A STUDY OF UNCONFINED SEEPAGE FROM PARALLEL CANALS

A THESIS

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By

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitles 'A STUDY OF UNCONFINED SEEPAGE FROM PARALLEL CANALS' in fulfilment of the requirement for the award of the Degree of Doctor of Philosophy, submitted in the Department of Hydrology of the University of Roorkee is an authentic record of my own work carried out during the period from March 1983 to June 1988 under the supervision of Prof.(Dr.) Satish Chandra, Dr. G.C. Mishra and Dr. Deepak Kashyap.

The matter embodied in this thesis has not been submitted by me for the award of any other degree.

(D.N. Bhargava)

This is to certify that the above statement made by the candidate is correct to the best of our knowledge.

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To my wife Pushpa

ABSTRACT

Irrigation canals are generally constructed in a deep pervious alluvium and are largely unlined. These canals alongwith their conveyance system are a major source of seepage from them which recharges the aquifer. The continuous recharging of aquifer leads to the rise of ground water table in the influence area of the canals. In order to plan and design an irrigation canal network or to manage efficiently an existing system of canals, a reasonably accurate assessment of seepage losses from canals is essential. When canals run continuously with a constant discharge, a steady state of seepage may be attained in course of time. However, generally the canals run in roster or with varying discharges, and the water table positions may vary with time due to which a steady state condition of seepage may not be reached at all. In a number of major canal systems in the state of Uttar Pradesh (India), parallel canals by the side of the existing main canals have been constructed to augment supplies in the command area. Since the parallel canals largely run intermittently the seepage from such canal systems would remain in unsteady state.

The canal systems may be constructed in regions

where the existing water table is too deep, or the canal may be located at high elevation, such that the canal is not hydraulically connected with the aquifer. In general, however, the irrigation canals are located in pervious alluvium with shallow water table positions and the canals are hydraulically connected with the aquifer.

At present analytical solutions of the unsteady seepage from parallel canals are not yet available. In the persent thesis a study has been made to analyse the unsteady seepage from a canal for deep as well as for shallow water table positions and it has then been extended to study the unsteady seepage from parallel canals and their interference. The analysis is based on linear theory of hydrologic system. Many complex ground water flow problems have been solved based on this theory. Ground water hydrology is a quantitative science and mathematics is its important dialect. Discrete kernel approach is comparatively new in its ambit. The discrete kernels are the properties of a linear system. Using the discrete kernel response functions, unsteady seepage from canals has been studied in this thesis.

From the study of a single canal, which is not hydraulically connected with the aquifer, and which runs intermittently, it is found that there is no reversal of flow in consequence of intermittent running of the canal.

For two parallel canals, which are not hydraulically connected with the aquifer, their interferrence relates to evolution of water table only. In case of two identical parallel canals, it has been found that in the beginning of recharge, two distinct water mounds are formed below the centre of each canal. With lapse of time, the points of maximum rise move towards each other; but they do not move beyond the respective recharging strip. With further lapse of time, a stagnant zone gets created between the canals, and the region between the two parallel canals takes the shape of a plateau. It has been found for the case of unequal parallel canals that, sometime after the onset of recharge, only one point of maximum rise under the larger canal is established.

When a canal is hydraulically connected with the aquifer, the seepage losses decrease with time. In the case of parallel canals, the seepage from each canal will be further reduced because of interference of one canal on the other one. The interference is the decrease in seepage loss of a canal due to the presence of the parallel canal. It has been taken as the difference between the seepage losses from a canal when it runs alone and when it runs alongwith the parallel canal. The seepage loss from canal may be linearly or non-linearly dependent on the potential difference between the canal and the aquifer. It has been found that for very shallow water table position below the canal bed the linear relationship can be used. However, as the potential difference between the canal and the aquifer increases or the width of canal increases, the non-linearity gets pronounced. The study of interference of parallel canals has been done assuming a linear relationship between the seepage and the potential difference. It is found that Herbert's formula for reach transmissivity is appropriate for use in canal-aquifer interaction studies.

The study of unsteady seepage from two parallel canals, when the water table is located at shallow depth below the bed of the canals, has been carried out for equal and unequal canals which run continuously. The study has been extended for the case in which one of the canals runs intermittently. It has been found that in case of two continuously running parallel canals, the reduction in seepage from one canal due to interference of the other is zero in the beginning of seepage. The interference increases with time, attains a maximum value, and then decreases. The decrease in inteference is monotonic at large time. The interference of parallel canals is found to decrease with increase in the spacing between the canals. For unequal parallel canals, the inteference of bigger canal on smaller canal is more than that of the smaller canal on the bigger one. If one of the parallel canals runs intermittently, it is found that the reduction in seepage from the continuously running canal, due to interference of the intermittently running canal, starts from zero, increases from cycle to cycle, reaches a maximum value, and then decreases. Also, the intermittently running canal in the parallel canal system acts as a drain during its closure period after a few cycles of running. It has been seen that in case of two continuously running equal parallel canals a stagnant zone is formed between the canals with lapse of time.

From the study of interference of two parallel canals, when one canal is situated on a high ridge and the other in the valley having hydraulic connection with the aquifer, it is found that interference of ridge canal increases with time and that reversal of flow to the valley canal is mainly controlled by the dimension of the ridge canal and its distance from the valley canal.

When there is a natural drainage in the vicinity of the canals it will influence the water table evolution and carry away part of the recharge after it gets activated. In the present thesis a solution has been given to find the time of activation of the drain located in the vicinity of two parallel ridge canals running continuously. Temporal variation of the return flow to the drainage channel has also been quantified.

It is hoped that this study will be helpful in understanding the interference of seepage of parallel canals.

NOTATIONS

The following notations have been used in this thesis. In chapter 2, which deals with review of literature, original notations have been used.

S.No.	Notation	Description	Dimension
1	a	Half width of the recharging strip	L
2	B	Width of canal at water surface	L
3	B_1, B_2	Width of the first (left) canal and width of the second (right) canal respectively, at the water surface	L
4	B_3	Width of drain	L
5	C_2, C_3	Constants appearing in the non-linear relationship between recharge rate and difference of potentials at the peri-phery of the canal and in the aquifer under the bed of canal	L^{-1}
6	D	Centre to centre distance between two parallel canals	L
7	D_i	Depth to impervious stratum measured from a high datum	L
8	D_1	The distance from the centre of the first ridge canal to the centre of the drainage channel	
9	D_2	The distance from the centre of the second ridge canal to the centre of the drainage channel	L
10	D_b	Depth to bed of channel from high datum	L
11	D_{b1}, D_{b2}	Depth to bed of the first and the second canal respectively measured from high datum	L

S.No.	Notation	Description	Dimension
12	$d_{r\rho}$	The distance from the centre of the r^{th} reach to the centre of the ρ^{th} reach	L
13	e	Saturated thickness of the aquifer below the bed of the canal	L
14	E	Saturated thickness of the aquifer	L
15	Erf(X)	Error function of X	
16	H	Depth of water in the canal	L
17	$\bar{H}$	Saturated thickness of the aquifer	L
18	$\bar{h}$	Weighted mean depths of saturation during the period of recharge	L
19	h(0,n)	Potential in the aquifer under the bed of the canal during time period, n.	L
20	h_o	Initial water table height	L
21	h_r	Ground water potential at the canal perimeter	L
22	K	Hydraulic conductivity of the aquifer material	$L\,t^{-1}$
23	l_r	Length of a canal reach	L
24	n,r,m,M	Time - steps	t
25	Q	Seepage rate per unit length of canal	L^2t^{-1}
26	q	Recharge rate per unit length of the line/strip source	L^2t^{-1}
27	Q_1, Q_2	Seepage loss from unit length of the first and the second canal respectively	L^2t^{-1}
28	$Q_2(\rho,\gamma)$	Seepage during γ^{th} unit time-step from the ρ^{th} reach of the second canal	L^3t^{-1}

S.No.	Notation	Description	Dimension
29	$Q_1(\rho,\gamma)$	Seepage during γ^{th} unit time step from ρ^{th} reach of first canal	L^3t^{-1}
30	$Q_{1_r}(n)$	Recharge rate from a canal reach of length l_r at various times	L^3t^{-1}
31	R	Number of identical canal reaches	L
32	r_r	Radius of the equivalent semi circular section of the canal appearing in the Herbert's formula for reach transmissivity	L
33	$S_1(r,n)$	The drawdown at the end of the n^{th} unit time step under the r^{th} reach of the first canal measured from a high datum	L
34	$S_2(r,n)$	The drawdown at the end of n^{th} unit time-step under the r^{th} reach of the second canal	L
35	$S(x,t)$	Rise in water table height at co-ordinate x and at time, t	L
36	T	Transmissivity of the aquifer	L^2t^{-1}
37	t	Time	t
38	W	Half width of the channel in the equation of Dillon and Liggett	
39	w	Uniform recharge rate of the strip source per unit area	$L\ t^{-1}$
40	W_p	Wetted perimeter of the canal section	L
41	x,y	Cartesian coordinates	L^2t^{-1}
42	α	T/ϕ	L^2t^{-1}
43	$\Gamma(r,n)$	Reach transmissivity of the r^{th} reach of the first canal during the n^{th} unit time step	L^2t^{-1}
44	Γ_r	Constant of proportionality between the exchange flow rate and difference in potentials at the canal and aquifer	L^2t^{-1}

S.No.	Notation	Description	Dimension
45	$\Gamma_2(r,n)$	Reach transmissivity of the r^{th} reach of the second canal during n^{th} unit time - step	L^2t^{-1}
46	$\delta(n)$	Discrete kernel coefficient	$L/(L^3/t)$
47	$\delta[r,1,r,1, B_1(\gamma), (n-\gamma+1)]$	Discrete kernel coefficient for water table rise at the end of $(n-\gamma+1)^{th}$ unit time - step under the r^{th} reach of the first canal in response to unit recharge during time - step γ from the r^{th} reach of the first canal when the canal width was $B_1(\gamma)$	$L/(L^3/t)$
48	$\delta(r,1,\rho,1,M)$ $\rho \neq r$	Discrete kernel Coefficient for water table rise at the end of M^{th} unit time - step under the r^{th} reach of the first canal in response to unit recharge during the first time period from the ρ^{th} reach of the first canal	$L/(L^3/t)$
49	$\delta(r,1,\rho,2,M)$	Discrete kernel coefficient for water table rise at the end of M^{th} unit time - step under the r^{th} reach of the first canal in response to unit recharge during the first unit time period from the ρ^{th} reach of the second canal	$L/(L^3/t)$
50	$\delta(r,2,\rho,1,M)$	Discrete kernel coefficient for water table rise at the end of M^{th} unit time - step under the r^{th} reach of the second canal in response to unit recharge during first time - step through the ρ^{th} reach of the first canal	$L/(L^3/t)$
51	$\delta(r,2,\rho,2,M)$ $\rho \neq r$	Discrete kernel coefficient for water table rise at the end of M^{th} unit time - step under the r^{th} reach of the second canal in response to unit recharge during first unit time period from the ρ^{th} reach of the second canal	$L/(L^3/t)$

S.No.	Notation	Description	Dimension
52	ϕ	Storage coefficient of the aquifer	
53	$\sigma_1(r,n)$	The drawdown to water level in the r^{th} reach of the first canal measured from the high datum	L
54	$\sigma_r(n)$	Drawdown of the water level in the canal measured from the high datum	L
55	$\sigma_2(r,n)$	The drawdown to water level in the r^{th} reach of the second canal measured from the high datum	L
56	ρ	Number of canal reaches	

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(D.N.Bhargava)

CONTENTS

CHAPTER 1

INTRODUCTION

Canals continue to be the major conveyance system for delivering water for irrigation in most parts of the world. The main canals, in irrigation projects, are designed keeping in view the water availability and the irrigation requirements in their command areas. It has been seen that in some canal systems in northern India, the capacities of main canals fall short of irrigation water requirements for paddy and sugarcane crops in the command areas during the months of May to October. Since, during this period, additional water is available in the rivers, from which the canals offtake, parallel canals have been constructed along the existing canals to augment supplies to the respective command areas. The irrigation canals, specially the parallel canals, generally run intermittently. Therefore, in most cases, the seepage from such parallel canal systems remains in unsteady state.

Estimation of seepage from irrigation canals and assessment of its impact on groundwater regime are required for a rational water resources management. The importance is particularly noticeable when large unlined canals are constructed in alluvial soil. Steady state seepage from a canal for various boundary conditions, that are encountered in nature, has been analysed by a number of investigators. The literature on steady seepage from canal and its impact on groundwater regime have been documented by Muskat (1946), Harr (1962), Polubarinova-Kochina (1962), Shestakov (1965), Bouwer (1969), Kovacs (1981) etc. The computations of the unsteady free seepage have been made by few authors. Averjanov (1950) has expressed the time dependent flow rate of seepage as a product of the water loss under a free steady state ($t=\infty$) and a factor greater than unity which has been determined analytically and by experiments. Dillon and Ligget (1983) have studied the stream acquifer system using a two dimensional numerical model based on boundary integral equation method. Hantush (1967) has derived a solution for the evolution of water table due to recharge from a rectangular source. If the dimension of length is increased to a very large value, the solution will correspond to that for a canal. However the solution involves numerical integration. Shestakov (1965) has tabulated special functions, using numerical integration, for determining water table rise due to recharge from a strip source of infinite length. Glover (1974) has analysed the evolution of water table due to recharge from a line source but has not considered the width of the recharging body in the analysis.

In many situations the water table below canal bed may be at a shallow depth and the canal may be hydraulically connected with the aquifer. The time variant recharge to ground water in such a situation is proportional to the difference of water levels in the canal and aquifer. Aravin and Numerov (1965), Streltsova (1974) and Morel-Seytoux et al.(1979), have assumed the exchange flow rate between stream and aquifer to be linearly dependent on the potential difference between the stream and the aquifer. The coefficient of proportionality depends on stream bed characteristics and aquifer parameters,and is recognised as reach transmissivity. Complex problems of stream aquifer interaction have been analysed by Morel- Seytoux and Daly (1975) who have used the reach transmissivity concept and discrete kernel theory. Non-linear relationships between flow to the aquifer and the potential difference between stream and aquifer have been proposed by Rushton and Redshaw (1979) which appear to give a fair representation to the true relationships. However, analytical treatment of seepage which considers nonlinear relationship is yet to be developed.

Analytical solutions for interference of two parallel canals are not available so far. In the present thesis, seepage from two parallel canals has been studied for unsteady state. Mathematical models have been developed for determining time variant recharge and temporal and spatial variations of water table, due to seepage from two parallel canals, for different boundary conditions and locations of water table.

Scheme of the work carried out in the present thesis is given below:

(i) In chapter 2, a review of literature has been presented pertaining to steady and unsteady seepage from canal, and evolution of water table for deep and shallow water table positions. The literature review also includes the reach transmissivity concepts given by various investigators and non-linear relationship between stream and hydraulically connected aquifer.

(ii) A study has been made in chapter 3 to analyse the evolution of water table due to recharge from a canal which has no hydraulic connection with the aquifer. In the analysis, the width of the canal and depth of water in it have been presered and a closed form solution for evolution of water table has been obtained. The analysis is based on Glover's basic solution of Boussinesq's equation for a line source of negligible width and method of superposition. The results have been compared with those of Shestakov. Results have been presented for increase in the saturated thickness of the aquifer at various distances from the canal due to its long and continous running. Making use of the above solution, the response functions of the system, which is linear, have been determined for a unit pulse excitation. Using the unit pulse response functions, the evolution of water table due to intermittent running of the canal has been studied. The analysis has been extended to find the evolution of water table due to seepage from two parallel canals for their different spacings and widths. The loci of points of maximum rise of water table have been determined for continuous running of parallel canals.

(iii) The unsteady seepage from a canal, which is nonlinearly dependent on the potential difference between the canal and aquifer, has been studied in chapter 4, making use of stream aquifer interaction model proposed by Morel- Seytoux. The non-linear relation proposed by Rushton and Redshaw (1979) has been used. The parameters involved in the nonlinear relationship have been derived analytically. The recharge rates pertaining to non-linear and linear relationships have been compared.

(iv) In chapter 5 the interference of seepage of two parallel canals, which are hydraulically connected with the aquifer, has been investigated using reach transmissivity concept and discrete kernel theory. The rise of water table at different times and locations along a transverse section of the parallel canals has been evaluated for different spacings and bed widths of the canals, and for various initial positions of water table below the bed level of the canals. Results of interference of parallel canals have been presented when one of the canals runs intermittently and the other runs continuously.

(v) In chapter 6, the study of interference of two parallel canals has been carried out when one of the canals is situated on a high ridge and the other is at much lower elevation. The higher canal is not hydraulically connected with the aquifer, whereas, the lower canal is hydraulically connected with the aquifer and its seepage rate is assumed to be linearly dependent on the difference of potentials in the lower canal and in the aquifer underneath. A mathematical model, for seepage studies from the two parallel canals, has been developed and the time variant seepage from the lower canal has been quantified for various spacings of the two canals. The temporal and spatial variations of water table due to seepage from the two canals have been determined. The model predicts the time at which the seepage from the lower canal, having hydraulic connection with the aquifer, reduces to zero. Thereafter, the lower canal starts functioning as a drain. The analysis presented quantifies the temporal variation of water entering into the lower canal.

(vi) The effect of a drainage channel on evolution of water table due to recharge from two parallel canals which are not hydraulically connected with the aquifer has been studied in chapter 7. The drainage channel activates when the water table rises above its bed level. The time at which the drain starts functioning has been determined and the drainage rate at various times has been quantified.

(vii) The general conclusions drawn from the studies are summarised in chapter 8.

CHAPTER 2

REVIEW OF LITERATURE

2.0 Introduction

A literature review of steady and unsteady seepage from canals in homogeneous and isotropic porous medium with water table either at shallow or at large depth has been made in this chapter. The aim of the present thesis is to analyse unsteady seepage from parallel canal system. Since solution of a single canal and aquifer interaction problem forms the basis for solving an aquifer and parallel canals interaction problem, literature review of seepage from a single canal has been carried out and included in this chapter.

2.1 Steady seepage from canals

Steady state seepage from a canal, when the water table is at large depth has been analysed by a number of investigators for various boundary conditions. Kozeny (1931) has shown that, if the shape of a ditch or canal conforms to the equation $[x- \frac{Q}{\pi K} .\cos^{-1}(y/H)]^2 + y^2= H^2$, where Q is the seepage rate, x and y are cartesian coordinates with origin at the centre of water surface, and H is the maximum depth of water in the canal, the maximum width of sheet of water seeping down into the porous medium is equal to (B+2H); B being the width of canal at water surface. According to Kozeny, the seepage quantity from such a canal is K(B+2H), where, K is the coefficient of permeability.The result holds good if the porous medium is of very large thickness so that the seeping water can maintain its vertical downward movement indefinitely. This requirement prohibits the applicability of the solution to cases where the ground water table is at shallow depth. Muskat (1946) has compared the values of seepage discharge for three different shapes of canals and quantified that the extreme variation in seepage,due to the effect of shape of canal or ditch,is about 10 percent. Wedernikov (1937) obtained an exact solution for seepage from channels of triangular and trapezoidal shape with ground water table at infinite depth. Solutions for various other simplified flow geometries were also obtained by Risenkampf (1940). Morel-Seytoux (1964) applied hodograph techniques, Schwarz - Christoffel transformations, and the Green - Neumann function to obtain solutions of seepage for the condition when water table is at large depth for channels of different geometry including shapes deviating from the standard rectangular, trapezoidal,and triangular cross sections. Jeppson (1968) solved the problem of seepage from canal to an underlying pervious stratum by the method of finite differences. All these derivations assume the water table to be at infinite depth.From the studies of these authors it is seen that, the variations in seepage discharge due to changes in shape of canal are of small order.

Seepage from canal in uniform soil, with shallow water table merging with the canal water surface, has been analysed by Dachler (1933). He derived a procedure in which both model experiments and an approximate analysis were combined for computing the seepage from a trapezoidal channel, embedded in a porous medium of finite depth, to a fully penetrating vertical drain at some distance away from the channel. Analysis of seepage from leveed rivers into the low lying adjoining lands was carried out by Todd and Bear (1961) with an electrical analog.

A detailed study was made by Bouwer (1965) to determine how seepage from canals or streams, in theory, is affected by the shape and depth of water in the channel, by the position of the groundwater table, and by the soil conditions. The multitude of soil conditions, that may be encountered in practice, were reduced for this study to three basic groups, i.e., (i) the channel is in uniform soil which is underlain by much more permeable material, designated as condition A; (ii) the channel is in uniform soil which is underlain by much less permeable material, designated as condition B, and (iii) the channel is surrounded by a thin, slowly permeable (clogged) layer along its wetted perimeter, designated as condition C. The case of seepage to a free draining, permeable layer is a special case of condition A and it is termed condition A'. For condition A, A', and B, solutions of steady state seepage systems were obtained with an electrical resistance network analog. The results were expressed in dimensionless graphs showing seepage in relation to the position of the ground-water table for different depths to the permeable or impermeable layer, and different water depths in the canal. The canal was taken as trapezoidal with 1:1 side slopes. The graphs showed that the effect of a permeable or impermeable layer on seepage becomes rather small when this layer lies below the channel bed at depth more than 5 times the bottem width, W_b, of the channel. Thus, soil explorations for new canals do not need to go deeper than 5 W_b below the projected bottom elevation. The graphs also showed that seepage rates increase with increasing depth to the groundwater table, but at a decreasing rate. If the water table at a distance of 10 W_b from the channel centre is at a depth more than 2.5 times the width of the channel at the water surface, the corresponding seepage is close to that which would occur if the water table is at infinite depth. For condition C, an equation is presented which gives the seepage as a function of the geometry of the channel (triangular, trapezoidal, or rectangular), the hydraulic impedance of the slowly permeable (clogged) layer, and the pressure condition in the unsaturated underlying material as determined by the unsaturated hydraulic conductivity characteristics of that material.

Analyses regarding the effect of channel shape on seepage showed that, for a given surface width and depth of the water in the channel, seepage increases from a triangular to a trapezoidal and from trapezoidal to a rectangular cross section. For most conditions, this increase is only moderate and less than the corresponding increase in hydraulic discharge capacity of the channel. Therefore, for a certain width and depth of the water, rectangular channels have lower relative water losses due to seepage than trapezoidal or triangular channels. An exception to this rule may be condition A' if the permeable drainage layer is at very small distance below the channel bottom.

According to Bouwer, the seepage from open channels increases with increasing water depth in the channel. Also the discharge in the channel increases with increasing water depth in the channel, if the flow is uniform.For all three soil conditions, however, the rate of increase in seepage was less than the rate of increase in discharge. Therefore, canals with uniform flow and uniform soil conditions along the wetted perimeter become more efficient conveyor of water with increasing water depth in the canal.

In addition to the above works, the literature on steady seepage from canal and its impact on ground water regime has been extensively documented by Muskat (1946), Harr (1962), Polubarinova Kochina (1962), Aravin and Numerov (1965), Bouwer (1969), Bear (1972), Kovacs (1981), Verrujit (1982), and Huisman and Olsthoorn (1983). The evaluation of seepage losses from parallel canals evoked attention of investigators quite late. It was not until 1960 that Hammad (1960) examined the problem of seepage from parallel canals. The analysis given by him was for a system consisting of a number of identical and equally spaced parallel canals having no particular geometrical shape. Hammad did not find solution for the loci of the free stream lines in his analysis; but instead assumed the free surface to be horizontal between canals. He also assumed the canals to be constructed in a semipervious clay layer of finite thickness, underlain by a free permeable layer of sand and gravel, in which the piezometric head is very near the canal water level. The vertical lines of symmetry between the canals were stream lines and, therefore, were replaced by impervious boundaries in the analysis to simplify the problem. Ahmed El Nimr (1963) considered the canal section to be trapezoidal and derived a solution for the unconfined flow problem by using inverse hodograph method. Bruch and Street (1967) studied the seepage from an infinite array of parallel triangular channels. The array of canals were assumed to be underlain by a drainage layer at a finite depth. In this study, the shape of the free surface was obtained from the analysis by using conformal mapping technique and inverse hodograph method allowing fully for the possibility of point of inflection along the free stream line. Charmonman (1967) studied the seepage flow from parallel canals with intermediate drains in coastal aquifers. The solution for the set of mathematical model of the flow pattern was obtained by numerical methods. Sharma and Chawla (1974) analysed the seepage from a canal into a drain which is parallel to the canal alignment. The analysis was made by making use of Zhukovsky's function and Schwartz-Christoffel transformation.

2.2 Unsteady seepage from canals

The literature review on unsteady seepage from canals for deep and shallow water table positions has been presented separately since the phenomena of flow in the two cases are quite different from each other.

2.2.1 Unsteady seepage from canals when water table is at large depth

When the water table is at large depth, the canal is not hydraulically connected with the aquifer, and the seepage from canal is independent of the location of water table. In such a situation, there are three main aspects of the flow process, viz., the movement of water through the unsaturated zone till it reaches the deep water table, recharge to the aquifer after the wetting front reaches the water table, and evolution of water table after the onset of recharge.

A simplified model which describes the flow from a rectangular water body to deep water table through a unsaturated zone has been given by Abdulrazzak and Morel-Seytoux (1983). The flow process envisaged by Morel-Seytoux is depicted in Fig. (2.1). If water becomes available in the basin at time zero, infiltration will proceed. Though the wetting front is shown as a line in Fig. (2.1), Morel-Seytoux (1985) has stated that zone of seperation between the originally dry soil and the significantly wetted soil has some thickness, as shown in Fig. (2.2). The water content

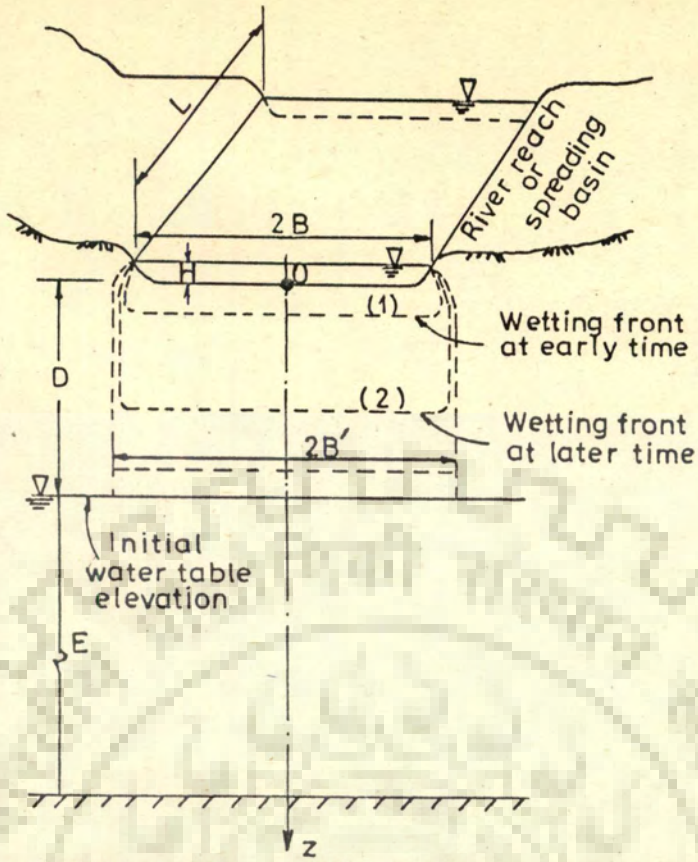

Fig. 2.1-Time evolution of the wetting front progression during infiltration from a riverbed or a recharge basin

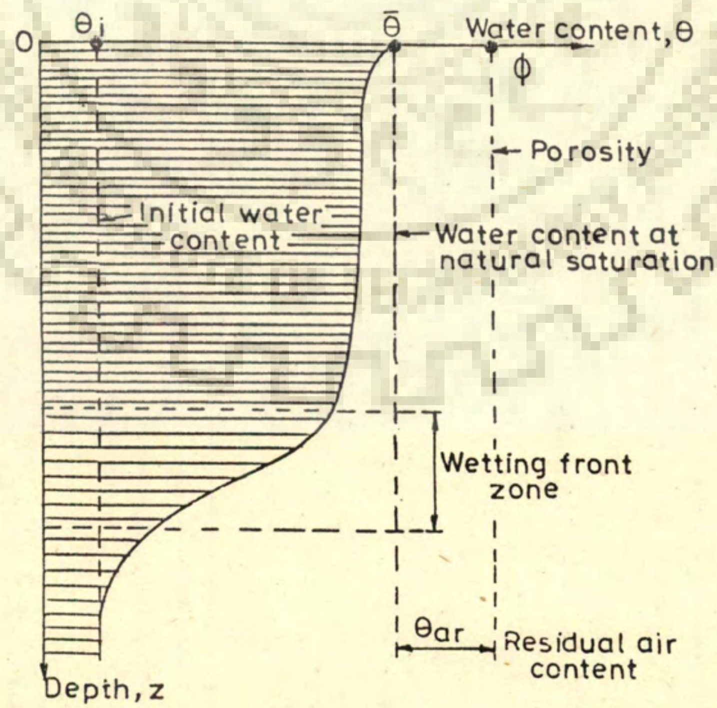

Fig.2.2-Variation of water content with distance from water source

behind the wetting front need not be at its highest possible value; but it is essentially constant behind the front for most of the profile. As the front propagates downward, it also spreads laterally. As it nears the water table, the width of the front is 2 B' [Fig (2.1)], somewhat greater than 2 B, the width of the basin. According to Morel-Seytoux and Khanji (1974), until the wetting front reaches the water table, the infiltration rate can be represented adequately by a modified Green Ampt equation given below:

$$I_{(t)} = \frac{K\{(\vartheta - \vartheta_1)[H(t) + H_c] + w(t)\}}{w(t)} \qquad \ldots(2.1)$$

where,

$H(t)$ = depth of water above soil,

$w(t)$ = cumulative volume of infiltration expressed as depth,

K = hydraulic conductivity at normal saturation,

H_c = effective capillary drive,

ϑ = water content at natural saturation, and

ϑ_1 = initial water content.

According to Abdul-razzak and Morel-Seytoux (1983), the seepage rate from a canal is not the recharge rate at the water table at all time. With the wetting front position between the canal bed and initial water table position, and for initially dry soil, the seepage rate varies in time, but the recharge rate is constant and zero. Even after hydraulic connection is established, unless the soil column between river bed and initial water table position is saturated, the two rates will be different. When the depth of water table from canal bed is large, by the time the wetting front reaches the saturated zone it travels at a velocity approximately equal to the saturated hydraulic conductivity, K. Bouwer (1969) states that for the case of seepage from a canal for the shape of channel given by Kozeny, the vertical downward flow and maximum width of the flow system are essentially reached at a depth of 1.5 (B+2H) below the bed of the canal. The rate of recharge at the time the wetting front meets the ground water table is equal to the seepage rate and remains constant equal to K(B+2H).

A time delay, t_d, for recharge to reach the water table after the onset of seepage flow from a stream has been obtained by Dillon and Ligett (1983) based on the Green and Ampt equation. The delay is given by:

$$t_d = \frac{\varphi}{K} \{y_m - D_i - (h_s-h_c).Log[(h_s-h_c+y_m-D_i)/(h_s-h_c)]\} \quad ...(2.2)$$

Where,

φ = effective porosity,

h_s = head in the stream,

y_m = elevation of the semi-permeable blanket,

h_c = capillary potential or suction head at the base of blanket,

K = hydraulic conductivity at saturation, and

D_i = the initial saturated aquifer thickness.

The flow of water from the canal through the unsaturated zone till it reaches the water table is not under the scope of the present study. The evolution of water table after the onset of recharge has been dealt in the present work. Therefore, a literature review pertaining to the research work on evolution of water table due to recharge from strip source, for a deep water table position, has been presented in the following paragraphs:

Theis (1935) has given the following equation for water table rise due to a unit impulse recharge from a vertical line source:

$$S(x,t) = \frac{1}{4\pi T} \frac{e^{-r^2/4\beta t}}{t} \quad \ldots(2.3)$$

in which, r is the distance between the point of recharge and the point of observation, and $\beta = T/\varphi$; T being the transmissivity of the aquifer and φ the storage coefficient. This solution is based on an analogous solution for conduction of heat in solids given by Carlslaw (1921). Making use of the above equation, Polubarinova - Kochina [1951, (Vide Polubarinova-Kochina, 1962)] has derived the expression for water table rise due to a line source and extended it to strip source. If unit impulse recharge per unit length is taking place from a line source, the water table rise at a distance, x, from the line source is given by:

$$S(x,t) = \frac{1}{4\pi T} \int_{-\infty}^{+\infty} \frac{e^{-(x^2+y^2)/4\beta t}}{t} \, .dy$$

This expression reduces to:

$$S(x,t) = \frac{1}{2\sqrt{(Tt\varphi\pi)}} \, e^{-x^2/4\beta t} \qquad \ldots(2.4)$$

If the recharge is taking place continuously through a strip source of width 2R, the expression of rise of water table has been obtained using convolution technique (Polubarinova - Kochina, 1951). The expression derived by Polubarinova - Kochina is given by:

$$S(x,t) = \int_{0}^{t} \frac{1.\, d\zeta}{2\sqrt{(T\pi\varphi)}\sqrt{(t-\zeta)}} \int_{-R}^{R} w[e^{-(x-\xi)^2/4\beta(t-\zeta)}]d\xi \qquad \ldots(2.5)$$

in which, w is the recharge rate per unit area of the strip source. After integration, Polubarinova - Kochina derived the expressions for water table rise below and beyond the recharging strip. The expressions derived by Polubarinova-Kochina (1962) have been given in the text book "Theory of Ground Water Movement", by P. Ya. Polubarinova - Kochina, translated from Russian by I.M. Roger De Wiest, and also in the book, "Dynamic Hydrology", by Eagleson (1970). The expressions given in the translated version of the book of Polubarinova - Kochina contains several printing errors. In the expression given in Eagleson's book, a part of the solution is missing.A complete derivation of water table evolution has been presented in Chapter 3.

Hantush (1967) has developed approximate expression for the rise and fall of the water table in an infinite unconfined aquifer in response to uniform recharge from a rectangular spreading basin. Hantush described the water table rise by the following expression:

$$h^2 = h_o^2 + \frac{(w\bar{h}t)}{2\varphi}\{F[(a+x)/2\surd(K\bar{h}t/\varphi),(b+y)/2\surd(K\bar{h}t/\varphi)]$$
$$+F[(a+x)/2\surd(K\bar{h}t/\varphi),(b-y)/2\surd(K\bar{h}t/\varphi)]$$
$$+F[(a-x)/2\surd(K\bar{h}t/\varphi),(b+y)/2\surd(K\bar{h}t/\varphi)]$$
$$+F[(a-x)/2\surd(K\bar{h}t/\varphi),(b-y)/2\surd(K\bar{h}t/\varphi)]\} \quad ...(2.6)$$

in which,

$\bar{h}$ = weighted mean of the depth of saturation during the period of flow (a constant of linearization), 2a, 2b are dimensions of the rectangular strip, w = constant rate of percolation, K is the coefficient of permeability, φ is the storage coefficient, h_o is the initial depth of saturation, and,

$$F(p,q) = {}_0\int^1 \mathrm{Erf}(p/\sqrt{\zeta}).\mathrm{Erf}(q/\sqrt{\zeta})d\zeta \qquad ...(2.7)$$

The following expression for height of water table due to recharge from a canal can be derived from Equation (2.6) given by Hantush.

$$h^2(x,t)-h_o^2 = \frac{w\bar{h}t}{2\varphi}\{2\,{}_0\int^1 \mathrm{Erf}[\frac{a+x}{2\sqrt{(K.\bar{h}t\zeta/\varphi)}}]\,d\zeta$$

$$+ 2\,{}_0\int^1 \mathrm{Erf}[\frac{a-x}{2\sqrt{(K\bar{h}t\,\zeta/\varphi)}}]\,d\zeta\} \qquad ...(2.8)$$

The solution of the above equation involves numerical integration.

Bianchi and Muckel (1970) have presented the water table rise due to recharge from a square basin. The results of mound height have been presented in the non-dimensional form for different non-dimensional time.

Shestakov (1965) has tabulated special functions numerically for determination of water table rise due to recharge from a strip source of width 2b and of infinite length. The expression for rise of water table given by Shestakov is as follows:

$$s(x,t) = wt/\varphi\,.F_w(\tau) \qquad ...(2.9)$$

in which, $s(x,t)$ = rise in water table at a distance, x, from the centre of the strip,

w = uniform percolation rate,

φ = storage coefficient, and

t = time reckoned from the onset of recharge.

For the centre of the strip (x=0), $F_w = F_w(\tau_o)$;

for $x < b$,

$$F_w = [F_w(\tau_x') + F_w(\tau_x'')]/2, \qquad ...(2.10)$$

and for $x > b$,

$$F_w = [F_w(\tau_x') - F_w(\tau_x'')]/2 \qquad ...(2.11)$$

$$\tau_x' = \frac{Tt}{\varphi(b+x)^2}, \quad \tau_x'' = \frac{Tt}{\varphi(b-x)^2}$$

$$\tau_o = T\tau/(\varphi . b^2)$$

The special functions tabulated by Shestakov are given in Table (2.1). It is evident from the table that for values of τ which are not given in table, the value of special function has to be interpolated.

Assuming the canal to be a line source, as given in Fig. (2.3), Glover (1974) analysed the evolution of water table due to recharge from a canal. The rise in water table at a distance x at time t, s(x,t), has been derived by Glover as given below:

$$s(x,t) = \frac{q\sqrt{4\pi\alpha t}}{2\pi KD}\left(\frac{x}{\sqrt{4\alpha t}}\right) \int_{\left(\frac{x}{\sqrt{4\alpha t}}\right)}^{\infty} \frac{e^{-u^2}du}{u^2} \qquad ...(2.12)$$

Table (2.1) - Shestakov's special functions for evaluation of water table rise due to recharge from a strip source.

τ	$F_w(\tau)$	τ	$F_w(\tau)$
0.00	1.000	14.0	0.268
0.05	1.000	16.0	0.253
0.10	0.994	18.0	0.240
0.20	0.963	20.0	0.229
0.30	0.924	25.0	0.207
0.40	0.884	30.0	0.190
0.50	0.849	35.0	0.178
0.60	0.818	40.0	0.167
0.70	0.790	50.0	0.151
0.80	0.764	60.0	0.138
0.90	0.741	70.0	0.128
1.00	0.720	80.0	0.120
1.50	0.639	90.0	0.113
2.00	0.583	100.0	0.108
2.50	0.537	156.3	0.089
3.00	0.503	200.0	0.078
4.00	0.451	278.0	0.062
5.00	0.413	400.0	0.054
6.00	0.384	625.0	-
7.00	0.361	2500.0	-
8.00	0.342	10000.0	-
9.00	0.325	-	-
10.00	0.311	-	-
12.00	0.287	-	-

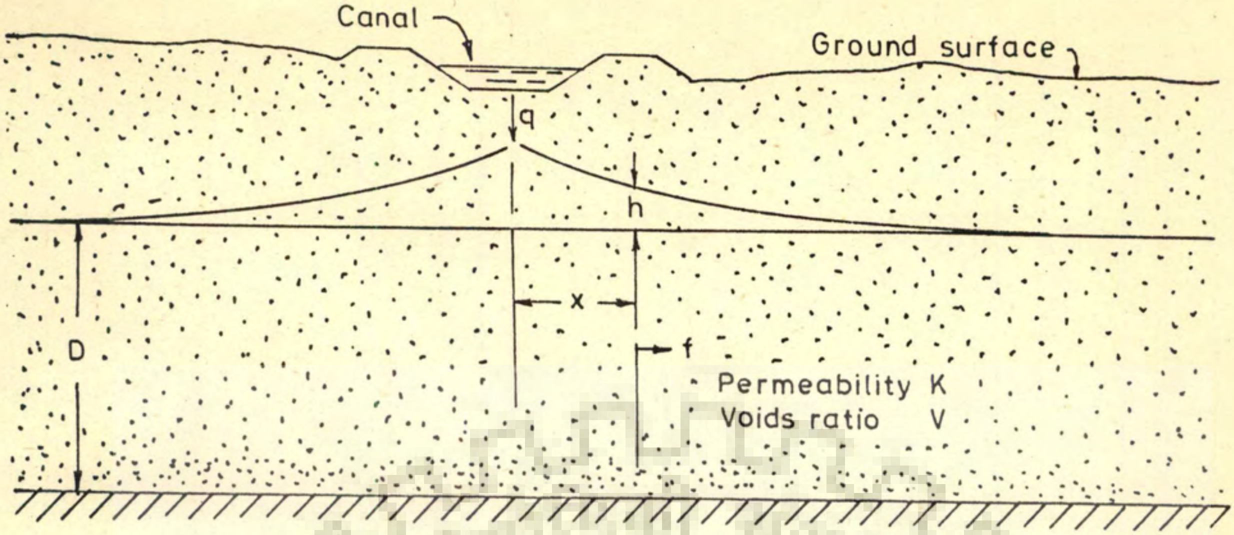

Fig.2.3–Line source

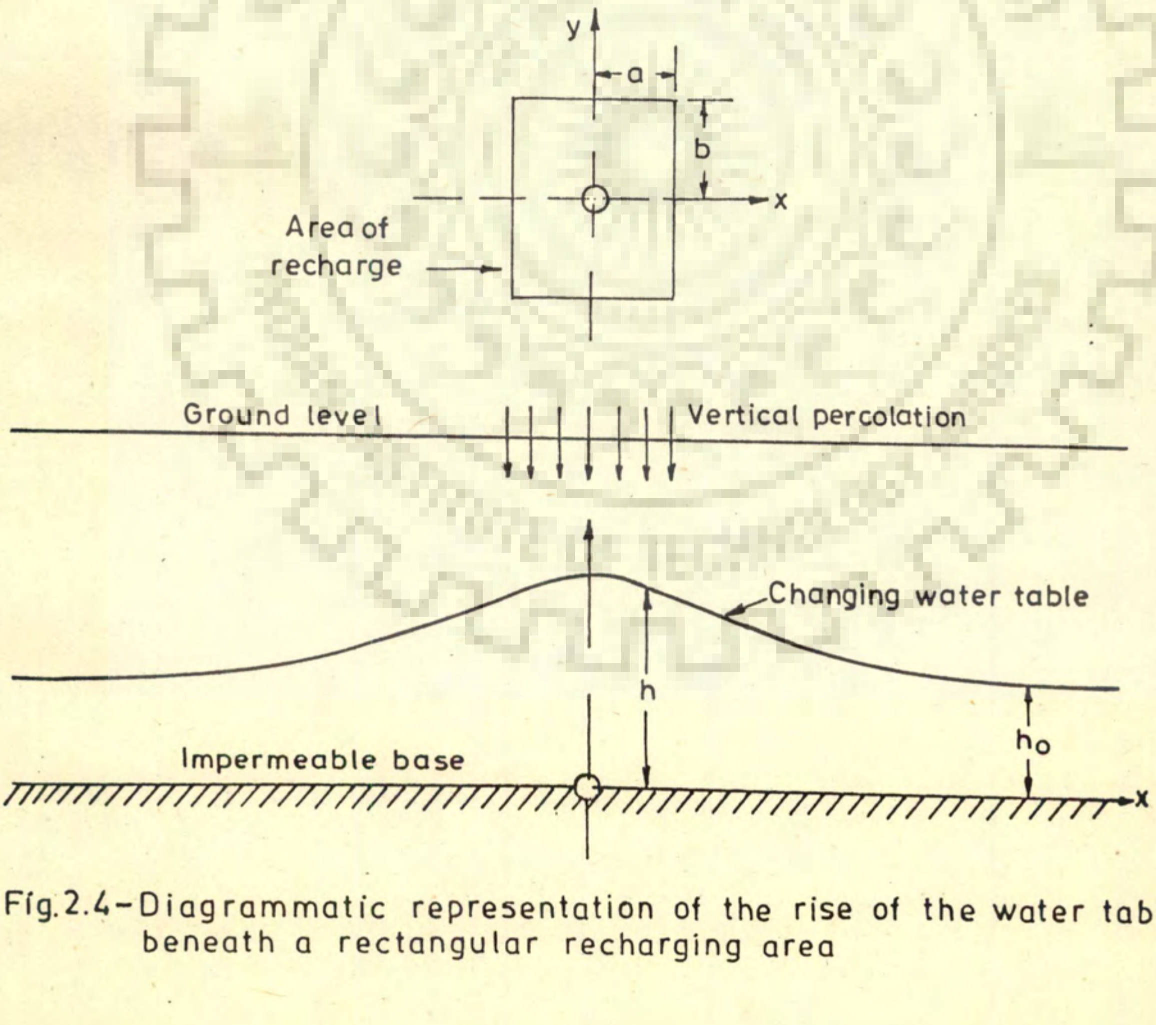

Fig.2.4–Diagrammatic representation of the rise of the water table beneath a rectangular recharging area

The integration appearing in Equation (2.12) has been evaluated by Glover as given below:

$$\int_{\left(\frac{x}{\sqrt{4\alpha t}}\right)}^{\infty} \frac{e^{-u^2}du}{u^2} = -\frac{e^{-u^2}}{u} - \int \frac{2ue^{-u^2}du}{u} \Bigg]_{\left(\frac{x}{\sqrt{4\alpha t}}\right)}^{\infty}$$

$$= \frac{\sqrt{4\alpha t}\; e^{-\left(\frac{x^2}{4\alpha t}\right)}}{x} - \sqrt{\pi} + \sqrt{\pi}\frac{2}{\sqrt{\pi}} \int_0^{\left(\frac{x}{\sqrt{4\alpha t}}\right)} e^{-u^2} du$$

The final expression derived by Glover is as follows:

$$s(x,t) = \frac{q\sqrt{(4\pi\alpha t)}}{2\,\pi\,KD} \cdot \frac{x}{\sqrt{(4\alpha t)}} \left[\frac{\sqrt{(4\alpha t)}e^{-\frac{x^2}{4\alpha t}}}{x} - \sqrt{\pi} + \sqrt{\pi}.\mathrm{Erf}\left(\frac{x}{\sqrt{(4\alpha t)}}\right)\right] \qquad \ldots(2.13)$$

The response of the aquifer to a unit impulse excitation in the form of a rectangular water table mound has been analysed by Moench and Kisiel (1970). The use of the response functions has been shown through convolution technique for finding out the behaviour of the aquifer for variable recharge rate.

A ground water recharge system has been analysed by Marino (1975) in which the recharging area is rectangular in shape as shown in Fig. (2.4). The unconfined aquifer, receiving the recharge, is assumed to be homogeneous, isotropic, and resting on a horizontal impermeable base. The aquifer parameters are assumed to be constant in time and space. The constant rate of recharge is considered to be small in comparison to the hydraulic conductivity. It is assumed that the vertically downward recharge is almost completely refracted in the direction of the slope of the water table. Marino obtained the numerical solution of the non-linear partial differential equation characterising the flow in the unconfined aquifer receiving the recharge, by an alternating direction implicit procedure with an inhomogeneous grid spacing. The method employs two different equations which are used in turn over successive time - steps. The first equation is implicit only in the x direction, and the second equation is implicit in the y direction. According to Marino, the method of solution is such that a change in spacing can be easily introduced at any stage of the progressing calculation. The numerical solution obtained by Marino has been compared with linearised solution given by Hantush for a similar flow system under identical conditions. The linearised solution by Hantush gives results that are smaller than those obtained by alternating direction implicit procedure. The maximum relative deviation found by Marino was two percent.

Rao and Sarma (1980) have derived solutions for rise of water table due to recharge from a strip source in an unconfined aquifer of finite lateral extent. The solutions to the non - linear differential equation were obtained using two different methods of linearisation, those of Baumann (1952) and Hantush (1967). The flow equation has been solved using Laplace transform techniques.

According to Rao and Sarma, both the linearisation procedures were found to yield results which have satisfactory agreement with the experimental results (within ±5 percent) upto 40 percent rise of the water table. Beyond this limit, the linearisation procedure suggested by Hantush gave a more satisfactory agreement. However, the Hantush's procedure involves computations of average height of the water table through successive iteration.

2.2.2 Unsteady seepage from canals when water table is at shallow depth

When the water table is at shallow depth, the canal gets hydraulically connected with the aquifer and the recharge rate is influenced by the change in water table position. The recharge rate in such situations is to be treated as time variant. The merger of the wetting front with the underlying saturated zone at shallow depth involves a complicated two dimensional transient flow pattern that has not received rigorous analyses,except through the use of numerical modelling techniques.The experimental and numerical studies carried out by Vauclin et al. (1979) deal with the development of a numerical model for predicting the response of a shallow water table to infiltration from the soil surface,taking into account the transfer of water in the unsaturated zone.The equations used is for a unified saturated-unsaturated flow system. It has been pointed out by Vauclin et al. that the tranfer of water through the unsaturated zone,in predicting the recharge of a water table aquifer, should not be neglected. According to them, from practical point of view, the most important items to know are the time of transfer and the volume of recharge joining the aquifer at a given time.

A simplified model has been proposed and verified by Abdulrazaak and Morel-Seytoux (1983) in which the influence of water table on recharge rate has been considered; an expression for time variant recharge rate to aquifer has been arrived at; and water table evolution has been determined for the derived recharge rate. Abdulrazzak and Morel - Seytoux (1983) used the flow net approach given by Muskat (1946) to obtain an approximate two dimensional recharge rate, q(t), which can be expressed by Darcy's law as:

$$q(t) = \frac{K \bar{A} \Delta h}{\bar{L}} \qquad ...(2.14)$$

where,

$\bar{A}$ is the average cross sectional area of the flow tubes,

Δh is the piezometric head drop and $\bar{L}$ is the average length of the flow tubes for the flow domain illustrated in Fig. (2.5). $\bar{A}$, $\bar{L}$ and Δh are given by:

$$\bar{A} = \frac{1}{2} \{W+[E+h(o,t)]\} \qquad ...(2.15)$$

where,

E is the initial saturated thickness of the aquifer, W is half width of the wetting front, and h(o,t) is the deviation of the ground water mound height from the initial water table elevation at the boundary.

$$\bar{L} = \{[D+E+W]+[D-h(o,t)]\}/2$$

$$= \frac{2D + E + W - h(o,t)}{2} \qquad ...(2.16)$$

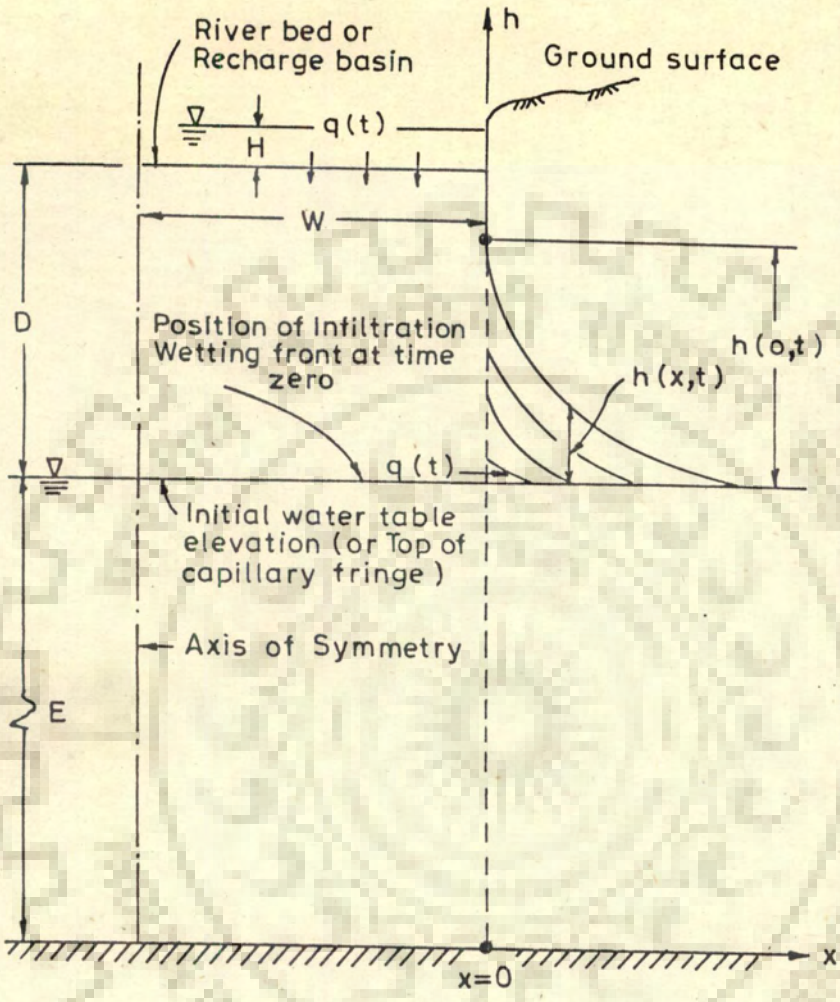

Fig.2.5- Evolution of the groundwater mound during recharge

in which, D is the depth from bed of river to initial water table position.

$$\Delta h = H+D-h(o,t)$$

where,

H is the depth of water in the river.

By substituting the values of $\bar{A}$, Δh and $\bar{L}$ in Equation (2.14), the approximate formula for recharge rate has been obtained as below:

$$q(t) = K\frac{[E+W+h(o,t)]}{[2D+E+W-h(o,t)]}.[H+D-h(o,t)] \qquad ...(2.17)$$

The recharge rate q(t) is also expressed as:

$$q(t) = q_o[1+\frac{h(o,t)}{E + W}].[1-\frac{h(o,t)}{2D+E+W}]^{-1}[1-\frac{h(o,t)}{D + H}] \qquad ...(2.18)$$

in which, q_o is the recharge rate at t = 0.

Abdulrazzak and Morel - Seytoux (1983) have verified the above solution by laboratory experiments and have concluded that as long as the ratio of river width over depth to water table (2 W/D) exceeds 2 and the ratio of initial saturated thickness over depth to water table (E/D) exceeds 2.5, there is good agreement between theoretical and observed results of recharge with time.

In the present study, a unified flow model encompassing both the unsaturated and saturated zone has not been considered. In the following paragraphs literature review of those studies, which do not consider the flow in the unsaturated zone while predicting the recharge from canal or water body to shallow water table, has been carried out.

Liggett and Dillon (1985) have analysed a stream aquifer interaction problem using boundary integral equation method (BIEM). They plotted the rise in water table due to recharge from two sets of parallel water courses, located on either side of a dry stream bed. Results have been presented depicting the time at which the stream activates, the rate at which the stream receives water thereafter, and the variation of seepage with time from the two sets of parallel water courses. The evolution of water table has also been presented. The results of the study given by Liggett and Dillon are shown in Figs. (2.6) and (2.7).

The analysis of unsteady recharge from river to aquifer has been dealt by Morel -Seytoux (1973), Morel - Seytoux and Daly (1975), Morel - Seytoux (1975), and Illangasekare and Morel - Seytoux (1982). An efficient yet accurate hydrologic model on the interaction between river (canal)and the alluvial aquifer has been developed by Morel - Seytoux and Daly. The flow from the river to the aquifer has been assumed to be linearly dependent on

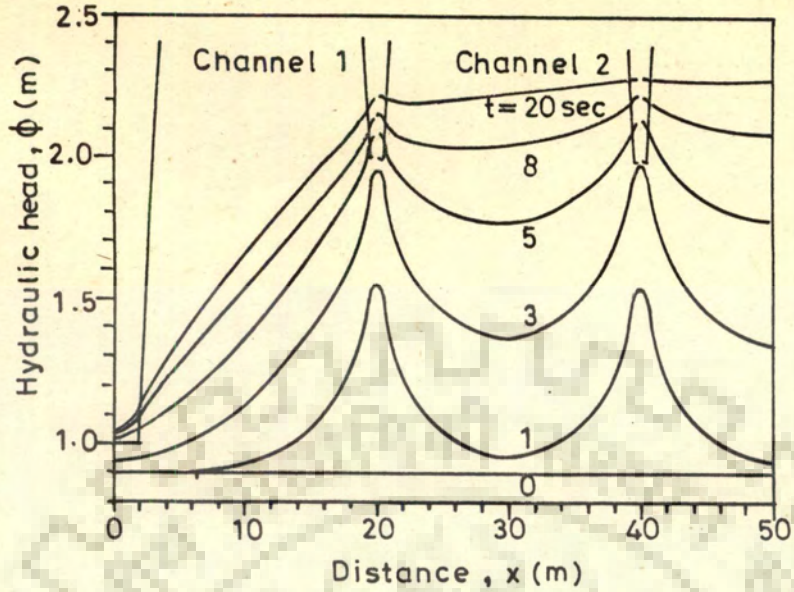

Fig.2.6 - Free surface profile, including hydraulic head beneath the water courses at various times

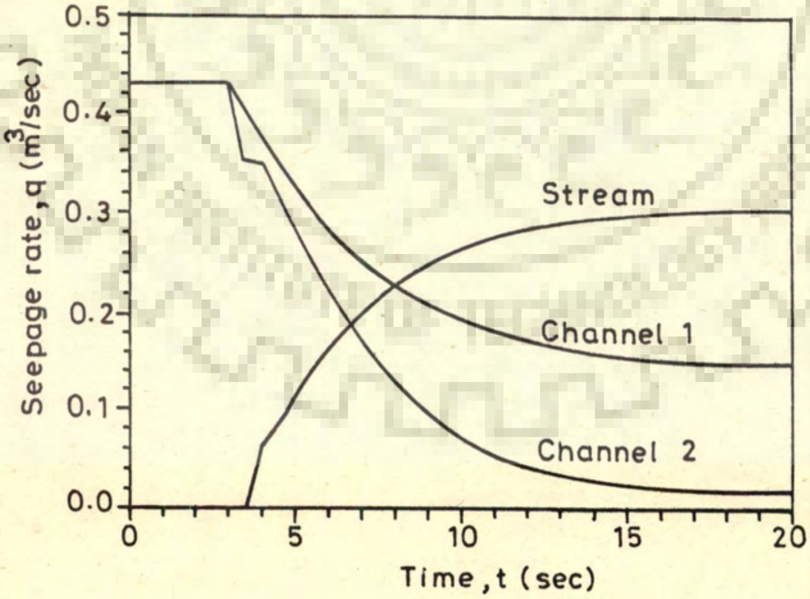

Fig.2.7 - Seepage rate variation with time for each watercourse

the difference of potentials at the periphery of the river and in the aquifer near the river. The following relation has been used by Morel - Seytoux :

$$Q_r(n) = \Gamma_r[\sigma_r(n) - S_r(n)] \qquad ...(2.19)$$

in which,

Γ_r = the constant of proportionality known as reach transmissivity,

$\sigma_r(n)$ = draw down of the water level in the river reach during n^{th} time period measured from a high datum, and

$S_r(n)$ = drawdown of the water table in the aquifer measured from the same datum in the vicinity of the river.

Making use of the equation, and relating the value of S_r (n) to all the excitations in the aquifer, such as pumping from the aquifer, q(t), and the return flow from the aquifer to the river, Q_r (t), the following integral equation, when return flow is coming to one single reach, has been given:

$$Q_r(t) + \Gamma_r \int_o^t Q_r(\zeta)k_{rr}(t-\zeta)\, d\zeta$$

$$= \Gamma_r[\sigma_r(t) - \sum_{p=1}^{P} \int_o^t q_p(\zeta)k_{rp}(t-\zeta)d\zeta] \qquad ...(2.20)$$

in which, P is the number of pumping wells.

For the case of several river reaches,

$$Q_r(t) + \Gamma_r \sum_{\rho=1}^{R} {}_o\int^t Q_\rho(\zeta) k_{r\rho}(t-\zeta) d\zeta$$

$$= \Gamma_r [\sigma_r(t) - \sum_{p=1}^{P} {}_o\int^t q_p(\zeta) k_{rp}(t-\zeta) d\zeta] \qquad \ldots(2.21)$$

where R, is the number of reaches. Equation (2.21) is a system of R integral equations to be solved simultaneously.

Discretising the time span into uniform time - steps and assuming that the excitations are uniform within each time - step but vary from time - step to time - step, the Equation (2.21) has been simplified to the form:

$$Q_r(n) + \Gamma_r \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n} \delta_{r\rho}(n-\gamma+1)\, Q_\rho(\gamma)$$

$$= -\Gamma_r \sum_{p=1}^{P} \sum_{\gamma=1}^{n} \delta_{rp}(n-\gamma+1) q_p(\gamma) + \Gamma_r \sigma_r(n) \qquad \ldots(2.22)$$

where δ_{rp}, δ_{rr} and $\delta_{r\rho}$ are given by:

$$\delta_{rp}(n) = {}_o\int^1 \frac{1}{4\pi T(n-\zeta)} . e^{R_{rp}^2/[4\beta(n-\zeta)]} . d\zeta$$

$$= \frac{1}{4\pi T} [E_1(\frac{R_{rp}^2}{4\beta n}) - E_1\{\frac{R_{rp}^2}{4\beta(n-1)}\}] \qquad \ldots(2.23)$$

in which, R_{rp} is the distance between the centre of r^{th} reach and p^{th} well,

$$\delta_{r\rho}(n) = {}_0\int^1 \frac{1}{4\pi T(n-\zeta)} e^{R^2_{r\rho}/[4\beta(n-\zeta)]} d\zeta$$

$$= \frac{1}{4\pi T}[E_1(\frac{R^2_{r\rho}}{4\beta n})-E_1\{\frac{R^2_{r\rho}}{4\beta(n-1)}\}] \qquad ...(2.24)$$

where, $R_{r\rho}$ is the distance between the centre of the r^{th} reach and ρ^{th} reach, and,

$$\delta_{rr}(n) = {}_0\int^1 \frac{1}{\varphi.a.b} \operatorname{erf}[\frac{a}{4\surd[\beta(n-\zeta)]}].\operatorname{erf}[\frac{b}{4\surd[\beta(n-\zeta)]}]d\zeta$$

in which, a and b are length and width of the r^{th} reach respectively,and erf( ) is the error function. R number of equations similar to Equations (2.22) can be written and the R number of unknown return flows at any time - step n can be solved in succession starting from time - step 1.

2.3 Reach transmissivity constant

For solving unsteady state stream-aquifer interaction problem, the use of reach transmissivity has been introduced by Morel - Seytoux and Daly (1975). The reach transmissivity has been defined as the constant of proportionality between the return flow to river and the difference of potentials at the periphery of the river and in the aquifer in the vicinity of the river.

The constant of proportionality has been obtained analytically by various investigators, e.g., Hammad (1959), Ernst (1962) Aravin and Numerov (1965), Bouwer (1965 a), Herbert (1970) and Streltsova (1974), for different aquifer and river geometry. According to Muskat (1946), and Bouwer (1969), an unsteady state can be treated as a succession of steady states.The validity of this assumption has been reasoned out by Muskat in detail [Muskat (1946), pp.621 - 625]. Based on the above principle, the reach transmissivity constant, though has been derived on the assumption of steady flow condition, has been used for analysis of unsteady state problems by Morel - Seytoux (1975 a, 1975 b, 1975 c, 1975 d, 1975 e). The reach transmissivity constant derived by various investigators for different canal and aquifer geometry has been reviewed in the following paragraphs:

The geometry of a channel constructed in an aquifer of finite depth which is underlain by an impermeable layer is shown in Figure (2.8). The channel is hydraulically connected with the aquifer. For a specific case in which the channel is rectangular and the bottom of the channel extends to the impermeable layer, the seepage loss is given by (Bouwer, 1965 b).

$$Q = 2K(H_w - 0.5\ D_w)/(L - 0.5\ W_b)\ D_w \qquad \ldots(2.25)$$

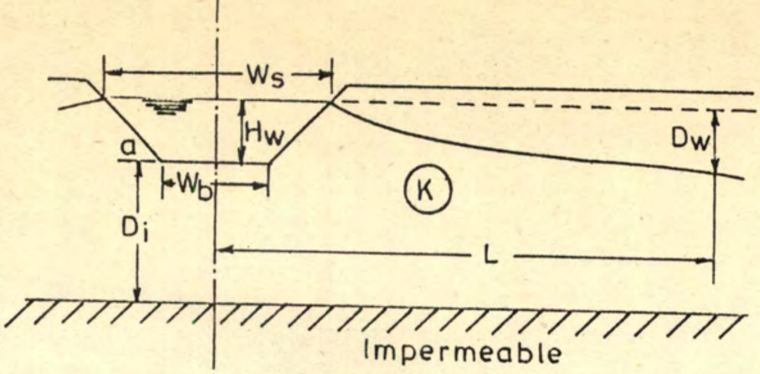

Fig.2.8-Geometry for channels in soil underlain by impermeable material

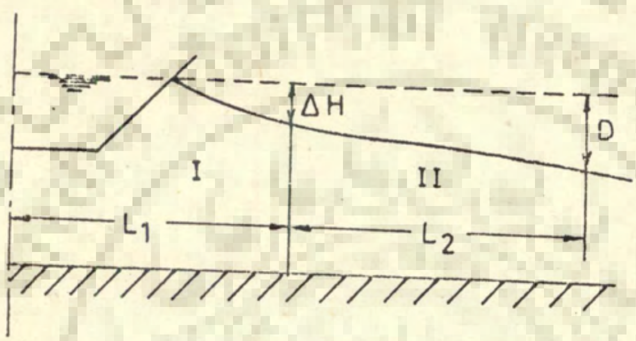

Fig.2.9-Division of flow system in regions I and II for Dachler's analysis

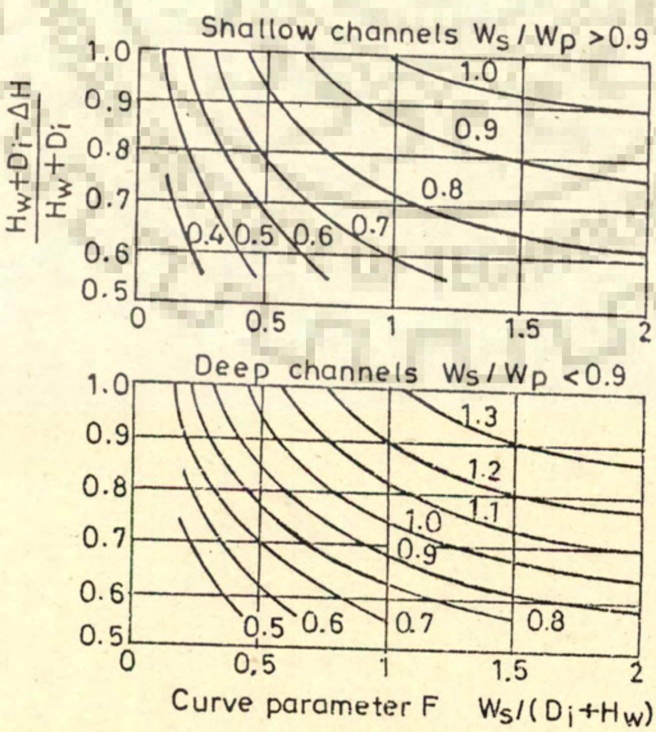

Fig.2.10- Dachler's values of F for shallow and for deep channels

The reach transmissivity for a fully penetrating canal of reach length L_r is, therefore, given by:

$$\Gamma_r = 2\,KL_r(H_w - 0.5\,D_w)/(L - 0.5\,W_b) \qquad \ldots(2.26)$$

L can be regarded as the distance of the observation well where the drawdown 'D_w' is observed.

An approximate expression for seepage from a partially penetrating channel shown in Figure (2.8) is given by (vide Bouwer, 1969)

$$Q = K2(H_w + D_i - 0.5\,D_w)/(L - 0.25\,W_b - 0.25\,W_s)D_w \ldots(2.27)$$

Hence, the approximate expression for reach transmissivity for a canal conforming to the configuration depicted in Figure (2.8) is,

$$\Gamma_r = 2KL_r(H_w + D_i - 0.5D_w)/(L - 0.25W_b - 0.25W_s) \ldots(2.28)$$

According to Bouwer (1969), the above expression is not exact and the error in Γ_r will increase with increasing D_i. The error in Equation (2.28) is due to the curvature and divergence of the stream lines in the vicinity of the channel.

Dachler (1936) had divided the flow system on the basis of model studies into a region with curvilinear flow (region I) and the other with Dupuit Forchheimer flow (region II) [Fig.(2.9)], the dividing line being at a distance, L_1, from the centre of the canal, where,

$$L_1 = \frac{W_s + H_w + D_i}{2} \qquad \ldots(2.29)$$

The flow in region I was analysed with an approximate equation for the potential and the stream line distribution under a plain source of finite width. A factor 'F' has been determined to estimate flow in region I as:

$$Q_I = 2 \cdot F \cdot K \cdot \Delta H \qquad \ldots(2.30)$$

where ΔH is the vertical distance between the water surface in the canal and the ground water table at the dividing line between the two flow regions. Values of F given by Dachler are presented in Figure (2.10).

The flow in region II has been expressed with Dupuit Forchheimer theory as:

$$Q_{II} = \frac{2K(D_w - \Delta H)}{L_2} \, [D_i + H_w - 0.5\,\Delta H - 0.5 D_w] \qquad \ldots(2.31)$$

Since it is required to calculate the seepage for a given value of D_w at a distance $(L_1 + L_2)$ from the channel centre, ΔH will not be known initially. ΔH is found by trial&error which satisfies the condition $Q_I = Q_{II}$. The reach transmissivity for a canal reach of length L_r will be given by:

$$\Gamma_r = \frac{2KL_r}{L_2}[1-\frac{\Delta H}{D_w}][D_i + H_w - 0.5\Delta H - 0.5 D_w] \qquad \ldots(2.32)$$

Bouwer (1969) has applied Ernst's approach to analyse seepage from a canal constructed in a porous medium of finite depth underlain by an impervious layer. Following Ernst's approximate solution for potential distribution pertaining to flow to a line sink, the head loss, h_r , due to radial flow in the vicinity of the canal, has been expressed by Bouwer as:

$$h_r = \frac{Q}{\pi K} \log_e \left(\frac{D_i + H_w}{W_p}\right) \qquad ...(2.33)$$

Hence, reach transmissivity for a canal reach of length L_r is given by:

$$\Gamma_r = \pi K \, L_r / \log_e \left(\frac{D_i + H_w}{W_p}\right) \qquad ...(2.34)$$

The head loss, h_h, due to horizontal flow in the region away from the canal has been expressed by Bouwer as:

$$h_h = \frac{Q}{2K} \cdot \frac{L}{D_i + H_w - 0.5 D_w} \qquad ...(2.35)$$

Since $D_w = h_r + h_h$, Bouwer has combined Equations (2.33) and (2.35) to obtain the relation:-

$$Q = \frac{K D_w}{\frac{1}{\pi} \log_e [(D_i + H_w)/W_p] + [\frac{0.5L}{D_i + H_w - 0.5 D_w}]} \qquad ...(2.36)$$

The reach transmissivity for a canal reach of length L_r from Equation (2.36) can be obtained as:

$$\Gamma_r = \frac{K L_r}{\frac{1}{\pi} \log_e [(D_i + H_w)/W_p] + [\frac{0.5L}{D_i + H_w - 0.5 D_w}]} \qquad ...(2.37)$$

Equation (2.33) was developed for semi circular channels of radious r, where the wetted perimeter is πr. The equation according to Bouwer (1969) can be used for channels of other shapes by substituting the actual wetted perimeter as shown in the above equation. For shallow channels ($W_s >> H_w$), the seepage rate can be more accurately estimated by the following expression:

$$Q = \frac{K\pi}{\log_e[\frac{4D_i+H_w}{\pi W_s}]} \cdot h_r$$

Hence,

the reach transmissivity for a canal reach of length L_r by Ernst modified formula would be given by:

$$\Gamma_r = K\pi L_r/\log_e[4(D_i+H_w)/\pi W_s] \qquad ...(2.38)$$

Using a simple potential theory Morel-Seytoux et al (1979) have derived the following expression of reach transmissivity for a canal embedded in a porous medium underlain by an impervious layer [Fig. (2.11)]:

$$\Gamma_r = \frac{TL_r}{e} \cdot \frac{0.5W_p + e}{5\ W_p + 0.5e} \qquad ...(2.39)$$

in which,

L_r = length of canal reach,

T = transmissivity of the aquifer,

W_p = wetted perimeter of the canal, and

e = saturated thickness below the canal bed.

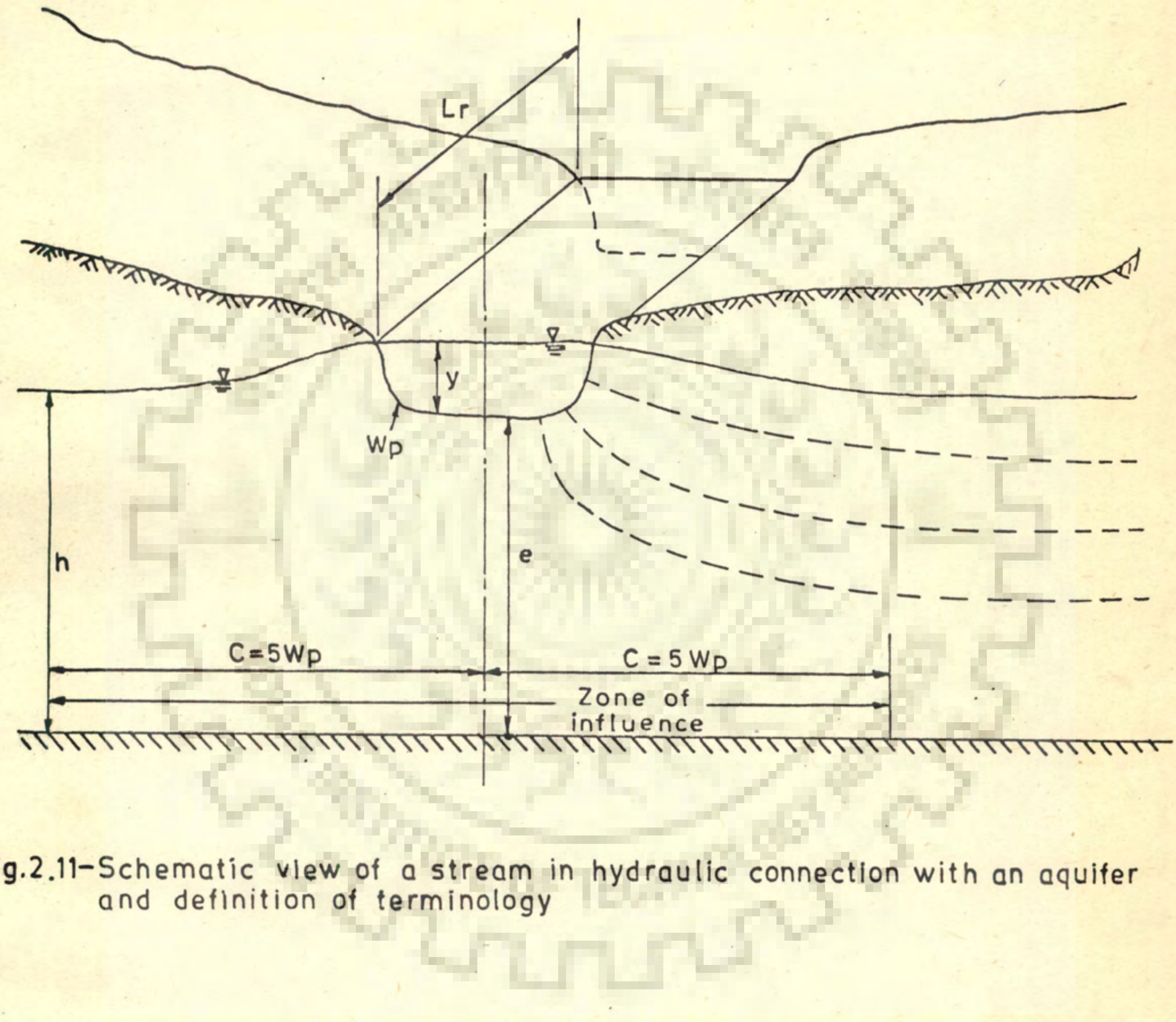

Fig.2.11–Schematic view of a stream in hydraulic connection with an aquifer and definition of terminology

Herbert (1970) has related the flow from a partially penetrating river, having semicircular cross section [Fig.(2.12)], to the potential difference between the river and in the aquifer below the river bed. The expression is given by:

$$Q_r = \pi L_r K (h_r - h_o)/\log_e (0.5m/r_r) \qquad ...(2.40)$$

in which,

L_r = length of river reach,

h_r = potential at the river boundary,

h_o = potential in the aquifer below the river bed,

m = saturated thickness of the aquifer, and

r_r = radius of the semicircular river cross section.

The reach transmissivity which could be obtained from equation (2.40) is:

$$\Gamma_r = \pi L_r K/\log_e (0.5m/r_r) \qquad ...(2.41)$$

For a rectangular channel shown in Fig.(2.13), Aravin (1965) has derived the following expression for flow to the channel:

$$Q = \frac{K(H+h)(H-h)}{L - \frac{B}{2}} + \frac{K(H-h)}{\frac{L}{2T} - \frac{1}{\pi}\log_e \sinh(\frac{\pi B}{4T})} \qquad ...(2.42)$$

The reach transmissivity for a canal reach of length L_r could be written as:

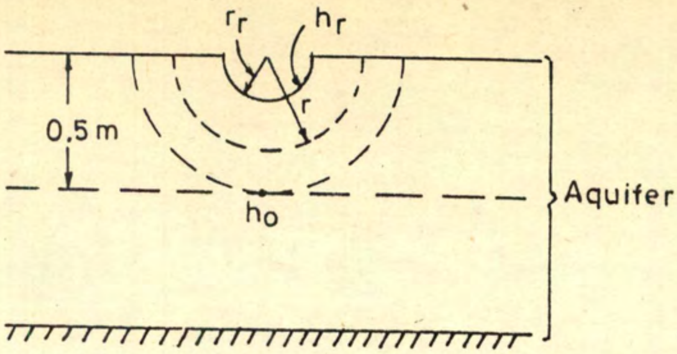

Fig.2.12-Representation of partially penetrating river

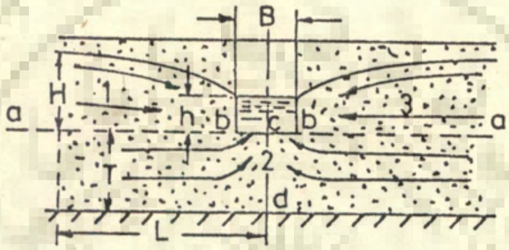

Fig.2.13-Flow to a rectangular ditch

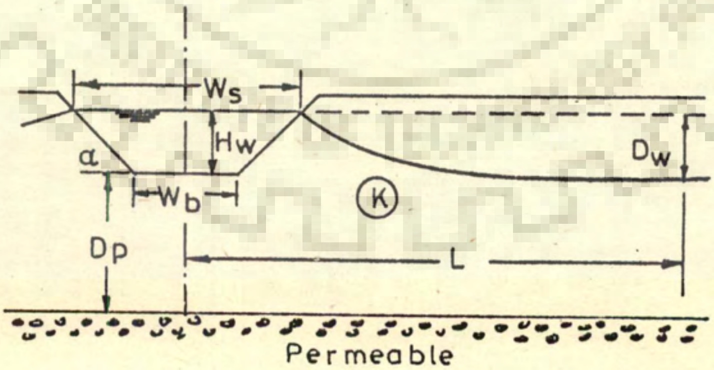

Fig.2.14-Geometry and symbols for channels in soil underlain by permeable material

$$\Gamma_r = \frac{KL_r(H+h)}{L-0.5B} + \frac{KL_r}{0.5\frac{L}{T} - \frac{1}{\pi}\log_e \sinh(\frac{\pi B}{4T})} \quad ...(2.43)$$

Seepage flow from a canal embedded in a porous medium of finite depth, underlain by a highly pervious layer, [Fig.(2.14)], has been analysed for simplified canal geometry by Hammad (1959). The analysis is valid for the situation in which the piezometric head in the underlying highly pervious layer is very near the canal water level. According to Hammad,

$$Q = KD_w \frac{2K_1}{K_1' - C} \quad ...(2.44)$$

in which,

K_1 and K_1' are the complete elliptic integral of the first kind corresponding to modulus k_1 and complementary modulus k_1' respectively. The moduli are defined as:

$$k_1 = 0.5[\frac{W_s'}{2} + (\frac{W_s'^2}{4} - 2H_w'^2)^{\frac{1}{2}}]$$

$$k_1' = (1-k_1^2)^{\frac{1}{2}}$$

The other constants are:

$$C = H_w' / k_1$$

$$H'_w = \tan\left[\frac{\pi H_w}{2(H_w+D_p)}\right], \text{ for } H_w < D_p$$

and

$$W'_s = 2 \tanh\left[\frac{\pi W_s}{4(H_w+D_p)}\right], \text{for } H_w < D_p$$

The reach transmissivity for a canal reach of length l_r can be written as:

$$\Gamma_r = KL_r\left[\frac{2K_1}{K'-C}\right] \qquad \ldots(2.45)$$

Aravin (1965) has analysed the seepage from a canal which has very shallow water depth in it. The water table lies above the highly permeable layer as shown in Fig. (2.15). The analysis has been carried out using Zhukovsky's function and conformal mapping. The seepage quantity is given by,

$$Q = K(T - H) K'_1 / K_1 \qquad \ldots(2.46)$$

in which, K_1 is the complete elliptical integral of first kind with modulus $k = \exp\{(\frac{-(b+Q/K)}{2H})\}$, K'_1 is complete elliptic integral of first kind with modulus k', where k' is given by,

$$k' = \sqrt{(1 - k^2)}$$

When k is very near to zero, the seepage rate is given by:

$$Q = K(T - H)(b + 0.882H)/T$$

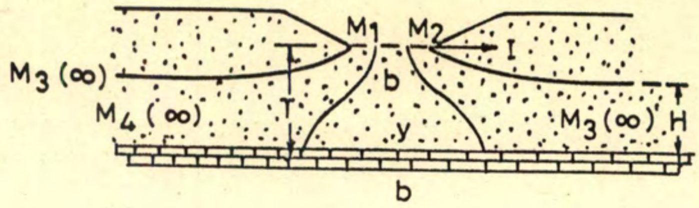

Fig.2.15- Seepage from a canal with shallow water depth embedded in a porous medium underlain by a highly permeable layer

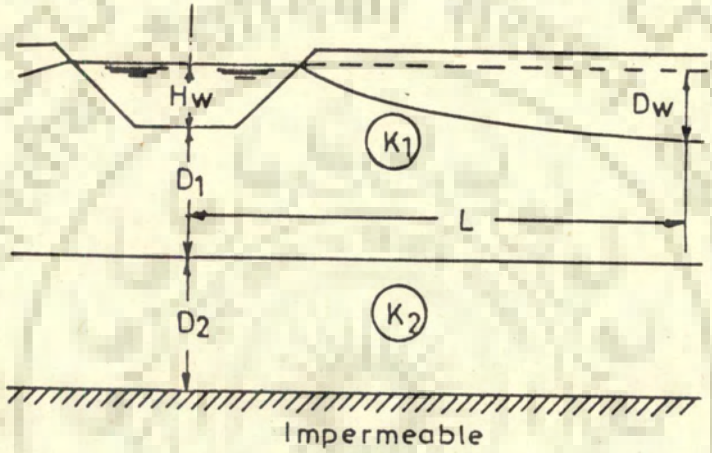

Fig.2.16—Canal in a two layered soil system

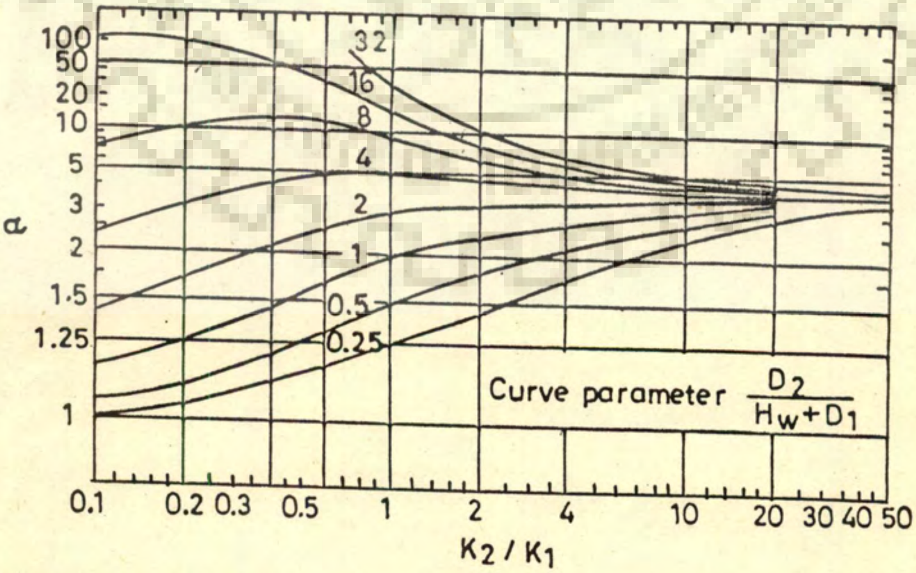

Fig.2.17- Parameter α for calculating seepage loss from a canal in a two layered soil system

Thus,

$$\Gamma_r = K L_r (b + 0.882H)/T \qquad ...(2.47)$$

The case of seepage from a canal in a two layered soil [Fig.(2.16)], underlain by an impermeable layer, has been analysed by Ernst (vide Bower, 1969). Following Ernst's solution, the reach transmissivity pertaining to a two layered soil system can be written as:

$$\Gamma_r = \frac{K_1 L_r}{\dfrac{0.5K_1 L}{K_1(D_1+H_w-0.5D_w)+K_2D_2} + \dfrac{1}{\pi}\ln\dfrac{\alpha(H_w+D_1)}{W_p}} \qquad ...(2.48)$$

in which K_1 and K_2 are permeabilities of the top and bottom layer respectively. The parameter α given by Van Beer (vide Bouwer, 1969), is shown in Fig. (2.17).

2.4 Non-linear relationship in stream aquifer interaction

The exchange flow rate between a stream and an aquifer hydraulically connected with the stream has been asasumed to be linearly proportional to the potential difference between the aquifer and the stream by various investigators as discussed above. However, there has been evidence that the process can be very non-linear [Rushton and Redshaw (1979), Dillon (1983 , 1984)]. Rushton and Redshaw (1979) have presented a typical non-linear relationship between the exchange flow rate and the potential difference between the river and aquifer.

According to Rushton and Redshaw (1979) the generalised non-linear relationships between flow to the aquifer and the potential difference between the aquifer and the river, which appear to give a fair representation, are as follows [Figs. (2.12) and (2.18)] :

$$Q = C_1(h_o-h_r)+C_2[1-e^{-C_3(h_o-h_r)}], \text{for } h_o \geq h_r \quad ...(2.49)$$

and

$$Q = 0.3\ C_2[e^{C_3(h_o-h_r)}-1], \text{for } h_o \leq h_r \quad ...(2.50)$$

where C_1, C_2, C_3 are constants which depend on field condition.

Because of the difficulty in determining the actual non- linear relationship, it has been a common practice with most of the investigators to use a linear relationship.

2.5 Conclusions

From the literature review it could be seen that study of seepage from parallel canals have been carried out for steady state condition and for simplified boundary conditions. Boundary integral equation method has been applied to study specific cases of interaction of a stream and two parallel channels of small dimensions. Though there have been evidence that the

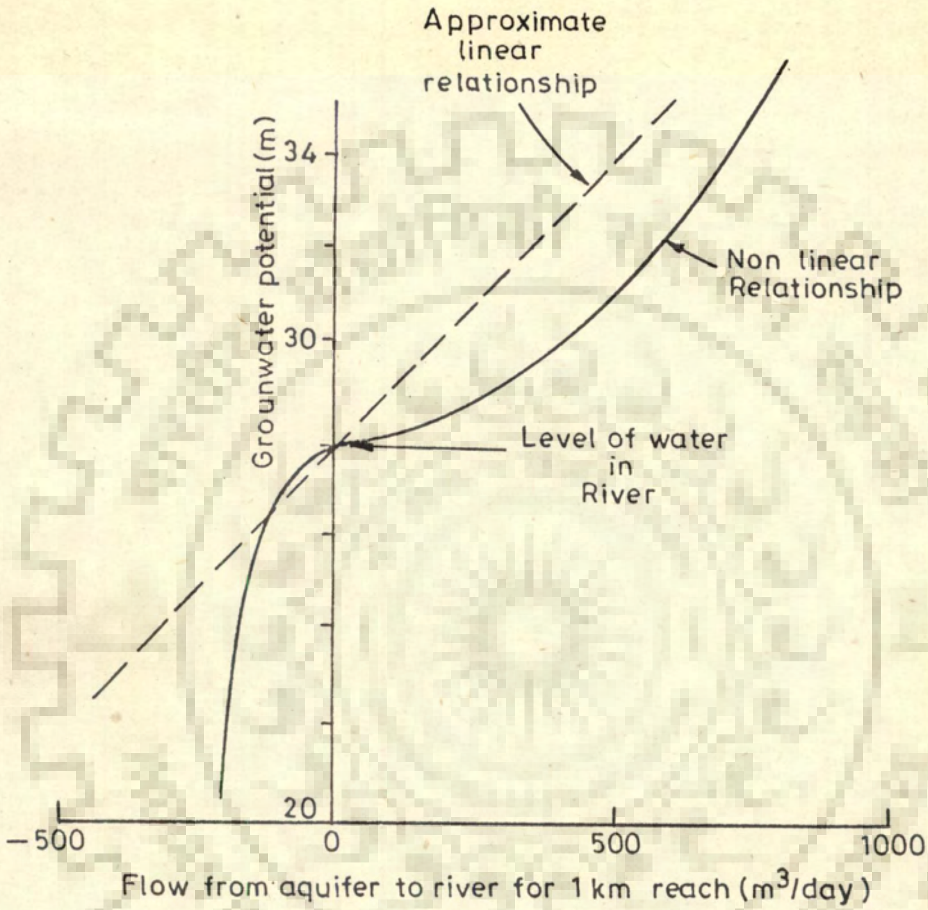

Fig. 2.18—Typical relationship between flow from aquifer to river and groundwater potential proposed by Rushton and Redshaw (1979)

exchange flow rate between the stream (canal) and aquifer is non - linearly dependent on the potential difference between them, the linear relationship is still in vogue. The studies pertaining to unsteady seepage from parallel canals and their interference are yet not available. There has not been any analytical study of unsteady seepage from stream or canal which consideres the non-linear relationship of stream aquifer interactions.

EVOLUTION OF WATER TABLE DUE TO SEEPAGE FROM TWO PARALLEL CANALS WHEN WATER TABLE IS AT LARGE DEPTH

3.0 Introduction

The process of seepage from a canal starts as soon as water is filled in it. As the time elapses, in the first stage, the soil layers around the canal get saturated. The saturated water front, in the next stage, moves slowly downwards and after a certain period of time it reaches the water table below the bed of the canal. During this downward propagation of seepage from the canal into the flow domain, the seepage water is used for the saturation of wetted zone, where the pores were previously filled with air. After reaching the water table only a part of the infiltrating water is stored within the extending saturated zone, whereas, the remaining part recharges the groundwater. The time for the seeping water to reach the water table, after the onset of seepage from the canal which is hydraulically unconnected from aquifer, can be obtained by using the Green and Ampt equation (Abdulrazzak, and Morel-Seytoux, 1983). It may be noted that seepage rate from the canal is not the recharge rate at the water table at all time. With wetting front position some where between the canal bed and initial water table position at very large depth, and for initially dry soil, the seepage rate varies in time, but the recharge rate is constant and zero. If the water content behind the wetting front is close to saturation,recharge rate rises abruptly from zero to the prevailing seepage rate at the time the saturation front encounters the water table (Abdul-razzak and Morel-Seytoux, 1983). Ultimately the recharge due to seepage from the canal results in rise of water table in the form of water mound. The study of seepage prior to the initiation of recharge to groundwater is not in the scope of the study presented.

Theoretically the steady state of seepage from a canal can develop only if the wetting front can propagate large depth without encountering the water table. For a canal having width 'B' at the water surface and maximum depth of water 'H', if the water table is lying at a depth more than 1.5 (B+2H), for all practical purposes, water table can be assumed to be at large depth (Bouwer, 1969). In practical situation, however, the water seeping down from canal will reach the normal groundwater at a finite depth and, in the process, the seepage flow gets influenced by the near horizontal flow of groundwater; thus forcing the stream lines to follow a horizontal rather than a vertical trend.

Many research workers have investigated the steady free seepage from a canal to deep water table. A number of theoretically well established results are known after Kozeny (1931), Wedernikov (1934), Riesenkampf (1938), Muscat

(1946) etc. Their investigations are equally based on the application of both hodograph and conformal mapping. The difference in their detailed analysis are, in general, due to difference in the profiles of the canal considered in their analysis. If a flow domain comprises homogeneous and isotropic material, for the shape of a canal given by Kozeny, the maximum or asymptotic width of the downwards seeping sheet of water is approximately given by (B+2H). The discharge flowing down as seepage is equal to K(B+2H), 'K' being coefficient of permeability of the medium. Small deviation in the shape of canal from that given by Kozeny, will not cause any appreciable error in the computation of seepage. The seepage from canal travelling downwards and meeting the deep groundwater table will raise a groundwater ridge from which groundwater can flow both ways. As the time passes the water table will continue to rise in the absence of drainage in the vicinity of flow domain.

In many canal systems the canal seldom carries a fixed discharge or have constant water depth in it. The discharge in a canal depends on availability of water and its demand in the command area. On account of fluctuating water level in canal, largely for the reasons mentioned above, the steady seepage condition is seldom reached. Also, in some command areas there may be two parallel canals running simultaneously or intermittently depending on requirements at various times. For instance, both the canals may run simultaneously in Kharif (Summer) crop season when adequate supplies for both the canals are available, whereas, when supplies in the river get diminished in Rabi season (Winter) only one canal may run in those periods. In all such cases the seepage from canals remains in unsteady state.

The computations of the unsteady free seepage have been dealt with by few authors. Starting from the solution given by Theis (1935) for water table rise due to a unit impulse recharge through a vertical line source, Polubarinova-Kochina (1951, vide Polubarinova Kochina - 1962) has derived the expression for water table rise due to a strip source making use of convolution technique. Shestakov (1965) has tabulated special functions by adopting numerical method to determine the rise in water table due to recharge from a strip source of infinite length. Hantush (1967) has derived an expression for rise in water table height due to recharge from a basin of finite length and width. If the dimension of length is increased to a very large value, the solution will correspond to rise in water table due to recharge from a canal. However, the solution involves numerical integration. Glover (1974) has analysed the evolution of water table due to recharge from a line source, but has not taken the width of recharge body into consideration. Rao and Sarma (1980) have obtained an analytical solution for evolution of water table rise in a finite aquifer due to recharge from strip source, using Laplace transformation technique.

In the study presented in this chapter, the evolution of water table due to recharge from a canal has been analysed when water table is at large depth. In the analysis the width of the canal and depth of water in it have been preserved. The expressions for rise of water table due to recharge from a strip source given by Polubarinova - Kochina have been rederived. The analysis is based on Glovers solution of Boussinesq's equation for a line source and method of superposition. The analysis has been extended to study the evolution of water table for an intermittently running canal and also to study the interference of two parallel canals.

3.1 Statement of the problem

Fig. [3.1(a)] shows a schematic section of a canal constructed in a homogeneous and isotropic pervious medium of infinite extent. 'B' is the width of the canal at the water surface and 'H' is the maximum depth of water in the canal. The water table is at large depth below the canal bed such that the canal is hydraulically unconnected with aquifer. The coefficient of permeability of the flow domain is 'K'. The thickness of the saturated depth of aquifer is 'E'. There is no drainage channel in the vicinity of the canal. It is required to determine the rise of water table at different locations across the canal due to its continuous or intermittent running. The rise of water table

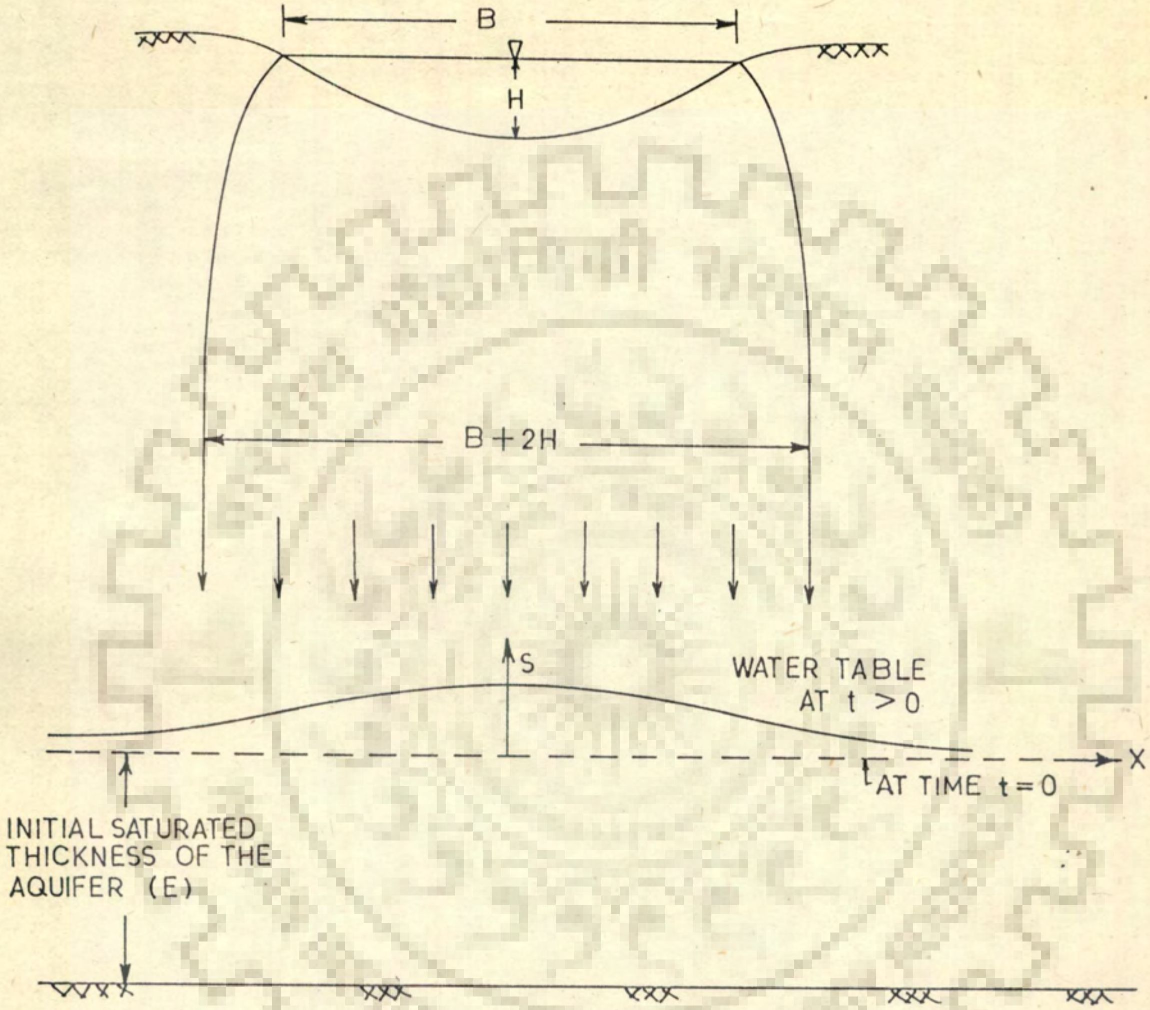

Fig.3.1 (a).– Schematic section of a canal

is to be evaluated in time, reckoned from the instant the water front reaches the groundwater table. It is also required to determine the evolution of water table due to recharges from two parallel canals located at a distance 'D' apart. A schematic section of two parallel canals has been depicted in Fig. [3.1(b)].

3.2 Analysis

3.2.1 Evolution of water table due to continuous recharge from a canal (strip source)

The assumptions made to carry out the analysis for evolution of water table are as follows :

(i) The aquifer is homogeneous, isotropic, and infinite , resting on a horizontal impermeable base.

(ii) The hydraulic properties of the aquifer remain constant with time and space.

(iii) The rate of seepage is constant with respect to time.

(iv) The flow due to seepage is vertically downward until it reaches the water table.

(v) The water table remains below the bottom of the recharging body.

(vi) The average head over the depth of saturation is approximately equal to the height of the water table above the base of the aquifer.

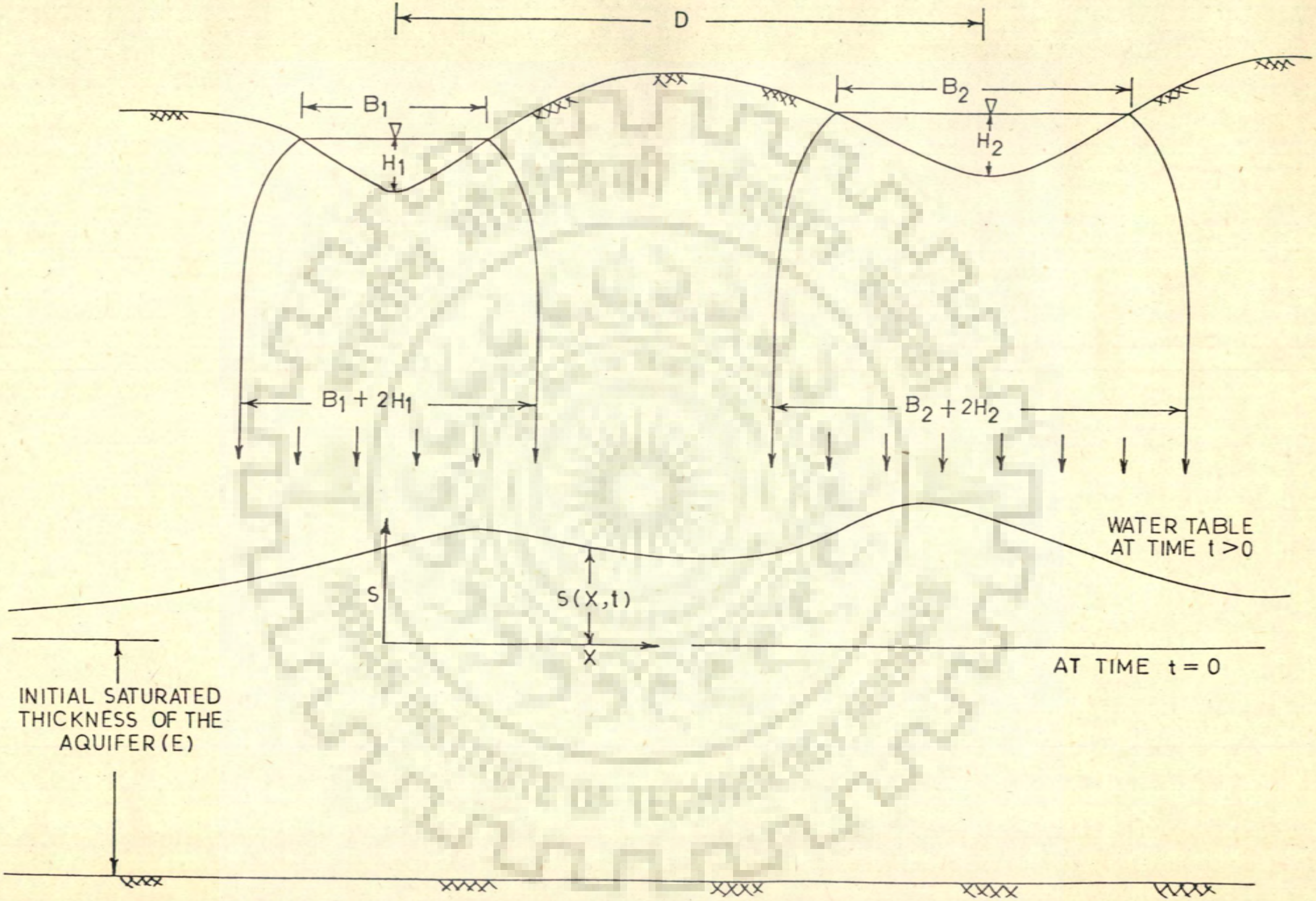

Fig.3.1 (b).—Schematic section of two parallel canals

(vii) The rise of water table relative to the initial depth of saturation is small.

(viii) Dupuit's assumptions are valid.

The differential equation which governs the saturated flow in a water table aquifer is the Boussinesq equation. For one dimensional flow the linearised Boussinesq's equation is :

$$\frac{\partial^2 s}{\partial x^2} = \frac{1}{\alpha} \frac{\partial s}{\partial t} \qquad \ldots(3.1)$$

in which 's' is the rise in water table, $\alpha = \frac{T}{\Phi}$; T and Φ are the transmissivity and storage coefficient of the aquifer respectively.

If the evolution of water table is due to recharge from a line source, the solution of the above equation is required to satisfy the following boundary conditions :

$$\text{At } x = 0, \; T\frac{\partial s}{\partial x} = -\frac{q}{2}$$

$$\text{and at } x = \infty, \; s(\infty, t) = 0$$

in which 'q' is the recharge rate per unit length of the line source.

The initial condition required to be satisfied is :

$$s(x, 0) = 0$$

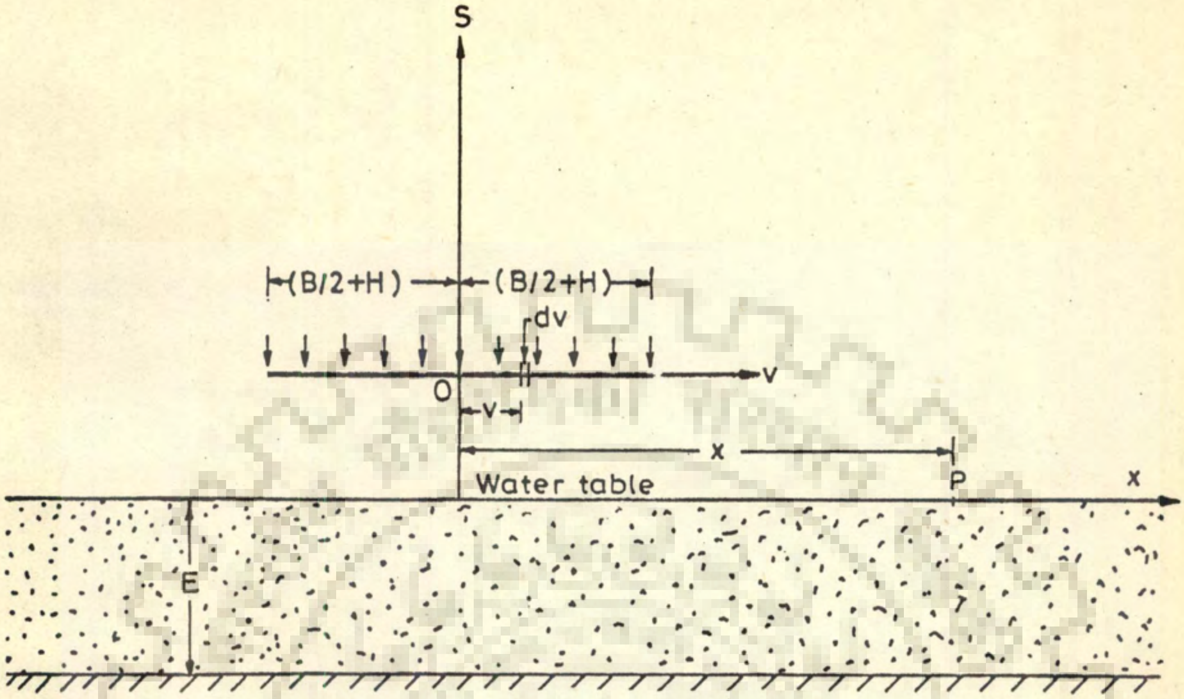

Fig.3.2(a).–Steps for deriving the rise in water table outside the recharging strip

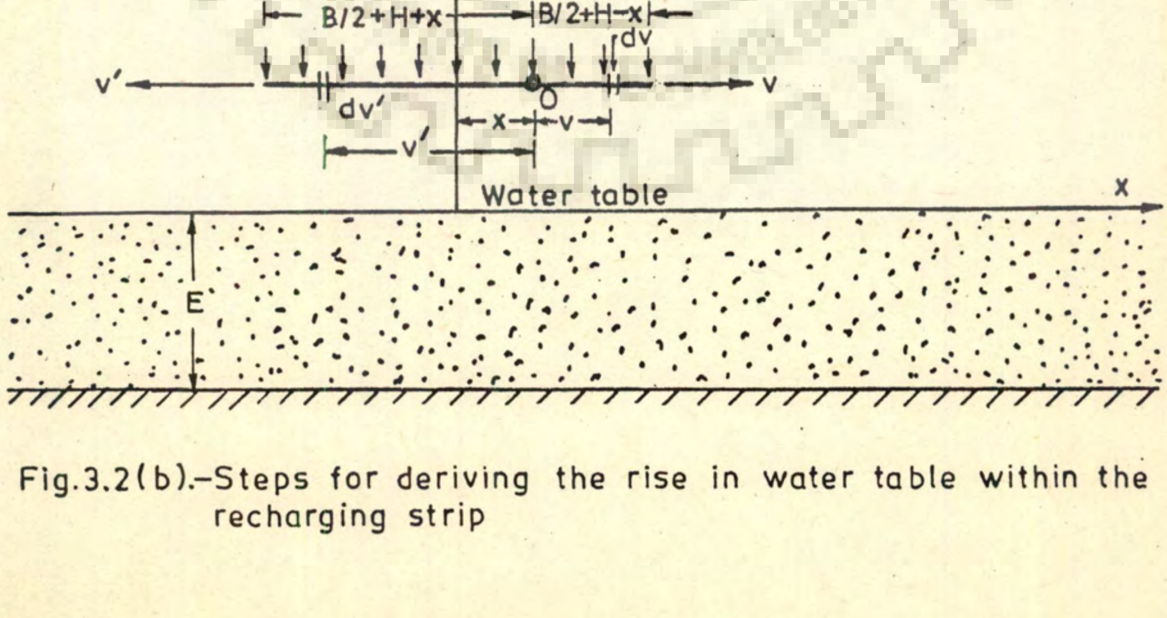

Fig.3.2(b).–Steps for deriving the rise in water table within the recharging strip

Solution to the differential equation (3,1), which satisfies the above initial and boundary conditions has been given by Glover (1974) as below:

$$s(x,t) = \frac{q\sqrt{4\pi\alpha t}}{2\pi T} \cdot \frac{x}{\sqrt{4\alpha t}} \int_{\frac{x}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u^2}\, du$$

$$= \frac{q\sqrt{4\pi\alpha t}}{2\pi T} \cdot \frac{x}{\sqrt{4\alpha t}} \left[\frac{\sqrt{4\alpha t} \cdot e^{-\frac{x^2}{4\alpha t}}}{x} - \sqrt{\pi} + \sqrt{\pi}\,\mathrm{Erf}\left(\frac{x}{\sqrt{4\alpha t}}\right)\right]$$

$$= \frac{q\sqrt{\alpha t}}{\sqrt{\pi}T}\, e^{-\frac{x^2}{4\alpha t}} - \frac{qx}{2T} + \frac{qx}{2T}\,\mathrm{Erf}\left(\frac{x}{\sqrt{4\alpha t}}\right) \qquad \ldots(3.2)$$

In actual case a canal has a certain finite width. Therefore, it would be appropriate to treat it as a strip source instead of a line source. If the water table is at very large depth below the bed of canal, the width of the strip can be taken approximately to be (B+ 2H). Also, if the water table lies at large depth, according to Kozeny the seepage rate per unit area of strip is 'K'. A strip source can be regarded to be consisting of a number of line sources. As the governing differential equation of flow is linear, the rise of water table due to seepage from a strip source can be obtained by integrating the rise of water table due to each line source.

Assuming that the strip source is of width (B + 2H) and seepage rate per unit area = K, the expression for rise of water table,s,at a distance x from the centre of the strip for $x \leq -(B/2+H)$ and $x \geq (B/2+H)$ is derived as follows:

$$s(x,t) = \int_{-(B/2+H)}^{(B/2+H)} \left[\frac{K\sqrt{4\pi\alpha t}}{2\pi T} \sqrt{\left\{ \frac{(x-v)^2}{4\alpha t} \right\}} \int_{\frac{x-v}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u^2} du \right] dv \qquad ...(3.3)$$

The variable, v, is measured from origin as shown in Fig.[3.2(a)].

According to Glover,

$$\int_{\frac{x-v}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u^2} du = \frac{\sqrt{4\alpha t}}{x-v} e^{-\frac{(x-v)^2}{4\alpha t}} - \sqrt{\pi} + \sqrt{\pi} \frac{2}{\sqrt{\pi}} \int_{0}^{\frac{x-v}{\sqrt{(4\alpha t)}}} e^{-u^2} du$$

$$= \frac{\sqrt{4\alpha t}}{x-v} e^{-\frac{(x-v)^2}{4\alpha t}} - \sqrt{\pi} + \sqrt{\pi}\,\mathrm{Erf}\left(\frac{x-v}{\sqrt{4\alpha t}}\right)$$

Making use of this relationship in Equation (3.3) and simplifying,

$$s(x,t) = \int_{-(B/2+H)}^{(B/2+H)} \frac{K\sqrt{4\pi\alpha t}}{2\pi T} \left[e^{-(x-v)^2/(4\alpha t)} - \sqrt{\pi}\sqrt{\left\{ \frac{(x-v)^2}{4\alpha t} \right\}} \right.$$

$$\left. + \sqrt{\pi}\sqrt{\left\{ \frac{(x-v)^2}{4\alpha t} \right\}} \mathrm{Erf}\left(\frac{x-v}{\sqrt{4\alpha t}}\right) \right] dv \qquad ...(3.4)$$

Splitting the limits of integration and integrating,

$$s(x,t) = \frac{K\sqrt{4\pi\alpha t}}{2\pi T} \left[\int_{-(B/2+H)}^{o} e^{-(x-v)^2/(4\alpha t)} dv \right.$$

$$\left. + \int_{o}^{(B/2+H)} e^{-(x-v)^2/(4\alpha t)} dv \right]$$

$$- \frac{K\sqrt{4\pi\alpha t}\cdot\sqrt{\pi}}{2\pi T} \left[\int_{-(B/2+H)}^{o} \frac{(x-v)}{\sqrt{4\alpha t}} dv + \int_{o}^{(B/2+H)} \frac{(x-v)}{\sqrt{4\alpha t}} dv \right]$$

$$+ \frac{K\sqrt{4\pi\alpha t}\sqrt{\pi}}{2\pi T} \left[\int_{-(B/2+H)}^{o} \frac{(x-v)}{\sqrt{4\alpha t}} \, Erf \, \frac{(x-v)}{\sqrt{4\alpha t}} \, dv + \int_{o}^{(B/2+H)} \frac{(x-v)}{\sqrt{4\alpha t}} \, Erf \, \frac{(x-v)}{\sqrt{4\alpha t}} \, dv \right]$$

$$= \frac{K\sqrt{4\pi\alpha t}}{2\pi T} \left[\sqrt{\pi\alpha t} \, Erf \left(\frac{x+B/2+H}{\sqrt{4\alpha t}}\right) - \sqrt{\pi\alpha t} \, Erf \left(\frac{x-B/2-H}{\sqrt{4\alpha t}}\right) \right]$$

$$- \frac{K\sqrt{4\pi\alpha t}.\sqrt{\pi}}{2\pi T} \cdot \frac{x(B+2H)}{\sqrt{4\alpha t}}$$

$$+ \frac{K\sqrt{4\pi\alpha t}.\sqrt{\pi}}{2\pi T} \left[\int_{-(B/2+H)}^{o} \left(\frac{x-v}{\sqrt{4\alpha t}}\right) Erf \left(\frac{x-v}{\sqrt{4\alpha t}}\right) dv + \int_{o}^{(B/2+H)} \frac{x-v}{\sqrt{4\alpha t}} \, Erf \left(\frac{x-v}{\sqrt{4\alpha t}}\right) dv \right] \qquad ...(3.5)$$

An integral of the form $\int Y.Erf(Y)\,dY$ has been derived as:

$$\int Y.Erf(Y)dY = \frac{Y^2}{2} \, Erf(Y) + \frac{Y}{2\sqrt{\pi}} \, e^{-Y^2} - \frac{1}{2\sqrt{\pi}} \int e^{-Y^2} dY + \text{constant}$$

Using this relationship in equation (3.5) and integrating,

$$s(x,t) = \frac{K\sqrt{4\pi\alpha t}}{2\pi T} \Bigg[\sqrt{\pi\alpha t} \, Erf \left(\frac{x+B/2+H}{\sqrt{4\alpha t}}\right) - \sqrt{\pi\alpha t} \, Erf \left(\frac{x-B/2-H}{\sqrt{4\alpha t}}\right)$$

$$- \frac{x\cdot\sqrt{\pi}(B+2H)}{\sqrt{4\alpha t}} + \frac{\sqrt{\pi}(x+B/2+H)^2}{2\cdot\sqrt{4\alpha t}} \, Erf \left(\frac{x+B/2+H}{\sqrt{4\alpha t}}\right)$$

$$+ \frac{x+B/2+H}{2} \cdot e^{-\frac{(x+B/2+H)^2}{4\alpha t}} - \frac{\sqrt{\pi\alpha t}}{2} \, Erf \left(\frac{x+B/2+H}{\sqrt{4\alpha t}}\right)$$

$$- \frac{\sqrt{\pi}(x-B/2-H)^2}{2\cdot\sqrt{4\alpha t}} \, Erf \left(\frac{x-B/2-H}{\sqrt{4\alpha t}}\right) - \frac{(x-B/2-H)}{2} \cdot e^{-\frac{(x-B/2-H)^2}{4\alpha t}}$$

$$+ \frac{\sqrt{\pi\alpha t}}{2} \, Erf \left(\frac{x-B/2-H}{\sqrt{4\alpha t}}\right) \Bigg] \qquad ...(3.6)$$

Simplifying and rearranging,

$$s(x,t) = \frac{K\alpha t}{2T} \, Erf \left(\frac{x+B/2+H}{\sqrt{4\alpha t}}\right) - \frac{K\alpha t}{2T} \, Erf \left(\frac{x-B/2-H}{\sqrt{4\alpha t}}\right)$$

$$+ \frac{K}{4T} (x+B/2+H)^2 \; Erf(\frac{x+B/2+H}{\sqrt{4\alpha t}}) + \frac{K\sqrt{\alpha t}}{2T\sqrt{\pi}} (x+B/2+H) \; e^{-\frac{(x+B/2+H)^2}{4\alpha t}}$$

$$- \frac{K}{2T} \frac{\sqrt{\alpha t}}{\sqrt{\pi}} (x-B/2-H) \; e^{-\frac{(x-B/2-H)^2}{4\alpha t}} - \frac{K(x-B/2-H)^2}{4T} \; Erf\;(\frac{x-B/2-H}{\sqrt{4\alpha t}})$$

$$- \frac{K\sqrt{(x^2)}\;(B+2H)}{2T} \qquad \ldots(3.7)$$

If x is replaced by -x, the above solution remains unchanged, indicating that water table rise is symmetrical about the 's' axis.

The above expression can be written as:

$$s(x,t) = F(x,B,H,t) - \frac{K\sqrt{(x^2)}\,(B+2H)}{2T} \qquad \ldots(3.8)$$

The expression for rise of water table, s(x,t), under the strip source at location x, for $-(B/2+H) \leq x \leq (B/2+H)$, is derived as follows:

$$s(x,t) = \int_{o}^{(B/2+H+x)} [\frac{K\sqrt{\alpha t}}{T\sqrt{\pi}} \; e^{-v'^2/4\alpha t} - \frac{Kv'}{2T} + \frac{Kv'}{2T} \; Erf(\frac{v'}{\sqrt{4\alpha t}})]dv'$$

$$+ \int_{o}^{(B/2+H-x)} [\frac{K\sqrt{\alpha t}}{T\sqrt{\pi}} \; e^{-v^2/4\alpha t} - \frac{Kv}{2T} + \frac{Kv}{2T} \; Erf(\frac{v}{\sqrt{4\alpha t}})]dv \qquad \ldots(3.9)$$

The variables, v, and v' are measured from the point, x, as shown in Fig.[3.2(b)]

Integrating and simplifying,

$$s(x,t) = \frac{K\alpha t}{2T} \; Erf\;(\frac{x+B/2+H}{\sqrt{4\alpha t}}) - \frac{K\alpha t}{2T} \; Erf\;(\frac{x-B/2-H}{\sqrt{4\alpha t}})$$

$$+ \frac{K}{4T} (x+B/2+H)^2 \; Erf\;(\frac{x+B/2+H}{\sqrt{4\alpha t}}) - \frac{K}{4T} (x-B/2-H)^2 \; Erf(\frac{x-B/2-H}{\sqrt{4\alpha t}})$$

$$+ \frac{K\sqrt{\alpha t}}{2T\sqrt{\pi}} (x+B/2+H) \;\; e^{-(x+B/2+H)^2/4\alpha t}$$

$$- \frac{K\sqrt{\alpha t}}{2T\sqrt{\pi}} (x-B/2-H) \;\; e^{-(x-B/2-H)^2/4\alpha t} - \frac{K}{2T} [x^2 + (B/2+H)^2] \qquad \ldots(3.10)$$

The above expression can be written as

$$s(x,t) = F(x,B,H,t) - \frac{K}{2T} [x^2 + (B/2+H)^2] \qquad ...(3.11)$$

The expressions for rise in water table given by Equations (3.7) and (3.10) are the basic solutions which can be used to derive the expressions for evolution of water table for an intermittently running canal and for parallel canal system.

3.2.2 The evolution of water table due to recharge from an intermittently running canal

Let during the running period of the canal, the depth of water in it be constant. This implies that the width of the canal at the water surface, B, is also constant. As already stated, the governing differential Equation (3.1) is linear. Therefore, the expressions given by Equations (3.7) and (3.10) are the response of a linear system due to a continuous uniform excitation. Let this response due to continuous uniform excitation be designated as U(t). Thus, for $x \leq -(B/2+H)$ and $x \geq (B/2+H)$,

$$U(t) = F(x,B,H,t) - \frac{K\sqrt{(x^2)}(B+2H)}{2T} \qquad ...(3.12)$$

and, for $-(B/2+H) \leq x \leq (B/2+H)$,

$$U(t) = F(x,B,H,t) - \frac{K[x^2 + (B/2+H)^2]}{2T} \qquad ...(3.13)$$

Let the time parameter be discretised with uniform time-step. If the canal runs for a unit time period and is closed thereafter, the rise in water table at the end of nth time-step, $\delta(n)$, is given by:

$$\delta(n) = U(n) - U(n-1)$$

Thus, for $x \leq -(B/2+H)$ and $x \geq (B/2+H)$, the rise in water table due to running of the canal during the first unit time period is given by:

$$\delta(n) = F(x, B, H, n) - F(x, B, H, n-1) \qquad \ldots(3.14)$$

In particular, the rise in water table at the end of time-step 1 is:

$$\delta(1) = F(x, B, H, 1) - K\sqrt{(x^2)}(B+2H)/2T \qquad \ldots(3.15)$$

For $-(B/2+H) \leq x \leq (B/2 + H)$,

$$\delta(n) = F(x, B, H, n) - F(x, B, H, n-1) \qquad \ldots(3.16)$$

and $\delta(1) = F(x, B, H, 1) - K[x^2 + (B/2+H)^2]/2T$ ...(3.17)

When the canal runs intermittently, the rise in water table at the end of m^{th} time-step is given by:

$$s(x, m) = \sum_{\gamma=1}^{m} R(\gamma)\delta(m-\gamma+1) \qquad \ldots(3.18)$$

Where,

$R(\gamma)$ is a factor which is equal to 1 for the time-steps during which the canal runs and is zero during the time steps the canal is closed.

3.2.3 Evolution of water table due to recharge from two parallel canals.

The evolution of water table due to seepage from any number of parallel canals can be obtained by method of superposition as the governing differential equation of flow is linear. However, the present analysis has been done for two parallel canals. The width of canals at the water surface and depth of water for the left and right canal are B_1, H_1 and B_2, H_2 respectively as shown in Fig. [3.1(b)]. The distance between centre to centre of the canals is D. The rise in water table at any point along a transverse section across the two parallel canals at time, t, can be computed by summing up the values of rise in water table for each canal with the help of Equations (3.7) and (3.10). For different locations of point x, as shown in Fig. [3.1(b)], the expressions for rise in water table are as follows:

For $x \leq -(B_1/2+H_1)$,

$$s(x,t) = F(x, B_1, H_1,t) + F(D-x,B_2,H_2,t) - [K\sqrt{x^2}(B_1+2H_1)]/2T$$

$$-K[\sqrt{(D-x)^2}(B_2+2H_2)]/2T \qquad \ldots(3.19)$$

for, $-(B_1/2+H_1) \leq x \leq (B_1/2+H_1)$,

$$s(x,t) = F(x,B_1,H_1,t)+F(D-x,B_2,H_2,t)- K[x^2+(B_1/2+H_1)^2]/2T$$

$$- K[\sqrt{(D-x)^2}\ (B_2+2H_2)]/2T \qquad \ldots(3.20)$$

for, $(B_1/2+H_1) \leq x \leq D-(B_2/2+H_2)$,

$$s(x,t) = F(x,B_1,H_1,t) + F(D-x,B_2,H_2,t) - K\ \sqrt{x^2}(B_1+2H_1)/2T$$

$$- [K\sqrt{(D-x)^2}(B_2+2H_2)]/2T \qquad \ldots(3.21)$$

for, $D-(B_2/2+H_2) \leq x \leq D + (B_2/2+H_2)$,

$$s(x,t) = F(x,B_1,H_1,t) + F(D-x,B_2,H_2,t) - [K\sqrt{x^2}(B_1+2H_1)]/2T - K[(D-x)^2 + (B_2/2+H_2)^2]/2T \qquad ...(3.22)$$

and for, $x \geq D + (B_2/2+H_2)$,

$$s(x,t) = F(x,B_1,H_1,t)+F(D-x,B_2,H_2,t)-K\sqrt{x^2}(B_1+2H_1)/2T - K\sqrt{(D-x)^2}(B_2+2H_2)/2T \qquad ...(3.23)$$

3.3 Results and Discussions

Numerical results for rise in water table have been presented for the following cases:

(i) A single canal running continuously with a constant depth of water,

(ii) A single canal running intermittently with a constant depth of water during the run, and

(iii) Two parallel canals running continuously with constant water depth in them.

The rise in water table has been evaluated for known aquifer parameters, viz., the coefficient of permeability, K, the storativity, φ, and the initial saturated thickness, E, of the aquifer. The transmissivity, T, of the aquifer has been taken as (K.E), since it has been assumed that there is no appreciable change in the saturated thickness of the aquifer due to recharge from a canal. The non-dimensional groups which have been formed to present the results are :

(i)	Time factor	$Kt/(2\ \varphi E)$
(ii)	Water table rise	$s(x,t)/E$
(iii)	Width of canal at water surface	B/E
(iv)	Depth of water in canal	H/E
(v)	Distance from centre of the canal to an observation point	x/E
(vi)	Distance between the canals	D/E

The error function appearing in the solution has been evaluated using the rational approximation given by Hastings (vide, Abramowitz and Stegun, 1970).

The percentage rise in water table elevations at various non-dimensional time across a canal are presented in Fig. (3.3), for $B/E = 0.03$ and $H/E = 0.003$. The results for canal of larger width, having $B/E = 0.06$ and $H/E = 0.003$, are presented in Fig. (3.4). It is found that for a canal with $B/E = 0.03$ and $H/E = 0.003$ the percentage rise in water table elevation below the centre of the canal, at non- dimensional time, $Kt/(2\varphi E)$, = 0.05, is 0.6262. At $x/E = 0.3$, the percentage rise at this time is 0.2417. For a canal with a larger non-dimensional width of 0.06, the corresponding percentage rises are 1.1241 and 0.4438 respectively. According to the assumption made in the present analysis, the recharge rate from a unit length

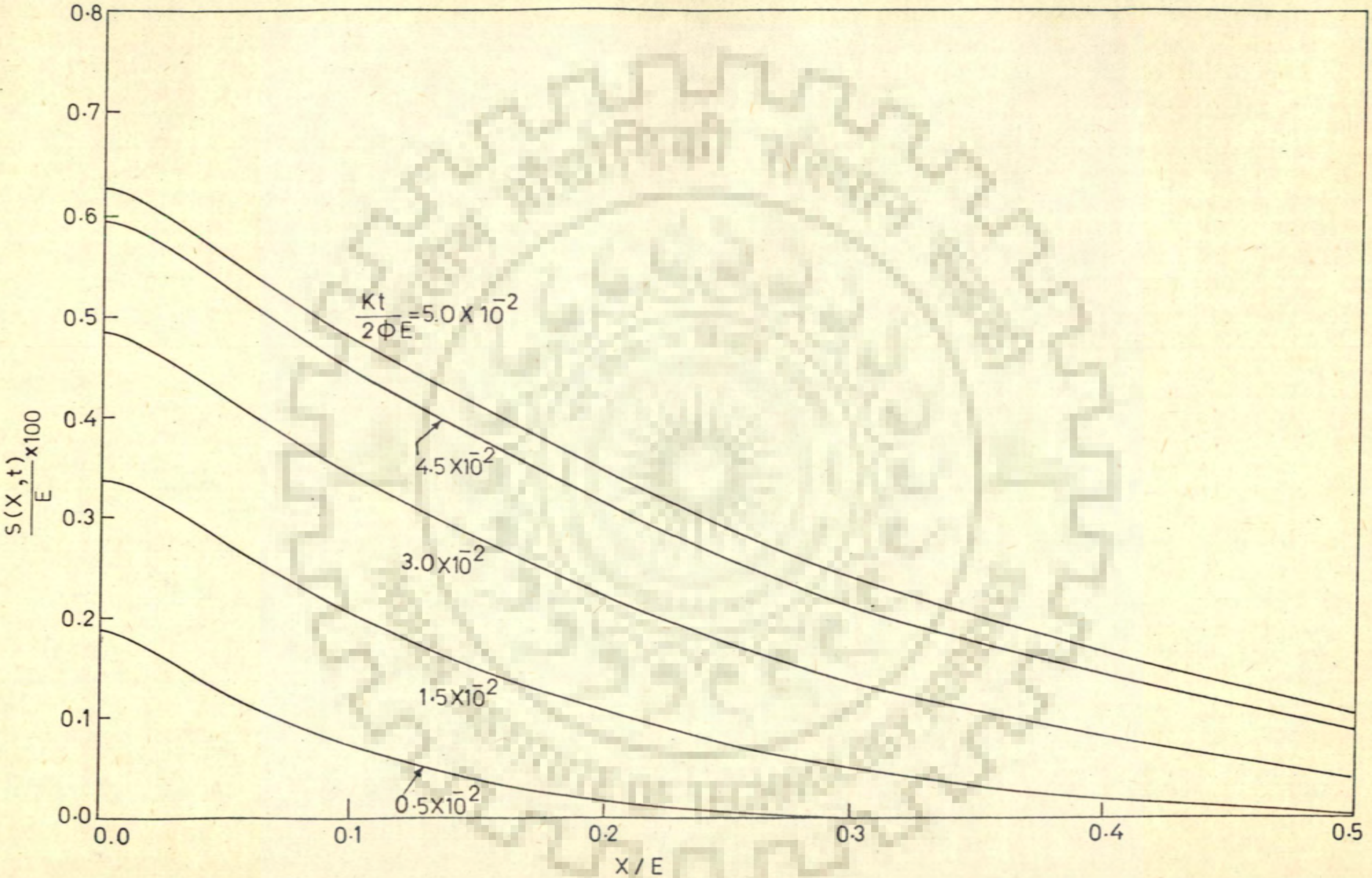

Fig.3.3–Variation of percentage rise of water table due to recharge from a canal for B/E = 0.03 and H/E = 0.003

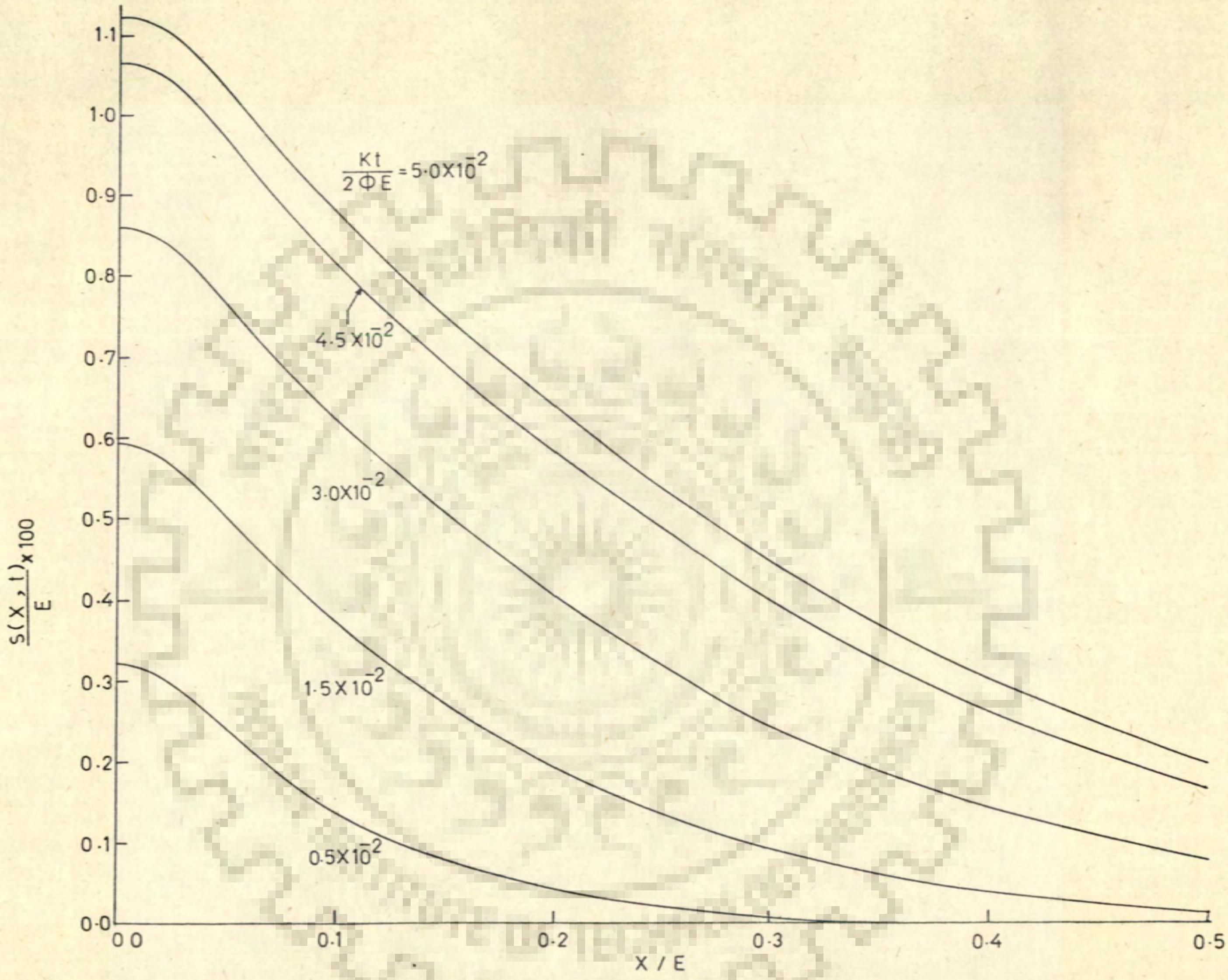

Fig.3.4.–Variation of percentage rise of water table due to recharge from a canal for B/E = 0.06 and H/E = 0.003

of canal is $K(B+2H)$ and the width of the recharging strip is $(B+2H)$. A comparison of rise of water table per unit recharge rate, $s(x,t)/[K(B+2H)]$, for two different canal sections is shown in Table (3.1). It is seen from the table that water table rise is not proportional to the recharge rate from canals of different cross sections. According to the analysis, the water table rise at a point depends not only on the recharge quantity but also on the distribution of recharge. Therefore, if the width of the canal is changed, there will not be a proportionate change in the elevation of water table, as the relationship between water table rise, $s(x,t)$, and the width of the recharging strip, $(B+2H)$, given by Equation (3.10) is non-linear. It is also observed that the water table rise below the centre of the canal, per unit recharge rate, decreases with increase in the width of the recharging strip. At observation points located beyond the recharging strip, the water table rise per unit recharge rate increases with increase in the width of the recharging strip.

The rise of water table, $s(x,t)/E$, at various non-dimensional time, $Kt/(2\varphi E)$, for observation points located under the canal, and at various distances from the centre of the canal, are presented in Figs.(3.5) and (3.6) for two sets of B/E and H/E. The non-dimensional

Table 3.1 - Water table rise per unit recharge rate for canals of different cross sections evaluated at $Kt/(2\varphi E) = 0.05$.

Sl.No.	B/E	H/E	x/E	s(x,t)/[K(B+2H)]
1.	0.03	0.003	0.00	0.17396/K
2.	0.06	0.003	0.00	0.17033/K
3.	0.03	0.003	0.05	0.15457/K
4.	0.06	0.003	0.05	0.15469/K
5.	0.03	0.003	0.10	0.13290/K
6.	0.06	0.003	0.10	0.13301/K
7.	0.03	0.003	0.30	0.06714/K
8.	0.06	0.003	0.30	0.06724/K
9.	0.03	0.003	0.50	0.02963/K
10.	0.06	0.003	0.50	0.02970/K

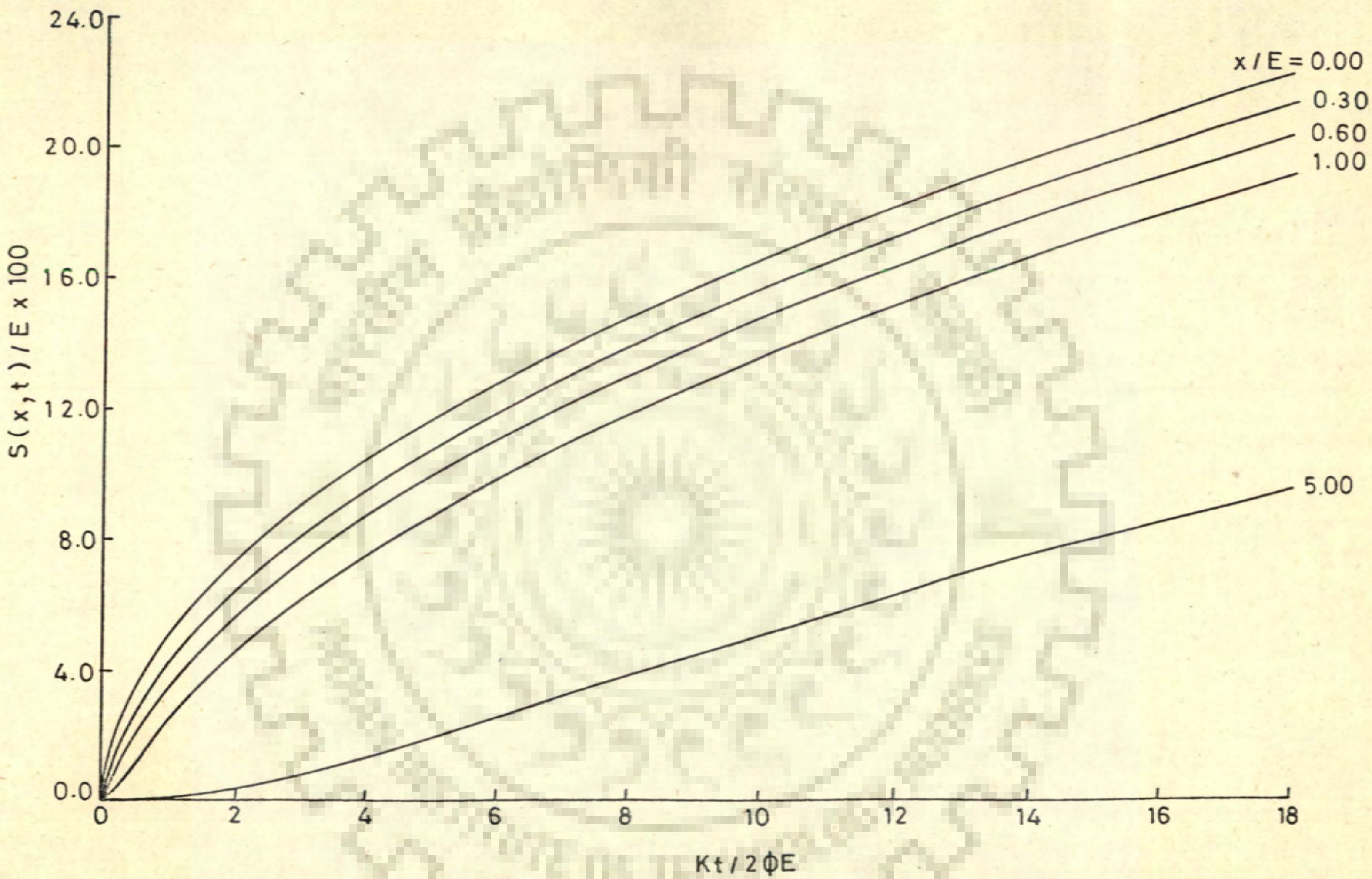

Fig.3.5–Percentage rise in water table elevation at various observation points due to recharge from a canal for B/E= 0.06 and H/E= 0.003

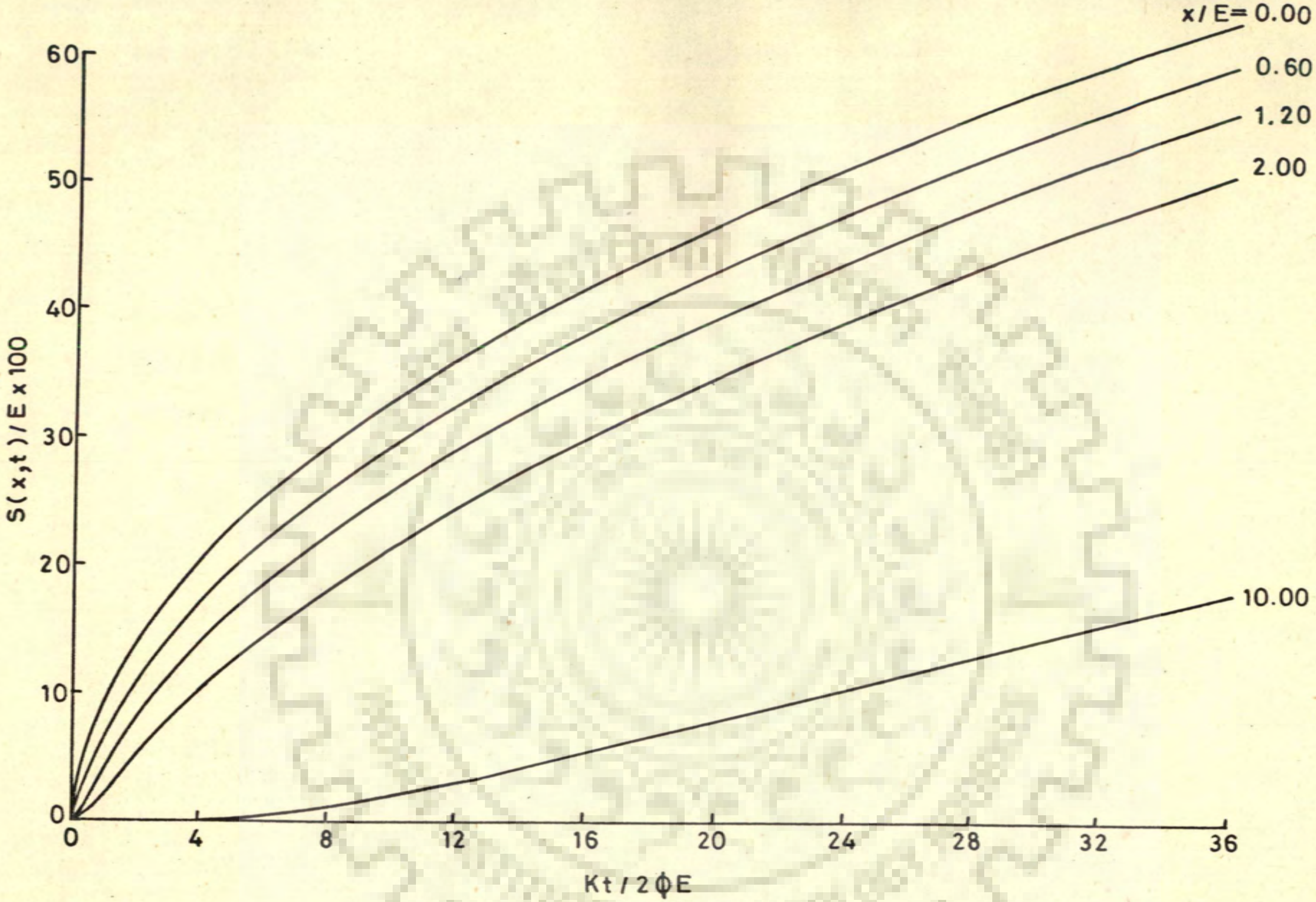

Fig.3.6.-Percentage rise in water table elevation at various observation points due to recharge from a canal for B/E = 0.12 and H/E=0.006

plot implies that if the geometric similarities are preserved, i.e., if the ratios of B/E and H/E remain unaltered, the variation of s(x,t)/E with $kt/2\varphi E$ at an observation point would follow a fixed graph for any value of K and φ. The water table rise at these observation points corresponding to various aquifer parameters, K and φ, can be obtained from these graphs for geometrically similar canals. A set of results for draw-down evaluated for transmissivity, T, ranging from 100 m^2 / day to 1000 m^2 / day and storage coefficient, φ ranging from 0.05 to 0.2 are presented in Table (3.2).

The expressions for gradient of the water table outside and below the recharging strip have been obtained from Equations (3.7) and (3.10) by differentiating s(x,t) with respect to x. The expression for gradient outside the recharging strip is given by :

$$\frac{\partial s(x,t)}{\partial x} = \frac{K\sqrt{(\alpha t)}}{T\sqrt{\pi}} e^{-(x+B/2+H)^2/(4\alpha t)} - \frac{K\sqrt{(\alpha t)}}{T\sqrt{\pi}} e^{-(x-B/2-H)^2/(4\alpha t)} + \frac{K}{2T}(x+B/2+H) \operatorname{Erf}\left[\frac{x+B/2+H}{\sqrt{4\alpha t}}\right] - \frac{K}{2T}(x-B/2-H) \operatorname{Erf}\left[\frac{x-B/2-H}{\sqrt{4\alpha t}}\right] - \frac{K(B+2H)}{2T} \quad \ldots(3.24)$$

Table 3.2 - Rise in Water table at the end of 180 days due to continuous recharge from canals having 30 m and 60 m width and 3.0 m depth of water, evaluated for various values of aquifer parameters

B	E (m)	T m²/day	x (m)	Rise in water table (m) for : φ=0.05	0.10	0.20
30	500	100	150	19.3546	12.3726	7.5437
	500	500	150	49.2709	33.3787	22.1912
	500	100	300	15.0824	8.5471	4.3151
	500	500	300	44.3804	28.6976	17.8028
	1000	100	150	9.6773	6.1862	3.7719
	1000	500	150	24.6354	16.6893	11.0956
	1000	1000	150	35.8982	24.6354	16.6893
	1000	100	300	7.5412	4.2735	2.1575
	1000	500	300	22.1902	14.3488	8.9014
	1000	1000	300	33.3780	22.1902	14.3488
60	500	100	300	27.6585	15.6795	7.9233
	500	500	300	81.3675	52.6170	32.6450
	500	100	600	15.8204	6.6082	1.9984
	500	500	600	65.2748	38.0075	20.0378
	1000	100	300	13.8292	7.8398	3.9617
	1000	500	300	40.6837	26.3085	16.3225
	1000	1000	300	61.1952	40.6837	26.3085
	1000	100	600	7.9102	3.3041	0.9992
	1000	500	600	32.6374	19.0037	10.0189
	1000	1000	600	52.6118	32.6374	19.0038

The expression for water table gradient below the reacharging strip is found to be:

$$\frac{\partial s(x,t)}{\partial x} = \frac{K\sqrt{(\alpha t)}}{T\sqrt{\pi}} e^{-(x+B/2+H)^2/(4\alpha t)} - \frac{K\sqrt{(\alpha t)}}{T\sqrt{\pi}} e^{-(x-B/2-H)^2/(4\alpha t)} + \frac{K}{2T}(x+B/2+H) \operatorname{Erf}\left[\frac{x+B/2+H}{\sqrt{(4\alpha t)}}\right] - \frac{K}{2T}(x-B/2-H) \operatorname{Erf}\left[\frac{x-B/2-H}{\sqrt{(4\alpha t)}}\right] - \frac{Kx}{T} \quad \ldots(3.25)$$

The water table gradients at various locations, lying outside and within the recharging strip, have been computed using Equations (3.24) and (3.25) respectively and are presented in Table (3.3). As seen from the table, the absolute value of the gradient at any point, except at the origin and at infinity, increases with time. It could be deduced from Equation (3.24) that, the absolute value of the gradient at any point outside the recharging zone attains a limiting value of $0.5(B/E + 2H/E)$ after a long duration of time. At any point within the recharging strip the limiting value of the gradient attained after a long time is x/E. If the gradient of the water table is known at a particular location outside a continuously running canal, when the water table evolution at this observation point has reached a near steady state condition, the saturated thickness of the aquifer can be ascertained by making use of the observed limiting gradient in the expression:

Limiting gradient = 0.5 (B/E + 2H/E).

Replacing the strip source by an equivalent line source, the water table evolution can be predicted by Glover's basic solution given in Equation (3.2). Assuming the strength of the line source to be K(B+2H) per unit length, and using Glover's solution, the water table positions at different time have been evaluated for different canal widths and are presented in Table (3.4). The water table positions predicted using the solution for strip source are also given in the Table for the purpose of comparison.It could be seen that Glover's method over-estimates the water table rise at all time below the centre of the canal. At other points within the recharging strip, in the beginning of recharge, the water table rise is under estimated by Glover's approach. But with lapse of time, the water table rise calculated using Glover's solution is found to be higher than the rise estimated by Equation (3.10). In the region outside the recharging strip Glover's solution under-estimates the water table rise

Table 3.3 - Water table gradient at various locations within and outside the recharging strip at various non-dimensional time

			$[-\partial s/\partial x].100$ ie., - gradient x 100 evaluated at $Kt/(2\varphi E)$ =					
H/E	B/E	x/E	0.050	0.100	0.300	0.600	0.900	9.00
0.003	0.03	0.005	0.484	0.488	0.493	0.495	0.496	0.498
		0.015	1.452	1.466	1.480	1.486	1.488	1.496
	0.06	0.010	0.941	0.958	0.976	0.983	0.986	0.995
		0.030	2.823	2.875	2.928	2.945	2.958	2.986
	0.03	0.150	1.327	1.462	1.604	1.661	1.686	1.764
		0.300	0.904	1.143	1.411	1.523	1.574	1.728
	0.06	0.300	1.659	2.097	2.588	2.793	2.885	3.168
		0.600	0.594	1.132	1.927	2.305	2.481	3.037

Table 3.4 - Comparison of results obtained from the present analysis with Glover's Solution

B/E	H/E	x/E	Kt/(2 φE)	s(x,t)/E obtained from present solution for strip source (10^{-3})	s(x,t)/E obtained from Glover's solution for equivalent line source (10^{-3})	Difference between Glover's solution and present solution (10^{-3})
0.03	0.003	0.000	0.00005	0.0919	0.2031	0.1112
			0.00050	0.4974	0.6422	0.1448
			0.00500	1.8745	2.0310	0.1565
			0.05000	6.2626	6.4228	0.1602
			0.50000	20.1498	20.3108	0.1610
			1.00000	28.5626	28.7238	0.1612
		0.015	0.00005	0.0645	0.0377	-0.0268
			0.00050	0.4198	0.4080	-0.0118
			0.00500	1.7734	1.7724	-0.0010
			0.05000	6.1537	6.1564	0.0027
			0.50000	20.0385	20.0419	0.0034
			1.00000	28.4517	28.4546	0.0029
		0.150	0.00500	0.3805	0.3773	-0.0032
			0.05000	4.0824	4.0807	-0.0017
			0.50000	17.7254	17.7249	-0.0005
			1.00000	26.1049	26.1045	-0.0004
0.06	0.003	0.000	0.00005	0.0995	0.3723	0.2728
			0.00050	0.7370	1.1775	0.4405
			0.00500	3.2128	3.7236	0.5108
			0.05000	11.2414	11.7752	0.5338
			0.50000	36.6960	37.2365	0.5405
			1.00000	52.1192	52.6603	0.5411
		0.015	0.00005	0.0959	0.0691	-0.0268
			0.00050	0.6848	0.7481	0.0633
			0.00500	3.1210	3.2495	0.1285
			0.05000	11.1355	11.2868	0.1513
			0.50000	36.5855	36.7436	0.1581
			1.00000	52.0081	52.1668	0.1587
		0.150	0.0050	0.7111	0.6918	-0.0193
			0.0500	7.4915	7.4814	-0.0101
			0.5000	32.4991	32.4957	-0.0034
			1.0000	47.8608	47.8584	-0.0024

marginally as seen from the table.

Hantush has derived an expression for water table rise due to recharge from a rectangular spreading basin. If the length of the rectangular spreading basin is extended to infinity, Hantush's solution would give an expression for water table rise due to recharge from a strip source. Assuming the length of the spreading basin to be infinite, the following expression for water table height, consequent to recharge from a strip source, has been deduced starting from Hantush's basic Equation (2.6).

$$h^2(x,t) = h_o^2 + \left(\frac{w\bar{h}t}{2\varphi}\right) . \left[2\int_o^1 \mathrm{Erf}\left(\frac{a+x}{2\sqrt{(K\bar{h}t\tau/\varphi)}}\right)d\tau + 2\int_o^1 \mathrm{Erf}\left(\frac{a-x}{2\sqrt{(K\bar{h}t\tau/\varphi)}}\right)d\tau\right] \qquad ...(3.26)$$

where,

- w = uniform recharge rate which has been assumed to be equal to K in the present analysis,
- a = half width of the recharging strip, which is equal to (B/2+H) for a canal,
- h_o = initial water table height, and
- $\bar{h}$ = weighted mean depth of saturation during the period of flow.

Incorporating the above assumptions, and assuming $\bar{h} = E$, the expression for water table rise is found to be:

$$s(x,t) = \{h_o^2+2(\frac{Tt}{2\varphi})\ [{}_o\int^1 Erf(\frac{B/2+H+x}{2\sqrt{(Tt\tau/\varphi)}})d\tau$$

$$+ {}_o\int^1 Erf\ (\frac{B/2+H-x}{2\sqrt{(Tt\tau/\varphi)}})\ d\tau]\}^{1/2}-h_o \quad ...(3.27)$$

The water table rises have been calculated using Equation (3.27) and are presented in Table (3.5). The corresponding water table rises calculated using Equations (3.7) and (3.10) are also given in Table (3.5) for the purpose of comparison. In the present study, the solution of Boussinesq's equation is based on Glover's linearisation technique. The small difference between the results of present study, and those obtained from Hantush's equation, is attributed to the difference in linearisation techniques of Boussinesq's equation adopted by Hantush and Glover, and to the numerical integration used in the Hantush's solution for evaluating the drawdown.

Shestakov has analysed the evolution of water table due to recharge from a strip source and has tabulated a special function to predict the water table rise at different locations in an aquifer within and outside the

Table 3.5 - Comparison of water table rises predicted by Hantush's solution and the present analytical method, evaluated for φ =0.1 and K=0.1 m/day.

B (m)	H (m)	E (m)	x/E	Kt/(2 φE)	s(x,t)/E predicted by present method (10^{-3})	s(x,t)/E predicted by Hantush's method (10^{-3})
30	3	1000	0.00	0.01	2.714	2.711
				0.02	3.903	3.895
				0.03	4.815	4.804
				0.04	5.584	5.569
				0.05	6.262	6.243
			0.15	0.01	0.947	0.957
				0.02	1.922	1.941
				0.03	2.736	2.763
				0.04	3.445	3.484
				0.05	4.082	4.124

strip source. The values of the special function for different values of a non-dimensional time parameter are given in Table (2.1). The water table rises computed using Equations (3.7) and (3.10) have been compared with the rises evaluated using Shestakov's special function. The comparison has been made for the following values of aquifer parameters and canal dimensions:-

$K = 0.1$ m/day,

$\varphi = 0.1$,

$E = 1000$ m,

$B = 14$ m, and

$H = 3$ m.

While computing water table rise by Shestakov's equations, it has been assumed that the width of the strip is (B+2H) and the recharge rate per unit area is K. The water table rises computed by both the methods are presented in Table(3.6). It is seen that the water table rises computed using Equations (3.7) and (3.10) compare well with those computed using Shestakov's special function.

The water table evolution for a continuously running canal has been presented in the preceding paragraphs. An Irrigation canal may run intermittently depending upon the availability of water as well as its need in the command area. The evolution of water table

Table 3.6 - Comparison of rise of water table computed by equations (3.7) and (3.10) and by Shestakov's special functions for B=14 m, H = 3 m, K =0.1 m/day, T=100 m^2/day, φ =0.1.

Location from centre of the canal (m)	Time (days)	Rise in water table computed by equations (3.7)&(3.10) (m)	Rise in water table computed by Shestakov's special functions (m)
0.0	1	0.3098	0.3110
	5	0.7492	0.7550
	10	1.0793	1.0800
5.0	1	0.2995	0.3010
	10	1.0675	1.0700
10.0	1	0.2686	0.2685
	10	1.0320	1.0350
15.0	1	0.2295	0.2305
	10	0.9857	0.9950
20.0	1	0.1946	0.1955
	10	0.9406	0.9500

near an intermittently running canal is, therefore, considered next. In order to predict the water table evolution near an intermittently running canal, the response of the aquifer at desired observation points to a continuous running of the canal is first determined. Making use of the response to the continuous running of the canal, the response of the aquifer, if the canal runs only for the first unit time period, is predicted by Equations (3.14) and (3.16) for the desired observation point. The response of the aquifer to a pulse excitation, imparted by running the canal for the first unit time period, has been designated as $\delta(n)$. The procedure for determining the $\delta(n)$ coefficients at different time-steps, n, for an observation point near a canal has been explained through an example presented in Table (3.7). $\delta(n)$ coefficients for different integer values of n, for various sets of aquifer parameters and canal dimensions, have been presented graphically in Figs. (3.7) through (3.10). Using these coefficients, the water table evolution for any running schedule of these canals could be predicted.

The evolution of water table has been predicted for an intermittently running canal whose running and closure periods are of equal duration. The water table evolutions have been presented in non-dimensional form

Table 3.7 - Calculation of $\delta(n)$ coefficients for an observation point at $x/E = 0.15$ evaluated for $T = 500\ m^2/day$, $\varphi = 0.1$, $B/E = 0.03$ and $H/E = 0.003$.

Time in days (n)	Rise in water table height if the canal would run continuously K(n)	δ (n) K(n)-K(n-1)
1	0.1080	0.1080
2	0.3805	0.2725
3	0.6692	0.2887
4	0.9473	0.2781
5	1.2108	0.2635
6	1.4603	0.2495
7	1.6971	0.2368
8	1.9227	0.2256
9	2.1384	0.2157
10	2.3453	0.2069

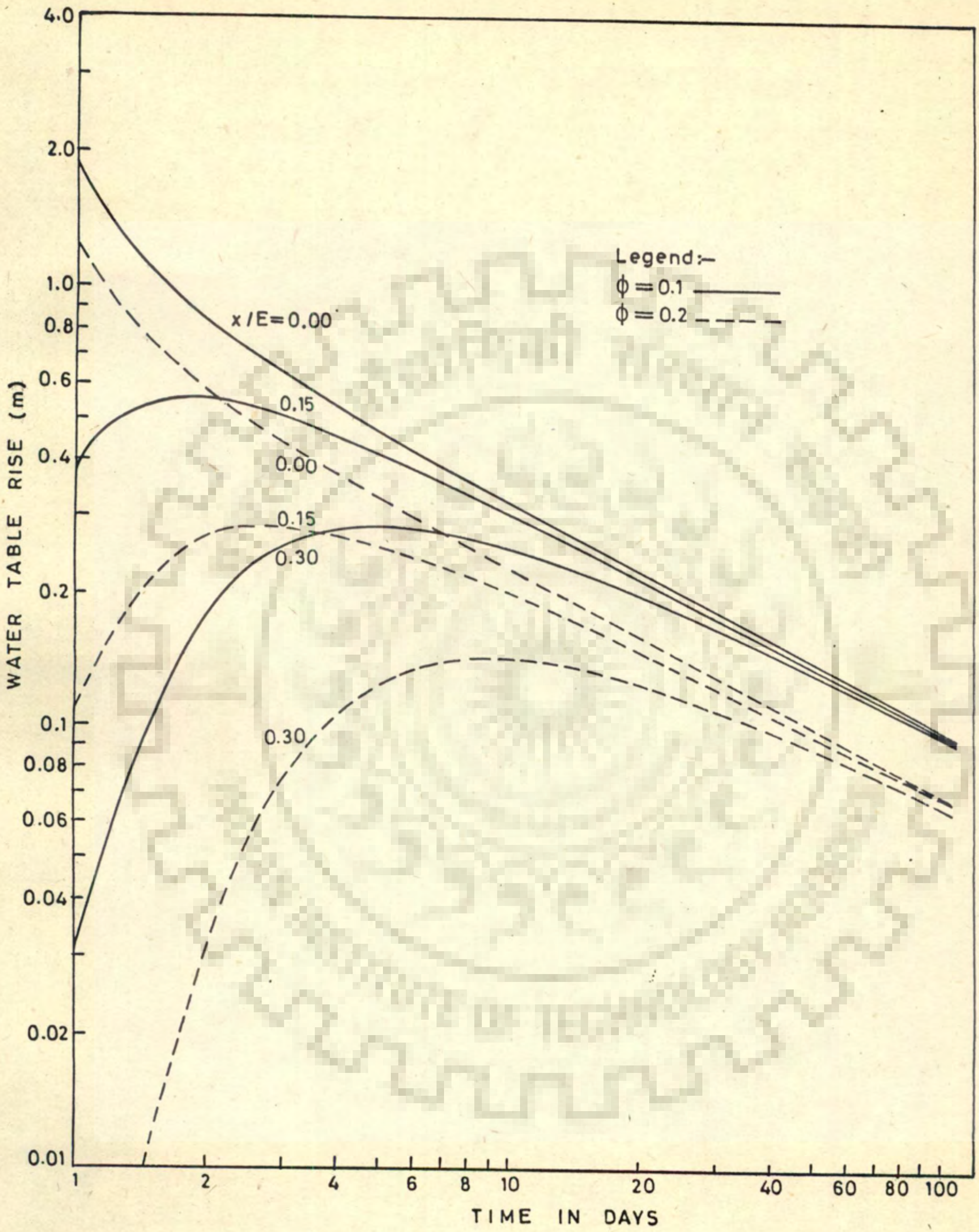

Fig.3.7-Response function coefficients at locations across canal for B/E=0.03, H/E=0.003, and K=1.0 m/day

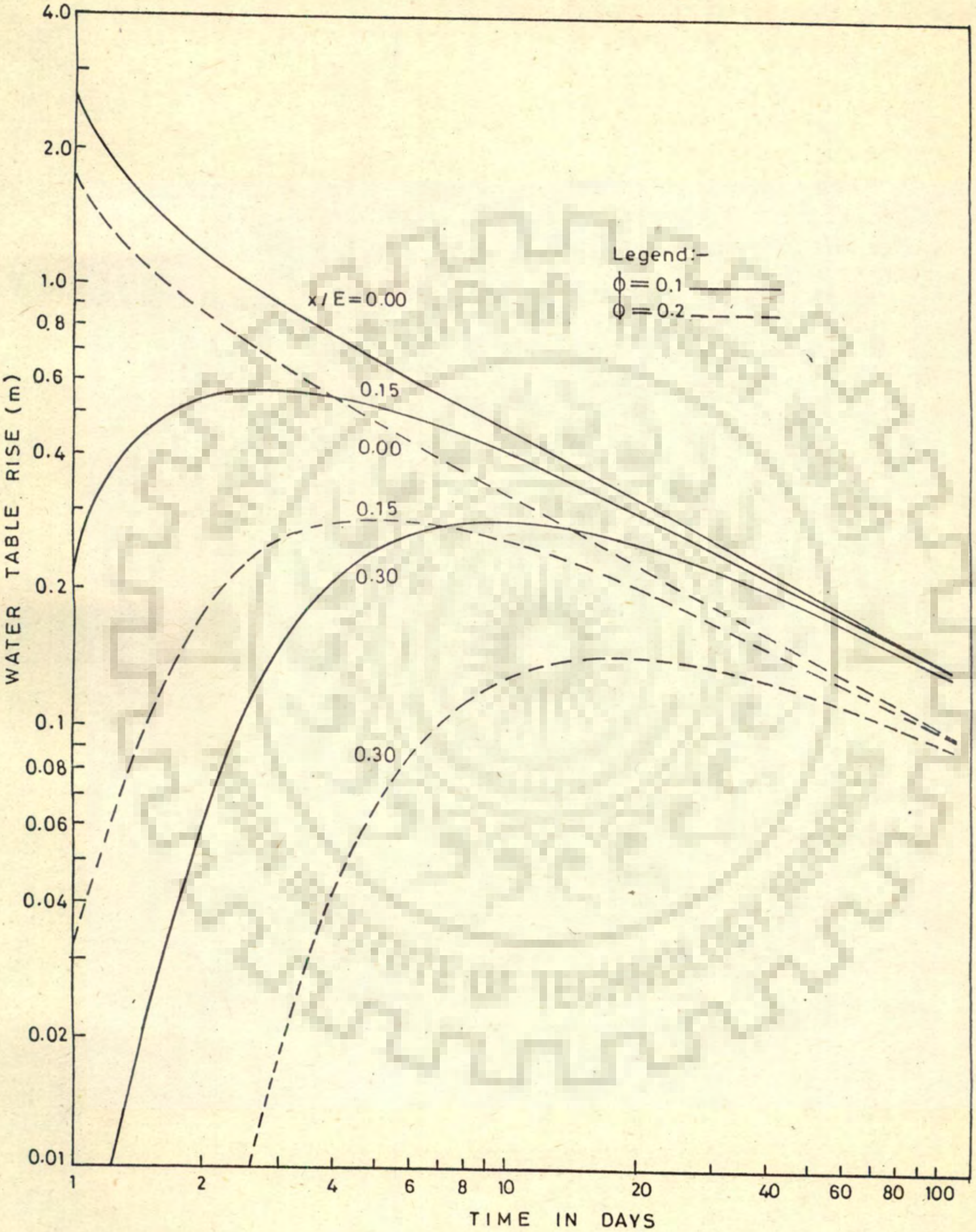

Fig.3.8.-Response function coefficients at locations across canal for B/E = 0.06, H/E = 0.006, and K = 1.0 m/day

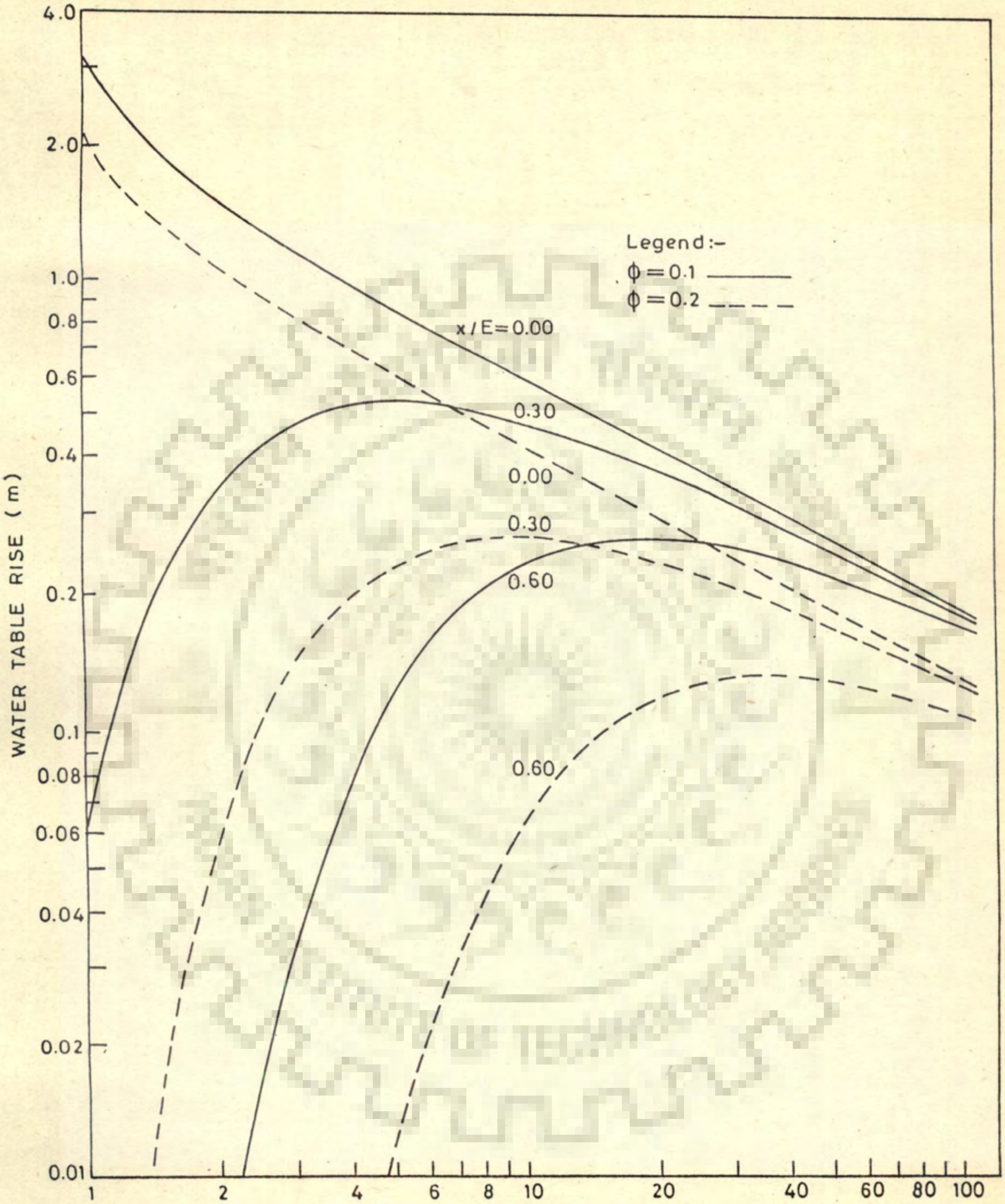

Fig.3.9.-Response function coefficients at locations across canal for B/E = 0.06, H/E = 0.003, and K = 1.0 m/day

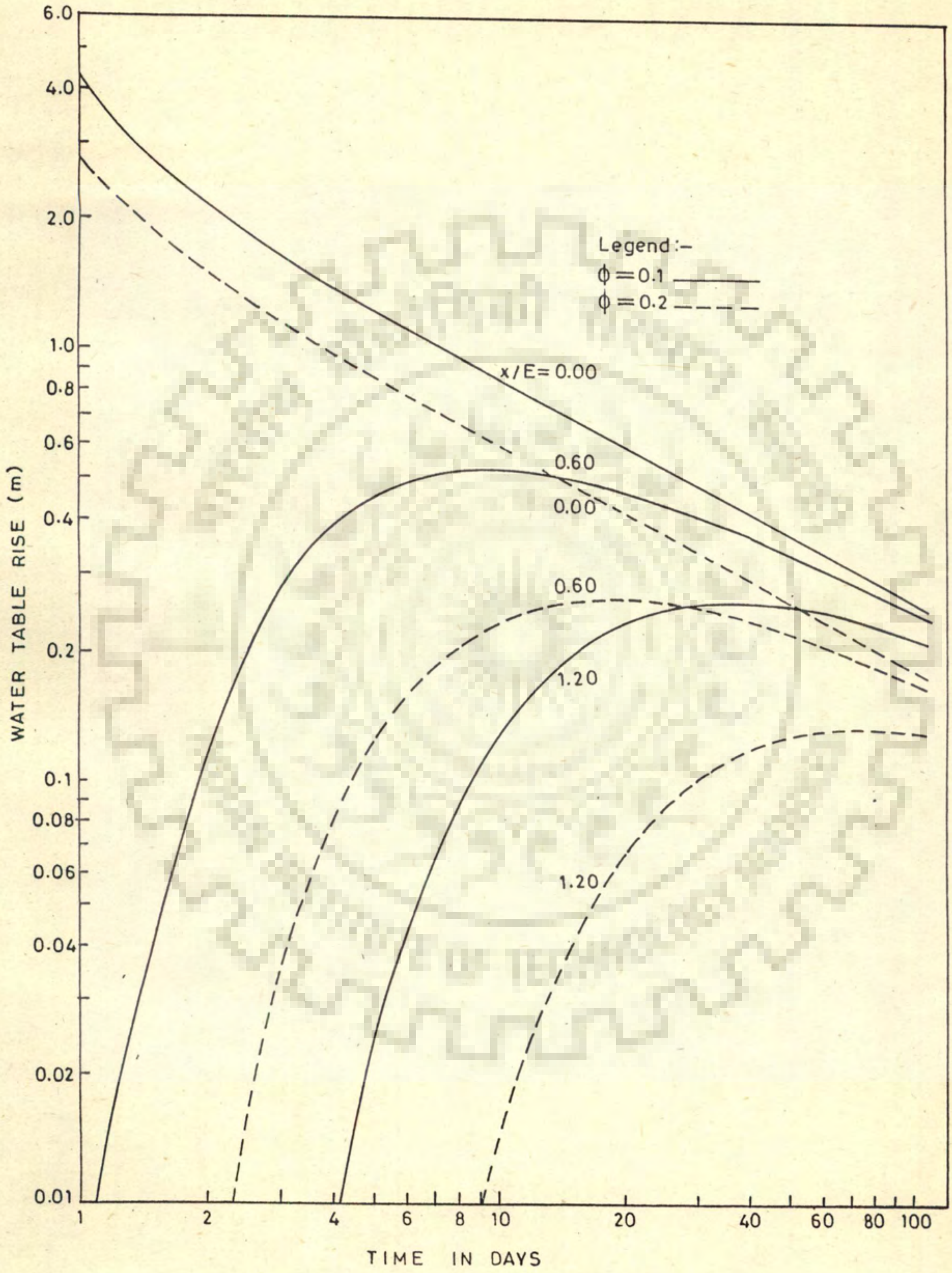

Fig.3.10.– Response function coefficients at locations across canal for B/E = 0.12 , H/E = 0.006, and K = 1.0 m/day

for the first three cycles of canal operation in Figs. (3.11) and (3.12) for two sets of canal dimensions. The variation of water table rise $s(x,t)/E$ with non-dimensional time factor, $Kt/(2\varphi E)$, at different dimensionless distances, x/E, are presented in Fig. (3.11) for an intermittently running canal with $B/E=0.06$ and $H/E=0.003$. It has been assumed that the canal first runs for a dimensionless time period of 9×10^{-2}. The canal then remains closed for an equal duration of time and runs again. This cycle of canal operation is repeated. If $K = 0.1$ m/day, $\varphi = 0.1$, and $E = 1000$m, a dimensionless running period of 9×10^{-2} would correspond to 180 days of canal running and the result would, therefore, depict the water table evolution for the running of canal for 180 days (six months). It would be seen from the figures that the water table height under the centre of the canal declines immediately after the closure of the canal. But, at locations beyond the recharging strip the water table continues to rise even after the closure of the canal. For example, in Fig.(3.11), at $x/E=0.4$, and at non-dimensional time $Kt/(2\varphi E) = 0.09$, the percentage rise in saturated thickness, $100.s(x,t)/E$ is 0.599. The water table continues to rise at this observation point even after the closure of the canal, and the percentage rise in

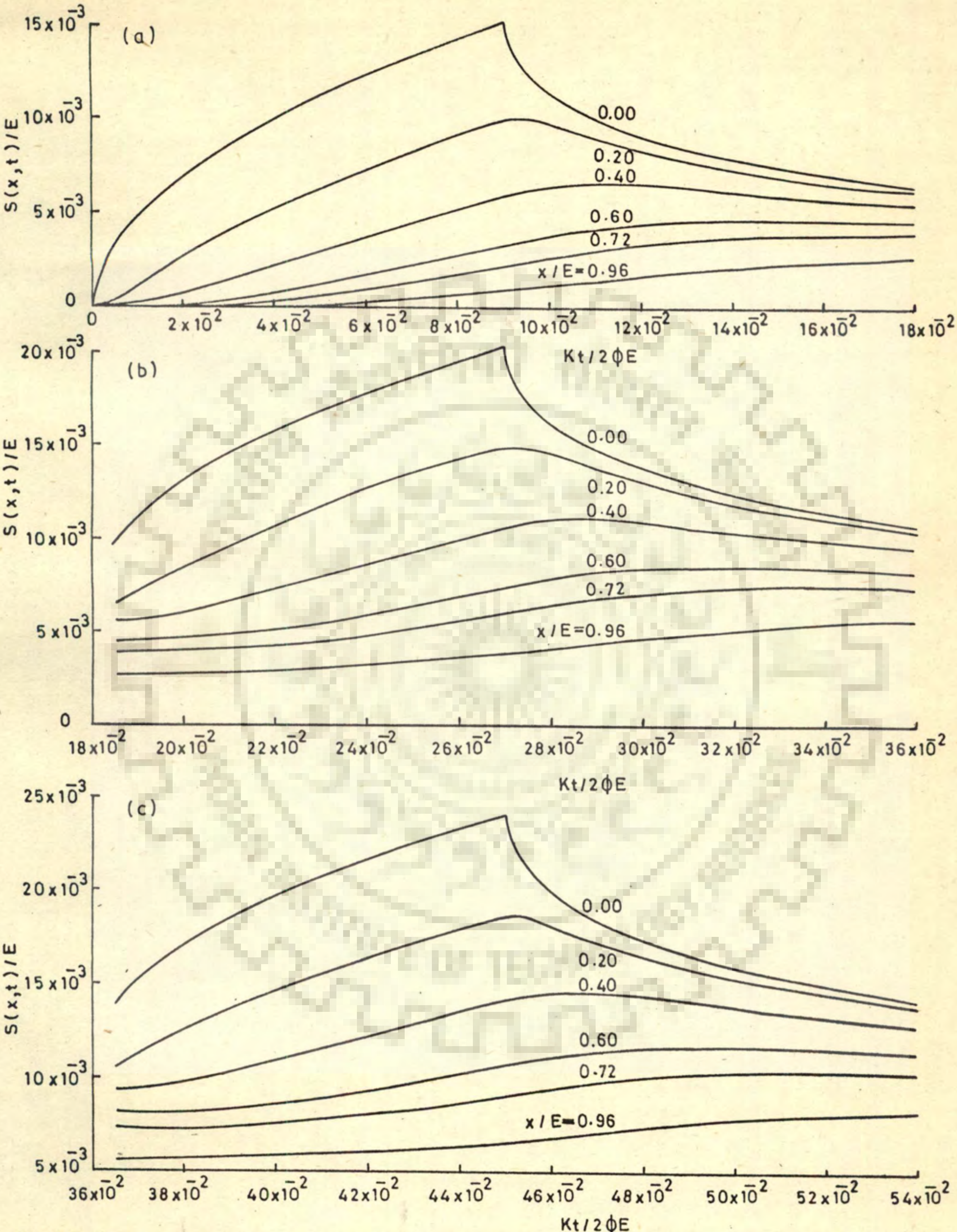

Fig.3.11.- Percentage rise in water table elevation at various observation points due to recharge from an intermittently running canal for B/E=0.06 and H/E=0.003

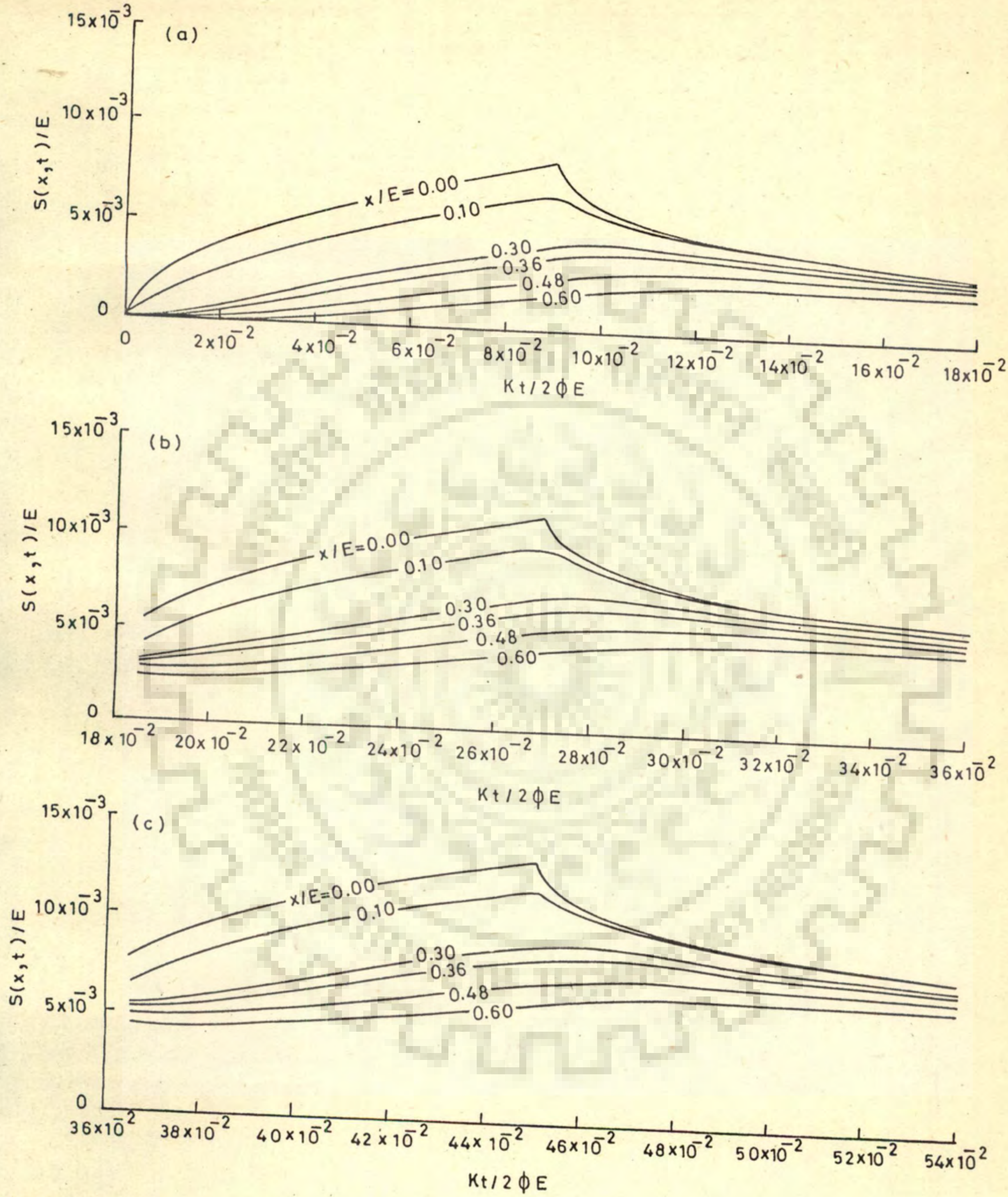

Fig.3.12–Percentage rise in water table elevation at various observation points due to recharge from an intermittently running canal for $B/E=0.03$ and $H/E=0.003$

water table height attains a maximum value of 0.668 at non -dimensional time of 0.11 and then starts declining in response to the first closure of the canal. In the region beyond the recharging strip, the aquifer exhibits a delayed response to the closure of the canal. It could be further seen that there is no reversal of flow any - where because of intermittent running of the canal. Therefore, at any time, the height of water table will be maximum at the centre of the canal and it would decrease as the distance from the centre of the canal increases. The water table evolution for canal of smaller width with B/E = 0.03, H/E = 0.003 is presented in Fig.(3.12). Comparing the results presented on Figs. (3.11) and (3.12) it is seen that for B/E =0.06 and H/E =0.003, the water table rise s (o,t)/E below the centre of the canal is 24×10^{-3} at the end of non-dimensional time $Kt/(2\varphi E) = 45 \times 10^{-2}$, whereas, for a canal of half the width, i.e., B/E=0.03 and H/E=0.003, the corresponding water table rise is 13.25×10^{-3} . Thus, the water table rise for the smaller canal gets reduced by 10.75×10^{-3}.

The water table evolutions at an observation point pertaining to continuous and intermittent running of a canal are depicted in Fig.(3.13) for comparison. It could be seen that if a canal with B/E=0.06 and H/E=0.006 runs

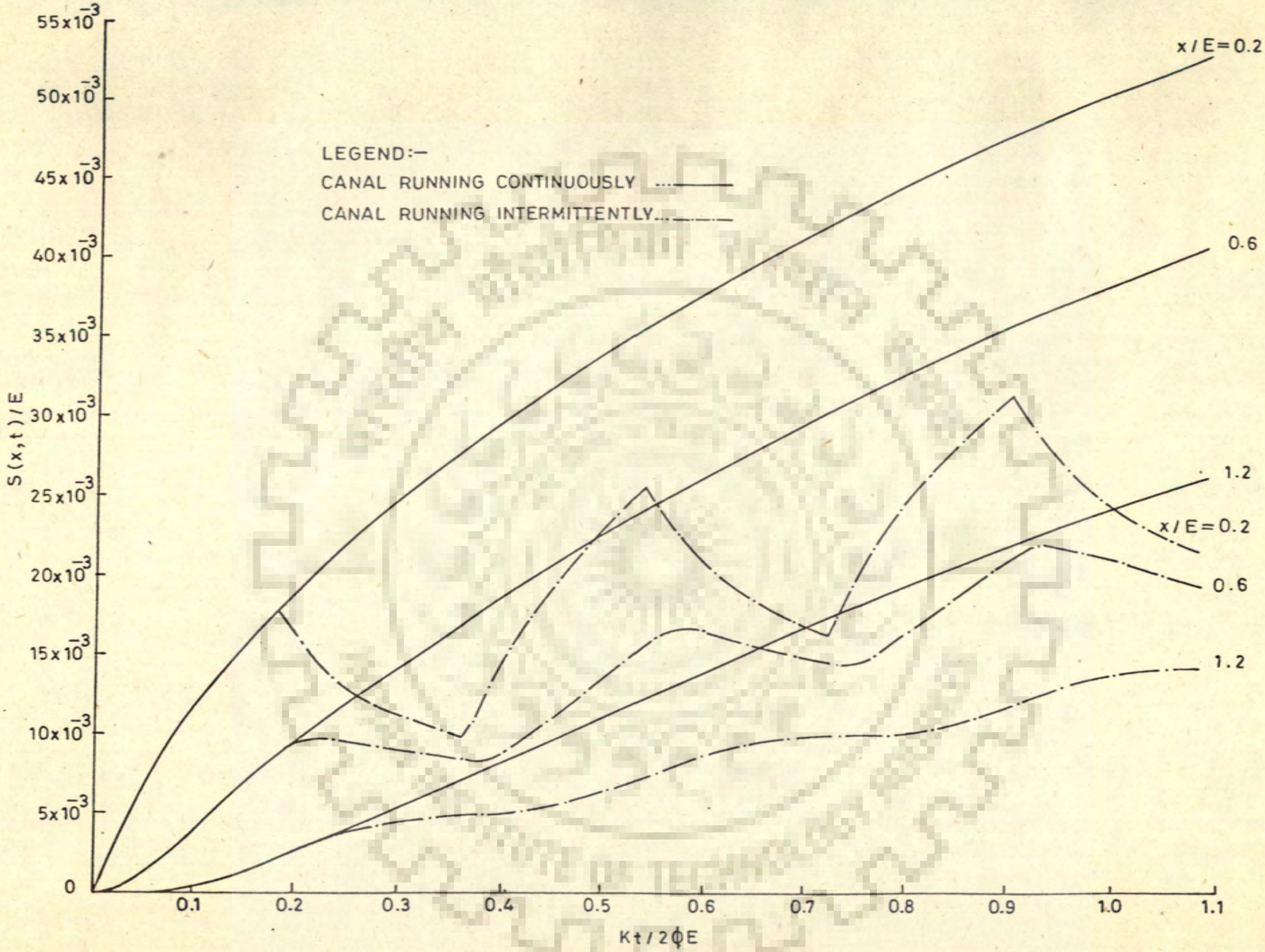

Fig.3.13—Water table evolution at observation points for continuous and intermittent running of a canal evaluated for $B/E = 0.06$, $H/E = 0.006$

for half of the time in a year (180 days) and is closed for the next half year, and this cycle is repeated, the percentage increase in water table height, 100.s(x,t)/E, at a distance x/E=1.2 from the canal in an aquifer with φ = 0.1, and K =0.1 m/day, is 1.426 after the end of 1080 days(3 years).If the canal had run continuously, the increase in water table height would have been 2.619 percent.

The evolution of water table due to recharge from parallel canal system is next considered. Water table evolution can be predicted for any number of parallel canals and for any running schedule of the canals by using method of superposition. In the present study, the water table evolution due to recharge from two parallel canals, which run continuously, has been determined for different spacings between them. The watertable rise at various times along a section across two identical parallel canals, each with 30m width and 3m depth of water,is presented in Figs.(3.14) and (3.15) for D=80 m, and 180m.The results presented in Figs.(3.16) and (3.17) are for canals with 60 m width. Though the water table evolution can be presented in non-dimensional form, the results for water table evolution have been presented with physical dimensions for an easy understanding of the process of evolution of water table.

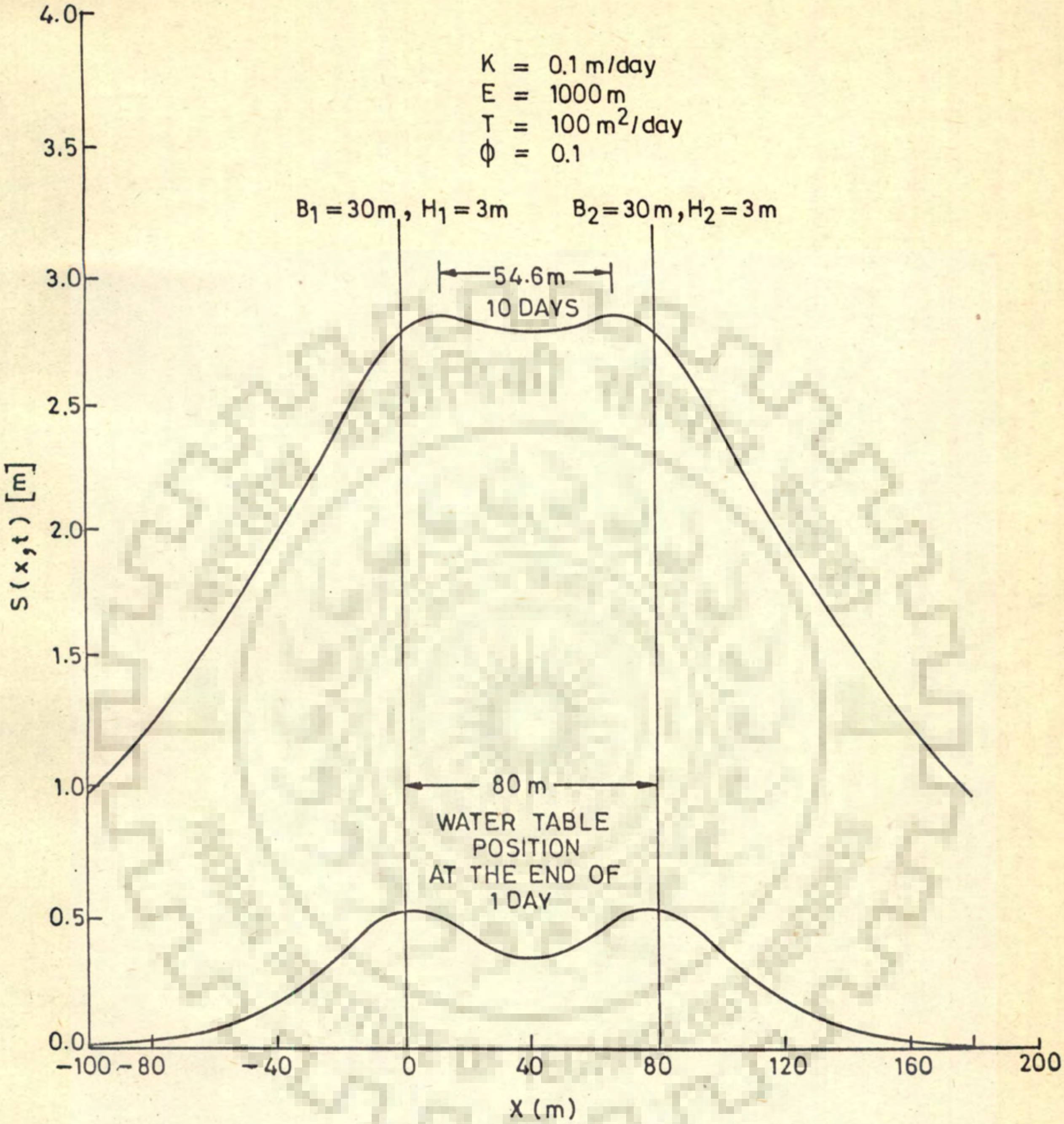

Fig.3.14 (a).- Water table rise due to recharge from two identical parallel canals of 30m width spaced at a distance of 80m apart

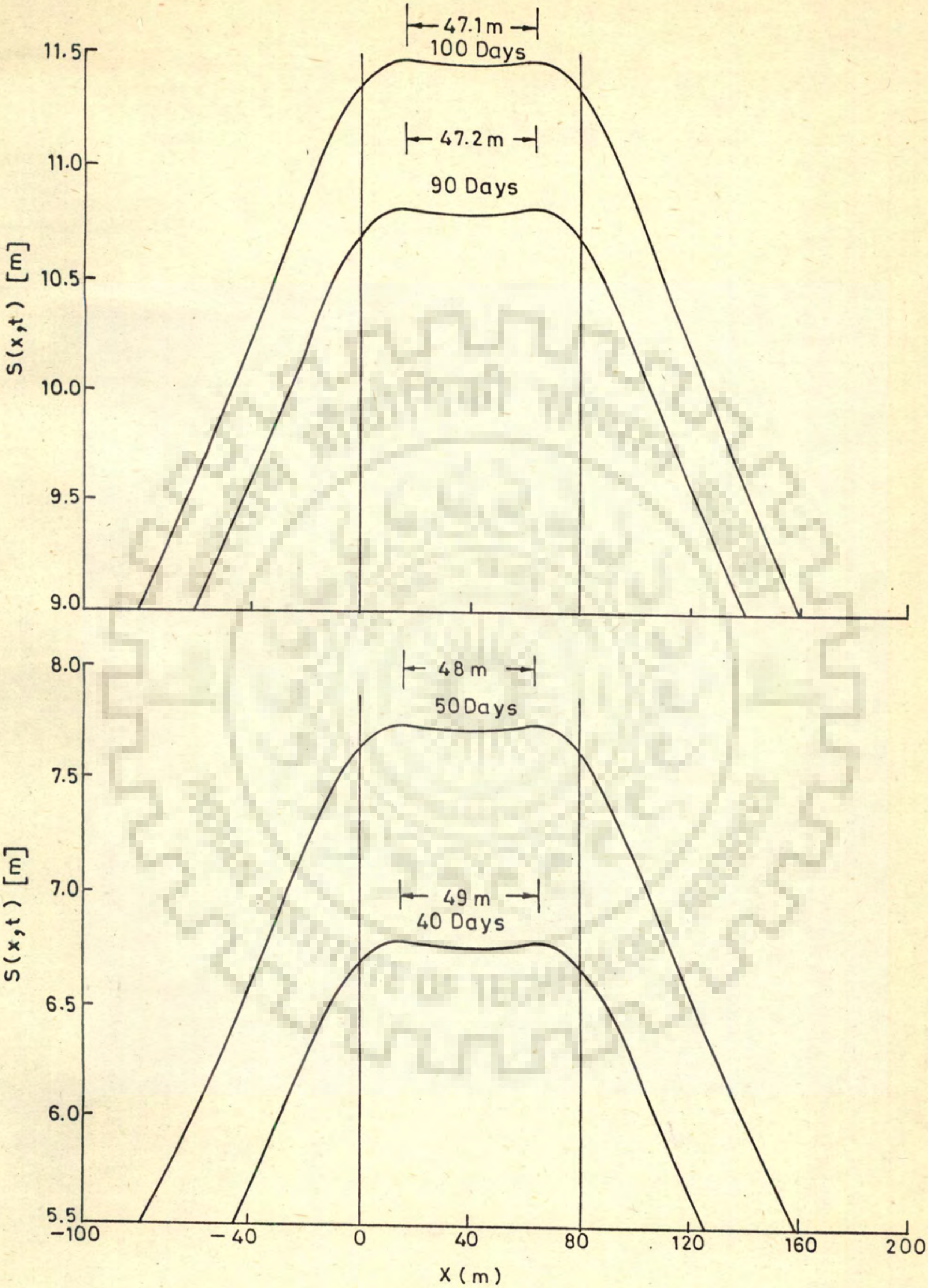

Fig.3.14 (b).- Water table rise due to recharge from two identical parallel canals of 30m width spaced at a distance of 80m apart

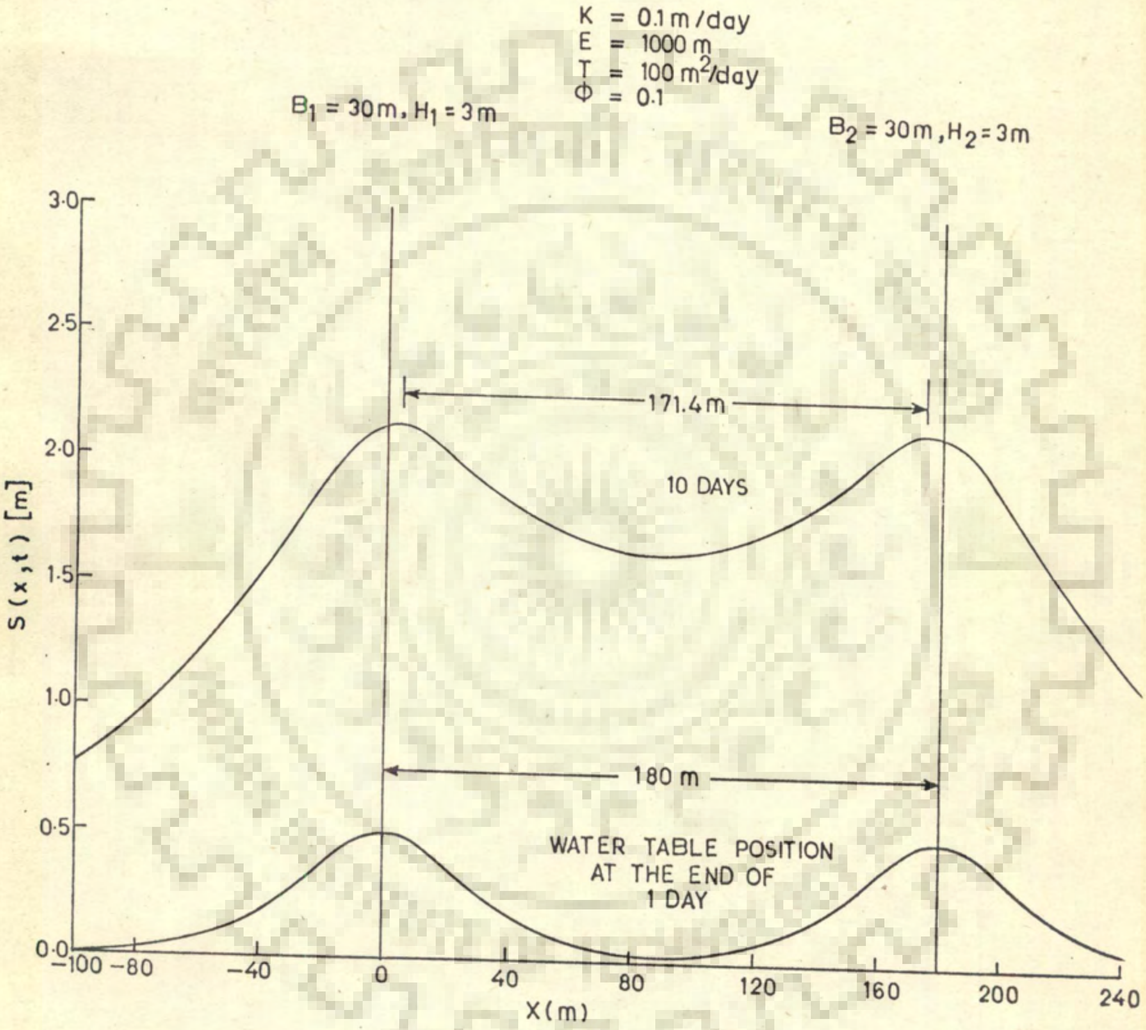

Fig.3.15 (a).-Water table rise due to recharge from two identical parallel canals of 30 m width spaced at a distance of 180 m apart

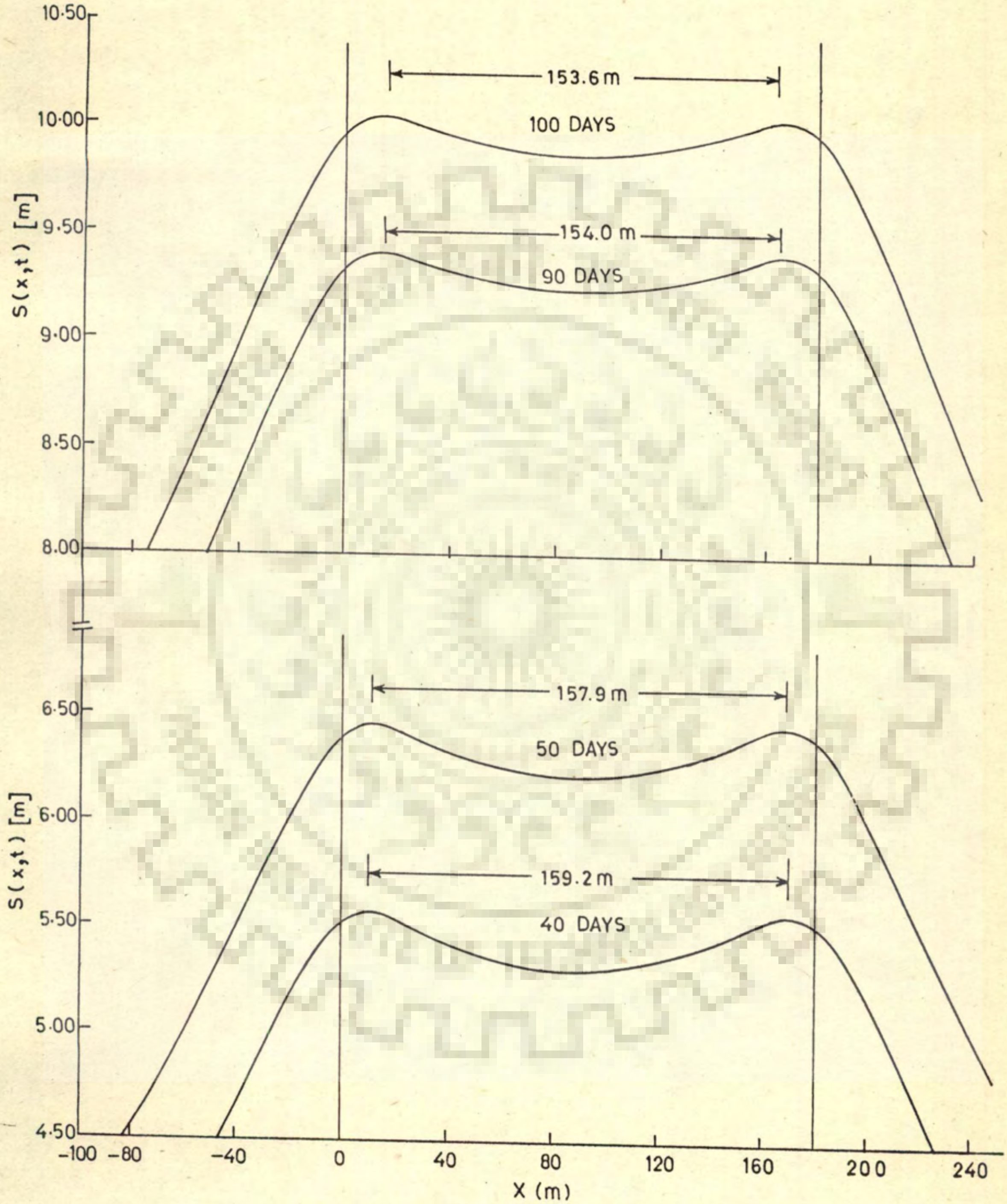

Fig.3.15 (b).- Water table rise due to recharge from two identical parallel canals of 30 m width spaced at a distance of 180 m apart

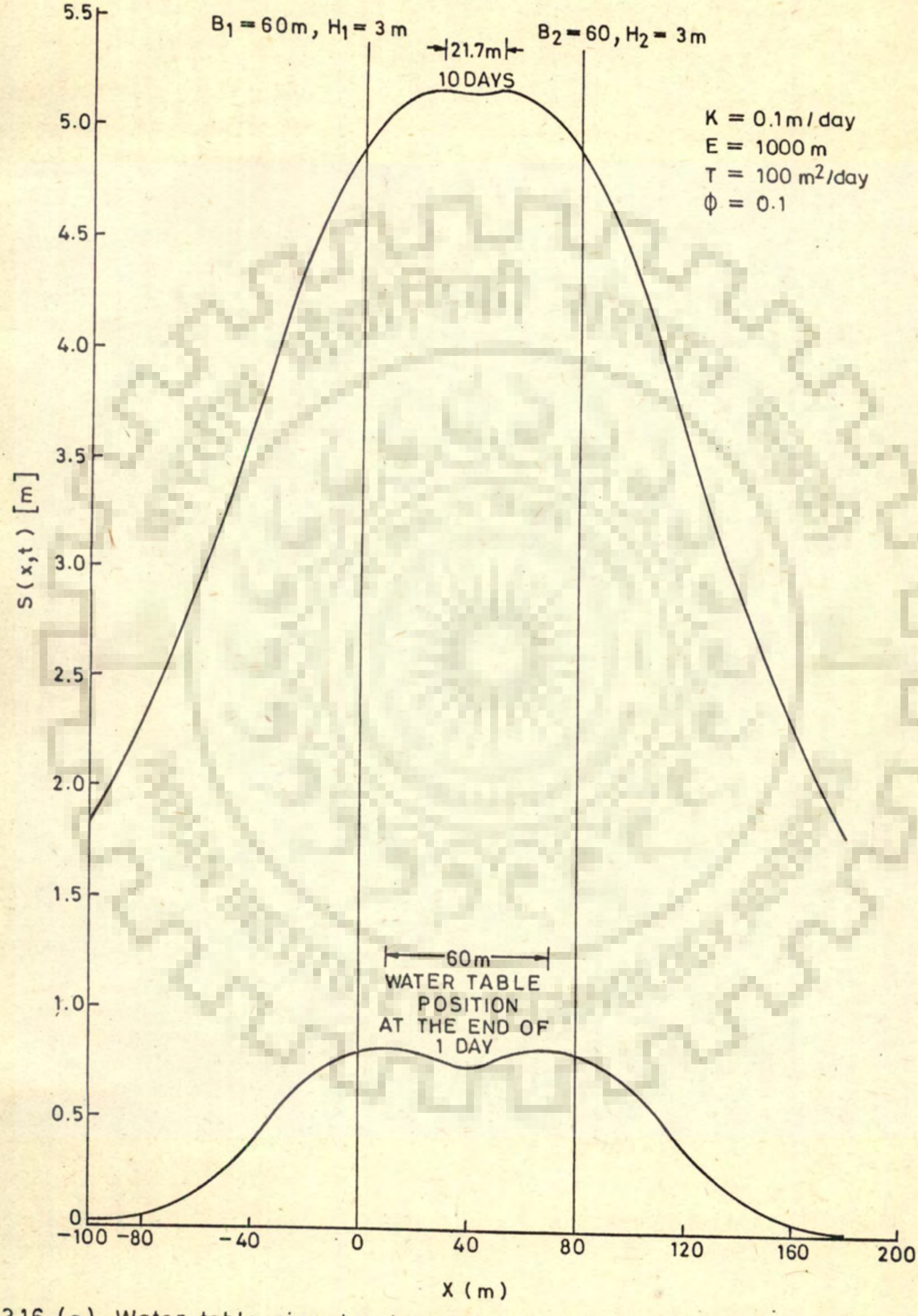

Fig.3.16.(a).–Water table rise due to recharge from two identical parallel canals of 60 m width spaced at a distance of 80 m apart

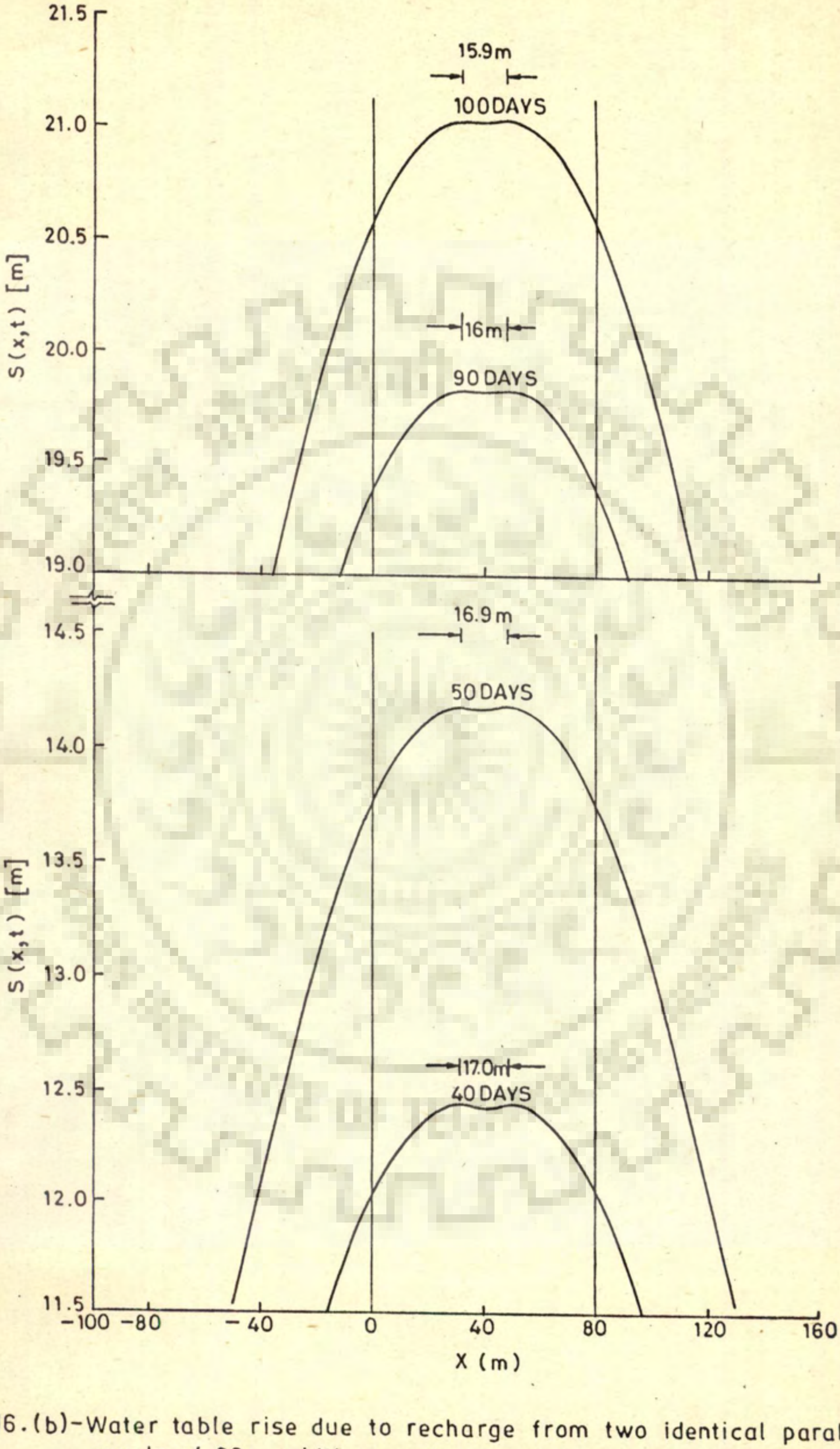

Fig.3.16.(b)-Water table rise due to recharge from two identical parallel canals of 60m width spaced at a distance of 80 m apart

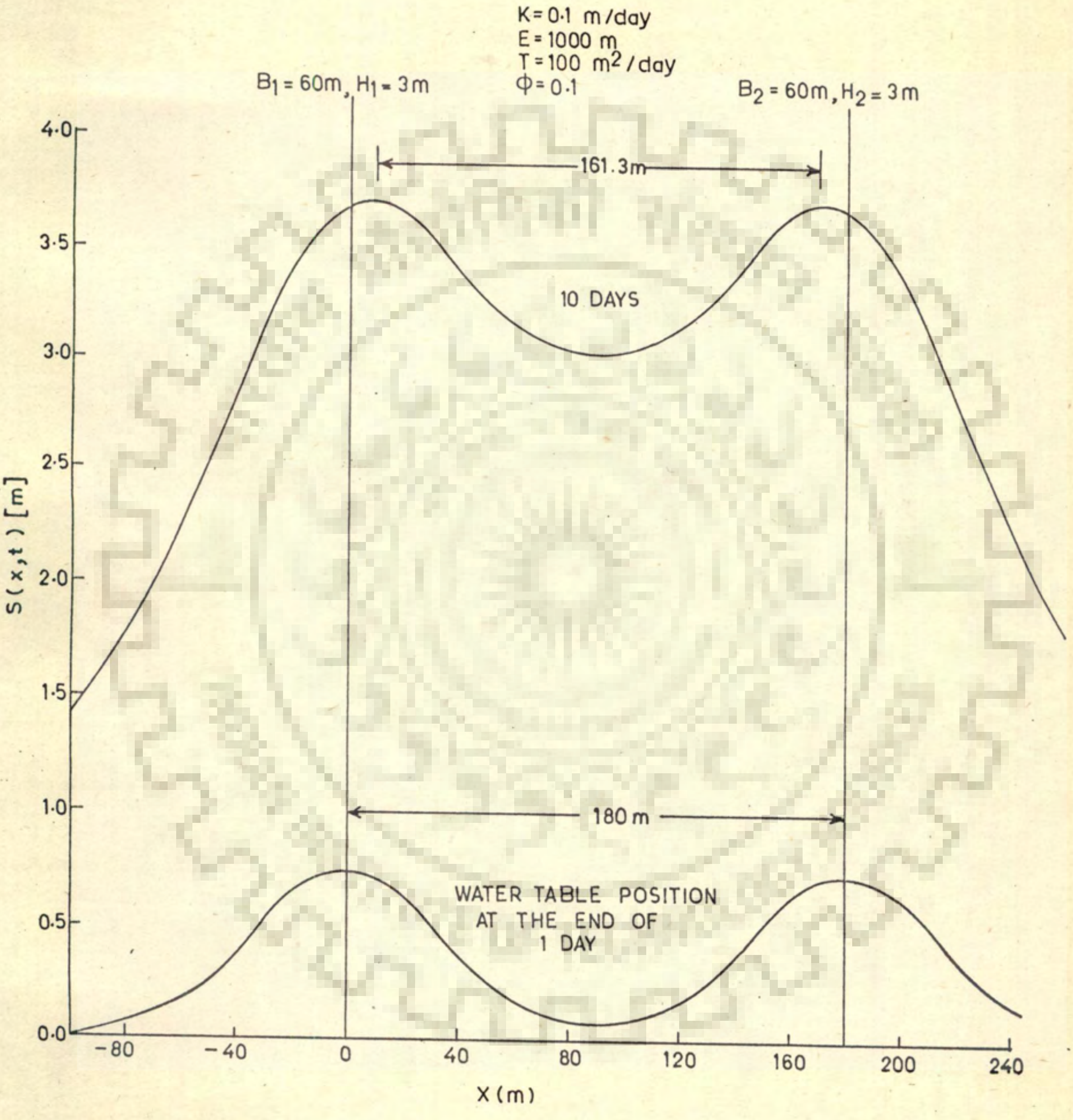

Fig3.17 (a).- Water table rise due to recharge from two identical parallel canals of 60 m width spaced at a distance of 180 m apart

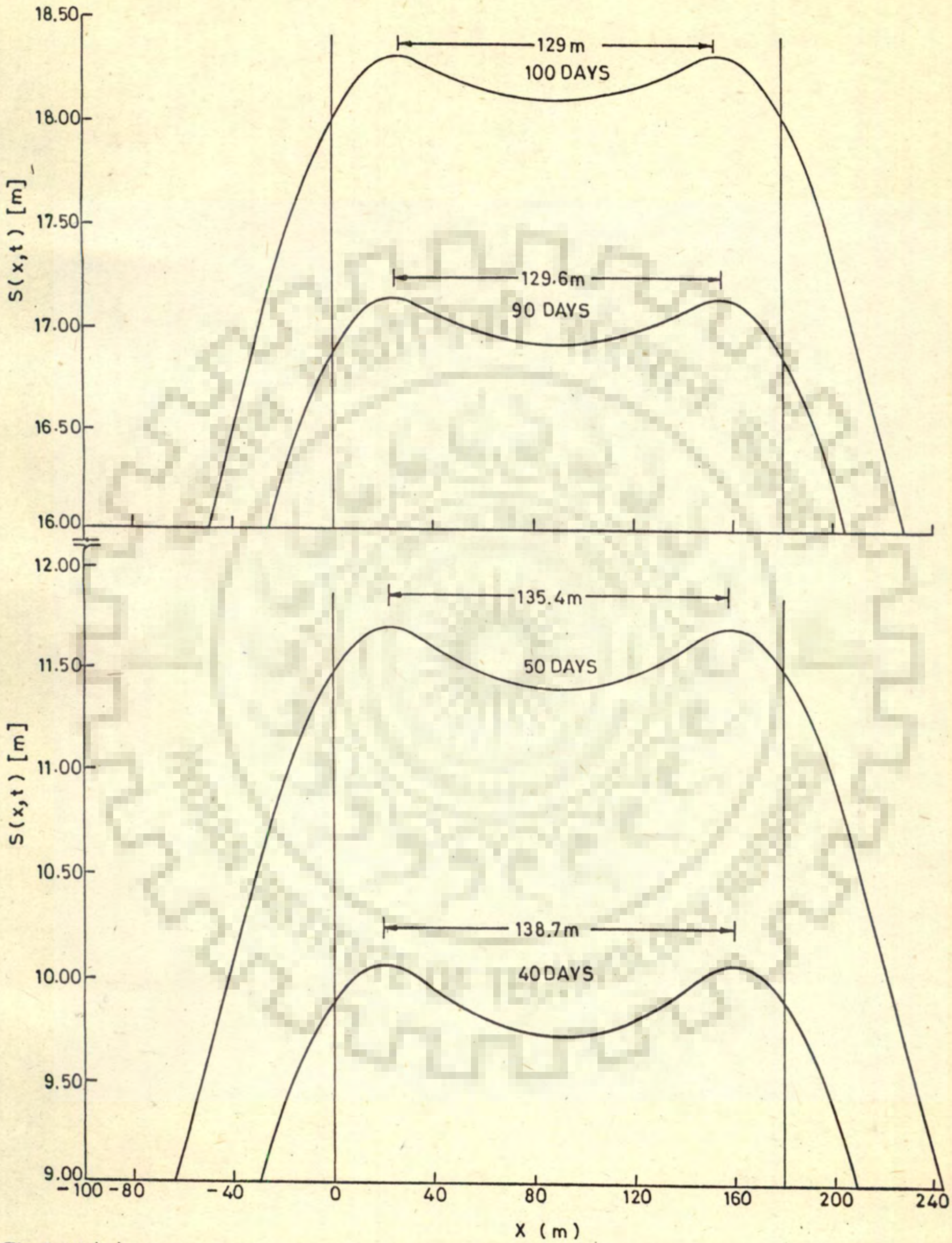

Fig.3.17 (b)–Water table rise due to recharge from two identical parallel canals of 60m width spaced at a distance of 180 m apart

It could be seen from the figures that, in the beginning of recharge, the two water table ridges are located at the centres of the recharging strips. As the recharge continues, the points of maximum rise move towards each other. The points of maximum rise, though continue to move towards each other, do not cross a distance of (B/2+H) from the centre of the canal, where B is the width of each of the parallel canals. In the very beginning, the recharge from each canal is equally divided in the aquifer,i.e.,under each canal, a quantity of $\frac{K}{2}.(B + 2H)$ flows in the aquifer in horizontal direction on either side of canal. The quantities of flow entering to the aquifer between the canals oppose each other with the result that a stagnant zone gets created between the canals. Therefore, with lapse of time, the quantity of recharge under each canal gets unequally divided, resulting in more than half of the recharge quantity from each canal flowing to the external sides. The quantity flowing to external side increases with time which is evident by the steeper slope of the water table at higher time. The points of maximum rise do not cross the recharging strip since a fraction of recharge from each canal would still be going in the zone in between the canals even after a very long duration of time.However, as time elapses, the water table in the zone in between the canals tends to become flat. It can also be seen from Figs. (3.14) and (3.15) (for 30 m width), and (3.16) and (3.17) for (60 m width), that at the end of a period of 100 days of continuous running of the parallel canals, the zone in between the canals becomes more flat when spacing is 80 m as compared to the case having 180 m spacing.

The percentage rise in water table (s/E) at different non-dimensional time for various locations across the parallel canals have been presented in Figs.(3.18) through (3.21) for different canal dimensions and spacings between them. Figs. (3.18) and (3.19) depict the rise in water table (s/E) at different locations (x/E) for canals with B/E=0.03 and H/E =0.003. It is seen from the results that percentage of rise of water table at x/E = -0.6 for D/E = 0.08, and 0.18 is 0.112, and 0.094 respectively at the end of non-dimensional time of 5×10^{-2}. The corresponding results for canals with B/E = 0.06 are 0.332, and 0.279 [Figs. (3.20 and (3.21)]. These results indicate that as the spacing between the canals, D/E, increases, the percent rise of water table at any point outside the parallel canals decreases.

The maximum rise of water table for different widths of parallel canals and spacings between them are

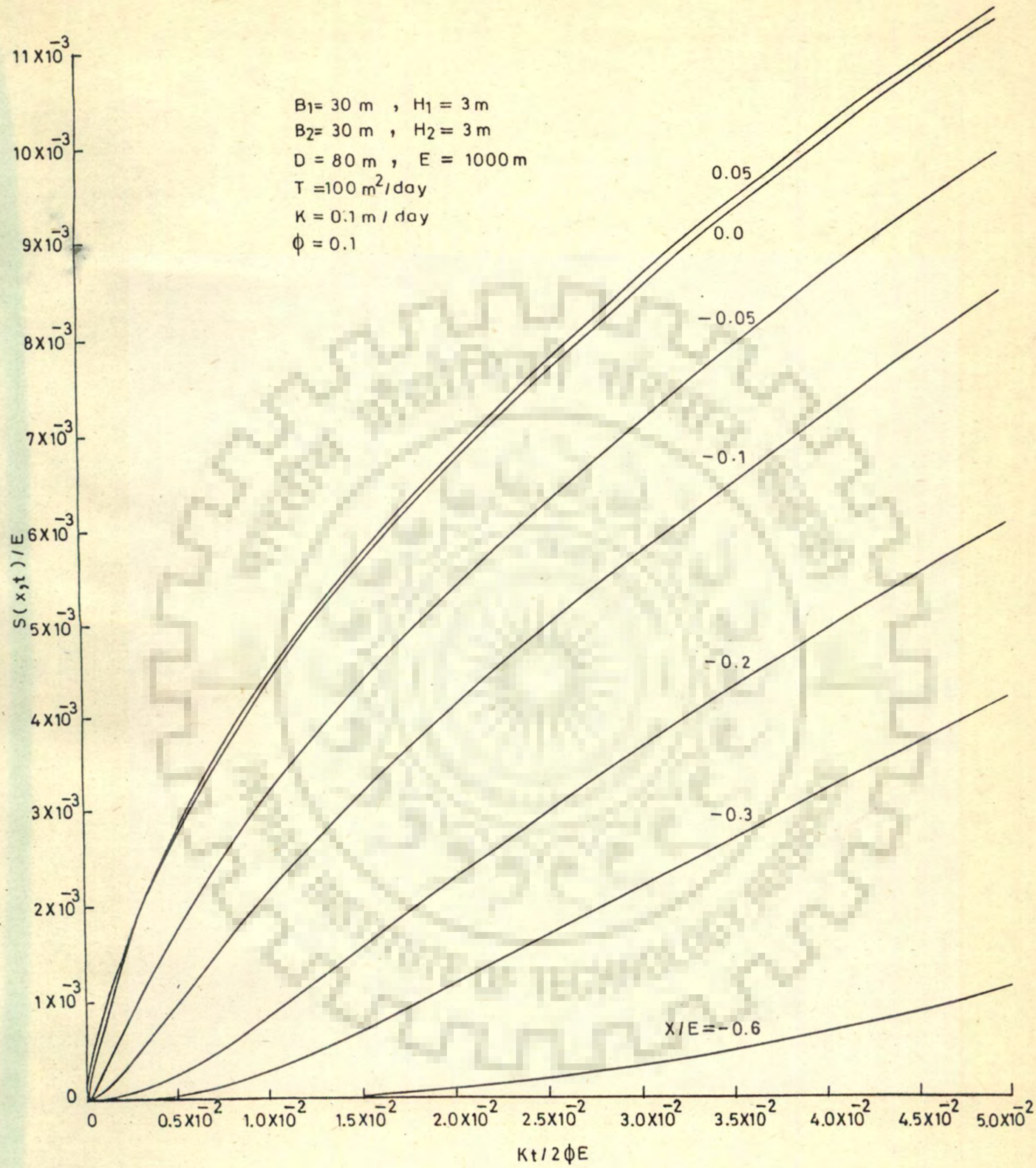

Fig.3.18 – Variation of water table rise due to recharge from two identical parallel canals of width B/E = 0.03 spaced at a distance of D/E = 0.080

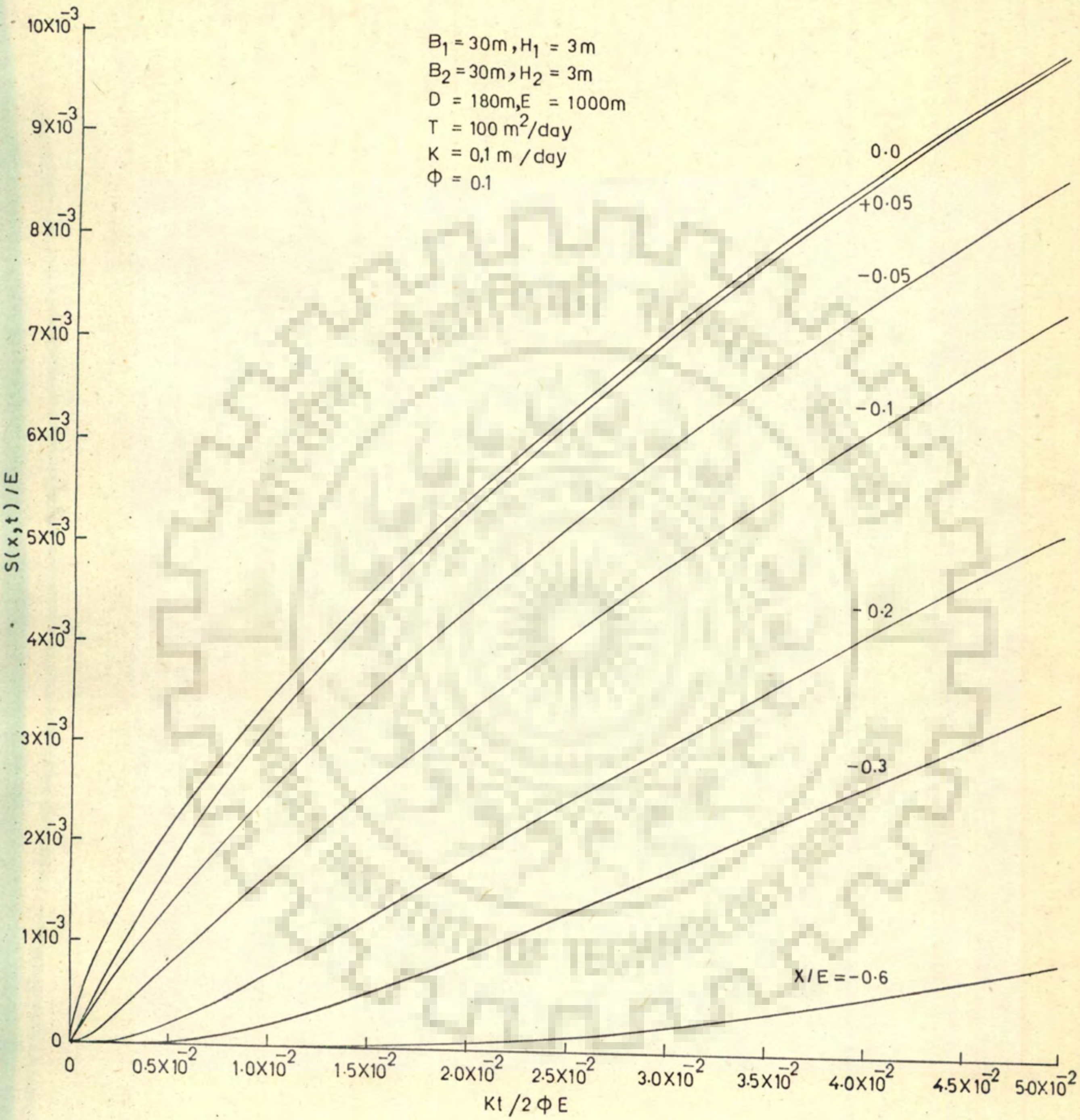

Fig. 3.19 — Variation of water table rise due to recharge from two identical parallel canals of width B/E = 0.03 spaced at a distance of D/E = 0.180

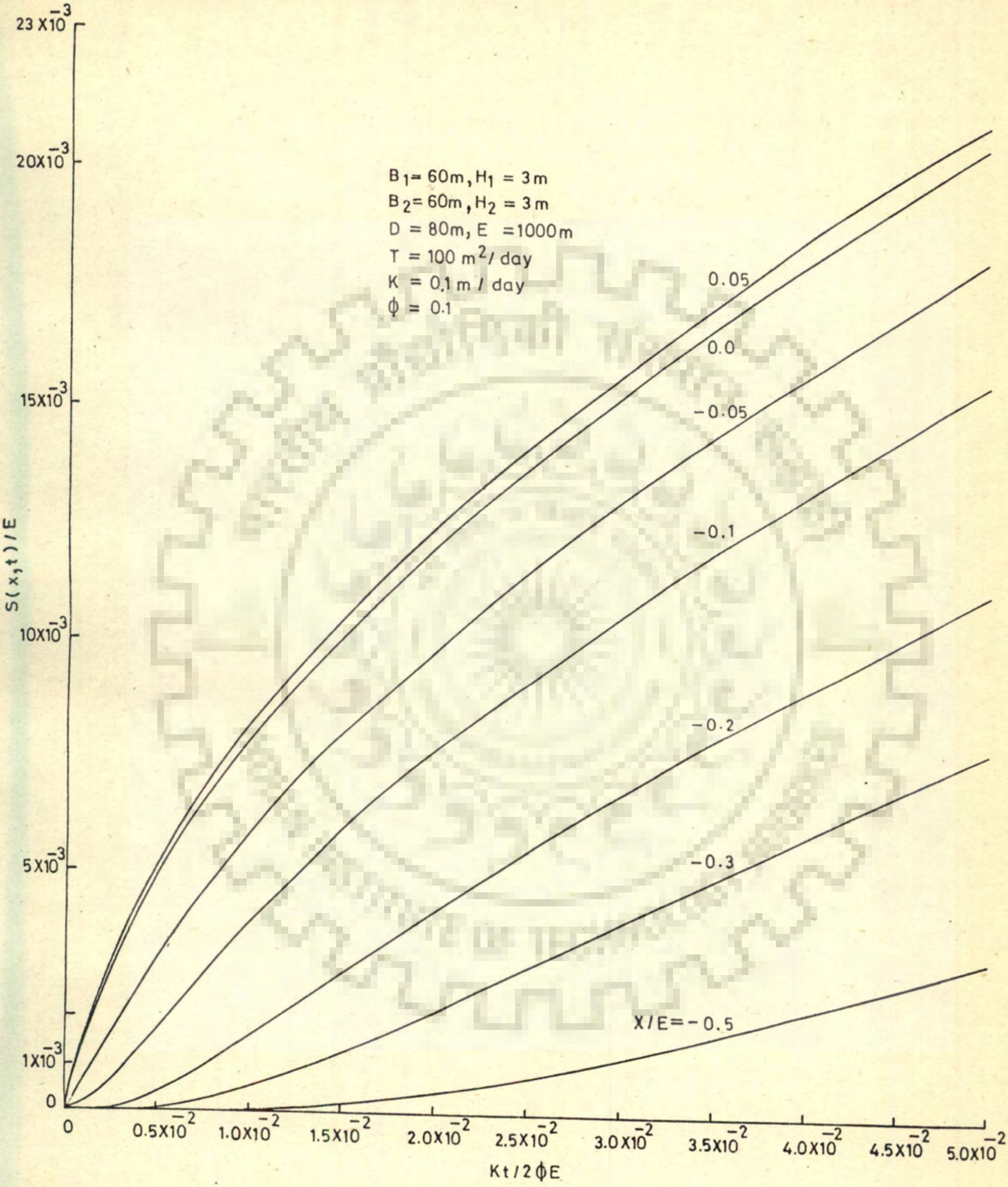

Fig.3.20 — Variation of water table rise due to recharge from two identical parallel canals of width $B/E = 0.06$ spaced at a distance of $D/E = 0.080$

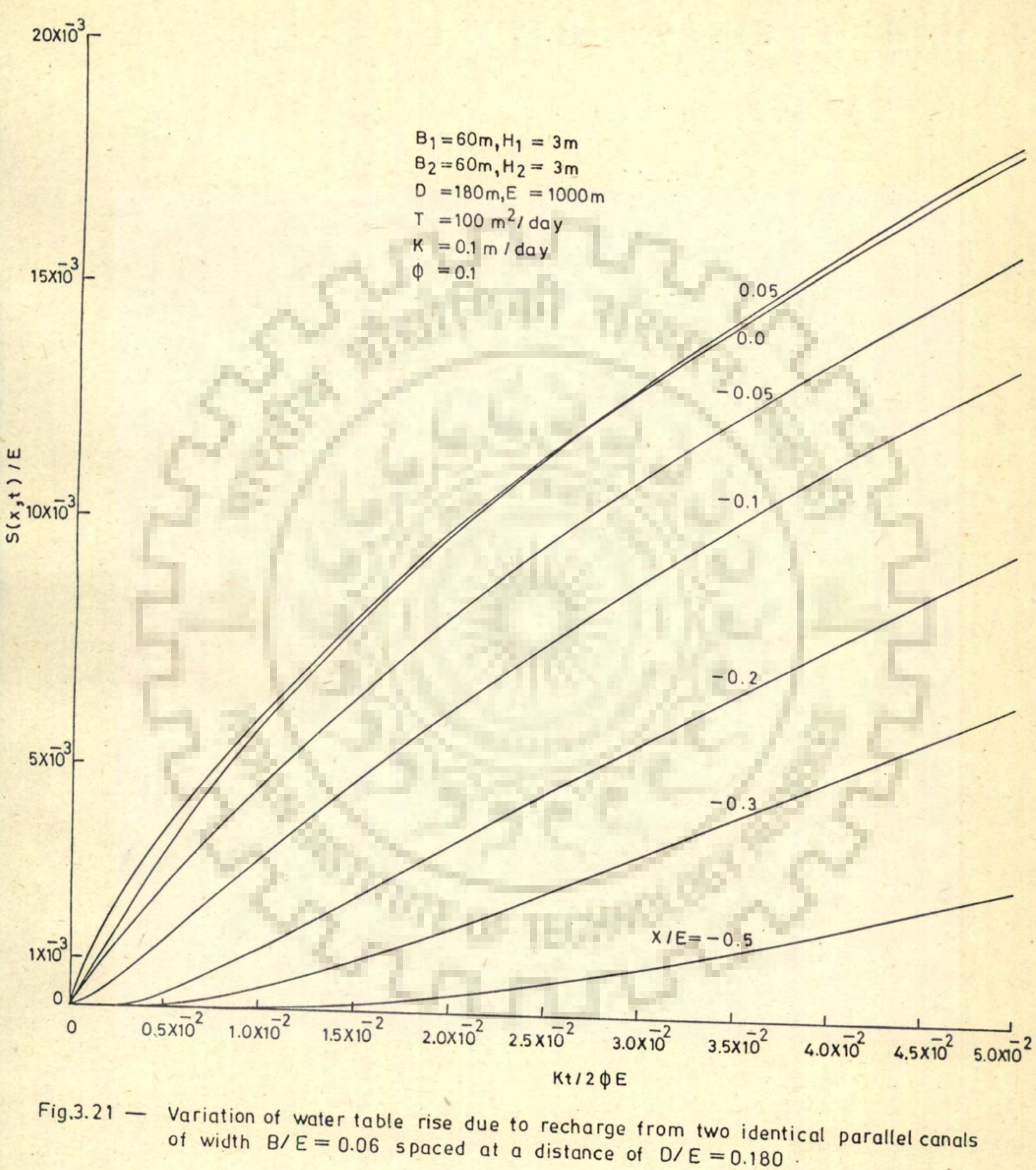

Fig.3.21 — Variation of water table rise due to recharge from two identical parallel canals of width $B/E = 0.06$ spaced at a distance of $D/E = 0.180$.

presented in Tables (3.8) and (3.9). The results show the reduction in maximum rise as spacing between the parallel canals is varied from 80m to 480m. It is seen from the table that for identical parallel canals for spacings, D/E, upto 0.48 and beyond a time factor 0.15, the difference in rise of water table at the middle point between the canals and maximum rise is insignificant. This shows that the region between the canals takes the shape of a plateau ultimately and becomes a stagnant zone.

A comparison of results of rise of water table due to recharge from a single canal,($D=\infty$),with 30m width and 3 m depth of water, and that due to parallel canals of the same dimensions spaced at 120 m apart, can be made from Table (3.8).It is seen that maximum percentage rise for single canal, (s/E),at the non-dimensional time factor of 0.045 is 0.593,whereas the maximum percentage rise for parallel canals is 1.022.If the spacing increases to 480 m, the corresponding percentage of rise is 0.693.This shows that for spacing of parallel canals of more than sixteen times the width of the canal, the interference of canals can be considered as insignificant upto a time factor 0.045.

In the results presented above, the two parallel canals were identical. The case of unequal parallel

Table 3.8 - Effect of spacing of parallel canals on maximum rise of water table determined for $B_1 = B_2 = 30m$, $H_1 = H_2 = 3.0$ m.

Spacing between canals, D (m)	Non-dimensional Kt/(2 φ E)	Maximum rise as percentage of saturated thickness, E	Percentage rise at middle point between the canals
80	0.015	0.573	0.570
	0.045	1.082	1.080
	0.150	2.085	2.084
120	0.015	0.520	0.509
	0.045	1.022	1.015
	0.150	2.019	2.016
180	0.015	0.459	0.427
	0.045	0.941	0.922
	0.150	1.926	1.916
240	0.015	0.414	0.355
	0.045	0.871	0.835
	0.150	1.840	1.820
480	0.015	0.344	0.153
	0.045	0.693	0.545
	0.150	1.557	1.467
∞	0.015	0.336	-
	0.045	0.593	-
	0.150	1.096	-

Table 3.9 - Effect of spacing of parallel canals on maximum rise of water table determined for $B_1 = B_2 = 60m$, $H_1 = H_2 = 3.0m$.

Spacing between canals, D (m)	Non-dimensional time $Kt/(2 \varphi E)$	Maximum rise as percentage of saturated thickness	Percentage rise at middel point between the canals
80	0.015	1.048	1.047
	0.045	1.983	1.982
	0.150	3.822	3.822
120	0.015	0.946	0.936
	0.045	1.868	1.863
	0.150	3.699	3.697
180	0.015	0.826	0.786
	0.045	1.715	1.692
	0.150	3.526	3.514
240	0.015	0.741	0.653
	0.045	1.581	1.533
	0.150	3.364	3.337
480	0.015	0.608	0.282
	0.045	1.249	1.000
	0.150	2.838	2.690
∞	0.015	0.592	-
	0.045	1.064	-
	0.150	1.986	-

canals is next considered to study the rise of water table. The results of rise of water table pertaining to two parallel canals, one of the canals with 60m width and 3m depth of water and the other with 30m width and 3 m depth of water, are depicted in Figs. (3.22) and (3.23) for spacings of 80 m and 180 m respectively between the canals. From the shape of the water table shown in the figures, it is indicated that in the beginning of recharge the water table mound is formed under each recharging strip. It could further be seen from the figures that with lapse of time the water table mound under the canal having less strength gets diminished. Ultimately the water table exhibits one maximum point where slope of water table is zero. This phenomenon happens quicker when the spacing between the canals is smaller. In the beginning of recharge, the recharge from each strip gets distributed in the aquifer on either side of each strip. It could be deduced from shape of the water table mound that with passage of time the recharge from right canal with lower strength, flows towards the right.

Fig.(3.22) shows that when spacing between the left canal of 60 m width,and right canal of 30 m width, is 80 m,the maximum point of rise of water table under the canal shifts by 17 m from the centre of the canal

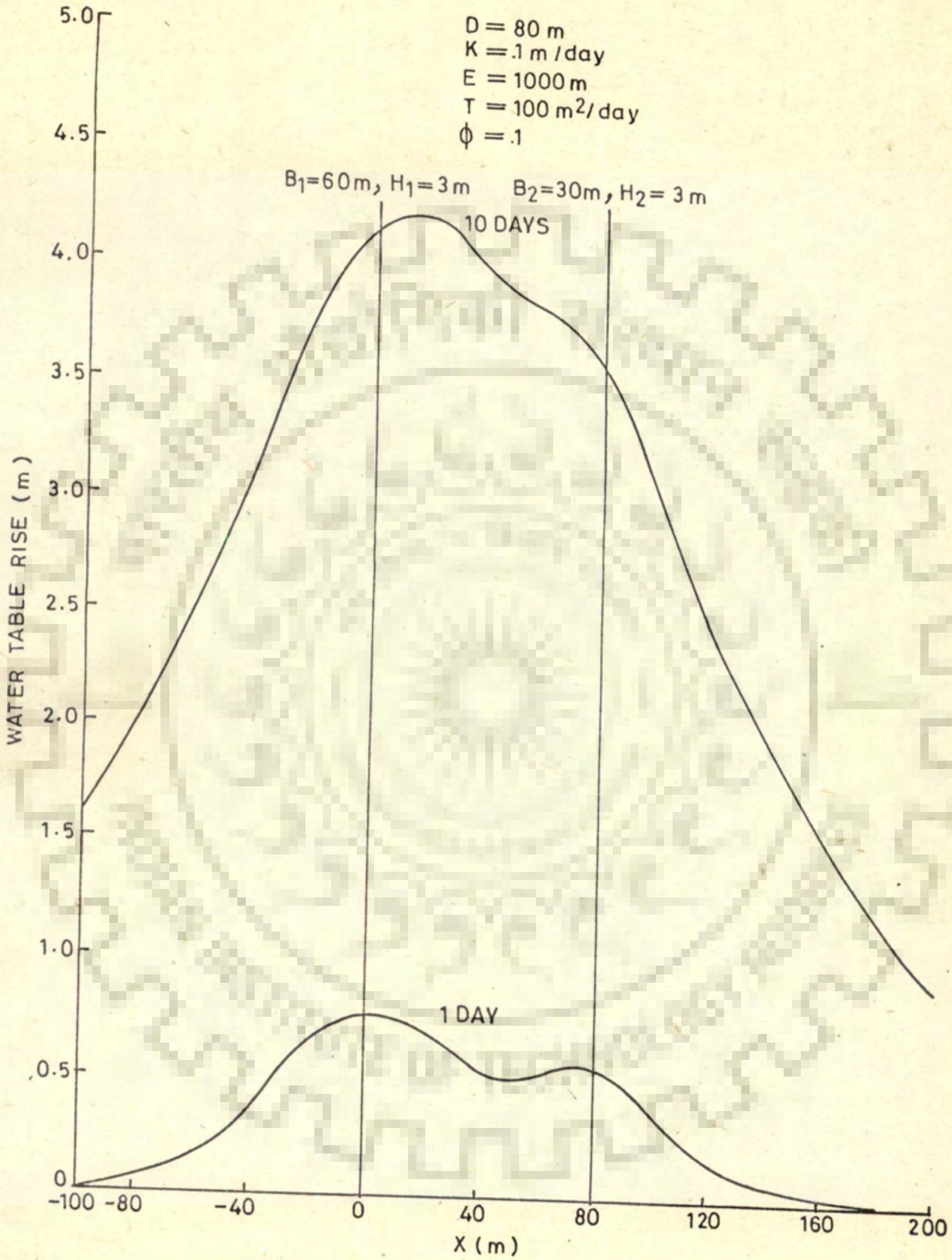

Fig.3.22(a)–Water table rise due to recharge from two unequal parallel canals spaced at 80m apart

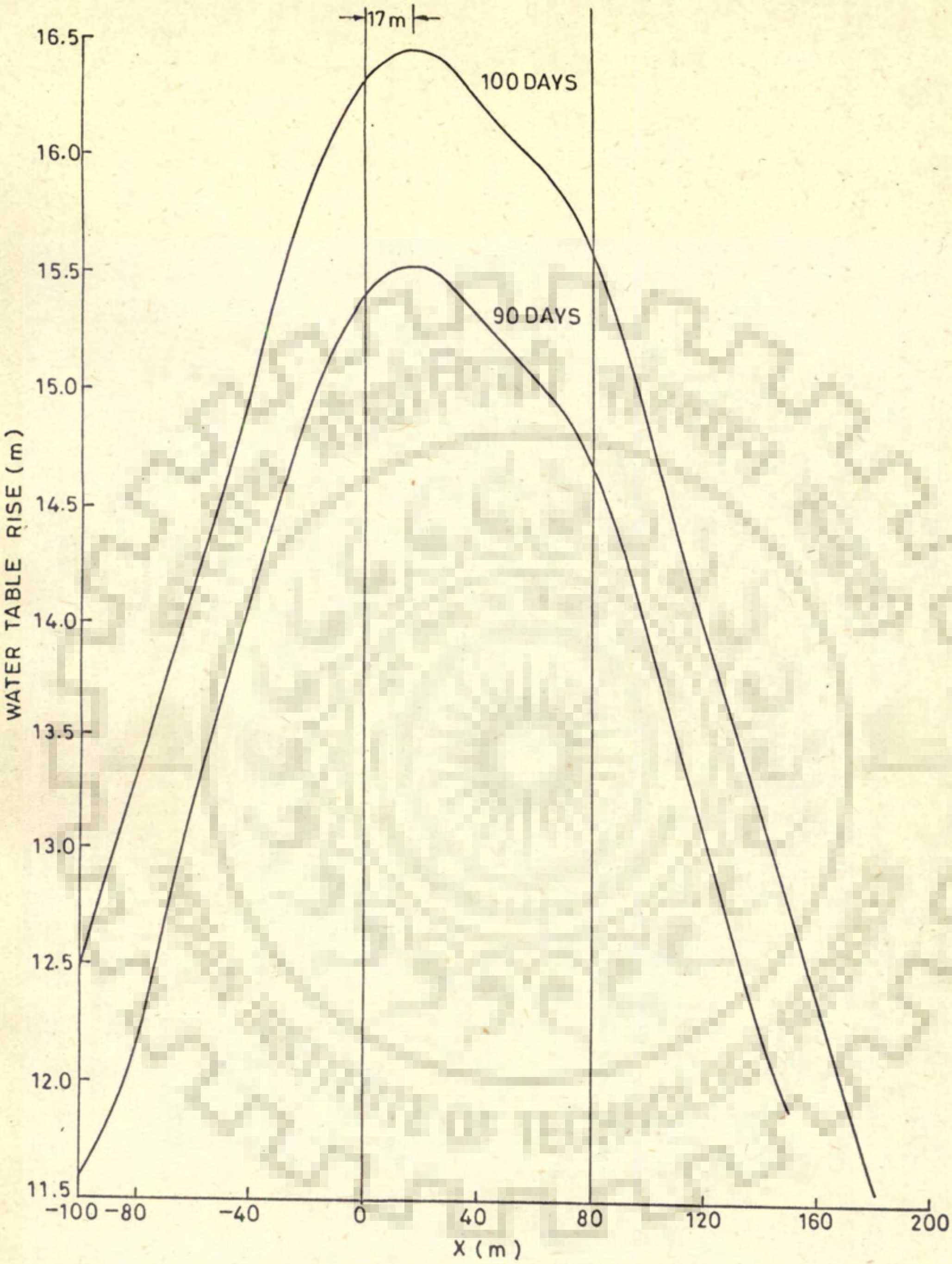

Fig.3.22 (b)-Water table rise due to recharge from two unequal parallel canals spaced at 80 m apart

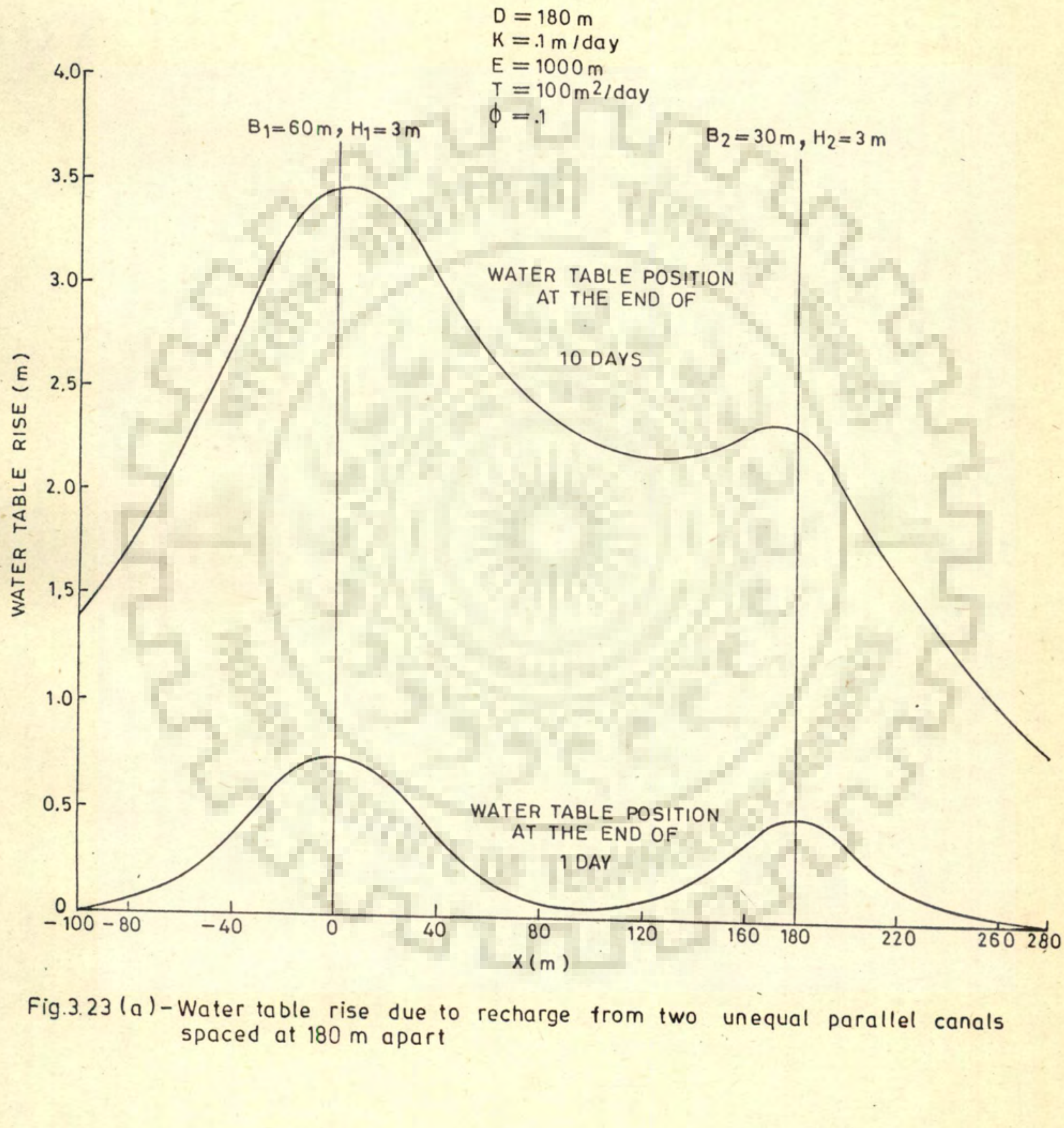

Fig.3.23 (a) - Water table rise due to recharge from two unequal parallel canals spaced at 180 m apart

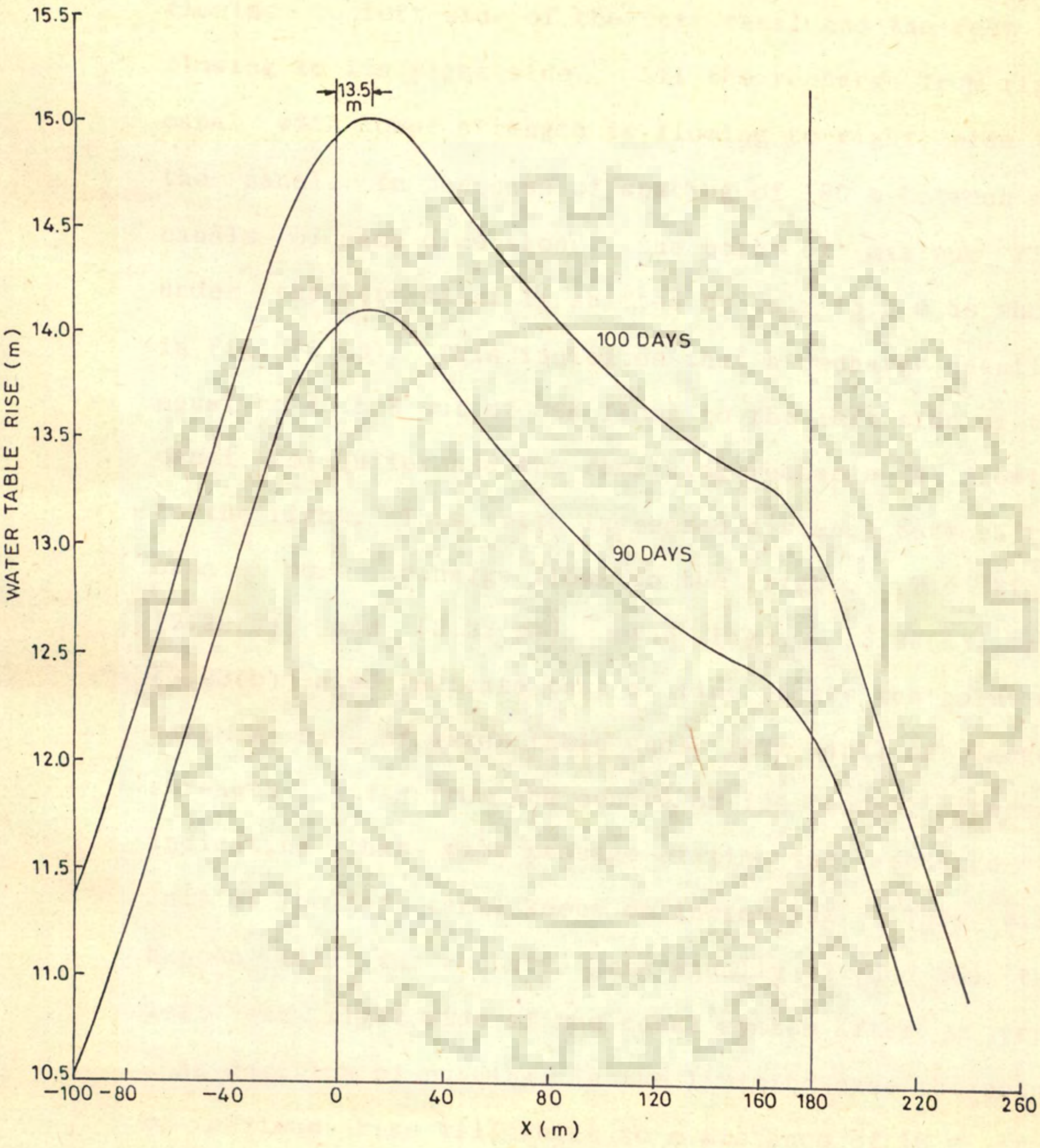

Fig.3.23 (b)—Water table rise due to recharge from two unequal parallel canals spaced at 180 m apart

3.4 Conclusions

Based on the study presented, the following conclusions are drawn:-

1. Using method of superposition, and starting from Glover's basic solution of one dimensional Boussinesq's equation for a line source, closed form solutions for evolution of water table within and outside a strip source have been derived. The solutions are identical to the solutions which have been given by Polubarinova-Kochina.

   Making use of the solutions, the evolutions of water table for intermittently running canal and for parallel canals have been obtained.

2. The results obtained by using the derived analytical solutions for a strip source compare well with the numerical solutions of Hantush and Shestakov.

3. Glover's method over estimates the water table rise at all times below the centre of the canal. At other points inside the recharging strip, in the beginning of recharge the Glover's approach under estimates the water table rise. But, with lapse of time, the water table calculated by Glover's solution is higher than the rise

after a period of 100 days of ruuning of the two canals. This shows that recharge quantity of 50 K out of 66 K is flowing to left side of the left canal and the rest is flowing to its right side. All the recharge from right canal with lower strength is flowing to right side of the canal. In the case of spacing of 180 m between the canals of same dimensions, the point of maximum rise under the left canal is shifted by only 13.5 m as shown in Fig. (3.23). This indicates that a recharge quantity equal to 46.5 K out of 66K flows to the left side of the canal to aquifer and the remaining recharge is flowing to the right. Thus, with increased distance between the canals more recharge flows to the right. The Figs. [3.22(a)] and [3.22(b)], and Figs. [3.23(a)] and [3.23(b)] also indicate that as time passes the point of maximum rise of water table under left canal of larger strength shifts from the centre of the canal, thereby indicating that with passage of time the recharge to left of the left canal keeps on increasing. This will happen till the recharge gets equally divided on the left and right side of the canal system after a very long duration of running. In the limiting case the point of maximum rise will shift to a distance of 18 m from the centre of left canal so that recharge at that stage is equally divided on both sides of the system.

calculated by the present solution. In the region outside the recharging strip, Glover's solution under- estimates the water table rise marginally.

4. It is found that, when the strip source is replaced by an identical line source, the water table evolution beyond the recharging strip could be predicted without any appreciable error using Glover's solution.

5. The absolute value of water table gradient at any point outside the recharging strip attains a limiting value of 0.5(B/E + 2H/E) after a long time. At any point within the recharging strip the limiting value of the gradient attained after a long time is x/E, where, x is the distance of the observation point from the centre of the canal. The saturated thickness of aquifer can be predicted by making use of the limiting value of the gradient of water table in the vicinity of the canal.

6. For a canal having B/E =0.06 and H/E = 0.006, the percentage rise of water table due to recharge, when the canal runs continuously is 2.62 at the end of non-dimensional time factor 1.08. If the canal runs intetmittently for a period of 0.18 and is closed for equal duration, and this cycle

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is repeated, the rise in water table at non-dimensional time of 1.08 is 1.43 percent. The water table rise is reduced considerably due to intermittent running of the canal.

7. If a canal runs intermittently, the water table below the canal declines immediately after the canal is closed. In the region beyond the recharging strip the aquifer exhibits a delayed response to the closure of the canal. There is, however, no reversal of flow any where on account of intermittent running of the canal, and, at any time the height of water table is maximum at the centre of the canal and it decreases as the distance from the centre of the canal increases.

8. In the case of two identical parallel canals, in the beginning of recharge, the two water table ridges are located at the centres of the recharging strips. However, as time elapses, the points of maximum rise move towards each other but they do not move beyond the respective recharging strip,i.e.,none of the points of maximum rise crosses a distance of (B/2+H) from the respective centre of the canal.With lapse of time the flows entering the aquifer between canals oppose each other due to which a stagnant zone

gets created between the canals. The region between the two parallel canals under continuous recharge takes the shape of a plateau in course of time. It is also found that for canal systems having smaller distance between them, the water table in the stagnant zone becomes flat at earlier time.

9. The results of rise of water table due to two parallel canals have shown that if the distance between the two canals is 16 times the width of the strip (canal), the interference of canals is insignificant upto a time factor of 0.045.

10. It is seen that in the case of unequal parallel canals, some time after the onset of recharge there is only one point of maximum rise, where the slope of water table is zero, and it lies under the canal of larger strength. With passage of time, the recharge from canal of smaller strength flows only to the right side of the canal.

Based on a part of the work reported in the Chapter, the following paper has been published:

1. Bhargava, D.N., Mishra, G.C., Chandra, S.(1987), "Evolution of water table due to seepage from two parallel canals", International Symposium on Groundwater Monitoring and Management, Dresden, GDR, Published by the Institute of Water Management, Berlin (GDR).

ANALYSIS OF UNSTEADY SEEPAGE FROM A CANAL WHICH IS NONLINEARLY DEPENDENT ON THE POTENTIAL DIFFERENCE BETWEEN THE CANAL AND THE AQUIFER

4.0 Introduction

In chapter 3 the evolution of water table due to recharge from a single strip source and from two parallel strip sources, when the water table is at large depth, has been analysed. The recharging strips were unconnected with the aquifer and, therefore, the recharge rates were assumed to be constant and independent of location of water table. In this chapter, the evolution of water table due to time variant recharge from a canal when water table is at shallow depth has been analysed. The canal is hydraulically connected with the aquifer and the recharge rate is dependent on the location of water table below the bed of the canal.

It has been often assumed for a stream (canal), which is hydraulically connected with the aquifer, that the exchange flow rate is linearly dependent on the potential difference between the stream

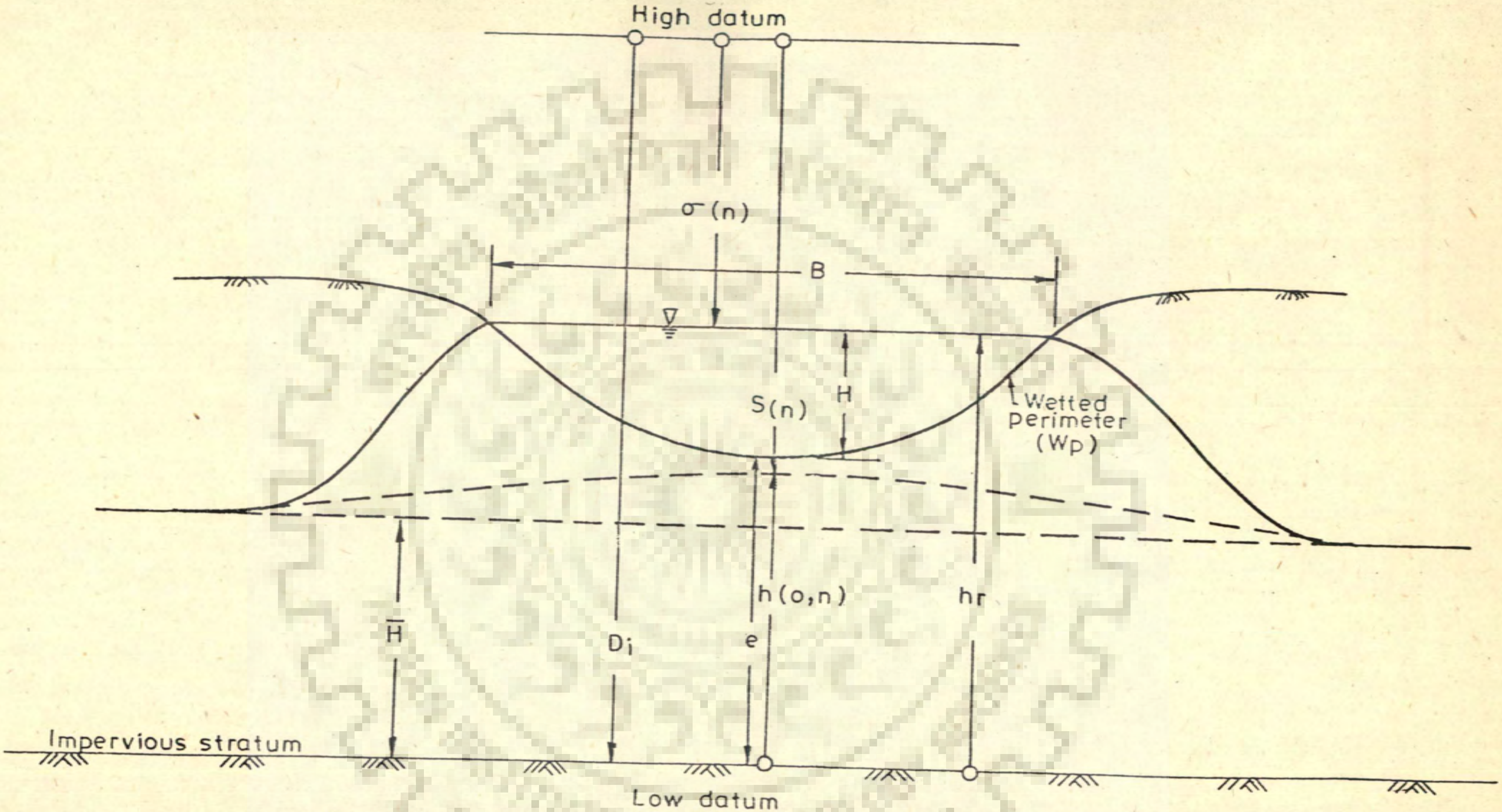

Fig.4.1– Schematic section of a canal hydraulically connected with the aquifer

and the aquifer (Ernst, 1962, Aravin and Numerov, 1965, Herbert, 1970, Morel-Seytoux, 1975, Besbes et al, 1978, Flug et al, 1980). It was shown by Bouwer (1969) that the recharge from a canal to an aquifer is proportional to the difference in the water levels in the canal and in the aquifer in the vicinity of the canal. The coefficient of proportionality, recognised as reach transmissivity, depends on the stream bed characteristics and shape of canal cross section (Morel-Seytoux, 1964, Bouwer, 1969). However, there have been evidences that the process of stream aquifer interaction can be very non-linear (Rushton and Redshaw, 1972, Dillon, 1983, 1984). In this chapter, assuming a nonlinear relationship between the exchange flow rate and the potential difference, a canal aquifer interaction problem has been solved.

4.1 Statement of the Problem

A canal having hydraulic connection with the underlying aquifer is depicted in Fig. (4.1). The water table is at a shallow depth below the canal bed. The recharge from the canal to the aquifer is assumed to have the following non-linear relationship with the potential difference between the canal and the aquifer which has been proposed by Rushton and Redshaw (1979):

$$Q_{l_r}(n) = 0.3\ C_2\ l_r\left[1-e^{-C_3\ \{h_r-h(o,n)\}}\right] \quad \ldots(4.1)$$

C_2 and C_3 are constants and l_r is the length of the canal reach, h_r is the ground water potential at the canal perimeter, and $h(o,n)$ is the potential in the aquifer under the bed of the canal during time period n , h_r and $h(o,n)$ are measured upwards from a low datum. The impervious bed of the aquifer has been selected as the low datum. Other aquifer parameters are as shown in Fig.(4.1). The hydraulic head, $h(o,n)$, in the aquifer during time period 'n' is governed by the recharges from all reaches of the canal which occur during time period 'n', besides those which took place from all reaches upto $(n-1)^{th}$ time period. It is required to find the recharge rate , $Q_{l_r}(n)$, from a reach of length l_r of the canal at various times after the onset of recharge.

4.2 Analysis

The following assumptions have been made in the analysis:

(i) The time parameter is discrete. Within each time-step the recharge rate is constant, but the recharge rate varies from time-step to time-step.

(ii) The water level in the canal is controlled by the supply source.

(iii) The Darcy's law and the Dupuit's hypothesis are valid for flow in the aquifer.

4.2.1 Evaluation of Constants C_2 and C_3

The constants C_2 and C_3, which appear in Equation (4.1), are evaluated by the following procedure:

According to Muskat, if the water table is at a depth of 1.5 (B + 2H) below the water surface in the canal, the water table can be considered to be at large depth. According to Kozeny, the seepage from a strip source of width B, and a depth of water H, when water table is at very large depth is given by:

$$Q_{1_r} = K(B+2H)1_r \qquad \ldots(4.2)$$

Applying the condition proposed by Muskat and Kozeny in Equation(4.1), [i.e., when $h_r - h(o,n)=1.5(B+2H)$, $Q_{1_r} = K(B+2H)1_r$] the following relation is obtained:

$$K(B+2H)1_r = 0.3C_2 1_r[1-\exp\{-1.5C_3(B+2H)\}] \qquad \ldots(4.3)$$

Hence, $C_2 = K(B+2H)/[0.3-0.3\exp\{-1.5C_3(B+2H)\}]$ ...(4.4)

Substituting C_2 in Equation (4.1),

$$Q_{1_r}(n) = \frac{K(B+2H)\ 1_r[1-\exp\{-C_3(h_r-h(o,n))\}]}{[1-\exp\ \{-1.5C_3\ (B+2H)\}]} \qquad \ldots(4.5)$$

For small difference between h_r and $h(o,n)$ the higher order terms of the polynomial expansion of the exponential term appearing in Equation (4.5) can be neglected and the recharge rate can be approximated to be:

$$Q_{l_r}(n) \simeq \frac{K(B+2H)\, l_r\, [1-1+C_3\{h_r-h(o,n)\}]}{[1-\exp\{-1.5\, C_3(B+2H)\}]}$$

or

$$Q_{l_r}(n) \approx \frac{K(B+2H)\, l_r\, C_3[h_r - h\,(o,n)]}{[1-\exp\{-1.5\, C_3\,(B+2H)\}]} \qquad \ldots(4.6)$$

It could be seen from Fig.(2.18) that for small difference in the potentials at the canal and aquifer, the exchange flow rate has a linear relation with the potential difference. The linear relationship proposed by various investigators is of the form:

$$Q_{l_r}(n) = \Gamma_r\, [h_r - h\,(o,n)] \qquad \ldots(4.7)$$

in which, Γ_r is the constant of proportionality. Though the constant of proportionality has been derived by various investigators for steady state condition, it has been applied to unsteady flow on the basis that an unsteady state can be approximated to be succession of steady state conditions. Equating Equations (4.6) and (4.7),

$$C_3 = \Gamma_r[1-\exp\{-1.5C_3(B+2H)\}]/[Kl_r(B+2H)] \qquad \ldots(4.8)$$

constant C_3 can be evaluated by an iteration procedure from the above equation. Knowing C_3, C_2 can be obtained from Equation (4.4).

If Muskat's condition is relaxed and if it is implied that flow from the canal becomes vertically downward at a very large depth below the canal, it could be derived from Equations (4.1) and (4.2) that $C_2 = K(B+2H)/0.3$ as the exponential term in Equation (4.1) would tend to zero for very large value of h_r-h(o,n). The other constant could be derived to be :-

$$C_3 = \Gamma_r / \{K(B+2H)l_r\}$$

4.2.2 Estimation of unsteady seepage from the canal

For a homogeneous aquifer, if the canal stages and canal section in different reaches are not varying from each other, the recharge rates from all canal reaches during a particular time period are equal. Let the time span be discretised by time-steps of equal duration and let during a particular time-step γ, the recharge rate from unit length of the canal be constant and denoted by Q(γ). With these assumptions the hydraulic head h(o,n) can be written as:

$$h(o,n) = \bar{H} + \sum_{\gamma=1}^{n} Q(\gamma)\,\delta(o,n-\gamma+1) \qquad \ldots(4.9)$$

in which $\delta(.,.)$ are discrete kernel coefficients for water table rise under the canal and $\overline{H}$ is the initial water table height before the onset of recharge. Discrete kernel coefficients $\delta(.,.)$ are computed as follows:

The width of the strip source from which the recharge is taking place is, B. If the recharge takes place at unit rate per unit length of the strip, the rate of recharge per unit area would be 1/B. Let the response of the aquifer due to continuous uniform excitation, at a rate of 1/B per unit area and per unit time, through the strip source of infinite length, be designated by U(x,t). The expression for U(x,t) can be obtained from Equation (3.7) and (3.10), replacing K by 1/B and (B/2+H) by B/2. Thus, for $x \leq -B/2$ and, $x \geq B/2$,

$$U(x,t) = F(x,B,t) - \frac{1}{2BT} \cdot [\sqrt{(x^2)}.B]$$

and for $-B/2 \leq x \leq B/2$,

$$U(x,t) = F(x,B,t) - \frac{1}{2BT} [x^2+B^2/4]$$

where,

$$F(x,B,T) = \frac{\alpha t}{2BT} \text{Erf} \left[\frac{x+B/2}{\sqrt{(4\alpha t)}}\right] - \frac{\alpha t}{2BT} \text{Erf} \left[\frac{x-B/2}{\sqrt{(4\alpha t)}}\right]$$

$$+ \frac{1}{4BT} (x+B/2)^2 \text{Erf}\left[\frac{x+B/2}{\sqrt{(4\alpha t)}}\right]$$

$$- \frac{1}{4BT} (x-B/2)^2 \text{Erf} \left[\frac{x-B/2}{\sqrt{(4\alpha t)}}\right]$$

$$+ \frac{\sqrt{(\alpha t)}}{2BT\sqrt{\pi}} (x+B/2)\exp[-(x+B/2)^2/(4\alpha t)]$$

$$- \frac{\sqrt{(\alpha t)}}{2BT\sqrt{\pi}} (x-B/2) \exp[-(x-B/2)^2/(4\alpha t)] \quad ...(4.10)$$

If unit recharge per unit length of the recharging strip takes place during the first unit time period and no recharge takes place thereafter, the response of the aquifer, $\delta(x,n)$, at the end of n^{th} time step is given by:

$$\delta(x,n) = U(x,n) - U(x,n-1), \text{ for } n>1$$

$$\text{and} \quad \delta(x,1) = U(x,1), \qquad \text{for } n=1$$

Thus,

$$\delta(x,n) = F(x,B,n)-F(x,B,n-1), \text{for } n>1 \quad \ldots(4.11)$$

$$\delta(x,1) = F(x,B,1)-\frac{1}{2T}\sqrt{(x)}^2, \quad \ldots(4.12)$$

$$\text{for } x \leq -B/2 \text{ and } x \geq B/2$$

$$= F(x,B,1) - \frac{1}{2BT}(x^2+B^2/4), \quad \ldots(4.13)$$

$$\text{for } -B/2 \leq x \leq B/2$$

The unsteady seepage problem could be solved with Muskat's condition imposed or relaxed. In the following paragraphs the solution to the seepage problem with Muskat's condition imposed has been derived as follows:

On substituting $h(o,n)$ from equation (4.9) in Equation (4.5), taking l_r to be equal to one, and simplifying:-

$$1-Q(n)\,[1-\exp\{-1.5\,C_3(B+2H)\}]\,/\,[K(B+2H)]$$

$$= \exp[-C_3\{D_i-\sigma_r(n)-\overline{H} - \sum_{\gamma=1}^{n} Q(\gamma)\delta(0,n-\gamma+1)\}] \quad \ldots(4.14)$$

in which D_i is depth to impervious stratum measured from a high datum and $\sigma_r(n)$ is drawdown of the water level in the canal measured from the same datum. Taking logarithm of terms on either side,

$$\log_e [1-Q(n)\{1-\exp(-1.5C_3(B+2H))\}/\{K(B+2H)\}]$$

$$= -C_3\{D_i-\sigma_r(n)-\bar{H}-\sum_{\gamma=1}^{n} Q(\gamma)\delta(o,n-\gamma+1)\} \quad \ldots(4.15)$$

Splitting the summation into parts and rearranging:

$$\log_e [1-Q(n)\{1-\exp(-1.5C_3(B+2H))\}/\{K(B+2H)\}]$$

$$-C_3\, Q(n)\delta(o,1)$$

$$= -C_3\{D_i-\sigma_r(n)-\bar{H}-\sum_{\gamma=1}^{n-1} Q(\gamma)\delta(0,n-\gamma+1)\} \quad \ldots(4.16)$$

Q(n) can be solved in succession starting from time step 1 by an iteration procedure. The following simplification can be adopted without much loss of accuracy when the water table is at very shallow depth below the canal bed. K(B+2H) being the maximum recharge rate per unit length of the canal when water table is at large depth, the ratio Q(n)/[K(B+2H)] is less than 1. Expanding the logarithmic term and neglecting higher order terms,

$$-Q(n)\{1-\exp(-1.5C_3(B+2H))\}/[K(B+2H)]$$

$$-1/2\,[Q(n)\{1-\exp(-1.5C_3(B+2H))\}/\{K(B+2H)\}]^2$$

$$-C_3Q(n)\delta(0,1)$$

$$= -C_3[D_i-\sigma_r(n)-\bar{H}-\sum_{\gamma=1}^{n-1} Q(\gamma)\delta(0,n-\gamma+1)] \quad \ldots.(4.17)$$

Equation (4.17) is a quadratic equation in Q(n) and can be written in the form:

$$a\,Q^2(n) + b\,Q(n) + c = 0$$

Hence,

Q(n) is given by ,

$$Q(n) = [-b + \sqrt{(b^2 - 4ac)}]/(2a) \qquad \ldots(4.18)$$

where,

$$a = 0.5\,[1-\exp\{-1.5\, C_3(B+2H)\}]^2/[K(B+2H)]^2$$

$$b = [1-\exp\{-1.5C_3(B+2H)\}]/[K(B+2H)]+C_3\delta(o,1), \text{and}$$

$$c = -C_3\,[D_i - \sigma_r(n) - \bar{H} - \sum_{\gamma=1}^{n-1} Q(\gamma)\,\delta\,(o,n-\gamma+1)]$$

The solution to the seepage problem with Muskat's condition relaxed is derived as follows:

When Muskat's condition is relaxed, the constants C_2 and C_3 are given by:

$$C_2 = K\,(B+2H)\,/\,0.3, \text{ and}$$

$$C_3 = \Gamma_r\,/[K(B+2H)L_r]$$

Substituting for C_2 in Equation (4.1) and considering the reach length, L_r, to be equal to 1, the following expression is obtained:

$$Q(n) = K(B+2H)[1-\exp\{-C_3(h_r - h(o,n))\}] \qquad \ldots(4.19)$$

Substituting for h (o,n) from Equation (4.9) in Equation (4.19), taking $h_r = [D_i - \sigma_r(n)]$, and rearranging the terms,

$$Q(n)/\{K(B+2H)\} = 1-\exp[-C_3\{D_i - \sigma_r(n) - \bar{H} - \sum_{\gamma=1}^{n} Q(\gamma)\,\delta\,(o,n-\gamma+1)\}]$$

or

$$1-Q(n)/\{K(B+2H)\} = \exp[-C_3\{D_i - \sigma_r(n) - \bar{H} - \sum_{\gamma=1}^{n} Q(\gamma)\delta\,(o,n-\gamma+1)\}]$$

Taking logarithm of terms on both sides, and simplifying,

$$\log_e [1- Q(n)/K(B+2H)] - C_3 Q(n)\, \delta(o,1)$$

$$= - C_3 [D_i - \sigma_r(n) - \overline{H} - \sum_{\gamma=1}^{n-1} Q(\gamma)\, \delta(o,n-\gamma+1)] \quad ...(4.20)$$

Q(n) can be solved in succession starting from time - step 1.

Expanding the logarithmic term in Equation (4.20) and neglecting higher order terms, the approximate equation for seepage loss is given by :

$$-Q(n)/[K(B+2H)] - 1/2[Q(n)/\{K(B+2H)\}]^2 - C_3\, Q(n)\, \delta(o,1)$$

$$= -C_3 [D_i - \sigma_r(n) - \overline{H} - \sum_{\gamma=1}^{n-1} Q(\gamma)\, \delta(o,n-\gamma+1)] \quad ...(4.21)$$

The above quadratic equation can be expressed as:

$$a_1 Q^2(n) + b_1 Q(n) + c_1 = 0, \text{ hence}$$

$$Q(n) = [-b_1 + \sqrt{(b_1^2 - 4a_1 c_1)}]/2a_1 \quad ...(4.22)$$

where,

$$a_1 = 0.5/[K(B+2H)]^2, \quad b_1 = 1/[K(B+2H)] + C_3\, \delta(o,1)$$

$$c_1 = -C_3 [D_i - \sigma_r(n) - \overline{H} - \sum_{\gamma=1}^{n-1} Q(\gamma)\, \delta(o,n-\gamma+1)]$$

Once the Q(n) values are obtained, the evolution of water table at any point can be evaluated using the following relation:-

$$S(x,n) = \sum_{\gamma=1}^{n} Q(\gamma)\, \delta(x, n-\gamma+1),$$

where, S(x,n) is the rise in water table at location, x, at the end of time step n.

If the relationship between the flow rate and the potential difference is linear, the seepage rate can be predicted by the stream aquifer interaction model of Morel-Seytoux and Daly (1975) as described below:-

The potential h(o,n) in the aquifer below the canal bed is given by Equation (4.9). Substituting for h(o,n) in Equation (4.7) and rearranging,

$$Q(n)/\Gamma_r = h_r - [\bar{H} + \sum_{\gamma=1}^{n} Q(\gamma)\ \delta(o,n-\gamma+1)] \quad \ldots(4.23)$$

$\delta(o,n-\gamma+1)$ are the discrete kernel coefficients computed from Equations (4.10) through (4.13). Splitting the summation into two parts,

$$Q(n)/\Gamma_r = h_r - [\bar{H} + Q(n)\delta(o,1) + \sum_{\gamma=1}^{n-1} Q(\gamma)\delta(o,n-\gamma+1)]$$

or,

$$Q(n) = [h_r - \{\bar{H} + \sum_{\gamma=1}^{n-1} Q(\gamma)\ \delta(o,n-\gamma+1)\}]/[\frac{1}{\Gamma_r} + \delta(o,1)] \quad \ldots(4.24)$$

Q(n) can be computed in succession starting from time step 1.

In particular for time step, 1,

$$Q(1) = (h_r - \bar{H})/[\frac{1}{\Gamma_r} + \delta(o,1)] \quad \ldots(4.25)$$

Once the Q(n) values are obtained, the evolution of water table at any point can be evaluated using the following relation:-

$$S(x,n) = \sum_{\gamma=1}^{n} Q(\gamma)\ \delta(x,n-\gamma+1)$$

4.3 Results and Discussions

The constant of proportionality, Γ_r, which relates the seepage rate from a canal to the small difference in the potentials at the canal and in the aquifer can be determined by formulae proposed by several investigators. A comparison of reach transmissivity values evaluated by various formulae has been made to select the appropriate one for the present study of canal aquifer interaction.

The computations of reach transmissivity by formulae proposed by Morel-Seytoux, Herbert and Ernst have been made for canals having vertical sides. Canals with vertical sides have been chosen in order to eliminate the dependence of width of water surface on the depth of water in the canal so that the individual effect of depth of water in the canal, and width of water surface on reach transmissivity could be ascertained. The reach transmissivity values for canals having different widths and depths of water are given in Table (4.1) for an assumed set of aquifer parameters. It is seen from the table that the reach transmissivity, evaluated by the formula given by Morel-Seytoux [Equation(2.39)] with characteristic length equal to $5w_p$, decreases with the increase in width of the canal. With all other parameters remaining the

Table (4.1) - Comparison of reach transmissivity computed by different formulae evaluated for: e=1000m, L_r(reach length)=10m, K=0.1m/day

B (m)	H (m)	W_p (m)	Reach transmissivity: $\frac{Kl_r(0.5W_p+e)}{5W_p+0.5e}$ (Morel-Seytoux's Formula)	$\pi Kl_r/\log[\frac{e+H}{W_p}]$ (Ernst's Formula)	$\pi Kl_r/\log[\frac{4(e+H)}{B\pi}]$ (Modified Ernst's Formula)	$\pi Kl_r/\log[\frac{e+H}{2W_p/\pi}]$ (Herbert's Formula)
15.0	3.0	21.0	1.6702	0.8126	0.7069	0.7276
15.0	1.0	17.0	1.7329	0.7708	0.7072	0.6939
15.0	0.1	15.2	1.7493	0.7504	0.7073	0.6773
30.0	3.0	36.0	1.4971	0.9442	0.8375	0.8314
30.0	1.0	32.0	1.5394	0.9125	0.8380	0.8067
30.0	0.1	30.2	1.5593	0.8976	0.8382	0.7950
60.0	3.0	66.0	1.2446	1.1545	1.0273	0.9906
60.0	1.0	62.0	1.2728	1.1294	1.0280	0.9717
60.0	0.1	60.2	1.2860	1.1180	1.0283	0.9632
90.0	3.0	96.0	1.0694	1.3389	1.1844	1.1228
90.0	1.0	92.0	1.0896	1.3161	1.1853	1.1068
90.0	0.1	90.2	1.0990	1.3058	1.1857	1.0995
120.0	3.0	126.0	0.9407	1.5144	1.3285	1.2437
120.0	1.0	122.0	0.9559	1.4926	1.3296	1.2290
120.0	0.1	120.2	0.9629	1.4828	1.3301	1.2223
180.0	3.0	186.0	0.7643	1.8644	1.6034	1.4704
180.0	1.0	182.0	0.7738	1.8428	1.6050	1.4570
180.0	0.1	180.2	0.7781	1.8331	1.6058	1.4508
210.0	3.0	216.0	0.7013	2.0460	1.7403	1.5810
210.0	1.0	216.0	0.7013	2.0460	1.7403	1.5810
210.0	0.1	212.0	0.7090	2.0240	1.7422	1.5679

same, the seepage loss from a canal would increase with increase in width of the canal. Therefore, the formula suggested by Morel-Seytoux cannot be applied to relate the seepage loss, and the difference in the potentials at the canal periphery and in the aquifer under the canal, without an appropriate assessment of the characteristic length which represents the zone of influence.

When reach transmissivity is calculated by the Ernst's modified formula [Equation(2.38)], which is valid for shallow water table condition (vide Bouwer, 1969), it is seen from the Table (4.1) that the reach transmissivity increases marginally with decrease in depth of water in the canal, with all other parameters remaining the same. This indication is contrary to the fact that with decrease in depth of water in a canal, all other parameters remaining same, the reach transmissivity should decrease.

The reach transmissivity computed by Ernst's formula [Equation(2.34)] and Herbert's formula [Equation(2.41)] are sensitive to change in depth of water in canal and the values of reach transmissivity increase with an increase in depth of water or in the width of water surface in the canal. Therefore, any of the two formulae viz. the one given by Ernst and the other by Herbert can be used for computing the reach transmissivity. However, a comparison of the results given in Table (4.1) shows that the results given by Ernst modified formula and by Herbert's formula are close to each other for the same aquifer and canal parameters. Therefore, the Herbert's formula for calculating reach transmissivity has been chosen for the analysis in the present study. The Herbert's formula is given by:

$$\Gamma_r = \pi K l_r / \log \left[\frac{0.5(e+H)}{r_r} \right] \qquad .....(4.26)$$

where,

- l_r = length of the reach,
- e = saturated thickness of aquifer below the bed of the canal,
- H = depth of water in the canal, and
- r_r = radius of the equivalent semi circular section of the canal.

The shape of canal for which Herbert's formula has been derived is shown in Fig.(4.2). While calculating the reach transmissivity for a trapezoidal shape of canal, the equivalent r_r has been taken W_p / π where, W_p is the wetted perimeter of the canal. However Herbert's formula can only be applied to cases for which $0.5(e+H) > r_r$, otherwise the reach transmissivity term would become negative.

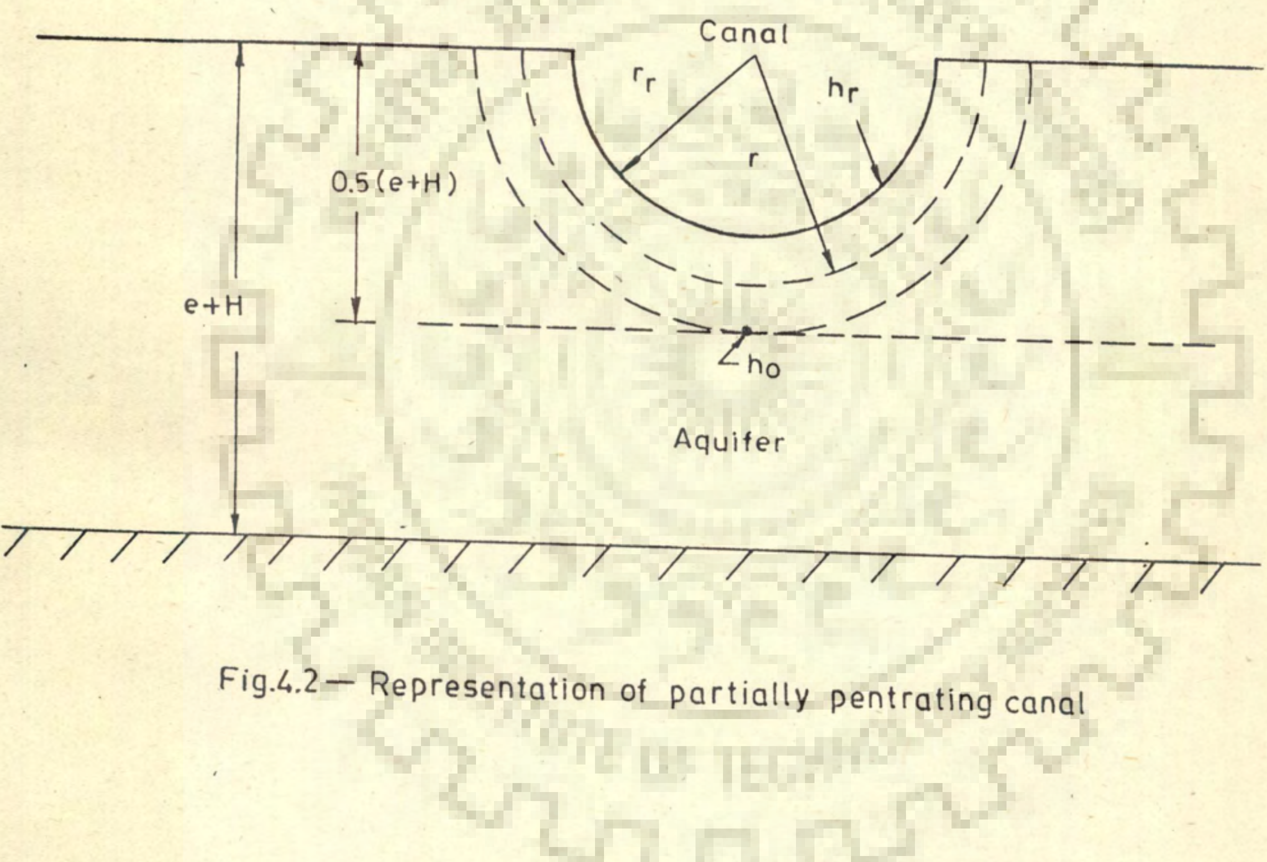

Fig.4.2— Representation of partially pentrating canal

Before computing the seepage from a canal, which is non-linearly dependent on the potential difference between the canal and the aquifer, the two procedures suggested in the analysis for determining the constants C_2 and C_3, appearing in the Equation (4.1), have to be examined. The parameters C_2 and C_3 are determined first without imposing Muskat's condition and the time variant seepage from a canal are found in succession starting from time step 1 by an iteration procedure, making use of Equation (4.20), for a known position of initially rest water table. The rates at which seepage would occur during the first time period corresponding to different initial potential differences are shown in Fig. (4.3). These seepage rates have been calculated assuming the hydraulic conductivity of the aquifer material to be 0.1m/day. Since the time-step size has been chosen to be one day, the seepage rates presented in Fig.(4.3) are the rates during the first day after the onset of seepage. It could be seen from the figure that corresponding to an initial potential difference of 100m, the seepage from the canal with 10m width and 2m depth of water is 1.38 m^3/day. According to Kozeny, if the water table is at a very large depth below the canal bed, for K=0.1m/day, the seepage

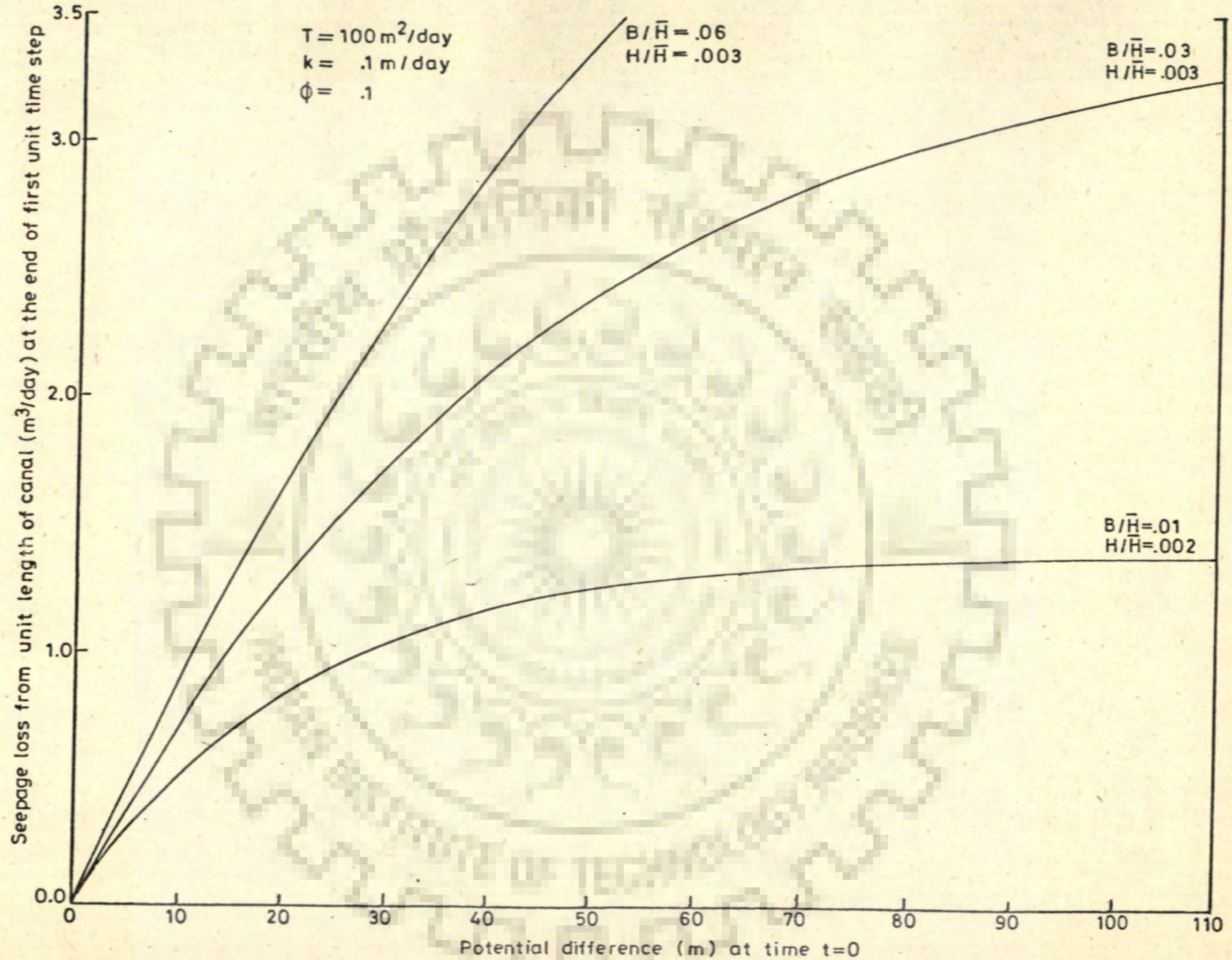

Fig.4.3 – Variation of seepage losses from unit length of canals with initial potential difference evaluated at the end of first unit time step if Muskat's condition is not imposed

from a canal with 10m width and 2 m depth of water will be $1.4m^3$/day. It is seen from the figure that seepage from the canal with 10m width and 2m depth of water becomes approximately constant if the potential difference is more than 8(B+2H). At potential difference equal to 21m, the seepage rate from this canal is about $0.86m^3$/day. According to the experimental finding reported by Muskat, the flow from a canal becomes constant if the water table is at a depth more than 1.5(B+2H) below the water surface in the canal. The constants C_2 and C_3 should therefore be evaluated imposing the condition that seepage from the canal attains a limiting value when the potential difference is equal to 1.5(B+2H).

Imposing the Muskat's condition i.e. the seepage from a canal would tend to K(B+2H) if the potential difference is equal to 1.5(B+2H), the constant C_2 and C_3 are determined by iteration. The exact seepage rates at different time have been computed using Equation (4.16) by an iteration procedure. The seepage that would occur during the first unit time period for different initial potential difference is shown in Fig.(4.4) It could be seen from the figure that the non-linearity gets pronounced with increase

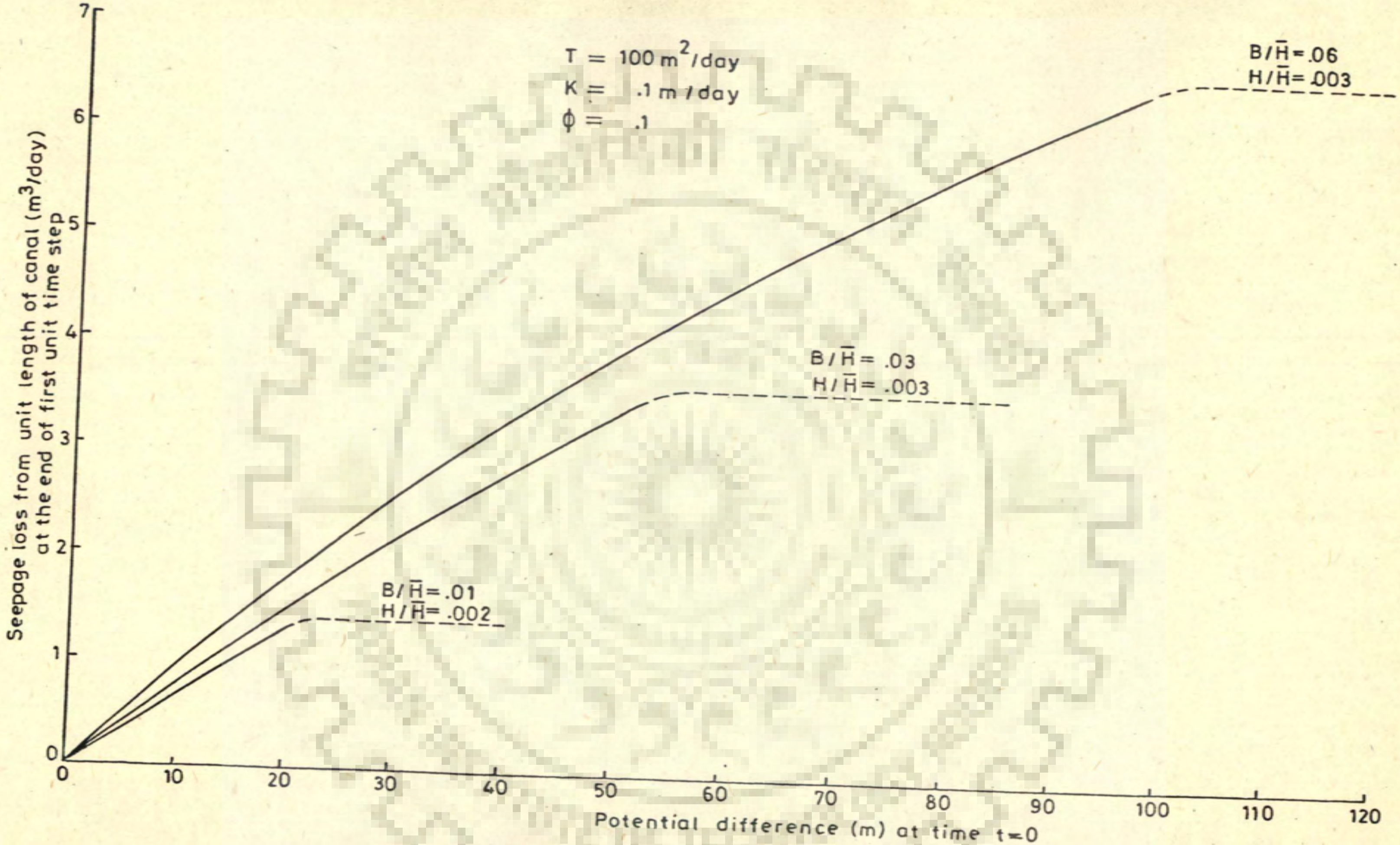

Fig.4.4 – Variation of seepage losses from unit length of canal at the end of first unit time step with initial potential difference, evaluated with Muskat's condition imposed.

in the width of the canal. It could also be noticed that at small potential difference the relationship between seepage and potential difference is very much linear. However, the range of lower potential difference over which the relationship is linear, decreases with increase in width of the canal.

The variations of the dimensionless seepage quantity, $Q(t)/(K\overline{H})$, from a trapezoidal canal with the dimensionless time factor, $Kt/(2\varphi\overline{H})$, have been presented in Figs. (4.5) and (4.6) for three different positions of the initially rest water table. The results presented in Fig. (4.5) have been computed for $B/\overline{H}=0.03$ and $H/\overline{H} = 0.003$ and those presented in Fig. (4.6) have been obtained for $B/\overline{H} = 0.06$. The canal banks have a 1:1 slope. It is seen from the figures that the seepage rate declines faster for a canal with deeper water table position. If the water table before the onset of seepage lies at a depth of 1 m below the canal bed i.e for $(h_r-\overline{H})/\overline{H} = 0.004$, the reduction in non-dimensional seepage rate during the dimensionless time interval from $Kt/(2\varphi\overline{H}) = 1 \times 10^{-2}$ to $Kt/(2\varphi\overline{H}) = 1 \times 10^{-1}$ is about 0.3×10^{-3}. If the water table lies initially at a depth of 5m below the canal bed i.e. for $(h_r-\overline{H})/\overline{H}=0.008$, the reduction in seepage rate during the above period is about 0.7×10^{-3}.It is also seen that the rate of decrease

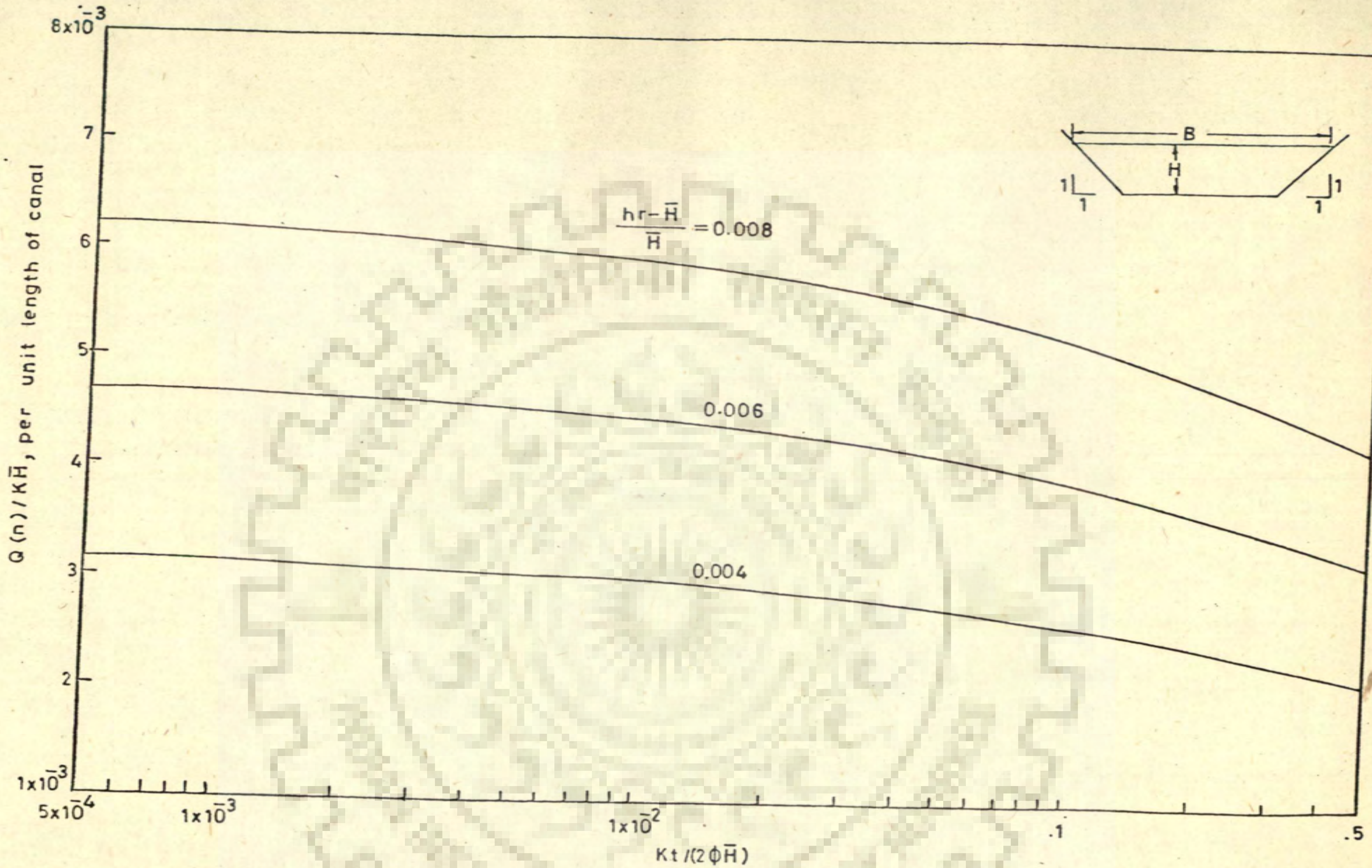

Fig.4.5 – Variation of non dimensional seepage losses per unit length of a trapezoidal canal with non-dimensional time for different initial potential difference between canal and aquifer for $B/\bar{H}=0.03$ and $H/\bar{H}=0.003$

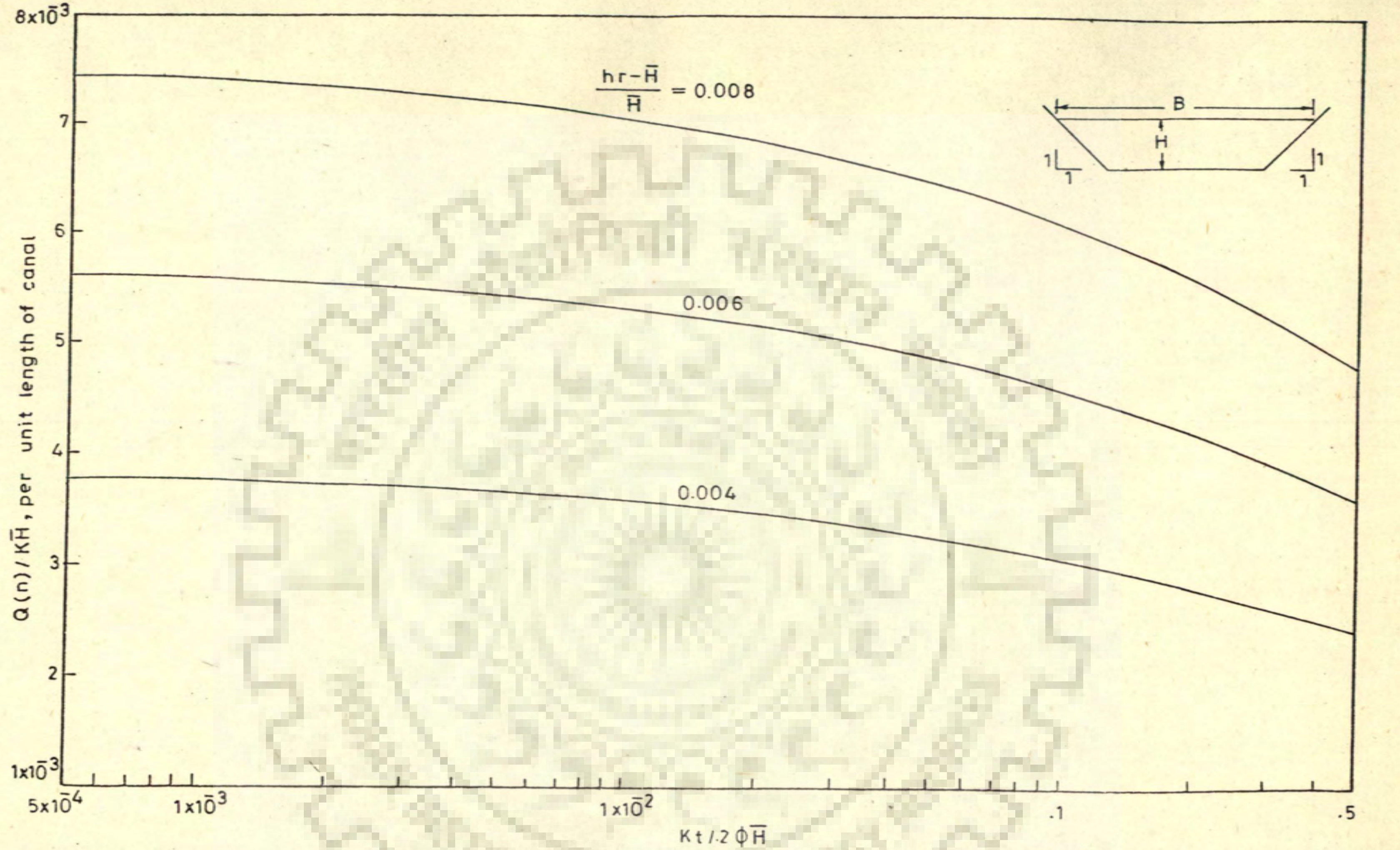

Fig.4.6 – Variation of non dimensional seepage losses per unit length of a trapezoidal canal with non-dimensional time for different initial potential difference between canal and aquifer for $B/\bar{H} = 0.06$ and $H/\bar{H} = 0.003$

increases with time for any initial position of the water table before onset of seepage.

A comparison of results obtained by exact method with Muskat's condition imposed, and by exact and approximate method without Muskat condition imposed has been made in Table (4.2). The corresponding results assuming linear relationship between seepage and potential difference have also been presented in the table. It could be seen from the table, that for water table lying within 5 m below canal bed, the linear relationship could be adopted to predict the time variant seepage loss from a canal. Also, if the water table lies within 5 m below a canal bed, one can predict the seepage loss which is non-linearly related to potential difference by the approximate method.

The water table evolution, if the seepage is non-linearly dependent on the potential difference, has been shown in Fig.(4.7). The corresponding water table position if the seepage loss obeys the linear relation is also shown in the figure. It could be seen that, if the water table lies within 5 m below the canal bed, there is no appreciable difference in water table evolutions whether the seepage rate obeys the linear or the non-linear law.

Table 4.2 Comparison of seepage rates which are nonlinearly and linearly dependent on potential difference between canal and aquifer estimated for B=30m, H=3m and $(h_r-\bar{H})$=4m.

Non-dimensional time factor $Kt/(2\phi\bar{H})$	Nondimensional seepage rate per unit length of canal $[Q(n)/(K\bar{H})]$			
	Nonlinearly dependent on potential difference estimated by approximate analysis with Muskat's condition relaxed	Nonlinearly dependent on potential difference estimated by exact analysis with Muskat's condition relaxed	Nonlinearly dependent on potential difference estimated by exact analysis with Muskat's condition imposed	Linearly dependent on potential difference
(10^{-1})	(10^{-3})	(10^{-3})	(10^{-3})	(10^{-3})
0.005	3.0699	3.0633	3.1533	3.1993
0.010	3.0534	3.0472	3.1356	3.1806
0.020	3.0301	3.0231	3.1108	3.1543
0.030	3.0123	3.0055	3.0911	3.1343
0.040	2.9975	2.9919	3.0757	3.1176
0.050	2.9845	2.9788	3.0618	3.1030
0.075	2.9570	2.9518	3.0317	3.0722
0.100	2.9342	2.9288	3.0071	3.0465
0.150	2.8964	2.8916	2.9669	3.0043
0.200	2.8651	2.8606	2.9333	2.9694
0.250	2.8380	2.8335	2.9043	2.9392
0.300	2.8138	2.8086	2.8782	2.9123
0.350	2.7918	2.7867	2.8543	2.8879
0.400	2.7716	2.7667	2.8326	2.8655
0.450	2.7528	2.7480	2.8126	2.8447
0.500	2.7352	2.7315	2.7940	2.8252
0.750	2.6614	2.6572	2.7144	2.7439
1.000	2.6006	2.5968	2.6497	2.6771
1.250	2.5487	2.5452	2.5947	2.6204
1.500	2.5032	2.5000	2.5466	2.5708
2.000	2.4254	2.4226	2.4646	2.4863
2.500	2.3600	2.3575	2.3958	2.4156
3.000	2.3032	2.3009	2.3362	2.3544
3.500	2.2529	3.2508	2.2835	2.3004
4.000	2.2077	2.2057	2.2362	2.2520
4.500	2.1664	2.1647	2.1932	2.2080
5.000	2.1286	2.1270	2.1537	2.1677

Table 4.2 Comparison of seepage rates which are nonlinearly and linearly dependent on potential difference between canal and aquifer estimated for B=30m, H=3m and $(h_r-\bar{H})=6$m.

Nondimensional time factor $Kt/(2\phi\bar{H})$	Nondimensional seepage rate per unit length of canal $[Q(n)/(K\bar{H})]$			
	Nonlinearly dependent on potential difference estimated by approximate analysis with Muskat's condition relaxed	Nonlinearly dependent on potential difference estimated by exact analysis with Muskat's condition relaxed	Nonlinearly dependent on potential difference estimated by exact analysis with Muskat's condition imposed	Linearly dependent on potential difference
(10^{-1})	(10^{-3})	(10^{-3})	(10^{-3})	(10^{-3})
0.005	4.5165	4.4940	4.6942	4.7965
0.010	4.4930	4.4716	4.6678	4.7685
0.020	4.4600	4.4392	4.6315	4.7291
0.030	4.4347	4.4146	4.6030	4.6992
0.040	4.4136	4.3937	4.5793	4.6741
0.050	4.3951	4.3746	4.5584	4.6522
0.075	4.3561	4.3363	4.5156	4.6060
0.100	4.3236	4.3055	4.4789	4.5676
0.150	4.2698	4.2526	4.4194	4.5043
0.200	4.2252	4.2090	4.3701	4.4520
0.250	4.1865	4.1706	4.3276	4.4067
0.300	4.1520	4.1356	4.2894	4.3664
0.350	4.1206	4.1059	4.2550	4.3299
0.400	4.0917	4.0763	4.2229	4.2963
0.450	4.0648	4.0506	4.1939	4.2651
0.500	4.0396	4.0262	4.1659	4.2359
0.750	3.9338	3.9207	4.0489	4.1141
1.000	3.8464	3.8345	3.9535	4.0140
1.250	3.7718	3.7609	3.8724	3.9290
1.500	3.7063	3.6961	3.8013	3.8546
2.000	3.5942	3.5853	3.6801	3.7280
2.500	3.4997	3.4918	3.5783	3.6221
3.000	3.4175	3.4103	3.4902	3.5304
3.500	3.3445	3.3381	3.4122	3.4494
4.000	3.2788	3.2729	3.3421	3.3768
4.500	3.2189	3.2135	3.2783	3.3108
5.000	3.1639	3.1588	3.2199	3.2504

Table 4.2 Comparison of seepage rates which are nonlinearly and linearly dependent on potential difference between canal and aquifer estimated for B=30m, H=3m and $(h_r - \bar{H})$=8m.

Non-dimensional time factor $Kt/(2\phi\bar{H})$	Nondimensional seepage rate per unit length of canal $[Q(n)/(K\bar{H})]$			
	Nonlinearly dependent on potential difference estimated by approximate analysis with Muskat's condition relaxed	Nonlinearly dependent on potential difference estimated by exact analysis with Muskat's condition relaxed	Nonlinearly dependent on potential difference estimated by exact analysis with Muskat's condition imposed	Linearly dependent on potential difference
(10^{-1})	(10^{-3})	(10^{-3})	(10^{-3})	(10^{-3})
0.005	5.9122	5.8623	6.2113	6.3921
0.010	5.8826	5.8331	6.1776	6.3549
0.020	5.8407	5.7926	6.1288	6.3024
0.030	5.8088	5.7618	6.0921	6.2625
0.040	5.7820	5.7367	6.0614	6.2291
0.050	5.7586	5.7142	6.0347	6.1999
0.075	5.7091	5.6664	5.9782	6.1384
0.100	5.6678	5.6260	5.9301	6.0872
0.150	5.5995	5.5589	5.8532	6.0029
0.200	5.5429	5.5050	5.7884	5.9333
0.250	5.4936	5.4569	5.7326	5.8729
0.300	5.4497	5.4145	5.6828	5.8192
0.350	5.4097	5.3757	5.6378	5.7705
0.400	5.3729	5.3396	5.5956	5.7258
0.450	5.3386	5.3059	5.5574	5.6843
0.500	5.3065	5.2747	5.5213	5.6454
0.750	5.1714	5.1419	5.3683	5.4832
1.000	5.0596	5.0328	5.2434	5.3498
1.250	4.9641	4.9394	5.1369	5.2365
1.500	4.8801	4.8571	5.0436	5.1374
2.000	4.7361	4.7159	4.8844	4.9688
2.500	4.6145	4.5961	4.7508	4.8276
3.000	4.5086	4.4923	4.6347	4.7055
3.500	4.4146	4.3998	4.5320	4.5976
4.000	4.3297	4.3161	4.4397	4.5008
4.500	4.2523	4.2398	4.3557	4.4130
5.000	4.1810	4.1694	4.2786	4.3324

Table 4.2 Comparison of seepage rates which are nonlinearly and linearly dependent on potential difference between canal and aquifer estimated for B= 60m, H=3m and $(h_r-\bar{H})$=4m.

Non-dimensional time factor $Kt/(2\phi\bar{H})$	Nondimensional seepage rate per unit length of canal $[Q(n)/(K\bar{H})]$			
	Nonlinearly dependent on potential difference estimated by approximate analysis with Muskat's condition relaxed	Nonlinearly dependent on potential difference estimated by exact analysis with Muskat's condition relaxed	Nonlinearly dependent on potential difference estimated by exact analysis with Muskat's condition imposed	Linearly dependent on potential difference
(10^{-1})	(10^{-3})	(10^{-3})	(10^{-3})	(10^{-3})
0.005	3.7438	3.7400	3.7882	3.8488
0.010	3.7199	3.7170	3.7630	3.8228
0.020	3.6855	3.6824	3.7280	3.7855
0.030	3.6592	3.6569	3.7008	3.7570
0.040	3.6371	3.6350	3.6775	3.7332
0.050	3.6178	3.6148	3.6577	3.7122
0.075	3.5770	3.5746	3.6158	3.6682
0.100	3.5432	3.5413	3.5804	3.6317
0.150	3.4874	3.4851	3.5232	3.5716
0.200	3.4414	3.4389	3.4759	3.5222
0.250	3.4016	3.3998	3.4451	3.4795
0.300	3.3662	3.3647	3.3987	3.4416
0.350	3.3342	3.3325	3.3652	3.4073
0.400	3.3047	3.3037	3.3351	3.3758
0.450	3.2775	3.2764	3.3070	3.3467
0.500	3.2520	3.2507	3.2809	3.3196
0.750	3.1462	3.1439	3.1713	3.2071
1.000	3.0595	3.0574	3.0823	3.1150
1.250	2.9861	2.9842	3.0072	3.0374
1.500	2.9221	2.9203	2.9417	2.9698
2.000	2.8138	2.8122	2.8310	2.8558
2.500	2.7235	2.7221	2.7390	2.7612
3.000	2.6459	2.6447	2.6599	2.6801
3.500	2.5776	2.5766	2.5904	2.6088
4.000	2.5166	2.5156	2.5283	2.5453
4.500	2.4615	2.4605	2.4723	2.4880
5.000	2.4111	2.4103	2.4211	2.4357

Table 4.2 Comparison of seepage rates which are nonlinearly and linearly dependent on potential difference between canal and aquifer estimated for B=60m, H=3m and $(h_r - \bar{H})$=6m.

Non-dimensional time factor $Kt/(2\phi\bar{H})$	Nondimensional seepage rate per unit length of canal $[Q(n)/(K\bar{H})]$			
	Nonlinearly dependent on potential difference estimated by approximate analysis with Muskat's condition relaxed	Nonlinearly dependent on potential difference estimated by exact analysis with Muskat's condition relaxed	Nonlinearly dependent on potential difference estimated by exact analysis with Muskat's condition imposed	Linearly dependent on potential difference
(10^{-1})	(10^{-3})	(10^{-3})	(10^{-3})	(10^{-3})
0.005	5.5398	5.5271	5.6335	5.7697
0.010	5.5053	5.4941	5.5976	5.7308
0.020	5.4557	5.4446	5.5456	5.6750
0.030	5.4177	5.4073	5.5061	5.6322
0.040	5.3858	5.3750	5.4722	5.5964
0.050	5.3579	5.3476	5.4432	5.5651
0.075	5.2990	5.2896	5.3819	5.4992
0.100	5.2500	5.2408	5.3306	5.4444
0.150	5.1693	5.1610	5.2455	5.3544
0.200	5.1026	5.0942	5.1760	5.2803
0.250	5.0450	5.0374	5.1164	5.2163
0.300	4.9936	4.9862	5.0627	5.1595
0.350	4.9471	4.9395	5.0139	5.1082
0.400	4.9044	4.8976	4.9695	5.0611
0.450	4.8649	4.8586	4.9280	5.0174
0.500	4.8279	4.8214	4.8904	4.9767
0.750	4.6739	4.6672	4.7290	4.8082
1.000	4.5474	4.5415	4.5979	4.6702
1.250	4.4404	4.4350	4.4872	4.5539
1.500	4.3469	4.3420	4.3906	4.4527
2.000	4.1885	4.1843	4.2271	4.2819
2.500	4.0563	4.0526	4.0911	4.1401
3.000	3.9425	3.9392	3.9740	4.0184
3.500	3.8422	3.8393	3.8711	3.9119
4.000	3.7526	3.7499	3.7792	3.8165
4.500	3.6714	3.6691	3.6961	3.7306
5.000	3.5973	3.5951	3.6202	3.6523

Table 4.2 Comparison of seepage rates which are nonlinearly and linearly dependent on potential difference between canal and aquifer estimated for B= 60m, H=3m and $(h_r-\bar{H})$=8m.

Non-dimensional time factor $Kt/(2\phi\bar{H})$	Nondimensional seepage rate per unit length of canal $[Q(n)/(K\bar{H})]$			
	Nonlinearly dependent on potential difference estimated by approximate analysis with Muskat's condition relaxed	Nonlinearly dependent on potential difference estimated by exact analysis with Muskat's condition relaxed	Nonlinearly dependent on potential difference estimated by exact analysis with Muskat's condition imposed	Linearly dependent on potential difference
(10^{-1})	(10^{-3})	(10^{-3})	(10^{-3})	(10^{-3})
0.005	7.2901	7.2613	7.4487	7.6883
0.010	7.2458	7.2191	7.4016	7.6364
0.020	7.1821	7.1562	7.3332	7.5621
0.030	7.1332	7.1083	7.2821	7.5052
0.040	7.0923	7.0676	7.2378	7.4575
0.050	7.0564	7.0318	7.2002	7.4158
0.075	6.9806	6.9578	7.1202	7.3280
0.100	6.9176	6.8955	7.0527	7.2551
0.150	6.8135	6.7929	6.9435	7.1352
0.200	6.7276	6.7074	6.8524	7.0365
0.250	6.6532	6.6339	6.7737	6.9513
0.300	6.5869	6.5685	6.7034	6.8757
0.350	6.5269	6.5100	6.6402	6.8072
0.400	6.4717	6.4552	6.5830	6.7445
0.450	6.4205	6.4046	6.5289	6.6864
0.500	6.3727	6.3571	6.4787	6.6322
0.750	6.1733	6.1585	6.2681	6.4077
1.000	6.0094	5.9962	6.0965	6.2239
1.250	5.8704	5.8584	5.9513	6.0690
1.500	5.7490	5.7381	5.8247	5.9342
2.000	5.5429	5.5336	5.6101	5.7066
2.500	5.3707	5.3626	5.4313	5.5177
3.000	5.2223	5.2151	5.2775	5.3557
3.500	5.0913	5.0849	5.1420	5.2135
4.000	4.9472	4.9684	5.0209	5.0867
4.500	4.8680	4.8628	4.9114	4.9723
5.000	4.7709	4.7662	4.8114	4.8679

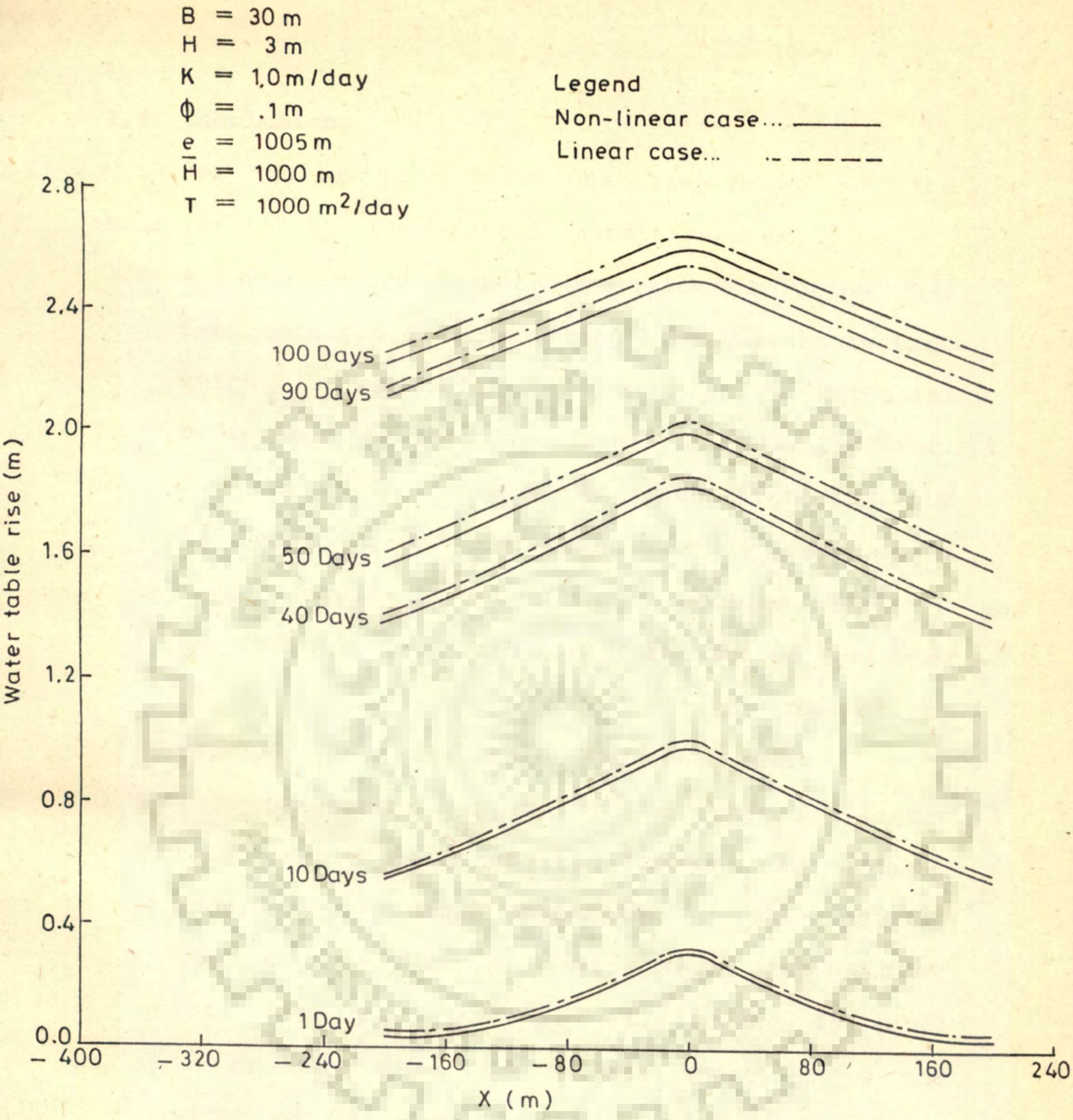

Fig.4.7 – Water table rise across a canal predicted using non-linear and linear relationships between seepage loss and potential difference

difference upto 8 m, the linear relationship between the seepage loss and the potential difference can be used to evaluate seepage loss from canals having non dimensional width, $B/\bar{H}$ upto 0.06.

4.4 Conclusions

The following conclusions are drawn on the basis of the study presented in this chapter:

1. A numerically tractable exact solution has been obtained for predicting time variant seepage which is non-linearly dependent on potential difference between the canal and the aquifer.
2. The Herbert's formula of reach transmissivity is found to be appropriate to be used in a canal aquifer interaction problem. However, this formula is applicable if the wetted perimeter of the canal is less than 1.5 times the thickness of the aquifer below the canal bed.
3. The parameters appearing in the non-linear relationship proposed by Rushton and Redshaw should be evaluated making use of the condition that seepage from a canal attains its maximum rate when the potential difference is equal to or more than 1.5 (B+2H), where, B is the width of the canal and H is the maximum depth of water in the canal.
4. With increase in the width of canal, the non-linearity in the canal aquifer interaction relationship gets pronounced. For potential

CHAPTER 5

INTERFERENCE OF SEEPAGE FROM TWO PARALLEL CANALS WHICH ARE HYDRAULICALLY CONNECTED WITH THE AQUIFER

5.0 Introduction

Evolution of water table due to seepage from two parallel canals, which were not hydraulically connected with the aquifer, has been analysed in chapter 3. In a canal command area the watertable is likely to be present at a shallow depth below the canal bed. Considering a nonlinear relationship between the seepage rate and the potential difference between the canal and the aquifer, a canal aquifer interaction problem for a shallow water table position has been analysed in chapter 4.The study presented in chapter 4 has shown that for shallow water table position the relationship is approximately linear. Using discrete kernel coefficients and reach transmissivity constant, a complex stream aquifer interaction problem has been analysed by Morel-Seytoux and Daly (1975) for finding an expression of recharge from a partially penetrating river. A linear relationship between exchange flow rate and the difference in potentials at the stream and in the aquifer has been assumed by them. In the present chapter using discrete kernel coefficients and reach transmissivity constant, interference of seepage from two parallel canals which are hydraulically connected with the aquifer has been studied.

5.1 Statement of the Problem

Two parallel canals have been constructed in a homogeneous, isotropic, porous medium of finite depth and infinite lateral extent. The widths of canals at water surface are $B_1(n)$ and $B_2(n)$ and the depths of water in the canals are $H_1(n)$ and $H_2(n)$ as shown in Fig.(5.1). The depths $H_1(n)$ and $H_2(n)$ may vary with time (n) depending on the supply from the source. The canal cross sections being trapezoidal, $B_1(n)$ and $B_2(n)$ vary with $H_1(n)$ and $H_2(n)$ respectively. The water table lies at a shallow depth below the canal beds. It is required to find the seepage from the canals with time and the evolution of watertable at a section across the canals.

5.2 Analysis

Let both the canals be divided into a number of identical reaches, say, R as depicted in Fig.(5.2). Let the seepage rate from a canal reach be linearly proportional to the difference in the potentials at the periphery of the canal reach and in the aquifer below the canal bed .

The seepage rate from the r^{th} reach of the first canal during the n^{th} unit time-step can be expressed as:

$$Q_1(r,n) = - \Gamma_1(r,n)[\sigma_1(r,n)-S_1(r,n)] \qquad ...(5.1)$$

where,

$\Gamma_1(r,n)$ = reach transmissivity of the r^{th} reach of the first (left) canal during the n^{th} unit time step,

$S_1(r,n)$ = the drawdown at the end of n^{th} unit time step under the r^{th} reach of the first canal measured from a high datum, and

$\sigma_1(r,n)$ = the drawdown to water level in the r^{th} reach of the first canal measured from the high datum.

The drawdown under any reach at the end of n^{th} unit time-step is caused due to seepage which occurred upto the end of n^{th} unit time-step from all the reaches of both the canals.

Hence,

$$S_1(r,n) = D_i - \bar{H} - \sum_{\substack{\rho=1 \\ \rho \neq r}}^{R} \sum_{\gamma=1}^{n} Q_1(\rho,\gamma)\delta(r,1,\rho,1,n-\gamma+1)$$

$$- \sum_{\gamma=1}^{n} Q_1(r,\gamma)\ \delta[r,1,r,1,B_1(\gamma),\ (n-\gamma+1)]$$

$$- \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n} Q_2(\rho,\gamma)\ \delta(r,1,\rho,\ 2,n-\gamma+1) \qquad ...(5.2)$$

in which,

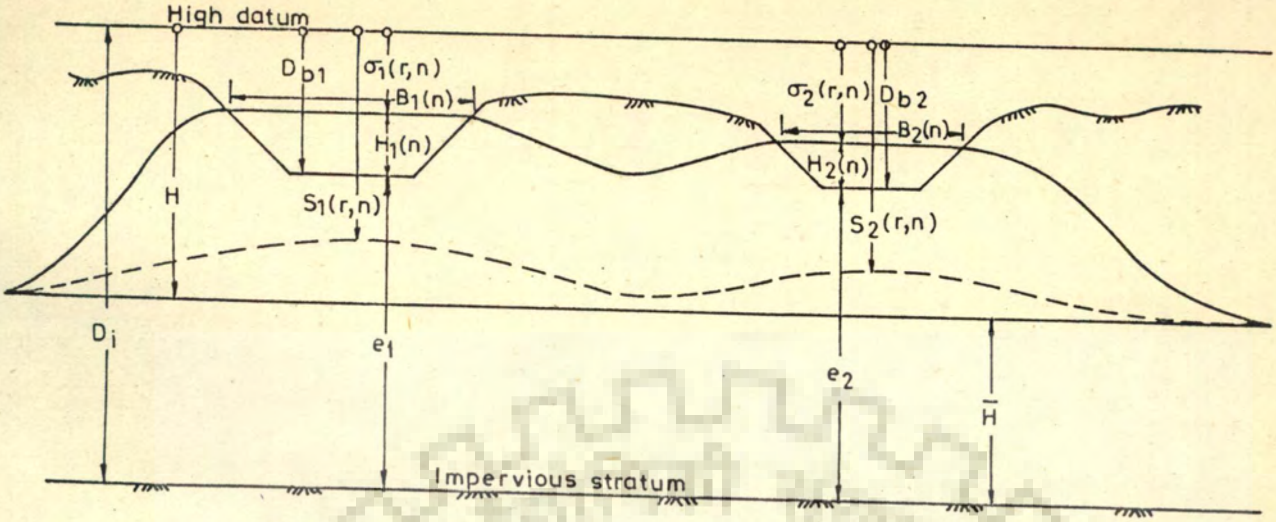

Fig.5.1-Schematic section of a parallel canal system

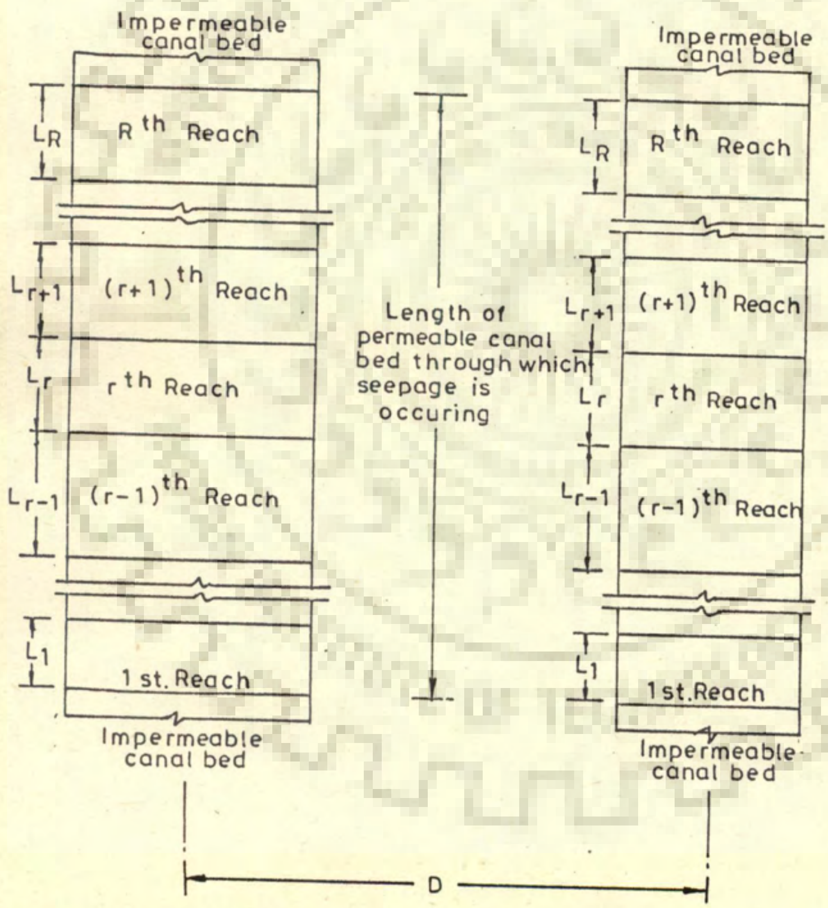

Fig.5.2-Division of the canals into identical reaches from which seepage is taking place

D_i = depth to impervious stratum measured from a high datum as shown in Fig.(5.1),

$\bar{H}$ = initial saturated thickness of the aquifer before the onset of recharge,

$Q_1(\rho,\gamma)$= seepage during γ^{th} unit time-step from the ρ^{th} reach of the first canal,

$Q_2(\rho,\gamma)$ = seepage during γ^{th} unit time-step from the ρ^{th} reach of the second canal,

$\delta[r,1,r,1,B_1(\gamma),(n-\gamma+1)]$ = discrete kernel coefficient for water table rise at the end of $(n-\gamma+1)^{th}$ unit time-step under the r^{th} reach of the first canal in response to unit recharge during time-step γ from the r^{th} reach of the first canal when the canal width was $B_1(\gamma)$,

$\delta(r,1,\rho,1,M)$ = discrete kernel coefficient for water
$\rho \neq r$
table rise at the end of M^{th} unit time-step under the r^{th} reach of the first canal in response to unit recharge during the first time period from the ρ^{th} reach of the first canal,

$\delta(r,1,\rho,2,M)$ = discrete kernel coefficient for water table rise at the end of M^{th} unit time-step under the r^{th} reach of the first canal in response to unit recharge during the first unit time period from the ρ^{th} reach of the second canal.

Substituting the expression for $S_1(r,n)$ in Equation (5.1) and rearranging,

$$\frac{Q_1(r,n)}{-\Gamma_1(r,n)} = \sigma_1(r,n) - D_i + \bar{H}$$

$$+ \sum_{\substack{\rho=1 \\ \rho \neq r}}^{R} \sum_{\gamma=1}^{n} Q_1(\rho,\gamma)\delta(r,1,\rho,1,n-\gamma+1)$$

$$+ \sum_{\gamma=1}^{n} Q_1(r,\gamma)\,\delta[r,1,r,1,B_1(\gamma),n-\gamma+1]$$

$$+ \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n} Q_2(\rho,\gamma)\delta(r,1,\rho,2,n-\gamma+1) \qquad \ldots(5.3)$$

Splitting the temporal summation into two parts, one part containing the summation upto $(n-1)^{th}$ term and the other part the n^{th} term, and rearranging, Equation (5.3) simplifies to:

$$Q_1(r,n)\{-1/\Gamma_1(r,n)-\delta[r,1,r,1,B_1(n),1]\}$$

$$- \sum_{\substack{\rho=1 \\ \rho \neq r}}^{R} Q_1(\rho,n)\,\delta(r,1,\rho,1,1)$$

$$- \sum_{\rho=1}^{R} Q_2(\rho,n)\delta(r,1,\rho,2,1)$$

$$=\sigma_1(r,n)-D_i+\bar{H}$$

$$+ \sum_{\substack{\rho=1 \\ \rho \neq r}}^{R} \sum_{\gamma=1}^{n-1} Q_1(\rho,\gamma)\delta(r,1,\rho,1,n-\gamma+1)$$

$$+\sum_{\gamma=1}^{n-1} Q_1(r,\gamma)\,\delta[r,1,r,1,B_1(\gamma),n-\gamma+1]$$

$$+\sum_{\rho=1}^{R}\sum_{\gamma=1}^{n-1} Q_2(\rho,\gamma)\,\delta(r,1,\rho,2,n-\gamma+1) \qquad \ldots(5.4)$$

The seepage loss from the r^{th} reach of the second canal during the n^{th} time-step is given by the following equation which is similar to Equation (5.1):

$$Q_2(r,n) = -\Gamma_2(r,n)\,[\sigma_2(r,n)-S_2(r,n)] \qquad \ldots(5.5)$$

where,

$\Gamma_2(r,n)$ = reach transmissivity of the r^{th} reach of the second canal during n^{th} unit-time step,

$S_2(r,n)$ = the drawdown at the end of n^{th} unit time-step under the r^{th} reach of the second canal,

$\sigma_2(r,n)$ = the drawdown to water level in the r^{th} reach of the second canal measured from the high datum.

The drawdown, $S_2(r,n)$ is given by the expression:

$$S_2(r,n) = D_i - \bar{H} - \sum_{\rho=1}^{R}\sum_{\gamma=1}^{n} Q_1(\rho,\gamma)\,\delta(r,2,\rho,1,n-\gamma+1)$$

$$-\sum_{\substack{\rho=1\\ \rho\neq r}}^{R}\sum_{\gamma=1}^{n} Q_2(\rho,\gamma)\,\delta(r,2,\rho,2,n-\gamma+1)$$

$$-\sum_{\gamma=1}^{n} Q_2(r,\gamma)\,\delta[r,2,r,2,B_2(\gamma),n-\gamma+1] \qquad \ldots(5.6)$$

where

$\delta(r,2,\rho,1,M)$ = discrete kernel coefficient for water table rise at the end of M^{th} unit time step under the r^{th} reach of the second canal in response to unit recharge during first time-step through the ρ^{th} reach of the first canal,

$\delta(r,2,\rho,2,M)$, $\rho \neq r$ = discrete kernel coefficient for water table rise at the end of M^{th} unit time step under the r^{th} reach of the second canal in response to unit recharge during first unit time period from the ρ^{th} reach of the second canal.

Incorporating the expression for $S_2(r,n)$ in Equation(5.5) and splitting the temporal summation into two parts and rearranging,

$$Q_2(r,n)\{-1/\Gamma_2(r,n) - \delta[r,2,r,2,B_2(n),1]\}$$

$$-\sum_{\substack{\rho=1 \\ \rho\neq r}}^{R} Q_2(\rho,n)\,\delta(r,2,\rho,2,1)$$

$$-\sum_{\rho=1}^{R} Q_1(\rho,n)\,\delta(r,2,\rho,1,1)$$

$$= \sigma_2(r,n) - D_i + \bar{H}$$

$$+\sum_{\rho=1}^{R}\sum_{\gamma=1}^{n-1} Q_1(\rho,\gamma)\,\delta(r,2,\rho,1,n-\gamma+1)$$

$$+ \sum_{\substack{\rho=1 \\ \rho \neq r}}^{R} \sum_{\gamma=1}^{n-1} Q_2(\rho, \gamma)\, \delta(r, 2, \rho, 2, n-\gamma+1)$$

$$+ \sum_{\gamma=1}^{n-1} Q_2(r, \gamma)\, \delta\,[r, 2, r, 2, B_2(\gamma), n-\gamma+1] \qquad \ldots(5.7)$$

According to Morel-Seytoux the discrete kernel coefficients are given by the following relations:

$$\delta(r, 1, \rho, 1, M) = \frac{1}{4\pi T} \{E_1[\frac{d_{r\rho}^2}{4\alpha M}] - E_1[\frac{d_{r\rho}^2}{4\alpha(M-1)}]\} \qquad (\rho \neq r)$$

$$\delta[r, 1, r, 1, B_1(N), M] = \frac{1}{\phi L_r B_1(N)} \int_0^1 \{\mathrm{Erf}\,[\frac{L_r}{4\sqrt{\alpha\,(M-Y)}}] \,.\, \mathrm{Erf}\,[\frac{B_1(N)}{4\sqrt{\alpha\,(M-Y)}}]\}\, dY$$

$$\delta(r, 2, \rho, 1, M) = \frac{1}{4\pi T} \{E_1[\frac{d_{r\rho}^2}{4\alpha M}] - E_1[\frac{d_{r\rho}^2}{4\alpha(M-1)}]\}$$

$$\delta(r, 1, \rho, 2, M) = \frac{1}{4\pi T} \{E_1[\frac{d_{r\rho}^2}{4\alpha M}] - E_1[\frac{d_{r\rho}^2}{4\alpha(M-1)}]\}$$

$$\delta(r, 2, \rho, 2, M) = \frac{1}{4\pi T} \{E_1[\frac{d_{r\rho}^2}{4\alpha M}] - E_1[\frac{d_{r\rho}^2}{4\alpha(M-1)}]\}$$

$$\delta[r, 2, r, 2, B_2(N), M] = \frac{1}{\phi L_r B_2(N)} \int_0^1 \mathrm{Erf}\,[\frac{L_r}{4\sqrt{\alpha(M-Y)}}]\,\mathrm{Erf}[\frac{B_2(N)}{4\sqrt{\alpha(M-Y)}}]\, dY$$

where,

$d_{r\rho}$ = the distance from the centre of the r^{th} reach to the centre of the ρ^{th} reach,

L_r = reach length of the canal,

T = Transmissivity of the aquifer,

ϕ = Storage coefficient, and

$\alpha = T/\phi$.

If each canal is divided into R reaches, there are 2 R unknown recharge quantites at each time-step. A set of 2 R equations, one equation for each reach for each of the two canals, can be written from which the unknowns, $Q_1(\rho, n)$ and $Q_2(\rho,n), \rho = 1,2,3,4.....R$, can be solved in succession starting from time step 1. The 2 R equations can be written in the following matrix notation:

$$[A]\ [B] = [C] \qquad ...(5.8)$$

The elements of the matrices, [A], [B], [C], are as given below:

$$
[A] = \begin{bmatrix}
\left\{\frac{1}{\Gamma_1(1,n)} + \delta(1,1,1,1,B_1(n),1)\right\}, & \delta(1,1,2,1,1) & \cdots & \delta(1,1,r,1,1) & \cdots & \delta(1,1,R,1,1), & \delta(1,1,1,2,1), & \delta(1,1,2,2,1) & \cdots & \delta(1,1,r,2,1) & \cdots & \delta(1,1,R,2,1) \\
\delta(2,1,1,1,1), & \left\{\frac{1}{\Gamma_1(2,n)} + \delta(2,1,2,1,B_1(n),1)\right\} & \cdots & \delta(2,1,r,1,1) & \cdots & \delta(2,1,R,1,1), & \delta(2,1,1,2,1), & \delta(2,1,2,2,1) & \cdots & \delta(2,1,r,2,1) & \cdots & \delta(2,1,R,2,1) \\
\vdots & \vdots & & \vdots & & \vdots & \vdots & \vdots & & \vdots & & \vdots \\
\delta(r,1,1,1,1), & \delta(r,1,2,1,1) & \cdots & \left\{\frac{1}{\Gamma_1(r,n)} + \delta(r,1,r,1,B_1(n),1)\right\} & \cdots & \delta(r,1,R,1,1), & \delta(r,1,1,2,1), & \delta(r,1,2,2,1) & \cdots & \delta(r,1,r,2,1) & \cdots & \delta(r,1,R,2,1) \\
\vdots & \vdots & & \vdots & & \vdots & \vdots & \vdots & & \vdots & & \vdots \\
\delta(R,1,1,1,1), & \delta(R,1,2,1,1) & \cdots & \delta(R,1,r,1,1) & \cdots & \left\{\frac{1}{\Gamma_1(R,n)} + \delta(R,1,R,1,B_1(n),1)\right\}, & \delta(R,1,1,2,1), & \delta(R,1,2,2,1) & \cdots & \delta(R,1,r,2,1) & \cdots & \delta(R,2,R,2,1) \\
\delta(1,2,1,1,1), & \delta(1,2,2,1,1) & \cdots & \delta(1,2,r,1,1) & \cdots & \delta(1,2,R,1,1), & \left\{\frac{1}{_2(1,n)} + \delta(1,2,1,2,B_2(n),1)\right\}, & \delta(1,2,2,2,1) & \cdots & \delta(1,2,r,2,1) & \cdots & \delta(1,2,R,2,1) \\
\delta(2,2,1,1,1), & \delta(2,2,2,1,1) & \cdots & \delta(2,2,r,1,1) & \cdots & \delta(2,2,R,1,1), & \delta(2,2,1,2,1), & \left\{\frac{1}{\Gamma_2(2,n)} + \delta(2,2,2,2,B_2(n),1)\right\} & \cdots & \delta(2,2,r,2,1) & \cdots & \delta(2,2,R,2,1) \\
\vdots & \vdots & & \vdots & & \vdots & \vdots & \vdots & & \vdots & & \vdots \\
\delta(r,2,1,1,2), & \delta(r,2,2,1,1) & \cdots & \delta(r,2,r,1,1) & \cdots & \delta(r,2,R,1,1), & \delta(r,2,1,2,1), & \delta(r,2,2,2,1) & \cdots & \left\{\frac{1}{\Gamma_2(r,n)} + \delta(r,2,r,2,B_2(n),1)\right\} & \cdots & \delta(r,2,R,2,1) \\
\vdots & \vdots & & \vdots & & \vdots & \vdots & \vdots & & \vdots & & \vdots \\
\delta(R,2,1,1,1), & \delta(R,2,2,1,1) & \cdots & \delta(R,2,r,1,1) & \cdots & \delta(R,2,R,1,1), & \delta(R,2,1,2,1), & \delta(R,2,2,2,1) & \cdots & \delta(R,2,r,2,1) & \cdots & \left\{\frac{1}{\Gamma_1(1,n)} + \delta(R,2,R,2,B_2(n),1)\right\}
\end{bmatrix}
$$

$$
[B] = [Q_1(1,n),\ Q_1(2,n)\ \cdots\ Q_1(r,n)\ \cdots\ Q_1(R,N),\ Q_2(1,n),\ Q_2(2,n)\ \cdots\ Q_2(r,n)\ \cdots\ Q_2(R,N)]^T
$$

The elements of the matrix (C) are as follows :

$$C(1,1) = -\{\sigma_1(1,n)-D_i+\bar{H}+\sum_{\substack{\rho=1\\ \rho\neq 1}}^{R}\sum_{\gamma=1}^{n-1} Q_1(\rho,\gamma)\partial(1,1,\rho,1,n-\gamma+1)$$
$$+\sum_{\gamma=1}^{n-1} Q_1(1,\gamma)\partial(1,1,1,1,B_1(\gamma),n-\gamma+1)+\sum_{\gamma=1}^{n-1}\sum_{\rho=1}^{R} Q_2(\rho,\gamma)\partial(1,1,\rho,2,n-\gamma+1)\}$$

$$C(2,1) = -\{\sigma_1(2,n)-D_i+\bar{H}+\sum_{\substack{\rho=1\\ \rho\neq 2}}^{R}\sum_{\gamma=1}^{n-1} Q_1(\rho,\gamma)\partial(2,1,\rho,1,n-\gamma+1)$$
$$+\sum_{\gamma=1}^{n-1} Q_1(2,\gamma)\partial(2,1,2,1,B_1(\gamma),n-\gamma+1)+\sum_{\gamma=1}^{n-1}\sum_{\rho=1}^{R} Q_2(\rho,\gamma)\partial(2,1,\rho,2,n-\gamma+1)\}$$

$$C(r,1) = -\{\sigma_1(r,n)-D_i+\bar{H}+\sum_{\substack{\rho=1\\ \rho\neq r}}^{R}\sum_{\gamma=1}^{n-1} Q_1(\rho,\gamma)\partial(r,1,\rho,1,n-\gamma+1)$$
$$+Q_1(r,\gamma)\partial(r,1,r,1,B_1(\gamma),n-\gamma+1)+\sum_{\gamma=1}^{n-1}\sum_{\rho=1}^{R} Q_2(\rho,\gamma)\partial(r,1,\rho,2,n-\gamma+1)\}$$

$$C(R,1) = -\{\sigma_1(R,n)-D_i+\bar{H}+\sum_{\substack{\rho=1\\ \rho\neq R}}^{R}\sum_{\gamma=1}^{n-1} Q_1(\rho,\gamma)\partial(R,1,\rho,1,n-\gamma+1)$$
$$+Q_1(R,\gamma)\partial(R,1,R,1,B_1(\gamma),n-\gamma+1)+\sum_{\gamma=1}^{n-1}\sum_{\rho=1}^{R} Q_2(\rho,\gamma)\partial(R,1,\rho,2,n-\gamma+1)\}$$

$$C(R+1,1) = -\{\sigma_2(1,n)-D_i+\bar{H}+\sum_{\gamma=1}^{n-1}\sum_{\rho=1}^{R} Q_1(\rho,\gamma)\partial(1,2,\rho,1,n-\gamma+1)$$
$$+\sum_{\substack{\rho=1\\ \rho\neq 1}}^{R}\sum_{\gamma=1}^{n-1} Q_2(\rho,\gamma)\partial(1,2,\rho,2,n-\gamma+1)+\sum_{\gamma=1}^{n-1} Q_2(1,\gamma)\partial(1,2,1,2,B_2(\gamma),n-\gamma+1)\}$$

$$C(R+2,1) = -\{\sigma_2(2,n)-D_i+\bar{H}+\sum_{\gamma=1}^{n-1}\sum_{\rho=1}^{R} Q_1(\rho,\gamma)\partial(2,2,\rho,1,n-\gamma+1)$$
$$+\sum_{\substack{\rho=1\\ \rho\neq 2}}^{R}\sum_{\gamma=1}^{n-1} Q_2(\rho,\gamma)\partial(2,2,\rho,2,n-\gamma+1)+\sum_{\gamma=1}^{n-1} Q_2(2,\gamma)\partial(2,2,2,2,B_2(\gamma),n-\gamma+1)\}$$

$$C(R+r,1) = -\{\sigma_2(r,n)-D_i+\bar{H}+\sum_{\gamma=1}^{n-1}\sum_{\rho=1}^{R} Q_1(\rho,\gamma)\partial(r,2,\rho,1,n-\gamma+1)$$
$$+\sum_{\substack{\rho=1\\ \rho\neq r}}^{R}\sum_{\gamma=1}^{n-1} Q_2(\rho,\gamma)\partial(r,2,\rho,2,n-\gamma+1)+\sum_{\gamma=1}^{n-1} Q_2(r,\gamma)\partial(r,2,r,2,B_2(\gamma),n-\gamma+1)\}$$

$$C(2R,1) = -\{\sigma_2(R,n)-D_i+\bar{H}+\sum_{\gamma=1}^{n-1}\sum_{\rho=1}^{R} Q_1(\rho,\gamma)\partial(R,2,\rho,1,n-\gamma+1)$$
$$+\sum_{\substack{\rho=1\\ \rho\neq R}}^{R}\sum_{\gamma=1}^{n-1} Q_2(\rho,\gamma)\partial(R,2,\rho,2,n-\gamma+1)+\sum_{\gamma=1}^{n-1} Q_2(R,\gamma)\partial(R,2,R,2,B_2(\gamma),n-\gamma+1)\}$$

Hence,

$$[B] = [A]^{-1}\ [C] \qquad \ldots(5.9)$$

If, the aquifer is homogeneous and isotropic, the section and depth of water in each canal do not vary from reach to reach over a large stretch, no other source of recharge or abstraction well exists near the canals, seepage is occuring from very large lengths of the canals, and the canals are divided into identical reaches of equal length, the seepage rates from all the reaches of a canal during a particular time step will be equal and there will be only two unknowns at a particular time-step. The two pertinent equations have been derived from Equations (5.4) and (5.7) as follows:

Let the canals be divided into reaches of equal length. Let the origin be chosen arbitrarily at the centre of a reach. Because of symmetry about the x axis, the reaches are numbered as shown in Fig.(5.3). Let the rise in water table height at any section across the canals be influenced by recharge from the nearest (2R-1) number of reaches of each canal and let the effect of recharge from the other distant reaches be negligible. Thus, the recharge from $(R+1)^{th}$ reach and beyond has negligible influence on the rise in water table height at the first

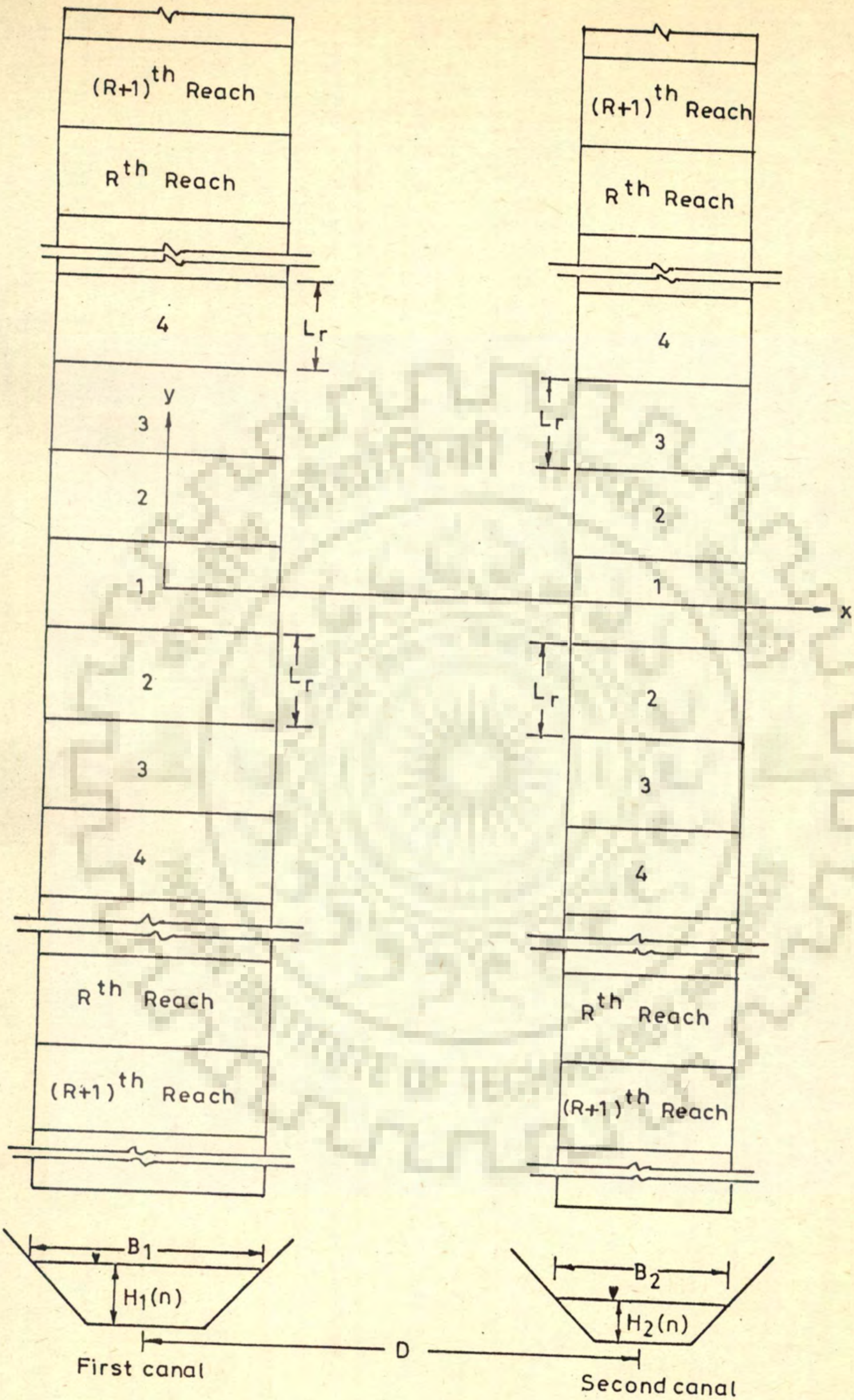

Fig.5.3 - Division of the canals into equal reaches and their numbering

reach. In accordance with these assumptions, Equation (5.4) reduces to:

$$Q_1'(n)\{-1/\Gamma_1'(n) - \delta[1,1,1,1,B_1(n),1] - 2\sum_{\rho=2}^{R}\delta(1,1,\rho,1,1)\}$$

$$+ Q_2'(n)[-\delta(1,1,1,2,1) - 2\sum_{\rho=2}^{R}\delta(1,1,\rho,2,1)]$$

$$= \sigma_1(n) - D_i + \bar{H} + \sum_{\gamma=1}^{n-1} Q_1'(\gamma)\,\delta[1,1,1,1\ B_1(\gamma),\ n-\gamma+1]$$

$$+ 2\sum_{\rho=2}^{R}\sum_{\gamma=1}^{n-1} Q_1'(\gamma)\,\delta(1,1,\rho,1,n-\gamma+1)$$

$$+ \sum_{\gamma=1}^{n-1} Q_2'(\gamma)\,\delta(1,1,1,2,n-\gamma+1)$$

$$+ 2\sum_{\rho=2}^{R}\sum_{\gamma=1}^{n-1} Q_2'(\gamma)\,\delta(1,1,\rho,2,n-\gamma+1) \quad \ldots(5.10)$$

Similarly, Equation (5.7) reduces to:-

$$Q_1'(n)[-\delta(1,2,1,1,1) - 2\sum_{\rho=2}^{R}\delta(1,2,\rho,1,1)] +$$

$$Q_2'(n)\{-1/\Gamma_2'(n) - \delta[1,2,1,2,B_2(n),1] - \sum_{\rho=2}^{R}\delta(1,2,\rho,2,1)\}$$

$$= \sigma_2(n) - D_i + \bar{H} + \sum_{\gamma=1}^{n-1} Q_1'(\gamma)\,\delta(1,2,1,1,n-\gamma+1)$$

$$+ 2 \sum_{\rho=2}^{R} \sum_{\gamma=1}^{n-1} Q_1'(\gamma)\, \delta(1,2,\rho,1,n-\gamma+1)$$

$$+ \sum_{\gamma=1}^{n-1} Q_2'(\gamma)\, \delta[1,2,1,2,B_2(\gamma),\ n-\gamma+1]$$

$$+2 \sum_{\rho=2}^{R} \sum_{\gamma=1}^{n-1} Q_2'(\gamma)\delta(1,2,\rho,2,n-\gamma+1) \qquad \ldots(5.12)$$

$B(1,1)=Q_1'(n)$, $B(2,1) = Q_2'(n)$, Hence,

$[B] = [A]^{-1}[C]$

If the flow characteristics do not vary from section to section across the canals, it is not necessary to divide the canals into reaches to solve for the seepage rates. The seepage rates could be determined without dividing the canal into reaches in the following manner: Let $Q_1(n)$ and $Q_2(n)$ be the seepage rates per unit length of the first and the second canal respectively. Let the origin be chosen at the centre of the first canal. Let $S(x,n)$ be the drawdown measured from a high datum at, x. Considering reach transmissivity for unit length of canal, the $Q_1(n)$ and $Q_2(n)$ are given by:-

$$Q_1(n)=-\Gamma_1(n)\ [\sigma_1(n) - S\ (o,n)] \qquad \ldots(5.13)$$

$$Q_2(n)=-\Gamma_2(n)\ [\sigma_2(n) - S\ (D,n)] \qquad \ldots(5.14)$$

where, $\Gamma_1(n)$ and $\Gamma_2(n)$ are the reach transmissivity values per unit length of the first and the second canal respectively and D is the distance between centre to centre of the canals.

The drawdown $S(o,n)$ and $S(D,n)$ can be expressed in terms of recharge as:

$$+ 2\sum_{\rho=2}^{R}\sum_{\gamma=1}^{n-1} Q'_1(\gamma)\delta(1,2,\rho,1,n-\gamma+1)$$

$$+ \sum_{\gamma=1}^{n-1} Q'_2(\gamma)\delta[1,2,1,2,B_2(\gamma), n-\gamma+1]$$

$$+ 2\sum_{\rho=2}^{R}\sum_{\gamma=1}^{n-1} Q'_2(\gamma)\delta(1,2,\rho,2,n-\gamma+1) \qquad \ldots(5.11)$$

The two unknowns $Q'_1(n)$ and $Q'_2(n)$ can be solved from Equations (5.10) and (5.11) in succession, starting from time step 1. The equation in matrix notation is given by [A].[B]=[C] where,

$$A(1,1) = \frac{-1}{\Gamma'_1(n)} - \delta[1,1,1,1,B_1(n),1]-2\sum_{\rho=2}^{R}\delta(1,1,\rho,1,1),$$

$$A(1,2) = -\delta(1,1,1,2,1) - 2\sum_{\rho=2}^{R}\delta(1,1,\rho,2,1),$$

$$A(2,1) = -\delta(1,2,1,1,1)- 2\sum_{\rho=2}^{R}\delta(1,2,\rho,1,1),$$

$$A(2,2) = -\frac{1}{\Gamma'_2(n)} - \delta[1,2,1,2,B_2(n),1] - 2\sum_{\rho=2}^{R}\delta(1,2,\rho,2,1),$$

$$C(1,1) = \sigma_1(n) - D_i+\bar{H} + \sum_{\gamma=1}^{n-1} Q'_1(\gamma)\delta[1,1,1,1,B_1(\gamma),n-\gamma+1]$$

$$+2\sum_{\rho=2}^{R}\sum_{\gamma=1}^{n-1} Q'_1(\gamma)\delta(1,1,\rho,1,n-\gamma+1)$$

$$+\sum_{\gamma=1}^{n-1} Q'_2(\gamma)\delta(1,1,1,2,n-\gamma+1)$$

$$+2\sum_{\rho=2}^{R}\sum_{\gamma=1}^{n-1} Q'_2(\gamma)\delta(1,1,\rho,2,n-\gamma+1)$$

$$C(2,1) = \sigma_2(n)-D_i+\bar{H} + \sum_{\gamma=1}^{n-1} Q'_1(\gamma)\delta(1,2,1,1,n-\gamma+1)$$

$$S(o,n) = D_i - \bar{H} - \sum_{\gamma=1}^{n} Q_1(\gamma)\,\delta_1[0,B_1(\gamma), n-\gamma+1]$$

$$- \sum_{\gamma=1}^{n} Q_2(\gamma)\,\delta_2[-D,B_2(\gamma), n-\gamma+1] \qquad ..(5.15)$$

$$S(D,n) = D_i - \bar{H} - \sum_{\gamma=1}^{n} Q_1(\gamma)\,\delta_1[D,B_1(\gamma), n-\gamma+1]$$

$$- \sum_{\gamma=1}^{n} Q_2(\gamma)\,\delta_2[0,B_2(\gamma), n-\gamma+1] \qquad ...(5.16)$$

Where, the discrete kernel coefficients for rise in water table are given by Equations (3.12) & (3.13) of chap. 3:

$$\delta_1[0,B_1(N), M] = F[0, B_1(N), M] - F[0,B_1(N),M-1], \text{ for } M \geq 2 \qquad ...(5.17)$$

$$\delta_1[0,B_1(N), 1] = F[0, B_1(N), 1] - B_1(N)/(8T) \qquad ...(5.18)$$

$$\delta_2[-D,B_2(N),M] = F[-D,B_2(N), M] - F[-D,B_2(N),M-1], \text{ for } M \geq 2 \qquad ...(5.19)$$

$$\delta_2[-D,B_2(N),1] = F[-D,B_2(N), 1] - \frac{1}{2T}\sqrt{(-D)^2} \qquad ...(5.20)$$

$$\delta_1[D,B_1(N),M] = F[D,B_1(N),M] - F[D, B_1(N), M-1], \text{ for } M \geq 2 \qquad ...(5.21)$$

$$\delta_1[D, B_1(N),1] = F[D,B_1(N),1] - \frac{1}{2T}\sqrt{(D^2)} \qquad ...(5.22)$$

$$\delta_2[0,B_2(N), M] = F[0,B_2(N), M] - F[0, B_2(N), M-1], \text{ for } M \geq 2 \qquad ...(5.23)$$

$$\delta_2[0,B_2(N),1] = F[0,B_2(N),1] - B_2(N)/(8T) \qquad ...(5.24)$$

Incorporating Equation (5.15) in Equation (5.13) and Equation (5.16) in Equation (5.14) the following equations are obtained:

$$Q_1(n) = -\Gamma_1(n)\{\sigma_1(n) - D_i + \bar{H} + \sum_{\gamma=1}^{n} Q_1(\gamma)\ \delta_1(O, B_1(\gamma), n-\gamma+1)$$

$$+ \sum_{\gamma=1}^{n} Q_2(\gamma)\ \delta_2[-D, B_2(\gamma), n-\gamma+1]\} \quad ..(5.25)$$

$$Q_2(n) = -\Gamma_2(n)\{\sigma_2(n) - D_i + \bar{H} + \sum_{\gamma=1}^{n} Q_1(\gamma)\ \delta_1[D, B_1(\gamma), n-\gamma+1)$$

$$+ \sum_{\gamma=1}^{n} Q_2(\gamma)\delta_2[O, B_2(\gamma), n-\gamma+1]\} \quad ...(5.26)$$

Splitting the temporal summation into two parts and rearranging, the above equations reduce to:

$$Q_1(n)\{-1/\Gamma_1(n) - \delta_1[O, B_1(n), 1]\} - Q_2(n)\delta_2[-D, B_2(n), 1)]$$

$$= \sigma_1(n) - D_i + \bar{H} + \sum_{\gamma=1}^{n-1} Q_1(\gamma)\delta_1\ [O, B_1(\gamma), n-\gamma+1)]$$

$$+ \sum_{\gamma=1}^{n-1} Q_2(\gamma)\ \delta_2[-D, B_2(\gamma), n-\gamma+1] \quad ...(5.27)$$

$$Q_2(n)\{-1/\Gamma_2(n) - \delta_2[O, B_2(n), 1]\} - Q_1(n)\delta_1[(D, B_1(n), 1]$$

$$= \sigma_2(n) - D_i + \bar{H} + \sum_{\gamma=1}^{n-1} Q_1(\gamma)\delta_1[D, B_1(\gamma), n-\gamma+1]$$

$$+ \sum_{\gamma=1}^{n-1} Q_2(\gamma)\ \delta_2[O, B_2(\gamma), n-\gamma+1] \quad ...(5.28)$$

Equations (5.27) and (5.28) can be written in the following matrix form:

$$[A].[B] = [C]$$

in which,

$$[B] = \begin{bmatrix} Q_1(n) \\ Q_2(n) \end{bmatrix}$$

The elements of matrices [A] and [C] are as follows:

$$A(1,1) = -1/\Gamma_1(n) - \delta_1 [0, B_1(n), 1]$$

$$A(1,2) = -\delta_2[-D, B_2(n), 1]$$

$$A(2,1) = -\delta_1 [D, B_1(n), 1]$$

$$A(2,2) = -1/\Gamma_2(n) - \delta_2 [0, B_2(n), 1]$$

$$C(1,1) = \sigma_1(n) - D_i + \bar{H} + \sum_{\gamma=1}^{n-1} Q_1(\gamma)\delta_1[0, B_1(\gamma), n-\gamma+1]$$

$$+ \sum_{\gamma=1}^{n-1} Q_2(\gamma)\, \delta_2 [-D, B_2(\gamma), n-\gamma+1]$$

$$C(2,1) = \sigma_2(n) - D_i + \bar{H} + \sum_{\gamma=1}^{n-1} Q_1(\gamma)\,\delta_1[D, B_1(\gamma), n-\gamma+1]$$

$$+ \sum_{\gamma=1}^{n-1} Q_2(\gamma)\, \delta_2[0, B_2(\gamma), n-\gamma+1]$$

$Q_1(n)$ and $Q_2(n)$ can be solved in succession starting from time step 1 using the following equation,

$$\begin{bmatrix} Q_1(n) \\ Q_2(n) \end{bmatrix} = [A]^{-1} [C] \qquad ...(5.29)$$

5.3 Results and Discussions

Numerical results quantifying the time variant seepage losses from two parallel canals and the consequent rise in water table have been presented for the following cases :

(a) Two parallel canals running continuously with constant depth of water in them; the canals may have different widths and their beds may be at different levels.

(b) Two identical parallel canals having their beds at the same level; one of the canals running continuously with constant depth of water and the other running intermittently with nominal depth of water during its closure period.

(c) Two identical parallel canals having their beds at the same level; one of the canals running continuously with constant depth of water and the other running intermittently, being completely dry during the closure period.

The results for seepage losses and water table rise have been obtained for assumed canal dimensions, aquifer parameters, K, $\overline{H}$, and φ , and initial potential difference between the canals and the aquifer. The discrete kernels for water table rise have been generated using the Equations (5.17) through (5.24) for the assumed values of canal dimensions, spacing between the canals, and the aquifer parameters. Since both the canals run continuously with constant depth of water, and the widths of the canals at water surface do not change with time for case (a), the discrete kernels for case (a) have been generated for constant canal widths. The reach transmissivity constant for each canal, which does not change with time for case (a), has been evaluated using Herbert's formula [Equation (2.41)]. Typical reach transmissivity values for various canal dimensions are given in Table (5.1).

In order to analyse the sensitivity of the solution to the size and number of time- steps, the seepage losses from a single canal and the consequent water table rise at selected observation points have been calculated with different time - step sizes, and are presented in Tables (5.2) and (5.3). The computations have been made for B_1= 60m, H_1= 3m, H = 9m, $\overline{H}$ = 1000m, T = 1000m²/day, and φ = 0.1. It can be observed from Table (5.2) that with any time-step size the error in the solution decreases with time. It is also seen that the rate of seepage loss from unit length of the canal at the end of one day is 5.567898m³/day, which has been calculated with time-step size of one day. The rates of seepage loss from unit length of the canal, calculated with time-step size of $1/10^{th}$ and $1/100^{th}$ of a day are 5.565095 m^3/day and 5.564659m³/day respectively. A comparison of the above

Table (5.1) - Reach transmissivity for trapezoidal canals of different cross sections with 1:1 side slopes, evaluated for aquifer parameters K=1 m/day, T=1000m²/day, and initial saturated thickness of aquifer, H=1000 m.

Width of canal at the water surface (m)	Depth of water in the canal (m)	Height of bed of canal above the initial water table (m)	Reach transmissivity for unit length of canal (m^2/day)
60	3	1	0.9731
60	3	3	0.9725
60	3	5	0.9719
30	3	1	0.8092
30	3	3	0.8087
30	3	5	0.8083
15	3	1	0.6978
15	3	3	0.6975
15	3	5	0.6972

Table (5.2) - Seepage loss, Q_1 (n), Calculated with different time - step sizes for B_1= 60m, H_1=3 m, H=9 m, $\bar{H}$ =1000 m, T=1000 m²/day and φ = 0.1.

Time (day)	Seepage loss $Q_1(n)$, calculated with time - step size:		
	1 day	$1/10^{th}$ of a day	$1/100^{th}$ of a day
1	5.567898	5.565095	5.564659
2	5.447787	5.444447	5.443994
3	5.358006	5.354448	5.353994
4	5.283993	5.280330	5.279878
5	5.220059	5.216343	5.215897
6	5.163275	5.159538	5.159097
7	5.111897	5.108155	5.107720
8	5.064792	5.061055	5.060625
9	5.021166	5.017442	5.017017
10	4.980447	4.976741	4.976322

Table 5.3 - Water table rise, $S(x,t)/\overline{H}$, calculated with different time-step sizes for $T = 1000\ m^2/day$, $\varphi = 0.1$, $\overline{H} = 1000m$, $B_1 = 60\ m$, $H_1 = 3\ m$, and $H = 9m$.

Time	Water table rise evaluated with time-step size:					
	1 day		$1/10^{th}$ of a day		$1/100^{th}$ of a day	
day	$x/\overline{H} = 0.0$	$x/\overline{H} = 0.05$	$x/\overline{H} = 0.0$	$x/\overline{H} = 0.05$	$x/\overline{H} = 0.0$	$x/\overline{H} = 0.05$
1	0.274727	0.196578	0.277609	0.199182	0.278057	0.199548
2	0.398233	0.316240	0.401667	0.319460	0.402133	0.319869
3	0.490551	0.407444	0.494210	0.410931	0.494677	0.411354
4	0.566656	0.483234	0.570423	0.486859	0.570887	0.487286
5	0.632398	0.548995	0.636217	0.552694	0.636676	0.553121
6	0.690787	0.607567	0.694629	0.611305	0.695083	0.611731
7	0.743616	0.660665	0.747464	0.664422	0.747911	0.664845
8	0.792053	0.709419	0.795895	0.713181	0.796337	0.713600
9	0.836912	0.754620	0.840741	0.758378	0.841178	0.758793
10	0.878782	0.796849	0.882593	0.800596	0.883024	0.801008

results shows that there is no significant difference in the seepage losses which have been computed with time - step size of $1/10^{th}$ and $1/100^{th}$ of a day ; whereas there is appreciable difference between seepage losses which have been calculated with time - step size of 1 day and $1/10^{th}$ of a day. Therefore, if it is required to predict the seepage rate at the end of one day, the time - step size of $1/10^{th}$ of a day is sufficient to predict the seepage loss with sufficient accuracy. Thus, a minimum number of 10 time-steps should be used to arrive at the seepage loss at a particular time.

The effect of time - step size on the accuracy of prediction of water table rise is shown in Table (5.3). It could be seen from the table that for computing the water table rise, a minimum of 10 time - steps should also be used. It can be seen that water table rise is more sensitive than the seepage loss to time discretisation.

The seepage losses have been determined in succession, starting from time-step 1, using Equations (5.27) and (5.28). The results have been presented in non-dimensional form. The non-dimensional groups which have been formed are given below:

(i)	Seepage losses per unit length of the first canal and the second canal respectively	$Q_1(t)/(KH_1)$, $Q_2(t)/(KH_2)$
(ii)	Time factor	$Kt/(2\ \varphi\ \overline{H})$
(iii)	Water table rise	$S(x,t)/\overline{H}$

(iv)	Width of canals at water surface	$B_1/\overline{H}$, $B_2/\overline{H}$
(v)	Depth of water in the canals	$H_1/\overline{H}$, $H_2/\overline{H}$
(vi)	Distance of observation point from centre of left canal	$x/\overline{H}$
(vii)	Centre to centre distance between the canals	$D/\overline{H}$
(viii)	Depth to water levels in the canals from a high datum	$\sigma_1/\overline{H}, \sigma_2/\overline{H}$
(ix)	Initial water table position measured from the high datum	$H/\overline{H}$
(x)	Depth of canal beds from the high datum	$D_{b1}/\overline{H}$, $D_{b2}/\overline{H}$

The variations of seepage losses with time from two parallel canals, evaluated for $B_1/\overline{H}=B_2/\overline{H} = 0.03$, $H_1/\overline{H} = H_2/\overline{H} = 0.003$, $D/\overline{H} = 0.18$, and for initial water table positions, $H/\overline{H} = 0.005$, 0.007, and 0.009, are presented in Fig. (5.4). Seepage losses from parallel canals, which have larger widths, are presented in Fig. (5.5) for $B_1/\overline{H} = B_2/\overline{H} = 0.06$. Seepage losses from canals for a closer spacing of $D/\overline{H} = 0.08$ are presented in Figs. (5.6) and (5.7). The seepage loss from each canal, at time $t = 0$, is given by the product of the corresponding reach transmissivity and the initial potential difference. Therefore, as shown in the above mentioned figures, the seepage loss is finite in the beginning and is equal to the product of corresponding reach transmissivity and initial potential difference. The seepage decreases as time elapses, the rate of

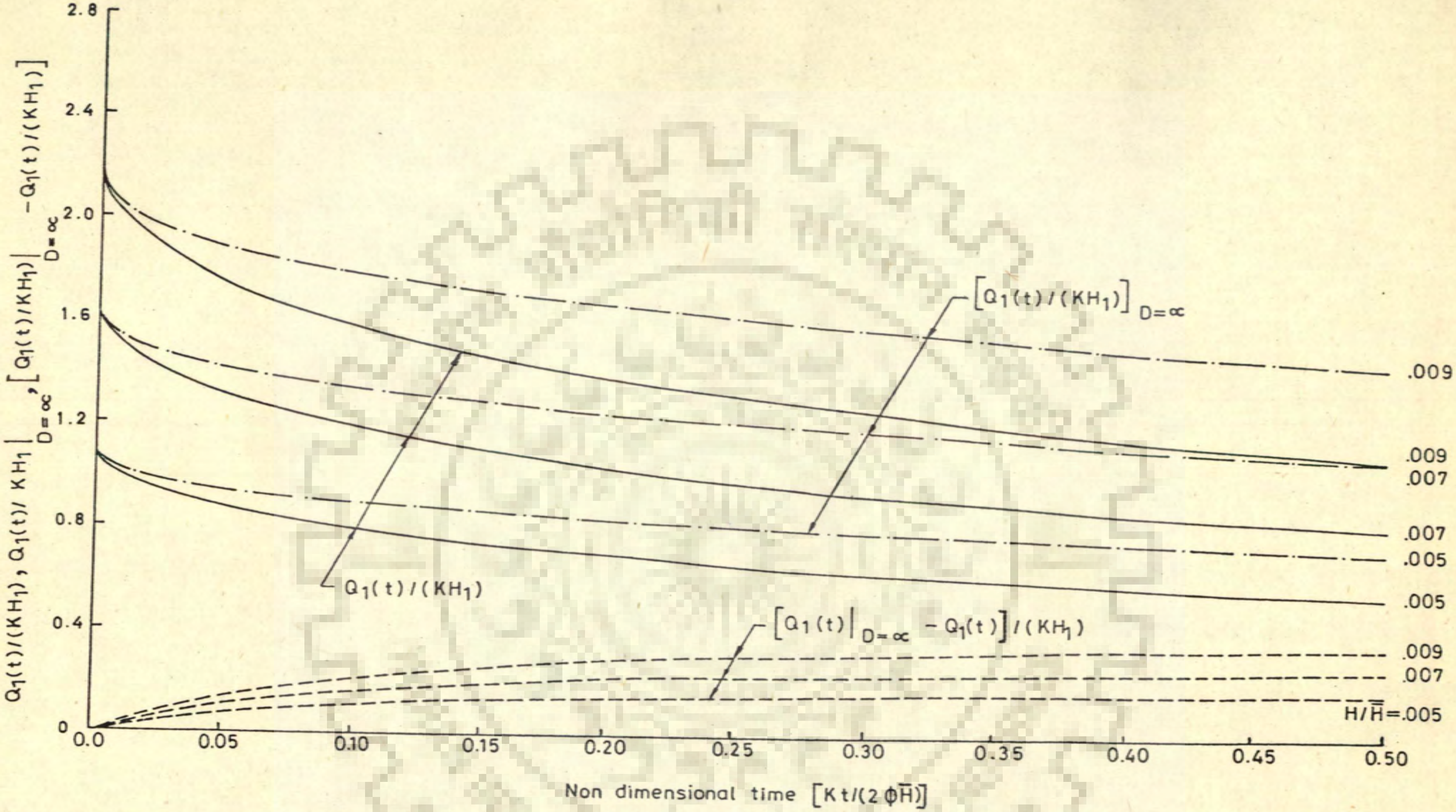

Fig.5.4–Interference of seepage losses from two continuously running identical parallel canals, evaluated for $D/\bar{H} = 0.18$, $B_1/\bar{H} = B_2/\bar{H} = 0.03$, $H_1/\bar{H} = H_2/\bar{H} = 0.003$, $\sigma_1/\bar{H} = \sigma_2/\bar{H} = 0.001$ and for initial water table positions $H/\bar{H} = 0.005$, 0.007 and 0.009

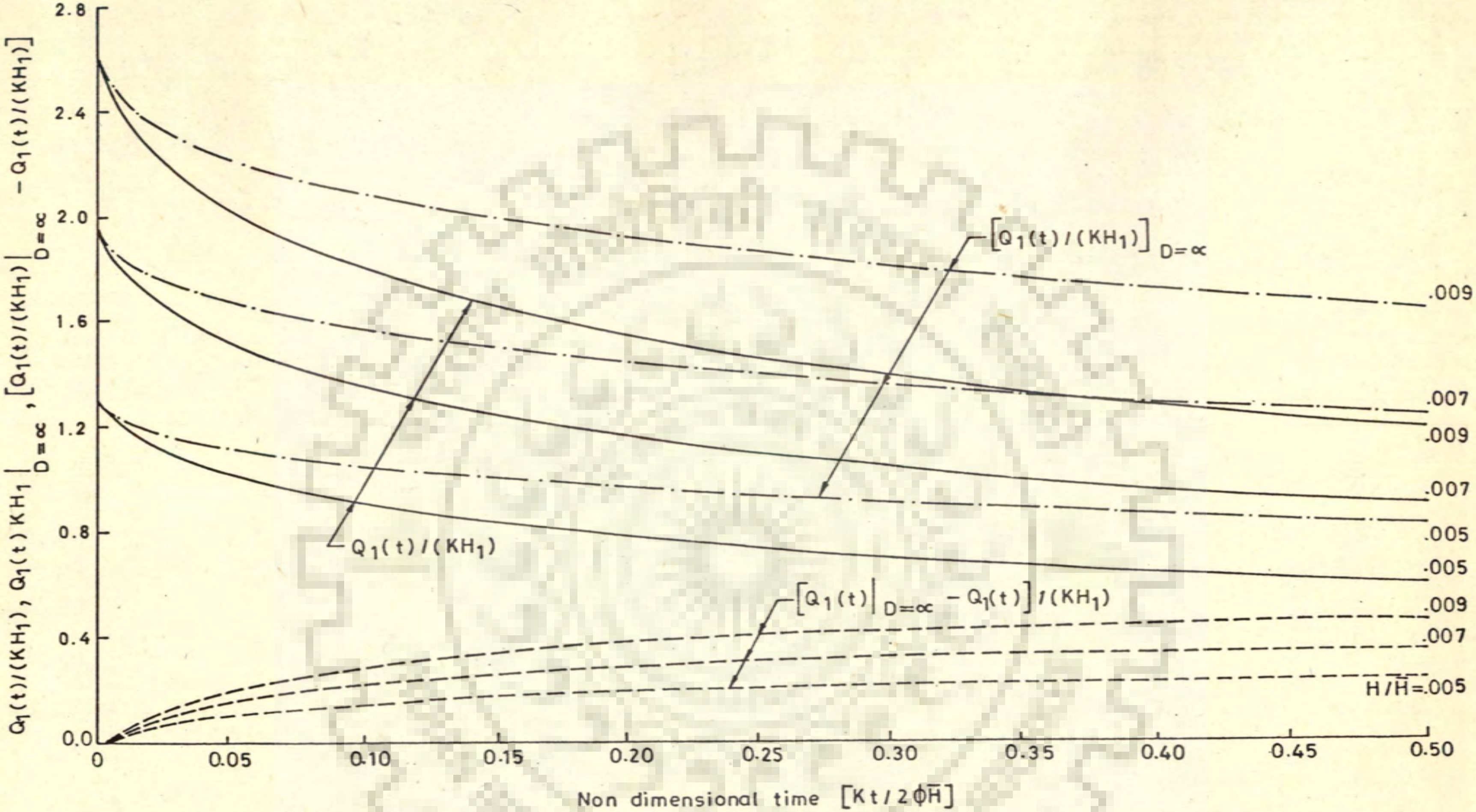

Fig.5.5 - Interference of seepage losses from two continuously running identical parallel canals, evaluated for $D/\bar{H}=0.18$, $B_1/\bar{H}=B_2/\bar{H}=0.06$, $H_1/\bar{H}=H_2/\bar{H}=0.003$, $\sigma_1/\bar{H}=\sigma_2/\bar{H}=0.001$ and for initial water table positions $H/\bar{H}=0.005$, 0.007 and 0.009

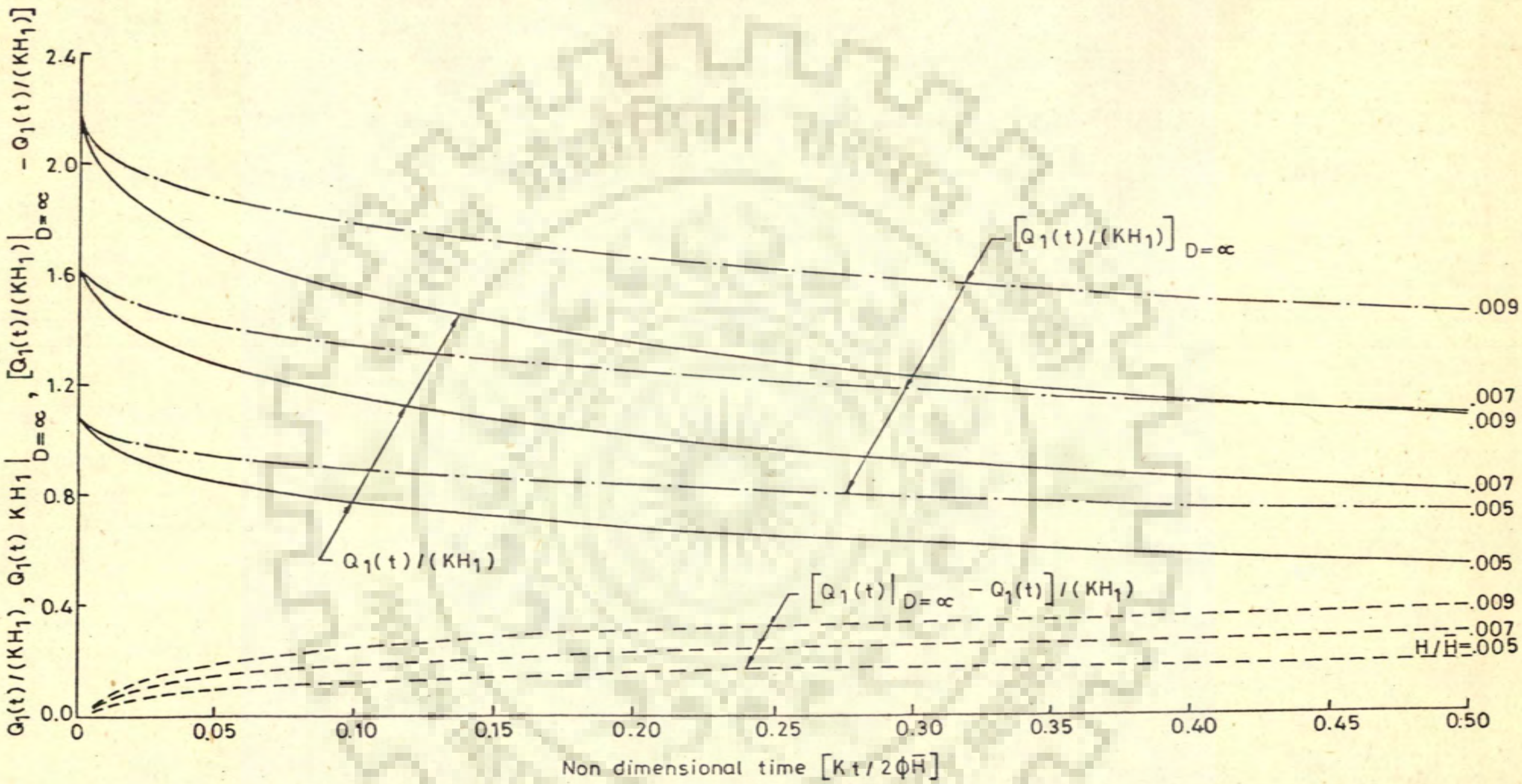

Fig.5.6 – Interference of seepage losses from two identical continuously running parallel canals, evaluted for $D/\bar{H}=0.08$, $B_1/\bar{H}=0.03$, $B_2/\bar{H}=0.03$, $H_1/\bar{H}=H_2/\bar{H}=0.003$, $\sigma_1/\bar{H}=\sigma_2/\bar{H}=0.001$ and for initial water table positions $H/\bar{H}=0.005, 0.007$ and 0.009

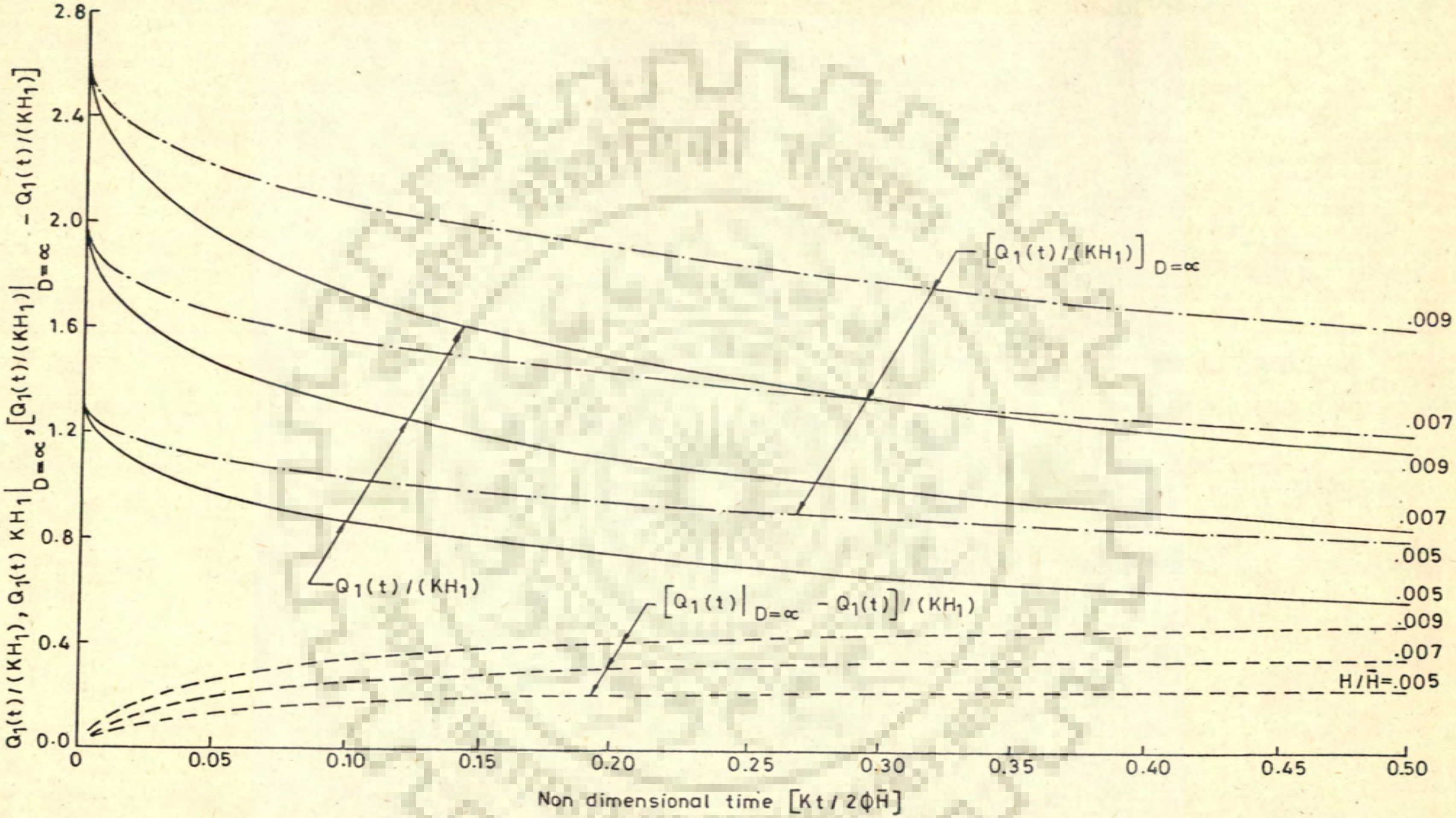

Fig 5.7 – Interference of seepage losses from two identical continuously running parallel canals, evaluted for $D/\bar{H}=0.08$, $B_1/\bar{H}=B_2/\bar{H}=0.06$, $H_1/\bar{H}=H_2/\bar{H}=0.003$, $\sigma_1/\bar{H}=\sigma_2/\bar{H}=0.001$ and for initial water table position $H/\bar{H}=0.005$, 0.007 and 0.009

decrease being dependent on: the width of the canal, the initial potential difference,and the spacing between the canals. Since the two parallel canals considered above are identical, their seepage losses will be equal.

The seepage loss from a single canal, that would occur if the other canal is located at infinity ($D=\infty$), has also been shown in the figures. The difference between seepage loss from a canal of the parallel canal system and the seepage loss from the same canal, if the other canal does not exist, quantifies the interference of the latter on the former in a parallel canal system.

The reduction in non-dimensional seepage loss from a canal due to interference of a similar parallel canal located at a known distance has also been shown in the Figs.(5.4) through (5.7). It could be seen from the figures, that in the beginning of seepage the interference is zero, i.e., the reduction in seepage in each canal due to the interference of the other is zero. Subsequently, as time elapses, the interference increases with time and attains a maximum value.

The variations in reduction of seepage with time due to interference for different values of initial potential difference, are also depicted in Figs. (5.4) through (5.7). It could be seen from the figures that the interference between the parallel canals, at any time, increases with increase in the initial potential difference. At non-dimensional time factor, $Kt/(2\ \varphi\ \bar{H}) = 0.50$; for $B_1/\bar{H} = B_2/\bar{H} = 0.03$, $D/\bar{H} = 0.18$, and $(H - \sigma_1)/\bar{H} = 0.004$, the reduction in seepage due to interference is 0.180. If the initial potential difference, $(H - \sigma_1)/\bar{H} = 0.008$, the corresponding reduction due to interference is 0.360. It could be seen that the interference is linearly proportional to the initial potential difference. At non-dimentional time factor 0.5, the reductions in seepage losses due to interference are 0.360, 0.270, and 0.180 for $(H - \sigma_1)/\bar{H} = 0.008$, 0.006, and 0.004 respectively. Thus, the ratio of the reduction in seepage due to interference and the corresponding initial potential difference is constant and equal to 0.045. At non-dimensional time factor 0.30, the reductions in seepage due to interference, corresponding to the above initial potential differences, are 0.320, 0.240, and 0.160 respectively and the ratio of reduction in seepage due to interference and corresponding initial potential difference is 0.04. The reduction in seepage from any canal due to interference of the other is linearly proportional to the initial potential difference.

At any particular time the reduction in seepage due to interference would be more for canals of larger width. It could be seen from Fig. (5.4) that for $B_1/\bar{H} = B_2/\bar{H} = 0.03$, $D/\bar{H} = 0.18$, and $H/\bar{H} = 0.009$, at non-dimensional time = 0.5, the reduction in seepage loss from either canal due to interference is 0.36; whereas, for $B_1/\bar{H} = B_2/\bar{H}=0.06$, as seen from Fig.(5.5), the corresponding reduction is 0.45.

The reduction in seepage loss due to interference at large times, for various distances between the two identical canals, has been shown in Figs.[5.8(a)] and [5.8(b)], in a semilog plot, for $B_1/\bar{H}=0.06$ and 0.03 respectively. It is seen from the figures that the reduction in seepage due to interference reaches a maximum value at very large time and then decreases. The reason for the decline in interference at large time is as follows :-

The seepage from a canal decreases as the difference in the potentials at the canal and in the aquifer under the canal decreases. In a parallel canal system the potential difference under a canal decreases with time partly due to its own seepage and partly due to seepage from the other canal. Ultimately, the seepage loss from a canal at large time would tend to zero whether it runs alone or it runs alongwith the other canal. Since the seepage loss tends to zero in either case, the reduction in seepage loss due to interference will also tend to zero. Because the interference ultimately tends to zero, it would decline after reaching a maximum value.

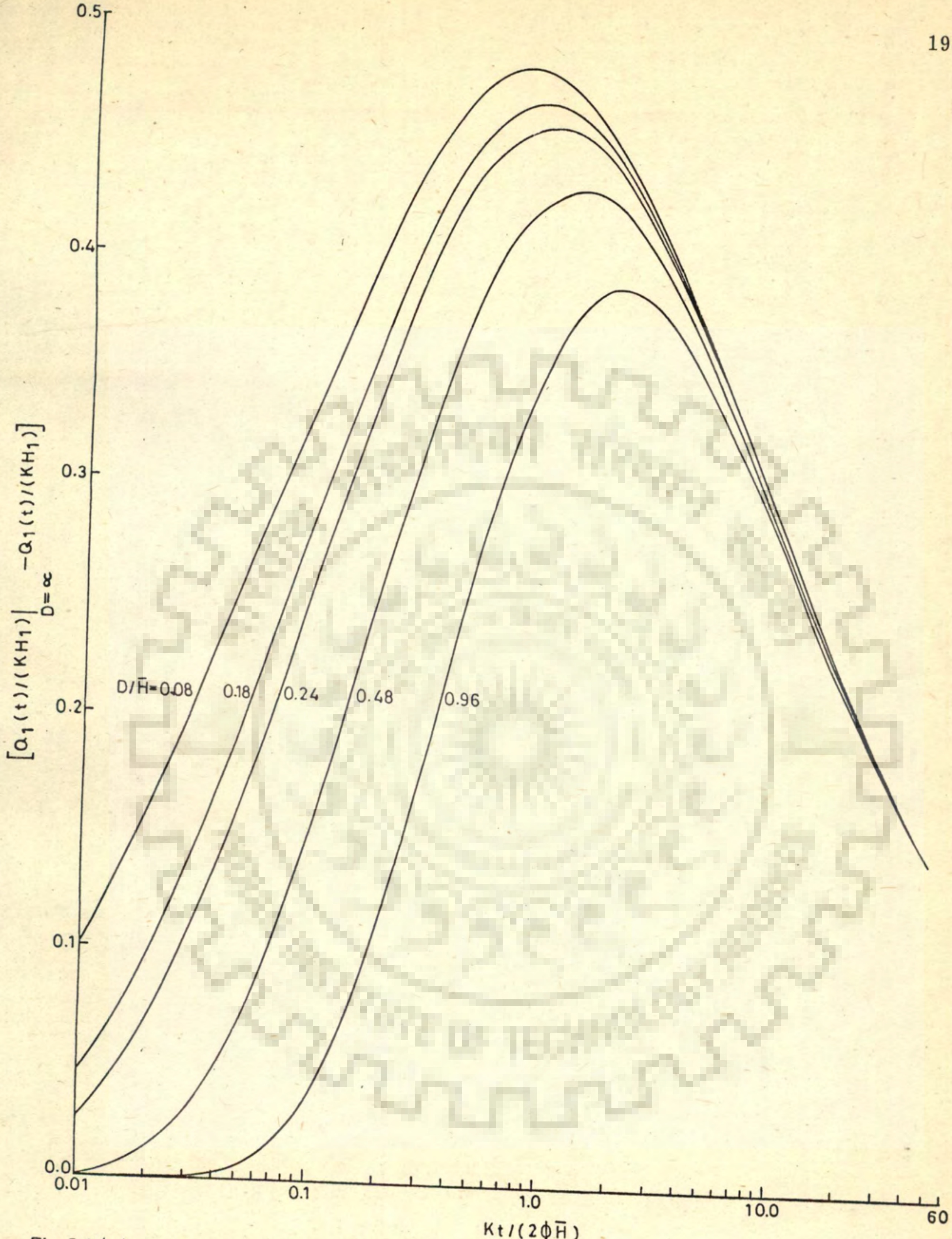

Fig.5.8 (a)—Reduction in seepage from a continuously running canal due to interference of an indentical parallel canal, evaluated for $B_1/\bar{H} = B_2/\bar{H} = 0.06$, $H_1/\bar{H} = H_2/\bar{H} = 0.003$, $\sigma_1/\bar{H} = \sigma_2/\bar{H} = 0.001$ and $H/\bar{H} = 0.009$ for various spacing between the canal

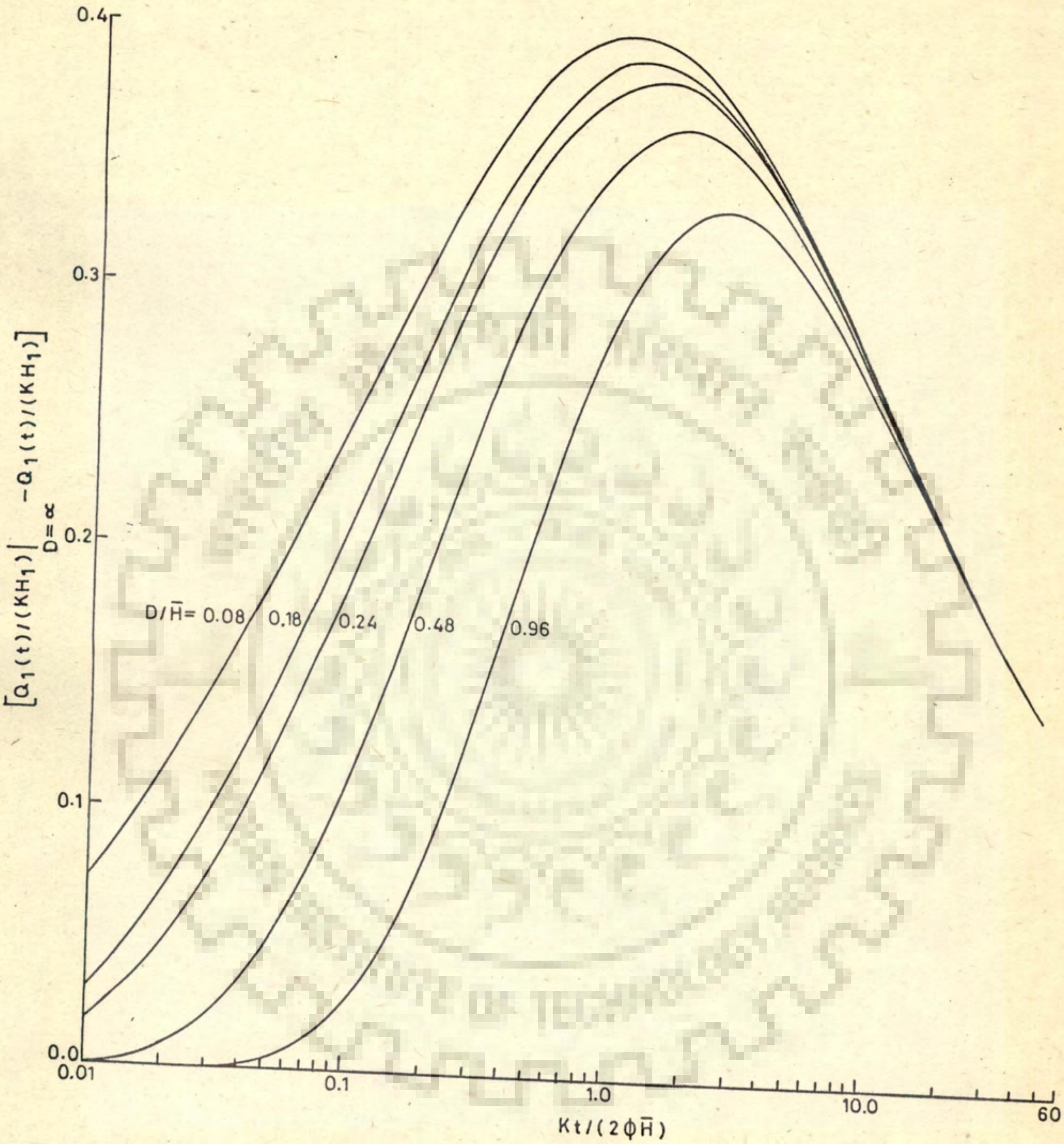

Fig. 5.8(b)–Reduction in seepage from a continuously running canal due to interference of an identical parallel canal, evaluated for $B_1/\bar{H} = B_2/\bar{H} = 0.03$, $H_1/\bar{H} = H_2/\bar{H} = 0.003$, $\sigma_1/\bar{H} = \sigma_2/\bar{H} = 0.001$ and $H/\bar{H} = 0.009$ for various spacing between the canal

The decrease in the interference is monotonic at large time. It could be seen from the Fig.[5.8(a)] that for the parallel canals with $B_1/\bar{H} = B_2/\bar{H} = 0.06$, $D/\bar{H}=0.18$, and $H/\bar{H}= 0.009$, during the non-dimensional time interval 2.0 to 3.0, the reduction in interference is 0.030. The reductions in interference during the time intervals 3.0 to 4.0 and 4.0 to 5.0 are 0.026 and 0.023 respectively.

It is seen from the Figs. [5.8(a)] and [5.8(b)] that the occurrence of maximum interference is delayed for larger spacing between the canals. For $B_1/\bar{H} = B_2/\bar{H} = 0.06$, $H/\bar{H} = 0.009$, and $D/\bar{H} = 0.08$, the maximum interference occurs at non-dimensional time, $Kt/(2\varphi\bar{H})$, = 0.80. If the spacing between the canals is 0.18, the maximum interference occurs at $Kt/(2\varphi\bar{H}) = 0.90$. The maximum value of interference declines with increase in spacing between the canals. It could be seen from Fig. [5.8(a)] that for $B_1/\bar{H} = B_2/\bar{H} = 0.06$ and $D/\bar{H} =0.08$, the maximum interference is 0.480 whereas for $D/\bar{H} = 0.48$, the maximum value of interference is 0.430.

It is seen from Figs. [5.8(a)] and [5.8(b)] that beyond a time factor of 10.0, for equal parallel canals with $B_1/\bar{H} = B_2/\bar{H} = 0.06$ or 0.03, the interference is approximately the same whether the canals are spaced at $D/\bar{H} = 0.08$ or 0.96.

The variations of seepage losses with time for unequal parallel canals have been evaluated for different initial potential difference for $B_1/\overline{H}$ =0.06, $B_2/\overline{H}$ = 0.03, $H_1/\overline{H}$ = $H_2/\overline{H}$ = 0.003, and $D/\overline{H}$ = 0.18. The results for $(H-\sigma_1)/\overline{H}$ = 0.004 and 0.008 are shown in Figs.(5.9) and (5.10) respectively. It could be seen that at non-dimensional time = 0.5, for initial non-dimensional potential difference, $(H-\sigma_1)/\overline{H}$, =0.004, the non-dimensional seepage losses are 0.615 and 0.516 from the larger and the smaller canal respectively. For an initial non-dimensional potential difference of 0.008 the corresponding seepage losses are 1.229 for the larger canal and 1.032 for the smaller canal. The seepage losses from the canals at any time are, thus, proportional to the initial potential difference, $(H-\sigma_1)$,that initiates the flow. The solution to the seepage problem of parallel canals is based on a solution of linearised Boussinesq's equation and method of superposition. This implies that the parallel canal system is a linear system. If the seepage losses and their reductions due to interference are determined for a unit initial potential difference, the seepage losses and their reductions due to interference for the actual value of initial potential difference could be obtained by multiplying the flow characteristics pertaining to unit initial potential difference with the actual potential difference, provided the flow conditions do not change with the change in the initial potential difference.

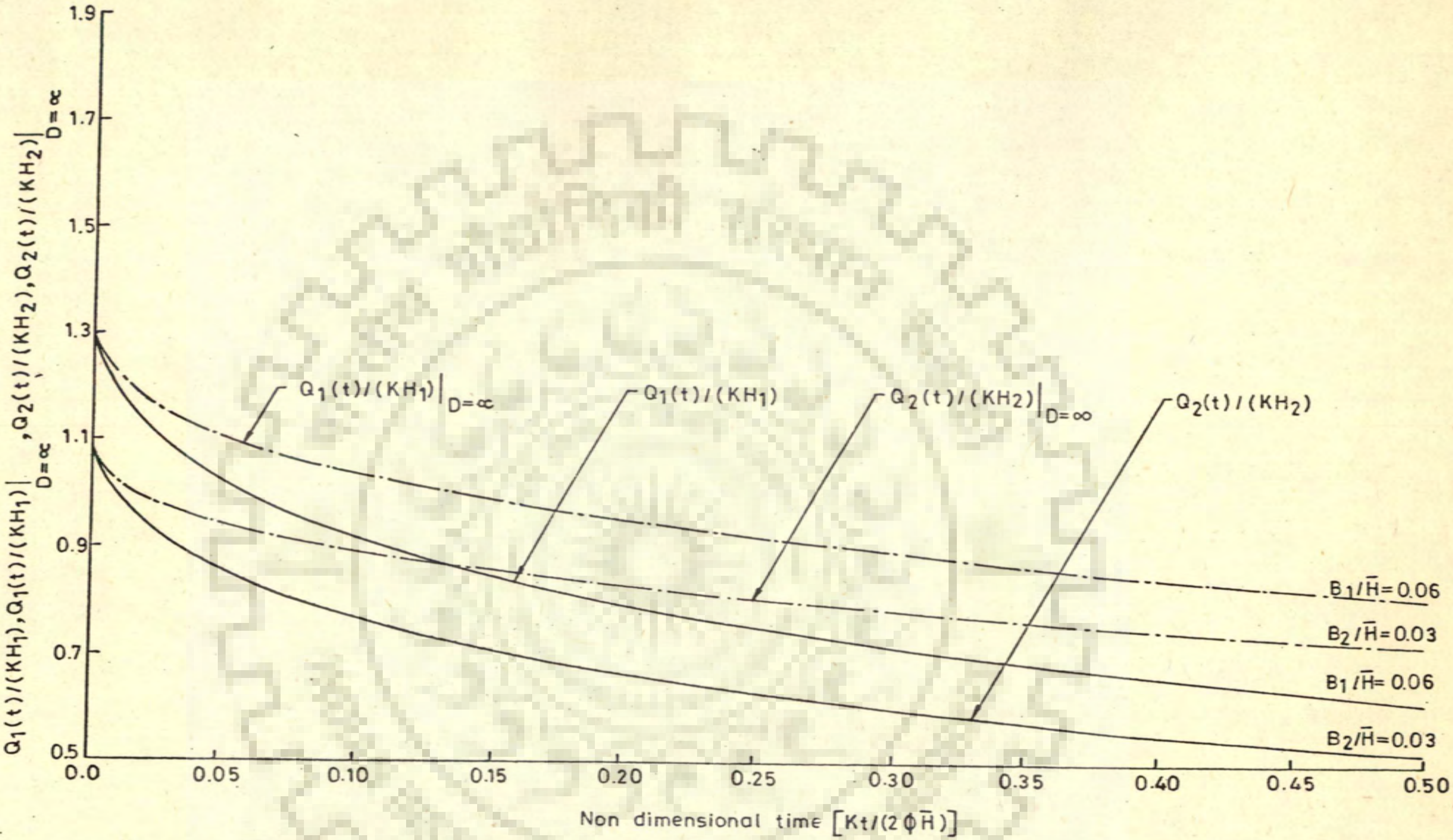

Fig.5.9 – Interference of seepage losses from two unequal continuously running parallel canals, evaluated for $D/\bar{H} = 0.18$, $B_1/\bar{H} = 0.06$, $B_2/\bar{H} = 0.03$, $H_1/\bar{H} = H_2/\bar{H} = 0.003$, $\sigma_1/\bar{H} = \sigma_2/\bar{H} = 0.001$ and $(H-\sigma_1)/\bar{H} = 0.004$

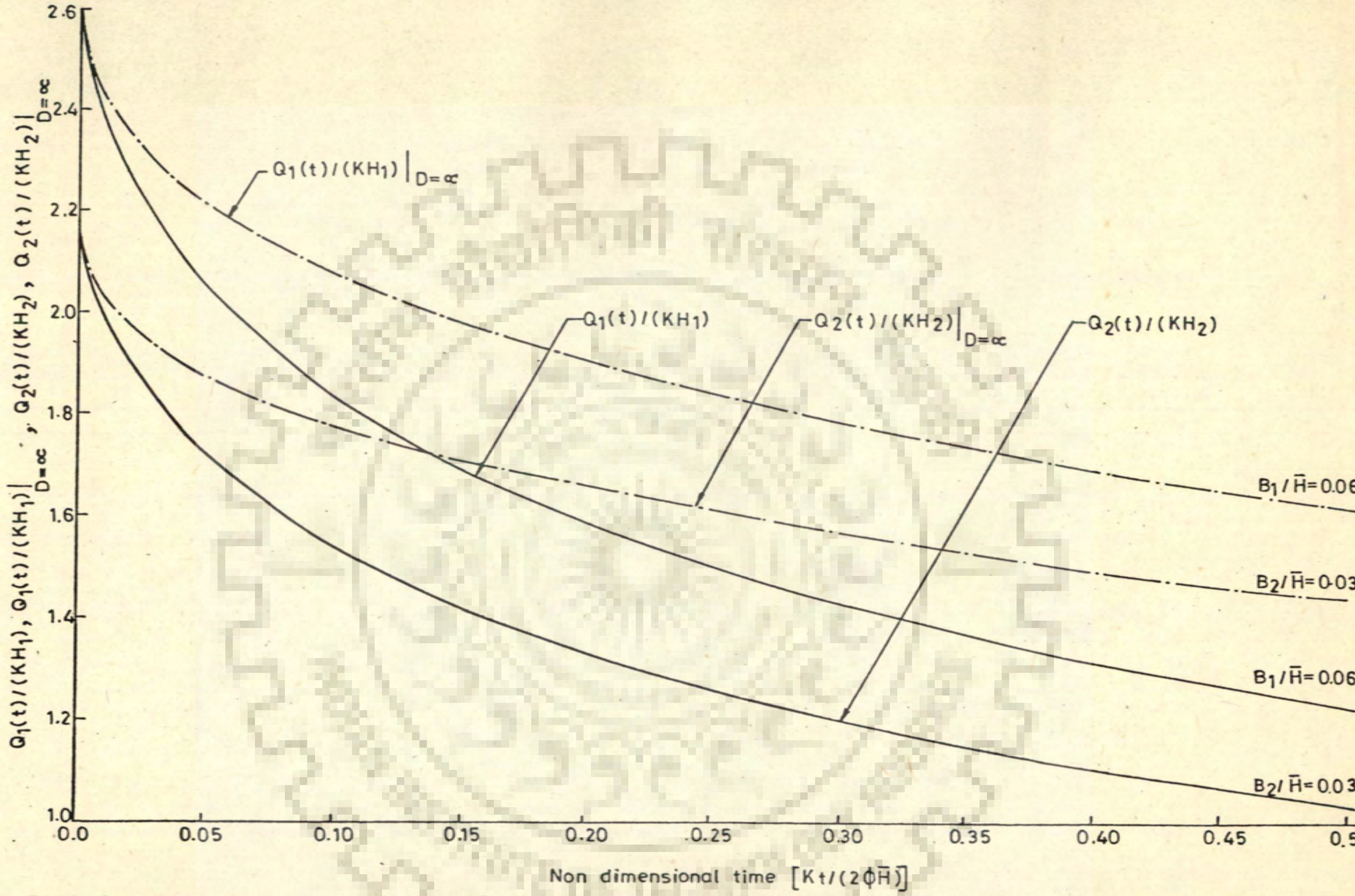

Fig. 5.10 – Interference of seepage losses from two unequal continuously running parallel canals, evaluted for $D/\bar{H}=0.18$, $B_1/\bar{H}=0.06$, $B_2/\bar{H}=0.03$, $H_1/\bar{H}=H_2/\bar{H}=0.003$, $\sigma_1/\bar{H}=\sigma_2/\bar{H}=0.001$ and $(H-\sigma_1)/\bar{H}=0.008$

Reductions in seepage due to interference of unequal parallel canals are presented in Table (5.4) for $H/\overline{H}$=0.009, $D/\overline{H}$=0.08,$H_1/\overline{H}=H_2/\overline{H}$=0.003, and $\sigma_1/\overline{H}= \sigma_2/\overline{H}$ = 0.001. It could be seen from the table that for $B_1/\overline{H}$=0.06 of the first canal,corresponding to $B_2/\overline{H}$=0.06, 0.03, and 0.015 of the second canal, the reductions in seepage from the first canal due to interference of the second canal are 0.473, 0.411, and 0.367 respectively at non-dimensional time=0.5. Thus, if the width of the interfering canal decreases, its interference on the other canal of larger width decreases; although the decrease is not proportional to the decrease in the width of the interfering canal.It could be seen from the table that for $B_1/\overline{H}$ = 0.06 and $B_2/\overline{H}$ = 0.015, at non-dimensional time 0.5, the reduction in seepage from the canal with smaller width due to interference of the larger canal is 0.403, whereas the reduction in seepage from the larger canal due to interference of the smaller canal is 0.367. Hence, interference of the larger canal on the smaller canal is more as compared to that of the smaller canal on the larger one.

The temporal variation of reduction in seepage due to interference for two unequal parallel canals, which run continuously for a long duration, has been presented in a semilog plot in Fig. (5.11) for different values of $D/\overline{H}$; for $B_1/\overline{H}$ = 0.06, $B_2/\overline{H}$ = 0.03, $H_1/\overline{H} = H_2/\overline{H}$ = 0.003, and $H/\overline{H}$ = 0.009. It could be

Table (5.4) - Interference of unequal parallel canals predicted for $D/\bar{H} = 0.08$, $H/\bar{H} = 0.009$, $\sigma_1/\bar{H} = \sigma_2/\bar{H} = 0.001$, $H_1/\bar{H} = H_2/\bar{H} = 0.003$

$\frac{B_1}{\bar{H}}$	$\frac{B_2}{\bar{H}}$	$\frac{Kt}{(2\varphi\bar{H})}$	$\frac{Q_1(t)}{(KH_1)}$	$\left.\frac{Q_1(t)}{(KH_1)}\right\vert_{D=\infty}$	$[\left.\frac{Q_1(t)}{(KH_1)}\right\vert_{D=\infty} - \frac{Q_1(t)}{(KH_1)}]$	$\frac{Q_2(t)}{(KH_2)}$	$\left.\frac{Q_2(t)}{(KH_2)}\right\vert_{D=\infty}$	$[\left.\frac{Q_2(t)}{(KH_2)}\right\vert_{D=\infty} - \frac{Q_2(t)}{(KH_2)}]$
0.060	0.060	0.005	2.416	2.473	0.057	2.416	2.473	0.057
		0.050	1.958	2.212	0.254	1.958	2.212	0.254
		0.150	1.599	1.978	0.379	1.599	1.978	0.379
		0.300	1.342	1.785	0.443	1.342	1.785	0.443
		0.450	1.189	1.657	0.468	1.189	1.657	0.468
		0.500	1.150	1.623	0.473	1.150	1.623	0.473
0.060	0.030	0.005	2.423	2.473	0.049	2.017	2.067	0.050
		0.050	1.997	2.212	0.215	1.664	1.883	0.219
		0.150	1.653	1.978	0.325	1.378	1.712	0.334
		0.300	1.402	1.785	0.383	1.169	1.568	0.399
		0.450	1.251	1.657	0.406	1.042	1.471	0.429
		0.500	1.211	1.623	0.411	1.010	1.444	0.434
0.060	0.015	0.005	2.432	2.473	0.041	1.749	1.791	0.042
		0.050	2.025	2.212	0.187	1.458	1.651	0.198
		0.150	1.692	1.978	0.286	1.219	1.520	0.301
		0.300	1.446	1.785	0.339	1.042	1.407	0.365
		0.450	1.295	1.657	0.362	0.934	1.329	0.395
		0.500	1.256	1.623	0.367	0.905	1.308	0.403
0.030	0.030	0.005	2.028	2.067	0.030	2.028	2.067	0.039
		0.050	1.698	1.883	0.185	1.698	1.883	0.185
		0.150	1.428	1.712	0.286	1.428	1.712	0.286
		0.300	1.223	1.568	0.345	1.223	1.568	0.345
		0.450	1.098	1.471	0.373	1.098	1.471	0.373
		0.500	1.066	1.444	0.378	1.066	1.444	0.378
0.030	0.015	0.005	2.033	2.067	0.034	1.756	1.791	0.035
		0.050	1.721	1.883	0.152	1.488	1.651	0.163
		0.150	1.460	1.712	0.252	1.263	1.520	0.257
		0.300	1.262	1.568	0.306	1.092	1.407	0.315
		0.450	1.139	1.471	0.332	0.985	1.329	0.344
		0.500	1.107	1.444	0.337	0.957	1.308	0.351
0.015	0.015	0.005	1.761	1.791	0.030	1.761	1.791	0.030
		0.050	1.509	1.509	0.142	1.509	1.651	0.142
		0.150	1.294	1.294	0.226	1.294	1.520	0.226
		0.300	1.127	1.127	0.280	1.127	1.407	0.280
		0.450	1.022	1.022	0.307	1.022	1.329	0.307
		0.500	0.995	0.995	0.313	0.995	1.308	0.313

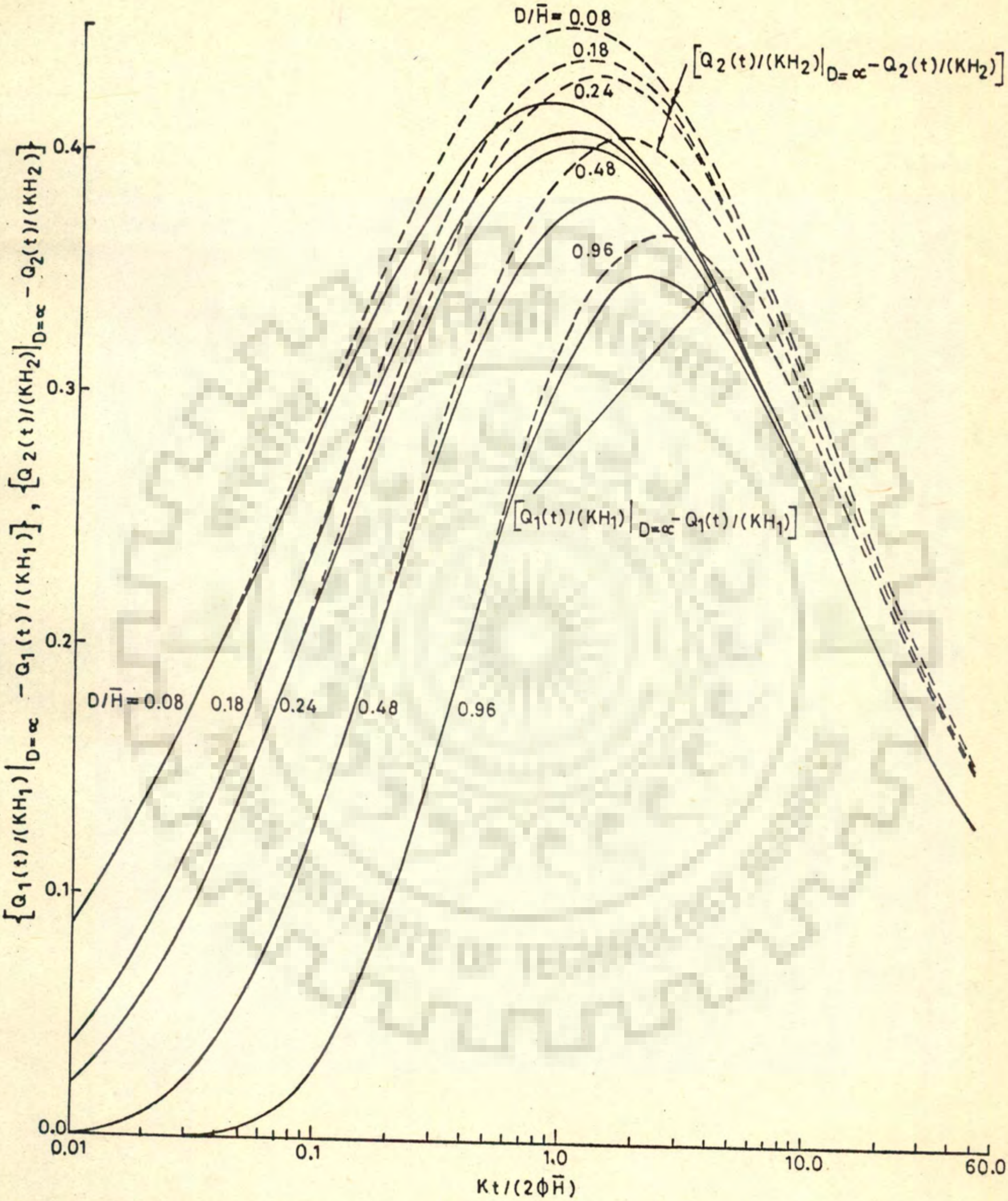

Fig.5.11– Reductions in seepage due to interference of two unequal continuously running parallel canals spaced at various distances, evaluated for $B_1/\bar{H}=0.06$, $B_2/\bar{H}=0.03$, $H_1/\bar{H}=H_2/\bar{H}=0.003$, $\sigma_1/\bar{H}=\sigma_2/\bar{H}=0.001$ and $H/\bar{H}=0.009$

seen from the figure that, at any time, the reduction in seepage from each canal due to interference is more for smaller spacing between the canals, as observed in the case of equal parallel canals. The results show that the reductions in seepage due to interference for each of the two canals are approximately equal during the earlier occurrence of seepage. The time, over which the interference of the two unequal canals is approximately equal, is longer for larger spacing of the canals. The reductions in seepage due to interference for both the canals would approach different maximum values at different times for a given spacing between the canals. After reaching a maximum value, the interference reduces with time. The occurrence of maximum reduction in seepage due to interference takes place at earlier time for the larger canal. For example, for $D/\bar{H} = 0.24$, the maximum reducion in seepage for the first canal, which is larger, occurs at $Kt/(2\varphi\bar{H})=1.1$, whereas, for the smaller canal the time of occurence of maximum reduction in seepage due to interference is 1.3. It is also seen that the effect of larger canal on the smaller canal is more than the effect of smaller canal on the larger one. For example, the maximum non-dimensional reduction in seepage from the larger canal, due to interference of the smaller canal, is 0.403 for $D/\bar{H} = 0.24$, whereas, the maximum reduction in seepage from the smaller canal , due to interference of the larger canal, is 0.432. It could also be seen from the figure that beyond a non-dimensional time = 10.0, for $D/\overline{H}$ upto 0.96, the interference is independent of the spacing between the canals.

The interference of parallel canals whose beds are at different levels has been presented in Table (5.5). The canals have equal width and equal depth of water in them. Seepage losses from each of the canals and their interference have been evaluated for $B_1/\overline{H} = B_2/\overline{H} = 0.06$, $H_1/\overline{H} = H_2/\overline{H} = 0.003$, $H/\overline{H} = 0.007$, $D_{b1}/\overline{H} = 0.004$, $D/\overline{H} = 0.08$ and 0.18, and $D_{b2}/\overline{H} = 0.004$, 0.005, and 0.006. The difference in $D_{b1}/\overline{H}$ and $D_{b2}/\overline{H}$ is the difference in the bed levels of the canals in non-dimensional form. It could be seen from the table that if the canal beds are at the same level, for $D/\overline{H} = 0.18$, the non-dimensional seepage loss from each canal is 1.029 at non-dimensional time = 0.3. If the difference in the bed levels is 0.001 $\overline{H}$, and the second canal is at a lower level, the corresponding seepage loss from the first canal is 1.094 and from the second canal is 0.793. The seepage loss from the second canal in the latter case has reduced from 1.029 to 0.793 because of reduction in the initial potential difference between the second canal and the aquifer. On the contrary, the seepage loss from the first canal has increased from 1.029 to 1.094, on account of decrease in the

Table (5.5) - Seepage loss from equal parallel canals whose beds are at different levels, evaluated for $B_1/\bar{H} = B_2/\bar{H} = 0.06$, $H_1/\bar{H} = H_2/\bar{H} = 0.003$, and $H/\bar{H} = 0.007$.

$D/\bar{H}$	Non-dimensional time $Kt/(2\varphi\bar{H})$	$Q_1(t)/(KH_1)$ evaluated for: [difference in bed levels)/$\bar{H}$] equal to: 0.0	0.001	0.002	$Q_2(t)/(KH_2)$ evaluated for: [(difference in bed levels)/$\bar{H}$] equal to: 0.0	0.001	0.002
0.18	0.050	1.514	1.540	1.565	1.514	1.236	0.958
0.18	0.100	1.344	1.383	1.422	1.344	1.080	0.817
0.18	0.150	1.232	1.280	1.329	1.232	0.978	0.725
0.18	0.200	1.156	1.204	1.259	1.156	0.902	0.656
0.18	0.250	1.083	1.144	1.205	1.083	0.842	0.601
0.18	0.300	1.029	1.094	1.159	1.029	0.793	0.556
0.18	0.350	0.984	1.052	1.121	0.984	0.751	0.518
0.18	0.400	0.944	1.016	1.088	0.944	0.715	0.485
0.18	0.450	0.909	0.984	1.059	0.909	0.683	0.457
0.18	0.500	0.878	0.956	1.033	0.878	0.655	0.431
0.08	0.050	1.469	1.504	1.540	1.469	1.189	0.910
0.08	0.100	1.306	1.354	1.403	1.306	1.040	0.774
0.08	0.150	1.199	1.257	1.314	1.199	0.942	0.685
0.08	0.200	1.121	1.184	1.248	1.121	0.870	0.619
0.08	0.250	1.058	1.127	1.196	1.058	0.813	0.567
0.08	0.300	1.007	1.080	1.153	1.007	0.766	0.525
0.08	0.350	0.963	1.040	1.117	0.963	0.726	0.488
0.08	0.400	0.925	1.005	1.085	0.925	0.691	0.457
0.08	0.450	0.892	0.975	1.058	0.892	0.661	0.429
0.08	0.500	0.863	0.948	1.033	0.863	0.634	0.405

interference of the second canal on the first canal. With the increase in the difference in bed levels, the interference of the second canal on the first canal would decrease and the seepage from the second canal would tend to zero. If the difference in bed levels of the canals is 0.002 $\overline{H}$, the non-dimensional seepage loss from the first and the second canal would be 1.159 and 0.556 respectively at non-dimensional time 0.30.

Inteference of unequal of parellel canals, whose beds are at different levels, has been presented in Table (5.6) for $B_1/\overline{H}$ = 0.06 and $B_2/\overline{H}$ = 0.03. Similar trend, i.e., increase in seepage loss from the first canal, whose bed is at a higher elevation, and reduction in seepage loss from the second canal with increase in the bed difference of the canals, is also noticed if the second canal has smaller dimension than that of the first canal.

Once the seepage losses at different times are obtained, the rise in water table can be computed by making use of Equations (5.15) or (5.16), after evaluating the discrete kernel coefficients for water table rise at desired x-coordinates. The rise in water table due to seepage from two equal parallel canals have been evaluated at different locations across the canals for $B_1/\overline{H} = B_2/\overline{H}$ = 0.06, $H_1/\overline{H} = H_2/\overline{H}$ = 0.003, initial potential difference, $(H - \sigma_1)/\overline{H}$, = 0.008, and $D/\overline{H}$= 0.18. The results are shown in Fig. (5.12). It could be

Table (5.6) - Seepage loss from unequal parallel canals whose beds are at different levels, evaluated for $B_1/\bar{H} = 0.06$, $B_2/\bar{H} = 0.03$, $H_1/\bar{H} = H_2/\bar{H} = 0.003$, $\sigma_1/\bar{H} = 0.001$, and $H/\bar{H} = 0.007$.

$D/\bar{H}$	Non-dimensional time $Kt/(2\varphi\bar{H})$	$Q_1(t)/(KH_1)$ evaluated for: [(difference in bed levels)/$\bar{H}$] equal to			$Q_2(t)/(KH_2)$ evaluated for: [(difference in bed levels)/$\bar{H}$] equal to:		
		0.0	0.001	0.002	0.0	0.001	0.002
0.18	0.050	1.537	1.558	1.580	1.289	1.052	0.815
0.18	0.100	1.375	1.409	1.442	1.153	0.928	0.702
0.18	0.150	1.268	1.310	1.351	1.064	0.845	0.627
0.18	0.200	1.189	1.236	1.283	0.997	0.784	0.570
0.18	0.250	1.124	1.176	1.229	0.944	0.734	0.525
0.18	0.300	1.071	1.128	1.184	0.899	0.693	0.487
0.18	0.350	1.026	1.086	1.145	0.861	0.658	0.455
0.18	0.400	0.987	1.050	1.112	0.829	0.628	0.428
0.18	0.450	0.953	1.018	1.083	0.800	0.602	0.403
0.18	0.500	0.922	0.990	1.057	0.774	0.578	0.381
0.08	0.050	1.499	1.528	1.558	1.249	1.011	0.773
0.08	0.100	1.343	1.384	1.426	1.120	0.891	0.664
0.08	0.150	1.241	1.290	1.339	1.034	0.813	0.591
0.08	0.200	1.164	1.219	1.274	0.970	0.754	0.537
0.08	0.250	1.103	1.162	1.222	0.919	0.707	0.494
0.08	0.300	1.052	1.115	1.179	0.877	0.667	0.458
0.08	0.350	1.009	1.075	1.142	0.841	0.634	0.428
0.08	0.400	0.971	1.041	1.110	0.810	0.605	0.401
0.08	0.450	0.938	1.010	1.082	0.782	0.580	0.378
0.08	0.500	0.909	0.983	1.057	0.757	0.557	0.357

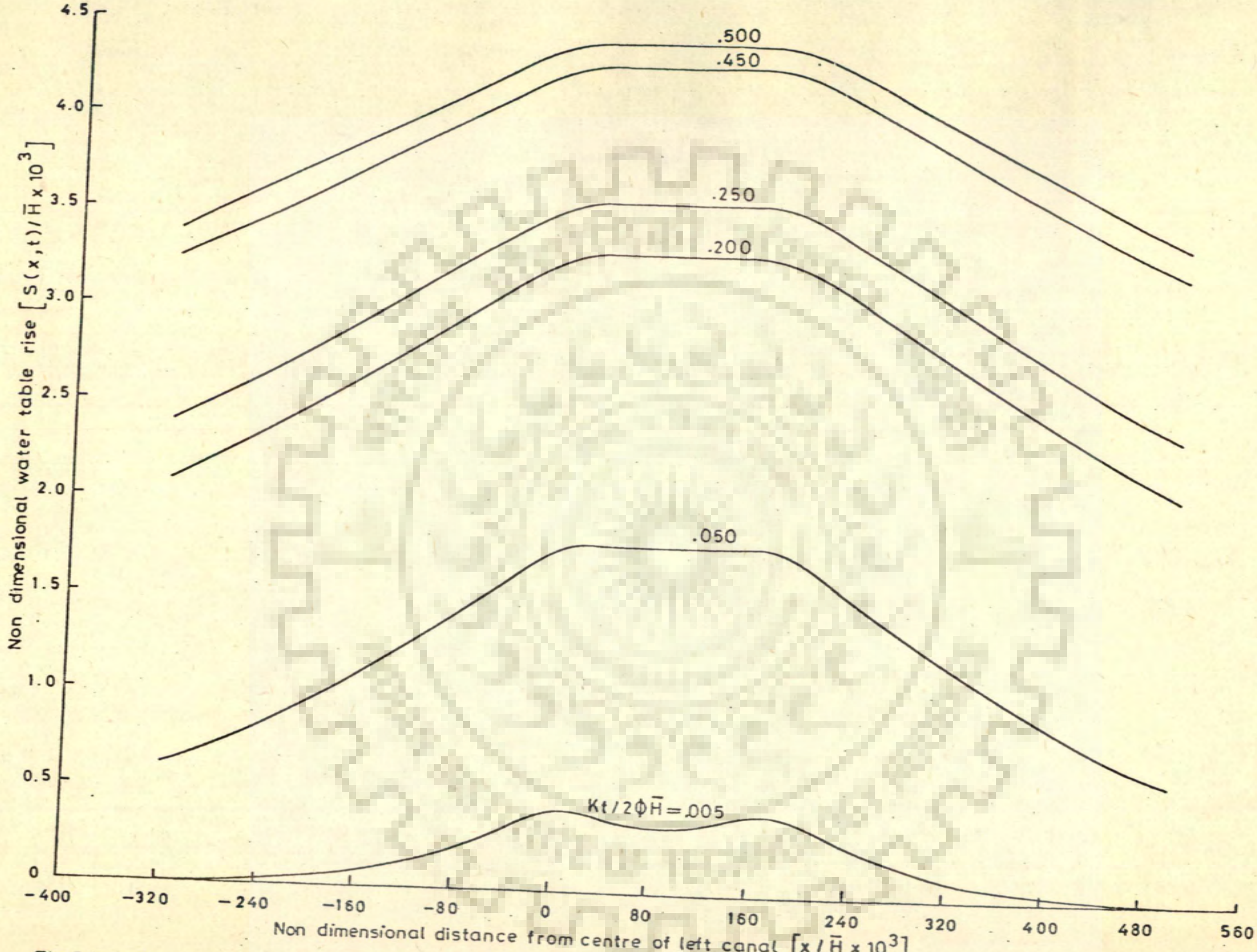

Fig.5.12 - Water table evolution due to seepage from two continuously running identical parallel canals predicted for $D/\bar{H} = 0.18$, $B_1/\bar{H} = B_2/\bar{H} = 0.06$, $H_1/\bar{H} = H_2/\bar{H} = 0.003$, $\sigma_1/\bar{H} = \sigma_2/\bar{H} = 0.001$ and $H/\bar{H} = 0.009$

seen from the figure that in the beginning of seepage, at non-dimensional time 0.005, well defined water mounds are formed under the centre of each canal. As the time passes, the ridges get dissipated and the points of maximum rise move towards each other, indicating a higher fraction of seepage flow from each canal going towards the outer sides of the canals. The points of maximum rise of water table, however, do not go beyond a distance of half the respective width of the canals at the water surface. For example, the point of maximum rise of water table under the left canal is located at $x/\bar{H}$ = 0.00827, 0.02421 , 0.02890, and 0.02986 at the end of non-dimensional time 0.005, 0.05, 0.25, and 0.5 respectively as shown in Fig.(5.12). The evolution of water table for equal parallel canals, which are spaced at a closer distance with $D/\bar{H}$ = 0.08, has been shown in Fig.(5.13). Comparison of the results shown in Figs. (5.12) and (5.13) indicates that the rise of water table is faster for closer spacing of canals. It could be seen from the Fig. (5.12) , that for $D/\bar{H}$ = 0.18, at non-dimensional time = 0.5, the maximum non-dimensional rise of water table is 4.408×10^{-3}. For $D/\bar{H}$ = 0.08, the corresponding maximum non-dimensional rise of water table is 4.475×10^{-3} as shown in Fig. (5.13). The water table in the zone between the canals tends to become flat and a stagnant zone is formed between the canals with passage of time.

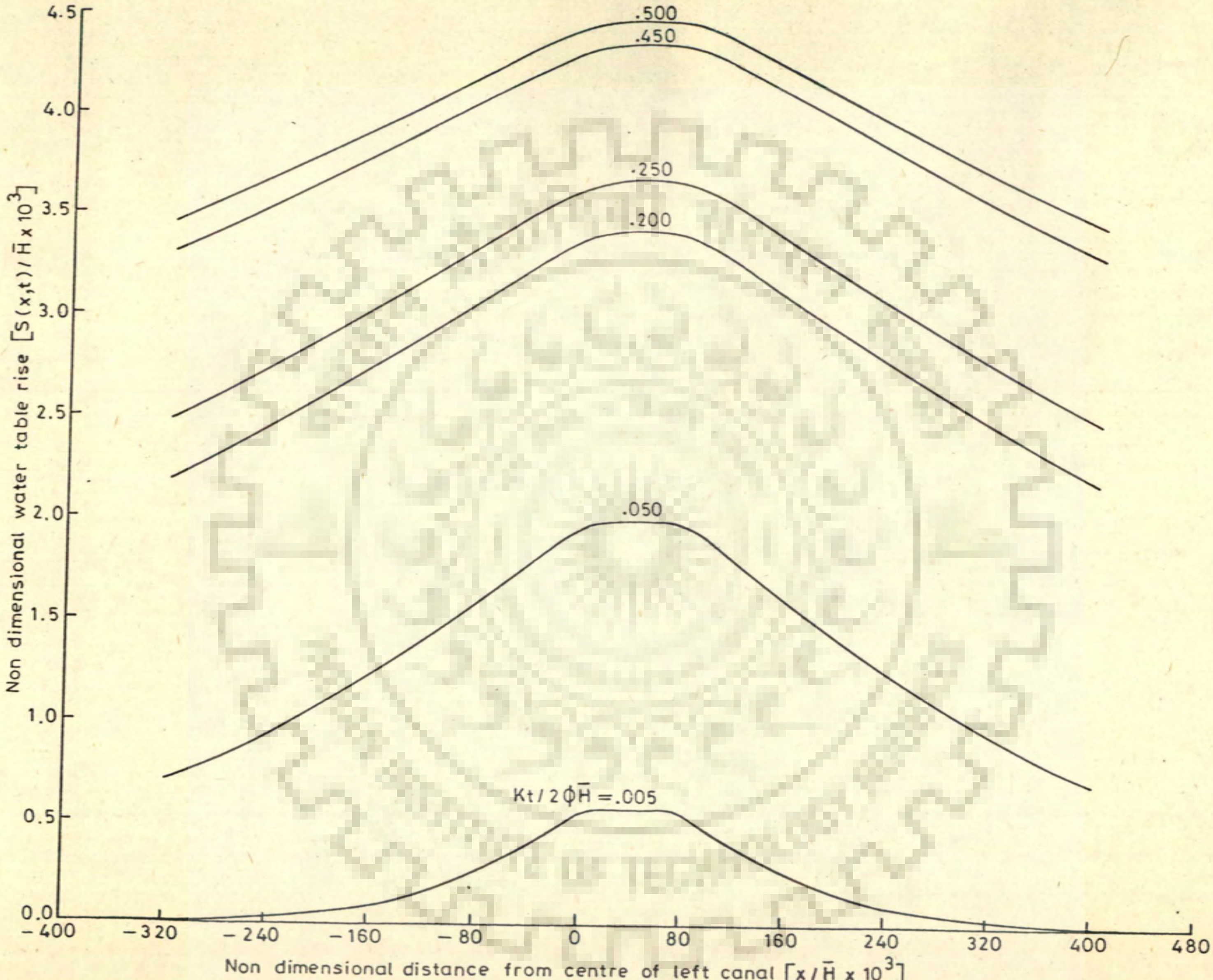

Fig. 5.13 – Water table evolution due to seepage from two continuously running identical parallel canals predicted for $D/\bar{H} = 0.08$, $B_1/\bar{H} = B_2/\bar{H} = 0.06$, $H_1/\bar{H} = H_2/\bar{H} = 0.003$, $\sigma_1/\bar{H} = \sigma_2/\bar{H} = 0.001$ and $H/\bar{H} = 0.009$

The water table rise at different locations due to seepage from two unequal parallel canals, whose beds are at the same level, has been predicted for $B_1/\overline{H} = 0.06$, $B_2/\overline{H}=0.03$, $H_1/\overline{H} = H_2/\overline{H} = 0.003$, $\sigma_1/\overline{H} = \sigma_2/\overline{H} = 0.001$, $H/\overline{H}= 0.009$, and $D/\overline{H}= 0.18$ .The water table positions across the canals at various times are shown in Fig.(5.14). It could be seen from the figure that distinct water mounds are formed under each canal in the beginning of seepage. The maximum height of the mound under the larger canal is more than that of the mound under the smaller canal.As the time elapses, the water mounds dissipate ; the dissipation of the mound under the bigger canal occurring at a faster rate. The water table between the canals becomes a flat plane, slightly dipping towards the right at large time, and a stagnant zone is formed similar to that formed for equal parallel canals. For large values of $H/\overline{H}$, when the seepage loss is not controlled by water table position, it was seen in Fig.(3.23) of Chapter 3 that a stagnant zone had not developed upto $Kt/(2\varphi\overline{H}) = 0.5$ in the case of unequal parallel canals with dimensions same as that mentioned above.

It could be seen from Fig. (5.14) that at the end of non-dimensional time 0.5, the maximum non-dimensional rise of water table under the larger canal is 4.223×10^{-3}. If the canals would have been of equal dimension with $B_1/\overline{H} = 0.06$, the maximum non-dimensional

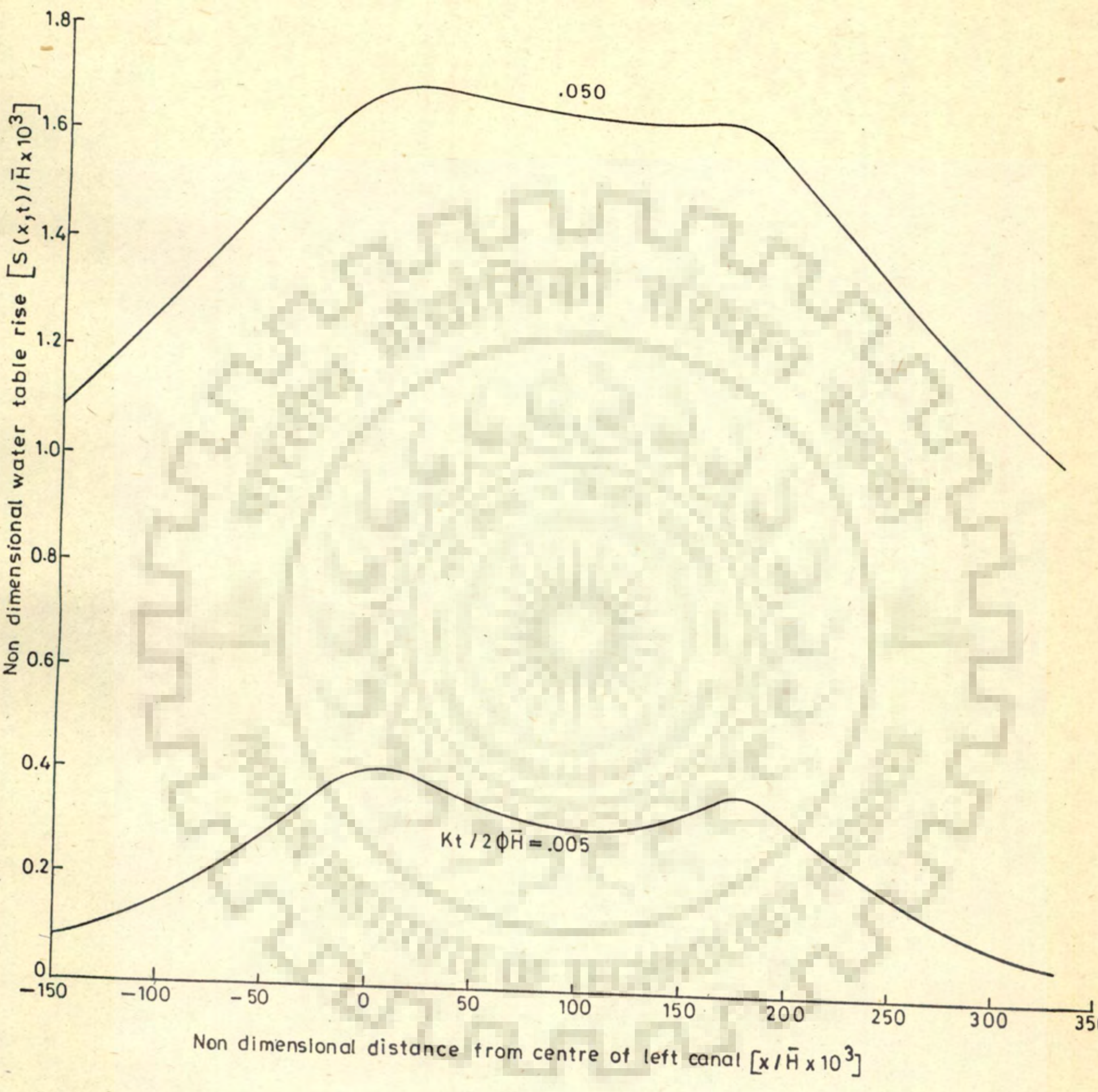

Fig.5.14 (a)–Water table evolution due to seepage from two continuously running unequal parallel canals predicted for $D/\bar{H}=0.18$, $B_1/\bar{H}=0.06$, $B_2/\bar{H}=0.03$, $H_1/\bar{H}=H_2/\bar{H}=0.003$, $\sigma_1/\bar{H}=\sigma_2/\bar{H}=0.001$ and $H/\bar{H}=0.009$

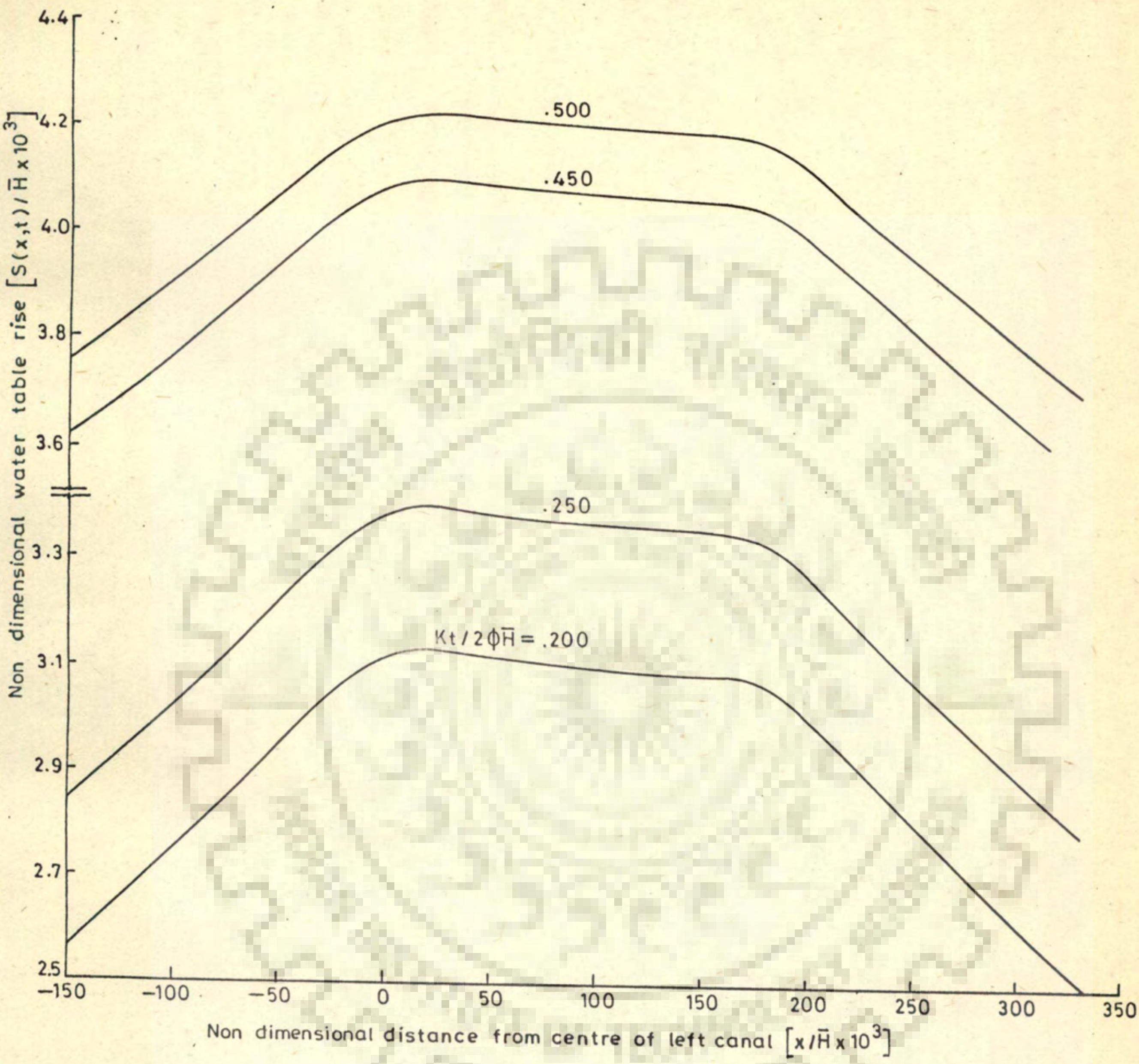

Fig.5.14 (b)-Water table evolution due to seepage from two continuously running unequal parallel canals predicted for $D/\bar{H}=0.18$, $B_1/\bar{H}=0.06$, $B_2/\bar{H}=0.03$, $H_1/\bar{H}=H_2/\bar{H}=0.003$, $\sigma_1/\bar{H}=\sigma_2/\bar{H}=0.001$ and $H/\bar{H}=0.009$

rise under each of the parallel canals at this time would have been 4.408×10^{-3}. The difference in the maximum non-dimensional water table rises for the two cases is 0.185×10^{-3} though the width of the right canal in the two cases differ significantly.

The water table rise due to seepage from two parallel canals, whose beds are at different levels, has been evaluated for $B_1/\overline{H} = B_2/\overline{H} = 0.06$, $H_1/\overline{H} = H_2/\overline{H} = 0.003$, $\sigma_1/\overline{H} = 0.001$, $\sigma_2/\overline{H} = 0.003$, $H/\overline{H} = 0.007$, and $D/\overline{H} = 0.18$. The water table positions at different times are shown in Fig.(5.15). It could be seen from the figure that, in the beginning of seepage, distinct water mounds are formed below the bed of each canal similar to that for equal and unequal parallel canals. As the time passes, the water mound under the right canal, whose bed is at a lower level than that of the left canal, gets dissipated more rapidly than the mound under the left canal. It could be seen that, at the end of non-dimensional time = 0.5, the water mound under the right canal has almost disappeared. The zone between the canals is also not stagnant. The right canal being at a lower level, the potential difference between the right canal and the aquifer is less in comparison to the difference in potential between the left canal and the aquifer. Consequently, the seepage loss from the right canal is less than that from the left canal. Also, the seepage from the right canal is further reduced due to

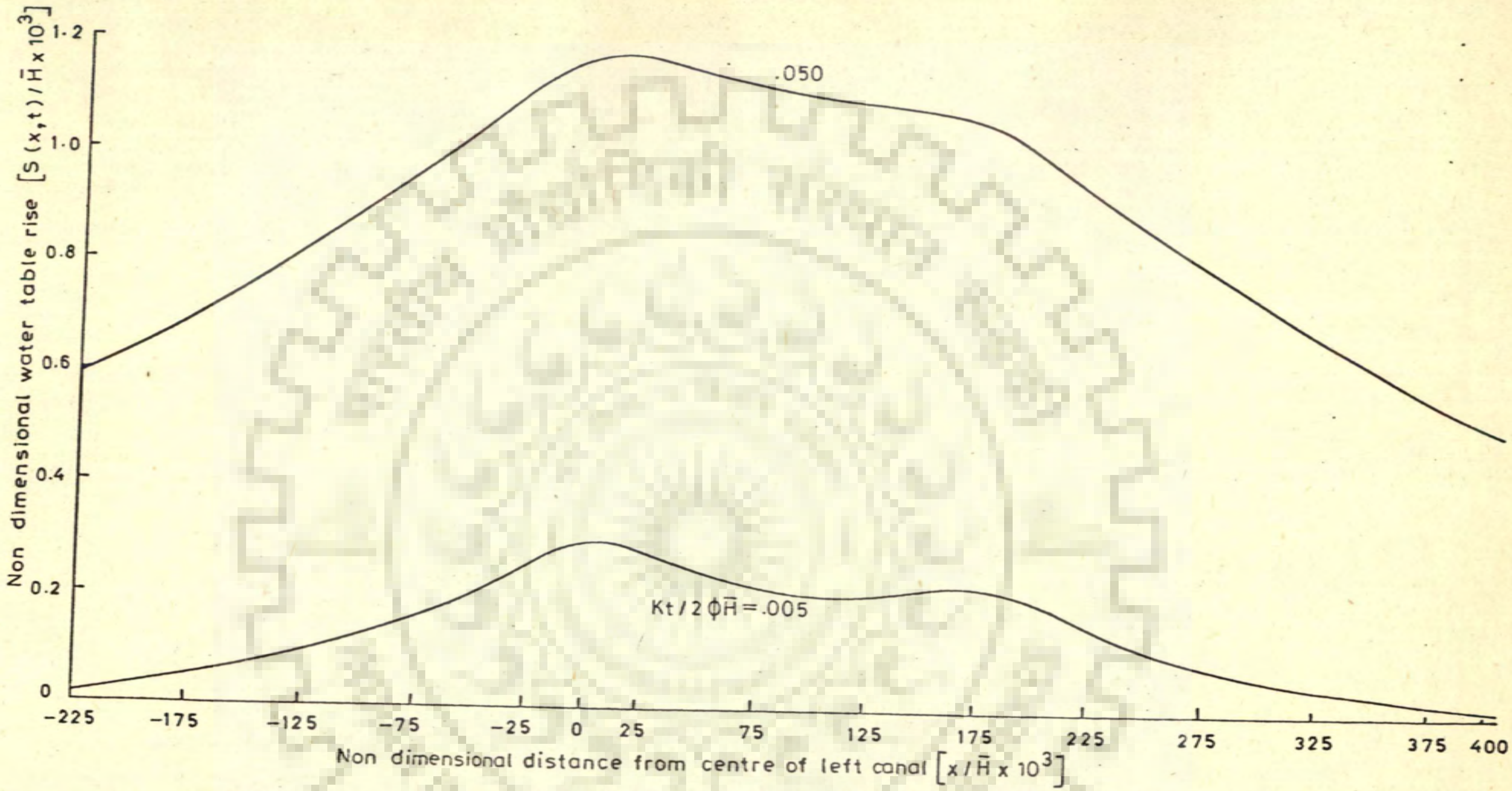

Fig.5.15 (a)– Water table evolution due to seepage from two continuously running identical parallel canals predicted for $D/\bar{H}=0.18$, $B_1/\bar{H}=B_2/\bar{H}=0.06$, $H_1/\bar{H}=H_2/\bar{H}=0.003$, $\sigma_1/\bar{H}=0.001$, $\sigma_2/\bar{H}=0.003$ and $H/\bar{H}=0.007$; canal beds are at different levels

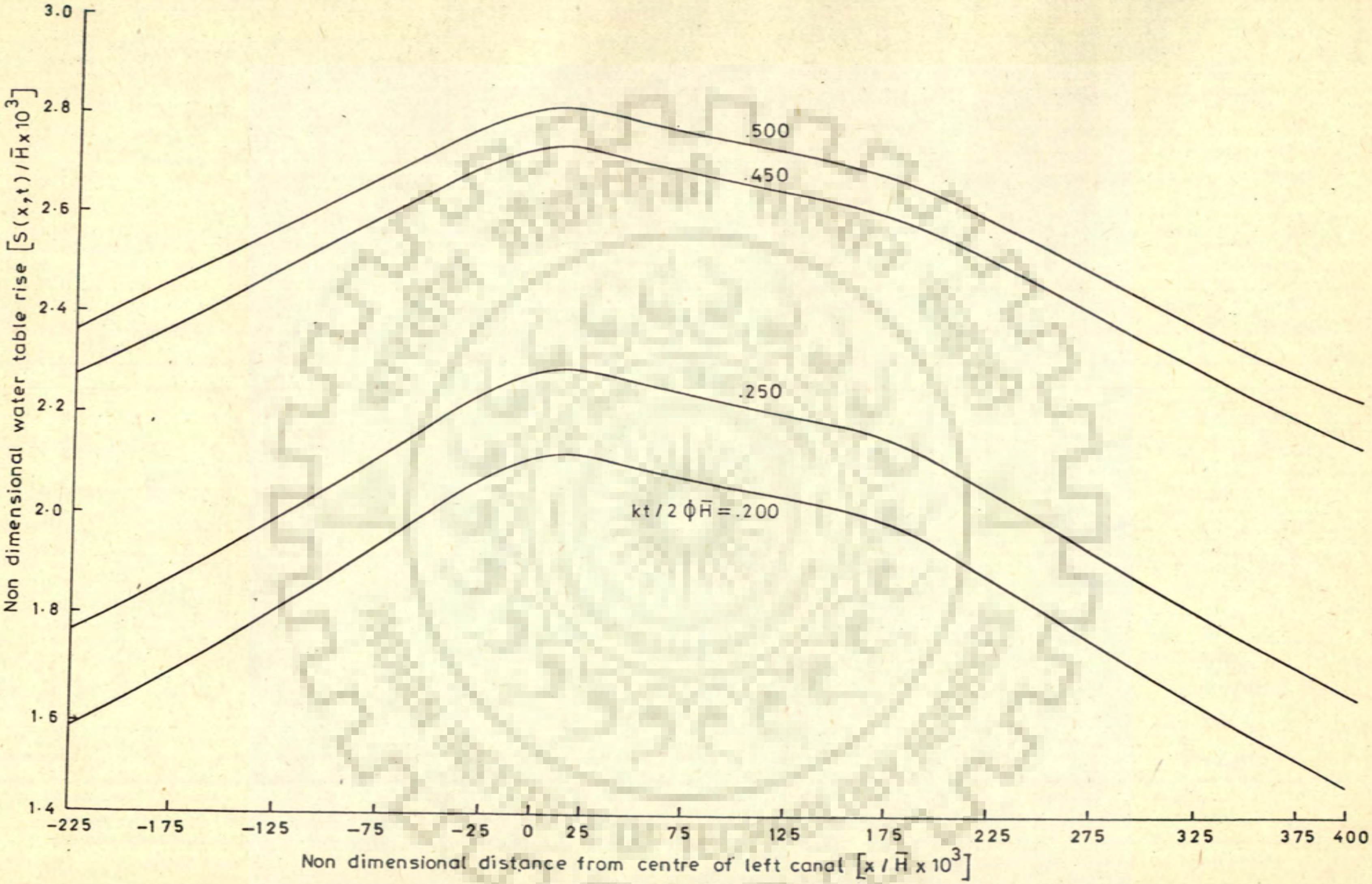

Fig. 5.15 (b)-Water table evolution due to seepage from two continuously running identical parallel canals predicted for $D/\bar{H}=0.18$, $B_1/\bar{H}=B_2/\bar{H}=0.06$, $H_1/\bar{H}=H_2/\bar{H}=0.003$, $\sigma_1/\bar{H}=0.001$, $\sigma_2/\bar{H}=0.003$ and $H/\bar{H}=0.007$; canal beds are at different levels

interference of the left canal. As the seepage loss from the right canal is greatly reduced, the subsequent water table evolution is governed by the left canal. Therefore, there is no formation of a stagnant zone in this case. This phenomenon is more prominent if the canals are more closely spaced as seen from the water table evolutions presented in Fig. (5.16) for $D/\overline{H}$ =0.08.

A typical calculation of seepage loss from two identical canals has also been made by using Equations (5.10) and (5.11). The derivation of Equations (5.10) and (5.11), which compute the seepage loss from two parallel canals, is based on discretisation of the effective length of the canals into various identical reaches and also discretisation of the time parameter. The effective length of the canals contributing towards seepage at any point has been assumed to be 495 m on either side of the point. This 990 m length of the canal has been divided into 99 equal reaches. The results obtained using equation (5.10) and (5.11) and those obtained using Equations (5.17) through (5.24) are presented in Table (5.7) for comparison. The results indicate that, in the beginning, the seepage loss from unit length of canal calculated by the above two approaches are nearly equal. As time elapses, the seepage loss from unit length of canal, if infinite length of canal strip is considered in the analysis, is less than the seepage loss from unit length of canal if

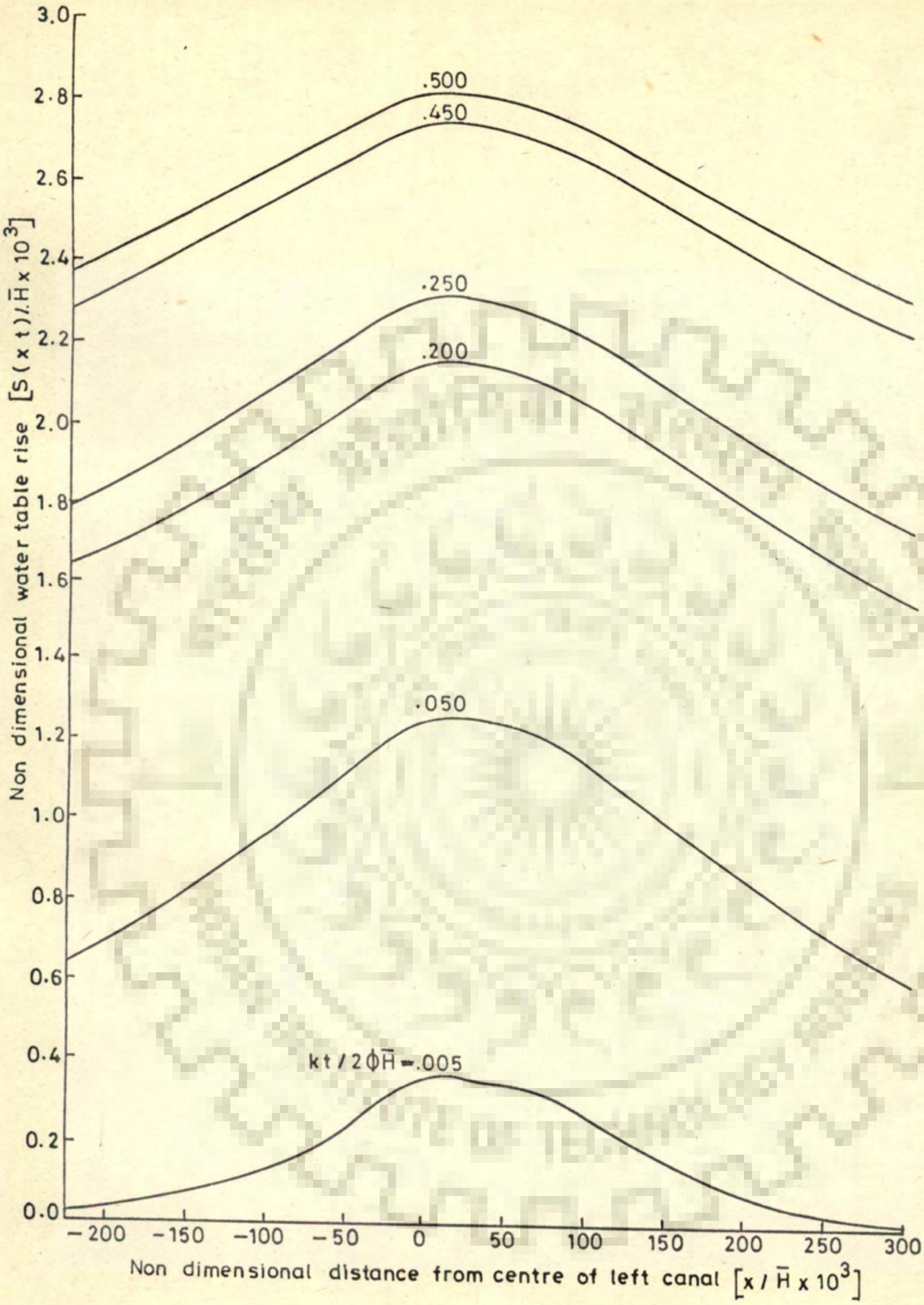

Fig.5.16 - Water table evolution due to seepage from two continuously running identical parallel canals predicted for $D/\bar{H}=0.08, B_1/\bar{H}=B_2/\bar{H}=0.06, H_1/\bar{H}=H_2/\bar{H}=0.003$, $\sigma_1/\bar{H}=0.001$, $\sigma_2/\bar{H}=0.003$ and $H/\bar{H}=0.007$; canal beds are at different levels

Table (5.7) - Non dimensional seepage loss for unit length for two identical parallel canals evaluated for $B_1/\bar{H} = B_2/\bar{H} = 0.06$, $H_1/\bar{H} = H_2/\bar{H} = 0.003$, $D/\bar{H} = 0.18$, $H/\bar{H} = 0.007$, $T = 1000\ m^2/day$, $\varphi = 0.1$ and $K = 1$ m/day.

Non-dimensional time $Kt/(2\varphi\bar{H})$	Seepage loss computed by assuming the number of canal reaches contributing towards seepage = 99; length of canal reach being 10 m. $Q_1(t)/(KL_r\bar{H}_1)$	Seepage loss computed by taking the entire canal into consideration $Q_1(t)/(KL_rH_1)$
0.005	1.836	1.842
0.010	1.774	1.779
0.015	1.725	1.729
0.020	1.685	1.686
0.025	1.652	1.649
0.050	1.545	1.514
0.075	1.480	1.418
0.100	1.436	1.344
0.125	1.402	1.283
0.150	1.376	1.232
0.175	1.354	1.188
0.200	1.336	1.149
0.225	1.320	1.114
0.250	1.306	1.083

a finite number of reaches are assumed to contribute towards seepage. The results given in the Table (5.7) indicate that after a non-dimensional time 0.025, the seepage loss from unit length, if a finite length of canal is considered, is more than that when the canal is taken as an infinite strip.

Having presented the interference of parallel canals which run continuously, the results for the seepage loss from parallel canals, one of which runs intermittently [case (b)], and the resulting water table evolutions are presented in the following paragraphs.

Let the first canal run continuously with a constant depth of water and let the width at the water surface be B_1. Let the second canal have a width B_2 at the water surface during the full supply periods and a width B_{21} at the water surface during the lean supply or closure periods. B_{21} will be approximately equal to the bottom width of the second canal since a nominal depth of water is maintained in the canal during its closure periods. Let the second canal run continuously for M units of time-steps and let it be closed for an equal time period and let the cycle of supply be repeated. Such a running schedule is pertinent if the second canal conveys water during half of the year and is closed for the second half. For such running schedule, the Equation (5.29) simplifies to the following expression :

For time step, $n \le M$,

$$\begin{bmatrix} Q_1\ (n) \\ Q_2\ (n) \end{bmatrix} = \begin{bmatrix} -\frac{1}{\Gamma_1} - \delta_1\ (0,B_1,1), & -\ \delta_2(-D,B_2,1) \\ -\ \delta_1(D,B_1,1), & -\ \frac{1}{\Gamma_2} - \delta_2(0,B_2,1) \end{bmatrix}^{-1} \cdot \begin{bmatrix} \{\sigma_1 - D_i + \overline{H} + \sum_{\gamma=1}^{n-1} Q_1\ (\gamma)\ \delta_1(0,B_1,n-\gamma+1) \\ + \sum_{\gamma=1}^{n-1} Q_2\ (\gamma)\ \delta_2(-D,B_2,n-\gamma+1)\} \\ \{\sigma_2 - D_i + \overline{H} + \sum_{\gamma=1}^{n-1} Q_1\ (\gamma)\ \delta_1(D,B_1,n-\gamma+1) \\ + \sum_{\gamma=1}^{n-1} Q_2\ (\gamma)\ \delta_2(0,B_2,n-\gamma+1)\ \} \end{bmatrix} \quad ...(5.30)$$

and for $M<n\le 2M$

$$\begin{bmatrix} Q_1\ (n) \\ Q_2\ (n) \end{bmatrix} = \begin{bmatrix} -\frac{1}{\Gamma_1} - \delta_1\ (0,B_1,1), & -\ \delta_2(-D,B_{21},1) \\ -\ \delta_1(D,B_1,1), & -\ \frac{1}{\Gamma_2'} - \delta_2(0,B_{21},1) \end{bmatrix}^{-1} \cdot \begin{bmatrix} \{\sigma_1 - D_i + \overline{H} + \sum_{\gamma=1}^{n-1} Q_1\ (\gamma)\ \delta_1(0,B_1,n-\gamma+1) \\ + \sum_{\gamma=1}^{M} Q_2\ (\gamma)\ \delta_2(-D,B_2,n-\gamma+1) \\ + \sum_{\gamma=M+1}^{n-1} Q_2\ (\gamma)\ \delta_2(-D,B_{21},n-\gamma+1)\} \\ \{\sigma_2' - D_i + \overline{H} + \sum_{\gamma=1}^{n-1} Q_1\ (\gamma)\ \delta_1\ (D,B_1,n-\gamma+1) \\ + \sum_{\gamma=1}^{M} Q_2\ (\gamma)\ \delta_2\ (0,B_2,n-\gamma+1) \\ + \sum_{\gamma=M+1}^{n-1} Q_2\ (\gamma)\ \delta_2(0,B_{21},n-\gamma+1)\} \end{bmatrix} \quad ...(5.31)$$

in which

$$\Gamma_1 = \pi K/\log \left[\frac{0.5\ (D_i - \sigma_1)}{B_1 + 2(D_{b1} - \sigma_1)(\sqrt{2} - 1)}\right] .. \quad (5.32)$$

$$\Gamma_2 = \pi K/\log \left[\frac{0.5\ (D_i - \sigma_2)}{B_2 + 2(D_{b2} - \sigma_2)(\sqrt{2} - 1)}\right] .. \quad (5.33)$$

and,

$$\Gamma_2' = \pi K/\log \left[\frac{0.5\ (D_i - \sigma_2')}{B_{21} + 2(D_{b2} - \sigma_2')(\sqrt{2} - 1)}\right] \quad ... \quad (5.34)$$

Γ_2' is the reach transmissivity of second canal during closure when there is a nominal depth of water in the canal. D_i is the depth to impervious bed measured from an assumed high datum as shown in Fig. (5.1). σ_1, σ_2 are the depths to water surfaces measured from the same datum for the first and second canal respectively; σ_2' is the depth to water surface in the second canal from the high datum during the closure. D_{b1}, D_{b2} are the depths to beds of the canals from high datum. The expressions of reach transmissivity given by Equations (5.32) and (5.33) have been obtained assuming the slopes of the canal banks to be 1:1.

In a similar manner, equations valid for other cycles could be written.

Numerical results for case (b) have been obtained assuming $M = 18$ and $KM/(2\varphi\overline{H}) = 0.09$. If the second parallel canal has been introduced to supplement water for irrigation for summer crops, in that case, the second parallel canal is likely to run for six months and would remain closed for almost the same length of time. Therefore, while obtaining numerical results, the durations of running and closure of second canal have been taken as equal. Assuming a unit time-step size of 10 days, the total number of time-steps for which the canal would run at a stretch would be 18. If unit time-step size =10 days, hydraulic conductivity = 1m/10 days, $\varphi = 0.1$, $\overline{H} = 1000$m, and M = 18, then, $KM/(2\varphi\overline{H}) = 0.09$.

Seepage losses from two parallel canals, one of which runs intermittently, have been evaluated for $B_1/\overline{H} = B_2/\overline{H} = 0.03$, $B_{21}/\overline{H} = 0.024$, and $D/\overline{H} = 0.18$.The non-dimensional depth of water in the left canal, $H_1/\overline{H}$, is 0.003. During periods of full supply, the non-dimensional depth of water in the right canal, $H_2/\overline{H}$, is 0.003. During the periods of closure, a dimensionless depth of 1.0×10^{-6} has been assumed to prevail. Numerical results have been obtained for two initial water table positions defined by $H/\overline{H} = 0.005$ and 0.009. The variation of seepage losses with time from each canal, corresponding to $H/\overline{H} = 0.005$, are presented in Fig.(5.17).The seepage losses from the canals had they run continuously, and the seepage loss from the left canal for $D/\overline{H} = \infty$, are also presented in Fig.(5.17) for comparison.To start with, seepage loss from each canal of the parallel canal system decreases with time. The instant the right canal is closed, the variation in

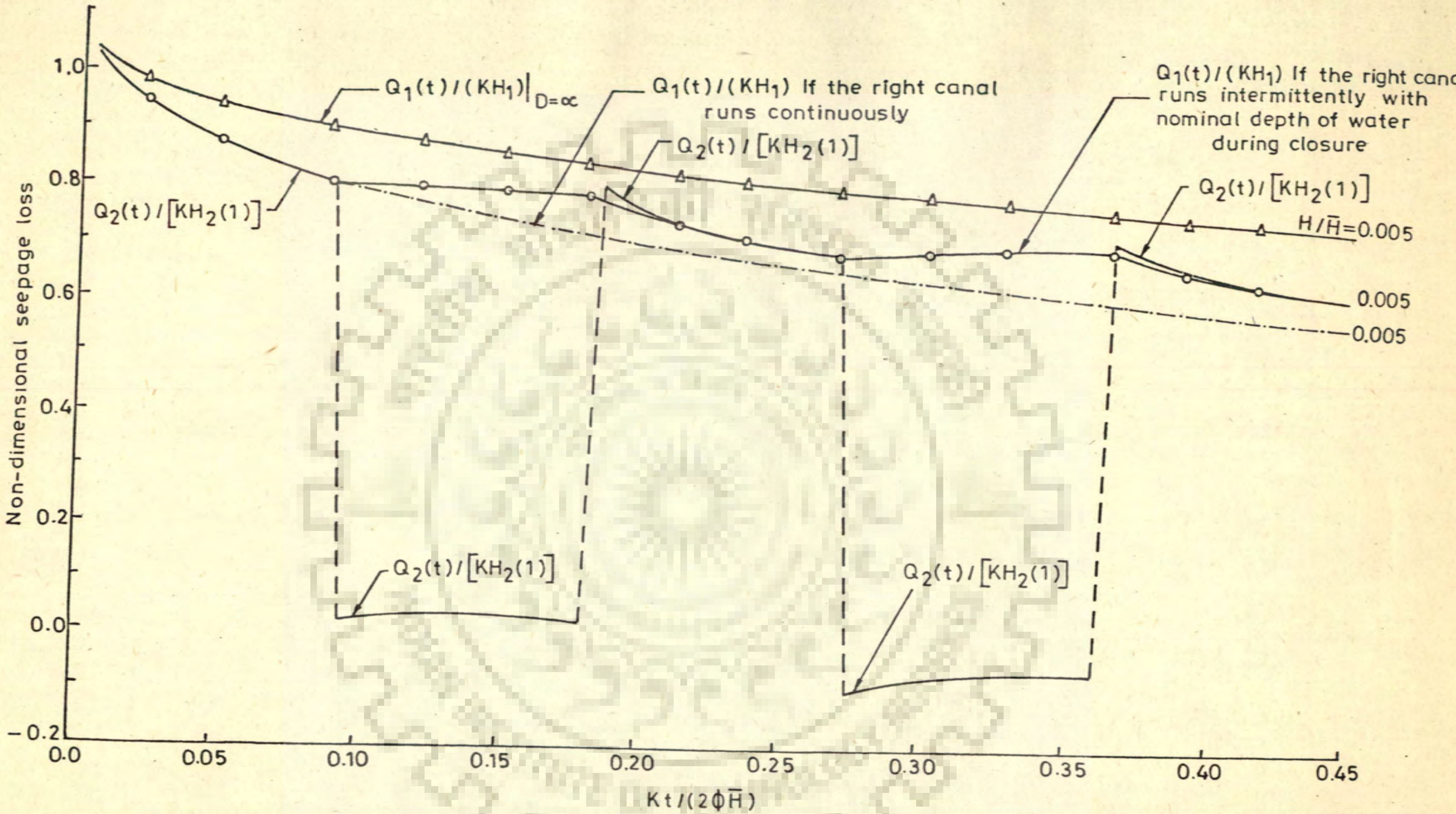

Fig. 5 17 - Seepage loss from two parallel canals, one of which runs intermittently, evaluted for $B_1/\bar{H}=B_2/\bar{H}=0.03$ $B_{2l}/\bar{H}=0.024$, $D/\bar{H}=0.18$, $H/\bar{H}=0.005$, $H_1/\bar{H}=0.003$, $H_2/\bar{H}=0.003$ during the running period and $H_2/\bar{H}=1\times10^{-6}$ during closure period seepage loss from the left canal deviates from its decreasing trend. It is seen that for $H/\overline{H}$ = 0.005, on closure of the right canal, the seepage loss from the left canal increases until the right canal runs again with full supply. The increase in seepage loss from the left canal is attributed to the decrease in the interference of the right canal consequent to its closure. Similar trend is observed during the second cycle of running. For $H/\overline{H}$ =0.009, it is seen from Fig. (5.18),that during the first cycle on closure of the second canal, the variation of seepage loss though deviates from its decreasing trend but the seepage loss continues to decrease at a reduced rate. However, during the second cycle,on closure of the second canal, seepage loss from the first canal increases until the second canal runs again with full supply. Thus, if the water table position is at a very shallow depth,the decreasing trend in seepage loss from the left canal is reversed and the seepage loss from the left canal starts increasing on closure of the right canal. If the water table is somewhat at large depth, on closure of the right canal the decreasing trend in seepage loss from the left canal though deviates, the seepage loss continues to decrease at reduced rate.

The following observations could be made from Figs. (5.17) and (5.18) in respect of seepage loss from the left canal when the right canal runs intermittently:

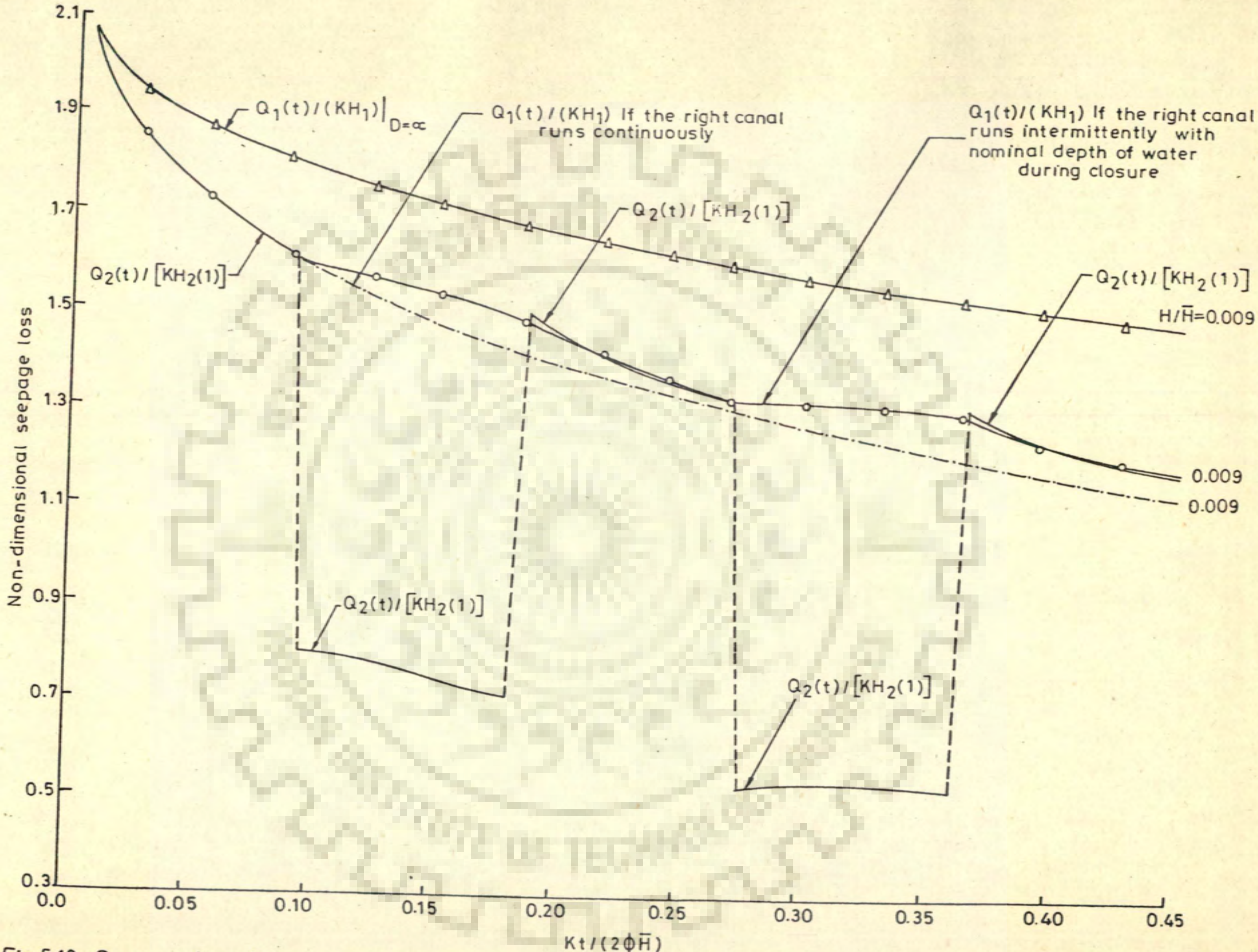

Fig.5.18 – Seepage loss from two parallel canals, one of which runs intermittently, evaluated for $B_1/\bar{H} = B_2/\bar{H} = 0.03$, $B_{2l}/\bar{H} = 0.024$, $D/\bar{H} = 0.18$, $H/\bar{H} = 0.009$, $H_1/\bar{H} = 0.003$, $H_2/\bar{H} = 0.003$ during the running period and $H_2/\bar{H} = 1 \times 10^{-6}$ during closure period

(i) The seepage loss from the left canal, had both the canals run continuously, forms the lower bound.

(ii) The seepage loss from the left canal, had the right canal been at infinity, forms the upper bound.

(iii) The seepage loss from the left canal, when the right canal runs intermittently, fluctuates between these two bounds during the first few cycles of running for which the results have been presented in these figures.

The variations of seepage loss with time from the right canal, which runs intermittently, are discontinuous at the closure and the reopening time. It is seen that for $H/\overline{H} = 0.005$, during the first closure of the right canal, at non-dimensional time 0.095 and 0.18, the non-dimensional seepage losses from the right canal are 0.018 and 0.022 respectively. However, during the closure in the second cycle, the right canal receives water and acts as a drain. At non-dimensional time 0.275, the non-dimensional flow to the right canal is 0.099 and at time 0.36, before the canal runs again with full supply, the flow to the canal is 0.061. The assumption that there exists a nominal depth of water in the right canal during its closure is appropriate for the situation in which the right canal would act as a drain during periods of its closure. If the water table is at a shallow depth, the right canal would act as a drain during its closure.

The decrease in seepage loss from the left canal, due to interference of the right canal which runs intermittently, has been shown in Figs. [5.19 (a)] and [5.19 (b)] for $B_1/\overline{H}=B_2/\overline{H}=0.03$, $B_{21}/\overline{H}=0.024$, $D/\overline{H}=0.18$, $H_1/\overline{H}=H_2(1)/\overline{H}=0.003$, $\sigma_1/\overline{H}=0.001$, and $H/\overline{H}=0.007$. It can be seen from Fig. [5.19 (a)] that the reduction in seepage due to interference increases from cycle to cycle, reaches a maximum value and then decreases as observed in the case of continuously running canals. The interference is maximum at $Kt/(2\varphi\overline{H}) = 0.81$ and the value of maximum interference is 0.214. With closure of the right canal, its interference decreases. The residual interference at the time the right canal runs again after the closure, increases from cycle to cycle; reaches a maximum value, and then decreases. The residual interference is maximum at $Kt/(2\varphi\overline{H}) = 0.72$ and the maximum value of residual interference is 0.158.

The interference that would prevail at large time has been presented in Fig. [5.19 (b)]. The trend indicates that if the right canal runs intermittently for an indefinite period, the interference would fluctuate about zero. It was found from Fig. (5.8) that if the right canal runs continuously, its interference at large time tends to zero. However, as seen from Fig. [5.19 (b)], if the right canal runs intermittently its

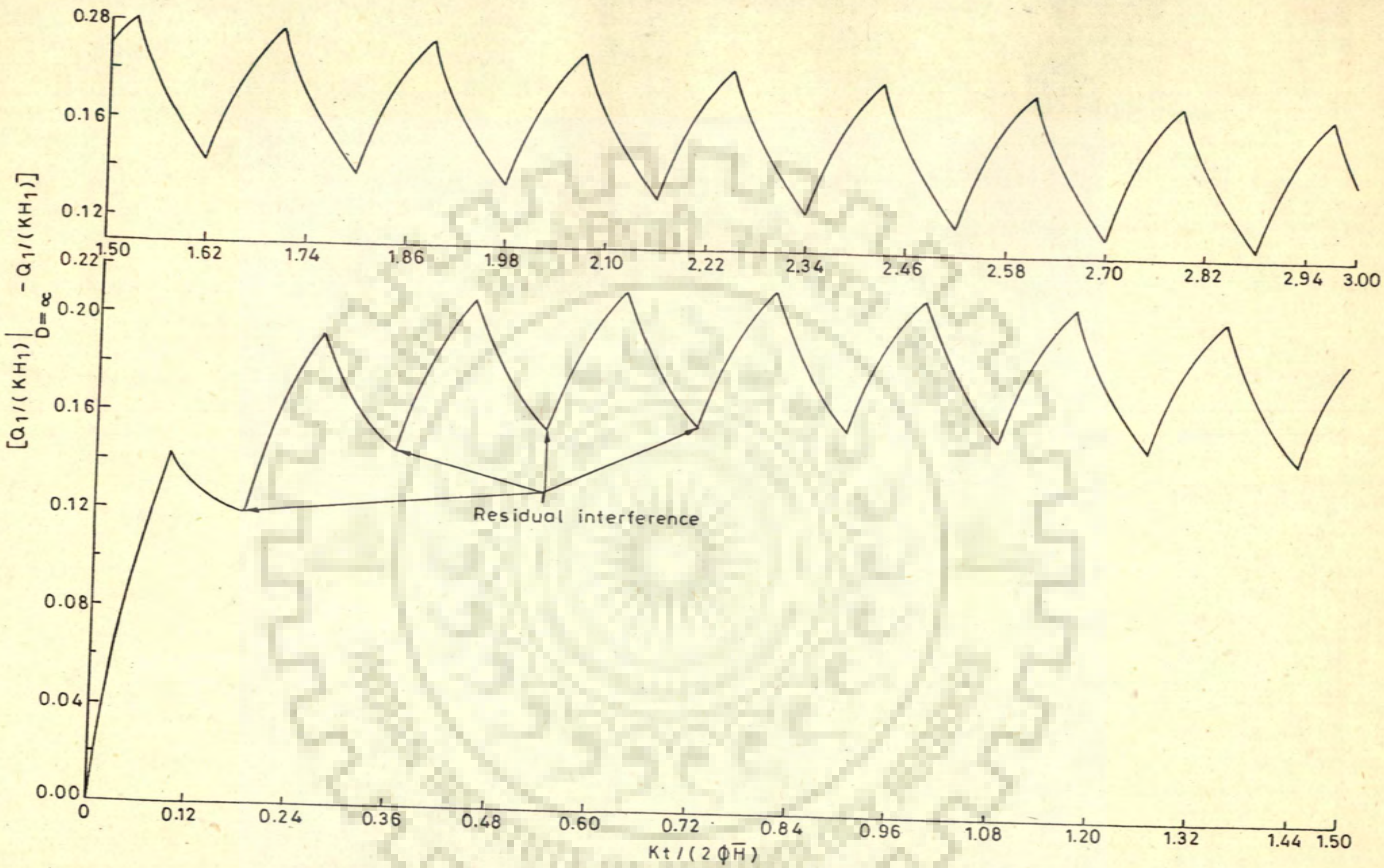

Fig.5.19 (a)–Reduction in seepage from left canal at large time due to interference of right canal which runs intermittently, evaluated for $B_1/\bar{H} = B_2/\bar{H} = 0.03$, $H_1/\bar{H} = H_2/\bar{H} = 0.003$, $B_{21}/\bar{H} = 0.024$, $D/\bar{H} = 0.18$ and $H/\bar{H} = 0.007$.

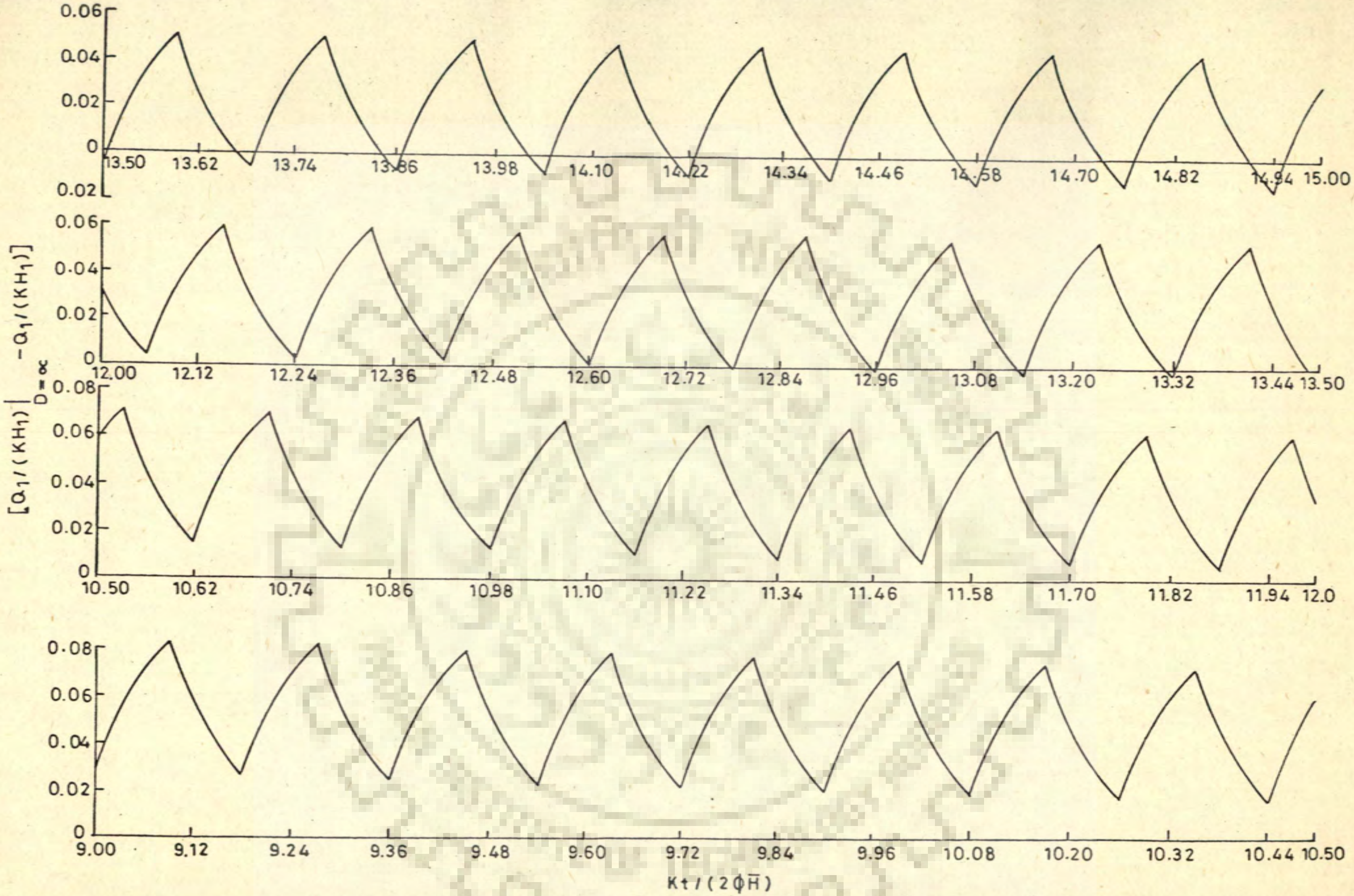

Fig.5.19 (b)—Reduction in seepage from left canal at large time due to interference of right canal which runs intermittently, evaluated for $B_1/\bar{H} = B_2/\bar{H} = 0.03$, $H_1/\bar{H} = H_2/\bar{H} = 0.003$, $B_{2l}/\bar{H} = 0.024$, $D/\bar{H} = 0.18$, $H/\bar{H} = 0.007$

interference fluctuates between positive and negative values at large time. Negative interference means that the right canal is inducing seepage from the left canal. At large time, the water table would come up to the water surface in the canals. During the closure of the right canal if the water level is brought to the bed level to impose the boundary condition, the right canal would act as a drain and would induce seepage from the left canal.

The seepage losses from two parallel canals for case (c) is next considered, in which the left canal runs continuously with a constant depth of water, $H_1/\bar{H}=0.003$, and the right canal runs intermittently, such that during the running periods $H_1/\bar{H} = H_2/\bar{H}=0.003$, and during closure the right canal is dry. The results have been obtained using the Equations (5.30) and (5.31) and imposing the condition $Q_2(n) = 0$ during the closure periods. The assumption of $Q_2(n)=0$ during the closure of right canal is appropriate for that situation in which the water table would remain below the bed of the right canal. The results for seepage losses for $B_1/\bar{H}=B_2/\bar{H}=0.03$, $D/\bar{H} = 0.18$, and $H/\bar{H} = 0.009$ are shown in Fig. (5.20). It could be seen from the figure that at $Kt/(2\varphi\bar{H}) = 0$, $Q_2(t)/KH_2=2.1$. The rate of seepage loss decreases with time and becomes zero abruptly at non-dimensional time $=0.09+\varepsilon$, where ε is a very small quantity, due to the

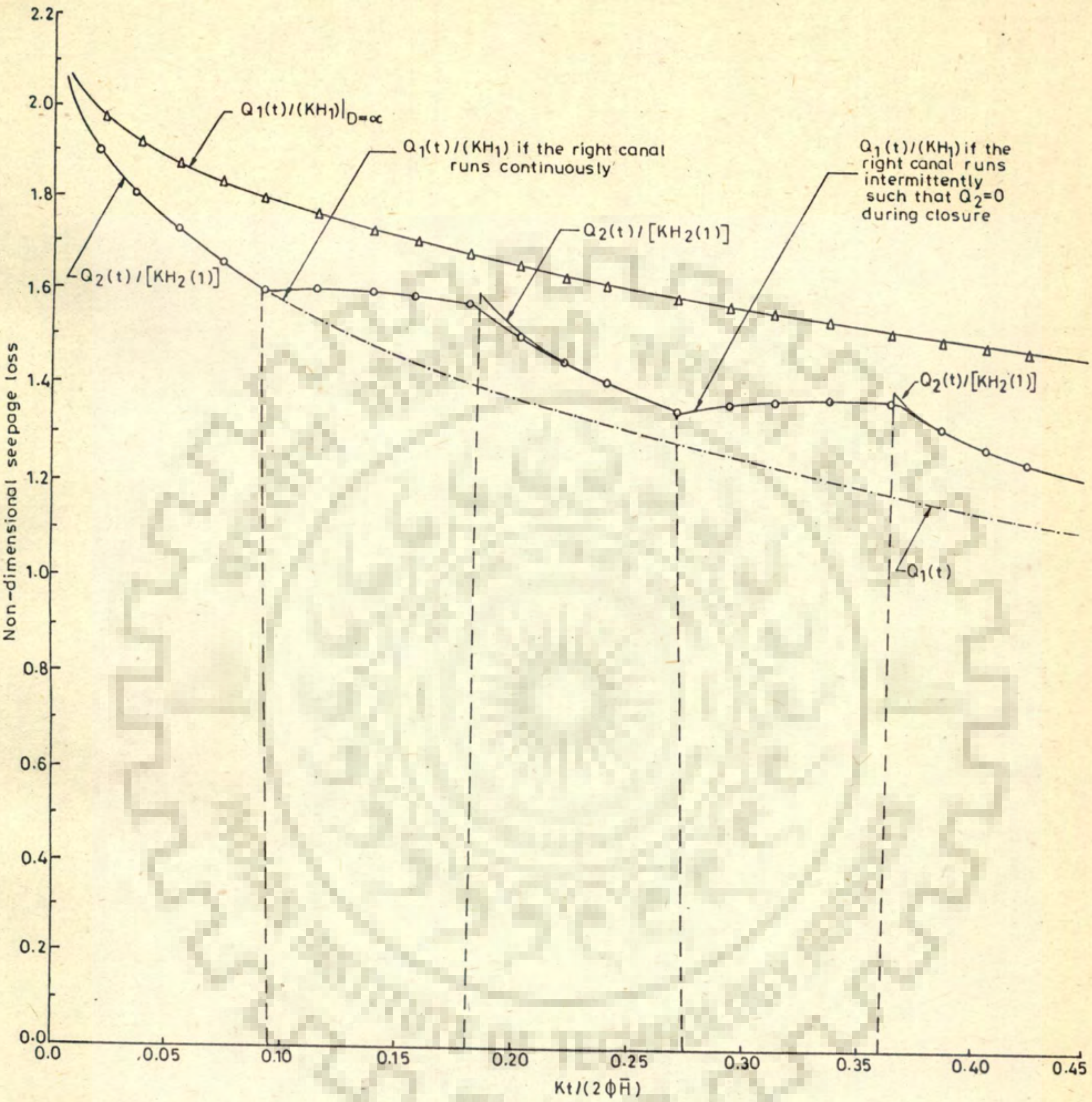

Fig.5.20 – Seepage loss from two parallel canals one of which runs intermittently, evaluated for $B_1/\bar{H}=0.03$, $B_2/\bar{H}=0.03$, $D/\bar{H}=0.18$, $H/\bar{H}=0.009$, $H_1/\bar{H}=0.003$ and $H_2/\bar{H}=0.003$ during running and $Q_2(t)=0$ during the closure period

boundary condition that has been imposed. During the second cycle of running, the seepage loss from the right canal does not start from the value 2.1; instead it starts from a value of 1.600 and then decreases. The starting value of seepage loss in the second cycle is less than that of the first cycle due to the fact that the initial potential difference has already been reduced on account of the running of the canals.

A comparison of the reductions in seepage from the left canal due to interference of the right canal, which may run in three different manners as prescribed in cases (a), (b), and (c), has been made in Table (5.8). The results presented in the table have been computed for $H/\overline{H} = 0.007$, $D/\overline{H} = 0.18$, $B_1/\overline{H} = 0.03$, $H_1/\overline{H} = 0.003$, $B_2/\overline{H} = 0.03$, and $H_2/\overline{H} = 0.003$ during the period the right canal runs with full supply [case (a)], $B_{21}/\overline{H} = 0.024$ and $H_2/\overline{H} = 1.0 \times 10^{-6}$ during the period the right canal runs with nominal depth [case (b)]. If both the canals run continuously the reductions in seepage from the left canal due to interference are 0.1488, 0.2328, and 0.2644 at the end of non-dimensional time 0.09, 0.27, and 0.45 respectively. If the right canal runs with a nominal depth of water during the closure, the corresponding reductions in seepage from the left canal due to interference are 0.1488, 0.1968, and 0.2115 respectively. If the seepage loss from the right canal is assumed to be zero during its closure, the

Table (5.8) - Interference of an intermittently or continuously running canal on seepage loss from a continuously running parallel canal, evaluated for $D/\overline{H} = 0.18$, $H/\overline{H} = 0.007$, $B_1/\overline{H} = B_2/\overline{H} = 0.03$, $H_1/\overline{H} = 0.003$, $H_2/\overline{H} = 0.003$ during running and $H_2/\overline{H} = 1 \times 10^{-6}$ during closure.

Time factor	Seepage loss from a single canal for $D = \infty$	Seepage loss from each of the canal when they run continuously		Interference of right canal on the left canal	Seepage loss from each canal when the right canal runs intermittently		Interference of right canal on the left canal	Seepage loss from each canal when the right canal runs intermittently with $Q_2(t)=0$		Interference of the right canal on the left canal
$\frac{Kt}{2 \varphi \overline{H}}$	$\frac{Q_1(t)}{KH_1}$	$\frac{Q_1(t)}{KH_1}$	$\frac{Q_2(t)}{KH_2}$		$\frac{Q_1(t)}{KH_1}$	$\frac{Q_2(t)}{KH_2(1)}$		$\frac{Q_1(t)}{KH_1}$	$\frac{Q_2(t)}{KH_2(1)}$	
0.005	1.5513	1.5431	1.5431	0.0082	1.5431	1.5431	0.0082	1.5431	1.5431	0.0082
0.090	1.3506	1.2018	1.2018	0.1488	1.2018	1.2081	0.1488	1.2018	1.2018	0.1488
0.095	1.3442	1.1912	1.1912	0.1530	1.1954	0.4081	0.1488	1.1976	0.0	0.1466
0.180	1.2586	1.0556	1.0556	0.2030	1.1370	0.3682	0.1216	1.1813	0.0	0.0773
0.185	1.2545	1.0494	1.0494	0.2051	1.1292	1.1471	0.1253	1.1726	1.2004	0.0819
0.270	1.1946	0.9618	0.9618	0.2328	0.9978	0.9996	0.1968	1.0175	1.0202	0.1771
0.275	1.1916	0.9575	0.9575	0.2341	0.9966	0.2146	0.1950	1.0169	0.0	0.1747
0.360	1.1448	0.8931	0.8931	0.2517	0.9974	0.2320	0.1474	1.0383	0.0	0.1065
0.365	1.1423	0.8898	0.8898	0.2525	0.9920	1.0107	0.1503	1.0322	1.0577	0.1101
0.450	1.1036	0.8392	0.8392	0.2644	0.8921	0.8944	0.2115	0.9139	0.9171	0.1897

corresponding reductions in seepage are 0.1488, 0.1771, and 0.1897. The non-dimensional times 0.09, 0.27, and 0.45 refer to the end of running of the right canal in the first, second, and the third cycle respectively. Reduction in seepage from the left canal due to interference of the right canal is highest for case (a) and lowest for case (c).

The water table evolutions due to seepage from two parallel canals, one of which runs intermittently [case (b)], have been predicted for $B_1/\bar{H}$ = 0.03, $H_1/\bar{H}$ = 0.003, $B_2/\bar{H}$ =0.03, and $H_2/\bar{H}$ =0.003 during running of the right canal; $B_2/\bar{H}$ =0.024 and $H_2/\bar{H} = 1 \times 10^{-6}$ during closure of the right canal, and $H/\bar{H}$ =0.005. The evolution of water table shown in Fig. (5.21) has been predicted for $D/\bar{H}$ = 0.08. The water table evolution presented in Fig. (5.22) is for $D/\bar{H}$ = 0.18. The right canal remains closed during non-dimensional time periods 0.09 to 0.18 and 0.27 to 0.36. The following observations could be made from Figs. (5.21) and (5.22):

(i) There is only one water table mound at non-dimensional time 0.18 and 0.36 under the left canal which runs continuously, and the mound under the right canal has disappeared.

(ii) For $D/\bar{H}$ = 0.08, the maximum height of the mound under the left canal at the end of non-dimensional time 0.18 is less than that of the mound at non-dimensional time 0.09.

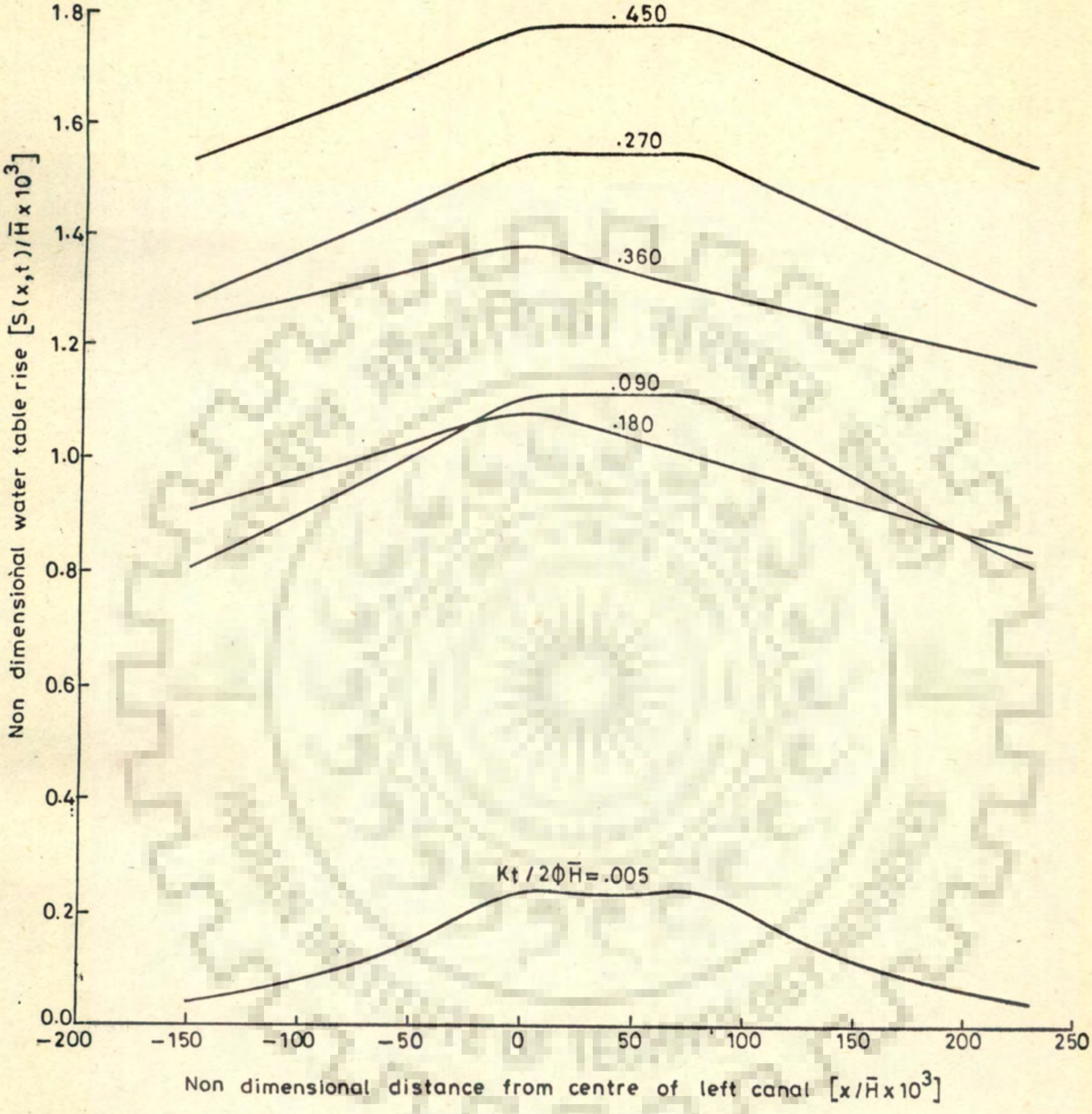

Fig. 5.21-Water table evolution due to seepage from two equal parallel canals one of which runs intermittently, evaluated for $D/\bar{H}=0.08$, $H/\bar{H}=0.005$, $B_1/\bar{H}=0.03$, $B_2/\bar{H}=0.03$, $H_1/\bar{H}=0.003$ and $H_2(t)/\bar{H}=0.003$ while running and $H_2(t)/\bar{H}=1\times10^{-6}$ when closed.

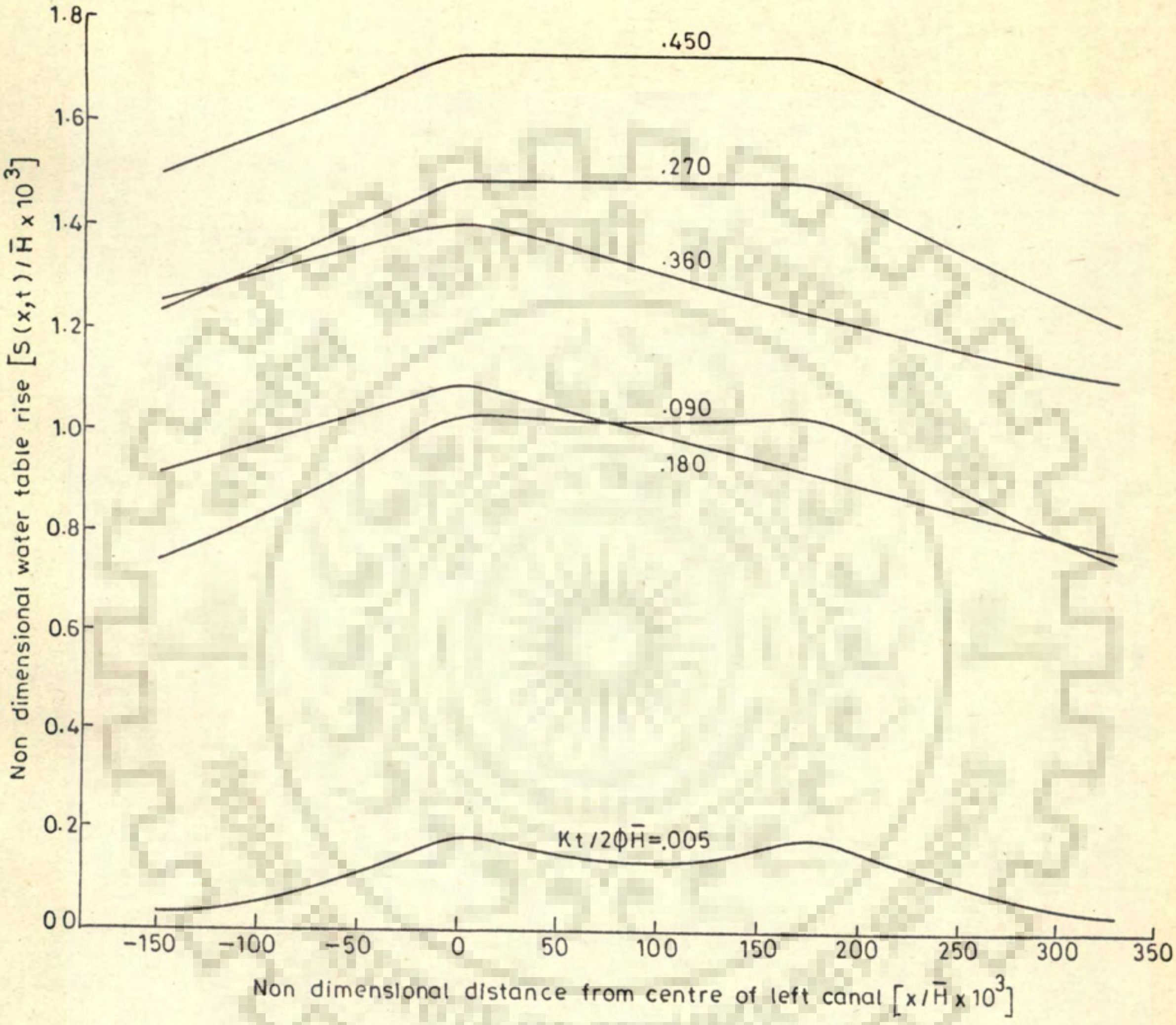

Fig.5.22– Water table evolution due to seepage from two equal parallel canals one of which runs intermittently, evaluated for $D/\bar{H} = 0.18$, $H/\bar{H} = 0.005$, $B_1/\bar{H} = 0.03$, $B_2/\bar{H} = 0.03$, $H_1/\bar{H} = 0.003$ and $H_2(t)/\bar{H} = 0.003$ while running and $H_2(t)/\bar{H} = 1 \times 10^{-6}$ when closed.

(iii) For $D/\overline{H}$=0.08 and 0.18, the maximum height of the mound under the left canal at the end of non-dimensional time 0.36 is less than that of the mound at non-dimensional time 0.27.

(iv) The height of the mound under the left canal declines during closure of the right canal as the seepage loss from the right canal is very little during its closure, or the right canal acts as a drain. For the same reason the mound disappears under the right canal.

The water table evolutions for the same canal system are shown in Figs.(5.23) and (5.24) for $H/\overline{H}$ =0.009. As the water table is at a comparatively deeper position, the seepage loss from the right canal during its closure is considerable and the right canal does not act as a drain. Hence, as seen from Fig. (5.23), the water table mound below the left canal progressively rises with time upto non-dimensional time 0.36. Similar trend could be observed from Fig. (5.24) when the canals are located at larger spacing with $D/\overline{H}$ = 0.18. Thus, the water table mound will go on rising with time below the left canal till the seepage loss from the right canal is insignificant or the canal acts as a drain.

The water table rise for case (c), in which the right canal runs intermittently such that $Q_2(n)$=0 during its closure, has been evaluated for $B_1/\overline{H}=B_2/\overline{H}$=0.03,

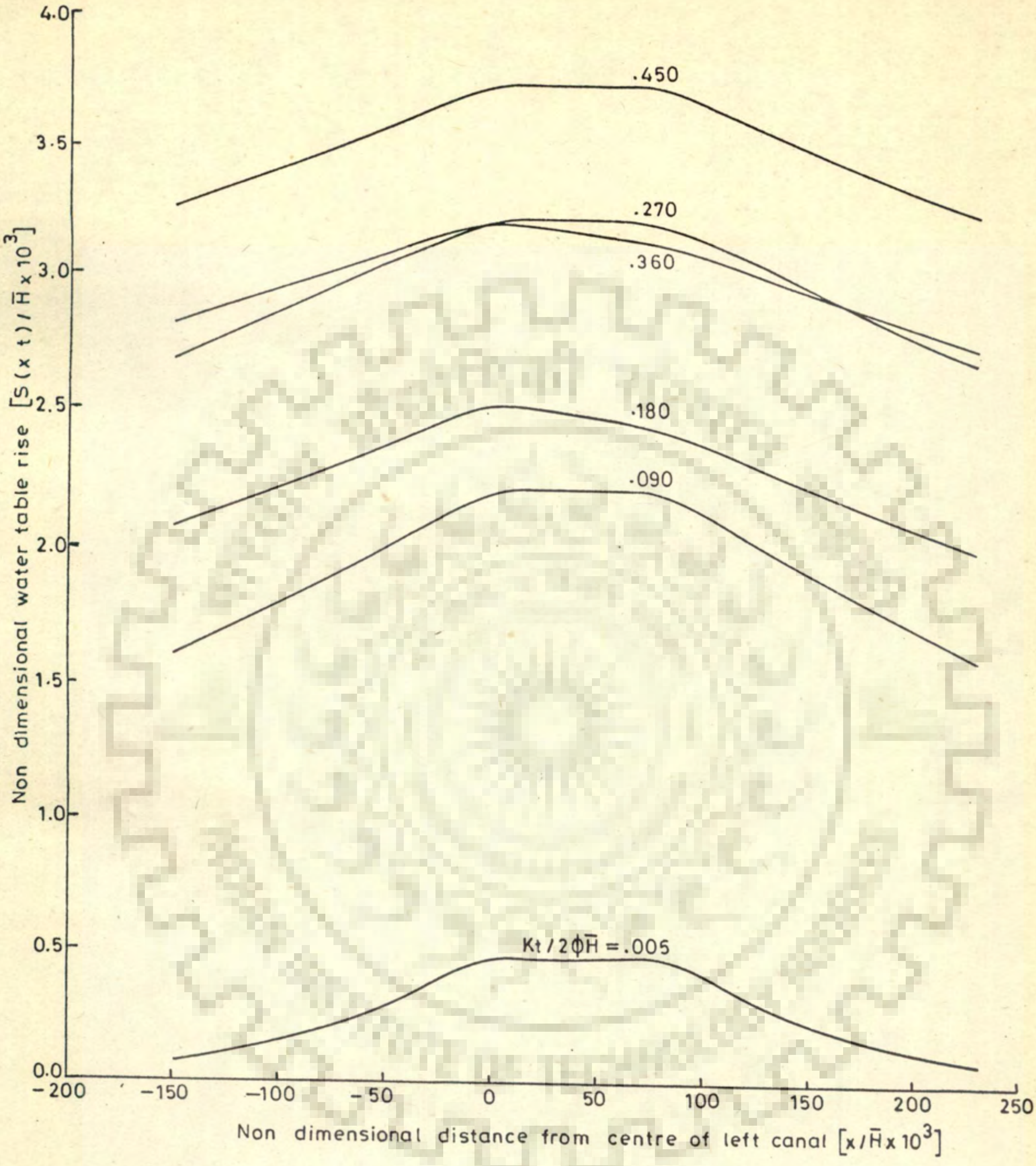

Fig.5.23-Water table evalution due to seepage from two equal parallel canals one of which runs intermittently, evaluated for $D/\bar{H} = 0.08$, $H/\bar{H} = 0.009$, $B_1/\bar{H} = B_2/\bar{H} = 0.03$, $H_1/\bar{H} = 0.003$ and $H_2(t)/\bar{H} = 0.003$ while running and $H_2(t)/\bar{H} = 1 \times 10^{-6}$ when closed.

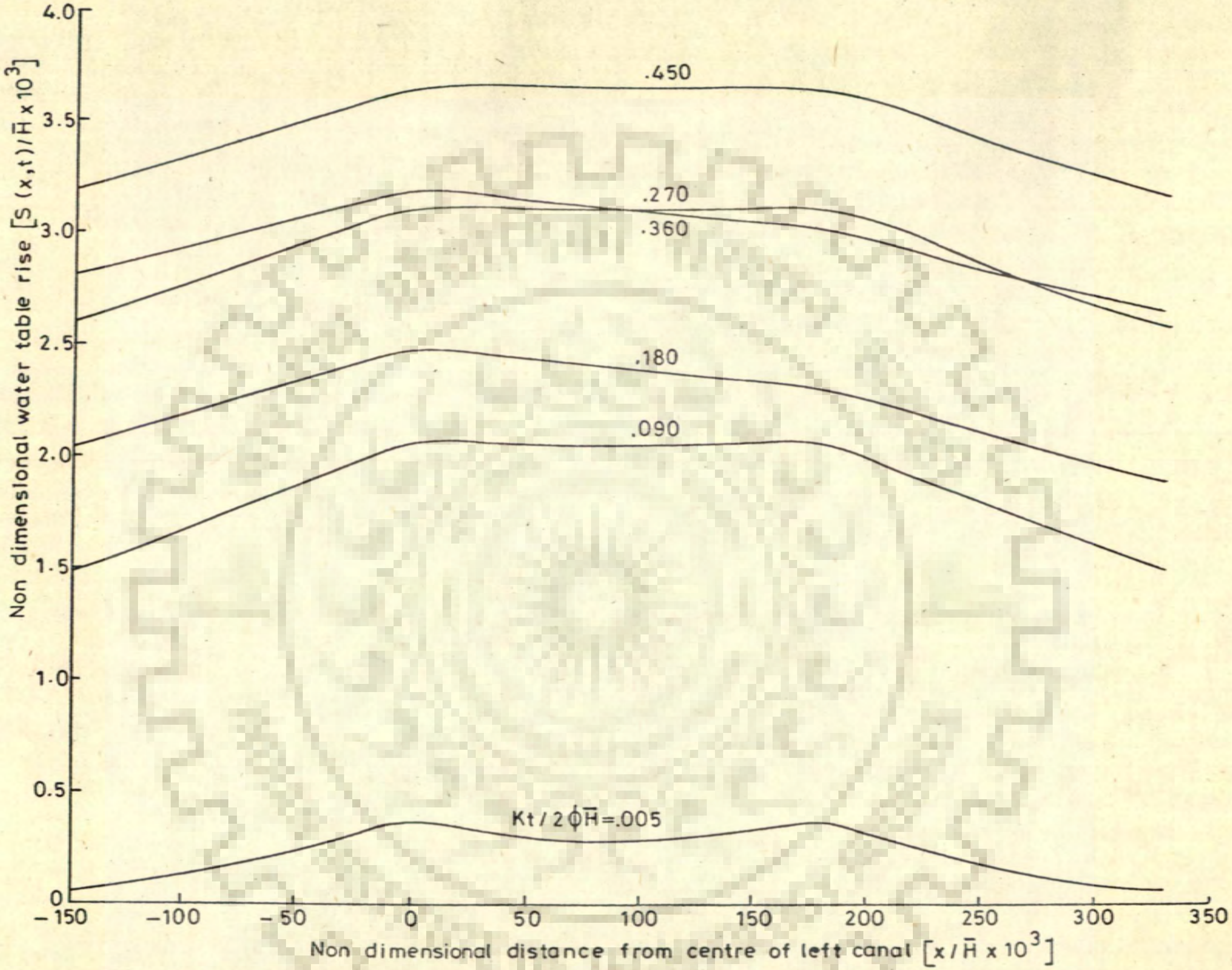

Fig.5.24–Water table evolution due to seepage from two equal parallel canals one of which runs intermittently, evaluated for $D/\bar{H}=0.18$, $H/\bar{H}=0.009$, $B_1/\bar{H}=B_2/\bar{H}=0.03$, $H_1/\bar{H}=0.003$ and $H_2(t)/\bar{H}=0.003$ while running and $H_2(t)/\bar{H}=1\times10^{-6}$ when closed.

$H_1/\overline{H}$ = 0.003, $H_2(t)/\overline{H}$ during running of the right canal = 0.003, and $H/\overline{H}$ = 0.009. Results for $D/\overline{H}$ = 0.08 and 0.18 have been presented in Figs. (5.25) and (5.26) respectively. It can be seen from Fig. (5.25) that the maximum height of water table mound below the left canal at non-dimensional time 0.18 is less than that of the mound at non-dimensional time 0.09. This is because the seepage loss from the right canal during its closure is zero. It can be seen from Fig. (5.26) that similar trend occurs only at a later time as the spacing between the canals is comparatively more.

5.4 Conclusions

Based on the results presented in this chapter, the following conclusions are drawn:

1. The unsteady seepage losses from the canals and the reduction in seepage due to interference are linearly proportional to the initial potential difference that initiates the flow.
2. In case of two continuously running parallel canals, the reduction in seepage from one canal, due to interference of the other, is zero in the beginning of seepage. The interference increases as the time passes and attains a maximum value and

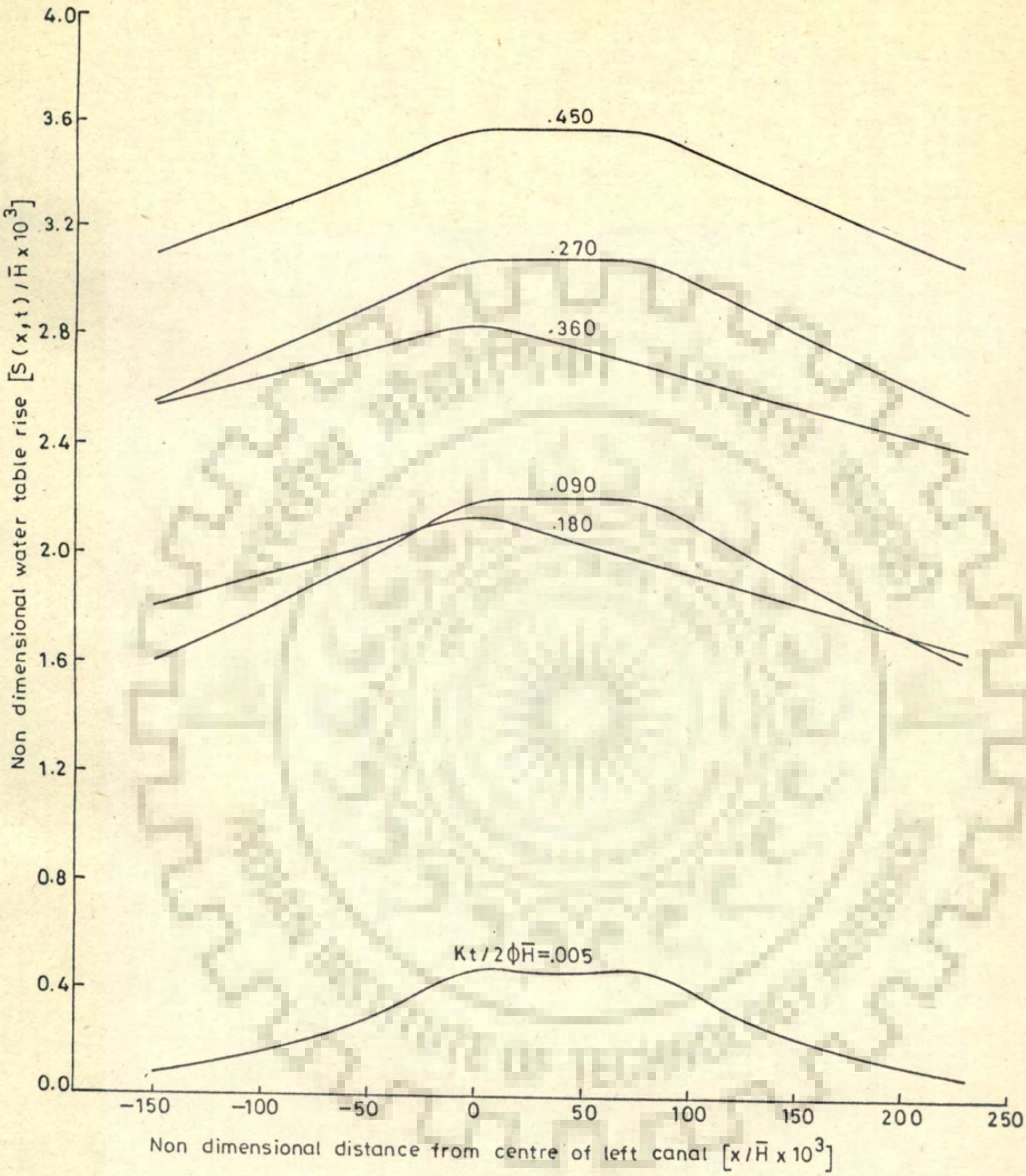

Fig.5.25– Water table evolution due to seepage from two equal parallel canals one of which runs intermittently, evaluated for $D/\bar{H}=0.08$, $H/\bar{H}=0.009$, $B_1/\bar{H}=B_2/\bar{H}=0.03$, $H_1/\bar{H}=0.003$, $H_2(t)/\bar{H}=0.003$ while running and $Q_2=0$ when canal is closed.

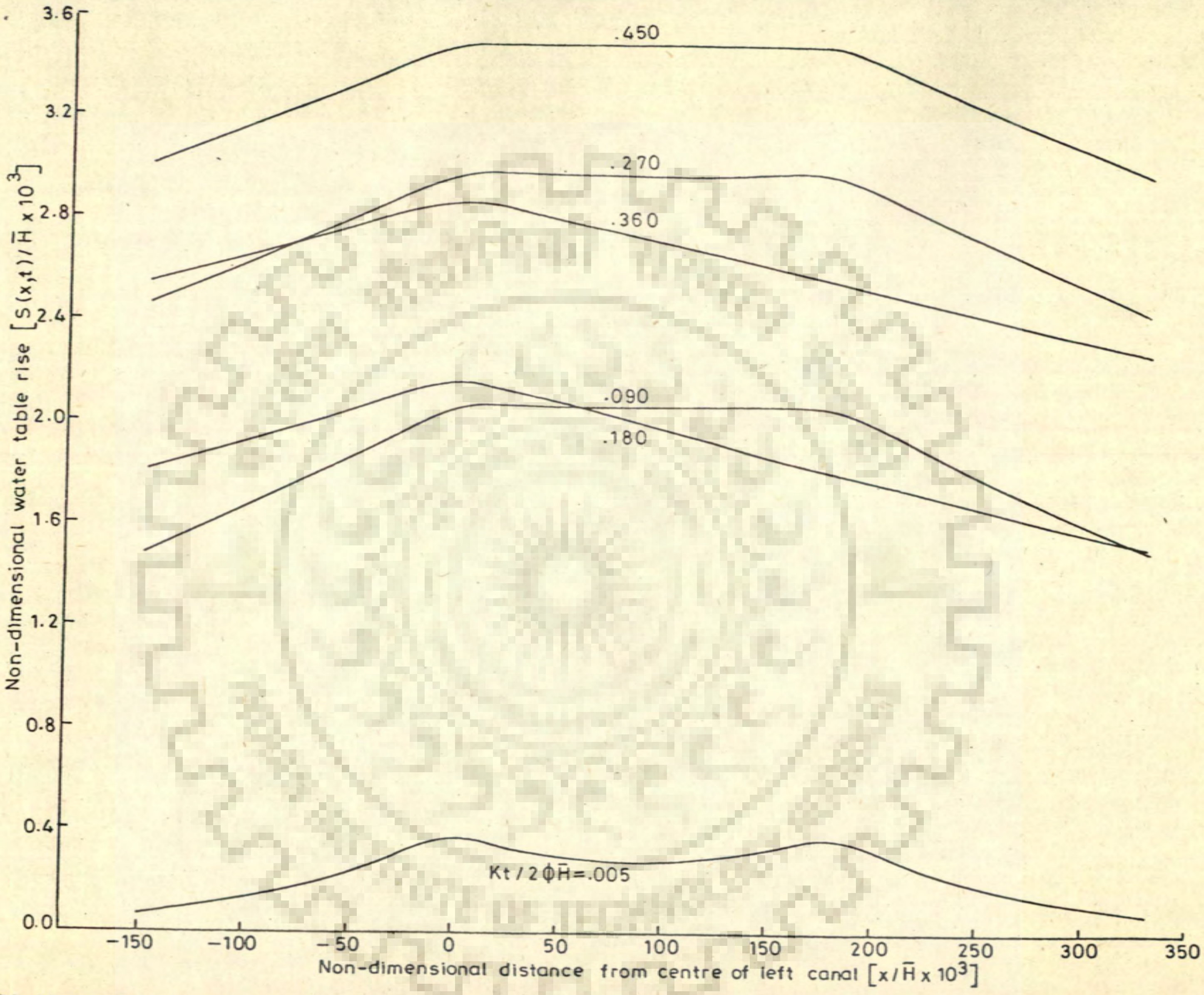

Fig.5.26– Water table evolution due to seepage from two equal parallel canals one of which runs intermittently, evaluated for $D/\bar{H}=0.18$, $H/\bar{H}=0.009$, $B_1/\bar{H}=B_2/\bar{H}=0.03$, $H_1/\bar{H}=0.003$, $H_2(t)/\bar{H}=0.003$ while running and $Q_2=0$ when canal is closed.

then decreases. The decrease is monotonic at large time.

3. The maximum reductions in seepage due to interference decreases with increase in the spacing between the canals. Also, the occurrence of maximum interference is delayed for canals having larger spacing.

4. The interference of a bigger canal on smaller canal is more than that of the smaller canal on the bigger one.

5. If two canals of equal dimensions are at different bed levels, the interference of the canal at lower elevation is less than that of the canal at higher elevation.

6. If one of the parallel canals runs continuously, and the other intermittently with equal durations of closure and running, it is found that, the reduction in seepage from the continuously running canal due to interference of the intermittently running canal starts from zero, increases from cycle to cycle, reaches a maximum value, and then decreases. If the intermittently running canal is operated indefinitely, it is found that its interference on the other would fluctuate about a zero mean value.

7. If one of the parallel canals runs intermittently, after some cycles of operation, the intermittently running canal would act as a drain during its closure period.

8. For the parallel canals of equal dimensions, distinct water mounds of equal height are formed under the canals. In the beginning of seepage, the ridges lie under the centre of the canals. With lapse of time, as seepage continues, the points of maximum water table height move towards each other; but they do not cross the width of the respective recharging strips. With passage of time, the zone in between the canals becomes a stagnant zone.

9. If the canals are located at different levels, the water table mound under the canal,which is at lower elevation, disappears with passage of time due to rapid reduction of seepage from the canal at lower elevation. Therefore, one ridge is established ultimately under the canal which is at higher elevation.

10. If two unequal parallel canals have their beds at the same level, the width of one canal is twice that of the other, and they run

continuously, the water table in between the canals takes the shape of a flat plane at large time dipping towards the canal of smaller strength.

INTERFERENCE OF SEEPAGE FROM TWO PARALLEL CANALS ONE OF WHICH HAS HYDRAULIC CONNECTION WITH THE AQUIFER

6.0 Introduction

The evolution of water table due to recharge from parallel canals, when water table is located at large depth, has been analysed in Chapter 3. In such a situation the canals are not hydraulically connected with the aquifer and, therefore, the seepage losses from the parallel canals are invariant and independent of location of the water table below the bed of canals. A two canal - aquifer interaction problem, when water table is located at shallow depth, such that, the canals are hydraulically connected with the aquifer, has been analysed in Chapter 5. In the field, a situation may arise in which a canal is constructed at a high elevation on a ridge as a contour canal to supply water for irrigation in the local command area or to a power house. There may also be a canal in the valley at lower elevation, running parallel to the ridge canal, and supplying water for irrigation in the valley. A study of interference of seepage from a parallel canal system has been made in this chapter in which, one canal situated in the valley is hydraulically connected with the aquifer and the other canal, on the high ridge, has no hydraulic connection with the aquifer.

6.1 Statement of the problem

Two parallel canals have been constructed in a homogeneous and isotropic porous medium of finite depth, and infinite lateral extent. The dimensions of the canals are as shown in fig. (6.1). One of the canals is situated on a high ridge and the other is at a much lower elevation. On account of large difference in the elevations of bed level of the ridge canal and the water table underneath, the ridge canal is hydraulically unconnected with the aquifer. Therefore, the seepage occurs at constant rate from the ridge canal. The bed of the lower canal is near to the ground water table and is hydraulically connected to the aquifer. Its seepage rate is controlled by the potential difference between the canal and the aquifer below the bed of the canal. The permeability of the aquifer material is K. The initial saturated thickness of the aquifer is $\bar{H}$ and the storage coefficient is φ . D_i is the depth to impervious base measured from the high datum. The initial potential difference between the valley canal and the aquifer is H. It is required to determine the seepage from the lower canal and the temporal and spatial variation of water table rise.

6.2 Analysis

The following assumptions have been made in the analysis:

(i) The time in which the seeping water from the ridge canal reaches the water table has been neglected.

(ii) The hydraulic properties of the aquifer remain constant with respect to time and space.

(iii) The flow due to seepage from ridge canal is vertically downwards until it reaches the water table.

(iv) The Dupuit's assumptions are valid.

(v) The time span is discretised by uniform time-step. Within each time-step the seepage from the lower canal is constant, but it varies from step to step.

The seepage from unit length of the hydraulically connected canal during time-step n is given by

$$Q_r\,(n) = -\,\Gamma_r\,\left[\sigma_r\,(n) - S_r\,(o,n)\right] \qquad \ldots(6.1)$$

where, $\sigma_r\,(n)$ and $S_r\,(o,n)$ are the depth to water surface in the canal and the depth to water table below the canal bed respectively measured from the same high

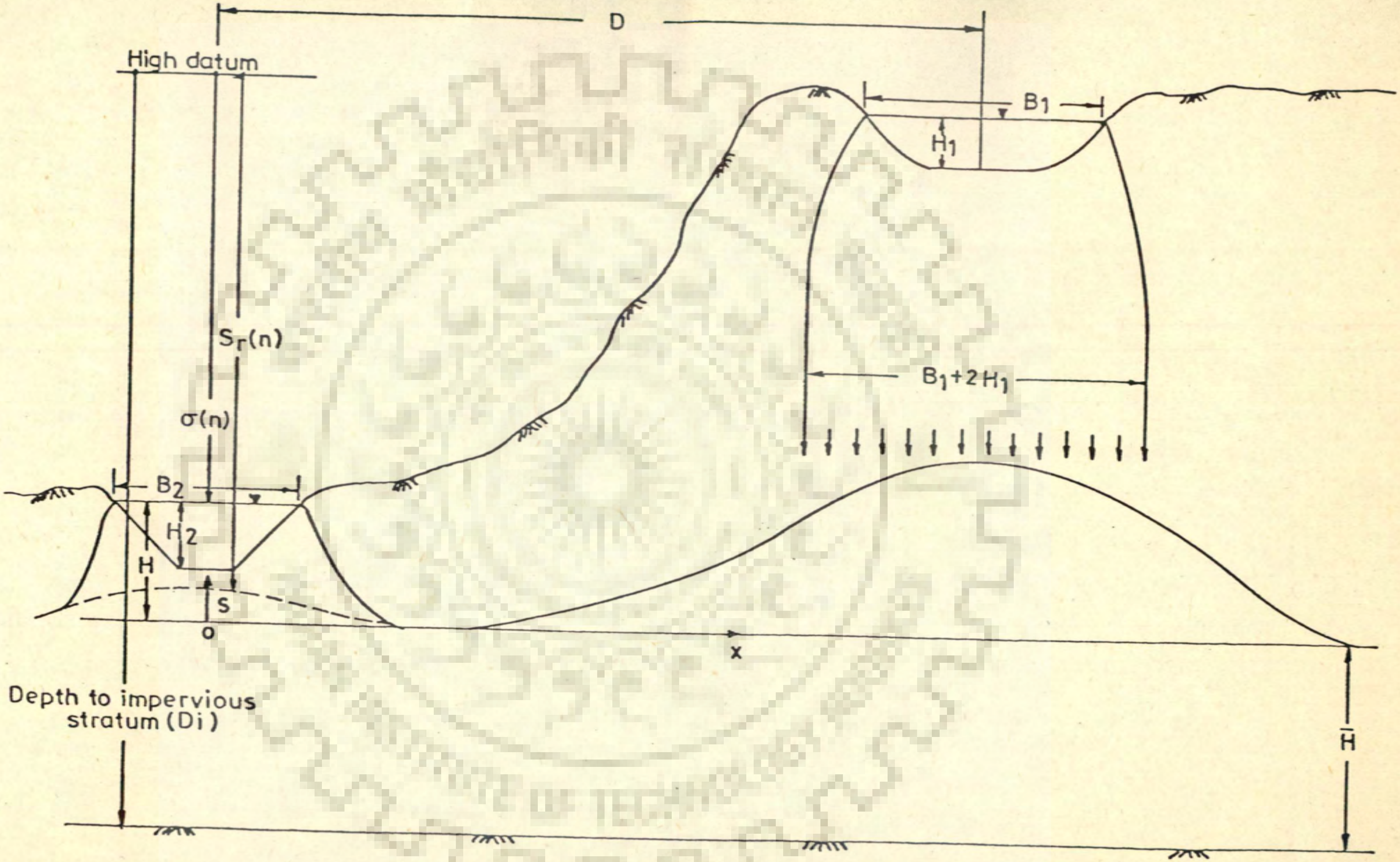

Fig.6.1-Schematic section of two parallel canals

datum during time-step n, and Γ_r is the reach transmissivity for unit length of the canal reach.

The depth to water table below the lower canal bed, S_r (o,n), consists of two parts, S_1 (o,n) and S_2 (o,n), where, S_1(o,n) is the rise on account of seepage from the ridge canal and S_2 (o,n) is the rise due to its own seepage.

The rise of water table under the lower canal due to seepage from the ridge canal at the end of time-step 'n' is given by,

$$S_1(o,n) = F(-D,B_1, H_1,n) - K\surd(-D)^2(B_1+2H_1)/(2T) \quad \ldots(6.2)$$

where, D is the distance between centre to centre of the two canals.

The rise in water table due to seepage from the lower canal which is hydraulically connected with the aquifer is given by:

$$S_2(o,n) = \sum_{\gamma=1}^{n} Q_r(\gamma)\delta\,(o,B_2,n-\gamma+1) \quad \ldots(6.3)$$

The discrete kernel coefficients $\delta(o,B_2,M)$ are given by Equations (4.11) to (4.13). Spliting the temporal summation, appearing in equation (6.3), into two parts,

$$S_2(o,n) = \sum_{\gamma=1}^{n-1} Q_r(\gamma)\,\delta\,(o,B_2,n-\gamma+1) + Q_r(n)\,\delta\,(o,B_2,1) \quad \ldots(6.4)$$

The depth to water table from the high datum below the centre of lower canal at the end of time-step n is given by,

$$S_r(o,n) = D_i - \bar{H} - S_1(o,n) - S_2(o,n) \qquad ...(6.5)$$

Incorporating $S_r(o,n)$ in Equation (6.1),

$$Q_r(n) = -\Gamma_r \left[\sigma_r(n) - D_i + \bar{H} + S_1(o,n) + S_2(o,n) \right] \qquad ...(6.6)$$

Substituting $S_1(o,n)$ and $S_2(o,n)$ from Equations (6.2) and (6.4) respectively in Equation (6.6), and rearranging,

$$Q_r(n) = \left[\frac{1}{\Gamma_r} + \delta(o,B_2,1)\right]^{-1} .\Big[D_i - \bar{H} - F(-D,B_1, H_1,n) + K\surd(-D)^2(B_1+2H_1)/(2T) - \sum_{\gamma=1}^{n-1} Q_r(\gamma)\,\delta(o,B_2,n-\gamma+1) - \sigma_r(n)\Big] \qquad ...(6.7)$$

The seepage rate during any time-step n can be found in succession starting from time - step 1, by using Equation (6.7).

In particular, for the time-step 1, the seepage rate is given by:

$$Q_1(n) = \frac{D_i - \bar{H} - F(-D,B_1, H_1,1) + 0.5K\surd(-D)^2(B_1+2H_1)/T - \sigma_r(1)}{1/\Gamma_r + \delta(o,B_2,1)} \qquad ...(6.8)$$

After solving for $Q_r(\gamma)$, $\gamma = 1,2,\ldots n$, the water table rise at any distance x from the centre of the hydraulically connected canal, at the end of time-step n, can be predicted using the formula:

$$S(x,n) = F[-(D-x),\ B_1,\ H_1,n] - K\sqrt{[(D-x)^2]}(B_1+2H_1)/(2T)$$

$$+ \Sigma\ Q_r(\gamma)\ \delta\ (x, B_2, n - \gamma + 1), \qquad \ldots(6.9)$$

$$\text{for } |x-D| \leq (B_1+2H_1)/2$$

$$= F[-(D-x), B_1,\ \ H_1, n]$$

$$- \frac{[(x-D)^2+0.25(B_1+2H_1)^2]K}{2\ T} \qquad \ldots(6.10)$$

$$\text{for } |x-D| \geq (B_1+2H_1)/2$$

The discrete kernel coefficients are given by Equations (4.11),(4.12) and (4.13). The function $F[-(D-x), B_1, H_1, n]$ can be evaluated using the relationship given at Equation (3.8).

6.3 Results and discussions

The valley canal being hydraulically connected with the aquifer, its seepage loss would vary with time. The seepage loss from the valley canal is governed by:

(i) The initial potential difference between the canal and the aquifer under the canal.

(ii) The spacing between the valley canal and the ridge canal.

(iii) Dimensions of the canals.

(iv) Hydraulic conductivity.

(v) Initial saturated thickness of the aquifer.

Out of the above factors, the influence of spacing between the canals and widths of the canals on seepage loss has been analysed. The variations of dimensionless seepage, $Q_2(t)/KH_2$, from the valley canal with non-dimensional time have been presented in Figs. (6.2) through (6.5), with $H/\overline{H}=0.006$, for four sets of canal widths and for various spacings between them. The results presented in Fig. (6.2) are for $B_1/\overline{H} = B_2/\overline{H} = 0.03$. It could be seen from the figure that for $D/\overline{H} = 0.08$, the seepage from the valley canal reduces to zero at non-dimensional time, $Kt/(2\varphi\overline{H})$, $= 6.1 \times 10^{-2}$. Thereafter, it receives water from the aquifer. The time to reversal of flow increases as the spacing between the canals increases. For example, for $D/\overline{H} = 0.96$, the canal receives water after $Kt/(2\varphi\overline{H}) = 3.55 \times 10^{-1}$.

The temporal variations of $Q_2(t)/(KH_2)$, for $D/\overline{H} = \infty$ have also been presented in Figs. (6.2) through (6.5). At any given time, the difference in the variation of $Q_2(t)/(KH_2)$ corresponding to any specific distance between the ridge and the valley canal, from that of the

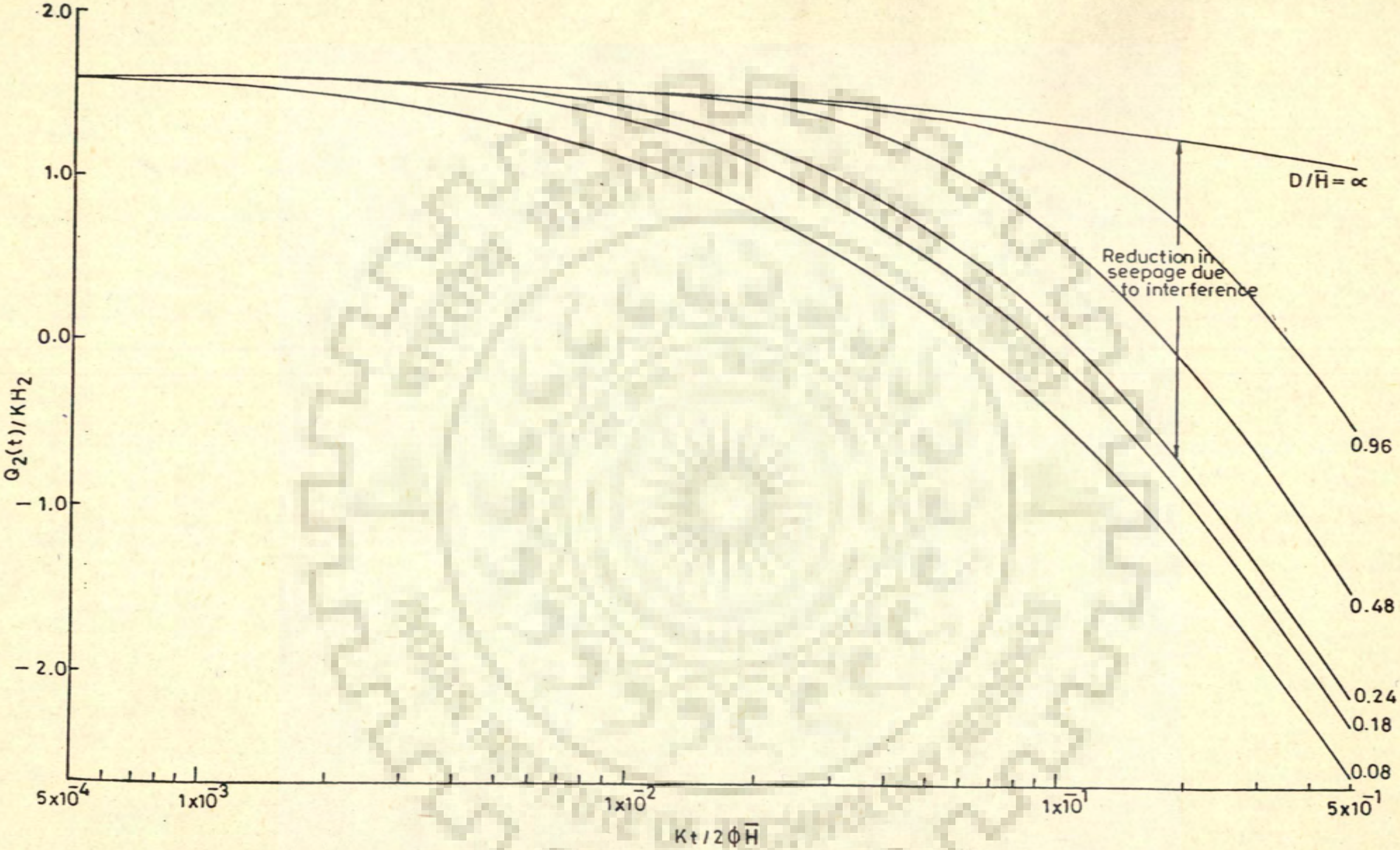

Fig. 6.2—Variation of seepage with time for the canal hydraulically connected with the aquifer, evaluated for $B_1/\bar{H} = B_2/\bar{H} = 0.03$, $H_1/\bar{H} = H_2/\bar{H} = 0.003$ for different values of $D/\bar{H}$

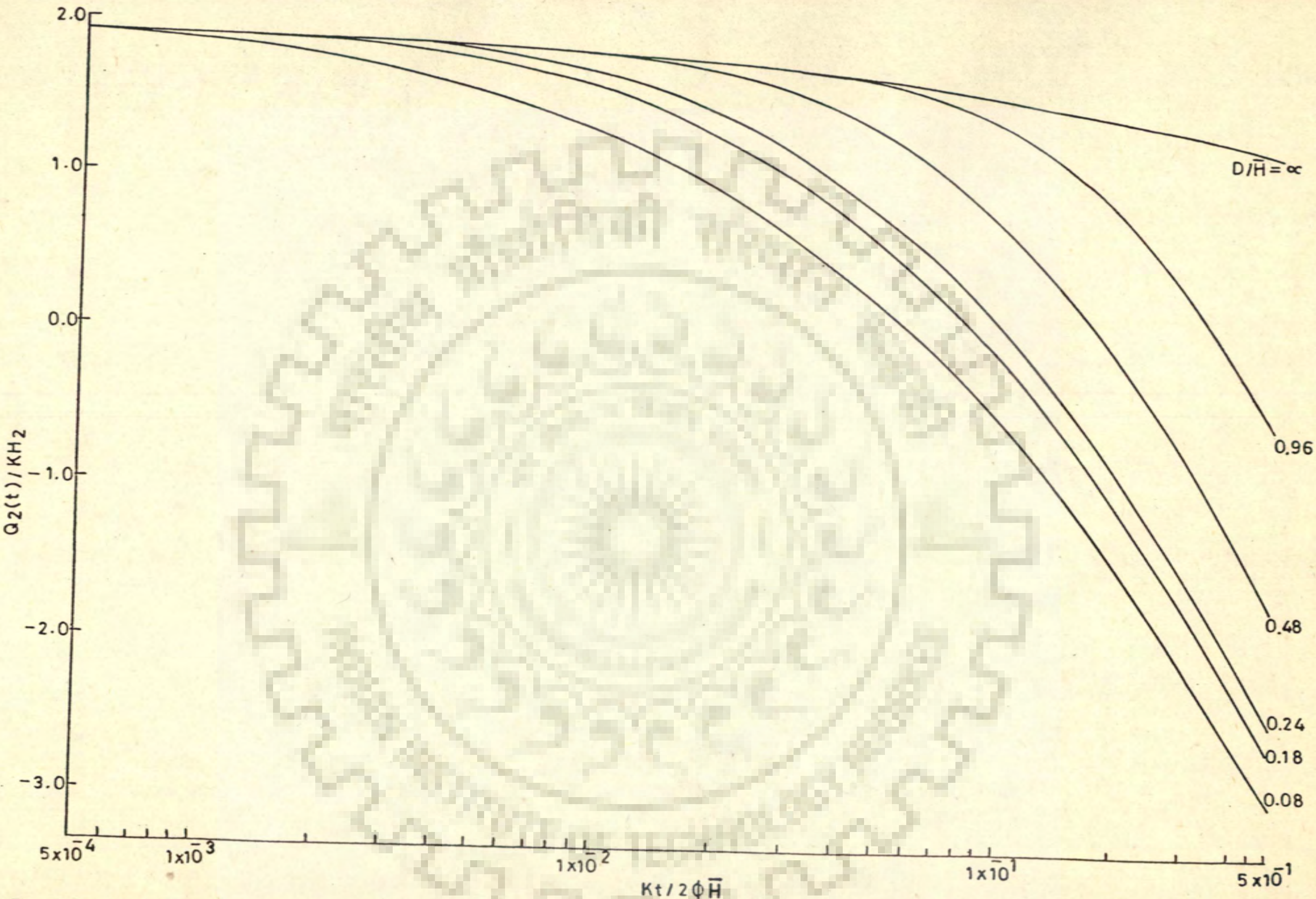

Fig.6.3 – Variation of seepage with time for the canal hydraulically connected with the aquifer, evaluated for $B_1/\bar{H}=0.03$, $B_2/\bar{H}=0.06$, $H_1/\bar{H}=H_2/\bar{H}=0.003$ for different values of $D/\bar{H}$

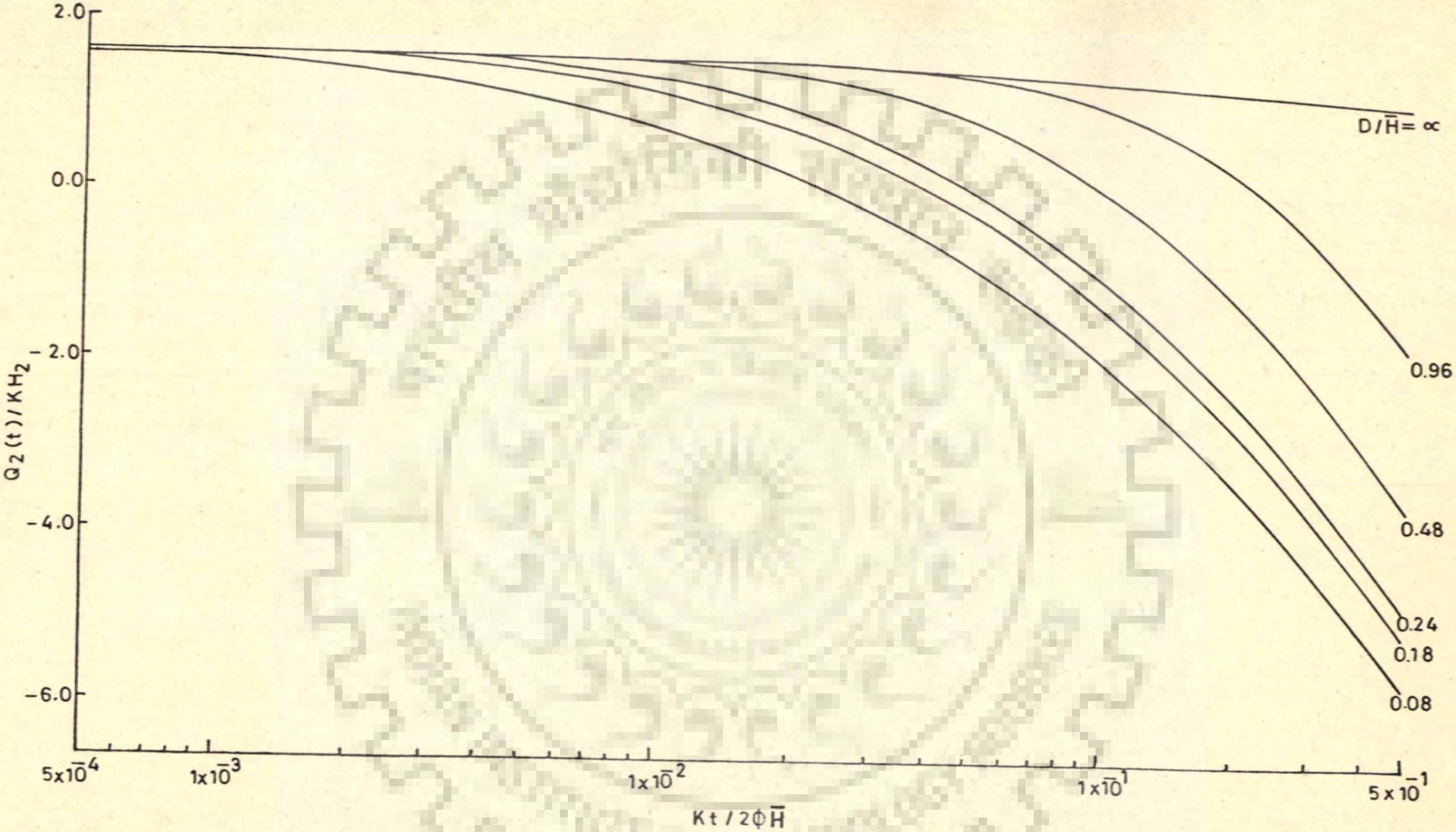

Fig.6.4 – Variation of seepage with time for the canal hydraulically connected with the aquifer, evaluated for $B_1/\bar{H}=0.06$, $B_2/\bar{H}=0.03$, $H_1/\bar{H}=H_2/\bar{H}=0.003$ for different values of $D/\bar{H}$

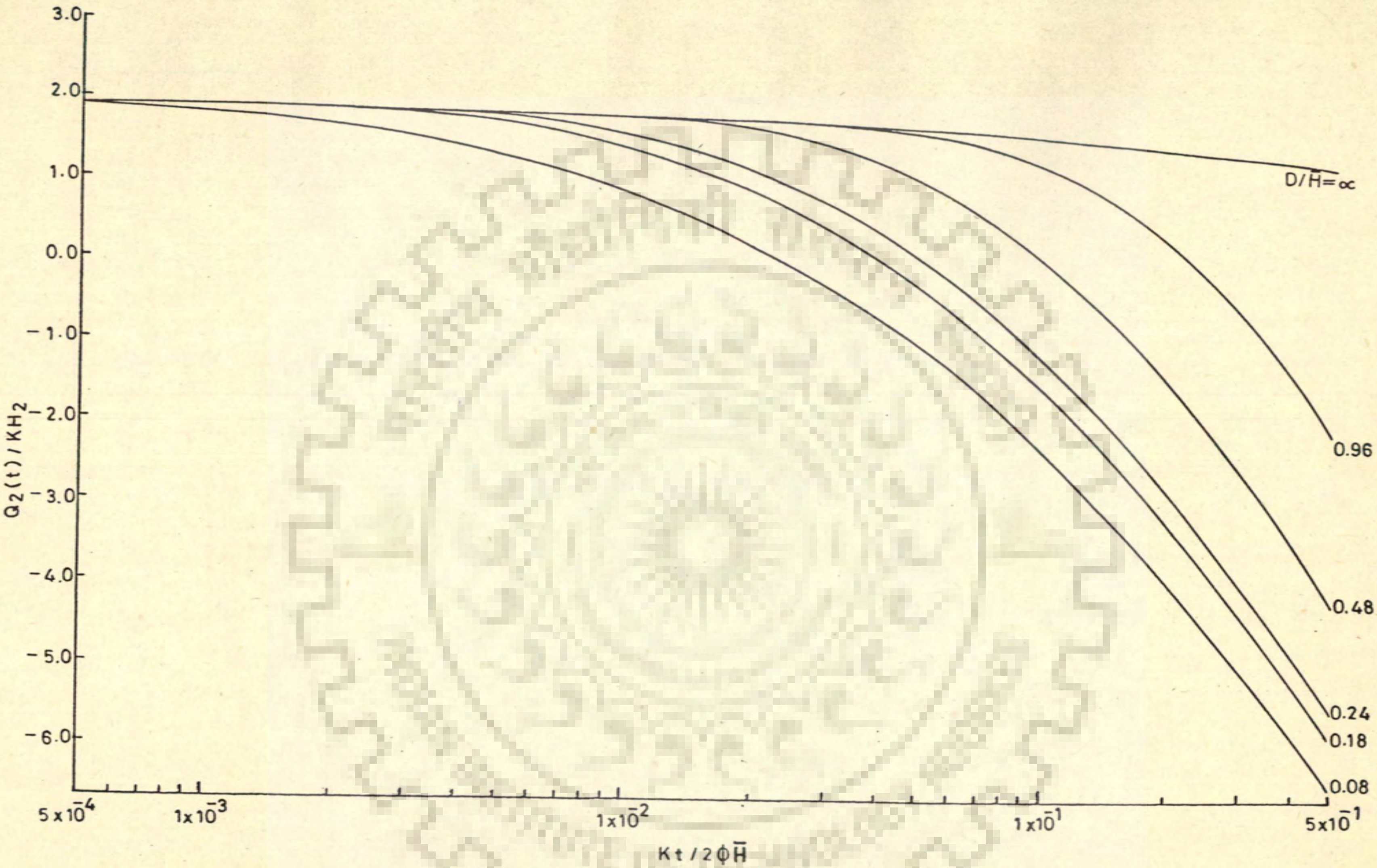

Fig.6.5 – Variation of seepage with time for the canal hydraulically connected with the aquifer, evaluated for $B_1/\bar{H}=B_2/\bar{H}=0.06$, $H_1/\bar{H}=H_2/\bar{H}=0.003$ for different values of $D/\bar{H}$

variation for $D/\overline{H} = \infty$, is the interference of the ridge canal on the valley canal. The reduction in non-dimensional seepage due to interference has been indicated in the Figures. It is seen from Fig. (6.2) that the interference increases with lapse of time. For $B_1/\overline{H} = B_2/\overline{H} = 0.03$ and $D/\overline{H} = 0.24$, at $Kt/(2\varphi\overline{H}) = 2 \times 10^{-2}$, the reduction in non-dimensional seepage due to interference is equal to 0.2857 whereas, at $Kt/(2\varphi\overline{H}) = 2 \times 10^{-1}$ the corresponding reduction in non-dimensional seepage due to interference is 1.9985. At any given time, the interference is less for larger spacing between the canals. For example, for $D/\overline{H} = 0.24$, at $Kt/(2\varphi\overline{H}) = 1 \times 10^{-1}$, the reduction in non-dimensional seepage loss due to interference is 1.2742. For $D/\overline{H} = 0.96$, the corresponding reduction is 0.1566.

The variation of $Q_2(t)/(KH_2)$ with $Kt/(2\varphi\overline{H})$ for $B_1/\overline{H} = 0.03$ and $B_2/\overline{H} = 0.06$ is shown in Fig.(6.3) for different values of $D/\overline{H}$. A comparison of the results shown in Figs.(6.2) and (6.3) indicates that for any given spacings the reversal of flow takes place in both the cases nearly at the same non-dimensional time, though the width of valley canal in one case is twice that of the valley canal in the other case. For example, as seen from Fig.(6.2), for $B_2/\overline{H} = 0.03$, $B_1/\overline{H} = 0.03$, and $D/\overline{H} = 0.48$, the reversal of flow takes place in the non-dimensional time interval from 1.80×10^{-1} to 1.85×10^{-1}. It could be seen from Fig.(6.3) that for $B_2/\overline{H} = 0.06$ and $B_1/\overline{H} = 0.03$, for the above spacing, the reversal of flow takes place in the non-dimensional time interval from 1.75×10^{-1} to 1.80×10^{-1}.

The influence of a ridge canal of larger width on seepage losses from valley canal has been presented in Figs.(6.4) and (6.5). It could be seen from Fig.(6.4) that for $B_2/\overline{H} = 0.03$, $B_1/\overline{H} = 0.06$, and $D/\overline{H} = 0.48$, the reversal of flow takes place in the non-dimensional time interval between 1.0×10^{-1} to 1.05×10^{-1}. For $B_2/\overline{H} = 0.06$, $B_1/\overline{H} = 0.06$ and $D/\overline{H} = 0.48$, the reversal of flow also takes place between the same non-dimensional time interval. Thus, the reversal of flow is not significantly governed by the dimension of the canal which is hydraulically connected with the aquifer. It is also seen that for same width of the valley canal, if the width of the ridge canal is more, the reversal of flow occurs at earlier time.

A typical set of water table positions have been presented in Fig. (6.6) for $B_1/\overline{H} = B_2/\overline{H} = 0.03$, $H_1/\overline{H} = H_2/\overline{H} = 0.003$, $H/\overline{H} = 0.006$, and $D/\overline{H} = 0.24$ at various non-dimensional time. The part of the phreatic line near the valley canal, shown by dotted lines, can not be predicted with sufficient accuracy by the present methodology as the

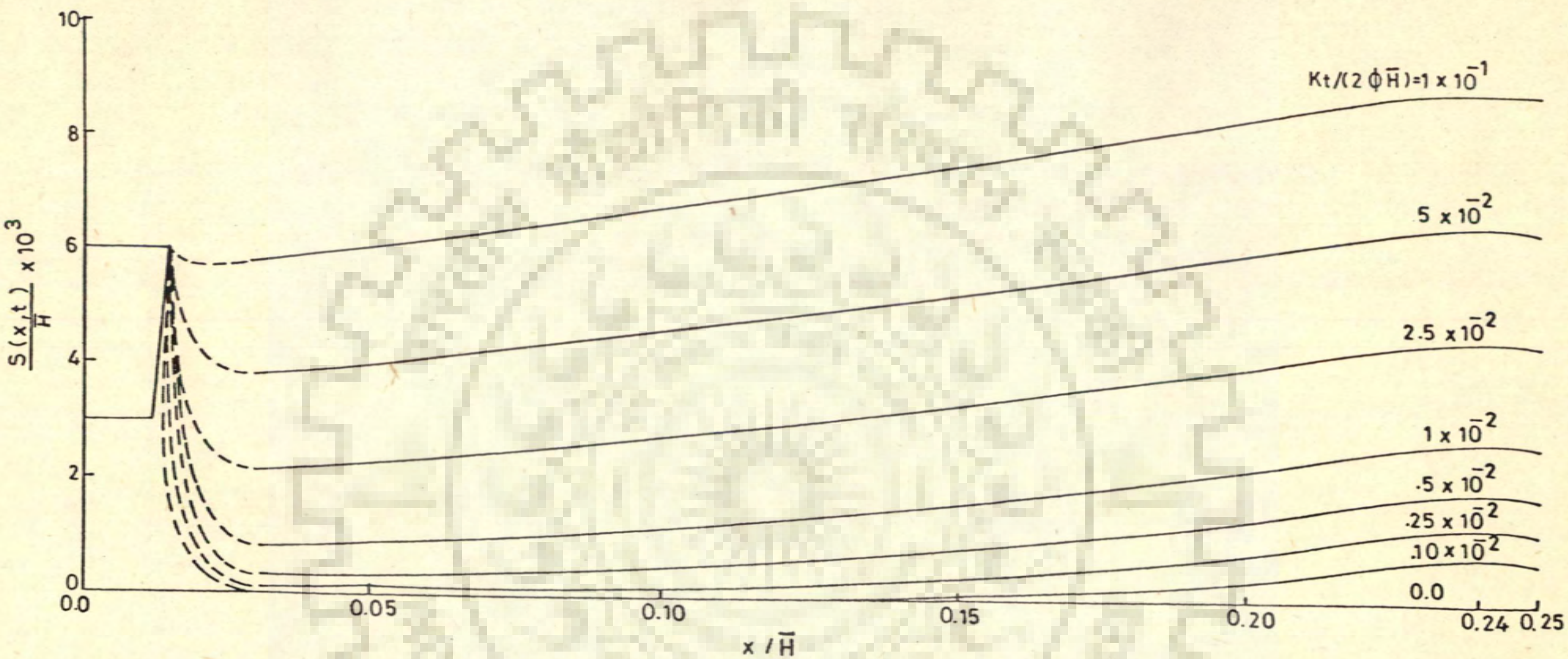

Fig.6.6 — Water table positions at different locations, evaluated for $B_1/\bar{H} = B_2/\bar{H} = 0.03$, $H_1/\bar{H} = H_2/\bar{H} = 0.003$ and $D/\bar{H} = 0.024$

flow in this region does not obey the Dupuit-Forchheimer assumptions. The water table evolution upto $Kt/(2\varphi\bar{H}) = 5 \times 10^{-2}$ corresponds to a period in which the valley canal looses water. The water table position presented for $Kt/(2\varphi\bar{H}) = 1 \times 10^{-1}$ corresponds to the time just prior to the occurrence of reversal of flow.

6.4 Conclusions

From the study of interference of ridge and valley canal, which run continuously, the following conclusions have been drawn:

1. The interference of a continuously running ridge canal increases with time for any given spacing between the canals.
2. The interference of a ridge canal at any time decreases if the spacing between the canals increases.
3. When the water table is at shallow depth, the reversal of flow in the valley canal is controlled by the dimension of the ridge canal and its distance from the valley canal.

EFFECT OF DRAINAGE CHANNEL ON EVOLUTION OF WATER TABLE DUE TO RECHARGE FROM TWO PARALLEL CANALS

7.0 Introduction

The irrigation canals are constructed in a basin where a natural drainage system generally exists. A part of the recharge due to seepage from the canals will be taken away from the basin through the natural drainage system. The evolution of water table due to seepage from canals, which are hydraulically connected or hydraulically unconnected with the aquifer, have been dealt with seperately in previous chapters in which the existence of drainage system was not considered. If a drainage channel exists in the vicinity of the canals, the water table will come up near the bed level of the drainage channel in course of time due to seepage from the canals. The drainage channel activates only when the water table reaches its bed. The return flow to the drainage channel will be governed by the difference in potentials at the periphery of the drainage channel and in the aquifer below its bed. The time of activation of drain and return flow to it are governed by:

i) the dimensions of the parallel canals,

ii) the spacing between the canals,

iii) distance of the drain from the centre of the nearest canal,

iv) hydraulic conductivity, storage coefficient, and

v) initial saturated thickness of the aquifer.

In this chapter, the evolution of water table in the presence of a drain running parallel to a canal system, and the performance of the parallel drain have been studied. The study has been made for parallel canals which are hydraulically unconnected with the aquifer. Such a situation may arise in case of ridge canals. The rate at which the water will be drained by the drainage channel is controlled by width of drain, depth of water in it, and its bed slope. In the present study, the width of the drain has been taken into consideration. The depth of water in the drain, after it receives water, has been assumed to be nominal and its variation has not been considered in the present analysis. The boundary condition that has been imposed is that the drain after receiving water acts as a constant head boundary.

7.1 Statement of the Problem

Two parallel canals have been constructed on high ridges in a homogeneous and isotropic porous medium of infinite areal extent. There exists a drainage channel in the vicinity of the canals. The dimensions and locations of the canals and the drainage channel are as shown in Fig. (7.1). On account of large difference in the elevation of bed level of the ridge canals and the water table underneath, the ridge canals are not hydraulically connected with the aquifer. Therefore, the seepage occurs at constant rate from the ridge canals. The drainage channel situated at a lower level gets activated when the piezometric surface rises above its bed level. Thereafter, it gets hydraulically conneccted to the aquifer. The rate of return flow to the drainage channel is controlled by the potential difference between the channel and the aquifer below the bed of the channel. The permeability of the aquifer is K. The initial saturated thickness of the aquifer is $\overline{H}$ and the storage coefficient is φ. It is required to determine the time at which seepage from the ridge canals enters the drainage channel, the temporal and spatial variation of water table rise and the rate of return flow.

7.2 Analysis

The assumptions made to carry out the analysis are as follows:

i) The hydraulic properties of the aquifer remain constant with respect to time and space.

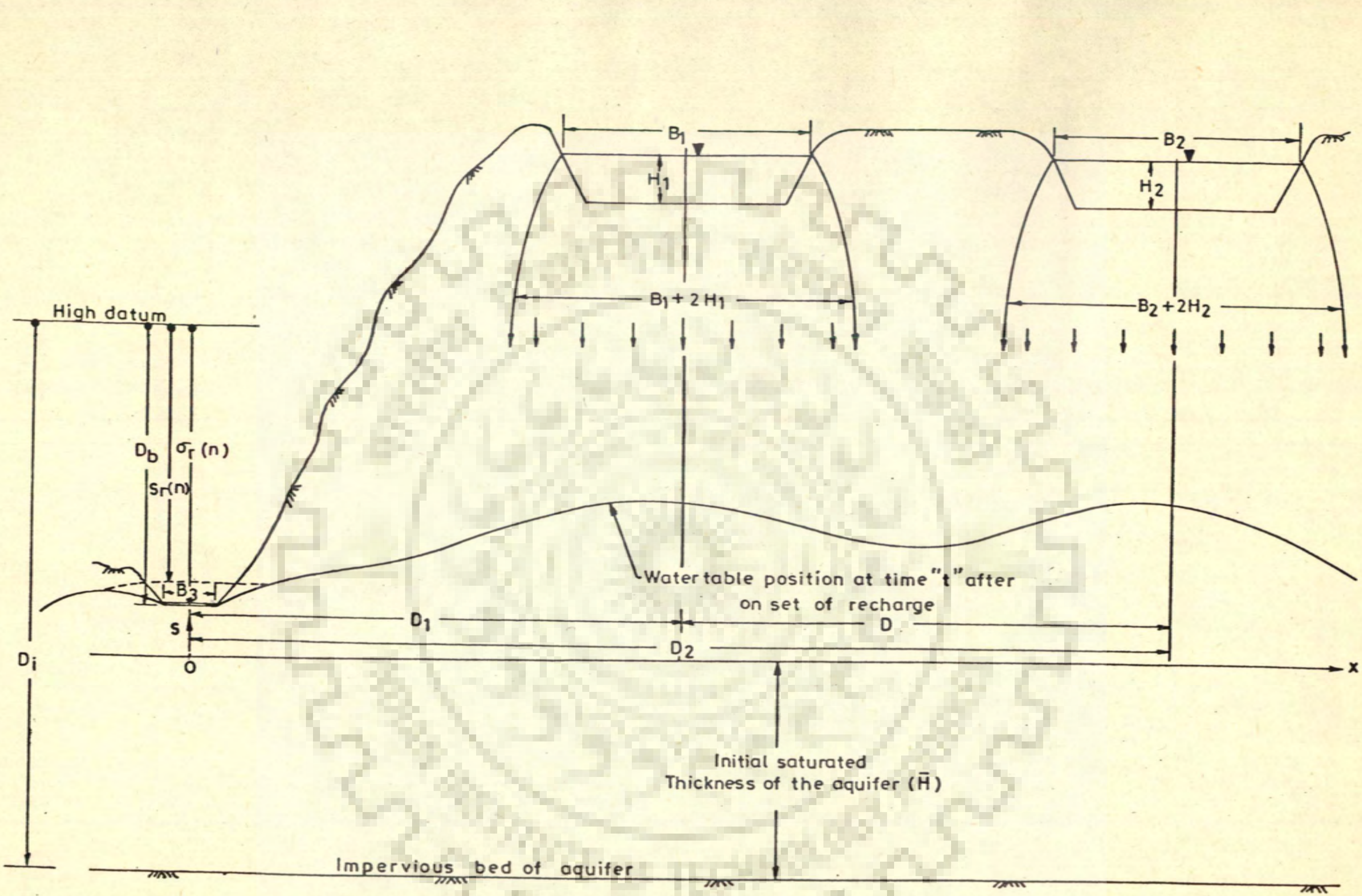

Fig.7.1– Schematic section of two parallel ridge canals and a drainage channel

ii) The flow due to seepage from ridge canals is vertically downwards until it reaches the water table.

iii) Dupuit's assumptions are valid.

iv) The time span is discretised by uniform time-step. Within each time-step the return flow to the drainage channel is constant, but it varies from step to step.

Let the return flow entering to the drainage channel from unit length during time-step n be Q_r (n). According to the linear relationship the return flow is given by:

$$Q_r(n) = \Gamma_r\,[\,\sigma_r(n) - S_r(o,n)] \qquad \ldots(7.1)$$

where $\sigma_r(n)$ is the drawdown to the water level in the drainage channel measured from a high datum during n^{th} time- step and $S_r(o,n)$ is the drawdown in the aquifer under the channel measured from the same datum during n^{th} time-step. The reach transmissivity coefficient, Γ_r, for the drainage channel is given by the Herbert's formula. Γ_r is the reach transmissivity for unit length of the drainage channel.

Let the longitudinal bed slope of the drainage channel be so steep that water in the drainage channel is disposed of quickly and a small depth of water is maintained in the channel. Under such situation, $\sigma_r(n)$

can be assumed to be equal to the depth to channel bed from the high datum (D_b).

The depth to water table below the channel bed, $S_r(o,n)$, comprises two parts, $S_1(o,n)$ and $S_2(o,n)$, where $S_1(o,n)$ is the rise on account of seepage from ridge canals and $S_2(o,n)$ is the decrease in height due to irrigation return flow.

The component of the water table rise at any time due to seepage from the ridge canals can be obtained using Equation (3.7). The component of water table rise under the drainage channel due to seepage from the ridge canals at the end of time-step 'n' is given by:

$$S_1(o,n) = F(-D_1,B_1,H_1,n) - K\sqrt{(-D_1)^2 (B_1+2H_1)/2T}$$

$$+F(-D_2,B_2,H_2,n) - K\sqrt{(-D_2)^2 (B_2+2H_2)/2T} \quad \ldots(7.2)$$

where D_1 and D_2 are the distances from the centre of the ridge canals to the centre of the drainage channel, D_2 being equal to (D_1 + D). The first two terms on right side of Equation (7.2) represent water table rise due to the first ridge canal and the last two terms represent the rise due to the second ridge canal.

Let water enter the drainage channel for the first time at the end of time-step, m.

Therefore,

$$D_i - D_b = \bar{H} + F(-D_1, B_1, H_1, m) - K\sqrt{(-D_1)^2}(B_1 + 2H_1)/2T$$

$$+F(-D_2, B_2, H_2, m) - K\sqrt{(-D_2)^2}(B_2 + 2H_2)/2T \quad ..(7.3)$$

The time-step, m, during which water enters the drainage channel,can be known by an iteration from this equation. The lowering in water table at the centre of the drainage channel, at the end of n^{th} unit time- step, S_2 (r,n), due to return flow which have entered the channel upto n^{th} time-step, is given by

$$S_2(o,n) = \sum_{\gamma=m+1}^{n} Q_r(\gamma)\ \partial(o, B_3, n-\gamma+1) \quad ...(7.4)$$

in which,

B_3 = width of the drainage channel.

The discrete kernel coefficients δ (o,B_3 ,M) are defined by Equations (4.11), (4.12) and (4.13). Splitting the temporal summation appearing in Equation (7.4) into two parts,

$$S_2(o,n) = \sum_{\gamma=m+1}^{n-1} Q_r(\gamma)\ \partial(o, B_3, n-\gamma+1)$$

$$+ Q_r(n)\ \partial(o, B_3, 1) \quad ...(7.5)$$

Referring to Fig. (7.1), the expression for depth to water table from the high datum at the centre of the drainage channel can be written as:

$$S_r(o,n) = D_i - \bar{H} - S_1(o,n) + S_2(o,n) \quad ...(7.6)$$

Substituting for $S_r(o,n)$ in Equation (7.1),

$$Q_r(n) = \Gamma_r\ [\sigma_r(n) - D_i + \overline{H} + S_1(o,n) - S_2(o,n)] \quad \ldots(7.7)$$

Substituting $S_1\ (o,n)$ and $S_2(o,n)$ from Equations (7.2) and (7.5)respectively in Equation (7.7) and rearranging,

$$\begin{aligned} Q_r\ (n) = [\frac{1}{\Gamma_r} + \delta(o,B_3,1)]^{-1} \cdot [&\sigma_r(n) - D_i + \overline{H} + F(-D_1,B_1,H_1,n) \\ &- K\sqrt{(-D_1)^2}(B_1 + 2H_1)/2T + F(-D_2,B_2,H_2,n) \\ &- K\sqrt{(-D_2)^2}\,(B_2 + 2H_2)/2T \\ &- \sum_{\gamma=m+1}^{n-1} Q_r(\gamma)\ \delta(o,B_3,n-\gamma+1)] \quad \ldots(7.8) \\ &\text{for } [\ n \geq (m+1)] \end{aligned}$$

In particular for the $(m+1)^{th}$ time-step the return flow rate is given by:

$$\begin{aligned} Q_r(m+1) = [&\sigma_r(m+1) - D_i + \overline{H} + F(-D_1,B_1,H_1,m+1) \\ &- 0.5K\sqrt{(-D_1)^2}\ (B_1 + 2H_1)/T + F(-D_2,B_2,H_2,m+1) \\ &- 0.5K\sqrt{(-D_2)^2}(B_2 + 2H_2)/T][\frac{1}{\Gamma_r} + \delta(o,B_3,1)]^{-1} \end{aligned}$$

$$\ldots(7.9)$$

7.3 Results and Discussions

Results have been presented for a drainage channel which is located to the left of the first canal. The time at which the drain gets activated, the rate at which the return flow enters the drainage channel, and loci of the water table at different time have been presented. The non-dimensional times, $Kt/(2 \varphi \bar{H})$, at which water first enters into the drainage channel, have been predicted for $B_1/\bar{H} = B_2/\bar{H} = 0.03$ and 0.06, $H_1/\bar{H} = H_2/\bar{H} = 0.003$, $B_3/\bar{H} = 0.02$, for various values of $D_1/\bar{H}$ and $D/\bar{H}$, and are shown in Table (7.1). It is seen that with larger spacings between the parallel canals, the time at which the drainage channel activates is delayed. For example, with $B_1/\bar{H} = B_2/\bar{H} = 0.03$ and $D_1/\bar{H} = 0.3$ the non-dimensional time for activation of drain is 0.028 when spacing between the ridge canals, $D/\bar{H}$, is 0.08. For $D/\bar{H} = 2.0$, the corresponding time is 0.042. It is also seen from the table that if the distance of the drain from the first canal, i.e., $D_1/\bar{H}$, is increased, the drain activates at a later time, other dimensions remaining same. For example, if $D/\bar{H} = 0.5$ and $D_1/\bar{H} = 0.3$, the drain activates at non-dimensional time = 0.0395 whereas, it activates at non-dimensional time = 0.0875, if $D_1/\bar{H} = 0.6$. It could also be seen from the table that for larger width of parallel canals, a drain would activate at earlier time, other dimensions remaining same. For example, for $B_1/\bar{H} = B_2/\bar{H} = 0.03$, $D_1/\bar{H} = 0.3$ and $D/\bar{H} = 0.5$, the drain activates at non-dimensional time= 0.0395 whereas, if $B_1/\bar{H} = B_2/\bar{H} = 0.06$, the

Table (7.1) - Non-dimensional time at which the water enters the drainage channel; evaluated for $D_i/\overline{H} = 1.1$, $\sigma(n)/\overline{H} = 0.098$ and $B_3/\overline{H} = 0.02$; the drain lies to the left of the first canal

Width of the canals $B_1/\overline{H}, B_2/\overline{H}$	Depth of water in the canals $H_1/\overline{H}, H_2/\overline{H}$	Spacing between the canals $D/\overline{H}$	Distance of the drain to first canal $D_1/\overline{H}$	Non-dimensional time at which the drain activates $Kt/(2\varphi\overline{H})$
0.030	0.003	0.08	0.30	0.0280
		0.18		0.0320
		0.50		0.0395
		1.00		0.0415
		2.00		0.0420
		0.08	0.60	0.0675
		0.18		0.0735
		0.50		0.0875
		1.00		0.0955
		2.00		0.0965
0.06	0.003	0.08	0.30	0.0190
		0.18		0.0215
		0.50		0.0250
		1.00		0.0255
		2.00		0.0256
		0.08	0.60	0.0490
		0.18		0.0535
		0.50		0.0620
		1.00		0.0650
		2.00		0.0651

corresponding time is 0.025.

The results of return flow to the drainage channel with time after the drain gets activated, are presented in Fig.(7.2) for $B_1/\overline{H} = B_2/\overline{H} = 0.03$, $H_1/\overline{H} = H_2/\overline{H} = 0.003$, $B_3/\overline{H} = 0.02$, and $D_1/\overline{H} = 0.3$ and 0.6, for various values of $D/\overline{H}$. Similar results for canal of larger width with $B_1/\overline{H} = B_2/\overline{H} = 0.06$ are presented in Fig. (7.3). It is seen from the figures that water enters the drain at an increasing rate. For example, for $B_1/\overline{H} = B_2/\overline{H} = 0.03$, $D_1/\overline{H} = 0.3$ and $D/\overline{H} = 0.5$, the non- dimensional flow $[Q_r(n)/(KH_1)]$ entering to the drainage channel per unit length is 1.2553 at non -dimensional time =0.1395. The flow rate to the drainage channel at non-dimensional time = 0.3995 is 3.5337. The predicted return flow to the drainage channel presented in the figures are valid only if the depth of water in the drainage channel is very nominal as has been assumed.

Typical results of evolution of water table with time, due to seepage from parallel ridge canals, in the presence of drainage channel, are depicted in Fig. (7.4) for $B_1/\overline{H} = B_2/\overline{H} = 0.06$, $H_1/\overline{H} = H_2/\overline{H} = 0.003$, $D/\overline{H} = 0.18$, and $D_1/\overline{H} = 0.3$. The water enters the drainage channel at the end of non-dimensional time = 0.0215. The water table evolutions shown in the figure are after the drainage channel has been activated. It could be seen

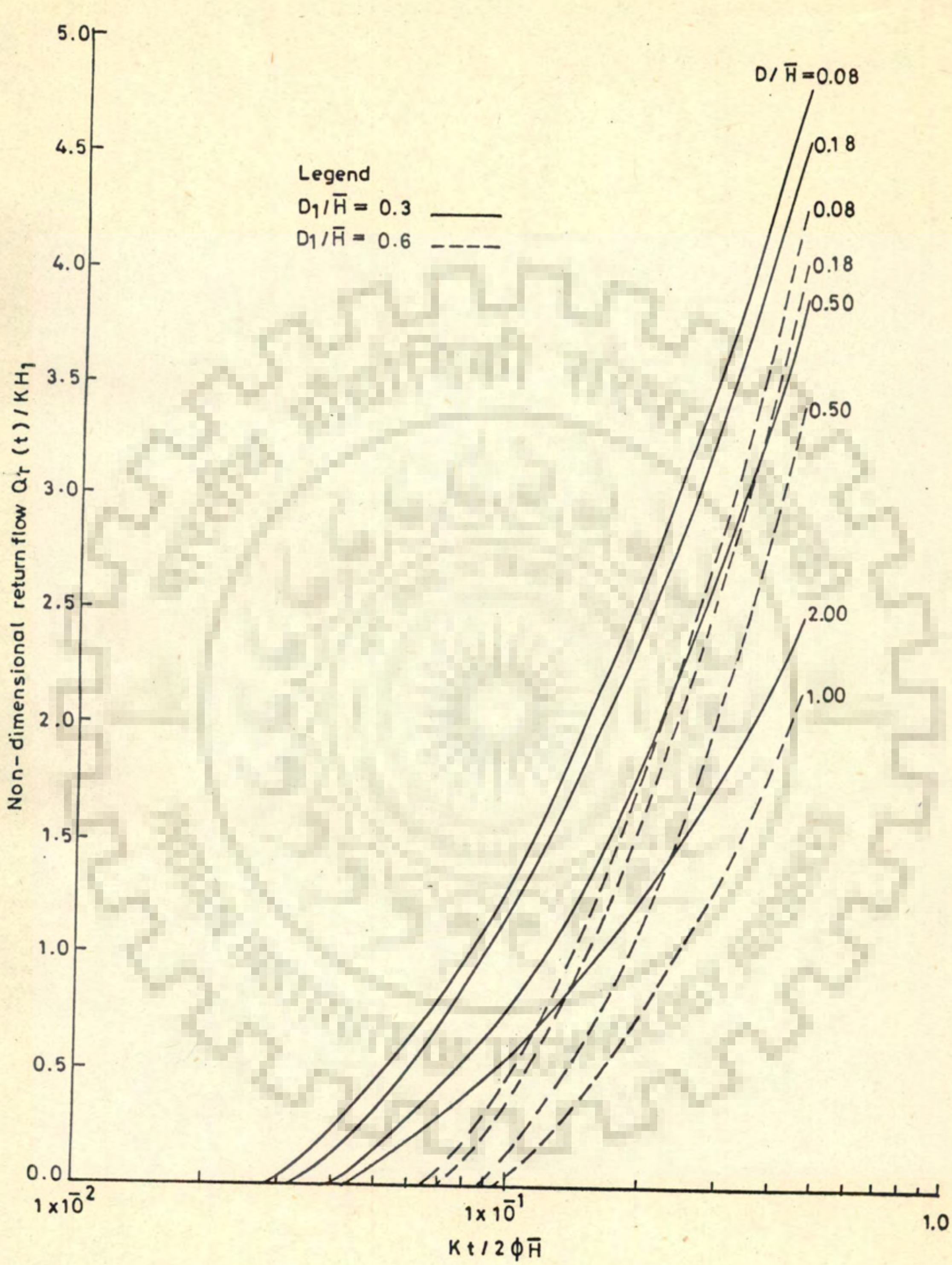

Fig.7.2 — Variation of the return flow to the drainage channel with time, evaluated for $B_1/\bar{H} = B_2/\bar{H} = 0.03$, $H_1/\bar{H} = H_2/\bar{H} = 0.003$, $B_3/\bar{H} = 0.02$, $\sigma(n)/\bar{H} = 0.098$, $D_i/\bar{H} = 1.1$, and $D_1/\bar{H} = 0.3$ and 0.6

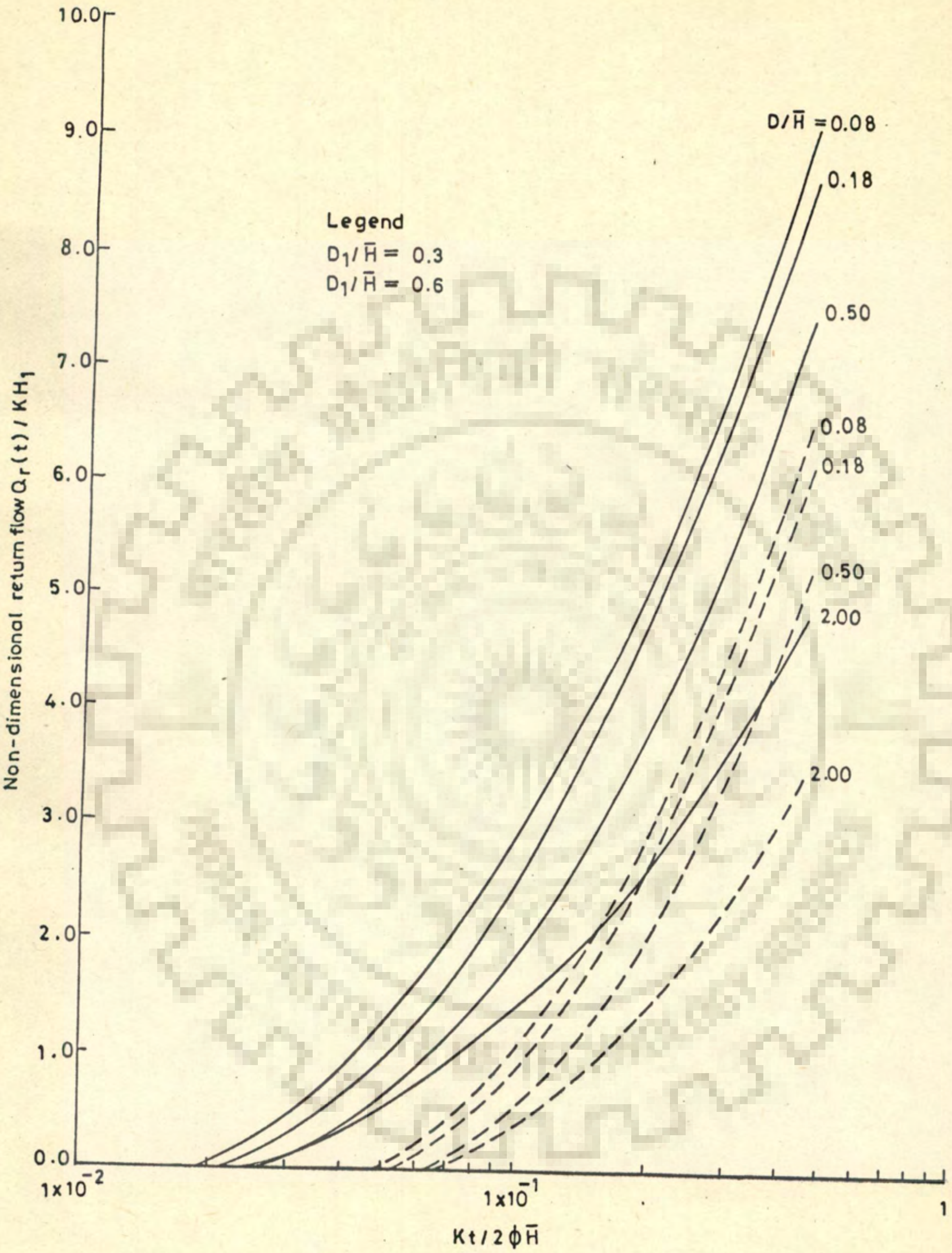

Fig. 7.3 – Variation of the return flow to the drainage channel with time, evaluated for $B_1/\bar{H} = B_2/\bar{H} = 0.06$, $H_1/\bar{H} = H_2/\bar{H} = 0.003$, $B_3/\bar{H} = 0.02$, $\sigma(n)/\bar{H} = 0.098$, $Di/\bar{H} = 1.1$, and $D_1/\bar{H} = 0.3$ and 0.6

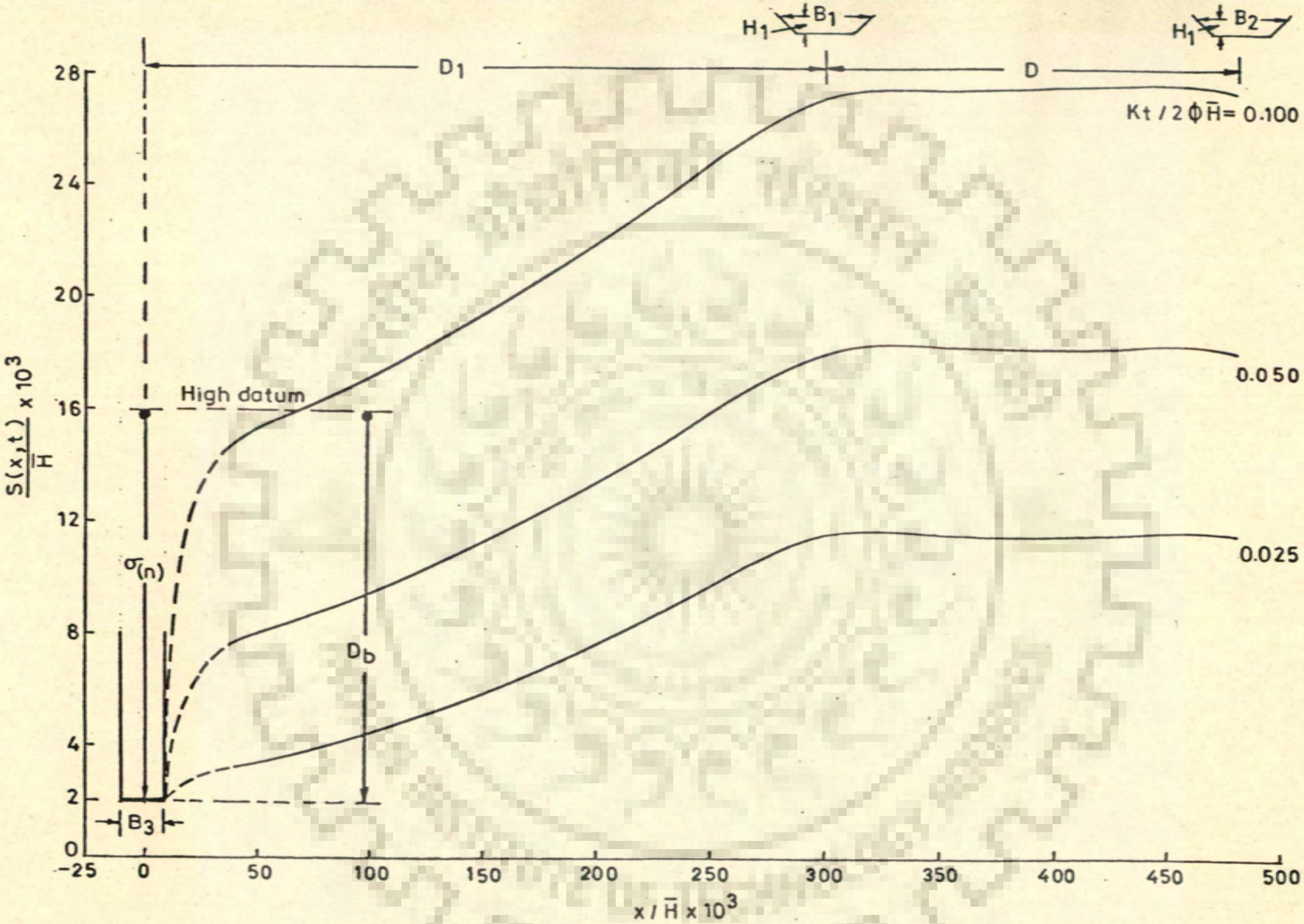

Fig.7.4—Typical water table positions at different times after the drainage channel gets activated evaluated for $B_1/\bar{H} = B_2/\bar{H} = 0.06$, $H_1/\bar{H} = H_2/\bar{H} = 0.003$, $D/\bar{H} = 0.18$, $D_1/\bar{H} = 0.3$, $\sigma(n)/\bar{H} = 0.098$, and $D_i/\bar{H} = 1.1$

that at non-dimensional time = 0.050, the water table rise, $s(x,t)/\overline{H}$, under the first canal, which is nearer to the drain, is 0.01798. Had there been no drainage channel nearby, the non-dimensional water table rise under this canal would have been 0.01803. [Table 7.2]. Thus, due to the presence of the drainage channel, the water table evolution under the canal has not been influenced appreciably upto non-dimensional time = 0.050. The results of water table rise at various times below the centre of the first canal, with and without the presence of drainage channel, are shown in Table (7.2). It could be seen that at non-dimensional time = 0.2, the water table rise below the centre of the first canal is 0.03924 when the drainage channel exists nearby. The water table rise without the drain would have been 0.04110.

7.4 Conclusions

The following conclusions have been drawn from the study presented above:

i) A drain activates at earlier time, for canals, having larger width, other parameters remaining the same.

ii) If the spacing between the parallel ridge canals is increased, the time for activation of drain is

Table 7.2 - Rise of water table below the centre of first canal with and without drainage channel nearby, evaluated for $B_1/\overline{H} = B_2/\overline{H} = 0.06$, $B_3/\overline{H} = 0.02$, $H_1/\overline{H} = H_2/\overline{H} = 0.003$, $D/\overline{H} = 0.18$, $D_1/\overline{H} = 0.30$, $\sigma(n)/\overline{H} = 0.098$, and $D_i/\overline{H} = 1.1$

Non-dimensional time $Kt/(2\varphi\overline{H})$	Water table rise $[S(x,t)/\overline{H}]$ below the centre of first canal	
	In the absence of drainage channel	In the presence of drainage channel
0.005	0.00368	0.00368
0.010	0.00609	0.00609
0.015	0.00812	0.00812
0.020	0.00990	0.00990
0.025	0.01151	0.01151
0.030	0.01299	0.01299
0.035	0.01436	0.01436
0.040	0.01565	0.01564
0.045	0.01687	0.01684
0.050	0.01803	0.01798
0.100	0.02751	0.02707
0.150	0.03481	0.03379
0.200	0.04110	0.03924
0.250	0.04661	0.04387
0.300	0.05160	0.04791
0.350	0.05619	0.05151
0.400	0.06047	0.05476
0.450	0.06449	0.05772

delayed.

iii) A drain activates at earlier time if its distance from the parallel canals is decreased.

iv) With the presence of the drainage channel, the maximum water table height under the left canal is marginally affected upto certain initial period.

CHAPTER 8

GENERAL CONCLUSIONS

In the present study, the interference of seepage from two parallel canals has been analysed both for deep as well as for shallow water table position below the bed of canals. The study of interference of canals for deep water table position, in which case the canals are not hydraulically connected with the aquifer, has been carried out using the Glover's basic solution of the one dimensional Boussinesq's equation for a line source and method of superposition. When water table is located at shallow depth below the bed of a canal, and the canal is hydraulically connected with the aquifer, the time variant seepage loss may be linearly or non-linearly dependent on the potential difference between the canal and the aquifer. A solution has been obtained to find seepage from a canal in which the seepage loss is non-linearly dependent on the potential difference between the canal and the aquifer, and the results have been compared with the solution which is based on linear relationship. Making use of a linear relation-ship between seepage loss and the potential difference, an analysis has been carried out using the reach transmissivity constant and discrete kernel coefficients to evaluate the time variant seepage, the water table rise at different time after the onset of recharge, and the interference of two parallel canals. Further, an anlysis of interference of seepage from two parallel canals, one of which is situated on a high ridge and the other in a valley has been carried out assuming the ridge canal to be hydraulically unconnected and the valley canal to be hydraulically connected with the aquifer. A study of return flow to a drainage channel due to seepage from two parallel canals, which are not hydraulically connected with the aquifer, has also been carried out.

The following general conclusions have been drawn from the above studies:

1. From the study of seepage from a canal, when the water table is at large depth below the canal bed, it is found that if the canal runs continuously for a long time, the absolute value of water table gradient at any point outside the recharging strip attains a limiting value equal to 0.5(B/E+2H/E) in which, B is width of canal at water surface, H is the maximum depth of water, and E is the saturated thickness of the aquifer. At any point within the recharging strip, the limiting value of the gradient attained after a long time is x/E, where

x is the distance of the observation point from the centre of the canal. The saturated thickness of aquifer can be predicted by observing the limiting value of the gradient of water table in the vicinity of the canal.

2. If a canal, which is not hydraulically connected with the aquifer, runs intermittently, the water table below the canal declines immediately after the canal is closed. In the region beyond the recharging strip,the aquifer exhibits a delayed response to the closure of the canal. There is, however, no reversal of flow any where on account of intermittent running of the canal, and, at any time the height of water table is maximum at the centre of the canal and it decreases as the distance from the centre of the canal increases.

3. If two identical parallel canals, which are not hydraulically connected with the aquifer, run continuously,in the beginning of recharge, the two water table ridges are located at the centres of the recharging strips. However, as time elapses, the points of maximum rise move towards each other but they do not move beyond the respective recharging strip. If the identical parallel canals run continuously, with lapse of time a

stagnant zone gets created between the canals, and the region between the two parallel canals takes the shape of a plateau.

4. For unequal parallel canals it is seen that, some time after the onset of recharge, there is only one point of maximum rise under the canal of larger strength where the slope of water table is zero; the ridge under the canal of smaller strength vanishes.

5. The non-linear relation proposed by Rushton and Redshaw for the stream - aquifer interaction, when water table is at shallow depth, is the only relationship known so far. The parameters appearing in the non-linear relationship proposed by Rushton and Redshaw can be evaluated making use of the condition proposed by Muskat, that the seepage from a canal attains its maximum value when potential difference between the canal and the aquifer is more than 1.5(B+2H), where, B is the width of the canal at water surface and H is the maximum depth of water in the canal.

6. Discrete kernel approach can provide a numerically tractable solution for predicting time variant seepage which is non-linearly dependent on the potential difference between the canal and the aquifer.

7. The Herbert's formula of reach transmissivity is found to be appropriate to be used in a canal aquifer interaction problem. However, this formula is applicable if the wetted perimeter of the canal is less than 1.5 times the thickness of the aquifer below the canal bed.

8. For very shallow water table position below the bed of canal (upto a depth of 5m below the bed of canal), the linear relationship between the seepage from the canal and the potential difference between the canal and the aquifer can be used for predicting seepage losses.

9. Non-linearity between seepage loss and potential difference gets pronounced with increase in width of the canal or with increase in the initial potential difference that initiates flow.

10. From the study of interference of parallel canals, which is based on a linear relationship between the seepage loss and the potential difference between the canals and the aquifer for shallow water table position, it is found that:

(i) The unsteady seepage losses from the canals and the reduction in seepage due to interference are linearly proportional to the initial potential difference that initiates the flow.

(ii) In case of two continuously running parallel canals, the reduction in seepage from one canal due to interference of the other is zero in the beginning of seepage. The interference increases as the time passes and attains a maximum value and then decreases. The decrease is monotonic at large time.

(iii) The maximum reduction in seepage due to interference decreases with increase in the spacing between the canals. Also, the occurrence of maximum interference is delayed for canals having larger spacing.

(iv) The interference of a bigger canal on smaller canal is more than that of the smaller canal on the bigger one.

(v) If two canals of equal dimensions are at different bed levels, the interference of the canal at lower elevation is less than that of the canal at higher elevation.

(vi) If one of the parallel canals runs continuously, and the other intermittently with equal durations of closure and running, it is found that the reduction in seepage from the continuously running canal, due to interference of the intermittently running canal, starts from zero, increases from cycle to cycle , reaches a maximum value, and then decreases. If the intermittently running canal is operated indefinitely, it is found that its interference on the other would fluctuate about a zero mean value.

(vii) If one of the parallel canals runs intermittently, after some cycles of operation the intermittently running canal would act as a drain during its closure period.

(viii) For the parallel canals of equal dimensions, distinct water mounds of equal height are formed under the canals. In the beginning of seepage, the ridges lie under the centre of the canals. With lapse of time, as seepage continues, the points of maximum water table height move towards each other; but they do not cross the width of the respective recharging strips. With passage of time, the zone between the canals becomes a stagnant zone.

11. From the study of interference of ridge and valley canals, which run continuously, it is found that:

(i) The interference of a ridge canal on seepage loss from a valley canal, which is hydraulically connected with the aquifer, increases with time for any spacing between the canals.

(ii) With increase in the spacing between the canals the interference is delayed.

(iii) The reversal of flow to a valley canal is mainly controlled by the dimension of the ridge canal and its distance from the valley canal.

12. From the study of effect of a drainage channel on the evolution of water table due to seepage from two parallel canals situated on high ridge it has been found that:

(i) A drain activates at earlier time for canals having larger width, other parameters remaining the same.

(ii) If the spacing between the parallel ridge canals is increased, the time for activation of drain is delayed.

(iii) A drain activates at earlier time if its distance from the parallel canals is decreased.

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