NUMERICAL MODELLING OF TWO-DIMENSIONAL TRANSIENT SUBSURFACE FLOW TO DITCHES

A THESIS

submitted in fulfilment of the
requirements for the award of the degree
of
DOCTOR OF PHILOSOPHY
in
HYDROLOGY

By

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TO

MY WIFE

CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled NUMERICAL MODELLING OF TWO-DIMENSIONAL TRANSIENT SUBSURFACE FLOW TO DITCHES in fulfillment of the requirement for the award of Degree of Doctor of Philosophy, submitted in the Department of Hydrology, University of Roorkee, is an authentic record of my own work carried out during a period from November 1987 to October 1992 under the supervision of Dr. B.S.Mathur and Dr. Deepak Kashyap.

The matter embodied in this thesis has not been submitted by me for the award of any other degree.

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ABSTRACT

Inadequate natural subsurface drainage in an agricultural area results in a rise of water table up to the root zone of cultivated plants. This restricts the oxygen supply to the plant root system. The high water table also reverses the benefits of leaching of soluble salts from the root zone. An appropriate artificial subsurface drainage system can maintain the water table at a permissible depth, depending upon the crop and the soil.

The prevalent theories of subsurface drainage, employing Dupuit-Forchheimer assumptions, ignore the loss of head due to vertical component of flow. This leads to an underestimation of water table rise and, thus, an overestimation of drain spacing. The effect is significant in the case of horizontally stratified soils, pipe drains and partially penetrating ditches due to higher vertical velocities. The vertical flow can be accounted for by numerical solution of the differential equation governing either two dimensional flow in a vertical plane or three dimensional flow.

In the present study two numerical models of two-dimensional subsurface drainage, one analysing only the saturated domain (saturated flow model) and another analysing the entire unsaturated-saturated domain (Total Response Model) have been developed.

In the saturated flow model (SFM) the two dimensional nature of the flow is accounted for by a finite differences based solution of the differential equation governing two-dimensional transient, unconfined saturated flow in a heterogeneous porous medium having vertical anisotropy (subjected to drainage boundary

conditions). The SFM requires among others the time variant distribution of recharge rate at the water table as input data and yields the time variant water table position. In the total response model (TRM), the two-dimensional flow is accounted for by a finite differences based solution of the differential equation governing two-dimensional transient unsaturated-saturated flow in a heterogeneous porous medium having vertical anisotropy (subjected to the drainage boundary conditions). The TRM requires among others, the time variant distribution of infiltration rate at ground surface as input data and yields the spatial and temporal distribution of capillary head (h_c). This in turn yields the time variant water table position defined by $h_c = 0$.

In chapter III the development of saturated flow model and total response model, along with their solution techniques have been presented.

The saturated flow model has been implicitly validated by comparing its response with Donnan and Kraijenhaff analytical solutions. The model computed water table rises are found to converge to these analytical solutions as the ideal conditions (negligible relative resistance to vertical flow, i.e., $K_z/K_x >> 1$) assumed in the analytical solutions are approached. However, under non-ideal conditions the analytical solutions are found to underestimate the water table rise. The model computed lateral flows (with $K_z/K_x >> 1$) into a ditch are also found to compare well with the Edelman solutions under different conditions, viz, (i) sudden lowering of water level in the ditch, (ii) constant lateral flow from aquifer to the ditch, (iii) linearly increasing lowering of water level in the ditch, and (iv) linearly increasing lateral flow to the ditch.

The computed rises by the saturated flow model and the total response model have been compared with the corresponding field data from Haryana, India, reported by Chhedi Lal (1986). The two models have reproduced the water table rises quite well. As expected, the reproduction by the total response model is better.

In chapter IV, the model validation, by comparing it with the analytical solutions and the reported field data, has been presented in detail.

The model solution for partially penetrating ditch systems has been presented in the form of dimensionless design curves. The ratio $\Delta h/\Delta h^*$, i.e., the water table rise at the midsection computed by the model (Δh) divided by Kraijenhoff solution (Δh^*), is expressed as a function of three dimensionless independent variables K_x/K_z, d/Y_0, and d/L. The design curves along with Kraijenhoff solution permit graphical estimation of the steady state rise of water table (accounting for the vertical flows) within a practical range of geometric dimensions and parameters (i.e. $20 \geq K_x/K_z \geq 0$, $1.0 \geq d/Y_0 \geq 0.25$, $0.5 \geq d/L \geq 0.075$).

The bank storage development and its subsequent release to a ditch has been studied by passing an assumed stage hydrograph of 7 days duration through the drain. For the case considered, it is found that for no infiltration on the ground surface 60% of the bank storage is released within a short period (20 days). The rest 40% is released slowly.

The total response model developed in the present study is capable of simulating the generation of perched water table condition (and associated throughflows to the drains) over an impeding layer in the unsaturated zone.

The throughflow development has been studied by considering a horizontal clay layer in the unsaturated zone of a ditch system consisting of uniform loam soil above and below the clay layer.

The applications of the two models, have been described in details in chapter V. The prominent conclusions drawn from the study have been presented in chapter VI.

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SYMBOL	SYMBOL DEFINITION	DIMENSION
(A_i)	Coefficient (of $h_{i-1jk+1}$) matrix of (NR)th order.	-
(A_j)	Coefficient (of $h_{ij-1k+1}$) matrix of (NC)th order.	-
b	width of the ditch	[L]
(B_i)	Coefficient (of h_{ijk+1}) matrix of (NR)th order	-
(B_j)	Coefficient (of h_{ijk+1}) matrix of (NC)th order	-
C=C(x,z)	Specific storage in the saturated zone	(L^{-1})
C=C(z,θ)	Specific Soil moisture capacity in the unsaturated zone.	(L^{-1})
C_{ij}	Value of 'C' at node (i,j)	(L^{-1})
(C_i)	Coefficient (of $h_{i+1jk+1}$) matrix of (NR)th order	-
(C_j)	Coefficient (of $h_{ij+1k+1}$) matrix of (NC)th order	-
d	depth of the ditch below drain level	(L)
D_c	depth to the top of clay layer	(L)
D_i	depth of impervious layer below ground surface	(L)
(D_i)	Coefficient matrix (of constants) of (NR)th order	-
(D_j)	Coefficient matrix (of constants) of (NC)th order	-
D_r	Depth of the ditch below ground surface	(L)
D_w	Depth of initial water table below ground surface	(L)
$DZ=DZ_{jk+1}$	Vertical spacing between top most row and preceding row at jth column at (k+1)th discrete	(L)

	time.	
e	base of natural (Napierian) logarithm	
$ET=ET(\theta)$	Actual evapotranspiration rate per unit soil Volume.	(LT^{-1})
F_c	field capacity	-
$h=h(x,z,\theta)$	hydraulic head above a datum (or sum of capillary head and elevation head)	(L)
$h_c=h_c(x,z,\theta)$	capillary head	(L)
h_b	bubbling pressure (air entry value)	(L)
h_{ijk+1}	computed 'h' value at node (i,j) at (k+1)th discrete time	(L)
i	indicative of row number	-
j	indicative of column number	-
$I=I(t)$	infiltration per unit time	(LT^{-1})
$Kx=K_{xx}$ $=K_{xx}(x,z)=K(x,\theta)$	Hydraulic conductivity in x-direction	(LT^{-1})
Kx_{ij}	hydraulic conductivity of the link between nodes (i,j+1) and (i,j)	(LT^{-1})
Kxsat	saturated hydraulic conductivity in x-direction.	(LT^{-1})
$Kz=K_{zz}$ $K_{zz}(x,z)=K(z,\theta)$	hydraulic conductivity in z-direction	(LT^{-1})
Kz_{ij}	hydraulic conductivity of the link between nodes (i+1,j) and (i,j)	(LT^{-1})
K_{zsat}	Saturated hydraulic conductivity in z-direction	(LT^{-1})
l	number of ADI iterations	-

L	Centre to centre spacing of drains	(L)
m	number of the ADIE iteration using known values of nth iteration	-
n	number of the ADIE iteration using known values of mth iteration.	-
np	number of subtime steps in a time step	-
NC	total number of columns	-
NR	total number of rows	-
p_f	fraction of available soil moisture after which ET=PET	-
PET	potential evapotranspiration rate per unit soil volume.	(LT^{-1})
q	rate of lateral flow from beneath the water table.	(L^2T^{-1})
q_1, q_2, q_3	rates of lateral flows as indicated in boundary conditions	(L^2T^{-1})
q_b	rate of lateral flow from aquifer to the drain or vice-versa.	(L^2T^{-1})
q_i	rate of throughflow from perched water table zone.	(L^2T^{-1})
$Q=\Sigma q\, dt$	cumulative lateral flow from beneath the water table.	(L^2)
$Q_b=\Sigma q_i . dt.$	cumulative lateral flow from aquifer to the drain	(L^2)
$Q_i=\Sigma q_i . dt.$	cumulative throughflow	(L^2)
Q_r	cumulative lateral flow from aquifer to the ditch when I(t) is taken zero.	(L^2)

$R=R(t)$	recharge per unit time	(LT^{-1})
s	seepage face of the ditch above initial water table	(L)
S_s	specific storage	(L^{-1})
S_{sij}	specific storage at node (i,j)	(L^{-1})
S_y	specific yield	
S_{yj}	specific yield at jth column	-
t	time	
t_c	thickness of clay layer	(T)
W_p	wilting point	(L)
x	horizontal coordinate direction	-
Y_o	depth of impervious layer below drain level.	(L)
U_{ijk+1}	horizontal velocity component in the domain (i,j) to (i,j+1)	(LT^{-1})
V_{ijk+1}	vertical velocity component in the domain (i,j) to (i+1,j)	(LT^{-1})
z	vertical coordinate direction	-
Δh	rise of water table at midsection	(L)
Δh^*	rise of water table at midsection computed by Kraijenhoff solution	(L)
Δx_j	horizontal spacing between colmuns j and (j+1)	(L)
Δt	discrete subtime step	(T)
Δz_i	vertical spacing between rows i and (i+1)	(L)
ε_1	convergence factor for ADIE iterations (in the saturated flow model)	(L)

ε_2	convergence factor for subtime step iterations	(L)
ε_3	convergence factor for DZ modifications	(L)
ε_4	Convergence factor for ADIE iterations in in the total response model	(L)
ε_5	convergence factor for subtime step iterations in the total response model	(L)
θ	volumetric soil moisture content	-
θ_r	residual soil moisture content	-
θ_t	a dummy variable defined by $\theta_t=F_c-p_f(F_c-W_p)$	-
ϕ	soil porosity	
$\xi = \xi(x,t)$	elevation of water table above impervious layer	(L)
ξ_{jk+1}	elevation of water table at column j at (k+1)th discrete time.	(L)

CHAPTER-I

INTRODUCTION

1.1 ARTIFICIAL SUBSURFACE DRAINAGE

Salt concentration in the root zone in irrigated areas can be maintained at a permissible level by leaching,i.e.,applying water in excess of the evapotranspiration requirements of the crops. The excess water percolates down to the water table and transports along with it a part of the salt content of the root zone.Thus,the leaching process is always associated with a recharge to the water table.Even when there is no leaching requirement,some recharge is inevitable as it is rarely practical to apply water exactly equal to the evapotranspiration requirement at the plant roots in large irrigation projects.

The recharge to the water table will cause a rise of the water table.In the areas of adequate natural drainage; i.e., the subsurface soil having good water-transmitting and storage properties, closely spaced subsurface natural drains;the rise may be insignificant. However, if the natural drainage is not adequate the water table rises substantially. If the water table reaches the root zone it will not only cause waterlogging but will also reverse the benefits of leaching.

The natural subsurface drainage, in such areas has to be supplemented by the artificial subsurface drainage to control the excessive water table rise. The water table rise can be effectively controlled either by pumping (vertical subsurface drainage) or by providing horizontal subsurface drains (horizontal drainage).

1.2 THEORIES OF SUBSURFACE HORIZONTAL DRAINAGE-CRITIQUE

Subsurface horizontal drainage can be accomplished by buried pipes or open ditches. The prevalent theories of subsurface drainage (e.g., Donnan 1946, Hooghoudt 1940, Ernst (1956, 62), Kraijenhoff 1958) are based on Dupuit-Forchheimer assumptions. Since these assumptions essentially imply horizontal flow, the design based upon these ignore the loss of head due to vertical component of flow. This means that the streamlines are assumed to be horizontal and parallel to each other. However, in practice all streamlines originate from the free surface and converge at the ditch. Such streamlines result from flow of the recharged water (occurring at the water table) towards the ditch. Thus, the flow has to be two-dimensional with a component of vertical flow even in case of fully penetrating ditches. This vertical flow will involve head loss, which may be quite significant in the case of horizontally stratified soils. This will cause an extra rise in the water table. The effect will be far more pronounced in the case of partially penetrating ditches and tile drains due to higher vertical velocities. Thus, the prevalent theories underestimate the water table rise and hence lead to an underdesign of the drainage system.

The vertical flow can be accounted for by numerical solutions of differential equations governing either two-dimensional flow in a vertical plane or three dimensional flow. Several numerical models of three dimensional flow (e.g., Freeze 1971, France 1974) have been developed for groundwater resources evaluation. In the field of subsurface drainage also, a few numerical models of the two-dimensional flow in the saturated zone (e.g., Gureghian and Youngs 1975) and in the unsaturated-saturated zone (e.g., Skaggs and Tang 1976, Tang and Skaggs 1977, Vauclin et al., 1979, Gureghian 1981, Merva et al. 1983) have been presented.

1.3 OBJECTIVES OF THE PRESENT STUDY

The present study has been carried out with the following objectives:

(i) To develop a numerical model of two-dimensional subsurface drainage in a saturated domain. Such a model would permit estimation of water table rise for a given rate of recharge at the water table.

(ii) To evaluate the currently used analytical solutions (based upon Dupuit-Forchheimer assumptions) by employing the numerical model.

(iii) To identify the conditions under which the existing solutions may be used without incurring significant errors. Further, to construct appropriate design curves for correcting the water table rise computed by the existing solutions.

(iv) To develop a Total Response Model i.e.,a numerical model of two-dimensional subsurface drainage in the entire unsaturated-saturated domain extending from the ground surface to the lower impervious layer. Such a model would permit estimation of water table rise for a given infiltration (and not recharge) rate.

(v) To use the total response model for studying,apart from the water table rise, the throughflow (lateral flow to a drain caused by an impeding layer in the unsaturated zone), bank storage development and its release.

1.4 THE PRESENT STUDY AT A GLANCE

In the present study, the two-dimensional nature of the flow has been accounted for by solving numerically the differential equation governing two-dimensional transient, saturated flow in a heterogeneous porous medium having vertical anisotropy. The resulting solution provides the steady state as well as unsteady state water table positions. The model solutions have been found to compare well with Donnan (1946) and Kraijenhoff (1958) solutions under ideal conditions (negligible relative resistance to vertical flow, i.e., $K_z/K_x >> 1$). These analytical solutions are, however, found to underestimate the water table rise when the vertical hydraulic conductivity is lower than the horizontal hydraulic conductivity ($K_z/K_x<1$) due to stratification. Thus, the proposed model can lead to a more rational design by accounting for the vertical flow and the associated head loss. Dimensionless curves for graphical designs have been generated by a systematic operation of the model. These curves along with Kraijenhoff solution permit graphical estimation of steady state rise of water table (neglecting the flows above the initial drain level) within a practical range of the geometric dimensions and the parameters. The curves yield higher water table elevation as compared to the steady state Kraijenhoff solution. However, the error in the Kraijenhoff solution is found to be low in case the soil is nearly isotropic, the ditches penetrate more than 75% saturated thickness and the ditch spacing is at least 15 times the ditch penetration below drain level.

The saturated flow model explained above requires the estimates of recharge at the water table as input data. However, the recharge is not a directly measurable quantity and, thus, has to be estimated from the data of inputs (i.e., infiltration from rainfall, applied irrigation etc.) at the ground surface. This may introduce some errors. The transfer of infiltrated water through the unsaturated zone and its subsequent flow towards the drains has been accounted for in the total response model, by considering the two-dimensional flow in the entire unsaturated-saturated domain extending from the ground surface to the lower impervious layer. The total response model also accounts for evapotranspiration. The soil water properties of the unsaturated zone are generated as part of the model solution by using Brooks and Corey (1964) relation.

The saturated flow and the total response models have been validated by comparing the computed water table rise (by the two models) with the corresponding field data reported by Chhedi Lal (1986). The experimental field a research station of Soil Salinity Research Institute, Karnal, is located at Sampla village (longitude $28^{O}46'$N, latitude $76^{O}46'$E) in Rohtak district of Haryana state (India). The area suffers with salinity and waterlogging. A subsurface drainage system laid out in a 10 hectare plot, to reclaim the highly saline land, consisted of three tile drain spacings of 25,50 and 75m, buried at an average depth of 1.75m. The soils are mainly loamy sand and sandy loam in texture. The water table rise computed by the saturated flow model is in good agreement with observed water table rise. As expected, the total response model results are in still better agreement with the observations.

In the presence of an impeding layer in the unsaturated zone, the percolating water tends to accumulate over the layer. This raises the soil moisture and ultimately may lead to the development of saturated conditions (generally termed as perched water table) over the impeding layer. This may initiate lateral flow towards the drain through the perched water table. Such flow is termed as throughflow. The drainage systems involving throughflow can not be analysed by the saturated flow model. The subsurface drainage system in such cases, can be designed by using the total response model. A typical example of throughflow development has been illustrated by model operation for a domain consisting of a clay layer in the unsatuarated zone and uniform loam soil above and below the clay layer. A few dimensionless curves to show the dependence of throughflow on the clay thickness and partial penetration have also been presented.

Application of the total response model in studying the bank storage development and its subsequent release to a drain has also been demonstrated, by passing an assumed stage hydrograph of 7 days duration through a ditch. In this particular case, it is found that in the absence of any infiltration from ground surface, about 60% of the bank storage is released to the ditch within a comparatively short period (20 days in the case considered), but the rest 40% release is expected to take a very very long time (as the gradient becomes insignificant after 60% release and it sill goes on decreasing). The case has also been analysed considering infiltration from the ground surface.

CHAPTER-II

LITERATURE REVIEW

2.1 DEVELOPMENT OF SUBSURFACE DRAINAGE AND A REVIEW OF FIELD EXPERIMENTS CONDUCTED IN INDIA

In 1865, for the first time the government of Punjab drew the attention of the Governor General to the problem of 'reh' and 'usur' (saline soils) in the canal commands. In 1876, the problem of soil salinity was reported from Uttar Pradesh and from Nira Canal Command in Maharashtra. In 1937, a Technical Report No.56 compiling observations and design techniques was submitted to Maharashtra Government (still referred to for drainage design in the state). A subsurface drainage research institute functioned at Ibban (now in Pakistan). In 1928, the Royal Commission on Agriculture mentioned, "Now lessons have been learnt and in all future irrigation projects drainage will form an essential component". In 1972, The Second National Irrigation Commission on Agriculture emphasized the concept of irrigation and drainage to go together. The agricultural scientists realized the importance of subsurface drainage in reclaiming the waterlogged and saline land, and various small scale experiments were carried out in parts of India (Table 2.1 and 2.2). This led to the establishment of Command Area Development Authorities for several projects. Thus, the usefulness of subsurface drainage has been proved and efforts are underway for provision of subsurface drainage network in more irrigation commands having problems of waterlogging and salinity.

TABLE 2.1: REPRESENTATIVE SUBSURFACE DRAINAGE EXPERIMENTS FOR WATERLOGGED/SALINE SOILS

Location	Type of drains	Major Conclusion	Reference
Manjri (Maharashtra)	Random combination of open and tile drains	Increased crop yield	R.P.Talati (1941)
Ludhiana (Punjab)	Interceptor and combination of open and tile drains	Significant increase in Kharif (maize), no signifícnat increase in Rabi (wheat)	Michael (1967)
Digod (Rajasthan)	Mole, plastic and asbestos drains	Mole drains are cheap and consume less time during installation.	Lovas (1972)
Sirugupa (Karnataka)	Open drains	Significant increase in crop yield over undrained areas.	Channabasiah (1972)
IARI (New Delhi)	Open drains	Significant increase in Sorghum and Wheat yield	Yadav (1975)
Indore (M.P.)	Tile drains	Significant increase in crop yield. Yield decreases as spacing increases	Yadav (1975)

TABLE 2.2: SUBSURFACE DRAINAGE WORKS AT VARIOUS CENTRES IN INDIA

Place, State reference	Soil type	Spacing m	Depth m	Type of drains	Major Conclusions
Karnal (Haryana), Jaiswal and Dhruva Narayana (1972).	Sandy/ Loam (Alkali)	10.30	1.5	Tile & Open drains	Horizontal drainage not desirable for alkali soils
Sampla (Haryana), S.K.Gupta (1979).	Sandy-loam Sandy-loam	20.0 50.0	1.5 1.5	Open drains Tile drains	Increased crop yield & reduced salinity, favourable salt and water balance
Sampla (Haryana), Rao and Pandey (1982).	Sandy-loam	25,50,75	1.75	Tile drains (cement concrete)	
Kailana Khas (Haryana), O.P.Singh (1982).	Sandy/ loam	58.5	1.5	Open drains	Increase crop yields, favourable salt and water balance
Canning(W.B.) Rao and Kamra (1984).	Silty-clay-loam	15-45	1.75	Open drains	15m spacing is beneficial for leaching and reducing resalinization
Parbhani (Mahrashtra), Holsambre et al (1982).	Vertisols (Clay soils)	13	1.5	Brick, stone and tile drains	under irrigated conditions net profit of Rs.6651 per hectare is envisaged.

Contd.Table 2.2

Place,State reference	Soil type	Spacing m	Depth m	Type of drains	Major Conclusions
Bidaj (Gujarat) Dhruva Narayana et al (1981)	Clay soils	15-25	1.50	Open drains	Overall improvement in water-logged & salinity situation

2.2 THEORIES OF SUBSURFACE DRAINAGE-A REVIEW

Most of the theories in practice for artificial horizontal subsurface drainage are based on Dupuit-Forchheimer (D.F.) assumptions. In these theories, loss of hydraulic head is ignored in the vertical direction and, accordingly, convergence loss and surfaces of seepage are ignored. The design of a drainage system requires knowledge of the requirement of the crops to be grown, evaluation of the appropriate soil properties, and incorporation of these data into the methodology adopted (analytical/numerical) for the determination of the proper depth and spacing of drains. Though, the input and calculation requirement for the analytical solutions is less, but the unsafe drainage design (especially in anisotropic soils) by these, poor effectivity in removing waterlogging and salinity; and greater demand for water, requiring most efficient water management; have necessitated the use of more exact methods. This has been possible by adopting the numerical methods. In this section, a brief literature review of the analytical and numerical solutions for solving steady state as well as non-steady state drainage problems is presented.

2.2.1 Steady State Theories

The steady state theories are based on the assumption that the recharge intensity equals the drain discharge rate and consequently that the water table remains in position.

2.2.1.1 Analytical Solutions

Hooghoudt (1936) and Donnan (1946) derived identical solution for one dimensional flow to parallel fully penetrating ditches resulting from uniform vertical recharge at the water table. According to this solution the spacing of ditches is given by the following equation:

$$L^2 = \frac{4K\Delta h\ (2Y_0+\Delta h)}{R} \qquad \ldots (2.1)$$

where, R is the recharge rate per unit surface area, K is the hydraulic conductivity of the soil, Y_0 is the thickness of aquifer below drain level, L is the drain spacing, and Δh is the rise of water table at midsection.

Hooghoudt (1940) accounted the radial flow empirically by considering two regions of flow i.e., radial flow in the vicinity of the pipe drain and horizontal flow away from it. He also introduced the concept of equivalent depth to transform a combination of horizontal and radial flow into an equivalent horizontal flow. His solution can be expressed as follows:

$$\Delta h = \frac{RL}{K} F_H \qquad \ldots (2.2)$$

and

$$F_H = \frac{L - Y_0\sqrt{2}}{8Y_0 L} + \frac{1}{\pi} \ln \frac{Y_0}{r_0\sqrt{2}} \qquad \ldots (2.3)$$

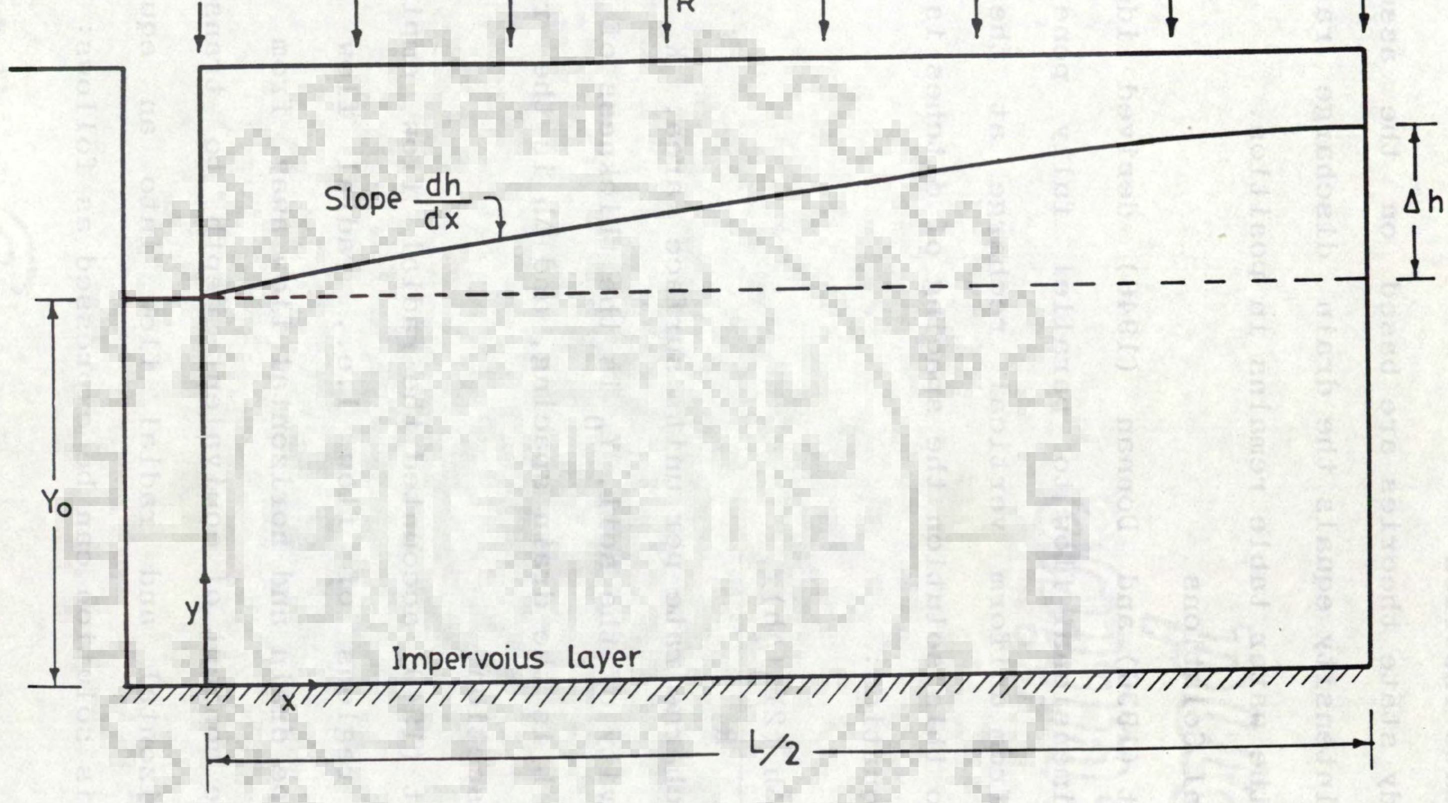


FIG. 2·1 GEOMETRY OF THE DONNAN (1946) MODEL FOR DRAINAGE BY FULLY PENETRATING PARALLEL DITCHES.

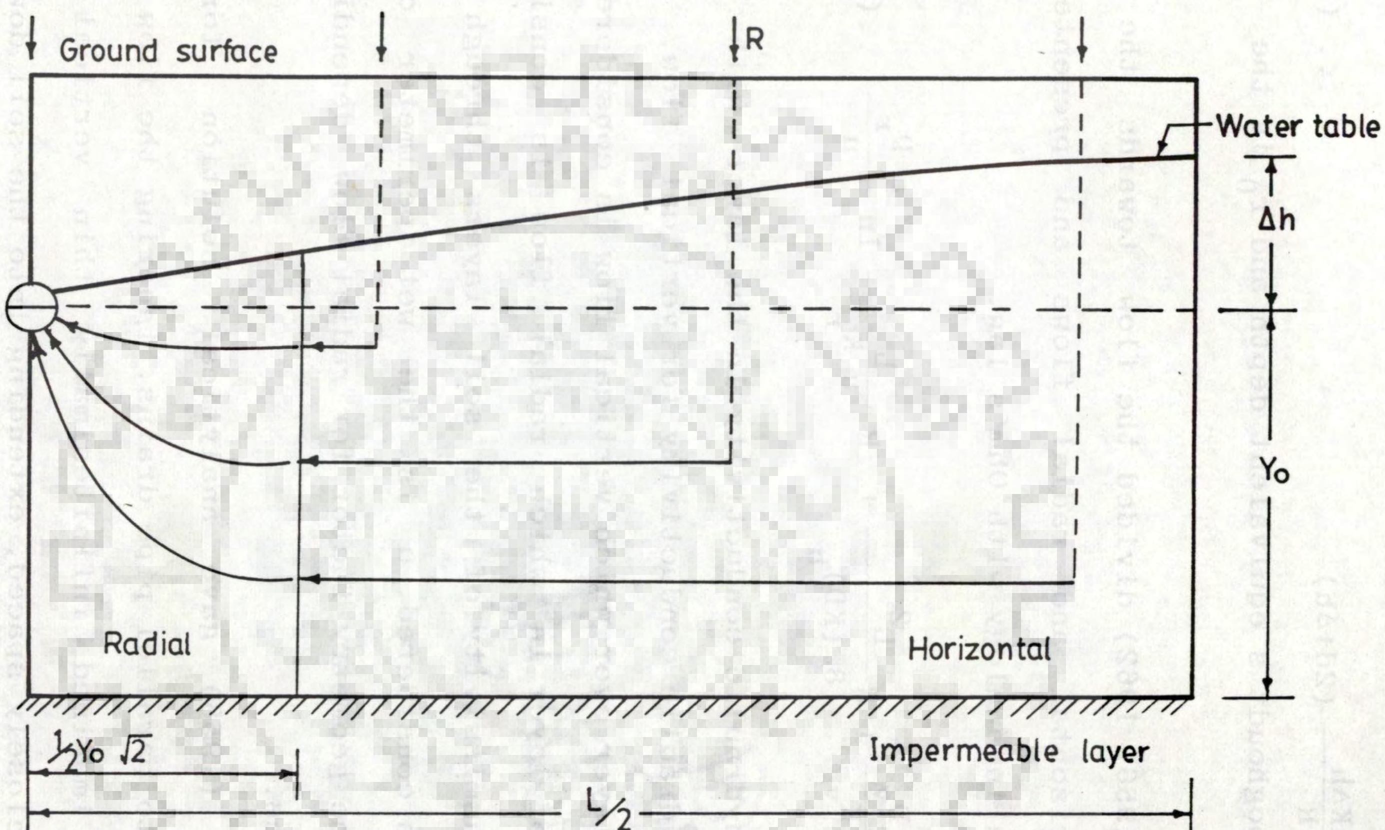


FIG. 2·2 GEOMETRY OF THE HOOGHOUDT'S (1940) MODEL ACCOUNTING CONVERGENCE NEAR THE DRAIN.

By equivalent depth concept

$$L^2 = \frac{4K\Delta h}{R}(2d+\Delta h) \quad \ldots (2.4)$$

where, d is the Hooghoudt's equivalent depth and r_0 is the radius of the drains.

Ernst (1956, 1962) divided the flow towards the drain into vertical, horizontal and radial flows and presented the following solution in analogy with Ohm's law.

$$\Delta h = R\frac{D_v}{K_v} + R\frac{L^2}{8\Sigma(KD)_h} + R\frac{L}{\pi K_r}\ln\frac{a D_r}{u} \quad ..(2.5)$$

where, K_r is the hydraulic conductivity in the layer with radial flow, K_v is the hydraulic conductivity for vertical flow, D_v is the thickness of layer over which vertical flow is considered, D_r is the thickness of layer in which radial flow is considered, $\Sigma(KD)_h$ is the transmissivity of the soil layers through which horizontal flow is considered, u is the wet perimeter of the drain, and a is the geometry factor for radial flow depending on the flow conditions.

Kirkham (1958) gave analytical solution for two dimensional flow to parallel pipe drains, ignoring the flow above drain level. He imagined infinitesimally thin vertical plane parallel strips, closely spaced, extending into the soil down to a horizontal plane passing through the lowest points of the water table. The strips forced the flow to be rectilinear in the region of water table arch. To ignore the head loss in the arched region he imagined gravel to be placed. He expressed the water table rise at midsection by the following equation.

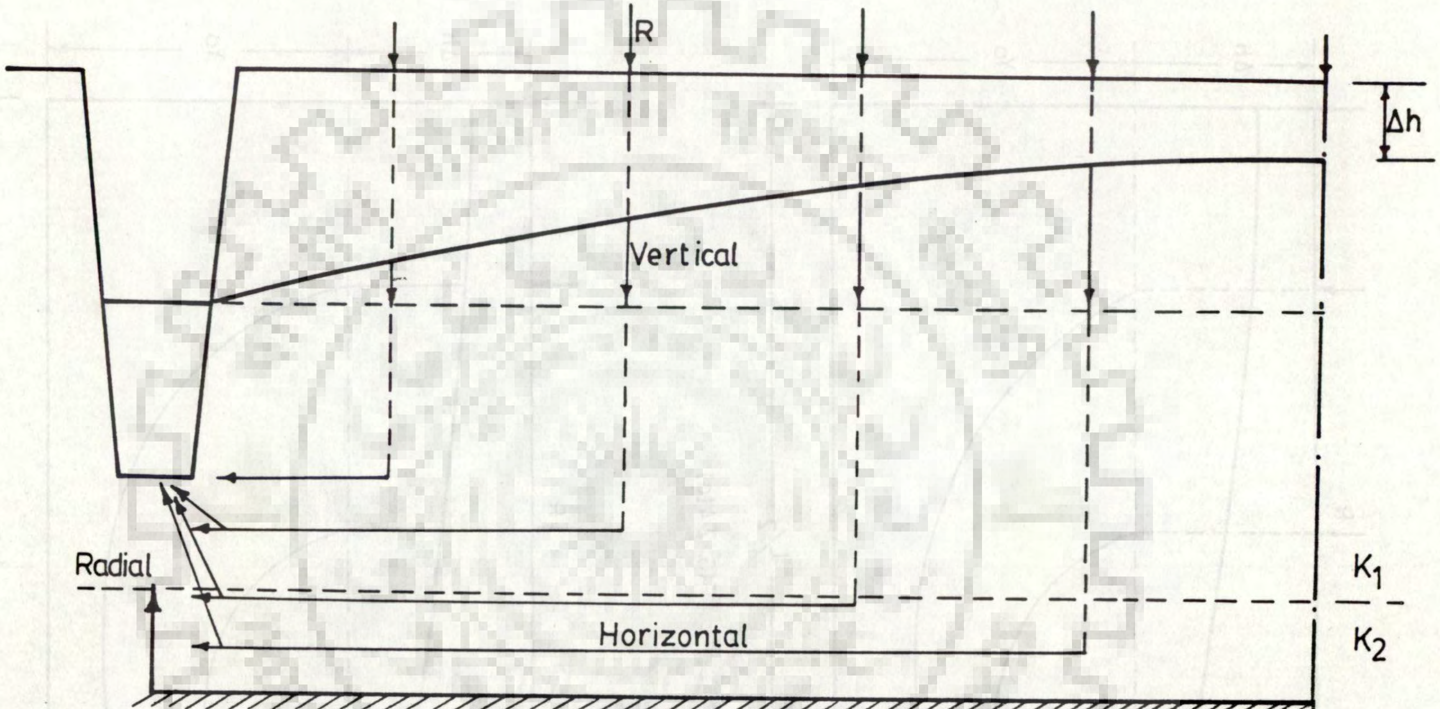


FIG. 2·3 GEOMETRY OF THE EARNST (1962) MODEL FOR FLOW TOWARDS DRAINS.

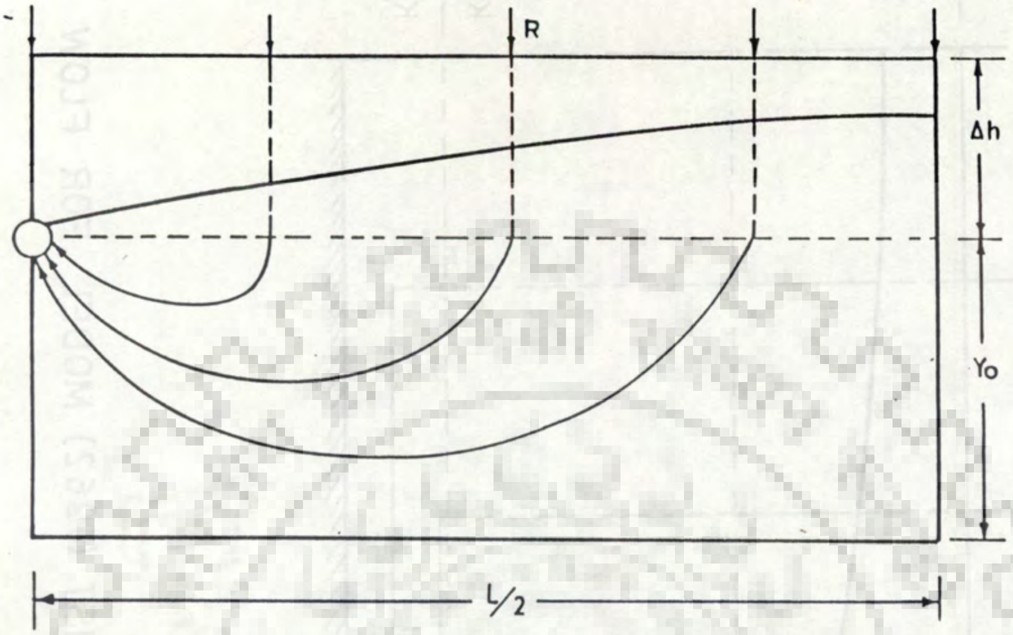


FIG. 2·4(a)

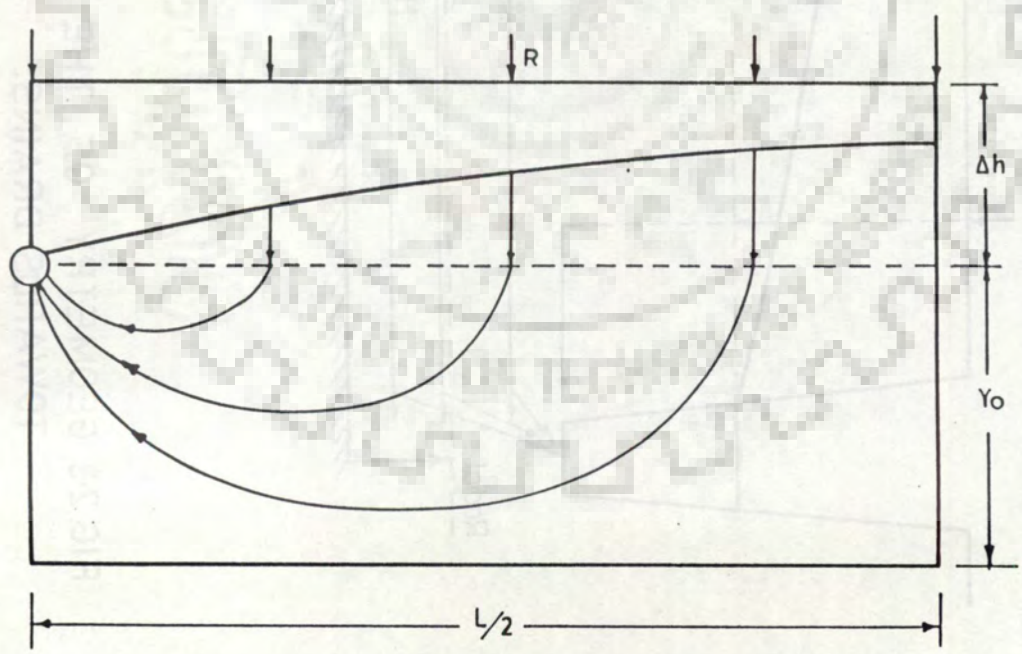


FIG. 2·4(b)

FIG 2·4(a) GEOMETRY OF THE KIRKHAM'S (1958) MODEL ACCOUNTING TWO-DIMENSIONAL FLOW BELOW DRAIN LEVEL (ISOTROPIC SOILS)

(b) KIRKHAM'S (1960) MODEL ACCOUNTING VERTICAL FLOW IN THE ARCHED REGION AND TWO-DIMENSIONAL FLOW BELOW DRAIN LEVEL (ISOTROPIC SOILS.)

$$\Delta h = \frac{RL}{K} F_K \qquad (2.6)$$

and

$$F_K = \frac{1}{\pi} \left[\ln \frac{L}{\pi r_0} + \sum_{n=1} \frac{1}{n} \left(\text{Cos} \frac{2n\pi r_0}{L} - \text{Cos} n\pi \right) \left(\text{Coth} \frac{2n\pi Y_0}{L} - 1 \right) \right] \qquad (2.7)$$

Kirkham (1960) modified his earlier solution, considering vertical flow in the arched region. Toksoz and Kirkham (1961) extended Kirkham's work and prepared practical nomographs for drainage design. Hammad (1963) obtained a function that satisfied approximately the free surface condition and gave an approximately uniform rain over this surface. In his model, the rain flux was not exactly uniformly distributed in the x-direction over the free surface. His solution can be expressed as follows:

$$\phi = \frac{Q}{4\pi K} \ln \left((\text{Sin} \frac{2\pi x}{L} \text{Cosh} \frac{2\pi y}{L} + \text{Cosh} \frac{2\pi D^2}{L} + \text{Cos}^2 \frac{2\pi x}{L} \text{Sinh}^2 \frac{2\pi y}{L} \right) + C' \qquad \ldots(2.8)$$

where, $\phi(=\eta + D + h)$ is the hydraulic head referred to the level of barrier; $y = \eta + D + h$, at free surface; Q is the quantity of water entering unit length of drain per unit time; and D is the thickness of aquifer below drain level.

Wesseling (1964) carried out extensive calculations of Kirkham's theory and compared these calculations with recomputed results of Hooghoudt's model. He showed that Hooghoudt's results did not vary more than approximately 5% from those of Kirkham for midpoint water table heights and for situations in which head losses in the water table arch were neglected. He also showed that a reinterpretation of the y-axis of the Toksoz-Kirkham chart for

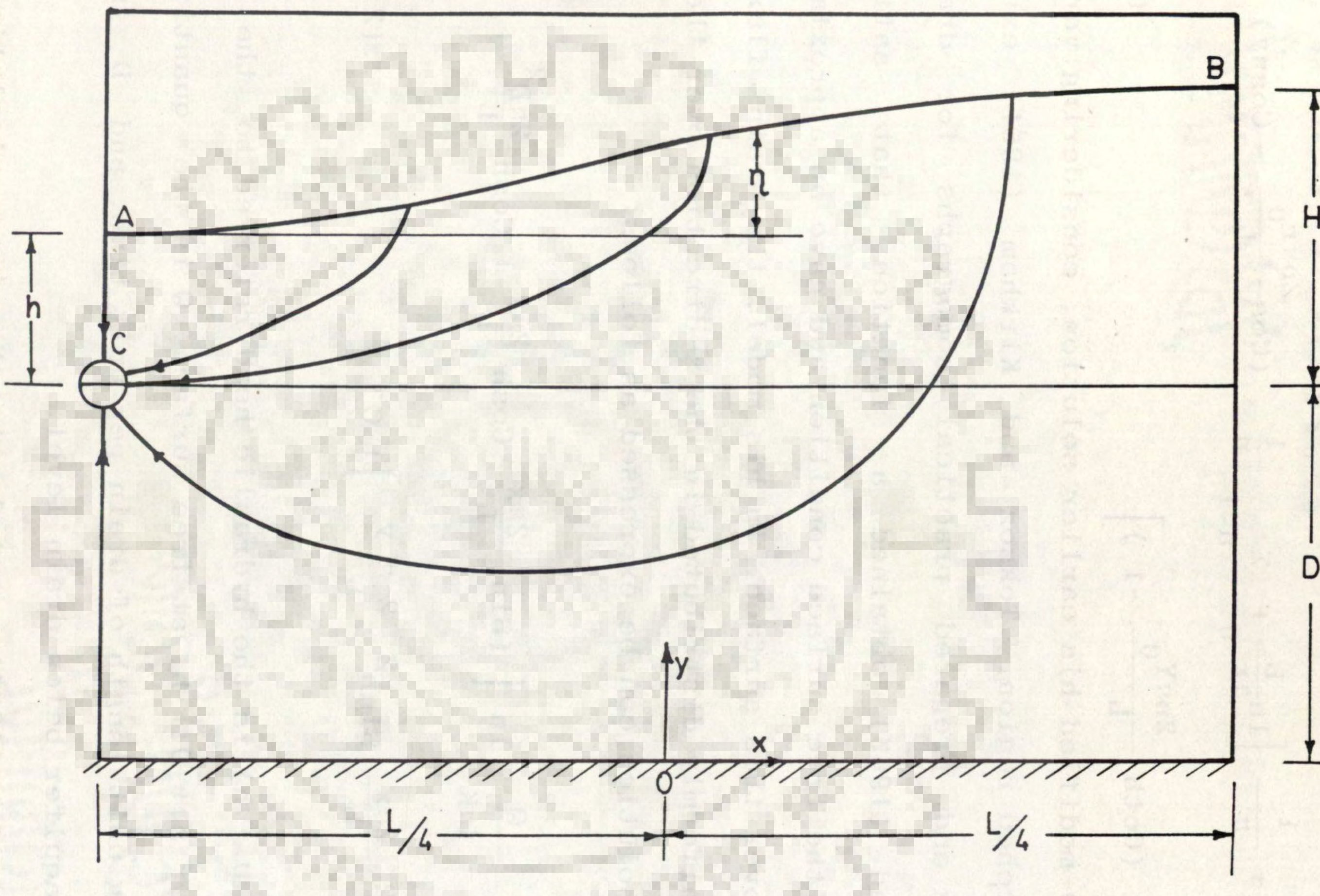


FIG. 2·5 GEOMETRY OF HAMMAD (1963) MODEL.

tile or ditch drainage of rainfall allowed the hydraulic conductivity of the soil in the water table arch to be accounted for when this conductivity differed from that of the rest of the soil.

Dagan (1964) imagined two-part flow region. In one part the boundary condition was linearized and in the other part horizontal flow was assumed. Dagan combined a line sink, confined flow potential with a potential to give a uniform steady rain. The potential for the uniform steady rain yielded a uniform rain along a horizontal line passing through the lowest points of the water table arch. This consideration of uniform rain along a horizontal line was termed `linearization´ as only the linear term of a power series was used in the analysis. Dagan´s solution is given by the following equation:

$$\Delta h = \frac{RL}{K} F_D \qquad \ldots(2.9)$$

and

$$F_D = \frac{1}{4}\left(\frac{L}{2D} - \beta\right) \qquad \ldots(2.10)$$

where

$$\beta = \frac{2}{\pi} \ln \left(2 \operatorname{Cosh} \frac{\pi r_0}{y_0} - 2\right) \qquad \ldots(2.11)$$

List (1964) assumed the tile drains as line sinks and considered the boundary conditions exactly, except the impermeable boundary. He obtained the impermeable boundary as an undulating surface rather than a horizontal plane. The undulating plane occurred in List´s theory because he used a quasi-image method in which a linear flux feeding a drain originated at a finite depth above the real drains, while for image drains the flux originated

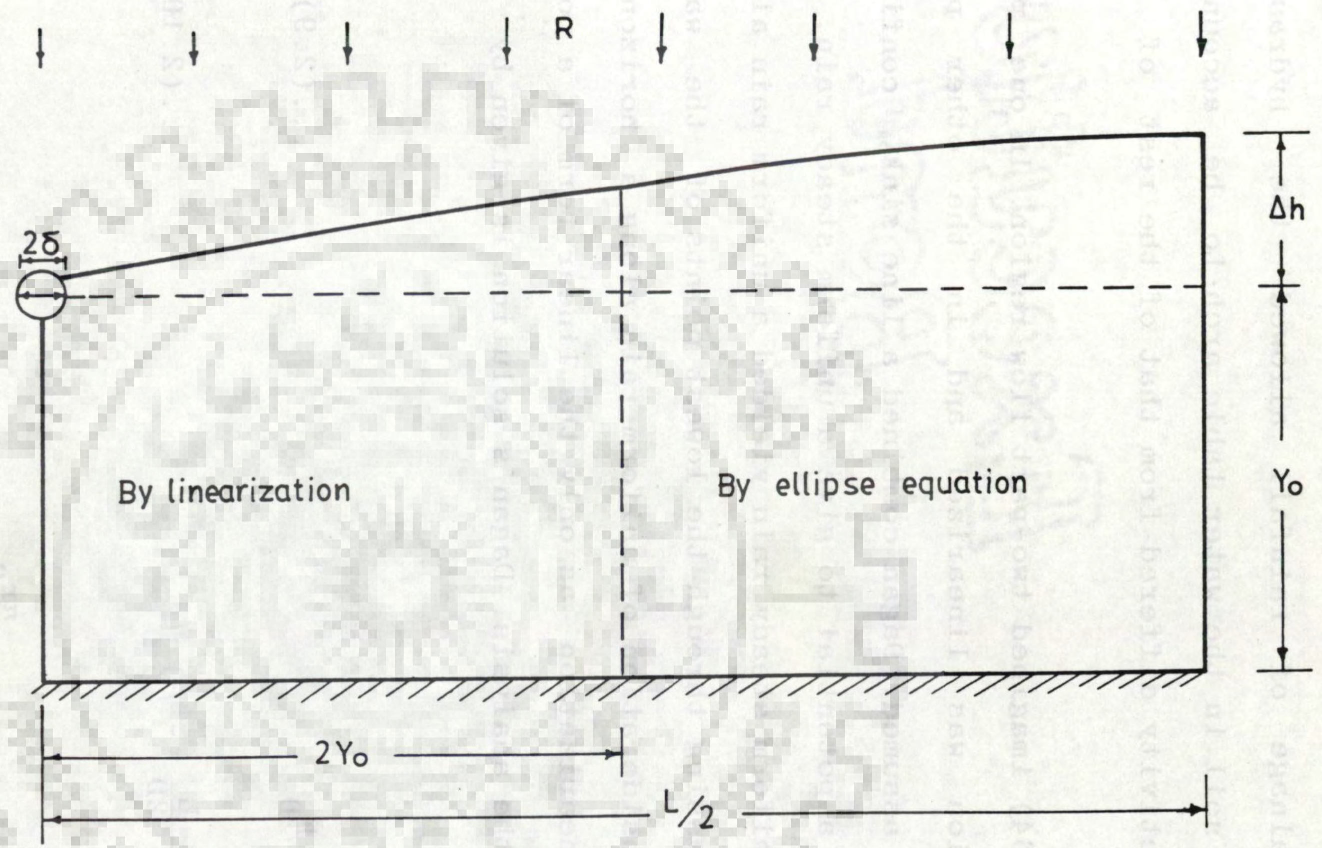
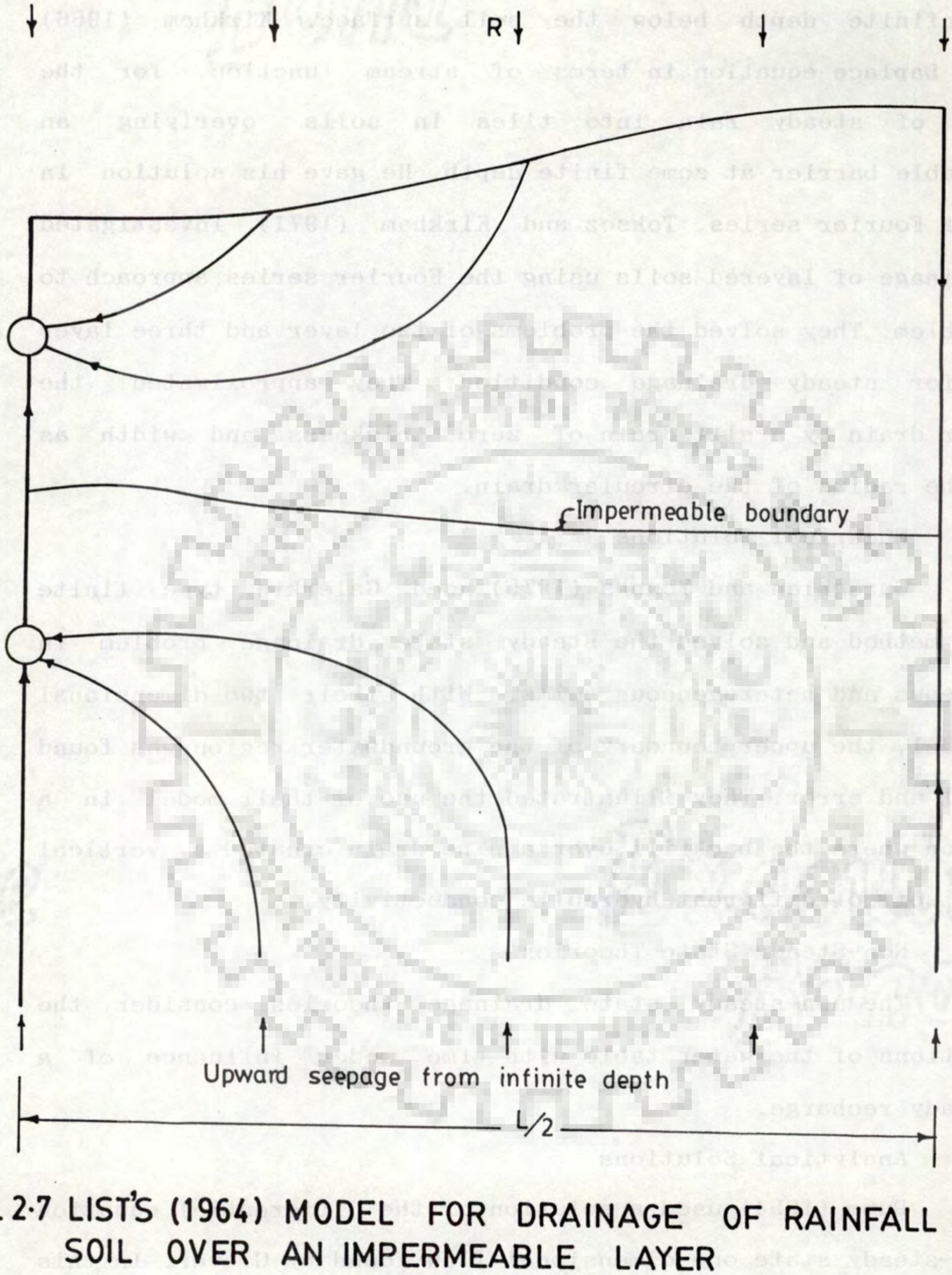


FIG. 2·6 GEOMETRY OF THE DAGAN'S (1964) MODEL FOR SUBSURFACE DRAINAGE.



IG. 2·7 LIST'S (1964) MODEL FOR DRAINAGE OF RAINFALL IN SOIL OVER AN IMPERMEABLE LAYER at an infinite depth below the soil surface. Kirkham (1966) solved, Laplace equation in terms of stream function, for the seepage of steady rain into tiles in soils overlying an impermeable barrier at some finite depth. He gave his solution in terms of Fourier series. Toksoz and Kirkham (1971) investigated the drainage of layered soils using the Fourier series approach to the problem. They solved the problems of two layer and three layer soils for steady drainage condition. They approximated the circular drain by a slit drain of zero thickness and width as twice the radius of the circular drain.

2.2.1.2 Numerical Solutions

Gureghian and Youngs (1975) used Galerkin type finite element method and solved the steady state drainage problem in homogeneous and heterogeneous soils. With their two-dimensional flow model, the upper boundary of the groundwater region was found by trial and error. They illustrated the use of their model in a situation where the backfill over a pipe drain created a vertical band of soil of different hydraulic conductivity.

2.2.2 Non-Steady State Theories

The non-steady state drainage theories consider the fluctuations of the water table with time under influence of a non-steady recharge.

2.2.2.1 Analytical Solutions

Dumm (1954) used a solution of the differential equation for non-steady state one-dimensional flow found by Glover. In this theory, the initial horizontal water table was considered the result of an instantaneous rise caused by rainfall or irrigation (recharging the groundwater instantaneously). He gave the following solution for spacing of ditches:

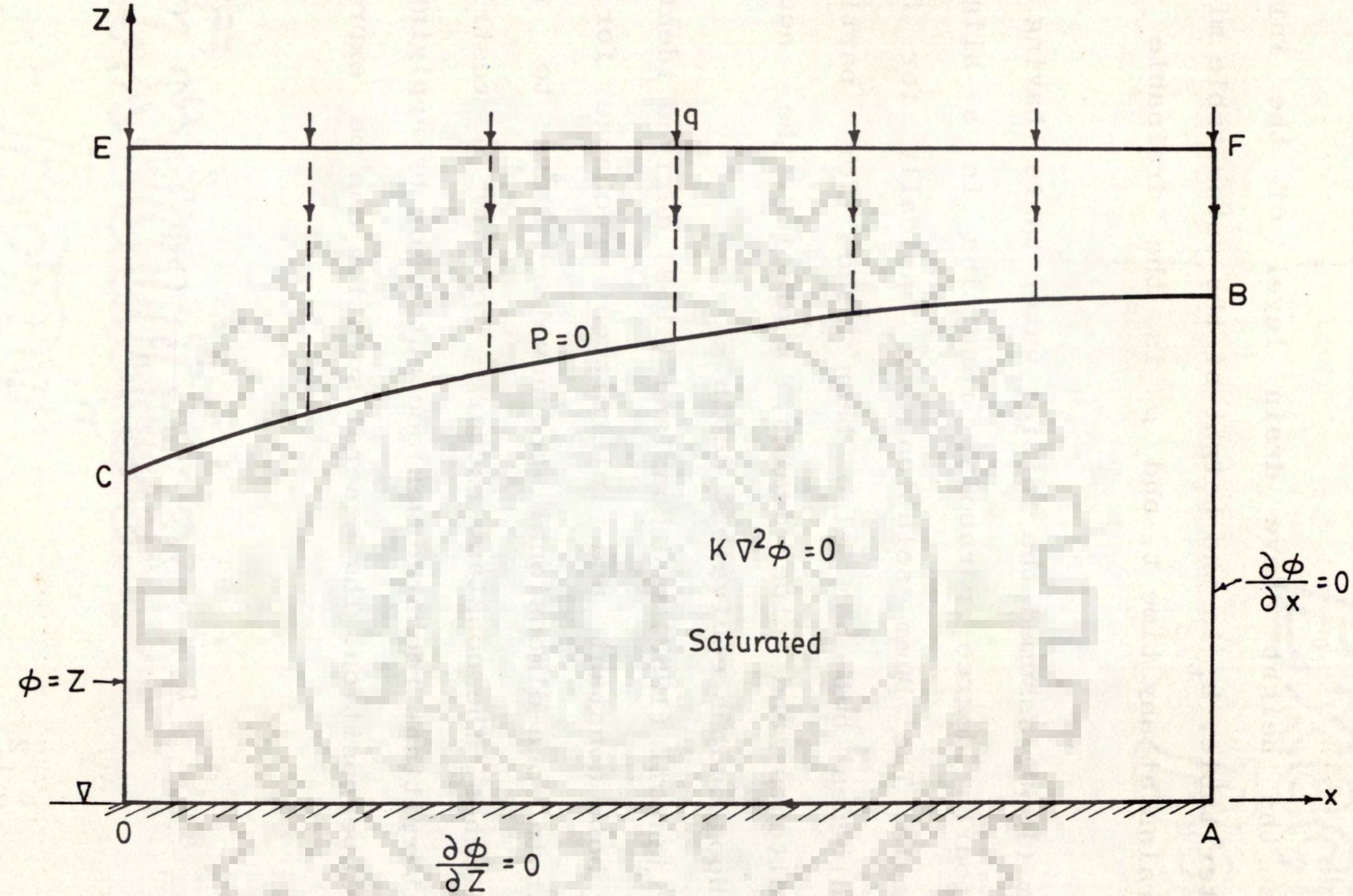


FIG. 2·8 GEOMETRY OF GUREGHIAN AND YOUNGS (1975) FINITE ELEMENT MODEL OF STEADY STATE TWO-DIMENSIONAL DITCH DRAINAGE.

$$L = \pi \left[\frac{K\, Y_0\, t}{\mu} \right] \left[\ln 1.27 \frac{h_0}{h_t} \right] \qquad \ldots (2.12)$$

where, h_0 is the height above drain level of the initial horizontal water table, h_t is the height of the water table midway between the drains at any time t, and μ is the drainable pore space.

Dumm (1960) assumed the initial water table having the shape of a fourth degree parabola, resulting in a slightly different formula. The Dumm solution was basically for fully penetrating ditches but has also been used for partially penetrating ditches and pipe drains, replacing the aquifer thickness by Hooghoudt's equivalent depth.

Kraijenhoff (1958) and Maasland (1959) derived identical solutions for both constant and intermittent recharge for the non-steady state one-dimensional groundwater flow to fully penetrating ditches. The constant recharge for only a restricted period and intermittent recharge was accounted for by applying the principle of superposition. The general solution can be expressed as follows:

$$h_t = \frac{4}{\pi} \frac{R}{\mu} j \sum_{n=1,-3,5\ldots} \frac{1}{n^3} (1-e^{-n^2 t/j}) \qquad (2.13)$$

where:

$$j = \frac{\mu L^2}{\pi^2 K Y_0} \qquad (2.14)$$

Jan Van Schilfgaarde (1963) presented a solution for design of a drainage system based on a specified rate of drop of the water table. The theory accounts for varying thickness of

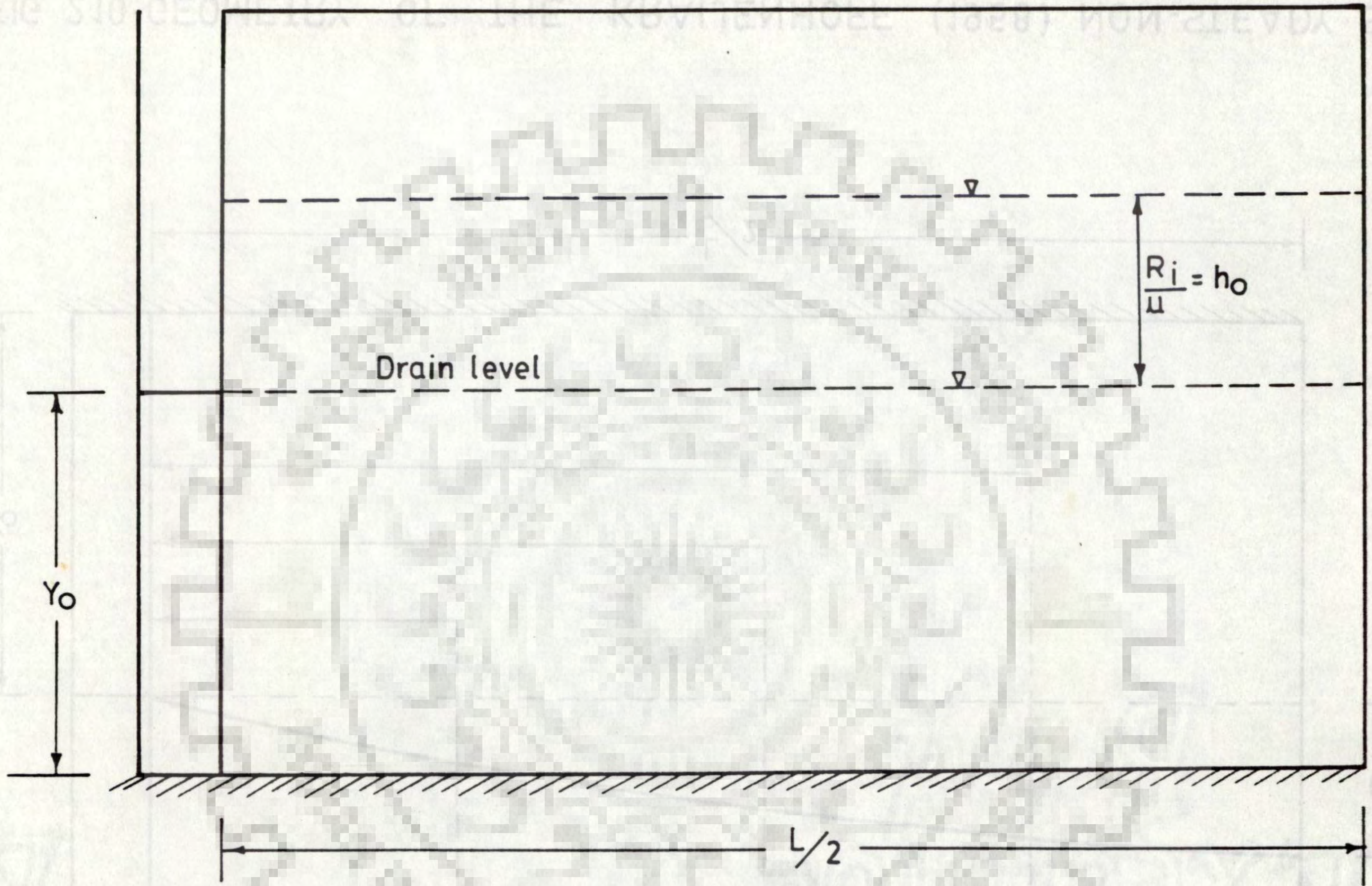


FIG. 2·9 GEOMETRY OF DUMM (1954) NON-STEADY STATE DRAINAGE MODEL.

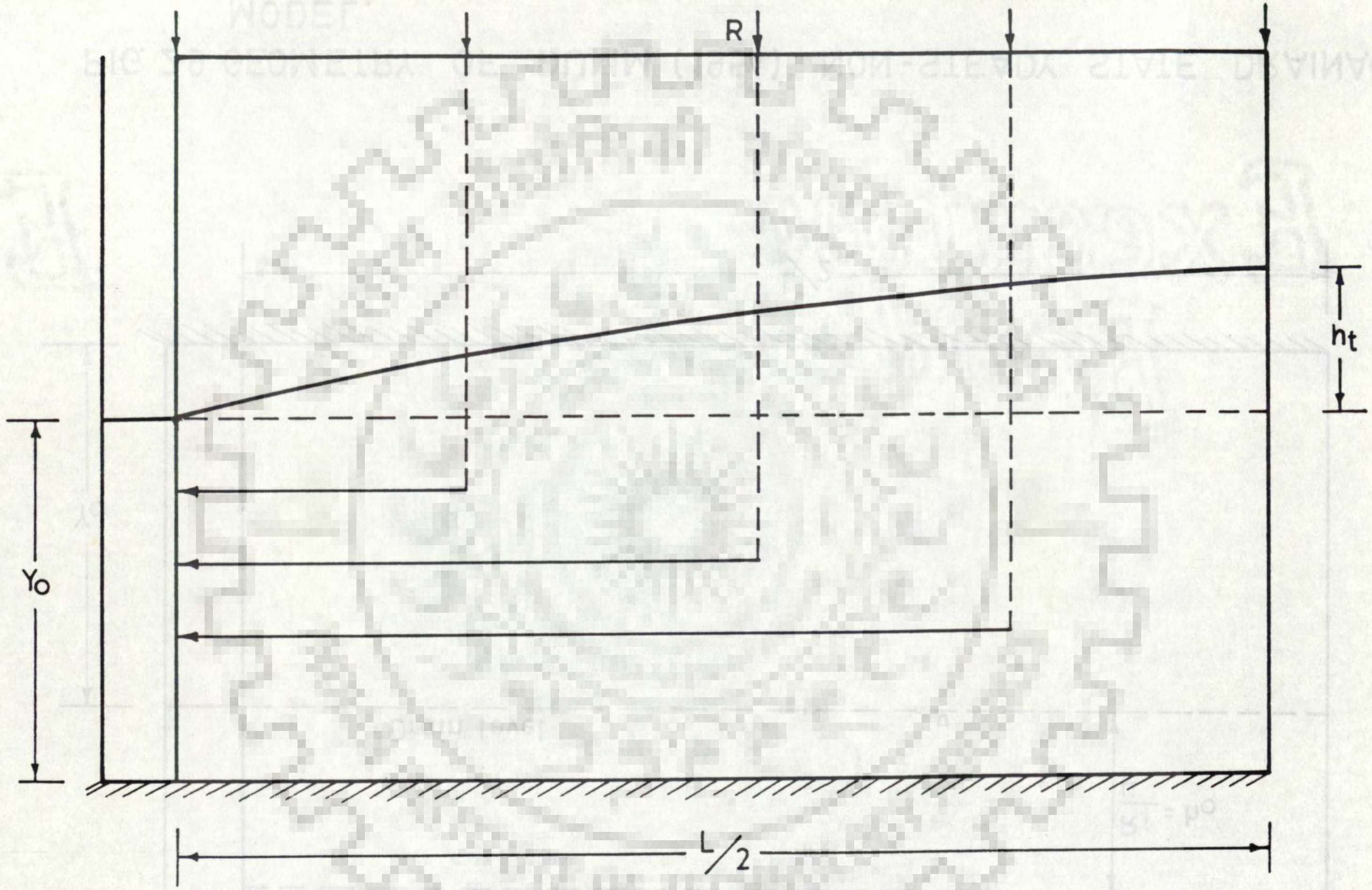


FIG. 2·10 GEOMETRY OF THE KRAIJENHOFF (1958) NON-STEADY STATE DRAINAGE MODEL.

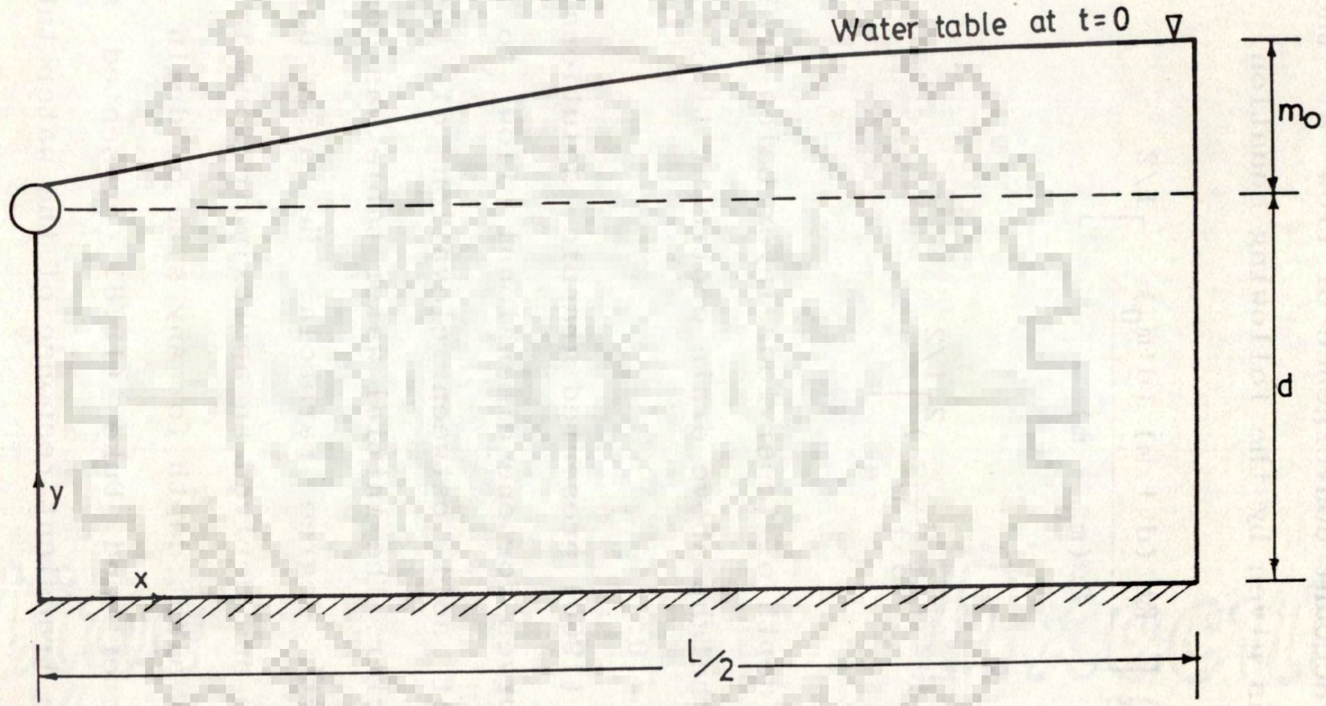


FIG. 2·11 GEOMETRY OF VAN SCHILFGAARDE'S (1963) MODEL water bearing stratum resulting from falling water table. He suggested the use of Hooghoudt's equivalent depth to replace the actual depth to account convergence of flow at the drains. The drain spacing was given by the following equation.

$$L = 3A \left[\frac{Kt\ (d + m)\ (d+m_0)}{2\mu(m_0 - m)} \right]^{1/2} \tag{2.15}$$

and

$$A = \left[1 - \left(\frac{d}{d + m_0} \right)^2 \right]^{1/2} \tag{2.16}$$

where, d is the depth of impervious layer below drain centre, m is the water table height above drain centre at time t, and m_0 is the value of m at t = 0.

Moody (1966) presented graphic solutions in terms of dimensionless curves for one dimensional flow to drains having finite depth of soil between elevation of the drains and impermeable barrier. The initial water table was considered in the shape of a fourth degree parabola. He also used Hooghoudt's equivalent depth and derived an approximate formula for computing Hooghoudt's equivalent depth for any size of drain.

McWhorter and Duke (1976) presented an analytical solution for the transient response of the water table to parallel fully penetrating ditches considering soil water above and below the water table. In this theory, the solutions of Dumm, Moody etc. were retained and used for the appropriate boundary and initial conditions and a correction was applied to account for nonlinearity due to decreasing flow depth, flow above the water table and the variation of the apparent specific yield with water table depth. Indices to the degree of importance of capillary storage, flow above the water table and nonlinearity were defined to enable one to judge the importance of the influences seperately.

2.2.2.2 Numerical Solutions

Transient numerical models considering the flows in saturated and unsaturated zones were pioneered by Rubin (1968). This was followed by many other two-dimensional applications to various specific problems (Hornberger et al (1969)), Taylor and Luthin (1969), Verma and Brutsaert, (1970)). Jeppson (1969) considered the saturated-unsaturated flow on a basin wide scale, but he restricted himself to a steady state treatment. Amerman (1969) presented finite difference solution of unsteady, two-dimensional, partially saturated porous media flow, in his Ph.D. thesis. Freeze (1971) presented a three-dimensional finite differences model for the treatment of saturated-unsaturated transient flow in small nonhomogeneous, anisotropic geologic basins.

Skaggs and Tang (1976) presented solutions to the two dimensional Richards equation for open ditch drainage using numerical method developed by Amerman and compared their solutions with solutions of the Boussinesq equation. They concluded that agreement between solutions to the Boussinesq and Richards equations can be improved by correcting the Bousinesq solutions for convergėnce near the drain and for a nonconstant drainable porosity. Tang and Skaggs (1977) modified the numerical method developed by Amerman (1969) to solve the Richards equation for ditch drainage and sub irrigation boundary conditions. They also conducted laboratory experiments using a large soil tank to test the validity of solutions to the Richards equation and Boussinesq equation for drainage and sub irrigation for a homogeneous soil. They concluded that the solutions to the Richards equation were in better agreement with experimental observations than the approximate solutions considered.

Vauclin et al.(1979) studied the transient two-dimensional water flow in relation to the recharge of a water table aquifer. Their approach was based on the physics of water transfer in the complete domain defined by the saturated and unsaturated zones of soils. They obtained experimental data in a slab of soil in which the changes of water content and water pressure occurring in the flow domain were measured throughout an artificial recharge event. Their numerical model was based on classical nonlinear parabolic equation. In the unsaturated zone the solution was obtained by using the alternating direction implicit (ADI) scheme, and in the saturated zone, where the nonlinear parabolic equation changed into a linear elliptic Laplace equation, the iterative numerical scheme was used. They found an excellent agreement between simulated and experimental results. They concluded that the problem of transient recharge of a water table aquifer can be correctly solved by considering a unified numerical treatment of unsaturated-saturated flow and that classical saturated approach was unable to determine the transfer time for water in the unsaturated zone.

Gureghian (1981) presented a two-dimensional finite element solution scheme for the saturated-unsaturated movement of water in homogeneous and nonhomogeneous aquifers drained by parallel equidistant ditches extending to an impermeable floor. Results obtained for the case of drainage with incident rainfall under steady state conditions for homogeneous and layered soils

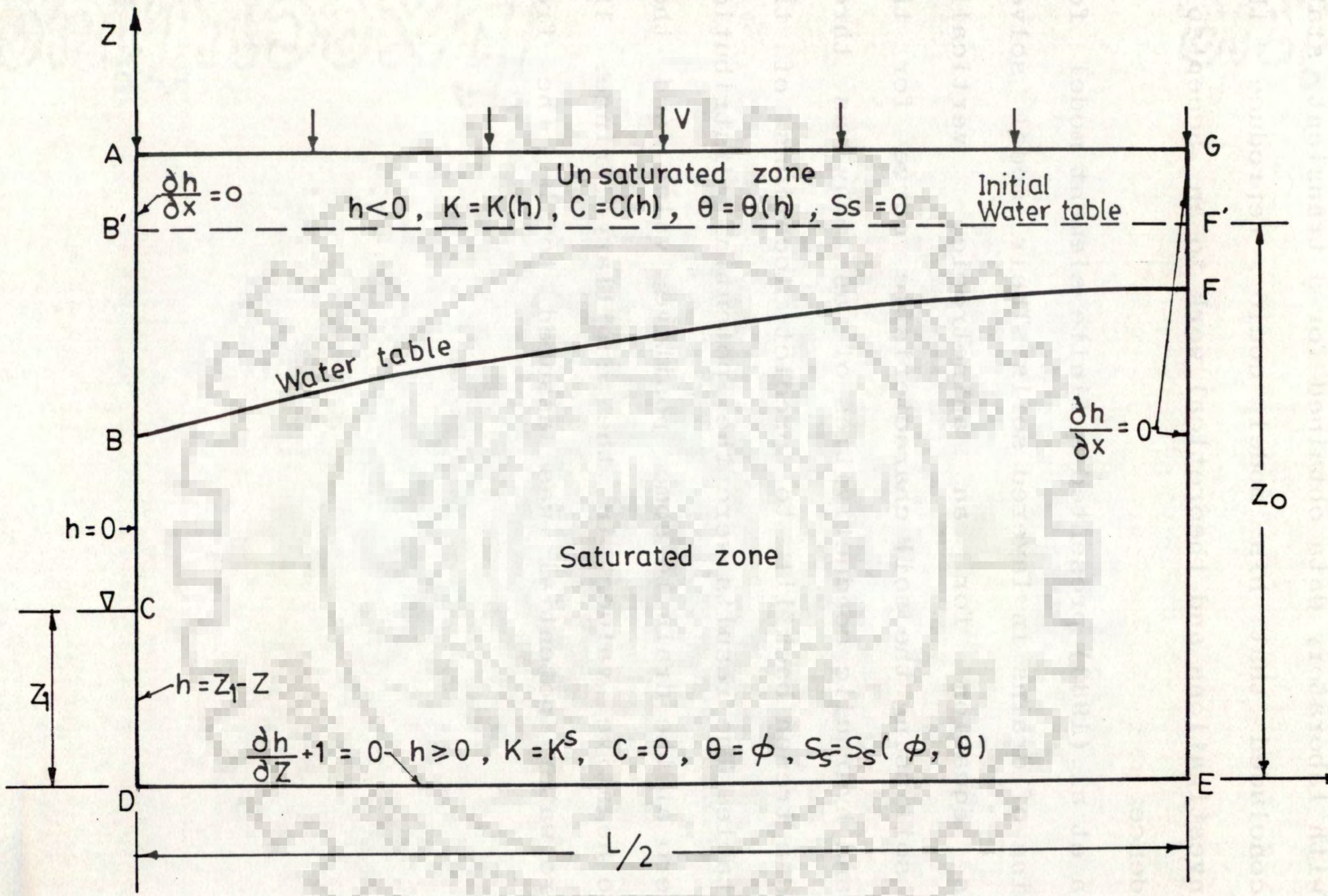


FIG. 2·12 GEOMETRY OF GUREGHIAN (1981) FINITE ELEMENT MODEL FOR SATURATED - UNSATURATED FLOWS TO DITCHES.

were compared with numerical solutions obtained in the case where only the saturated region was considered. He also compared his model results with laboratory data obtained for a transient state problem. He concluded that his model could reproduce the experimental investigations and theoretical work to an acceptable level of confidence.

Merva et al.(1983) presented a finite element model for depth and spacing of drains in layered soils. Their model solved the Laplace's equation for an anisotropic, vertically nonhomogeneous soil using the soil characteristic curve for the surface layer and hydraulic conductivities of as many as three soil layers and a trench backfill to predict locations of the falling water table. The results were the potential distribution ranging from zero at the drain to some positive value and then through zero to negative values in the capillary fringe. The position of zero water potential was assumed to be the free surface.

CHAPTER III

THE MODEL DEVELOPMENT

Two numerical models of two dimensional sub surface drainage,one analyzing only the saturated domain (saturated flow model) and another analyzing the entire unsaturated-saturated domain (total response model) have been developed .The saturated flow model (SFM) requires among others ,the time variant distribution of recharge rate at the water table as input data and yields the time variant water table position .The total response model (TRM) requires similar distribution of infiltration at ground surface and yields the spatial and temporal distribution of capillary head (h_c).This in turn yields the time variant water table position defined by $h_c = 0$.

3.1 SATURATED FLOW MODEL (NEGLECTING THE FLOW ABOVE DRAIN LEVEL)

The saturated flow model essentially involves numerical solution of the differential equation governing two-dimensional flow in an anisotropic and heterogeneous porous medium by finite differences .Initially the solution is obtained in a flow domain bounded by a horizontal impervious boundary at the bottom,the horizontal drain level at the top and two parallel drains on the sides (Fig.3.1).

3.1.1 The Flow Equation

The differential equation governing two dimensional transient flow in an anisotropic and heterogeneous porous medium in x-z plane (Bear, 1979) can be written as follows:

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right) = S_s\frac{\partial h}{\partial t} \qquad \ldots (3.1)$$

where, x(L) and z(L) are the coordinates along principal

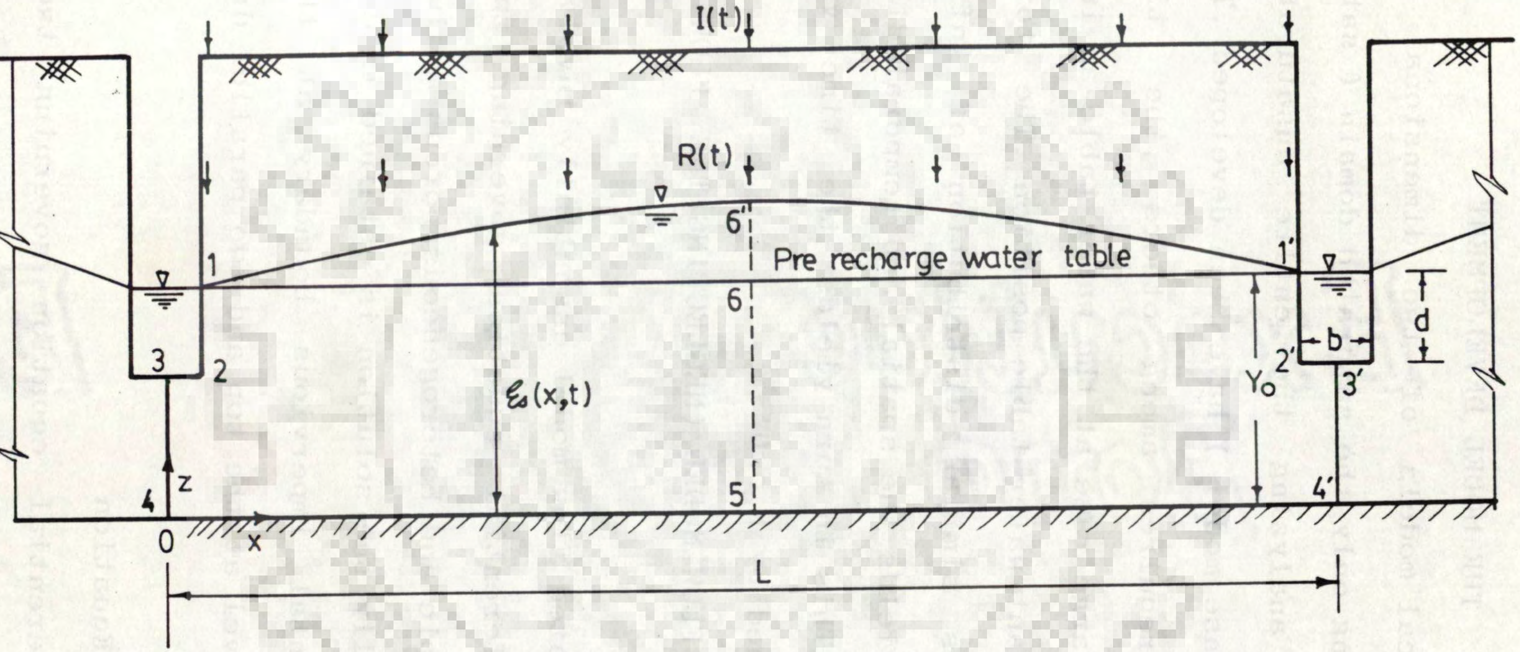


FIG. 3·1-SUBSURFACE DITCH DRAINAGE (Definition sketch) permeability directions in a vertical plane (x in horizontal direction and z in vertical direction), K_{xx} (LT^{-1}) and K_{zz}(LT^{-1}) are the hydraulic conductivities in x and z directions respectively, h (L) is the head above a datum, S_s(L^{-1}) is the specific storage. For the domain shown in Fig.3.1, the boundary and initial conditions considered are as follows:

3.1.2 Boundary Conditions

(A) Boundary 1-2-3 and 1´-2´-3´:

Boundary 1-2-3 is the boundary of the ditch/drain below drain level. Water level in the drain, and thus, the head at the drain boundary is considered to be known. At this boundary the following boundary condition is assigned.

$$h = Y_0,\ Y_0-d \le z \le Y_0,\ 0 \le x \le \frac{b}{2},\ t \ge 0 \quad \ldots (3.2a)$$

Similarly at boundary 1´-2´-3´ the following boundary condition is assigned.

$$h = Y_0,\ Y_0-d \le z \le Y_0,\ L-\frac{b}{2} \le x \le L,\ t \ge 0 \quad \ldots (3.2b)$$

(B) Boundary 3-4 and 3´-4´:

The boundary 3-4 is the vertical plane extending from the centre of the drain bottom to the impervious layer. Considering no flow across the vertical plane, the boundary condition at 3-4 is assigned as follows:

$$\frac{\partial h}{\partial x} = 0,\ 0 < z < Y_0-d,\ x = 0,\ t > 0 \quad \ldots (3.3a)$$

Similarly at boundary 3´-4´ the following boundary condition is assigned

$$\frac{\partial h}{\partial x} = 0,\ 0 < z < Y_0-d,\ x = L,\ t > 0 \quad \ldots (3.3b)$$

(c) Boundary 4-4´:

The boundary 4-4´ is the impervious boundary at a finite depth (Y_0) below drain level. Considering no flow across the impervious layer, the boundary condition is assigned as follows:

$$\frac{\partial h}{\partial z} = 0,\ z = 0,\ 0 \le x \le L\ ,\ t > 0 \qquad \ldots (3.4)$$

(D) Boundary 1-1′:

The boundary 1-1′ is the initial drain level. At this boundary the following boundary condition is assigned.

$$Kz\ \frac{\partial h}{\partial z} + S_y\ \frac{\partial h}{\partial t} = R,\ z = Y_0\ ,\frac{b}{2} \le x \le L-\frac{b}{2}\ ,t>0 \ \ldots (3.5)$$

where, $S_y = S_y(x)$ is the specific yield: $R = R(t)$ is the recharge to water table per unit time (LT^{-1}); L is the spacing of the ditches; b is the width and d is the depth of the ditch below drain level.

If the flow is symmetrical about the midsection then there would exist a water divide at the midsection and the differential equation (equation 3.1) may be solved only in half the domain ($0 \le x \le L/2$). This may lead to considerable saving in the computational efforts. For such a solution the boundary of the solution domain is 1-2-3-4-5-6-1. The boundary condition at 5-6 will be assigned as follows.

(E) Boundary 5-6:

The midsection between the drains is the boundary 5-6. Considering no flow across the midsection, the boundary condition is assigned as follows:

$$\frac{\partial h}{\partial x} = 0,\ 0 \le z \le Y_0,\ x = \frac{L}{2}\ ,\ t > 0 \qquad \ldots (3.6)$$

3.1.3 Initial Condition

Initially a horizontal water table coinciding with the initial drain level is considered. The initial condition is assigned as follows:

$$h = Y_0,\ 0 \le z \le Y_0,\ 0 \le x \le L\ ,\ t = 0 \qquad \ldots (3.7)$$

3.1.4 Finite Differences Approximation

Without any loss of generality, the finite differences solution described in the following paragraph pertains to half the

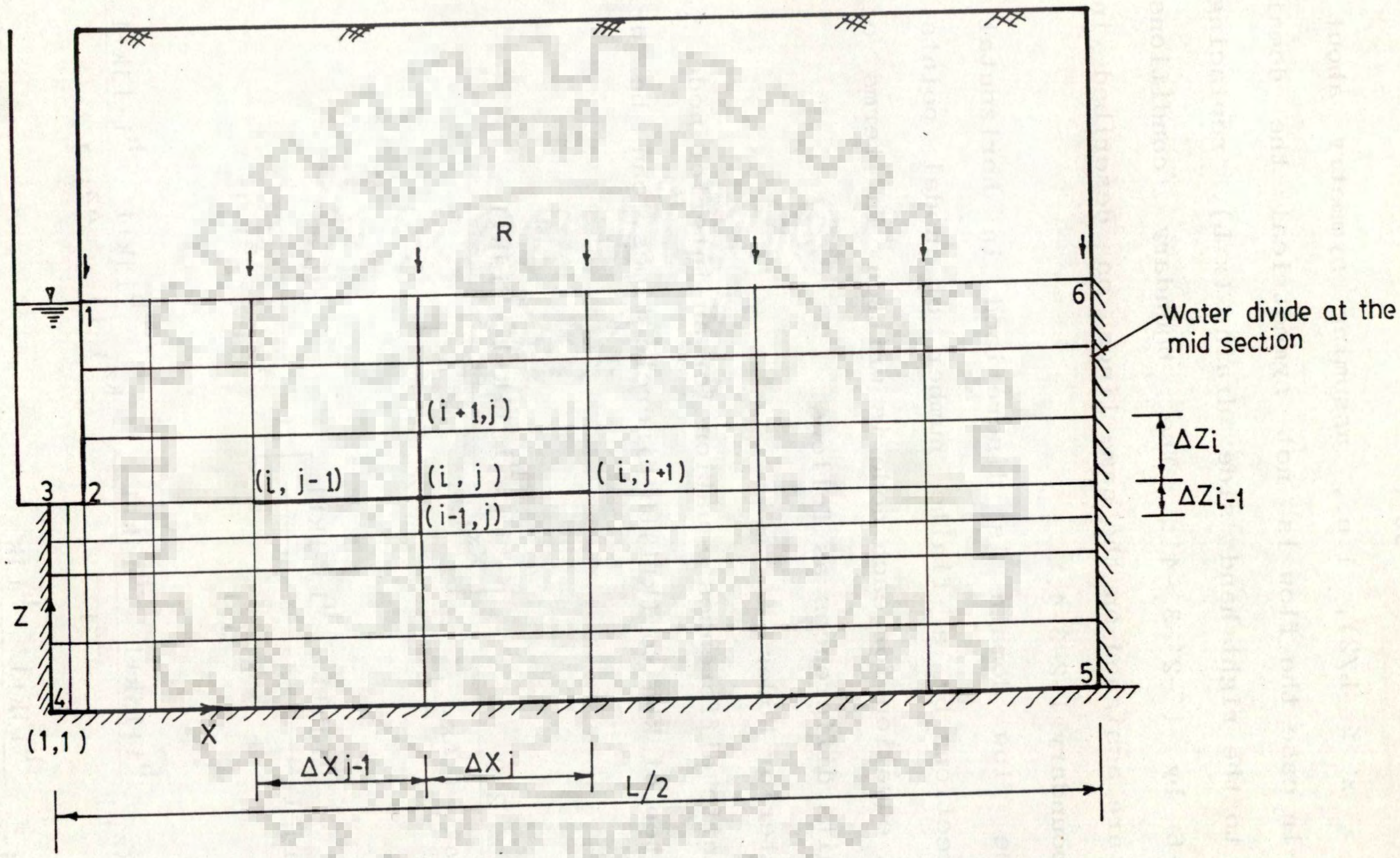


FIG.3.2- FINITE DIFFERENCES MESH AND BOUNDARIES (NEGLECTING THE FLOW ABOVE DRAIN LEVEL)

domain ($0 \le x \le L/2$), i.e., assuming symmetry about the midsection. In case the flow is not symmetrical the domain is extended up to the right hand side drain ($x=L$), replacing the boundary 5-6 by 1´-2´-3´-4´. The boundary conditions at 1´-2´-3´-4´ are assigned on the same lines as described in the context of boundary 1-2-3-4.

The flow domain is discretized in horizontal and vertical directions by a finite number of nodal points. The differential equation for each node is written in terms of the implicit finite differences as follows.

3.1.4.1 Interior Nodes

Finite differences equation for an interior node (i,j), for a time step of kth to (k+1)th discrete times, can be written as follows:

$$\frac{2.0}{\Delta x_j + \Delta x_{j-1}} \left[Kx_{ij} \frac{h_{ij+1k+1} - h_{ijk+1}}{\Delta x_j} - Kx_{ij-1} \frac{h_{ijk+1} - h_{ij-1k+1}}{\Delta x_{j-1}} \right] + \frac{2.0}{\Delta z_i + \Delta z_{i-1}} \left[Kz_{ij} \frac{h_{i+1jk+1} - h_{ijk+1}}{\Delta z_i} - Kz_{i-1j} \frac{h_{ijk+1} - h_{i-1jk+1}}{\Delta z_{i-1}} \right] = S_{sij} \frac{h_{ijk+1} - h_{ijk}}{\Delta t} \quad \ldots (3.8)$$

Where, h_{ijk+1} is the piezometric head at the nodal point (i,j); Kx_{ij} is the hydraulic conductivity of the link between nodes (i,j) and (i,j+1); Kz_{ij} is the hydraulic conductivity of the link between nodes (i,j) and (i+1,j); Δx_j is the spacing between nodes (i,j) and (i,j+1); Δz_i is the spacing between nodes (i,j) and (i+1,j); and Ss_{ij} is the specific storage at node (i,j) (refer Fig.3.2)

3.1.4.2 Boundary Nodes

Assuming symmetry and considering only the left half of the flow domain, the boundaries [Figs. 3.3(i) - (vii)] are assigned as follows:

(A) Boundary 1-2-3

Drichlet boundary condition at boundary 1-2-3 is assigned in accordance with equation (3.2)

i.e., $$h_{ijk+1} = Y_0 \qquad ...(3.9)$$

(B) Boundary 3-4

Neuman type of boundary condition in accordance with equation (3.3) is assigned ensuring water balance.

i.e., for a node (i,1) at 3-4,

$$q_1 + q_2 + q_3 = S_s \frac{\partial h}{\partial t} \frac{\Delta x_j}{2} . \frac{\Delta z_i + \Delta z_{i-1}}{2} \qquad (3.10)$$

(C) Boundary 4-5

Neuman type of boundary condition in accordance with equation (3.4) is assigned ensuring water balance.

i.e., for a node (1,j) at boundary 4-5,

$$q_1 + q_2 + q_3 = S_s \frac{\partial h}{\partial t} . \frac{\Delta x_j + \Delta x_{j-1}}{2} . \frac{\Delta z_i}{2} \qquad (3.11)$$

(D) Boundary 5-6

The vertical plane passing through the mid point between the ditches is boundary 5-6. Neuman type of boundary condition in accordance with equation (3.6) is assigned ensuring water balance.

i.e., for a node (i,j) at boundary 5-6,

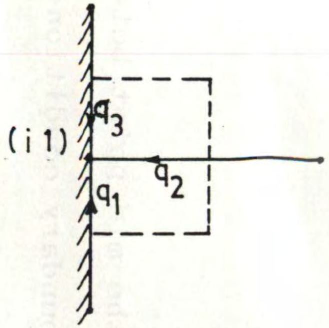


FIG 3.3 (I)
BOUNDARY 3-4

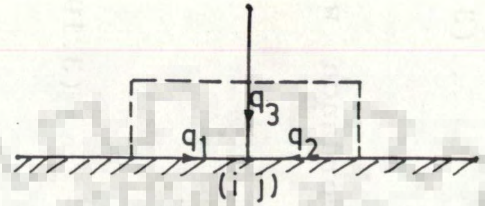


FIG 3.3 (II)
BOUNDARY 4-5

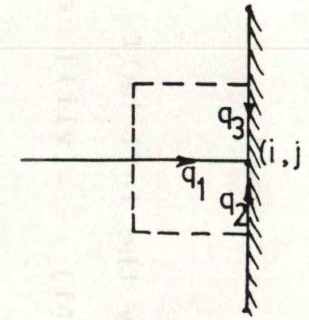


FIG 3.3 (III)
BOUNDARY 5-6

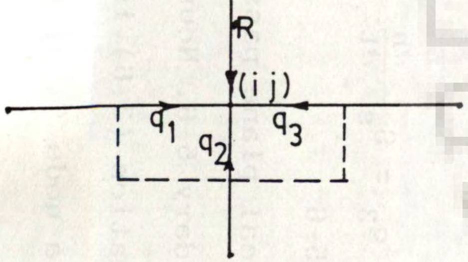


FIG 3.3 (IV)
BOUNDARY 6-1

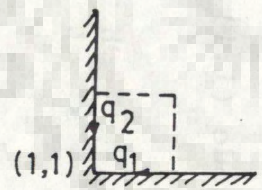


FIG 3.3 (V)
POINT 4

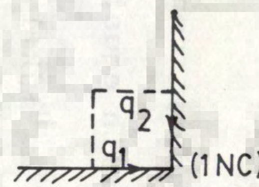


FIG 3.3 (VI)
POINT 5

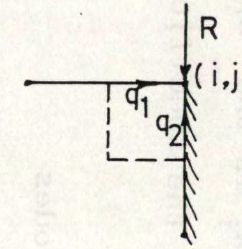


FIG 3.3 (VII)
POINT 6

FIG. 3.3 (I)-(VII) BOUNDARY NODES

$$q_1 + q_2 + q_3 = S_s \frac{\partial h}{\partial t} \frac{\Delta x_{j-1}}{2} \cdot \frac{\Delta z_i + \Delta z_{i-1}}{2} \quad (3.12)$$

(E) Boundary 6-1

In this section the head loss for flow above the drain level is neglected. Boundary 6-1 is the assumed initial horizontal water table at the drain level. Neuman type of boundary condition is assigned in accordance with equation (3.5). i.e., for a node (i,j) at 6-1,

$$R.\frac{\Delta x_j + \Delta x_{j-1}}{2} + q_1 + q_2 + q_3 = S_y \frac{\partial h}{\partial t} \frac{\Delta x_j + \Delta x_{j-1}}{2}$$

$$+ S_s \frac{\partial h}{\partial t} \cdot \frac{\Delta x_j + \Delta x_{j-1}}{2} \cdot \frac{\Delta z_{i-1}}{2} \quad \ldots (3.13)$$

At points 1,2 and 3 boundary 1-2-3 is applicable. At point 4, Neuman type of boundary condition is assigned ensuring water balance.

i.e., for node (1,1)

$$q_1 + q_2 = S_s \frac{\partial h}{\partial t} \frac{\Delta x_j}{2} \cdot \frac{\Delta z_i}{2} \quad \ldots (3.14)$$

Similarly at point 5, the Neuman type of boundary condition ensuring water balance is assigned as follows:

$$q_1 + q_2 = S_s \frac{\partial h}{\partial t} \frac{\Delta x_{j-1}}{2} \cdot \frac{\Delta z_i}{2} \quad \ldots (3.15)$$

At point 6, Neuman type of boundary condition is assigned ensuring water balance as follows:

$$q_1 + q_2 + R \frac{\Delta x_{j-1}}{2} = \left[S_s \frac{\Delta x_{j-1}}{2} \cdot \frac{\Delta z_{i-1}}{2} + S_y \frac{\Delta x_{j-1}}{2} \right] \frac{\partial h}{\partial t} \quad (3.16)$$

In the above equations for boundary conditions, the flows q_1, q_2 and q_3 are evaluated in accordance with the Darcy's Law.

3.1.5 Solution of the Finite Differences Equation

The differential equations written in terms of the implicit finite differences discussed in the above paragraphs are linear. These are solved by the Iterative Alternating Direction Implicit Explicit (IADIE) scheme (Remson, Hornberger and Molz, 1971) proposed by Peaceman and Rachford (1955).

3.1.6 IADIE Scheme

The finite differences equation for a node (i,j) for a time step of kth to (k+1)th discrete times and lth iteration can be written as follows:

$$\frac{2.0}{\Delta x_j + \Delta x_{j-1}} \left[Kx_{ij} \frac{h^{(m)}_{ij+1k+1} - h^{(m)}_{ijk+1}}{\Delta x_j} - Kx_{ij-1} \frac{h^{(m)}_{ijk+1} - h^{(m)}_{ij-1k+1}}{\Delta x_{j-1}} \right] + \frac{2.0}{\Delta z_i + \Delta z_{i-1}} \left[Kz_{ij} \frac{h^{(n)}_{i+1jk+1} - h^{(n)}_{ijk+1}}{\Delta z_i} - Kz_{i-1j} \frac{h^{(n)}_{ijk+1} - h^{(n)}_{i-1jk+1}}{\Delta z_{i-1}} \right]$$

$$= S_{sij} \frac{h^{(m)}_{ijk+1} - h_{ijk}}{\Delta t} \qquad \ldots(3.17)$$

Where, m = l and n = l - 1 for solving implicitly along x-direction and explicitly along z-direction; m = l-1 and n=l for solving implicitly along z-direction and explicitly along x-direction; $h^{(0)}_{ijk+1} = h_{ijk}$ is the initial head at the beginning of time step Δt at the nodal point (i,j). The other terms are as defined for equation (3.8).

3.1.7 Matrices for Solving Implicitly along x-direction

While solving implicitly along x-direction the equation

(3.17) is rearranged as follows:

$$A_j h_{ij-1k+1} + B_j h_{ijk+1} + C_j h_{ij+1k+1} = D_j \qquad \ldots (3.18)$$

Thus, (A_j), (B_j), (C_j) and (D_j) are the matrices of (NC)th order (NC being the number of columns of the finite differences mesh). These are completely defined in terms of known spatial and temporal step sizes, aquifer parameters and the piezometric heads computed in the preceding iteration, as follows.

(a) INTERIOR NODES

$$A_j = \frac{2}{\Delta x_j + \Delta x_{j-1}} \cdot \frac{Kx_{ij-1}}{\Delta x_{j-1}} \qquad \ldots (3.19)$$

$$B_j = -\frac{2}{\Delta x_j + \Delta x_{j-1}}\left(\frac{Kx_{ij}}{\Delta x_j} + \frac{Kx_{ij-1}}{\Delta x_{j-1}}\right) - \frac{2}{\Delta z_i + \Delta z_{i-1}}\left(\frac{Kz_{ij}}{\Delta z_i} + \frac{Kz_{i-1j}}{\Delta z_{i-1}}\right) - \frac{S_{sij}}{\Delta t} \qquad (3.20)$$

$$C_j = \frac{2}{\Delta x_j + \Delta x_{j-1}} \cdot \frac{Kx_{ij}}{\Delta x_j} \qquad \ldots (3.21)$$

$$D_j = \frac{-2}{\Delta z_i + \Delta z_{i-1}} \cdot \left[Kz_{ij}\frac{h^{(l-1)}_{i+1jk+1}}{\Delta z_i} + Kz_{i-1j}\frac{h^{(l-1)}_{i-1jk+1}}{\Delta z_{i-1}}\right] - S_{sij}\frac{h_{ijk}}{\Delta t} \qquad \ldots (3.22)$$

(b) NODES ON BOUNDARY 1-2-3

$$A_j = 0.0 \qquad \ldots (3.23)$$

$$B_j = 1.0 \qquad \ldots (3.24)$$

$$C_j = 0.0 \qquad \ldots (3.25)$$

$$D_j = Y_0 \qquad \ldots (3.26)$$

(C) NODES ON BOUNDARY 3-4

$$A_j = 0.0 \qquad \ldots (3.27)$$

$$B_j = -\left[\frac{2Kx_{ij}}{(\Delta x_j)^2} + \frac{S_{sij}}{\Delta t}\right]$$

$$- \frac{2}{\Delta z_i + \Delta z_{i-1}}\left[\frac{Kz_{ij}}{\Delta z_i} + \frac{Kz_{i-1j}}{\Delta z_{i-1}}\right] \qquad \ldots (3.28)$$

$$C_j = \frac{2Kx_{ij}}{(\Delta x_j)^2} \qquad \ldots (3.29)$$

$$D_j = -S_{sij}\frac{h_{ijk}}{\Delta t} - \frac{2}{\Delta z_i + \Delta z_{i-1}}\left[Kz_{ij}\frac{h^{(l-1)}_{i+1jk+1}}{\Delta z_i}\right]$$

$$- \frac{2}{\Delta z_i + \Delta z_{i-1}}\left[Kz_{i-1j}\frac{h^{(l-1)}_{i-1jk+1}}{\Delta z_{i-1}}\right] \qquad \ldots (3.30)$$

(d) NODES ON BOUNDARY 4-5

$$A_j = \frac{2}{\Delta x_j + \Delta x_{j-1}} \cdot \left[\frac{Kx_{ij-1}}{\Delta x_{j-1}}\right] \qquad \ldots (3.31)$$

$$B_j = -\frac{2}{\Delta x_j + \Delta x_{j-1}}\left(\frac{Kx_{ij}}{\Delta x_j} + \frac{Kx_{ij-1}}{\Delta x_{j-1}}\right) - \frac{S_{sij}}{\Delta t}$$

$$- \frac{2Kz_{ij}}{(\Delta z_i)^2} \qquad \ldots (3.32)$$

$$C_j = \frac{2}{\Delta x_j + \Delta x_{j-1}} \cdot \left[\frac{Kx_{ij}}{\Delta x_j}\right] \quad \ldots (3.33)$$

$$D_j = S_{sij}\frac{h_{ijk}}{\Delta t} - 2Kz_{ij} \frac{h_{i+1jk+1}^{(l-1)}}{(\Delta z_i)^2} \quad \ldots (3.34)$$

(e) NODES ON BOUNDARY 5-6

$$A_j = \frac{2Kx_{ij-1}}{(\Delta x_{j-1})^2} \quad \ldots (3.35)$$

$$B_j = -\frac{2Kx_{ij-1}}{(\Delta x_{j-1})^2} - \frac{S_{sij}}{\Delta t} - \frac{2}{\Delta z_i + \Delta z_{i-1}}$$

$$\left[\frac{Kz_{ij}}{\Delta z_i} + \frac{Kz_{i-1j}}{\Delta z_{i-1}}\right] \quad \ldots (3.36)$$

$$C_j = 0.0 \quad \ldots (3.37)$$

$$D_j = -S_{sij}\frac{h_{ijk}}{\Delta t} - \frac{2}{\Delta z_i + \Delta z_{i-1}}\left[Kz_{ij}\frac{h_{i+1jk+1}^{(l-1)}}{\Delta z_i}\right]$$

$$-\frac{2}{\Delta z_i + \Delta z_{i-1}}\left[Kz_{i-1j}\frac{h_{i-1jk+1}^{(l-1)}}{\Delta z_{i-1}}\right] \quad \ldots (3.38)$$

(f) NODES ON BOUNDARY 6-1

$$A_j = \frac{Kx_{ij-1}}{2} \quad \frac{\Delta z_{i-1}}{\Delta x_{j-1}} \quad \ldots (3.39)$$

$$B_j = -\frac{Kz_{i-1j}}{2} \quad \frac{\Delta x_j + \Delta x_{j-1}}{\Delta z_{i-1}}$$

$$- \frac{Kx_{ij-1}}{2} \cdot \frac{\Delta z_{i-1}}{\Delta x_{j-1}} - \frac{Kx_{ij}}{2} \cdot \frac{\Delta z_{i-1}}{\Delta x_j}$$

$$- \frac{S_{yj}}{2} \cdot \frac{\Delta x_j + \Delta x_{j-1}}{\Delta t}$$

$$- \frac{S_{sij}}{\Delta t} \cdot \frac{\Delta x_j + \Delta x_{j-1}}{2} \quad \frac{\Delta z_{i-1}}{2} \qquad \text{... (3.40)}$$

$$C_j = \frac{Kx_{ij}}{2} \cdot \frac{\Delta z_{i-1}}{\Delta x_j} \qquad \text{... (3.41)}$$

$$D_j = - R \frac{\Delta x_j + \Delta x_{j-1}}{2} - Kz_{i-1j} \frac{h^{(l-1)}_{i-1jk+1}}{\Delta z_{i-1}}$$

$$- (S_{yj} + S_{sij} \frac{\Delta z_i}{2})(\frac{h_{ijk}}{2} \quad \frac{\Delta x_j + \Delta x_{j-1}}{\Delta t}) \qquad \text{...(3.42)}$$

(g) NODE (1,1)

$$A_j = 0.0 \qquad \text{... (3.43)}$$

$$B_j = - \frac{Kx_{ij}}{(\Delta x_j)^2} - \frac{Kz_{ij}}{(\Delta z_i)^2} - \frac{S_{sij}}{2\Delta t} \qquad \text{... (3.44)}$$

$$C_j = \frac{Kx_{ij}}{(\Delta x_j)^2} \qquad \text{... (3.45)}$$

$$D_j = - Kz_{ij} \frac{h^{(l-1)}_{i+1jk+1}}{(\Delta z_i)^2} - S_{sij} \frac{h_{ijk}}{2\Delta t} \qquad \text{... (3.46)}$$

(h) NODE (1,NC)

$$A_j = \frac{Kx_{ij-1}}{(\Delta x_{j-1})^2} \qquad \text{.. (3.47)}$$

$$B_j = -\frac{Kx_{ij-1}}{(\Delta x_{j-1})^2} - \frac{Kz_{ij}}{(\Delta z_i)^2} - \frac{Ss_{ij}}{2\Delta t} \qquad \ldots (3.48)$$

$$C_j = 0.0 \qquad \ldots (3.49)$$

$$D_j = -Kz_{ij}\frac{h^{(l-1)}_{i+1jk+1}}{(\Delta z_i)^2} - S_{sij}\frac{h_{ijk}}{2\Delta t} \qquad \ldots (3.50)$$

(i) NODE (NR,NC)

NR = total number of rows of the finite differences mesh

$$A_j = Kx_{ij-1}\cdot\frac{\Delta z_{i-1}}{2\Delta x_{j-1}} \qquad \ldots (3.51)$$

$$B_j = -Kz_{i-1j}\cdot\frac{\Delta x_{j-1}}{2\,\Delta z_{i-1}} - Kx_{ij-1}\frac{\Delta z_{i-1}}{2\Delta x_{j-1}}$$

$$-S_{yj}\frac{\Delta x_{j-1}}{2\Delta t} - S_{sij}\frac{\Delta x_{j-1}}{2\Delta t}\cdot\frac{\Delta z_{i-1}}{2} \qquad \ldots (3.52)$$

$$C_j = 0.0 \qquad (3.53)$$

$$D_j = -R\frac{\Delta x_{j-1}}{2} - Kz_{i-1j}\frac{h^{(l-1)}_{i-1jk+1}\cdot\Delta x_{j-1}}{2\Delta z_{i-1}}$$

$$-S_{yj}\frac{h_{ijk}}{\Delta t}\frac{\Delta x_{j-1}}{2} - S_{sij}\frac{h_{ijk}}{\Delta t}\frac{\Delta x_{j-1}}{2}\frac{\Delta z_{i-1}}{2}$$

$$\ldots (3.54)$$

.1.8 Matrices for Solving Implicitly along z - direction.

While solving implicitly along z-direction the equation (3.17) is rewritten as follows

$$A_i h_{i-1jk+1} + B_i h_{ijk+1} + C_i h_{i+1jk+1} = D_i \qquad \ldots (3.55)$$

Thus, (A_i), (B_i), (C_i) and (D_i) are the matrices of (NR)th order. These are completely defined in terms of known spatial and temporal step sizes, aquifer parameters and piezometric heads computed in the preceding iteration, as follows:

(a) INTERIOR NODES

$$A_i = \frac{2}{\Delta z_i + \Delta z_{i-1}} \cdot \frac{Kz_{i-1j}}{\Delta z_{i-1}} \qquad \ldots (3.56)$$

$$B_i = -\frac{2}{\Delta z_i + \Delta z_{i-1}} \left[\frac{Kz_{ij}}{\Delta z_i} + \frac{Kz_{i-1j}}{\Delta z_{i-1}} \right]$$

$$- \frac{2}{2\Delta x_j + \Delta x_{j-1}} \left[\frac{Kx_{ij}}{\Delta x_j} + \frac{Kx_{ij-1}}{\Delta x_{j-1}} \right] - \frac{S_{sij}}{\Delta t}$$

$$\ldots (3.57)$$

$$C_i = \frac{2}{\Delta z_i + \Delta z_{i-1}} \frac{Kz_{ij}}{\Delta z_i} \qquad \ldots (3.58)$$

$$D_i = -\frac{2}{\Delta x_j + \Delta x_{j-1}} \left[Kx_{ij} \frac{h^{(l-1)}_{ij+1k+1}}{\Delta x_j} + Kx_{ij-1} \frac{h^{(l-1)}_{ij-1k+1}}{\Delta x_{j-1}} \right]$$

$$- S_{sij} \frac{h_{ijk}}{\Delta t} \qquad \ldots (3.59)$$

(b) NODES ON BOUNDARY 1-2-3

$$A_i = 0.0 \qquad \ldots (3.60)$$

$$B_i = 1.0 \qquad \ldots (3.61)$$

$$C_i = 0.0 \qquad \ldots (3.62)$$

$$D_i = Y_0 \qquad \ldots (3.63)$$

(c) NODES ON BOUNDARY 3-4

$$A_i = \frac{2}{\Delta z_i + \Delta z_{i-1}} \cdot \frac{Kz_{i-1j}}{\Delta z_{i-1}} \quad \text{... (3.64)}$$

$$B_i = -\frac{2}{\Delta z_i + \Delta z_{i-1}} \left[\frac{Kz_{ij}}{\Delta z_i} + \frac{Kz_{i-1j}}{\Delta z_{i-1}} \right] - \frac{S_{sij}}{\Delta t}$$

$$- \frac{2Kx_{ij}}{(\Delta x_j)^2} \quad \text{... (3.65)}$$

$$C_i = \frac{2}{\Delta z_i + \Delta z_{i-1}} \frac{Kz_{ij}}{\Delta z_i} \quad \text{...(3.66)}$$

$$D_i = - 2Kx_{ij} \frac{h^{(l-1)}_{ij+1k+1}}{(\Delta x_j)^2} - S_{sij} \frac{h_{ijk}}{\Delta t} \quad \text{...(3.67)}$$

(d) NODES ON BOUNDARY 4-5

$$A_i = 0.0 \quad \text{...(3.68)}$$

$$B_i = - \frac{2Kz_{ij}}{(\Delta z_i)^2} - \frac{2}{\Delta x_j + \Delta x_{j-1}} \left[\frac{Kx_{ij}}{\Delta x_j} + \frac{Kx_{ij-1}}{\Delta x_{j-1}} \right]$$

$$- \frac{S_{sij}}{\Delta t} \quad \text{...(3.69)}$$

$$C_i = \frac{2Kz_{ij}}{(\Delta z_i)^2} \quad \text{...(3.70)}$$

$$D_i = - \frac{2}{\Delta x_j + \Delta x_{j-1}} \left[Kx_{ij-1} \quad \frac{h^{(l-1)}_{ij-1k+1}}{\Delta x_{j-1}} \right]$$

$$- \frac{2}{\Delta x_j + \Delta x_{j-1}} \left[Kx_{ij} \quad \frac{h^{(l-1)}_{ij+1k+1}}{\Delta x_j} \right] - S_{sij} \frac{h_{ijk}}{\Delta t} \quad \text{(3.71)}$$

(e) NODES ON BOUNDARY 5-6

$$A_i = \frac{2}{\Delta z_i + \Delta z_{i-1}} \left[\frac{Kz_{i-1j}}{\Delta z_{i-1}} \right] \quad \ldots (3.72)$$

$$B_i = - \frac{2}{\Delta z_i + \Delta z_{i-1}} \left[\frac{Kz_{ij}}{\Delta z_i} + \frac{Kz_{i-1j}}{\Delta z_{i-1}} \right] - \frac{2Kx_{ij-1}}{(\Delta x_{j-1})^2}$$

$$- \frac{S_{sij}}{\Delta t} \quad \ldots (3.73)$$

$$C_i = \frac{2}{\Delta z_i + \Delta z_{i-1}} \left[\frac{Kz_{ij}}{\Delta z_i} \right] \quad \ldots (3.74)$$

$$D_i = - 2Kx_{ij-1} \frac{h^{(l-1)}_{ij-1k+1}}{(\Delta x_{j-1})^2} - S_{sij} \frac{h_{ijk}}{\Delta t} \quad \ldots (3.75)$$

(f) NODES ON BOUNDARY 6-1

$$A_i = Kz_{i-1j} \frac{\Delta x_j + \Delta x_{j-1}}{2\Delta z_{i-1}} \quad \ldots (3.76)$$

$$B_i = - Kz_{i-1j} \frac{\Delta x_j + \Delta x_{j-1}}{2\Delta z_{i-1}} - Kx_{ij-1} \frac{\Delta z_{i-1}}{2\Delta x_{j-1}}$$

$$- Kx_{ij} \frac{\Delta z_{i-1}}{2\Delta x_j} - S_{yj} \frac{\Delta x_j + \Delta x_{j-1}}{2\Delta t}$$

$$- S_{sij} \frac{\Delta x_j + \Delta x_{j-1}}{2} \frac{\Delta z_{i-1}}{2\Delta t} \quad \ldots (3.77)$$

$$C_i = 0.0 \quad \ldots (3.78)$$

$$D_i = - R \frac{\Delta x_j + \Delta x_{j-1}}{2} - Kx_{ij-1} \frac{h^{(l-1)}_{ij-1k+1}}{2}$$

$$\frac{\Delta z_{i-1}}{\Delta x_{j-1}} - Kx_{ij} \frac{h^{(l-1)}_{ij+1k+1}}{2} \frac{\Delta z_{i-1}}{\Delta x_{j}}$$

$$- \left[S_{yj} + S_{sij} \frac{\Delta z_{i-1}}{2} \right] \left[\frac{h_{ijk}}{\Delta t} \frac{\Delta x_{j} + \Delta x_{j-1}}{2} \right]$$

... (3.79)

(g) NODE (1,1)

$$A_i = 0.0 \qquad \text{... (3.80)}$$

$$B_i = - \frac{Kz_{ij}}{(\Delta z_i)^2} - \frac{Kx_{ij}}{(\Delta x_j)^2} - \frac{S_{sij}}{2\Delta t} \qquad \text{... (3.81)}$$

$$C_i = \frac{Kz_{ij}}{(\Delta z_i)^2} \qquad \text{... (3.82)}$$

$$D_i = - Kx_{ij} \frac{h^{(l-1)}_{ij+1k+1}}{(\Delta x_j)^2} - S_{sij} \frac{h_{ijk}}{2\Delta t} \qquad \text{... (3.83)}$$

(h) NODE (1,NC)

$$A_i = 0.0 \qquad \text{... (3.84)}$$

$$B_i = - \frac{Kz_{ij}}{(\Delta z_i)^2} - \frac{Kx_{ij-1}}{(\Delta x_j)^2} - \frac{S_{sij}}{2\Delta t} \qquad \text{... (3.85)}$$

$$C_i = \frac{Kz_{ij}}{(\Delta z_i)^2} \qquad \text{... (3.86)}$$

$$D_i = - Kx_{ij-1} \frac{h^{(l-1)}_{ij-1k+1}}{(\Delta x_{j-1})^2} - S_{sij} \frac{h_{ijk}}{2\Delta t} \qquad \text{... (3.87)}$$

(i) NODE (NR,NC)

$$A_i = Kz_{i-1j} \left[\frac{\Delta x_{j-1}}{2\Delta z_{i-1}} \right] \qquad \text{... (3.88)}$$

$$B_i = - Kz_{i-1j} \frac{\Delta x_{j-1}}{2\Delta z_{i-1}} - Kx_{ij-1} \frac{\Delta z_{i-1}}{2\Delta x_{j-1}}$$

$$- S_{yj} \frac{\Delta x_{j-1}}{2\Delta t} - \frac{S_{sij}}{\Delta t} \frac{\Delta z_{i-1}}{2} \frac{\Delta x_{j-1}}{2} \qquad (3.89)$$

$$C_i = 0.0 \qquad (3.90)$$

$$D_i = - R \frac{\Delta x_{j-1}}{2} - Kx_{ij-1} \frac{h^{(l-1)}_{ij-1k+1}}{\Delta x_{j-1}} \frac{\Delta z_{i-1}}{2}$$

$$- S_{yj} \frac{h_{ijk}}{\Delta t} \cdot \frac{\Delta x_{j-1}}{2} - S_{sij} \frac{h_{ijk}}{\Delta t} \cdot \frac{\Delta z_{i-1}}{2} \cdot \frac{\Delta x_{j-1}}{2}$$

... (3.91)

The coefficient matrices (3.19) to (3.54) are substituted in equation (3.18) and the coefficient matrices (3.56) to (3.91) are substituted in equation (3.55). The equations (3.18) and (3.55) are then solved using the Thomas Algorithm (Remson, Hornberger and Molz, 1971) as the governing coefficient matrices ((3.19) to (3.54) and (3.56) to (3.91)) are tridiagonal and positive definite. These matrices contain coefficients derived from numerical approximation to spatial derivatives. Moreover, they are defined such that only the derivatives in one space variable are represented in each matrix. The equations ((3.18)/(3.55)) are solved implicitly in one space dimension (x/z) using the known values in other space dimension (z/x). This sequential sweeping of the matrices is done first horizontally (equation (3.18)) and then vertically (equation (3.55)). This leads to the computation of piezometric heads at all the finite differences nodes and at the discrete times considered.

3.1.9 Convergence Criteria

The differences of the (h_{ijk+1}) values of two successive iterations (l) and (l-1) for all the nodes are summed up. This sum is then compared with a prestipulated convergence factor (say $\varepsilon 1$). The ADIE iterations are repeated until this sum attains a value less than $\varepsilon 1$, i.e.,

$$\sum | h_{ijk+1}^{(l)} - h_{ijk+1}^{(l-1)} | < \varepsilon 1 \qquad ...(3.92)$$

The piezometric heads are computed at each node by the above procedure at the discrete times separated by nonuniform time steps. Smaller time steps were assigned in the early stage when piezometric heads are known to change faster. Later each time step is divided into appropriate number of subtime steps so that the computed piezometeric heads at the end of the time step does not change significantly with further increase in the number of subtime steps (Ahmad et al, 1990), i.e.,

$$\sum |(h_{ijk+1})_{np} - (h_{ijk+1})_{np/2}| \leq \varepsilon 2 \qquad ...(3.93)$$

where, $(h_{ijk+1})_{np}$ is the piezometric head calculated with np number of subtime steps, $(h_{ijk+1})_{np/2}$ is the piezometric head calculated with np/2 number of subtime steps at the end of (k+1)th discrete time and $\varepsilon 2$ is the prestipulated convergence factor.

3.2 SATURATED FLOW MODEL (ACCOUNTING FOR THE FLOW ABOVE DRAIN LEVEL)

In this section the flow domain of the model (refer para 3.1, Fig.3.2) is extended up to the transient water table position by modifying the boundary conditions at the upper boundary and at the mid section as follows

(a) Upper Boundary at the Water Table Position (6′-1)

(refer Fig 3.4)

$$\xi(x,0) = Y_0 \text{ (initially horizontal water table)} \quad ..(3.94)$$

$$Kz \frac{\partial h}{\partial z} + S_y \frac{\partial \xi}{\partial t} = R, \; z = \xi, \; \frac{b}{2} \le x \le \frac{L}{2}, t > 0 \quad ... (3.95)$$

$$\xi(x,t) = h \left[x, \xi(x,t), t\right], \; z = \xi, \; \frac{b}{2} \le x \le \frac{L}{2} \quad (3.96)$$

(b) Boundary at the mid section (5-6′)

(refer Fig 3.4)

$$\frac{\partial h}{\partial x} = 0, \; 0 \le z \le \xi \left(\frac{L}{2}, t\right), \; x = \frac{L}{2}, \; t > 0 \quad ..(3.97)$$

where, $\xi = \xi\ (x,t)$ is the elevation of the water table above the lower impervious layer. These modified boundary conditions (equations 3.94 - 3.97) are numerically implemented as follows.

3.2.1 Solution of the Differential Equation for the Modified Boundary Conditions.

The boundary condition at the free surface (equation 3.95) includes the variables $h(x,\xi,t)$ and $\xi(x,t)$, which are implicitly related to each other in accordance with equation (3.96). Thus, a solution for these two variables is not feasible by conventional use of numerical techniques like finite differences. An iterative solution of $\xi(x,t)$ based upon repetitive finite differences application is adopted (Ahmad et al., 1991).

3.2.2 Solution Strategy

The implicit nature of the relation between ξ and h (equation 3.96) is approximated by the following explicit relation,

$$\hat{\xi}^{(p)}_{(x,t)} = h\left(x, \hat{\xi}^{(p-1)}_{(x,t)}, t\right) \quad .. (3.98)$$

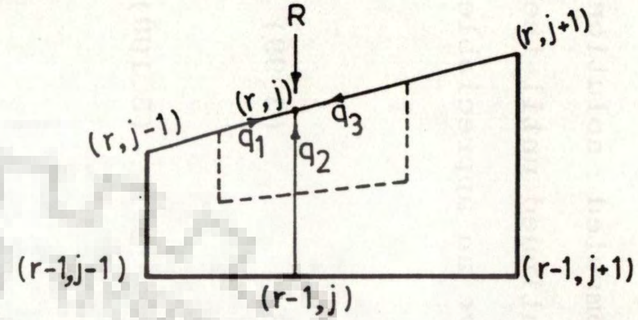


FIG 3·4 (II)

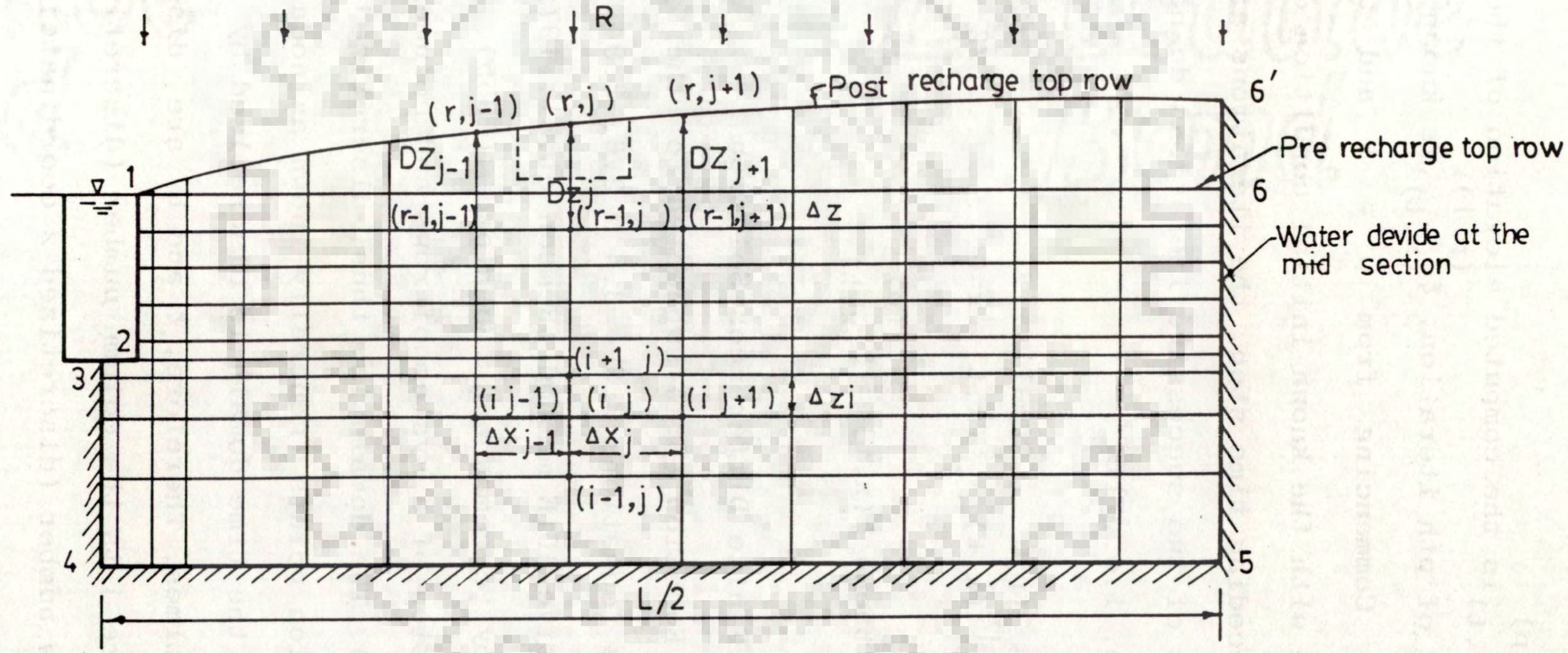


FIG. 3·4 (I)

FIG 3·4 FINITE DIFFERENCES MESH AND BOUNDARIES (ACCOUNTING FOR THE FLOW ABOVE DRAIN LEVEL) where, $\hat{\xi}^{(p)}_{(x,t)}$ is the computed elevation of the water table at (x,t) at the end of pth iteration; $\hat{\xi}^{(p-1)}_{(x,t)}$, is known from the preceding iteration. Commencing from p = 1 and assigning $\hat{\xi}^{(0)}(x,t)$ in accordance with the known initial condition or computed solution at the preceding time step, the iterations are continued until the ($\hat{\xi}$) values of two successive iterations do not have an appreciable difference, i.e., if

$$\sum \left| \hat{\xi}^{(p)}_{(x,t)} - \hat{\xi}^{(p-1)}_{(x,t)} \right|_{x=0.........L/2} \leq \varepsilon 3 \qquad \text{... (3.99)}$$

$$\text{then } \xi(x,t) = \hat{\xi}^{(p)}_{(x,t)} \qquad \text{... (3.100)}$$

3.2.3 Finite Differences Solution

Equation (3.1) subjected to the initial and boundary conditions (equations 3.2 - 3.4 & 3.6, 3.94 - 3.97) is solved by the iterative ADIE method (Remson et.al. 1971; Ahmad et. al. 1990) proposed by Peaceman and Rachford (1955). The method involves superposition of a system of rows and columns over the flow domain (Fig. 3.4). The domain is, thus, discretized by a finite number of intersection points (generally known as nodal points or nodes). Similarly the time domain is discretized by a finite number of discrete times. Therefore, ξ and h are discretized as ξ_{jk} and h_{ijk}; where j is the column number (discretized x coordinate), i is the row number (discretized z coordinate) and k is the discrete time (similar to those explained in para 3.1.4 and Fig.3.2).

Knowing h_{ijk}, the iterative ADIE techniques can provide h_{ijk+1}. At the top most row (i = NR), the piezometric head equals the water table elevation, i.e.,

$$\xi_{jk+1} = h_{NRjk+1} \qquad \ldots (3.101)$$

3.2.4 Accounting of Free Surface

In the ADIE solution the time variant position of the water table is discretized by the top most row (Fig.3.4). Thus, the top most row is successively redefined at the end of each ADIE solution in accordance with the following discretized form of equation (3.98).

$$Y_0 + DZ^{(P)}_{jk+1} - \Delta z = \hat{\xi}^{(P-1)}_{jk+1} \qquad (3.102)$$

$$\text{or} \quad DZ^{(P)}_{jk+1} = \Delta z + \hat{\xi}^{(P-1)}_{jk+1} - Y_0 \qquad \ldots (3.103)$$

where, $DZ^{(P)}_{jk+1}$ is the spacing between nodes (NR,j) and (NR-1,j) at (k+1)th discrete time during pth ADIE solution; $\xi^{(P-1)}_{jk+1}$ is the known water table elevation computed in the (p-1)th ADIE solution; Δz is the initial spacing between the (NR)th and (NR-1)the rows. The vertical spacing between the (NR)th and (NR-1)th row at the jth column is modified in accordance with equation (3.103).

The boundary condition at the free surface (equation 3.95) is applied, ensuring water balance as follows (refer Fig. 3.4).

$$q_1 + q_2 + q_3 + R\left(\frac{\Delta x_j + \Delta x_{j-1}}{2}\right)$$

$$= S_y \frac{\partial \xi}{\partial t}\left(\frac{\Delta x_j + \Delta x_{j-1}}{2}\right) + S_s \frac{\partial \xi}{\partial t}\left(\frac{\Delta x_j + \Delta x_{j-1}}{2}\right)$$

$$\left(\frac{DZ_{j-1} + 2DZ_j + DZ_{j+1}}{8}\right) \qquad \ldots (3.104)$$

3.2.5 Matrices for the Modified Boundary Conditions

3.2.5.1 For Solving Implicitly along x-direction

The solution is proceeded in the same way as explained in paragraphs (3.1.5 - 3.1.6). All the coefficient matrices will remain the same (equations 3.19-3.38, 3.43-3.50) except the matrices for nodes of the top most row.

(a) MATRICES FOR NODES OF THE TOP MOST ROW

(Boundary 6-1, Excluding Node (NR,NC))

$$A_j = Kx_{ij-1} \frac{DZ_j + DZ_{j-1}}{4DX_{j-1}} \quad \ldots (3.105)$$

$$B_j = -Kz_{i-1j} \frac{\Delta x_j + \Delta x_{j-1}}{2DZ_j} - Kx_{ij-1} \frac{DZ_j + DZ_{j-1}}{4DX_{j-1}} - Kx_{ij} \frac{DZ_j + DZ_{j+1}}{4DX_j} - S_{yj} \frac{\Delta x_j + \Delta x_{j-1}}{2\Delta t} - S_{sij} \frac{\Delta x_j + \Delta x_{j-1}}{2\Delta t} \frac{DZ_{j-1} + 2DZ_j + DZ_{j+1}}{8} \quad \ldots (3.106)$$

$$C_j = Kx_{ij} \frac{DZ_j + DZ_{j+1}}{4DX_j} \quad \ldots (3.107)$$

$$D_j = -R \frac{\Delta x_j + \Delta x_{j-1}}{2} - Kz_{i-1j} \frac{h^{(l-1)}_{i-1jk+1} (\Delta x_j + \Delta x_{j-1})}{2DZ_j} - S_{yj} \frac{h_{ijk}(\Delta x_j + \Delta x_{j-1})}{2\Delta t} - S_{sij} \frac{h_{ijk}}{\Delta t} \frac{\Delta x_j + \Delta x_{j-1}}{2} \frac{(DZ_{j-1} + 2DZ_j + DZ_{j+1})}{8} \quad \ldots (3.108)$$

(b) MATRICES FOR NODE (NR,NC)

$$A_j = Kx_{ij-1} \frac{DZ_{j-1} + DZ_j}{4DX_{j-1}} \quad \ldots (3.109)$$

$$B_j = -Kz_{i-1j} \frac{\Delta x_{j-1}}{2DZ_j} - Kx_{ij-1} \frac{DZ_{j-1} + DZ_j}{4DX_j}$$

$$- S_{yj} \frac{\Delta x_{j-1}}{2\Delta t} - S_{sij}\, \Delta x_{j-1} \frac{DZ_j + DZ_{j-1}}{8\Delta t} \quad . (3.110)$$

$$C_j = 0.0 \quad \ldots (3.111)$$

$$D_j = -R \frac{\Delta x_{j-1}}{2} - Kz_{i-1j} \frac{h^{(l-1)}_{i-1jk+1} \cdot \Delta x_{j-1}}{2DZ_j}$$

$$- S_{yj}\, h_{ijk} \cdot \frac{\Delta x_{j-1}}{2\Delta t} - S_{sij}\, h_{ijk}\, \Delta x_{j-1} \frac{DZ_{j-1} + DZ_j}{8\Delta t}$$

$$\ldots (3.112)$$

where, $DX_j = \left[(\Delta x_j)^2 + (DZ_{j+1} - DZ_j)^2 \right]^{1/2}$ .. (3.113)

and $DX_{j-1} = \left[(\Delta x_{j-1})^2 + (DZ_j - DZ_{j-1})^2 \right]^{1/2}$. (3.114)

3.2.5.2 For Solving Implicitly along z-direction

(a) MATRICES FOR NODES OF THE TOP MOST ROW (BOUNDARY 6′-1, EXCLUDING NODE (NR,NC))

$$A_i = Kz_{i-1j} \frac{\Delta x_j + \Delta x_{j-1}}{2DZ_j} \quad \ldots (3.115)$$

$$B_i = -Kz_{i-1j} \frac{\Delta x_j + \Delta x_{j-1}}{2DZ_j} - Kx_{ij-1} \frac{DZ_j + DZ_{j-1}}{4DX_{j-1}}$$

$$- Kx_{ij} \frac{DZ_j + DZ_{j+1}}{4DX_j} - S_{yj} \frac{\Delta x_j + \Delta x_{j-1}}{2\Delta t}$$

$$- \frac{S_{sij}}{\Delta t} \frac{\Delta x_j + \Delta x_{j-1}}{2} \frac{DZ_{j-1} + 2DZ_j + DZ_{j+1}}{8} \quad \ldots (3.116)$$

$$C_i = 0.0 \quad \ldots (3.117)$$

$$D_i = - R \frac{\Delta x_j + \Delta x_{j-1}}{2} - Kx_{ij-1} \frac{h_{ij-1k+1}^{(1-1)} (DZ_j + DZ_{j-1})}{4DX_{j-1}}$$

$$- Kx_{ij} . h_{ij+1k+1}^{(1-1)} . \frac{DZ_j + DZ_{j+1}}{4DX_j}$$

$$- S_{yj} . h_{ijk} \frac{\Delta x_j + \Delta x_{j-1}}{2\Delta t}$$

$$- S_{sij} \frac{h_{ijk}}{\Delta t} . \frac{(\Delta x_j + \Delta x_{j-1})}{2} . \frac{DZ_{j-1} + 2DZ_j + DZ_{j+1}}{8} \quad \ldots (3.118)$$

(b) MATRICES FOR NODE (NR,NC)

$$A_i = Kz_{i-1j} \frac{\Delta x_{j-1}}{2DZ_j} \quad \ldots (3.119)$$

$$B_i = - Kz_{i-1j} \frac{\Delta x_{j-1}}{2DZ_j} - Kx_{ij-1} \frac{DZ_j + DZ_{j-1}}{4DX_{j-1}}$$

$$- S_{yj} \frac{\Delta x_{j-1}}{2\Delta t} - S_{sij} \Delta x_{j-1} \frac{DZ_j + DZ_{j-1}}{8\Delta t} \quad \ldots (3.120)$$

$$C_i = 0.0 \quad \ldots (3.121)$$

$$D_i = - R \frac{\Delta x_{j-1}}{2} - Kx_{ij-1} h_{ij-1k+1}^{(1-1)} \frac{DZ_j + DZ_{j-1}}{4DX_{j-1}}$$

$$- S_{yj} h_{ijk} . \frac{\Delta x_{j-1}}{2\Delta t} - S_{sij} . h_{ijk} \frac{DZ_{j-1} + DZ_j}{8\Delta t} \Delta x_{j-1} \quad \ldots (3.122)$$

3.2.6 Convergence

At the end of each ADIE solution (DZ), and hence the domain, is modified in accordance with equation (3.103). The modification of (DZ) is continued until the spacings at the end of two consecutive ADIE solutions are

$$\Sigma \ |DZ^{(P)}_{jk+1} - DZ^{(p-1)}_{jk+1} \ | \leq \varepsilon \ 3 \qquad \ldots (3.123)$$

for all j's (Ahmad et al.1991).

Equation (3.123) is the finite differences equivalent of equation (3.99). The convergence of piezometric heads with respect to ADIE iterations and time step iterations are achieved in accordance with the criteria explained in para 3.1.9. (equations 3.92 and 3.93).

3.3 THE TOTAL RESPONSE MODEL

The total response model essentially involves numerical solution of the differential equation governing two-dimensional flow (x-z plane) in an anisotropic and heterogeneous porous medium by finite differences.The solution is obtained in a flow domain bounded by a horizontal impervious boundary at the bottom ,the horizontal ground surface at the top and two parallel drains on the sides .

3.3.1 The Flow Equation

The differential equation governing two-dimensional transient unsaturated -saturated flow in an anisotropic and heterogeneous porous medium (in x-z plane) can be written as follows.

$$C \ \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left(Kx \ \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial z} \left(Kz \ \frac{\partial h}{\partial z}\right) - ET \qquad \ldots (3.124)$$

where,

$$C = \frac{d\theta}{dh_c} \quad , \theta < \phi \qquad (3.125)$$

$$Kx = Kx(Kxsat,\theta) \quad , \theta < \phi \qquad (3.126)$$

$$Kz = Kz(Kzsat,\theta) \quad , \theta < \phi \qquad (3.127)$$

$$C = S_s \quad , \theta = \phi \qquad (3.128)$$

$$Kx = Kxsat \quad , \theta = \phi \qquad (3.129)$$

$$Kz = Kzsat \quad , \theta = \phi \qquad (3.130)$$

$$ET = f(PET,\theta) \quad , z \geq D_i\text{-droot} \qquad (3.131)$$

$$ET = 0 \quad , z < D_i\text{-droot} \qquad (3.132)$$

Here, h is the algebraic sum of capillary head $[(h_c(\theta)]$ and the elevation head z , (θ being the volumetric soil moisture content); C equals the specific storage/specific soil moisture capacity $(d\theta/dh_c)$; ET (=ET(θ)) is the actual evapotranspiration rate per unit soil volume; Kx and Kz are capillary conductivities in x and z directions respectively. Kxsat and Kzsat are the saturated hydraulic conductivities of the medium in x and z directions respectively ;ϕ is the soil porosity ;PET is the potential evapotranspiration rate ;D_i is the depth of impervious layer below ground surface ;droot is the root zone depth. Thus, equation (3.124), by adopting appropriate values of parameters, represents the Richards equation in the unsaturated zone and classical groundwater flow equation (equation 3.1) in the saturated zone below water table. Referring to Fig. 3.5, the initial and boundary conditions for drainage(considering symmetry about the midsection) are assigned as follows:

$$h = Y_0,\ 0 \leq z \leq D_i,\ 0 \leq x \leq \frac{L}{2},\ t = 0 \qquad (3.133)$$

$$h = Y_0,\ Y_0 - d \leq z \leq Y_0,\ 0 \leq x \leq \frac{b}{2}, t > 0 \ \ldots \qquad (3.134)$$

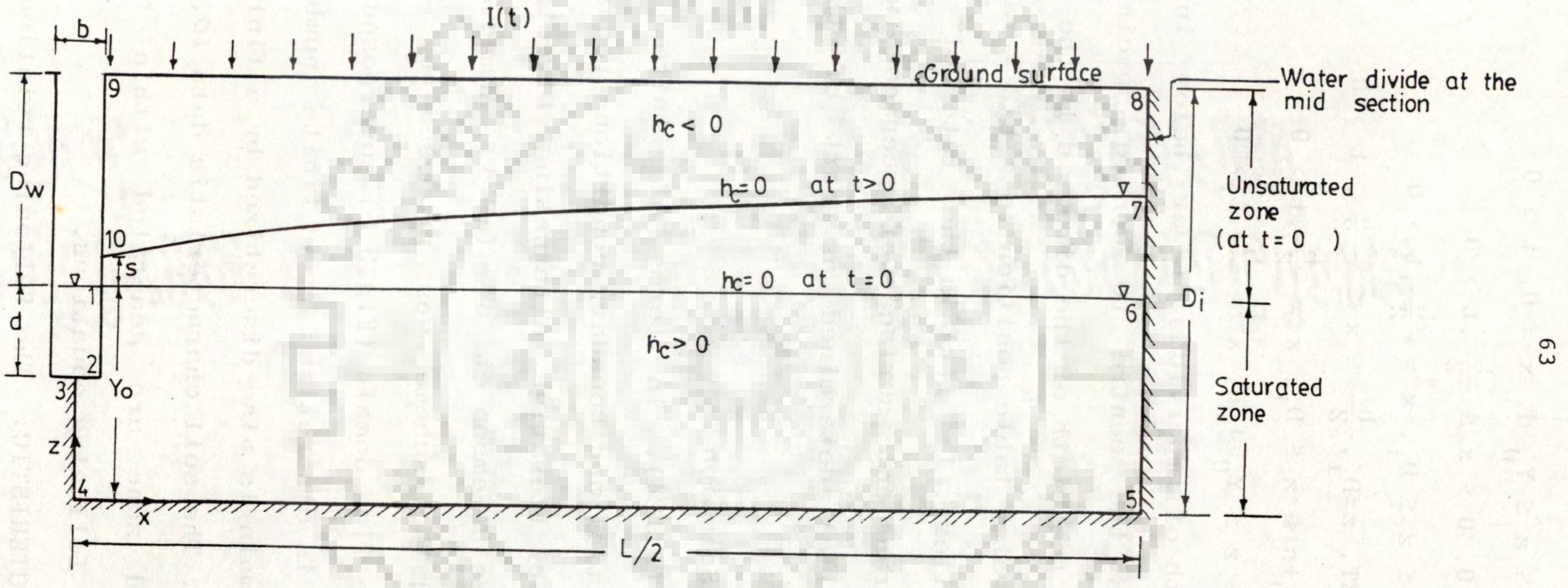


FIG.3.5- SUBSURFACE DITCH DRAINAGE
(DEFINITION SKETCH FOR THE TOTAL RESPONSE MODEL)

$$\frac{\partial h}{\partial x} = 0,\ 0 \le z \le Y_0 - d,\ x = 0,\ t > 0 \quad \ldots \quad (3.135)$$

$$\frac{\partial h}{\partial z} = 0,\ z=0,\ 0 \le x \le \frac{L}{2}, t > 0 \quad \ldots \quad (3.136)$$

$$\frac{\partial h}{\partial x} = 0,\ 0 \le z \le D_i,\ x = \frac{L}{2}, t > 0 \quad \ldots \quad (3.137)$$

$$Kz\ \frac{\partial h}{\partial z} = I - ET,\ z=D_i, \frac{b}{2} \le x \le \frac{L}{2},\ t > 0 \quad .. \quad (3.138)$$

$$\frac{\partial h}{\partial x} = 0,\ Y_0 + s \le z \le D_i,\ x = \frac{b}{2}, t > 0 \quad .. \quad (3.139)$$

$$h = z,\ Y_0 \le z \le Y_0 + s,\ x = \frac{b}{2}, t > 0 \quad \ldots \quad (3.140)$$

where, Y_0 is the depth of impervious layer below initial water table position;L is the centre to centre spacing of the ditches/drains, b is the width of the ditch, d is the depth of the ditch below initial water table position (d = b/2 = radius of the tile, for circular tiles). I = I(t) is the rate of infiltration and s = s(t) is transient surface of seepage (water height standing at the tile) determined as part of the numerical solution.

3.3.2 Numerical Solution

Equation (3.124), in terms of finite differences, subjected to the initial and boundary conditions (equations 3.133 to 3.140) is solved by Iterative Alternating Direction Implicit Explicit(ADIE) scheme [(Remson et. al. (1971), Ahmad et.al (1990), Ahmad et.al. (1991)].The scheme was proposed by Peacemen and Rachford (1955). The flow domain (Fig.3.5) superposed by a system of rows and columns is discretized by a finite number of nodal points. The time domain is also discretized by a finite number of discrete time steps. The soil characteristic data (θ, Kx, Kz and ET) for unsaturated zone are generated within the model in accordance with the following equations.

(i) K - CHARACTERISTIC: The following relations (equations 3.141 to 3.144) proposed by Brooks and Corey (1964) are adopted.

$$K(x,\theta) = Kxsat \left[\frac{\theta - \theta_r}{\phi - \theta_r} \right]^4, \quad \theta \geq \theta_r \quad \ldots (3.141)$$

$$= 0 \qquad \theta < \theta_r \quad \ldots (3.142)$$

$$K(z,\theta) = Kzsat \left[\frac{\theta - \theta_r}{\phi - \theta_r} \right]^4, \quad \theta \geq \theta_r \quad \ldots (3.143)$$

$$= 0 \qquad \theta < \theta_r \quad \ldots (3.144)$$

(ii) θ - CHARACTERISTIC:

The following relations (Mohan Rao et al. (1986)) are adopted

$$\theta = e^{(\ln(\phi - 2\theta_r) \cdot \frac{h_c}{h_b})} + \theta_r, \quad h \geq h_b \quad \ldots (3.145)$$

$$= \phi - \frac{\theta_r}{h_b} \cdot h_c \quad , \quad 0 \leq h_c \leq h_b \quad \ldots (3.146)$$

(iii) RELATION FOR ET:

The following relations (Doorenbos et al.(1979),Mohan Rao et al. (1990)) for calculation of evapotranspiration are adopted.

$$ET = \frac{PET(\theta - W_p)}{(F_c - W_p)(1-p_f)}, \quad W_p < \theta \leq \theta_t \quad \ldots (3.147)$$

$$= PET, \quad \theta_t < \theta \leq \phi - \theta_r \quad \ldots (3.148)$$

$$= 0, \quad 0 \leq \theta \leq W_p \quad \ldots (3.149)$$

where, θ_r is the residual soil moisture content, h_b is the bubbling pressure, e is the base of natural (Napierian) logarithm, F_c is the field capacity, W_p is the wilting point, $(F_c - W_p)$ is the available soil moisture content, p_f is the fraction of available soil moisture after which ET = PET, $\theta_t [=(F_c - p_f(F_c - W_p)]$ is a dummy variable.

The solution provides h and h_c values at different times at all the nodes of the domain. The points of zero pressure at each column of the domain (Fig.3.5) are obtained by interpolating between two such successive rows where h_c changes its sign. These points are joined by curved line to get the water table position.

3.3.3 Finite Differences Approximation

The flow domain is discretized in horizontal and vertical directions by a finite number of nodal points. The differential equation for each node is written in terms of the implicit finite differences as follows.

3.3.3.1 Interior Nodes

Finite differences equation for an interior node (i,j) for a time step of kth to (k+1)th discrete times can be written as follows:

$$\frac{2.0}{\Delta x_j + \Delta x_{j-1}} \cdot \left[Kx_{ij} \frac{h_{ij+1k+1} - h_{ijk+1}}{\Delta x_j} - Kx_{ij-1} \frac{h_{ijk+1} - h_{ij-1k+1}}{\Delta x_{j-1}} \right] + \frac{2.0}{\Delta z_i + \Delta z_{i-1}} \left[Kz_{ij} \frac{h_{i+1jk+1} - h_{ijk+1}}{\Delta z_i} - Kz_{i-1j} \frac{h_{ijk+1} - h_{i-1jk+1}}{\Delta z_{i-1}} \right] - ET_i = C_{ij} \frac{h_{ijk+1} - h_{ijk}}{\Delta t} \qquad \ldots (3.150)$$

Where, i,j and k have already been defined in the saturated flow model (Section 3.1 and 3.2), h_{ijk+1} is the value of 'h' at the nodal point (i,j) at the end of (k + 1)th discrete time; Kx_{ij} = the capillary conductivity of the link between nodes (i,j) and (i, j+1); Kz_{ij} is the capillary conductivity of the link between nodes (i,j) and (i+1,j); Δx_j is the spacing between nodes (i,j) and (i,j+1), Δz_i is the spacing between nodes (i,j) and (i+1,j);

C_{ij} is the specific soil moisture capacity (in the unsaturated zone) or specific storage (in the saturated zone) at node (i,j); ET_i is rate of actual evapotranspiration per unit soil volume in the ith row; and Δt is size of the subtime step.

3.3.3.2 Boundary Nodes for Total Response Model

For the left half of the flow domain (Fig.3.5) the boundary conditions (Fig. 3.6(i) - (ix)) have been considered as follows:

(a) BOUNDARY 1-2-3

Drichlet boundary condition at boundary 1-2-3 is assigned in accordance with equation (3.134) i.e., for a node on boundary 1-2-3,

$$h_{ijk+1} = Y_0 \qquad \ldots(3.151)$$

(b) BOUNDARY 3-4

Neuman type of boundary condition in accordance with equation (3.135) is assigned ensuring water balance

i.e., for a node on boundary 3-4,

$$q_1 + q_2 + q_3 = C \frac{\partial h}{\partial t} \cdot \frac{\Delta x_j}{2} \frac{\Delta z_i + \Delta z_{i-1}}{2}$$

$$- ET \cdot \frac{\Delta x_j}{2} \cdot \frac{\Delta z_i + \Delta z_{i-1}}{2} \qquad \ldots(3.152)$$

(c) BOUNDARY 4-5

Neuman type of boundary condition in accordance with equation (3.136) is assigned ensuring water balance.

i.e., for a node at boundary 4-5

$$q_1 + q_2 + q_3 = C \frac{\partial h}{\partial t} \cdot \frac{\Delta z_i}{2} \cdot \frac{\Delta x_j + \Delta x_{j-1}}{2}$$

$$- ET \cdot \frac{\Delta z_i}{2} \cdot \frac{\Delta x_j + \Delta x_{j-1}}{2} \qquad \ldots(3.153)$$

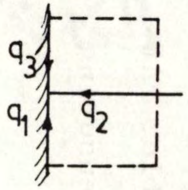


FIG 3.6 (I)
BOUNDARY 3-4

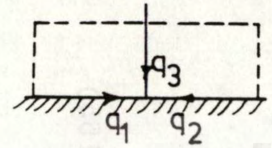


FIG 3.6 (II)
BOUNDARY 4-5

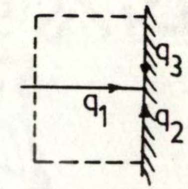


FIG 3.6(III)
BOUNDARY 5-8

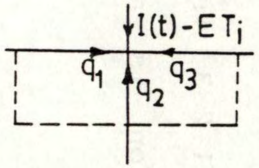


FIG 3.6 (IV)
BOUNDARY 8-9

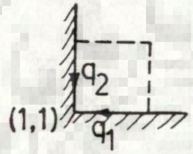


FIG 3.6(V)
POINT 4

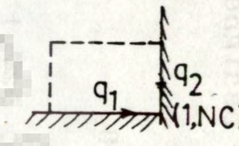


FIG 3.6(VI)
POINT 5

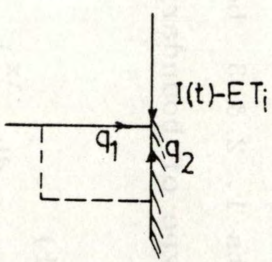


FIG 3.6(VII)
POINT 8

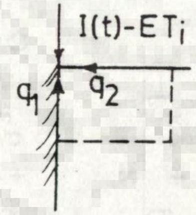


FIG 3.6(VIII)
POINT 9

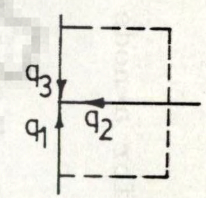


FIG 3.6(IX)
BOUNDARY 9-10

FIG. 3.6 (I)-(IX) BOUNDARY NODES FOR TOTAL RESPONSE MODEL

(d) BOUNDARY 5-8 (or 5-6-7-8)

The vertical plane passing through the midpoint between the drains is boundary 5-8. Neuman type of boundary condition in accordance with equation (3.137) is assigned ensuring water balance.

i.e. for a node at boundary 5-8,

$$q_1 + q_2 + q_3 = C \frac{\partial h}{\partial t} \cdot \frac{\Delta x_{j-1}}{2} \cdot \frac{\Delta z_i + \Delta z_{i-1}}{2}$$

$$- ET \cdot \frac{\Delta x_{j-1}}{2} \cdot \frac{\Delta z_i + \Delta z_{i-1}}{2} \qquad \ldots(3.154)$$

(e) BOUNDARY 8-9

Neuman type of boundary condition in accordance with equation (3.139) is assigned ensuring water balance.

$$I \cdot \frac{\Delta x_j + \Delta x_{j-1}}{2} - ET \cdot \frac{\Delta z_{i-1}}{2} \cdot \frac{\Delta x_j + \Delta x_{j-1}}{2}$$

$$+ q_1 + q_2 + q_3 = C \cdot \frac{\partial h}{\partial t} \cdot \frac{\Delta z_{i-1}}{2} \cdot \frac{\Delta x_j + \Delta x_{j-1}}{2} \qquad \ldots (3.155)$$

(f) BOUNDARY 9-10

At the ditch face, the following Neuman type of boundary condition is assigned:

$$q_1 + q_2 + q_3 = C \cdot \frac{\partial h}{\partial t} \cdot \frac{\Delta x_j}{2} \cdot \frac{\Delta z_i + \Delta z_{i-1}}{2}$$

$$- ET \cdot \frac{\Delta x_j}{2} \cdot \frac{\Delta z_i + \Delta z_{i-1}}{2} \qquad (3.156)$$

At points 1, 2 & 3 boundary 1-2-3 is applicable. At point 4, Neuman type of boundary condition is assigned ensuring water balance.

i.e. for node (1,1)

$$q_1 + q_2 = C \frac{\partial h}{\partial t} \cdot \frac{\Delta x_j}{2} \frac{\Delta z_i}{2} - ET \frac{\Delta x_j}{2} \frac{\Delta z_i}{2} \qquad (3.157)$$

At point 5, Neuman type of boundary condition is assigned ensuring water balance.

i.e. , for Node (1,NC)

$$q_1 + q_2 = C \frac{\partial h}{\partial t} \cdot \frac{\Delta x_{j-1}}{2} \frac{\Delta z_i}{2} - ET \frac{\Delta z_i}{2} \frac{\Delta x_{j-1}}{2} \quad (3.158)$$

At point 8, Neuman type of boundary condition is assigned ensuring water balance.

i.e. for Node (NR,NC)

$$q_1 + q_2 + I \frac{\Delta x_{j-1}}{2} - ET \frac{\Delta x_{j-1}}{2} \frac{\Delta z_{i-1}}{2}$$

$$= C \frac{\partial h}{\partial t} \cdot \frac{\Delta x_{j-1}}{2} \frac{\Delta z_{i-1}}{2} \quad (3.159)$$

At point 9, Neuman type of boundary condition is assigned ensuring water balance.

i.e., for Node (NR,1)

$$I \cdot \frac{\Delta x_j}{2} - ET \frac{\Delta z_{i-1}}{2} \frac{\Delta x_j}{2} + q_1 + q_2$$

$$= C \frac{\partial h}{\partial t} \cdot \frac{\Delta x_j}{2} \frac{\Delta z_{i-1}}{2} \quad \ldots (3.160)$$

In the above equations for boundary conditions, the flows q_1, q_2 and q_3 are evaluated in accordance with Darcy's Law.

3.3.4 Solution of the Finite Differences Equations

The differential equation (3.150) written in terms of the implicit finite differences is linear. This is solved, subjected to boundary conditions (equations 3.151-3.160), by the iterative ADIE scheme (Remson et al. 1971, Ahmad et al.1991) proposed by Peaceman and Rachford (1955).

3.3.4.1 Iterative Alternating Direction Implicit Explicit Scheme

The finite differences equation for a node (i,j) for a time step of (k)th to (k+1)th discrete times and lth iteration can be written as follows.

$$\frac{2}{\Delta x_j + \Delta x_{j-1}} \left[Kx_{ij} \frac{h^{(m)}_{ij+1k+1} - h^{(m)}_{ijk+1}}{\Delta x_j} - Kx_{ij-1} \right.$$

$$\left. \frac{h^{(m)}_{ijk+1} - h^{(m)}_{ij-1k+1}}{\Delta x_{j-1}} \right] + \frac{2}{\Delta z_i + \Delta z_{i-1}} \left[Kz_{ij} \frac{h^{(n)}_{i+1jk+1} - h^{(n)}_{ijk+1}}{\Delta z_i} \right.$$

$$\left. - Kz_{i-1j} \frac{h^{(n)}_{ijk+1} - h^{(n)}_{i-1jk+1}}{\Delta z_{i-1}} \right]$$

$$= C_{ij} \frac{h^{(m)}_{ijk+1} - h_{ijk}}{\Delta t} + ET_i \qquad \ldots (3.161)$$

where, $m = l$ and $n = l-1$ for solving implicitly along x - direction and explicitly along z-direction; $m = l-1$ and $n=l$ for solving implicitly along z- direction and explicitly along x-direction, $h^{(0)}_{ijk+1} = h_{ijk}$ is the initial h value at the beginning of time step Δt at nodal point (i,j).

3.3.4.2 Matrices for Solving Implicitly along x - Direction

Equation (3.161) and equations for boundary nodes in terms of finite differences are arranged as follows to solve implicitly along x-direction.

$$A_j\, h_{ij-1k+1} + B_j\, h_{ijk+1} + C_j\, h_{ij+1k+1} = D_j \qquad \ldots (3.162)$$

In equation (3.162) the (A_j), (B_j), (C_j) and (D_j) are the matrices of (NC)th order. These are completely defined in terms of known spatial and temporal step sizes, aquifer parameters, soil water properties, and $\left[h^{(l-1)}_{ijk+1}\right]$ computed in preceding iteration as follows.

(a) INTERIOR NODES

$$A_j = \frac{2}{\Delta x_j + \Delta x_{j-1}} \quad \frac{Kx_{ij}}{\Delta x_{j-1}} \qquad \ldots (3.163)$$

$$B_j = -\frac{2}{\Delta x_j + \Delta x_{j-1}}\left[\frac{Kx_{ij}}{\Delta x_j} + \frac{Kx_{ij-1}}{\Delta x_{j-1}}\right]$$

$$-\frac{2}{\Delta z_i + \Delta z_{i-1}}\left[\frac{Kz_{ij}}{\Delta z_i} + \frac{Kz_{i-1j}}{\Delta z_{i-1}}\right] - \frac{C_{ij}}{\Delta t} \qquad (3.164)$$

$$C_j = \frac{2}{\Delta x_j + \Delta x_{j-1}} \quad \frac{Kx_{ij}}{\Delta x_j} \qquad . (3.165)$$

$$D_j = \frac{-2}{\Delta z_i + \Delta z_{i-1}}\left[Kz_{ij}\frac{h^{(l-1)}_{i+1jk+1}}{\Delta z_i} + Kz_{i-1j}\right.$$

$$\left.\frac{h^{(l-1)}_{i-1jk+1}}{\Delta z_{i-1}}\right] - C_{ij}\frac{h_{ijk+1}}{\Delta t} + ET_i \qquad (3.166)$$

(b) NODES ON BOUNDARY 1-2-3

$$A_j = 0.0 \qquad \ldots (3.167)$$

$$B_j = 1.0 \qquad \ldots (3.168)$$

$$C_j = 0.0 \qquad \ldots (3.169)$$

$$D_j = Y_0 \qquad \ldots (3.170)$$

(c) NODES ON BOUNDARY 3-4

$$A_j = 0.0 \qquad \ldots (3.171)$$

$$B_j = -\left[\frac{2Kx_{ij}}{(\Delta x_j)^2} + \frac{C_{ij}}{\Delta t}\right]$$

$$-\frac{2}{\Delta z_i + \Delta z_{i-1}}\left[\frac{Kz_{ij}}{\Delta z_i} + \frac{Kz_{i-1j}}{\Delta z_{i-1}}\right] \qquad \ldots (3.172)$$

$$C_j = \frac{2Kx_{ij}}{(\Delta x_j)^2} \qquad \ldots (3.173)$$

$$D_j = \left[- C_{ij} \frac{h_{ijk}}{\Delta t} + ET_i \right.$$

$$\left. - \frac{2}{\Delta z_i + \Delta z_{i-1}} \left[Kz_{i-1j} \frac{h^{(l-1)}_{i-1jk+1}}{\Delta z_{i-1}} + Kz_{ij} \frac{h^{(l-1)}_{i+1jk+1}}{\Delta z_i} \right] \right] \quad \ldots (3.174)$$

(d) NODES ON BOUNDARY 4-5

$$A_j = \frac{2}{\Delta x_j + \Delta x_{j-1}} \cdot \frac{Kx_{ij-1}}{\Delta x_{j-1}} \quad \ldots (3.175)$$

$$B_j = - \frac{2}{\Delta x_j + \Delta x_{j-1}} \left[\frac{Kx_{ij}}{\Delta x_j} + \frac{Kx_{ij-1}}{\Delta x_{j-1}} \right] - \frac{C_{ij}}{\Delta t} \quad (3.176)$$

$$C_j = \frac{2}{\Delta x_j + \Delta x_{j-1}} \left[\frac{Kx_{ij}}{\Delta x_j} \right] \quad \ldots (3.177)$$

$$D_j = - C_{ij} \frac{h_{ijk}}{\Delta t} + ET_i - 2Kz_{ij} \frac{h^{(l-1)}_{i+1jk+1}}{(\Delta z_i)^2} \quad \ldots (3.178)$$

(e) NODES ON BOUNDARY 5-8

$$A_j = \frac{2Kx_{ij-1}}{(\Delta x_{j-1})^2} \quad \ldots (3.179)$$

$$B_j = - \frac{2Kx_{ij-1}}{\Delta x_{j-1}} - \frac{C_{ij}}{\Delta t} - \frac{2}{\Delta z_i + \Delta z_{i-1}} \left[\frac{Kz_{ij}}{\Delta z_i} + \frac{Kz_{i-1j}}{\Delta z_{i-1}} \right] \quad \ldots (3.180)$$

$$C_j = 0.0 \quad \ldots (3.181)$$

$$D_j = - C_{ij} \frac{h_{ijk}}{\Delta t} - \frac{2}{\Delta z_i + \Delta z_{i-1}} \left[Kz_{ij} \frac{h^{(l-1)}_{i+1jk+1}}{\Delta z_i} + Kz_{i-1j} \frac{h^{(l-1)}_{i-1jk+1}}{\Delta z_{i-1}} \right] + ET_i \qquad \ldots(3.182)$$

(f) NODES ON BOUNDARY 8-9

$$A_j = \frac{Kx_{ij-1}}{2} \frac{\Delta z_{i-1}}{\Delta x_{j-1}} \qquad \ldots(3.183)$$

$$B_j = - \frac{Kz_{i-1j}}{2} \cdot \frac{\Delta x_j + \Delta x_{j-1}}{\Delta z_{i-1}} - \frac{Kx_{ij-1}}{2} \cdot \frac{\Delta z_{i-1}}{\Delta x_{j-1}} - \frac{Kx_{ij}}{2} \cdot \frac{\Delta z_{i-1}}{\Delta x_j} - \frac{C_{ij}}{\Delta t} \frac{(\Delta x_j + \Delta x_{j-1})}{2} \frac{(\Delta z_{i-1})}{2} \qquad \ldots(3.184)$$

$$C_j = \frac{Kx_{ij}}{2} \cdot \frac{\Delta z_i}{\Delta x_j} \qquad \ldots(3.185)$$

$$D_j = - I . \frac{\Delta x_j + \Delta x_{j-1}}{2} + ET_i \frac{\Delta x_j + \Delta x_{j-1}}{2} \cdot \frac{\Delta z_{i-1}}{2} - Kz_{i-1j} \cdot \frac{h^{(l-1)}_{i-1jk+1}}{\Delta z_i} \frac{\Delta x_j + \Delta x_{j-1}}{2} - C_{ij} \cdot \frac{\Delta z_{i-1}}{2} \cdot \frac{\Delta x_j + \Delta x_{j-1}}{2} \cdot \frac{h_{ijk}}{\Delta t} \qquad \ldots(3.186)$$

(g) NODES ON BOUNDARY 9-10

$$A_j = 0.0 \qquad (3.187)$$

$$B_j = - \frac{Kx_{ij}}{2} \cdot \frac{\Delta z_i + \Delta z_{i-1}}{\Delta x_j} - \frac{Kz_{ij}}{2} \cdot \frac{\Delta x_j}{\Delta z_i}$$

$$- \frac{Kz_{i-1j}}{2} \frac{\Delta x_j}{\Delta z_{i-1}} - \frac{C_{ij}}{\Delta t} \frac{\Delta x_j}{2} \frac{\Delta z_i + \Delta z_{i-1}}{2} \quad \ldots \quad (3.188)$$

$$C_j = Kx_{ij} \cdot \frac{\Delta z_i + \Delta z_{i-1}}{2\Delta x_j} \quad \ldots (3.189)$$

$$D_j = - Kz_{ij} \frac{h^{(l-1)}_{i+1jk+1}}{\Delta z_i} \cdot \frac{\Delta x_j}{2}$$

$$- Kz_{i-1j} \frac{h^{(l-1)}_{i-1jk+1}}{\Delta z_i} \cdot \frac{\Delta x_j}{2}$$

$$- C_{ij} \frac{h_{ijk}}{\Delta t} \cdot \frac{\Delta x_j}{2} \cdot \frac{\Delta z_i + \Delta z_{i-1}}{2}$$

$$- ET_i \cdot \frac{\Delta x_j}{2} \cdot \frac{\Delta z_i + \Delta z_{i-1}}{2} \quad \ldots (3.190)$$

(h) NODES ON BOUNDARY 10-1

$A_j = 0.0$ ...(3.191)

$B_j = 1.0$ ...(3.192)

$C_j = 0.0$ ...(3.193)

$D_j = z$ ...(3.194)

(i) NODE (1,1)

$A_j = 0.0$ ...(3.195)

$$B_j = - \frac{Kx_{ij}}{(\Delta x_j)^2} - \frac{Kz_{ij}}{(\Delta z_i)^2} - \frac{C_{ij}}{2\Delta t} \quad \ldots (3.196)$$

$$C_j = \frac{Kx_{ij}}{(\Delta x_j)^2} \quad \ldots (3.197)$$

$$D_j = -C_{ij} \frac{h_{ijk}}{2\Delta t} - \frac{ET_i}{2} - Kz_{ij} \frac{h^{(l-1)}_{i+1jk+1}}{(\Delta z_i)^2} \quad (3.198)$$

(j) NODE (1,NC)

$$A_j = \frac{Kx_{ij-1}}{(\Delta x_{j-1})^2} \qquad \text{...(3.199)}$$

$$B_j = - \frac{Kx_{ij-1}}{(\Delta x_{j-1})^2} - \frac{Kz_{ij}}{(\Delta z_i)^2} - \frac{C_{ij}}{2\Delta t} \qquad \text{...(3.200)}$$

$$C_j = 0.0 \qquad \text{(3.201)}$$

$$D_j = - Kz_{ij} \frac{h^{(l-1)}_{i+1jk+1}}{(\Delta z_i)^2} + \frac{ET_i}{2} - C_{ij} \frac{h_{ijk}}{2\Delta t}$$

...(3.202)

(k) NODE (NR,NC)

$$A_j = Kx_{ij-1} \frac{\Delta z_{i-1}}{2\Delta x_{j-1}} \qquad \text{...(3.203)}$$

$$B_j = -Kz_{i-1j} \frac{\Delta x_{j-1}}{2\Delta z_{i-1}} - Kx_{ij-1} \frac{\Delta z_{i-1}}{2\Delta x_{j-1}}$$

$$- C_{ij} \frac{\Delta x_{j-1}}{2} \frac{\Delta z_{i-1}}{2\Delta t} \qquad \text{...(3.204)}$$

$$C_j = 0.0 \qquad \text{...(3.205)}$$

$$D_j = -I \frac{\Delta x_{j-1}}{2} + ET_i \frac{\Delta z_{i-1}}{2} \frac{\Delta x_{j-1}}{2}$$

$$- Kz_{i-1j} \frac{h^{(l-1)}_{i-1jk+1}}{2} \cdot \frac{\Delta x_{j-1}}{\Delta z_{i-1}} - C_{ij} \frac{h_{ijk}}{\Delta t}$$

$$\frac{\Delta z_{i-1}}{2} \cdot \frac{\Delta x_{j-1}}{2} \qquad \text{... (3.206)}$$

(1) NODE AT POINT 9

$$A_j = 0.0 \qquad \ldots (3.207)$$

$$B_j = -Kx_{ij} \frac{\Delta z_{i-1}}{2\Delta x_j} - Kz_{i-1j} \frac{\Delta x_j}{2\Delta z_{i-1}}$$

$$- C_{ij} \Delta x_j \frac{\Delta z_{i-1}}{4\Delta t} \qquad \ldots (3.208)$$

$$C_j = Kx_{ij} \frac{\Delta z_{i-1}}{2\Delta x_j} \qquad \ldots (3.209)$$

$$D_j = -I \frac{\Delta x_j}{2} + ET_i \frac{\Delta x_j}{2} \frac{\Delta z_{i-1}}{2} - Kz_{i-1j}$$

$$\frac{h^{(l-1)}_{i-1jk+1}}{\Delta z_{i-1}} \cdot \frac{\Delta x_j}{2} - C_{ij} \frac{h_{ijk}}{\Delta t} \frac{\Delta x_j}{2} \frac{\Delta z_{i-1}}{2} \qquad \ldots (3.210)$$

3.3.4.3 Matrices for Solving Implicitly along z-Direction

Equation (3.161) and equations for boundary nodes (3.151-3.160) in terms of finite differences are arranged as follows to solve implicitly along z-direction.

$$A_i h_{i-1jk+1} + B_i h_{ijk+1} + C_i h_{i+1jk+1} = D_i \qquad \ldots (3.211)$$

In equation (3.211) the (A_i), (B_i), (C_i) and (D_i) are the matrices of (NR)th order. These are completely defined in terms of known spatial and temporal step sizes, aquifer parameters, soil hydraulic properties, and h computed in the preceding iteration $\left[\text{i.e. } h^{(l-1)}_{ijk+1}\right]$ as follows.

(a) INTERIOR NODES

$$A_i = \frac{2}{\Delta z_i + \Delta z_{i-1}} \quad \frac{Kz_{i-1j}}{\Delta z_{i-1}} \qquad \ldots (3.212)$$

$$B_i = - \frac{2}{\Delta z_i + \Delta z_{i-1}} \cdot \left[\frac{Kz_{ij}}{\Delta z_i} + \frac{Kz_{i-1j}}{\Delta z_{i-1}} \right] - \frac{C_{ij}}{\Delta t}$$

$$- \frac{2}{\Delta x_j + \Delta x_{j-1}} \cdot \left[\frac{Kx_{ij}}{\Delta x_j} + \frac{Kx_{ij-1}}{\Delta x_{j-1}} \cdot \right] \qquad \ldots (3.213)$$

$$C_i = \frac{2}{\Delta z_i + \Delta z_{i-1}} \quad \frac{Kz_{ij}}{\Delta z_i} \qquad \ldots (3.214)$$

$$D_i = - \frac{2}{\Delta x_j + \Delta x_{j-1}} \left[Kx_{ij} \frac{h_{ij+1k+1}^{(l-1)}}{\Delta x_j} + Kx_{ij-1} \right.$$

$$\left. \frac{h_{ij-1k+1}^{(l-1)}}{\Delta x_{j-1}} \right] - C_{ij} \frac{h_{ijk}}{\Delta t} + ET_i \qquad \ldots (3.215)$$

(b) NODES ON BOUNDARY 1-2-3

$$A_i = 0.0 \qquad \ldots (3.216)$$

$$B_i = 1.0 \qquad \ldots (3.217)$$

$$C_i = 0.0 \qquad \ldots (3.218)$$

$$D_i = Y_0 \qquad \ldots (3.219)$$

(c) NODES ON BOUNDARY 3-4

$$A_i = \frac{2}{\Delta z_i + \Delta z_{i-1}} \cdot \frac{Kz_{i-1j}}{\Delta z_{i-1}} \qquad \ldots (3.220)$$

$$B_i = -\frac{2}{\Delta z_i + \Delta z_{i-1}} \cdot \left[\frac{Kz_{ij}}{\Delta z_i} + \frac{Kz_{i-1j}}{\Delta z_{i-1}}\right]$$

$$-\frac{C_{ij}}{\Delta t} - \frac{2Kx_{ij}}{(\Delta x_j)^2} \quad \text{...(3.221)}$$

$$C_i = \frac{2}{\Delta z_i + \Delta z_{i-1}} \cdot \frac{Kz_{ij}}{\Delta z_i} \quad \text{...(3.222)}$$

$$D_i = -2Kx_{ij} \frac{h^{(l-1)}_{ij+1k+1}}{(\Delta x_j)^2} - C_{ij} \frac{h_{ijk}}{\Delta t} + ET_i \quad \text{...(3.223)}$$

(d) NODES ON BOUNDARY 4-5

$$A_i = 0.0 \quad \text{...(3.224)}$$

$$B_i = -\frac{2Kz_{ij}}{(\Delta z_i)^2} - \frac{2}{\Delta x_j + \Delta x_{j-1}} \left[\frac{Kx_{ij}}{\Delta x_j} + \frac{Kx_{ij-1}}{\Delta x_{j-1}}\right]$$

$$-\frac{C_{ij}}{\Delta t} \quad \text{...(3.225)}$$

$$C_i = \frac{2Kz_{ij}}{(\Delta z_i)^2} \quad \text{...(3.226)}$$

$$D_i = -C_{ij} \frac{h_{ijk}}{\Delta t} + ET_i - \frac{2}{\Delta x_j + \Delta x_{j-1}} \left[Kx_{ij} \frac{h^{(l-1)}_{ij+1k+1}}{\Delta x_j}\right.$$

$$\left. + Kx_{ij-1} \frac{h^{(l-1)}_{ij-1k+1}}{\Delta x_{j-1}}\right] \quad \text{...(3.227)}$$

(e) NODES ON BOUNDARY 5-8(or 5-6-7-8)

$$A_i = \frac{2}{\Delta z_i + \Delta z_{i-1}} \cdot \frac{Kz_{i-1j}}{\Delta z_{i-1}} \quad \text{...(3.228)}$$

$$B_i = -\frac{2}{\Delta z_i + \Delta z_{i-1}} \cdot \left[\frac{Kz_{ij}}{\Delta z_i} + \frac{Kz_{i-1j}}{\Delta z_{i-1}}\right]$$

$$-\frac{C_{ij}}{\Delta t} - \frac{2Kx_{ij-1}}{(\Delta x_{j-1})^2} \qquad ..(3.229)$$

$$C_i = \frac{2}{\Delta z_i + \Delta z_{i-1}} \cdot \frac{Kz_{ij}}{\Delta z_i} \qquad ...(3.230)$$

$$D_i = -2Kx_{ij-1} \cdot \frac{h^{(l-1)}_{ij-1k+1}}{(\Delta x_{j-1})^2} - C_{ij}\frac{h_{ijk}}{\Delta t} + ET_i \qquad ..(3.231)$$

(f) NODES ON BOUNDARY 8-9

$$A_i = Kz_{i-1j}\frac{\Delta x_j + \Delta x_{j-1}}{2\Delta z_{i-1}} \qquad ...(3.232)$$

$$B_i = -Kz_{i-1j} \cdot \frac{\Delta x_j + \Delta x_{j-1}}{2\Delta z_{i-1}} - Kx_{ij-1} \cdot \frac{\Delta z_{i-1}}{2\Delta x_{j-1}}$$

$$- Kx_{ij} \cdot \frac{\Delta z_{i-1}}{2\Delta x_j} - \frac{C_{ij}}{\Delta t} \cdot \frac{\Delta x_j + \Delta x_{j-1}}{2} \cdot \frac{\Delta z_{i-1}}{2}$$

...(3.233)

$$C_i = 0.0 \qquad (3.234)$$

$$D_i = -I \cdot \frac{\Delta x_j + \Delta x_{j-1}}{2} - Kx_{ij-1}\frac{h^{(l-1)}_{ij-1k+1}}{2} \cdot \frac{\Delta z_{i-1}}{\Delta x_{j-1}}$$

$$- Kx_{ij} \frac{h^{(l-1)}_{ij+1k+1}}{2} \Bigg] . \frac{\Delta z_{i-1}}{\Delta x_j}$$

$$- C_{ij} \frac{h_{ijk}}{\Delta t} \left(\frac{\Delta x_j + \Delta x_{j-1}}{2} \right) . \frac{\Delta z_{i-1}}{2}$$

$$+ ET_i \left(\frac{\Delta x_j + \Delta x_{j-1}}{2} \right) . \frac{\Delta z_{i-1}}{2} \qquad \ldots(3.235)$$

(g) NODES ON BOUNDARY 9-10

$$A_j = Kz_{i-1j} \frac{\Delta x_j}{2 \Delta z_{i-1}} \qquad \ldots(3.236)$$

$$B_i = -Kx_{ij} . \frac{\Delta z_i + \Delta z_{i-1}}{2 \Delta x_j} - C_{ij} . \Delta x_j . \frac{\Delta z_i + \Delta z_{i-1}}{4 \Delta t}$$

$$- Kz_{ij} . \frac{\Delta x_j}{2 \Delta z_i} - Kz_{i-1j} . \frac{\Delta x_j}{2 \Delta z_{i-1}} \qquad (3.237)$$

$$C_i = Kz_{ij} \frac{\Delta x_j}{2 \Delta z_i} \qquad (3.238)$$

$$D_i = - Kx_{ij} . \frac{h^{(l-1)}_{ij+1k+1}}{\Delta x_j} . \frac{\Delta z_i + \Delta z_{i-1}}{2}$$

$$+ ET_i . \Delta x_j \left(\frac{\Delta z_i + \Delta z_{i-1}}{4} \right)$$

$$- C_{ij} . h_{ijk} . \Delta x_j \left(\frac{\Delta z_i + \Delta z_{i-1}}{4 \Delta t} \right) \qquad (3.239)$$

(h) NODES ON BOUNDARY 10-1

$$A_i = 0.0 \qquad \ldots(3.240)$$

$$B_j = 1.0 \qquad \ldots(3.241)$$

$$C_i = 0.0 \qquad \ldots(3.242)$$

$$D_i = z \qquad \ldots(3.243)$$

(i) NODE (1,1)

$$A_i = 0.0 \quad \ldots(3.244)$$

$$B_i = -\frac{Kz_{ij}}{(\Delta z_i)^2} - \frac{Kx_{ij}}{(\Delta x_j)^2} - \frac{C_{ij}}{2\Delta t} \quad \ldots(3.245)$$

$$C_i = \frac{Kz_{ij}}{(\Delta z_i)^2} \quad \ldots(3.246)$$

$$D_i = -Kx_{ij}\frac{h_{ij+1k+1}}{(\Delta x_j)^2} - C_{ij}\frac{h_{ijk}}{2\Delta t} + \frac{ET_{ij}}{2} \quad \ldots(3.247)$$

(j) NODE (1,NC)

$$A_i = 0.0 \quad \ldots(3.248)$$

$$B_i = -\frac{Kz_{ij}}{(\Delta z_i)^2} - \frac{Kx_{ij-1}}{(\Delta x_{j-1})^2} - \frac{C_{ij}}{2\Delta t} \quad \ldots(3.249)$$

$$C_i = \frac{Kz_{ij}}{(\Delta z_i)^2} \quad \ldots(3.250)$$

$$D_i = -Kx_{ij-1}\frac{h^{(l-1)}_{ij-1k+1}}{(\Delta x_{j-1})^2} - C_{ij}\frac{h_{ijk}}{2\Delta t} + \frac{ET_{ij}}{2} \quad \ldots(3.251)$$

(k) NODE (NR,NC)

$$A_i = Kz_{i-1j}\frac{\Delta x_{j-1}}{2\Delta z_{i-1}} \quad \ldots(3.252)$$

$$B_i = -Kz_{i-1j}\frac{\Delta x_{j-1}}{2\Delta z_{i-1}} \quad Kx_{ij-1}\frac{\Delta z_i}{2\Delta x_{j-1}}$$

$$- C_{ij}.\Delta x_{j-1}\left(\frac{\Delta z_{i-1}}{4\Delta t}\right) \quad \ldots(3.253)$$

$$C_i = 0.0 \qquad (3.254)$$

$$D_i = -I \frac{\Delta x_{j-1}}{2} + - Kx_{ij-1} \frac{h_{ij-1k+1}^{(l-1)}}{\Delta x_{j-1}} . \frac{\Delta z_{i-1}}{2} - C_{ij}(\frac{h_{ijk}}{\Delta t}) (\frac{\Delta x_{j-1}}{2}) . (\frac{\Delta z_{i-1}}{2}) + ET_i . \Delta x_j . \frac{\Delta z_i}{4} \qquad (3.255)$$

(l) NODE AT POINT 9

$$A_i = Kz_{i-1j} \frac{\Delta x_j}{2\Delta z_{i-1}} \qquad (3.256)$$

$$B_i = -Kz_{i-1j} \frac{\Delta x_j}{2\Delta z_{i-1}} - Kx_{ij} \frac{\Delta z_{i-1}}{2\Delta x_j} - C_{ij} \Delta x_j \frac{\Delta z_{i-1}}{4\Delta t} \qquad (3.257)$$

$$C_i = 0.0 \qquad (3.258)$$

$$D_i = -I \frac{\Delta x_j}{2} + ET_i \frac{\Delta x_j}{2} \frac{\Delta z_{i-1}}{2} - Kx_{ij} \frac{h_{ij+1k+1}^{(l-1)}}{\Delta x_j} \frac{\Delta z_{i-1}}{2} - C_{ij} \frac{h_{ijk}}{\Delta t} \frac{\Delta x_j}{2} \frac{\Delta z_{i-1}}{2} \qquad (3.259)$$

The coefficient matrices (3.163) to (3.210) are substituted in equation (3.162) and the coefficient matrices (3.212) to (3.259) are substituted in equation (3.211). The equations (3.162) and (3.211) are then solved using the Thomas Algorithm (Remson et al.1971) as the governing coefficient matrices [(3.163) to (3.210) and (3.212) to (3.259)] are tridiagonal and positive definite. These matrices contain coefficients derived from numerical approximation to spatial derivatives. Moreover, they are defined such that only the derivatives in one space variable are represented in each matrix. The equation (3.162)/(3.211) are solved implicitly in one space dimension (x/z) using the known values in other space dimension (z/x). This sequential sweeping of the matrices is done first horizontally (equation (3.162) and then vertically (equation (3.211)). This leads to the computation of 'h' values at all the finite differences nodes and at the discrete times considered. Thereafter, the h_c values at all the nodes, are calculated. The water table position ($h_c = 0$) at each column is obtained by interpolating between two such successive rows where h_c changes its sign.

3.3.5 Convergence Criteria

At the end of a discrete time (say k+1) the difference of h values of two successive iterations at each of the finite differences nodes is computed. The ADIE iterations are continued until the biggest of these differences is less than a prestipulated convergence factor, $\epsilon 4$. Thus, for ADIE solution the convergence criteria is as follows.

$$\text{Max}\left[\text{Abs}\left(h^{(l)}_{ijk+1} - h^{(l-1)}_{ijk+1}\right)\right] < \epsilon 4 \qquad \ldots (3.260)$$

where, $h^{(l)}_{ijk+1}$ is the h value at a node (i,j) after lth ADIE iteration, $h^{(l-1)}_{ijk+1}$ is the h value at the node after (l-1)th iteration.

The h values are computed at each node by the above procedure at the discrete times separated by non-uniform time steps. Later each time step is divided into appropriate number of subtime steps so that the computed h value at the end of the time step does not change significantly with further increase in the number of subtime steps (Ahmad et.al., 1991); i.e.,

$$\text{Max}\left[\text{Abs}\ (h_{ijk+1})_{np} - (h_{ijk+1})_{np/2}\right] < \varepsilon 5 \quad \ldots (3.261)$$

where, $(h_{ijk+1})_{np}$ is the h value computed with np number of subtime steps and $(h_{ijk+1})_{np/2}$ is computed with np/2 number of subtime steps at the end of (k+1)th discrete time.

3.4 COMPUTATION OF VELOCITIES AND OUTFLOW TO DRAINS

The horizontal and vertical velocities have been calculated as follows:

$$U_{ijk+1} = \left(\frac{h_{ij+1k+1} - h_{ijk+1}}{\Delta x_j} \right) \cdot Kx_{ij} \quad \ldots (3.262)$$

$$V_{ijk+1} = \left(\frac{h_{i+1jk+1} - h_{ijk+1}}{\Delta z_i} \right) \cdot Kz_{ij} \quad \ldots (3.263)$$

Where 'h', 'Kx' and 'Kz' bear different meanings in saturated flow model(SFM) and the total response model(TRM) and are considered accordingly (refer section 3.1 and 3.2 for saturated flow model and 3.3 for total response model). While computing by SFM the Kx and Kz are the saturated hydraulic conductivities, whereas, in the TRM computation the Kx and Kz are defined by equations 3.126-3.127 and 3.129-3.130 depending upon θ ($\theta < \phi$ or $\theta = \phi$). For $\theta < \phi$ the Kx and Kz are computed as part of the TRM solution in accordance with equations (3.141 - 3.144) proposed by Brooks and Corey (1964).

Here, U_{ijk+1} is the horizontal velocity component in the domain (i,j) to (i, j+1) at the $(k+1)^{th}$ discrete time; V_{ijk+1} is the vertical velocity component in the domain (i,j) to (i+1,j) at the $(k+1)^{th}$ discrete time; h_{ijk+1} is the computed 'h' at node (i,j) at (k+1)th time; Δz_i is the spacing between nodes (i,j) and (i+1,j); Kx_{ij} and Kz_{ij} are the hydraulic conductivities of the domain (i,j+1)to (i,j) and (i+1,j) to (i,j) respectively.

An integration of normal velocity over the drain boundary leads to the computation of discharge q (L^2T^{-1}) entering the drain per unit length. The integration is performed numerically by trapezoidal rule. Thus q is given by:

$$q = \sum U_{ij} \frac{\Delta z_{i-1} + \Delta z_i}{2} + \sum V_{ij} \frac{\Delta x_j + \Delta x_{j-1}}{2} \tag{3.264}$$

3.5 SUBSURFACE DRAINAGE BY TILES

The two models, the saturated flow model and the total response model, explained in sections 3.2 and 3.3 have been presented for drainage by parallel ditches. For tile drainage the boundary conditions will be modified as follows:

3.5.1 Saturated Flow Model

The boundary conditions [equations 3.2,3.3 and 3.5(or 3.95] will be modified by replacing the width of the ditch (b) by the diameter of the tile (2r) and the depth of the ditch (d) below drain level by the radius (r) of the tile (i.e., b/2 or d = r).

3.5.2 Total Response Model

The boundary conditions (equations 3.133-3.140) will be modified by replacing d and b/2. In equations 3.134-3.135, d and b/2 will be replaced by radius of the tile (r). In equations 3.138-3.140 , b/2 will be replaced by zero.

3.6 COMPUTER CODE

The computer codes, for performing the calculations of the saturated flow model and the total response model have been written in FORTRAN IV.

3.6.1 Computer Code of the SFM.

The programme consists of one subroutine and a main programme. Role of the main programme and the subroutine is described briefly in the following paragraphs.

3.6.1.1 Main Programme

The following tasks are performed.

(A) Reading of all the input data

The details of the READ statement are as follows:

(i) OWT(LTM): Variation with time of the observed water table rise at mid section, NT: number of time steps, NR: number of rows, NC: number of columns, KOUNT: maximum number of iterations for convergence of additional domain above drain level (spacing between NRth and (NR-1)th rows at Jth column, i.e., DDZ(J)), ALW: convergence factor for ADIES, NRD: number of the row passing through the lowest point of the drain, NCD1: number of the column passing through the meeting point of drain face of first drain and initial horizontal water table (drain level), NCD2: number of the column passing through the meeting point of drain face of second drain and initial drain level.

(ii) JD(I): number of the column in Ith row (I = NRD....NR) passing through the drain boundary, EPS: convergence factor of piezometric head with respect to subtime steps, KTM: maximum number of reductions allowed for discretization of a time step into subtime steps. ISYM (=0): complete domain between the drains is analyzed, ISYM (=1): considering symmetry only left half of the domain is analysed, NCONF (=0):flow above drain level is accounted for, NCONF (=100) flow above drain level is neglected, EPS1: convergence factor for DDZ(J), KOUNT1: maximum number of iterations for DDZ(J) modification.

(iii) DT(IT): time step size for (IT)th time step, IT: number of the time step varying from 1 to NT.

(iv) HH(I,J):piezometric head at the beginning of a time step at a node (I,J) (i.e., piezometric head distribution).

(v) SS(I,J): specific storage distribution

(vi) SY(J): specific yield distribution

(vii) DX(J): horizontal grid spacing

(viii) DZ(I): vertical grid spacing

(ix) AKX (I,J): distribution of hydraulic conductivity in x-direction.

(x) AKZ(I,J): distribution of hydraulic conductivity in z-direction.

(xi) H1: piezometric head at the left hand side drain, H2: piezometric head at the right hand side drain, R(IT): distribution of recharge rate.

(B) computations of the distribution of piezometric head.

(C) computations of the distribution of horizontal velocity, VX(I,J).

(D) computations of the distribution of vertical velocities, VZ (I,J).

(E) computation of lateral flows to the drain

(F) comparison of the computed and observed water table elevations at midsection.

(G) computations of the model efficiency

(H) printing the computed results.

3.6.1.2 Subroutine Called: STN

STN: In this subroutine, the matrix generated by the finite differences approximation is solved using the Thomas Algorithm (Remson et al, 1971).

3.6.2 Computer Code of the TRM

The programme consists of six subroutines and a main programme. Role of the main programme and each subroutine is described briefly in the following paragraphs.

3.6.2.1 Main Programme

The following tasks are performed

(A) Reading of all the input data

The input data of the saturated flow model are also read in the total response model (refer 3.6.1.1 (A)). In addition there are some additional input data in TRM. The details of corresponding additional READ statements are as follows.

(i) ICLAY1: number of the row representing the bottom of the clay layer, ICLAY2: number of the row representing the top of the clay layer, NRWD(IT): number of the row at water level in the drain , DL: depth of impervious layer below drain level, NRW: number of the row at the drain level. AL: spacing between the drains.

(ii) WP: wilting point, THR: residual soil moisture content, PET: potential evapotranspiration rate, PF: fraction of available soil moisture after which ET=PET, where, ET is the actual evapotranspiration rate, POR: soil porosity, FFC: field capacity, AEV: air entry value (bubbling pressure), POW: power factor (taken as 4) in Brooks and Corey (1964) relation, E: evaporation rate, NRZ: number of the row representing bottom of root zone.

(B) computations of distribution of capillary pressure head, P(I,J), and water table elevations (P(I,J)=0.0).

(C) computations of the distributions of hydraulic properties of soil in the unsaturated zone.

(D) computations of the distribution of horizontal velocities, VX(I,J).

(E) computations of the distribution of vertical velocities, VZ(I,J).

(F) computations of lateral flows to the drain

(G) comparison of the computed and observed water table elevations at midsection.

(H) computations of model efficiency

(I) Printing the computed results.

3.6.2.2 Subroutines Called

Transfer of data from main programme to the subroutines and in between the subroutines, is made by labelled COMMON blocks.

STN: In this subroutine the matrix generated by the finite differences approximation is solved using the Thomas Algorithm (Remson et al, 1971).

THETA: In this subroutine the θ for a given P (i.e., h_c) is calculated.

COND: In this subroutine conductivity is calculated (for the link) for the specified θ(or P) in the unsaturated zone.

DIFU: In this subroutine, the specific moisture capacity is calculated. Either numerical or closed form method should be programmed in this subroutine.

EVPT: In this subroutine, the actual evapotranspiration rate at all the nodes in the root zone is calculated. Necessary data is made available through a COMMON block. The actual ET, potential ET (PET) and θ relation needs to be programmed.

BIG: In this subroutine largest difference of P values of two consecutive iterations (ADIE/subtime step) is calculated.

In annexure II listings of the computer codes (of SFM and TRM) have been presented.

CHAPTER IV

MODEL VALIDATION

The saturated flow model (SFM) has been validated by comparing the model solutions with analytical solutions and with reported field observations. The total response model (TRM) has been validated by comparing the model solutions with reported field observations.

4.1 COMPARISON WITH ANALYTICAL SOLUTIONS

The water table elevations computed by the saturated flow model (SFM) have been compared with Kraijenhoff's (1958) unsteady state solution and Donnan's (1946) steady state solution. The transient lateral outflows computed by SFM have been compared with Edelman's (1947) solution. These comparisons are explained in details in the following paragraphs.

4.1.1 Comparison with Kraijenhoff Solution

Kraijenhoff (1958) presented an analytical solution (refer section 2.2.2.1) for the unsteady state water table elevations midway between two parallel fully penetrating ditches, assuming the flow to be one dimensional horizontal. The solution is based upon the following differential equation ("Drainage Principles" 1983, Vol. II).

$$Kx \; Y_0 \frac{\partial^2 h}{\partial x^2} + R = S_y \frac{\partial h}{\partial t} \qquad (4.1)$$

with $h(x,0) = Y_0$, $h(0,t) = Y_0$ and $h(L,t) = Y_0$

where, Kx is the hydraulic conductivity of the medium in x direction, Y_0 is the depth of the impervious layer below drain level, h is the piezometric head, R is the recharge depth per unit time at the water table, S_y is the specific yield of the medium, and t is the time. The Kraijenhoff solution ignores the vertical component of flow & the associated head loss, the flow above the initial drain level, and the compressibility of the soil.

The assumption of one dimensional horizontal flow holds good provided the vertical hydraulic gradient is insignificant in comparison to the horizontal hydraulic gradient. However, the transfer of recharge occurring at the water table into the flow domain requires a relatively significant component of velocity. The vertical component of velocity can be significant under insignificant hydraulic gradient only if $Kz \gg Kx$. This requires the saturated flow model solution,with the storage coefficient assigned as zero and neglecting the flow above the drain level, to converge to the Kraijenhoff solution at $Kz \gg Kx$. Based upon this logic the saturated flow model solution, neglecting the flow above drain level (refer section 3.1), has been compared with the Kraijenhoff solution. The details of the space & time domains in which the model and Kraijenhoff's solutions were obtained are as follows.

4.1.1.1 Space Domain

The flow domain between two parallel fully penetrating ditches, 30 meters apart, is bounded by the drain level as upper boundary and by an impervious base at 5 meters below the drain level as lower boundary (i.e., $L = 30$ m, $Y_0 = 5$ m). Owing to symmetry, only the left half of the flow domain is analysed. This domain is discretised by 11 rows and 16 columns. The spacings of these rows and columns are non-uniform. The spacing of the columns near the ditch is 0.1 metre, which is increased gradually, reaching a maximum value of 2.4m at mid section. Similarly, the spacing between the rows near the drain level is 0.2m, increasing gradually towards the impervious layer to a maximum value of 0.8m.

4.1.1.2 Time Domain

A time domain of 5.79 days is discretised by 15 discrete non-uniform time steps, starting from a minimum value of 0.01 days and subsequently increasing gradually to reach a maximum value of 1.0 day. The discretisation of each time step into appropriate number of subtime steps is done automatically as per the provision made within the model (refer section 3.1.9 convergence criteria).

4.1.1.3 Solution by Saturated Flow Model

A uniform recharge of 0.1 metre per day is assumed to occur at the initial position of water table at drain level.The flow domain(para 4.1.1.1) bounded by a horizontal impervious boundary at the bottom and the horizontal drain level at the top is considered.Thus,the flow above the drain level is neglected.The model solutions are obtained for varying vertical hydraulic conductivities (Kz= 0.02, 0.2, 2.0 and 2000.0 m/day) and uniform hydraulic conductivity in horizontal direction (Kx = 2.0m/day).The specific yield (S_y) is taken as 0.02. The midpoint water table elevations computed by the model at the discrete times are plotted in Fig.4.1.

4.1.1.4 Kraijenhoff Solution

The water table elevation at midpoint of the flow domain described in section (4.1.1.1) at the discrete times described in section (4.1.1.2), has also been computed by the Kraijenhoff solution (refer equation 2.13 in section 2.2.2.1), assigning identical values for recharge and horizontal hydraulic conductivity (R = 0.1 m/day, Kx = 2.0 m/day).

The computed elevations are plotted along with model

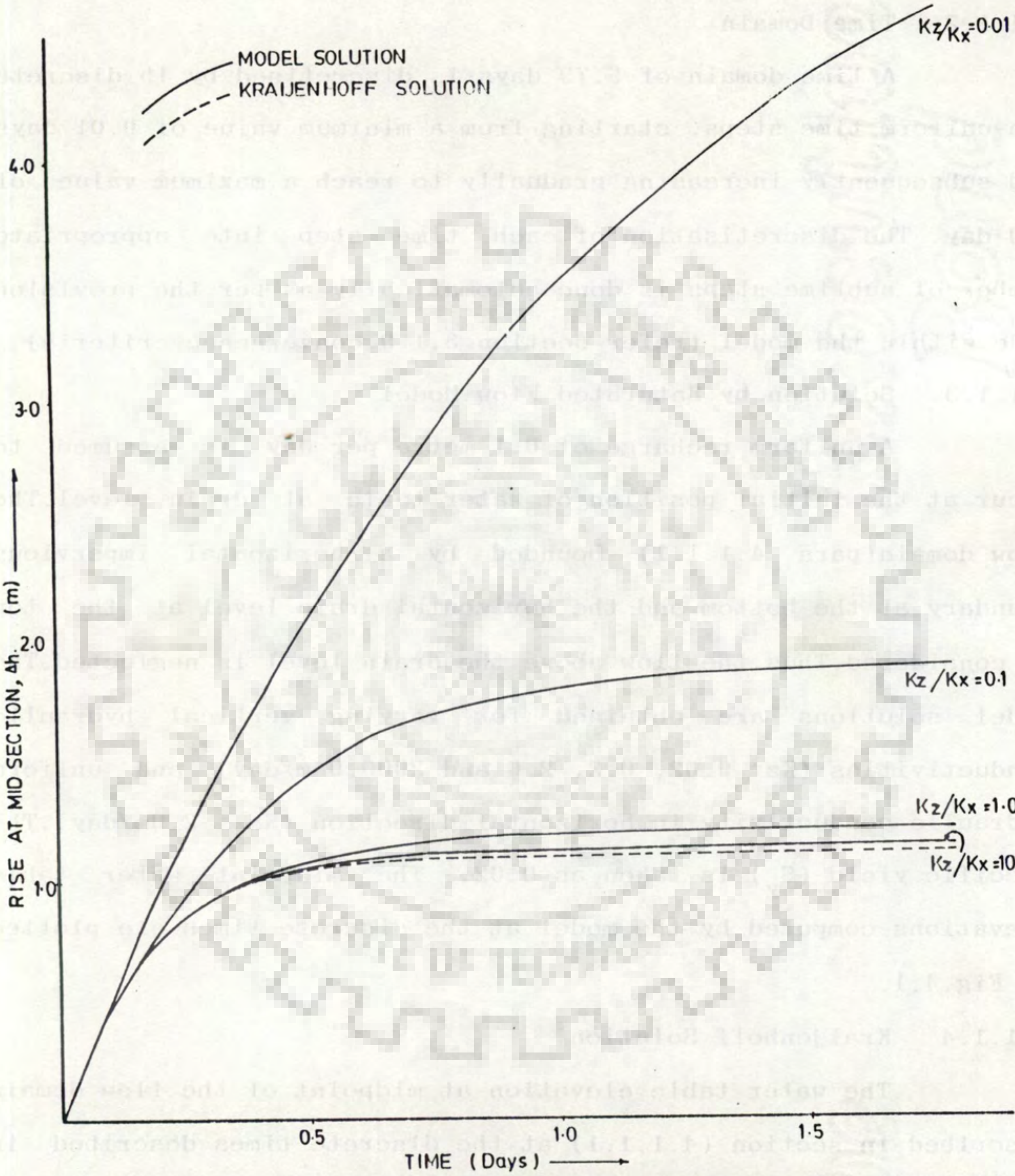


FIG.4.1 COMPARISON OF THE MODEL SOLUTION (NEGLECTING THE FLOW ABOVE DRAIN LEVEL) WITH KRAIJENHOFF SOLUTION.

solution (Fig. 4.1). It is found that for Kz= 2000.0 m/day, i.e., at Kz/Kx=1000.0, the model solution converges to the Kraijenhoff's solution. For Kz < Kx the Kraijenhoff solution is found to underestimate the water table rise.

4.1.2 Comparison with Donnan Solution

Donnan (1946) presented an analytical solution (refer equation 2.1, section 2.2.1.1) for the steady state rise of water table midway between two fully penetrating parallel ditches. The solution is based upon the following equation governing one dimensional horizontal, steady state flow in a porous medium ("Drainage Principles" 1983, Vol.I).

$$K_x \, \xi \, \frac{d\xi}{dx} = R \left(\frac{L}{2} - x \right) \tag{4.2}$$

with $\xi(x,0)$, $= Y_0$, $\xi\ (0,t)$, $= Y_0$, $\xi\ (L,\ t)$ $= Y_0$ and $\xi = \xi\ (x,t)$ at $t > 0.0$. Where ξ is the elevation of the water table above the impervious layer. The other symbols are as defined in section (4.1.1). Thus unlike Kraijenhoff's solution, the Donnan solution accounts for the flow in the arched region of the water table. The Donnan solution can be expressed as follows.

$$\xi^2 = Y_0^2 + \frac{R.x}{K_x} \left(L - x \right) \tag{4.3}$$

For similar reasons as explained in the context of the Kraijenoff solution (section 4.1.1), the saturated flow model solution (accounting for the flows in the arched region of the water table section 3.2) at steady state, with Kz >> Kx, should converge to the Donnan solution.

The saturated flow model solution as well as the Donnan's solution are obtained for the flow domain described in section (4.1.1.1) for identical recharge and parameter values. While obtaining the model solution the flow in the additional

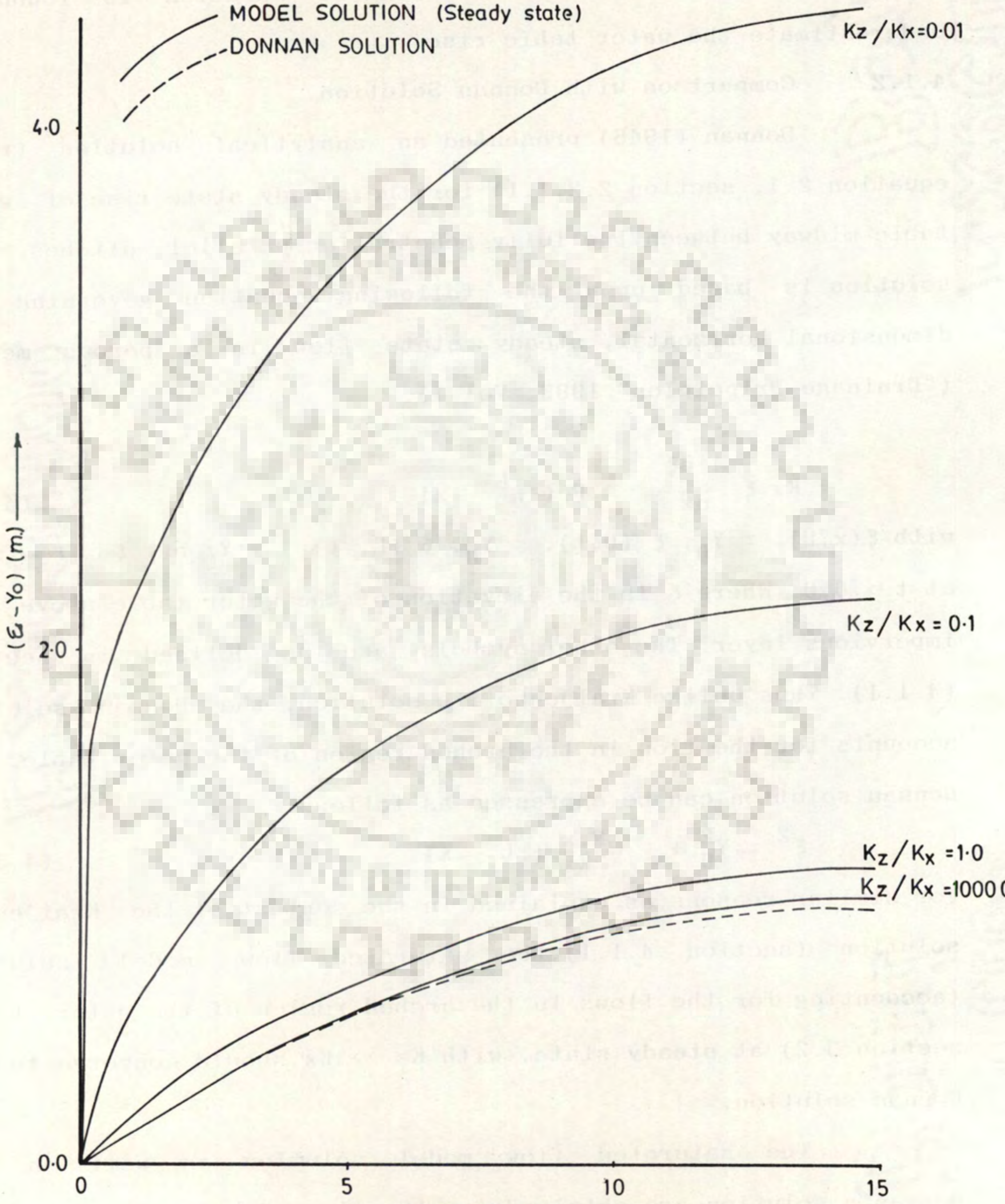


FIG. 4·2 COMPARISON OF THE MODEL SOLUTION WITH DONNAN SOLUTION domain between the drain level and the risen water table is accounted for in accordance with the logic described in section 3.2. The model has been operated for a long enough time to reach the steady state. The model computed steady state water table profiles for different vertical hydraulic conductivities have been plotted in Fig. 4.2. The steady state water table profile computed by Donnan solution is also plotted in Fig. 4.2. It is noticed that for Kz/Kx = 1000.0, the model solution converges to the Donnan's solution. For Kz<Kx, the Donnan solution is found to underestimate the water table rise at all the points between the drains.

4.1.3 Comparison with Edelman Solution

Edelman (1947) presented analytical solutions for transient lateral outflows from a homogeneous unconfined aquifer of infinite lateral extent to a fully penetrating ditch (Fig. 4.3).He considered four different boundary conditions at the ditch ,i.e.,sudden lowering of the water level in the ditch by an amount Δ,linearly increasing lowering of water level in the ditch by an amount α t,constant groundwater discharge into the ditch in a magnitude q_0,linearly increasing groundwater discharge into the ditch of magnitude β t (Huisman,1982).The solutions are based on the following differential equation.

$$\frac{\partial^2 s}{\partial x^2} = \frac{S_y}{Kx\ Y_0}\ \frac{\partial s}{\partial t} \qquad (4.4)$$

where, s is the water table drawdown caused by lateral flow from aquifer to the ditch, the definitions of other variables follows from the explanation presented earlier in this chapter.The model (neglecting the flow above drain level, section. 3.1) was operated to estimate the outflow rate to the ditch under each of the boundary conditions .The model computed outflows have been

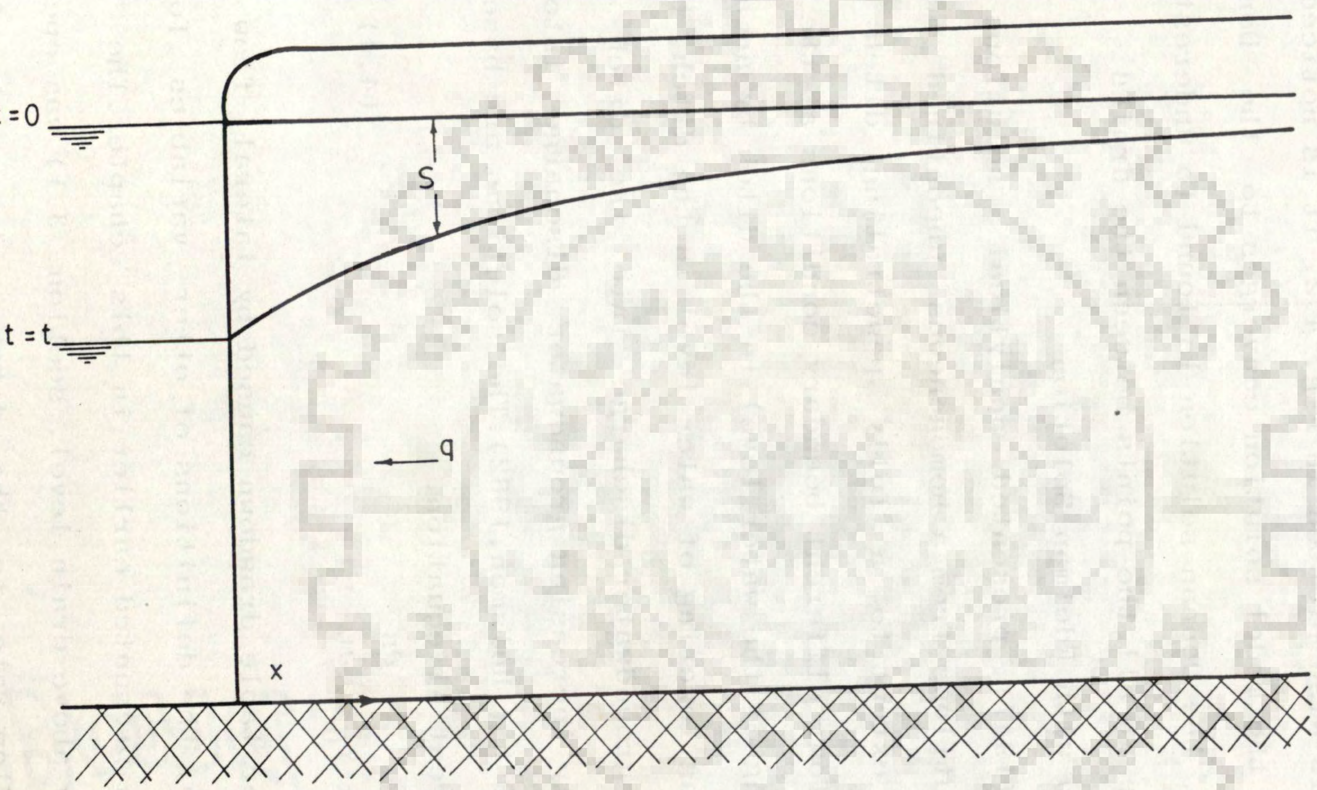


FIG. 4·3-UNSTEADY ONE-DIMENSIONAL FLOW OF GROUNDWATER IN A SEMI-INFINITE UNCONFINED AQUIFER WITHOUT RECHARGE.

compared with the corresponding Edelman solutions .For model solutions the space and time domains are discretised by non-uniform step sizes (similar to the ones explained in paragraphs 4.1.1.1 and 4.1.1.2).The details of the Edelman's solutions ,model solutions ,and comparison for each case of the boundary conditions are as follows .

(a) Sudden lowering of water level in the boundary ditch by an amount Δ (refer Fig. 4.4)

i.e.,at x = 0,s_0=Δ ,the Edelman solution for q_0,the lateral outflow ,is as follows

$$q_0 = s\left[\frac{S_y\ Kx\ Y_0}{\pi . t}\right]^{0.5} \qquad (4.5)$$

where, s_0 is the drawdown at the ditch face (taken as 0.5 m) and q_0 is the lateral outflow from aquifer to the ditch.

The solution was obtained for a soil medium (uniform loam soil) having an impervious boundary at 9.5 m below drain level. The soil properties are considered as (S_s= 0.0001,S_y = 0.434, Kx= 0.3168 m/day).Since the saturated flow model can not be operated for an infinite lateral extent (x =∞), the requirement of infinite lateral dimension has been approximated by assigning a constant head boundary condition at a distance of 200 metres from the ditch, thus making x >> Y_0. The constant head boundary condition has been numerically implemented by assigning very high values to the storage coefficient and specific yield (S_s= 1000.0/m, S_y= 1000) at all the nodes of the last column (representing the constant head boundary) of the domain.

The computed variations of the outflow rate (q_0) by Edelman´s solution (equation 4.5) and by SFM (with Kz = 0.03168, 0.3168, 316.8 m/day and Kx= 0.3168 m/day) are plotted with time in Fig. 4.5. It is found that the SFM solution converges to the

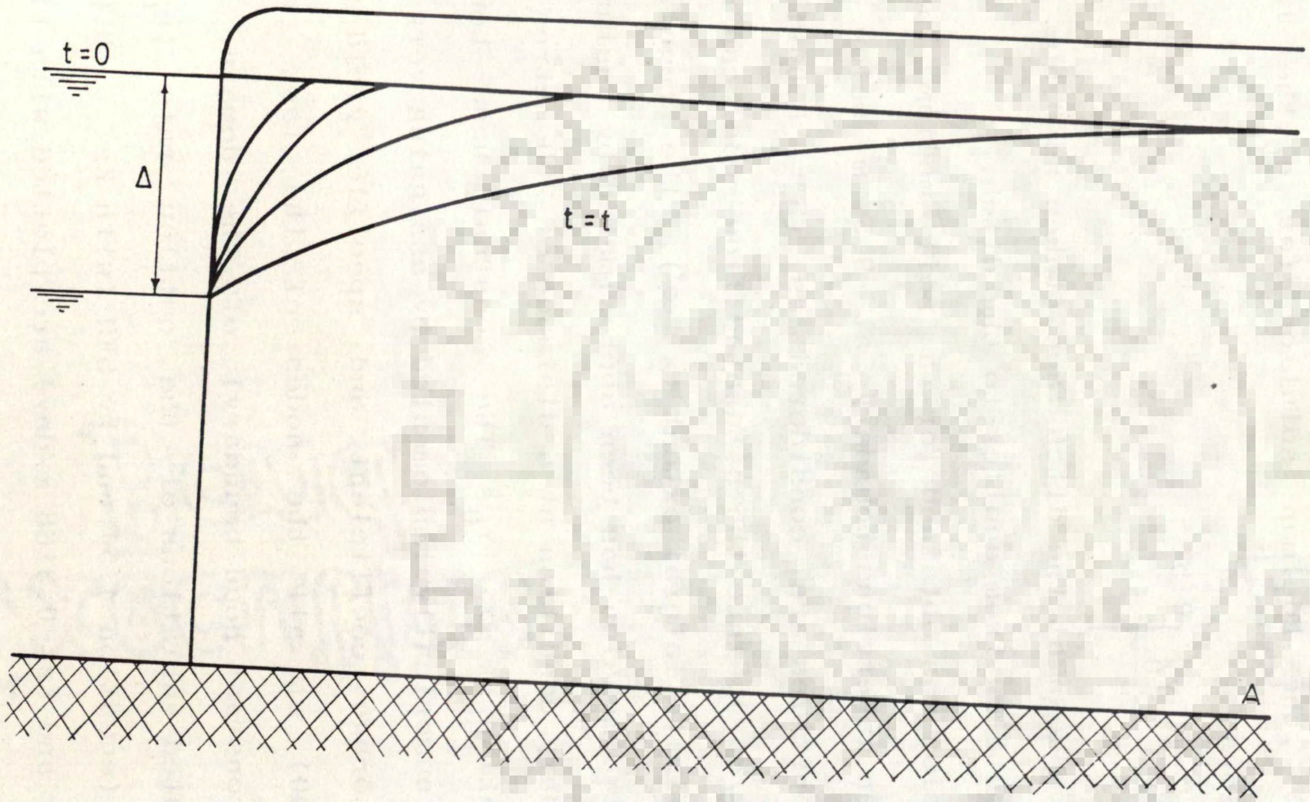


FIG. 4·4-SUDDEN LOWERING OF THE WATER LEVEL IN THE BOUNDARY DITCH

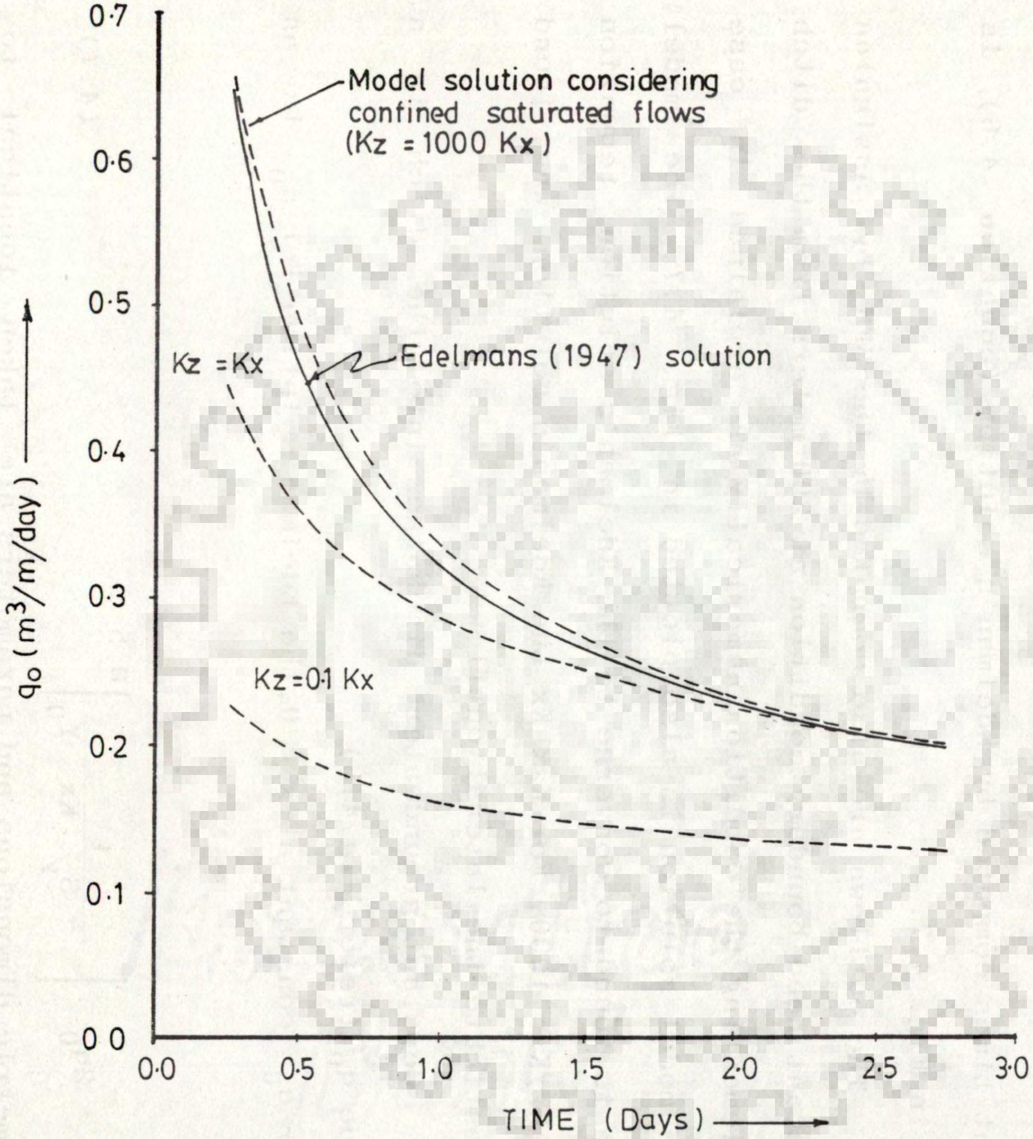


FIG 4·5 RATE OF LATERAL FLOWS CAUSED BY SUDDEN LOWERING OF WATER LEVEL IN THE FULLY PENETRATING BOUNDARY DITCH

Edelman's solution at Kz/Kx = 316.8 m/day). For Kz<Kx the Edelman's solution was found to overestimate the lateral outflows.

(b) Linearly increasing lowering of water level in the ditch by an amount αt (refer Fig. 4.6)

i.e.,at x = 0,$s_0 = \alpha t$,the Edelman's solution for q_0 is as follows.

$$q_0 = 2\alpha \left[\frac{1}{\pi} S_y \; Kx. \; Y_0. \; t \right]^{0.5} \qquad (4.6)$$

where, α is taken as 0.1m/day. The geometric dimensions and parameters are taken identical to case (a).The outflow at different times computed by Edelman's solution (equation 4.6) is plotted in Fig. 4.7.

The outflows are also computed by the SFM by assigning the constant head boundary condition at 200 meters from the ditch and implementing the condition numerically as explained in case (a). The model solution is also plotted in Fig. 4.7. The model solution is found to converge to the Edelman's solution (equation 4.6) at Kz/Kx= 1000. For Kz< Kx the Edelman's solution was found to overestimate the lateral outflows.

(c) Constant groundwater discharge into the ditch in a magnitude q_0 (refer Fig.4.8)

i.e.,for q_0= constant at x = 0,the Edelman solution for s_0 is as follows.

$$s_0 = 2q_0. \left[\frac{t}{\pi \; S_y. \; Kx. \; Y_0} \right]^{0.5} \qquad (4.7)$$

The geometric dimensions and parameters are taken identical to case (a). The values of $s_0$are computed at different times for q_0 = 0.08 m^3/m/day. The water level in the ditch ($Y_0 - s_0$) at different times computed by Edelman's solution (equation 4.7) are presented in Fig.4.9. The constant outflow (q_o) is plotted in Fig. 4.10.

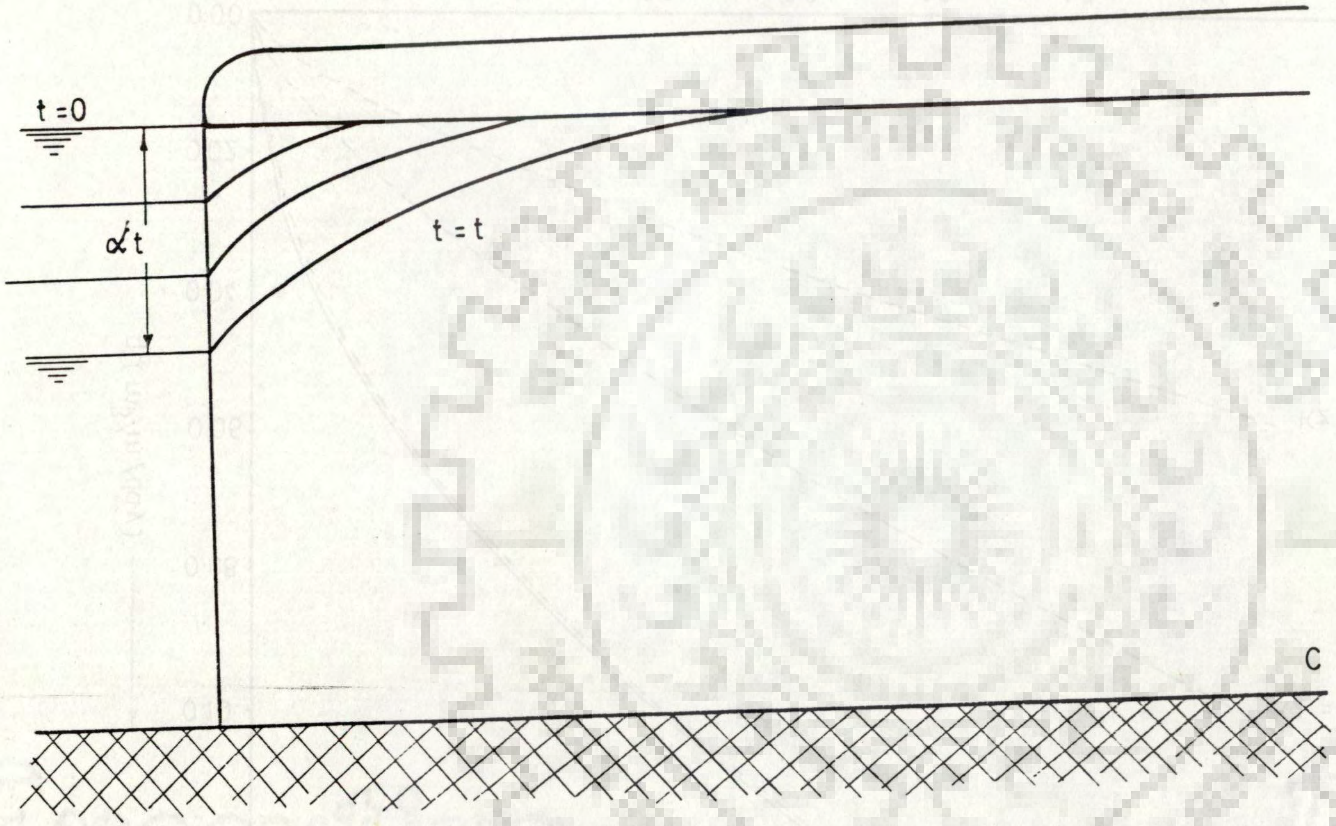


FIG. 4·6-LINEARLY INCREASING LOWERING OF THE WATER LEVEL IN THE DITCH

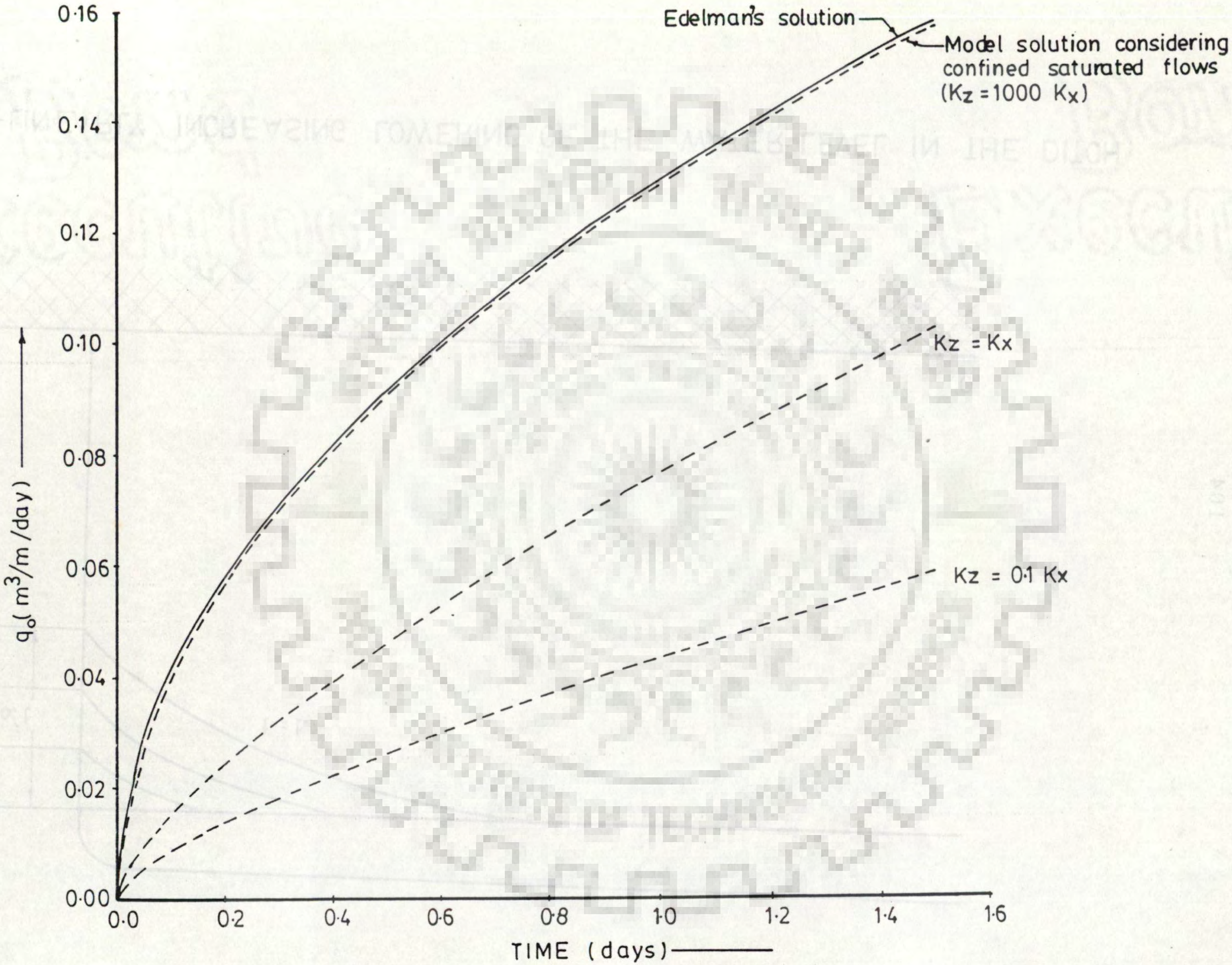


FIG 4·7 RATE OF LATERAL FLOW CAUSED BY LINEARLY INCREASING LOWERING OF THE WATER LEVEL IN THE FULLY PENETRATING BOUNDARY DITCH

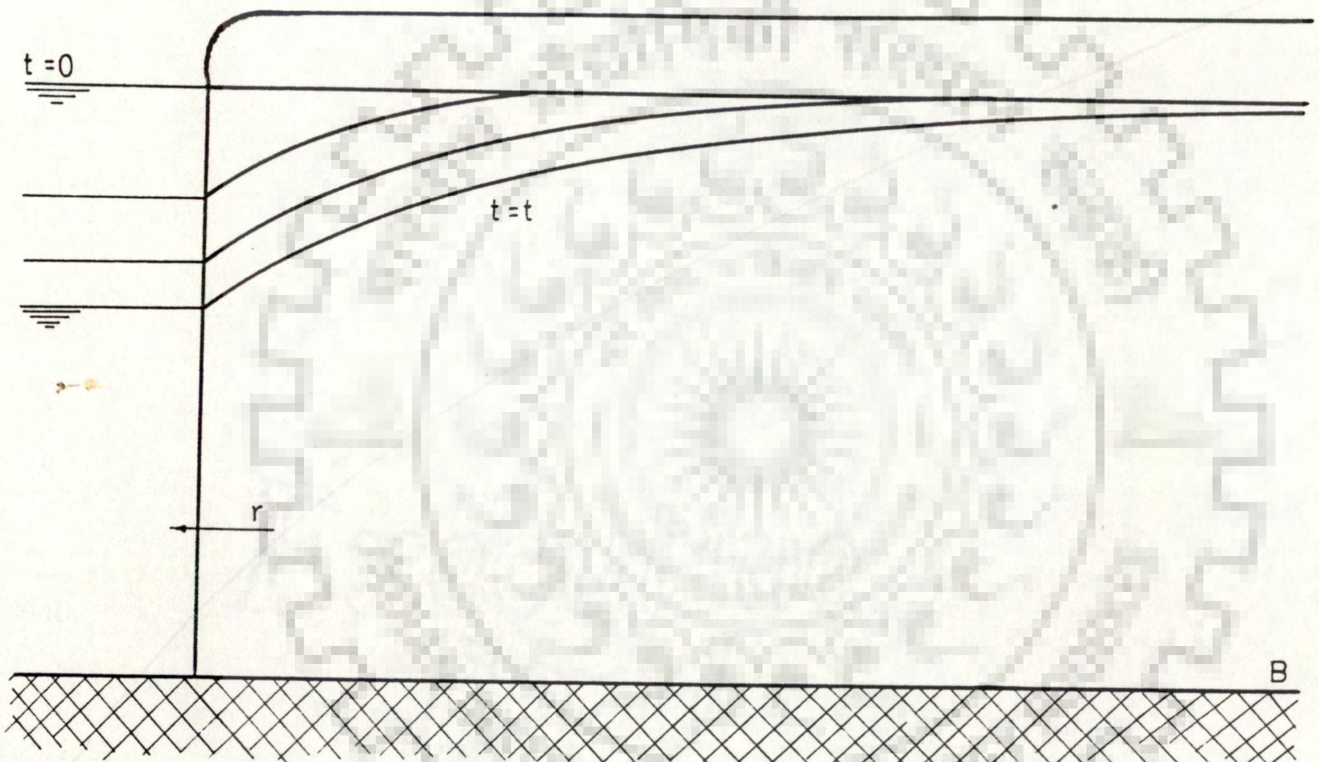


FIG 4·8 CONSTANT GROUNDWATER DISCHARGE INTO THE DITCH

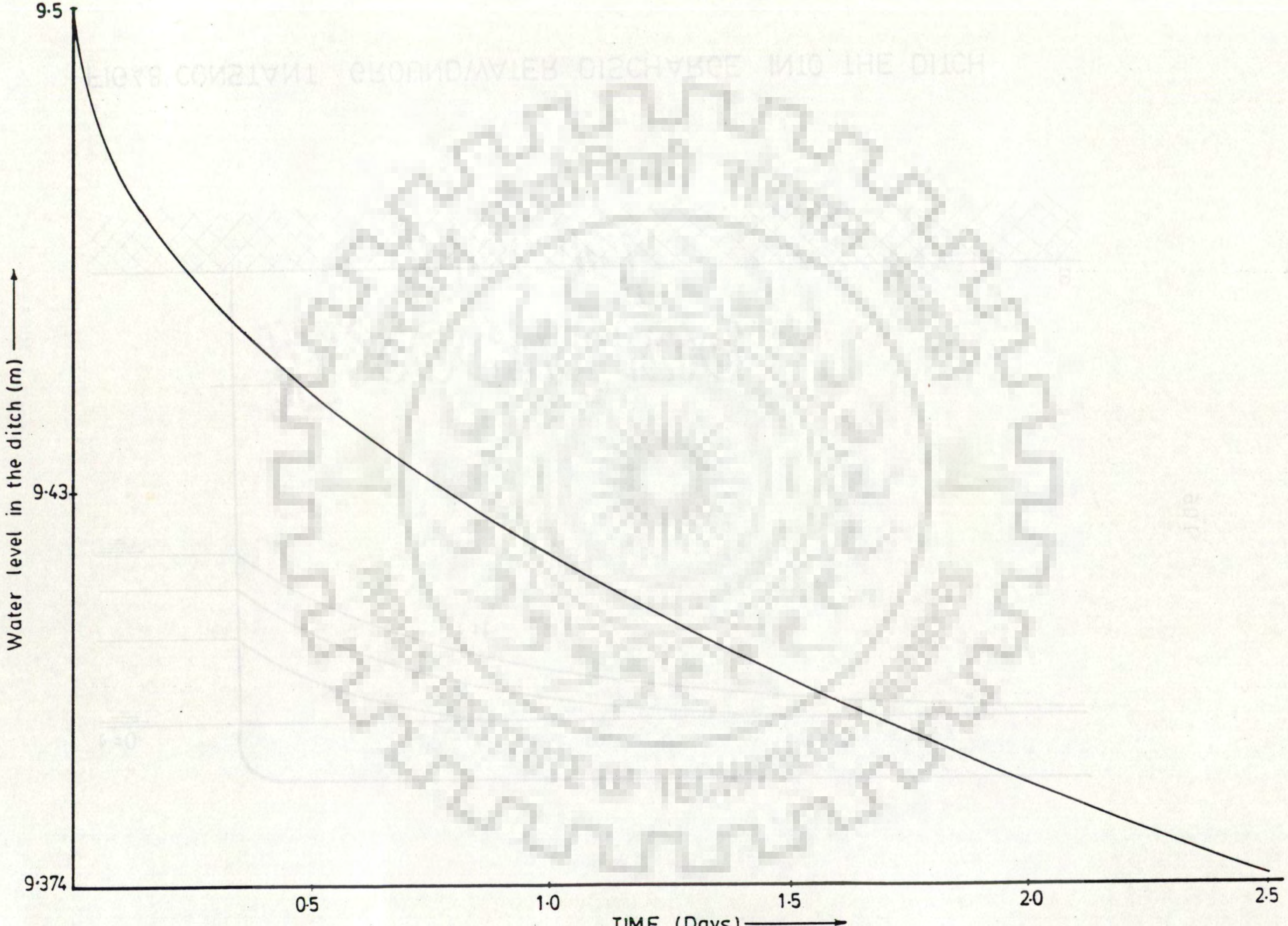


FIG. 49 FALLING WATER LEVEL IN THE DITCH CAUSED BY CONSTANT GROUNDWATER DISCHARGE INTO THE DITCH

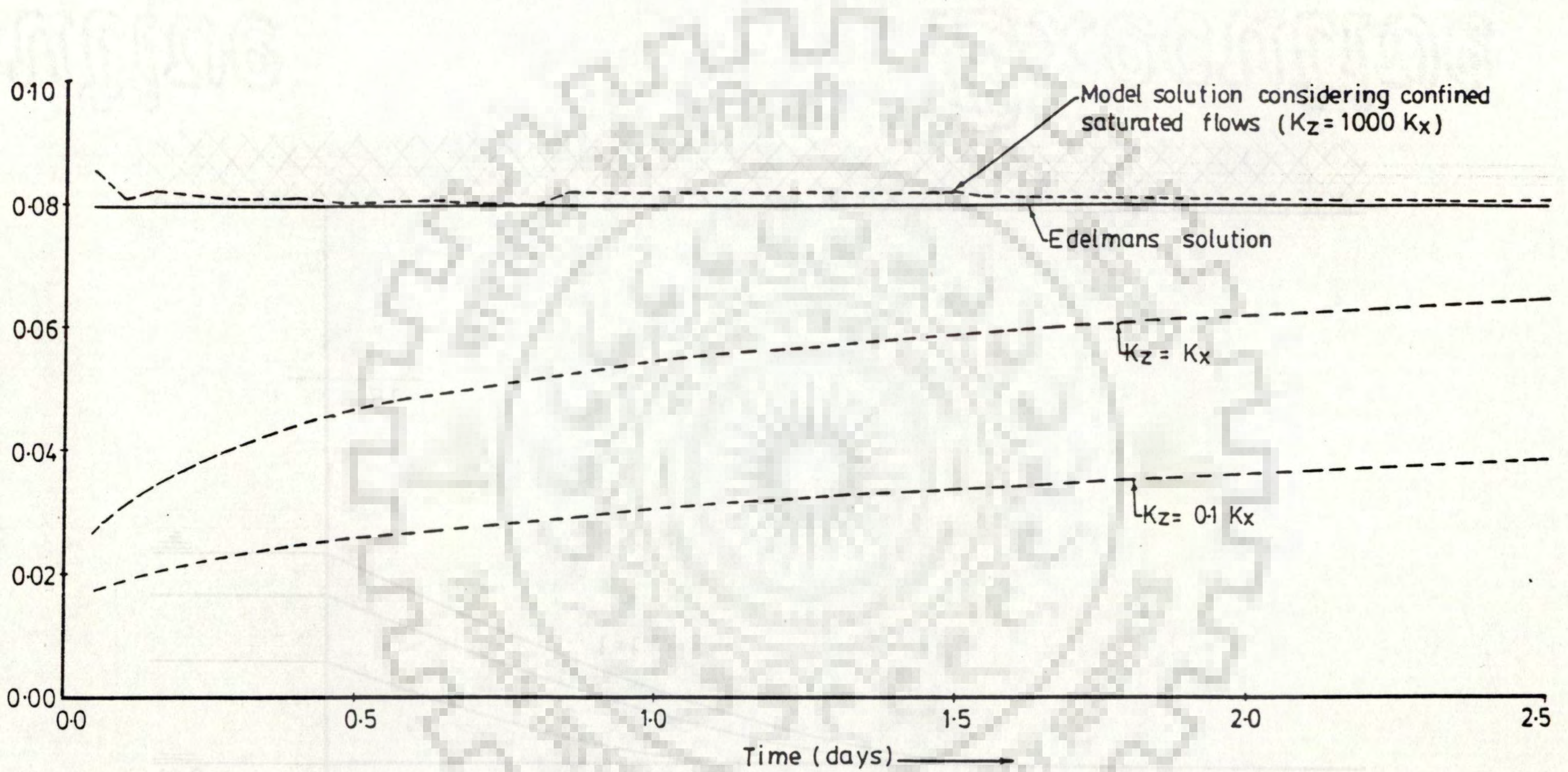


FIG 4.10 CONSTANT LATERAL FLOW INTO THE DITCH AS PER EDELMAN SOLUTION AND CORRESPONDING MODEL COMPUTED LATERAL FLOW .

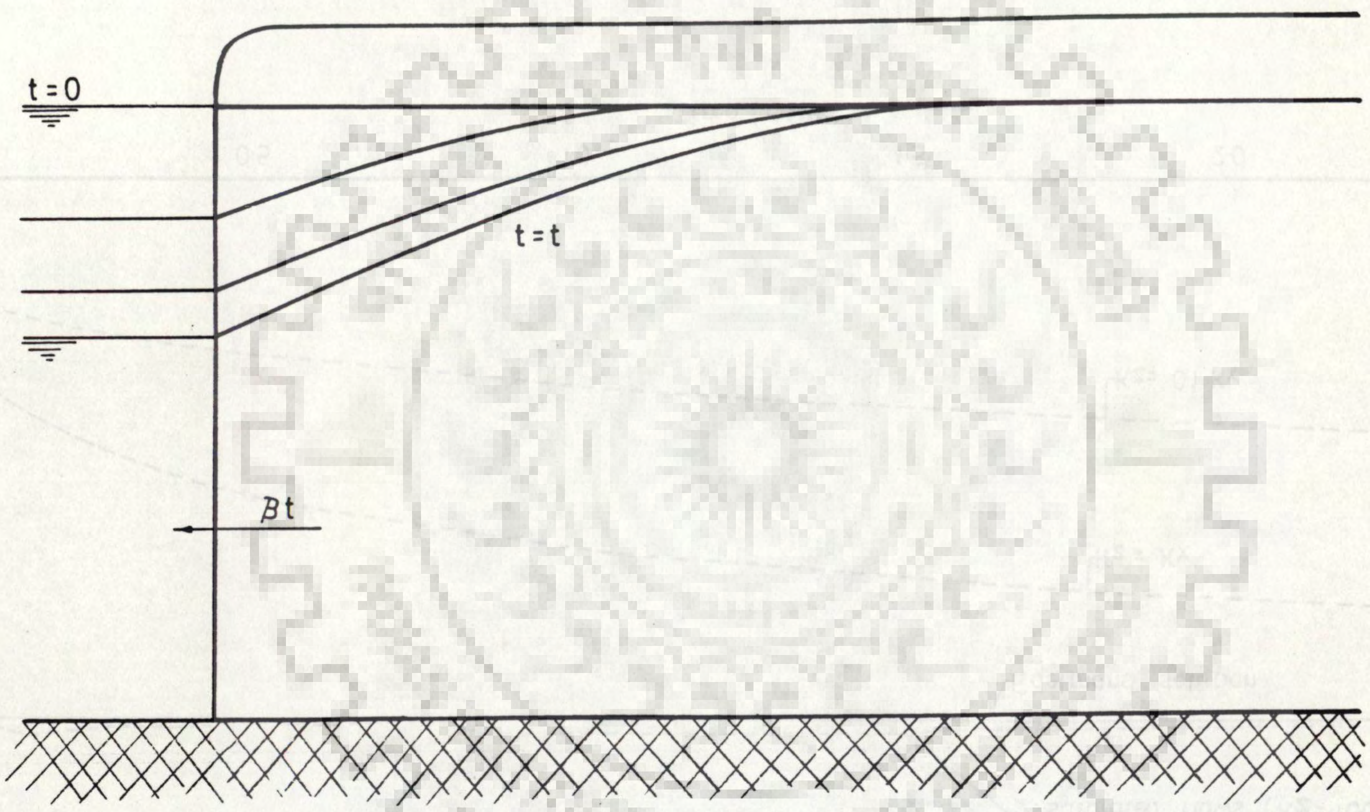


FIG 4·11 LINEARLY INCREASING GROUNDWATER DISCHARGE INTO THE DITCH

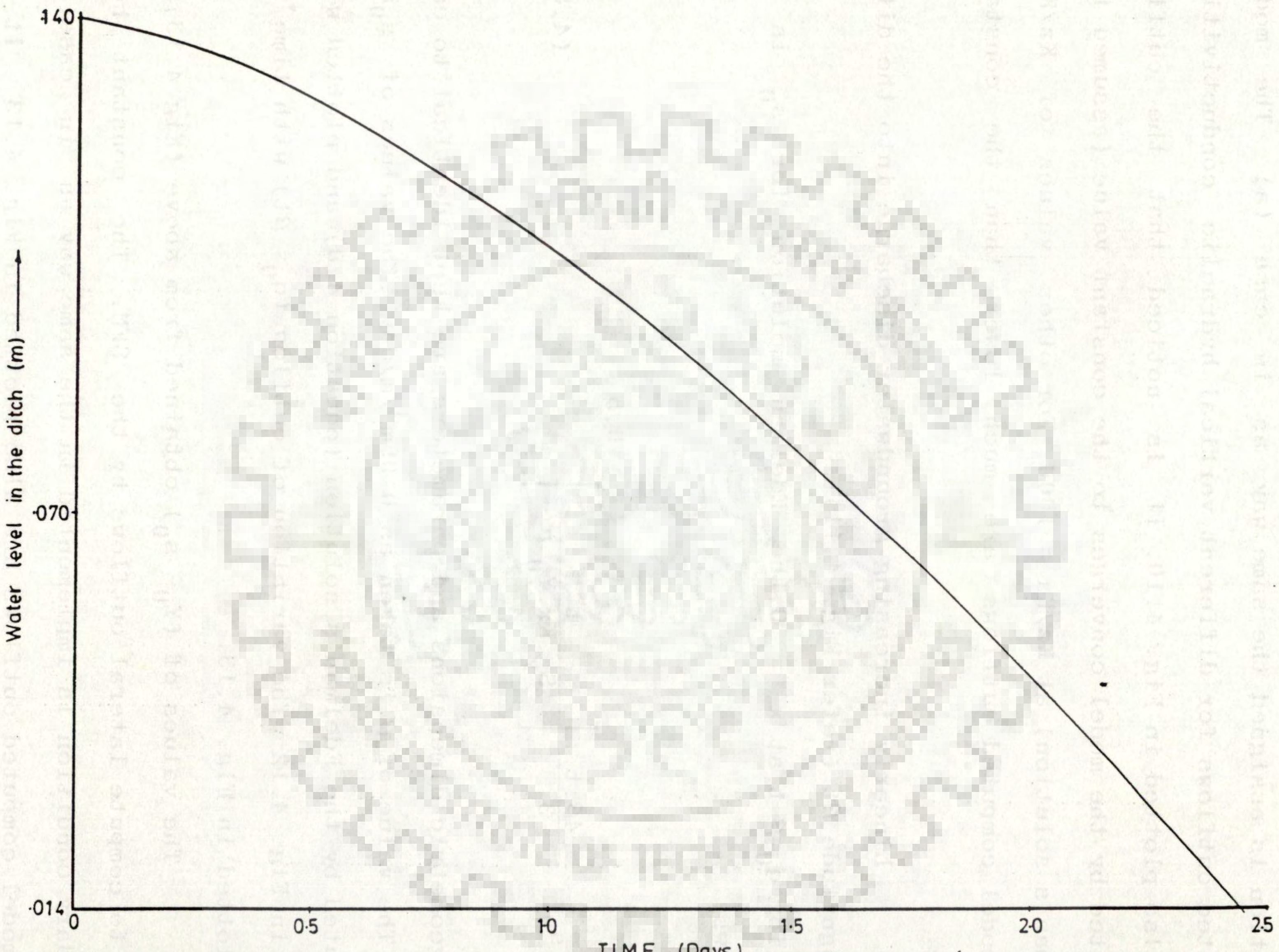


FIG.4·12-FALLING WATER LEVEL IN THE DITCH CAUSED BY LINEARLY INCREASING GROUNDWATER DISCHARGE INTO THE DITCH

The values of $(Y_0 - s_0)$ obtained from above (Fig 4.9)are used to compute q_0 by the SFM. The constant head boundary condition is assigned the same way as in case (a). The model computed outflows for different vertical hydraulic conductivities are also plotted in Fig. 4.10. It is noticed that the outflow computed by the model converges to the constant value (assumed for Edelman's solution) at Kz/Kx = 1000. For other values of Kz/Kx, the model computed outflows are much lower than the constant value.

(d) Linearly increasing groundwater discharge into the ditch of magnitude βt (refer Fig. 4.11)

i.e.,for $q_0 = \beta$ t at x = 0,the Edelman solution for s_0 is as follows.

$$s_0 = \frac{4\beta . t}{3} \left[\frac{t}{S_y . Kx . Y_0 . \pi} \right]^{0.5} \tag{4.8}$$

The geometric dimensions and parameters are kept identical to case (a). The value of β is taken as $0.05 m^3/m/day^2$.The values of $s_0$are computed by the Edelman's solution (equation 4.8) and plotted with time in Fig. 4.12. The variation of outflow ($q_0 = \beta t$) with time (t) is plotted in Fig. 4.13.

The values of $(Y_0 - s_0)$ obtained from above (Fig 4.13)are used to compute lateral outflows by the SFM. The constant head boundary condition is implemented in the same way as in case(a). The model computed outflows are also plotted in Fig. 4.13. It is found that q_0 computed by the model converges to the q_0 $(=\beta t)$ values assumed for Edelman's solution at Kz/Kx = 1000 . For other cases, Kz<Kx, the Edelman solution is found to overestimate the

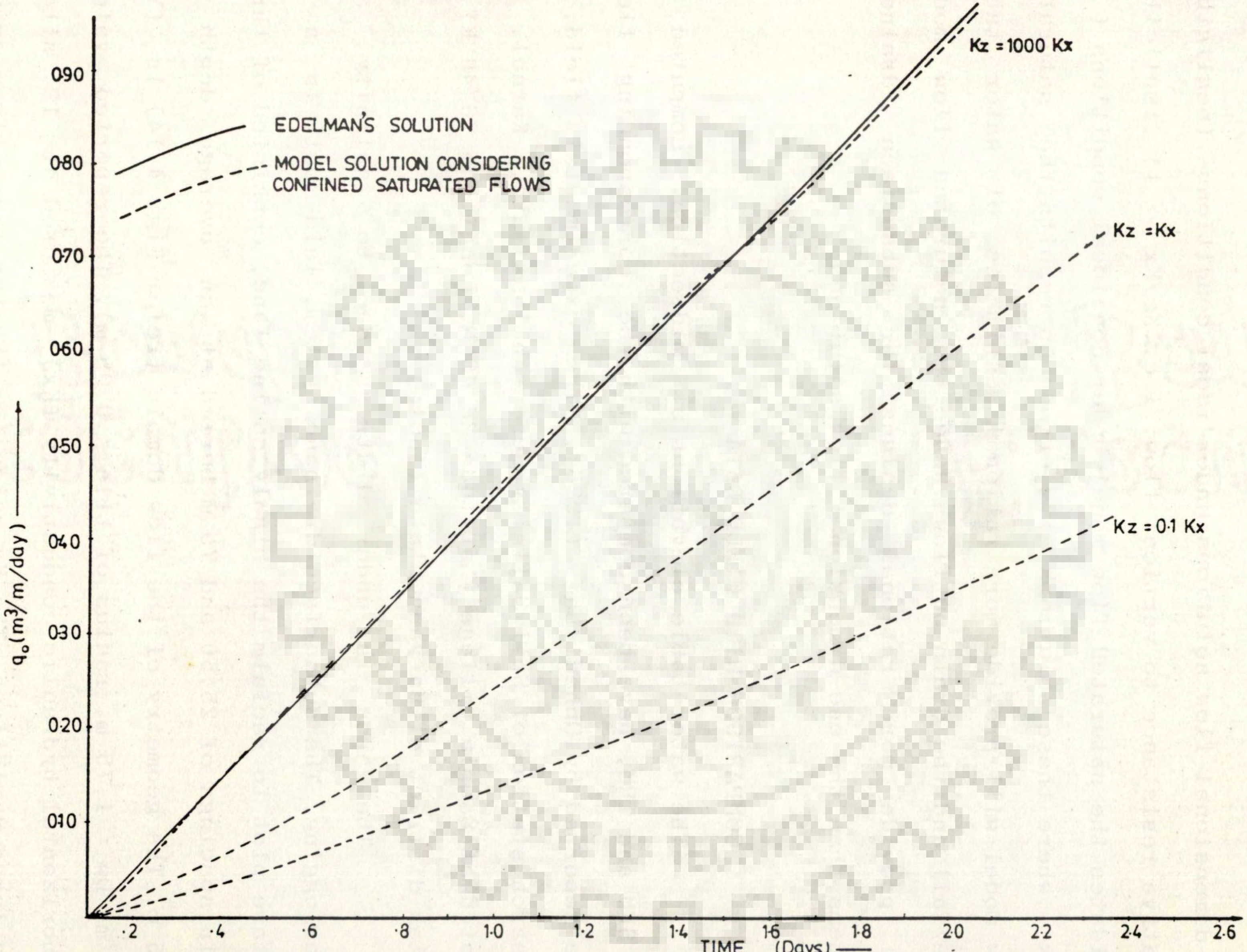


FIG. 4·13 LINEARLY INCREASING LATERAL FLOW AS PER EDELMAN SOLUTION AND CORRESPONDING MODEL COMPUTED LATERAL FLOW lateral outflows.

4.1.4 Inference

The SFM solution's convergence to the analytical one-dimensional flow solutions under ideal conditions (negligible relative resistance to vertical flow, i.e.,Kz/Kx >> 1) implicitly validates the saturated flow model.Under non-ideal conditions (Kz < Kx),where these solutions give erroneous results,the saturated flow model will provide more rational estimates of water table rise/fall and the drain outflows.Hence,the saturated flow model will provide more rational designs for subsurface drainage systems,than the one-dimensional flow solutions.

4.2 COMPARISON WITH FIELD DATA

The water table elevations at midsection, computed by the SFM and TRM, have been compared with the corresponding field data reported by Chhedi Lal (1986). The experimental field, a research station of Soil Salinity Research Institute, Karnal, is located at Sampla village (Latitude 28°46'N, Longitude 76°46'E) in Rohtak district of Haryana state (India).

The area is under distress due to salinity and waterlogging. The subsurface drainage system, laid out in a 10 hectare plot to reclaim the highly saline land, consisted of three drain spacings of 25,50 and 75 m buried at an average depth of 1.75 m. The geometry of the flow domain (refer Fig. 4.14) is (Y_0= 1.2 m, Dw = 1.75 m, radius of tile = 0.05 m). The reported values of horizontal hydraulic conductivity (Kx), measured at 12 sites, have large variations. The Kx value for the top layer (from ground surface to 1.1 m depth) ranges from 0.06 m/day to 10 m/day, for the middle layer (at 1.1 m to 1.8 m from ground surface) from 0.05

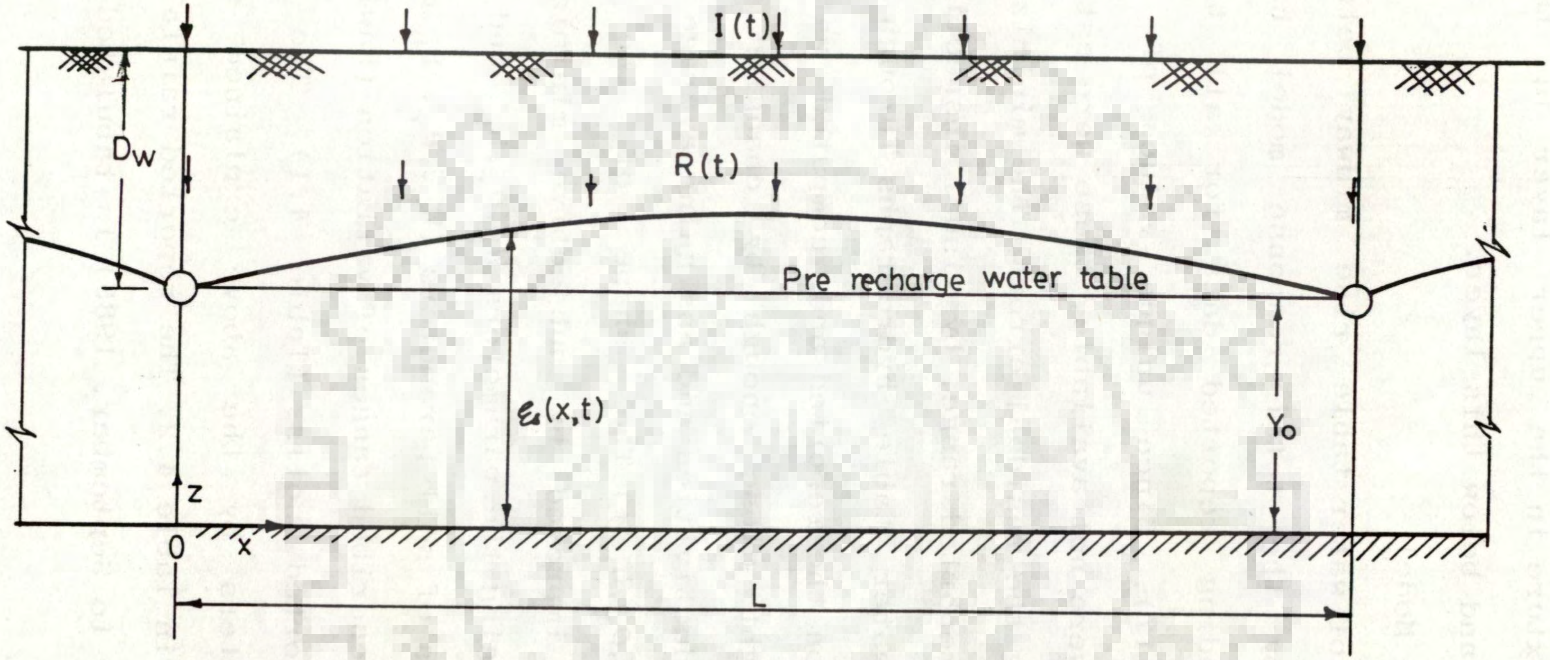


FIG. 4.14 - SUBSURFACE DRAINAGE BY TILES

m/day to 2.44 m/day and for the bottom layer (at 1.8 to 3 m from ground surface) from 1.27 to 22.50 m/day. It is reported that the soil is sandy loam in texture in the upper layer up to 1.2 m depth,and sand & loamy sand below this layer.

4.2.1 Total Response Model

The estimates of water table rise midway between the drains as arrived at by the total response model have been compared with corresponding reported data for all the three spacings (Figs. 4.15-4.17). Since unique values of saturated hydraulic conductivity were not available, these were estimated by subjectively minimising the residual errors. The minimization was carried out within the reported range, by visual inspection of the computed and reported water table hydrographs.Through rigorous numerical experimentation the reported and computed water table hydrographs were brought close enough by changing only the saturated hydraulic conductivities and assigning average values to other parameters (W_p,θ_r,h_b,ϕ).The shape of the computed hydrographs was further improved by subjective minimization of residual errors by changing the parameters W_p,θ_r,h_b and ϕ, only one at a time.The values of soil parameters (W_p, θ_r, h_b and ϕ) are estimated within the prescribed ranges of variation (Rawls et al 1981, 1982) for the reported soils (Table 4.1). The estimated values of soil parameters by the above explained subjective optimization are shown in Table 4.2. The reported rainfall for the period considered (July to September, 1985) is tabulated in Table 4.3.

The computed and reported water table hydrographs are found in good agreement. The model efficiencies for 25,50 and 75m spacings have been estimated as 0.87, 0.87 and 0.89 respectively.

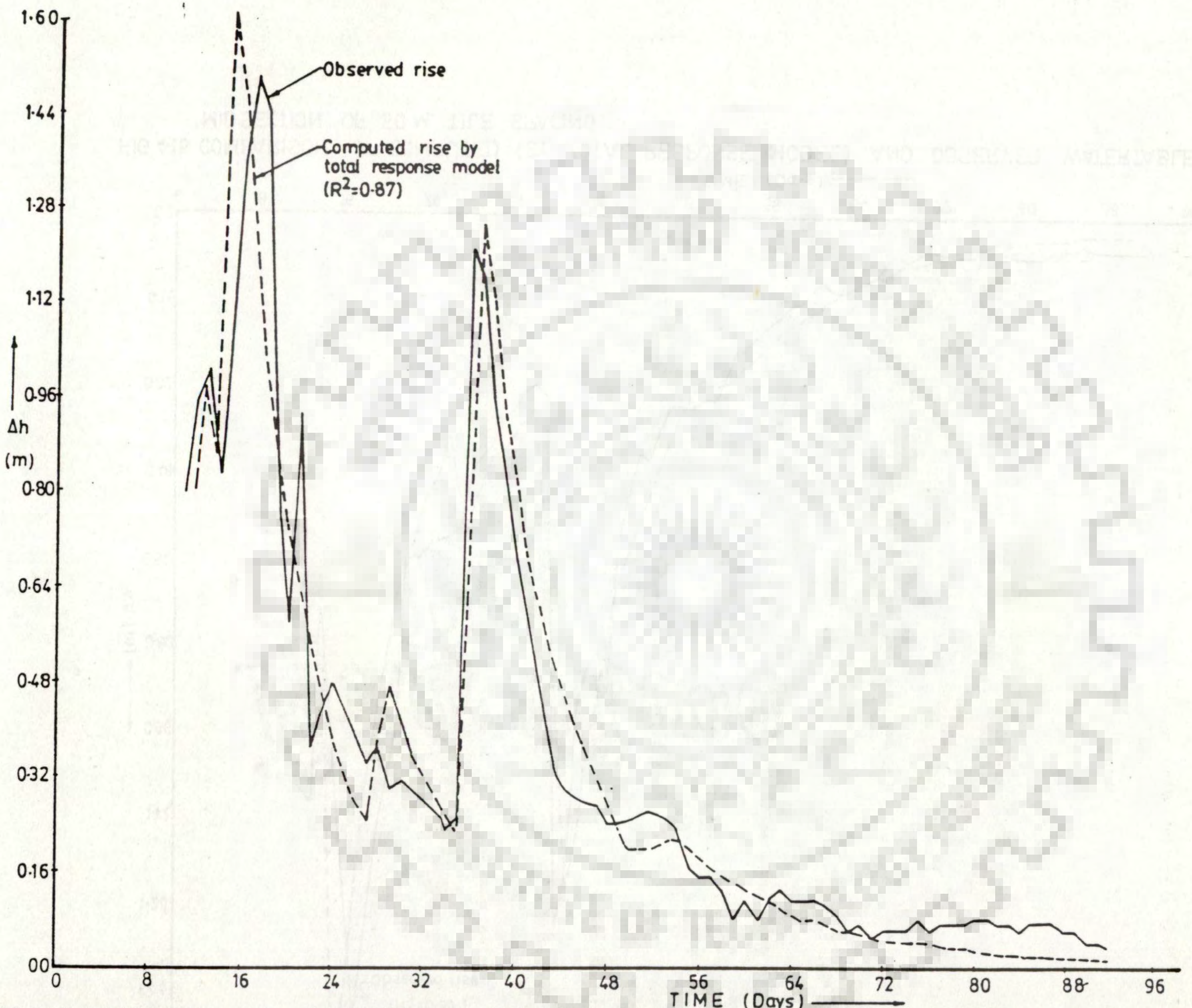


FIG. 4·15-COMPARISON OF COMPUTED (BY TOTAL RESPONSE MODEL) AND OBSERVED WATERTABLE RISE AT MIDSECTION OF 25 M. TILE SPACING.

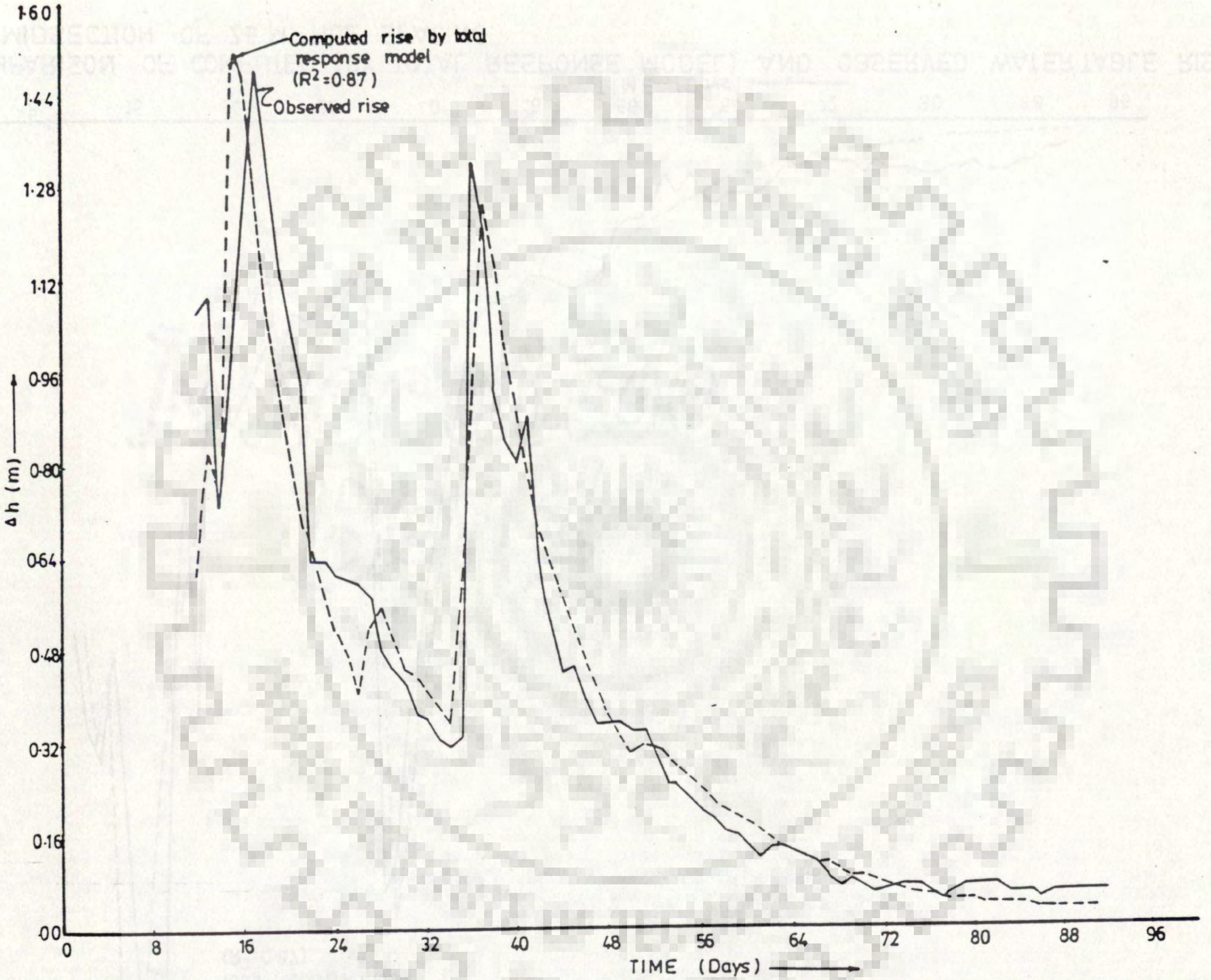


FIG 4·16 COMPARISON OF COMPUTED (BY TOTAL RESPONSE MODEL) AND OBSERVED WATERTABLE RISE AT MIDSECTION OF 50 M. TILE SPACING.

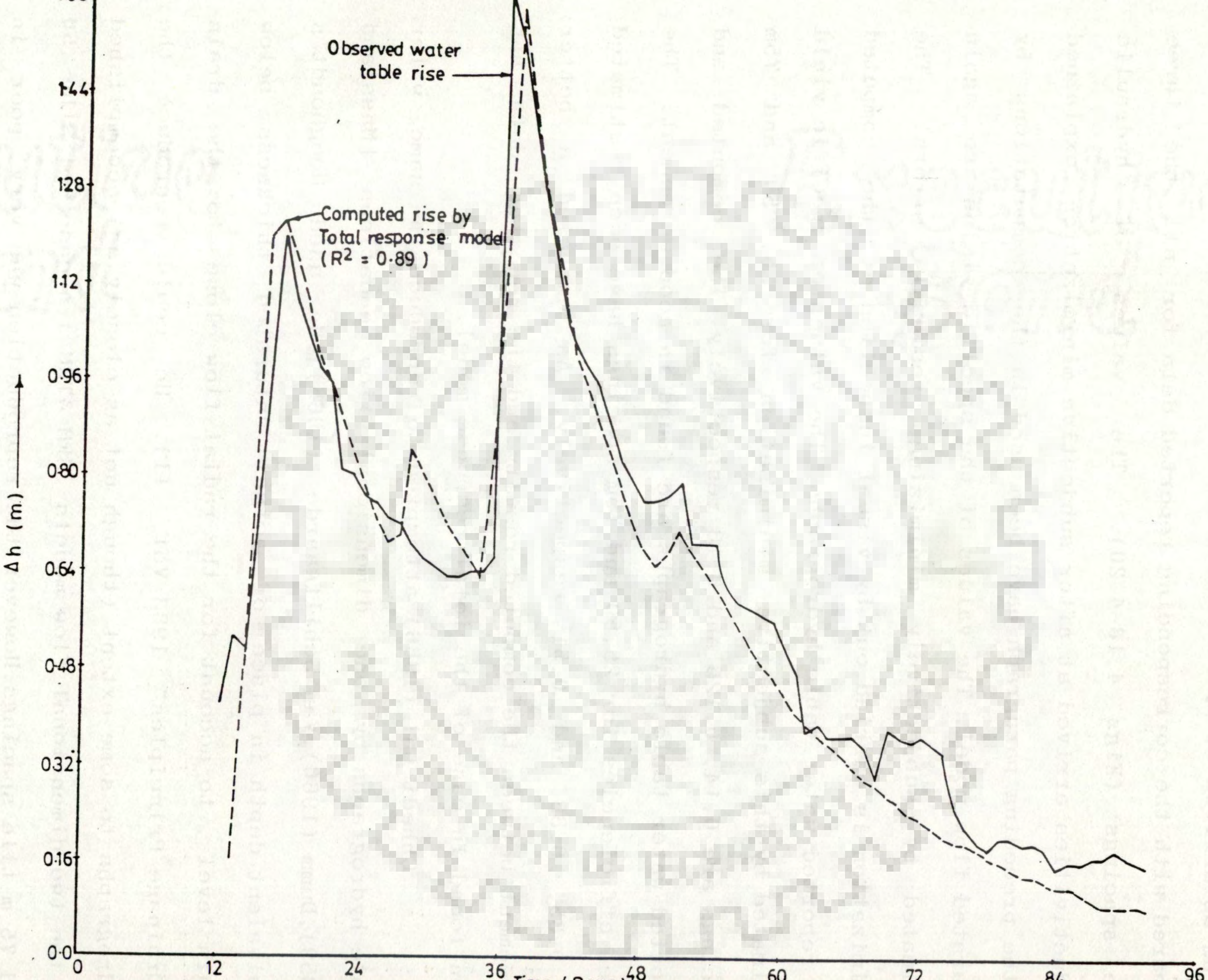


FIG.4.17 COMPARISON OF COMPUTED (BY TOTAL RESPONSE MODEL) AND OBSERVED WATER TABLE RISE AT MIDSECTION OF 75 M TILE SPACING.

4.2.2 Saturated Flow Model

The estimates of water table rise midway between the drains as arrived at by the saturated flow model have also been compared with the corresponding reported data for all the three tile spacings (Figs 4.18-4.20). The values of hydraulic conductivities arrived at after subjective minimization explained in the preceding paragraph have been used in the computations by saturated flow model. The values of the specific yields are again estimated by subjectively minimizing residual errors. The minimization is carried out by visual inspection of the computed and reported water table hydrographs. The values of specific yield estimated by this subjective minimisation for 25, 50 and 75m spacings are 0.14, 0.125 and 0.18 respectively. The computed and reported water table hydrographs are found in good agreement. The model efficiencies for 25,50 and 75m spacings have been estimated as 0.85, 0.86 and 0.51 respectively. As expected a better agreement between the computed results and the reported data has been found in case of the total response model.

Chhedi Lal (1986) attempted to reproduce the same water table hydrographs by one dimensional flow equations [Massland (1959),Dumm (1960),Van Schilfgaarde (1965)].He used Hooghoudt's equivalent depth in place of actual saturated thickness below drain level , to account for the radial flow close to the drain ("Drainage Principles" 1983-Vol. II). He could reproduce the hydrographs to some extent (though not as closely as accomplished by the two dimensional flow models ,SFM &TRM)in case of the 50 and 75 m tile spacings.However,the reproduction was very poor in case of the 25 m tile spacing.This can be explained by considering the effect of drain spacing on the head loss.In case of the larger

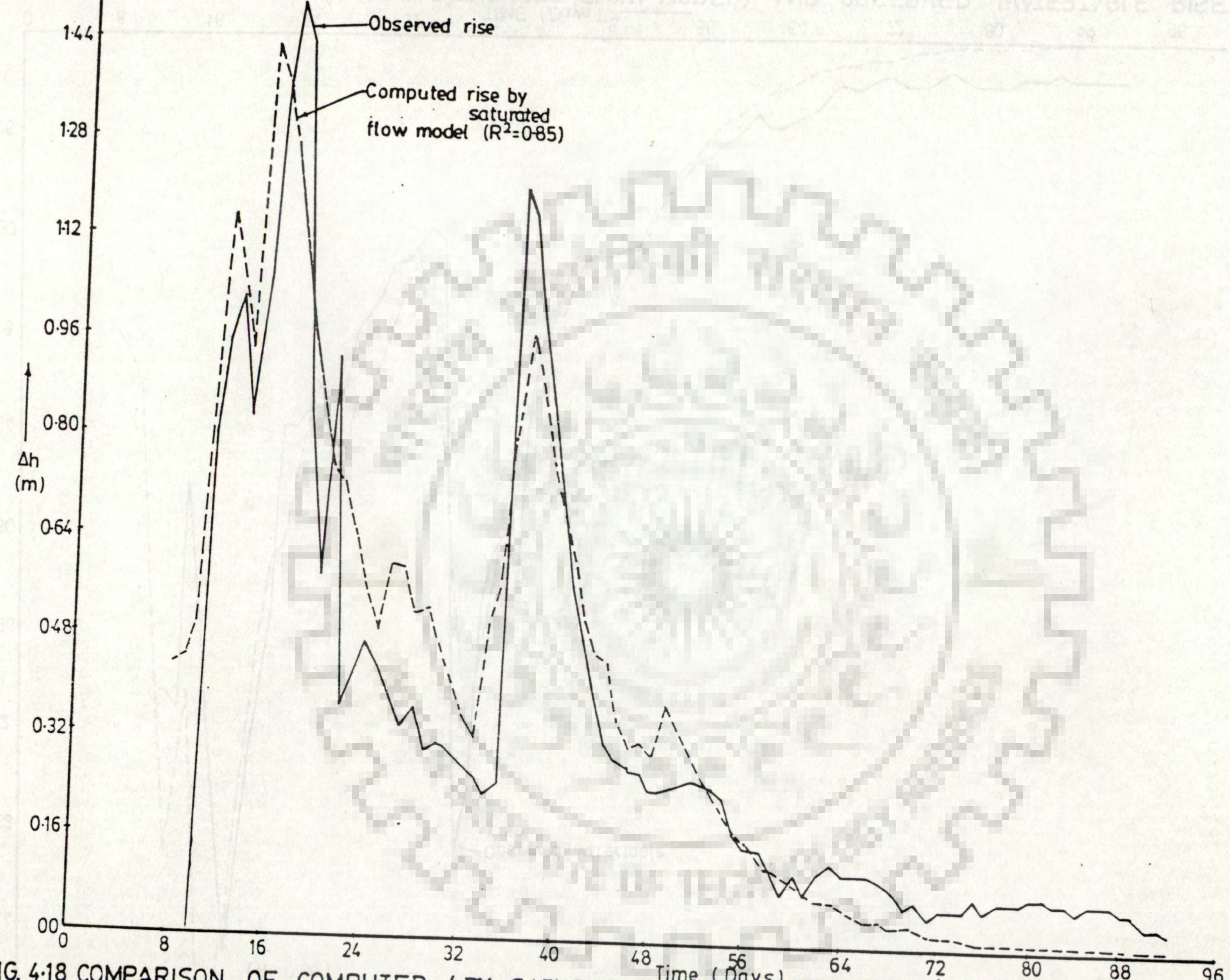


FIG. 4·18 COMPARISON OF COMPUTED (BY SATURATED FLOW MODEL) AND OBSERVED WATER TABLE RISE AT MIDSECTION OF 25 M. TILE SPACING.

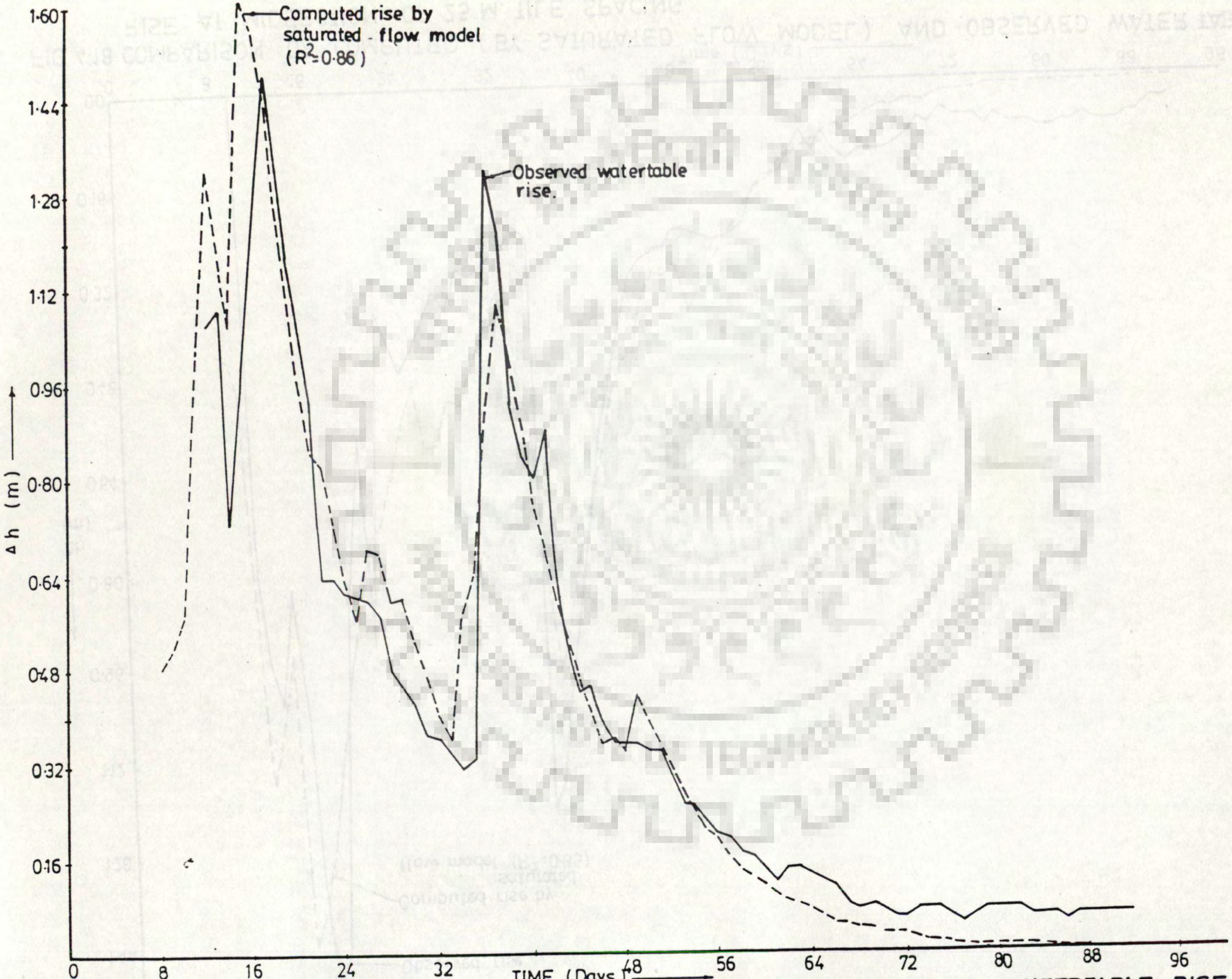


FIG 4.19 COMPARISON OF COMPUTED (BY SATURATED FLOW MODEL) AND OBSERVED WATERTABLE RISE

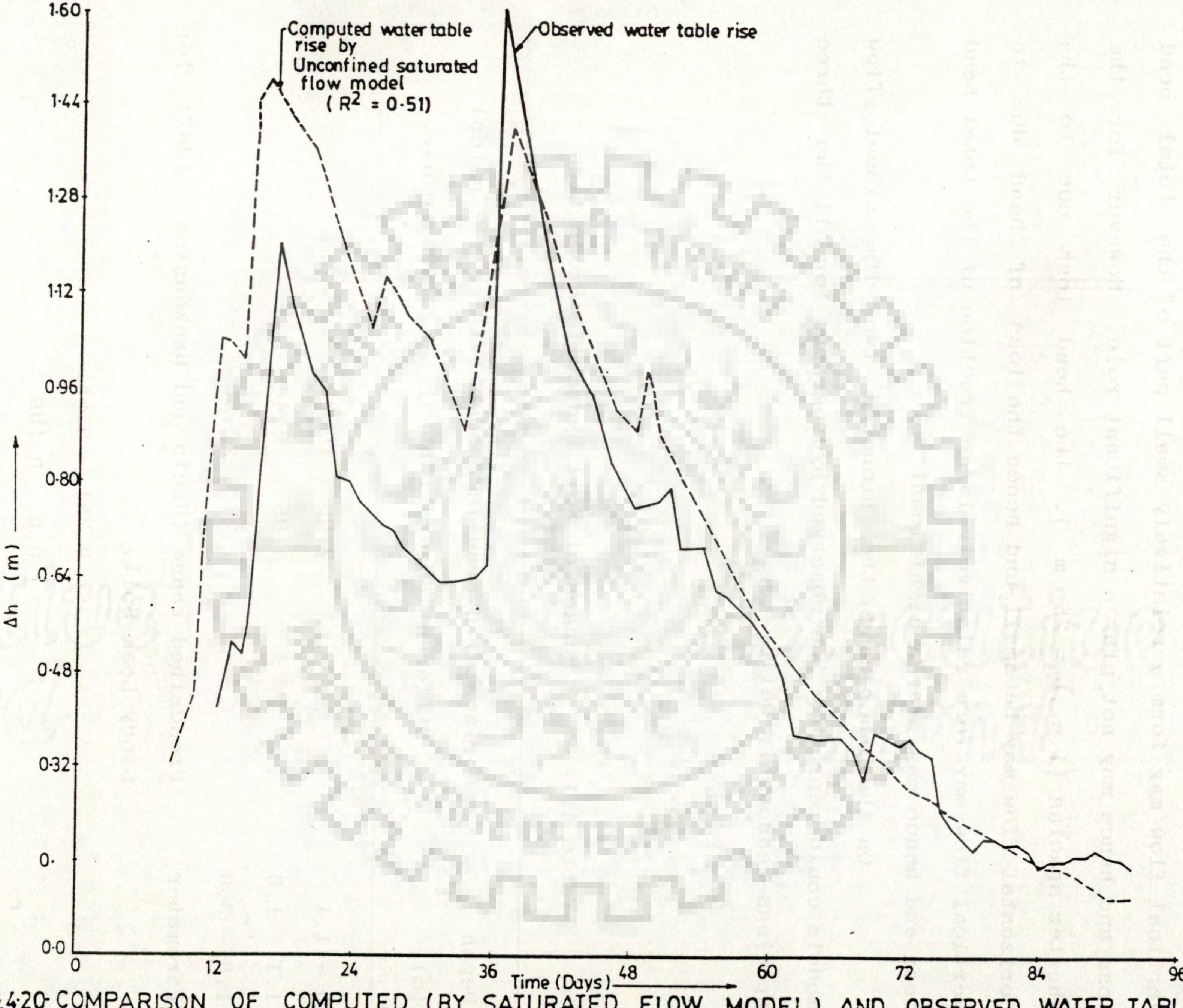


FIG.4·20- COMPARISON OF COMPUTED (BY SATURATED FLOW MODEL) AND OBSERVED WATER TABLE RISE AT MIDSECTION OF 75 M TILE SPACING.

spacings (i.e., L = 50 m and 75 m) the head loss due to the horizontal flow may be large and thus, the loss of head due to vertical flow may form a relatively small part of the total head loss and hence may not play a significant role. However for the shorter spacing (i.e., L = 25 m), the head loss due to the horizontal flow may be small and hence the loss of head due to vertical flow may form relatively large fraction of the total head loss and hence may play a significant role .

On the other hand the proposed two dimensional flow models could reproduce the observed hydrographs for all the three spacings reasonably well.

Table 4.1: Ranges of Soil Hydraulic Properties

Depth (m)	Horizontal Hydraulic Conductivities (12 sites) of Different Layers (Chhedi Lal, 1986) m/day.
0 - 1.1	0.06 - 10.00
1.1 - 1.8	0.05 - 2.44
1.8 - 3.0	1.27 - 22.50
Parameter	Prescribed range (Rawls and Brakensiek, 1982) for Sandy Loam Soil.
ϕ	0.351 - 0.555
θ_r	0.0 - 0.106
h_b (m)	0.0 0.302

Table 4.2: Adopted Values of Soil Hydraulic Properties

Parameter	Spacing = 25 m	Spacing = 50 m	Spacing = 75 m
Saturated-	0.2	0.6	0.75
hydraulic-	0.1	0.32	0.40
Conductivity	1.3	4.22	5.00
(Kx or Kz)			
W_p	0.035	0.055	0.055
θ_r	0.030	0.035	0.035
h_b	0.220	0.222	0.180
ϕ	0.50	0.50	0.50

Table 4.3: Observed Rainfall from July to September, 1985

Dates	Total Rainfall (cm.)	Dates	Total Rainfall (cm.)
7.7.85	0.474	26.7.85	2.48
8.7.85	5.61	27.7.85	0.69
9.7.85	0.66	29.7.85	0.96
10.7.85	1.37	3.8.85	3.14
11.7.85	5.94	4.8.85	1.62
12.7.85	5.18	5.8.85	4.33
15.7.85	8.65	6.8.85	3.27
16.7.85	1.34	16.8.85	0.69
17.7.85	0.57	17.8.85	0.24
22.7.85	0.98	18.8.85	1.67

CHAPTER V

MODEL APPLICATION

5.1 SATURATED FLOW MODEL

The saturated flow model solution for partially and fully penetrating ditch systems has been presented in the form of dimensionless design curves. These curves along with Kraijenhoff solution permit graphical estimation of steady state rise of water table within a practical range of geometric dimensions and the parameters.

5.1.1 Design Curves for Ditch Drainage

The proposed model can provide more realistic designs in comparison to the Donnan and Kraijenhoff solutions. It has been shown earlier (paragraphs 4.1.1 and 4.1.2) that the model computed water table elevations compare well with the analytical solutions provided Kz is very large in comparison to Kx (Ahmad et al. 1990,1991). However, in practice Kz may be much less than Kx due to stratification. Such anisotropic conditions cause an excessive water table rise. For example, for Kz/Kx = 0.1, the Donnan solution leads to an underestimation of steady state water table rise by as much as 50% and can be as much as 75% for Kz/Kx =0.01 (refer Fig. 4.2). Similarly the Kraijenhoff solution gives an underestimation of the unsteady state water table rise (refer Fig. 4.1). Thus, the saturated flow model gives a more realistic and rational design for subsurface drainage systems. Recognizing that a designer may not have access to a computer and the necessary software, dimensionless design curves have been generated for graphical designs (Fig. 5.1–5.4).

5.1.1.1 Dimensional Analysis

The model computed steady state rise of the water table (Δh) at the midsection (in a partially penetrating ditch drainage system) can be expressed as a function of the physical quantities involved.

$$f_1(\Delta h,\ Kx\ ,\ Kz\ ,\ L,\ b,\ d,\ Y_0,\ R) = 0 \qquad (5.1)$$

In accordance with Buckinghum π theorem (Streeter and Benjamin, 1987) equation (5.1) can be completely defined in terms of dimensionless variables as follows:

$$\frac{\Delta h}{Y_0} = f_2\left(\frac{Kx}{Kz}, \frac{d}{y_0}, \frac{b}{d}, \frac{d}{L}, \frac{R}{Kz}\right) \qquad (5.2)$$

Denoting the steady state rise in accordance with the Kraijenhoff solution as Δh^*, the ratio ($\Delta h^*/y_0$) can be written as:

$$\frac{\Delta h^*}{y_0} = f_3\left(\frac{Kx}{Kz}, \frac{d}{y_0}, \frac{b}{d}, \frac{d}{L}, \frac{R}{Kz}\right) \qquad (5.3)$$

Dividing equation (5.2) by equation (5.3) we get:

$$\frac{\Delta h}{\Delta h^*} = f_4\left(\frac{Kx}{Kz}, \frac{d}{y_0}, \frac{b}{d}, \frac{d}{L}, \frac{R}{Kz}\right) \qquad (5.4)$$

The number of curves required to completely describe the dependence of $\Delta h/\Delta h^*$ on the five independent dimensionless variables (equation 5.4) will be prohibitively large. It has been found by numerical experimentation, that b/d does not affect

$\Delta h/\Delta h^*$ appreciably. Further, $\Delta h/\Delta h^*$ becomes independent of R/Kz when the flow above the initial drain level is neglected. This reduces the number of dimensionless variables to three and equation (5.4) can be written as:

$$\frac{\Delta h}{\Delta h^*} = f\left(\frac{Kx}{Kz}, \frac{d}{y_0}, \frac{d}{L}\right) \qquad (5.5)$$

5.1.1.2 Preparation of Design Curves for Ditch Drainage

The model is operated for fully penetrating and partially penetrating ditches under isotropic and anisotropic soil conditions ignoring the flow above drain level. A finite differences grid is superposed over the flow domain (Fig. 3.2). Non-uniform grid spacing are adopted, assigning in general, lower spacings in the zones of higher anticipated velocities. The model operation, commencing from the initial condition, is continued until the steady state. The piezometric heads are computed at the discrete times separated by nonuniform time steps. Smaller time steps are assigned in the early stage when piezometric heads are known to change faster. Later, each time step (Δt) is divided into two equal subtime steps and the piezometric heads are calculated after two time steps of $\Delta t/2$. The two different values of piezometric heads each at the end of time step Δt are then compared. The differences are summed up and compared with a prestipulated convergence factor. If this sum of differences exceeds the convergence factor, then the time step is divided into four equal subtime steps. The procedure of doubling the number of subtime steps is continued until there is no appreciable change in the computed heads (at the end of the time step) by the subsequent doubling.

Typical values are assigned to the variables L and Kz (L= 30 m, Kz= 2 m/day). The model is operated to estimate Δh for various discrete values of other independent variables (b,d,y_0 and Kx). The discrete values are selected to cover a practical range of variation of each of the three independent dimensionless variables i.e., Kx/Kz from 0.1 to 20, d/y_0 from 0.25 to 1.0 and d/L from 0.075 to 0.5. The resulting design curves show that $\Delta h/\Delta h^*$ approaches unity (i.e., Kraijenhoff solution holds good) as Kx/Kz decreases (relatively higher vertical conductivity), d/y_0 increases (higher ditch penetration) and d/L decreases (larger ditch spacing). Thus, for isotropic soils (i.e., Kx/Kz = 1), the error in Kraijenhoff solution is negligible provided the ditches penetrate at least 75% thickness (i.e., $d/y_0 \geq 0.75$) and the ditch spacing is att least 13.3 (say 15) times the ditch penetration (i.e., $d/L \leq 0.075$). For anisotropic soils the error is restricted to 5% provided Kx/Kz does not exceed 5, the ditches are fully penetrating and the ditch spacing is at least 15 times the ditch penetration. Use of these curves may require interpolation with respect to d/y_0 and d/L.

5.1.1.4 Illustration

Consider an area drained by a parallel partially penetrating rectangular ditch system with ditch geometry and soil parameters as follows: L = 50 m, y_0=6 m, d =4 m, Kx = 1 m/day, Kz= 0.1 m/day, R = 0.01 m/day (refer Fig. 3.1). The steady state rise of water table at the mid section is estimated graphically as follows:

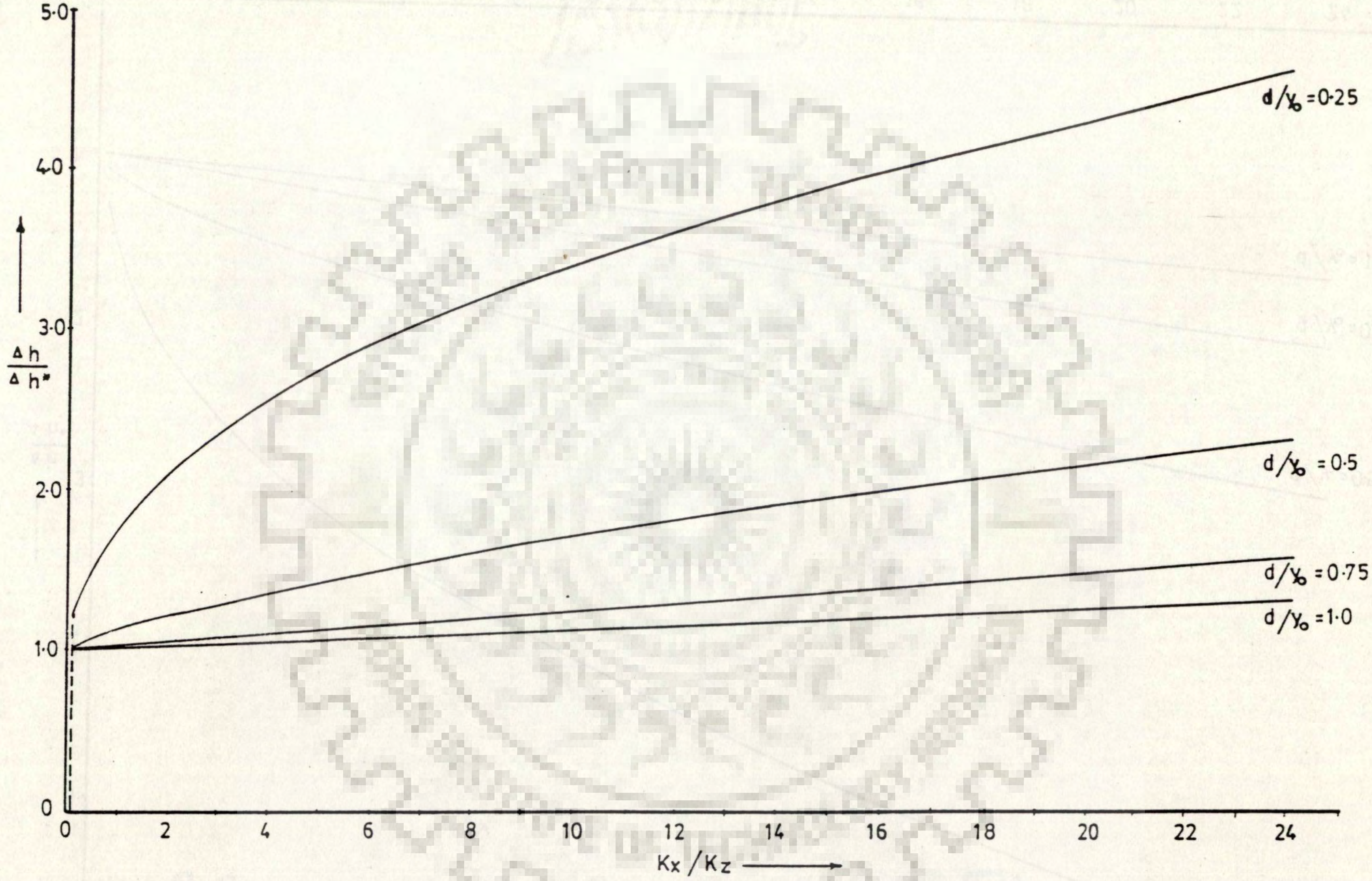


FIG. 5·1 DESIGN CURVES SHOWING VARIATION OF $\Delta h/\Delta h^*$ WITH K_x/K_z AND d/y_0 AT $d/L = 0{\cdot}075$

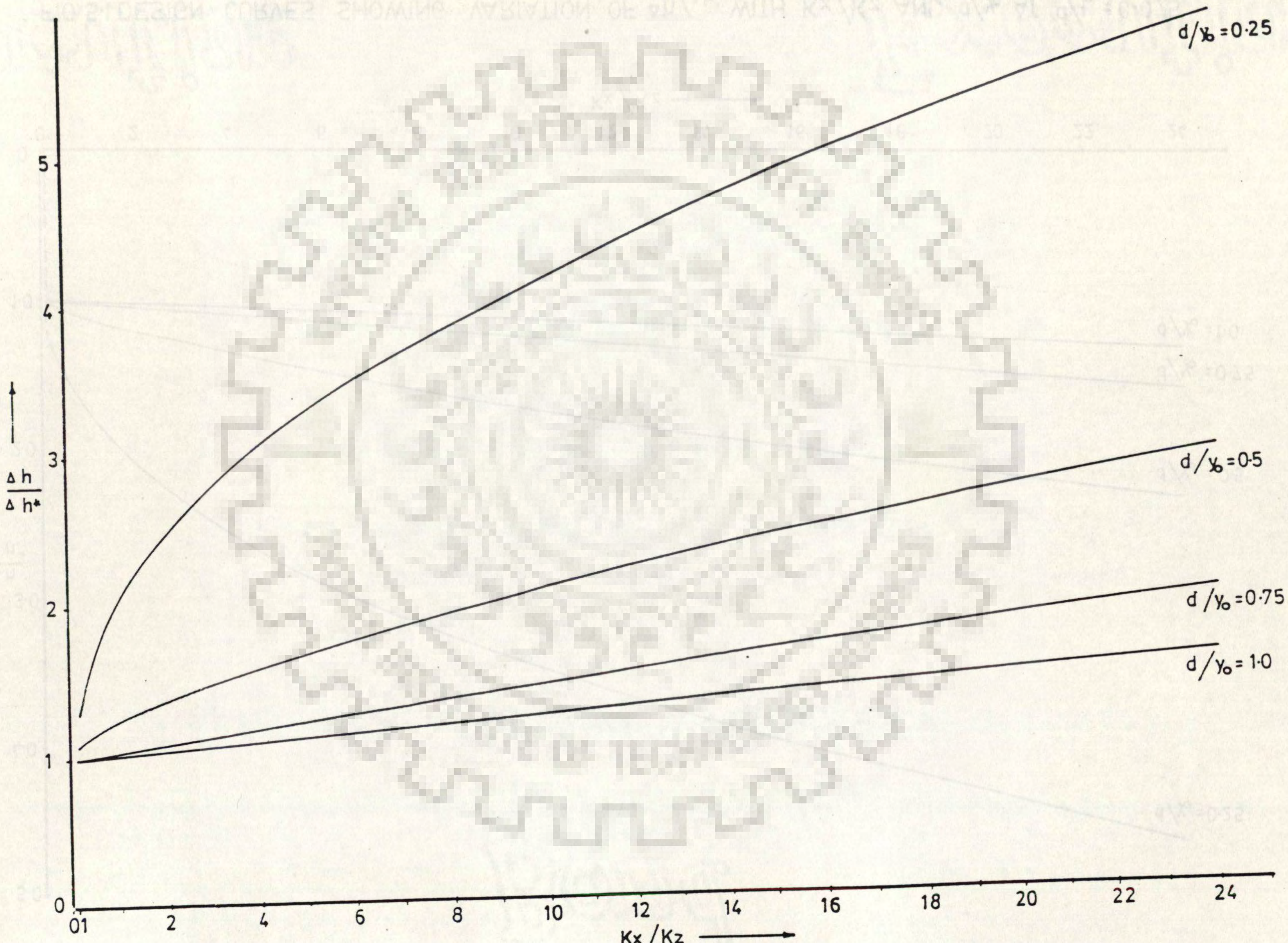


FIG. 5·2-DESIGN CURVES SHOWING VARIATION OF $\Delta h/\Delta h^*$ WITH Kx/Kz AND d/y_o AT d/L =0·1

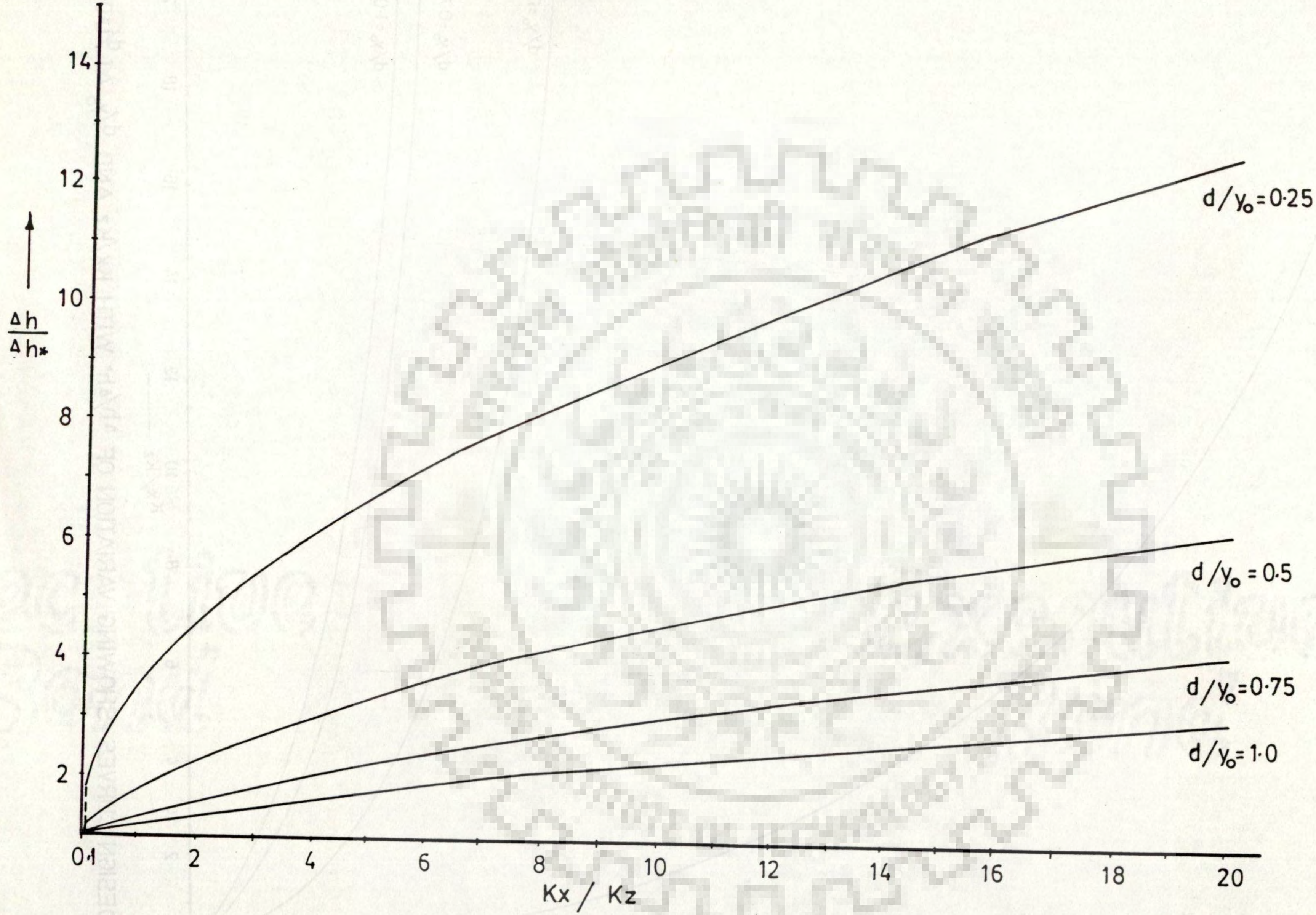


FIG. 5·3 DESIGN CURVES SHOWING VARIATION OF $\Delta h / \Delta h^{*}$ WITH Kx / Kz AND d/y_0 AT $d/L = 0.25$

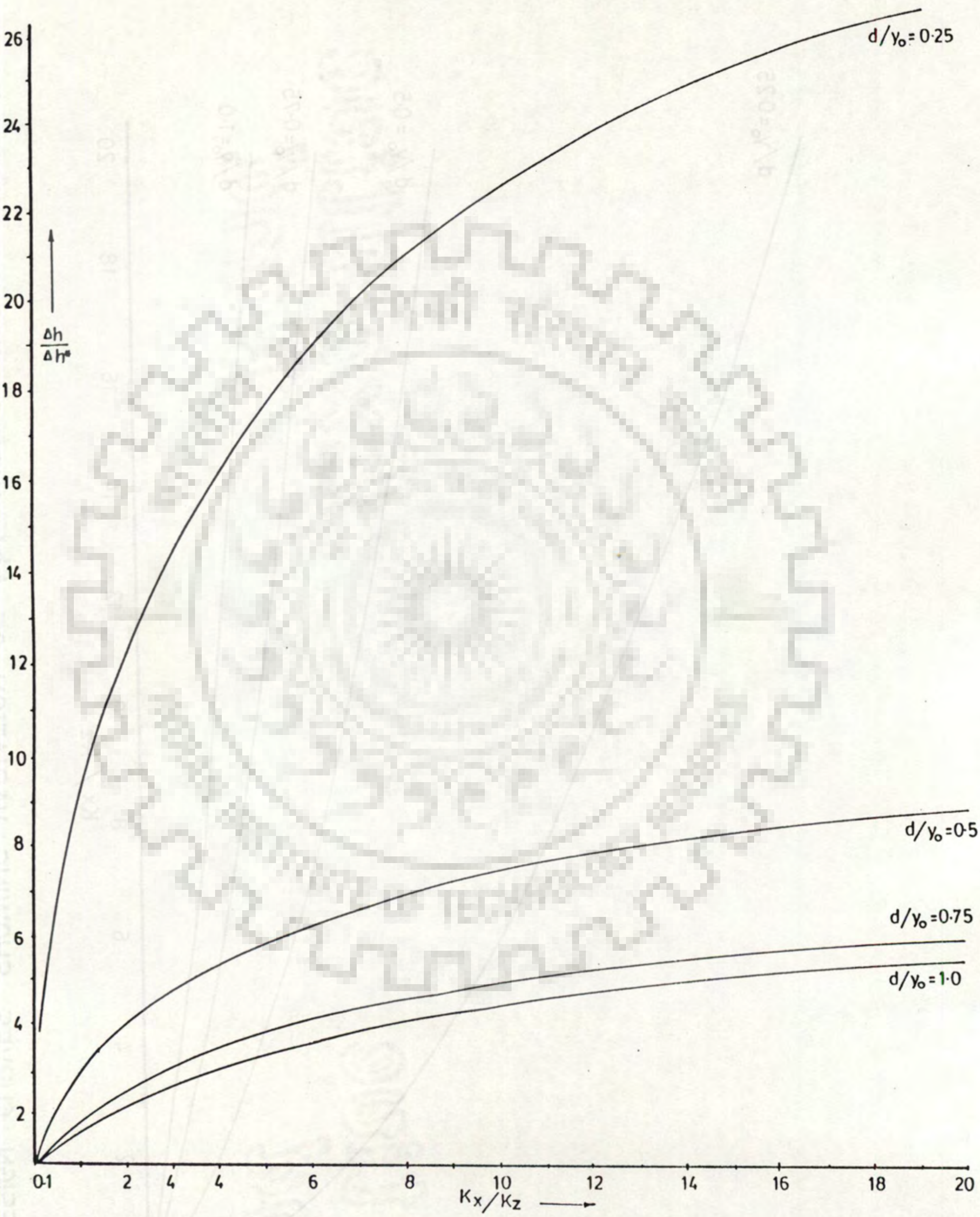


FIG.5·4 DESIGN CURVES SHOWING VARIATION OF $\Delta h/\Delta h^{*}$ WITH Kx/Kz AND d/y_o AT d/L=0·5

(i) CALCULATION OF ΔH^*:

The steady state Kraijenhoff solution (Drainage Principles and Applications, Vol.II, 1983) yields

$$\Delta h^* = \frac{RL^2}{8Ky_o} = \frac{0.01x50x50}{8x1x6} = 0.5208 \text{ m.}$$

(ii) GRAPHICAL ESTIMATION OF $\Delta h/\Delta h^*$:

For the present problem independent dimensionless variables are:

$$\frac{d}{L} = 0.08, \; d/y_0 = 0.67, \; \frac{Kx}{Kz} = 10$$

since $\Delta h/\Delta h^*$ for these values of the independent variables is not directly available in any of the curves, an interpolation is called for. For the present problem d/L lies in between d/L ratings (0.075 and 0.1) of two sets of curves given in Fig. 5.1 and Fig. 5.2; d/y_0 lies in between d/y_0 ratings (0.5 and 0.75) of the middle two curves of each set. Thus, $\Delta h/\Delta h^*$ can be estimated by two way interpolation from the four corners having d/L and d/y_0 coordinates as (0.075, 0.5); (0.075, 0.75); (0.1, 0.5); (0.1,0.75) (Fig. 5.5). The interpolated values of $\Delta h/\Delta h^*$ at (0.075, 0.67) and (0.1, 0.67) are 1.401 and 1.665 respectively. Thus, the interpolated value of $\Delta h/\Delta h^*$ for the problem in hand (d/L = 0.08, d/y_0 = 0.67) is 1.453.

Therefore, the rise at midsection,

Δh = 1.453 x 0.5208 = 0.757 m.

The rise computed by direct operation of the model is 0.750 m.

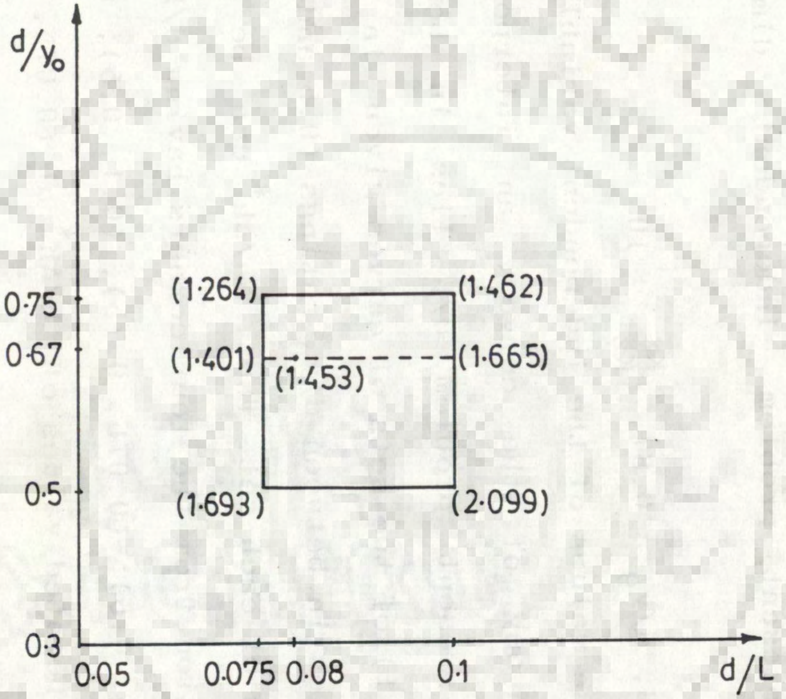


FIG 5·5 GRAPHICAL ESTIMATION OF $\frac{\Delta h}{\Delta h^*}$

($\frac{\Delta h}{\Delta h^*}$ Values shown in parentheses)

5.1.2 Computation of Velocity Distribution

For solving transport problems the spatial and temporal distribution of the horizontal and vertical velocities need to be known. These can be computed by the application of the saturated flow model or the total response model (refer section 3.4). A typical distribution of horizontal and vertical velocities computed by the saturated flow model have been shown in Figs (5.6 - 5.13).

5.2 TOTAL RESPONSE MODEL

The total response model can be used for stream aquifer interaction problems in situations where the flow in the aquifer system can be considered two dimensional (x-z plane), instead of three dimensional. The application of the total response model in studying the throughflow and bank storage has been explained in the following paragraphs.

5.2.1 Study of the Development of Throughflow

Consider an unsaturated zone with an impeding layer. The percolating water tends to accommodate over the layer. This raises the soil moisture and ultimately may lead to the development of saturated condition (generally termed as perched water table) over the impeding layer. This may initiate lateral flow towards the drain through the perched water table. Such flow is termed as throughflow.

In layered soils, if the unsaturated zone above water table consists of layer(s) of very low hydraulic conductivity, some of the infiltrated water may find its way to the ditch as throughflow from above the low hydraulic conductivity layer(s).

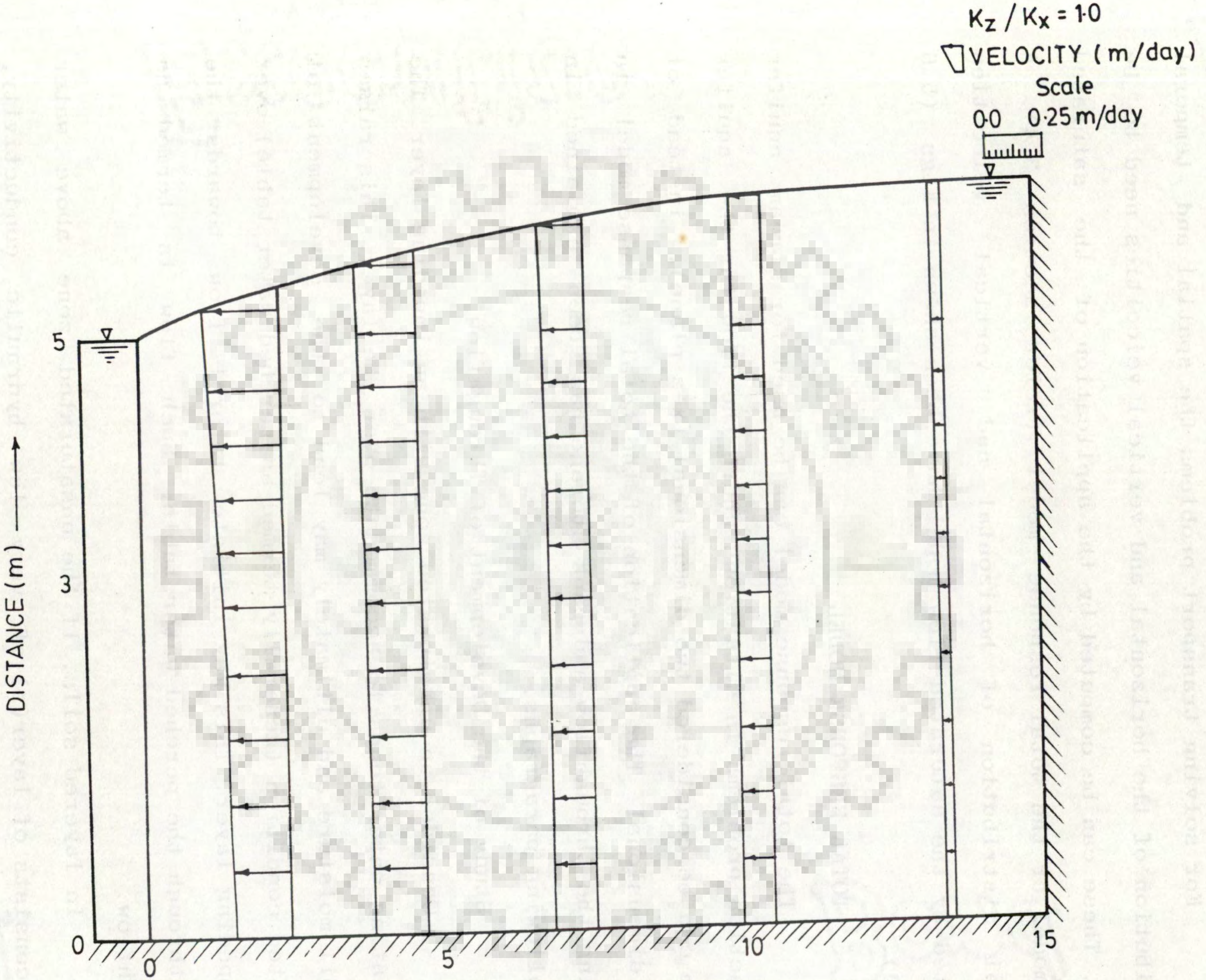


FIG 5·6 HORIZONTAL VELOCITIES TOWARDS A FULLY PENETRATING DITCH (AT STEADY STATE)

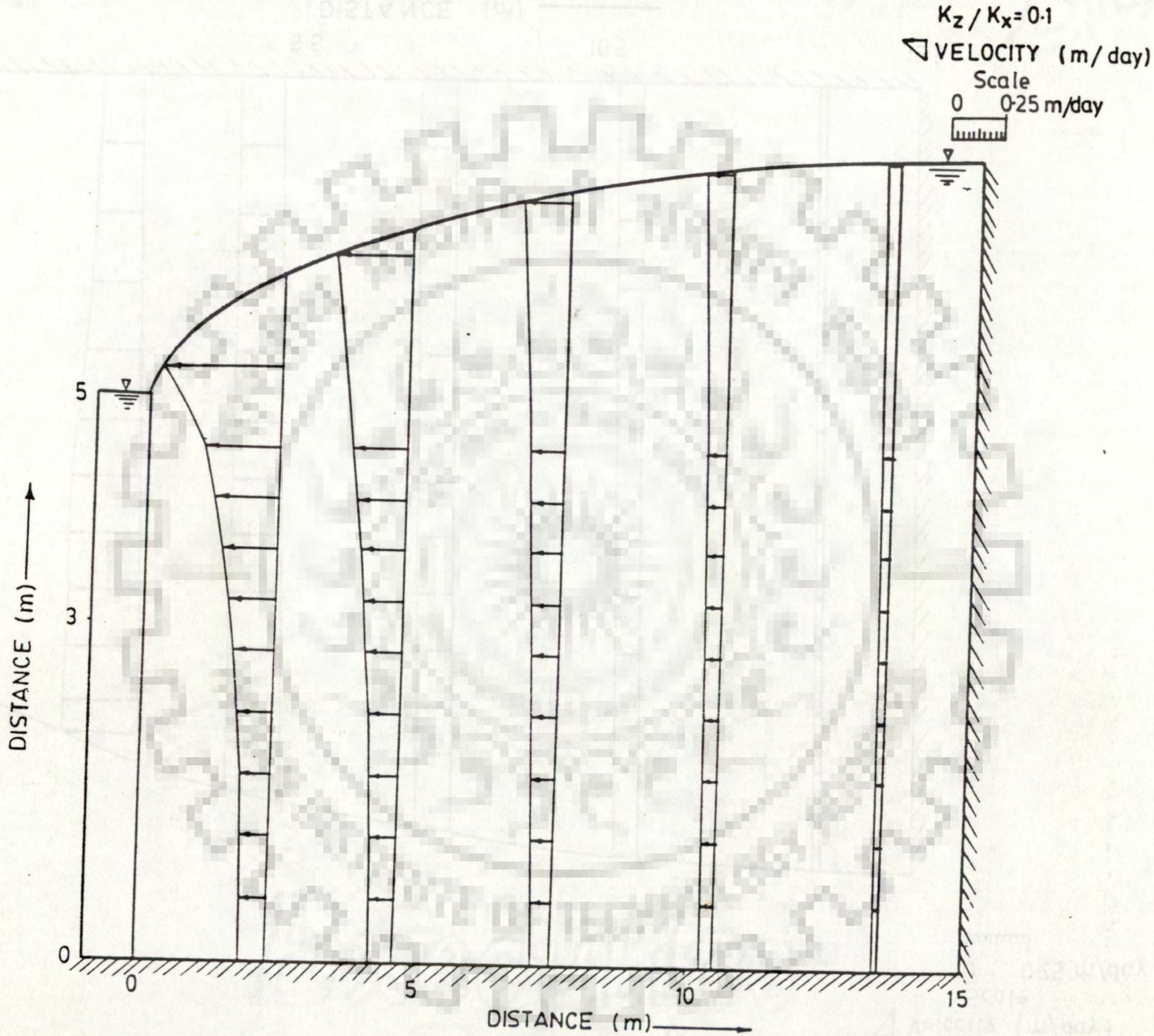


FIG.-5.7 -HORIZONTAL VELOCITIES TOWARDS A FULLY PENETRATING DITCH (AT STEADY STATE)

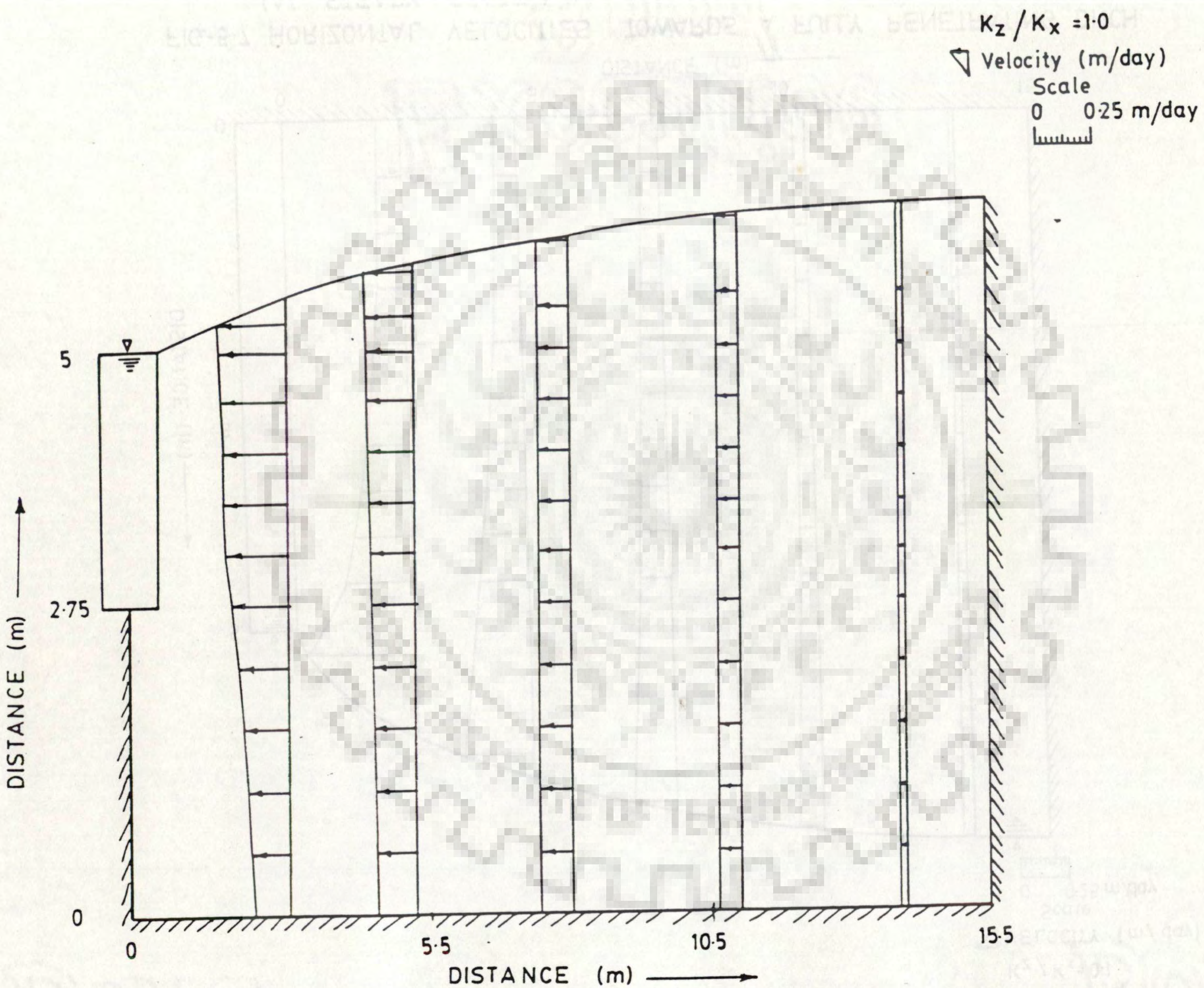


FIG.5·8 - HORIZONTAL VELOCITIES TOWARDS A PARTIALLY PENETRATING DITCH

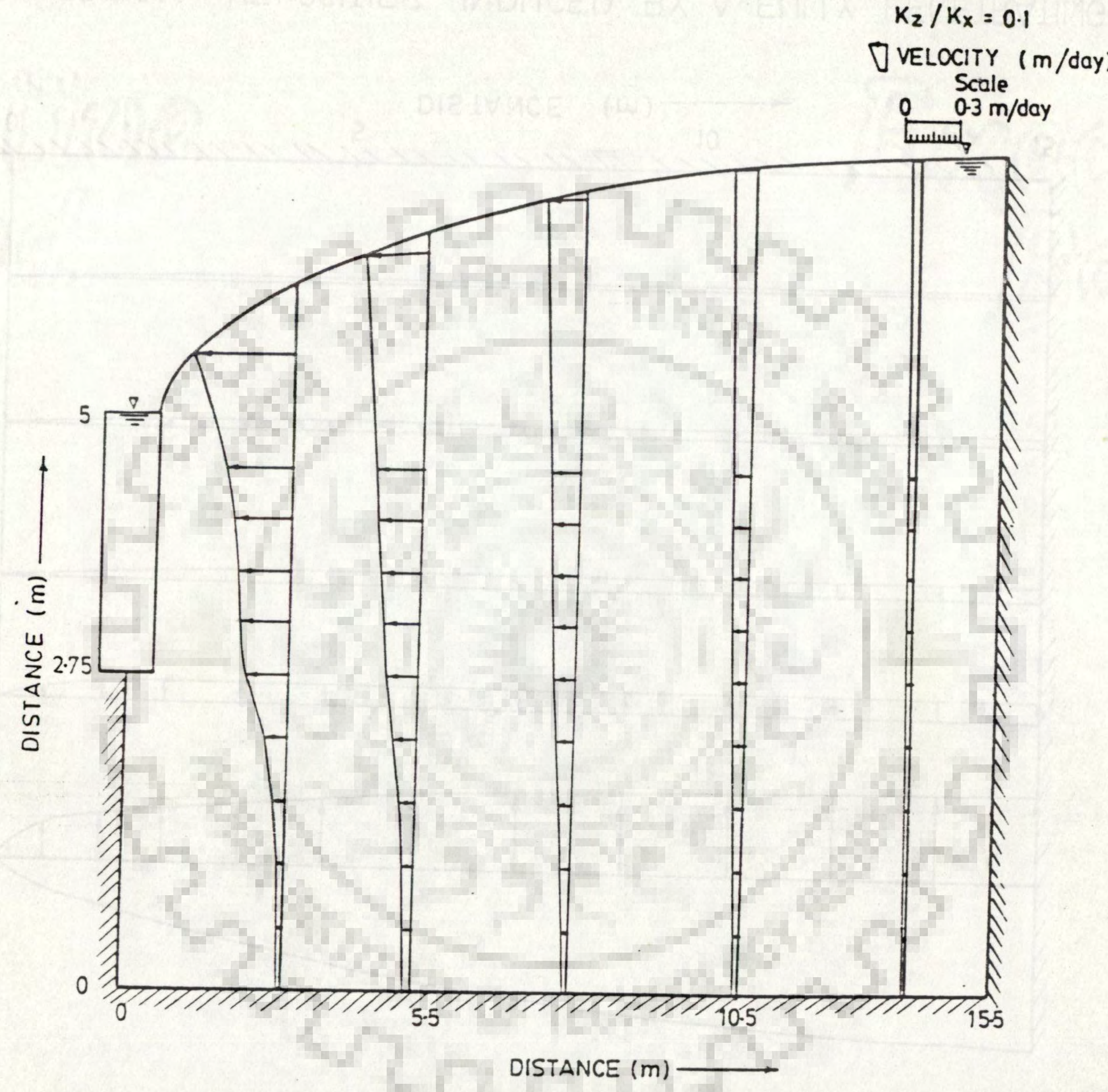


FIG 5·9 HORIZONTAL VELOCITIES TOWARDS A PARTIALLY PENETRATING DITCH (AT STEADY STATE)

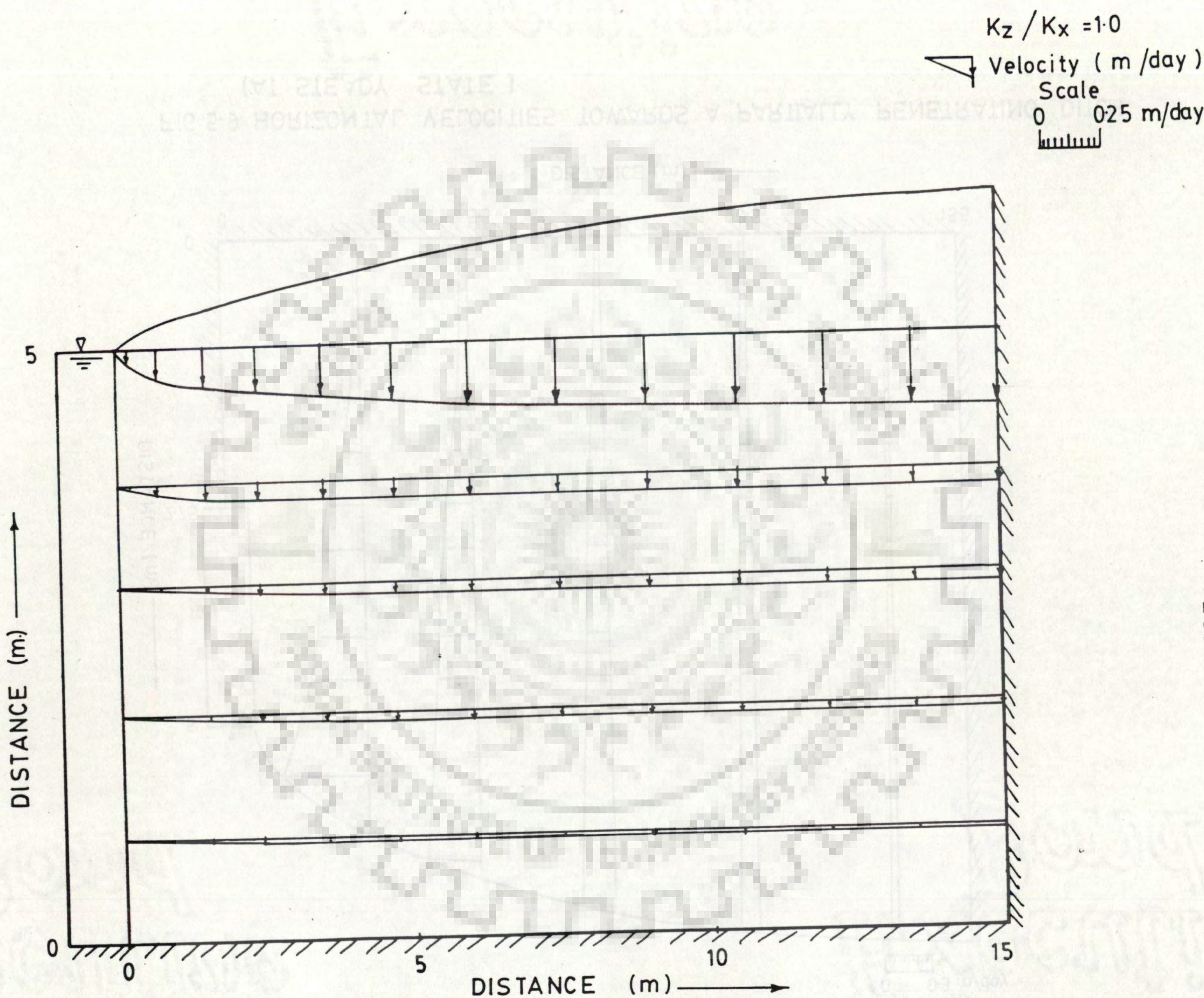


FIG. 5·10 - VERTICAL VELOCITIES INDUCED BY A FULLY PENETRATING DITCH

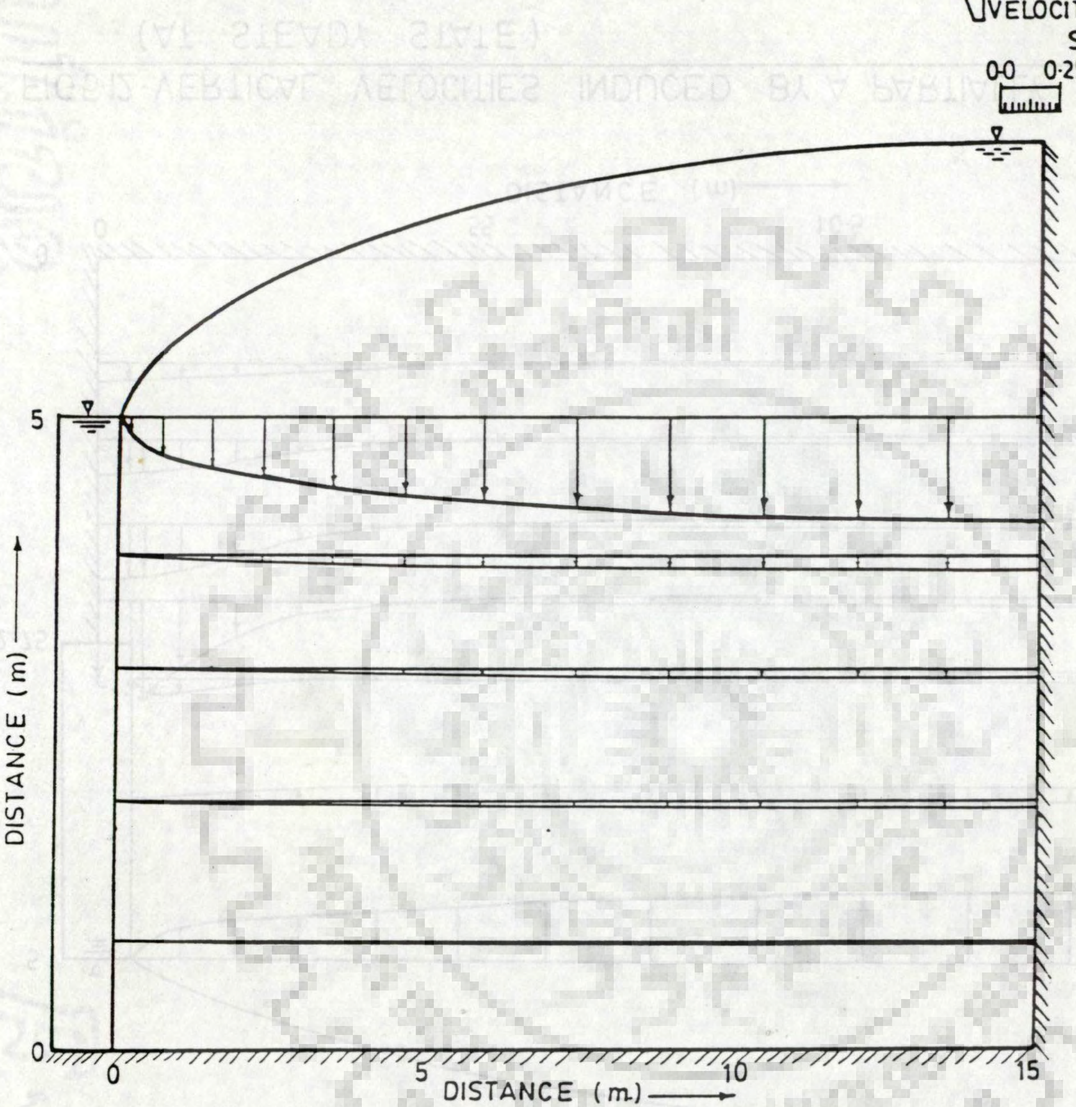


FIG. 5·11 -VERTICAL VELOCITIES INDUCED BY A FULLY PENETRATING DITCH
(AT STEADY STATE)

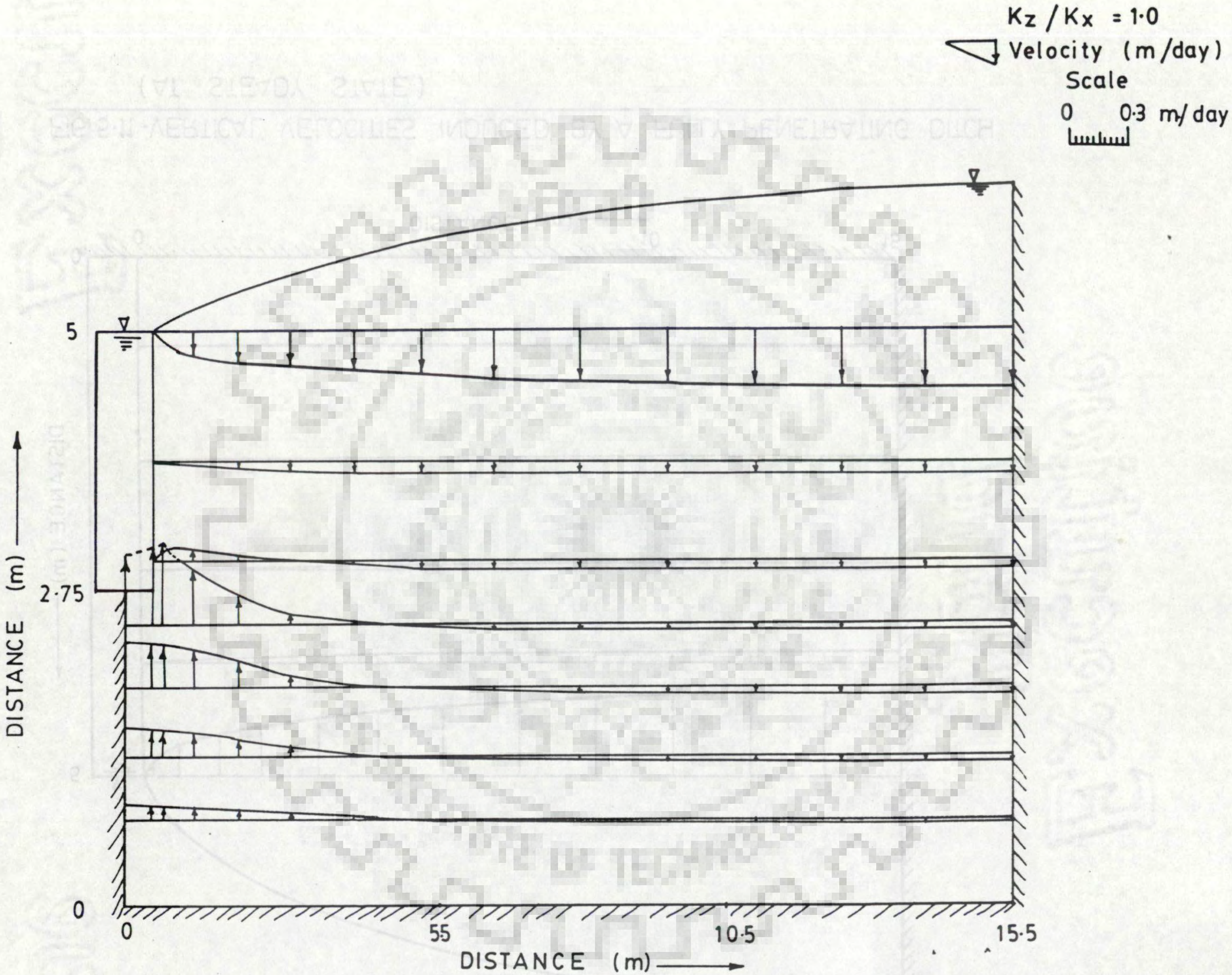


FIG.5·12 - VERTICAL VELOCITIES INDUCED BY A PARTIALLY PENETRATING DITCH (AT STEADY STATE)

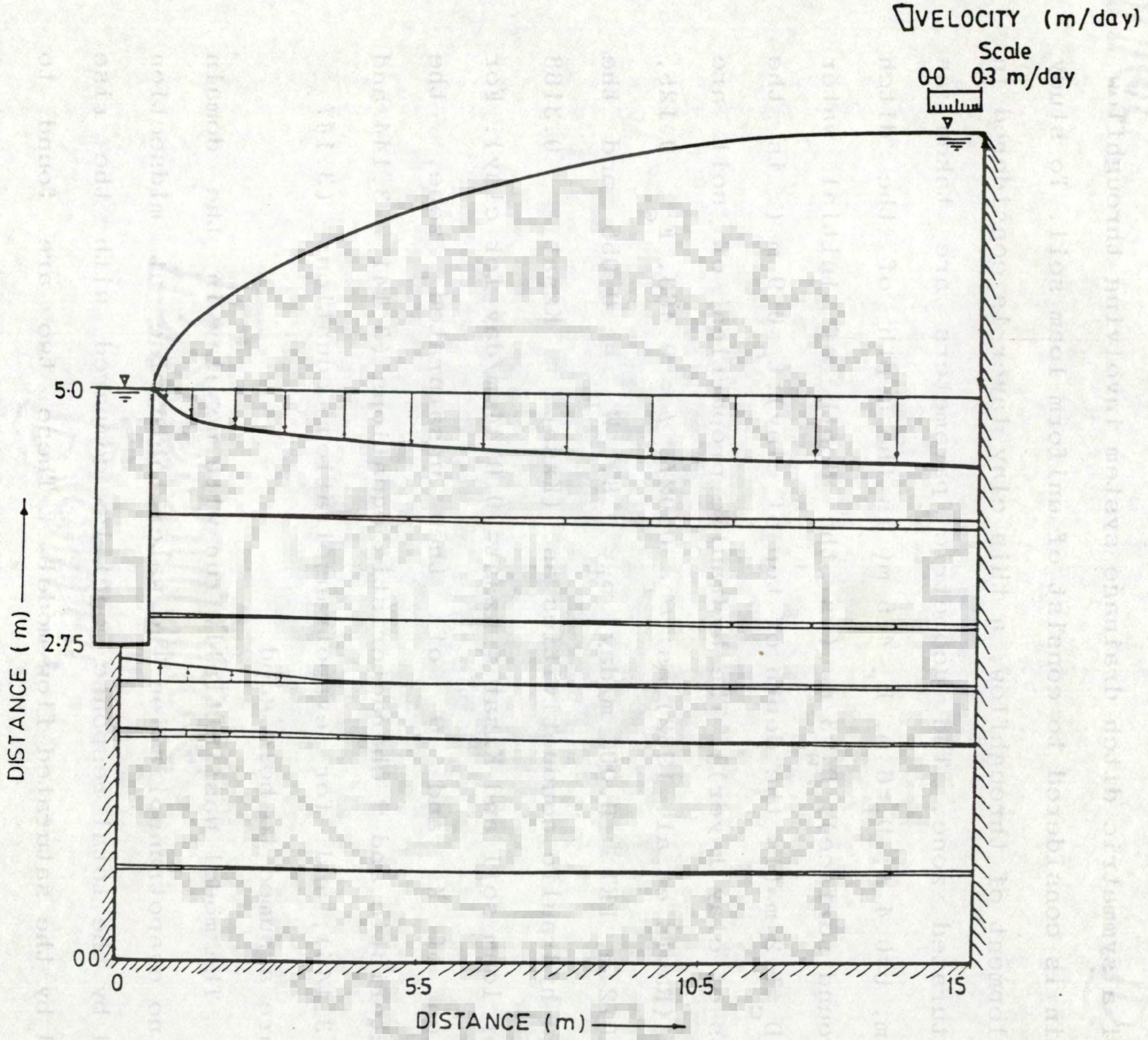


FIG. 5·13-VERTICAL VELOCITIES INDUCED BY A PARTIALLY PENETRATING DITCH. (AT STEADY STATE)

This throughflow can be computed by using the total response model.

Fig (5.14) represents schematic diagram of the flow domain of a symmetric ditch drainage system involving throughflow. The domain is considered to consist of uniform loam soil. To study the development of throughflow, a thin clay layer is considered in the unsaturated zone. The geometric parameters are taken as [L=10.4 m; b=0.4 m; D_i=6 m; D_r=4.5 m(is the depth of the ditch below ground surface); D_w=3 m (is the depth of initial water table); D_c=1.2 m (is the depth of top of clay);t_c=0.9 m (is the thickness of clay layer)]. The hydraulic properties of soil are assigned (Rawls et al. 1982) as: θ_r= 0.027, ϕ = 0.463, F_c= 0.125, h_b = 0.4012 m, PET = 0.001 m/day, p_f= 0.8, W_p = 0.055 and the saturated hydraulic conductivities as (Kxsat = Kzsat = 0.3168 m/day for loam soil and Kxsat = Kzsat= 0.0144 m/day for clay). For estimation of K and θ of the unsaturated zone, the K-characteristic and θ-characteristic equations (3.141 - 3.144 and 3.145 - 3.146); and for evapotranspiration equations (3.147 - 3.149) are assumed to hold good.

The model was initially run with no clay in the domain and for no evapotranspiration.The water table rise at midsection computed by the total response model is compared with the rise computed by the saturated flow model . These two are found to match at steady state (Fig. 5.15). This fulfills the theoretical requirement for the TRM, as the total response model accounts the transfer of flow through unsaturated zone. At steady state, the recharge at the water table should be equal to the infiltration

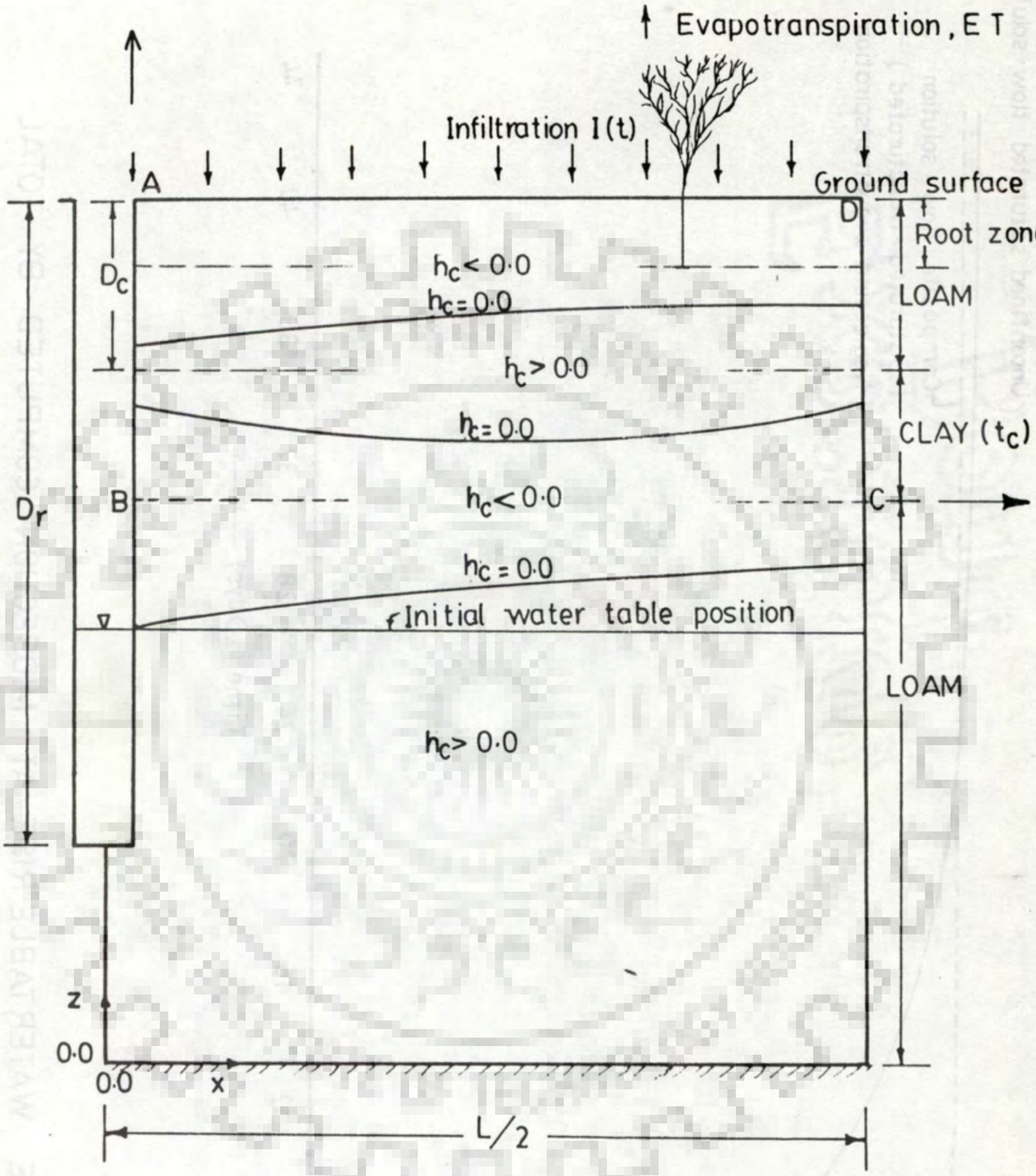


FIG.5·14 -SCHEMATIC DIAGRAM OF DITCH DRAINAGE WITH A CLAY LAYER IN THE UNSATURATED ZONE.

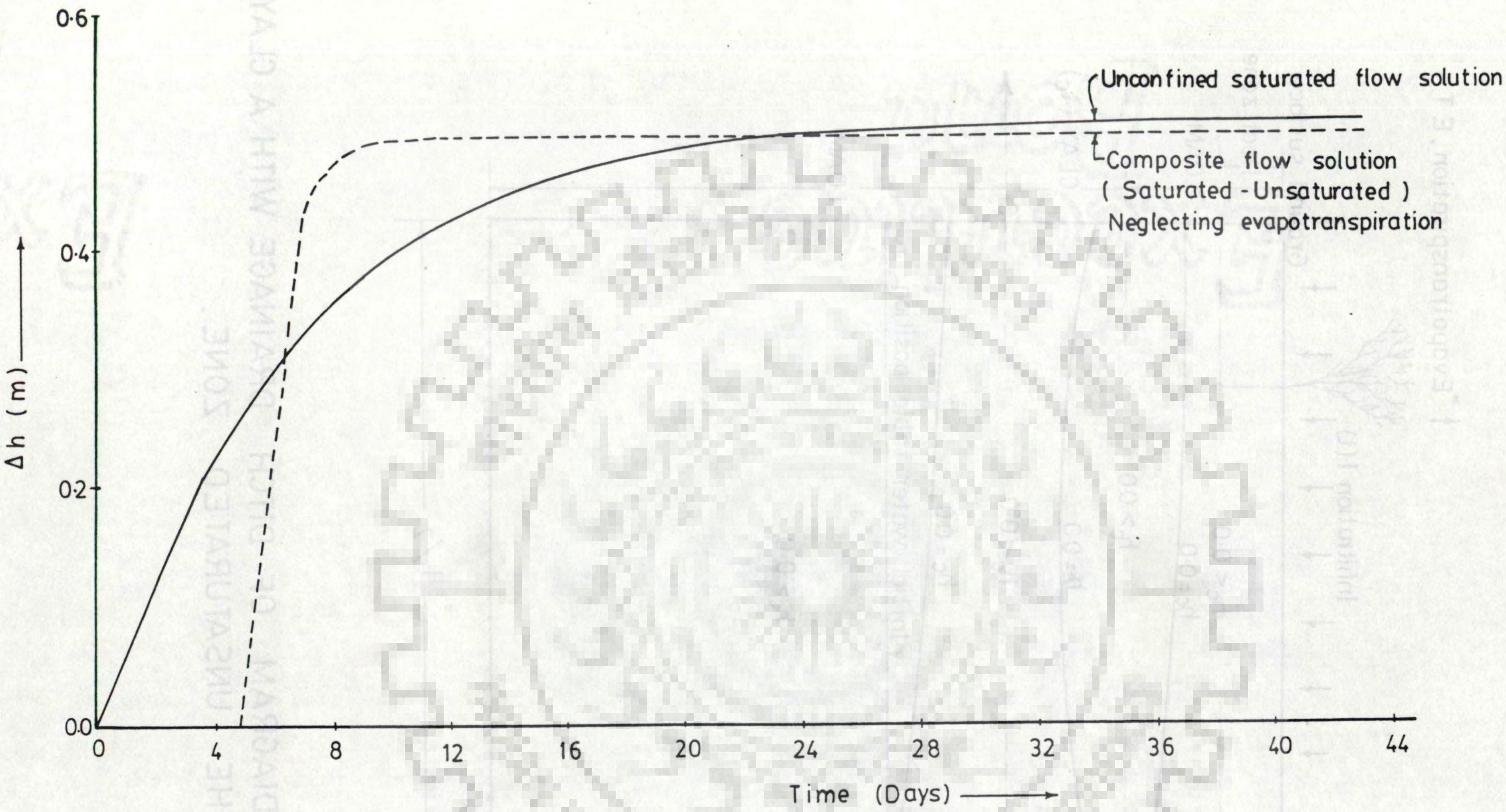


FIG. 5·15 - COMPARISON OF THE WATERTABLE RISE AT MIDSECTION COMPUTED BY TOTAL RESPONSE MODEL (PET = 0·0) AND SATURATED FLOW MODEL.

assigned at the ground surface (for no evapotranspiration) and hence the water table rise computed by SFM and TRM should match.

The total response model is then run with and without a clay layer in the unsaturated zone assigning a potential evapotranspiration of 0.001 m/day . In the presence of a 0.9 m thick horizontal clay layer the initiation of rise is lagged by about 2 days, and the maximum rise is reduced by about 0.06 m (Fig. 5.16). The infiltration is assigned for a period of 12 days. The computed water table is found to rise till 12 days, beyond which it starts receding. The period of recession is longer in the presence of clay layer. The portion of Fig. (5.14) marked as ABCD is redrawn in Fig. (5.17) to explain the build up of perched saturated zone and the development of throughflow (q_i). It is seen that the perched saturated zone's thickness is maximum on 9th day, though, a near steady state reaches much later (12th day). Fig. (5.18) shows the rate of lateral inflow (q) to the ditch from the saturated zone below water table and the rate of throughflow (q_i) from the perched saturated zone. It is observed that the maximum rate of throughflow is also on the 9th day, which reaches a near steady state rate on 12th day. Fig. (5.19) shows the decrease of the perched saturated zone after ceasure of the infiltration. It is noticed that on 13th day, though, there exists a perched saturated zone but there is no throughflow. (Figs 5.18 and 5.19). Seepage face for throughflow existed even at the 30th day. The period of throughflow in general, however, depends upon many factors (such as rate of infiltration, thickness of clay, depth to the top of clay etc.), and it may continue for many days even

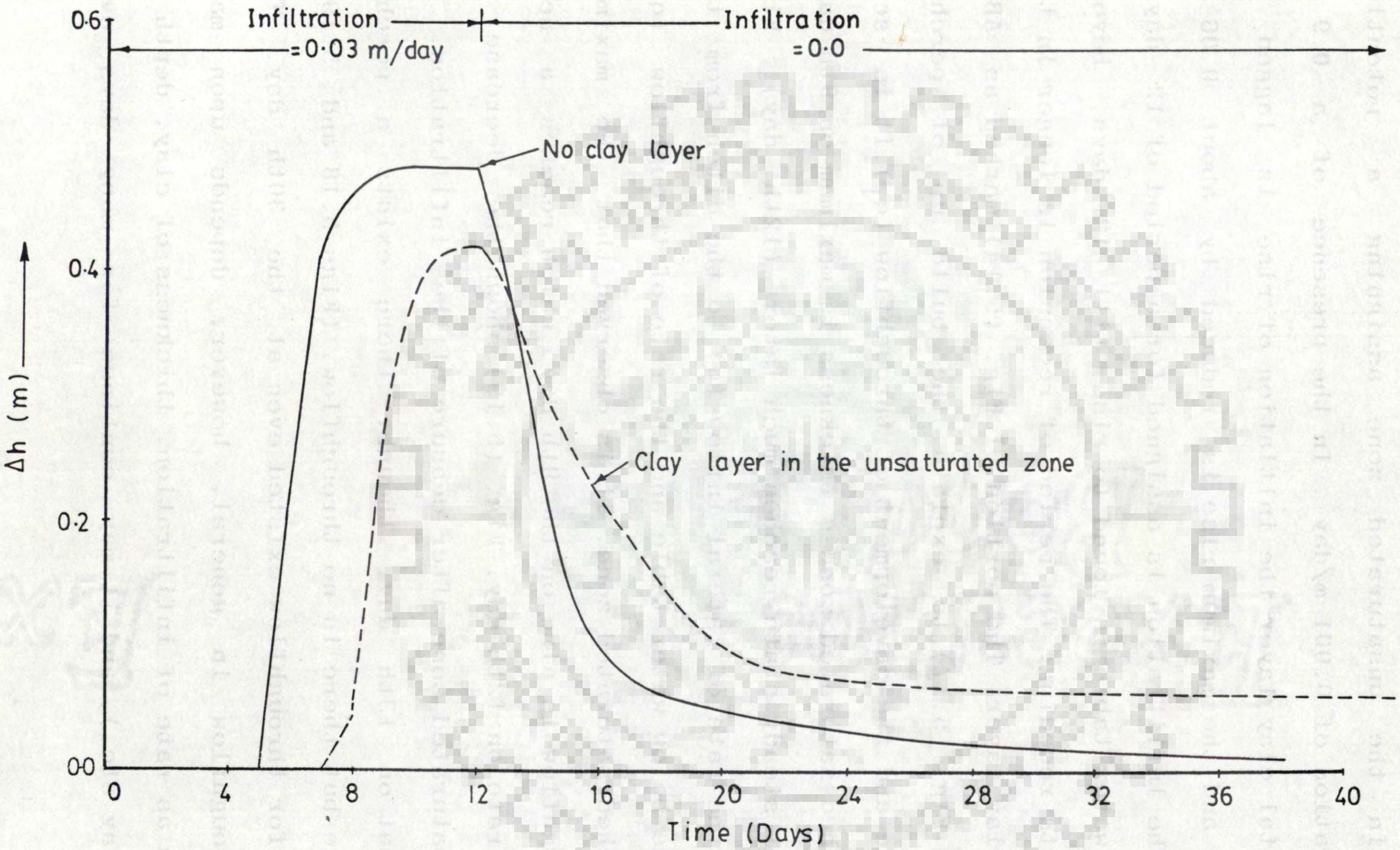


FIG. 5·16 - WATER TABLE RISE (Δh) AT MIDSECTION COMPUTED BY TOTAL RESPONSE MODEL (PET = 0·001 m/day)

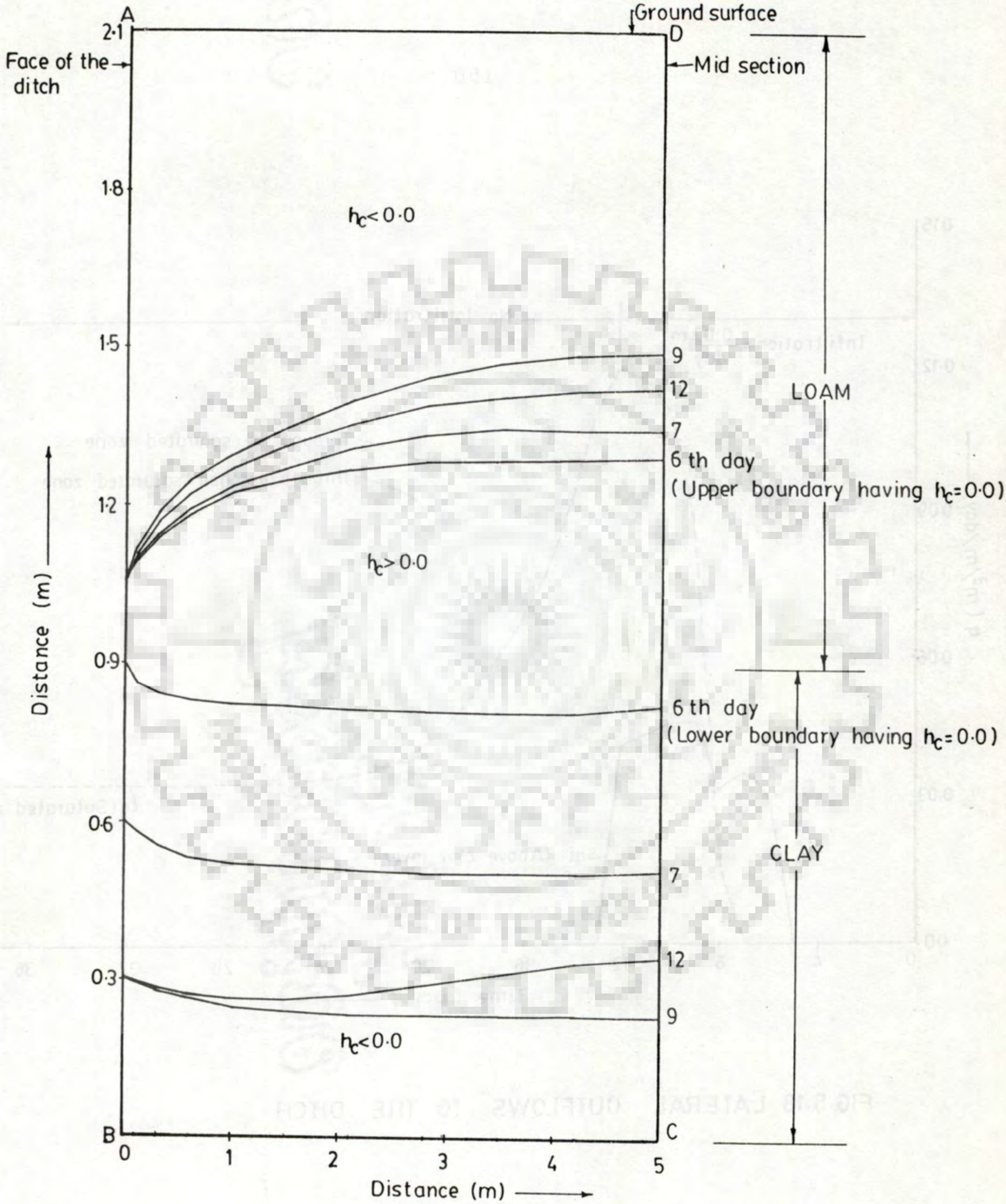


FIG.5·17 - DEVELOPMENT OF THROUGH-FLOW ABOVE A CLAY LAYER IN THE UNSATURATED ZONE DURING INFILTRATION.

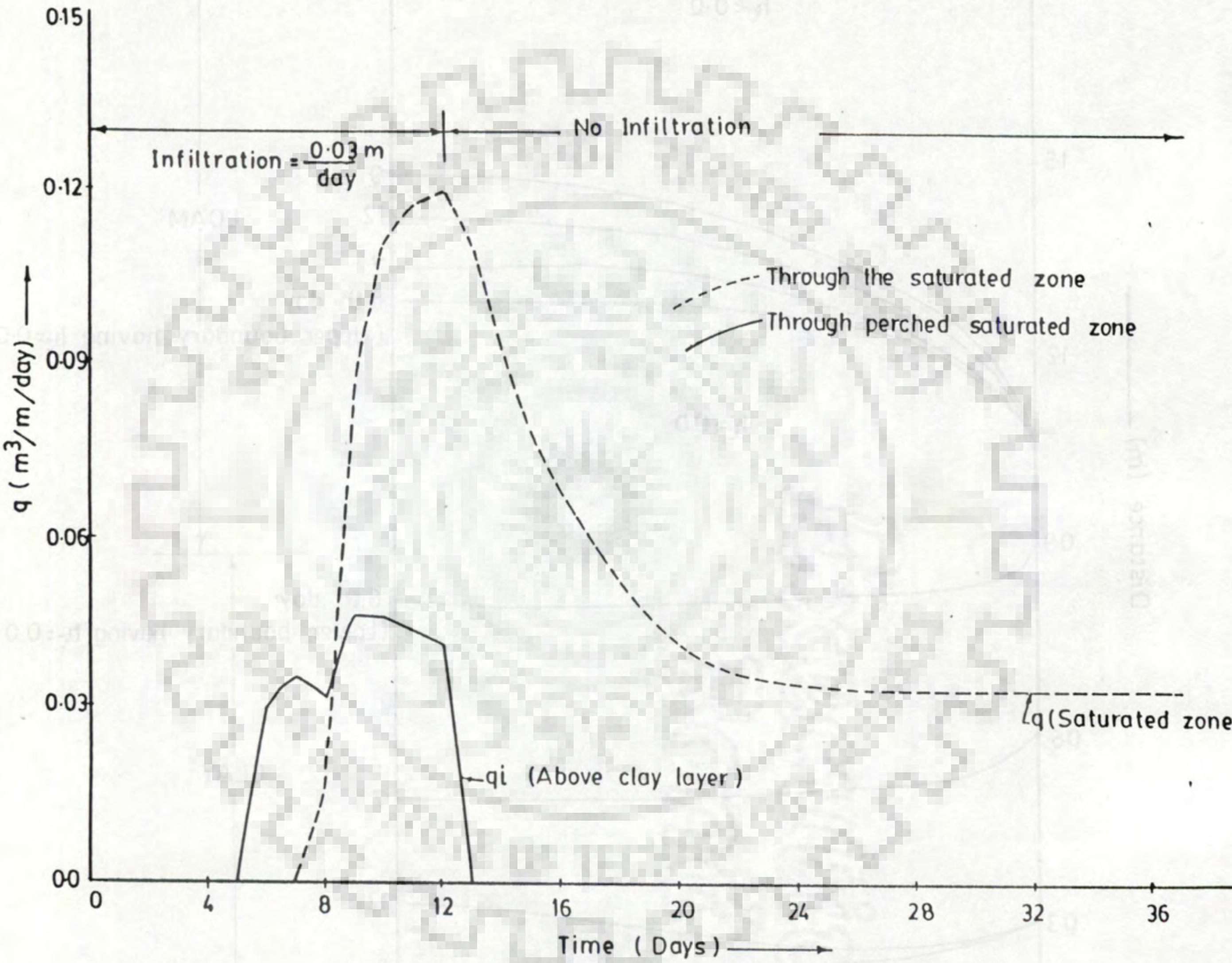


FIG 5.18 LATERAL OUTFLOWS TO THE DITCH

after infiltration is ceased.

The effect of clay thickness and the effect of partial penetration on the throughflow is studied (Fig. 5.20) in terms of the dimensionless parameters (D_w/t_c, D_r/D_i and Q_i/Q). Where $Q_i = \Sigma q_i dt$ and $Q = \Sigma q.dt$ are respectively the cumulative throughflow from perched saturated zone and the cumulative saturated flows from beneath the water table in 60 days. The geometric and other parameters are considered as [L = 10.4 m, D_i = 6.3 m, D_w = 3.3 m, I(t) = 0.05 m/day (only for 20 days), b = 0.4 m, D_c = 1.5 m, PET = 0.0]. The thickness of clay and the depth of the ditch has been varied as (t_c = 0.3 , 0.6 , 0.9 , 1.2 , 1.5 m, and D_r = 3.3 , 3.9 , 4.5 , 5.1, 5.7, 6.3m). Dimensionless curves are plotted taking D_r/D_i on horizontal axis and Q_i/Q on vertical axis, for varying clay thickness. It is observed that Q_i/Q increases with increasing clay thickness and with decreasing ditch penetration (Fig. 5.20). These dimensionless curves are, however, found to change with change in other parameters (e.g., spacing L; rate of infiltration I(t)). On the other hand, it is not even possible to draw unique sets of dimensionless curves due to nonlinearity of the problem and a large number of parameters involved.

5.2.2 Bank Storage Development and Release

Bank storage build up and its subsequent release to the ditch is studied by passing an assumed stage hydrograph (Fig. 5.21 of 7 days duration through the ditch. This shape of the stage hydrograph is chosen to suit the time step size (0.25 days) and less number of rows considered. However, the model can be used to execute any shape of the stage hydrograph by considering

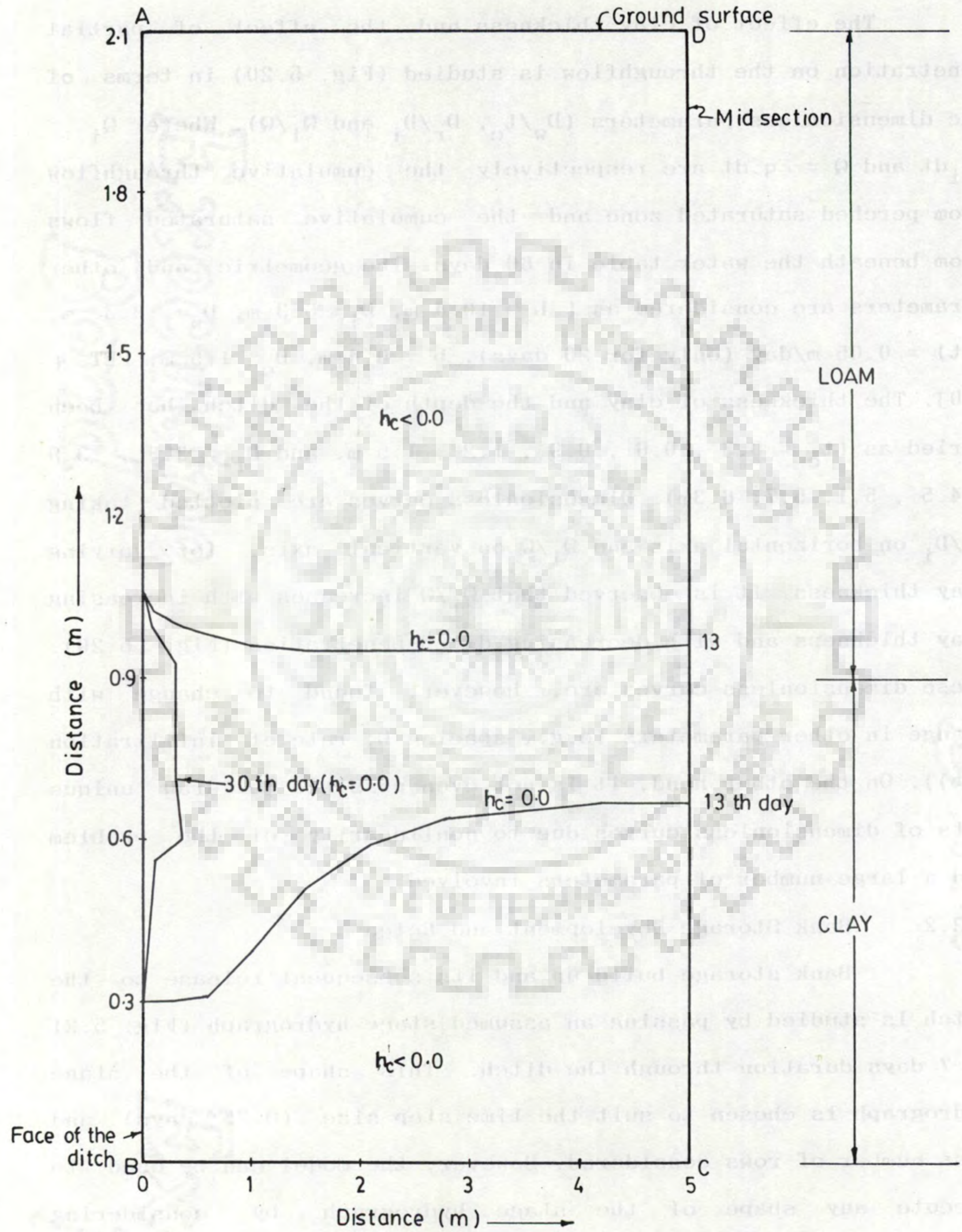


FIG. 5.19-DEPLETION OF THE PERCHED SATURATED ZONE. AFTER CEASURE OF THE INFILTRATION

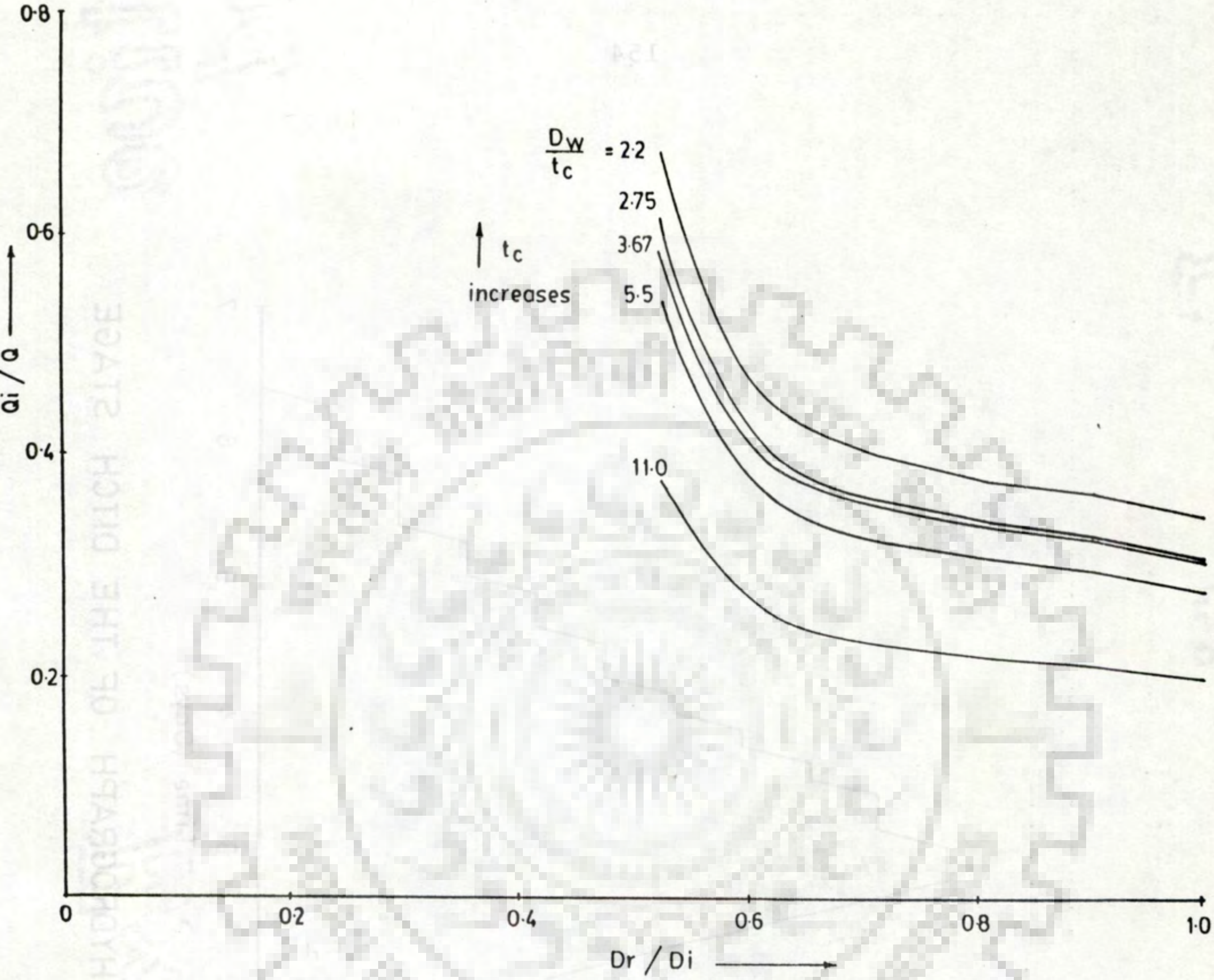


FIG.5.20 DEPENDENCE OF THROUGH-FLOW ON CLAY THICKNESS AND DITCH PENETRATION.

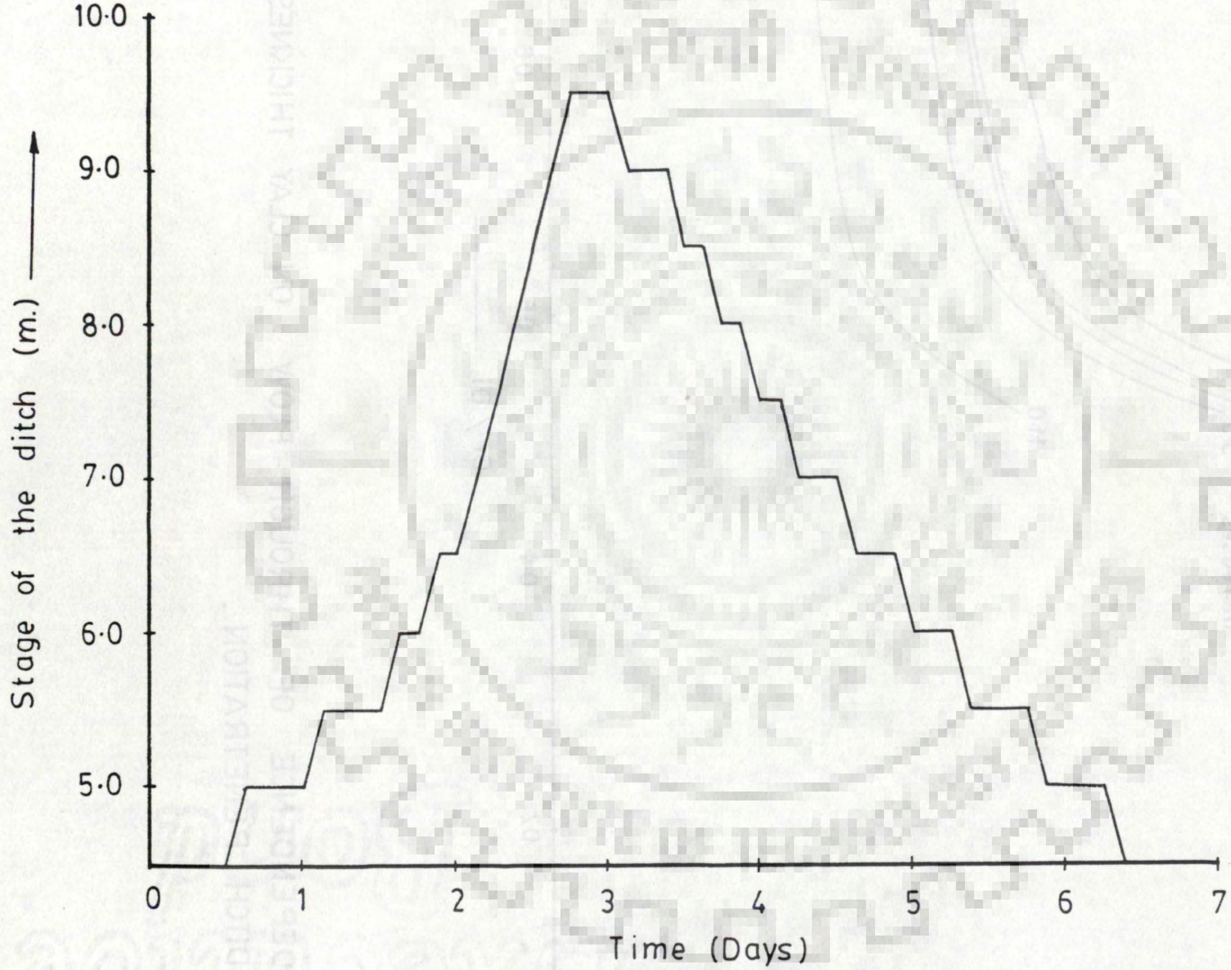


FIG. 5·21 - ASSIGNED HYDROGRAPH OF THE DITCH STAGE appropriate time step sizes and suitable discrete spatial sizes. The parameters considered here are (L = 6000 m, b =60 m, D_i = 10.5 m, D_w = 6.0 m, D_r = 7.5 m). Soil is heterogeneous in horizontal direction (up to 102 m sand, 102 to 312 m loamy sand, 312 to 1014 m sandy loam, 1014 to 3000 m loam). The hydraulic properties of soil are taken from Rawls et al (1982). For properties of the unsaturated zone Brooks and Corey (1964) relations (equations 3.141 - 3.144) and equations (3.145 - 3.149) are considered to hold good. Two cases; i.e., with infiltration, I(t) = 0.0 and I(t) = 0.1m/day are considered. Fig. (5.22 - 5.24) and Figs (5.25 - 5.27) represents the cases of no infiltration (I(t) = 0.0) and with infiltration (I(t) = 0.1 m/day) respectively. Fig. (5.22) represents the rate of lateral flows (q_b) from ditch to the aquifer (when water level is higher in the ditch). Thus negative of these ($-q_b$) represents the rate of lateral flows from aquifer to the ditch. Fig. (5.23) represents the cumulative flow (Q_b) to the aquifer, the maximum being at 4.5 days. Fig. (5.24) shows the cumulative flows released to the ditch. In the specific problem considered, it is seen that about 60% of the maximum cumulative flows stored in the banks are released within 20 days. The rest 40% may take a very very long time as the gradient of flow has become significantly low and it is still decreasing. The Figs (5.25-5.27) are corresponding to the infiltration rate, I(t) = 0.1 m/day.

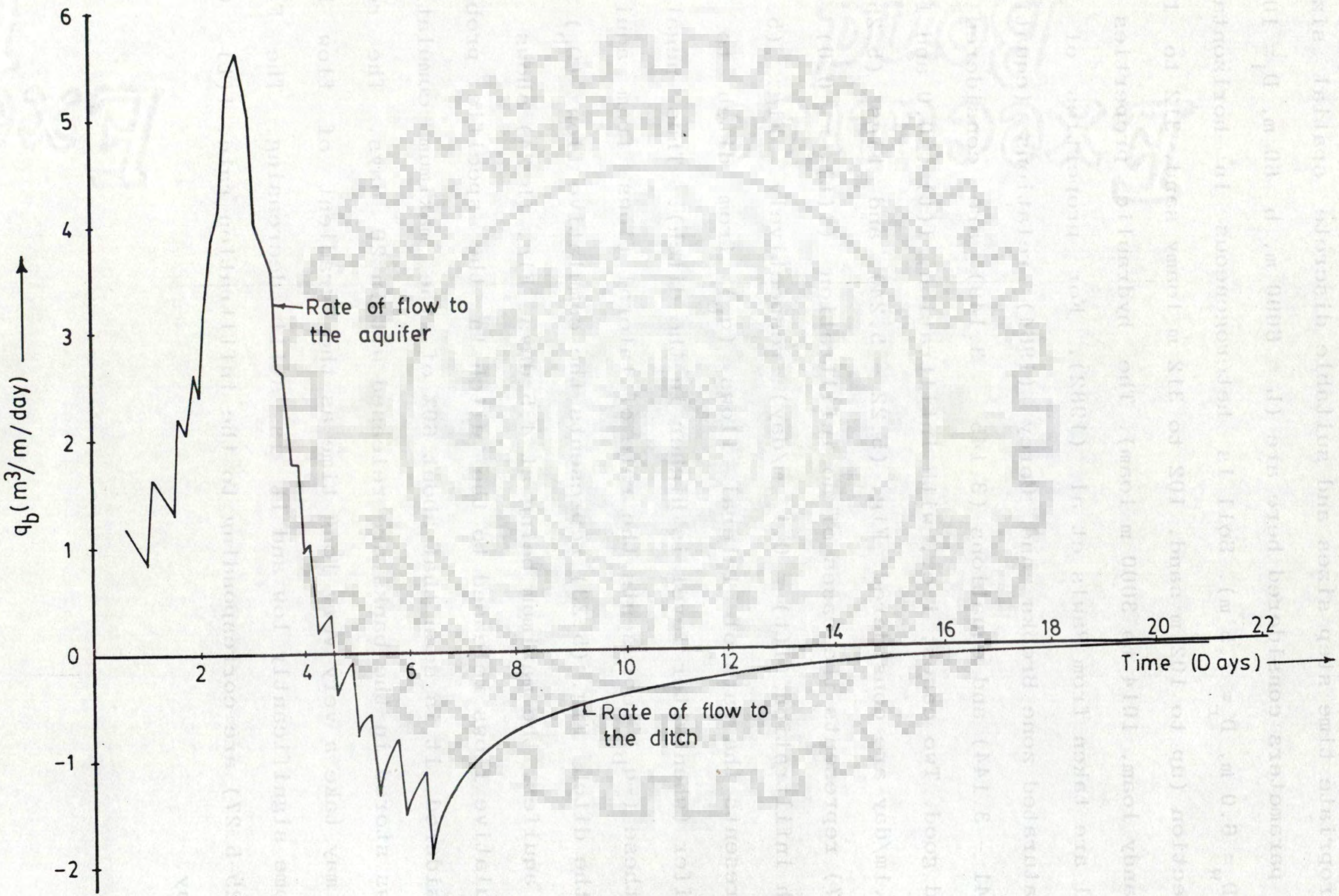


FIG. 5·22 DITCH - SOIL INTERFLOW

(I (t) =0·0 , PET =0·0)

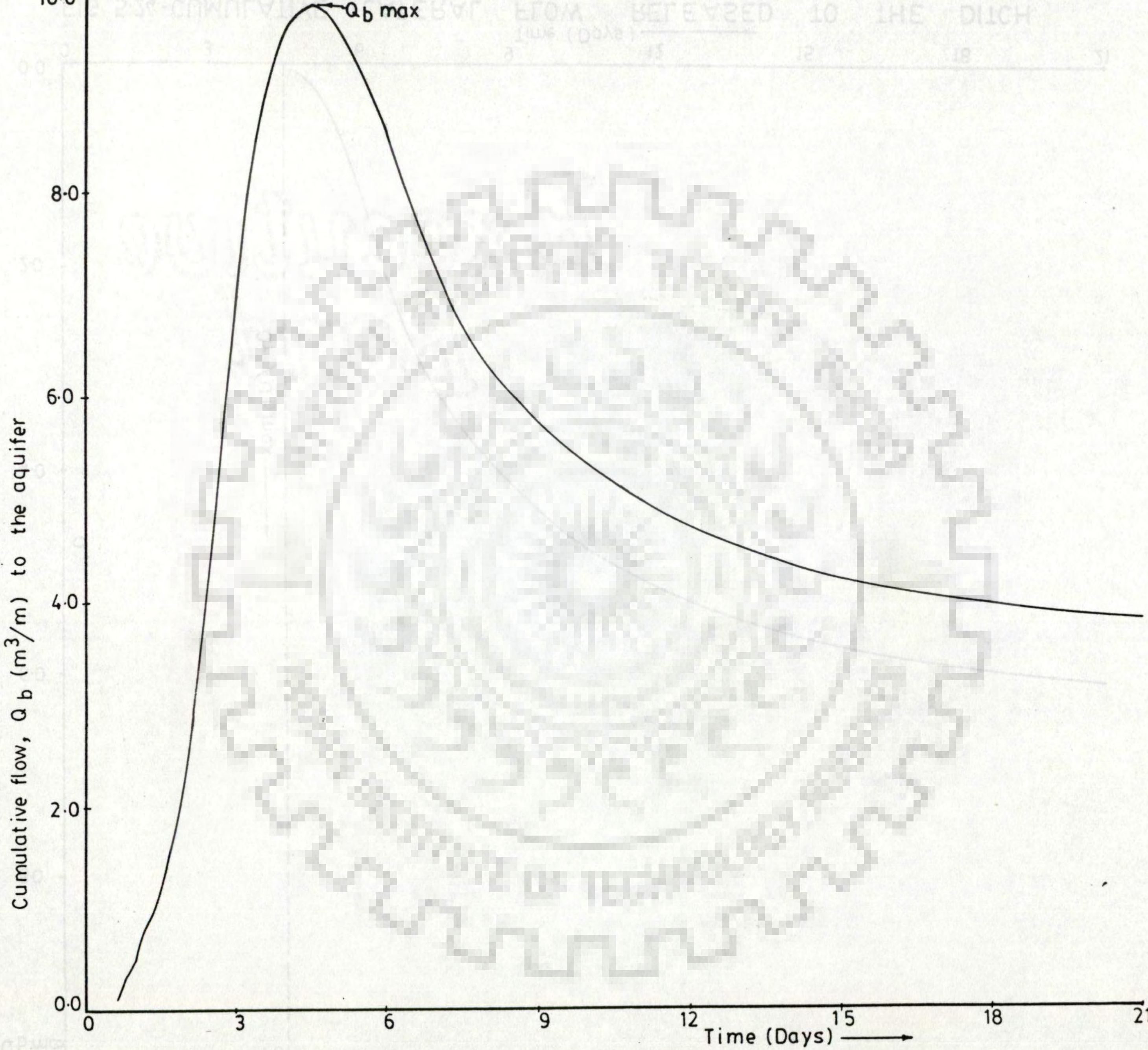


FIG. 5·23 CUMULATIVE LATERAL FLOW (Q_b) TO THE SOIL
(I (t) = 0·0 , PET = 0·0)

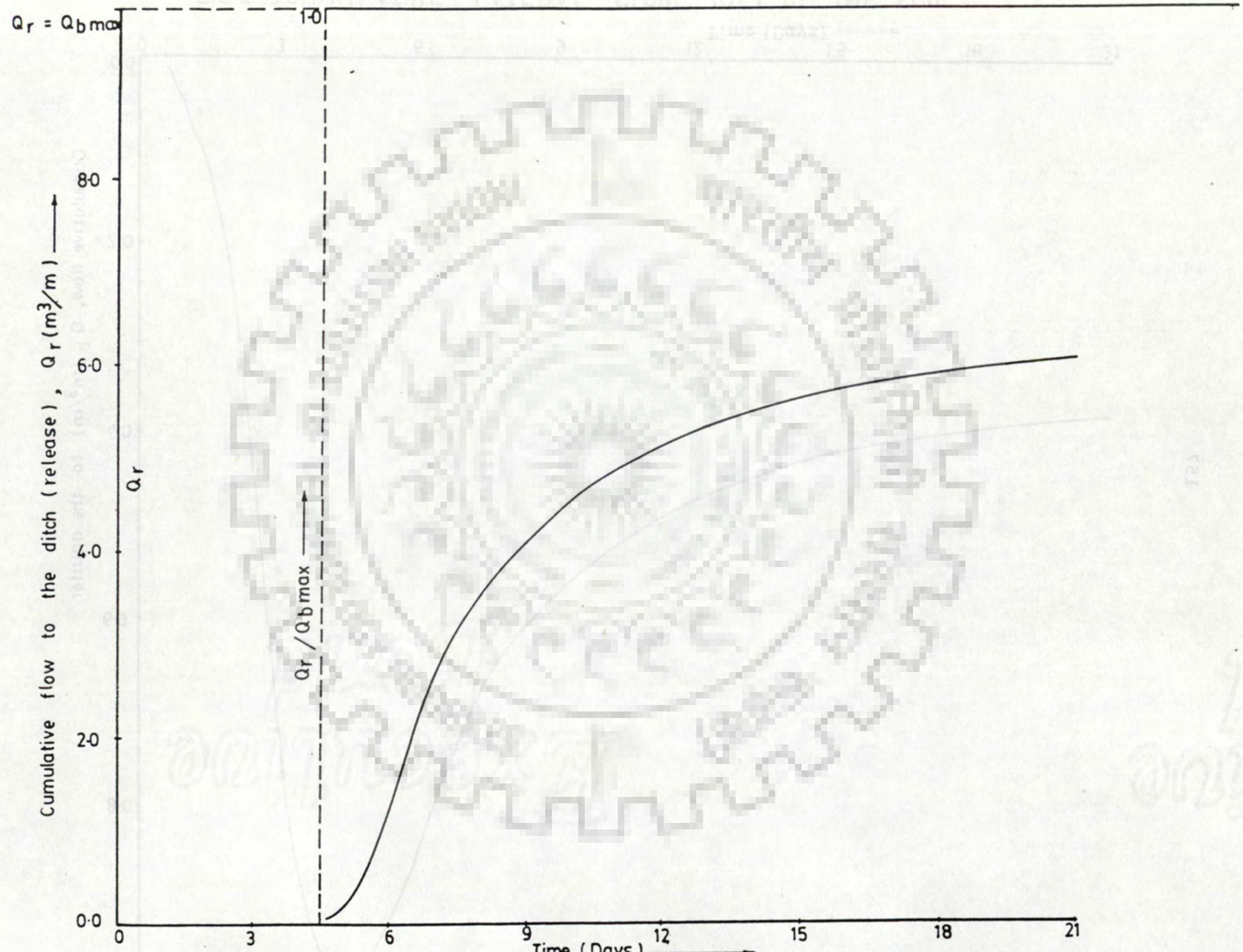


FIG. 5·24-CUMULATIVE LATERAL FLOW RELEASED TO THE DITCH

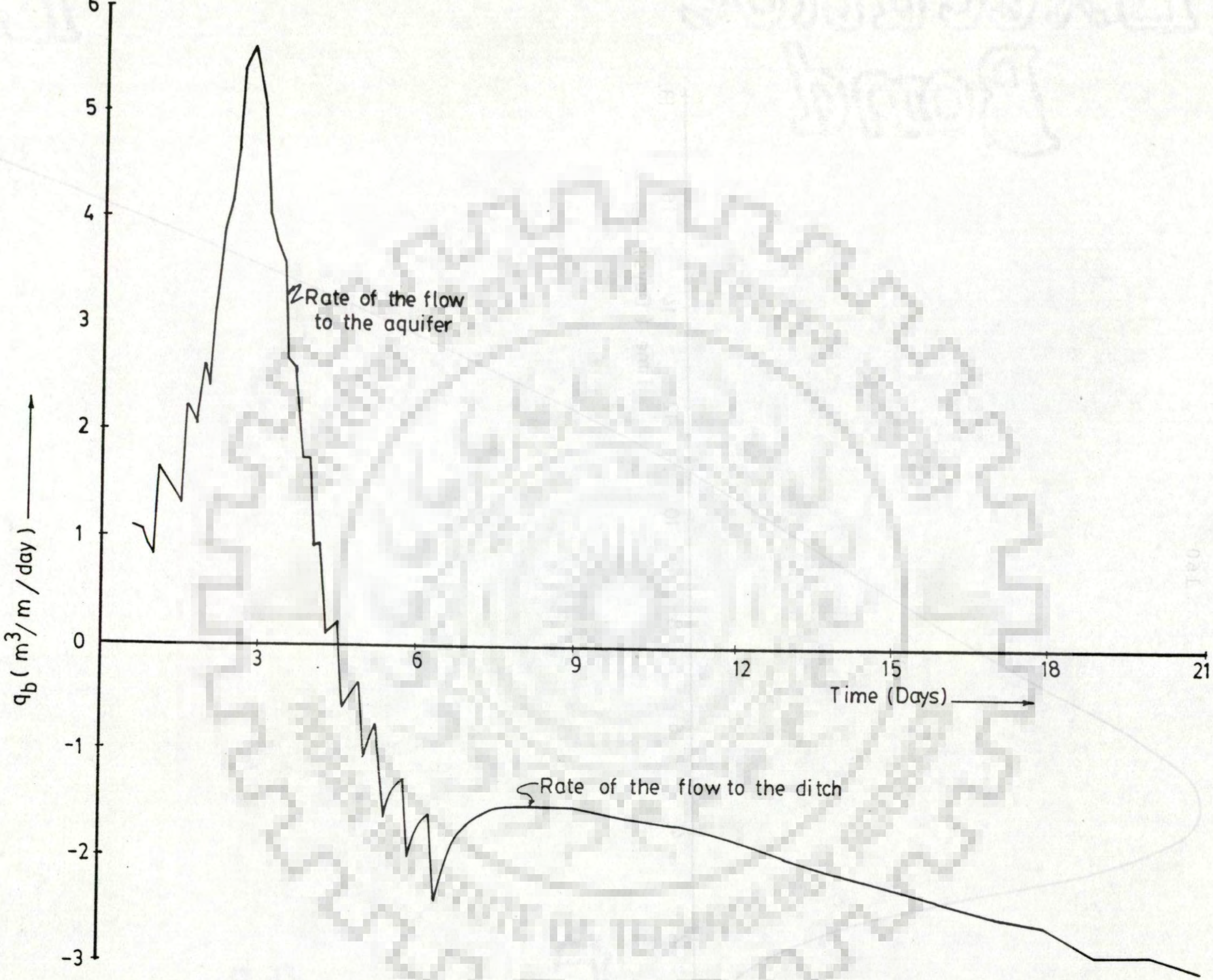


FIG 5·25 DITCH - SOIL INTERFLOW

(I (t) = 0·1 m/day, PET = 0·0)

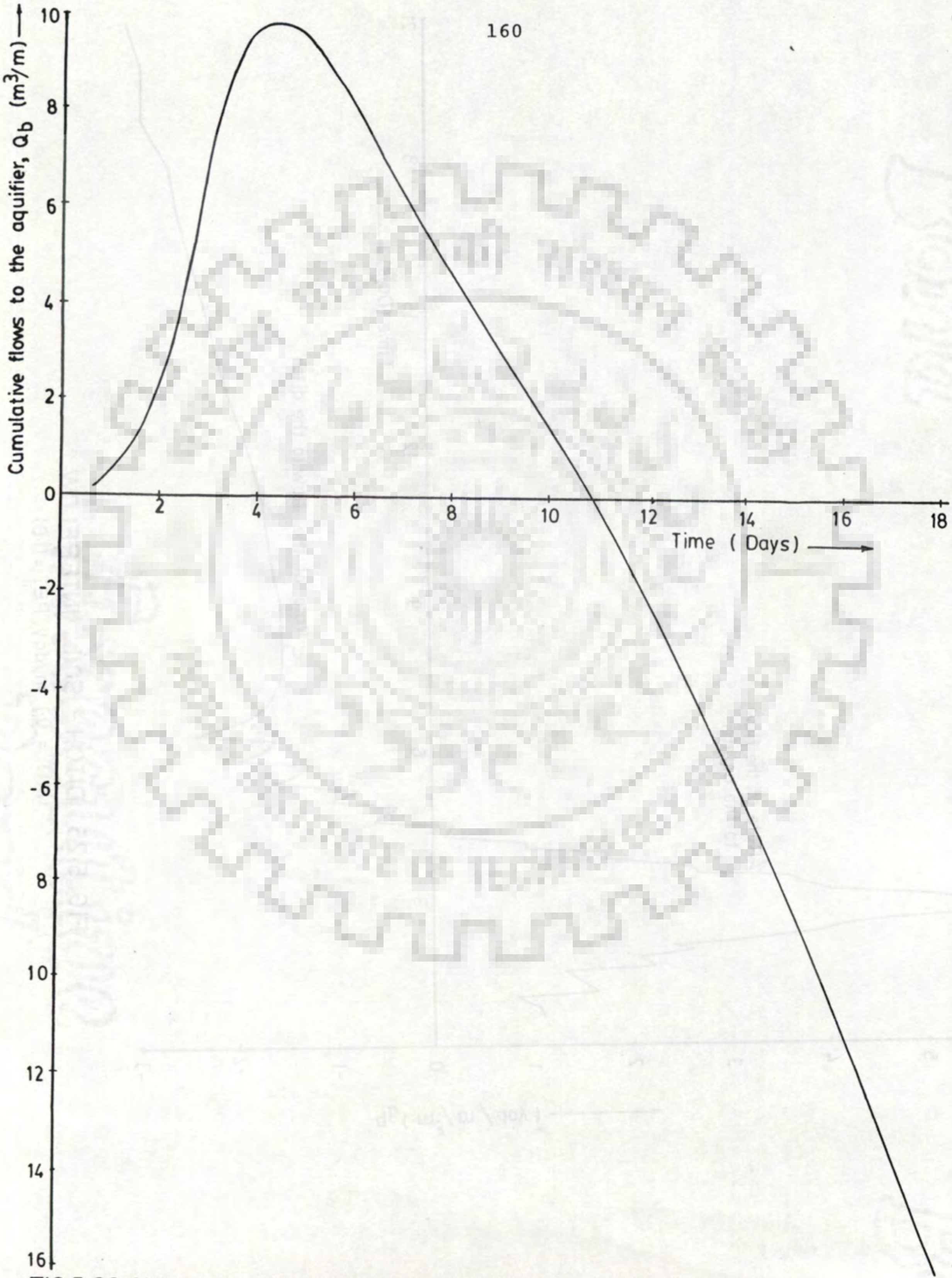


FIG. 5.26 CUMULATIVE LATERAL FLOW (Q_b) TO THE SOIL
($I(t) = 0.1$ m/day, PET = 0.0)

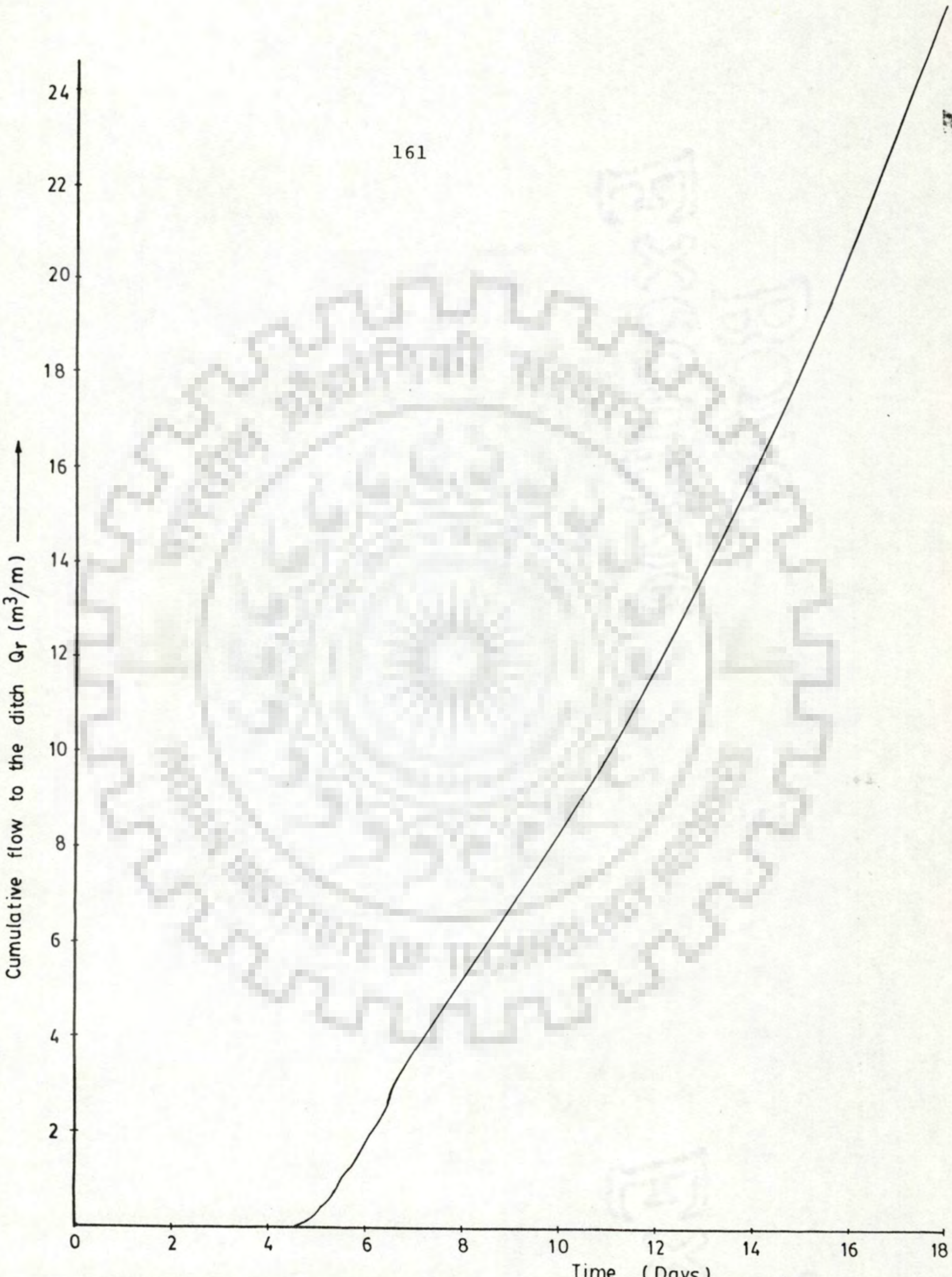


FIG.5·27-CUMULATIVE LATERAL FLOW (Q_r) RELEASED TO THE DITCH.
($I(t) = 0{\cdot}1$ m/day, PET = 0·0)

CHAPTER-VI

CONCLUSIONS

Two numerical models of two dimensional subsurface drainage have been developed .The first one (saturated flow model) analyses the flow in the saturated flow domain bounded by the impervious layer at the bottom ,water table at the top and two parallel drains at the sides. The second one (total response model)analyses the flow in the entire unsaturated-saturated domain bounded by the impervious layer at the bottom ,ground surface at the top and two parallel drains at the sides.The saturated flow model (SFM) requires among others, the time variant distribution of recharge rate at the water table as input data and yields the time variant water table position. The total response model (TRM) requires similar distribution of infiltration at ground surface and yields the spatial and temporal distribution of capillary head (h_c). This in turn yields the time variant water table position(defined by $h_c = 0$).

Following are the prominent conclusions of the study.

1a. Transfer of recharge occurring at the water table requires a vertical component of velocity. This component gets more prominent if the ditches are partially penetrating. One dimensional flow theories (e.g. Donnan's and Kraijenhoff's solutions) of subsurface drainage neglect the head loss associated with this vertical flow. This head loss may be quite significant in case of anisotropic soils with $Kz < Kx$. This leads to an underestimation of water table rise by such theories. Both the models corroborate the vertical flows. The SFM accounts for two-dimensional flows in the saturated domain below water table, whereas the TRM models these flows in the entire unsaturated-saturated domain. The two models yield higher water table rise than one dimensional flow solutions of Donnan, Kraijenhoff etc (chapter IV,section 4.1 and Figs 4.1-4.2).

1b. The two models can account for the vertical flows, the partial penetration of drains, vertical anisotropy, flow above the initial drain level and associated head losses.

2. The time lag between the occurrence of infiltration at the ground surface and recharge at the water table influences the transient water table rise, though the steady state rise is not affected. Thus, for the unsteady state drainage designs, it is necessary to account for the lag rationally. However, this may not be possible unless the flow through the intervening medium (i.e., the unsaturated zone) is modelled realistically. The proposed total response model, models the flow through both the (saturated as well as unsaturated) zones and thus implicitly accounts for the time lag.

3. The TRM is capable of simulating the generation of perched water table condition (and associated throughflow to the drains) over an impeding layer in the unsaturated zone. The throughflow was found to increase with, the increase in thickness of the clay layer, decrease in ditch penetration, and increase in the rate of infiltration (chapter V,section 5.2 and Fig 5.20).

4. The total response model can be used to estimate the build up and depletion of bank storage, provided the subsurface flow is predominantly normal to the drain (chapter V,section 5.2 and Figs 5.21-5.27).

5a. The saturated flow model has been implicitly validated by comparing the SFM solution with one dimensional flow solutions of Donnan and Kraijenhoff. The computed water table rise by SFM has been found to converge to these analytical solutions when resistance to vertical flow is made negligible by numerically assigning a very high vertical hydraulic conductivity (Kz >> Kx) [chapter IV,section 4.1,and Figs 4.1-4.2].However, similar validation of the total response model could not be taken up, since no analytical solution in the unsaturated-saturated domain was accessible.

5b. The SFM computed outflows to drains are found to be lower than the outflows computed by the Edelman's one dimensional flow solutions except at Kz >> Kx (where the SFM solutions are found to converge to the Edelman's solutions) [chapter IV,section 4.1,and Figs 4.5,4.7,4.10,4.13].

5c. Both the models (SFM & TRM) have also been validated by comparing their solutions with a set of reported experimental data. The experimental field, a research station of Soil Salinity Research Institute, Karnal, is located at Sampla village in Rohtak District of Haryana State, India. The area is under distress due to salinity and waterlogging. The subsurface drainage system, laid out in a 10 hectares plot to reclaim the highly saline land, consisted of three tile drain spacings of 25, 50 and 75 metres buried at an average depth of 1.75 m. The computed and reported water table hydrographs are found in good agreement. As expected, the agreement is better in case of the total response model (chapter IV,section 4.2,and Figs 4.15-4.20).

6a. The saturated flow model solution has been presented in the form of dimensionless design curves. These curves along with

Kraijenhoff solution permit graphical estimation of the steady state rise of water table, accounting for the vertical flows, within a practical range of the geometric dimensions and the parameters (chapter V, section 5.1, and Figs 5.1-5.4).

6b. The design curves reveal that the extra rise of water table on account of vertical flows is insignificant provided all of the following conditions hold good,

(i) the ditch spacing is more than 15 times the ditch penetration,

(ii) the ditch penetration is more than 75% saturated thickness,

(iii) the vertical hydraulic conductivity is not less than 20% the horizontal hydraulic conductivity.

Thus, under these conditions the prevalent one dimensional flow theories [Kraijenhoff(1958), Donnan (1946), Dumm (1954) etc.] may be employed without incurring any significant error. However, these theories may lead to significant overestimation of ditch spacing if any one of the above conditions is violated .

ANNEXURE - I

PUBLICATIONS

Ahmad,S.,Kashyap,D.,and Mathur,B.S.(1990)."Role of vertical flows in subsurface horizontal drainage." HYDROLOGY Journal of IAH,India,Vol. XIII,No.1,pp.1-12.

Ahmad,S.,Kashyap,D.,and Mathur,B.S.(1991)."Numerical modelling of two dimensional transient flow to ditches ." Journal of the Irrigation and Drainage Engineering, ASCE, 117(6), 839-851.

Ahmad,S.,Kashyap,D.,and Mathur,B.S.(1993)."Mathematical modelling of saturated-unsaturated flow to drains." Journal of Irrigation and Drainage Engineering,ASCE, (in press for February 1993 issue).

ANNEXURE-II

COMPUTER CODE

```
C     ***********************************************************
C       SATURATED FLOW MODEL
C        PROGRAMME FOR COMPUTATION OF RISE /FALL OF WATER TABLE
C     ***********************************************************
C       NT:NUMBER OF TIME STEPS
C       NR:NO.OF ROWS FOR FINITE DIFFERENCE MESH/TOP MOST ROW
C       I=1:BOTTOM MOST ROW OF THE MESH
C       NC:NO.OF COLUMNS FOR THE MESH
C       KOUNT:NO.OF ITERATIONS FOR ALTERNATE DIRECTION IMPLICIT
C       1 EXPLICIT SCHEME
C       ALW:CONVERGENCE FOR ADIES
C       NRD:SERIAL NO.OF THE ROW AT THE DRAIN BOTTOM
C       NCD1:SERIAL NO.OF THE COLUMN AT THE FACE OF FIRST DRAIN
C       NCD2:SERIAL NO.OF THE COLUMN AT THE FACE OF SECOND DRAIN
C       EPS:CONVERGENCE WITH RESPECT TO SUBTIMESTEPS
C       KTM:NO.OF ALLOWABLE SUBTIME STEP REDUCTIONS
C       DT(IT):TIME INCREMENTS(DAYS)
C       DELT:SUBTIME STEP INCREMENT
C       TM:TOTAL TIME OF SIMULATION(DAYS)
C       HH(I,J):INITIAL PIEZ.HEAD AT THE NODE AT ITH ROW AND
C       1 JTH COLUMN(METER)
C       SS(I,J):SPECIFIC STORAGE AT NODE (I,J)  (1/METER)
C       SY(J):SPECIFIC YIELD AT JTH COLUMN IN NRTH(TOP)ROW
C       DX(J):SPACING BETWEEN COLUMNS (J-1)& J (METER)
C       DZ(I):SPACING BETWEEN ROWS (I-1)& I (METER)
C       AKX(I,J):HYDRAULIC CONDUCTIVITY OF THE LINK BETWEEN
C       1 NODES (I,J-1)&(I,J) (M/DAY)
C       AKZ(I,J):HYDRAULIC CONDUCTIVITY OF THE LINK BETWEEN
C       1 NODES (I-1,J)&(I,J) (M/DAY)
C       H1:WATER LEVEL IN FIRST DRAIN(METER)
C       H2:WATER LEVEL IN SECOND DRAIN(METER)
C       R:RATE OF RECHARGE(M/DAY)
C       H(I,J):SIULATED PIEZOMET. HEAD AT NODE (I,J)  (M)
C       HIN(J):INITIAL PIE.HEAD IN TOPMOST ROW AT JTH COLUMN (M)
C       HSA(I,J):PIEZOMET.  HEAD AT THE BEGINNING OF TIME STEP
C       1 AT NODE (I,J) (M)
C       NTD:NO.OF TOTAL SUBTIME STEPS
C       HP(I,J):PIEZOMET. HEAD IN PREVIOUS ITERATION AT
C       1 NODE(I,J) (M)
```

```
C      HPT(I,J):PIEZOMET.HAD IN PREVIOUS ITERATION OF SUBTIME
C      1 STEP AT NODE(I,J) (M)
C      VX(I,J):VELOCITY IN X-DIRECTION IN THE  LINK BETWEEN
C      1 (I,J-1)&(I,J) (M/DAY)
C      VZ(I,J):VELOCITY IN Z-DIRECTION IN THE LINK BETWEEN
C      1 (I-1,J)&(I,J) (M/DAY)
C      Q:COMPUTED LATERAL FLOW IN A ROW/COLUMN (M**3/DAY/M)
C      SQ1:TOTAL HORIZONTAL FLOW TO THE DRAIN (M**3/DAY/M)
C      SQ2:TOTAL VERTICAL FLOW TO THE DRAIN (M**3/DAY/M)
C      SQ:TOTAL LATERAL FLOW  TO THE DRAIN (M**3/DAY/M)
C      NTD:NO.OF TOTAL SUBTIME STEPS
C      IMD:INDEX TO USE AVERAGE DDZ(J)
C      KOUNT1:NO.OF ITERATIONS FOR CONVERGENCE W.R.T.DDZ(J)
C      DDZM(J):MODIFIED DDZ(J)  M
C      EPS1:CONVERGENCE W.R.T.DDZ
C      NRD=1:FULLY PENETRATING DITCH
C      ISYM=0:NO SYMMETRY
C      ISYM=1:SYMMETRIC DRAINS
C      NCONF=0:UNCONFINED AQUIFER
C      NCONF=100:CONFINED AQUIFER
C      OWT(ITM):OBS.WATERTABLE RISE AT MIDSECTION AT TIME ITM
C      FO:VARIANCE OF OBSERVED WATERTABLE RISE*TOTAL NO.OF DAYS
C      FM:VAR. USING COMP. AND OBS. WATERTABLE RISE*TOTAL DAYS
C      FOM:MEAN OF OBSERVED WATERTABLE RISE VALUES
C      SOM:SUM OF OBSERVED WATERTABLE RISE VALUES
C      SDFM:UNEXPLAINED VARIANCE USING(OBSERVED-COMPUTED)RISE
C      SDFO:VARIANCE OF OBSERVED WATERTABLE RISE VALUES
C      DIRSQ:DETERMINATION INDEX R SQUARE(EFFICIENCY)
C      KT1(=1):SUBTIME STEP REDUCTION INITIATED
C      NTD(=1):NO.OF SUBTIME STEPS IN A TIME STEP TAKEN AS 1
C      KTM:NO.OF REDUCTIONS FOR SUBTIME STEP DISCRETISATION
C      HH(I,J):PIEZ.HEAD AT THE BEGINNING OF SUBTIME STEP
C      HSA(I,J):PIEZ.HEAD AT THE BEGINNING OF TIME STEP
C      H(I,J):COMPUTED PIEZOMETRIC HEAD
C      IMPLICIT REAL*8(A-H,O-Z)
       DIMENSION H(30,51),AKX(30,51),AKZ(30,51),A(51)
       DIMENSION B(51),C(51),D(51),DX(51),DZ(51)
C      DIMENSION SS(30,51),SY(51),HH(30,51),R(120),H1(51)
       DIMENSION HS(51),DT(120),HP(30,51),H1N(51)
```

```
      DIMENSION VX(30,51),VZ(30,51),HPT(30,51),JD(51),OWT(100)
      DIMENSION HSA(30,51),DH(51),DDZ(51),DXX(51),DDZM(51)
C     OPEN(UNIT=1,DEVICE='DSK',FILE='YS25.DAT')
      OPEN(UNIT=1,DEVICE='DSK',FILE='YS75.DAT')
C     OPEN(UNIT=2,DEVICE='DSK',FILE='WTR75.DAT')
C     OPEN(UNIT=2,DEVICE='DSK',FILE='WTR25.DAT')
      OPEN(UNIT=3,DEVICE='DSK',FILE='OBWT75.DAT')
C     OPEN(UNIT=1,DEVICE='DSK',FILE='YS50.DAT')
C     OPEN(UNIT=2,DEVICE='DSK',FILE='WTR75.DAT')
C     OPEN(UNIT=3,DEVICE='DSK',FILE='OBWT50.DAT')
C     OPEN(UNIT=1,DEVICE='DSK',FILE='DL.DAT')
C     OPEN(UNIT=3,DEVICE='DSK',FILE='OBWT25.DAT')
      READ(3,*)(OWT(LTM),LTM=1,83)
      READ(1,*)NT,NR,NC,KOUNT,ALW,NRD,NCD1,NCD2
      READ(1,*)(JD(I),I=NRD,NR)
      READ(1,*)EPS,KTM,ISYM,NCONF,EPS1,KOUNT1
      READ(1,*)(DT(IT),IT=1,NT)
      TM=6.0
      FM=0.0
      FO=0.0
C     QLI=0.000263157
      READ(1,*) ((HH(I,J),J=1,NC),I=1,NR)
      READ(1,*)((SS(I,J),J=1,NC),I=1,NR)
C     ACCEPT*,(SS(I,NC),I=1,NR)
      READ(1,*)(SY(J),J=1,NC)
C     ACCEPT*,(SY(NC))
      READ(1,*)(DX(J),J=1,NC-1)
      READ(1,*)(DZ(I),I=1,NR-1)
      READ(1,*)((AKX(I,J),J=1,NC-1),I=1,NR)
      READ(1,*)((AKZ(I,J),J=1,NC),I=1,NR-1)
C     READ(1,*)(HO,H2,(R(IT),IT=1,NT))
C     H1=HO
C     READ(1,*)(H1(IT),IT=1,NT)
C     READ(2,*)(R(IT),IT=1,NT)
      READ(1,*)(H1,H2,(R(IT),IT=1,NT))
      DO 15 J=1,NC
      DO 10 I=1,NR
      H(I,J)=HH(I,J)
10    CONTINUE
```

```
      HIN(J)=HH(NR,J)
15    CONTINUE
      DO 26 J=1,NC
      DDZ(J)=DZ(NR-1)
26    CONTINUE
      JTM=1
      DO 1000 IT=1,NT
      DO 500I=1,NR
      DO 505 J=1,NC
      HSA(I,J)=HH(I,J)
505   CONTINUE
500   CONTINUE
      KT1=1
      NTD=1
6     DO 510I=1,NR
      DO 515 J=1,NC
      HH(I,J)=HSA(I,J)
515   CONTINUE
510   CONTINUE
      DELT=DT(IT)/FLOAT(NTD)
      IF(NCONF.EQ.100)GO TO 453
      DO 452 J=1,NC
      DDZ(J)=DZ(NR-1)+HSA(NR,J)-HIN(J)
452   CONTINUE
453   DO 1050 ITD=1,NTD
      DO 88 ID=1,KOUNT1
      DO 16 IK=1,KOUNT
      DO 100 I=1,NR
      DO 110 J=1,NC
      IF(I.EQ.NR)GO TO 25
      IF(I.EQ.NR-1)GO TO 36
      DZI=DZ(I)
      IF(I.EQ.1)GO TO 35
      DZI1=DZ(I-1)
      GO TO 35
25    DZI1=DDZ(J)
      GO TO 35
36    DZI=DDZ(J)
      DZI1=DZ(I-1)
```

```
35      CONTINUE
        IF(I.GE.NRD.AND.J.LE.JD(I))GO TO 160
C       IF(I.GE.NRD.AND.J.LE.JD(I))GO TO 161
        IF(ISYM.EQ.1)GO TO 801
        IF(I.GE.NRD.AND.J.GE.NCD2)GO TO 170
801     IF(I.EQ.NR.AND.J.EQ.NC.AND.ISYM.EQ.1)GO TO 1120
        IF(I.EQ.NR) GO TO 120
        IF(I.EQ.1) GO TO 130
        IF(I.LT.NRD.AND.J.EQ.1) GO TO 140
        IF(I.LT.NRD.AND.J.EQ.NC) GO TO 150
        IF(ISYM.EQ.1.AND.J.EQ.NC)GO TO 150
        IF(I.EQ.1.AND.J.EQ.NC)GO TO 175
        IF(DZI1.GT.DZI)DA=DZI
        IF(DZI1.LE.DZI)DA=DZI1
        IF(DX(J-1).GT.DX(J))DB=DX(J)
        IF(DX(J-1).LE.DX(J))DB=DX(J-1)
        A(J)=(AKX(I,J-1)/DX(J-1))/DB
        B(J)=-((AKX(I,J)/DX(J)+AKX(I,J-1)/DX(J-1))
        1/DB+SS(I,J)/DELT)
        1-(AKZ(I,J)/DZI+AKZ(I-1,J)/DZI1)/DA
        C(J)=(AKX(I,J)/DX(J))/DB
        D1=-(H(I-1,J)*AKZ(I-1,J)/DZI1)/DA
        D3=-(H(I+1,J)*AKZ(I,J)/DZI)/DA
        D4=-HH(I,J)*SS(I,J)/DELT
        D(J)=D1+D3+D4
        GO TO 110
120     DXX(J)=(DX(J)**2+(DDZ(J+1)-DDZ(J))**2)**0.5
        DXX(J-1)=(DX(J-1)**2+(DDZ(J)-DDZ(J-1))**2)**0.5
        A(J)=(DDZ(J-1)+DDZ(J))*AKX(I,J-1)/(4.0*DXX(J-1))
        B(J)=-(DX(J)+DX(J-1))*AKZ(I-1,J)/(2.0*DDZ(J))-(DDZ(J-1)+
        1DDZ(J))*
        1AKX(I,J-1)/(4.0*DXX(J-1))-(DDZ(J+1)+DDZ(J))*AKX(I,J)/(4
        1.0*DXX(J))-
        1SY(J)*(DX(J)+DX(J-1))/(2.0*DELT)-SS(I,J)*(DX(J)+
        1DX(J-1))*(DDZ(J)+(DDZ(J+1)+DDZ(J-1))/2.0)/(8.0*DELT)
        C(J)=(DDZ(J+1)+DDZ(J))*AKX(I,J)/(4.0*DXX(J))
        D(J)=-((DX(J)+DX(J-1))/2.0)*(R(IT)+H(I-1,J)*AKZ(I-1,J)/DDZ(J
        1))-(HH(I,J)*(DX(J)+DX(J-1))/(2.0*DELT))*(SY(J)
        1+SS(I,J)*((DDZ(J)+(DDZ(J+1)+DDZ(J-1))/2.0)/4.0))
```

```
      GO TO 110
1120  DXX(J)=(DX(J)**2+(DDZ(J+1)-DDZ(J))**2)**0.5
      DXX(J-1)=(DX(J-1)**2+(DDZ(J)-DDZ(J-1))**2)**0.5
      A(J)=(DDZ(J-1)+DDZ(J))*AKX(I,J-1)/(4.0*DXX(J-1))
      B(J)=-DX(J-1)*AKZ(I-1,J)/(2.0*DDZ(J))-(DDZ(J-1)+DDZ(J))*
      1AKX(I,J-1)/
      1(4.0*DXX(J-1))-SY(J)*DX(J-1)/(2.0*DELT)-SS(I,J)*
      1(DDZ(J)+DDZ(J-1))*DX(J-1)/(8.0*DELT)
      C(J)=0.0
      D(J)=-R(IT)*DX(J-1)/2.0-H(I-1,J)*DX(J-1)*AKZ(I-1,J)/(2.0*D
      1DZ(J))-HH(I,J)*SY(J)*DX(J-1)/(2.0*DELT)-HH(I,J)*SS(I,J)
      1*(DDZ(J-1)+DDZ(J))*DX(J-1)/(8.0*DELT)
      GO TO 110
C160  IF(IT.GT.1)GO TO 161
160   A(J)=0.0
      B(J)=1.0
      C(J)=0.0
C     D(J)=H1(IT)
      D(J)=H1
      GO TO 110
      GO TO 163
161   IF(I.EQ.NR)GO TO 162
      A(J)=0.0
      B(J)=-AKX(I,J)*(DZ(I)+DZ(I-1))/4.0-(AKZ(I,J)+AKZ(I-1,J))
      1*DX(J)/4.0-SS(I,J)*DX(J)*(DZ(I)+DZ(I-1))/(4.0*DELT)
      C(J)=AKX(I,J)*(DZ(I)+DZ(I-1))/4.0
      D(J)=-H(I+1,J)*AKZ(I,J)*DX(J)/4.0-H(I-1,J)*AKZ(I-1,J)*D
      1X(J)/4.0+QLI-HH(I,J)*SS(I,J)*DX(J)*(DZ(I)+DZ(I-1))/(4.0
      1*DELT)
      GO TO 110
162   A(J)=0.0
      B(J)=-AKX(I,J)*DZ(I-1)/4.0-AKZ(I-1,J)*DX(J)/4.0
      1-SS(I,J)*DX(J)*DZ(I-1)/(4.0*DELT)-SY(J)*DX(J)/(2.0*DELT)
      C(J)=AKX(I,J)*(DZ(I)+DZ(I-1))/4.0
      D(J)=-H(I-1,J)*AKZ(I-1,J)*DX(J)/4.0+QLI-SS(I,J)*HH(I,J)
      1*DX(J)*DZ(I-1)/(4.0*DELT)-SY(J)*HH(I,J)*DX(J)/(2.0*DELT)
      GO TO 110
163   CONTINUE
170   A(J)=0.0
```

```
      B(J)=1.0
      C(J)=0.0
      D(J)=H2
      GO TO 110
130   IF(NRD.GT.1.AND.J.EQ.1) GO TO 165
      IF(NRD.GT.1.AND.J.EQ.NC) GO TO 175
      IF(I.EQ.1.AND.NRD.GT.1.AND.J.EQ.NC) GO TO 175
      IF(ISYM.EQ.1.AND.J.EQ.NC)GO TO 175
      A(J)=(AKX(I,J-1)/DX(J-1))*(2.0/(DX(J)+DX(J-1)))
      B(J)=-((AKX(I,J)/DX(J)+AKX(I,J-1)/DX(J-1))*
     1(2.0/(DX(J)+DX(J-1)))+SS(I,J)/DELT)
     1-2.0*(AKZ(I,J)/DZI**2)
      C(J)=(AKX(I,J)/DX(J))*(2.0/(DX(J)+DX(J-1)))
      D(J)=-HH(I,J)*SS(I,J)/DELT
     1-2.0*H(I+1,J)*(AKZ(I,J)/DZI**2)
      GO TO 110
165   A(J)=0.0
      B(J)=-(AKX(I,J)/DX(J)**2+SS(I,J)/(2.0*DELT))
     1-(AKZ(I,J)/DZI**2)
      C(J)=AKX(I,J)/DX(J)**2
      D(J)=-H(I+1,J)*AKZ(I,J)/DZI**2
     1-HH(I,J)*SS(I,J)/(2.0*DELT)
      GO TO 110
175   A(J)=AKX(I,J-1)/DX(J-1)**2
      B(J)=-(AKX(I,J-1)/DX(J-1)**2+SS(I,J)/(2.0*DELT))
     1-(AKZ(I,J)/DZI**2)
      C(J)=0.0
      D(J)=-H(I+1,J)*AKZ(I,J)/DZI**2
     1-HH(I,J)*SS(I,J)/(2.0*DELT)
      GO TO 110
140   A(J)=0.0
      B(J)=-((2.0*AKX(I,J)/DX(J)**2)+SS(I,J)/DELT)
     1-(AKZ(I,J)/DZI+AKZ(I-1,J)/DZI1)
     1*(2.0/(DZI+DZI1))
      C(J)=2.0*AKX(I,J)/DX(J)**2
      D(J)=-HH(I,J)*SS(I,J)/DELT-H(I+1,J)*(AKZ(I,J)/DZI)
     1*(2.0/(DZI+DZI1))
     1-H(I-1,J)*(AKZ(I-1,J)/DZI1)
     1*(2.0/(DZI+DZI1))
```

```
      GO TO 110
150   IF(I.EQ.1)GO TO 175
      IF(I.EQ.NR)GO TO 1120
      A(J)=(2.0*AKX(I,J-1))/DX(J-1)**2
      B(J)=-((2.0*AKX(I,J-1))/DX(J-1)**2+SS(I,J)/DELT)
      1-(AKZ(I,J)/DZI+AKZ(I-1,J)/DZI1)
      1*(2.0/(DZI+DZI1))
      C(J)=0.0
      D(J)=-HH(I,J)*SS(I,J)/DELT-H(I+1,J)*(AKZ(I,J)/DZI)
      1*(2.0/(DZI+DZI1))
      1-H(I-1,J)*(AKZ(I-1,J)/DZI1)*(2.0/(DZI+DZI1))
110   CONTINUE
      CALL STN(NC,A,B,C,D,HS)
      DO 20 J=1,NC
      H(I,J)=HS(J)
20    CONTINUE
100   CONTINUE
      DO 200 J=1,NC
      DO 210 I=1,NR
      IF(I.EQ.NR)GO TO 251
      IF(I.EQ.NR-1)GO TO 361
      DZI=DZ(I)
      IF(I.EQ.1)GO TO 351
      DZI1=DZ(I-1)
      GO TO 351
251   DZI1=DDZ(J)
      GO TO 351
361   DZI=DDZ(J)
      DZI1=DZ(I-1)
351   CONTINUE
      IF(J.LE.JD(I).AND.I.GE.NRD)GO TO 260
C     IF(J.LE.JD(I).AND.I.GE.NRD)GO TO 261
      IF(ISYM.EQ.1)GO TO 805
      IF(J.GE.NCD2.AND.I.GE.NRD)GO TO 270
805   IF(J.EQ.NC.AND.I.EQ.NR.AND.ISYM.EQ.1)GO TO 1240
      IF(I.LT.NRD.AND.J.Q.1) GO TO 220
      IF(I.LT.NRD.AND.J.EQ.NC) GO TO 230
      IF(ISYM.EQ.1.AND.J.EQ.NC)GO TO 230
      IF(I.EQ.NR) GO TO 240

      IF(I.EQ.1) GO TO 250
```

```
      IF(I.EQ.1.AND.J.EQ.NC)GO TO 275
      IF(DZI1.GT.DZI)DA=DZI
      IF(DZI1.LE.DZI)DA=DZI1
      IF(DX(J-1).GT.DX(J))DB=DX(J)
      IF(DX(J-1).LE.DX(J))DB=DX(J-1)
      A(I)=(AKZ(I-1,J)/DZI1)/DA
      B(I)=-((AKZ(I,J)/DZI+AKZ(I-1,J)/DZI1)
      1/DA+SS(I,J)/DELT)
      1-(AKX(I,J)/DX(J)+AKX(I,J-1)/DX(J-1))/DB
      C(I)=(AKZ(I,J)/DZI)/DA
      D(I)=-(H(I,J+1)*AKX(I,J)/DX(J))/DB
      1-HH(I,J)*SS(I,J)/DELT-H(I,J-1)*(AKX(I,J-1)/DX(J-1))/DB
      GO TO 210
220   IF(I.EQ.1) GO TO 265
      A(I)=(AKZ(I-1,J)/DZI1)*(2.0/(DZI+DZI1))
      B(I)=-((AKZ(I,J)/DZI+AKZ(I-1,J)/DZI1)
      1*(2.0/(DZI+DZI1))+SS(I,J)/DELT)
      1-(2.0*AKX(I,J)/DX(J)**2)
      C(I)=(AKZ(I,J)/DZI)*(2.0/(DZI+DZI1))
      D(I)=-H(I,J+1)*2.0*AKX(I,J)/DX(J)**2
      1-HH(I,J)*SS(I,J)/DELT
      GO TO 210
C260  IF(IT.GT.1)GO TO 261
260   A(I)=0.0
      B(I)=1.0
      C(I)=0.0
C     D(I)=H1(IT)
      D(I)=H1
      GO TO 210
      GO TO 263
261   IF(I.EQ.NR)GO TO 262
      A(I)=AKZ(I-1,J)*DX(J)/4.0
      B(I)=-AKX(I,J)*(DZ(I)+DZ(I-1))/4.0-(AKZ(I,J)+AKZ(I-1,J))
      1*DX(J)/4.0-SS(I,J)*DX(J)*(DZ(I)+DZ(I-1))/(4.0*DELT)
      C(I)=AKZ(I,J)*DX(J)/4.0
      D(I)=-H(I,J+1)*AKX(I,J)*(DZ(I)+DZ(I-1))+QLI-HH(I,J)*
      1SS(I,J)*DX(J)*(DZ(I)+DZ(I-1))/(4.0*DELT)
      GO TO 210
262   A(I)=AKZ(I-1,J)*DX(J)/4.0
```

```
      B(I)=-AKX(I,J)*DZ(I-1)/4.0-AKZ(I-1,J)*DX(J)/4.0-SS(I,J)
     1*DX(J)*DZ(I-1)/(4.0*DELT)-SY(J)*DX(J)/(2.0*DELT)
      C(I)=0.0
      D(I)=-H(I,J+1)*AKX(I,J)*DZ(I-1)/4.0+QLI-HH(I,J)*SS(I,J)*
     1DX(J)*DZ(I-1)/(4.0*DELT)-HH(I,J)*SY(J)*DX(J)/(2.0*DELT)
      GO TO 210
263   CONTINUE
265   A(I)=0.0
      B(I)=-(AKZ(I,J)/DZI**2+SS(I,J)/(2.0*DELT))
     1-(AKX(I,J)/DX(J)**2)
      C(I)=AKZ(I,J)/DZI**2
      D(I)=-H(I,J+1)*AKX(I,J)/DX(J)**2
     1-HH(I,J)*SS(I,J)/(2.0*DELT)
      GO TO 210
230   IF(I.EQ.1) GO TO 275
      A(I)=(AKZ(I-1,J)/DZI1)*(2.0/(DZI+DZI1))
      B(I)=-((AKZ(I,J)/DZI+AKZ(I-1,J)/DZI1)
     1*(2.0/(DZI+DZI1))+SS(I,J)/DELT)
     1-((2.0*AKX(I,J-1))/DX(J-1)**2)
      C(I)=(AKZ(I,J)/DZI)*(2.0/(DZI+DZI1))
      D(I)=-(2.0*H(I,J-1))*AKX(I,J-1)/DX(J-1)**2
     1-HH(I,J)*SS(I,J)/DELT
      GO TO 210
270   A(I)=0.0
      B(I)=1.0
```

```
      1(I,J-1)/(4.0*DXX(J-1))-(DDZ(J+1)+DDZ(J))*AKX(I,J)/(4.0*
      1DXX(J))-SY(
      1J)*(DX(J)+DX(J-1))/(2.0*DELT)-SS(I,J)*(DX(J)+DX(J-1))*
      1(DDZ(J)+(DDZ(J-1)+DDZ(J+1))/2.0)/(8.0*DELT)
      C(I)=0.0
      D(I)=-R(IT)*(DX(J)+DX(J-1))/2.0-H(I,J-1)*(DDZ(J-1)+DDZ(J))*A
      1KX(I,J-1)/(
      14.0*DXX(J-1))-H(I,J+1)*(DDZ(J+1)+DDZ(J))*AKX(I,J)/(4.0*
      1DXX(J))-(HH
      1(I,J)*(DX(J)+DX(J-1))/(2.0*DELT))*(SY(J)+SS(I,J)
      1*(DDZ(J)+(DDZ(J-1)+DDZ(J+1))/2.0)/4.0)
      GO TO 210
1240  DXX(J)=(DX(J)**2+(DDZ(J+1)-DDZ(J))**2)**0.5
      DXX(J-1)=(DX(J-1)**2+(DDZ(J)-DDZ(J-1))**2)**0.5
      A(I)=DX(J-1)*AKZ(I-1,J)/(2.0*DDZ(J))
      B(I)=-DX(J-1)*AKZ(I-1,J)/(2.0*DDZ(J))-(DDZ(J-1)+DDZ(J))*
      1AKX(I,J-1)/
      1(4.0*DXX(J-1))-SY(J)*DX(J-1)/(2.0*DELT)-SS(I,J)*
      1(DDZ(J)+DDZ(J-1))*DX(J-1)/(8.0*DELT)
      C(I)=0.0
      D(I)=-R(IT)*DX(J-1)/2.0-H(I,J-1)*(DDZ(J-1)+DDZ(J))*AKX(I,J-1
      1)/(4.0*DXX(
      1J-1))-SY(J)*DX(J-1)*HH(I,J)/(2.0*DELT)-SS(I,J)*DX(J-1)*
      1(DDZ(J)+DDZ(J-1))*HH(I,J)/(8.0*DELT)
      GO TO 210
250   A(I)=0.0
      B(I)=-(2.0*AKZ(I,J)/DZI**2+SS(I,J)/DELT)
      1-(AKX(I,J)/DX(J)+AKX(I,J-1)/DX(J-1))
      1*(2.0/(DX(J)+DX(J-1)))
      C(I)=2.0*AKZ(I,J)/DZI**2
      D(I)=-H(I,J-1)*(AKX(I,J-1)/DX(J-1))
      1*(2.0/(DX(J)+DX(J-1)))
      1-H(I,J+1)*(AKX(I,J)/DX(J))*(2.0/(DX(J)+DX(J-1)))
      1-HH(I,J)*SS(I,J)/DELT
210   CONTINUE
      CALL STN (NR,A,B,C,D,HS)
      DO 30 I=1,NR
      H(I,J)=HS(I)
30    CONTINUE
```

```
200   CONTINUE
      IF(IK.EQ.1) GO TO 400
      SM=0.0
      DO 350 I=1,NR
      DO 370 J=1,NC
      SM=SM+ABS(H(I,J)-HP(I,J))
370   CONTINUE
350   CONTINUE
      IF(SM.LT.ALW) GO TO 50
400   DO 380 I=1,NR
      DO 390 J=1,NC
      HP(I,J)=H(I,J)
390   CONTINUE
380   CONTINUE
16    CONTINUE
C     PRINT 65,SM
65    FORMAT(5X,'CONVERGENCE NOT ACHIEVED',E16.7)
      STOP
50    CONTINUE
      IF(NCONF.EQ.100)GO TO 45
      DO 44 J=1,NC
      DDZM(J)=DZ(NR-1)+((H(NR,J)-HIN(J))+(HH(NR,J)-HIN(J)))*0.5
44    CONTINUE
      DO 810 J=1,NC
      IF(ABS(DDZ(J)-DDZM(J)).GT.EPS1)GO TO 820
810   CONTINUE
      GO TO 830
820   DO 840 J=1,NC
      DDZ(J)=DDZM(J)
840   CONTINUE
      DO 77 I=1,NR
      DO 99 J=1,NC
      H(I,J)=HH(I,J)
99    CONTINUE
77    CONTINUE
88    CONTINUE
      PRINT850
850   FORMAT(5X,'CONVERGENCE W.R.T.DDZ NOT ACHIEVED')
      PRINT*,(DDZ(J),J=1,NC)
```

```
        PRINT*,(DDZM(J),J=1,NC)
830     CONTINUE
        DO 89 J=1,NC
        DDZ(J)=DZ(NR-1)+H(NR,J)-HIN(J)
89      CONTINUE
45      DO 551 I=1,NR
        DO 661 J=1,NC
        HH(I,J)=H(I,J)
661     CONTINUE
551     CONTINUE
1050    CONTINUE
        PRINT*,IT,NTD,DELT
        TYPE*,IT,NTD,DELT
C       DO 305 I=1,NR
C       PRINT*,(H(I,J),J=1,NC)
C       TYPE*,(H(I,J),J=1,NC)
C       PRINT*,(H(NR,J),J=1,NC)
        TYPE*,(H(NR,J),J=1,NC)
C305    CONTINUE
        IF(NTD.EQ.1)GO TO 7
        SM1=0.0
        DO 1 I=1,NR
        DO 2 J=1,NC
        SM1=SM1+ABS(H(I,J)-HPT(I,J))
2       CONTINUE
1       CONTINUE
        IF(SM1.LT.EPS)GO TO 5
7       DO 3 I=1,NR
        DO 4 J=1,NC
        HPT(I,J)=H(I,J)
4       CONTINUE
3       CONTINUE
        IF(KT1.LE.KTM)GO TO 67
C       PRINT66,SM1
66      FORMAT(5X,'NO.OF PERMISSIBLE TIME REDUCTIONS EXCEEDED',
        1E16.7)
        STOP
67      KT1=KT1+1
        NTD=NTD*2
```

```
      GO TO 6
5     TM=TM+DT(IT)
C     QLI=QLI+0.00526315*TM
C     H1=HO-(0.1*TM)
      PRINT*,TM
C     DO 300 I=1,NR
C     PRINT *,(H(I,J),J=1,NC)
C     TYPE *,(H(I,J),J=1,NC)
      PRINT*,(H(NR,J),J=1,NC)
      TYPE*,(H(NR,J),J=1,NC)
C300  CONTINUE
      DO 55 I=1,NR
      DO 60 J=1,NC
      HH(I,J)=H(I,J)
60    CONTINUE
55    CONTINUE
C     HORIZONTAL VELOCITIES
      PRINT 101
101   FORMAT(1X'VX VALUES')
      DO 11 I=1,NR
      DO 12 J=1,NC-1
      VX(I,J)=-AKX(I,J)*(H(I,J+1)-H(I,J))/DX(J)
12    CONTINUE
C     PRINT*,I,(VX(I,J),J=1,NC-1)
11    CONTINUE
      SQ1=0.0
      DO 700 I=NRD,NR
      J=NCD1
      IF(I.EQ.NR)GO TO 702
      IF(I.EQ.NRD)GO TO 703
      Q=-VX(I,J)*((DZ(I-1)+DZ(I))/2.0)
      GO TO 704
702   Q=-VX(I,J)*(DZ(I-1)/2.0)
      GO TO 704
703   Q=-VX(I,J)*DZ(NRD)*0.5
704   SQ1=SQ1+Q
700   CONTINUE
C     VERTICAL VELOCITIES
      PRINT 201
```

```
201     FORMAT(1X'VZ VALUES'/)
        DO 21 I=1,NR-1
        DO 22 J=1,NC
        VZ(I,J)=-AKZ(I,J)*(H(I+1,J)-H(I,J))/DZ(I)
22      CONTINUE
C       PRINT*,I,(VZ(I,J),J=1,NC)
21      CONTINUE
        SQ2=0.0
        DO 707 J=1,NCD1
        I=NRD
        IF(J.EQ.1)GO TO 705
        IF(J.EQ.NCD1)GO TO 706
        Q=VZ(I-1,J)*((DX(J)+DX(J-1))/2.0)
        GO TO 708
705     Q=VZ(I-1,J)*(DX(J)/2.0)
        GO TO 708
706     Q=VZ(I-1,J)*DX(NCD1-1)*0.5
708     SQ2=SQ2+Q
707     CONTINUE
        SQ=SQ1+SQ2
        PRINT*,SQ1,SQ2,SQ
        TYPE*,SQ1,SQ2,SQ
C       IF(NCONF.EQ.100)GO TO 1000
C       DO 44 J=1,NC
C       DDZ(J)=DDZ(J)+(H(NR,J)-HSA(NR,J))
C44     CONTINUE
        PRINT*,(DDZ(J),J=1,NC)
        SOM=0.0
        DO 1111 LTM=1,83
        SOM=(SOM+OWT(LTM))
1111    CONTINUE
        FOM=SOM/83.0
        IF(TM.LE.10.0)GO TO 1000
        TMM=TM-IFIX(TM)
        IF(TMM.NE.0.0)GO TO 1000
        JTM=JTM+1
        FM=FM+(H(NR,NC)-OWT(JTM))**2.0
        FO=FO+(OWT(JTM)-FOM)**2.0
        TYPE*,TM,JTM,FOM,FO,FM
```

```
      PRINT*,TM,JTM,FOM,FO,FM
1000  CONTINUE
      SDFM=FM/82.0
      SDFO=FO/82.0
      DIRSQ=1-(SDFM/SDFO)
      TYPE*,DIRSQ
      PRINT*,DIRSQ
      STOP
      END
C     ***********************************************************
C     SUBROUTINE TO SOLVE TRIDIAGONAL MATRIX
C     ***********************************************************
      SUBROUTINE STN(N,A,B,C,D,H)
C     IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(400),B(400),C(400),D(400)
      DIMENSION AL(400),BT(400),Y(400),H(100)
      AL(1)=B(1)
      BT(1)=C(1)/B(1)
      DO500 I=2,N
      AL(I)=B(I)-A(I)*BT(I-1)
C     TYPE*,AL(I)
C     PRINT*,AL(I)
      BT(I)=C(I)/AL(I)
500   CONTINUE
      Y(1)=D(1)/AL(1)
      DO 520I=2,N
      Y(I)=(D(I)-A(I)*Y(I-1))/AL(I)
520   CONTINUE
      H(N)=Y(N)
      DO 530 I=2,N
      II=N-I+1
      H(II)=Y(II)-BT(II)*H(II+1)
530   CONTINUE
      RETURN
      END
```

```
C     ************************************************************
C      TOTAL RESPONSE MODEL
C       PROGRAMME FOR COMPUTATION OF RISE /FALL OF WATER TABLE
C     ************************************************************
C      NT:NUMBER OF TIME STEPS
C      NR:NO.OF ROWS FOR FINITE DIFFERENCE MESH/TOP MOST ROW
C      I=1:BOTTOM MOST ROW OF THE MESH
C      NC:NO.OF COLUMNS FOR THE MESH
C      KOUNT:NO.OF ITERATIONS FOR ALTERNATE DIRECTION IMPLICIT
C      1 EXPLICIT SCHEME
C      ALW:CONVERGENCE FOR ADIES
C      NRD:SERIAL NO.OF THE ROW AT THE DRAIN BOTTOM
C      NCD1:SERIAL NO.OF THE COLUMN AT THE FACE OF FIRST DRAIN
C      NCD2:SERIAL NO.OF THE COLUMN AT THE FACE OF SECOND DRAIN
C      EPS:CONVERGENCE WITH RESPECT TO SUBTIMESTEPS
C      KTM:NO.OF ALLOWABLE SUBTIME STEP REDUCTIONS
C      DT(IT):TIME INCREMENTS(DAYS)
C      DELT:SUBTIME STEP INCREMENT
C      TM:TOTAL TIME OF SIMULATION(DAYS)
C      PP(I,J):INITIAL P.HEAD AT THE NODE AT ITH ROW AND JTH
C      1 COLUMN(METER)
C      SS(I,J):SPECIFIC STORAGE AT NODE (I,J)  (1/METER)
C      SY(J):SPECIFIC YIELD AT JTH COLUMN IN NRTH(TOP)ROW
C      DX(J):SPACING BETWEEN COLUMNS (J-1)& J (METER)
C      DZ(I):SPACING BETWEEN ROWS (I-1)& I (METER)
C      AKX(I,J):HYDRAULIC CONDUCTIVITY OF THE LINK BETWEEN
C     1 NODES (I,J-1)&(I,J) (M/DAY)
C      AKZ(I,J):HYDRAULIC CONDUCTIVITY OF THE LINK BETWEEN
C      1 NODES (I-1,J)&(I,J) (M/DAY)
C      H1:WATER LEVEL IN FIRST DRAIN(METER)
C      H2:WATER LEVEL IN SECOND DRAIN(METER)
C      R(IT):RATE OF INFILTRATION (M/DAY)
C      P(I,J):SIMULATED CAPILLARY HEAD AT NODE (I,J)  (M)
C      PIN(J):INITIAL CAP.HEAD IN TOPMOST ROW AT JTH COLUMN (M)
C      PSA(I,J):CAPILLARY  HEAD AT THE BEGINNING OF TIME STEP
C      1 AT NODE (I,J) (M)
C      NTD:NO.OF TOTAL SUBTIME STEPS
C      PIP(I,J):CAPILLARY HEAD IN PREVIOUS ITERATION AT
C      1 NODE(I,J) (M)
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C       PPT(I,J):CAPILLARY HEAD IN PREVIOUS ITERATION OF SUBTIME
C       1 STEP AT NODE(I,J) (M)
C       VX(I,J):VELOCITY IN X-DIRECTION IN THE LINK BETWEEN
C       1 (I,J-1)&(I,J) (M/DAY)
C       VZ(I,J):VELOCITY IN Z-DIRECTION IN THE LINK BETWEEN
C       1 (I-1,J)&(I,J) (M/DAY)
C       Q:COMPUTED LATERAL FLOW IN A ROW/COLUMN (M**3/DAY/M)
C       SQ1:TOTAL HORIZONTAL FLOW TO THE DRAIN (M**3/DAY/M)
C       SQ2:TOTAL VERTICAL FLOW TO THE DRAIN (M**3/DAY/M)
C       SQ:TOTAL LATERAL FLOW  TO THE DRAIN (M**3/DAY/M)
C       SQ3:DIRECT DOWNWARD FLOW INTO THE DRAIN
C       WTR:TOTAL DOWNWARD FLOW AT WATER TABLE(RECHARGE)
C       WTRS(J):WATER TABLE RISE AT COLUMN J
C       OWT(ITM):OBS.WATERTABLE RISE AT MIDSECTION AT TIME ITM
C       FO:VARIANCE OF OBSERVED WATERTABLE RISE*TOTAL NO.OF DAYS
C       FM:VAR. USING COMP. AND OBS. WATERTABLE RISE*TOTAL DAYS
C       FOM:MEAN OF OBSERVED WATERTABLE RISE VALUES
C       SOM:SUM OF OBSERVED WATERTABLE RISE VALUES
C       SDFM:UNEXPLAINED VARIANCE USING(OBSERVED-COMPUTED)RISE
C       SDFO:VARIANCE OF OBSERVED WATERTABLE RISE VALUES
C       DIRSQ:DETERMINATION INDEX R SQUARE(EFFICIENCY)
C       KT1(=1):SUBTIME STEP REDUCTION INITIATED
C       NTD(=1):NO.OF SUBTIME STES IN A TIME STEP TAKEN AS 1
C       KTM:NO.OF REDUCTIONS FOR SUBTIME STEP DISCRETISATION
C       PP(I,J):CAP.HEAD AT THE BEGINNING OF SUBTIME STEP
C       PSA(I,J):CAP.HEAD AT THE BEGINNING OF TIME STEP
C       P(I,J):COMPUTED CAP.HEAD
C       PF:FACTOR 'P' IN DOORENBOS ET AL.(1979)'ET'RELATION
C       SAT:SATURATED CAPILLARY CONDUCTIVITY
C       WP:WILTING POINT
C       THR:THETA-R
C       POR:POROSITY
C       SSM:AIR ENTRY VALUE
C       PET:POTENTIAL ET OF CROP
C       THE:VECTOR OF MOISTURE CONTENT
C       NRD=1:FULLY PENETRATING DITCH
C       ISYM=0:NO SYMMETRY
C       ISYM=1:SYMMETRIC DRAINS
C       NCONF=0:UNCONFINED AQUIFER
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```
C       NCONF=100:CONFINED AQUIFER
        DIMENSION AKXM(20,20),AKZM(20,20),SSM(20,20),DP(50,50)
        DIMENSION TP(50,50),RECH(120),OWT(100),AP(20,20)
        DIMENSION H(20,20),AKX(20,20),AKZ(20,20),A(30),WTRS(20)
        DIMENSION B(30),C(30),D(30),DX(20),DZ(20),EZ(20)
        DIMENSION SS(20,20),PP(20,20),ET(30,20),JD(40),R(250)
        DIMENSION PS(30),DT(250),PIP(30,20),PIN(30),Z(20)
        DIMENSION VX(20,20),VZ(20,20),PPT(20,20),P(20,20)
        DIMENSION PSA(20,20),DH(20),DDZ(20),DXX(20),DDZM(20)
        COMMON/CONST/THR,POR,AM,POW,AC,AEV,SAT,PET,PF,WP,FFC
C       OPEN(UNIT=1,DEVICE='DSK',FILE='Y75.DAT')
C       OPEN(UNIT=2,DEVICE='DSK',FILE='OBWT75.DAT')
C       OPEN(UNIT=3,DEVICE='DSK',FILE='WTR75.DAT')
C       OPEN(UNIT=1,DEVICE='DSK',FILE='Y50.DAT')
C       OPEN(UNIT=2,DEVICE='DSK',FILE='OBWT50.DAT')
C       OPEN(UNIT=3,DEVICE='DSK',FILE='WTR50.DAT')
        OPEN(UNIT=1,DEVICE='DSK',FILE='Y25.DAT')
        OPEN(UNIT=2,DEVICE='DSK',FILE='OBWT25.DAT')
C       OPEN(UNIT=3,DEVICE='DSK',FILE='WTR25.DAT')
        READ(2,*)(OWT(LTM),LTM=1,83)
        READ(1,*)NT,NR,NC,KOUNT,ALW,NRD,NCD1,NCD2
        READ(1,*)(JD(I),I=NRD,NRW)
        READ(1,*)EPS,KTM,ISYM,NCONF,EPS1,KOUNT1
        READ(1,*)(DT(IT),IT=1,NT)
        TM=6.0
        FM=0.0
        FO=0.0
        READ(1,*) DL,NRW,AL
        READ(1,*)((SS(I,J),J=1,NC),I=1,NR)
        READ(1,*)(DX(J),J=1,NC-1)
        READ(1,*)(DZ(I),I=1,NR-1)
        READ(1,*)((AKX(I,J),J=1,NC-1),I=1,NR)
        READ(1,*)((AKZ(I,J),J=1,NC),I=1,NR-1)
        READ(1,*) (R(IT),IT=1,NT),WP,THR,PET,PF,POR,FFC,AEV,POW,E,NRZ
C       READ(1,*)((PP(I,J),J=1,NC),I=1,NR)
C       DO 111 I=1,NR
C       DO 222 =1,NC
C       PP(I,J)=-AP(I,J)
C       TYPE*,PP(I,J)
```

```
C222    CONTINUE
C111    CONTINUE
        AM=-THR/AEV
        AC=-ALOG(AEV*AM+POR-THR)/AEV
        Z(1)=DL
        DO 405 I=1,NR-1
        Z(I+1)=Z(I)-DZ(I)
405     CONTINUE
        EZ(1)=0.0
        DO 109 I=1,NR-1
        EZ(I+1)=EZ(I)+DZ(I)
109     CONTINUE
        DO 505 I=1,NR
        DO 605 J=1,NC
        PP(I,J)=Z(I)
605     CONTINUE
505     CONTINUE
        DO 15 J=1,NC
        DO 10 I=1,NR
        P(I,J)=PP(I,J)
10      CONTINUE
        PIN(J)=PP(NR,J)
15      CONTINUE
        DO 26 J=1,NC
        DDZ(J)=DZ(NR-1)
26      CONTINUE
        JTM=1
        DO 1000 IT=1,NT
        DO 500I=1,NR
        DO 905 J=1,NC
        PSA(I,J)=PP(I,J)
905     CONTINUE
500     CONTINUE
        KT1=1
        NTD=1
6       DO 510I=1,NR
        DO 515 J=1,NC
        PP(I,J)=PSA(I,J)
515     CONTINUE
```

```
510     CONTINUE
        DELT=DT(IT)/FLOAT(NTD)
453     DO 1050 ITD=1,NTD
        DO 90 I=1,NR
        DO 1 J=1,NC
        IF(PP(I,J))92,93,93
93      AKZM(I,J)=AKZ(I,J)
        AKXM(I,J)=AKX(I,J)
        SSM(I,J)=SS(I,J)
        GO TO 91
C92     PPC=-PP(I,J)
C       THE=THETA(PPC)
C       SAT=AKX(I,J)
C       AKXM(I,J)=COND(THE)
C       SAT=AKZ(I,J)
C       AKZM(I,J)=COND(THE)
92      SSM(I,J)=-DIFU(PPC)
91      CONTINUE
        IF(I.LT.NRZ)GO TO 94
        ET(I,J)=EVPT(THE)
        GO TO 90
94      ET(I,J)=0.0
90      CONTINUE
        DO 501 I=1,NR-1
        DO 502 J=1,NC-1
        P1=PP(I,J)
        P2=PP(I+1,J)
        P3=PP(I,J+1)
        IF(P1.GE.0.0)GO TO 503
        TH1=THETA(-P1)
        GO TO 504
503     TH1=POR
504     IF(P2.GE.0.0)GO TO 511
        TH2=THETA(-P2)
        GO TO 506
511     TH2=POR
506     IF(P3.GE.0.0)GO TO 507
        TH3=THETA(-P3)
        GO TO 508
```

```
507     TH3=POR
508     CONTINUE
        TH4=(TH1+TH3)/2.0
        TH5=(TH1+TH2)/2.0
        SAT=AKX(I,J)
        AKXM(I,J)=COND(TH4)
        SAT=AKZ(I,J)
        AKZM(I,J)=COND(TH5)
        IF(I.LT.NRZGO TO 509
        ET(I,J)=EVPT(TH1)
        GO TO 502
509     ET(I,J)=0.0
502     CONTINUE
501     CONTINUE
        DO 95 I=1,NR-1
        J=NC
        P1=PP(I,J)
        P2=PP(I+1,J)
```

```
      P3=PP(I,J+1)
      IF(P1.GE.0.0)GO TO 296
      TH1=THETA(-P1)
      GO TO 297
296   TH1=POR
297   IF(P3.GE.0.0)GO TO 298
      TH3=THETA(-P3)
      GO TO 299
298   TH3=POR
299   CONTINUE
      TH4=(TH1+TH3)/2.0
      SAT=AKX(I,J)
      AKXM(I,J)=COND(TH4)
      IF(I.LT.NRZ)GO TO 495
      ET(I,J)=EVPT(TH1)
      GO TO 396
495   ET(I,J)=0.0
396   CONTINUE
295   CONTINUE
      DO 88 ID=1,KOUNT1
      DO 16 IK=1,KOUNT
      DO 100 I=1,NR
      DO 110 J=1,NC
      IF(I.EQ.NR)GO TO 25
      IF(I.EQ.NR-1)GO TO 36
      DZI=DZ(I)
      IF(I.EQ.1)GO TO 35
      DZI1=DZ(I-1)
      GO TO 35
25    DZI1=DDZ(J)
      GO TO 35
36    DZI=DDZ(J)
      DZI1=DZ(I-1)
35    CONTINUE
C     IF(I.GT.NRW.AND.J.EQ.1.AND.PP(I,J).GE.0.0)GO TO 209
      IF(I.GE.NRD.AND.I.LE.NRW.AND.J.LE.ND1)GO TO 160
C     IF(I.GT.NRW.AND.J.EQ.NCD1)GO TO 709
C     IF(I.GT.NRW.AND.J.LT.NCD1)GO TO 209
      IF(I.GT.NRW.AND.J.EQ.1)GO TO 709
```

```
      IF(ISYM.EQ.1)GO TO 801
      IF(I.GT.NRW.AND.J.EQ.NCD2)GO TO 809
      IF(I.GT.NRW.AND.J.GT.NCD2)GO TO 309
      IF(I.GE.NRD.AND.J.GE.NCD2)GO TO 170
801   IF(I.EQ.NR.AND.J.EQ.NC.AND.ISYM.EQ.1)GO TO 1120
      IF(I.EQ.NR) GO TO 120
      IF(I.EQ.1) GO TO 130
      IF(I.LT.NRD.AND.J.EQ.1) GO TO 140
      IF(I.LT.NRD.AND.J.EQ.NC) GO TO 150
      IF(ISYM.EQ.1.AND.J.EQ.NC)GO TO 150
      IF(DZI1.GT.DZI)DA=DZ
      IF(DZI1.LE.DZI)DA=DZI1
      IF(DX(J-1).GT.DX(J))DB=DX(J)
      IF(DX(J-1).LE.DX(J))DB=DX(J-1)
      A(J)=(AKXM(I,J-1)/DX(J-1))/DB
      B(J)=-((AKXM(I,J)/DX(J)+AKXM(I,J-1)/DX(J-1))
      1/DB+SSM(I,J)/DELT)
      1-(AKZM(I,J)/DZI+AKZM(I-1,J)/DZI1)/DA
      C(J)=(AKXM(I,J)/DX(J))/DB
      D1=-(P(I-1,J)*AKZM(I-1,J)/DZI1)/DA
      D2=-AKZM(I,J)*((EZ(I+1)-EZ(I))/DZI)/DA
      D3=-(P(I+1,J)*AKZM(I,J)/DZI)/DA
      D4=-PP(I,J)*SSM(I,J)/DELT+ET(I,J)
      D5=AKZM(I-1,J)*((EZ(I)-EZ(I-1))/DZI1)/DA
      D(J)=D1+D2+D3+D4+D5
      GO TO 110
120   DXX(J)=(DX(J)**2+(DDZ(J+1)-DDZ(J))**2)**0.5
      DXX(J-1)=(DX(J-1)**2+(DDZ(J)-DDZ(J-1))**2)**0.5
      A(J)=(DDZ(J-1)+DDZ(J))*AKXM(I,J-1)/(4.0*DXX(J-1))
      B(J)=-(DX(J)+DX(J-1))*AKZM(I-1,J)/(2.0*DDZ(J))-(DDZ(J-1)
      1+DDZ(J))*
      1AKXM(I,J-1)/(4.0*DXX(J-1))-(DDZ(J+1)+DDZ(J))*AKXM(I,J)/
      1(4.0*DXX(J))-
      1SSM(I,J)*(DX(J)+
      1DX(J-1))*(DDZ(J)+(DDZ(J+1)+DDZ(J-1))/2.0)/(8.0*DELT)
      C(J)=(DDZ(J+1)+DDZ(J))*AKXM(I,J)/(4.0*DXX(J))
      D(J)=-((DX(J)+DX(J-1))/2.0)*(R(IT)+P(I-1,J)*AKZM(I-1,J)/DD
      1Z(J))-(PP(I,J)*(DX(J)+DX(J-1))/(2.0*DELT))*(
      1SSM(I,J)*((DDZ(J)+(DDZ(J+1)+DDZ(J-1))/2.0)/4.0))
```

```
      1-AKZM(I-1,J)*(DX(J)+DX(J-1))*(EZ(I-1)-EZ(I))/(2.0*DDZ(J))
      1+ET(I,J)*(DX(J)+DX(J-1))*DDZ(J)/4.0
      GO TO 110
1120  DXX(J)=(DX(J)**2+(DDZ(J+1)-DDZ(J))**2)**0.5
      DXX(J-1)=(DX(J-1)**2+(DDZ(J)-DDZ(J-1))**2)**0.5
      A(J)=(DDZ(J-1)+DDZ(J))*AKXM(I,J-1)/(4.0*DXX(J-1))
      B(J)=-DX(J-1)*AKZM(I-1,J)/(2.0*DDZ(J))-(DDZ(J-1)+DDZ(J))
      1*AKXM(I,J-1)/
      1(4.0*DXX(J-1))-SSM(I,J)*
      1(DDZ(J)+DDZ(J-1))*DX(J-1)/(8.0*DELT)
      C(J)=0.0
      D(J)=-R(IT)*DX(J-1)/2.0-P(I-1,J)*DX(J-1)*AKZM(I-1,J)/(2.0*
      1DDZ(J))-PP(I,J)*SSM(I,J)
      1*(DDZ(J-1)+DDZ(J))*DX(J-1)/(8.0*DELT)
      1+ET(I,J)*DDZ(J)*DX(J-1)/4.0
      1-AKZM(I-1,J)*DX(J-1)*(EZ(I-1)-EZ(I))/(2.0*DDZ(J))
      GO TO 110
160   A(J)=0.0
      B(J)=1.0
      C(J)=0.0
      D(J)=Z(I)
      GO TO 110
170   A(J)=0.0
      B(J)=1.0
      C(J)=0.0
      D(J)=Z(I)
      GO TO 110
209   A(J)=0.0
      B(J)=10
      C(J)=0.0
      D(J)=0.0
      GO TO 110
309   A(J)=0.0
      B(J)=1.0
      C(J)=0.0
      D(J)=0.0
      GO TO 110
709   IF(I.EQ.NR)GO TO 718
      A(J)=0.0
```

```
      B(J)=-AKXM(I,J)*(DZI+DZI1)/(2.0*DX(J))
      1-AKZM(I,J)*DX(J)/(2.0*DZI)
      1-AKZM(I-1,J)*DX(J)/(2.0*DZI1)
      1-SSM(I,J)*DX(J)*(DZI+DZI1)/(4.0*DELT)
      C(J)=AKXM(I,J)*(DZI+DZI1)/(2.0*DX(J))
      D(J)=-P(I+1,J)*AKZM(I,J)*DX(J)/(2.0*DZI)
      1-P(I-1,J)*AKZM(I-1J)*DX(J)/(2.0*DZI1)
      1+ET(I,J)*DX(J)*(DZI+DZI1)/4.0
      1-(EZ(I+1)-EZ(I))*AKZM(I,J)*DX(J)/(2.0*DZI)
      1-(EZ(I-1)-EZ(I))*AKZM(I-1,J)*DX(J)/(2.0*DZI1)
      1-PP(I,J)*SSM(I,J)*DX(J)*(DZI+DZI1)/(4.0*DELT)
      GO TO 110
809   IF(I.EQ.NR)GO TO 808
      A(J)=AKXM(I,J-1)*(DZI+DZI1)/(2.0*DX(J-1))
      B(J)=-AKXM(I,J-1)*(DZI+DZI1)/(2.0*DX(J-1))
      1-AKZM(I,J)*DX(J-1)/(2.0*DZI)
      1-AKZM(I-1,J)*DX(J-1)/(2.0*DZI1)
      1-SSM(I,J)*DX(J-1)*(DZI+DZI1)/(4.0*DELT)
      C(J)=0.0
      D(J)=-P(I+1,J)*AKZM(I,J)*DX(J-1)/(2.0DZI)
      1-P(I-1,J)*AKZM(I-1,J)*DX(J-1)/(2.0*DZI1)
      1+ET(I,J)*DX(J-1)*(DZI+DZI1)/4.0
      1-(EZ(I+1)-EZ(I))*AKZM(I,J)*DX(J-1)/(2.0*DZI)
      1-(EZ(I-1)-EZ(I))*AKZM(I-1,J)*DX(J-1)/(2.0*DZI1)
      1-SSM(I,J)*PP(I,J)*DX(J-1)*(DZI+DZI1)/(4.0*DELT)
      GO TO 110
130   IF(NRD.GT.1.AND.J.EQ.1) GO TO 165
      IF(NRD.GT.1.AND.J.EQ.NC) GO TO 175
      IF(ISYM.EQ.1.AND.J.EQ.NC)GO TO 175
      A(J)=(AKXM(I,J-1)/DX(J-1))*(2.0/(DX(J)+DX(J-1)))
      B(J)=-((AKXM(I,J)/DX(J)+AKXM(I,J-1)/DX(J-1))*
      1(2.0/(DX(J)+DX(J-1)))+SSM(I,J)/DELT)
      1-2.0*(AKZM(I,J)/DZI**2)
      C(J)=(AKXM(I,J)/DX(J))*(2.0/(DX(J)+DX(J-1)))
      D(J)=-PP(I,J)*SSM(I,J)/DELT+ET(I,J)
      1-2.0*P(I+1,J)*(AKZM(I,J)/DZI**2)
      1-2.0*AKZM(I,J)*(EZ(I+1)-EZ(I))/DZI**2
      GO TO 110
165   A(J)=0.0
```

```
      B(J)=-(AKXM(I,J)/DX(J)**2+SSM(I,J)/(2.0*DELT))
     1-(AKZM(I,J)/DZI**2)
      C(J)=AKXM(I,J)/DX(J)**2
      D(J)=-P(I+1,J)*AKZM(I,J)/DZI**2+ET(I,J)/2.0
     1-PP(I,J)*SSM(I,J)/(2.0*DELT)
     1-AKZM(I,J)*(EZ(I+1)-EZ(I))/DZI**2
      GO TO 110
175   A(J)=AKXM(I,J-1)/DX(J-1)**2
      B(J)=-(AKXM(I,J-1)/DX(J-1)**2+SSM(I,J)/(2.0*DELT))
     1-(AKZM(I,J)/DZI**2)
      C(J)=0.0
      D(J)=-P(I+1,J)*AKZM(I,J)/DZI**2+ET(I,J)/2.0
     1-PP(I,J)*SSM(I,J)/(2.0*DELT)
     1-AKZM(I,J)*(EZ(I+1)-EZ(I))/DZI**2
      GO TO 110
140   A(J)=0.0
      B(J)=-((2.0*AKXM(I,J)/DX(J)**2)+SSM(I,J)/DELT)
     1-(AKZM(I,J)/DZI+AKZM(I-1,J)/DZI1)
     1*(2.0/(DZI+DZI1))
      C(J)=2.0*AKXM(I,J)/DX(J)**2
      D(J)=-PP(I,J)*SSM(I,J)/DELT-P(I+1,J)*(AKZM(I,J)/DZI)
     1*(2.0/(DZI+DZI1))
     1-P(I-1,J)*(AKZM(I-1,J)/DZI1)
     1*(2.0/(DZI+DZI1))
     1-AKZM(I,J)*((EZ(I+1)-EZ(I))/DZI)
     1*(2.0/(DZI+DZI1))
     1-AKZM(I-1,J)*((EZ(I-1)-EZ(I))/DZI1)
     1*(2.0/(DZI+DZI1))+ET(I,J)
      GO TO 110
150   A(J)=(2.0*AKXM(I,J-1))/DX(J-1)**2
      B(J)=-((2.0*AKXM(I,J-1))/DX(J-1)**2+SSM(I,J)/DELT)
     1-(AKZM(I,J)/DZI+AKZM(I-1,J)/DZI1)
     1*(2.0/(DZI+DZI1))
      C(J)=0.0
      D(J)=-PP(I,J)*SSM(I,J)/DELT-P(I+1,J)*(AKZM(I,J)/DZI)
     1*(2.0/(DZI+DZI1))
     1-P(I-1,J)*(AKZM(I-1,J)/DZI1)*(2.0/(DZI+DZI1))
     1-AKZM(I,J)*((EZ(I+1)-EZ(I))/DZI)
     1*(2.0/(DZI+DZI1))
```

```
      1-AKZM(I-1,J)*((EZ(I-1)-EZ(I))/DZI1)
      1*(2.0/(DZI+DZI1))+ET(I,J)
      GO TO 110
718   A(J)=0.0
      B(J)=-AKXM(I,J)*DZ(I-1)/(2.0*DX(J))
      1-AKZM(I-1,J)*DX(J)/(2.0*DZ(I-1))
      1-SSM(I,J)*DX(J)*DZ(I-1)/(4.0*DELT)
      C(J)=AKXM(I,J)*DZ(I-1)/(2.0*DX(J))
      D(J)=-R(IT)*DX(J)/2.0+ET(I,J)*DX(J)*DZ(I-1)/4.0
      1-P(I-1,J)*AKZM(I-1,J)*DX(J)/(2.0*DZ(I-1))
      1-(EZ(I-1)-EZ(I))*AKZM(I-1,J)*DX(J)/(2.0*DZ(I-1))
      1-PP(I,J)*SSM(I,J)*DX(J)*DZ(I-1)/(4.0*DELT)
      GO TO 110
808   A(J)=AKXM(I,J-1)*DZ(I-1)/(2.0*DX(J-1))
      B(J)=-AKXM(I,J-1)*DZ(I-1)/(2.0*DX(J-1))
      1-AKZM(I-1,J)*DX(J-1)/(2.0*DZ(I-1))
      1-SSM(I,J)*DX(J-1)*DZ(I-1)/(4.0*DELT)
      C(J)=0.0
      D(J)=-P(I-1,J)*AKZM(I-1,J)*DX(J-1)/(2.0*DZ(I-1))
      1-R(IT)*DX(J-1)/2.0
      1+ET(I,J)*DX(J-1)*DZ(I-1)/4.0
      1-(EZ(I-1)-EZ(I))*AKZM(I-1,J)*DX(J-1)/(2.0*DZ(I-1))
      1-PP(I,J)*SSM(I,J)*DX(J-1)*DZ(I-1)/(4.0*DELT)
110   CONTINUE
      CALL STN(NC,A,B,C,D,PS)
      DO 20 J=1,NC
      P(I,J)=PS(J)
20    CONTINUE
100   CONTINUE
      DO 200 J=1,NC
      DO 210 I=1,NR
      IF(I.EQ.NR)GO TO 251
      IF(I.EQ.NR-1)GO TO 361
      DZI=DZ(I)
      IF(I.EQ.1)GO TO 351
      DZI1=DZ(I-1)
      GO TO 351
251   DZI1=DDZ(J)
      GO TO 351
```

```
361       DZI=DDZ(J)
          DZI1=DZ(I-1)
351        CONTINUE
C         IF(J.EQ.1.AND.I.GT.NRW.AND.PP(I,J).GE.0.0)GO TO 1209
          IF(J.LE.NCD1.AND.I.GE.NRD.AND.I.LE.NRW)GO TO 260
C         IF(I.GT.NRW.AND.J.LT.NCD1)GO TO 1209
C         IF(I.GT.NRW.AND.J.EQ.NCD1)GO TO 1709
          IF(I.GT.NRW.AND.J.EQ.1)GO TO 1709
          IF(ISYM.EQ.1)GO TO 805
          IF(J.GE.NCD2.AND.I.GE.NRD)GO TO 270
          IF(I.GT.NRW.AND.J.GT.NCD2)GO TO 1309
          IF(I.GT.NRW.AND.J.EQ.NCD2)GO TO 1809
805       IF(J.EQ.NC.AND.I.EQ.NR.AND.ISYM.EQ.1)GO TO 1240
          IF(I.LT.NRD.AND.J.EQ.1) GO TO 220
          IF(I.LT.NRD.AND.J.EQ.NC) GO TO 230
          IF(ISYM.EQ.1.AND.J.EQ.NC)GO TO 230
          IF(I.EQ.NR) GO TO 240
          IF(I.EQ.1) GO TO 250
         IF(DZI1.GT.DZI)DA=DZI
          IF(DZI1.LE.DZI)DA=DZI1
          IF(DX(J-1).GT.DX(J))DB=DX(J)
          IF(DX(J-1).LE.DX(J))DB=DX(J-1)
          A(I)=(AKZM(I-1,J)/DZI1)/DA
          B(I)=-((AKZM(I,J)/DZI+AKZM(I-1,J)/DZI1)
          1/DA+SSM(I,J)/DELT)
          1-(AKXM(I,J)/DX(J)+AKXM(I,J-1)/DX(J-1))/DB
          C(I)=(AKZM(I,J)/DZI)/DA
          D(I)=-(P(I,J+1)*AKXM(I,J)/DX(J))/DB+ET(I,J)
          1-AKZM(I,J)*((EZ(I+1)-EZ(I))/DZI)/DA
          1+AKZM(I-1,J)*((EZ(I)-EZ(I-1))/DZI1)/DA
          1-PP(I,J)*SSM(I,J)/DELT-P(I,J-1)*(AKXM(I,J-1)/DX(J-1))/DB
          GO TO 210
220       IF(I.EQ.1) GO TO 265
          A(I)=(AKZM(I-1,J)/DZI1)*(2.0/(DZI+DZI1))
          B(I)=-((AKZM(I,J)/DZI+AKZM(I-1,J)/DZI1)
          1*(2.0/(DZI+DZI1))+SSM(I,J)/DELT)
          1-(2.0*AKXM(I,J)/DX(J)**2)
          C(I)=(AKZM(I,J)/DZI)*(2.0/(DZI+DZI1))
          D(I)=-P(I,J+1)*2.0*AKXM(I,J)/DX(J)**2
```

```
      1-PP(I,J)*SSM(I,J)/DELT
      1-AKZM(I,J)*((EZ(I+1)-EZ(I))/DZI)
      1*(2.0/(DZI+DZI1))
      1-AKZM(I-1,J)*((EZ(I-1)-EZ(I))/DZI1)
      1*(2.0/(DZI+DZI1))+ET(I,J)
      GO TO 210
260   A(I)=0.0
      B(I)=1.0
      C(I)=0.0
      D(I)=Z(I)
      GO TO 210
265   A(I)=0.0
      B(I)=-(AKZM(I,J)/DZI**2+SSM(I,J)/(2.0*DELT))
      1-(AKXM(I,J)/DX(J)**2)
      C(I)=AKZM(I,J)/DZI**2
      D(I)=-P(I,J+1)*AKXM(I,J)/DX(J)**2
      1-PP(I,J)*SSM(I,J)/(2.0*DELT)
      1-AKZM(I,J)*(EZ(I+1)-EZ(I))/DZI**2+ET(I,J)/2.0
      GO TO 210
230   IF(I.EQ.1) GO TO 275
      A(I)=(AKZM(I-1,J)/DZI1)*(2.0/(DZI+DZI1))
      B(I)=-((AKZM(I,J)/DZI+AKZM(I-1,J)/DZI1)
      1*(2.0/(DZI+DZI1))+SSM(I,J)/DELT)
      1-((2.0*AKXM(I,J-1))/DX(J-1)**2)
      C(I)=(AKZM(I,J)/DZI)*(2.0/(DZI+DZI1))
      D(I)=-(2.0*P(I,J-1))*AKXM(I,J-1)/DX(J-1)**2
      1-PP(I,J)*SSM(I,J)/DELT
      1-AKZM(I,J)*((EZ(I+1)-EZ(I))/DZI)
      1*(2.0/(DZI+DZI1))
      1-AKZM(I-1,J)*((EZ(I-1)-EZ(I))/DZI1)
      1*(2.0/(DZI+DZI1))+ET(I,J)
      GO TO 210
270   A(I)=0.0
      B(I)=1.0
      C(I)=0.0
      D(I)=Z(I)
      GO TO 210
1209  A(I)=0.0
      B(I)=1.0
```

```
      C(I)=0.0
      D(I)=0.0
      GO TO 210
1309  A(I)=0.0
      B(I)=1.0
      C(I)=0.0
      D(I)=0.0
      GO TO 210
1709  IF(I.EQ.NR)GO TO 1708
      A(I)=AKZM(I-1,J)*DX(J)/(2.0*DZI1)
      B(I)=-AKXM(I,J)*(DZI+DZI1)/(2.0*DX(J))
     1-AKZM(I,J)*DX(J)/(2.0*DZI)
     1-AKZM(I-1,J)*DX(J)/(2.0*DZI1)
     1-SSM(I,J)*DX(J)*(DZI+DZI1)/(4.0*DELT)
      C(I)=AKZM(I,J)*DX(J)/(2.0*DZI)
      D(I)=-P(I,J+1)*AKXM(I,J)*(DZI+DZI1)/(2.0*DX(J))
     1+ET(I,J)*DX(J)*(DZI+DZI1)/4.0
     1-(EZ(I+1)-EZ(I))*AKZM(I,J)*DX(J)/(2.0*DZI)
     1-(EZ(I-1)-EZ(I))*AKZM(I-1,J)*DX(J)/(2.0*DZI1)
     1-PP(I,J)*SSM(I,J)*DX(J)*(DZI+DZI1)/(4.0*DELT)
      GO TO 210
1809  IF(I.EQ.NR)GO TO 1808
      A(I)=AKZM(I-1,J)*DX(J-1)/(2.0*DZI1)
      B(I)=-AKXM(I,J-1)*(DZI+DZI1)/(2.0*DX(J-1))
     1-AKZM(I,J)*DX(J-1)/(2.0*DZI)
     1-AKZM(I-1,J)*DX(J-1)/(2.0*DZI1)
     1-SSM(I,J)*DX(J-1)*(DZI+DZI1)/(4.0*DELT)
      C(I)=AKZM(I,J)*DX(J-1)/(2.0*DZI)
      D(I)=-P(I,J-1)*AKXM(I,J-1)*(DZI+DZI1)/(2.0*DX(J-1))
     1+ET(I,J)*DX(J-1)*(DZI+DZI1)/4.0
     1-(EZ(I+1)-EZ(I))*AKZM(I,J)*DX(J-1)/(2.0*DZI)
     1-(EZ(I-1)-EZ(I))*AKZM(I-1,J)*DX(J-1)/(2.0*DZI1)
     1-SSM(I,J)*PP(I,J)*DX(J-1)*(DZI+DZI1)/(4.0*DELT)
      GO TO 210
275   A(I)=0.0
      B(I)=-(AKZM(I,J)/DZI**2+SSM(I,J)/(2.0*DELT))
     1-(AKXM(I,J-1)/DX(J-1)**2)
      C(I)=AKZM(I,J)/DZI**2
      D(I)=-P(I,J-1)*AKXM(I,J-1)/DX(J-1)**2
```

```
      1-PP(I,J)*SSM(I,J)/(2.0*DELT)
      1-AKZM(I,J)*(EZ(I+1)-EZ(I))/DZI**2+ET(I,J)/2.0
      GO TO 210
240   IF(J.EQ.NCD1)GO TO 1708
      IF(J.LT.NCD1)GO TO 1209
      DXX(J)=(DX(J)**2+(DDZ(J+1)-DDZ(J))**2)**0.5
      DXX(J-1)=(DX(J-1)**2+(DDZ(J)-DDZ(J-1))**2)**0.5
      A(I)=(DX(J)+DX(J-1))*AKZM(I-1,J)/(2.0*DDZ(J))
      B(I)=-(DX(J)+DX(J-1))*AKZM(I-1,J)/(2.0*DDZ(J))-(DDZ(J-1)
      1+DDZ(J))*AKXM
      1(I,J-1)/(4.0*DXX(J-1))-(DDZ(J+1)+DDZ(J))*AKXM(I,J)/(4.0
      1*DXX(J))-
      1SSM(I,J)*(DX(J)+DX(J-1))*
      1(DDZ(J)+(DDZ(J-1)+DDZ(J+1))/2.0)/(8.0*DELT)
      C(I)=0.0
      D(I)=-R(IT)*(DX(J)+DX(J-1))/2.0-P(I,J-1)*(DDZ(J-1)+DDZ(J))
      1*AKXM(I,J-1)/(
      14.0*DXX(J-1))-P(I,J+1)*(DDZ(J+1)+DDZ(J))*AKXM(I,J)/(4.0
      1*DXX(J))-(PP
      1(I,J)*(DX(J)+DX(J-1))/(2.0*DELT))*(SSM(I,J)
      1*(DDZ(J)+(DDZ(J-1)+DDZ(J+1))/2.0)/4.0)
      1+ET(I,J)*(DX(J)+DX(J-1))*DDZ(J)/4.0
      1-AKZM(I-1,J)*(DX(J)+DX(J-1))*(EZ(I-1)-EZ(I))/(2.0*DDZ(J))
      GO TO 210
1240  DXX(J)=(DX(J)**2+(DDZ(J+1)-DDZ(J))**2)**0.5
      DXX(J-1)=(DX(J-1)**2+(DDZ(J)-DDZ(J-1))**2)**0.5
      A(I)=DX(J-1)*AKZM(I-1,J)/(2.0*DDZ(J))
      B(I)=-DX(J-1)*AKZM(I-1,J)/(2.0*DDZ(J))-(DDZ(J-1)+DDZ(J))
      1*AKXM(I,J-1)/
      1(4.0*DXX(J-1))-SSM(I,J)*DX(J-1)*DDZ(J)/(4.0*DELT)
      C(I)=0.0
      D(I)=-R(IT)*DX(J-1)/2.0-P(I,J-1)*(DDZ(J-1)+DDZ(J))*AKXM(I,
      1J-1)/(4.0*DXX(
      1J-1))-SSM(I,J)*DX(J-1)*PP(I,J)*DDZ(J)/(4.0*DELT)
      1+ET(I,J)*DDZ(J)*DX(J-1)/4.0
      1-AKZM(I-1,J)*DX(J-1)*(EZ(I-1)-EZ(I))/(2.0*DDZ(J))
      GO TO 210
250   A(I)=0.0
      B(I)=-(2.0*AKZM(I,J)/DZI**2+SSM(I,J)/DELT)
```

```
      1-(AKXM(I,J)/DX(J)+AKXM(I,J-1)/DX(J-1))
      1*(2.0/(DX(J)+DX(J-1)))
      C(I)=2.0*AKZM(I,J)/DZI**2
      D(I)=ET(I,J)-P(I,J-1)*(AKXM(I,J-1)/DX(J-1))
      1*(2.0/(DX(J)+DX(J-1)))
      1-P(I,J+1)*(AKXM(I,J)/DX(J))*(2.0/(DX(J)+DX(J-1)))
      1-PP(I,J)*SSM(I,J)/DELT
      1-2.0*AKZM(I,J)*(EZ(I+1)-EZ(I))/DZI**2
      GO TO 210
1708  A(I)=AKZM(I-1,J)*DX(J)/(2.0*DZ(I-1))
      B(I)=-AKXM(I,J)*DZ(I-1)/(2.0*DX(J))
      1-AKZM(I-1,J)*DX(J)/(2.0*DZ(I-1))
      1-SSM(I,J)*DX(J)*DZ(I-1)/(4.0*DELT)
      C(I)=0.0
      D(I)=-R(IT)*DX(J)/2.0+ET(I,J)*DX(J)*DZ(I-1)/4.0
      1-P(I,J+1)*AKXM(I,J)*DZ(I-1)/(2.0*DX(J)
      1-(EZ(I-1)-EZ(I))*AKZM(I-1,J)*DX(J)/(2.0*DZ(I-1))
      1-PP(I,J)*SSM(I,J)*DX(J)*DZ(I-1)/(4.0*DELT)
      GO TO 210
1808  A(I)=AKZM(I-1,J)*DX(J-1)/(2.0*DZ(I-1))
      B(I)=-AKXM(I,J-1)*DZ(I-1)/(2.0*DX(J-1))
      1-AKZM(I-1,J)*DX(J-1)/(2.0*DZ(I-1))
      1-SSM(I,J)*DX(J-1)*DZ(I-1)/(4.0*DELT)
      C(I)=0.0
      D(I)=-P(I,J-1)*AKXM(I,J-1)*DZ(I-1)/(2.0*DX(J-1))
      1-R(IT)*DX(J-1)/2.0
      1+ET(I,J)*DX(J-1)*DZ(I-1)/4.0
      1-(EZ(I-1)-EZ(I))*AKZM(I-1,J)*DX(J-1)/(2.0*DZ(I-1))
      1-PP(I,J)*SSM(I,J)*DX(J-1)*DZ(I-1)/(4.0*DELT)
210   CONTINUE
      CALL STN (NR,A,B,C,D,PS)
      DO 30 I=1,NR
      P(I,J)=PS(I)
30    CONTINUE
200   CONTINUE
      IF(IK.EQ.1) GO TO 400
C     SM=0.0
      DO 350 I=1,NR
      DO 370 J=1,NC
```

```
C       SM=SM+ABS(P(I,J)-PIP(I,J))
C       SM=ABS(P(I,J)-PIP(I,J))
        DP(I,J)=ABS(P(I,J)-PIP(I,J))
370     CONTINUE
350     CONTINUE
        CALL BIG(NR,NC,DP,DPBIG)
C       IF(SM.LT.ALW) GO TO 50
        IF(DPBIG.LT.ALW)GO TO 50
400     DO 380 I=1,NR
        DO 390 J=1,NC
        PIP(I,J)=P(I,J)
390     CONTINUE
380     CONTINUE
16      CONTINUE
        PRINT 65,DPBIG
65      FORMAT(5X,'CONVERGENCE NOT ACHIEVED',E16.7)
        STOP
50      CONTINUE
        IF(NCONF.EQ.100)GO TO 45
        DO 44 J=1,NC
        DDZM(J)=DZ(NR-1)+((P(NR,J)-PIN(J))+(PP(NR,J)-PIN(J)))*0.5
44      CONTINUE
        DO 810 J=1,NC
        IF(ABS(DDZ(J)-DDZM(J)).GT.EPS1)GO TO 820
810     CONTINUE
        GO TO 830
820     DO 840 J=1,NC
        DDZ(J)=DDZM(J)
840     CONTINUE
        DO 77 I=1,NR
        DO 99 J=1,NC
        P(I,J)=PP(I,J)
99      CONTINUE
77      CONTINUE
        PRINT850
850     FORMAT(5X,'CONVERGENCE W.R.T.DZ NOT ACHIEVED')
C       PRINT*,(DDZ(J),J=1,NC)
C       PRINT*,(DDZM(J),J=1,NC)
830     CONTINUE
```

```
         DO 89 J=1,NC
         DDZ(J)=DZ(NR-1)+P(NR,J)-PIN(J)
 89      CONTINUE
 88      CONTINUE
 45      DO 551 I=1,NR
         DO 661 J=1,NC
         PP(I,J)=P(I,J)
 661     CONTINUE
 51      CONTINUE
 1050    CONTINUE
         PRINT*,IT,NTD,DELT
         TYPE*,TM,IT,NTD,DELT
C        DO 305 I=1,NR
C        PRINT*,(P(I,J),J=1,NC)
         PRINT*,(P(I,NC),I=1,NR)
C305     CONTINUE
         IF(NTD.EQ.1)GO TO 7
C        SM1=0.0
         DO 1 I=1,NR
         DO 2 J=1,NC
C        SM1=SM1+ABS(P(I,J)-PPT(I,J))
C        SM1=ABS(P(I,J)-PPT(I,J))
         TP(I,J)=ABS(P(I,J)-PPT(I,J))
2        CONTINUE
1        CONTINUE
         CALL BIG(NR,NC,TP,TPBIG)
C        IF(SM1.LT.EPS)GO TO 5
         IF(TPBIG.LT.EPS)GO TO 5
         DO 3 I=1,NR
         DO 4 J=1,NC
         PPT(I,J)=P(I,J)
         CONTINUE
         CONTINUE
         IF(KT1.LE.KTM)GO TO 67
         PRINT66,TPBIG
6        FORMAT(5X,'NO.OF PERMISSIBLE TIME REDUCTIONS EXCEEDED',E16.7)
         STOP
         KT1=KT1+1
         NTD=NTD*2
```

```
      GO TO 6
5     TM=TM+DT(IT)
      PRINT*,TM
C     DO 300 I=1,NR
C     PRINT *,(P(I,J),J=1,NC)
      PRINT *,(P(I,NC),I=1,NR)
C     TYPE *,(P(I,J),J=1,NC)
      TYPE *,(P(I,NC),I=1,NR)
C300  CONTINUE
      DO 55 I=1,NR
      DO 60 J=1,NC
      PP(I,J)P(I,J)
60    CONTINUE
55    CONTINUE
      DO 205 I=1,NR
      DO 105 J=1,NC
      H(I,J)=P(I,J)+EZ(I)
105   CONTINUE
205   CONTINUE
C     HORIZONTAL VELOCITIES
      PRINT 101
101   FORMAT(1X'VX VALUES')
      DO 11 I=1,NR
      DO 12 J=1,NC-1
      VX(I,J)=-AKXM(I,J)*(H(I,J+1)-H(I,J))/DX(J)
 12   CONTINUE
C     PRINT*,I,(VX(I,J),J=1,NC-1)
11    CONTINUE
      SQ1=0.0
      DO 700 I=NRD,NRW+1
      J=NCD1
      IF(I.EQ.NRW+1)GO TO 702
      IF(I.EQ.NRD)GO TO 703
      Q=-VX(I,J)*((DZ(I-1)+DZ(I))/2.0)
      GO TO 704
702   Q=-VX(I,J)*(DZ(I-1)/2.0)
      GO TO 704
703   Q=-VX(I,J)*DZ(NRD)*0.5
704   SQ1=SQ1+Q
```

```
700     CONTINUE
C       VERTICAL VELOCITIES
        PRINT 201
201     FORMAT(1X'VZ VALUES'/)
        DO 21 I=1,NR-1
        DO 22 J=1,NC
        VZ(I,J)=-AKZM(I,J)*(H(I+1,J)-H(I,J)
        1)/DZI
22      CONTINUE
C       PRINT*,I,(VZ(I,J),J=1,NC)
21      CONTINUE
        SQ2=0.0
        DO 707 J=1,NCD1
        I=NRD
        IF(I.EQ.1)GO TO 707
        IF(J.EQ.1)GO TO 705
        IF(J.EQ.NCD1)GO TO 706
        Q=VZ(I-1,J)*((DX(J)+DX(J-1))/2.0)
        GO TO 708
705     Q=VZ(I-1,J)*(DX(J)/2.0)
        GO TO 708
706     Q=VZ(I-1,J)*DX(NCD1-1)*0.5
708     SQ2=SQ2+Q
707     CONTINUE
        SQ3=0.0
        DO 888 J=1,NCD1
        I=NRW
        IF(J.EQ.1)GO TO 786
        IF(J.EQ.NCD1)GO TO 787
        Q=-VZ(I,J)*((DX(J)+DX(J-1))/2.0)
        GO TO 777
786     Q=-VZ(I,J)*(DX(J)/2.0)
        GO TO 777
787     Q=-VZ(I,J)*DX(NCD1-1)*0.5
777     SQ3=SQ3+Q
888     CONTINUE
        SQ=SQ1+SQ2+SQ3
        PRINT*,SQ1,SQ2,SQ3,SQ
        WTR=0.0
```

```
      DO 999 J=1,NC
      I=NRW
      IF(J.EQ.1)GO TO 333
      IF(J.EQ.NCD1)GO TO 444
      IF(J.EQ.NC)GO TO 555
      Q=-VZ(I,J)*((DX(J)+DX(J-1))/2.0)
      GO TO 666
333   Q=-VZ(I,J)*(DX(J)/2.0)
      GO TO 666
444   Q=-VZ(I,J)*DX(NCD1-1)*0.5
      GO TO 666
555   Q=-VZ(I,J)*(DX(J-1)/2.0)
666   WTR=WTR+Q
999   CONTINUE
      RECH(IT)=WTR/(0.5*AL)
      TYPE*,TM,WTR,R(IT),RECH(IT),VZ(NRW,NC)
      PRINT*,TM,WTR,R(IT),RECH(IT),VZ(NRW,NC)
      DO 152 J=1,NC
      DO 151 I=NRW,NR-1
      IF(P(I,J).GE.0.0.AND.P(I+1,J).LT.0.0)GO TO 153
      GO TO 151
153   WTRS(J)=(P(I,J)*DZ(I)/(P(I,J)-P(I+1,J)))+(EZ(I)-EZ(NRW))
C     PRINT*,VZ(I,J)
      TYPE*,VZ(I,J)
151   CONTINUE
152   CONTINUE
      PRINT*,(WTRS(J),J=1,NC)
      SOM=0.0
      DO 1111 LTM=1,83
      SOM=(SOM+OWT(LTM))
1111  CONTINUE
      FOM=SOM/83.0
      IF(TM.LE.10.0)GO TO 1000
      TMM=TM-IFIX(TM)
      IF(TMM.NE.0.0)GO TO 1000
      JTM=JTM+1
      FM=FM+(WTRS(NC)-OWT(JTM))**2.0
C     FOM=(OWT(JTM)+OWT(JTM-1)/2.0
      FO=FO+(OWT(JTM)-FOM)**2.0
```

```
      TYPE*,TM,JTM,FOM,FO,FM
      PRINT*,TM,JTM,FOM,FO,FM
1000  CONTINUE
      WRITE(3,*)(RECH(IT),IT=1,NT)
      DO 395 IT=1,NT
C     WRITE(3,*)(VZ(NRW,J),J=1,NC)
395   CONTINUE
      SDFM=FM/82.0
      SDFO=FO/82.0
      DIRSQ=1-(SDFM/SDFO)
      PRINT*,DIRSQ
      STOP
      END
C     ***************************************************
C     SUBROUTINE TO SOLVE TRIDIAGONAL MATRIX
C     ***************************************************
      SUBROUTINE STN(N,A,B,C,D,P)
      DIMENSION A(20),B(20),C(20),D(20)
     DIMENSION AL(20),BT(20),Y(20),P(20)
      AL(1)=B(1)
      BT(1)=C(1)/B(1)
      DO500 I=2,N
      AL(I)=B(I)-A(I)*BT(I-1)
C     TYPE*,AL(I)
C     PRINT*,AL(I)
      BT(I)=C(I)/AL(I)
500   CONTINUE
      Y(1)=D(1)/AL(1)
      DO 520I=2,N
      Y(I)=(D(I)-A(I)*Y(I-1))/AL(I)
520   CONTINUE
      P(N)=Y(N)
      DO 530 I=2,N
      II=N-I+1
      P(II)=Y(II)-BT(II)*P(II+1)
530   CONTINUE
      RETURN
      END
C     ***************************************************
```

```
C       SUBROUTINE TO CALCULATE HYD.COND.FOR COMP.CAP.HEADS
        FUNCTION COND(TUF)
C       ************************************************************
        COMMON/CONST/THR,POR,AM,POW,AC,SSM,SAT,PET,PF,WP,FFC
        COND =SAT*((TUF-THR)/(POR-THR))**POW
        RETURN;END
C       ********************************************
C       SUBROUTINE TO CALCULATE VOL.MOIS.CONT. FOR COMP.P(I,J)
        FUNCTION THETA(TUF)
C       ****************************************************************
        COMMON/CONST/THR,POR,AM,POW,AC,SSM,SAT,PET,PF,WP,FFC
        IF(TUF.GT.SSM)GO TO 100
        THETA=AM*TUF+POR
        GO TO 101
100     THETA=EXP(-AC*TUF)+THR
101     IF(THETA.GT.POR)THETA=POR
        RETURN;END
C       *******************************************************
C       SUBROUTINE TO CALCULATE SPECIFIC SOIL MOISTURE CAPACITY
        FUNCTION DIFU(TUF1)
C       ******************************************************
        COMMON/CONST/THR,POR,AM,POW,AC,SSM,SAT,PET,PF,WP,FFC
        IF(TUF1.EQ.SSM) GO TO 887
        IF(TUF1.GT.SSM) GO TO 1000
        DIFU=AM
        RETURN
1000    DIFU=EXP(-AC*TUF1)*(-AC)
        IF(DIFU.GE.0.0) STOP'ERROR-SPEC'
        RETURN
887     CONTINUE
        A1=THETA(TUF1+0.001)/0.002
        A2=THETA(TUF1-0.001)/0.002
        DIFU=A1-A2
        IF(DIFU.GE.0.0) STOP'ERROR-SPEC';RETURN;END
C       *******************************************************
C       SUBROUTINE TO CALCULATE EVAPOTRANSPIRATION RATE
        FUNCTION EVPT(THETA)
C       **************************************************************
        COMMON/CONST/THR,POR,AM,POW,AC,SSM,SAT,PET,PF,WP,FFC
```

```
      IF(THETA.LT.WP)GO TO 10
      IF(THETA.GE.(FC-PF*(FC-WP)))GO TO 20
      EVPT=PET*(THE-WP)/((FC-WP)*(1.0-PF))
      RETURN
 10   EVPT=0.0
      RETURN
 20   EPT=PET
      RETURN
      END
C     ****************************************************
C     SUBROUTINE TO FIND OUT LARGEST DIFF.OF P(I,J) COMP.IN
C     TWO CONSECUTIVE ITERATIONS(OF ADIE/SUBTIME STEPS)
      SUBROUTINE BIG(NR,NC,DP,XBIG)
C     ****************************************************
      DIMENSION DP(50,50)
      XBIG=DP(1,1)
      DO 333 I=1,NR
      DO 444 J=1,NC
      IF(XBIG-DP(I,J))555,444,444
 555  XBIG=DP(I,J)
 444  CONTINUE
 333  CONTINUE
      RETURN
      END
```

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