Vibration of Cylindrical Shells Partially or Completely Filled with Water

A Thesis
submitted in partial fulfilment
of the requirements for the Degree
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MASTER OF ENGINEERING
in
Civil Engineering
with Specialization in
EARTHQUAKE ENGINEERING

By

ATUL CHANDRA GOYAL

DEPARTMENT OF CIVIL ENGINEERING
UNIVERSITY OF ROORKEE
ROORKEE
(INDIA)

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CERTIFICATE

Certified that the thesis entitled "Vibration of Cylindrical Shells partially or completely filled with water" which is being submitted by Sri Atul Chandra Goyal in partial fulfilment for the award of the Degree of Master of Engineering in Civil Engineering with specialisation in Earthquake Engineering of the University of Roorkee is a record of student's own work carried out by him under our supervision and guidance. The matter embodied in this thesis has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for a period of seven months from February 1968 to July 1968 and from September 1968 to October 1968 for preparing this thesis for Master of Engineering Degree at this University.

S.K. Thakkar
(S.K.Thakkar)
Lecturer,
School of Research and
Training in Earthquake Engg.
University of Roorkee,
Roorkee.

A.S. Arya
(A.S.Arya)
Professor and Asstt.Director
School of Research and Training
in Earthquake Engineering
University of Roorkee
Roorkee.

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SYNOPSIS

This analytical and experimental investigation is carried out to study the dynamic behaviour of cylindrical shells without water and with water filled inside them. The shell model is tested to see its behaviour under shock type of loading at base.

In the experimental study, two shells of different length to radius ratios have been studied and the water depth is varied in stages. Free vibration tests are conducted under condition of initial displacement by pulling and releasing the shell itself and also by tapping the shell at different points. Forced vibration study is carried out by mounting a mechanical oscillator on the shaking table.

The theoretical verification of the experimental results is done by analysing the shell as fixed at the base and free at the top.

Some strange behaviour of the shell has been observed under forced vibrations and conclusions have been drawn which shall be helpful in further investigations.

CONTENTS

CHAPTER - I

INTRODUCTION

1.1 INTRODUCTORY

The problem of dynamic interaction between liquid motions and elastic deformations of the walls of a container is of fundamental interest and importance with respect to a variety of applications. For example, there is a problem of bending oscillations in long tubes containing static or flowing liquids as encountered in many piping systems, the effect of non-rigid walls on the transmission of acoustic or pressure pulses in liquids or gases in long tubes, and the effect of a free surface on overall dynamic response of a liquid-tank system.

Most of the concrete or steel tanks (water containers or fuel storage tanks) in civil engineering practices are fixed at the base and free at the top.

Much work has been done on the vibration characteristics (frequencies and mode shapes under various combination of axial and circumferential waves) of circular cylinders similar to those used in large liquid-propellent rockets. But so far little attention has been paid to the type of tanks used in civil engineering works. The main aim of the thesis is to study the dynamic characteristics of such containers so that their earthquake response could be investigated.

Different parameters involved in the earthquake engineering problem of a cylindrical container are as follows:-

(i) The ground motion record i.e. its amplitude and frequency characteristics.

(ii) Geometrical properties of the shell i.e. length to radius ratio, radius to thickness ratio, end conditions at top and bottom.

(iii) Physical properties of the material of the shell i.e. density and Poisson's ratio.

(iv) Properties of the infilled liquid i.e. its density and level inside the shell.

Complete vibration study involves the consideration of all the above parameters. The present study is mainly concerned with shells clamped at base and free at top of two different L/a ratios, and also the influence of varying water depths.

1.2 OBJECT AND SCOPE

The investigations were made with the following objectives,

(i) To derive a method for the determination of the frequency of vibration of empty cylindrical shells having various boundary conditions, fixed at the base and free at the top.

(ii) To study the effect of filled fluid inside the shell, and

(iii) To study the free vibrations and steady state vibration of model tanks and to compare the results with those obtained from the theory under empty and water filled conditions.

The energy method using basic functions for freely vibrating beams has been used and applied to the case of fixed end at bottom and free end at top. It can be applied similarly to other end conditions. The effect of water has been considered only in an approximate manner by making assumptions regarding its virtual mass. In the experimental investigations, the model could be tested only in the fundamental mode of vibration. Results for higher number of axial and circumferential nodes could not be obtained because of limitations of the equipment.

1.3 OUTLINE OF THESIS

This work has been divided mainly into four parts. Chapter II gives a historical review of the work done by the various other investigators in the past. The analytical approach and experimental techniques of some authors, which have a bearing on the present work have been presented in more detail in this chapter. Theoretical derivation of frequency determinant based on Energy method has been given in Chapter III. Chapter IV includes the experimentation on the shell under free and forced vibration conditions. Finally all the results have been discussed in Chapter V and the main conclusions are drawn therein.

In the end a bibliography on the vibration of cylindrical shells has been given.

Computer programmes for determination of frequency has been given in the Appendix.

1.4 NOTATIONS

a	radius of the shell,
c	a factor in the expression of Weingarten formula for frequency $\left[c^2 = \frac{h^2}{12(1-\sigma^2)\,a^2} \right]$
E	Young's modulus of the elasticity of the material of the shell.
g	acceleration due to gravity.
h	thickness of the shell.
K	a factor in frequency determinant $\left[K = \frac{h^2}{12a^2} \right]$
L	length of the shell.
m	number of axial half waves (number of axial nodes=m-1)

n	number of circumferential waves (number of circumferential nodes=2n)
u	shell displacement in longitudinal direction.
v	shell displacement in circumferential direction.
w	shell displacement in radial direction.
γ	a fraction representing the ratio of water depth to total length of the shell.
Δ	frequency parameter $\left\| = \frac{(1-\sigma^2)\omega^2}{E g} \right\|$
σ	Poisson's ratio.
ρ	mass density of the material of the shell.
ρ'	mass density of the fluid inside the shell.
Ω	frequency parameter in Weingarten expression $\left\| = 12\rho a^4(1-\sigma^2)\frac{\omega^2}{Eh^2} \right\|$
ω	circular natural frequency of vibration of shell in radians per second.
f	frequency in cycles per second.

All these notations and others used, if any, have also been defined in the text wherever they appear first.

CHAPTER - II

REVIEW OF PREVIOUS WORK

2.1 HISTORICAL REVIEW IN GENERAL

Most of the investigators in this field have confined themselves to the free vibration of empty cylindrical shells. Some have considered the influence of fluid sloshing in the structures similar to those used in the aircraft's fuel storage tanks. The boundary conditions in the water storage tanks resting on ground are somewhat different than these and very few investigators have taken up this problem. The various methods used earlier for vibration analysis of cylindrical shells under different conditions are briefly reviewed here.

In the theoretical field Rayleigh (1894) [1] derived an expression for the frequencies of thin cylinders in which the motion of all cross-sections was identical. This corresponds to the fundamental axial form for a free-ended cylinder and is only of minor interest in the present problem. The general case of flexural vibrations of cylinders was later investigated by Love (1927) [2] who included both bending and the extensional deformations, though he did not include frequency equations for any specified end conditions. Flugge (1934) by a similar approach succeeded in obtaining a frequency equation for a cylindrical with freely supported ends. The roots of his equation defined three natural frequencies for any given nodal pattern; each frequency had a unique arrangement of amplitude ratios for the three component directions of strain X,Y and Z.

The problem was further investigated by Arnold and Warburton (1949)|3| in attempting to explain certain strange frequency phenomena observed in experiments with thin cylinders. It was found that the natural frequencies of thin cylinders with freely supported ends were arranged in a somewhat unexpected order which had little relation to the complexity of the nodal pattern.

Arnold and Warburton (1953)|4| further worked on the same lines as their work in 1949. Theoretical expressions were developed for the natural frequencies of cylinders with freely-supported and fixed ends and a comparison was made with the frequencies obtained experimentally. To make possible the estimation of such frequencies, a method was devised in which an equivalent wave-length factor was used. This factor represented the wavelength of the freely-supported cylinders that would have the same frequency as the cylinder under consideration when vibrating in the same mode. The results of experimental investigations with various end thicknesses and flange dimensions were recorded and from these the equivalent factors were derived.

Sets of curves calculated for cylinders with freely-supported ends and covering a range of cylinder thicknesses were given in their paper. From these it was possible to obtain close approximation to the frequencies of cylinders under other end conditions by the use of an appropriate factor.

This approach started from the very fundamentals and, therefore, gave quite accurate results. The only assumption was made that radial deflection varied along the axis in the same form as that of the similarly end conditioned vibrating beam. But the time required to derive the expressions and thereafter the evaluation of the values of frequencies and modeshapes was very great.

Recent investigations have concentrated on simplifying the method of analysis of vibrating cylindrical shells. By means of a number of approximations, Yu (1955) |5| was able to obtain a simple expression for the radial frequencies of a clamped or simply supported cylinder vibrating in a mode consisting of a number of circumferential waves that is large compared to the number of axial waves. Simplified frequency equations were also obtained by Vlasov(1958), Breslavskii(1953) and Reissner(1955) by neglecting the circumferential and axial inertia forces of the shell. Further, the simplifications of Breslavskii and Yu were combined by Rapport(1960) to yield frequency equations for a shell with various boundary conditions. A method similar to Rapport has been used by Weingarten(1964) [6]. An experimental investigation of the frequency spectra and mode shapes of a clamped-free cylinder were also performed by him. The experimental data are found to be in good agreement with theory. The results from this theory and that of Arnold and Warburton agreed quite closely at higher number of circumferential nodes. At lower nodes the difference was as great as 15%.

In the above mentioned investigations, no effort was made to include the effect of an internal liquid on the frequencies of the shell. Succeeding investigations have extended the studies of cylindrical shell vibration to include internal pressurization and improved measurement techniques. Fung, Sechler and Kaplan (1957) [7] studied pressurization effects using a loudspeaker as the excitation source and a number of capacitance probes mounted inside the cylinder to record wall displacements. Their results showed that the resonant frequencies and, particularly, the order in which the lowest resonant frequencies occur depend significantly upon the internal pressure. Berry and Reissner(1958) [8] had given a simplified expression for vibrating shell based on shallow shell theory. Thereafter they added one more term known as apparant mass factor to take into account the effect of fluid inside the shell. Gottenberg (1960) [9] extended the

the resonant bending frequency of the tank as compared with the tank having the same total mass of liquid but with the sloshing suppressed.

All these investigators noted a large number of resonant frequencies present and the need for careful identification of each resonance with the proper mode shape. This becomes increasingly difficult because the order of the resonant frequencies does not follow from the relative complexity of the mode shape i.e. a mode with a large number of circumferential nodes may have a lower resonant frequency than one with fewer circumferential nodes.

For this reason, for a cylindrical shell, there occurred several resonant breathing frequencies lower than the fundamental bending frequency.

All the work mentioned so far was more or less connected with vibration problems in large rocket propellents.

Jacobsen and Ayre (1951) |13| presented, for the first time, a treatment for liquid filled rigid cylinders when subjected to impulse at the base. By mounting dynamometers at the base of the tank they found out the equivalent mass and overturning moment due to the fluid. Their study, however, dealt principally with the nature of wave profiles and the location of maximum wave heights. In 1962 Baron and Skalak |14| presented an analytical treatment of the problem and studied the simply supported-free cylinders. The mode shapes of the empty shells were used as generalised coordinates of the shell fluid interaction problem. This approach may be considered as the first step of an iterative solution. It permits the evaluation of the influence of different heights of water inside the tank quite accurately. But the method is highly mathematical and evaluation of the frequencies and mode shapes for different boundary conditions is a very tedious job.

Perusal of the work done by all these and various other investigators, it becomes evident that little attention has been paid to the behaviour of the shells when subjected to a random motion at the base. This aspect needs further investigations.

The details of some of the investigations which are closely related to the present problem are described in the following paragraphs.

2.2 ARNOLD AND WARBURTON (1949) |3|

For freely supported ends, they derived frequency equations based on strain relations due to Timoshenko (1940) and were able to verify the experimental results with considerable accuracy. It was found, for example, that the natural frequencies of thin cylinders with freely supported ends were arranged in a somewhat unexpected order which had little relation to the complexity of the nodal pattern. Thus, for short cylinders with very thin walls the natural frequency may actually decrease as the number of circumferential nodes (2n) increases. This was shown theoretically to be due to the proportion of strain energy contributed respectively by bending and stretching; the latter was sometimes predominant for the simpler nodal patterns. Typical curves illustrating this phenomenon have been drawn in which frequency factor $\sqrt{\Delta}$ is plotted to a base of wave length factor λ .

It is observed from these non-dimensional curves that for a cylinder in which λ is 3.0, the configuration with four nodes (n=2) has a frequency approximately 50 percent greater than that with 24 nodes (n=12).

2.3 ARNOLD AND WARBURTON (1953) |4|

With freely supported and fixed ends both the theoretical analysis follows a somewhat similar pattern. Mathematical expressions were first derived for the component strains of an element of cylinder situated at the middle surface in terms of its rectangular displacements, u, v and w in directions X,Y and Z. These relations defined the possible ways, in which an element might deform elastically. Thereafter an attempt was made to find expressions for u,v and w which were not only compatible with elastic strain but also satisfied the specified end conditions. This, unfortunately, was not always possible by the introduction of known simple functions, but provided the shape of the assumed vibration form was approximately correct and the end conditions were not violated, the resulting expression for frequency would be close to truth. This follows from Rayleigh principle.

After the desired wave-forms had been obtained, the strain energy and kinetic energy of the cylinder were derived respectively in terms of displacements and rate of change of displacement, the latter involving the unknown frequency. Lagrange's equations were then written for the three independent displacements u,v and w and after elimination of the arbitrary amplitude constants, a cubic equation was eventually obtained. The roots of this equation defined frequencies associated with a given nodal arrangement.

The frequency equation derived for freely-supported cylinder is of the form

$$\Delta^3 - K_2\Delta^2 + K_1\Delta - K_o = 0$$

where the coefficients K_o, K_1 and K_2 are constants for a given cylinder under a given nodal configuration and the vibration frequency is given by

$$f = \frac{1}{2\pi a}\sqrt{\frac{E\,g\,\Delta}{\rho(1-\sigma^2)}} \qquad \ldots (2.1)$$

Different curves were plotted connecting wavelength factor with frequency factor $\sqrt{\Delta}$.

To make possible to calculate the frequency of vibration of cylinders with different end conditions from the same equation as for freely supported ends, an equivalent wavelength factor λ_e was defined. The expression for λ_e is,

$$\lambda_e = (m+c)\,\frac{\pi a}{L} \qquad \ldots (2.2)$$

where,

m = number of axial half waves,

c = a function of m, n and the cylinder dimension,

a = radius of cylinder,

L = length of cylinder.

If the ends are flanged, then,

$$\lambda_e = m+0.3\,e^{-q(h/d)}\,\frac{\pi a}{L} \qquad \ldots (2.3)$$

where,

q = constant,

h = thickness of cylinder

d = thickness at end.

2.4 LINDHOLM, KANA AND ABRAMSON (1962) |11|

They considered the shell simply supported at both ends. The frequency equation has been derived in the form,

$$4\pi^2 a^2 \frac{\rho}{E} f_{mn}^2 = \left| 1 + \frac{a}{h} \cdot \frac{\rho'}{\rho} \frac{I_n(\lambda_m)}{m\, I_n'(\lambda_m)} \right|^{-1} \cdot \left| \frac{(h/a)^2}{12(1-\sigma^2)} \left({}^{2}_{m} + n^2 \right)^2 + \frac{\lambda_m^4}{(\lambda_m^2 + n^2)^2} \right| \quad \ldots (2.4)$$

where,

a = radius of the shell,

ρ = mass density of the shell,

E = Young's modulus of elasticity

f_{mn} = frequency,

h = wall thickness,

ρ' = mass density of the fluid,

I_n = modified Bessel's function of the first kind of order n,

λ_m = characteristic root = $\frac{m\pi a}{L}$

σ = Poisson's ratio.

From here it is evident that as either m or n increases i.e. as the effective wavelengths decrease- the contribution of the apparent liquid mass to the total vibrating mass decreases. Thus, the resonant frequencies of the higher order modes are decreasingly affected by the presence of the liquid.

Their experiments on shells (on which there was a radial restraint at the ends but no restraint axially) showed that the presence of liquid did not affect the symmetry of the circumferential wave form, as expected, but did diminish the amplitude response. The liquid level had a marked effect on mode shapes. In general, the position of the axial nodes and anti-nodes got shifted towards the bottom or filled portion of the shell, the shift being greater the lower the liquid level. However, when the level was very low , as in the 1/4-full case, the nodes in the upper or unfilled portion of the shell tended to return to their normal positions. Also, the amplitude response of that portion of the shell in contact with the liquid was appreciably decreased.

Based on their experimentation, the frequency transition from the empty shell to the full shell is indicated in figure 2.1. These curves clearly show that the order in which the resonances occur depends upon the fluid level. For instance, for the empty shell $f_{1,2}$ is greater than $f_{1,5}$ whereas this order is reversed for the full shell.

They also observed that large number of frequencies were possible for various combination of axial and circumferential nodes, and they lay very close to each other, Fig. (22).

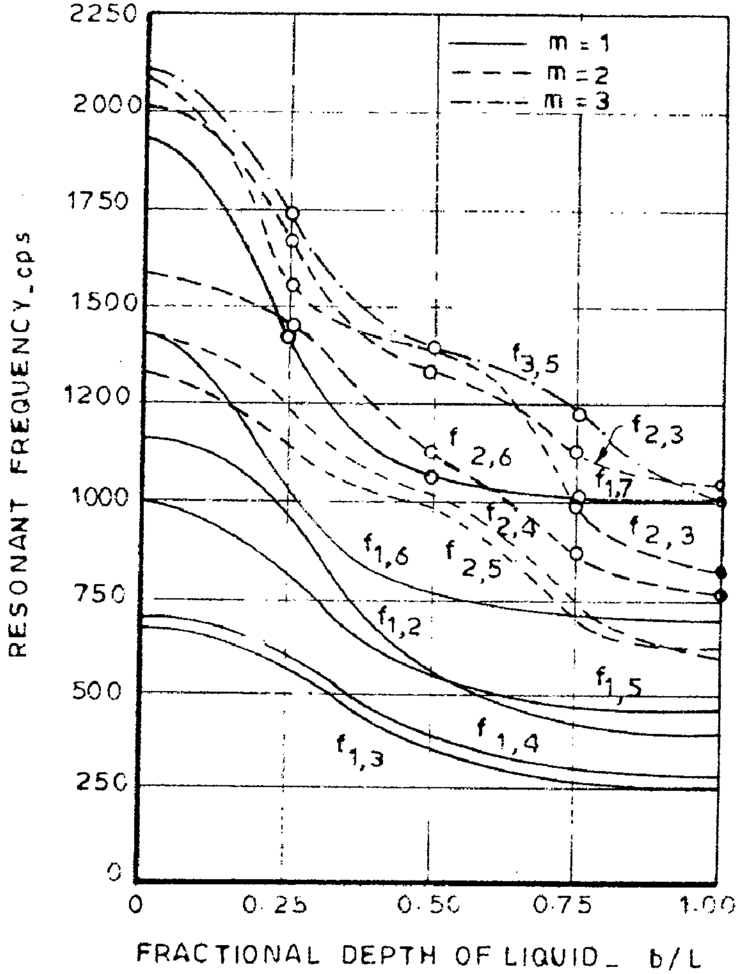


FIG. 2.1 – VARIATION OF RESONANT FREQUENCY WITH LIQUID DEPTH FOR SEVERAL BREATHING MODES

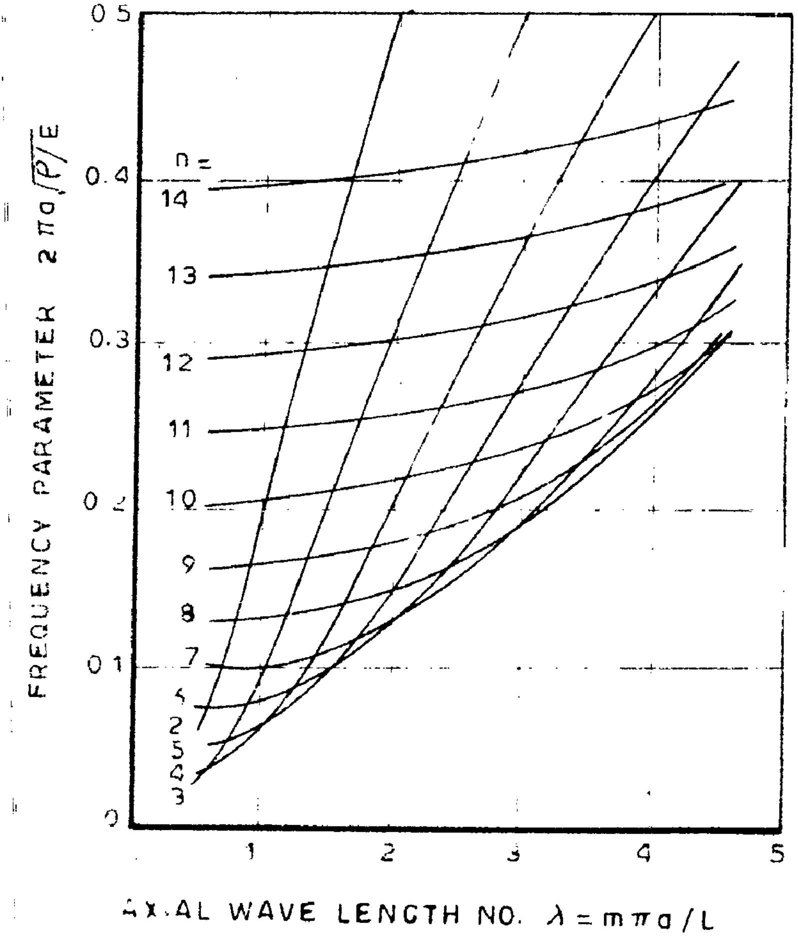


FIG.2.2 – VARIATION OF FREQUENCY WITH VARIOUS COMBINATION OF AXIAL AND CIRCUMFERENTIAL MODES

2.5 WEINGARTEN (1964) [6]

He derived the frequency equation from the well known Donnell's differential equation of a circular cylindrical shell. This equation involved a frequency parameter Ω given by

$$\Omega = \frac{12\rho a^4(1-\sigma^2)\omega^2}{Eh^2} = \frac{(\lambda_{kmn}+n^2)^4+\left|\frac{1}{c^2}\cdot\lambda^4_{knm}\right|}{(\lambda^2_{kmn}+n^2)^2} \quad \ldots (2.5)$$

where,

ρ = density of the shell material,

a = radius of the cylinder,

σ = Poisson's ratio of the material of the shell,

ω = Natural frequency of vibration of the shell in radians per second.

E = Young's modulus of cylinder material,

h = thickness of the cylinder,

n = number of circumferential waves,

c^2 = geometry parameter = $\dfrac{h^2}{12(1-\sigma^2)a^2}$

λ_{kmn} = characteristic roots.

Values of λ_{kmn} were obtained by substituting the appropriate boundary conditions for a vibrating beam in equation,

$$w = (c_1 \sin\lambda_{mn}\xi + c_2 \cos\lambda_{mn}\xi + c_3 \sinh\lambda_{mn}\xi + c_4 \cosh\lambda_{mn}\xi)\cos n\theta . \sin\omega t \quad .. (2.6)$$

where,

ξ - non-dimensional axial coordinate (x/a)

2.6 BARON AND SKALAK [14]

Two sets of modes were considered consisting of three and five constants respectively. Three constant modes were of the form,

$$\left.\begin{aligned} u(x,\phi) &= C_n \frac{U}{W} . \frac{x}{a} . \cos n\phi \\ v(x,\phi) &= C_n \frac{V}{W} . \frac{x}{a} . \sin n\phi \\ w(x,\phi) &= C_n \frac{x}{a} . \cos n\phi \end{aligned}\right| \quad .. (2.7)$$

and five-constant modes of the form

$$\left.\begin{aligned} u(x,\phi) &= C_n \left| \frac{U}{W} . \frac{x}{a} + \frac{X}{W} \left(\frac{x^2}{a^2} - \frac{3}{4} . \frac{L.x}{a^2}\right)\right| \cos n\phi \\ v(x,\phi) &= C_n \left| \frac{V}{W} . \frac{x}{a} \right| \sin n\phi \\ w(x,\phi) &= C_n \left| \frac{Y}{W} + \left(\frac{x}{a} - \frac{L}{2a}\right)\right| \cos n\phi \end{aligned}\right\} \quad .. (2.8)$$

where, a is radius of the shell, C_n is normalisation coefficient, L the height of shell, n the number of circumferential waves of a mode of vibration, ϕ and x are cylindrical coordinates, U,V,W,X,Y are coefficients depending on the mode shapes of free vibration of the shell.

It was seen that whereas three-constant modes gave accurate values of the frequency only, the five-constant modes gave accurate frequencies and mode shapes both for use in forced vibration problems.

To take the effect of water, a velocity potential function θ was chosen such that it satisfied the Laplace's equation,

$$\nabla^2\theta = \frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \frac{1}{r^2}\frac{\partial^2\theta}{\partial\phi^2} + \frac{\partial^2\theta}{\partial x^2} = 0$$

and at the same time it also satisfied the following boundary conditions:

(i) The radial velocity of the fluid to be equal the radial velocity of the shell on the surface $r = a$.

(ii) the vertical velocity at the bottom of the shell to be zero.

(iii) the pressure on the free surface of the liquid at $x = \frac{\gamma L}{2}$ to be zero.

Once the potential function ϕ had been determined the K.E. of the fluid was evaluated from either of the following relations,

$$T_{fluid} = -\frac{\rho'}{2}\iint_s \theta\,\frac{\partial\theta}{\partial n}\,.ds \qquad .. \quad (2.9)$$

where, the integration is taken over all surface s of the fluid and $\frac{\partial\theta}{\partial n}$ is the derivative of θ normal to s. ρ' is the mass density of the fluid.

or

$$T_{fluid} = \frac{\rho'}{2} \int_0^a \int_0^{2\pi} \int_0^{L} \left| \left(\frac{\partial\theta}{\partial r}\right)^2 + \left(\frac{\partial\theta}{\partial x}\right)^2 + \frac{1}{r^2}\left(\frac{\partial\theta}{\partial\phi}\right)^2 \right| r\,dr\,d\phi\,dz \qquad .. (2.10)$$

The potential function θ was evaluated in three parts, the sum of which satisfied all requirements on θ. For this purpose, the functions θ_k, K=1,2,3 were defined where each function θ_k could be expressed as summation of components in the n^{th} modes,

$$\theta_k = \sum_{n=1}^{\infty} \theta_{kn}.(r,\phi,x,t) \qquad .. (2.11)$$

Substituting the appropriate values of θ_n and its space derivatives into eq.(2.10) the kinetic energy of the fluid in the nth mode took the form

$$T_{n\ fluid} = D\, q_n^2 + \sum_{i=1}^{\infty} D_{1i}\, q_n^2 + \sum_{i=1}^{\infty} D_{2i}\, q_n\, A_{ni} + \sum_{i=1}^{\infty} D_{3i}\, A_{ni}^2 \qquad .. (2.12)$$

where D, D_{1i}, D_{2i}, D_{3i} are the quantities depending upon the shell dimensions and the water depth.

$q_{n(t)}$ and A_{ni} are the generalised coordinates of nth mode.

The kinetic and potential energies of the shell in the nth mode could also be written as

$$\left.\begin{aligned} T_{n\,shell} &= \frac{1}{2} M_n \dot{q}_n^2 \\ \text{and} \quad V_{nshell} &= \frac{1}{2} \bar{K}_n q_n^2 \end{aligned}\right\} \quad .. \ (2.13)$$

in which M_n is the generalised mass of the empty shell and $\bar{K}_n$ is the elasticity coefficients of the empty shell. The equation of motion governing the generalised coordinates $q_n(t)$ and $A_{ni}(t)$ were then obtained from Lagrange's equation.

$$\left.\begin{aligned} \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_n}\right) + \frac{\partial V}{\partial q_n} &= Q_n \\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{A}_{ni}}\right) + \frac{\partial V}{\partial A_{ni}} &= Q_{ni} \end{aligned}\right\} \quad .. \ (2.14)$$

where, T, the total kinetic energy of the system in nth mode was given by the sum of equations (2.10) and (2.13).

Solution of these equations implies that the kinetic energy of the fluid in nth mode may be written as

$$T_{nfluid} = \frac{\pi}{6} C_n^2 \gamma^3 L^3 \rho' \epsilon_n \dot{q}_n^2(t) \quad .. \ (2.15)$$

where ϵ_n is a virtual mass coefficient given by

$$\epsilon_n = \left\langle \frac{1}{n} \left| 1 - \sum_{i=1}^{\infty} \frac{6a^2 n \left| 1 - \frac{2a}{\alpha_{ni} \gamma L} \tanh\left(\frac{\alpha_{ni}\gamma L}{2a}\right) \right|}{\alpha_{ni}^2 \gamma^2 L^2 (\alpha_{ni}^2 - n^2)} \right| \right.$$

$$- \sum_{i=1}^{\infty} \frac{6 \tanh\left(\alpha_{ni} \frac{\gamma L}{a}\right)}{\alpha_{ni} \frac{\gamma L}{a} (\alpha_{ni}^2 - n^2)} \left| 1 - \frac{1}{2\alpha_{ni} \frac{\gamma L}{a} \tanh\left(\alpha_{ni} \frac{\gamma L}{a}\right)} \right.$$

$$\left. \left. + \frac{1}{2\alpha_{ni} \frac{\gamma L}{a} \sinh\left(\alpha_{ni} \frac{\gamma L}{a}\right)} - \frac{\tanh\left(\alpha_{ni} \frac{\gamma L}{2a}\right)}{2\alpha_{ni} \frac{\gamma L}{a}} \right|^2 \right\rangle \quad \text{.. (2.16)}$$

Here, α_{ni} are the roots of the equation $J_n'(\alpha_{ni}) = 0$

For convenience in forced vibration problems, the virtual mass was defined such that kinetic energy could be expressed as,

$$T_{nfluid} = \frac{m_{vn}}{2} \int_0^{2\pi} \int_0^{\gamma L} \dot{w}_n^2 (x, \phi, t)\, a\, d\phi\, dx \quad \text{.. (2.17)}$$

Here, m_{vn} is the mass per unit area due to water.

Substituting the value of w_n and equating the result to eq.(2.15) the virtual mass of the fluid in nth mode becomes,

$$m_{vn} = \epsilon_n \rho' a \quad \text{.. (2.18)}$$

The potential energy of the combined system is not affected by the fluid in the shell.

The frequency determinant can now be evaluated using Rayleigh-Ritz method.

The frequency of vibration is given by

$$\omega_n = \left| \frac{M\,E\,h}{m\;a^2(1-\sigma^2)} \right|^{1/2} \qquad \ldots (2.19)$$

where, lowest frequency number M can be determined from the determinant. Here m is the mass per unit area of the empty shell.

Problems investigated by above mentioned investigators are somewhat similar to the present problem. Hence, use will be made of their results in the present paper wherever necessary.

CHAPTER -III

FREQUENCY DETERMINANT FOR CYLINDERS FIXED AT BASE FREE AT TOP

3.1 STEPS IN THE DERIVATION

For deriving the frequency determinant of the cylindrical containers with fixity at base and free end at top, the following steps have been taken:

(i) To derive mathematical expressions for component strains of an element of cylinder situated at the middle surface in terms of u,v and w;

(ii) to find expressions for u,v and w which are not only compatible with elastic strains but also satisfy end conditions;

(iii) to derive kinetic energy and potential energy of the cylinder in terms of the displacements and their derivatives;

(iv) to consider a virtual mass factor for taking into account the effect of water on the vibrations of the shell;

(v) to write Lagrange's equation for the three displacements u,v, and w; and

(vi) to obtain a frequency determinant from the three equations derived in step(v).

3.2 DETAILS OF DERIVATION

A cylindrical shell of length L, thickness h, and mean radius a, is considered, an element of which is shown in figure (3.1). This element is bounded by two parallel planes perpendicular to the axis and distance δx apart, and by two radial planes subtending an angle $\delta\phi$ at the axis. The direct stresses acting on the element parallel to the X, Y and Z axes are p_x, p_y and p_z respectively; the shear stress acting on the face perpendicular to the X axis in the direction Y is p_{xy}; the shear stresses p_{yx}, p_{zx}, p_{yz} being similarly defined. The corresponding direct strains are e_x, e_y and e_z and the shear strains e_{xy}, e_{yz} and e_{zx}.

The total strain energy of the deformed shell, neglecting the trapezoidal form of the faces perpendicular to the X-axis may be written as,

$$S = \int_0^{2\pi}\int_0^{L}\int_{-h/2}^{h/2} \frac{1}{2}\left| p_x\, e_x + p_y e_y + p_{xy} e_{xy} \right| a\, d\phi\, dx\, dz \qquad \ldots (3.1)$$

In this it is assumed that, as is usual in the first approximation, the direct stresses p_z and the shear strains e_{yz} and e_{zx} are zero.

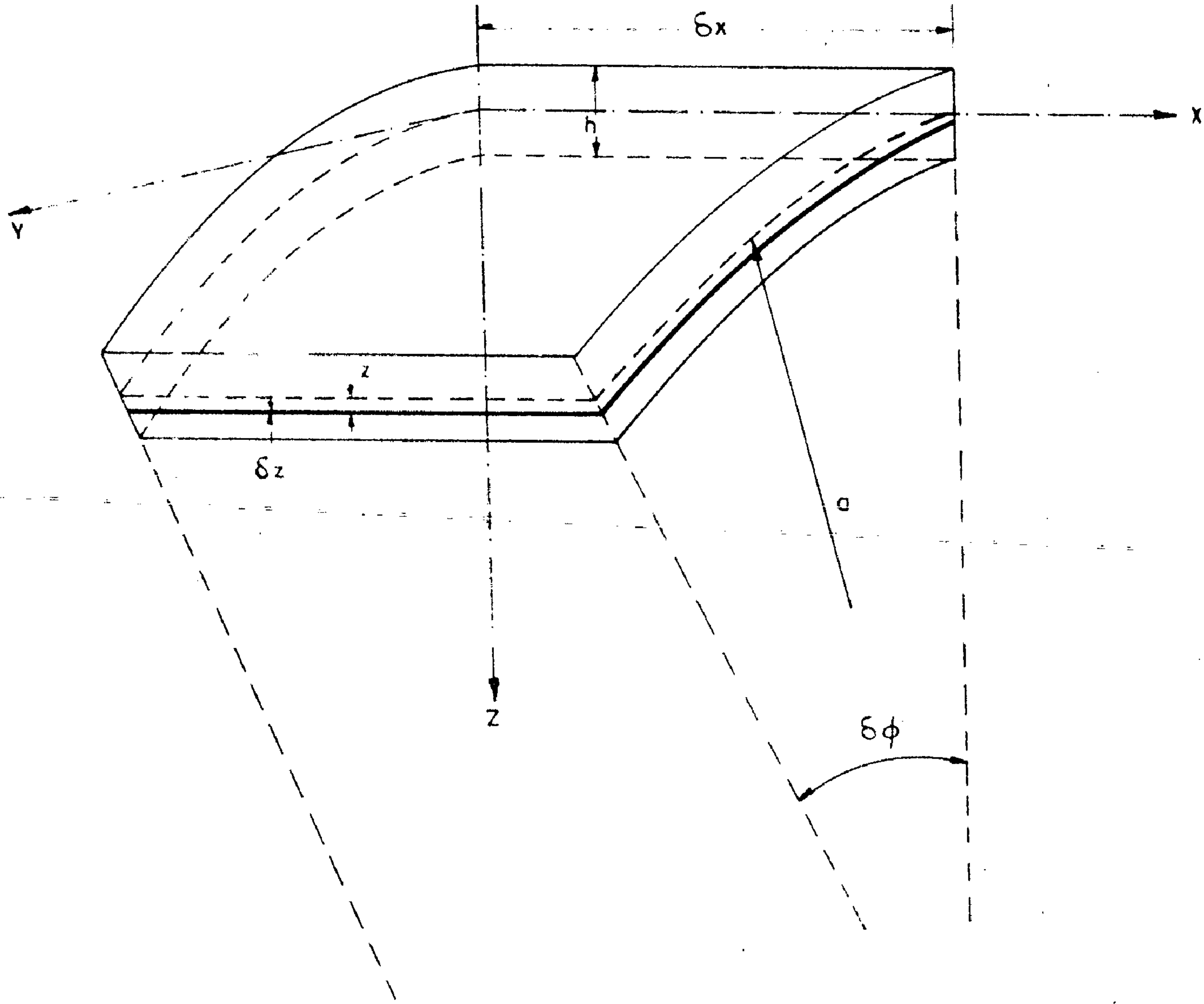


FIG. 3.1 _ SMALL ELEMENT OF THE SHELL

From Hooke's law

$$\left.\begin{aligned} p_x &= \frac{E}{1-\sigma^2}(e_x + \sigma . e_y) \\ p_y &= \frac{E}{1-\sigma^2}(e_y + \sigma . e_x) \\ p_{xy} &= \frac{E}{2(1+\sigma)} e_{xy} \end{aligned}\right\} \quad .. \quad (3.2)$$

Thus,

$$S = \frac{E}{2(1-\sigma^2)} \int_0^{2\pi} \int_0^{L} \int_{-h/2}^{+h/2} \left| e_x^2 + e_y^2 + 2\sigma . e_x e_y + \frac{1}{2}(1-\sigma) e_{xy}^2 \right| a d\phi dx dz \quad .. \quad (3.3)$$

The following symbols refer to the middle surface,

$\left.\begin{array}{l} \epsilon_1 \\ \epsilon_2 \end{array}\right\}$ strains in direction X and Y

K_1, K_2 changes of curvature in direction X and Y.

γ shear strain

τ twist

The strains at a distance Z from the middle surface of the deformed shell may then be expressed approximately,

$$\left.\begin{aligned} e_x &= \epsilon_1 - Z K_1 \\ e_y &= \epsilon_2 - Z K_2 \\ e_{xy} &= \gamma - 2Z\tau \end{aligned}\right\} \quad .. \quad (3.4)$$

If u, v, and w are the instantaneous displacements in the direction X, Y and Z of a point on the middle surface, the strains and changes of curvature are given in terms of the displacements and their derivatives by

$$\epsilon_1 = \frac{\partial u}{\partial x} \quad ; \quad \epsilon_2 = \frac{1}{a}\frac{\partial v}{\partial \phi} - \frac{w}{a}$$

$$K_1 = \frac{\partial^2 w}{\partial x^2} \quad ; \quad K_2 = \frac{1}{a^2}\frac{\partial^2 w}{\partial \phi^2} + \frac{1}{a^2}\frac{\partial v}{\partial \phi}$$

$$\gamma = \frac{\partial v}{\partial x} + \frac{1}{a}\frac{\partial u}{\partial \phi} \qquad \tau = \frac{1}{a}\frac{\partial^2 w}{\partial x \partial \phi} + \frac{1}{a}\frac{\partial v}{\partial x} \qquad \text{.. (3.5)}$$

where, ϕ defines the angular position of the point considered.

A convenient vibration form must now be assumed to satisfy the following end conditions,

At $x = 0$

$$u = 0, \quad v = 0, \quad w = 0, \quad \frac{\partial w}{\partial x} = 0$$

At $x = L$

$$\frac{\partial^2 w}{\partial x^2} = 0, \quad \frac{\partial^3 w}{\partial x^3} = 0, \quad p_x = 0, \quad p_{x\phi} = 0$$

For this, the variation of w along the axis of the cylinder is assumed to be of the same form as that of a cantilever beam during flexural vibrations.

Thus we can choose the axial, circumferential, and radial displacements as follows:

$$u = U\left|(\sinh\lambda x + \sin\lambda x) - k(\cosh\lambda x - \cos\lambda x)\right|\cos n\phi$$

$$v = V\left|(\cosh\lambda x - \cos\lambda x) - k(\sinh\lambda x - \sin\lambda x)\right|\sin n\phi$$

$$w = W\left|(\cosh\lambda x - \cos\lambda x) - k(\sinh\lambda x - \sin\lambda x)\right|\cos n\phi \qquad \ldots (3.6)$$

where U, V and W are function of time only and the constant k is given by

$$k = \frac{\cosh\lambda L + \cos\lambda L}{\sinh\lambda L + \sin\lambda L}$$

in which various values of are given by

$$\lambda L = 1.875,\ 4.694,\ 7.855,\ 10.996,\ 14.137,\ 17.279$$

corresponding to order of axial modes, that is first, second, third etc. respectively.

Substituting various values as defined above in equation (3.4) we get,

$$e_x = \left|U\lambda - Z\,W\lambda^2\right|\left|(\cosh\lambda x + \cos\lambda x) - k(\sinh\lambda x + \sin\lambda x)\right|\cos n\phi$$

$$e_y = \left|\frac{1}{a}(V.n - W) - \frac{Z}{a^2}(-W.n^2 + V.n)\right|\left|(\cosh\lambda x - \cos\lambda x) - k(\sinh\lambda x - \sin\lambda x)\right|\cos n\phi$$

$$= \left|\frac{V.n}{a}\left(1 - \frac{Z}{a}\right) - \frac{W}{a}\left(1 - \frac{Zn^2}{a}\right)\right|\left|(\cosh\lambda x - \cos\lambda x) - k(\sinh\lambda x - \sin\lambda x)\right|\cos n\phi$$

$$e_{xy} = \left|V\lambda\left(1 - \frac{2Z}{a}\right) - \frac{Un}{a} + \frac{2Z}{a}.n.\lambda.W\right|\left|(\sinh\lambda x + \sin\lambda x) - k(\cosh\lambda x - \cos\lambda x)\right|\sin n\phi \qquad \ldots (3.7)$$

Substituting these values in the expression of strain energy (equation 3.1) we get,

$$S = \frac{Ea\ \pi}{2(1-\sigma^2)} \int_0^L \Big\{ \Big| U^2 \lambda^2 h + W^2 \lambda^4 \frac{h^3}{12} \Big| \Big| (\cosh \lambda x + \cos \lambda x)$$

$$- k(\sinh \lambda x + \sin \lambda x) \Big|^2 + \Big| \frac{h}{a^2} (Vn - W)^2$$

$$+ \frac{h^3}{12a^4} (Vn - Wn^2)^2 \Big| \Big| (\cosh \lambda x - \cos \lambda x)$$

$$- k(\sinh \lambda x - \sin \lambda x) \Big|^2 + 2\sigma \Big| \frac{U \lambda h}{a} (Vn - W)$$

$$+ \frac{h^3}{12a^2} W \lambda^2 (Vn - Wn^2) \Big| \Big| (\cosh^2 \lambda x - \cos^2 \lambda x)$$

$$- k(\sinh \lambda x . \cosh \lambda x) - \sin \lambda x \cos \lambda x$$

$$+ \sinh\ x . \cosh\ x - \cos\ x . \sin\ x)$$

$$+ k^2 (\sinh^2 \lambda x - \sin^2 \lambda x) \Big| + (\frac{1-\sigma}{2}) \Big| h\ V^2 \lambda^2$$

$$+ 4 \frac{h^3 V^2 \lambda^2}{12a^2} + h \frac{U^2 n^2}{a^2} + \frac{4h^3}{12a^2} n^2 \lambda^2 W^2$$

$$- \frac{2VUn\lambda}{a} h - \frac{8h^3}{12a^2} n \lambda^2 VW \Big| \Big| \sinh \lambda x + \sin \lambda x)$$

$$- k(\cosh \lambda x - \cos \lambda x) \Big|^2 \Big\} dx \qquad .. \ (3.8)$$

Since,

$$\int_{-h/2}^{h/2} dZ = h; \quad \int_{-h/2}^{h/2} Zdz = 0\ ; \quad \int_{-h/2}^{h/2} Z^2 dz = \frac{h^3}{12}$$

$$\int_0^{2\pi} \cos^2 n\phi\ d\phi = \int_0^{2\pi} \sin^2 n\phi d\phi = \pi$$

Taking h outside the integral sign in equation and putting $\frac{h^2}{12a^2} = K$, we get

$$S = \frac{E\pi a h}{2(1-\sigma^2)} \int_0^L \Big\langle \Big| a^2\lambda^2\Big(\frac{U^2}{a^2} + W^2\lambda^2 K\Big)\Big| (\cosh\lambda x + \cos\lambda x)$$

$$-k(\sinh\lambda x + \sin\lambda x)\Big|^2 + \frac{1}{a^2}(V^2n^2 + W^2 - 2nVW)$$

$$+\frac{1}{a^2}K(V^2n^2 + W^2n^4 - 2n^3VW)\Big|\Big|(\cosh\lambda x - \cos\lambda x)$$

$$-k(\sinh\lambda x - \sin\lambda x)\Big|^2 + 2\sigma\Big|\frac{U\lambda}{a}(Vn-W) + K W^2$$

$$(Vn - Wn^2)\Big|\Big|(\cosh^2\lambda x - \cos^2\lambda x) - 2k(\sinh\lambda x.\cosh\lambda x$$

$$-\cos\lambda x.\sin\lambda x) + k^2(\sinh^2\lambda x - \sin^2\lambda x)\Big|$$

$$+\Big(\frac{1-\sigma}{2}\Big)\Big|V^2\lambda^2 + 4V^2\lambda^2 K + \frac{U^2n^2}{a^2} + 4Kn^2\lambda^2W^2$$

$$-\frac{2VUn\lambda}{a} - 8Kn\lambda^2VW\Big|\Big|(\sinh\lambda x + \sin\lambda x)$$

$$-k(\cosh\lambda x - \cos\lambda x)\Big|^2\Big\rangle dx \qquad .. \quad (3.9)$$

For the sake of convenience let us use the following notations,

$$\left.\begin{aligned} \alpha &= \frac{\sinh 2\lambda L}{4}; & \varepsilon &= \underline{1}.\cosh\lambda L.\cos\lambda L \\ \beta &= \frac{\cosh 2\lambda L}{4}; & \eta &= \underline{1}.\cosh\ L.\sin\ L \\ \gamma &= \frac{\sin 2\lambda L}{4}; & \chi &= \underline{1}.\sinh\lambda L.\sin\lambda L \\ \delta &= \frac{\cos 2\lambda L}{4}; & \zeta &= \underline{1}.\sinh\lambda L.\cos\lambda L \end{aligned}\right\} \quad .. \quad (3.10)$$

Thus various integrals become,

$$\int_0^L \cosh^2\lambda x\, dx = \frac{L}{2} + \alpha$$

$$\int_0^L \sinh^2\lambda x \; dx = \alpha - \frac{L}{2}$$

$$\int_0^L \sinh\lambda x . \cosh\lambda x \; dx = \beta - \frac{1}{4\lambda}$$

$$\int_0^L \cos^2\lambda x \; dx = \frac{L}{2} + \gamma$$

$$\int_0^L \sin^2\lambda x \; dx = \frac{L}{2} - \gamma$$

$$\int_0^L \sin\lambda x . \cos\lambda x \; dx = -\delta + \frac{1}{4\lambda}$$

$$\int_0^L \sin\lambda x . \sinh\lambda x \; dx = \frac{1}{2}|\psi - \xi|$$

$$\int_0^L \sin\lambda x . \cosh\lambda x \; dx = \frac{1}{2}|\chi - \xi + \frac{1}{\lambda}|$$

$$\int_0^L \cos\lambda x . \sinh\lambda x \; dx = \frac{1}{2}|\xi + \chi - \frac{1}{\lambda}|$$

$$\int_0^L \cos\lambda x . \cosh\lambda x \; dx = \frac{1}{2}|\zeta + \psi|$$

Also,

$$\int_0^L \left| (\cosh\lambda x + \cos\lambda x) - k(\sinh\lambda x + \sin\lambda x) \right|^2 dx$$

$$= (L + \alpha + \gamma + \psi + \zeta) + k^2(\alpha - \gamma + \psi - \zeta)$$

$$- 2k(\beta + \chi - \delta)$$

$$= M_1 \text{ (say)}$$

$$\int_0^L \left| (\cosh\lambda x - \cos\lambda x) - k(\sinh\lambda x - \sin\lambda x) \right|^2 dx$$

$$= (L + \alpha + \gamma - \psi - \zeta) + k^2(\alpha - \gamma - \psi + \zeta)$$

$$- 2k(\beta - \chi - \delta)$$

$$= M_2 \text{ (say)}$$

$$\int_0^L \left| (\cosh^2\lambda x - \cos^2\lambda x) - 2k(\sinh\lambda x.\cosh\lambda x - \cos\lambda x.\sin\lambda x) + k^2(\sinh^2\lambda x - \sin^2\lambda x) \right| dx$$

$$= (\alpha - \gamma) - 2k(\beta + \delta - \frac{1}{2\lambda}) + k^2(\alpha + \gamma - L)$$

$$= M_3 \text{ (say)}$$

$$\int_0^L \left| (\sinh\lambda x + \sin\lambda x) - k(\cosh\lambda x - \cos\lambda x) \right|^2$$

$$= (\alpha - \gamma + \psi - \zeta) + k^2(L + \alpha + \gamma - \zeta - \psi) - 2k(\beta - \xi + \delta + \frac{1}{2\lambda})$$

$$= M_4 \quad \text{(say)}$$

The terms M_1, M_2, M_3 and M_4 have again been introduced for the sake of convenience.

Substituting all these values in equation (3.9) we get the expression for potential energy as follows,

$$S = \frac{E\pi ah}{2(1-\sigma^2)} \left\{ \left| a^2\lambda^2(\frac{U^2}{a^2} + W^2\lambda^2 K) \right| M_1 + \frac{1}{a^2} \left| V^2 n^2(1+K) + W^2(1+Kn^4) - 2nVW(1+Kn^2) \right| M_2 + 2\sigma \left| \frac{n\lambda}{a} UV - \frac{\lambda}{a} UW + K\lambda^2 nVW - Kn^2\lambda^2 W^2 \right| M_3 + \frac{1-\sigma}{2} \left| V^2\lambda^2(1+4K) + \frac{U^2 n^2}{a^2} + 4Kn^2\lambda^2 W^2 - \frac{2n\lambda}{a} UV - 8Kn\lambda^2 VW \right| M_4 \right\} \quad .. (3.11)$$

Kinetic Energy is given by

$$T = \frac{\rho}{2g} \int_0^{2\pi} \int_0^L \int_{-h/2}^{h/2} \left| (\frac{\partial u}{\partial t})^2 + (\frac{\partial v}{\partial t})^2 + (\frac{\partial w}{\partial t})^2 \right| a d\phi \, dx \, dz$$

$$= \frac{\rho \pi ah}{2g} \left| \dot{U}^2 M_4 + (\dot{V}^2 + \dot{W}^2) M_2 \right| \quad .. (3.12)$$

Here, $\dot{U}$, $\dot{V}$ and $\dot{W}$ represent the derivatives of U,V and W with respect to time.

Since, U,V and W are independent variables, the Lagrange's equation is applicable,

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{U}}\right) - \frac{\partial T}{\partial U} = - \frac{\partial s}{\partial U} \qquad .. \ (3.13)$$

and two similar expressions in V and W.

Putting,

$$U = A \cos \omega t$$

$$V = B \cos \omega t$$

$$W = C \cos \omega t,$$

and writing

$$\Delta = \frac{(1-\sigma^2)\rho \omega^2}{E \ g}$$

where, ω is the natural frequency in radians per second we get following three equations,

$$\left| \lambda^2 M_1 + \frac{1-\sigma}{2} \frac{n^2}{a^2} M_4 - \Delta M_4 \right| A + \left| \frac{\sigma n \lambda}{a} M_3 - \left(\frac{1-\sigma}{2}\right) \frac{n\lambda}{a} M_4 \right| B$$
$$- \left| \frac{\sigma \lambda}{a} \cdot M_3 \right| C = 0$$

$$\left| \frac{\sigma n \lambda}{a} M_3 - \frac{1-\sigma}{2} \cdot \frac{n\lambda}{a} \cdot M_4 \right| A + \left| \frac{n^2}{a^2}(1+K)M_2 + \frac{1-\sigma}{2} \cdot {}^2(1+4K)M_4 \right.$$
$$\left. - \Delta M_2 \right| B + \left| -\frac{n}{a^2}(1+n^2K)M_2 + \sigma k \lambda^2 n M_3 + \right.$$
$$\left. + \frac{1-\sigma}{2} \cdot 4 Kn \lambda^2 M_4 \right| C = 0$$

$$\left|-\frac{\sigma\lambda}{a} M_3\right| A + \left|-\frac{n}{a^2}(1+n^2K)M_2+\sigma K\lambda^2 nM_3 - \frac{1-\sigma}{2} 4K.n\lambda^2 M_4\right| B$$

$$+\left|a^2\lambda^4 KM_1 + \frac{1}{a2}(1+n^4K)M_2 - \sigma 2Kn^2\lambda^2 M_3 + \frac{1-\sigma}{2} 4Kn^2\lambda^2 M_4\right| C=0 \qquad .. (3.13)$$

From these equations (3.13) one may obtain the value of natural frequency of vibration of the system by eliminating three constants A,B and C. To take the effect of fluid inside the shell the approach similar to that given by Baron and Skalak (see **Art.**2.6) shall be used.

It is convenient , for application to forced vibration problems to define a virtual mass of the fluid such that the kinetic energy of the fluid can be expressed in terms of the radial displacements of the shell only, that is,

$$T_{nfluid} = \frac{m_{vn}}{2}\int_0^{2\pi}\int_0^{\gamma L} \dot{w}^2 \, a \, d\phi \, dx$$

$$= \frac{m_{vn}}{2}\int_0^{2\pi}\int_0^{\gamma L} \dot{W}^2 \left|(\cosh\lambda x-\cos\lambda x)-k(\sinh\lambda x -\sin\lambda x)\right|^2 \cos^2 n\phi \, a d\phi \, dx$$

$$= \frac{m_{vn}.\dot{W}^2.a\pi}{2}\int_0^{\gamma L}\left|\cosh\lambda x-\cos\lambda x) -k(\sinh\lambda x-\sin\lambda x)\right|^2 dx$$

$$= \frac{m_{vn}\,\dot{W}^2\,a\,\pi}{2} M_5 \text{ (say)} \qquad .. (3.14)$$

Here m_{vn} is the virtual mass per unit area which has to be added to the mass per unit area of the shell to take the effect of fluid inside the shell into account. The virtual mass of the fluid may thus be considered to be an additional tank mass moving only in the radial direction.

At this stage, an assumption is made that the virtual mass in case of clamped-free cylinder is not different from that of simply supported-free cylinder. This, in reality, will not be very far off from the actual conditions for higher modes, because of lesser influence of the boundary conditions, on the frequency of higher modes. For fundamental and other lower modes, however, the results will have some inaccuracy due to this assumption. However, with this assumption, the kinetic energy of the fluid may directly be taken from Eq.(2.15)

$$T_{n\ fluid} = \frac{\pi}{6} C_n^2 \gamma^3 L^3 \epsilon_n \dot{W}^2 \qquad .. (3.15)$$

where, ϵ_n is the virtual mass coefficient whose value may be taken up directly from equation (2.16) page 24 . Values of α_{ni} occurring in that expression may be obtained from Table 3.1.

Equating equations (3.14) and (3.15),

$$\frac{\pi}{6} \gamma^3 L^3 \varrho' \epsilon_n \dot{W}^2 = \frac{m_{vn} \dot{W}^2 a \pi}{2} M_5$$

Table 3.1

Values of α_{ni} (in equation 2.16) which are roots of equation $J'_n(\alpha_{ni}) = 0$

	n=0	1	2	3	4	5	6
i = 1	3.832	1.841	3.053	4.20	5.31	6.40	7.50
2	7.016	5.332	6.707	7.89	9.04	10.52	11.74
3	10.17	8.536	9.970	11.17	12.33	13.99	15.27
4	13.32	11.710	13.170	14.37	15.53	17.27	18.60
5	16.47	14.860	16.310	17.52	18.79	20.53	21.88

From where the expression for virtual mass is obtained as,

$$m_{vn} = \frac{\gamma^3 L^3 \rho'}{3\ a\ M_5}\ \epsilon_n \qquad .. (3.16)$$

The different curves of values of ϵ_n for different water heights and number of circumferential modes for the experimental shell have been drawn in figure (3.2).

Now the total kinetic energy of the shell-fluid system becomes,

$$T = \frac{\rho h}{2g} \int_0^{2\pi} \int_0^{L} (\dot{U}^2+\dot{V}^2+\dot{W}^2) a d\phi dx + \frac{m_{vn}}{2g} \int_0^{2\pi} \int_0^{\gamma L} W^2 a d\phi dx$$

Substituting the values in Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{W}}\right) - \frac{\partial T}{\partial W} = - \frac{\partial S}{\partial W}$$

we get,

$$\frac{\pi \rho h a}{2 g} \left| 2\ddot{W} M_2 + \frac{\gamma^3 L^3 \rho' \epsilon_n}{3\ a \rho h M_5} 2W.M_5 \right|$$

$$= - \frac{E\pi a h}{2(1-\sigma^2)} \left\{ 2a^2 \lambda^4 K M_1 W + \frac{1}{a^2} \left| 2(1+n^4K)W \right.\right.$$

$$\left. -2n(1+n^2K)V \right| M_2 + 2\sigma\left(-\frac{\lambda}{a} U\right.$$

$$\left. +K \lambda^2 nV - 2Kn^2 \lambda^2 W\right) M_3$$

$$\left. + \frac{1-\sigma}{2} (8Kn^2 \lambda^2 W - 8Kn \lambda^2 V) M_4 \right\}$$

or,

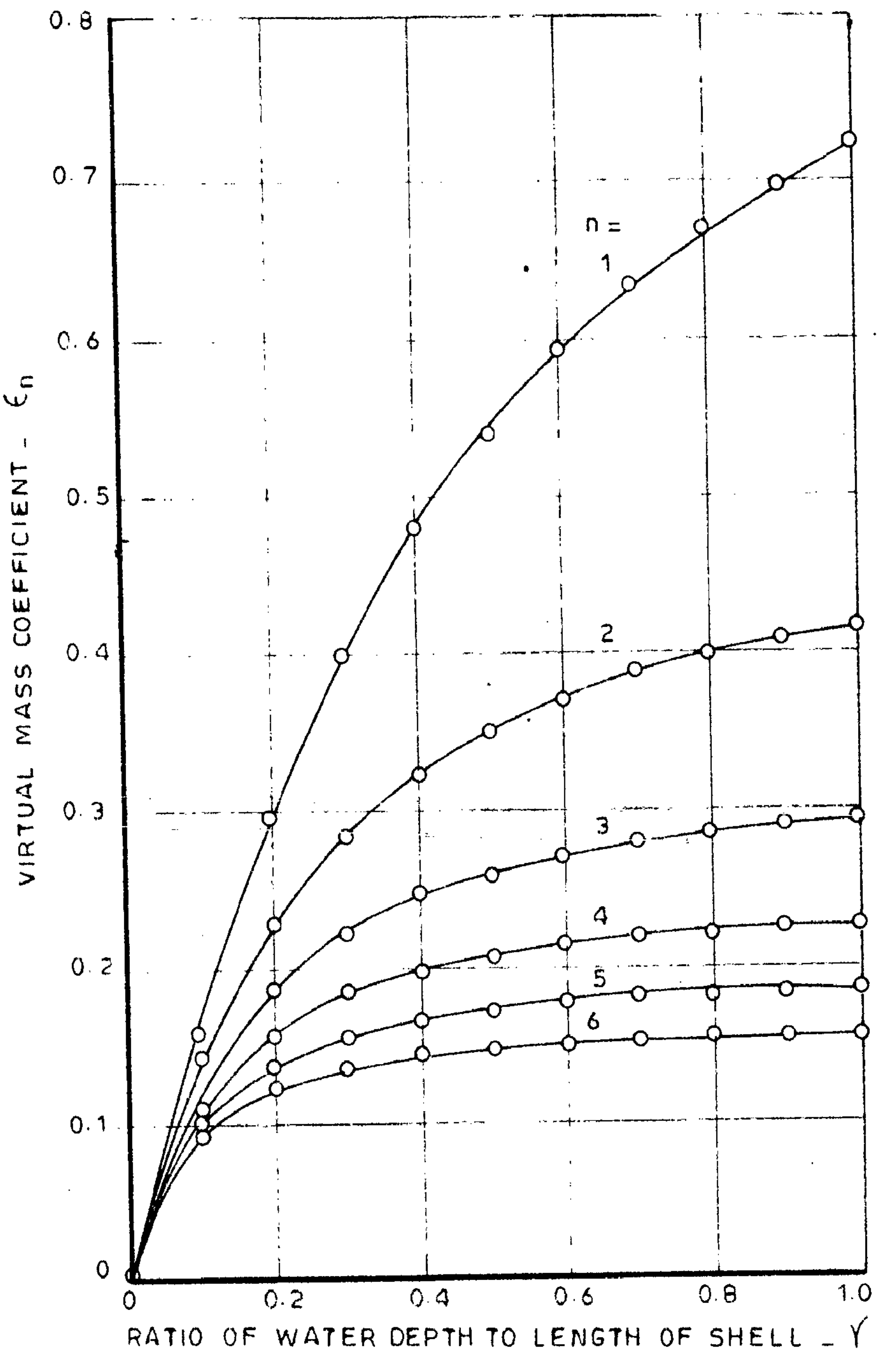


FIG. 3.2 – VARIATION OF VIRTUAL MASS COEFFICIENT ϵ_n WITH THE DEPTH OF WATER IN THE TANK TAKING $i = 1$ TO 4 FOR $L/a = 4.38$

$$\left|-\frac{\sigma\lambda}{a}M_3\right|A+\left|-\frac{n}{a^2}(1+n^2K)M_2+\sigma K\lambda^2 nM_3-\left(\frac{1-\sigma}{2}\right)4Kn\lambda^2M_4\right|B$$
$$+\left|a^2\lambda^4KM_1+\frac{1}{a^2}(1+n^4K)M_2-\sigma 2Kn^2\lambda^2M_3\right.$$
$$\left.+\frac{1-\sigma}{2}.4Kn^2\lambda^2M_4-\triangle\left(M_2+\frac{\sqrt{3}L^3\varrho'}{3ha\varrho}\epsilon_n\right)\right|C\quad=0$$

The first two equations of eq.(3.13) will remain unaltered as the presence of water will only have a effect in radial displacement terms.

Therefore, now the frequency determinant becomes,

$$\begin{vmatrix} \left|\lambda^2M_1+\frac{1-\sigma}{2}.\frac{n^2}{a^2}.M_4-\triangle M_4\right| & \left|\frac{\sigma n\lambda}{a}M_3\frac{1-\sigma}{2}.\frac{n\lambda}{a}.M_4\right| & \left|-\frac{\sigma\lambda}{a}M_3\right| \\ \left|\frac{\sigma n\lambda}{a}M_3-\frac{1-\sigma}{2}.\frac{n\lambda}{a}.M_4\right| & \left|\frac{n^2}{a^2}(1+K)M_2+\frac{1-\sigma}{2}.\lambda^2.(1+4K)M_4-\triangle M_2\right| & \left|-\frac{n}{a^2}(1+n^2K)M_2+\sigma K\lambda^2nM_3-\frac{1-\sigma}{2}.4Kn\lambda^2M_4\right| \\ \left|-\frac{\sigma\lambda}{a}M_3\right| & \left|-\frac{n}{a^2}(1+n^2K)M_2+\sigma K\lambda^2nM_3-\frac{1-\sigma}{2}.4Kn\lambda^2M_4\right| & \left|a^2\lambda^4KM_1++\frac{1}{a^2}(1+n^4K)M_2-2\sigma K n^2\lambda^2M_3+\frac{1-\sigma}{2}.4Kn^2\lambda^2M_4-\triangle\left(M_2+\frac{\sqrt{3}L^3\varrho'}{3ah\varrho}\epsilon_n\right)\right| \end{vmatrix}=0$$

.. (3.17)

Various terms occurring in determinant (3.17) have the following values,

$$\alpha = \frac{\sinh 2\lambda L}{4\lambda} \qquad \xi = \frac{1}{\lambda}\left|\cosh\lambda L.\cos\lambda L\right|$$

$$\beta = \frac{\cosh 2\lambda L}{4\lambda} \qquad \psi = \frac{1}{\lambda}\left|\cosh\lambda L.\sin\lambda L\right|$$

$$= \frac{\sin 2\lambda L}{4\lambda} \qquad \chi = \frac{1}{\lambda}\left|\sinh\lambda L.\sin\lambda L\right|$$

$$\delta = \frac{\cos 2\lambda L}{4\lambda} \qquad \zeta = \frac{1}{\lambda}\left|\sinh\lambda L.\ \cos\lambda L\right|$$

$$\Delta = \frac{(1-\sigma^2)\rho w^2}{E\ g} \text{ (frequency parameter)} \qquad K = \frac{h^2}{12a^2}$$

$$k = \frac{\cosh\lambda L+\cos\lambda L}{\sinh\lambda L+\sin\lambda L}$$

L = 1.875, 4.694, 7.855, 10.996, 14.137, 17.279 corresponding to number of axial modes = 1,2,3,4,5, 6,7,8

$$M_1 = (L+\alpha+\gamma+\psi+\zeta)+k^2(\alpha-\gamma+\psi-\zeta)-2k(\beta+\chi-\delta)$$

$$M_2 = (L+\alpha+\gamma-\psi-\zeta)+k^2(\alpha-\gamma-\psi+\zeta)-2k(\beta-\chi-\delta)$$

$$M_3 = (\alpha-\gamma)+k^2(\alpha+\gamma-L) - 2k(\beta+\delta-\frac{1}{2\lambda})$$

$$M_4 = (\alpha-\gamma+\psi-\zeta) + k^2(L+\alpha+\gamma-\zeta-\psi)-2k(\beta-\xi+\delta+\frac{1}{2\lambda})$$

For given values of λ, n, L, a, h and γ, the lowest frequency parameter Δ, can be determined from the determinant 3.17 and the frequencies of vibration of the fluid filled shell calculated from the equation,

$$\omega = \sqrt{\frac{\Delta E g}{\rho(1-\sigma^2)}} \qquad .. (3.18)$$

in which h is the thickness of the shell.

Once the frequencies are evaluated, the corresponding mode shape ratios may be evaluated from the system of homogeneous equations, the determinant of which is given in Eq.3.17. Knowing frequencies and mode shapes, the modal analysis of the shell may be carried out.

3.3 NUMERICAL RESULTS

Numerical results, from the derived expressions for frequency and mode shapes, have been obtained for the shell which was used for experimental investigations. This shell had the following characteristics. (For details see Chapter IV).

Length of the shell	$L = 60.2$ cms.
Radius of the shell	$a = 13.725$ cms.
Thickness of the shell	$h = 0.0795$ cms.
Poisson's ratio of the material of the shell	$= 0.345$
Young's modulus of elasticity of the material	$E = 7.2\text{x}10^5$ kg/cm^2
Density of the material	$= 8.0$ gm/cm^3
Density of the fluid	$= 1.0$ gm/cm^3

Thus for the shell,

$L/a = 4.38$

$a/h = 173$

Numerical computation may be divided into three parts ,

(i) Evaluation of virtual mass coefficient,

(ii) Evaluation of natural frequency of vibration, and

(iii) Evaluation of mode shapes.

We shall take these items one by one.

(i) Evaluation of virtual mass coefficients

Equation (2.16) was used to obtain the values of ϵ_n. A computer programme was made which has been given in Appendix (see A.2). The variation of ϵ_n with various water depths and number of circumferential waves has been plotted in figure (3.2), page 41

The computations include terms upto and including i=4 in the summation in ϵ_n.

(ii) Evaluation of frequency of vibration

To determine the frequency parameter Δ from the determinant (Eq. 3.17) for known values of other variables and then the frequency from the equation 3.18, a computer programme was written as given in Appendix A.3. The procedure followed for evaluation of frequency is as follows.

An approximate value of the frequency is assumed and the value of the determinant is worked out. This is repeated with increments of frequency given at regular intervals. The value of frequency at which the value of the determinant becomes zero gives the requency of vibration of the system. For a particular set of axial and circumferential nodes, there are three possible frequencies at which the determinant is zero. Only the lowest of these will give the true natural frequency of vibration for the set of nodes as the other two frequencies will be far removed from the lowest value. For any frequency , the computation is stopped when the ratio of the difference between two consecutive frequencies for which the determinant changes sign to one of them becomes less than the pre-specified accuracy. The accuracy in these computations is 0.0001.

Results obtained have been shown in Table 3.2 and curves for first mode has been drawn in figure 3.3, where m is the number of half waves (nodes = m-1) in the axial direction and n is the number of waves (nodes = 2n) in the circumferential direction.

Results for Weingarten method

For comparison purpose, frequencies have also been calculated from the expression derived by Weingarten(Eq. 2.5). The results have been plotted in Fig. 3.3 and 3.4 for both the shells (L/a = 4.38 and 2.63). To show the effect of fixity at the base ,

Table 3.2

Frequencies obtained theoretically

length of the shell (L)	Number of axial half wave (m)	Number of circumferential waves. (n)	Water depth (cms)	Frequency of vibration by (Energy Method) (cps)	Frequency of vibration by Weingarten method (cps)
60.2 cms. h/a = 4.38	1	1	0	383.20	519.92
	1	2	0	141.70	148.79
	1	3	0	83.65	87.27
	1	4	0	97.50	106.14
	1	5	0	148.60	156.24
	1	6	0	214.10	222.40
	1	3	0.332	58.150	-
	1	3	0.535	40.450	-
	1	3	0.93	33.120	-

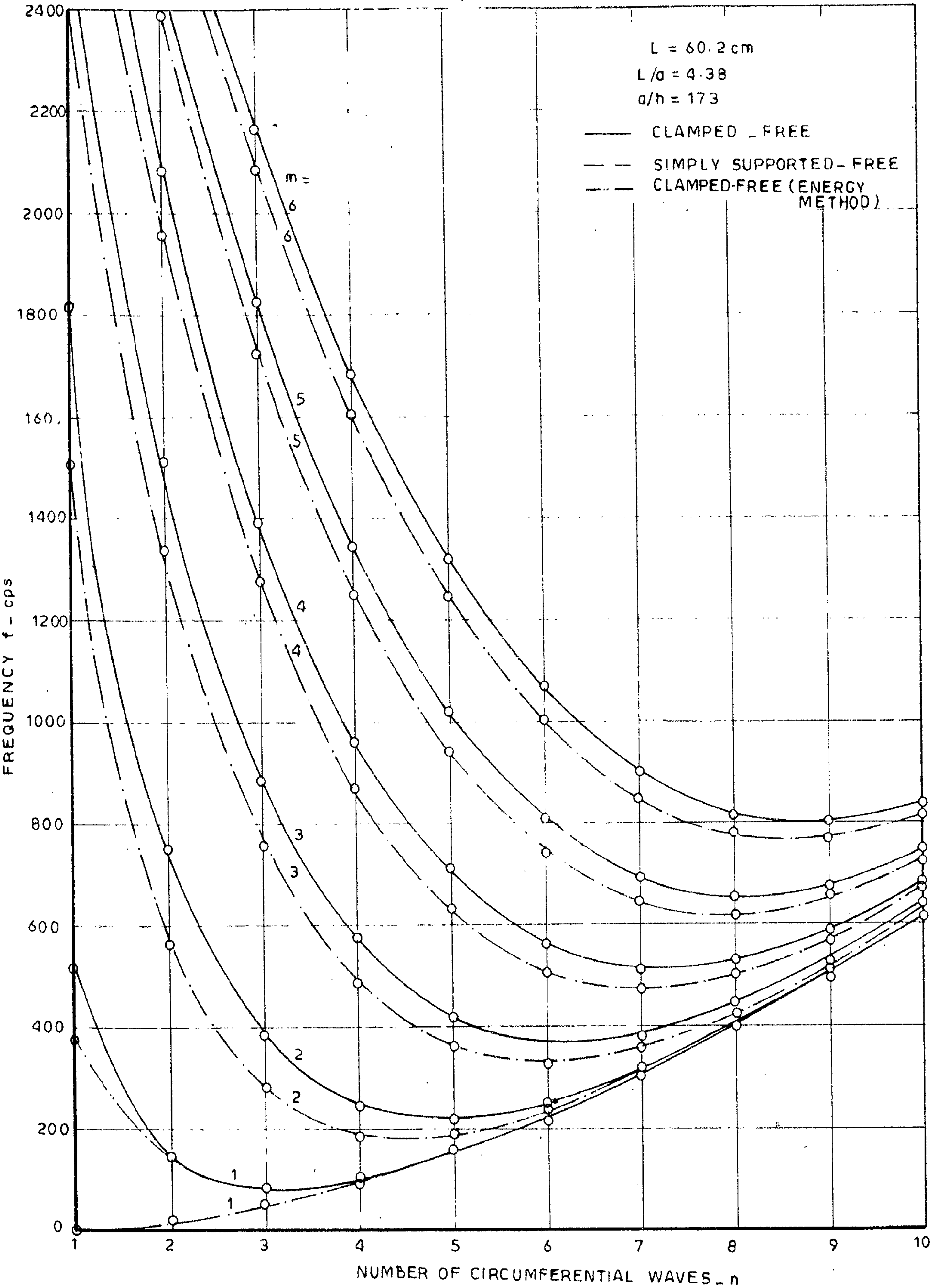


FIG.3.3 _VARIATION OF FREQUENCY WITH NUMBER OF WAVES IN AXIAL AND CIRCUMFERENTIAL DIRECTION

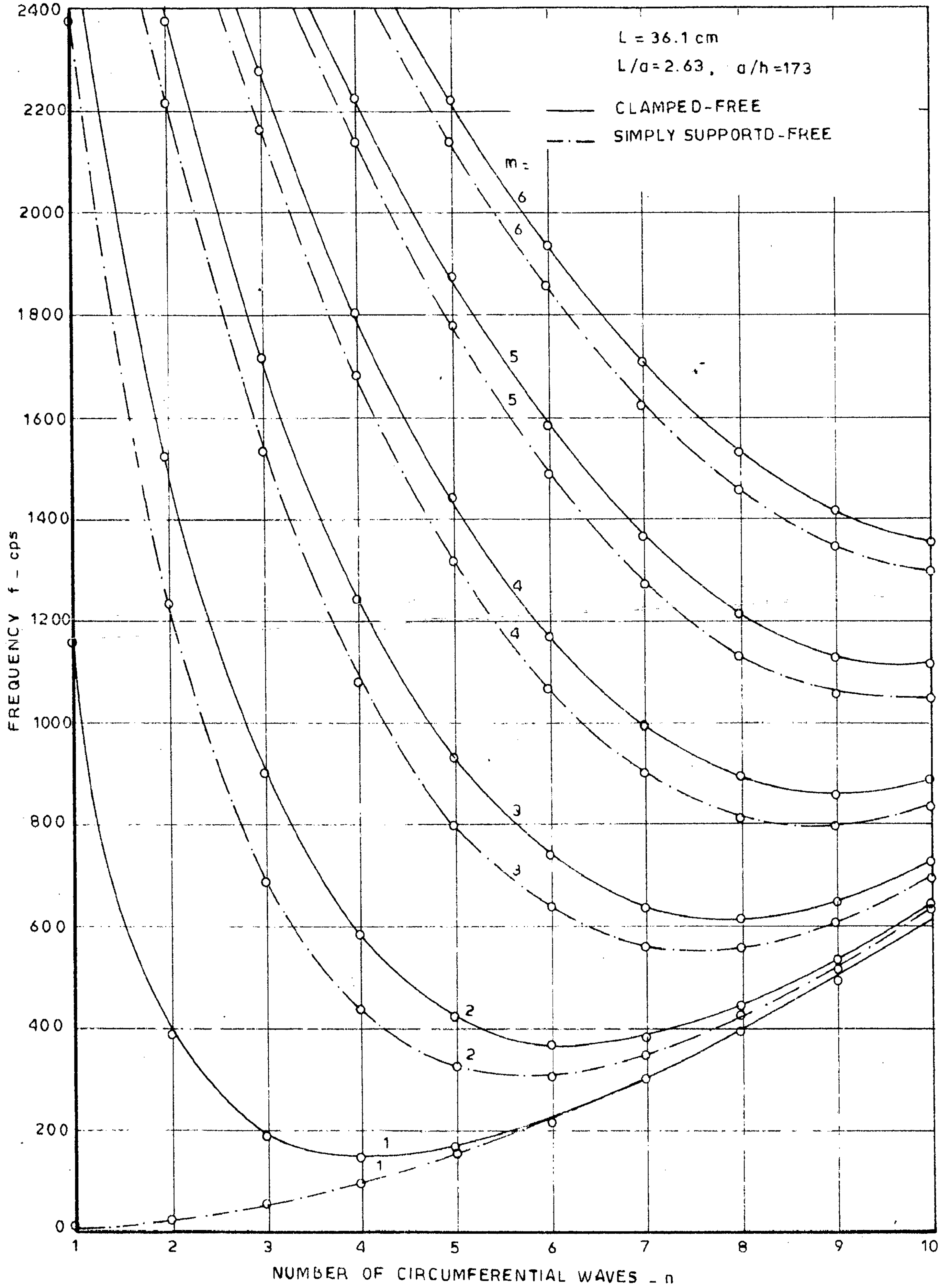


FIG. 3.4 _ VARIATION OF FREQUENCY WITH NUMBER OF WAVES IN AXIAL AND CIRCUMFERENTIAL DIRECTION

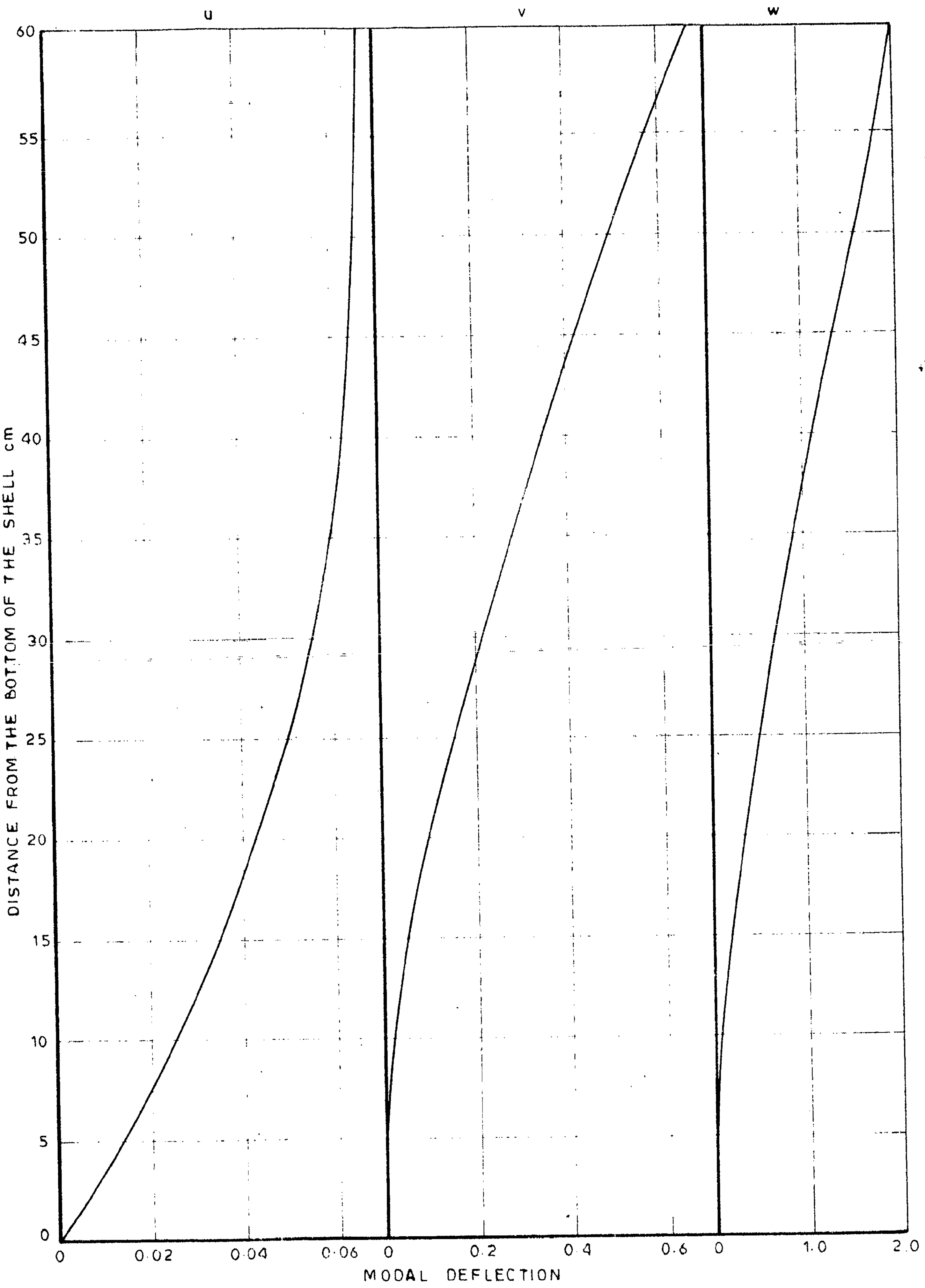


FIG. 3.5 _ MODE SHAPES OBTAINED ANALYTICALLY

curves for shell simply supported at base have also been drawn in the same figures.

EVALUATION OF MODE SHAPES

For known values of frequency, number of axial and circumferential nodes, properties of the shell, the fractional depth of water, all the elements in equations(3.13) are known except the constant terms A,B and C. By letting one of them to unity, we can find out the values of other two by solving the equations simultaneously. Substituting back these values of constants in equation (3.13) the three displacements of the shell (u,v, and w) can be determined. These have been obtained for the experimental model of the shell for the first mode of vibration for 3 numbers of circumferential waves. The results obtained have been plotted in Figure 3.5. The computer programme is given in Appendix A.3

3.4 OBSERVATIONS ON THE THEORETICAL RESULTS

From figures 3.3 , 3.4 and 3.5 and tables 3.1 , 3.2 following observations can be made:

(1) Frequency of vibration is high at lower circumferential nodes (say at two or three). Its value diminishes as number of circumferential waves increases achieving a minimum value at certain

value of n, after which it again starts increasing with increase in n.

(2) Frequency is lower for lower number of axial nodes and higher for larger number of have waves in the axial direction.

(3) Except at m=1, the frequency curves for clamped-free shell and simply supported-free shell follow each other closely. The difference is large (of the order of 16%) at lower nodes and small (of the order of 4%) at higher nodes.

(4) Frequencies obtained from the energy method and from the Weingarten method agree quite closely with each other. At n=1, the difference between the two is nearly 23% . But at higher number of circumferential waves, the difference is only 8% or even less.

(5) The effect of water is to reduce the natural frequency of vibration.

(6) The virtual mass coefficient varies greatly with depth of water at lower number of circumferential waves. At higher values of n, the variation in ϵ_n is very little with increasing water depths. It means that at higher nodes, the frequency will not alter appreciably with different water depths.

CHAPTER - IV

EXPERIMENTAL INVESTIGATIONS

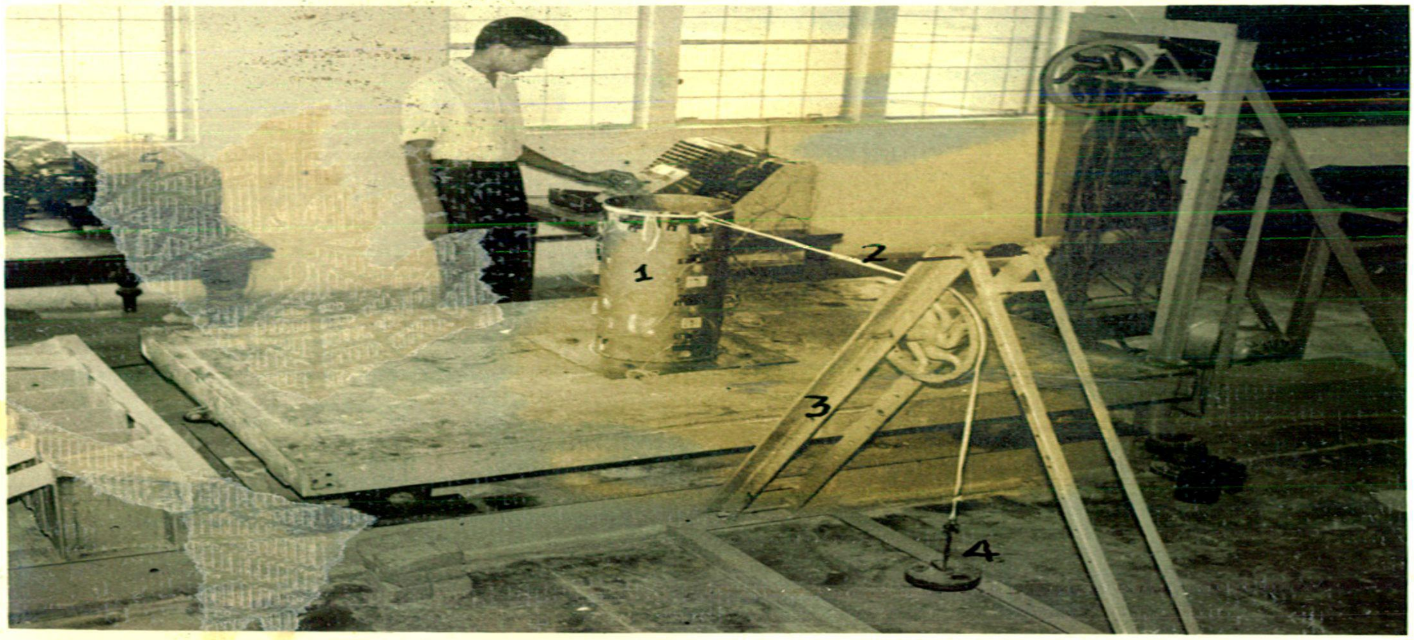
4.1 THE MODEL USED IN EXPERIMENTAL INVESTIGATIONS (See Fig. 4.7)

The model was made by rolling a thin brass sheet sheet of thickness 0.0795 cms. over a wooden form-work. The main fabrication was done in Earthquake Engineering School Workshop and the joint along the axial length was silver soldered in Mechanical Engineering Workshop. The diameter of the cylinder was 27.45 cms and the length 60.2 cms. Top end was kept free while the bottom one was clamped. The clampness was provided by tigtening the cylinder at its lower end with several screws passing all around its periphery as is clear from a look on figure (4.7). A brass plate of 3/8" thickness of the size 46 cms x 46 cms was used as a base for this model. Some important features of the model are as follows:-

Material of the shell	Brass
Density of the material of the shell (ρ)	8.0 gms./cm^3
Poisson's ratio (σ)	0.345
Young's modulus of Elasticity of the material (E).	7.2×10^5 kg/cm^2

Fig. 4.1. The General View of the Model

1. Brass cylinder; 2. Brass base plate, 3. Strain gages, 4. Wooden blocks for fixing pickups.



ig. 4.2 Test Arrangement for Free Vibration by pulling and releasing.
1. Model; 2. Cotton tape,
3. Triangular loading frame,
4. Loading hanger,
5. Universal amplifier.

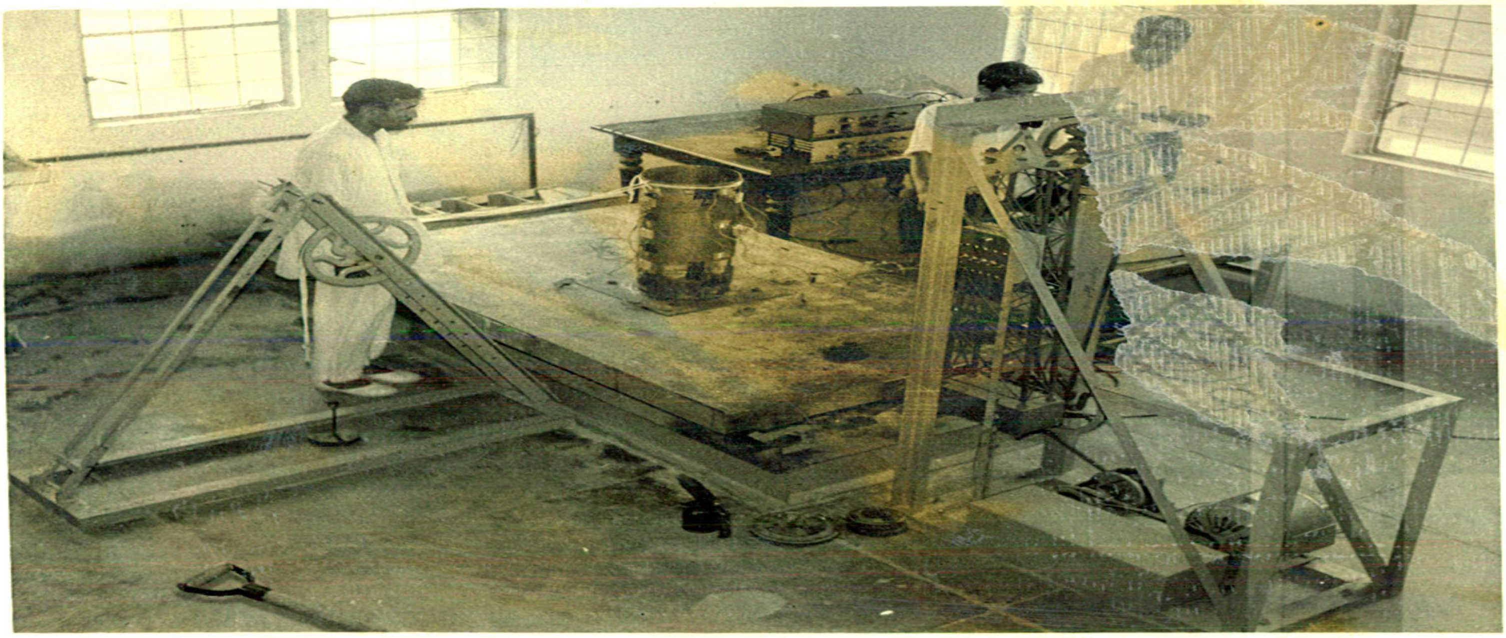


Fig. 4.3 General setup for the Free Vibration Test

g. 4.4 Arrangement for Steady State
Vibrations.
.Model, 2.Lazan Oscillator,3. -D.C.
otor, 4.Shaking Table,5. Wooden
locks for pick-ups,6.Speed Control
nit,7.Universal Amplifiers,8.Brush
en recorders.9.Transformer,10.Weight
or counteracting vert.vibrations.

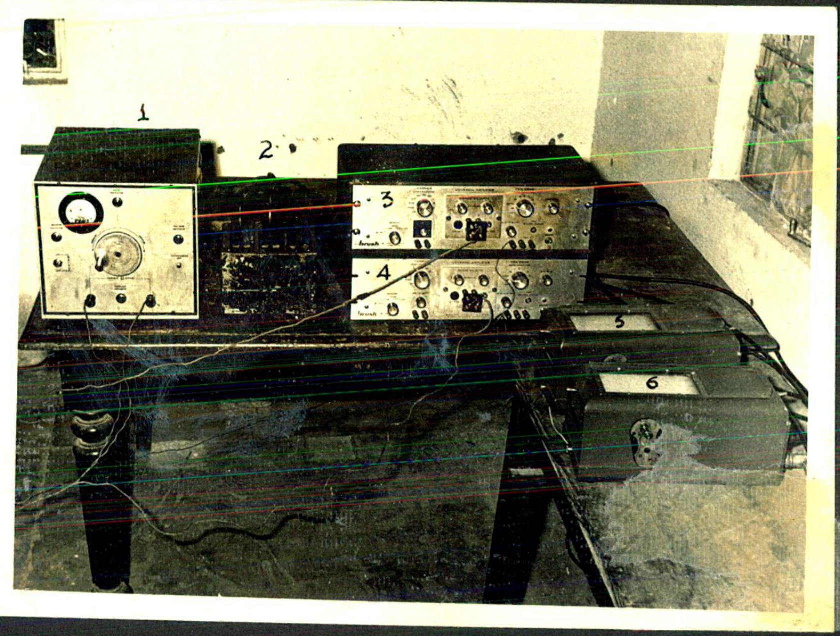
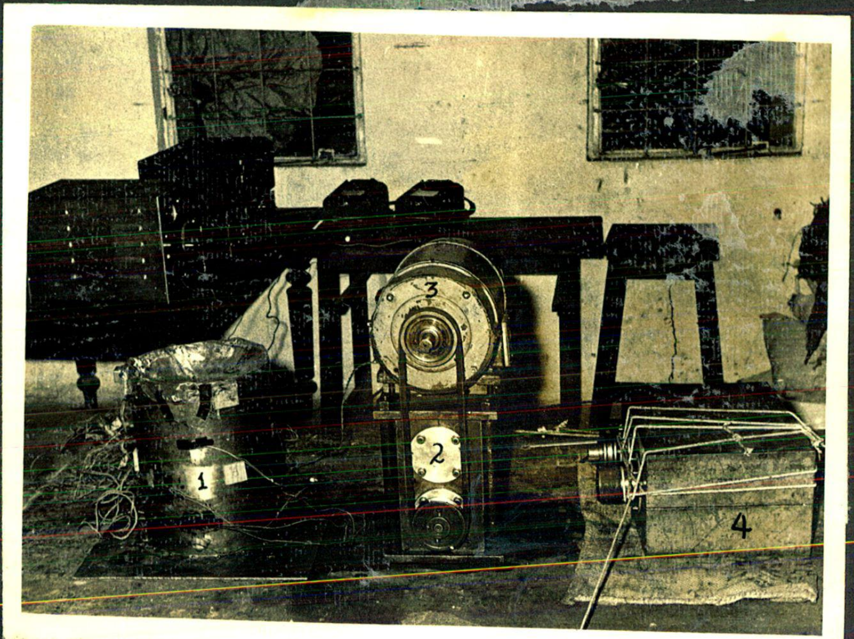


Fig. 4.5 Instrumentation for Steady
State Test.
1. Speed Control Unit,2.Transformer
3, and 4. Universal Amplifiers,
5 and 6. Pen recorders.

Fig.4.6-Details on Shaking
Table
1-Model; 2-Lazan Oscillator;
3-Motor; 4-Weight to counter-
act vertical vibrations



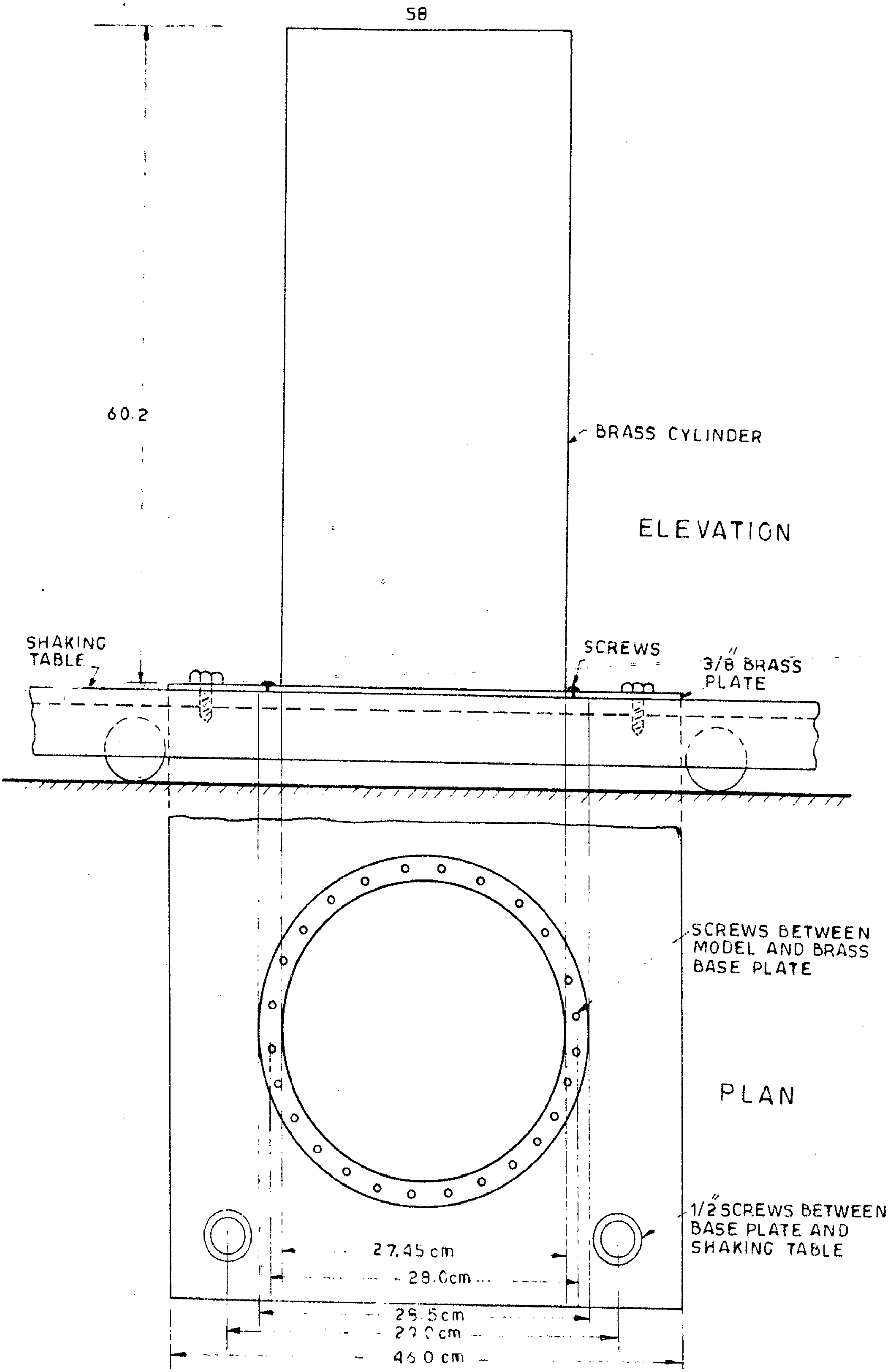


FIG. 4.7 THE MODEL

Density of the fluid (water) which was filled inside the shell (ρ') 1.0 gm/cm^3.

Radius of the cylinder (a) 13.725 cms.

The minimum diameter measured was 27.25 cms and the maximum diameter was 27.65 cms. The average value was 27.45 cms. Thus a discrepancy of about $\pm$ 0.20 cms in diameter from the mean value was present in the shell.

Length of the cylinder (L)

Experiments were done with two different lengths

(i) L=60.2 cms. L/a = 4.38 , a/h = 173

(ii) L=36.1 cms. L/a = 2.63, a/h = 173

The value of E was found from the vibration test on a cantilever beam of the brass (some specimen as used for model). A brass strip of 2.55 cms. width, 12.8 cms. length and .0795 cms thickness, fixed at the base and free at the top was pulled with hand and then released. The natural frequency of vibration of the beam was measured from the pen recorder record of the vibration. The value of E was then obtained from the well known formula for the vibration of cantilever beam

$$f = \frac{1}{2\pi} \cdot \frac{3.516}{L^2} \sqrt{\frac{E\,I\,g}{A\,\rho}}$$

The cylinder was made water tight at the bottom by a rubber solution and after that with an application of Araldite on inside and outside both. This arrangement was very successful and no problem of the leaking of water was faced during the experiment.

4.2 EXPERIMENTAL PROCEDURE

The experimental work was conducted under various sets of conditions as follows:

(A) Free Vibration Test

(a) By pulling and releasing,
(b) By tapping the model at various places.

Records were taken with empty shell and with varying water depths.

(B) Forced Vibration Test

Model was mounted on shaking table and vibration records, for steady state forced vibrations, were taken with Miller pickup at various positions along the length of the shell as well as around the circumference. The resonance records were obtained without water and with varying water depths.

The details of each one of tests are given below:

(A) Free Vibration Test

Free vibration test was conducted under different ways e.g. by pulling and releasing arrangement and by tapping on the model with hand, to see the behaviour of the shell under these conditions. The aim was to obtain the natural frequency of vibration in each case and to see if there was any discrepancy in various records of vibration of the shell.

An other method by radial pulling and releasing was also tried but it did not give any good record from which any conclusion could be drawn and, therefore, was abandoned.

The strain gages were pasted at different points on the shell to know the nature of strains produced and to calculate from their records, the natural frequency of vibration. At one or two places, in some cases, Miller pickup was mounted on a wooden block of 1/2" thickness and the records were taken in the same manner as with strain-gages.

The records were obtained on Brush ink writing oscillograph by connecting the pick-up (or strain gages as the case may be) with D.C. Amplifier and recorder.

The general set-up of the experiment appears in Fig. (4.3).

(a) Free Vibration Test by Pulling and Releasing

Because of the flexibility of the shell, the clutch system which is most widely used in earthquake engineering practices for conducting the free vibration tests was found to be inconvenient and unreliable under present circumstances. A light cotton tape was wound all around the periphery and on its one end was placed a hanger over which weights could be placed. This tape was made to pass over a pulley to ensure the horizontal loading (See Fig. 4.2).

The condition of sudden releasing was achieved by burning the tape near its end with a gas burner. The flame of the burner was kept bright enough so that no slackness in the tension in the tape occurs during the burning.

Three loads of 5 kg, 7 kg and 9 kg were put on the hanger successively and the records were taken with various strain gages. The aim was to see the effect of initial strain level on the vibration conditions. Loads were not increased beyond these values because of the danger of permanent deformations near the top.

(b) Tapping the Cylinder

Cylinder was tapped with a blow by hand at different heights in the axial direction and also around the circumference. The records were obtained with several strain gages to study the behaviour of the shell.

(b) Forced Vibration Test

The model was subjected to steady state vibrations on shaking table with the help of Lazan oscillator. The later was driven by a d.c. motor, the speed of which could be varied with the help of speed control unit. Varying the eccentricity of the masses in the oscillator or and the speed of the motor any desired amount of sinusoidal force within the limits of each instrument, could be given. The specifications of oscillator and the motor were as follows.

Lazan Oscillator

Capacity ± 1600 lb. at 1800 rpm.

or

4000 inch-lbs.at 1800 rpm.

Maximum speed = 3600 rpm.

D.C. Motor

H.P.	3
Volts	220
Amps	13
Phase	D.C.
R.P.M.	2000

Wooden blocks were pasted on the shell at different places (see figure 4.8) in order to enable the pickups to be screwed at different positions. The aim was to have a relative idea of the amplitudes occurring at various points and also to have an idea of the behaviour of shell in one particular direction when it is subjected to a motion in the same or in some other direction. The output of the pickup was fed to universal amplifier circuit and vibration records were obtained from the pen recorder. Two sets of amplifier and pen recorders were needed-one for pickup of the model and the other for the pickup on the shaking table. The latter gave an idea of the magnitude of the base acceleration. Thus the ratio of model acceleration to base acceleration could be obtained at any stage of vibration.

Experimental setup for this case can be seen in figures 4.4 ,4.5 and 4.6.

The experiment was started with a low frequency of the order of 5 cycles per second and the corresponding vibration records from the pick-up on model and on shaking table were taken simultaneously by putting the recorder at on or off position at the same instant. Thus, frequency measured from both the records will be same and their calibrated amplitudes will give us the acceleration to which they have been subjected by the forced vibrations.

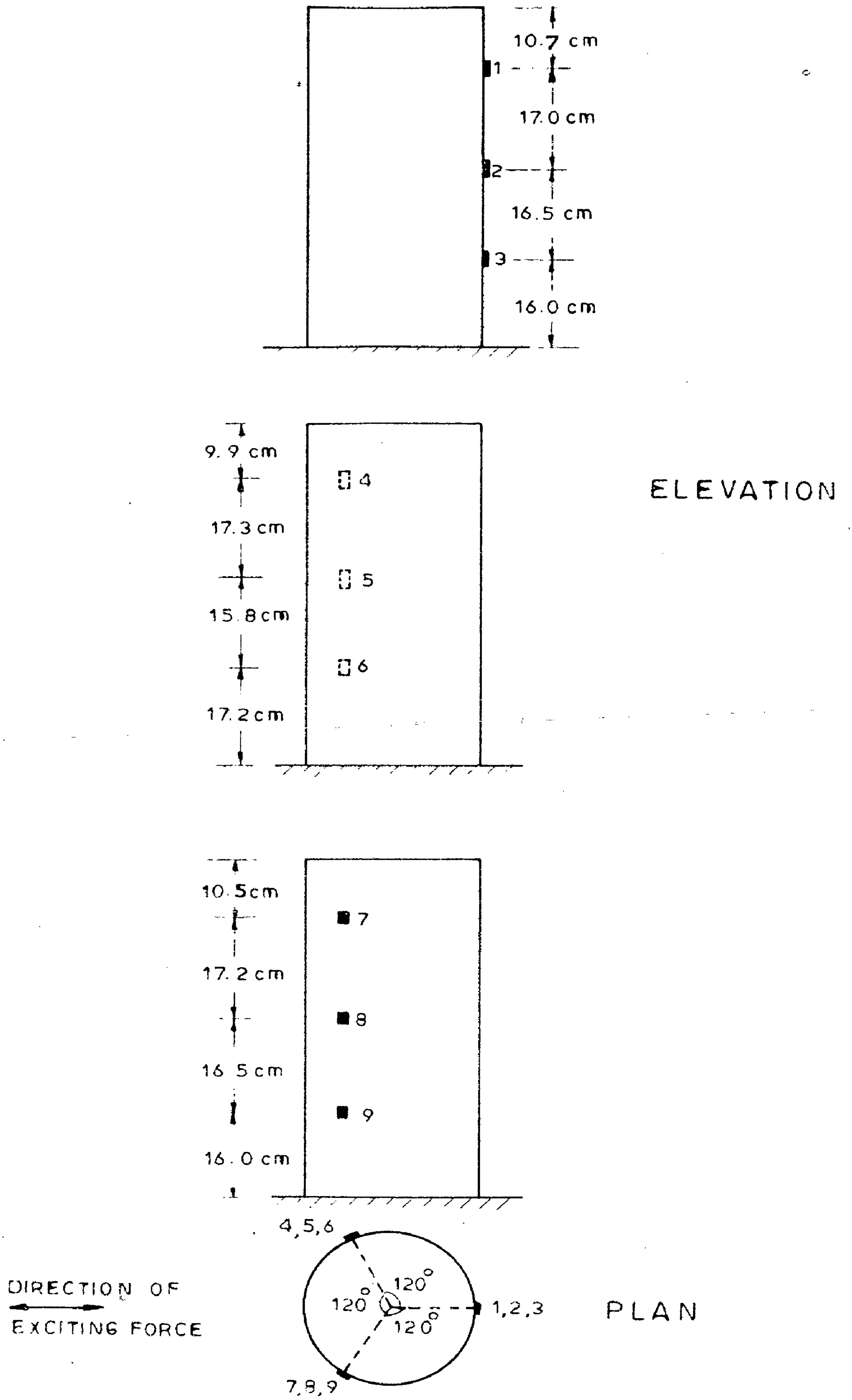


FIG. 4.8 _ POSITION OF VARIOUS PICKUPS ON THE SHELL THE NO. INDICATES THE POSITION AS MENTIONED IN THE PAPER

Slowly the frequency was increased with the help of speed control unit and the corresponding vibration records taken every time. As an average the frequency was increased by 0.5 cps every time. At some frequency for a particular condition the amplitudes measured were very large and after it they subdided again. This indicates the occurrence of resonance and this frequency will be the natural frequency of vibration. However, the various limitations of this will be discussed later. The maximum frequency which could be reached with the available instruments was between 35 and 40 cps, though in some cases higher frequency of even 48 cps was achieved.

Near resonance, there occurred vertical vibrations of the table which interfered with the natural mode of vibration. To remedy this a heavy weight was tied down on the table (Fig.4.4) and this solved the problem to a great extent.

Similar to free vibration tests, this test was also carried out for two lengths of the cylinder (L=60.2 cms. and L=36.1 cms). In each case the records were obtained with varying water depths. Miller pickups' position was also varied as has already been explained in article 4.2. Table 4.1 summarises the way in which the test was conducted.

Table 4.1

Statement showing procedure for forced vibration test (a/h=173)

Length of cylinder (cms)	Ratio of length to radius of the cylinder	Depth of water inside the cylinder (cms)	Ratio of water depth to length of the cylinder	Pick-up positions on which records were taken *
		0	0	1,2,3,4,6,7
60.2 cms.	L/a=4.38	20.0	0.332	1,2,3,7
		32.2	0.535	1,3,7
		0	0	2,3,5
36.1 cms.	L/a=2.63	17.80	0.494	2,
		31.60	0.875	2,5,8

* For various pickup positions please see figure 4.8.

4.3 EXPERIMENTAL RESULTS

(A) Free Vibration Tests

(a) By pulling and Releasing:

Following observations have been recorded in this case:-

1. Vibration of empty shell,
2. Vibration of shell with varying water depths.

These observations were made for two lengths of the shell (L=60.2 cms. and L=36.1 cms).

Typical records have been shown in Fig. (4.9A)

(b) By Tapping:

Observations were made for the same conditions as with pulling and releasing. Object was also to compare the records taken with these two different methods of free vibration. Typical records have been shown in Figures (4.9B, 4.10 and 4.11).

The variation of frequency with depth of water is given in Table (4.2).

(B) Forced Vibration Tests

Records have been obtained for the following observations:-

1. Vibration of empty shell with pick-up at various location.

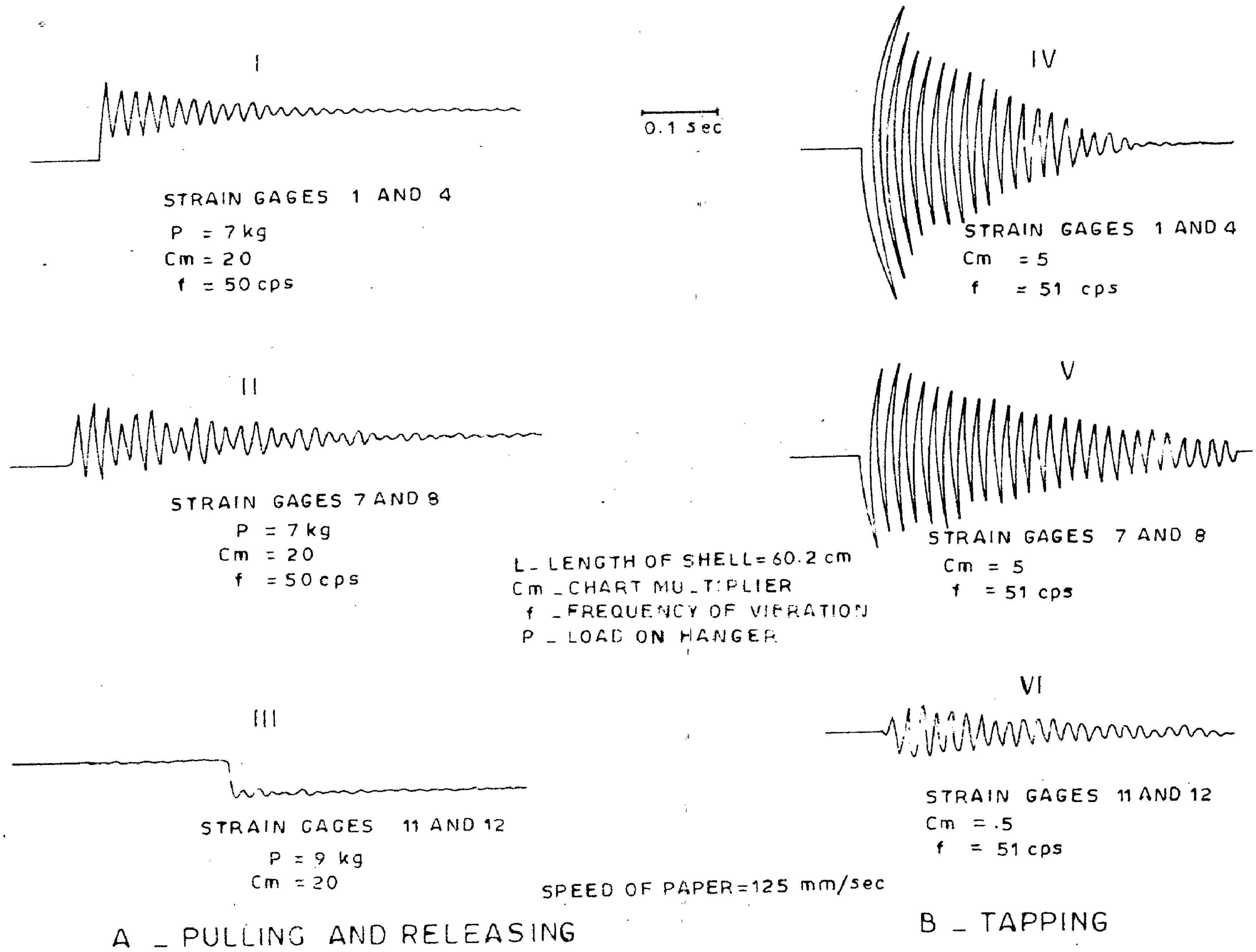


FIG.4.9 _ FREE VIBRATION RECORDS

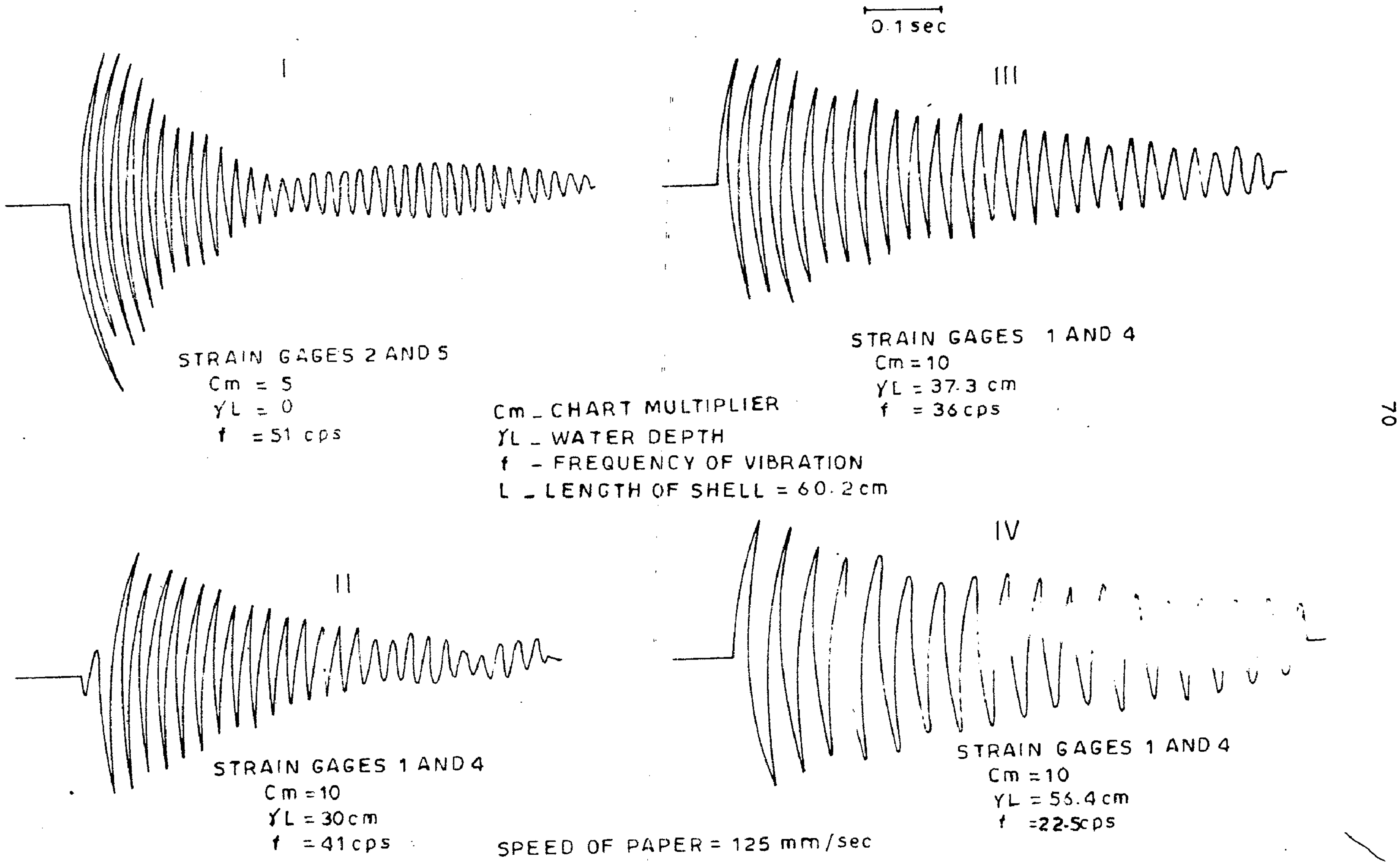


FIG. 4.10 _ FREE VIBRATION RECORDS WITH TAPPING (WITH VARYING WATER DEPTH)

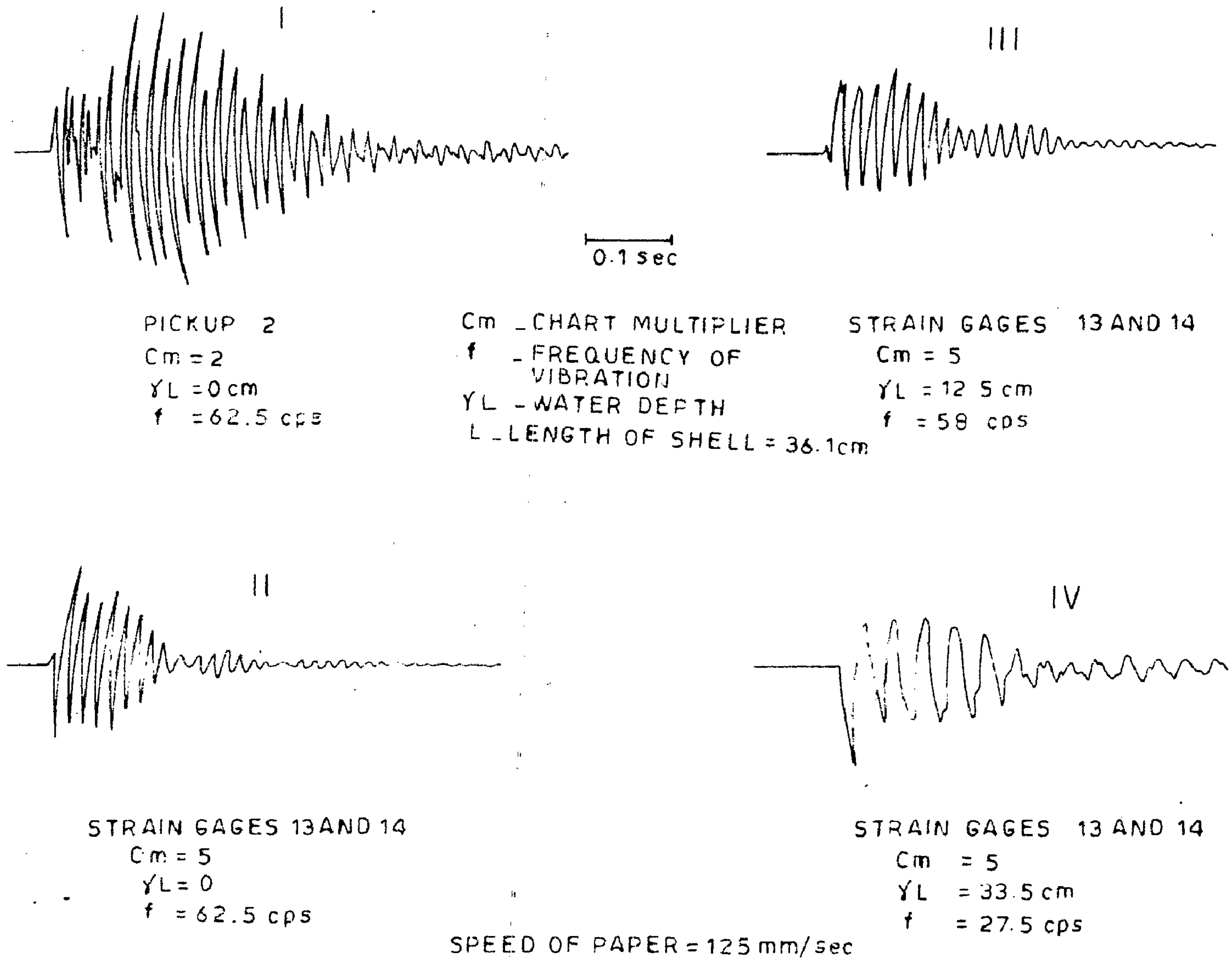


FIG.4.11 _ VIBRATION WITH VARYING WATER DEPTH

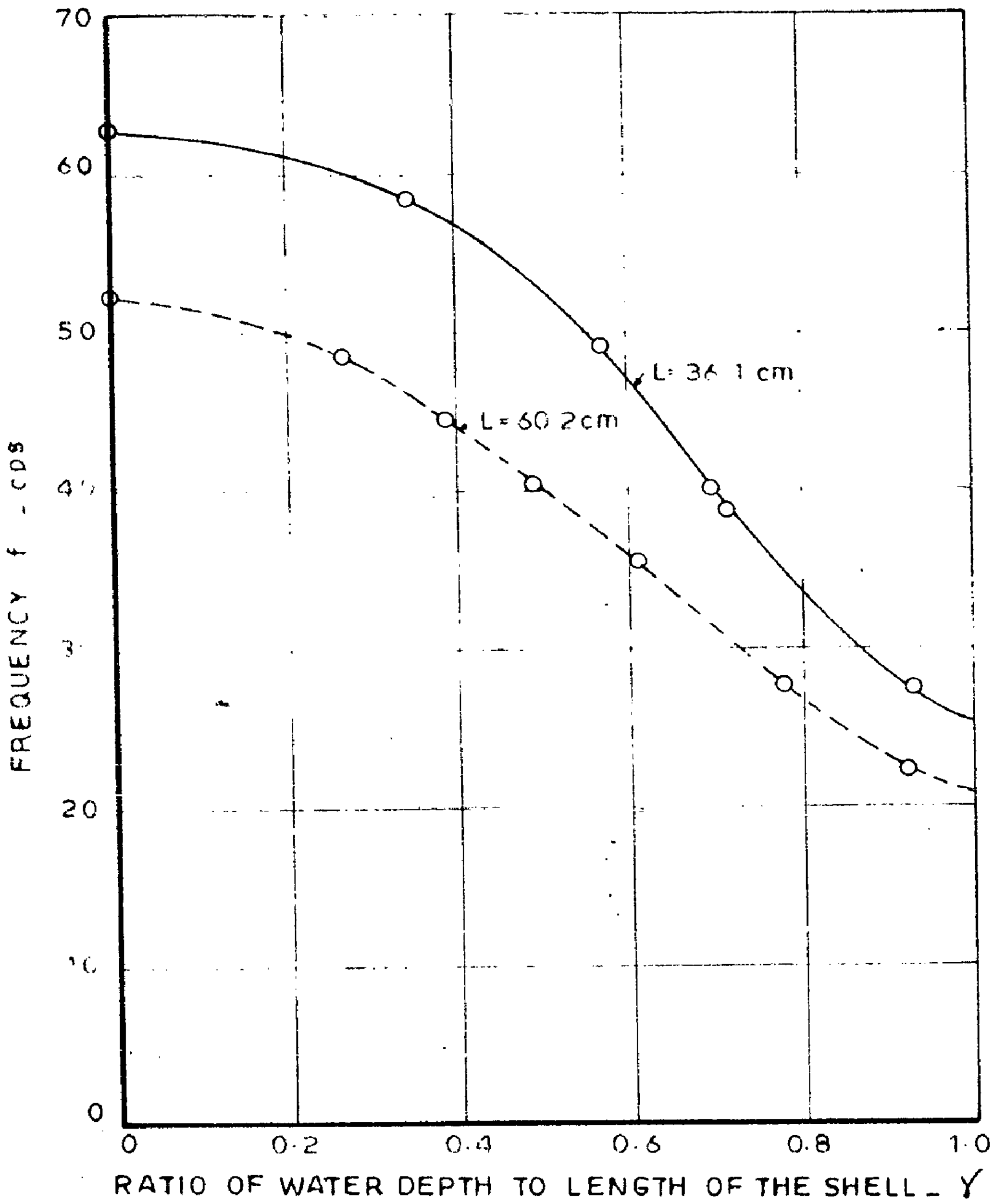


FIG.4·12 _ VARIATION OF THE NATURAL FREQUENCY OF VIBRATION OF THE SHELL WITH VARYING WATER DEPTHS INSIDE THE SHELL

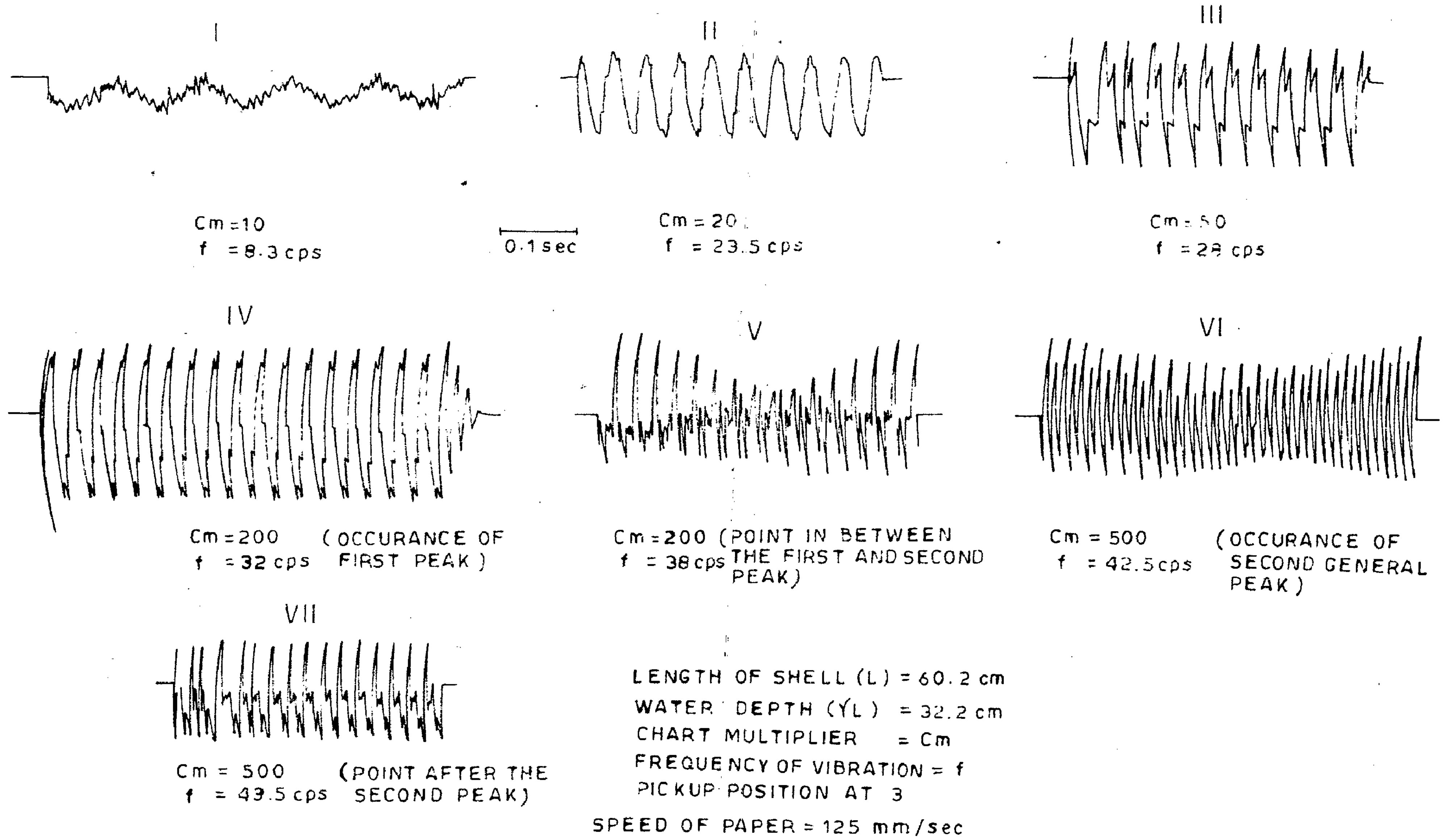


FIG. 4.13 _FORCED VIBRATION RECORDS

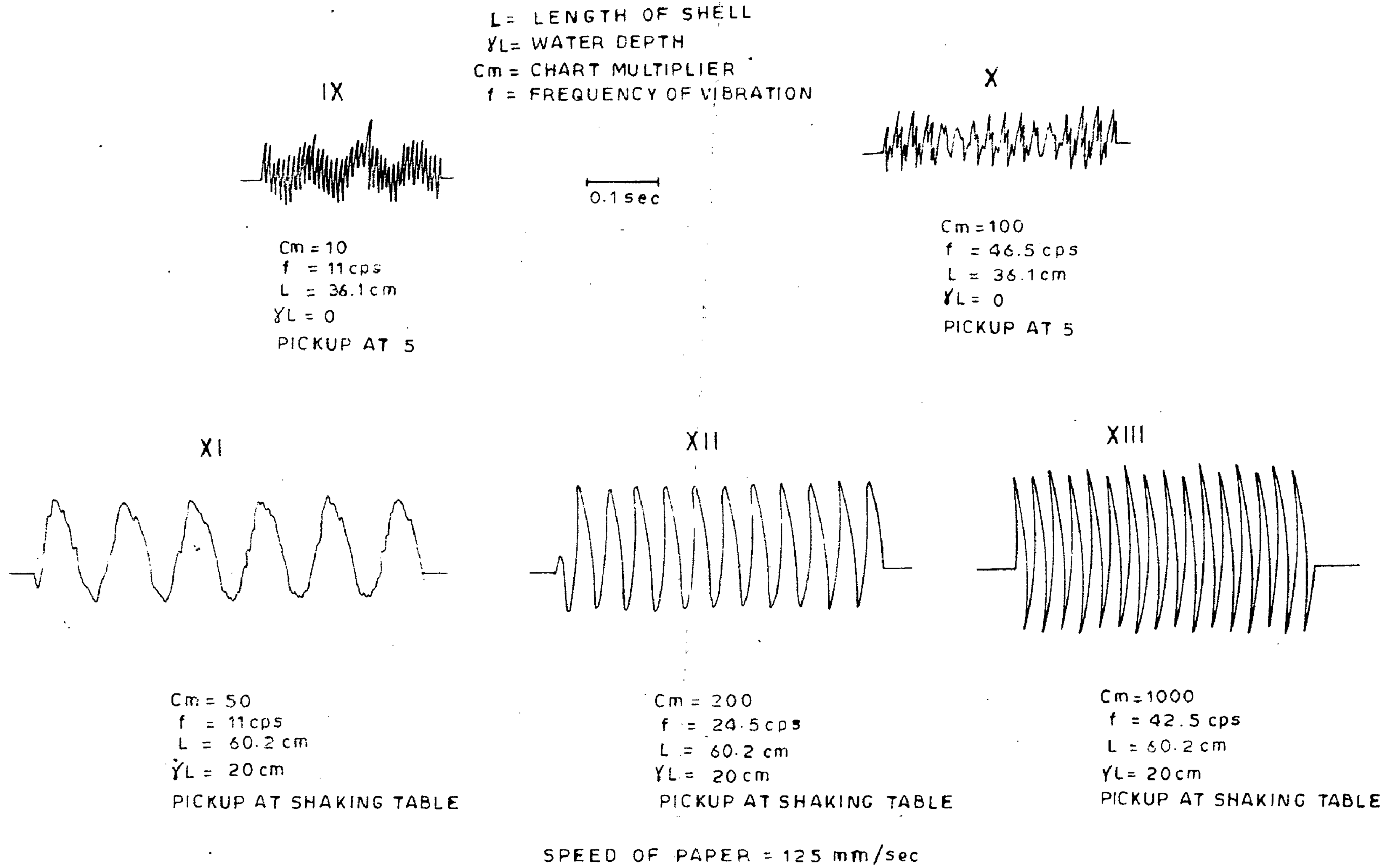
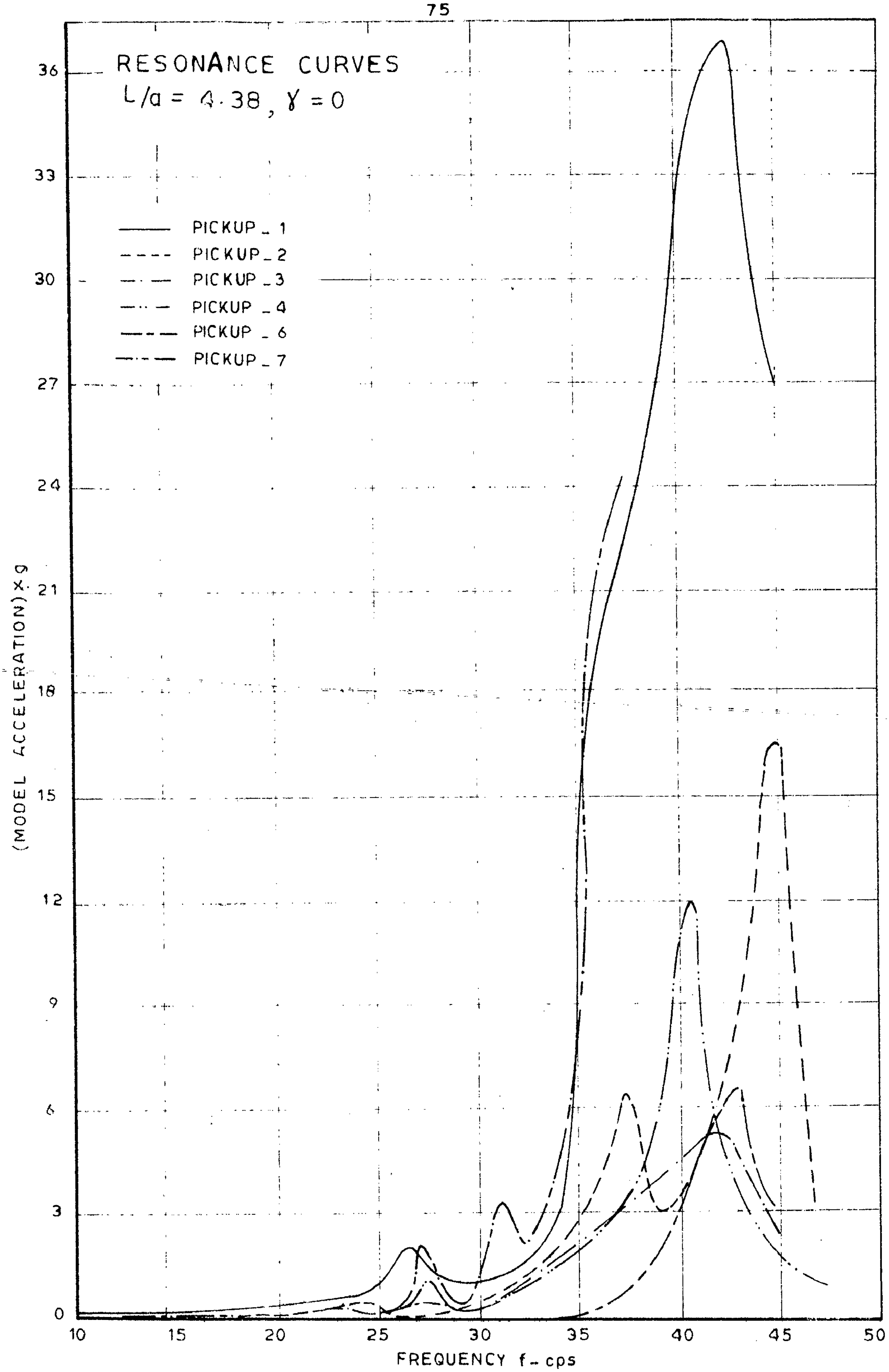


FIG. 4.13 (CONTD.)_ FORCED VIBRATION RECORDS



RESONANCE CURVES
L/a = 4.38, γ = 0
PICKUP _ 1
PICKUP _ 2
PICKUP _ 3
PICKUP _ 4
PICKUP _ 6
PICKUP _ 7
(MODEL ACCELERATION) x g
0
3
6
9
12
15
18
21
24
27
30
33
36
10
15
20
25
30
35
40
45
50
FREQUENCY f – cps

FIG. 4.14

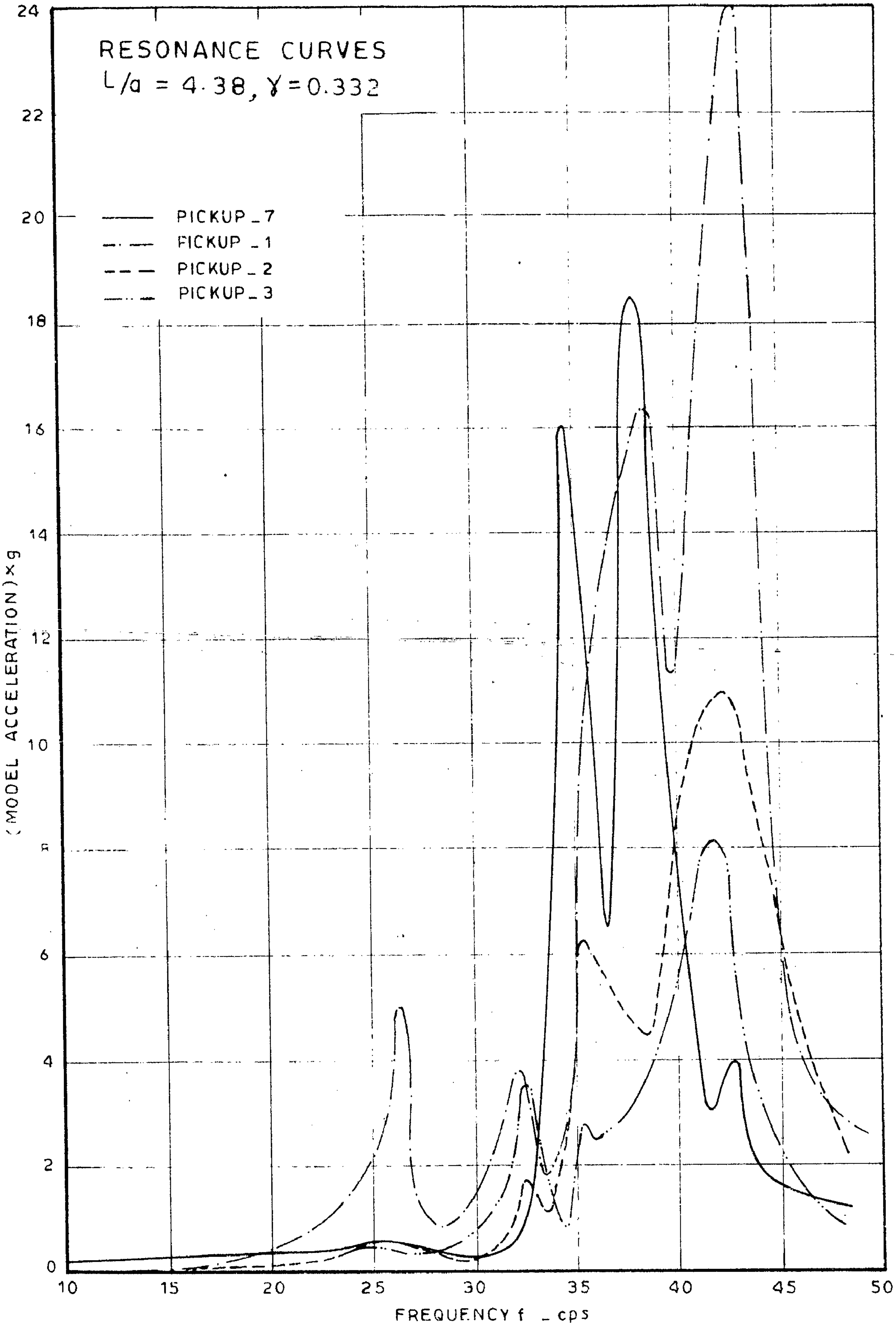

RESONANCE CURVES
L/a = 4.38, γ=0.332
PICKUP _ 7
PICKUP _ 1
PICKUP _ 2
PICKUP _ 3
(MODEL ACCELERATION) x g
FREQUENCY f _ cps
0
2
4
6
8
10
12
14
16
18
20
22
24
10
15
20
25
30
35
40
45
50

FIG. 4.15

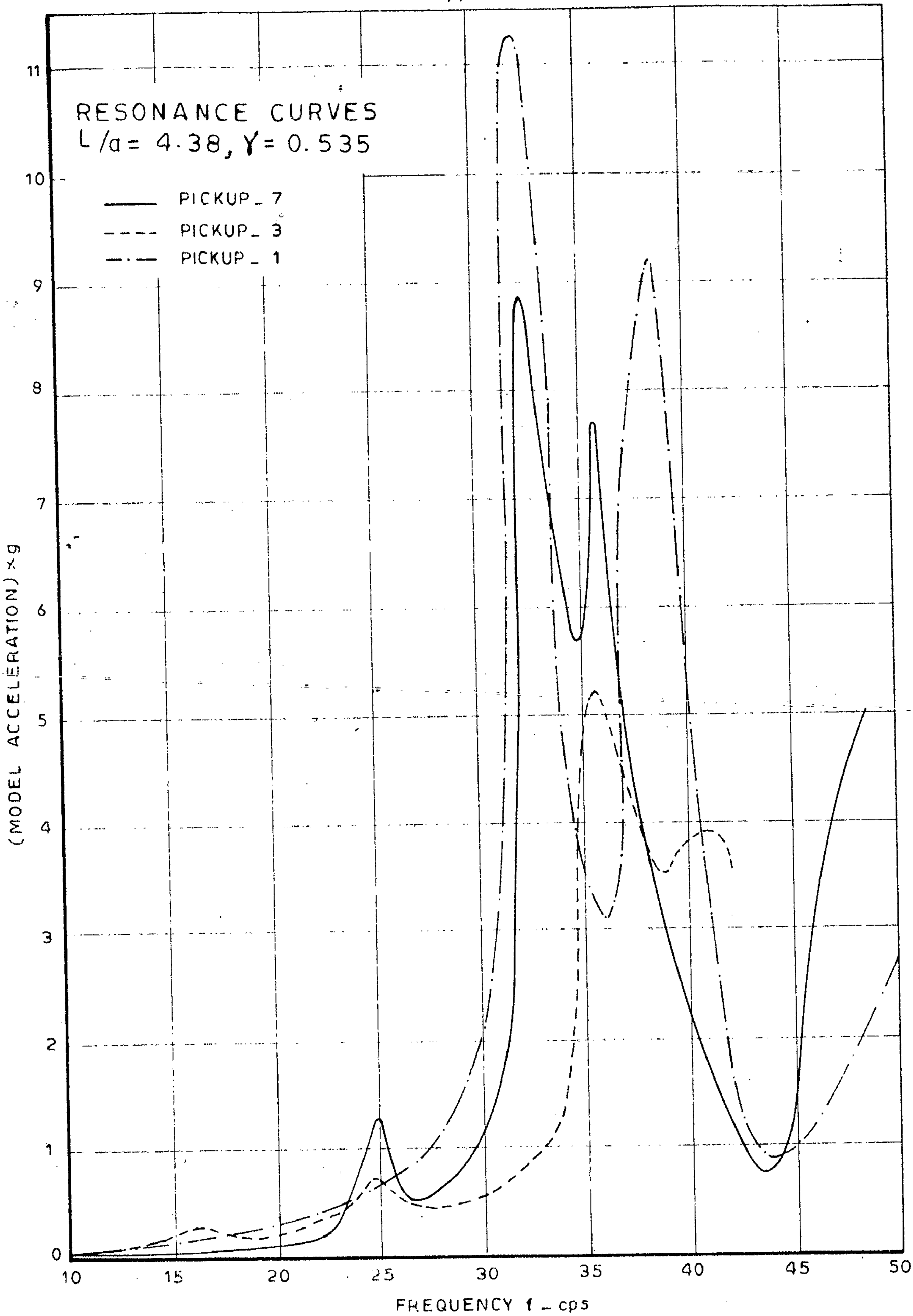

RESONANCE CURVES
L/a = 4.38, γ = 0.535
PICKUP_7
PICKUP_3
PICKUP_1
(MODEL ACCELERATION) × g
FREQUENCY f _ cps
0
1
2
3
4
5
6
7
8
9
10
11
10
15
20
25
30
35
40
45
50

FIG. 4.16

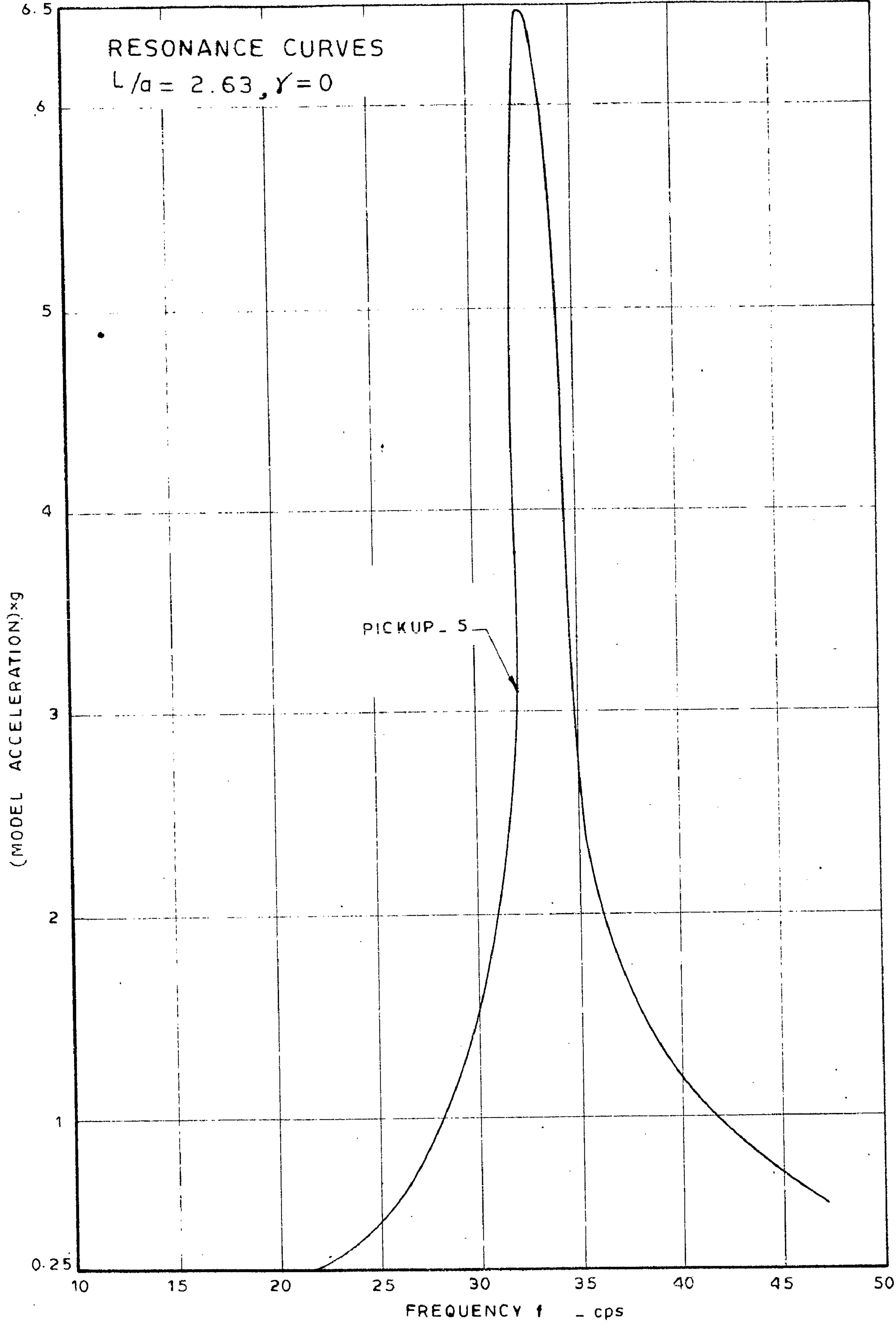

RESONANCE CURVES
L/a = 2.63, γ = 0
PICKUP _ 5
(MODEL ACCELERATION)×g
6.5
6
5
4
3
2
1
0.25
10
15
20
25
30
35
40
45
50
FREQUENCY f _ cps

FIG. 4·17

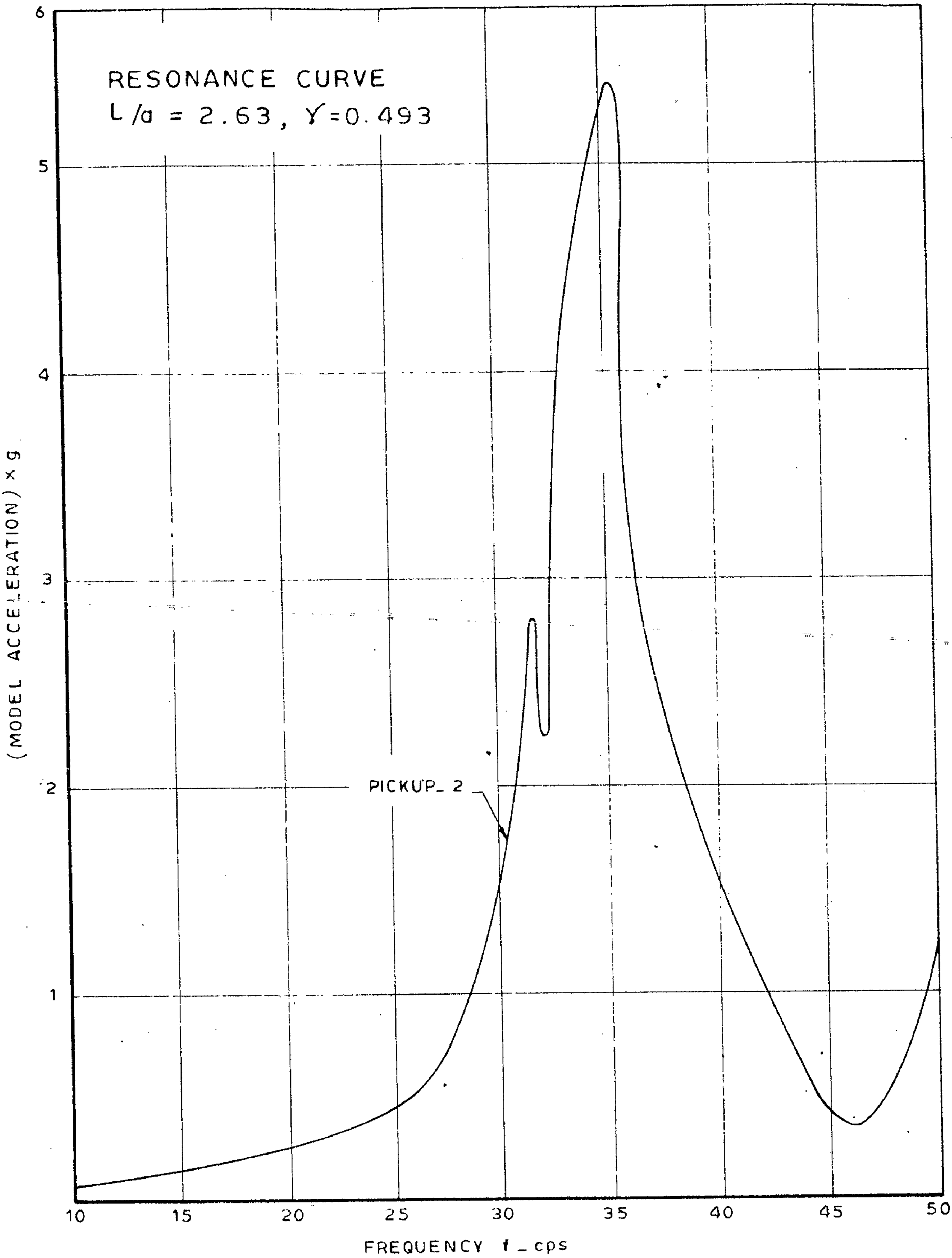

RESONANCE CURVE
L/a = 2.63, γ=0.493
(MODEL ACCELERATION) × g
FREQUENCY f _ cps
PICKUP_ 2
6
5
4
3
2
1
10
15
20
25
30
35
40
45
50

FIG. 4.18

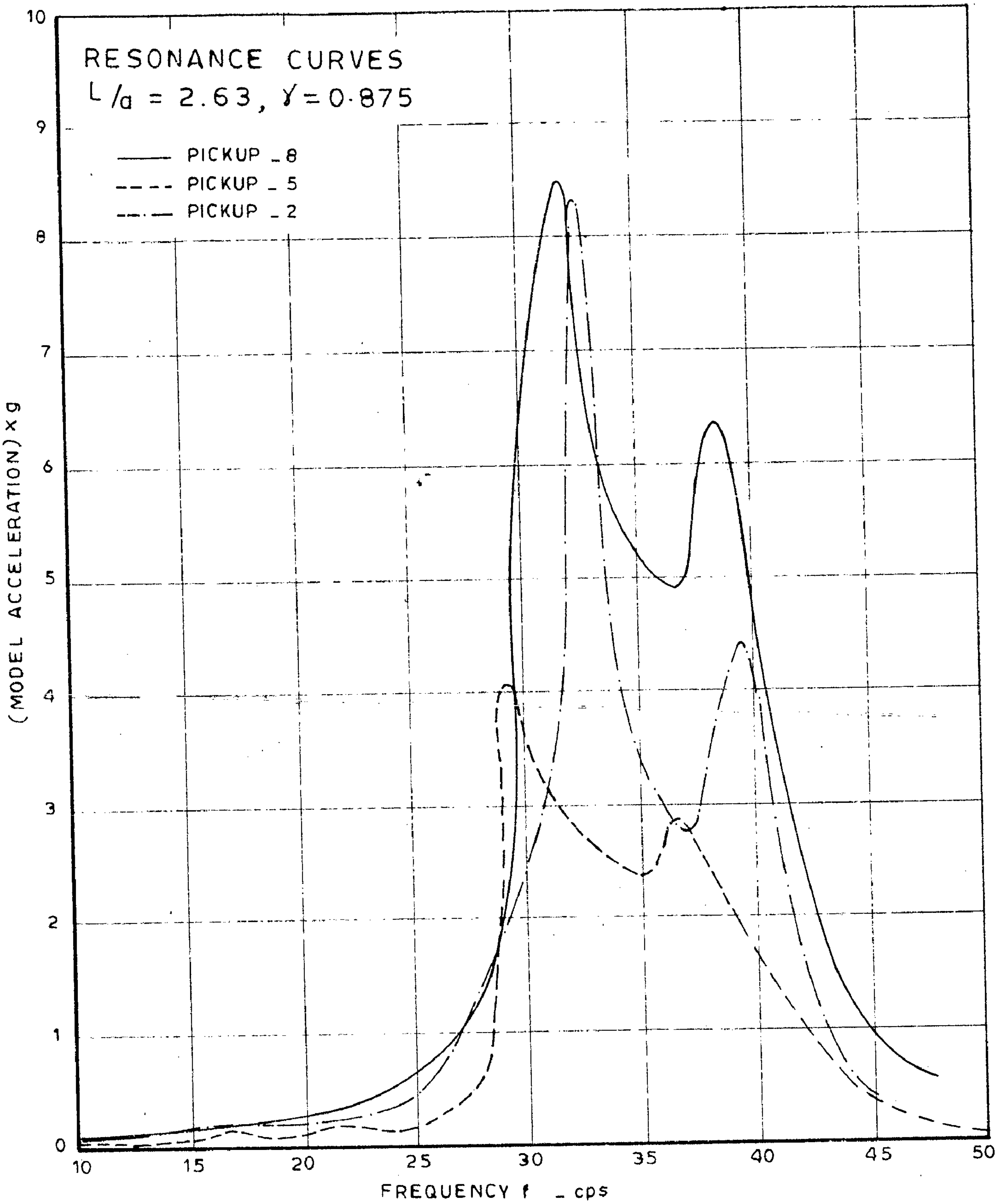

RESONANCE CURVES
L/a = 2.63, γ = 0.875
PICKUP _ 8
PICKUP _ 5
PICKUP _ 2
(MODEL ACCELERATION) x g
FREQUENCY f _ cps
0
1
2
3
4
5
6
7
8
9
10
10
15
20
25
30
35
40
45
50

FIG. 4.19

Table 4.2

Frequencies observed in Free Vibration test with various water depths

Length of the shell (cms) (L)	Ratio of water depth to length of the cylinder	Frequency of vibrations (cps)
	0.0	51.5
	0.270	48.0
	0.388	44.5
60.2	0.491	40.5
	0.611	35.5
	0.708	31.0
	0.786	27.5
	0.924	22.5
	0.0	62.5
	0.345	58.5
	0.570	49.0
36.1 cms.	0.717	38.8
	0.93	27.5

2. Vibration of the shell with varying water depths, with pickup at various locations.

Typical records are shown in Fig.(4.13). Resonance curves have been drawn taking model acceleration as ordinate and frequency as abscissa. These are shown from Figure 4.14 to 4.19 for different conditions.

4.4. OBSERVATIONS ON EXPERIMENTAL RESULTS

(A) Free Vibration Tests

From the vibration records of figures 4.9 , 4.10 , 4.11 and 4.12 , following observations can be made.

1. Records with circumferential strain gages were better than those taken with axial strain gages perhaps because the strains in circumferential gauges were more than those in axial gauges.

2. Vibration records with strain gages near the bottom were very poor (Fig.4.9A iii). They did not give any idea of the frequency of vibration. Strains in the circumferential gages will be small at this level because of restraint on the deformation of the section and therefore local high frequency components were being superimposed on the fundamental frequency. Perhaps the noise of the instrument was as important as these vibrations resulting in

unclear records.

3. Frequency obtained from all the records, for example from all the gages with pulling and releasing and from tapping at different points, was nearly the same. The average value may be taken as 51.5 cps. It shows that each time the shell was vibrating in the same mode.

4. In some cases, in records with tapping, some sort of beat phenomenon was observed (Fig.4.10-1). It may be due to the fact that depending upon the position of the place where the shell was tapped and the magnitude of the force, two frequencies lying very close to each other were excited.

5. Frequency of vibration reduces as the water depth increases. This is the type of behaviour which is expected in a vibrating system due to added virtual mass.

6. The vibration records became more distinct as the depth of water inside the shell increased.

7. Calculating the values of the virtual mass from the expression already derived we get the values shown in Table 4.3

It shows that whole of the weight of the water is not effective in producing stresses in the

Table 4.3

Variation of Virtual mass with water depth

Fractional water depth L	Virtual mass coefficient ϵ_n	Virtual mass (kg)	wt. of water (Kg.)	virtual mass expressed as percentage of total water weight
0	0	0	0	0
0.2	0.18565	0.595	7.12	8.35
0.3	0.22077	2.400	10.72	22.35
0.4	0.24339	5.400	14.25	37.90
0.5	0.25879	9.300	17.80	52.30
0.6	0.26981	13.520	21.37	63.50
0.7	0.27805	17.650	24.90	70.90
0.8	0.28443	21.750	28.50	76.40
0.9	0.28949	25.600	32.05	79.90
1.0	0.29362	29.500	35.55	83.20

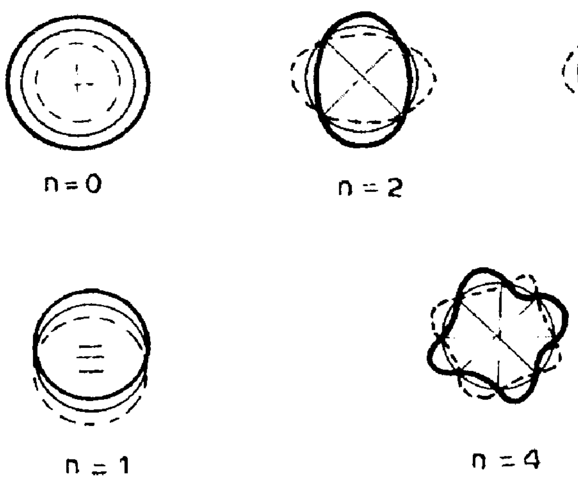
shell during the vibration. As the water depth goes on increasing, percentage contribution of the total water weight also increases.

(B) Steady State Vibration Tests

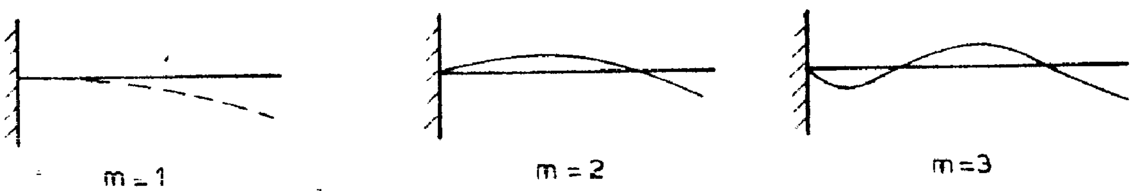
Behaviour of shell during steady state vibration conditions was observed to be very peculiar. Following are the observations made from the figure s 4.13 to 4.19 .

1. The nature of vibration throughout the frequency range was not uniform. (See Fig. 4.13). At the lower imparted frequency of say 15 cps the amplitudes observed were very small and the motion of the shell was more or less a rigid body motion. For this forcing frequency on other very high frequency of the order of 140 cps was also observed to have occurred, (Fig. 4.13 ix) simultaneously. This phenomenon was more pronounced in the shorter shell.

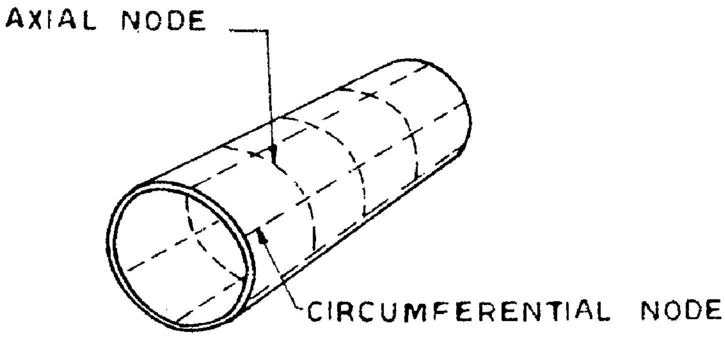
2. At some stages the records were very smooth (Fig. 4.13-ii) while at some stages there appears to have occurred superposition of some more number of modes. At some value of frequencies some strange records were obtained (Fig. 4.13-v and x) where even under the steady conditions, the amplitude varied sinusoidally at the same frequency. The possible explanation for this behaviour may be the shape of the shell. Being circular



CIRCUMFERENTIAL NODAL PATTERN



AXIAL NODAL PATTERN



NODAL ARRANGEMENT FOR
m=3, m=4.

FIG. 4.20_NODAL PATTERNS

in plan there were not only the flexural vibrations but vibrations of some other kind also.

The general character of the various mode shapes is indicated in Figure 4.20. In vibrating shells two type of nodal patterns exist together. The circumferential nodal pattern denoted by letter n(n=number of circumferential waves; circumferential nodes=2n) and the axial nodal pattern denoted by letter m(m is number of half waves in axial direction; axial nodes are m-1). Occurrence of these two type of waves will affect the vibration behaviour of the shell to a great extent.

3. Vibration record of the pick-up on shaking table was very smooth throughout the frequency range.(Fig. 4.13-x, xi, xii). Frequency could be measured very accurately at any stage. This shows that motion was entirely sinusoidal. Any irregularity or pecularity in the record of the pick-up on model was due to some peculiar behaviour of the model itself and not due to some haphazard motion of the shaking table.

4. In some cases, the resonance peaks could not be obtained because of the high frequency of vibration of the shell. The maximum frequency which could be reached by the mechanical oscillator was 45 cps.

5. For any one given case of a shell having the same water depth, the resonance peak was observed to have occurred at different frequencies (Table 4.4, and Figs. 4.14 to 4.19) when recorded at different points of the shell. This may be explained with the help of the manner in which a shell vibrates (Fig. 4.20) From Fig. (2.2), it is seen that for various combinations of axial and circumferential modes there are possible several resonant frequencies. For one particular mode, say fundamental mode, the frequency of vibration will change with the number of circumferential nodes. In a vibrating shell it is very difficult to say which of the combinations of these modes is present at the time of resonance.

Due to the limitations of the availability of instrumentation, only one pick-up could be used at a time on the model. Hence, for one particular case the experiment had to be repeated as many number of times as were the pick-up locations desired on the shell. Therefore, it was not possible to bring the same conditions every time. Under these conditions, it was possible that at one time some combination of nodal lines was occurring while at other time some other combination was occurring. This might have shifted the peak sometime at lower frequency and sometimes at higher frequency.

Another very interesting phenomenon was also observed. There occurred not only one resonance peak but several peaks in the frequency range of the experiment. These peaks varied from three to four in number. However, there was one general peak at which the amplitude was maximum of all other ones and this was assumed to represent actual resonant frequency. Sometimes the smaller peaks were formed before this peak and sometimes after it and sometimes one or two were before the main peak and one or two after it. The relative amplitudes of these different peaks can be seen in figures 4.14 to 4.19.

There does not seem to be some definite reasoning behind this behaviour of the shell. However, two explanations may be advanced as follows:

(i) Because of the various combinations of axial and circumferential modes, large number of resonances may occur close to each other. Importance of exact location of different nodes axially and circumferentially both has been mentioned by the other authors also. It is rather a very difficult task and more sensitive instruments will have to be developed.

(ii) As desribed in article 4.1 , the model could not be made to exact circular shape. Also, the material is never homogeneous and of the same density as we assume in theory. These all deviations from the ideal conditions play a very important role in the vibration conditions of the structure. This has been explained by Tobias [15] is some details. According to him these different peaks are the result of initial imperfections or deviation from the rotational symmetry of the circular cylinders, such as variation in radius, wall thickness, or physical properties of the shell. If we are concerned with varying forces, problem of fatigue arises and then the dimensional or surface imperfections will make a considerable difference between the calculated and practical results. He observed that for any vibrating surface of revolution, there occurred two planes such that, if the body was made to vibrate in those directions, only one peak will occur. These planes have been named as preferential planes. If the exciting force acts outside the preferential plane then both configurations are excited at the same time and the amplitudes we measure are due to the superimposition of both preferential configurations with regard to their phase angles relative to the exciting force. This is the more probable reason for several peaks occurring in the resonance tests.

7. Resonance curves for empty shell are comparatively smooth (with only one peak or two) while the amplitude fluctuates from one value to other in the case of shell with water. This may be due to the

sloshing of water in the latter case.

8. Amplitude of vibration at resonance was more at top and less at bottom. The reason is quite obvious as the shell was fixed at base and free at the top.

9. The acceleration to which the model was subjected at resonance varied with the height of the cylinder and water depth. The acceleration was observed to be less when water was filled inside the shell.

4.5 COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

Theoretical and experimental results have been compared in Table 4.4. From this table we observe the following.

1. Difference between the analytical and experimental natural frequency of vibration in fundamental node is 38 per cent.

2. For partially full or completely full shell the difference between the theoretical and analytical frequency is relatively small (of the order of 21%). The difference reduced to 10% when the water depth was increased still more.

3. It has been proved both theoretically and experimentally that frequency decreases as water depth increases.

Table 4.4

Experimental results and their comparison with Theory

Radius to thickness ratio a/h	Length to radius ratio L/a	Fractional water depth γ	Pickup Position (see fig.	Resonant frequency obtained from steady state vib. test (cps)	Natural frequency obtained from free vib. test. (cps)	Freq. from Energy Method (Eq. 3.18) (cps)	Frequency from Weingarten method (Eq. 2.5 for clamped-free shell. (cps)	Frequency from Weingarten method for simply supported free shell (cps)
173	4.38	0	1	42.5	51.5	83.20	87.27	6.129
			2	45.0				
			3	42.0				
			4	40.5				
			6	43.0				
			7	-				
		0.332	1	43.3	46.5	58.15	-	-
			2	43.0				
			3	42.2				
			7	38.2				
		0.535	1	32.2	38.5	40.45	-	-
			3	35.8				
			7	32.2				
	2.63	0	2	-	62.5	-	146.48	6.129
			3	-				
			5	32.5				
		0.494	2	35.5	53.75	-	-	-
		0.875	2	32.5	30.75	-	-	-
			5	29.6				
			8	32.0				

CHAPTER -V

C O N C L U S I O N S

On the basis of the results reported in the previous chapters, the following conclusions can be drawn.

1. The energy method used here is fairly accurate for predicting the frequencies of a vibrating shell. The results agree with those of other investigators.

The method may be applied to any set of boundary conditions by properly selecting the basic functions.

2. The effect of mass of water can be taken into consideration by considering a virtual mass determined for the mode shapes considered either experimentally or from analysis.

3. Analytical results indicate that frequency of vibration will increase with decrease in length to radius ratio of the shell. Also, the frequency decreases with increase in water depth.

4. The frequencies at the higher order modes are decreasingly affected by the presence of the liquid.

5. Percentage contribution of the water mass increases with water depth. When the tank is fully filled the contribution of water mass is 83 per cent.

6. Behaviour of the shell under forced vibration conditions is a peculiar one and occurrence of several peaks is observed. Further detailed study is needed to investigate into the characteristics of the various peaks.

7. Results from the experiment and analysis agree closely at higher water depths.

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APPENDICES

A.1 Notations used in computer programmes

AL	-	length of the shell (L)
GL	-	Height of water inside the shell (γL)
A	-	Radius of the shell (a)
H	-	Thickness of the cylinder (h)
ROH.	-	Density of the material (ρ)
ROHD	-	Density of the fluid (ρ')
GAMA	-	Ratio of the water depth to length of the shell (γ)
AN	-	Number of circumferential waves (n)
M	-	Number of axial waves (m)
ALEMD	-	Characteristic root (λ)
ABSIN	-	Virtual mass coefficient (ϵ_n)
ALPHN	-	The root α_{ni} used in the expressions.
CIGMA	-	Poisson's ratio of the material (σ)
YM	-	Young's modulus of elasticity (E)
FREQ	-	Frequency of vibrations (f)

(A.3) PROGRAMME FOR EVALUATION OF FREQUENCIES AND MODE SHAPES

```
C  C   EVALUATION OF DETERMINANT ATUL UOR   Z
5      READ 1,ALEMD,AN,ABSLN,GL
       READ 2,AL,CIGMA,A,G,ROH,ROHD,H
       READ 9,YM
       READ 300,FREQ,CFREQ,ACC
1      FORMAT(4F10.0)
2      FORMAT(7F9.0)
9      FORMAT(F10.0)
300    FORMAT(3F10.0)
100    RT1=0.
101    RT2=0
102    FREQ1=FREQ
       SAN=AN*AN
       SLEMD=ALEMD*ALEMD
       X=ALEMD*AL
       XN=2.*ALEMD*AL
       EX=EXPF(X)
       EXN=EXPF(XN)
       Y=1./EX
       YN=1./EXN
       Z=1./ALEMD
       SA=A*A
       CK=(H*H)/(12.*SA)
       P=(EX+Y)+2.*COSF(X)
       Q=(EX-Y)+2.*SINF(X)
       AK=P/Q
       SAK=AK*AK
10     ALPHA=.125*(EXN-YN)*Z
       BETA=.125*(EXN+YN)*Z
       GAMA=.25*SINF(XN)*Z
       DELTA=.25*COSF(XN)*Z
       ZAI=.5*(EX+Y)*COSF(X)*Z
       SAI=.5*(EX+Y)*SINF(X)*Z
       AKAI=.5*(EX-Y)*SINF(X)*Z
       ZETA=.5*(EX-Y)*COSF(X)*Z
11     X1=ALPHA+GAMA+SAI+ZETA
       X2=ALPHA-GAMA+SAI-ZETA
       X3=BETA+AKAI-DELTA
       X4=ALPHA+GAMA-SAI-ZETA
       X5=ALPHA-GAMA-SAI+ZETA
       X6=BETA-AKAI-DELTA
       X7=ALPHA-GAMA
       X8=ALPHA+GAMA-AL
       X9=BETA+DELTA-0.5*Z
       X10=ALPHA-GAMA+SAI-ZETA
       X11=AL+ALPHA+GAMA-SAI-ZETA
       X12=BETA-ZAI+DELTA+0.5*Z
```

```
 12      AM1=AL+X1+SAK*X2-2.*AK*X3
         AM2=AL+X4+SAK*X5-2.*AK*X6
         AM3=X7+SAK*X8-2.*AK*X9
         AM4=X10+SAK*X11-2.*AK*X12
         CDELT=((1.-CIGMA*CIGMA)*ROH*FREQ1*FREQ1)/(YM*G)
 13      A11=SLEMD*AM1+(1.-CIGMA)*0.5*SAN*AM4/SA
         ALL=ALL-CDELT*AM4
         A12=(ALEMD*AN*(CIGMA*AM3-0.5*(1.-CIGMA)*AM4))/A
         A13=(-CIGMA*ALEMD*AM3)/A
         A221=(SAN*(1.+CK)*AM2)/SA
         A221=A221+0.5*(1.-CIGMA)*SLEMD*(1.+4.*CK)*AM
         A222=-CDELT*AM2
         A22=A221+A222
         A231=(-AN*(1.+SAN*CK)*AM2)/SA
         A231=A231+CIGMA*SLEMD*AN*CK*AM3
         A232=-2.*(1.-CIGMA)*SLEMD*AN*CK*AM4
         A23=A231+A232
         A331=SA*SLEMD*SLEMD*CK*AM1+((1.+SAN*SAN*CK)*AM2)/SA
         A332=-2.*CIGMA*CK*SAN*SLEMD*AM3
         A332=A332+c.*(1.-CIGMA)*CK*SAN*SLEMD*AM4
         A333=(GL**3).)*ROHD*ABSLN/3.*ROH*H*A)
         A333=AM2+A333
         A333=-CDELT*A333
         A33=A331+A332+A333
         DET1=A11*(A22*A33-A23*A23)-A12*(A12*A33-A13*A23)
         DET2=A13*(A12*A23-A13*A22)
         DET=DET1+DET2
 59      IF(DET)60,66,61
 60      RT1=DET
         Q1=FREQ
         GO TO 62
 61      RT2=DET
         Q2=FREQ
 62      IF(RT1)63,64,64
 63      IF(RT2)64,64,65
 64      FREQ=FREQ=CFREQ
         GO TO 102
 65      FREQ=Q1+RT1*(Q2-Q1)/(RT1-RT2)
         FREQ1=ABSF(FREQ-FREQ1)  ----> FREQ2=FREQ1/FREQ.
         IF(FREQ2-ACC)60,66,102
 66      F=0.5*FREQ/3.14159265
         AW=(A11*A22-A12*A12)
         IF(AW)350,320,350
 350     A=(A23*A12-A13*A22)/AW
         B=-(A13+A11*A)/A12
         DO7 LX=1,61,2
         AX=LX
         ALX=ALEMD*AX
         ELX=EXPF(ALX)
         ELXI=1./ELX
         SHLX=0.5*(ELX-ELXI)
         COLX=0.5*(ELX+ELXI)
         SLX=SINF(ALX)
         CLX=COSF(ALX)
```

```
      U=A*((SHLX+SLX)-AK*(COLX-CLX))
      W=((COLX-CLX)-AK*(SHLX-SLX))
      PUNCH 310,AX,U,V,W
310   FORMAT(4E16.8)
7     CONTINUE
320   PUNCH 40,F,CDELT,DET
      GO TO 5
200   FORMAT(2E16.8)
40    FORMAT(3E16.8)
      STOP
      END
```