CASTING DESIGN BY COMPUTER

A DISSERTATION
submitted in partial fulfilment of
the requirements for the award of the degree
of
MASTER OF ENGINEERING
in
METALLURGICAL ENGINEERING
(PHYSICAL METALLURGY)

By
KRISHAN GOPAL GARG

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UNIVERSITY OF ROORKEE
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CERTIFICATE

Certified that the dissertation entitled, 'CASTING DESIGN BY COMPUTER', which is being submitted by Shri Krishan Gopal Garg in partial fulfilment for the award of the Degree of MASTER OF ENGINEERING in Metallurgical Engineering (Physical Metallurgy) of the University of Roorkee, Roorkee (India), is a record of his own work carried out by him under our supervision and guidance from January, 1977 to June, 1977.

The matter embodied in this dissertation has not been submitted for the award of any other degree.

T.V. Rajan
(T.V.Rajan)
Associate Professor
Supervisor

(M.N. Saxena)
Professor and Head,
Supervisor,

Department of Metallurgical Engg.,
University of Roorkee,
Roorkee(India)

Dated July 22, 1977

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K. G. Garg

July 22 ,1977 Krishna Gopal Garg

ABSTRACT

A three dimensional heat transfer model is formulated and a computer programme using 360/44 digital computer to solve heat transfer equations is proposed, so that casting design engineer can analyse critical areas while a proposed design is still on the drafting board. Shrinkage free designs can be rapidly assessed by using this programme to calculate, in advance, solidification pattern in small and large castings.

Computer simulation of solidification was based on explicit finite difference approximation method to the equations describing three dimensional conduction heat-transfer from the casting. The casting cross-section was divided into a number of small control volumes using a cartesian grid system. Simulation output was in the form of grid node temperatures at specified times. Cooling curves of specified cross-sectional locations, and temperature distribution along specified paths were obtained from this output.

Cooling curves of specified locations and temperature distribution along specified path were obtained by monitoring experimentally the output of thermocouples placed at known cross-sectional locations in an aluminium casting in a dry sand mold.

Satisfactory agreement exists between simulated and experimental results.

CONTENTS

LIST OF SYMBOLS

T	Temperature of the element of the body
T_s	Temperature of sand
T_m	Temperature of metal
T_{om}	Initial metal temperature as it enters mold cavity
T_{os}	Initial sand temperature (ambient temp.)
T_I	Initial interface temperature
T_o	Initial temperature of corner node.
T_{st}	Temperature at start of solidification.
T_f	Temperature at finish of solidification.
T'	Temperature of the node after time Δt.
t	Time
Δt	Time increment
K	Thermal conductivity of the body
K_m	Thermal conductivity of metal
K_s	Thermal conductivity of sand
ρ	Density of the body
ρ_m	Density of metal
ρ_s	Density of sand
C_p	Specific heat of the body
C_{pm}	Specific heat of metal
C_{ps}	Specific heat of sand
α	Temperature diffusivity of the body

erf	error function
L_m	Latent heat of metal
V	Volume of the body
ΔX_x, ΔX_y, ΔX_z, ΔX	Nodal grid spacing in the casting/mold
ΔH_f	Heat of fusion of metal-latent heat approximation.

CHAPTER I ——— INTRODUCTION

The solidification of an alloy is affected by the pouring temperature, solidification range of the alloy, thickness of sand mold and its thermal conductivity, density and specific heat of both the melt and the mold. Formerly, analytical[1,2,3], electrical analogue[2,3], 'pour out' or 'bleeding'[2], and temperature measurement techniques[2] were available for the analysis of solidification problems.

'Analytical techniques' attempt to study the thermal aspects of solidification by analyzing exact heat transfer equations describing the phenomenon. However, to solve the equations, it is often necessary to make simplifying assumptions such as: one dimensional heat flow, semi-infinite mold material, constant interface temperature, or thermal properties which do not vary with temperature. As a result the solutions obtained, while helpful, are limited in applications.

'Analogue computers' have been used with advantage to study the solidification of castings. The analogue computer simulates the flow of heat during solidification by appropriate substitution of equivalent electrical characteristics. This is a very accurate method and when the technique is fully developed, it will prove invaluable in elucidating solidification problems.

In 'pour out' or 'bleeding' method, a number of identical molds are cast and then bled at predetermined intervals after casting, the thickness of the solidified metal in the mold being subsequently measured. Its use is limited to alloys with short freezing range.

The 'measurement of temperatures' in the mold and in mold requires minimum of high temperature data. Measurement of temperature at various points at the same time permits the calculation of the temperature gradients existing in both casting and mold as well as allows progress of solidification to be followed. Thus, it is the best and most versatile experimental method of all the methods described above.

The last decade has seen an exponential increase in the use of computers to aid in the solution of engineering problems. One of the most rapidly advancing fields is the use of numerical methods of approximating the solution to mathematical formulations of problems that are too complex to be solved analytically. The solidification of castings is an example of such problems. The purpose of present study is to design, computer simulate and experimentally monitor a casting. This work involves three-dimensional numerical simulation of a stepped casting from the point of view of heat-transfer. Hence, a three-dimensional heat transfer model is developed to study the interrelationship between the thermophysical properties of the casting material and mold material, the solidification rate of the casting, and the temperature distribution developed in the casting during solidification using explicit finite difference approximation method.

Cooling curves of specified locations and temperature distribution along specified path are obtained, by monitoring the output of thermocouples placed at these locations, and by computer output in the form of grid node temperature at specified times for an aluminium casting.

The numerically simulated and experimentally obtained temperature distributions during the solidifcation of aluminium castings have beem compared.

CHAPTER 2 —— LITERATURE SURVEY

2.1 SOLIDIFICATION

Our knowledge about the various aspects of solidification is growing and a considerable number of publications on this subject are available[2,4-10]. From the point of view of foundry-man, the most important feature is the effect of the rate of solidification on the soundness and the mechanical properties of the casting.

For most practical purposes the ideal casting is one which is sound, has a fine structure (grain size and dendritic cell size), and is free from macroscopic segregation. However, the factors that the foundryman has to consider in attempting to accomplish this ideal, such as gating, risering, chilling, section thickness, complexity of shape, etc., cannot be taken independently because they affect more basic variables, such as cooling rate, direction of heat flow, and thermal gradients at various points in the casting. In broad terms, the cooling rate determines the fineness of the structure and the other variables affect the soundness.

2.2 HEAT TRANSFER MODEL

Most of the literature on heat transfer model dealing with the solidification of metal castings is concerned with one-dimensional problems because of the complexity which other cases would introduce. In general, the following methods are available for the solution of the heat transfer problems associated with solidification of metal casting.

2.21 Analytical Method[1,2,3]

Analytical methods attempt to study the thermal aspect of solidification by analysis of exact heat transfer equations describing the freezing process. Since solidification of a metal in a mold is essentially a problem in unsteady heat flow, the fundamental equation which applies is the well known Fourier relationship. The three-dimensional heat conduction equation in a homogeneous, isotropic solid is given by

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \qquad \ldots (2.1)$$

If conduction in only one dimension is considered, the equation reduces to

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2}\right) \qquad \ldots (2.2)$$

where α = the temperature diffusivity of the body

$= \frac{k}{\rho c}$

Solution to equation (2.2) can be found by specifying initial and boundary conditions characteristic of each particular problem.

It may be shown[1,2] that if the plane boundary of a semi-infinite solid body, initially at a uniform temperature T_o is instantaneously raised to a temperature T_1 at time $t = o$, then after time t the temperature T of any point whose perpendicular distance from the boundary is x is given by,

$$T = T_o + (T_1 - T_o)\ \mathrm{erf}_c \frac{x}{2\sqrt{\alpha t}} \qquad \ldots (2.3)$$

where $\mathrm{erf}_c = 1 - \mathrm{erf} \frac{x}{2\sqrt{\alpha t}}$.

$$\therefore \frac{\partial T}{\partial x} = -(T_1 - T_o) \frac{d}{dx} \left(\mathrm{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)\right)$$

$$\text{or} \quad \frac{\partial T}{\partial x} = -(T_1 - T_o) \cdot \frac{1}{\sqrt{\pi \alpha t}} \qquad \ldots (2.4)$$

Thus rate of influx of heat into the body $\frac{\partial Q}{\partial t}$ is given by,

$$\frac{\partial Q}{\partial t} = -KA\left(\frac{\partial T}{\partial x}\right)_{x=o} = \frac{KA(T_1 - T_o)}{\sqrt{\pi \alpha t}}$$

Total amount of heat absorbed by the body:

$$Q = \int_o^t - KA \frac{(T_1 - T_o)}{\sqrt{\pi \alpha t}} \cdot dt$$

$$Q = \frac{2KA(T_1 - T_o)}{\sqrt{\pi \alpha}} \cdot \sqrt{t} \qquad \ldots (2.5)$$

Total amount of heat released from the metal

$$Q = \text{Mass} \cdot \text{specific heat} \cdot \text{Temperature} + \text{Mass} \cdot \text{latent heat}$$

$$= (\rho_m V_m)\ [C_{pm})(T_p - T_1) + L] \qquad \ldots (2.6)$$

From Equations (2.5) and (2.6) we get

$$X = \left[\frac{2k_{sm}(T_i - T_o)}{\rho_s \cdot \sqrt{\pi \alpha_{mold}}\,(L_h + c_{ps}(T_p - T_i))}\right] \cdot \sqrt{t} \quad \ldots (2.7)$$

This treatment has been used by Chvorinov[2,10] who claims good agreement with experiments.

The problem of freezing was originally solved[1] by F. Neumann in 1860's and then modified by Stefan in 1891 and by Lightfoot in 1929. However, to solve the equations it is often necessary to make simplifying assumptions such as: one dimensional heat flow, semi-infinite mold material, constant interface temperature, or thermal properties which do not vary with temperature. As a result the answers obtained while helpful, are limited in applications.

A very serious disadvantage of all the analytical methods is that they are only applicable to materials which freeze at constant temperatures and are not capable of dealing with alloys solidifying over a range of temperature.

2.12 Electrical Analogue Method[2,3,11]

The method utilizes the close analogy between the heat flow in a heat exchange system and the flow of electricity in an appropriate circuit. The heat conduction equation (2.2) is compared with electrical energy flow equation:

$$\frac{\partial V}{\partial t} = \frac{1}{RC_E} \cdot \frac{\partial^2 V}{\partial x^2} \quad \ldots (2.8)$$

where R is resistivity and C_E is capacitance.

If, therefore, the various parts of the heat-exchange system are represented by condensers and resistances whose values are proportional to the thermal properties of the materials they represent and which are linked together in a circuit in a manner corresponding with the structure of the heat exchange system, and if the appropriate voltages and electrical charges are initially impressed on the electrical system, the resulting current flow is analogous to the heat flow in the system under study. Similarly the electrical equivalents of the various thermal properties used are:

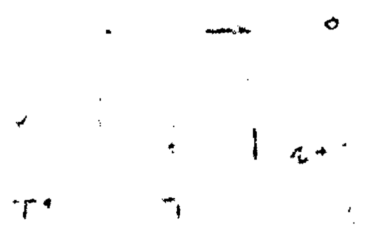
Thermal Property	Electrical Equivalent
Thermal capacity	Electrical capacity
Thermal resistivity	Electrical resistivity
Temperature	Voltage
Rate of heat flow	Current
Heat content	Electrical charge

The electrical analogue is a very accurate method of solving problems in unsteady heat flow and that, when the techniques are fully developed, it will prove invaluable in elucidating problems of both theoretical and practical interest involved in metal casting.

2.23 Digital Computer Method[3]

The heat conduction equation in one dimension in a homogeneous, isotropic solid is:

$$\frac{dT}{dt} = \frac{k}{\rho c}\left(\frac{d^2T}{dx^2}\right) \qquad \ldots (2.9)$$



General node in one dimension.

Using finite difference approximation method, the first derivative of temperature with respect to time is approximated using forward difference, and second derivative of temperature with respect to distance is approximated using central difference, we get:

$$\frac{T'-T}{\Delta t} = \frac{k}{\rho c}\frac{1}{\Delta x^2}(T1-2T+T2)$$

$$\therefore \quad T' = \frac{k}{\rho c}\frac{\Delta t}{\Delta x^2}(T1-2T+T2) + T \qquad \ldots (2.10)$$

Equation (2.10) expresses the future temperature of the node i in terms of the present temperature of the node i and the present temperature of the surrounding node i-1 and i+1, all of which are known. Similar equations are developed for all the nodes. To use digital computer following scheme is followed:

1. Divide the geometry of the problem into nodes.

2. Specify the coordinate dimensions and initial conditions of each node on computer input sheet.

3. State the boundary conditions as functions of time or temperature and

4. List the thermal conductivity and specific a heat of materials as functions of temperature.

This information is then transferred to key punch cards and used as input to the computer which solves 'n' simultaneous equations in a minimum and tabulates the temperature for all desired times at each node centre for convenient analysis by the casting design engineer.

2.3 Other Experimental Methods

2.3.1 'Pour-out' or Bleeding Technique[2]:

In this method a number of identical molds are cast and then bled at pre-determined intervals after casting, the thickness of the solid shells of metal remaining in the mould being subsequently measured. This method, although very restricted in so far as it only enables one quantity, the rate of skin formation, to be measured, has given very valuable results with steel. Its application to non-ferrous material having long freezing ranges would, however, yield little information of value, since the formation of a solid skin takes place at some unknown point between the liquidus and solidus, and the results of such experiments would throw no light on either the beginning or end of solidification.

2.2.2 Temperature Measurement Technique[2]

In this method, the temperature is measured at specified locations of the mold and casting as the solidification is progressing. This method requires minimum of high temperature data, enables the progress of both the beginning and end of solidification to be followed, and shows the behaviour in the zone where the metal is at temperatures between the liquidus and the solidus, i.e. the zone in which feeding occurs. This method is quicker than the pourout method. In addition, the method enables measurement of the temperature at various points in the mold to be taken at the same time, thus permitting the calculation of the temperature gradients existing in both casting and mold as well as allowing the progress of solidification to be followed. It is, therefore, clear that the temperature measurement method is the best technique for use with non-ferrous metals, especially those with long freezing ranges. In recent years this method has been extensively used by research workers.

2.3 Heat Transfer Models A General Review

Carslaw and Jaeger[1] have several exact solutions to the freezing front problem for homogeneous boundary conditions. Since sand casting does not involve constant heat fluxes i.e. the sand does not remain at a constant temperature, exact solutions are not applicable. Even most recent studies by Tien and Koump[12] on the effect of density change on the solidification of alloys, and by Tien and Geiger[13], on the solidification of a binary eutectic system use homogeneous boundary conditions. The closest exact solution for heat transfer through the casting is given by Tien and Geiger[14] in their model for unidimensional solidification of a binary eutectic system with a time dependent surface temperature. Although the temperature of the casting/mold interface can be thought of as a time-dependent function, it is determined by the combined properties of the metal being cast and mold material being used. Therefore, this method applies only to situations where the researcher can control the interface temperature. Murray and Landis[15] present several numerical solutions to problems of melting and freezing. Moreover their paper provides a comprehensive review of most previously published models for freezing. In that paper the authors developed a unique finite difference model for one dimensional heat flow which permits a continuous determination of the frozen front travel and a method for determining the temperature distribution in the solidifying mass at any given time increment. However, they did not apply it to any practical casting problem.

2.3 CASTING DESIGN BY COMPUTER

During the last decade, there has been a tremendous increase in the use of computers to aid in the solution of engineering problems. One of the most rapidly advancing fields is the use of numerical methods of approximating the solution to mathematical formulations of problems that are too complex to be solved analytically. The solidification of castings is an example of such problems. By this it is possible to predict temperature distributions within the casting and the mold. The prediction of temperature distribution during solidification is the starting stage for casting design by computer.

A good account of work has been given by Jonar[16], Roshan et al.[17] for determining temperature fields in the mold during casting and solidification for one dimensional heat flow using digital computers.

Marrone, Wilkes and Pehlke[18,19] carried out computer simulation of solidification of several casting geometries and compared their results with the experimental results available in the literature for these geometries. To eliminate the problem of the limited accuracy with which thermal properties of the mold materials were specified in the literature Kirt and Pehlke[20] developed computer techniques for determining mold material thermal properties as a function of temperature using the thermal properties obtained by these techniques, solidification of a 3.5 inch square bar in dry sand mold was simulated for commercially pure Al[2]. In all these studies, the latent heat of fusion was taken into account by appropriately increasing the specific heat of the metal in the solidification range. Jeyarajan and Pehlke[22] carried out computer simulation of solidification of castings in the form of a series of connected discs or cylinders and experimentally monitored to demonstrate the capability of the computer in predicting satisfactory and unsatisfactory casting design. They further developed[23] a computer program to simulate the freezing of axisymmetric castings against an end chill with airgap formation across the chill metal interface and tested the case and applied the simulation to various chill materials.

Pehlke and coworkers study was limited in all the above cases for two dimensional heat transfer computer simulation.

3.1 Casting design includes art, experience and empirical rules. Many times, this involves costly and time consuming experimentation. Design of castings using the digital computer, based on numerical simulation of solidification, would eliminate experimentation and improve yield, reliability and accuracy of the casting process. The prediction of temperature distribution during solidification is the starting stage for casting design by computer.

Most of the literature on computer simulation of heat transfer models dealing with solidification of castings is concerned with two dimensional problems because of the complexity which three dimensional heat transfer computer simulation would introduce. But in actual practice, in most of the cases, the shape of the casting is such that the heat transfer is three dimensional and in such cases the results obtained on two dimensional model may not be valid. In any case a separate computer program is necessary for three dimensional heat transfer simulation.

In this light, a three-dimensional heat transfer model is formulated and a computer programme using a digital computer 360/44 system to solve heat transfer equations is proposed.

A three step casting has been selected for the present study because it has external and internal corners and edges and different types of surfaces. These provide complexities which are usually present in industrial castings and offer a challenge to the heat transfer simulation.

Computer simulation of solidification was based on explicit finite difference approximation method to the equations describing three dimensional conduction heat transfer from the casting. Explicit finite difference approximation method has been used because of its accuracy and stability.

CHAPTER 4 — MATHEMATICAL ANALYSIS [illegible]

4.1 [illegible]

This chapter is an analysis of the process of solidification in the casting from the [illegible] point of heat transfer. The model is primarily developed to study the inter-relationship between the thermophysical properties of the casting material and mould material, the solidification rate of the casting, and the temperature distribution developed in the casting during solidification.

4.11 Model Assumptions:

The model developed for this work is formulated based on the following assumptions:

1. The liquid metal has filled the mold cavity instant[illegible] and is stagnant (not moving).

2. There are no natural convective currents present in the liquid metal (no convective mass and energy transport in the liquid phase was considered).

3. All materials are homogeneous and exhibit isotropic physical properties.

4. Freezing occurs at a single front for pure metals (In case of alloys, there is a well defined

solidus and liquidus temperature).

5. Air gap does not form between the mold and casting.

Assumptions 1,2, and 3 are valid approximations because of extremely rapid filling times and small solidification times actually found in case of most of the materials.

Assumption 4 is made to simplify the freezing front movement analysis. From a heat transfer standpoint when an alloy forms a 'mushy zone' during solidification, it is difficult to formulate model since the latent heat of fusion no longer is released at the melting point but rather it is given up over a freezing zone between the liquidus and solidus temperatures. The latent heat of fusion has been accounted for by arbitrarily incorporating it into the specific heat values.

Assumption 5 is in line with the usual practice for carrying out such analysis.

4.12 Mathematical Analysis

Conduction in both casting and mold is described by Fourier's general parabolic heat conduction equation. The differential equation of heat flow in a homogeneous, isotropic solid is:

$$\frac{\partial T}{\partial t} = \frac{K}{\rho c}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \quad \ldots (4.1)$$

The solution of this equation gives temperature distribution in the casting and mold walls. A set of initial and boundary conditions should be specified to define the problem.

4.121 Initial Conditions

$$\text{At } t = 0, \quad \text{For casting region} \quad T = T_{om}$$
$$\text{For sand region} \quad T = T_{os} \qquad \ldots (4.2)$$

To find a reasonable approximation to the initial interface temperature following methods are used:

FIRST METHOD

To calculate interface temperature at $t = 0$

As per Prof. R.D. Pehlke[21]:

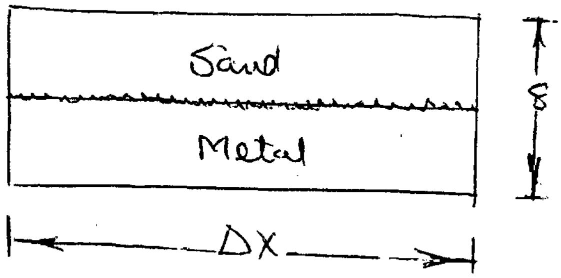


Fig.4.1 Control Volume considered when calculating the initial sand/metal interface temperature

The thermal energy contained in this system is

$$\frac{1}{2}\delta.\Delta x(\rho_m.C_{pm}.T_{om} + \rho_s C_{ps}.T_{os})$$

Allow the system to come adiabatically to equilibrium. The adiabatic assumption is reasonable since as $\theta \to 0$, whatever happens to the system is assumed to happen instantly. Also, by definition of this initial condition, no heat flux has yet been established, hence no heat is transferred to or from the system which is adiabatic by definition. There are three possible final states for the system depending on the relative values of ρ_m, c_{pm}, ρ_c, T_{cm}, T_{co} and the heat of fusion of the metal.

Case I: None of the metal solidifies

Equating the thermal energy in the system initially to that in the system at equilibrium yields

$$\tfrac{1}{2}\lambda\Delta m(\rho_m c_{pm} T_{cm} + \rho_c c_{pc} T_{co}) = \tfrac{1}{2}\lambda\Delta m(\rho_m c_{pm} + \rho_c c_{pc}) T_x$$

$$\therefore \quad T_x = \frac{\rho_m c_{pm} T_{cm} + \rho_c c_{pc} T_{co}}{\rho_m c_{pm} + \rho_c c_{pc}} \qquad \ldots (4.3)$$

The properties of the sand and metal in the numerator are at T_{cm} and T_{co}, initially assuming a value for T_x, take the sand and metal properties for the denominator at this T_x and calculate a new T_x, then continue the iteration. The same iterative process must also be used for Cases II and III.

Case II: All the metal solidifies

The thermal energy released when the metal solidifies is $(\tfrac{1}{2}\lambda\Delta m)\cdot\rho_m\cdot(\Delta H_f)$

Conservation of energy yields

$$(\tfrac{1}{2}\Delta x)(\rho_L c_{pL} T_{oL} + \rho_s c_{ps} T_{os}) + \tfrac{1}{2}(\rho\Delta x)\rho_L(\Delta H_f)$$
$$= (\tfrac{1}{2}\Delta x)(\rho_L c_{pL} + \rho_s c_{ps}) T_I$$

Hence $$T_I = \frac{\rho_L c_{pL} T_{oL} + \rho_s c_{ps} T_{os} + \rho_L \Delta H_f}{\rho_L c_{pL} + \rho_s c_{ps}} \quad \ldots (4.4)$$

Case III: <u>An alloy partially solidifies</u>

If the liquidus and solidus of a binary system are approximated by straight lines, the fraction of liquid solidified[5] as a function of temperature can be expressed as

$$f_s = \text{Fraction solidified} = \frac{1}{(1-D) + \frac{A}{T_{oL}-T}} \quad \ldots (4.5)$$

where,

$A = T_{oL} - T_L$

$D = (\tan\theta)(\cot\phi)$

θ and ϕ are the angles between the liquidus and solidus and an isotherm.

<u>For a pure metal</u>, $D = 1$

$$f_s = \frac{T_{oL}-T}{T_{oL}-T_L} \quad \ldots (4.6)$$

Equating thermal energy in the system initially to that in the system at equilibrium yields:

$$(\tfrac{1}{2}\delta\Delta x)(\rho_m C_{pm} T_{om} + \rho_c C_{pc} T_{oc}) + \frac{T_{mt} - T_i}{T_{mt} - T_s}\,\rho_m \Delta H_f (\tfrac{1}{2}\delta\Delta x)$$

$$= (\tfrac{1}{2}\delta\Delta x)(\rho_m C_{pm} + \rho_c C_{pc}) T_i \qquad \text{... (4.7)}$$

Hence, $$T_i = \frac{\rho_m C_{pm} T_{om} + \rho_c C_{pc} T_{oc} + (T_{mt}/(T_{mt} - T_s))\cdot\rho_m \Delta H_f}{\rho_m C_{pm} + \rho_c C_{pc} + \left(\frac{\rho_m \Delta H_f}{T_{mt} - T_s}\right)} \qquad \text{....(4.8)}$$

In the denominator of equation (4.8) the value of C_{pm} is found by taking the weighted average of the liquid metal C_{pm} at T_{mt} and C_{pm} at T_s with respect to the fraction solidified.

To determine which equation should be used to calculate the initial interface temperature, it is necessary to assume a final state for the metal (liquid, mush, or solid), use the appropriate equation to calculate T_i, and check to make sure the calculated T_i falls within the temperature range initially assumed.

For the initial temperature of the casting corner node, the equations are developed in an analogous manner and are listed below:

TCN

No solidification:

$$T_o = \frac{3\rho_c C_{pc} T_{oc} + \rho_m C_{pm} T_{om}}{3\rho_c C_{pc} + \rho_m C_{pm}} \qquad \text{... (4.9)}$$

Complete solidification:

$$T_o = \frac{S\rho_s C_{ps} T_{os} + \rho_L C_{pL} T_{oL} + \rho_L \Delta H_f}{S\rho_s C_{ps} + \rho_L C_{pL}} \quad \ldots (4.10)$$

Partial solidification:

$$T_o = \frac{S\rho_s C_{ps} T_{os} + \rho_L C_{pL} T_{oL} + \left(\frac{T_{oL}}{T_{oL} - T_L}\right)\rho_L \Delta H_f}{S\rho_s C_{ps} + \rho_L C_{pL} + \rho_L\left(\frac{\Delta H_f}{T_{oL} - T_L}\right)} \quad \ldots (4.11)$$

<u>For a binary alloy:</u>

$$f_o = \frac{1}{(1-k) + \frac{\Delta T}{T_{oL} - T_L}}$$

Then, the initial interface temperature is the solution of the following quadratic equation:

$$(1-k).D.T_i^2 - \left[(1-k)(Q+T_{oL}D) + \rho_L \Delta H_f + k.D.[?]T_L \Delta T\right] + T_{oL}\left[(1-k)Q + \rho_L \Delta H_f\right] = 0 \quad \ldots (4.12)$$

where $D = (\rho_L C_{pL} + S\rho_s C_{ps})$

$Q = (\rho_L C_{pL} T_{oL} + S\rho_s C_{ps} T_{os})$

For copper mold, equation (4.12) is the same however,

$D = (\rho_L C_{pL} + S\rho_s C_{ps})$

and $Q = (\rho_L C_{pL} + S\rho_s C_{ps})$

Flow chart of logic for the calculation of interface temperature as per Prof. R.D.Pehlke approach is given in Fig.2.

Second Method (Modified): To calculate interface temperature at t = 0.

Initially temperature of interface is temperature of metal as it enters the mold, then

$$T_I = T_m \qquad \ldots (4.13)$$

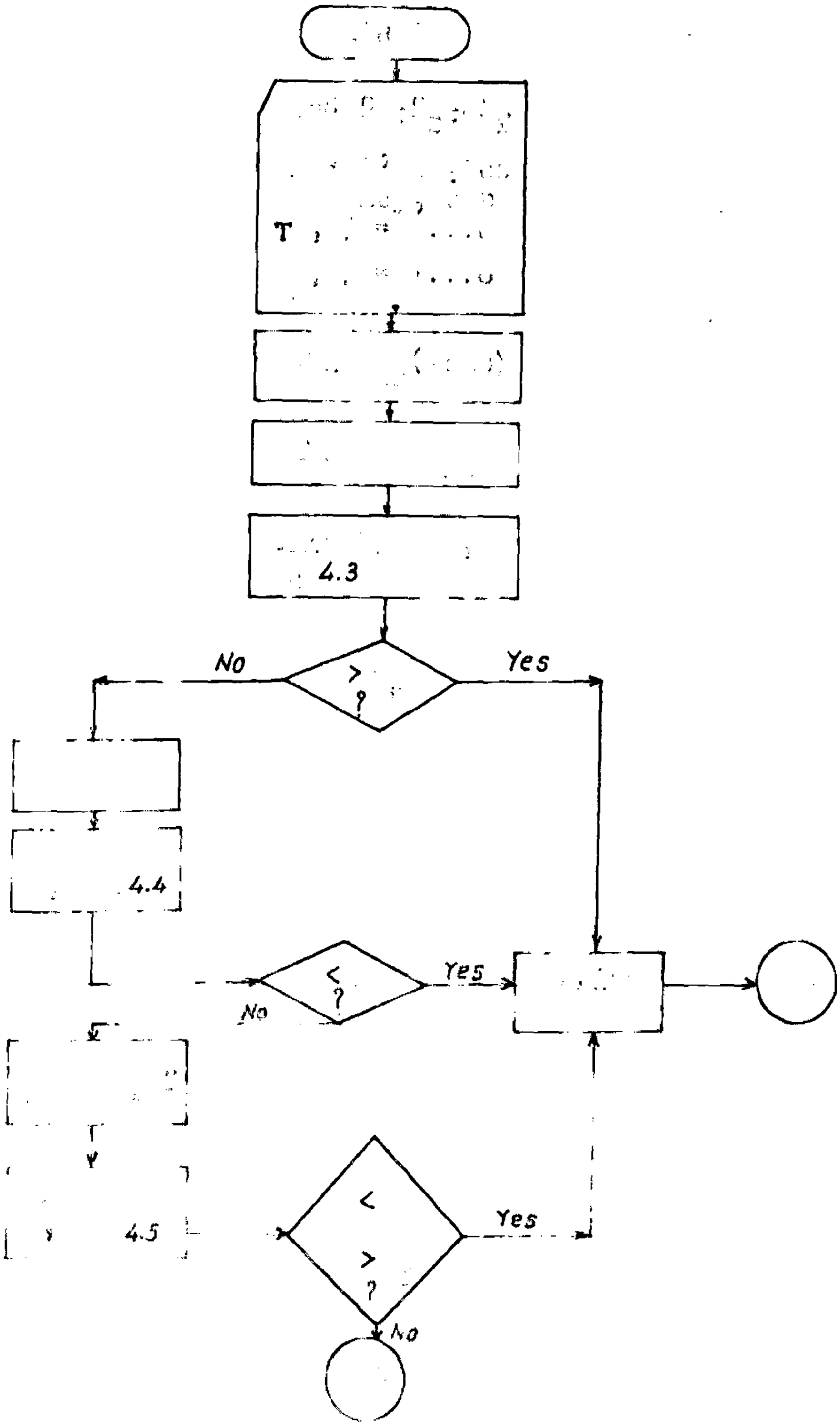
Boundary Condition

At flask wall i.e. sand/air interface heat is lost by convection and radiation. But in the present analysis it has been assumed that for the period of solidification the flask wall temperature i.e. sand/air interface temperature remains the ambient temperature.

4.2 TWO DIMENSIONAL CASTING SIMULATION

4.21 Casting Geometry

The casting simulated is a 3.5 in (89 mm) square bar surrounded on all sides by 3.25 in (82 mm) of molded sand as shown in Fig.3. The casting is of sufficient length that heat transfer at the half-length plane is essentially two dimensional. Using symmetry, this square cross-section can be described completely by the section shown in Fig.4. Also shown in Fig.4 is a cartesian system of nodes which can be established on the cross-section.


4.3
No
Yes
4.4
Yes
No
4.5
Yes
No

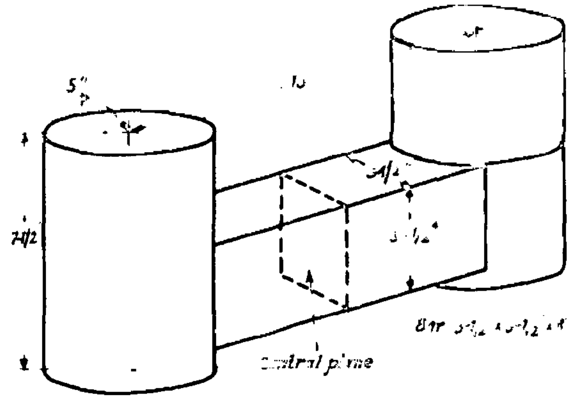


Fig. 3 - Experimental bar casting.

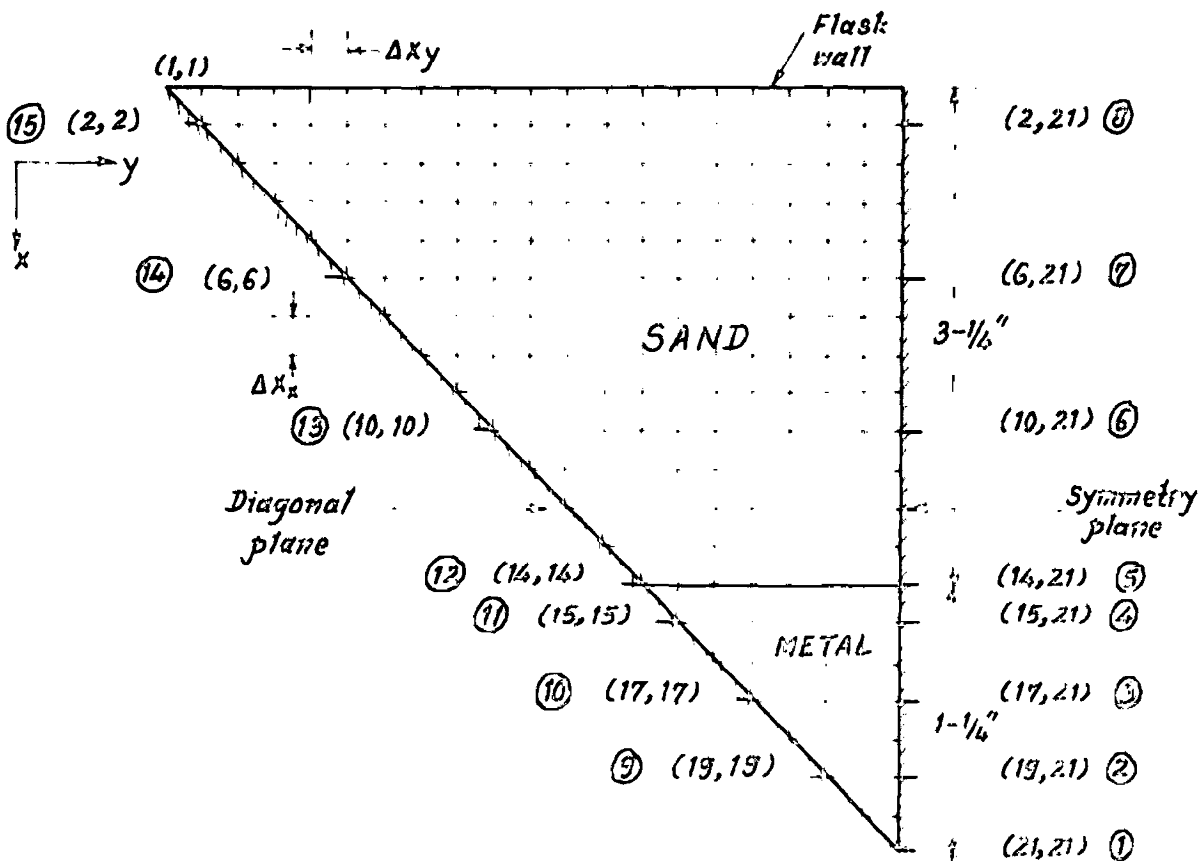


Fig. 4 - Illustrative grid point presentation of casting & mold showing symmetry of half quadrant for casting of bar.

The physical process of the solidification involved several key assumptions (1 to 5 stated in 4.11) concerning the cooling process in order to formulate a heat conduction model in a form suitable for numerical simulation.

4.12 Mathematical Analysis

Heat conduction equation for two dimensional heat flow is given by

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \qquad \ldots (4.24)$$

Consider a general node as shown:

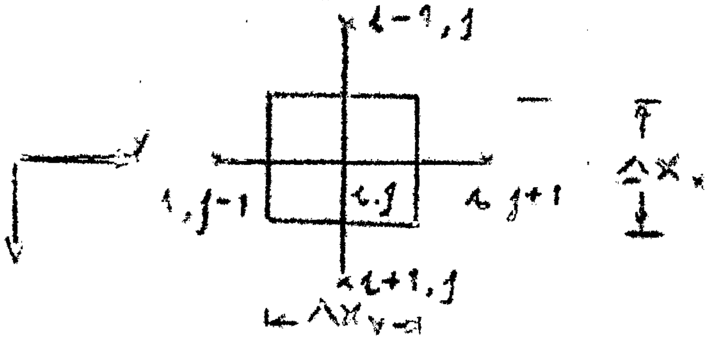


Fig.5: General node

$\Delta X_x = \Delta X_y = \Delta X$

In equation (4.24), the first derivative of temperature with respect to time is approximated using the forward difference method, and the second derivative of temperature with respect to distance is approximated using a central difference method.

$$\frac{T'_{i,j} - T_{i,j}}{\Delta t} = \frac{k}{\rho c_p} \cdot \frac{1}{\Delta X^2}\left[T_{i-1,j} + T_{i+1,j} + T_{i,j-1} + T_{i,j+1} - 4T_{i,j}\right]$$

or $$T'_{i,j} = \frac{k}{\rho c_p} \cdot \frac{\Delta t}{\Delta X^2}\left[T_{i-1,j} + T_{i+1,j} + T_{i,j-1} + T_{i,j+1} - 4T_{i,j}\right] + T_{i,j} \qquad \ldots (4.25)$$

Equation (4.19) expresses the future temperature of node i,j in terms of the present temperature of node i,j and the present temperature of the surrounding nodes, all of which are known.

Similar equations can be developed for the specialized nodes. For example, consider the node on the corner of the casting shown:

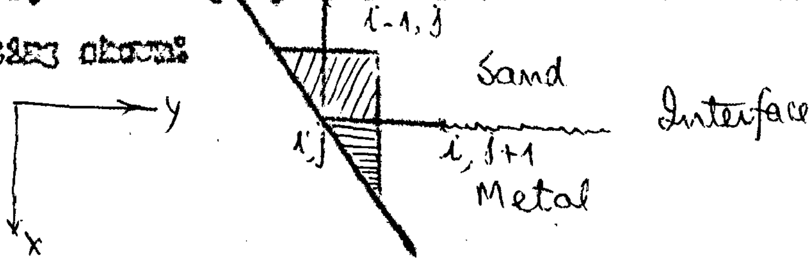


Fig. 6: Corner Casting Node

The following are known:

Volume of sand $= \frac{3}{8}\Delta x^2,$

Volume of metal $= \frac{1}{8}\Delta x^2$

Net rate of thermal energy increase in sand $= (\frac{3}{8}\Delta x^2)(\rho_s c_{ps}) \frac{T'_{i,j} - T_{i,j}}{\Delta t}$

Net rate of thermal energy increase in metal $= (\frac{1}{8}\Delta x^2)(\rho_m c_{pm}) \frac{T'_{i,j} - T_{i,j}}{\Delta t}$

Heat influx from node i-1,j $= k_s \Delta x . \frac{T_{i-1,j} - T_{i,j}}{\Delta x}$

Heat influx from node i,j+1 $= k_s(\frac{1}{2}\Delta x) \frac{T_{i,j+1} - T_{i,j}}{\Delta x} + k_m(\frac{1}{2}\Delta x) \frac{T_{i,j+1} - T_{i,j}}{\Delta x}$

Conservation of energy yields:

$$T'_{i,j} = \frac{4}{(3\rho_s c_{ps} + \rho_m c_{pm})} \frac{\Delta t}{\Delta x^2} \Big[2k_s T_{i-1,j} + (k_s + k_m) T_{i,j+1} + (k_m + 3k_s) T_{i,j}\Big] + T_{i,j} \quad \ldots (4.20)$$

Thermal properties of the materials are assumed to be those corresponding to the present temperature of node i,j.

Equations of all the nodes are summarized below:

1. Metal General

$$T'_{i,j} = \frac{k}{\rho_m c_{pm}} \frac{\Delta t}{\Delta x^2}\left[T_{i,j+1} + T_{i,j-1} + T_{i+1,j} + T_{i-1,j} - 4T_{i,j}\right] + T_{i,j} \quad \ldots (4.17)$$

2. Metal Center

$$T'_{i,j} = \frac{k}{\rho_m c_{pm}} \cdot \frac{\Delta t}{\Delta x^2}\left[T_{i-1,j} - 4T_{i,j}\right] + T_{i,j} \quad \ldots (4.18)$$

3. Metal Diagonal Boundary

$$T'_{i,j} = \frac{k}{\rho_m c_{pm}} \cdot \frac{\Delta t}{\Delta x^2}\left[2T_{i,j+1} + 2T_{i-1,j} - 4T_{i,j}\right] + T_{i,j} \quad \ldots (4.19)$$

4. Metal Plane Boundary

$$T'_{i,j} = \frac{k}{\rho_m c_{pm}} \frac{\Delta t}{\Delta x^2}\left[2T_{i,j-1} + T_{i+1,j} + T_{i-1,j} - 4T_{i,j}\right] + T_{i,j} \quad \ldots (4.20)$$

5. Air General

$$T'_{i,j} = \frac{k}{\rho_a c_{pa}} \cdot \frac{\Delta t}{\Delta x^2}\left[T_{i,j+1} + T_{i,j-1} + T_{i+1,j} + T_{i-1,j} - 4T_{i,j}\right] + T_{i,j} \quad \ldots (4.21)$$

6. [illegible]

[illegible]

$$T'_{i,j} = T_{i,j} = \text{Ambient temperature} = 25^\circ C = 77^\circ F \quad \ldots (4.22)$$

7. Cool Element Symmetry

$$T'_{i,j} = \frac{K_n}{\rho_n C_{pn}} \cdot \frac{\Delta\theta}{\Delta X^2}\left[2T_{i,j+1} + 2T_{i-1,j} - 4T_{i,j}\right] + T_{i,j} \quad \ldots (4.23)$$

8. Cool Plane Symmetry

$$T'_{i,j} = \frac{K_n}{\rho_n C_{pn}} \cdot \frac{\Delta\theta}{\Delta X^2}\left[2T_{i,j-1} + T_{i-1,j} - 4T_{i,j}\right] + T_{i,j} \quad \ldots (4.24)$$

9. Interface General

$$T'_{i,j} = \frac{2}{\rho_s C_{ps} + \rho_n C_{pn}} \cdot \frac{\Delta\theta}{\Delta X^2}\Big[2K_s T_{i-1,j} + 2K_n T_{i+1,j} + (K_s + K_n)T_{i,j-1} + (K_s + K_n)T_{i,j+1} - 4(K_n + K_s)T_{i,j}\Big] + T_{i,j} \quad \ldots (4.25)$$

10. Interface Corner

$$T'_{i,j} = \frac{4}{3\rho_s C_{ps} + \rho_n C_{pn}} \cdot \frac{\Delta\theta}{\Delta X^2}\left[2K_s T_{i-1,j} + (K_s + K_n)T_{i,j+1} - (K_n + 3K_s)T_{i,j}\right] + T_{i,j} \quad \ldots (4.26)$$

11. Interface Plane Symmetry

$$T'_{i,j} = \frac{2}{\rho_s C_{ps} + \rho_n C_{pn}} \cdot \frac{\Delta\theta}{\Delta X^2}\Big[K_s T_{i-1,j} + K_n T_{i+1,j} + (K_s + K_n)T_{i,j-1} - 2(K_s + K_n)T_{i,j}\Big] + T_{i,j} \quad \ldots (4.27)$$

The expressions are used for the solution of heat conduction equations.

(1) Boundary Definition Approach

At $\theta = 0$ for cooling nodes $T = T_{cn}$

for coal nodes $T = T_{co}$

for interface nodes $T = T_X$ as calculated from 4.121

(2) Modified Approach

At $t = o$ For casting nodes $T = T_{co}$

For sand nodes $T = T_{co}$

For interface nodes $T = T_{co}$

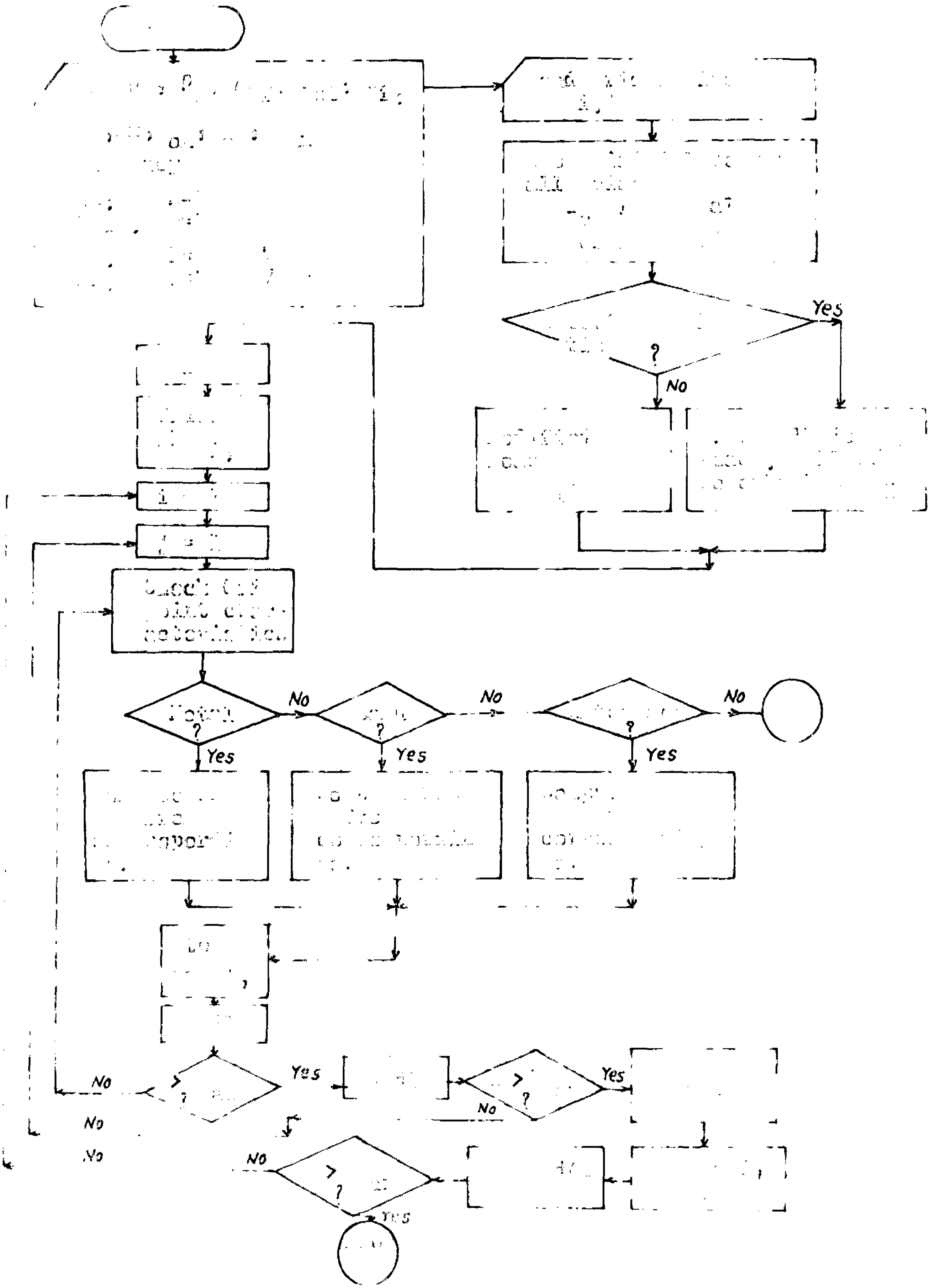
The initial temperature of every node now is specified. Using equations (4.17) to (4.27) the temperature of each node at time = Δt can be calculated. When all the node temperatures at time = Δt have been determined, temperature at time = Δt becomes the present temperatures and node temperature at time = $2\Delta t$ can be calculated. Similarly node temperatures can be calculated to any time = $n\Delta t$ where $n = 1,2,3.....$.

Flow chart of logic for the two dimensional heat transfer computer programme for the two approaches (Prof.R.D. Pehlke and modified) 4.221 and 4.222 is given in Fig.7. Computer programmes for 360/44 system for the two approaches 4.221 and 4.222 are is given in Appendix III and Appendix IV respectively.

4.3 THREE DIMENSIONAL COMPUTER SIMULATION

4.31 Casting Design

The casting shape was designed with the requirement that the casting should be suitable for computer simulation with a reasonable computer time requirement, should provide complexities which are usually present in industrial castings and offer a challenge to the heat transfer simulation. The



.7 [illegible]

test casting selected for experimental and computer simulation is shown in Fig.8, Fig.9 and Fig.10. Wlodawer's Modulus method[23] was employed to design the shape of the casting. He defines the solidification modulus of a section of a casting as the ratio of the volume of the section to the cooling area of that section. The shape is designed such that the section moduli increases from Section I towards the riser. This arrangement ensures that section II is capable of feeding section I, Section III is capable of feeding Section II and that the riser which feeds section III will solidify last. The design of riser and sprue is as per Flinn[77] and Heine, Looper and Rosenthal[9]. This shape also provides an opportunity to employ three dimensional heat transfer computer simulation. Furthermore, the presence of external and internal edges and corners, different types of surfaces provide the complexities which are usually present in industrial castings and they offer a challenge to the heat transfer simulation. Figure 10 is a cartesian system of nodes which can be established on the cross-section. The physical description of the solidification process involved requires several key assumptions (as stated in 4.11) concerning the casting process in order to formulate a heat conduction model in a form suitable for numerical solution.

4.12 Mathematical Analysis

Heat conduction equation for three dimension heat

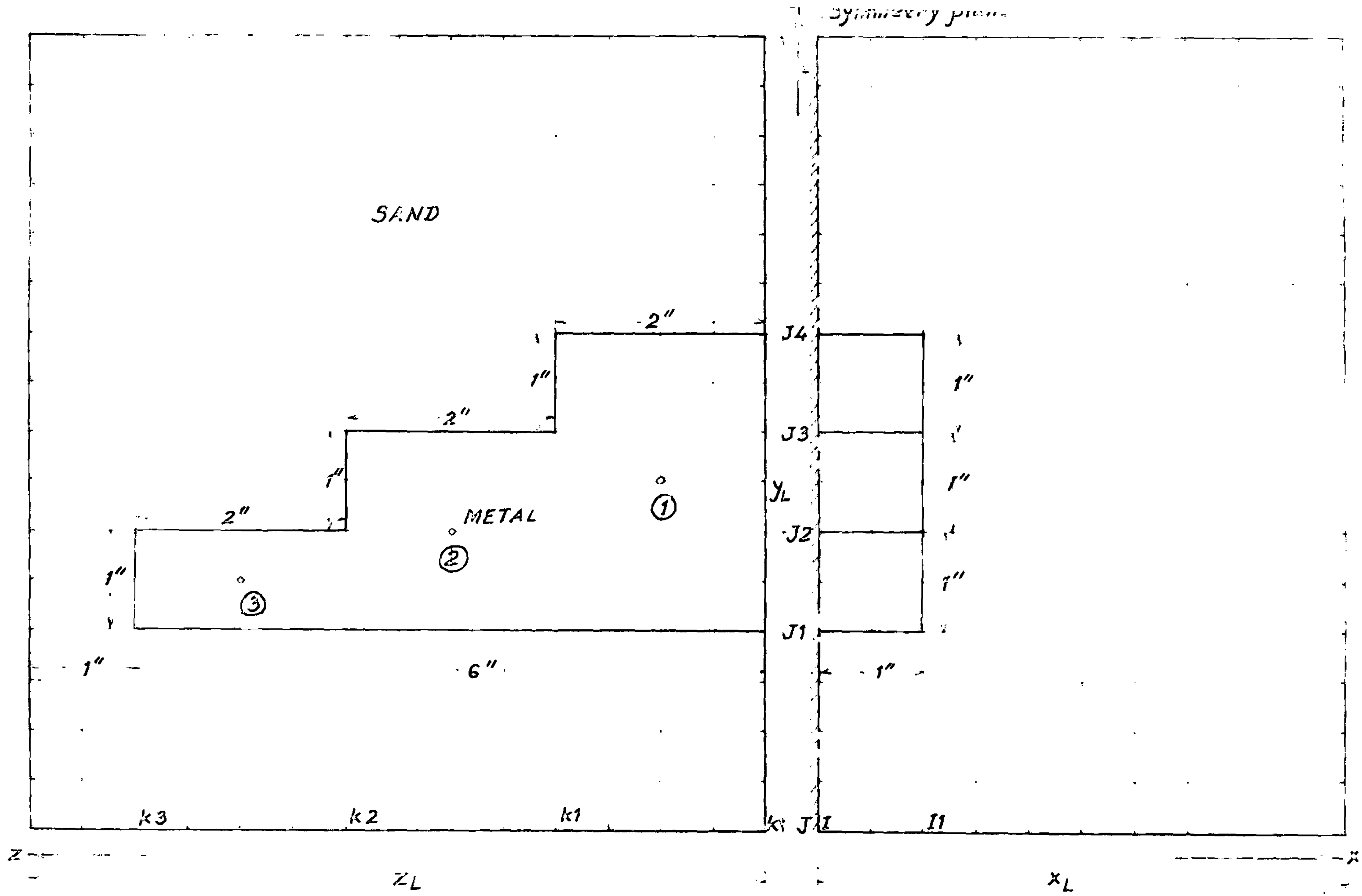


Fig.10_Illustrative Grid point presentation of experimental Casting & mold.

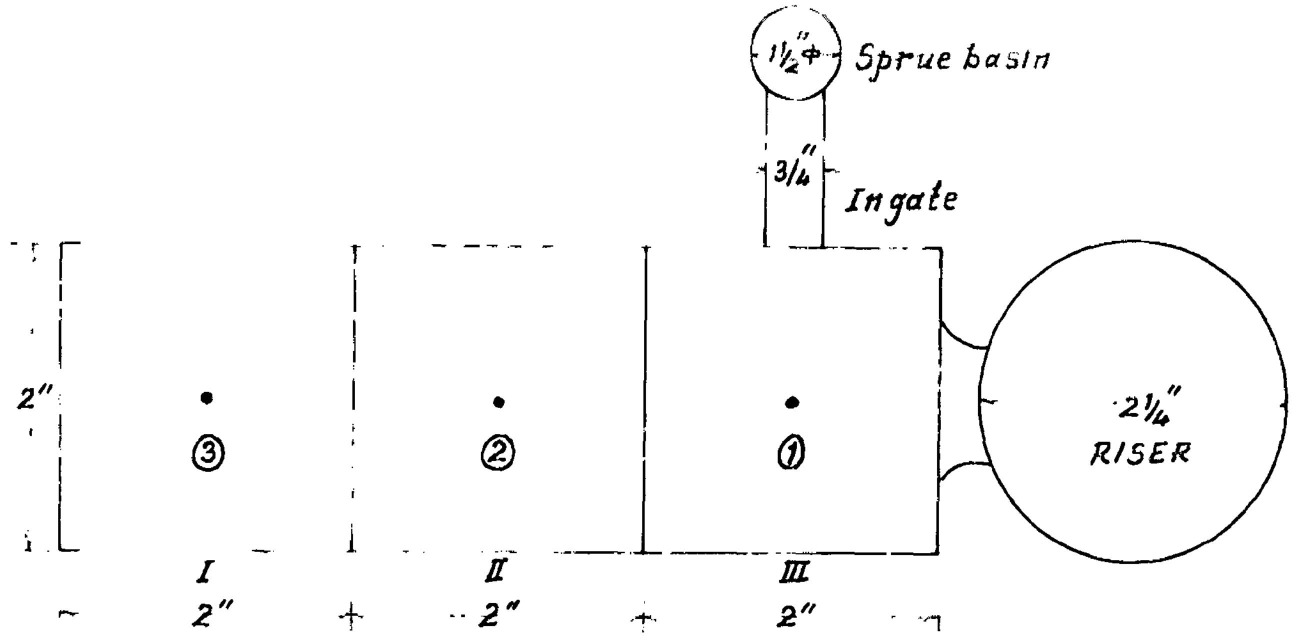


Fig. 8- Plan view of sound experimental casting showing sprue and gate.

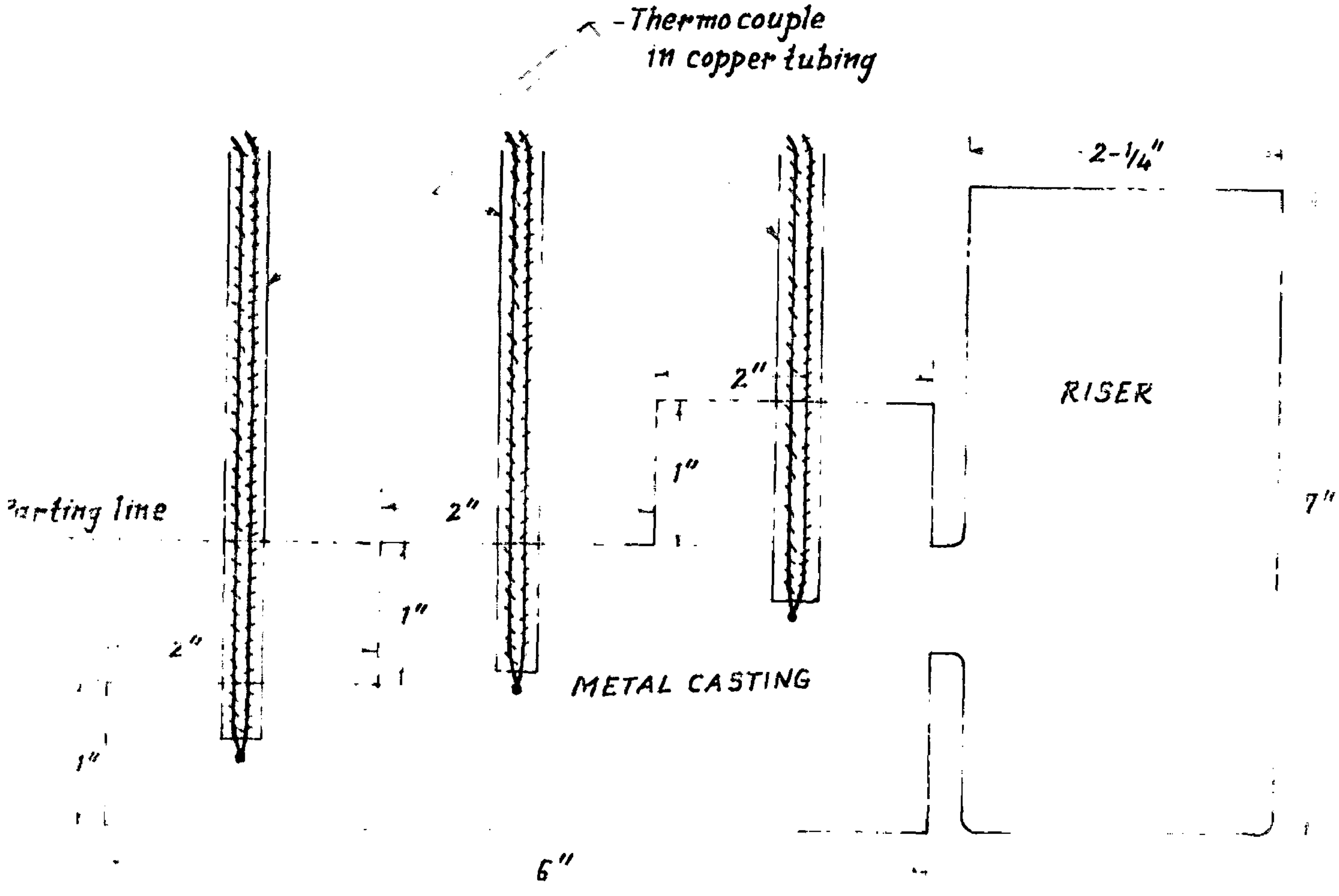


Fig. 9- Cross-section elevation view of sound casting.

Then is,

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \qquad \ldots (4.28)$$

Using explicit finite difference method scheme the first derivative of temperature with respect to time is approximated using the forward difference method, and the second derivative of temperature with respect to distance is approximated using a central difference method. The equation (4.28) will be

$$\frac{T'_{i,j,k} - T_{i,j,k}}{\Delta t} = \frac{k}{\rho c_p}\cdot\frac{1}{\Delta x^2}\Big[T_{i-1,j,k} + T_{i+1,j,k} + T_{i,j-1,k} + T_{i,j+1,k} + T_{i,j,k-1} + T_{i,j,k+1} - 6T_{i,j,k}\Big]$$

$$T'_{i,j,k} = \frac{k}{\rho c_p}\frac{\Delta t}{\Delta x^2}\Big[T_{i-1,j,k} + T_{i+1,j,k} + T_{i,j-1,k} + T_{i,j+1,k} + T_{i,j,k-1} + T_{i,j,k+1} - 6T_{i,j,k}\Big] + T_{i,j,k} \qquad \ldots (4.29)$$

Equation (4.29) expresses the future temperature of node i,j,k in terms of the present temperature of node i,j,k and the present temperature of the surrounding nodes, all of which are known. Equations of all the nodes are now summarized below:

1. <u>Bottom Front Corner</u>

$$T'_{i,j,k} = \frac{8}{\rho_s c_{ps} + \rho_m c_{pm}}\cdot\frac{\Delta t}{\Delta x^2}\Big[k_s(T_{i+1,j,k} + T_{i,j-1,k} + T_{i,j,k+1} - 3T_{i,j,k}) + 0.5(k_s + k_m)(T_{i-1,j,k} + T_{i,j+1,k} + T_{i,j,k-1} - 3T_{i,j,k})\Big] + T_{i,j,k} \qquad \ldots (4.30)$$

2. [illegible]

[illegible] ... (4.31)

3. [illegible]

[illegible] ... (4.32)

4. [illegible]

[illegible] ... (4.33)

5. [illegible]

[illegible] ... (4.34)

6. [illegible]

[illegible] ... (4.35)

7. [illegible]

[illegible] ... (4.36)

8. [illegible]

[illegible] ... (4.37)

9. [illegible]

[illegible] ... (4.38)

10. [illegible]

$$[illegible]$$ ... ([illegible])

11. [illegible]

$$[illegible]$$ ... ([illegible])

12. [illegible]

$$[illegible]$$ ... ([illegible])

13. [illegible]

$$[illegible]$$ ... ([illegible])

Initially at $t = 0$

For casting nodes $T = T_{cm}$

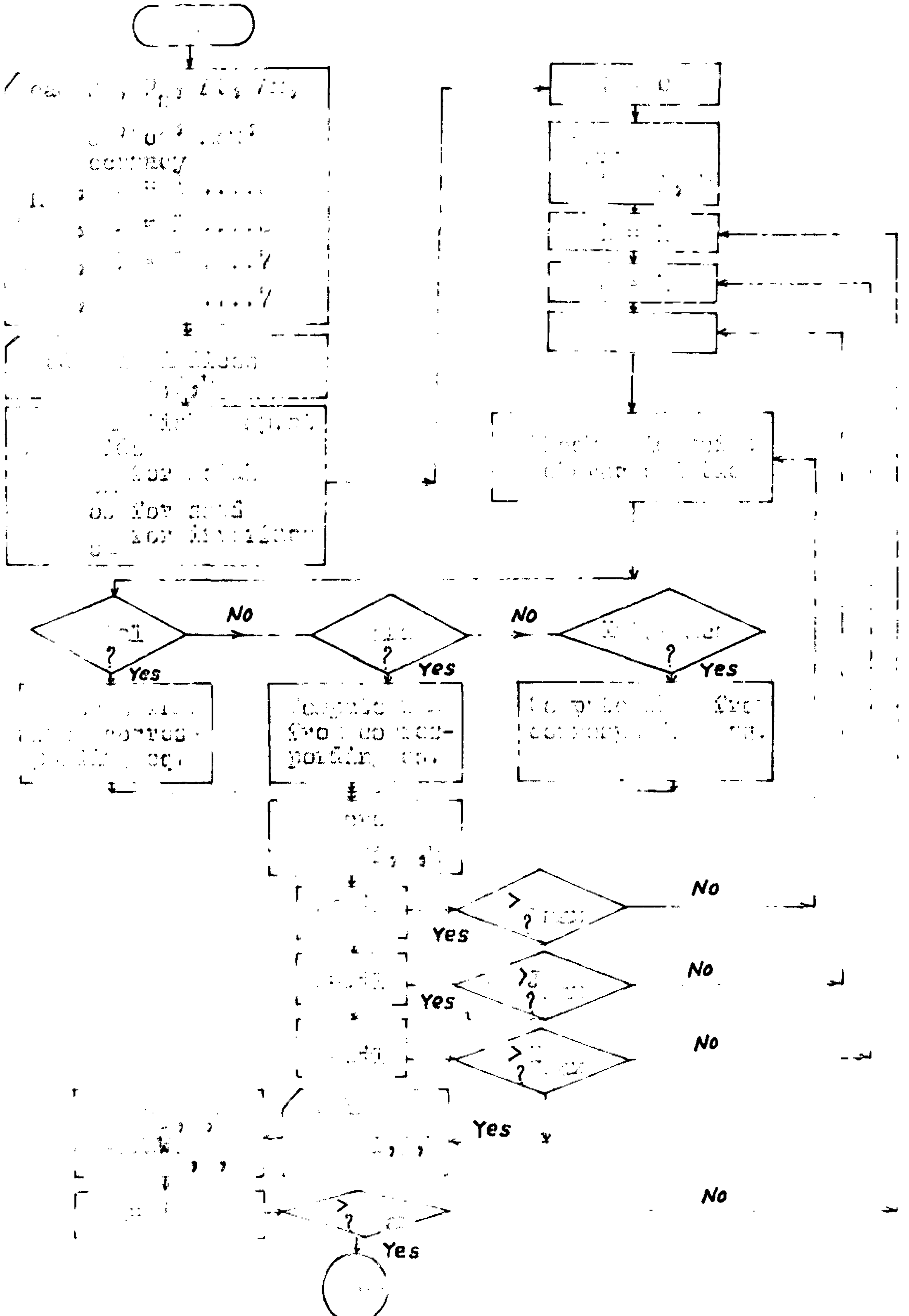
For sand nodes $T = T_{so}$

For interface nodes $T = T_I = T_{cm}$

The initial temperature of every node now is specified. Using equations (4.30) to (4.42), the temperature of each node at time = Δt can be calculated. When all the node temperatures at time = Δt have been determined, temperatures at time = Δt become the present temperatures and node temperatures at time = $2\Delta t$ can be calculated. Similarly node temperatures can be calculated to any time = $n\Delta t$ where $n = 1,2,3......$. Flow chart for logic for the three dimensional heat transfer computer programme is given in Fig.11 and computer programme for 360/44 system is given in Appendix V.

4.4 Thermal Properties of Materials

The method outlined above requires that the thermal properties involved be known as functions of temperature. The temperature dependence of the thermal conductivity and specific heat of aluminium is well established[26,27,28]. Referenced data are approximated by a series of straight line segments. This is done to save computer time required to calculate thermal properties of materials at different


No
No
Yes
Yes
Yes
No
Yes
No
Yes
No
Yes
Yes
No
Yes

temperatures and also because the data lay close to straight line segments. The thermal conductivity and specific heat of pure aluminium used in numerical simulation of solidification are shown in Fig.12 and Fig.13. The area under the C_p curve above this base line C_p between the liquidus and solidus temperature is numerically equal to the heat of fusion. In order to account for the heat of fusion in this manner, an arbitrarily small, but artificial, freezing range is utilized for pure Al. The density of pure Al^{27} is taken to be 169 lb/ft^3 (2.7 gm/cm^3). The specific heat of molding sand is not a strong function of the type of sand, processing or ramming. Hence, the specific heats of molding sands do not exhibit a great deal of scatter and can be fit reasonably well by a simple curve as in Fig.14. Thermal conductivity of molding sand, on the other hand, is strongly dependent on processing and ramming variable[20]. For this reason reported thermal conductivities of molding sands show a wide scatter. The thermal conductivity of the particular molding sand used for this work is shown in Fig.15. The flow chart and the computer programmes for material properties are given in Fig.16 and Appendix I respectively.

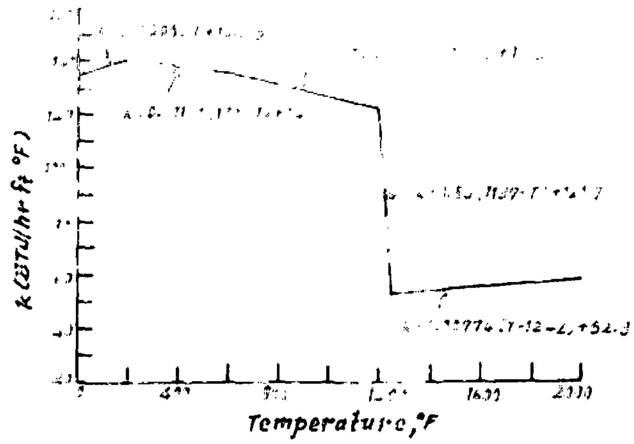


Fig.12. Data & functional relationship for thermal conductivity of aluminium.

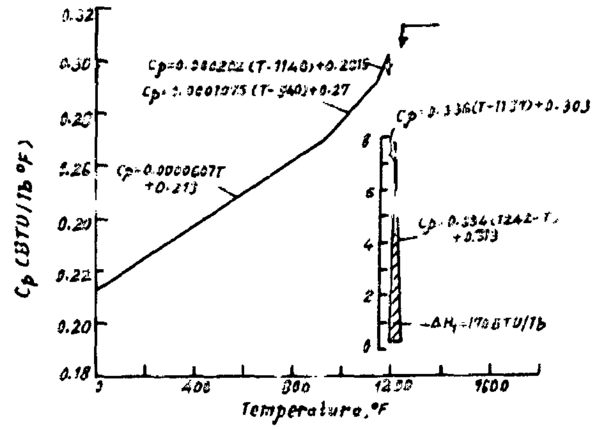


Fig.13. Data & functional relationships for specific heat of aluminium, including latent heat approximation.

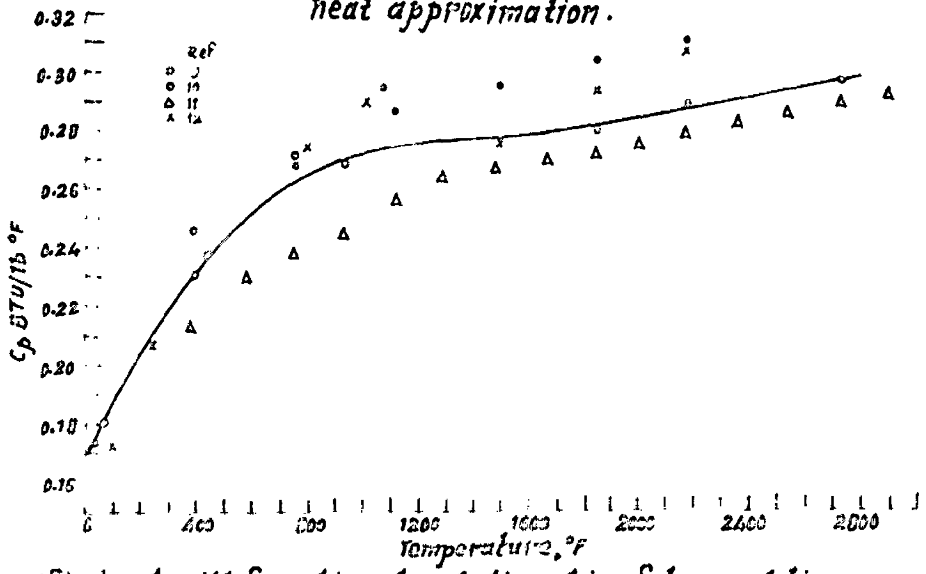


Fig.14. Specific heat with functional relationship of dry molding sand.

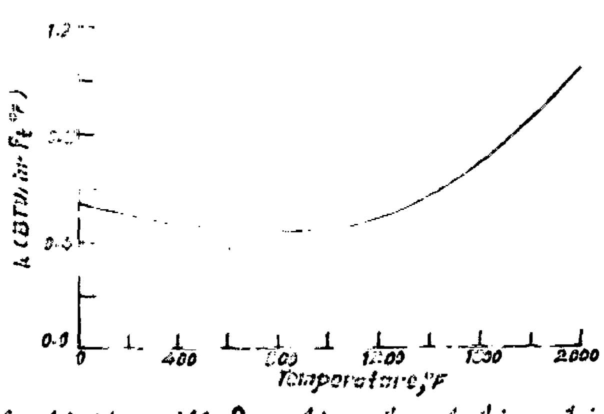
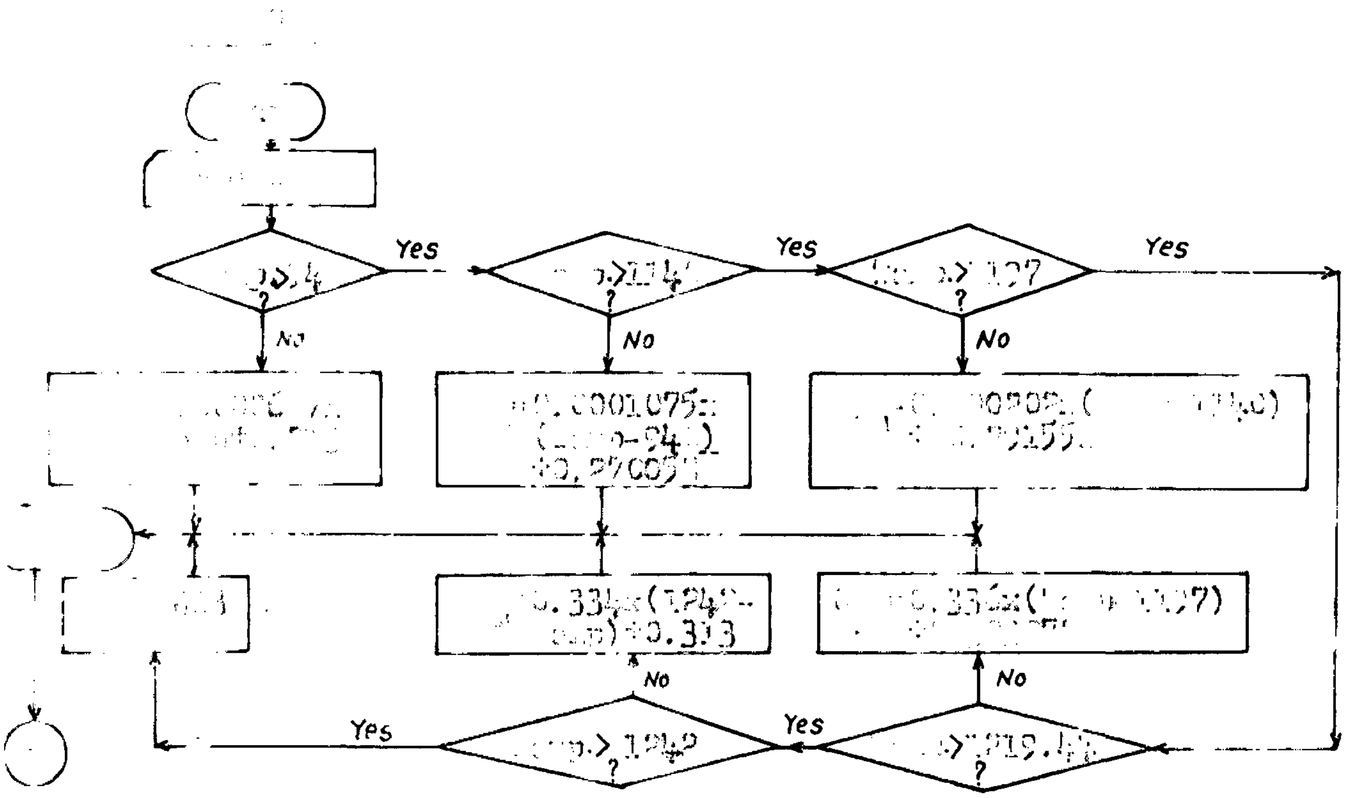


Fig.15. Thermal conductivity with functional relationship of dry molding sand.


Yes
Yes
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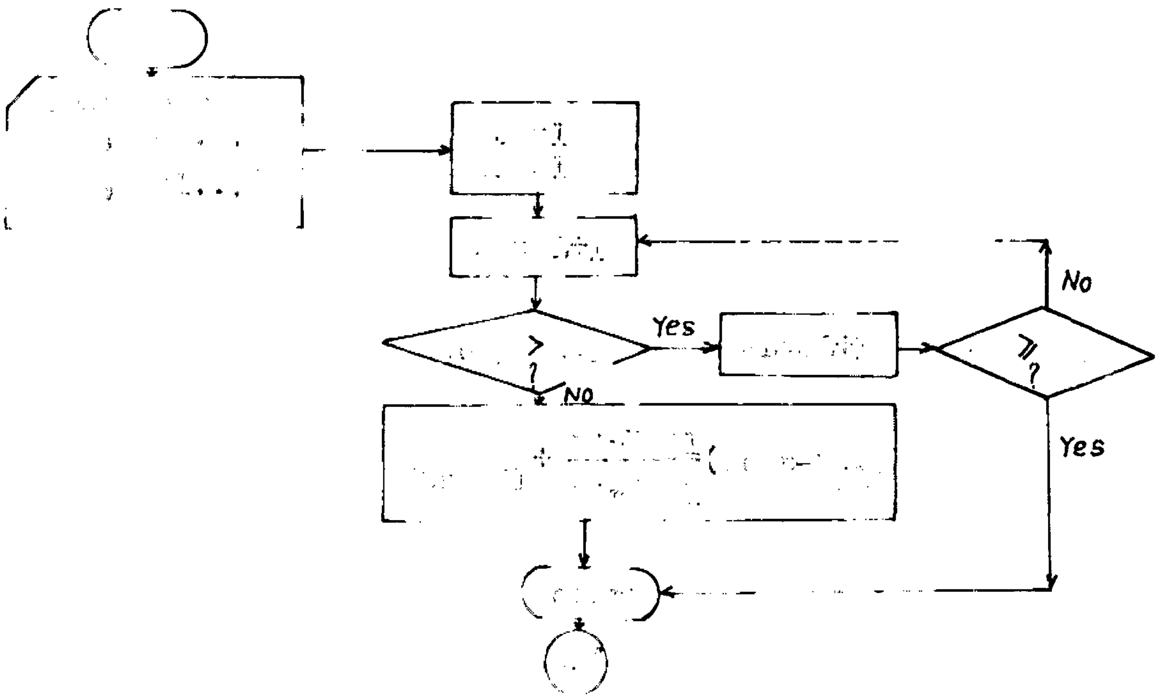
No
Yes
Yes


No
Yes
No
Yes

CHAPTER 5 ——— EXPERIMENTAL SET UP AND PROCEDURE

5.1 EXPERIMENTAL SET UP

To measure the output of thermocouples at specified locations for the complete solidification time, in a stepped Al casting in dry sand mold, experimental set up must consist of:

1. Wooden pattern, riser, sprue and a sand mold.
2. Chromel-Alumel thermocouples protected by ceramic beads and placed in copper tubings after proper calibration.
3. Melting furnace of range 1000°C.
4. Aluminium of commercial purity.
5. Graphite crucible no.11 for melting.

Wooden pattern, riser and sprue

The pattern, riser and sprue are made of teak wood. The design of these parts is as per Flinn[6] and Heine, Loper and Rosenthal[9].

Sand Mold

Silica sand was used for the sand mold. Sand mix was of the following composition.

Silica sand		: 30 kg.
Water	: 5%	: 1.5 kg.
Bentonite	: 4%	: 1.2 kg.

The sand mix was tested as per ISI Standard 1918:1966 for the following properties and the values are reported:

AFS mesh	: 60
Green Compressive Strength	: 0.46 kg/cm^2
Green Hardness	: 75-80.

After drying for 2 hours at 150°C,

Density	: 1.6 gm/cm^3 (100 lb/cu.ft)
Dry Hardness	: 90.
Dry compressive Strength	: 4.2 kg/cm^2

The composition and properties of sand mix were maintained for each mix.

Sand molds were prepared by hand ramming in 10"x10"x5" flasks and dried for 6 hours.

Chromel-Alumel Thermocouples

Chromel-Alumel thermocouples of 30 AWG was taken and protected by ceramic beads for insulation. They were placed in copper tubings for correct positioning in the mold.

The calibration of these thermocouples was carried out in Pb molten bath.

7.2 EXPERIMENTAL PROCEDURE

Aluminium was cut to required piece and placed in a graphite crucible no.12. Electric furnace was used for melting this charge. Melt quality is the most important aspect for the production of sound casting. The main factors controlling melt quality are: careful choice of metal charge, cleaning of melt from non-metallic inclusions and through degassing, effective grain refining, protective covering of melt, avoid turbulence, strict temperature control during melting, avoidance of long holding time, and use of proper pouring temperature and proper pouring practice.

The following steps were followed for melting and temperature recording for Al casting in dry sand molds:

1. Preheat crucible.
2. Place charge (3.5 kg) in the crucible.
3. Keep temperature of the furnace at 750°C and increase it to 800°C just 10 minutes before pouring.
4. Use minimum holding time.
5. Connect thermocouples at locations (1), (2) and (3) to potentiometers.
6. Standardize potentiometers.

7. Take out the charge from furnace, skim the charge while pouring.

8. Pour at about 750°C (in all cases) into the mold cavity, care is taken to avoid flow disturbance, gas entrainment, mold erosion etc.

9. The pouring time is kept minimum possible in the present case it was about 15 seconds. This is the initial condition of temperature distribution in the casting at different locations.

10. Measure e.m.f. output by potentiometers at an interval of 30 seconds for locations (1), (2) and (3).

After cooling castings were sectioned and found to be satisfactory.

Experimental Casting with Sprue and Riser

CHAPTER 6 ——— RESULTS

6.1 TWO DIMENSIONAL COMPUTER SIMULATION

Computer simulation based on explicit finite difference method to the equations describing two dimensional conduction heat transfer as per

1. R.D.Pehlke approach,
2. Modified approach.

was carried out for a square bar (Fig.8). The computer programmes for system 360/44 are listed in Appendix III and Appendix IV respectively. Simulation output was in the form of grid node temperatures at time increment of 0.0001 hr (0.36 sec.). Temperature along symmetry plane and along diagonal for 0 sec, 36 sec, 72 sec, 108 sec. 144 sec., 360 sec. 540 sec, and 720 sec. only are given in table 1 to table 4 for both the approaches.

6.2 Three dimensional computer simulation

Computer simulation was based on explicit finite difference method to the equations describing three dimensional conduction heat transfer as per modified approach from the casting (Fig.8 and 9). The computer programme is given in Appendix V.

Again, in this case simulation output was in the form of grid node temperatures at time increment of 0.0001 hrs. (0.36 sec.). Temperatures at locations (1), (2) and (3) (Fig.9 and 10) for 0 secs., 36 sec, 72 sec, 108 sec, 144 secs., 180 secs., 360 secs., 540 secs. and 720 secs. are given in table 5.

6.3 [illegible] SOLIDIFICATION EXPERIMENTAL RESULTS

Temperatures at three locations (1), (2), and (3) (Fig.8 and 9) at an interval of 30 seconds were obtained by monitoring the output of Chromel-Alumel thermocouples for two aluminium castings in a dry sand mold. Temperature output is reported in table 6 and 7.

Table 1[illegible]: [illegible] Computer [illegible] Approach
[illegible] PLANT

Δt = .0001 hrs.

Location / Solidification Time	(21,21) T(°C) 1	(19,21) T(°C) 2	(17,21) T(°C) 3	(15,21) T(°C) 4	(13,21) T(°C) 5	(10,21) T(°C) 6	(6,21) T(°C) 7	(2,21) T(°C) 8
1. 0.0 hrs	1350(732)	1350(732)	1350(732)	1350(732)	1207(653)	77(25)	77(25)	77(25)
2. 0.01 hrs (36 sec)	1260(682)	1258(681)	1252(678)	1243(673)	1238(670)	80(27)	77(25)	77(25)
3. 0.02 hrs. (72 sec.)	1240(671)	1240(671)	1238(670)	1234(668)	1230(666)	97(36)	77(25)	77(25)
4. 0.03 hrs (108 sec.)	1237(669)	1237(669)	1235(668)	1231(666)	1228(664)	123(51)	77(25)	77(25)
5. 0.04 hrs (144 sec.)	1235(668)	1235(668)	1233(667)	1230(665)	1226(663)	154(68)	78(25)	77(25)
6. 0.05 hrs (180 sec.)	1233(667)	1233(667)	1231(666)	1227(665)	1225(663)	185(85)	80(27)	77(25)
7. 0.1 hrs (360 sec) 6 minutes	1225(663)	1225(663)	1224(662)	1221(661)	1220(660)	320(160)	102(39)	78(25)
8. 0.15 hrs (540 sec.) 9 minutes	1222(661)	1221(661)	1220(660)	1218(659)	1216(658)	437(225)	137(58)	82(28)
9. 0.2 hrs. (720 sec.) 12 minutes	1218(659)	1217(658)	1216(658)	1214(657)	1213(656)	487(250)	175(79)	88(31)

Table 8: Two Dimensional Computer Simulation : Modified Approach

[illegible]

Δt = 0.0001 hrs

Sim. Time Hr. (Elapsed Time)	(21, 22) °F(°C)	(19, 22) °F(°C)	(17, 22) °F(°C)	(15, 21) °F(°C)	Interface (14, 21) °F(°C)	(13, 21) °F(°C)	(6, 21) °F(°C)	(2, 21) °F(°C)
1. 0.00 hrs.	1350(732)	1350(732)	1350(732)	1350(732)	1350(732)	77(25)	77(25)	77(25)
2. 0.01 hrs (36 secs.)	1300(709)	1296(703)	1290(703)	1287(697)	1280(693)	80(27)	77(25)	77(25)
3. 0.02 hrs (72 secs.)	1250(682)	1258(681)	1253(678)	1246(674)	1241(672)	97(35)	77(25)	77(25)
4. 0.03 hrs (108 secs.)	1241(672)	1240(671)	1239(671)	1235(668)	1232(667)	125(52)	77(25)	77(25)
5. 0.04 hrs. (144 secs.)	1233(670)	1236(670)	1233(669)	1232(667)	1230(666)	150(69)	78(25)	77(25)
6. 0.05 hrs. (180 secs.)	1235(669)	1235(668)	1234(668)	1230(666)	1230(665)	187(83)	80(27)	77(25)
7. 0.1 hrs (360 secs.)	1229(665)	1230(665)	1225(663)	1223(662)	1222(661)	322(161)	102(39)	78(25)
8. 0.15 hrs (540 secs.)	1223(662)	1223(662)	1222(661)	1220(660)	1218(659)	419(215)	130(50)	82(25)
9. 0.2 hrs. (720 secs.) 12 minutes	1220(660)	1220(660)	1218(659)	1216(658)	1215(657)	4[illegible](25[illegible])	175(79)	88(31)

Table 9 Two dimensional computer simulation: Schultz Approach Along Diagonal Plane

Δt = 0.0001 hrs

Location / Cell designation / Time	(21, 21) °F(°C)	(19, 19) °F(°C)	(17, 17) °F(°C)	(15, 15) °F(°C)	Interface (14, 14) °F(°C)	(10, 10) °F(°C)	(6, 6) °F(°C)	(2, 2) °F(°C)
1. 0.0 hrs.	1350(732)	1350(732)	1350(732)	1350(732)	845(452)	77(25)	77(25)	77(25)
2. 0.01 hrs (36 secs.)	1250(677)	1253(678)	1245(674)	1231(666)	1217(658)	77(25)	77(25)	77(25)
3. 0.02 hrs (72 secs.)	1240(671)	1239(670)	1235(668)	1225(663)	1215(657)	77(25)	77(25)	77(25)
4. 0.03 hrs (108 secs.)	1237(669)	1236(669)	1232(667)	1222(661)	1213(656)	79(26)	77(25)	77(25)
5. 0.04 hrs. (144 secs.)	1235(668)	1234(668)	1230(666)	1221(660)	1212(656)	83(28)	77(25)	77(25)
6. 0.05 hrs. (180 secs.) 3 minutes	1233(667)	1232(667)	1229(665)	1219(659)	1211(655)	88(31)	77(25)	77(25)
7. 0.1 hrs. (360 secs.) 6 minutes	1225(663)	1223(663)	1222(661)	1215(657)	1209(654)	127(53)	78(25)	77(25)
8. 0.15 hrs (540 secs.) 9 minutes	1222(661)	1221(661)	1218(659)	1212(655)	1205(652)	170(77)	81(27)	77(25)
9. 0.2 hrs. (720 secs.) 12 minutes	1218(659)	1217(658)	1214(657)	1209(654)	1204(651)	208(98)	86(30)	77(25)

Table 6 Two dimensional Computer Simulation: Modified Approach
ALONG DIAGONAL PLANE

Δt = 0.0001 hrs.

Location / Cell- / Time	(21,21)	(19,19)	(17,17)	(15,15)	Interface (14,14)	(10,10)	(6,6)	(2,2)
	°F(°C)	°F(°C)	°F(°C)	°F(°C)	°F(°C)	°F(°C)	°F(°C)	°F(°C)
1. 0.0 hrs.	1350(732)	1350(732)	1350(732)	1350(732)	1350(732)	77(25)	77(25)	77(25)
2. 0.01 hrs (36 sec.)	1300(705)	1303(706)	1283(693)	1252(683)	1240(671)	77(25)	77(25)	77(25)
3. 0.02 hrs (72 sec.)	1260(682)	1257(680)	1243(676)	1237(669)	1225(663)	77(25)	77(25)	77(25)
4. 0.03 hrs. (108 sec.)	1241(672)	1240(671)	1237(669)	1223(665)	1219(659)	79(26)	77(25)	77(25)
5. 0.04 hrs (144 sec.)	1238(670)	1237(667)	1234(668)	1223(663)	1216(658)	82(28)	77(25)	77(25)
6. 0.05 hrs (180 sec.) 3 minutes	1236(669)	1235(668)	1231(666)	1223(662)	1214(657)	88(31)	77(25)	77(25)
7. 0.1 hrs (360 sec.) 6 minutes	1229(665)	1227 (664)	1225(663)	1217(658)	1211(655)	123(51)	73(25)	77(25)
8. 0.15 hrs (540 sec.) 9 minutes	1224(662)	1223(662)	1220(660)	1214(657)	1203(653)	170(77)	81(27)	77(25)
9. 0.2 hrs (720 sec.) 12 minutes	1220(660)	1219(659)	1216(658)	1211(655)	1206(652)	209(98)	86(30)	77(25)

Table 5- Three dimensional Computer Simulation Results

Solidi- fication Location (time)	$T(^oC)$ 1	$T(^oC)$ 2	$T(^oC)$ 3
1. 0.0 km. = 0.0 sec.	1350(732)	1350(732)	1350(732)
2. 0.01 km.= 36 sec.	1306(708)	1298(703)	1280(693)
3. 0.02 km = 72 sec.	1252(678)	1249(676)	1236(669)
4. 0.03 km = 108 sec.	1240(671)	1234(668)	1216(658)
5. 0.04 km = 144 sec.	1225(663)	1224(662)	1200(649)
6. 0.05 km.= 180 sec. (3 min)	1228(665)	1213(657)	1190(640)
7. 0.1 km = 360 sec. (6 min)	1220(660)	1210(654)	1170(632)
8. 0.15 km = 540 sec. (9 min)	1212(656)	1202(650)	1165(628)
9. 0.20 km = 720 sec. (12 min)	1203(652)	11[illegible](6[illegible]3)	1115(602)

Table 6: Experimental Measurements of Solidification of Al casting

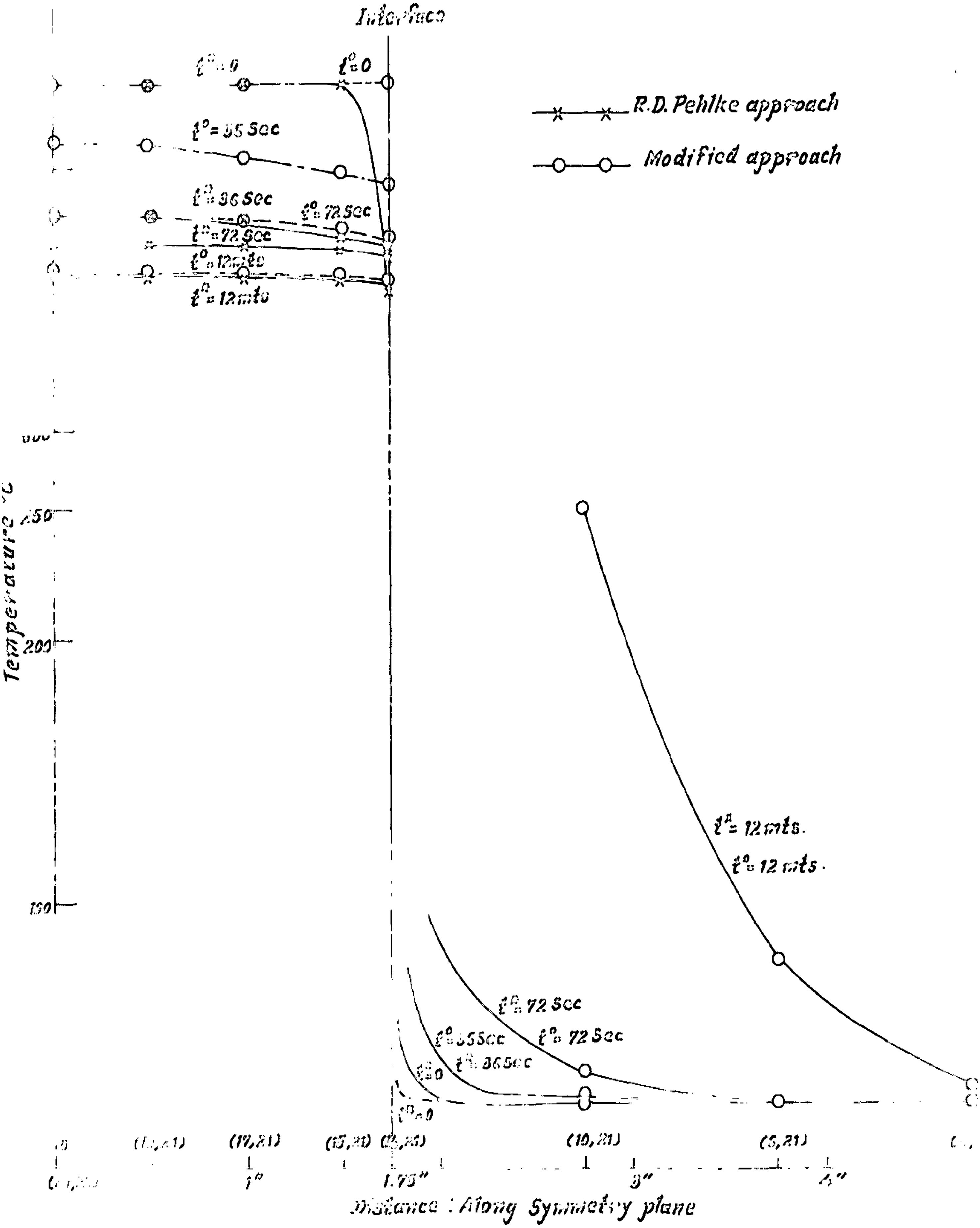
FIRST CASTING — Room Temp = 31°C

Thermo-couple / Solidification Time	mv = °C+R.T = Temp.°C 1	mv = °C+R.T.=Temp°C 2	mv = °C+ R.T. = Temp°C 3
1. 0 sec.	-	-	-
2. 30 sec.	26.8 = 645+31 = 676	26.6 = 640+31 = 671	26.3 = 633+31 = 664
3. 60 sec.(1)	26.4 = 635+31 = 666	26.2 = 630+31 = 661	26.3 = 633+31 = 664
4. 90 sec.	26.2 = 630+31 = 661	26.2 = 630+31 = 661	26.3 = 633+31 = 664
5. 120 sec.(2)	26.2 = 630+31 = 661	26.2 = 630+31 = 661	26.9 = 623+31 = 654
6. 150 sec.	26.2 = 630+31 = 661	26.2 = 630+31 = 661	25.6 = 616+31 = 647
7. 180 sec.(3)	26.2 = 630+31 = 661	26.2 = 630+31 = 661	25.3 = 609+31 = 640
8. 210 sec.	26.2 = 630+31 = 661		25.0 = 602+31 = 633
9. 240 sec.(4)	26.2 = 630+31 = 661	26.1 = 628+31 = 659 ↑	24.6 = 593+31 = 624
10. 270 sec.	26.2 = 630+31 = 661	26.0 = 626+31 = 657	24.3 = 586+31 = 617
11. 300 sec.(5)	26.1 = 628+31 = 659	25.9 = 623+31 = 654	24.0 = 579+31 = 610
12. 330 sec.	26.0 = 626+31 = 657	25.8 = 621+31 = 652	23.7 = 572+31 = 603
13. 360 sec.(6)	25.9 = 623+31 = 654	25.8 = 621+31 = 652	23.4 = 565+31 = 586
14. 390 sec.	25.8 = 621+31 = 652	25.7 = 619+31 = 650	23.2 = 560+31 = 591
15. 420 sec.(7)	25.7 = 619+31 = 650	25.6 = 616+31 = 647	23.0 = 555+31 = 586
16. 450 sec.	25.7 = 619+31 = 650	25.5 = 614+31 = 644	22.2 = 537+31 = 568
17. 480 sec.(8)	25.6 = 616+31 = 647	25.4 = 612+31 = 643	21.8 = 527+31 = 558
18. 510 sec.	25.5 = 614+31 = 645	25.3 = 609+31 = 640	21.0 = 508+31 = 539
19. 540 sec.(9)	25.4 = 612+31 = 643	25.2 = 607+31 = 638	20.6 = 499+31 = 53D
		25.1 = 605+31 = 636	

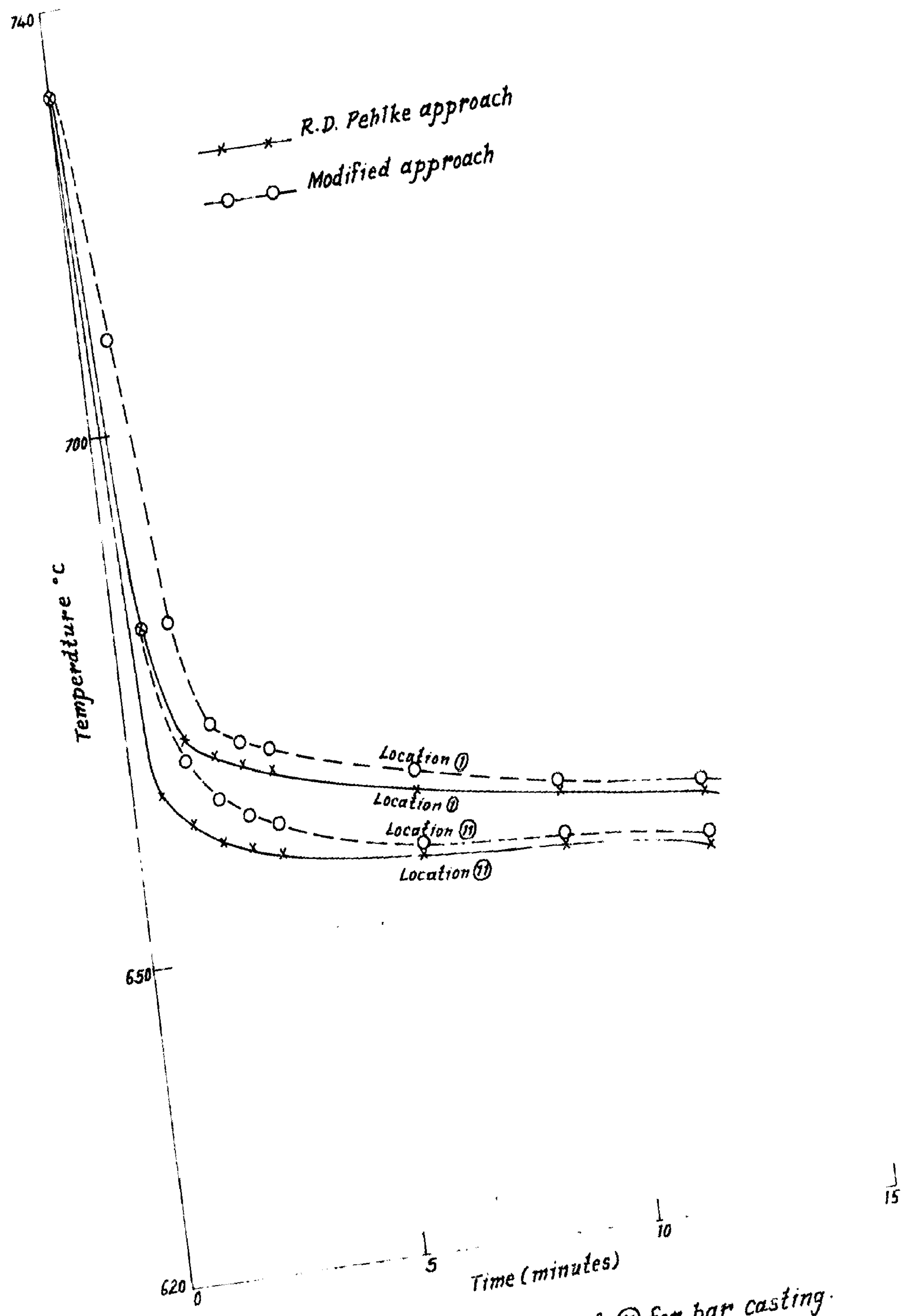
Table 7c Experimental Measurements of Solidification of Al Casting

Second Casting Room Temp. = 30 °C (R.T.)

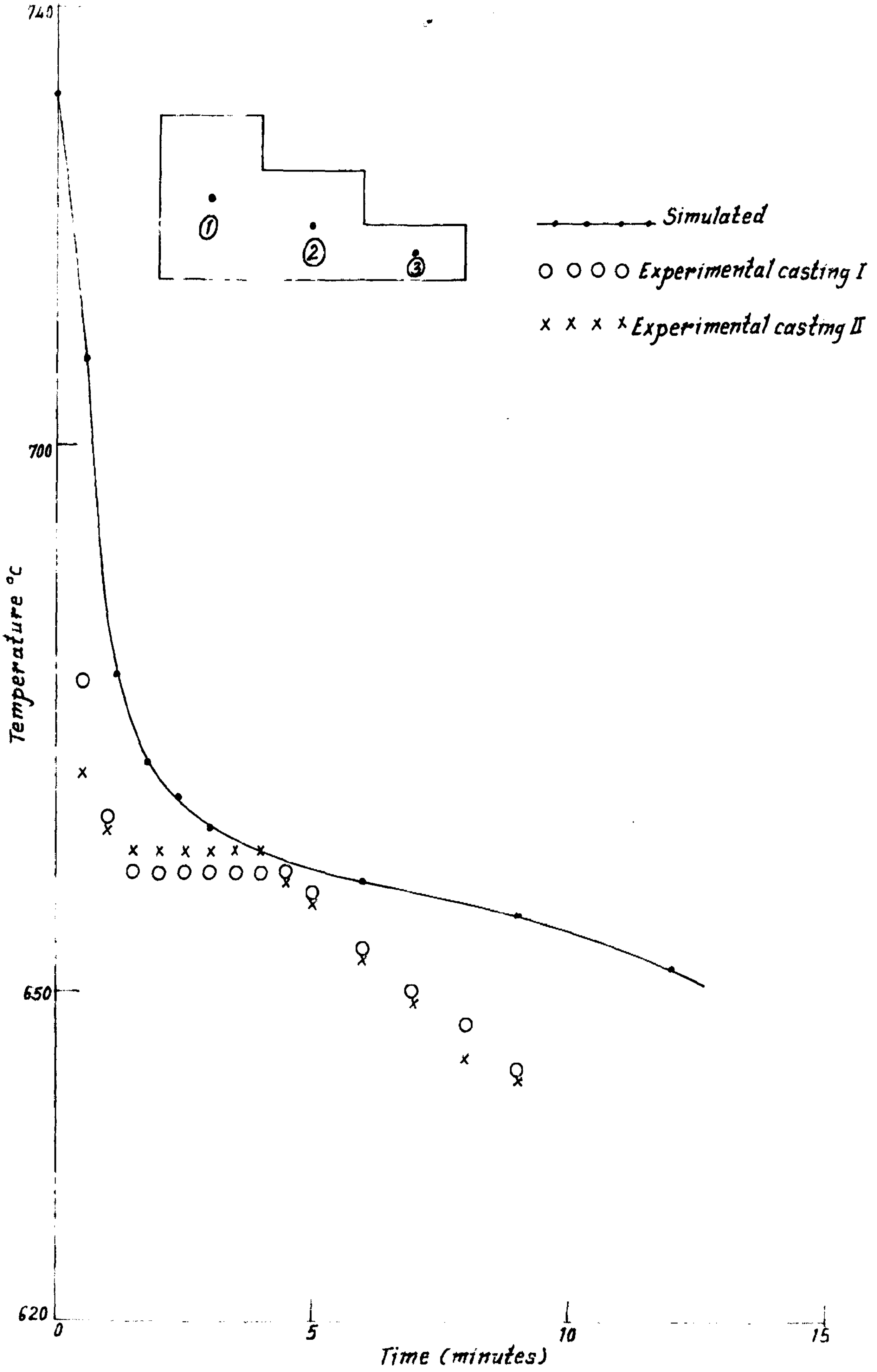
Sl. No. / Time in secs.	mV = °C + RT = Temp. °C 1	mV = °C+R.T.= Temp. °C 2	mV = °C+ R.T. = Temp. °C 3
1. 0 secs.	-	-	-
2. 30 secs.	25.6 = 640+30 = 670	25.5 = 638+30 = 668	25.3 = 633+30 = 663
3. 60 secs.(1)	25.4 = 635+30 = 665	25.3 = 633+30 = 663	25.3 = 633+30 = 663
4. 90 secs.	25.3 = 633+30 = 663	25.3 = 633+30 = 663	25.3 = 633+30 = 663
5. 120 secs.(2)	25.3 = 633+30 = 663	25.3 = 633+30 = 663	25.1 = 628+30 = 658
6. 150 secs.	25.3 = 633+30 = 663	25.3 = 633+30 = 663	24.8 = 621+30 = 651
7. 180 secs.(3)	25.3 = 633+30 = 663	25.2 = 630+30 = 660	24.5 = 614+30 = 644
8. 210 secs.	25.3 = 633+30 = 663	25.2 = 630+30 = 660	24.2 = 607+30 = 637
9. 240 secs.(4)	25.3 = 633+30 = 663.	25.1 = 628+30 = 658	24.0 = 602+30 = 632
10. 270 secs.	25.2 = 630+30 = 660	25.0 = 626+30 = 656	23.7 = 595+30 = 625
11. 300 secs. (5)	25.1 = 628+30 = 658	24.9 = 623+30 = 653	23.3 = 586+30 = 616
12. 330 secs.	25.0 = 626+30 = 656	24.8 = 621+30= 651	23.0 = 579+30 = 609
13. 360 secs. (6)	24.9 = 623+30 = 653	24.7 = 619+30 = 649	23.7 = 572+30 = 602
14. 390 secs.	24.8 = 621+30 = 651	24.0 = 602+30 = 632	23.4 = 565+30 = 595
15. 420(7)	24.7 = 619+30 = 649	24.6 = 616+30 = 646	23.0 = 555+30 = 585
16. [illegible] 450 Sec.	24.6 = 616+30 = 646	24.4 = 612+30 = 642	22.8 = 551+30 = 581
17. 480(8)	24.5 = 614+30 = 644	24.3 = 609+30 = 639	22.6 = 546+30 = 576
18. 510 Sec.	24.4 = 612+30 = 642	24.2 = 607+30 = 637	22.4 = 541+30 = 571
19. 540(9)	24.3 = 609+30 = 639	24.1 = 605+30 = 635	22.0 = 532+30 = 562
20. 570 Sec.	24.2 = 607+30 = 637	24.1 = 605+30 = 635	22.0 = 532+30 = 562
21. 600(10)	24.1 = 605+30 = 635	24.0 = 602+30 = 632	21.8 = 527+30 = 557



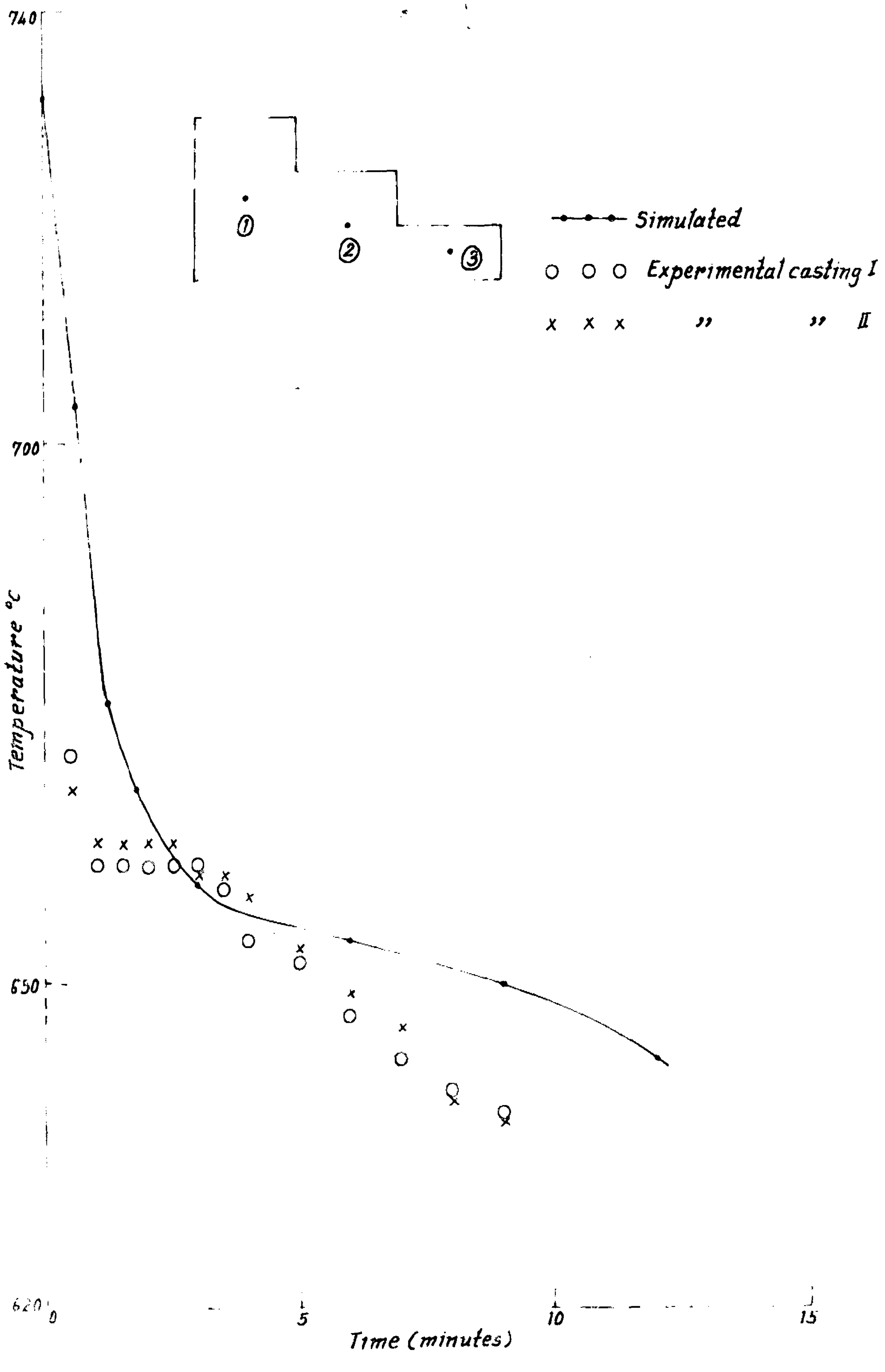
Graph 1 : Solidification profiles along symmetry plane for bar casting.



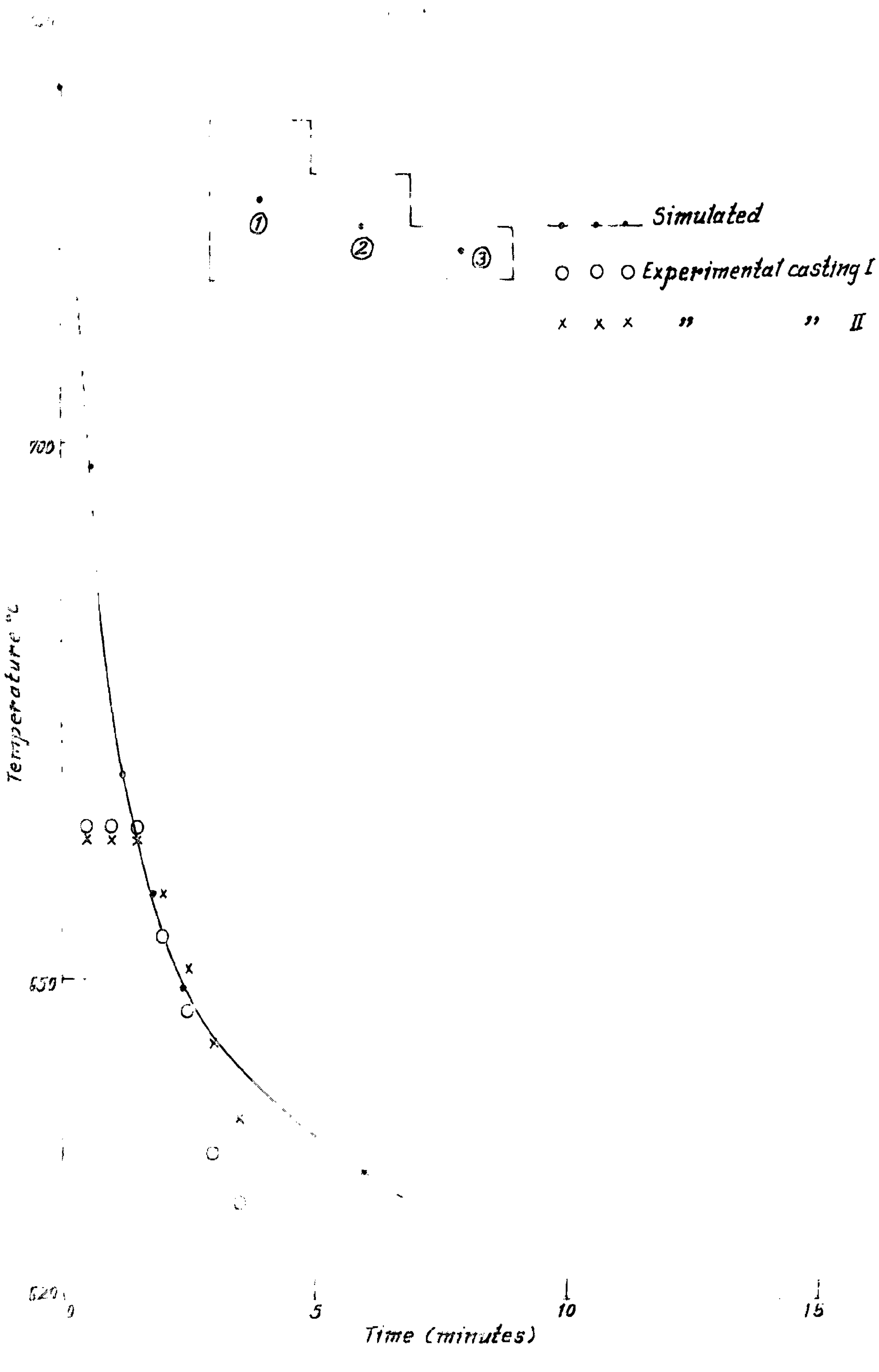
Graph 2 : Cooling curve for Location ① & ⑪ for bar casting.



Graph 3 : Cooling curve for location ① for experimental casting.



Graph 4 : Cooling curve for location ② for experimental casting.



Graph 5 : Cooling curve for location ③ for experimental casting.

CHAPTER -7 ——— DISCUSSION

7.1 TWO DIMENSIONAL CONDUCTION SOLIDIFICATION

Computer simulation for two dimensional conduction heat transfer was carried out as per (1) R.D. Pehlke approach and (2) Modified approach. The temperature along symmetry plane and cooling curves at two specified locations (at the centre of casting (21,21) and near the corner edge (15,15) are plotted in graph I and graph II for both the approaches.

It is noted from graph I that there is a temperature difference of the order of 25° to 27°C from the metal/mold interface to the centre of the casting for 36 seconds solidification profile. Further for 72 seconds solidification profile, the difference of temperature is of the order of only 6°C to 11°C from the metal/mold interface to the centre of the casting. While for more than 72 second solidification profiles, the difference in the temperatures at difference location from metal/mold interface to the centre of the casting is just 2°C to 3°C only. It is also noted that practically there is no difference in temperatures at different locations in the sand mold for the complete solidification time.

Thus we find that it is only the period of about 72 seconds, where there is a difference of temperatures at different

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locations in the two approaches. The initial period of 72 seconds is very small in comparison to the complete solidification time of 12 to 15 minutes.

Here it may be pointed out that the procedure of calculating interface temperatures used in Prof. Pehlke approach is not very simple, and is certainly very difficult for complicated castings. In the modified approach these calculations are not required and in addition computer time is also saved. Moreover this additional exercise of calculating interface temperature in no way improves the results for the major part of solidification period.

Thus, it is concluded that metal/sand interface temperatures should be the initial metal temperature as it enters the mold.

It is seen from the results that the assumption of taking mold/air interface temperature as ambient temperature is valid for the complete solidification time.

7.2 [illegible]

Computer simulation for three dimensional conduction heat transfer was carried out as per modified approach. Cooling curves of specified locations (1), (2) and (3) (Fig.8 and 9) are obtained by noting the out-put of thermo-couple placed at these locations. The computer simulated and experimentally obtained temperature distributions at 1, 2 and 3 are plotted in graph 3, graph 4 and graph 5 respectively.

The cooling curves for location 1, 2 and 3 are in line with usual temperature profiles for pure metal and compare satisfactorily with the experimental cooling curves.

It has been observed that it is difficult to measure the thermocouple output for location 3 because of fast rate of cooling of this small cross-section.

7.3 SUGGESTIONS FOR FUTURE WORK

The following topics are suggested for further study for three dimensional computer simulation.

1. An analysis of the heat transfer model considering the formation of air gap between the mold and metal.

2. A study of the effect of different chill materials on the solidification profiles by computer simulation.

3. Determination of the correct size and shape of chills by computer simulation.

CHAPTER 8 ——— CONCLUSIONS

1. Initial metal/sand interface temperature should be the initial metal temperature as it enters the mold.

2. The explicit finite difference approximations to the differential equations describing three dimensional conduction heat transfer provide an accurate method of determining temperature distribution at different locations at different times in the castings.

3. The computer simulation can forecast whether a casting will solidify with shrinkage defects or not. Using this technique changes can be made in the casting design to provide an optimum arrangements for production of a sound casting.

4. It is possible to find thermal gradient in the casting by computer simulation, which controls the structure of casting.

5. Satisfactory agreement was obtained between thermocouple measurements on the solidifying casting and the computer simulation using independently determined specific heat and conductivity of aluminium and sand.

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C                            *** COMPUTER PROGRAM ***

C      CONDUCTIVITY METAL
C  C   CNMTEM/TEMP/CNM
  100 CONTINUE
      READ101,TEMP
  101 FORMAT(F20.0)
      IF(TEMP)999,201,201
  201 IF(TEMP-180.)202,202,203
  202 CNM=0.03090*TEMP+136.5
      GOTO901
  203 IF(TEMP-610.)204,204,205
  204 CNM=0.01161*(180.-TEMP)+140.
      GOTO901
  205 IF(TEMP-1197.)206,206,207
  206 CNM=0.02203*(610.-TEMP)+135.
      GOTO901
  207 IF(TEMP-1242.)208,208,209
  208 CNM=1.53*(1197.-TEMP)+121.7
      GOTO901
  209 CNM=0.00774*(TEMP-1242.)+52.9
      GOTO901
  901 PUNCH102,TEMP,CNM
  102 FORMAT(2F20.8)
      GOTO100
  999 STOP
      END
C                            *** COMPUTER PROGRAM ***

C      SPECIFICHEAT METAL
C  C   CPMTEM/TEMP/CPM
  100 CONTINUE
  101 FORMAT(F20.0)
      READ101,TEMP
      IF(TEMP)999,201,201
  201 IF(TEMP-960.)202,202,203
  202 CPM=0.0000607*TEMP+0.213000
      GOTO901
  203 IF(TEMP-1140.)204,204,205
  204 CPM=0.0001079*(TEMP-960.)+0.270090
      GOTO901
  205 IF(TEMP-1197.)206,206,207
  206 CPM=0.0002020*(TEMP-1140.)+0.291550
      GOTO901
  207 IF(TEMP-1219.06)208,208,209
  208 CPM=0.0060*(TEMP-1197.)+0.303072
      GOTO901
  209 IF(TEMP-1242.)210,210,211
  210 CPM=0.0040*(1242.-TEMP)+0.313
      GOTO901
  211 CPM=0.313
      GOTO901
  901 PUNCH102,TEMP,CPM
  102 FORMAT(2F20.0)
      GOTO100
  999 STOP
      END
```

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C                          ooo   COMPUTER  PROGRAM  ooo
C      SPECIFICHEAT SAND
C  C   CPSTEM/TEMP/CPS//JMAX,T,C
       DIMENSIONT(20),C(20)
       READ101,JMAX
   101 FORMAT(I5)
       READ102,(T(J),J=1,JMAX)
   102 FORMAT(8F10.4)
       READ103,(C(J),J=1,JMAX)
   103 FORMAT(8F10.4)
   100 CONTINUE
       READ104,TEMP
   104 FORMAT(F20.0)
       IF(TEMP-T(1))999,601,601
   601 JL=1
   602 JR=JL+1
       IF(TEMP-T(JR))603,603,605
   603 SLOPE=(C(JR)-C(JL))/(T(JR)-T(JL))
       CPS=C(JL)+SLOPE*(TEMP-T(JL))
       GOTO801
   605 JL=JL+1
       IF(JL-JMAX)602,999,999
   801 PUNCH202,TEMP,CPS
   202 FORMAT(2F20.0)
       GOTO100
   999 STOP
       END
     8
     0.       300.       500.       700.       900.      1100.      1500.      200
      .170      .222      .260      .255      .266      .272      .276
C                          ooo   COMPUTER  PROGRAM  ooo
C      CONDUCTIVITY SAND
C  C   CNSTEM/TEMP/CNS//JMAX,T,C
       DIMENSION T(20),C(20)
       READ101,JMAX
   101 FORMAT(I5)
       READ102,(T(J),J=1,JMAX)
   102 FORMAT(7F10.4)
       READ103,(C(J),J=1,JMAX)
   103 FORMAT(7F10.4)
   100 CONTINUE
       READ104,TEMP
   104 FORMAT(F20.0)
       IF(TEMP-T(1))999,601,601
   601 JL=1
   602 JR=JL+1
       IF(TEMP-T(JR))603,603,605
   603 SLOPE=(C(JR)-C(JL))/(T(JR)-T(JL))
       CNS=C(JL)+SLOPE*(TEMP-T(JL))
       GOTO801
   605 JL=JL+1
   801 PUNCH202,TEMP,CNS
   202 FORMAT(2F20.0)
       GOTO100
   999 STOP
       END
     7
     0.       300.       700.      1000.      1300.      1600.      1800.
     0.35      0.40      0.42      0.45      0.52      0.60      0.84
```

```
C                    *** COMPUTER PROGRAM ***
C     INITIAL INTERFACE TEMPERATURE
C  C  TINTO
      DIMENSIONTCPS(20),CCPS(20)
      READ11,ROHM,ROHS,DHF,TSTR,TFIN
   11 FORMAT(8F10.6)
      READ11,TEMM,TEMS,AORC
      READ12,JCPS
      READ11,(TCPS(J),J=1,JCPS)
      READ11,(CCPS(J),J=1,JCPS)
   12 FORMAT(16I5)
      CPMM=FCPMT(TEMM)
      CALL CPST(TEMS,JCPS,TCPS,CCPS,CPSS)
      A=ROHM*CPMM*TEMM+ROHS*CPSS*TEMS
      TOLD=TSTR
   61 CPMI=FCPMT(TOLD)
      CALL CPST(TOLD,JCPS,TCPS,CCPS,CPSI)
      TNEW=A/(ROHM*CPMI+ROHS*CPSI)
      TNEW=A/(ROHM*CPMI+ROHS*CPSI)
      PUNCH11,TNEW,TOLD
      IF(TNEW-TSTR)23,21,21
   21 IF(ABSF(TNEW-TOLD)-AORC)51,51,22
   22 TOLD=TNEW
      GOTO61
   23 B=ROHM*DHF
      TOLD=TFIN
   62 CPMI=FCPMT(TOLD)
      CALL CPST(TOLD,JCPS,TCPS,CCPS,CPSI)
      TNEW=(A+B)/(ROHM*CPMI+ROHS*CPSI)
      PUNCH11,TNEW,TOLD
      IF(TNEW-TFIN)24,26,26
   24 IF(ABSF(TNEW-TOLD)-AORC)51,51,25
   25 TOLD=TNEW
      GOTO62
   26 C=(TSTR*B)/(TSTR-TFIN)
      D=B/(TSTR-TFIN)
      TOLD=(TSTR+TFIN)/2.
   63 CPMI=FCPMT(TOLD)
      CALL CPST(TOLD,JCPS,TCPS,CCPS,CPSI)
      TNEW=(A+C)/(ROHM*CPMI+ROHS*CPSI+D)
      IF(ABSF(TNEW-TOLD)-AORC)51,51,27
   27 PUNCH11,TNEW,TOLD
      TOLD=TNEW
      GOTO63
   51 PUNCH11,TNEW
   99 STOP
      END
```

```
C                    *** COMPUTER PROGRAM ***

C     TWO DIMENSIONAL SOLIDIFICATION  R D PEHLKE APPROACH
//KGARG    JOB     ,RU278J K,15,50,00
//K3TRAN EXEC FORTRAN(BCD,MAP)
C  C  HEAT TRANSFER TWO DIMENSIONAL SOLIDIFICATION
      DIMENSION TG(21,21),TN(21,21),TCPS(20),CCPS(20),TCNS(20),CCNS(20)
      COMMON ROMM,ROMS,DHF,TSTR,TFIN,AQRC
      COMMON JCPS,TCPS,CCPS
 1001 FORMAT(8F10.4)
 1002 FORMAT(16I5)
 0001 FORMAT(8H TIME IS,F10.5)
      READ1001,ROMM,ROMS,DHF,TSTR,TFIN,TSTR
      READ1001,DT,DX,TEMM,TEMS,AQRC,TMAX
      READ1002,IMAX,ISAN,JCPS,JCNS
      READ1001,(TCPS(N),N=1,JCPS)
      READ1001,(CCPS(N),N=1,JCPS)
      READ1001,(TCNS(M),M=1,JCNS)
      READ1001,(CCNS(M),M=1,JCNS)
      TIME=0.
      TBEG=TSTR
      DX=DX/12.
      DO3001I=1,ISAN
      DO3001J=1,IMAX
 3001 TG(I,J)=TEMS
      DO3002I=ISAN,IMAX
      DO3002J=1,IMAX
 3002 TG(I,J)=TEMM
      CALL TINT(TEMM,TEMS,TNEW)
      DO3003J=ISAN,IMAX
 3003 TG(ISAN,J)=TNEW
      ROMS=3.*ROMS
      CALL TINT(TEMM,TEMS,TNEW)
      TG(ISAN,ISAN)=TNEW
      ROMS=ROMS/3.
      PRINT0001,TIME
      PRINT1001,TG(21,21),TG(19,21),TG(17,21),TG(13,21),TG(14,21),TG(10,
     121),TG(6,21),TG(2,21)
      PRINT1001,TG(21,21),TG(19,19),TG(17,17),TG(13,13),TG(14,16),TG(10,
     110),TG(6,6),TG(2,2)
      PRINT1001,TG(17,17),TG(13,17),TG(14,17),TG(10,17),TG(6,17),TG(2,17
     1)
      TAX=DT/(DX*DX)
 5001 TIME=TIME+DT
      DO4001I=1,IMAX
      DO4001J=1,IMAX
      TIJ=TG(I,J)
      IF(I-1)9999,2002,2021
 2021 IF(I-ISAN)2022,2023,2024
 2022 IF(J-1)9999,2003,2023
 2023 IF(J-IMAX)2001,2004,9999
 2023 IF(J-1)9999,2010,2026
 2026 IF(J-IMAX)2009,2011,9999
 2024 IF(I-IMAX)2027,2006,9999
 2027 IF(J-1)9999,2007,2028
 2020 IF(J-IMAX)2009,2008,9999
```

```
 2001 CALL CPST(TIJ,JCPS,TCPS,CCPS,CPSG)
      CALL CNST(TIJ,JCNS,TCNS,CCNS,CNSG)
      UBRT=(CNSG*TXX)/(ROMS*CPSG)
      DRTD=TG(I,J+1)+TG(I,J-1)+TG(I+1,J)+TG(I-1,J)-4.*TIJ
      GO TO 4001
 2002 UBRT=0.
      DRTD=0.
      GO TO 4001
 2003 CALL CPST(TIJ,JCPS,TCPS,CCPS,CPSG)
      CALL CNST(TIJ,JCNS,TCNS,CCNS,CNSG)
      UBRT=(CNSG*TXX)/(ROMS*CPSG)
      DRTD=2.*TG(I,J+1)+2.*TG(I-1,J)-4.*TIJ
      GO TO 4001
 2004 CALL CPST(TIJ,JCPS,TCPS,CCPS,CPSG)
      CALL CNST(TIJ,JCNS,TCNS,CCNS,CNSG)
      UBRT=(CNSG*TXX)/(ROMS*CPSG)
      DRTD=2.*TG(I,J-1)+TG(I+1,J)+TG(I-1,J)-4.*TIJ
      GO TO 4001
 2005 CPMG=FCPMS(TIJ)
      CNMG=FCNMT(TIJ)
      UBRT=(CNMG*TXX)/(ROMM*CPMG)
      DRTD=TG(I,J+1)+TG(I,J-1)+TG(I+1,J)+TG(I-1,J)-4.*TIJ
      GO TO 4001
 2006 CPMG=FCPMS(TIJ)
      CNMG=FCNMT(TIJ)
      UBRT=(CNMG*TXX)/(ROMM*CPMG)
      DRTD=4.*TG(I-1,J)-4.*TIJ
      GO TO 4001
 2007 CPMG=FCPMS(TIJ)
      CNMG=FCNMT(TIJ)
      UBRT=(CNMG*TXX)/(ROMM*CPMG)
      DRTD=2.*TG(I,J+1)+2.*TG(I-1,J)-4.*TIJ
      GO TO 4001
 2008 CPMG=FCPMS(TIJ)
      CNMG=FCNMT(TIJ)
      UBRT=(CNMG*TXX)/(ROMM*CPMG)
      DRTD=2.*TG(I,J-1)+TG(I+1,J)+TG(I-1,J)-4.*TIJ
      GO TO 4001
 2009 CPMG=FCPMS(TIJ)
      CNMG=FCNMT(TIJ)
      CALL CPST (TIJ,JCPS,TCPS,CCPS,CPSG)
      CALL CNST (TIJ,JCNS,TCNS,CCNS,CNSG)
      UBRT=TXX/(ROMS*CPSG+ROMM*CPMG)
      DRTD=2.*CNSG*TG(I-1,J)+2.*CNMG*TG(I+1,J)+(CNSG+CNMG)*TG(I,J-1)+
     1(CNSG+CNMG)*TG(I,J+1)-4.*(CNSG+CNMG)*TIJ
      GO TO 4001
 2010 CPMG=FCPMS(TIJ)
      CNMG=FCNMT(TIJ)
      CALL CPST (TIJ,JCPS,TCPS,CCPS,CPSG)
      CALL CNST (TIJ,JCNS,TCNS,CCNS,CNSG)
      UBRT=(4.*TXX)/(3.*ROMS*CPSG+ROMM*CPMG)
      DRTD=2.*CNSG*TG(I-1,J)+(CNSG+CNMG)*TG(I,J+1)-(CNMG+3.*CNSG)*TIJ
      GO TO 4001
 2011 CPMG=FCPMS(TIJ)
      CNMG=FCNMT(TIJ)
      CALL CPST (TIJ,JCPS,TCPS,CCPS,CPSG)
      CALL CNST (TIJ,JCNS,TCNS,CCNS,CNSG)
      UBRT=(2.*TXX)/(ROMS*CPSG+ROMM*CPMG)
      DRTD=CNSG*TG(I-1,J)+CNMG*TG(I+1,J)+(CNSG+CNMG)*TG(I,J-1)-2.*(CNSG+
     1CNMG)*TIJ
```

```
4001 TN(I,J)=UDRT*URTD+TIJ
     IF(TIME-TDEG)6003,6001,6002
6002 CONTINUE
     PRINT0001,TDEG
6001 TDEG=TDEG+TETR
     PRINT1001,TG(21,21),TG(19,21),TG(17,21),TG(15,21),TG(14,21),TG(10,
    121),TG(6,21),TG(2,21)
     PRINT1001,TG(21,21),TG(19,19),TG(17,17),TG(15,15),TG(14,14),TG(10,
    110),TG(6,6),TG(2,2)
     PRINT1001,TG(17,17),TG(15,17),TG(14,17),TG(10,17),TG(6,17),TG(2,17
    1)
6003 CONTINUE
     DO3004I=1,IMAX
     DO3004J=1,IMAX
3004 TG(I,J)=TN(I,J)
     IF(TIME-TMAX)5001,9999,9999
9999 STOP
     END

     SUBROUTINE TINT(TEMM,TEMS,TNEW)
     DIMENSIONTCPS(20),CCPS(20)
     COMMON ROMM,ROMS,DHF,TSTR,TFIN,AORC
     COMMON JCPS,TCPS,CCPS
     CPMM=FCPMT(TEMM)
     CALL CPST(TEMS,JCPS,TCPS,CCPS,CPSS)
     A=ROMM*CPMM*TEMM+ROMS*CPSS*TEMS
     TOLD=TSTR
  41 CPMI=FCPMT(TOLD)
     CALL CPST(TOLD,JCPS,TCPS,CCPS,CPSI)
     TNEW=A/(ROMM*CPMI+ROMS*CPSI)
     IF(TNEW-TSTR)23,21,21
  21 IF(ABS (TNEW-TOLD)-AORC)51,51,22
  22 TOLD=TNEW
     GOTO41
  23 B=ROMM*DHF
     TOLD=TFIN
  42 CPMI=FCPMT(TOLD)
     CALL CPST(TOLD,JCPS,TCPS,CCPS,CPSI)
     TNEW=(A+B)/(ROMM*CPMI+ROMS*CPSI)
     IF(TNEW-TFIN)24,26,26
  24 IF(ABS (TNEW-TOLD)-AORC)51,51,25
  25 TOLD=TNEW
     GOTO42
  26 C=(TSTR*B)/(TSTR-TFIN)
     D=B/(TSTR-TFIN)
     TOLD=(TSTR+TFIN)/2.
  43 CPMI=FCPMT(TOLD)
     CALL CPST(TOLD,JCPS,TCPS,CCPS,CPSI)
     TNEW=(A+C)/(ROMM*CPMI+ROMS*CPSI+D)
     IF(ABS(TNEW-TOLD)-AORC)51,51,27
  27 TOLD=TNEW
     GOTO43
  51 CONTINUE
     RETURN
     END

     FUNCTION FCPMT(TEMP)
     IF(TEMP)999,201,201
 201 IF(TEMP-940.)202,202,203
 202 CPM=0.0000607*TEMP+0.213000
```

```
      GOTO901
  203 IF(TEMP-1160.)204,204,205
  204 CPM=0.0001075*(TEMP-940.)+0.270050
      GOTO901
  205 IF(TEMP-1242.)206,206,211
  206 CPM=0.0002020*(TEMP-1160.)+0.291550
      GOTO901
  211 CPM=0.313
  901 FCPMT=CPM
  999 RETURN
      END

      SUBROUTINE CPST(TEMP,JMAX,T,C,CPS)
      DIMENSIONT(20),C(20)
      IF(TEMP-T(1))999,601,601
  601 JL=1
  602 JR=JL+1
      IF(TEMP-T(JR))602,602,603
  602 SLOPE=(C(JR)-C(JL))/(T(JR)-T(JL))
      CPS=C(JL)+SLOPE*(TEMP-T(JL))
      GOTO999
  603 JL=JL+1
      IF(JL-JMAX)602,999,999
  999 RETURN
      END

      SUBROUTINE CNST(TEMP,JMAX,T,C,CNS)
      DIMENSION T(20),C(20)
      IF(TEMP-T(1))999,601,601
  601 JL=1
  602 JR=JL+1
      IF(TEMP-T(JR))602,602,603
  602 SLOPE=(C(JR)-C(JL))/(T(JR)-T(JL))
      CNS=C(JL)+SLOPE*(TEMP-T(JL))
      GO TO 999
  603 JL=JL+1
      IF(JL-JMAX)602,999,999
  999 CONTINUE
      RETURN
      END

      FUNCTION FCPMS(TEMP)
      IF(TEMP)999,201,201
  201 IF(TEMP-940.)202,202,203
  202 CPM=0.0000607*TEMP+0.213000
      GOTO901
  203 IF(TEMP-1160.)204,204,205
  204 CPM=0.0001075*(TEMP-940.)+0.270050
      GOTO901
  205 IF(TEMP-1197.)206,206,207
  206 CPM=0.0002020*(TEMP-1160.)+0.291550
      GOTO901
  207 IF(TEMP-1219.44)208,208,209
  208 CPM=0.3360*(TEMP-1197.)+0.309072
      GOTO901
  209 IF(TEMP-1242.)210,210,211
  210 CPM=0.3340*(1242.-TEMP)+0.313
      GOTO901
  211 CPM=0.313
  901 FCPMS=CPM
  999 CONTINUE
```

```
      RETURN
      END

      FUNCTION FCNMT(TEMP)
      IF(TEMP)999,201,201
  201 IF(TEMP-180.)202,202,203
  202 CNM=0.03058*TEMP+134.5
  203 IF(TEMP-610.)204,204,205
  204 CNM=0.01161*(180.-TEMP)+140.
      GOTO501
  205 IF(TEMP-1197.)206,206,207
  206 CNM=0.0?263*(610.-TEMP)+135.
      GOTO501
  207 IF(TEMP-1242.)208,208,209
  208 CNM=1.53*(1197.-TEMP)+121.7
      GOTO501
  209 CNM=0.00774*(TEMP-1242.)+52.9
      GOTO501
  501 FCNMT=CNM
  999 CONTINUE
      RETURN
      END
/*
// EXEC LNKEDT(MAP)
/*
// EXEC
 169.       100.       170.       1242.      1197.         .0005
   .0001       .25    1350.         77.        1.          .25
   21   14     8    7
   0.       300.       500.       700.       900.      1100.      1500.      280(
   .170        .222       .240       .235       .266       .272       .276
   0.       300.       700.       1000.      1300.     1600.      1800.
   0.55        0.48       0.42       0.45       0.52       0.68       0.84
/*
/*
```

```
C                          *** COMPUTER PROGRAM ***

C     TWO DIMENSIONAL SOLIDIFICATION  MODIFIED APPROACH
//KGARG    JOB      ,RU270JK,15,50,00
//MSTRAN EXEC FORTRAN(BCD,MAP)
C  C  HEAT TRANSFER TWO DIMENSIONAL SOLIDIFICATION
      DIMENSION TG(21,21),TN(21,21),TCPS(20),CCPS(20),TCNS(20),CCNS(20)
 1001 FORMAT(8F10.4)
 1002 FORMAT(16I5)
 8001 FORMAT(8H TIME IS,F10.5)
      READ1001,ROMM,ROMS,DHF,TSTR,TFIN,TETR
      READ1001,DT,DX,TEMM,TEMS,AGRC,TMAX
      READ1002,IMAX,ISAN,JCPS,JCNS
      READ1001,(TCPS(N),N=1,JCPS)
      READ1001,(CCPS(N),N=1,JCPS)
      READ1001,(TCNS(M),M=1,JCNS)
      READ1001,(CCNS(M),M=1,JCNS)
      TIME=0.
      TBEG=TETR
      DX=DX/12.
      DO3001I=1,ISAN
      DO3001J=I,IMAX
 3001 TG(I,J)=TEMS
      DO3002I=ISAN,IMAX
      DO3002J=I,IMAX
 3002 TG(I,J)=TEMM
      PRINT8001,TIME
      PRINT1001,TG(21,21),TG(19,21),TG(17,21),TG(15,21),TG(14,21),TG(10,
     121),TG(6,21),TG(2,21)
      PRINT1001,TG(21,21),TG(19,19),TG(17,17),TG(15,15),TG(14,14),TG(10,
     110),TG(6,6),TG(2,2)
      PRINT1001,TG(17,17),TG(15,17),TG(14,17),TG(10,17),TG(6,17),TG(2,17
     1)
      TKK=DT/(DX*DX)
 5001 TIME=TIME+DT
      DO4001I=1,IMAX
      DO4001J=I,IMAX
      TIJ=TG(I,J)
      IF(I-1)9999,2002,2021
 2021 IF(I-ISAN)2022,2023,2024
 2022 IF(J-I)9999,2005,2025
 2025 IF(J-IMAX)2001,2004,9999
 2023 IF(J-I)9999,2010,2026
 2026 IF(J-IMAX)2009,2011,9999
 2024 IF(I-IMAX)2027,2006,9999
 2027 IF(J-I)9999,2007,2028
 2028 IF(J-IMAX)2005,2008,9999
 2001 CALL CPST(TIJ,JCPS,TCPS,CCPS,CPSG)
      CALL CNST(TIJ,JCNS,TCNS,CCNS,CNSG)
      UORY=(CNSG*TKK)/(ROMS*CPSG)
      CRTD=TG(I,J+1)+TG(I,J-1)+TG(I+1,J)+TG(I-1,J)-4.*TIJ
      GO TO 4001
```

```
2002 UDRT=0.
     DRTD=0.
     GO TO 6001
2003 CALL CPST(TIJ,JCPS,TCPS,CCPS,CPSG)
     CALL CNST(TIJ,JCNS,TCNS,CCNS,CNSG)
     UDRT=(CNSG*TXX)/(ROHS*CPSG)
     DRTD=2.*TG(I,J+1)+2.*TG(I-1,J)-4.*TIJ
     GO TO 6001
2004 CALL CPST(TIJ,JCPS,TCPS,CCPS,CPSG)
     CALL CNST(TIJ,JCNS,TCNS,CCNS,CNSG)
     UDRT=(CNSG*TXX)/(ROHS*CPSG)
     DRTD=2.*TG(I,J-1)+TG(I+1,J)+TG(I-1,J)-4.*TIJ
     GO TO 6001
2005 CPMG=FCPMS(TIJ)
     CNMG=FCNMT(TIJ)
     UDRT=(CNMG*TXX)/(ROHM*CPMG)
     DRTD=TG(I,J+1)+TG(I,J-1)+TG(I+1,J)+TG(I-1,J)-4.*TIJ
     GO TO 6001
2006 CPMG=FCPMS(TIJ)
     CNMG=FCNMT(TIJ)
     UDRT=(CNMG*TXX)/(ROHM*CPMG)
     DRTD=4.*TG(I-1,J)-4.*TIJ
     GO TO 6001
2007 CPMG=FCPMS(TIJ)
     CNMG=FCNMT(TIJ)
     UDRT=(CNMG*TXX)/(ROHM*CPMG)
     DRTD=2.*TG(I,J+1)+2.*TG(I-1,J)-4.*TIJ
     GO TO 6001
2008 CPMG=FCPMS(TIJ)
     CNMG=FCNMT(TIJ)
     UDRT=(CNMG*TXX)/(ROHM*CPMG)
     DRTD=2.*TG(I,J-1)+TG(I+1,J)+TG(I-1,J)-4.*TIJ
     GO TO 6001
2009 CPMG=FCPMS(TIJ)
     CNMG=FCNMT(TIJ)
     CALL CPST (TIJ,JCPS,TCPS,CCPS,CPSG)
     CALL CNST (TIJ,JCNS,TCNS,CCNS,CNSG)
     UDRT=TXX/(ROHS*CPSG+ROHM*CPMG)
     DRTD=2.*CNSG*TG(I-1,J)+2.*CNMG*TG(I+1,J)+(CNSG+CNMG)*TG(I,J-1)+
    1(CNSG+CNMG)*TG(I,J+1)-4.*(CNSG+CNMG)*TIJ
     GO TO 6001
2010 CPMG=FCPMS(TIJ)
     CNMG=FCNMT(TIJ)
     CALL CPST (TIJ,JCPS,TCPS,CCPS,CPSG)
     CALL CNST (TIJ,JCNS,TCNS,CCNS,CNSG)
     UDRT=(4.*TXX)/(3.*ROHS*CPSG+ROHM*CPMG)
     DRTD=2.*CNSG*TG(I-1,J)+(CNSG+CNMG)*TG(I,J+1)-(CNMG+3.*CNSG)*TIJ
     GO TO 6001
2011 CPMG=FCPMS(TIJ)
     CNMG=FCNMT(TIJ)
     CALL CPST (TIJ,JCPS,TCPS,CCPS,CPSG)
     CALL CNST (TIJ,JCNS,TCNS,CCNS,CNSG)
     UDRT=(2.*TXX)/(ROHS*CPSG+ROHM*CPMG)
     DRTD=CNSG*TG(I-1,J)+CNMG*TG(I+1,J)+(CNSG+CNMG)*TG(I,J-1)-2.*(CNSG+
    1CNMG)*TIJ
6001 TN(I,J)=UDRT*DRTD+TIJ
     IF(TIME-TUSO)6003,6001,6002
6002 CONTINUE
```

```
      PRINT0001,TDEG
 6001 TDEG=TDEG+TETR
      PRINT1001,TG(21,21),TG(19,21),TG(17,21),TG(15,21),TG(14,21),TG(10,
     121),TG(6,21),TG(2,21)
      PRINT1001,TG(21,21),TG(19,19),TG(17,17),TG(15,15),TG(14,14),TG(10,
     110),TG(6,6),TG(2,2)
      PRINT1001,TG(17,17),TG(15,17),TG(14,17),TG(10,17),TG(6,17),TG(2,17
     1)
 6000 CONTINUE
      DO3006I=1,IMAX
      DO3006J=1,IMAX
 3006 TG(I,J)=TN(I,J)
      IF(TIME-TMAX)3001,9999,9999
 9999 STOP
      END

      SUBROUTINE CPST(TEMP,JMAX,T,C,CPS)
      DIMENSIONT(20),C(20)
      IF(TEMP-T(1))999,601,601
  601 JL=1
  602 JR=JL+1
      IF(TEMP-T(JR))602,602,603
  602 SLOPE=(C(JR)-C(JL))/(T(JR)-T(JL))
      CPS=C(JL)+SLOPE*(TEMP-T(JL))
      GOTO999
  603 JL=JL+1
      IF(JL-JMAX)602,999,999
  999 RETURN
      END

      SUBROUTINE CNST(TEMP,JMAX,T,C,CNS)
      DIMENSION T(20),C(20)
      IF(TEMP-T(1))999,601,601
  601 JL=1
  602 JR=JL+1
      IF(TEMP-T(JR))602,602,603
  602 SLOPE=(C(JR)-C(JL))/(T(JR)-T(JL))
      CNS=C(JL)+SLOPE*(TEMP-T(JL))
      GO TO 999
  603 JL=JL+1
      IF(JL-JMAX)602,999,999
  999 CONTINUE
      RETURN
      END

      FUNCTION FCPMS(TEMP)
      IF(TEMP)999,201,201
  201 IF(TEMP-960.)202,202,203
  202 CPM=0.0000607*TEMP+0.219000
      GOTO901
  203 IF(TEMP-1160.)204,204,205
  204 CPM=0.0001079*(TEMP-960.)+0.270050
      GOTO901
  205 IF(TEMP-1197.)206,206,207
  206 CPM=0.0002020*(TEMP-1160.)+0.291990
      GOTO901
  207 IF(TEMP-1219.44)208,208,209
  208 CPM=0.0360*(TEMP-1197.)+0.303072
      GOTO901
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```
209 IF(TEMP-1242.)210,210,211
210 CPM=0.394*(1242.-TEMP)+0.313
    GOTO901
211 CPM=0.313
901 FCPMS=CPM
999 CONTINUE
    RETURN
    END

    FUNCTION FCNMT(TEMP)
    IF(TEMP)999,201,201
201 IF(TEMP-100.)202,202,203
202 CNM=0.08090*TEMP+134.5
203 IF(TEMP-610.)204,204,205
204 CNM=0.01161*(100.-TEMP)+140.
    GOTO901
205 IF(TEMP-1197.)206,206,207
206 CNM=0.02269*(610.-TEMP)+135.
    GOTO901
207 IF(TEMP-1242.)208,208,209
208 CNM=1.53*(1197.-TEMP)+121.7
    GOTO901
209 CNM=0.00774*(TEMP-1242.)+52.9
    GOTO901
901 FCNMT=CNM
999 CONTINUE
    RETURN
    END
/
/ EXEC LNKED1(MAP)
/
/ EXEC
169.       100.      170.      1242.     1197.        .0005
  .0001      .25   1350.        77.        1.          .25
 21   16     8    7
  0.       300.      500.      700.      900.      1100.      1500.      2000.
  .170       .222      .240      .255      .266      .272      .276       .293
  0.       300.      700.     1000.     1300.      1600.     1800.
  0.39     0.40      0.42      0.45      0.52      0.60      0.64
/
/
```

```
C                         ===   COMPUTER  PROGRAM  ===

C      THREE DIMENSIONAL SOLIDIFICATION EXPERIMENTAL CASTING
//KGARG   JOB     ,RU278JK,15,50,00
//M3TRAN EXEC FORTRAN(BCD,MAP)
C  C   HEAT TRANSFER THREE DIMENSIONAL SOLIDIFICATION
       DIMENSION TOLD(20,31,27),TNEW(20,31,27),ROH(2),JS(3),KS(3)
       DIMENSION TCPS(20),CCPS(20),TCNS(20),CCNS(20)
 1001 FORMAT(16I5)
 1002 FORMAT(8F10.4)
 1003 FORMAT(1H0,8H TIME IS,F10.5)
 1004 FORMAT(1H ,1X,15F7.1)
 1005 FORMAT(1X,I5,4E16.8)
 1006 FORMAT(3F20.8)
       READ1001,NX,NY,NZ,I1,J1,J2,J3,J4,K1,K2,K3,LP,LM
       PRINT1001,NX,NY,NZ,I1,J1,J2,J3,J4,K1,K2,K3,LP,LM
       READ1001,(JS(NS),KS(NS),NS=1,3)
       PRINT1001,(JS(NS),KS(NS),NS=1,3)
       READ1001,JCPS,JCNS
       PRINT1001,JCPS,JCNS
       READ1002,TSAN,TMET
       PRINT1002,TSAN,TMET
       READ1006,DT,DX,TMAX
       PRINT1006,DT,DX,TMAX
       READ1002,(TCPS(N),N=1,JCPS)
       PRINT1002,(TCPS(N),N=1,JCPS)
       READ1002,(CCPS(N),N=1,JCPS)
       PRINT1002,(CCPS(N),N=1,JCPS)
       READ1002,(TCNS(M),M=1,JCNS)
       PRINT1002,(TCNS(M),M=1,JCNS)
       READ1002,(CCNS(M),M=1,JCNS)
       PRINT1002,(CCNS(M),M=1,JCNS)
       READ1002,ROH(1),ROH(2)
       PRINT1002,ROH(1),ROH(2)
C      ONE FOR SAND,TWO FOR METAL
C      INITIAL TEMPERATURE CALCULATION
       DX=DX/12.
       TIME=0.
       DO3001 I=1,NX
       DO3001 J=1,NY
       DO3001 K=1,NZ
       IF(I.GT.I1)GO TO 2001
       IF(J.LT.J1.OR.J.GT.J4) GO TO 2001
       IF(K.GT.K3) GO TO 2001
       IF(K.GT.K2.AND.J.GT.J2) GO TO 2001
       IF(K.GT.K1.AND.J.GT.J3) GO TO 2001
       TOLD(I,J,K)=TMET
       GO TO 3001
 2001 TOLD(I,J,K)=TSAN
 3001 CONTINUE
       PRINT1003,TIME
       DO3002 NS=1,3
       JP=JS(NS)
       KP=KS(NS)
       PRINT1004,(TOLD(1,JP,K),K=1,NZ)
       PRINT1004,(TOLD(1,J,KP),J=1,NY)
       PRINT1004,(TOLD(I,JP,KP),I=1,NX)
```

```
 3002 CONTINUE
      CON=DT/(DX*DX)
      L=0
 1000 CONTINUE
      L=L+1
C     LOGIC TO CALCULATE FURTHER TEMPERATURES
      TIME=TIME+DT
      IF(TIME.GE.TMAX) GO TO 9999
      PRINT1005,L,CON,DT,DX,TIME
      DO3003 I=1,NX
      DO3003 J=1,NY
      DO3003 K=1,NZ
      UC=TOLD(I,J,K)
      IF(I.GT.I1) GO TO 2006
      IF(J.LT.J1.OR.J.GT.J4) GO TO 2006
      IF(K.GT.K3) GO TO 2006
      IF(K.GT.K2.AND.J.GT.J2) GO TO 2006
      IF(K.GT.K1.AND.J.GT.J3) GO TO 2006
      MAT=2
      GO TO 2007
 2006 MAT=1
 2007 CONTINUE
      CALL CONDY(UC,MAT,JCNS,TCNS,CCNS,CN)
      CALL SPHIT(UC,MAT,JCPS,TCPS,CCPS,CP)
      IF(MAT.EQ.1) GO TO 2005
C     INTERFACE START
  100 CONTINUE
      IF(I.NE.I1) GO TO 201
  101 IF(J.EQ.J4.AND.K.EQ.K1) GO TO 5
      IF(J.EQ.J4.AND.K.LT.K1) GO TO6
      IF(J.LE.J3) GO TO 102
      IF(K.EQ.K1) GO TO3
      IF(K.LT.K1) GO TO 4
      GO TO 2005
  102 IF(J.NE.J3) GO TO 103
      IF(K.EQ.K2) GO TO 5
      IF(K.LT.K2.AND.K.GT.K1) GO TO 6
      IF(K.EQ.K1) GO TO 7
      IF(K.LT.K1) GO TO4
      GO TO 2005
  103 IF(J.LE.J2) GO TO 104
      IF(K.EQ.K2) GO TO 3
      IF(K.LT.K2) GO TO 4
      GO TO 2005
  104 IF(J.NE.J2) GO TO 105
      IF(K.EQ.K3) GO TO 5
      IF(K.LT.K3.AND.K.GT.K2) GO TO 6
      IF(K.EQ.K2) GO TO 7
      IF(K.LT.K2) GO TO 4
      GO TO 2005
  105 IF(J.LE.J1) GO TO 106
      IF(K.EQ.K3) GO TO 3
      IF(K.LT.K3) GO TO4
      GO TO 2005
  106 IF(J.NE.J1) GO TO 2005
      IF(K.EQ.K3) GO TO 1
      IF(K.GT.K3) GO TO 2
      GO TO 2005
  201 IF(I.GT.I1) GO TO 2005
      IF(J.EQ.J4.AND.K.EQ.K1) GO TO 11
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```
      IF(J.EQ.J4.AND.K.LT.K1) GO TO 12
      IF(J.LE.J3) GO TO 202
      IF(K.EQ.K1) GO TO 10
      IF(K.LT.K1) GO TO2003
      GO TO 2003
  202 IF(J.NE.J3) GO TO 203
      IF(K.EQ.K2) GO TO 11
      IF(K.LT.K2.AND.K.GT.K1) GO TO 12
      IF(K.EQ.K1) GO TO 13
      IF(K.LT.K1) GO TO 2003
      GO TO 2003
  203 IF(J.LE.J2) GO TO 204
      IF(K.EQ.K2) GO TO 10
      IF(K.LT.K2) GO TO 2003
      GO TO 2003
  204 IF(J.NE.J2) GO TO 205
      IF(K.EQ.K3) GO TO 11
      IF(K.LT.K3.AND.K.GT.K2) GO TO 12
      IF(K.EQ.K2) GO TO 13
      IF(K.LT.K2) GO TO 2003
      GO TO 2003
  205 IF(J.LE.J1) GO TO 206
      IF(K.EQ.K3) GO TO 10
      IF(K.LT.K3) GO TO 2003
      GO TO 2003
  206 IF(J.NE.J1) GO TO 2003
      IF(K.EQ.K3) GO TO 8
      IF(K.GT.K3) GO TO 9
      GO TO 2003
    1 CONTINUE
      CALL CONDY(UC,1,JCNS,TCNS,CCNS,CNS)
      CALL SPHIT(UC,1,JCPS,TCPS,CCPS,CPS)
      VAR=8./(7.*ROH(1)*CPS+ROH(2)*CP)
      U1=CNS*(TOLD(I+1,J,K)+TOLD(I,J-1,K)+TOLD(I,J,K+1)-3.*UC)
      U2=0.5*(CNS+CN)*(TOLD(I-1,J,K)+TOLD(I,J+1,K)+TOLD(I,J,K-1)-3.*UC)
      UTOT=CON*(U1+U2)
      TNEW(I,J,K)=VAR*UTOT+UC
      GO TO 3003
    2 CONTINUE
      CALL CONDY(UC,1,JCNS,TCNS,CCNS,CNS)
      CALL SPHIT(UC,1,JCPS,TCPS,CCPS,CPS)
      VAR=4./(3.*ROH(1)*CPS+ROH(2)*CP)
      U1=CNS*(TOLD(I+1,J,K)+TOLD(I,J-1,K)-2.*UC)
      U2=0.5*(CNS+CN)*(TOLD(I,J,K-1)+TOLD(I-1,J,K))
      U3=0.5*(CNS+CN)*(TOLD(I,J+1,K)+TOLD(I,J,K+1)-4.*UC)
      UTOT=CON*(U1+U2+U3)
      TNEW(I,J,K)=VAR*UTOT+UC
      GO TO 3003
    3 CONTINUE
      CALL CONDY(UC,1,JCNS,TCNS,CCNS,CNS)
      CALL SPHIT(UC,1,JCPS,TCPS,CCPS,CPS)
      VAR=4./(3.*ROH(1)*CPS+ROH(2)*CP)
      U1=CNS*(TOLD(I+1,J,K)+TOLD(I,J,K+1)-2.*UC)
      U2=0.5*(CNS+CN)*(TOLD(I-1,J,K)+TOLD(I,J-1,K))
      U3=0.5*(CNS+CN)*(TOLD(I,J+1,K)+TOLD(I,J,K-1)-4.*UC)
      UTOT=CON*(U1+U2+U3)
      TNEW(I,J,K)=VAR*UTOT+UC
      GO TO 3003
    4 CONTINUE
```

```
      CALL CONDY(UC,1,JCNS,TCNS,CCNS,CNS)
      CALL SPHIT(UC,1,JCPS,TCPS,CCPS,CPS)
      VAR=2./(ROH(1)*CPS+ROH(2)*CP)
      U1=CN*(TOLD(I-1,J,K)-UC)
      U2=CNS*(TOLD(I+1,J,K)-UC)
      U3=0.5*(CNS+CN)*(TOLD(I,J-1,K)+TOLD(I,J+1,K))
      U4=0.5*(CNS+CN)*(TOLD(I,J,K-1)+TOLD(I,J,K+1)-4.*UC)
      UTOT=CON*(U1+U2+U3+U4)
      TNEW(I,J,K)=VAR*UTOT+UC
      GO TO 3003
    5 CONTINUE
      CALL CONDY(UC,1,JCNS,TCNS,CCNS,CNS)
      CALL SPHIT(UC,1,JCPS,TCPS,CCPS,CPS)
      VAR=8./(7.*ROH(1)*CPS+ROH(2)*CP)
      U1=CNS*(TOLD(I+1,J,K)+TOLD(I,J+1,K)+TOLD(I,J,K+1)-3.*UC)
      U2=0.5*(CNS+CN)*(TOLD(I-1,J,K)+TOLD(I,J-1,K)+TOLD(I,J,K-1)-3.*UC)
      UTOT=CON*(U1+U2)
      TNEW(I,J,K)=VAR*UTOT+UC
      GO TO 3003
    6 CONTINUE
      CALL CONDY(UC,1,JCNS,TCNS,CCNS,CNS)
      CALL SPHIT(UC,1,JCPS,TCPS,CCPS,CPS)
      VAR=4./(3.*ROH(1)*CPS+ROH(2)*CP)
      U1=CNS*(TOLD(I+1,J,K)+TOLD(I,J+1,K)-2.*UC)
      U2=0.5*(CNS+CN)*(TOLD(I-1,J,K)+TOLD(I,J-1,K))
      U3=0.5*(CNS+CN)*(TOLD(I,J,K-1)+TOLD(I,J,K+1)-4.*UC)
      UTOT=CON*(U1+U2+U3)
      TNEW(I,J,K)=VAR*UTOT+UC
      GO TO 3003
    7 CONTINUE
      CALL CONDY(UC,1,JCNS,TCNS,CCNS,CNS)
      CALL SPHIT(UC,1,JCPS,TCPS,CCPS,CPS)
      VAR=8./(7.*ROH(1)*CPS+ROH(2)*CP)
      U1=CNS*(TOLD(I+1,J,K))
      U2=0.5*(CNS+CN)*(TOLD(I-1,J,K)+TOLD(I,J-1,K))
      U3=0.5*(CNS+CN)*(TOLD(I,J+1,K)+TOLD(I,J,K-1)+TOLD(I,J,K+1)-5.*UC)
      UTOT=CON*(U1+U2+U3)
      TNEW(I,J,K)=VAR*UTOT+UC
      GO TO 3003
    8 CONTINUE
      CALL CONDY(UC,1,JCNS,TCNS,CCNS,CNS)
      CALL SPHIT(UC,1,JCPS,TCPS,CCPS,CPS)
      VAR=4./(3.*ROH(1)*CPS+ROH(2)*CP)
      U1=CNS*(TOLD(I,J-1,K)+TOLD(I,J,K+1)-2.*UC)
      U2=0.5*(CNS+CN)*(TOLD(I-1,J,K)+TOLD(I+1,J,K))
      U3=0.5*(CNS+CN)*(TOLD(I,J+1,K)+TOLD(I,J,K-1)-3.*UC)
      UTOT=CON*(U1+U2+U3)
      TNEW(I,J,K)=VAR*UTOT+UC
      GO TO 3003
    9 CONTINUE
      CALL CONDY(UC,1,JCNS,TCNS,CCNS,CNS)
      CALL SPHIT(UC,1,JCPS,TCPS,CCPS,CPS)
      VAR=2./(ROH(1)*CPS+ROH(2)*CP)
      U1=CN*(TOLD(I,J-1,K)-UC)
      U2=CNS*(TOLD(I,J+1,K)-UC)
      U3=0.5*(CNS+CN)*(TOLD(I-1,J,K)+TOLD(I+1,J,K))
      U4=0.5*(CNS+CN)*(TOLD(I,J,K+1)+TOLD(I,J,K-1)-4.*UC)
      UTOT=CON*(U1+U2+U3+U4)
```

```
      TNEW(I,J,K)=VAR*UTOT+UC
      GO TO 3000
   10 CONTINUE
      CALL CONDY(UC,1,JCNS,TCNS,CCNS,CNS)
      CALL SPHIT(UC,1,JCPS,TCPS,CCPS,CPS)
      VAR=2./(ROM(1)*CPS+ROM(2)*CP)
      U1=CN*(TOLD(I,J,K-1)-UC)
      U2=CNS*(TOLD(I,J,K+1)-UC)
      U3=0.5*(CNS+CN)*(TOLD(I-1,J,K)+TOLD(I+1,J,K))
      U4=0.5*(CNS+CN)*(TOLD(I,J+1,K)+TOLD(I,J-1,K)-4.*UC)
      UTOT=CON*(U1+U2+U3+U4)
      TNEW(I,J,K)=VAR*UTOT+UC
      GO TO 3000
   11 CONTINUE
      CALL CONDY(UC,1,JCNS,TCNS,CCNS,CNS)
      CALL SPHIT(UC,1,JCPS,TCPS,CCPS,CPS)
      VAR=4./(3.*ROM(1)*CPS+ROM(2)*CP)
      U1=CNS*(TOLD(I,J+1,K)+TOLD(I,J,K+1)-2.*UC)
      U2=0.5*(CNS+CN)*(TOLD(I-1,J,K)+TOLD(I+1,J,K))
      U3=0.5*(CNS+CN)*(TOLD(I,J-1,K)+TOLD(I,J,K-1)-4.*UC)
      UTOT=CON*(U1+U2+U3)
      TNEW(I,J,K)=VAR*UTOT+UC
      GO TO 3000
   12 CONTINUE
      CALL CONDY(UC,1,JCNS,TCNS,CCNS,CNS)
      CALL SPHIT(UC,1,JCPS,TCPS,CCPS,CPS)
      VAR=2./(ROM(1)*CPS+ROM(2)*CP)
      U1=CN*(TOLD(I,J-1,K)-UC)
      U2=CNS*(TOLD(I,J+1,K)-UC)
      U3=0.5*(CNS+CN)*(TOLD(I-1,J,K)+TOLD(I+1,J,K))
      U4=0.5*(CNS+CN)*(TOLD(I,J,K+1)+TOLD(I,J,K-1)-4.*UC)
      UTOT=CON*(U1+U2+U3+U4)
      TNEW(I,J,K)=VAR*UTOT+UC
      GO TO 3000
   13 CONTINUE
      CALL CONDY(UC,1,JCNS,TCNS,CCNS,CNS)
      CALL SPHIT(UC,1,JCPS,TCPS,CCPS,CPS)
      VAR=4./(3.*ROM(1)*CPS+ROM(2)*CP)
      U1=CN*(TOLD(I,J-1,K)+TOLD(I,J,K-1)-2.*UC)
      U2=0.5*(CNS+CN)*(TOLD(I-1,J,K)+TOLD(I+1,J,K))
      U3=0.5*(CNS+CN)*(TOLD(I,J+1,K)+TOLD(I,J,K+1)-4.*UC)
      UTOT=CON*(U1+U2+U3)
      TNEW(I,J,K)=VAR*UTOT+UC
      GO TO 3000
C     INTERFACE COMPLETE
 2000 CONTINUE
      IF(I.EQ.1)ULR=2*TOLD(2,J,K)
      IF(I.GT.1.AND.I.LT.NX)ULR=TOLD(I-1,J,K)+TOLD(I+1,J,K)
      IF(I.EQ.NX)ULR=TOLD(NX-1,J,K)+TSAN
      IF(J.EQ.1)UUD=TOLD(I,2,K)+TSAN
      IF(J.GT.1.AND.J.LT.NY)UUD=TOLD(I,J-1,K)+TOLD(I,J+1,K)
      IF(J.EQ.NY)UUD=TOLD(I,NY-1,K)+TSAN
      IF(K.EQ.1) GO TO 2002
      IF(K.GT.1.AND.K.LT.NZ)UFB=TOLD(I,J,K-1)+TOLD(I,J,K+1)
      IF(K.EQ.NZ)UFB=TOLD(I,J,NZ-1)+TSAN
      GO TO 2003
 2002 IF(J.GE.J1.AND.J.LE.J4) GO TO 2003
      UFB=TOLD(I,J,2)+TSAN
      GO TO 2003
 2003 IF(I.GE.1.AND.I.LE.I1) GO TO 2004
```

```
      UFB=TOLD(I,J,2)+TSAN
      GO TO 2009
 2004 UFB=TOLD(I,J,2)+TMET
 2009 CONTINUE
      UTOT=CON*(ULR+UUD+UFB-6.*UC)
      VAR=CN/(ROH(MAT)*CP)
      TNEW(I,J,K)=VAR*UTOT+UC
 3003 CONTINUE
      PRINT1005,K,VAR,UTOT,UC,TNEW(I1,J1,K3)
      PRINT1005,MAT,ROH(MAT),TSAN,TMET,TNEW(I1,J3,K2)
      IF(L.GE.LP) GO TO 2008
  500 DO3004 I=1,NX
      DO3004 J=1,NY
      DO 3004 K=1,NZ
 3004 TOLD(I,J,K)=TNEW(I,J,K)
      GO TO 600
 2008 PRINT 1003,TIME
      DO3005 NS=1,3
      JP=JS(NS)
      KP=KS(NS)
      PRINT1004,(TNEW(1,JP,K),K=1,NZ)
      PRINT1004,(TNEW(1,J,KP),J=1,NY)
      PRINT1004,(TNEW(I,JP,KP),I=1,NX)
 3005 CONTINUE
      L=0
      GO TO 500
  600 CONTINUE
      GO TO 1000
 9999 STOP
      END

      SUBROUTINE CONDY(UC,MAT,JMAX,T,C,CN)
      DIMENSION T(20),C(20)
      GO TO (10,20),MAT
   10 IF(UC-T(1))699,601,601
  601 JL=1
  802 JR=JL+1
      IF(UC-T(JR))602,602,603
  602 SLOPE=(C(JR)-C(JL))/(T(JR)-T(JL))
      CN=C(JL)+SLOPE*(UC-T(JL))
      GO TO 699
  603 JL=JL+1
      IF(JL-JMAX)802,699,699
  699 CONTINUE
      RETURN
   20 IF(UC)999,201,201
  201 IF(UC-180.)202,202,203
  202 CN=0.03058*UC+134.5
  203 IF(UC-610.)204,204,205
  204 CN=0.01161*(180.-UC)+140.
      GO TO 999
  205 IF(UC-1197.)206,206,207
  206 CN=0.02263*(610.-UC)+135.
      GO TO 999
  207 IF(UC-1242.)208,208,209
  208 CN=1.53*(1197.-UC)+121.7
      GO TO 999
  209 CN=0.00774*(UC-1242.)+52.9
      GO TO 999
  999 CONTINUE
```

```
      RETURN
      END

      SUBROUTINE SPHIT(UC,MAT,JMAX,T,C,CP)
      DIMENSION T(20),C(20)
      GO TO (10,20),MAT
   10 IF(UC-T(1))699,601,601
  601 JL=1
  802 JR=JL+1
      IF(UC-T(JR))602,602,603
  602 SLOPE=(C(JR)-C(JL))/(T(JR)-T(JL))
      CP=C(JL)+SLOPE*(UC-T(JL))
      GO TO 699
  603 JL=JL+1
      IF(JL-JMAX)802,699,699
  699 CONTINUE
      RETURN
   20 IF(UC)999,201,201
  201 IF(UC-940.)202,202,203
  202 CP=0.0000607*UC+0.213
      GO TO 999
  203 IF(UC-1140.)204,204,205
  404 CP=0.0001075*(UC-940.)+0.270058
      GO TO 999
  205 IF(UC-1197.)206,206,207
  206 CP=0.000202*(UC-1140.)+0.291558
      GO TO 999
  207 IF(UC-1219.44)208,208,209
  208 CP=0.336*(UC-1197.)+0.303072
      GO TO 999
  209 IF(UC-1242.)210,210,211
  210 CP=0.334*(1242.-UC)+0.313
      GO TO 999
  211 CP=0.313
  999 CONTINUE
      RETURN
      END
/*
// EXEC LNKEDT,MAP*
/*
// EXEC
/*
/+
```