OPTIMUM TEMPERATURE PROFILE OF THE BLAST FURNACE

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[EXTRACTIVE METALLURGY]

By

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CERTIFICATE

Certified that the dissertation entitled, OPTIMUM TEMPERATURE PROFILE OF THE BLAST FURNACE, which is being submitted by Shri Yogeshwar Sahai in partial fulfilment for the award of the Degree of MASTER OF ENGINEERING in Metallurgical Engineering (Extractive Metallurgy) of the University of Roorkee, Roorkee (India), is a record of his own work carried out by him under my supervision and guidance from January, 1973 to September, 1973.

The matter embodied in this dissertation has not been submitted for the award of any other degree.

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October 12,1973. (Yogeshwar Sahai)

ABSTRACT

The importance of the Blast Furnace can be understood from the fact that in 1971 about 98 per cent of the total pig iron produced in the world was smelted in blast furnaces. Metallurgical coke, which is produced from the good quality coking coal, is used in the blast furnace as fuel and reductant. The shortage of good quality coking coal has led the researchers to think about the methods to reduce the coke rate of the blast furnace. Many methods are being used successfully to reduce the coke rate of the furnace.

In this work, a temperature profile, called the optimum temperature profile, has been calculated theoretically, which corresponds to the minimum coke rate. The rates of the following reactions have been considered in the derivation:

(i) Reduction of iron oxide by carbon monoxide.
(ii) Gasification of carbon.
(iii) Dissociation of lime stone.

The optimum temperature profile is calculated from room temperature, at which the raw materials are charged to about 1200°C. Above 1200°C the iron oxide is assumed to be reduced directly.

This work, presented here, is divided in five chapters. First chapter reviews the physical chemistry of the blast furnace and the developments in the mathematical models of the furnace.

Second and third chapters consist of formulation and derivation of the problem. In derivation the rates of the reactions mentioned above are considered and the mass balance around a section of small thickness of bed has been taken.

In chapter fourth the optimization method has been discussed and used, to obtain six differential and one algebraic equations. The computational sequence has also been given to solve these equations. The data needed in the calculation is given, and some parameters needed in computation are calculated.

Last chapter gives the results obtained. These results are discussed and it has been shown that the optimum temperature profile, if employed in the blast furnace reduces the coke rate.

The Runge Kutta method has been used to solve these equations. This method is given in detail in the Appendix.

CONTENTS

LIST OF SYMBOLS

a - rate of change of equilibrium pressure with temperature, atms/°K.

D - diffusion coefficient of CO_2 through calcium carbonate, m^2/hr.

Dj - effective diffusion coefficient of j th species in porous solid, m^2/hr.

Dr - driving rate of blast, moles/m^2.hr.

Gj - mass flow rate of j th component, kg-mole/m^2.hr.

Gj° - initial mass flow rate of j th component, kg-mole/m^2.hr.

h - heat transfer coefficient to surface of the calcium carbonate, Kcal/m^2.hr.°K.

H - molar heat of decomposition of calcium carbonate, Kcal/kg-mole.

ΔH - heat of reaction Kcal/kg-mole.

Keo - equilibrium constant for iron oxide-iron equilibrium with CO/CO_2.

kr - specific rate constant for equation (1),m/hr.

Kct - rate of constant for carbon gasification, m/hr.

Kec - equilibrium constant for formation of CO from CO_2.

km - mass transfer coefficient in porous iron layer m/hr.

K - thermal conductivity of porous lime,Kcal/m.hr.°K.

$\dot{n}$j - rate of reaction of j th species, kg-mole/hr.

nj - number of particles of j th component per unit bed volume, $(m^3)^{-1}$.

pt - total pressure in the furnace, atm.

pj - partial pressure of j th species in the bulk gas, atm.

$(pco_2)_{eq}$ - partial pressure of CO_2 in equilibrium with lime and calcium carbonate at the temperature of the furnace gases, atm.

R - gas constant, m^3. atm/kg-mole°K.

$\dot{r}j$ - rate of removal of j th species per unit bed volume, kg-mole/m^3. hr.

S - area of cross section of the furnace, m^2.

Tj - temperature of j th component, °K.

v - linear velocity of the charge in the furnace, m/hr.

xo^j - initial radius of any particle of j th component, m.

xi^j - radius of unreacted core of j th component, m.

Z - height of the furnace, increasing downwards, m.

α - a constant defined as

$$= \frac{Keo\ km\ (co)\ \ km\ (co2)}{Keo\ \ km(co_2)+km(co)}$$

- a constant defined as

$$\beta = \frac{keo\ Dco\ Dco2}{keo\ Dco_2+Dco}$$

γ - mass transfer coefficient to surface of calcium carbonate, m/hr.

ρj - density of j the species, kg-mole/m^3.

Subscripts:

g - gas
s - solid
o - ore
c - coke
l - lime stone

Superscripts:

o - ore
c - coke
l - lime stone

CHAPTER - I

LITERATURE REVIEW

LITERATURE REVIEW

1.1 INTRODUCTION

Iron was known even in the prehistoric times, though the method of producing was known only around 2000 B.C. From 2000 B.C. to 1300 A.D., reduction of iron was carried out in a relatively simple manner using charcoal as fuel in a crude hearth which was eventually superseded by furnaces of various designs having a strong family resemblance. Iron was first melted using charcoal as a fuel and air under pressure in 1350 A.D. The blast furnace is in continuous use and development since that time. In 1600, one tenth of the charcoal was replaced by the use of mineral Anthracite coal. It was only after the utilization of coke in about 1709 that earliest form of modern blast furnace came into being. Later in 1824, the advent of hot blast and in 1860, the use of regeneration principle brought the blast furnace to sounder footing.

It is interesting that first half of the present century was the period, where though no major development in the blast furnace technology took place, but a large amount of effort was put to study, the physical chemistry of the process, stock descent and distribution, which contributed to the later developments. In a short period of five years (1945-1950), the concept of high top pressure, oxygen blast, high blast temperature were given. While the period 1950-1960 saw the advents of developments like agglomeration, steam injection etc. It seems that the last decade 1960-1970 was by far the revolutionary period, where the advantages of oil injection, and other modifications were fully realized. In the same period, for the purpose of improving the actual operations of blast furnace, a number of attempts have been made to develop mathematical models for analysing the productivity of the furnace or estimating the situations of process variables in the furnace. Thus in the last decade there has been a substantial rise in the capacity of the blast furnaces.

The present status of the blast furnace could be judged from the fact that in 1971 about 98 per cent of the total 423 million tons of pig iron produced in the world was smelted in the blast furnaces. Blast furnace uses metallurgical coke as the fuel and the reductant. Metallurgical coke is produced from coking coal, of which reserves in the world are depleting very fast. With the increasing demand and restricted availability of such good quality coking coal, its cost has been increasing at a fast rate. Various attempts which have been made successfully to reduce the coke rate of the furnaces are (i) high temperature of the blast, (ii) humidified blast, (iii) oxygen enriched blast, (iv) fuel injection at the tuyeres, (v) preparation of raw materials (vi) high top pressure etc.

1.2 PHYSICAL CHEMISTRY OF THE BLAST FURNACE PROCESS

Fundamentally the blast furnace is a counter current apparatus in which descending iron oxide, coke and slag making materials remove heat from an ascending stream of hot reducing gases composed mainly of nitrogen, carbon monoxide, carbon dioxide. Chemical reduction of the iron oxide also occurs as the charge descends and finally in the bosh fusion of the reduced iron, some unreduced ironoxide and slag making materials takes place. In the hearth of the furnace, the liquid slag and metal collect, and also carbon is burnt at the tuyeres to produce heat and carbon monoxide which serves as the reducing agent for the whole process.

1.21 Reduction of iron oxides[1] : There are three main oxides of iron: ferric oxide or hematite (Fe_2O_3), magnetite (Fe_3O_4) and ferrous oxide or Wustite (FeO). The first two of these occur abundantly in nature, but Wustite, being stable only at temperatures above 570°C, has been reported only in a few localities. However, it does play an important role as an intermediate product in the reduction of iron ores to metallic iron. (Although the formula for Wustite is usually designated as FeO, the crystal lattice of this oxide is always deficient in iron and a more realistic formula would be $Fe_{0.95}O$.)

1.22 <u>Reduction by carbon monoxide</u>[1] : The reduction of hematite by carbon monoxide is a gas-solid reaction and takes place in three steps according to following three reactions:

$3\ Fe_2O_3 + CO = 2\ Fe_3O_4 + CO_2$ 1.1

$\Delta H = -\ 12636$ cal/mole of CO*

$Fe_3O_4 + CO = 3\ FeO + CO_2$ 1.2

$\Delta H = +\ 8664$ cal/mole of CO

$FeO + CO = Fe + CO_2$ 1.3

$\Delta H = -\ 4136$ cal/mole of CO

The reactions 1.1 and 1.3 are exothermic whereas 1.2 is an endothermic reaction.

The reduction process is one of diffusion in which the reducing gas, carbon monoxide diffuses towards the centre of iron ore particles while the gaseous products diffuse outwardly.

The overall reduction of ferric oxide to metallic iron can be written as follows by adding 1.1, 1.2 & 1.3 reactions:

$Fe_2O_3 + 3\ CO = 2\ Fe + 3\ CO_2$ 1.4

$\Delta H = -6708$ cal.

The overall reduction is exothermic .

---

* All ΔH values are for 25°C and are taken from ref. 2.

1.23 Reduction Equilibria[1] : All the above reduction reactions are reversible and the reactions will proceed only from left to right if the CO/CO_2 ratio is greater than equilibrium ratio. These equilibrium ratios vary with temperature and are illustrated in figure 1.

1.24 Carbon as fuel and reductant[1]: Carbon, the most important constituent in the coke, functions both as fuel and reductant. For production of heat, carbon combines with oxygen to form carbon-dioxide and this is an exothermic reaction as follows:

$$C + O_2 = CO_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1.5$$

$\Delta H = -94052$ cal/mole of carbon.

At high temperatures and in an oxygen deficient atmosphere, the CO_2 produced in the reaction 1.5 reacts with carbon to produce carbon monoxide as follows:

$$C + CO_2 = 2\,CO \quad \ldots\ldots\ldots\ldots\ldots\ldots 1.6$$

$\Delta H = + 41220$ cal/mole of carbon.

The reaction 1.6 is known as Boudouard reaction or solution loss reaction or endothermic gasification of carbon (EGC). The dependence of Endothermic Gasification reaction on temperature is shown in figure 2.

The overall effect of the reactions 1.5 and 1.6 is as follows:

$$2C + O_2 = 2\,CO \quad \ldots\ldots\ldots\ldots\ldots\ldots 1.7$$

$\Delta H = -26416$ cal/mole of carbon.

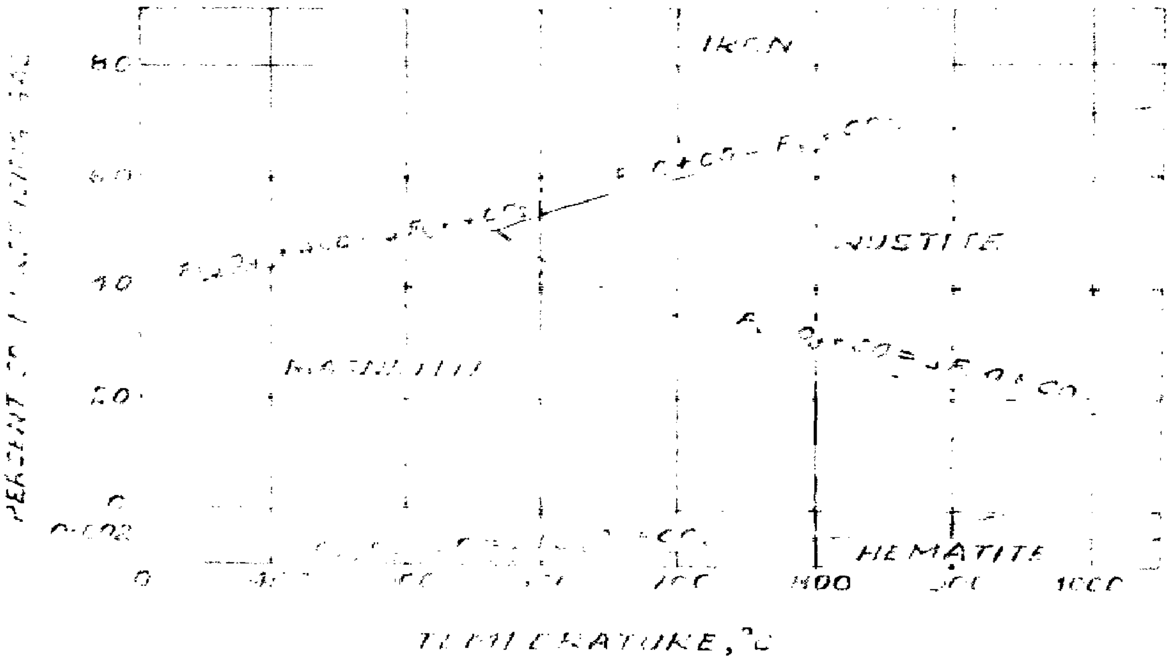
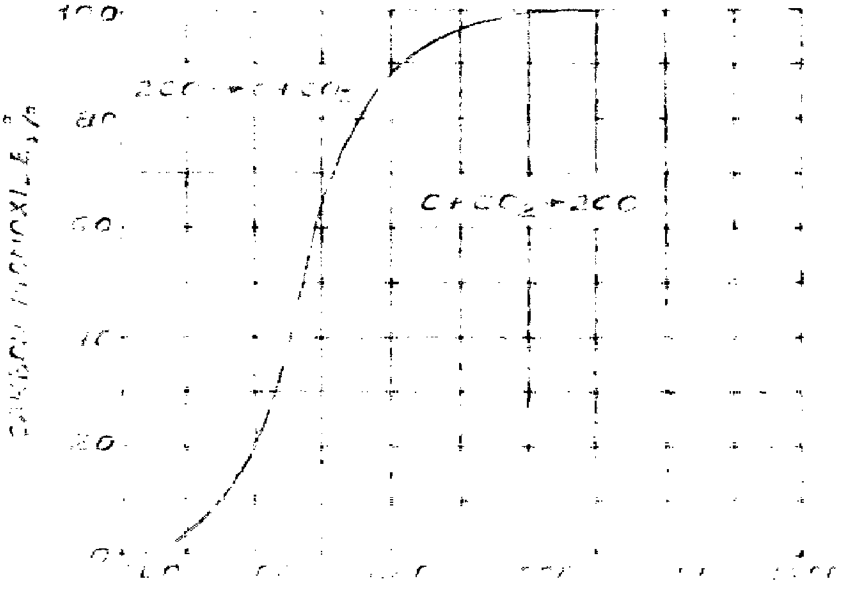


FIG. 1. [illegible] REDUCTION EQUILIBRIA.



[illegible]

This exothermic reaction might be considered as the ultimate reaction that takes place when the carbon is burnt to generate heat in the combustion zone of the blast furnace.

As a reductant carbon can reduce iron oxide in two ways:

$Fe_nO_m + mC = nFe + mCO$ 1.8

$Fe_nO_m + mCO = nFe + mCO_2$.............1.9

Reaction 1.8 is commonly referred to as 'direct reduction' while reaction 1.9 as 'indirect reduction'. The indirect reduction is more important because the diffusion of the gaseous reactant carbon monoxide can take place at a much faster rate than the solid state diffusion of solid reductant carbon with iron oxide. At high temperatures above 1100°C, when the iron oxide is reduced according to reaction 1.9, the CO_2 is converted to CO by Boudouard reaction . The resultant of these reactions, 1.9 & 1.6, is given by reaction 1.8 - the direct reduction reaction. It is, therefore, believed that even in the direct reduction, the effective reductant is CO.

1.3 MATHEMATICAL MODELS

Mathematical model is described by a set of differential and algebraic equations, which can be numerically solved by computer. From a mathematical model the variations of the situations in the furnace associated with the changes of operating conditions can be predicted precisely, and so, for carrying out the efficient operation and the appropriate process control of blast furnace it is necessary to derive the mathematical model.

The existing models can be classified into three groups:

(i) Blast furnace is divided into several zones and the chemical equilibrium is kept in each zone.

(ii) The furnace is divided into several zones and the temperature in the individual one is assumed to be invarient and the reaction rates are included in the model.

(iii) Basic equations are derived by taking heat and material balances around a differential height of the bed at an arbitrary level in the furnace in which the rates of heat transfer between fluid and solid particles and the reaction rates are taken into account.

Most important ones, constituting first group are those of Reichardt[3], Ridgion[4], Wartmann[5] and Staib et.al.[6].

Reichardt[3] divided the Blast furnace in five ranges of temperature and obtained longitudinal distributions of temperature of gas and solid particles.

Ridgion[4] set up calculus of the model and analysed the change in sensible heat of solid particles. He also calculated the effect of oil injection on coke rate.

Wartmann[5] divided the furnace into four zones such as pre-heating, indirect reduction, direct reduction and hearth. In this model, heat and material balances in each zone were combined with overall material balance taken around a whole furnace, and the quantity of heat transferred from a zone to another, the production rate of pig iron, the discharge rate of slag and the consumption of coke were determined numerically.

Staib et. al[6] conceived the existence of chemical reserve zone and a thermal reserve zone. The latter is one where temperature of solid particles almost equals to that of gas while former is the lower part of the latter where the reduction of iron oxide might not occur. Taking account of these zones and also of pre-heating and processing zone, the model for calculating the carbon ratio and the production rate of pig iron under the given composition of top gas was developed.

Contribution to the second group is that of Flierman et.al.[7]. He divided the blast furnace into five zones such as shaft, melting zone, zone containing molten materials, tuyere zone and hearth. The reaction rates of indirect reduction of iron ore by CO and by H_2, solution loss reaction, and decomposition of limestone were taken into account. Longitudinal distributions of temperature of gas and solid particles and those of molar fractions CO and CO_2 were estimated in terms of this model.

The models which belong to third group are of Koump et.al.[8], Lahiri et.al.[9], Shimotsuma et.al.[10], Wakabayashi et.al.[11], Okabe et.al.[12] and Yagi & Muchi[13].

Koump[8] treated the stack region of the blast furnace as a steady state adiabatic, one dimensional, countercurrent, heterogeneous chemical reactor in which two components in the solid state are reacting with a gas. This model consists of six differential equations. These equations were derived by considering the reaction rates of indirect reduction of iron ore by carbon monoxide and carbon gasification reaction, and several transport equations. These transport equations were derived by taking mass, energy and momentum balances around a differential height of bed at any level in the stack region with the following assumptions:

(i) Axial dispersion of mass and energy

(ii) Constant pressure throughout the reactor volume.

(iii) Uniform temperature of solid particles through out the particle volume.

These assumptions do not correspond to the actual conditions in Blast furnace, however, overcome much of the mathematical complexity.

By solving these equations with the aid of computer, he obtained the longitudinal distributions of temperature of gas and the solid particles, fractional reduction of iron ore and the partial pressures of carbon monoxide and carbon dioxide.

Lahiri et.al.[9] took up more realistic situation to simulate the actual blast furnace conditions wherein compositions and temperatures of gas and solid may vary both axially and radially. He developed an improved mathematical model for the stack region.

Process analysis was carried out for four different cases to find out the general behaviour of the process and the effect of the operating variables on the temperature and concentration profiles in the furnace. The following conclusions could be drawn on the basis of the analysis.

(i)* 'In the stack region, the temperature and composition of solid and gas vary along the radial direction even when the charge and gas distribution are uniform.

(ii) Temperature and concentration profiles are affected by the change in the operating variables.

(iii) Thermal and chemical reserve zones exist in the furnace stack region.

(iv) There are two regions in the furnace where rate of oxygen removal is maximum.

(v) The pressure varies almost linearly with the height of the furnace.

(vi) The chemical and thermal state of the furnace can be predicted by the top gas temperature and gas composition.

For (i)* average temperature and pressure values are computed and corresponding average rates of reduction and gasification are determined and Koump's model is thereby corrected for thse values.

Shimotsuma[10] analysed the longitudinal distributions of temperatures of gas and Wakabayashi et.al.[11] developed a model, which consisted of total balance and stepwise models and analysed the operations of high top pressure and oxygen enrichment.

Okabe's[12] model consisted of six ordinary differential equations and determined the longitudinal distributions of process variables.

Model by Yagi & Muchi[13] is the latest. It predicts the theoretical estimation on the longitudinal distribution of process variables in the blast furnace. Rates of the following reactions were considered in deriving the model:

(i) Indirect reduction of iron ore by carbonmonoxide.
(ii) Carbon gasification reaction.
(iii) Direct reduction of Wustite by coke.
(iv) Decomposition of lime stone.
(v) Reduction of iron ore by hydrogen.
(vi) Reaction between coke and steam.
(vii) Watér gas shift reaction.
(viii) Direct reduction of Silica in slag by solid coke.

Equations were derived by taking mass, heat and momentum balances for gas and solid particles around the differential height of bed at an arbitrary position from the top level of the bed. The model is comprised of ten differential equations and three algebraic equation. Numerical solution of these equations is obtained by Runge-Kutta-Gill method with the aid of computer.

This model determines (i) Carbon ratio (ii) Production rate of pig iron and (iii) Longitudinal distribution of process variables in the stack and bosh regions under arbitrary operating conditions. Yagi and Muchi also studied following factors which could increase production rate of pig iron and decrease carbon ratio:

(i) High top pressure. (ii) Charging of iron ore of small size. (iii) High temperature blast. (iv) Steam injection. (v) Oxygen enrichment. (vi) Pre-reduction of charged materials.

All these models assume steady state conditions. Recently some dynamic models have also been proposed. Fielden et.al.[14] have estimated the transitional variations of temperature and composition of pig iron and those of top gas. Dynamic behaviour of process variables in the zone between melting and tuyere levels have been determined by Yagi and Muchi[15] for the stepwise changes of blast furnace conditions.

CHAPTER - II

FORMULATION OF PROBLEM

FORMULATION OF THE PROBLEM

So far the temperature profile of the Blast furnace has been considered as fixed and the various reactions in the furnace are forced to take place in these existing conditions of temperature. For the sake of discussion let the furnace be divided into two zones:

i) From room temperature, at which the raw materials are charged to 1150°C.

ii) Above 1150°C.

Reactions taking place in the first zone are

(a) Reduction reaction - Hematite is reduced to magnetic and finally to wustite in the upper stack. This oxygen removal is rather easy and can take place at relatively lower temperature without large excess of reducing gas, as the rate of reduction is high. Further reduction of wustite to metallic iron is slow and considerable excess of reducing gas is required. Therefore, only a part of the oxygen content in the wustite can be removed economically in this zone.

(b) Gasification of Carbon - This reaction is highly temperature dependent. At high temperatures carbon monoxide is the stable oxide of carbon whereas at lower temperature the carbondioxide is the stable oxide, as shown in figure 2. So the carbon gasified at any level in the blast furnace will depend upon the temperature prevailing there.

(c) Dissociation of Lime Stone - The lime stone dissociates readily as the charge descends a few meters from the top.

The reactions taking place in the second zone i.e. at temperatures above 1150°C are-

(a) The direct reduction of the remaining wustite.
(b) The reduction of some of SiO_2, MnO etc. by carbon.

In this zone the fusion of solid starts and finally liquid metal and slag forms.

Since the carbon gasification reaction greatly depends upon temperature, so a temperature profile could be thought of, which would minimise the gasification of carbon consistent with the maximum reduction of iron oxides, or the temperature profile which would optimise the gasification of carbon. Hence this temperature profile is termed as 'optimum temperature profile'. The optimum temperature profile would correspond to minimum coke rate. The optimum temperature profile is calculated from room temperature, at which the solids are charged, to about 1200°C (solid temperature). The reasons, that 1200°C is chosen as the upper limit of temperature, are as follows:

(a) Above 1200°C, strictly speaking above 1130° -1150°C, the fusion of solids starts and then the available kinetic data for reduction would not be valid.

(b) Iron oxide (wustite) retained above 1200°C may be reasonably assumed to be reduced by carbon only (FeO+C = Fe + CO). So above 1200°C the gas will consist of CO & N_2.

DERIVATION OF THE PROBLEM

The optimum temperature profile has been calculated from room temperature to 1200°C by considering the rates of the three reactions given below and the mass balance around a differential height of bed at any level in the furnace:

(i) Reduction of Iron oxide (Hematite) by carbon monoxide to metallic Iron.

$$Fe_2O_3 + CO \longrightarrow Fe + CO_2$$

(ii) Gasification of carbon

$$CO_2 + C \longrightarrow 2CO$$

(iii) Dissociation of lime stone

$$CaCO_3 \longrightarrow CaO + CO_2$$

Reduction of dense Hematite pellet with CO.

$$Fe_2O_3 + CO \longrightarrow Fe + CO_2 \text{ ----------------- } (1)$$

The rate of reduction[16] is given by

$$\left[\frac{1}{4\pi\alpha\, x_o^{o2}} + \frac{x_o^o - x_i^o}{4\pi\beta\, x_i^o\, x_o^o} + \frac{1}{k_r 4\pi\, x_i^{o2}}\right](-\dot{n}_o) = \frac{p_t}{R\,T_g}\left[p_{co} - p_{co_2}/K_{eo}\right] \text{ ---------- } (2)$$

$$-\dot{n}_o = \frac{\dfrac{p_t}{R\,T_g}\left[p_{co} - p_{co_2}/K_{eo}\right]}{\dfrac{1}{\alpha\, 4\pi\, x_o^{o2}} + \dfrac{x_o^o - x_i^o}{\beta\, 4\pi\, x_i^o\, x_o^o} + \dfrac{1}{k_r 4\pi\, x_i^{o2}}} \text{ ---------- } (3)$$

Volume of the unreacted core of one particle at any time = $4/3\,\pi\,x_i^{o\,3}$

Amount of oxygen in one particle = $4/3\,\pi\,x_i^{o\,3}\,\rho_o$

Change in oxygen with time = $-\dot{n}_o = -\frac{d}{dt}\left(4/3\,\pi\,x_i^{o\,3}\,\rho_o\right)$

Change in oxygen per unit volume per unit time= $-\dot{n}_o\,n_o$

$$= -\frac{d}{dt}\left(4/3\,\pi\,x_i^{o\,3}\,\rho_o\,n_o\right)$$

$$= -\frac{v}{v}\,\frac{d}{dt}\left(4/3\,\pi\,x_i^{o\,3}\,\rho_o\,n_o\right)$$

$$-\dot{n}_o\,n_o = -\frac{d}{dz}\left(4/3\,\pi\,x_i^{o\,3}\,\rho_o\,n_o\,v\right) \quad \text{———}(4)$$

Mass of oxygen per unit bed volume = $4/3\,\pi\,x_i^{o\,3}\,\rho_o\,n_o$

Mass flow rate of oxygen per unit area = $G_o = 4/3\,\pi\,x_i^{o\,3}\,\rho_o\,n_o\,v$

From Equation (4) $\quad -\frac{d}{dz}(G_o) = -\dot{n}_o\,n_o$

From Equation (3)

$$-\frac{d}{dz}(G_o) = \frac{\frac{n_o\,p_t}{R\,T_g}\left[p_{co} - p_{co_2}/K_{eo}\right]}{\frac{1}{4\pi\alpha\,x_o^{o\,2}} + \frac{x_o^o - x_i^o}{4\pi\beta\,x_i^o\,x_o^o} + \frac{1}{k_r\,4\pi\,x_i^{o\,2}}} \text{-------}(5)$$

$$\left.\begin{array}{l} G_o = 4/3\,\pi\,x_i^{o\,3}\,\rho_o\,n_o\,v \\ G_o^o = 4/3\,\pi\,x_o^{o\,3}\,\rho_o\,n_o\,v \end{array}\right\} \quad \text{———}(6)$$

$$\frac{G_o}{G_o^o} = \left(\frac{x_i^o}{x_o^o}\right)^3 = f_o \quad \text{———}(7)$$

$$-\frac{d}{dz}(G_o) = \left[\frac{\frac{n_o\,p_t}{R\,T_g}\left[p_{co} - p_{co_2}/K_{eo}\right]}{\frac{1}{4\pi\alpha\,x_o^{o\,2}} + \frac{1}{4\pi\beta}\left[\frac{1}{x_i^o} - \frac{1}{x_o^o}\right] + \frac{1}{4\pi\,k_r\,x_i^{o\,2}}}\right]$$

$$-\frac{d}{dz}(f_o) = \frac{3\,p_t}{R\,T_g\,x_o^o\,\rho_o\,v}\left\{\frac{p_{co} - p_{co_2}/K_{eo}}{1/\alpha + x_o^o/\beta\left[f_o^{-1/3} - 1\right] + f_o^{-2/3}/k_r}\right\} \quad \text{———}(8)$$

Gasification of Carbon:

$$C + CO_2 \longrightarrow 2CO \quad \text{-----------------------} (9)$$

Rate of gasification[9] of the coke particle is given by:

$$-\dot{n}_c = D_{CO_2} p_{CO_2} \, p_t \, \frac{4\pi x_o^{c\,2}}{R\,T_g} \left[\frac{\lambda_o^c x_o^c \cosh(\lambda_o^c x_o^c) - \sinh(\lambda_o^c x_o^c)}{x_o^c \sinh(\lambda_o^c x_o^c)} \right] \text{--} (10)$$

$$\text{Where} \quad \lambda_o^{c\,2} = \frac{\rho_c}{D_{CO_2}} \; \frac{K_{ct}}{p_{CO_2} + p_{CO}/K_{eq}} \; R\,T_g \quad \text{———} (11)$$

$$\text{Let } A = \frac{\lambda_o^c x_o^c \cosh(\lambda_o^c x_o^c) - \sinh(\lambda_o^c x_o^c)}{\sinh(\lambda_o^c x_o^c)} \text{—} (12)$$

$$-\dot{n}_c = \frac{D_{CO_2} \; p_{CO_2} \, 4\pi x_o^c \; A \; p_t}{R\,T_g} \quad \text{———} (13)$$

Volume of a particle at any time $= \frac{4}{3}\pi x_i^{c\,3}$

Mass of carbon in one particle $= \frac{4}{3}\pi x_i^{c\,3} \rho_c$

Change in carbon content with time $= -\dot{n}_c = -\frac{d}{dt}\left(\frac{4}{3}\pi x_i^{c\,3} \rho_c\right)$

Change in carbon content per unit volume per unit time

$$= -\dot{n}_c n_c = -\frac{d}{dt}\left(\frac{4}{3}\pi x_i^{c\,3} \rho_c n_c\right)$$

$$-\dot{n}_c n_c = -\frac{v}{v}\frac{d}{dt}\left(\frac{4}{3}\pi x_i^{c\,3} \rho_c n_c\right)$$

$$-\dot{n}_c n_c = -\frac{d}{dz}\left(\frac{4}{3}\pi x_i^{c\,3} \rho_c n_c v\right) \quad \text{———} (14)$$

Mass flow rate of carbon per unit area $= G_c = \frac{4}{3}\pi x_i^{c\,3} \rho_c n_c v$

From equation (14)

$$-\frac{d}{dz}(G_c) = -\dot{n}_c n_c$$

From equation (13)

$$-\frac{d}{dz}(G_c) = \frac{D_{CO_2} \; p_{CO_2} \, 4\pi x_o^c \, n_c \, A \, p_t}{R\,T_g} \quad \text{-------} (15)$$

Decomposition of lime stone:

$$CaCO_3 \longrightarrow CaO + CO_2 \qquad \text{-----------------(16)}$$

The rate of decomposition[17] is given by:

$$-\dot{n}_l = \frac{(p_{CO_2})_{eq} - p_{CO_2}}{\frac{1}{4\pi x_o^l p_t}\left\{\frac{R\,T_g}{D}\left[\frac{x_o^l}{x_i^l} - 1 + \left(\frac{D}{\gamma x_o^l}\right)\right] + \frac{a_H}{k}\left[\frac{x_o^l}{x_i^l} - 1 + \frac{K}{h\,x_o^l}\right]\right\}} \quad \text{---(17)}$$

Volume of an undecomposed solid particle = $4/3\,\pi\,x_i^{l\,3}$

Decomposition of lime stone with time $= -\dot{n}_l = -\frac{d}{dt}\left(\frac{4}{3}\pi\,x_i^{l\,3}\ell_l\right)$

Decomposition of lime stone per unit volume per unit time=

$$-\dot{n}_l n_l = -\frac{d}{dt}\left(\frac{4}{3}\pi\,x_i^{l\,3}\,n_l\,\ell_l\right)$$

$$= -\frac{u}{u}\,\frac{d}{dt}\left(\frac{4}{3}\pi\,x_i^{l\,3}\,n_l\,\ell_l\right)$$

$$= -\frac{d}{dz}\left(\frac{4}{3}\pi\,x_i^{l\,3}\,n_l\,\ell_l u\right)$$

Mass flow rate of lime stone per unit area $= G_l = \frac{4}{3}\pi\,x_i^{l\,3}\ell_l n_l u$

$$-\frac{d}{dz}(G_l) = -\dot{n}_l n_l$$

$$-\frac{d}{dz}(G_l) = \frac{4\pi x_o^l p_t n_l\left[(p_{CO_2})_{eq} - p_{CO_2}\right]}{\left\{\frac{R\,T_g}{D}\left[\frac{x_o^l}{x_i^l} - 1 + \frac{D}{\gamma x_o^l}\right] + \frac{a_H}{K}\left[\frac{x_o^l}{x_i^l} - 1 + \frac{K}{h\,x_o^l}\right]\right\}} \quad \text{---(18)}$$

$$f_l = \frac{G_l}{G_l^o} = \left(\frac{x_i^l}{x_o^l}\right)^3$$

$$-\frac{d}{dz}(G_l) = \frac{4\pi x_o^l p_t n_l\left[(p_{CO_2})_{eq} - p_{CO_2}\right]}{\left\{\frac{R\,T_g}{D}\left[\frac{x_o^l}{x_i^l} - 1 + \frac{D}{\gamma x_o^l}\right] + \frac{a_H}{K}\left[\frac{x_o^l}{x_i^l} - 1 + \frac{K}{h\,x_o^l}\right]\right.}$$

$$-\frac{d}{dz}(G_l) = \frac{4\pi x_o^l p_t n_l\left[(p_{CO_2})_{eq} - p_{CO_2}\right]}{\frac{R\,T_g}{D}\left[f_l^{-1/3} - 1 + \frac{D}{\gamma x_o^l}\right] + \frac{a_H}{K}\left[\frac{x_o^l}{x_i^l} - 1 + \frac{K}{h\,x_o^l}\right]} \quad \text{---(19)}$$

Derivation of the expressions for partial pressures of carbonmonoxide and carbon dioxide in the furnace:

The solid are moving downwards in positive Z direction whereas the gases are moving upwards in the negative Z direction (Fig.3).

Considering the mass balance over a section of thickness ΔZ.

Rate of Mass in - Rate of Mass out + Rate of formation = 0

Considering the mass balance for oxygen

$$Go\Big|_{Z} - Go\Big|_{Z+\Delta Z} = -\text{ rate of formation}$$

$$Go\Big|_{Z} - Go\Big|_{Z+\Delta Z} = +\, \dot{r}o\,\Delta Z$$

$$S(Go\Big|_{Z} - Go\Big|_{Z+\Delta Z}) = +\, \dot{r}oS\Delta Z$$

On dividing by ΔZ and taking limit

$$-\frac{d}{dZ}(Go) = \dot{r}o \qquad \text{--------------}(20)$$

Similar expressions for carbon and calcium carbonate can be written as-

$$-\frac{d}{dZ}(Gc) = \dot{r}c \qquad \text{--------------}(21)$$

$$-\frac{d}{dZ}(Gl) = \dot{r}l \qquad \text{--------------}(22)$$

The gases are moving upwards, so their mass balance equations can be written as-

Rate of mass in - Rate of mass out + Rate of formation = 0

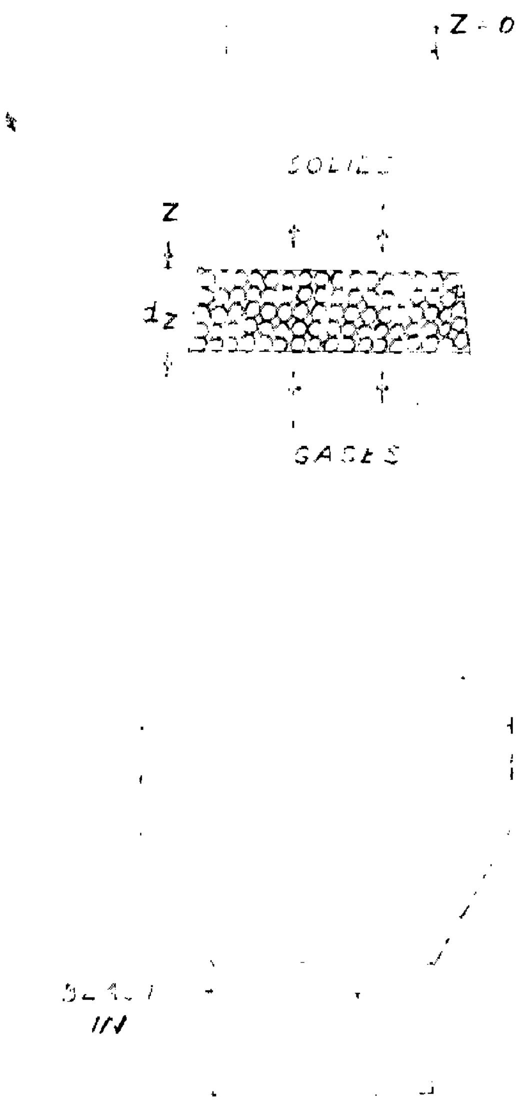


FIG. 3(a)

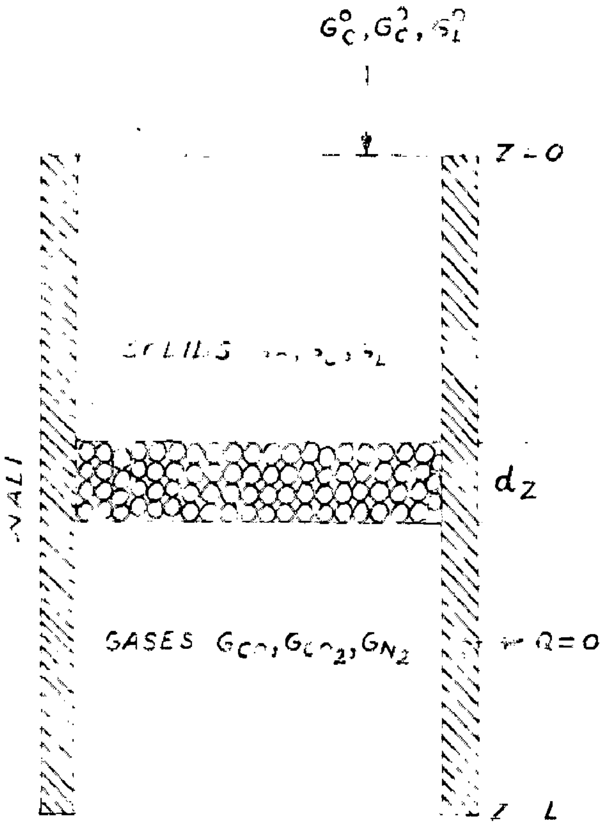


FIG. 3(b)

BLAST FURNACE SECTION OF SMALL THICKNESS.

$$S \left[Gco \Big|_{Z+\Delta Z} - Gco \Big|_{Z} \right] = \dot{r}co \; S \, \Delta Z$$

On dividing by ΔZ and taking limit

$$\frac{d}{dZ} (Gco) = \dot{r}co \quad \text{-----------------------(23)}$$

Similarly

$$\frac{d}{dZ} (Gco_2) = \dot{r}co_2 \quad \text{-----------------------(24)}$$

$$\frac{d}{dZ} (Gg) = \dot{r}g \quad \text{-----------------------(25)}$$

where $g = CO + CO_2 + N_2$ i.e. total gases.

The basic reactions taking place in the blast furnace are-

$$O + CO \xrightarrow{\dot{r}o} CO_2 \quad \text{-----------(26)}$$

$$C + CO_2 \xrightarrow{\dot{r}c} 2CO \quad \text{-----------(27)}$$

$$CaCO_3 \xrightarrow{\dot{r}l} CaO + CO_2 \quad \text{--------(28)}$$

Equation (26) is the removal of oxygen by carbonmonoxide so its rate of the forward reaction would be $\dot{r}o$. Similarly rates of reactions (27) and (28) would be $\dot{r}c$ and $\dot{r}l$.

From equations (26),(27) and (28)

$$\dot{r}co = \dot{r}o - 2\,\dot{r}c \quad \text{------------(29)}$$

$$\dot{r}co_2 = +\dot{r}o + \dot{r}c - \dot{r}l \quad \text{------------(30)}$$

On adding equations (29) and (30)

$$\dot{r}g = \dot{r}co + \dot{r}co_2 = -\,\dot{r}c - \dot{r}l \quad \text{---(31)}$$

since there is no removal of nitrogen gas.

On substituting the values of the rates of removals of various species from equations (20) to (25) in equations (29), (30) and (31)-

$$\frac{d}{dZ}(Gco) = -\frac{d}{dZ}(Go) + 2\frac{d}{dZ}(Gc) \text{ ---------(32)}$$

$$\frac{d}{dZ}(Gco_2) = \frac{d}{dZ}(Go) - \frac{d}{dZ}(Gc) + \frac{d}{dZ}(G1) \text{ -----(33)}$$

$$\frac{d}{dZ}(Gg) = \frac{d}{dZ}(G1) + \frac{d}{dZ}(Gc) \text{ ---------------(34)}$$

On integrating equations (32), (33) and (34).

$$Gco = -Go + 2Gc + C1 \text{ ------------------------(35)}$$

$$Gco_2 = Go - Gc + G1 + C2 \text{ ----------------------(36)}$$

$$Gg = G1 + Gc + C3 \text{ ----------------------(37)}$$

where C1, C2 and C3 are the constants of integration, and can be evaluated by boundary conditions.

Boundary conditions are-

at 1200°C

$$\text{let}\quad \left.\begin{aligned} G1 &= 0 \\ Gco_2 &= 0 \\ Go &= Go^* \\ Gc &= Gc^* \\ Gco &= Gco^* \\ Gg &= Gg^* \end{aligned}\right\} \text{ --------------------(38)}$$

So constants of integration from equations (35),(36) & (37) will be-

$$C1 = Gco^* + Go^* - 2Gc^* \text{ ----------------} (39)$$

$$C2 = 0 - Go^* + Gc^* - 0 = Gc^* - Go^* \text{ -------} (40)$$

$$C3 = Gg^* - Gc^* \text{ ----------------} (41)$$

On substituting the values of C1,C2 & C3 in equations (35),(36) and (37)-

$$Gco = - Go + 2Gc + Gco^* + Go^* - 2Gc^* \text{ ----} (42)$$

$$Gco_2 = Go - Gc + Gl + Gc^* - Go^* \text{ ----------} (43)$$

$$Gg = Gl + Gc + Gg^* - Gc^* \text{ ---------------} (44)$$

Consider the lower portion of the blast furnace. The blast enters the furnace through the tuyeres at tuyeres' level. The oxygen of the blast is used for burning the carbon. The unreduced wustite (FeO) gets reduced by carbon directly at temperatures greater than 1200°C. Some carbon is dissolved in the pig iron and the rest of the carbon is burnt to carbon monoxide at tuyeres' level.

Let $\bar{G}c$ be the remaining carbon at tuyeres' level for burning with incoming blast. This carbon is burnt to give carbon monoxide by the reaction-

$$2C + O_2 = 2CO$$

$$\text{Molar Volume of CO} = \bar{G}c \text{ -----------------} (45)$$

$$\text{Molar Volume of oxygen gas needed for combustion} = \frac{\bar{G}c}{2} \text{ --} (46)$$

Molar Volume of nitrogen gas associated with this oxygen =

$\frac{\overline{G}c}{2} \times \frac{79}{21}$ ----------------(47)

Gases after combustion at the tuyeres' level would be CO and N_2 . The total volume of gases would be-

$\overline{G}c + \frac{\overline{G}c}{2} \times \frac{79}{21}$

$= \overline{G}c \left[1 + \frac{79}{42}\right]$ ----------------(48)

Carbon used for direct reduction of Go* oxygen associated with FeO is Go* according to reaction

$C + O(FeO) \longrightarrow CO$

Carbon going to pig iron = Gc* - $\overline{G}c$ - Go* -----(49)

Gases generating from tuyeres' level to 1200°C level (since only direct reduction is taking place) = Go* moles of CO --(50)

So total gases at 1200°C are (adding (48) &(50)) -

$\overline{G}c \left[1 + \frac{79}{42}\right] + Go^* = Gg^*$ --------------(51)

Total carbon monoxide at 1200°C is

$\overline{G}c + Go^* = Gco^*$ --------------------(52)

Let Dr be the Driving rate of blast

Oxygen in the blast = .21 Dr.

Combustion reaction of carbon at tuyeres' level is

$C + \frac{1}{2} O_2 (g) = CO$

$$\text{Hence } \overline{Gc} = .21\ Dr \times 2 = 0.42\ Dr \quad \text{--------(53)}$$

Substituting the values of $\overline{Gc}$ from equation (53) in equations (51) and (52)

$$Gg^{*} = 0.42\ Dr \left[1 + \frac{79}{42}\right] + Go^{*} \quad \text{----------(54)}$$

and

$$Gco^{*} = 0.42\ Dr + Go^{*} \quad \text{--------------(55)}$$

Substituting the values of Gg^{*} and Gco^{*} in equations (42), (43) & (44) –

$$Gco = -\ Go + 2Gc + Go^{*} - 2Gc^{*} + 0.42\ Dr + Go^{*} \text{---(56)}$$

$$Gco_2 = Go - Gc + Gl + Gc^{*} - Go^{*} \quad \text{---------------(57)}$$

$$Gg = Gl + Gc - Gc^{*} + 0.42\ Dr \left(\frac{121}{42}\right) + Go^{*} \text{---(58)}$$

$$pco = \frac{Gco}{Gg} = \frac{-Go+2Gc + 2Go^{*} - 2Gc^{*} + 0.42\ Dr}{Gl + Gc + Go^{*} - Gc^{*} + 0.42\ Dr(121/42)} \quad \text{---------------(59)}$$

$$pco_2 = \frac{Gco_2}{Gg} = \frac{Go-Gc+Gl+Gc^{*} - Go^{*}}{Gl + Gc + Go^{*}-Gc^{*}+0.42Dr(121/42)} \text{----(60)}$$

The numerator and denominator of equations (59) & (60) could be made dimensionless by dividing $Go^{\ominus}$.

$$pco = \frac{-\frac{Go}{Go^{\ominus}} + \frac{2Gc}{Go^{\ominus}} + \frac{2Go^{*}}{Go^{\ominus}} - \frac{2Gc^{*}}{Go^{\ominus}} + \frac{Dr}{Go^{\ominus}} \times 0.42}{\frac{Gl}{Go^{\ominus}} + \frac{Gc}{Go^{\ominus}} + \frac{Go^{*}}{Go^{\ominus}} - \frac{Gc^{*}}{Go^{\ominus}} + \frac{Dr}{Go^{\ominus}} \times 1.21} \text{------(61)}$$

$$pco_2 = \frac{\frac{Go}{Go^{\ominus}} - \frac{Gc}{Go^{\ominus}} + \frac{Gl}{Go^{\ominus}} + \frac{Gc^{*}}{Go^{\ominus}} - \frac{Go^{*}}{Go^{\ominus}}}{\frac{Gl}{Go^{\ominus}} + \frac{Gc}{Go^{\ominus}} + \frac{Go^{*}}{Go^{\ominus}} - \frac{Gc^{*}}{Go^{\ominus}} + \frac{Dr}{Go^{\ominus}} \times 1.21} \text{-------(62)}$$

Equations (61) and (62) give the partial pressures of carbon monoxide and carbon dioxide at any level in the furnace from room temperature to 1200°C.

Integral to be maximised:

The aim here is to reduce the maximum possible iron oxide with the minimum coke consumption between room temperature and 1200°C. Also the initial input of ore should be maximum. By incorporating these conditions the integral which is to be maximised can be written as follows:

$$I' = G_o^\circ + \int_{Z=o}^{Z=L} (\dot{r}o - \dot{r}c)\, dZ$$

The limits Z = 0 and Z=L correspond to room temperature and 1200°C.

$$\text{or } I' = 1 + \int_{Z=o}^{Z=L} \left(\frac{\dot{r}o}{G_o^\circ} - \frac{\dot{r}c}{G_o^\circ} \right) dZ$$

$$\text{or } I' = 1 + \int_{Z=o}^{Z=L} (f_1 - f_2)\, dZ \text{ ----------(63)}$$

$$\text{where } f_1 = \frac{\dot{r}o}{G_o^\circ} \text{ and } f_2 = \frac{\dot{r}c}{G_o^\circ}$$

So the integral which is to be maximised can be written as follows:

$$I = \int_{Z=o}^{Z=L} (f_1 - f_2)\, dZ \text{ ---------------(64)}$$

$$\text{or } I = \int_{Z=o}^{Z=L} f_o \, dZ \text{ ----------------(65)}$$

$$\text{where } f_o = f_1 - f_2 \text{ ----------------------------(66)}$$

The problem, therefore, is to find out mathematically that temperature profile which would maximise the integral given by equation (65).

CHAPTER - IV

MATHEMATICAL ANALYSIS & COMPUTATION

MATHEMATICAL ANALYSIS & COMPUTATION

For maximising the integral given by equation (65), let the equation (8) be rewritten and equations (15) and (19) be divided by Go°:

$$-\frac{d}{dz}\left(\frac{G_o}{G_o^o}\right) = \frac{3p_t}{R T_g x_o^o \rho_o v}\left\{\frac{p_{co} - p_{co_2}/K_{eo}}{1/\alpha + x_o^o/\beta\left[f_o^{-1/3} - 1\right] + f_o^{-2/3}/R_r}\right\} \text{---}(8)$$

$$-\frac{d}{dz}\left(\frac{G_c}{G_c^o}\right) = \frac{3 p_t D_{co_2} p_{co_2} x_o^c n_c A}{R T_g x_o^{o3} \rho_o n_o v} \quad \ldots\ldots\ldots(67)$$

$$-\frac{d}{dz}\left(\frac{G_l}{G_o^o}\right) = \frac{3 p_t x_o^l n_l\left[(p_{co_2})_{eq} - p_{co_2}\right]}{\rho_o x_o^{o3} n_o v\left\{\frac{R T_g}{D}\left[f_l^{-1/3} - 1 + \frac{D}{\gamma x_o^l}\right] + \frac{a_H}{K}\left[f_l^{-1/3} - 1 + \frac{K}{k x_o^l}\right]\right.}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(68)$$

Since $-\frac{d}{dZ}(Go) = \dot{r}o$, from equation (20)

Hence $-\frac{d}{dZ}\left(\frac{Go}{Go^\circ}\right) = \frac{\dot{r}o}{Go^\circ} = f_1$

Similarly $-\frac{d}{dZ}\left(\frac{Gc}{Go^\circ}\right) = \frac{\dot{r}c}{Go^\circ} = f_2$

and $-\frac{d}{dZ}\left(\frac{Gl}{Go^\circ}\right) = \frac{\dot{r}l}{Go^\circ} = f_3$

$$\text{Let}\quad \left.\begin{aligned} \frac{Go}{Go^\circ} &= X_1 \\ \frac{Gc}{Go^\circ} &= X_2 \\ \frac{Gl}{Go^\circ} &= X_3 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(69)$$

Equations (8), (67) & (68) are equivalent to:

$$\frac{dX_1}{dZ} = - f_1 \qquad \ldots\ldots\ldots\ldots\ldots(70)$$

$$\frac{d\ X_2}{dZ} = - f_2 \qquad \ldots\ldots\ldots\ldots\ldots(71)$$

$$\frac{d\ X_3}{dZ} = - f_3 \qquad \ldots\ldots\ldots\ldots\ldots(72)$$

Let a new component X_o be defined as:

$$\frac{d\ X_o}{dZ} = - f_o \qquad \ldots\ldots\ldots\ldots\ldots(73)$$

with the inlet conditions

$$X_o = 0 \quad \text{at } Z = 0$$

Optimization Method[18]:

Here is defined a set of new terms known as 'adjoint functions' (λi) which satisfies a set of coupled differential equations:

$$\frac{d\lambda_j}{dZ} = \sum_{i=0}^{n} \frac{\partial f_i}{\partial X_j} \lambda i \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(74)$$

Where $j = 0, 1, 2, \ldots, n$.

So, for this problem, the adjoint functions can be written as follows:

$$\frac{d\lambda_o}{dZ} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(75)$$

$$\frac{d\lambda_1}{dZ} = \left[\frac{\partial f_o}{\partial X_1}\lambda_o + \frac{\partial f_1}{\partial X_1}\lambda_1 + \frac{\partial f_2}{\partial X_1}\lambda_2 + \frac{\partial f_3}{\partial X_1}\lambda_3\right] \ldots(76)$$

$$\frac{d\lambda_2}{dZ} = \left[\frac{\partial f_o}{\partial X_2}\lambda_o + \frac{\partial f_1}{\partial X_2}\lambda_1 + \frac{\partial f_2}{\partial X_2}\lambda_2 + \frac{\partial f_3}{\partial X_2}\lambda_3\right] \ldots(77)$$

$$\frac{d\lambda_3}{dZ} = \left[\frac{\partial f_o}{\partial X_3}\lambda_o + \frac{\partial f_1}{\partial X_3}\lambda_1 + \frac{\partial f_2}{\partial X_3}\lambda_2 + \frac{\partial f_3}{\partial X_3}\lambda_3\right] \ldots(78)$$

The optimum temperature profile for the maximisation of the integral given by equation (65) in one, which satisfies:

$$\sum_{i=o}^{n} \lambda i \frac{\partial f_i}{\partial T} = 0$$

$$\text{or } \lambda_o \cdot \frac{\partial f_o}{\partial T} + \lambda_1 \cdot \frac{\partial f_1}{\partial T} + \lambda_2 \cdot \frac{\partial f_2}{\partial T} + \lambda_3 \cdot \frac{\partial f_3}{\partial T} = 0 \quad \ldots(79)$$

Where

$$\left.\begin{aligned} \frac{\partial f_o}{\partial X_1} &= \frac{\partial f_1}{\partial X_1} - \frac{\partial f_2}{\partial X_1} \\ \frac{\partial f_o}{\partial X_2} &= \frac{\partial f_1}{\partial X_2} - \frac{\partial f_2}{\partial X_2} \\ \frac{\partial f_o}{\partial X_3} &= \frac{\partial f_1}{\partial X_3} - \frac{\partial f_2}{\partial X_3} \\ \frac{\partial f_o}{\partial T} &= \frac{\partial f_1}{\partial T} - \frac{\partial f_2}{\partial T} \end{aligned}\right) \quad \ldots\ldots\ldots\ldots(80)$$

and λi , given by equations (75) to (78), are undefined functions of position which are restricted by the following Boundary Conditions:

$$\left.\begin{aligned} &\lambda_o(L) = 1 \\ &\lambda_1(L) = \lambda_2(L) = \lambda_3(L) = 0 \end{aligned}\right) \quad \ldots\ldots\ldots\ldots(81)$$

Equation (75) can be integrated to give

$$\lambda_o = \text{constant} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(82)$$

On substituting the Boundary Condition from equation (81)

At $Z=L$, constant $= 1$

On substituting the value of constant in equation (82)

$$\lambda_o = 1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(83)$$

On substituting the value of λ_o from equation (83) and expressions of f_o from equations (80), equations (76),(77) and (78) can be written as:

$$\frac{d\lambda_1}{dZ} = \left[\frac{\partial f_1}{\partial X_1} - \frac{\partial f_2}{\partial X_1} + \frac{\partial f_1}{\partial X_1}\lambda_1 + \frac{\partial f_2}{\partial X_1}\lambda_2 + \frac{\partial f_3}{\partial X_1}\lambda_3\right]$$

$$\frac{d\lambda_1}{dZ} = \left[\frac{\partial f_1}{\partial X_1}(\lambda_1 + 1) + \frac{\partial f_2}{\partial X_1}(\lambda_2 - 1) + \frac{\partial f_3}{\partial X_1}\lambda_3\right]$$

$$\frac{d\lambda_2}{dZ} = \left[\frac{\partial f_1}{\partial X_2}(\lambda_1 + 1) + \frac{\partial f_2}{\partial X_2}(\lambda_2 - 1) + \frac{\partial f_3}{\partial X_2}\lambda_3\right]$$

$$\frac{d\lambda_3}{dZ} = \left[\frac{f_1}{X_3}(\lambda_1 + 1) + \frac{\partial f_2}{\partial X_3}(\lambda_2 - 1) + \frac{\partial f_3}{\partial X_3}\lambda_3\right]$$

$$\left.\begin{aligned} &\text{Let } (\lambda_1 + 1) = \overline{\lambda_1} \\ &\text{and } (\lambda_2 - 1) = \overline{\lambda_2} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(84)$$

On differentiating equations (84)

$$\frac{d\lambda_1}{dZ} = \frac{d\overline{\lambda_1}}{dZ}$$

$$\frac{d\lambda_2}{dZ} = \frac{d\overline{\lambda_2}}{dZ}$$

Hence $$\frac{d\overline{\lambda_1}}{dZ} = \left[\frac{\partial f_1}{\partial X_1}\cdot\overline{\lambda_1} + \frac{\partial f_2}{\partial X_1}\cdot\overline{\lambda_2} + \frac{\partial f_3}{\partial X_1}\cdot\lambda_3\right] \text{ ---------(8}$$

$$\frac{d\overline{\lambda_2}}{dZ} = \left[\frac{\partial f_1}{\partial X_2}\cdot\overline{\lambda_1} + \frac{\partial f_2}{\partial X_2}\cdot\overline{\lambda_2} + \frac{\partial f_3}{\partial X_2}\cdot\lambda_3\right] \text{ ---------(86)}$$

$$\frac{d\lambda_3}{dZ} = \left[\frac{\partial f_1}{\partial X_3}\cdot\overline{\lambda_1} + \frac{\partial f_2}{\partial X_3}\cdot\overline{\lambda_2} + \frac{\partial f_3}{\partial X_3}\cdot\lambda_3\right] \text{ ---------(87)}$$

Equations (85), (86), (87) contain modified adjoint functions $\overline{\lambda_1}$, $\overline{\lambda_2}$, which are defined by equations (84).

So modified Boundary Conditions are:

$$\left.\begin{array}{l} \overline{\lambda_1}(L) = 1 \\ \overline{\lambda_2}(L) = -1 \\ \lambda_3(L) = 0 \end{array}\right) \text{ ------------------(88)}$$

On substituting the value of λ_o from equation (83) and expression of f_o from equation (80), equation (79) can be written as:

$$\frac{\partial f_1}{\partial T} - \frac{\partial f_2}{\partial T} + \lambda_1\cdot\frac{\partial f_1}{\partial T} + \lambda_2\cdot\frac{\partial f_2}{\partial T} + \lambda_3\cdot\frac{\partial f_3}{\partial T} = 0$$

or $$\frac{\partial f_1}{\partial T}(\lambda_1+1) + \frac{\partial f_2}{\partial T}(\lambda_2 - 1) + \lambda_3 \frac{\partial f_3}{\partial T} = 0$$

or $$\frac{\partial f_1}{\partial T}\overline{\lambda_1} + \frac{\partial f_2}{\partial T}\overline{\lambda_2} + \frac{\partial f_3}{\partial T}\lambda_3 = 0 \text{ ------------(89)}$$

$\frac{\partial f_i}{\partial X_j}$ and $\frac{\partial f_i}{\partial T}$, where $i = 1,2,3$ and $j = 1,2,3$ occurring in equations (85), (86), (87) and (89) can be obtained by partial differential of the right hand side term of equations (8), (67) and (68).

On comparing equations (8) and (70):

$$f_1 = \frac{3\,p_t}{R\,T_g\,x_0^o\,\rho_0\,\upsilon}\left\{\frac{p_{co} - p_{co_2}/K_{eo}}{1/\alpha + \frac{x_0^o}{\beta}\left[(X_1)^{-1/3} - 1\right] + (X_1)^{-2/3}/k_r}\right\} \text{———} (90)$$

For convenience of writing the lengthy expressions, some symbols are defined, to denote the expressions, as follows:

Pco = (A)/ (B) from equation (61)

P_{co_2}= (C)/(B) from equation (62)

$$(S) = \frac{3\,p_t}{R\,T_g\,x_0^o\,\rho_0\,\upsilon}$$

$$(D) = \left\{ 1/\alpha + \frac{x_0^o}{\beta}\left[(X_1)^{-1/3} - 1\right] + (X_1)^{-2/3}/k_r \right\}$$

Equation (90) can be written conveniently as:

$$f_1 = (S)\left[\frac{(A)/(B) - (C)/(B)/\,Keo}{(D)}\right] \text{-----------------} (91)$$

On comparing equations (67) and (71):

$$f_2 = \frac{3\,p_t\,D_{co_2}\,x_0^c\,n_c\,A\,p_{co_2}}{R\,T_g\,x_0^{o3}\,\rho_0\,n_0\,\upsilon} \qquad \text{———} \qquad (92)$$

$$= \frac{3\,p_t\,D_{co_2}\,x_0^c\,n_c\,A\,(C)/(B)}{R\,T_g\,x_0^{o3}\,\rho_0\,n_0\,\upsilon} \qquad \text{———} \qquad (93)$$

On comparing equations (68) and (72):

$$f_3 = \frac{3 p_t x_0^l n_e \{ (p_{co_2})_{eq} - p_{co_2} \}}{\rho_o x_0^{o3} n_o v \left\{ \frac{R T_g}{D} \left[f_l^{-1/3} - 1 + \frac{D}{\gamma x_0^l} \right] + \frac{a H}{K} \left[f_l^{-1/3} - 1 + \frac{K}{h x_0^l} \right] \right\}} \quad \ldots\ldots(94)$$

$$(G) = 3 p_t x_0^l n_e \{ (p_{co_2})_{eq} - p_{co_2} \}$$

$$(P) = \rho_o x_0^{o3} n_o v \left\{ \frac{R T_g}{D} \left[f_l^{-1/3} - 1 + \frac{D}{\gamma x_0^l} \right] + \frac{a H}{K} \left[f_l^{-1/3} - 1 + \frac{K}{h x_0^l} \right] \right\}$$

$$f_3 = (G)/(P) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(95)$$

On differentiating partially, the equations (91),(93) & (95) with respect to X_1, X_2 and X_3:

$$\frac{\partial f_1}{\partial X_1} = \frac{(S)}{(D)^2} \left\{ \left(-\frac{1}{(B)} - \frac{1}{(B) K_{eo}} \right)(D) - (p_{co} - p_{co_2}/K_{eo}) \left[\frac{x_0^o}{\beta} \left(-\frac{1}{3} X_1^{-\frac{4}{3}} \right) - \frac{2}{3} \frac{X_1^{-\frac{5}{3}}}{RT} \right] \right\} \quad \text{———— (96)}$$

$$\frac{\partial f_2}{\partial X_1} = \frac{3 p_t D_{co_2} x_0^c n_c A}{R T_g x_0^{o3} \rho_o n_o v} \cdot \frac{1}{(B)} \quad \text{———— (97)}$$

$$\frac{\partial f_3}{\partial X_1} = \frac{3 p_t x_0^l n_e \{ (p_{co_2})_{eq} - 1/(B) \}}{(P)} \quad \text{———— (98)}$$

$$\frac{\partial f_1}{\partial X_2} = \frac{(S)}{(D)} \left\{ \frac{2(B) - (A)}{(B)^2} - \frac{-(B) - (C)}{(B)^2 K_{eo}} \right\} \quad \text{———— (99)}$$

$$\frac{\partial f_2}{\partial X_2} = \frac{3 p_t D_{co_2} x_0^c n_c A}{R T_g x_0^{o3} \rho_o n_o v} \left\{ \frac{-(B) - (C)}{(B)^2} \right\} \quad \text{———— (100)}$$

$$\frac{\partial f_3}{\partial X_2} = \frac{3 p_t x_0^l n_e}{(P)} \left\{ (p_{co_2})_{eq} - \left[\frac{-(B) - (C)}{(B)^2} \right] \right. \quad \text{———— (101)}$$

$$\frac{\partial f_1}{\partial X_3} = \frac{(S)}{(D)}\left\{-\frac{(A)}{(B)^2} - \frac{(B)-(C)}{(B)^2 K_{eo}}\right\} \quad \ldots\ldots\ldots\ldots\ldots(102)$$

$$\frac{\partial f_2}{\partial X_3} = \frac{3 p_t D_{CO_2} x_o^c n_c A}{R T_g x_o^{o3} \rho_o n_o v}\left[\frac{(B)-(C)}{(B)^2}\right] \quad \ldots\ldots\ldots\ldots\ldots(103)$$

$$\frac{\partial f_3}{\partial X_3} = \frac{3 p_t x_o^{\ell} n_\ell \left\{(p_{CO_2})_{eq} - \frac{(B)-(C)}{(B)^2}\right\}(P)}{(P)^2} -$$

$$-\frac{(G)}{(P)^2}\left[\rho_o x_o^{o3} n_o v\left(-\frac{1}{3} X_3^{-\frac{4}{3}}\right)\left(\frac{x_o^{o3} \rho_o n_o}{x_o^{\ell 3} \rho_\ell n_\ell}\right)^{-1/3}\left\{\frac{R T_g}{D} + \frac{a \cdot H}{K}\right\}\right]$$

$$\ldots\ldots\ldots\ldots\ldots(104)$$

On differentiating partially, equations (90), (92) & (94), with respect to temperature (assuming that temp. of solids and temp. of gas is same i.e. $(T_s = T_g = T)$:

$$\frac{\partial f_1}{\partial T} = \frac{3 p_t}{R x_o^o \rho_o v}\left(-\frac{1}{T_g^2}\right)\left\{\frac{p_{CO} - p_{CO_2}/K_{eo}}{(D)}\right\} + (S)\left[\frac{-p_{CO_2}\frac{\partial}{\partial T}\left(\frac{1}{K_{eo}}\right)\cdot(D)}{(D)^2} -\right.$$

$$\left. - \frac{p_{CO} - p_{CO_2}/K_{eo}}{(D)^2}\left\{\frac{\partial}{\partial T}\left(\frac{1}{\alpha}\right) + x_o^o\left(X_1^{-1/3} - 1\right)\frac{\partial}{\partial T}\left(\frac{1}{\beta}\right) + X_1^{-\frac{2}{3}}\frac{\partial}{\partial T}\left(\frac{1}{k_r}\right)\right\}\right]$$

$$\ldots\ldots\ldots\ldots\ldots(105)$$

$$\frac{\partial f_2}{\partial T} = \frac{3 p_t x_o^c n_c p_{CO_2}}{R x_o^{o3} \rho_o n_o v T_g}\left[\left\{A \cdot \frac{\partial}{\partial T}(D_{CO_2}) + D_{CO_2} \cdot \frac{\partial A}{\partial T}\right\} - \frac{D_{CO_2} \cdot A}{T_g}\right]$$

$$\ldots\ldots\ldots\ldots\ldots(106)$$

$$\frac{\partial f_3}{\partial T} = \frac{3\, p_t\, x_0^{\ell}\, n_\ell\, \frac{\partial}{\partial T}(p_{CO_2})_{eq}}{(P)} - \frac{(G)}{(P)^2}\left[\ell_0 x_0^{o3} n_0 v \left\{ R\left[\left(x_3 \frac{x_0^{o3} \ell_0 n_0}{x_0^{\ell 3} \ell_\ell n_\ell}\right)^{-\frac{1}{3}} - 1\right]\right.\right.$$

$$\left.\left. \left(\frac{1}{D} - \frac{T_g}{D^2}\frac{\partial D}{\partial T}\right) + \frac{R}{\gamma x_0^{\ell}} + \frac{H}{K}\left[\left(x_3 \frac{x_0^{o3} \ell_0 n_0}{x_0^{\ell 3} \ell_\ell n_\ell}\right)^{-1/3} - 1 + \frac{K}{h x_0^{\ell}}\right]\frac{\partial a}{\partial T} \right\}\right] \quad \ldots\ldots\ldots\ldots\ldots(107)$$

In equation(107) heat-transfer coefficient & mass transfer coefficient to surface of calcium carbonate have been assumed to be independent of temperature.

The terms occuring in above equations can be expressed as:

Let $K_{eo} = L_1 e^{-Q_1/RT}$

$$1/K_{eo} = \frac{1}{L_1} e^{Q_1/RT}$$

$$\frac{\partial}{\partial T}\left(\frac{1}{K_{eo}}\right) = \frac{1}{L_1}\left(-\frac{Q_1}{RT^2}\right) e^{Q_1/RT} \quad \ldots\ldots\ldots\ldots\ldots\ldots(108)$$

$$\alpha = \frac{K_{eo}\, k_{m(CO)}\, k_{m(CO_2)}}{K_{eo}\, k_{m(CO_2)} + k_{m(CO)}}$$

Taking $k_{m(CO)} = k_{m(CO_2)} = k_m$

$$\alpha = \frac{K_{eo}}{K_{eo}+1} \cdot k_m$$

Let $k_m = M T^m$

$$\frac{1}{\alpha} = \frac{K_{eo}+1}{K_{eo}\cdot k_m} = \frac{1}{k_m} + \frac{1}{K_{eo}\cdot k_m}$$

$$= \frac{1}{M} T^{-m} + \frac{1}{L_1 M} T^{-m} e^{Q_1/RT}$$

$$\frac{\partial}{\partial T}\left(\frac{1}{\alpha}\right) = -\frac{m}{M} T^{(-m-1)} + \frac{1}{L_1 M}\left(-m T^{(-m-1)} \cdot e^{Q_1/RT} + \left(-\frac{Q_1}{RT^2}\right) T^{-m} e^{Q_1/RT}\right)$$

$$\frac{\partial}{\partial T}\left(\frac{1}{\alpha}\right) = -\frac{m}{M} T^{(-m-1)} + \frac{1}{L_1 M} e^{Q_1/RT}\left(-m T^{(-m-1)} - \frac{Q_1}{RT^2} T^{-m}\right)$$

...............(109)

$$\beta = \frac{K_{eo}\, D_{CO}^{eff}\, D_{CO_2}^{eff}}{K_{eo}\, D_{CO_2}^{eff} + D_{CO}^{eff}}$$

Taking $D_{CO}^{eff} = D_{CO_2}^{eff} = D^{eff}$

Let $D^{eff} = N T^{n}$

$$\beta = \frac{K_{eo}\, D^{eff}}{K_{eo} + 1}$$

$$1/\beta = \frac{K_{eo} + 1}{K_{eo}\, D^{eff}} = \frac{1}{D^{eff}} + \frac{1}{K_{eo} \cdot D^{eff}}$$

$$= \frac{1}{N} T^{-n} + \frac{1}{L_1 N} T^{-n} e^{Q_1/RT}$$

$$\frac{\partial}{\partial T}\left(\frac{1}{\beta}\right) = -\frac{n}{N} T^{(-n-1)} + \frac{1}{L_1 N} e^{Q_1/RT}\left(-n T^{(-n-1)} - \frac{Q}{RT^2} T^{-n}\right)$$

.............(110)

Let $k_r = L_2\, e^{-Q_2/RT}$

$$1/k_r = 1/L_2\, e^{Q_2/RT}$$

$$\frac{\partial}{\partial T}\left(1/k_r\right) = 1/L_2 \left(-\frac{Q_2}{RT^2}\right) e^{Q_2/RT}$$

............(111)

Let $D_{CO_2} = N_1 T^{n_1}$

$$\frac{\partial}{\partial T}(D_{CO_2}) = N_1 n_1 T^{n_1 - 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(112)$$

$$A = \frac{\lambda_0^c x_0^c \cosh(\lambda_0^c x_0^c) - \sinh(\lambda_0^c x_0^c)}{\sinh(\lambda_0^c x_0^c)}$$

$$= \lambda_0^c x_0^c \coth(\lambda_0^c x_0^c) - 1$$

$$\frac{\partial A}{\partial T} = \coth(\lambda_0^c x_0^c) \cdot \frac{\partial}{\partial T}(\lambda_0^c x_0^c) + (\lambda_0^c x_0^c) \cdot \frac{\partial}{\partial T} \coth(\lambda_0^c x_0^c)$$

$$\frac{\partial A}{\partial T} = \coth(\lambda_0^c x_0^c) \cdot \frac{\partial}{\partial T}(\lambda_0^c x_0^c) + (\lambda_0^c x_0^c)(-\operatorname{cosech}^2(\lambda_0^2 x_0^2)) \cdot \frac{\partial}{\partial T}(\lambda_0^2 x_0^2)$$

$$\frac{\partial A}{\partial T} = \frac{\partial}{\partial T}(\lambda_0^c x_0^c)\left[\coth(\lambda_0^c x_0^c) - (\lambda_0^c x_0^c) \cdot \operatorname{cosech}^2(\lambda_0^c x_0^c)\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(113)$$

where $\lambda_0^c x_0^c = \left[\frac{l_c}{D_{CO_2}} \cdot \frac{K_{ct}}{p_{CO_2} + p_{CO}/K_{ec}} \cdot x_0^{c\,2} R\, T_g\right]^{1/2}$ ...(114)

Let $K_{ct} = L_3\, e^{-Q_3/RT}$

$K_{ec} = L_4\, e^{-Q_4/RT}$

$D_{CO_2} = N_1 T^{n_1}$

$$\lambda_0^c x_0^c = \left[\frac{l_c x_0^{c\,2} R \cdot L_3 e^{-Q_3/RT} \cdot L_4 e^{-Q_4/RT}}{N_1 T^{(n_1 - 1)}(L_4 p_{CO_2} e^{-\frac{Q_4}{RT}} + p_{CO})}\right]^{1/2}$$

$$= \left(\frac{l_c x_0^{c\,2} R L_3 L_4}{N_1 \; T^{(n_1-1)}}\right)^{1/2} \frac{e^{-\left(\frac{Q_3 + Q_4}{2RT}\right)}}{(L_4 p_{CO_2} e^{-Q_4/RT} + p_{CO})^{1/2}}$$

Let $L_5 = \left(\frac{l_c x_0^{c\,2} R L_3 L_4}{N_1}\right)^{1/2}$ & $Q_5 = \frac{Q_3 + Q_4}{2}$

Hence $$\lambda_0^c \chi_0^c = \frac{L_5}{T^{\frac{n_1-1}{2}}} \frac{e^{-Q_5/RT}}{(L_4 p_{CO_2} e^{-Q_4/RT} + p_{CO})^{1/2}}$$

$$\frac{\partial}{\partial T}(\lambda_0^c \chi_0^c) = L_5 \left[\frac{\frac{Q_5}{RT^2} \quad e^{-\frac{Q_5}{RT}}}{T^{\frac{n_1-1}{2}}(L_4 p_{CO_2} e^{-Q_4/RT} + p_{CO})^{1/2}} - \frac{e^{-\frac{Q_5}{RT}} \cdot L_6}{T^{(n_1-1)}(L_4 p_{CO_2} e^{-Q_4/RT} + p_{CO})} \right. \qquad \text{..............(115)}$$

where $$L_6 = \frac{n_1-1}{2} T^{\left(\frac{n_1-3}{2}\right)} \left(L_4 p_{CO_2} e^{-\frac{Q_4}{RT}} + p_{CO}\right) +$$
$$+ T^{\frac{n_1-1}{2}} \cdot \tfrac{1}{2} \cdot \left(L_4 p_{CO_2} e^{-\frac{Q_4}{RT}} + p_{CO}\right)^{-1/2} \cdot p_{CO_2} L_4 \frac{Q_4}{RT^2} e^{-\frac{Q_4}{RT}}$$

Let $$(p_{CO_2})_{eq} = L_7 e^{-\frac{Q_7}{T}}$$

$$\frac{\partial}{\partial T}(p_{CO_2})_{eq} = L_7 \frac{Q_7}{T^2} e^{-\frac{Q_7}{T}} \qquad \text{..................(116)}$$

Let $$D = N_2 T^{n_2}$$

$$\frac{\partial D}{\partial T} = N_2 n_2 T^{n_2-1} \qquad \text{..................(117)}$$

Let $$a = \frac{N_3}{T^{n_3}} e^{-\frac{Q_7}{T}}$$
$$= N_3 T^{-n_3} e^{-\frac{Q_7}{T}}$$

$$\frac{\partial a}{\partial T} = N_3(-n_3) T^{-n_3-1} \cdot e^{-\frac{Q_7}{T}} + N_3 T^{-n_3} \frac{Q_7}{T^2} e^{-\frac{Q_7}{T}}$$

$$\frac{\partial a}{\partial T} = N_3 e^{-\frac{Q_7}{T}} \left[-n_3 T^{-n_3-1} + Q_7 T^{-n_3-2} \right] \text{...(118)}$$

For finding the optimum temperature profile equations (8),(67),(68),(85),(86), (87) and (89) should be solved simultaneously. The analytical solution of these equations is not possible. Therefore, these equations are solved numerically.

Computational Scheme:

The scheme, for solving the above mentioned equations, numerically, is outlined in the following sequence of steps:

1. An arbitrary temperature profile has been assumed along the furnace height. This profile chosen here is approximately the existing temperature profile*. The temperatures at the intervals of 0.5 m, from the top, along the furnace height, have been determined.

2. The concentrations of various materials charged in the furnace have been computed along the furnace height by solving equations (8), (67) & (68) numerically by Runge-Kutta[19] method, (Numerical method is discussed in Appendix).

3. After establishing the concentrations, the adjoint functions have been computed by solving equations (85), (86) and (87) numerically again by Runge-Kutta[19] method.

4. For the determination of a better approximation for the optimum temperature, equation (89) has been used. The concentration values determined in step 2 and adjoint functions determined in step 3 have been substituted in equation (89) and a better value of temperature is computed.

5. A new temperature profile has been taken as an arithmetic mean of the assumed temperature and the temperature computed in step 4.

6. Steps 2, 3, 4 & 5 are repeated with the new temperature profiles until the temperature profile remains unchanged.

* The approximate existing temperature profile[20] used for starting the calculations is given by the following equation:

$$T = 273 + 1000 \left[1 - \exp(-0.25Z) + \exp(0.5625Z - 12.7)\right] \qquad \text{---------(119)}$$

Data Used in Computation

Equilibrium constants and Rate data:

Equilibrium constant[21] for

$Fe_2O_3 + CO \longrightarrow Fe + CO_2$

For $Ts < 848°K$

for $(1-X_1) < 0.111$: $Keo = \exp(4.91 + 6235/Ts)$

for $(1-X_1) > 0.111$: $Keo = \exp(-0.7625 + 543.3/Ts)$

For $Ts > 848°K$

for $(1-X_1) < 0.111$: $Keo = \exp(4.91 + 6235/Ts)$

for $0.111 < (1-X_1) < 0.333$: $Keo = \exp(2.13 - 2050/Ts)$

for $(1-X_1) > 0.333$: $Keo = \exp(-2.642 + 2164/Ts)$

Specific rate constant[22] for

$Fe_2O_3 + CO \longrightarrow Fe + CO_2$

$kr = 347 \exp(-3460/Ts)$

Equilibrium constant[9] for formation of CO

$Kec = \exp(10.957 - 15300/Ts)$

Rate Constant[9] for carbon gasification

$Ts < 1273°K$: $Kct = \exp(20.35 - 30590/Ts)$

$Ts > 1273°K$: $Kct = \exp(8.482 - 15400/Ts)$

Other parameters used:

$R = 0.0826\ m^3.\ atm/Kg\text{-}mole°K$

$Xo° = 0.02\ m$

Xo^{c} = 0.035 m

Xo^{l} = 0.017 m

v = 3 m/hr.

pt = 1 atm.

H = 42.5×10^{3} Kcal/kg-mole.

K[22] = 10.2×10^{-2} Kcal/m.hr.°K.

γ[24] = 129.6 m/hr.

h[24] = 71.6 Kcal/m^{2} . hr.°K.

Void fraction of bed = 0.4

Diffusion coefficient of CO in porous solid[22] = $2.592 \times 10^{-6}\ Ts^{1.78}$ m^{2}/hr.

Diffusion coefficient of CO_2 in porous solid[22] = $2.236 \times 10^{-6}\ Ts^{1.78}$ m^{2}/hr.

Density of iron ore = 4.33×10^{3} Kg/m^{3}

Density of coke = 1.24×10^{3} Kg/m^{3}

Density of lime stone = 2.5×10^{3} Kg/m^{3}

Porosity in iron ore[23] = Ep = .20

Porosity of lime stone = Epl= .15

Porosity of coke[9] = Ec = .50

Production rate of the furnace = 1500 tons/24 hrs.

Time of descent of raw material from top to tuyeres' level = 8 hrs.

Height of the furnace from top to tuyeres' level = 24 m

Average diameter of the furnace = 9 m.

Raw materials needed for one ton of pig iron:

1600 Kg of ore $\simeq$ 1370 Kg of Fe_2O_3.

800 Kg of coke

450 Kg of lime stone.

Driving rate = 150000 N m^3/hr.

Calculations of some parameters:

Driving rate:

$$\text{Driving rate} = 150000 \text{ N } m^3/hr.$$

$$= \frac{150000}{22.4} = 6700 \text{ Kg-moles/hr.}$$

Area of cross section = πr^2

$$= 3.14 \times 4.5 \times 4.5 = 63.6 \; m^2$$

$$Dr = \frac{6700}{63.6} = 105 \text{ Kg-moles}/m^2 . \text{ hr.}$$

Mass flow rates:

Iron oxide

1370 Kg of Fe_2O_3 $= \frac{1370 \times 48}{160} = 411$ Kg oxygen.

$$= \frac{411}{16} = 25.6 \text{ Kg-moles of oxygen/ton of metal}$$

G_o° = Kg moles/m^2/hr.

$$= \frac{25.6 \times 1500}{24 \times 63.6} = 25.4 \text{ kg moles}/m^2/hr.$$

Coke

800 Kg of coke $\simeq$ 640 Kg of carbon

$= \frac{640}{12} = 53.33$ Kg-moles of carbon/ton of metal.

Lime stone

450 Kg of calcium carbonate =

$\frac{450}{100}$ = 4.5 Kg-moles of $CaCO_3$/ton of metal.

$$X_1\Big|_{Z=o} = \left(\frac{Go}{Go^\circ}\right)_{Z=o} = \frac{25.4}{25.4} = 1.0$$

$$X_2\Big|_{Z=o} = \left(\frac{Gc}{Go^\circ}\right)_{Z=o} = \frac{53.33}{25.6} = 2.04$$

$$X_3\Big|_{Z=o} = \left(\frac{Gl}{Go^\circ}\right)_{Z=o} = \frac{4.5}{25.6} = 0.171$$

Number of particles per unit bed volume:

Let no, vo and do be the number of particles, volume of one particle and density of one particle of ore. Similarly nc, vc, dc for coke and nl, vl and dl for lime stone.

no vo = Total volume of iron ore particles in unit bed volume.
nc vc = Total volume of coke particles in unit bed volume.
nl vl = Total volume of lime stone particles in unit bed vol.

no vo + nc vc + nl vl = V
where V is the volume occupied by solids in one cu.m.

Void fraction of bed = 0.4
hence V = 0.6
no vo + nc vc + nl vl = 0.6 ....(a)

no vo do = Weight of iron ore in unit bed volume.

nc vc dc = Weight of coke in unit bed volume.

nl vl dl = Weight of lime stone in unit bed volume.

$$\frac{no\ vo\ do}{nc\ vc\ dc} = k_1 = \frac{1600}{800} = 2.0 \ldots\ldots\ldots\ldots (b)$$

$$\frac{no\ vo\ do}{nl\ vl\ dl} = k_2 = \frac{1600}{450} = 3.56 \ldots\ldots\ldots\ldots (c)$$

Radius of iron ore particle = 0.02 m

Radius of coke particle = 0.035 m

Radius of lime stone particle=0.017 m

Volume of one iron ore particle $= \frac{4}{3} \times 3.14 \times 2 \times 2 \times 2 \times 10^{-6} m^3$

$= 33.49 \times 10^{-6}\ m^3$

Volume of one coke particle $= \frac{4}{3} \times 3.14 \times 3.5 \times 3.5 \times 3.5 \times 10^{-6} m^3$

$= 180.075 \times 10^{-6}\ m^3$

Volume of one lime stone particle= $\frac{4}{3} \times 3.14 \times 1.7 \times 1.7 \times 1.7 \times 10^{-6} m^3$

$= 20.63 \times 10^{-6}\ m^3$

Density of ore = 4.33×10^3 Kg/m^3

Density of coke = 1.24×10^3 Kg/m^3

Density of lime stone = 2.5×10^3 Kg/m^3

From equation (b)

$$nc\ vc = \frac{no\ vo\ do}{dc\ k_1}$$

$$= \frac{no \times 33.49 \times 10^{-6} \times 4.33 \times 10^3}{1.24 \times 10^{+3} \times 2}$$

From equation (c)

$$nl\ vl = \frac{no\ vo\ do}{dl\ k_2}$$

$$= \frac{no \times 33.49 \times 10^{-6} \times 4.33 \times 10^3}{2.5 \times 10^3 \times 3.56}$$

Substituting the values of nc vc, nl vl, no vo in equation (a)

$$no \left[\frac{33.49\text{x}10^{-6}}{1} + \frac{33.49\text{x}10^{-6}\text{x}4.33}{1.24 \times 2} + \frac{33.49\text{x}10^{-6}\text{x}4.33}{2.5 \times 3.56} \right] = 0.6$$

$$no . 33.49 \times 10^{-6} \cdot\cdot \left[1 + \frac{4.33}{2.48} + \frac{4.33}{2.5\text{x}3.56} \right] = 0.6$$

$$no = \frac{0.6}{33.49 \times 10^{-6}} \left[\frac{1}{1 + \frac{4.33}{2.48} + \frac{4.33}{2.5 \times 3.56}} \right] \quad \text{.........(d)}$$

$$nc = \frac{no \times 33.49 \times 10^{-6} \times 4.33}{180.075 \times 10^{-6} \times 1.24 \times 2}$$

$$nc = \frac{no \times 33.49 \times 4.33}{180.075 \times 2.48} \quad \text{..............................(e)}$$

$$nl = \frac{no \times 33.49 \times 10^{-6} \times 4.33 \times 10^3}{20.63 \times 10^{-6} \times 2.5 \times 10^3 \times 3.56}$$

$$nl = \frac{no \times 33.49 \times 4.33}{20.63 \times 2.5 \times 3.56} \quad \text{........................(f)}$$

Equations (d), (e) & (f) can be solved for number of particles of ore, coke and lime stone in unit bed volume.

Molar Density of oxygen in iron ore (ρ_o):

Density of iron ore = 4.333×10^3 Kg/m^3

$$\text{Density of oxygen} = \frac{4.333 \times 10^3 \times 48}{160} \text{ Kg/m}^3$$

$$\text{Molar density of oxygen} = \frac{4.333 \times 10^3 \times 48}{160 \times 16}$$

$$= 81.25 \text{ Kg-mole/m}^3$$

Molar density of lime stone $(\rho 1)$:

$$\text{Density of lime stone} = 2.5 \times 10^3 \text{ Kg/m}^3$$

$$\text{Molar density} = \frac{2.5 \times 10^3}{100}$$

$$= 25.0 \text{ Kg-mole/m}^3$$

Effective diffusion coefficients of CO & CO_2:

Diffusion coefficients of CO & CO_2 are almost equal.

Effective diffusion coefficient of CO or CO_2 in iron ore is given by[22]:

$$Dco = 2.592\text{x}10^{-6}\ Ts^{1.78}(0.53+0.47\ Ep)\ (0.238\ Ep+0.04)$$

where Ep = porosity of iron ore.

$$\text{Hence } Dco = 2.592\text{x}10^{-6}\ Ts^{1.78}(0.53+0.47\text{x}.2)(0.283\text{x}.2+0.04)$$

Effective diffusion coefficient of CO_2 in coke (Dco_2, used in equation 15) is given by[22]:

$$Dco_2 = 2.236\text{x}10^{-6}\ Ts^{1.78}\text{x}0.45\ (0.04 + 0.238\ Ec)$$

where Ec = porosity of coke

$$\text{Hence } Dco_2 = 2.236\ \text{x}10^{-6}\ Ts^{1.78}\ \text{x}\ 0.45\ (0.04 + 0.238\text{x}0.5)$$

Effective diffusion coefficient of CO_2 in lime stone is given by[22] :

$$D = 2.236 \times 10^{-6} \; Ts^{1.78} \; (0.702 \; Epl + 0.298)^{1.41}$$

where Epl= porosity of lime stone

Hence $D = 2.236 \times 10^{-6} \; Ts^{1.78} \; (0.702 \times .15 + 0.298)^{1.41}$

Mass transfer coefficient (km) in iron layer is given by[22]:

$$Sh = 2.0 + 0.55 \; (Re)^{\frac{1}{2}} \; (Sc)^{\frac{1}{3}}$$

where $Sh = \frac{km \; 2 \; xo^{\circ}}{Dco}$

$Re = \frac{2 \; xo^{\circ} \; v\rho}{\mu}$

$Sc = \frac{\mu}{\rho \; Dco}$

v - linear velocity

ρ - density of blast furnace gas

μ - viscosity of gas

$$km = \frac{Dco}{2 \; xo^{\circ}} \left[2.0 + 0.55 \; \left(\frac{2 \; xo^{\circ} \; v \rho}{\mu} \right)^{\frac{1}{2}} \left(\frac{\mu}{\rho \; Dco} \right)^{\frac{1}{3}} \right]$$

$$km = \frac{Dco}{2 \; xo^{\circ}} \left[2.0 + 0.55 \; (2 \; xo^{\circ} \; v)^{\frac{1}{2}} \; \mu^{-1/6} \; \rho^{1/6} \; Dco^{-\frac{1}{3}} \right]$$

where $\mu = 4.960 \times 10^{-3} \; Tg^{3/2} \; / \; (Tg+103)$

$\rho = 1.2507 + 0.7261 \times 0.09$

Equilibrium partial pressure of CO_2 with lime and calcium carbonate:

$$\langle CaCO_3 \rangle = \langle CaO \rangle + (CO_2)$$

Free energy of the above reaction is given by[21]:

$$\Delta G^\circ = 40250 - 34.4\ T$$

$$\Delta G^\circ = -RT\ \ln K$$

$$\ln (pco_2)_{eq} = -\frac{40250 - 34.4\ T}{1.987\ T}$$

$$(pco_2)_{eq} = \exp\ (17.3 - 20220/T)$$

Rate of change of equilibrium partial pressure of CO_2 with temperature (denoted by a):

$$\frac{d}{dT}(pco_2)_{eq} = \frac{d}{dT}\exp.\ (17.3 - 20220/T).$$

$$= \frac{20220}{T^2}\ \exp\ (17.3 - 20220/T).$$

CHAPTER - V

RESULTS AND DISCUSSIONS

RESULTS & DISCUSSIONS

Equations (8), (67), (68), (85), (86), (87) and (89) are solved with the aid of computer using the computational sequence given in the last chapter. The initial temperature profile is calculated from equation (119), values of which are given in table 1. The initial temperature profile (ITP), which closely resembles the temperature profile of the Blast Furnace, is shown in figure 4.

During the execution of programme, each successive iteration gave refinement in the initial temperature profile. They are given in tables 2 to 9 . The iterations were finally stopped when the temperature profile remained unchanged. The temperature profile so obtained is given in the table 10, and shown in figure 5. This temperature profile called the optimum temperature profile corresponds to the minimum coke rate, consistent with the maximum reduction. Figure 6 shows the existing temperature profile & the optimum temperature profile, for the sake of comparison.

The amounts of iron oxide reduced, carbon gasified, and lime stone decomposed are calculated, at existing and the optimum temperature profiles. The values of, Go/Go°, which gives the amount of iron oxide reduced and, Gc/Go°, which gives the amount of carbon gasified, at the existing temperature profile are given in table 11 and shown in figure 7. These values at the optimum temperature profile are given in table 12 and shown in figure 8.

It can be seen from figure 7 or table 11 that the values of Go/Go° at Z=20 m. is 0.332 with existing temperature profile. The same value with optimum temperature profile is reduced to 0.170 (figure 8 or table 12). So if furnace is operated at optimum temperature profile the value of Go/Go° at Z=20 m., is reduced from 0.332 to 0.170 i.e. by an amount = 0.162. On the other hand, the carbon gasified is increased. At Z=20 m., Gc/Go° value with existing temperature profile is 1.6732 (table 11), and Gc/Go° value with optimum temperature profile is 1.5992 (table 12). Value of Gc/Go° increased due to the optimum temperature profile is 0.0740.

From the above discussion, it is obvious that due to the optimum temperature profile,

decrease in Go/Go° = 0.162

and increase in Gc/Go° = 0.0740

Hence decrease in Go = 0.162 x 25.4 = 4.1148 kg-moles/m^2/hr.

With the existing temperature profile, this oxygen would have needed an equal amount of carbon at temperatures above 1200°C. This is because only direct reduction takes place above 1200°C and one mole of carbon is needed to reduce one mole of wustite. So the amount of carbon required to reduce 4.11 kg-moles of oxygen is 4.11 kg-moles of carbon per square meter per hour.

Now considering the increase in Gc/Go.

$$
\begin{aligned}
Gc &= 0.0740 \times 25.4 \\
&= 1.8796 \\
&\simeq 1.88 \text{ kg-moles/m}^2\text{/hr.}
\end{aligned}
$$

So saving in carbon is $4.11 - 1.88$

$$
\begin{aligned}
&= 2.23 \text{ kg-moles/m}^2\text{/hr.} \\
&= 2.23 \times 12 \text{ kg/m}^2\text{/hr.} \\
&= 26.76 \text{ kg/m}^2\text{/hr.}
\end{aligned}
$$

$$
\begin{aligned}
\text{Saving in coke} &= \frac{26.76 \times 100}{80} \\
&= 33.45 \text{ kg/m}^2\text{/hr.}
\end{aligned}
$$

Saving in coke per ton of pig iron.

$$
\begin{aligned}
&= \frac{33.45 \times 63.6 \times 24}{1500} \\
&= 34.05 \text{ kg.}
\end{aligned}
$$

Thus, by operating the furnace at optimum temperature profile about 34 kg. of coke per ton of metal can be saved in reduction of iron oxide.

It should be pointed out here that the direct reduction of wustite is an endothermic reaction. So by operating the furnace at optimum temperature profile, the amount of wustite, to be reduced directly in high temperature region, can be reduced. Thus, the thermal load of the hearth can be decreased.

It is, therefore, suggested that auxiliary burners can be provided in the upper portion of the blast furnace, so that the temperature may approximate the optimum temperature profile.

TABLE 1.

0.0	273.003
.5	389.144
1.0	491.797
1.5	582.528
2.0	662.723
2.5	733.605
3.0	796.255
3.5	851.632
4.0	900.579
4.5	943.844
5.0	982.089
5.5	1015.896
6.0	1045.785
6.5	1072.211
7.0	1095.580
7.5	1116.250
8.0	1134.514
8.5	1150.735
9.0	1165.085
9.5	1177.816
10.0	1189.131
10.5	1199.216
11.0	1208.240
11.5	1216.361
12.0	1223.732
12.5	1230.500
13.0	1236.817
13.5	1242.843
14.0	1248.752
14.5	1254.746
15.0	1261.061
15.5	1267.986
16.0	1275.881
16.5	1285.199
17.0	1296.527
17.5	1310.619
18.0	1328.462
18.5	1351.344
19.0	1380.959
19.5	1419.529
20.0	1469.985

TABLE 2.

0.0	391.615
.5	571.420
1.0	719.090
1.5	822.016
2.0	893.870
2.5	946.559
3.0	987.735
3.5	1018.762
4.0	1043.417
4.5	1062.502
5.0	1079.485
5.5	1094.504
6.0	1107.837
6.5	1119.681
7.0	1130.211
7.5	1139.580
8.0	1147.915
8.5	1155.371
9.0	1162.011
9.5	1167.967
10.0	1173.314
10.5	1178.132
11.0	1182.494
11.5	1186.901
12.0	1191.823
12.5	1197.188
13.0	1202.507
13.5	1208.238
14.0	1215.070
14.5	1221.645
15.0	1229.537
15.5	1241.722
16.0	1254.965
16.5	1270.402
17.0	1288.941
17.5	1313.682
18.0	1341.351
18.5	1372.135
19.0	1406.386
19.5	1448.104
20.0	1495.074

TABLE 3.

0.0	442.619
.5	649.798
1.0	816.826
1.5	924.996
2.0	993.263
2.5	1038.130
3.0	1070.072
3.5	1090.627
4.0	1104.837
4.5	1113.525
5.0	1121.366
5.5	1128.305
6.0	1134.519
6.5	1140.093
7.0	1145.102
7.5	1149.612
8.0	1153.677
8.5	1157.364
9.0	1160.680
9.5	1163.732
10.0	1166.513
10.5	1169.066
11.0	1171.423
11.5	1174.233
12.0	1173.103
12.5	1182.863
13.0	1187.753
13.5	1193.358
14.0	1200.587
14.5	1207.412
15.0	1215.982
15.5	1230.428
16.0	1245.971
16.5	1264.040
17.0	1285.679
17.5	1314.999
18.0	1340.894
18.5	1381.075
19.0	1417.319
19.5	1460.392
20.0	1505.862

TABLE 4.

0.0	464.550
.5	683.501
1.0	858.853
1.5	967.277
2.0	10 6.002
2.5	1077.505
3.0	1105.476
3.5	1121.530
4.0	1131.248
4.5	1135.465
5.0	1139.375
5.5	11 2.839
6.0	11 5.993
6.5	11 8.871
7.0	1151.505
7.5	1153.926
8.0	1156.155
8.5	1158.221
9.0	1160.121
9.5	1161.911
10.0	1163.588
10.5	1165.168
11.0	1166.662
11.5	1168.785
12.0	1172.203
12.5	1176.704
13.0	1181.409
13.5	1186.960
14.0	1194.359
14.5	1201.291
15.0	1210.153
15.5	1225.572
16.0	1242.104
16.5	1261.304
17.0	1284.276
17.5	1315.566
18.0	1349.277
18.5	1384.919
19.0	1422.020
19.5	1465.675
20.0	1510.501

TABLE 5.

0.0	473.981
.5	697.993
1.0	876.924
1.5	988.318
2.0	1054.380
2.5	1094.436
3.0	1120.700
3.5	1134.818
4.0	1142.605
4.5	1144.899
5.0	1147.118
5.5	1149.089
6.0	1150.926
6.5	1152.645
7.0	1154.259
7.5	1155.781
8.0	1157.220
8.5	1158.590
9.0	1159.876
9.5	1161.128
10.0	1162.331
10.5	1163.492
11.0	1164.615
11.5	1166.443
12.0	1169.666
12.5	1174.055
13.0	1178.681
13.5	1184.209
14.0	1191.681
14.5	1198.659
15.0	1207.647
15.5	1223.493
16.0	1240.441
16.5	1260.128
17.0	1283.673
17.5	1315.809
18.0	1350.302
18.5	1386.572
19.0	1424.042
19.5	1467.947
20.0	1512.496

TABLE 6.

0.0	479.780
.5	706.905
1.0	888.036
1.5	1000.026
2.0	1062.282
2.5	1101.717
3.0	1127.246
3.5	1140.531
4.0	1147.488
4.5	1148.955
5.0	1150.448
5.5	1151.777
6.0	1153.048
6.5	1154.267
7.0	1155.442
7.5	1156.578
8.0	1157.678
8.5	1158.748
9.0	1159.771
9.5	1160.791
10.0	1161.790
10.5	1162.771
11.0	1163.735
11.5	1165.436
12.0	1168.575
12.5	1172.916
13.0	1177.508
13.5	1183.025
14.0	1190.530
14.5	1197.528
15.0	1206.559
15.5	1222.535
16.0	1239.725
16.5	1259.622
17.0	1283.414
17.5	1315.914
18.0	1350.742
18.5	1387.282
19.0	1424.911
19.5	1468.924
20.0	1513.723

TABLE 7.

0.0	480.529
.5	708.057
1.0	889.473
1.5	1001.540
2.0	1065.681
2.5	1104.847
3.0	1130.061
3.5	1142.988
4.0	1149.588
4.5	1150.700
5.0	1151.880
5.5	1152.932
6.0	1153.960
6.5	1154.965
7.0	1155.952
7.5	1156.921
8.0	1157.875
8.5	1158.816
9.0	1159.726
9.5	1160.646
10.0	1161.557
10.5	1162.461
11.0	1163.357
11.5	1165.003
12.0	1168.106
12.5	1172.427
13.0	1177.004
13.5	1182.517
14.0	1190.035
14.5	1197.041
15.0	1206.106
15.5	1222.199
16.0	1239.418
16.5	1259.404
17.0	1283.302
17.5	1315.959
18.0	1350.932
18.5	1387.588
19.0	1425.285
19.5	1469.344
20.0	1513.949

TABLE 8.

0.0	480.852
.5	708.552
1.0	890.091
1.5	1002.191
2.0	1067.142
2.5	1106.193
3.0	1131.272
3.5	1144.045
4.0	1150.491
4.5	1151.450
5.0	1152.495
5.5	1153.429
6.0	1154.352
6.5	1155.266
7.0	1156.171
7.5	1157.069
8.0	1157.960
8.5	1158.846
9.0	1159.707
9.5	1160.584
10.0	1161.458
10.5	1162.328
11.0	1163.194
11.5	1164.817
12.0	1167.904
12.5	1172.216
13.0	1176.787
13.5	1182.298
14.0	1189.822
14.5	1196.832
15.0	1205.906
15.5	1222.033
16.0	1239.286
16.5	1259.311
17.0	1283.254
17.5	1315.978
18.0	1351.013
18.5	1387.720
19.0	1425.446
19.5	1469.525
20.0	1513.991

TABLE 9.

0.0	481.095
.5	708.926
1.0	890.557
1.5	1002.682
2.0	1067.770
2.5	1106.772
3.0	1131.792
3.5	1144.499
4.0	1150.879
4.5	1151.773
5.0	1152.760
5.5	1153.643
6.0	1154.521
6.5	1155.395
7.0	1156.265
7.5	1157.132
8.0	1157.996
8.5	1158.858
9.0	1159.698
9.5	1160.557
10.0	1161.415
10.5	1162.270
11.0	1163.124
11.5	1164.737
12.0	1167.818
12.5	1172.125
13.0	1176.693
13.5	1182.204
14.0	1189.730
14.5	1196.742
15.0	1205.821
15.5	1221.962
16.0	1239.229
16.5	1259.270
17.0	1283.234
17.5	1315.987
18.0	1351.049
18.5	1387.776
19.0	1425.515
19.5	1469.602
20.0	1513.999

TABLE 10.

0.0	481.095
.5	708.926
1.0	890.557
1.5	1002.682
2.0	1068.244
2.5	1107.209
3.0	1132.185
3.5	1144.842
4.0	1151.172
4.5	1152.016
5.0	1152.960
5.5	1153.804
6.0	1154.648
6.5	1155.492
7.0	1156.336
7.5	1157.180
8.0	1158.024
8.5	1158.868
9.0	1159.692
9.5	1160.537
10.0	1161.382
10.5	1162.227
11.0	1163.071
11.5	1164.676
12.0	1167.752
12.5	1172.057
13.0	1176.623
13.5	1182.132
14.0	1189.661
14.5	1196.674
15.0	1205.756
15.5	1221.908
16.0	1239.186
16.5	1259.240
17.0	1283.218
17.5	1315.993
18.0	1351.075
18.5	1387.819
19.0	1425.567
19.5	1469.661
20.0	1514.001

TABLE 11.

HEIGHT	TEMP.	X1	X2	X3
0.0	273.003	1.000	2.0400	.171
.5	389.144	.998	2.0400	.167
1.0	491.797	.938	2.0400	.159
1.5	582.528	.890	2.0399	.148
2.0	662.723	.849	2.0398	.131
2.5	733.605	.811	2.0396	.121
3.0	796.255	.774	2.0393	.106
3.5	851.632	.742	2.0390	.071
4.0	900.579	.707	2.0388	.026
4.0	900.579	.707	2.0388	.000
4.5	943.844	.678	2.0388	.000
5.0	982.089	.655	2.0388	.000
5.5	1015.896	.634	2.0388	.000
6.0	1045.785	.613	2.0388	.000
6.5	1072.211	.598	2.0387	.000
7.0	1095.580	.581	2.0385	.000
7.5	1116.250	.567	2.0380	.000
8.0	1134.514	.551	2.0372	.000
8.5	1150.735	.538	2.0359	.000
9.0	1165.085	.527	2.0341	.000
9.5	1177.816	.518	2.0317	.000
10.0	1189.131	.509	2.0287	.000
10.5	1199.216	.501	2.0250	.000
11.0	1208.240	.496	2.0206	.000
11.5	1216.361	.487	2.0155	.000
12.0	1223.732	.480	2.0097	.000
12.5	1230.500	.474	2.0031	.000
13.0	1236.817	.468	1.9958	.000
13.5	1242.843	.462	1.9876	.000
14.0	1248.752	.456	1.9786	.000
14.5	1254.746	.450	1.9687	.000
15.0	1261.061	.444	1.9577	.000
15.5	1267.986	.438	1.9455	.000
16.0	1275.881	.432	1.9316	.000
16.5	1285.199	.426	1.9157	.000
17.0	1296.527	.420	1.8971	.000
17.5	1310.619	.414	1.8757	.000
18.0	1328.462	.404	1.8503	.000
18.5	1351.344	.389	1.8189	.000
19.0	1380.959	.371	1.7781	.000
19.5	1419.529	.350	1.7330	.000
20.0	1469.985	.332	1.6732	.000

TABLE 12.

HEIGHT	TEMP.	X1	X2	X3
0.0	481.095	1.000	2.0400	.171
0.5	708.926	.966	2.0400	.164
1.0	890.557	.892	2.0399	.152
1.5	1002.682	.838	2.0397	.127
2.0	1068.244	.789	2.0393	.087
2.5	1107.209	.744	2.0388	.021
3.0	1132.135	.701	2.0383	.000
3.5	1144.842	.658	2.0378	.000
4.0	1151.172	.619	2.0372	.000
4.5	1152.016	.583	2.0365	.000
5.0	1152.960	.554	2.0348	.000
5.5	1153.804	.525	2.0329	.000
6.0	1154.648	.498	2.0309	.000
6.5	1155.492	.474	2.0281	.000
7.0	1156.336	.450	2.0255	.000
7.5	1157.180	.431	2.0221	.000
8.0	1158.024	.413	2.0183	.000
8.5	1158.868	.397	2.0145	.000
9.0	1159.692	.385	2.0106	.000
9.5	1160.537	.373	2.0065	.000
10.0	1161.382	.364	2.0020	.000
10.5	1162.227	.356	1.9969	.000
11.0	1163.071	.350	1.9903	.000
11.5	1164.676	.342	1.9832	.000
12.0	1167.752	.335	1.9758	.000
12.5	1172.057	.328	1.9661	.000
13.0	1176.623	.322	1.9582	.000
13.5	1182.133	.317	1.9503	.000
14.0	1189.651	.312	1.9398	.000
14.5	1196.674	.306	1.9290	.000
15.0	1205.756	.302	1.9181	.000
15.5	1221.908	.294	1.9047	.000
16.0	1239.186	.285	1.8896	.000
16.5	1259.240	.272	1.8721	.000
17.0	1283.218	.260	1.8506	.000
17.5	1315.993	.248	1.8273	.000
18.0	1351.075	.234	1.7972	.000
18.5	1387.819	.217	1.7557	.000
19.0	1425.567	.208	1.7075	.000
19.5	1469.631	.189	1.6498	.000
20.0	1514.001	.170	1.5992	.000

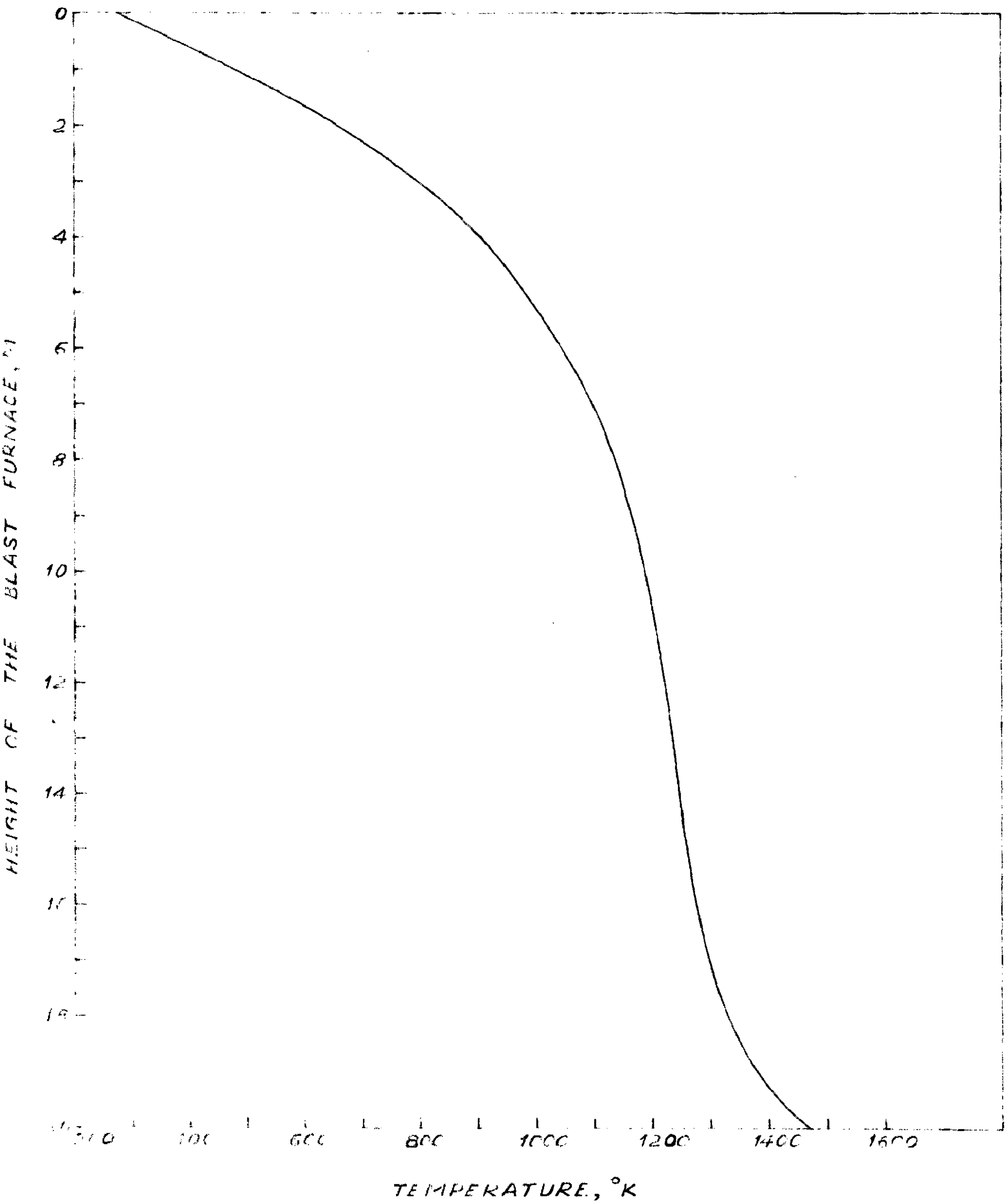


FIG. 4 THE ASSUMED TEMPERATURE PROFILE (APPROXIMATE EXISTING TEMPERATURE PROFILE)

[illegible] TEMPERATURE PROFILE

(a) APPROXIMATE [illegible] TEMPERATURE [illegible]
(b) [illegible]

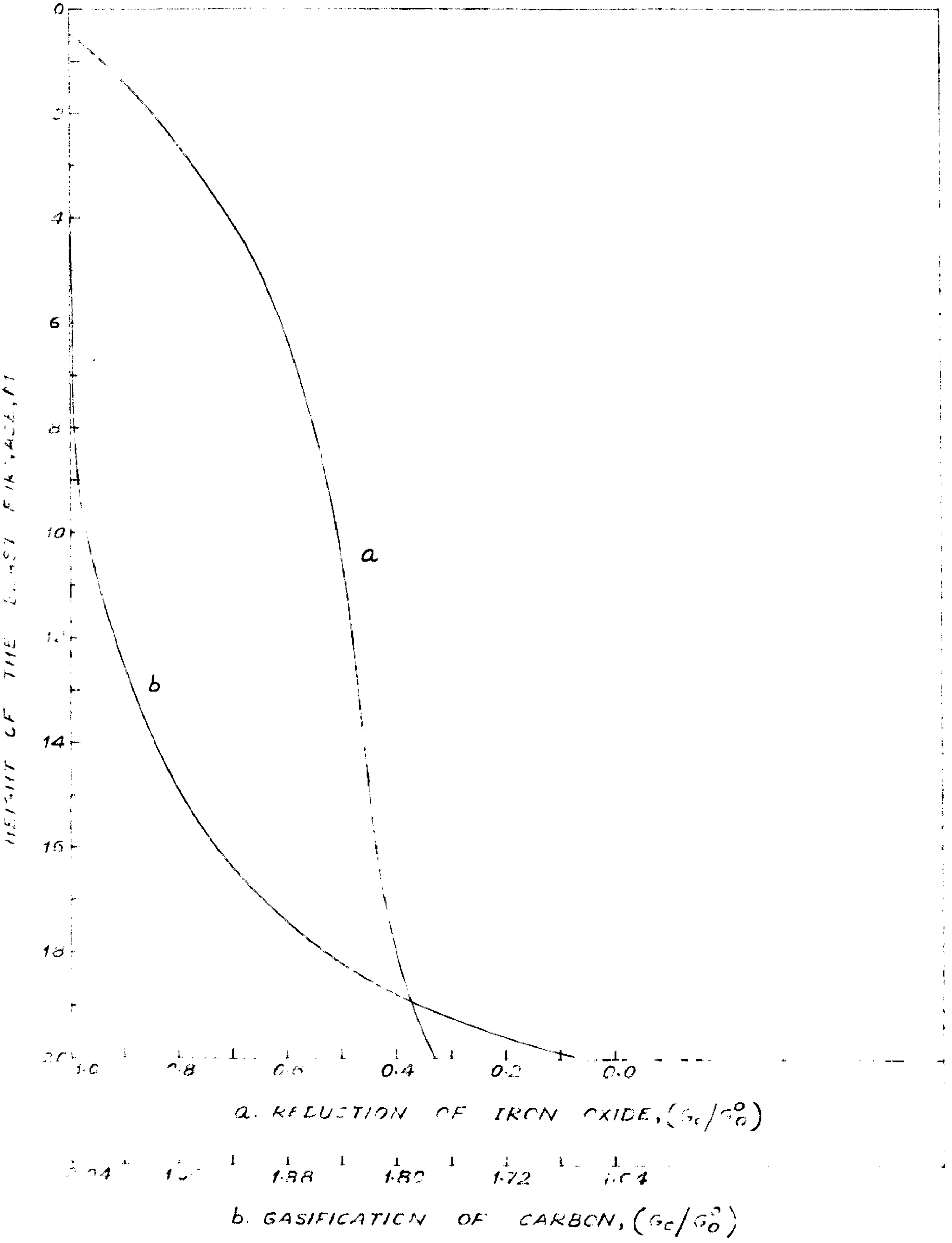


FIG.7 REDUCTION OF IRON OXIDE AND GASIFICATION OF CARBON AT ASSUMED TEMPERATURE PROFILE.

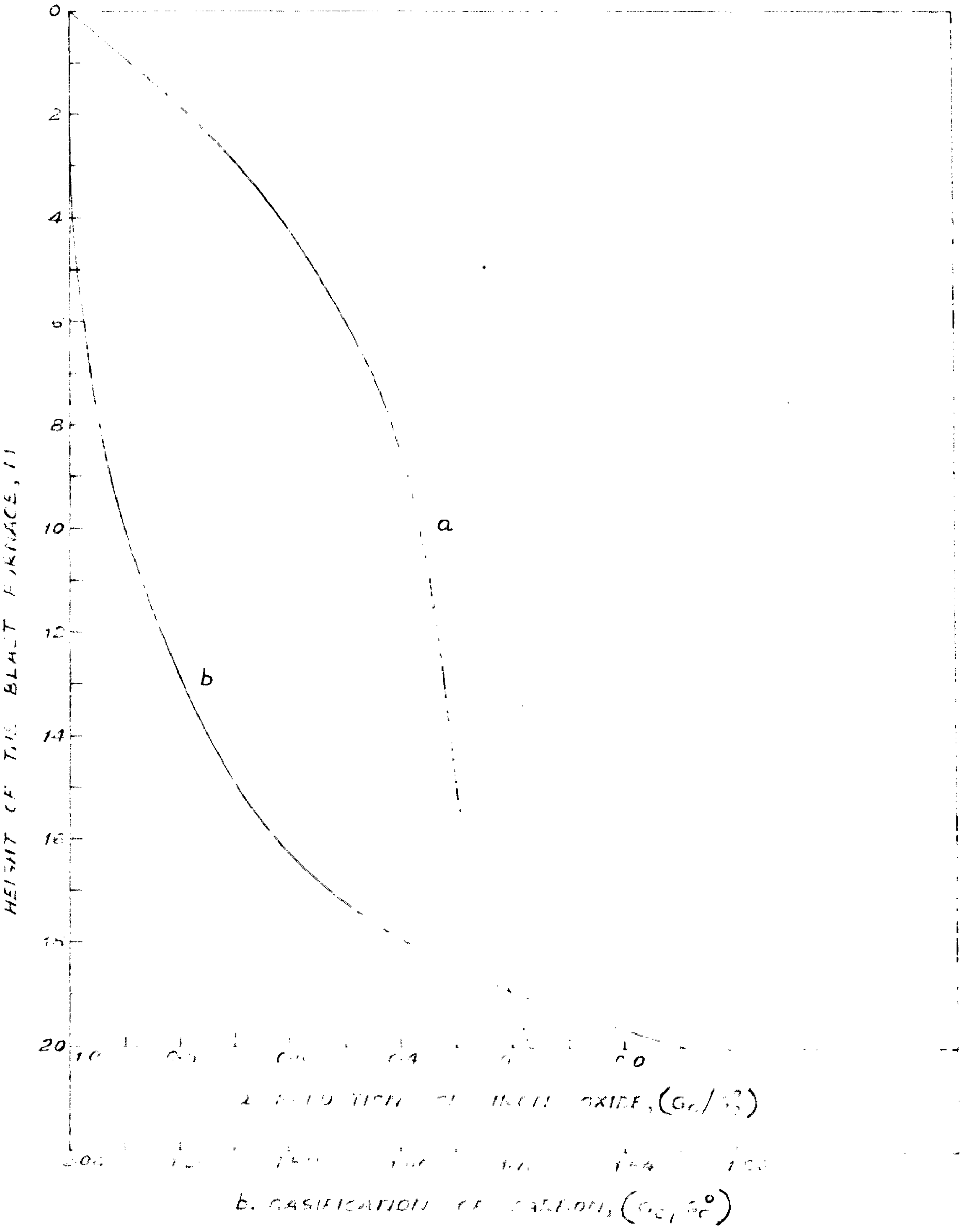


FIG. 8 REDUCTION OF IRON OXIDE AND GASIFICATION OF CARBON AT OPTIMUM TEMPERATURE PROFILE.

CONCLUSIONS

1. The temperature profile, which corresponds to the minimum coke rate, called the optimum temperature profile is shown in figure 5.

2. Coke consumption, about 34 kg. per ton of metal, can be decreased, in reduction of iron oxide, by employing optimum temperature profile in the blast furnace.

3. Operation of the blast furnace with optimum temperature profile reduces the thermal load of the high temperature zone.

APPENDIX

Runge - Kutta Method

This numerical method has been used for solving equations (8), (67), (68) and then equations (85), (86) & (87). The method is given below:

The given equations are of the type:

$$\frac{d\,X_1}{dZ} = F_1$$

$$\frac{d\,X_2}{dZ} = F_2$$

$$\frac{d\,X_3}{dZ} = F_3$$

where F_1, F_2 & F_3 are functions of X_1, X_2 & X_3.

Step 1: At $Z = Zo$, X_1, X_2, X_3 are known

Store X_1 as X_{1o},
X_2 as X_{2o},
X_3 as X_{3o}.

Calculate F_1, F_2, F_3 and store them as F_{10}, F_{20}, F_{30}.

Step 2: At $Z = Zo + h/2$, find X_1, X_2, X_3, where

$$X_1 = X_{1o} + 0.5\ (h\ F_{10})$$
$$X_2 = X_{2o} + 0.5\ (h\ F_{20})$$
$$X_3 = X_{3o} + 0.5\ (h\ F_{30})$$

and then calculate F_1, F_2, F_3, store them as F_{11}, F_{21}, F_{31}.

Step 3: At $Z = Zo + h/2$, find X_1, X_2, X_3, where

$$X_1 = X_{1o} + 0.5\ (h\ F_{11})$$
$$X_2 = X_{2o} + 0.5\ (h\ F_{21})$$
$$X_3 = X_{3o} + 0.5\ (h\ F_{31})$$

and then calculate F_1, F_2, F_3, store them as F_{12}, F_{22}, F_{32}.

Step 4 : At $Z = Zo + h$, find X_1, X_2, X_3 where

$$X_1 = X_{1o} + h\ F_{12}$$
$$X_2 = X_{20} + h\ F_{22}$$
$$X_3 = X_{3o} + h\ F_{32}$$

and then calculate F_1, F_2, F_3 and store them as F_{13}, F_{23}, F_{33}.

Step 5: At $Z = Zo + h$, the values of X_1, X_2 & X_3 will be

$$X_1 = X_{1o} + \frac{h}{6}(F_{1o} + 2\ F_{11} + 2F_{12} + F_{13})$$
$$X_2 = X_{2o} + \frac{h}{6}(F_{2o} + 2F_{21} + 2F_{22} + F_{23})$$
$$X_3 = X_{3o} + \frac{h}{6}(F_{3o} + 2F_{31} + 2F_{32} + F_{33})$$

These values of X_1, X_2 & X_3 are used for next calculations.

Source Programme for **solving** the algebraic equation (89)

The algebraic equation is of the type:

function (T) = 0

Programme

```
        TMX   = 1550.
        DINCM= 1.
        ITER  = 0
        DINCR= 50.
        T     = 270.
40      CONTINUE
        FUNY  = function (T)
        IF(ITER-1) 21,22,24
21      ITER  = 1
25      FUNY1= FUNY
        GO TO (27,26), ITER
26      DINCR= DINCR* .5
27      T     = T + DINCR
        IF(T - TMX) 40, 40, 200
22      IF (FUNY1* FUNY) 23, 190, 25
23      ITER  = 2
        DINCR= DINCR* .5
        T     = T - DINCR
        GO TO 40
24      IF (DINCR - DINCM) 190, 190, 22
190     PUNCH T
200     STOP
```

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