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HYDRODYNAMIC PRESSURE DUE TO EARTHQUAKE

(ON DAMS, CONTAINERS AND PIERS)

by

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HYDRODYNAMIC PRESSURE DUE TO EARTHQUAKE

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CERTIFICATE

Certified that the dissertation entitled "Hydrodynamic Pressure due to Earthquake (On Dams, Containers and Piers)" which is being submitted by Shri Pramod Kumar Agrawal in partial fulfilment for the award of the Degree of Master of Engineering in STRUCTURAL ENGINEERING INCLUDING CONCRETE TECHNOLOGY of the University of Roorkee is a record of students own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

It is further to certify that Shri Agrawal has worked for a period from 1st May, 1962 to 14th December, 1962 at the University for preparing this dissertation.

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C O N T E N T S

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S Y N O P S I S

This dissertation makes a critical review of the various methods, analytical and experimental, for the determination of hydrodynamic pressures on dams, containers and piers due to an earthquake. A comparative study of all the available methods concludes that Werner and Sundquist's approach to the problem is the best but for all practical purposes Housner's approximate so-lutions can be used safely without involving appreciable error. The experimental work conducted on an Electrical Analogy Tray apparatus for determination of hydrodynamic pressure due to an earthquake on eleven profiles of dams in 4' x 6" x 2" (deep) tray, supports Zanger's work. It is concluded that this method affords a very easy and cheap method for experimental determination of hydrodynamic pressure distribution on a dam of any profile for a two dimensional case only. Study has also been made to show that how the various assumptions of several approaches to the problem effect the distribution of hydrodynamic pressure. The conclusions drawn from the critical study of the problem have been summarised in the end. In the case of tanks for a economic design in seismic region, a open design is recommended. The recommendations of I.S.I. and U.S.B.R. have also been incorporated in the end.

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C H A P T E R. I.

I N T R O D U C T I O N .

1.1. The dynamic fluid pressures developed during an earth -- quake are of importance in the design of structures such as dams, water tanks etc. The present work relates to the critical study of the methods, that have so far been evolved, with a view to find suitablity or other-wise of each under different circumstances and for different structures. This work will, it is hoped, make the task of the designer easier and will also bring forth the work for the research worker for further investigation. Available literature in this field has provided various solutions to the problem in which additional hydrodynamic pressures - impulsive and convective - have been worked out and increase in the effective mass of the fluid in the liquid containing or retaining structures, has been found. A critical review of these methods helps in finding the rationality and applicability of each method. The different solutions to the problem differ essentially due to difference in basic assumptions, the validity of which is proposed to be studied here. How these different theoretical approaches agree with the experimental results, has been studied here. The solutions to the problem are more or less same in character and can be applied to the problem in the field. This, however, needs examining as to which method will lead to correct results without appreciable error and how much is the effort involved in applying it to the problem in the field. The first solution to the problem of determining the hydrodynamic pressure in liquid retaining structures during earthquake, was that by Westergaard (1) (1933), who determined the pressures on a triangular, vertical dam subjected to horizontal acceleration. Jacobsen (2)(1949) suggested a solution of the corresponding problem for a cylindrical pier surrounded by fluid. Werner and Sandquist (3) (1949) extendend Jacobsen's work to include a rectangular container , a semicircular trough, a triangular trough and a hemisphere. Graham and Rodrigues (4) (1952) gave a very thorough analysis of the impulsive and convective pressure in a rectangular container. Hoskins and Jacobsen (5) (1934) determined impulsive fluid pressures experimentally, and Jacobsen and Ayre (6) (1951) gave the result of similar measurements. Zanger (7) (1953) presented the pressures on dam faces as measured on the electric analogy tray for different profiles of dam sections. Experimental work of Zanger (7) agrees well with that of Westergaard (1). Housner (8) (1957) gave simplified formulae for containers having two fold symmetry, for dams with sloping faces and for flexibleretaining walls. The analysis includes both impulsive and convective fluid pressures. In 1959 Seima Kotsubo (9) gave so--lutions to arch dams and has evaluated a solution for irregular earthquake. Prior to Kotsubo (9) the problem has been analysed with a two dimensional analysis but Kotsubo (9) has extended his solution to three dimensional analysis. Ambrasseys (10) has given hydrodynamic pressure coefficients for a faulted basin in the seventh general meeting of A.I.R.H., from his experiments on an electrical analogy tray.

1.2. The problem of determining the dynamic fluid pressures during earthquakes in liquid retaining structures is gaining importance these days due to development of the seismic areas in the country. The construction of massive gravity dams, concrete, masonary or earthen, and tanks containing water or oil, has necessitated a thorough and detailed study of this problem for safe

-guarding against failures due to the earthquakes and fire that usually breakout as a consequence. An arbitrary design procedure or neglecting relevant factors is not recommended.

1.3. Experimental work has been carried out to verfy Zanger's (7) work with different profiles of dam sections. In all eleven cases has been studied. Experimental curves obtained from the electric tray analogy, for pressure distribution coefficients are quite close to the Zanger's (7) experimental curves , which shows that the eheoretical approach to the problem is justified within limitations. Experiments have been conducted on a tray made of sheet plastic of size 36" x 6" and 1" deep. The different profiles of dams 6" high, were constructed by binding nichrome wire giving linear potential drop along its face. From the flow nets obtained from the electrical analogy tray, pressure co-efficients have been determined. There is scope of further investigation and research of this problem.

1.4. After an analytical investigation and a comparative study of various methods, it was found the solutions presented by Hous-ner (8) are more simplified and easy to apply. Errors in the solutions are not of appreciable magnitudes in all practical cases and are confined to 5 to 6% but in most of the cases it is with in 2 to 3%. The compressibility of water can be neglected in all the practical cases as error caused on this account in the solutions is not excessive. Housner (8) work has also been reproduced in the form of curves to enable the designers to apply the results in practical case.

CHAPTER II

CRITICAL REVIEW, COMPARATIVE STUDY AND DISCUSSIONS.

2.1. The analytical as well as experimental model used for the solution of the problem of determining the distribution of hydrodynamic pressures on liquid containing or retaining structures like dams, containers etc., during earthquake, is of a highly idealised type and it must be admitted that in some ways there is no correspondence whatever with the prototype, specially when for the sake of simplicity the choatic motion of the near earthquake is assumed to be of a simple harmonic nature. The same applies equally well to all attempts made to analyse the effect of hydrodynamic water pressures on dams during a earthquake.

2.2. Some of the other factors, to be taken into consideration if a rigrous solution of the problem is desired, are the compressibility of the water, the flexibility of the dam, the three dimensional seismic disturbances, the effect of gravity upon the free deformation of the surface of water, the effect of sides slopes in the case of canyons, the dam not beeing straight in plan, faulting of the basin, and the more important factor of irregularity in time, magnitude of the seismic disturbances and its duration. These factors imply that a rigrous solution of the problem is almost impossible. Furthermore it is questionable whether a steady state of oscillation can ever be established in a reservoir, and whether resonance or even in phase motion between the Fourier components of the seismic spectrum and the natural modes of the water can ever be attained. It has been found (11) that a long series of the waves of the same period and amplitude is not required for setting up reasonance, and that two or more impulses of quite different intensity timed at right interval, can accomplish this, phenomenon of resonance. But again it is questionable whether the right interval can even be attained.

2.3. So the characterestics involved in the calculation of the hydrodynamic pressures greatly complicate and make doutful the implied accuracy and security of any rigrous solution as a sole criteria. Therefore it is justified to balance the factors that can possibly be involved in a mathematical or experimental treatment of the problem. The accuracy in simulating the conditions to be encountered during earthquake must compare with the accuracy within which a prediction or estimate can be made on the seismic factors. A higher value of the seismic coefficient will increase the cost of structure considerably. The cost of structure increases more rapidly with the increase in seismic coefficient. A thorough investigation should be carried in deciding the seismic coefficient in all the major projects in seismic zones. In the treatment of the problem, a simple solution is obtained only on the basis of macroscopic information on the involved quantities.

2.4. The theoretical treatment of the problem produces great difficulties if the boundry conditions are functions of the either of variable to be used. A simple solution of the problem is obtained from the Laplaces equation on the assumption that water is incompressible, with which suitable boundry conditions define a monochromatic solution for the hydrodynamic pressures.

2.5. The hydrodynamic pressures during earthquake will be studied for two types of structures,namely,"dams" and "containers including cincular pier ".

PART ONE - D A M S.

2.6. Considering "dams" first we know that the additional stresses in a dam during an earthquake can be accounted due to :-

(a) The acceleration of the mass of the dam i.e., inertia forces.
(b) The change of water pressure i.e., hydrodynamic pressure.
(c) Increase in the horizontal vibrations of the dam due to resonance.
(d) Due to the movement of silt and ice on upstream face of the dam and,
(e) Due to the movement on faults.

2.7. During seismic orcillations, the volume of water on the upstream side of the dam would be energised and consequently compressional pulses thus generated, inside the mass would be impacting against the wall of the dam and thus extra hydrodynamic pressure is impressed on the upstream face of the dam, over and above static water pressure already existing. The process and nature of generation of extra hydrodynamic pressure is principally the same as those for sound waves in water or in any fluid. The velocity (1) of these compressional waves in water has been found to be equal to $\sqrt{\frac{gk}{w}}$.

2.8. The pioneer work of determining the hydrodynamic pressures on the body of a dam is due to H.M. Westergaad (1) who first presented an analytical solution to the problem in 1933.

2.9. The water pressure on the upstream face of the dam or due to the tail water is modified during an earthquake. The exact nature of the variation is highly involved due to the complex character of the vibratory forces set up during an earthquake. A simplified approach to the problem has been suggested by Westergaard(1) . Other significant contributions in this direction are due to the following :-

(1) Von Karman (11)
(2) Jacobsen and Hoskins (5)
(3) Werner and Sanquist (3)
(4) Zanger (7)
(5) Ambraseys (10)
(6) Housner (8)
(7) Seima Kotsubo (9)

We shall now examine each one of the available work in detail.

2.10. (1) <u>H.M.Westergaard (1)</u>

Westergaard's analytical solution to the problem is based on the following assumptions:-

(1) The dam body is straight along its lenth.
(11) The upstream face of the dam is vertical.
(111) The vibrations in the earthquakes are assumed horizontal in a direction perpendicular to the dam.
(1v) All points of the foundation have the same displacement, velocity and acceleration at the same time.
(v) The displacement of a particle of water (u,v) in the directions of X and Y respectively are small.
(v1) The dam is subjected to a simple harmonic motion during the earthquakes with a maximum acceleration equal to αg where α is the seismic coefficient and is in % of acceleration due to gravity.
(v11) The dam body is rigid.
(v111) The water is compressible.
(1X) The reservoir is of infinite length.
(X) There is no overtopping due to wave heights due to seismic agitation of the reservoir.
(X1) The water can escape vertically.
(X11) The effect of hill side slopes in the case of canyon has not been considered.
(X111) There is no faulting in the basin of reservoir.

All the above assumptions will now be discussed to examine the validity of the solution and its limitations. An attempt has been made to determine the influence of these assumption on the result.

2.11. "The dam body is straight along its length", is usually the case in masonary, earthen or concrete dams except in the case of arch dams to which the given solutions are not applicable. The straight length is adopted for economy and ease in analysis, design and construction. The behaviour of the structure can also easily be predicted experimentally under various forces if the dam body is straight along its length. If the geological features of the proposed dam site, are such that the suitable fondation rocks are not available in a straight line, then for the stability and the economy infoundations, one bend or more may have to be provided in the dam section along its length. No work seems to have been done for determining the hydrodynamic pressures on the dams which have bends in plan, as such cases are not encountered usually. The development of the seismic zones may encounter this problem. It is expected that the hydrodynamic pressures will increase near the bends when it is towards downstream side and vice versa. This can be shown by approximation a arch dam into a isoseles triangle. When the dam body is not straight along its length, the problem becomes a three dimensional one and to get analytical solution to the problem is difficult. The case of arch dams has been analysed for the hydrodynamic pressure by Seima Kotsubo (2). He has presented a three dimensional analysis for symmetrical arch dams.

2.12. Westergaard (1) considered in his solution only the vertical upstream face of the dam. But in most of the gravity dams the section has a sloping upstream face. The effect of upstream slope of the dam on the hydrodynamic pressure during earthquake has been considered by Zanger (7), Housner (8) and Venti Chou(12). Zanger solved the problem experimentally by the electrical tray analogy and has givena set of curves for pressure coefficients for various upstream slopes and various slopes of sections. Housner has given approximate solutions to the problem. It is observed that with the decrease in upstream slopes, the hydrodynamic pressure decreases.

2.13. The earthquake shocks are irregular in nature and can-- not be represented mathematically. An earthquake wave can have components in all the three directions. The body of the dam and reservoir water will be subjected to all the three components - horizontal acceleration perpendicular to dam body, horizontal acceleration along the dam body and the vertical component of acceleration. Out of these components which is distrastous to dam in determining the hydrodynamic pressure on the dam body, is horizontal component perpendicular to the dam body. This component will set waves in reservoir water which will act against the dam causing increase in water pressure known as hydrodynamic pressure. The vertical component has no overturning effect upon a gravity dam, because the weight of the water, and, therefore, the water pressure on the dam is increased in the same proportion and at the same time as the mass of the dam and water, for ordinary static condition will be changed to $(1 + \alpha v)$ g acting upon both dam and water during upward acceleration. A downward acceleration also does not changed moment conditions. The vertical acceleration affects the sliding factor and should be investigated in each case. The component along the dam is not of any significance in determining the hydrodynamic pressure, as the dam is rigidly fixed between two firm banks. Therefore for determining the hydrodynamic pressure, the only horizontal component perpendicular to dam section will be considered giving the worst condition while the increase in pressure due to vertical component will be determined by simple statics.

2.14. It has been assumed that all points of the foundation have the same displacement, velocity and acceleration. This assumption is not true in practice as the earthquake waves are very complex and cannot be represented mathematically. So all points under the base of the dam will not have the same displacement, velocity and acceleration, because the base of the dam has enough width and all points along width and length will not subjected to uniform motion. This non-uniform motion of the foundation will cause torsional stress, in the dam. The effect of which has not yet been investigated. For simplicity in the analysis it is assumed that all points of the foundation are subjected to uniform motion otherwise the problem will become very complicated and impracticable to solve.

2.15. The analytical solution to the problem has been derived on the basis that the displacement of the water particle are very small as the solution is based on the formula from elasticity for stress and strain, which are valid only for small displacements. The displacements of the second order has not been considered as no appreciable change in results can be obtained. Robinson Rowe (13) has calculated the maximum value of u as .1460 ft. at $x=0$. for $\alpha = .10$.

2.16. For making the solution to the problem simple, it is assumed that earthquake waves consist of a group of simple harmonic waves represented by a Fourier series, with maximum amplitude, αg, where α is the seismic coefficent and g is the acceleration due to the gravity. The earthquake shocks consist of waves of varying amplitude and time period which cannot be represented mathematically but can be approximated to a summation of simple harmonic motions, sine and cosine series, with varying time periods and amplitudes. It is very difficult to consider an irregular earthquake either in an analytical solution or in an experimental work. We only consider the first term of the sinusoidal motion series, which gives satisfactory results for our purpose. For more exact results we can consider other terms of this series but their contribution is very little. Seima Kotsubo (9) has considered the effect of irregular earthquake. He has represented the earthquake tremor by the following equation :-

$$\text{Acceleration} = \alpha g \int_0^t \psi(\tau)\,d\tau \quad .$$

The value under integral can be found by velocity spectrum. As it is impossible to evaluate strictly the actual transient phenome-non of ever changing dynamic water pressure it is reasonable to assume that the dam is subjected to a simple hormonic motions during earthquake.

2.17. In the case of the dams it is quite reasonable to assume that the dam body is rigid and moves along with the foundation as it is a massive structure either of earth, masonary or concrete between two firm rigid banks. All theories are based on this assumption. The flexibility of the dam has been considered by

John H.A. Bralitz and Carl Heilborn (14). Since the period of free vibration is usually a small fraction of a second, while the period of vibration of the earthquake may be assumed to be not le -ss than a second in most of the cases, the resonance need not be expected ordinarily, although the possibility of part resonance may well be investigated in some cases of high concrete or masona -ry dams. The part resonance is caused both by the hydrodynamic pressure and by inertia effect of the weight of the dam and as it amplifies bo-th the hydrodynamic pressures as well as the ine -rtia forces, it is impossible to seperate the two effects and both should be considered. The hydrodynamic pressures are first taken and the inertia forces are found as if the dam were rigid, the deflections of the dam due to these forces are found. The displacement of the dam is now taken as the displacement of its base plus the deflection. The hydrodynamic pressures and the inertia forces are recalculated on the basis of new displacements and the process is repeated if a closer approximation is desired. If the dam is flexible only the base moves with the prescribed motion and at other places it will be some relation like uo =f(y,t) (y,t). Considering this effect, the equation to p is modified to the form :-

$$p = \frac{4\pi}{T^2} \cdot \frac{w}{g} \sum_{1,3,5\ldots}^{n} E_n \sin \frac{n\pi}{2h} y \qquad (1)$$

Where En is a constant given by relation

$$E_n = \frac{8h}{n^2\pi^2 C_n} \left[n + \delta_0 + (-1)^{\frac{n+1}{n}} \cdot \frac{2h}{n\pi} \cdot \frac{\delta_0}{H} \right] \qquad (2)$$

and uo is of the form

$$u_0 = - r \frac{\cos \gamma \gamma_1 y}{\cos \gamma \gamma_1 H} \cos \gamma t \qquad (3)$$

in which,

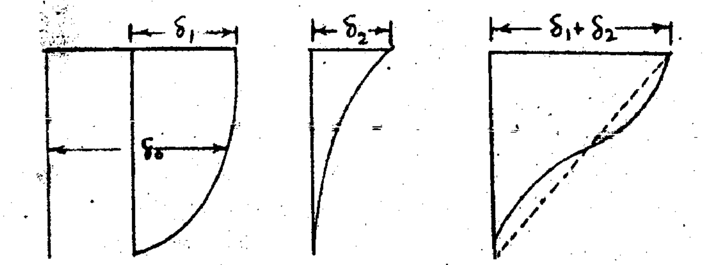
$$\gamma = \frac{2\pi}{T}$$

$$\gamma_1 = \sqrt{\frac{\rho}{Gg}}$$

and $c_n = \sqrt{1 - \frac{16 w h^2}{n^2 g k T^2}}$

H = Total depth of the dam

δ_0 = maximum deflection at the top due to dynamic forces i.e. summation of shear and moment diflection.



(a) SHEAR DEFLECTION CURVE. (b) MOMENT DEFLECTION CURVE. (c) TOTAL DEFLECTION CURVE.

FIG. NO. 1.

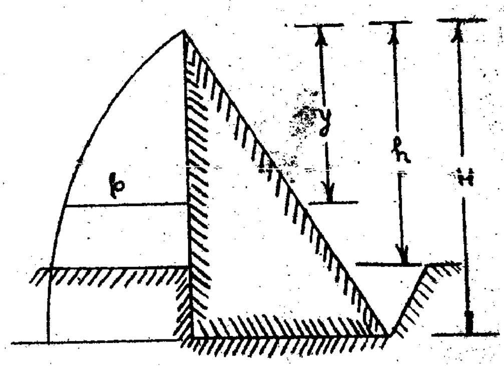


FIG. NO. 2.

Taking the total deflection curve as a triangle, the inertia coefficient α_1 at any part of the dam will be given by :-

$$\alpha_1 = \alpha \left[1 + \frac{\delta_0}{r} \cdot \frac{H - y}{H} \right] \qquad (4)$$

Curves for pressure considering the deflection due to the shear and moment for a dam having h=300', H= 375' , α =.10 and T= 1 sec. has been drawn as shown in figure no. 3 and 4.

Top deflection due to shear and moment was found respectively 0.220 inches and 0.105 inches therefore total deflection at top δo=.325 inch. It was found that increase in hydrodynamic pressure on this account is nearly 14% at the bottom of the dam for this height. For height more than 300 ft. the increase in the hydrodynamic pressure will be larger. Similar curves can be drawn to other heights. For all practical case this increase in pressure on this account can be neglected.

Housner (8) also considered the stiffness of a rectangular wall. Considering this effect on the flexibility of the wall, he found that pressure p is given by the following equation :-

$$p = wh\alpha \frac{4\pi^2}{T^2}\sqrt{3}\cdot\sqrt{\frac{1-1.68\beta+1.18\beta^2}{1+2.44\beta+1.63\beta^2}}\left[(1-\beta)\left\{\frac{y}{h}-\frac{1}{2}\left(\frac{y}{h}\right)^2\right\}+\frac{4}{\pi^2}\beta\sin\frac{\pi y}{2h}\right] \quad (5)$$

and total pressure

$$P = \frac{wh^2\alpha}{\sqrt{3}}\cdot\frac{4\pi^2}{T^2}\sqrt{\frac{1-1.68\beta+1.18\beta^2}{1+2.44\beta+1.63\beta^2}}\,(1-0.22\beta) \quad (6)$$

where

$$\beta = \frac{P}{\alpha g}\cdot\frac{h^3}{(\frac{\pi}{4})^4 EI}$$

T = natural period of vibration of wall

β will be zero for a rigid wall as EI= ∞.

Housner (8) found that the total force on the wall is reduced by wall flexibility due to the stiffness of the section. Figure No. 5 shows how the total force on the wall is reduced by wall flexibility. For a rigid, EI=∞, the above equation overestimates P by 6 %.

Westergaard (1) also considered the compressibility of water which in most of the other analysis has been neglected. If compressibility of water is neglected the coefficient Cn, would become equal to one. Considering the compressibility of water, the coefficients for pressure, shear and moment change by 17.5%, 15.4% and 14% respectively for a height of 800 ft., otherwise coefficient would have been independent of the height and the vibration period of the earthquake.

2.18. How the boundary of the reservoir changes the distribution of hydrodynamic pressure has been considered in detail in the following pages.

2.19. In the light of existing evidence no special provision is made for waves excited in the reservoir by causes other than winds. The effect of earthquake on the formation of water waves in the reservoir has not ever been considered in the design of dams, and there appears to be no literature available on this subject. The magnitudes of such waves during earthquakes, generated in the basin of the Steeg Sur l'ouhed Fodda dam, in North Africa, during the earthquake of Orleansirlle of the 9th Sep. 1954 reported in the "Terres at Eav ". During this earthquake water waves were excited, leaving their traces on the face of the dam. From these traces a maximum amplitude of the wave of 50 Cm was measured. Oscillations of water in a reservoir, observed in the case of Kita Mino Earthquake on August 19, 1961 have wave height only 40 Cm. Results from a model test, with a limited number of numerical datas, supported bytheoretical considerations, were given by Mononobe (15), in the discussion of Westergaards paper. Mononobe was inclined to think that the increase in water pressure was due to the standing waves formed in the basin rather than the factors involved in Westergaards' theory. N.N.Ambraseys

(10) analysed the problem theoretically and also carried out experiments. He found that the vibrations of the dam and basin of the reservoir proper cannot excite waves of measurable height. For reservoir of very small size, some waves may be observed but their amplitude bears no harmful effect on the dam. So it appears quite reasonable that the seismic disturbances on a large mass of water cannot excite waves of any considerable amplitude. The impact of the face of the dam upon the mass of the water and the passage of a surface wave does not provide waves of appreciable heights but disturbances caused by the sudden displacement along a fault line of the bottom of reservoir or a land slide, gives rise to the most destructive type of waves. Hence it can be concluded that if it is most unlikely that the seismic disturbances can excite water waves in reservoirs. Therefore in the design of a dam the free board to take care of any overtopping effects, is not considered.

2.20. Westergaard (1) found that if it is supposed that water does not heave when pressed by the dam, which would move in simple harmonic motion during an earthquake, the maximum pressure obtained uniform throughout the height is given by

$$p = \frac{\alpha T}{2\pi} \sqrt{g k w} \qquad (7)$$

then for $\alpha = \cdot 1$, $T = 1$ sec.

$$p = 5270 \text{ lbs/sq ft.}$$

where $k = 3 \times 10^5$ psi.

which gives a very high value. Same results are obtained if impulsive effect of wave is considered. Bakhmeteff (16) obtained similar results considering water hammer phenomenon in conduits. Be ing a fluid, water is freeto escape vertically and the pressures are actually much less, and arenot uniformly distributed throughout the depth of reservoir. It is therefore, assumed that p=0 at y=0 and v=0 at y = h.

2.21. The effect of hill side slopes in the case of a canyon and the faulting of the reservoir basin, on the hydrodynamic pressure during an earthquake has beendiscussed in detail in the following pages.

2.22. Westergaard (1) considered an element of water subjected to horizontal acceleration given by

$$u_0 = -\frac{\alpha g}{4\pi^2} T^2 \cos \frac{2\pi t}{T} \qquad (8)$$

Then from the stress strain relation ship as given by Lamb (17), for very small displacement and considering the compressibility of water into account, the following equations are obtained

$$K\frac{\partial^2 u}{\partial x^2} = \frac{w}{g}\frac{\partial^2 u}{\partial t^2} = -\frac{\partial p}{\partial x} \qquad (9)$$

$$K\frac{\partial^2 v}{\partial y^2} = \frac{w}{g}\frac{\partial^2 v}{\partial t^2} = -\frac{\partial p}{\partial y} \qquad (10).$$

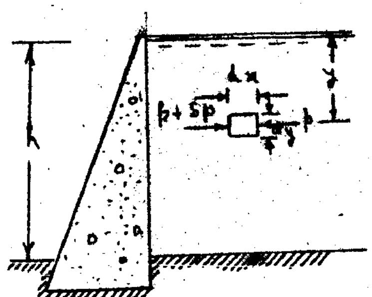


FIG. NO. 6.

Assuming that water can escape vertically and p=0 at x=0, the following equation was obtained for hydrodynamic pressu

-ure during horizontal earthquake on a vertical rigid wall.

$$p = \frac{8\alpha wh}{\pi^2} \cos\frac{2\pi t}{T} \sum_{1,3,5\ldots}^{n} \frac{1}{n^2 c_n} e^{-q_n} \sin\frac{n\pi y}{2h} \qquad (11)$$

where $$c_n = \sqrt{1 - \frac{16\, wh^2}{n^2 g K T^2}} = \sqrt{1 - \frac{\cdot 7189}{n^2}\left[\frac{h \text{ sec}}{1000T}\right]^2} \qquad (12)$$

and, $$q_n = \frac{n\pi c_n x}{2h} \qquad (13)$$

The maximum value of pressure p, of the water on the dam will be occuring at t=0,T, 2T,................given by

$$p = \frac{8\alpha wh}{\pi^2} \sum_{1,3,5\ldots}^{n} \frac{1}{n^2 c_n} \sin\frac{n\pi y}{2h} \qquad (14)$$

The greatest pressure will occure at the bottom of dam at y=h

$$p_o = \frac{8\alpha wh}{\pi^2} \sum_{1,3,5\ldots}^{n} \frac{(-1)^{\frac{n-1}{2}}}{n^2 c_n} \qquad (15)$$

The curve for the pressure distribution is obtained as shown in figure no.

If only the horizontal motion of water is considered then maximum pressure is given by

$$p = (2\beta - 1)\frac{\alpha T}{2\pi}\sqrt{gkw} \qquad (16)$$

varying uniformly throughout the depth. If β=1 i.e., all waves moving away from the dam then for α=.1 and T=1sec.

p = 5270 lbs/sq.ft. for K = 3×10^5 psi.

which gives a very high value.

Total shears or pressure, P and overturning moments, M at any depth y are given by the following equations

$$P = \frac{16\alpha w h^2}{\pi^3}\left[\sum_{1,3,5..}^{n}\frac{1}{n^3 C_n} - \sum_{1,3,5..}^{n}\frac{1}{n^3 C_n}\cos\frac{n\pi y}{2h}\right] \tag{17}$$

and,

$$M = \frac{16\alpha w h^2}{\pi^3}\left[y\sum_{1,3,5..}^{n}\frac{1}{n^3 C_n} - \frac{2h}{\pi}\sum_{1,3,5..}^{n}\frac{1}{n^4 C_n}\sin\frac{n\pi y}{2h}\right] \tag{18}$$

The above relations are rather complicated being in a summation form, so for making calculation easier. Westergaard gave the following approximate formulae :-

$$p = C\alpha\sqrt{hy} \tag{19}$$

$$P = \frac{2}{3}C\alpha y\sqrt{hy} \tag{20}$$

and,

$$M = \frac{4}{15}C\alpha y^2\sqrt{hy} \tag{21}$$

Where value of C is obtained by equating total overturning moments from approximate and correct equation and is given by

$$C = \frac{15 M_o}{4\alpha h^3} \tag{22}$$

C varies with the height of dam, although the variation is very slight. An approximate relationship for the value of C is given as

$$C = \frac{51}{\sqrt{1 - 0.72\left(\frac{h \text{ sec}}{1000 T \text{ ft}}\right)^2}} \quad lb/ft^3 \tag{23}$$

Curve has been drawn to show the variation in the approximate and correct relation as shown in figure No. 7. Curves have been drawn between h and C for various values of T as shown in figure 8.

The width of the body of water moving with the dam, at any depth y is given by equation

$$b = \frac{7}{8}\sqrt{hy} \qquad (24)$$

2.23. THEODOR VON KARMAN (11)

Von Karman (11) gave a very simple solution to the problem while discussing Westergaard's (1) paper. He ignored the compressibility of the water as its effects are comparatively small, so that the value of apparent mass multiplied by the maximum horizontal acceleration gives a first approximation for the excess pressure due to the earthquake. He derived the width of apparent mass of the fluid at any height y' from the bottom of reservoir of infinite length, in which full acceleration develops and gave the following equation

$$b = \cdot 707\sqrt{h^2 - y'^2} \qquad (25)$$

$$\text{or} \quad b = \cdot 707\sqrt{y(2h-y)} \qquad (26)$$

Hence, with this approximation the shape of the fluid body moving with the dam is shown to be a quadrant of an ellipse. One half the axis equal $b_o = \cdot 707 h$, the corresponding pressure.

$$p_o = \cdot 707\, \alpha w h \qquad (27)$$

and

$$P_o = \cdot 555\, \alpha w h^2 \qquad (28)$$

While the values given by Westergaard (1) are

$$p_0 = \cdot 743\, \alpha w h \tag{29}$$

$$\text{and, } P_0 = \cdot 543\, \alpha w h^2 \tag{30}$$

Comparing these values we find, that the values for increase in pressure differs by 4 to 5%. Total pressure load is close to approximate value.

The values given by Zanger for vertical face are, at the base

$$p_0 = \cdot 735\, \alpha w h \tag{31}$$

$$\text{and } P_0 = \cdot 535\, \alpha w h^2 \tag{32}$$

Showing an error of 4% in p_0 and 8% in P_0. Therefore the effect of compressibility is not much and can be neglected.

2.24 <u>Jacobsen and Hoskins</u> (5)

Jacobsen and Hoskins extended Westergaard's hypothetical ideas of reservoir of infinite fetch to an actual case of bounded reservoir. Assuming similar hydrodynamical equations as of Westergaard under similar boundry conditions existing in a vibrating reservoir, Jacobsen found

$$p = \frac{8h\,\alpha w}{\pi^2}\left[\frac{\cos\frac{\pi(h-y)}{2h}\sinh\frac{\pi(l-2x)}{4h}}{\cosh\frac{\pi l}{4h}} - \frac{\cos\frac{3\pi(h-y)}{2h}\sinh\frac{3\pi(l-2x)}{4h}}{3^2\cosh\frac{3\pi l}{4h}}\cdots\right] \tag{33}$$

Hence end pressures at the sides of the vibrating reservoir is, for p at x = 0 and x= l

$$p_{x=0} = -p_{x=l}$$

$$p = \frac{8h\alpha w}{\pi^2}\left[\cos\frac{\pi(h-y)}{2h}\tanh\frac{\pi l}{4h} - \frac{1}{3^2}\cos\frac{3\pi(h-y)}{2h}\tanh\frac{3\pi l}{4h}\cdots\right] \tag{34}$$

$$\text{and } P = \frac{16h^2\alpha w}{\pi^3}\left[\tanh\frac{\pi l}{4h} + \frac{1}{3^2}\tanh\frac{3\pi l}{4h}\cdots\cdots\right] \tag{35}$$

and for a reservoir of infinite length

$$p = \frac{8h\alpha w}{\pi^2}\left[e^{-\frac{\pi x}{2h}}\cos\frac{\pi(h-y)}{2h} - \frac{1}{3^2}e^{-\frac{3\pi x}{2h}}\cos\frac{3\pi(h-y)}{2h} \cdots\cdots\right] \quad (36)$$

and $$P = \frac{16h^2\alpha w}{\pi^3}\left[1 + \frac{1}{3^3} + \frac{1}{5^3} + \cdots\cdots\right] \quad (37)$$

Comparing equation (34) and (36) it is seen that only beyond $l = 4h$ the boundries of the reservoir has no effect on the total pressures exerted during hydrodynamic change.

Jocobsen and Hoskins considered noncompressibility of water and no provision has been made for resonance in his results. This seems to be fundamental draw-back in his analysis which is later perfected by Werner and Sundquist (3).

2.25. Werner and Sandquist (3)

Werner and Sandquist (3) treated the problem of horizontal earthquake on the basis of general differential equations for the dynamic pressure and the solutions are both for two and three dimensional cases. Some of the solutions involve the use of Bessel functions (Watson,1944) and spherical surface harmonics (Frank and Mises,1930), which however may be rather conveniently evaluated by means of the available tables. Werner and Sandquist (3) also considered the compressibility of water.Where compressibilityhas not been considered graphical solutions have been given, which are valid for any arbitrary sections. The boundry of the reservoir has also been taken into consideration and rest of the assumptions are same as that of Westergaard(1). The solutions has certain points of singularity where the displacements are infinite in a infinitesinal area and it does impar the validity of the solution. It is of great interest when the free water body vibration period and earthquake coincides with that of earthquake, resonance takes place, and the pressure and the displacements are theoretically infinite.

For a case of two dimensional flow in a basin with square section moving in the direction of x-axis, the hydrodynamic pressure is given by the following expression

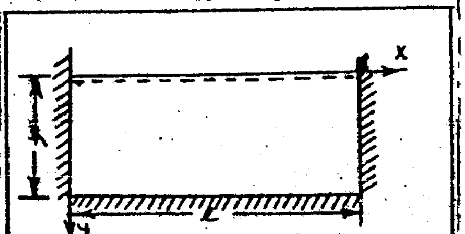


FIG. NO. 9.

$$p = \frac{8h}{\pi^2} \alpha w \cos\frac{2\pi t}{T} \sum_0^\infty \frac{1}{(2n+1)} \cdot \frac{\sinh \chi x + \sinh \chi(l-x)}{\sinh \chi l} \cdot \sin\frac{(2n+1)\pi y}{2h} \quad (38)$$

in which

$$\chi = \lambda (2n+1)\frac{\pi}{2h}$$

and $$\lambda = \sqrt{1 - \left[\frac{4h}{(2n+1)KT}\right]^2}$$

When λ is real i.e. $h < \frac{KT}{4}$, the flow is stable under all conditions. If λ is imaginary the denominator becomes Zero for certain values of χ and resonance occurs. If $\chi l = \mu\pi i$ resonance will occure, with $n = 0$, at

$$h = \frac{KT}{4} \cdot \frac{1}{\sqrt{1 - \left(\frac{\mu KT}{2l}\right)^2}} , \quad \text{when } l > \frac{\mu KT}{2} \quad (39)$$

For a infinite reservoir $h = \frac{KT}{4}$, which is the same value as derived by Westergaard (1).

Total dynamic pressure in this case is given by expression

$$P = \pm \frac{16 h^2}{\pi^3} \alpha w \cos\frac{2\pi t}{T} \sum_0^\infty \frac{1}{\lambda (2n+1)^3} \cdot \tanh\frac{\chi l}{2} \quad (40)$$

Eq.(38)+(40) have been evaluated in figure 10 and 11 respectively for different values of l/h .

In the case when the end wall of the reservoir is fixed, following solutions were obtained for the moving wall,

$$p = -\frac{8h}{\pi^2}\alpha w \cos\frac{2\pi t}{T}\sum_0^{\infty}\frac{1}{\lambda(2n+1)}\cdot\frac{\cosh \chi(l-x)}{\sinh \chi l}\sin(2n+1)\frac{\pi y}{2h} \qquad (41)$$

and $$P = -\frac{16h^2}{\pi^3}\alpha w \cos\frac{2\pi t}{T}\sum_0^{\infty}\frac{1}{\lambda(2n+1)^3}\coth \chi l \qquad (42)$$

In this case occurance of resonance is independent of the reservoir length.

Eq (41) and (42) have been evaluated in figure 12 and 13 in order to show the influence of the compressibility of the water, Curves in figure 11 and 13 have been plotted for different values of $\frac{h}{KT}$ and it is found that there is reduction in the hydrodynamic pressure if the compressibility of water is neglected.

2.26. <u>Carl N. Zanger (7)</u>

Based on the same assumptions and lines as Westergaard's (1) analytical solution, Zanger (7) obtained equation for continuity of flow as follows

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{K^2}\frac{\partial^2 p}{\partial t^2} \qquad (43)$$

in three dimensional flow.

He considered only a two dimensional flow and neglected the effect of compressibility of water i.e., $K = \infty$,. The above equation reduces to Laplaces equation

$$\nabla^2 p = 0 \qquad (44)$$

which governs the steady state flow of electricity. Zanger (7) used the electrical analogy tray to solve this equation and obtained flow nets for studying horizontal earthquake effects on dams of various upstream faces,. He found that the pressure distribution due to a earthquake is given by the following simple equation for various upstream constant slopes including vertical face -

$$p = C\alpha w h \qquad (45)$$

This equation is also applicable to upstream faces with compound slopes.

In the above equation, C is the pressure coefficient, defining the magnitude and distribution of pressures which are determined by equipotential lines in the flow net. The pressure coefficient C is a function of the shape of the dam and reservoir and is unaffected by the intensity of the quake.

In order to make this study of general value to the designers, earthquake pressures were determined for several shapes of dams. Dams studied by Zanger (7) were those with constant upstream slopes, θ of 0°, 15°, 30°, 45°, 60°, and 75° and 14 cases of compound slopes. All his work has been shown in the form of curves for each case and have been reproduced in figure 15 to 17. To permit rapid use of these by designers, the experimentally determined pressure curves are represented by a family of parabolas which closely approximate the experimental curves for constant slope. The parabolic distribution is given by equation

$$C = \frac{C_m}{2}\left[\frac{y}{h}\left(2-\frac{y}{h}\right) + \sqrt{\frac{y}{h}\left(2-\frac{y}{h}\right)}\right] \qquad (46)$$

Where Cm is the maximum value of C obtained from figure no. 14 . Therefore for constant slopes, pressure is given by

$$p = \frac{1}{2}\alpha w h C_m\left[\frac{y}{h}\left(2-\frac{y}{h}\right) + \sqrt{\frac{y}{h}\left(2-\frac{y}{h}\right)}\right] \qquad (47)$$

as shown in figure 15 which closely resembles the experimental curves. Total shear and moment is given by

$$P = 0{\cdot}726\, p y \qquad (48)$$

$$M = 0{\cdot}299\, p y^2 \qquad (49)$$

obtained analytically.

The assumption that water is incompressible, is not conservative, however a comparision with Westergaard's (1) analytical solution for dams with upstream veŧtical face shows that the % of errors are very small for dams under 400 ft. in height and are not excessive for dams as high as 800 ft. The error has been calculated for T=1 sec. and T= $\frac{4}{3}$ = sec and is shown in tables below

T=0.8 sec

Quantity	100 ft.	200 ft.	400 ft.	600 ft.	800 ft.
p	-.9	-1.9	-.5.6	-11	-25.4
P	-1.7	-2.4	--5.2	-10.6	-24.2
M	+1.1	+0.2	-2.4	-7.1	-18.7

T=1 sec.

Quantity	100 ft.	200 ft.	400 ft.	600 ft.	800 ft.
p	-.9	-1.9	-5.4	-9.8	-18.4
P	-1.7	-2.4	-5.1	-9.5	-17.4
M	+1.1	+0.2	-2.3	-6.3	-13.5

$T = \frac{4}{3}$ sec

Quantity	100 ft.	200 ft.	400 ft.	600 ft.	800 ft.
p	-0.9	-1.9	-5.2	-9.1	-15.6
P	-1.7	-2.4	-4.9	-8.8	-14.7
M	+1.1	+0.2	-2.1	-5.8	-11.4

The difference between the two results, is due to the fact that Zangers pressure coefficient C is independent of the vibration period of earthquake while the constant C of Westergaard (1) depends upon the vibration period of the shock. The error in the Zangers (7) result increases with smaller vibration period as seen from the above table, however the difference is not of any significance in determination of hydrodynamic pressure as it is only 15% of hydrostatic pressure for $\alpha = 0.15$. Westergaard and Zanger considered the earthquake vibration period not less than 1 sec. In case of near shocks it may be less than 1 sec. For practical purposes this difficiency can be over looked.

In the design of dams as recommended by U.S.B.R., the value of T is taken 1 sec. Therefore the assumption that water is incompressible holds good without any appreciable error.

No data exist giving hydrodynamic pressure as a function of upstream slope. Mathematical methods may be used, but they are complicated and time consuming. The pressure coefficient C, varies almost line-arly from ·735 for a dam with vertical face $\theta = 0°$, to .165 for $\theta = 75°$, so the curves for pressure coefficient for constant slopes can be grouped in the following simple relation

$$C = \cdot 3675 \left[\frac{y}{h}\left(2-\frac{y}{h}\right) + \sqrt{\frac{y}{h}\left(2-\frac{y}{h}\right)} \right] - 0\cdot 12 \cdot \frac{\theta^\circ}{15}\sqrt{\frac{y}{h}} \qquad (50)$$

Curves represented by the above equation closely resembles with the family of parabolas given by Zanger (7) shown in figure 18

It is observed from the experimental work of Zanger(7) that the increase in pressure on dam with slopes is less than on dams with vertical faces, the flatter the slope the smaller the increase in pressure and except for a vertical face, the maximum pressure coefficient occurs at some distance above the base of the dam.

The electrical tray analogy affords an easy and simple method for obtaining the increase in water pressure on the dam of any profile, due to horizontal earthquake and can be utilised for any shape of reservoir. For the field study this method is very simple and little time consuming.

It is concluded from Zanger's work that this method is rapid, inexpensive and accurate for determining the increase in water pressure due to horizontal earthquake and can be applied to any shape of dam and vessel without appreciable error.

2.27. G.W.Housner (8)

Housner (8) in 1957 gave solutions for the hydrodynamic pressures developed when a rectangular dam with sloping upstream faces is subjected to horizontal earthquake. Analysis given by Housner(8) are based on the assumptions that water is incompressible and fluid displacements are small and therefore finding a solution of Laplaces' equation, $\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0$, that satisfies the boundry conditions with these known solutions as checks on accuracy, simplified solutions were obtained suitable to engineering application. Following are the equations for the hydrodynamic pressure

$$p = \alpha w \frac{h}{\cos\theta}\left[\left\{\left(\frac{y}{h}\right) - \frac{1}{2}\left(\frac{y}{h}\right)^2\right\}\sqrt{3} - \frac{y}{h}\sin\theta\right] \qquad (51)$$

and the resultant horizontal force on dam is

$$P = \alpha w \frac{h^2}{\cos\theta}\left[\frac{1}{\sqrt{3}} - \frac{\sin\theta}{2}\right] \qquad (52)$$

For $0 \leqslant \theta \leqslant 35^\circ$,equation (52) overestimates P by 6.5% and for $\theta > 55^\circ$ the accuracy of the preceding equation decreases. A different approximate solution is given as follows.

$$p = \alpha w h\left[\left\{\left(\frac{y}{h}\right) - \frac{1}{2}\left(\frac{y}{h}\right)^2\right\}\psi + q\frac{y}{h}\sin\theta\right] \qquad (53)$$

and the resultant horizontal force exerted against the inclined face of dam is

$$P = \alpha w h^2\left[\frac{\psi}{3} + \frac{q}{2}\sin\theta\right] \qquad (54)$$

where $\gamma = \sqrt{1 + 3\cot^2\theta} - 1$

$$q = -\frac{\frac{\gamma}{\gamma+2}}{2\beta\cot^2\theta + \sin\theta - \frac{\sin\theta}{\cot\theta}\cdot\frac{\gamma}{\gamma+2}}$$

$$\beta = \left(\frac{1}{\sqrt{3}} - \frac{\sin\theta}{2}\right)$$

and $\psi = \frac{\gamma}{\cot\theta}\left(1 - q\frac{\sin\theta}{\cot\theta}\right)$

Equation (54) overestimates P, with a maximum error of 6% at $\theta = 65^\circ$.

To make the above equations easy to apply in the determination of hydrodynamic pressures, curves have drawn between θ verses γ, q, β and ψ as shown in figure No. 19. Curve has also been drawn between θ and P in the same figure. A comparision of the above results has been made with Zangers as shown in figure 20, for constant upstream slopes, and it is found that for vertical faces Housner results give more value for pressure coefficients than Zangers. For other slopes C -

difference between the curves is not of any appreciable magnitude.

2.28. Ven Te Chow (11).

Ven Te Chow (11) in 1951 gave the formula of the following form for the hydrodynamic pressures due to horizontal earthquake, perpendicular to the face of structure.

$$p = \alpha \beta \gamma y^{\frac{1}{2}} \text{ in psf.} \qquad (55)$$

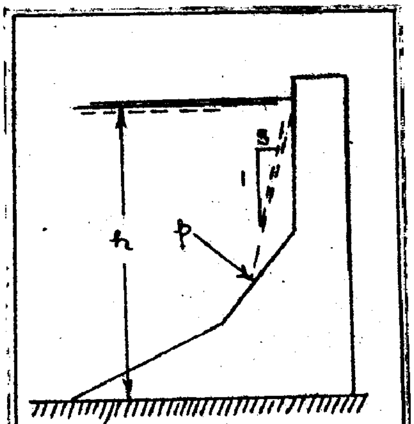


FIG. NO. 21.

in which β = factor depending on

(1) The slope, $\frac{1}{S}$, of the line joining the intersecting point of the water surface and the face of the structure to the point where p is to be found and

(11) The period of horizontal earthquake vibrations.

γ = factor depending on the total depth of water in feet.

He gave charts for γ v/s h and β V/s $\frac{1}{S}$ for a vibration period of earthquake of 1 sec.

Chow's (11) work is based on that of Westergaard (1). His method could not get any importance due to overlooking of basic hydrodynamic principle. Since the face of a dam is a stream line and no water can escape across it, the velocity of a particle normal to the surface of the dam is zero, and therefore particle can have no displacement in that direction. Thus, the assumption $\beta = f(S)$ is incorrect. The hydrodynamic pressure due to earthquake acceleration is not the same as if the area between the sloping line, $\frac{1}{S}$, and the face of the dam were filled with concrete. Values for p given by Chow's method are much below than others.

2.29. Creager, Justin and Hinds (19)

They have adopted the results of Westergaards(1), in their "Engineering for Dams", Volume II, for vertical face. For sloping faces of the dams they have introduced a multiplier $\cos\theta$, in the Westergaard's (1) results for a vertical face. As their results are not based on any theoretical approach and are therefore of little importance.

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ARCH DAMS :

2.30. Siema Kotsubo (9)

Theoretical solution has been deduced on the dynamic water pressure on arch dams during earthquake, by which the dynamic water pressure is calculated for various cases of central angle of arch, U/S radius of dams and intersection angles subtended by both banks. These results have been testified by model ex-periments. Dynamic water pressure during irregular earthquake has also been evaluated. A three dimensional approach to the prblem has been evolved. Seima Kotsubo's results are quite different from Westergaards.

The ground features of dam sites are generally too complicated to obtain any precise theoretical solution. Kotsubo has derived solution for constant radius arch dams built in U shaped canyons situated symmetrically, which are very complicated. Fig. 22 shows the result of evaluation of the vertical distribution of dynamic water pressure at crown and abutment of the dam of $h = 100\,m$, $r_e = 50\,m$ $+ 2\theta_o = 90°$ with both banks in radial direction. It is seen from figure that the vertical distribution of dynamic water pressure is closely resembled to that of two dimensional dynamic water pressure in the vibration in the direction of river course while in a perpendicular direction, it is considerably smaller than the usual value from two-dimensional dynamic water pressure.

Figure 23 shows the horizontal distribution of dynamic water pressure at the bottom of dam with respect to various cases of U/S radius of dam, central angle of arch and intersection angle of both banks. From the figure, the dynamic water pressure is strikingly different from the usual value. For vibration in the

the direction of river course the minimum value occurs at crown increasing towards abutment and is independent of the intersection angle of both banks or central angle of arch. If the width of river is much more than height, the value approaches to usual value. For the vibration in perpendicular direction to the river course the dynamic water pressure is much smaller in the case of both banks in radial direction, and becomes negative with the decrease of intersection angle of both banks, and adds to the stability of dam against the earthquake in this direction.

In case of rectangular canyons, the theoretical value has been confirmed to correspond to that of model.

For a two - dimensional dynamic water pressure due to Irregular Earthquake (   ) the following solution, will be obtained after satisfying the boundry conditions.

$$p = \sum_{m=0}^{\infty} \frac{4\alpha w_0 (-1)^m \cos \lambda_m z}{(2m+1)\pi} \int_{\frac{x}{v}}^{t} \psi(t-\tau) J_0 \left(\lambda_m \sqrt{v^2\tau^2 - x^2}\right) d\tau \qquad \left(t > \frac{x}{v}\right) \quad (56)$$

where $\lambda_m = \frac{(2m+1)}{2h}\pi$, $v = \sqrt{\frac{gE_0}{w_0}}$.

and resonance period of dynamic water pressure will be obtained by equation

$$T_m = \frac{2\pi}{\lambda_m}\sqrt{\frac{\omega}{gE_0}} \qquad (57)$$

Figure 24 is an example of an earthquake records. In figure 25 is given the result of calculations made on the dynamic water pressure caused by such earthquakes and also shows that obtained from Westergaard (1). The following facts are observed.

1) The dynamic water pressure at a given time is not always proportional to the corresponding earthquake acceleration, but different in time of resonance and in magnitude accordingly to the relation between T_m and T .

2) For $h = 100m.$ and $75m.$ the maximum value of dynamic water pressure is considerably larger than that based on Westergaards.

3) When the $T > T_m$,the phase of dynamic water pressure is equivalent to that of earthquake accleration and when $T < T_m$, the dynamic water pressure is delayed from that of earthquake acceleration by 90^0 .

4) Even after the suspension of tremor, the dynamic water pressure is not reduced to zero all of a sudden, but gradually dimnishes.

5) In the whole dynamic water pressure, greater part of them is occupied by the pressure of primary mode of vibration.

6) From the Kotsubo(9)theory, the dynamic water pressure can also be obtained while $T = T_m$ and explains the phenomenon.

From the results of Kotsubo, the dynamic water pressure for the given section of canyon will be expressed approximately by the following equation for two dimensional case for a gravity or arch dam.

$$p = \alpha w h \sqrt{1 - \frac{y}{h}} \cdot \frac{4v}{\pi h} \int_0^t \psi(t-\tau)\, J\left(\frac{2\pi\tau}{T_1}\right) d\tau \qquad (58)$$

$$K = \frac{4v}{\pi h} \int_0^t \psi(t-\tau)\, J\left(\frac{2n\tau}{T_1}\right) d\tau$$

where, T_1 is the primary resonance period of given section and will be expressed approximately by the following formula

$$T_1 = T\sqrt{\frac{h_m}{h}} \qquad (59)$$

in which, h is the maximum depth of water of given section, h_m mean depth of water of the given section, T two - dimensional primary resonance period for the maximum depth and where K is the magnifying factor for irregular earthquake and can be taken about 1.5, though it will be clarified when numerous records of earthquake accn at dam site have been obtained.

When the earthquake period is shorter than the resonance period of dynamic water pressure, the dynamic water pressure is delayed from the phase of inertial force of dam body by 90^o, and the dynamic water pressure caused by the elastic displacement acting upon the dam as a kind of damping force. In such a case it seems that dams are comparatively safe.

For an arch dam it is almost impossible to explain theoretical magnitude of dynamic water pressure caused by irregular earthquake. Accordingly, if the results obtained by stationary vibration theory is adopted for dynamic water pressure as from eq. (58), the results from figure no. 26 can be applied to arch dam. It is, however, expected that if the intersection angles subtended by both banks increases, time function will be different, and we have to depend upon future studies.

2.31. Studying in detail all the theoretical and experimental methods of determination of hydrodynamic pressure due to earthquake on the U/S face of the dam, we find that in the consideration of stable equilibrium it has been assumed that the dam moves as a whole with foundation, but it is realised that due to the self-vibration of the dam itself under an impressed force, independent of the foundation, the procedure for determination of critical condition of equilibrium may not be fully representative of the entire phenomenon. In short one has to consider it as a dynamic case with a force varying with time at the base in addition to the forced vibrating motion of the dam itself. But for approximate value, without going into such complicacy, the above consideration holds good at least in case of overturning, where the maximum value of the force is taken into consideration. But in case of hydrodynamic effect on the upstream face of the dam, the vibration of the dam should be taken into consideration to evaluate the hydrodynamic pressure change on it instead of considering it to be moving as a whole without any deformation in itself. Westergaard and others have not considered this phenomeno, while considering the hydrodynamic pressure change on the upstream face due to the volume of reservoir water during seismic oscillations. It is realised that such a detailed consideration is not useful in the present case, as the profile has already been considered to be a strictly triangular , and also, the transverse vibration of the dam and consequent hydrodynamic pressure changes, when superimposed on the oscillation on the water mass of the reservoir, may be difficult to solve hydrodynamically. It is all the more physically probable that there may be considerable additional increase in the hydrodynamic pressure on the U/S face of the dam, when the mass of the water, and the body of the dam are not moving in phase, such an out of phase movement of the dam and the mass of water can be feasible only when self vibration of the dam is also considered over and above to and fro movements of the watermass and dam in phase with the foundation and will cause enormous increase in hydrodynamic loads. Westergaards(1) assumption though not physically sound from the conception of vibration of the dam is ignored, however, an exact theory is not possible due to complicacy involved. There seems to be fundamental draw back in the analysis of Jacobsens (5) analysis that C_n does not allow for infinite length of reservoir and compressibility of water.

2.32. For $l = \infty$, the analysis of Werner and Sundquist(3) and Westergaard (1) are same but both differ fundamentally from Jacobsen (5), Zangers(7) and Housner (8) as shown earlier due to neglecting of compressibility of water. This is incompleteness of later works.

2.33. In Werner and Sandquists(3) analysis, it is seen that, in the value of p and P, l appears and consequently in case of canyons, which do not have long straight fetch just immediate to U/S, the value of l , for practical purposes, may be taken to be equal to the amount of straight fetch, thus available, plus certain fraction of l depending on topography of the canyon and other fractions also. It is seen in case of Jacobsens(5) analysis that for small values of larger amount of hydrodynamic pressure appears during earthquake.

2.34. Westergaard(1), Jacobsen(5), Werner and Sandquist (3), Zanger worked with force of the nature of simple harmonic variation action on the foundation of the dam and reservoir. However, though Jacobsen, maintains an indeterminate nature of force functions $f(t)$ in his orginal paper yet it is realised that in case of force varying in a complicated manner, more of the analysis of can be relied upon because immediate response of the force in the systems of dam and immediate portion of water will commence. Thus it is easily visualised, that, in case of near earthquakes, whose vibrations are all of highly complicated nature, such simple picture of the effect of total earthquake force on the dam may be far from truth due to relative response.

2.35. The solutions of Westergaard(1) and Werner (3) have restricted to small value for displacements can not be easily seem to be applicable at the point of singularity: ($x=0, y=0$) but it can be neglected as it is in a small part in comparision with entire face concerned.

2.36. Figure No. 25 and 26 shows the distribution of hydrodynamic water pressure during earthquakes on dams with vertical U/S face due to all the available methods. A comparision of the available method can be made with the help of above figures.

2.37. Figure No. 28 and 29 shows a comparision of distribution of hydrodynamic water pressure during earthquakes on dams with sloping U/S face and between the experimental curves of Zangers (7) and analytical approximate solutions of Housner (8). Two curves compare very well.

2.38. Though the Zangers (7) experimental method for determination of hydrodynamic pressure on any profile of dam affords an easy and practical method but due to consideration of compressibility of water and infinite length of reservoir his results do not represent an actual case. Also the results of Zangers are independent of the vibration period of earthquake which is also incompleteness of his work. His results a-re comparable to Westergaards for $T = \frac{4}{3}$ secs. while for $T < \frac{4}{3}$ secs. the error in his results increases. This incompleteness of his work can be made good by considering the actual case and applying a correction factor to his result as his method only affords solution to actual profiles of dam.

2.39. Though Housner (8) results are based on approximate analytical solutions of Laplaces equation but affords an easy and convenient method for determination of hydrodynamic pressure on dams during earthquake, without much error as seen earlier and is recommended for all cases.

2.40. Westergaards(1) theory which is not faulty mathematically but also inapplicable in case when the period of earthquake is shorter than the resonance period of dynamic water. Seima Kotsubo, Werner and Sandquist (3), Tadeshi H atano studied these deficiencies to amend and obtain a more precise solution. As a result of it study on dynamic water pressure has progressed and the phenomena have been better understood . Seima Kotsubo(9) first presented a three dimensional solution to the constant radius arch dams and he has also considered the effect of irregular earthquakes and shape of canyon on dynamic water pressure.

2.41. From the above considerations, the theory proposed by Werner and Sundquist (3) seems to be better as far as hydrodynamic water pressure is concerned, than of Jacobsens(5), Westergaard(1), Zanger(7), Housner(8) etc. but for all practical purposes solutions of Housner are very simple and easy to be applied with maximum error less than 5 to 6% and recommended for all practical cases where above two methods are not applicable Zangers (7), experimental determination of pressure coefficient by Electric Analogy Tray apparatus affords a easy method and can be used or use of his experimental curves is proposed.

For all cases where boundry conditions do not agree with the assumptions of theory the results should appropriately be modified by multiplying by a connection factor.

A three dimensional solution of the problem is urged which will afford the various facts and characteristics of dynammic water pressure unknown so far.

2.42. Hydrodynamic Pressure in a finite reservoir.

Analysis given by Westergaard (1) has considered the reservoir of infinite length. If the reservoir is to be considered infinitely long the assumption that $p = 0$ when x is large or infinite is replaced by u=0 at x=l. At x =l an immovable vertical wall is considered, but in an actual case any surface at the far end of a reservoir would probably vibrate with the earth -quake so that a better assumption would be that both ends of the reservoir move with the same amplitude and period which can be represented mathematically by $u = -\frac{\alpha g}{4\pi} \cdot T^2 \cos \frac{2\pi t}{T}$ when x=l. Value for the correction factor to the total pressure P is given by Brahtz and Heilbron (14) for various $\frac{l}{h}$ ratios. A curve (Fig.No.30) has been plotted between correction factor and $\frac{l}{h}$ ratios. Correction factor is to be multiplied to the value calculated for a infinitely long reservoir. From the graph it is found that the error in assuming the reservoir infinitly long is negligible if the length of the reservoir is more than three times its depth. When l is small, the pressure reduces to $p = \frac{1}{2}\alpha wh$ representing a condition in an open well during an earth quake.

Jacobsen (5) and Sundquist (3) extended Westergaards (1) hypothetical idea of reservoir of infinite feteh to an actual case of bounded reservoir and found that for $\frac{l}{h} > 4$ the boundry of the reservoir has no effect on the total pressure exerted during hydrodynamic change. Most reservoir satisfy this, thus boundry effect is need not be emphasised. The effect is much ex -aggerated when $l < 4h$ and effect is three times as large as for

$l = \frac{h}{2}$ as for $l = \infty$. This consideration of a restricted reservoir is important in case of high dam in a gorge, like Bhakra and Kosi dams, which may not naturally have straight fetch for a considerable distance.

2.43. Influence of the side slopes of the reservoir on the hydrodynamic pressure on a dam during earthquake.

The crest length of Bhakra and Kosi dams is only slightly more than twice the maximum height and the hill sides are so inclined that the whole valley has a triangular shape. Thus the possible effects of the sloping sides on the hydrodynamic pressu-res on the face of the dam need be considered. In the case of Hoover dam (U.S.A.) Westergaard(1) found that weight of water vibrating with the dam during earthquake is equal to $\cdot 525 h^3$ while with average width is equal to $\cdot 583 h^3$. Thus it is evident that the effect on hydrodynamic pressure during earthquake due to sloping sides of canyons is to reduce it.

According to the results of model experiments of Seima Kotsubo (9) for constant angle arch dams, it has been revealed that the dynamic water pressure is not very different from that of rectangular section and that the horizontal distribution of dynamic water pressure approaches nearer to uniform distribution when the vibration is in the direction of river course, but extremely small in the variation when the direction is at right angle to the river so un necessary to calculate the stresses of arch dam in this case. As it is very difficult to get a theoretical solution for a given section of canyon, but from model experiment for triangular section, the magnitude of dynamic water pressure reduces to 70% of that in the case of rectangular section.

Following Werner and Sundquist(3), it can be shown that, considering a canyon to be roughly a semi circle in shape, the critical depth for resonance is greater than in case of a recta-angular section. Same observations were taken by Kotsubo (9). Thus, in a canyon, the possibility of resonance is less than in a rectangular reservoir of similar magnitude. But partial resonance may take place in a deep gorge if it is crossed by deeper faults on the upstream side.

PART TWO - CONTAINERS AND PIERS.

2.44. Other important engineering structures which are subjected to hydrodynamic pressures during earthquakes are tanks containing water or oil, sea shore, levees, piers and light house towers etc. The rectangular vessels with vertical or sloping walls can be treated as a dam with bounded reservoir and all the results and discussion applicable to rectangular dams are also applicable to rectangular vessels satisfying the required boundry condition and assumptions as taken in the analysis.

2.45. Both from a geophysical and from an engineering point of view it is of value to know the hydrodynamic earthquake effects on structures retaining water or inside water. Although, apparently, with no reference to earthquake action, the vibration of water vessels attracted the attention of scientists at least as early as in the middle of the ninteenth century. In a paper on waves, Rayleigh (1876) investigated, on the theory of velocity of potential, the waves occuring in certain enclosed water bodies due to vibrations. Rayleigh in his paper also referred to some practical tests, which showed an excellent agreement with the general theory with special reference to Earthquake shocks. Tests carried out by Hoskins and Jacobsen (5) (1954) shows a satisfactory verification of Westergaards(1) theory. In 1949, Jacobsen (2) presented a theoretical investigation pertaining to the inpulsive hydrodynamic pressure of an incompressible fluid inside a cylindrical tank and outside a cylindrical pier and he gave effective hydrodynamic masses and mass centroids for the above two cases. In the same year Werner and Sandquist (3) also gave solutions for hydrodynamic pressure in the case of semicircular section, triangular section, the walls sloping 45°, vertical cylinder, cylindrical pier and semi-spherical vessel. In 1951, Jacobsen and Ayre (6) gave experimental results from 6" to 4 ft. diameter tanks, subjected to transient horizontal ground motion. Housner (7) in 1957 gave simplified formulae for containers having two-fold symmetry like cylindrical and elliptical tank. In 1951, Graham and Rodriguez (4) gave simple mechanical systems equivalent to the fluid. These spring mass systems respond to motions of the tanks walls in the same fashion as the liquid, producing indentical forces and moments. The use of such mechanical analogies should simplify in many cases the analysis of the complete dynamic system.

The tanks and cylindrical pier will be discussed here in detail.

2.46. Lydik S Jacobsen (2)

To evaluate the hydrodynamic mass and overturning effect on partly filled cylindrical tanks as well as cylindrical piers surrounded by water, Jacobsens analysis is based on the following assumptions:

(i) The tanks and piers experiences translatory, impulsive but unspecified ground displacements in hotizontal direction only.

(ii) The circular, cylindrical boundries of the tanks and piers are not deformed as a consequence of their motion.

(iii) The fluid is incompressible and non-viscous.

(iv) The fluid motion generated by the impulsive motion of the boundries is treated by simplifying the hydrodynamic equations so as to exclude the effect of gravity waves subsequently set up.

In determining the effect of an earthquake on the increase in the water pressure, only the horizontal component is considered which gives the worst effect and it is taken that it is independent of the earthquake duration. This assumption is supported by the experimental verification for a cylindrical tank of diameter of 23.12 inches, subjected to an impulsive, horizontal acceleration of maximum value of 46 inches/sec/sec.(12% of g).Fig. 31 shows the theoretical curves and the experimental curves for moments and effective hydrodynamic masses. The agreement between the theory and the experiment is satisfactory.

The boundaries of the circular or cylindrical tanks and piers are taken rigid so that they are not deformed as a consequence of their motion. Flexibility of the boundries of such types of vessels have not been accounted in any of the work except for a vertical rectangular face. A little ellipticity changes the pattern of wave formation in the tank.

Jacobsen's(2) analysis is based on the incompressibility of fluid. The compressibility of fluid does not effect the results appreciably for tanks of usual shapes and heights as seen in the case of dams. Werner and Sandquist(3) has considered this effect.

The viscosity of the fluid has not been considered for the hydrodynamic pressures during earthquake on dams and containers upto this time. However it will be conservative due to the drag between the particles, not to account it. Changes in the pressure due to this account has not yet been known. Water and other light oils like petroleum etc. can be taken non-viscous as their viscosity is very small. In the case of dense oil storage taks the study should be made for viscous effect.

In the analysis, the change in the formation of waves on account of gravity has not been considered. Experimental observation supports this fact as the observed wave height is very insignificant and change in the results if any, on this account will be very small.

For an incompressible fluid, tank must satisfy the following Laplacian equation in polar coordinates,

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2\phi}{\partial\theta^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \qquad (60)$$

where ϕ is the velocity potential. When the tank is subjected to small impulsive translation in the x-direction, the fluid initially at rest will be set into motion by the tank. The impulsive translation of the cylindrical boundry is given by an unspecified function of time.

$$x = f(t) \qquad (61)$$

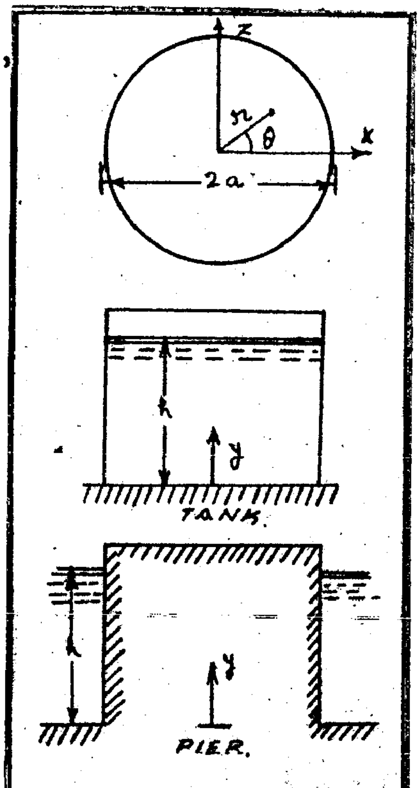


FIG. NO. 52.

To obtain a solution of the above equation the following boundry conditions should be satisfied :-

(i) at $y = 0$, vertical velocity w, $\left(\frac{\partial\phi}{\partial y}\right)_{y=0} = w_{y=0}$

(ii) at $y = h$, $\phi = 0$ (62)

(iii) when $r = a$, the translatory velocity u in the x-direction must be equal to $f'(t)$, consequently the radial velocity component

$$\left(\frac{\partial\phi}{\partial r}\right)_{r=a} = f'(t)\cos\theta \qquad (63)$$

Taking $\phi = R\Theta Y f(t)$ as the solution of the above equation, where Y and Θ are functions of velocity potential and is given by

$$Y = \cos\frac{n\pi}{2h}y \qquad (64).$$

and $\Theta = \cos\theta$

Therefore $\phi = R \cos\theta \cos \frac{n\pi}{2h} y . f'(t)$

From equation (60)

$$\left[\frac{\partial^2 R}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial R}{\partial r} - \left(\frac{n^2\pi^2}{4h^2} + \frac{1}{r^2} \right) R \right] \cos\theta \cos \frac{n\pi}{2h} y . f'(t) = 0 \qquad (65)$$

For any value of θ

$$\frac{\partial^2 R}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial R}{\partial r} - \left(\frac{n^2\pi^2}{4h^2} + \frac{1}{r^2} \right) R = 0 \qquad (66)$$

which can be reduced to a Bessel Equation of first order having a solution for

$$R = A_n J_1 \left(i \frac{n\pi}{2h} r \right) + B_n Y_1 \left(i \frac{n\pi}{2h} r \right) \qquad (67)$$

Therefore inside a tank

$$\phi_1 = f'(t) \cos\theta \sum_{1,3,5\ldots}^{\infty} \cos \frac{n\pi}{2h} y . A_n J_1 \left(i \frac{n\pi}{2h} r \right) \qquad (68)$$

and outside a pier

$$\phi_2 = f'(t) \cos\theta \sum_{1,3,5\ldots}^{\infty} \cos \frac{n\pi}{2h} y . B_n Y_1 \left(i \frac{n\pi}{2h} r \right) \qquad (69)$$

Considering only the impulse part of the acceleration for a small interval

$$p = - \frac{w}{g} \cdot \frac{\partial \phi}{\partial t} \qquad (70)$$

Therefore, the hydrodynamic pressure inside a tank

$$p_1 = - \frac{w}{g} f''(t) \cos\theta \sum_{1,3,5\ldots}^{\infty} \cos \frac{n\pi}{2h} y . A_n J_1 \left(i \frac{n\pi}{2h} r \right) \qquad (71)$$

and outside a pier

$$p_2 = - \frac{w}{g} f''(t) \cos\theta \sum_{1,3,5\ldots}^{\infty} \cos \frac{n\pi}{2h} y . B_n Y_1 \left(i \frac{n\pi}{2h} r \right) \qquad (72)$$

Where
$$A_n = \frac{\frac{8h}{n^2\pi^2}}{J_0 \left(i \frac{n\pi}{2h} a \right) - \frac{2h}{n\pi a} \cdot J_1 \left(i \frac{n\pi}{2h} a \right)} \qquad (73)$$

A_n is +ve for $n = 1, 5, 9 \ldots$

A_n is −ve for $n = 3, 7, 11 \ldots$

and
$$B_n = \frac{\frac{8h}{n^2\pi^2}}{Y_0 \left(i \frac{n\pi}{2h} a \right) + \frac{2h}{n\pi a} Y_1 \left(i \frac{n\pi}{2h} a \right)} \qquad (74)$$

B_n is +ve for $n = 1, 5, 9 \ldots$

B_n is −ve for $n = 3, 7, 11 \ldots$

The total hydrodynamic forces acting in the direction of x

$$P = 2\int_0^y \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (p)_{r=a} \cos\theta \, r \, d\theta \, dy \qquad (75)$$

which gives

$$P_1 = -\frac{w}{g} f''(t)\, ha\left[A_1 J_1\left(i\frac{\pi a}{2h}\right) - \frac{1}{3} A_3 J_1\left(i\frac{3\pi a}{2h}\right) + \frac{1}{5} A_5 J_1\left(i\frac{5\pi a}{2h}\right) - \cdots\right] \qquad (76)$$

and,

$$P_2 = -\frac{w}{g} f''(t)\, ha\left[B_1 Y_1\left(i\frac{\pi a}{2h}\right) - \frac{1}{3} B_3 Y_1\left(i\frac{3\pi a}{2h}\right) + \frac{1}{5} B_5 Y_1\left(i\frac{5\pi a}{2h}\right) - \cdots\right] \qquad (77)$$

The centioids of these forces are located $\bar{y}_1$ and $\bar{y}_2$ above xz plane.

$$\bar{y} = \frac{2}{P}\int_0^y \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} (p)_{r=a} \cos\theta \, r \, d\theta \, dy \qquad (78)$$

The moments of the hydrodynamic impulsive pressure $p_{y=0}$ about the yz planeare given by

$$N_1 = 2\int_0^r \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (-p_1)_{y=0}\, r^2 \cos\theta \, d\theta \, dr \qquad (79)$$

and,

$$N_2 = 2\int_r^{\infty} \int_{-\pi/2}^{\pi/2} (p_2)_{y=0}\, r^2 \cos\theta \, d\theta \, dr \qquad (80)$$

and total hydrodynamic impulsive moments will be given by $M_1 + N_1$ for tank filled with and M_2 for cylindrical pier, the moment $-N_2$ acting on the xz planeoutside the tank, being left out of consideration.

Following figures illustrate the works of Jacobsen. (i) figure no. 33 shows the ratios of the effective hydrodynamic masses m_1, m_2 referred to the total mass of the fluid within a tank or outside the tank, dividing equation (76) and (77) by $\frac{w}{g} f''(t)\, \pi^2 a^2 h^2$.

(ii) Figure No. 34 shows the ratios of the effective hydrodynamic mass centroids referred to h.

(iii) Figure no. 35 shows the ratio of the effective mass moments acting on the bottom of tank, obtained by dividing equation (79) and (80) by $-\frac{w}{g} f''(t)\, a^2 h^2$.

(iv) Figure no. 31 shows the theoretical and the experimental curves for cylindrical tank of diameter a = 11.56" and maximum horizontal acceleration 1/2 g which shows a close agreement between theory and experiment.

2.47. <u>Werner and Sundquist.</u>(3)

Werner and Sundquist theory has been developed assuming simple harmonic motion, taking the compressibility of the water into consideration and finite boundry for tanks or reservoir. The solutions given for rectangular sections under the head of dams are also applicable to the rectangular tanks.

For tanks withsemi-circular ends, there may be two modes of vibration

I. <u>Circular section vibrating in the direction of generating axis.</u>

The dynamic water pressure on end wall will be

$$p = \frac{8}{\pi} \alpha w r_0 \cos \frac{2\pi t}{T} \cdot \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \frac{a_{mn} \cosh \chi x - \cosh \chi (l - x)}{\lambda (2n+1) \tau_m \sinh \chi l} \cdot J_{2n+1}(\beta_m r) \sin (2n+1) \varphi \quad (81)$$

where

$$a_{mn} = \frac{\int_0^{\tau_m} \tau . J_{2n+1}(\tau) d\tau}{\left[\tau_m^2 - (2n+1)^2\right] J_{2n+1}^2(\tau_m)}$$

$$\lambda = \sqrt{1 - \left(\frac{2\pi r_0}{\tau K T}\right)^2}$$

$$\chi = \lambda \frac{\tau}{r_0} = \lambda \beta \quad \text{and} \quad K = \sqrt{\frac{gE}{w}}$$

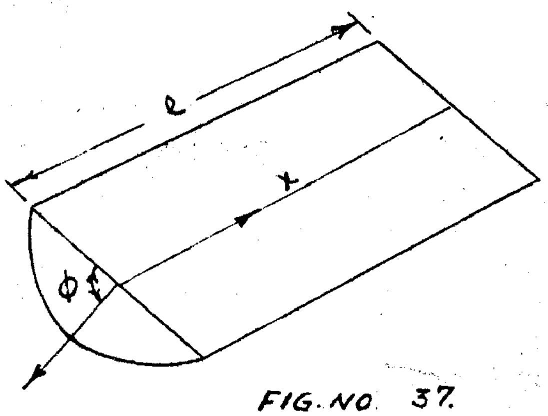


FIG. NO 37.

The dynamic water pressure on the end wall, calculated according to eqn. (81) for $\frac{l}{r_0} = \infty$ and with $\lambda = 1$ is shown in fig. 38.

II <u>Vibrating in a direction transverse to the generating</u>

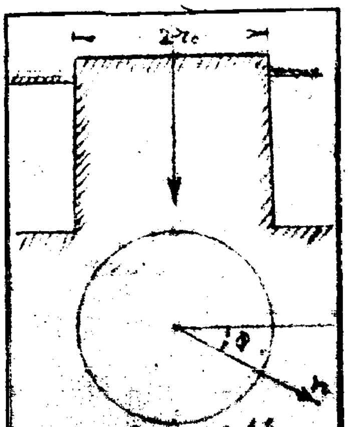


FIG. No 45

$$p = -\frac{8h}{\pi^2}\alpha\omega\cos\varphi\cos\frac{2\pi t}{T}\cdot\sum_0^\infty\frac{1}{\lambda(2n+1)^2}\cdot\frac{K_1(\chi r)}{K_1'(\chi r_0)}\sin(2n+1)\frac{\pi y}{2h} \quad (86)$$

and the resultant dynamic pressure is given by

$$P = \pm\frac{8h^2 r_0}{\pi^2}\alpha\omega\cos\frac{2\pi t}{T}\sum_0^\infty\frac{1}{\lambda(2n+1)^3}\cdot\frac{K_1(\chi r_0)}{K_1'(\chi r_0)} \quad (87)$$

where χ & λ are same as in the previous case of vertical cylinder and K is the Kankel function of purely imaginary argument.

Equations (86) & (87) have been evaluated for $\lambda = 1$ in fig. (46) & (47).

In a semi spherical vessel vibrating in a dirdction $\phi = 0$, the dynamic water pressure is given by

$$p = \alpha\omega r_0\cos\varphi\cos\frac{2\pi t}{T}\cdot\sum_1^\infty a_n\frac{4n+1}{8n^2(2n+1)}\cdot\left(\frac{r}{r_0}\right)^{2n}\cdot P_{2n}^{(1)}\cos\psi \quad (88)$$

where

$$a_n = \int_0^\pi P_n^{(1)}(\cos\psi)\cdot\cos\psi\sin\psi\,d\psi$$

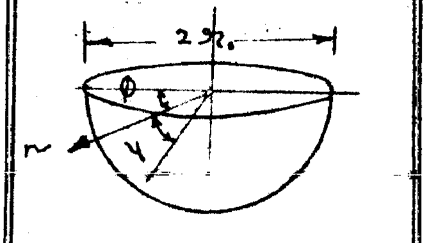


FIG. No 48

and $P_n^{(1)}$ is spherical surface harmonic function of first order of degree n. Figure No. (49) shows the dynamic pressure on the wall from e.g., (88). A comparision with figure No. (38) shows that the pressure in a spherical vessel is somewhat lower than in a circular one.

2.48. Jacobsen and Ayre (6)

Jacobsen and Ayre (6) carried out experimental investigation on four tanks, from 4" to 4 ft. in diameter, subjected to transient hotozontal "ground motions" of simplified type. The important parameters, in addition to size of tank, were depth of fluid and frequency, duration and amplitude of ground motion. The

(iv) Figure no. 37 shows the theoretical and the experimental curves for cylindrical tank of diameter a = 11.56" and maximum horizontal acceleration k_{g} which shows a close agreement between theory and experiment.

2.47. Werner and Sundquist.(3)

Werner and Sundquist theory has been developed assuming simple harmonic motion, taking the compressibility of the water into consideration and finite boundry for tanks or reservoir. The solutions given for rectangular sections under the head of dams are also applicable to the rectangular tanks.

For tanks withsemi-circular ends, there may be two modes of vibration

I. Circular section vibrating in the direction of generating axis.

The dynamic water pressure on end wall will be

$$p = \frac{8}{\pi} \alpha w r_0 \cos\frac{2\pi t}{T} \cdot \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} \frac{a_{mn} \cosh \chi x - \cosh \chi (l-x)}{\lambda(2n+1)\tau_m \sinh \chi l} \cdot J_{2n+1}(\beta_m r) \sin(2n+1)\varphi \qquad (81)$$

where

$$a_{mn} = \frac{\int_0^{\tau_m} \tau . J_{2n+1}(\tau)d\tau}{[\tau_m^2 - (2n+1)^2]\, J^2_{2n+1}(\tau_m)}$$

$$\lambda = \sqrt{1-\left(\frac{2\pi r_0}{\tau kT}\right)^2}$$

$$\chi = \lambda\frac{\tau}{r_0} = \lambda\beta \quad \text{and } k = \sqrt{\frac{gE}{w}}$$

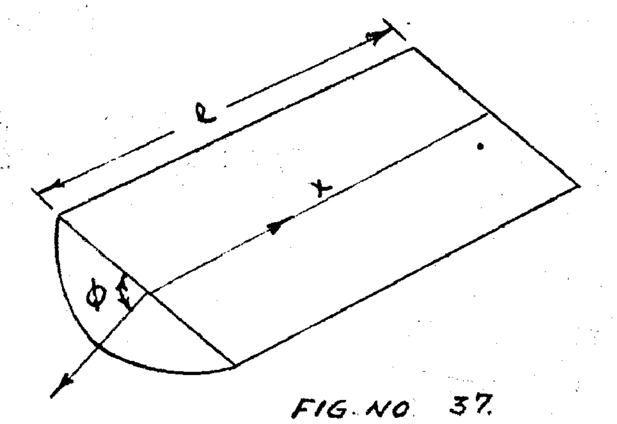


FIG. NO 37

The dynamic water pressure on the end wall, calculated according to eqn. (81) for $\frac{l}{r_0} = \infty$ and with $\lambda = 1$ is shown in fig. 38.

II Vibrating in a direction transverse to the generating

<u>axis</u> :

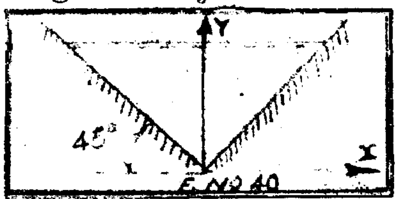
The dynamic water pressure on the wall is given by

$$p = \frac{4}{\pi}\alpha w r_0 \cos\frac{2\pi t}{T}\sum_1^\infty \frac{(-1)^{n-1}}{4n^2-1}\left(\frac{r}{r_0}\right)^{2n}\sin 2n\varphi \qquad (82)$$

Figure No. (39) shows the distribution of the hydrodynamic pressure.

For a case with triangular section, the walls sloping 45^0, vibrating in a transverse direction, p is given by

$$p = \alpha w h \cos\frac{2\pi t}{T}\left[\frac{x}{h} - \frac{2}{\pi}\sum_1^\infty \frac{(-1)^{n-1}}{n\sinh n\pi}\cdot\left\{\sinh\frac{n\pi x}{h}\sin n\pi\frac{y}{h} + \sin\frac{n\pi x}{h} - \sin n\pi\frac{y}{h}\right\}\right] \qquad (83)$$



F. NO 40

Figure no. 41 shows the distribution of dynamic pressure on wall.

On a vertical cylinder of circular cross section the dynamic water pressure p is given by

$$p = \frac{8h}{\pi^2}\alpha w\cos\varphi\cos\frac{2\pi t}{T}\sum_0^\infty\frac{1}{\lambda(2n+1)^2}\cdot\frac{I_1(\chi r)}{I_1'(\chi r_0)}\sin(2n+1)\frac{\pi y}{2h} \qquad (84)$$

where $I_1(\chi r)$ is a Bessel function of purely imaginary argument, in which

$$\chi = \lambda(2n+1)\frac{\pi}{2h}$$

and $$\lambda = \sqrt{1-\left\{\frac{4h}{(2n+1)KT}\right\}^2}$$

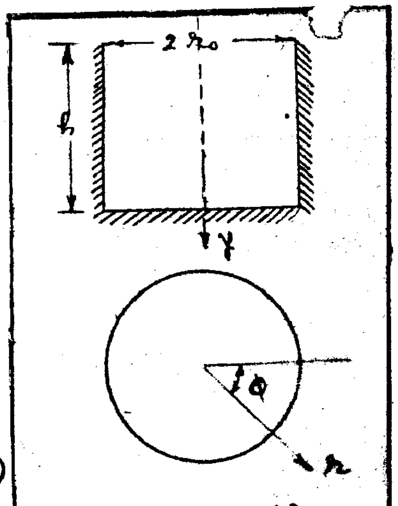


FIG. NO. 42

The resultant dynamic water pressure on wall is

$$P = \pm\frac{8h^2}{\pi^2}r_0\alpha w\cos\frac{2\pi t}{T}\sum_0^\infty\frac{1}{\lambda(2n+1)^3}\cdot\frac{I_1(\chi r_0)}{I_1'(\chi r_0)} \qquad (85)$$

Equations (84)+(85) have been evaluated in figure (43) and (44).

In the case of vertical pier of circular cross section vibrating in a reservoir infinitely extended in a horizontal plane, the dynamic pressure is given by

$$p = -\frac{8h}{\pi^2}\,\alpha w \cos\varphi \cos\frac{2\pi t}{T}\,.\sum_0^{\infty}\frac{1}{\lambda(2n+1)^2}\cdot\frac{K_1(\chi r)}{K_1'(\chi r_0)}\sin(2n+1)\frac{\pi y}{2h} \qquad (86)$$

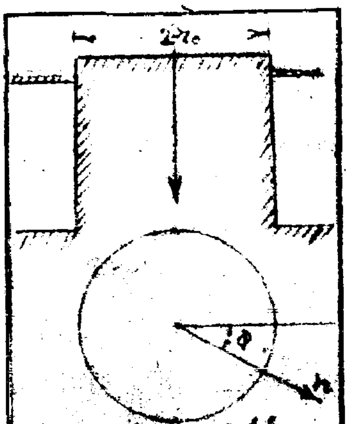


FIG. NO 45

and the resultant dynamic pressure is given by

$$P = \pm\frac{8h^2 r_0}{\pi^2}\,\alpha w \cos\frac{2\pi t}{T}\sum_0^{\infty}\frac{1}{\lambda(2n+1)^3}\cdot\frac{K_1(\chi r_0)}{K_1'(\chi r_0)} \qquad (87)$$

where χ + λ are same as in the previous case of vertical cylinder and K is the Kankel function of purely imaginary argument.

Equations (86) + (87) have been evaluated for $\lambda = 1$ in fig. (46) + (47).

In a semi spherical vessel vibrating in a direction $\phi = 0$, the dynamic water pressure is given by

$$p = \alpha w r_0 \cos\varphi \cos\frac{2\pi t}{T}\,.\sum_1^{\infty} a_n \frac{4n+1}{8n^2(2n+1)}\cdot\left(\frac{r}{r_0}\right)^{2n}\cdot P_{2n}^{(1)}\cos\psi \qquad (88)$$

where

$$a_n = \int_0^{\pi} P_n^{(1)}(\cos\psi)\,.\cos\psi\,\sin\psi\,d\psi$$

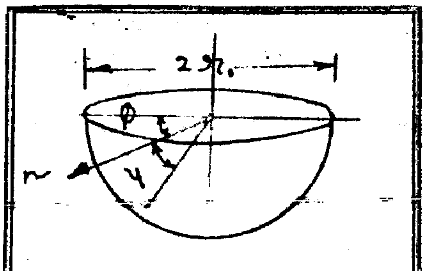


FIG. NO 48

and $P_n^{(1)}$ is spherical surface harmonic function of first order of degree n. Figure No. (49) shows the dynamic pressure on the wall from e.g., (88). A comparision with figure No. (38) shows that the pressure in a spherical vessel is somewhat lower than in a circular one.

2.48. Jacobsen and Ayre (6)

Jacobsen and Ayre (6) carried out experimental investigation on four tanks, from 4" to 4 ft. in diameter, subjected to transient hotozontal "ground motions" of simplified type. The important parameters, in addition to size of tank, were depth of fluid and frequency, duration and amplitude of ground motion. The tanks have taken as elastically supported. This fact has been supported from the experimental work of Aurther C-Ruge, with the result that a coupling exists between the fluid system and the tan-k tower system. The existence of this coupling has been demonstrated on a full scale basis in experiments conducted by D.S.Carter, on actual tanks. The photographic studies had been made on wave formation, maximum wave heights and the locations of these maxima and the fluid damping coefficients. Equivalent mass and overturning moment due to the fluid have been shown for various cases. The tanks may be either covered or open. In case of open tank of diameter $2a$, for the effective mass and overturning moment due to the behaviour of the fluid, the aggrement between theory and experiment is good. For practical purposes the ratio of effective hydrodynamic mass to actual mass of the fluid can be calculated within 5% of error from the following simple relations:

$$\left.\begin{aligned} \frac{m_1}{m} &= 0{\cdot}57\,\frac{h}{a}, \quad \text{for } 0 < \frac{h}{a} < 1 \\ \frac{m_1}{m} &= 0{\cdot}39 + {\cdot}18\,\frac{h}{a}, \quad \text{for } 1 < \frac{h}{a} < 2{\cdot}5 \end{aligned}\right\} \qquad (89)$$

The centre of gravity of effective hydrodynamic mass is given by same degree of approximation by

$$\frac{\bar{z}_1}{h} = 0{\cdot}36 + {\cdot}027\sqrt{\frac{h}{a}} \quad \text{from the bottom}$$

of tank.

In the case of tanks with rigid cover the removal of a small proportion of the fluid makes the tank effectively an open one. The overturning moments reduces even more rapidly, than the effective mass does, since it is function not only of the effective mass but also of the effective centroidal hieght. It has been concluded that wave studies are applicable to fluids other than water as

(i) Natural modes are independent of fluid density.

(2) Viscous damping is small and,

(3) Surface tension effects are secondary.

The effect of static fluid depth is a complicated one and comparision of tanks of different sizes should be made on equal values of $\frac{h}{a}$. The maximum wave height can, for most practical purpose, be assumed to vary as the square root of tank diameter $2a$. Well defined resonance occurs only with the fundamental mode. The higher natural frequencies are closely spaced, with the result that very sharp tuning is required for resonance. Waves of significant height apparently are not to be expected in large tanks subjected to earthquake on the otherhand, large waves may occur in small tanks when subjected to

(a) large amplitudes motions near the top of building, or

(b) large motions arising from elastic support of the tank on a moving ground (elevated storage tanks).

Figure No. (50)+(51) shows the influence of rigid cover on the effective hydrodynamic mass of the fluid and overturning moments due to fluid respectively. The maximum value of impulsive acc^n was 0.12 g and for full tank $\frac{h}{a} = 1.5$.

From an example of a rigid tank of 120' inside diameter, containing water to a depth of 90 ft., subjected to horizontal of 0.20 of -

(i) For open tank:-

Weight of water (mg) $= 63.6 \times 10^6$ lbs.

$$\bar{z}_1 = .393\,h$$
$$= 35.4'$$

Hydrodynamic couple acting at the bottom of tank from the curve figure No. (50)

$$= 62.5 \times .20 \times 60^2 \times 90^2 \times .39$$

$$= 142 \times 10^6 \text{ lb.ft.}$$

$$\therefore M_1 = .682 \times 63.6 \times 10^6 \times .20 \times 35.4$$

$$= 307 \times 10^6$$

due to horizontal force

$$\therefore \text{Total Moment} = 449 \times 10^6 \text{ lb.ft.}$$

which is 59% of the hydrostatic pressure.

(ii) For covered tanks

$$\frac{m_1}{m_2} = 1 \quad \text{and } \bar{Z} = .5h = 45'$$

$$\therefore M = 63.6 \times 10^6 \times .20 \times 45 = 571 \times 10^6 \text{ lb.ft.}$$

This shows that open tanks have smaller moments. Clearance required to reduce the effective mass to the open tank it was found that for this case only 10" clearance is needed which is very small in relation to the dimensions of the tank. If the tank has been disigned as shallow tank with $\frac{h}{a} = 0.5$, total moments would have much smaller.

For this case of design $2a = 173'$ and $h = 43.3'$

$$\frac{m_1}{m_2} = .305 \ , \ \bar{Z} = .38h = 16.5'$$

$$\therefore M_1 = 63.6 \times 10^6 \times .305 \times .20 \times 16.5$$

$$= 64 \times 10^6 \text{ lb.ft.}$$

couple at bottom :-

$$N = 62.5 \times .20 \times 86.5^2 \times 43.3^2 \times 1.1$$

$$= 19.3 \times 10^6 \text{ lb.ft.}$$

$$\therefore \text{Total moment} = 83.3 \times 10^6 \text{ lb.ft.}$$

which shows a great reduction in total hydrodynamic forces resulting from the change to a tank of shallower design.

Therefore, for counter acting of hydrodynamic pressure moments, shallower designs of tanks, should be done.

2.49. H.W.Housner (8)

Housner presented simplified solutions for hydrodynamic pressure during earthquake for various shapes of liquid containers. His solutions are based on the assumption that the water is incompressible and the fluid displacements are small, therefore finding the solution of Laplaces equation, satisfying the boundry conditions. Based on the known solutions he derived simplified approximate solutions within permissible errors, otherwise the solutions are very complicated as we saw above. These solutions avoid partial differential equations and infinite series and are in very simple form. The approximate method appeals to physical intuition and makes it easy to visualize the fluid motion and therefore particularly suitable for engineering applications. Housner seperated hydrodynamic pressure into two parts, (1) Convective, and (11) impulsive. The impulsive pressures are caused by inertia forces and are directly proportional to the acceleration of the container walls while convective pressures, are produced by oscillation of the fluid and thus results into impulsive pressure.

Considering a cylindrical tank as shown in figure No.52, subjected to a horizontal acceleration αg and let the fluid be constrained between fixed membranes parallel to the x-axis. Jacobsen(1949)(2), has shown that an impulsive acceleration does not generate a velocity component in the fluid so that in this case the membranes do not introduce any constraint. Each slice may thus be treated as if it were a narrow rectangular tank. The pre-ssure exerted against the wall of the tank is given by equation

$$p = -\alpha w h\left[\frac{y}{h}-\frac{1}{2}\left(\frac{y}{h}\right)^2\right]\sqrt{3}\tanh\left(\sqrt{3}\frac{R}{h}\cos\phi\right) \quad (91)$$

The pressure on the bottom of the tank is

$$p_b = -\alpha w h \frac{\sqrt{3}}{2}\frac{\sinh\sqrt{3}\frac{x}{h}}{\cosh\sqrt{3}\,R/h} \quad (92)$$

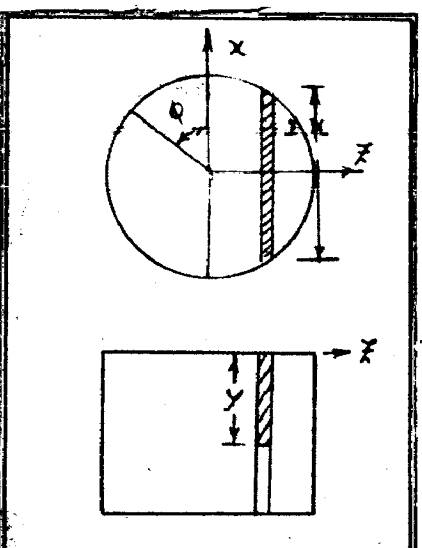


FIG NO. 52.

The above expressions are not convenient for calculating the total force exerted b-y the fluid. The following modifications gives very accurate values for $\frac{R}{h}$ small and som-ewhat overestimates the pressure when $\frac{R}{h}$ is not small.

$$p = -\alpha w h\left[\frac{y}{h}-\frac{1}{2}\left(\frac{y}{h}\right)^2\right]\sqrt{3}\cos\phi\tanh\sqrt{3}\frac{R}{h} \quad (93)$$

and resultant pressure exerted on the wall

$$P = \int_0^h\int_0^{2\pi} p\cos\phi\, R\,d\phi\,dy = -\alpha w\pi R^2 h\frac{\tanh\sqrt{3}\frac{R}{h}}{\sqrt{3}\frac{R}{h}} \quad (94)$$

from which it is seen that the force exerted is the same as if an equivalent mass M_o were moving with the tank, where

$$M_o = M\frac{\tanh\sqrt{3}\frac{R}{h}}{\sqrt{3}\frac{R}{h}} \quad (95)$$

Comparing with Jacobsen (1949) it is found that eq.95 overestimates M o with a maximum error less than 4%.

To exert a moment equal to that exerted by the fluid pressure on the wall, the mass M o should be at a height above the bottom

$$h_o = \frac{3}{8}h \quad \left(\frac{h}{R} \lesssim 1\cdot 5\right) \quad (96)$$

If the moment exerted by the pressures on the tank bottom are included, the equivalent mass, Mo, must be at a height

$$h_o = \frac{3}{8}h\left[1+\frac{4}{3}\left(\frac{\sqrt{3}\frac{R}{h}}{\tanh\sqrt{3}\frac{R}{h}}\right)\right] \quad \left(\frac{h}{R} \lesssim 1\cdot 5\right) \quad (97)$$

to produce the proper total moment on the tank which underestim-ates h_o with Jacobsen(1949)with a maximum error of 6%.

For the case of elliptical tank and composite tanks, the impulsive pressure on the wall is given by equation by proceeding in the same way as for the cylindrical tank

$$p = \alpha w h \left[\frac{y}{h} - \frac{1}{2}\left(\frac{y}{h}\right)^2 \right] \sqrt{3} \tanh \sqrt{3} \frac{l}{h} \qquad (98)$$

where l is the length of tank.

Similar expressions can be obtained for acceleration in other direction.

Ray W. Clough (20)

2.50. The dynamic system in contact with fluid give rise to increase in the apparent mass of the system. The hydrodynamic equations required to evaluate this added mass effect is very complex and phenomenon can be explained on energy principles. The problem of added mass has been evaluated for various cases in various field for small amplitude harmonic motions. However, there is considerable doubt that these results may be applied indiscriminately to the earthquake response of structures under water. The specific objectives of Clough's study were to determine added mass values pertinent to simple, prismatic forms of structures considering the effects of

(1) irregular earthquakes

(2) Flexural deformation of the structure.

Two series of models were tested on shaking table. For details of model and apparatus reference can be made to second world conference on earthquake Engg. Vol.II, Japan 1960, p. 815 and results are reported here.

Test series I.

In this series values of the added mass coefficients for rigid bodiesunder water were obtained. For a rigid circular section of infinite length, hydrodynamic theory gives a value of unity for the added mass coefficient, i.e., the added mass is equal to the weight displaced by the model. Resilts show good agreement with theory. It is difficult to get added mass coefficient for non circular section. The results shows that the added mass values obtained by free vibration studies were consequently lower than the values obtained from the earthquake tests by several % . Difference can be ignored and coefficient can be used. The damping effects of water are quite negligible.

Test series II.

In this series results were obtained in reduction of added mass coefficients where a flexible vertical model replaced the rigid horizontal model. The reduction may be due to three dimension flow pattern in case of flexible models and two-dimension in the rigid case. In some cases this reduction may be as great as 30%. Agreement with theretical and experimental values of bending moments obtained in the vertical model is satisfactory.

CHAPTER III.

EXPERIMENTAL WORK.

3.1 The experimental work consists of varification of Zanger's (7) experimental work on an electrolytic analogy tray for determination of hydrodynamic pressures on dam surfaces of various profiles during earthquakes, by solving Laplaces equation, $\nabla^2\phi = 0$.

3.2 THEORY AND ASSUMPTIONS :-

(i) The dam body is straight along its length.

(ii) The water is incompressible.

(iii) The vibrations in the earthquakes are assumed horizontal in a direction perpendicular to the dam.

(iv) All points of the foundation have the same displacement, velocity and acceleration at the same time.

(v) The displacement of a particle of water (u, v) in the directions of x and y respectively are small.

(vi) The dam is subjected to a simple harmonic motion during the earthquakes.

(vii) The reservoir is of infinite length.

(viii) The effect of hill side slopes in the case of canyon has not been considered.

(ix) There is no faulting in the case of canyon.

(x) The dam wall is rigid.

All the above assumptions have been discussed under Westergaard (1).

First, third and ninth assumptions make the problem two dimensional. If the second assumption, i.e. water is incompressible, is assumed, an electric analogy may be used to determine the magnitude and distribution of water pressure on a dam, of any profile which are caused by a horizontal earthquake. This assumption is not conservative, however a comparision has been made previously.

When the compressibility of water is considered in the hydrodynamic effect of horizontal earthquake, it is convenient to assume that the earthquake manifests itself a harmonic motion. Analytical solutions are also based upon the assumption that the dam is a rigid wall which moves a unit with the foundation. The di-splacement are assumed to be small and may be expressed by

$$\epsilon_0 = -\frac{\alpha g T^2}{4\pi^2} \cos \frac{2\pi t}{T} \qquad (99)$$

Assuming again that the displacements of the water body are small, the differential equation in rectangular corrdinates expressing the relationships of pressure p, time t, and three orthoganel displacements, u, v and s are

$$\left.\begin{aligned} \frac{\partial p}{\partial x} &= \frac{w}{g} \cdot \frac{\partial^2 u}{\partial t^2} \\ \frac{\partial p}{\partial y} &= \frac{w}{g} \cdot \frac{\partial^2 v}{\partial t^2} \\ \frac{\partial p}{\partial z} &= \frac{w}{g} \cdot \frac{\partial^2 s}{\partial t^2} \end{aligned}\right\} \qquad (100)$$

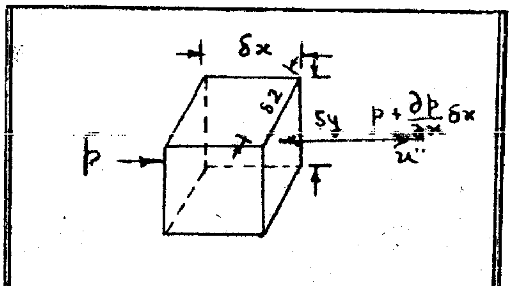


FIG. NO. 53.

with these assumptions for a compressible fluid the conditions of continuity are given by the equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial s}{\partial z} = \frac{p}{E} \qquad (101)$$

From equations (100) & (101), the following differential equation for the pressure in three dimensional flow is obtained.

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z} = \frac{1}{K^2} \cdot \frac{\partial^2 p}{\partial t^2} \qquad (102).$$

For two dimensional flow the problem resolves itself into determining solution for the differential equations (101) and (102) which also satisfy the boundry conditions. The general conditions to be met at any boundry may be written after consideration of fig. (54). For the two dimensional case, the displacement at the face are

$$\left.\begin{aligned} u &= \epsilon_0 - \bar{u} \\ v &= \bar{V} \end{aligned}\right] \quad (103).$$

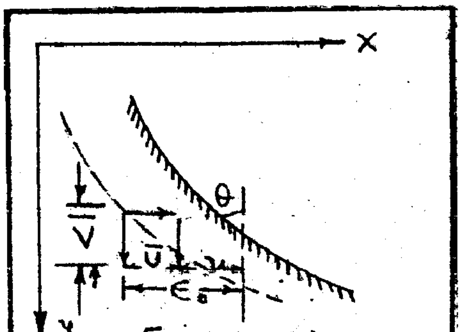


FIG. NO. 54.

The top indices refers to the movement of the water, relative to the dam. The displacement component perpendicular to any point on the face of the dam must be zero since the face is a streamline. Therefore, the following equation may be written:

$$u + v \tan \theta = \epsilon_0 \quad (104).$$

If water is considered as incompressible, E and hence K becomes infinite. Therefore, for two dimensional flow and incom-pressible fluid (Basis on which electric analogy solution is ba-sed), equation (102) then becomes.

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0 \quad (105)$$

This is Laplaces equation in two dimension. The exper-imental solution of problem, based on ideal and viscous incompr-essible fluid, involve the solution of this equation. The math-matical treating of such problems, in the spheres of hydraulics and aerodynamics, leads to partial differential equations which can only be evaluated when the necessary number of boundry requ-irements for the defined case are given. Generally, they can be obtained only for geometrically simple arrangements, correspond-ing rarely with those of technical practice. For practical re-search, use is therefore made of the analogy of all potential fields. The electric current field, as a natural potential field, is specially suited for accurate experimental evaluations.

There are several hydrodynamic analogies available in the literature. The electric analogy tray apparatus may be one of the solutions to the problem by solving Laplaces equation, $\nabla^2\phi = 0$,where ϕ is potential, and satisfying the boundry conditions. As the Laplaces equation governs the steady state flow of electricity, therefore, electric analog tray may be used to obtain flow nets for studing horizontal earthquake effects on dam of various upstream shapes.

The flow net is an orthogonal system which consists of two sets of curves, one representing stream lines and the other equipotential lines. If one of these lines of flow net are determined, other lines can be plotted from the properties of flow nets for continuous steady flow.

(i) $d\phi$ = constant

(ii) $\frac{ab}{bc} = \frac{cd}{da}$

(iii) lines of flow net cut perpendicular to each other.

In the electric analogy tray apparatus equipotentials lines are surveyed and are plotted on grapth. From (ii) + (iii) properties of flow nets, stream lines can be plotted for a particular dam profile.

Once the flow net is obtained, the proper scale of the pressures in the net must be determined. The pressure scale is easily determined by the following considerations :-

(i) Reservoir depth, h, is divided into n equal parts.

(ii) As the dam body is assumed to be rigid, therefore, same quantity of water flow through each element of flow net.

(iii) No water can escape across the stream lines.

Applying equation of motion and continuity to an elem-

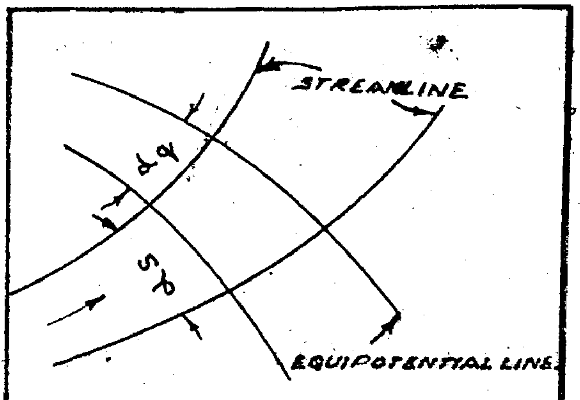


FIG. No. 55

ment of flow net we get

$$\frac{dp}{d\varphi}.ds = \alpha g \frac{wh}{gn} \cos\frac{2\pi t}{T}$$
$$= \alpha \frac{wh}{n} \cos\frac{2\pi t}{T} \quad (106)$$
$$\therefore \; dp = \alpha \frac{wh}{n} . \frac{d\varphi}{ds} \cos\frac{2\pi t}{T} \quad (107).$$

This equation determines the pressure scale. This equation can further be simplified if the flow net is made into squares so that $d\varphi = ds$. Also, the maximum pressure increase is only impotant which occurs at t=T. Therefore, equation (107) becomes.

$$dp = \alpha \frac{wh}{n} \quad (108)$$
$$\therefore \; p = \alpha C \tilde{w} h \quad (109)$$

where C is pressure constant.

To obtain flow net for a profile of a dam electric analogy tray is made as described in detail below.

3.3. DESCRIPTION AND SETUP OF APPARATUS.

(i) GENERAL: The "Electrolytic Tray" is the device to execute such measurements, being a container filled with electrolyte. This is a spacy extended conductor to which the current is led through one or several electrodes. At flow through of current through the electrolytic, a potential drop is produced. By means of a probe, adjusted at the calibration potentiometer, Balancing of this tension compensation bridge can be observed at an indica -tion instrument. In such a manner, the equipotential lines over the electrolytic surface can be found point by point, corresponding to the adjusted tension at the calibration potentiometer.

(ii) DESCRIPTION OF APPARATUS:

The electric analogy tray apparatus consist of the following :-

(a) 1 Tray sheet plastic ($\frac{1}{4}$"), size 6"x36"x2" (deep).

(b) 1 Projection platform, size 150x100 cms.

(c) 1 Light indicator (8 v) directly coupled with probe, mounted on coordination carriage.

(d) 1 Precision potentiometer 0-100% with zero galvanometer and rectifier, mounted on coordination carriage.

(e) 1 Potentiometer box with 50 potentiometers.

(f) 1 High resistance voltmeter, with voltmeter pole changing switch, on rotatable stand.

(g) 1 Transformer for projection lamp (8v).

(h) 1 Transformer for potentiometer.

(i) 220v, 50 c/s supply.

The movement of the probe over the electrolyte surface is effected by means of two carriages, each mounted on rollers ball bearings. The movement along the length of the tray is eff-ected by means of bridge carriage, which also carries the switch board with the elements for the service of the tension-compensation. The movement of the bridge carriage is effected by a knob. Above this knob there is another knob which moves the probe carriage in transverse direction of the tray. At the lower part of the probe carriage, the probe can be inserted in such a manner that within certain limits, an adjustment in vertical direction may be effected. The position of the probe is marked by means of a projected light point on a drawing board of glass mounted over the tray, then the position of the probe can be located and marked on a graph paper.

The connection of the apparatus may be seen from connection sketch No. (56 & 57). The feeding takes place from the light net 220-230 V 50 cps. The switch board allows 50 different potentials to be taken from each of the 2 pairs of bushes. Every second bush (counted from the left) of each pair possesses between them same potential, whereas all first bushes are connected with the sliding connection of the potentiometers, 1-50, and which can then be set on a potential between o and 20 volts at will. Main electrodes and calibration potentiometer are fed from two parallel connected pairs of bushes. The voltage as selected by means, of the potentiometer can be read off for each pair of bushes by setting the selecting switch on the respective bush number on the control instrument.

The connection to the net is effected over a net cable with 2 poles-plug with earth connection, second bush pair of the connec-tion, connection is made by means of a two conductor cable to the bushes of the two stretched brass wires and brushes and also with the calibration potentiometer. The feed for lighting the projection lamp is effected by means of a one conductor cable with red bananaplug. The voltage amounts to 8 volts.

The electric analogy method consists of constructing a tray geometrically similar to the dam and reservoir area. As we saw above that hydrodynamic pressure is directly proportional to the height of dam, a linearly varying electric potential is placed along the boundry representing the upstream face of the dam. This has been done by making various dam profiles (6" heightx 1" length) of a non conducting heat resistant material (like asbes tous sheet) representing upstream boundry at one end of the tray. This boundry was wound with nichrome wire (24 SWG) to give a linear drop in potential. The linear drop of the boundry was assured by checking by a voltmeter. A constant electric potential is placed along the boundry representing the bottom of the reservoir (making constant hydrodynamic pressure boundry at the bottom of dam). This was done by placing a copper strip (1" wide) at the bottom of dam (see fig. No. 57). Free surface of the reservoir is of plas-tic strip representing (0% pressure). The tray was then filled with tap water, which is an electrolyte.

This tray was placed under projection platform and adjusted such that probe can move longitudinaly along the longer edge of tray. The graph paper of the size of tray was placed over the projection platform and with the help of projection lan-tern and by moving probe longitudinally and transverly shape of the dam profile was drawn on graph paper. Electric connections were made as shown in figure No. (56). Three photograph will c-clear the set up and connection of the Electric Analogy Tray as shown in figure No. 59,60 and 61.

3.4. <u>Preparation of Test and Measurements</u>.

When all the connections has been completed, the tray is filled with tap water to such an extent that probe can touch its surface. The calibration potentiometer has a resistance of 1000 ohms.(10 V linear drop was allowed along the U/S face of dam) in order to attain maximum sensibility of the bridge, the commutation switch is switched to position '1'. In this position the "adjustment" until the galvanometer shows readings at its maximum sensibility. This adjustment has to be carried out but once only. After-wards it is brought to position II. In this position the probe is connected over the rectfier arrangement with measuring potentiometer. This potentiometer is now set to a certain desired value. (In % of the allowed drop as read on dial on bridge carriage). The probe is shifted until the galvanometer reads zero. By connecting the solocated points of equal potentials over graph paper, the desired equipotentials lines are ob-tained.

Equipotentials lines of 95,90 to 10% with in an interval of 10% were drawn.

3.5. OBSERVATIONS:-

In order to make this study, hydrodynamic pressure due to horizontal earthquakes were determined for several shapes of dams. In all eleven cases were studied. Dams studied were those with constant U/S slope of 0^o, 30^o, 45^o, 60^o and six other cases as shown in figure No. 62.

After surveying the equipotential lines as shown in figure No. 58, the flow net is completed. The typical flow nets so obtained for two cases of dams have been shown in figure No. 66 and 67. The pressure coefficients are then determined as explained earlier and curves are plotted for each profile of dam.

3.6. RESULTS :-

The curves for hydrodynamic pressure distribution for various profiles of dams have been drawn in figure No. 62,63 and 64. All the experimental curves so obtained have been compared with experimental curves of Zangers (7) and it is concluded that curves are in close aggrement with that of Zangers and therefore are reproducible and thoroughly reliable.

C H A P T E R IV

C O N C L U S I O N S .

4.1 A comparative study of various approaches to the problem of determining hydrodynamic pressure during earthquake on dams and containers and experimental verification of Zanger's (7) work led to the following broad conclusions for dams and containers.

4.2. DAMS:

(1) Westergaards theory is faulty. It is not applicable in case when the period of earthquake is shorter than the resonance period of dynamic water. Later theories of Werner and Sundquist, Tadashi Hatano etc. studied the problem to amend the deficiencies of Westergaards solutions as well as to obtain the right solution considering the reservoir limits, gorge section etc. As a result, later theories on the dynamic water pressure have progressed and the phenomena have been greatly made known.

(2) Since all of theories except Kotsubo, discussed have assumed the earthquake as stationary, simple harmonic motion and dynamic water pressure become infinitly large when the earthquake period corresponds with the resonance period of dynamic water pressure. Kotsubo considered the effect of irregular earthquake for a two dimensional case and found that the dyna-mic water pressure is not always proportional to the corresponding earthquake acceleration and can be even found in case of resonance period of dynamic water pressure. The dynamic pressure so found is much larger than Westergaard's. Kotsubo is first man to give a three dimentional solution to constant radius arch dams. The magnifying power of dynamic water pressure over Westergaards is 1.6 for dams about 100 m. high. For vibrations in the direction of river course, the dynamic water pressure acting upon a gravity dam or an arch dam will be given by

$$p = \alpha w h \sqrt{1 - \frac{y}{h}} \cdot K \qquad (110)$$

where, K is the magnifying power of dynamic water pressure, which will be expressed by the following equation

$$K = \frac{4v}{\pi h} \int_0^t \psi(t-\tau)\, J_0\left(\frac{2\pi\tau}{T_1}\right) d\tau \qquad (111)$$

(3) From all considerations, the theory proposed by Werner and Sundquist seems to be better, as far as hydrodynamic pressure is considered, than both of Jacobsens and Westergaards as they have considered the fetch of reservoir as well as the compressibility of water.

(4) As Zanger neglected the compressibility of water, his solution to Laplace equation by analogy tray method do not contain any factor for vibration period of earthquake and his solutions are comparable with those of Westergaards for $T=\frac{4}{3}$ secs. For shorter period of vibration of earthquake his experimental curves are of no use and can only be applied for $T > 1$ sec. Zangers work is quite useful for inclined U/S faces and other profiles of dam as no other theory has considered this effect. Electric analogy tray method affords a very easy, convenient and cheaper method for experimental determination of dynamic water pressure due to horizontal earthquake.

(5) The increase in pressure on dams with sloping faces is less than on dams with vertical face, the flatter the slope the smaller the increase in pressure.

(6) The maximum pressure for sloping faces of dams occur at some distance above the base of the dam except for a vertical face and distribution of hydrodynamic pressure due to horizontal

can approximately represented by a parabolic curve and can be expressed by equation

$$p = C \alpha w h \qquad (112)$$

where C is the pressure coefficient. The maximum value of pressure coefficient was found to .735 at the base of a dam with vertical U/S face. The approximate pressure distribution for dams with constant slopes can be obtained by equation

$$p = \frac{1}{2} \alpha w h \, c_m \left[\frac{y}{h}\left(2-\frac{y}{h}\right) + \sqrt{\frac{y}{h}\left(2-\frac{y}{h}\right)} \right] \qquad (113)$$

and total horizontal pressure and overturning moment are given by following expressions

$$P = 0.726 \, py \quad (114)$$

and $$M = 0.299 \, py^2 \quad (115)$$

The above expressions of Zanger are supported well by experimental verification by Electric Tray Analogy method. The results obtained are quite near to those of Zangers (7).

(7) For practical purposes dams having U/S faces, vertical for more than half of the height can be treated as having vertical U/S face and subjected to the same intensity of earthquake.

(8) For irregular earthquakes which is the practical case, the dynamic water pressure at a given time is not always proportional to the corresponding earthquake acceleration but different in time of resonance and in magnitude according to the relation between the resonance period of dynamic water pressure and the earthquake and pressure is considerably larger than that based on simple harmonic earthquake and for a dam about 100 m. high, the magnifying power of pressure is as large as approximately 1.6.

(9) Total force on the wall is reduced by wall flexibility

due to stiffness of the section.

(10) The assumption that water is incompressible, is not conservative but errors so caused for T=1, compared with Wester gaard are small for dams under 400 ft. in height and are not ex -cessive for dams as high as 800 ft. and error in moment for this height is 13.5%. For earthquake period less than 1 second ($T < 1$)error will be increasing with Zangers results.

(11) For a finite reservoir with far end fixed the hydrodyn- amic pressure increases. For $\frac{l}{h} > 3.0$ the reservoir can be taken as infinite (Heilton) while from the results of Sundquist ratio $\frac{l}{h}$ is > 4.0 for infinite reservoir. Either a correction factor as given in figure No.(28) should be used or Werner and Sundqui -st results should be used for a case of finite reservoir.

(12) The magnitude and primary resonance period of dynamic water pressure become smaller as the section of canyon approaches nearer to the triangular form from rectangular one and dynamic water pressure is 70% that in case of rectangular section.

(13) From the above considerations, the theory proposed by Werner and Sundquist(3) seems to be more justified than any oth -er proposed theories as far as hydrodynamic water pressure is concerned. But for all practical purposes solutions given by Housner (8) are very simple and easy to be applied with maximum error 5 to 6% and recommended for all practical purposes. Zanger (7) experimental determination of pressure coefficients by Ele- ctric Analogy Tray apparatus affords a easy and cheaper method for any profile of dam for a two dimensional case and results are applicable for all practical problems.

4.3. <u>CONTAINER AND PIERS</u>:-

(1) The compressibility of fluid does not effect the

results appreciably for tanks of usual shapes and heights.

(2) It will be conservative not to consider the viscosity of fluid due to the drag between the particles.

(3) For practical purposes the ratio of effective hydrodynamic mass to actual mass of the fluid can be calculated within 5% of error from the following simple relations

$$\frac{m_1}{m} = 0.57\frac{h}{a}, \quad \text{for } 0 < \frac{h}{a} < 1 \qquad (116)$$

and $$\frac{m_1}{m} = 0.39 + .18\frac{h}{a}, \quad \text{for } 1 < \frac{h}{a} < 2.5 \qquad (117).$$

The centre of gravity of effective hydrodynamic mass is given by same degree of approximation by $\frac{\bar{x}_1}{h} = 0.36 + .027\sqrt{\frac{h}{a}}$, (118) from the bottom of tank

(4) Surface tension effects are secondary.

(5) The tanks with rigid cover, the removal of a small proportion of the fluid makes the tank effectively an open one. The overturning moments reduces more rapidly than effective hydrodynamic mass.

(6) Housner results are much simpler for a case of tanks and overestimates M with a maximum error less than 4%.

(7) From all considerations, the theory proposed by Werner and Sundquist(3) seems to be better as far as hydrodynamic water pressure is concerned than any other proposed theory but for all practical purposes solutions of Housner (8) are very simple and easy to be applied with error not more than 5 to 6% and recommended for all practical purposes.

4.4 A study of three dimensional analysis both experimentally and analytically is desirable to provide more facts and characterstics of dynamic water pressure so far unknown and for a theoretical study of the problem.

C H A P T E R   V

RECOMMENDATIONS OF I.S.I. OF INDIA.

5.1 A comprehensive set of recommendations for the design of earthquake - resistant structures has recently been published by the Indian Standards Institution. The standard is also appli -cable to dams of minor importance, and retaining walls. A major portion of the standard is devoted to the determination of the seismic coefficient, which is defined as the ratio of accler-ation due to earthquake to that due to gravity as the seismic forces are directly proportional to it and cost of structure ri-ses considerable with its rise. Though there are many factors which affect the value of the seismic coefficient the standard has in the main related it to the type of structure and its loc-ation and to the nature of soil on which it is founded i.e., its geology. For this purpose the country has been divided into seven zones and soils into three catogories- hard, average and soft. For each zone and each catogory of soil a basic seismic coefficient has been specified as shown in figure No. 65. For major projects a futher investigation of past recorded earthquakes and geology of thearea is required to specify the seismic coeff-icient. In the case of bridges and dams the hydrodynamic effect of an earthquake has to be considered. Standard recommends the use of Zangers(7) curves for various U/S profiles of dams for the gravity dams, concrete, masonary or earthen; and arch dams.

5.2 The standard requires design to be based on the assump-tions that resonance is not likely to occur and that the earthq-uake and wind forces etc. not act simultaneously. A 33% increa-se in the permissible stresses of materials is allowed whenever earthquake forces are considered along with other normal forces. At the same time, the bearing pressure on the foundation can be increased upto 50% depending upon the type of soil.

5.3. United States Bureau of Reclammation also recommends the use of Zangers Curves and coefficients for the case of Gravity and arch dams in their chapters on 'Dams'.

increased upto 50% depending upon the type of soil.

5.3. United States Bureau of Reclammation also recommends the use of Zangers Curves and coefficients for the case of Gravity and arch dams in their chapters on 'Dams'.

C H A P T E R VI.

SUGGESTIONS FOR FURTHER STUDY OF THE PROBLEM

6. From practical point of view a further study of the pr--oblem is not of any particular importance as the extra hydrodynamic pressure caused on the U/S face of the dam during earthquakes is only a small part of the hydrostatic pressure. For a case of dam in a seismic region with the extra hydrodynamic pressure will be only about 15% of hydrostatic pressure. Therotetically a further study of the problem will help in finding the various unknown characteristics of the problem and will also lead to a rational solution of the problem.

6.2. The study of the problem upto this time has been confined to two dimensional problems. A three-dimensional study both experimentally and analytically is urged. A three dimensional study will solve the problems involving various U/S profiles of the dams and non-straightness of the dam axis. The consideration of side slopes and reservoir shape should be incorporated in the experimental tests. A experimental study is also required for a case when either dam body or basin is under resonance.

6.3. In the case of earthen gravity dams, the hydrodynamic effect on the pore water pressure is also to be investigated and therefore stability of the slopes is to be found.

NOTATIONS

g	=	Acceleration due to gravity
K	=	Bulk modulus of liquid.
ω	=	Unit weight of liquid.
x, y, z	=	Axes.
u, v, s	=	Displacements along x,y and z axes respectively.
α	=	Seismic coefficient (% of g) in horizontal direction.
α_v	=	Seismic coefficient in vertical direction.
t	=	Time.
T	=	Period of earthquake wave.
T_1	=	Primary resonance period of dam.
Tm	=	Resonance period of dynamic water pressure.
E	=	Elastic modulus.
p	=	Hydrodynamic pressure at a depth y.
p_b	=	Hydrodynamic pressure at the bottom.
n	=	An integar.
h	=	Depth of reservoir.
H	=	Total depth of the dam.
ρ	=	Density of concrete.
P	=	Total pressure at depth y.
M	=	Overturning moment at depth y.
p_o, P_o and M_o	=	Corresponding values at the bottom of reservoir.
b	=	Width of body of water.
C	=	Pressure coefficient.
θ	=	Angle which the U/S face of the dam makes with vertical.

(r, θ) = Polar coordinates.

a = Radius of circular cylinder and cylindrical pier.

R = Radius of cylinder or sphere.

J = Jacobian function.

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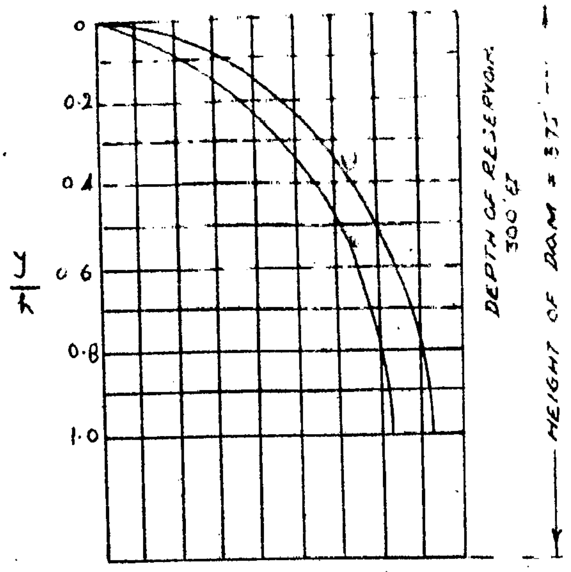
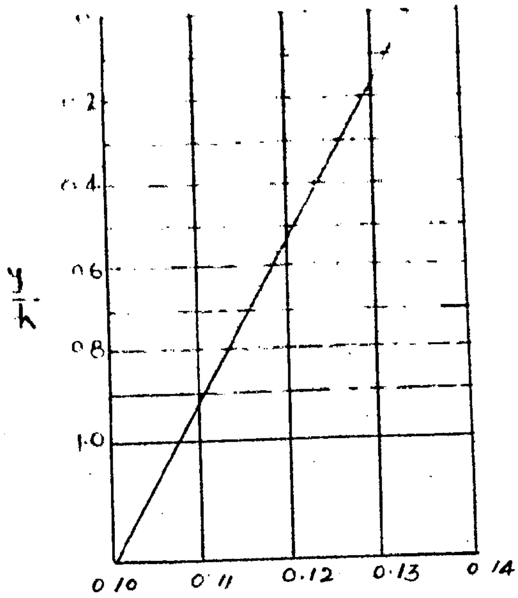
F I G U R E S.



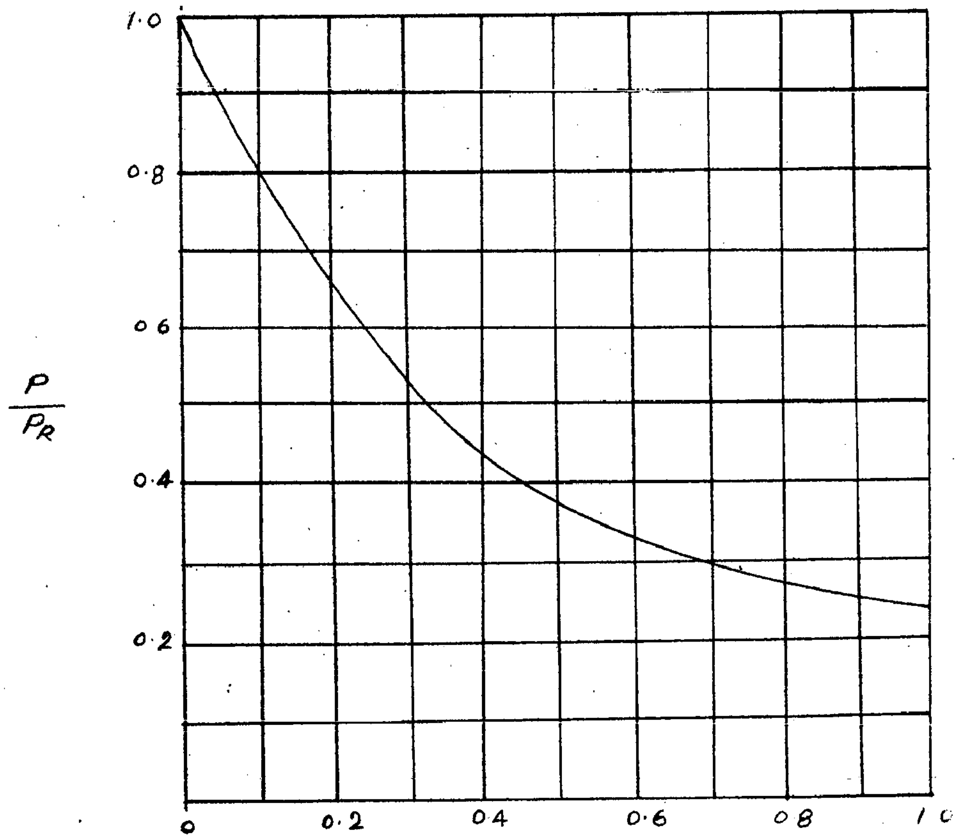
HYDRODYNAMIC PRESSURE IN KIPS/SQ FT.

FIG NO. 3



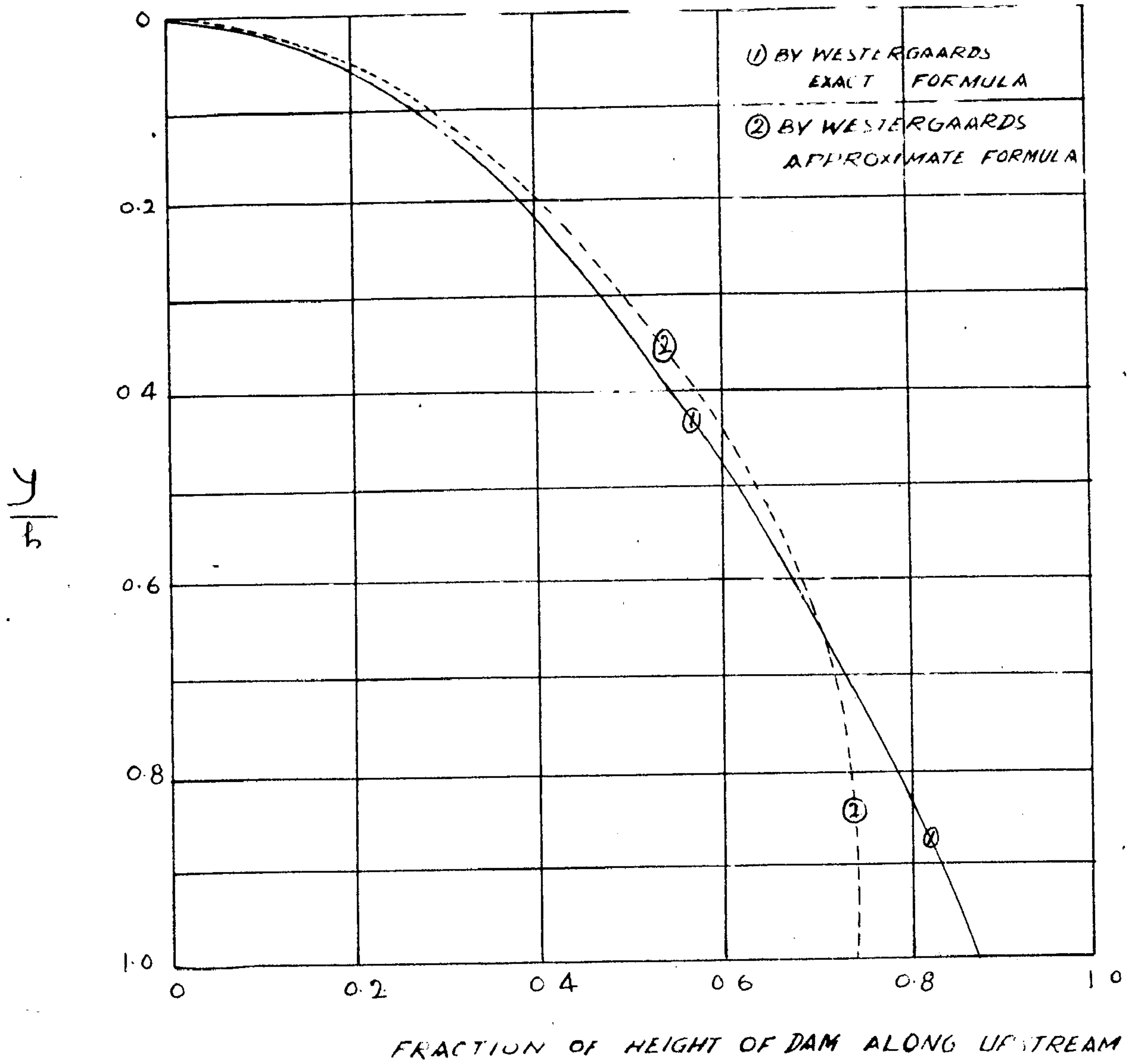
INERTIA COEFFICIENTS

FIG NO. 4



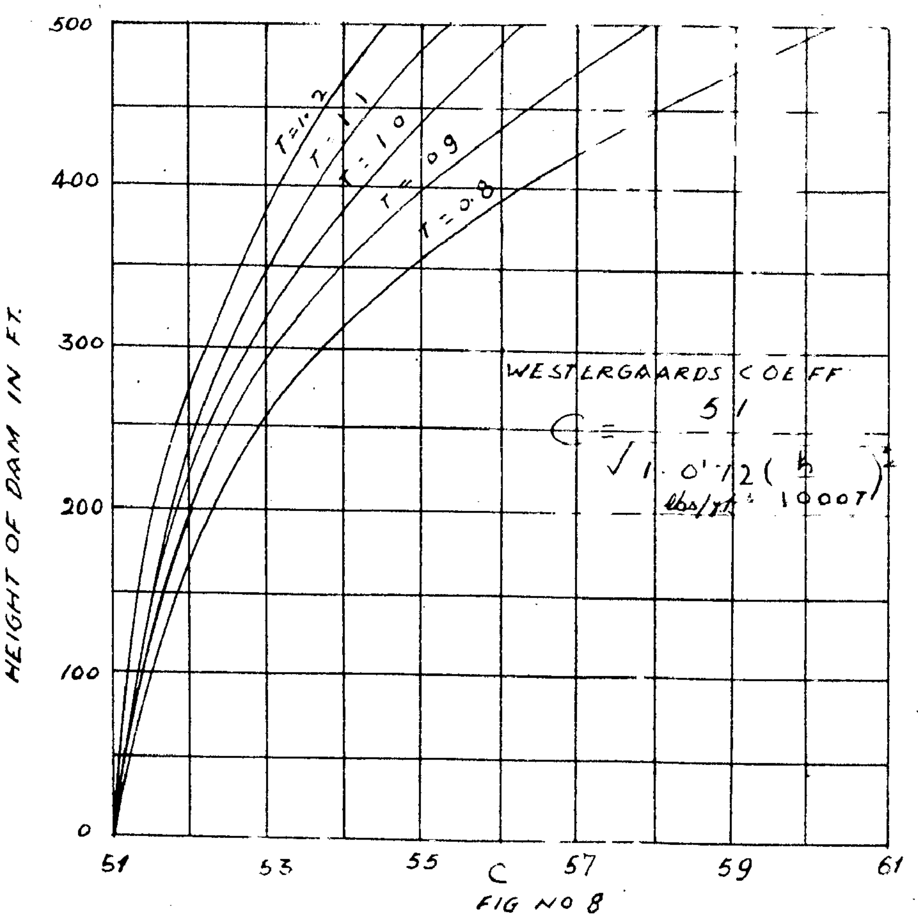
$\frac{P}{P_R} = \frac{\text{TOTAL PRESSURE ON A RIGID WALL}}{\text{TOTAL PRESSURE ON A FLEXIBLE WALL}}$.

FIG. NO 5.



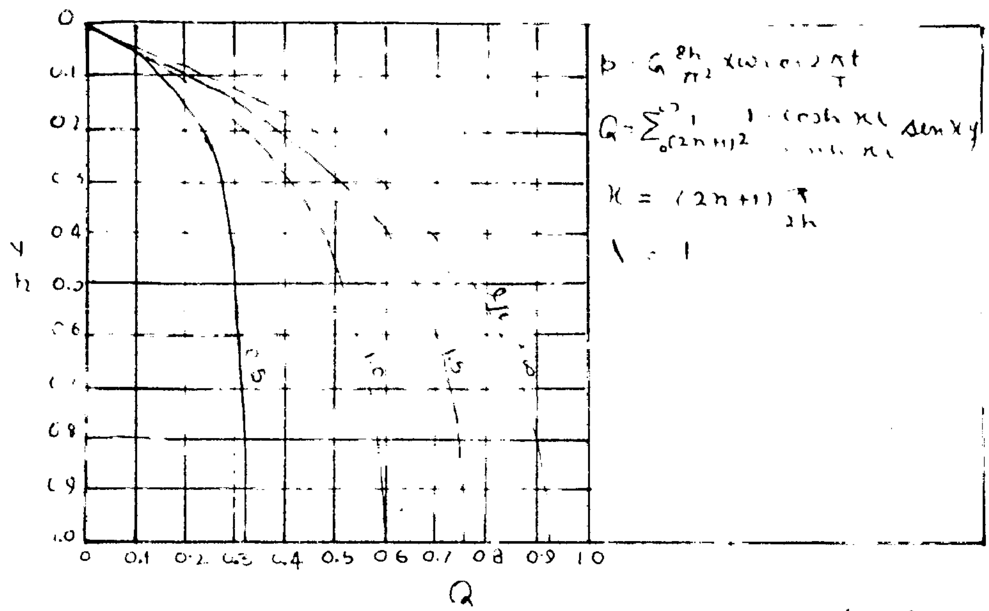
COMPARISION OF WESTERGAARDS EXACT AND APPROXIMATE FORMULA FOR D.W.P DISTRIBUTION

FIG. N 7.



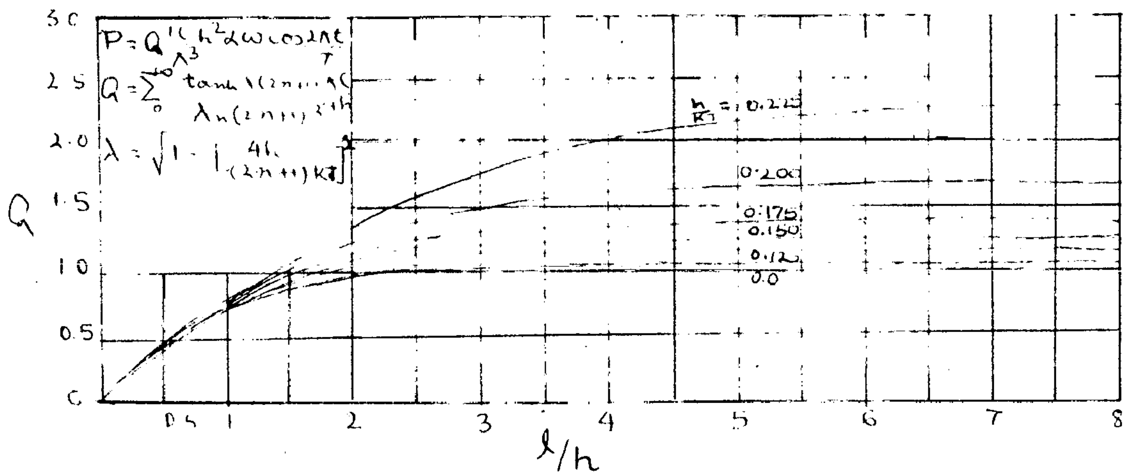
$$C = \frac{51}{\sqrt{1 - 0.72\left(\frac{h}{1000T}\right)^2}}$$

FIG NO 8



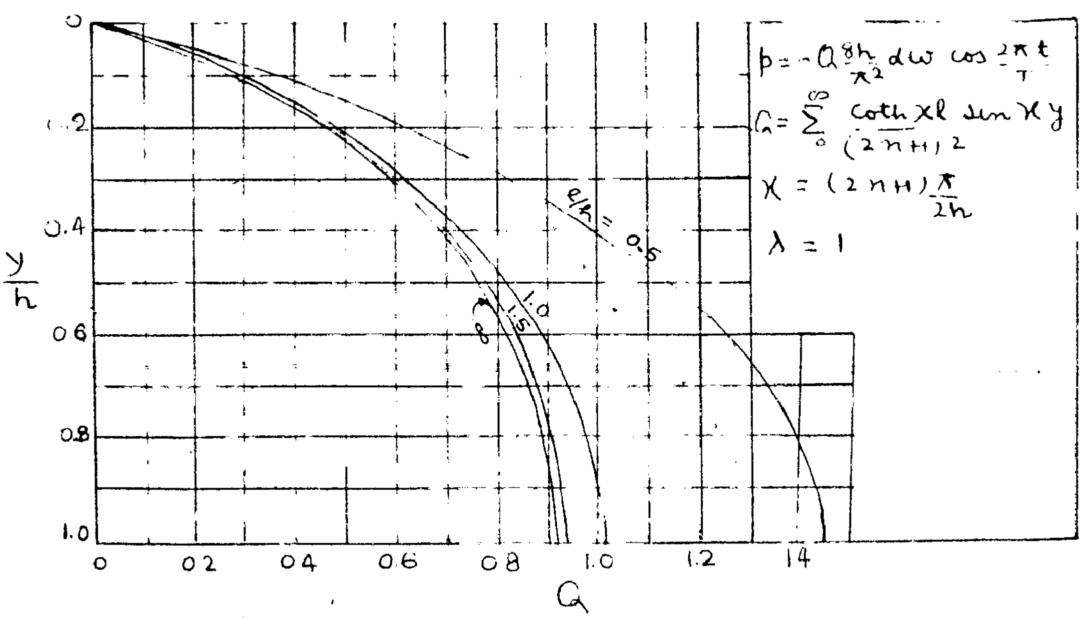
EVALUATION OF EQUATION (38)

FIG No. 10.



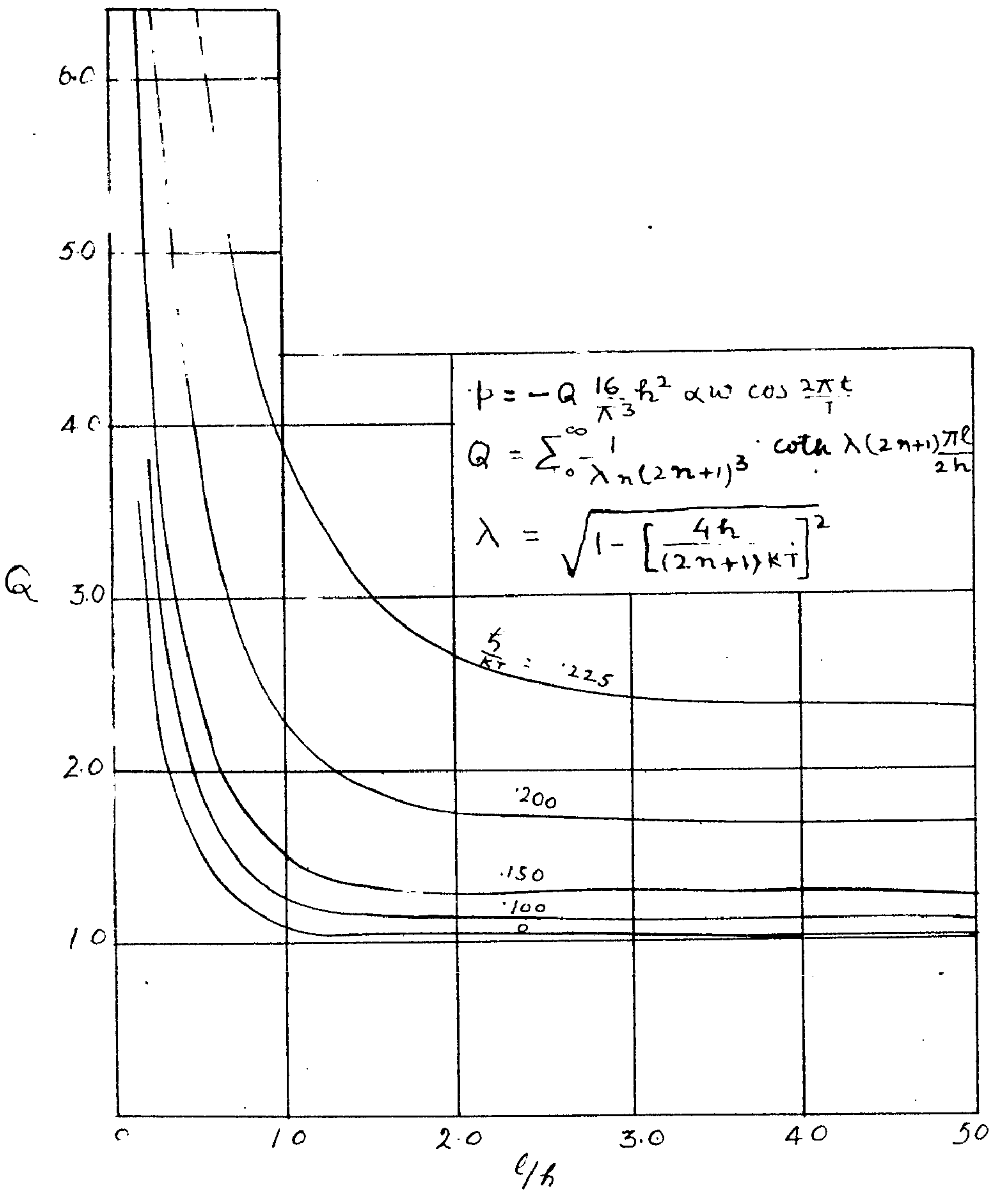
EVALUATION OF EQUATION (40)

FIG. No. 11.



EVALUATION OF EQUATION (41)

FIG. No. 12.



EVALUATION OF E. Q NO.(42)

FIG NO. 15.

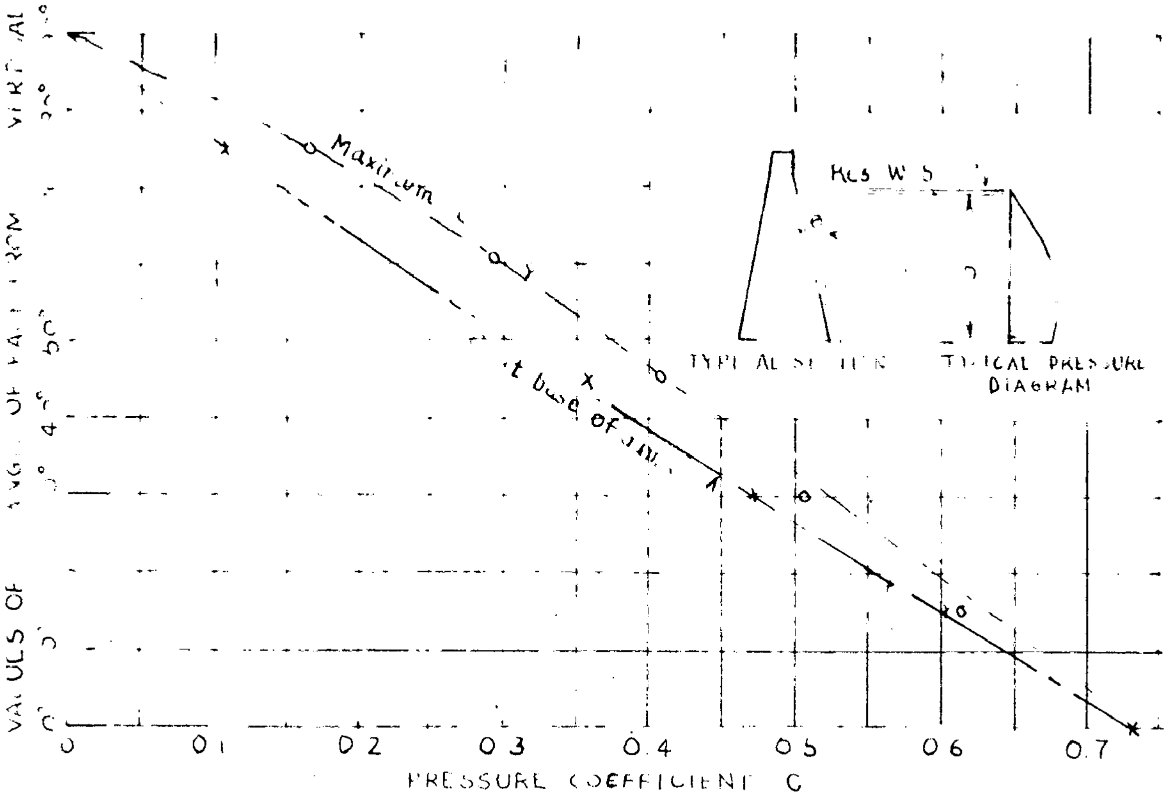


FIGURE 14. PRESSURE COEFFICIENTS FOR CONSTANT SLOPING FACES.

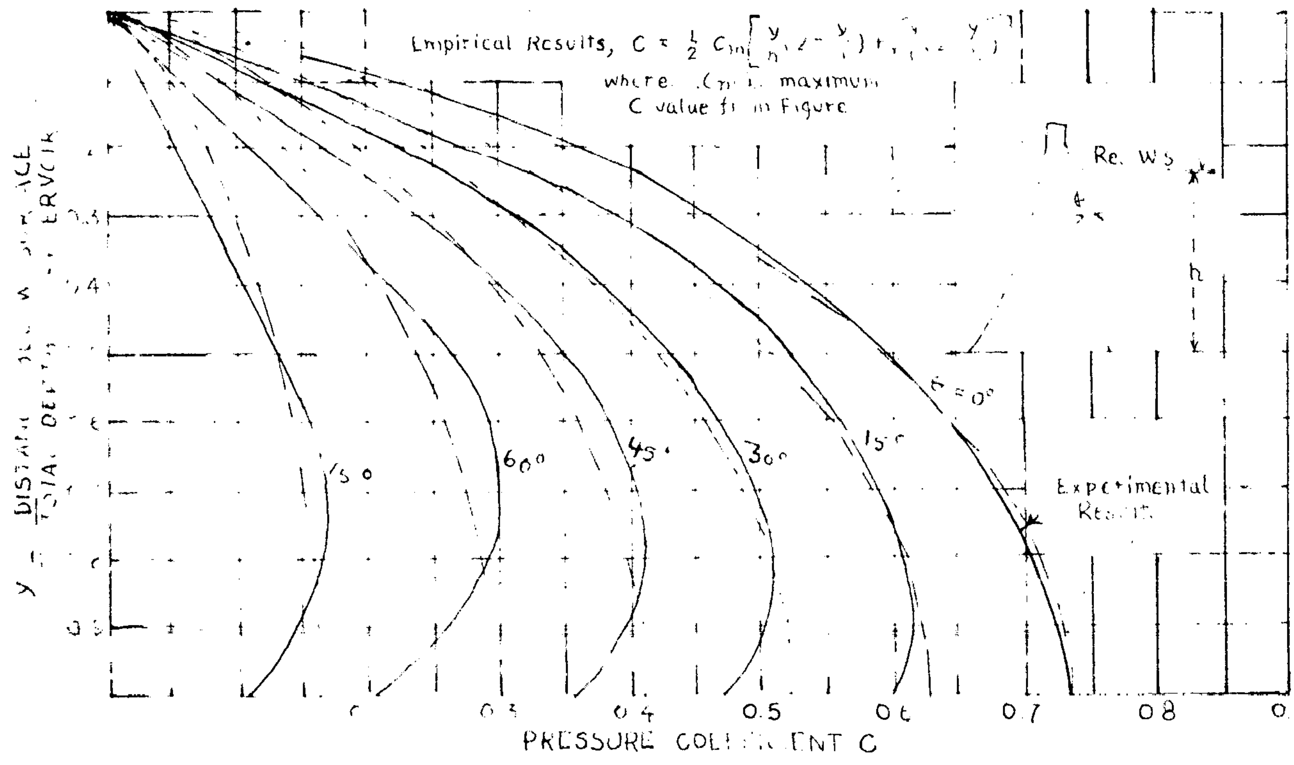
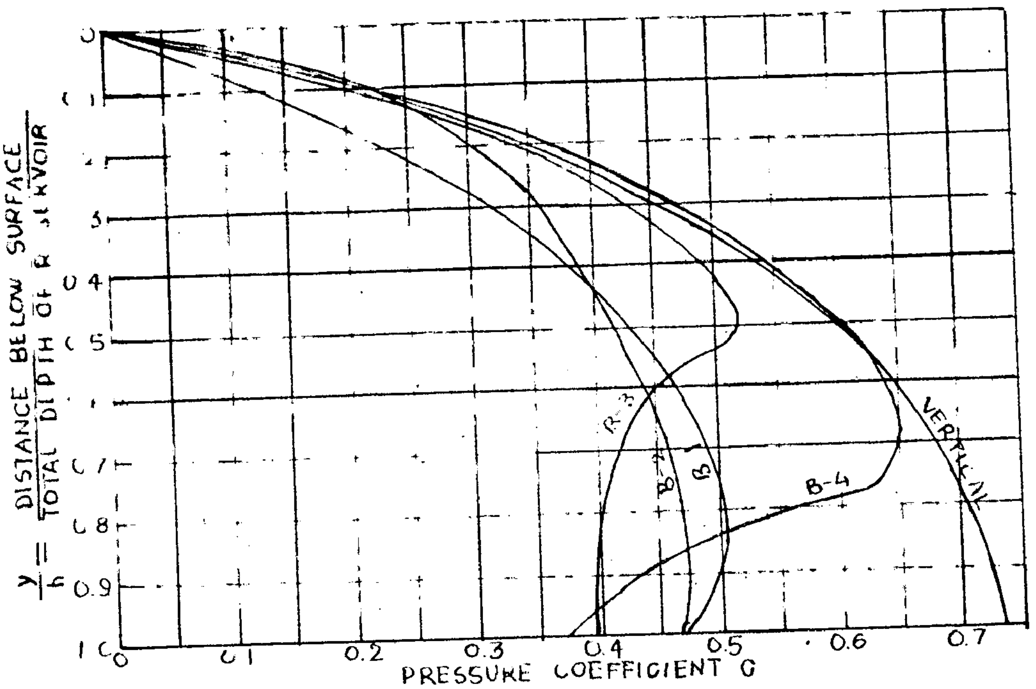
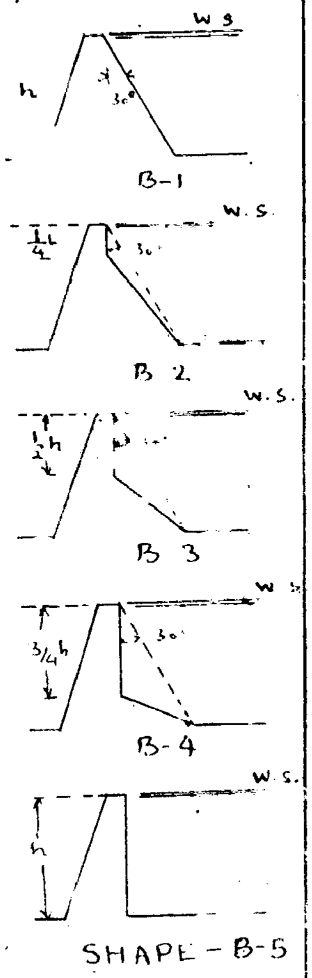


FIGURE 15. PRESSURE COEFFICIENT DISTRIBUTION. COMPARISON OF EXPERIMENTAL AND EMPIRICAL CURVES.

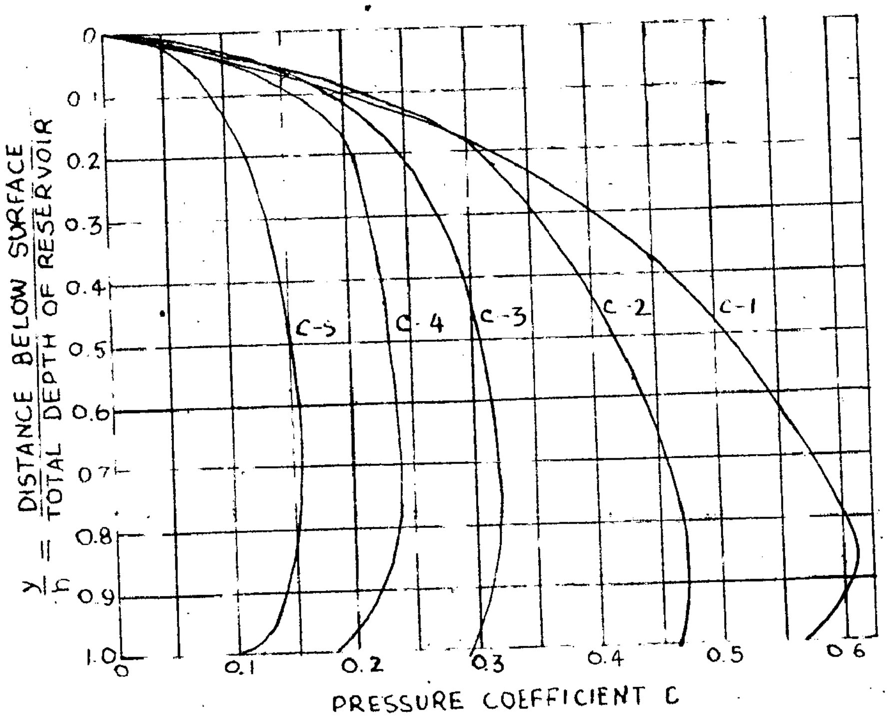


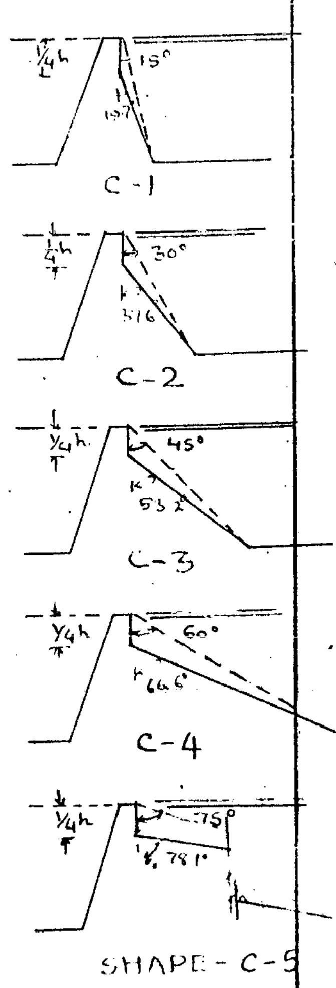


VALUES OF PRESS. COEFF. C FOR SHAPES SHOWN

VERTICAL PORTION OF U/S FACE OF DAM VERTICAL

FIG. No. 16.

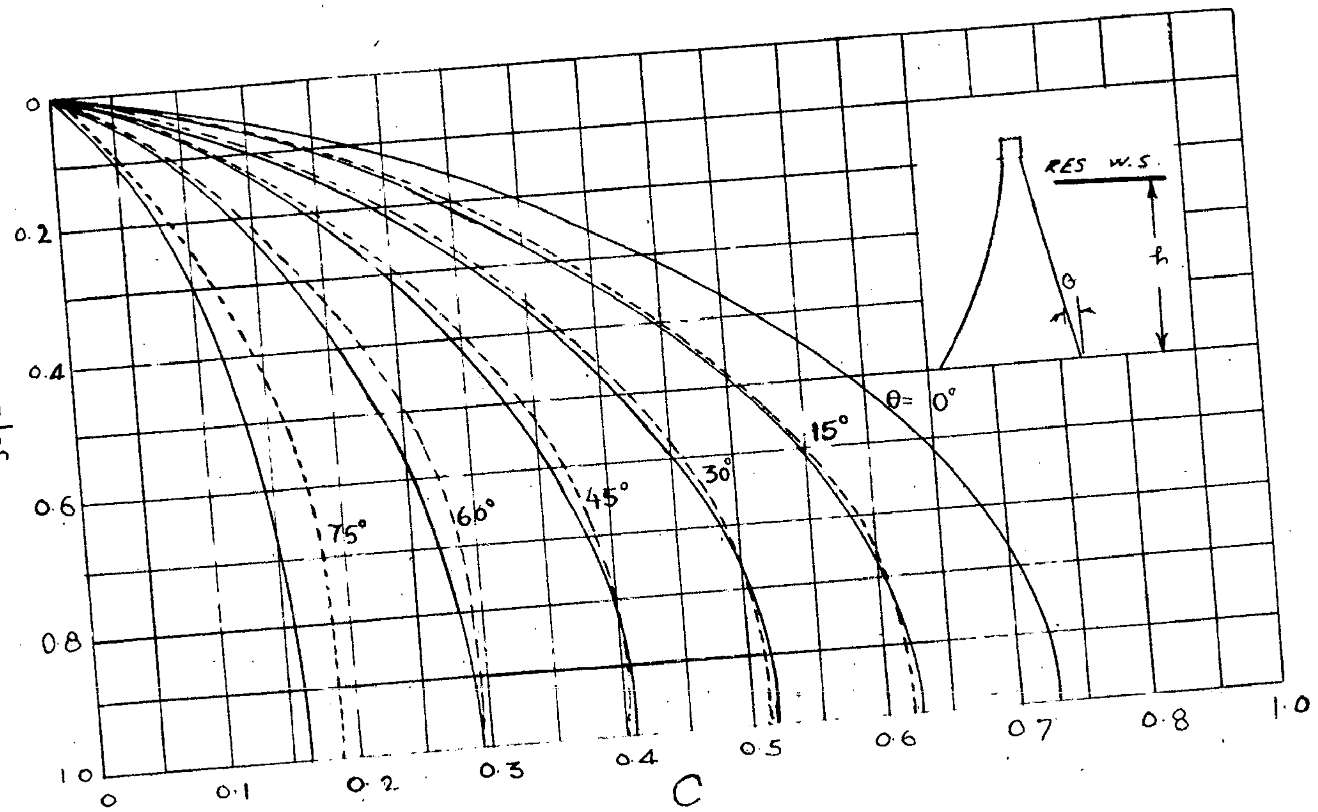




VALUES OF PRESS. COEFF. C FOR SHAPES SHOWN

VERTICAL PORTION OF U/S FACE OF DAM = $\frac{1}{4}h$

FIG. No. 17.



VALUES OF PRESSURE COEFF. C FOR SLOPING U/S FACES OF DAM

———— = PRESSURE COEFF. FROM ZANGERS EXPERIMENTAL CURVES

- - - - - - C - PRESSURE COEFFICIENTS GIVEN BY EQUATION.

$$C = \cdot 3675 \left[\frac{Y}{h}\left(2 - \frac{Y}{h}\right) + \sqrt{\frac{Y}{h}\left(2 - \frac{Y}{h}\right)} \right] - \cdot 12 \frac{\theta^{\circ}}{15} \sqrt{\frac{Y}{h}}$$

FIG. NO. 18.

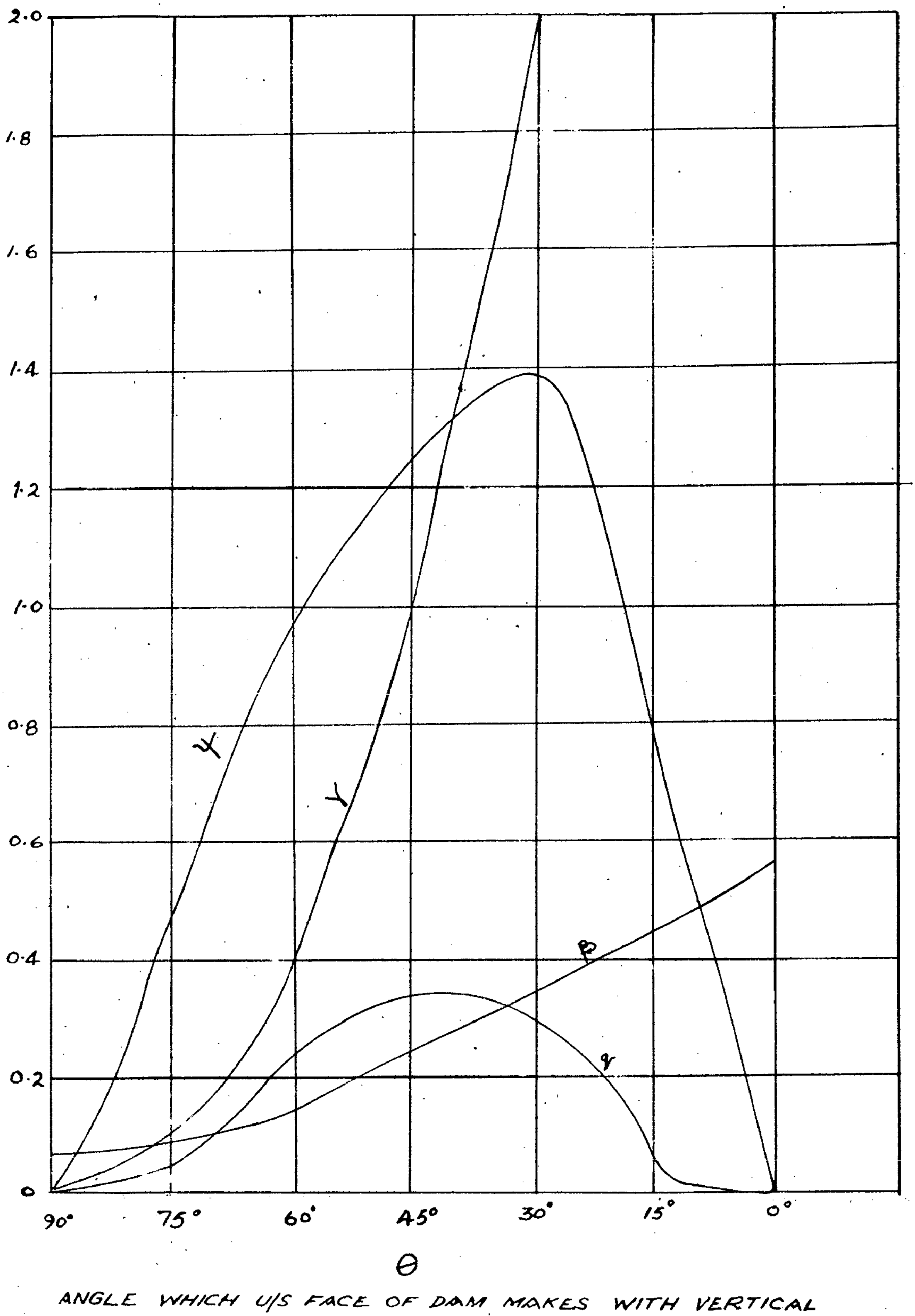
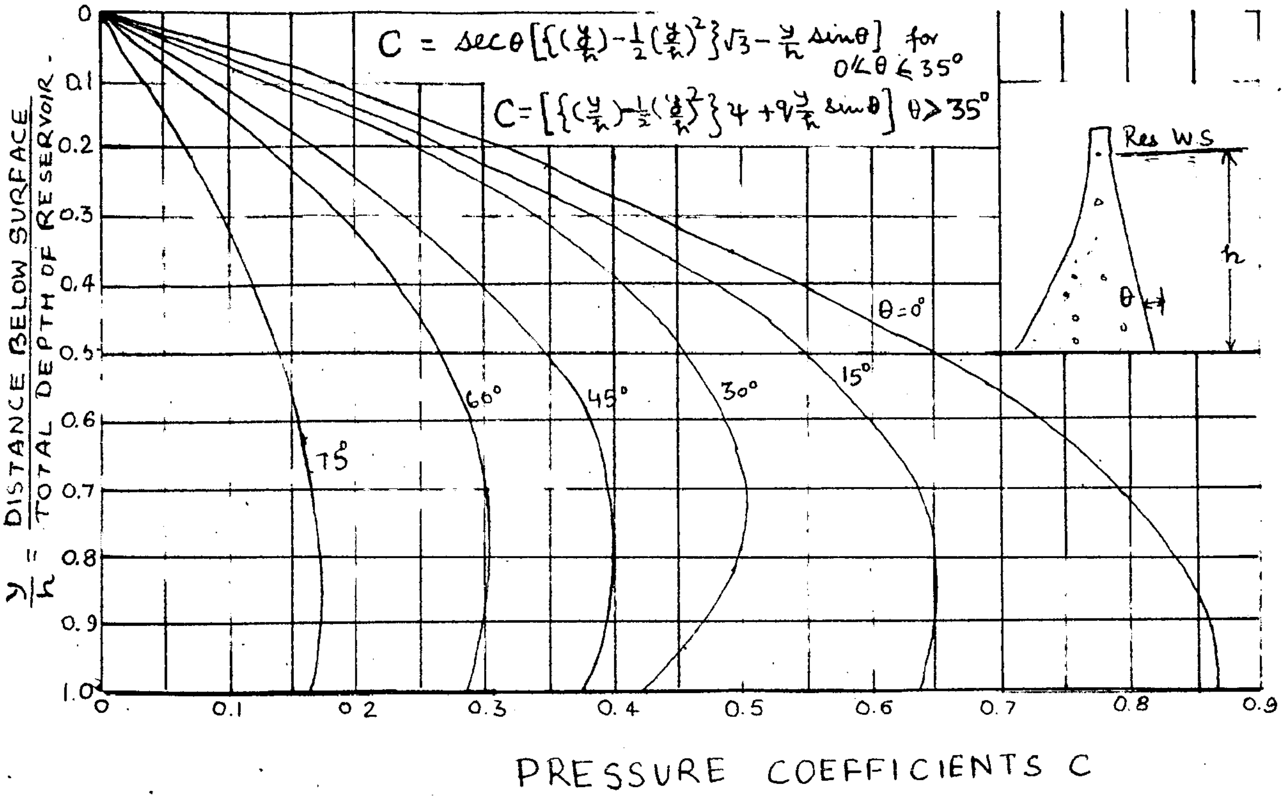

2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
90°
75°
60°
45°
30°
15°
0°
θ
ANGLE WHICH U/S FACE OF DAM MAKES WITH VERTICAL

FIG. NO. 19.



PRESSURE COEFFICIENT DISTRIBUTION – (HOUSNER)
ON SLOPING U/S FACES OF DAM

FIG. No. 20

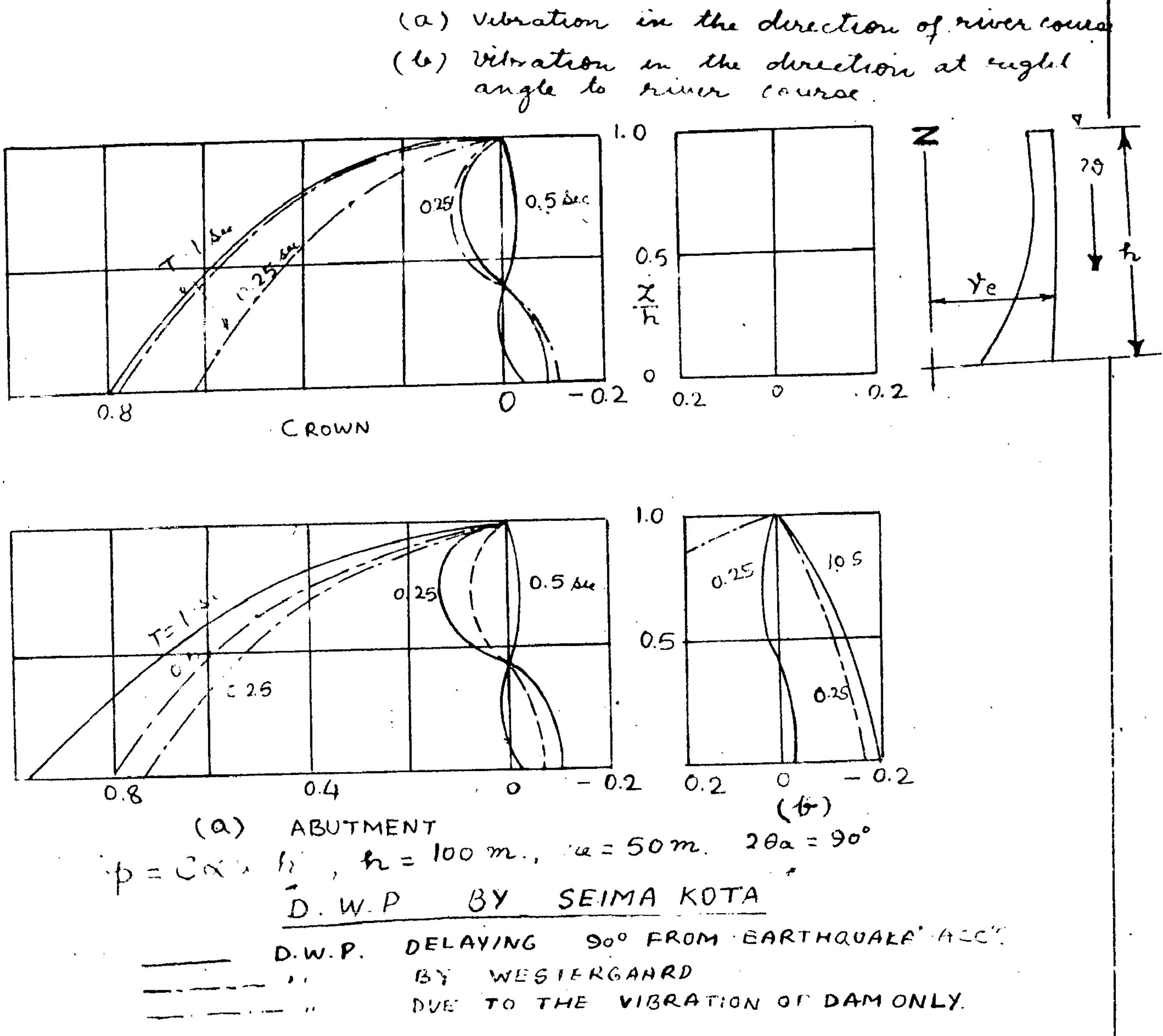


FIG. No. 22. VERTICAL DISTRIBUTION OF D.W.P.

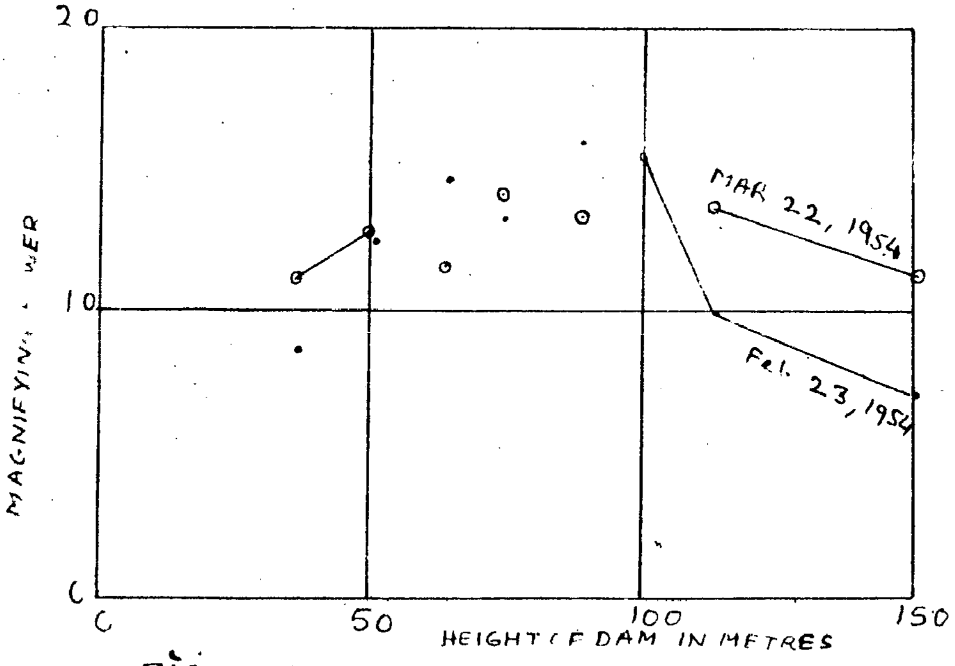
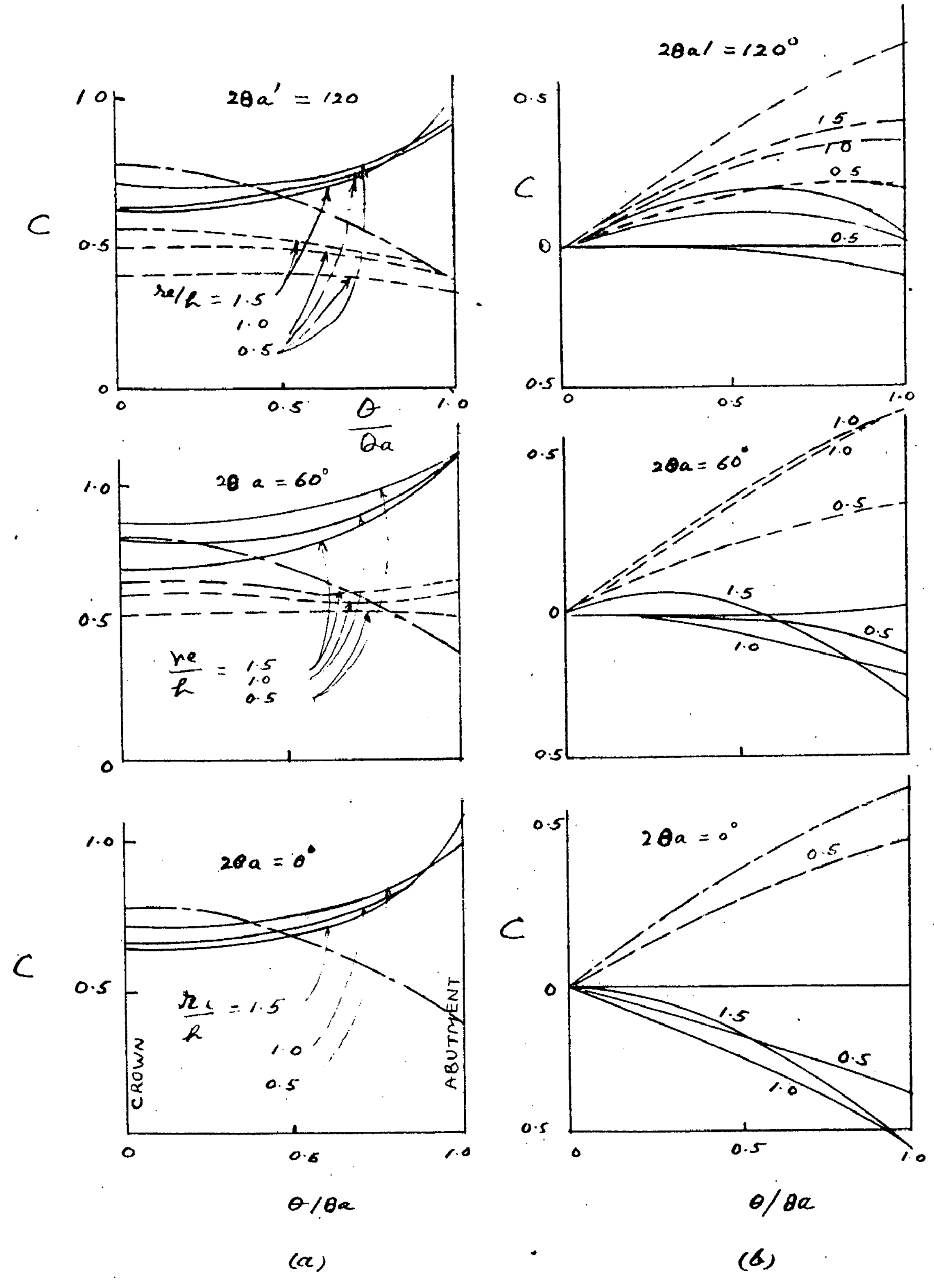


FIG. No. 26. MAGNIFYING POWER OF D.W.P



HORIZONTAL DISTRIBUTION OF D.W.P.

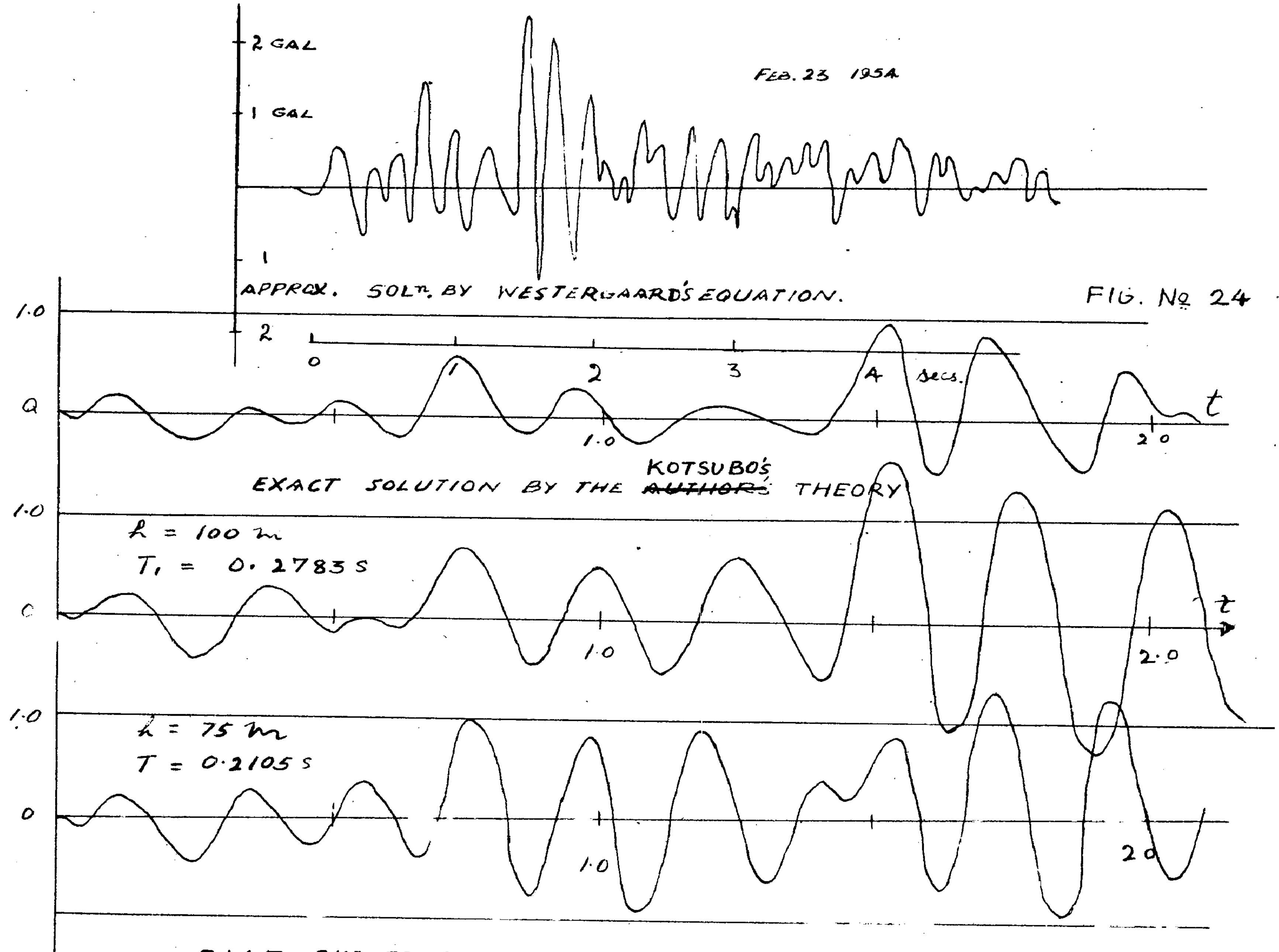
H = 100 m, 2θa = 120°, T = 1.05

W = ANGULAR VELOCITY OF EARTHQUAKE

θa = 1/2 OF CENTRAL

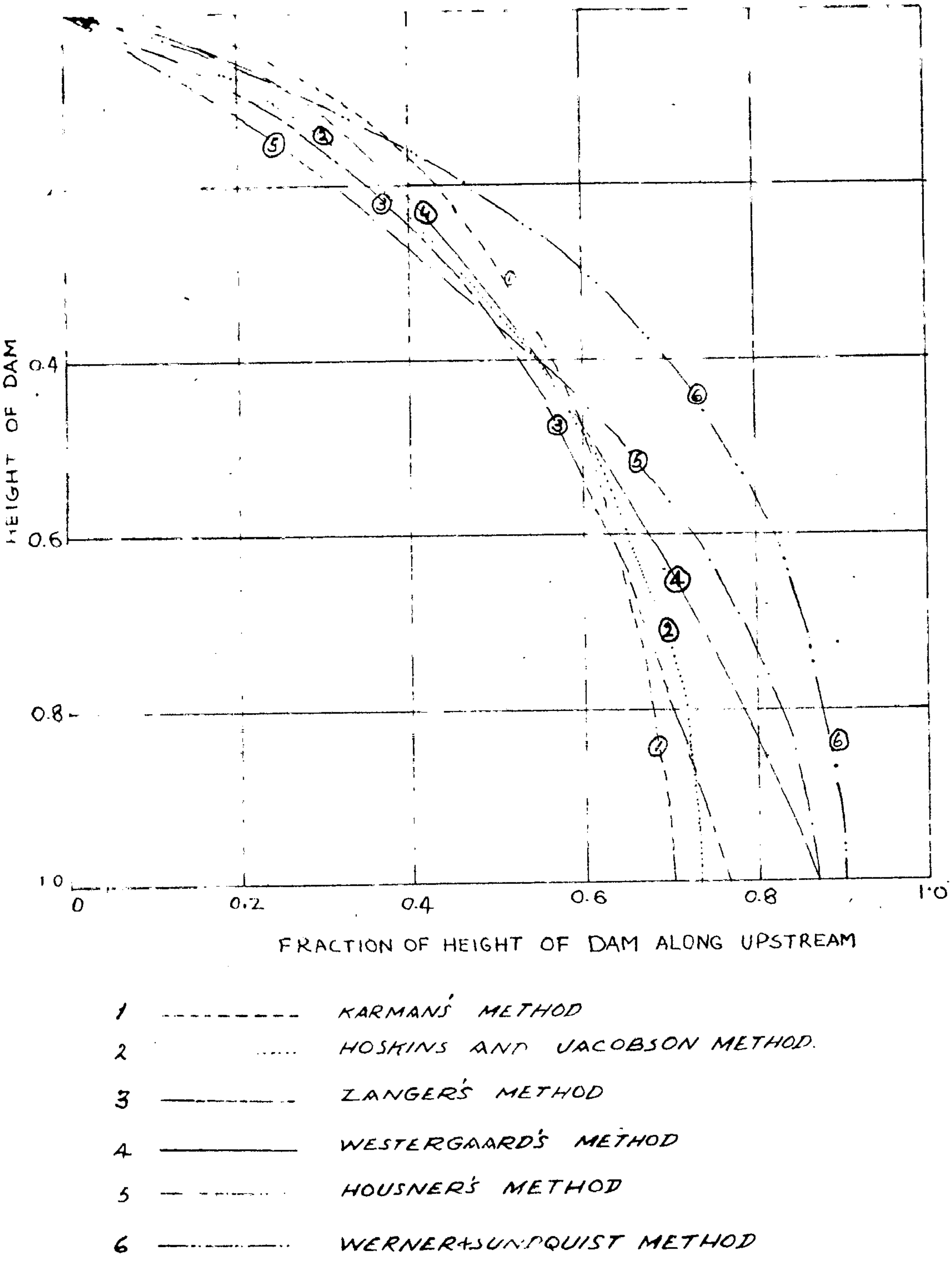
Φ = VELOCITY POTENTIAL.

FIG. No 23



D.W.P DUE TO IRREGULAR EARTHQUAKE

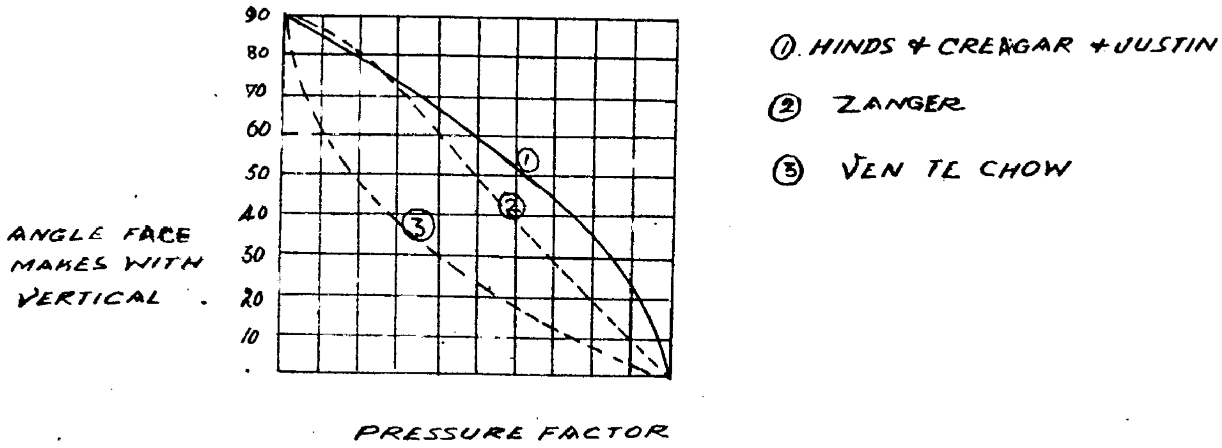
FIG 25. FEB. 23, 1939 JAPAN



PROFILE OF WATER SUPPOSED TO BE VIBRATING WITH A DAM FOR $l/h = \infty$

COMPARISION OF VARIOUS METHODS FOR U/S OF DAM VERTICAL.

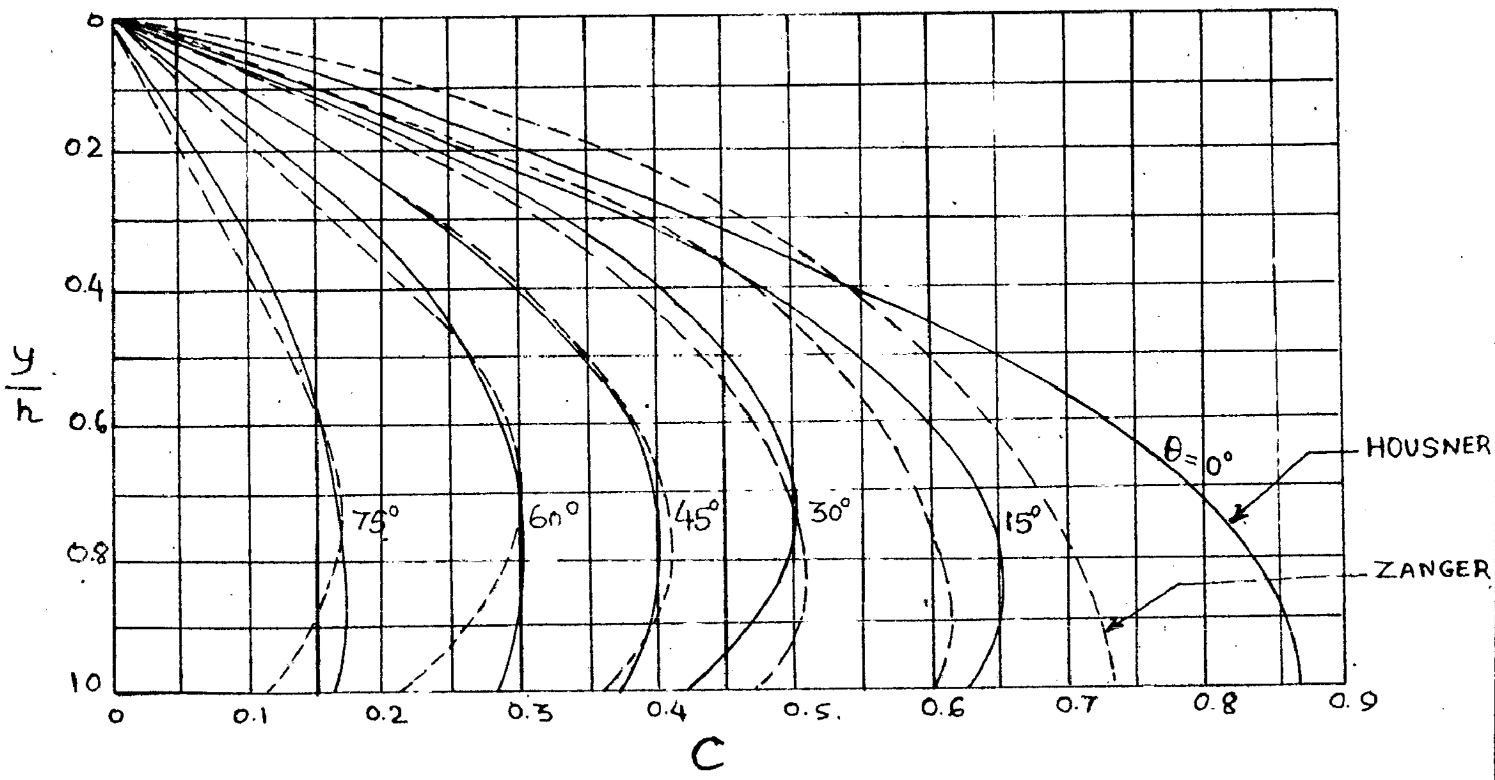
FIG. NO. 27.



$f = \frac{\text{MAX}^m \text{ PRESSURE ON SLOPING FACE}}{\text{MAX}^m \text{ PRESSURE ON VERTICAL FACE}}$

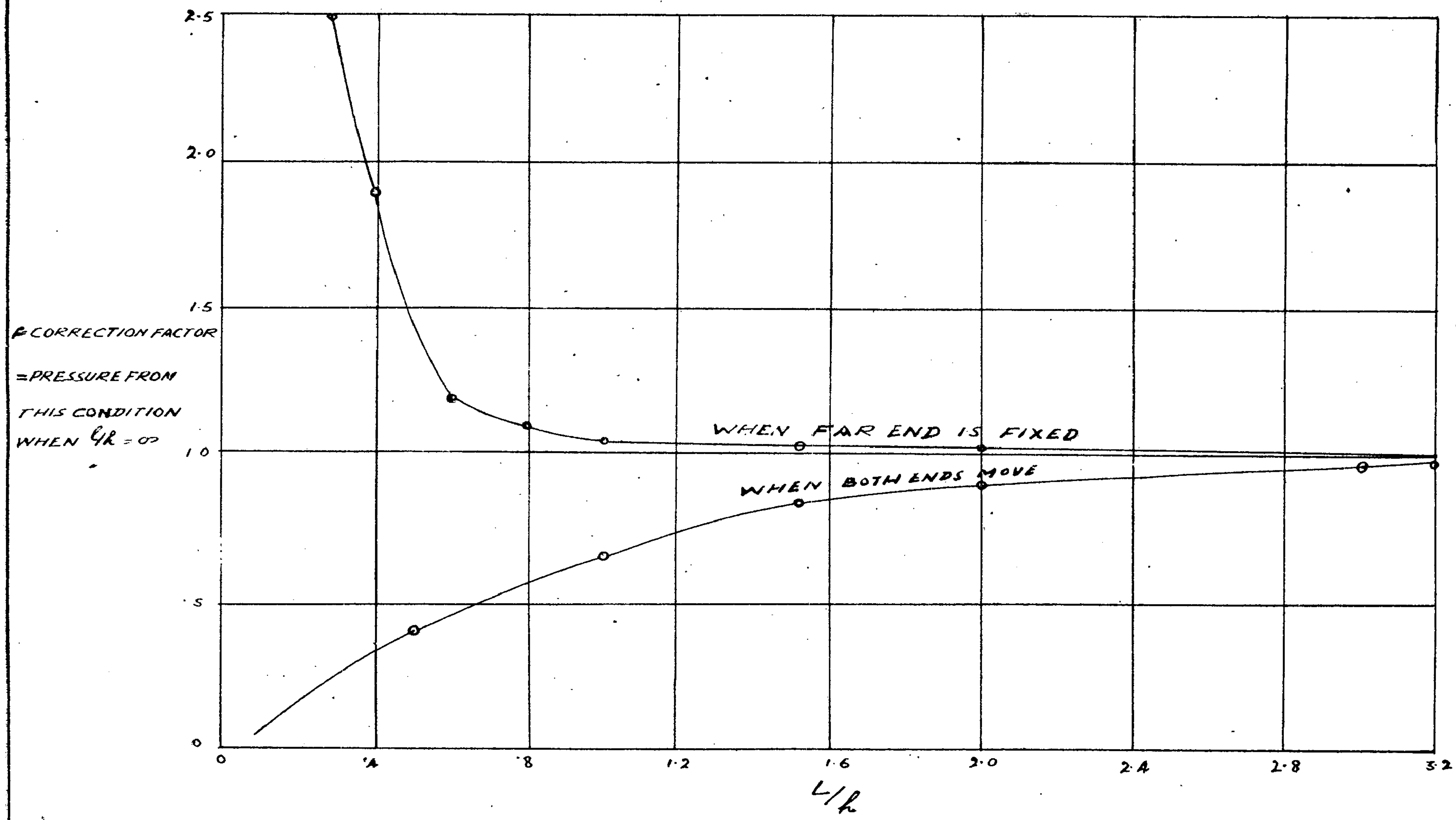
COMPARISION OF VARIOUS METHODS FOR DETERMINATION OF D.W.P. ON SLOPING U/S FACES OF DAMS

FIG. NO. 28



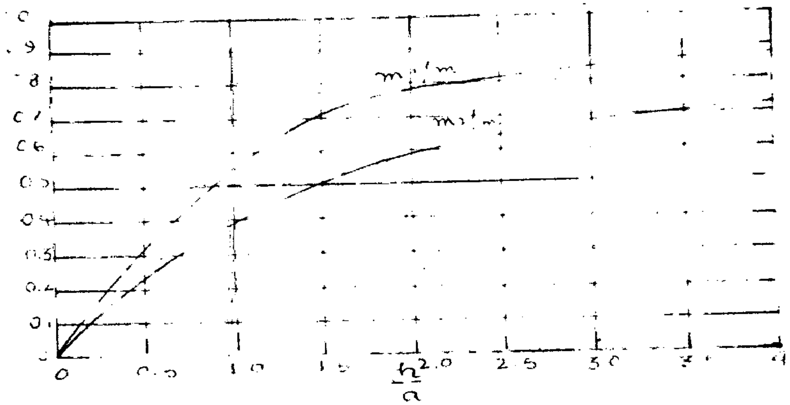
COMPARISION OF HOUSNER AND ZANGER'S RESULTS FOR D.W.P. DISTRIBUTION ON SLOPING U/S FACES OF DAMS

FIG. NO. 29



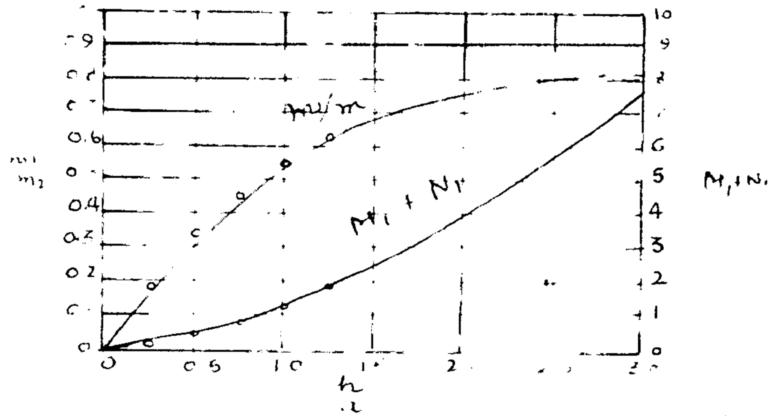
(BRAHTZ & HEILTON.)

FIG 30



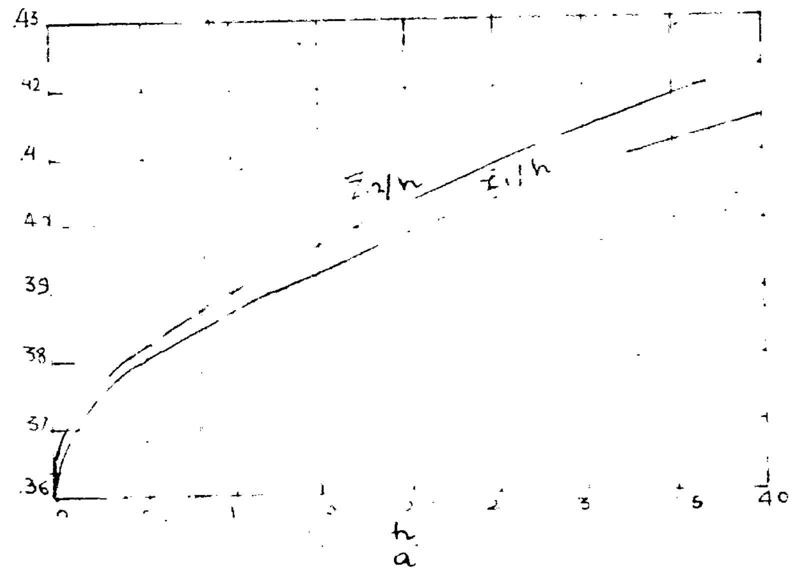
RATIOS OF EFFECTIVE HYDRODYNAMIC MASS m_1 AND m_2 TO TOTAL MASS OF THE FLUID WITHIN A TANK [illegible]

FIG. No. 33



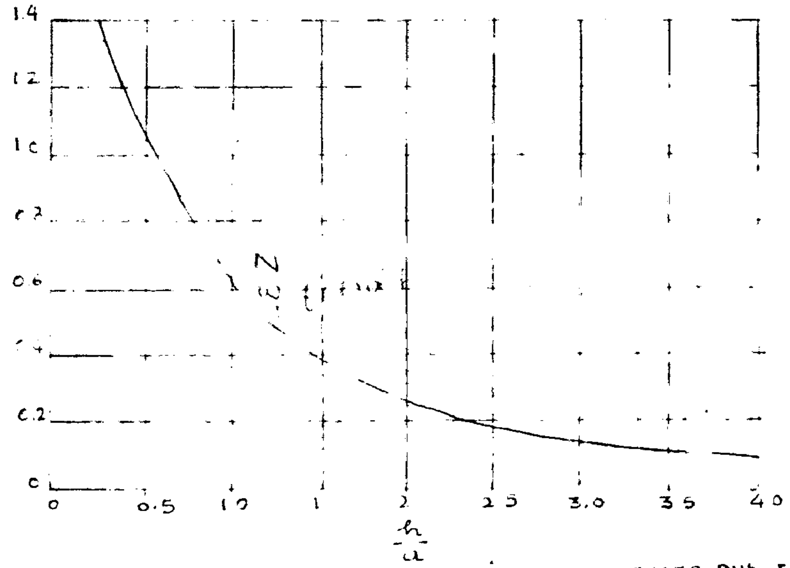
EXPERIMENTAL POINTS ON THE THEORETICAL CURVES FOR A CYLINDRICAL TANK OF a = 11.56", $f''(t)$ = 12 g

FIG No. 31.



RATIOS OF EFFECTIVE MASS CENTERS [illegible] REFFERED TO THE DEPTH h + H

FIG. No. 34.



RATIO OF EFFECTIVE MASS MOMENTS DUE TO THE FLUID AND FICTITIOUS MASS MOMENTS $\frac{1}{3}f''(t)a^2h^2$ ACTING ON THE BOTTOM OF THE TANK.

FIG. No. 35.

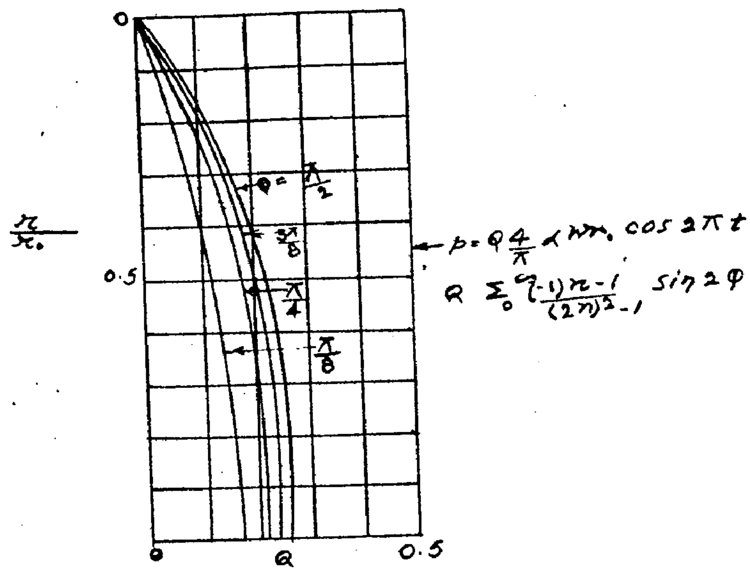
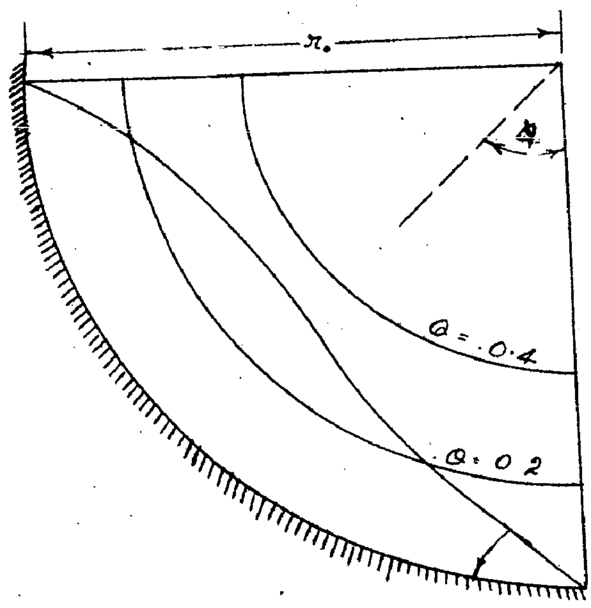


FIG. NO. 39

EVALUATION OF EQ. NO. 82.



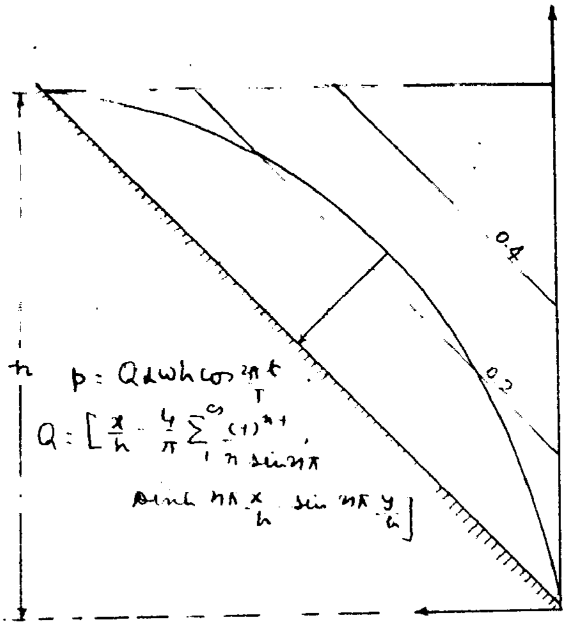
$$p = Q \frac{8}{\pi} \lambda w r_0 \cos \frac{2\pi t}{T}$$

$$Q = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \frac{a_{mn}}{(2n+1) c_m} J_{2n+1} (\beta_m r) \sin (2n+1) \varphi$$

EVALUATION OF EQ. NO. 81.

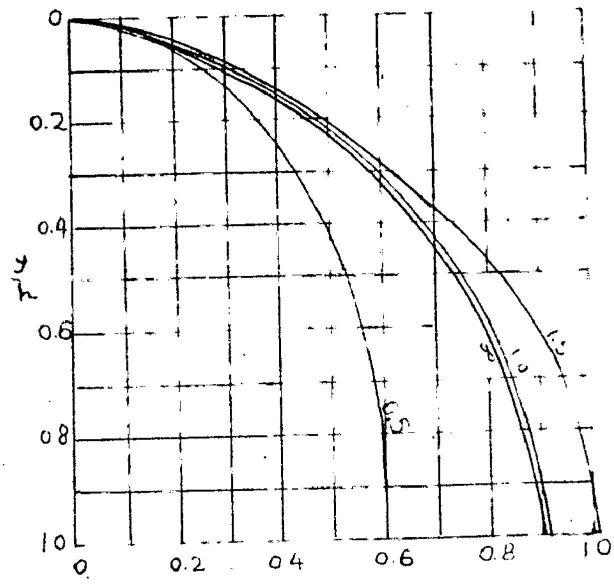
$l/r_0 = \infty \quad 1 = 1.$

FIG. NO. 38



EVALUATION OF EQ. NO 83.

FIG. No. 41.



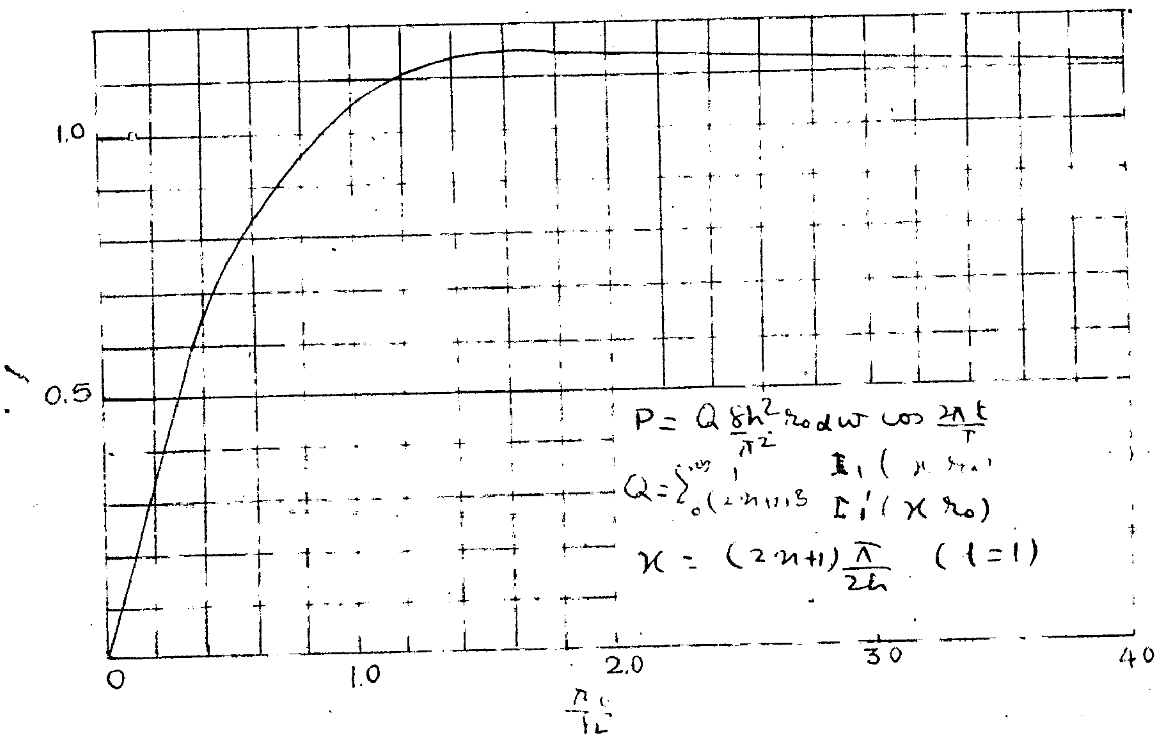
$$p = Q \frac{8h}{\pi^2} \rho \omega \cos \varphi \cos \frac{2\pi t}{T}$$

$$Q = \sum_{0}^{\infty} \frac{1}{(2n+1)^2} \frac{I_1(\chi r_0)}{I_1'(\chi r_0)} \sin \chi y$$

$$\chi = (2n+1) \frac{\pi}{2h} \quad (l = 1)$$

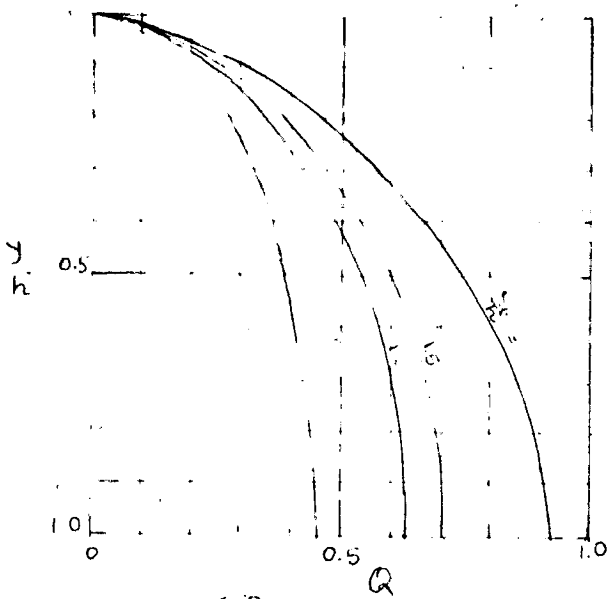
EVALUATION OF EQ. No 84.

FIG. No. 43.



EVALUATION OF EQ. No. 85.

FIG. No. 44.



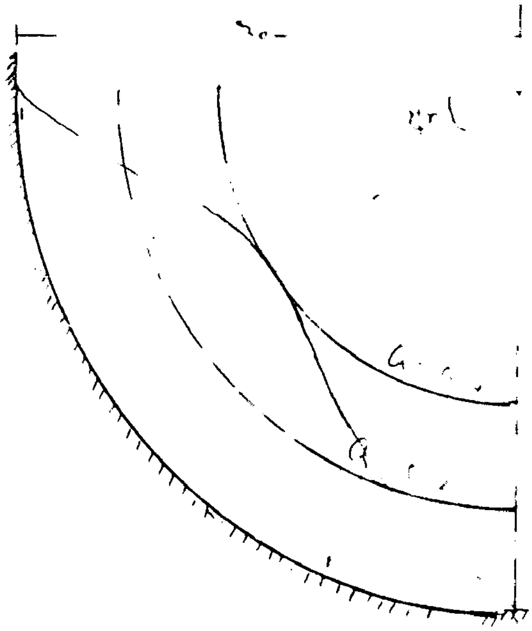
$$p = Q\frac{8h}{\pi^2}\alpha w \cos\varphi \cos\frac{2\pi t}{T}$$

$$G = \sum_0^\infty \frac{1}{(2n+1)^2}\frac{K(\varkappa r_0)}{K_1'(\varkappa r_0)}\sin\varkappa y$$

$$\varkappa = (2n+1)\frac{\pi}{2h} \quad (1 = N)$$

EVALUATION OF EQ. No. 86

FIG. No. 46.

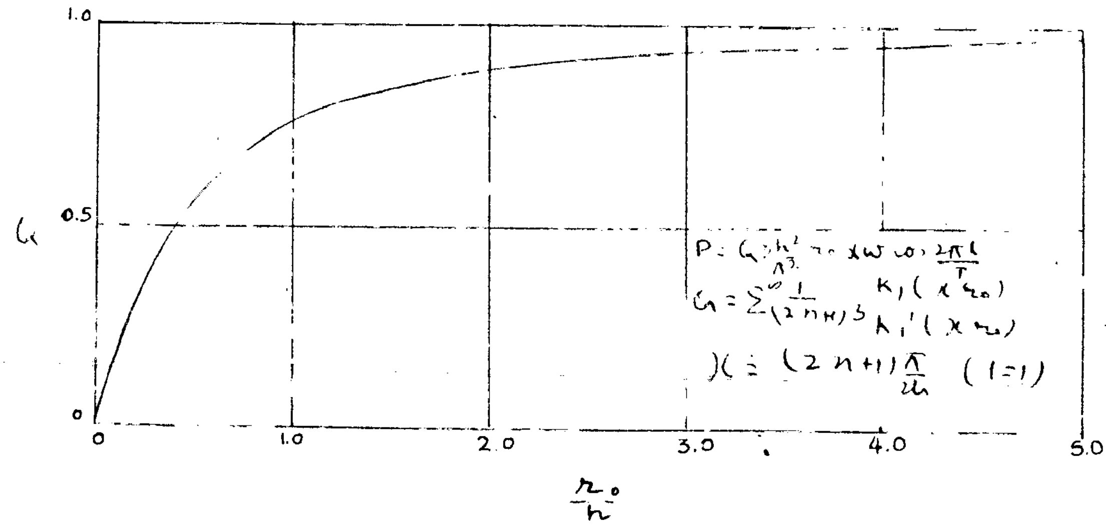


$$p = G\alpha w r_0 \cos\varphi \cos\frac{2\pi t}{T}$$

$$G = \sum_1^\infty a_n \frac{4n+1}{4n(2n+1)} P_n(\cos\varphi)$$

EVALUATION OF EQ. No. 88

FIG. No. 49.



$$p = G\frac{8h^2}{\pi^3} r_0 \alpha w \cos\frac{2\pi t}{T}$$

$$G = \sum_0^\infty \frac{1}{(2n+1)^3}\frac{K_1(\varkappa r_0)}{K_1'(\varkappa r_0)}$$

$$\varkappa = (2n+1)\frac{\pi}{2h} \quad (1=1)$$

EVALUATION OF EQ. No. 87

FIG. No. 47.

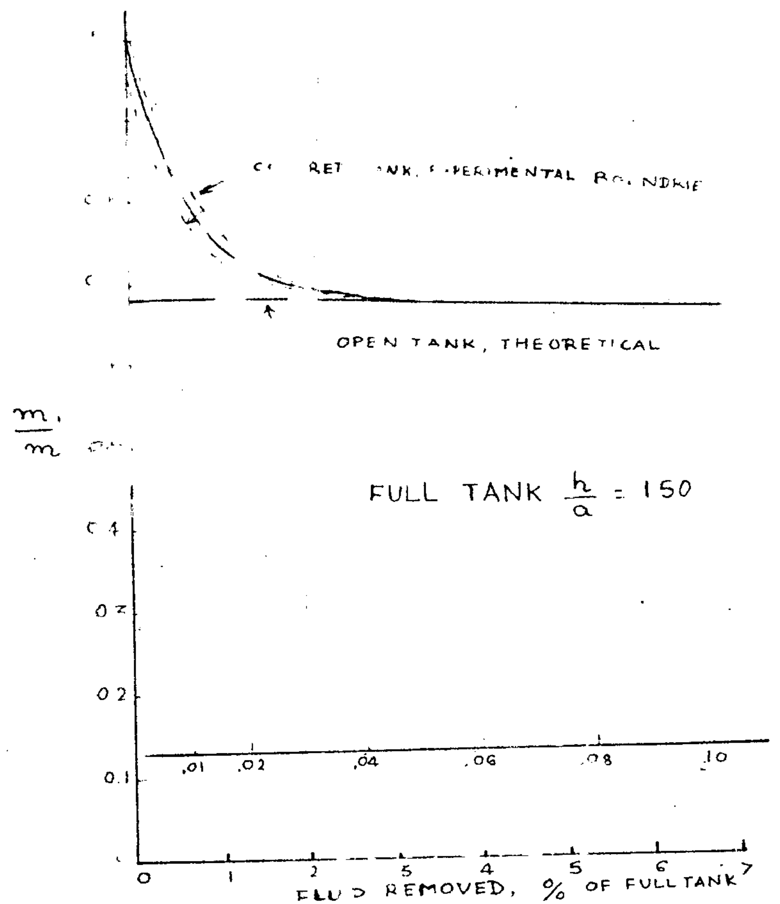


FIGURE NO. 50.

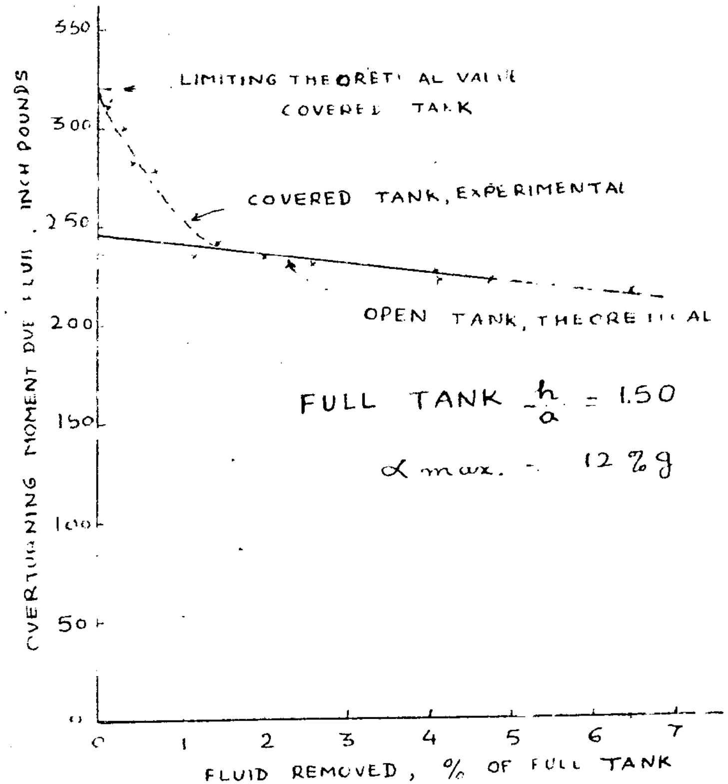
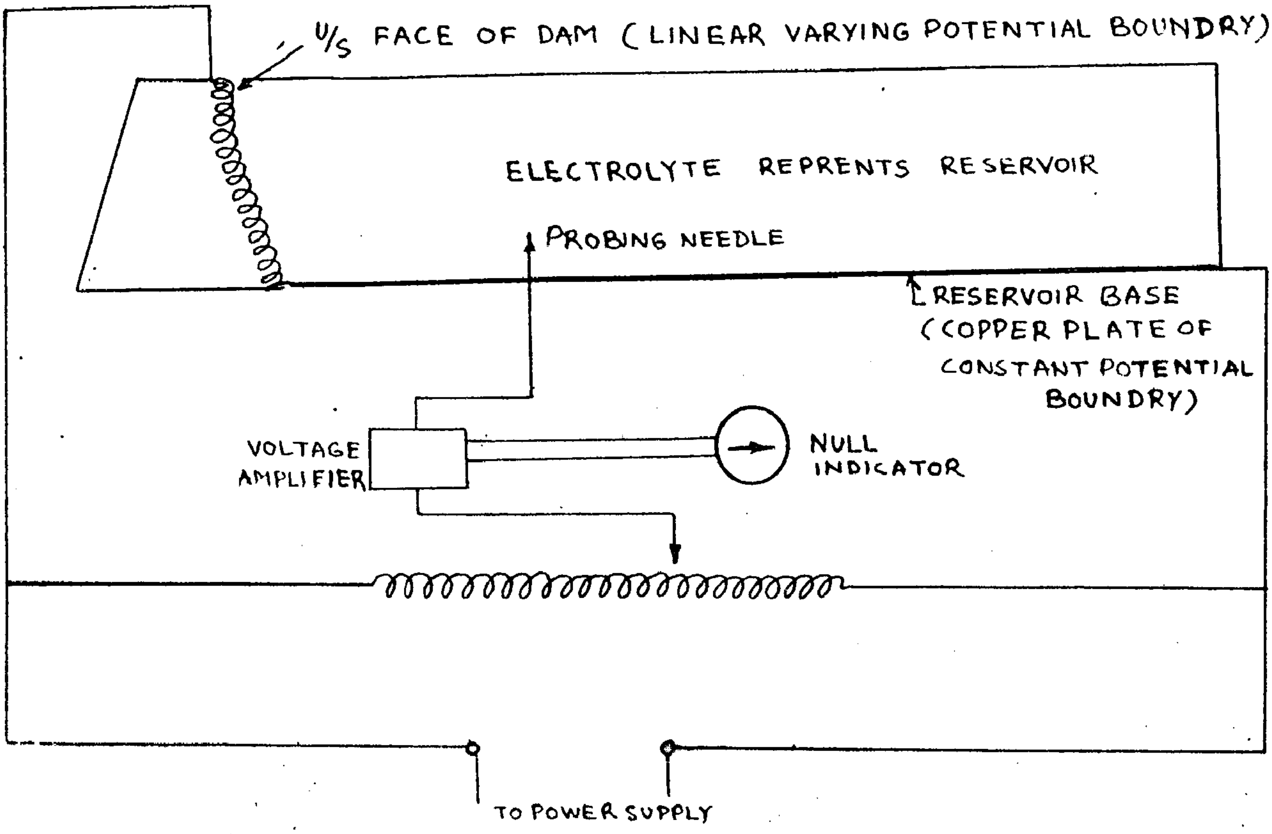


FIGURE NO. 51.

INFLUENCE OF RIGID COVER ON THE EFFECTIVE HYDRODYANAMIC MASS AND OVERTURNING MOMENT DUE TO FLUID IN A CIRCULAR TANK

[JACOBSEN AND AYRE]



ELECTRIC ANALOGY TRAY MODEL
DIAGRAMMATIC LAYOUT
FIG. No. 56.

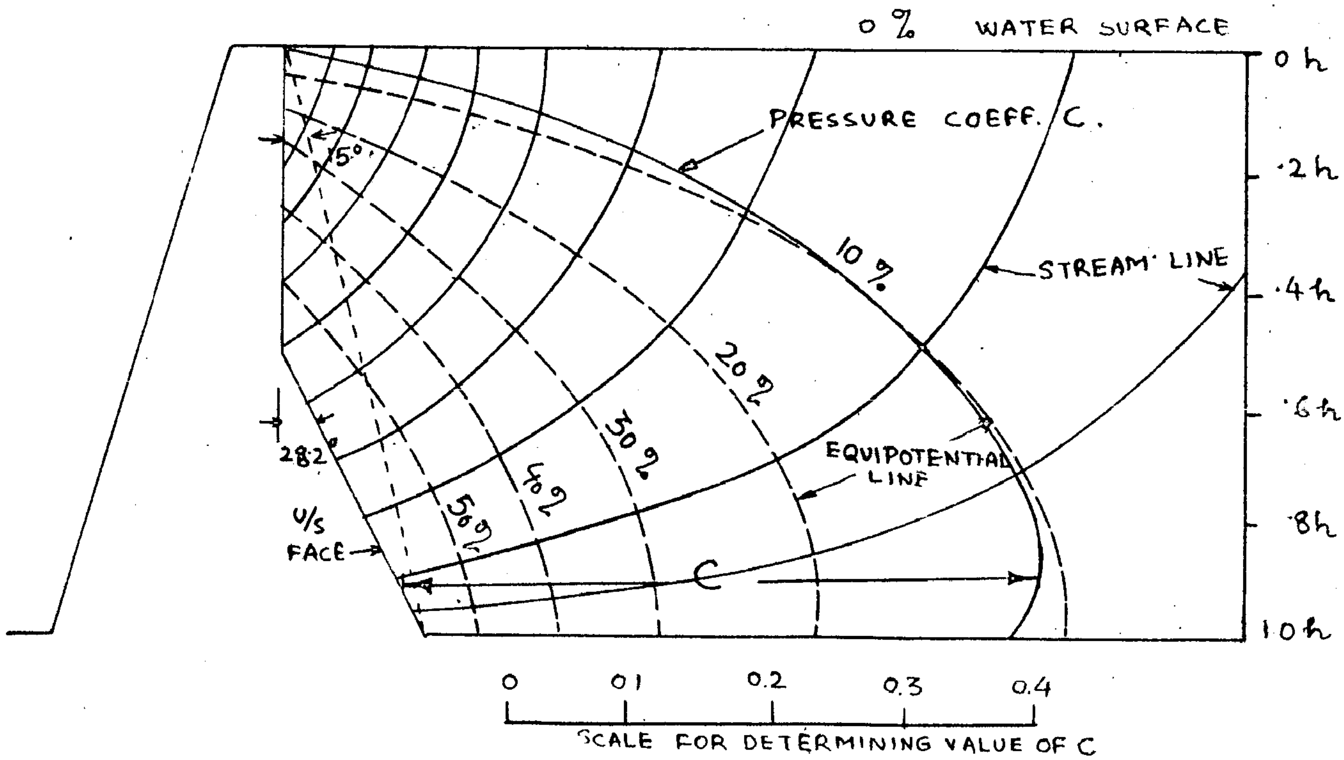
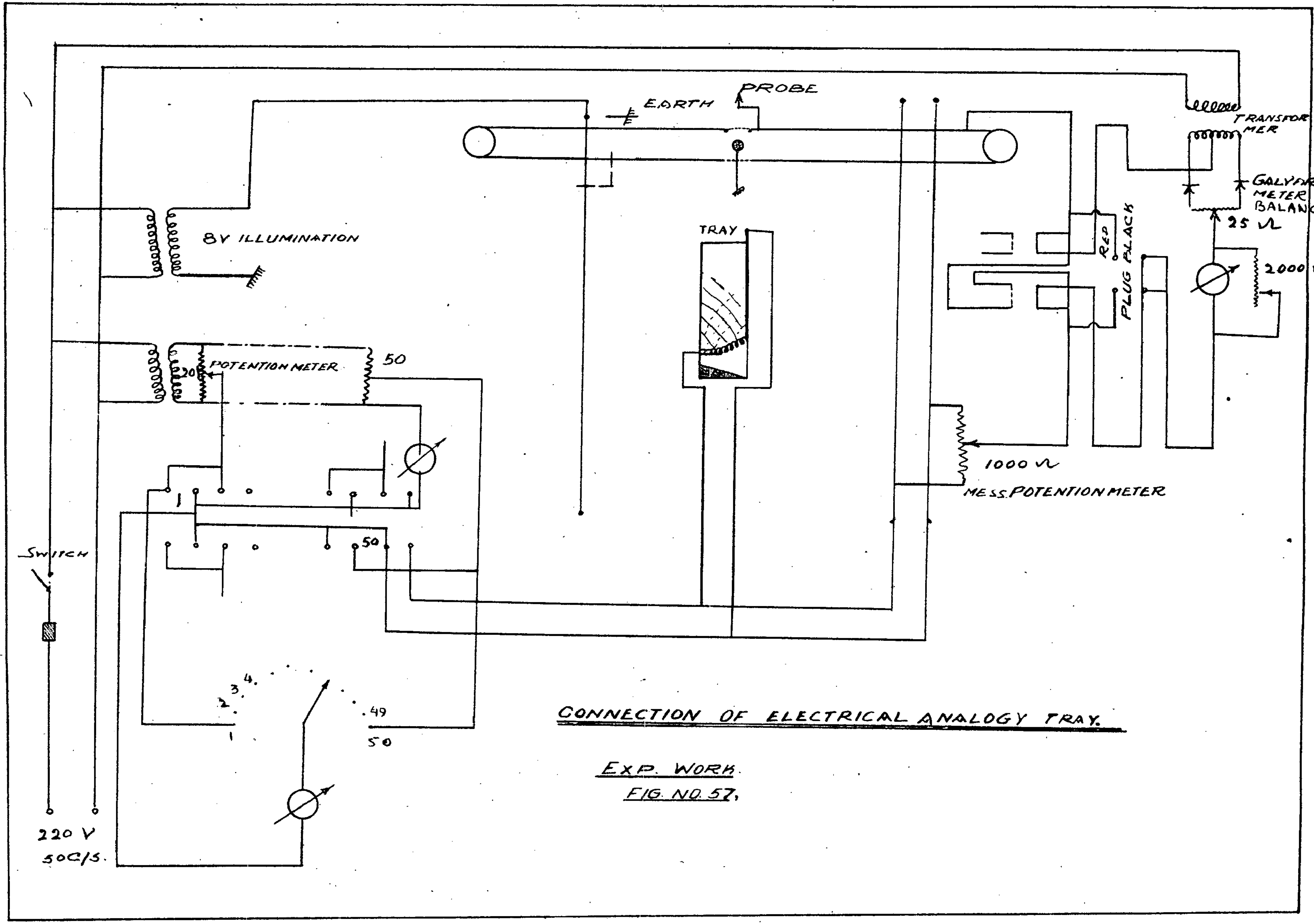


FIG. No. 58. TYPICAL FLOW NET



CONNECTION OF ELECTRICAL ANALOGY TRAY.

EXP. WORK.
FIG. NO. 57.

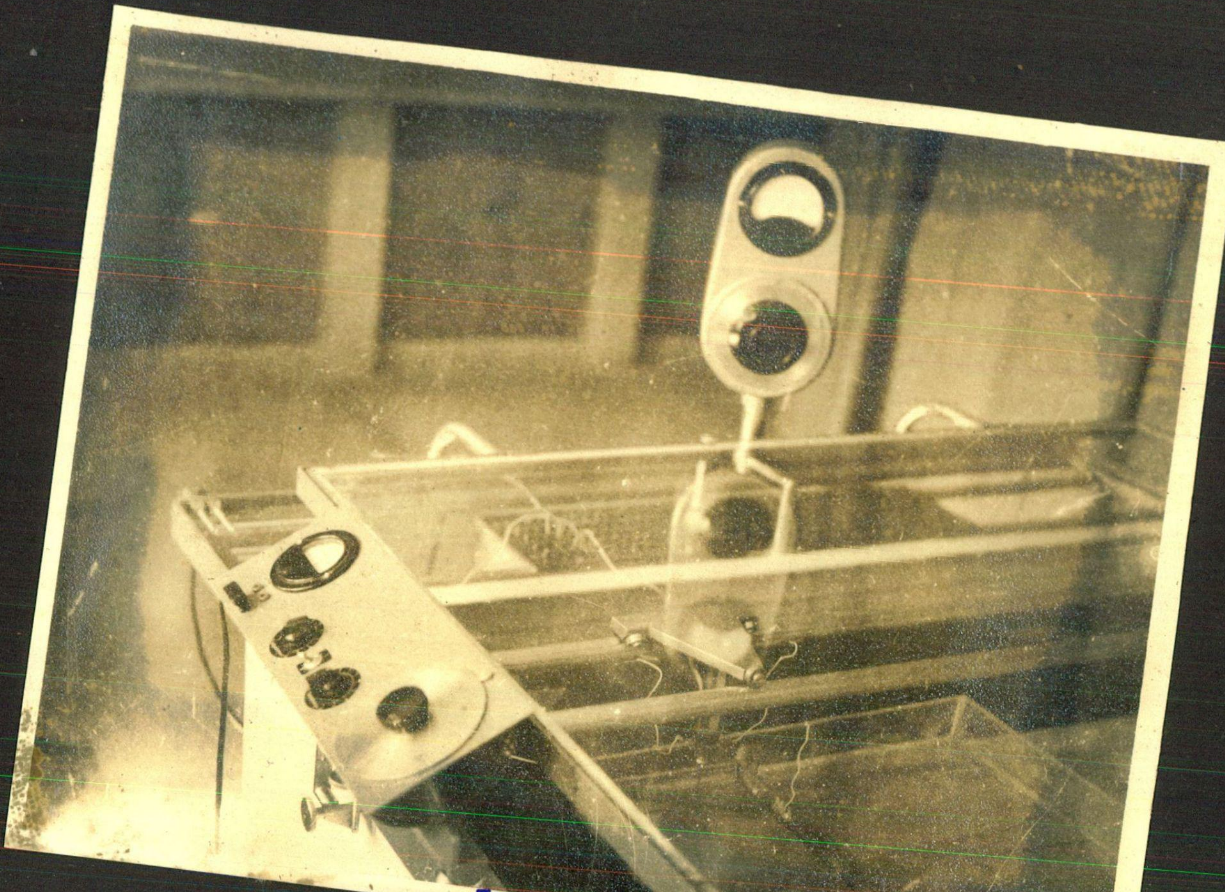
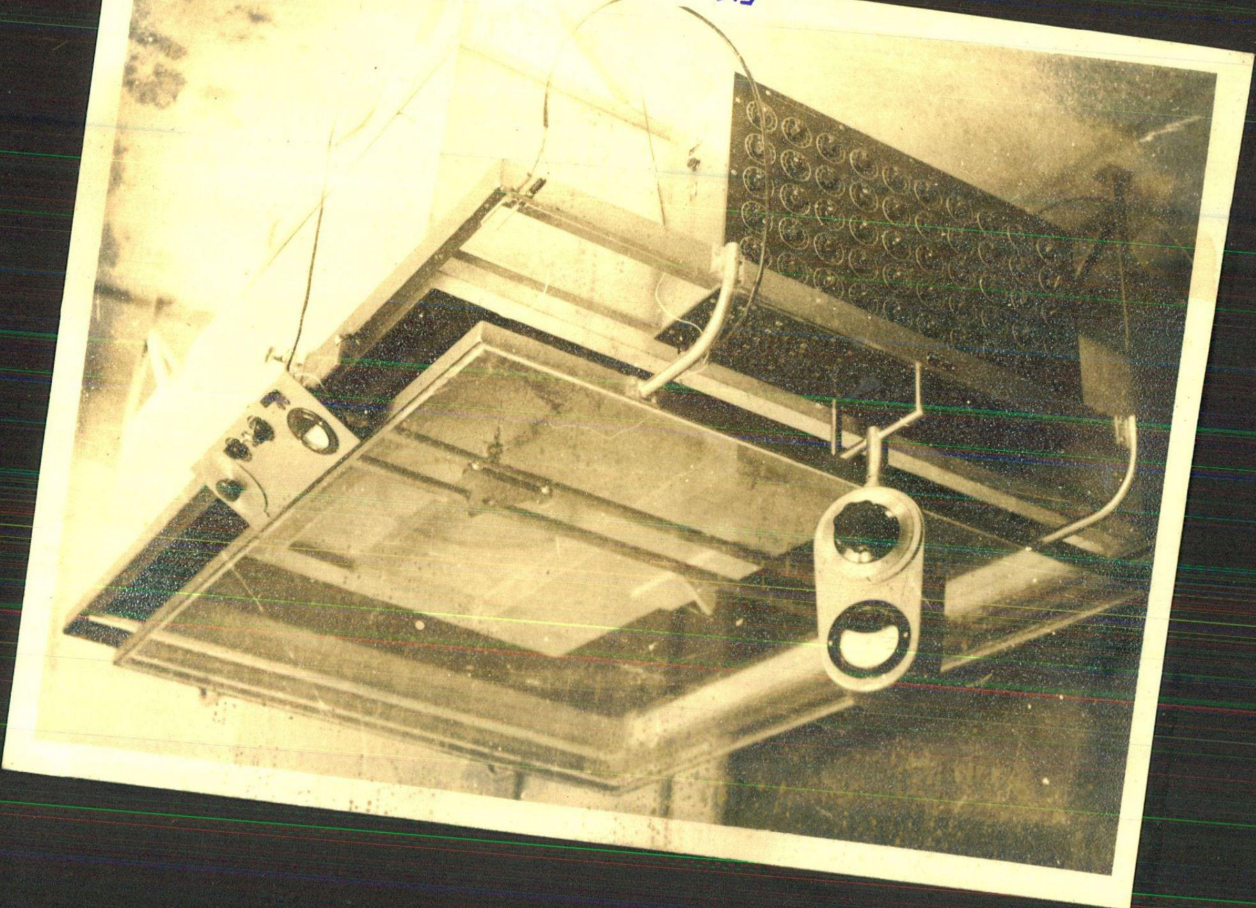


FIG 59

FIG 60



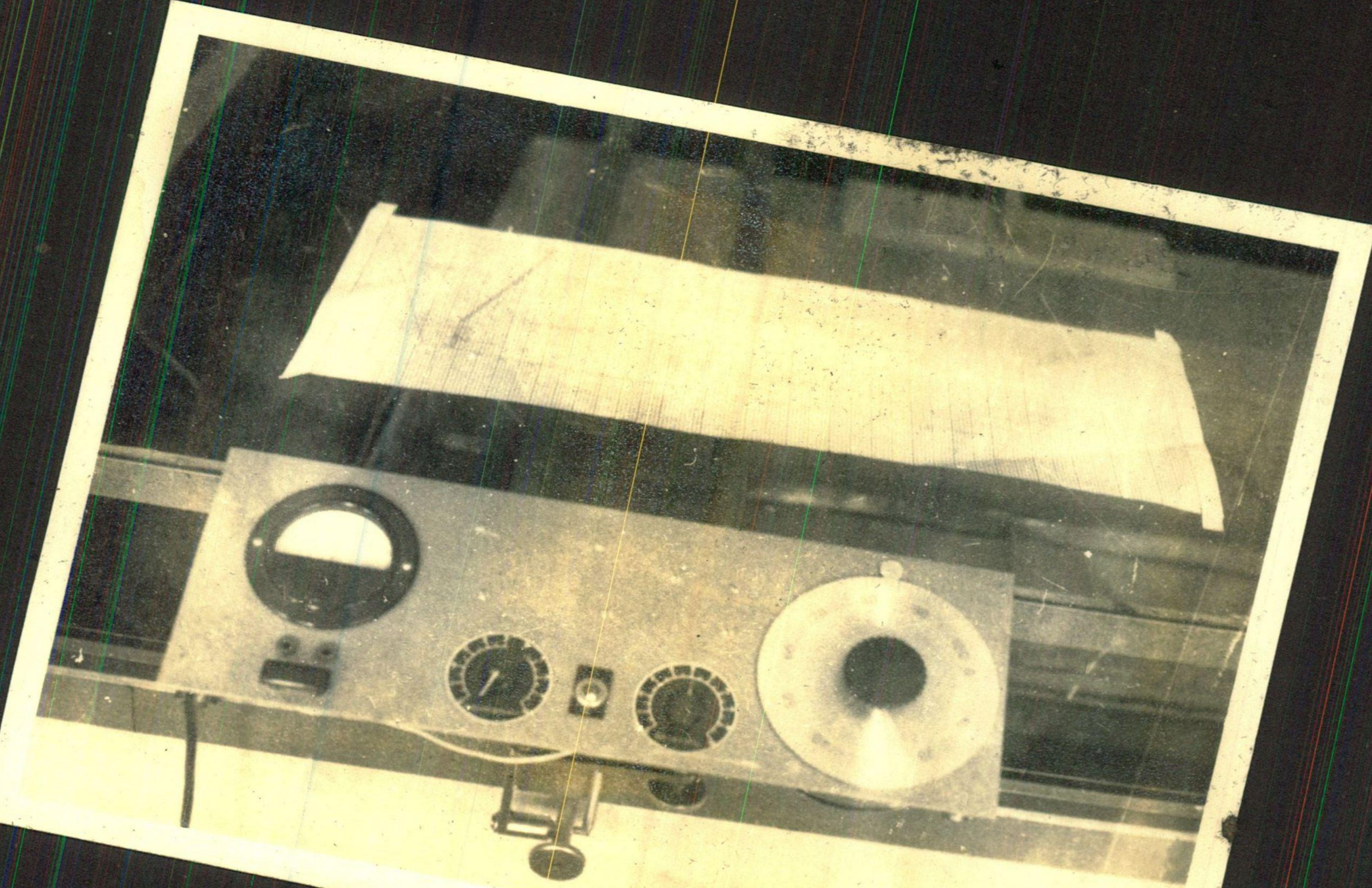


FIG 61

EXPERIMENTAL RESULTS

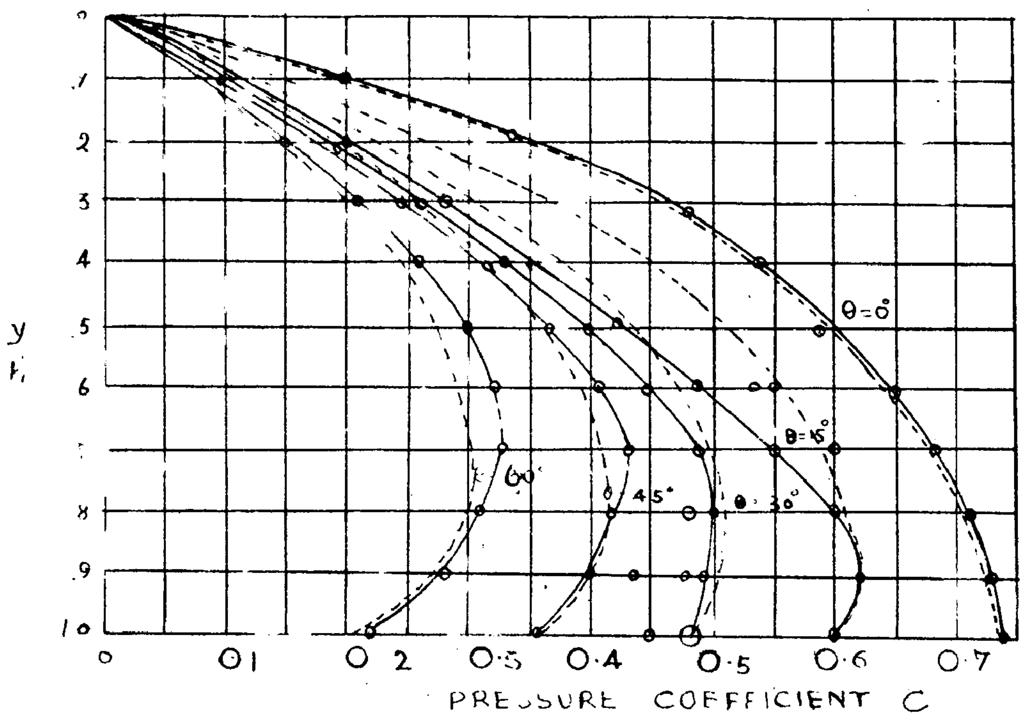


FIG. NO. 62.

COMPARISION OF EXPERIMENTAL AND ZAGER'S RESULTS FOR SHAPES OF DAMS SHOWN

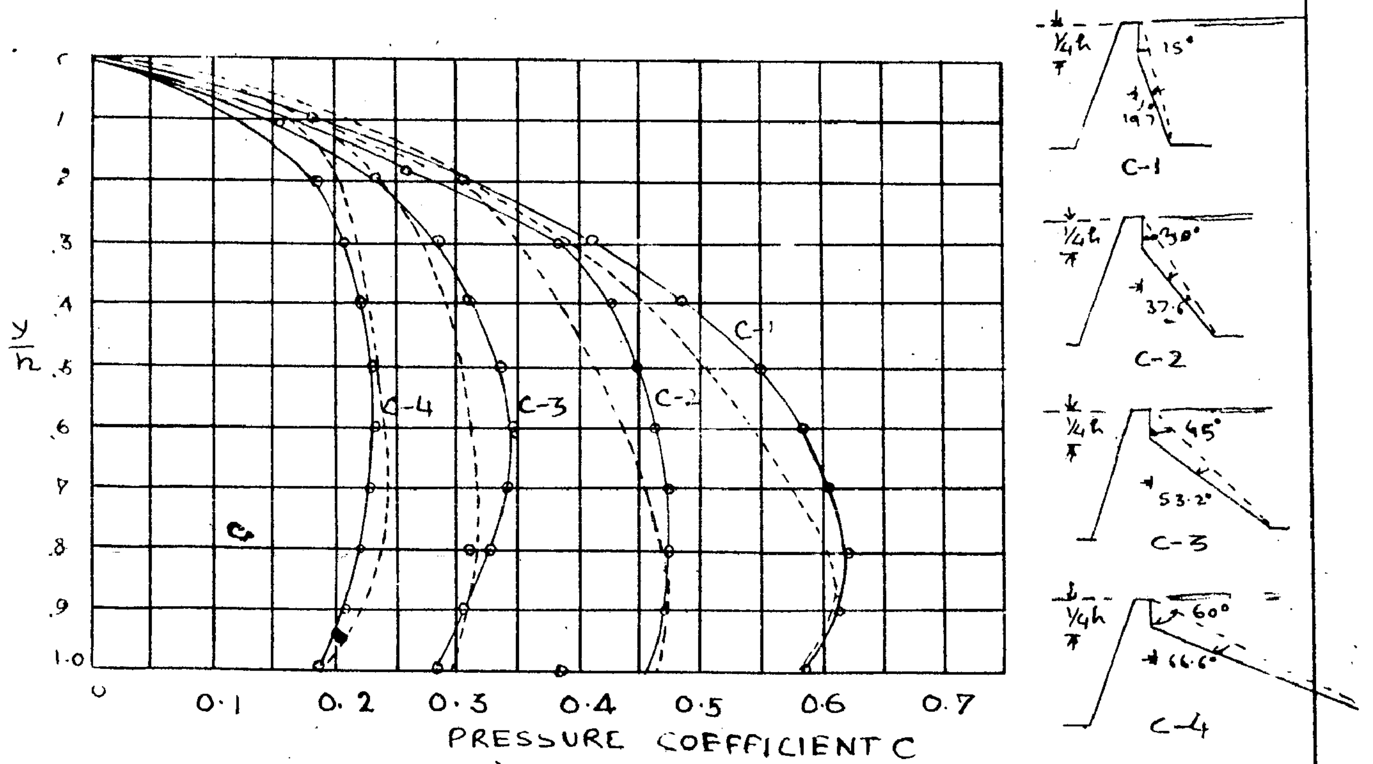


FIG. NO. 63.

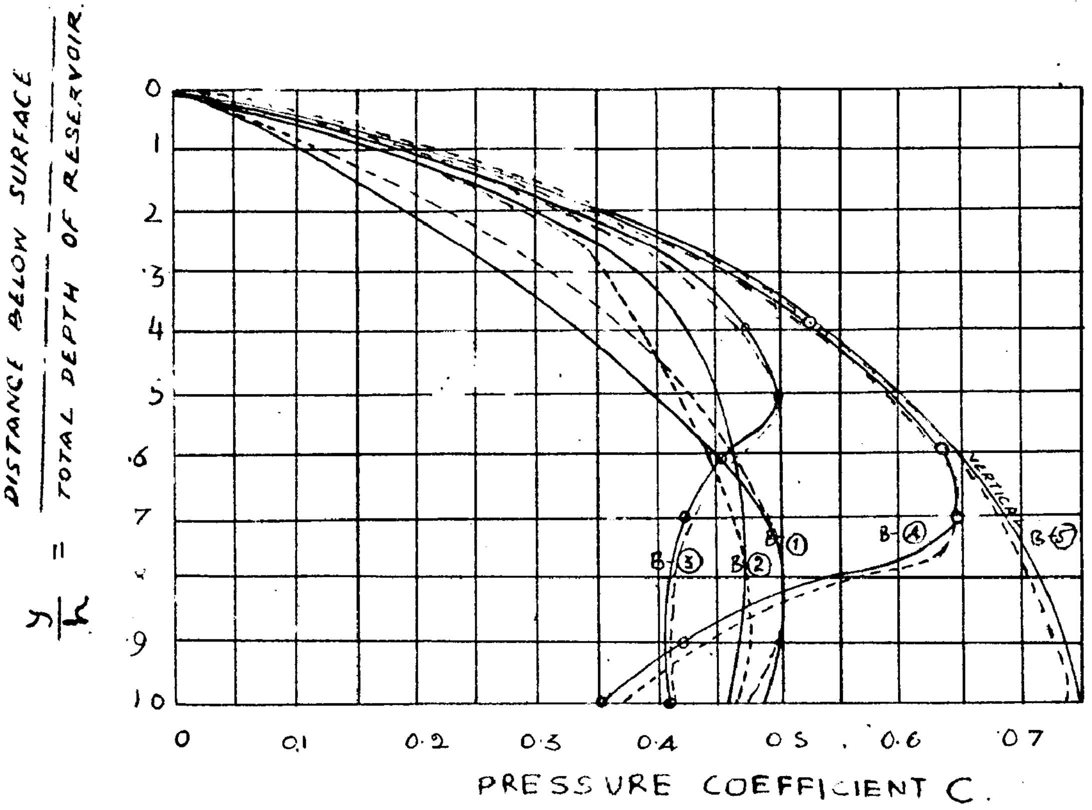
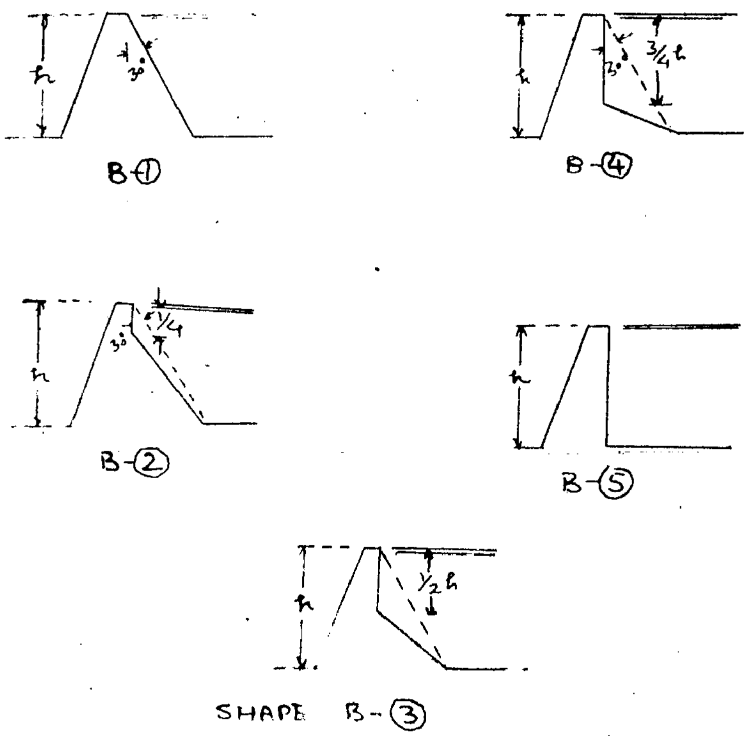


FIG. NO. 64.

COMPARISION OF EXPERIMENTAL AND ZANGER'S RESULTS FOR SHAPES OF DAMS SHOWN.



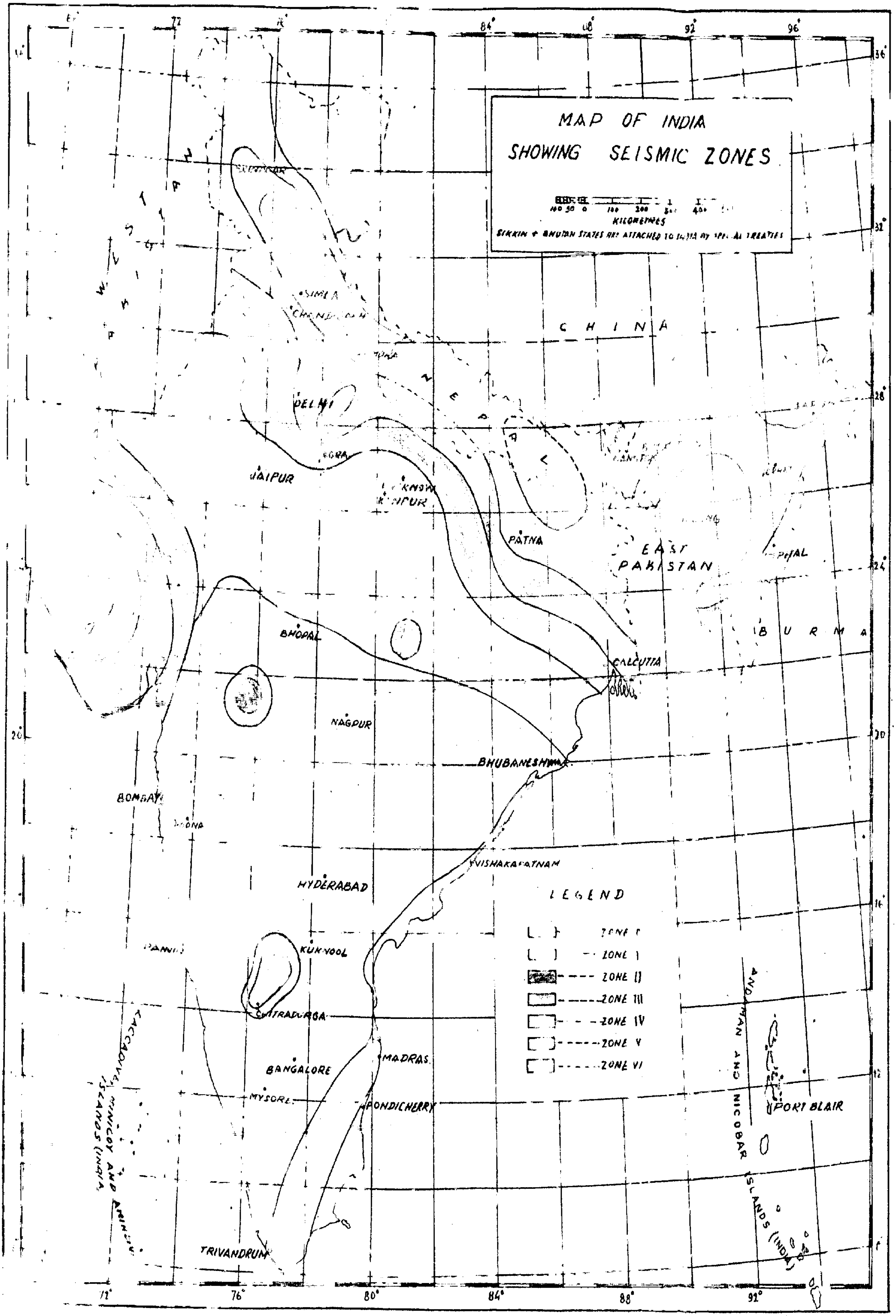

MAP OF INDIA
SHOWING SEISMIC ZONES
KILOMETRES
CHINA
NEPAL
EAST PAKISTAN
BURMA
SIMLA
DELHI
AGRA
JAIPUR
PATNA
CALCUTTA
BHOPAL
NAGPUR
BHUBANESHWAR
BOMBAY
POONA
VISHAKAPATNAM
HYDERABAD
KURNOOL
CHITRADURGA
MADRAS
BANGALORE
MYSORE
PONDICHERRY
TRIVANDRUM
PORT BLAIR
ANDAMAN AND NICOBAR ISLANDS (INDIA)
LACCADIVE, MINICOY AND AMINDIVI ISLANDS (INDIA)
LEGEND
ZONE I
ZONE II
ZONE III
ZONE IV
ZONE V
ZONE VI
72°
76°
80°
84°
88°
92°
96°
36°
32°
28°
24°
20°
16°
12°

FIG. No. 65