=1,JMAX
c     S(J)=STORC
c     END DO
C
C     GRID SIZES ARE BEING READ
C
      READ(1,*)  (DELX(J),J=1,JMAX)
      WRITE(2,54)(DELX(J),J=1,JMAX)
C
c     DO J=1,JMAX
c     DELX(J)       =DELTAX
c     END DO
C     WRITE(2,60)
60    FORMAT(2X,'JMAX',5X,'DELTAX')
C     WRITE(2,51)JMAX,DELTAX
51    FORMAT(2X,I2,4X,F8.2)
```

```
      WRITE(2,70)
70    FORMAT(2X,'NTIME',6X,'DELT')
      WRITE(2,51)NTIME,DELT

      WRITE(2,61)
61    FORMAT(5X,'BOUNHL', 5X,  'BOUNHR', 5X, 'HEADI')
      WRITE(2,52)BOUNHL,BOUNHR,HEADI
52    FORMAT(3F10.2)

      DO J=1,JMAX
      H(J,0)=HEADI
      END DO

      SRIVR(0)=BOUNHR
C     SRIVL(0)=BOUNHL
      SRIVL(0)=HEADI

      DO K=1,NTIME
      SRIVR(K)=BOUNHR
      SRIVL(K)=BOUNHL
      END DO
C
      M=JMAX
      DO I=1,M
      DO J=1,M
      A(I,J)=0.
      END DO
      END DO

      K=1

      W2=0.5
      W3=0.5

      J=1

      HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))

      DELXD(J+1)=(DELX(J)+DELX(J+1))/2.
      DELXD(J+2)=(DELX(J+1)+DELX(J+2))/2.

      D11=DELXD(J+1)**2
      D12=DELXD(J+1)**3
      D21=(DELXD(J+1)+DELXD(J+2))**2
      D22=(DELXD(J+1)+DELXD(J+2))**3

      DETERM1=D11*D22-D12*D21

      D11=DELXD(J+1)
      D12=DELXD(J+1)**3
      D21=DELXD(J+1)+DELXD(J+2)
      D22=(DELXD(J+1)+DELXD(J+2))**3

      DETERM2=D11*D22-D12*D21

      D11=DELXD(J+1)
      D12=DELXD(J+1)**2
      D21=DELXD(J+1)+DELXD(J+2)
      D22=(DELXD(J+1)+DELXD(J+2))**2
```

```
      DETERM3=D11*D22-D12*D21

      DETER=(-DELX(J)/2.)*DETERM1-(DELX(J)**2/4.)*DETERM2+
     1   (-DELX(J)**3/8.)*DETERM3

      D11=DELXD(J+1)**2
      D12=(DELXD(J+1)+DELXD(J+2))**2
      D21=DELXD(J+1)**3
      D22=(DELXD(J+1)+DELXD(J+2))**3

      D41=(D11*D22-D12*D21)/DETER

      D11=DELX(J)**2/4.
      D12=(DELXD(J+1)+DELXD(J+2))**2
      D21=-DELX(J)**3/8.
      D22=(DELXD(J+1)+DELXD(J+2))**3

      D42=(D11*D22-D12*D21)/DETER

      D11=DELX(J)**2/4.
      D12=DELXD(J+1)**2
      D21=-DELX(J)**3/8.
      D22=DELXD(J+1)**3

      D43=(D11*D22-D12*D21)/DETER

      D11=DELXD(J+1)
      D12=DELXD(J+1)+DELXD(J+2)
      D21=DELXD(J+1)**3
      D22=(DELXD(J+1)+DELXD(J+2))**3

      D44=(D11*D22-D12*D21)/DETER

      D11=-DELX(J)/2.
      D12=DELXD(J+1)+DELXD(J+2)
      D21=-DELX(J)**3/8.
      D22=(DELXD(J+1)+DELXD(J+2))**3

      D45=(D11*D22-D12*D21)/DETER

      D11=-DELX(J)/2.
      D12=DELXD(J+1)
      D21=-DELX(J)**3/8.
      D22=DELXD(J+1)**3

      D46=(D11*D22-D12*D21)/DETER

      D11=DELXD(J+1)
      D12=DELXD(J+1)+DELXD(J+2)
      D21=DELXD(J+1)**2
      D22=(DELXD(J+1)+DELXD(J+2))**2

      D47=(D11*D22-D12*D21)/DETER

      D11=-DELX(J)/2.
      D12=DELXD(J+1)+DELXD(J+2)
      D21=DELX(J)**2/4.
      D22=(DELXD(J+1)+DELXD(J+2))**2

      D48=(D11*D22-D12*D21)/DETER
```

```
 D11=-DELX(J)/2.
 D12=DELXD(J+1)
 D21=DELX(J)**2/4.
 D22=DELXD(J+1)**2

 D49=(D11*D22-D12*D21)/DETER


 FC22(J)=W3*(-T(J)*(D43+D46*DELX(J)+D49*.75*DELX(J)**2)+
1HMT2*(D43-D46*DELX(J)+D49*.75*DELX(J)**2))

 FC2(J)=W3*(-T(J)*(-D42-D45*DELX(J)-D48*.75*DELX(J)**2)+HMT2*
1(-D42+D45*DELX(J)-D48*.75*DELX(J)**2))

 FC1(J)=W3*(-T(J)*(-D41+D42-D43-(D44-D45+D46)*DELX(J)+
1(-D47+D48-D49)*.75*DELX(J)**2)+HMT2*(-D41+D42-D43+
2(D44-D45+D46)*DELX(J)+(-D47+D48-D49)*.75*DELX(J)**2))-
3S(J)*DELX(J)/DELT
 DO J=2,JMAX-2
 HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
 HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))

 DELXD(J)   =(DELX(J-1)+DELX(J))   /2.
 DELXD(J+1)=(DELX(J)   +DELX(J+1))/2.
 DELXD(J+2)=(DELX(J+1)+DELX(J+2))/2.

 D11=DELXD(J+1)**2
 D12=DELXD(J+1)**3
 D21=(DELXD(J+1)+DELXD(J+2))**2
 D22=(DELXD(J+1)+DELXD(J+2))**3
 DETERM1=D11*D22-D12*D21

 D11=DELXD(J+1)
 D12=DELXD(J+1)**3
 D21=DELXD(J+1)+DELXD(J+2)
 D22=(DELXD(J+1)+DELXD(J+2))**3
 DETERM2=D11*D22-D12*D21

 D11=DELXD(J+1)
 D12=DELXD(J+1)**2
 D21=DELXD(J+1)+DELXD(J+2)
 D22=(DELXD(J+1)+DELXD(J+2))**2
 DETERM3=D11*D22-D12*D21

 DETER=-DELXD(J)*DETERM1-DELXD(J)**2*DETERM2-DELXD(J)**3*DETERM3

 D11=DELXD(J+1)**2
 D12=(DELXD(J+1)+DELXD(J+2))**2
 D21=DELXD(J+1)**3
 D22=(DELXD(J+1)+DELXD(J+2))**3

 D1=(D11*D22-D12*D21)/DETER

 D11=DELXD(J)**2
 D12=(DELXD(J+1)+DELXD(J+2))**2
 D21=-DELXD(J)**3
 D22=(DELXD(J+1)+DELXD(J+2))**3

 D2=(D11*D22-D12*D21)/DETER

 D11=DELXD(J)**2
```

```
      D12=DELXD(J+1)**2
      D21=-DELXD(J)**3
      D22=DELXD(J+1)**3

      D3=(D11*D22-D12*D21)/DETER

      D11=DELXD(J+1)
      D12=DELXD(J+1)+DELXD(J+2)
      D21=DELXD(J+1)**3
      D22=(DELXD(J+1)+DELXD(J+2))**3

      D4=(D11*D22-D12*D21)/DETER

      D11=-DELXD(J)
      D12=DELXD(J+1)+DELXD(J+2)
      D21=-DELXD(J)**3
      D22=(DELXD(J+1)+DELXD(J+2))**3

      D5=(D11*D22-D12*D21)/DETER

      D11=-DELXD(J)
      D12=DELXD(J+1)
      D21=-DELXD(J)**3
      D22=DELXD(J+1)**3

      D6=(D11*D22-D12*D21)/DETER

      D11=DELXD(J+1)
      D12=DELXD(J+1)+DELXD(J+2)
      D21=DELXD(J+1)**2
      D22=(DELXD(J+1)+DELXD(J+2))**2

      D7=(D11*D22-D12*D21)/DETER

      D11=-DELXD(J)
      D12=DELXD(J+1)+DELXD(J+2)
      D21=DELXD(J)**2
      D22=(DELXD(J+1)+DELXD(J+2))**2

      D8=(D11*D22-D12*D21)/DETER

      D11=-DELXD(J)
      D12=DELXD(J+1)
      D21=DELXD(J)**2
      D22=DELXD(J+1)**2

      D9=(D11*D22-D12*D21)/DETER


      FC22(J)=W3*(-HMT4*(D3+D6*DELX(J)+D9*.75*DELX(J)**2)+
     1HMT2*(D3-D6*DELX(J)+D9*.75*DELX(J)**2))

      FC2(J)=W3*(-HMT4*(-D2-D5*DELX(J)-D8*.75*DELX(J)**2)+
     1HMT2*(-D2+D5*DELX(J)-D8*.75*DELX(J)**2))

      FC1(J)=W3*(-HMT4*(-D1+D2-D3-(D4-D5+D6)*DELX(J)+
     1(-D7+D8-D9)*.75*DELX(J)**2)+
     2HMT2*(-D1+D2-D3+(D4-D5+D6)*DELX(J)+
     3(-D7+D8-D9)*.75*DELX(J)**2))-S(J)*DELX(J)/DELT

      FC4(J)=W3*(-HMT4*(D1+D4*DELX(J)+D7*.75*DELX(J)**2)+
```

```
1HMT2*(D1-D4*DELX(J)+D7*.75*DELX(j)**2))
 END DO

 J=JMAX-1
 HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
 HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))

 DELXD(J-1)=(DELX(J-2)+DELX(J-1))/2.
 DELXD(J)=(DELX(J-1)+DELX(J))/2.
 DELXD(J+1)=(DELX(J)+DELX(J+1))/2.

 D11=DELXD(J)**2
 D12=-DELXD(J)**3
 D21=DELXD(J+1)**2
 D22=DELXD(J+1)**3

 DETERM1=D11*D22-D12*D21

 D11=-DELXD(J)
 D12=-DELXD(J)**3
 D21=DELXD(J+1)
 D22=DELXD(J+1)**3

 DETERM2=D11*D22-D12*D21

 D11=-DELXD(J)
 D12=DELXD(J)**2
 D21=DELXD(J+1)
 D22=DELXD(J+1)**2
 DETERM3=D11*D22-D12*D21

 DETER=-(DELXD(J)+DELXD(J-1))*DETERM1-(DELXD(J)+DELXD(J-1))**2
1*DETERM2-(DELXD(J)+DELXD(J-1))**3*DETERM3

 D11=DELXD(J)**2
 D12=DELXD(J+1)**2
 D21=-DELXD(J)**3
 D22=DELXD(J+1)**3

 D31=(D11*D22-D12*D21)/DETER

 D11=(DELXD(J)+DELXD(J-1))**2
 D12=DELXD(J+1)**2
 D21=-(DELXD(J)+DELXD(J-1))**3
 D22=DELXD(J+1)**3

 D32=(D11*D22-D12*D21)/DETER

 D11=(DELXD(J)+DELXD(J-1))**2
 D12=DELXD(J)**2
 D21=-(DELXD(J)+DELXD(J-1))**3
 D22=-DELXD(J)**3

 D33=(D11*D22-D12*D21)/DETER

 D11=-DELXD(J)
 D12=DELXD(J+1)
 D21=-DELXD(J)**3
 D22=DELXD(J+1)**3

 D34=(D11*D22-D12*D21)/DETER
```

```
      D11=-(DELXD(J)+DELXD(J-1))
      D12=DELXD(J+1)
      D21=-(DELXD(J)+DELXD(J-1))**3
      D22=DELXD(J+1)**3

      D35=(D11*D22-D12*D21)/DETER

      D11=-(DELXD(J)+DELXD(J-1))
      D12=-DELXD(J)
      D21=-(DELXD(J)+DELXD(J-1))**3
      D22=-DELXD(J)**3

      D36=(D11*D22-D12*D21)/DETER

      D11=-DELXD(J)
      D12=DELXD(J+1)
      D21=DELXD(J)**2
      D22=DELXD(J+1)**2


      D37=(D11*D22-D12*D21)/DETER

      D11=-(DELXD(J)+DELXD(J-1))
      D12=DELXD(J+1)
      D21=(DELXD(J)+DELXD(J-1))**2
      D22=DELXD(J+1)**2

      D38=(D11*D22-D12*D21)/DETER

      D11=-(DELXD(J)+DELXD(J-1))
      D12=-DELXD(J)
      D21=(DELXD(J)+DELXD(J-1))**2
      D22=DELXD(J)**2

      D39=(D11*D22-D12*D21)/DETER


      FC2(J)=W3*(-HMT4*(D33+D36*DELX(J)+D39*.75*DELX(J)**2)+
     1HMT2*(D33-D36*DELX(J)+D39*.75*DELX(J)**2))

      FC1(J)=W3*(-HMT4*(-D31+D32-D33-(D34-D35+D36)*DELX(J)+
     1(-D37+D38-D39)*.75*DELX(J)**2)+HMT2*(-D31+D32-D33+
     2(D34-D35+D36)*DELX(J)+(-D37+D38-D39)*.75*DELX(J)**2))-
     3S(J)*DELX(J)/DELT

      FC4(J)=W3*(-HMT4*(-D32-D35*DELX(J)-D38*.75*DELX(J)**2)+
     1HMT2*(-D32+D35*DELX(J)-D38*.75*DELX(J)**2))

      FC44(J)=W3*(-HMT4*(D31+D34*DELX(J)+D37*.75*DELX(J)**2)+
     1HMT2*(D31-D34*DELX(J)+D37*.75*DELX(J)**2))
      J=JMAX
      HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))
      DELXD(J-1)=(DELX(J-2)+DELX(J-1))/2.
      DELXD(J)=(DELX(J-1)+DELX(J))/2.

      D11=DELXD(J)**2
      D12=-DELXD(J)**3
      D21=DELX(J)**2/4.
      D22=DELX(J)**3/8.
```

```
DETERM1=D11*D22-D12*D21

D11=-DELXD(J)
D12=-DELXD(J)**3
D21=DELX(J)/2.
D22=DELX(J)**3/8.

DETERM2=D11*D22-D12*D21

D11=-DELXD(J)
D12=DELXD(J)**2
D21=DELX(J)/2.
D22=DELX(J)**2/4.

DETERM3=D11*D22-D12*D21

DETER=-(DELXD(J)+DELXD(J-1))*DETERM1-(DELXD(J)+DELXD(J-1))**2*
1DETERM2-(DELXD(J)+DELXD(J-1))**3*DETERM3

D11=DELXD(J)**2
D12=DELX(J)**2/4.
D12=-DELXD(J)**3
D21=DELX(J)**3/8.

D51=(D11*D22-D12*D21)/DETER

D11=(DELXD(J)+DELXD(J-1))**2
D12=DELX(J)**2/4.
D21=-(DELXD(J)+DELXD(J-1))**3
D22=DELX(J)**3/8.

D52=(D11*D22-D12*D21)/DETER

D11=(DELXD(J)+DELXD(J-1))**2
D12=DELXD(J)**2
D21=-(DELXD(J)+DELXD(J-1))**3
D22=-DELXD(J)**3

D53=(D11*D22-D12*D21)/DETER

D11=-DELXD(J)
D12=DELX(J)/2.
D21=-DELXD(J)**3
D22=DELX(J)**3/8.

D54=(D11*D22-D12*D21)/DETER

D11=-(DELXD(J)+DELXD(J-1))
D12=DELX(J)/2.
D21=-(DELXD(J)+DELXD(J-1))**3
D22=DELX(J)**3/8.

D55=(D11*D22-D12*D21)/DETER

D11=-(DELXD(J)+DELXD(J-1))
D12=-DELXD(J)
D21=-(DELXD(J)+DELXD(J-1))**3
D22=-DELXD(J)**3

D56=(D11*D22-D12*D21)/DETER
```

```
      D11=-DELXD(J)
      D12=DELX(J)/2.
      D21=DELXD(J)**2
      D22=DELX(J)**2/4.

      D57=(D11*D22-D12*D21)/DETER

      D11=-(DELXD(J)+DELXD(J-1))
      D12=DELX(J)/2.
      D21=(DELXD(J)+DELXD(J-1))**2
      D22=DELX(J)**2/4.

      D58=(D11*D22-D12*D21)/DETER

      D11=-(DELXD(J)+DELXD(J-1))
      D12=-DELXD(J)
      D21=(DELXD(J)+DELXD(J-1))**2
      D22=DELXD(J)**2

      D59=(D11*D22-D12*D21)/DETER

      FC1(J)=W3*(-HMT4*(-D51+D52-D53-(D54-D55+D56)*DELX(J)+
     1(-D57+D58-D59)*.75*DELX(J)**2)+T(J)*(-D51+D52-D53+
     2(D54-D55+D56)*DELX(J)+(-D57+D58-D59)*.75*DELX(J)**2))-
     3S(J)*DELX(J)/DELT

      FC4(J)=w3*(-HMT4*(-D52-D55*DELX(J)-D58*.75*DELX(J)**2)+
     1T(J)*(-D52+D55*DELX(J)-D58*.75*DELX(J)**2))

      FC44(J)=w3*(-HMT4*(D51+D54*DELX(J)+D57*.75*DELX(J)**2)+
     1T(J)*(D51-D54*DELX(J)+D57*.75*DELX(J)**2))
      I=1
      A(I,I)=FC1(I)
      A(I,I+1)=FC2(I)
      A(I,I+2)=FC22(I)

      DO I=2,JMAX-2
      A(I,I-1)=FC4(I)
      A(I,I)=FC1(I)
      A(I,I+1)=FC2(I)
      A(I,I+2)=FC22(I)
      END DO
      I=JMAX-1
      A(I,I+1)=FC2(I)
      A(I,I)=FC1(I)
      A(I,I-1)=FC4(I)
      A(I,I-2)=FC44(I)
      I=JMAX
      A(I,I)=FC1(I)
      A(I,I-1)=FC4(I)
      A(I,I-2)=FC44(I)
C     WRITE(2,*)'ELEMENT OF THE MATRIX FOR K=1'
      DO I=1,M
C     WRITE(2,197)(A(I,J),J=1,M)
      END DO

C     STORING OF THE MATRIX ELEMENT
      DO I=1,M
      DO J=1,M
      AA(I,J)=A(I,J)
      END DO
```

```
      END DO

C     CALL MATRIX INVERSE
C     WRITE(2,*)'WRITING THE MATRIX ELEMENT BEFORE INVERSION'
C     DO I=1,M
C     WRITE(2,197)( A(I,J),J=1,M)
C     END DO
197   FORMAT(7E10.4)
C
C
      CALL MATIN (A,M)
C
C
C     WRITE(2,*)'MATRIX ELEMENT AFTER INVERSION'
C     DO I=1,M
C     WRITE(2,197)(A(I,J),J=1,M)
C     END DO

C     WRITE(2,*)'CHECKING THE MATRIX INVERSION'
      KK=0
      DO I=1,M
      SUM=0
      KK=KK+1
      DO J=1,M
      SUM=SUM+AA(I,J)*A(J,I)
      END DO
      AAAA(I,KK)=SUM
      END DO

      DO I=1,M
C     WRITE(2,197)(AAAA(I,J),J=1,M)
      END DO

C     SOLUTION FOR THE FIRST TIME STEP


C     FC6(J,1)  AT  INTERIOR NODES

      K=1
      J=1
      HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))


        FC6(J,K)=W2*(-T(J)*(D43+D46*DELX(J)+D49*.75*DELX(J)**2)
     1 +HMT2*(D43-D46*DELX(J)+D49*.75*DELX(J)**2))*H(J+2,K-1)+
     2  W2*(-T(J)*(-D42-D45*DELX(J)-D48*.75*DELX(J)**2)+HMT2*
     3  (-D42+D45*DELX(J)-D48*.75*DELX(J)**2))*H(J+1,K-1)+
     4  W2*(-T(J)*(-D41+D42-D43-(D44-D45+D46)*DELX(J)+
     5 (-D47+D48-D49)*.75*DELX(J)**2)+HMT2*(-D41+D42-D43+
     6 (D44-D45+D46)*DELX(J)+(-D47+D48-D49)*.75*DELX(J)**2))*H(J,k-1)+
     7 W2*(-T(J)*(D41+D44*DELX(J)+D47*.75*DELX(J)**2)+HMT2*(D41-D44*
     8 DELX(J)+D47*.75*DELX(J)**2))*SRIVL(K-1)+
     9 S(J)*DELX(J)/DELT*H(J,K-1)
      FC6(J,K)=FC6(J,K)+ W3*(-T(J)*(D41+D44*DELX(J)
     1 +D47*.75*DELX(J)**2)+HMT2*
     2 (D41-D44*DELX(J)+D47*.75*DELX(J)**2))*SRIVL(K)


       DO J=2,JMAX-2
      HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
```

```
      HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))

      A1=W2*(-HMT4*(D3+D6*DELX(J)+D9*.75*DELX(J)**2)+
     1       HMT2*(D3-D6*DELX(J)+D9*.75*DELX(J)**2))*H(J+2,K-1)+
     2       W2*(-HMT4*(-D2-D5*DELX(J)-D8*.75*DELX(J)**2)+
     3       HMT2*(-D2+D5*DELX(J)-D8*.75*DELX(J)**2))*H(J+1,K-1)+
     4 W2*(-HMT4*(-D1+D2-D3-(D4-D5+D6)*DELX(J)+
     5 (-D7+D8-D9)*.75*DELX(J)**2)+
     6       HMT2*(-D1+D2-D3+(D4-D5+D6)*DELX(J)+
     7 (-D7+D8-D9)*.75*DELX(J)**2))*H(J,K-1)+
     8 W2*(-HMT4*(D1+D4*DELX(J)+D7*.75*DELX(J)**2)+
     9       HMT2*(D1-D4*DELX(J)+D7*.75*DELX(j)**2))*H(J-1,K-1)
      FC6(J,K)= A1+S(J)*DELX(J)/DELT*H(J,K-1)
      END DO

      J=JMAX-1

      HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
      HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))

      FC6(J,k)=W2*(-HMT4*(D33+D36*DELX(J)+D39*.75*DELX(J)**2)+
     1 HMT2*(D33-D36*DELX(J)+D39*.75*DELX(J)**2))*H(J+1,K-1)+
     2 W2*(-HMT4*(-D31+D32-D33-(D34-D35+D36)*DELX(J)+
     3 (-D37+D38-D39)*.75*DELX(J)**2)+HMT2*(-D31+D32-D33+
     4 (D34-D35+D36)*DELX(J)+(-D37+D38-D39)*.75*DELX(J)**2))*H(J,K-1)+
     5 W2*(-HMT4*(-D32-D35*DELX(J)-D38*.75*DELX(J)**2)+
     6 HMT2*(-D32+D35*DELX(J)-D38*.75*DELX(J)**2))*H(J-1,K-1)+
     7 W2*(-HMT4*(D31+D34*DELX(J)+D37*.75*DELX(J)**2)+
     8 HMT2*(D31-D34*DELX(J)+D37*.75*DELX(J)**2))*H(J-2,K-1)+
     9 S(J)*DELX(J)/DELT*H(J,K-1)

      J=JMAX
      HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))

      A2=W2*(-HMT4*(-D51+D52-D53-(D54-D55+D56)*DELX(J)+
     1 (-D57+D58-D59)*.75*DELX(J)**2)+T(J)*(-D51+D52-D53+
     2 (D54-D55+D56)*DELX(J)+(-D57+D58-D59)*.75*DELX(J)**2))*H(J,K-1)+
     3 w2*(-HMT4*(-D52-D55*DELX(J)-D58*.75*DELX(J)**2)+
     4       T(J)*(-D52+D55*DELX(J)-D58*.75*DELX(J)**2))*H(J-1,K-1)+
     5 w2*(-HMT4*(D51+D54*DELX(J)+D57*.75*DELX(J)**2)+
     6 T(J)*(D51-D54*DELX(J)+D57*.75*DELX(J)**2))*H(J-2,K-1)+
     7       W2*(-HMT4*(D53+D56*DELX(J)+D59*.75*DELX(J)**2)+
     8 T(J)*(D53-D56*DELX(J)+D59*.75*DELX(J)**2))*SRIVR(K-1)+
     9       S(J)*DELX(J)/DELT*H(J,K-1)

      FC6(J,K)=A2+W3*(-HMT4*(D53+D56*DELX(J)+
     1 D59*.75*DELX(J)**2)+T(J)*(D53-D56*DELX(J)+D59*.75*DELX(J)**2))*
     2 SRIVR(K)

C
      INDEX=0
      DO J=1,JMAX
      INDEX=INDEX+1
      RIGHT(INDEX)=-FC6(J,K)
      END DO

C     WRITE(2,*)'K=',K
C     WRITE(2,*)'RIGHTHAND COLUMN MATRIX'
C     WRITE(2,197)(RIGHT(J),J=1,JMAX)
```

```
      DO I=1,M
      HSOL(I)=0.
      DO J=1,M
      HSOL(I)=HSOL(I)+A(I,J)*RIGHT(J)
      END DO
      END DO
C
      INDEX=0
      DO J=1,JMAX
      INDEX=INDEX+1
      H(J,K)=HSOL(INDEX)
      END DO

C     SOLUTION FOR K=2
      M=JMAX

      DO I=1,M
      DO J=1,M
      A(I,J)=0.
      END DO
      END DO
      K=2
      W1=-1./12.
      W2=2./3.
      W3=5./12.

      J=1
      HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))

      FC22(J)=W3*(-T(J)*(D43+D46*DELX(J)+D49*.75*DELX(J)**2)+
     1HMT2*(D43-D46*DELX(J)+D49*.75*DELX(J)**2))

      FC2(J)=W3*(-T(J)*(-D42-D45*DELX(J)-D48*.75*DELX(J)**2)+HMT2*
     1(-D42+D45*DELX(J)-D48*.75*DELX(J)**2))

      FC1(J)=W3*(-T(J)*(-D41+D42-D43-(D44-D45+D46)*DELX(J)+
     1(-D47+D48-D49)*.75*DELX(J)**2)+HMT2*(-D41+D42-D43+
     2(D44-D45+D46)*DELX(J)+(-D47+D48-D49)*.75*DELX(J)**2))-
     3S(J)*DELX(J)/DELT



      DO J=2,JMAX-2

      HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
      HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))

      FC22(J)=W3*(-HMT4*(D3+D6*DELX(J)+D9*.75*DELX(J)**2)+
     1HMT2*(D3-D6*DELX(J)+D9*.75*DELX(J)**2))

      FC2(J)=W3*(-HMT4*(-D2-D5*DELX(J)-D8*.75*DELX(J)**2)+
     1HMT2*(-D2+D5*DELX(J)-D8*.75*DELX(J)**2))

      FC1(J)=W3*(-HMT4*(-D1+D2-D3-(D4-D5+D6)*DELX(J)+
     1(-D7+D8-D9)*.75*DELX(J)**2)+
     2HMT2*(-D1+D2-D3+(D4-D5+D6)*DELX(J)+
     3(-D7+D8-D9)*.75*DELX(J)**2))-S(J)*DELX(J)/DELT

      FC4(J)=W3*(-HMT4*(D1+D4*DELX(J)+D7*.75*DELX(J)**2)+
     1HMT2*(D1-D4*DELX(J)+D7*.75*DELX(j)**2))
```

```
      END DO

      J=JMAX-1

     HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
     HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))

      FC2(J)=W3*(-HMT4*(D33+D36*DELX(J)+D39*.75*DELX(J)**2)+
     1 HMT2*(D33-D36*DELX(J)+D39*.75*DELX(J)**2))

      FC1(J)=W3*(-HMT4*(-D31+D32-D33-(D34-D35+D36)*DELX(J)+
     1 (-D37+D38-D39)*.75*DELX(J)**2)+HMT2*(-D31+D32-D33+
     2 (D34-D35+D36)*DELX(J)+(-D37+D38-D39)*.75*DELX(J)**2))-
     3 S(J)*DELX(J)/DELT

      FC4(J)=W3*(-HMT4*(-D32-D35*DELX(J)-D38*.75*DELX(J)**2)+
     1       HMT2*(-D32+D35*DELX(J)-D38*.75*DELX(J)**2))

      FC44(J)=W3*(-HMT4*(D31+D34*DELX(J)+D37*.75*DELX(J)**2)+
     1       HMT2*(D31-D34*DELX(J)+D37*.75*DELX(J)**2))

      J=JMAX

      HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))

      FC1(J)=W3*(-HMT4*(-D51+D52-D53-(D54-D55+D56)*DELX(J)+
     1 (-D57+D58-D59)*.75*DELX(J)**2)+T(J)*(-D51+D52-D53+
     2 (D54-D55+D56)*DELX(J)+(-D57+D58-D59)*.75*DELX(J)**2))-
     3 S(J)*DELX(J)/DELT

      FC4(J)=w3*(-HMT4*(-D52-D55*DELX(J)-D58*.75*DELX(J)**2)+
     1       T(J)*(-D52+D55*DELX(J)-D58*.75*DELX(J)**2))

      FC44(J)=w3*(-HMT4*(D51+D54*DELX(J)+D57*.75*DELX(J)**2)+
     1 T(J)*(D51-D54*DELX(J)+D57*.75*DELX(J)**2))
      I=1
      A(I,I)=FC1(I)
      A(I,I+1)=FC2(I)
      A(I,I+2)=FC22(I)

      DO I=2,JMAX-2
      A(I,I-1)=FC4(I)
      A(I,I)=FC1(I)
      A(I,I+1)=FC2(I)
      A(I,I+2)=FC22(I)
      END DO
      I=JMAX-1
      A(I,I+1)=FC2(I)
      A(I,I)=FC1(I)
      A(I,I-1)=FC4(I)
      A(I,I-2)=FC44(I)

      I=JMAX
      A(I,I)=FC1(I)
      A(I,I-1)=FC4(I)
      A(I,I-2)=FC44(I)

C     WRITE(2,*)'ELEMENT OF THE MATRIX FOR K=2'
```

```
      DO I=1,M
C     WRITE(2,197)(A(I,J),J=1,M)
      END DO

C     STORING OF THE MATRIX ELEMENT
      DO I=1,M
      DO J=1,M
      AA(I,J)=A(I,J)
      END DO
      END DO

C     CALL MATRIX INVERSE
C     WRITE(2,*)'WRITING THE MATRIX ELEMENT BEFORE INVERSION'
C     DO I=1,M
C     WRITE(2,197)( A(I,J),J=1,M)
C     END DO

      CALL MATIN (A,M)

C     WRITE(2,*)'MATRIX ELEMENT AFTER INVERSION'
C     DO I=1,M
C     WRITE(2,197)(A(I,J),J=1,M)
C     END DO
C     WRITE(2,*)'CHECKING THE MATRIX INVERSION'
      KK=0
      DO I=1,M
      SUM=0
      KK=KK+1
      DO J=1,M
      SUM=SUM+AA(I,J)*A(J,I)
      END DO
      AAAA(I,KK)=SUM
      END DO
      DO I=1,M
C     WRITE(2,197)(AAAA(I,J),J=1,M)
      END DO

C     SOLUTION FOR THE 2ND TIME STEP ONWARD
1000  CONTINUE
      J=1
      A1=W1*(-T(J)*(D43+D46*DELX(J)+D49*.75*DELX(J)**2)+
     1 HMT2*(D43-D46*DELX(J)+D49*.75*DELX(J)**2))*H(J+2,K-2)+
     2 W1*(-T(J)*(-D42-D45*DELX(J)-D48*.75*DELX(J)**2)+HMT2*
     3 (-D42+D45*DELX(J)-D48*.75*DELX(J)**2))*H(J+1,K-2)+
     4 W1*(-T(J)*(-D41+D42-D43-(D44-D45+D46)*DELX(J)+
     5 (-D47+D48-D49)*.75*DELX(J)**2)+HMT2*(-D41+D42-D43+
     6 (D44-D45+D46)*DELX(J)+(-D47+D48-D49)*.75*DELX(J)**2))*H(J,k-2)+
     7 W1*(-T(J)*(D41+D44*DELX(J)+D47*.75*DELX(J)**2)+HMT2*(D41-D44*
     8 DELX(J)+D47*.75*DELX(J)**2))*SRIVL(K-2)
       A2=S(J)*DELX(J)/DELT*H(J,K-1)
       A3= W3*(-T(J)*(D41+D44*DELX(J)+D47*.75*DELX(J)**2)+HMT2*
     1 (D41-D44*DELX(J)+D47*.75*DELX(J)**2))*SRIVL(K)
       A4=W2*(-T(J)*(D43+D46*DELX(J)+D49*.75*DELX(J)**2)+
     1 HMT2*(D43-D46*DELX(J)+D49*.75*DELX(J)**2))*H(J+2,K-1)+
     2 W2*(-T(J)*(-D42-D45*DELX(J)-D48*.75*DELX(J)**2)+HMT2*
     3 (-D42+D45*DELX(J)-D48*.75*DELX(J)**2))*H(J+1,K-1)+
     4 W2*(-T(J)*(-D41+D42-D43-(D44-D45+D46)*DELX(J)+
     5 (-D47+D48-D49)*.75*DELX(J)**2)+HMT2*(-D41+D42-D43+
     6 (D44-D45+D46)*DELX(J)+(-D47+D48-D49)*.75*DELX(J)**2))*H(J,k-1)+
     7 W2*(-T(J)*(D41+D44*DELX(J)+D47*.75*DELX(J)**2)+HMT2*(D41-D44*
     8 DELX(J)+D47*.75*DELX(J)**2))*SRIVL(K-1)
```

```
 FC6(J,K)=A1+A2+A3+A4
 DO J=2,JMAX-2
 A1=W1*(-HMT4*(D3+D6*DELX(J)+D9*.75*DELX(J)**2)+
1 HMT2*(D3-D6*DELX(J)+D9*.75*DELX(J)**2))*H(J+2,K-2)+
2 W1*(-HMT4*(-D2-D5*DELX(J)-D8*.75*DELX(J)**2)+
3 HMT2*(-D2+D5*DELX(J)-D8*.75*DELX(J)**2))*H(J+1,K-2)+
4 W1*(-HMT4*(-D1+D2-D3-(D4-D5+D6)*DELX(J)+
5 (-D7+D8-D9)*.75*DELX(J)**2)+
6 HMT2*(-D1+D2-D3+(D4-D5+D6)*DELX(J)+
7 (-D7+D8-D9)*.75*DELX(J)**2))*H(J,K-2)+
8 W1*(-HMT4*(D1+D4*DELX(J)+D7*.75*DELX(J)**2)+
9 HMT2*(D1-D4*DELX(J)+D7*.75*DELX(j)**2))*H(J-1,K-2)

 A2= S(J)*DELX(J)/DELT*H(J,K-1)

 A3=W2*(-HMT4*(D3+D6*DELX(J)+D9*.75*DELX(J)**2)+
1 HMT2*(D3-D6*DELX(J)+D9*.75*DELX(J)**2))*H(J+2,K-1)+
2 W2*(-HMT4*(-D2-D5*DELX(J)-D8*.75*DELX(J)**2)+
3 HMT2*(-D2+D5*DELX(J)-D8*.75*DELX(J)**2))*H(J+1,K-1)+
4 W2*(-HMT4*(-D1+D2-D3-(D4-D5+D6)*DELX(J)+
5 (-D7+D8-D9)*.75*DELX(J)**2)+
6 HMT2*(-D1+D2-D3+(D4-D5+D6)*DELX(J)+
7 (-D7+D8-D9)*.75*DELX(J)**2))*H(J,K-1)+
8 W2*(-HMT4*(D1+D4*DELX(J)+D7*.75*DELX(J)**2)+
9 HMT2*(D1-D4*DELX(J)+D7*.75*DELX(j)**2))*H(J-1,K-1)
 FC6(J,K)=A1+A2+A3
 END DO
 J=JMAX-1
 FC6(J,k)=W1*(-HMT4*(D33+D36*DELX(J)+D39*.75*DELX(J)**2)+
1 HMT2*(D33-D36*DELX(J)+D39*.75*DELX(J)**2))*H(J+1,K-2)+
2 W1*(-HMT4*(-D31+D32-D33-(D34-D35+D36)*DELX(J)+
3 (-D37+D38-D39)*.75*DELX(J)**2)+HMT2*(-D31+D32-D33+
4 (D34-D35+D36)*DELX(J)+(-D37+D38-D39)*.75*DELX(J)**2))*H(J,K-2)+
5 W1*(-HMT4*(-D32-D35*DELX(J)-D38*.75*DELX(J)**2)+
6 HMT2*(-D32+D35*DELX(J)-D38*.75*DELX(J)**2))*H(J-1,K-2)+
7 W1*(-HMT4*(D31+D34*DELX(J)+D37*.75*DELX(J)**2)+
8 HMT2*(D31-D34*DELX(J)+D37*.75*DELX(J)**2))*H(J-2,K-2)+
9 S(J)*DELX(J)/DELT*H(J,K-1)
 FC6(J,k)=FC6(J,K)+W2*(-HMT4*(D33+D36*DELX(J)+D39*.75*DELX(J)**2)
1 +HMT2*(D33-D36*DELX(J)+D39*.75*DELX(J)**2))*H(J+1,K-1)+
2 W2*(-HMT4*(-D31+D32-D33-(D34-D35+D36)*DELX(J)+
3 (-D37+D38-D39)*.75*DELX(J)**2)+HMT2*(-D31+D32-D33+
4 (D34-D35+D36)*DELX(J)+(-D37+D38-D39)*.75*DELX(J)**2))*H(J,K-1)+
5 W2*(-HMT4*(-D32-D35*DELX(J)-D38*.75*DELX(J)**2)+
6 HMT2*(-D32+D35*DELX(J)-D38*.75*DELX(J)**2))*H(J-1,K-1)+
7 W2*(-HMT4*(D31+D34*DELX(J)+D37*.75*DELX(J)**2)+
8 HMT2*(D31-D34*DELX(J)+D37*.75*DELX(J)**2))*H(J-2,K-1)

 J=JMAX

  A1=W1*(-HMT4*(-D51+D52-D53-(D54-D55+D56)*DELX(J)+
1  (-D57+D58-D59)*.75*DELX(J)**2)+T(J)*(-D51+D52-D53+
2  (D54-D55+D56)*DELX(J)+(-D57+D58-D59)*.75*DELX(J)**2))*H(J,K-2)+
3  w1*(-HMT4*(-D52-D55*DELX(J)-D58*.75*DELX(J)**2)+
4  T(J)*(-D52+D55*DELX(J)-D58*.75*DELX(J)**2))*H(J-1,K-2)+
5  w1*(-HMT4*(D51+D54*DELX(J)+D57*.75*DELX(J)**2)+
6  T(J)*(D51-D54*DELX(J)+D57*.75*DELX(J)**2))*H(J-2,K-2)+
7  W1*(-HMT4*(D53+D56*DELX(J)+D59*.75*DELX(J)**2)+
8  T(J)*(D53-D56*DELX(J)+D59*.75*DELX(J)**2))*SRIVR(K-2)
 A2= S(J)*DELX(J)/DELT*H(J,K-1)
```

```
      A3= W3*(-HMT4*(D53+D56*DELX(J)+
     1 D59*.75*DELX(J)**2)+T(J)*(D53-D56*DELX(J)+D59*.75*DELX(J)**2))*
     2 SRIVR(K)
      A4=W2*(-HMT4*(-D51+D52-D53-(D54-D55+D56)*DELX(J)+
     1 (-D57+D58-D59)*.75*DELX(J)**2)+T(J)*(-D51+D52-D53+
     2 (D54-D55+D56)*DELX(J)+(-D57+D58-D59)*.75*DELX(J)**2))*H(J,K-1)+
     3 w2*(-HMT4*(-D52-D55*DELX(J)-D58*.75*DELX(J)**2)+
     4 T(J)*(-D52+D55*DELX(J)-D58*.75*DELX(J)**2))*H(J-1,K-1)+
     5 w2*(-HMT4*(D51+D54*DELX(J)+D57*.75*DELX(J)**2)+
     6 T(J)*(D51-D54*DELX(J)+D57*.75*DELX(J)**2))*H(J-2,K-1)+
     7 W2*(-HMT4*(D53+D56*DELX(J)+D59*.75*DELX(J)**2)+
     8 T(J)*(D53-D56*DELX(J)+D59*.75*DELX(J)**2))*SRIVR(K-1)
      FC6(J,K)=A1+A2+A3+A4
      INDEX=0
      DO J=1,JMAX
      INDEX=INDEX+1
      RIGHT(INDEX)=-FC6(J,K)
      END DO
C     WRITE(2,*)'K=',K
C     WRITE(2,*)'RIGHTHAND COLUMN MATRIX'
C     WRITE(2,*)(RIGHT(J),J=1,JMAX)
      DO I=1,M
      HSOL(I)=0.
      DO J=1,M
      HSOL(I)=HSOL(I)+A(I,J)*RIGHT(J)
      END DO
      END DO
C
      INDEX=0
      DO J=1,JMAX
      INDEX=INDEX+1
      H(J,K)=HSOL(INDEX)
      END DO
      K=K+1
      IF(K.LE.NTIME)GO TO 1000
C     WRITE(2,*)'COMPUTATION OVER'
C     WRITE(2,*)'K=',K
C     WRITE(2,*)'K=',K
C     WRITE(2,*) 'HEAD VALUES ARE WRITTEN HERE'
      DO K=1,NTIME
      WRITE(2,*)'TIME STEP=',K
      WRITE(2,222)(H(J,K), J=1,JMAX)
      END DO
222   FORMAT(10F8.3)
      DO K=1,NTIME
      DO J=1,JMAX
      RISE(J,K)=H(J,K)-HEADI
      END DO
      END DO
      WRITE(2,*)'RISE IN PIEZOMETRIC SURFACE'
      DO K=1,NTIME
      WRITE(2,333)(RISE(J,K),J=1,JMAX)
      END DO
333   format(10f8.4)
      x1=DELX(1)/2.
      beta=t(1)/s(1)
C     WRITE(2,*)'RISE AT FIRST NODE'
      DO N=1,NTIME
      AN=N
      x=x1/((4.*beta*AN*DELT)**0.5)
      call erf(x,erfx)
```

```
      trise(1,n)=(BOUNHL-HEADI)*(1.-erfx)
      END DO
      x1=DELX(1)+DELX(2)/2.
      beta=t(1)/s(1)
      DO N=1,NTIME
      AN=N
      x=x1/((4.*beta*AN*DELT)**0.5)
      call erf(x,erfx)
      trise(2,n)=(BOUNHL-HEADI)*(1.-erfx)
      END DO
C     write(2,333)(trise(1,k),k=1,ntime)
C     WRITE(2,*)'RISE AT SECOND NODE'
C     write(2,333)(trise(2,k),k=1,ntime)
      STOP
      END


      SUBROUTINE MATIN (AAA,MMM)
      DIMENSION AAA(25,25),B(25),C(25)
      NN=MMM-1
      AAA(1,1)=1./AAA(1,1)
      DO 8 M=1,NN
      K=M+1
      DO 3 I=1,M
      B(I)=0.0
      DO 3 J=1,M
3     B(I)=B(I)+AAA(I,J)*AAA(J,K)
      D=0.0
      DO 4 I=1,M
4     D=D+AAA(K,I)*B(I)
      D=-D+AAA(K,K)
      AAA(K,K)=1./D
      DO 5 I=1,M
 5    AAA(I,K)=-B(I)*AAA(K,K)
      DO 6 J=1,M
      C(J)=0.0
      DO 6 I=1,M
6     C(J)=C(J)+AAA(K,I)*AAA(I,J)
      DO 7 J=1,M
7     AAA(K,J)=-C(J)*AAA(K,K)
      DO 8 I=1,M
      DO 8 J=1,M
8     AAA(I,J)=AAA(I,J)-B(I)*AAA(K,J)
      RETURN
      END
      SUBROUTINE ERF(X,ERFX)
      XINDEX=X
      X1=X
      IF(X)4,5,5
4     X1=-X
5     CONTINUE
      IF(X1-15.)1,2,2
1     CONTINUE
      T=1.0/(1.0+0.3275911*X1)
      ERFX=1.0-(0.25482959*T-0.28449673*T**2+1.42141374*T**3-1.
     1  45315202*T**4+1.06140542*T**5)*EXP(-X1**2)
        GO TO3
2       ERFX=1.
3       CONTINUE
        IF(XINDEX)6,7,7
6       ERFX=-ERFX
```

```
7     CONTINUE
C     WRITE(2,52)X,ERFX
C52   FORMAT(2F10.5)
      RETURN
      END
```

APPENDIX - 2C

```
$DebuG
C     IN THIS PROGRAMME THE HEADS AT VARIOUS NODES HAVE BEEN FOUND
C     USING ANALYTICAL SOLUTION

      DIMENSION DIST(10),HEAD(10,10),RISE(10,10)
      OPEN(1, FILE= 'SREST3.DAT', STATUS= 'OLD')
      OPEN(2, FILE= 'SREST3.OUT', STATUS='UNKNOWN')
      READ(1,*)DELX,TRANS,PHI,HEADI,SIGMA0,NTIME,DISTI

      PAI=3.14159265
      BETA=TRANS/PHI

      WRITE(2,20)
20    FORMAT(5X,'TRANSMISSIVITY',8X,'STORATIVITY',9X,
     1       'HYDRAULIC DIFFUSIVITY')
      WRITE(2,21)TRANS,PHI,BETA
21    FORMAT(2X,F10.2,12X,F10.4,13X, F10.1)
      WRITE(2,22)
22    FORMAT(2X,'INITIAL HEAD',5X,'SIGMA0')
      WRITE(2,23)HEADI,SIGMA0
23    FORMAT(2F12.2)
      WRITE(2,24)
24    FORMAT(2X,'TIME    ',4X,'DISTI',8X,'DELX')
      WRITE(2,25)NTIME,DISTI,DELX
25    FORMAT(I5,2F12.2)

      TIME=NTIME
      DIST(1)=DISTI+0.5*DELX
      DIST(2)=DISTI+1.5*DELX
      DIST(3)=DISTI+2.5*DELX
      DIST(4)=DISTI+3.5*DELX

      DO I=1,4
      X=SQRT(DIST(I)**2/(4.*BETA*TIME))
      CALL ERF(X,ERFX)
      RISE(I,NTIME)=SIGMA0*(1.-ERFX)
      HEAD(I,NTIME)=HEADI+RISE(I,NTIME)
      END DO
      WRITE(2,26)
26    FORMAT(2X,'HEAD AT 4 CONSECUTIVE NODES COMPUTED ANALYTICALLY:')
      WRITE(2,*)(HEAD(I,NTIME),I=1,4)
      WRITE(2,27)
27    FORMAT(2X,'RISE AT 4 CONSECUTIVE NODES COMPUTED ANALYTICALLY:')
      WRITE(2,*)(RISE(I,NTIME),I=1,4)

C     POLYNOMIAL EXPRESSION : H= A+B*X+C*X**2+D*X**3
C     A,B,C,D ARE OBTAINED

      N=NTIME
      I=2
      A=HEAD(I,N)
      B=(-HEAD(I+2,N)+6.*HEAD(I+1,N)-3.*HEAD(I,N)-2.*HEAD(I-1,N))
     1    /(6.*DELX)
      C=(HEAD(I+1,N)-2.*HEAD(I,N)+HEAD(I-1,N))/(2.*DELX**2)
      D=(HEAD(I+2,N)-3.*HEAD(I+1,N)+3.*HEAD(I,N)-HEAD(I-1,N))
     1    /(6.*DELX**3)
      WRITE(2,32)
32    FORMAT(8X,'A',16X,'B',16X,'C',16X,'D')
      WRITE(2,33)A,B,C,D
33    FORMAT(F14.5,3E17.4)
```

```
      WRITE(2,28)

      HEADL=A-B*DELX/2.+C*(DELX/2.)**2-D*(DELX/2)**3

      X1=DISTI+DELX
      X=SQRT(X1**2/(4.*BETA*TIME))
      CALL ERF(X,ERFX)
      RISEL=SIGMA0*(1.-ERFX)
      HEADLC=RISEL+HEADI
      WRITE(2,29)X1,HEADLC,HEADL

      HEADR=A+B*DELX/2.+C*(DELX/2.)**2+D*(DELX/2)**3
      X1=DISTI+2.*DELX
      X=SQRT(X1**2/(4.*BETA*TIME))
      CALL ERF(X,ERFX)
      RISER=SIGMA0*(1.-ERFX)
      HEADRC=RISER+HEADI
      WRITE(2,28)
28    FORMAT(2X,'DISTANCE FROM STREAM',4X,'ANALYTICAL HEAD'
     1      ,4X,'COMPUTED HEAD')
      WRITE(2,29)X1,HEADRC,HEADR
29    FORMAT(2XF10.2,11X,F10.4,10X,F10.4)


C     COMPUTATION OF DIFFERENTIAL

      CENTDL=(HEAD(2,NTIME)-HEAD(1,NTIME))/DELX
      CENTDR=(HEAD(3,NTIME)-HEAD(2,NTIME))/DELX
      DIFFL=B+2.*C*(-DELX/2.)+3.*D*(DELX/2.)**2.
      DIFFLC=-SIGMA0*2/SQRT(PAI)*EXP(-(DELX+DISTI)**2/(4.*BETA*TIME))*
     1 1./SQRT(4.*BETA *TIME)
      DIFFR=B+2.*C*(DELX/2.)+3.*D*(DELX/2.)**2.
      DIFFRC=-SIGMA0*2/SQRT(PAI)*EXP(-(DISTI+2.*DELX)**2
     1      /(4.*BETA*TIME))/SQRT(4.*BETA*TIME)
      WRITE(2,30)
30    FORMAT(2X,'COMPUTED DIFFERENTIAL',2X,'ANALYTICAL DIFFERENTIAL',
     1 2X,'CENTRAL DIFFERENTIAL' )
      WRITE(2,31)DIFFL,DIFFLC,CENTDL
      WRITE(2,30)
      WRITE(2,31)DIFFR,DIFFRC,CENTDR
31    FORMAT(3E20.4)

      WRITE(2,*)'CHECKING THE END DERIVATION'


      DIST(1)=0.5*DELX
      DIST(2)=1.5*DELX
      DIST(3)=2.5*DELX

      DO I=1,3
      X=SQRT(DIST(I)**2/(4.*BETA*TIME))
      CALL ERF(X,ERFX)
      RISE(I,NTIME)=SIGMA0*(1.-ERFX)
      HEAD(I,NTIME)=HEADI+RISE(I,NTIME)
      END DO
      WRITE(2,126)
126   FORMAT(2X,'HEAD AT 3 CONSECUTIVE NODES COMPUTED ANALYTICALLY:')
      WRITE(2,*)(HEAD(I,NTIME),I=1,3)
      WRITE(2,127)
127   FORMAT(2X,'RISE AT 3 CONSECUTIVE NODES COMPUTED ANALYTICALLY:')
```

```
      WRITE(2,*)(RISE(I,NTIME),I=1,3)
      N=NTIME
      I=1
      A=HEAD(I,N)
      B=(-3.*HEAD(I+2,N)+20.*HEAD(I+1,N)+15.*HEAD(I,N)-32.*
     1 (SIGMA0+HEADI)) /(30.*DELX)
       C=(-HEAD(I+2,N)+10.*HEAD(I+1,N)-25.*HEAD(I,N)+16.*(SIGMA0+HEADI)
     1        )/(10.*DELX**2)
       D=(6.*HEAD(I+2,N)-20.*HEAD(I+1,N)+30.*HEAD(I,N)-16.*
     1 (SIGMA0+HEADI))/(30.*DELX**3)

      WRITE(2,32)
      WRITE(2,33)A,B,C,D
      X=DELX/2.
      HEADR1=A+B*X+C*X**2+D*X**3
      X1=DELX
      X=SQRT(X1**2/(4.*BETA*TIME))
      CALL ERF(X,ERFX)
      RISER1=SIGMA0*(1.-ERFX)
      HEADRC1=RISER1+HEADI
      WRITE(2,29)X1,HEADRC1,HEADR1

      X=1.5*DELX
      HEADR2=A+B*X+C*X**2+D*X**3
      X1=2.0*DELX
      X=SQRT(X1**2/(4.*BETA*TIME))
      CALL ERF(X,ERFX)
      RISER2=SIGMA0*(1.-ERFX)
      HEADRC2=RISER2+HEADI
      WRITE(2,28)
      WRITE(2,29)X1,HEADRC2,HEADR2

      STOP
      END


      SUBROUTINE ERF(X,ERFX)
      XINDEX=X
      X1=X
      IF(X)4,5,5
4     X1=-X
5     CONTINUE
      IF(X1-15.)1,2,2
1     CONTINUE
      T=1.0/(1.0+0.3275911*X1)
       ERFX=1.0-(0.25482959*T-0.28449673*T**2+1.42141374*T**3-1.
     1 45315202*T**4+1.06140542*T**5)*EXP(-X1**2)
        GO TO3
2      ERFX=1.
3      CONTINUE
       IF(XINDEX)6,7,7
6      ERFX=-ERFX
7      CONTINUE
C      WRITE(2,52)X,ERFX
C52    FORMAT(2F10.5)
       RETURN
       END
```

APPENDIX - 2D

```
$DebuG
C      FLOW FROM THE RIVER IS COMPUTED NUMERICALLY STARTING FROM THE 1ST
C      TIME STEP
C      IN THIS PROGRAMME THE HEADS AT VARIOUS NODES HAVE BEEN FOUND
C      DIRECTLY USING MATRIX INVERSION METHOD. THIS IS A PROGRAMME
C      IN WHICH  HEAD IS OBTAINED USING CRANK NICHOLSON IMPLICIT
C      SCHEME
C      IN THIS THE RESULT HAS BEEN  OBTAINED FOR RAMP AND STEP INPUT

       DIMENSION  DELX(25),T(25),S(25)

       DIMENSION SRIVR(0:100),H(25,0:100),SRIVL(0:100)

       DIMENSION  FC1(25),FC2(25),FC4(25),
      1              FC6(25,100),HSOL(25),
      2              A(25,25),RIGHT(25)
C      DIMENSION AA(25,25),AAAA(25,25), RISE(25,100)

       OPEN(1, FILE= 'SREST4.DAT', STATUS= 'OLD')
       OPEN(2, FILE= 'SREST4.OUT', STATUS='UNKNOWN')
       READ(1,*)TRANS,PHI,DELXJ,JMAX,HEADL,HEADR,HEADI
       READ(1,*) DELT,NTIME
       READ(1,*)W11,W22,W33
       READ(1,*)W111,W222,W333
       WRITE(2,*)'THE RESULTS ARE FOR A STEP INPUT'
       WRITE(2,*)'THE INFLOW AT X=0 IS OBTAINED NUMERICALLY'
       WRITE(2,41)
41     FORMAT(7X,'DELT')
       WRITE(2,42)DELT
42     FORMAT(F10.3)

       WRITE(2,40)
40     FORMAT(2X,'TRANSMISSIVITY',3X,'STORATIVITY',5X,'DELXJ',4X,
      1       'INITIALHEAD',3X,'HEADL ',6X,'HEADR')
       WRITE(2,44)TRANS,PHI,DELXJ,HEADI,HEADL,HEADR
44     FORMAT(2X,F8.1,2X,F12.3,2X,2F13.1,2X,2F10.2)

C      READ(1,*) (DELX(J),J=1,JMAX)
C      WRITE(2,40)
C40    FORMAT(4X,'J',8X,'DELX(J)')
C      DO J=1,JMAX
C      WRITE(2,51) J,DELX(J)
C      END DO
C51    FORMAT(I5,5X,F8.2)
C
       DO J=1,JMAX
       DELX(J)=DELXJ
       END DO

       DO J=1,JMAX
       H(J,0)=HEADI
       END DO

       SRIVR(0)=HEADR
       SRIVL(0)=HEADL

C      SRIVL(0)=HEADI
C      IF SRIVL(0)=HEADI, IT CORRESPONDS TO RAMP INPUT
C      IF SRIVL(0)=HEADL, IT CORRESPONDS TO STEP INPUT
```

```
C
      DO K=1,NTIME
C     RIVER STAGE FOR RAMP INPUT
C     AK=K
C     SRIVL(K)=(HEADL-HEADI)*AK*DELT+HEADI
      SRIVL(K)=HEADL
      SRIVR(K)=HEADR
      END DO
      WRITE(2,*)'BOUNDARY CONDITION AT X=0; SRIVL(K)'
      WRITE(2,55)(SRIVL(K),K=1,NTIME)
55    FORMAT(5F10.2)
C
C
C     TRANSMISSIVITY VALUES ARE BEING READ
C
C     WRITE(2,*)'TRANSMISSIVITY VALUES'
C     READ(1,*)  (T(J),J=1,JMAX)
C     WRITE(2,54)(T(J),J=1,JMAX)
C54   FORMAT(6F10.2)
      DO J=1,JMAX
      T(J)=TRANS
      END DO
C
C     STORAGE COEFFICIENT VALUES ARE BEING READ
C
      RTRIVL=2.*T(1)/DELX(1)
      RTRIVR=2.*T(JMAX)/DELX(JMAX)
C     RTRIVR=0.
C     RTRIVL=0.
C     WRITE(2,*)'STORATIVITY VALUES'
C     READ(1,*)  (S(J),J=1,JMAX)
C     WRITE(2,54)(S(J),J=1,JMAX)
      DO J=1,JMAX
      S(J)=PHI
      END DO
C
C     COEFFICIENTS OF HEADS AT INTERIOR NODES
C     IN FINITE DIFFERENCE EQUATION
C
      K=1
      W1=W11
      W2=W22+W11
      W3=W33
      WRITE(2,*)'W2=',W2
      WRITE(2,*)'W3=',W3
      J=1
      HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
      TERML2=2./((DELX(J)+DELX(J+1))*DELX(J))
      TERM4=RTRIVL/DELX(J)*W3
      FC2(J)=HMT2*TERML2*W3
      FC4(J)=0.
C
C     FLOW FROM RIVER HAS BEEN CONSIDERED NUMWRICALLY
C

      FC1(J)=-S(J)/DELT-(FC2(J)+TERM4)

      DO J=2,JMAX-1

      HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
      HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))
```

```
      TERML2=2./((DELX(J)+DELX(J+1))*DELX(J))
      TERML4=2./((DELX(J)+DELX(J-1))*DELX(J))

      FC2(J)=HMT2*TERML2*W3
      FC4(J)=HMT4*TERML4*W3

      FC1(J)=-S(J)/DELT-(FC2(J)+FC4(J))
      END DO

      J=JMAX
      HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))
      TERML4=2./((DELX(J)+DELX(J-1))*DELX(J))

      TERM2=RTRIVR/DELX(J)*W3
      FC2(J)=0.
      FC4(J)=HMT4*TERML4*W3
      FC1(J)=-S(J)/DELT-(TERM2+FC4(J))
C
C     GENERATION OF MATRIX COEFFICIENTS
      M=JMAX
      DO I=1,M
      DO J=1,M
      A(I,J)=0.
      END DO
      END DO
C
C     ELEMENT OF THE SPARCE BANDED MATRIX
C
      DO I=1,M
      INDEX=0

      DO J1=1,JMAX
      INDEX=INDEX+1
      IF(I.EQ.INDEX) GO TO  1
      GO TO 6
1     CONTINUE
      A(I,INDEX)=FC1(J1)
      IF(J1.EQ.JMAX) GO TO 2
      A(I,INDEX+1)=FC2(J1)
2     CONTINUE
      IF(J1.EQ.1) GO TO 3
      A(I,INDEX-1)=FC4(J1)
3     CONTINUE
6     CONTINUE
      END DO
      END DO

C     WRITE(2,*)'ELEMENT OF THE MATRIX'
C     DO I=1,M
C     WRITE(2,197)(A(I,J),J=1,M)
C     END DO

C     STORING OF THE MATRIX ELEMENT
C
C     DO I=1,M
C     DO J=1,M
C     AA(I,J)=A(I,J)
C     END DO
C     END DO
C
C
```

```
C      CALL MATRIX INVERSE
C
C      WRITE(2,*)'WRITING THE MATRIX ELEMENT BEFORE INVERSION'
C      DO I=1,M
C      WRITE(2,197)( A(I,J),J=1,M)
C      END DO

197    FORMAT(10F7.3)
C
C
C
       CALL MATIN (A,M)

C      WRITE(2,*)'MATRIX ELEMENT AFTER INVERSION'
C      DO I=1,M
C      WRITE(2,197)(A(I,J),J=1,M)
C      END DO

C      WRITE(2,*)'CHECKING THE MATRIX INVERSION'
C      KK=0
C            DO I=1,M
C      SUM=0
C      KK=KK+1
C      DO J=1,M
C      SUM=SUM+AA(I,J)*A(J,I)
C      END DO
C      AAAA(I,KK)=SUM
C      END DO
C      DO I=1,M
C      WRITE(2,197)(AAAA(I,J),J=1,M)
C      END DO
C
C      SOLUTION FOR THE FIRST TIME STEP
C
C
C
C      FC6(1,K) FOR  NODES NEAR LEFT BOUNDARY
C
       J=1

C      FLOW FROM STREAM HAS BEEN CONSIDERED NUMERICALLY

       FC6(J,K)=S(J)/DELT*H(J,K-1)
     1  +W3*RTRIVL/DELX(J)*SRIVL(K)
     2  +W2*RTRIVL/DELX(J)*(SRIVL(K-1)-H(J,K-1))
     3  +W2/W3*FC2(J)*(H(J+1,K-1)-H(J,K-1))

C
C
C      FC6(J,K)   AT   INTERIOR NODES
C

       DO J=2,JMAX-1
       FC6(J,K)=S(J)/DELT*H(J,K-1)
     1  +W2/W3*FC2(J)*(H(J+1,K-1)-H(J,K-1))
     2  +W2/W3*FC4(J)*(H(J-1,K-1)-H(J,K-1))

       END DO

C
C      FC6(JMAX,K) FOR  NODES NEAR RIGHT BOUNDARY
```

```
C
      J=JMAX
      FC6(J,K)=S(J)/DELT*H(J,K-1)
     1  +W3*RTRIVR/DELX(J)*SRIVR(K)
     2  +W2*RTRIVR/DELX(J)*(SRIVR(K-1)-H(J,K-1))
     3  +W2/W3*FC4(J)*(H(J-1,K-1)-H(J,K-1))
C
      INDEX=0
      DO J=1,JMAX
      INDEX=INDEX+1
      RIGHT(INDEX)=-FC6(J,K)
      END DO
C     WRITE(2,*)'TIME STEP K=',K
C     WRITE(2,*)'RIGHTHAND COLUMN MATRIX'
C     WRITE(2,*)(RIGHT(J),J=1,JMAX)
      DO I=1,M
      HSOL(I)=0.
      DO J=1,M
      HSOL(I)=HSOL(I)+A(I,J)*RIGHT(J)
      END DO
      END DO
C
      INDEX=0
      DO J=1,JMAX
      INDEX=INDEX+1
      H(J,K)=HSOL(INDEX)
      END DO
C     SOLUTION FOR FIRST TIME IS OVER
      W1=W111
      W2=W222
      W3=W333
      WRITE(2,*)'W1=',W1
      WRITE(2,*)'W2=',W2
      WRITE(2,*)'W3=',W3
      J=1
      HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
      TERML2=2./((DELX(J)+DELX(J+1))*DELX(J))
      FC2(J)=HMT2*TERML2*W3
      TERM4=RTRIVL/DELX(J)*W3
      FC4(J)=0.
      FC1(J)=-S(J)/DELT-(FC2(J)+TERM4)
      DO J=2,JMAX-1
      HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
      HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))
      TERML2=2./((DELX(J)+DELX(J+1))*DELX(J))
      TERML4=2./((DELX(J)+DELX(J-1))*DELX(J))
      FC2(J)=HMT2*TERML2*W3
      FC4(J)=HMT4*TERML4*W3
      FC1(J)=-S(J)/DELT-(FC2(J)+FC4(J))
      END DO
      J=JMAX
      HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))
      TERML4=2./((DELX(J)+DELX(J-1))*DELX(J))
      TERM2=RTRIVR/DELX(J)*W3
      FC2(J)=0.
      FC4(J)=HMT4*TERML4*W3
      FC1(J)=-S(J)/DELT-(FC4(J)+TERM2)
C
C     GENERATION OF MATRIX COEFFICIENTS
```

```
      M=JMAX
      DO I=1,M
      DO J=1,M
      A(I,J)=0.
      END DO
      END DO
C
C     ELEMENT OF THE SPARCE BANDED MATRIX
C
      DO I=1,M
      INDEX=0
      DO J1=1,JMAX
      INDEX=INDEX+1
      IF(I.EQ.INDEX) GO TO  199
      GO TO 699
199   CONTINUE
      A(I,INDEX)=FC1(J1)
      IF(J1.EQ.JMAX) GO TO 299
      A(I,INDEX+1)=FC2(J1)
299   CONTINUE
      IF(J1.EQ.1) GO TO 399
      A(I,INDEX-1)=FC4(J1)
399   CONTINUE
699   CONTINUE
      END DO
      END DO
C     WRITE(2,*)'ELEMENT OF THE MATRIX'
C     DO I=1,M
C     WRITE(2,197)(A(I,J),J=1,M)
C     END DO
C
C     STORING OF THE MATRIX ELEMENT
C
C     DO I=1,M
C     DO J=1,M
C     AA(I,J)=A(I,J)
C     END DO
C     END DO
C
C
C     CALL MATRIX INVERSE
C
C     WRITE(2,*)'WRITING THE MATRIX ELEMENT BEFORE INVERSION'
C     DO I=1,M
C     WRITE(2,197)( A(I,J),J=1,M)
C     END DO
C
      CALL MATIN (A,M)
C     WRITE(2,*)'MATRIX ELEMENT AFTER INVERSION'
C     DO I=1,M
C     WRITE(2,197)(A(I,J),J=1,M)
C     END DO

C     WRITE(2,*)'CHECKING THE MATRIX INVERSION'
C     KK=0
C     DO I=1,M
C     SUM=0
C     KK=KK+1
C     DO J=1,M
C     SUM=SUM+AA(I,J)*A(J,I)
C     END DO
```

```
C      AAAA(I,KK)=SUM
C      END DO
C      DO I=1,M
C      WRITE(2,197)(AAAA(I,J),J=1,M)
C      END DO
C
       K=2
1000   CONTINUE
C      SOLUTION FOR THE 2ND TIME STEP ONWARD
C
C      FC6(1,K) FOR  NODES NEAR LEFT BOUNDARY
C
       J=1
       FC6(J,K)=S(J)/DELT*H(J,K-1)
     1   +W3*RTRIVL/DELX(J)*SRIVL(K)
     2   +W2*RTRIVL/DELX(J)*(SRIVL(K-1)-H(J,K-1))
     3   +(W2/W3)*FC2(J)*(H(J+1,K-1)-H(J,K-1))
     4   +W1*RTRIVL/DELX(J)*(SRIVL(K-2)-H(J,K-2))
     5   +(W1/W3)*FC2(J)*(H(J+1,K-2)-H(J,K-2))
C
C
C      FC6(J,K)   AT   INTERIOR NODES
C

       DO J=2,JMAX-1
       FC6(J,K)=S(J)*H(J,K-1)/DELT
     1   +W2/W3*FC2(J)*(H(J+1,K-1)-H(J,K-1))
     2   +W2/W3*FC4(J)*(H(J-1,K-1)-H(J,K-1))
     3   +W1/W3*FC2(J)*(H(J+1,K-2)-H(J,K-2))
     4   +W1/W3*FC4(J)*(H(J-1,K-2)-H(J,K-2))
       END DO


C
C      FC6(JMAX,K)  FOR  NODES NEAR RIGHT BOUNDARY
C

       J=JMAX

       FC6(J,K)=S(J)/DELT*H(J,K-1)
     1   +W3*RTRIVR/DELX(J)*SRIVR(K)
     2   +W2*RTRIVR/DELX(J)*(SRIVR(K-1)-H(J,K-1))
     3   +W2/W3*FC4(J)*(H(J-1,K-1)-H(J,K-1))
     4   +W1*RTRIVR/DELX(J)*(SRIVR(K-2)-H(J,K-2))
     5   +W1/W3*FC4(J)*(H(J-1,K-2)-H(J,K-2))

C
       INDEX=0
       DO J=1,JMAX
       INDEX=INDEX+1
       RIGHT(INDEX)=-FC6(J,K)
       END DO

C      WRITE(2,*)'TIME STEP K=',K
C      WRITE(2,*)'RIGHTHAND COLUMN MATRIX'
C      WRITE(2,*)(RIGHT(J),J=1,JMAX)

       DO I=1,M
       HSOL(I)=0.
       DO J=1,M
       HSOL(I)=HSOL(I)+A(I,J)*RIGHT(J)
```

```
      END DO
      END DO
C
      INDEX=0
      DO J=1,JMAX
      INDEX=INDEX+1
      H(J,K)=HSOL(INDEX)
      END DO
      K=K+1
      IF(K.LE.NTIME) GO TO 1000
      WRITE(2,*) 'HEAD (metre) AT DIFFERENT TIMES AND NODES '
      DO K=1,NTIME
C     WRITE(2,*)'K=',K
      WRITE(2,222) (H(J,K), J=1,JMAX)
222   FORMAT(8F8.2)
      END DO
      STOP
      END

      SUBROUTINE MATIN (AAA,MMM)
      DIMENSION AAA(25,25),B(25),C(25)
        NN=MMM-1
        AAA(1,1)=1./AAA(1,1)
        DO 8 M=1,NN
        K=M+1
        DO 3 I=1,M
        B(I)=0.0
        DO 3 J=1,M
3       B(I)=B(I)+AAA(I,J)*AAA(J,K)
        D=0.0
        DO 4 I=1,M
4       D=D+AAA(K,I)*B(I)
        D=-D+AAA(K,K)
        AAA(K,K)=1./D
        DO 5 I=1,M
 5      AAA(I,K)=-B(I)*AAA(K,K)
        DO 6 J=1,M
        C(J)=0.0
        DO 6 I=1,M
6       C(J)=C(J)+AAA(K,I)*AAA(I,J)
        DO 7 J=1,M
7       AAA(K,J)=-C(J)*AAA(K,K)
        DO 8 I=1,M
        DO 8 J=1,M
8       AAA(I,J)=AAA(I,J)-B(I)*AAA(K,J)
        RETURN
        END

        SUBROUTINE ERF(X,ERFX)
        XINDEX=X
      X1=X
        IF(X)4,5,5
4       X1=-X
5       CONTINUE
        IF(X1-15.)1,2,2
1       CONTINUE
        T=1.0/(1.0+0.3275911*X1)
        ERFX=1.0-(0.25482959*T-0.28449673*T**2+1.42141374*T**3-1.
     1  45315202*T**4+1.06140542*T**5)*EXP(-X1**2)
        GO TO3
2       ERFX=1.
```

```
3       CONTINUE
        IF(XINDEX)6,7,7
6       ERFX=-ERFX
7       CONTINUE
C       WRITE(2,52)X,ERFX
C52     FORMAT(2F10.5)
        RETURN
        END
```

APPENDIX - 2E

```
$DebuG
C       FLOW FROM RIVER HAS BEEN CONSIDERED ANALYTICALLY FOR THE 1ST
C       TIME STEP
C       IN THIS PROGRAMME THE HEADS AT VARIOUS NODES HAVE BEEN FOUND
C       DIRECTLY USING MATRIX INVERSION METHOD. THIS IS A PROGRAMME
C       IN WHICH  HEAD IS OBTAINED USING CRANK NICHOLSON IMPLICIT
C       SCHEME
C       IN THIS THE RESULT HAS BEEN  OBTAINED FOR RAMP AND STEP INPUT
        DIMENSION  DELX(25),T(25),S(25)
        DIMENSION SRIVR(0:100),H(25,0:100),SRIVL(0:100)

        DIMENSION  FC1(25),FC2(25),FC4(25),
     1             FC6(25,100),HSOL(25),
     2             A(25,25),RIGHT(25)
C       DIMENSION AA(25,25),AAAA(25,25), RISE(25,100)

        OPEN(1, FILE= 'SREST5.DAT', STATUS= 'OLD')
        OPEN(2, FILE= 'SREST5.OUT', STATUS='UNKNOWN')
        READ(1,*)TRANS,PHI,DELXJ,JMAX,HEADL,HEADR,HEADI
        READ(1,*) DELT,NTIME
        READ(1,*)W11,W22,W33
        READ(1,*)W111,W222,W333
        WRITE(2,41)
 41     FORMAT(7X,'DELT')
        WRITE(2,42)DELT
 42     FORMAT(F10.3)

        WRITE(2,40)
 40     FORMAT(2X,'TRANSMISSIVITY',3X,'STORATIVITY',5X,'DELXJ',4X,
     1       'INITIALHEAD',3X,'HEADL ',6X,'HEADR')
        WRITE(2,44)TRANS,PHI,DELXJ,HEADI,HEADL,HEADR
 44     FORMAT(2X,F8.1,2X,F12.3,2X,2F13.1,2X,2F10.2)

C       READ(1,*) (DELX(J),J=1,JMAX)
C       WRITE(2,40)
C40     FORMAT(4X,'J',8X,'DELX(J)')
C       DO J=1,JMAX
C       WRITE(2,51) J,DELX(J)
C       END DO
C51     FORMAT(I5,5X,F8.2)
C
        DO J=1,JMAX
        DELX(J)=DELXJ
        END DO

        DO J=1,JMAX
        H(J,0)=HEADI
        END DO

        SRIVR(0)=HEADR
        SRIVL(0)=HEADL
C       SRIVL(0)=HEADI
C       IF SRIVL(0)=HEADI, IT CORRESPONDS TO RAMP INPUT
C       IF SRIVL(0)=HEADL, IT CORRESPONDS TO STEP INPUT
C
        DO K=1,NTIME
C
C       AK=K
C       SRIVL(K)=(HEADL-HEADI)*AK*DELT+HEADI
        SRIVL(K)=HEADL
```

```
      SRIVR(K)=HEADR
      END DO

      WRITE(2,*)'SRIVL(K)'
      WRITE(2,55)(SRIVL(K),K=1,NTIME)
55    FORMAT(5F10.2)
C
C
C     TRANSMISSIVITY VALUES ARE BEING READ
C
C     WRITE(2,*)'TRANSMISSIVITY VALUES'
C     READ(1,*)  (T(J),J=1,JMAX)
C     WRITE(2,54)(T(J),J=1,JMAX)
C54   FORMAT(6F10.2)
      DO J=1,JMAX
      T(J)=TRANS
      END DO
C
C     STORAGE COEFFICIENT VALUES ARE BEING READ
C
      RTRIVL=2.*T(1)/DELX(1)
      RTRIVR=2.*T(JMAX)/DELX(JMAX)

C     RTRIVR=0.
C     RTRIVL=0.
C     WRITE(2,*)'STORATIVITY VALUES'
C     READ(1,*)  (S(J),J=1,JMAX)
C     WRITE(2,54)(S(J),J=1,JMAX)
      DO J=1,JMAX
      S(J)=PHI
      END DO
C
C     COEFFICIENTS OF HEADS AT INTERIOR NODES
C     IN FINITE DIFFERENCE EQUATION
C

      K=1
      W1=W11
      W2=W22+W11
      W3=W33
      WRITE(2,*)'W3=',W3
      WRITE(2,*)'W2=',W2
      J=1
      HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
      TERML2=2./((DELX(J)+DELX(J+1))*DELX(J))
      FC2(J)=HMT2*TERML2*W3
      FC4(J)=0.
      FC1(J)=-S(J)/DELT-FC2(J)
      DO J=2,JMAX-1
      HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
      HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))
      TERML2=2./((DELX(J)+DELX(J+1))*DELX(J))
      TERML4=2./((DELX(J)+DELX(J-1))*DELX(J))
      FC2(J)=HMT2*TERML2*W3
      FC4(J)=HMT4*TERML4*W3
      FC1(J)=-S(J)/DELT-(FC2(J)+FC4(J))
      END DO
      J=JMAX
      HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))
      TERML4=2./((DELX(J)+DELX(J-1))*DELX(J))
```

```
      TERM2=RTRIVR/DELX(J)*W3
      FC2(J)=0.
      FC4(J)=HMT4*TERML4*W3
      FC1(J)=-S(J)/DELT-(TERM2+FC4(J))
C
C     GENERATION OF MATRIX COEFFICIENTS
      M=JMAX
      DO I=1,M
      DO J=1,M
      A(I,J)=0.
      END DO
      END DO
C
C     ELEMENT OF THE SPARCE BANDED MATRIX
C
      DO I=1,M
      INDEX=0
      DO J1=1,JMAX
      INDEX=INDEX+1
      IF(I.EQ.INDEX) GO TO  1
      GO TO 6
1     CONTINUE
      A(I,INDEX)=FC1(J1)
      IF(J1.EQ.JMAX) GO TO 2
      A(I,INDEX+1)=FC2(J1)
2     CONTINUE
      IF(J1.EQ.1) GO TO 3
      A(I,INDEX-1)=FC4(J1)
3     CONTINUE
6     CONTINUE
      END DO
      END DO
C     WRITE(2,*)'ELEMENT OF THE MATRIX'
C     DO I=1,M
C     WRITE(2,197)(A(I,J),J=1,M)
C     END DO
C     STORING OF THE MATRIX ELEMENT
C
C     DO I=1,M
C     DO J=1,M
C     AA(I,J)=A(I,J)
C     END DO
C     END DO
C
C
C     CALL MATRIX INVERSE
C
C     WRITE(2,*)'WRITING THE MATRIX ELEMENT BEFORE INVERSION'
C     DO I=1,M
C     WRITE(2,197)( A(I,J),J=1,M)
C     END DO
197   FORMAT(10F7.3)
C
C
C
      CALL MATIN (A,M)
C     WRITE(2,*)'MATRIX ELEMENT AFTER INVERSION'
C     DO I=1,M
C     WRITE(2,197)(A(I,J),J=1,M)
C     END DO
```

```
C       WRITE(2,*)'CHECKING THE MATRIX INVERSION'
C       KK=0
C       DO I=1,M
C       SUM=0
C       KK=KK+1
C       DO J=1,M
C       SUM=SUM+AA(I,J)*A(J,I)
C       END DO
C       AAAA(I,KK)=SUM
C       END DO
C       DO I=1,M
C       WRITE(2,197)(AAAA(I,J),J=1,M)
C       END DO
C
C       SOLUTION FOR THE FIRST TIME STEP
C
C
C
C       FC6(1,K) FOR  NODES NEAR LEFT BOUNDARY
C
        PAI=3.14159265
        J=1
C       FLOW FROM STREAM HAS BEEN CONSIDERED ANALYTICALLY
        RISE=HEADL-HEADI
        FC6(J,K)=S(J)/DELT*H(J,K-1)+2.*SQRT(T(1)*S(1)/PAI)*RISE/
       1      (SQRT(DELT)*DELX(1))+W2/W3*FC2(J)*(H(J+1,K-1)-H(J,K-1))
C
C
C
C       FC6(J,K)  AT  INTERIOR NODES
C

        DO J=2,JMAX-1
        FC6(J,K)=S(J)/DELT*H(J,K-1)
       1  +W2/W3*FC2(J)*(H(J+1,K-1)-H(J,K-1))
       2  +W2/W3*FC4(J)*(H(J-1,K-1)-H(J,K-1))

        END DO
C
C       FC6(JMAX,K) FOR  NODES NEAR RIGHT BOUNDARY
C
        J=JMAX
        FC6(J,K)=S(J)/DELT*H(J,K-1)
       1  +W3*RTRIVR/DELX(J)*SRIVR(K)
       2  +W2*RTRIVR/DELX(J)*(SRIVR(K-1)-H(J,K-1))
       3  +W2/W3*FC4(J)*(H(J-1,K-1)-H(J,K-1))


C
        INDEX=0
        DO J=1,JMAX
        INDEX=INDEX+1
        RIGHT(INDEX)=-FC6(J,K)
        END DO
C       WRITE(2,*)'TIME STEP K=',K
C       WRITE(2,*)'RIGHTHAND COLUMN MATRIX'
C       WRITE(2,*)(RIGHT(J),J=1,JMAX)
        DO I=1,M
        HSOL(I)=0.
        DO J=1,M
        HSOL(I)=HSOL(I)+A(I,J)*RIGHT(J)
        END DO
```

```
      END DO
C
      INDEX=0
      DO J=1,JMAX
      INDEX=INDEX+1
      H(J,K)=HSOL(INDEX)
      END DO
C     SOLUTION FOR FIRST TIME IS OVER
      W1=W111
      W2=W222
      W3=W333
      WRITE(2,*)'W1=',W1
      WRITE(2,*)'W2=',W2
      WRITE(2,*)'W3=',W3
      J=1
      HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
      TERML2=2./((DELX(J)+DELX(J+1))*DELX(J))
      FC2(J)=HMT2*TERML2*W3
      TERM4=RTRIVL/DELX(J)*W3
      FC4(J)=0.
      FC1(J)=-S(J)/DELT-(FC2(J)+TERM4)
      DO J=2,JMAX-1
      HMT2=(DELX(J)+DELX(J+1))/(DELX(J)/T(J)+DELX(J+1)/T(J+1))
      HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))
      TERML2=2./((DELX(J)+DELX(J+1))*DELX(J))
      TERML4=2./((DELX(J)+DELX(J-1))*DELX(J))
      FC2(J)=HMT2*TERML2*W3
      FC4(J)=HMT4*TERML4*W3
      FC1(J)=-S(J)/DELT-(FC2(J)+FC4(J))
      END DO
      J=JMAX
      HMT4=(DELX(J)+DELX(J-1))/(DELX(J)/T(J)+DELX(J-1)/T(J-1))
      TERML4=2./((DELX(J)+DELX(J-1))*DELX(J))
      TERM2=RTRIVR/DELX(J)*W3
      FC2(J)=0.
      FC4(J)=HMT4*TERML4*W3
      FC1(J)=-S(J)/DELT-(FC4(J)+TERM2)
C
C     GENERATION OF MATRIX COEFFICIENTS
      M=JMAX
      DO I=1,M
      DO J=1,M
      A(I,J)=0.
      END DO
      END DO
C
C     ELEMENT OF THE SPARCE BANDED MATRIX
C
      DO I=1,M
      INDEX=0

      DO J1=1,JMAX
      INDEX=INDEX+1
      IF(I.EQ.INDEX) GO TO  199
      GO TO 699
199   CONTINUE
      A(I,INDEX)=FC1(J1)
      IF(J1.EQ.JMAX) GO TO 299
      A(I,INDEX+1)=FC2(J1)
299   CONTINUE
      IF(J1.EQ.1) GO TO 399
```

```
      A(I,INDEX-1)=FC4(J1)
399   CONTINUE
699   CONTINUE
      END DO
      END DO
C     WRITE(2,*)'ELEMENT OF THE MATRIX'
C     DO I=1,M
C     WRITE(2,197)(A(I,J),J=1,M)
C     END DO
C
C     STORING OF THE MATRIX ELEMENT
C
C     DO I=1,M
C     DO J=1,M
C     AA(I,J)=A(I,J)
C     END DO
C     END DO
C
C
C     CALL MATRIX INVERSE
C
C     WRITE(2,*)'WRITING THE MATRIX ELEMENT BEFORE INVERSION'
C     DO I=1,M
C     WRITE(2,197)( A(I,J),J=1,M)
C     END DO
C
      CALL MATIN (A,M)

C     WRITE(2,*)'MATRIX ELEMENT AFTER INVERSION'
C     DO I=1,M
C     WRITE(2,197)(A(I,J),J=1,M)
C     END DO
C     WRITE(2,*)'CHECKING THE MATRIX INVERSION'
C     KK=0
C     DO I=1,M
C     SUM=0
C     KK=KK+1
C     DO J=1,M
C     SUM=SUM+AA(I,J)*A(J,I)
C     END DO
C     AAAA(I,KK)=SUM
C     END DO
C     DO I=1,M
C     WRITE(2,197)(AAAA(I,J),J=1,M)
C     END DO
C
      K=2
1000  CONTINUE
C     SOLUTION FOR THE 2ND TIME STEP ONWARD
C     FC6(1,K) FOR  NODES NEAR LEFT BOUNDARY
C
      J=1
      FC6(J,K)=S(J)/DELT*H(J,K-1)
     1  +W3*RTRIVL/DELX(J)*SRIVL(K)
     2  +W2*RTRIVL/DELX(J)*(SRIVL(K-1)-H(J,K-1))
     3  +(W2/W3)*FC2(J)*(H(J+1,K-1)-H(J,K-1))
     4  +W1*RTRIVL/DELX(J)*(SRIVL(K-2)-H(J,K-2))
     5  +(W1/W3)*FC2(J)*(H(J+1,K-2)-H(J,K-2))
C
C
C     FC6(J,K)   AT   INTERIOR NODES
```

```
C
      DO J=2,JMAX-1
      FC6(J,K)=S(J)*H(J,K-1)/DELT
     1  +W2/W3*FC2(J)*(H(J+1,K-1)-H(J,K-1))
     2  +W2/W3*FC4(J)*(H(J-1,K-1)-H(J,K-1))
     3  +W1/W3*FC2(J)*(H(J+1,K-2)-H(J,K-2))
     4  +W1/W3*FC4(J)*(H(J-1,K-2)-H(J,K-2))
      END DO
C
C     FC6(JMAX,K) FOR  NODES NEAR RIGHT BOUNDARY
C
      J=JMAX
      FC6(J,K)=S(J)/DELT*H(J,K-1)
     1  +W3*RTRIVR/DELX(J)*SRIVR(K)
     2  +W2*RTRIVR/DELX(J)*(SRIVR(K-1)-H(J,K-1))
     3  +W2/W3*FC4(J)*(H(J-1,K-1)-H(J,K-1))
     4  +W1*RTRIVR/DELX(J)*(SRIVR(K-2)-H(J,K-2))
     5  +W1/W3*FC4(J)*(H(J-1,K-2)-H(J,K-2))
C
      INDEX=0
      DO J=1,JMAX
      INDEX=INDEX+1
      RIGHT(INDEX)=-FC6(J,K)
      END DO
C     WRITE(2,*)'TIME STEP K=',K
C     WRITE(2,*)'RIGHTHAND COLUMN MATRIX'
C     WRITE(2,*)(RIGHT(J),J=1,JMAX)
      DO I=1,M
      HSOL(I)=0.
      DO J=1,M
      HSOL(I)=HSOL(I)+A(I,J)*RIGHT(J)
      END DO
      END DO
C
      INDEX=0
      DO J=1,JMAX
      INDEX=INDEX+1
      H(J,K)=HSOL(INDEX)
      END DO
      K=K+1 IF(K.LE.NTIME) GO TO 1000

      WRITE(2,*) 'HEAD (metre) AT DIFFERENT TIMES AND NODES '
      DO K=1,NTIME
C     WRITE(2,*)'K=',K
      WRITE(2,222) (H(J,K), J=1,JMAX)
222   FORMAT(8F8.2)
      END DO
      STOP
      END
      SUBROUTINE MATIN (AAA,MMM)
      DIMENSION AAA(25,25),B(25),C(25)
        NN=MMM-1
        AAA(1,1)=1./AAA(1,1)
        DO 8 M=1,NN
        K=M+1
        DO 3 I=1,M
        B(I)=0.0
        DO 3 J=1,M
3       B(I)=B(I)+AAA(I,J)*AAA(J,K)
        D=0.0
```

```
      DO 4 I=1,M
4     D=D+AAA(K,I)*B(I)
      D=-D+AAA(K,K)
      AAA(K,K)=1./D
      DO 5 I=1,M
5     AAA(I,K)=-B(I)*AAA(K,K)
      DO 6 J=1,M
      C(J)=0.0
      DO 6 I=1,M
6     C(J)=C(J)+AAA(K,I)*AAA(I,J)
      DO 7 J=1,M
7     AAA(K,J)=-C(J)*AAA(K,K)
      DO 8 I=1,M
      DO 8 J=1,M
8     AAA(I,J)=AAA(I,J)-B(I)*AAA(K,J)
      RETURN
      END

      SUBROUTINE ERF(X,ERFX)
      XINDEX=X
     X1=X
      IF(X)4,5,5
4     X1=-X
5     CONTINUE
      IF(X1-15.)1,2,2
1     CONTINUE
      T=1.0/(1.0+0.3275911*X1)
      ERFX=1.0-(0.25482959*T-0.28449673*T**2+1.42141374*T**3-1.
     1 45315202*T**4+1.06140542*T**5)*EXP(-X1**2)
      GO TO3
2     ERFX=1.
3     CONTINUE
      IF(XINDEX)6,7,7
6     ERFX=-ERFX
7     CONTINUE
C    WRITE(2,52)X,ERFX
C52  FORMAT(2F10.5)
      RETURN
      END
```

APPENDIX - 3
DATA INPUT

APPENDIX - 3A

```
8      10.
100.   0.02
.1       10
15.    10.    10.
```

APPENDIX - 3B

```
8      10.
0.01   10
15.    10.    10.
100.   50.    50.    50.    50.    50.    50.    100.
0.02   0.01   0.01   0.01   0.01   0.01   0.01   0.02
10.    20.    20.    20.    20.    20.    20.    10.
```

APPENDIX - 3C

```
10.    300.   .02    10.    5.     1      0.
50.    10.    0.001  50.    5.0    1      0.
```

APPENDIX - 3D

```
100.   0.02   10.    8      15.    10.    10.
.05    10
0.     0.     1.
0.     0.     1.
```

APPENDIX - 3E

```
100.   0.02   10.    8      15.    10.    10.
.05    10
0.     0.     1.
0.     0.     1.
```

APPENDIX - 4
OUTPUT DATA

OUTPUT DATA OF PROGRAMME-2A (CASE 1)

```
TRANS=     100.000000
STORC=   2.000000E-02
 JMAX      DELTAX
  8        10.00
 NTIME   DELT
 10          .10
    BHL      BHR      HI
    15.00      10.00      10.00
THE RESULTS ARE FOR A STEP INPUT
FIRST TIME STEP
W2=    0.000000E+00
W3=        1.000000
```

13.999	12.562	11.637	11.039	10.649	10.388	10.206	10.064
14.849	13.987	12.988	12.114	11.423	10.897	10.491	10.156
14.269	13.623	13.123	12.553	11.942	11.344	10.783	10.257
15.106	14.185	13.274	12.571	11.979	11.417	10.855	10.286
14.066	13.841	13.466	12.835	12.145	11.488	10.875	10.289
15.528	14.189	13.259	12.681	12.145	11.562	10.944	10.316
13.474	13.955	13.636	12.886	12.155	11.510	10.904	10.302
16.438	14.080	13.144	12.731	12.223	11.599	10.950	10.314
12.124	14.164	13.811	12.853	12.111	11.516	10.930	10.315
18.462	13.750	12.952	12.813	12.292	11.599	10.926	10.302

RISE IN PIEZOMETRIC SURFACE COMPUTED NUMERICALLY

3.9993	2.5618	1.6366	1.0388	.6487	.3884	.2057	.0642
4.8494	3.9865	2.9877	2.1137	1.4233	.8972	.4909	.1558
4.2692	3.6233	3.1229	2.5532	1.9423	1.3436	.7828	.2567
5.1060	4.1850	3.2737	2.5707	1.9787	1.4174	.8549	.2858
4.0661	3.8415	3.4657	2.8349	2.1450	1.4882	.8749	.2887
5.5283	4.1886	3.2593	2.6806	2.1447	1.5618	.9443	.3156
3.4741	3.9546	3.6359	2.8860	2.1546	1.5096	.9036	.3016
6.4382	4.0804	3.1438	2.7309	2.2227	1.5994	.9499	.3141
2.1241	4.1642	3.8107	2.8532	2.1106	1.5158	.9304	.3146
8.4623	3.7497	2.9519	2.8132	2.2924	1.5995	.9262	.3023

OUTPUT DATA OF PROGRAMME-2A (CASE2)

```
TRANS=      100.000000
 STORC=   2.000000E-02
  JMAX     DELTAX
   8       10.00
  NTIME  DELT
  10         .10
     BHL     BHR      HI
     15.00      10.00      10.00
 THE RESULTS ARE FOR A RAMP INPUT
 FIRST TIME STEP
 W2=   0.000000E+00
 W3=        1.000000
  10.400  10.256  10.164  10.104  10.065  10.039  10.021  10.006
  10.839  10.578  10.390  10.257  10.165  10.101  10.054  10.017
  11.294  10.954  10.689  10.485  10.329  10.209  10.116  10.037
  11.761  11.343  11.007  10.739  10.522  10.345  10.196  10.063
  12.221  11.744  11.343  11.008  10.727  10.490  10.282  10.092
  12.699  12.145  11.679  11.283  10.941  10.642  10.373  10.122
  13.151  12.552  12.023  11.561  11.156  10.795  10.465  10.153
  13.643  12.954  12.363  11.842  11.374  10.950  10.558  10.184
  14.076  13.366  12.710  12.121  11.591  11.106  10.651  10.215
  14.598  13.762  13.048  12.404  11.811  11.262  10.744  10.246
 RISE IN PIEZOMETRIC SURFACE COMPUTED NUMERICALLY
  .3999  .2562  .1637  .1039  .0649  .0388  .0206  .0064
  .8391  .5781  .3897  .2574  .1655  .1012  .0543  .0171
 1.2937  .9541  .6891  .4846  .3286  .2094  .1157  .0369
 1.7614 1.3434 1.0074  .7385  .5221  .3451  .1960  .0635
 2.2207 1.7438 1.3429 1.0076  .7273  .4897  .2820  .0921
 2.6989 2.1452 1.6791 1.2828  .9411  .6416  .3726  .1221
 3.1511 2.5519 2.0232 1.5609 1.1559  .7950  .4649  .1529
 3.6434 2.9541 2.3626 1.8415 1.3745  .9503  .5575  .1837
 4.0764 3.3655 2.7096 2.1209 1.5913 1.1060  .6514  .2151
 4.5984 3.7624 3.0485 2.4039 1.8112 1.2618  .7443  .2460
  .3858  .8337 1.2937 1.7599 2.2302 2.7033 3.1785 3.6555 4.1339 4.6134
 RISE AT SECOND NODE
  .2200  .5673  .9488 1.3485 1.7599 2.1797 2.6058 3.0370 3.4724 3.9112
```

OUTPUT DATA OF PROGRAMME-2B

```
100.00   50.00   50.00   50.00   50.00   50.00
   50.00  100.00
     .02     .01     .01     .01     .01     .01
     .01     .02
   10.00   20.00   20.00   20.00   20.00   20.00
   20.00   10.00
  NTIME      DELT
  10         .01
     BOUNHL      BOUNHR      HEADI
     15.00      10.00      10.00
 TIME STEP=           1
  11.805  10.185  10.010  10.001  10.000  10.000  10.000  10.000
 TIME STEP=           2
  14.161  10.531  10.050  10.004  10.000  10.000  10.000  10.000
 TIME STEP=           3
  13.942  10.969  10.129  10.013  10.001  10.000  10.000  10.000
 TIME STEP=           4
  14.229  11.273  10.239  10.032  10.004  10.000  10.000  10.000
 TIME STEP=           5
  14.254  11.546  10.359  10.061  10.008  10.001  10.000  10.000
 TIME STEP=           6
  14.328  11.767  10.484  10.099  10.016  10.002  10.000  10.000
 TIME STEP=           7
  14.369  11.956  10.609  10.145  10.027  10.004  10.000  10.000
 TIME STEP=           8
  14.408  12.118  10.730  10.195  10.042  10.008  10.001  10.000
 TIME STEP=           9
  14.440  12.258  10.847  10.250  10.060  10.012  10.002  10.000
 TIME STEP=          10
  14.468  12.380  10.958  10.308  10.081  10.018  10.003  10.000
 RISE IN PIEZOMETRIC SURFACE
  1.8053   .1854   .0103   .0006   .0000   .0000   .0000   .0000
  4.1610   .5305   .0495   .0037   .0003   .0000   .0000   .0000
  3.9422   .9688   .1287   .0132   .0011   .0001   .0000   .0000
  4.2287  1.2731   .2389   .0323   .0035   .0003   .0000   .0000
  4.2543  1.5458   .3589   .0615   .0083   .0009   .0000   .0000
  4.3283  1.7670   .4843   .0994   .0162   .0022   .0001   .0000
  4.3685  1.9563   .6089   .1446   .0274   .0043   .0004   .0000
  4.4084  2.1177   .7304   .1954   .0422   .0075   .0008   .0001
  4.4402  2.2576   .8470   .2504   .0602   .0120   .0016   .0002
  4.4682  2.3799   .9580   .3083   .0815   .0180   .0028   .0003
```

OUTPUT DATA OF PROGRAMME-2C (CASE1)

TRANSMISSIVITY STORATIVITY HYDRAULIC DIFFUSIVITY
□
300.00 .0200 15000.0
□
INITIAL HEAD SIGMA0
10.00 5.00
TIME DISTI DELX
1 .00 10.00
HEAD AT 4 CONSECUTIVE NODES COMPUTED ANALYTICALLY:
14.884850 14.654940 14.426170 14.199300
RISE AT 4 CONSECUTIVE NODES COMPUTED ANALYTICALLY:
4.884851 4.654937 4.426169 4.199300
A B C D
14.65494 -.2295E-01 .5732E-05 .1254E-06
DISTANCE FROM STREAM ANALYTICAL HEAD COMPUTED HEAD
10.00 14.7698 14.7698
20.00 14.5404 14.5404
COMPUTED DIFFERENTIAL ANALYTICAL DIFFERENTIAL CENTRAL DIFFERENTIAL
-.2299E-01 -.2299E-01 -.2299E-01
COMPUTED DIFFERENTIAL ANALYTICAL DIFFERENTIAL CENTRAL DIFFERENTIAL
-.2288E-01 -.2288E-01 -.2288E-01
CHECKING THE END DERIVATION
HEAD AT 3 CONSECUTIVE NODES COMPUTED ANALYTICALLY:
14.884850 14.654940 14.426170
RISE AT 3 CONSECUTIVE NODES COMPUTED ANALYTICALLY:
4.884851 4.654937 4.426169
A B C D
14.88485 -.2302E-01 .1936E-05 .1265E-06
DISTANCE FROM STREAM ANALYTICAL HEAD COMPUTED HEAD
10.00 14.7698 14.7698
20.00 14.5404 14.5404

OUTPUT DATA OF PROGRAMME-2C (CASE-2

TRANSMISSIVITY STORATIVITY HYDRAULIC DIFFUSIVITY
10.00 .0010 10000.0
INITIAL HEAD SIGMA0
50.00 5.00
TIME DISTI DELX
1 .00 50.00
HEAD AT 4 CONSECUTIVE NODES COMPUTED ANALYTICALLY:
54.298420 52.979420 51.883800 51.079620
RISE AT 4 CONSECUTIVE NODES COMPUTED ANALYTICALLY:
4.298419 2.979415 1.883795 1.079625
A B C D
52.97942 -.2437E-01 .4468E-04 .9075E-07
DISTANCE FROM STREAM ANALYTICAL HEAD COMPUTED HEAD
50.00 53.6184 53.6152
DISTANCE FROM STREAM ANALYTICAL HEAD COMPUTED HEAD
100.00 52.3975 52.3994
COMPUTED DIFFERENTIAL ANALYTICAL DIFFERENTIAL CENTRAL DIFFERENTIAL
-.2644E-01 -.2650E-01 -.2638E-01
COMPUTED DIFFERENTIAL ANALYTICAL DIFFERENTIAL CENTRAL DIFFERENTIAL
-.2197E-01 -.2197E-01 -.2191E-01
CHECKING THE END DERIVATION
HEAD AT 3 CONSECUTIVE NODES COMPUTED ANALYTICALLY:
54.298420 52.979420 51.883800
RISE AT 3 CONSECUTIVE NODES COMPUTED ANALYTICALLY:
4.298419 2.979415 1.883795
A B C D
54.29842 -.2772E-01 .1799E-04 .1779E-06
50.00 53.6184 53.6193

DISTANCE FROM STREAM	ANALYTICAL HEAD	COMPUTED HEAD
100.00	52.3975	52.3953

OUTPUT DATA OF PROGRAMME-2D

THE RESULTS ARE FOR A STEP INPUT

THE INFLOW AT X=0 IS OBTAINED NUMERICALLY

DELT
.050

TRANSMISSIVITY	STORATIVITY	DELXJ	INITIALHEAD	HEADL	HEADR
100.0	.020	10.0	10.0	15.00	10.00

BOUNDARY CONDITION AT X=0; SRIVL(K)

15.00	15.00	15.00	15.00	15.00
15.00	15.00	15.00	15.00	15.00

W2= 0.000000E+00
W3= 1.000000
W1= 0.000000E+00
W2= 0.000000E+00
W3= 1.000000

HEAD (metre) AT DIFFERENT TIMES AND NODES

13.49	11.87	11.00	10.54	10.29	10.15	10.07	10.02
14.18	12.80	11.80	11.12	10.68	10.39	10.20	10.06
14.39	13.27	12.33	11.59	11.05	10.65	10.35	10.11
14.50	13.53	12.66	11.93	11.34	10.87	10.48	10.16
14.56	13.69	12.89	12.18	11.57	11.05	10.60	10.19
14.59	13.80	13.05	12.36	11.74	11.18	10.69	10.22
14.62	13.87	13.16	12.48	11.86	11.29	10.75	10.25
14.64	13.93	13.23	12.57	11.95	11.36	10.80	10.27
14.65	13.96	13.29	12.64	12.02	11.42	10.84	10.28
14.66	13.99	13.33	12.69	12.06	11.46	10.87	10.29

OUTPUT DATA OF PROGRAMME-2E

```
DELT
      .050
 TRANSMISSIVITY    STORATIVITY       DELXJ     INITIALHEAD    HEADL          HEADR
    100.0            .020            10.0          10.0        15.00         10.00
SRIVL(K)
    15.00      15.00      15.00      15.00      15.00
    15.00      15.00      15.00      15.00      15.00
W3=       1.000000
W2=   0.000000E+00
W1=   0.000000E+00
W2=   0.000000E+00
W3=       1.000000
HEAD (metre) AT DIFFERENT TIMES AND NODES
  14.13   12.22   11.19   10.64   10.34   10.18   10.08   10.02
  14.30   12.98   11.95   11.23   10.75   10.44   10.22   10.07
  14.43   13.35   12.42   11.68   11.11   10.69   10.38   10.12
  14.51   13.58   12.73   12.00   11.40   10.91   10.51   10.16
  14.57   13.72   12.93   12.23   11.61   11.08   10.62   10.20
  14.60   13.82   13.07   12.39   11.77   11.21   10.70   10.23
  14.63   13.89   13.18   12.51   11.88   11.31   10.77   10.25
  14.64   13.94   13.25   12.59   11.97   11.38   10.81   10.27
  14.66   13.97   13.30   12.65   12.03   11.43   10.85   10.28
  14.66   14.00   13.34   12.70   12.07   11.46   10.87   10.29
```