HYGROSCOPIC EFFECT ON THE BUCKLING BEHAVIOUR OF COMPOSITE LAMINATED PLATES

A DISSERTATION

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requirements for the award of the degree
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MECHANICAL ENGINEERING
(With Specialization in Machine Design)

By

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FEBRUARY, 1993

DEDICATED

TO

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in this dissertation entitled **"HYGROSCOPIC EFFECT ON THE BUCKLING BEHAVIOUR OF COMPOSITE LAMINATED PLATES"** in partial fulfilment of the requirements for the award of the degree of **Master of Engineering** in Mechanical Engineering with specialization in **MACHINE DESIGN**, submitted to the Department of Mechanical and Industrial Engineering, University of Roorkee, Roorkee, India is an authentic record of my own work carried out for a period from June 1992 to February 1993 under the guidance of **Dr. B. K. MISHRA**, Lecturer in Department of Mechanical and Industrial Engineering.

The matter embodied in this dissertation has not been submitted by me for the award of any other degree or diploma.

DATED:- 18-2-93

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This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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ABSTRACT

The present work deals with the study of effect of moisture absorption on the buckling of composite laminated plates.

Composite materials are being used in many critical application where they are subjected to adverse environmental condition. As such, a knowledge of the deterioration in the performance of these structures is a subject of great importance.

In the present work the composite laminate has been modelled as a thin plate. Moisture diffusion in the plate is assumed to be governed by Fick's law. The hygroscopic stress resultant due to moisture absorption is incorporated in the formulation of the plate buckling problem. A finite element formulation with three noded triangular element is carried out to form the governing equation. Results have been presented for simply supported and clamped square plate of T 300/5208 graphite epoxy laminate. Stacking sequences are taken to be (0 /± 45/90)4s, (0/90)8s and (± 45)8s. Variation of buckling load of the laminate with moisture absorption time has been plotted for different equilibrium moisture concentration in the laminate.

Results presented in this work indicate that the effect of moisture absorption on buckling load of laminates is quite significant. For low values of moisture concentration a steady state value of buckling load is obtained whereas for higher values of moisture concentration required buckling load drops to zero,

indicating plate buckling due to hygroscopic stress resultants alone.

From the work presented, it is concluded that with moisture diffusion, performance of composite plates subjected to buckling loads deteriorates and there is a need for accounting the hygroscopic effect in a realistic design of composite structures subjected to adverse environmental condition.

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NOMENCLATURE

D	Coeff. of moisture diffusion
E_1	Modulus of elasticity in 1-direction
E_2	Modulus of elasticity in 2-direction
G_{12}	Modulus of rigidity
h	Thickness of plate
$[k]$	Stiffness Matrix for a particular element in level co-ordinate system.
$[k^H_D]$	Hygroscopic stiffness Matrix for a particular element in local co-ordinate system.
$[k^H_{DG}]$	Hygroscopic stiffness Matrix for a particular element in ~~local~~ global co-ordinate system.
$[\bar{k}_D]$	Diferential stiffness matrix due to in-plane loads in local co-ordinate.
$[K^H_{DG}]$	Hygroscopic stiffness Matrix for a particular element in global co-ordinate system.
$[\bar{K}_D]$	Diferential stiffness matrix due to in-plane loadsin global co-ordinate.
$M_{o(z)}$	Initial moisture concentration in the plate
M_∞	Moisture concentration at the plate surfaces due to environment.
N_x, N_y, N_{xy}	Total inplane stress Resultants
$\bar{N}_x, \bar{N}_y, \bar{N}_{xy}$	Stress resultants due to applied inplane loads
$[Q_o]$	Stiffness matrix for a lamina in the principle material direction

[Q]	Stiffness matrix for a lamina with respect to x-y co-ordinate system.
t	Time
t^{*}	Non-dimensional time.
[T]	Transformation matrix
u, v, w	Mid-plane displacement of the plate in x, y, z direction
U	Strain Energy density function
U_i	Strain Energy for ith lamina
$\beta_1, \beta_2, \beta_6$	Coefficient of hygroscopic expansion in 1,2 and 1-2 direction respectively
σ	Stress in the plate
δ	Variational del operator
$\{\Delta^e\}$	Noded Variable matrix
π	Total potential energy
$[\lambda]$	Local to Global Transformation Matrix.

CHAPTER - 1

INTRODUCTION

1.1 INTRODUCTION

Composite materials are engineering combination of two or more materials essentially insoluble into each other and such that the properties of the whole are better than the properties of each constituent, taken separately. Composite materials have high strength and stiffness coupled with low density. In addition they have good vibration, fatigue and corrosion resistance, low heat conductivity, good electrical insulation and favourable cost effectiveness. Manufacture of composites requires less labour and generates less waste. It has the additional advantage of easy processibility [1,2]. Due to above mentioned advantages composite materials are being extensively used and they are fast replacing conventional materials in many critical applications. The polymeric composites, however, have a serious disadvantages. They are very sensitive to environmental conditions. For structural applications, most of the environmental degradation in composites is due to exposure to a combination of temperature and moisture. Thermal and moisture absorption results in swelling, residual stresses, enhanced viscoelasticity, reduction in material properties and damage by debondings at the fibre matrix interfaces. All the above phenomenon occur concurrently and interact with each other. Recent developments have shown that significant changes in the mechanical properties of fibre-reinforced polymeric composites may result from the combined effects of absorbed moisture and thermal environment. Since advanced material systems are being considered for flight critical airframe application in aero-industries, it is imperative that a fundamental analysis of moisture or thermal environment be undertaken in order to assess the nature and extent of their deleterious effects upon performance.

The influence of the thermal/moisture environment upon the behaviour of polymeric composites can be divided into three basic problem areas. First, the physics of moisture diffusion in an anisotrapic, heterogeneous medium is not well understood. Second, the absorption of moisture has been shown to reduce the glass transition temperature of the polymeric matrix and thereby diminish the matrix influenced strength properties of the composite at elevated temperatures. Finally, moisture absorption of the matrix results in swelling of the composite. Since swelling of the composite lamina is restrained in the fibre direction, significant residual stresses are introduced in the multi-layered laminate by moisture absorption [3].

In addition to these basic problem areas there is a need to investigate the changes in static as well as dynamic performance of different structural components. This type of investigation can be carried out to a good level of confidence only after understanding the above mentioned three basic problems. It is essential to have a knowledge of moisture diffusion as well as the resultant stress distribution in composites, and then only the static and dynamic performance of composite structures can be investigated.

1.2 LITERATURE REVIEW

It is well recognised that high performance polymeric composites absorb moisture from high humidity environment [4]. The simplest theory governing moisture diffusion in a material is the Ficks law of moisture diffusion. It has been experimentally verified that weight gain due to moisture absorption is predicted accurately by classical Fick's model [5]. The classical Fickean model does not consider the anisotropy and heterogenity of the composite material and hence it can't predict accurately the actual moisture distribution in the composites [6]. However an accurate moisture distribution is needed only when stress values near some crack or interlaminar boundary are required. If bulk properties such as stability or vibrational characteristic of the laminate is required then the simple Fick's law is adequate.

Many researchers have found that significant change in the mechanical properties of fibre reinforced polymeric composites may result from the combined effects of absorbed moisture and thermal environment [7-9].

It has been shown that mechanical property degradation depends mainly on the moisture content in the composite [4,10]. Moisture diffusion studies on graphite fibre composite at different temperatures and relative humidities have been carried out by several investigators [5,6,11]. It has been shown by Shen and Springer [8] and McKague [9] that an increase in moisture concentration and/or temperature will reduce the elastic modulus of composite materials, induce internal stresses and degrade the strength. They have also shown that the physical changes due to hygrothermal effects in the polymetric matrix results in decreasing tensile properties, transverse and shear modulus. Wang [12] have shown that interlaminar stresses are greatly affected by moisture distribution.

In a composite laminate, deformation of matrix materials due to absorption of heat and moisture is restrained by the fibres, while laminate deformation due to which severe stresses develop at the edges and interlaminar boundaries [12]. Studies on moisture induced residual stresses were conducted by Pipes, Vinson , Chon [3] and they found that hygroscopic stresses are identical and analogus to thermal stresses.

Investigation of changes in the static and dynamic performance of composite laminates subjected to adverse environmental condition is a subject of great technical interest. A considerable amount of work has been done in investigating the buckling and vibration problem of composite plates [13-20].

The vibration problem of composite plates is one of the important aspects for the design of structural members. Srinivas Rao and Rao [13] have analysed vibration in a simply supported homogeneous and laminated thick rectangular plates. Bert and Chen [14] investigated the effect of shear deformation on vibration of antisymmetric angle ply laminated rectangular plates. A finite element analysis for the free and forced vibration of a laminated rectangular composite plate under steady state hygrothermal conditions were performed by L.W. Chen and Y.M Chen [15]. They found that thermal and moisture loading reduces the natural frequency of the composite plate. Virender Kumar [16] has studied the free vibration problem in composite laminated plates subjected to unsteady hygroscopic condition and he found that natural frequency decreases with moisture absorption time. He concluded that as the moisture concentration is above some limits then natural frequency finally drops down to zero indicating the presence of hygroscopic buckling.

The buckling problem of composite plates is very important for the structural designers. A considerable amount of research work has appeared in the literature on the stability of structural members made of composite materials [17-22]. Chrislos C. Chamis [17] has studied the buckling of Anisotropic composite plates. He has concluded that the buckling load of both homogeneous and layered anisotropic plates is quite sensitive to plate aspect ratio, degree of anisotropy and load conditions. It has been found by Phan and Reddy [18] that the transverse shear deformation effect on the stability of plate should be considered even for thin plate, if the shear modulus is much smaller than normal modulus. Flaggs and Vinson [19] have shown that hygrothermal loads significantly reduce the applied surface tractions necessary to buckle a composite structure. Studies were carried out by S.Y. Lee and W.J. Yen [20] to find out the hygrothermal effect on the stability of a cylindrical composite panel. They found that the extent of degradation in the buckling load is influenced by the degree of moisture concentration and due to the effect of reduction of elastic modulii resulting from hygrothermal change.

Composite laminated plates are being extensively used in aerospace and marine structures. As such, a knowledge of the reduction in buckling load with moisture exposure time is vital for design. So, the present work aims at predicting the buckling load of a composite laminated plate, subjected to unsteady hygroscopic condition.

1.3 SCOPE OF PRESENT WORK

Aim of the present work is to predict the change in the applied in -plane load required to buckle the composite laminated plate subjected to unsteady hygroscopic condition.

For this purpose it is assumed that Fick's law of moisture absorption holds good for the composite laminate. At any time, moisture distribution in the plate is calculated using Fick's law. From the moisture distribution thus hygroscopic stress resultants on the plate is calculated. The plate buckling problem is formulated using the hygroscopic as well as applied in-plane loads.

By finite element formulation the governing equations are converted into an algebraic eigen value problem. Solution of this problem yields applied in-plane load necessary to buckle the plate.

Results have been provided for three laminate configurations :- Quasi-isotropic (0, ± 45,90)4s, cross plied (0,90)8s, and angle plied (± 45)8s. Two different support conditions-simply supported and clamped have been investigated.

Results indicate that the externally applied load required for buckling the composite plate decreases with continued exposure to moisture. For low moisture concentration buckling load converges to some value corresponding to the steady state hygroscopic condition. But for high moisture concentration, buckling load drops down to zero even before the steady state hygroscopic condition is achieved.

The present study with some refinements can be helpful in predicting the useful life of a composite laminated plate subjected to different environmental condition.

CHAPTER-2

GOVERNING EQUATIONS AND FORMULATION

2.1 INTRODUCTION

When a composite laminated plate is subjected to hygroscopic loading then the external in-plane forces required to buckle the plate becomes smaller. This change has an important effect on the performance of composite structures. In this chapter, equations governing the buckling of composite laminated plate is developed. For this purpose, expression for the total potential energy of the composite laminate is developed by obtaining the total strain energy of the laminate and work done by hygrothermal stress resultants and applied external force.

The different steps in the formulation of the above mentioned problem are presented in the following articles.

2.2 EXPRESSION FOR STRAIN ENERGY OF A COMPOSITE PLATE

For modelling the composite plate a thin plate theory based on Kirchoff's hypothesis is considered. A composite plate made of n laminae as shown in Fig.(2.1) is considered. Z-co-ordinate is measured from the plate middle surface. i^{th} lamina of the laminate is considered to be bound between $h_{i-1}<z<h_i$. Neglecting the effect of transverse shear, the displacement field u,v,w at any point Z distance from the middle surface of the plate is assumed as

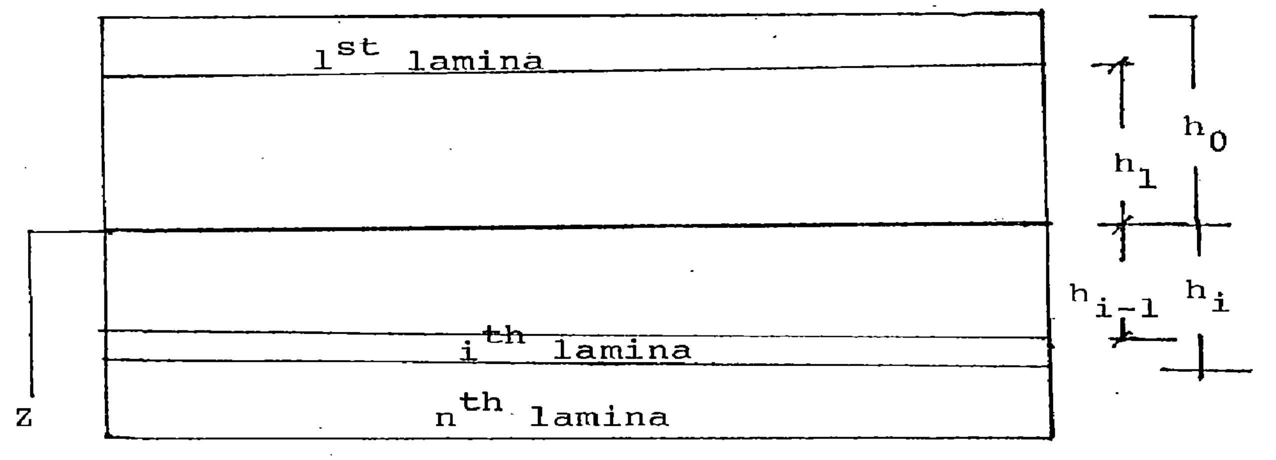

FIG.2.1 SCHEMATIC DIAGRAM OF A COMPOSITE LAMINATE

$$u(x,y,z) = -Z.\ \frac{\partial w}{\partial x}$$
$$v(x,y,z) = -Z.\ \frac{\partial w}{\partial y} \qquad \ldots(2.1)$$
$$w(x,y,z) = w$$

Then the strain-displacement relations are expressed as,

$$\varepsilon_x = -Z.\ \frac{\partial^2 w}{\partial x^2}$$

$$\varepsilon_y = -Z.\ \frac{\partial^2 w}{\partial y^2}$$

$$\gamma_{xy} = -2Z\ \frac{\partial^2 w}{\partial x \partial y}$$

$$\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0$$

In matrix form,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = -Z \begin{Bmatrix} \partial^2 w/\partial x^2 \\ \partial^2 w/\partial y^2 \\ 2\ \dfrac{\partial^2 w}{\partial x \partial y} \end{Bmatrix}$$

$$= -Z \left\{\varepsilon_1\right\} \qquad \ldots(2.2)$$

$$\text{where } \varepsilon_1 = \begin{Bmatrix} \partial^2 w/\partial x^2 \\ \partial^2 w/\partial y^2 \\ 2\ \dfrac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \qquad \ldots(2.2a)$$

The stiffness matrix $[Q_o]$ For a Particular Lamina in the principle material direction is given by

$$[Q_o] = \begin{bmatrix} \dfrac{E_1}{(1-\upsilon_{12}\upsilon_{20})} & \dfrac{\upsilon_{12}\,E_2}{(1-\upsilon_{12}\upsilon_{21})} & 0 \\ \dfrac{\upsilon_{12}\,E_2}{(1-\upsilon_{12}\upsilon_{21})} & \dfrac{E_2}{(1-\upsilon_{12}\upsilon_{21})} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \qquad \ldots(2.3)$$

where

E_1 , E_2 are the youngs moduli in 1 & 2 directions respectively.

G_{12} is the shear moduli in 1-2 plane.

υ_{12} and υ_{21} are the poisson's ratio.

The stiffness matrix of an off-axis lamina with reference to xy co-ordinate system can be expressed as [2]

$$[Q] = [T]^{-1}\,[Q_o]\,[T]^{-T} \qquad \ldots(2.4)$$

$$\text{where } [T] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta-\sin^2\theta \end{bmatrix} \qquad \ldots(2.5)$$

where θ is the angle between x-direction and the material direction given by 1.

Now the stress strain relationship for a lamina with reference to x-y Co-ordinate system can be given as,

$$\{\sigma\} = [Q]\,\{\varepsilon\} \qquad \ldots(2.6)$$

$$\text{where } \{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

Strain energy density function can be expressed as ,

$$\begin{aligned} \bar{u} &= \int \sigma \, d\,\varepsilon \\ &= \int \{\varepsilon\}^T [Q] \, d\varepsilon \\ &= 1/2 \, \{\varepsilon\}^T [Q] \, \{\varepsilon\} \end{aligned} \qquad \ldots(2.7)$$

For particular lamina (i^{th} lamina) located between $Z=hi-1$ and hi, strain energy can be expressed as

$$U_i = \int_{V_i} \bar{u} \, dv_i$$

$$= 1/2 \int_A \int_{hi-1}^{hi} \{\varepsilon\}^T [Q] \, \{\varepsilon\} \, dz \, dA \,. \qquad \ldots(2.8)$$

So the total strain energy for a composite laminate consisting of n laminae can be written as,

$$U = \sum_{i=1}^{n} \frac{1}{2} \int_A \int_{hi-1}^{hi} \{\varepsilon\}^T [Q] \, \{\varepsilon\} \, dz \, dA \qquad \ldots(2.9)$$

Using equn. (2.2),

$$U = \sum_{i=1}^{n} \left[\frac{1}{2} \int_A \int_{hi-1}^{hi} z^2 \left\{\varepsilon_1\right\}^T \left[Q\right] \left\{\varepsilon_1\right\} dz \, dA \right. \qquad \ldots(2.10)$$

$$= \frac{1}{2} \int_A \left\{\varepsilon_1\right\}^T \left[D\right] \left\{\varepsilon_1\right\} dA \qquad \ldots(2.11)$$

where $[D] = \sum_{i=1}^{n} \int_{hi-1}^{hi} Z^2 \, [Q] \, dz$ ...(2.12)

2.3 WORK DONE BY INPLANE LOADS

The composite laminate is subjected to inplane stress resultants N_x, N_y and N_{xy} as shown in Fig.(2.2). If equilibrium of the composite plate is considered in the deformed position, work done by these inplane forces can be expressed as [21,22]

$$W.D = -\int_A \left[\frac{1}{2} N_x \left(\frac{\partial w}{\partial x} \right)^2 + \frac{1}{2} N_y \left(\frac{\partial w}{\partial y} \right)^2 + N_{xy} \frac{\partial w}{\partial x} \cdot \frac{\partial w}{\partial y} \right] dA$$

$$= - \int \frac{1}{2} \begin{Bmatrix} \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial y} \end{Bmatrix}^T \begin{bmatrix} N_x & N_{xy} \\ N_{xy} & N_y \end{bmatrix} \begin{Bmatrix} \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial y} \end{Bmatrix} dA$$

$$= - \frac{1}{2} \int \left\{ \varepsilon_2 \right\}^T \left[N \right] \left\{ \varepsilon_2 \right\} dA \qquad \text{...(2.13)}$$

where $\left\{ \varepsilon_2 \right\} = \begin{Bmatrix} \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial y} \end{Bmatrix}$ ...(2.14)

and $\left[N \right] = \begin{bmatrix} N_x & N_{xy} \\ N_{xy} & N_y \end{bmatrix}$ ...(2.15)

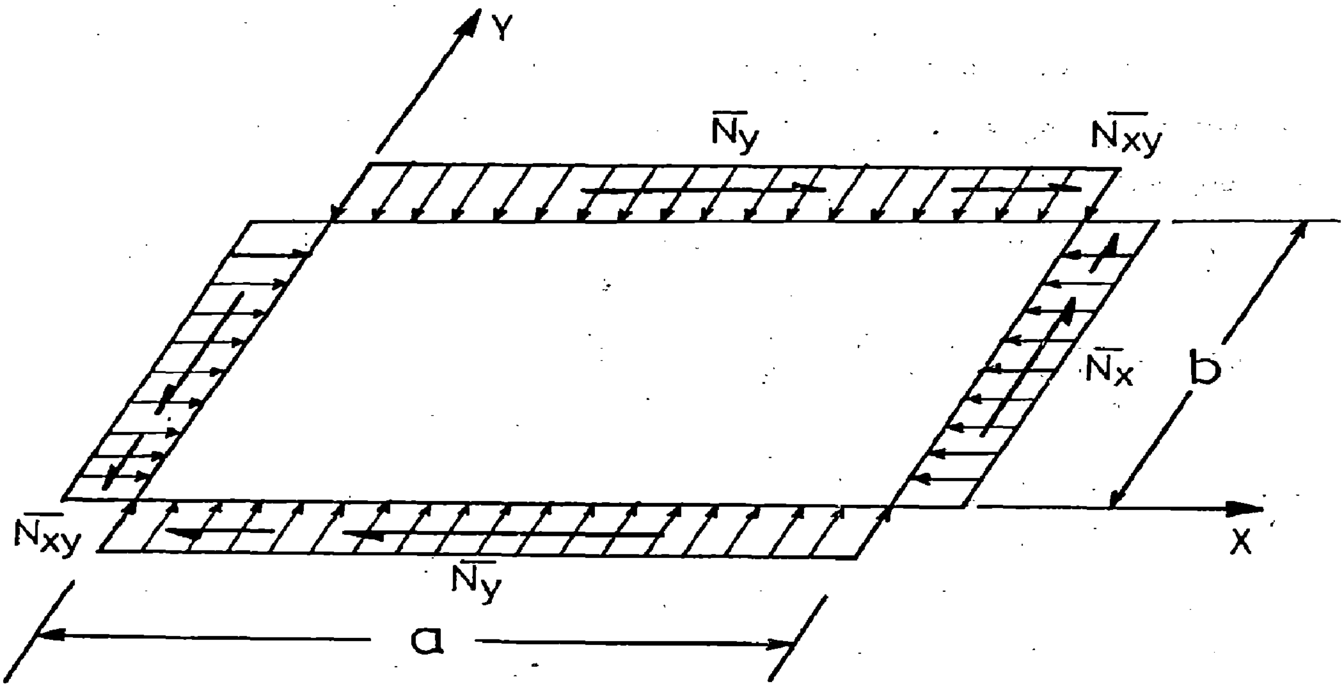

FIG. 2·2 – IN-PLANE LOAD ON A PLATE

For the present problem the total inplane stress resultants $\lfloor N_x \; N_y \; N_{xy} \rfloor^T$ are made up of two components, namely the stress resultants due to applied inplane loads and stress resultants due to restrictions imposed on hygroscopic strains and thus

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{Bmatrix} \bar{N}_x \\ \bar{N}_y \\ \bar{N}_{xy} \end{Bmatrix} + \begin{Bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{Bmatrix} \qquad \ldots(2.16)$$

Where the first term on the right hand side is stress resultants due to applied loads and the other terms is due to hygroscopic stresses developed in the laminate.

2.4 HYGROSCOPIC STRESS RESULTANTS

Moisture diffusion in a composite plate can be described by Fick's law of moisture diffusion. A composite plate subjected to moisture environment on both sides is considered. It is assumed that moisture concentration on both sides of the plate ($z = \pm h/2$) is M_∞ and initially moisture concentration in plate is given by $M_{o(z)}$. Then, the governing equation, boundary and initial conditions are :

$$\frac{\partial M}{\partial t} = \frac{\partial}{\partial z}\left(D \frac{\partial M}{\partial z}\right) \qquad \ldots(2.17)$$

$$M(\pm h/2, t) = M_\infty \qquad \ldots(2.18)$$

$$M(z, o) = M_o(z) \qquad \ldots(2.19)$$

D - moisture diffusivity of the material of the plate

t - time

Solution of the equation (2.17-19) is given by [5]

$$\frac{M(z,t)-M_o}{M_\infty - M_o} = 1-2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{P_n} \cos\left(P_n \frac{2z}{h}\right) \exp\left(-P_n^2 t^*\right) \quad \ldots(2.20)$$

where $P_n = (2n-1)\pi/2$

$M(z,t)$ = Moisture concentration in the plate

$t^* = 4Dt/h^2$

Once the moisture distribution in the composite plate is obtained, the resultant hygroscopic strain in the principal directions can be found by the following expressions,

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}^H = \begin{Bmatrix} \beta_1 \\ \beta_2 \\ \beta_6 \end{Bmatrix} M(z,t) \quad \ldots(2.21)$$

where β_1 and β_2 are coefficients of hygroscopic expansion in directions 1 and 2 respectively. β_6 relates shear strain in 1-2 plane with moisture concentration. 1 and 2 signify directions parallel and perpendicular to the fibre direction.

Hygroscopic stress in the principal material directions can be expressed as

$$\begin{Bmatrix} \sigma_1^H \\ \sigma_2^H \\ \tau_{12}^H \end{Bmatrix} = [Q_o] \begin{Bmatrix} \varepsilon_1^H \\ \varepsilon_2^H \\ \gamma_{12}^H \end{Bmatrix} \quad \ldots(2.22)$$

The stress resultants along x, y co-cordinate system can be expressed as,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} \sigma_1^H \\ \sigma_2^H \\ \tau_{12}^H \end{Bmatrix}$$

$$= [T]^{-1} \quad [Q_o] \; \{\beta\} \; M\,(z,t) \qquad \ldots(2.23)$$

Inplane stress resultants developed due to hygroscopic effect can, now be expressed as,

$$\begin{Bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{Bmatrix} = \sum_{i=1}^{n} \int_{hi-1}^{hi} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz$$

$$= \sum_{i=1}^{n} \int_{hi-1}^{hi} [T]^{-1}[Q_o] \; \{\beta\} \; M(z,t) \; dz \qquad \ldots(2.24)$$

2.5 TOTAL POTENTIAL ENERGY OF LAMINATE

Using eq (2.11) and (2.13), the expression for total potential energy can be expressed as

$$\pi = \frac{1}{2} \int_A \{\varepsilon_1\}^T \; [D] \; \{\varepsilon_1\} \; dA - \frac{1}{2} \int_A \{\varepsilon_2\}^T \; [N] \; \{\varepsilon_2\} \; dA \qquad \ldots(2.24)$$

To determine the buckling load the variation of total potential energy is equated to zero i.e. $\delta\pi = 0$

$$\delta\pi = \int_A \left\{ \delta\varepsilon_1 \right\}^T [D] \left\{\varepsilon_1\right\} dA - \int_A \left\{\delta\varepsilon_2\right\}^T [N] \left\{\varepsilon_2\right\} dA = 0 \qquad \ldots(2.25)$$

2.6 FINITE ELEMENT FORMULATION

For the finite element formulation a proper plate element has to be selected. An inspection of Eq. (2.25) reveals that the continuity of $w, \frac{\partial w}{\partial y}, \frac{\partial w}{\partial x}$, and completeness of $W, \frac{\partial w}{\partial y}, \frac{\partial w}{\partial x}, \frac{\partial^2 w}{\partial y^2}, \frac{\partial^2 w}{\partial y \partial x}, \frac{\partial^2 w}{\partial x^2}$ are required. In View of this a three noded triangular element with three degrees of freedom at each node $\left[w, \frac{\partial w}{\partial y}, -\frac{\partial w}{\partial x}\right]$ has been selected [23]. The assumed polynomial for the displacement w is

$$w = \left[a_1 + a_2 x + a_3 y + a_4 x^2 + a_5 xy + a_6 y^2 + a_7 x^3 + a_8 (x^2 y + xy^2) + a_9 y^3\right] \qquad \ldots(2.26)$$

Now a local Co-ordinate system for a particular element is as shown in Fig.(2.3). Origin of the local Co-ordinate system is at the 1st node, local y axis is along the line joining node 1 and 2, local X-axis normal to the local y-axis in the direction of node 3. ψ is the angle between the global x-axis and local y-axis. So working in the local Co-ordinate system, w for a particular element can be expressed as,

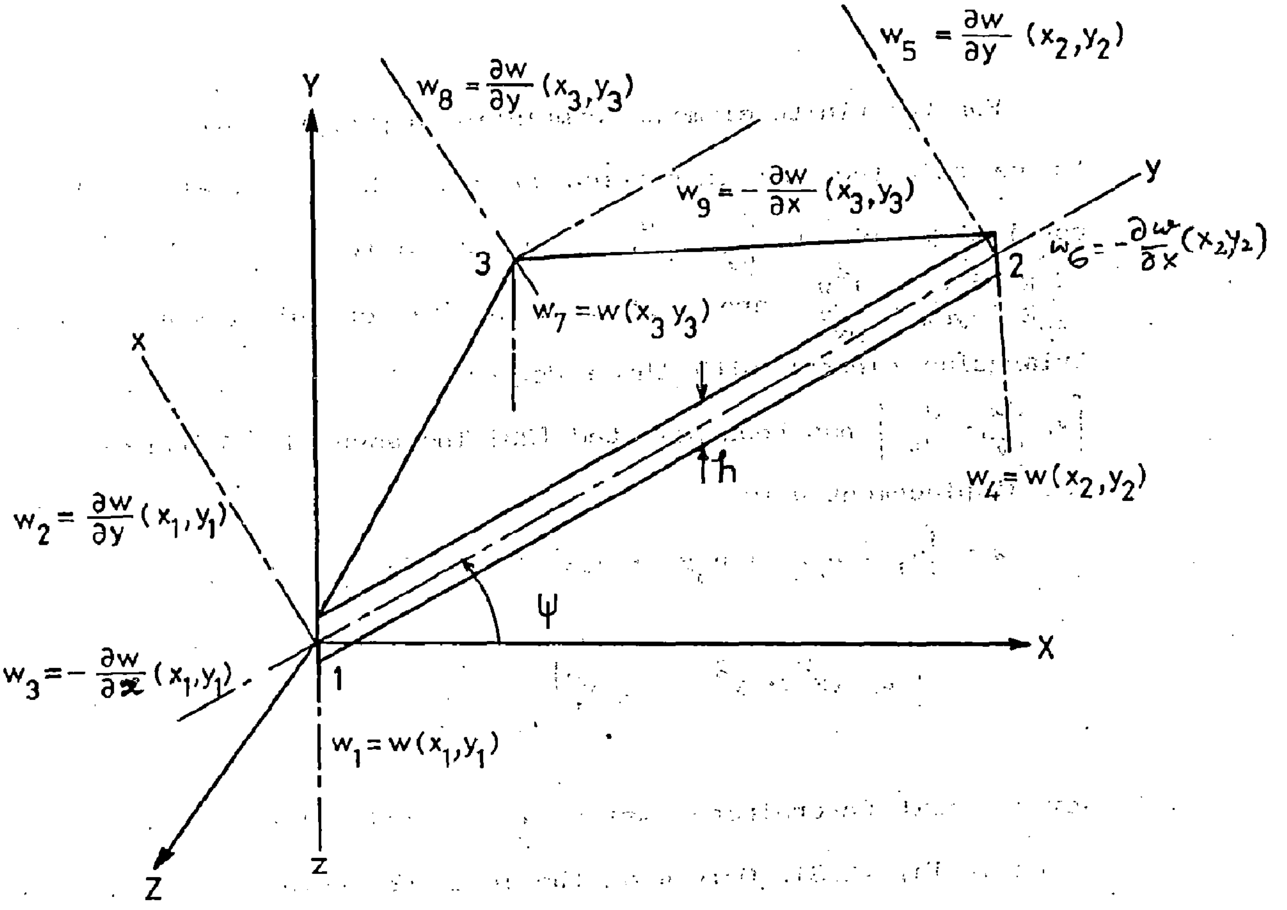

FIG. 2·3– LOCAL CO-ORDINATE SYSTEM AND DEGREES OF FREEDOM OF A FINITE ELEMENT

$$w = \lfloor \bar{\eta} \rfloor \, [J]^{-1} \{ \Delta^e \} \qquad \ldots(2.27)$$

$$= \lfloor \eta \rfloor \{ \Delta^e \} \qquad \ldots(2.28)$$

where $\lfloor \eta \rfloor = \lfloor \bar{\eta} \rfloor \, [J]^{-1}$

$$[J] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & y_2 & 0 & 0 & y_2^2 & 0 & 0 & y_3^3 \\ 0 & 0 & 1 & 0 & 0 & 2y_2 & 0 & 0 & 3y_2^2 \\ 0 & -1 & 0 & 0 & -y_2 & 0 & 0 & -y_2^2 & 0 \\ 1 & x_3 & y & x_3^2 & x_3y_3 & y_3^2 & x_3^3 & x_3^2y_3+x_3y_3^2 & y_3^3 \\ 0 & 0 & 1 & 0 & x_3 & 2y_3 & 0 & x_3^2+2x_3y_3 & y_3^3 \\ 0 & -1 & 0 & -2x_3 & -y_3 & 0 & -3x_3^2 & -2x_3y_3-y_3^2 & 0 \end{bmatrix}, \quad \{\Delta^e\} = \begin{Bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ w_8 \\ w_9 \end{Bmatrix}$$

From equation (2.2a) ε_1 can be expressed as,

$$\{\varepsilon_1\} = \begin{Bmatrix} \dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial^2 w}{\partial y^2} \\ \dfrac{\partial^2 w}{\partial x \partial y} \end{Bmatrix}$$

$$= [\beta] \{\Delta^e\} \qquad \ldots(2.29)$$

where $[B] = \{H\} [J]^{-1}$

$$\text{and } [H] = \begin{bmatrix} 0 & 0 & 0 & 2 & 0 & 0 & 6x & 2y & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2x & 6y \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 4(x+y) & 0 \end{bmatrix}$$

From equation (2.14), $\{\varepsilon_1\}$ can be expressed as

$$\{\varepsilon_2\} = \begin{Bmatrix} \dfrac{\partial w}{\partial x} \\ \dfrac{\partial w}{\partial y} \end{Bmatrix}$$

$$= [B_D]\ \{\Delta^e\} \qquad \ldots(2.30)$$

Where $[B_D] = [G]\ \{J\}^{-1}$ and

$$[G] = \begin{bmatrix} 0 & 1 & 0 & 2x & y & 0 & 3x^2 & 2x+y^2 & 0 \\ 0 & 0 & 1 & 0 & x & 2y & 0 & x^2+2xy & 3y^2 \end{bmatrix}$$

Therefore eq.(2.25) can be written as,

$$\left\{\delta\Delta^e\right\}^T \left[\int_A B^T D\ B\ dA\right]\left\{\Delta^e\right\} - \left\{\delta\Delta^e\right\}^T\left[\int_A B_D^T\ N\ B_D\ dA\right]\ \left\{\Delta^e\right\} = 0 \qquad \ldots(2.31)$$

Now [N] can be expressed as,

$$\begin{bmatrix} N_x & N_{xy} \\ N_{xy} & N_y \end{bmatrix} = \begin{bmatrix} \bar{N}_x & \bar{N}_{xy} \\ \bar{N}_{xy} & \bar{N}_y \end{bmatrix} + \begin{bmatrix} N^H_x & N^H_{xy} \\ N^H_{xy} & N^H_y \end{bmatrix} \qquad \ldots(2.32)$$

If it is assumed that stress resultants due to applied loads $\bar{N}_x, \bar{N}_{xy}$ and $\bar{N}_y$ are in a certain ratio given by $C_1 = \bar{N}_y / \bar{N}_x$ and $C_2 = \bar{N}_{xy} / \bar{N}_x$ then eq. (2.32) can be expressed as,

$$[N] = \bar{N}_x \begin{bmatrix} 1 & C_2 \\ C_2 & C_1 \end{bmatrix} + \begin{bmatrix} N_x^H & N_{xy}^H \\ N_{xy}^H & N_y^H \end{bmatrix}$$

$$= \bar{N}_x \; [C] + [N^H] = [\bar{N}] + [N^H] \qquad \ldots (2.33)$$

Where $[C] = \begin{bmatrix} 1 & C_2 \\ C_2 & C_1 \end{bmatrix}$

Using eq. (2.33) equation (2.31) can be rewritten as,

$$\left[\int_A [B]^T [D][B] dA - \int_A [B_D]^T [N]^H [B_D] dA - N_x \int_A [B_D]^T [C][B_D] dA \right] \left\{ \Delta^e \right\} = 0 \qquad \ldots (2.34)$$

Now defining,

$$[k] = \int_A [B]^T \; [D] \; [B] \; dA$$

$$[k_D^H] = \int_A [B_D]^T \; [N]^H \; [B_D] \; dA$$

$$[\bar{k}_D] = \int_A [B_D]^T \; [C] \; [B_D] \; dA$$

Equation (2.34) can be written as

$$\left[[k] - [k_D^H] - \bar{N}_x [\bar{k}_D] \right] \{\Delta^e\} = 0 \qquad \ldots(2.35)$$

To get the element equation in global co-ordinates system the various matrices like $[k]$, $[k_D^H]$, $[\bar{k}_D]$ must be transformed to global co-ordinate system.

The resulting equation is ,

$$[[K] - [K_D^H] - \bar{N}_x [\bar{K}_D]] \cdot \{\Delta\} = 0$$

where $[K] = [\lambda]^T [k] [\lambda]$

$[K_D^H] = [\lambda]^T [k_D^H] [\lambda]$

$[\bar{K}_D] = [\lambda]^T [\bar{k}_D] [\lambda]$

where,

$$[\lambda] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\text{Sin}\ \psi & \text{Cos}\ \psi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \text{Cos}\ \psi & \text{Sin}\ \psi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\text{Sin}\ \psi & \text{Cos}\ \psi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \text{Cos}\ \psi & \text{Sin}\ \psi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\text{Sin}\ \psi & \text{Cos}\ \psi \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \text{Cos}\ \psi & \text{Sin}\ \psi \end{bmatrix}$$

The angle ψ is as shown in Fig.2.3

Now assembling all the element equations the system equation governing the hygroscopic buckling of the plate can be obtained as,

$$\left[[K_G] - [K_{DG}^H] - \bar{N}_x [\bar{K}_{DG}] \right] \{\Delta\} = 0 \qquad \ldots(2.36)$$

$[K_G]$ is the global stiffness matrix of the composite laminated plate. $[K_{DG}^H]$ can be termed the differential stiffness matrix of the plate due to hygroscopic absorption. So for a given ambient moisture concentration and for a given moisture absorption time $[K_{DG}^H]$ can be obtained. As such the total global stiffness of the plate, $\left[[K_G] - [K_{DG}^H]\right]$ becomes lower with increasing time. $[K_{DG}]$ is the differential stiffness matrix due to applied inplane loads. As such it is clear from equation (2.36) that with moisture absorption time, applied in-plane loads required to buckle the plate will become smaller.

Before attempting to solve eqn. (2.36), the boundary conditions of the plate are in corporated and the modified systems equations become an algebric eigen value problem in $\bar{N}_x$. The eigen value problem can be solved to obtain $\bar{N}_y$. Then from the known values of $\bar{N}y/\bar{N}x$, and $\bar{N}_{xy}/\bar{N}_x$ all the buckling loads can be found out.

CHAPTER - 3

RESULTS AND DISCUSSIONS

3.1 INTRODUCTION

In the present chapter the effect of moisture absorption on the buckling behaviour of polymeric composite laminates has been highlighted through numerical results. For this purpose composite laminates made of T 300/5208 graphite-epoxy material system have been analysed. Three different laminate configuration with same ply thickness and number of plies - eg. quasi-isotropic (0,± 45,90)4s, cross plied (0,90)8s and angle plied (±45)8s have been analysed. Buckling load for square plates with support conditions (i) all edges simply supported and (ii) all edges clamped have been investigated. The results have been obtained for various equilibrium moisture concentrations.

3.2 RESULTS AND DISCUSSIONS

The plate support conditions are either (i) all edges simply supported or (ii) all edges clamped. The finite element mesh along with the boundary conditions are shown in Fig. 3.1. The computer program has been validated by comparing the results with exact solution for buckling load of Isotropic plate with both type of edge conditions. i.e simply supported and clamped. The results have been matched with the buckling load at steady state condition as reported by Flaggs and Vinson [19].

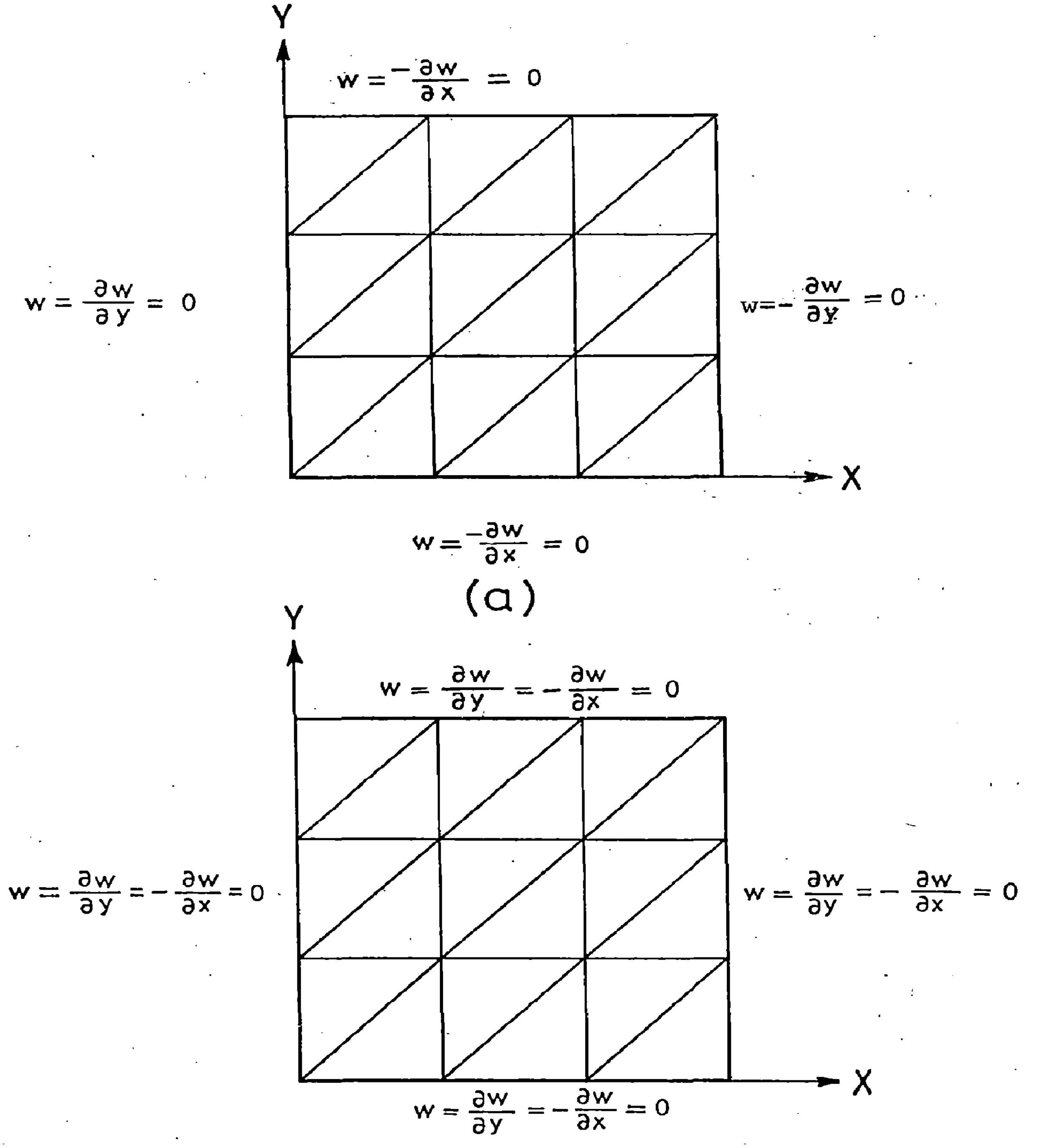

FIG. 3·1 - SQUARE PLATE WITH FINITE ELEMENT MESH

a) BOUNDARY CONDITION FOR SIMPLY SUPPORTED PLATE

b) BOUNDARY CONDITION FOR CLAMPED PLATE

The result have been presented for a square plate of side 0.3048m and thickness 0.00447m made from T300/5208 graphite-epoxy laminae. Material properties for T300/5208 lamina in principle material direction at $21^{o}C$ are as following [19].

$E_1 = 1.44 \times 10^{11}\ N/m^2$ $\qquad \nu_{12} = 0.21$

$E_2 = 1.1176 \times 10^{10}\ N/m^2$ $\qquad G_{12} = 4.485 \times 10^{9}\ N/m^2$

$\beta_1 = 0.0$ $\qquad \beta_2 = \beta_6 = 0.00667$

Lamina thickness = $1.396 \times 10^{-4} m$

$D = 2.63 \times 10^{-10}\ m^2/hr.$

Results for three different stacking sequence namely quasi-isotropic (0,±45,90)4s, cross plied (0,90)8s and angle plied (±45)8s have been obtained. The externally applied buckling load is assumed to be only in X-direction, ie, the assumed in plane loading is such that

$$\bar{N}_x \neq 0, \quad \bar{N}_y = \bar{N}_{xy} = 0$$

Results have been presented through Figs (3.2 to3.7). In all the Figures variation of the buckling load with respect to time is shown. Parameter of the curves shown in Fig (3.2 - 3.7) is the equilibrium moisture concentration (% wt/wt) of the composite plate. The result presented in this chapter refers to T 300/5208 laminate at 21°C as such the elevated temperature results may be radically different those presented herein.

Fig. (3.2) shows the variations of buckling load ($\bar{N}_x$) with moisture absorption time for a simply supported quasi-isotropic (0, ± 45,90) 4s plate. It can be seen from Fig. (3.2) that $\bar{N}_x$ decreases significantly with moisture absorption. For lower values of equilibrium moisture concentration the buckling load asymptotically approaches a value corresponding to the steady state hygroscopic condition. This steady state hygroscopic condition is obtained in about 1000 hrs for the material system considered. Even for a very low equilibrium moisture concentration (M_∞ = 0.1) where the net weight gain of the composite plate due to moisture absorption is only one in Thousand. The buckling load decreases by almost 40%. For a higher value of equilibrium moisture concentration (M_∞ = 0.3) the steady state hygroscopic condition is never achieved and the buckling load drops down to zero, much earlier than t = 1000 hrs.

Fig. (3.3) shows the variations of buckling load with time for a clamped quasi-isotropic plate. It is observed that the buckling load for a clamped plate is much higher than the same for a simply supported plate. The buckling load is less sensitive to moisture concentration. For an equilibrium moisture concentration (M_∞ = 0.2) the decrease in buckling load is only 30% as compared to a decrease of almost 80% for the case of simply supported plate. Here also it is observed that for M_∞ = 0.2 and 0.4 the buckling load reaches a steady state value, but for M_∞ = 0.6, the required buckling load drops down to zero.

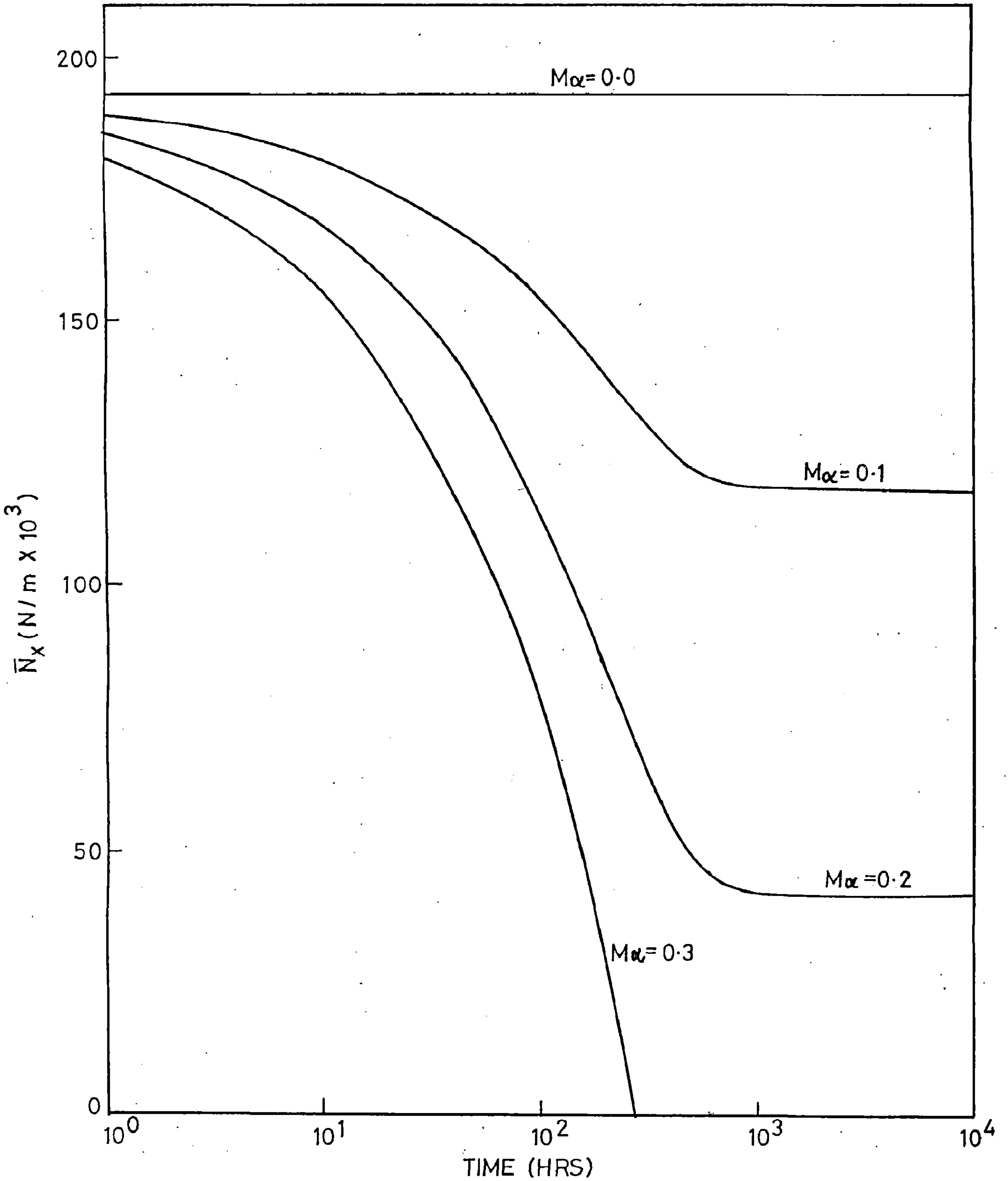

FIG. 3·2- VARIATION OF BUCKLING LOAD WITH MOISTURE ABSORPTION TIME; WITH MOISTURE AS PARAMETER, FOR A SIMPLY SUPPORTED QUASI ISOTOPIC $(0,\pm45,90)_{4S}$ PLATE

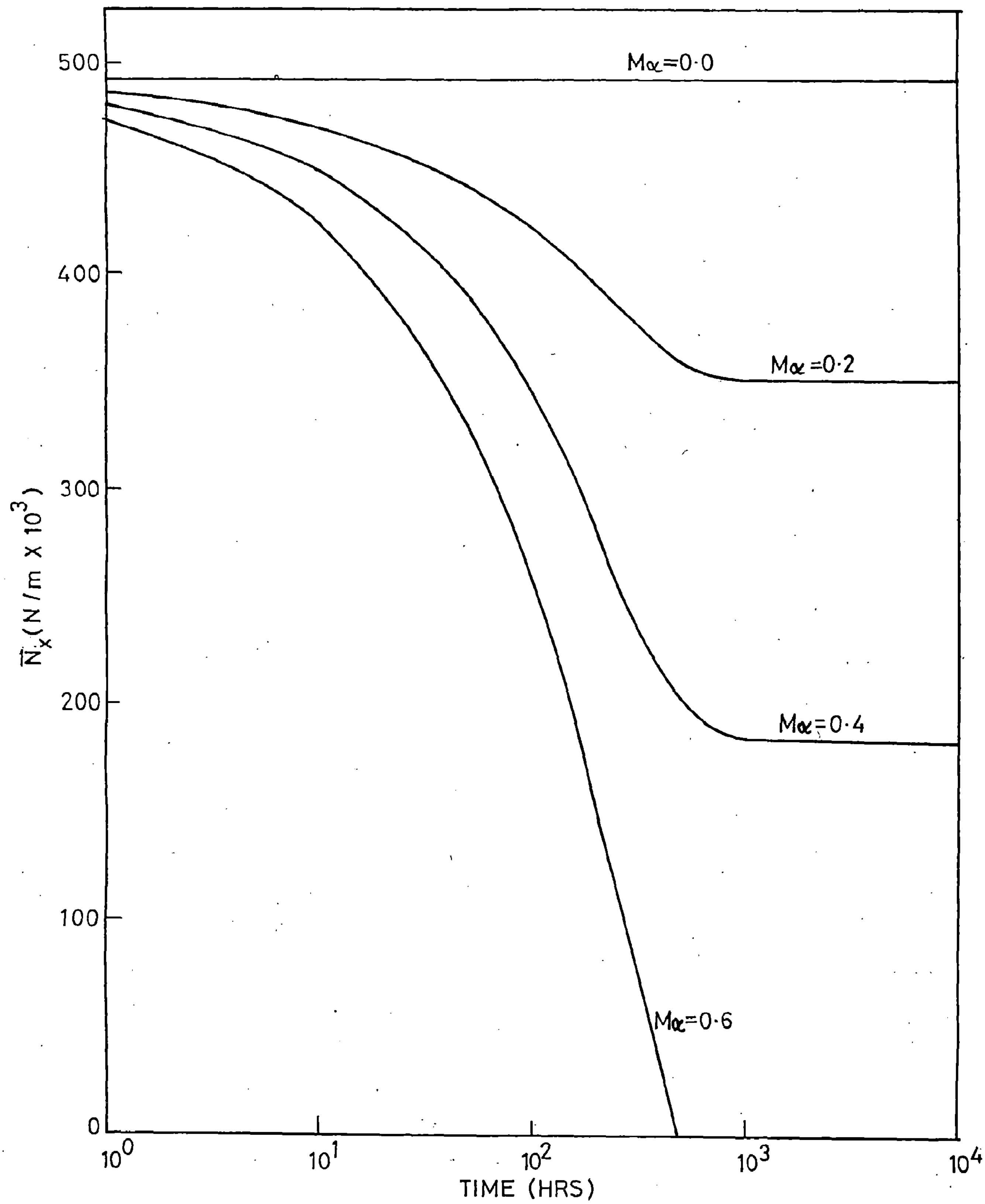

FIG. 3·3 – VARIATION OF BUCKLING LOAD WITH MOISTURE ABSORPTION TIME; WITH MOISTURE AS A PARAMETER FOR A CLAMPED QUASI ISOTOPIC $(0, \pm 45, 90)_{4S}$ PLATE

The results for the cross ply laminates (0/90)8s are presented in Fig. 3.3 and 3.4. The same for the angle plied laminate (± 45)8s are presented in Figs (3.6 and 3.7). It is observed from these plots (Figs. 3.4–3.7) that nature of the buckling load variation for cross ply as well as angle ply laminate is same as that discussed for quasi-isotropic laminates through Fig. 3.2–3.3.

The stiffness of the plate depends on the [D] matrix and the geometry and boundary conditions of the plate. Since the [D] matrix for the three different laminate systems are not much different so the buckling load for these laminates are not much different. The $[K^H_{DG}]$ matrix is dependent on moisture distribution $M(z,t)$, hygroscopic expansion coefficients $\{\beta\}$, and the laminate property $[T]^{-1}$ $[Q_o]$. Since diffusivity, hygroscopic expansion coefficients are lamina properties and for the three different laminate systems considered $[T]^{-1}$ $[Q]$ are also much different. So the decrease in the effective stiffness of laminate is almost for the three different laminate systems considered. In short, if laminates are selected such that their bending stiffness matrix [D] is almost same and the hygroscopic stress resultants are also same then the buckling load will not be sensitive to the choice of laminates. So, with these constraint a choice of laminate systems is available and the final selection may be made on the basic of other design considerations.

From the results presented through Figs 3.2–3.7 it is clearly seen that effect of moisture absorption on the buckling behaviour of the composite laminate is significant. It is observed

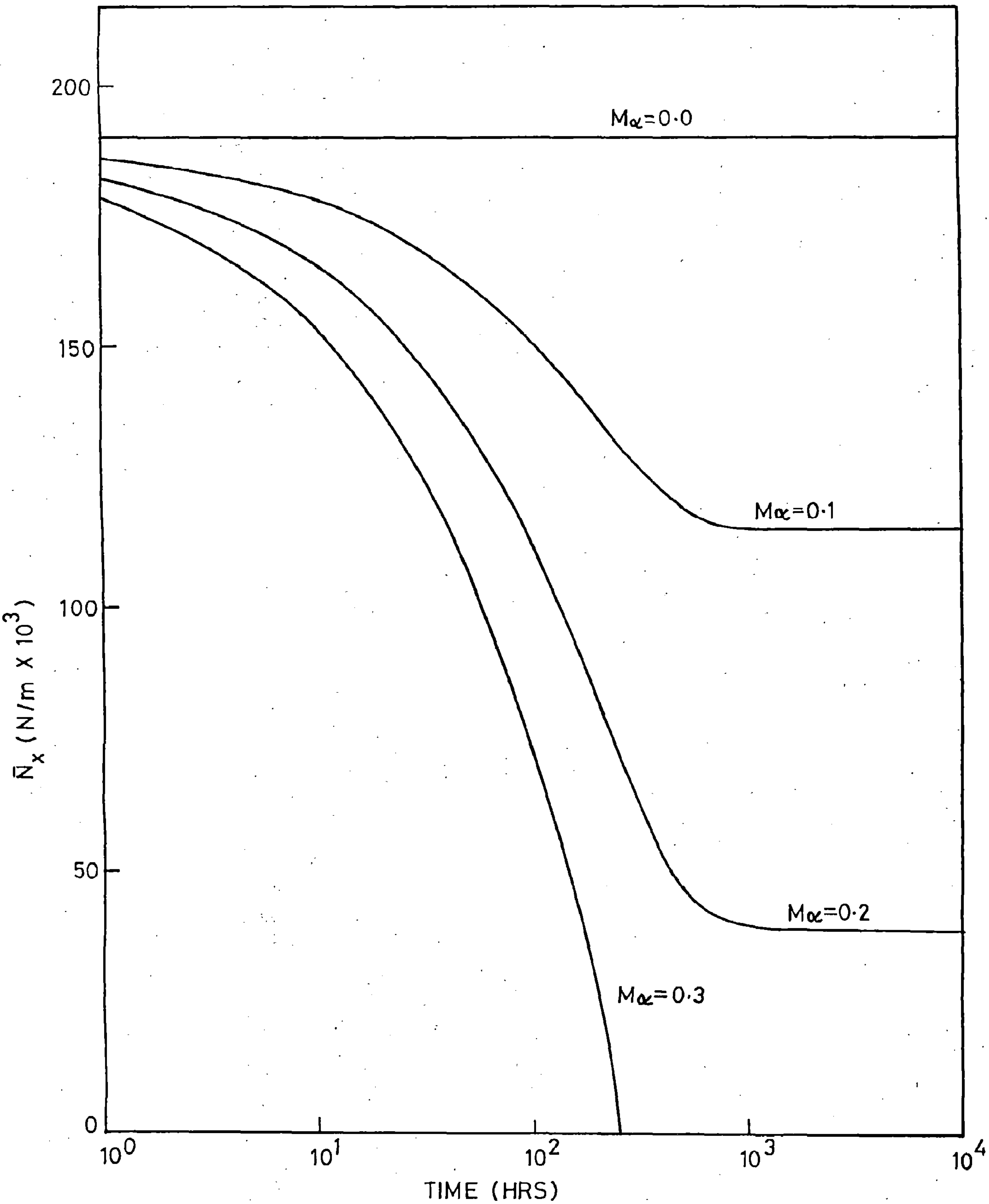

FIG. 3·4 – VARIATION OF BUCKLING LOAD WITH MOISTURE ABSORPTION TIME; WITH MOISTURE AS A PARAMETER FOR A SIMPLY SUPPORTED CROSS PLIED $(0,90)_{8S}$ PLATE

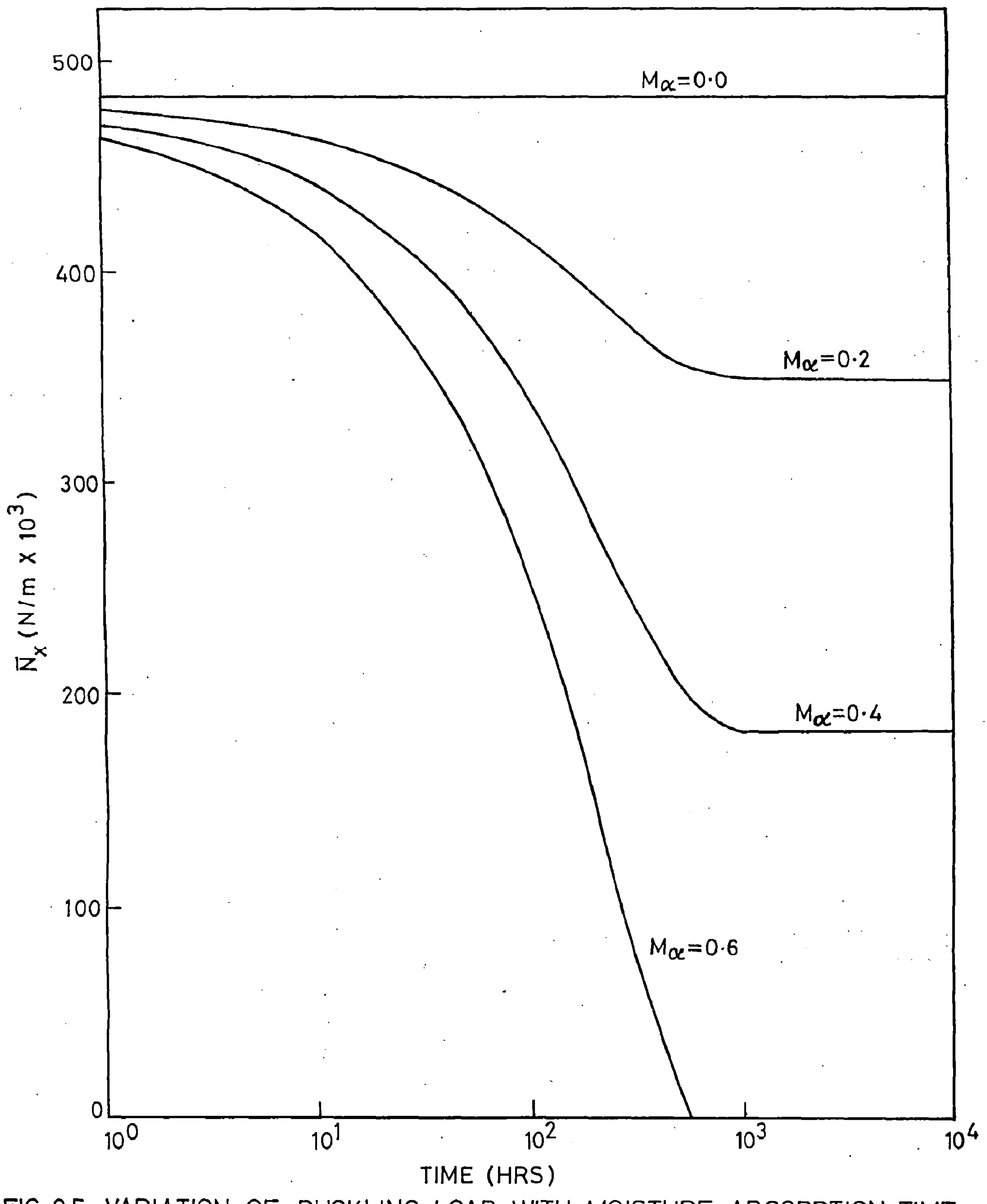

FIG. 3·5 – VARIATION OF BUCKLING LOAD WITH MOISTURE ABSORPTION TIME; WITH MOISTURE AS A PARAMETER FOR A CLAMPED CROSS PLIED $(0, 90)_{8S}$ PLATE

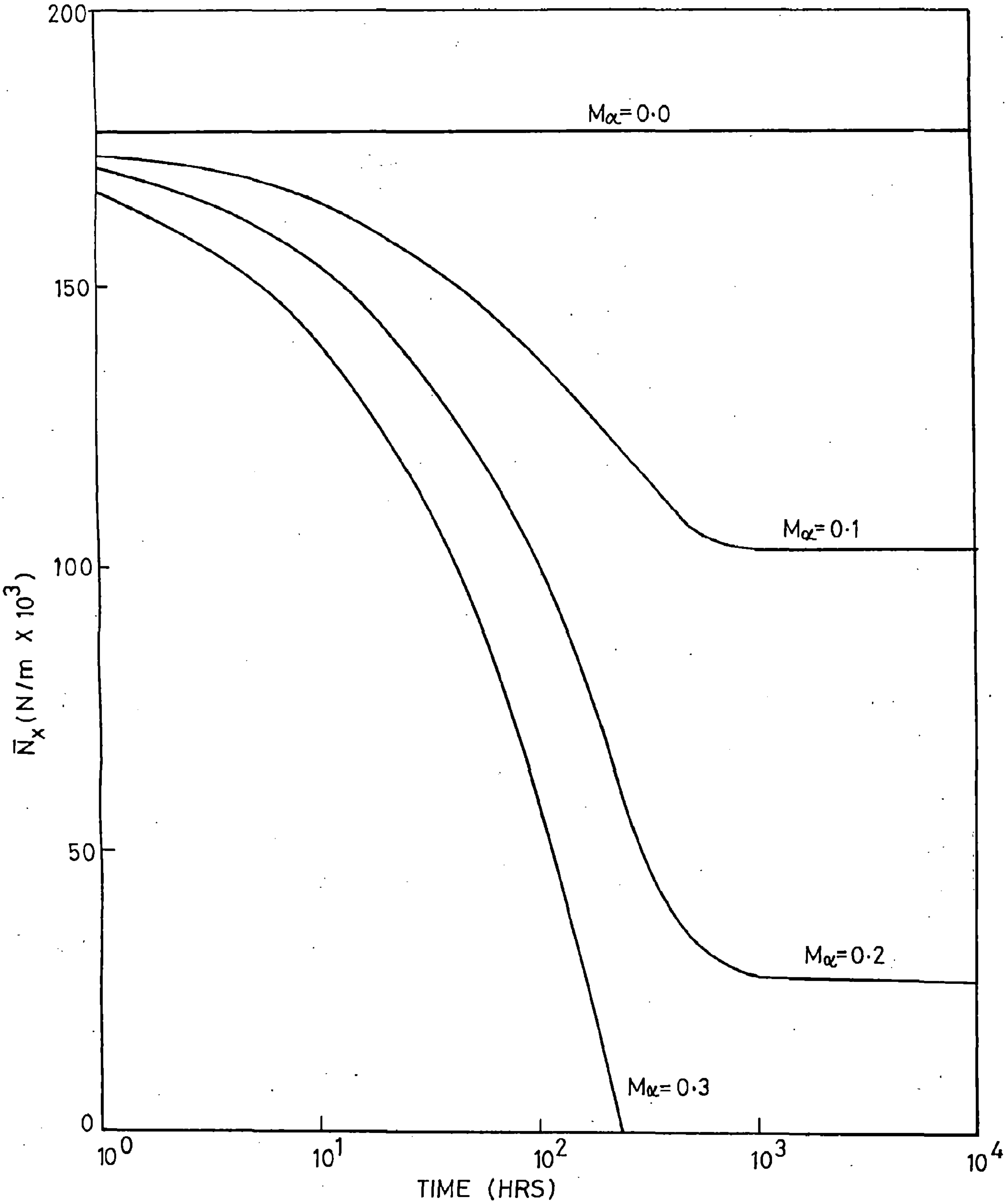

FIG. 3·6 - VARIATION OF BUCKLING LOAD WITH MOISTURE ABSORPTION TIME; WITH MOISTURE (R) AS A PARAMETER, FOR A SIMPLY SUPPORTED ANGLE PLIED $(\pm 45)_{8S}$ PLATE

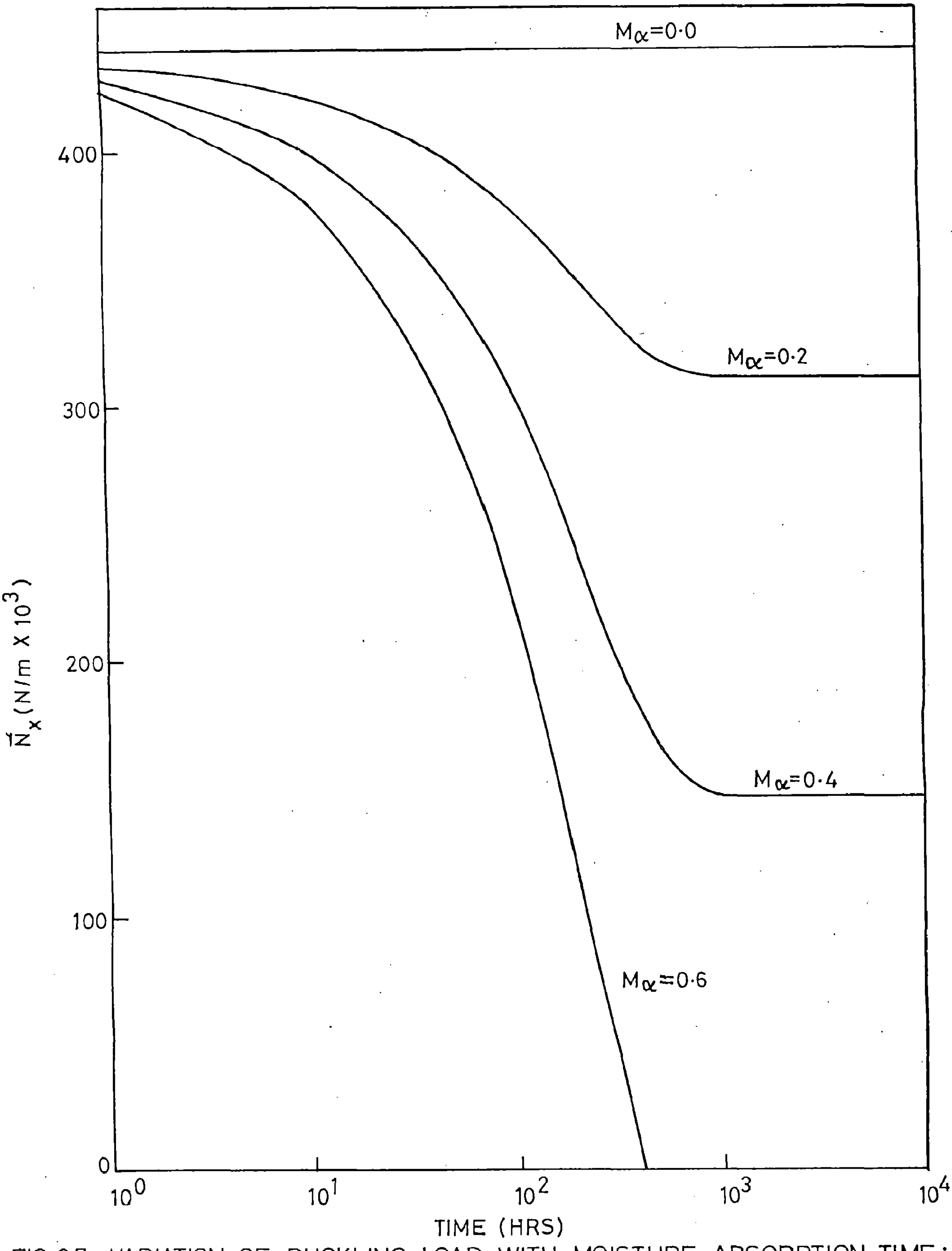

FIG. 3·7 - VARIATION OF BUCKLING LOAD WITH MOISTURE ABSORPTION TIME; WITH MOISTURE AS A PARAMETER FOR A CLAMPED ANGLE PLIED (± 45)$_{8S}$ PLATE that clamped edge conditions is less sensitive to moisture absorption. Also, it is observed that for a smaller values of equilibrium moisture concentration, the buckling load approaches a steady state hygroscopic value.

So it is possible to design a structure with these steady state buckling load. But for higher values of equilibrium moisture concentration, the buckling load drops down to zero in finite time. Here the buckling is entirely due to hygroscopic stress resultants.

From the results and discussions presented above, it is seen that effect of moisture absorption on the performance of composite laminate is significant and any realistic design of composite structures must taken into account, the stress resultants due to moisture absorption. The present study indicates that a recommendation for the maximum value of moisture concentration to which a laminate can be subjected, can be made. Also for an equilibrium moisture concentration below this, the buckling load corresponding to any particular expected life of the structure can be obtained, and this value of buckling load can be used for design purpose.

CHAPTER - 4

CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

4.1 CONCLUSIONS

Based on the results and discussion presented in chapter-3 following conclusions can be made :

(i) Effect of moisture absorption significantly reduces the buckling load ($\bar{N}_x$). So for a realistic design of composite structures subjected to adverse environmental condition, effect of moisture absorption must be considered.

(ii) For a particular laminate and support condition there is a threshold moisture concentration level. If the moisture concentration is less than this threshold then buckling load finally approaches a value corresponding to steady state hygroscopic condition. This value of buckling load may be used as a design basis for the composite structure.

(iii) For moisture concentration higher than the threshold discussed above, buckling load of the composite laminate drops down drastically with time and finally becomes zero even before steady-state hygroscopic condition is achieved. Zero value of buckling load implies complete loss of stiffness and so this condition corresponds to hygroscopic, i.e. buckling due to moisture absorption alone.

(iv) It can be concluded that clamped plates are less sensitive to moisture diffusion.

4.2 SUGGESTIONS FOR FUTURE WORK

Environmental degradation of composite laminate is a subject of great technical interest and it is a very active research area. As such, lot of work is being done in this field. Only a few suggestions for future work directly linked with the present work is presented here.

(i) As a direct extension of the present work, plates of different shapes and edge conditions should be analysed.

(ii) The present formulation ignores the effect of shear deformation and rotary inertia of the plate. These effects can be significant if the shear modulus of laminate is very small compared to longitudinal and transverse modulus. So a higher level plate theory should be used such that above mentioned effects are incorporated.

(iii) The results presented in this work refers to a temperature of $21^{\circ}C$. In many spacecraft applications temperatures may be very high due to atmospheric friction and as such there is a need to analyse the laminate behaviour at higher temperature.

(iv) Composite laminates are used either as plates, curved panels or shells. A similar analysis for curved panels and shells is much in order.

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APPENDIX

The Computer Program can be subdivided into following modules. These modules are briefly explained below :

(i) INPUT modules : In this all relevent informations are read into the program.

(ii) Module for calculating bending stiffness matrix D : This module calculates the bending stiffness matrix[D] of the laminate.

(iii) Hygroscopic stress resultant module : In this module hygroscopic stress resultants are calculated.

(iv) Moment area module : Here moment area of the form $\int_A x^m y^n dA$ is calculated.

(v) Strain Energy module : Using module (ii) the strain energy of the plate is calculated.

(vi) Hygroscopic stiffness module : Using moment area module, the hygroscopic stiffness is calculated.

(vii) In-plane module : The work done by in-plane load is calculated in this module.

(viii) Assembly module : Here the total potential energy of the composite plate is assembled.

(ix) Modification module : This module modifies the assembled system of equations by incorporating the boundary conditions.

(x) Eigen value module : In this module eigen value of the modified system of equations is calculated and the buckling load is given as output.