STRENGTH OF COHESIVE SOILS

A Dissertation
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By
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CERTIFICATE

Certified that the dissertation entitled "Strength of Cohesive Soils" which is being submitted by Sri G.N. Yoganarasimhan, in partial fulfilment for the award of the degree of Master of Engineering in Water Resources Development of the University of Roorkee is a record of the student's own work carried out by him under our supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked from 1-4-1967 to 1-7-68 for preparing this dissertation at Birla Institute of Technology and Science, Pilani.

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(G.N. YOGANARASIMHAN)

STRENGTH OF COHESIVE SOILS

CONTENTS

SUMMARY

1. Strength of cohesive soils cannot be explained in terms of clear cut parameters, as it depends on various factors which are inter-dependent. Efforts have been made by different research workers to explore important individual factors such as, soil type and structure, time effect (creep and relaxation), influence of intermediate principal stress, influence of anisotropy, dilatancy effect, progressive failure process, influence of disturbance in sampling, scale effect of specimen, etc. In clays adsorbed water layer plays an important role, Their grain constituent, shape and structural arrangement give soils compressibility and resilience. Because of low permeability they retain neutral stresses for a long time. The various possible effects of adsorbed water, compressibility, neutral stresses and low permeability provide clays with al-most unlimited variations in shear strength.

2. For the simplified assumption of plane strain, the yield criteria (i.e., the relation existing between the stresses at the limiting condition) combined with the two equations of motion renders the problem statically determinate. But the difficulty is to determine the mathematical nature of the failure criterion that prevails for soils. The two approaches viz., the tacit assumption of an yield criterion, and the failure criteria based on test results have their own limitations.

3. All studies of cohesive soils sheared to failure present empirical relationship between the various factors at failure whose numerical values hold only for the soil or soils tested.

4. Of the various theoretical failure criteria put forward the one suggested by Mohr-Colomb and that of Tresca (which is in accord with that of Mohr) find favour in the field of mechanics of cohesive soils. Though majority incline towards these criteria based on certain test results, the opinion on this is still divergent.

5. When shear strengths are determined by various techniques those methods which entail the least amount of handling of material tend to give the largest values of strength. In applying shear strength data to practical problems many details must be considered, among them

(a) the appropriate rate of shearing strain,

(b) the possibility of progressive failure associated with weakening which the soil experiences after subjection to large deformation.

(c) The non-uniform distribution of shearing stresses within the soil mass.

6. The division of shear strength of clays into cohesion and frictional strength is an over simplification of doubtful utility. It is better to consider in every case, the total shear strength and to establish the principle of its variation for given conditions, and then to apply it ~~to~~ in actual analysis.

7. To provide correspondence between the test data and the field strength, the sample of the soil under consideration should be as representative as possible of the soil in the field. In addition every effort should be made to preserve the degree and distribution of the density, moisture content and consolidation

characteristics of the soil. Moreover, the drainage conditions during the test should correspond to those anticipated in the field.

7. Existing research on strength theories can be classified into two types as below -

(1) Microscopic researches on the behaviour of absorptive film and structural skeleton of clay, and (2) Macroscopic researches on the mechanical behaviour of clay mass in relation to various external factors and strength. It has not yet been possible to establish a substantial theory based on the microscopic properties as the structural skeleton and structural degree of the soil is extremely complicated. Such relations to-date are only qualitative.

8. Of the several methods and mechanism for laboratory evaluation of shear strength parameters the ultimate choice depends on the availability of facilities and the particular type of analysis, which one prefers. The C and Ø parameters obtained from evaluation of test results are only demonstrated mechanical properties peculiar to the test system and methods used for analysis of results. The relationship between these mechanical properties and actual microscopic properties is not easily defined, since there does not exist common agreement as to what friction and cohesion means in clay soils. Further, test systems developed to-date, are themselves based on, and hence biased to some concept of strength parameters.

1 INTRODUCTION AND IMPORTANCE OF THE STUDY

1.1 Cohesive soils are essential ingredients in earth dams and levees, because of their ability to resist the passage of water. These soils are also used extensively in the construction of fills for highways, rail-roads and airports. Because of the high cost of transporting large volumes of soil over great distances, earth construction virtually requires utilization of materials available close to the site of the structure. In many cases, cohesive soils are used for fills even when imperviousness is not a design criteria or requirement because no other materials are economically available.

The mechanical properties of compacted cohesive soils such as compressibility and shearing strength must be given renewed attention in view of project requirements for embankments of unprecedented height. Earthen dams of 500 ft. height are under active consideration in our country, while considerably higher dams are already being built abroad. The soils constituting the water barrier zones of these structures will be subjected to stresses well beyond those for which prototype performance data are available. This emphasizes the need for better understanding of the factors involved in transforming loose soil into structure behaviour and the mechanics of soils subjected to stresses.

Unlike any other materials, the strength of cohesive soils cannot be defined or categorized in terms of any one specific property such as ultimate fracture or yield strength. The nature of the particle interaction renders it difficult to establish a definite strength quantity, except in terms of certain conditions and limitations. Clay water interaction cannot be easily described in "cause and effect" terms since a clay soil is not ideally plastic or elastic, it becomes difficult to apply conventional theories of mechanics to explain and evaluate cohesive soil strength.

In the majority of troublesome stability problems the soils involved are cohesive materials. Therefore it may be stated that the importance of an understanding of the fundamentals of shearing strength will apply in greatest degree to cohesive soils. In fact no physical property of cohesive soils is more complex than the shearing strength. This property depends on many factors, and the individual factors are themselves complicated, but in addition they are inter-related to such a degree that it is extremely difficult to understand their combined action.

Considerable effort has been expended in studying the strength of saturated clays in connection with the important problem of bearing capacity. Although much has yet to be learnt about soft clay foundations, significant advances have been made in this subject by means of devices designed to determine the in-situ strength directly. On the other hand the soil mechanics of unsaturated soils is still a subject of much conjucture, misconception and controversy. It has been seriously questioned as to whether the pore pressures is a reality in strength characteristics of unsaturated soil is concerned. The reliability of pore pressure measurement obtained during laboratory shear tests on unsaturated clays has been doubted. The criteria of failure in laboratory shear tests of compacted cohesive soils in which pore pressures are measured has been the subject of diverse opinion.

1.2 The purpose of this dissertation is to bring out the phenomenon of shearing resistance in cohesive soils as it is understood to date, to show how this varies with the influencing factors and to trace the developments in defining the strength of cohesive soils.

It is a simple task to place a clay specimen in a shearing apparatus and cause a shear failure. A numerical value of shearing strength, which has acceptable precision may readily be obtained if proper technique, a representative sample and satisfactory apparatus are used. The point which has too often been insufficiently appreciated by testing engineers is that the shearing strength, both in the laboratory specimen and the clay in nature is dependent on a number of variables. Before meaning can be attached to shearing strengths determined in the laboratory the engineer who is to interpret the test results must have at his command an understanding of the factors or variables on which the strength is dependent, and he must make adjustments for every factor which occurs differently in the test than in nature. Of course the test conditions should be chosen to reproduce natural conditions as closely as possible. However, exact reproduction of all natural conditions is not obtainable.

Actually, test conditions and natural conditions are often too involved to permit complete comparison. Fundamental research on shearing phenomenon during the last two decades or so has led to a much improved understanding of the various phases of the subject, but to-day it must be admitted that certain factors are still only partially understood. The problem of estimating from laboratory test data the shear strength which exists under natural conditions is one that probably will always bo among the most complox in Soil Engineoring and its solution will always be subjoct to many pitfalls. However it may now b- claimod that most of the important phenomena affocting shear strength are sufficiently well understood to allow the presentation of a rational outline of such phenomena.

Understanding of the basic factors affecting shear strength is the first pre-requisite to an ability to deal with shear strength problem.

The other aspect of the study of strehgth of soils is the failure criteria which are used to express the results of strength tests and which reflect the influence, if any. Of the intermediate principal stress and the behaviour of soils under the high stresses implied by the greatly increased height of earth and rock fill dams now under construction.

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2 REVIEW OF LITERATURE (10)

The most important single contribution to the problem of shearing strength of soils is found in an essay of Coulomb published in 1773 and entitled "Essai sur une application des regles de maximis et minimis a quelques problems de stalique, relatifs a L architecture" (22). In this essay Coulomb deals with the earth pressure theory, and introduces an equation to determine the resistance of soil to shear.

This equation, $S = C + \sigma \tan \phi$ states that the total shear resistance of a soil can be considered a sum of (a) cohesive resistance, a constant and (b) friction, a figure increasing proportionally to the normal pressure acting on the considered plane. Ever since its appearance, this law of Coulomb has served as the basis for any stability computation of soil masses or scientific investigation of shear strength problems.

In spite of the uncomplicated form of this equation a determination of soil coefficients introduced by Coulomb creates a number of difficulties, especially where cohesive soils are concerned. Much progress was not achieved in the field during the 18th and 19th centuries for want of the understanding of the physical properties of soils and the phenomena which determines the strength.

The question of shear strength was soon taken up and a direct shear apparatus was constructed after the appearance of a report in 1922 in Stockhom publishing the results of a thorough investigation of a number of slides by the commission of the Swedish State railways and the publication of "Erdbaumechanik" in

1925 by Terzaghi giving the fundamental basis for the new science of Soil Engineering.

A number of publications appeared in which attempts were made to explain the phenomena and give rules for application of shear tests in practice. However important factors such as pore water pressures were often ignored.

Real progress in the knowledge of fundamental shear strength properties was not made until systematic investigations were carried out at Terzaghi's Soil Mechanics Laboratory in Vienna in order to clear up some more important questions concerning shear strength. During the year 1934-37 Hvorslev carried out a large number of direct shear tests, and the results were subjected to statistical treatment. Hvorslev confirmed in principle the law of Coulomb, but as cohesion was found to be a function of water content he extended Coulomb's equation

$$S = \bar{C} + \bar{\sigma} \tan \bar{\phi}$$

in which $\bar{\sigma}$ is the normal effective stress on the failure plane, $\bar{\phi}$ is the "true angle of internal friction" and $\bar{C}$ is the true cohesion which was found to depend on the water content. This dependence can be expressed by introduction of the equivalent consolidation pressure σ_e

$$S = K \sigma_e + \bar{\sigma} \tan \bar{\phi}$$

The significance of the result of Hvorslev's investigations in 1937 is very profound. The two main results, dependency of the cohesion on water content alone, and recognition of the angle of internal friction as a soil characteristic which determines the strength of even rather plastic clays, were most important steps in the advancement of knowledge of the fundamental strength properties of soils.

In some later papers Terzaghi called attention to the importance of the results attained through Hvorslev's investigations and suplemented them by pointing out failure conditions and the effect of pore water pressures.

The systematic investigation of the fundamental strength properties of cohesive soils started in Vienna by Hvorslev, was continued by Rendulic, who made the first attempt to measure the pore water pressure set up by a triaxial shear test. Rendulic pointed out the relation between the pore water pressure and the deformation properties and utilising this relationship he carried out an experimental investigation of the volume change of samples submitted to stresses in a triaxial apparatus.

Simultaneously with these European investigations a systematic shear research was started in U.S.A. In 1934 Jurgenson made an important contribution to the understanding of shear strength problems. In his paper one of the first triaxial shear tests can be found and in a subsequent discussions of Jurgenson's paper, A. Casagrande made the first attempt to analyse the effective stresses acting in a specimen subjected to triaxial shear at constant volume. These considerations, which Casagrande later extended and published give an analysis of the undrained shear test and demonstrate the affinity between the undrained shear strength and deformation properties.

The year just before the Second World War were characterized by improvement of the shear apparatus. The ring shear machine was developed by Hvorslev, Tiedemann, Grunner, Hoefeli, and the triaxial shear apparatus was improved.

In 1939, the Corps of Engineers U.S. Army initiated a soil mechanics Fact Finding Survey, and one of the problems which was first subjected to investigation was the triaxiol shear method of testing soils. In co-operation three institutions, Harward University, Massachusetts Institute of Technology and U.S. Waterways Experimental Station took up for treatment with admirable thoroughness, the construction of triaxial shear machine, the technique of testing and methods of interpreting the shear test results. A great number of unpublished progress reports dealt with the results of the research work and in 1944 a review was prepared by Rutledge and published in a comprehensive report. The first accomplishment of the programme was an evaluation based on the results of triaxial tests of testing technique and the technical details in the testing machines. For cohesionless soils a consistent and well-supported analysing method was developed and proved by tests. For an interpretation of triaxial shear tests with clays, two hypotheses were investigated. The first one was the working hypothesis put forward by A. Casagrande (16). The second one which was described by Rutledge (68) considers the strength problems from quite a new point of view. The tests carried out in the research programme indicate that the strength of saturated clay depends only on the water content at failure and is independent of either the minor principal stress or the pore water pressure or of the method of testing. Consequently Rutledge proposed the use of a diagram in which logarithm of the compressive stress is plotted against the water content. The resulting curve runs roughly parallel to the semilogarithmic pressure water content curve (Fig. 2.1), plotted from a standard consolidation test. Thus according to Rudledge the magnitude of pore water pressures need not be considered in practical

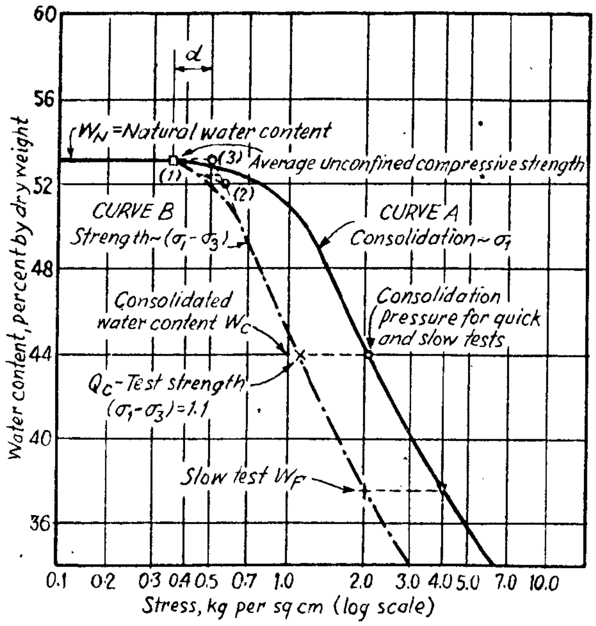


Fig. 2.1

application and in addition, the question of the angle of internal friction is evaded.. Another important investigation included in the research program is the measurement of pore water pressure.

The results of this co-operative triaxial shear research programme contribute essentially to the progress of soil mechanics and the detailed publications of test datas and information on the construction of apparatus is of great assistance to other soil mechanics laboratories. However no final conclusions were drawn and the problem of the fundamental properties of clay was not solved.

Rutledge expresses the opinion that it is doubtful if saturated cohesive soils can be considered to have an angle of internal friction, a point of view not shared by Taylor who ascribes the total strength of normally consolidated clay to friction.

In the 1st number of the periodical "Geotechnique"

Skempton (76) published an outstanding paper which discussed the shear strength of different clays. In this paper Skempton uses the definitions of cohesion and friction as given by Coulomb-Hvorslev and through an introduction of the co-efficient of elasticity of compression and expansion he succeeds in obtaining an expression for the compressive strength found by undrained triaxial tests. That is to say he expresses quantitatively through formulae and figures the qualitative considerations advanced by A. Casagrande in the triaxial shear review. In a number of later papers Skempton has elaborated this method of analysis.

At the 2nd International Conference of Soil Mechanics held in Rotterdam (63) in 1948 there was a great difference of opinion regarding the nature of shear strength of clays. Some research workers interpreted the strength of clay as due merely to cohesion, others as due exclusively to friction. Finally the water content strength diagram was opposed to the common Mohr's diagram. In the proceedings of the Conference numerous papers concerning strength and stability problems can be found. Of these the English contributions formulating and testing in practice the so-called "$\phi = 0$ analysis" deserve special notice.

In June 1950 a "shear strength conference" was arranged in London and in a number of papers the methods of measuring the shear strength as practised in the different European countries were presented and compared. These papers and discussions are printed in Geotechnique (Dec. 1950, June 1951).

The principle of effective stress has made possible the application of rational principles to many soil engineering problems that could not be treated in an empirical manner. The relationship between the behaviour of soil tested under undrained condition in triaxial test and the strength characteristics expressed in terms of effective stress depends on the magnitude of pore pressures set up in the test. To obtain a clear picture of how pore pressures respond to the different combinations of applied stress, the concept of pore pressure parameters introduced by Skempton (77) (1954) is found to be convenient. This concept serves not only to explain the relationship between different types of triaxial test, but also provide a basis for estimating pore pressures to be encountered in practical problems.

For soils with single fluid, either water or air the equation $\sigma' = \sigma - u$ is true to a high degree. However, in partly saturated soils the pore space contains water and air, at pressures which differ considerably due to surface tension. A tentative expression $\overline{\sigma} = (\sigma - u_a) + X(u_a - u_w)$ in general case was put forward by Bishop (5) in 1954, where u_a denotes the pressure in air in vapour phase and u_w denotes pressure in pore water. The work of Hilf (1956) (34) in particular drew attention to the large difference between pore water and pore air pressures encountered in soils compacted on dry side of optimum water content and led to the suggestion that the effective cohesion intercept measured in conventional drained, undrained tests on such materials might be largely accounted for by this difference. Developments in the triaxial apparatus at Imperial College have enabled the necessary experimental data to be obtained.

In contribution respectively to the discussion at the Conference on pore pressure and suction in soil and to the ASCE shear research conference Colemann (1960) (21) and Lambe (1960)(46) derived the equation of effective stress put forward by Bishop from considerations of thermodynamics on the one hand and a physical model on the other. The relationship between X and S_r the degree of saturation obtained from tests by Donald (1961) and tests by Blight (1961) (8) on compacted soils also show the dependence of X on S_r th~~r~~ough simple common relationship for all soil types is not apparent at this stage.

The factors that influence soil strength viewed on microscopic molecular and atomic scales referred to by the term soil structure and application of such concepts to the solutions of Engineering Problems was initiated by Lamb (1959) (45).

Two Conferences on Shear Strength of Cohesive Soils (1960) and ASTM Strength of Soils (1963) point up the gaps in the knowledge in the field. Notable articles are "Failure hypotheses for soils" by Newmark (58) and "A mechanistic picture of shear strength in clay" by T.W. Lambe (47) among other articles by the pioneers in the field of soil mechanics.

The progress of work in the line between 1960-66 is summarized at the Sixth International Conference on soil mechanics.

Laboratory techniques have continued to improve and new developments have been reported in the design of both triaxial and direct shear apparatuses. Analysis of testing such as the effects of non-uniformity and distribution of both effective stresses and pore pressures and details of the influences derived from stress history, stress paths and rate of strain were made by various techniques of testing. Design of apparatuses for testing soils under high pressures, new devices for pore pressure measurements and methods for testing under anisotropic states of stress are probably the subjects that received more attention. Particularly important progress was made in the design and construction in Mexico and Germany of triaxial equipment to handle very large samples under pressure comparable to those existing in high earth and rock-fill dams and the developments in England of a method to eliminate friction at the end caps restraining the sample in triaxial tests.

Fundamental studies of the shear strength behaviour of soils were continued. Some of them pursue a general theory which would give the mechanism of the ultimate shear strength behaviour for remoulded clays.

Other studies refer to methods of obtaining the components of shear strength of clays, either based on Hvorshev's criteria or supported by new ideas. In addition, investigations were made of frictional characteristics of soil minerals and of shear strength of chemically purified clay minerals.

The representativeness of laboratory undrained testing of samples of saturated clays was the subject of two important papers published in 1963, one by Skempton and Sowa (81) in Geotechnique and the other by Ladd and Lambe (42) in the proceedings of the Ottawa Symposium on laboratory shear testing of soils. In them analysis were made of the reliability of unconsolidated undrained test results obtained from undisturbed samples in relation to shear strength of soil in the ground arriving at somewhat conflicting conclusions, as to the importance on the obtained laboratory strength, of soil disturbance coming from stress change produced by sampling. The effect of varying the stress path and especially that produced by reaching failure under so-called anisotropic state of stress, with particular reference to plane strain occupied the attention of many investigators. The study of this problem seems to have made a good start.

Shear strength behaviour of soils under high confining pressure has been one of the favourite subjects of research. Great progress was made on the study of long term strength of overconsolidated fissured clays particularly through the fourth Rankine lecture which elucidated the mechanism of the problem and supported results with a large number of case records.

Very little was published, by contrast on the strength characteristics of non-saturated soils, an important problem in soil mechanics practice, about which the present knowledge is limited. The few papers published on the subject refer mainly to the application of effective stress to such soils, but practically no progress was made on representative testing in relation to field soil behaviour.

In the proceedings of the Sixth International Conference on soil mechanics and foundation engineering it was concluded under the chairmanship of Prof. Rosenquist that research on physico-chemical aspects of soil engineering has been of much value to give a mechanistic understanding of what is happening when phase changes occur in the solids, gas, water system which will help in understanding and solving practical problem.

A description of the different kinds of equipment commonly used in soil engineering practice and research was given by Sowers (1963) (82). The advantages and disadvantages of each type of equipment were discussed whereas most are strain controlled apparatuses, each is capable of straining a soil sample whose shape is different in each type of test.

3 DEFINITIONS AND BASIC CONCEPTS

3.1 Cohesive Soils

Cohesive soils are those which contain sufficient qualities of silt or clay to affect significantly their engineering properties. Such soils vary in texture from pure clay and silts (grain size smaller than 0.074 mm) to mixtures containing more than 75% by weight of sand and gravel sizes. The fine grained components of soil exhibit to various degrees, the property of plasticity within certain ranges of moisture content.

The highest water content that a soil may have without flowing when jarred in a standard device is called the 'liquid limit'. The lowest water content at which a soil can be rolled into threads of 1/8" Ø without crumbling is called 'plastic limit'. The difference between those two water contents is known as 'plasticity index' which represents the range of water content within which the soil is plastic. The liquid limit of a soil and its plasticity index were used by Casagrande (16) to differentiate between silts and clays and between types of silts and clays.

An important characteristic of cohesive soils is the fact that compaction improves their engineering properties. Compaction of cohesive soils has been shown to follow the principle stated by Proctor.

All studies of cohesive soils sheared to failure present empirical relationships between various factors at failure whose numerical values hold only for the soil or soils tested. It is postulated in general that the same qualitative relationship hold for other cohesive soils. Frequently other materials will exhibit some differences in their qualitative behaviour also. No relation holding for all soils have been developed on the basis of fundamental parameters such as minerological content and environmental conditions. These basic properties are difficult to measure in the laboratory but to some extent they are indicated by simple experiments such as Atterberg limit tests. Most general relationships have therefore been evaluated in terms of the liquid or plastic limit or plasticity index. Skempton (80) introduced for such purposes the concept of the "activity" of a soil which he defined to be the ratio of the plasticity index to the pecentage by weight of soil particles of diameter smaller than 2 microns. Thus the mineral type determined by plasticity index and the amount of clay fraction present are both recognised and the importance of the contribution of the finer clay mineral particles to the behaviour of a soil is emphasized.

3.2 RHEOTROPY

Many clay soils exhibit the property of rheotropy at water contents above the liquid limit, and also to a lesser degree at water contents in the plastic range. This is the change to a more fluid consistency on stirring or disturbance; when the disturbance has ceased the system reverts to its less fluid or more rigid condition. This is often called thixotropy, although the restricted definition of thixotropy is a reversible, isothermal sol-gel transformation. A sol, by definition has no yield value, while a gel has rigidity. The change in clay-water system is generally from a system with higher yield value to one with a lower yield value. In the engineering sense, one may consider a sol as a colloidal dispersion. This restricts sols to liquid like behaviour. When hardening of the sol occurs, a gel is formed. This requires a change of state from a liquid like substance to a semi-solid.

There are several ways of measuring the rheotropy of clay soils. It can be measured in certain types of shear apparatus, for example the vane shear, or at higher water contents with a viscometer.

A loss in shear strength of clay soils on remoulding is usually observed. If such a soil is tested at increasing intervals of time after remoulding, an increase in strength with time is generally measured. This is illustrated in Fig. 3.1. (Page.24)

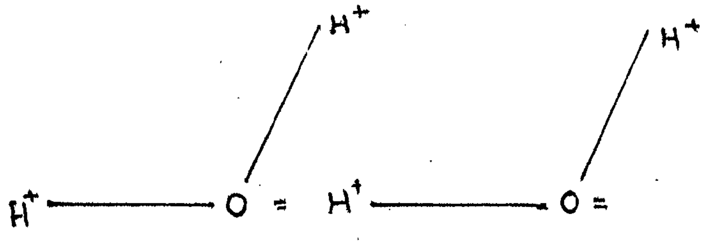


Fig 3.3

The entire undisturbed strength may not be regained. This strength regain has been called "thixotropic regain" but is more properly termed age-hardening or rest hardening. 'Sensitivity' is the ratio of undisturbed to remolded shear strength.

The property of rest hardening has been explained either by changes in particle rearrangement and interparticle forces, or by changes in adsorbed water. On stirring, the particles are rearranged and the bonds between the particles are broken. Also the structure of the adsorbed water is broken up, and the clay will flow. After rest, the particles rearrange themselves into positions of minimum energy with maximum attraction between particles. The adsorbed water also regains its quasi-crystalline form to give the system sufficient rigidity to have a yield value.

3.3 SOIL STRUCTURE

The factors that influence the behaviour of soils viewed on microscopic, molecular and atomic scales, are referred to by the term soil structure. This term includes considerations of the minerological composition and electrical properties of the solid particles, as well as their shape and orientation with respect to each other and properties of soil water and its ionic composition and their absorption complexes.

The concepts of soil structure are concerned primarily with very small particles - about 2µ in size or smaller. The two types of forces between the particles are gravitational and surface boundary forces. Unless the surface area is large compared with the volume, the combined effect of Bonding forces due to area and character of surface will be small when compared with gravitational effects. The nature of surface bonding forces is not understood completely. Certain types of bonding forces are recognized and these will be discussed briefly.

Atoms bonding to atoms forming molecules (intramolecular bonds) are called primary valence bonds. These bonds are sometimes considered in terms of atoms sharing or exchanging their outer election shells. They are sufficiently strong so that they are seldom broken in engineering work.

Atoms in one molecules bonding to atoms in another molecule are called secondary valence bonds. These bonds are of two types:

1. Vander Waal forces
2. Hydrogen bonds

Although a molecule is electrically neutral the centre of gravity of positive and negative charges may not coincide. An electric moment is thus developed, and the system is referred to as being polar. For example Van der Waals forces are generally attributed to electrical or electromagnetic attractions between systems of molecules. The structure of molecule may be polar and attractive Van der Waal's forces will develop depending upon orientation of particles. Many types of orientations are possible with unsymmetrical water molecules. These possibilities for simple dipoles are illustrated in Fig. 3.2.

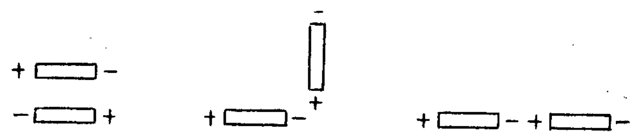


Figure – 3.2

Because all electrons oscillate, the centre of gravity of the negative charges is constantly changing, resulting in the formation of temporary dipoles. The attractive Van der Waals forces resulting from these fluctuating dipoles are referred to as dispersion effects. When a molecule is placed in an electric field, slight displacement of electron occur,causing non-polar molecules to be polarised. The forces resulting from such induced polarization are referred to as induction effects. The relative magnitude of the attractive forces resulting from these effects may be assessed from the following breakdown of energies in the Van der Waals forces between water molecules.

Orientation - 77%

Dispersion - 19%

Induction - 4%

It is apparent that orientation of water molecules has a dominating influence on the Van der Waals attractive forces and that the changes in such orientations will be reflected in important changes in the observed microscopic behaviour.

When an atom of hydrogen is attracted by two other atoms, the hydrogen atom cannot decide with which of the other atom it wants to bond and shares its bond between them, the resulting attractive force is referred to as a hydrogen bond. Hydrogen bonds are effective in orienting water molecules as illustrated.

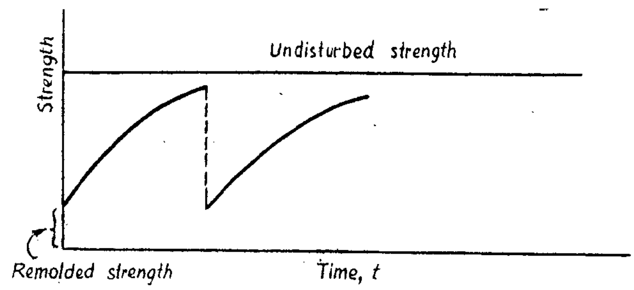


Figure-3·1. Influence of Time of Set on Strength Increase of a Remolded Clay

The orientation of water molecules resulting from hydrogen (or other) bonds into particular patterns has been called water structure. This structure plays a primary role in the behaviour of soil water system.

The attractive forces between electrostatic charges (such as might develop between two small ions or between two charged plates) are referred to as Coulombic forces. Ions orient water molecules, and their radius of (strong) influence is referred to as the hydration radius. They can disrupt the structural pattern of the soil water, thereby altering the soil behaviour to a great extent than can be attributed to the change in the force field between soil particles that they cause.

The variation in attractive force between atoms, molecules and ions is critically dependent on the distance r between them, as evidenced by the following tabulation.

Attractive force	Inversely proportional to
Ion to ion	r^2
Ion to dipole	r^3
Ion to non-polar molecule	r^5
Dipole to non-polar molecule	r^7

Particle spacing is therefore one of the critical factors affecting the engineering properties of clay. For a given type of particle orientation, the void ratio (or water content in saturated soils) adequately reflects particle spacing, but it should be remembered that different orientations can exist at the same void ratio; thus there can be no direct relation between void ratio and particle spacing unless the nature of the particle orientations is defined.

The attractive forces between the charged particles are dependent upon the nature of the medium that separates them. A vacuum is taken as a standard, and the ratio of the attractive force in a vacuum to that in a particular medium is called the dielectric constant of the medium. Bulk water has an unusually high dielectric constant whose value can vary with changes in water structure. Thus changes in ionic concentration and replacement of water with other fluids have important influence on bonding forces between soil particles.

3.4 SOLID PARTICLES IN SOIL

Many investigations have shown that fine grained soils are composed predominantly of crystalline minerals and that the amorphous materials that may be present have little if any effect on the soil behaviour. Although small in size some of the minerals present in fine-grained soils have such low surface activities that they do not contribute appreciable plasticity or cohesion. These minerals are referred to as non-clay minerals. The crystalline minerals whose surface activity is such that they develop cohesion and plasticity are called clay minerals.

The most important clay minerological groups are kaolin, montmorillonite, illite grous which are discussed briefly.

Clay minerals are alumino silicates i.e., oxides of aluminium and silicon with smaller amounts of metal ions substituted within the crystal. The aluminium oxygen and silicon oxygen combinations are the basic structural units which are bonded together in such a way that sheets of each one result. The stacking of these sheets into layers, the bonding between layers, and the substitution of other ions for aluminium and silicon account for the different minerals. This substitution occurs for ions of approximately same size, and is therefore called isomorphous substitution.

The silica unit consists of a silicon atom, Si, surrounded by four oxygen atoms, O, equidistant from the silicon. These oxygen atoms are arranged at the corners of a tetra-hedron with each of the three oxygens at the base of the tetrahedron shared by two silicons of adjacent units. This sharing forms a sheet with the tetra-units packed in such a way that the sheet has hexagonal holes. Since the silicon is smaller than the oxygens, the sheet can be visualized as two layers of oxygen atoms with silicon atoms fitting in the holes between. This sheet has a thickness of 4.93 A (A denotes Angstrom units) in clay minerals. The O-O distance is 2.55 A, leaving a hole within the tetrahedron of 0.55 A radius into which the silicon of 0.5A can fit without distortion. Silicon has a positive valence of 4 and oxygen a negative valence of 2. With each silicon having one oxygen atom and sharing three other oxygents, this unit has a negative charge of 1. When top oxygen takes on a hydrogen with a positive valence of 1 to become hydroxyl, OH, the unit is neutral. Oxygen and hydroxyl have about the same radius in clay minerals.

The alumina unit is an aluminium atom, Al, equidistant from six oxygens or hydroxyl in octahedral co-ordination. Each oxygen is shared by two aluminium ions, forming sheets of two layers of oxygen (or hydroxyl) in close packing, but only two thirds of the possible octahedral centres are occupied by aluminium, This sheet is 5.05 A thick in clay minerals. When all oxygens are hydroxyls, this is called gibbsite, with a chemical formula $Al_2(OH)_6$. If Magnesium, Mg is present in place of aluminium, all the octahedral positions are filled and the mineral is called brucite with the formula $Mg_3(OH)_6$. The radius of aluminium is 0.55 A and of magnesium 0.65 A. The O-O distance in octahedral co-ordination is 2.60 A and OH-OH is 2.94 A, leaving an octahedral space of 0.61 A radius. Clay minerals in which two thirds of octahedral positions are filled are called dioctahedral, when all the positions are filled they are termed trioctahedral.

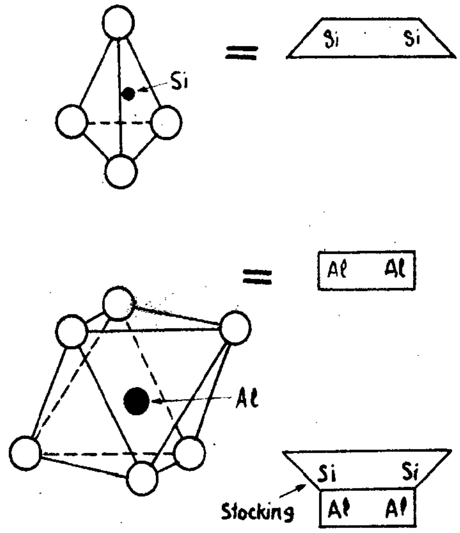


Figure - 3·4 Schematic Representation of Silica Tetrahedron and Alumina Octahedron

Kaolinite: It is the most common mineral of kaolin group. Its basic structural unit is an octahedral sheet (with aluminium as the enclosed atom) with a parallel superimposed silica sheet inter-grown in such a way that the tips of the silica sheet and one of the layers of octahedral sheet form a common layer. This unit can be represented by the symbol - (Fig 3.5)

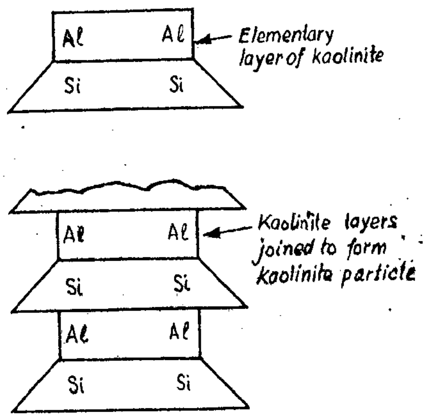


Figure - 3·5 Schematic Representation of Typical Kaolinite Structure

Since hydrogen bonds are comparatively strong, the kaolinite crystal consists of many sheet stackings (often 100 or more) that are difficult to dissociate. The mineral is therefore stable and water cannot enter between the sheets to expand or shrink the unit cell.

Kaolinites are found in soils that have undergone considerable weathering in warm, moist climates. They have a low liquid limit and a low activity.

Halloysite is similar to kaolinite, being composed of the same basic 7A thick structural units, but the successive units are more randomly packed and may be separated by a single molecular layer of water. If the monomolecular layer of water is removed (by drying), the mineral exhibits different properties; hence the properties of fine grained soils containing halloysite will be radically altered if the soil is first dried sufficiently to remove the monomolecular water layer, since the process is not reversible. A further structural feature of halloysite is that the particles appear to take the form of elongated units (tubes or rods) as opposed to the platey shape of kaolinite.

Montmorillonite is the most common mineral of the monmorillonite group. Its basic structural unit is an octahedral sheet sandwiched between two silica sheets as represented by the symbol. The thickness of unit is about 10A, and as in the case of kaolinite the dimensions in the other two directions are indefinite. Isomorphous substitution occurs mainly in the alumina sheet, with magnesium or iron substituting for aluminium in the dioctahedral minerals. The different montmorillonite minerals have different substitutions. Water enters easily between layers.

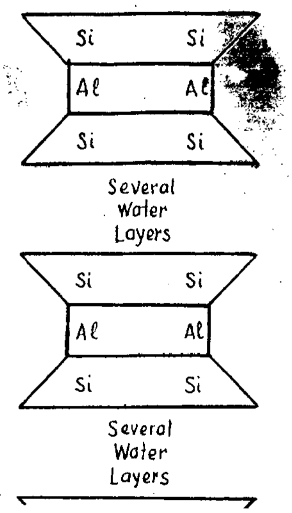


Fig.3.6

The montmorillonite clays have a high activity and high liquid limit. The most obvious characteristic is the swelling of clay to several times its dry volume when placed in contact with water. This swelling is due to water absorbed between layers pushing the layers apart.

Montmorillonites occur in sediments of semi-arid regions and are the main minerals in bentonite rock. They are formed when volcanic ash weathers in marine water or under conditions of restricted drainage.

Illite: Illite is the most common mineral of illite group. The basic structure is similar to that of montmorillonite, except that there is always a substantial replacement of silicon by Aluminium atoms in the silica sheet, resulting in a residual negative charge somewhat larger than that of montmorillonite unit. However, a substantial fraction of this negative charge is balanced by non-exchangeable potassium ions, which provide the primary link between the illite units that form the illite crystal, thus:

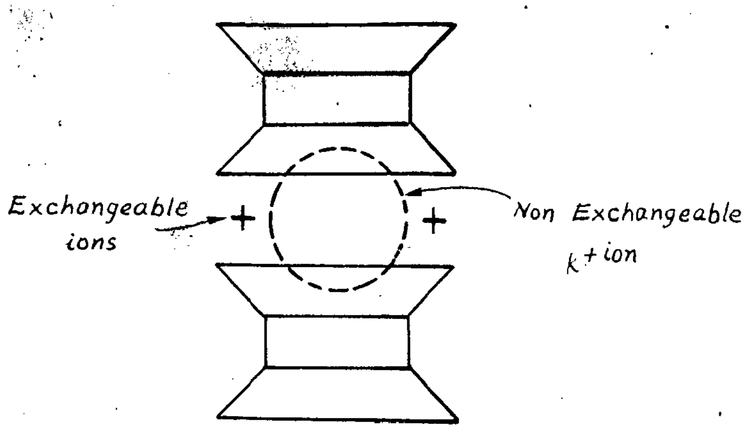


Figure – 3.7

The bonds with the non-exchangeable K^+ ions are weaker than the hydrogen bonds that link the units of the kaolinite crystals, but they are much stronger than the exchangeable ionic bonds that form the monmorillonite crystal. This is due, in part, to the fact that the non-exchangeable K^+ ions just fit in between the silica sheet surfaces and therefore are much closer to the atoms in the sheet than the exchangeable ions. Accordingly, the illite structure does not swell because of the movement of water between the sheets as is the case for montmorillonite.

3.5 ELECTRIC CHARGE

Substitution of one ion for another in the clay crystal lattice and imperfections at the surface, especially at the edges, lead to negative electric charges on clay particles. Cations from the pore water are attracted to the particles to maintain electro-neutrality. These are the exchangeable cations and their number is the cation exchange capacity or the amount of negative charge per unit weight or per unit surface area of clay.

Charge arising in the clay particle from isomorphous substitution of an ion by another of nearly equal size but lower

ence has been discussed already. This occurs during crystal-
ation or formation of the mineral. If the substituting ion has
ower positive valence than the substituted ion, then the lattice
left with a net negative charge. The main substitution formed
aluminium for silicon in the silica sheet, and ions such as
nesium, iron or lithium substituting for aluminium in the
mina sheet.

A second source of electric charge on clay particles is
atisfied valence charges at the edges of particles. These are
erred to as broken bond charges. The clay crystal lattice is
tinuous in two directions, but at the edges there must be broken
ds between oxygen and silicon and between oxygen and aluminium.
amount of this charge per unit weight of clay increases with
ecrease in particle size, because the proportion of edge area
total area is increase charge due to broken bonds would also
se if pieces were broken out of the flat surfaces.

These bromen bonds attract hydrogen or hydroxyl ions from
pore water. The hydrogen can be exchanged for other cations.
term hydrogen ion is used, although H^+ as such does not exist
solution. It is usually referred to as hydronium ion, H^+, H_2O
H_3O^+. The ease with which the hydrogen can be exchanged
reases as the pH of the pore water increases i.e. as the
rogen ion concentration of the pore water decreases. Therefore
charge due to broken bonds increases as pH increases.

The kind and number of exchangeable cations have an
ortant influence on the behaviour of soils. Monovalent cations
as sodium increase the activity of clay, its swelling etc.

3.6 WATER ADSORPTION AT CLAY SURFACES

Clay particles are always hydrated i.e., surrounded by layers of water molecules called adsorbed water. These water molecules should be considered part of the clay surfaces when the behaviour of clay soils is considered. The properties of clays change as the thickness of this hydration shell changes and consequently the engineering characteristics of soils change.

The forces holding water molecules to the clay surface arise both from the water and from the clay. Water is a dipolar molecule, with a separation of centres of positive and negative charges. This means that water will be attracted by the charges on the clay surface. Further the hydrogen ions of water will lead to hydrogen bonding of water molecules to the exposed oxygen atoms of the clay mineral surface. Hence the clay contributes both the negative charge and the oxygen or hydroxyl surface to attract water molecules. Cations in water are always hydrated. Therefore the exchangeable cations held near the negatively charged surface hold some of the water at surface as water of hydration of ions. Since cations are hydrated to different degrees, the hydration of the surface would vary depending upon the cation present.

The main force bonding water to the surface is due to the hydrogen bond. The first layer of water molecules is held to the 1st again by hydrogen bonding, but the force become weaker with distance as the orienting influence of the surface of the water molecules decreases. Each of the successive layer is held less strongly, and the bonding quickly reduces to that of free water.

The properties of this water close to the clay surface differ from those of free water. The density of adsorbed water is higher than that of free water. Values Vp to 1.4 g/c.c. have been measured for the 1st layer of water molecules. The density decreases as further layers are added, dropping to 0.97 g/c.c. at about four water layers and then increases to 1.0 g/c.c. for free water. The viscosity of this water, as measured by the diffusion of ions near the surface, is greater than that of free water. In the 1st water layers it may be a hundred times greater. The dielectric constant near the surface is about one tenth that of free water.

Water adsorbed on the clay mineral surface can best be visualized as composed of water molecules which are relatively free to move in the two directions parallel to the clay surface but are restricted in their movement perpendicular to, or away from the surface. Movement parallel to the surface is a transfer from one bonded position to another. The thermodynamic properties of this water are not the same as those of ice where movement of water molecule is restricted in all the three directions.

There is lack of agreement as to the thickness of the hydration layer. The forces holding water become gradually weaker farther from the surface. So there is no sharp division between water of hydration and free water. It is agreed that the 1st two of three layers of water molecules are bonded to the surface and that properties of adsorbed water differ from those of free water to distance of 15 A from the surface.

3.7 EXCHANGEABLE CATIONS

These are positively charged ions from salts in the pore water which are attracted to the surface of clay particles to balance the negative charge. They are termed exchangeable because one cation can be readily replaced by another of equal valence, or by two of one-half the valence of original one. The process is called cation exchange or base exchange. The ability of a clay to absorb ions on its surface or edges is called its base exchange capacity which is a function of the mineral structure of clay and the size of the particles.

The exchangeable cations are not all held in a layer right at the clay surface, but are present at some average distance from the surface. The electrical force between negatively charged surface and positively charged ions attracts the cations to the surface, but their thermal energy makes them diffuse away from this space with a high ion concentration. The balance of coulomb electrical attraction and thermal diffusion leads to a diffuse layer of cations with the concentrations highest at the surface and gradually decreasing with distance from the surface. This is often called the diffuse double layer, one layer being the negative charges in the clay crystal or at its surface and the other being the diffuse ion-layers of adjacent particles gives an explanation for the properties of swelling, plasticity and water retention of clays.

The main assumptions in the derivation of the theoretical distribution in a diffuse layer are that the clay particle can be considered a simple charged plate for which the electric field is described by Poissons equation, and thatthe distribution of ions in this field is described by the Boltzman equation.

The theoretical distribution of ions at a negatively charged surface was worked out by Gouy (28) and later by Chapman (20).

The resulting equation for cation is:-

$$n_+ = n_o\left(\coth \frac{x}{2} \sqrt{\frac{8\pi e^2 z^2 C_o N}{kT}}\right)^2$$

$$= n_o(\coth 0.16\, z \sqrt{C_o}\, x)^2$$

where

n_+ = No. of cations per unit volume at any distance x from the surface

n_o = No. of cations/unit volume in the pore water away from the influence of the surface.

z = Valence of cations

C_o = Concentration of cations in moles/litre away from the influence of the surface.

x = distance from the surface in Angstrom.

The ion distribution for interacting clay particles rather than that around a single particle is of interest in interpreting the soil behaviour. The solution of Langmuir for such cases is

$$Y_c = 2 \log_e \frac{\pi}{0.32\, z\sqrt{C_o}\, x}$$

where

Y_c = electric potential in the line midway between parallel interacting particles

Since $\quad Y_c = \log_e \dfrac{n_c}{n_o}$

$$n_c = n_o \left(\frac{\pi}{0.32\, z\sqrt{C_o}\, x}\right)^2$$

where n_c = No. of ions per unit volume at the mid plane between particles.

Schematic diagram of clay water with adsorbed water layer and some cations at the surface and the remainder of the exchangeable cations. (Fig 3.8)

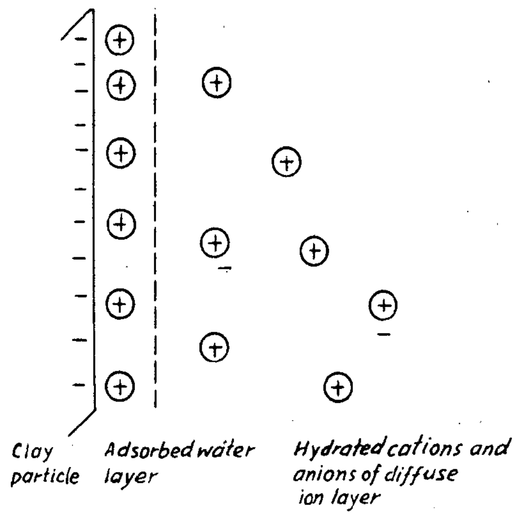


Figure - 3.8. Schematic Diagram of Clay Particle with Adsorbed Water Layer and Some Cations at the Surface, and the Remainder of the Exchangeable Cations in the Diffuse Ion-Layer

These calculations of electric potential and resulting ion distribution are not quantitatively applicable to most clay soils. An improvement is made by applying stern correction, by which some of the ions are held in a layer near the surface and the remainder are in the diffuse layer.

<u>Interaction of clay particles:</u> Clay particles interact through the layers of adsorbed water, through the diffuse layers of exchangeable cations and in some cases through direct particle contact.

Repulsion results from inter-penetration of diffuse ion-layers of adjacent particles, and from adsorption of water on surfaces of adjacent particles. When two clay particles are less than 15 A apart, the exchangeable ions are uniformly distributed in the inter-particle space and do not separate into two diffuse layers, one associated with each surface. Under these conditions there is a net attraction between particles. However, when the inter-particle distance exceeds about 15A, diffuse ion layers form, with a resulting net repulsion. This repulsion can again be visualized as being due to water attracted between the particles forcing them apart. In this case the water moves due to osmotic activity of the ions between particles rather than due to properties of surface.

Several different forces must be considered in describing attraction between clay particles. First there is attraction between molecules and atoms described by London-Van der Waals theory.

At inter-particle distances less than 15A there is a net force of attraction between clay particles when their exchangeable cations are in the inter-particle space.

Under certain conditions electrical forces inversely proportional to the square of the distance acts between particles of clay.

Cementation bonds may be due to inorganic bonding materials such as carbonates or oxides precipitated between particles. In natural soils these may be quite common. Bonding from organic matter can arise between the negative charge on a clay particle surface and the positive charge on the organic matter. In partially saturated soils, surface tension forces arising from the curved air water interface can hold particles together.

3.8 SEDIMENTATION PROCESS AND RESULTING STRUCTURE

Clay particles of colloidal size are subject to chance impacts by water molecules, which cause the clay platelets to move randomly in the suspension (Brownian movement). Such random movements, together with grosser fluctuations due to water currents, will from time to time bring clay particles together, to distances within the range of inter-particle forces.

If the net force is repulsive, the particles will be kept separate and future random movements will separate them even more. The process which takes place under these conditions during sedimentation is known as dispersion and the soil so produced is called dispersed soil.

If a small quantity of electrolyte is added, the chance approach of particles in suspension may bring them together, so the attractive forces tend to bring them still closer. In this case the force of attraction increases as the distance diminishes, so that the end result of the chance encounter will be co-agulation of two particles.

Rosenquist (1959) (66) has reviewed the development of particle orientation concepts. The different types of structures are:

(a) Random

(b) Oriented

(c) Flocculated

Natural clay sediments will have more or less flocculated particle orientation depending upon whether they were deposited in a fresh water environment. Marine clays generally have a more open

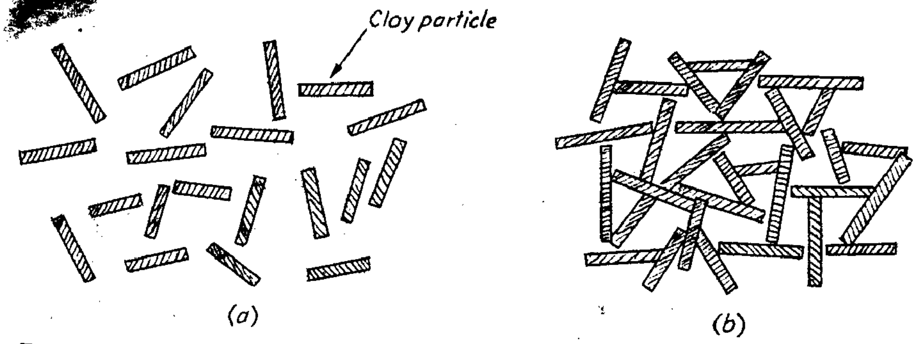


Figure-3·9. Idealized Clay Structures. (a) Random Structure (b) Flocculated Structure

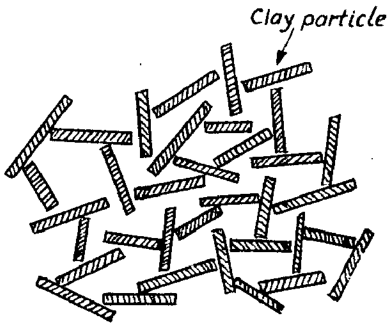


Figure-3·10. Typical Edge-to-Surface Flocculated Structure Characteristic of Seawater Deposition

structure than fresh water clays. Actually the particles are arranged in more random three dimensional orientations. Photomicrographs obtained with an electron microscope have demonstrated visually that particles in Norwegian Marine clays do actually have the orientations visualized by Tan (1957) (83) and that the schematic diagrams proposed by Lambe (1953) (43) are a general valid general concept of particle arrangement.

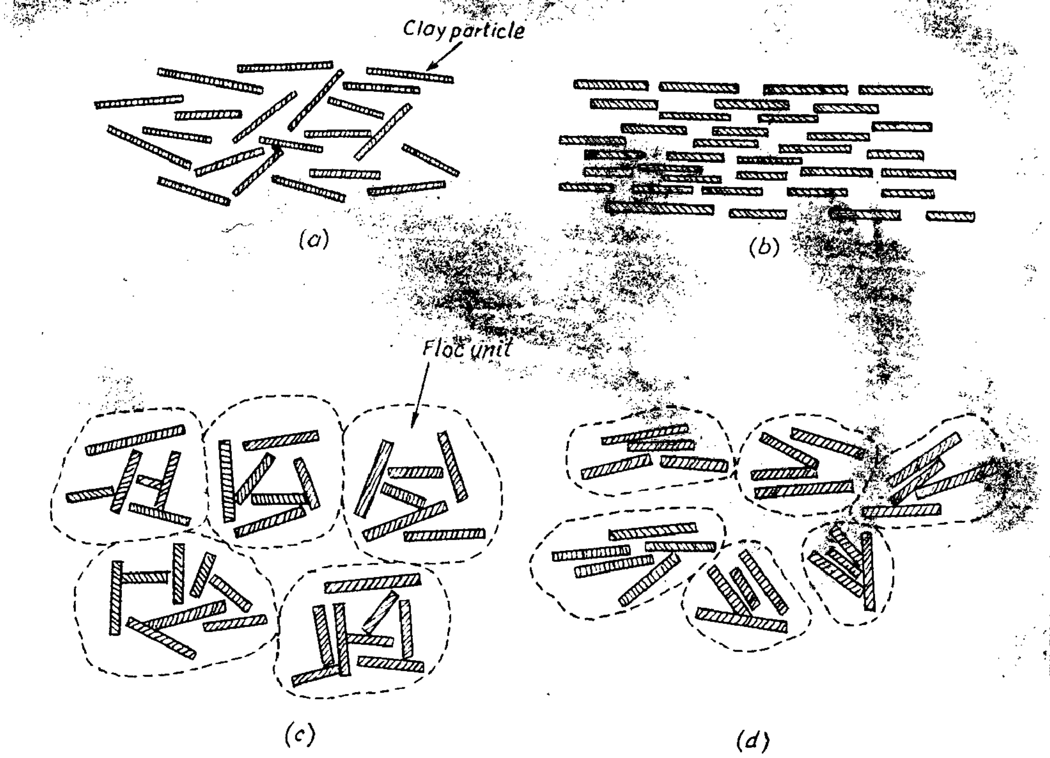


Figure-3.9. Idealized Clay Structures. (a) Partially-Oriented Structure (b) Fully-Oriented Structure (c) Floc Units in Random Orientation (d) Packet Flocs

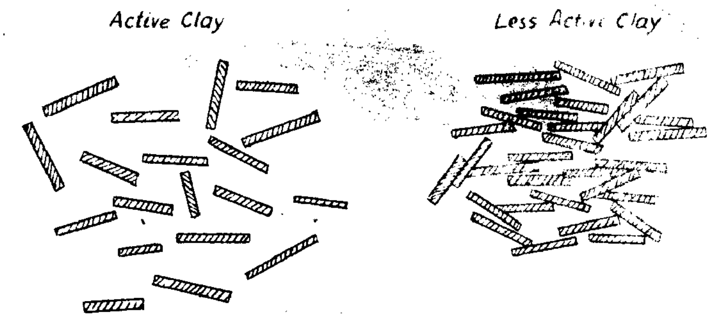


Figure-3.10. Structure of Clay Deposited in Fresh Water

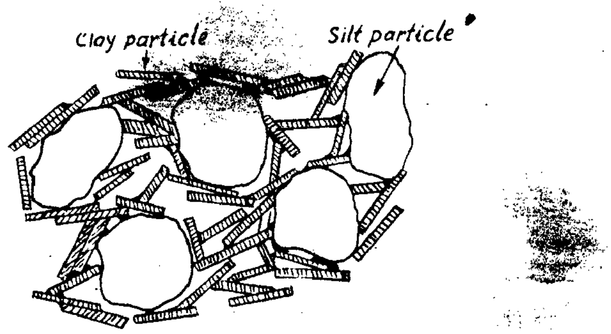


Figure-3.10. Fresh-Water Deposition of Clay and Silt Particles

Consolidation tends to orient particles into the dispersed arrangement; in the case of one dimensional consolidation, all the particles would eventually orient themselves into substantially parallel planes (Hvorslev, 1938 (35); Lambe, 1958 (44)). Remoulding results in clusters of parallel-oriented particles arranged at random which is similar to the arrangement suggested by Michaels for disturbed dry clay.

Shear strains also tend to arrange particles in the dispersed type of orientation (Seed and Chan 1959)(73). These concepts of particle orientations will be of considerable utility for later explanations of the phenomenon involved in strength characteristics of clay.

When a clay is compacted at water contents somewhat lower than the optimum water content, a randomly oriented structure is obtained. Increasing the molding water will reduce the randomness in particle arrangement. If one compacts at water contents above optimum the compacted clay soils would have a partial oriented structure. The greater the molding water content the more oriented the structure would be. The arrangement of particles as influenced by molding water content ~~and water content~~ and Compaction is shown in Fig. 3.11.

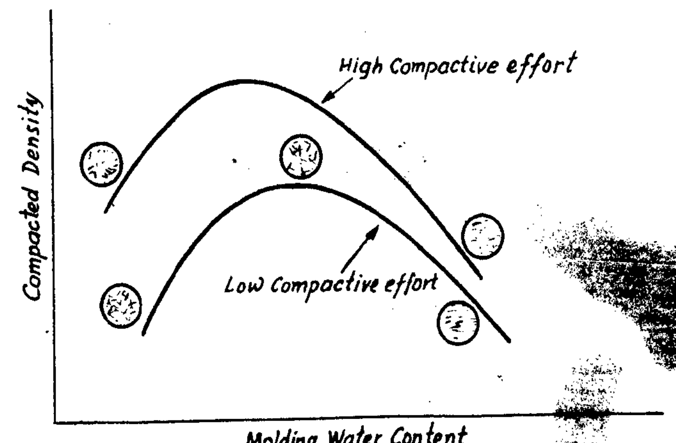


Figure - 3.11. Arrangement of Particles as Influenced by Compaction and Molding Water Content (from Lambe, 1960)

There is general agreement among all the investigators on soil structure relative to the basic soil structural models. The exact terminology and definition of the types and nature of the bonds involved may differ between individuals, but in essence the models remain the same. Notable among these is Rosenquists (66) concept of clay water structure. He is in close agreement with Lambe (44), Mitchell (54) and Michaels (52) as to the arrangement of clay particles. However, the nature of forces between clay particles proposed by Rosenquist differs to some degree. The explanation suggested by Rosenquist for adhesion between clay particles is based upon the difference in surface energy of the adsorbed water and the liquid pore water. Thus the establishment of inter-facial tension between the two types of water could be the cause for the cohesion observed in saturated clays.

The concepts of Trollope and Chan (88) are similar to those of Lambe. Their proposal of a card house structure is based upon the establishment of equilibrium between adjacent particles shown schematically in Fig. 3.12.

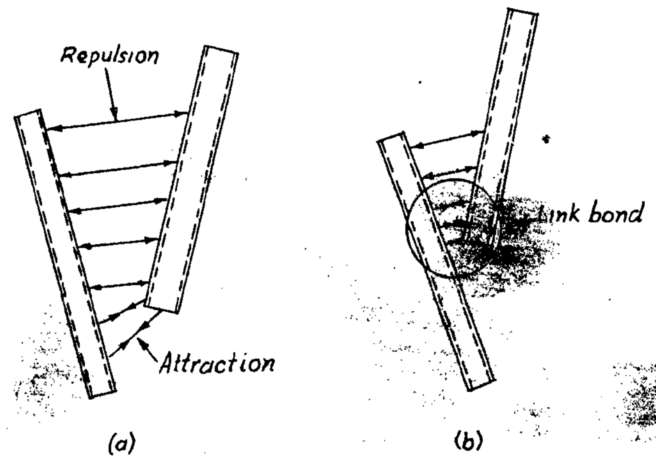


Figure-3.12. (a) Equilibrium Arrangement of Clay Particles in Idealized Model; (b) Formation of Link Bond Due to Edge-Surface Proximity (Trollope and Chan, 1959)

3.9 PORE WATER PRESSURE

Components - Fully Saturated Soil

The calculation of total and component water potentials has been discussed by Bolt and Miller (14). The total pore water pressure U may be broken down into individual components as follows:

(a) U_p = Pore water pressure resulting from reaction to applied external pressure

(b) U_π = Pore water pressure arising from forces originating from the dissolved solutes in the liquid phase i.e. excluding osmotic pressure arising from the concentration of exchangeable ions

(c) U_h = Pore water pressure arising due to the position of soil mass in the gravitational field

(d) U_a = Pressure in the pore water arising from forces originating in the particles. This includes adsorptive forces holding the 1st few layers of water. Surface tension forces holding water in coarse grained soils and osmotic forces from exchangeable cations in swelling fine grained soils.

(e) U_f = The hydrostatic pore water pressure which could result from flow in the water under external head in water itself.

In many instances, it is not possible to differentiate between all these component pore water pressures nor to evaluate the separate components

$$U = U_p + U_\pi + U_h + U_a + U_f$$

The component U_a is dependent upon structure, exchangeable ions and orientation of particles in the soil mass. The components U_π and U_a may be incorporated into a component pore pressure term called U_{cf} which is resulting from total soil structure.

In high swelling soils, the component U_{cf} is related to swelling pressure of the test sample and U_{cf} is thus a negative quantity. The amount of pressure required to sustain constant volume for a high swelling soil in the presence of available water represents the force or pressure that must be exerted on the available water to keep it from being drawn into the soil sample.

In non-swelling or low swelling clays pore pressures equal applied all round pressures if the sample is completely saturated. For saturated high swelling clays, the net pore pressure will be less than the applied confining pressure if consolidation is incomplete.

Partly Saturated System

Consider a random arrangement of particles. In the point located in the free water region, the pore pressure components are

$$U_1 = U_{\pi 1} + U_{h1} + U_p$$

where U_1 = total pressure

$U_{h1} = h\, r_w$

$U_{\pi 1}$ = osmotic pressure.

A point just outside a curved air water interface outside the range of the bound water layers surrounding the clay particle

$$U_2 = U_{h2} + U_{\pi 2} = (U_{air} - \frac{2T}{r_2}) + U_{\pi 2}$$

where U_{air} = air pressure in voids

T = Surface tension (considered +ve)

r_2 = radius of curvature of the meniscus at the point

Since both points considered are out of the range of the forces originating in the particles and forces originating from the dissolved solutes in the fluid phase, the ionic concentration at these points may be taken to be equal. $U_{\pi 1} = U_{\pi 2}$ and since adsorptive forces are zero, $U_{\pi 1} = U_{\pi 2} = 0$.

At equilibrium $U_1 = U_2$

$$U_{h1} + U_p = U_{h2} = U_{air} - \frac{2T}{r_2}$$

Hence

$$U_{h1} = U_{air} - \frac{2T}{r_2} - U_p$$

Consider a point which lies just outside the curved air water interface but is within the diffuse ion layer

$$U_3 = U_{air} - \frac{2T}{r_3} + U_{\pi 3} + U_{a3}$$

U_a may be taken to be zero as a 1st approximation.

At the interface $U_{a3} = 0$

But at equilibrium

$$U_1 = U_2 = U_3$$

Hence $$U_{h1} + U_p = U_{air} - \frac{2T}{r_2} = U_{air} - \frac{2T}{r_3} + U_{\pi 3}$$

and thus $$U_{\pi 3} = 2T\left(\frac{1}{r_3} - \frac{1}{r_2}\right)$$

This indicates that at equilibrium, the osmotic pressure component at a point within the diffuse ion layer must equal the difference in hydrostatic components due to different curvatures of menisci between the point within the diffuse ion layer and a point in the bulk or free water.

The curvature and ion concentration at any point are governed by factors such as arrangement of particles, Size pore spaces, degree of saturation structure, etc. The free energy of the system at equilibrium is at a minimum. For partly saturated soils, the capillary and osmotice pressure effects will, in all probability, vary in equal but unlike manner from one point to another in the entire system in order to maintain equilibrium.

3.10 PRINCIPLE OF EFFECTIVE STRESS

Classical soil mechanics has been concerned chiefly with soil in which voids are completely filled with water. It was for such soils that Terzaghi formulated his principle of effective stress. It is true to say that the principle of effective stress lies at the foundation of most modern soil mechanics theory and practice. The fundamental role played by principle of effective stress in the prediction of behaviour of saturated soil makes its extension to partly saturated soils attractive and important.

The principle of effective stress as stated by Terzaghi (1936) (86) consists of two statements:

1. In saturated soils the effective stress $\bar{\sigma}$, which has "its seat exclusively in the solid phase of the soil " is equal to the total stress σ minus the pore pressure U

$$\text{i.e.,} \quad \bar{\sigma} = \sigma - U. \qquad \text{---} \quad (1)$$

2. All the measurable effects of a change of stress, such as compression, distortion, and a change of shearing resistance are exclusively due to change in effective stress.

The principle since its enunciation has been the subject of intensive as well as extensive study. It is now recognized that various other factors such as void ratio, stress history, soil structure, etc., also influence the engineering behaviour of soil. The reluctance to tamper with the concept of effective stress is because it continues to be "the best available parameter to express certain aspects of soil behaviour notably compression and strength [Lambe (1962)] (48) and because its intrinsic simplicity makes it amenable to use in solving engineering problems.

For saturated soils a physical interpretation of the stress which is equal to the difference between the total stress and the pore water pressure has been developed by considering the equilibrium of the forces across a "wavy plane" through the soil. The "wavy plane" is such that it does not cut across any soil grain. The stress difference ($\sigma - U$), can then be to be equal to the force transmitted at grain contacts per unit area of "wavy plane", and is often termed the inter-granular stress.

$$\sigma_{ig} = \sigma - U \qquad .. (2)$$

The equations (1) and (2) show the algebraic equivalency of inter-granular stress and effective stress, which has led, unfortunately to the terms often being used interchangeably. It is important to note the distinction between the two. Inter-granular stress originates from the force transmitted at grain contacts. Effective stress is the stress which controls soil behaviour. In soil systems that are fully saturated the numerical value of the effective stress is equal to the value of the inter-granular stress except as indicated by Skempton (1960) (78). In soil systems that are only partially saturated the effective stress may or may not be equal to the inter-granular stress.

Partially saturated soil is characterised by the presence of all the three phases - solid, liquid and gaseous. It is convenient to consider three stages of partial saturation in a soil.

The 1st is at high degree of saturation when air exists in the form of occluded bubbles in the pore water and the air water menisci envelop the soil by forming on the outer extremities of the soil sample, hereafter referred to as enveloped stage. The second: at low degree of saturation, when air in the pore space exists in interconnected channels, when water exists in lenses around particles contacts and the air water menisci form around particle contacts - referred to as Lenticular stage. The third: the transition stage between the two. The physics of the equilibrium condition between the liquid and gaseous phases has been discussed at length by Hilf (1956) (34). He observed that

1. As a consequence of the surface tension at the air water interface, the pressure in the water is lower than the pressure in the air, and

2. At equilibrium the air and water pressures, though different, are constant throughout a soil sample.

In one of the very 1st attempts to extend the effective stress principle to partially saturated soils Hilf (1956) (34) suggested that the behaviour of partially saturated soil is a function of the stress as given by equation 1. With the recognition that the value of the pore water pressure may be negative Members on the Staff of U.S.B.R. seemingly continue to hold this view (Gibbs, Hilf, Holtz, Walker (1960) (27)),(Gibbs 1963 (26)).

Bishop (1959) (6) suggested that equation 1 ought to be modified since "a point will be reached (as the degree of saturation is reduced) when the soil particle will cease to be surrounded by the liquid phase. The pressure in the liquid phase will then act only over a reduced area. The effective stress equation may then be written

$$\overline{\sigma} = \sigma - U_a - X(U_w - U_a) \qquad .. (3)$$

where U_a denotes the pressure in the gaseous phase

U_w denotes the pressure in the liquid phase

and X denotes a parameter which equals unity for saturated soils and decrease with degree of saturation".

In a later paper (Bishop, Alpen, Blight, Donald 1960 (8)) a method to determine the value of the parameter X was presented.

Equation (2) is an equation of statics for saturated soil. Lambe (1960) presented an equation of statics for the most generalised soil system which took into account the presence of two phases in the voids, as well as the physico-chemical forces around soil particles. In a discussion (Lambe 1960 (46)) he reduced the generalized equation to the following form for partly saturated soils.

$$\sigma_{ig} = \sigma - U_a(1 - a_w) - U_w a_w \qquad .. (4)$$

where a_w is the solid liquid and liquid liquid contact area per unit area of wavy plane and demonstrated the algebraic equivalency of X in eqn. (3) and a_w in eqn. (4). It is suggested that the stress as given by the equation (4) is the effective stress for partly saturated soil. It is not possible to evaluate the value of a_w with the present state of knowledge and has a range from 0.0 to 1.0 for partially saturated soil. Consequently one cannot yet test whether the stress as defined in equation (4) is effective in controlling the soil behaviour.

Aitchison (1956) (1) noted that the inter-granular stress in partially saturated soils has two components; one which results from applied pressure and the other which originates as a consequence of the tension in pore water.

$$\sigma_{ig} = (\sigma - U_a) + \psi(U_a - U_w) \qquad .. (5)$$

where ψ is the parameter which transforms the negative pore water pressure to its contribution to the intergranular stress (ψ is algebraically equivalent to X and a_w).

Some data is available which indicates that the stress computed from eqn. (5) is indeed the stress which controls volume change and shear strength (Aitchison and Donald, 1956) (2).

The effective stress for partly saturated soils can be written as

$$\bar{\sigma} = \sigma - U^*$$

where U* is the equivalent pore pressure and is = $XU_w - (1-X)\,U_a$. If the soil is saturated, X = 1 and the Terzaghi law results. Thus the effective stress σ' in a partly saturated soil is defined as the excess of the total applied stress σ over the equivalent pore pressure U*. This definition is in close agreement with Skempton's who stated that an effective stress is that controlling changes in volume of soil.

To date most authors have tacitly assumed that the principle of effective stress is valid for partly saturated soils.

3.11 BASIC CONCEPTS OF STRESS AND STRAIN

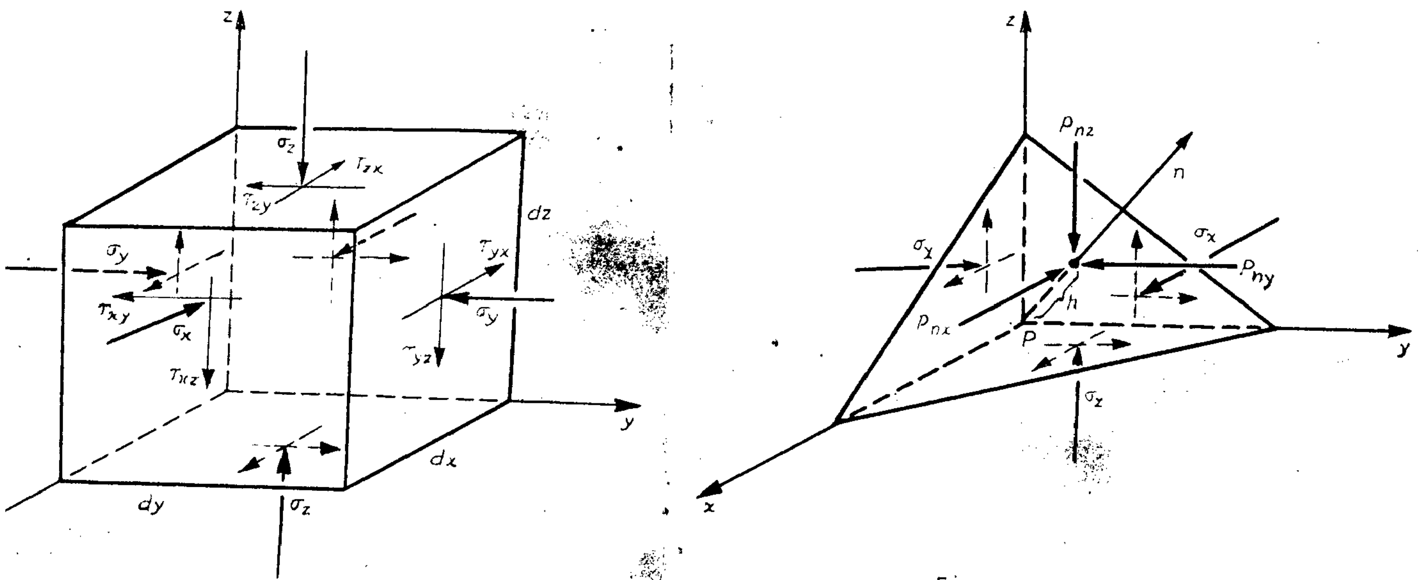


Figure – 3.13.

For most conditions of interest in soil mechanics, loadings are compressive, hence, it is expedient to take normal stresses as positive if so directed. The 1st subscript of shear stresses denotes the direction of a normal stress to the plane on which it acts. A shear stress is considered positive if it is directed (second subscript) in the same sense of co-ordinate direction as the normal stress in its plane. The defined positive, normal and shear stresses acting on the surfaces of an elemental cube at a point P within a soil mass are shown. (Fig 3.13)

The components of resultant stress P_n on the inclined plane with a directed normal n are: (Ref. Fig. 3.13)

$$P_{nx} = \sigma_x \cos(n,x) + \tau_{yx} \cos(n,y) + \tau_{zx} \cos(n,z)$$

$$P_{ny} = \tau_{xy} \cos(n,x) + \sigma_y \cos(n,y) + \tau_{xy} \cos(n,z)$$

EQN. 1

$$P_{nz} = \tau_{xz} \cos(n,x) + \tau_{yz} \cos(n,y) + \sigma_z \cos(n,z)$$

or in the matrix form

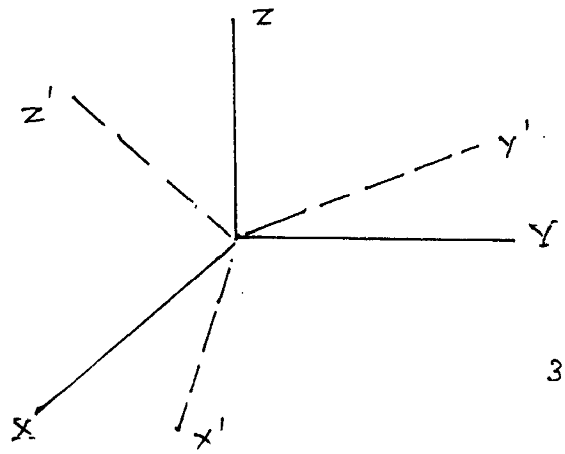
$$\begin{bmatrix} P_{nx} \\ P_{ny} \\ P_{nz} \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \begin{bmatrix} \cos(n,x) \\ \cos(n,y) \\ \cos(n,z) \end{bmatrix}$$

where $\cos(n,x)$ is the cosine of the angle between n and x direction etc.

3.12 GENERAL LAW OF TRANSFORMATION FROM ONE SYSTEM OF CO-ORDINATES TO THE OTHER

i.e. P_x, P_y, P_z of P are known w.r.t. x, y, z

To obtain components of P w.r.t. x', y', z'



3.14 (a)

Table of direction cosines

	x	y	z	x	y	z
x'	a_{11}	a_{12}	a_{13}	cos(x',x)	cos(x',y)	cos(x',z)
y'	a_{21}	a_{22}	a_{23}	cos(y',x)	cos(y',y)	cos(y',z)
z'	a_{31}	a_{32}	a_{33}	cos(z',x)	cos(z',y)	cos(z',z)

Components of P in x' direction i.e., $P_{x'}$ are

$$\begin{bmatrix} P_{x'x} \\ P_{x'y} \\ P_{x'z} \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \end{bmatrix}$$

Components $P_{y'x'}$, $P_{y'y'}$, $P_{y'z}$ of P_y

and $P_{z'x'}$, $P_{z'y'}$, $P_{z'z}$ of P_z

can be written by substituting direction cosines of a_{21}, a_{22}, a_{23} and a_{31}, a_{32}, a_{33} respectively. Now noting that the components of $P_{x'}$ in the direction of x' are the normal stress $\sigma_{x'}$ and that the components in the y' and z' directions are the respective shear stresses $\tau_{x'y'}$ and $\tau_{x'z'}$, we find

$$\begin{bmatrix} \sigma_{x'} \\ \tau_{x'y'} \\ \tau_{x'z'} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{33} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} P_{x'x} \\ P_{x'y} \\ P_{x'z} \end{bmatrix}$$

Repeating for $P_{y'}$ and $P_{z'}$ and combining their equivalents we obtain

$$\begin{bmatrix} \sigma_{x'} & \tau_{y'x'} & \tau_{z'x'} \\ \tau_{x'y'} & \sigma_{y'} & \tau_{z'y'} \\ \tau_{x'z'} & \tau_{y'z'} & \sigma_{z'} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

or $S_1 = ASA^T$

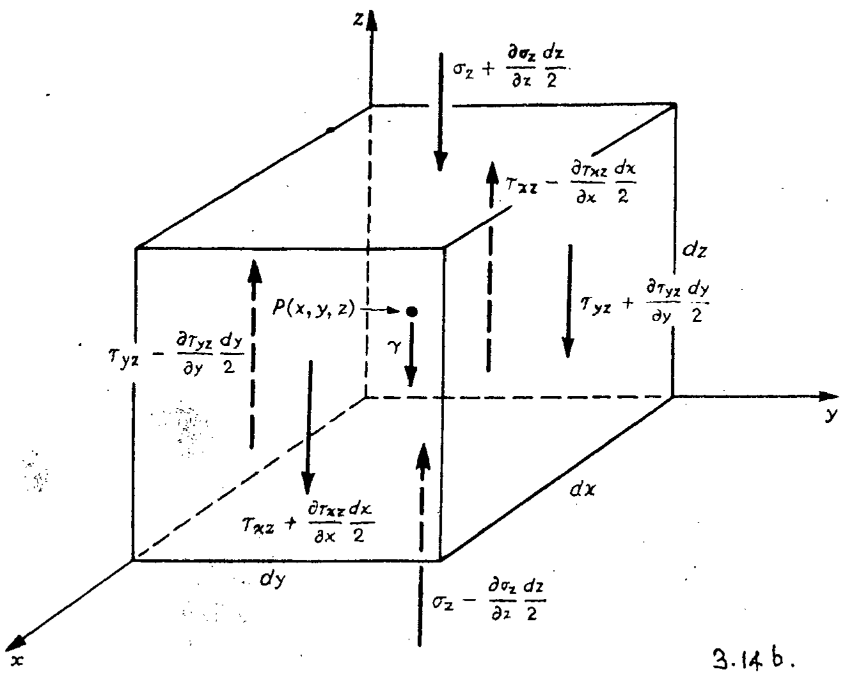
where S is the stress matrix w.r.t. x,y,z co-ordinate system

S_1 is the stress matrix w.r.t. x',y',z' co-ordinate system

A is the direction consie matrix

A^T is its transpose.

3.12 EQUATIONS OF EQUILIBRIUM-EQUATION OF MOTION



3.14 b.

Fig. 3.14(b) shows the variation of stresses acting on the sides of a cubical element that contribute to its equilibrium in the Z-direction. Summing the forces in this direction, we find the third of the following relations. The 1st and 2nd are obtained by similar arguments in the x and y directions.

$$\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + \frac{\partial\tau_{zx}}{\partial z} = 0 = \left(\frac{r}{g}\ \frac{\partial^2 u}{\partial t^2}\right)$$

$$\frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\sigma_y}{\partial y} + \frac{\partial\tau_{zy}}{} = 0 = \left(\frac{r}{g}\frac{\partial^2 v}{\partial t^2}\right) \qquad ..\ (2)$$

$$\frac{\partial\tau_{xz}}{\partial x} + \frac{\partial\tau_{yz}}{\partial y} + \frac{\partial\sigma_z}{\partial z} + r = 0 = \left(\frac{r}{g}\ \frac{\partial w}{\partial t^2}\right)$$

Equations (2) are called the equations of equilibrium. If the inertia forces (u,v,w are the component displacement of point P in x,y,z, directions respectively) cannot be neglected, the above equation with the R.H.S. as in the parenthesis is called the equations of motion.

The equation of equilibrium taking seepage pressures into consideration can be written as

$$\frac{\partial \bar{\sigma}_x}{\partial x} + \frac{\sigma \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + (r_w \frac{\partial h}{\partial x}) = 0$$

$$\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \bar{\sigma}_y}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + (r_w \frac{\partial h}{\partial y}) = 0$$

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \bar{\sigma}_z}{\partial z} + (r_w \frac{\partial h}{\partial z} + r') = 0$$

where r' = submerged unit weight.

3.13 STRESS INVARIENTS

A principal stress is defined as the normal stress on a plane on which there is no shear stress.

If σ_n is the principal stress, with n as the direction normal to the plane, the components $P_{nx} = \sigma_n \cos(n,x)$, $P_{ny} = \sigma_n \cos(n,y)$ and $P_{nz} = \sigma_n \cos(n,z)$

∴ Equn. (1) can be written as

$$(\sigma_x - \sigma_n) \cos(n,x) + \tau_{yz} \cos(n,y) + \tau_{xz} \cos(n,z) = 0$$

$$\tau_{xy} \cos(n,x) + (\sigma_y - \sigma_n) \cos(n,y) + \tau_{yz} \cos(n,z) = 0$$

$$\tau_{zx} \cos(n,x) + \tau_{yz} \cos(n,y) + (\sigma_z - \sigma_n) \cos(n,z) = 0$$

and also $\cos^2(n,x) + \cos^2(n,y) + \cos^2(n,z) = 1$

∵ $\cos(n,x), \cos(n,y), \cos(n,z) \neq 0$ it follows

$$\begin{vmatrix} \sigma_x - \sigma_n & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y - \sigma_n & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z - \sigma_n \end{vmatrix} = 0$$

Expanding the determinant, we find the functional relationship called the characteristic equation.

$$\phi(\sigma_n) = \sigma_n^{3} - I_1\, \sigma_n^{2} + I_2\, \sigma_n - I_3 = 0$$

where $\quad I_1 = \sigma_x + \sigma_y + \sigma_z$

$$I_2 = \begin{vmatrix} \sigma_x & \tau_{xz} \\ \tau_{xy} & \sigma_z \end{vmatrix} + \begin{vmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{vmatrix} + \begin{vmatrix} \sigma_y & \tau_{yz} \\ \tau_{yz} & \sigma_z \end{vmatrix}$$

$$= \sigma_x\, \sigma_y + \sigma_x\, \sigma_z + \sigma_y\, \sigma_z - {}_{xy}{}^{2} - {}_{xz}{}^{2} - {}_{yz}{}^{2}$$

$$I_3 = \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & \sigma_z \end{vmatrix} = \sigma_x\, \sigma_y\, \sigma_z - \sigma_x\, \tau_{yz}{}^{2} + \sigma_y\, \tau_{xz}{}^{2} + \sigma_2\, \tau_{xy}{}^{2} + 2\, \tau_{xy}\, \tau_{xz}\, \tau_{yz}$$

I_1, I_2 and I_3 are independent of the direction cosines and are therefore independent of co-ordinate axes.

3.14 OCTAHEDRAL STRESSES

Assume that principal stresses directions (1,2,3) coincide with x,y,z co-ordinate axes, n is the direction normal to an octahedral plane.

$$\cos(n,1) = \cos(n,2) = \cos(n,3) = \cos 54^{\circ}44' = \pm \frac{1}{\sqrt{3}}$$

$\therefore$ Normal stress on octahedral plane

$$\sigma_{oct} = \left[\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right] \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}$$

$$= \tfrac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

$$\tau^{2}_{oct} = P_n^{2} - \sigma_{oct}^{2}$$

where P_n^2 is the resulting stress

$$= \left[\sigma_1 \cos(n,1)\right]^2 + \left[\sigma_2 \cos(n,2)\right]^2 + \left[\sigma_3 \cos(n,3)\right]^2$$

$$\therefore \quad \text{oct} = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_1-\sigma_3)^2}$$

It is immediately apparent that octahedral stresses are related to the stress invariants as

$$\sigma_{oct} = \frac{1}{3} I_1 = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z)$$

$$\tau_{oct}^2 = \frac{2}{9}(I_1^2 - 3I_2) = \frac{1}{9}\Big[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_x - \sigma_z)^2 + 6(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2)\Big]$$

3.15 STRESS SPACE

The state of stress on a soil element can be represented by a point in the stress space. Such a point is called a stress point. During a test the stress acting on an element varies and the stress point them traces a curve in stress space which is called a stress path.

3.16 YIELD CRITERIA

It is a common hypothesis of the theory of plasticity that if any material is tested to determine the various combinations of principal stress under which it yields and these states of compound stress are represented as points in principle stress space the points will define a surface called a "yield surface in principle stress space". Various forms of surface have been shown to correspond to various yield criteria.

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4 FACTORS AFFECTING THE SHEAR STRENGTH OF SOIL

4.1 The number of fundamental parameters which affect the shearing behaviour of cohesive soil is large. They are usually difficult to measure and the relative importance of their individual influences on the results of testing is difficult to assess. In the broadest term the shearing strength of cohesive soil is a function of structure of the soil, the void ratio or the average spacing of particles and the rate of shear, all measured in the plane or zone of failure. It must be recognised that there may be factors still unknown that affect the shear strength. Taylor(84) admitting that no outline of the factors on which shearing strength depends could be set-up which would be simple in all respects and satisfactory, has given a reference list of the factors which are classified as:

(i) Fundamental factors affecting shear strength
(ii) Alternate or inter-related factors
(iii) Types of apparatus and tests
(iv) Other subjects such as fissures in clay and non-isotropic effects.

Though it is not possible to define the effect of these individual factors quantitatively, which are inter-dependent, it is possible to give a qualitative picture as to,how these factors might affect the measured strength of a cohesive soil. The different factors are discussed under the following broad based groups

(i) Physical and physico-chemical factors

(ii) Stress history

(iii) Particle interaction

(iv) Time effect or progressive failure

(v) Methods of test evaluation and strain rate.

4.2 PHYSICAL AND PHYSICO-CHEMICAL FACTORS

The physical components governing development of shear strength of soils arise primarily from the resistance to sliding movement of one particle on another and to interlock between particles. Rosenquist (66) divides the interlocking phenomena into two distinct types.

1. Macroscopic interlocking of particles requiring appreciable movement of particles, normal to the failure plane thereby giving to an increase in volume prior to failure.

2. Microscopic interlocking because of surface roughness of particles resulting in small movements of particles normal to the failure plane.

These physical factors are proportional to the effective normal stress on the failure plane and are characteristic of grannular materials. To what extent these factors are valid in cohesive soils depends on the inter-particle forces of attraction and repulsion associated with clay water system. If the inter-particle forces are small and do not inhibit particle contact, then the physical factors become significant.

Frictional resistance may sometimes be considered as a physico-chemical phenomenon. Because surfaces of particles are not absolutely smooth, when two particles are brought into contact with each other under stress, the contact points will deform elastically or plastically by an amount sufficient to sustain the applied effective stress. The close proximity of the contacting areas gives rise to adhesion derived from the electrical forces of attraction. The adhesive forces which must be overcome for sliding to occur give rise to shear resistance. The shear resistance is proportional to the strength of adhesive bond, which is a function of distance between atoms in the containing surfaces and of the compositin of the material.

In dense materials where particles are packed in a close configuration, it is necessary for particles to move over adjacent particles for shear displacement. The interlocking that occurs because of packing will cause the resistance to motion and consequently volumetric expansion must occur to accommodate particle displacement. In undrained triaxial tests on saturated soils, volumetric expansion causes decrease in pore water pressure and an increase in effective stress, thus resulting in an increase in measured strength. The magnitude of friction and interlocking effects depends on soil composition, nature of minerals present, surface characteristics, packing or structure, test condition etc., since particle size, shape, size distribution and packing are function of composition. Volumetric expansion may be related directly to composition and associated factors such as void ratio and arrangement of particles.

If in a clay water system the interaction of clay particles is such that there is minimum of actual physical particle to particle contact, the definition of friction as such must be altered. Friction can no longer be accepted as that property derived from particle contact if the interaction of clay particles is through the adsorbed water layer and the layers of exchangeable ions. The physical demonstraction of friction as a parameter concerned with the shear strength of clay soils will depend on constraints, determined by the test technique, and method used for evaluating the test results.

The structural factors of clay can roughly be divided into two groups: dispersed (oriented) and flocculated (unoriented clays).

Dispersed Clays: The soil is dispersed as a result of the environmental conditions at the time of deposition and the clay minerals present. Subsequent leaching may have altered the pore water properties between the time of deposition and the time of stressing by an engineering structure. Both the structures of the soil and its behaviour under shear stressing develop in part as a result of these conditions.

In dispersed clay soil, or soil whose particles do not touch, a threshold shearing strength is not expected to exist, and strictly speaking, the soil will possess no angle of internal friction. However, even if the grains are an oriented parallel to one another, an increase in external stress will change the spacing of the grains at equilibrium, and a higher shear stress will be required to cause a given rate of shear. In this case shear strength will increase by increase in the rate of shear, since the shear is due almost entirely to viscous effects.

Temperature which has also an effect on shear stress ordinate at given rate of strain acts in a complex way. An increase in temperature decreases the thickness of double layer so that at a constant external stress the void ratio will decrease, which might be expected to raise the shearing stress.

However an increase in temperature reduces the pore fluid viscosity and this reduction would have the opposite effect on shearing stress. It has been established from the tests that viscosity of clay suspensions and therefore shearing strength at a given rate of shear, decreases with temperature at approximately the same rate as the viscosity of ordinate[24] water. In dispersed clays the electrolytic nature of the pore water and the exchangeable ions present will also affect the void ratio and the shearing stress required to cause a given rate of shear.

Flocculated clays: The behaviour of the material under stress will depend both on the structure of the soil as formed in the original environment from clay minerals and on subsequent leaching to give the properties of the pore fluid at the time of stressing by an engineering structure. As with dispersed clays, the dielectric constant of the pore fluid, the valence and concentration of the electrolyte present, temperature and time elasped since deposition play a role in soils behaviour.

In addition, the structure of flocculated clays at the time of shearing has developed to a greater degree as a direct result of the history of stressing of the soil than is the case in already strongly oriented clays. If previously applied stresses normal to a possible failure surface exceed the stresses existing on the surface at failure, the soil is prestressed, or over consolidated. Thus the structure as well as the inter-particle spacing or void ratio in the failure zone depend on stress history. The presence of a wide range of grain sizes will modify the shearing strength of the soil. Since the shearing strength of the soil depends on the number of intergranular contacts in the shearing zone and the contacts reflect the stress history of the material, in particular, in that they increase in number with increasing effective stress, the shearing strength will exhibit a dependence on effective stress. The relationship, although analogous to that characterised by the friction angle of granular soils, arises, as we have pointed out from different mechanism.

All the above consideration have been dealt with from the point of view of a saturated soil. If the soil is not completely saturated, the degree of saturation will play a part in the shearing behaviour. In this case, further capillary effects arise through the stresses developed in the soil by the menisci at the gas water interfaces.

It was noted that both the void ratio and the structure of the soil in the failure zone influenced the shearing strength. Normal effective stresses act to change principally the void ratio and, to a lesser extent the structure. Shearing stresses, on the other hand, have their greatest effect on the structure of the soil in the shearing zone, orienting the particles parallel to the direction of shear. Since in this orientation, the soil is much less able to resist normal effective stresses on the failure zone, the void ratio can more easily be changed by the normal effective stresses. It follows that a combination of shearing and normal stresses on a particular zone of the soil is a more efficient agent in decreasing void ratio than normal effective stress alone.

4.3 NON-ISOTROPIC EFFECTS

It is likely that most stratified soils have somewhat smaller shearing strength parallel to the strata than they do across the strata. In some varved clays this difference may be large. However in clays having only a moderate degree of stratification there is little evidence of much strength variation on different planes.

4.4 STRESS HISTORY

Observed failure characteristics in cohesive soils present a complex picture. For initial considerations, the drained shear test can be used to illustrate certain phenomena in laboratory tests on clays subjected to various stress histories. The following nomenclature will be used in discussing these properties:

$\bar{\sigma}_c$ = Consolidating pressure = effective consolidating stress $\bar{\sigma}_c$

$\bar{\sigma}_c = \bar{\sigma}_{cf}$ at failure

$\bar{\sigma}_f$ = normal stress on failure plane at failure

In the consolidated drained shear test, if the test sample is fully consolidated prior to shear, and if the shearing process is conducted slowly so that pore pressures are not generated, the stress developed in the soil are effective stresses.

For a condition of $\bar{\sigma}_c$ > previous load, the relationship between shear stress τ_f and consolidation load at failure, $\bar{\sigma}_{cf}$ is essentially linear (Fig. 4.1). The importance of load history

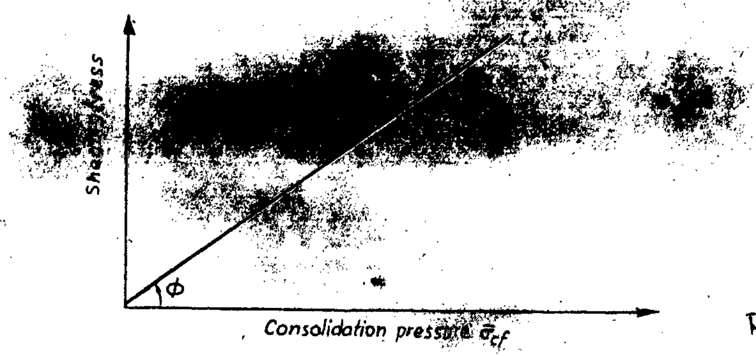


Fig. 4.1

in shearing strength can be demonstrated with over consolidated samples which are subsequently subjected to the drained shear test at confinements other than the over consolidating load. This phenomenon is shown in Fig. 4.2.

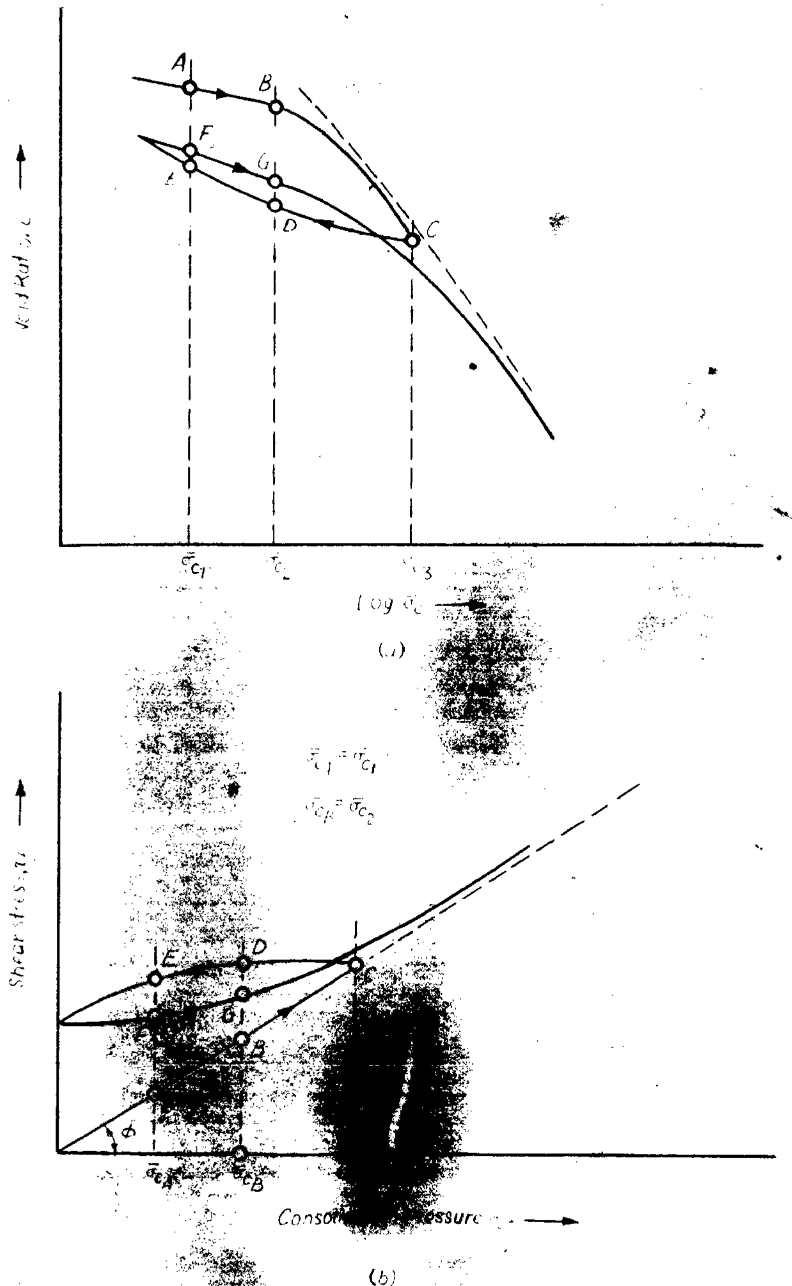
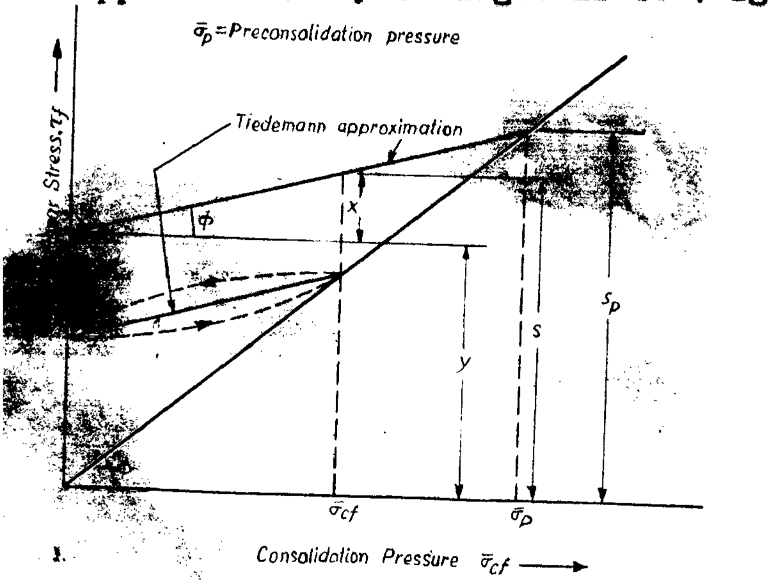


Figure – 4.2 (a) e-Log σ_c Curve for Samples Consolidated for Drained Shear Test, (b) Drained Shear Test Results for the Consolidated Samples

The drained shear strength of samples A, B and C, normally consolidated as shown in Fig. at varying consolidation loads shows a linear relationship between strength and preconsolidation pressure. Following preconsolidation to a terminal pressure, the drained shear strength is much higher than that of a companion sample sheared without pre-loading if rebound to a lower consolidating or confining pressure is allowed. This is demonstrated in the shear strength of samples E and D. The characteristics of shear following rebound and reloading are shown by samples F and G. The property of residual strength or "cohesion" following preloading to a terminal load has led to examination of shear strength of cohesive soils in terms of initial structure and orientation of particles.

For simplicity in analysis, the hysteresis loop can be approximated by straight lines (Fig. 4.3).



ure-4·3. Tiedemann Approximation of Rebound Shear Curves

from the graph are C'

ϕ' are effective stress Parameters.

$$\tan \phi' = \frac{S_p - y}{\bar{\sigma}_p}$$

$$\text{or} \quad \bar{\sigma}_p \tan \phi' - S_p = -y$$

$$y = \bar{\sigma}_p \left(\frac{S_p}{\bar{\sigma}_p} - \tan \phi' \right)$$

But $\dfrac{S_p}{\bar{\sigma}_p} = \tan \phi$

$$\therefore\ y = \sigma_p (\tan \phi - \tan \phi')$$

From Fig.

$$x = \bar{\sigma}_{cf} \tan \phi'$$

and $\tau = x + y$

$$= \bar{\sigma}_p (\tan \phi - \tan \phi') + \bar{\sigma}_{cf} \tan \phi'$$

$$\tau = K\, \bar{\sigma}_p + \bar{\sigma}_{cp} \tan \phi'$$

where $K = \tan \phi - \tan \phi'$

The equation given for shear strength τ shows that the cohesion term is dependent on the stress history and preconsolidation pressure of the clay soil.

4.5 PARTICLES INTER-ACTION AND SHEAR STRENGTH

We have shown that prestress history affects shear strength. This may be examined on the basis of inter-action of the particles during shear and their importance in the development of ultimate strength of the soil sample. In the previous chapter, random, flocculated and oriented structure are discussed.

The end result of prestressing or preloading is to reorient the initial arrangement or structure of particles to some final configuration. The development of shear resistance within the soil depends on the interaction of the soil particles. If the final configuration results in increased resistance to displacement of soil particles under the action of an applied shear force, then greater strength may be found. It is possible on this basis to formulate a working hypothesis which would provide the mechanism for shear and answer some questions concerning pore pressure, inter-particle forces and particle interaction.

Consider the case of a pure clay where on the soil particles are plate like in shape. These particles will be oriented in some particular configuration, i.e., in flocculated or random structure.

The three types of bonding that can occur as a result of inter-particle and surface forces may be classified as follows.

(a) Edge-to-surface bonds in the edge to surface arrangement.

(b) Edge to edge bonds in the edge to edge arrangement.

(c) Surface to surface bonds in parallel orientation.

The edge to edge and edge to surface bonds can include both cementing bonds and those resulting from inter-particle forces. Denoting these as es, ee, and ss bonds in the order given above, the ss bonds tend to be the weakest for the same effective particle spacing. The differentiation between ee and es bonding strength must depend upon the type of minerals present, the cementing agents and the specific charges on minerals themselves. The dominant bonding forces in a flocculated structure would likely be the es forces. Under the action of shear displacement the primary component of shear resistance is derived from destruction or breakdown of the es bonds. Strain curves for this type of sample is shown in Fig. 4.4.

For comparison, Fig. 4.4 also give stress strain curve for a randomly oriented structure. Because of the weaker ss bonding there will be a flatter slope in the stress-strain curve. In many instances these samples are characterised by the absence of a peak point. Particle interference during relative particle displacement also contributes to the shape of the curves. A higher degree of physical interference is found in the focculated structure.

In drained strength tests where no pore pressures are generated, the breakdown of bonds and the consequent reorientation of particles together with physical particle interference will define the stress strain curve. In an undrained test where pore pressures may be generated, restricted particle orientation due to the absence of volume change will influence total particle interaction and consequently yield a different set of parameters for shear strength. This is most important and must be considered in the evaluation of these shear strength parameters. It is important to realise that in the laboratory testing of soils for evaluation of shear strength, the pore pressure simulated must be that anticipated in the field. If a drained condition is expected in the field under actual loading conditions, this type of test must be performed in the laboratory. In this case shear strength development is a function of physical arrangement of particles without generation of pore pressures. The significant property is that arising from physical breakdown of es bonds. This manifests itself in the conventional Mohr-Coulomb analysis of test results as the friction parameter. For undrained tests, resistance to particle rearrangement may be provided through total pore pressure response, which would produce different yield characteristics of the soil samples. If one is able to measure accurately the pressures in the fluid in the soil sample, it would

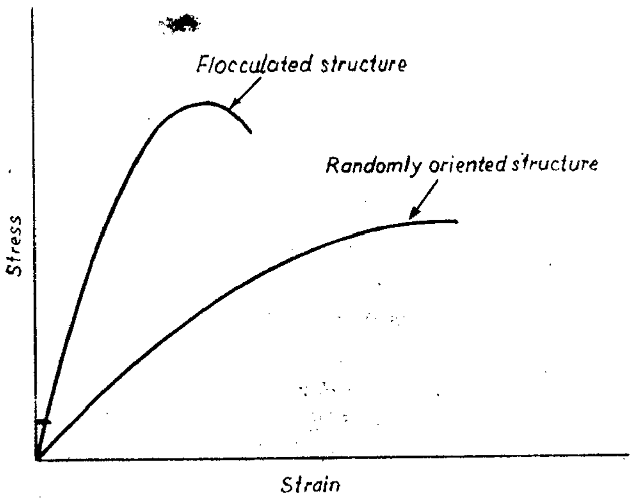


Figure - 4·4 Influence of Structure on Stress-Strain Relationships be possible to compute what pressures might be due to physical breakdown and reorientation of particles. This would provide a more realistic evaluation of soil performance.

4.6 PROGRESSIVE FAILURE

In the concluding the third Terzaghi lecture on progressive failure in slopes of over-consolidated plastic clay and shales L. Bjerrum (11) has remarked, "a land slide devil seems to laugh at the human incompetency" quoted from a report describing the tremendous land slides in the heavily over consolidated clay shales in Japan.

In a most significant contribution to this subject presented in the fourth Rankine lecture in 1964 Skempton (79) reviewed our knowledge on Residual Shear strength and compared its value with shear strength computed from slides in over consolidated clay. In this paper Skempton used the term stiff fissured clay. However it is changed by Bjerrum to over-consolidated plastic clay as fissures are not believed to be essential to the conclusions drawn therein.

Skempton drew three important conclusions:

1. The residual shear strength parameter ϕ'_r is independent of the original strength of clay and such factors as water content and liquidity index. The value of ϕ'_r of clay seems to depend only on the size, shape and minerological composition of the constituent particles.

2. The average shear stress along the failure surface computed from a number of slides in over consolidated plastic clays bears much more resemblance to the residual shear strength

than to peak strength and an analysis of some slides in natural slope showed that the shear strength at failure were nearly equal to the residual shear strength.

3. As a consequence of this finding, Skempton concluded that the slides in over-consolidated clays were preceded by a progressive development of a failure surface. In natural slopes where sufficient time has been available for the development of sliding surfaces by progressive failure the ultimate stability depends on the residual shear strength only.

Now it is a well-known fact that if in a drained shear test the sample is strained beyond failure its strength will decrease and ultimately reach a certain value the residual strength, which will remain constant for further straining.

For over-consolidated plastic clays the drop in shear strength after failure is very significant and the residual shear strength is only a fraction of the peak strength. The residual shear resistance has a frictional character although the value of $\emptyset_r$ is not necessarily independent of the normal pressure. A distinction is made between shear strength expressed in units of stress, and shear resistance which is a ratio of shear strength to effective normal stress.

The powerful agent capable of bringing even the hardest clay to failure is to be described with a single name, it should probably be "recoverable strain energy".

It is concluded that consideration of the properties of over-consolidated clays thus tells us that the potential for a progressive failure is not equal in all over-consolidated clays, but greatly depends on the time at which the stored recoverable strain energy is liberated.

It is clear from the study of recoverable strain energy that in some clay the strain energy is recovered simultaneously with a change of stress, whereas in others it is "locked in" and is thus not immediately available. The explanation of why strain energy is stored in some clays proved to be that in these clays Diagenetic bonds were formed when the clays were carrying their maximum consolidation load. These diagenetic bonds have the character of a welding of contact points between particles, thus preventing the bent particles from straightening when load is reduced. The diagenetic bonds are however gradually destroyed when the clay near the surface becomes exposed to the various agents of weathering. During weathering the "locked in" strain energy is gradually liberated. The presence of diagenetic bonds and their gradual destruction thus proved to be an important factor in the discussion of progressive failure because of the clay in liberation of the recoverable strain energy. There is consequently an essential difference in behaviour of over consolidated clays with weak and strong diagenetic bonds.

4.7 <u>METHODS OF TEST EVALUATION - RATE OF STRAIN</u>

The methods of test evaluation are discussed in full in Chapter 7, where shear strengths are determined by various techniques those methods which entail the least amount of handling of material tend to give the largest values of strength.

4.8 RATE OF STRAIN

Investigations of this parameter are complicated by the presence and behaviour of the pore water in the soil. If one establishes the criterion that a test should be run sufficiently slowly so that no excess pressures are generated in the pore water, the variation in shear strength of the soil alone can be evaluated. The results of such tests indicate that shearing strength defined by the peak of a stress-deformation curve at very slow rates of strain, say 10^{-4}% per min. may be 15 to 20% lower than that at the rates of about 1% per min. In general, the strength is more affected by excess pore pressures generated during shear.

Undrained tests at different rates of strain were performed on unconfined specimens of cohesive soil by Casagrande and Wilson (18) and on both unconfined and confined samples at more rapid rates by Whiteman.(90a) The results in general of these tests are shown in Fig. 4.5, and it can be seen that the strength changes

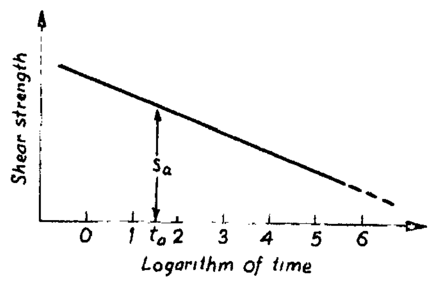


Figure - 4.5 Undrained strength versus duration of test

considerably at the more rapid loading rates. The effect varies with different soils since it is controlled by viscous effects as well as development of excess pore water pressures.

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5 YIELD AND FAILURE - DIFFERENT CRITERIA

5.1 BEHAVIOUR OF SOIL UNDER STRESS

Consider a body of soil under the action of applied effective stress, which in soil may be either hydrostatic or deviator stress or both. The stress will be applied to the body and removed so that one can discriminate among the various effects. The effective stresses may be considered to be applied to the body in such a way that all the elemental volumes of the body are subjected to the same stress system simultaneously. Hence the soil is said to be stressed homogeneously. Figure 5.1 shows the displacements which take place in the body as a function of time during and after load application. They may be considered to be length changes.

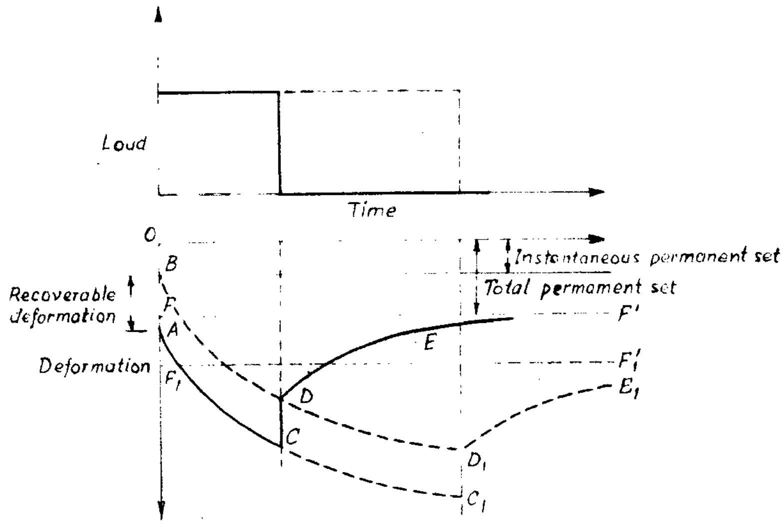

Figure_ 5.1. General soil deformation behavior.

When load is first applied there is an instantaneous movement OA. If the load were immediately removed the point studied would return to a position indicated by point B, leaving a permanent set of amount OB, while the recovered displacement AB can be considered to result from elastic strains in the material. The permanent set comes about as a result of local overstressing of soil grains which move under unbalanced forces. On removal of the applied stress, there is no mechanism by which the disturbed grains can return to their original positions, and the resultant movement is irrecoverable. Because of micro stress discontinuities in soils, additional permanent sets will accrue from repeated loadings, but they become smaller, tending towards zero, as individual particles reach stable positions at the applied stress level. Should the stress be increased at some subsequent loading cycle, a new permanent set will, of course, be established. Thus, in general, repetetive loadings of a soil at stresses well below the macroscopic failure intensity cause the instantaneous response of soil to become essentially elastic. This effect is borne out by experiment.

If the stress, once applied is allowed to remain on the soil for some time the displacement will continue along the curve of exponential form illustrated by AC. The part of the displacement time curve causes plastic flow produced by

(a) relaxation process which causes plastic flow of soils, i.e., individual points of contact of grains change position gradually at a rate dependent on the stress level and the environmental temperature, and

(b) the stress readjustments that distribute themselves through the mass from individual particle movements under load imbalances.

The material deforms in this stage as a non-linear viscous fluid.

At the time and displacement represented by point C the stress is presumed to be instantaneously removed. The point under study will immediately return to the displacement indicated by point D, where CD is the elastic deformation of the mass and is approximately equal to AB. If the load were to be reapplied immediately, the point would return to a position represented by point C. Otherwise, should the soil remain unloaded, the point will gradually move exponentially in time, producing the curve DE, which eventually becomes tangential to the line FF'. Thus under the application and removal of the applied effective stress, the point comes to rest, the extent of permanent displacement being OF.

The position of the movement shown by DE is ascribed to an ELASTIC AFTER EFFECT and may be explained as follows. The original and continued application of effective stress result in the development of stresses in soil grains which cause viscous flow in adjacent pore water or move adjacent soil grains in time. When the load is removed after some time, the viscous deformations which have taken place are no longer compatible with the tendency of elastically stressed grains to return to their original shape or position; ~~residual stresses exist in some grains to return to their original shape or position;~~ residual stresses exist in some grains on removal of the applied stress. These residual stresses gradually relax in the course of time as the pore fluid or adjacent grains again move viscously and in this case, the relaxation process tends to restore the displaced point to its original position. However, when permanent stresses have relaxed completely, some portion of the first viscous flow which occurred during the period of load application remains in the form of permanent set, in addition to the original instantaneous set, consequently the longer the load is applied the greater are the deformations resulting form viscous flow and the larger is the ultimate permanent set BF resulting from these deformations.

This situation is illustrated in Figure 5.1 in which the originally applied load is maintained to the time represented by point C_1. Removal of the stress at this time again yields the recoverable portion of the movement C_1D_1 (approximately equal to AB and CD) and a slow change in position of the point, resulting in the curve D_1E_1, which is assymtotic to F_1F_1'. The permanent set BF_1 resulting from the length of the period of the load application is greater than the set BF due to the 1st shorter period. In general, the constant (which is representative of the viscosity of the material in flow) describing the exponential unloading curve DE will not be same as that of the loading curve AC and both constants will be affected by the number of stress cycles to which the soil has been submitted, since the structure of the soil material which determines its response to stress, can be altered to a great degree by progressive deformations.

The behaviour has been described for soils in general, and may be considered to apply to cohesive soils.

5.2 In practice, the hypotheses of a linear elastic behaviour without time effects for soils is used as a basis for calculations which extend the assumption far beyond the reasonable limits. The model is always considered to be isotropic, homogeneous, ideally elastic solid exhibiting a linear relationship between stress and strain, and in terms of shearing stress and shearing strains.

Three sets of relation are required in general to determine the stress distribution in a material. They are

1. Equations of equilibrium
2. Equations of compatibility
3. Equations describing the behaviour of the material.

When the loads applied to a soil mass are gradually increased, these equations and the boundary conditions determine the increasing stresses.

5.3 Interest in the possible application of plastic theory to soil mechanics has quickened during the last few years and the need for further fundamental information on the properties of soils has become apparent.

Although some of the 1st workers in plasticity were primarily interested in soil mechanics such as Coulomb in 1773 (22), work within the frame work of modern plasticity theory dates from 1951 when Drucker and Prager (25) made a study of possible application of limit design in this area. This work has been followed by a series of theoretical studies by Shields (74)(mixed boundary value problems in soil mechanics and on Coulomb's law of failure in soils).

Chandwick and others, assume that soil can be regarded as ideally plastic material which follows Coulomb's yield criteria at all stress states. Direct evidence in support of these assumptions on the nature of the material has been lacking. As far as the yield condition is concerned, only the results from the standard triaxial test and from torsion test have been available.

These two tests examine the states of simple shear and of simple torsion superimposed on a hydrostatic pressure. The only experimental information available on intermediate stress states appears to be a short series on a sand reported by Habib (29).

As regards the flow rule even less information is available. In an ideally elastic body, a dilation would be expected. In soils a small dilation is observed but it is far less than would correspond to an ideally plastic material. This has led to speculations about the use of strain hardening model.

However, the stresses in soil "at failure" imply that soil behaves like an ideally plastic material, with the stress deformation behaviour exemplified in Figure 5.2.

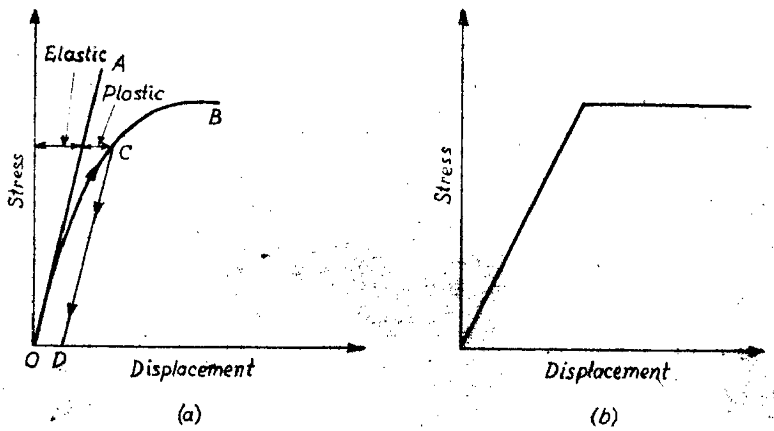

Figure-5·2. Idealized plastic-flow characteristic: curved strength envelope. (a) Real case. (b) Idealization.

In this context, the actual behaviour of the material in time, after the flow stress has been reached, is not specified; failure alone is sufficiently important. When time is involved, the behaviour is characterised as visco-elastic rather than plastic. In some soils, the recoverable deformations which take place prior to failure are small in magnitude, particularly in comparison with the movements consequent upon the attainment of the limiting stress, and hence may be neglected. A soil possessing this property is called rigid plastic and behaviour is shown in figure above. When recoverable deformations are so large that they play an important part in any stress analysis performed, the soil involved is called elastic plastic. As usual in soil mechanics, the relatively precise meanings which can be given to these terms in other branches of applied mechanics, even in terms of real materials, are difficult to grasp. Irrecoverable or plastic deformations take place at all stress levels in a typical soil as a result of the accumulation of individual grain movements that are brought about by both macroscopic, hydrostatic and shearing stresses. In the typical stress deformation pattern shown for a soil, separate contributions of recoverable deformations, OA and irrecoverable deformations AB are noted. If the soil is stressed upto a point C, and the load is removed the soil recovers to a permanent deformation represented by point D. The work done on the soil in this cycle of loading is represented by the area OCD in the figure and since the work done to produce the recoverable deformation is regained, the area represents the work lost to the soil, i.e., the work which produces the permanent deformation (assumed to be homogeneous) in the volume of the soil. Work is therefore stored in the structure, lost or dissipated in the process of producing irrecoverable deformations in a material. Since loading alone is insufficient to determine the separate contributions of elastic and plastic deformations to the total, the initial elastic response of the material at low stresses as presented by the line OA is obtained by drawing a parallel line to the unloading line CD. The material whose behaviour is indicated in figure above is called a 'work hardening solid' in the terminology of applied mechanics.

5.4 The main object of laboratory tests on a soil is to determine the stress deformation or failure characteristics of material under homogeneous stress conditions. Ideally it should then be possible to assume that these characteristic properties will also be exhibited by the same soil in the field, where the stresses applied to it may vary from point to point.

5.5 The investigation of the limiting behaviour of different soils under applied stress reveal that it is possible to prescribe a stress state at which the soil would just begin to deform continuously, provided that certain properties of the given materials were known. The state of static equilibrium cannot exist at stresses in excess of those determined. These conditions are referred to as limiting or failure condition and can be described in terms of deformation as well as stress. In fact, for a completely rigorous analysis of the behaviour of soil under applied loads, we must use the deformation criteria since the stress distributions that develop in soils adjacent to structures are extremely sensitive to both the gross and relative movements of the structures. Thus, in most practical soil engineering problems, boundaries will be stipulated along which both stress and displacement conditions must be observed. This class of problems possess "mixed" boundary conditions and the resulting stress distributions in the soil are statically indeterminate in both elastic and limit condition analysis.

Should stresses alone be specified along a soil boundary, the boundary conditions may be considered to be statically defined, and although the elastic problem may still be indeterminate, the solution at the limiting or failure condition is statically determinate. The difference in this case occurs because a complete analysis of the elastic problem requires that the compatibility equations be satisfied by the strains arising from any proposed solution. The elastic stresses arising, in the medium under study must be continuous throughout the medium although boundary discontinuities occur. However, in the development of failure or limiting stresses, unlimited flow or strain takes place when the limiting value is reached in a zone or area of the region being examined; thus the specification of a limiting stress imposes no deformation condition, and the solution does not involve a compatibility requirement in the same sense as does the problem in elasticity although a continuity conditions must be adhered to.

For a plane strain we can write

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = \frac{r}{g} \cdot \frac{\partial^2 u}{\partial t^2} \qquad (1)$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} = 0 \qquad (2)$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_z}{\partial z} - r = \frac{r}{g} \frac{\partial^2 w}{\partial t^2}$$

also $\epsilon_x = \frac{\partial u}{\partial x}$, $\epsilon_z = \frac{\partial w}{\partial z}$, $\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$

$$\epsilon_y = \gamma_{xy} = \gamma_{yz} = 0,$$

where u and w are displacements in x and z direction.

These expressions provide the starting point for all subsequent considerations of stability problems in soil mechanics for which plane strain can be postulated. Of considrable importance in this array is that three equations of motion are required rather than the two equations of equilibrium usually assumed.

To reduce the number of equations, some assumptions must be made relating shearing stresses and shearing strains. The usual procedure is to assume tacitly the validity of elastic theory or simply to ignore the existence of the third dimension. Either assumption will render $\tau_{xy} = \tau_{yz} = 0$, thus eliminating eqn. (2), and reducing the three equations to the two expressions below -

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = \frac{r}{g} \cdot \frac{\partial^2 w}{\partial t^2}$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_z}{\partial z} - r = \frac{r}{g} \frac{\partial^2 w}{\partial t^2}$$

For conditions existing prior to the initial development of a discontinuity, the inertia terms in the above equations vanish. The same is true once a steady state velocity field is established. However for conditions existing in the time span from the initiation of motion to the evolution of a steady state, due consideration must be given to the dynamic problem and progressive failure. Although some work has been done in allied fields on matters dealing with the nature of moving cracks, to date only very simple boundary conditions have been amenable to theoretical considerations. To obtain a procedure with some expression could provide a measure of the circumstances governing the motion of a part of the soil, it is said to be a yield or failure criterion. Since it would be required to furnish a relationship valid for both simple laboratory tests and complex state of stress in the field, it is reasonable to expect that, should a yield criterion exist, it should be a function of invariants of stress.

$f(I_1, I_2, I_3) = 0$. Since the invariants of stress can be defined in terms of the three principal stresses (assumption of homogenity and isotropy will render them independent of direction) may also be expressed in the form

$$F(\sigma_1, \sigma_2, \sigma_3) = 0$$

where the principal stresses σ_1, σ_2, and σ_3 are the roots of the cubic equation

$$\sigma^3 - I_1\,\sigma^2 - I_2\,\sigma - I_3 = 0$$

From the mathematical view point the functional relationship in eqn., $F(\sigma_1, \sigma_2, \sigma_3) = 0$ represents a surface in principal stress space with axes of co-ordinates along σ_1, σ_2 and σ_3. In as much as any combination of stresses falling on this surface represents a limiting condition or state of failure the determination of its shape is of paramount importance.

The successful determination of a yield surface will obviate the need for consideration of strains or volume changes. The implications is that, for any yield criterion to be valid, it must of necessity reflect these considerations in the relationships it specifies among principal stresses for limiting conditions.

hope of success it is necessary to limit considerations to equilibrium conditions. That is, it will be assumed in subsequent considerations that equilibrium prevails and that the boundary loading is just capable inducing failure. The philosophy of failure associated with this simplification is said to be one of limiting equilibrium.

The two main limit theorems for any body or assemblage of bodes of elastic-perfectly plastic material are:

(i) Collapse will not occur if any state of stress can be found which satisfies equilibrium and the boundary conditions on stress and for which all stress points lie inside the yield surface.

(ii) Collapse must occur if for any flow pattern considered as plastic only, the rate at which the external forces do work on the bodies equals or exceeds the rate of internal dissipation.

For the limiting condition we can write,

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = 0 \qquad \text{(a)}$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_z}{\partial z} = r \qquad \text{(b)}$$

Equations (a) and (b) demonstrate that conditions of equilibrium alone are not sufficient to provide the stress field within the soil body yielding only two relationships among the three unknown stresses. Should another independent expression exist that could relate these three stresses in a unique way, the resulting system would fulfil our needs for assessing the adequacy of a structure subject to loading. Because such an expression could provide a measure of the circumstances governing the motion of a part of the soil, it is said to be a yield or failure criterion. Since it would be required to furnish a relationship valid for both simple laboratory tests and complex state of stress in the field, it is reasonable to expect that, should a yield criterion exist, it should be a function of invariants of stress.

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5.6 THEORIES OF FAILURE

Here a consistent set of laws for plastic flow will be discussed . The main requirements are that the volume of the material remains constant under plastic deformation, that a hydrostatic pressure does not cause yielding and further that the hydrostatic component of a complex state of stress does not influence the point at which yielding occurs. We also need some criterion that will predict when yielding starts under a complex state of stress, given only the yield stress under a simple stress (e.g., uniaxial tension or compression) determined by experiment.

A satisfactory failure criterion must express with reasonable accuracy -

1. The relationship between the principal stresses when the soil is in limiting equilibrium.

2. It should (for practical purposes) express their relationship in terms of parameters which can be used in the solution of problems of stability, bearing capacity active and passive pressures etc., and which can form the currency of exchange of information about soil properties.

5.6.1 The Criterion of Yield under Complex Stress

Any criterion must be such that hydrostatic stress has no influence on yielding; yield must also be independent of the directions of the areas chosen to define the system because the material is assumed isotopic and yielding will be related to the intensity of stress. Therefore yielding must be a function of the invariants of the stress tensor which are invariant to transformation of co-ordinates. Moreover if yielding is to be unaffected by the hydrostatic stress $\overline{\sigma} = I_1/3$, then it depends only on the deviatoric or reduced stress components. This means that the criterion of yield must be a function of I_2 and I_3.

It is also assumed that there is no Bauschinger effect; put mathematically that means that if σ_{ij} cause yielding $-\sigma_{ij}$ will cause yielding also. The invariant I_3 will change sign if all the stresses are reversed and it follows that the function must be an even function of I_3.

This is as far as we can go mathematically in deducing an yield criterion from the assumption made about the ideal plastic material. Appeal must now be made to experiment and convenience, but some clarification follows if the geometrical form of the deviatoric stress components is considered.

A hydrostatic stress state implies the equality of all principal stresses or $\sigma_1 = \sigma_2 = \sigma_3$. Hence its locus in principal stress space is said to be given by a line called the hydrostatic axis that pass through the origin and makes equal angles of $\cos^{-1}(\frac{1}{\sqrt{3}}) = 54^{\circ}44'$ with each of the three principal stress reference axes.

Since failure cannot be induced by hydrostatic pressures, the yield surface may not intersect the hydrostatic axis for positive (compressive) stresses.

It is convenient at this point to introduce the principal stress space on auxiliary plane perpendicular to the hydrostatic axis. We shall provide for the transformation of stresses from the principal stress system to a σ_x, σ_y, σ_z system wherein σ_z

axis coincides with the hydrostatic axis and the other two stresses lie in the auxiliary plane. The correspondence between the systems with corresponding direction cosines is shown. Another representation "sighting" along the hydrostatic axis so that the auxiliary plane is in the plane of the paper is also shown.

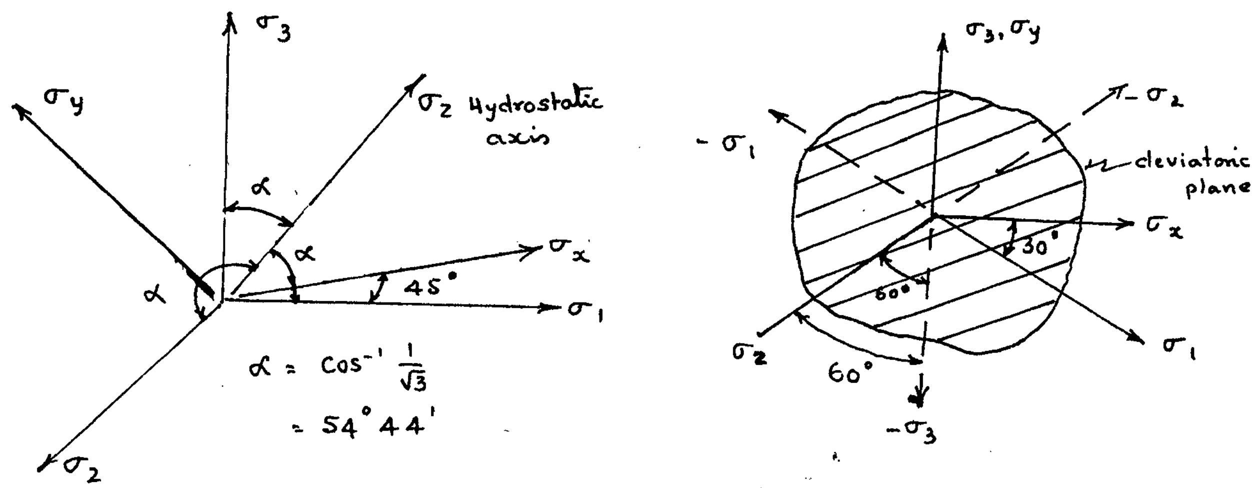


Fig.5.3.

The transformation between the system is seen to be

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\end{bmatrix} = \begin{bmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \sqrt{\frac{2}{3}}\\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}\end{bmatrix}\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\end{bmatrix}$$

whence

$$\sigma_x = \frac{\sigma_1 - \sigma_2}{\sqrt{2}}$$

$$\sigma_y = \frac{(\sigma_3 - \sigma_1) + (\sigma_3 - \sigma_2)}{\sqrt{6}}$$

$$\sigma_z = \frac{\sigma_1 + \sigma_2 + \sigma_3}{\sqrt{3}}$$

The resultant stress $\sigma_r = \sqrt{\sigma_x^2 + \sigma_y^2}$

$$= \frac{1}{\sqrt{3}} \sqrt{(\sigma_1-\sigma_3)^2 + (\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2}$$

and the angle θ is given by

$$\tan \theta = \frac{1}{\sqrt{3}} \frac{(\sigma_3 - \sigma_1) + (\sigma_3 - \sigma_2)}{(\sigma_1 - \sigma_2)}$$

Clearly, the above equations indicate that the auxiliary plane is an octahedral plane the equation of which is $\sigma_1+\sigma_2+\sigma_3$ = constant.

Because all points in the plane represent deviational state of stress, the plane is called a deviatoric plane.

It is evident that an increase in pore water pressure (of the hydrostatic component of stress) will move a point along the hydrostatic axis σ_z but will leave the stresses in the deviatoric plane (σ_x, σ_y) unchanged. Thus all stresses represented by points in the deviatoric plane may be considered to be effective stresses.

A point on the failure locus represents the stress state at incipient failure, while a point completely within the failure locus corresponds to a stable condition. In as much as the deviatoric component is seen to determine the proximity of failure, it appears that a functional relationship for a failure criterion that contains principal stress difference has much to recommend it. Such a function may be written as

$$F(\sigma_1 - \sigma_2, \sigma_2 - \sigma_3, \sigma_3 - \sigma_1) = 0$$

As there is no preference associated with the principal stress differences, the failure locus should display symmetry about all projections of positive and negative principal stress axes in the deviatoric plane i.e., the shape of the surface in each 60° segment such as $-\sigma_2\ 0'\ \sigma_1$ should be repeated around the plane. Thus it is necessary to investigate the nature of the yield surface in only one segment.

5.6.2 Perhaps the simplest form of yield surface that satisfies the functional relationships discussed above is the one advanced by TRESCA (87) in 1868. It assumes that failure occurs for a given material when the maximum shear stress equals a critical constant value, i.e., yielding occurs when the maximum shear stress in the material equals the maximum shear stress at the yield point in simple tension.

The maximum shear stress in a material under some general state of stress ($\sigma_1 > \sigma_2 > \sigma_3$) is $(\sigma_1 - \sigma_3)/2$ and the maximum shear in the tension test is equal to half the normal stress $\sigma_{yp}/2$. The condition for yielding is thus given as $(\sigma_1 - \sigma_3) = \sigma_{yp}$. It is a direct consequence of Coulomb's theory for a frictionless material. The maximum shear stress theory (Coulomb theory) has been extended by Navier to account for pressures normal to the failure plane, and is known as the Coulomb-Navier theory. The concept of maximum shear stress to explain a fracture type failure in a cohesive soil appears in the work of Collins.

In its most useful form the theory may be stated as follows

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \text{const.}$$

In uniaxial tension $\sigma_1 = \sigma_o$, $\sigma_2 = \sigma_3 = 0$, $\tau_{max} = \frac{\sigma_o}{2}$

In uniaxial compression $\sigma_1 = \sigma_2 = 0$, $\sigma_3 = -\sigma_o$ thus

$$\tau_{max} = -\sigma_o/2$$

$\therefore$ yield criterion requires that $\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_2) = \frac{\sigma_o}{2}$

The above expression requires that the yield stress of the material in either simple tension or compression must be equal and the "slip lines" which appear at the onset on plastic flow should be inclined at an angle 45^o with respect to the directions of the principal stresses σ_1 and σ_3, i.e., coincident with the directions of maximum shearing stress.

The condition of flow does not contain the intermediate principal stress σ_2 which can have any value between $\sigma_1 > \sigma_2 > \sigma_3$. The flow condition in its most general form may be expressed by three equations.

$$\sigma_1 - \sigma_3 = \pm \sigma_{yp}; \quad \sigma_2 - \sigma_1 = \pm \sigma_{yp}; \quad \sigma_3 - \sigma_2 = \pm \sigma_{yp}$$

where σ_{yp} is the absolute value of yield stress in tension or compression. Thus the surface of yielding corresponding to the maximum shear stress theory consists of three sets of parallel planes which define a straight hexagonal prism in σ_1, σ_2, σ_3 space whose axis coincides with the space diagonal $\sigma_1 = \sigma_2 = \sigma_3$ i.e. in the positive quadrant of axes. Cross section of the prism are regular hexagons (Fig. 5.4).

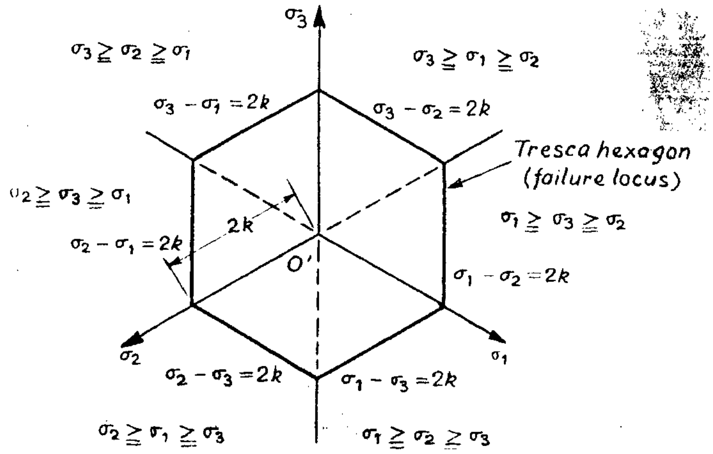


Figure - 5.4

5.6.3 Another simple functional form for the yield surface is variously attributed to Huber, Hencky and Von Mises (53).

This theory states that plastic yielding begins when the strain energy of distortion given by W_D,

$$W_D = \frac{1+\mu}{6E}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$

reaches a critical value. For a material with a pronounced yield point in simple tension σ_{yp} we have $\sigma_1 = \sigma_{yp}$ and $\sigma_2 = \sigma_3 = 0$. Substitution into the above formula gives

$$W_D = \frac{1+\mu}{3E}(\sigma_{yp})^2$$

Thus the condition of yielding based on the distortion energy theory is

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2(\sigma_{yp})^2$$

A useful form of the theory is obtained by considering the octahedral shearing stress. An octahedral plane is obtained by passing a plane through the unit points on the principal axes. Thus it is normal to a space diagonal in σ_1, σ_2, σ_3 space.

The normal octahedral stress

$$\sigma_{oct} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = \frac{1}{3} I_1$$

where I_1 is the first stress invariant.

The octahedral shearing stress is

$$\tau_{oct} = \frac{1}{3}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

Thus $$9(\ _{oct})^2 = 2(\sigma_{yp})^2$$

and hence $$\tau_{oct} = \frac{\sqrt{2}}{3}\sigma_{yp}$$

This equation is thus a statement of the maximum energy of distortion theory. The theory further shows that at the plastic limit the octahedral shearing stress in the material is constant, which depends on the yield point of the material in simple tension or compression. The yield stresses in simple tension and compression are thus assumed to be equal.

The yielding surface defined by this theory is a straight circular cylinder whose axis coincide with the space diagonal $\sigma_1 = \sigma_2 = \sigma_3$. Since planes normal to the axis of cylinder are octahedral planes, the radius of the cylinder equals the octahedral shearing stress. The radius of the cylinder is therefore $\frac{\sqrt{2}}{3\cancel{\sigma_{yp}}} \cdot \sigma_{yp}$

5.6.4 The failure theory proposed by Mohr (56) followed the earlier work of Coulomb and Navier which considered the state of failure as a shear failure. As it turns out, both the Coulomb-Navier theory and the extended maximum shear stress theory are special cases of the Mohr's theory.

The theory, which considers failure by both yielding and fracture (assuming slippage as a mode of failure), provides a functional relationship between normal and shear stresses on the failure plane i.e.,

$$\tau = f(\sigma)$$

where τ = shear stress along the failure plane

σ = normal stress across the failure plane.

From the two parameters nature of the theory, the curve defined by this functional relationship may be plotted on τ, σ plane. Since changing the sign of τ merely changes the direction of failure but not the condition for it, the curve must be symmetrical about the σ axis. The curve so obtained, which is termed the Mohr rupture envelope, represents the locus of all points defining the limiting values of both components of stress (τ and σ) in the slip planes under the different state of stress σ_1, σ_2, σ_3 to which the material may be subjected. The Mohr envelop thus reflects a property of the material which is independent of the stresses imposed on the material. The theory is attractive for use in studying the shear strength of soils since there is no requirement that the material obey Hooke's law or that Poisson's ratio be constant; also the strength and stiffness of the material in tension and compression need not be equal.

Mohr's hypothesis states that failure depends upon the stresses on the slip planes and failure will take place when the obliquity of the resultant stress exceeds a certain maximum value. It is also stated that "the elastic limit and the ultimate strength of materials are dependent on the stresses acting on the slip planes".

The Mohr representation of stresses acting on the three principal planes are shown in Figure 5.5 . Stresses on any plane within the body must lie within the shaded area.

The slope of the line joining the origin and point A gives the obliquity of stress. The maximum inclination of stress will be given by the tangents to the largest circle. Failure occurs on planes where stresses are represented by points B and C. These stresses act on planes which are parallel to the diameter of the intermediate principal stress. Therefore the diameter of the largest Mohr circle and the magnitude of the stresses at points B and C are independent of the intermediate principal stress, σ_2.

With the assumption that σ_2 is the intermediate principal stress, the limiting state of stress will be of diameter $(\sigma_1-\sigma_3)$ and the centred $(\sigma_1+\sigma_3)/2$ along the σ axis, taking due account of the algebraic sign of the stresses. Since the two parallel sets of slip planes which occur where an isotropic specimen has been stressed slightly beyond plastic yielding by a state of homogeneous stress are symmetrically inclined with respect to

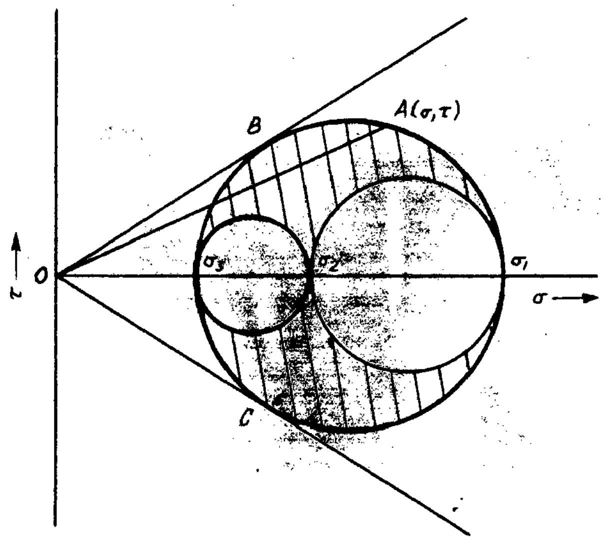


Figure-5·5. Properties of a Straight Line Mohr Rupture Envelope

the directions in which the major and minor principal stresses act, and the two plane systems intersect each other along the direction in which the intermediate principal stress acts. Mohr assumed that the intermediate principal stress is without influence on the failure of a material. Accordingly, some point on theperimeter of the circle of diameter $(\sigma_1-\sigma_3)$ must represent the limiting stress conditions. The theory thus affords a method of devising a failure theory for a specific material, i.e., establishing its Mohr rupture envelope, from actual test results.

In practice, a series of similar specimens is subjected to different stresses and brought to failure. The various Mohr stress circles are plotted for the limiting states of stress, and the unique failure stress on the failure plane for each test is taken as the point of common tangency between a smooth limiting curve, or envelope, and the various $(\sigma_1-\sigma_3)$ circles. By taking the points of common tangency as representative of the σ, τ stresses on the failure plane, a state of homogeneous stress in an isotropic material is assumed. Due to experimental shortcomings however, one may not necessarily obtain a state of homogeneous stress and thus the inferred stresses at the point of common tangency for the envelope, obtained experimentally, will not in all likelihood represent the actual stresses on the failure plane. It follows that the predicted location of the failure plane based on the common tangency points might be in error. The possible discrepancy between the actual Mohr rupture envelope and an experimentally obtained envelope is shown in Figure 5.6.

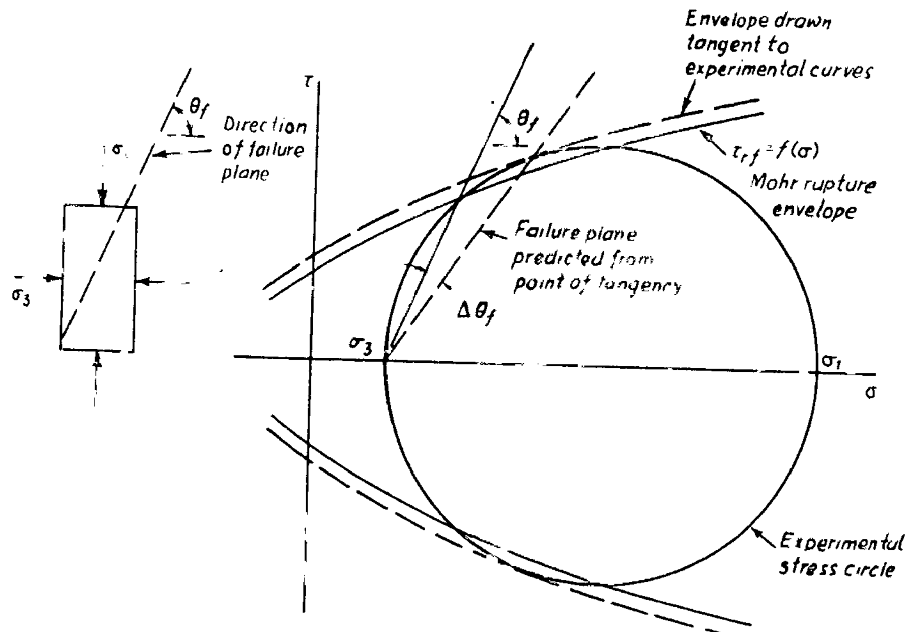
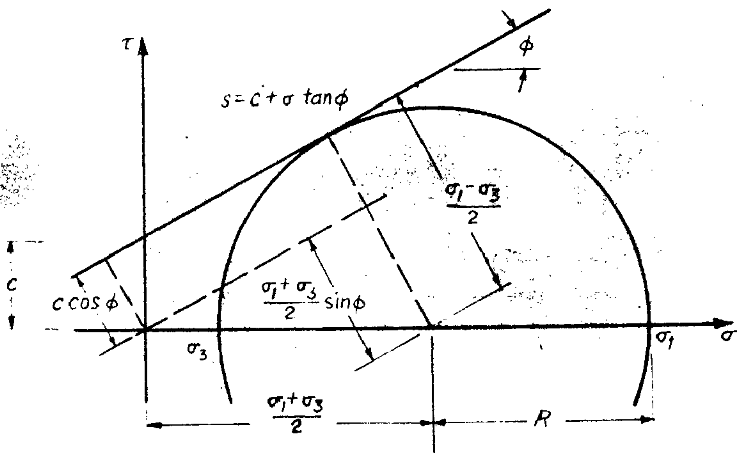


Figure – 5.6 Mohr's Failure Theory Plotted in the τ-σ Plane

Actual Mohr rupture envelopes are often curves. However, for soils the curvature is usually not great and it has proved useful to approximate the envelope by a str aight line, at least over a limited range of normal pressure. The equation of a straight line in the τ, σ-plane is the Coulomb's equation

$$= S = C + \sigma \tan \phi$$



Notes 1. σ & σ3 are limiting effective stresses at failure
2. Compressive stresses considered positive ~~in deriving~~ Eq.(a·42)

Figure - 5·7. Mohr Representation of Stresses in Three-Dimensional System

From the Figure 5.7, $R = \frac{\sigma_1 - \sigma_3}{2} = C \cos \phi + (\frac{\sigma_1 + \sigma_3}{2}) \sin \phi$ -(1)

The parameters C and ϕ refer to the cohesion and "angle of internal friction" of the soil. These shear strength parameters have been subject of much research.

The equation (1) may be manipulated in many ways to state the failure criterion in various forms. By adding $\sigma_3 \sin \phi$ to both sides of Eqn. and rearranging we get

$$\frac{1}{2}(\sigma_1 - \sigma_3)(1 - \sin \phi) = C \cos \phi + \sigma_3 \sin \phi$$

This was used by Skempton and Bishop, gives a straight line when $\frac{1}{2}(\sigma_1 - \sigma_3)$ is plotted against σ_3.

By multiplying both sides of Eqn. (1) by 2 and rearranging terms we get,

$$\sigma_1(1 - \sin \phi) = 2C \cos \phi + \sigma_3(1 + \sin \phi)$$

which gives straight lines when σ_1 is plotted against σ_3. This equation has been used as a plotting method by Rendulic and more recently by Henkel.

Expressed in its most general form the failure surface corresponding to the Mohr-Coulomb condition of failure is

$$\left[(\sigma_1 - \sigma_3)^2 - (2C \cos \phi + (\sigma_1 + \sigma_2) \sin \phi)^2\right]$$
$$\times \left[(\sigma_2 - \sigma_3)^2 - (2C \cos \phi + (\sigma_2 + \sigma_3) \sin \phi)^2\right]$$
$$\times \left[(\sigma_3 - \sigma_1)^2 - (2C \cos \phi + (\sigma_3 + \sigma_1) \sin \phi)^2\right] = 0$$

The failure surface defined by this equation is a pyramid with the space diagonal $\sigma_1 = \sigma_2 = \sigma_3$ as axis and a cross-section which is an irregular hexagon with non-parallel sides of equal length. The projection of this irregular hexagon on the plane $\sigma_1 + \sigma_2 + \sigma_3 =$ const. is shown in Fig. 5.8.

5.7 The theories of Tresca and Von Mises may be extended to include the effect of the intermediate principal stress. Thus the extended Tresca condition can be represented by the three dimensional equation:

$$\left\{(\bar{\sigma}_1 - \bar{\sigma}_2)^2 - \left[\bar{C} + \frac{K_z}{3}(\bar{\sigma}_1 + \bar{\sigma}_2 + \bar{\sigma}_3)\right]^2\right\}$$
$$\times \left\{(\bar{\sigma}_2 - \bar{\sigma}_3)^2 - \left[\bar{C} + \frac{K_z}{3}(\bar{\sigma}_1 + \bar{\sigma}_2 + \bar{\sigma}_3)\right]^2\right\}$$
$$\times \left\{(\bar{\sigma}_3 - \bar{\sigma}_1)^2 - \left[\bar{C} + \frac{K_z}{3}(\bar{\sigma}_1 + \bar{\sigma}_2 + \bar{\sigma}_3)\right]^2\right\} = 0$$

where $\bar{\sigma} = (\sigma - u)$ i.e., effective stress,

K_z = constant related to sin ϕ

C = effective cohesion

Following a similar development from the two dimensional case, the extended Von Mises condition is represented by the three dimensional equation

$$(\bar{\sigma}_1 - \bar{\sigma}_2)^2 + (\bar{\sigma}_2 - \bar{\sigma}_3)^2 + (\bar{\sigma}_3 - \bar{\sigma}_1)^2$$
$$= \left[\bar{C} + \frac{K_1}{3}(\bar{\sigma}_1 + \bar{\sigma}_2 + \bar{\sigma}_3)\right]^2$$

where K_1 is constant related to sin ϕ

C = effective cohesion

The shape of the right section of the three dimensional failure surfaces in σ_1, σ_2, σ_3 co-ordinates is shown in Figure 5.8

The shape of the section corresponding to the Mohr-Coulomb condition of failure is an irregular hexagon AC' BA' CB' and the surface in space is a hexagon pyramid with its apex at the origin of the system of co-ordinates.

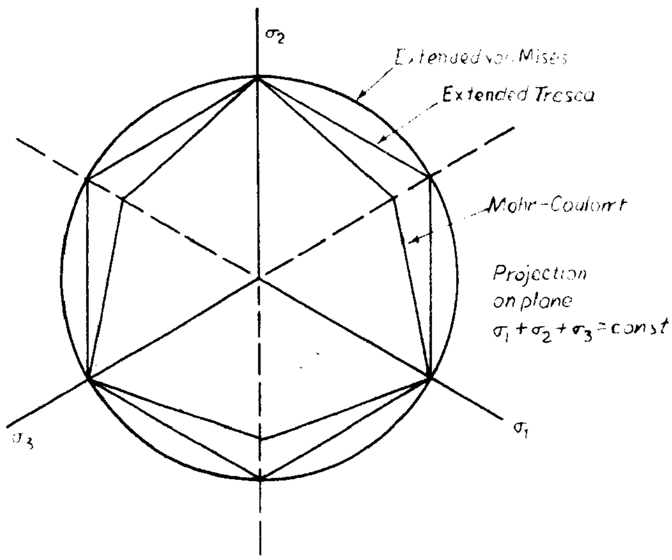


Figure 5·8. Failure Surfaces—Mohr Coulomb and Extended Yield Criteria

The surface given by the extended Tresca condition is a hexagonal pyramid and has its apex at the origin and its axis along the space diagonal. Right sections ofthe surface are regular hexagons and coincide with the Mohr-Coulomb surfaces at A, B and C.

The extended Von Mises condition gives a failure surface represented by a cone with its apex at the origin and its axis lying along the space diagonal. Right sections of the surfaces are circles and coincide with the Mohr-Coulomb surface acting through major principal stress directions at A, B and C.

5.8 RECENT INVESTIGATIONS IN STRENGTH THEORIES

The proposal by Jennings and Kirchman (38) to replace the current Mohr-Coulomb theory with a modified form of the extended Von Mises theory requires two assumptions:

1. The elastic Modulus E is a linear function of the mean principal stress. σ_m in the material.

2. Yielding takes place in material when the energy of distortion reaches a critical value which is constant for the material.

Then the critical strain energy eqn. is

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = \frac{6W_d E}{1+\mu}$$

$$= \frac{6W_d}{1+\mu}\left[\frac{a}{3}(\sigma_1 + \sigma_2 + \sigma_3) + b\right]$$

where a and b are constants.

With this theory, experiments with a mixture of Kolin and sand showed a good agreement between friction angles measured on the rupture envelope and on the failure plane.

The very substantial difference in result obtained by various investigators emphasises the difficulties encountered in performing triaxial extension tests. The theory by Jennings and Kirchman has not been tested sufficiently to warrant general application.

5.9 Assuming time dependence, size effects relative to continuity, homogeneous and isotropy we have presented so far the various failure criteria which define the statical determinacy of the complex state of stress. Thus it is now left for us to determine which of these could be applied to soils. To define the statical determinacy there are two approaches possible.

(1) To arbitrarily accept one of the above discussed failure criteria to be applicable to soils and then to obtain from simple lab. tests consistent numerical values of parameters of the soil in accordance with the chosen criteria.

(2) To subject the soil specimens to known states of loading, varying the loading. Determine the boundary loadings at which failure takes place or yield will just impend. Correlation and analysis of data so obtained to establish failure criteria.

It would be optimistic to expect that in both approaches the invariant nature of yield criterion should prevail, and with experiments under the simple conditions which can be realised in the laboratory, we should be able to define the behaviour of soils under complex field situations. This necessitates additional stipulation to ensure the invariance of an yield criterion. These are matters that have long taxed workers in soil mechanics and even at present are not completely understood. To overcome this difficulty i.e., to provide correspondence, the soil specimen under study should be as representative as possible of the soil in the field and the test should be conducted under the same hysteresis and drainage conditions as is actually found in the field. Every effort should be made to preserve the degree and distribution of density, moisture content and consolidation characteristics of the soil.

The various tests to determine the strength parameters are discussed separately.

5.11 For discussion of the applicability of any of the failure criteria discussed already the test data of Kirkpatrick (39) with hollow cylinder test is excellent. The experiments using the hollow cylinder conducted by Wu, Loh and Malvern in 1963 (93) further support the data of Kirkpatrik. The yield surface obtained by these test datas are shown to correspond to values obtained from standard triaxial tests and the Mohr-Coulomb criterion.

Figure 5.9 shows the results obtained by Wu et al.

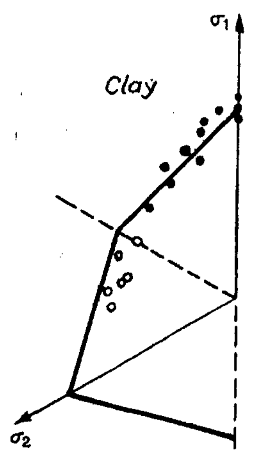


Fig.5.9

Based on the experimental data, several modifications to the three existing yield criteria have been suggested. These are shown in Figure 5.10.

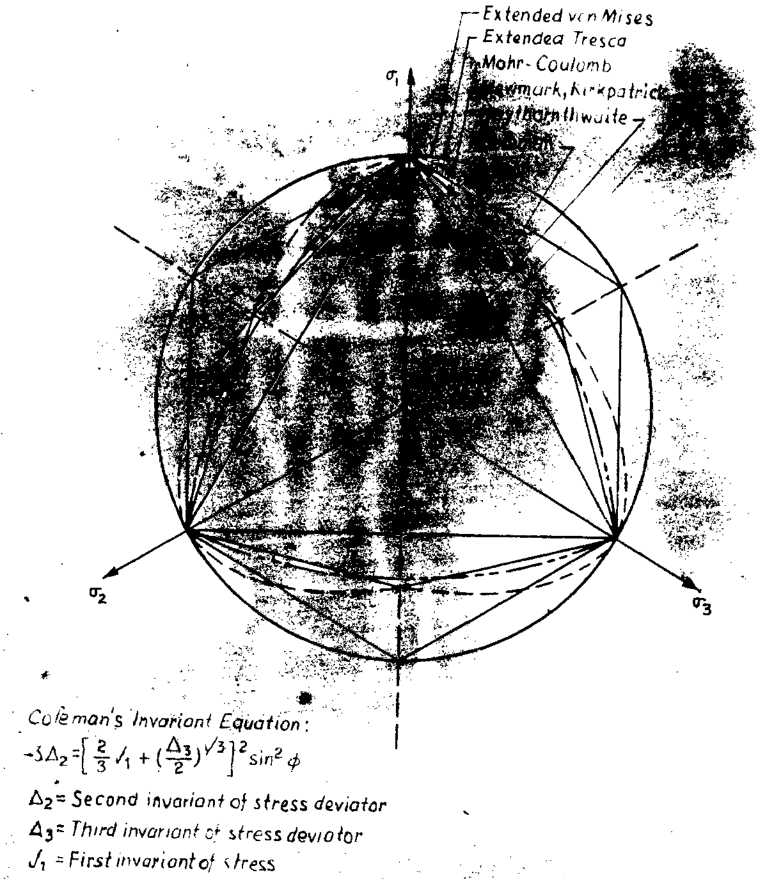


Figure 5.10. Yield Criteria Suggested for Soils Plotted on an Octahedral Plane

Newmarks (58) and Kirkpatrick (39) suggest a slightly rounded shape for the yield surface, following quite closely the outline of the Mohr-Coulomb criterion, which would thus become a three parameter criterion. Haythornthwaite (31) suggests a triangular section as the limiting shape which still satisfies the condition of convexity of the yield surface required by plastic theory. Coleman (21) suggests a yield surface that incorporates three stress invariants. However the yield surface is not entirely convex and thus fails to satisfy strain conditions if the material were to behave plastically.

All hellow cylinders test results are seen to deviate but slightly from the yield surface predicted by the Mohr-Coulomb theory.

A.W. Bishop (7) in his sixth Rankine lecture "On strength of soils as engineering materials" has stated that the experimentál results strongly supported Mohr-Coulomb criteria, hence concluded that Mohr-Coulomb criteria as the simple criteria of reasonable generality applicable to soils.

However some experts in the field are not very happy about this recommendation.

Hankel, Roscoe et al., have established certain criteria based on the experimental results on weald clay in a triaxial apparatus. These are discussed separately in Chapter 7.

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6 STRENGTH THEORIES - STRENGTH PARAMETERS

6.1 Strength theory can be classified into two types of researches:

(i) Macroscopic researches on the mechanical behaviour of clay mass in relation with various external factors and strength.

(ii) Microscopic researches on the behaviour of adsorptive film and structural skeleton of clay.

The former have been developed by Terzaghi, Casagrande, Taylor, Rendulic, Rutledge, Hvorslev, Skempton, Bjerrum, Bishop, Henkel, Whiteman, etc., the latter by Lamb and Rosenquist.

Macroscopic rules constitute necessary and sufficient condition when applied in stability calculation of slope, earth pressure, bearing capacity and others, while the microscopic physico-chemical rules constitute merely the background of macroscopic rules.

6.2 As such strength theories have been developed by macroscopic researches to set up stability analysis, the form of Coulombs strength formula $S = C + \sigma \tan \phi$ shall be the first aim to be reached which have been the basis of all calculation methods since a long time ago, such as earth pressure calculation by Coulomb and Rankine, slope stability analysis by slip circle method and bearing capacity calculation by Terzaghi (85), Meycroft (51) and others. In other words the value of C and ϕ are the foci of discussion.

The practical conclusion will be expressed that the "values of C and Ø will be obtained by shear test with same stress hysteresis and drainage condition as in field of which general trends are shown in Fig. 6.1.

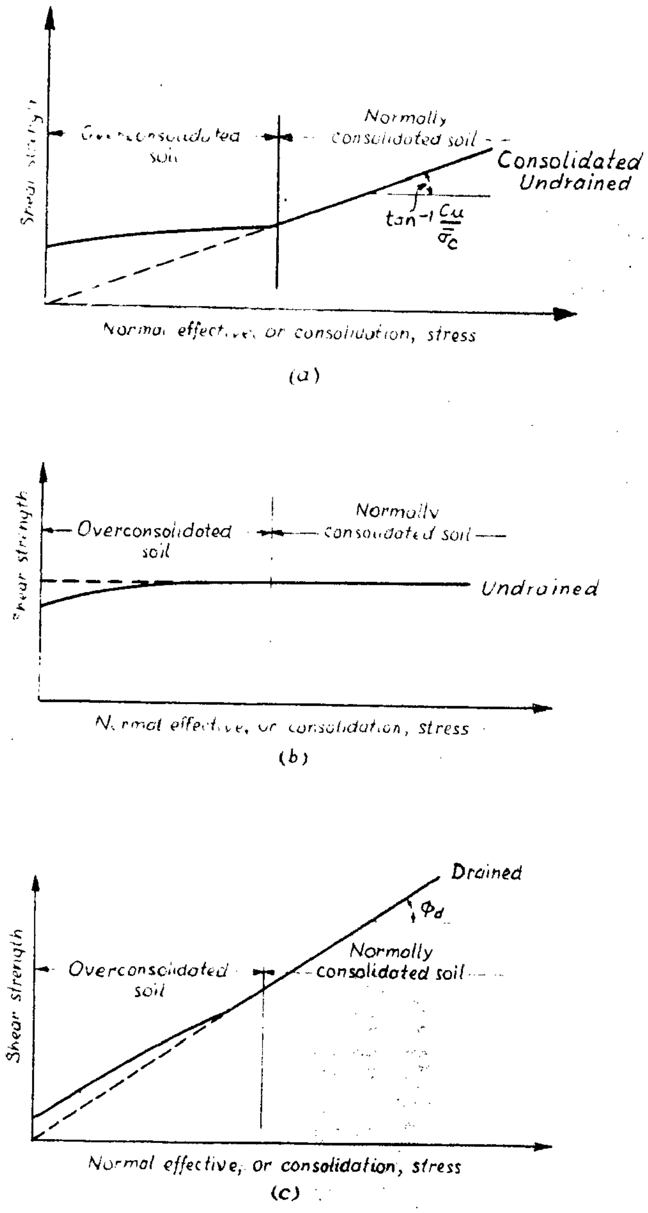

Figure 6.1 Comparison of strengths obtained in consolidated undrained, undrained, and drained tests.

However it will not be easily agreed to call C and Ø determined in each case as cohesion and angle of internal friction respectively.

It seems unreasonable that the shear strength of same clay can be differently divided depending on different points of view into cohesion and internal friction. In some cases strength of clay turns to cohesion only, or angle of internal friction only, and in other cases partly to cohesion and internal friction. This feeling is natural and thought to be the starting point of most researches.

6.3 Most of the research workers have agreed to call various combinations of C and ϕ as apparent cohesion and apparent angle of internal friction respectively under corresponding drainage conditions and on the other hand to call C_e and ϕ_e which are thought constant and proper to a given clay under a given condition as true cohesion and true angle of internal friction, on the assumption that shear strength of clay can be divided into constant cohesion C_e and a constant frictional resistance $\sigma' \tan \phi_e$ where σ' denotes effective normal stress working on shear plane of failure. These theories can be summarized as follows:

(i) Taylor (84), Casagrande and Wilson (18), assume that ϕ_d at drained shear test is the true angle of internal friction. Therefore true cohesion becomes zero and the shear strength is consibred to be replaced by frictional resistance only

$$\tau_f = \tan \phi_d.$$

(ii) Rutledge (68) assumes that shear strength of clay may be considered to be caused by cohesion only, because he has found a linear relation between void ratio e and log τ_f at shear failure of normally consolidated clays regardless of type of test.

(iii) Krey and Tiedemman (40) assume that drained shear curve render the pressure less than pre-loading pressure σ_p may be replaced by straight line and this line will give cohesion C and the angle of internal friction ϕ of clay considered under σ_p.

(iv) Hvorslev assumed that shear strength of clay τ_f will be determined by void ratio at shear failure e_f and effective normal stress σ' and he called cohesion determined by e_f and angle of internal friction determined by σ' axis as true cohesion and true frictional resistance $\sigma' \tan \phi_e$.

$$\tau_f = C_e + \sigma' \tan \phi_e$$

This concept explains well the similarity of hysteresis curve of $\sigma - e$ and $\sigma - \tau_f$ below pre-loading pressures.

(v) Terzaghi (85) believes that the angle of shear plane determined by unconfined and triaxial tests determine the true angle of internal friction ϕ_e and therefore agrees with Hvorslev's true angle of internal friction.

What will determine the strength of clay will make clear respective characters of the theories.

(i) Taylor and Casagrande (in case of drained test) ...... effective normal stress at failure.

(ii) Rutledge (in case of normally consolidated clay ...... void ratio (or moisture content) at failure.

(iii) Krey and Tiedmaunn (in case of drained strength) ...... pre-loading pressure and effective normal stress.

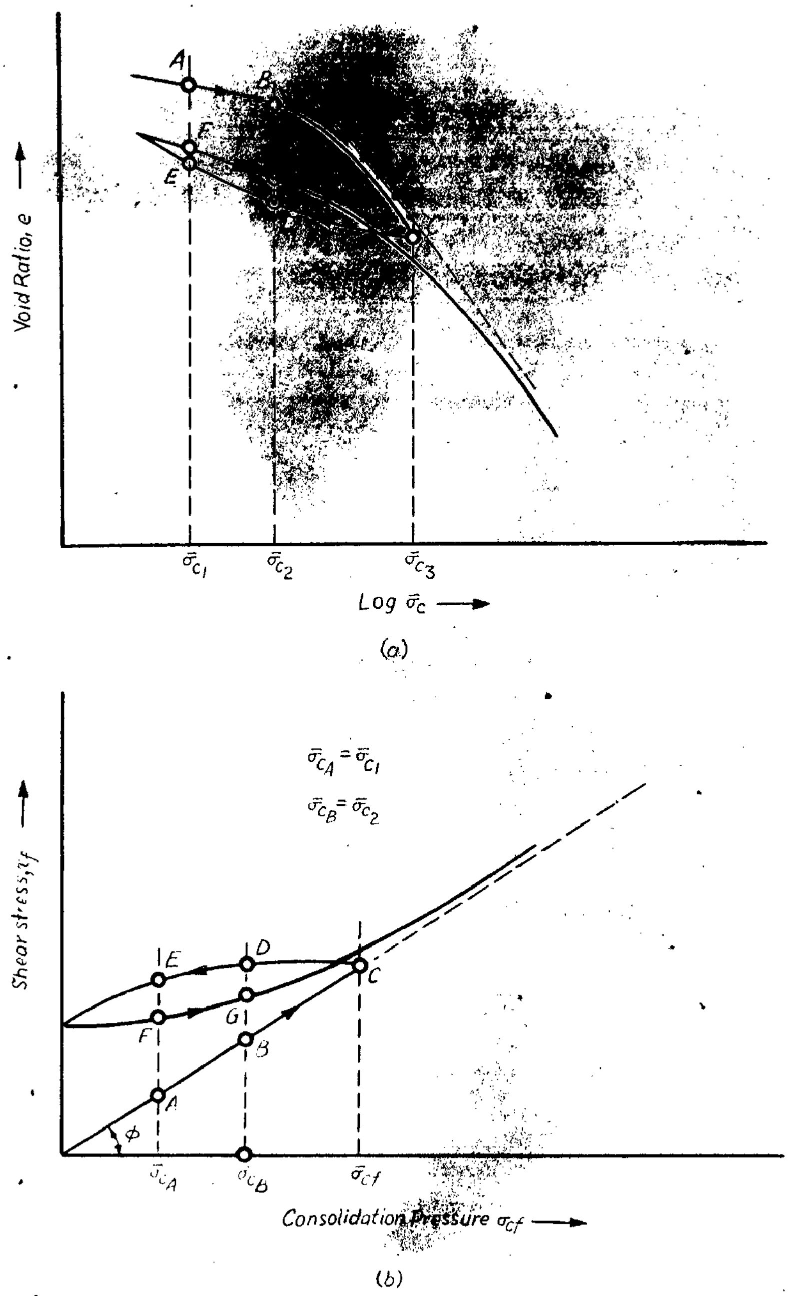

Figure - 4.2. (a) e-Log $\bar{\sigma}_c$ Curve for Samples Consolidated for Drained Shear Test, (b) Drained Shear Test Results for the Consolidated Samples .

(iv) Hvorslev (in all cases) ..... void ratio at failure and effective normal stress.

As judged from the applicable scopes in the parenthesis above, only Hvorslev's theory is the generalised rule. This theory has been widely agreed because of such generality, trueness and Terzaghi's agreement.

6.4 SHEAR STRENGTH PARAMETERS

Theories of strength of clay had long been far away from practical use sticking to the problem of cohesion and frictional force. But $C'-\phi'$ analysis has been developed for practical method based on the principle of effective stress.

Fundamental formula of this analysis is,

$$\tau_f = C' + \sigma' \tan \phi'$$

or $$\tau_f = C' + (\sigma - u) \tan \phi'$$

where C' = effective cohesion intercept

ϕ' = effective angle of shearing resistance or friction

σ = Total normal stress

u = pore pressure

In terms of total stresses the equation takes the form

$$\tau_f = C_u + \sigma \tan \phi_u$$

where C_u = apparent cohesion

and ϕ_u = apparent angle of shearing resistance or friction.

The defintion and the basis for the expression of effective stress parameters may be summarized.

"The fact that C and ϕ vary with drain condition is all attributable to the variation of effective stress" and $C'-\phi'$ analysis has the concept that "shear strength of clay is determined by effective normal pressure on shear plane at failure.

The effective stress parameters defined above are not unique. The most important factors which affect them are (1) void ratio at failure, (2) stress history, (3) structure. The effective stress parameters can be determined by a consolidated undrained test with pore pressure measurement or a drained test in the triaxial or a direct shear machine.

For normally consolidated clays the envelope is a straight line passing through the origin and $C' = 0$. This shows that for clays which are consolidated from a slurry the shear strength does not depend on water content but only on the effective stress. For over consolidated clays the intercept gives the value of C'. For some clays the value of ϕ' is not affected by structure, and it has the same value for remoulded and undisturbed stages and is not influenced by the manner of consolidation isotropic or unisotropic. But the evidence of Hirsohfied as reported by Whiteman (90) shows that ϕ' determined from undrained tests is affected by remoulding.

The strength envelops determined from consolidated undrained and drained tests are practically the same for normally consolidated clays, provided comparable rates of testing are used. Simons (75) has however reported difference in results between drained and undrained strengths.

The effective stress parameters do not take into account the effect of over consolidation and different vales are obtained for different values of over consolidation ratios.

While the shear strength parameters defined by the three types of shear test according to drainage and loading condition in the field are very well suited for practical use in Engineering problems, the fundamental strength property of soils are sometimes studied in terms of Hvorslev strength parameters.

6.5 Hvorslev carried out investigation on the variation of shear strength of soil at a constant void ratio at failure assuming the structure to be same. He has defined more fundamental parameters as follows

$$\tau_f = C_e + \sigma' \tan \phi_e$$

where C_e is the "true cohesion" corresponding to void ratio at failure and ϕ_e is the "true angle of shear resistance".

C_e has been found to be function of void ratio and ϕ_e to change only slightly witn void ratio.

Where C_e = true cohesion and defined by

$$\frac{dC_e}{dW_f} = \left(\frac{\partial \tau}{\partial W_f}\right)_{\sigma'}$$

and ϕ_e = true angle of shearing resistance defined as the rate of increase of strength with effective normal stress as constant water content.

$$\tan \phi_e = \left(\frac{\partial \tau_f}{\partial \sigma_1}\right)_{W_f}$$

Actually ϕ_e decreases slightly with the increase of moisture content. At any water content W, the true cohesion is directly proportional to the equivalent consolidation pressure σ_e'

$$C_e = K \sigma_e'$$

where K = Hvorslev coefficient of cohesion and σ_e' is the pressure on virgin consolidation curve at void ratio corresponding to the water content at failure. Therefore the equation can be written as

$$\tau_f = K \sigma_e' + \sigma' \tan \phi_e \,.$$

The true cohesion plotted against water content gives a unique straight line independent of the initial water content, stress history or method of test. Instead of the water content,

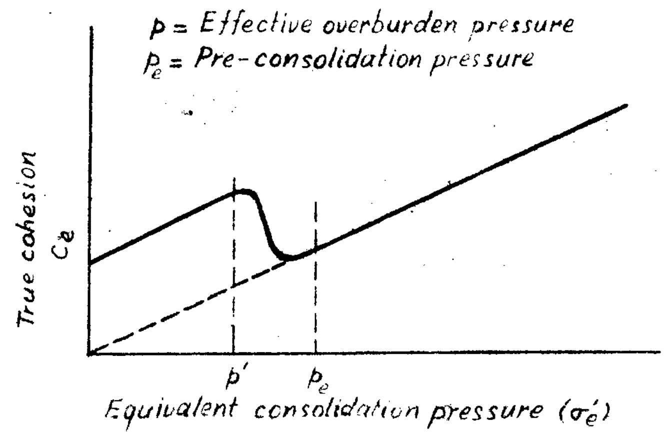

Figure - 6 3.

the equivalent consolidation pressure can be used. In this case the curve of true cohesion plotted against equivalent consolidation pressure is a straight line passing through the origin. This however is true only for clay consolidated from a high initial water content below the liquid limit. Bjerrum (10) has found that $C_e = C_o + K\, \sigma_e'$ where C_o is the initial cohesion for zero consolidation pressure. Deviation from this relation has been observed by Bjerrum and Wu (13) in the case of Lila Edet clay. There is a sudden reduction in true cohesion for decreasing water content in the range of the pre-consolidation stress. This observation has been explained as the threshold effect of rigid bonds between particles which are susceptible to break down for pressures exceeding a critical value. A correlation is found to exist between threshold effect and pre-consolidation effect observed in consolidation tests.

Crawford (23) suggests that true cohesion is due to an equivalent "intrinsic pressure" σ_1 acting through the true angle of internal friction ϕ_e

$$C_e = \sigma_1 \tan \phi_e$$

The intrinsic pressure is not immediately reversible but decreases with time. Thus true cohesion may be time dependent as shown by the decrease of cohesion in the laboratory and field.

A good correlation has been found to exist between the true angle of internal friction and the plasticity index of soils. The curves are different for undisturbed and remolded samples.

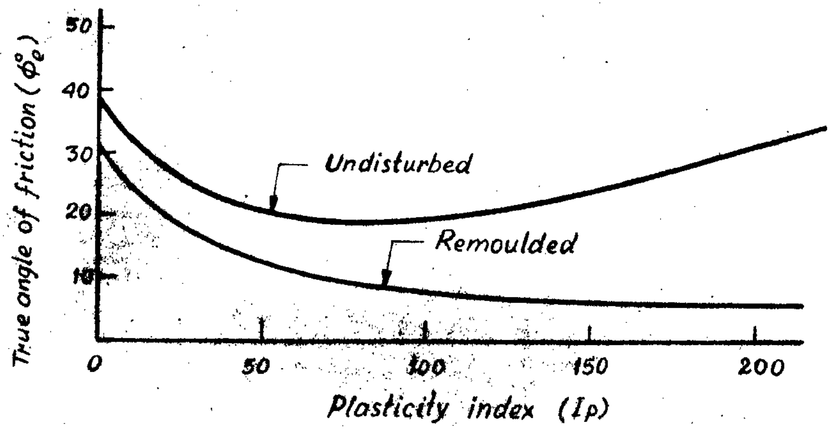

Figure_6.4

6.6 Hvorslev's parameter give a convenient link between soil samples of different stress history. They are not so varying as either the undrained shear strength or effective stress parameters and could be used as fundamental parameters for a particular clay. Ladd and Lambe (42) have suggested a method for getting the correct undrained shear strength of a soil sample (truly undisturbed) from the results on samples with some disturbance making use of Hvorslev's parameters. Noorany and Seed (59) have also similarly suggested expressions making use of these functions to get the insitu shear strength from the results of laboratory tests on samples.

6.7 The application of Hvorslev's parameters to assess the shear strength of soil is, however, more involved than application of effective stress parameters. These require the determination of the water content or void ratio at failure as well as the effective stresses. The failure effective stresses and void ratios can be found from initial conditions and nature of stress path by the use of Henkel's curves (33). These curves have been derived only for remolded soils and their applicability to undisturbed soils has not yet been proved.

Another setback with Hvorslev's parameters is their determination in triaxial tests, because of the difficulty of defining a failure criterion and accurate determination of the final void ratio of the sample. The choice of the equivalent consolidation pressure σ_e' is not easy because of the varying initial conditions of the soil. The values of the parameters are hypersensitive to experimental errors.

6.8 Experiments by Bjerrum and Simons (12) and Lo (50) indicate that the Hvorslev parameters have widely different values for different states. The values of ϕ_e for undisturbed soils are higher than those for remolded soils. Olson (60) expresses doubt about this. Wu (91) has found that the true cohesion is greater for undisturbed soils than remolded ones. Newland and Alleby (57) have reported that clay acquires a sensitivity as a result of consolidation and have explained the difference in undrained strength of the consolidated and remolded samples to be partly due to the difference in the true parameters for the unconsolidated and remolded states. They have suggested that true cohesion of the as consolidated sample may be higher than that of the remolded sample. Parry (62) suggested that there may not be such a difference in the true parameters and the difference may be explained by the pore pressure developed in the different states causing different values of effective stresses. This observation has been supported by the experiments of Skempton and Sowa (81) who have found that the sensitivity could be explained by the different pore pressure behaviour of remolded and consolidated samples. It has been observed that the cohesion of a flocculant structure exceeds that of the dispersed structure even at the same void ratio and this is more true of sensitive clays. The separation of the shearing resistance into Hvorslev components involves the assumption that two different effective stresses are possible in a soil of the same void ratio and structur e. Different effective stresses existing in a sample at a given void ratio are associated with different structures and the existence of identical structures at different effective stresses may be an impossible condition specially for flocculated soils - Mitchel (55).

Figure (6.5) explains this aspect (Scott,(72)). The curves 1 and 3 show the variation of void ratio with effective stress for the random and oriented states respectively, whereas curves 2, 4 and 5 for other intermediate states of structure. The ultimate state of soil under all tests conforms to curve 3. Large

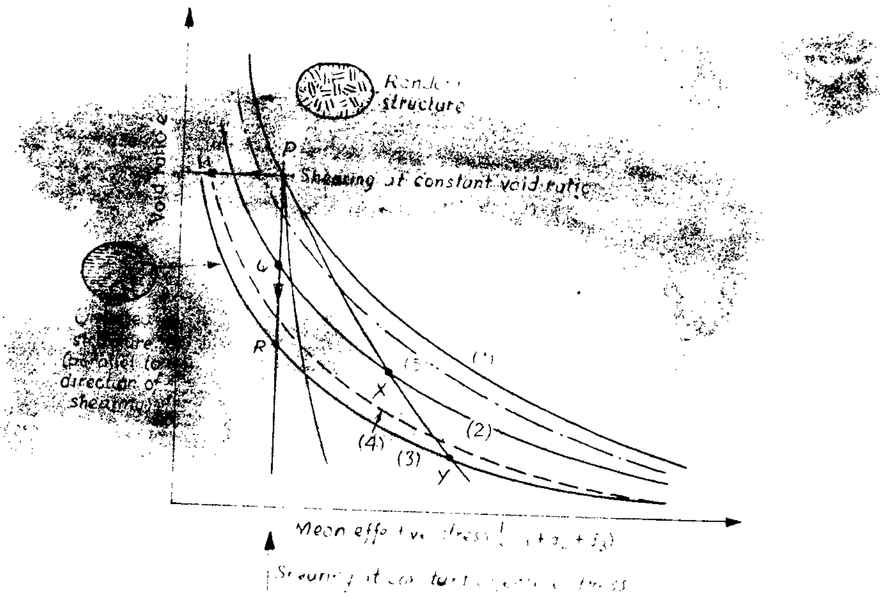

Figure-6.5. Change of structure and void ratio under shearing and normal effective stresses

deformations prior to achieving maximum shear stress, will mean that the failure curve will be near curve 3 like curve 4. If only slight deformation occurs before failure, the failure curve will be near curve 1, like curve 5. Thus the structure of the soil at failure will be dependent on the degree of deformation. The reported invariance of the friction angle for the consolidated and remolded states by Skempton and Sowa may be due to the high failure strain of the samples and is the case with Hvorslev's experiments with Vienna and Little Belt clays. The consolidated and remolded soils essentially attain the same structure at failure on account of high failure strain. However the matter deserves further study.

6.9 GENERALISED COMPONENTS OF SHEAR RESISTANCE

Schmertmann (69) has developed a new type of triaxial compression test known as cohesion-friction-strain test or C.F.S. test, which is intended to determine the cohesion and friction components of the total shear resistance at selected values of strain during a compression test. He found that the components cohesion and friction are strain dependent as a result of his experiments using C.F.S. technique.

In the C.F.S. test, the shear resistance of soil is assumed to be made up of two components, namely D_{ϵ} and I_{ϵ}. The D_{ϵ} is the dependent component of shear resistance mobilised on any plane and at any strain ϵ. This component is dependent on the effective stress on that plane according to the equation

$$D = \left[\sigma' \left(\frac{\Delta \tau}{\Delta \sigma'}\right)\right]_{\epsilon} \quad {}_{\sigma' \rightarrow 0} = \left[\sigma' \left(\frac{d\tau}{d\sigma'}\right)\right]_{\epsilon}$$

where ~~the~~ at strain ϵ the shear resistance on that plane changes by $\Delta\tau$ due to $\Delta\sigma'$. As there must be no change in structure, $\Delta\sigma'$ must approach zero. The independent component I is defined by

$$I_{\epsilon} = \tau_{\epsilon} - D_{\epsilon}$$

where τ_{ϵ} is the total shear resistance mobilised at ϵ on the plane considered.

These components can be obtained by one specimen test which is a unique advantage while testing undisturbed soils. These components are expected to be related to fundamental cohesion and friction.

The evaluation of these components is done by obtaining the stress strain curve, subjecting identical samples of soils to two effective stresses. These stresses are only slightly different in order to have essentially the same structure. The effective stresses are held constant by manually adjusting the pore pressure corresponding to different values of principal stress difference. This gives the shear resistance mobilised at same values of strain under two effective stresses for essentially same structure. The practical difficulty of getting identical specimens has been obviated by the use of a "curve hopping" technique.

In the technique the induced pore pressure is varied to maintain a constant, preselected value of $\bar{\sigma}_1$. In the figure 6.6 points a and b are obtained during strain at σ_1 (high), after which the induced pore pressure is changed to maintain a new constant value of $\bar{\sigma}_1$ (low). Points c and d may then be recorded for this value of $\bar{\sigma}_1$ (low).

~~Points c and d may then be recorded for this value of~~ ~~σ_1 (low)~~. This procedure of altering between two chosen values of σ_1 permits the derivation of two stress strain curves from one test. Values of $(\sigma_1 - \sigma_3)/2$ and σ_3 are plotted to obtain straight line relations at various strains. If we assume Mohr Coulomb failure criterion for any one strain, rewriting K σ_p as C' both C' and ϕ' may then be obtained mathematically from the expression

$$\frac{\sigma_1 - \sigma_3}{2} = \left(\frac{\sin \phi'}{1 - \sin \phi'}\right) \bar{\sigma}_3 + C'\left(\frac{\cos \phi'}{1 - \sin \phi'}\right)$$

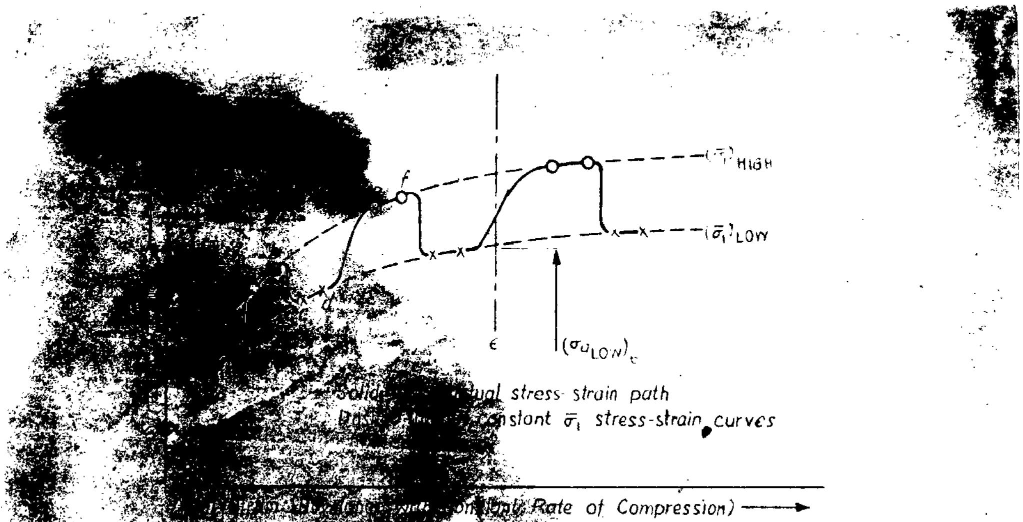

Figure – 6.5 [illegible] ique for Derivation of Strength Parameters
([illegible] nn and Hall, 1961)

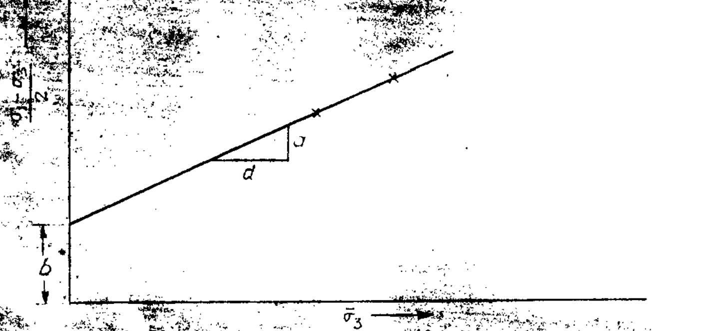

Figure – 6.6 – Computation for Cohesion and Friction Parameters
(Schmertmann and Osterberg, 1960)

The linear relationship described may be redrawn to show $(\sigma_1 - \sigma_3)/2$ and σ_3 axes thus providing

$$\text{Friction } \phi' = \sin^{-1}\left(\frac{a}{a+d}\right)$$

$$\text{Cohesion } c' = b\left(\frac{1 - \sin \phi'}{\cos \phi'}\right)$$

The variation of cohesion and friction parameters with axial compressive strains is shown. (Fig 6-7) With the curve hopping technique, we note that the cohesion component generally develops to its maximum value at a very low compressive strain, while the friction component requires a much greater strain to reach its maximum value.

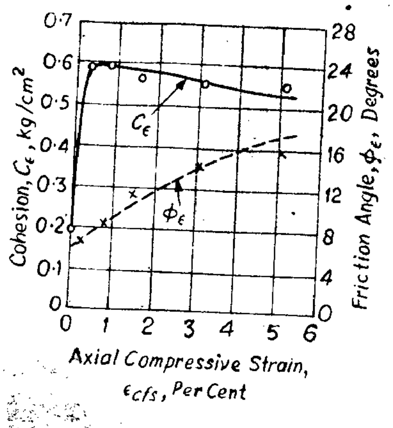

Figure – 6·7. Derivation of ϕ and C with Relation to Strain. (Schmertmann and Hall, 1961)

A comparison of these components with Hvorslev's parameters is not possible on account of differences in failure strain of normally consolidated and over-consolidated samples and failure criteria. But preliminary conclusions by Wu (92) show that D_{max} and tan ϕ_e are nearly equal.

6.10 ROWE'S STRESS DILATANCY PARAMETERS

Rowe et al. (67) proposed stress dilatancy concept and introduced the inter-particle shearing parameters C_f and ϕ_f. This is based on the fact that during loading rearrangement of particles takes place and energy corrections have to be made to separate the shear resistance. The values are obtained by making use of the equation

$$\frac{\sigma_1'}{1 + \frac{dV'}{V\epsilon_1'}} = \sigma_3' \tan^2(45 + \frac{\phi_f}{2}) + 2C_f \tan(45 + \frac{\phi_f}{2})$$

where σ_1' = Major principal stress, σ_3' = Minor principal stress

$\frac{dV'}{V}$ - Instantaneous change in volume of particle mass

ϵ_1 - Change of major principal stress.

The value of $C_f = 0$ except for anisotropically consolidated samples and the value of ϕ_f depends on the particular state of soil. Hence the value of ϕ_f is not unique. The value of ϕ_f also changes with pore fluid tremendously. The values C_f and ϕ_f are thus not fundamental even though they constitute an attempt to separate energy components from inter-particle shear behaviour.

6.11 Lamb (47) divides shear resistance into three basic components; cohesion, dilatancy and friction. The exact separation in terms of local contribution from the three components is not too distinct. Qualitatively, this is shown in Fig. 6.8.

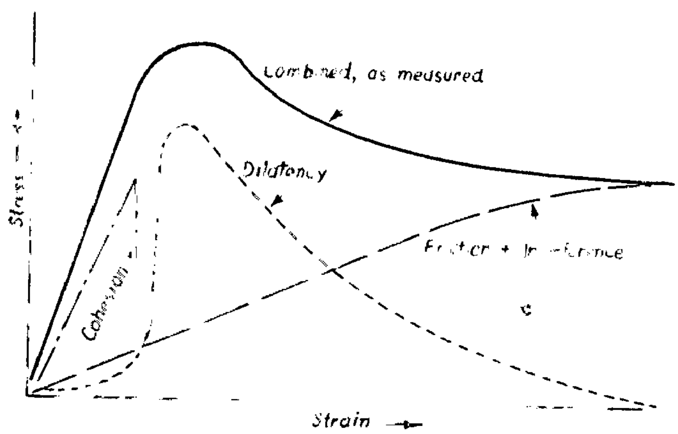

Figure - 6 8 Components of Shear Resistance (Lambe, 1960)

Cohesion is mobilized at very small strains and soon ceases to be operative. Dilation increases from zero to a maximum value, but after particle interference is overcome, this component must decrease. When the stress-strain curve approaches an asymtotic value, since cohesion and dilation are no longer important contributors to shear strength, we must conclude that friction is responsible for the continued shear strength of the soil.

6.12 The stress locus techniques used by Yong and Vey(94), which is an extension of the vector techniques used by Casagrande and Hirschfeld (17) interprets the effective strength parameters of normally consolidated clays in terms of the stress on the potential failure plane. If one defines friction as that property in the shear stress vs. normal stress, region wherein the shear stress increases with the effective normal stress, and the cohesion parameter, $\bar{C}$, as the parameter in the same region where the shear stress increases despite a decrease in the effective normal stress, the action of these parameters will be evident from an examination of the stress locus.

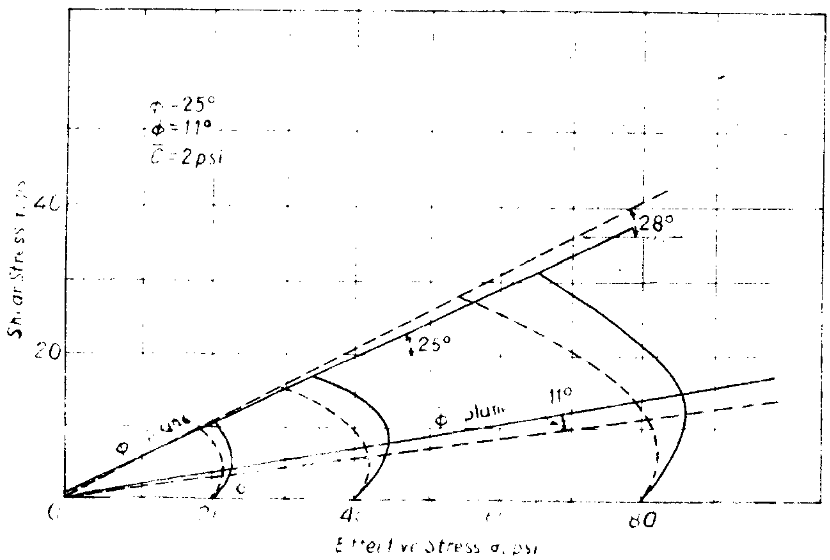

Figure – 6.9. Stress Locus for Chicago Clay (Yong and Vey, 1962)

Extensive tests by various techniques has led to the finding that cohesion and friction are dependent on strain with

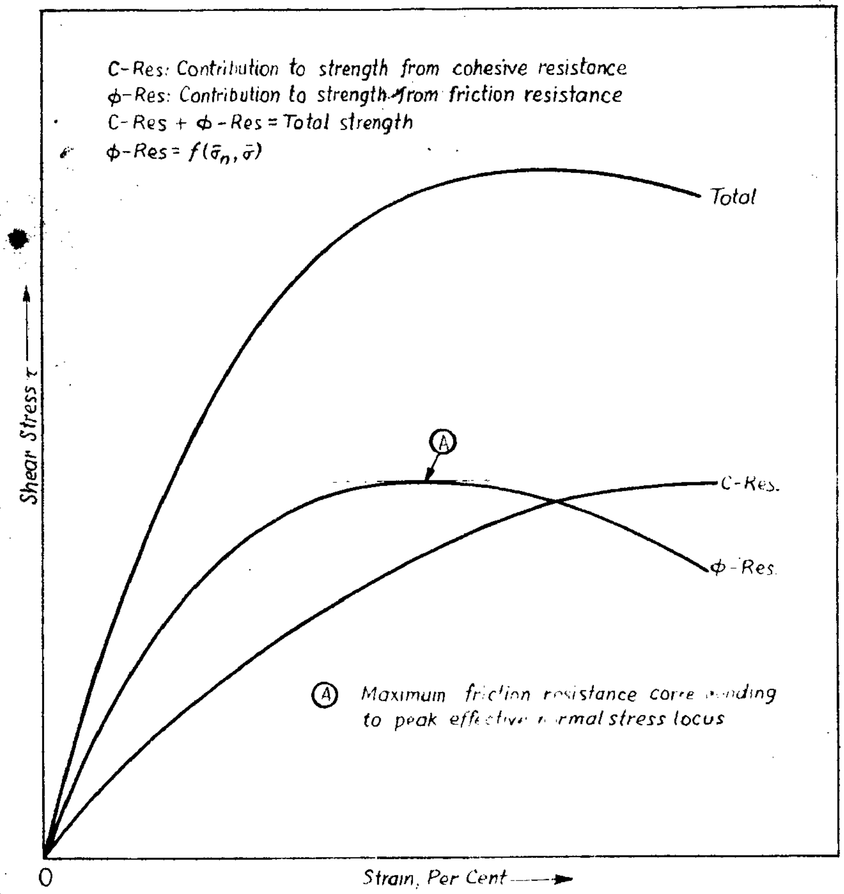

Figure - 6·10. Strength Parameter Contributions to Total Strength (Yong and Vey, 1962)

cohesion developing at very small strain and then decreasing while friction develops gradually at a higher strain. This is also in accordance with the mechanistic picture of clays given by Lambe (47). Wu (94) has explained the behaviour of clays under various conditions of loading by means of a rheological model. Bea (4) has also found that there is a cohesion to friction time transfer of stress. This observation may help to explain the failure criterion of different soils. It is likely that a sensitive clay may fail at a low strain because of exceeding cohesion and other clays fail at higher strain, When the total cohesion and frictional strength is exceeded. The above observations lead to doubts regarding the feasibility of separating the shear resistance into the components cohesion and friction.

Schmidt (71) suggests a new approach which considers the shear strength as dependent on water content of soil and duration of the load. It has been recognised that many problems which are analysed at present by strength theories are in reality problems in which the load deformation-time response plays an important role. The development of realistic stress-strain-time relationships for soils, which can be incorporated into any mathematical theory in one of the greatest hurdles at the moment. Kondes (41) has suggested one possible approach which is given in the next article.

6.13 The explicit form of the stress-strain relationship used herein to represent the soil response in triaxial compression is a two constant hyperbolic equation of the form

$$\sigma_1 - \sigma_3 = \frac{\epsilon}{a + b\epsilon}$$

in which $(\sigma_1 - \sigma_3)$ is the normal stress difference which is usually referred to as deviator stress, ϵ denotes the axial strain, a as well as b represent function of the rate of axial strain, preconsolidation pressure, rebound stress and soil under consideration which have been explicitely expressed in the article. The hyperbolic stress strain is such that the ultimate shear strength of soil is contained within the formulation and appears in the mathematical limit of the stress as the strain becomes excessive.

Thus it has been shown that

$$\lim_{\epsilon \longrightarrow \infty} (\sigma_1 - \sigma_3) = \frac{1}{b} = K(\sigma_1 - \sigma_3)_f$$

in which $(\sigma_1 - \sigma_3)_f$ is the ultimate or failure value of the deviator stress and K is a correlation coefficient. This has been quantitatively demonstrated for a remoulded cohesive soil tested in consolidated undrained triaxial compression at various rates of axial strain. History effects are included in terms of the over-consolidated ratio. The failure relations given therein have been shown to degenerate into conventional Mohr-Coulomb failure envelope in a two dimensional stress space.

6.14 MECHANISM OF SHEAR STRENGTH BASED ON INTER-PARTICLE FORCES

In postulating a mechanism for shear strength based on inter-particle forces of attraction and repulsion, Lambe (47) lists the following forces acting between particles

F_m = force where contact is mineral to mineral

F_a = force where contact is air-mineral

F_w = force where contact is water mineral or water-water

R' = electric replusion between particles

A' = electrical attraction between particles.

Equating the sum of the forces acting across the potential plane of failure to the combined mineral stress and expressing the equation in terms of stresses rather than forces, one obtains the equation for the combined normal stress across the 'failure plane σ_n as:

$$\sigma_n = \bar{\sigma}\, a_m + P_a\, a_a + U a_w + R - A$$

where $\bar{\sigma}$ = effective stress between minerals derived from and associated with F_m.

a_m = unit contact area for mineral-mineral

P_a, a_a = unit pressure and unit area directly associated with F_a

a_w = unit area directly associated with F_w.

R, A = unit electrical forces of replusion and attraction respectively.

Lambe's equation represents an idealized breakdown of the distribution of stresses induced in the soil (and acting across the potential shear plane) as a result of the applied normal stresses. It is difficult to measure any of the particular items precisely.

Measurements of resistance to direct translatory shear on clay samples of known composition, with known difference in inter-particle forces, may be used to relate shear strength to inter-particle forces. From the relationship, it is possible to determine the inter-particle forces responsible for shear strength and to set up a model for their action. Swelling and non-swelling clays behave differently. Swelling clays, in which repulsion can be the dominant inter-particle force, retain strength even when wet. When non-swelling clays are wetted to the same water content, the samples fall apart.

Shear strength increases as net repulsion increases. This increase in repulsion is accompanied by an increase in soil suction, so that a greater strength would be predicted from the effective stresses. However, this does not explain the existence of shear strength in a clay where the swelling shows that a net repulsion exists between particles.

During shear, particles in the failure plane become oriented parallel to each other, and any force resisting this orientation should contribute to shear strength of remolded clays. Figure (6.11) suggests how inter-particle repulsion can play this role for high swelling clays.

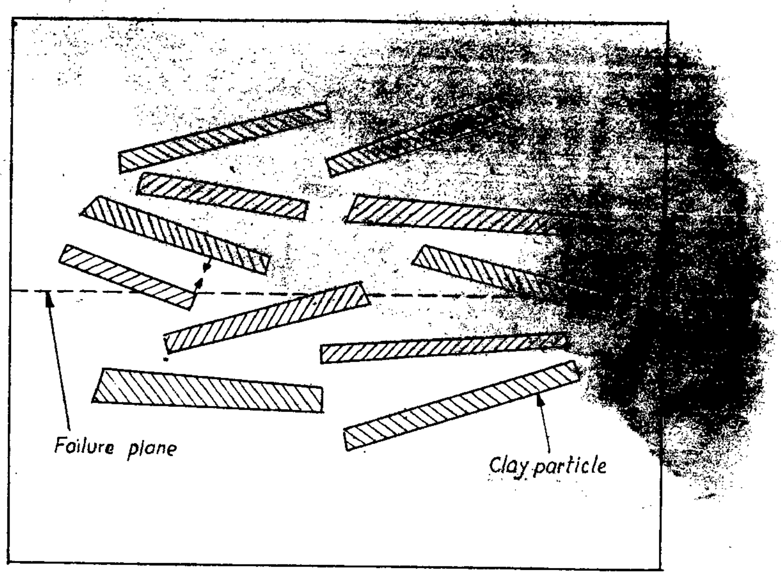

Figure- 6-11. Schematic Diagram for Model of Interacting Clay Plates with Interparticle Repulsion Resisting Particle Rearrangement During Development of Failure Plane (Warkentin and Yong, 1962)

Formation of the failure plane requires particle movement, in many instances one particle must move closer to another. Inter-particle repulsion resists this movement, accounting qualitatively for measured strength values. It is difficult to apply this concept qualitatively because the original particle orientation is not known and it generally changes as void ratio changes. Particle orientation becomes more parallel as the void ratio decreases. The amount of movement required in the failure plane cannot be specified.

The configuration at failure of soil partifles in a clay soil requires minimal particle interference along the failure plane. Thus an arrangement parallel or near parallel to the failure plane is favoured. The energy or force required to achieve the parallel arrangement accounts for the major component of strength.

Highest shear strength is measured for flocculated clays where attraction is the dominant inter-particle force. The random clay with net repulsion has a lower strength at the same void ratio.

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7 LABORATORY DETERMINATION OF SHEAR STRENGTH OF COHESIVE SOILS

The more common techniques used for determining cohesive soil strength are

1. Compressive test
   (a) triaxial compression
   (b) unconfined compression
2. Direct shear test
3. Vane test

Less common tests

1. Hollow cylinder tests
2. Cone penetration test
3. Ring shear tests.

7.1 TRIAXIAL SHEAR TEST

In a triaxial shear test a soil specimen is subjected to three principal stresses and the soil is failed in shear by increasing or sometimes decreasing one of the stresses.

The specimen is cylindrical in shape and lateral pressure are applied by a fluid (usually water) under pressure which acts on all sides of the specimen. To fail the specimen additional vertical stress is applied axially on top of the specimen. Under these conditions the vertical axial stress is the major principal stress and intermediate and minor principal stresses are both equal to the confining fluid pressure.

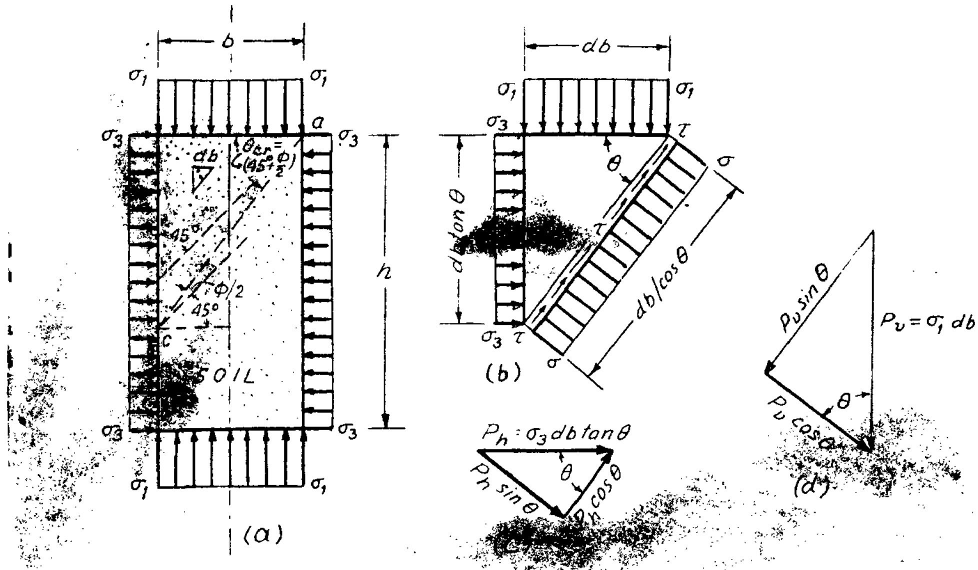


Figure — 7-1 Sketch illustrating equilibrium of external pressures and of internal stresses during a triaxial shear test.

Consider the equilibrium of a small prism of soil of width db. First let us find the inclination of the plane along which there will be the least resistance to shearing, that is the value of the angle θ_{cr}.

The force normal to an inclined plane forming the angle θ with the horizontal is

$$P_n = P_h \sin\theta + P_r \cos\theta$$

or in the terms of unit stresses

$$\sigma \cdot \frac{db}{\cos\theta} = \sigma_3 \, db \tan\theta \sin\theta + \sigma_1 \, db \cdot \cos\theta$$

$$\sigma = \sigma_3 \sin^2\theta + \sigma_1 \cos^2\theta$$

$$= \sigma_3 + \cos^2\theta(\sigma_1 - \sigma_3)$$

Similarly, the force tangential to the plane forming the angle θ with the horizontal is

$$P_t = P_r \sin\theta - P_n \cos\theta$$

$$\frac{db}{\cos\theta} = \sigma_1 \, db \sin\theta - \sigma_3 \, db \tan\theta \cdot \cos\theta$$

$$= \sigma_1 \sin\theta\cos\theta - \sigma_3 \sin\theta\cos\theta$$

$$= \sin 2\theta\,(\sigma_1 - \sigma_3) \times \tfrac{1}{2} \qquad (1)$$

In case of a purely cohesive frictionless soil the shearing resistance along any plane will be independent of the normal pressure σ against it and will therefore be governed only by the intensity of the shearing stress τ along it. τ will reach maximum (from eqn. 1) when sin 2θ equals its maximum value of unity, i.e. when $\theta = 45^\circ$

$$\tau_{max} = \tfrac{1}{2}(\sigma_1 - \sigma_3)$$

If the shearing resistance S of the soil depends on both friction and cohesion, sliding failure will occur in accordance with the Coulomb's equation

$$\tau = S = C + \sigma \tan\phi$$

$$\sigma_1 \sin\theta\cos\theta - \sigma_3 \sin\theta\cos\theta = C + \sigma_3 \tan\phi + \cos^2\theta\,\sigma_1 \tan\phi - \cos^2\theta\,\sigma_3 \tan\phi$$

Solving for σ_1

$$\sigma_1(\sin\theta\cos\theta - \cos^2\theta\tan\phi) = C + \sigma_3\tan\phi + \sigma_3(\sin\theta\,.\,\cos\theta - \cos^2\theta \times \tan\phi)$$

$$\sigma_1 = \sigma_3 + \frac{C + \sigma_3\tan\phi}{\sin\theta\cos\theta - \cos^2\theta\,.\,\tan\phi} \qquad (2)$$

The plane with the least resistance to shear along it will correspond to the minimum value of σ_1 which can produce failure in accordance with the above equation. σ_1 will be at a minimum when the denominator in the second member of the equation is at a maximum i.e. when $\frac{d}{d\theta}(\sin\theta\cos\theta - \cos^2\theta\tan\phi) = 0$.

Differentiating, we obtain

$$\cos^2 \theta_{cr} - \sin^2 \theta_{cr} + 2 \tan \phi \cos \theta_{cr} \cdot \sin \theta_{cr} = 0$$

$$\text{or } \cos 2\theta_{cr} = - \tan \phi \sin 2\theta_{cr}$$

$$\cot 2\theta_{cr} = - \tan \phi = \cot(90 + \phi)$$

$$\theta_{cr} = (45^\circ + \frac{\phi}{2})$$

Substituting this in the denominator of Eqn.

$$\tan \phi = \cot(90 - \phi) = - \cot(90 + \phi) = - \cot 2\theta_{cr}$$

$$= \frac{1}{2}(\tan \theta_{cr} - \cot \theta_{cr})$$

We obtain

$$\sin \theta_{cr} \cos \theta_{cr} - \cos^2 \theta_{cr} \tan \phi$$

$$= \sin \theta_{cr} \cdot \cos \theta_{cr} - \cos^2 \theta_{cr}\left(\frac{\sin \theta_{cr}}{\cos \theta_{cr}} - \frac{\cos \theta_{cr}}{\sin \theta_{cr}}\right) \frac{1}{2}$$

$$= \frac{\cos \theta_{cr}}{2 \sin \theta_{cr}} (\sin^2 \theta_{cr} + \cos^2 \theta_{cr}) = \frac{1}{2 \tan \theta_{cr}}$$

$\therefore$ Eqn. :

$$\sigma_1 = \sigma_3 \tan^2(45^\circ + \frac{\phi}{2}) + 2C \tan(45 + \frac{\phi}{2}) \qquad - \text{II}$$

If $C = 0$,

$$\sigma_1 = \sigma_3 \tan^2(45 + \frac{\phi}{2})$$

If $\phi = 0$,

$$\sigma_1 = \sigma_3 + 2C$$

The above results can be derived from Mohr's circle also. When the stresses in a soil mass are in accordance with eqn. II, the soil is said to be in a state of plastic equilibrium.

A number of other relationships can be presented by means of the Mohr's circle which are already presented under a different heading i.e., Mohr-Coulomb failure criteria.

Failure of the specimen in a triaxial test is usually defined when the deviator stress or the compressive strength $(\sigma_1 - \sigma_3)$ or in terms of effective stresses $(\sigma_1' - \sigma_3')$ reaches maximum value. Alternatively, the compressive strength per unit of effective minor principal stress at failure, i.e.

$$\frac{\sigma_1' - \sigma_3'}{\sigma_3'} \quad \text{or} \quad \frac{\sigma_1'}{\sigma_3'} - 1$$

at failure is used as a measure of shear resistance and the failure is defined at the maximum value of $\frac{\sigma_1'}{\sigma_3'}$ called the effective principal stress ratio. The maximum deviator stress $(\sigma_a)_{max}$ and the minimum principal stress ratio $\left(\frac{\sigma_1'}{\sigma_3'}\right)_{max}$ may or may not occur at the same strain and hence the shear strength may be different according to the criterion of failure.

7.2 A. Casagrande (15) proposed three types of tests for evaluating the shear resistance of clays - "Qufck", "Consolidated Quick" and "Slow" tests. As the terms "Quick" and "Slow" are relative, and as the essential aspects of the imposed conditions are either no drainage or full drainage, these terms have been replaced by the terms undrained and drained respectively and are defined as follows.

Undrained Shear Stress. Tests in which no drainage of water is permitted during the entire test. Excess pore pressures (either positive or negative) will result from the application of both normal and shear stresses.

Consolidated-Undrained Shear Tests. Tests in which drainage is permitted and full primary consolidation is allowed to take place under the initially applied normal stresses only. No drainage is permitted during the subsequent applications of either normal or shear stresses.

<u>Drained Shear Tests.</u> Tests in which drainage is permitted during the application of both normal and shearing stresses. No excess pore pressures (either positive or negative) are allowed to develop at any time during the test.

As drainage conditions in many practical problems closely approximate one or the other of the limiting drainage conditions imposed by the tests described above. These tests are widely used to evaluate the shearing resistance of soils.

7.3 DIAGRAMATIC REPRESENTATION OF STRESS CONDITION

Since there is radial symmetry in the tests, the three dimensional stress space may be reduced to an equivalent two

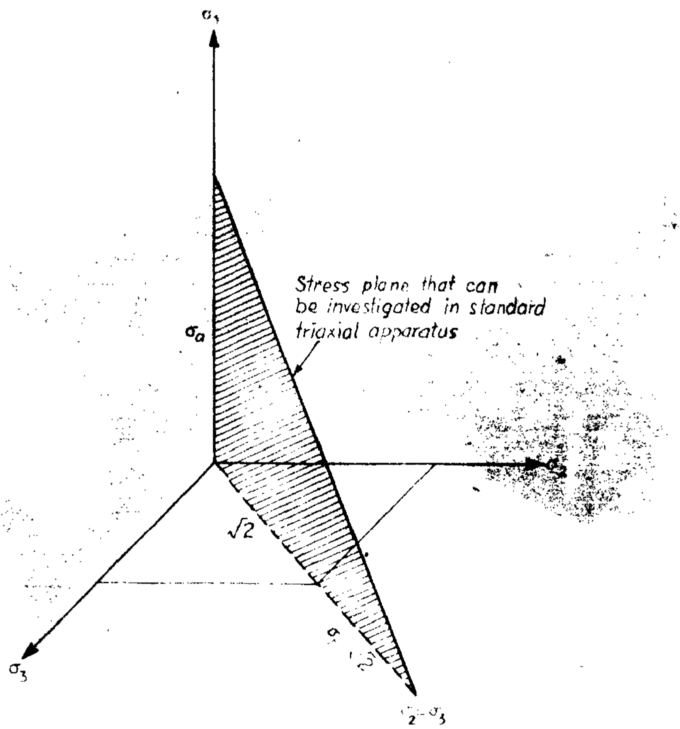


Figure – 7.2. Stress Plane for Triaxial Investigation (Henkel, 1960) dimensional space by considering the plane passing through the axial stress plane equidistant from the two radial stress planes (i.e., at 45 degrees).

The vector sum of the two radial stresses must be plotted along the radial stress axis. The space diagonal in this plot defines its true angle with the axial stress axis, namely $\cos^{-1} \frac{1}{\sqrt{3}}$. A right section to the space diagonal (an octahedral plane) can be drawn by drawing a line at 90° to the space diagonal.

If the results of drained and undrained tests are plotted in the stress plane given by the diagram, the stress paths for undrained

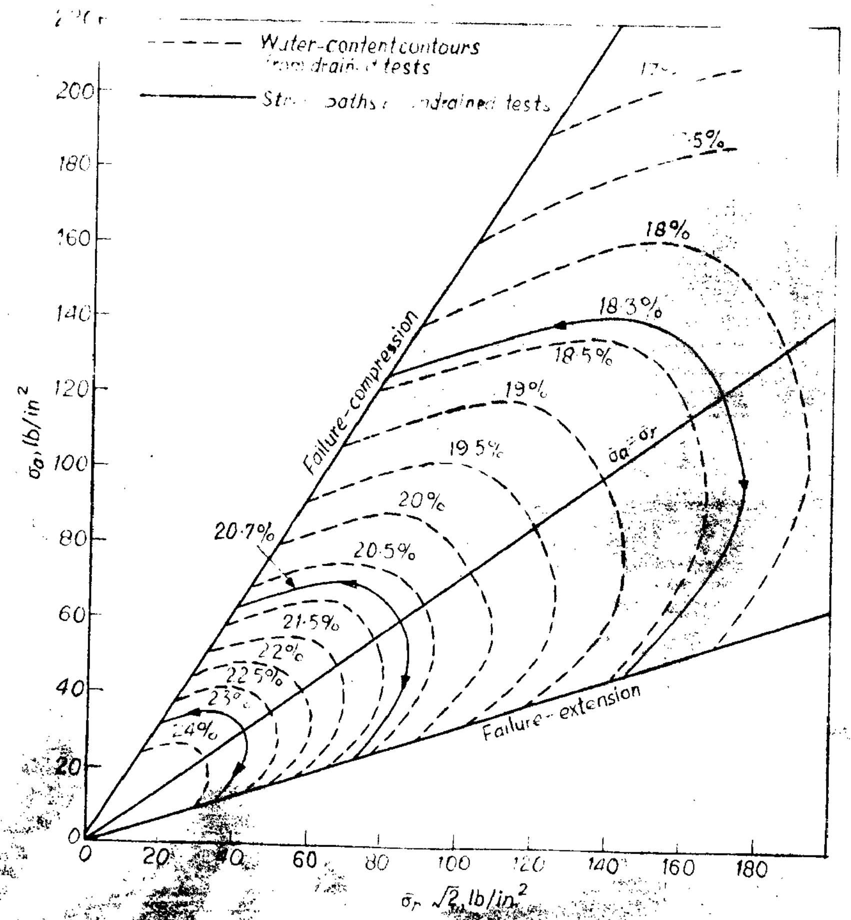


Figure Stress Vectors for Normally Consolidated Specimens (Henkel, 1960) tests on normally consolidated clays and over consolidated clays will be shown for compression and extension tests as in Figures 7.3 and 7.4 respectively.

Contours of constant water content similar in form to the stress paths may be constructed from drained compression and extension tests for both normally consolidated and over consolidated clays. Using this diagram known as Rendulic diagram the unique relationship between the effective stress and the water content becomes evident.

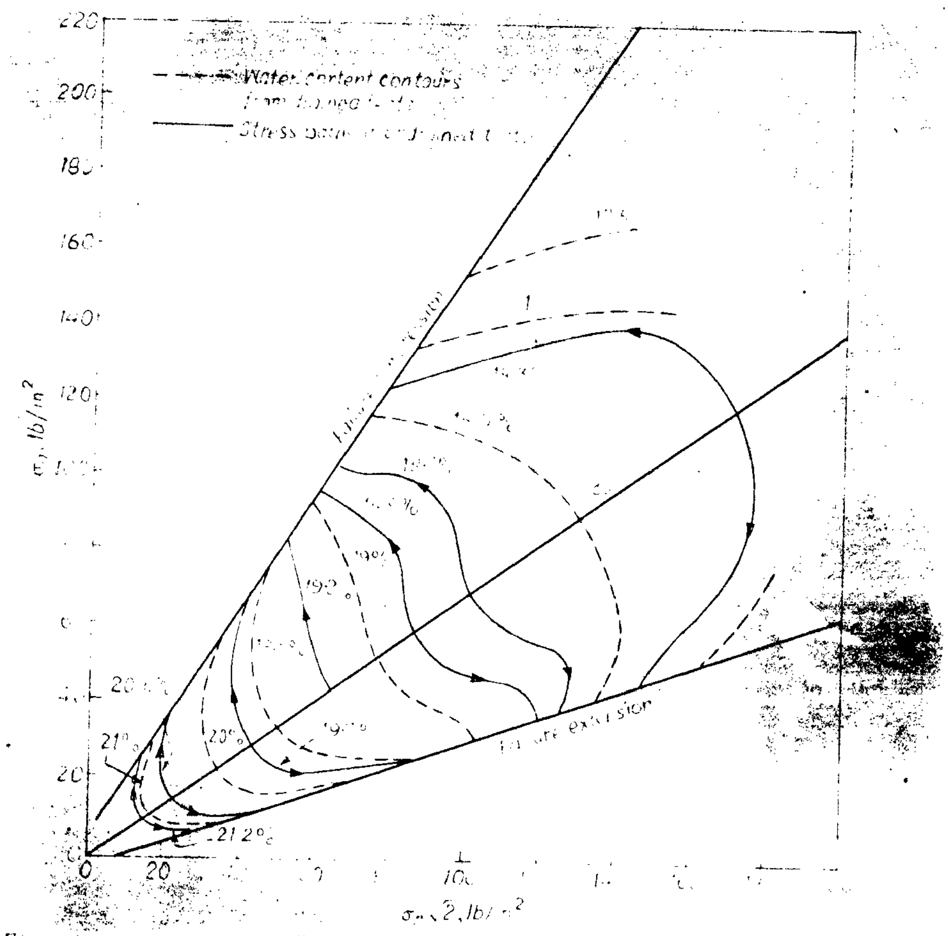


Figure 7.4 . Stress Vectors for [illegible] S[illegible]cimers (Henkel, 1960

However the main difference between normally consolidated and over consolidated clays lies in the shape of stress paths. For the former, the stress paths for compression and extension tests are concave towards the origin within the boundaries of the compression and extension failure envelopes with the stress paths perpendicular to the hydrostatic pressure line, whereas for the latter, the stress paths are convex towards the origin. The pore pressure coefficient A, which is a controlling factor in the derivation of the effective stress, determines the convexity. The greater the convexity, the more consolidated is the soil.

The above illustration is the work of Henkel (32) who continued the work of Rendulic (64) who examined the relationship between shearing stresses, effective stresses and void ratio or water content in saturated cohesive soils. He has presented his results in the form of paths in the stress plane. The data are given separately for normally consolidated and over consolidated saturated cohesive soils. The end of failure points obtained from the testing program were found to be independent of the stress paths followed to failure and are shown for normally consolidated and over consolidated samples of remolded weald clay respectively.

The properties of the clay were

L.L = 43% P.L = 18% Activity = 0.6

and was initially remolded to a water content of 34%. No minerological analysis is given in the papers except the fact that the clay is said to be esturine.

The procedure for preparing the curves are:

1. Sample of soil is 1st normally consolidated to an

hydrostatic stress.

2. The total stress path is determined by the conventional undrained axial compression test.

3. The pore pressures developed during the experiment is measured and effective stress path is determined.

The water content can be changed by consolidating the sample and experiment repeated for various water contents.

When the soil is first consolidated and sheared in a drained test, the vertical lines represent total as well as effective stress and of course the soil volume decreases during deformation. Knowledge of the initial and final water contents, together with the volume of sample enabled Henkal to mark points on a vertical stress paths corresponding to different water contents of the sample. Carrying out a number of such drained tests enabled him to draw contours of constant water contents.

The same procedure was followed in the extension tests and also in tests on soil consolidated under unequal principal stresses. All the data gave the consistent curves.

From these curves it is claimed, the behaviour of soil in any type of test can be predicted.

The relationships expressed hold only for increasing stresses because of the irreversible effect on soil structure. The effect of shear stress reversal was not studied.

The curves drawn apply only to the batch of the soil used by Henkel, the conditions under which it was remolded, the time of storage, the technique of specimen preparation, the time allowed for consolidation and rebound, the maximum consolidation pressure, the rate of stressing and temperatures of the different stages of the investigation. The list may not be complete but indicate the complexity of such a study.

However such a diagram therefore gives a graphic representation of the stress system during the entire test, not only at the failure conditions, and is of great value when general correlations are sought.

7.4 Another method for depicting graphically the stress conditions during shear tests is the "vector" curve proposed by

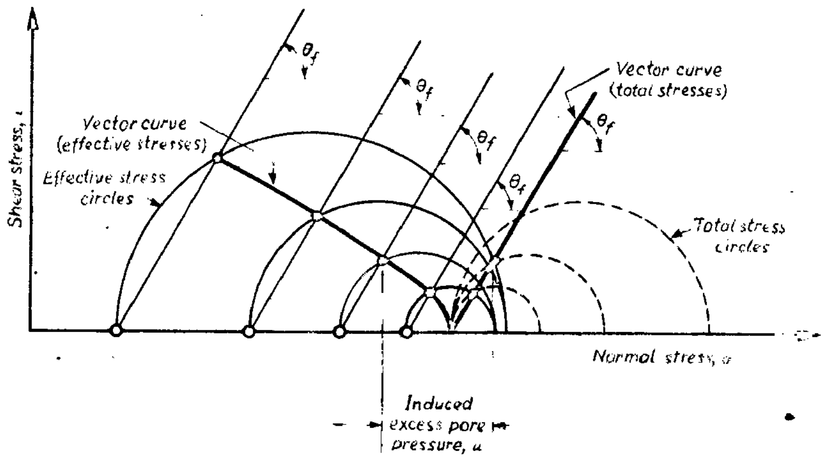


Figure 7-5 Definition of Vector curves.

Casagrande (19) and Wilson. The procedure involves drawing the effective and total stress circles as the test proceeds on the conventional Mohr diagram, and drawing a locus through the points on these circles representing the states of stress on the ultimate failure plane.

These loci are the vector curves for the effective and total stresses, respectively and show graphically the variation of effective normal stress and shear stress and of total normal stress and shear stress on the ultimate failure plane. The horizontal distance between the two vector curves is the excess pore pressure v developed as a result of the application of shear stresses. While this representation is useful for certain purposes, it is limited by the fact that it is a two-dimensional, representation and by the fact that it is necessary to assume the direction of the ultimate failure plane and the latter procedure can lead to serious errors.

7.5 For any one cohesive soil, either the void ratio or the structure of the soil, or both, change during the shearing process and void ratio, of course, varies with the effective stress under hydrostatic stresses.

If shear stress, effective stress and void ratio are determined at failure, a failure surface will be defined in space for the remolded weald clay used by Henkel Such a study has been carried out by Roscoe, Scofield and Wroth (65).

With the limitations to compressive tests in the triaxial equipment and the emphasis on shearing strengths, the investigators used stress axes of mean principal effective stress $p = \frac{1}{3}(\bar{\sigma}_1 + 2\bar{\sigma}_3)$ and principal stress difference $q = (\bar{\sigma}_1 - \bar{\sigma}_3)$ or $\sigma_1 - \sigma_3$ corrected for expansion or contraction effect. Instead of the void ratio they took water content which is an equivalent for saturated soil. The failure surface is shown. The yield surface incorporates a change in state.

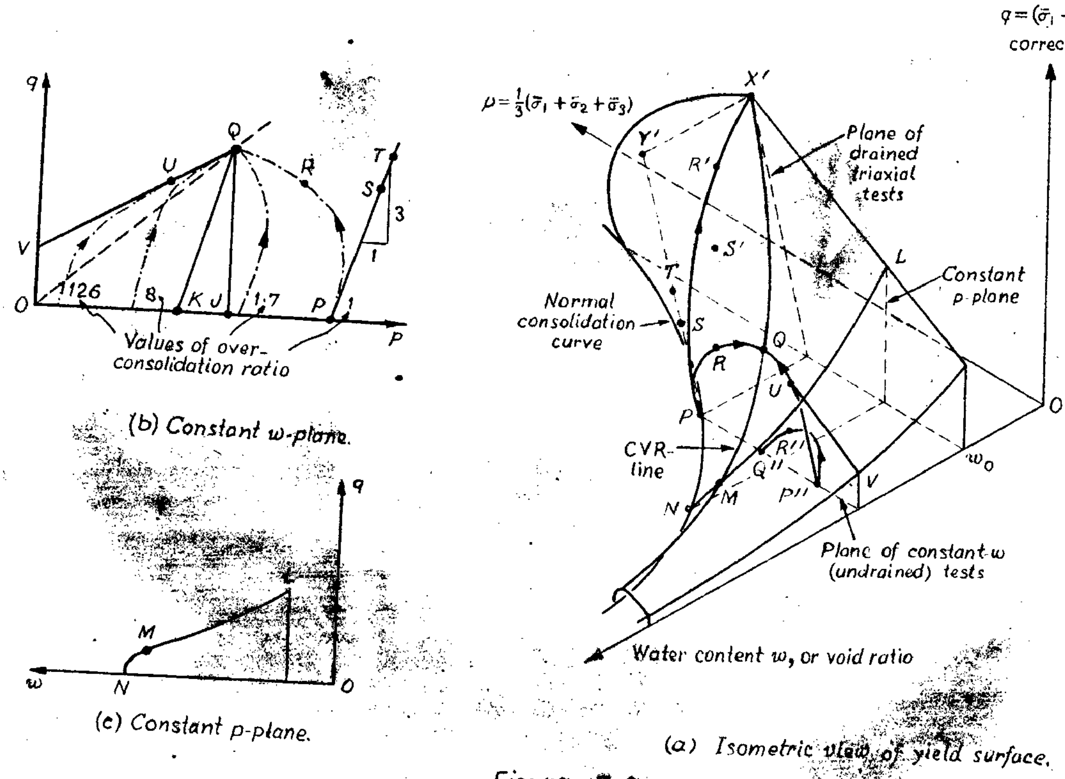


Figure- 7.6.

The boundary of the failure surfaces are water content vs. p in pw plane, p-o and q = 0.

It was found that the loading paths from all tests in the normally consolidated clay defined the same curved surface passing through PQR or PR'X'. In these tests the maximum principal stress difference or shearing strength, and the ultimate stress difference or ultimate strength were the same. The soil thus exhibits a stable stress vs. displacement characteristic of the type shown. The failure and ultimate conditions in this case both plot along a single curve in the stress void ratio space which the investigators call the line of critical void ratio (CVR). The CVR line may be assumed to be one on which no further structural or void ratio change occurs in the soil on continued shearing.

In an undrained test on an over-consolidated soil sample which is at equilibrium at the water content of the plane and the mean effective stress at point K in Fig. 7.6, the applied stress path will begin at K and end at Q. Since Q is on the CVR line, the loading path will also end at Q and therefore in this test the ultimate excess pore pressure will be zero. In a drained test beginning at point K, the applied stress and loading paths will also end at Q and sample will suffer no net water content change during the test. The projection of the CVR line on PW plane differs from normal consolidation curve by a constant water difference and so the trace of the point K from the projected CVR line on this plane. This curve is called the critical over consolidation ratio line.

Were shear tests to be carried out in which mean effective principal stress remained constant the critical over consolidation line would be coincident with the CVR line.

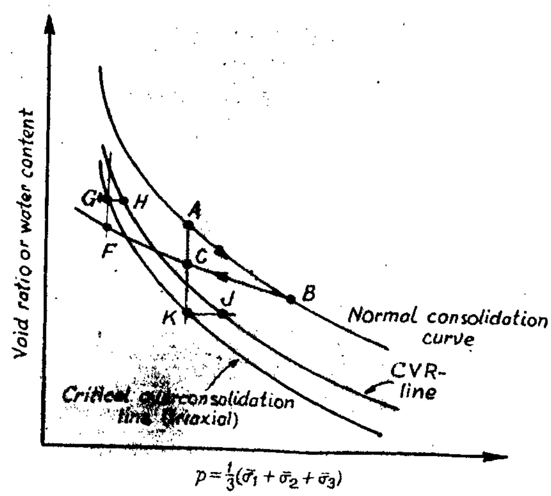


Figure 7.7 Relationships of void ratio versus mean effective stress.

The three curves of normal consolidation critical void ratio and critical over consolidation (for triaxial apparatus) projected on to the PW plane are shown in Fig. 7.7. These figures may be used to deduce the results of the tests. If the initial conditions of effective stress and void ratio of a clay sample are such that the appropriate point plots on the diagram in the zone above the over consolidation line, the soil will contract on shearing.

If the initial condition of the soil is represented by a point below the critical over consolidation expansion will take place on shearing.

7.6 STRESS - STRAIN CURVE FOR SOILS OBTAINED FROM TRIAXIAL TEST

The sample is studied in a three dimensional stress state of radial symmetry in which the radially directed applied stress is the same over the entire cylindrical surface. It is therefore not possible to investigate the soil specimen under general conditions of triaxial stress with all the three principal stresses different.

To make provision for the application of vertical load and to permit possible drainage of sample during consolidations, the soil cylinder is capped top and bottom by porous stones. Because of the difference between the rigidity of the capping stones and the soil, the latter is not subjected to a uniform vertical stress over its end surfaces. In addition, radially directed shear stress will always be generated at these surfaces, while the cylinder surfaces of the sample are subjected to normal stresses only, it can be seen that the soil is not tested in a homogeneous stress state.

Balla (3) has shown theoretically that friction on the loading caps creates nonuniform state of stress in the triaxial specimen. Shear stresses due to friction act radially inwards at the ends of specimen thus causing restraint at the ends. This end restraint allows higher deviator stress to be measured than that on the failure plane. Further work by Zeleman shows that compressed cores are formed during compression of the specimen which cause development of tensile radial stresses thus undermining the shear strength of the specimen. As both of abovementioned conditions oppose each other, the resulting uncertainty in determination of deviator stress on failure plane may not be high.

The behaviour of soil under stress in such an apparatus therefore develops as a result not only of the properties of soil under stress, the proper subject of the experiment, but as a consequence of the geometrical proportions of sample and the amount of restraint imposed on the deformation process by the equipment. Thus information of details of the stress behaviour of a soil obtained in the course of a triaxial test can be accorded only by qualitative value.

7.7 Considerable attention has been given to the nature of the pore pressure build up during triaxial testing of soils.

The pore pressure measurement in a conventional triaxial test are affected by (i) time lag (ii) non-equalisation of initially non-uniform pore pressure and temperature (iii) base pore pressure measurement is unrealiable even in conventional tests.

For rapidly applied total stress the volume of water displaced by the deflection of the upper surface of the porous disc due to difference between the total vertical σ and the pore pressure u in the measuring system and the volume of water entering the measuring system under a pore pressure change Δu due to compression of water and expansion of tubing valves etc., must be equal and the pore pressure change observed after a total stress change $\Delta\sigma$ must be directly proportional.

In the laboratory the immediate value of Δu is much less than $\Delta\sigma$.

For compressible soils the time taken for the observed pore pressure to approximate to that within the sample is negligible except for transient loads. However low compressibility coupled with entrapped air in the null indicator system can lead to very serious time lag. The lag can be minimised by the use of a back pressure high enough to drive all the entrapped air into solution and by the use of side drains of filter paper.

In undrained test the extent to which equalisation of pore pressures occur within the sample depends on the permeability of the sample, its dimension and rate of testing. The distribution of pore pressure before equalisation is difficult to calculate at all accurately, but the general trend can be inferred from the derived equations.

In the cylindrical compression test end restraint tend to increase slightly the major principal stress increment $\Delta\sigma_1$ and reduce considerably the deviator stress increment $(\Delta\sigma_1-\Delta\sigma_3)$ near the ends of the specimen relative to their values at the middle. Both these factors will lead to a high pore pressure at the ends of specimen unless A exceeds unit or unless the value of A increases fairly rapidly as $(\Delta\sigma_1 - \Delta\sigma_3)$ increases (A is a component of the equation $\Delta u = B\left[\Delta\sigma_1 - (1-A)(\Delta\sigma_1 - \Delta\sigma_3)\right]$.

7.8 It has been shown by Lamb that changes in temperature lead to marked changes in pore water pressure within undrained samples of saturated clays. It is thus necessary to run these tests at a constant temperature.

7.9 The practical significance of the two different criteria of failure is usually negligible. In a test in which the stress strain curve does not become horizontal and thus not indicating ultimate strength, the strength at 15% strain is always designated as the ultimate strength.

7.10 In an isotropic soil failure is predicted by Mohr Coulomb criterion to occur on planes at an angle of $(45 + \frac{\phi}{2})$ to the major principal plane. In practice it is observed that failure takes place on planes with smaller angles. If the failure envelope is redrawn through the point corresponding to the practical failure angle it would be slightly lower than the Mohr-Coulomb envelop. However many soils are naturally unisotropic and unisotropy can also develop in the course of triaxial test. The angle of failure plane is very sensitive to deviations from isotropic conditions so that the question of correct failure line is still unsettled.

7.11 <u>RATE OF STRAIN.</u> Before consideration is given to any other variables, attention must be paid to the speed at which the shearing process is carried out. Investigations of this are complicated by the presence of and behaviour of the pore water in the soil. If one establishes a criterion that a test should be run sufficiently slowly so that no excess pore pressures are generated in the pore water, the variation of shear strength alone can be evaluated.

The results of tests have indicated that shearing strength defined by the peak of a stress deformation at very low rates of strain say 10^{-4}% per min. may be 15 to 20%, lower than at rates of about 1% per min. In general the strength is more affected by excess pore pressure generated during shear.

7.12 THE UNCONFINED TEST

The unconfined test (also designated as U test) is a special case of the triaxial compression test in which a cylindrical specimen is failed under the axial compressive stress only with zero lateral load ($\sigma_2 = \sigma_3 = 0$). The test is based on the assumption that no moisture is lost from the specimen during the test. Hence it is considered an undrained shear test.

The unconfined compression test is generally applicable to intact saturated clays for which the apparent angle of shearing resistance ϕ_u = zero (or almost zero)

when $\sigma_3 = 0$

$$\sigma_1 = 2C_u \tan \theta_{cr} = 2C_u \tan(45 + \frac{\phi_u}{2})$$

Putting $\phi_u = 0$ and $\theta_{cr} = 45^o$

$$\sigma_1 = 2C_u$$

The major principal stress at failure in an unconfined compression test is called the unconfined compressive strength q_u of the soil.

$$q_u = 2\,C_u$$

The unconfined shear strength of a saturated clay ($\phi_u = 0$) may thus be expressed as

$$\tau_f = C_u = \frac{q_u}{2}$$

The above relation can also be obtained by drawing a Mohr circle of failure which passes through the origin of stress ($\sigma_3 = 0$) and has its radius $= \frac{\sigma_1}{2}$.

Since $\phi_u = 0$, the failure envelope is horizontal. It touches the circle at top most point. It can be actually observed

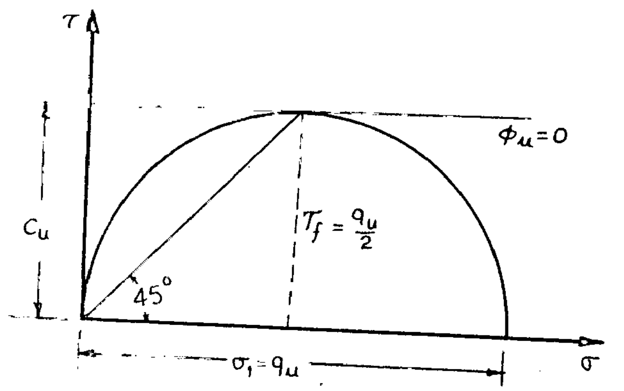


Fig. 7.8

that the inclination to the horizontal of failure planes in a saturated clay specimen is always greater than 45°. Although $\phi_u = 0$, the true angle of internal friction ϕ_0 is greater than zero. Therefore the failure angle θ_f is always greater than 45°.

The unconfined compression test is a simple and quick test. It is normally employed for measuring in-situ strength of fully saturated or nearly saturated cohesive soils in the field. The test is also used for studying the decrease in shear strength due to remolding. The result may be misleading in case of fissured clays for which the undrained shear strength may be 30 per cent below the value as measured in an undrained triaxial test under the overburden pressure. Undrained triaxial tests with cell pressure exceeding the overburden pressure should be used for testing fissured clays. The test is also misleading for soils in which the apparent angle of shearing resistance ϕ_u is not equal to zero, as unless the value of ϕ_u is known, the strength envelope cannot be drawn from a test result. In view of the above the results obtained from unconfined compression tests are considered as approximate.

7.13 DIRECT SHEAR TEST

In a direct shear test the plane of shear failure is predetermined (which is horizontal). The test is usually carried out in a box split in two halves.

All the three types of tests based on drainage can be conducted in this instrument. The stress conditions on a Mohr circle is presented.

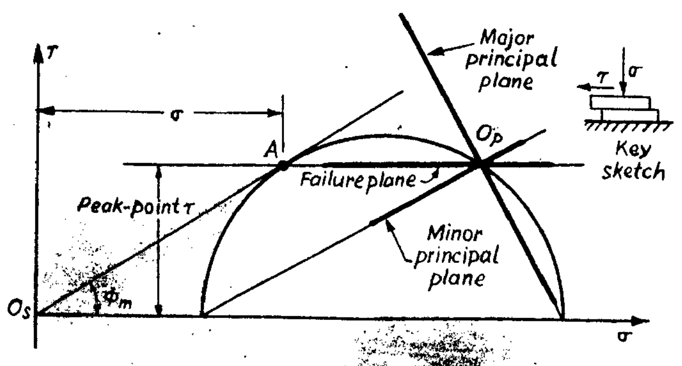


Figure-7.9. Mohr diagram for a direct shear test at failure.

A shear box test is simple and can be performed relatively rapidly as compared to a triaxial compression test. Since a thin sample is used in the test, quick drainage of pore water is facilitated. The test has however, a number of inherent disadvantages.

(i) There is an unequal distribution of shear stresses over the potential surfaces of sliding; the stress is more at the edges and less inthe centre. This results in progressive failure of the specimen i.e., the entire strength of the soil is not mobilised simultaneously.

(ii) With the progress of the test, the area under shear gradually decreases.

(iii) There is little control on the drainage of the soil. Undrained tests can be carried out only with clayey soils. It is usually impossible to completely prevent the escape of pore water from saturated specimens of relatively high permeability.

(iv) The plane of shear failure is predetermined which may not necessarily be the weakest one.

A constant volume, direct shear test termed the R_o test, has recently been developed by O'Neil (61) for the rapid determination of total and effective strength parameters for soils, including clay. It is a consolidated undrained test in which the development of pore pressure during constant volume shear is theoretically prevented by changing the vertical load on the specimen to hold the vertical dial reading constant. The development of R_o test is noteworthy.

7.14 SHEAR VANE TEST

A direct measurement of undrained shear strength of soft cohesive soils can be made by a shear vane test. This test can be performed on a soil sample in a laboratory or into the undisturbed soil in situ at the bottom of the bore hole.

A shear vane essentially consists of four steel plates, called vanes, welded orthogonally to a steel rod. After pushing the vane gently into the soil, it is rotated at a uniform speed (usually at 1^{o} per minute or 0.1^{o} per second) by applying a turning moment (torque) through the steel torque rod. The rotation of the vane shears the soil along a cylindrical surface. If the top end of the vane is embedded deep into the soil, both the top and bottom ends also partake in shearing the soil. Let τ_f be the unit shear strength of soil. H be the height of vane and d the diameter of vane. If it is further assumed that the shear resistance of soil develops uniformly on the cylindrical surface and on the top and the bottom of the sheared cylinder, the maximum total shear resistance at failure equals

$$\pi . d . H . \tau_f + 2 \int_0^{d/2} (2\pi \, r \, dr) \tau_f$$

where r is the radius of the sheared surface. The maximum moment of the total shear resistance about the axis of torque and equal the torque T at failure

$$T = \pi . d . H . \tau_f \times \frac{d}{2} + 2 \int_0^{d/2} (2\pi r . dr . \tau_f) \, r$$

$$T = \pi . \tau_f \left[\frac{d^2 H}{2} + \frac{d^3}{6} \right]$$

If only the bottom end of the vane partakes in shearing the soil

$$T = \pi \tau_f \left[\frac{d^2 H}{2} + \frac{d^3}{12} \right]$$

For saturated clay $_f$ is replaced by C_u or $\frac{q_u}{2}$.

A vane shear test of a cohesive soil usually gives a value of shear strength about 15% greater than an unconfined compressive strength (89).

7.15 THE LESS COMMON METHODS

The less common methods are

(a) Hollow cylinder tests

(b) Cone penetration

(c) Ring shear test.

7.16 METHODS AVAILABLE FOR DETERMINATION OF HVORSHEV'S PARAMETERS

(a) Measurement of inclination of rupture plane in unconfined compression test.

(b) Hvorslev's method

(c) Bjerrums method

(d) Crawfords method

(e) Method for sensitive clays by Noorany and Seed.

These methods are reviewed by Jagdish Narain and T.S.R. Ayyar (37).

(a) Measurement of inclination of rupture plane in unconfined compression test: Terzaghi (85) showed that in an isotropic soil, the angle between rupture plane and direction of major principal stress is $(45 - \phi_e/2)$ where ϕ_e is the true angle of internal friction. This has been experimentally confirmed and makes it possible to determine ϕ_e from the inclinations of rupture planes of soil in compression tests. This method is useful only for an approximate value of ϕ_e as (i) the value is affected by an anisotropy of the soil affecting failure angle to be different from $(45 - \phi_e/2)$ (ii) the deformation of soil sample affecting the measured slope of failure plane after the test, and (iii) the difficulty in accurately measuring the inclination of rupture plane in a failed specimen.

(b) Hvorslev's method (65): This method can be used for the determination of true parameters both for undisturbed and remolded soils. Tests are conducted with normally consolidated and over-consolidated samples using identical samples. Plots are made of the failure water content (or void ratio) against effective normal stress on failure plane and shear stress against the same effective stress as shown in Fig. 7.10.

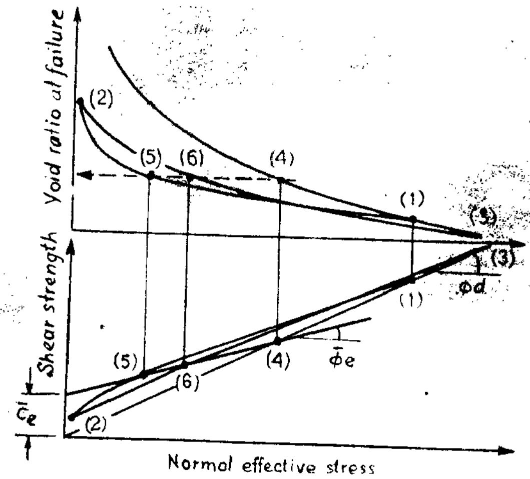


Figure-7.10. Friction and cohesion components, Determination of friction and cohesion components at constant void ratio.

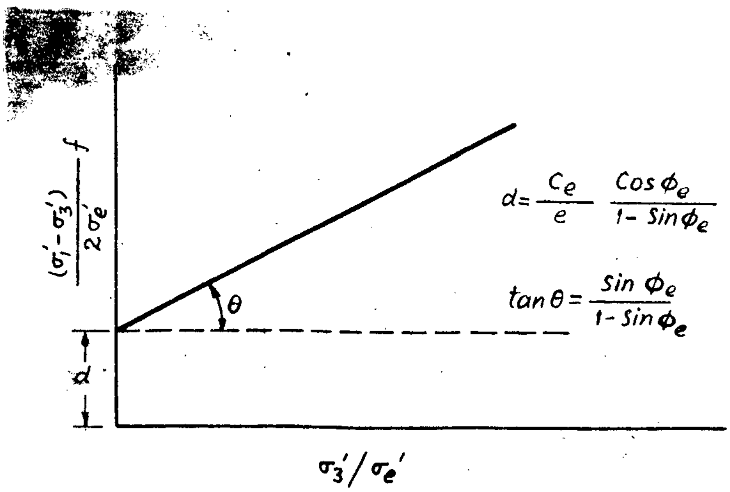
As the difference in shear strength of normally consolidated and over consolidated samples at thesame water content at failure is due to the difference in effective stress, the true angle of internal friction and true cohesion can be found out. Hvorslev originally used drained direct shear tests. Triaxial shear tests can also be used. In this case it is preferable to use a drained or consolidated undrained test with pore pressure measurement. From Mohr's theory, it can be seen that

$$\frac{(\sigma_1 - \sigma_3)_f}{2} = C_e \frac{\cos \phi_e}{1-\sin \phi_e} + \sigma_3' \frac{\sin \phi_e}{1 - \sin \phi_e}$$

where $(\sigma_1 - \sigma_3)_f$ is the principal stress difference at failure and σ_3' effective minor principal stress at failure. This equation can be made non-dimensional by using the equivalent consolidation pressure σ_e'

$$\frac{(\sigma_1 - \sigma_3)_f}{2\sigma_e'} = \frac{C_e}{\sigma_e'} \frac{\cos \phi_e}{1 - \sin \phi_e} + \frac{\sigma_3'}{\sigma_e'} \frac{\sin \phi_e}{1 - \sin \phi_e}$$

If a plot is made of $(\sigma_1 - \sigma_3)_f/2\sigma_e$ against σ_3'/σ_e' a straight line is obtained as shown in Fig. 7·11



Figure– 7·11·

C_e and ϕ_e can then be calculated. The samples used should be identical and uniform. Since this method uses the difference in shear strength between a normally consolidated and over consolidated sample, only highly plastic clays give good results. The determination of these parameters has been simplfied by the use of a direct shear test apparatus with a device for maintaining constant specimen thickness during shear.

(c) Bjerrum's (10) method: This method uses the fact that clays consolidated from different initial water contents give two

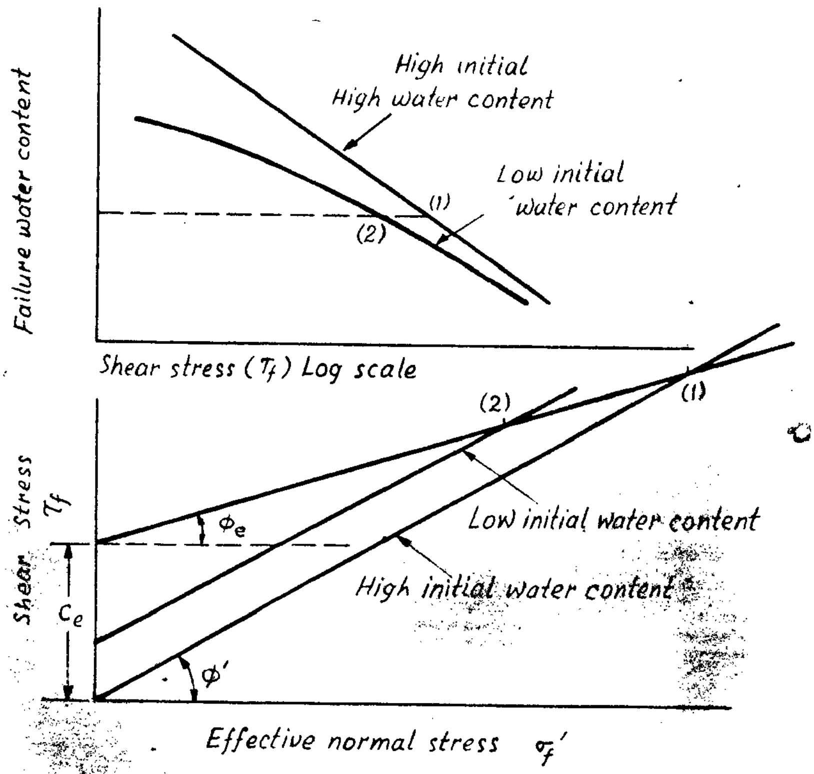


Figure 7·12. Determination of Hvorslev's parameters.

different consolidation curves and have different shear strengths for the same failure water content. Drained direct shear tests were used originally. It is also applicable to use triaxial tests drained or consolidated undrained as in the previous method. Water content-shear strength and shear strength-effective stress diagrams can be drawn for the two sets of damples consolidated from two different initial water contents. By comparing the shear strength of samples for the same failure water content, the true angle of internal friction and true cohesion can be found. The accuracy is very high for plastic soils which can be remoulded and placed without air bubbles at a wide range of water content.

(d) Crawford's method: Crawford has developed a simple method of finding fundamental parameters believed to Hvorslev's parameters. This is based on a series of experiments on Leda clay in which the computed angle of shearing resistance ϕ' increased with strain. In this soil which was normally consolidated, the application of the last 10% deviator stress altered ϕ' by as much as 15°. The method of computation is as follows. It is assumed that the maximum principal stress difference is considered to represent failure and $C' = 0$. For example, when one half of the maximum principal stress difference is applied, 50% of the true frictional resistance is mobilised. Tan ϕ' is thus twice the ratio of the principal stress differance to the normal stress at this stage of loading. This procedure is computed for various values of principal stress difference and can be plotted as shown in Fig. 7.13 It is found that the value ϕ' is constant upto about half the principal stress difference and the value is independent of the consolidation pressure. Crawford, therefore suggests that this value is of fundamental nature and equivalent to Hvorslev's friction angle.

It is assumed that for maximum deviator stress, the friction is fully mobilised and a line is drawn tangential to the

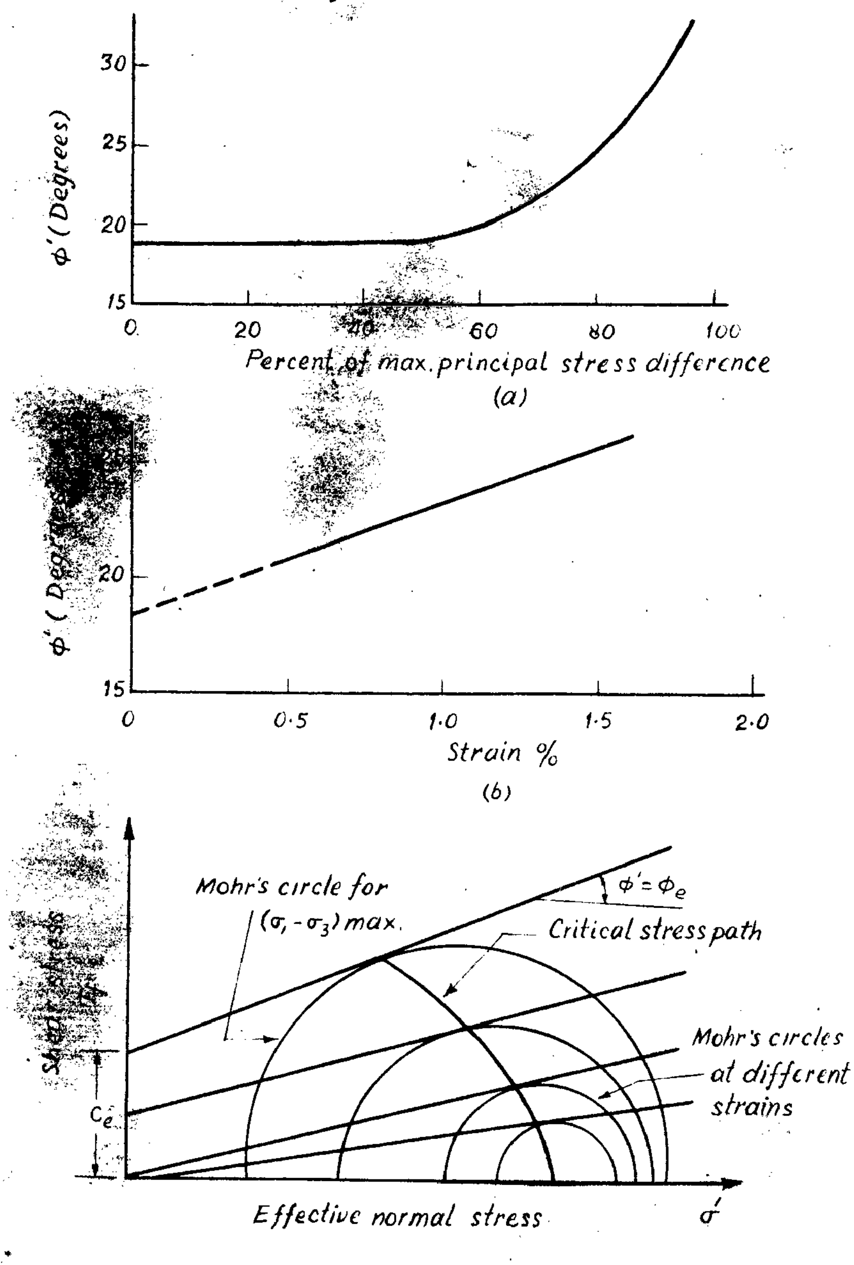


Figure_7.13. Crawford's method

circle for strain corresponding to max. $(\sigma_1 - \sigma_3)$ having the slope $\phi' = \phi_e$. The intercept on the shear stress axis is taken as the true cohesion C_e. It is mobilized at very low strain and true cohesion as required during stress release. This suggestion seems incompatible with the findings of Schmertmann (107) and Osterberg and the theoretical picture of Lambe and further study is needed.

(e) Method for sensitive clays: Noorany and Seed (59) have suggested a new method in which the necessity of having over consolidated sample is avoided. The procedure involves the use of several pairs of samples consolidated anisotropically. In each pair one sample is sheared under undrained conditions while in the other, the anisotropical condition is removed by unloading and then the sample is sheared. This obviously results in two samples of same failure water content but having different effective stresses. From a comarison of the two effective stress, the values of C_e and ϕ_e can be found.

This procedure, while avoiding over-consolidated samples, necessitates the use of anisotropic loading system. Another disadvantage is that unless the soil is very sensitive the effective stresses of failure of the two samples may not be sufficiently apart to get reliable values of C_e and ϕ_e.

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