A REVIEW OF THE BEHAVIOUR AND DESIGN OF MACHINE FOUNDATIONS

By
UMESH KUMAR BHATIA

A

THESIS

Submitted in Partial Fulfilment of the Degree of
MASTER OF ENGINEERING
(Soil Mechanics and Foundation Engineering)

DEPARTMENT OF CIVIL ENGINEERING,
UNIVERSITY OF ROORKEE,
ROORKEE U.P.
JUNE 1963,

CERTIFICATE

Certified that the thesis entitled "A REVIEW OF THE BEHAVIOUR AND DESIGN OF MACHINE FOUNDATIONS" which is being submitted by Shri Umesh Kumar Bhatia in partial fulfilment of the requirements for the degree of Master of Engineering in Soil Mechanics and Foundation Engineering of University of Roorkee is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this thesis has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for a period of 7 months from 1st Nov. 1962 to 1st June 1963, for preparing the thesis for Master of Engineering Degree at this University.

Dated June 8, 1963.

S. Prakash 8/6

(Shamsher Prakash)
Reader in Civil Engineering,
University of Roorkee,
Roorkee.

*****

ACKNOWLEDGEMENTS

The author wishes to express his deep and sincere gratitude to Dr. Shamsher Prakash, Reader in Civil Engineering, School of Research and Training in Earthquake Engineering, University of Roorkee for his valuable guidance, suggestions, keen interest in the work and encouragement at every stage during the preparation of the thesis.

The author is highly indebted to Shri H.A. Balakrishna Rao, Senior Scientific Officer (I), Central Building Research Institute, Roorkee for creating author's interest in the subject and help given from time to time.

-:0:-

TABLE OF CONTENTS.

CONTENTS (Contd.)

NOTATIONS

Symbol.		Units
A	- Area of the Foundation Base	L^2
a	- Dimensionless frequency factor $\omega r_o\sqrt{\rho/G}$	-
a_o	- Value of 'a' at resonance.	-
B	- Width of foundation or vibrator	L
b	- Mass ratio $= m_o/\rho r_o^3$	-
b_i	- Inertia ratio $= I_o/\rho r_o^5$	-
b_q	- Mass ratio due to quinlan $= m_o/2\rho r_o^3$	-
b_q'	- Mass ratio for infinite strip loading $= m_o'/\rho (B/2)^2$	-
C	- Ratio of W_s to W_f	-
c	- Damping coefficient	$FL^{-1}T$
C_m	- Mass factor.	-
C_i	- Inertia factor	-
d	- Diameter of circular vase $= 2r_o$	L
E	- Modulus of elasticity (Dynamic)	FL^{-2}
E(z)	Value of modulus of elasticity at depth Z	FL^{-2}
e	- Eccentricity factor $m_1 e/m_o$	L
F	- Resulting force acting on piston	F
F_o	- Maximum magnitude of dynamic force	F
F_o'	- Maximum magnitude of dynamic force per unit length (For strip footing)	FL^{-1}
F_y	-Vertical component of inertia force	F
F_x	-Horizontal component of inertia force	F
F'.	-Primary force	F
F"	-Secondary force	F

F_1, F_2 - Functions of f_1 and f_2, $F_1 = \dfrac{-f_1}{f_1^2 + f_2^2}$ -

$F_2 = f_2 / f_1^2 + f_2^2$

f - Operating frequency of machine T^{-1}

f_{nr} - Reduced natural frequency $F^{1/2}$ L^{-1} T^{-1}

f_o - Resonant frequency of machine foundation soil system T^{-1}

f_1, f_2 Functions of (ν , a) for circular vibrator -

f_3, f_4 - Functions of (ν,a) for infinite strip vibrator. -

G - Modulus of Rigidity FL^{-2}

g - Acceleration due to gravity LT^{-2}

H - Height of the c.g of machine above ground or height of the Cylindrical vibrator. L L

h - Equivalent surcharge $= \dfrac{\sigma_{st}}{\gamma}$ L

I_o - Mass moment of inertia. FLT^2

K - Spring constant FL^{-1}

K_x - K_y, K_z - Spring factor for displacement in x,y,z directions respectively. FL^{-1}

K_{yz}, K_{xz}, K_{xy} - Spring factor for rotation about x,y,z axes respectively. FL

K' - Dynamic modulus of subgrade reaction. FL^{-3}

k_o - Coefficient of earth pressure at rest. -

L - Distance between 2 Crank shafts. L

L - Length of the base of machine foundation L

L_p - Power input requirement $FL\ T^{-1}$

$L_p(1)$ Dimensionless power factor $= \dfrac{\rho^{5/2} \quad r_o^6}{2\ m_1^2\ e^2\ G^{3/2}} L_p$ -

$L_p(2)$ -Dimensionless power factor $\dfrac{2r_o^2\sqrt{\rho G}}{F_o^2} L_p$ -

L_T - Work performed in one cycle FLT^{-1}

l -	Length of connecting rod	L
l -	eccentricity of rotating masses	L
M -	Maximum magnitude of exciting couple	FL
m -	Shape factor depending upon L/B	-
m_o -	Mass of machine foundation	$FL^{-1}\ T^2$
m_o' -	Vibrator mass per unit length (infinite strip vibrator)	$FL^{-2}T^2$
m_1 -	Eccentric masses	$FL^{-1}\ T^2$
m_1' -	Eccentric masses per unit length (infinite strip vibrator).	$FL^{-2}\ T^2$
m_e -	Mass of crank pin (Rotating parts, including a part of connecting rod)	$FL^{-1}\ T^2$
m_r -	Mass of piston (Reciprocating parts including other part of connecting rod)	$FL^{-1}\ T^2$
m_s -	Soil mass (equivalent)	$FL^{-1}T^{-2}$
N -	Revolutions per minute	T-1
N_1 -	Magnification factor	-
p -	Ratio of Rayleight wave velicty to shear wave velocity - A function of Poisson's ratio.	-
P_d -	Damping force	F
P_a -	Dynamic force transmitted to ground.	F
Q -	Sum of static and dynamic loads	F
q_a -	Allowable bearing capacity of soil	FL^{-2}
r -	Ratio of length to breadth of the foundation base	-
-	Crank Radius	L
r_o -	Radius of circular vibrator	L
r_1 -	Radius of density pressure bulb.	L
S -	Dynamic stiffness (Dynamic load per unit deflection).	FL^{-1}

s - $\alpha h/B$ (Pauw 1953) -

U_b, V_b, W_b - Translatory displacements of the centroid of base of vibrator in x,y,z directions respectively. L

U_g, V_g, W_g - Translatoy displacement of the c.g. of machine foundation in x,y,z, directions respectively. L

U_n - Summation of all unbalances in the machine. FT^2

U_n = Dynamic force $/\omega^2$

V_c, V_R, V_s - Velocities of propogation of compressive wave, Rayleight wave, and shear wave respectively. LT^{-1}

W_o - Weight of machine and foundation. F

W_f - Weight of foundation. F

W_m - Weight of machine. F

W_p - Weight of piston (reciprocating parts including a part of connecting rod). F

W_s - Equivalent weight of soil participating in vibration. F

X, Y, Z- Maximum amplitude of vibrations in x direction (Horizontal) y direction (Horizontal) and z direction (vertical) L

x, y, z, The coordinares of centroid of base with respect to the three coordinate axes passing through c.g. of the machine foundation. L

x_c - Displacement of crankpin perpendicular to line of stroke. L

y_c - Displacement of crankpin parallel to line of stroke. L

y_p - Displacement of piston parallel to line of stroke. L

$Z^{(1)}$ - Non dimensional amplitude factor

$$= \frac{\rho r_o^3}{m_1} Z.$$ -

$z^{(2)}$ - Non dimensional amplitude factor

$= \frac{Gr_o}{F_o} Z$ -

α - Twice the slope of truncated spring.

β - Rate of increase of modulus of elasticity with depth FL^{-3}

γ - Unit weight of soil FL^{-3}

$\gamma_x, \gamma_y, \gamma_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}$ - Spring factors (Pauw) for horizontal contact surface (xy) for three translatory and three rotary modes of vibration.

θ - Angular rotational displacement of piston

$\theta_x, \theta_y, \theta_z$ The rotational displacements of machine foundation about x,y,z axes respectively.

λ - Damping factor T^{-1}

λ_1, λ_2 - Constants of equation (3.40) -

μ - Decay factor. L^{-1}

ν - Poisson's ratio. -

ρ - Mass density of soil. $FL^{-4} T^2$

σ - Stress FL^{-2}

$\sigma_{st.}$ - Static stress FL^{-2}

σ_{dyn} - Dynamic stress FL^{-2}

τ - Period of vibrations. T

Φ - Angular displacement of crankpin -

ϕ - Amplitude of rotation.

$\phi_{x,y,z,}$ - Amplitude of rotation about x,y,z, axes respectively. -

$\phi^{(1)}_{max}$ - Max. amplitude factor $= \frac{Gr_o^3}{M} \phi_{max}$

$\phi(\nu)$ - Function of Poisson's ratio,.

ψ - Phase angle difference between the dynamic force applied and the resulting vibrations.

ω - Angular velocity or circular frequency T^{-1}

ω_o - Natural circular frequency. T^{-1}

LIST OF FIGURES.

LIST OF FIGURES (contd.)

LIST OF TABLES

LIST OF TABLES (contd.

......

SYNOPSIS

A machine foundation differs from any other type of foundation, because of the dynamic nature of loads. Till about 1930, a machine foundation was designed based upon emperical methods. These methods did not take into account the properties of the underlying soil. In the years to follow, attempts have been made to understand the problem scientifically.

Characteristics of the underlying soil strata affect the resonance of the system with the machine. The available literature on the subject is scattered and no systematic investigation, covering the present trend is available. The present investigation of the behaviour and design of machine foundation is intended to make a systematic study of available literature and is believed to lead to a better understanding of the problems connected. The various approaches to the design of machine foundations are critically reviewed, compared with each other and their limitations discussed. A simple emprical equation for determination of resonant frequency of the system is developed. Suggestions for further research have also been made.

*****

CHAPTER - I

INTRODUCTION

1.1 GENERAL :

The function of a machine foundation similar to any other foundation, is to transmit the imposed loads safely on to the soil, on which it is placed. Its special feature, however, is that in addition to the satic load, due to the weight of machine, and the foundation, vibrating or pulsating forces varying with time have to be considered. Such forces may be of short duration, such as shock or impact forces in forging hammers or may vary periodically as in reciprocating and rotating machines with unbalance masses. As a result, waves or steady vibrations are set up in the foundation soil. If the natural frequency of the m/c foundation soil system happens to coincide with or lie close to the frequency of the exciting forces generated by the machine, excessive vibration amplitudes may occur, which may lead to the structural damage or the operational failure of the machine.

1.2 ROLE OF SOIL MECHANICS :

Problems connected with machine foundations, were not considered to be important till the advent of the thirties, when the greater use of heavy industrial plants, and consistent failure of machine foundations, attracted attention of designers (Tschebotarioff 1951). Before that, the design of machine foundations was purely empirical, the simplest being to provide heavy foundation blocks. It was considered adequate to rest the machine on a rigid foundation so as to avoid excessive amplitudes of vibration. It was believed that natural frequency of the rigid foundation would be higher than the operating frequency of the machine. But invariably the rigid foundation has to rest on the ground. The result is that rigid foundation transmits the vibrations to the ground, which is relatively elastic, so that danger of resonance is still there. The importance of this fact was realized, when even after the provision of rigid foundation, excessive vibrations due to resonance were caused, which led to operational failures of the machine. With the advent of soil mechanics, and the science of structural dynamics, the problem of machine foundation has been tackled more rationally and scientifically. The investigations, both theoretical and experimental have led to better understanding of the behaviour of machine foundations, resulting in economy. At the same time, the number of failures of machine foundations, which are mostly due to excessive amplitudes of vibrations, have been minimized.

1.3 CONCEPT OF RESONANT FREQUENCY :

Experimental studies on the phenomena of ground vibrations, were first systematically conducted by Degebo (Deustche Gesellsectioft für Boden-mechanik) organisation in Germany from 1928 to 1939. Their earlier experiments were with a vibrator without any variations in dynamic force and static forces. Their results showed that soil at any site has a natural frequency or self frequency of vibrations, depending only on type of soil (Lorenz 1934). This was further substantiated by Andrews and Crockett (1945) who independently determined the natural frequency by a study of resonance between heavy industrial plant, and the ground. At about the same time as the Degebo's experiments with vibrator on the ground, Vios, the Institute for Engineering Foundation Research, performed similar tests in Russia (Barkan 1936).

Later experimental and analytical studies (Lorenz 1934, 1953, Reissner 1936, Sung 1953, Quinlan 1953, Richart 1960, Tschebotarioff, 1948) have shown that the natural frequency of soil as such is meaningless because the resonant frequency is not only dependent upon the type of underlying soil, but also on the area and shape of contact, magnitude of dynamic and static loads. Hence it will be proper to use the term, resonant frequency of the machine, foundation & soil system, or simply the resonant frequency of the system.

The problem of determining the resonant frequency of the system has been tackled both by theoretical methods and empirical methods based on past data of resonance. The theoretical approaches which have been developed are based on the assumption of (a) soil as an elastic solid and (b) Soil as a spring, (usually elastic.)

1.4 SEMI-INFINITE ELASTIC SOLID :-

One of the theoretical approaches is concerned with the "Dynamic Boussinesq's Problem" . This approach considers the machine foundations resting on the ground and oscillating on the surface of the semi-infinite elastic, isotropic and homogeneous medium. Prominent contributions based on this concept are those of Reissner, (1936), Quinlan (1953), Sung (1953), for vertical vibrations and those of

Arnold, Bycroft, and Warburton (1955), Bycroft (1959) and Hseih (1962) for other modes of vibration viz., horizontal and rotational modes.

This analysis of course will necessitate the determination of the dynamic response of the ground in terms of dynamic soil constants that is modulus of elasticity or modulus of rigidity, and Poisson's ratio. Dynamic testing of soils is done by measuring the velocities of propogation in the medium. Rayleigh (1885), Lamb (1904), Leet (1950) and others have analysed the velocity of wave propagation in the semi-infinite elastic homogeneous and isotropic medium. Bergstrom and Linderholm (1946) Bernhard and Finelli (1953), Jones (1955, 1958) Bernhard (1958) Vanderpoel (1951), Nijboer and Vanderpoel (1953), Nijboer (1959) Heukelom and Foster, (1960) and many others have given the analysis applicable to soils for determining in-situ dynamic constants.

1.5 MASS SPRING SYSTEM :

The other theoretical approach is to assume the ground to be a spring, with or without, damping. In initial studies (Rausch 1926) spring was assumed to be weightless and linear. But experimental studies by Degebo (Lorenz 1934), Vois (Barkan 1936), and their subsequent analysis have shown that some soil mass also oscillates with the machine foundation. This mass of soil was found to be 4 to 10 times the vibrator mass by Degebo studies. Vios concluded it to be relatively insignificant and have neglected it to obtain the dynamic modulus of subgrade reaction.

? But indirectly it was accepted that soil mass could not be neglected and that its value must lie between 2/3 and 1½ times that of foundation. The above two statements seem to be contrary. Terzaghi (1943) recommends the soil mass to be 3 times the dynamic force transmitted to the ground. Anderews and Crockett (1945) Crockett and Hammond (1947, 48, 49) suggested that mass of soil which vibrates with the foundation must bear some relation to the bulb of stress, which gives the stress distribution under a uniformly loaded area on an elastic medium. None of them gave any precise relationship. It was, however, regarded that this mass must vary with the area of contact and the dynamic unbalance forces. Balakrishna Rao (1960, 61, 62) further advanced the concept of stress bulb and has suggested that the mass oscillating should be taken as the mass of the soil within the pressure bulb of the same intensity (lb./sq.ft.) as the density of the soil (lb./cu.ft.).

The value of the spring constant has been taken as the load required per unit reversible deflection (Barkan 1936; Newcomb 1951); Lorenz (1934), has obtained k' dynamic modulus ? of subgrade reaction and W_s the soil weight, from the two vibrator tests under different loading conditions and with different areas, by assuming both k' and W_s to be constant. But this assumption is not justified, as the tests have shown that both spring constant and soil weight vary with different vibrator sizes and loading conditions even on the same type of soil.

Pauw (1953) by assuming soil as truncated spring has given the expressions for spring factors and mass factors for different modes of vibration.

1.6 USE OF EXPERIMENTAL BEHAVIOUR :

Another approach uses the past records of resonant frequencies observed. Empirical relations have been developed. Tschebotarioff (1948, 51, 53) has obtained logirthmic relationship between reduced natural frequency and the contact area. Newcomb has plotted the resonant frequency versus the static pressures.

Another approach which considers soil as sublinear spring, uses the resonance curves obtained from the test vibrator, to plot the sub-linear characteristic of soil, has been put forward by Lorenz (1953), and Alpan (1961).

1.7 SCOPE OF STUDY :

The problem of machine foundation has been receiving importance since the thirties of this century. The importance of underlying soil strata has been realized, in respect of the resonance phenomena. The available literature is scattered and no systematic investigation covering the present trend is available. It is felt that this investigation of the behaviour and design of machine foundation, based on a systematic study of available literature, will lead to better understanding of the problems connected. The nature of the problem consists of a system to be analysed (Alpan 1961). This system consists of the machine, the foundation and the soil, and involves the following procedure :-

a) Weight and operating frequency of the machine and the magnitude of the dynamic forces, is given.
b) The properties and the dynamic response of the foundation soil are to be determined, or assumed.

c) The foundation is designed based on soil properties. The general shape and dimensions of which may be assumed for preliminary design.

The type of the foundation considered in this investiga- is massive block resting directly on the ground. Isolators and shock absorbers are not considered.

Review of the available literature on resonant frequency, leads to an interesting observation. Spring constant in all the approaches can be expressed as a simple multiple of G ro, while the mass factor a simple multiple of $\rho\ ro^3$,

where G is the modulus of rigidity of underlying soil,

ρ is the mass density of underlying soil,

and ro is the radius of the foundation base, in contact with soil.

Out of the available approaches to resonant frequency determination, the theory of vibrator resting on semi-infinite isotropic, homogeneous, elastic medium, is recommended for use in cases where soil can be assumed to have fairly uniform modulus of elasticity. This, if coupled with Hseih's transformation, gives easy way to calculate resonant frequencies. For soils, where modulus of elasticity can be assumed to increase linearly with depth Pauw's (1953) analysis is recommended. Limitations of the applicability of theoretical analysis to the machine problem have been discussed.

Almost all the experimental investigation reported in literature have been carried out for vertical vibrations. The approaches already existing should be verified for other modes of vibrations as well. Pressure distribution under machine foundation have

have not been investigated. Information on effect of dynamic load on bearing capacity is not available. Based on these observations, suggestions for further research have been made .

It is felt that the dynamic behaviour of soil be evaluated by observing the resonant frequency of a test vibrator under different loading conditions, and contact areas. The information obtained then can be used for determination of resonant frequencies of the foundation soil system.

CHAPTER - 2.

BEHAVIOUR OF MACHINE FOUNDATIONS

2.1 GENERAL :

Machine foundations are important substructures. For the safety of operation of every factory, dependable foundations of its machines, are essential. If the foundation is not properly designed, not only the machine gets damaged but the adjoining structures may also be damaged. In addition for proper working conditions in a factory, the vibrations produced by the machine should be such as not to interfere with the worker's comfort. For a proper design of machine foundation it is essential that its behaviour be understood. The discussion to follow has been prepared from evidence reported in literature from time to time. Also it is essantial to know the magnitude of the dynamic forces and its frequency. The dynamic loads as produced by a reciprocating machine is dealt in detail. The other cases have been briefly referred to. Finally the requirements of machine foundation based on its behaviour are discussed.

2.2 BEHAVIOUR OF A MACHINE FOUNDATION :-

A machine foundation is different from other foundations, mainly because this is subjected to a dynamic load which is usually periodic. Under the influence of this load, the foundation starts vibrating. For every system, there is a natural frequency, which is defined as the frequency with which it will vibrate, when subjected to free vibrations. For a body with spring stiffness as k and mass m_0, the natural frequency ω_0 is given by (neglecting damping)

$$\omega_0 = \sqrt{\frac{K}{m_0}} \qquad \text{................(2.1)}$$

Under forced vibrations, as in machine foundations the phenomena of resonance occurs, if the operating frequency coincides with this natural frequency. For no damping, the amplitude at resonance tends to infinity. If damping is included in the system, the amplitude of vibration is still maximum close to resonance, though of finite value. The ratio of actual amplitude to free amplitude (the static deflection of spring due to dynamic load) is called magnification factor N_1 (Denhartog 1947). At frequency ratio (the ratio of operating frequency to the resonant frequency) of 1.0, this magnification factor is maximum.

The transmissibility is defined as the ratio of force transmitted to the dynamic force applied. For small damping, transmissibility is maximum at frequency ratio of 1.0. For the machines having dynamic loads independent of frequency, it is maximum at frequency ratio of 1.0, for large damping as well. But for the machines having dynamic load proportional to the square of the frequency (which is true for rotating and reciprocating machines), transmissibility is maximum for higher frequency ratios and for large damping (Mykelstad 1956). In such cases it is preferable to keep frequency ratio much lower than 1.0.

In addition, at resonance, the power required to keep the system oscillating is maximum. This has been observed experimentally by Lorenz (1934), Crockett and Hammond (1948) and analytically by Reissner (1936), Sung (1953), Quinlan (1953). It is seen that at resonance, the amplitude of vibration, the force transmitted, and the power input requirement of the machine, are maximum. Hence resonance has to be avoided.

In an attempt to avoid resonance, the foundation was made rigid and firm. The natural frequency of such a rigid body is very high. The value of spring constants of a rigid body like mass concrete foundation is very high. The equivalent spring constant (K) is (Timoshenko 1937)

$$K \propto \frac{E}{1 - \nu^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2.2)$$

where E is modulus of elasticity.

and ν is the Poisson's ratio.

Values of E and ν for the concrete are of the order of 2 to 5×10^6 psi and 0.15 respectively. This would give a very high natural frequency of the foundation, with hardly any chance of resonance with machine's operating frequency. But it has been observed that even these massive foundations start vibrating and sometimes the amplitudes become quite large. The answer lies in the fact, that though the foundation is rigid in itself, it is resting on the ground. The ground is not so rigid and is relatively elastic. The value of E for soils is of the order of 10 to 15×10^3 lbs/sq.in., and poisson's ratio of 0.3 to 0.4. This means that soil and foundation are in series (two springs in series), with the soil characteristic predominating. That is why phenomena of resonance can be noticed even after providing rigid foundation. Therefore, the term resonant frequency should mean natural frequency of machine foundation and soil system.

Consider a rigid, concrete foundation block, which supports a steam engine with speed of 250 r.p.m. resting on ground. Assumethat resonant frequency of the system is 300 r.p.m. This will lead to fairly excessive amplitude of vibration, as the frequency ratio is close to unity. If this foundation is made 'stronger' by adding more concrete mass to the foundation block, the value of m_0, increases. This will lead to the decreased resonant frequency of the system, and the frequency ratio approaches closer to unity leading to still more severe vibration amplitudes. This example shows that the quality of the foundation does not necessarily improve with the mass of the foundation block.

The forced vibrations are transmitted though the ground even after some distance, if some adjoining foundation has a natural frequency equal to the frequency of transmitted vibrations, the resonance may occur, leading to its damage. For this reason, the foundation under heavy machines and forging hammers are isolated, and shock absorbers are used. The study of these shock absorbers and isolators is beyond the scope of this investigation.

2.3 RECIPROCATING MACHINES :

The machines considered in this section are those which transform the rotational motion to, reciprocating motion and vice versa. The machine may be driven by the reciprocating motion, as an internal cumbustion engine, or by the rotational motion, as air compressor. The essential moving elements of such a machine are a piston, a crank, and the connection rod. Vibration of the machine may result from the gas pressure applied periodically to the piston, and from the inertia forces associated with the moving parts. It may be possible to balance the inertia forces, and couples in certain types of multi-cylinder machines, but this cannot be achieved in one or two cylinder machines. The gas pressure acting upon the piston reacts as the foundation of the machine in the form of a couple which is transmitted to its support.

2.3.1 KINEMATICS :

The Kinematics of the mechanism being considered is illustrated by Fig. 2.1. The crank rotates in a counter clockwise direction with constant angular velocity ω . The piston is constrained to move along a vertical line in a manner determined by the crank radius r, and length of connecting rod ℓ . The upper most position of the piston is taken as the co-ordinate reference, and downward displacement y_p of the piston is taken as positive. The piston displacement is then given by ;

$$y_p = r + \ell - r\cos\omega t - \ell\cos\theta \qquad \text{..............(2.3)}$$

Now $\sin\theta = \frac{r}{\ell}\sin\omega t.$

$$\therefore \quad \cos\theta = \sqrt{1 - (r/\ell)^2\sin^2\omega t.}$$

This can be expanded by the bionomal theorem as follows :-

$$\cos\theta = (1 - \tfrac{1}{2}(r/\ell)^2\sin^2\omega t - \tfrac{1}{8}(r/\ell)^4\sin^4\omega t + \text{-----})$$

Now $\sin^2\omega t = \frac{1}{2}(1 - \cos 2\omega t)$, and dropping all powers of r/ℓ , greater than the second, equation (2.3) becomes,

$$y_p = r(1 + \frac{r}{4\ell}) - r(\cos\omega t + \frac{r}{4\ell}\cos 2\omega t)$$

$$\text{................(2.4)}$$

Expression for the velocity $\dot{y}_p$ and acceleration is $\ddot{y}_p$ of the piston are :-

$$\dot{y}_p = r\omega(\sin\omega t + \frac{r}{2\ell}\sin 2\omega t) \qquad \text{............(2.5)}$$

$$\ddot{y}_p = r\omega^2(\cos\omega t + \frac{r}{\ell}\cos 2\omega t) \qquad \text{.........(2.6)}$$

The crank pin moves in a circular path with the axis of the crank shaft as a center. Taking the same system of co-ordinates equations that define the vertical and horizontal components of the crankpin motion are readily written as follows :-

$$y_c = \ell + r(1 - \cos\omega t),$$

$$\dot{y}_c = r\omega\sin\omega t. \qquad \text{.................(2.7)}$$

$$\ddot{y}_c = r\omega^2\cos\omega t.$$

$$x_c = r \sin \omega t,$$
$$\dot{x}_c = r\omega \cos \omega t, \qquad \text{.........................(2.8)}$$
$$\ddot{x}_c = -r\omega^2 \sin \omega t$$

where y_c and x_c are vertical and horizontal components, respectively, of the crank pin displacement.

The motion of the connecting rod is rather complicated. A simplification which is usually adequate, is obtained by assuming the connecting rod to consist of (1) a concentrated mass whose motion corresponds to that of the piston and (2) a second concentrated mass, whose motion corresponds to that of the crank pin, both the masses joined by a massless strut. Designating the mass of the piston and crank pin (including the connecting rod) by mp and mc respectively. The vertical component F_y is obtained from the product of these masses and their respective accelerations as given by equations above.

$$F_y = mp\ \ddot{yp} + mc\ \ddot{yc}$$
$$= (mp + mc)\ r\,\omega^2 \cos \omega t + mp\ r\ (r/\ell)\,\omega^2 \cos 2\omega t$$
$$\text{...............(2.9-a)}$$

The horizontal component Fx of the inertia force results only from the rotating mass mc (crank pin + part of connecting rod) and is readily written as ;

$$Fx = m_c\ \ddot{x}_c = -m_c\ r\,\omega^2 \sin \omega t \qquad \text{..........(2.9 b)}$$

Thus we see that inertia forces Fy along the line of stroke consists of a force with a frequency of ω (the same as that of rotation), called primary force, and a force with a frequency of 2ω, which is called secondary force. In addition there is an inertia force Fx, perpendicular to the line of stroke with a frequency of ω .

The crank shaft can be counter-balanced so that the mass m_c is substantially zero. Thus Fx (horizontal component of the inertia force) is eliminated, but the vertical force resulting from the single reciprocating piston is a source of severe unbalance. Therefore, equation (2.9a) becomes ;

$$F_y = m_p \; r\,\omega^2 \cos \omega t + r\,(r/\ell)^2 \cos 2\,\omega t$$

$$= m_p\, r\, \omega^2 (\cos \omega t + (r/\ell) \cos 2\,\omega t) \quad ....(2.10\text{-}a)$$

Changing m_p, in to the weight of the piston in lbs. taking r in inches, and $\omega = 2\pi N/60$ where N is number of revolution per minutes, we get

$$Fy = .0000284\; W_p\, r\, N^2 (\cos \phi + \cos 2\phi \frac{r}{\ell}) \quad .....(2.10\text{-}b)$$

where $\phi = \omega t$.

If higher terms of r/ℓ were not neglected, we would have obtained (Newcomb . 1951)

$$F_y = .0000284\, W_p\, r\; N^2 (\cos \phi + \cos 2\phi + B \cos 4\phi + C \cos 6\phi)$$

$$= .0000284\, W_p\, rN^2 \times (\text{constant}) \quad ....(2.10\text{-}c)$$

Higher harmonics, which are of usually negligible magnitude can be excluded without much effect on the unbalance dynamic force.

2.3.2 MULTI-CYLINDER ENGINE :

In a multicylinder engine, some or all of the inertia forces and the couples resulting therefrom, may be balanced by proper arrangement of the cranks. The condition necessary for such balancing are indicated by reference to figure 2.2. The cranks are numbered, and the angular position of each is indicated by ϕ_n, referred to the position of crank 0. The position of each crank along the shaft is indicated by the distance L_n from the crank 0. If the reciprocating and rotating mass for each cylinder are respectively equal, the following conditions for balance of inertia forces are obtained.

$$\begin{rcases} \sum F_y = 0 \qquad \sum \cos \phi_n = 0 \text{ and } \sum \cos 2\phi_n = 0 \\ \sum F_x = 0 \qquad \sum \sin \phi_n = 0 \end{rcases} \text{.........(2.11-a)}$$

In a multicylinder engine, the inertia forces defined by equations (2.9-a) and (2.9 -b) create couples about the horizontal and vertical transverse axes. For convenience, the couples are taken with respect to axes through the 0 crank as indicated in Fig. 2.4. The following conditions for balance of inertia couples are obtained,

$$\sum L_n F_y = 0 \qquad \sum L_n \cos \phi_n = 0 \text{ and } \sum L_n \cos 2\phi_n = 0$$

$$\sum L_n F_x = 0 \qquad \sum L_n \sin \phi_n = 0 \qquad \text{..............(2.11-b)}$$

For example, consider the four cylinder engine, (Crede, 1951) whose crank angles are 0, 90, 270, 180 degrees, and where cranks are spaced apart equal distances L along the shaft. Table 2.1 is now established in accordance with equations (2.11-a) and (2.11-b) It is evident that the primary and secondary forces are balanced because $\sum \cos \phi = \sum \cos 2\phi = \sum \sin \phi = 0$. Furthermore, the secondary couples are balanced because $\sum L \cos 2\phi = 0$. However, $\sum L \cos \phi \neq 0$, and $\sum L \sin \phi \neq 0$, the primary couples are, therefore, not balanced. The engine will thus tend to vibrate in a rotational made about a transverse axis.

Table 2.2 gives the summary of unbalanced inertia forces and couples for different crank arrangements, as given by Newcomb, (1951).

2.4 PUNCH PRESS :

Machinary used for forming metal by shearing, drawing or punching is a frequent source of disturbance in industrial plants. The machine most commonly used for these operations is the punch press illustrated in Fig. 2.3. Such a machine generally embodies a relatively heavy, rigid lower portion carrying the stationary plattern, and a vertically reciprocating head carrying the moving platten. The moving platten is usually driven by a crank and connecting rod, and in some types of presses it moves in a line inclined to the vertical.

A punch press is a machine of conservative momentum. There is no addition of momentum from an external source, and the machine cannot permanently acquire a velocity, although it may acquire a displacement. In other words, if the press were supported by some means that offered no constraint to its movement, it would move intermittently with a short step at each cycle of operation. If supported a rigid foundation which prevents appreciable movement, the forces that tend to cause this stepwise displacement are transmitted directly to the foundation. The foundation in this case has to be rigid and capable of suffering impact caused by the punching press (Barkan 1963).

2.5 LOOMS :

One of the most troubles ome machines with regard to vibration and shock is the cloth weaving loom. The principle features that cause vibration and shock are illustrated schematically in Fig.2.4.

They lay a relatively heavy member is driven with a horizontally reciprocating motion by a pair of cranks and connecting rods. A shuttle travels alternately in opposite directions, across the lay, from one shuttle box to the other by means of a mechanism.

The two principal sources of vibration and shock resulting from loom operation are :-

a) The inertia forces created by the reciprocating motion of the lay is suatantial. This is almost a pure harmonic force acting in a horizontal direction, and the reaction upon the frame of the loom is at the crank shaft. When looms are installed on the upper floors of mills, the entire building may sway at the frequency of the lay motion. It is characteristic of textile mills that the amplitude of sway continually increases and decreases as the many looms operating nominally at the same but actually at slightly different speeds, change phase relations. There is some evidence that the magnitude of sway is occasionally increased by resonance of the building with the looms (Crede 1951). The floors that support the looms are caused to weave or bend under the influence of the moment resulting from the lay force acting upon the loom frame at the height of the crank shaft above the floor.

b) The force that propels the shuttle is in the nature of an impact. The complexity of mechanism employed for this purpose makes the exact nature and direction of this impact uncertain. Although the picking action occurs with the same period as the lay movement, the actual force is induced for only a small fraction of period. Since the period of application of picking force is smaller than the period of the lay force, the associated frequencies are higher.

2.6 ELECTRIC MOTORS :

An electric motor is comprised of two principal parts, a startor, and a rotor. The torque delivered by the motor results from the attraction of a magnetic field on current - carrying conductors of the rotor. The magnetic field is created by electric current flowing through the winding of the startor. Any variation in the current is reflected in the strength of the field, and consequently in the torque. In a single phase 60-cycle alternating current motor for example, the current in the windings of the startor passes through zero 120 times per second. There is thus variation in torque at a frequency of 120 c.p.s.

The vibration created by the operation of an electric motor may result from :-

a) the periodic variation in the strength of the magnetic field and

b) inertia forces associated with unbalanced rotating parts.

The power output of a motor is manifested in a torque applied to the motor shaft, and an equal and opposite torque reaction is exerted upon the startor. The motor support (foundations) thus experiences, in the form of vibration, the periodic fluctuation in the strength of the magnetic field. This disturbance is fundamentally torsional in nature. Vibrations resulting from the unbalance of rotating parts should be treated in the same manner (i.e. these are of the same nature) as the vibration of any rotating machine.

2.7 FANS AND BLOWERS :

The term fan is used to designate a machine having a rotor with several blades arranged to cause a flow of air gas axially of the fan, the term blower designates the so called squirrel-cage blower in which the air flows into the blower in an axial direction and out in a radial direction. Machines of these types operate at many different speeds, depending upon the size and the type of the service. In general, the operating speed decreases as the size of the fan or blower increased. Predominant sources of the vibration are rotor balance, bearing, gear, and belt forces, motor impulses and aero-dynamic forces.

2.8 PUMPS :

This class of equipment is comprised of both centri-fugal and reciprocating pumps. The former usually includes a multi-vane rotor which operates at a relatively high speed. Vibration may be expected at the rotational frequency, as a result of mass unbalance of the rotor, at the vane frequency, because the moving vanes pass in close proximity to the fixed vanes, and at the random frequencies usually relatively high, as a result of forces created by turbulant flow of liquid within the pump.

Reciprocating pumps are used for causing a flow of fluid, for the compressing air and gas, and for creating a vacuum. These are characteristically low speed machines, and the number of cylinders is usually small. Unbalanced rotating and reciporcating parts and torsional or torque impulses associated with the work being done as the fluid are predominant sources of vibration. Minor sources of vibrations are gears and bearings, motor impulses and aerodynamic or hydrodynamic forces resulting from turbulent flow of fluids through and around structural member of the pump.

2.9 FORGING HAMMERS :

The term forging hammer is used to designate a machine in which a relatively heavy hammer is caused to fall freely against an anvil. According to Anderews and Crockett (1945) the largest unit in operation at that time had a 25 ton hammer (tup) and the foundation weighed 3000 tons. The principal problem from the stand point of shock arised from the fact that the momentum of the falling hammer is transferred to the body of the machine. The machine thus tends to acquire a downward velocity and to carry its foundations with it (Crockett and Hammond 1958, Barkan 1963).

2.10 CLASSIFICATION OF DYNAMIC LOADS :

From section 2.3 to 2.9, some of the important classes of machinary have been considered. The nature of vibrations and dynamic forces associated with these machine types have been shown. In general the dynamic loads can be classified as :-

a) Shock loads occuring at regular intervals e.g. vertical loads as in punching press, forging hammers and horizontal shock loads, as in looms.

b) Vibratory loads, which repeat after a particular period and are cyclic in nature. These may include the vibrations caused in any of the six degrees of freedom (for a single mass) that is three translatory load and three rotational torques (Fig. 2.5).

Figure 2.5 indicates the possible six modes of vibration for a foundation. These include three translatory vibrations, viz. vertical, longitudinal and lateral along three coordinate axes, x, y, z , and three rotational vibrations. Rotation about vertical axis (z-axis) is called ya wing, while rocking is rotation about longitudinal axis (y - axis) and pitching is rotational about lateral axis (x - axis). For symmetrical foundations, vertical vibrations and ya wing can exist independently, but rocking is associated with lateral vibrations and pitching is associated with longitudinal vibrations.

In most of the machine foundations, the vibrations occur in vertical direction, or in rocking.

2.11 REQUIREMENTS OF MACHINE FOUNDATIONS :

A properly designed foundation for a machine must first of all meet the general requirements for all foundation for the particular load transmitted to the ground. These are as follows (Tschebotarioff 1951) :-

1. The loads of the structure should be transferred to soil layers capable of supporting them without a shear failure.
2. The deformation of the soil layers underlying the foundation should be compatible with those which the foundation itself in super structure, as well as adjoining existing structures can safely undergo.
3. The construction operations should not endanger adjoining existing structures.

Usually static loads play only a minor part as compared with the dynamic loads produced by the moving parts of the machinery.

Besides these, the machine foundation must meet the following additional requirements, which are characteristics of dynamic loading :-

a) Vibrational Amplitude :

It is not possible to eliminate the oscillating motion completely from a foundation which is subjected to significant dynamic impulses. The designer can only attempt to reduce the foundation vibration to a magnitude which is tolerable at the operating frequency for the design conditions. In general, the permissible amplitude of vibration decreases as the frequency of operation increases (Richart 1960). Thus no value of allowable amplitude should be considered as design criterion, unless the frequency of operation is also specified. Again the amplitude of vibration may be limited by the other machines acting on the same ground nearby. In many cases, some fine milling machines with low tolerance limits have to stand idle, while nearby forging hammer is operating which produces vibrations in ground, and as such the accurate milling machine cannot function to the required degree of tolerance. (Crockett & Hammond 1958).

In the absence of the design specifications for limiting vibrations, either of the following recommendations may be used as a guide. One of these is originally suggested by Rausch, (1936) and reported in English by Converse (1962). According to this, the permissible amplitude is given by ;

$$\text{Permissible amplitude} = \frac{9.54}{f} \quad \text{for frequencies less than 1800 r.p.m.}$$

$$= \frac{17,600}{f^2} \quad \text{for frequencies more than 1800 r.p.m.}$$

....................(2.12)

Richart (1960) has recommended the use of curves to obtain permissible amplitude of vibration. These were obtained by him on the basis of published records of machine foundations, and given in figure 2.6, which is self explanatory.

The vibration amplitude which is designated as the limit for machines and machine foundation is approximately 100 times that which is barely perceptible or noticeable to human beings.

B. Resonanace :

It is necessary to avoid resonance between machine and the foundation soil system. Resonance phenomena will usually lead to excessive amplitudes of vibrations, larger loads transmitted to foundations. Hence resonant frequency of the system, should be far from the operation frequency of the machine.

a) Shock Loads :- The force occurs at equal time intervals, even though it is actually applied but for a fraction of the loading cycle. The entire machine experiences vibrations of large amplitudes if the time interval between successive strokes of the machine equals the resonant period, of the machine foundation soil system. This is a condition of resonance even though the exciting force is not harmonic.

b) Low Frequency Machines : In order to avoid resonance, and to control the amplitude of vibration, the frequency of operation of machine must be considerably lower than the resonant frequency of the system. The suggested frequency ratio is 0.5, in order to ensure small amplitudes of vibration. The exact ratio may depend upon the accuracy with which resonant frequency of the system, can be predicted.

c) High Frequency Machines : In order to avoid resonance, the resonant frequency of the system should be substantially lower than the operating frequency of operation of machine. The frequency ratio suggested in this case for frequency ratio is 2.0 or more. These types of machine will usually have to pass through the resonance condition which should last for as small a time as possible.

3. Height Base Ratio :

In the case of machines subjected to rotary motion as in a compressor, foundation is subjected to rocking. In such a case it may be necessary to control amplitudes of rocking so as to avoid damage to the machine. This can be achieved by decreasin the ratio of height to base dimension in the direction of rocking (Newcomb 1951) resulting in increase of general stability of the foundation block. This implies that the rocking machine should be as near to the foundation or ground, as possible.

CHAPTER - 3.

RESONANT FREQUENCY.

3.1 GENERAL :

In the previous chapter, it has been stated as to why it is necessary to avoid resonance for the satisfactory functioning of the machine foundations. This leads to the necessity of determining the resonant frequency of the system.

Chiefly the designer is concerned with the vibrations in ~~all~~ mode usually vertical or rocking. Various workers have mostly concentrated on vertical vibrations. Theoretical analysis have been developed for the other modes of vibrations also.

Broadly, the methods for determining the critical frequency can be divided into - i) theoretical methods and ii) Experimental methods. Theoretical methods involve application of two basic concepts, viz., a) vibrating source (machine) resting on the semi-infinite, elastic, isotropic and homogeneous medium, (soil), and b) vibrating source resting on elastic spring (soil). In theoretical methods the properties of soil are E Dynamic modulus of elasticity and ν - the poisson's ratio in case (a) and spring constant or dynamic modulus of subgrade reaction and weight of soil participating in case (b). These are to be suitably assumed or may have to be determined experimentally.

Experimental approaches consist of developing certain empirical relations, or the response curves from the experimental data, and using these to determine the resonant frequency of the actual system.

For convenience, this chapter is divided into three main heads, which will indicate the basic approaches towards the problem of determining the resonant frequency.

RESONANT FREQUENCY - Soil as Elastic, Isotropic, semi-infinite & homogenous medium.

RESONANT FREQUENCY - Soil as Spring.

RESONANT FREQUENCY - Experimental approaches or miscellaneous methods.

RESONANT FREQUENCY - SOIL AS ELASTIC MEDIUM.

3.2 ELASTIC WAVES :

An actual foundation to which vibratory motion is imparted by a periodic force, becomes the source of periodic impulses which proceed into the subgrade in radial directions, similar to a sound wave. In the course of transmitting waves, the particles of subgrade also undergo periodic motion, but only at particular locations does this motion correspond to the motion of foundations. Directly, below the foundation base, the subgrade material moves with the foundation and is "in phase" with the foundation motion. At a greater distance a zone of subgrade moves opposite to the foundation and may be designated as "180° out of phase". Fig. 3.1 illustrates this concept of phase relations of zones of subgrade with the shaded areas representing the "inphase" zones. The spacing between the centres of these zones is determined by wave length, which in turn, is established by the velocity of propogation of elastic wave in the subgrade and the frequency of load application.

In an infinite elastic, isotropic, homogeneous, body, disturbances may be propagated by compression waves or push waves or P-wave (The displacement of the particle is in the direction of the propogation of wave,) and by shear wave or transverse wave or S-wave (The displacement of the particle is in a direction at right angles to the direction of propogation of wave). The compression and shear waves also transmit disturbances throughout the interior of a semi-infinite, elastic, isotropic, homogeneous body. But because of free surface, a third type of wave appears. This wave has been designed as the surface or Rayleight wave or R-wave, after Lord Rayleigh (1885) who investigated the behaviour of surface waves in an elastic "half space". His solution of surface elastic wave equation is now known as "Rayleight free wave solution". There are other types of surface wave, such as Love Waves, but it has been shown that for a circular vibrator operating normal to the surface of a semi-infinite medium, a large part of its power is radiated as Rayleigh waves (Miller and Pursey 1955). In Fig. 3.2, relationship between V/V_S and poisson's ratio has been plotted, where V is the velocity of propogation of P, S, and R - waves. The equations for V are given for different values of poisson's ratio, ν , for the above three waves.

3.3 VERTICAL IMPULSES AT THE SURFACE :

Lamb (1904) analysed the effects produced by a single impulse which acted at the surface of a semi-infinite isotropic, homogeneous, elastic solid. He considered primarily the effects produced at the surface and found that the disturbance produced by the impulse, spreads in the form of a symmetrical annular wave system. He also studied the effects produced by the periodic vertical and horizontal forces applied at a point, or distributed along a line on the surface of semi-infinite solid. He established relation between the diplacements and stress, within the soil mass. This is now known as the "dynamic analogue of Boussinesq's problem".

3.4 PERIODIC LOAD OVER CIRCULAR AREA :

More recently in 1936 Reissner (Sung 1953, Lorenz 1959) presented an analytical solution for the oscillation of a vibrator resting upon the surface of a semi-infinite, isotropic, homogeneous elastic body by integration of the effects of the periodic vertical point load over a circular area. The vibrater was represented by a system of vertical periodic forces, uniformly distributed over a circular area on the surface. The displacement amplitude at the centre of circular vibrator mass, was obtained. The expression is also obtained for the power requirement of a given vibrator soil installation. Also the phase difference between the dynamic force and the amplitude of vibration was determined analytically. The expressions for amplitude of vibration, power in put and phase difference, as obtained are given below :-

$$Z = \frac{F_o}{Gr_o} \sqrt{\frac{f_1^2 + f_2^2}{(1+b_a{}^2 f_1)^2 + (b_a{}^2 f_2)^2}} \quad \ldots\ldots\ldots(3.1a)$$

$$L_p = \frac{F_o^2}{2r_o^2\sqrt{\rho G}} \cdot \frac{a\,f_2}{(1+b_a^2 f_1)^2 + (b_a^2 f_2)^2} \quad \text{......(3.1b)}$$

$$\text{and } \tan\psi = \frac{-\,f_2}{f_1 + b_a^2\,(f_1^2 + f_2^2)} \quad \text{.........(3.1 c)}$$

where, Z is the vertical amplitude of oscillation,

G is the modulus of rigidity.

r_o is the radius of circular oscillator.

F_o is the maximum magnitude of the dynamic force applied.

b is the dimensionless quantity known as mass ratio $= m_o/\rho r_o^3$

m_o is the mass of oscillator (foundation & machine)

ρ is the mass density of soil medium.

a is the dimensionless frequency term,
$= \omega r_o\sqrt{\rho/G} = 2\pi f r_o\sqrt{\rho/G}$

f is the frequency of the forced vibrations.

ψ is the phase difference between dynamic force and the vibration amplitude of machine foudndation.

f_1, f_2 is function of ν and a, $F(\nu, a)$ in which ν is the posisson's ratio.

L_p is the power input, required to drive the vibrator at the amplitude Z.

The resonant frequency of the system is defined as the frequency at which maximum amplitude of vibrations or max.

power consumption occurs. Other criterion is that phase difference between dynamic force, and vibrations is $\pi/2$ or 90 deg. Reissner has taken the criterion of resonant frequency as phase difference, $\psi = \pi/2$ in which case equation (3.1 c) gives ;

$$f_1 + b a_0^2 (f_1^2 + f_2^2) = 0$$

or $$a_0^2 = - f_1 / b(f_1^2 + f_2^2) \quad \ldots\ldots\ldots\ldots\ldots (3.1\ d)$$

where a_0 is the value of dimensionless frequency term at resonance.

In order to make the analysis, applicable to every case of vibrator, Reissner has introduced the use of dimensionless amplitude $Z^{(2)}$ and power requirement $L_p^{(2)}$. The expressions are ;

$$Z^{(2)} = \frac{G r_0}{F_0} Z = \sqrt{\frac{f_1^2 + f_2^2}{(1+b_a^2 f_1)^2 + (b_a^2 f_2)^2}} \quad \ldots\ldots\ldots\ldots\ldots (3.2\ a)$$

$$L_p^{(2)} = \frac{2 r_0^2 \sqrt{\rho G}}{F_0^2} L_p$$

$$= \frac{a f_2}{(1 + b_a^2 f_1)^2 + (b_a^2 f_2)^2} \quad \ldots\ldots\ldots (3.2\ b)$$

Curves have been plotted between this dimensionless amplitude factor $Z^{(2)}$ and dimensionless power requirement factor $L_p^{(2)}$ versus the dimensionless frequency 'a' for different values of mass ratio (b), and poisson's ratio (ν), (As f_1 and f_2 both depend upon ν and ao), Curves have also been plotted between $Z^{(2)}_{max}$. and a_0 for different values of mass ratio (b) and poisson's ratio (ν).

Equation (3.1 a) demonstrates that a dynamically excited body on a homogeneous semi-infinite space, represents a system capable of vibration, as a single degree of freedom system.

An equally important result is that the amplitude $z^{(2)}_{max.}$ at resonant frequency of the system a_0 assumes finite values even though no damping factor was introduced throughout Reissner's calculations.

Terzaghi (1943) commented that no attempt has yet been made to apply the results of this analysis to the practice of vibrator investigations. The main difficulties which restricted the use of his results are due to the uncertainties concerning (a) the effect of a change of the oscillating pressure from the uniform distribution which he assumed (b) the effects of a change of shape of loaded area or region and, (c) the effects produced by deviations of the behaviour of the ideal elastic body.

While the item (c) is the general drawback of all such approaches which assume the soil to be ideal solid, and as such must be left as a problem to be decided at each installation, and item (b), is usually taken into account by assuming some equivalent circular area, item (a) has been studied in detail recently by Sung (1953) Quinlan (1958) and further on by Richart (1953, 1960).

3.5 CONTACT PRESSURES – QUINLAN AND SUNG :

Quinlan (1953) and Sung (1953) obtained the above solution for amplitude, power requirement and the phase difference, independently, for the circular vibrator with various types of contact pressure distributions. The ground pressure distribution, considered are rigid, uniform and parabolic distribution illusrated in Fig. 3.3 The final equations are same in both cases and correspond with the equations given by Reissner (Equ. 3.1) with the difference that functions f_1 and f_2 are different for different types of load distributions, in other words these functions depend not only on (a, ν) but also on the type of load distribution.

The mass ratio assumed in Quinlan's analysis is $b_q = m_o/2\rho r_o^3 = b/2$, and this gives the corresponding difference in values of f_1 and f_2 as given by the two workers. But the final result is same, though the mathematical approach is different.

While Quinlan finds the resonant frequency of the system by assuming that at resonance phase difference between the dynamic force and the amplitude vibration produced is $\pi/2$ (He has concluded that result is quite accurate for Degebo type oscillators), Sung, considers that resonace occurs at the frequency where amplitude of vibrations and power requirement are maximum.

Quinlan has given functions f_1 and f_2 for $\nu = \frac{1}{2}$, 1/3, and 1/4 for different values of 'a' (dimensionless frequency) for rigid base approximation. Similar curves can be set up for uniform and parabolic distribution of contact pressures. Also as at resonance tan $\psi = \pi/2 = \infty$, equation (3.1 b) reduces to ;

$$f_1 + b_q \ a_o^2 (f_1^2 + f_2^2) = 0$$

or $$a_o^2 = \frac{-f_1}{b_q\,(f_1^2 + f_2^2)} \qquad \ldots\ldots\ldots\ldots(3.1\ d)$$

He has plotted the value of b_q and a_o, for different values of poisson's ratio, ($\nu = 1/2, 1/3, 1/4$) for rigid base approximation. Assuming pressure distribution, for a particular value of b_q (mass ratio,) and poisson's ratio, the value of resonant frequency a_o can be calculated, or read from the graph. By substituting the value of a_o, and corresponding functions f_1 and f_2 in equation (3.1 a) the maximum amplitude at resonance (a_o) can be determined. He has also given the solution for the long vibrator, with the same formulae except that b_q is per unit length of vibrator, and the functions f_1 and f_2 change into f_3 and f_4 and dimensionless frequency term 'a' is given by ;

$$a = \frac{B}{2}\sqrt{\rho\ G}$$

F' is taken as the magnitude of dynamic force per unit length $= m_1' \ell \omega^2$ where m_1' is the eccentric mass per unit length. The similar curves as in circular vibrator have been plotted by Quinlan, for long vibrator as well.

For a Degebo type vibrator with total eccentric masses m_1 at eccentricity ℓ, the value of maximum magnitude of dynamic force $F_o = m_1\ \ell\ w^2$. Fig. 3.4 gives the diagrametic sketch of Degebo type vibrator.

The dimensionless amplitude and power requirement as given by Reissner (Equ. 3.2 a and b) are modified by Sung to suit the Degebo type vibrator, as below ;

$$Z^{(1)} = \frac{Gro\ a^2}{F_o} Z = \frac{\rho\ ro^3}{m_1} = a^2\sqrt{\frac{f_1^2 + f_2^2}{(1+b_a^2 f_1)^2 + (b_a^2 f_2)^2}} \quad \ldots\ldots\ldots\ldots\ldots(3.3\ a)$$

and $$L_p^{(1)} = \frac{2\ ro^2 \sqrt{\rho G}\ a_2^4\ Lp}{F_o^2} = \frac{2\ \rho^{5/2}\ r_o^6\ Lp}{m_1^2\ \ell^2\ G^{1/2}}$$

$$= \frac{a^5\ f_2}{(1+b_a^2 f_1)^2 + (b_a^2 f_2)^2} \quad \ldots\ldots\ldots\ldots(3.3\ b).$$

Sung has plotted this dimensionless amplitude and dimensionless power input versus 'a' (the dimensionless frequency) for various values of b (the mass ratio), poisson's ratio ν and the type of load distribution. Out of these curves the dimensionless frequency a_o at maximum amplitude (dimensionless) $Z^{(1)}_{max}$ have been plotted for different types of load distributions, poisson's ratio and b - value. He has also plotted the maximum value of $Lp^{(1)}$ and corresponding a_o, for different kinds of load distribution poisson's ratio and b-value.

In Fig. 3.5, the variation of amplitude factors $A^{(1)}$ and $A^{(2)}$ versus dimensionless frequency term 'a', are shown for several constant values of the mass ratio b. The diagrams of Fig. 3.5 result from the assumptions of poisson's ratio of 0.25 for the subgrade material and a distribution of contact pressure corresponding to that produced by a rigid circular base. By assembling the values of 'a', corresponding to peak amplitude, for a particular value of b, the relationship between ao and b has been established as shown in Fig. 3.6(a). Also by taking the value of the peak amplitude factor and plotting it against the corresponding values of b, the curves of Fig. 3.6 (b) were obtained (Richart 1960).

Curves given in Figures 3.6 a and 3.6-b can be used to read off the value of 'ao' and the maximum amplitude factor, for a particular value of mass ratio b, and poisson's ratio ν , on the assumption of pressure distribution. These curves sum up the bulk work of calculation and are extremely useful for practice.

3.6 CONCEPT OF EFFECTIVE RADIUS :

Richart (1953) commenting on Sung's paper introduces the concept of "effective radius" for each pressure distribution , which corresponds to an equivalent uniformly distributed load. This is illustrated in Fig. 3.7.

The centroid of the stress diagram included between ro and o (Fig. 3.7 -b) is at $r_o/2$. Consequently a statically equivalent loading consists of a lime load, which acts along the circumference of a circle ix of radius $r_o/2$. The centroid for the rigid base distribution is at 0.6366 r_o and that for parabolic distribution is at 3/8 r_o. From this, the radii for equivalent uniform distributions of load are 1.273 r_o and 0.75 r_o respectively for the rigid base and parabolic distribution. These effective radii may be used for the cases where the theoretical curves regarding various types of load distribution are not available. Suppose only data regarding the uniform distribution is available, and we want to find for rigid distribution, then in all the calculations of 'b', 'a' we use the radius 1.273 ro and the result will be the same as for rigid distribution.

It is common experience that with increase in dynamic load, the resonant frequency decreases. Lorenz (1934) BalaKrishna Rao (1961) observed this behaviour ~~from~~ experimentally. The explanation in terms of effective radius may be given as follows, (Richart 1953). As the dynamic force increases, the pressure tends to become more intense near the centre of the oscillater, which means that effective radius becomes less. The curves plotted by Sung indicate the variation of the maximum amplitude of oscillation as a function of frequency for three types of pressure distributions. In these plots every thing else except pressure distribution is same. The peak of these resonance curves show graphically that the amplitude of oscillation increases and the resonant frequency decreases as the pressure is concentrated nearer the centre of the oscillator base. But no quantative information is available as to the effect of the increase in dynamic force as the decrease of resonant frequency and increase of amplitude or the change in the effective radius.

3.7 HORIZONTAL AND ROTARY MODES OF VIBRATIONS:

Arnold, By Croft and Warburton (1957) Bycroft (1959) have extended the analysis to the other types of the vibratory modes, so as to include all the translatory and rotationry vibrations. They have considered circular vibrator with rigid base distribution. As the circular vibrator is the case of axial symmetry, in reality the vibrating mass has four degrees of freedom, i.e., translation horizontally and vertically and rotation about horizontal and vertical axes. The values of f_1 and f_2 for different modes of vibration (4 degrees of freedom) have been given. The functions of rotation about vertical axis are independent of poisson's ratio, while functions for other modes of vibration depend upon poisson's ratio. The expression for amplitudes of translatory motion are the same as that obtained by Reissner (1936), Sung (1953) and Quinlan (1953) while for rotating modes the expression for angular displacement is as follows :-

$$\phi_x \phi_y \phi_z = \frac{M}{G r_o^3} \sqrt{\frac{f_1^2 + f_2^2}{(1+b_1 a^2 f_1)^2 + (b_1 a^2 f_2)^2}} \quad .....(3.4)$$

where $\phi_{z,x,y}$ is the amplitude of vertical or horizontal rotation.

M is the maximum magnitude of exciting couple (about vertical axis or horizontal axis)

f_1, f_2 are the corresponding functions for particular mode.

b_i is non dimensional moment of inertia

$$= \frac{I_o}{\rho r_o^5}$$

r_o is the radius of circular base plate.

I_o is the mass moment of inertia of the oscillator.

It has been suggested by Bycroft (1959) that for rectangular contact area, r_o should be taken as the radius of a circle of an area equal to that of the rectangle. Z_r the corresponding amplitude for rectangular base is given by $Z_r = m \times Z$ where m is a factor depending upon the ratio of longer to shorter side (L/B). He has recommended values of shape factor based on theory of elasticity for static case given by Timoshenko(1937) and given in Table No. 3.1.

Richart (1960) has plotted the amplitude ϕ versus frequency for b_i = 2,5, 10 and 20 for rocking oscillations for $\nu = 0$, calculated from the values given by the above authors. Fig. 3.8, gives (a) the maximum amplitude factor $\phi^{(1)}_{max.} = \phi \frac{Gr_o^3}{M}$ versus 'a' the dimensionless frequency factor. Also shown is the envelope curve which is tangent to each curve of amplitude versus frequency for different values of b_i. The point of tangency is close to the point of maximum amplitude and this tangent curve is used to define the relation between frequency at maximum amplitude and the value of inertia ratio b_i, which is shown in Fig. 3.8 (b).

The corresponding curves for horizontal oscillation are shown in Fig. 3.9(a) and (b) as developed from the above author's analysis by Richart (1960) which show (a) the maximum amplitude factor $X^{(1)}_{max.}$ for horizontal displacement versus 'a' the frequency term, and (b), the mass ratio, (?) versus a_o, from the consideration of touching envelope for Fig. 3.9 (a).

3.8 <u>EQUATIONS OF MOTION</u> :

The work done by Reissner, Quinlan, Sung and Arnold Bycroft and Warburton, dealt with one of the six modes of vibration at a time and therefore, is limited to the case, where the six modes exist independently. It is not possible to evaluate the equations of motion directly from the above theories. An interesting transformation suggested by Hseih (1962) makes it possible to find the equations of motion. For simplicity only the transformation achieved for vertical displacement will be given.

For the waves radiating from a source at the surface of semi-infinite homogeneous isotropic elastic body created from a circular vibrator with dynamic force $F_o \; e^{i\omega t}$ and the ground pressure $Pe^{i\omega t}$, it has been shown by Reissner, Sung (1953, eq. 45) that the displacement $Z_b e^{i\omega t}$ of the base of vibrator is given by ;

$$w_b = Z_b e^{i\omega t} = \frac{P}{Gro}(f_1 + if_2)\, e^{i\omega t} \quad \text{......(3.5 a)}$$

Differentiating equation 3.5-a w.r.t., t, we get,

$$\frac{d\,w_b}{dt} = \frac{P}{Gro}(if_1 - f_2)\, e^{i\omega t} \quad \text{.......(3.5 b)}$$

Multiplying equation 3.5 -a by f_1 and equation (3.5 b) by $- f_2$, and adding, we get,

$$f_1 \omega\, w_b - f_2 \frac{dw_b}{dt} = \frac{P}{Gro}(f_1^2 + f_2^2)\, e^{i\omega t}$$

$$\text{or} \qquad P\, e^{i\omega t} = -\frac{Gro}{\omega}\,\frac{f_2}{f_1^2 + f_2^2}\,\frac{dw_b}{dt} + Gro\,\frac{f_1}{f_1^2 + f_2^2}\, w_b$$

$$\text{..............(3.5 -c)}$$

which is of the same form as the pressure transmitted to ground by a spring and dash pot, where,

$$\frac{Gro}{\omega}\,\frac{f_2}{f_1^2 + f_2^2} = \frac{Gro^2}{a}\sqrt{\rho/G}\,\frac{f_2}{f_1^2 + f_2^2} = \sqrt{G\rho r_o^2} \times \frac{f_2}{a(f_1^2 + f_2^2)}$$

$$= \sqrt{G\rho} \quad r_o^2 / F_2 = c \text{ (equivalent damping coefficient)}, \ldots\ldots (3.6\text{-a})$$

represents the dash pot, or damping of the system, and

$$- Gro \frac{f_1}{f_1^2 + f_2^2} = G_{ro} \quad F_1 = \quad K \quad \ldots\ldots\ldots (3.6\text{-b})$$

represents the spring constant of the system. In these equations,

$$F_1 = \frac{-f_1}{f_1^2 + f_2^2}$$

$$F_2 = \frac{f_2}{a(f_1^2 + f_2^2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.6\text{-c})$$

and have been evaluated by Hseih for different modes of vibration, including rotational modes.

The similar transformations are possible for other modes of vibration. From ground reactions, (Equation 3.5-c) the equations of the motion can be derived, for example, for vertical, vibrations,

$$m_o \frac{d^2 w_g}{dt^2} = P e^{i\omega t} + F_o e^{i\omega t}$$

$$= -K w_b - c \frac{dw_b}{dt} + F_o e^{i\omega t} \quad \ldots\ldots\ldots\ldots (3.7)$$

where w_g is the displacement of c.g. of the machine foundation. Now w_b can be converted into w_g as

$$w_b = w_g + x\theta_y - y\theta_x ,$$

where x, y, z are the coordinates of the centroid of the contract surface, with respect to the coordinate axes passing through the combined c.g. of the foundation and machine (Fig.3.10) θ_y and θ_x, are the rotational displacements about y, x axis respectively.

Five more equations similar to equation (3.7) can beset up for other modes of vibrations, and from these six simultaneous second order differential equations , u_g, v_g, w_g θ_x, θ_y, θ_z the three translatory and three rotational displacements can be calculated by means of analogue computors.

But if $\bar{x}$ and $\bar{y}$ are zero, (that is the machine foundation is symmetrical about contact base), then the vertical translation and rotation about vertical axis exist independently while the horizontal translation is coupled with rotation about horizontal axis, (u_g and θ_y are coupled, v_g and θ_x are coupled, w_g and θ_z are decoupled).

If the centroid of contact area and c.g. of the machine foundation coincide ($\bar{z} = \bar{x} = \bar{y} = 0$) all the six degrees of freedom are decoupled. This is the hypothetical case and is not possible in practice.

The same conclusions are reached by Pauw (1953) by considering the equation of motion, which are obtained by the soil spring analogy.

This method offers a correlation between the two theoretical approaches viz., soil as elastic solid, and soil as spring, which will be discussed later on.

3.9. CONSERVATIVE SYSTEM :

Ford and Haddow (1960) has obtained the natural frequency of machine foundation based on Rayleigh's principle for rigid foundations. For a conservative system, according to Rayleigh's principle, the maximum strain energy is equal to the maximum kinetic energy. It is based mainly on the following assumptions,

(a) Vertical Vibrations.

1. The system may be considered as conservative in order to determine the natural frequency.
2. Dynamic pressure is transmitted through soil contained in a solid formed by the base of foundation and the surface $y = f(z)$, $y = -f(z)$, $x = \phi(z)$ and $x = -\phi(z)$ as shown in Fig. 3.11.
3. The dynamic stress at depth z is uniformly distributed over a section parallel to the base of foundation.

The last assumption is inaccurate, but is useful for the development of relation.

(b) Horizontal Vibrations.

The same assumption as for vertical vibrations are made with additional assumption that the dynamic shearing stress is uniformly distributed over a section of the solid parallel to x , y plane.

It is further assumed that amplitude of vibration of a layer of thickness dz at depth z, decreases with depth as

$$Zo = Zo_f \; e^{-\mu z} \qquad \ldots\ldots\ldots (3.8)$$

where Zo is the amplitude to vibration of layer at depth z .

Zof is the amplitude of vibration of foundation.

μ is the decay factor, dimensionally equivalent to L^{-1}

Equating the kinetic energy of the soil and machine foundation to the maximum strain energy of the soil, the author has obtained the vertical frequency (resonant frequency) and horizontal natural frequency of vibrations. The expressions obtained are,

$$f_o \text{ (vertical)} = \frac{1}{2\pi}\sqrt{\frac{2\,G\,(1+\nu)\,.\,\mu g}{\gamma/\mu + \sigma_{st.}}} \qquad \ldots(3.8a)$$

$$\text{and } f_o \text{ (Horizontal)} = \frac{1}{2\pi}\sqrt{\frac{G\,.\,\mu g.}{\gamma/\mu + \sigma_{st.}}} \qquad \ldots\ldots\ldots(3.8b)$$

where γ is the density of soil.

$\sigma_{st} = \frac{m_o}{L \times B}$ and is the static pressure exerted by machine foundation.

The decay factor μ is determined from the equation

$$= \frac{B_1}{m\sqrt{A}\ (1-\nu^2)} \qquad \ldots(3.8c)$$

where m is a constant depending upon L/B and is given Table 3.1

B_1 is a constant, and is taken as 2.0 for sands and 1.5 for clays or may be determined from dynamic tests, by noting the resonant frequency for particular δ_{st} and working back for B_1 .

The authors have stated that value of B_1 for horizontal vibration may be different than as taken for vertical vibrations but no values have been recommended.

3.10 DISCUSSION :

Reissner's analysis (1936) forms the basis of the subsequent analysis given by Quinlan (1958) Sung (1953) Hseigh (1962) Richart (1953, 1960) Bycroft (1959) and Arnold, Bycroft and Warburton (1955). It forms a sound basis as long as soil can be assumed semi-infinite, homogeneous, elastic and isotropic solid. Reissner's analysis assumes the distribution under the circular base as uniform, which obviously is not the case. It is evidenced from the experiments that the resonant frequency decreases and maximum amplitude of vibration increases with the increase in the exciter forces as indicated by Lorenz (1934), 1953, 1959) and Balkrishna Rao (1961). Reissner's analysis does not give the varying frequency of resonance for change in the exciter frequency and as such deviates from the experimental data.

The modification by Sung (1953) and Quinlan (1953) for the different load distribution (Parabolic, uniform Rigid), show that as the pressures tend to concentrate nearer the centre of circular base, the resonant frequency decreases and amplitude of vibration increases. The increase in dynamic force, may qualitatively be assumed to be associated with the change in pressure distribution or the decrease in 'effective radius' corresponding to Richart's foundation coincides with the centroid of the contact area, all motions are decoupled (This case is not feasible, practically).

Ford and Haddow (1960) obviously have given a basically different approach to the problem by considering the machine foundation soil as the conservative system. But this gives the resonant frequency which is independent of the dynamic load, which certainly is not correct as shown by the experiment of Lorenz (1934, 53, 59) and Balakrishna Rao (1961).

For the foundations other than circular ones, which have been considered in the theoretical approach following modifications have been suggested for various investigators.

1. For translatory motion, use an equivalent radius which gives the area of circle equal to that of the contact area of the foundation with ground (Sung 1953, Richart 1960, Hseih 1962).
2. For rotational motion, use an equivalent radius which gives the moment of inertia of the circle equal to that of the contact area of the foundation about the axis of rotation (Hseih 1962).
3. Bycroft (1959) has suggested that if Z is the amplitude for equivalent circular base and Zr for the rectangular base, then $Zr = m \times Z$, where m is a shape factor and may be taken as for static case given by Timoshenko (1937).
4. As the distribution assumed by Bycroft (1959) is the rigid base distribution, the concept of

effective radius is suggested to find the values of frequency and amplitude for other type of distribution.

In all the above theoretical methods based on the homogeneous semi-infinite, isotropic, elastic solid, certain values of G (modulus of rigidity) and ν (Poisson's ratio), have to be estimated. The values of modulus of rigidity G vary with depth and so does the poisson's ratio ν . The test on oscillator, will no doubt give certain value of G and ν , but it is valid only for the depth, which may be taken at best equal to three times the base width of test vibrator. With increase in prototype area, the value of G and ν should be valid for depth upto three times the foundation width, and these values will obviously be different than those in test vibrator.

Another short-coming is the assumption of pressure distribution to be assumed, in a solution of particular problem. No data is available from field records, regarding the actual distribution and change in distribution with increase in dynamic loads. It has further been found from the survey of available literature that while the ratio of dynamic to static force in the prototype is about 4% to 5%, its value in the model vibrator is any where from about 25% to 90%. The distribution in two cases may be different, but no quantative approach is available.

RESONANT FREQUENCY - SOIL AS SPRING

3.11 WEIGHTLESS SPRING :

The first known approach to analyse the foundation vibrations considered the vibrating system to behave as a single mass supported by a weightless spring and subjected to viscous damping (Lorenz 1934, and Barkan 1936). It will, therefore, be useful to review briefly the simple case of damped forced vibrations of a single mass supported by a weightless spring. The theoretical model which Hertwig (1933) (quoted by Lorenz 1934, 1959) considered during his first investigations for Degebo is shown in Fig. 3.12 . The periodic vertical exciting force is defined by $F_0 \sin\omega t$, where F_0 is maximum magnitude of the exciting force.

If Z denotes the vertical displacement of the block at time t, the equilibrium condition for vertical oscillation requires that ;

$$m_0 \ddot{Z} + 2\Lambda m_0 \dot{Z} + Z . K = F_0 \sin \omega t \quad ......(3.9\text{-}a)$$

where m_0 is the mass of the supported block (machine and foundation)

Λ (Sec^{-1}) denotes the damping factor such that $2\Lambda m_0 = c$ = damping coefficient.

K is spring constant.

Natural frequency ω_0 of free vibration of the mass spring system is given by

$$\omega_0 = \sqrt{\frac{K}{m_0}} \quad ....(3.9\text{-}b)$$

Defining Z_s the static displacement as would be caused by a force of magnitude F_o acting on the spring, $Z_s = F_o/k$, the equation (3.9-a) becomes ;

$$\ddot{Z} + 2\lambda \dot{Z} + \omega_o^2 Z = Z_s \, \omega_o^2 \sin \omega t \quad \text{.........(3.9-c)}$$

The solution of which is given as (Denhartog 1947) ;

$$Z = Z_s \frac{1}{\sqrt{\left\{1 - \left(\frac{\omega}{\omega_o}\right)^2\right\}^2 + (2\lambda/\omega_o)^2(\omega/\omega_o)^2}} \sin(\omega t - \psi)$$

$$= Z_s N_1 \sin(\omega t - \psi) \quad \text{............(3.9-d)}$$

where $\psi = \tan^{-1} \dfrac{2\omega\lambda}{\omega_o^2 - \omega^2}$ ....(3.9- e)

(3.9-d) is the equation of a simple harmonic vibration with frequency equal to the frequency of the impulse (ω). The value N_1 represents the magnification factor. The amplitude of forced vibration lags behind the impulse, by phase angle ψ.

The magnification factor N_1 is shown as the ordinate in Fig. 3.13 (a) and the curves on this diagram show the manner in which N_1 varies as a function of frequency ratio ω/ω_o for different values of damping ratio λ/ω_o,

For rotating machinery with unbalanced weights, the exciting force is the function of square of exciting frequency.

For Degebo vibrator,

$$F_o = m_1 \ell \, \omega^2 \quad \text{.................(3.10)}$$

where m_1 is the mass of unbalanced rotating part, and l is the eccentric radius from the c.g. of unbalanced mass to the centre of rotation.

By introducing the value of F_0 as defined above, the solution for the displacement can be determined. The results are as shown in Fig. 3.13(b) in which the ordinate is equal to the ordinate of Fig. 3.13(a) multiplied by $(\omega/\omega_0)^2$. The value of maximum vibrational amplitude is ;

$$Z = N_1 (\omega/\omega_0)^2 \cdot \frac{m_1 l}{m_0} \quad \text{.............(3.11)}$$

It should be noted that on Fig. 3.13(b) all curves approach an ordinate value of 1.0 as the frequency ratio (ω/ω_0) becomes very large and is independent of damping. The force transmitted through the spring

$$P_s = Z_{max} \cdot K_s \sin(\omega t - \psi)$$

$$P_s = \frac{(\omega/\omega_0)^2}{\sqrt{\left\{1-(\omega/\omega_0)^2\right\}^2 + \left(\frac{2\lambda}{\omega_0} \cdot \frac{\omega}{\omega_0}\right)^2}} \frac{m_1 l}{m_0} \cdot K_s \sin(\omega t - \psi) \quad \text{.......(3.12-a)}$$

The force transmitted through the dash pot,

$$P_d = 2\lambda m_0 \dot{Z} = 2\lambda m_0 \omega \cdot Z_{max} \cos(\omega t - \psi) \quad \text{......(3.12b)}$$

There is a phase difference of 90° between (3.12a) and (3.12-b). The resulting force transmitted at any time is, therefore, (Myklestad-1956) ;

$$= \frac{m_1 l}{m_0} K_s (\omega/\omega_0)^2 \sqrt{\frac{1+(2\lambda/\omega_0 \cdot \omega/\omega_0)^2}{\left\{1-(\omega/\omega_0)^2\right\}^2 + \left\{\frac{2\lambda}{\omega_0} \cdot \frac{\omega}{\omega_0}\right\}^2}}$$

$$\text{...............................(3.12c)}$$

This for large values of (ω/ω_o) and negligible damping reduces to $\frac{m_1 \ell}{m_o} K_s$.

For undamped forced vibrations, resonance occurs at $f/f_o = 1.0$. When the exciting force has a constant amplitude regardless of frequency, the curves representing damped forced vibrations in Fig. 3.13(a) show the maximum amplitude magnification at values of f/f_o less than 1.0. For small values of damping the amplitude peaks occur at frequency ratios so close to $\omega/\omega_o = 1.0$ that the difference is usually negligible. However, when the damping ratio $\lambda/\omega_o = 0.5$, the peak is at $f/f_o = 0.707$ In other case for which the exciting force is a function of the exciting frequency, the peak amplitude occurs at a value of f/f_o greater than 1.0, which for the damping ratio $\lambda/\omega_o = 0.5$ results in a peak of $\omega/\omega_o = 1.415$. This shows the effect of damping in shifting the frequency for maximum amplitude of vibration away from the "natural frequency of the foundation. However, for all practical purposes, the resonance occurs at frequency ratio of unity, since the damping ratio (λ/ω_o) of the ground is of the order of 0.166 (Alpan 1961). The variation of phase angle as given by equation (3.9 e) with frequency ratio ω/ω_o can be plotted for different values of damping ratio as shown in Fig. 3.14 - a.

Another possible variable with frequency ratio besides the amplitude and the phase angle as given above is the work per unit of time required to operate the vibrator. This work consists of two parts (Terzaghi 1943) One part is used up in overcoming the friction in bearings and other resistances within the mechanism. It has been found that this part increases approximately in direct proportion to the square of frequency. The second part is consumed by the viscous resistance of soil against periodic deformation. The damping force Pd is determined by (Equation 3.12 -b).

The work performed in overcoming the damping force during one complete cycle with the period $\tau = 1/f$ is

$$L_\tau = \int_0^\tau P_d \frac{dz}{dt} \qquad \text{........ (3.13 a)}$$

and work per unit time is

$$L_p = f.\,L = f \int_0^\tau P_d \frac{d_z}{dt} \qquad \text{.......(3.13 b)}$$

substituting for P_d from equation 3.12(b) and Z from equation (3.11) in equation (3.13 b) and integrating, we get

$$L_p = 32\,\pi^6\,(m_1 \ell)^2\, N_1^2\, \frac{c}{K^2}\, f^6 \qquad \text{.......(3.13 c)}$$

(Terzaghi 1943)

The variation of the work rate with the frequency ratio f/fo or ω/ω_0 is given in Fig. 3.14 b.

3.12 <u>EXPERIMENTS BY DEGEBO</u> :

From 1930 onwards, the work was carried at the Deutshe Forschungs-gesellchoft für Bodenmechanik (Degebo) (Lorenz 1934) The standard experimental set up consisted of weight of vibrator 2700 kgm base area 1 sq. meter eccentricity 10° (moment of inertia: eccentric weight x eccentricity = 30.4 x 1.02 = 31.0 kgm. cm)

Experiments were conducted on different sites using the above vibrator. Amplitude of vibration, the phase angle between the exciting force and the resulting vibrations, and the power requirement of the vibrator, were determined at various frequencies. The reasonant frequency is determined where maximum amplitude occurs, and checked with the frequency, where maximum power is required and the frequency where phase difference Ψ is $\pi/2$. Comparison of the experimental plots with those in Fig. No. 3.14 give the value for damping factor.

The values for the frequency which correspond to individual soil types and hence to bearing capacity according to Lorenz (1934) are given in Table 3.2. From the table a general trend is observed i.e. the higher the natural frequency, higher the safe soil pressure. λ the damping factor was found to have the following significance. A value in excess of about 3 to 4 sec^{-1} combined with an important settlement of the base was considered an indication of high compressibility, and sensitivity to vibrations (Lorenz 1934).

During the series of experiments it was found that W in equation (3.9 b) is not the weight of vibrator alone, but also includes the weight of the soil vibrating with it.
Natural frequency becomes ;

$$\omega_o = \sqrt{\frac{K.g}{W_o + W_s}} \quad \text{............(3.14 a)}$$

where W_s = is equivalent soil weight which is assumed to be concentrated at the c.g. of foundation mass, In order to determine W_s, the weight of vibrator was increased by means of surcharge, and the test repeated. The natural circular frequency of system decreases from ω_o to $\omega_{o'}$, where ω_o and $\omega_{o'}$ are the frequencies obtained for different weights of vibrator. Assuming for the sake of simplicity that the increase of the weight of the vibrator has no effect on W_s, two equations are obtained which make it possible to determine W_s .

Another suggested method is to increase the area of the vibrator base, keeping weight of the vibrator same. Replacing k by k'.A in equation (3.14 a), ω_o is obtained as;

$$\omega_o = \sqrt{\frac{k'.A.g}{W_o + W_s}} \qquad \text{..........(3.14 b)}$$

where A is the area of base plate, and

k' is the modulus of dynamic subgrade reaction.

Assuming value of k' to be same from one test to another, value of W_s can be determined.

By increasing the weight of vibrator from 1.8 to 3.4 metric tons, the value W_s is found to be 12.5 tons (Lorenz 1934). Similarly keeping the vibrator weight at 2700 kgm (2.7 metric tons), and changing the area from 1/4 sq. meter to 1 sq. meter, the value of W_s for the same site was found to be again 12.5 tons.

In another set of the test mentioned , W_s was equal to 1 metric ton, when the weight of vibrator was increased from 2060 kgm. to 2700 kgm. These results indicate that value of W_s is likely to vary between wide limits.

For change in eccentricity (increase in dynamic loads), the natural frequency was found to decrease.

3.13 EXPERIMENTS BY VIOS :

At about the same time, the independent tests were carried out in Russia which are reported by Barkan (1936). The theory is based on free vibrations, discussed above.

For the vibrations, so produced as to give both gyration (rotation) and translatory displacement the system is two degrees of freedom, the resonant frequencies are coupled, and there will be two resonant frequencies which were noted.

The experimental foundations weighed upto 30 tons and having an area at bottom upto 8 m. sq.

Value of $K = k'.A$ was determined by the statical tests (reversible displacement x k' = normal stress). k' was determined for areas 2, 4 and 8 m. sq. From the determined values of k', the frequencies of the vertical vibrations only, were calculated. The foundation was subjected to forced vertical vibration with the aid of vibrating machine and resonance diagrams recorded. In nearly all the cases frequencies differed but little from the theoretically calculated ones.

In the analysis the weight of the soil participating has been neglected. In fact, no note seems to have been taken of it. Also k' was determined by static tests and not by dynamic tests. But their extension to the problem of displacement and gyratory motion is worth noting. The two resonant frequencies were observed and their values confirmed, from theory. In a particular case, these two frequencies were 40 c.p.s. and 160 cps. Neglecting the soil weight, should have led to some discrepancies between the experimental and theoretical values. Probably the explanation lies in that the value of k' taken was from the static tests. The two errors may have compensated. Latter experiments in Sweden (Bergstrom and Linderholm 1946) have shown that for large base plates (of order of 3 m. sq.) the value of subgrade reaction (k') corresponds to the values of obtained from dynamic as determined from the wave velocity measurements.

Another conclusion which have been proved wrong in todate experiments, is that resonance frequency is independent of the dynamic load. In fact Barkan (1936), has reported the resonant frequency of 11 cps for the eccentricities of 225 mm, 17.5 mm, 6.5 mm. This again seems erronous conclusion as the resonant frequency has been found to decrease with increase in eccentricity or the dynamic loading (Lorenz 1934, Crockett and Hammond 1948, 1949, Lorenz, 1953).

3.14 EXPERIMENTS BY CROCKETT AND HAMMOND :

Andrews and Crockett (1945), and Crockett and Hammond (1948, 1949, 1958) have also measured natural frequencies using a vibrograph to pick up the oscillations in the vicinity of large hammers. These frequencies are roughly the same as those reported by Degebo (Lorenz 1934) and these are given in Table 3.3. Crockett and Hammond (1948) also stated that for any particular type of ground they got the same natural frequency irrespective of the size of the foundation, the largest foundation tested had an area of 2500 sq.ft. But for reasons given below, it does not seem logical that all foundations whatever their size and weight should be having the same natural frequency. For example, it would be necessary that the spring stiffness of ground is constant for all different widths. This is very unlikely, since the soil mass must behave at least partially like an elastic mass to set up the oscillations. As is well known, a foundation or an elastic mass stresses soil to a depth proportional to the foundation width. This would cause the spring stiffness per unit area k', to decrease as the foundation size increased. In addition, if a very narrow foundation is considered it would be necessary for the effective mass of soil vibrating with it to be very large if natural frequency were to remain constant, whether the footing were loaded with 3 ton/sq.ft. or virtually unloaded. This would mean that the soil must be highly stressed to a depth many times the width of the foundations which is contrary to common knowledge (Eastwood 1953).

But all workers have agreed that mass of soil which vibrates with the foundation must bear some relation to bulb of stress, which gives the stress distribution under a uniformally loaded area on an elastic medium. As shown in Fig. 3.15(Crockett and Hammond 1948, 1949) the active ground weight is assumed to be within a certain bulb of pressure. But no relationship has been indicated.

3.15 TRUNCATED PYRAMID OF SOIL SPRINGS - PAUW :

Pauw (1953) has given an analytical procedure whereby the dynamic soil constants required for the prediction of natural frequencies of a foundation soil system may be determined. The foundation soil system is treated by considering the foundation mass to be supported by a truncated pyramid of "soil springs".

Based on the concept that the modulus of elasticity is approximately proportional to the shearing strength, Pauw made the following assumptions :-

1. For cohesionless soils the modulus of elasticity is proportional to the effective depth which equals the actual depth plus equivalent surcharge.
2. For cohesive soils the modulus of elasticity is constant. Intermediate soil conditions may be interpolated on the basis of coloumb's law. How?
3. The distribution of stress takes place within a truncated pyramid.
4. The soil pressure below the foundation and also at any depth is uniform.

Consider a rectangular area of length L and width B loaded with a uniform load σ_{st} (Fig. 3.16). The effective zone assumed is the volume of the truncated pyramid defined by the surface area LB and the planes sloping at an angle

$\tan^{-1} \alpha/2$.

Value of E at any depth z, according to assumptions (1) and (2) is ;

$E(z) = E$ for cohesive soils ....(3.15 a)

$E(z) = \beta(h + z)$ for cohesionless soils ....(3.15b)

where β is the rate at which modulus of elasticity increases with depth.

h is the equivalent surcharge such that

$h = \sigma_{st}/\gamma$.

σ_{st} is the static soil pressure.

and γ is the density of soil.

Spring factor is defined as the force or moment exerted on a system when the system is displaced a unit distance, or rotated through a unit angle, from the equilibrium position. For a foundation with six degrees of freedom, six spring constants are required for each surface in contact with soil. Apparent mass of soil vibrating with the foundation is estimated by equating the kinetic energy of an equivalent concentrated mass at the surface to the total kinetic energy in the effective zone. Author has given these factors for horizontal and vertical surface.

Spring factors for horizontal contact surface (for cohesive and cohesionless soils) are reproduced in Fig. 3.16. The mass factors for horizontal contact surface are given in Figure 3.17. The integral for mass factor in case of translatory vibrations for cohesive soils does not yield to a converging solution . In above figures the following dimension less parameters are introduced :-

$$s = \frac{\alpha . h}{B},$$

$$r = L/B.$$

Author has also considered the equations of motion for a symmetrical foundation (e.g. of machine foundation is directly above centroid of the contact surface) and found that only vertical vibrations and rotation about vertical axis exist independently, the horizontal translatory motion is coupled with rotation about horizontal axis.

The attempt deserves credit as a rational method ensues to determine the two variables K and W_s for different modes of vibration. The variation of E with respect to depth for cohesionless soils is not necessarily linear, as assumed. (How? Substantiate this?) The author has verified experimentally, the theory advanced by him. This method suffers from disadvantage that no frequency variation is obtained with dynamic loads.

3.16 BULB OF PRESSURE CONCEPT :

Balakrishna Rao and Nagraj (1960) and Balakrishna Rao (1961, 1962) have developed further the concept of oscillation of Bulb of pressure as advanced by Crockett and Hammond (1948, 1949). This has been modified to the density pressure bulb concept. The equation for resonant frequency is ;

$$\omega_0 = \sqrt{\frac{K.g}{W_0 + W_s}} \quad ....(3.14 -a)$$

The weight of the soil mass participating in vibration is estimated by taking the weight of the soil contained in a definite pressure bulb, this pressure bulb is obtained by considering the sum of static and maximum positive dynamic load of the machine and the foundation block to act as a concentrated load at the mass centre of the foundation block. The reason advanced for adding the dynamic load is that the additional static stresses are developed by dynamic load (Nagraj and Balakrishna Rao, 1959). The boundary of this pressure bulb is supposed to be given by pressure intensity of $|\gamma|$ lbs per sq.ft. where γ is the density of the soil mass in lbs per cft.

Thus if Q is the total vertical load (static + dynamic) i.e. $Q = W_o + F_o$, where F_o is the maximum magnitude of dynamic loading, γ is the required density and intensity of pressure, r_1 the radius of the pressure bulb (sphere) then,

$$(2r_1)^2 = 0.4775 \frac{Q}{|\gamma|}$$ (Boussinesq's theory of pressure distribution)

....................(3.16 -a)

∴ Volume of soil contained in this sphere is ;

$$\frac{4}{3}\pi r_1^3 = \frac{4}{3}\pi \left(\frac{0.4475}{4} \frac{Q}{|\gamma|}\right)^{3/2} \qquad ....(3.16\text{-b})$$

and the weight of soil oscillating is,

$$W_s = \frac{4}{3}\pi\gamma \left(\frac{0.4775}{4} \frac{Q}{|\gamma|}\right)^{3/2} \qquad ....(3.16\text{-c})$$

The resonant frequency is then determined by substituting W_s in equation (3.14 -b) ;

$$f_o = \frac{1}{2\pi}\sqrt{\frac{K.g}{W_o + Wdyn + W_s}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{K \cdot g}{W_o + F_o + W_s}} \qquad \ldots\ldots\ldots\ldots(3.16 - d)$$

The above authors have suggested to take the value of K or spring constant according to that given by Pauw, or for that matter from the dynamic soil tests. Knowing the value of K and W_s, the natural frequency can be determined from equation (3.16 - d) . The approach is significant in as much as it considers the pressure bulb for the combined static plus dynamic load and a specific value to pressure bulb is given. This explains the phenomenon of decreased resonant frequency for higher dynamic load as it assumes that soil mass will increase. In most of his calculations, he assumes the static and dynamic loads to act as a concentrated load. The pressure bulb for distributed load based on equivalent sphere did not give much different results.

He has calculated W_s on the assumption of soil as uniform, homogenous, elastic medium, (having same value of E at various depths). The spring constant K is calculated on the assumption that value of E increases with depth (Pauw's approach). That is, the value of W_s and K are calculated on the basis of two contrary assumptions. But it offers a good empirical means to evaluate the effect of changed dynamic force.

He has verified the resonant frequency as calculated by pressure bulb concept with the published results of Converse (1953) and Eastwood (1953).

RESONANT FREQUENCY - MISCELLANEOUS METHODS

3.17 GENERAL :

Under the subhead will be considered the empirical approach to the problem of determining resonant frequency, the resonant frequency of soil as linear spring, and the attempted co-relation between the two approaches is given.

3.18 REDUCED NATURAL FREQUENCY METHOD :

Tschebotarioff and Ward (1948) Tschebotarioff(1951, 1953) have suggested that there is a logarithmic relation between the area of foundation and the reduced natural frequency. The resonant frequency is given by ;

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k'.A.g}{W_o + W_s}} \qquad \text{..........(3.14)}$$

$$= \frac{1}{2\pi}\sqrt{\frac{k'.g}{1+W_s/W_o}} \cdot \sqrt{A/W_o} \qquad \text{.......(3.17a)}$$

W_o/A is equal to the static load on foundation per unit area (σst). Thus at unit static pressure, the frequency would be ;

$$f_{nr} = \frac{1}{2\pi}\sqrt{\frac{k'.g}{1 + W_s/W_o}} \cdot \sqrt{1} \qquad \text{.....(3.17b)}$$

This is termed as the reduced natural frequency.

$$f_o = \frac{j}{\sqrt{W_o}} \quad A^{0.248} \quad \text{............(3.18 d)}$$

The contact area in Tschebotraioff diagram ranges from 1 to 1000 sq.meters (10 to 10,000 sq.ft). For smaller areas, results of laboratory experiments with model footings are available (Eastwood 1953). These tests were made to investigate the factors influencing the resonance frequency on dry and inundated sand. The oscillations was generated by impact. The sand employed in Eastwood's tests (1953) had a dry density of 1.74 g.c.c and a void ratio as 0.525.

Alpan (1961) plotted the results of Eastwood in terms of reduced natural frequency f_{nr} versus area A on log log paper and compared it with extrapolated values as obtained by Tschebotarioff's plot for sands. It will be seen from Fig. 3.19 that the lines obtained by Alpan are quite different than that of Tschebotariff's. This raises the question if these straight lines donot represent an over simplification of relation between contact area and reduced natural frequency. Actually the points are anything upto 100% high or 50% low, the errors being masked by the log log scale (Eastwood's comments 1953). From the data which Tschebotarioff has used for his plot (summarized in Table 1) Tschebotarioff 1953) it will be seen that the resonant frequency was obtained by forced vibration test and Shock or impact and exciting force was either vertical or horizontal and vertical or, only horizontal. But it is a known fact that the nature of the vibrations and the method by which they are induced materially affect the frequency response of the ground. Hence Tschebotarioff's plot is not the true picture of frequencies.

Eastwood's (1953) tests show that for the same applied load per unit area, the natural frequency of a 12" x 3" model footing is the same as that of for a 24" x 3" model footing. Thus they will also have the same reduced natural frequency even though area of one is twice that of the other. He has suggested a possible relation between reduced natural frequency and the least dimension of footing.

Refer equation (3.17 b) for resonant frequency. To obtain same values of f_{nr} (for same area), whatever be the applied load, $\sqrt{k'.g/1+W_s/W_o}$ has to be constant for different areas. This means that either W_s must increase at exactly the same ratio as W_o or alternatively that W_s is always negligible compared to W_o. The latter is impossible and the former extremely unlikely (Eastwood 1953).

Alpan (1961) has made an attempt to analyze from first principles, the relation between frequency and area.

Spring constant

$$k = \frac{E\sqrt{A}}{m(1-\nu^2)} \quad \ldots\ldots\ldots\ldots\ldots(3.19\ a)$$

(for uniformly distributed load i.e. flexible base according to Timoshenko 1937).

where E is modulus of elasticity of soil,

and ν is the poisson's ratio.

m is the shape factor previously described depending upon L/B.

The propogation velocity of Rayleigh waves (Jones 1958)

(Henkelam and Foster, 1960) is given by ;

$$V_R^2 = p \frac{E}{2(1+\nu)\rho} \qquad \text{............(3.19 b)}$$

where ρ is the mass density

and p is a factor depending on .

(Henkelom and Foster 1960).

A table of values of p for a range of from 0.2 to 0.5 is given in Jones (1958) and given in Fig. 3.2

$$\text{Now } f_o = \sqrt{\frac{k.g}{W_o}} \times \frac{1}{2\pi} \qquad \text{............(3.19 c)}$$

Substituting the value of E, from equation (3.19b) in the values of K in equation (3.19 a), which in turn is substituted in equation (3.19 c), Alpan (1961) obtains,

$$f_o = \frac{1}{2\pi} \cdot \frac{1}{p} \sqrt{\frac{2}{m(1-\nu)}} \quad \frac{V_R . \sqrt{\gamma}}{\sqrt{W_o}} \quad A^{0.25}$$

where γ is the unit weight of soil.

Now for a particular type of soil, p, ν, γ, V_R are constant, leading to,

$$f_o = \frac{\text{constant}}{\sqrt{m}} \cdot \frac{A^{0.25}}{\sqrt{W_o}} \qquad \text{.........(3.20b)}$$

which differs from equation (3.18 d) in only that a shape factor m is involved, and that exponental power of A is 0.25 instead of 0.248.

Now shape factor is not only dependent upon the length/width ratio, but may depend also on the type of the load distribution. This probably may be able to remove the discre-

-pancies in the plot of Tschebotarioff. For example, Eastwood(1953) obtains the same natural frequency f_o , for 24" x 3" and 12" x 3" foundation models, for the same static load intensity, though the area is twice. This can be explained by introduction of shape factor. Further work has to be done along these lines. Shape factor which is derived from Pauw's analysis (1953) for cohesive soils is given by ;

$$m = \frac{\log L/B}{L/B - 1} \qquad \ldots\ldots\ldots\ldots(3.21)$$

It will be interesting to use the value of this 'm' in the theoretical analysis equation (3. 20b) to see how Eastwood's result fit in. Equations (3. 20b) for two footings 24" x 3" and 12" x 3" gives,

$$(f_o)\ 24"\times 3" = \frac{\text{const.}}{\sqrt{(m)\text{for } L/B=8}} \times \frac{(2A)^{0.25}}{(2W_o)^{0.5}} \qquad \ldots\ldots(3.22a)$$

$$(f_o)12"3" = \frac{\text{const}}{\sqrt{(m)\text{for } L/B = 4}} \times \frac{A^{0.25}}{(W_o)} \qquad \ldots\ldots(3.22b)$$

Dividing (3. a) by (3. b) we get,

$$\frac{(f)\ 24"\times 3"}{(f_o)12"\times 3"} = \frac{\sqrt{0.463}}{\sqrt{.301}} \cdot \frac{1}{(2)^{0.25}}$$

$$= 1.047 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3.22\ c)$$

That is the frequencies should be almost same which is as was observed by Eastwood (1953).

3.19 EMPERICAL EQUATION DUE TO CONVERSE :

From the results of the field testing programme mainly conducted to note the effect of various parameters, on the compaction of sand by vibration in a test pit 6' deep, 10' squares, an empirical equation was developed (Converse 1953) for predicting the resonance frequency of a vibrator sand mass system. As a first approximation, the system is assumed to be analogous to a simple harmonic vibration system of a single degree of freedom.

Since the field tests indicated that resonant frequency is influenced both by the dead load, W_o and the dynamic force F_o, the spring modulus k is non linear. By using the data from 28 field tests for vibrator plate diameter as 19.2", converse calculated k as ;

$$k = 44.3\, W_o + 16000 - 27\, F_o \qquad \text{..........(3.23 a)}$$

$$\text{Hence } f_o = \frac{\sqrt{g}}{2\pi} \sqrt{44.3 + \frac{16000}{W_o} - \frac{27\, F_o}{W_o}} \qquad \text{.....(3.23b)}$$

In order to make above equation dimensionally consistent, the term under the radical must have the dimensions of K/W (that is L^{-1}) , Converse (1953) further changed the natural frequency expressions as ;

$$f_o = \frac{\sqrt{g}}{2\pi} \sqrt{1380\, \frac{\gamma}{G} + 0.55\, \frac{Gro}{W_o} - 840\, \frac{F_o}{W_o}\, \frac{\gamma}{G}}$$

............(3.23 c)

$$\text{or} \quad f_o = \frac{\sqrt{g}}{2\pi} \sqrt{840\, \frac{\gamma}{G}\,(1.64 - F_o/W_o) + 0.55\, Gro/W_o}$$

............(3.23 d)

$$\text{where} \qquad o < \frac{F_o}{W_o} < 1 .$$

which in terms of unit loads , will be ;

$$f_o = 313 \sqrt{840\, \frac{\gamma}{G}\left(1.64 - \frac{\sigma_{dyn}}{\sigma_{st}}\right) + 0.18\, \frac{G}{\sigma_{st}\, ro}}$$

...............(3.23 e)

-cular soil type. But before adopting this, much experimental confirmation is required for different types of soils.

3.20 DEPARTURE FROM THEORY OF HARMONIC OSCILLATING POINT :

Lorenz (1953) has noted that the following three phenomena donot agree with the theory of a harmonic oscillations is unable to explain the following experimental behaviour of the system :-

a) Increase in the exciter forces leads to decrease in natural frequency.

b) Keeping the exciter forces and static soil loads constant, and increasing the contact area the resonant frequency increases.

c) The damping constant increases with the contact area, consequently lower amplitudes are obtained if the contact area is increased.

The resonant frequency of a harmonic system depends only upon the spring constant and mass. Hence the dynamic force does not have influence on the natural frequency. However, this may be explained by assuming the Ws participating in vibration increases with increase in dynamic load (Balakrishna Rao 1960, 61, 62).

The effect of increase in area, for the same dynamic and static load, on the natural frequency may be explained as below:-

Since W_s, the soil weight participating in vibrations, based on concept of pressure bulb below the uniformly loaded area, increases with area, the natural frequency decreases.

where σ dyn = unit dynamic force lb/sq.in.

and σ st = unit dead load lb/sq.in.

The units in equations (3.23) are lb.inch units.

Converse has verified the resonant frequency based on the above formula with that of the field test results obtained with base plates 15.7, 19.2, 24.0 and 45.0 inch in diameter.

The development of the empirical equation is significant, as it involves not only the soil constants, but also the vibrator dimensions (ro), weight (W_o) and dynamic force (F_o) But the equation is developed only for one type of soil and it is only reasonable to expect that this will vary with the type of soil and as such the equation is not universal in its nature.

The equation takes into account that

a) increasing the dynamic load, the natural frequency will decrease

b) increasing the contact area, but keeping exciter forces and the static weight constant, that is F/W_o constant, W_o - constant, ro - increasing, the resonant frequency increases.

(a) and (b) agree with the experimental behaviour as observed by Lorenz (1934) 1953, 1959), Balakrishna Rao (1961).

The approach is significant but the constants will vary with the type of soil and as such this may not be of much help . It may, however, be suggested that keeping the same parameters, the constants may be determined at the site for a par-

But the soil spring constant K increases with area. Therefore, there is a tendency for net increase in the natural frequency.

The third phenomenon has been explained by Ehlers (Lorenz (1953,1959) who introduced the concept that loss of energy is caused by waves radiating into the soil. This is equivalent to an additional damping, called system damping. It appears probable that this loss of energy increases with the contact area. The results of Ehler's theory are represented here (Lorenz 1959) In particular the simple formulae for amplitudes and phase displacements are dependent on a spring factor, and a damping factor, which are both related to the surface areas as well as to the propogation velocity and Elasticity modulus of the soil.

According to Ehler's, the system damping is proportional to E, A and inversly proportional to the velocity of compression wave.

i.e. $$c = \frac{E.A.}{V_c} \qquad \text{...........(3.24) -a}$$

where c is damping coefficient

V_c is the propogation velocity of compression wave, and is $\sqrt{E/\rho \cdot \phi(\nu)}$,

$\phi(\nu)$ is the function of poisson's ratio.

$$\therefore \quad c = \frac{E.\ A}{\sqrt{E / \phi(\nu)}} \sqrt{\rho} = \sqrt{E} \sqrt{\frac{\rho}{\phi(\nu)}}\ A. \qquad \text{..........(3.24 b)}$$

Ehler has further shown that with the increase in area, the value of damping will increase, and after a certain limit the damping will be over critical.

The result of Ehler's investigation is so significant, because we are forced to conclude from it that a dynamically affected structure shows perceptible amplitudes only within a certain surface area. If the surface area increases beyond the limit, the vibration becomes a periodic, that is no amplitudes are discernible. The same conclusions can be derived with the help of Hseih's transformation (Hseih 1962) equation (3.6 a). The system is equivalent to the simple dashpot and spring system, with the following values of k and c.

$$k = G r_o F_1, \text{ and } c = \sqrt{G\rho}\; r_o^2 F_2$$

where F_1 and F_2 depend upon dimensionless frequency term 'a', poisson's ratio ν, and loading distribution.

For rigid base distribution, Hseih (1962) has calculated F_1 and F_2 as given below for vertical translation ($o < a < 1.5$)

$$\left.\begin{aligned} \nu = 0, \quad & F_1 = 4.0 - 0.5\, a^2 \\ & F_2 = 3.3 + 0.4\, a \\ \nu = \tfrac{1}{4}, \quad & F_1 = 5.3 - 1.0\, a^2 \\ & F_2 = 4.4 + 0.8\, a \\ \nu = \tfrac{1}{2}, \quad & F_1 = 8.0 - 2.0\, a^2 \\ & F_2 = 6.9 \end{aligned}\right\} \quad \ldots\ldots\ldots (3.25)$$

Value of c is of the same form as that obtained by Ehler's and increases with area A, so it is possible by increasing area (which will be too large) to obtain a periodic motion (to reach critical damping).

3.21 SOIL AS SUBLINEAR SPRING :

The decrease in natural frequency with increase in dynamic loads and increase in contact area may also be explained from the concept of sublinear spring (Lorenz 1953a, Alpan 1961). A sublinear spring is defined as the spring, in which the spring stiffness decreases as the deflection increases. Fig. 3.20 shows 3 amplitude curves, obtained with an oscillator (Lorenz 1953a) Total weight of oscillator was 675 kg contact area 0 0.25 sq.meter static pressure was 0.27 kg/sq.cm. The total eccentric weight was about 24 kg. and 3 tests shown in the figure, run with l = 1, 2 3 cm., approximately. The corresponding eccentricity factors were given as $\varepsilon = m_1 l / m_0$ = 0.037, 0.074, 0.111 cm. respectively. The shape of amplitude curves is particularly, for ε = 0.111 cm. is characteristic of damped forced vibrations of a system with a sublinear (soft) spring. This type of spring exhibits greater strains at higher stresses. Based on a method developed by Denhartog (1947), and modified by Lorenz (1953) it is possible to derive the nonlinear spring characteristics from experimental amplitude curves. The procedure illustrated in Fig. 3.21 is briefly as follows :-

A point is chosen on the ascending branch of the amplitude curve, say point A (ω, z) in figure 3.20. The spring characteristic is computed as follows :-

In equation (3.9a) neglecting damping, value K(z) is substituted instead of K . K(Z) indicates that spring constant depends upon amplitude.

$$m_o \ddot{z} + K(z)\, z = F_o \sin \omega t. \quad \text{........(3.9a)}$$

Let the maximum amplitude (b) Z, then

$z = Z \sin \omega t.$

$F_o = m_1 \ell \omega^2 = m_o \varepsilon \omega^2$ (as $m_1 \ell = m_o \varepsilon$)

Substituting equation (3.9a) we obtain ;

$$-m_o Z \omega^2 + K(Z)\, Z = m_o \varepsilon \omega^2$$

or $k(Z).Z = m_o \omega^2 \; (\varepsilon + Z)$

Now $K(Z) . Z = \sigma_{dyn}$ A. (By definition) .....(3.26a)

$$\therefore \quad \sigma_{dyn} = \frac{mo\, \omega^2 \; (\varepsilon + Z)}{A} \quad \text{...... ....(3.26 b)}$$

From point A, ( )dyn. can be computed and a plot is obtained between Z and σ dyn. The slope of this curve at any point defines the characteristics of the sublinear spring. Equation (3.26 b) can also be plotted graphically as shown in Fig. 3.21 which is self explanatory. It must be emphasized here that the method breaks down for points chosen too near the resonant frequency. In many cases, the spring characteristic so obtained may be extrapolated for higher values of σdyn and Z.

This may be used in Eq. (3.9) to give the solution for amplitude and frequency for any other foundation soil system. This becomes complicated. Alpan (1961) has suggested a simple method by which knowing the spring characteristics, fo and Zo (resonant frequency and resonant amplitude), may be determined for any value of ε and m_o

ALPAN'S APPROACH (1961) :

is it an assumption?

First simplifying assumption that Alpan makes is that "it is considered permissible to analyze the forces with the help of a phase diagram at no great sacrifice of accuracy".

For the case of forcing frequency being equal to natural frequency of the system i.e. $\omega = \omega_o$, the damping force is 180 deg. out of phase with the driving force, and so is the restoring spring force with the acceleration force. The direction of these pairs of forces are normal to each other, The vector diagram of Fig. (3.22 a) gives ;

$$\left.\begin{aligned} m_1 \ell\, w_o^2 &= c\,\omega_o Z_o \\ \text{and} \quad kZ_o &= m_o \omega_o^2 Z_o \end{aligned}\right\} \quad \text{.............(3.27 a)}$$

These two equations give ;

$$\left.\begin{aligned} \omega_o &= \frac{c.Z_o}{m_1 \ell} = \frac{c\,Z_o}{m_o \varepsilon} \quad \text{since } m_1 \ell = m_o \varepsilon \\ \text{and} \quad w_o^2 &= k/m_o \quad \text{respectively} \end{aligned}\right\} \quad \text{....(3.27b)}$$

Eliminating ω_o from equations (3.27b), we get,

$$Z_o^2 = k\, m_o (\varepsilon/c) \quad \text{.......... ....(3.28 a)}$$

Also from the equation (3.26 a),

$$k = A.\, \sigma\, \text{dyn.} / Z_o \quad \text{.........(3.26 a)}$$

By eliminating k from equations (3.28 a) and (3.26 a)

$$Z_o^3 = A\, m_o (\varepsilon/c)\, \sigma\, \text{dyn.} \quad \text{...........(3.28 b)}$$

The spring characteristic (determined in Fig. 3.21) is represented by M/A curve (or σdyn/Z) . The cubic parabola of equation (3.28 b) intersects this curve at a point which gives the value of Z_o , the amplitude at resonance (Fig. 3.22 b) but cubic parabola can be constructed only when values of c is known. Actually to a certain extent, this damping constant varies with ε or the exciter force. But this variation may be neglected and determined from Equation (3.26), which gives,

$$c = \frac{m_o \ \varepsilon \ \omega_o}{Z_o} \qquad \text{..............(3.29)}$$

Experimental curves such as shown in Fig. 3.20, may be used to evaluate c from equation (3.29).

Having assigned to the soil an average value of c, the cubic parabola equation(3.28 b) can be constructed for any ε and m_o . The intersection with characteristic (M/A or σdyn/Z) curve gives the value of Z_{01} for $\varepsilon = \varepsilon_1$. The value of Z_{o_1} found graphically is used in equation (3.27 b) to determine ω_{o_1} the resonant frequency at $\varepsilon = \varepsilon_1$.

The method described to determine the soil spring characteristic by graphical means is a simple one. Alpan gives a simple method of using this characteristic, to determine for any exciter force (ε), the value of resonant frequency and the amplitude. Alpan has checked the method and found it sufficiently accurate, for varying dynamic loads. However, it seems likely that if the contact area A is changed, spring characteristic will change, and hence for changing area this cannot be applied.

3.22 BASIC SIMILARITY OF VARIOUS APPROACHES :

In all the methods for predicting resonant frequency, which assume soil to be homogeneous, an interesting similarity is pointed out. Based on this an empirical formula is developed, which will help in simplifying the calculations .

For static loads, the value of spring constant for various distributions are ;

$$k = \frac{Gro}{(1 - \nu)} \qquad \text{.......................(3.30 a)}$$

for rigid plate condition. (Snedd∆n 1951).

$$k = \frac{\pi \; Gro}{1 - \nu} \qquad \text{..........(3.30 b)}$$

for uniform stress condition, (Baussinseq's 1885).

$$k = \frac{3\pi}{4} \; \frac{Gro}{1-\nu} \qquad \text{..................(3.30 c)}$$

for parabolic stress distribution. (Frohlich 1934)

Equations (3.30) show the spring constant is multiple of Gro. Now consider the theory of vibration resting on elastic homogeneous, semi-infinite, isotropic medium (soil). In the analysis, mass ratio 'b' and dimensionless frequency term 'a' is used, where,

$$b = \frac{m_o}{\rho r_o^3} \quad \text{and} \quad a_o = \omega_o \, r_o \sqrt{\rho / G} \qquad \text{.....(3.31 a)}$$

$$\therefore \quad b{a_o}^2 = \frac{m_o}{\rho r_o^3} \cdot {\omega_o}^2 \; {r_o}^2 \cdot \frac{\rho}{G} \qquad \text{..............(3.31 b)}$$

which after re-arranging gives,

$$\omega_o^{\,2} = \frac{Gro}{m_o} \, (b{a_o}^2) \qquad \text{.............(3.31 c)}$$

Value of equivalent spring constant is, therefore, (Assuming $\omega_o^2 = K/m_o$)

$$K = \frac{Gro}{\nu} \cdot (ba_o^2) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3.31\ d)$$

Equation (3.31 d) based on the criterion of resonance at phase angle = $\pi/2$, gives (Reissner 1936, Quinlan 1953).

$$(ba_o^2) = \frac{-f_1}{f_1^2 + f_2^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3.32\ e)$$

(ba_o^2) is, therefore, a function of load distribution Poisson's ratio and 'a'. Jones (1958) has plotted value of (ba_o^2) versus (b) for different load distributions and poisson's ratio from Sung's (1953) calculations.

Hseih's transformation (1962) gives spring constant as (Equation 3.6 a)

$$K = G\, ro\ F_1 \qquad \ldots\ldots\ldots\ldots\ldots(3.6\ a)$$

where F_1 has been evaluated for rigid distribution and different poisson's ratio in terms of 'a', given in equation(3.25).

$$\therefore \omega_o^2 = \frac{Gro\ F_1}{m_o} \qquad \ldots\ldots\ldots\ldots\ldots(3.32)$$

Putting value of F_1 from equation (3.25) we get (for $\nu = 1/2$)

$$\omega_o^2 = \frac{Gro\ (\ 8 - 2.0\ a_o^2\ )}{m_o} \qquad \ldots\ldots\ldots(3.32\ b)$$

Substituting value of $'a'_o = \omega_o r_o \sqrt{\rho/G}$, and rearranging we, obtain,

$$\omega_o^2 = \frac{8\ Gro}{m_o + 2\rho\, ro^3} \qquad \ldots\ldots\ldots\ldots(3.33\ a)$$

Similarly for $\nu = 1/4$,

$$\omega_o^2 = \frac{5.3 \; G r_o}{m_o + \rho \, r_o^3} \qquad \ldots\ldots\ldots\ldots(3.33\ b)$$

For $\nu = 0$,

$$\omega_o^2 = \frac{4.0 \; G r_o}{m_o + 0.5 \rho \, r_o^3} \qquad \ldots\ldots\ldots(3.33\ c)$$

Values of spring constant from equation (3.30) for rigid base distribution is $4Gro/1-\nu$, which for $\nu = 1/2, 1/4,$ 0 is 8 Gro, 5.33 Gro and 4.0 Gro respectively. This tallies with spring constants in equations (3.33).

Let us take Ford and Haddow's analysis (1960). In this analysis, value of resonant frequency (equation 3.8 a) is

$$\omega_o^2 = \frac{2G\,(1 + \nu)\ \mu\ .\ g}{\gamma/\mu + \sigma_{st.}} \qquad \ldots\ldots\ldots(3.8\ a)$$

in which μ is the decay factor and is given by equations (3.8 c)

$$= \frac{B_1}{m/\sqrt{A}\,(1 - \nu^2)} \qquad \ldots\ldots\ldots\ldots(3.8\ c)$$

Changing σ_{st} in to $m_o/\pi\, ro^2$, area A into $\pi\, ro^2$ and substituting value of μ from equation (3.8 c) in equation,(3.8a) we obtain,

$$\omega_o^2 = \frac{2G\,(1 + \nu) \times B_1\, ro\, /\sqrt{\pi}}{(1 - \nu^2)} \cdot \frac{1}{\left(m_o + \dfrac{\pi^{3/2} \rho\, r_o^3 (1 - \nu^2)}{B_1}\right)}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots(3.34\ a)$$

$$= \frac{G_{ro} \times 2B_1\ /\sqrt{\pi}}{(1 - \nu)} \cdot \frac{1}{m_o + \rho ro^3\ \dfrac{\pi^{3/2}(1-\nu^2)}{B_1}}$$

$$\ldots\ldots\ldots\ldots\ldots(3.34\ b)$$

Therefore, the equivalent spring constant is a multiple of G_{ro}, and the mass of soil participating in vibration is a multiple of ρro^3.

Now consider the emperical plot of Tschebotarioff (1948, 51, 53). The equation of the plot (Equation 3.18 d) is

$$f_o = \frac{j}{\sqrt{W_o}} A^{0.248} \quad \text{.........(3.18 d)}$$

Substituting $A = \pi ro^3$ in equation (3.18 d), we obtain,

$$f_o = \text{constant} \times \frac{r_o^{0.496}}{\sqrt{W_o}} \quad \text{.........(3.35 a)}$$

$$= \text{constant} \times \sqrt{\frac{r_o^{0.992}}{W_o}} \quad \text{........(3.35 b)}$$

where the constant depends upon soil type, (and hence an value of G).

Hence spring constant may be taken as a multiple of Gro. Note very small power difference between 1.0 and 0,992.

Converse (1953) has given an emperical equation for resonant frequency of sand vibrator system (Equation 3.23 d)

$$f_o = \frac{\sqrt{g}}{2\pi} \sqrt{840 \frac{Y}{G} (1.64 - F_o/W_o) + 0.55 \frac{Gro}{W_o}} \quad \text{..................(3.23d)}$$

For large values of G, equation (3.23 d) reduces to,

$$f_o = \sqrt{\frac{Gro}{W_o}} \times \text{a constant} \quad \text{..........(3.36)}$$

Pauw (1953) has evaluated the spring constants for the cohesive soils for which values of E or G can be assumed to be constant. Though it was not possible to evaluate the mass factor, it will be of interest to see nature of spring constant by him. Expression for spring constant for vertical vibration is

$$k = E\alpha \quad B \quad \gamma_z \quad \text{..........(3.37 a)}$$

where $\tan^{-1}\alpha/2$ denotes the angle of pressure distribution, and γ_z is a factor depending upon L/B ratio.

For circular vibration B = 2 ro, and $\gamma_z = 1.0$ (Pauw 1953),

$$\therefore \quad k = E.\alpha.(2\,ro).\,1 = 2\,G(1+\nu)\,.\alpha.(2ro)$$

$$= Gro\left\{4\alpha(1+\nu)\right\} \quad \text{...............(3.37 b)}$$

i.e. the spring constant is a multiple of Gro.

Experiments by Nijboer (1953, 1959) Vander Poel (1951, 1953) Heukelom (1959), Heukelom and Foster (1960), obtained the value of spring constant as approximately 7.6 to 7.7 Gro, in their dynamic tests.

$$\text{i.e.} \quad k = 7.6 \; Gro \quad \text{..............(3.38)}$$

From consideration of equations (3.30), (3.31), (3.32a) (3.33a,b,c) (3.34 b), (3.35b), (3.36), (3.37b) and (3.38), a simplified form of the natural frequency expression is suggested viz.,

$$\omega_o^2 = \frac{\lambda_1 \times G_{ro}}{m_o + \lambda_2 \rho ro^3} \quad \text{............(3.39)}$$

Interoducing the shape factor m in equation (3.39), we obtain,

$$\omega_o^2 = (\lambda_1/m)\cdot\frac{Gro}{m_o + \lambda_2\,\rho\, ro^3} \quad \text{.........(3.40)}$$

where λ_1, λ_2 are constants for the system and m is the shape factor depending upon L/B ratio.

The effect of change in dynamic load upon λ_1 and λ_2 will have to be investigated experimentally.

CHAPTER - 4.

DYNAMIC TESTING OF SOILS

4.1 GENERAL :

In Chapter - 3 various methods for predicting the resonant frequency of foundation soil system have been discussed. It was, however, assumed, while considering the methods, that dynamic constants of soil stratum were known. In analysis of machine, foundations by Elastic wave approach, soil properties are of prime importance. These are the (i) modulus of elasticity, (E), or modulus of rigidity (G), (ii) the poisson's ratio (ν) and (iii) the density (γ). In mass spring analogy again three soil properties are of significance. These are (i) spring constant (K) (ii) damping factor (λ), (iii) soil weight (W_s) Pauw (1953) Balakrishna Rao (1960, 1961, 1962) have shown that W_s depends upon γ. Pauw (1953) has related the spring constant with (E) or its increase with depth (β). Ehler (Lorenz 1959), Hseih (1962), have attempted a correlation of spring constant damping factor with E, ν, and γ - Lorenz (1953) and Alpan (1961), have used the nonlinear characteristics of the spring to determine the resonant frequency. Tschebotarioff (1948, 1951, 1953) has given an empirical plot between contact area and reduced natural frequency. All these approaches try to use the soil data as obtained from vibrator test on the soil and from that data analyse the soil behaviour.

As far as the value of ' γ ' is concerned, it offers no difficulty, Ordinarily, sampling will serve the purpose. Also Poisson's ratio varies only over a small range for most soils as long as the applied loads do not stress the particular soil excessively. It is not a property that appreciably varies even under a dynamic load (Fisher and Winter 1962). In many cases it may be estimated or assumed with sufficient accuracy to be used in analyses. For greater accuracy laboratory tests may be used to measure poisson's ratio directly (Tschebotarioff 1951). By measurement of the coefficient of earth pressure at rest (ko) in a triaxial test, poisson's ratio may be found by the following relationship ($\nu = ko/1 + ko$). Typical values of poisson's ratio are given in Table No. 4.1.

Therefore, the principal problem lies in evaluating either the modulus of elasticity or the modulus of rigidity. These properties vary not only in different soils, but depend upon the imposed loading condition. Measurements of the modulus of elasticity can be made by ;

i) direct measurement of stress and strain in a confined compression test,

ii) using the first part of a stress strain curve on a triaxial,

iii) using a straight line portion of a triaxial compression test, after several repetetions of load and reload cycles have been made.

These methods may be considered to be standard test procedures for evaluating static elastic modulii. Dynamic elastic modulii with which we are concerned is not the same as obtained from static tests, and hence the static tests cannot be used for finding the dynamic behaviour of soils.

4.2 SEISMIC METHODS :

Seismic methods of exploration for mineral prospecting and oil explorations, are based on the velocity of propogation, and any change in velocity of propagation of wave(when a wave travels through media having different densities). An artificial impulse produces chiefly compression waves, and if seismographs are employed, the subsequent wave to arrive is shear wave. From these velocities E and ν can be calculated by using equations given in Fig. (3.2). Disadvantage was that same force or impact may not be generated. Also as the explosion produces a single impulse, there is no possibility of determining the thickness of individual soil strata by means of observing interference phenomena on the surface of ground.

Recent developments have made it possible to adopt seismic methods to the local sites within the economical range. Also the impact or shock is produced by a sledge hammer blow. These units are suitable for exploring subsurface conditions down to 25 to 50 ft. below the ground surface, which is usually sufficient, to define the dynamic characteristics of subsurface, materials below vibratory equipment foundations. A typical unit is shown in Fig.4.1 (Fisher and Minter). While the operations of these shallow depth seismographs is relatively simple, the electronic circuit is somewhat involved. This helps in evaluating the time difference between the time when the impact is imparted and when the geophone picks the first wave arrival. Knowing the distance between source of disturbance and geophone and the time taken for the wave to arrive at geophone, velocity of propagation is calculated. This is mainly the compressive wave and from its velocity the value of E dynamic may be determined by assuming suitable value of ν .

4.3 EARLY STUDIES :

Studies with mechanical vibrator, was first suggested in a report of Indian Railway Bridge stress Committee in 1921 (Bernhard 1949) in which the idea of testing with a machine of two mass vibrator was advanced. Very few experiments were made till 1930 in Germany for the purposes of studying vibrations produced by industrial machinery. These earlier experiments have been reviewed by Heinrich (Bergstrom and Linderholm 1946, 1949, Subbarao 1962). In early investigations amplitudies were observed at various distances from the source of initial disturbance. Heinrich found a relation of the following type :-

$$Z_n = Z_1 \sqrt{r_1/r_n} \; e^{-\mu (r_n - r_1)} \qquad \ldots\ldots\ldots\ldots(4.1)$$

where Z_n is the amplitude in a vertical direction at a distance r_n from the source of initial disturbance,

Z_1 is the amplitude in a vertical direction at a distance r_1 from the source of initial disturbance.

μ is a decay coefficient depending upon characteristics of soil.

Similar expression has been given by Bernhard (1958) Barkan (1963).

The constant μ characterized the properties of the soil to a certain extent, but the accuracy of the method was not sufficient to admit of a classification of various types of soil.

4.4 DYNAMIC CONSTANTS BY RESONANT FREQUENCY :

The beginning of the application of dynamic testing for the evaluation of the soil properties can be traced back to 1930's when the research workers of Degebo started the systematic investigations of soil vibrations. The first attempt was by analysing the behaviour of soil from the response curves -

a) frequency vs amplitude,

b) frequency vs power point and

c) frequency vs phase difference, between the periodic disturbing force and the vibrations produced by this force -

- obtained by means of a mechanical oscillator with two revolving eccentric masses (Lorenz 1934). These response curves can be interpreted to obtain resonant frequency, and damping of the system, and consequently spring constant k and soil weight participating in vibration with system as explained in section 3.12. In this analysis, modulus of subgrade reaction was assumed to be independent of area and dynamic load, which is not true. Theoretically k' is not a f^n of area

Jones (1958) has also given a method, which determines the values of modulus of rigidity by observing resonant frequency. He has applied the theory of vibrator as semi-infinite, elastic homogeneous and isotropic solid. He has simplified the calculations by using a simplified equation (3.31 c) in which he has plotted value of (bao^2) versus different values of b, for different poisson's ratio and load distribution(Fig. 4.2) For different values of 'b', and fo, the value of G is calculated by equation (3.31 c) (section 3.22).

It is shown that shear modulus deduced from the measurement of the resonant frequency, at low amplitudes of vibration was in good agreement with the shear modulus deduced from the phase velocity measurements. In this comparison, the stress distribution, was assumed to be either uniform or that arising from a rigid plates. It appears that when the soil was firm and dry the first assumption was justified, where as on soft wet soils the rigid plate condition applied. The results indicate that the uncertainty in the shear modulus due to unknown distribution of stress beneath the vibrator is likely to be about ± 11% Reference ?

The method seems to be more useful for machine foundations, as the behaviour of the soil is obtained at resonant frequency. In fact, by determining the resonant frequency, any particular soil constant representative of the dynamic behaviour of soil (e.g. decay factor in Ford and Haddow's analysis, rate of increase of modulus of elasticity with depth in Pauw's spring analogy, or the spring constant) will be more applicable than from the empirical tables set up. The uncertainty due to load distribution, seems at present stage of knowledge difficult to remove. Which method ?

This method has been applied by Central Building Research Institute in in-situ dynamic tests conducted for a wind tunnel design at Bangalore. No

4.5 WAVE PROPAGATION :

This report has not been published

Subsequent investigations by Degebo (Kohler, Ramspeck 1936, Hertwig 1936) reported by Linderholm and Bergrstorm, 1946, 1949, Jones, 1958, Subbarao 1962), comprised the measurement of velocity of wave propagation in the soil. The velocity of wave propagation is independent of the size and mass of the vibrator and can, therefore, be used for characterizing the properties of soil. If the soil is regarded as homogeneous, elastic semi-infinite body, the modulus of elasticity in tension, E, and modulus of rigidity, G, of the semi-infinite body can be computed from the velocity of wave propagation. Actually, the soil is not homogeneous, and the calculated values of E, and G have to be regarded only as statistical values or effective modulii.

The above work showed that on many sites, the velocity of propagation of vibrations decreased with increase in frequency. The variation was attributed to the variation of soil properties with depth, and attempts were made to give a theoretical explanation to fite the actual thickness and properties at bore holes. However, the type of wave propagation was not definitely established. To quote Jone (1958) some workers considered it to be of the Rayleigh wavetype while others thought the waves were vertical polarized Love waves. However, in view of the theoretical work by Miller and Pursey (1955) it now appears that the first opinion was correct.

The German work, (frequency range 10 to 60 cps) also showed that within the frequency range of 20 to 25 cps, the velocity of propagation was related to the strength of the soil (E or G). However, Jones (1958) has shown that on sites with a shallow surface layer, the frequency range of the mechanical vibrator does not always extend high enough to obtain a phase velocity that is representative of top layer. In fact he has shown that the dynamic shear modulus and the form of its variations with depth can be ascertained from measurements of the phase velocity of the surface vibrations with in the frequency range of 35 to 400 cps. He further used the dynamic testing method for the concrete pavements which are relatively thin, with sustained vibrations of 40 to 60 kc/sec., in order to get the dynamic modulii for both pavement and subgrade Jones (1955,1959).

An investigation of the effects of soil stratification made by Ramspeck (1936) is of great interest. This subject has also been dealt with in several seismological papers (Oosterbeek 1948). It has been shown that the amplitude frequency curve has several maximum and minimum which are due to interference of waves passing through different soil layers. This conclusion was confirmed empirically. In some cases the agreement between theory and experiment was very close, while in some other cases the agreement was unsatisfactory. Jones (1958), used the electrodynamic vibrator weighing 85 lbs, which he had used earlier for the concrete pavement testing. It will be interesting to note that the complete vibrator generator, is such as to produce frequencies between 35 cps to 60 kc/s. (Jones 1955, 1959).

Jones (1958) has indentified, surface waves as Rayleigh waves, based on the theoretical analysis by Miller and Pursey (1955)., who have shown that a vibrator on a circular base, operating normal to the surface of a semi infinite elastic solid ($\nu = 0.25$) radiates 67.4% of the power as a surface wave. The surface waves in which the soil particles have displacements at right angles to surface and also in the direction of propagation of wave are Rayleigh waves. If however, the arrangement, is such as to produce and detect vibrations in a horizontal direction, transverse to the line of measurement, the waves are shear or love wave in the soil. Love waves are basically horizontally polarized shear waves (i.e. TH waves) which have a particle displacement parallel to the surface and transverse to the direction of propagation.

What is phase velocity ; Define this How can phase velocity be used in finding G-Value

Natural soil formations are rarely uniform with depth, and this is reflected in the vibrational measurements by a variation in phase velocity with wave length. However, as the wave length tends to zero (at high frequencies), the phase velocity reflects the behaviour at top surface in which we are interested. In such case, the data is extrapolated for zero wave length, phase velocity which gives the shear modulus. The shear modulus so obtained refers to the soil nearest to the surface, which can be used in predicting the dynamic soil behaviour. In a particular case, for a surface layer of 4'-10", silty clay over stratum of the gravel, the phase velocity approahed a constant magnitude V_R , at frequencies greater than 150 cps.

Jones has also analysed the phase velocity of Love waves for a surface layer over a stratum of higher shear modulus and has shown by experiments, a very good correlation between the theory and experiments. Phase velocity of vibrations become velocity of shear wave in top medium, when the thickness of surface layer, becomes equal or greater than the wave length of vibrations.

He has also analysed and interpreted from these data the thickness and the elastic constants of underlying medium. Jones (1959).

4.6 SWEDISH EXPERIMENTS :

Similar to Degebo experiments in Sweden (Bergstrom, Linderholm 1946, 1949) used the phase velocity at resonant frequency of vibrator soil system to arrive at dynamic soil constants. The vibrator used was 75 kg in weight, 600 mm. in diameter (base area 0.282 m.sq) eccentric weight of 0.46 kg at an eccentricity radius of 43.5 mm, with a frequency range of 10 to 60 cycles per sec. The general arrangement of the equipment used for displacement and wave length measurements is shown in fig. 4.3. Velocity of propagation, was obtained from wave length (min. distance between pick ups so that phase difference is 2π) and the frequency of sustained vibration. This was assumed to be Rayleigh wave (according to an analysis by Sezawa and Kanai 1935). Values of E and G were determined for the two Poisson ratios $\nu = 0.25$, and 0.50. The site seems to be uniform, as indicated by no significant variation in the phase velocity between frequency range of 14 to 32 c/s. (39.1 to 44.1 meters /sec. was corresponding range of velocity of propagation). If the soil behaves elastically, an empirical soil constant has been defined, by the above authors which is related to E ang G as follows :-

$$\text{Soil constant} = \frac{E}{1-\nu^2} = \frac{2G}{1-\nu} \quad \ldots\ldots\ldots\ldots\ldots (4.2)$$

? $E = 2G(1+\mu)$ This is a well [illegible] eq[illegible] What he means is Soil Constant $= \frac{E}{1-\nu^2}$ used in plate theory

This constant is related to modulus of subgrade reaction k' as ;

$$\text{Soil constant} = k' \frac{\pi r_o}{2}$$

where $k' = k/\pi r_o^2$ (Bergstrom and Linderholm 1946).

The field data has shown that the value of this constant derived from the plate bearing tests on large plates (r_o = 3 meters) was in extremely good agreement with the values obtained from vibration experiments. Smaller plates (R_o=0.14 to 0.56 mm) gave poor agreement. Another conclusion drawn from the results in this case was that dynamic method gave a relatively correct idea of the behaviour of the soil under the action of distributed load if the maximum values of ν = 0.5 were used.

4.7 DYNAMIC STIFFNESS :

In post war period (1948 till date), Vender Poel and Nijboer (1953) Vander Poel (1953) developed the technique of testing the road pavement and its subgrade with mechanical oscillators. This has been applied to the subgrade testing, and the interpretation applies to the soils in general. This oscillator, consists of eccentric masses on 3 synchronized shafts, so that the centrifugal forces from the masses get mutually cancelled in the horizontal direction, but will get added in t he vertical direction, giving sinusoidal vertical forces. This machine which has been developed further by Vander Poel and Nijboer (1953), is commonly known as 'Road Vibrating Machine' (R.V.M.) Nijboer (1959), similar machine has also been developed in Bundesanstalt für strassenwesen at Coogne (Germany) Baum (1959). The only difference between the two is that former has frequency range of 5 to 60 cps, while the latter has a range of 10 to 80 cps. R.V.M. is supposed to duplicate the condition of wheelload of 4 tons. Vander Poel (1951) introduced the concept of dynamic stiffness of the subgrade which is defined by the ratio of amplitudes of forces generated and deflection of the strata. The displacement was measured by feeding the signals from a displacement pick up just below the centre of the vibrator, on to the oscilloscope. The results have shown that the dynamic stiffness is dependent on the frequency at which it is determined (Heukelan (1959,1960). The data has been analysed on the basis of elastic spring theory. According to this theory, k, m_o, c are independent of frequency. This has been found to be true at frequencies low enough for the wave length to be large as compared with the radius of the body of soil in which greater part of deformation takes place. At relatively high frequencies, however, the wave length become rather short and parts of distributed mass of soil vibrate in opposite direction. It has been found that this can still be represented by the above simple model provided mass of soil acting is assumed to be a value inversly proportional to the third power of the frequency.

$$\text{Dynamic stiffness } S = \frac{F_o}{Z} \Big/ \sqrt{\left(K - 4\pi^2 f^2 \left(\frac{W_o + W_s}{g}\right)\right)^2 + 4\pi^2 f^2 c^2} \quad \ldots\ldots\ldots\ldots\ldots(4.4)$$

and phase difference ψ is given as

$$\tan \psi = \frac{2\pi f . c.}{k - 4\pi^2 f^2 \left(\frac{W_o + W_s}{g}\right)} \quad \ldots\ldots\ldots(4.5)$$

Equation (4.4) and (4.5) yield,

$$S \cos\psi = k - 4\pi^2 f^2 \left(\frac{W_o + W_s}{g}\right) \quad \ldots\ldots\ldots(4.6a)$$

$$\text{and} \quad S \sin\psi = 2\pi f . c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.6b)$$

If both the dynamic stiffness and the phase angle are measured at few different frequencies, the three quantities k, W_s and c in equations (4.6) can be determined from experiment. The damping c follows directly from equation (4.6 b). Equation (4.6a) shows that a linear relationship exists between S cos ψ and f^2, provided that k and W_s are independent of the frequency. A plot of S cos ψ versus f^2 from test data is obtained. The straight line, through the experimental points intersects the vertical axis (S cos ψ - axis) giving the value of k. The slope of the line is equal to $-4\pi^2 \frac{(W_o + W_s)}{g}$ from which W_s can be calculated. (See Fig. 4.4 between S cos ψ and f^2, and notethe excellent agreement till frequency = 35 c.p.s.) In low frequency range of upto

20 to 40 c/s, the relations have been found to apply. Another interesting feature is that k is invariably found to be constant times ro x G . The value is 7.6 roG as quoted by Lorenz (1959), 7.7 roG as given by Henkelom (1959). (See Section 3.22).

In moderate frequency range (value upto about twice the low frequency range), the mass of the soil participating is found to be inversly proportional to the third power of the frequency. This is shown in (Fig. 4.5). The agreement between suggested mass of soil participating, and the experimental points is excellent.

4.8 LABORATORY METHODS :

Fisher and Winter (1962) have been suggested an interesting laboratory method to find the elastic modulus (dynamic) by measuring the velocity of wave propagation, which is essentially compression wave, on selected undisturbed soil sample. The procedure is relatively simple. A pulse is introduced into one end of an undisturbed soil sample, and the time taken by this pulse to travel to other end of sample is accurately measured. Knowing the length of the sample and the time taken, the velocity of wave propagation is estimated. From this velocity, soils constants (E) can be determined. To duplicate field conditions, the sample is subjected to a confining pressure in the same manner as in a triaxial compression test. However, several problems enter into the lab. device for measuring wave velocity which are as follows :-

1. Extremely accurate time measurement must be made.
2. Field conditions must be duplicated as for as possible.

Fisher and Winter (1962) have devised the apparatus, which they name as Shock Scope, which will meet the above criteria.

Oosteybeck (1948 and Wilson and Dietrich (1960) have computed E_{dyn} from theories of elasticity based on laboratory determination of resonant frequency of soil specimens subjected to forced vibrations.

The restraint for the specimen being vibrated has been verified to correspond to lower end clamped and the upper end free (specimen was kept in triaxial cell) For this condition at resonance (neglecting poisson's ratio (ν) as the effect is negligible),

$$E = \frac{16\, f_o^2\, \ell^2 \cdot \gamma}{g} \qquad \text{.........(4.7)}$$

where ℓ is length of sample,
γ is unit weight of sample
f_o is resonant frequency.

4.9 CORRELATION OF E_{DYN} WITH C.B.R. VALUE :

As E (dynamic) represents the characteristics or strength of soil, attempts have been made to correlate E_{dyn} with California Bearing Ratio. By introducing the diameter of the C.B.R. piston (3 sq.in. area, and 1.96 inches in dia.,) in the Boussineg equation for the surface deflection under the centre of a uniform circular load, and the C.B.R. at 0.1 in deflection equal to the applied load on piston in lb/sq.in. divided by 1000, the relation between E and C.B.R. can be obtained as follows :-

There seems to be some mistake E = 10 CBR or 100 CBR Verify?

The spring constant is given as (Equation 3.30 b)

$$k = \frac{\pi G r_o}{(1 - \nu)} = \frac{\pi \times E \times r_o}{2\,(1 + \nu)\,(1 - \nu)} \quad \text{........(3.30 b)}$$

By defination k = force/deflection which gives,

$$= \frac{1000 \times C.B.R.}{100} \times \frac{\pi\, r_o^2}{0.1} \quad \text{.....(4.8)}$$

Equating (3.30 b) and (4.8) putting $\nu = 0.5$,

E = 100 CBR

E = 10 C.B.R. (kg./sq.cm.)

Actual soils, however, behave differently from elastic materials, for instance, after deformation under the C.B.R. piston, the soil does not completely rebound on removal of load. A considerable part of the deformation (is) plastic and only a small percentage of the deflection is elastic. Under dynamic loading conditions, only elastic deformation is recorded after a certain number of loading cycles. The dynamic modulus E, is found to be higher than 10 C.B.R. The ratio between plastic and elastic deformations varies for various soils and also may depend upon loading, so that correlation between Edyn and C.B.R. shows a considerable scatter. Jones (1958), Heukelon and Foster (1960), have found that as an average Edyn = 110 C.B.R. The factor varies betwen 50 to 200 for individual soils. Figure 4.6 shows the points for various soils as they lie on Edyn in kg./cm.sq. versus C.B.R. value.

This correlation only goes to show the wide scatter which is possible, and as such may be used only for the soil for which correlation has been established, which means that Edyn has to be found by other methods discussed. Any way approximate range may be fixed from the C.B.R. value, and in absence better data, this correlation may be used.

4.10 DISCUSSION :

The oldest method of the dynamic testing of subsurface is by seismic methods. These methods have been extensively used, in mineral prospecting. By knowing the velocity of propogation it is possible to find the dynamic modulii, and poisson's ratio. Also refraction methods have been used to calculate the depth of stratum.

The next group of methods uses one or other type of vibrator to generate the waves. Various methods of interpretation and their development are described in this chapter. By means of vibrator a stationery wave pattern can be obtained and with interference effects from the various underlying strata, it is possible to calculate the depths and dynamic behaviour of soil layers. The vibrator method can be used to find the resonant frequency of vibrator soil system, which in turn may be used to find the constants depicting dynamic behaviour of soils. Laboratory methods to find the wave of propogation have been described. The method offers the solution of problem by laboratory testing of samples. It suffers from the same limitations as ordinary static loading tests viz. the simulation of field condition in the laboratory. A concept of stiffness of foundation soil leads us to believe that below a certain critical frequency, the system may be assumed to be linear, and after that the non linearity may be accounted for by a decrease in soil mass. But unfortunately in that approach nothing has been done to see if the stiffness is independent of the dynamic force. (The tests are done with one type of machine).

The best method available for predicting the dynamic behaviour of soils under machine foundations seem to be to observe the resonant frequencies with various vibrator parameters and evaluate the response from the experimental data. But there is one hitch in this also as to what distribution of pressure be assumed i.e. uniform, rigid or parabolic. This leads to a variation of $\pm$ 11% in the dynamic constants. The finding that spring constant is about 7.6 to 7.7 roG is important if the system is assumed to be mass spring system, and the mass factor may be changed to account for non-linearity seems to be promising approach. However, much further work needs to be done in this connection.

CHAPTER - 5

DESIGN OF MACHINE FOUNDATIONS.

5.1 GENERAL :

In the present chapter, design procedures for machine foundations will be summarized. The earlier methods were completely empirical, and the design of foundation depended upon the manufacturer's recommendation, A heavy foundation was believed to be trouble free. These methods obviously did not account for soil properties.

The first rational approach to the problem is giv by Hool and Kinne (1943), who obtained the weight of foundation by considering its stabilizing action, so as to get the permissi- ble amplitude of vibrations. A value for soil weight is assumed empirically. This has been modified later on by Balakrishna Rao (1961), who assigns a definite value to the soil weight. In the above two approaches resonance was not at all considered though significance of the same was understood as early as 1926 (Rausch 1926).

The next group of methods used the published data, to give the empirical relationship between ;

a) soil bearing pressure and the resonant frequency (Newcomb 1951) and
b) contact area dn reduced natural frequency, (Tschbotarioff and Ward 1948), Tschebotarioff, (1951, 1953), for difference types of soil.

These curves are used in the design of foundation.

The present state of knowledge lays stress on avoiding the resonance and limiting the amplitude of vibration. These methods use the various approaches to determine the resonant frequency. All these methods are described in the following sections.

5.2 EMPERICAL METHODS :

The basic principle of these methods is that the dimensions or the weight, or the amount of concrete used in the foundation is dependent upon the type of machine, its operating frequency or its break horse power. This is based mainly on the recommendations given by the manufacturers, who observed from their experience that certain designs were satisfactory for the working of machine. In these methods obviously the type of soil, was not considered to be of much significance except that its bearing capacity should not be exceeded and that these should not be excessive settlement.

The basic idea was to use heavy machine foundations which were considered as means of providing resistance to the dynamic reactions arising from the moving parts of the machine. An adequate mass of the foundations is also necessary in order to absorb vibrations and to prevent resonance between machine and the soil foundation system. Several recommendations, sometime contradictory are available to achieve this end.

Table 5.1, gives the representative recommendation as to size of foundations to be adopted for various engines.

5.3 TERZAGHI'S RECOMMENDATIONS :

Terzaghi (1943) has given the following suggestions for the design of machine foundations.

1. The dynamic pressure transmitted to the ground by an oscillating force $F_o \sin wt$, is given by $F_o \times N_1$, where N_1 is the magnification factor, which in turn depends upon the frequency ratio. Hence, the total pressure on the base of the foundation varies periodically between $W_o + P_a$ and $W_o - P_a$. According to Terzaghi the dynamic pressure should be multiplied by a factor of 3 to obtain equivalent static load. Hence, the

Why did Terzaghi suggest 3Pa. In crane wires 10Pa is suggested, why?

greatest total load on the soil is equal to

$W_o + 3\,P_a$

The greatest unit pressure is $= \dfrac{W_o + 2P_a}{A}$ per unit of area. This load should not exceed the allowable bearing value q_a as in static case for the soil, where

$$A = \frac{W_o + 3P_a}{q_a} \quad .$$

2. For low frequency machines ω/ω_0 or frequency ratio is to be less than 0.5, which means that ω_0 should be more than twice the operating frequency.

Now
$$\omega_o = \sqrt{\frac{K.g}{W_o + W_s}} \quad \text{...........(3.9 b)}$$

As such the value of soil mass participating, if neglected will give more natural frequency than actual one and this will give the false sense of security to the designer. Hence the value of W_s, should not be neglected.

3. For high frequency machines, ω_o should be less than half the operating frequency. In such a case if W_s is neglected, we get a higher ω_o, and if this satisfies the requirement of $\omega_o = 0.5\omega$, then we are all the more safe.

4. For high frequency machines, there will be transition through resonant frequency, when the machine builts upto operating machine. Care should be taken to limit this resonant amplitude.

5.4 HOOLE AND KINNE'S METHOD :

A fairly rational approach to the problem of design of machine foundations is to equatethe stablizing force obtained by the weight of machine, the foundation block, and a certain portion of the soil to the disturbing forces caused by the reciprocating and rotating masses.

Invariably the maximum magnitude of dynamic disturbing forces of the machine can be written as ;

$$F_o = U_n \omega^2 \quad \text{.............................(5.1)}$$

where U_n is summation of all unbalances, including resciprocating and rotating parts and is constant for a particular machine .

ω is the operating frequency.

Let W_m be the weight of the machine, and W_f be the weight of foundation, so that $W_o = W_m + W_f$. Hool and Kinne (1943) have assumed that the weight of soil participating is a constant times weight of foundation, that is $W_s = C.W_f$.

Further they have assumed that all these masses (Machine, foundation and soil) are accelerated to the same amount. Since the frequency of the large masses is periodic, and is the same as the operating frequency of the machine, the assumption is made that motion is harmonic, in which acceleration equals $Z\omega^2$, where Z is limiting amplitude of displacement of this harmonic motion.

The forces of small masses (Machine parts) with large accelerations are balanced by the forces of large masses with small acceleration (inertia forces).

i.e. $$\frac{W_m + W_s + W_f}{g} \times (Z\omega^2) = U_n\ \omega^2 \qquad \text{..........(5.2 a)}$$

Substituting $W_s = C.W_f$, the value of W_f is obtained as,

$$W_f = \left(\frac{U_n.g}{Z} - W_m\right) \frac{1}{1 + C} \qquad \text{..............(5.2 b)}$$

So that knowing U_n, W_m for a particular machine and Z the limiting amplitude, and assuming a suitable value of C, the wieght of foundation required can be found out.

Thus Hool and Kinne have assumed that mass of soil oscillating is proportional to the mass of foundation,(roughly 10 times). The soil mass participating in vibrations is probably proportional to the mass of foundation and machine and not foundation only since they are likely to act as one unit.

The phenomenon of possible resonance is not even considered while estimating the weight of foundation. This means that the foundation as designed by this method should be checked for the frequency ratio, as recommended by Terzaghi (1943).

5.5 DENSITY PRESSURE BULB CONCEPT :

Balakrishna Rao (1961) has given a method to determine the value of W_s or C, for use in Hool and Kinne's method. The method employed for the design is same as suggested by Hool and Kinne, but the value of constant (W_s/W_f) is determined on the assumption that W_s is weight of soil contained within the pressure bulb intensity of 'γ' lb/sq. ft. where γ is the unit weight of the soil. The intensity bulb, being calculated on the basis of static weight plus dynamic weight acting as the concentrated load at the centre of the area of contact. The procedure as given by Balakrishna Rao (1961) is summarized here. Weight of foundation as obtained by Hool and Kinne is

$$W_f = \frac{1}{1 + C} \left(\frac{U_n \times g}{Z} \right) - W_m \quad \ldots\ldots\ldots\ldots(5.2\ b)$$

1. For a particular machine, portion in the brackets is constant. Assume arbitrary value of C, and corresponding value of weight of foundation is determined. Let this be W_{f_1}. Then the weight of soil participating will be $W_{s_1} = W_{f_1} \times C$.

2. The maximum magnitude of unbalanced inertia force is determined F_o. Then the equivalent concentrated load is $W_m + W_{f_1} + F_o$. Let γ be the density of soil in lbs/cu.ft.

3. Then radius of density pressure bulb is given by radius

$$r_1 = \sqrt{\frac{(W_m + W_{f_1} + F_o) \cdot 0.4775}{\gamma \qquad 4}} \quad \ldots\ldots\ldots\ldots\ldots(5.3\ a)$$

4. Volume of soil contained in this pressure bulb is equal to $\frac{4}{3} \pi \left(\frac{W_m + W_{f_1} + F_o}{\gamma} \times \frac{0.4775}{4} \right)^{3/2}$ ....(5.3 b)

Hence weight of soil can be calculated by multiplying the above volume by γ . Let this be W_{s2} .

If $W_{s_1} = W_{s_2}$, assumed volume of C is correct.

5. If not, assume, $C = \frac{W_{s2}}{W_{f_1}}$, with the new value of constant the steps are repeated. By trial and error, correct value of constant can be evaluated.

6. Then W_f can be determined by the final value of the constant .

An alternative method is suggested by the same author which results in a cubic equation in C. This cubic equation has only one positive real root which gives the unique value for the constant. The cubic equation is obtained by equating the value of W_s as obtained by a constant times W_f, and as obtained by the weight of soil contained in density pressure bulb when W_f itself is expressed in terms of machine constants, and the constant C.

The method suggested is the modification of Hool and Kinne's method. A rational approach to determine the value of constant C is given. Except for this, this method also suffers from the limitations of the parent method, viz., the phenomenon of possible resonance is not considered. It may have been preferable if the soil weight were assumed to be a function of both machine

obviously, because it is only meant for finding C weight and foundation weight i.e. W_s = constant x W_o where W_o is the weight of machine and its foundation, though it is realized that final value of W_f will remain same.

5.6 NEWCOMB'S METHOD :

Newcomb (1951) from the past data of resonant frequencies in the engine and compressor foundation, has plotted the natural frequency versus the static soil pressure, and has obtained two distinot curves for hard and soft clay shown in Fig. 5.1. He has compared these curves with the result obtained from static deflection tests as follows. Resonant freqeuncy for a weightless spring is given by ;

$$f_o = \frac{1}{2\pi}\sqrt{\frac{K.\,g}{W_o}} \qquad \ldots\ldots\ldots\ldots\ldots(3.9\text{ b})$$

The term W_o/k is called the static deflection which is the deflection of ground under the load W_o. Substituting $W_o/K = \delta$ in the expression for resonant frequency, we get ;

$$f_o = \frac{1}{2\pi}\sqrt{\frac{g.}{\delta}}\ \text{c.p.s.} \quad = \frac{1}{188}\sqrt{\frac{1}{\delta}}\ \text{cpm} \qquad \ldots\ldots\ldots\ldots(5.4)$$

From the load settlement curves for a particular value of deflection δ , static pressure = $\frac{\text{Load from curve}}{\text{Area}}$, and frequency is given by equation (5.4). These have also been plotted in figure 5.1.

The dashed lines represent the natural frequency, as obtained from static deflection tests. The correlation is good for the static pressure higher than 1000 lb/sq.ft.

The design of the foundation is done in the following manner :

1. Determine the magnitude and frequency of the unbalanced forces. This information is generally supplied by the manufacturer of the machine or can be readily calculated (Denhartog 1947). The natural frequency of the system should be at least twice the frequency of any substantial unbalanced force, i.e. the frequency ratio should less than 0.5 .

2. Knowing the natural frequency of the system, the static pressure is read from figure 5.1, for particular soil type. Soils other than those in Fig. 5.1, a similar relationship between σ_{st} and natural frequency should first be determined from vibrator tests.

3. The weight of the foundation is assumed in accordance with manufacturer's specifications, or general empirical recommendation for the particular type of engine. The area of the machine foundation is then fixed by knowing σ_{st} from item (2). It is assumed that no resonance would occur now.

4. Amplitude of vibration is then calculated on the basis of weight less spring (Section 3.11) and can be read from Fig. 3.13 (b) In most of the cases the damping ratio may be neglected or a suitable value (Newcomb suggests damping ratio as 0.25) can be assumed.

5. Amplitude Z can be checked for the machine data. If it is within permissible limits, the weight chosen is all right. If not revise the calculations for the weight and area in

accordance with item (2) and (3).

This method suffers from the following limitations.

1. There are only two lines, each for hard and soft clay (Fig. 5.1). There is no indication about the position for various other types of soil. ? Further lines are available in Worthington's pamphlet
2. The plot in Fig. 5.1, is questionable, as our present knowledge shows that, the resonant frequency is not solely dependent upon static stress. It is a complex factor of the type of soil, static pressures, dynamic loads, area of contact. The plot is infact an over-simplification of the problem. In any case , if such a relationship is to be obtained for a particular problem, the ratio of dynamic to static loads should be kept approximately the same as in an actual machine.
3. The weight of soil participating is ignored.

However, the method offers a rational basis of the engine foundation, as it takes into account the phenomena of resonance and tends to limit the value of amplitude of vibration.

5.7 <u>REDUCED NATURAL FREQUENCY METHOD</u> :

This method uses, Tschebotarioff's plot between reduced natural frequency and the contact area, discussed in section 3.18. The plot is reproduced in Fig. 3.18, The method consists of the following steps.

1. A suitable foundation weight and area of the foundation block is assumed, depending on the machine dimensions, and

available space for the foundation.

2. The type of the foundation soil is analysed, and an estimated line for the corresponding soil type is plotted (interplotted) on Tschebotarioff's plot (Fig. 3.18), of reduced natural frequency versus the contact area.

3. Reduced natural frequency is estimated from the plot for the assumed contact area.

4. The resonant frequency of the system is then calculated from $f_o = f_{nr} \cdot \sqrt{\frac{A}{W_o}}$ ....(3.17)

5. Knowing the operational frequency of machine, the frequency ratio can be calculated, and checked if resonance is avoided. Also for the frequency ratio, the amplitude of vibration can be checked as in Newcomb's method).

6. If within required limit, the design is safe, otherwise assume another foundation weight and / or area.

Its main limitation lies in the validity of the assumed plot between reduced natural frequency and the contact area. This has been discussed in section 3.18.

5.8 METHODS EMPLOYING VARIOUS APPROACHES TO RESONANT FREQUENCIES:

The general method of designing the foundation is to determine the natural frequency of the system based on analytical methods discussed in Chapter - 3. The common steps in these methods are :-

1. The dynamic unbalanced forces, and their frequencies of operation are calculated, or these may be supplied by the manufacturers.

2. The dimensions of the foundation block are assumed, taking care that allowable soil pressures (which are less than in case of static loads only) are not exceeded.

3. The soil type is analysed by borings, and sampling. To analyse the dynamic behaviour of the soil, in-situ vibrator tests, should be conducted. In these tests, either the velocity of wave propogation is determined (which will give value of E and ν), or the resonant frequency and amplitude of vibration, are determined for different base plate areas, and for different combination of static and dynamic weights. On the basis of vibrator data, E, ν can be determined by applying Sung's theory (described in section 4.4) From these dynamic tests the value of β (rate of increase of Young's modulus with depth) can be calculated with the help of Pauws' analysis (section 3.15). The value of decay factor, or B, a constant is Ford and Haddow's analysis can also be calculated(Section 3.9). The sub-linear characteristics of soil spring can be determined graphically by a method given by Lorenz (1953) and developed by Alpan (1961), and described in section 3.21. It is recommended that tests with at least 3 different areas of base of vibrator be performed. Balakrishna Rao (1962) has suggested a linear variation of β with area A, based on analytical results of Ford and Haddows. Pauw (1953) himself has assumed constant value of β . However, this is not confirmed. The

β cannot be calculated

Wrong $\log\beta \propto \log A$

dynamic constants evaluted for three different areas will give a better idea of variation with areas and in some cases, it may be possible to extrapolate the constants for prototype from test vibrator.

4. Knowing the above soil properties, the resonant frequency can be calculated by any one of the methods described in Chapter - 3.

5. Resonant frequency is then checked with operational frequency of the machine, and if the frequency ratio is in safe limits (less than 0.5, or more than 2.0), the design may be checked for amplitude of vibrations.

6. Usually the amplitude of vibrations, can be determined with sufficient accuracy by assumption of a simple spring, in which the damping value may be neglected or a reasonable value may be assumed (Fig. 3.13 b).

7. If this is found to be within permissible limits, which can be tolerated by the machine and the structure, then design is safe.

8. If not, assume another preliminary design and repeat the above steps.

Usually the vertical vibrations exist independently. If the vibrations occur in more than one degree of freedom, the frequencies will be coupled as shown by Pauw (1953) and Hseih (1962) Then the only possible approach as to the determination of resonant frequencies are (i) Pauw's method of truncated springs, which gives spring factors and man factors, and corresponding equations of motion, and (ii) from the theory of vibrator on elastic soils, the transformation is made to the spring factor and damping, and then equations of motion can be obtained, and corresponding resonant frequencies obtained. But Pauw's method is applicable only for coehesionless soils, as for cohesive soils, mass factors are not obtained.

For fairly homogeneous soils, the theory of vibrator on elastic soils can be applied.

5.9 SHORT COMINGS OF VARIOUS METHODS :-

In the emprical methods of design of foundation soil type is usually not considered. Emphasis is on providing rigid and heavy foundations. These methods do not take apparently into account the phenomena of reconance and the excess amplitude of vibrations. The past experience is relied on, but every machine is an individual case and the design of foundation cannot be generalized.

The second group of methods (Sections 5.4 and 5.5) consider the stabilizing action of static weights of foundation machine and the soil participating to the dynamic forces produced (Hool and Kinne 1943, Balakrishna Rao 1961). But these methods do not take into account the increased power input requirement of machines at resonance, which provide the energy required for excessive vibrations.

The third group of methods uses the published records of resonant frequencies, and emprical plots obtained from them. (Tschebotarioff 1948,1951,1953) and(New Comb 1951). These methods

Why should variation of E affect pressure distribution. After all vert. pr. distribution is independent of E & u
How was Burmister's theory developed?

Advantage of it is that the shape or ratio L/B is taken into account.

Ford and Haddow (1960) have treated the soil to be conservative system and have obtained the frequency expression for vertical and horizontal vibrations. This method also suffers from the limitation that no variation in resonant frequency is given with change in dynamic load.

Balakrishna Rao (1960, 61, 62) has developed the concept of density bulb of pressure, the mass within which is taken as the mass factor. As this varies with dynamic load, as well, it explains the reduction in natural frequency with increase in dynamic loads as observed experimentally. The results obtained by this inclusion of mass factor are in good agreement with those obtained experimentally by Converse (1953). But it is not understood if this same mass factor can be taken for the soil, for which the value of E can be assumed to be uniform.

Hseih (1962) has given the transformation from the theory of vibrator resting on elastic solid to the spring with viscous damping. This would remove the necessity for tedious calculations and graphs involved in the theoretical approach.

Dynamic characteristics have been determined by assuming the soil to be sublinear spring (Lorenz 1953, Alpan 1961), This method though will give the resonant frequency for any dynamic load for the constant area, suffers from the limitation that the spring characteristics depend on area. Hence it is not possible to extrapolate for different areas.

CHAPTER - 6.

SUGGESTIONS FOR FURTHER RESEARCH.

In view of the study made, the following suggestions for further research are made :-

1. A simple equation forthe natural frequency of the system is evolved (Section 3.22), by considering the basic similarities of various analytical approaches. An experiment investigation regarding variation of parameters , λ_1 , λ_2 is recommended. The factors affecting λ_1 and λ_2 need be studied systematically.
2. Shape factors (m) that is the effect of shape of base area (characterized by L/B ratio) needs to be investigated experimentally. There are reasons to believe that an emprical shape factor based on Pauw's analysis for cohesive soils, may give reasonably good results. However, experimental investigation have to be made.
3. Balakrishna Rao (1960, 61, 62) has evolved density pressure bulb concept. It accounts for the change in the natural frequency of system with change in dynamic loads. This approach has given good results when Pauw's spring factor for cohesionless soils (E increasing with depth) is used. Experimental verification of this approach for cohesive soils should be tried.
4. Experimental investigations of bearing capacity of soil under dynamic loads are being undertaken (A.S.T.M. 1961). Till now no definite conclusions have been drawn, as to how bearing capacity is affected by the dynamic loads. The practice is to take an equivalent static load for dynamic

load, and bearing capacity as in static case is taken. Proper study of the problem will prove extremely useful to the designer of machine foundations.

5. The problem of machine foundations on piles has not been tackled at all. Experimental and analytical investigations of this problem need be made.

6. No data from the actual machine foundations in India is available. It is suggested that a questionaire be prepared and sent to various industries, designers, and research workers in the field of machine foundations. The purpose will be to undertake a systematic analysis of field data regarding actual behaviour of machine foundations. This will help in co-ordinating the efforts of various workers and will consequently lead to a standard practice for the design of machine foundations.

7. Richart (1953), has suggested a possible variation in pressure distribution due to change in dynamic loads. The effect of the change in dynamic load is not yet sufficiently accounted for. Further work will have to be undertaken for experimental determination of pressure distribution for different dynamic loads. In this regard, it may be useful to study effect of ratio of dynamic load to static load on the natural frequency of system and on the amplitude of vibration.

8. Though the experiments have been performed for vibratory loads, most of these are confined to vertical vibrations. The existing theories have yet to be verified for other modes of vibration. Much work will have to be done in

Why doubt?

this field, before designers can be sure of the accuracy of the prediction of behaviour of actual machine foundation.

9. Russians have performed the dynamic tests on relatively large footings (8 sq. meter or so) - Barkan (1936,1963),. A study of Russian literature, which is usually not available in English, regarding machine foundations, may throw a great deal of light on the problem. A suitable study in this connection will be most desirable.

Now it is available as translation (1965 [illegible])

CHAPTER - VII.

CONCLUSIONS.

From the study of the available literaturein the preceeding chapters, it has been observed that none of the methods for design of machine foundation is absolutely reliable. Following recommendations are however, made :-

Resonance phenomena cannot be ignored in machine foundations and design should account for it.

For soils, where it can be assumed that value of E, increases linearly with depth (for sand, and normally loaded clays), Pauw's method can be adopted with the modification that mass factor may be calculated by Balakrishna Raow's density pressure bulb concept. This modification in mass factor, makes the method complete for the vertical vibrations and other translatory modes of vibrations.

For the rotary modes of vibrations, since the corresponding mass inertia factors have not been modified, the mass inertia factors given by Pauw only can be used. This method takes into account the shape of machine foundation, and hence can be applied for any foundation shape. The value of β should be determined from vibrator tests. The variation of β, with area can be taken into account by performing tests with different areas and β versus corresponding area plotted on log log paper. The actual β for the machine area can then be extrapolated by extending the straight line as recommended by Balakrishna Rao(1962).

For cohesive soils for which value of E, can be assumed to be uniform, Pauw's method cannot be applied. In this case, the theoretical analysis by Sung (1953), can be made use of. To make the interpretation from vibrator test consistent with machine foundation, it is recommended that ratio of dynamic to static load be kept approximately the same as in case of the protype. The excess dynamic load leads to jumping of vibrator (in which value of acceleration, $z\omega^2$ is greater than g) which certainly is not the case with heavy foundation prototype.

For other modes of vibration, Bycroft's (1959) analysis can be used and Hseih's transformation eases the calculation . He has considered only rigid base distribution. For other types of pressure distribution, concept of effective radius can be used without any sacrifice of accuracy. For any shape other than circular, the value of r_0 is so chosen as to have equal area of base in case of translatory vibrations, and to have equal moment of inertia in case of rotary vibrations. A shape factor may be taken as in static case (Timoshenko 1937) or from Pauw's analysis for cohesive soils which may be expressed as ;

$$m = \log L/B \Big/ L/B-1 .$$

As regards the amplitude of vibration, if it is determined by the model tests, the test vibrator should be arranged to have the same dimensionless frequency 'a' and mass ratio 'b' as the proposed foundation. This can be achieved by keeping the ratio of operating frequencies of the model to prototype as inversly proportional to the ratio of their linear dimensions and by selecting the ratio of the masses of the model to the prototype as directly proportional to the cube of ratio of their linear dimensions respectively. Then the amplitudes of vibrations in prototype will be directly proportional to $\frac{(F_0/r_0)Prot.}{(F_0/r_0)mod.}$ (Bycroft, 1959).

Usually the test for amplitude determinations is not resorted to. In most cases the amplitude of vibration can be determined with sufficient accuracy by simple mass spring analogy (Section 3.11).

A simple equation, based on the general similiarity of analytical approache is developed, for determination of the natural frequency of the system. The parameters involved (λ_1, λ_2) can be determined by vibrator test in the field.

the transformation is made to the spring factor and damping, and then equations of motion can be obtained, and corresponding resonant frequencies obtained. But Pauw's method is applicable only for coehesionless soils, as for cohesive soils, mass factors are not obtained.

For fairly homogeneous soils, the theory of vibrator on elastic soils can be applied.

5.9 SHORT COMINGS OF VARIOUS METHODS :-

In the emprical methods of design of foundation soil type is usually not considered. Emphasis is on providing rigid and heavy foundations. These methods do not take apparently into account the phenomena of reconance and the excess amplitude of vibrations. The past experience is relied on, but every machine is an individual case and the design of foundation cannot be generalized.

The second group of methods (Sections 5.4 and 5.5) consider the stabilizing action of static weights of foundation machine and the soil participating to the dynamic forces produced (Hool and Kinne 1943, Balakrishna Rao 1961). But these methods do not take into account the increased power input requirement of machines at resonance, which provide the energy required for excessive vibrations.

The third group of methods uses the published records of resonant frequencies, and emprical plots obtained from them. (Tschebotarioff 1948,1951,1953) and(New Comb 1951). These methods recognise phenomena of resonance, but the plots represent over simplification of the facts and generalizations from limited data is questionable. An emprical equation for the resonant frequencies of sand has been developed by Converse (1953). This type of equation, is applicable only for the particular site, It is not economical to develope similar emprical expressions for subsoil on which the machine is constructed, as it will involve a large number of vibrator tests.

The fourth group of methods gives due prominance to the phenomena of resonance, and uses any one of the analytical methods discussed in Chapter - 3, to predict the resonant frequency of the system.

The theory of vibrator resting on elastic, semi-infinite homogeneous, and isotropic soil, which is developed by Reissner (1936), Sung (1953), Quinlan (1953), Richart (1953,60), Bycroft (1959), Hseih (1962 and others, suffers from the limitation that real soils do not have the above properties. This limitation is in common with any theory of elasticity when applied to soils. The value of dynamic soil constants E and ν , can be obtained from the vibrator tests, or by the velocity of wave propagation measurements. The pressure distribution below the base is unknown. Also it may vary with the magnitude of dynamic loads. For the rigid foundation and mats, the rigid pressure distribution can be assumed. The uncertainty in pressure distribution may result in an error of the order of ± 11% in modulus of elasticity of soils (E) (Jones 1958). Though it is an experimental fact that resonant frequency of the system, decreases with increase in dynamic loads, the theory does not give any quantitative results. It may qualitatively be explained by assuming change of the pressure distribution. The theory is applicable for an axially symmetrical case (that is the circular machine or vibrator). The results are likely to be different for different shapes of base plate of vibrator, or the shape of the machine foundations. A shape factor will have to be introduced, about which the present state of knowledge is not adequate.

The other theoretical approach is to treat the soil as weightless spring, with the mass of vibrator, or, machine foundation, and some mass of soil, oscillating in simple harmonic motion as one system. But in this both the spring constant and the soil mass vary with the contact area, static and dynamic loads. The value of spring constants in static case as derived by Timoshenko (1937) vary with the pressure distribution well. These spring constants explain at least one thing that for increase in radius, the resonant frequency of the system, increases with contact area, which has been observed experimentally also (Lorenz 1953).

Pauw (1953) assumes the soil to be truncated pyramid of spring, and has given the expressions for the spring factors and mass factors for different modes of vibrations for cohesionless soil in which E is assumed to increase linearly with the depth. For cohesive soils though the spring factors are obtained, mass factor could not be obtained due to divergent integral form. The assumption in case of cohesive soils was that E was constant with depth. The method suffers from the limitation that no variation in resonant frequency with respect to dynamic load is obtained.

BIBLIOGRAPHY

Alpan, I., (1961) "Mahine Foundations and Soil Resonance" Geotechnique Vol. 11, pp. 95-113.

A.S.T.M. (1961) "Symposium on Soil Dynamics"

Andrews, W.C. & J.H.A. Crockett (1945) "Large Hammers and their Foundations" The Structural Engineer, Vol. 3, Oct., 1945, pp.453-492.

Arnold, R.N., G.N.Bycroft & G.B.Warburtan (1955) "Forced Vibrations of a Body on an Infinite Elastic Solid" Journal of applied Mechanics, Trans. A.S.M.E. Vol. 77, pp. 391-401.

Balakrishna Rao., H.A. & C.N.Nagaraj (1960) "A new Method for predicting the Natural Frequency of Foundation-Soil System", The Structural Engineer, vol.38, Oct., Pp.310-316.

Balakrishna Rao, H.A.(1961), "Design of Machine Foundation related to the Bulb of Pressure" Proc.5th Int.Conf. S.M.F.E. Vol. I, pp.563-568.

Balakrishna Rao, H.A.(1962), "A New Method of Predicting Resonant Frequency for Square Footing" Second Symp. Earthquake Engg. University of Roorkee, Roorkee, Nov., 1962, (Under print).

Barkan D.D., (1936), "Field Investigations of the Theory of Vibrations of Massive Foundations under Machims" Ist Int.Conf.S.M.F.E.Vol.II, pp.285-288.

Barkan, D.D., (1963) "Dynamics of Base & Foundations" McGraw Hill Co.

Baum G., (1939) "Dynamic Investigations of Roads in Germany" Symp. on Vibration Testing of Roads & Runways, Paper No.VII,.

Bergstrom, S.G. & S.Linderholm (1946), "A Dynamic Method for Determining Average Elastic Properties of Surface Soil Layer" Handlinger No.7, Svenska Forskings, Institutet fur cement och Betong vid. Kungl, Teknishka Hogskolan i Stockholm.

Bergstrom, S.G., EFromen, and S.Linderholm, (1949), "Investigation of Wheel Load Stresses in Concrete Pavements" Handlinger No.13, Svenska Fors.igs -Institutet fur cement och Betong vid Kungl.Tekniska Hogskolan, i, Stockholm.

Bernhard, R.K. (1937) "Dynamic Tests by Means of induced vibrations" Proc. A.S.T.M.Vol.37, Part 2, pp.634-645.

Bernhard, R.K. (1939) "Highway investigations by means of Induced Vibrations" Bull No.49, Eng.Expt. Station, Pensylvania State College.

Bermhard, R.K. (1949) "Study on Mechanical Vibrators" Proc. A.S.T.M. Vol.49, pp. 1106.

Bernhard, R.K. and Finelli (1953), "Pilot Studies on Soil Dynamics" A.S.T.M. Tech. Pub.No.156, Symp. on Dynamic Testing of Soils" pp.211-253.

Bernhard R.K. (1956), "Microscisms" A.S.T.M. Sp.Tech.Pub.No.206, Paper on Soils, pp. 80-102.

Bernhard, R.K. (1958), "A Study of Wave Propagation" Proc. H.R.B. pp.618-646.

Biot, N. (1942) "Analytical and Experimental Methods in Engg. Seismology" Proc.A.S.C.E.Jan., 1942.

Boussinesq, J., (1885), "Application des potentiels letude de l'equilibre et due mouvement des solides elastiques" Application of Potentials to the study of the Equilibrium and movement of Elastic Solids) Gauthier Villars, Paris.

Bycroft, G.N. (1959), "Machine Foundation Vibrations" Proc.I.M.E. (Lond) Vo.173, No.18. pp 469-73.

where?

Cadambe, V. (1954) "Vibrations in Foundations" Journal of the Institution of Engineers (India) Sept., 1954, pp.80-86.

Consverse, F.J. (1953) "Compaction of Sand at Resonant Frequency" A.S.T.M. Sp. Tech.Pub.No.156, pp.124-137.

Converse F.J. (1956) "Compaction of Cohesive Soil by Low Frequency Vibrations" A.S.T.M. Sp.Tech. Pub. No, 206, pp.70-81.

Converse, F.J. (1962), "Foundation Subjected to Dynamic Forces" Foundation Engineering Edited by G.A. Leonards.

Crandell, F.J. (1949) "Ground Vibrations due to Blasting and its Effect upon Structures" Contribution to Soil Mechanics (1940-1953) Also Journal Boston Society of Civil Engineers, April, 1949.

Crede (1951) "Vibration and Shock Isolation" John Wiley & Sons Inc.

Crockett, J.H.A. and B.E.R. Hammond (1947), "Reduction of Ground vibrations into Structures, Institution of Civil Engineering, Structural paper No.18.

Crockett, J.H.A., and R.E.R. Hammond (1948), " Natural Oscillation of Ground and Industrial Foundations" Proc.2nd, Int. Conf. S.M.F.E. Rotterdam, Vol.3, pp.88-93.

Crockett , J.H.A. and R.E.R., Hammond (1949), "The Dynamic Principles of Machine Foundations and Ground" Proc. I.M.E. London, Vol.160. pp.512-531.

Crockett, J.H.A. (1958), "Foging Hammer Foundations" Parts I to II, June to Sept., 1958, Civil Engg. and Public Works Review (London)

Denttartog J.A. (1947), "Mechanical Vibrations" McGraw Hill .

Eastwood, W. (1953), "Vibrations in Foundations" Structural Engineer, March, pp. 82-93.

Eastwood W., (1953), "The Factors which affect the Natural Frequency of Vibrations of Foundations and the effect of Vibrations on the Bearing Power of Foundations on Sand" Proc.3rd Int.Conf.S.M.F.E. Voo.1, pp.118-22.

Ehlers, G.(1942), "The Soil as Spring in Oscillating Systems" Beton und Eisen Vol.41, pp.197.

Fisher, Josheph,A. and Winter J.D.(1962) "Evaluation of Dynamic Soil Properties" Paper sent to A.S.C.E.

Ford and Haddow (1960), "Determining the Machine Foundation Natural Frequency by Analysis" The Engg. Journal Vol.43, No.12, Dec.

Forhlich, O.K. (1934), "Druckvertcihung in Bangrunde" (Pressure Distributions in Foundations' Springer,Vienna.

Goyal, S.C. and Alam Singh, (1960), "Soil Engineering Aspects of Design of Machine Foundations" Bull.Ind.Natu.Soc. Soil Mech.No.6, New Delhi.

Heiland, C.R., (1946), "Geophysical Exploration" Prentice Halline, New York.

Henery, F.D.C. (1956), "The Design and Construction of Engineering Foundation" Span Civil Engg. Series.

Henkelom, W.,(1959), "Dynamic Stiffness of Soils and Pavements" Paper No.VI, Symp.Vibration Testing of Roads and Runways, Amasterdam.

Henkelom, W. and C.R.Foster (1960), "Dynamic Testing of Pavement" Proc.A.S.C.E. Feb.,1960, S.M.I.

Hool and Kinne (1943) "Foundations,Abutments and Footings" McGraw Hill Book Co.pp.317-320.

Hertwig, A., G. Fruh, and H. Lorenz "Determination by means of Forced Vibbations of Soil Properties of Particular Importance for Construction Work" Veroffentlichunger der Degebo, Heft 1, Berlin, (1933.)

Hwukelom, W. (1962), "Analysis of Dynamic Deflexions of Soils and Pavements" Geotechnique Vol.11, Pp.224-243.

Hseih, T.K. (1962) "Foundation Vibrations" Proc.I.C.E. Paper No.6571, June, 1962, pp.211-225.

Jones, R. (1958) "A Vibration Method For Measuring The Thickness of Concrete Road Slabs Insitu" Con. Res. 7(20) pp.97-102.

Jones, R. (1958), "Insite Measurement of Dynamic Properties of Soil by Vibration Methods", Geotechnique Vol.VII, pp.1-21.

Jone, R. (1959), "Interpretation of Surface Vibration Measurements" Paper No.V, Symp. on Vibration Testing of Roads and Runways, April, 1959, Amasterdom.

Jacobsen, L.S. and B.S. Ayre (1958) "Engg. Vibrations" McGraw Hill Book Co., Inc. New York.

Kondner, R.L. (1962) "Vibratory Response of a Cohesive Soil in Uniaxial Compression", Symp. on Earthquake Engineering, University of Roorkee, Roorkee.

Lamb, H. (1904) "On the Propagation of Termors Over the Surface of an Elastic Solid" Trans. Royal Soc. (London) (A), Vol.203, pp.1-42.

Leet, L.D. "Earthwaves" (1950) "Harvard University Press,

Lichty, "Internal Combustion Engines"

Lorenz, H. (1934) "New Results Obtained from Dynamic Foundation Soil Tests" National Research Council of Canada Technical Translation TT-421-1955.

Lorenz, H. (1953) "Elasticity and Damping Effects of Oscillating bodies on Soil" A.S.T.M. Sp.Tech.Pub.No. 156, pp.113-122.

Lorenz, H. (1953) "The Determination of Dynamical Characteristics of Soils, A good help in the Calculation of Dynamically excited foundations" Proc.3rd, Int. Conf. Soil Mech. Zurich, p.400-408.

Lorenz, H. (1959) "Vibration Testing of Soils" Paper III, Symp. on Vibration Testing of Soils, Amasterdum.

Magomi, T. and K.Kubo, (1953) "The Behaviour of Soils During Vibrations" Proc.3rd Int.Conf.S.M.F.E.Vol.1, pp. 152-155

Miller E.F. and Pursey, H.(1955),"On the Partition of Energy between Elastic Waves in a Semi-Infinite Solid".Proc.Ray.Soc.4,Vol.233,pp.55-69.

Morse, F.T. (1942) "Power Plant, Engg.Design" Mykelstad, (1956).

Newcomb, W.K. (1951) "Principles of Foundation Design for Engines and Compressors" Trans.A.S.M.E.Vol.73, April,1951,pp.307-309.

Nijboer,L.W. and C.VenderPoel,(1953), "A Study of Vibration Phenomena in Asphaltic Road Constructions" Proc.Ams.Asph.Pav.Tech.22, 197-231.

Nijboer,L.W. (1959), "Vibration Testing of Roads" Paper No.IV, Symp.on Vibration Testing of Roads and runways, Amasterdom.

Oosterbeck Jr.H.J.(1948),"Soil Vibrations" Reprint Proc.2nd, Int.Conf.S.M.F.E.Rotterdem (1948).

Pauw Adrian (1953), "A Dynamic Enalogy for foundation soil Systems" A.S.T.M.156, pp.89-112.

Pauw A., (1953) "Discussion on Performance Records of Engine Foundations by Tschebotarioff" ASTM Sp.Tech.Pub.No.156, pp.169-171.

Quinlan, P.M. (1953) "The Elastic Theory of Soil Dynamics" A.S.T.M. Sp.Tech.Pub.No.156, pp.3-34.

Rao,S.R.(1954), "Machine Foundations" J.I.E.(India),Vol.35, No.1, pp.43-62, Sept.,1954.

Rayleigh (1885), "On waves propagated along the Plane Surface of an elastic solid" Proc.london Math.Society Vol. 17, p. 4.

Richart, F.E.(1953) "Discussion on "Vibrations in Semi-Infinite Solids due to Periodic surface Loading by Sung" A.S.T.M. Sp.Tech.Pub.No.156,pp.64-68.

Richart,F.E. (1960) "Foundation Vibrations" Proc.A.S.C.E.Jour. of S.M.F.E. Aug.,1960,pp.1-34.

Rausch,E., (1936), "Hammer Foundations" Der Bauingenieur,17(33/34) Aug.1936,pp.342-344. CBRI. Translation No.27, May, 1961.

Rausch,E. (1926), "Machine Foundations" Ref.Der Bauingeneiur, 7(44), Oct.,1926,C.B.R.I.Library Trans.No.49, Part I.

Rausch, E. (1926), "Machine Foundations" C.B.R.I.Library Translation No.50, part II.

Reissner, E.(1936), "Stationare, axialsymmetrische durch eine schutteinde Masse erregte Schwingungen eines homogenen elastischen Halbraumes",Ingenieur-Archiv Vol.7 Pt.6,Dec.1936 ,pp.381.

Slade, J.J. Jr. (1953) "A Discontinuous Model for the Problems of Soil Dynamics" A.S.T.M. Sp.Tech.Pub.No.156, pp. 69-96.

Sneddon, I.N. (1953) "Fourier Transforms" McGraw Hill- Ist Ed.p.485.

Sridharan, A. (1962) "Settlement Studies on a Model Footing Under Dynamic Loads" Proc.Second Semp.Earthquake Engg. Roorkee University, U.P. Nov.,1962, (Under Print), India.

Sung, Tse Yund, (1953) "Vibrations in Semi-Infinite Solids due to a periodic Surface Loading" A.S.T.M. Sp.Tech.Pub. No.156, pp.35 - 63.

Subbarao, V.V.(1962) "Dynamic Stresses in Concrete Pavements" Final Report (Unpublished), University of Roorkee, Roorkee.

Terzaghi, K. (1943) "Theoretical Soil Mechanics"JohnWiley & Sons Inc.New York, Chap. XIX, pp.434-479.

Timoshenko (1937), "Theory of Elasticity"Chap.11, McGraw Hill New York.

Timoshenko S.P. (1955) "Vibration Problems in Engg." 3rd Ed. D.Van Mostrad Inc.New York.

Tschebotarioff, G.P. and E.R.Ward (1948) "The Resonance of Machine Foundations and the Soil Coefficients which affect it" Proc.2nd Int.Conf.S.M.F.E., Rotterdam, Vol.1,pp.309-313.

Tschebotarioff, G.P. (1951), "Soil Mechanics, Foundations, and Earth Structures" McGraw Hill Book Co.New York, Chapt. 18, pp.568-595.

Tschebotariff, G.P. (1953) "Performance Records of Engine Foundations" A.S.T.M. Sp.Tech.Pub.No.156, pp.163-168.

Vander,P.E., (1951) "Dynamic Testing of Road Construction" J.App. Chem. 1(7), 281-290.

Vander P.C., (1953) "Vibration Research on Road Construction" A.S.T.M. Sp.Tech.Pub.No.156, pp.174-185.

Worburton, G.M. (1957), "Forced Vibrations on an Elastic Stratum" Journal of App.Mech.Vol.24,pp.55-58.

Wildon, S.D. and R.J.Dictrich (1960) "Effect of Consolidation Pressure on Elastic and Strength Properties of Clay" Research Conference on Shear Strength of Cohesive Soils" A.S.C.E. June, 1960 . pp. 419-435.

Winterkorn, H.F. (1953), "Macrometric Liquids" A.S.T.M. Sp. Tech. Pub. No. 156, pp. 77-89.

Zeller, W. (1934), "Principles for laying Hammer Foundations" Der Bauingeniur,12(41/42),Oct.,1934, pp. 402-406. C.B.R.I. Library Translation, No.28,May,1961.

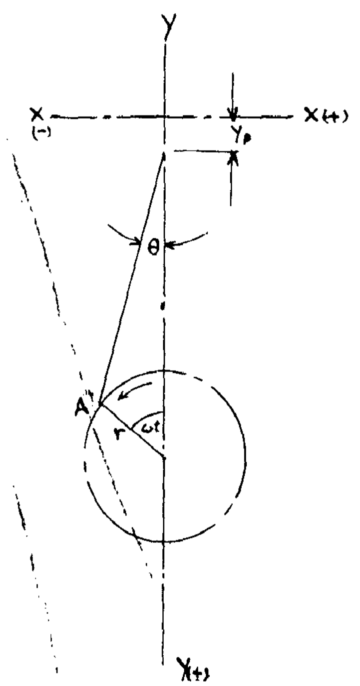


Fig.2.1 Kinematics of Piston,Crank and Connecting rod motion.

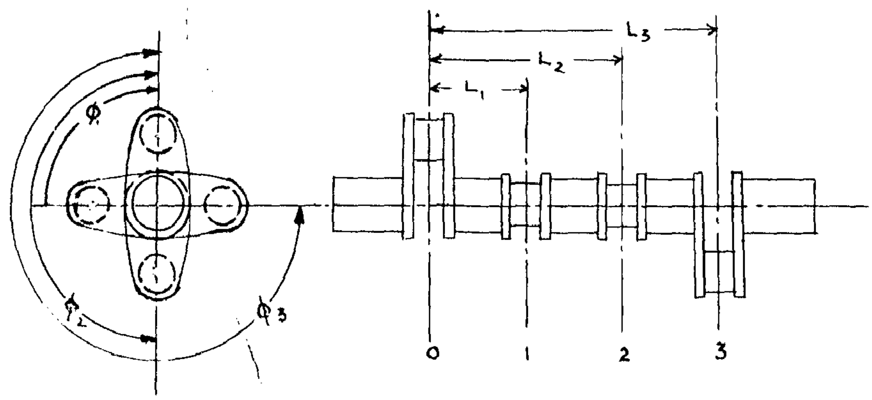


Fig.2.2 Positions of Cranks on the Crank Shaft of a Multi-cylinder Engine (Crede 1959)

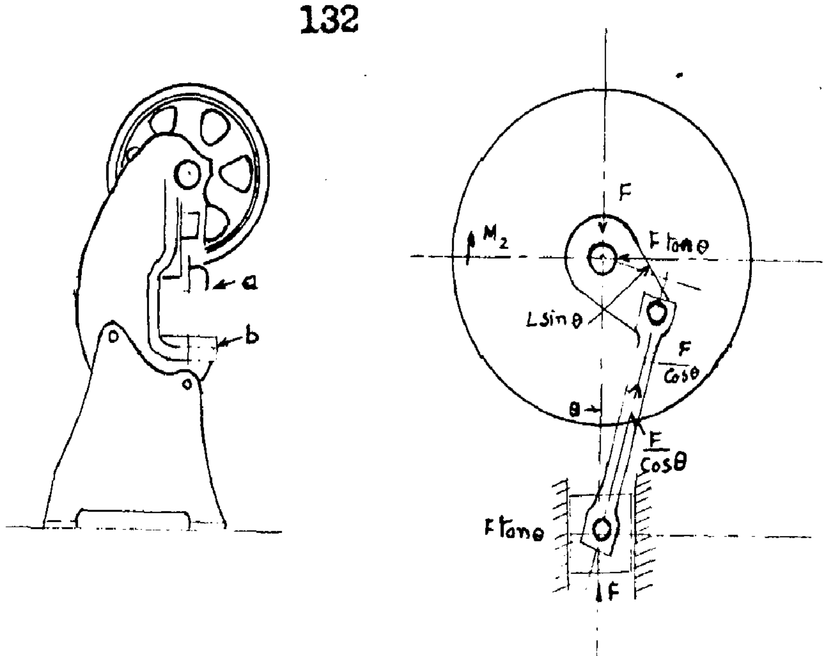


Fig.2.3 Typical Punch Press with Vertically Moving Platten. (Crede,1959)

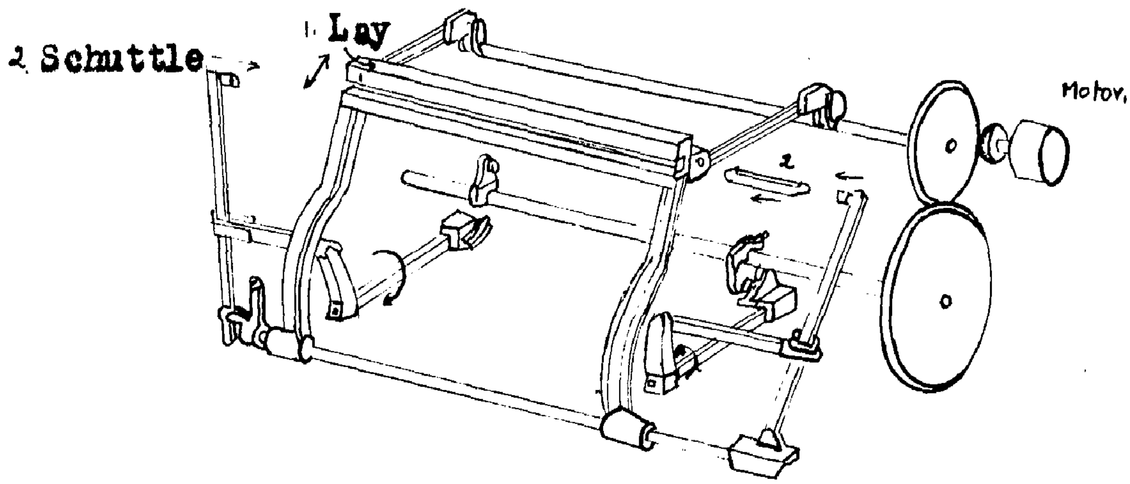


Fig. 2.4 Schematic Diagram of Cloth-weaving Loom.

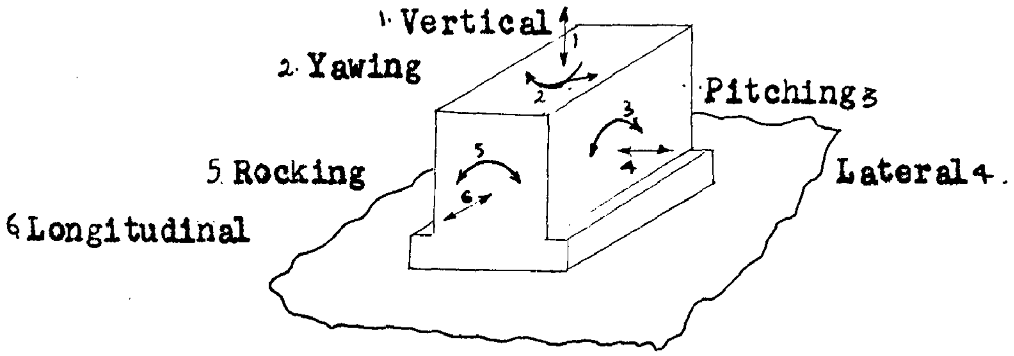


Fig. 2.5 Six Modes of Vibrations for a Foundation.
(Richart, 1960)

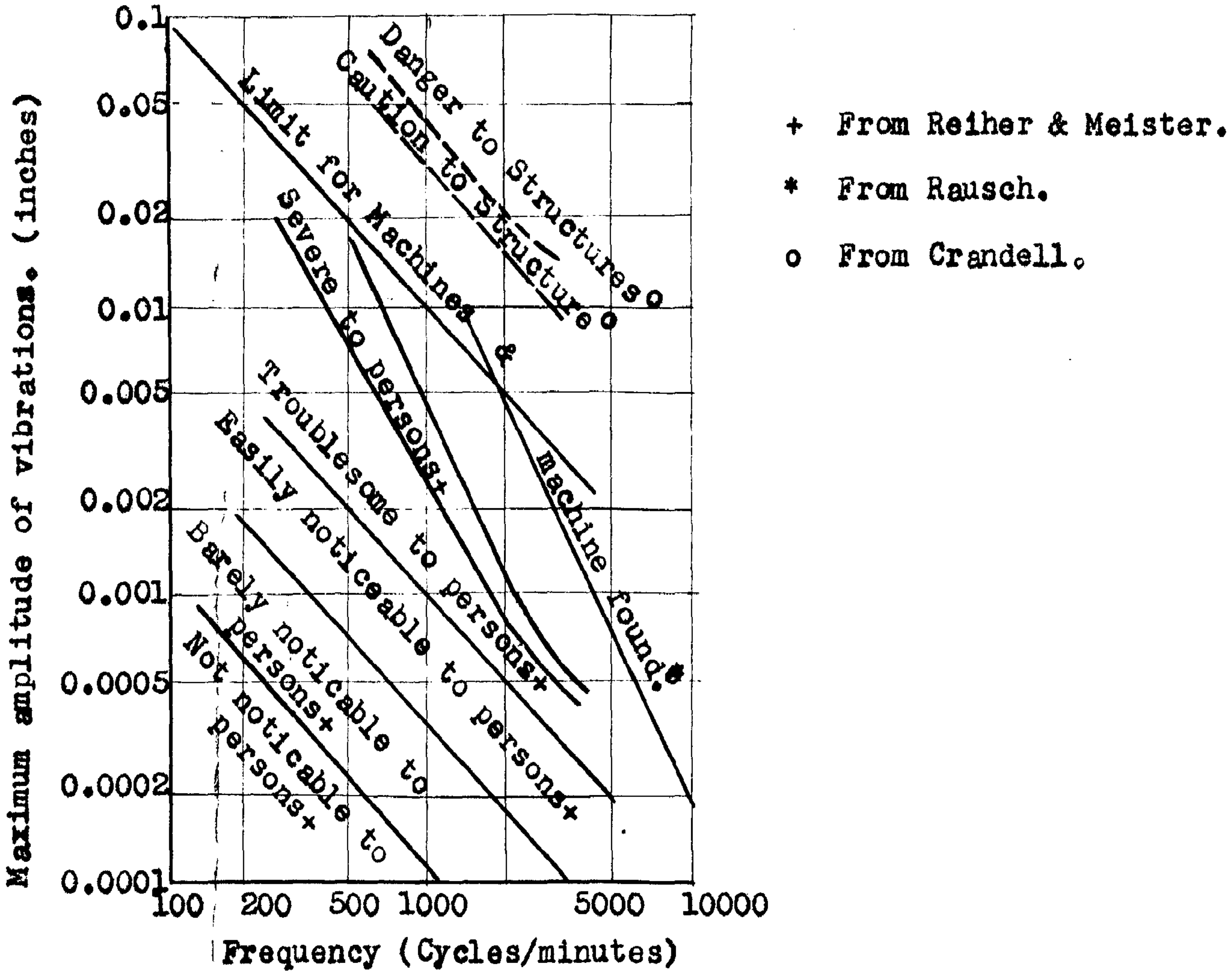


Fig.2.6 Maximum permissible amplitude vs. operating frequency.

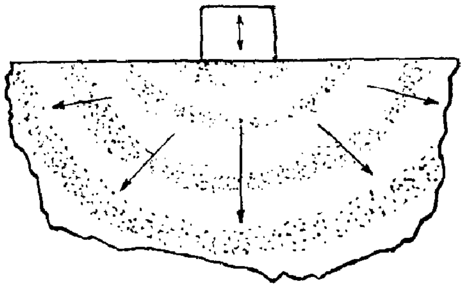


Fig. 3.1 Propagation of elastic waves into the soil beneath an oscillator. (Richart 1960)

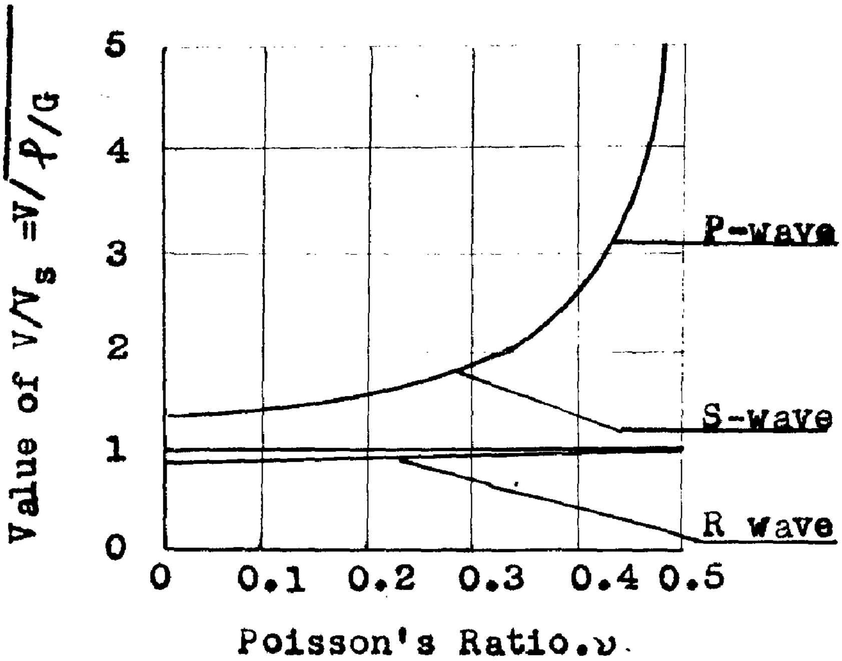


Fig. 3.2 Relation of V_R, V_c and V_s vs. poisson's ratio.

$V_s^2 = G/\rho$, $V_R^2 = p^2 V_s^2$, p is a function of ν.

ν	0.2,	0.3,	0.4	0.5
p	0.911	0.928	0.942	0.955

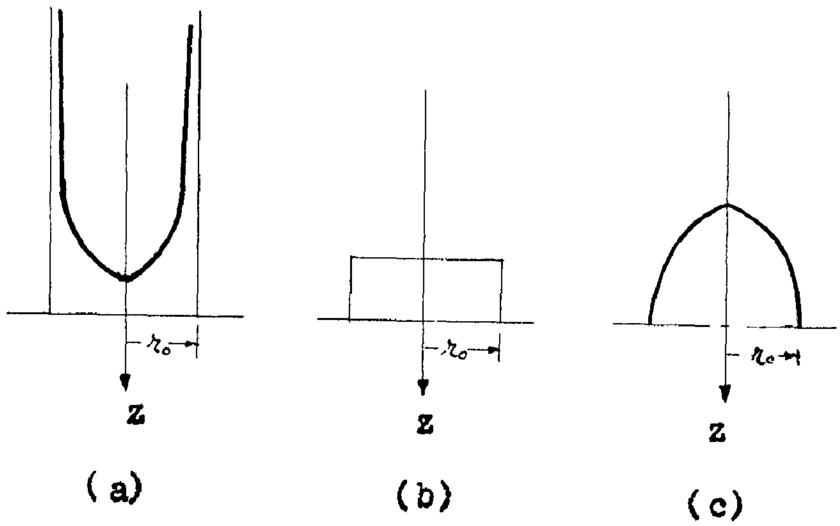


Fig.3.3 Pressure Distribution assumed by Sung (1953) Quinlan(1953) over a circular Region.

a) Rigid base distribution.
b) Uniform loading.
c) Parabolic loading (After Sung,1953)

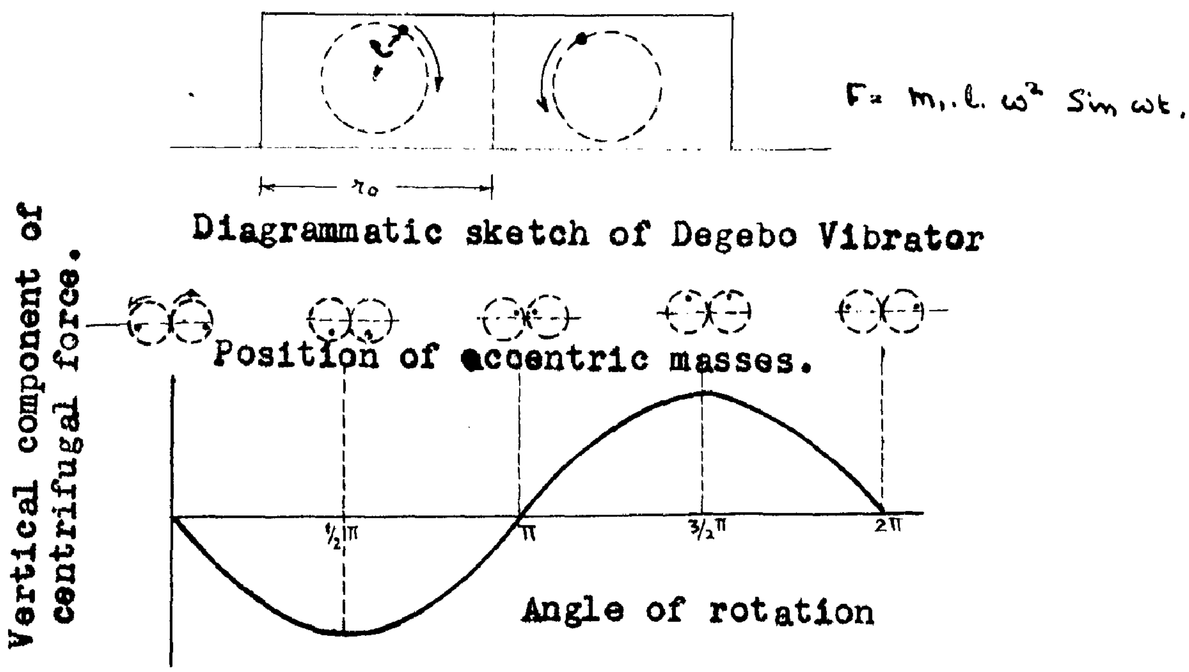
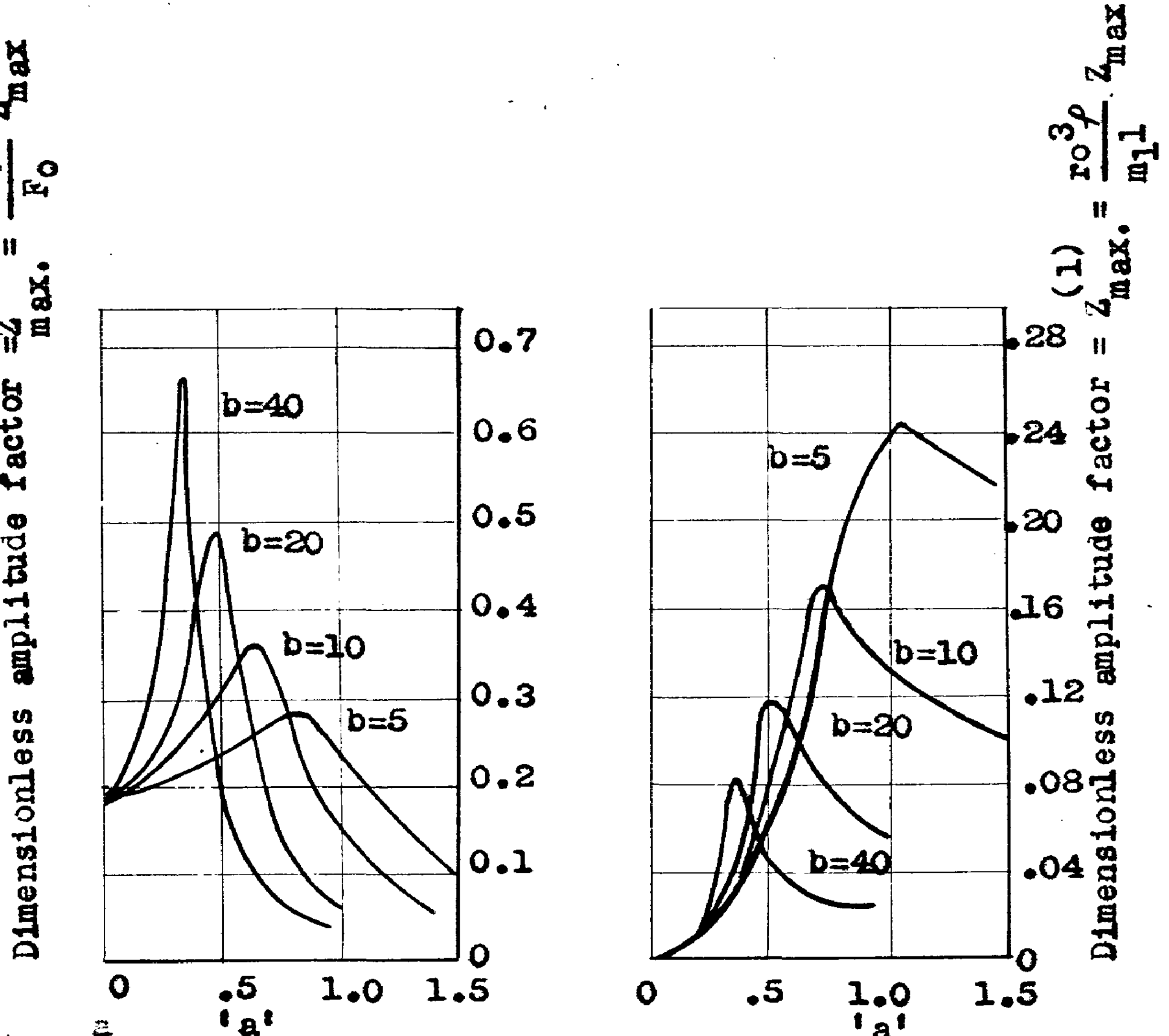


Fig.3.4 Degebo Type Vibrator and its operating principle (After Lorenz 1953)



a) For constant amplitude of exciting force F_o=Const.

b) For exciting force amplitude dependent upon the exciting frequency $F_o = m_1 l \omega^2$

Fig.3.5 Amplitude versus frequency relations for rigid base oscillator resting semi-infinite body(ν =.25) (Richart 1960)

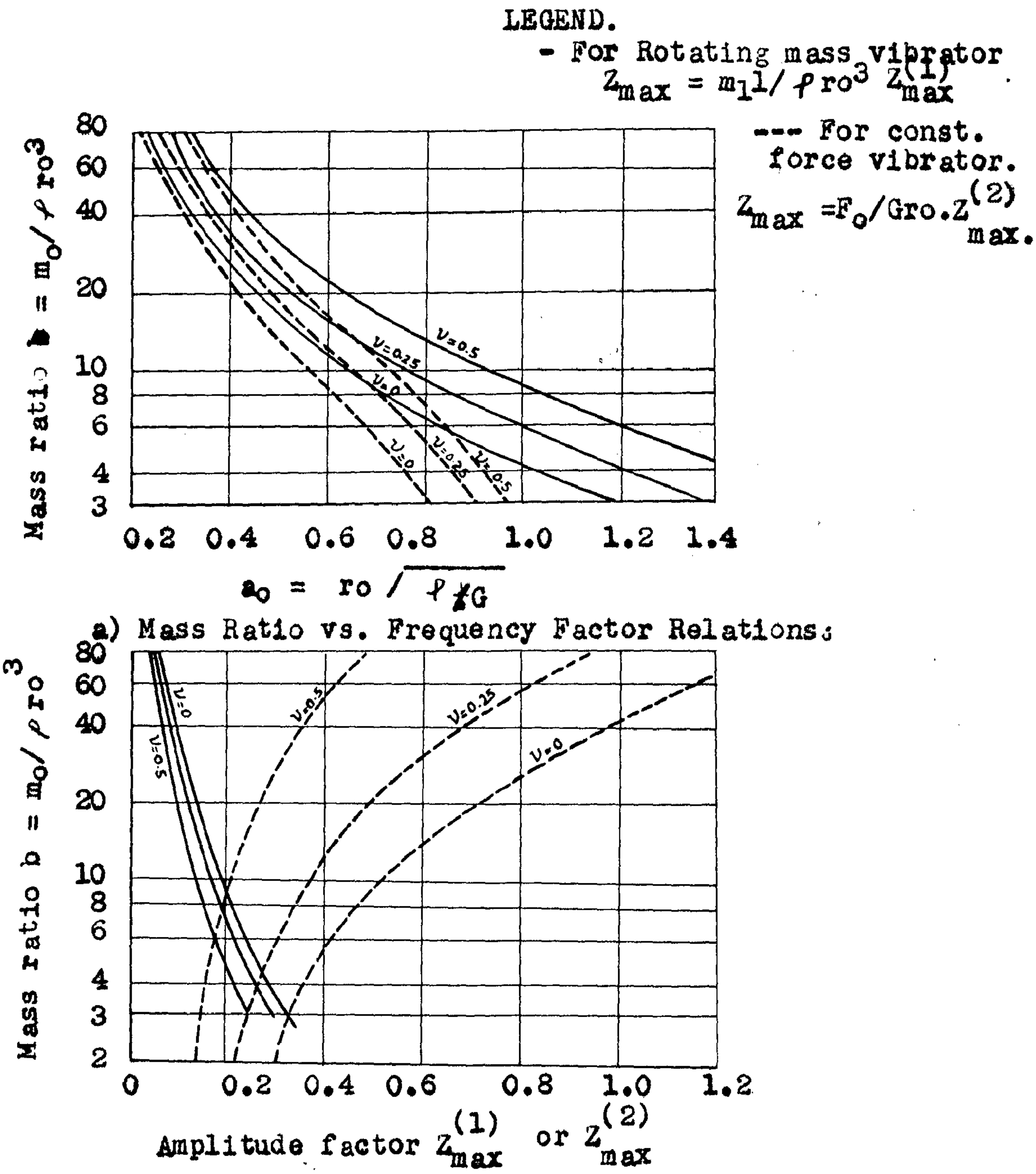
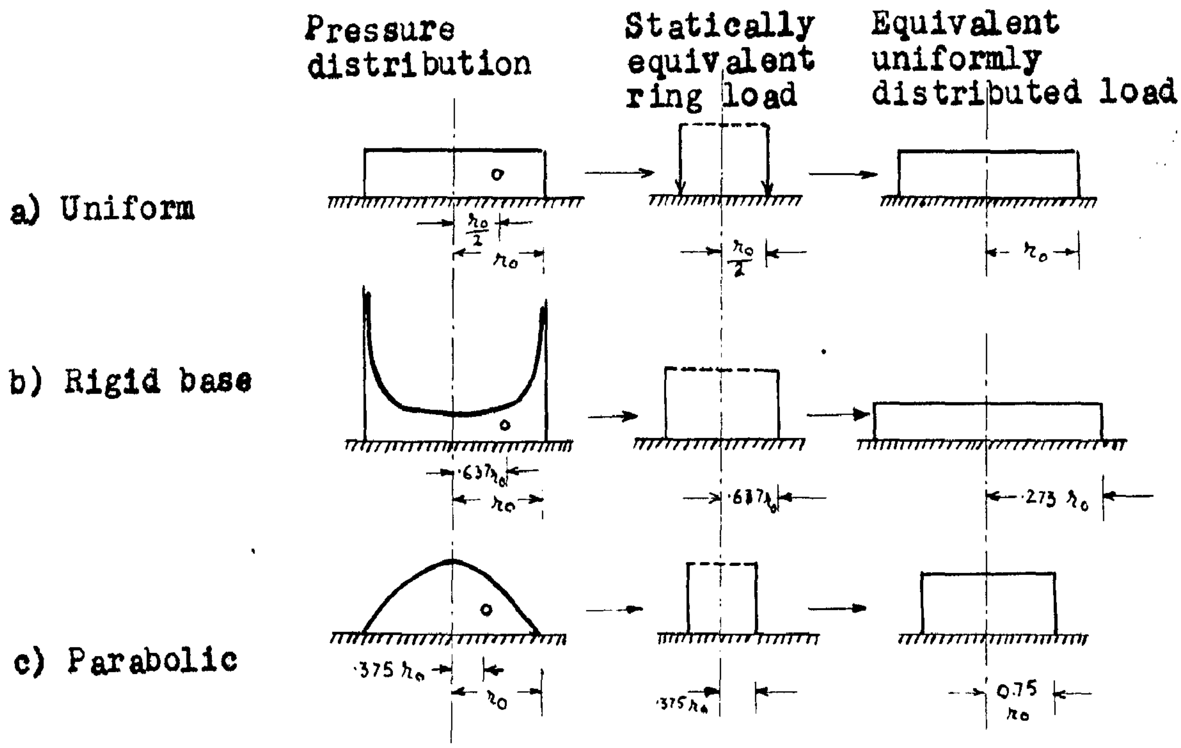
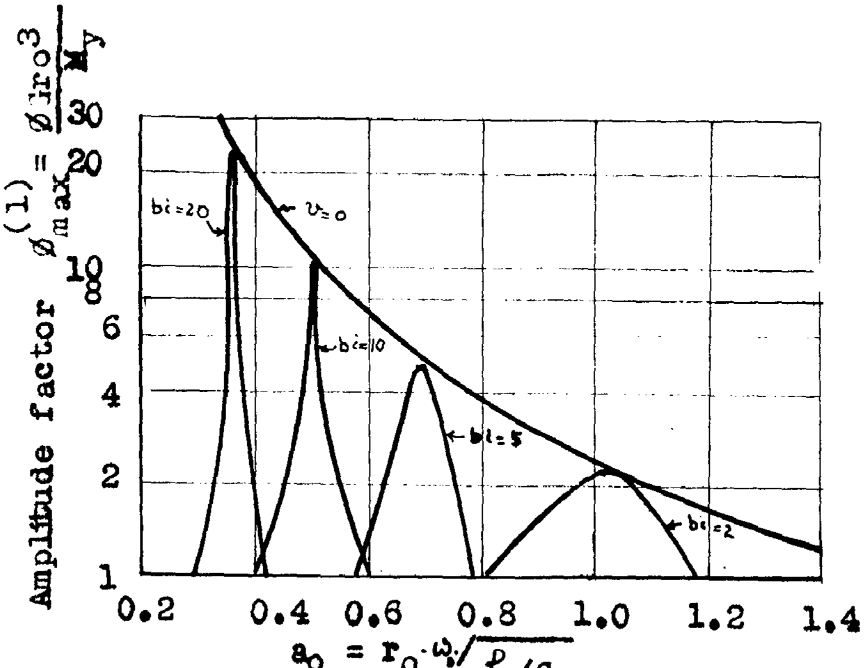


Fig. 3.6 Characteristics of vertical oscillations for an oscillator with a rigid circular base resting on a semi-infinite elastic medium. (Richart 1960).

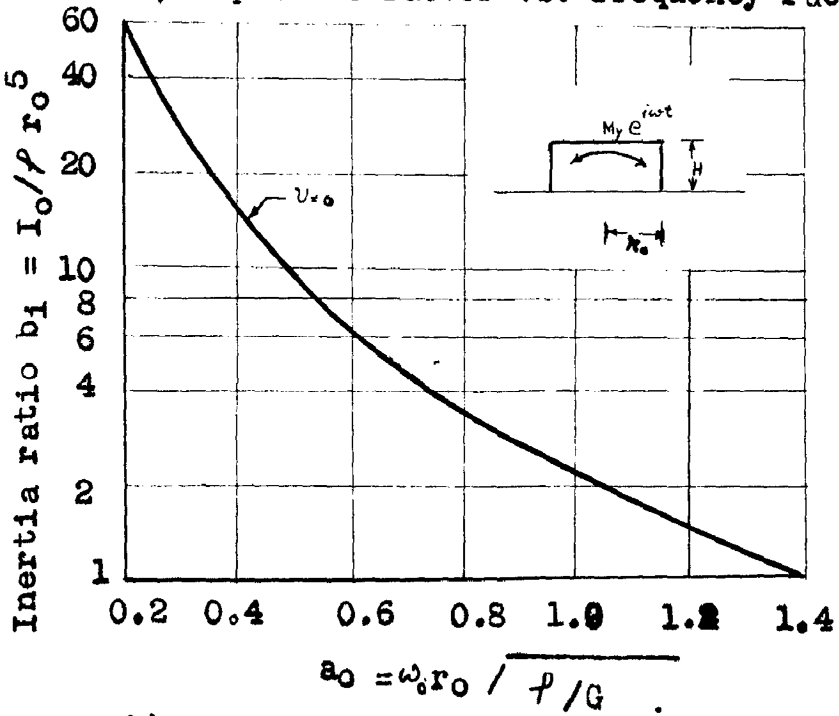


	UNIFORM	RIGID BASE	PARABOLIC
Eq.Uniform	ro	1.273 r_o	0.75 r_o
Eq.Rigid base	0.78r_o	r_o	.58 r_o
Eq.Parabolic	1.33 r_o	1.693 r_o	r_o

Fig. 3.7 Concept of Equivalent or Effective Radius (Richart 1953)

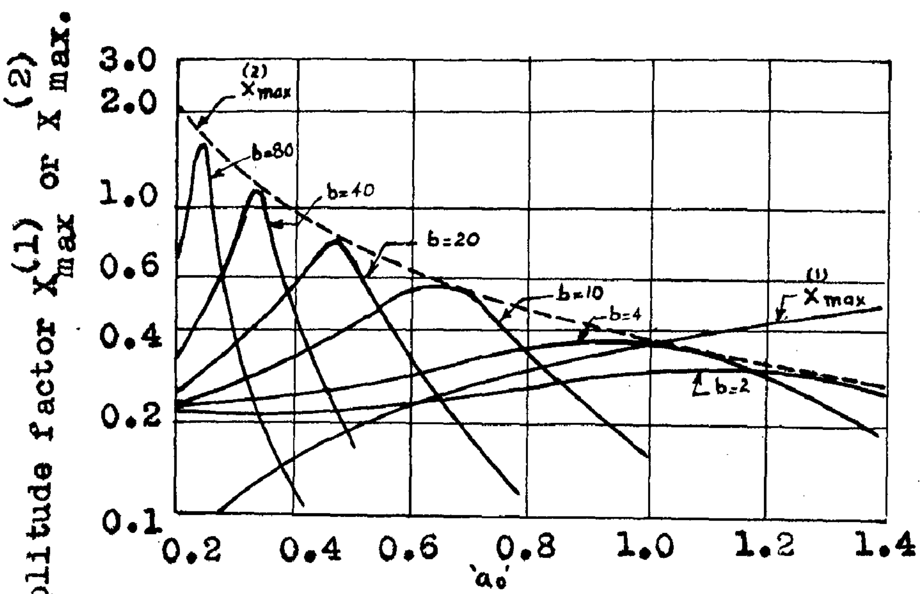


a) Amplitude factor vs. frequency factor.

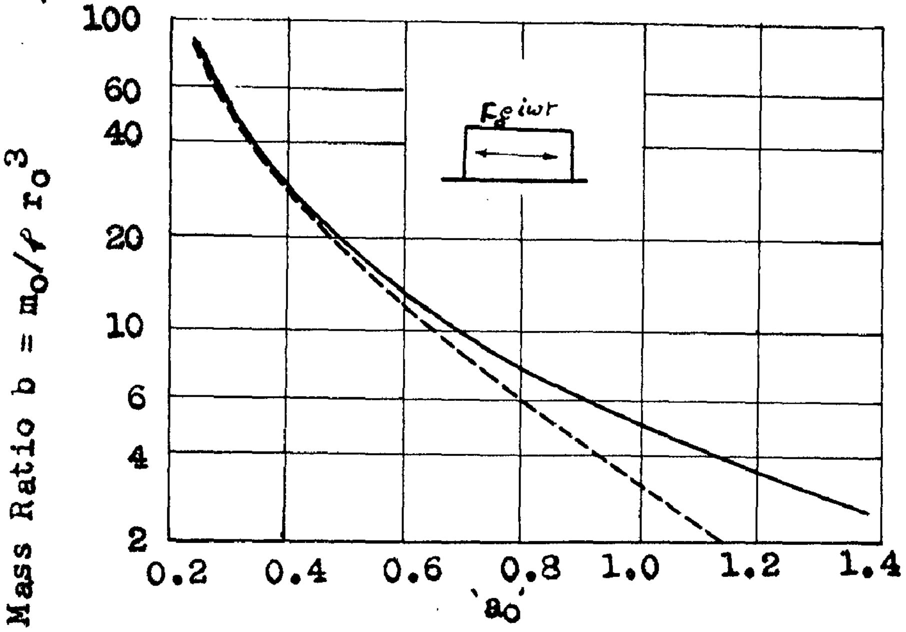


b) Inertia ratio vs. frequency factors.

Fig.3.8 Characteristics of Rocking Oscillation for an oscillator with a rigid circular base resting on semi-infinite elastic medium (Richart 1960).



a) Amplitude factor vs. frequency factor



b) Mass ratio vs. frequency factor.

Fig.3.9 Characteristics of Sliding Oscillation for an Oscillator with rigid circular base resting on a semi-infinite elastic medium (Richart 1960)

LEGEND

———— For Oscillator with horizontal force $F_o = m_1 l \omega^2$

- $X_{max} = m_1 l / \rho r_o^3 \cdot X^{(1)}_{max.}$

------ For oscillator with constant horizontal force

$X_{max} = F_o / G r_o \cdot X^{(2)}_{max.}$

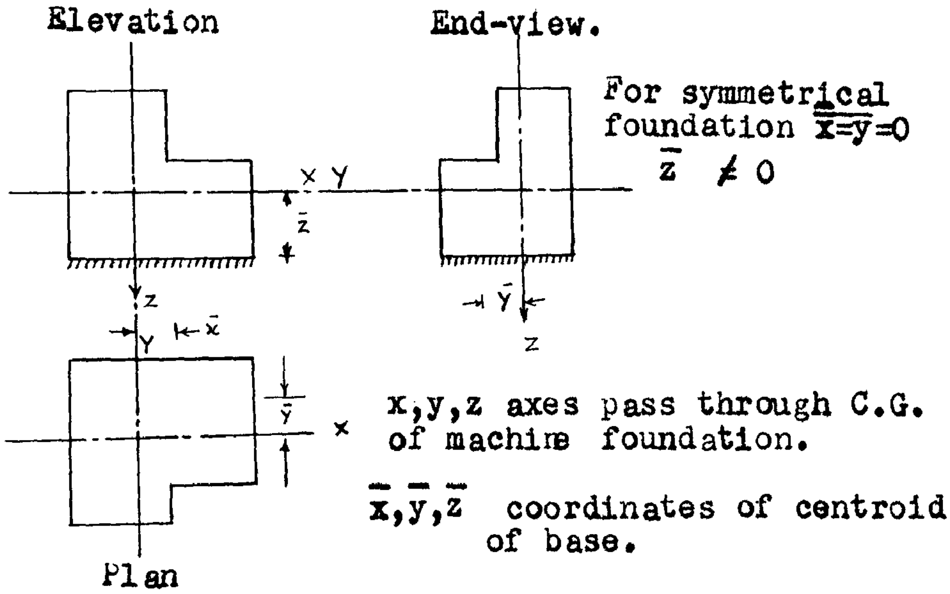
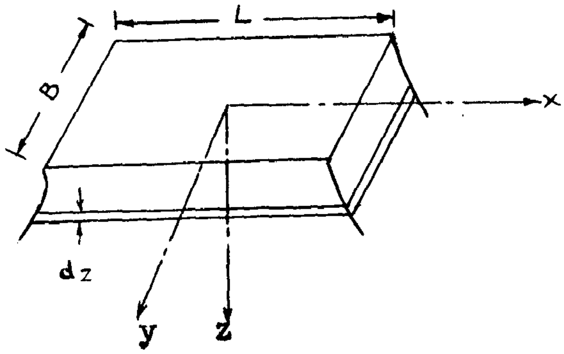


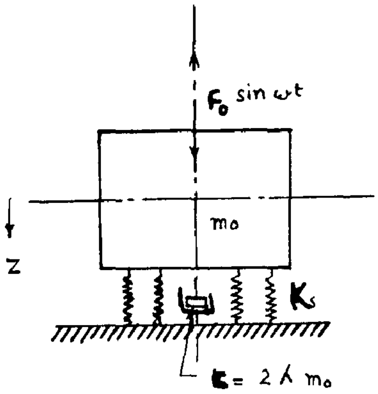
Fig.3.10 Generalized foundation co-ordinates (Hseih,1962)



Dynamic pressure is transmitted through soil containing a solid formed by the base and surfaces

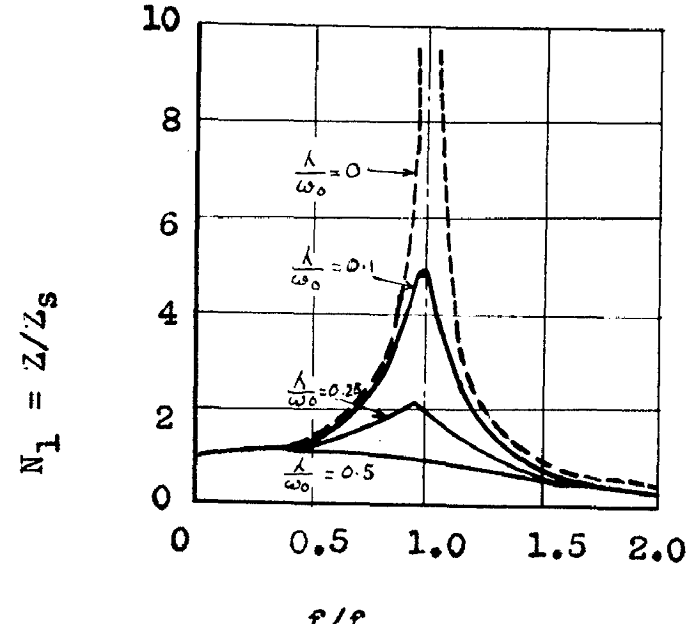
$y = f(z)$, $y = f(-z)$, $x = \emptyset(z)$, and $x = -\emptyset(z)$

Fig.3.11 Soil as a conservative medium (Ford and Haddow 1960)



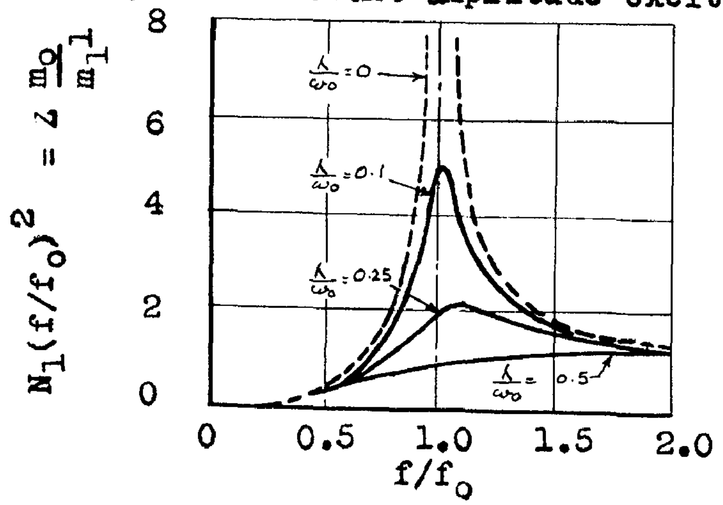
$$m_0 \ddot{z} + 2\lambda m_0 \dot{z} + K.z = F_0 \sin \omega t$$

Fig. 3.12 Mass Spring Analogy.



$$N_1 = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_0}\right)^2\right]^2 + \left(\frac{2\lambda}{\omega_0}\cdot\frac{\omega}{\omega_0}\right)^2}}$$

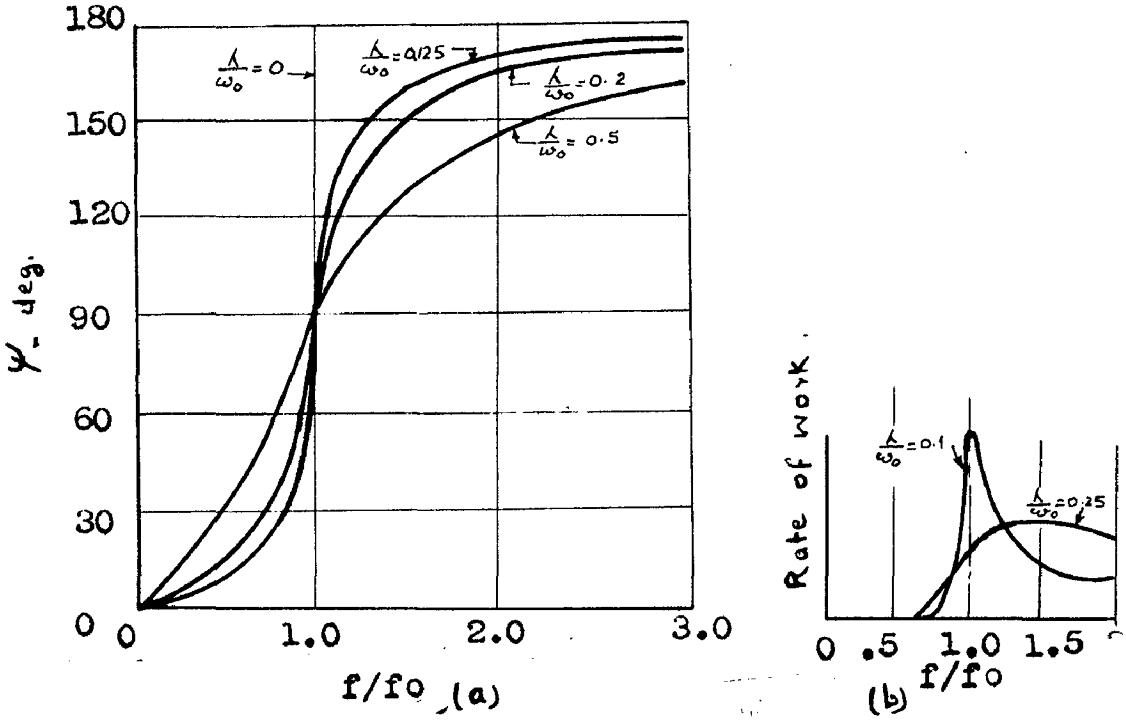
a) For constant amplitude exciting force F_0= const.

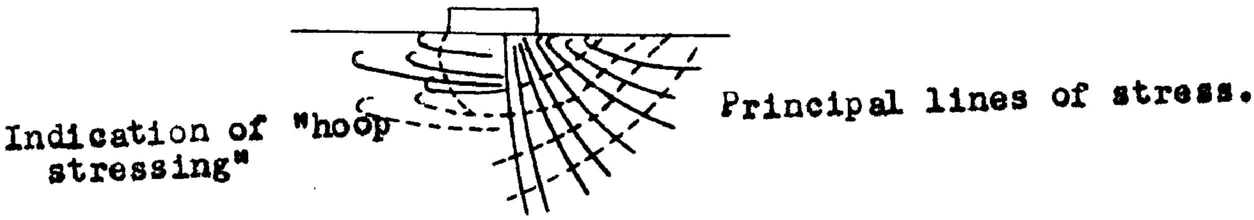


b) For exciting force depending upon frequency $F_0 = m_1 l \omega^2$

Fig. 3.13 Amplitude frequency relations for damped forced vibration of a mass spring system.

Fig.3.14 a) Phase angle between force and displacement (ψ)
b) Rate of work as a function of frequency for various values of damping.
a) (DenHartog 1947) b) Terzaghi 1943)





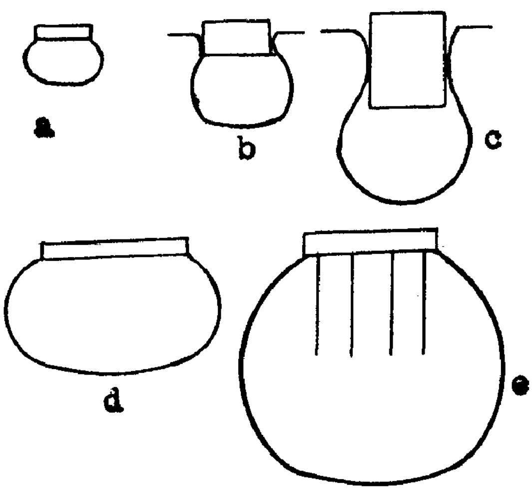
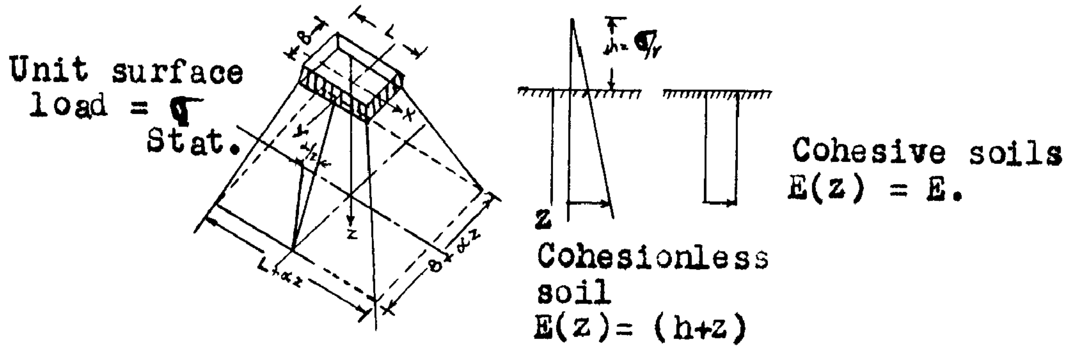
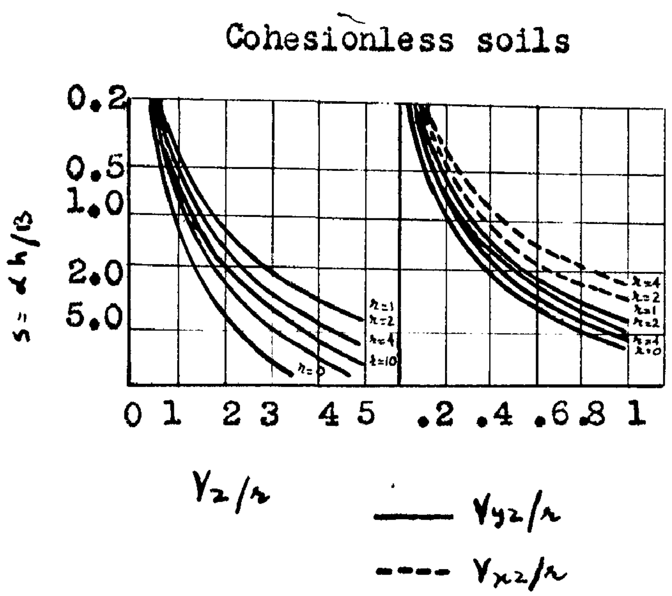
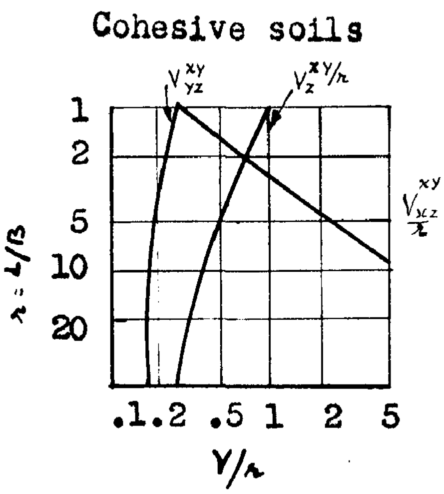


Fig.3.15 Proposed method of obtaining the active ground weight from the bulb of pressures.
(Crockett & Hammond 1949)



$r = L/B. \quad s = \alpha h/B \quad \beta' = \dfrac{\beta}{2(1+\nu)} \quad G = \dfrac{E}{2(1+\nu)}$





$K_z = \beta B^2 \gamma_z$ $\quad K_{yz} = \beta B^4 \gamma_{yz}$

$K_y = K_x = \beta' B^2 \gamma_z$ $\quad K_{xz} = \beta B^4 \gamma_{xz}$

$K_{xy} = \beta' B^4 (\gamma_{yz} + \gamma_{xz})$

$K_z = E \alpha B \gamma_z$

$K_x = K_y = G \alpha B \gamma_z$

$K_{yz} = E \alpha B^3 \gamma_{yz}$

$K_{xz} = E \alpha B^3 \gamma_{xz}$

$K_{xy} = G \alpha B^3 (\gamma_{yz} + \gamma_{xz})$.

K_x, K_y, K_z spring constants for vibrations in x,y,z directions.
K_{yz}, K_{xz}, K_{xy} spring constants for rotating modes about x,y,z axes.

Fig. 3.16 Equivalent Soil Spring Constants For Horizontal Contact Surface (xy) (Pauw 1953)

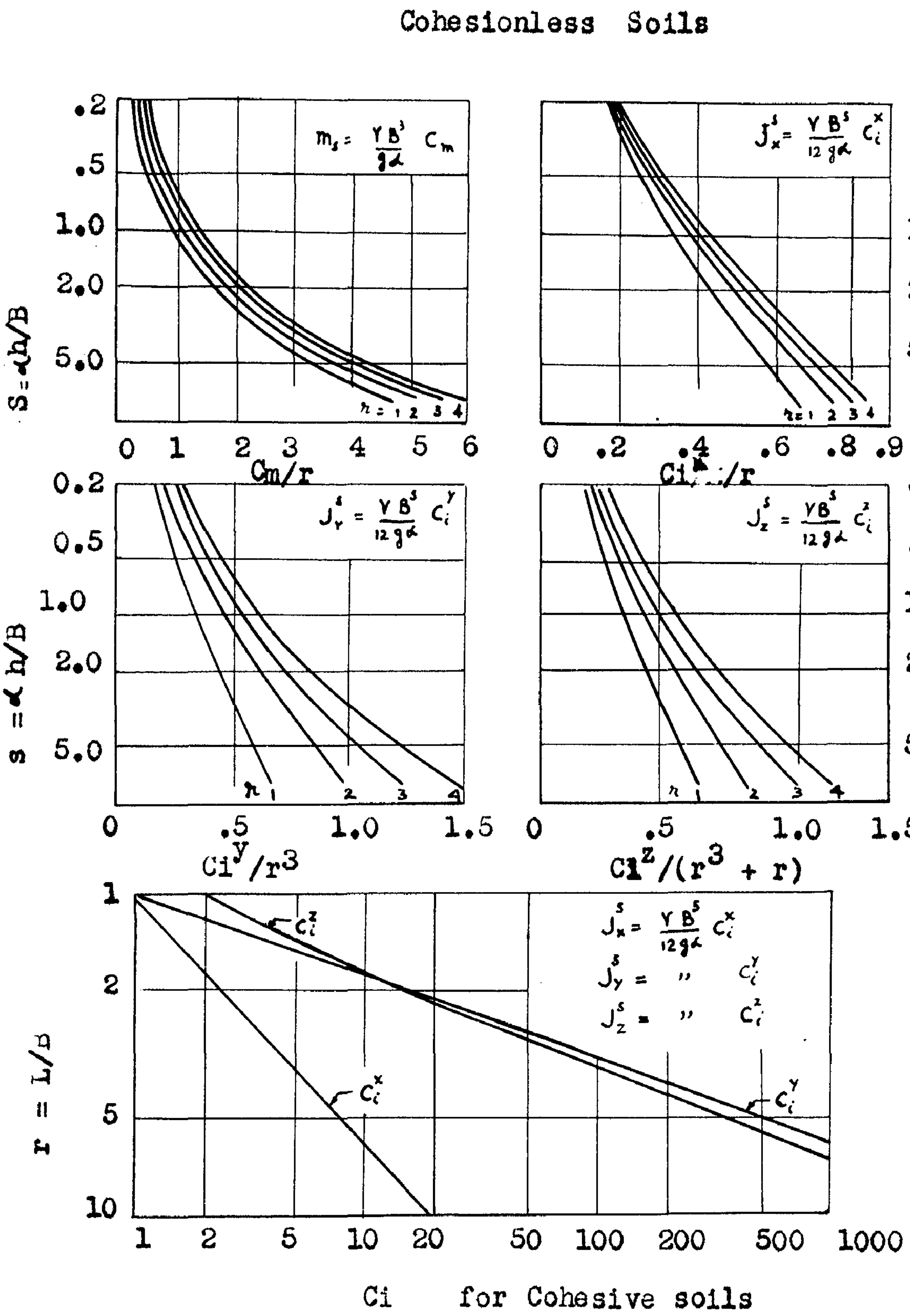


Fig. 3.17 Apparent mass factors for horizontal contact surface (Pauw 1953)

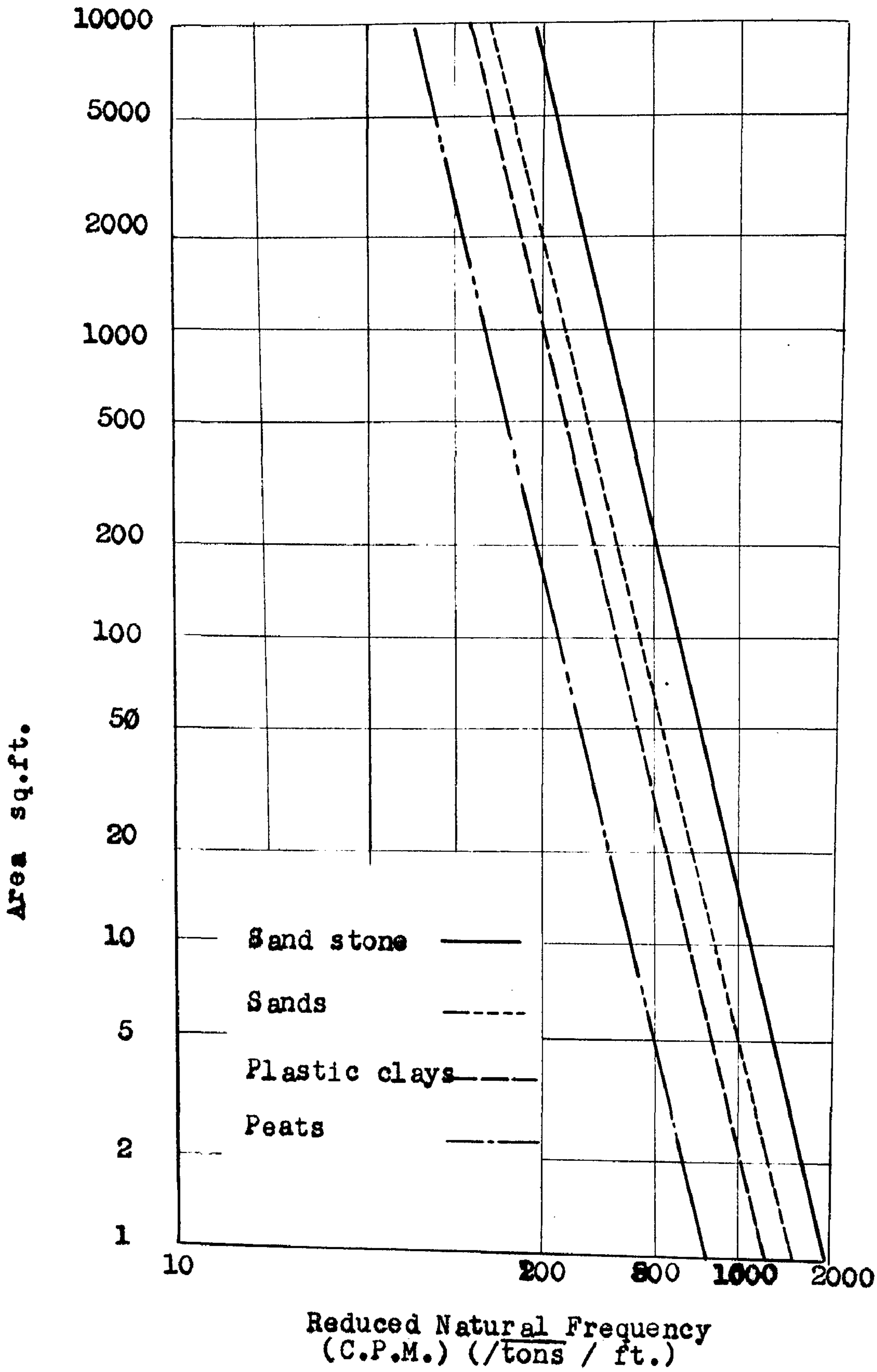


Fig. 3.18 Reduced Natural Frequency Plotted against the area of foundation (Tschebotorioff 1953)

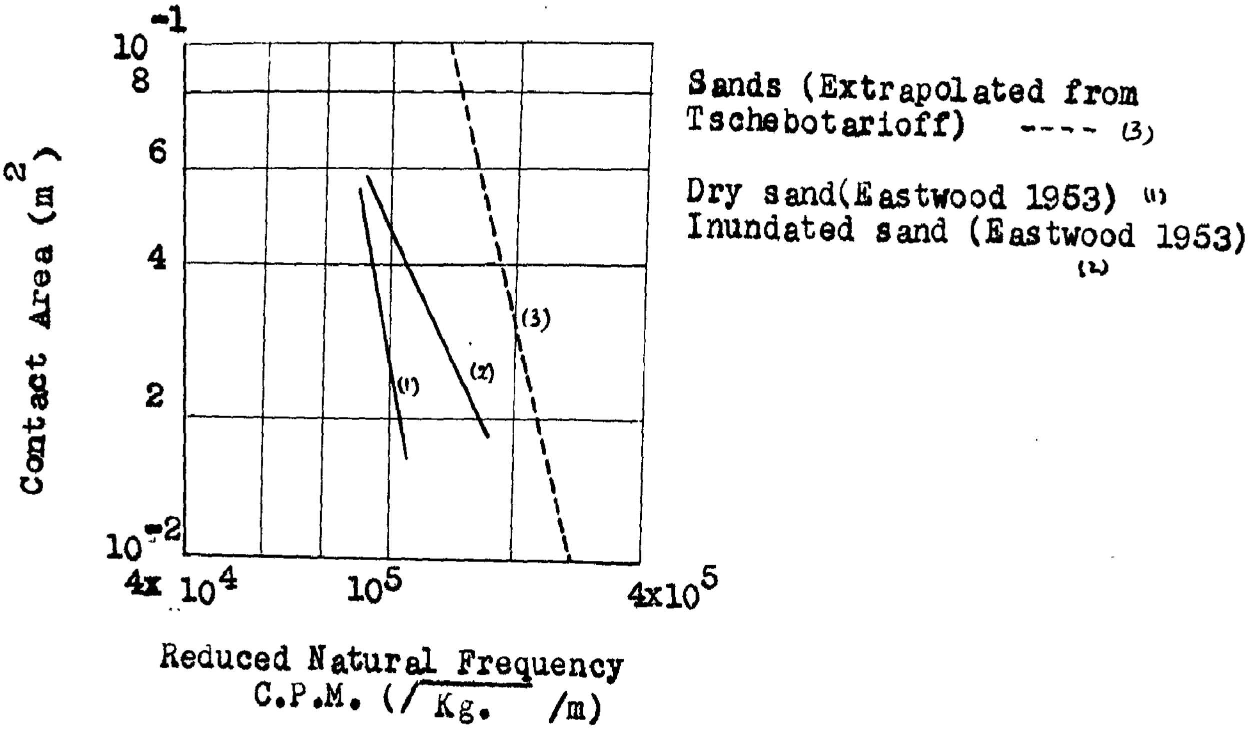


Fig. 3.19 Eastood's Model Tests plotted on Area vs. reduced natural frequency plot extrapolated from Tschebotarioff (Alpan 1961).

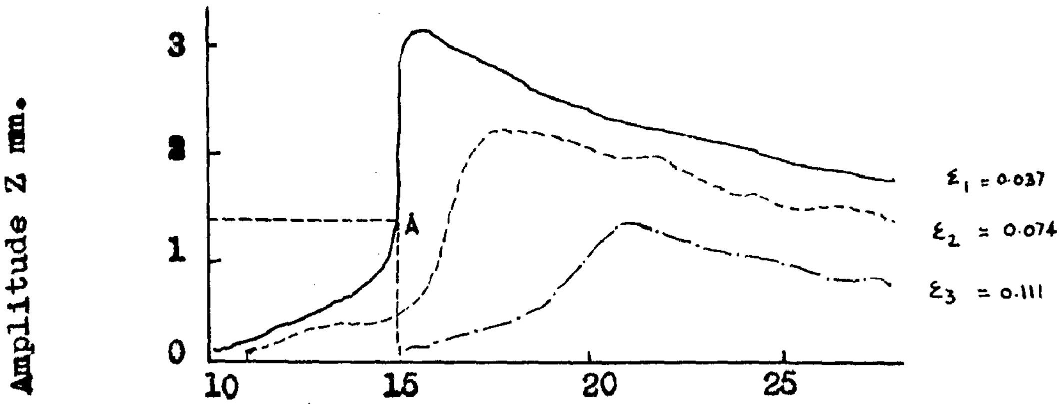


Fig.3.20 Resonance curves of a vibrator on a soil for different eccentricities. (Lorenz 1953)

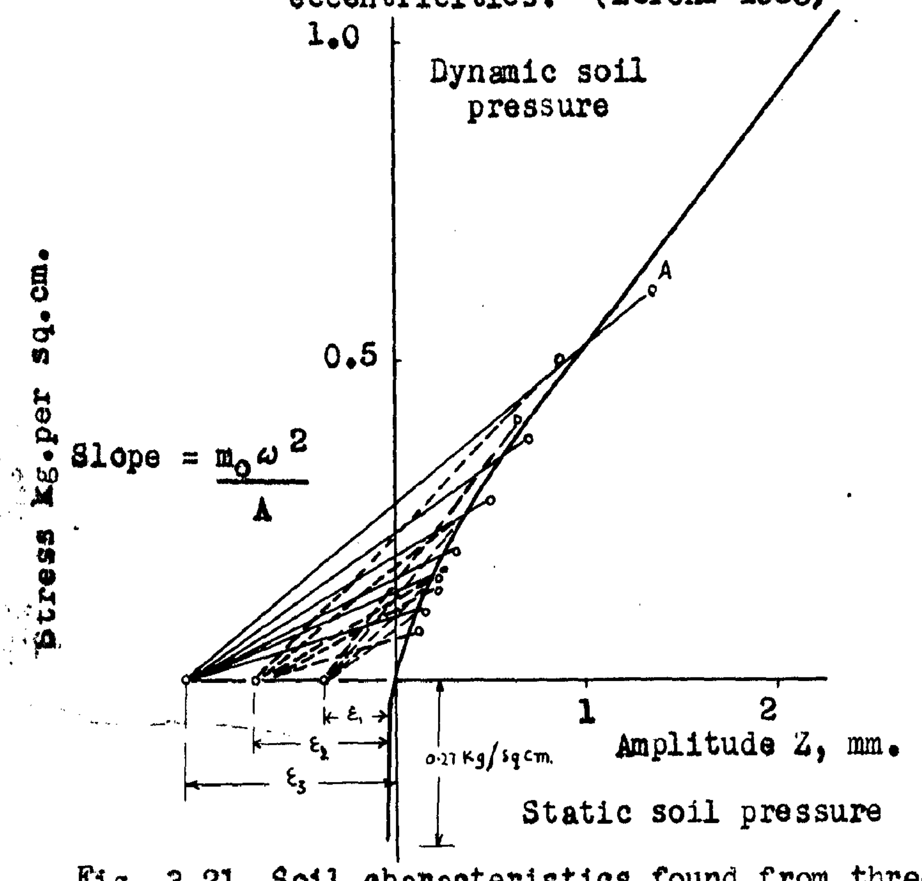


Fig. 3.21 Soil characteristics found from three resonance curves by graphical methods (Lorenz 1953)

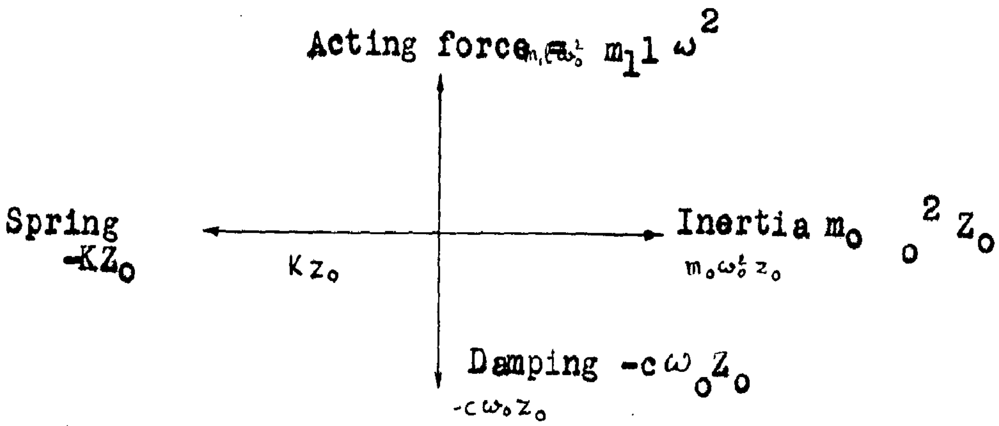


Fig. 3.22 a) Vector Diagram of Harmonic Motion.

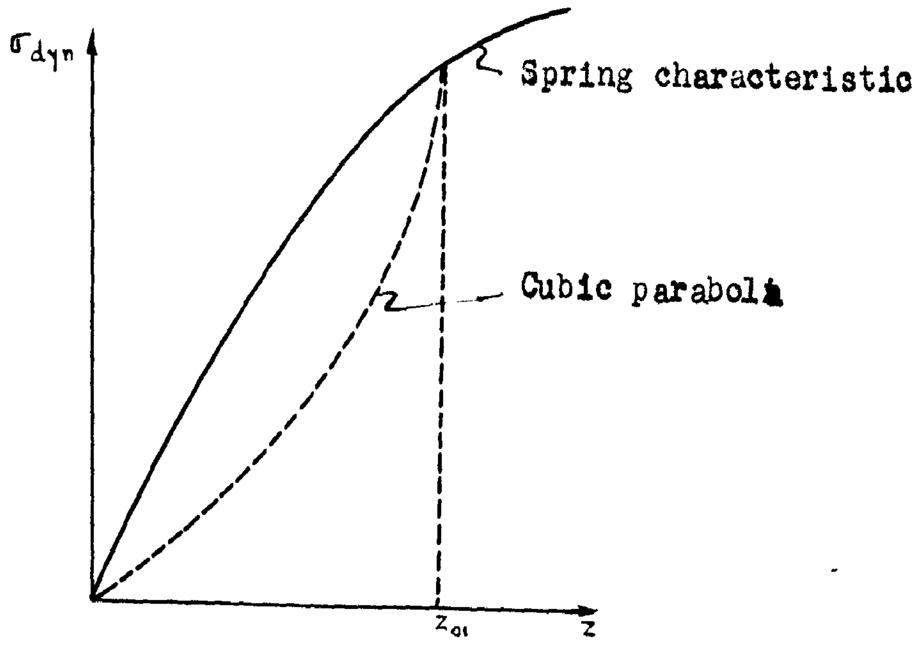


Fig. 3.22 b) Graphical method of determining the resonant amplitude (Z_{o1}) at $\varepsilon = \varepsilon_1$ and $m_o = m_{o1}$ (Alpan 1961)

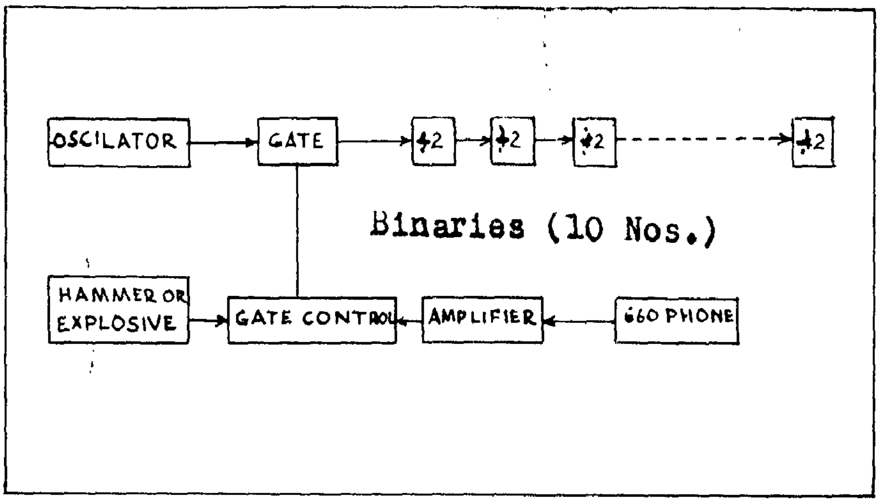


Fig. 4.1 Instrument operation diagram, Seismic methods (Fisher & Winter,1962)

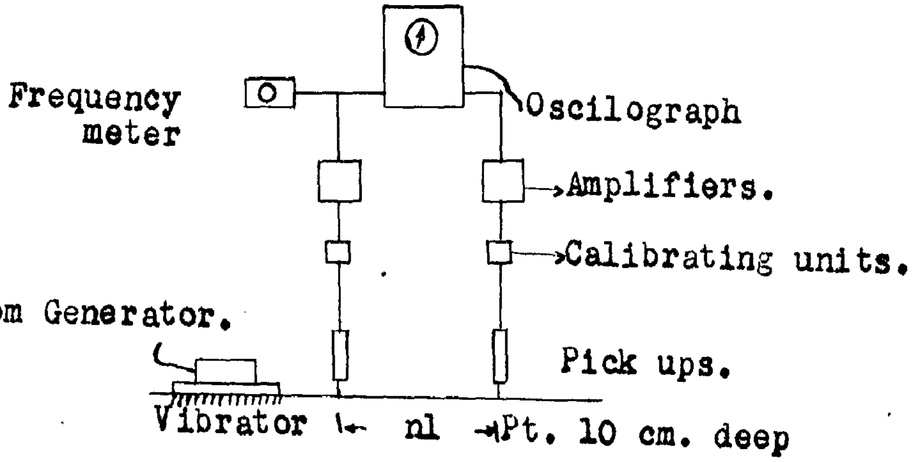


Fig. 4.3 General Arrangement of Equipment for Amplitude and wave length measurements. (Bergstrom and Linderholm 1946)

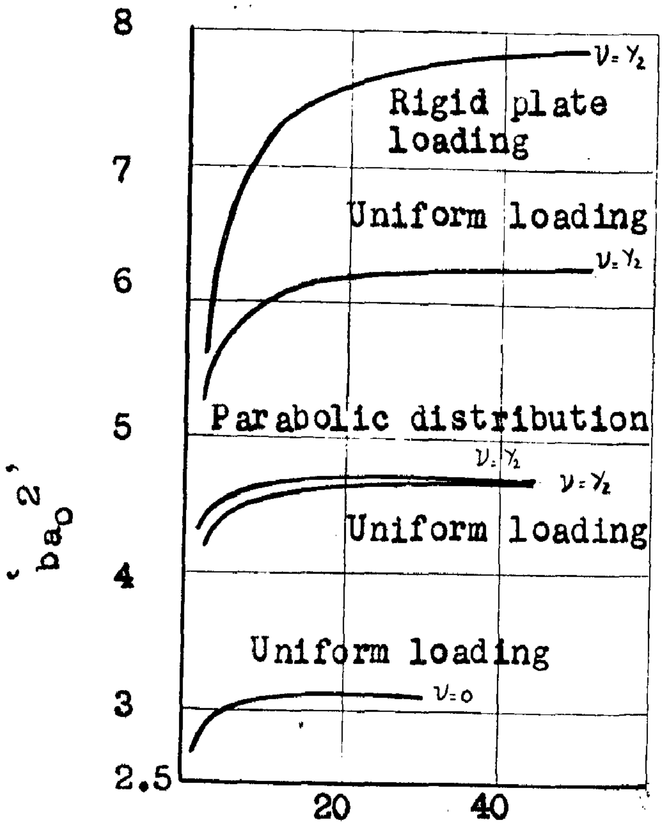
Pickups - Philips type GM 5520 electrodynamic pickups (pulsation in voltage propotional to velocity)

Calibrating unit - Integrates electrically so as to obtain a voltage proportional to amplitude. Also supplies a reference voltage of sixty cycles/second.

Amplifiers - Two type Fvs-138, Svensta Radio Bologet

Oscillograph - Philips Type GM 3156 Cathode ray Oscillograph.

Frequency meter. Type FMD 838 Svaska Radio Bologet.



(b)

Fig. 4.2 Effect of Stress distribution and Poisson's ratio on the non-dimensional quantity ba_o^2 (Jones 1958)

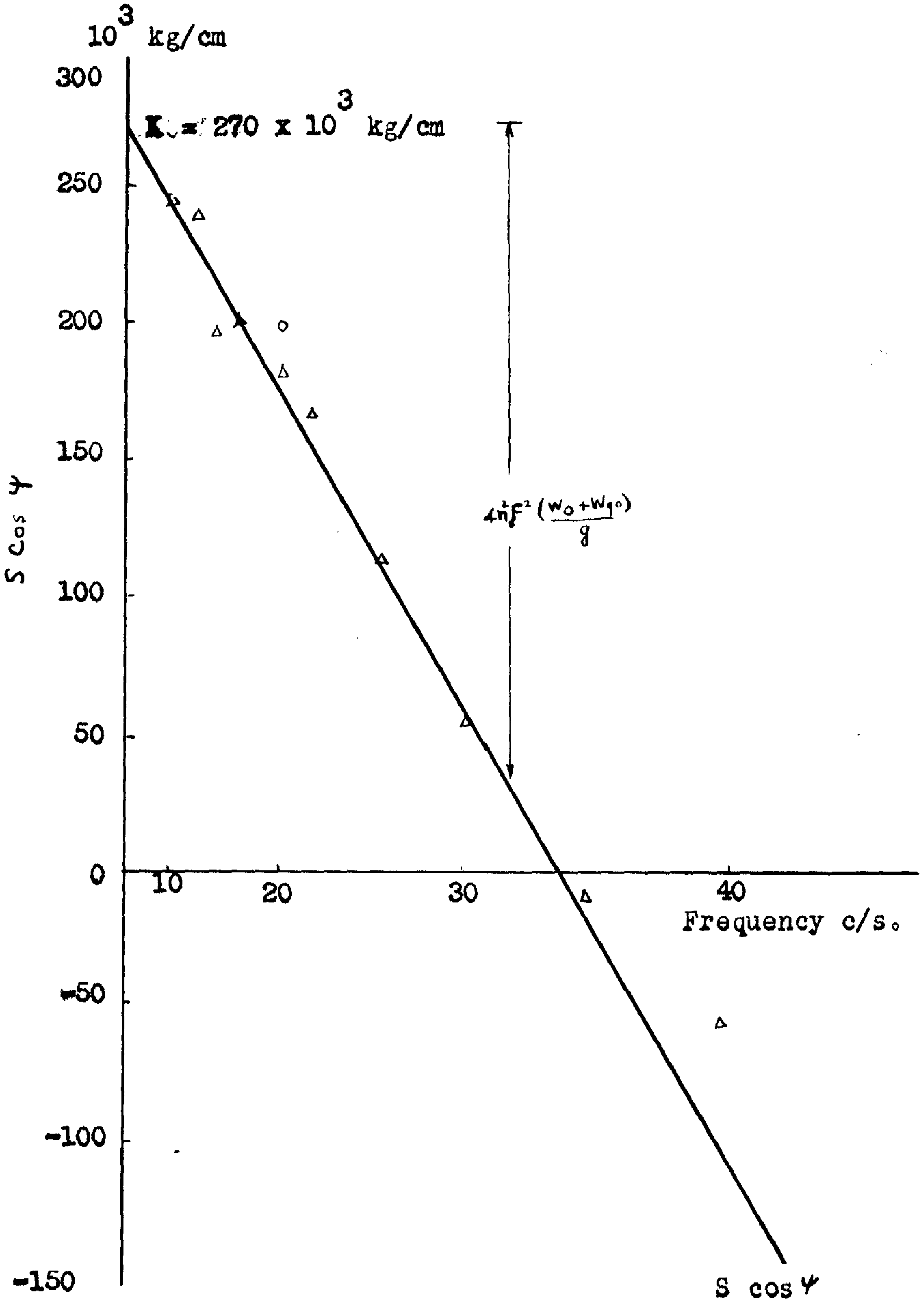


Fig. 4.6 Linear relationship between S cos ψ and f^2 at low frequencies ~~is~~ used to determine the values of K and Wso .

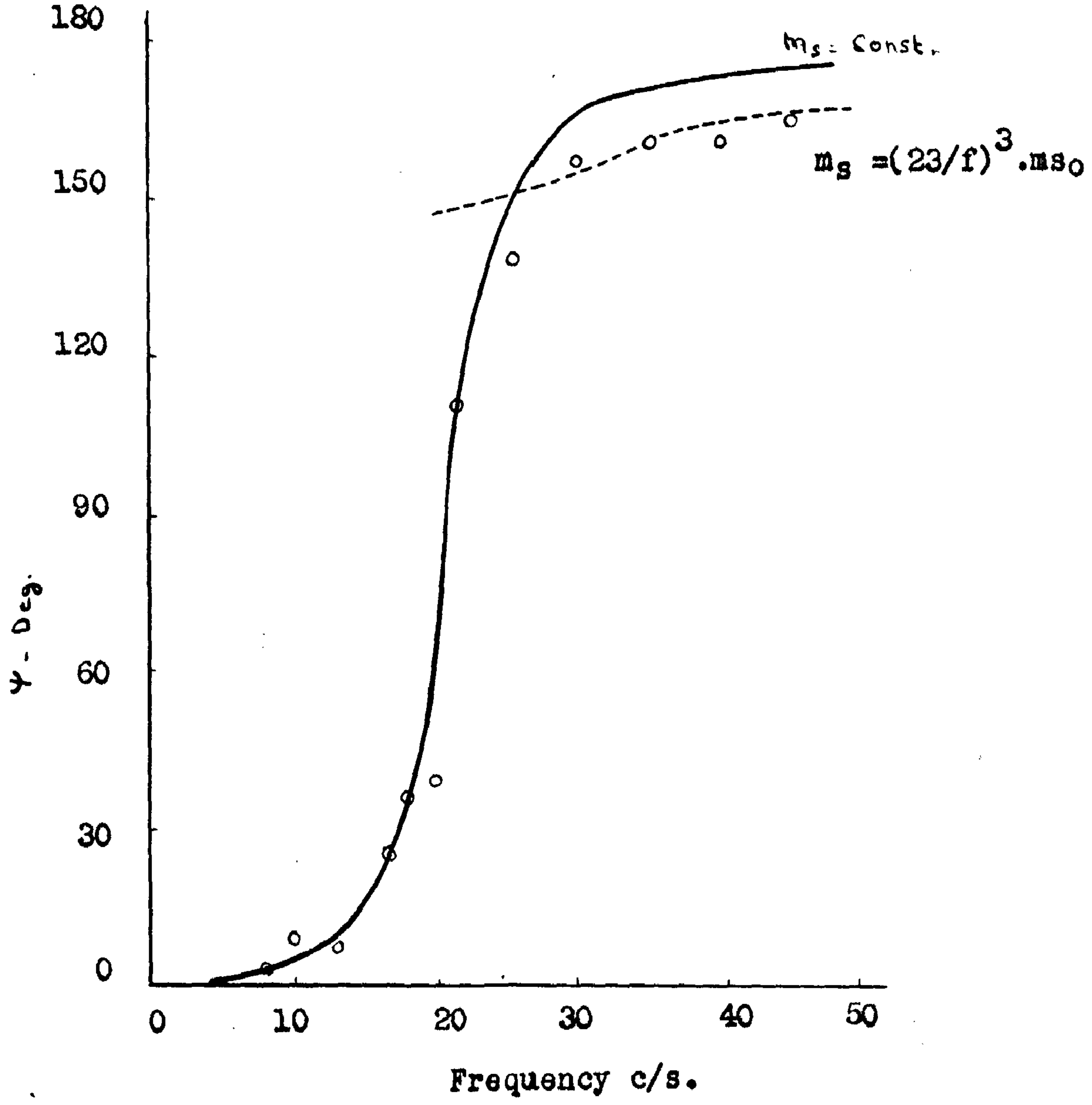


Fig. 4.5 Phase angle ψ versus frequency of vibrations.

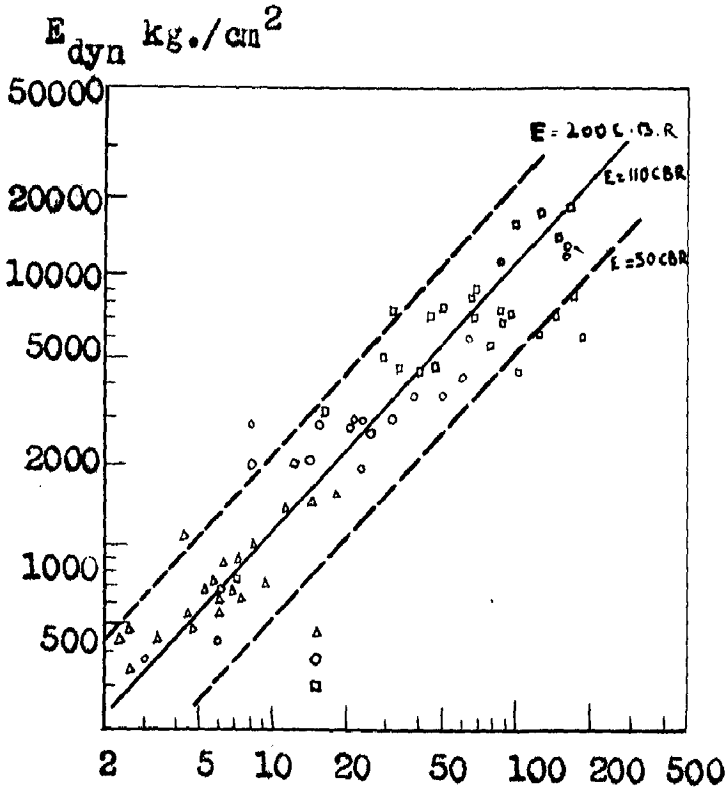


Fig. 4.6 E dyn. versus California Bearing Ratio.

(Heukelom and Foster , 1960)

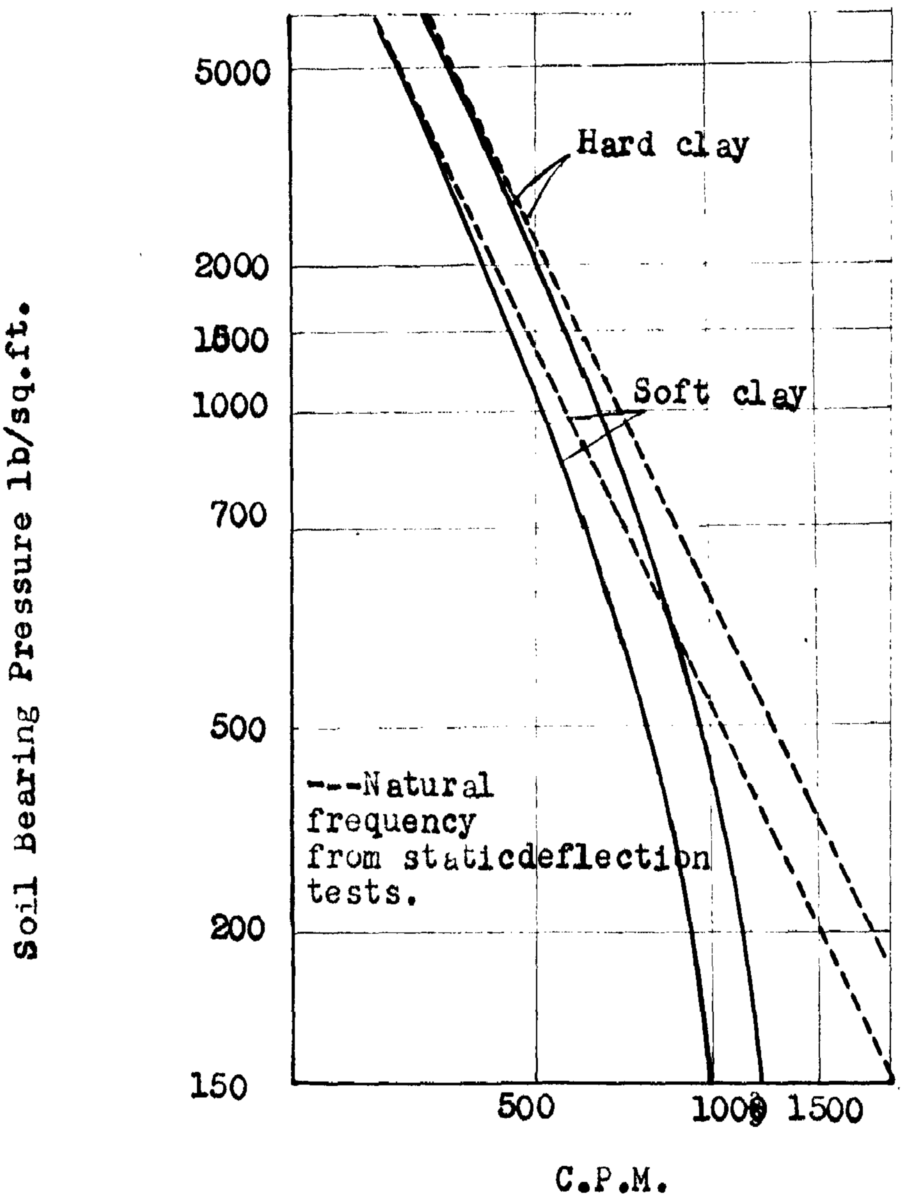


Fig. 5.1 Natural Frequency of Foundation on Two Types of Soils (New Comb - 1951).

APPENDIX.(TABLES)

TABLE - 2.1

SUMMARY OF INERTIA FORCES AND COUPLES IN FOUR-CYLINDER ENGINE.

Crank	Ø	2Ø	cosØ	cos2Ø	sinØ	L cosØ	L cos2Ø	L sin Ø
0	0	0	1	1	1	0	0	0
1	90	180	0	-1	0	0	- L	0
2	270	540	0	-1	-1	0	-2L	-2L
3	180	360	-1	1	0	-3L	3L	0
Summation			0	0	0	-3L	0	-2L

i.e. Primary force, secondary forces = 0, while primary couples are not balanced.

TABLE -2.2

SUMMARY OF UNBALANCED INERTIA FORCES AND COUPLES FOR DIFFERENT CRANK ARRANGEMENTS. (N[illegible] , 1951)

$F' = .0000284\ W_p r N^2 \cos \phi$ (Primary)

$F'' = .0000284\ W_p r N^2 \cos 2\phi$ (Secondary)

L = Distance between cylinder centers.

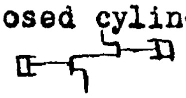
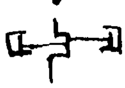
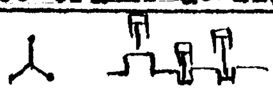
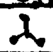
Crank Arrangements	Forces. Primary	Forces. Secondary	Couples Primary	Couples Secondary
Single crank 	F' without counter-wt. F'½ with counter-wt.	F"	none	none
Two cranks at 180°				
In line cylinders 	Zero	2F"	F'L without counter-wt. F'L/2 with counter-wts.	none
Opposed cylinders. 	Zero	Zero	Nil	Nil
Two cranks at 90° 	1.41 F' without counter-wts. or 0.707 with counter-wts.	Zero	1.41 F'L without counter-wt. 0.707 F'L with counter-wts.	F"L
Two cylinders on one crank.				
Cylinder at 90° 	F' without counter-wts. 0 with counter-wts.	1.41 F"	none	Nil
Opposted cylinders. 	2F' without counter-wts. F' with counter-wts.	Zero	none	Nil
Three cranks at 120° 	Zero	Zero	3.46F'L with counter-wts. 1.73 F'L with counter-wts.	3.4 F"L
Four cylinders.				
Cranks at 180° 	Zero	4F"	Zero	Zero
Cranks at 90° 	Zero	Zero	1.41F'L without counter-wts. 0.707F'L with counter-wts.	4.0F"L
Six Cylinders. 	Zero	Zero	Zero	Zero.

TABLE - 3.1

VALUE OF SHAPE FACTOR 'm' FOR STATIC CASE (Timoshenko).

L/B	1.0	1.5	2	3	5	10	100.
m	0.95	0.94	0.92	0.88	0.82	0.71	0.37

TABLE - 3.2

RELATION BETWEEN NATURAL AND SAFE SOIL PRESSURE. (LORENZ 1934)

Soil Type	Cps	Safe soil pressure kgm/sq.cm.
1.5 m peat bag on sand	12.5	-
1.5 m old filling: medium sand with peat residues.	19.1	1.0
Gravelly sand with clay lenses.	19.4	-
Old, stamped dam slag filling	21.3	1.5
Very old, stamped dam filling of loamy sand.	21.7	2.0
Tertinary clay wet.	21.8	-
Biam clay wet.	23.8	-
Very homogeneous, yellow medium sand. (so called stettin sand.	24.1	3.0
Homogeneous gravel	26.2	4.5
Non homogeneous, closely packed sand.	26.7	4.5
Absolutely dry tertiary clay	26.7	-
Closely packed medium gravel	28.1	4.5
Shell lime (bed rock)	30.0	2/3 of the admissible compression strain.
Mottled sand stone (bed rock)	34.0	-do-

TABLE - 3.3

RELATION BETWEEN NATURAL AND SAFE SOIL PRESSURE (CROCKETT & HAMMOND 1943)

		Tons sq/ft.
Peat	7.5	-
Water logged estuarine silt	10	0.7
Very light soft clay	12	1.0
Light water logged sand	15	1.5
Medium clay	15	2.0
Hard peat and sand layers	17	2.0
stiff clay	19	3.0
Silt and sand mixed	23.3	3.0
Sand and rubable loosely compated	23.5	3.0
Lime stone	30.0	-
Granite	40	-

TABLE - 4.1

TYPICAL VALUES OF POISSON'S RATIO FOR DIFFERENT SUBGRADE MATERIAL

Soil OR Rock	Poisson's Ratio	
Loose sand saturated	0.32	
Dense sand saturated	0.27	
Organic silty clay	0.36	
Sandy clay	0.30	
Clay remoulded	0.42	
Sand stone	0.11	0.12
Slates	0.10	0.12

TABLE - 5.1

VARIOUS RECOMMENDATIONS FOR SIZE OF M/C FOUNDATIONS.

Engine Description.	Ratio of weight of foundation to Engine weight.
COUZERIS (CONVERSE 1962)	
Gas Engine	
1. Cylinder	3.0
2. "	3.0
4. "	2.75
6. "	2.0
8. "	
DIESEL ENGINES.	
2. Cylinder	2.75
4. "	2.4
6. "	2.1
8. "	1.9
Rotary converter.	0.5 to 0.75
Vertical compound steam engine coupled to generator.	3.8
Vertical triple expansion steam engine coupled.	3.5
Horizontal cross compound to generator.	3.25
Horizontal steam turbine coupled to generator.	3.0 to 4.0
Vertical Diesel Engine coupled to Generator.	3.5
Vertical Diesel Engine coupled to Generator.	2.6
LIGHTY (gas engines)	Concrete
1. Horizontal engine without outboard.	14 to 18 cft/BHP
2. Horizontal engine with outboard bearing.	19 to 22 cft/BHP.
3. Vertical engine without outboard bearing.	7.7 to 8.8 cft/BHP
4. Vertical engine with outboard bearing.	9.8 to 10.5 cft/BHP.

MORSE (1942)	Weight of foundation.
Multicylinder engines.	
1. Gas engines.	1600 lb/BHP
2. Diesel engines.	1250 lb/BHP
3. Steam engines.	500 lb/BHP

For single cylinder engines the above should be increased by about 40 to 60 percent.

BOYER

Engines running at 400 cpm or less	wt. of foundation.
3. Cylinder engine	550 lb/BHP
8. Cylinder engine.	335/ lb/BHP

-.-.-.-.-.-

V I T A

NAME	Umesh Kumar Bhatia
BIRTH.	5th March, 1938, Ubbavro (Sind now in W.Pakistan)
EDUCATION,	Govt. Sindhi High School, Rajendra Nagar, New Delhi, Hindu College, Delhi. D.S.D. College, Gurgaon, (Punjab). Indian Institute of Technology, Kharagpur. University of Roorkee, Roorkee, U.P.

QUAEIFICATIONS.	High School	Ist Class	1954
	Inter. Science.	Ist Class	1956
	B.Tech.(Hons)	Ist Class	1960
	P.G. Diploma.	Ist Class	1962.

EXPERIENCE.	Technical Teacher Trainee, University of Roorkee, Roorkee.	3 years.
PUBLICATIONS.	Nil.	

*****